INFORMALITY AND INEQUALITY DYNAMICS IN A TWO-SECTOR RAMSEY-TYPE GROWTH MODEL

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DECLARATION OF ORIGINALITY

I, Mustafa Kaba, certify that

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- this thesis contains no material that has been submitted or accepted for a degree or diploma in any other educational institution;
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ABSTRACT

Informality and Inequality Dynamics in a Two-sector Ramsey-type Growth Model

This thesis develops a two-sector Ramsey type growth model with heterogeneous agents differing in terms of their initial endowment of capital. The informality level and income/wealth distributions are generated endogenously in the model, given the tax rates for capital and labor, and tax enforcement rate. We aim to investigate the effect of informality on wealth and income inequality dynamics. We find that economies with higher informality level end up with lower capital stock and less inequality. We also introduce a skill heterogeneity to this model and investigate the effects of skill heterogeneity under different informality levels.

ÖZET

İki Sektörlü Ramsey Tipi Büyüme Modelinde Kayıtdışılık ve Eşitsizlik Dinamikleri

Bu tezde dağılımsal dinamikleri inceleme amacıyla heterojen ajanlı, kayıtlı ve kayıtdışı olmak üzere iki sektörlü Ramsey büyüme modeli geliştirilmektedir. Heterojenliğin kaynağı ajanların başlangıç sermaye seviyeleri arasındaki farklılıklardır. Kayıtdışılık oranı ve gelir/servet dağılımları modelde içsel olarak belirlenmektedir. Çalışmanın önemli bulgularından birisi artan kayıtdışılığın denge halinde daha düşük sermaye birikimlerine yol açtığı ama aynı zamanda gelir ve servet eşitsizliğini de azalttığı yönündedir. Ayrıca modele ajanlar arası kabiliyet farklılıkları eklenerek kabiliyetten kaynaklanan heterojenliğin eşitsizliğe etkisi de incelenmektedir.

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CHAPTER 1

INTRODUCTION

Informal economy has become an important research topic in the field of macroeconomics in the last two decades. It is a very common problem especially in the developing countries. It even reaches a level of 60% of GDP in some developing countries. It has significant outcomes on a wide range of areas including the economy, social life, political life and distribution of resources.

Informality has various definitions depending on the measurement methodology, however a widely used definition by Schneider & Enste (2000) state that informal activities are ”all economic activities that contribute to the officially calculated (or observed) gross national product but are currently unregistered”. A more detailed definition by the same study is that ”unreported income from the production of legal goods and services, either from monetary or barter transactions, hence all economic activities that would generally be taxable when they are reported to the tax authorities.” (Schneider & Enste, 2000).

It is widely agreed on that informality has a significant effect on taxation since it hinders governments’ ability to raise tax revenue. Remembering that taxation is one of the most important revenue sources for developing countries in the times of trade liberalization, there is a greater emphasis on the domestic revenue sources such as taxation (Toye, 2000). Whereas informality has a direct effect on taxes collected, taxation itself can be a redistributive instrument with a direct effect on the distribution of resources by affecting the public good provisions and government activities for poverty alleviation. The effect of informality on tax collection appears in most of the developing countries with a substantial shadow economy because it is difficult to raise tax revenue from personal income taxes in these countries. Therefore, most of the time, tax rate is increased to collect a higher amount of revenue. However, when the tax rate is increased, economic activity is very likely to move from formal sector to informal sector, hence, the tax base shrinks and the revenue decreases. This leads to further increases in the tax rates, which in turn, leads to further shrinkage of tax base, which is known as the ”Recursive Fiscal Dilemma” (Enste & Schneider, 2000).

Informal economy is also thought to have tight linkages with the inequality in the economy. This idea stems from the fact that informal sector constitutes a difficulty for a government to raise tax revenue, therefore, decreases government's public services to alleviate inequality. However, informal economy may also serve as an alternative economic area for those who cannot afford operating in the formal sector because of all the regulations, taxes and standards required. As being alternative to formal sector for those who are less skilled and/or financially less well off, informal sector can diminish inequality in the economy by enabling a part of the population to work who are not able to work otherwise in the formal sector.

Literature Review

Loayza (1996) builds a two-sector model of informality within an endogenous growth model framework. The factors contributing to informality -tax burden and regulations- and effects of informality are analysed. Their findings state that, first, informality level in an economy is positively related to tax burden and the labour market regulations/restrictions. Secondly, informality level is negatively related to the quality of the institutions. Thirdly, Loayza (1996) shows that informality harms the growth rate of an economy by decreasing the public services for the people in that economy.

A simple but dynamic model of informality has been developed by Ihrig & Moe (2004). The informality level is linked to tax rates as in Loayza (1996) and tax enforcement rates. The representative agent allocates its time between formal sector, informal sector and leisure while considering the tax rates and tax enforcement policies in the economy. In this two-sector model, formal sector has been characterized by high productivity whereas informal sector has lower productivity. Moreover, the firms in the formal sector have access to capital market while the firms in the informal sector has no access to capital markets. Only advantage for the firms operating in the informal sector is the tax avoidance. The government taxes formal sector fully however it can tax only a fraction of the informal sector activities. They show that both sectors can exist with positive employment levels in the equilibrium.

The question of informality-inequality relationship has been investigated by both theoretical and empirical studies. According to empirical works, the empirical evidence falls short to be conclusive about the sign of relationship between informality and inequality.

A significant amount of empirical and theoretical studies in the literature finds a positive relationship between informality and inequality. Among these studies there are Rosser, Rosser & Ahmed (2000, 2003), Chong & Gradstein (2007), Del'Anno (2016).

Rosser, Rosser & Ahmed (2000) find a positive relationship between informality and inequality by using a dataset for 16 countries between 1987 to 1989 and 1993 to 1994. They argue there is a two way causality between informality and inequality because when informality increases, the tax revenue decreases and consequently social safety expenditures of government decreases. At the end, inequality increases. On the other hand, when inequality arises, it depreciates the social solidarity and trust among people hence increases the informality level of an economy. Rosser, Rosser & Ahmed (2003) also document that increases in informality are positively related to income inequality by using a dataset for 18 transition economies.

Chong & Gradstein (2007) propose a theoretical framework for analysing the determinants of informality and its relationship with income inequality. In their model, three factors which affect the size of informal sector are considered as ”institutional quality, overall economic development and income inequality”. It is found that agents with less financial wealth are more likely to move into informal sector, implying that inequality increases the informality level of the economy. The mechanism which drives this result is as follows: It is assumed that the agents operating in the formal sector are in competition for technological opportunities. However, to look for technological opportunities an agents needs to be able to reach credit markets. In the model, the wealthier agents do have access to credit markets whereas the poorer agents do not. Those who cannot access credit markets move into the informal sector. This is how inequality drives informality in the model. The mechanism is similar to the one in the model proposed in this paper in the sense that informal sector is hosting those who cannot afford being formal or those who find being informal more profitable.

A recent study by Del'Anno (2016) decomposes the effect of inequality on informality ratio into two effects which are, first, its direct effect on informal output, secondly, the indirect one, which is the effect of inequality on official output. He uses a cross-section of 118 countries and concludes that inequality increases the informality ratio of the economy by decreasing the official output rather than increasing informal output.

Another empirical study by Krstic & Sanfey (2011) investigate the informality and inequality relationship in the Serbian labour market for 2002-2007 by using an OLS based methodology. They document that the level of informality has increased significantly for this period. They also find that income inequality has been more or less constant in this period. This paper suggests that transition economies may reveal different dynamics regarding the evolution of informality and inequality than developed economies.

Del'Anno & Solomon (2014) show that under certain circumstances, agents may choose to operate less in the formal sector and this can decrease income inequality. Theoretically, they show that under weak (low-quality) institutions, the productivity of the formal sector is reduced. The informal sector serves as an alternative income source for those who experience a fall in their incomes, hence, it can reduce income inequality. They use a panel data of 16 transition economies and show that the sign of the relationship between informality and inequality remains ambiguous.

Gutierrez-Romero (2007) documents that the correlation between informality and inequality is 0.25 for developing economies, whereas it is -0.14 for developed economies. Our results for the model calibrated to US economy also reveals negative correlation between informality and inequality. Moreover, according to Schneider and Enste (1998), a larger social safety expenditure by governments (less inequality) may help informality to prevail because of diminished incentives to work in the formal sector which provides job security, minimum wage requirement, and so on.

Our model borrows heavily from Turnovsky & Garcia Penalosa (2008) which investigate the income and wealth distributions within a Ramsey type growth model with heterogeneous agents in terms of their initial endowments of capital. The labour is supplied elastically in this model and has a significant effect on the long-run distributions of income and wealth. We extend this model by introducing informality specification.

We make a second extension to Turnovsky & Garcia-Penalosa (2008) by introducing skill heterogeneity between agents. Garcia-Penalosa & Turnovsky (2013) investigate inequality in a Ramsey type growth model with heterogeneous agents in terms of their initial wealth and ability. We incorporate the skill heterogeneity in this model to our model in order to see the distributional effects of skill heterogeneity and initial endowment heterogeneity at the same time in a single theoretical framework.

The thesis proceeds as follows: Chapter 2 gives the analytical structure and the results of the benchmark model, Chapter 3 introduces the model with skill heterogeneity, finally Chapter 4 summarizes and concludes.

CHAPTER 2

BENCHMARK MODEL

2.1 Analytical framework

We build a two-sector Ramsey type growth model with heterogeneous agents in terms of their initial endowments of capital in order to analyse the income and wealth distributions. The economy consists of two sectors which are formal and informal sector and one single good which can be produced in both sectors.

2.1.1 Production structure

We have two sectors which produce the same good with different factors and factor intensities. In the formal sector, the production function is a standard Cobb-Douglas production function with

$$Y_{j,f} = \theta_f K_j^{\alpha} L_{j,f}^{1-\alpha} \tag{1}$$

where K_j denotes the amount of capital used by firm j in formal sector production, $L_{j,f}$ is the amount of labour employed by firm j in formal sector production and θ_f denotes the total factor productivity of the formal sector. The share of capital in the total output is denoted by α. This production function satisfies the standard Neoclassical properties.

The production in the informal sector by firm j is formulated by

$$Y_{j,i} = \theta_i L_{j,i}^{\phi} \tag{2}$$

where $L_{j,i}$ denotes the amount of labour employed by the firm j in informal sector production, θ_i denotes the total factor productivity of the informal sector. Informal sector production also satisfies positive and diminishing marginal product properties.

Since all the firms are identical and they will see the identical conditions in formal and informal sector, they will all choose the same capital and labour amount. Formal sector firms choose $K_j = K$ and $L_{j,f} = L_f$. Therefore, the wage rate and rental rate of the capital are respectively as follows in the formal sector

$$\begin{aligned} w_f(K, L_f) &= \theta_f(1-\alpha)\left(\frac{K}{L_f}\right)^{\alpha} \\ r(K, L_f) &= \theta_f\alpha\left(\frac{K}{L_f}\right)^{\alpha-1}. \end{aligned}$$

In the informal sector, firms choose $L_{j,i} = L_i$ and the wage rate in informal sector is given by

$$w_i(L_i) = \theta_i \phi L_i^{\phi-1}.$$

2.1.2 Consumers

We have a mass 1 of infinitely-lived agents indexed by i. Agents are identical except the fact that they have different initial endowments of capital, which is denoted by K_{i0}. The agent allocates its one unit of time between formal sector labour, informal sector labour and leisure. This yields the time allocation constraint of the individual which is

$$l_i = 1 - L_{i,f} - L_{i,i}.$$

We define relative capital as the share of individual i in the total capital stock and denote it by

$$k_i \equiv \frac{K_i}{K}.$$

The consumer maximizes its expected lifetime utility according to capital accumulation equation and time allocation constraint by choosing the consumption and leisure level. The utility function is an isoelastic utility function. The consumer's maximization problem is given by

$$\begin{aligned}
\max \quad & \int_0^\infty \frac{1}{\gamma}(C_i l_i^\eta)^\gamma e^{-\beta t} dt, \\
& with -\infty < \gamma < 1, \;\; \eta > 0, \;\; 1 > \gamma(1+\eta) \\
& \text{subject to} \\
& \dot{K}_i = (1-\tau_k) r K_i + (1-\tau_w) w_f L_{i,f} + (1-\rho\tau_w) w_i L_{i,i} - C_i + V_i \\
& l_i = 1 - L_{i,f} - L_{i,i}
\end{aligned}$$

where the inequalities regarding γ and n ensures the concavity of the utility function. $\frac{1}{1-\gamma}$ denotes the intertemporal elasticity of substitution and β is the time discount rate. η is defined as the elasticity of leisure in the utility function.

In the capital accumulation equation τ_k and τ_w stand for the tax rate on capital and labour income respectively. ρ stands for the tax enforcement rate, which is an exogenous parameter of the ability of governments' to tax informal sector. V_i is the transfers done by the government. We assume that these transfers are made in a way that it has no any effect on the distributional issues.

The Hamiltonian of this continuous maximization problem can be written as follows:

$$H = \frac{1}{\gamma}(C_i l_i^{\eta})^{\gamma} e^{-\beta t} + \lambda_i e^{-\beta t} *$$

$$[(1-\tau_k) r K_i + (1-\tau_w) w_f L_{i,f} + (1-\rho\tau_w) w_i L_{i,i} - C_i + V_i - \dot{K_i}].$$

First order conditions with respect to C_i, $L_{i,f}$ and $L_{i,i}$, and the Euler Condition with respect to state variable K_i is given below with capital accumulation equation.

$$\begin{aligned}
C_i^{\gamma-1}[1 - L_{i,f} - L_{i,i}]^{\eta\gamma} &= \lambda && (3)\\
nC_i^{\gamma}[1 - L_{i,f} - L_{i,i}]^{\eta\gamma-1} &= \lambda(1-\tau_w)w_f && (4)\\
nC_i^{\gamma}[1 - L_{i,f} - L_{i,i}]^{\eta\gamma-1} &= \lambda(1-\rho\tau_w)w_i && (5)\\
\frac{\dot{\lambda(t)}}{\lambda(t)} &= \beta - r(1-\tau_k) && (6)\\
\dot{K_i} + C_i - V_i &= (1-\tau_k) r K_i + (1-\tau_w) w_f L_{i,f} + (1-\rho\tau_w) w_i L_{i,i}
\end{aligned}$$

Labour market always clears since $\sum_i l_i = l$, $\sum_i L_{i,f} = L_f$, $\sum_i L_{i,i} = L_i$ and

$$L_f + L_i + l_i = 1$$

2.2 Deriving macroeconomic equilibrium

In this section, we will derive the aggregate macroeconomic equilibrium from the first order conditions of the utility maximization problem and we will show that the aggregate variables are independent of the individual characteristics, the distribution of the initial endowment of capital in this particular model.

We start with the equalization of the marginal gain from supplying one extra unit of labour to formal and informal sector. Combining Eq.(4) and Eq.(5), we show that

$$(1-\tau_w)w_f = (1-\rho\tau_w)w_i. \tag{7}$$

To derive the aggregate macroeconomic equilibrium, we need to aggregate our first order conditions over individuals in order to capture how aggregate variables behave. To make this aggregation, we first need to show that the growth rate of consumption and growth rate of leisure is the same for all individuals regardless of their different initial endowments of capital.

To this end, dividing Eq.(4) by Eq.(3) yields

$$\eta\frac{C_i}{K_i} = (1-\tau_w)w_f\frac{l_i}{K_i}. \tag{8}$$

Since we are interested in the evolution of our variables, we take the time derivative of Eq.(8) and find

$$\frac{\dot{C}_i}{C_i} - \frac{\dot{l}_i}{l_i} = \alpha\frac{\dot{K}}{K} - \alpha\frac{\dot{L}_f}{L_f} \tag{9}$$

Moreover, we take the time derivative of Eq.(3) and show that

$$(\gamma-1)\frac{\dot{C}_i}{C_i} + \eta\gamma\frac{\dot{l}_i}{l_i} = \beta - r(1-\tau_k) \tag{10}$$

If we consider Eq.(9) and Eq.(10) for the individuals i and k, we can derive the result that growth rates of consumption and leisure are same across the individuals

$$\frac{\dot{C}_i}{C_i} = \frac{\dot{C}_k}{C_k} \ and \ \frac{\dot{l}_i}{l_i} = \frac{\dot{l}_k}{l_k}. \tag{11}$$

Then we can immediately conclude that

$$\frac{\dot{C}_i}{C_i} = \frac{\dot{C}}{C} \ and \ \frac{\dot{l}_i}{l_i} = \frac{\dot{l}}{l}. \tag{12}$$

for all individuals.

Before aggregating the individual accumulation equation, we will combine it with Eq.(8) and Eq.(7) to eliminate C_i:

$$\frac{\dot{K}_i}{K_i} = (1-\tau_k)r + (1-\tau_w)\frac{w_f}{K_i}(1-l_i-\frac{l_i}{\eta}) = 0 \tag{13}$$

Aggregating Eq.(13) over individuals yields the aggregate capital accumulation equation:

$$\frac{\dot{K}}{K} = (1-\tau_k)r + (1-\tau_w)\frac{w_f}{K}(1-l-\frac{l}{\eta}). \tag{14}$$

Aggregating Eq.(8) over individuals gives economy-wide consumption-capital ratio which is given by

$$\eta\frac{C}{l} = (1-\tau_w)w_f \tag{15}$$

Finally, aggregating Eq.(10) over individuals gives us the aggregate Euler equation

$$(\gamma - 1)\frac{\dot{C}}{C} + \eta\gamma\frac{\dot{l}}{l} = \beta - r(1 - \tau_k). \tag{16}$$

It is important to notice that there is no individual characteristic in any of the aggregate capital accumulation, economy-wide consumption-capital ratio and the aggregate Euler equations. This result implies that distribution of wealth does not have any effect on the steady-state level of the economy. This result is very crucial to the analytical tractability of the model. This kind of model is first developed by Caselli & Ventura (2000). They construct a model in which the heterogeneous agents act as an average single consumer. Therefore any heterogeneity among agents does not affect the aggregate equilibrium.

The independence of aggregate equilibrium is also shown in Garcia-Penalosa & Turnovsky (2006) with an endogenous growth model with knowledge spillovers, in Turnovsky & Garcia-Penalosa (2008) with a Ramsey type growth model and in Garcia-Penalosa & Turnovsky (2013) in a Ramsey type model with two sources of heterogeneity which are initial endowments of capital and skill heterogeneity.

In order to derive the macroeconomic equilibrium we need at least 3 equations in which the only endogenous variables are K, L_f and L_i. The first one is chosen as aggregate capital accumulation equation. Second one is the time derivative of Eq.(7):

$$(1 - \tau_w)\theta_f(1 - \alpha)\alpha K^{\alpha-1}\dot{K} = (1 - \rho\tau_w)\theta_i\phi L_i^{\phi-2}L_f^{\alpha-1}[(\phi - 1)L_f\dot{L}_i + \alpha L_i\dot{L}_f].$$

Third equation for deriving steady-state comes from the combination of Eq.(9) and Eq.(10). Imposing the steady-state conditions which are

$$\begin{aligned} \dot{K} &= 0 \\ \dot{L}_f &= 0 \\ \dot{L}_i &= 0 \end{aligned}$$

and solving for K, L_f and L_i yields us the steady-state expressions of K, L_f and L_i in terms of model parameters:

$$\widetilde{K} = \frac{(\alpha-1)(\tau_w-1)\left(\frac{\beta}{\alpha\theta_f-\alpha\theta_f\tau_k}\right)^{\frac{1}{\alpha-1}}}{(\rho\tau_w-1)(-\alpha+\eta+(\alpha-1)\tau_w+1)} \; *$$

$$\left[\theta_f^{\frac{1}{\phi-1}}(\eta-\rho\tau_w+1)\left(-\frac{(\alpha-1)(\tau_w-1)\left(\frac{\beta}{\alpha-\alpha\tau_k}\right)^{\frac{\alpha}{\alpha-1}}}{\phi\theta_i(\rho\tau_w-1)}\right)^{\frac{1}{\phi-1}}+\rho\tau_w-1\right],$$

the level of total capital stock at the steady-state.

$$\widetilde{L}_i = \theta_f^{\frac{1}{\phi-1}}\left(\frac{(1-\alpha)(1-\tau_w)\left(\frac{\beta}{\alpha(1-\tau_k)}\right)^{\frac{\alpha}{\alpha-1}}}{\phi\theta_i(1-\rho\tau_w)}\right)^{\frac{1}{\phi-1}},$$

the employment level in the informal sector.

$$\widetilde{L_f} = \frac{(\alpha-1)(\tau_w-1)\left(\theta_f^{\frac{1}{\phi-1}}(\eta-\rho\tau_w+1)\left(-\frac{(\alpha-1)(\tau_w-1)\left(\frac{\beta}{\alpha-\alpha\tau_k}\right)^{\frac{\alpha}{\alpha-1}}}{\phi\theta_i(\rho\tau_w-1)}\right)^{\frac{1}{\phi-1}}+\rho\tau_w-1\right)}{(\rho\tau_w-1)(-\alpha+\eta+(\alpha-1)\tau_w+1)},$$

the employment level in the formal sector.

Since this is a two-sector model, the analytical expressions (partial derivatives of $\dot{K}$, $\dot{L}_f$ and $\dot{L}_i$ with respect to K, L_f and L_i) in the Jacobian matrix are complicated and it is difficult to identify the sign of these expressions analytically. We'll use certain parameter sets to guarantee the stability of the dynamic system, i.e. to ensure that the equilibrium is saddle path stable. Ensuring stability in a 3-variable dynamic system with 1 state variable requires 1 negative eigenvalues which we'll show that it is the case for our numerical exercises, in the Appendix A.

However, for analysing the distributional issues in this model, we need to be able to track the evolution of relative capital stock $k_i(t)$ over time and we also want to track the evolution of our endogenous variables K, L_f and L_i. Therefore, before jumping to distributional analysis, we will derive the stable solution for $k_i(t)$ and find the time-paths of K, L_f and L_i.

To analyse the stability of the dynamic system, we will linearise it around the steady-state. The linearisation is as follows:

$$\begin{pmatrix} \dot{K} \\ \dot{L}_f \\ \dot{L}_i \end{pmatrix} = \begin{pmatrix} F_{11} & F_{11} & F_{11} \\ F_{11} & F_{11} & F_{11} \\ F_{11} & F_{11} & F_{11} \end{pmatrix} \begin{pmatrix} K-\tilde{K} \\ L_f-\tilde{L_f} \\ L_i-\tilde{L_i} \end{pmatrix} = \begin{pmatrix} \frac{\partial \dot{K}}{\partial K} & \frac{\partial \dot{K}}{\partial L_f} & \frac{\partial \dot{K}}{\partial L_i} \\ \frac{\partial \dot{L_f}}{\partial K} & \frac{\partial \dot{L_f}}{\partial L_f} & \frac{\partial \dot{L_f}}{\partial L_i} \\ \frac{\partial \dot{L_i}}{\partial K} & \frac{\partial \dot{L_i}}{\partial L_f} & \frac{\partial \dot{L_i}}{\partial L_i} \end{pmatrix} \begin{pmatrix} K-\tilde{K} \\ L_f-\tilde{L_f} \\ L_i-\tilde{L_i} \end{pmatrix}$$

Assume that the eigenvalues corresponding to above dynamic system are μ_1, μ_2 and μ_3 with $\mu_1 < 0$, $\mu_2 > 0$ and $\mu_3 > 0$. In the Appendix A, we will show that this is actually the case with the particular parameters sets chosen. The stable solutions can be written in the following way:

$$K(t) = A_1 \exp^{\mu_1 t} + A_2 \exp^{\mu_2 t} + A_3 \exp^{\mu_3 t} \tag{17}$$

$$L_f(t) = B_1 \exp^{\mu_1 t} + B_2 \exp^{\mu_2 t} + B_3 \exp^{\mu_3 t} \tag{18}$$

$$L_i(t) = C_1 \exp^{\mu_1 t} + C_2 \exp^{\mu_2 t} + C_3 \exp^{\mu_3 t} \tag{19}$$

Notice that transversality condition,

$$\lim_{t \to \infty} \lambda(A_1 \exp^{\mu_1 t} + A_2 \exp^{\mu_2 t} + A_3 \exp^{\mu_3 t}) \exp^{-\beta t} \tag{20}$$

implies $A_2 = A_3 = 0$ because A_2 and A_3 are the coefficients of the positive exponential term with positive eigenvalues. In the case that these two coefficients are not zero, there is not going to be stable solution to this dynamic system.

Now, let the homogeneous solution to $\dot{K}$ be $K(t) = A_1 \exp^{\mu_1 t}$. Using the initial condition $K(0) = K_0$, the homogeneous solution can be written as follows:

$$K_h(t) = (K_0 - \tilde{K}) \exp^{\mu_1 t}.$$

Let the particular solution to the dynamic system be $K_p = \tilde{K}$, $L_{fp} = \tilde{L_f}$, and $L_{ip} = \tilde{L_i}$. Given that we have the homogeneous and particular solutions, we can write the generalized solution as follows:

$$K(t) = \tilde{K} + (K_0 - \tilde{K}) \exp^{\mu_1 t} . \tag{21}$$

Using the steady-state conditions and the solution $K(t)$, we can also write L_f and L_i by a few simple algebraic operations:

$$L_f(t) = \tilde{L_f} + B_1 \exp^{\mu_1 t} \tag{22}$$

$$L_i(t) = \tilde{L_i} + C_1 \exp^{\mu_1 t} \tag{23}$$

where

$$B_1 = \frac{(K_0 - \tilde{K})(-F_{21} - \frac{(\mu_1 - F_{11})F_{23}}{F_{13}})}{F_{22} - \mu_1 - \frac{F_{12}F_{23}}{F_{13}}}$$

$$C_1 = -\frac{F_{21}}{F_{23}}(K_0 - \tilde{K}) - \frac{(F_{22} - \mu_1)B_1}{F_{23}}$$

The numerical values of B_1 and C_1 will be calculated in the numerical exercises. $K(t)$ and $l(t)$'s evolutions are governed by the Eqs.(21), (22) and (23).

2.3 Distribution of wealth and income

2.3.1 Wealth dynamics

Analysing distributional issues requires us to track the evolution of relative capital stock of individual i, $k_i(t) = \frac{K_i(t)}{K_(t)}$. We assume that $\frac{V_i}{K_i} = \frac{V}{K}$. This ensures that transfers don't have any distributional effects. Combining the individual capital accumulation equation and aggregate capital accumulation equation, we find:

$$\begin{aligned} \dot{k_i}(t) &= k_i(t)\left(\frac{\dot{K_i}(t)}{K_i(t)} - \frac{\dot{K}(t)}{K(t)}\right) \\ &= (1-\tau_w)\frac{w_f}{K(t)}\left[\left(1 - l_i - \frac{l_i}{\eta}\right) - k_i(t)\left(1 - l - \frac{l}{\eta}\right)\right] \end{aligned} \quad (24)$$

The initial level of relative capital stock of individual i is given by k_{i0}. At steady-state, we know that the relative capital stock of individual i will be constant, i.e. $\dot{k_i}(t) = 0$. Imposing this condition we obtain:

$$1 - \tilde{l_i} - \frac{\tilde{l_i}}{\eta} = \tilde{k_i}(t)\left(1 - \tilde{l} - \frac{\tilde{l}}{\eta}\right)$$

Subtracting $\left(1 - \tilde{l} - \frac{\tilde{l}}{\eta}\right)$ from both sides,

$$\tilde{l_i} - \tilde{l} = \left(\tilde{l} - \frac{\eta}{\eta+1}\right)\left(\tilde{k_i}(t) - 1\right) \quad (25)$$

for each i. This equation has a very important implication depending on the sign of $\tilde{l} - \frac{\eta}{\eta+1}$. To determine the sign of this expression, we go back to Eq.(14), the aggregate capital accumulation equation and write it as follows:

$$\frac{\dot{K}}{K} = (1-\tau_k)r + (1-\tau_w)\frac{w_f}{\tilde{K}}\left(1-\tilde{l}-\frac{\tilde{l}}{n}\right) = 0 \tag{26}$$

at the steady-state. Since we know that $r(1-\tau_K) > 0$, it is clear that

$$(1-\tau_w)\frac{w_f}{\tilde{K}}\left(1-\tilde{l}-\frac{\tilde{l}}{\eta}\right) < 0.$$

Hence it immediately follows

$$\left(1-\tilde{l}-\frac{\tilde{l}}{\eta}\right) < 0 \tag{27}$$

which can be written as:

$$\tilde{l} > \frac{\eta}{\eta+1}.$$

Having determined the sign of $\tilde{l} - \frac{\eta}{\eta+1}$, we can conclude that if the agent ends up with a higher relative capital stock, then he supplies less labour and buys more leisure. Therefore, the endogenous labour supply has a mitigating effect on the income and wealth inequality.

In order to track the evolution of the individual relative capital stock, $k_i(t)$, we need to linearise it around the steady-state. Before diving into the linearisation, we will derive an equation regarding the share of leisure of individual i in the total amount of leisure with the aim of making linearisation easier.

Since we know that $\frac{\dot{l}_i}{l_i} = \frac{\dot{l}}{l}$ from Eq.(12), we can conclude that the share of leisure of individual i in the total amount of leisure will be constant during the transition to steady-state and also during the steady-state. Therefore, we can define the share of leisure of individual i in the total amount of leisure as ψ_i which satisfies

$$\begin{aligned} l_i &= \psi_i l \\ \int_0^1 \psi_i di &= 1. \end{aligned}$$

Using this and rewtiting Eq.(24) yields:

$$\dot{k_i(t)} = (1-\tau_w)\frac{w_f}{K(t)}\left[1-\psi_i l\left(1+\frac{1}{\eta}\right)-k_i(t)\left(1-l-\frac{l}{\eta}\right)\right] \quad (28)$$

We linearize Eq.(28) around $\tilde{K}$, $\tilde{L_f}$, $\tilde{l}$ and $\tilde{k_i}$. The lienarized version is as follows:

$$\dot{k_i(t)} = (1-\tau_w)\frac{w_f}{K(t)}\left[\left(1+\frac{1}{\eta}\right)(\tilde{k_i}-\psi_i)(l-\tilde{l})+\left(\tilde{l}\left(1+\frac{1}{\eta}\right)-1\right)(k_i-\tilde{k_i})\right] \quad (29)$$

The stable solution to linearised version of $k_i(t)$ is as follows:

$$k_i(t) = \tilde{k_i} + \frac{[(1-\tau_w)\frac{w_f}{\tilde{K}}[(1+\frac{1}{\eta})(k_i-\psi_i)(l(0)-\tilde{l})]]\exp^{\mu_1 t}}{\mu_1-(1-\tau_w)\frac{w_f}{\tilde{K}}\left(\tilde{l}\left(1+\frac{1}{\eta}\right)-1\right)} \quad (30)$$

Please see Appendix B for the details of obtaining stable solution.

Setting $t=0$ in Eq.(30) we obtain the k_{i0}:

$$k_{i0} = \tilde{k_i} + \frac{[(1-\tau_w)\frac{w_f}{\tilde{K}}[(1+\frac{1}{\eta})(k_i-\psi_i)(l(0)-\tilde{l})]]}{\mu_1-(1-\tau_w)\frac{w_f}{\tilde{K}}\left(\tilde{l}\left(1+\frac{1}{\eta}\right)-1\right)} \quad (31)$$

Now turning back to the Eq.(25), i.e. the relationship between labour supply and relative capital stock for individual i, and substituting $l_i = \psi_i l$ and solving for ψ_i yields us the following:

$$\psi_i = \tilde{k}_i + \frac{\eta}{(1+\eta)\tilde{l}}(1 - \tilde{k}_i).$$

Substituting ψ_i in Eq.(30) and Eq.(31) gives:

$$k_i(t) = \tilde{k}_i + \frac{1}{\mu_1 - B}\frac{w_f}{\tilde{K}}\left(\tilde{k}_i - 1\right)\left(\frac{l_t}{\tilde{l}} - 1\right) \tag{32}$$

and

$$k_{i0} = \tilde{k}_i + \frac{1}{\mu_1 - B}\frac{w_f}{\tilde{K}}(\tilde{k}_i - 1)\frac{l(0) - \tilde{l}}{\tilde{l}} \tag{33}$$

where

$$B = (1 - \tau_w)\frac{w_f}{\tilde{K}}\left(\tilde{l}\left(1 + \frac{1}{\eta}\right) - 1\right).$$

Eq.(32) shows the evolution of relative capital stock of individual i over time. This is crucial for analysing the distribution of capital during the transition and in the equilibrium. Eq.(33) shows the initial relative capital stock of individual i in terms of the model parameters and steady-state values of our variables. By these two equations, now it is possible to understand the relationship between the initial wealth distribution, wealth distribution at time t and wealth distribution at the steady-state.

We can write the Eq.(32) in the following way:

$$k_i(t) - 1 = \delta(t)(\tilde{k}_i - 1)$$

where

$$\delta(t) \equiv 1 + \frac{1}{\mu_1 - B}\frac{w_f}{\tilde{K}}\left(\frac{l(t)}{\tilde{l}} - 1\right).$$

Doing the same thing for $t = 0$ yields us the relationship between the initial relative capital stock and the steady-state relative capital stock for individual i:

$$k_{i0} - 1 = \delta(0)(\tilde{k}_i - 1).$$

Now, we can write Eq.(34) and Eq.(36) in terms of deviations:

$$\sigma_{k(t)} = \delta(t)\sigma_{\tilde{k}}$$

and

$$\sigma_{k_{i0}} = \delta(0)\sigma_{\tilde{k}}.$$

The function $\delta(t)$ is of high importance in determining the relationship between the initial relative capital stock and the steady-state relative capital stock for individual i. The value of this function will be determined endogenously in the model according to the selected parameter values and corresponding steady-state levels and negative eigenvalue of the system.

Finally we express this relationship as follows:

$$\sigma_{k(t)} = \frac{\delta(t)}{\delta(0)} \sigma_{k_{i0}} \tag{34}$$

2.3.2 Income inequality dynamics

To analyse the income distribution, we first define individual income, aggregate income and relative income of individual i, respectively:

$$\begin{aligned} Y_i(t) &= (1-\tau_K)r(t)K_i(t) + (1-\tau_w)w_f(1-\psi_i l(t)), \\ Y(t) &= (1-\tau_K)r(t)K(t) + (1-\tau_w)w_f(1-l(t)), \\ y_i(t) &= \frac{Y_i(t)}{Y(t)}. \end{aligned}$$

Denoting the share of capital in the total output as $s = \frac{(1-\tau_k)rK}{Y}$, we can write the relative income of individual i as follows:

$$\begin{aligned} y_i(t) - 1 &= \frac{Y_i(t)}{Y(t)} - \frac{Y(t)}{Y(t)} \\ &= s(t)(k_i(t)-1) + (1-s(t))\frac{l(t)}{1-l(t)}(1-\psi_i) \\ &= s(t)(k_i(t)-1) + (1-s(t))\frac{l(t)}{1-l(t)}\left(1 - \frac{1}{l(t)}\frac{\eta}{1+\eta}\right)(k_i(t)-1) \end{aligned}$$

by using Eq.(25) to substitute $(1-\psi_i)$. Now, it's straightforward to write this equation in terms of deviations:

$$\begin{aligned} y_i(t)-1 &= \kappa(t)(k_i(t)-1) \\ \sigma_{y(t)} &= \kappa(t)\sigma_{k(t)} \end{aligned} \tag{35}$$

where $\kappa(t)$ is shown to be less than 1 by using $l(t) > \frac{n}{n+1}$. This implies that the income is more equally distributed compared to wealth which will be the case in the numerical exercises.

The relationship between the initial relative income and relative income at time t can be found by setting $t=0$ in Eq.(35) and dividing Eq.(35) by the time 0 distribution relationship. It is given by:

$$\sigma_{y(t)} = \frac{\kappa(t)}{\kappa(0)} \frac{\sigma_{k(t)}}{\sigma_{k_0}} \sigma_{y_0}. \tag{36}$$

2.4 Numerical exercise

2.4.1 Calibration

Table 1. Parameter Assignments for Calibration

Parameter	Description	Value
ϕ	Factor intensity of labour in the informal sector production	0.5
ρ	Tax enforcement rate	0.5
η	Elasticity of leisure in the utility function	0.5
β	Time discount rate	0.04
α	Factor intensity of capital in the formal sector production	0.35
θ_f	Formal Sector TFP	1.1
θ_i	Informal Sector TFP	0.8 - 1.5
τ_w	Tax on labour income	0.16
τ_k	Tax on capital income	0.16
$\frac{1}{1-\gamma}$	Intertemporal Elasticity of Substitution	0.4

These parameters values are chosen to describe two model economies with different levels of informality. The only difference within these two economies is the total factor productivity of informal sector. In the benchmark model, informal TFP is chosen as 0.8 whereas formal sector TFP is 1.1. This calibration yields a 9 percent informal sector in the total GDP. Increasing informal sector TFP to 1.5, whereas all other parameters remain the same, informality level reaches to 30 percent of the total GDP.

2.4.2 Results for two economies

Table 2. Results of the Numerical Simulation

	Baseline Scenario	High Informality
K	12.125	9.913
L_f	0.487	0.398
L_i	0.044	0.154
$L_f + Li$	0.531	0.552
Y_f	1.650	1.349
Y_i	0.167	0.589
$Y = Y_f + Y_i$	1.817	1.937
Y_i/Y	0.092	0.304
K/Y	6.673	5.117
$\sigma_{\tilde{k}}/\sigma_{k_0}$	0.829	0.599
$\sigma_{\tilde{y}}/\sigma_{\tilde{k}}$	0.159	0.130
$\sigma_{\tilde{y}}$	0.132	0.078

The numerical simulation results above show how two economies differ from each other when we increase the TFP of informal sector from 0.8 to 1.5.

One effect of this increase can be seen in the employment levels in the formal and informal sectors. Although we do not observe any significant change in the total labour supply at the steady-state, we see that the composition of total labour supply changes significantly. The employment in the formal sector decreases by almost 20 percent whereas the informal employment increases to almost 3 times of its initial level. The reason behind the almost constant total labour supply is that there is a return equalization in the model which equalizes the return of supplying one unit of extra labour to each sector. Therefore, the total labour supply does not change much but the composition of it is sensitive to sectoral characteristics.

An interesting result from these simulations is the negative relationship between the informality level and inequality. This is mainly caused by the decrease in the rate of economic development, which is proxied by the total amount of capital in this model. Higher level of informality leads to a lower level of steady-state capital in the economy. This result is somewhat in line with the studies that argue economic growth is positively related to inequality. Increase of informality level from 9 percent to 30 percent leads to a 20 percent decrease in the total amount of capital. When the expansion of capital decreases in the economy, the dispersion of the distribution of it also decreases as seen in the results. Given that the initial standard deviation of the capital distribution in both models are equal, it is observed that the standard deviation of capital distribution at the steady-state is lower for the economy with high informality.

We have mentioned that income is more equally distributed than the wealth. It is also observed in the results. Moreover, the higher informality leads also less income inequality because of the decrease in the total capital stock of the economy.

The transitional dynamics regarding the two economies reveal highly different trends.

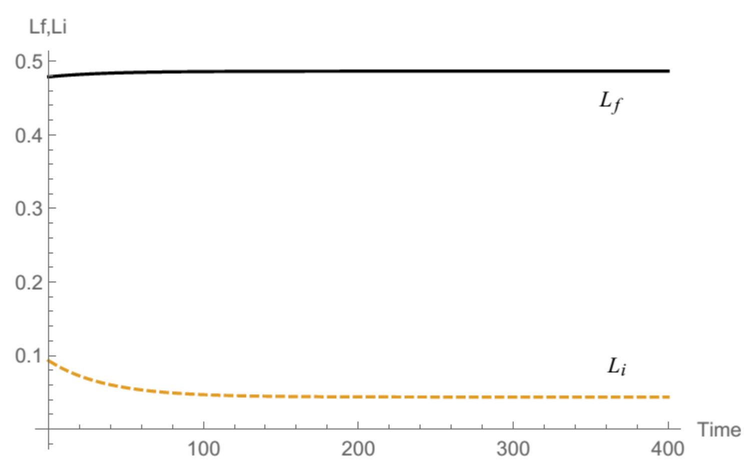

Figure 1. Formal and informal employment in baseline scenario

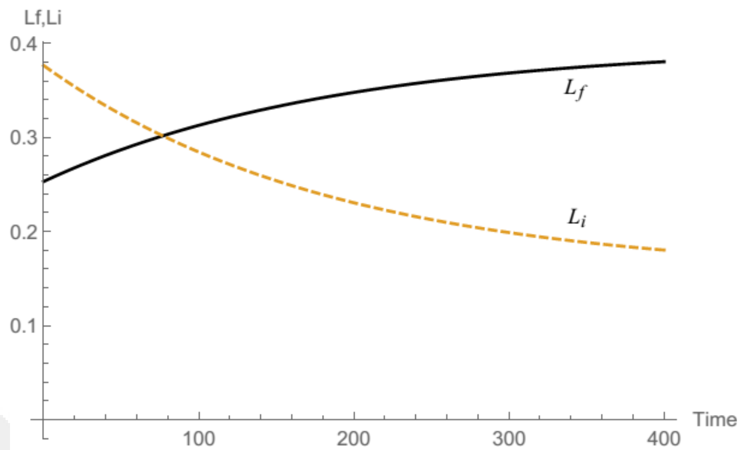

Figure 2. Formal and informal employment in high informality economy

The reason behind these different trends in employments stems from the fact that informal sector has no access to capital market. In the model economy with high informality, informal sector is more attractive in the beginning of the economy since there is not much capital stock. However, while the capital accumulates in the economy, the return to capital increases (although in a diminishing manner), therefore supplying labour to formal sector becomes more attractive. Towards the steady-state informal employment decreases whereas the formal employment increases. However, the high informal employment in the earlier periods harms the capital accumulation process and the economy with high informality ends up with less total capital stock.

2.4.3 Shock to formal sector productivity

Table 3. Baseline Scenario vs. 10% Shock to Formal Sector TFP

	Baseline Scenario	10% Shock to Formal TFP
K	12.125	14.217
L_f	0.487	0.493
L_i	0.044	0.036
$L_f + Li$	0.531	0.529
Y_f	1.650	1.934
Y_i	0.167	0.152
$Y = Y_f + Y_i$	1.817	2.086
Y_i/Y	0.092	0.073
K/Y	6.673	6.814
$\sigma_{\tilde{k}}/\sigma_{k_0}$	0.829	0.839
$\sigma_{\tilde{y}}/\sigma_{\tilde{k}}$	0.159	0.161
$\sigma_{\tilde{y}}$	0.132	0.135

After a 10% increase in the total factor productivity of formal sector, it is observed that the total employment in the economy does not change significantly. However, since the productivity of capital increases, more labour moves to formal sector and more capital accumulates until the steady-state. An interesting result is that both the standard deviations of income and wealth distributions do not change significantly following this shock.

10% increase in the productivity of formal sector brings the economy to a level of higher capital stock. Therefore, the economy starts initially with a capital lower than its new steady-state level. A reduction in the wealth inequality accompanies the transition when the economy starts with a capital lower than its steady-state level. The vice versa is also true as we show it in the next shock.

The transitional dynamics followed the shock are given below:

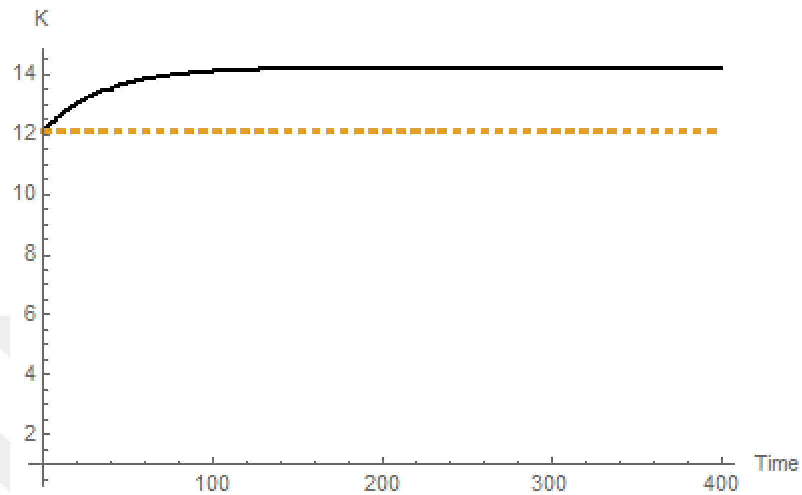

Figure 3. Evolution of capital after the shock

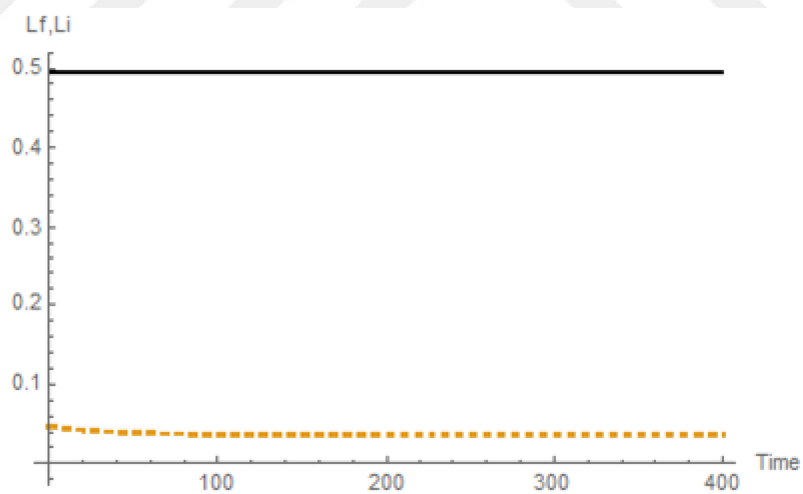

Figure 4. Evolution of employments after the shock

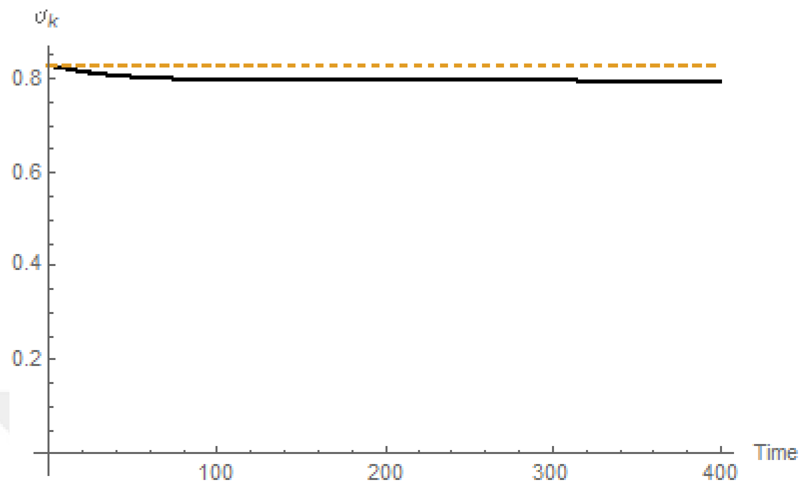

Figure 5. Evolution of capital distribution after the shock

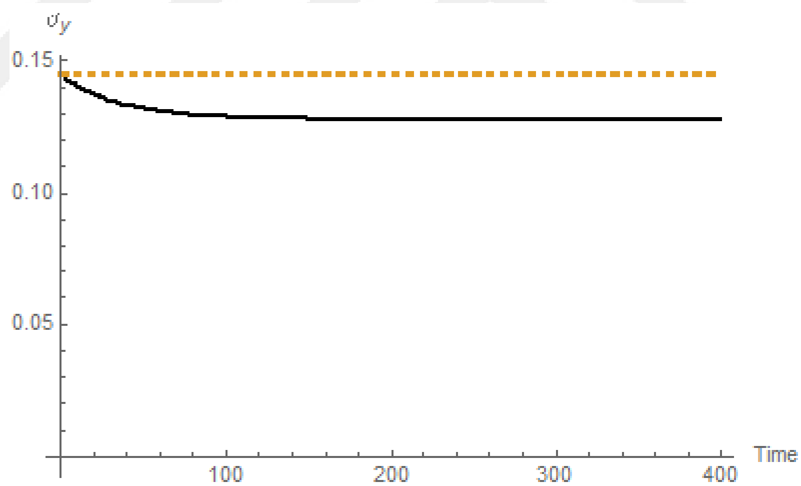

Figure 6. Evolution of income distribution after the shock

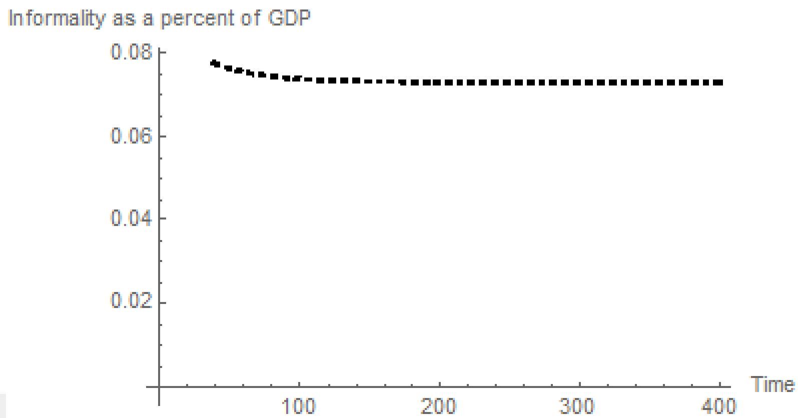

Figure 7. Evolution of informality level after the shock

2.4.4 Shock to elasticity of leisure

Table 4. Baseline Scenario vs. Shock to Elasticity of Leisure

	Baseline Scenario	Shock to Elasticity of Leisure
K	12.125	9.661
L_f	0.487	0.388
L_i	0.044	0.044
$L_f + Li$	0.531	0.432
Y_f	1.650	1.314
Y_i	0.167	0.167
$Y = Y_f + Y_i$	1.817	1.482
Y_i/Y	0.092	0.113
K/Y	6.673	6.519
$\sigma_{\tilde{k}}/\sigma_{k_0}$	0.829	1.094
$\sigma_{\tilde{y}}/\sigma_{\tilde{k}}$	0.159	0.108
$\sigma_{\tilde{y}}$	0.132	0.098

Assuming that economy is in the baseline scenario equilibrium initially, after a 50 percent increase in the elasticity of leisure parameter, first thing we observe is that individuals buy more leisure since the marginal utility of consuming one extra unit of leisure increases. Total labour supply decreases by 19%. Increase in the elasticity of leisure brings the economy to a steady-state capital level lower than its initial capital level. The capital level decreases by almost 20%. The total output also decreases by 18%. The interesting result is that the decrease in the total labour supply occurred only in the formal sector. Informal sector employment remained constant.

During the transition, the wealth inequality increases by 32% whereas the income inequality decreases by 26%.

The transitional dynamics following the shock are given below:

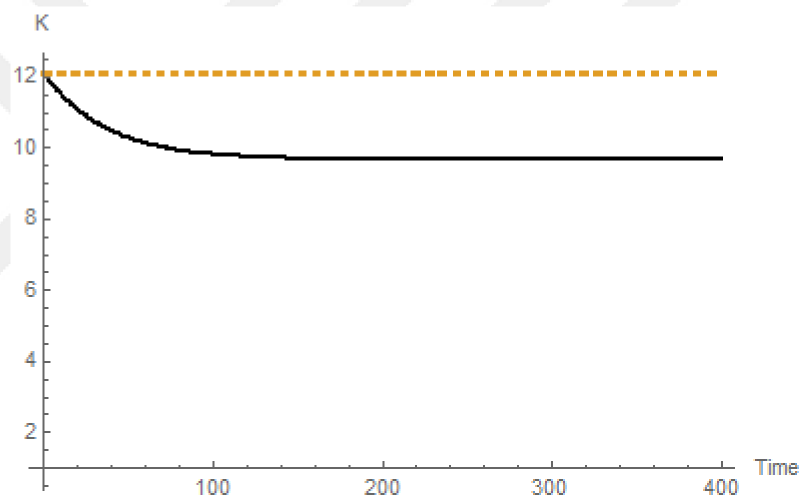

Figure 8. Evolution of capital after the shock

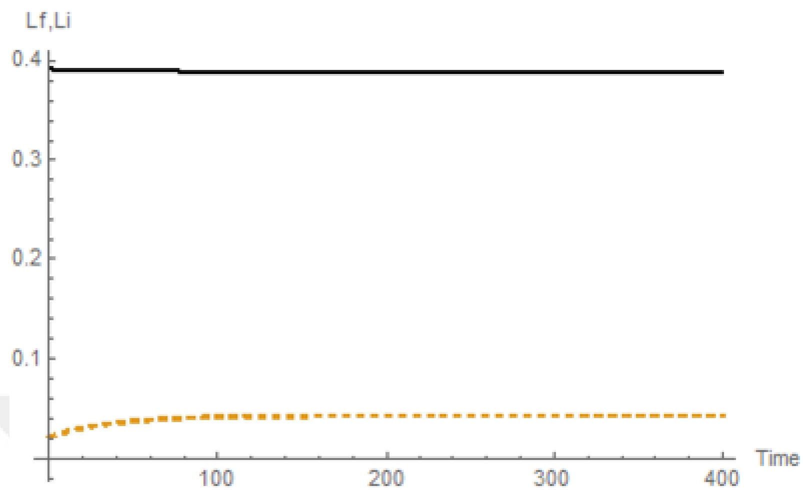

Figure 9. Evolution of employments after the shock

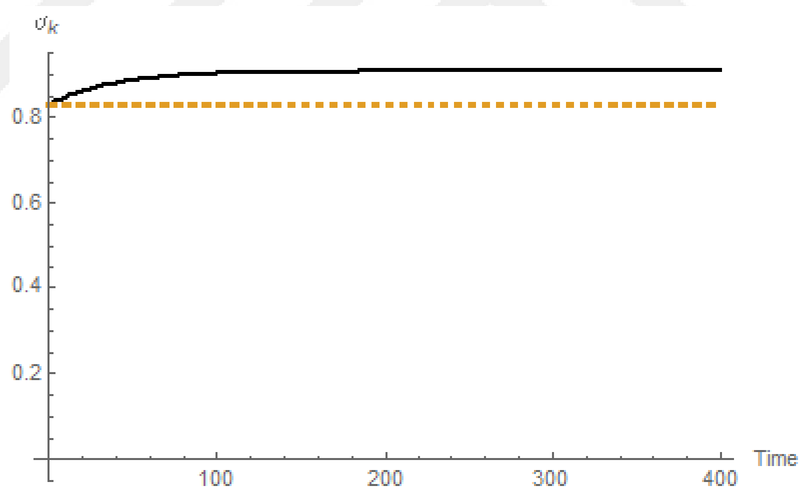

Figure 10. Evolution of capital distribution after the shock

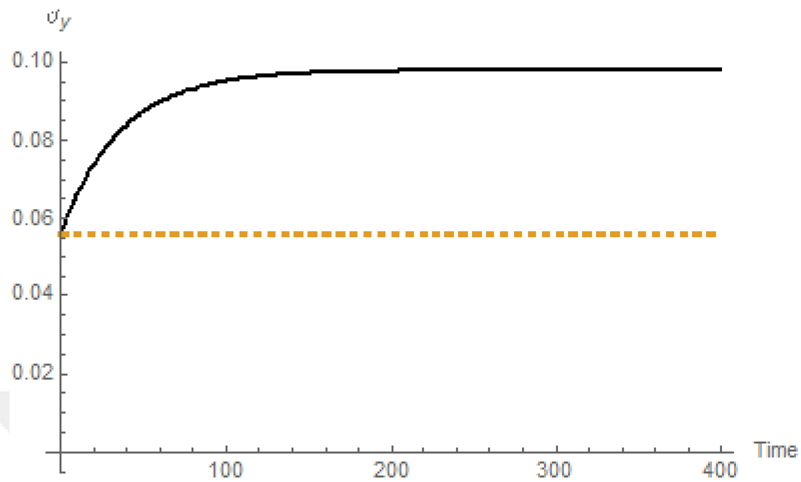

Figure 11. Evolution of income distribution after the shock

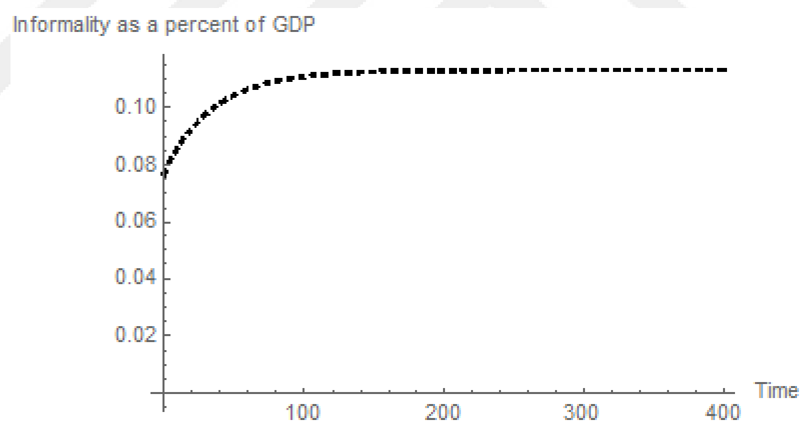

Figure 12. Evolution of informality level after the shock

CHAPTER 3

SKILL HETEROGENEITY MODEL

3.1 Analytical framework and deriving the macroeconomic Equilibrium

In this chapter, we extend the benchmark model in the Chapter 2 by introducing a second source of heterogeneity which is relative skills, denoted by s_i. Now, it is assumed that agents start their lives with both different initial endowments of capital and different abilities. Our aim is to understand how initial wealth heterogeneity and skill heterogeneity affect the outcome distributions of wealth and income at the steady-state.

Average relative skills in the economy is 1 since there is a mass of 1 individuals in the economy and $\sum_i s_i = 1$. We assume that s_i's are constant. The standard deviation of the distribution of s_i is defined as σ_s. Moreover, it is assumed that there is a constant correlation between initial wealth and skills which is denoted by $\Omega_{k0,s}$.

Introduction of skill heterogeneity creates wage differences between agents:

$$\begin{aligned} w_{f,i}(K, L_f) &= s_i \theta_f (1-\alpha) \left(\frac{K}{L_f}\right)^{\alpha} = s_i w_f \\ w_{i,i}(L_i) &= s_i \theta_i \phi L_i^{\phi-1} = s_i w_i, \end{aligned}$$

where w_f and w_i are average wage rates in the formal and informal sectors, respectively.

We define $\psi \equiv \frac{l_i}{l}$ as in the Chapter 2, and we define

$$\nu \equiv \sum_i s_i \psi_i \tag{37}$$

as Garcia-Penalosa & Turnovsky has defined in their 2013 dated working paper. The definition is of crucial important for the aggregation over individuals.

The consumer's utility maximization problem changes slightly as given below:

$$
\begin{aligned}
\max \quad & \int_0^\infty \frac{1}{\gamma}(C_i(t) l_i^\eta)^\gamma e^{-\beta t} dt, \\
& with -\infty < \gamma < 1, \;\; \eta > 0, \;\; 1 > \gamma(1+\eta) \\
& \text{subject to} \\
& \dot{K}_i = (1-\tau_k) r K_i + (1-\tau_w) s_i w_f L_{i,f} + (1-\rho\tau_w) s_i w_i L_{i,i} - C_i + V_i \\
& l_i = 1 - L_{i,f} - L_{i,i}
\end{aligned}
$$

We assume that V_i does not affect the distributional issues in the model, as we assumed in our benchmark model in the Chapter 2. This is done by $\frac{V_i}{K_i} = \frac{V}{K}$.

Writing Hamiltonian and taking first order conditions of this maximization problem yield us the necessary equations for the characterization of equilibrium. Thanks to the definition in Eq.(37), the characterization of equilibrium of this model does not differ from that of the model in the Chapter 2. The steady-state analytical expressions for K, L_f, L_i, and stability analysis are all valid for this model too. Therefore we directly begin with analysing the distributional dynamics.

3.2 Wealth inequality

We begin with writing the relative capital stock of individual i and taking the time derivative of it as in the Chapter 2:

$$\begin{aligned}
\dot{k_i}(t) &= k_i(t)\left(\frac{\dot{K}_i(t)}{K_i(t)} - \frac{\dot{K}(t)}{K(t)}\right) \\
&= (1-\tau_w)\frac{w_f}{K(t)}\left[\left(s_i - s_i\psi_i l\frac{\eta+1}{\eta}\right) - k_i(t)\left(1 - l\frac{\eta+1}{\eta}\right)\right] \qquad (38)
\end{aligned}$$

We know that in the equilibrium, relative capital stock of individual i will be constant, i.e. $\dot{k_i}(t) = 0$:

$$\left(1 - \psi_i\tilde{l}\frac{\eta+1}{\eta}\right) = \left(1 - \tilde{l}\frac{\eta+1}{\eta}\right)\frac{\tilde{k}_i}{s_i} \qquad (39)$$

Subtracting $\left(1 - \tilde{l}\frac{\eta+1}{\eta}\right)$ from both sides yields us the relationship between the labour supply, initial endowment of wealth and relative skills, as follows:

$$\tilde{l}_i - \tilde{l} = \left(\tilde{l} - \frac{\eta}{\eta+1}\right)\left(\frac{\tilde{k}_i}{s_i} - 1\right) \qquad (40)$$

for each i.

Now, we linearise the $\dot{k_i}(t)$ around $\tilde{K}$, $\tilde{L_f}$, $\tilde{l}$ and $\tilde{k}_i$ and obtain the following

$$\dot{k_i}(t) = (1-\tau_w)\frac{w_f}{\tilde{K}}\left[\left(1+\frac{1}{\eta}\right)(\tilde{k}_i - s_i\psi_i)(l-\tilde{l}) + \left(\tilde{l}\left(1+\frac{1}{\eta}\right) - 1\right)(k_i - \tilde{k}_i)\right] \qquad (41)$$

Stable solution to $\dot{k_i}(t)$ is given by:

$$k_i(t) = \tilde{k}_i + \frac{(1-\tau_w)\frac{w_f}{\tilde{K}}\left[\left(1+\frac{1}{\eta}\right)(\tilde{k}_i - s_i\psi_i)(l(0)-\tilde{l})\right]\exp^{\mu_1 t}}{\mu-(1-\tau_w)\frac{w_f}{\tilde{K}}\left(\tilde{l}\left(1+\frac{1}{\eta}\right)-1\right)} \tag{42}$$

Setting $t = 0$ in Eq.(42) we obtain:

$$k_{i0} = \tilde{k}_i + \frac{(1-\tau_w)\frac{w_f}{\tilde{K}}\left[\left(1+\frac{1}{\eta}\right)(\tilde{k}_i - s_i\psi_i)(l(0)-\tilde{l})\right]}{\mu-(1-\tau_w)\frac{w_f}{\tilde{K}}\left(\tilde{l}\left(1+\frac{1}{\eta}\right)-1\right)} \tag{43}$$

To have an understanding of the distributional dynamics we need to express them in terms of standard deviations or variances. To make it easier, we will use Eq.(40) to substitute $s_i\psi_i$ in Eq.(42). Doing a few algebraic operations on Eq.(40), we may write the following:

$$\tilde{k}_i - \psi_i s_i = \frac{1}{\tilde{l}}\frac{\eta}{\eta+1}\left(\tilde{k}_i - s_i\right) \tag{44}$$

Substituting Eq.(44) into Eq.(42) and arranging it yields:

$$k_i(t) = \tilde{k}_i\left(1+\frac{1}{\mu_1-B}(1-\tau_w)\frac{w_f}{\tilde{K}}\frac{l(t)-\tilde{l}}{\tilde{l}}\right) + s_i\left(\frac{1}{\mu_1-B}(1-\tau_w)\frac{w_f}{\tilde{K}}\frac{l(t)-\tilde{l}}{\tilde{l}}\right) \tag{45}$$

where

$$\pi(t) = \frac{1}{\mu_1 - B}(1-\tau_w)\frac{w_f}{\tilde{K}}\frac{l(t)-\tilde{l}}{\tilde{l}},$$
$$B = (1-\tau_w)\frac{w_f}{\tilde{K}}\left(\tilde{l}\left(1+\frac{1}{\eta}\right)-1\right).$$

Now, it is straightforward to show that:

$$k_{i0} = (1+\pi(0))\tilde{k_i} + (-\pi(0))s_i$$
$$and$$
$$k_t = (1+\pi(t))\tilde{k_i} + (-\pi(t))s_i$$

We will use these two equations to write down the relationships between the variances of $k_0, \tilde{k_i}, k_i(t)$ and s_i. Correlations will be denoted by Ω. We assume that the correlation between the initial wealth distribution and skill distribution, $\Omega_{k0,s}$, is known. There are four equations below with one unknown variable each and these four equations allow us to find the variances of $k_0, \tilde{k_i}, k_i(t)$, s_i and correlations between them:

$$\sigma_{\tilde{k}}{}^2 = \frac{1}{(1+\pi(t))^2}[\sigma_{k0}{}^2 + \pi^2(t)\sigma_s{}^2 + 2\pi(0)\sigma_{k0}\sigma_s\Omega_{k0,s}] \quad (46)$$
$$\sigma_{k0}{}^2 = (1+\pi(t))^2\sigma_{\tilde{k}}{}^2 + \pi^2(t)\sigma_s{}^2 + 2(1+\pi()0)(-\pi(0))\sigma_{\tilde{k}}\sigma_s\Omega_{\tilde{k},s} \quad (47)$$
$$\sigma_{k(t)}{}^2 = (1+\pi(t))^2\sigma_{\tilde{k}}{}^2 + \pi^2(t)\sigma_s{}^2 + 2(1+\pi(t))(-\pi(t))\sigma_{\tilde{k}}\sigma_s\Omega_{\tilde{k},s} \quad (48)$$
$$\sigma_{\tilde{k}}{}^2 = \frac{1}{(1+\pi(t))^2}[\sigma_{k(t)}{}^2 + \pi^2(t)\sigma_s{}^2 + 2\pi(t)\sigma_{k(t)}\sigma_s\Omega_{k_i(t),s}] \quad (49)$$

3.3 Income inequality

In order to analyse the distribution of income we will again focus on relative income of individual i. We, as in the Chapter 2, denote the share of output going to capital as $s = \frac{(1-\tau_k)rK}{Y}$. First we write the individual income and aggregate income in the economy:

$$\begin{aligned} Y_i(t) &= (1-\tau_K)r(t)K_i(t) + (1-\tau_w)s_i w_f(1-\psi_i l(t)), \\ Y(t) &= (1-\tau_K)r(t)K(t) + (1-\tau_w)w_f(1-l(t)), \\ y_i(t) &= \frac{Y_i(t)}{Y(t)}. \end{aligned}$$

Relative income equation can be arranged as follows:

$$\begin{aligned} y_i(t) - 1 &= \frac{Y_i(t)}{Y(t)} - \frac{Y(t)}{Y(t)} \\ y_i(t) &= sk_i(t) + \frac{(1-s)(1-l_i(t))}{(1-l(t)}s_i - s + \frac{(1-s)(-1+l(t))}{1-l(t)} \\ y_i(t) &= sk_i(t) + \frac{(1-s)}{1-l(t)}s_i + \frac{(s-1)l(t)}{1-l(t)}s_i\psi_i \end{aligned} \tag{50}$$

The last term in the last equation contains individual characteristics which are ψ_i and s_i at the same time. We will treat $\psi_i s_i$ as if it is a single variable with its own distribution and we will find its variance first before moving to variance of $y_i(t)$.

$$\sigma_{\psi_i s_i} = {\sigma_{\psi_i}}^2 + {\sigma_{s_i}}^2 + 2Cov(\psi_i, s_i)$$

We assume the correlation between the ψ_i and s_i is equal to the correlation between s_i and $\tilde{k}_i$ because $\tilde{k}_i$ and s_i are linear in each other.

Since we know that $Cov(\psi_i, s_i) = Corr(\psi_i, s_i)\sigma_{\psi_i}\sigma_s$, the covariance term between ψ_i and s_i can be calculated.

Rewriting Eq.(50) in terms of variances yields us the following:

$$\begin{aligned}
{\sigma_{y_i(t)}}^2 &= s^2(t){\sigma_{k_i(t)}}^2 + \frac{(1-s)^2}{(1-l(t))^2}{\sigma_s}^2 \\
&+ \frac{(s-1)^2 l^2(t)}{(1-l(t))^2}({\sigma_{\psi_i}}^2 + {\sigma_{s_i}}^2 + 2Cov(\psi_i, s_i)) \\
&+ \frac{2s(1-s)}{1-l(t)}\sigma_{k(t)}\sigma_s + \frac{2s(s-1)l(t)}{1-l(t)}\sigma_{k(t)}\sigma_{\psi_i s_i} \\
&+ \frac{2(s-1)(1-s)l(t)}{(1-l(t))^2}\sigma_s\sigma_{\psi_i s_i}
\end{aligned} \tag{51}$$

Eq.(51) expresses the income inequality in terms of variances of other distributions.

3.4 Calibration

The parameter assignments used for calibration are given below. The standard deviations of initial capital distribution and ability distribution, and also the correlation between these two have been taken from Garcia-Penalosa & Turnovsky (2013). The high informality corresponds to 30% informal sector. Low inequality of skills implies a standard deviation of 2 for skill distribution whereas high inequality of skills corresponds to a standard deviation of 4 for skill distribution.

Table 5. Parameter Assignments for Calibration

Parameter	Description	Value
ϕ	Factor intensity of labour in the informal sector production	0.5
ρ	Tax enforcement rate	0.5
η	Elasticity of leisure in the utility function	0.5
β	Time discount rate	0.04
α	Factor intensity of capital in the formal sector production	0.35
θ_f	Formal Sector TFP	1.1
θ_i	Informal Sector TFP	0.8-1.5
τ_w	Tax on labour income	0.16
τ_k	Tax on capital income	0.16
$1/(1-\gamma)$	Intertemporal Elasticity of Substitution	0.4
σ_{k0}	St.Dev. of initial capital distribution	14
σ_s	St.Dev. of skill distribution	2-4
$\Omega_{k0,s}$	Correlation between initial capital and skills	0.33-0.66

3.5 Numerical exercises

3.5.1 Results for $\Omega_{k0,s} = 0.33$

Assuming a low correlation (0.33) between the initial capital distribution and skill distribution, we simulate four economies with low/high informality and low/high inequality of skills.

It is observed that high informal economy is more equal than the low informal economy in terms of both wealth and income distribution. Moreover, skill heterogeneity has an immense effect on the income distribution whereas it has minimal effects on the wealth distribution. The effect of skill heterogeneity is magnified when the economy has a higher informality level.

Table 6. Wealth Distribution in the Case $\Omega_{k0,s} = 0.33$

Wealth Inequality		
	Low Informality	High Informality
Low Inequality of Skills	149.91	95.43
High Inequality of Skills	152.23	100.66

Table 7. Income Distribution in the Case $\Omega_{k0,s} = 0.33$

Income Inequality		
	Low Informality	High Informality
Low Inequality of Skills	14.73	11.32
High Inequality of Skills	29.84	25.16

3.5.2 Results for $\Omega_{k0,s} = 0.66$

When the correlation between the initial capital distribution and skill distribution is increased, wealth inequality increases in all four cases (low informality-low inequality of skills, low informality-high inequality of skills, high informality-high inequality of skills, high informality-low inequality of skills) compared to the lower correlation case. However, income inequality decreases in all four cases.

Table 8. Wealth Distribution in the Case $\Omega_{k0,s} = 0.66$

Wealth Inequality		
	Low Informality	High Informality
Low Inequality of Skills	152.02	99.44
High Inequality of Skills	156.46	108.68

Table 9. Income Distribution in the Case $\Omega_{k0,s} = 0.66$

Income Inequality		
	Low Informality	High Informality
Low Inequality of Skills	12.72	9.91
High Inequality of Skills	25.66	22.37

3.5.3 Shock to formal sector productivity

Table 10. Baseline Scenario vs. Shock to Formal Sector Productivity

	Baseline Scenario	Shock to θ_f
K	12.125	14.217
L_f	0.487	0.493
L_i	0.044	0.036
$L_f + Li$	0.531	0.529
Y_f	1.650	1.934
Y_i	0.167	0.152
$Y = Y_f + Y_i$	1.817	2.086
Y_i/Y	0.092	0.073
K/Y	6.673	6.814
${\sigma_k}^2$	149.905	140.106
${\sigma_y}^2$	14.731	14.327

Assuming that the economy is initially at the steady-state which is the baseline scenario equilibrium, after a 10% increase in the formal sector productivity, it is observed that the economy ends with a higher capital level because of increased productivity. The allocation of labour between formal and informal sector changes in favour of formal sector slightly. However, the total output rises by almost 18%.

An interesting result from the this exercise is that a 10% productivity shock to formal sector increases steady-state capital level, decreases informality level by 2%

and decreases the wealth equality significantly. Income inequality does not change significantly.

In the benchmark model, an increase in formal sector productivity increases the steady-state capital level and decreases the informality level as in this model, however, wealth inequality increases in response to total factor productivity rise in the benchmark model.

The transitional dynamics followed the shock are given below:

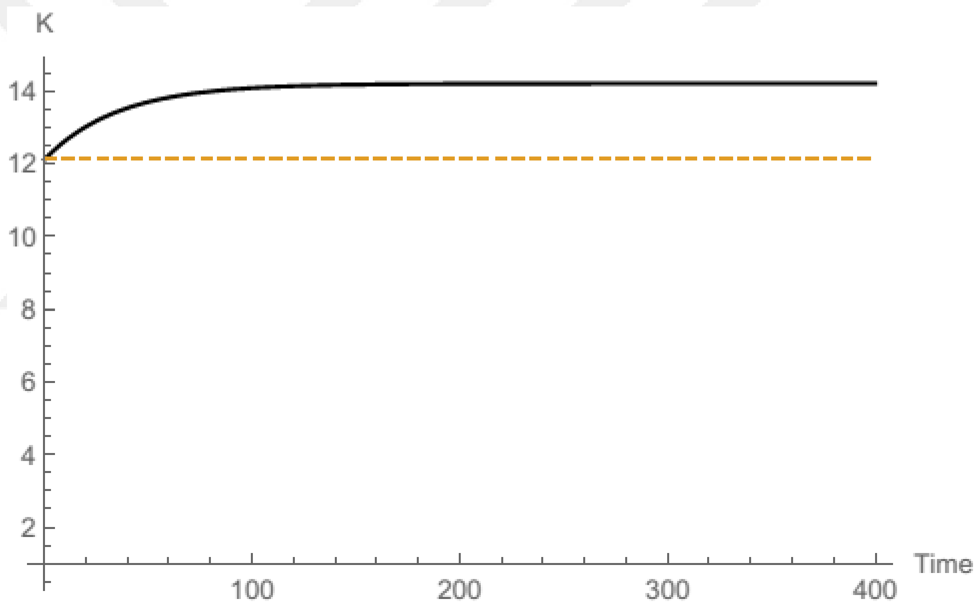

Figure 13. Evolution of capital after the shock

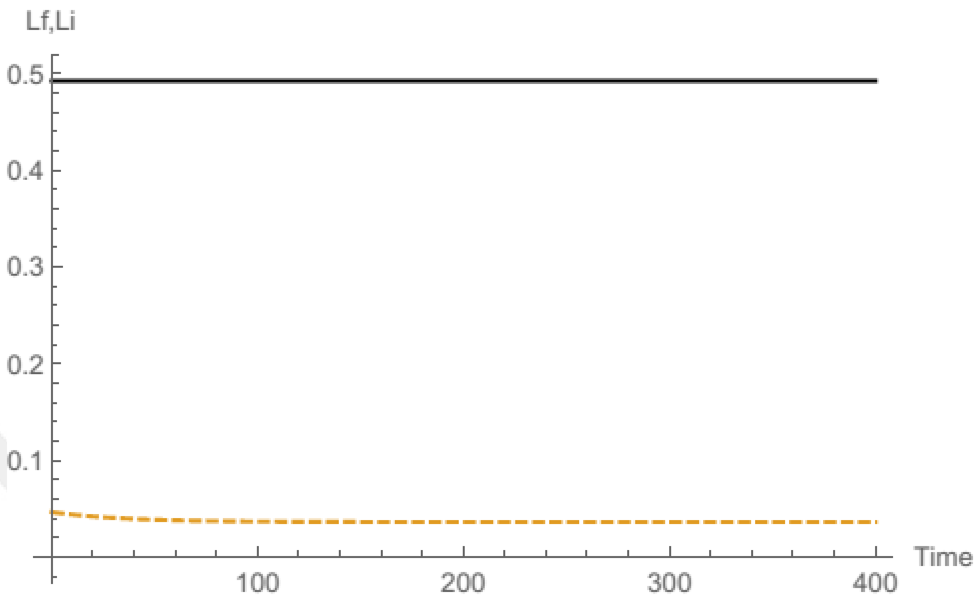

Figure 14. Evolution of employments after the shock

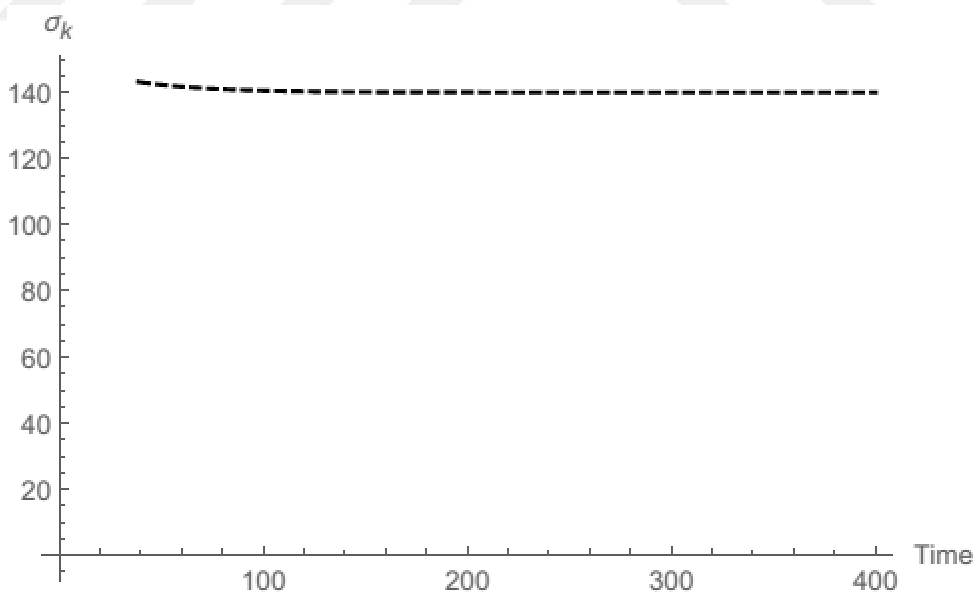

Figure 15. Evolution of capital distribution after the shock

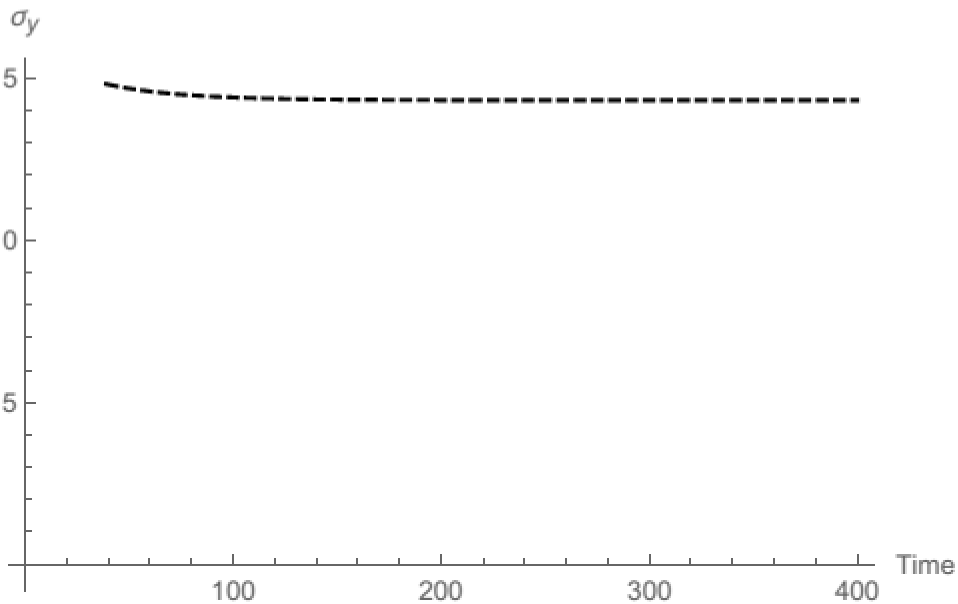

Figure 16. Evolution of income distribution after the shock

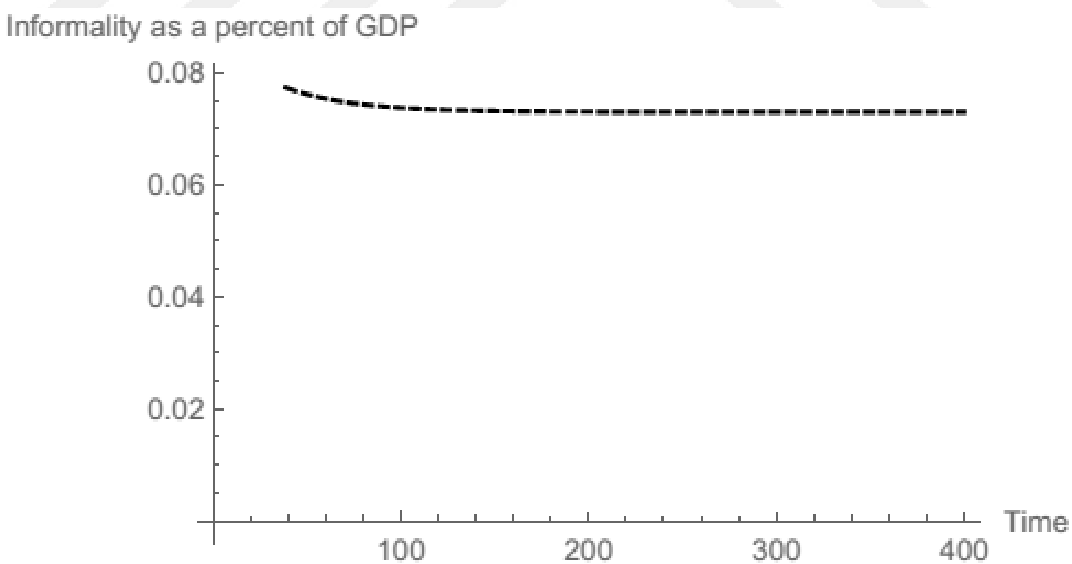

Figure 17. Evolution of informality level after the shock

3.5.4 Shock to elasticity of leisure

Table 11. Baseline Scenario vs. Shock to Elasticity of Leisure

	Baseline Scenario	Shock to η
K	12.125	9.661
L_f	0.487	0.388
L_i	0.044	0.044
$L_f + Li$	0.531	0.432
Y_f	1.650	1.314
Y_i	0.167	0.167
$Y = Y_f + Y_i$	1.817	1.482
Y_i/Y	0.092	0.113
K/Y	6.673	6.519
${\sigma_k}^2$	149.905	172.850
${\sigma_y}^2$	14.731	12.735

After a 10% increase in the elasticity of leisure, the economy ends up with a 25% lower steady-state capital level and the labour supply also decreases significantly. Although informal employment does not increase the share of informal sector in total output increases due to reduction in the formal sector employment.

When the elasticity of leisure increases, the wealth inequality increases very significantly. This is because the utilization of labour supply-wealth relationship decreases when the elasticity of leisure increases. In the new economy, relatively poor agents do not supply more labour to economy which has a mitigating effect on the wealth inequality, instead they prefer to buy more leisure.

The transitional dynamics followed the shock are given below:

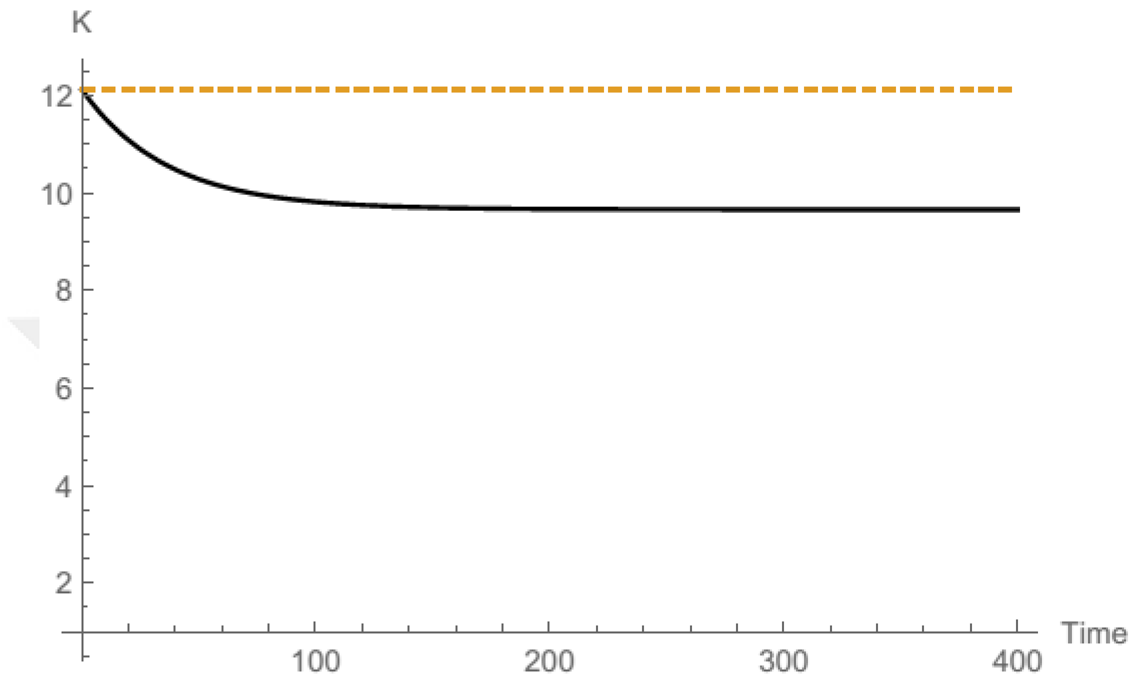

Figure 18. Evolution of capital after the shock

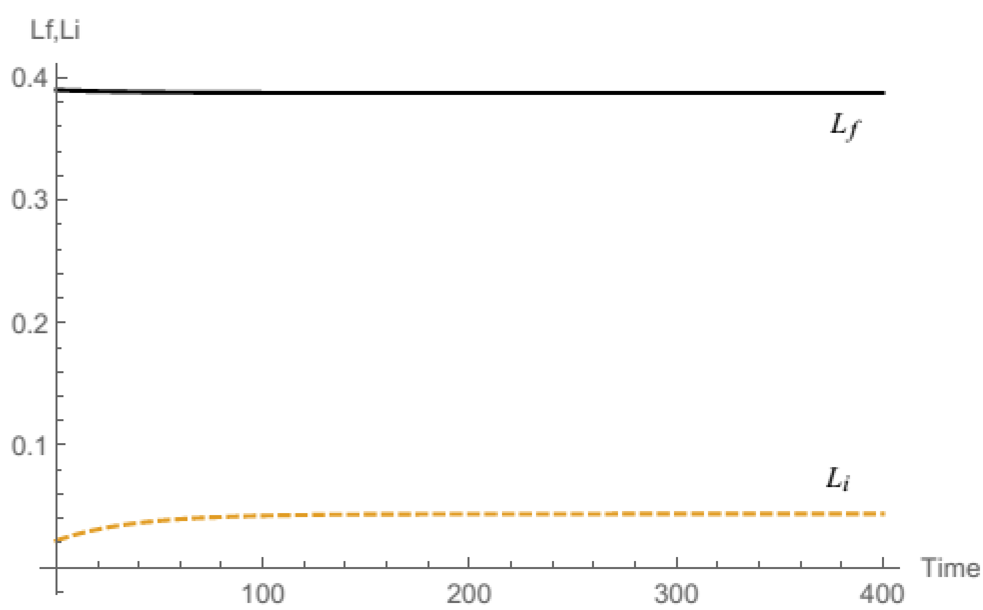

Figure 19. Evolution of employments after the shock

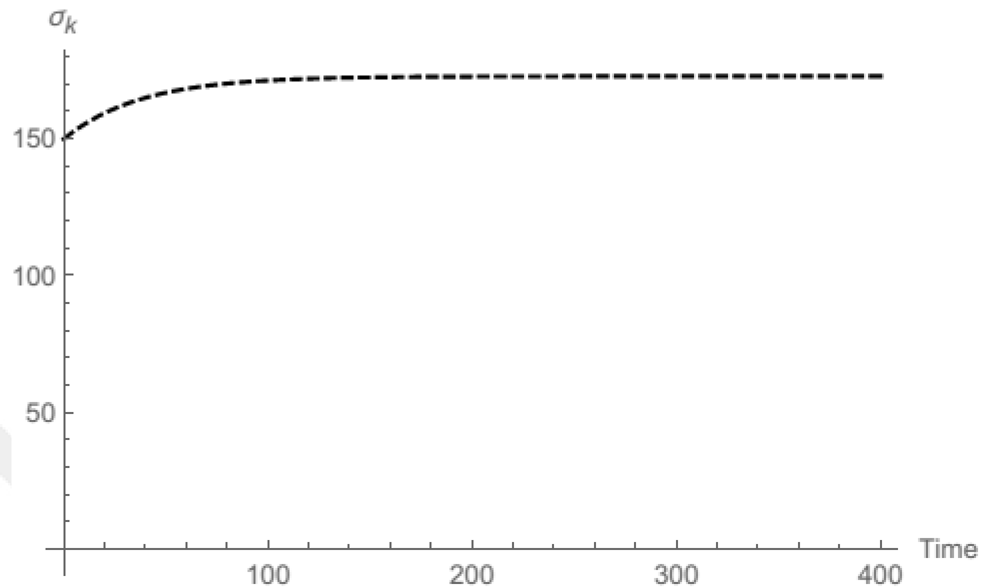

Figure 20. Evolution of capital distribution after the shock

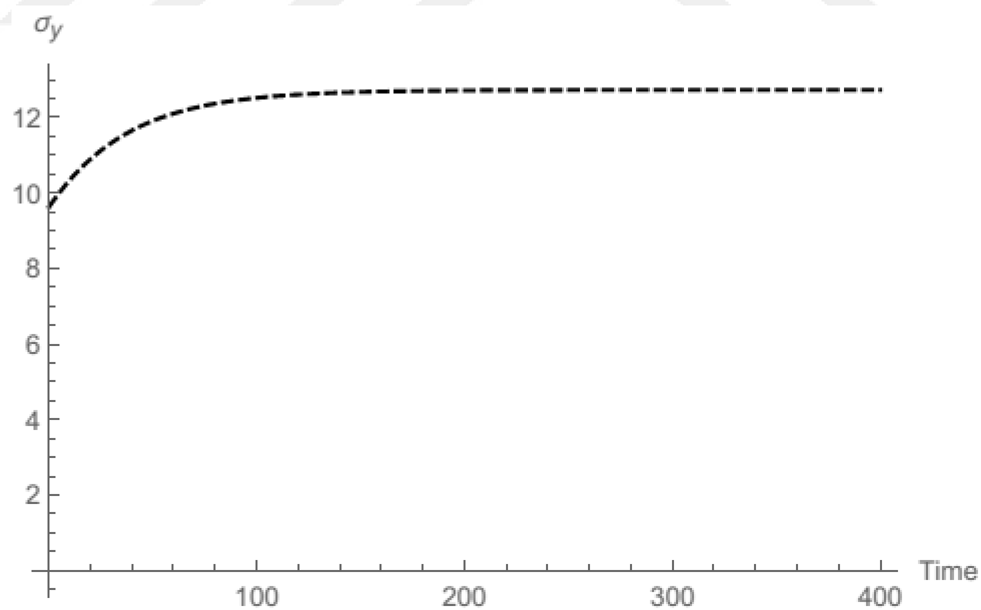

Figure 21. Evolution of income distribution after the shock

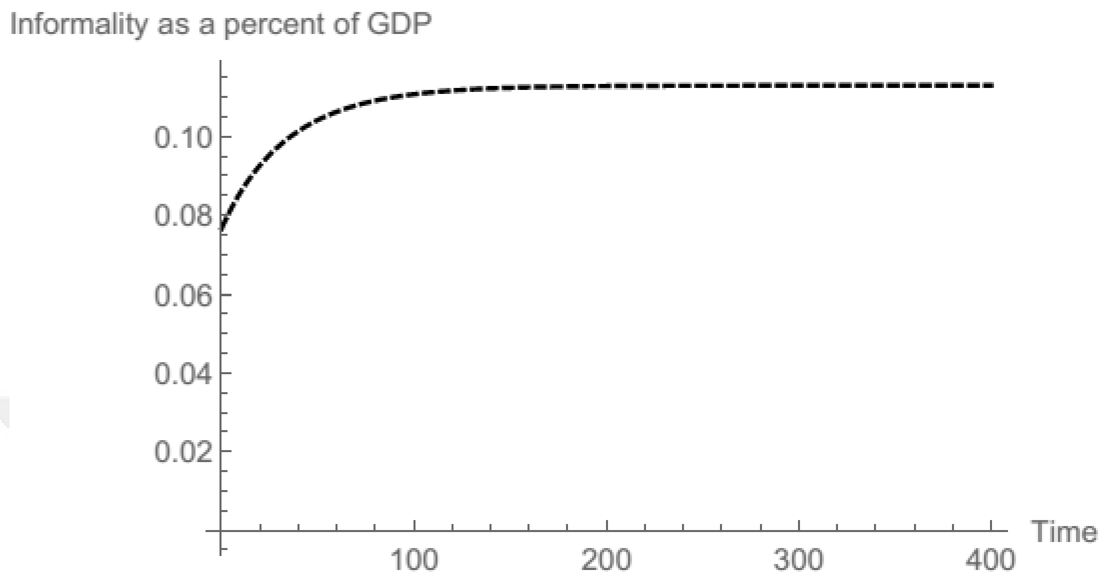

Figure 22. Evolution of informality level after the shock

CHAPTER 4

CONCLUSION

We build a two-sector model of economy in Ramsey type growth model with heterogeneous agents in terms of their initial endowments of capital. We model the informality very similar to two-sector model of Ihrig & Moe (2004). We have dual production structure which has formal and informal sector production. Formal sector is where the producers have access to capital market whereas informal sector producers have no access to capital markets. However, informal sector has the possibility of tax avoidance.

The endogenous labour supply is assumed in the model. This assumption is of crucial importance in terms of distributional issues since agents choose their level of labour supply considering the taxation policies and also the level of capital they hold. The outcome distributions of wealth and income are endogenously determined in the model.

We investigate how the inequality dynamics change with the informality level of the economy. To this end, we simulate two economies one with an approximately 10% of informality and the other with a 30% of informality. Our model indicates that the economy with a high informality level ends up with a lower capital stock whereas two economies have similar employment levels in total. However, the inequality in wealth and income decreases significantly if the economy has a high informality level. This is largely caused by the relatively lower capital stock and it is in line with the empirical evidence which suggests that there is a positive relationship between the inequality and economic growth. In this model, the informality is the channel which leads to a lower total stock of capital.

In Chapter 3, we introduce a second source of heterogeneity, which is relative skills, in addition to initial endowments of capital. We simulate the model economy for two main cases which are low correlation and high correlation between the initial endowments of capital and relative skills. In both simulation cases we see that the economy with a higher informality level ends up with less inequality in terms of both income and wealth.

The effect of skill heterogeneity differs in magnitude for wealth inequality and income inequality. The simulations show that an increase in the skill heterogeneity causes the income inequality to double at least, whereas it has relatively minimal increasing effects on the wealth inequality.

An interesting result drawn from the simulations is that while the wealth inequality levels in the low correlation case are higher than the high correlation case, the income inequality is lower in the high correlation case. The mechanism behind is that when the skill distribution is more correlated with initial endowments of capital, the endogenous labour supply mechanism works in favour of those who are relatively poor. Hence, the poor supplies more labour whereas the rich supplies relatively less labour which makes the income distribution more equal.

APPENDIX A

STABILITY OF THE NUMERICAL EXERCISES

After the calibration of the model in Chapter 2, we show that one of the eigenvalues of the dynamic system and the determinant of the Jacobian matrix are negative which indicates that the system is saddle-path stable.

Table 12. Stability of Dynamic System

	Baseline Scenario	High Informality
Determinant of Jacobian Matrix	-0.0757067	-0.00317751
Eigenvalue-1	12.7064	8.03745
Eigenvalue-2	0.219044	0.0741782
Eigenvalue-3	-0.0272007	-0.00532957

APPENDIX B

OBTAINING THE STABLE SOLUTION TO $\dot{k_i}(t)$

We begin by rewriting Eq.(29) as in the following form:

$$
\begin{aligned}
& \dot{k_i}(t) - (1-\tau_w)\frac{w_f}{K(t)}\left(\tilde{l}\left(1+\frac{1}{\eta}\right)-1\right)k_i(t) \\
= \; & (1-\tau_w)\frac{w_f}{K(t)}\left[\left(1+\frac{1}{\eta}\right)(\tilde{k_i}-\psi_i)(l-\tilde{l}) + \left(\tilde{l}\left(1+\frac{1}{\eta}\right)-1\right)\tilde{k_i})\right]
\end{aligned}
$$

where

$$
\begin{aligned}
B &= (1-\tau_w)\frac{w_f}{\tilde{K}}\left(\tilde{l}\left(1+\frac{1}{\eta}\right)-1\right) \\
Z &= (1-\tau_w)\frac{w_f}{K(t)}\left(1+\frac{1}{\eta}\right)(\tilde{k_i}-\psi_i)(l-\tilde{l}).
\end{aligned}
$$

Using B and Z, we can rewrite Eq.(29) as follows:

$$
\begin{aligned}
\dot{k_i}(t) - Bk_i(t) &= Z\exp^{\mu_1 t} + B\tilde{k_i} \\
\int [k_i(t) - Bk_i(t)]\exp^{-Bt}\,dt &= \int [Z\exp^{\mu_1 t} + B\tilde{k_i}]\exp^{-Bt}\,dt
\end{aligned}
$$

Taking this integral yields the following:

$$
k_i(t) = \frac{Z}{\mu_1 - B}\exp^{\mu_1 t}\tilde{k_i} + Constant\exp^{Bt}
$$

Since $Y > 0$, we know that the constant should be equal to 0, because otherwise $k_i(t)$ would grow exponentially. Hence the stable solution to $\dot{k_i}(t)$ is:

$$k_i(t) = \tilde{k_i} + \frac{Z \exp^{\mu_1 t}}{\mu_1 - B}.$$

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