DO BUBBLES SPILL OVER?

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2010

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Thesis submitted to the

Institute for Graduate Studies in the Social Sciences

in partial fulfillment of the requirements of for the degree of

Master of Arts

In

Economics

By

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Bogaziçi University

2010

Do Bubbles Spill Over?

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June 2010

Thesis Abstract

Osman Onur Uyar, “Do Bubbles Spill Over? “

We document the existence of rational bubbles in emerging markets by employing a structural state space model. The high correlation of stock price indices among a relatively large number of emerging markets indicates rational bubbles might spill over. We employ a newly developed Unscented Kalman Filtering technique to estimate the rational bubbles in stock markets. The bubbles mentioned here are assumed to be stochastic and feature time-variable parameters. Most of the variations of the stock prices which include rational bubbles in various sizes are captured by the model.

Tez Özeti

Osman Onur Uyar, “Bubblelar pazarlar arası geçiş yapar mı?“

Rasyonel bubbleların gelişmekte olan ülke pazarlarındaki varlığını model uygulayarak dokumante edilmiştir. Birçok gelişmekte olan ülke pazarlarının hisse senedi endekslerinin kendi içindeki yüksek korelasyonu bubble'ların geçiş yapabileceği ihtimalini doğurdu. Rasyonel Bubbleların tahmin edilebilmesi için yeni geliştirilen Unscented Kalman Filtresi kullanılmıştır. Modelin zaman içinde oluşan bir çok değişkenliği yakalayabildiği görülmüştür.

ACKNOWLEDGMENTS

My special thanks go to my thesis advisor Yrd. Doç. Dr.Ozan Hatipoğlu, who kept me focused on the task at hand. He has been a source of inspiration and a guiding light. I am sincerely grateful to all the economics faculty members for their kind supports. Furthermore, I shall be forever indebted to my dear wife Ayten Gülin Uyar, who showed confidence even through times when I had little faith in myself. Without her help I could never have balanced my life, faced with studies and work.

Finally, I offer special thanks to my parents, my sister and my wife, who encouraged me in every field of life and tried to help whenever I needed.

CONTENTS

CHAPTER 1

INTRODUCTION

Financial markets are becoming more integrated which cause frequent comovements of stock price indices in both emerging and developed countries (Shiller, 1989; Thomas W.Epps, 1979; Bekaert G., Hodrick J. R. and Zhang X., 2009; Morana C., 2008) We also observe high correlation of stock price indices among emerging markets (Martin Scheicher, 2001). Since fundamentals differ substantially between emerging markets and developed nations the frequent comovements and high correlation of asset prices might be due to factors that cannot be explained by the fundamentals and it is important to identify the source of this correlation.

In an efficient market, it is expected that the present value of the expected future dividends determines the fundamental value of the share only if the stock prices are realized with respect to the response to new information about change in fundamentals. If the investors purchase stocks only because of the future dividend expectations, this means that, the fundamentals are the driver factors for the stock prices. If the market dynamics are changing due to the non-fundamental speculative factors, the stock price does not represent its fundamental value. This divergence from the fundamental value of a stock price is called rational bubble.

The disastrous experiences of the 1996-2000 Internet Bubbles Burst or the 1987 US Stock Market Crash are cases in point. The origins and formations of rational bubbles have been extensively analyzed by both academic and finance professionals regarding their implications for monetary and regulatory policies. Although there have been many literature studies which addresses this issue, the exploration and tests of rational speculative bubbles are, however, still challenging for several reasons. First, the correct detection of market bubble is a difficult task mainly because of the uncertainty related to the determination of the fundamental value of a security. Second, the test of rational speculative bubbles can become very complicated since they might exist even with rational investors and take all kinds of shapes (Blanchard, 1979). Thirdly, the possible stochastic feature of market fundamentals makes harder the detection of rational bubbles.

Although the integration of markets with each other results in a bigger consumption basket for all of the individuals, it also enables transferring of the positive and negative effects of market sentiments to each other via the closed chains of asset positions.

The bubble spill-overs are in fact a product of the shifts in the chains of asset positions in between the agents of the markets. Because of that the bubbles spill over to the other markets i.e. via the shifts of asset positions in between the markets. This causes the bubbles to float on more than one market agents' preferences and the spillover of the bubbles to other markets.

CHAPTER 2

LITERATURE REVIEW

A number of authors (Campbell and Shiller, 1988; Diba and Grossman, 1988; Timmermann, 1995; Nasseh and Strauss, 2003; Koustas and Serletis, 2005; Cunado et al 2005) have investigated the rational bubbles in a number of developed markets by investigating the relation of stock prices and dividends. However, Blanchard and Watson (1982) define rational bubbles slightly differently as self-fulfilling expectations that push stock prices towards expected price level, which is unrelated to changes in the fundamentals of the stock price. In addition, some economists attribute rational bubbles to the presence of a large number of investors reacting simultaneously to new information so that an overreaction in aggregate is created.

The existence of speculative bubbles in the stock markets has always been an obstacle to the validity about the consistency of bubbles with the rationality assumption on theoretical grounds. Empirically, partly because of the lack of power of testing procedures, a general specification test for stock market bubbles does not give exact results. For example, Rappoport and White [1993; 1994] and West [1987] reject the null hypothesis of no bubbles, while Dezhbakhsh and Demirguc- Kunt [1990] and Diba and Grossman [1988b] report the opposite results. Flood and Garber [1980], Hamilton and Whiteman [1985] and Hamilton [1986] criticize these bubble tests such that bubbles are observationally in accordance with the regime changes in market fundamentals which cannot be observed by the econometrician. Furthermore, Evans [1991] showed by Monte-Carlo simulations that an important class of rational bubbles cannot be determined by these tests even though the variability of bubbles is explosively high.

There are two versions of the rational markets theory. The fundamental value analysis version allows current stock prices to be temporarily above or below their equilibrium (Intrinsic value) levels. According to the efficient markets hypothesis, stock prices are always at their intrinsic value levels because the market captures all the information.

Intrinsic values are discounted values of expected future profits and competitive market forces automatically move stock prices to equilibrium. Speculative markets theories of Keynes and Galbraith reject the notion that market forces move stock prices toward intrinsic values rather complex psychological and institutional factors moves.

In the aftermath of the 1987 crash, it is started to be argued that there can be “rational” bubbles. A “rational” bubble can occur if ‘rational’ agents believe there is a probability of p of a positive deviation from ‘intrinsic’ value in the next period’s price (Glickman, 1994, p. 339).

In The General Theory, Keynes explains that the stock prices are an important factor in the theory of aggregate demand through their influence on the levels of consumption and investment. The wealth effect of rising stock prices increases the marginal propensity to consume (Keynes, 1936, p. 319). Rising stock prices affects the investment in the same manner as a decrease in the interest rate or an increase in the marginal efficiency of capital.

Keynes’ explanation of speculative markets stock prices includes three major factors: human nature, the problem of intractable uncertainty about the economic future and the institutional features of modern stock exchanges which are reasonably well-organized and orderly spot markets with low transactions costs.

Davidson has explained that the neoclassical efficient markets theory depends on the ergodic axiom, which implies that the future must be statistically reliably calculated from past and present market data (2002, p. 43). Keynes rejected the ergodic axiom and recognized that the economic processes are nonergodic in defining uncertainty as meaning that “we simply do not know” about the future (Davidson, 2002, p. 52; Keynes, 1937, p. 214).

In line with Davidson’s explanation, Greenspan–Bernanke doctrine on stock market bubbles says that:

“Stock market bubbles do occur but cannot be detected until after they burst, and perhaps not even then.”

Regarding the past studies, it can be concluded that, divergent findings have been produced about the rational bubbles. Some of the studies that found evidence of rational bubbles comprise, among others, McQueen and Thorley (1994), Cunado et al. (2005), and Engsted (2006), whereas the hypothesis of rational bubbles is rejected in Wu (1995), Chan et al. (1998), and Koustas and Serletis (2005).

CHAPTER 3

RATIONAL BUBBLES MODELS

According to the literature, existing rational bubbles models can be classified into two main classes: models of exogenous bubbles and endogenous bubbles. While the first class of models treats bubbles independently from changes in asset's fundamental value fluctuations, the second class does take into account the impact of changes in fundamentals on the process of bubble formation.

Exogenous bubbles can be divided into deterministic, stochastic and periodically collapsing bubbles. First analyzed in Blanchard and Watson (1982), the deterministic bubble is simply modeled using an exponential function of time, as shown in Equation (1):

$$B_{t+i} = B_t(1+r)^i \qquad (1)$$

Where B_t is referred to as a rational bubble and r is the rate of return on a risk-free asset.

Under this structure, a rational bubble is greatly amplified through time, leading to an explosive divergence between stock price and its fundamental value. But, the model is unrealistic because it implicitly assumes a perpetual growth of stock prices. This result, leads Blanchard and Watson (1982) to introduce stochastic bubbles with a probability of bursting.

According to their specification, once the bubbles exist, they have an exponential growth, but they are likely to burst over the period. If they burst at a given time, their reformulation is not possible. Moreover, these rational bubbles can only exist, if they do, at the time of stock issuance. They grow, then collapse, and finally disappear (Diba and Grossman, 1988). In a related study, Evans (1991) defined a new class of bubbles, called periodically collapsing bubbles, which can deflate without bursting and grow again thereafter. The periodically collapsing bubbles can have multiple regimes due to its construction.

Second type of bubbles is endogenous bubbles which essentially include intrinsic and state bubbles. Major studies on testing for endogenous bubbles in asset returns and prices mainly point out that market fundamentals have significant effects on both the stock's fundamental value and bubble formation.

When market fundamentals change, stock price can overreact because the bubble term effects the price movement. In addition, important divergences can be created by the intrinsic bubbles as well as they can remain stable over certain periods depending on the firm's dividend policy. Obviously, the intrinsic bubbles model, with a certain possibility, explains why stock prices are highly volatile compared with the dividends, as pointed out by Shiller (1981).

Second, a state bubbles model has been developed in discrete time which differs from that of Froot and Obstfeld (1991), in the sense that rational bubbles depend both on time and dividends paid.

Empirical Methods

According to the literature, to test the existence of the bubbles two sets of tests—both direct and indirect tests of rational bubbles— can be applied.

Indirect Tests of Stock Market Bubbles

As far as the indirect tests are concerned, the stationarity test developed by Dickey and Fuller (1981) and by Phillips and Perron (1988), and the cointegration test in the sense of Engle and Granger (1987) can be used to detect asset bubbles. The same procedure was employed by Diba and Grossman (1987 and 1988), and Hamilton and Whiteman (1985), among others.

Diba and Grossman (1988) show that in the absence of rational bubbles, dividend and stock price series are cointegrated. It then follows that the cointegration technique can be used to prove the existence of bubbles if they do exist.

Direct Tests of Stock Market Bubbles

Unlike the indirect tests, the use of direct tests in detecting asset bubbles requires a complete specification and estimation of economic parameters. Using direct tests, past studies have showed that market bubbles do exist when asset prices have not deviated from the real economic conditions (see, e.g., Flood and Garber, 1980; Shiller, 1981; and West, 1987). Therefore, models of stock market bubbles related to this research stream compare the observed stock prices with the prices that should be based on the fundamentals. An observation of significant difference in between these shows that the hypothesis of asset bubbles cannot be rejected.

Related Assumptions

Despite these historical findings, this thesis specifies and estimates one type of rational bubble using Unscented Kalman Filtering Technique. A brief description of

Kalman Filter and detailed information about the Unscented Kalman Filter is already given in the Appendix.

The bubbles mentioned here are assumed to be stochastic and feature time-variable parameters. The state space model defined by Yangru Wu (1997) is used as the basis for the development of new model. But the difference from his model assumptions is that we decided to indicate some of the parameters to be time-variable. As a result of this our model becomes the nonlinear version of the state space model defined by Yangru Wu (1997).

The bubbles are assumed to be stochastic and features time variable parameters. Bubbles are set as they can either be positive or negative. While in some bull markets, where the participants are eager to buy, the stocks may be overvalued, there may be times that in some bear markets the stocks may be undervalued. As a result of this the non-negativity constraint is not imposed to the model estimation. So the bubbles are left free of having only positive values. A general ARIMA (p, 1, q) process is assumed for the log values of the dividends. The stock prices, bubble parameter, dividend ARIMA process and the bubble formations result the structural state-space models for the Kalman Filtering Model.

The remainder of the paper is organized as follows: In the next section the data used in the Kalman filter iterations is introduced. In the following three sections, the model is described, the results are explained and the conclusions are discussed respectively. In the appendix, the Unscented Kalman Filter concept is explained in more detail.

CHAPTER 4

DATA

The data employed for USA in this paper have been taken from the Shiller’s online web page. Real stock prices are the nominal Standard and Poor's (S&P) 500 indexes, deflated by the Consumer Price Index (CPI). Real dividends are the nominal dividends for the S&P deflated by the CPI. The data employed for Turkey and for the World has been taken from DataStream database. All data is consolidated on monthly basis.

CHAPTER 5

MODEL

The model of Yangru Wu has been taken as the basis in order to forecast the bubble amounts. The bubble considered by Yangru Wu is treated as an unobserved state vector in the state-space model. This model is extended by specifying some parameters as time–variables. So the new model is nonlinear and represents rational bubbles more accurately as suggested by Santos & Woodford (1997) and Battalio and Schultz (2006).

This thesis specifies and estimates one type of rational bubble using Unscented Kalman Filtering Technique. The bubbles mentioned here are assumed to be stochastic and feature time-variable parameters. The state space model defined by Yangru Wu (1997) is used as the basis for the development of new model specifications. Some of the parameters of this model are set to be time-variable and so the model has become nonlinear which is more acceptable in the real world. The basic model structure of Yangru Wu (1997), who expressed the stock price equation, the parametric bubble process and the dividend process in a state-space form, is as follows:

$$\Delta p_t = \Delta d_t + M\,\Delta Y_t + \Delta b_t \qquad (2)$$

$$Y_t = U + AY_{t-1} + v_t \qquad (3)$$

$$b_t = (1/\omega)b_{t-1} + \eta_t \qquad (4)$$

Where,

M becomes a time variable in the new model because ω variable is assumed to follow random walk distribution.

P_t= the real stock price at time t; $p_t=\ln(P_t)$;

D_t= the real dividend paid at time t; $d_t= \ln(D_t)$;

ω: the average ratio of the stock price to the sum of the stock price and the dividend, $0< \omega <1$;

$$Y_t = (\Delta d_t, \Delta d_{t-1}, \Delta d_{t-2}, ..., \Delta d_{t-h+1})',$$
$$U = (\mu , 0, 0, ..., 0)'$$
$$v_t = (\delta_t, 0, 0, ..., 0)'$$

These are all h-vectors and

$$A = \begin{pmatrix} \varphi_1 & \varphi_2 & \varphi_3 & \cdots & \varphi_{h-1} & \varphi_h \\ 1 & 0 & 0 & \dots & 0 & 0 \\ 0 & 1 & 0 & \dots & 0 & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix}$$ is an hxh matrix,

$g = (1, 0, 0, ..., 0)$ is an h-row vector; and $M = g.A.(I - A)^{-1}.[I - (1-\omega).(I - \omega.A)^{-1}]$ is an h-row vector and I is an h × h identity matrix.

Where the innovation η is assumed to be serially uncorrelated and have zero mean and finite variance σ^2_{η}. It is also assumed that η is uncorrelated with the dividend innovation, δ.

From this state space form, Yangru Wu has transformed the equation system to the below format by subtracting b_{t-1} and Y_{t-1} from the parametric bubble process equation and the dividend process equation respectively in order to set the state space equation system as below:

$$\Delta p_t = \Delta d_t + M.\Delta Y_t + \Delta b_t \quad (5)$$
$$\Delta Y_t = U + (A - I).Y_{t-1} + v_t \quad (6)$$
$$\Delta b_t = (1/\omega - 1).b_{t-1} + \eta_t \quad (7)$$

This format completes the equation system so that it is applicable with the Kalman Filtering Model. But as already mentioned this system is a linear equation system. In line with the assumption of time varying parameters, this model has been modified to the nonlinear form via setting the ω parameter as a random walk and M parameter as a time variable by the equation including the random walk distributed outcomes of ω. The modified functions of these parameters can be seen as below:

$$M_t = g.A.(I - A)^{-1}.[I - (1 - \omega_t).(I - \omega_t.A)^{-1}] \quad (8)$$

$$\omega_t = \omega_{t-1} + \xi \quad (9)$$

where
$\xi \sim N(0, 0.01)$

The logic behind these assumed modifications can be explained as; because the relation between the changes of preceding dividends and the preceding market prices should be time variable which integrates also one of the real world conditions to the model, the assumption that in the model the ω term is set as a time variable has been applied. As a result, the M parameter which represents the relation between the changes of preceding dividends and the changes of preceding market prices, as M is defined in Equation (8), becomes a time variable also.

It is assumed that the log dividends follow an ARIMA (h, 1, 0) process. To determine the autoregressive order h, the log dividend process is estimated by the maximum likelihood method for various choices of h, and compute both the Akaike information criterion (AIC) and the Schwartz information criterion for each h. The finding is that both criteria are at the minimum when h = 2 for all the series. The Kalman Filter Model therefore has three constant unknown parameters. Table 1. shows the point estimates of these unknowns.

Table 1. ARIMA(2,1,0) Parameters

	USA	Turkey	World
μ	0,001	-0,005	0,002
φ_1	0,828	0,104	0,05
φ_2	-0,102	-0,063	-0,036

CHAPTER 6

ESTIMATION RESULTS

By implementing the state space equation system composed of the equation(5), equation(6) and equation(7) and the related data of the parameters, the Unscented Kalman Filter is applied within the MATLAB program.

In order to observe the performance of the unscented Kalman Filter with the non-linear model with respect to the other results stated in Yangru Wu's paper, the RMSE of the results have been calculated. The RMSE values can be seen in Table 2.

Table 2. Root Mean Square Error: Comparison with Alternative Models

	This Paper	Yangru Wu	Intrinsic Bubbles	Simple Present Value
RMSE(%)	3,42	4,33	21,83	39,97

These RMSE values showed us that the new non-linear model performance is much better compared to the other alternative models.

Figure 1, Figure 2 and Figure 3, display the outcomes for United States, Turkey and World price indices respectively, in these figures the price estimates of the filter and the actual prices are shown at the same time.

Figure 2, Figure 4 and Figure 6 display the outcomes for United States, Turkey and World price indices respectively, in these figures the price estimates of the filter and the actual prices are shown at the same time with the relevant crisis that the Unscented Kalman Filter model has captured.

In Figure 2, it is observed that the 1973–1974 stock market crash, the Black Monday (1987) and the financial crisis of 2007–2010 has been captured by the model. In Figure 4, it is observed that the 2001 Economic Crisis and the financial crisis of 2007–2010 has been captured by the model. In Figure 6, it is observed that the 1973–1974 stock market crash, the Black Monday (1987) and the financial crisis of 2007–2010 has been captured by the model.

Figure 7 shows some of the financial episodes in more detail for United States with the bubble estimations of the model covering the crisis described as below:

- Wall Street Crash of 1929, followed by the Great Depression – the largest and most important economic depression in the twentieth century
- 1973 – 1973 oil crisis – oil prices soared, causing the 1973–1974 stock market crash and Secondary banking crisis of 1973–1975 – United Kingdom
- 1987 – Black Monday (1987) – the largest one-day percentage decline in stock market history and 1989–91 – United States Savings & Loan crisis
- 2007–10 – Financial crisis of 2007–2010, followed by the late 2000s recession

It can be concluded from Figure 7 that the filter has captured the crisis described above by the amounts of bubble estimates and the price estimates at relevant points.

Figure 8 shows the financial episodes time intervals in more detail only for Turkey with the bubble estimations of the model covering the crisis described as below:

- Late 2000s recession and the 2001 economic crisis of Turkey.
- Financial crisis of 2007–2010

It can be concluded from Figure 8 that the filter has captured these specific crisis by amounts of bubble estimates and the price estimates at the relevant points.

The results showed that, most of the time as the prices reaches to minimum level; a significant increase is observed in the bubble part of the prices which implies that the expectations rise from some point determined by the market agents' preferences so that the crisis has been captured and monitored by the filter with respect to each shift of the market agents' preferences.

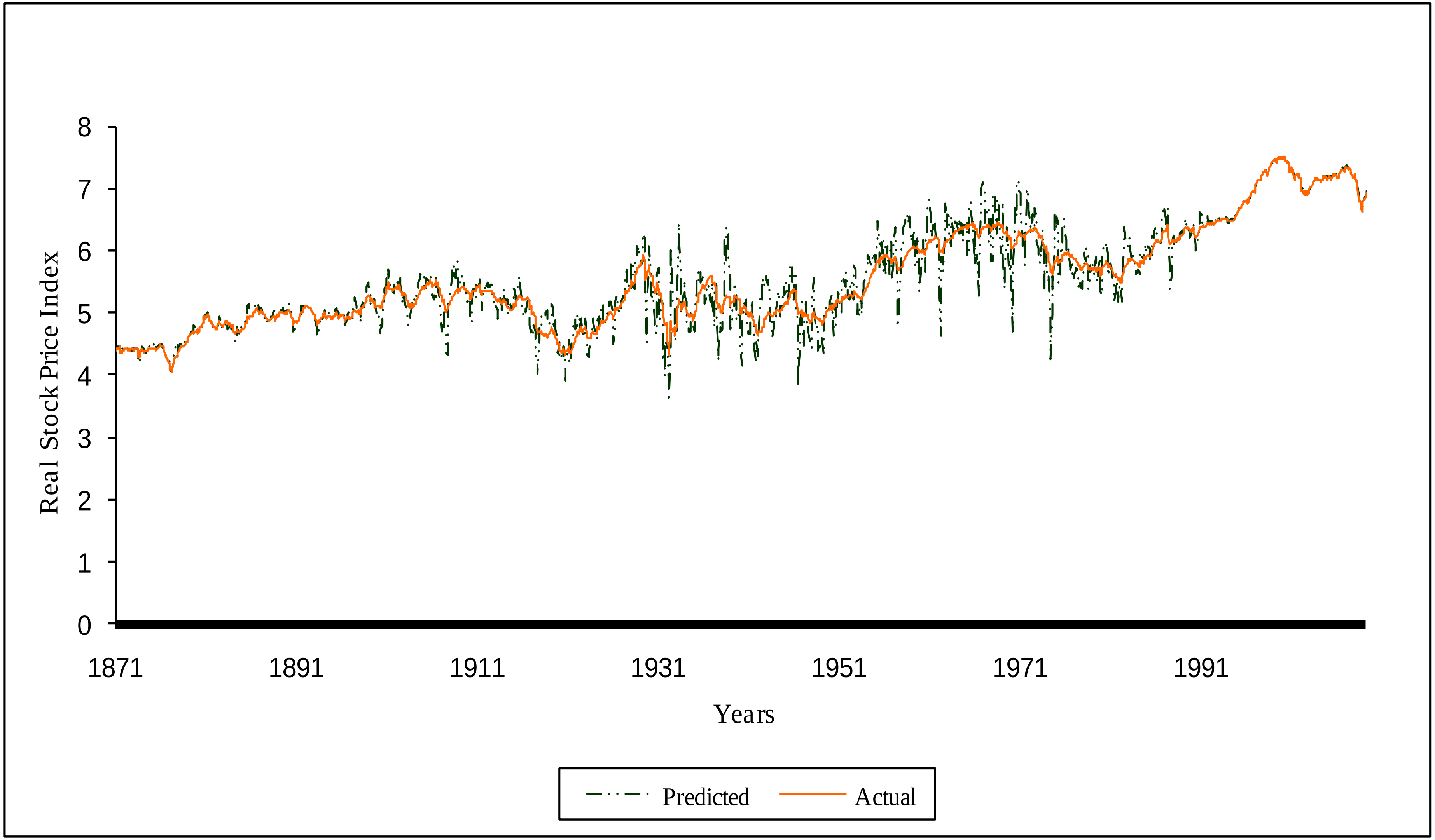

Figure 1. Predicted vs. Actual US

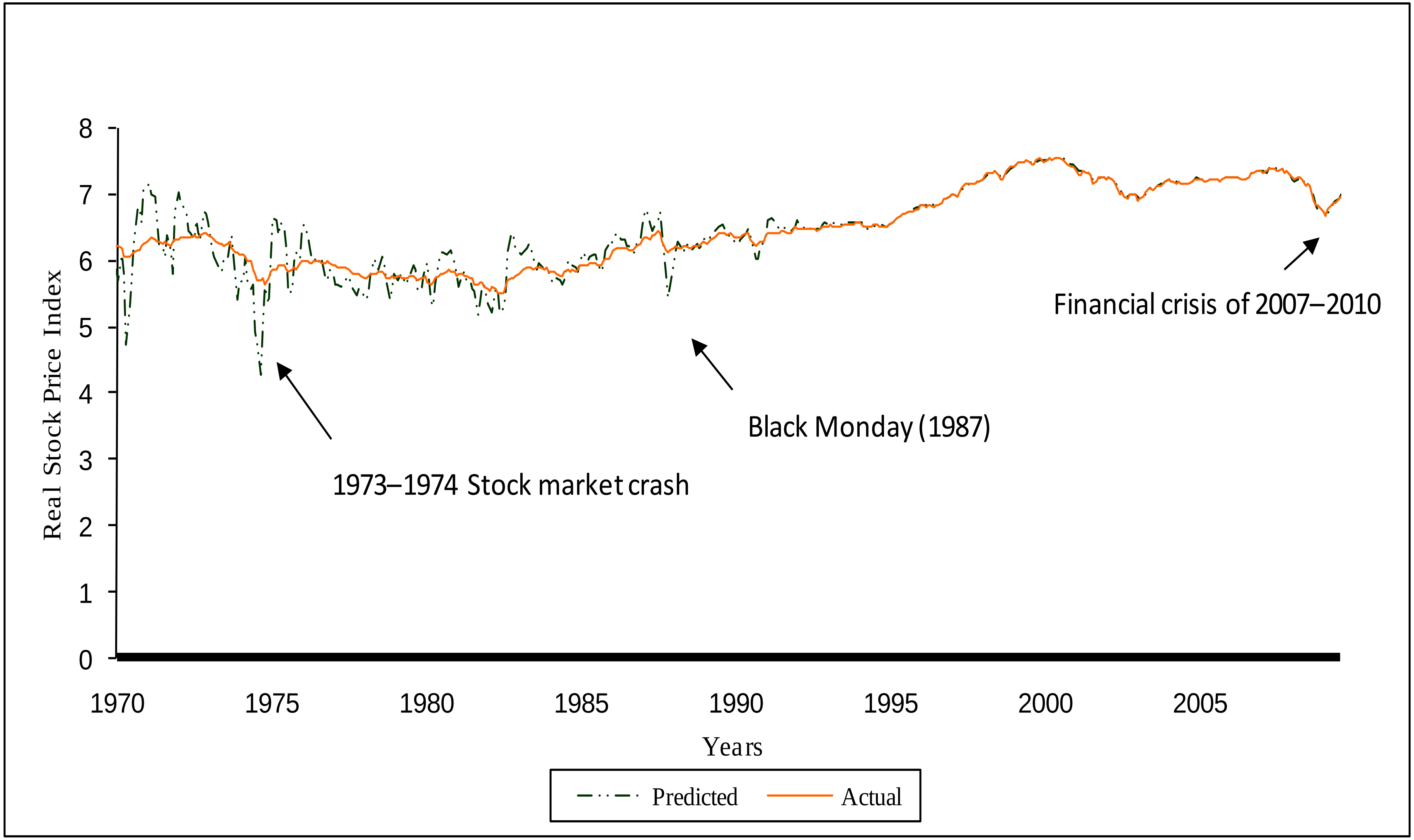

Figure 2. Recent Financial Crisis US

Figure 3. Predicted vs. Actual TR

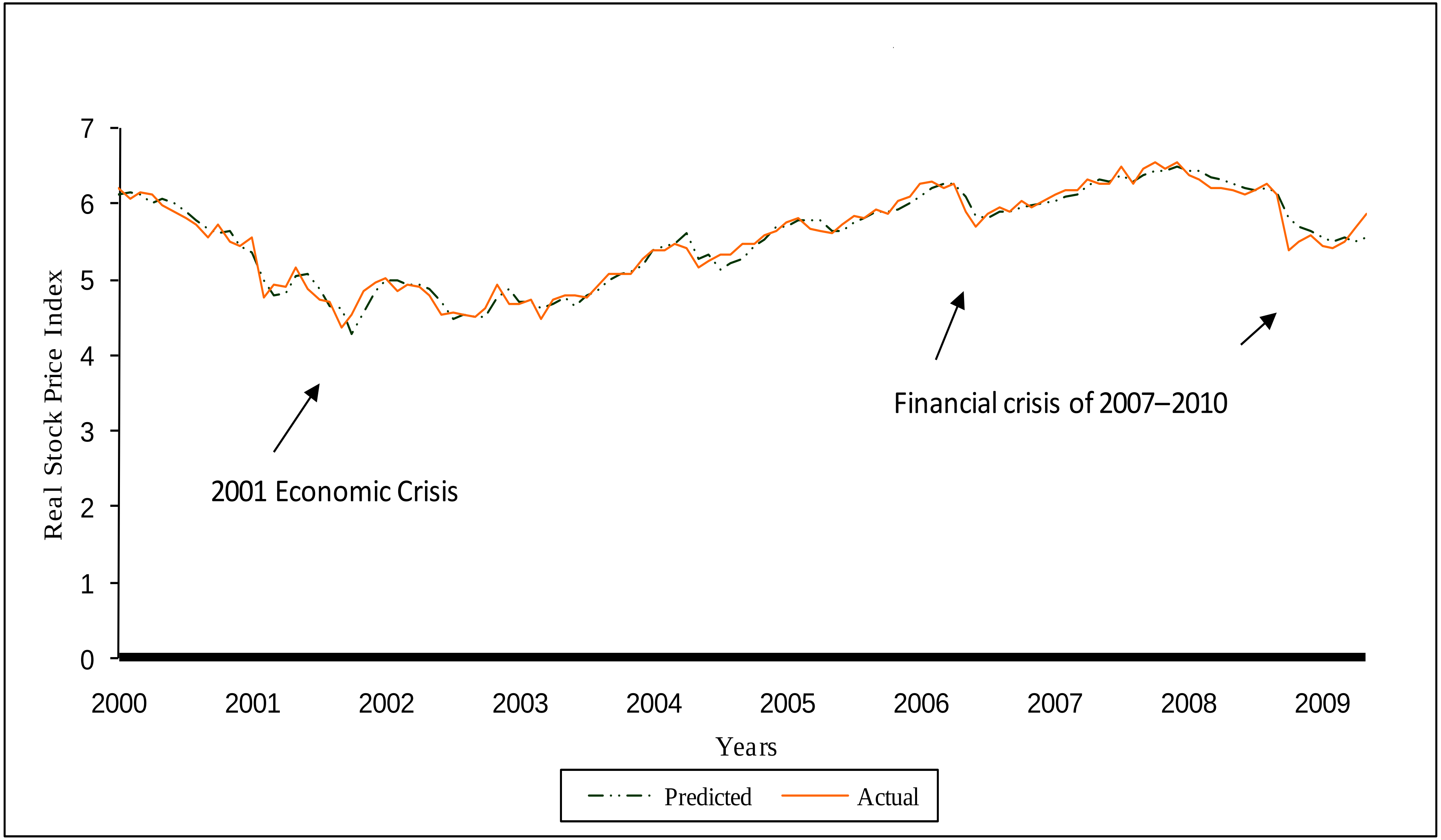

Figure 4. Recent Financial Crisis TR

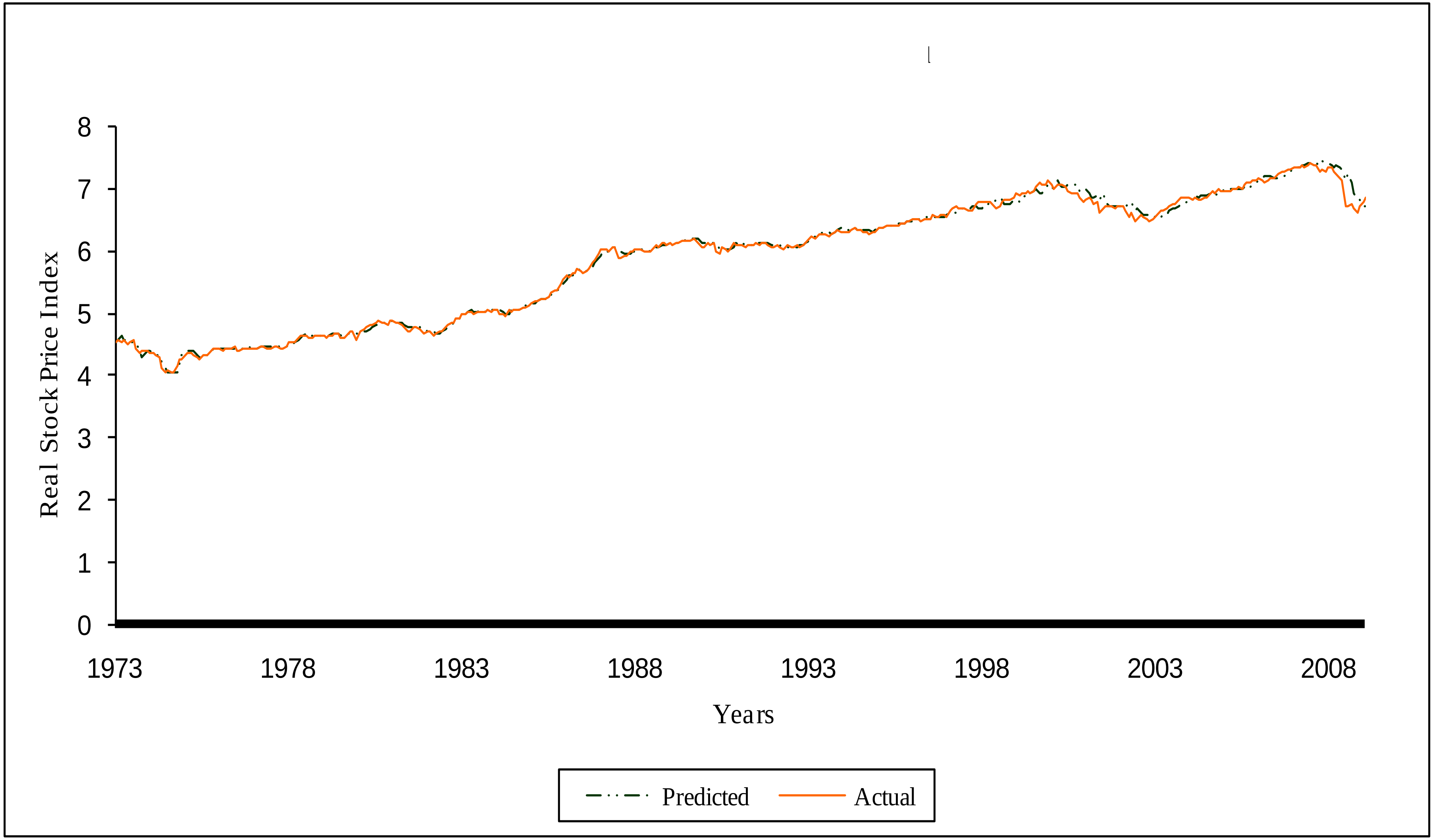

Figure 5. Predicted vs. Actual World

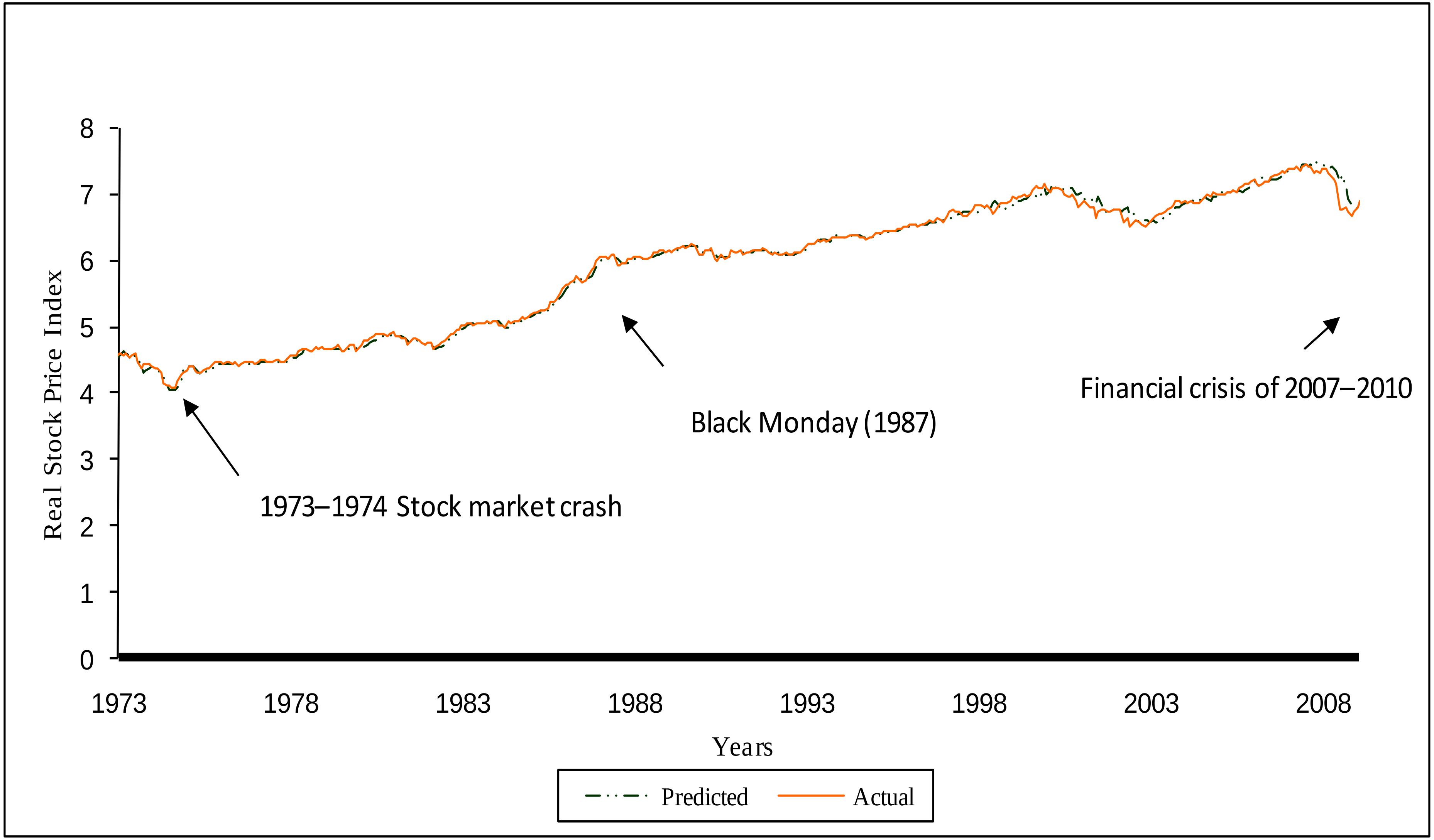

Figure 6. Recent Financial Crisis

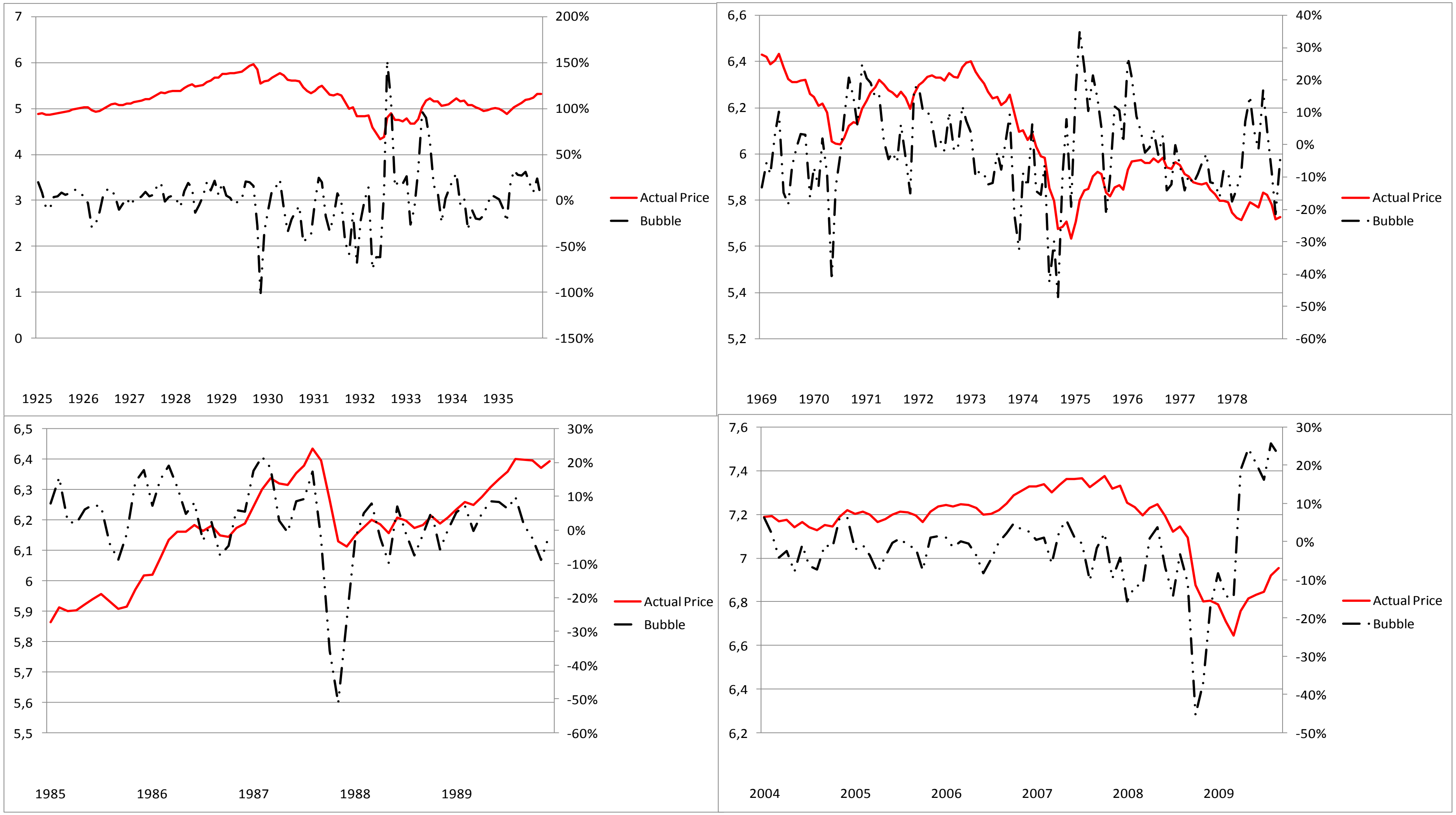

Figure 7. Actual prices vs. bubble/actual price percentages for US

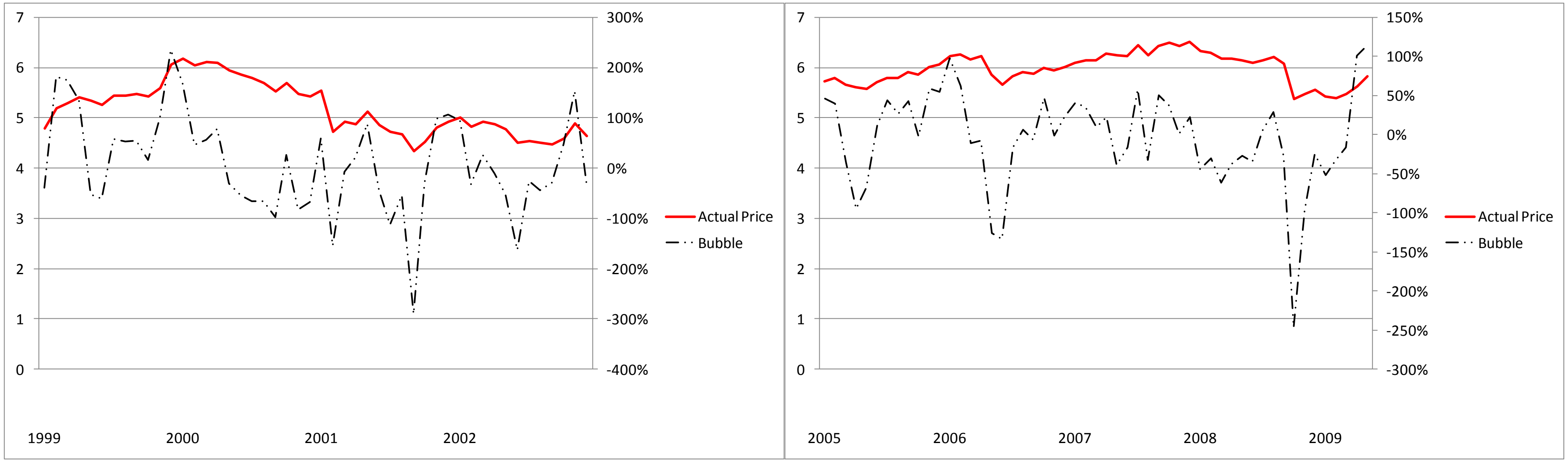

Figure 8. Actual prices vs. bubble/actual price percentages for Turkey

Granger Causality Tests

To determine if bubbles in one stock market have some explanatory power in predicting bubbles in another stock market Granger Causality tests are run using the estimated bubble data in the previous section.

First run ADF tests are run on bubble data. No unit roots are found in none of the estimated series. Table 3. reports the findings of the linear causality tests. The bubbles originating in US help explain bubbles in Turkey with a lag of 10 but fail to cause any changes in bubbles with a lag less than 7. Similarly, a bubble originating in the world causes bubbles in USA and in Turkey but not vice versa.

As compared to the Yangru Wu results, the granger causality relations between bubbles are calculated as shown in Table 3.

Table 3. Linear Granger Causality Tests: p-values.

Null Hypothesis: Lags	10	5	1
BTR does not Granger Cause BUSA	0,791	0,574	0,451
BUSA does not Granger Cause BTR	0,002	0,026	0,028
BWORLD does not Granger Cause BUSA	0,020	0,024	0,009
BUSA does not Granger Cause BWORLD	0,372	0,112	0,056
BWORLD does not Granger Cause BTR	0,010	0,021	0,068
BTR does not Granger Cause BWORLD	0,916	0,991	0,984

CHAPTER 7

CONCLUSION

To summarize the results, the estimated bubble components are obtained for a portion of different stock market prices, especially during several major bull and bear markets. In particular, significantly high estimates of positive bubbles are generally observed during the bull markets and significantly low estimates of negative bubbles are generally observed before the crisis in the bear markets, which is expected. Overall, the nonlinear rational stochastic bubble model does a credible job in characterizing the stock markets data.

As a result of the study, regarding the Granger causality test results, significant relations of the bubbles in between different stock market indices are found. And also the RMSE values show that Unscented Kalman Filter was giving better results for the estimations of the bubbles compared to other alternatives. And also the figures of the results displayed, show that, the crisis can be guessed before they happen by observing the bubble estimations such that bubble estimates seems to fluctuate earlier than the real values. This may be due to that asset positions are more sensitive to the economic indicators at that time.

As mentioned at the introduction the bubble spill-overs are in fact a product of the shifts in the chains of asset positions in between the agents of the markets. By these results, apart from the Granger Causality tests results, we conclude that the bubbles have the tendency to spill over to the other markets during the twentieth century and the beginning of twenty first century. Because a single bubbles can float on more than one market agents' preferences and the spillover of the bubbles will be on more than one market. Next topic may be how can we distinguish these bubbles?

APPENDICES

A. UNSCENTED KALMAN FILTER:

The Kalman filter is a mathematical method named after Rudolf E. Kalman. Its purpose is to use observed measurements over time that contain noise (random variations) and other inaccuracies, and produce values that tend to be closer to the true values of the measurements and their associated calculated values. The Kalman filter has many applications in technology, and is an important part of the development of space and military technology. Some of the Kalman Filter application areas are listed as below:

- Attitude and Heading Reference Systems
- Autopilot
- Battery state of charge (SoC) estimation
- Brain–computer interface
- Chaotic signals
- Dynamic positioning
- Economics, in particular macroeconomics, time series, and econometrics
- Inertial guidance system
- Radar tracker
- Satellite navigation systems
- Simultaneous localization and mapping
- Speech enhancement
- Weather forecasting
- Navigation Systems
- 3D-Modelling

The Unscented Kalman Filter belongs to a bigger class of filters called Sigma-Point Kalman Filters or Linear Regression Kalman Filters. This family of filters is using the statistical linearization technique which estimates a nonlinear function of a random variable through a linear regression between n points drawn from the prior distribution of the random variable.

Extended Kalman Filter propagates the state distribution through the first order linearization of the nonlinear system. As a result of that the posterior mean and covariance could be corrupted. UKF uses a deterministic sampling approach so this problem is eliminated naturally. In addition, the Unscented Kalman filter is a derivative free alternative to EKF.

Sigma points are a set of selected sample points via the state distribution in order to represent the state distribution. UKF also consists of the two classical Kalman filter steps: prediction and measurement steps. But exceptionally they now start with the selection of sigma points.

The UKF is founded on the intuition of Julier & Uhlmann,(2004) that it is easier to approximate a probability distribution that it is to approximate an arbitrary nonlinear function or transformation. Because the selected sigma points have to represent the state distribution the sigma points are derived from the distribution having the mean and covariance exactly to be x^a_{k-1} and P_{k-1}. Each selected sigma point is then entered to the nonlinear function in order to approximate the nonlinear function via the selected sample distribution. As a result a new sample of points is used to represent the estimated distribution. This process is called the unscented transformation. The unscented transformation is used to calculate the statistics of a random variable that is to be transformed via any nonlinear function.

Fredrik Orderud compares the relative estimation accuracy of UKF compared to EKF for linear state space models with nonlinear measurements. The relative advantage of using UKF does therefore seem to increase with the degree of nonlinearity in the measurement model. The estimation error distribution plots show that the two estimators yield quite similar results for both models, with the most significant exception being the amount of estimates having severely large errors. This leads us to the conclusion of UKF being a more robust estimator than EKF.

S. Konatowski & A. T. Pieniężny in their study summarized that; the Kalman filter (KF) is an optimal linear estimator when the process noise and the measurement noise can be modeled by white Gaussian noise. The KF only utilizes the first two moments of the state (mean and covariance) in its update rule. In situations when the problems are nonlinear or the noise that distorts the signals is non-Gaussian, the Kalman filters provide a solution that may be far from optimal. Nonlinear problems can be solved with the extended Kalman filter (EKF). This filter is based upon the principle of linearization of the state transition matrix and the observation matrix with Taylor series expansions. Exploiting the assumption that all transformations are quasi-linear, the EKF simply makes linear all nonlinear transformations and substitutes Jacobian matrices for the linear transformations in the KF equations. The linearization can lead to poor performance and divergence of the filter for highly non-linear problems. An improvement to the extended Kalman filter is the unscented Kalman filter (UKF). The UKF approximates the probability density resulting from the nonlinear transformation of a random variable. It is done by evaluating the nonlinear function with a minimal set of carefully chosen sample points. The posterior mean and covariance estimated from the sample points are accurate to the second order for any nonlinearity.

Most of the comparison studies of EKF vs. UKF have proved that UKF performs much better estimates.

The representation of the steps of the Unscented Kalman Filter with the general form of the unscented transformation is as below:

$$\Delta p_t = \Delta d_t + M.\Delta Y_t + \Delta b_t \quad \textbf{(10)}$$

$$\Delta Y_t = U + (A - I).Y_{t-1} + v_t \quad \textbf{(11)}$$

$$\Delta b_t = (1/\omega - 1).b_{t-1} + \eta_t \quad \textbf{(12)}$$

In our study, the above representation can be solved and rewritten as below:

$$p_t = M.U + p_{t-1} + (1/\omega + 1).b_t + [M.(A - I)].Y_{t-1} + \Delta d_t \quad \textbf{(13)}$$

$$Y_t = U + A.Y_{t-1} + v_t = \begin{pmatrix} u \\ 0 \end{pmatrix} + \begin{pmatrix} h_1 & h_2 \\ 1 & 0 \end{pmatrix} . \begin{pmatrix} \Delta d_{t-1} \\ \Delta d_{t-2} \end{pmatrix} + \begin{pmatrix} \delta_t \\ 0 \end{pmatrix} \quad \textbf{(14)}$$

$$b_t = (1/\omega).b_{t-1} + \eta_t \rightarrow \Delta b_t = (1/\omega - 1).b_{t-1} + \eta_t \quad \textbf{(15)}$$

$$x_k = \begin{pmatrix} b_k \\ p_k \\ Y_k \end{pmatrix},$$

(16)

$$f(x_k, u_k, t_k) = \alpha . x_{k-1} + \beta . u_k$$

Where

$$\alpha = \begin{pmatrix} 1/\omega & 0 & 0 \\ 1/\omega + 1 & 0 & M.(A - I) \\ 0 & 0 & A \end{pmatrix}$$

$$\beta = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\varepsilon = \begin{pmatrix} \eta_k \\ M.v_k + \eta_k \\ v_k \end{pmatrix}$$

$$h(x_k, t_k) = H.x_k \qquad \textbf{(17)}$$

where

$H = I,$

B. THE UNSCENTED KALMAN FILTER FLOW

The process is set up with respect to the following steps:

$$x_{k+1} = f(x_k, u_k, t_k) + \omega_k \quad \textbf{(17)}$$

$$y_k = h(x_k, t_k) + v_k \quad \textbf{(18)}$$

$$\omega_k \sim (0, Q_k)$$

$$v_k \sim (0, R_k)$$

1-Initiation

$$\hat{x}_o^+ = E(x_o) \quad \textbf{(19)}$$

$$P_o^+ = E[(x_o - \hat{x}_o^+).(x_o - \hat{x}_o^+)^T] \quad \textbf{(20)}$$

2- Sigma Point Estimator and Prediction Phase

$$\hat{x}_{k-1}^{(i)} = \hat{x}_{k-1}^{(i)} + \tilde{x}^{(i)} \quad , i = 1, \dots, 2n \quad \textbf{(21)}$$

$$\tilde{x}^{(i)} = \left(\sqrt{n.P_{k-1}^+}\right)_i^T \quad , i = 1, \dots, n \quad \textbf{(22)}$$

$$\tilde{x}^{(n+i)} = -\left(\sqrt{n.P_{k-1}^+}\right)_i^T \quad , i = 1, \dots, n \quad \textbf{(23)}$$

$$\hat{x}^{(i)} = f(\hat{x}_{k-1}^{(i)}, u_k, t_k) \quad (24)$$

$$\hat{x}_k^- = \frac{1}{2n} \sum_{i=1}^{2n} \hat{x}_k^{(i)} \quad (25)$$

$$P_k^- = \frac{1}{2n}\sum_{i=1}^{2n}\left(\hat{x}_k^{(i)} - \hat{x}_k^-\right).\left(\hat{x}_k^{(i)} - \hat{x}_k^-\right)^T + Q_{k-1} \tag{26}$$

3-Sigma Point Estimator and Measure Phase

$$\hat{x}_k^{(i)} = \hat{x}_k^- + \overset{\approx}{x}^{(i)}, i = 1,\dots,2n \tag{27}$$

$$\overset{\approx}{x}^{(i)} = \sqrt{n.P_k^-}\,{}_i^T, i = 1,\dots,n \tag{28}$$

$$\overset{\approx}{x}^{(n+i)} = -\sqrt{n.P_k^-}\,{}_i^T, i = 1,\dots,n \tag{29}$$

$$\hat{y}_k^{(i)} = h(\hat{x}_k^{(i)}, t_k) \tag{30}$$

$$\hat{y}_k = \frac{1}{2n}\sum_{i=1}^{2n}\hat{y}_k^{(i)} \tag{31}$$

$$P_y = \frac{1}{2}\sum_{i=1}^{2n}\left(\hat{y}_k^{(i)} - \hat{y}_k\right).\left(\hat{y}_k^{(i)} - \hat{y}_k\right)^T + R_k \tag{32}$$

$$P_{xy} = \frac{1}{2n}\sum_{i=1}^{2n}\left(\hat{x}_k^{(i)} - \hat{x}_k^-\right).\left(\hat{y}_k^{(i)} - \hat{y}_k\right)^T \tag{33}$$

$$K_k = P_{xy}.P_y^{-1} \quad \textbf{(34)}$$

$$\hat{x}_k^+ = \hat{x}_k^- + K_k.\left(y_k - \hat{y}_k \right) \quad \textbf{(35)}$$

$$P_k^+ = P_k^- - K_k.P_y.K_k^T \quad \textbf{(36)}$$

C. MATLAB PROGRAM CODE

```
%VECTORS
% X = state vector estimate. In the input struct, this is the
%       "a priori" state estimate (prior to the addition of the
%       information from the new observation). In the output struct,
%       this is the "a posteriori" state estimate (after the new
%       measurement information is included).
% Z = observation vector
% U = input control vector, optional (defaults to zero).
%
% MATRIX VARIABLES:
%
% alfa = state transition matrix (defaults to identity).
% P = covariance of the state vector estimate. In the input struct,
%       this is "a priori," and in the output it is "a posteriori."
%       (required unless autoinitializing as described below).
% beta = input matrix, optional (defaults to zero).
% QQ = process noise covariance (defaults to zero).
% RR = measurement noise covariance (required).
% H  = observation coefficient matrix (defaults to identity).
%
% Algorithm:
% (1) define all state definition fields: alfa,beta,HH,QQ,RR
% (2) define intial state estimate: X,P
% (3) obtain observation and control vectors: Z
% (4) call the filter to obtain updated state estimate: X,P
% (5) return to step (3) and repeat

%%
clear all;
%-----------------------------------------------------------------------
%Variable identifications
Y=[];p=[];d=[];
X=[];
Z=[];
A=[];
g=[];
I=[];
zero=[];
M=[];WW=[];
alfa=[];
beta=[];
xtilda=[];
xsigma=[];
XPred=[];
```

```
ZPredsigma=[];
H=[];
SUMZpredsigma=[];
ZPred=[];
Z=[];
PZPred=[];
RR=[];%Measurement cov.
QQ=[];%Process cov.
XPredsigma=[];
Kal=[];%Kalman Coeff.
Xcor=[];
Pcor=[];
PPred=[];
PXZPred=[];
PPred=[];
U=[];
SUMDIF=[];
%------------------------------------------------------------------------
%USA data formation
%load bubble33.txt;
%p=bubble33(:,1)';
%d=bubble33(:,2)';
%delta_d1=bubble33(:,3)';
%delta_d2=bubble33(:,4)';
%bubblemeasures=bubble33(:,5)';%Bubble tan?mlamas? buna göre=b=pt-pt-1

%WORLD data formation
%load bubble33WW.txt;
%p=bubble33WW(:,1)';
%d=bubble33WW(:,2)';
%delta_d1=bubble33WW(:,3)';
%delta_d2=bubble33WW(:,4)';
%bubblemeasures=bubble33WW(:,5)';%Bubble tan?mlamas? buna göre=b=pt-pt-1

%TURKEY data formation
%load bubble33TU.txt;
%p=bubble33TU(:,1)';
%d=bubble33TU(:,2)';
%delta_d1=bubble33TU(:,3)';
%delta_d2=bubble33TU(:,4)';
%bubblemeasures=bubble33TU(:,5)';%Bubble tan?mlamas? buna göre=b=pt-pt-1

%delta_p=bubblemeasures;
%------------------------------------------------------------------------
%Variables initiation
%------------------------------------------------------------------------
Y=[delta_d1;delta_d2];
```

```
n=4;%#of first dimensions of X matrix when this is changed all the program
dimensions change.
W=0.954;u=0.011;h1=0.048;h2=-0.129;
A=[h1 h2;1 0];
g=[1 0];I=eye(size(A));
zero=zeros(size(A,1),1);
M=g*A*inv(I-A)*(I-(1-W)*inv(I-W*A) );
alf =[(1/W)      0          zero'       ;
      (1/W-1)    1         (M*(A-I))    ;
       0         0          A(1,:)      ;
       0         0          A(2,:)]     ;
alfa=cat(1, cat(2,alf,zeros(4,n-4)),zeros(n-4,n));

beta=eye(n);

%time=size(p,2);
time=138;
c=1; %Display variable number in X matrix

%Z matrix formation
Z(:,1)=[0;p(:,1);Y(:,1);zeros(n-4,1)];
for m=1:time%-1
    Z(:,m)=[bubblemeasures(:,m);p(:,m);Y(:,m);zeros(n-4,1)];
end


for t=1:time
    if t==1,
%-----------------------------------------------------------------------
       %TIME UPDATE EQUATIONS
%-----------------------------------------------------------------------
       %Initialization
       Xcor(:,:,t)=ones(n,1);%Only used to choose the sigma points
       PPred(:,:,t)=100*eye(n);%Only used to choose the sigma points

       %Obtain the measurement variance
       RR(:,:,t)=[cov([bubblemeasures' p' delta_d1' delta_d2' zeros(size(p,2),n-
4)])];%measurement noise
       WW(:,t)=0.954;
       W=WW(:,t);u=0.11;
       h1=0.048;h2=-0.129;
       A=[h1 h2;1 0];
       g=[1 0];I=eye(size(A));
       zero=zeros(size(A,1),1);
       M=g*A*inv(I-A)*(I-(1-W)*inv(I-W*A) );
       alf =[(1/W)      0          zero'       ;
             (1/W-1)    1         (M*(A-I))    ;
```

```
        0       0       A(1,:)     ;
        0       0       A(2,:)]    ;
    alfa=cat(1, cat(2,alf,zeros(4,n-4)),zeros(n-4,n));
    U(:,t)=cat(1,[0;(delta_d1(:,t)+M*[u;0]);u;0],zeros(n-4,1));
%------------------------------------------------------------------------
    %MEASUREMENT UPDATE EQUATIONS
%------------------------------------------------------------------------
    %Choose Sigma Points
    np=chol(n*PPred(:,:,t));
    for i=1:n
        xtilda(:,i)=np(i,:)';
        xtilda(:,i+n)=-(np(i,:))';
        xsigma(:,i)=Xcor(:,t)+xtilda(:,i);
        xsigma(:,i+n)=Xcor(:,t)+xtilda(:,i+n);
    end
    %Use nonlinear system to transform Sigma points into Xk(i) vectors
    SUMXpredsigma=zeros(n,1);
    for i=1:(2*n)
        XPredsigma(:,i)=alfa*xsigma(:,i)+beta*U(:,t);
        SUMXpredsigma=SUMXpredsigma+XPredsigma(:,i);
    end
    %Obtain a priori estimate at time t
        XPred(:,t)=(1/(2*n))*SUMXpredsigma;

    %Obtain the process variance
    QQ(:,:,t)=0.01*eye(size(XPred,1));

    %Use Measurement Equation to transform Sigma points into Zk(i) vectors
    H=alfa;
    %H(1,:)=0;
    SUMZpredsigma=zeros(n,1);
    for i=1:(2*n)
       ZPredsigma(:,i)=H*xsigma(:,i);
       SUMZpredsigma=SUMZpredsigma+ZPredsigma(:,i);
    end
    %Obtain Predicted Measurement
    ZPred(:,t)=(1/(2*n))*SUMZpredsigma+beta*U(:,t);

    %Estimate the cov of the Predicted Measurement(ZPred)
    SUMDIF=zeros(n,n);
    for i=1:(2*n)
       SUMDIF=SUMDIF+(ZPredsigma(:,i)-ZPred(:,t))*(ZPredsigma(:,i)-
ZPred(:,t))';
    end
    SUMDIF=diag(diag(SUMDIF));
    PZPred(:,:,t)=(1/(2*n))*SUMDIF+RR(:,:,t);
```

```
    %Estimate the cross cov between XPred and ZPred
    SUMDIF=zeros(n,n);
    for i=1:(2*n)
       XPredsigma(:,i)=alfa*xsigma(:,i)+beta*U(:,t);
       SUMDIF=SUMDIF+(XPredsigma(:,i)-XPred(:,t))*(ZPredsigma(:,i)-
ZPred(:,t))';
    end
    SUMDIF=diag(diag(SUMDIF));
    PXZPred(:,:,t)=(1/(2*n))*SUMDIF;
    %The Measurement update of the state estimate
    Kal(:,:,t)=PXZPred(:,:,t)*inv(PZPred(:,:,t));

    Z(:,t)=[1;p(:,t);Y(:,t);zeros(n-4,1)];

    Xcor(:,t)=XPred(:,t)+Kal(:,:,t)*(Z(:,t)-ZPred(:,t));
    Pcor(:,:,t)=PPred(:,:,t)-Kal(:,:,t)*PZPred(:,:,t)*Kal(:,:,t)';
    Pcor(:,:,t)=abs(Pcor(:,:,t));
%------------------------------------------------------------------------
  else
%------------------------------------------------------------------------
    %Initialization
    RR(:,:,t)=[cov([bubblemeasures' p' delta_d1' delta_d2' zeros(size(p,2),n-
4)])];%measurement noise
    QQ(:,:,t)=cov([bubblemeasures'])*eye(n);%process noise

    WW(:,t)=WW(:,t-1)+normrnd(0,0.01,1);%Random Shock given to W;
    W=WW(:,t);
    u=0.011;h1=0.048;h2=-0.129

    A=[h1 h2;1 0];
    g=[1 0];I=eye(size(A));
    zero=zeros(size(A,1),1);
    M=g*A*inv(I-A)*(I-(1-WW(:,t))*inv(I-WW(:,t)*A) );
    alf =[(1/WW(:,t))      0          zero'       ;
         ((1/WW(:,t))-1)    1         (M*(A-I))    ;
          0         0          A(1,:)      ;
          0         0          A(2,:)]     ;
    alfa=cat(1, cat(2,alf,zeros(4,n-4)),zeros(n-4,n));

    U(:,t)=cat(1,[0;(delta_d1(:,t)+M*[u;0]);u;0],zeros(n-4,1));
%------------------------------------------------------------------------
    %TIME UPDATE EQUATIONS
%------------------------------------------------------------------------
    %choose sigma points
    Pcor
    np=chol(n*Pcor(:,:,t-1));
    xsigma=[];
```

```
xtilda=[];
for i=1:n
     xtilda(:,i)=np(i,:)';
     xtilda(:,i+n)=-np(i,:)';
     xsigma(:,i)=Xcor(:,t-1)+xtilda(:,i);
     xsigma(:,i+n)=Xcor(:,t-1)+xtilda(:,i+n);
end
%Use nonlinear system to transform Sigma points into Xk(i) vectors
SUMXpredsigma=zeros(n,1);
for i=1:(2*n)
    XPredsigma(:,i)=alfa*xsigma(:,i)+beta*U(:,t);
    SUMXpredsigma=SUMXpredsigma+XPredsigma(:,i);
end
%Obtain a priori estimate at time t
   XPred(:,t)=(1/(2*n))*SUMXpredsigma;

%Estimate the a priori cov matrix
SUMDIF=zeros(n,n);
for i=1:(2*n)
    SUMDIF=SUMDIF+(XPredsigma(:,i)-XPred(:,t))*(XPredsigma(:,i)-
XPred(:,t))';
end
SUMDIF=diag(diag(SUMDIF));
PPred(:,:,t)=(1/(2*n))*SUMDIF+QQ(:,:,t-1);

%-----------------------------------------------------------------------
%MEASUREMENT UPDATE EQUATIONS
%-----------------------------------------------------------------------
%Choose Sigma Points
np=chol(n*PPred(:,:,t));
xsigma=[];
xtilda=[];
for i=1:n
     xtilda(:,i)=np(i,:)';
     xtilda(:,i+n)=-np(i,:)';
     xsigma(:,i)=XPred(:,t)+xtilda(:,i);
     xsigma(:,i+n)=XPred(:,t)+xtilda(:,i+n);
end

%Use Measurement Equation to transform Sigma points into Zk(i) vectors
H=eye(size(n));
SUMZpredsigma=zeros(n,1);
for i=1:(2*n)
    ZPredsigma(:,i)=H*xsigma(:,i);
    SUMZpredsigma=SUMZpredsigma+ZPredsigma(:,i);
end
%Obtain Predicted Measurement
```

```
    ZPred(:,t)=(1/(2*n))*SUMZpredsigma;

    %Estimate the cov of the Predicted Measurement(ZPred)
    SUMDIF=zeros(n,n);
    for i=1:(2*n)
      SUMDIF=SUMDIF+(ZPredsigma(:,i)-ZPred(:,t))*(ZPredsigma(:,i)-
ZPred(:,t))';
    end
    SUMDIF=diag(diag(SUMDIF));
    PZPred(:,:,t)=(1/(2*n))*SUMDIF+RR(:,:,t);

    %Estimate the cross cov between XPred and ZPred
    SUMDIF=zeros(n,n);
    for i=1:(2*n)
      SUMDIF=SUMDIF+(XPredsigma(:,i)-XPred(:,t))*(ZPredsigma(:,i)-
ZPred(:,t))';
    end
    SUMDIF=diag(diag(SUMDIF));
    PXZPred(:,:,t)=(1/(2*n))*SUMDIF;

    %The Measurement update of the state estimate
    Kal(:,:,t)=PXZPred(:,:,t)*inv(PZPred(:,:,t));
    Xcor(:,t)=XPred(:,t)+Kal(:,:,t)*(Z(:,t)-ZPred(:,t));
    Pcor(:,:,t)=PPred(:,:,t)-Kal(:,:,t)*PZPred(:,:,t)*Kal(:,:,t)';
%-----------------------------------------------------------------------
  end

end
%%
%------------%---------------------------------------------------------
%Plot the diagram
%-----------------------------------------------------------------------
t=[];k=[];
%ZZ=zeros(4,time);
for m=1:time
  k=[k;m];
%   ZZ(:,m)=Z(:,m);
end
%ZZ


a=1;
b=time;%time-1;
c=1;
t=k(a:b,1)
XPredPlot=[];
XcorPlot=[];
```

```
XPredPlot=XPred(:,a:b);
XcorPlot=Xcor(:,a:b);
ZPlot=Z(:,a:b);
ZPlot
XcorPlot'
XPredPlot'

plot(t,ZPlot(2,:)','r--',t,XPredPlot(c,:)','b',t,XcorPlot(c,:)','g');%red=measures-blue=Predictions-Green=correccted values)
%plot(t,ZPlot(c,:)','r--');
%plot(t,XPredPlot(c,:)','b');
%plot(t,XcorPlot(c,:)','g');
%plot(t,XPred(1,:),'b',t,Xcor(1,:),'g');%red=measures-blue=Predictions-Green=correccted values)
%plot(t,Xcor(1,:),'g',t,XPred(1,:),'b');

XPred=XPred';
Xcor=Xcor';
Z=Z';

xlabel('No. of samples');
ylabel('Output');
title('Response with time-varying Unscented Kalman filter') ;
%%
```

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