SECONDARY SCHOOL PLACEMENT PROBLEM IN TURKEY

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Secondary School Placement Problem in Turkey

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Thesis Abstract

Orhan Aygün, "Secondary School Placement Problem in Turkey"

In this paper, I study the placement mechanism, a centralized student placement via standardized test, which is used for matching eight grade students and high schools, in Turkey within a many-to-one matching framework. The placement mechanism used is the two-stage segmented system with multi-category serial dictatorship. I show that this system is fair, but fails to satisfy non-wastefulness, strategy-proofness, efficiency and respecting improvements. I further show that, under the constraint of placing students to private and state schools in separate markets, there exists no fair and non-wasteful placement mechanism that satisfies strategy-proofness, efficiency and respecting improvements. I, then introduce two restrictions on the students' preference profiles; blocked preferences and common preferences. I show that; in those restricted matching environments using Gale and Shapley Student Optimal Deferred Acceptance Algorithm instead of multicategory serial dictatorship makes the system be the best placement mechanism among all stable matching mechanisms.

Tez Özeti

Orhan Aygün, "Türkiye'deki Ortaöğretim Yerleştirme Problemi"

Bu tezde, Türkiye'deki 8. Sınıf öğrencileri ve liseler arasında çoğa-bir eşleşme sistemini baz alarak sınav ile merkezi öğrenci yerleştirmesini çalıstım. Kullanılan yerleştirme sistemi, çok kategorili sıralı diktatörlük kullanan iki aşamalı ayrışmış bir sistemdir. Bu sistemin adil olduğunu ancak kaynakları boşa harcamama, stratejilere dayanıklılık, verimlilik ve gelişmeleri ödüllendirme özelliklerini sağlamadığını gösterdim. Buna ek olarak, öğrenciler özel ve devlet okullarına iki ayrı pazarda yerleştirildiğinde hiçbir adil ve kaynakları boşa harcamayan yerleştirme mekanizmasının stratejilere dayanıklılık, verimlilik ve gelişmeleri ödüllendirme özelliklerini sağlamadığını gösterdim. Daha sonra öğrenci tercihlerine, blok ve ortak tercihler olmak üzere iki sınırlama getirdim. Çok kategorili sıralı diktatörlük yerine sınırlamalı tercihler olduğunda, Gale ve Shapley Öğrenci Uygun Ertelenmiş Kabul Algoritması kullanıldığında sistmemin diğer bütün istikrarlı eşleşme mekanizmaları arasında en iyi sonuçları verdiğini gösterdim.

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I dedicate this thesis to my family who always supported and encouraged me in my whole life.

CONTENTS

CHAPTER 1

INTRODUCTION and LITERATURE REVIEW

As competition is increasing in every aspects of life, need for getting a high quality education is becoming more essential. As a result, demand for schools with high standards is higher than their capacities. Therefore planning the most effective placement mechanism for Turkish secondary school market is not only an economic problem to study but also an important milestone for education policies. Since, there are Turkey-specific restrictions of this placement system defining its problems and improving its deficiencies, if there is any, is worth studying.

The allocation of discrete resources like school seats is investigated using the tools of two sided matching theory which was first introduced by Gale and Shapley (1962). Over the years, this theory has been used for understanding and improving the allocation of resources in various markets. Marriage (Gale and Shapley, 1962), hospital-intern (Roth, 1984), entry-level labor (Crawford and Kelso, 1982), house allocation (Hylland and Zeckhauser, 1979), kidney exchange (Roth, Sönmez and Ünver, 2004), school choice (Abdulkadiroglu, Pathak and Roth, 2005) and college admission (Mumcu and Sağlam, 2007) markets are the most recognized examples of the kind.

The problem I am investigating is a school placement problem which differs from the school choice problem that has been widely studied for New York and Boston school systems.[1] The school choice problem deals with placing students to primary schools according to students' priorities, while school placement deals with placing students to schools according to students' test scores. One of the recent papers about school placement by Balinski and Sönmez (1999) which investigated college admission problem in Turkey.

In their paper, Balinski and Sönmez study the college placement mechanism in

[1] See Abdulkadiroglu, Pathak and Roth (2005).

Turkey. They show that the placement machanism used is a multicategory serial dictatorship. This mechanism has serious deficiencies and does not satisfies some of the properties a desirable placement mechanism should have.[2] Also they show that the only mechanism satisfying five important properties is Gale-Shapley Student Optimal Deferred Acceptance Algorithm. In Turkey, secondary school market is similar to college market, since in both markets students are placed to schools according to their test scores and students' preferences. However, there are some important differences which makes it worthwhile to study the secondary school placement problem.

In Turkey, secondary school placement is administered centrally each year. In this placement, students are assigned to three different types of schools according to their abilities. These schools are Private High Schools and two types of state schools, namely Anatolian High Schools and Science High Schools. Unlike the last two, Private High Schools' placement is managed by Private School Association (OOB), while the Ministry of National Education (MEB) has the authority on placement to the state schools. Schools' ranking of students is determined by students' test scores from centrally administered exams. Until 2008, the ranking of the students were determined by the exam called Secondary School Institutions Examination (OKS). As of 2009, OKS is replaced by a dual examination system, namely Level Determining Examination-Private Schools Examination (SBS-OOS). While the authorized institution to prepare and perform OKS and SBS is MEB, the authority for OOS is OOB.

In the OKS system, each student taking the test receives different types of scores calculated by weighting different parts of the exam differently. The score types are TM (Turkish-Mathematics), F (Science) and O (Private school score). TM and F type scores are calculated by MEB and used for placement to Anatolian

[2] These properties are fairness, non-wastefulness, strategy proofness, efficiency and respecting improvements.

and Science High Schools, respectively, whereas O type score is calculated by OOB and used for Private High School placement. Unlike OKS, for both types of state schools (Anatolian and Science High Schools), new placement system, SBS, uses one score that is calculated by using weighted average of three test scores that students take in the 6^{th}, 7^{th} and 8^{th} grades. On the other hand, Turkish private schools system uses raw SBS score and foreign private schools system uses OOS score. As the placement system remains the same under both OKS and SBS-OOS system, I will concentrate on OKS system in this study.

Upon receiving their scores, students submit two disjoint preference lists for state and private schools, separately. The placement mechanism assigns the students to both state and private schools independently based on their scores and preference lists. As a result of these independent assignments, there is a possibility for some students to receive more than one offer (e.g. one from private and one from state schools). At the end of the first assignment stage, students with more than one offer are obliged to accept at most one offer and decline the rest. Due to the fact that some students reject one of their offers, schools may have empty quotas. Subsequently, the system fills empty quotas of state schools in a second placement stage. On the other hand, private schools are not allowed to participate to the second stage and empty quotas of private schools remain empty, if there is any.[3] However, since the number of students who are placed in the aftermarket is less than two percent, it does not affect the placement system outcome by a considerable extent. Therefore, the decentralized aftermarket will be ignored in this study.

In this two-period many-to-one matching market, the two sets of agents are students and schools. Each school has a finite quota. Each student has a preference relation over the set of schools and being unmatched. Preference profiles of

[3] In practice, private schools fill their empty quotas in a decentralized aftermarket system by waitlists.

students, students' test scores and capacity of schools constitute a matching market. Matching is a symmetric relation between set of students and schools, which binds students to their assigned schools. A matching mechanism is a correspondence which picks a matching outcome for any matching market.
The Secondary School Placement System in Turkey uses Multicategory Serial Dictatorship (MSD, hereafter) mechanism as its basic placement tool. MSD mechanism works as follows:

In each step, for every score type, the mechanism assigns, starting from the highest ranked student, each student to her most preferable school available in that score type tentatively. Once the tentative matching is done for each score type, there will be some students with more than one assigned school. At the end of each step, mechanism updates students' preferences in the following way: If a student is not assigned to a school then the mechanism does not change her preference list, otherwise it adds a cut-off point (no school option) below the best assigned school. MSD mechanism continues until every student gets at most one offer. Once every student receives at most one offer, the mechanism matches the students to schools permanently.

Although MSD is used in the system in each market, the placement system differs from MSD in two major ways. First, since the placement to state and private schools are done independently, the market is segmented. Second, the empty quotas that arises at the end of the first placement stage, due to students rejecting one of the two offers they receive, the system calls for a second placement stage, but private schools are not allowed to participate to the second stage. Because of these extensions, I call this placement system as 2-Stage Segmented System with Multicategory Serial Dictatorship (2SSS-MSD, hereafter). 2SSS-MSD works as follows:

In the first stage of 2SSS-MSD, the algorithm assigns students in the state and private markets by using MSD mechanism and obtains independent set of matches for state schools and private schools.

At the end of the first stage, students with two offers choose at most one of them. In the second stage the algorithm runs MSD only for the residual state school market. In this stage, participants are the unmatched students and rejected state schools of the first stage.

In this study, I will first study whether 2SSS-MSD satisfies the following five criteria; fairness, non-wastefulness, strategy proofness, efficiency and respecting improvements. Then, I will look for ways to improve the deficiencies of the system. A matching is *fair* if for any student, preferring a school to her initial match implies that all students assigned to that school have higher test scores for that school. A matching system is fair if for any matching market, system chooses a fair outcome. A matching is *non-wasteful* if for any student, preferring a school to her initial match means that the school does not have an empty slot. A matching system is non-wasteful if for any matching market, system chooses a non-wasteful outcome. A matching system is *strategy proof* if no student can ever benefit by announcing different preference profile. A matching is *efficient* if there is no another matching that assigns every student to at least as good schools as students' initial match and makes some students better off. A matching system is efficient if the system always points an efficient outcome. A matching system *respects improvements* means that for any matching market, if a student increases her test scores then the system assigns her to at least as good school as her initial match.

I find that the placement system I studied satisfies only fairness property. Moreover, there is no fair and non-wasteful segmented system which is efficient or strategy proof or respects improvements.

Next, having found an impossibility result for the universal domain of preferences, I introduce some restrictions on students' preferences; blocked preferences and common preferences. A preference profile satisfies *blocked preferences* restriction if all students prefer any private school to any state school. A preference profile satisfies *common preferences* restriction if all students have identical preference lists.

I show that under blocked preferences restriction, the two-stage segmented system satisfies the five properties if one uses Gale-Shapley Student Optimal Deferred Acceptance Algorithm (GS mechanism hereafter) instead of MSD. Furthermore, under common preferences restriction, the segmented system can be improved by using GS mechanism while increasing the number of stages the market reopens and allowing private schools to enter all stages.

I also show that under 2SSS-MSD some private schools are better off while some of them worse off. Then, I conclude that private schools, collectively, has no incentive for keeping the segmented placement system.

The study is organized as follows: Chapter 2 introduces the components and the structure of the placement mechanism. Chapter 3 investigates properties of the system. Chapter 4 attempts to improve the deficiencies of the system and present my first result. Chapter 5 investigates the placement system under different restrictions on student preferences. Chapter 6 analyses the reason for using the segmented system. Chapter 7 concludes.

CHAPTER 2

THE MODEL

Basic Structure

Model Components

A matching environment is denoted by the list (C, S, q, R_s, T, f, t) that involves the following fixed components:

Market Participants: The first two components of a matching environment are non-empty, finite and disjoint sets of schools $C = c_1, c_2, ..., c_m$ and students $S = s_1, s_2, ..., s_n$. I will denote generic student by s, and generic school by c. There are two types of high schools; state and private which are denoted by C_{st} and C_{pr}, respectively.

Capacities of Schools: The third component is a vector of positive natural numbers $q = (q_{c_1}, ..., q_{c_m})$, where q_c is the total capacity of school c.

Preferences: List of student preference relations is $R_S = (R_{s_1}, ..., R_{s_n})$. For any $s \in S$, R_s is a binary preference relation that is a linear order on $\Sigma_s = \{\{c_1\}, \{c_2\}, ..., \{c_m\}, \emptyset\}$. The element $\emptyset$ is interpreted by both schools and students as the prospect of being unassigned. Let $\Re_s$ denote the set of all preference relations for $s \in S$. Define $\Re = \times_{s \in S} \Re_s$. Also $\forall s \in S, P_s$ is strict preference for s such that $\forall c, c' \in C$, cP_sc' implies cR_sc' and *not* $c'R_sc$.

Types of Scores (skill categories): Set of scores is $T = \{t_1, t_2, \ldots, t_l\}$. There are l types of scores, each of which is obtained by weighting different set of questions.

Test Scores: List of test scores $f = (f^{s_1}, ..., f^{s_n})$. For any student $s \in S$,

$f^s = (f^s_{t_1}, f^s_{t_2}, \ldots, f^s_{t_l})$ is a vector which gives the test score of student s in each category.

School Type Function: A function $t : C \to T$ where $t(c)$ is the score type required by school c.

I will assume that there are more students than available seats, i.e., $|S| > \sum_{i=1}^{m} q_{c_i}$. Moreover, there are no ties in the test scores, that is $\forall s, s' \in S$, $f^s_{t_i} = f^{s'}_{t_i}$ for $i = 1, 2, \ldots, l$ if and only if $s = s'$.

First-Stage Market

For any matching environment (C, S, q, R_S, T, f, t), there are two first stage matching markets: *state school market* and *private school market*. State school and private school markets are denoted as (R_S, f, q^{st}) and (R_S, f, q^{pr}), respectively. The last components are vectors of nonnegative numbers $q^{st} = (q^{st}_{c_1}, ..., q^{st}_{c_m})$ and $q^{pr} = (q^{pr}_{c_1}, ..., q^{pr}_{c_m})$, where q^{st} and q^{pr} are the capacity vectors for state and private school markets, respectively. For state school market, for any $c \in C_{st}$, $q^{st}_c = q_c$ and $c \notin C_{st}$, $q^{st}_c = 0$. Similarly, for state school market, for any $c \in C_{pr}$, $q^{pr}_c = q_c$ and $c \notin C_{pr}$, $q^{pr}_c = 0$.

First-Stage Market Matchings

Given the matching environment (C, S, q, R_S, T, f, t) and a first stage market (R_S, f, q^{γ}) where $\gamma \in \{st, pr\}$, a matching μ^{γ}_1 is a correspondence from the set $C_{\gamma} \cup S$ into $C_{\gamma} \cup S \cup \{\emptyset\}$ such that $\mu^{\gamma}_1(s) \in C_{\gamma} \cup \{\emptyset\}$, $\mu^{\gamma}_1(c) \in 2^S$ and $|\mu^{\gamma}_1(c)| \leq q_c$ also $\mu^{\gamma}_1(s) = c$ if and only if $s \in \mu^{\gamma}_1(c)$ for any student s and school c. Let S_1 be the set of students who receive offers from a private school, i.e., $S_1 = \{s \in S : \mu^{pr}_1(s) \neq \emptyset\}$.

Students who rejects their offers participate in the second stage matching. In practice this decision is taken by each student simultaneously. However, I assume that the placement system makes this decision on behalf of students. The set of students who rejects private school offer can be of two types. First, a student s may prefer her state school offer to the private one, $\mu_1^{st}(s) P_s \mu_1^{pr}(s)$. Second, a student who prefers her private school offer to the state one, i.e., $\mu_1^{pr}(s) P_s \mu_1^{st}(s)$, may reject her both offers with the hope that she will get a better state school match in the second stage.

Second-Stage Market

Let S_2 be the set of students who rejects her private school offer and students s such that $\mu_1^{pr}(s) = \emptyset$. Given the two first stage markets, (R_S, f, q^{st}) and (R_S, f, q^{pr}), and first-stage market matchings μ_1^{st} and μ_1^{pr}, a second-stage market is the list (S_2, R_S, f, q_2^{st}) where $q_{2,c}^{st}$ is equal to the number of remaining quotas for state schools and equal to 0 for private schools.

Second-Stage Market Matchings

Given the matching environment (C, S, q, R_s, T, f, t) and a second stage market (S_2, R_S, f, q_2^{st}), a matching μ_2 is a correspondence from the set $C_{st} \cup S_2$ into $C_{st} \cup S_2 \cup \{\emptyset\}$ such that $\mu_2^{st}(s) \in C_{st} \cup \{\emptyset\}$ and $\mu_2^{st}(c) \in 2^{S_2}$, also $\mu_2^{st}(s) = c$ if and only if $s \in \mu_2^{st}(c)$ for any student s and school c.

Matching Systems

Let S_{pr} be the set of students who accept her private school offer in the first stage, $S_{pr} = S \setminus S_2$. Given matching environment (C, S, q, R_s, T, f, t), a matching system is a correspondence from the set $C \cup S$ into $C \cup S \cup \{\emptyset\}$ such that:

$$\mu(s) = \begin{cases} \mu_1^{pr}(s) & s \in S_{pr} \\ \mu_2^{st}(s) & s \in S \setminus S_{pr} \end{cases}$$

and:

$$\mu(c) = \begin{cases} \mu_2^{st}(c) & c \in C_{st} \\ \mu_1^{pr}(c) \cap S_{pr} & c \in C_{pr} \end{cases}$$

Next, I define some properties of matching systems. A matching is immune to *individual blocking* if any student prefers her match to being unmatched. A matching is immune to *pairwise blocking* if $\nexists(s,c) \in S \times C$ such that the student s prefers c to her match and either c has an empty slot or one of the matches of c has lower score than s in the initial score type. A matching system is *stable* if for any preference and score profile, system picks a matching that is immune to individual and pairwise blocking.

Placement Mechanism: 2-Stage Segmented System with Multicategory Serial Dictatorship

The secondary school placement system in Turkey uses *Multicategory Serial Dictatorship* (MSD, hereafter) in each stage which I will refer as *2-Stage Segmented System with Multicategory Serial Dictatorship* (2SSS-MSD, hereafter). The MSD mechanism is previously studied in Balinski and Sonmez (1999). $2SSS - MSD$ uses MSD mechanism as its placement tool. However, the placement system differs than MSD in two ways. First, since the placement to state and private schools are done independently, the market is segmented. Second, the empty quotas that arises at the end of the first placement stage, due to students rejecting one of the two offers they receive, calls for a second placement stage. In order to fill the empty quotas, the system practices second stage placement. Because of these extensions, I call this placement system as 2-Stage Segmented System with Multicategory Serial Dictatorship. Before explaining how $2SSS - MSD$ works, I first introduce the structure of MSD mechanism.

MSD mechanism is simply the application of following recursive algorithm to any placement problem (P_S, f, q). The mechanism is applied in n steps and for each step k, mechanism uses updated market (P_S^k, f, q) with $(P_S^1, f, q) = (P_S, f, q)$. MSD mechanism works as follows:

In each step, for score type t, mechanism assigns highest ranked students to her most preferable school available with type t tentatively. Then, all the students for a particular score type is assigned. This is done for each score type. After assigning students based on every score type t, there will be some students with more than one assigned school. At the end of each step, mechanism updates students' preferences in the following way: If a student s is not assigned to a school then $P_s^k = P_s^{k+1}$, otherwise generate P_s^{k+1} by moving s (no school option) below the best assigned school.

MSD mechanism continues until every student gets at most one offer. Once every student receives at most one offer, the mechanism assigns students to the schools permanently.

Next, I show how $2SSS - MSD$ works. $2SSS - MSD$ performs the following algorithm:

In the first stage, algorithm assigns students in the state and private markets by using MSD mechanism and obtains independent set of matching outcomes for state school market (μ_1^{st}) and private school market (μ_1^{pr}). At the end of the first stage, students with 2 offers choose at most one of them. In the second stage algorithm runs MSD only for the residual state school market. In this stage, participants are the unmatched students and rejected state schools of the first stage. Hereafter I will use S_2 for the set of students participating in the second stage. The example below demonstrates how the $2SSS - MSD$ algorithm works.

Example 1: Let $S = \{s_1, s_2, s_3, s_4, s_5\}$, $C = \{c_1, c_2, c_3\}$ where $c_1, c_2 \in C_{st}$ and

$c_3 \in C_{pr}$, $q = \{q_1, q_2, q_3\} = (2, 1, 2)$, $T = \{t_1, t_2, t_3\}$, $t(c_1) = t_1$, $t(c_2) = t_2$, $t(c_3) = t_3$. Let preferences $R_S = \{R_{s_1}, R_{s_2}, R_{s_3}, R_{s_4}, R_{s_5}\}$ and the test scores $f = \{f^{s_1}, f^{s_2}, f^{s_3}, f^{s_4}, f^{s_5}\}$ be as follows:

$$
\begin{array}{llcl}
c_3 P_{s_1} c_1 P_{s_1} c_2 P_{s_1} \emptyset & f^{s_1} & = & (f^{s_1}_{t_1}, f^{s_1}_{t_2}, f^{s_1}_{t_3}) = (90, 90, 90) \\
c_1 P_{s_2} c_2 P_{s_2} c_3 P_{s_2} \emptyset & f^{s_2} & = & (f^{s_2}_{t_1}, f^{s_2}_{t_2}, f^{s_2}_{t_3}) = (80, 50, 60) \\
c_3 P_{s_3} c_2 P_{s_3} c_1 P_{s_3} \emptyset & f^{s_3} & = & (f^{s_3}_{t_1}, f^{s_3}_{t_2}, f^{s_3}_{t_3}) = (70, 80, 80) \\
c_1 P_{s_4} c_2 P_{s_4} c_3 P_{s_4} \emptyset & f^{s_4} & = & (f^{s_4}_{t_1}, f^{s_4}_{t_2}, f^{s_4}_{t_3}) = (60, 70, 70) \\
c_3 P_{s_5} c_2 P_{s_5} c_1 P_{s_5} \emptyset & f^{s_5} & = & (f^{s_5}_{t_1}, f^{s_5}_{t_2}, f^{s_5}_{t_3}) = (50, 60, 50)
\end{array}
$$

These scores make following rankings in categories t_1, t_2 and t_3:

$$
\begin{array}{llccccc}
t_1 & : & s_1 & s_2 & s_3 & s_4 & s_5 \\
t_2 & : & s_1 & s_3 & s_4 & s_5 & s_2 \\
t_3 & : & s_1 & s_3 & s_4 & s_2 & s_5
\end{array}
$$

Since there is just one private school, the second preference lists of students include only c_3. In the first stage students will be placed to state and private schools separately. Although this processes are simultaneous, I will show these two markets dynamics separately. Let's explore state market first.

The tentative placements of step 1 ($\mu^{st}_{1,2}$) is the following:

$$\mu^{st}_{1,1}(s_1) = (c_1, c_2) \quad \mu^{st}_{1,1}(s_2) = (c_1) \quad \mu^{st}_{1,1}(s_3) = (\emptyset) \quad \mu^{st}_{1,1}(s_4) = (\emptyset) \quad \mu^{st}_{1,1}(s_5) = (\emptyset)$$

First step implies the following path. Consider t_1, there is only one school, c_1, in this category and it has two available slots. Then s_1, the highest ranked student in t_1, *engages* to c_1, since c_1 is the most preferred t_1 type of school in her preference list. The second student s_2 also *engages* to c_1 which is still available for s_2. Since there is no more available quota in type t_1 school, algorithm continues with t_2. Like

t_1, there is only one school, c_2 and it has one available slot. This type starts with s_1 again, since she is the first student in t_2. c_2 is the most preferred type t_2 school in her list, therefore she *engages* to c_2.

At the end of step 1, the preference lists are updated as follows::

$$
\begin{array}{llcl}
c_1 P_{s_1} \emptyset P_{s_4} c_2 & f^{s_1} & = & (f^{s_1}_{t_1}, f^{s_1}_{t_2}) = (90, 90) \\
c_1 P_{s_2} \emptyset P_{s_4} c_2 & f^{s_2} & = & (f^{s_2}_{t_1}, f^{s_2}_{t_2}) = (80, 50) \\
c_2 P_{s_3} c_1 P_{s_3} \emptyset & f^{s_3} & = & (f^{s_3}_{t_1}, f^{s_3}_{t_2}) = (70, 80) \\
c_1 P_{s_4} c_2 P_{s_4} \emptyset & f^{s_4} & = & (f^{s_4}_{t_1}, f^{s_4}_{t_2}) = (60, 70) \\
c_2 P_{s_5} c_1 P_{s_5} \emptyset & f^{s_5} & = & (f^{s_5}_{t_1}, f^{s_5}_{t_2}) = (50, 60)
\end{array}
$$

In the second step, with new preferences, MSD mechanism is applied. The process is similar to the step 1, except cut-off points. Since c_2 is not available for s_1 and s_2 and every student receives at most one offer, this is the last step for this market and gives the $\mu_1^{st}(s) = \mu_{1,2}^{st}(s)$. Finally, state market generates following matching:

$$\mu_1^{st}(s_1) = (c_1) \quad \mu_1^{st}(s_2) = (c_1) \quad \mu_1^{st}(s_3) = (c_2) \quad \mu_1^{st}(s_4) = (\emptyset) \quad \mu_1^{st}(s_5) = (\emptyset)$$

In the private school market, process ends in one step and gives the following result:

$$\mu_1^{pr}(s_1) = (c_3) \quad \mu_1^{pr}(s_2) = (\emptyset) \quad \mu_1^{pr}(s_3) = (c_3) \quad \mu_1^{pr}(s_4) = (\emptyset) \quad \mu_1^{pr}(s_5) = (\emptyset)$$

At the end of the stage 1 the result is:

$$
\begin{array}{lcl}
\mu_1(s_1) & = & (c_1, c_3) \\
\mu_1(s_2) & = & (c_1, \emptyset) \\
\mu_1(s_3) & = & (c_2, c_3) \\
\mu_1(s_4) & = & (\emptyset, \emptyset) \\
\mu_1(s_5) & = & (\emptyset, \emptyset)
\end{array}
$$

At this point, the mechanism places each student to her highest ranked offer and declines the other offer.[4] Since c_3 is the first school in preference lists of s_1 and s_3, mechanism places them c_3. So, in the second stage quotas are
$q_C^2 = (q_1^2, q_2^2, q_3^2) = (2, 1, 0)$ and since s_1 and s_3 accepted their offers, $\mu(s_1)$ and $\mu(s_3)$ are already known.

Therefore, at the end of the second stage, the final result is:

$$\begin{aligned} \mu(s_1) &= (c_3) \\ \mu(s_2) &= (c_1) \\ \mu(s_3) &= (c_3) \\ \mu(s_4) &= (c_1) \\ \mu(s_5) &= (c_2) \end{aligned}$$

[4] In practice this decision is taken by each student simultaneously. But for simplicity I add this process into the system.

CHAPTER 3

PROPERTIES OF 2SSS-MSD

In this section properties of placement system in Turkey will be investigated. The five criteria I will look for are fairness, non-wastefulness, strategy proofness, respecting improvements, and efficiency. Since these five desiderata are the most basic properties, a desirable placement system should have, any improvement for these properties has a positive effect on the education policy of Turkey.

Fairness

Since student placement is done by considering students' test scores, fairness is one of the most essential properties. A desirable system should assign a student with higher score to her better choice. Fairness is defined as, if a student cannot match a school that she wanted more than her initial match, then all students assigned to that school have higher test scores for that school. Formal definition of fairness is given below.

Definition 1 (F): A matching mechanism μ satisfies fairness if $\forall s \in S$ and $\forall c \in C$, $cP_s\mu(s)$ implies $f^{s'}_{t(c)} > f^{s}_{t(c)}$, $\forall s' \in \mu(c)$.

It is proven by Balinski and Sonmez (1999) that Multicategory Serial Dictatorship is a fair mechanism. Here, I show that $2SSS - MSD$ also satisfies F, in spite of segmentation.[5]

Proposition 1: 2SSS-MSD satisfies fairness property.

Proof: Assume 2SSS-MSD is not fair. So, $\exists (s, c) \in S \times C$ such that $cP_s\mu(s)$ and $\exists s' \in S$ such that $s' \in \mu(c)$ but $f^{s'}_{t(c)} < f^{s}_{t(c)}$. But since MSD is fair, c can not be matched to s' in the first stage. If c is a state school and matched to s' in the second stage, since student s knows that there is an empty quota of c, she participates to

[5] The assumption that the placement system makes rejection decision at the end of the first stage on behalf of students guarantees the fairness of the system.

the second stage too, then a fair MSD mechanism does not matches c to s'. So, this contradicts with the initial assumption. Hence, 2SSS-MSD satisfies fairness.

Non-Wastefulness

Since private schools try to fill their empty quotas in the aftermarket by waitlists, it is not likely to satisfy fairness if there is any wasted quota in the system. So, non-wastefulness affects both efficiency and fairness of the system. A system is non-wasteful when a school does not have an empty slot while there is a student preferring that school to her match. Formally;

Definition 2 (NW): A matching mechanism satisfies non-wastefulness if $\forall s \in S$ and $\forall c \in C$, $cP_s\mu(s)$ implies $|\mu^{-1}(c)| = q_c$.

Although MSD is a non-wasteful mechanism, the following proposition demonstrates that due to nature of the second stage, $2SSS - MSD$ fails to satisfy NW property.

Proposition 2: 2SSS-MSD fails to satisfy non-wastefulness.

Proof: Let $S = \{s_1, s_2, s_3, s_4, s_5\}$, $C = \{c_1, c_2, c_3\}$ where $c_1, c_2 \in C_{st}$ and $c_3 \in C_{pr}$, $q = \{q_1, q_2, q_3\} = (2, 1, 2)$, $T = \{t_1, t_2, t_3\}$, $t(c_1) = t_1$, $t(c_2) = t_2$, $t(c_3) = t_3$. Let preferences $R_S = \{R_{s_1}, R_{s_2}, R_{s_3}, R_{s_4}, R_{s_5}\}$ and the test scores $f = \{f^{s_1}, f^{s_2}, f^{s_3}, f^{s_4}, f^{s_5}\}$ be as follows:

$$
\begin{array}{lrcl}
c_1P_{s_1}c_2P_{s_1}c_3P_{s_1}\emptyset & f^{s_1} & = & (f^{s_1}_{t_1}, f^{s_1}_{t_2}, f^{s_1}_{t_3}) = (90, 70, 80) \\
c_1P_{s_2}c_2P_{s_2}c_3P_{s_2}\emptyset & f^{s_2} & = & (f^{s_2}_{t_1}, f^{s_2}_{t_2}, f^{s_2}_{t_3}) = (80, 80, 90) \\
c_2P_{s_3}c_1P_{s_3}c_3P_{s_3}\emptyset & f^{s_3} & = & (f^{s_3}_{t_1}, f^{s_3}_{t_2}, f^{s_3}_{t_3}) = (70, 90, 50) \\
c_2P_{s_4}c_1P_{s_4}c_3P_{s_4}\emptyset & f^{s_4} & = & (f^{s_4}_{t_1}, f^{s_4}_{t_2}, f^{s_4}_{t_3}) = (60, 50, 60) \\
c_3P_{s_5}c_2P_{s_5}c_1P_{s_5}\emptyset & f^{s_5} & = & (f^{s_5}_{t_1}, f^{s_5}_{t_2}, f^{s_5}_{t_3}) = (50, 60, 70)
\end{array}
$$

These scores make following rankings in categories t_1, t_2 and t_3:

$$\begin{array}{lcccccc} t_1 & : & s_1 & s_2 & s_3 & s_4 & s_5 \\ t_2 & : & s_3 & s_2 & s_1 & s_5 & s_4 \\ t_3 & : & s_2 & s_1 & s_5 & s_4 & s_3 \end{array}$$

At the end of the stage 1 the result is:

$$\begin{aligned} \mu_1(s_1) &= (c_1, c_3) \\ \mu_1(s_2) &= (c_1, c_3) \\ \mu_1(s_3) &= (c_2, \emptyset) \\ \mu_1(s_4) &= (\emptyset, \emptyset) \\ \mu_1(s_5) &= (\emptyset, \emptyset) \end{aligned}$$

The final result of 2SSS-MSD is:

$$\begin{aligned} \mu(s_1) &= (c_1) \\ \mu(s_2) &= (c_1) \\ \mu(s_3) &= (c_2) \\ \mu(s_4) &= (\emptyset) \\ \mu(s_5) &= (\emptyset) \end{aligned}$$

This example shows that 2SSS-MSD mechanism is wasteful. At the end of the placement there are still 2 empty quotas which are more preferred to $\emptyset$ by s_4 and s_5 who are unmatched.

Respecting Improvements

In any placement system based on test scores, if the only criteria for schools to rank students is their test scores, then for any student, increasing her test score should make her better off. Let $f_t'^s \geq f_t^s$ for any score type t, f^{-s} be test scores of all students but s, and $\mu(s|f^s, f^{-s})$ be the match of student s given the test scores f^s and f^{-s}.

Definition 3 (RI): A matching mechanism μ respects improvements in students' test scores if $\forall s \in S$, $\mu(s|f'^s, f^{-s}) R_s \mu(s|f^s, f^{-s})$.

Proposition 3: 2SSS-MSD does not respect improvements.

Proof: Let $S = \{s_1, s_2, s_3\}$, $C = \{c_1, c_2, c_3\}$ where $c_1, c_2 \in C_{st}$ and $c_3 \in C_{pr}$, $q = \{q_1, q_2, q_3\} = (1, 1, 1)$, $T = \{t_1, t_2, t_3\}$, $t(c_1) = t_1$, $t(c_2) = t_2$, $t(c_3) = t_3$. Let preferences $R_S = \{R_{s_1}, R_{s_2}, R_{s_3}\}$ and the test scores $f = \{f^{s_1}, f^{s_2}, f^{s_3}\}$ be as follows:

$$
\begin{array}{lrcl}
c_2 P_{s_1} c_1 P_{s_1} c_3 P_{s_1} \emptyset & f^{s_1} & = & (f_{t_1}^{s_1}, f_{t_2}^{s_1}, f_{t_3}^{s_1}) = (70, 80, 70) \\
c_1 P_{s_2} c_2 P_{s_2} c_3 P_{s_2} \emptyset & f^{s_2} & = & (f_{t_1}^{s_2}, f_{t_2}^{s_2}, f_{t_3}^{s_2}) = (80, 90, 80) \\
c_3 P_{s_3} c_2 P_{s_3} c_1 P_{s_3} \emptyset & f^{s_3} & = & (f_{t_1}^{s_3}, f_{t_2}^{s_3}, f_{t_3}^{s_3}) = (60, 70, 90)
\end{array}
$$

These scores make following rankings in categories t_1, t_2 and t_3:

$$
\begin{array}{lcccc}
t_1 & : & s_2 & s_1 & s_3 \\
t_2 & : & s_2 & s_1 & s_3 \\
t_3 & : & s_3 & s_2 & s_1
\end{array}
$$

At the end of the stage 1 the result is:

$$
\begin{array}{lcl}
\mu(s_1) & = & (c_2, \emptyset) \\
\mu(s_2) & = & (c_1, \emptyset) \\
\mu(s_3) & = & (\emptyset, c_3)
\end{array}
$$

Then At the end of the second stage the final result is:

$$\begin{aligned}
\mu(s_1) &= (c_2) \\
\mu(s_2) &= (c_1) \\
\mu(s_3) &= (c_3)
\end{aligned}$$

Now let assume student 1 improved her score while other student's are constant. According to respecting improvements criterion she should assign c_2 again. Let preferences $R_S = \{R_{s_1}, R_{s_2}, R_{s_3}\}$ and the test scores $f = \{f^{s_1}, f^{s_2}, f^{s_3}\}$ be as follows:

$$\begin{aligned}
c_2 R_{s_1} c_1 R_{s_1} c_3 R_{s_1} \emptyset \qquad f^{s_1} &= (f^{s_1}_{t_1}, f^{s_1}_{t_2}, f^{s_1}_{t_3}) = (90, 80, 70) \\
c_1 R_{s_2} c_2 R_{s_2} c_3 R_{s_2} \emptyset \qquad f^{s_2} &= (f^{s_2}_{t_1}, f^{s_2}_{t_2}, f^{s_2}_{t_3}) = (80, 90, 80) \\
c_3 R_{s_3} c_2 R_{s_3} c_1 R_{s_3} \emptyset \qquad f^{s_3} &= (f^{s_3}_{t_1}, f^{s_3}_{t_2}, f^{s_3}_{t_3}) = (60, 70, 90)
\end{aligned}$$

These scores make following rankings in categories t_1, t_2 and t_3:

$$\begin{aligned}
t_1 &: s_1 \;\; s_2 \;\; s_3 \\
t_2 &: s_2 \;\; s_1 \;\; s_3 \\
t_3 &: s_3 \;\; s_2 \;\; s_1
\end{aligned}$$

At the end of the stage 1 the result is:

$$\begin{aligned}
\mu(s_1) &= (c_1, \emptyset) \\
\mu(s_2) &= (c_2, \emptyset) \\
\mu(s_3) &= (\emptyset, c_3)
\end{aligned}$$

Then, at the end of the second stage the final result is:

$$\begin{aligned}\mu(s_1) &= (c_1)\\ \mu(s_2) &= (c_2)\\ \mu(s_3) &= (c_3)\end{aligned}$$

The example above shows that $2SSS - MSD$ does not respect improvements in students' test scores. At the end of the placement student 1 matched to her worse choice.

Strategy Proofness

Strategy proofness increases the credibility of both the system and trust on Ministry of National Education policies. In any system providing this criteria, there is no room for manipulation. Non-manipulability means that every student should reveal their true preferences. Let R_s be preference list of student s and R_{-s} be preference profile of the rest of the students and let $\mu(s|R_s, R_{-s})$ be the match of student s given the preference lists R_s and R_{-s}.

Definition 4 (SP) : A matching mechanism μ is strategy proof if $\forall s \in S$, for any announced preference profile R'_s, $\mu(s|R_s, R_{-s}) R_s \mu(s|R'_s, R_{-s})$.

Proposition 4: 2SSS-MSD fails to satisfy strategy proofness.

Proof: Let $S = \{s_1, s_2, s_3\}$, $C = \{c_1, c_2, c_3\}$ where $c_1, c_2 \in C_{st}$ and $c_3 \in C_{pr}$, $q = \{q_1, q_2, q_3\} = (1, 1, 1)$, $T = \{t_1, t_2, t_3\}$, $t(c_1) = t_1$, $t(c_2) = t_2$, $t(c_3) = t_3$. Let preferences $R_S = \{R_{s_1}, R_{s_2}, R_{s_3}\}$ and the test scores $f = \{f^{s_1}, f^{s_2}, f^{s_3}\}$ be as follows:

$$\begin{aligned}
&c_1 P_{s_1} c_2 P_{s_1} c_3 P_{s_1} \emptyset \qquad & f^{s_1} &= (f^{s_1}_{t_1}, f^{s_1}_{t_2}, f^{s_1}_{t_3}) = (80, 90, 70)\\
&c_2 P_{s_2} c_1 P_{s_2} c_3 P_{s_2} \emptyset \qquad & f^{s_2} &= (f^{s_2}_{t_1}, f^{s_2}_{t_2}, f^{s_2}_{t_3}) = (90, 80, 80)\\
&c_3 P_{s_3} c_2 P_{s_3} c_1 P_{s_3} \emptyset \qquad & f^{s_3} &= (f^{s_3}_{t_1}, f^{s_3}_{t_2}, f^{s_3}_{t_3}) = (70, 70, 90)
\end{aligned}$$

These scores make following rankings in categories t_1, t_2 and t_3:

$$\begin{array}{lllll} t_1 & : & s_2 & s_1 & s_3 \\ t_2 & : & s_1 & s_2 & s_3 \\ t_3 & : & s_3 & s_2 & s_1 \end{array}$$

The final result is:

$$\begin{aligned} \mu(s_1) &= (c_2) \\ \mu(s_2) &= (c_1) \\ \mu(s_3) &= (c_3) \end{aligned}$$

Now, assume student 1 announced her preferences as $c_1 P_{s_1} \emptyset P_{s_1} c_2 P_{s_1} c_3$. According to new preference lists, the setup will be:

$$\begin{array}{lcll} c_1 P_{s_1} \emptyset P_{s_1} c_2 P_{s_1} c_3 & f^{s_1} & = & (f^{s_1}_{t_1}, f^{s_1}_{t_2}, f^{s_1}_{t_3}) = (80, 90, 70) \\ c_2 P_{s_2} c_1 P_{s_2} c_3 P_{s_2} \emptyset & f^{s_2} & = & (f^{s_2}_{t_1}, f^{s_2}_{t_2}, f^{s_2}_{t_3}) = (90, 80, 80) \\ c_3 P_{s_3} c_2 P_{s_3} c_1 P_{s_3} \emptyset & f^{s_3} & = & (f^{s_3}_{t_1}, f^{s_3}_{t_2}, f^{s_3}_{t_3}) = (70, 70, 90) \end{array}$$

Then, the final result is:

$$\begin{aligned} \mu(s_1) &= (c_1) \\ \mu(s_2) &= (c_2) \\ \mu(s_3) &= (c_3) \end{aligned}$$

The first student manipulates the mechanism by announcing different preferences over schools. Therefore, at the end of the second stage student 1 matched to her better choice. That means, the system is open to be manipulated.

Hence, $2SSS - MSD$ is not strategy-proof.

Pareto Efficiency

School seats are scare resources. Therefore allocation of these resources effectively is one of the important missions of central authority. In matching literature, Pareto criterion is the most used criterion for measuring the efficiency.

Definition 5: A matching μ Pareto dominates μ' if $\mu(s) R_s \mu'(s)$, $\forall s \in S$ and $\mu(s) P_s \mu'(s)$ for some s.

Definition 6 (PE): A matching μ is Pareto Efficient if $\nexists \mu'$ such that μ' Pareto dominates μ.

Proposition 5: 2SSS-MSD fails to satisfy Pareto Efficiency.

Proof: Let $S = \{s_1, s_2, s_3\}$, $C = \{c_1, c_2, c_3\}$ where $c_1, c_2 \in C_{st}$ and $c_3 \in C_{pr}$, $q = \{q_1, q_2, q_3\} = (1, 1, 1)$, $T = \{t_1, t_2, t_3\}$, $t(c_1) = t_1$, $t(c_2) = t_2$, $t(c_3) = t_3$. Let preferences $R_S = \{R_{s_1}, R_{s_2}, R_{s_3}\}$ and the test scores $f = \{f^{s_1}, f^{s_2}, f^{s_3}\}$ be as follows:

$$
\begin{array}{lrcl}
c_1 P_{s_1} c_2 P_{s_1} c_3 P_{s_1} \emptyset & f^{s_1} & = & (f^{s_1}_{t_1}, f^{s_1}_{t_2}, f^{s_1}_{t_3}) = (80, 90, 70) \\
c_2 P_{s_2} c_1 P_{s_2} c_3 P_{s_2} \emptyset & f^{s_2} & = & (f^{s_2}_{t_1}, f^{s_2}_{t_2}, f^{s_2}_{t_3}) = (90, 80, 80) \\
c_3 P_{s_3} c_2 P_{s_3} c_1 P_{s_3} \emptyset & f^{s_3} & = & (f^{s_3}_{t_1}, f^{s_3}_{t_2}, f^{s_3}_{t_3}) = (70, 70, 90)
\end{array}
$$

These scores make following rankings in categories t_1, t_2 and t_3:

$$
\begin{array}{lcccc}
t_1 & : & s_2 & s_1 & s_3 \\
t_2 & : & s_1 & s_2 & s_3 \\
t_3 & : & s_3 & s_2 & s_1
\end{array}
$$

The final result is:

$$\begin{aligned}
\mu(s_1) &= (c_2) \\
\mu(s_2) &= (c_1) \\
\mu(s_3) &= (c_3)
\end{aligned}$$

Now, let another matching mechanism (e.g, Gale and Shapley Student Optimal) μ' such that:

$$\begin{aligned}
\mu'(s_1) &= (c_1) \\
\mu'(s_2) &= (c_2) \\
\mu'(s_3) &= (c_3)
\end{aligned}$$

It is clear that first two students are better off while the third student is indifferent. Therefore, the $2SSS - MSD$ outcome is pareto dominated by μ'. Hence, $2SSS - MSD$ is not Pareto Efficient.

CHAPTER 4

CAN A BETTER SYSTEM BE FOUND?

To find a system that satisfies non-wastefulness, strategy proofness, efficiency, and respecting improvements is not as easy as it seems. If the markets were not segmented one can solve these problems by applying *Gale-Shapley Student Proposing Deferred Acceptance Algorithm* (GS hereafter) instead of MSD. But given the markets are segmented markets applying GS mechanism in a two stage segmented market does not improve all the above criteria. Next I show that although *Gale-Shapley Student Proposing DAA* satisfies five properties we are looking for, *Two-Stage Segmented System with Gale-Shapley Student Proposing DAA* (2SSS-GS hereafter) does not satisfy non-wastefulness, efficiency, strategy proofness, and does not respect improvements.

Proposition 6: 2SSS-GS is wasteful and is not efficient.

Proof: Let $S = \{s_1, s_2, s_3\}$, $C = \{c_1, c_2, c_3\}$ where $c_1, c_2 \in C_{st}$ and $c_3 \in C_{pr}$, $q = \{q_1, q_2, q_3\} = (1, 1, 1)$, $T = \{t_1, t_2, t_3\}$, $t(c_1) = t_1$, $t(c_2) = t_2$, $t(c_3) = t_3$. Let preferences $R_S = \{R_{s_1}, R_{s_2}, R_{s_3}\}$ and the test scores $f = \{f^{s_1}, f^{s_2}, f^{s_3}\}$ be as follows:

$$
\begin{array}{lrcl}
c_1 P_{s_1} c_2 P_{s_1} c_3 P_{s_1} \emptyset & f^{s_1} & = & (f^{s_1}_{t_1}, f^{s_1}_{t_2}, f^{s_1}_{t_3}) = (80, 90, 90) \\
c_2 P_{s_2} c_1 P_{s_2} c_3 P_{s_2} \emptyset & f^{s_2} & = & (f^{s_2}_{t_1}, f^{s_2}_{t_2}, f^{s_2}_{t_3}) = (90, 80, 70) \\
c_3 P_{s_3} c_2 P_{s_3} c_1 P_{s_3} \emptyset & f^{s_3} & = & (f^{s_3}_{t_1}, f^{s_3}_{t_2}, f^{s_3}_{t_3}) = (70, 70, 80)
\end{array}
$$

These scores make following rankings in categories t_1, t_2 and t_3:

$$
\begin{array}{lcccc}
t_1 & : & s_2 & s_1 & s_3 \\
t_2 & : & s_1 & s_2 & s_3 \\
t_3 & : & s_1 & s_3 & s_2
\end{array}
$$

At the end of the first stage:

$$\begin{aligned}\mu_1(s_1) &= (c_1, c_3)\\ \mu_1(s_2) &= (c_2, \emptyset)\\ \mu_1(s_3) &= (\emptyset, \emptyset)\end{aligned}$$

Since s_1 prefers c_1 to c_3, s_1 rejects c_3. So, final result is:

$$\begin{aligned}\mu(s_1) &= (c_1)\\ \mu(s_2) &= (c_2)\\ \mu(s_3) &= (\emptyset)\end{aligned}$$

Since $c_3 P_{s_3} \emptyset$ and $|\mu^{-1}(c_3)| < q_{c_3}$, the system is wasteful. Also, let μ' be another matching outcome as $(\mu'(s_1), \mu'(s_2), \mu'(s_3)) = (c_1, c_2, c_3)$. Since $\forall s \in S$, $\mu'(s) R_s \mu(s)$, the system is also not efficient.

Proposition 7: 2SSS-GS does not satisfy strategy proofness and does not respect improvements.

Proof: Let $S = \{s_1, s_2, s_3\}$, $C = \{c_1, c_2, c_3\}$ where $c_1, c_2 \in C_{st}$ and $c_3 \in C_{pr}$, $q = \{q_1, q_2, q_3\} = (1, 1, 1)$, $T = \{t_1, t_2, t_3\}$, $t(c_1) = t_1$, $t(c_2) = t_2$, $t(c_3) = t_3$. Let preferences $R_S = \{R_{s_1}, R_{s_2}, R_{s_3}\}$ and the test scores $f = \{f^{s_1}, f^{s_2}, f^{s_3}\}$ be as follows:

$$\begin{aligned}&c_1 P_{s_1} c_2 P_{s_1} c_3 P_{s_1} \emptyset \qquad & f^{s_1} &= (f^{s_1}_{t_1}, f^{s_1}_{t_2}, f^{s_1}_{t_3}) = (80, 70, 90)\\ &c_1 P_{s_2} c_2 P_{s_2} c_3 P_{s_2} \emptyset \qquad & f^{s_2} &= (f^{s_2}_{t_1}, f^{s_2}_{t_2}, f^{s_2}_{t_3}) = (90, 80, 70)\\ &c_3 P_{s_3} c_2 P_{s_3} c_1 P_{s_3} \emptyset \qquad & f^{s_3} &= (f^{s_3}_{t_1}, f^{s_3}_{t_2}, f^{s_3}_{t_3}) = (70, 90, 80)\end{aligned}$$

These scores make following rankings in categories t_1, t_2 and t_3:

$$\begin{array}{lllll} t_1 & : & s_2 & s_1 & s_3 \\ t_2 & : & s_3 & s_2 & s_1 \\ t_3 & : & s_1 & s_3 & s_2 \end{array}$$

The final result is:

$$\begin{aligned} \mu(s_1) &= (c_3) \\ \mu(s_2) &= (c_1) \\ \mu(s_3) &= (c_2) \end{aligned}$$

Now, assume student 1 announced her preferences as $c_1 P_{s_1} c_2 P_{s_1} \emptyset P_{s_1} c_3$. According to new preference lists the setup will be:

$$\begin{array}{llll} c_1 P_{s_1} c_2 P_{s_1} \emptyset P_{s_1} c_3 & f^{s_1} & = & (f^{s_1}_{t_1}, f^{s_1}_{t_2}, f^{s_1}_{t_3}) = (80, 70, 90) \\ c_1 P_{s_2} c_2 P_{s_2} c_3 P_{s_2} \emptyset & f^{s_2} & = & (f^{s_2}_{t_1}, f^{s_2}_{t_2}, f^{s_2}_{t_3}) = (90, 80, 70) \\ c_3 P_{s_3} c_2 P_{s_3} c_1 P_{s_3} \emptyset & f^{s_3} & = & (f^{s_3}_{t_1}, f^{s_3}_{t_2}, f^{s_3}_{t_3}) = (70, 90, 80) \end{array}$$

At the end of the first stage, s_3 will choose c_3. So, final result will be:

$$\begin{aligned} \mu(s_1) &= (c_2) \\ \mu(s_2) &= (c_1) \\ \mu(s_3) &= (c_3) \end{aligned}$$

The example above shows that 2SSS-GS is not strategy-proof which means it is open to be manipulated. The first student manipulated the mechanism by announcing different preferences over schools. So, at the end of the placement student 1 matched to her better choice. Next assume, $f^{s_1} = (80, 70, 70)$ instead of $f^{s_1} = (80, 70, 90)$. Now, the outcome in the first stage becomes:

$$\begin{aligned}
\mu_1(s_1) &= (\emptyset, \emptyset) \\
\mu_1(s_2) &= (c_1, \emptyset) \\
\mu_1(s_3) &= (c_2, c_3)
\end{aligned}$$

At the end of the first stage s_3 accepts c_3 and does not enter the second stage, since c_3 is her top choice. Therefore, final outcome will be:

$$\begin{aligned}
\mu(s_1) &= (c_2) \\
\mu(s_2) &= (c_1) \\
\mu(s_3) &= (c_3)
\end{aligned}$$

By this new scores, s_1 assigned her higher choice. So, when she improved her scores, 2SSS-GS makes her worse off. Hence, the system does not respect improvement.

As we see above, due to second stage and segmented market structure, GS mechanism fails to satisfy non-wastefulness, efficiency, strategy-proofness and respecting improvements. Next, I ask the following question: Given the segmented matching environment is there any mechanism that makes the placement system satisfy the five properties? The proposition below will demonstrate the difficulty of our problem better, since to satisfy fairness and non-wastefulness, one should sacrifice efficiency, strategy-proofness and respecting improvements.

Proposition 8: In a 2 stage segmented matching environment, there is no fair and non-wasteful mechanism that is used in stage 1 and stage 2 consecutively, and at the same time efficient or strategy proof or respecting improvement.

Proof: Let $S = \{s_1, s_2, s_3\}$, $C = \{c_1, c_2, c_3\}$ where $c_1, c_2 \in C_{st}$ and $c_3 \in C_{pr}$,

$q = \{q_1, q_2, q_3\} = (1, 1, 1)$, $T = \{t_1, t_2, t_3\}$, $t(c_1) = t_1$, $t(c_2) = t_2$, $t(c_3) = t_3$. Let preferences $R_S = \{R_{s_1}, R_{s_2}, R_{s_3}\}$ and the test scores $f = \{f^{s_1}, f^{s_2}, f^{s_3}\}$ be as follows:

$$\begin{array}{llll} c_1 P_{s_1} c_2 P_{s_1} c_3 P_{s_1} \emptyset & f^{s_1} & = & (f^{s_1}_{t_1}, f^{s_1}_{t_2}, f^{s_1}_{t_3}) = (90, 70, 70) \\ c_2 P_{s_2} c_1 P_{s_2} c_3 P_{s_2} \emptyset & f^{s_2} & = & (f^{s_2}_{t_1}, f^{s_2}_{t_2}, f^{s_2}_{t_3}) = (70, 80, 90) \\ c_3 P_{s_3} c_2 P_{s_3} c_1 P_{s_3} \emptyset & f^{s_3} & = & (f^{s_3}_{t_1}, f^{s_3}_{t_2}, f^{s_3}_{t_3}) = (80, 90, 80) \end{array}$$

These scores make following rankings in categories t_1, t_2 and t_3:

$$\begin{array}{lllll} t_1 & : & s_1 & s_3 & s_2 \\ t_2 & : & s_3 & s_2 & s_1 \\ t_3 & : & s_2 & s_3 & s_1 \end{array}$$

The only outcome that satisfies fairness and non-wastefulness is:

$$\begin{array}{lll} \mu_1(s_1) & = & (c_1, \emptyset) \\ \mu_1(s_2) & = & (\emptyset, c_3) \\ \mu_1(s_3) & = & (c_2, \emptyset) \end{array}$$

At the end of stage 1, since all students have only one assigned school, there will not be any rejected school. In the second stage, the set of students who participate may be one of the 8 possible student set. The only two possible fair and non-wasteful outcomes for the second
stage are listed below. But both μ' and μ'' are dominated by
$\mu^* = (\mu^*(s_1), \mu^*(s_2), \mu^*(s_3)) = (c_1, c_2, c_3)$. So there is no fair and non-wasteful

mechanism that is efficient.

For $S_2 \in \{\{\emptyset\}, \{s_1\}, \{s_3\}, \{s_1, s_3\}\}$ For $S_2 \in \{\{s_2\}, \{s_1, s_2\}, \{s_2, s_3\}, \{s_1, s_2, s_3\}\}$

$$\begin{aligned} \mu'(s_1) &= (c_1) \qquad \mu''(s_1) = (c_1) \\ \mu'(s_2) &= (c_3) \qquad \mu''(s_2) = (\emptyset) \\ \mu'(s_3) &= (c_2) \qquad \mu''(s_3) = (c_2) \end{aligned}$$

Next I will show that the mechanism is open to manipulation. In the same setup, assume that s_2 announces a different preference list,

$$c_2 P_{s_2} c_1 P_{s_2} \emptyset P_{s_2} c_3$$

With new preference profile, the only fair and non-wasteful outcome in the first stage is:

$$\begin{aligned} \mu_1(s_1) &= (c_1, \emptyset) \\ \mu_1(s_2) &= (\emptyset, \emptyset) \\ \mu_1(s_3) &= (c_2, c_3) \end{aligned}$$

At the end of the first stage s_3 accepts c_3 and does not enter the second stage, since c_3 is her top choice. Therefore, in the second stage only possible set of non-wasteful and fair outcome is:

For $S_2 \in \{\{\emptyset\}, \{s_1\}, \{s_2\}, \{s_1, s_2\}\}$

$$
\begin{aligned}
\mu(s_1) &= (c_1) \\
\mu(s_2) &= (c_2) \\
\mu(s_3) &= (c_3)
\end{aligned}
$$

So, s_2 can manipulate the system by announcing a different preference list. Hence, there is no fair and non-wasteful mechanism that is strategy-proof.

Finally, I show that any fair and non-wasteful mechanism does not respect improvements in 2 stage segmented matching environment. In the original example, now assume that $f^{s_2} = (70, 80, 70)$ instead of $f^{s_2} = (70, 80, 90)$. The only fair and non-wasteful outcome in the first stage becomes:

$$
\begin{aligned}
\mu_1(s_1) &= (c_1, \emptyset) \\
\mu_1(s_2) &= (\emptyset, \emptyset) \\
\mu_1(s_3) &= (c_2, c_3)
\end{aligned}
$$

At the end of the first stage s_3 accepts c_3 and does not enter the second stage, since c_3 is her top choice. Therefore, the only possible outcome will be:

$$\text{For } S_2 \in \{\{\emptyset\}, \{s_1\}, \{s_2\}, \{s_1, s_2\}\}$$

$$
\begin{aligned}
\mu(s_1) &= (c_1) \\
\mu(s_2) &= (c_2) \\
\mu(s_3) &= (c_3)
\end{aligned}
$$

By this new scores, s_2 assigned her higher choice. So, when she improved her scores, no fair and non-wasteful mechanism can guarantee her a better school. Hence, there is no fair and non-wasteful mechanism that respects improvement.

CHAPTER 5

2-STAGE SEGMENTED SYSTEM UNDER RESTRICTED PREFERENCE DOMAINS

Chapters 3 and 4 show that $2SSS - MSD$ has serious deficiencies and there does not exist any fair and non-wasteful placement mechanism to use instead of MSD mechanism to improve the system's deficiencies. Since I have found an impossibility result for the universal set of preferences, in this section I will study two restrictions, namely blocked preferences and common preferences restrictions. The former restriction is defined as students preferring any private school to any state school. The latter one implies students having identical preference lists. Let the set of all preference profiles satisfying blocked preference restriction be $\Re^{bp} \subset \Re$ and let the set of all preference profiles satisfying common preference restriction be $\Re^{c} \subset \Re$.

When compared to most of the state schools, it can be said that private schools offer broader opportunities for students like higher education quality, an international profile, and well established facilities etc. This fact makes the Blocked Preferences restriction legitimate for Turkey.

Statistics over the years show that minimum scores and quality of the schools do not show big differences. As a result, it is not illogical to assume that students have common preferences. Based on this assumption, we can improve the placement system by increasing the number of stages, allowing the private schools to enter all the stages as well as state schools and using GS algorithm instead of MSD.

I show that under these restricted preference profiles segmented system with GS is fair, non-wasteful, strategy-proof, respects improvements and is the most efficient mechanism among all stable mechanisms.

Blocked Preferences

Definition 7: A preference profile satisfies blocked preferences if $\forall s \in S$ and $\forall c, c' \in C$, $t(c) = t_3$ and $t(c') \in \{t_1, t_2\}$ implies cP_sc'.

Let $\mu^{GS}(s)$ be Gale-Shapley Student Optimal DA outcome for student s and let $\mu^{2GS}(s)$ be 2-Stage Segmented System with Gale-Shapley Student Optimal DA outcome for student s. Let's call the set of students who get an offer from a private school under $2SSS - GS$, S_1. Also let $C_{pr} = \{c \in C : t(c) = t_3\}$. Under this constraint any student with a private school offer will be placed to that school. Because they prefer any private school to state schools and since it is not possible to be assigned to a better private school, mechanism do not allow them to attend in the second stage.

Lemma 1: For any economy (R_s, f, q) such that $R_S \in \Re^{bp}$, $\nexists s \in S_1$ such that $\mu^{GS}(s) \notin C_{pr}$.

Proof: Assume $\exists s_i \in S_1$, such that $\mu^{GS}(s_i) \notin C_{pr}$. Then $\exists s_j \in S \setminus S_1$, such that $\mu^{GS}(s_j) \in C_{pr}$, due to non-wastefulness of μ^{GS}. So, $(s_i, \mu^{GS}(s_j))$ is a blocking pair since $f^{s_i}_{t(\mu^{GS}(s_j))} > f^{s_j}_{t(\mu^{GS}(s_j))}$ and $\mu^{GS}(s_j)P_{s_i}\mu^{GS}(s_i)$. This is a contradiction for stability of μ^{GS}. Hence, $\nexists s \in S_1$, such that $\mu^{GS}(s) \notin C_{pr}$. So, S_1 is also the set of students who receive an offer from a private school under Gale-Shapley Student Optimal DA.

Lemma 2: For any economy (R_s, f, q) such that $R_S \in \Re^{bp}$, $\forall s \in S_1$, $\mu^{GS}(s) = \mu^{2GS}(s)$.

Proof: Let $S' = S_1 \setminus \{s : \mu^{GS}(s) = \mu^{2GS}(s)\}$ and $C' = C_{pr} \setminus \{c : \mu^{GS}(c) = \mu^{2GS}(c)\}$. Assume that S' and C' are non-empty. Since Gale-Shapley Student Optimal DAA that we used in the first stage is efficient, $\forall s \in S', \mu^{2GS}(s)R_s\mu^{GS}(s)$. Now define μ' as:

$$\mu'(s) = \begin{cases} \mu^{GS}(s) & s \in S \setminus S' \\ \mu^{2GS}(s) & s \in S' \end{cases}$$

But $\forall s \in S$, $\mu'(s) R_s \mu^{GS}(s)$ contradicts with efficiency of Gale-Shapley Student Optimal DAA. Hence, $\forall s \in S_1$, $\mu^{GS}(s) = \mu^{2GS}(s)$.

Now let C_{st} be set of state schools and S_2 be set of students placed a state school under $2SSS - GS$, formally; $C_{st} = C \setminus C_{pr}$, $S_2 = S \setminus (S_1 \cup \{s : \mu^{2GS}(s) = \emptyset\})$.

Lemma 3: For any economy (R_s, f, q) such that $R_S \in \Re^{bp}$, $\nexists s \in S_2$, such that $\mu^{GS}(s) \notin C_2$.

Proof: Assume $\exists s_i \in S_2$, such that $\mu^{GS}(s_i) \notin C_{st}$. Then $\exists s_j \in S \setminus (S_1 \cup S_2)$, such that $\mu^{GS}(s_j) \in C_{pr}$, due to Lemma 1 and non-wastefullness of μ^{GS}. So, $(s_i, \mu^{GS}(s_j))$ is a blocking pair since $f^{s_i}_{t(\mu^{GS}(s_j))} > f^{s_j}_{t(\mu^{GS}(s_j))}$ and $\mu^{GS}(s_j) P_{s_i} \mu^{GS}(s_i)$. This is a contradiction for stability of μ^{GS}. Hence, $\nexists s \in S_1$, such that $\mu^{GS}(s) \notin C_{pr}$.

So, S_1 is also the set of students who receive an offer from a private school under Gale-Shapley Student Optimal DA.

Lemma 4: For any economy (R_s, f, q) such that $R_S \in \Re^{bp}$, $\forall s \in S_2$, $\mu^{GS}(s) = \mu^{2GS}(s)$.

Proof: Let $S'' = S_2 \setminus \{s : \mu^{GS}(s) = \mu^{2GS}(s)\}$ and $C'' = C_{st} \setminus \{c : \mu^{GS}(c) = \mu^{2GS}(c)\}$. Assume that S'' and C'' are non-empty. Since GS that we used in the second stage is efficient, $\forall s \in S'', \mu^{2GS}(s) R_s \mu^{GS}(s)$. Now define μ'' as:

$$\mu''(s) = \begin{cases} \mu^{GS}(s) & s \in S \setminus S'' \\ \mu^{2GS}(s) & s \in S'' \end{cases}$$

But $\forall s \in S$, $\mu''(s) R_s \mu^{GS}(s)$ contradicts with efficiency of GS. Hence, $\forall s \in S_2$, $\mu^{GS}(s) = \mu^{2GS}(s)$.

These four lemmas will help to prove the following proposition.

Proposition 9: In any student placement problem (R_S, f, q) where $R_S \in \Re^{bp}$, 2-Stage Segmented System with Gale-Shapley Student Optimal DA Algorithm is equivalent to Gale-Shapley Student Optimal DA Algorithm..

Proof: Four lemmas above show that the outcome of GS and $2SSS - GS$ are the same. Therefore, these two algorithms are equivalent to each other.

Next, I should check the properties of the system. Corollary 1 shows that under blocked preferences $2SSS - GS$ satisfies properties (F), (NW), (E), (RP), (SP) I defined before.

Corollary 1: Under blocked preferences restriction $2SSS - GS$ satisfies fairness, non-wastefulness, strategy proofness, respecting improvements and is the most efficient stable mechanism.

The main argument depends on the equivalence of the $2SSS - MSD$ and GS mechanisms under blocked preferences. It is proven by Balinski and Sonmez (1999) that GS satisfy fairness, non-wastefulness, strategy proofness, respecting improvements and is the most efficient stable mechanism. Hence under blocked preferences constraint $2SSS - GS$ satisfy all 5 properties.

Common Preferences

Definition 8: A preference profile satisfies common preferences if $\forall s, s' \in S$ and $\forall c, c' \in C$, $cP_sc' \iff cP_{s'}c'$.

The following system is called n-Stage Segmented System with Gale-Shapley Student Optimal Deferred Acceptance Algorithm ($nSSS - GS$, hereafter):

System runs for $n \geq m$ stage. In each stage, system assigns students in the state and private markets by using GS independently. At the end of the each stage, students with more than one offer accept at most one and reject rest of the offers. When a student accepts her offer she is placed to that school and leaves the system.

Let preferences of students, without loss of generality, be $c_1P_sc_2P_s \dots P_s$ $c_{m-1}P_sc_m$ and let $\forall c_i \in C$, $C_i = \{c_i, \dots, c_{k_i}\}$ where $k_i = \sup\{j : \forall a$ st. $i \leq a \leq j, t(c_a) = t(c_i)\}$.

It is obvious that, in the first stage, for any student who receives an offer from a school in C_1, mechanism places them to their match in the first stage and they left the system. In the second stage, for students with an offer from $c \in C_{k_1+1}$ mechanism places them to their match in the second stage and so on. Since I assumed that $|S| > \sum_{i=1}^{m} q_{c_i}$ before and $n \geq m$, the system continues until every acceptable school fills their quotas.

Proposition 10: Under common preferences constraint, $nSSS - GS$ is equivalent to the GS.

Proof: First for any acceptable school c, $\forall s \in S$, $cP_s\mu(s)$ implies $|\mu^{-1}(c)| = q_c$. Hence $nSSS - GS$ satisfies property NW. Secondly, assume $nSSS - GS$ does not satisfy F. Then, $\exists s, s' \in S$ and $\exists c \in C$, s.t. $s' \in \mu(c)$, $cP_s\mu(s)$ but $f^s_{t(c)} < f^{s'}_{t(c)}$. Let b be the stage number that c filled its quota. If s accepted an offer before stage b then $\mu(s)P_sc$. In stage b, since we applied GS (which satisfies F), matching between s' and c means that s has an offer from a school at least as good as c. So this contradicts with property F of GS mechanism. Hence $nSSS - GS$ satisfies property F. Since $\forall c \in C$ any student is acceptable, and in any stage GS assigns students to an acceptable schools ($nSSS - GS$ immune to individual blocking) and $nSSS - GS$ satisfies both NW and F (also immune to pairwise blocking), $nSSS - GS$ is stable. It is obvious that due to common preferences constraint there is a unique stable outcome which coincides with GS outcome. Hence, $nSSS - GS$ is equivalent to GS under common preferences restriction.

Like the common preference restriction, it is easy to check the properties of the system.

Corollary 2: Under common preferences restriction $nSSS - GS$ satisfies fairness, non-wastefulness, strategy proofness, respecting improvements and is the most efficient stable mechanism.

The intuition behind Corollary 2, like Corollary 1, is the equivalence of $2SSS - MSD$ and GS mechanisms. Hence if student preference profile satisfies common preferences restriction one can improve the placement system's properties by using GS mechanism instead os MSD while increasing the number of stages and allowing private schools to enter all stages.

CHAPTER 6

DO PRIVATE SCHOOLS BENEFIT FROM SEGMENTED SYSTEM?

The analysis above shows that the segmented structure of the market is the main reason for the failure of the finding the desirable placement mechanism. In Turkey by law, the state school placement is done by centrally by the Ministry of National Education. However, private schools chose to participate in the system or not. Over the years private schools have preferred to place their students independently, creating the segmented market structure in the secondary school market problem. As a result, the following question occurs in mind: Why does the private schools keep using this segmented system? One of the possible rationality behind this segmentation is that the segmented system increases the quality of student intake for the private schools. In this section, I show that the segmentation provides some private schools with higher qualified students while making some of them loses their students.

Let μ^{MSD} be the outcome of MSD mechanism (mechanism with one non-segmented market), and let μ^{2MSD} be outcome of $2SSS-MSD$. The example below highlights the above observation.

Example 2: Let $S=\{s_1,s_2,s_3,s_4\}$, $C=\{c_1,c_2,c_3,c_4\}$ where $c_1,c_2\in C_{st}$ and $c_3,c_4\in C_{pr}$, $q=\{q_1,q_2,q_3,q_4\}=(1,1,1,1)$, $T=\{t_1,t_2,t_3\}$, $t(c_1)=t_1$, $t(c_2)=t_2$, $t(c_3)=t(c_4)=t_3$. Let preferences $R_S=\{R_{s_1},R_{s_2},R_{s_3},R_{s_4}\}$ and the test scores $f=\{f^{s_1},f^{s_2},f^{s_3},f^{s_4}\}$ be as follows:

$$\begin{array}{lrcl}
c_1P_{s_1}c_3P_{s_1}c_4P_{s_1}c_2P_{s_1}\emptyset & f^{s_1} & = & (f^{s_1}_{t_1},f^{s_1}_{t_2},f^{s_1}_{t_3})=(90,70,90)\\
c_3P_{s_2}c_1P_{s_2}c_2P_{s_2}c_4P_{s_2}\emptyset & f^{s_2} & = & (f^{s_2}_{t_1},f^{s_2}_{t_2},f^{s_2}_{t_3})=(80,80,80)\\
c_3P_{s_3}c_4P_{s_3}c_1P_{s_3}c_2P_{s_3}\emptyset & f^{s_3} & = & (f^{s_3}_{t_1},f^{s_3}_{t_2},f^{s_3}_{t_3})=(60,60,60)\\
c_2P_{s_4}c_3P_{s_4}c_2P_{s_4}c_1P_{s_4}\emptyset & f^{s_4} & = & (f^{s_4}_{t_1},f^{s_4}_{t_2},f^{s_4}_{t_3})=(70,90,70)
\end{array}$$

These scores make following rankings in categories t_1, t_2 and t_3:

$$
\begin{aligned}
t_1 &: s_1 \quad s_2 \quad s_4 \quad s_3 \\
t_2 &: s_4 \quad s_2 \quad s_1 \quad s_3 \\
t_3 &: s_1 \quad s_2 \quad s_4 \quad s_3
\end{aligned}
$$

The final result of 2SSS-MSD is:

$$
\begin{aligned}
\mu^{2MSD}(c_1) &= (s_1) \\
\mu^{2MSD}(c_2) &= (s_4) \\
\mu^{2MSD}(c_3) &= (\emptyset) \\
\mu^{2MSD}(c_4) &= (s_2)
\end{aligned}
$$

Now, assume MSD mechanism is used instead of 2SSS-MSD. Now the final result is:

$$
\begin{aligned}
\mu^{MSD}(c_1) &= (s_1) \\
\mu^{MSD}(c_2) &= (s_4) \\
\mu^{MSD}(c_3) &= (s_2) \\
\mu^{MSD}(c_4) &= (s_3)
\end{aligned}
$$

The example above shows that $2SSS - MSD$ makes c_4 better off since the system assigns s_2 instead of s_3 who has the lowest score. On the other hand, $2SSS - MSD$ makes c_3 worse off, since no one is assigned to that school while MSD mechanism assigns an acceptable student s_2.

CHAPTER 7

CONCLUSION

In this thesis, I studied secondary school placement system in Turkey. I first showed that while the secondary school placement system satisfies fairness, it does not satisfy four important criteria namely; non-wastefulness, strategy proofness, efficiency, respecting improvements. I then analyzed improvements for the deficiencies of the system. I proved that there exist no fair and non-wasteful mechanism to use in the system that makes the placement system efficient or strategy proof or respecting improvements.

Next, I introduced two restrictions on students' preferences; blocked preferences and common preferences. Then I showed that under blocked preferences, 2-stage segmented system can be improved by using Gale-Shapley Student Optimal DAA instead of MSD mechanism in each market. I also showed that under common preferences, if one use nSSS-GS instead of MSD mechanism then the outcome of the system becomes equivalent to GS outcome. The final result that I found is that changing the system from MSD to 2SSS-MSD increases student quality of some private schools while decreasing student quality of other private schools.

The analysis above shows that the segmented structure of the market is the main reason for the failure of the finding the desirable placement mechanism. I have also show that segmented system is not helping out all of the private schools to get matched with better students. Hence the private schools have no incentive to manipulate the placement outcome by separating their market from that of state schools. In fact it has been argued by the Private Schools Association that the tests offered by the Ministry of National Education (OKS and SBS) which is used to rank the students in the placement is not well-designed for their needs. Hence, they prefer to stay out from the system. However, the issue here is not how to rank the students but how to place them. For the system to have desirable properties the students should be placed any type of schools within the same markets and such a system can accommodate schools ranking students with different test scores.

In a market where private and state schools ranking students according to different test scores, Gale-Shapley Student Proposing Deferred Acceptance Algorithm is the best available mechanism. GS mechanism as the best mechanism is not only easily applicable, but also it is the only fair and non-wasteful mechanism that satisfies strategy proofness (Alcalde and Barberà, 1994) or respecting improvements (Balinski and Sonmez, 1999).

The properties I proposed as properties of a desirable placement mechanism are not only theoretical contentions, they have sensible motives. A desirable mechanism does not punish student because of increasing her test score and should reward revealing her true preferences. Hence strategy proofness and respecting improvement properties are necessary in practice too. Also distributing school seats effectively and treating students equally justify efficiency, non-wastefulness and fairness.

The findings in this study demonstrated the instrumental role of integration of the school markets and placement mechanisms in increasing students' welfare. Further studies may demonstrate either accuracy of my restrictions or increase in welfare by integrating two school markets by simulations of student data. Also, using imperfect information setting at the end of first stage placement is worth to study.

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