

HUMAN RESOURCE SCHEDULING : AN APPLICATION OF  
LINEAR PROGRAMMING IN AN AUDIT FIRM

by

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## ABSTRACT

In a developing business society, human resources become the most valuable asset of the firms day by day. Considering this fact, most of the companies do not hesitate to make huge investments in this area. On the other hand, effective and efficient utilization of these rare and expensive resources creates problems for managements.

In this study, I aim to solve human resource scheduling problems that a worldwide auditing firm face by structuring a linear programming model.

In general, auditing firms perform audit work on companies' financial statements as at year-end or any time in order to prepare a report which indicates whether the company's records were done in accordance with

Generally Accepted Accounting Principles and legal requirements. At each

step of the audit work, an intensive involvement of human resources from

different levels is necessary. Due to the importance of human resources'

contribution to the work done, auditing firms invest big amounts for

human resources and related systems. Effective and efficient utilization

of human resources which is considered an important management problem

for service firms, can be achieved by planning the allocation of resources to works for specific periods through out the whole working period. In an auditing firm, efficiency of any resource is measured by chargeability percentage which can be calculated for a period by dividing the total working hours of the resource that can be invoiced to the client by the total hours that the auditing firm paid for the resource. Auditing firm management aims to maximize chargeability for each one of its resources. In this study, a 0-1 Linear Integer Programming (IP) model which aims to maximize chargeability was formulated. The integer programming problem was solved by an implicit enumeration algorithm after certain modifications. A computer program in Fortran 77 was written for the algorithm and executed on a CDC computer system. At the end of the process, jobs that maximize the chargeability when they are scheduled were determined.



## OZET

Gun gectikce rekabete yonelen is dunyasinda, ozellikle hizmet sektorunde insan kaynaklarinin onemi her gecen gun biraz daha artmaktadir.Bu onemi anlayarak sirketler insan kaynaklarina buyuk yatirimlar yapmaktadirlar.

Diger yandan bu az bulunur ve pahali kaynaklarin verimli ve etkin kullanilmasi sirket yonetimlerinin karsilastigi onemli problemlerden biridir.

Bu calismada dunya capinda hizmet veren bir bagimsiz denetim firmasinin insan kaynaklari planlanmasinda karsilastigi problemin , bir lineer tamsayi programlama modeli olusturularak cozulmesi amaclanmistir.

Bagimsiz denetim firmalari,sirketlerin genelde sene sonu veya herhangi bir tarih itibariyle mali kayitlarini denetler ve bu kayitlarin gerek

uluslararası kabul gormus muhasebe ilkelerine gerekse de yasal

yukumluluklere uygunlugu konusunda fikir bildiren bir rapor hazirlar.

Butun bu kayitlarin incelenmesi, cesitli analizlerin ve testlerin

yapilmasi, raporun hazirlanmasi cesitli seviyedeki insan kaynaklarinin

katilimlariyla basarilabilir.Yapilan is bireylerin ve sistemin gelismis

olmasını gerektirdiğinden, şirketler bu konudaki yatırımlardan kaçınmamaktadırlar. Eldeki kaynakların kit ve pahalı olmasından dolayı bu kaynakların en verimli ve etkin şekilde kullanımı önem kazanmıştır. Etkin ve verimli kullanım ise, bütün çalışma dönemi boyunca hangi kaynakların hangi işlerde ve hangi zamanlarda kullanılacağına planlanmasıyla başarılabilir. Bağımsız denetim firmalarında kaynakların verimliliği, herhangi bir dönemde kaynağın firmaya para kazandıran çalışma saatlerinin, kaynağın firmadan ücretini aldığı saatlere bölümüyle bulunur. Bağımsız denetim firma yönetimlerinin amacı, her bir çalışanları için verimliliği maksimize etmektir.

Bu çalışmada amaç fonksiyonu verimliliği maksimize eden bir 0-1 tamsayı programlama modeli oluşturulmuştur. Donald R. Plane ve Claude McMillan, Jr. tarafından geliştirilen Ortuk Birerleme (Implicit Enumeration) algoritması çeşitli düzenlemelerle modele uygulanmıştır. Algoritma için Fortran 77 dilinde yazılan bilgisayar programı CDC bilgisayar sisteminde çalıştırılmış ve sonuçta çalışma dönemine yerleştirildiğinde maksimum verimliliği sağlayan işlerin seçimi yapılmıştır.

## INFORMATION ABOUT THE BUSINESS OF THE FIRM

### Definition of an Audit:

An audit is an examination of financial statements or other data derived from a client's accounting processes for the purpose of expressing an opinion on certain characteristics of that data such as fair presentation and accuracy. Most commonly, auditors examine:

- A client's balance sheet as of one or more dates.
- The related statements of income and cash flows for the period(s) included in the balance sheets.

### Purpose of an audit:

Testing financial statement assertions:

Users of an entity's financial statements--investors, creditors, lenders, regulatory agencies, ect.--normally want assurance from an independent party that the company's financial representations are credible.

The primary purpose of an audit is to provide this assurance by testing the six financial statement assertions that are made by management as follows:

**Existence or occurrence:**

The assets and liabilities exist, and the transactions have occurred as of or for the period ended on the dates specified. In other words, the company owns the assets and owns the liabilities as of the dates specified.

**Completeness:**

The financial statements are complete. All of the transactions and accounts that should be presented are included in the financial statements.

**Accuracy:**

Transactions and account balances are accurately recorded.

**Valuation:**

Assets are stated at realizable value, and liabilities are stated at expected settlement amounts.

**Rights and obligations:**

Assets are the rights, and liabilities are the obligations of the entity at the given date.

**Presentation and disclosure:**

Particular components of the financial statements have been properly classified, described, and disclosed.

## History of Auditing:

Audit comes from the Latin word "audire" which means "to hear". Literally, auditing has existed for a long time. Before 1500, Byzantine rulers used two scribes to keep independent records of transactions. This made it easy to detect defalcations or embezzlements by comparing the two sets of records. Accuracy was not really an issue; the ruler was primarily concerned with fraud, a concern which was the focus of auditing until the twentieth century. During the early Middle Ages, merchants of Italian city states needed assurance about the transactions performed by the captains of their ships. The merchants hired auditors to ensure that money or goods were properly accounted for.

The Industrial Revolution began in the 1800s, and companies grew in size. Owners hired other people to manage the companies. These owners became concerned about the protection and growth of their investments. In Britain, in particular, groups of stockholders visited their companies to verify the accounting records. As a result of these visits, it became apparent that auditing required specialized knowledge and training.

Following the U.S. Civil War, the American economy expanded rapidly. As a

result, Europeans invested heavily in U.S. companies. There was no accounting profession in the U.S. at that time. (Although there were accountants, there was no professional organization or set of ethical standards to govern accounting practices.) European investors, mostly British, sent their own accountants to the U.S. to examine companies in which they had invested. This provided a starting point for many U.S. accounting firms, as Americans became exposed to this form of business and as some British firms opened permanent U.S. operations.

Around 1900, U.S. commercial banks began to use public accountants to audit companies obtaining bank financing. In 1903, United States Steel Corporation became the first American company to publish financial statements accompanied by an auditors' report. Other large companies followed this example.

The role of the auditor in the U.S. became even more important after the Great Depression and the passage of the Securities Exchange Laws of 1933 and 1934. These laws required audits of companies which were offering or had outstanding public securities and debt.

A similar evolution has occurred throughout the industrialized world as

third-party reporting has become more important in the capital formation process. Nowadays, in many countries laws require audits of all companies, both public and private, and also regulate the degree of financial statement disclosure and the form of the audit report.

In recent years, there has been an increasing interest in the detection of fraud and in internal controls, including internal controls for electronic data processing (EDP). More emphasis has also been placed on the auditor's social and public responsibility to the users of the financial statements.

The large international accounting firms not only perform audits, but they also provide many other services. Accounting firms assist clients in tax matters; provide management information consulting; perform studies and evaluations of internal controls, including EDP-related internal controls; and provide services related to financial planning, such as developing financial projections. These are just a few examples of the many types of services provided by accounting firms today.

#### Audit Team:

Each audit is planned and performed by a team under the overall supervision of the engagement partner (the audit partner). The size, composition and

experience levels of the team are consistent with the nature, size and complexity of the assignment. Each member of the team is assigned tasks that he or she has adequate training and experience to complete. In addition to the engagement partner, an audit team typically consists of an audit manager, an in-charge person, and one or more staff accountants. An additional audit manager or senior who is a member of the Computer Audit Speciality Team (CASTS) is assigned to the audit team for clients that have complex computer systems.

**Engagement Partner:**

The engagement partner has overall responsibility for the quality of an audit and for the resulting Auditor's Report. In fulfilling these responsibilities the partner participates actively in the determination of the scope of the audit.

**Engagement Manager:**

The engagement manager plays a key role in assisting the engagement partner in discharging the firm's responsibility to a client and to the public.

One of his or her major responsibilities is to participate in and supervise the progress of the audit. This is to ensure that potential audit problems



are addressed and are brought to the attention of the engagement partner on a prompt basis.

**In charge:**

Usually a senior or a semi-senior is designated as being "in charge" of the day-to-day operations of the audit engagement. The in-charge normally participates in the development of the overall audit plan. He or she also supervises and assists the staff accountant in fulfilling their daily work responsibilities.

**Staff Accountant:**

A staff accountant, works with--and under the direction of the in-charge.

Experienced assistants and assistants comprise the two level of staff accountant classification. In general, the staff accountants perform the following jobs:

- They meet with the client personnel to determine the current status of the client's procedures
- They perform test of controls on a number of the client's accounting functions.
- They perform substantive tests of certain transactions and account

balances.

--They prepare working-papers and documentation.



## HUMAN RESOURCE SCHEDULING PROBLEM OF THE FIRM

### Definition of the Problem:

The firm renders audit services over 100 clients per year by preparing financial reports. In the preparation of financial reports, an intensive involvement of human resources from different levels is required. Due to the importance of rare and expensive human resources required in the preparation of financial reports, the management faces difficulties in the assignment of resources to the jobs. In other words, management has to decide which jobs will be accepted and when they will be performed by utilizing resources at a maximum efficiency.

In order to make the problem more understandable, let us briefly describe how the jobs are handled by the audit firm:

In general, each audit is achieved in 3 stages,

1-Planning

2-Field

3-Reporting

The audit of year-end financial statements covers the majority of the

work done by an audit firm during the year. The audit firm also performs audits at any time by the request of the client.

In the planning stage; engagement partner, engagement manager and previous year senior, budget the current year audit considering previous year experiences ; if the audit is the "first time through" type of an audit, the data of another firm in the same sector are considered for planning purposes. At the end of the planning stage for a job, expected partner, manager, senior and staff hours for the related audit work are determined.

Within the firm, every level has a hourly per diem which is adjusted periodically. By multiplying expected hours with the related level's per diems and adding the level totals, the approximate price for the related job is calculated; and a proposal is made considering this price. When the audit work takes more than the expected (budgeted) completion time per client, the firm accumulates a loss in return of these excess work hours which were not charged to the client.

After a planning stage, proper audit teams and a schedule which indicates the beginning dates of the jobs are prepared. Audit work performed at the client's office or at the factory is called as "field work". Field work is

completed in 2 stages as follows:

1)Interim

2)Final

For interim work there is no need for year-end financial statements of the client, so interim can be performed before the end of the year. After the client prepared and presented year-end financial statements, the audit team performs an audit on these financial statements beginning at a planned date after year end. At the end of the field work, the senior writes down the audit report under the supervision of engagement manager. After detailed controls and corrections of engagement manager and engagement partner the report is typed and signed by engagement partner and presented to the client.

#### Current Application of Scheduling within the Firm:

The firm makes proposals indicating prices to all previous year clients and potential first time through clients during the summer season just before the intensive work period which begins on September. Considering clients' requests such as due dates ,the jobs are scheduled manually for a two month period. At the end of each period, the scheduling is adjusted in order to allow the uncompleted jobs to continue and new jobs to begin such

that they would be completed by their due dates. Since the whole year audit period is not considered in advance and the scheduling is not achieved in line with it, during some periods the human resources are not enough to complete the jobs by regular daily work hours, whereas in other periods idle human resources may exist. The management has periodically prepared chargeability reports. Chargeability of any employee within the firm is defined for a period as the total hours spent by the employee for clients (which could be charged to the client) over the total working hours (which should be paid to the employee) for that period. The management aims to reach maximum chargeability for every employee within the firm.

## **THE SOLUTION PROCEDURE**

### **The Improved Model:**

In the present system the firm accepts all jobs in advance and then tries to schedule the jobs through out the year. In the improved model job selection is done such that chargibility is maximized when the jobs are scheduled. In other words acceptance and scheduling stages are combined to reach maximum chargibility. Assumptions made for the model are as follows:

- 1) Since the firm earns the same margin for all jobs, we assume that all jobs have equal preferences.
- 2) Since interim work can easily be performed in one or two weeks, there is no human resource capacity problem for the interims. Therefore only final works are considered.
- 3) As the year-end financial statement audits comprise the majority of the work done, we consider only year end audits. Considering the fact that most of the year-end audits are required to be completed until June, the model schedules the jobs for a period of 5 months (20 weeks) extending from January to the end of May.

4) Data related to the times spent by senior, experienced assistant and assistant for each job are received in hours and converted into weeks assuming 40 hours regular work per week. We also assume that all jobs would be completed with integer number of weeks.

5) Field work of each audit is performed by a team. The team consists of a senior, experienced assistants, and assistants. In the model assignments of the seniors have the priority.

6) Managers and partners are not included in the model since they are basically involved in supervisory work. Most of the operations and paper work are performed by the senior, experienced assistants and assistants.

7) The number of human resources at each level are kept constant throughout the whole audit period.

8) It is assumed that every audit would be completed within the budgeted time limits.

#### Problem Formulation:

A linear programming (LP) model is constructed for the human resource scheduling of an audit firm. Each audit is performed by a team of engagement partner, engagement manager, senior, experienced assistant and



assistant. Since engagement manager and partner are not included in scheduling, every job is divided into 3 groups such as senior works, experienced assistant works and assistant works.

The decision variables of the model are as follows:

$$S_{i,j} = \begin{cases} 1 & \text{if } i\text{'th job is performed by a senior during } j\text{'th week} \\ 0 & \text{otherwise} \end{cases}$$

$$E_{i,j} = \begin{cases} 1 & \text{if } i\text{'th job is performed by an experienced assistant} \\ & \text{during } j\text{'th week} \\ 0 & \text{otherwise} \end{cases}$$

$$A_{i,j} = \begin{cases} 1 & \text{if } i\text{'th job is performed by an assistant during } j\text{'th week} \\ 0 & \text{otherwise} \end{cases}$$

where,

$$i = 1, 100$$

$$j = 1, 20$$

### Constraints:

There are basically 5 group of restrictions in the model:

1-) Human resource capacity restrictions

2-) Audit completion time restrictions

3-) Structural restrictions

Human resource capacity and audit completion time restrictions are considered as explicit constraints of a binary integer programming problem. The structural restrictions are considered by assigning proper values to these variables from a given data set.

1) Human resource is the most important asset of an audit firm as the firm requires service of its personnel in the preparation of financial reports.

Considering the fact that the audit firm has limited human resources at each level, the total number of jobs performed by each level are limited

due to the capacity constraints. For example, the total number of senior works performed at week  $j$  can not be more than the total number of seniors at

week  $j$ . Due to the assumption that the number of resources at each level are kept constant during the audit period, the resources that will be allocated

to the jobs should be less than the related level's upper bound at any week.

So the human resource capacity constraint for each level can be stated as follows:

For the senior level;

$$\sum_{i=1,100} S_{Ai} * S_{i,j} \leq SB_j \quad j=1,20$$

For the experienced assistant level;

$$\sum_{i=1,100} E_{Ai} * E_{i,j} \leq EB_j \quad j=1,20$$

For the assistant level;

$$\sum_{i=1,100} A_{Ai} * A_{i,j} \leq AB_j \quad j=1,20$$

where  $S_{Ai}, E_{Ai}, A_{Ai}$  denote number of resources required for the  $i$ 'th job from senior, experienced assistant and assistant levels respectively.

$SB_j, EB_j, AB_j$  denote capacities for senior, experienced assistant and assistant levels respectively.

2)As it is expected that every audit would be completed in a certain amount of time based on previous experiences,the following constraints are included in the model:

For the senior level;

$$\sum_{j=1,20} Si,j = SCi \quad i=1,100$$

For the experienced assistant level;

$$\sum_{j=1,20} Ei,j = ECI \quad i=1,100$$

For the assistant level;

$$\sum_{j=1,20} Ai,j = ACi \quad i=1,100$$

where  $SCi, ECI, ACi$  denote the budgeted number of weeks for the  $i$ 'th job for senior, experienced assistant and assistant works respectively.

3) Every audit work is performed by a team of senior, experienced assistants and assistants in consecutive weeks. Audit team begins field work together and experienced assistant and/or assistant may complete their works before senior as it is already planned. Due to the reasons such as Central Bank or Capital Market Board requirements, or companies' annual meetings, etc., each audit report should be completed up to a specific date requested by the client. These team work, consecutive weeks and due date requirements are satisfied using a data set instead of explicit constraint functions in the model. Data set includes data related to number of weeks which should

be spent by each level for each job and due dates of each job. For example if the  $i$ 'th job is planned to be completed in 4 senior weeks, 3 experienced assistant weeks and 2 assistant weeks, and if  $i$ 'th job is planned to begin in week  $j$ , the program checks whether it is suitable for the  $i$ 'th job to begin in week  $j$  in order to be completed until the due date. If the answer is yes, the following variables are raised to 1 by retrieving the related information from the data set:

$$\begin{array}{lll}
 S_{i,j} = 1 & E_{i,j} = 1 & A_{i,j} = 1 \\
 S_{i,j+1} = 1 & E_{i,j+1} = 1 & A_{i,j+1} = 1 \\
 S_{i,j+2} = 1 & E_{i,j+2} = 1 & \\
 S_{i,j+3} = 1 & & 
 \end{array}$$

Objective Function:

As the chargeability is the most important point that concerns management it is aimed to increase chargeability by maximizing the number of jobs which can be performed within a 20 week period. So our objective function will be as follows:

$$\text{Max } Z = \sum_{i=1,100} \sum_{j=1,20} S_{i,j} + \sum_{i=1,100} \sum_{j=1,20} E_{i,j} + \sum_{i=1,100} \sum_{j=1,20} A_{i,j}$$

**In summary, the objective is to schedule as many jobs as possible without violating constraints of the problem.**



## SOLVING ZERO-ONE LINEAR PROGRAMMING PROBLEMS USING IMPLICIT ENUMERATION

### Review of Implicit Enumeration:

Implicit enumeration refers to a class of branch and bound algorithms designed specially for problems with binary decision variables. An exhaustive search of all possible solutions in a  $n$  variable 0-1 LP problem requires the enumeration of  $2^n$  possible solutions. This is almost an impossible task to carry out in large problems. Implicit enumeration algorithm overcomes this problem by considering only some of the  $2^n$  possible combinations, excluding the others which are concluded to be infeasible and/or suboptimal. These exclusions within the selected search employed, are conducted according to certain decision rules and tests. Although several refinements have been done by various authors, the original version of the implicit enumeration algorithm is credited to Balas (1). This original implicit enumeration algorithm developed by Balas(1) is inefficient in solving problems where the number of variables exceeds 30. This is largely because of the fact that Balas'(1) procedure has

extensive bookkeeping requirements involving a large storage space and tedious data handling routines. Glover(3) has proposed some exclusion tests and applied the "backtracking" concept to implicit enumeration. Geoffrion (2) reformulated the Balas'(1) algorithm by representing the tree in vector form, which greatly improved the bookkeeping and computational efficiency of the algorithm. In this formulation, the path  $P_k$  from the initial node of the tree to node  $k$  is stored as a vector and uniquely determines the remaining enumeration required. Variable indices appear in the vector if they correspond to assigned variables. The order of the indices represents the level in the tree. A positive subscript indicates that the variable has been assigned the value one, and a negative subscript indicates that the variable has been assigned the value zero. In backtracking, the rightmost positive element is changed to a negative number and all negative elements to the right of it are dropped. The enumeration is complete when all remaining elements are negative. Petersen(6) has applied the implicit enumeration algorithm of Balas(1) with the modifications of Geoffrion(2) and Glover(3), to the selection of research and development projects. Tuan(7) has worked out a more flexible backtracking



procedure in the tree search. Finally Plane and McMillan(8) applied Geoffrion's(2) algorithm for solving zero-one problems considering exclusion tests developed by Fleischmann(5), Glover(3), Glover and Zionts(4) and Petersen(6). It is accepted that implicit enumeration is a powerful technique for zero-one problems due to the fact that it only employs additions and subtractions and these operations can be done very efficiently by a computer. Moreover, the technique does not require slack or surplus variables as in the simplex algorithm and the number of constraints do not grow as in the other integer programming algorithms. Finally, since the method is free of rounding errors, less storage space is needed in the solution of a given problem.

Concepts:

In order to fully understand the process, it is first necessary to define the following concepts;

**Partial solution:** In the implicit enumeration tree each node represents a "partial solution" where a subset of decision variables is assigned a value, either zero or one.

**Free variables:** Variables which are not included in the partial solution are

are called "free variables".

**Completion of the partial solution:** A solution formed by assigning binary values to all of the free variables is called a "completion" of the partial solution. In an  $n$  variable 0-1 LP problem, if a partial solution contains  $s$  variable, there are  $2^{n-s}$  possible completions.

**Fathoming a partial solution:** In the implicit enumeration procedure for solving 0-1 LP problems, if a partial solution cannot be completed in such a way as to avoid violating one or more of the constraints or if it becomes evident that all possible completions of the present partial solution cannot improve the objective function value it is said that that partial solution has been "fathomed", meaning that all completions of that partial solution are implicitly enumerated and can be ignored.

**Backtracking:** When a partial solution is fathomed, in order to examine other sets of solutions which have not yet been enumerated, a form of backtracking is required. Backtracking should be done such that we do not miss any of the possible combinations while making sure that we do not reevaluate any of the solutions we have implicitly enumerated before.

### Standart Form of the Zero-One LP Problem:

The implicit enumeration algorithm developed by Plane and McMillan(8) requires the 0-1 LP problem to be formulated as follows:

- 1) All constraints are expressed as a " $\geq$ " constraint.
- 2) All coefficients in the objective function are positive or zero.
- 3) The value of the objective function is aimed to be minimized.

The first attribute enables us to identify quickly which variables, in a violated constraint, could help bring about feasibility by being raised to 1.

The second and third attributes enable us to deduce that no completion of a feasible partial which required raising additional variables to 1 would be attractive.

Reformulating the constraints in conformance with 1 above is easy. We simply put all elements to the left of the inequality sign (including the constant, if any). If necessary we then multiply by -1 to change  $\leq$  to  $\geq$ . If we have equality constraints we replace them with inequality constraints as follows:

Given:  $g_1 = c_1$

$$g_2 = c_2$$

: :

: :

: :

$$g_n = c_n$$

Replace:  $g_1 \geq c_1$

$$g_2 \geq c_2$$

: :

: :

: :

$$g_n \geq c_n$$

and add the following constraint,

$$g_1 + g_2 + \dots + g_n \leq c_1 + c_2 + \dots + c_n$$

Note that with this formulation the number of constraints is only increased by 1.

Reformulating an objective function where necessary to cause all coefficients to be positive or zero (2) is also straight forward. We simply replace each variable ( $X_j$ ) which has a negative coefficient ( $C_j$ ) in the objective function with a new variable which is one less the original variable. For example, if  $X_j$  has a negative  $C_j$ , a new variable  $Y_j$  is defined such that  $Y_j = 1 - X_j$ . Substitution of  $1 - Y_j$  for  $X_j$  will convert  $C_j$  to

a positive value. This substitution should be realized in the constraints as well as the objective function of the problem. If the objective function is aimed to be maximized, we multiply the objective function by -1 and then proceed to minimize it. Any negative coefficient which this multiplication produces is transformed as suggested above.

The maximization objective function is multiplied by -1 to convert the problem into a minimization problem as a result of which all objective function coefficients became negative. This is corrected by proper substitution of the variables with new ones. Finally the " $\leq$ " and " $=$ " constraints of the problem were converted to " $\geq$ " constraints as suggested by Plane and McMillan. The standartized form of the objective function and constraints are as follows:

Objective Function:

$$\text{Min } Z = \sum_{i=1,100} \sum_{j=1,20} NS_{i,j} + \sum_{i=1,100} \sum_{j=1,20} NE_{i,j} + \sum_{i=1,100} \sum_{j=1,20} NA_{i,j}$$

Constraints:

For the senior level;

$$\sum_{i=1,100} S_{Ai} * N_{Si,j} \geq 100 - S_{Bj} \quad j=1,20$$

$$\sum_{j=1,20} - N_{Si,j} \geq S_{Ci} - 20 \quad i=1,100$$

$$\sum_{i=1,100} \sum_{j=1,20} N_{Si,j} \geq 2000 - \sum_{i=1,100} S_{Ci}$$

For the experienced assistant level;

$$\sum_{i=1,100} E_{Ai} * N_{Ei,j} \geq 100 - E_{Bj} \quad j=1,20$$

$$\sum_{j=1,20} - N_{Ei,j} \geq E_{Ci} - 20 \quad i=1,100$$

$$\sum_{i=1,100} \sum_{j=1,20} N_{Ei,j} \geq 2000 - \sum_{i=1,100} E_{Ci}$$

For the assistant level;

$$\sum_{i=1,100} A_{Ai} * N_{Ai,j} \geq 100 - A_{Bj} \quad j=1,20$$

$$\sum_{j=1,20} - N_{Ai,j} \geq A_{Ci} - 20 \quad i=1,100$$

$$\sum_{i=1,100} \sum_{j=1,20} NA_{i,j} \geq 2000 - \sum_{i=1,100} AC_i$$

where,

$$NS_{i,j} = 1 - S_{i,j}$$

$$NE_{i,j} = 1 - E_{i,j}$$

$$NA_{i,j} = 1 - A_{i,j}$$

Reformulating the objective function and constraints in conformance with the algorithm's requirements we increased the total number of constraints by 3 because of the equality constraints.

#### Exclusion Tests and Methods:

Many tests for reducing the enumeration have been proposed. However, these tests can reduce the enumeration at the expense of added calculation, and they should be applied as long as the extra calculations do not offset the benefits of reduced enumeration. Balas(1) brought forward three criteria for exclusion in an attempt to improve efficiency of the algorithm.

Balas' first test: If for any constraint the infeasibility can not be diminished even though all the variables with negative coefficients are entered, then that node is fathomed.

Balas' second test: If for any free variable, assignment of value 1 will make the present value of the objective function value less than the lower bound so far computed, that node is fathomed.

Balas' third test: If for all violated constraints, the coefficients of the entering variable are positive, that node is fathomed since it can not help to achieve feasibility.

In a note following Balas'(1) paper, Glover(3) also suggested modifications to increase the efficiency of the algorithm. Some other tests were suggested by Geoffrion(2), Fleischmann(5), Glover(3), Glover and Zionts(4) and Petersen(6). Plane and McMillan(8) summarized these tests as two methods for implicitly enumerating solutions as follows:

1) A group of solutions is implicitly enumerated by infeasibility if it is not possible to satisfy every constraint by any completion of the current partial solution.

2) A group of solutions is implicitly enumerated by feasibility when the partial solution being considered is feasible when it is completed by setting all remaining variables equal to zero. It is known that it would never be attractive to set additional variables equal to one, because this



would never decrease the value of the objective function. This is because all of the objective function coefficients are positive or reformulated as positive numbers.

### The General Procedure Improved by Plane and McMillan for Solving Zero-One LP Problems Via Implicit Enumeration:

Before reviewing the procedure improved by Plane and McMillan(8), let us briefly describe the symbolic notation used for easier reference;

$f$  :It represents the value of the objective function.

$S_k$  :It represents the set of variables which have been given specific values in a partial solution at step  $k$ . The elements stored at  $S_k$  will be the subscripts of the variables which have been assigned specific values in the  $k$ 'th partial solution. If a subscript has a minus sign in front of it this means that the variable which that subscript specifies has a value of 0 in that partial solution; otherwise it has a value of 1. The variables are also stored at  $S_k$  from left to right in the order in which they are assigned. It is also helpful to define a free variable as any variable which is not in  $S_k$ .

$V_k$  :It represents the set of constraints which are violated when the

partial solution is completed by setting to zero all free variables.

$T_k$  :It represents the set of variables which are both free variables and have positive coefficients in one or more constraints in  $V_k$ .

$X$  :It represents the best feasible solution thus far found-that is the feasible solution which thus far yields the smallest value for  $f$ .

$Z$  :It represents the value of  $f$  when the objective function is evaluated at solution  $X$ .

The general procedure is portrayed in the flow diagram of Fig.1 .The procedure begins by setting all variables to zero in order to check whether the solution "all  $X$ 's = 0" is feasible.If it is so the problem is solved otherwise an upper bound is calculated by evaluating  $f$  at setting all variables to 1.After the determination of starting value for  $Z$ , the algorithm cycle begins with  $S$  empty.The algorithm identifies the set of violated constraints, $V$ .It stores in  $T$  the variables which, by being raised to 1,might bring about feasibility.If it possible to make every constraint in  $V$  feasible by adding variables in  $T$ ,the algorithm adds to  $S$  that variable in  $T$  which is most "helpful" as measured by the sum of its coefficients in all of the constraints.This process is continued

until a partial solution is fathomed, either by feasibility or infeasibility.

When backtracking is applied, the general systematic procedure will be to complement the rightmost element in  $S$  which has not yet been complemented; that is, the rightmost positive element in  $S$  is found and replaced with its negative complement. At this step it is necessary to drop any element in  $S$  to the right of the variable currently being complemented and being again to fathom the new partial solution. The procedure continues until all of the elements in  $S$  are negative. The incumbent solution, if any, is an optimal solution. If no incumbent solution has been found, there is no feasible solution better than the solution corresponding to the best upper bound which was already found.

#### The Modified Implicit Enumeration Algorithm:

We have modified the implicit enumeration algorithm improved by Plane and McMillan(8) in order to solve 0-1 integer programming problem formulated for the human resource scheduling in an audit firm. The final form of the algorithm is obtained by adding the following improvements to the previous procedure:

- 1-) Although the variables are processed one by one in the previous

algorithm, the new procedure considers a group of variables since it is almost impossible to include these restrictions in the constraints. These restrictions are satisfied in advance using a data set instead of explicit constraint functions. The variable groups that satisfy the following restrictions are processed throughout the algorithm:

-- When a variable related to seniors is assigned, variables related to experienced assistants and assistants should be assigned since every audit is performed by a team of senior, experienced assistants and assistants.

-- When any variable related to seniors, experienced assistants and assistants is assigned, consecutive variables in the same categories should be assigned since every audit is performed in consecutive weeks.

-- Assignment of variables are restricted by due dates since every audit should be completed up to a specific date due to the reasons such as Central Bank and Capital Market Board requirements and companies' annual meetings.

Data set includes data related to number of weeks which should be spent by each level for each job and due dates of each job. Considering the above

mentioned restrictions and retrieving the related information from the data data set, the program makes assignments for variable groups.

Variables related to seniors are considered as the representative of the variable groups and when any of these variables is assigned, the corresponding group of variables is assigned by the help of the data set.

2-) In the general enumeration algorithm, the variable with the greatest coefficient sum is added to the partial solution. In the modified algorithm, a variable group determined by the data set is added to the partial solution without any priority.

These modifications have simplified the model and resulted in a decrease in computational time.

## ANALYSIS OF RESULTS

Application of the modified implicit enumeration algorithm to the integer programming problem yields the selection of jobs which will be performed during a period of 5 months extending from January to the end of May.

Omitting overtime capacity it was possible to select and schedule 69 jobs out of 100 jobs performed in 1989. The optimum value of the objective function, which is the total number of assigned resources, is 730 men-weeks.

Since the model is based on 1989 working period data, we can compare the number of jobs selected and chargibility obtained for each resource with actual values of 1989.

TABLE 1.  
Summary of 1989 Performance

	Seniors	Exp.Assistants	Assistants
Total capacity for the period(hrs)	16,800	8,800	12,000
Chargable hours			
Normal	14,670	6,230	9,310
Overtime	7,250	2,930	3,810
Chargibility %	79%	69%	74%

Number of jobs performed with overtime = 100

TABLE 2.

Summary of Model Outputs Without Overtime

		Seniors	Exp.Assistants	Assistants
Total capacity for the period(hrs)		16,800	8,800	12,000
Chargable hours	Normal	15,320	5,460	8,840
Chargibility %		91%	64%	74%

Number of jobs scheduled without overtime = 69

As can be seen from a comparison of TABLE 1. and TABLE 2.,chargibility percentage for seniors increased from 79% to 91% while chargibility percentage for experienced assistants decreased from 69% to 64% and chargibility percentage for assistants remained constant.

These results are highly satisfactory since the cost of human resources for the senior category is much more higher than the other levels and overtime is not desired by the management due to the bid price effect in such a competitive market.

On the other hand, in order to achieve an accurate comparison of model

outputs with actual results, the program was run by increasing capacities as much as overtimes realized in 1989 and the results were presented in TABLE 3. as follows:

TABLE 3.

Summary of Model Outputs With Overtime

		Seniors	Exp.Assistants	Assista
Total capacity for the period(hrs)		16,800	8,800	12,00
Chargable hours	Normal	16,760	7,880	11,320
	Overtime	5,120	1,280	1,720
Chargibility %		89%	85%	89%
Number of jobs scheduled with overtime = 100				

As can be seen from a comparison of TABLE 1. and TABLE 3., not only the chargibility percentages were increased but also the required overtime hours were decreased by considerable amounts. The chargibility percentage for seniors increased from 79% to 91% , the chargibility percentage for experienced assistants increased from 69% to 85% and the chargibility percentage for assistants increased from 74% to 89% . Since the model



**scheduled the jobs more efficiently throughout the working period, less overtime is required in order to complete all jobs.**



## CONCLUSION

The application of a linear programming model to an audit firm yields an overall human resource scheduling over a planning period such that the chargibility or the utilization of resources is maximized.

The determination of jobs and scheduling of human resources in an audit firm is achieved with 2 major contributions as follows:

1-) We have formulated a model that combines selection and scheduling stages. In the model, job selection is done such that chargibility is maximized when the jobs are scheduled.

2-) We have modified the implicit enumeration algorithm developed by Plane and McMillan. In order to simplify the problem and to decrease computational time, the modified algorithm processes variables in groups instead of considering them individually.

Computational experience resulted with the decision of which jobs will be accepted and when they will be performed by utilizing resources at a maximum efficiency.

The model helps the management to decide which jobs to accept and advises

a schedule that will utilize resources with maximum efficiency.

The application of the model does not induce any significant cost to the firm. Because of the simplicity of application, once the data are entered the application can easily be performed.



## APPENDIX

### Definition of Variables Used in the Program:

**SS(I,J)** :Partial solution matrix to keep track whether the corresponding variable related to seniors is free(=0),assigned one(=11) or assigned zero (=10)

**EE(I,J)** :Partial solution matrix to keep track whether the corresponding variable related to experienced assistants is free(=0),assigned one(=11) or assigned zero (=10)

**AA(I,J)** :Partial solution matrix to keep track whether the corresponding variable related to assistants is free(=0),assigned one(=11) or assigned zero (=10)

**S(I,J)** :Matrix containing the value of the corresponding SS(I,J).If

SS(I,J) is assigned to 11, S(I,J) will be assigned to 1 otherwise to 0.

**E(I,J)** :Matrix containing the value of the corresponding EE(I,J).If

EE(I,J) is assigned to 11, E(I,J) will be assigned to 1 otherwise to 0.

**A(I,J)** :Matrix containing the value of the corresponding AA(I,J).If

AA(I,J) is assigned to 11, A(I,J) will be assigned to 1 otherwise to 0.

**SAA(I,J):** Binary problem constraint coefficients matrix for senior category.

**EAA(I,J):** Binary problem constraint coefficients matrix for experienced assistant category.

**AAA(I,J):** Binary problem constraint coefficients matrix for assistant category.

**SB(I)** : Binary problem right hand side array for senior category.

**EB(I)** : Binary problem right hand side array for experienced assistant category.

**AB(I)** : Binary problem right hand side array for assistant category.

**CS(I,J)** : Binary problem objective function coefficients matrix for senior category.

**CE(I,J)** : Binary problem objective function coefficients matrix for experienced assistant category.

**CA(I,J)** : Binary problem objective function coefficients matrix for assistant category.

**VS(I,J)** : Matrix containing the values when the partial solution is completed by setting to zero all free variables. If **SS(I,J)** is assigned to zero, **VS(I,J)** will be assigned to 0 otherwise to **S(I,J)**.

**VE(I,J)** :Matrix containing the values when the partial solution is completed by setting to zero all free variables.If **EE(I,J)** is assigned to zero, **VE(I,J)** will be assigned to 0 otherwise to **E(I,J)**.

**VA(I,J)** :Matrix containing the values when the partial solution is completed by setting to zero all free variables.If **AA(I,J)** is assigned to zero, **VA(I,J)** will be assigned to 0 otherwise to **A(I,J)**.

**SD(I)** :Array containing the value of the left hand side of each constraint evaluated at **VS(I,J)**.

**ED(I)** :Array containing the value of the left hand side of each constraint evaluated at **VE(I,J)**.

**AD(I)** :Array containing the value of the left hand side of each constraint evaluated at **VA(I,J)**.

**SDD(I)** :Array of flags indicating whether the corresponding constraint related to seniors is violated(=1) or not(=0).

**EDD(I)** :Array of flags indicating whether the corresponding constraint related to experienced assistants is violated(=1) or not(=0).

**ADD(I)** :Array of flags indicating whether the corresponding constraint related to assistants is violated(=1) or not(=0).

**TDD** :Number of violated constraints.

**ZI** :Value of objective function evaluated at  $PS(I,J), PE(I,J), PA(I,J)$

**FP** :Value of objective function evaluated at  $VS(I,J), VE(I,J), VA(I,J)$

**Z** :Limit for variable selection.

**STT(I)** :Array of flags indicating which constraints should be considered for T storages for senior category.

**ETT(I)** :Array of flags indicating which constraints should be considered for T storages for experienced assistant category.

**ATT(I)** :Array of flags indicating which constraints should be considered for T storages for assistant category.

**TTS(I,J)**:Matrix containing the values when partial solution is completed by setting to 1 all free variables.If  $SS(I,J)$  is assigned to 0, $TTS(I,J)$  will be assigned to 1 otherwise to  $S(I,J)$ .

**TTE(I,J)**:Matrix containing the values when partial solution is completed by setting to 1 all free variables.If  $EE(I,J)$  is assigned to 0, $TTE(I,J)$  will be assigned to 1 otherwise to  $E(I,J)$ .

**TTA(I,J)**:Matrix containing the values when partial solution is completed by setting to 1 all free variables.If  $AA(I,J)$  is assigned to 0, $TTA(I,J)$

will be assigned to 1 otherwise to  $A(I,J)$ .

$ST(I)$  :Array containing the value of the left hand side of each constraint evaluated at  $TTS(I,J)$ .

$ET(I)$  :Array containing the value of the left hand side of each constraint evaluated at  $TTE(I,J)$ .

$AT(I)$  :Array containing the value of the left hand side of each constraint evaluated at  $TTA(I,J)$ .

$TS(I)$  :Array to keep track whether the corresponding constraint for senior category have any variable or not.

$TE(I)$  :Array to keep track whether the corresponding constraint for experienced assistant category have any variable or not.

$TA(I)$  :Array to keep track whether the corresponding constraint for assistant category have any variable or not.

$PS(I,J)$  :Matrix containing the best feasible solution thus far found for variables related to seniors.

$PE(I,J)$  :Matrix containing the best feasible solution thus far found for variables related to experienced assistants.

$PA(I,J)$  :Matrix containing the best feasible solution thus far found for



variables related to assistants.

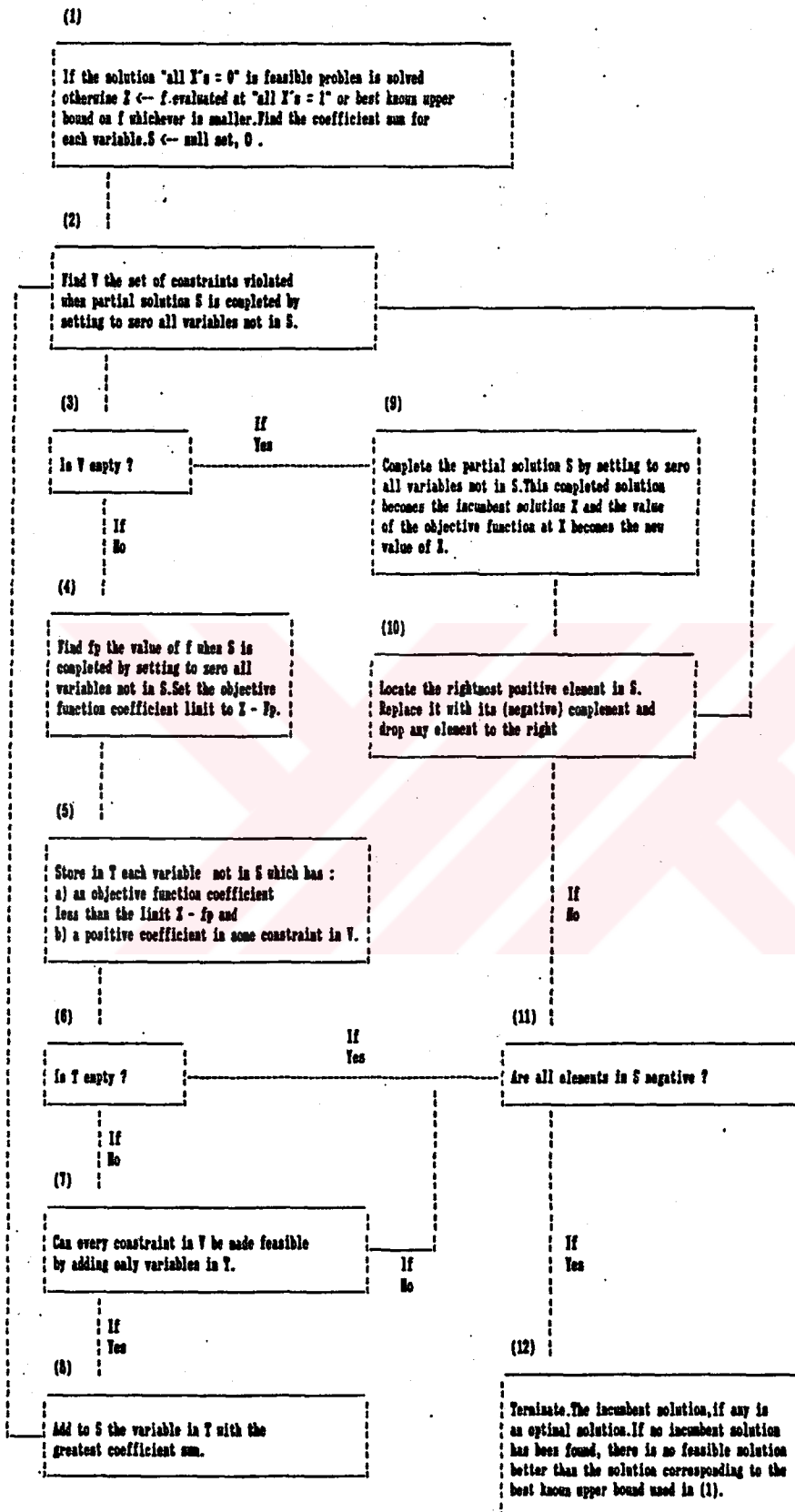
**DAT(I,J)**:Data matrix containing audit times per each employee category for each job.

**MAX(I)** :Data array indicating maximum starting week of each job.

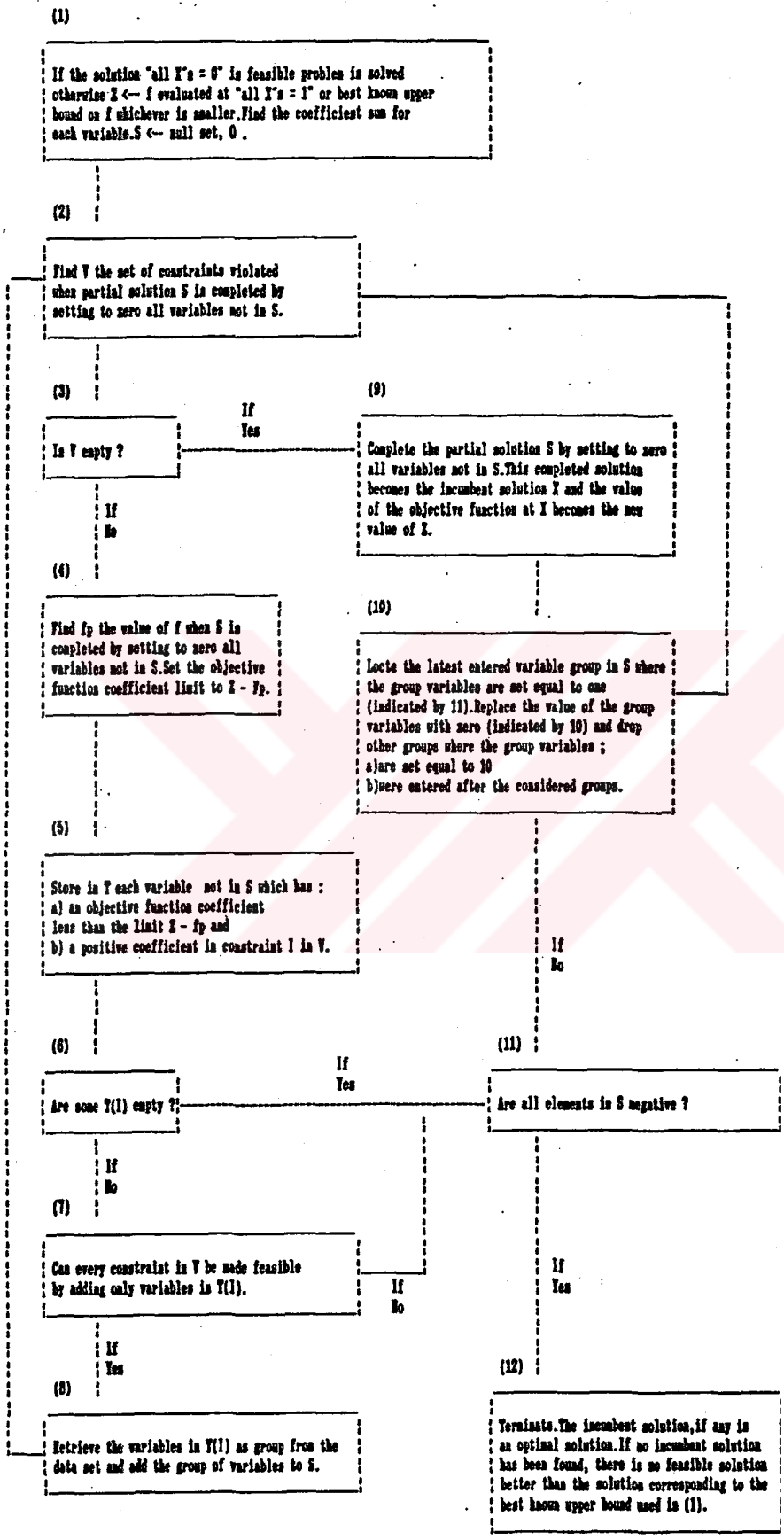
**K** :Clock to keep track iteration numbers.



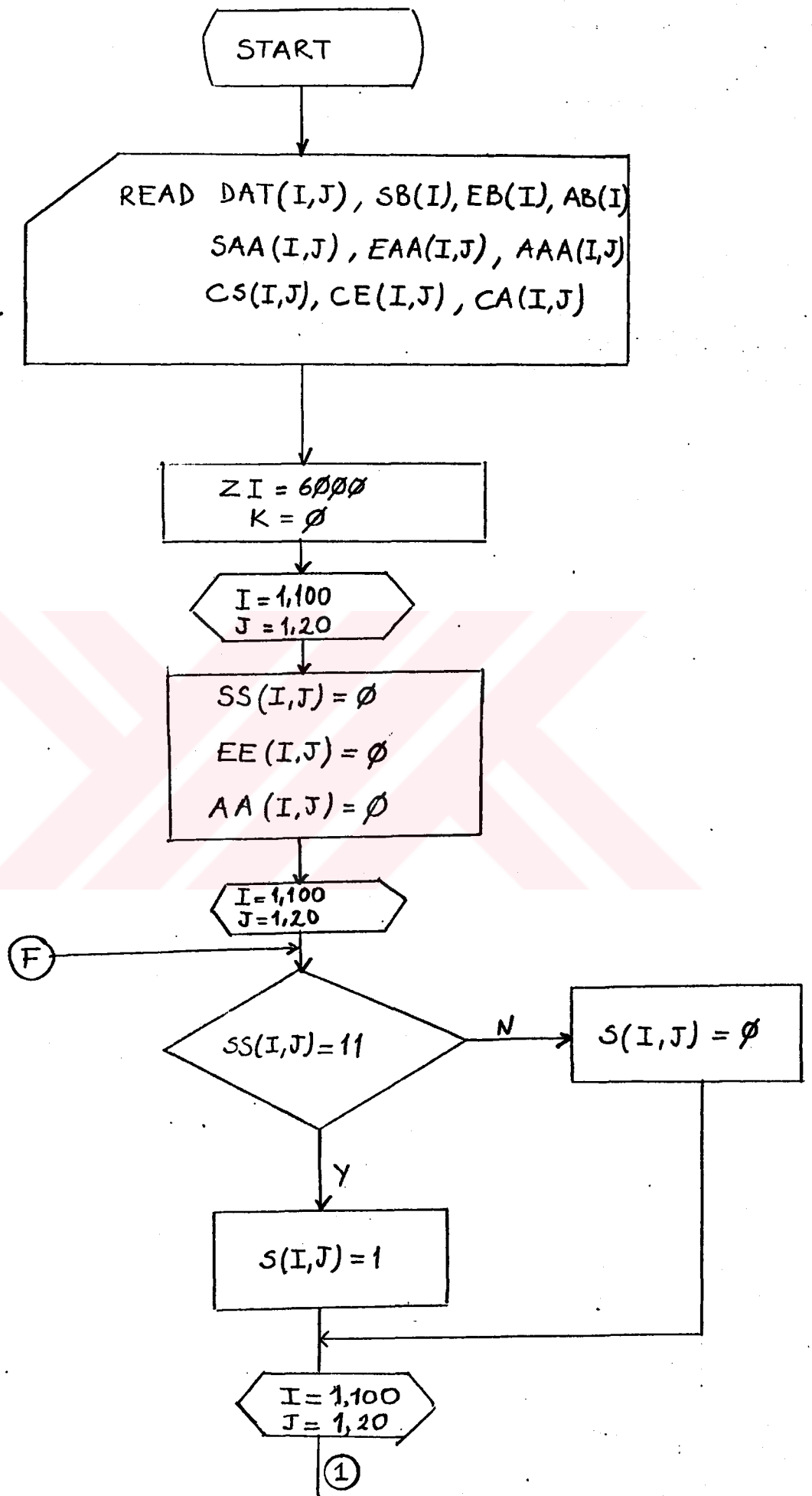
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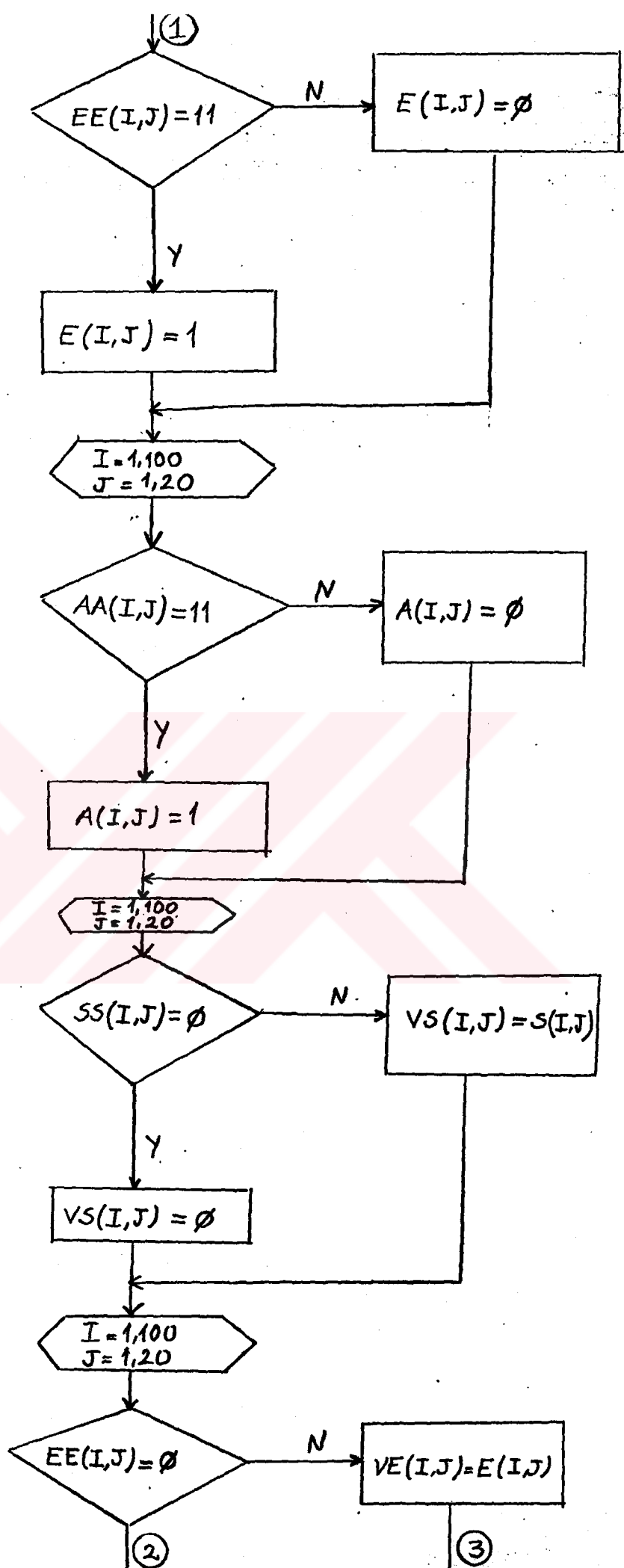


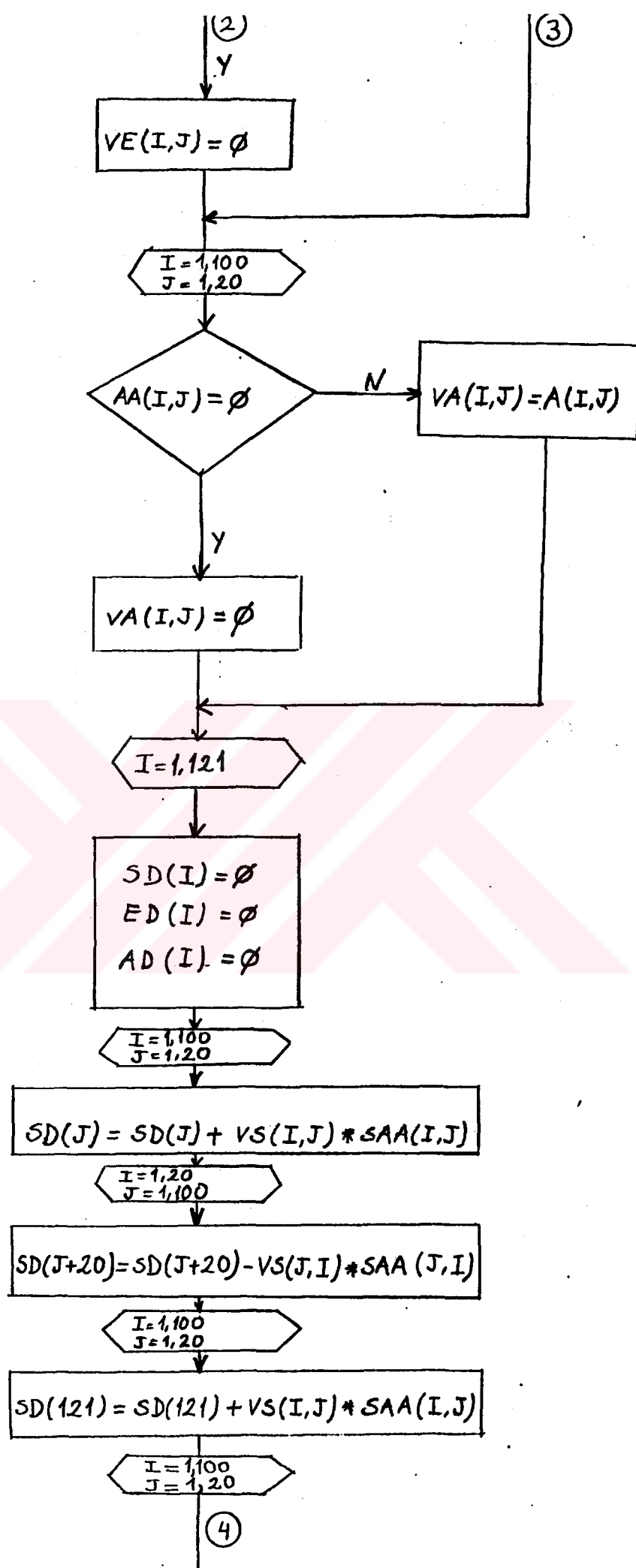
FLOWCHART OF MODIFIED IMPLICIT ENUMERATION ALGORITHM

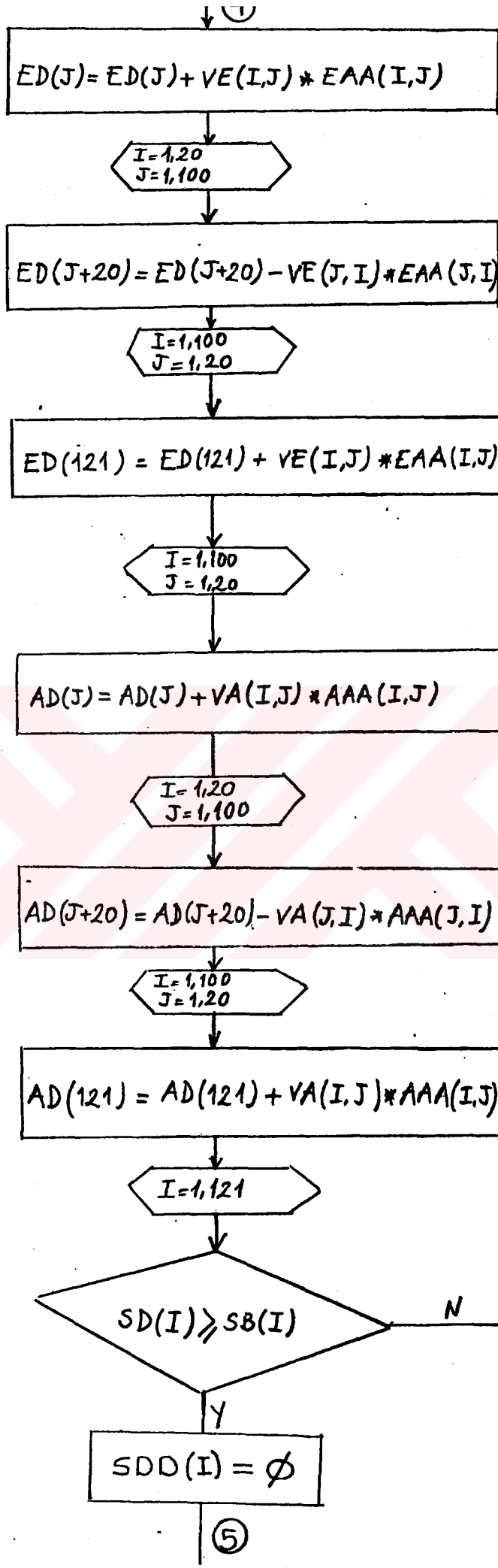


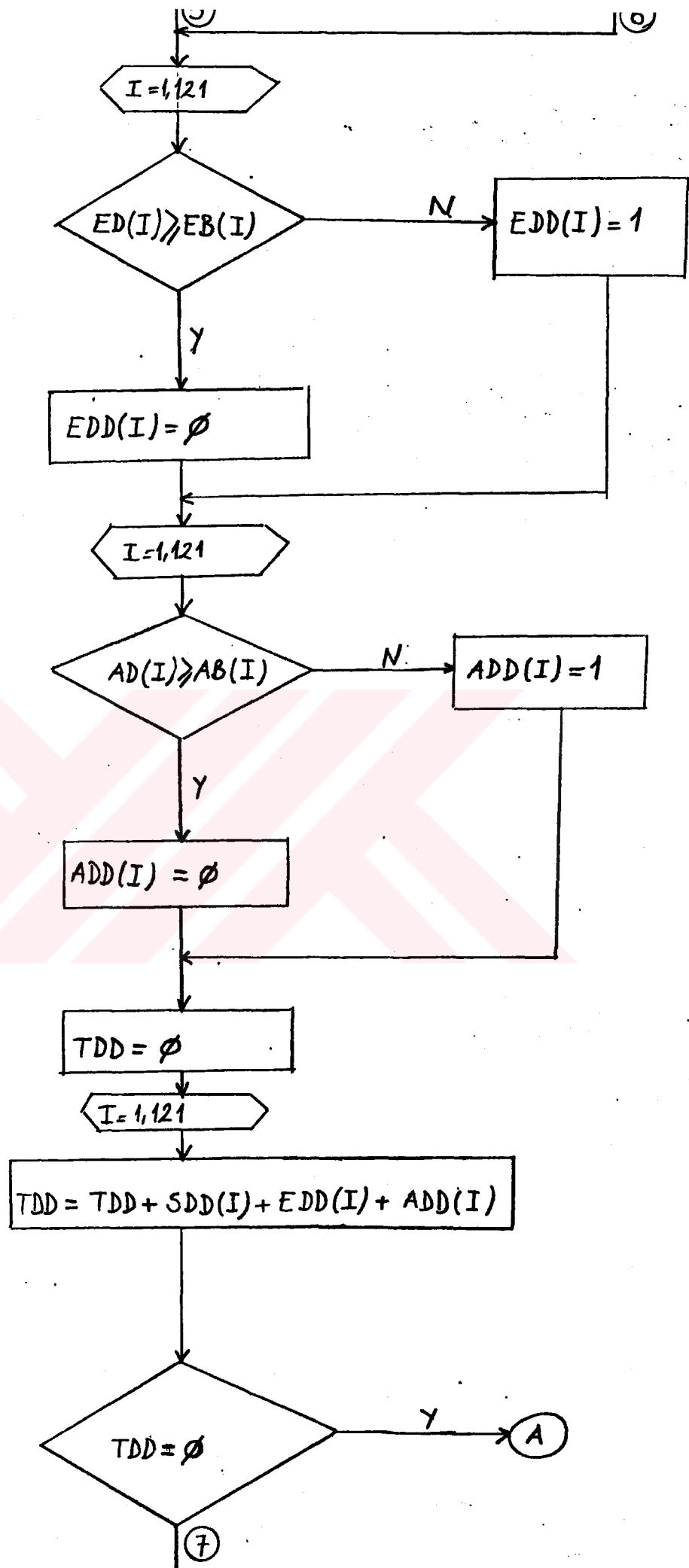
# COMPUTER FLOWCHART OF THE MODIFIED ALGORITHM



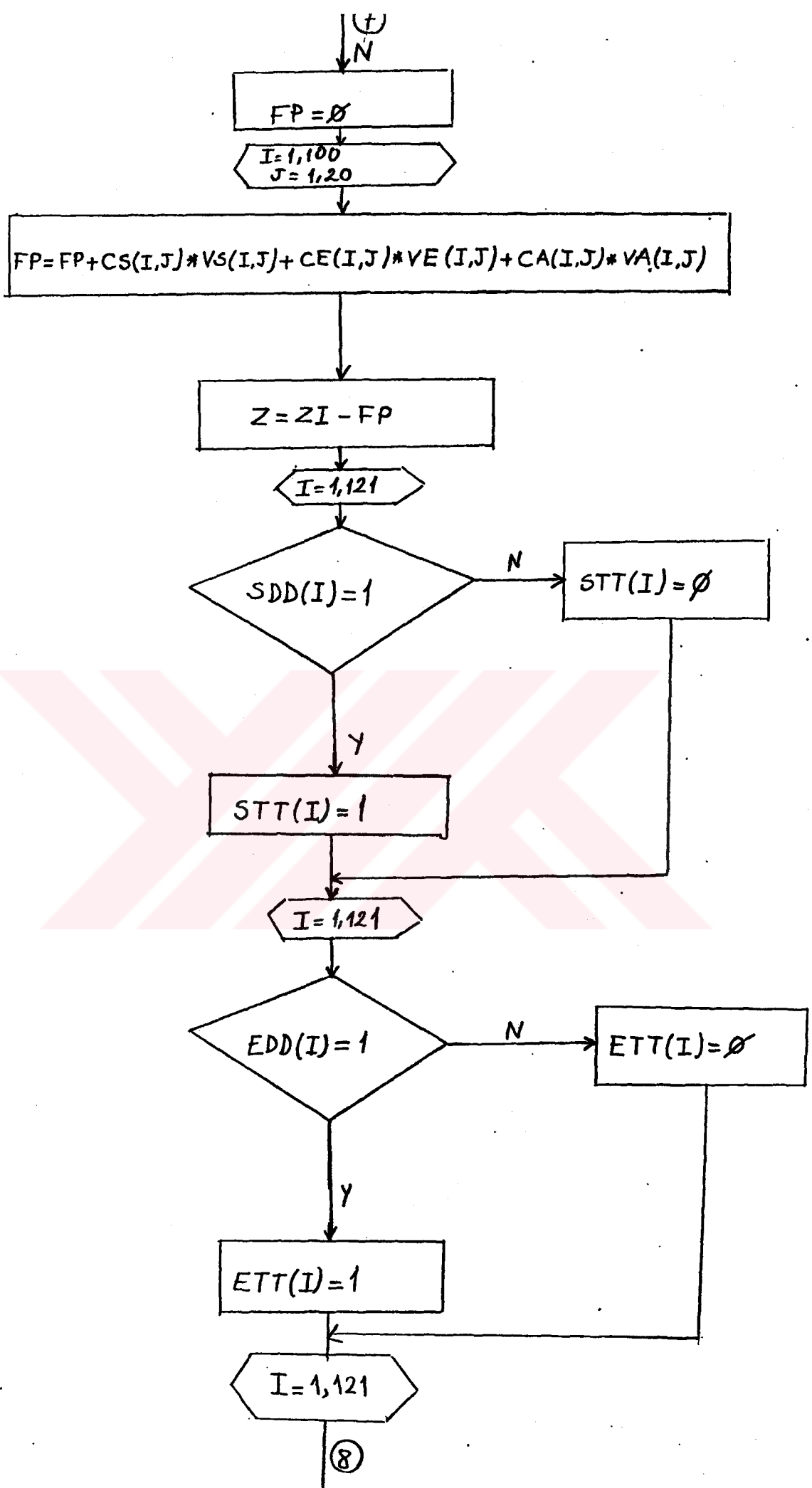


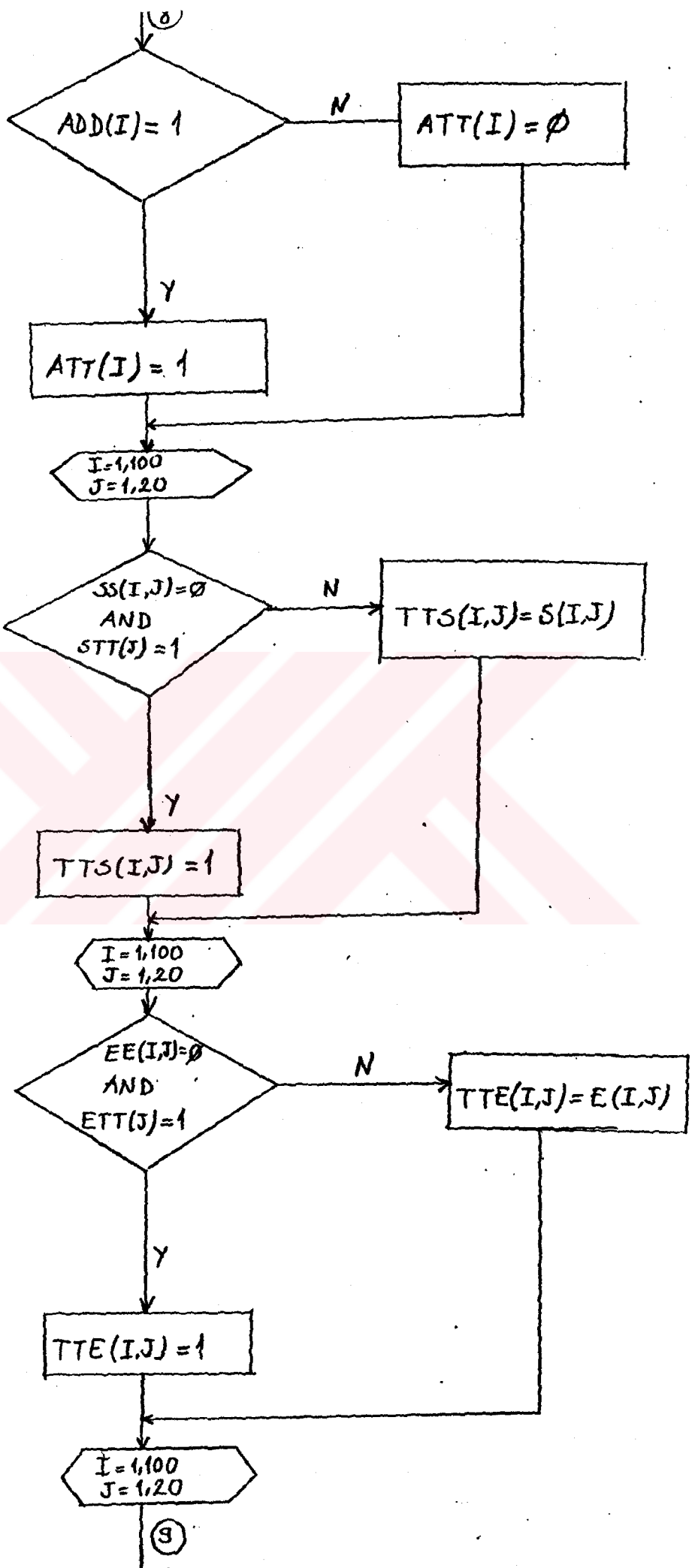


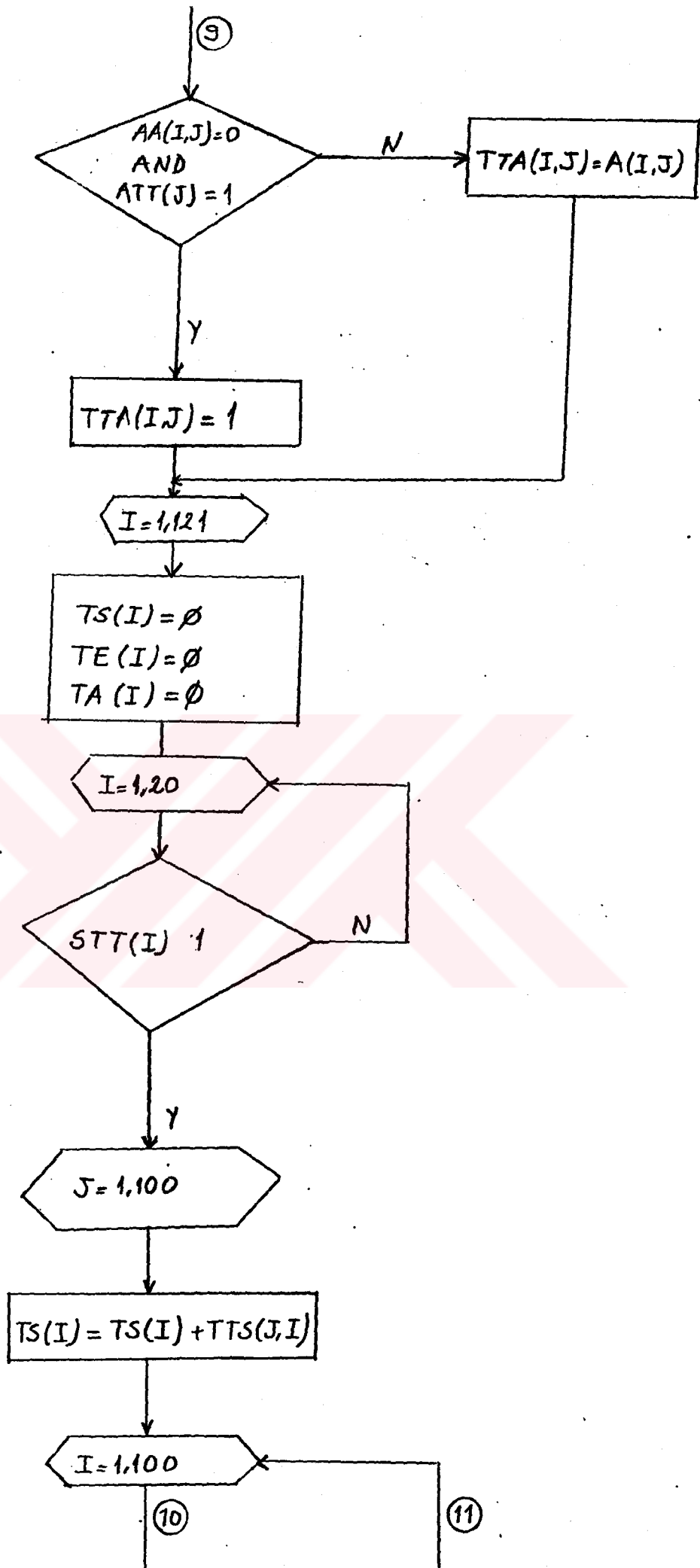


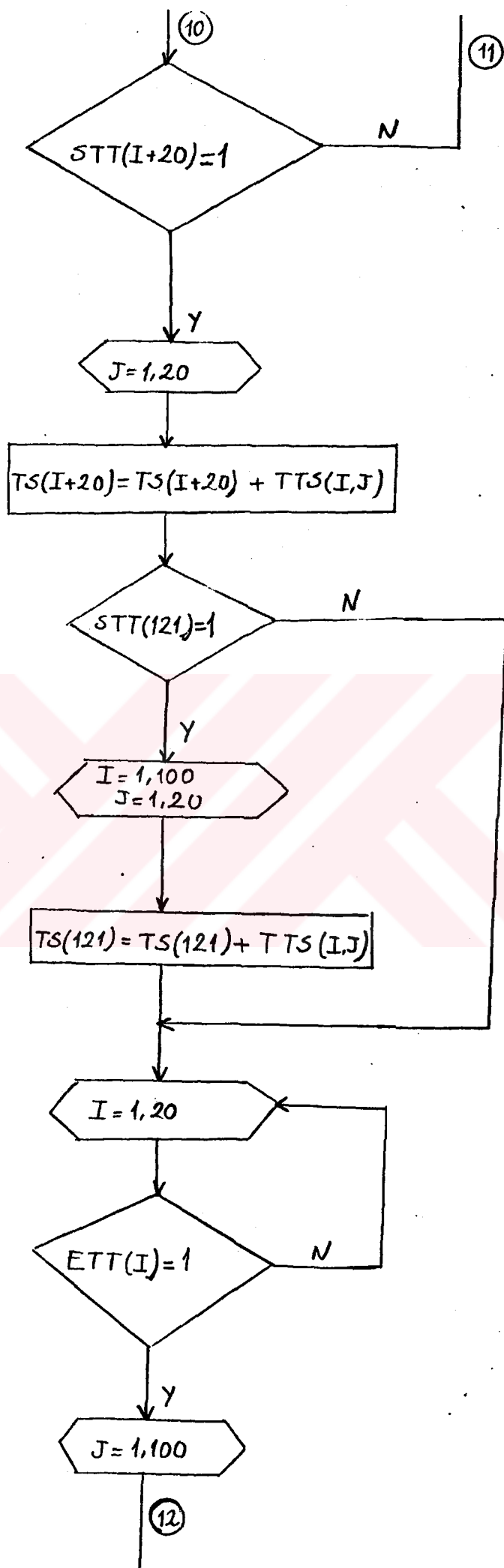


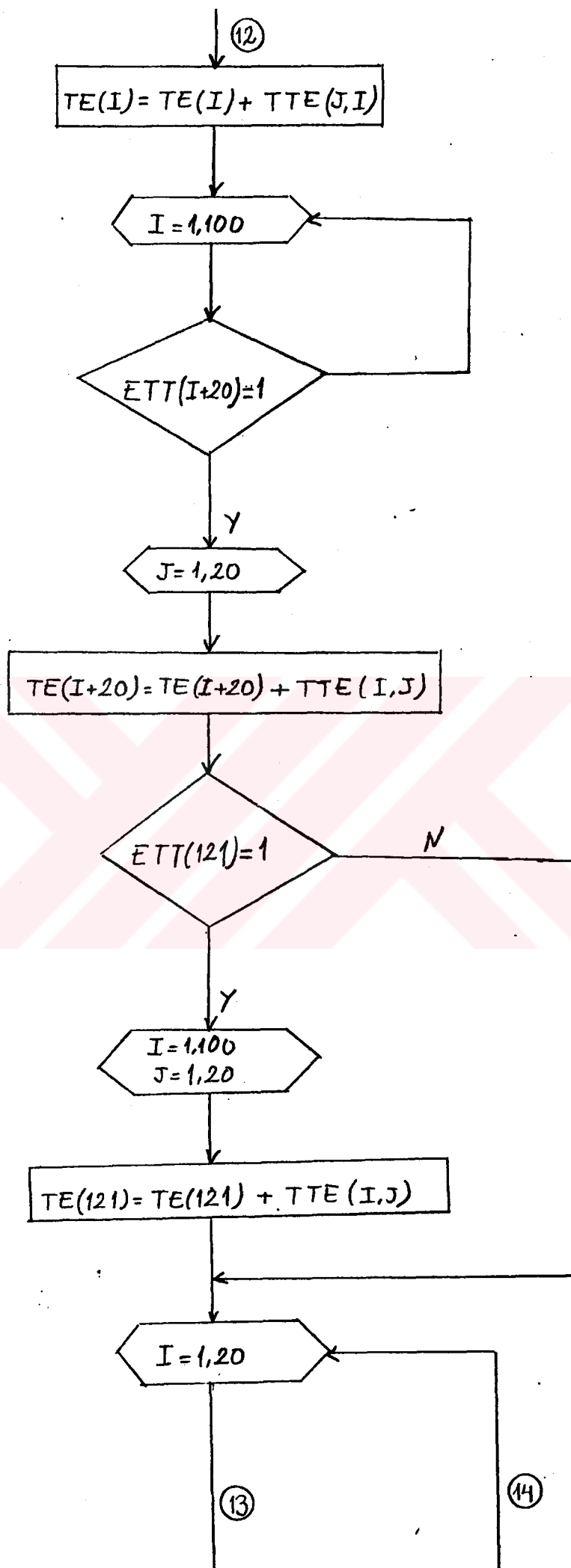


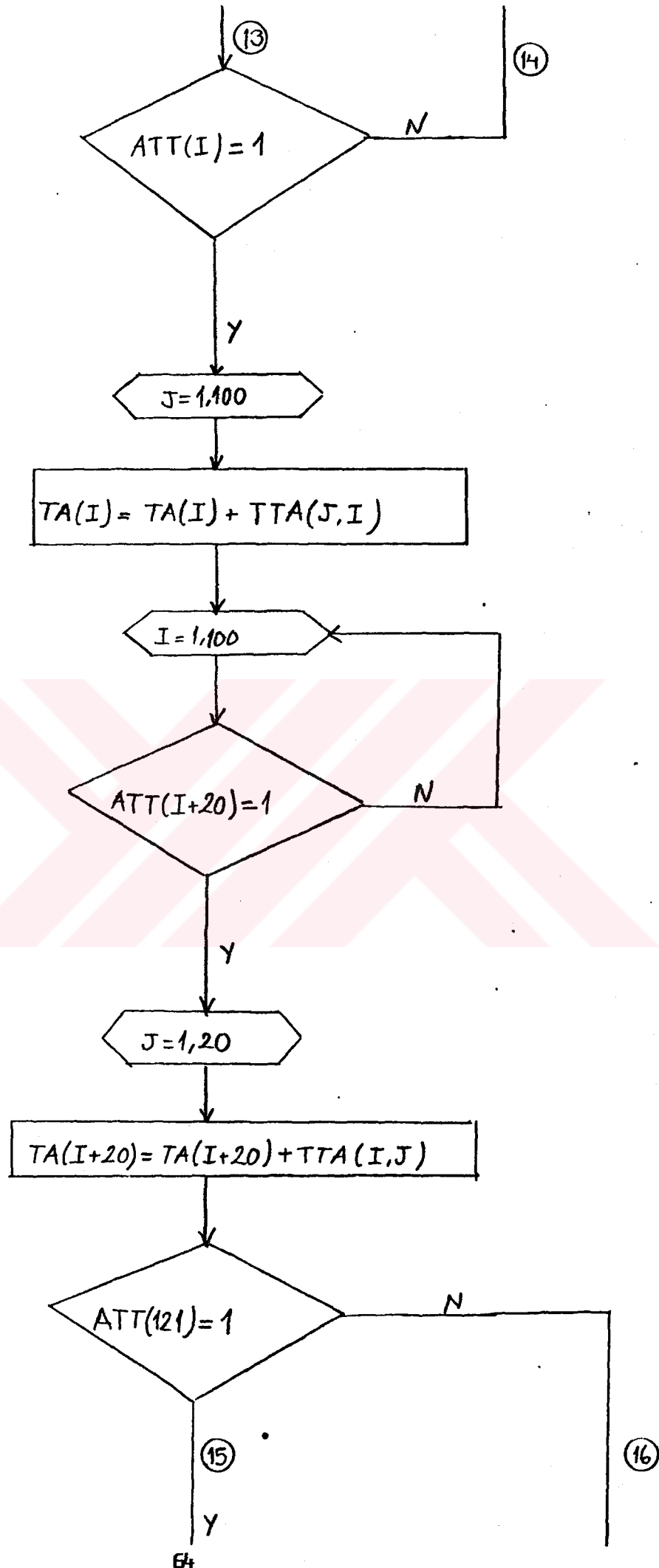


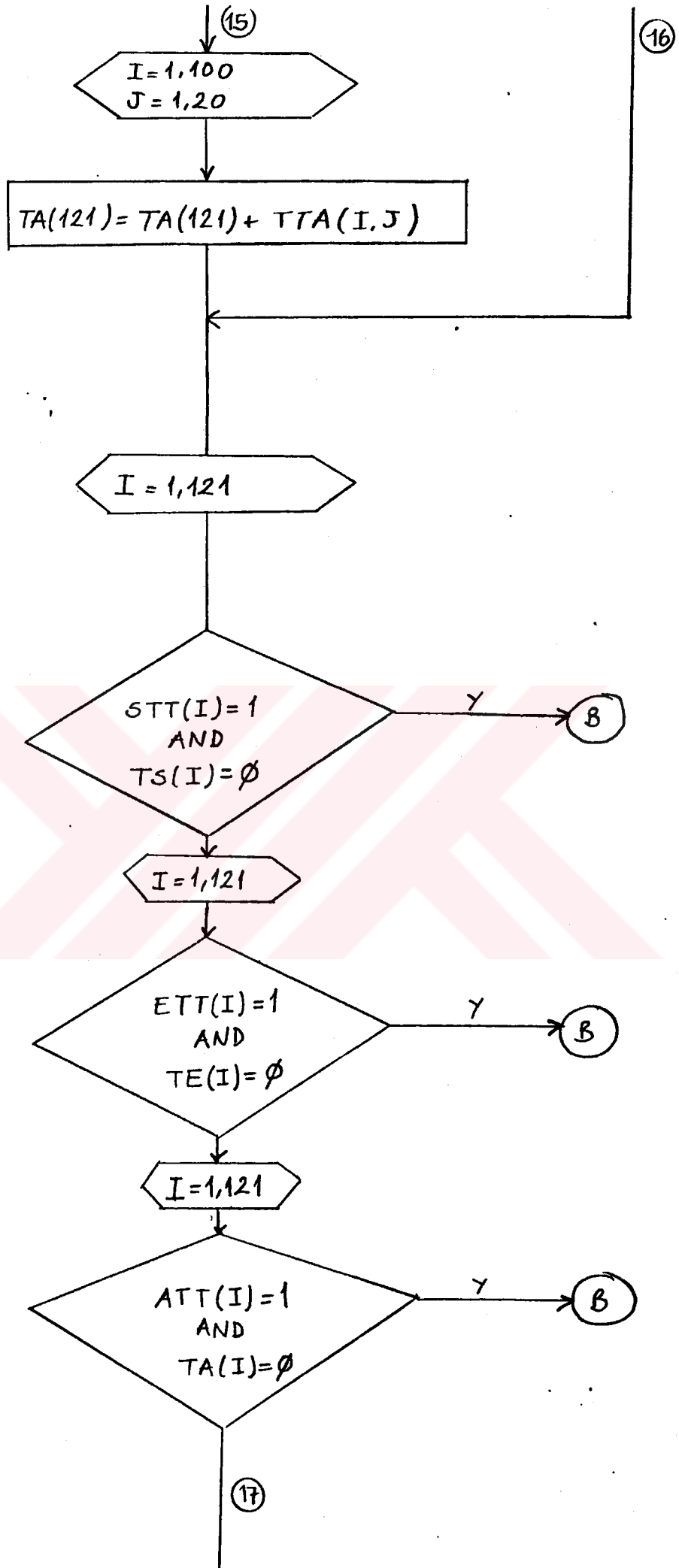


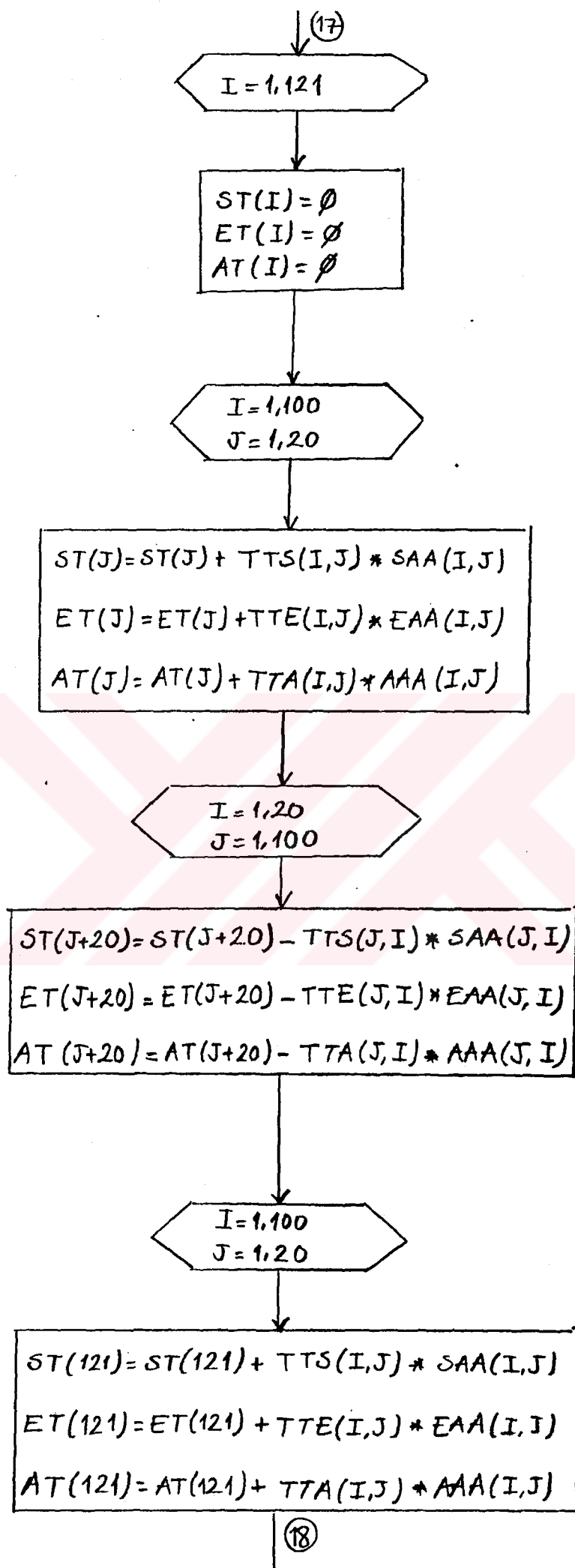




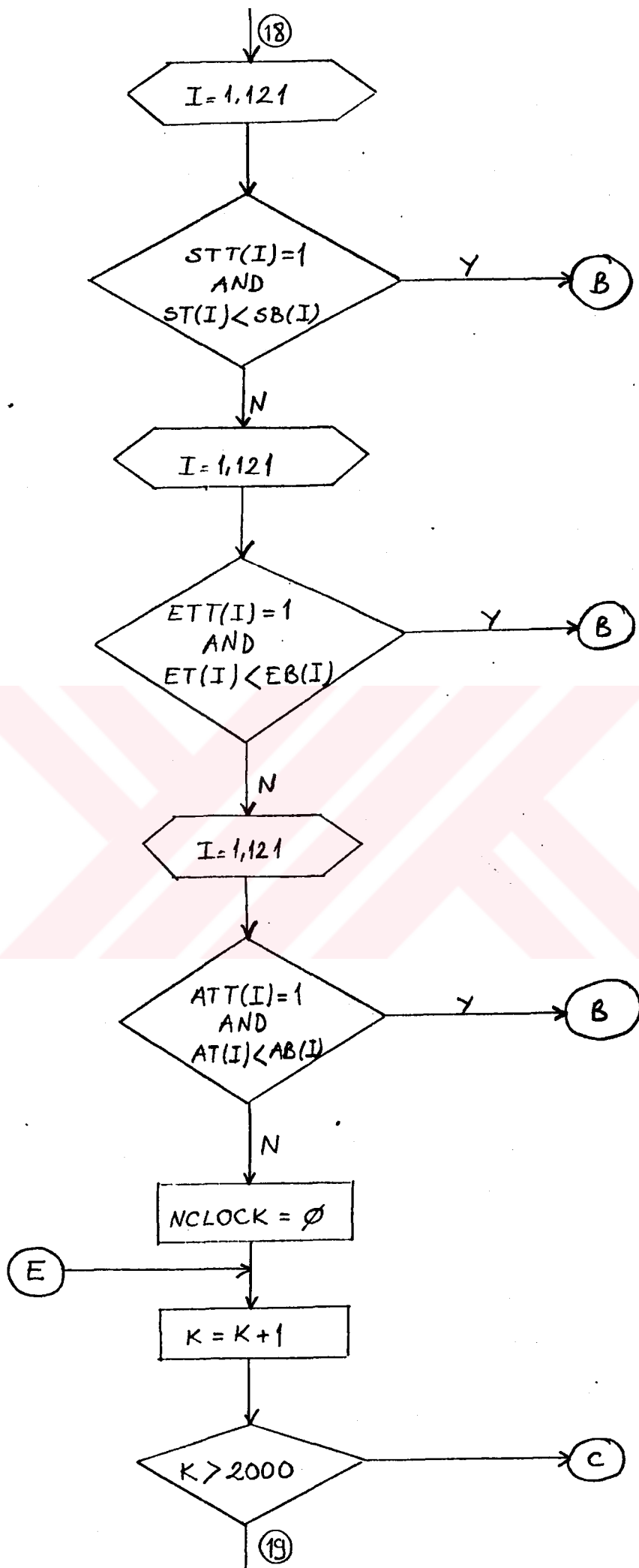


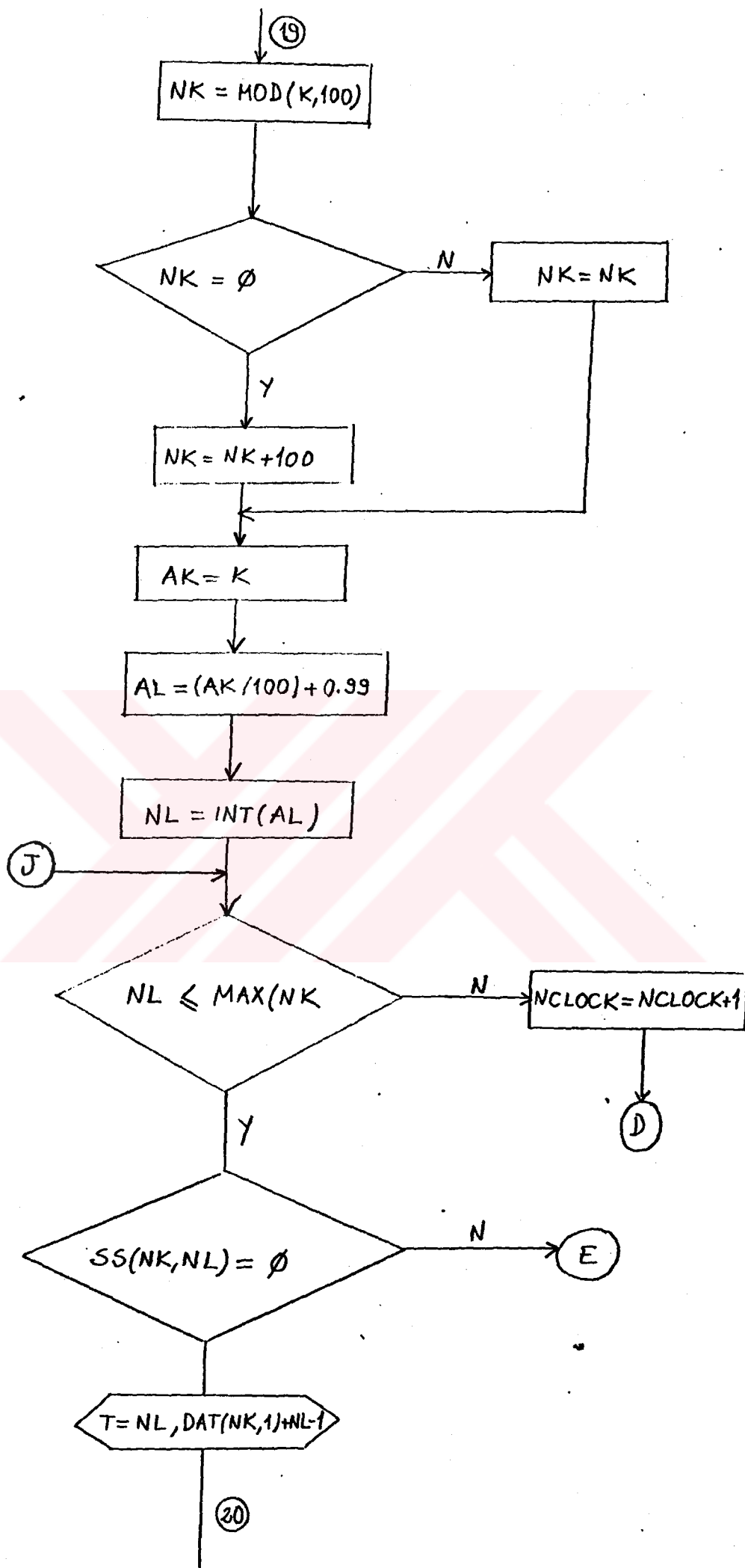


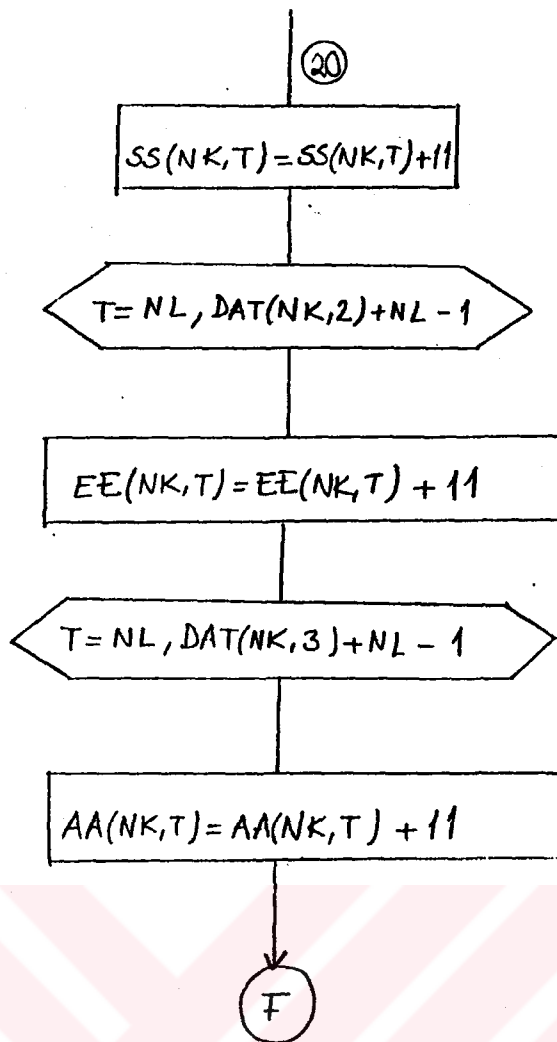


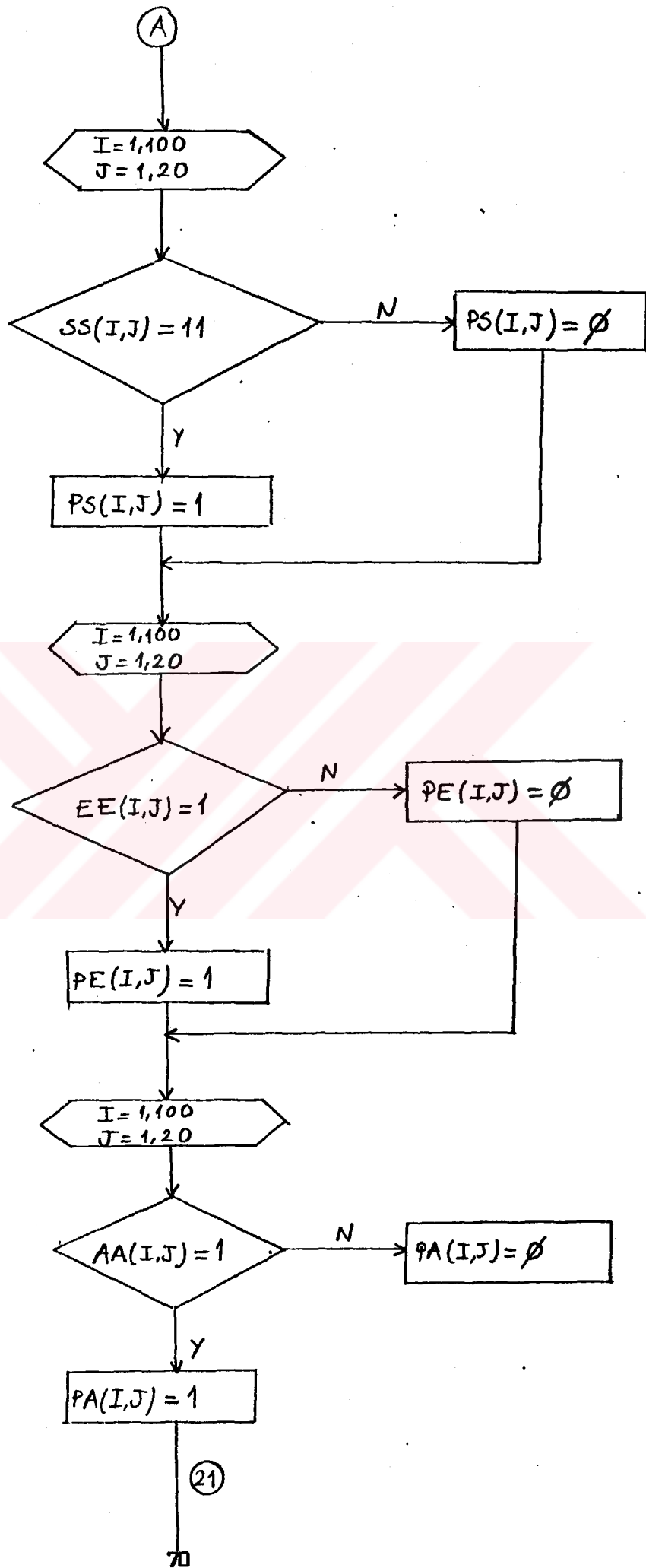


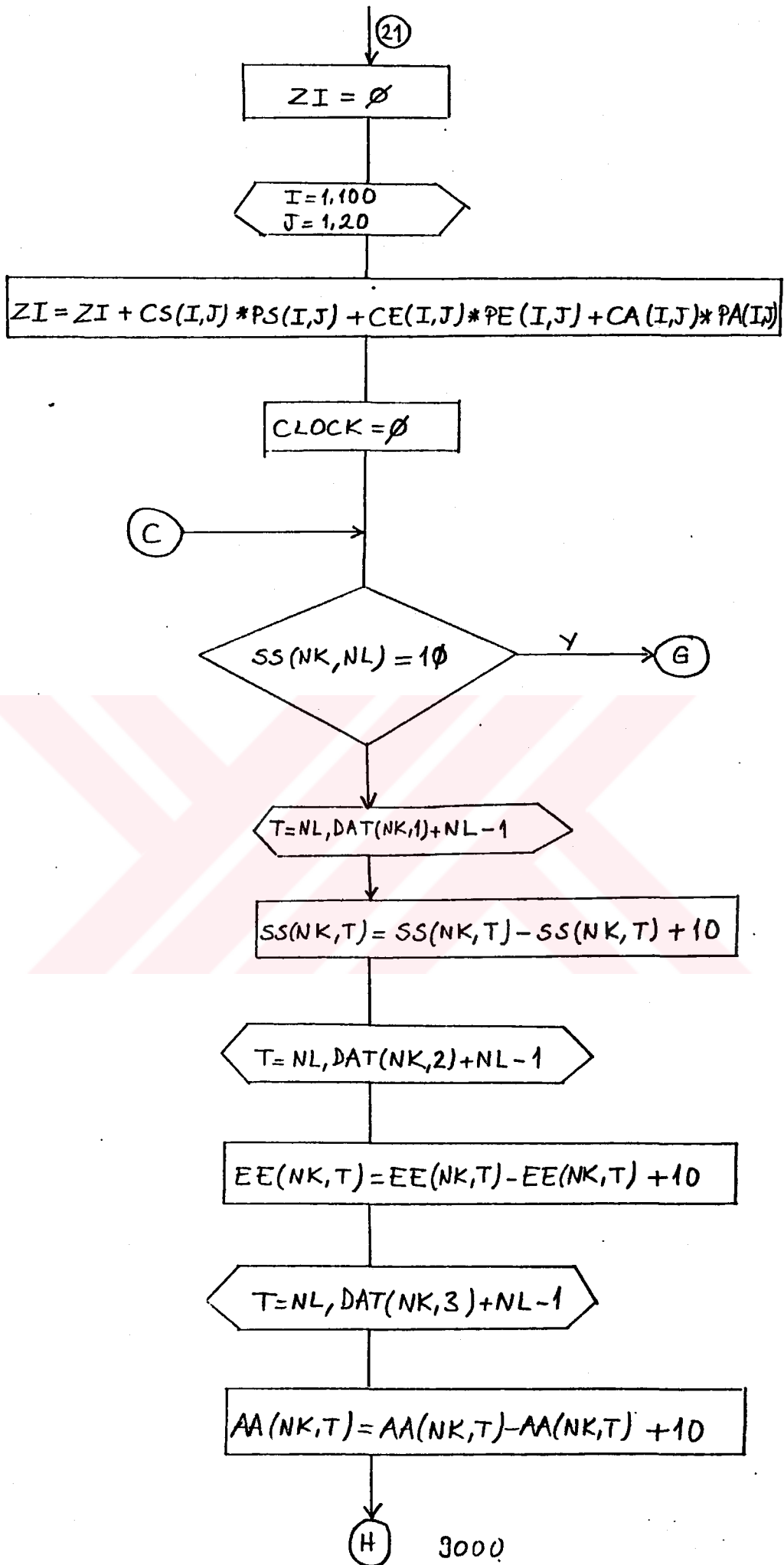


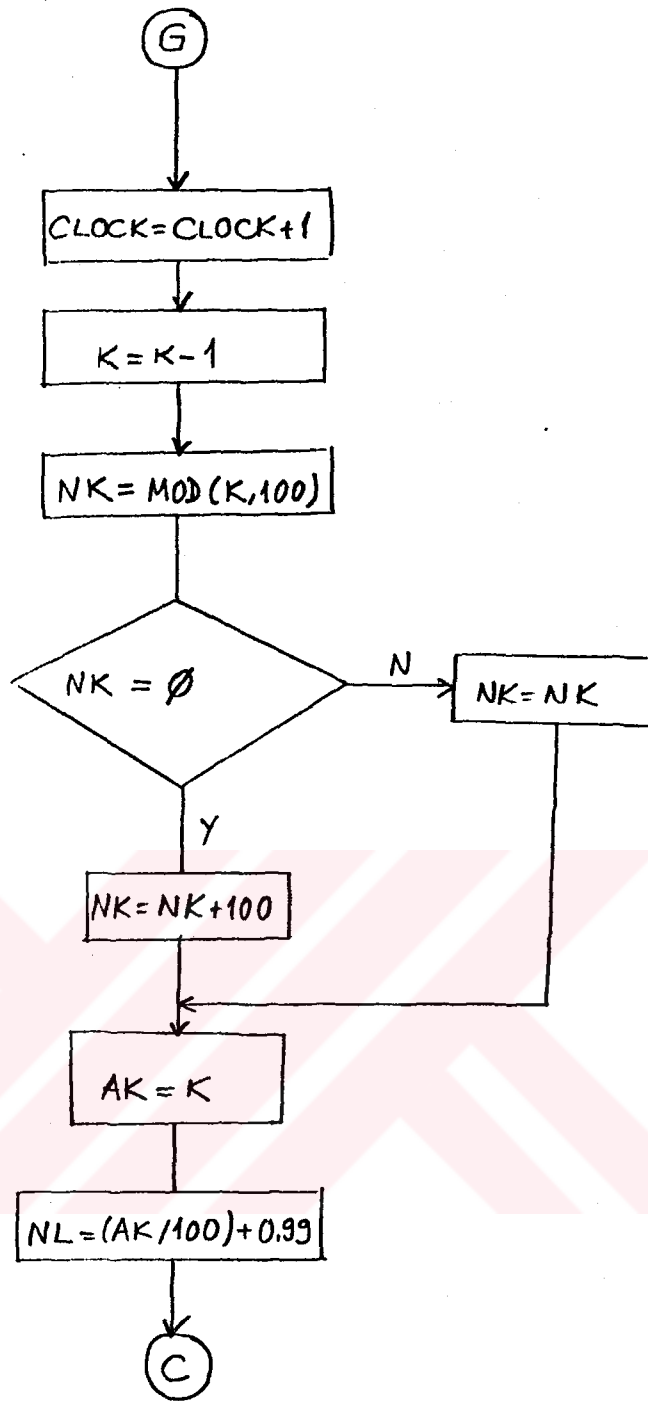


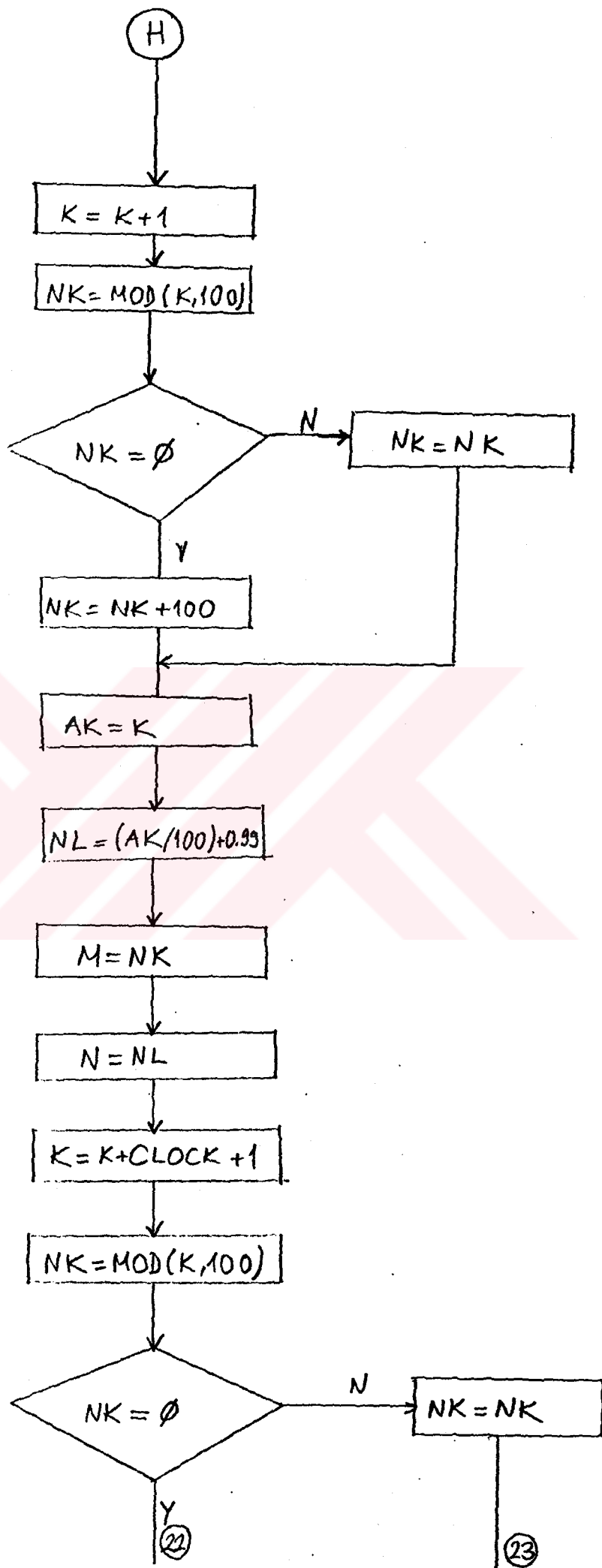


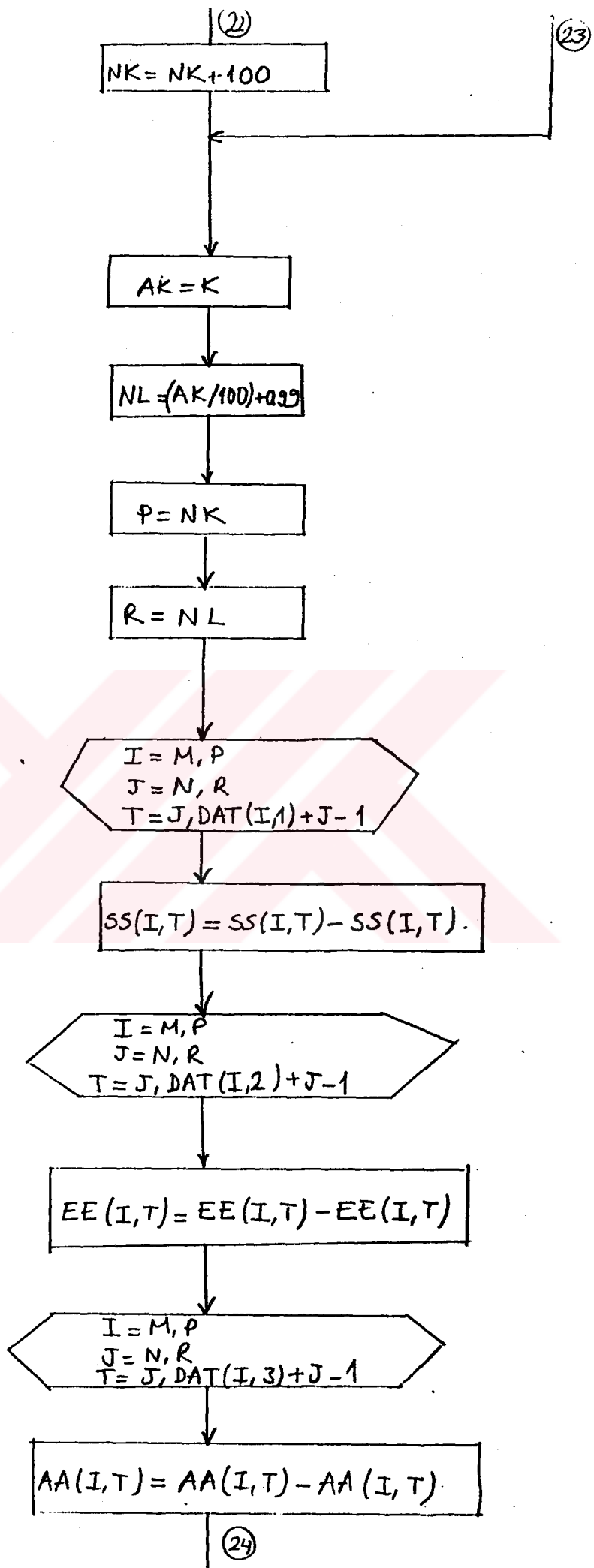




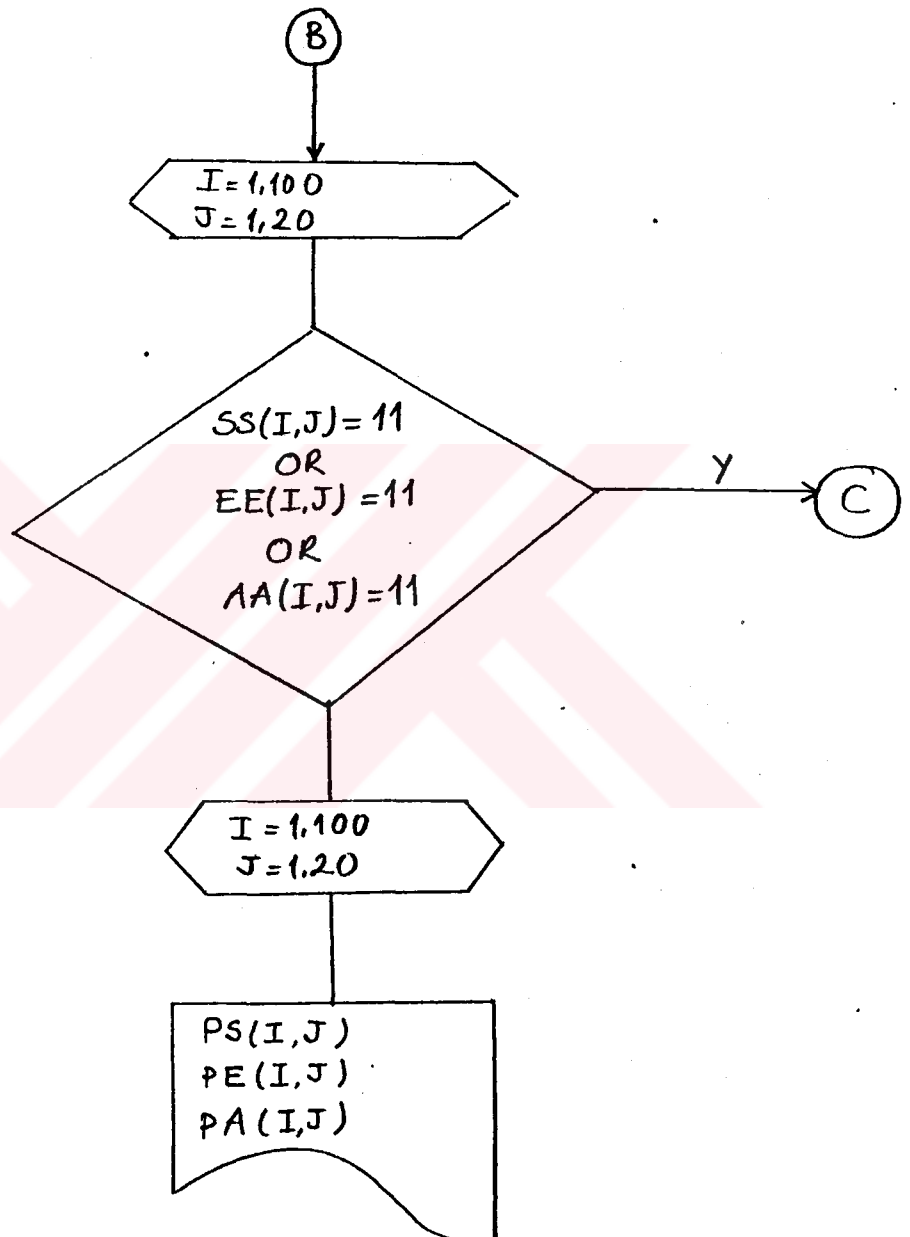
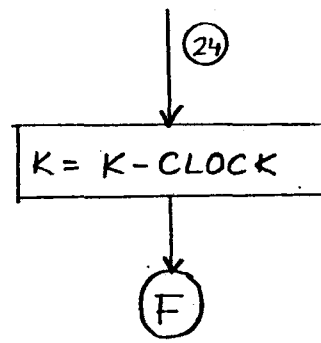


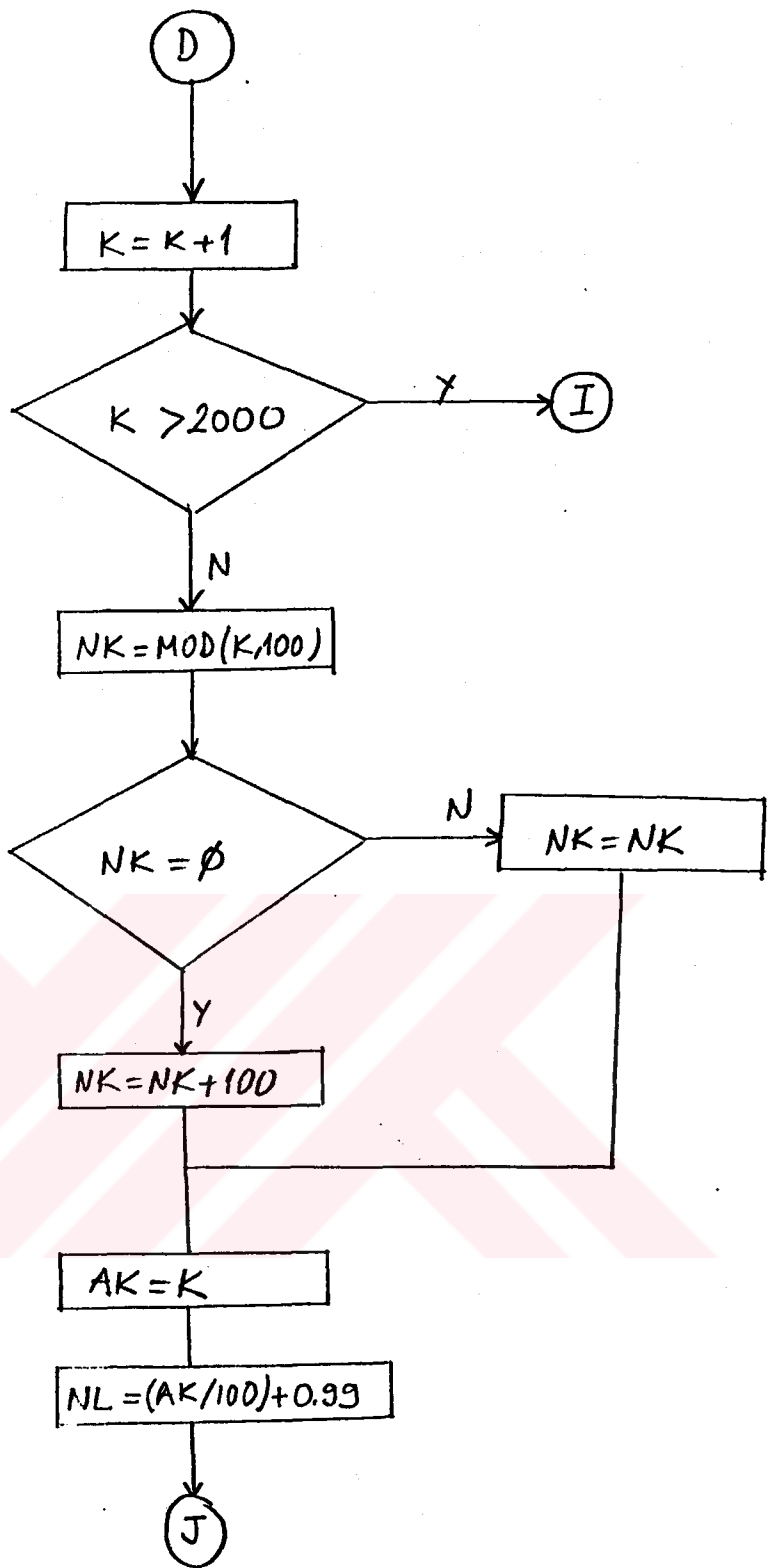












```

1 C
2 SOLVING 0-1 LP PROBLEM VIA IMPLICIT ENUMERATION ALGORITHM.
3 PROGRAM TEZ1(TD2,T03,TAPE7=TD2,TAPE6=T03)
4 INTEGER S(100,20),EA(100,20),A(100,20),DAT(100,3)
5 & SAA(100,20),EAA(100,20),AAA(100,20),CS(100,20),CE(100,20)
6 & CA(100,20),MAX(100),SS(100,20),EE(100,20),AA(100,20)
7 & VS(100,20),VE(100,20),VAC(100,20),SE(121),EB(121),AB(121)
8 & SD(121),ED(121),AD(121),SDP(121),EDP(121),ADD(121),STT(121)
9 & ETT(121),ATT(121),TTS(100,20),TTE(100,20),TTA(100,20),ST(121)
10 & ET(121),AT(121),TS(121),TE(121),TA(121),PS(100,20),PE(100,20)
11 & PA(100,20),NK,NL
12 READ(7,1) (DAT(I,J),J=1,3),I=1,100
13 READ(7,4) (SB(I),I=1,121)
14 READ(7,4) (EB(I),I=1,121)
15 READ(7,4) (AB(I),I=1,121)
16 READ(7,6) ((SAA(I,J),J=1,20),I=1,100)
17 READ(7,6) ((EAA(I,J),J=1,20),I=1,100)
18 READ(7,6) ((AAA(I,J),J=1,20),I=1,100)
19 READ(7,6) ((CS(I,J),J=1,20),I=1,100)
20 READ(7,6) ((CE(I,J),J=1,20),I=1,100)
21 READ(7,6) ((CA(I,J),J=1,20),I=1,100)
22 READ(7,2) (MAX(I),I=1,100)
23 1 FORMAT (11,1A,11,1X,11)
24 2 FORMAT (13)
25 4 FORMAT (35)
26 5 FORMAT ((15X,19(11,1X),14))
27 7 FORMAT (36)
28 II=6000
29 K=0
30 DO 10 J=1,20
31 DO 20 I=1,100
32 SS(I,J)=0
33 IF(I,J)=0
34 AA(I,J)=0
35 20 CONTINUE
36 10 CONTINUE
37 5 DO 30 J=1,20
38 DO 40 I=1,100
39 IF(SS(I,J)-11) 31,32,32
40 30 TO 40
41 31 5(I,J)=0
42 40 CONTINUE
43 30 CONTINUE
44 DO 50 J=1,20
45 DO 60 I=1,100
46 IF(E5(I,J)-11) 51,52,52
47 50 TO 60
48 51 5(I,J)=0
49 60 CONTINUE
50 CONTINUE
51 DO 70 J=1,20
52 DO 80 I=1,100
53 IF(AA(I,J)-11) 71,72,72
54 70 TO 80
55 71 7(I,J)=1

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56 GO TO 80
57 A(I,J)=0
58 CONTINUE
59 CONTINUE
60 C FIND V, THE SET OF CONSTRAINTS VIOLATED WHEN PARTIAL SOLUTION
61 C S IS COMPLETED BY SETTING TO ZERO ALL VARIABLES NOT IN S.
62 DO 90 J=1,20
63 DO 100 I=1,100
64 IF(SS(I,J)-0) 91,92,91
65 VS(I,J)=0
66 GO TO 100
67 91 VS(I,J)=S(I,J)
68 100 CONTINUE
69 CONTINUE
70 DO 110 J=1,20
71 DO 120 I=1,100
72 IF(EE(I,J)-0) 111,112,111
73 VC(I,J)=0
74 GO TO 120
75 111 VC(I,J)=E(I,J)
76 CONTINUE
77 110 CONTINUE
78 DO 130 J=1,20
79 DO 140 I=1,100
80 IF(EA(I,J)-0) 131,132,131
81 VA(I,J)=0
82 GO TO 140
83 131 VC(I,J)=A(I,J)
84 CONTINUE
85 130 CONTINUE
86 DO 150 I=1,121
87 SD(I)=0
88 ED(I)=0
89 AD(I)=0
90 CONTINUE
91 DO 160 J=1,20
92 DO 170 I=1,100
93 SD(J)=SD(J)+VS(I,J)+SAA(I,J)
94 170 CONTINUE
95 160 CONTINUE
96 DO 181 J=1,100
97 DO 182 I=1,20
98 SD(J+20)=SD(J+20)-VS(J,I)+SAA(J,I)
99 CONTINUE
100 CONTINUE
101 DO 163 I=1,100
102 DO 164 J=1,20
103 SD(121)=SD(121)+VS(I,J)+SAA(I,J)
104 CONTINUE
105 163 CONTINUE
106 DO 180 J=1,20
107 DO 180 I=1,100
108 ED(J)=ED(J)+VE(I,J)+EAA(I,J)
109 CONTINUE
110 CONTINUE
111 DO 181 J=1,100
112 DO 182 I=1,20

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113 ED(J+20)=ED(J+20)-VE(J,I)*EAA(J,I)
114 CONTINUE
115 DO 183 I=1,100
116 DO 184 J=1,20
117 ED(121)=ED(121)+VE(I,J)*EAA(I,J)
118 CONTINUE
119 DO 200 J=1,20
120 DO 201 I=1,100
121 AD(J)=AD(J)+VA(I,J)*AAA(I,J)
122 CONTINUE
123 DO 202 I=1,20
124 AD(J+20)=AD(J+20)-VA(J,I)*AAA(J,I)
125 CONTINUE
126 DO 203 I=1,100
127 DO 204 J=1,20
128 AD(121)=AD(121)+VA(I,J)*AAA(I,J)
129 CONTINUE
130 DO 200 I=1,121
131 IF(SD(I).GE.SB(I)) THEN
132 SD(I)=0
133 ELSE
134 SD(I)=1
135 ENDIF
136 DO 230 I=1,121
137 IF(ED(I).GE.EB(I)) THEN
138 ED(I)=0
139 ELSE
140 ED(I)=1
141 ENDIF
142 DO 240 I=1,121
143 IF(AD(I).GE.AB(I)) THEN
144 AD(I)=0
145 ELSE
146 AD(I)=1
147 ENDIF
148 DO 250 I=1,121
149 TDD=TDD+SD(I)+EDD(I)+ADD(I)
150 IF(I=1) THEN
151 TDD=0
152 ELSE
153 TDD=TDD-1
154 ENDIF
155 DO 270 I=1,100
156 DO 271 J=1,20
157 FP=FP+CS(I,J)*VS(I,J)*GE(I,J)*VE(I,J)*CA(I,J)*VA(I,J)
158 CONTINUE
159 IS V EMPTY ?
160 IF(TDD.EQ.0) GO TO 7500
161 SPT THE OBJECTIVE FUNCTION COEFFICIENT LIMIT TO ZBAP-FP
162 FP=0
163 DO 290 J=1,20
164 DO 291 I=1,100
165 FP=FP+CS(I,J)*VS(I,J)*GE(I,J)*VE(I,J)*CA(I,J)*VA(I,J)
166 CONTINUE
167 DO 290 I=1,100
168 DO 291 J=1,20
169 FP=FP+CS(I,J)*VS(I,J)*GE(I,J)*VE(I,J)*CA(I,J)*VA(I,J)

```

170 Z=ZI-EP  
 171 C STOPE IN THE SET T(I) FOR I INCLUDED IN V EACH VARIABLE NOT  
 172 C IN S WHICH HAS:  
 173 C 1-)AN OBJECTIVE FUNCTION COEFFICIENT LESS THAN THE LIMIT.  
 174 C 2-)A POSITIVE COEFFICIENT IN CONSTRAINT I IN V.  
 175 DO 280 I=1,121  
 176 IF(SDD(I).EQ.1) THEN  
 177 STT(I)=1  
 178 ELSE  
 179 STT(I)=0  
 180 ENDIF  
 181

280 CONTINUE  
 DO 290 I=1,121  
 182 IF(EDD(I).EQ.1) THEN  
 183 ITT(I)=1  
 184 ELSE

185 ETT(I)=0  
 186 ENDIF  
 187  
 290 CONTINUE  
 DO 300 I=1,121  
 188 IF(ADP(I).EQ.1) THEN  
 189 ATT(I)=1  
 190 ELSE

191 ATT(I)=0  
 192 ENDIF  
 193  
 300 CONTINUE  
 DO 310 J=1,20  
 194  
 195  
 196  
 197 DO 320 I=1,100  
 198 IF(STT(J).EQ.1.AND.SS(I,J).EQ.0) THEN  
 199 TTS(I,J)=1  
 200 ELSE

201 TTS(I,J)=S(I,J)  
 202 ENDIF  
 203  
 320 CONTINUE  
 310 CONTINUE  
 DO 330 J=1,20  
 204  
 205 DO 340 I=1,100  
 206 IF(ETT(J).EQ.1.AND.EE(I,J).EQ.0) THEN  
 207 TTE(I,J)=1  
 208 ELSE

209 TTE(I,J)=E(I,J)  
 210 ENDIF  
 211  
 340 CONTINUE  
 330 CONTINUE  
 DO 350 J=1,20  
 212  
 213 DO 360 I=1,100  
 214 IF(ATT(J).EQ.1.AND.AA(I,J).EQ.0) THEN  
 215 TTA(I,J)=1  
 216 ELSE

217 TTA(I,J)=A(I,J)  
 218 ENDIF  
 219  
 360 CONTINUE  
 350 CONTINUE  
 DO 370 J=1,20  
 220  
 221 DO 380 I=1,100  
 222 IF(STT(121).EQ.1.AND.SS(I,J).EQ.0) THEN  
 223 TTS(I,J)=1  
 224

225  
 226

```

227 1 ELSE
228 1 TTS(I,J)=S(I,J)
229 1 ENDIF
230 353 CONTINUE
231 352 CONTINUE
232 DO 354 J=1,20
233 DO 355 I=1,100
234 IF(ETT(121).EQ.1.AND.EE(I,J).EQ.0) THEN
235 TTE(I,J)=1
236 ELSE
237 TTE(I,J)=E(I,J)
238 ENDIF
239 355 CONTINUE
240 354 CONTINUE
241 DO 356 J=1,20
242 DO 357 I=1,100
243 IF(ATT(121).EQ.1.AND.AA(I,J).EQ.0) THEN
244 TTA(I,J)=1
245 ELSE
246 TTA(I,J)=A(I,J)
247 ENDIF
248 357 CONTINUE
249 356 CONTINUE
250 DO 351 I=1,121
251 TS(I)=0
252 TE(I)=0
253 TA(I)=0
254 351 CONTINUE
255 DO 370 I=1,20
256 IF(STT(I).EQ.1) GO TO 1000
257 GO TO 370
258 1000 DO 375 J=1,100
259 TS(I)=TS(I)+TTS(J,I)
260 375 CONTINUE
261 370 CONTINUE
262 DO 371 I=1,100
263 IF(STT(I+20).EQ.1) GO TO 1005
264 GO TO 371
265 1005 DO 376 J=1,20
266 TS(I+20)=TS(I+20)+TTS(I,J)
267 376 CONTINUE
268 371 CONTINUE
269 IF(STT(121).EQ.1) GO TO 1010
270 GO TO 1020
271 1010 DO 372 I=1,100
272 DO 373 J=1,20
273 TS(121)=TS(121)+TTS(I,J)
274 373 CONTINUE
275 372 CONTINUE
276 1020 DO 330 I=1,20
277 IF(ETT(I).EQ.1) GO TO 1500
278 GO TO 359
279 1500 DO 355 J=1,100
280 TE(I)=TE(I)+TTE(J,I)
281 355 CONTINUE
282 359 CONTINUE
283 DO 381 I=1,100

```

```

284 IF(ETT(I+20).EQ.1) GO TO 1505
285 GO TO 381
286 DO 386 J=1,20
287 TE(I+20)=TE(I+20)+TTE(I,J)
288 CONTINUE
289 IF(FTT(121).EQ.1) GO TO 1510
290 GO TO 1520
291 DO 382 I=1,100
292 DO 383 J=1,20
293 TE(121)=TE(121)+TTE(I,J)
294 CONTINUE
295 CONTINUE
296 CONTINUE
297 DO 390 I=1,20
298 IF(ATT(I).EQ.1) GO TO 2000
299 GO TO 390
300 DO 395 J=1,100
301 TA(I)=TA(I)+TTA(I,J)
302 CONTINUE
303 CONTINUE
304 DO 391 I=1,100
305 IF(ATT(I+20).EQ.1) GO TO 2005
306 GO TO 391
307 DO 396 J=1,20
308 TAC(I+20)=TAC(I+20)+TTA(I,J)
309 CONTINUE
310 CONTINUE
311 IF(ATT(121).EQ.1) GO TO 2010
312 GO TO 2020
313 DO 392 I=1,100
314 DO 393 J=1,20
315 TA(121)=TA(121)+TTA(I,J)
316 CONTINUE
317 CONTINUE
318 ARE SOME T(I) EMPTY ?
319 DO 400 I=1,121
320 IF(STT(I).EQ.1.AND.TS(I).EQ.0) GO TO 5000
321 CONTINUE
322 DO 410 I=1,121
323 IF(ETT(I).EQ.1.AND.TE(I).EQ.0) GO TO 5000
324 CONTINUE
325 DO 420 I=1,121
326 IF(ATT(I).EQ.1.AND.TA(I).EQ.0) GO TO 5000
327 CONTINUE
328 DO 430 I=1,121
329 ST(I)=0
330 ET(I)=0
331 AT(I)=0
332 CONTINUE
333 DO 440 J=1,20
334 DO 450 I=1,100
335 ST(I)=ST(I)+TTS(I,J)+SAA(I,J)
336 ET(I)=ET(I)+TTE(I,J)+EAA(I,J)
337 AT(I)=AT(I)+TTA(I,J)+AAA(I,J)
338 CONTINUE
339 CONTINUE
340 DO 441 J=1,100

```





```

398 IF(SS(I,J).EQ.11) THEN
399 PS(I,J)=1
400 ELSE
401 PS(I,J)=0
402 ENDIF
403 CONTINUE
404 CONTINUE
405 DO 540 J=1,20
406 DO 550 I=1,100
407 IF(EE(I,J).EQ.11) THEN
408 PE(I,J)=1
409 ELSE
410 PE(I,J)=0
411 ENDIF
412 CONTINUE
413 CONTINUE
414 DO 560 J=1,20
415 DO 570 I=1,100
416 IF(AA(I,J).EQ.11) THEN
417 PACI(J)=1
418 ELSE
419 PACI(J)=0
420 ENDIF
421 CONTINUE
422 CONTINUE
423 I=0
424 DO 580 J=1,20
425 DO 590 I=1,100
426 ZI=ZI+SS(I,J)*PS(I,J)+CE(I,J)+CA(I,J)+PA(I,J)
427 CONTINUE
428 CONTINUE
429 CLOCK=CLOCK+1
430 CONTINUE
431 LOCATE THE RIGHTMOST ELEMENT IN S. REPLACE IT WITH ITS
432 (NEGATIVE) COMPLEMENTS AND DROP ANY ELEMENTS TO THE RIGHT.
433 IF(SS(NK,ML).EQ.10) GO TO 8500
434 DO 600 T=ML,DAT(NK,1)+NL-1
435 SS(NK,T)=SS(NK,T)+10
436 CONTINUE
437 DO 610 T=ML,DAT(NK,2)+NL-1
438 EE(NK,T)=EE(NK,T)-EE(NK,1)+10
439 CONTINUE
440 DO 620 T=ML,DAT(NK,3)+NL-1
441 AA(NK,T)=AA(NK,T)-AA(NK,1)+10
442 CONTINUE
443 DO 630 T=ML,DAT(NK,4)+NL-1
444 K=K+1
445 CONTINUE
446 K=MOD(K,100)
447 IF(MK.EQ.0) THEN
448 MK=MK+100
449 ELSE
450 MK=MK
451 ENDIF
452 AK=K
453 NL=(AK/100)+0.99
454 GO TO 6500
455 K=K+1

```

```

455 NK=MOD(K,100)
456 IF CNK.EQ.0 THEN
457 NK=NK+100
458 ELSE
459 NK=NK
460 ENDIF
461 AK=K
462 NL=(CAK/100)+0.99
463 M=NK
464 M=NL
465 K=K+CLOCK-1
466 NK=MOD(K,100)
467 IF CNK.EQ.0 THEN
468 NK=NK+100
469 ELSE
470 NK=NK
471 ENDIF
472 AK=K
473 NL=(CAK/100)+0.99
474 PF=NK
475 R=NL
476 DO 655 I=M,P
477 DO 640 J=H,R
478 DO 641 T=J/DAT(I,1)+J-1
479 SS(I,T)=SS(I,T)-SS(J,T)
480 CONTINUE
481 CONTINUE
482 CONTINUE
483 DO 700 I=M,P
484 DO 710 J=H,R
485 DO 711 T=J/DAT(I,2)+J-1
486 EF(I,T)=EF(I,T)-EE(J,T)
487 CONTINUE
488 CONTINUE
489 CONTINUE
490 DO 720 I=M,P
491 DO 730 J=H,R
492 DO 721 T=J/DAT(I,3)+J-1
493 AA(I,T)=AA(I,T)-AA(J,T)
494 CONTINUE
495 CONTINUE
496 CONTINUE
497 K=K+CLOCK
498 GO TO 5
499 ARE ALL ELEMENTS IN S NEGATIVE ?
500 DO 650 K=1,20
501 DO 600 L=1,100
502 IF (SS(L,K).EQ.11.OR.EE(L,K).EQ.11.OR.AA(L,K).EQ.11) GO TO 8000
503 TERMINATE THE INCUMBENT SOLUTION SOLUTION IF ANY IS AN OPTIMAL
504 SOLUTION. IF NO INCUMBENT SOLUTION HAS BEEN FOUND THERE IS NO
505 FEASIBLE SOLUTION BETTER THAN THE SOLUTION CORRESPONDING TO
506 THE BEST KNOWN UPPER BOUND.
507 CONTINUE
508 CONTINUE
509 DO 651 J=1,20
510 DO 652 I=1,100
511 PS(I,J)=1-PS(I,J)

```

```

512 PE(I,J)=1-PE(I,J)
513 PA(I,J)=1-PA(I,J)
514 CONTINUE
515 CONTINUE
516 WRITE(6,6)
517 WRITE(6,6) ((PS(I,J),J=1,20),I=1,100)
518 WRITE(6,9)
519 WRITE(6,6) ((PE(I,J),J=1,20),I=1,100)
520 WRITE(6,11)
521 WRITE(6,6) ((PA(I,J),J=1,20),I=1,100)
522 GO TO 6500
523 3 FORMAT (5X,"SENIOR JOBS",12X,"WEEKS")
524 9 FORMAT (4X,"EXP.ASS.JOBS",12X,"WEEKS")
525 14 FORMAT (8X,"ASS.JOBS",12X,"WEEKS")
526 K=K+1
527 K=MOD(K,100)
528 IF(NK.EQ.0) THEN
529 NK=NK+100
530 ELSE
531 NK=NK
532 ENDIF
533 AK=K
534 NL=(AK/100)+0.99
535 GO TO 6100
536 K=K+1
537 IF(K.GT.3000) GO TO 9500
538 NK=MOD(K,100)
539 IF(NK.EQ.0) THEN
540 NK=NK+100
541 ELSE
542 NK=NK
543 ENDIF
544 AK=K
545 NL=(AK/100)+0.99
546 GO TO 4650
547 6500 STOP
548 END
    
```

VARIABLE NAME	MAP	(LOW-H)	ADDRESS	BLOCK	PROPERTIES	TYPE	SIZE	ADDRESS	BLOCK	PROPERTIES	TYPE	SIZE
A	143150					INTEGER	2000	10376B			INTEGER	2000
AA	37250B					INTEGER	2000	24632B			INTEGER	2000
AAA	39252B					INTEGER	2000	100147B			INTEGER	121
AD	100349B					INTEGER	121	100722B			INTEGER	121
AD	101113B					INTEGER	121	101475B			INTEGER	121
ADD	101365B					INTEGER	121	54326D			INTEGER	2000
AK	137379B					REAL		116603B			INTEGER	121
AL	133577B					REAL		102250B			INTEGER	121
AT	116774B					INTEGER	121	133612B			REAL	
ATT	102441B					INTEGER	121	133522B			INTEGER	2000
CA	44572B					INTEGER	2000	133523B			INTEGER	2000
CE	40412B					INTEGER	2000	133543B			INTEGER	2000
CLOCK	133714B					REAL		133726B			INTEGER	100
CS	34472B					INTEGER	2000	133720B			INTEGER	100
DAT	22236B					INTEGER	300	50252B			INTEGER	100

NAME	ADDRESS	SIZE	TYPE	PROPERTIES	DEF
U	133721D		INTEGER		
U	133675B		INTEGER		
U	133520D		INTEGER		
U	133571D		INTEGER		
P	133722B		REAL		
PA	12760JB	2000	INTEGER		
PE	123660B	2000	INTEGER		
PS	117740B	2000	INTEGER		
P	133723B		REAL		
S	4456B		INTEGER		
S	20712B	2000	INTEGER		
SAA	7775D	2000	INTEGER		
SB	100531D	121	INTEGER		
SD	191504D	121	INTEGER		
SDD	50416B	2000	INTEGER		
SS			REAL		

PROCEDURES	CLASS	ARGS
1	GENERIC	1
2	GENERIC	2

STATEMENT LABELS	DEF	PROPERTIES	DEF	PROPERTIES	DEF
1	4275B	FORMAT	22		
2	4301B	FORMAT	23		
4	4303B	FORMAT	24		
5	373B		25		
6	4305B	FORMAT	25		
7	4319B	FORMAT	26		
8	4312B	FORMAT	523		
9	4317B	FORMAT	524		
10	INACTIVE	DO-TERM	35		
11	4324B	FORMAT	525		
20	INACTIVE	DO-TERM	34		
30	INACTIVE	DO-TERM	43		
31	420R		41		
32	INACTIVE	DO-TERM	39		
40	424R	DO-TERM	42		
50	INACTIVE	DO-TERM	51		
51	461B		47		
52	INACTIVE		47		
60	465B	DO-TERM	50		
70	INACTIVE	DO-TERM	59		
71	523B		57		
72	INACTIVE		55		
80	570B	DO-TERM	53		
90	INACTIVE	DO-TERM	69		
91	562B		57		
92	INACTIVE		65		
100	567B	DO-TERM	83		
110	INACTIVE	DO-TERM	77		
111	623B		75		
112	INACTIVE		75		

NAME	ADDRESS	SIZE	TYPE	PROPERTIES	DEF
ST	116412B		INTEGER		
STT	102057B		INTEGER		
T	133700B		INTEGER		
TA	117547B		REAL		
TBD	133610B		REAL		
TE	117356B		INTEGER		
TS	117165B		INTEGER		
TTA	112472B		INTEGER		
TTE	106552B		INTEGER		
TTS	102632B		INTEGER		
VA	74036B		INTEGER		
VE	70116B		INTEGER		
VS	64176B		INTEGER		
Z	133615B		REAL		
ZI	13542B		REAL		

ENTRY POINTS--(CLO=0)	NAME--ADDRESS--ARGS--	PROPERTIES--	DEF	INACTIVE	DO-TERM	PROPERTIES--	DEF	INACTIVE	DO-TERM	PROPERTIES--	DEF
392	INACTIVE	DO-TERM	317	550	INACTIVE	DO-TERM	412	1010	2160B	271	271
393	INACTIVE	DO-TERM	316	560	INACTIVE	DO-TERM	422	1020	2207B	276	276
395	INACTIVE	DO-TERM	302	570	INACTIVE	DO-TERM	421	1417	4063B	516	516
396	INACTIVE	DO-TERM	309	580	INACTIVE	DO-TERM	428	1500	2217B	279	279
400	INACTIVE	DO-TERM	321	590	INACTIVE	DO-TERM	427	1505	2253B	286	286
410	INACTIVE	DO-TERM	324	600	INACTIVE	DO-TERM	435	1510	2303B	292	292
420	INACTIVE	DO-TERM	327	610	INACTIVE	DO-TERM	438	1520	2332B	297	297
430	INACTIVE	DO-TERM	332	620	INACTIVE	DO-TERM	441	2000	2342B	300	300
440	INACTIVE	DO-TERM	339	630	INACTIVE	DO-TERM	441	2005	2376B	307	307
441	INACTIVE	DO-TERM	346	640	INACTIVE	DO-TERM	482	2010	2426B	315	315
442	INACTIVE	DO-TERM	345	641	INACTIVE	DO-TERM	481	2020	2455B	319	319
443	INACTIVE	DO-TERM	353	650	INACTIVE	DO-TERM	508	3000	2775B	366	366
444	INACTIVE	DO-TERM	322	651	INACTIVE	DO-TERM	515	4500	3033B	381	381
450	INACTIVE	DO-TERM	338	652	INACTIVE	DO-TERM	514	4550	3024B	377	377
460	INACTIVE	DO-TERM	358	680	INACTIVE	DO-TERM	507	5000	3770B	500	500
470	INACTIVE	DO-TERM	361	700	INACTIVE	DO-TERM	489	6500	4263B	547	547
480	INACTIVE	DO-TERM	364	710	INACTIVE	DO-TERM	488	7500	3142B	396	396
490	INACTIVE	DO-TERM	354	711	INACTIVE	DO-TERM	487	7650	4234B	536	536
500	INACTIVE	DO-TERM	357	720	INACTIVE	DO-TERM	496	8000	3340B	432	432
510	INACTIVE	DO-TERM	370	721	INACTIVE	DO-TERM	494	8500	3447B	443	443
520	INACTIVE	DO-TERM	404	730	INACTIVE	DO-TERM	495	9000	3476B	454	454
530	INACTIVE	DO-TERM	403	1000	INACTIVE	DO-TERM	258	9500	4210B	526	526
540	INACTIVE	DO-TERM*	413	1005	INACTIVE	DO-TERM	265				

ENTRY POINTS--(CLO=0)  
NAME--ADDRESS--ARGS--

TEST 203 0

I/O UNITS--(CLO=0)  
NAME-- PROPERTIES

TAPES FMT/SEQ  
TAPE FMT/SEQ

STATISTICS--

PROGRAM-UNIT LENGTH 133750B = 47080  
CM STORAGE USED 97500B = 26460  
COMPILE TIME 5.350 SECONDS













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## CONFIGURATION OF THE COMPUTER SYSTEM USED

### Hardware:

Network Operating System (NOS) Version 2.0

Batch processing on Control Data Cyber 170 Computer System.

524 K Memory

2\*Line and 8\*Matrix Control Data Cyber 9334 Printer.

2\*Magnetic Disk Drive 883-6

2\*Magnetic Tape Drive 679-5


### Software:

Fortran 5 Compiler

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