AN OLG MODEL OF PRODUCTION WITH CASH-IN-ADVANCE CONSTRAINTS

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ABSTRACT

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This thesis shows the existence of the unique competitive equilibrium in a cash-in-advance economy involving overlapping generations of producers and workers. Producers have decreasing returns to scale technologies, and both producers and workers face liquidity constraints in the factor and good markets. Necessary and sufficient conditions for the existence of the monetary equilibrium are stated and the equilibrium is fully characterized. Effects of changes in the time preference of the producers, the number of producers, the number of workers and the money growth rate on the equilibrium are analyzed. The analysis reveals that money is not superneutral and that anticipated increases in the growth rate of money may lead to expansionary or contractionary impacts on the economy depending on the allocation of money transfers between young and old producers.

Keywords: Cash-in-advance, decreasing returns, limited participation, overlapping generations, superneutrality of money.

KISA ÖZET

Peşin Para Kısıtlı

Bir Çakışan Nesiller Üretim Modeli

Ayşe Müjde Sürel

Bu tezde, çakışan nesillere sahip işçi ve üreticilerden oluşan, peşin para kısıtı altındaki bir ekonomide rekabetçi dengenin varlığı ve tekliği gösterilmiştir. Modelde, üreticiler ölçeğe göre azalan verimler teknolojisine sahiptir ve hem üreticiler hem de tüketiciler emek ve mal piyasalarında likidite kısıtı ile karşı karşıyadır. Parasal dengenin varlığı için gerekli ve yeterli koşullar sunulmuş ve ekonomideki denge karakterize edilmiştir. İşçilerin ve üreticilerin sayılarındaki, üreticilerin zaman tercihindeki ve paranın artış hızındaki değişimlerin denge üzerindeki etkileri incelenmiştir. Analiz, paranın süper-nötr olmadığını ve paranın artış hızındaki beklenen yükselişlerin parasal transferlerin genç ve yaşlı üreticiler arasındaki dağılımına göre, ekonomiyi genişletici yada daraltıcı etkilere yol açabileceğini ortaya koymuştur.

Anahtar Kelimeler: Azalan verimler, çakışan nesiller, kısıtlı katılım, paranın süper-nötrlüğü, peşin para kısıtı.

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1. Introduction

Cash-in-advance constraints are mainly utilized to model the transactions demand for money since they require that agents hold cash in order to finance some or all of their transactions. These constraints can also be seen as a means to introduce valued fiat money and to study its effects.

Clower (1967) was the first to present a basic cash-in-advance model in which individuals were required to hold money to purchase consumption goods. He concluded that the classical results continued to hold under such a constraint and he also established the superneutrality of money. Thus, anticipated inflation was observed to have no welfare consequences. Lucas (1980, 1984, 1990) and Lucas and Stokey (1983, 1987) similarly impose liquidity constraints on consumers' purchases of a subset of commodities or assets. In this class of studies in which firms do not face any liquidity constraints, real wages are equal to the marginal product of labor and positive pure profits totally erode. However, superneutrality result seizes to hold since there exists a negative relation between money inflation and the quantity of cash goods consumed.

Other strand of the literature comprises of studies that analyze the effect of cash-in-advance constraints on production economies within an infinite-horizon framework. Some examples are by Fuerst (1992), Carlstrom and Fuerst (1995), Christiano, Eichenbaum and Evans (1997, 1998), and Basci and Saglam (2000, 2002, 2003). All of these studies establish the presence of a working capital premium as the gap between real wage and productivity.

Basci and Saglam (2000) explore an infinite-horizon cash-in-advance production economy with decreasing returns to scale (DRTS) technologies. Their study reveals that fully anticipated changes in money growth rate are negatively related to equilibrium real wage rate and it has an effect on output and income distribution. They also show that the rate of deflation suggested by the Friedman rule is limited by the subjective discount rate of the most patient agent in the economy. Moreover, they emphasize that the timing of the distribution of monetary transfers affects equilibrium outcomes.

The aim of this thesis is to contribute to the preceding literature by analyzing the superneutrality of money in a liquidity constrained *production* economy. Our model considers a two-period overlapping generations of consumer-producers and consumer-workers where producers own DRTS technologies in each period that they transfer as bequest to the next generation before they die. The agents carry out their transactions in factor and good markets using the cash holdings they are endowed with; in other words, existing commodities in the economy are cash commodities. It is also assumed that the factor market opens before the good market in each period[1]. Hence, each producer and worker face severe liquidity constraints in both factor and good market transactions. It is shown that for the values of the money growth rate above a certain level, there exists a monetary competitive equilibrium of this financially constrained economy. For a definition of monetary equilibrium where all nominal variables are assumed to be stationary when normalized with respect to the money growth rate, it is possible to characterize a unique competitive equilibrium with nonstationary real variables and with

[1] Basci and Saglam (2003) show that if the good market opens before the factor market, the competitive outcomes are the same as those obtained under the absence of such cash constraints. Liquidity constraints have some real effects only if the factor market opens before the good market; so sequencing of the markets matters.

the usual wage-productivity gap.

It is important to note that the incorporation of cash-in-advance constraints into an overlapping generations framework is in fact an attempt to model 'monetary' economies. In traditional overlapping generations models with money, agents choose to hold money as an asset that helps to smooth out consumption between periods and it is possible that money is not valued when the rate of return on other assets is greater than that of money. Therefore, it is not easy to classify these models as 'monetary' and there arises an incentive to introduce cash-in-advance constraints into OLG models to analyze the effects of liquidity. The study carried out by Crettez, Michel and Wigniolle (1999) is one of the most recent examples of this endeavor. They consider a Diamond OLG model with cash-in-advance constraints only in the good market transactions and study the conditions under which money is neutral and is not neutral. They also characterize the monetary policy that implements the optimal allocation of resources. It is expected that this study adds a dimension to this class of studies with its concentration on the effect of liquidity constraints on both production and consumption behaviours in an economy.

In this thesis, besides showing the existence of the monetary competitive equilibrium, several comparative statics are conducted to analyze neutrality and superneutrality of money. Some unconventional results are reached, it is observed that while the quantity of money in circulation has no real impact on the equilibrium, the rate of money growth affects the real variables, i.e. money is neutral but not superneutral. Interestingly, increasing inflation rate above its lower limit that supports the equilibrium does not necessarily harm the society. In a severely liquidity constrained DRTS production economy with overlapping generations of agents, letting inflation grow may either lead to a rise or to a fall in the real wage rate, output and employment depending on the allocation of money transfers between the producers of two different generations in the economy.

The thesis proceeds as follows. Section 2 introduces the basic settings of the model. Section 3 defines and characterizes the monetary competitive equilibrium for decreasing returns to scale technologies. Section 4 provides an analysis of the equilibrium and discusses some monetary policy implications. Section 5 contains some concluding remarks. Finally, proofs are relegated to the Appendix.

2. The Model

2.1. Basic Settings

The overlapping generations framework used to model a production economy can be described as follows.

Agents: There are two types of identical agents, 'workers' and 'producers', distinguished by the superscript $i = w, p$. Each agent of every generation lives for two periods. The subscript $\{1, t\}$ stands for a 'young' agent of generation t and $\{2, t\}$ an 'old' agent of generation $t-1$ who live in period t. There is no population growth. Additionally, at any period t, there are equal numbers of young and old agents in the economy which are denoted by N^i for $i = w, p$. Here, it is implicitly assumed that the populations of the two types of agents are large enough to maintain a competitive economy.

Commodities: There are two commodities at each period: a factor of production, labor, and a nonstorable consumption good that can be produced using labor as the only input.

Factor Endowments: A typical worker owns equal amounts of labor endowments in the two periods of his lifetime: $\bar{L}_1^w = \bar{L}_2^w = \bar{L}^w > 0$. Producers do not have any labor endowments.

Valuation of Leisure: Workers value leisure while the valuation is measured in terms of the consumption good by the function $v^w(.)$. Producers do not value leisure.

Production Technology: In both periods of his lifetime, each producer of generation t owns a DRTS production technology represented by the function $f^p(.)$ that was inherited from a predecessor of generation $t-2$. In other words, technology transfers are costless.

Utilities: A representative worker and a representative producer born at any time t have the lifetime utilities $U^w(c_{1,t}^w + v^w) + \beta^w U^w(c_{2,t+1}^w + v^w)$ and $U^p(c_{1,t}^p) + \beta^p U^p(c_{2,t+1}^p)$, respectively.

Here, $c_{1,t}^i$ denotes the consumption of a 'young' type-i agent at time t and $c_{2,t+1}^i$ the consumption of the same agent when old.

It is assumed that $U^i(.), v^w(.), f^p(.)$ are twice continuously differentiable, increasing and strictly concave. We also assume:

$v^{w\prime}(0) = \infty$, $v^{w\prime}(\bar{L}_1^w) = v^{w\prime}(\bar{L}_2^w) = v^{w\prime}(\bar{L}^w) = 0$ and

$U^{i\prime}(0) = \infty$, $U^{i\prime}(\infty) = 0$.

Money and the Government: The economy operates under the existence of fiat money. M_t denotes the aggregate money stock at the end of period t and the total money stock evolves over time according to the relation:

$M_{t+1} = (1+\alpha)M_t$, where $\alpha > -1$.

Government changes the money stock in the economy through lump-sum transfers/taxes at the beginning of each period. While none of the workers receives any transfers from the government, at the beginning of any period t each newborn producer is endowed with $X^p_{1,t}$ units of currency and each old producer is either endowed with or taxed by the amount $X^p_{2,t}$, where $X^p_{j,t} = \alpha_j M_{t-1}/N^p$, $j = 1,2$. It is assumed that $\alpha_1 + \alpha_2 = \alpha$ and $\alpha_1 > 0$, $\alpha_2 > -1$. These assumptions guarantee that the control of monetary inflation or deflation is through the changes in the lump-sum transfers/taxes applied, such that the change in the money supply between periods $t-1$ and t is $\alpha M_{t-1} = N^p(X^p_{1,t} + X^p_{2,t})$.

Old agents begin period t with an amount of currency that is equal to their end-of- period $t-1$ money balances. Throughout the thesis, let $M^i_{1,t}$ denote the end-of-period t money holding of a 'young' type-i agent and $M^i_{2,t}$ the end-of-period t money holding of an 'old' type-i agent.

The following table summarizes the basic structures of the model:

Table 1
Summary of the Basic Settings

	Workers		Producers	
	Young	Old	Young	Old
Labor Endowment	$\bar{L}^w_1 = \bar{L}^w > 0$	$\bar{L}^w_2 = \bar{L}^w > 0$	$\bar{L}^p_1 = 0$	$\bar{L}^p_2 = 0$
Valuation of Leisure	v^w (Concave)	v^w (Concave)	$v^p = 0$	$v^p = 0$
Production Technology	$f^w = 0$	$f^w = 0$	f^p (DRTS)	f^p (DRTS)
Period$-t$ Money Transfer	$X^w_{1,t} = 0$	$X^w_{2,t} = 0$	$X^p_{1,t} = \frac{\alpha_1 M_{t-1}}{N^p}$	$X^p_{2,t} = \frac{\alpha_2 M_{t-1}}{N^p}$

2.2. Trade Institution

A trade institution for a given *economy* $\mathcal{E} = \{ N^i, \beta^i, U^i, v^i, f^i, \bar{L}_1^i, \bar{L}_2^i, X_{1,t}^i, X_{2,t}^i | \, i = w, p \}$ is the description of choice variables for each type of agents, price variables, constraints on the given choice variables determined by the given prices, and a feasibility requirement for the collective choices of agents.

Choice variables of type-i agents when they are *young* and *old* respectively are:

$c_{1,t}^i$, $c_{2,t+1}^i$: consumption

$q_{1,t}^i$, $q_{2,t+1}^i$: (+) commodity demand, (-) commodity supply

$L_{1,t}^i$, $L_{2,t+1}^i$: (+) labor demand, (-) labor supply

$M_{1,t}^i$, $M_{2,t+1}^i$: end-of-period money holding

Money prices are:

ω_t, ω_{t+1} : nominal wage rates per unit of labor

p_t, p_{t+1} : money prices per unit of good

All prices and wages are expressed in terms of money as the price of money is taken as numeraire.

Timing of Transactions: Under the assumption that the factor market opens before the commodity market, the sequence of transactions that take place in the economy at period t can be listed under four subperiods, as follows.

T1. Type-i agent begins period t with a money balance that is equal to the sum of the government transfers/taxes and the balance carried from the end of period $t-1$. (For each newborn agent this sum is equal to $X_{1,t}^i$ and for each of the old agents it is $X_{2,t}^i + M_{1,t-1}^i$, where $M_{1,t-1}^i$ represents the end-of-period $t-1$ money holding.)

T2. Labor market opens. Factor trade takes place at the nominal wage rate ω_t

and all wage bills are paid.

T3. Commodity production occurs with the employed labor, $L^p_{1,t}$ and $L^p_{2,t}$.

T4. Good market opens. Commodity trade takes place at the nominal price p_t. Hence, the end-of-period t money balances are realized as

$$M^i_{1,t} = X^i_{1,t} - \omega_t L^i_{1,t} - p_t q^i_{1,t}$$

$$M^i_{2,t} = X^i_{2,t} + M^i_{1,t-1} - \omega_t L^i_{2,t} - p_t q^i_{2,t}$$

for each of the young and old agents, respectively.

Agents' Problems: A representative agent of type-i faces the following lifetime utility maximization problem given his endowment structure and strictly positive prices $\{\omega_t, \omega_{t+1}, p_t, p_{t+1}\}$:

$$\max \quad U^i(c^i_{1,t} + v^i(\bar{L}^i_1 + L^i_{1,t})) + \beta^i U^i(c^i_{2,t+1} + v^i(\bar{L}^i_2 + L^i_{2,t+1}))$$

subject to:

$$M^i_{1,t} = X^i_{1,t} - \omega_t L^i_{1,t} - p_t q^i_{1,t} \tag{1}$$

$$M^i_{2,t+1} = M^i_{1,t} + X^i_{2,t+1} - \omega_{t+1} L^i_{2,t+1} - p_{t+1} q^i_{2,t+1} \tag{2}$$

$$-\bar{L}^i_1 \leqslant L^i_{1,t} \leqslant \frac{X^i_{1,t}}{\omega_t} \tag{3}$$

$$-\bar{L}^i_2 \leqslant L^i_{2,t+1} \leqslant \frac{M^i_{1,t} + X^i_{2,t+1}}{\omega_{t+1}} \tag{4}$$

$$-f^i(\bar{L}^i_1 + L^i_{1,t}) \leqslant q^i_{1,t} \leqslant \frac{X^i_{1,t} - \omega_t L^i_{1,t}}{p_t} \tag{5}$$

$$-f^i(\bar{L}^i_2 + L^i_{2,t+1}) \leqslant q^i_{2,t+1} \leqslant \frac{M^i_{1,t} + X^i_{2,t+1} - \omega_{t+1} L^i_{2,t+1}}{p_{t+1}} \tag{6}$$

$$c^i_{1,t} = f^i(\bar{L}^i_1 + L^i_{1,t}) + q^i_{1,t} \tag{7}$$

$$c^i_{2,t+1} = f^i(\bar{L}^i_2 + L^i_{2,t+1}) + q^i_{2,t+1} \tag{8}$$

Equations (1) and (2) both describe the end-of-period cash holdings during the lifetime of a representative agent i. Constraints (3) and (4) include the cash-in-advance requirements stemming from the fact that the factor market opens first. The maximum amount of labor demand is determined by cash available to agents at the beginning of the relevant period and labor supply cannot exceed labor endowment. Similarly, the right hand side of the inequalities (5) and (6) involve cash-in-advance limits on commodity purchases that become effective because of the fact that good market opens after the labor market. Finally, equalities (7) and (8) state that consumption of an agent, when young and old respectively, is the sum of the quantities of the good produced and purchased.

In the next section, the definition and characterization of the monetary competitive equilibrium are presented.

3. Monetary Competitive Equilibrium

Following definition of the monetary competitive equilibrium is utilized for characterizing the solution of the model.

Definition: The list $\{p_t, \omega_t, p_{t+1}, \omega_{t+1}, c^i_{1,t}, c^i_{2,t+1}, L^i_{1,t}, L^i_{2,t+1}, q^i_{1,t}, q^i_{2,t+1}, M^i_{1,t}, M^i_{2,t+1}, \mid i = w, p\}$ is a *Monetary Competitive Equilibrium* (MCE) for the economy $\mathcal{E}$, if $\omega_t, \omega_{t+1}, p_t, p_{t+1} > 0$ and

(i) $\{c^i_{1,t}, c^i_{2,t+1}, q^i_{1,t}, q^i_{2,t+1}, M^i_{1,t}, M^i_{2,t+1}, L^i_{1,t}, L^i_{2,t+1}\}$ solves the maximization problem for each i under $\{\omega_t, \omega_{t+1}, p_t, p_{t+1}\}$

(ii) $N^w L^w_{1,t} + N^p L^p_{1,t} + N^w L^w_{2,t} + N^p L^p_{2,t} = 0$

(iii) $N^w q^w_{1,t} + N^p q^p_{1,t} + N^w q^w_{2,t} + N^p q^p_{2,t} = 0$

(iv) $N^w M^w_{1,t} + N^p M^p_{1,t} + N^w M^w_{2,t} + N^p M^p_{2,t} = M_t$

(v) $\omega_{t+1}/\omega_t = p_{t+1}/p_t = M_{t+1}/M_t = 1+\alpha$

(vi) $c^i_{1,t+1}/c^i_{1,t} = c^i_{2,t+1}/c^i_{2,t} = L^i_{1,t+1}/L^i_{1,t} = L^i_{2,t+1}/L^i_{2,t} = q^i_{1,t+1}/q^i_{1,t} = q^i_{2,t+1}/q^i_{2,t} = 1$

(vii) $M^w_{1,t} = M^w_{2,t+1} = 0.$

Condition (i) states that representative agents make their optimal choices under perfect foresight of future prices and price taking behaviour. Conditions (ii)-(iv) denote the equilibrium in the three markets. The fifth condition is the stationarity of the nominal variables as normalized by the money growth rate. The sixth condition denotes the symmetry of real variables across generations. Finally, condition (vii) is a requirement for workers not to hold any end-of-period money balances after the good market transactions which ensures that every unit of currency in the economy can get its proper use as a working capital in the factor market in every period.

In order to characterize the solution of the model, we first obtain the reduced form problem of each type-i agent. For this purpose, $c^i_{1,t}$, $c^i_{2,t+1}$ and $q^i_{1,t}$, $q^i_{2,t+1}$ are eliminated from the respective maximization problems, using the equality constraints (1),(2) and (7),(8). Then, we restrict ourselves to the clearing of the labor and money markets alone, since the good market will automatically clear as well, thanks to a version of Walras' law applicable to our case.

Noting $X^w_{1,t} = X^w_{2,t+1} = f^w(.) = 0$, the reduced form of the lifetime utility maximization problem of each 'worker' born at the beginning of period t is:

$$\max_{\{M^w_{1,t}, M^w_{2,t+1}, L^w_{1,t}, L^w_{2,t+1}\}} U^w\left(\frac{-M^w_{1,t} - \omega_t L^w_{1,t}}{p_t} + v^w(\bar{L}^w_1 + L^w_{1,t})\right) +$$

$$\beta^w U^w\left(\frac{M^w_{1,t} - M^w_{2,t+1} - \omega_{t+1} L^w_{2,t+1}}{p_{t+1}} + v^w(\bar{L}^w_2 + L^w_{2,t+1})\right)$$

subject to:

$$0 \leqslant M^w_{1,t} \leqslant -\omega_t L^w_{1,t} \tag{9}$$

$$0 \leqslant M^w_{2,t+1} \leqslant M^w_{1,t} - \omega_{t+1} L^w_{2,t+1} \tag{10}$$

$$-\bar{L}^w_1 \leqslant L^w_{1,t} \leqslant 0 \tag{11}$$

$$-\bar{L}^w_2 \leqslant L^w_{2,t+1} \leqslant \frac{M^w_{1,t}}{\omega_{t+1}} \tag{12}$$

Similarly, the reduced form of the lifetime utility maximization problem of each 'producer' born at the beginning of period t is:

$$\max_{\{M^p_{1,t}, M^p_{2,t+1}, L^p_{1,t}, L^p_{2,t+1}\}} U^p\left(\frac{X^p_{1,t} - M^p_{1,t} - \omega_t L^p_{1,t}}{p_t} + f^p(\bar{L}^p_1 + L^p_{1,t})\right) +$$

$$\beta^p U^p\left(\frac{M^p_{1,t} + X^p_{2,t+1} - M^p_{2,t+1} - \omega_{t+1} L^p_{2,t+1}}{p_{t+1}} + f^p(\bar{L}^p_2 + L^p_{2,t+1})\right)$$

subject to:

$$0 \leqslant M^p_{1,t} \leqslant X^p_{1,t} - \omega_t L^p_{1,t} + p_t f^p(\bar{L}^p_1 + L^p_{1,t}) \tag{13}$$

$$0 \leqslant M^p_{2,t+1} \leqslant M^p_{1,t} + X^p_{2,t+1} - \omega_{t+1} L^p_{2,t+1} + p_{t+1} f^p(\bar{L}^p_2 + L^p_{2,t+1}) \tag{14}$$

$$0 \leqslant L^p_{1,t} \leqslant \frac{X^p_{1,t}}{\omega_t} \tag{15}$$

$$0 \leqslant L^p_{2,t+1} \leqslant \frac{M^p_{1,t} + X^p_{2,t+1}}{\omega_{t+1}} \tag{16}$$

Proposition 1: *Monetary Competitive Equilibrium* $\{p_t, \omega_t, p_{t+1}, \omega_{t+1}, c^i_{1,t}, c^i_{2,t+1}, L^i_{1,t}, L^i_{2,t+1}, q^i_{1,t}, q^i_{2,t+1}, M^i_{1,t}, M^i_{2,t+1}, | i = w, p\}$ *of the economy* $\mathcal{E}$

(i) exists if and only if the following conditions are satisfied:

$$1 + \alpha \geqslant \max\{\beta^w, \beta^p U^{p'}(c^{*p}_{2,t+1})/U^{p'}(c^{*p}_{1,t})\}$$

$$1 + \alpha_2 > \beta^p \frac{f^{p'}(\bar{L}^p_2 + L^{*p}_{2,t+1})L^{*p}_{2,t+1}}{f^p(\bar{L}^p_1 + L^{*p}_{1,t})} \frac{U^{p'}(c^{*p}_{2,t+1})}{U^{p'}(c^{*p}_{1,t})}$$

(ii) is uniquely characterized by (17)-(32) for all t:

$$\frac{\omega^*_t}{p^*_t} = \frac{\beta^p}{1+\alpha} f^{p'}(\bar{L}^p_2 + L^{*p}_{2,t+1}) \frac{U^{p'}(c^{*p}_{2,t+1})}{U^{p'}(c^{*p}_{1,t})} \tag{17}$$

$$\frac{\omega^*_t}{p^*_t} = v^{w'}(\bar{L}^w_1 + L^{*w}_{1,t}) \tag{18}$$

$$\frac{\omega^*_{t+1}}{p^*_{t+1}} = v^{w'}(\bar{L}^w_2 + L^{*w}_{2,t+1}) \tag{19}$$

$$N^w L^{*w}_{1,t} + N^w L^{*w}_{2,t} = -N^p \frac{X^p_{1,t}}{\omega^*_t} - N^p \frac{M^{*p}_{1,t-1} + X^p_{2,t}}{\omega^*_t} \tag{20}$$

$$q^{*w}_{1,t} = -\frac{\omega^*_t}{p^*_t} L^{*w}_{1,t} \tag{21}$$

$$q^{*w}_{2,t+1} = -\frac{\omega^*_{t+1}}{p^*_{t+1}} L^{*w}_{2,t+1} \tag{22}$$

$$q^{*p}_{1,t} = -\frac{M^{*p}_{1,t}}{p^*_t} \tag{23}$$

$$q^{*p}_{2,t+1} = 0 \tag{24}$$

$$L^{*p}_{1,t} = \frac{X^p_{1,t}}{\omega^*_t} \tag{25}$$

$$L^{*p}_{2,t+1} = \frac{M^{*p}_{1,t} + X^p_{2,t+1}}{\omega^*_{t+1}} \tag{26}$$

$$c^{*i}_{1,t} = q^{*i}_{1,t} + f^i(\bar{L}^i_1 + L^{*i}_{1,t}), \qquad i = w, p \tag{27}$$

$$c^{*i}_{2,t+1} = q^{*i}_{2,t+1} + f^i(\bar{L}^i_2 + L^{*i}_{2,t+1}), \qquad i = w, p \tag{28}$$

$$M^{*w}_{1,t} = 0 \tag{29}$$

$$M^{*w}_{2,t+1} = 0 \tag{30}$$

$$M_{1,t}^{*p} = \frac{M_t}{N^p} \tag{31}$$

$$M_{2,t+1}^{*p} = 0 \tag{32}$$

Proof: See Appendix.

An important observation is that there is a lower limit on the level of inflation for the existence of an equilibrium as it is stated in part (i) of Proposition 1. That is, $1 + \alpha$ cannot be below the time preference of workers β^w at a stationary consumption plan with $c_{1,t}^w = c_{2,t+1}^w$. Therefore, in equilibrium, workers do not choose to carry cash balances across periods. Instead, they spend their entire wage earnings in the good market before the next period starts. On the other hand, the producers spend their beginning-of-period money balances in purchasing labor in the factor market and the whole money supply in the economy is held by the young producers in equilibrium in order to pay wage bills at the beginning of the next period.

The unique monetary competitive equilibrium described in Proposition 1 is nonstationary. It is shown that the producers have no incentive to hold positive money balances at the end of the second period since this is the end of their lives. Therefore, they make no transactions in the commodity market in that period and the quantity of the goods supplied by the old producers is nil. Indeed, stationarity of real variables is not possible as it would mean that the producers will not find it optimal to supply goods and hold cash in the first period of their lives either, which is a conclusion that would render money worthless and contradict with the existence of a monetary competitive equilibrium. Therefore, the nonstationarity is an important property of the unique competitive equilibrium that is characterized.

It is also crucial to note that while the labor demands of a young and an old producer are given by the equations (25) and (17), the labor supply equations of a young and an old worker are given by (18) and (19), respectively. The last two equations represent the workers' optimal labor supply decisions, $L_{1,t}^{*w}$ and $L_{2,t+1}^{*w}$, as a function of the real wage rate ω_t^*/p_t^*, and furthermore imply a conventional upward sloping aggregate labor supply curve due to the strict concavity of the leisure function. Additionally, since workers are endowed with a constant amount of labor throughout their lives, the two supply functions together imply the equality of the equilibrium levels of labor supplied and the equations (21), (22), (27) and (28) altogether imply the equality of the consumption levels for workers in each period t.

It is also immediate to verify that the real wage rate is below the marginal product of labor for the rates of inflation above the lower limit that is stated in part (i) of Proposition 1. This traditional finding of wage-productivity gap can be interpreted as the working capital premium accruing to the producers since in our model cash is actually used (for the payment of the wage bills) as a working capital for the viability of the production activities in the economy. As implied by the word 'traditional', the result that workers are paid less than their marginal productivity is in accordance with the findings of the previous literature on liquidity constrained competitive economies.

4. Analysis of Equilibrium

This section explores the effects of the changes in the patience level of producers, the number of workers/producers and the growth rate of money on the equilibrium outcome in this economy. Throughout this analysis, the superscript (*) that marks equilibrium variables will be suppressed, for ease of notation.

Recall that, as it was noted in the previous section, aggregate labor supply function is increasing in the real wage rate. This result can directly be inferred from equalities (18) and (19). We see that at the steady state equilibrium, the choices of labor supply are optimized at a level where the marginal utilities from leisure are equated to the real wage rate prevailing in the economy; in other words, liquidity constraints are not binding on workers in both periods. An increase in the real wage rate, due to the strict concavity of the leisure function, leads to a rise in the amount of labor supplied. Another observation which directly follows from equations (18) and (19) is that neither the level nor the growth rate of money stock has an effect on the aggregate labor supply.

For the purpose of the analysis, we need to rewrite the labor demand function in equation (17). Since $\bar{L}_1^p = \bar{L}_2^p = 0$ and $q_{2,t+1}^p = 0$, $q_{1,t}^p = -M_{1,t}^p/p_t$ in equilibrium, equation (17) becomes:

$$\frac{\omega_t}{p_t} = \frac{\beta^p}{1+\alpha} f^{p'}(L_{2,t+1}^p) \frac{U^{p'}(f^p(L_{2,t+1}^p))}{U^{p'}([-L_{2,t+1}^p(\omega_t/p_t)(1+\alpha)/(1+\alpha_2)] + f^p(L_{1,t}^p))} \quad (33)$$

It is worth noticing that the labor demand of a young producer $L_{1,t}^p$ is a function of the money growth rate because $L_{1,t}^p = X_{1,t}^p/\omega_t = \alpha_1 M_{t-1}/(N^p\omega_t)$ in equilibrium. Since $L_{2,t+1}^p = (M_{1,t}^p + X_{2,t+1}^p)/\omega_{t+1} = (1+\alpha_2)M_t/(N^p\omega_{t+1}) = (1+\alpha_2)M_{t-1}/(N^p\omega_t)$, an important intertemporal relation immediately follows:

$$L_{1,t}^p = \left(\frac{\alpha_1}{1+\alpha_2}\right) L_{2,t+1}^p$$

Note that the equilibrium level of the labor demanded by a typical young producer is positively related to the amount demanded by an old producer since $\alpha_1/(1+\alpha_2) > 0$ due to the feasibility conditions that are imposed.

However, because of the complex nature of equation (33), both the slope of the labor demand curve and the effect of a change in the money growth rate on the real wage rate cannot easily be traced unless some functional forms are assumed for the production and utility functions. For the sake of simplicity, it is suitable to assume

$f^p(L) = L^\gamma$, where $\gamma \in (0,1)$,

$U^p(c) = ln(c)$.

Then, the labor demand function in equation (33) becomes

$$\frac{\omega_t}{p_t} = \frac{\beta^p \gamma}{1+\alpha_2+\beta^p\gamma} \frac{1+\alpha_2}{1+\alpha} \frac{(L^p_{1,t})^\gamma}{L^p_{2,t+1}}. \tag{34}$$

Recalling that $L^p_{1,t} = [\alpha_1/(1+\alpha_2)]L^p_{2,t+1}$ in equilibrium, the labor demand function of an *old* producer further reduces to

$$\frac{\omega_t}{p_t} = \frac{\beta^p \gamma}{1+\alpha_2+\beta^p\gamma} \frac{\alpha_1^\gamma(1+\alpha_2)^{1-\gamma}}{(1+\alpha)} \frac{1}{(L^p_{2,t+1})^{1-\gamma}}. \tag{35}$$

Similarly, the labor demand function of a *young* producer is

$$\frac{\omega_t}{p_t} = \frac{\beta^p \gamma}{1+\alpha_2+\beta^p\gamma} \frac{\alpha_1}{(1+\alpha)} \frac{1}{(L^p_{1,t})^{1-\gamma}}. \tag{36}$$

Finally, the *aggregate* labor demand function is obtained as

$$\frac{\omega_t}{p_t} = \frac{\beta^p \gamma}{1+\alpha_2+\beta^p\gamma} \frac{\alpha_1^\gamma}{(1+\alpha)^\gamma} \frac{(N^p)^{1-\gamma}}{(L^d_t)^{1-\gamma}}, \tag{37}$$

where $L^d_t = N^p[L^p_{1,t} + L^p_{2,t+1}] = N^p[(1+\alpha)/(1+\alpha_2)]L^p_{2,t+1}$, i.e. the aggregate labor demand, and it can now easily be verified that the total labor demand is decreasing in the real wage rate.

Now, we are ready to state the effects of a change in the patience level of the producers.

Corollary 1: *As the producers' patience level β^p increases, the equilibrium real wage rate, employment and output increase.*

First, note that the term $\beta^p\gamma/(1+\alpha_2+\beta^p\gamma)$ is increasing in β^p. Therefore, the higher the patience level of the producers, the higher the aggregate labor demand (37) for all levels of real wage rate while initially there is no change in the aggregate labor supply. Hence, the new equilibrium is reached at a higher real wage rate ω_t/p_t and employment level. For the decomposition of the rise in the employment, we observe that an increase in β^p creates similar effects on the individual labor demands in (35) and (36). Therefore, we see that in the new equilibrium, the individual labor demands $L^p_{2,t+1}$, $L^p_{1,t} = [\alpha_1/(1+\alpha_2)]L^p_{2,t+1}$ and the individual labor supplies $-L^w_{1,t}$, $-L^w_{2,t+1}$ increase. Moreover, while the change in $c^p_{1,t}$ is ambiguous, all of the variables $c^p_{2,t+1}$, $c^w_{1,t}$, $c^w_{2,t+1}$ increase.

As a result, an increase in the patience level of the producers about the future level of utilities they will obtain yields an overall expansion in the economy. Accompanying the increases in output and employment, the price level decreases more than the nominal wage, consequently leading to a rise in the real wage rate.

The next corollary states the effects of an increase in the competitiveness of the labor market where the competitiveness is measured by the size of the labor force.

Corollary 2: *As the number of workers increases, the equilibrium real wage rate decreases whereas the equilibrium employment and output increase.*

As the competition increases in the factor market by an increase in the number of workers N^w, the aggregate labor supply curve shifts to right, forcing the equilibrium real wage rate to decrease and the employment level to increase. As the real wage rate falls after a rise in N^w, the individual labor supplies by the workers $-L^w_{1,t}$ and $-L^w_{2,t+1}$ decrease, while the individual labor demands $L^p_{2,t+1}$ and $L^p_{1,t}$ increase. This leads to a decrease in the consumption levels of the workers, $c^w_{1,t}$, $c^w_{2,t+1}$ as implied by (21), (22), (27), (28) and an increase in $c^p_{2,t+1}$, while the change in $c^p_{1,t}$ remains ambiguous.

A rise in the competitiveness of the labor market drives the real wage rate to a lower level, hence it has a negative impact on the individual consumption levels of workers. Although the economy expands, there occurs a change in the income distribution in the society that works against the physical benefits of the workers. Additionally, the direction of the change in the aggregate price level is indeterminate.

A natural question at this point is whether a similar increase in the competitiveness of the commodity market would yield symmetric results for the producers. Corollary 3 is an attempt to answer this question.

Corollary 3: *As the number of producers increases, the equilibrium real wage rate, employment and output increase.*

As the competition increases in the economy by an increase in the number of producers N^p, once more the aggregate labor demand increases at all levels of the real wage rate while initially there is no change in the *individual* labor demand and supply decisions. As a result, the excess demand therefore occurred in the economy pushes the equilibrium real wage rate and the employment to higher levels. By a similar argument, as N^p increases, $L^p_{1,t}$, $L^p_{2,t+1}$ decrease while $-L^w_{1,t}$, $-L^w_{2,t+1}$ increase. This leads to an increase in the consumption levels of the workers, $c^w_{1,t}$, $c^w_{2,t+1}$, and a decrease in $c^p_{2,t+1}$, while the change in $c^p_{1,t}$ is ambiguous.

Following the expansion in the economy due to an increase in the competitiveness of the good market, we observe an improvement in the lifetime consumption levels of the workers thanks to an eventual rise in the real wage rate. However, it is not easy to conclude about the resulting welfare of a representative producer because of the ambiguity of the change in the consumption when young.

Overall, a positive change in any one of parameters β^p, N^w and N^p, results in an increase in the output level, although each of them have distinct distributional effects on the agents in the economy as we have briefly discussed above. In the next section, we explore the impact of a change in the inflation rate and try to figure out the optimal monetary policy.

4.1. Effects of Monetary Policy

Our immediate observation is that the quantity of the money in the economy has no effects on the equilibrium real wage rate, employment and output since neither the aggregate labor supply nor the aggregate labor demand depends upon the level of money stock. In other words, money is neutral.

The second issue investigated is the superneutrality of money. For this aim, the impact of a rise in the money growth rate, i.e. money inflation, on the equilibrium is analyzed under two different distributions of money transfers. The following proposition summarizes the results obtained.

Proposition 2:

(i) When α_1, the ratio of young producers' transfers to money stock, is held constant, an increase in the growth rate of money supply, α, decreases the equilibrium real wage rate, aggregate employment and young producers' labor demand.

(ii) When α_2, the ratio of old producers' transfers to money stock, is held constant, an increase in the growth rate of money supply, α, increases the equilibrium real wage rate, aggregate employment and young producers' labor demand.

Proof: See Appendix.

On the workers' side, it is easy to observe that an increase in the inflation rate causes the amounts of labor supplied and goods consumed by each of young and old workers to decrease if that rise resulted in a fall in the equilibrium real wage rate and to increase them otherwise.

Considering the two transfer rules in Proposition 2 and the equilibrium relation $L^p_{2,t+1} = [(1+\alpha_2)/\alpha_1]L^p_{1,t}$, it is seen that young producers' labor demand $L^p_{1,t}$ and the term $[(1+\alpha_2)/\alpha_1]$ change in opposite directions. Hence, the effect of money inflation on the old producers' labor demand, output and therefore on the aggregate output is ambiguous.

Consequently, the change in the young producers' consumption $c^p_{1,t}$ is also indeterminate. In order to see this, it is enough to consider the following form of the labor demand equation:

$$\frac{\omega_t}{p_t} = \frac{\beta^p \gamma}{(1+\alpha)L^p_{2,t+1}}[(L^p_{1,t})^\gamma - L^p_{2,t+1}\frac{(1+\alpha)}{(1+\alpha_2)}\frac{\omega_t}{p_t}] = \frac{\beta^p \gamma}{(1+\alpha)L^p_{2,t+1}}(c^p_{1,t})$$

While ω_t/p_t decreases, the direction of the change in $(1+\alpha)L^p_{2,t+1}$ is ambiguous. Hence, the change in $c^p_{1,t}$ remains indeterminate.

In order to be able to overcome all those difficulties and to trace the relation between money inflation and output, the model is simulated for the following parameter values

$\beta^p = \beta^w = 0.98, \gamma = 0.5, \bar{L}^w = 1, N^p = N^w = 100$ and using the following leisure function $v^w(x) = x^{0.5} - x/[2(\bar{L}^w)^{0.5}]$.

First, α_1 is fixed to 0.25 and α_2 is changed between -0.27 and 3.00 in a way that supports the equilibrium conditions we stated in part(i) of Proposition 1. The accompanying changes on aggregate labor demand, real wage rate, young producers' labor demand and output, old producers' labor demand and output, aggregate output, individual labor supply and young producers' consumption are presented in Table 2 below.

In accordance with Proposition 2, the rise in the money inflation holding α_1 constant, is seen to decrease the real wage rate, aggregate labor demand L^d_t and the young producers' labor demand $L^p_{1,t}$. Correspondingly, the young producers' output $f^p(L^p_{1,t})$ is observed to decrease with the rising inflation, too.

The effect of a rise in money inflation on old producers' labor demand is contractionary as it is observed in the column titled '$L^p_{2,t+1}$', yielding a similar negative relationship between money inflation and the aggregate output level Q_t. Moreover, as it is seen in the tenth column of the table, consumption of young producers falls with the rising inflation in this simulated economy.

Table 2

Effects of Money Inflation on the Equilibrium Outcome
(When Young Producers' Transfer Share, α_1, is Constant)

α_2	L_t^d	ω_t/p_t	$L_{1,t}^p$	$L_{2,t+1}^p$	$f^p(L_{1,t}^p)$	$f^p(L_{2,t+1}^p)$	Q_t	$L_{1,t}^w = L_{2,t+1}^w$	$c_{1,t}^p$
-0.27	99.06	0.20	0.25	0.74	0.50	0.86	136.17	-0.50	0.30
-0.20	95.02	0.19	0.23	0.72	0.48	0.85	132.65	-0.48	0.29
-0.10	89.77	0.17	0.20	0.70	0.44	0.84	127.99	-0.45	0.29
0.00	85.04	0.16	0.17	0.68	0.41	0.82	123.71	-0.43	0.28
0.10	80.77	0.15	0.15	0.66	0.39	0.81	119.79	-0.40	0.27
0.20	76.89	0.14	0.13	0.64	0.36	0.80	116.19	-0.38	0.26
0.30	73.36	0.13	0.12	0.62	0.34	0.78	112.84	-0.37	0.25
0.40	70.14	0.12	0.11	0.60	0.33	0.77	109.74	-0.35	0.24
0.50	67.18	0.11	0.10	0.58	0.31	0.76	106.86	-0.34	0.23
0.60	64.45	0.11	0.09	0.56	0.30	0.75	104.18	-0.32	0.23
0.73	61.22	0.10	0.08	0.53	0.28	0.73	100.94	-0.31	0.22
0.78	60.06	0.10	0.07	0.53	0.27	0.73	99.77	-0.30	0.21
0.88	57.87	0.09	0.07	0.51	0.26	0.71	97.53	-0.29	0.21
0.99	55.63	0.09	0.06	0.49	0.25	0.70	94.84	-0.28	0.20
1.28	50.48	0.08	0.05	0.45	0.22	0.67	89.78	-0.25	0.18
1.48	47.44	0.07	0.04	0.43	0.21	0.66	86.49	-0.24	0.17
1.88	42.34	0.06	0.03	0.39	0.18	0.62	80.81	-0.21	0.16
2.28	38.23	0.06	0.03	0.36	0.16	0.60	76.06	-0.19	0.14
2.48	36.45	0.05	0.02	0.34	0.16	0.58	73.95	-0.18	0.14
3.00	32.53	0.05	0.02	0.31	0.14	0.55	69.16	-0.16	0.12

Next, α_2 is fixed to 0.25 and α_1 is changed between 0.10 and 2.48. The results reported in Table 3 below are totally in accordance with Proposition 2 in that all the equilibrium real wage rate, total employment and young producers' labor demand increase with money inflation. The table also shows that increasing the money growth rate stimulates the old producers' labor demand and the aggregate output at least up to a certain level. A similar positive impact of inflation is also observed on young producers' consumption level $c_{1,t}^p$ in the last column.

Table 3

Effects of Money Inflation on the Equilibrium Outcome
(When Old Producers' Transfer Share, α_2, is Constant)

α_1	L_t^d	ω_t/p_t	$L_{1,t}^p$	$L_{2,t+1}^p$	$f^p(L_{1,t}^p)$	$f^p(L_{2,t+1}^p)$	Q_t	$L_{1,t}^w = L_{2,t+1}^w$	$c_{1,t}^p$
0.10	60.45	0.10	0.04	0.56	0.21	0.75	95.97	-0.30	0.15
0.15	66.83	0.11	0.07	0.60	0.27	0.77	104.01	-0.33	0.19
0.20	71.47	0.12	0.10	0.62	0.31	0.78	109.89	-0.36	0.23
0.25	75.09	0.13	0.13	0.63	0.35	0.79	114.48	-0.38	0.25
0.30	78.03	0.14	0.15	0.63	0.39	0.79	118.18	-0.39	0.28
0.35	80.49	0.15	0.18	0.63	0.42	0.79	121.26	-0.40	0.30
0.40	82.59	0.15	0.20	0.63	0.45	0.79	123.85	-0.41	0.32
0.45	84.41	0.16	0.22	0.62	0.47	0.79	126.05	-0.42	0.34
0.50	86.01	0.16	0.25	0.61	0.50	0.78	127.96	-0.43	0.36
0.60	88.71	0.17	0.29	0.60	0.54	0.77	131.05	-0.44	0.39
0.73	91.48	0.18	0.34	0.58	0.58	0.76	134.07	-0.46	0.42
0.78	92.38	0.18	0.35	0.57	0.60	0.75	135.00	-0.46	0.43
0.88	93.98	0.19	0.39	0.55	0.62	0.74	136.57	-0.47	0.45
0.99	95.47	0.19	0.42	0.53	0.65	0.73	137.94	-0.48	0.47
1.28	98.49	0.20	0.50	0.49	0.71	0.70	140.35	-0.49	0.51
1.48	100.05	0.21	0.54	0.46	0.74	0.68	141.34	-0.50	0.53
1.88	102.38	0.22	0.62	0.41	0.78	0.64	142.38	-0.51	0.56
2.28	104.05	0.22	0.67	0.37	0.82	0.61	142.67	-0.52	0.59
2.48	104.72	0.22	0.70	0.35	0.83	0.59	142.67	-0.52	0.60

4.2. Intuition for Proposition 2

The negative relation between money inflation and real wage rate, expressed in part (i) of Proposition 2, is merely a restatement of some of the findings of previous cash-in-advance literature. However, it is interesting to see in part (ii) that higher inflation rates may also have expansionary real effects on the equilibrium in a liquidity constrained economy. Although it seems surprising to observe the positive response of the real wage rate and employment to higher inflation rates, after a careful inspection of the model, these results can be shown to be intuitive.

First of all, the two distinct results stated in the two parts of the proposition are mainly related to the distinct preferences of young and old producers regarding the good supply. In this overlapping generations framework, at any time t, young producers are the only suppliers of the commodity market since old producers optimally choose to produce for their private consumptions and not to supply the market in the final period of their lives. In other words, the production activity is bounded by the amount of the currency held in-advance by the young producers that is, $X_{1,t}^{p} = \alpha_1 M_{t-1}/N^p$. Due to these facts, an increase in the rate of inflation α via a rise in young producers' share of the total transfers α_1, plays a stimulating role on the production side of the economy by relaxing the young producers' liquidity constraints in the labor market.

Additionally, after the trade in the labor market is completed, the whole money supply M_t in the economy is held by the workers to be spent totally in the good market. This means, a rise in money inflation that relaxes the labor market will necessarily lead to a similar relaxation in the good market through its effect on workers' liquidity constraints. Therefore, an increase in the growth rate of money will put the nominal prices in both labor and good markets under an upwards pressure.

In the case where money inflation relieves the liquidity constraints of only old producers (as in part(i) of Proposition 2), the good price rises faster than the nominal wage. This is due to the fact that, although the additional money injected into the economy is totally reflected in the good demand by workers, young producers' supply decisions are adversely affected by the rise in the nominal wage while old producers continue to produce for their private consumption. Hence, the equilibrium real wage rate is attained at a lower level.

Whereas, in the case where money inflation relieves the liquidity constraints of solely young producers (as in part(ii) of Proposition 2), there occurs an expansionary effect on the supply side of the good market alleviating the upwards pressure on the good price. In other words, the equilibrium real wage rate increases accompanying a rise in the employment. The unexpected positive relation between money inflation and aggregate employment is hence justified under the structure and assumptions of this overlapping generations model.

4.3. Policy Implications and Optimal Money Growth Rate

The analysis revealed that even though the quantity of currency in circulation has no real effects (i.e. money is neutral), there is room for affecting the real wage rate, output and the employment positively or negatively by increasing the growth rate of the money in a liquidity constrained DRTS economy.

In the light of Proposition 2, it is easy to infer that the optimal money growth rule in such an economy is closely related to the allocation method of transfers between the members of two different generations.

As part (i) of Proposition 1 implies, the optimal monetary policy is always to choose the inflation rate high enough to satisfy $1+\alpha \geqslant \max\{\beta^w, \beta^p U^{p'}(c^{*p}_{2,t+1})/U^{p'}(c^{*p}_{1,t})\}$ for the existence of the equilibrium. Together with part (i) of Proposition 2, it follows that the optimal monetary policy for a society having a government injecting the additional money into the economy through a rise in the transfer rate of *old* producers, is to set the money inflation $1+\alpha$, at the lowest possible level, $\max\{\beta^w, \beta^p U^{p'}(c^{*p}_{2,t+1})/U^{p'}(c^{*p}_{1,t})\}$. This policy is the familiar 'Friedman Rule' in the literature that states the optimal money supply rule is deflation. Table 2 actually presents a numeric illustration of this statement since both the aggregate employment and the aggregate output take their maximum values at the lowest possible inflation rate that supports the competitive equilibrium and this rate is -0.02 (since $\alpha = \alpha_1 + \alpha_2 = 0.25 - 0.27 = -0.02$) implying a deflation.

On the contrary, part (ii) of Proposition 2 signals that the optimal money supply rule, in this case, is to raise the growth rate of money in circulation unboundedly. However, it is worth noticing that there exists a limit on this positive effect of the money growth rate on real variables because of the limits on the amount of the labor that can be supplied/demanded. Therefore, as α grows getting closer to infinity, the aggregate labor demand ceases to be responsive to money inflation as equation (37) implies. Moreover, the indirect costs of inflation still exists even if they are not considered in this model explicitly.

5. Conclusions and Extensions

This study incorporated cash-in-advance constraints imposed in both factor and good markets within an OLG framework. It was shown that monetary competitive equilibrium with nonstationary real variables exists if and only if the rate of money growth is sufficiently high. The well-known wage-productivity gap as a working capital premium was also established. Next, a series of comparative statics was carried out for analyzing the impacts of changes in some of the parameters of the model, namely, the time preference of producers, the number of workers and the number of producers, on the equilibrium outcome. All of these parameters were found to be positively related to the aggregate output and employment.

It was observed that very low inflation rates leads to a breakdown of the monetary equilibrium. While the quantity of money was observed to be neutral as it had no real effects on the economy, the rate of money growth was not found to be superneutral. Although this result is expected if one is familiar with the models in the cash-in-advance literature, it carries some nontraditional characteristics. Stated more explicitly, as a departure from the findings of the models in literature, it was observed that a rise in the anticipated money inflation above its prevailing level might either curb or stimulate the aggregate employment and output in this model, depending upon the allocation of money transfers between young and old producers. Since the supply of the commodity is actually limited by the amount of the cash alloted to the young producers for labor purchases, a rise in the money endowments of these producers helps reducing the magnitude of the excess demand in the good market alleviating the upwards pressure on the good price, subsequently leading to relative improvements in the levels of real wage rate, employment and output.

In the light of all these findings, some policy implications and the optimal monetary policy rules were discussed. Since different ways of monetary injection leads to completely opposite consequences on the equilibrium values of real variables, it was implied that a monetary authority had the chance to choose not only a target level of inflation but also a method of allocation for monetary transfers. While for one of the extreme allocation rules optimal money supply rule could be deflation, for the other extreme, it could be unbounded money creation.

The overlapping generations model presented in this thesis has also certain advantages that may facilitate its further use in other infinite-horizon studies with cash-in-advance constraints. A major advantage of the overlapping generations framework over the usual infinite-horizon approach is the ability to span the infinite-horizon by adopting a finite (mainly, two-period) scope. This technical ease renders this model promising in providing an infinite-horizon approach to the solution of many problems in relevant issues.

The obstacles faced by Basci and Saglam during their endeavor of presenting liquidity constraints as a cure to the famous problem of nonexistence of the competitive equilibrium under increasing returns to scale technologies (IRTS), best demonstrate the need for an OLG model with cash-in-advance constraints. Basci and Saglam (2002) introduce the innovation that with producers facing cash-in-advance constraints in the labor market, the demand for labor is no longer unbounded under increasing returns to scale as in the classical Arrow-Debreu economies. However, the nonconvexity of the production set (nonconcavity of production technology) still substantially narrows down the framework in which the equilibrium exists. Due to some technical limitations, Basci and Saglam (2002) unavoidably consider a two-period representative agent model, though with milder restrictions on the set of utility and production functions. The finiteness of the horizon necessitates terminal money taxes to make the money desirable for agents. It is expected that the OLG setup that presented in this study for a DRTS economy can also be applied for an IRTS economy to extend the result by Basci and Saglam (2002) to a much wider framework with infinite-horizon that does not need terminal money taxes by the very nature of OLG models.

This thesis could also be of some interest for a future study of social security system under liquidity constraints. One may seek to find how cash-in-advance constraints imposed in the social security market affect equilibrium real wage rate and consumption stream. In fact, it is quite promising that all the existing OLG models that studied bequests, transfers, social securities and Ricardian equivalence can now be revised under liquidity constraints possibly with some striking differences in the main results.

6. Appendix

6.1. Proof of Proposition 1

We will consider the two parts of the proposition separately.

Part (i): To prove the 'only if' part, consider the reduced form maximization problem of workers:

$$\max_{\{M^w_{1,t},M^w_{2,t+1},L^w_{1,t},L^w_{2,t+1}\}} U^w\left(\frac{-M^w_{1,t}-\omega_t L^w_{1,t}}{p_t}+v^w(\bar{L}^w_1+L^w_{1,t})\right)+$$
$$\beta^w U^w\left(\frac{M^w_{1,t}-M^w_{2,t+1}-\omega_{t+1}L^w_{2,t+1}}{p_{t+1}}+v^w(\bar{L}^w_2+L^w_{2,t+1})\right)$$

The first-order conditions for $L^w_{1,t}$ and $L^w_{2,t+1}$ yield the respective labor supply curves $\omega_t/p_t = v^{w\prime}(\bar{L}^w_1+L^w_{1,t})$ and $\omega_{t+1}/p_{t+1} = v^{w\prime}(\bar{L}^w_2+L^w_{2,t+1})$. The cash-in-advance constraint is not binding for workers.

Note that if $1+\alpha \geqslant \beta^w$, the Euler condition associated with the control $M^w_{1,t}$ becomes

$$-\frac{1}{p_t}U^{w\prime}\left(c^w_{1,t}+v^w(\bar{L}^w_1+L^w_{1,t})\right)+\frac{\beta^w}{p_{t+1}}U^{w\prime}\left(c^w_{2,t+1}+v^w(\bar{L}^w_2+L^w_{2,t+1})\right)\leqslant 0,$$

since (18) and (19) together imply $L^w_{1,t}=L^w_{2,t+1}$; equations (21), (22), (27), (28) imply $c^w_{1,t}=c^w_{2,t+1}$ and $p_{t+1}=p_t(1+\alpha)$. So, the money holding plan $M^w_{1,t}=0$ is optimal if $1+\alpha\geqslant\beta^w$.

If, on the contrary, $1+\alpha<\beta^w$, the Euler condition becomes

$$-\frac{1}{p_t}U^{w\prime}\left(c^w_{1,t}+v^w(\bar{L}^w_1+L^w_{1,t})\right)+\frac{\beta^w}{p_{t+1}}U^{w\prime}\left(c^w_{2,t+1}+v^w(\bar{L}^w_2+L^w_{2,t+1})\right)>0.$$

Then, by slightly increasing $M^w_{1,t}$ over initial money endowment of $X^w_{1,t}=0$ (hence slightly increasing $c^w_{2,t+1}$ over $c^w_{1,t}$) workers can be better off. In that case, the plan $M^w_{1,t}=0$ would not be optimal.

The necessity of the condition

$$1+\alpha \geqslant \beta^p \frac{U^{p\prime}(c^{*p}_{2,t+1})}{U^{p\prime}(c^{*p}_{1,t})}$$

follows from that the real wage rate is less than the marginal product of labor in equilibrium. The proof of the 'if' statement in part *(i)* of Proposition 1 is implicit in the proof in part *(ii)-(b)*.

Finally, the equilibrium consumption $c^p_{1,t} = [-L^p_{2,t+1}(\omega_t/p_t)(1+\alpha)/(1+\alpha_2)] + f^p(L^p_{1,t})$ is positive if and only if the second condition

$$1+\alpha_2 > \beta^p \frac{f^{p\prime}(L^p_{2,t+1})L^p_{2,t+1}}{f^p(L^p_{1,t})} \frac{U^{p\prime}(c^p_{2,t+1})}{U^{p\prime}(c^p_{1,t})}$$

is satisfied.

Part (ii): The proof consists of two parts: (a) Every MCE satisfies (17)-(32); (b) the plan (17)-(32) is a MCE.

(a) Let $\{\omega_t, \omega_{t+1}, p_t, p_{t+1}, c^i_{1,t}, c^i_{2,t+1}, L^i_{1,t}, L^i_{2,t+1}, q^i_{1,t}, q^i_{2,t+1}, M^i_{1,t}, M^i_{2,t+1}, \mid i = w, p\}$ be a MCE. In part (i) of the proof, we showed that the real wage rate and labor supply of each worker must satisfy (18) and (19). Labor market clearing implies (20). Equations (21),(22),(23),(24) follow from (1) and (2) given the optimal choices of money holding, while (27) and (28) are restatements of (7) and (8) in equilibrium.

To derive the rest of the MCE plan, consider the reduced form maximization problem of producers:

$$\max_{\{M^p_{1,t}, M^p_{2,t+1}, L^p_{1,t}, L^p_{2,t+1}\}} U^p\left(\frac{X^p_{1,t} - M^p_{1,t} - \omega_t L^p_{1,t}}{p_t} + f^p(\bar{L}^p_1 + L^p_{1,t})\right) +$$

$$\beta^p U^p\left(\frac{M^p_{1,t} + X^p_{2,t+1} - M^p_{2,t+1} - \omega_{t+1} L^p_{2,t+1}}{p_{t+1}} + f^p(\bar{L}^p_2 + L^p_{2,t+1})\right)$$

The first-order conditions for $L^p_{1,t}$ and $L^p_{2,t+1}$, under the assumption that $\omega_t/p_t \leqslant f^{p'}$, yield $L^p_{1,t} = X^p_{1,t}/\omega_t$ and $L^p_{2,t+1} = (M^p_{1,t}+X^p_{2,t+1})/\omega_{t+1}$. That is cash-in-advance constraint is binding for producers. The objective of producers, then, reduces to

$$\max_{\{M^p_{1,t},M^p_{2,t+1}\}} U^p\left(\frac{-M^p_{1,t}}{p_t} + f^p\left(\bar{L}^p_1 + \frac{X^p_{1,t}}{\omega_t}\right)\right) +$$
$$\beta^p U^p\left(\frac{-M^p_{2,t+1}}{p_{t+1}} + f^p\left(\bar{L}^p_2 + \frac{M^p_{1,t}+X^p_{2,t+1}}{\omega_{t+1}}\right)\right).$$

The Euler condition associated with the control $M^p_{1,t}$ becomes

$$-\frac{1}{p_t}U^{p'}\left(q^p_{1,t} + f^p(\bar{L}^p_1 + L^p_{1,t})\right) + \frac{\beta^p f^{p'}}{\omega_{t+1}}U^{p'}\left(q^p_{2,t+1} + f^p\left(\bar{L}^p_2 + \frac{M^p_{1,t}+X^p_{2,t+1}}{\omega_{t+1}}\right)\right) = 0.$$

From the stationarity condition $\omega_{t+1} = (1+\alpha)\omega_t$, it follows that

$$\frac{\omega_t}{p_t} = \frac{\beta^p}{1+\alpha}f^{p'}(\bar{L}^p_2 + L^p_{2,t+1})\frac{U^{p'}(c^p_{2,t+1})}{U^{p'}(c^p_{1,t})}.$$

(b) We have to prove that the plan (17)-(32) is optimal, individually feasible, stationary, symmetric across generations and satisfies aggregate feasibility (market clearing) conditions.

(b-i) Optimality: We will check that both types of agents optimize under the proposed prices and plans of action. The optimality of $L^{*w}_{1,t}$ and $L^{*w}_{2,t+1}$ were shown in part *(i)* of the proof. Now, we have to verify that lifetime utility is concave in $M^i_{1,t}$, $M^i_{2,t+1}$ for $i = w, p$. For ease of notation, suppress the superscript $(*)$ in equilibrium prices and wages, hereafter.

Denote the objective function of type-i agents as $V^i(M^i_{1,t}, M^i_{2,t+1})$ for $i = w, p$. Define $V^i_1(M^i_{1,t}, M^i_{2,t+1}) = \partial V^i(M^i_{1,t}, M^i_{2,t+1})/\partial M^i_{1,t}$. First consider

$$V^w(M^w_{1,t}, M^w_{2,t+1}) = U^w\left(-\frac{M^w_{1,t}}{p_t} - \frac{\omega_t}{p_t}L^w_{1,t} + v^w(\bar{L}^w_1 + L^w_{1,t})\right)$$
$$+\beta^w U^w\left(\frac{M^w_{1,t} - M^w_{2,t+1}}{p_{t+1}} - \frac{\omega_{t+1}}{p_{t+1}}L^w_{2,t+1} + v^w(\bar{L}^w_2 + L^w_{2,t+1})\right).$$

Then

$$V_1^w = -\frac{1}{p_t}U^{w\prime}\left(c_{1,t}^w + v^w(\bar{L}_1^w + L_{1,t}^w)\right)$$
$$+\frac{\beta^w}{p_{t+1}}U^{w\prime}\left(c_{2,t+1}^w + v^w(\bar{L}_2^w + L_{2,t+1}^w)\right) \leqslant 0,$$

since $1+\alpha \geqslant \beta^w$ and $c_{1,t}^w + v^w(\bar{L}_1^w + L_{1,t}^w) = c_{2,t+1}^w + v^w(\bar{L}_2^w + L_{2,t+1}^w)$.

$$V_2^w = -\frac{\beta^w}{p_{t+1}}U^{w\prime}\left(c_{2,t+1}^w + v^w(\bar{L}_2^w + L_{2,t+1}^w)\right) < 0$$

and

$$V_{11}^w = \frac{1}{p_t^2}U^{w\prime\prime}\left(c_{1,t}^w + v^w(\bar{L}_1^w + L_{1,t}^w)\right)$$
$$+\frac{\beta^w}{p_{t+1}^2}U^{w\prime\prime}\left(c_{2,t+1}^w + v^w(\bar{L}_2^w + L_{2,t+1}^w)\right) < 0$$
$$V_{22}^w = \frac{\beta^w}{p_{t+1}^2}U^{w\prime\prime}\left(c_{2,t+1}^w + v^w(\bar{L}_2^w + L_{2,t+1}^w)\right) < 0$$
$$V_{12}^w = V_{21}^w = -\frac{\beta^w}{p_{t+1}^2}U^{w\prime\prime}\left(c_{2,t+1}^w + v^w(\bar{L}_2^w + L_{2,t+1}^w)\right) > 0.$$

Hence, the Hessian matrix is negative semi-definite. Therefore, $V^w(M_{1,t}^w, M_{2,t+1}^w)$ is jointly concave in $M_{1,t}^w$ and $M_{2,t+1}^w$.

At the MCE prices the objective function of a representative producer is

$$V^p(M_{1,t}^p, M_{2,t+1}^p) = U^p\left(-\frac{M_{1,t}^p}{p_t} + f^p(\bar{L}_1^p + L_{1,t}^p)\right)$$
$$+\beta^p U^p\left(-\frac{M_{2,t+1}^p}{p_{t+1}} + f^p(\bar{L}_2^p + \frac{M_{1,t}^p + X_{2,t+1}^p}{\omega_{t+1}})\right)$$

and we have

$$V_1^p = -\frac{1}{p_t}U^{p\prime}\left(q_{1,t}^p + f^p(\bar{L}_1^p + L_{1,t}^p)\right)$$
$$+\frac{\beta^p f^{p\prime}}{\omega_{t+1}}U^{p\prime}\left(q_{2,t+1}^p + f^p(\bar{L}_2^p + \frac{M_{1,t}^p + X_{2,t+1}^p}{\omega_{t+1}})\right) = 0$$
$$V_2^p = -\frac{\beta^p}{p_{t+1}}U^{p\prime}\left(q_{2,t+1}^p + f^p(\bar{L}_2^p + \frac{M_{1,t}^p + X_{2,t+1}^p}{\omega_{t+1}})\right) < 0$$

$$V_{11}^p = \frac{1}{p_t^2} U^{p''} \left(q_{1,t}^p + f^p(\bar{L}_1^p + L_{1,t}^p) \right) + \frac{\beta^p}{\omega_{t+1}^2} \left(f^{p''} U^{p'} + (f^{p'})^2 U^{p''} \right) < 0$$

$$V_{22}^p = \frac{\beta^p}{p_{t+1}^2} U^{p''} \left(q_{2,t+1}^p + f^p(\bar{L}_2^p + \frac{M_{1,t}^p + X_{2,t+1}^p}{\omega_{t+1}}) \right) < 0$$

$$V_{12}^p = V_{21}^p = -\frac{\beta^p f^{p'}}{\omega_{t+1} p_{t+1}} U^{p''} \left(q_{2,t+1}^p + f^p(\bar{L}_2^p + \frac{M_{1,t}^p + X_{2,t+1}^p}{\omega_{t+1}}) \right) > 0.$$

So, the Hessian matrix is negative semi-definite. Therefore, $V^p(M_{1,t}^p, M_{2,t+1}^p)$ is jointly concave in $M_{1,t}^p$ and $M_{2,t+1}^p$.

As the implied values and/or signs of V_1^w, V_2^w, V_1^p, V_2^p at the proposed plans of action suggest, the plan (17)-(32) satisfies the Euler conditions.

(b-ii) Individual feasibility: On the workers' side, two period money demands in equilibrium in (29) and (30) satisfy the constraints (9) and (10) at the lower bounds, respectively. Conditions (11) and (12) are satisfied in the interior.

On the producers' side, constraint (13) is reduced to $M_{1,t}^p \leqslant p_t f^p(\bar{L}_1 + L_{1,t}^p)$ at (25). While (14) holds at the lower boundary, the constraint (15) holds at the upper bound. Finally, the equilibrium (26) satisfies the condition (16) at the boundary.

(b-iii) Aggregate feasibility: Equation (20) is consistent with labor market clearing. The plans (21),(22),(23) and (24) clear the good market, and money holding plans (29), (30),(31) and (32) are consistent with the money market clearing.

(b-iv) Symmetry and stationarity: One can easily verify that the nominal variables in the plan (17)-(32) are stationary and the real variables are symmetric across generations as defined. *Q.E.D.*

6.2. Proof of Proposition 2

Part (i): First, let us define

$$\Omega = \beta^p \gamma \alpha_1^\gamma / [(1+\alpha_2+\beta^p\gamma)(1+\alpha)^\gamma]$$

$$\Omega_1 = \beta^p \gamma \alpha_1 / [(1+\alpha_2+\beta^p\gamma)(1+\alpha)].$$

Since $\alpha_1 + \alpha_2 = \alpha$, the terms Ω in equation (37) and Ω_1 in equation (36) can be rewritten as

$\beta^p\gamma\alpha_1^\gamma/[(1+\alpha-\alpha_1+\beta^p\gamma)(1+\alpha)^\gamma]$ and $\beta^p\gamma\alpha_1/[(1+\alpha-\alpha_1+\beta^p\gamma)(1+\alpha)]$, respectively.

Since $d\Omega/d\alpha < 0$ when α_1 is constant, an increase in α causes aggregate labor demand L_t^d to decrease for all values of the real wage rate as it can be seen in equation (37). In other words, the aggregate labor demand curve (37) shifts downward while the aggregate supply curve remains the same. The resulting excess supply for labor is eliminated by a lower real wage rate ω/p in equilibrium. Hence, the aggregate employment decreases, too.

With the increase in the money growth rate α, the young producers' individual labor demand $L_{1,t}^p$ will also decrease. The result directly follows from equation (36) since it is easy to verify that the term Ω_1 is decreasing in α.

Part (ii): Using $\alpha_1+\alpha_2 = \alpha$, the terms Ω in equation (37) and Ω_1 in equation (36) can be rewritten as

$\beta^p\gamma(\alpha-\alpha_2)^\gamma/[(1+\alpha_2+\beta^p\gamma)(1+\alpha)^\gamma]$ and $\beta^p\gamma(\alpha-\alpha_2)/[(1+\alpha_2+\beta^p\gamma)(1+\alpha)]$, respectively.

Since $d\Omega/d\alpha > 0$ when α_2 is constant, an increase in α causes aggregate labor demand L_t^d to increase for all values of the real wage rate since the term Ω in equation (37) is increasing in the money growth rate. Following the shift in the aggregate labor demand curve (37), the excess demand for labor is eliminated by a higher real wage rate in equilibrium. This leads to a rise in aggregate employment.

With the increase in α, the individual labor demand by the young producers $L^p_{1,t}$, will increase. The result follows from the observation that Ω_1 in equation (36) increases with the money growth rate. *Q.E.D.*

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