ON THE NUMBER OF BRACKETS IN PIECEWISE LINEAR INCOME TAX SYSTEMS: AN APPLICATION

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On The Number of Brackets in Piecewise Linear

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DECLARATION OF ORIGINALITY

I, Elif Bodur, certify that

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ABSTRACT

On The Number of Brackets in Piecewise Linear

Income Tax Systems: An Application

This thesis aims to explore the welfare impacts of using different number of brackets to collect a specific amount of tax revenue in a piecewise linear income tax system. It does this by analyzing the effect of the number of brackets on optimal cut-off income levels as well as marginal tax rates for each bracket. Individuals with different abilities (wages) have a standard utility function defined over a consumption good and labor hours, whereas the social planner has increasing and strictly concave social welfare function. The model is simulated for Turkey using the 2014 Household Labor Force Survey Data of the Turkish Statistical Institute. Taking into account labor supply responses of each individual into account, the simulations using one, two, and three-brackets searched for the optimal cut-off income levels and marginal tax rates for each bracket to raise the same income tax revenue collected from a reference sample in 2014. Total social welfare in all simulations are higher than the current four-bracket piecewise linear tax system achieved.

ÖZET

Parçalı Lineer Gelir Vergisi Sistemlerinde
Vergi Dilimlerinin Sayısı Üzerine Bir Uygulama

Bu tez parçalı lineer gelir vergisi sisteminde, belirlenen miktarda vergi geliri toplamak için farklı vergi dilimleri sayıları kullanmanın refah etkisini araştırmayı amaçlamaktadır. Araştırma, vergi dilimleri sayılarının optimal dilim sınırlarına ve dilimlerdeki optimal vergi oranlarına etkisi analiz edilerek gerçekleştirildi. Farklı beceri düzeyleri olan (farklı ücret kazanan) hanehalkları tüketim ve çalışma saati üzerine tanımlanan standart bir fayda fonksiyonuna, sosyal planlayıcı ise artan ve tam konkav bir sosyal refah fonksiyonuna sahiptir. Model Türkiye İstatistik Kurumu Hanehalkı İşgücü Anketi 2014 verileri kullanılarak simüle edildi. Bir, iki ve üç dilimli vergi sistemleri hanehalklarının işgücü arzı tepkisi göz önünde bulundurularak simüle edildi ve 2014 yılında referans örneklemden toplanan gelir vergisi gelirini elde eden optimal dilim sınırları ile dilimlerdeki optimal marjinal vergi oranları bulundu. Tüm simülasyonlarda ulaşılan sosyal refah seviyesinin 2014 yılında uygulamada olan dört dilimli vergi sisteminin ulaştığı sosyal refah seviyesinden daha yüksek olduğu görüldü.

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CHAPTER 1

INTRODUCTION

Mirrlees' pioneering study (1971) unfolds the structure of a general optimal nonlinear income tax structure. Mirrless, under the assumption that the cost of administering the optimum tax schedule is negligible and an assumption of lognormal distribution of wages, found out that the optimal nonlinear income tax function is approximately linear, exhibiting decreasing marginal tax rates as income increases [1]. Following up on Mirrlees' finding and adopting his general approach, a large number of studies have analyzed the single rate linear income taxation, known as flat tax, as characterized in Stern (1976). However, neither the general nonlinear nor the flat tax systems is used virtually in any country[2].

Slemrod et al. (1994), using a model with a utility function $U = C - 0.5L^2$ and two types of individuals that differ only in terms of wage, show that linear flat tax schedule is always Pareto inferior to the two-bracket tax schedule. Apps et al. (2013) perform a more detailed and transperant analysis on the subject. Their study adopts the model utilized by Sheshinski (1989) with continuous type of individuals and carries out a numerical analysis with 100 type of individuals. The results of their numerical simulations also point out that that the two-bracket system Pareto dominates the flat tax. Adrienko et al. (2016), as a follow up and extended version of Apps et al. (2013), formulate the problem with the number of brackets $n \geq 2$ as a choice variable, and also allow explicitly for the the determination of tax rates in each

[1] See Tuomala (1990) for a review of the optimal tax theory, and Saez (2001) for a more recent application

[2] Some eastern European countries such as Russia, Lithuania, Serbia, and Ukraine are exceptional users of the flat tax system.

bracket as well as the bracket limits. In their numerical calculations, they search for the optimal parameters, given a social welfare function that allows different degrees of inequality aversion. For each degree of inequality aversion, an increase in number of brackets, which is four at most in their study, results in an increase in social welfare.

There is a substantial amount of research on whether the optimal marginal tax rates are progressive or regressive. Progressivity in a piecewise linear tax system refers to a structure where the higher the income bracket, the higher the marginal tax rate imposed in the bracket is (the "convex" case in Sheshinski, 1989); while regressivity refers to decreasing marginal rates over brackets (the "nonconvex" case in Sheshinski, 1989). Slemrod et al. (1994) points out the discontinuity of tax revenue function in the nonconvex case and refutes the claim by Sheshinski (1989) that the optimal tax structure is increasing in marginal tax rates. The analysis inititated by Slemrod et al. (1994) focusing on the discontinuity of tax revenue function in the nonconvex case was extended by Apps et al. (2013), and they show that the wage distributions adopted in Slemrod et al. (1994) is behind the finding that decreasing marginal tax rates are optimal. The wage distribution used by Slemrod et al. (1994) does not capture the increase in wage inequality of the recent decades. Their numerical results using the Pareto wage distribution show that increasing marginal rates produce higher social welfare. However, they do not study global optima and their results are also not robust to the assumption regarding the distribution of wages.

Despite the differences over the issue in the literature, in almost all countries, income tax systems show considerable marginal rate progressivity, which presumably reflects, in line with the optimal taxation theory, the amount of required revenue, distribution of abilities and income, and equity concerns of the government. Under the assumption of positive labor supply elasticity, progressive tax systems aim to decrease the tax burden on low income groups, and an increase in the number of brackets in a piecewise linear tax systems work towards that end. On the other hand, a progressive marginal tax structure leads to distortion among ’high’ income earners, hence to a decrease in the tax base. Sadka (1976) and Seade (1977) show that marginal tax rates should be zero at top income levels if the income distribution function is bounded from above. On the other hand, Dahlby (1998) shows that marginal cost of revenue raised may be very high when only a small fraction of the taxpayers’ income is subject to the top marginal tax rate. That occurs because the burden of raising the required amount of tax revenue is shared among small group of individuals.

Governments are typically not able to impose optimal non linear income tax schedules. The obstacle primarily stems from not having perfect information about the ’potential’ earning ability of each individual. Even if the government has perfect information on abilities, imposing different marginal tax rates for each income level would be unrealistic to implement and also it would create high administrative costs (Slemrod, 1994). Therefore, personal income taxes are collected through piecewise linear tax systems in many countries including Turkey. The income scale is divided into groups, whereby each category of income levels at margin are taxed with different tax rates. In 1995, the Turkish income tax system had seven brackets, with increasing marginal rates for each consecutive bracket. The number of tax brackets was reduced to six in 2002, and since 2006 the progressive piecewise linear tax schedule used involves four different marginal tax rates. The cut-off levels for each bracket are adjusted each year. Table 1 below presents the schedule employed in 2014, the year for which numerical simulations will be carried out in this study.

This study aims to answer the following question: "What would be the welfare effects of collecting the same amount of revenue raised by the current four-bracket

Table 1. Turkey Labor Income Tax System in 2014

Income Brackets	Rates
Until 11,000 TL	15%
1,650 TL for 11,000 TL of 27,000 TL, for more	20%
4,850 TL for 27,000 TL of 97,000 TL, for more	27%
23,750 TL for 97,000 TL of more than 97,000 TL, for more	35%

Source: Gelir İdaresi Başkanlığı (Revenue Administration)

schedule in Turkey by imposing a tax schedule with a one-bracket, two-bracket, or a three-bracket schedule, where the tax brackets (cutoff income levels) as well as marginal tax rates for each bracket are chosen optimally?" The welfare effects of these revenue-equivalent tax schedules are studied both at the society level and for different income groups separately. The simulations will allow comparison of the one-bracket, two-bracket, and three-bracket schedules with each other as well as each of them with the four-bracket schedule currently in use in Turkey. The effect of inequality aversion (of the social planner) is also investigated.

The steps followed in the analysis are as follows. Given the current tax rates, a formula for labor supply responses of individuals is derived on average for each possible tax rate and cutoff income level. This requires estimating the labor supply elasticity of individuals. The labor supply elasticity estimated using the household data for individuals earning only labor income yielded a labor supply elasticity of -0.058.

The thesis is organized as follows. Chapter 2 gives details and descriptive statistics about the data used, Chapter 3 explains the model used in numerical analyses. Chapter 4 presents the simulation results under the preferred parameter values for the parameters of the model used, carries out counterfactual comparions, and also explores the effect of inequality aversion of the social planner on total welfare. Chapter 5 discusses the results and provides concluding remarks.

CHAPTER 2

DATA

Following the procedure used by Apps et al. (2014), the model (described below in Chapter 3) is simulated for the "reference" wage distribution derived from the refined sample of Household Budget Survey Micro Data Set 2014 of the Turkish Statistical Institute. The sample is constructed using data for individuals above age 15, currently employed as registered workers and working more than 30 hours in a week. Individuals receiving non-labor income of any kind, including welfare payments, are not included.

The sample reduced as above contains 10,207 observations. The descriptive statistics for weekly working hours, annual net income, and net hourly wage are provided in Table 2. Figure A1, A2 and A3 in the Appendix illustrate the distribution of these variables.

Table 2. Descriptive Statistics for Variables of Interest

	Mean	Standard Deviation	Min	Max
Weekly working hours	50.03	11.94	30	99
Annual net income	23,741.94	17,705.17	198	387,925
Net hourly wage	10.72	8.23	0.54	156.62

In the household budget survey, individuals report their annual net income after taxes. Individuals also report their weekly working hours and how many months they have been working on their current job. Net wage was calculated by dividing annual net income with annual working hours reported. The minimum hourly net wage in the data is 0.54 TL, while the maximum is 156.62 TL, and the mean hourly net wage turns out to be 10.72 TL.

Gross annual income and the brackets in which individuals choose their labor supply were obtained by using the tax schedule and annual net income figures for 2014. In the data set of 10,207 observations, individuals are positioned within the tax brackets as follows: 1,033 (10.12%) in the first bracket (with 15% as the marginal tax rate), 4,759 (46.62%) in the second bracket (with 20% as the marginal tax rate) , 4,244 (41.57%) in the third bracket (with 27% as the marginal tax rate), and only 171 (1.67%) of households in the fourth bracket (with 35% as the marginal tax rate).

Following Apps et al. (2014), the "reference" wage distribution is constructed by smoothing annual working hours across the data[3]. Smoothed hourly gross wage percentiles is extracted from the new variable (annual gross income divided by smoothed annual working hours). Figure 1 displays the "reference" gross wage distribution [4].

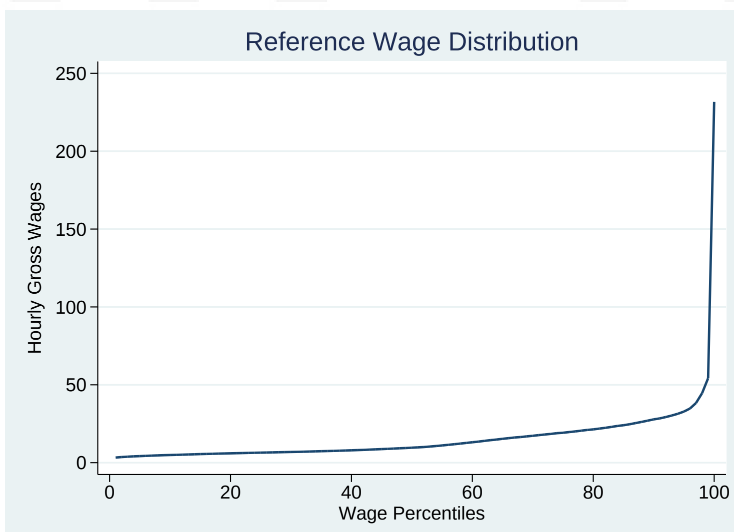

Figure 1. Reference wage distribution

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[3]Lowess method is used to smooth the annual working hours. The bandwidth is taken as 0.3.

[4]The shape of the distribution is very similar to UK, US, Australia data reported in Apps et al. (2014).

The remarkable feature of the gross wage distribution is the jump at the 98th percentile. While the wage levels are first almost flat and then increase slowly until the 98th percentile, there is a sudden jump from 38.67 TL to 160.84 TL at the 99th percentile. In the reference wage distribution consisting of 100 observations, 5 wage earners earn income falling in the first bracket, while 47 of them fall in the second bracket, 45 in the third bracket, and only 2 wage earners fall in the last bracket.

CHAPTER 3

MODEL

In the baseline model, individuals are identical in preferences but differ only in terms of abilities, which is captured by a wage rate w_i, with $w_i \subset R_+$, $i \in \{1, ... , N\}$. Consumption is denoted by c, and labor supply is denoted by l. The choice problems of individuals in Apps et al. (2014) are employed in this study except that individuals do not receive any lump-sum transfer in this model. Utility functions are $u = c - D(l)$, where $D(l) = kl^{\alpha}, \;\; k, l > 0$, with $D' > 0$ and $D'' > 0$. Gross income is given by y=wl. In choosing c and l, an individual with income y faces the following set of inequalities as budget constraint:

$$c \leq (1 - t_1)\, y \qquad\qquad if \quad y \leq \hat{y}_1 \tag{1}$$

$$c \leq (1 - t_n)\, y + \sum_{k=2}^{n-1} (t_k - t_{k-1})\, \hat{y}_{k-1} \qquad\qquad if \quad \hat{y}_{k-1} \leq y \leq \hat{y}_k \tag{2}$$

where t_k is the tax rate in k^{th} bracket, $\hat{y}_k$ is the cut-off income level between k^{th} and $(k+1)^{th}$ bracket. Figure 2 shows an example of an income-consumption space for three-bracket piecewise linear tax system with $t_3 > t_2 > t_1$.

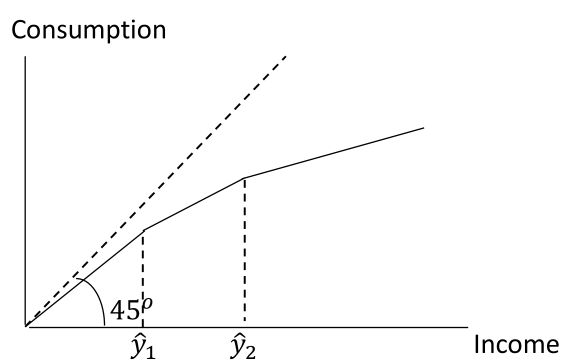

Figure 2. Income consumption space in three-bracket schedule

Given the specifications above, an individuals' choice problem becomes

$$\max_{c,l} \quad u = c - kl^{\alpha}$$

$$s.t. \quad c \le (1-t_1)\, y \qquad\qquad if \quad y \le \hat{y}_1$$

$$c \le (1-t_n)\, y + \sum_{k=2}^{n-1} (t_k - t_{k-1})\, \hat{y}_{k-1} \qquad\qquad if \quad \hat{y}_{k-1} \le y \le \hat{y}_k$$

where $y = wl$, w is the gross net wage rate. The household labor supply is given by

$$\ell = \left(\frac{\hat{w}}{\alpha k} \right)^{1/(1-\alpha)}, \tag{3}$$

and the labor supply elasticity becomes

$$\varepsilon = \frac{d\ell}{dw} \frac{w}{\ell} = \frac{1}{1-\alpha}. \tag{4}$$

Suppose that individual i chooses the m^{th} bracket, $\hat{y}_{m-1} < y_i^* < \hat{y}_m$, then the indirect utility function for i will be $v_i(t_1, ..t_m, \hat{y}_1, ..., \hat{y}_{m-1})$. The planner has a social welfare function (SWF) given by

$$\sum_{i=1}^{100} [\, v_i(.)^{1-\rho}]^{1/(1-\rho)}, \tag{5}$$

and chooses the optimal parameters $(t_1, t_2, ..., t_n, \hat{y}_1, \hat{y}_2, ..., \hat{y}_{n-1})$ to raise revenue R. To illustrate, amount of revenue raised from individuals preferring to be placed on m^{th} bracket, group i_m in this case, is

$$R_m = \sum_{i_m} [\, t_m y_{i_m} + \sum_{p=1}^{m} \hat{y}_p \, (t_p - t_{p+1}) \,]. \tag{6}$$

For example, amount of revenue raised from third-bracket, in a tax schedule with four-bracket, is expressed as $\sum_i [\, t_3 y_i + (t_2 - t_3)\, \hat{y}_2 + (t_1 - t_2)\, \hat{y}_1 \,]$.

First, we will calculate the amount of hypothetical revenue collected from our reference distribution by means of tax schedule in year 2014. Throughout the simulations of the one-bracket, the two-bracket and the three-bracket piecewise linear tax schedules, we search for parameters raising the tax revenue in the $\pm$2 TL interval of the hypothetical revenue.

Instead of deriving the weekly working hours by using wage levels and parameters, another method is preferred in the simulations. The actual values of weekly working hours computed using the labor supply expression above hold in average but fails to reflect the actual values on individual basis. Instead, labor supply responses are calculated as follows. If individual chooses to be in the first bracket in the actual as well as the hypothetical scheme, the new labor supply decision becomes

$$\ell^{'} = \left[(1-\varepsilon) \frac{t - 0.15}{0.15} \right] \ell \,, \tag{7}$$

where $\ell^{'}$ is the labor supply under new scheme and ℓ is the realized labor supply in 2014 under the actual scheme. For $\hat{y} \leq 11,000$, if an individual chooses to be in the first bracket in 2014 and in the second bracket under the hypothetical scheme the labor supply can be computed using

$$\ell' = \{1 - \left[\varepsilon\left(\frac{t_1 - 0.15}{0.15}\right)\frac{\hat{y}}{y} + \varepsilon\left(\frac{t_2 - 0.15}{0.15}\right)\frac{11000 - \hat{y}}{y} + \varepsilon\left(\frac{t_2 - 0.20}{0.20}\right)\frac{y - 11000}{y}\right]\}\ell, \tag{8}$$

where $\hat{y}$ is the cutoff income in the hypothetical tax scheme and y is the actual earned gross income in 2014.

We calculated the labor supply reaction of individuals to a proportional change in their wage levels by using the elasticity formula (4). For this purpose, it is enough to include only tax rates because gross wages are assumed to remain the same. Then, this reaction is added to old labor supply ℓ. For example, equation (3) is the new labor supply of individuals whose income y fell into the first-bracket under the actual tax schedule in 2014 and income decision fell into first-bracket also in the simulation. Equation (4) is the expression for the labor supply decision of individuals whose income y fell into the first-bracket under the actual tax schedule in 2014, whereas their income fell into the second-bracket in the simulation. In this case, the formula provides a weighted labor supply reaction in proportion to income that falls into separate income brackets using appropriate marginal tax rates in each bracket. For the sake of simplicity, the other formulas are not included, yet the logic followed is the same.

CHAPTER 4

RESULTS

Numerical simulations rely on the main assumption that individuals are identical and have the same constant labor supply elasticity, which is independent from wage levels. Therefore, individuals have the same values of parameters *k* and α which is directly related to labor supply elasticity[5]. The preferred value for the inequality aversion parameter, ρ, used in the simulations is 0.2[6]. Since there is no reliable estimate of labor supply elasticity for Turkey that can be drawn upon for this study, we carry out an estimation for the group under consideration using the data at hand.

4.1 Estimation of labor supply elasticity

To estimate the labor supply elasticity using the data at hand, the following model is used:

$$ln(week\_hour)_i = \beta_0 + \beta_1\, ln(wage)_i + \beta_2\, sex_i + \beta_3\, age_i + \beta_4\, educ_i + u_i,$$

where $ln(week\_hour)$ is logarithm of annual working hours, *ln(wage)* is logarithm of hourly wage, *age* is the age of an individual, and *educ* is a categorical variable for education.

Table A1 in the Appendix provides various the results of estimations using various versions of the labor supply model above. The preferred estimation is Regression (1) in Table A1, which gave an estimated elasticity of labor supply equal to -0.057 (the coefficient of *ln(wage)*).[7]. This value is close to the "consensus"

[5] *k* is a function of α and wages. Hence, if the value of α is obtained, the value of k can be derived as well.

[6] The value of ρ does not affect the calculation of revenue, but it affects social welfare level.

[7] Note that Regression 3 in Table A1 includes years of work experience instead of age. Age and work experience are highly correlated, and the estimated labor supply elasticity in Regression 3 -0.067. This

estimate of -0.1 that is used in similar studies (MaCurdy et. al, 1990).[8] The value of parameter k is calibrated from the data so that the average weekly income in the sample yields the same average hours of labor supplied using the labor supply expression (3).

4.2 Simulation results

We simulated the model using the reference distribution and the one-bracket, the two-bracket and the three-bracket tax schedule separately. In each of the simulations, we searched for the marginal tax rates and cutoff income levels parameters which raises the hypothetical revenue collected from the reference distribution through tax schedule in year 2014 (which is 800,758.84 TL). We searched for optima by increasing the marginal tax rates by 0.01 in the interval [0,1], as well as increasing the cutoff income levels by 500 TL in annual earnings. Because of the computational limitations, cutoff income levels are incremented by 5,000 TL in the three-bracket tax schedule. First, we calculated the amount of labor hours for each possible combination of grids. By doing that we obtained both the possible gross and the possible net income of individuals. For the two-bracket and the three-bracket schedules we determined the utility maximizing bracket in which each individual prefers from their income-consumption space. Note that the income-consumption decisions are structured by combinations of each marginal tax rates and each cutoff income levels. These informations allow us to calculate the amount of tax paid by each individual for each t's and ˜s. By adding up them, we determine the parameters

value is not significantly different from the elasticity value obtained in Regression (1), and the difference does not affect the results of the simulation qualitatively.

[8] The simulations were also conducted for 'consensus' labor supply elasticity estimate, -0.1. The results are not qualitatively different from the results reported in this study.

collecting in ± 2 TL interval of targeted revenue, since the Laffer curve effect implies the possibility of collecting the same amount of revenue by two different tax rates and also that we have a discrete wage distribution. The cutoff income levels and the marginal tax rates that maximize total social welfare under each scheme are presented in Table 3.

Table 3. Simulation Results

	t_1	t_2	t_3	$\hat{y}_1$	percentile	$\hat{y}_2$	percentile	R (TL)	SWF
1-Bracket	0.19	-	-	-	-	-	-	799,216.55	$17*10^4$
2-Bracket	0.21	0.08	-	106,500	99	-	-	800,760.64	$50*10^4$
3-Bracket	0.51	0.46	0.02	15,000	9	25,000	40	800,759.40	$88*10^4$

We note that social welfare achieved under each of the three hypothetical schemes considered is higher than $16*10^4$, which is the total social welfare level under the current four-bracket piecewise linear schedule[9].

4.2.1 Numerical solution for the one-bracket piecewise linear tax schedule

We applied the grid search algorithm with grid size of 0.01 over t in the interval [0, 1]. It turned out that t = 0.19 collects the exact amount of R = 800,758.84 TL. The Laffer curve for this case is given in Figure A4 in the Appendix.

Figure 3 illustrates the changes in the weekly working hours if the optimally chosen one-bracket tax scheme were to be used instead of the actual four-bracket schedule used in 2014.

[9]Note that the individuals are assumed to receive zero benefit from the collected revenue under both the actual and the hypothetical schemes.

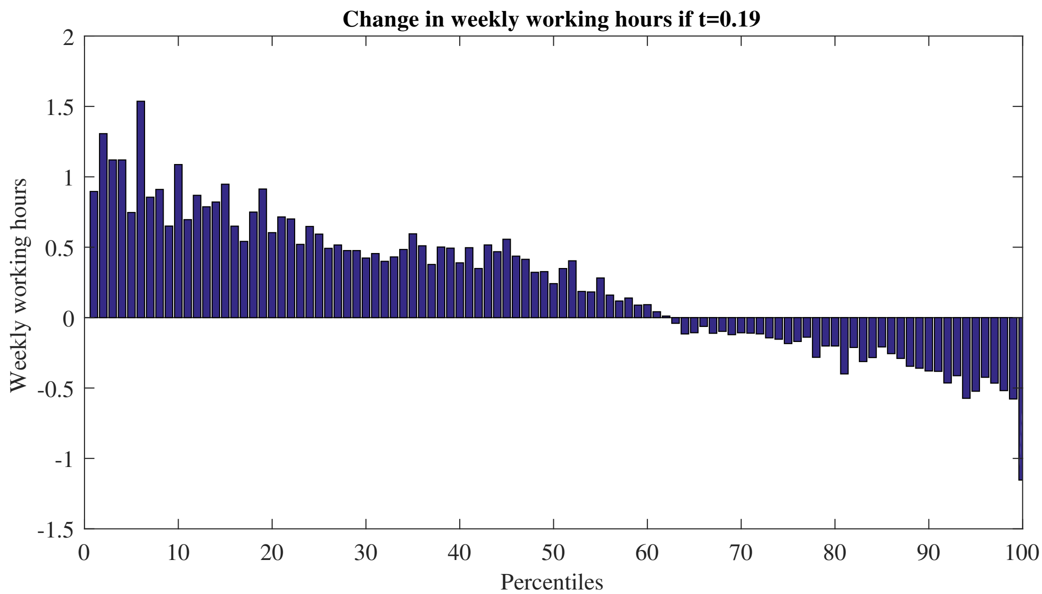

Figure 3. Change in weekly working hours if $t = 0.19$

It is observed that the change is positive until the 61^{st} percentile. Due to increase in the average tax rate for these individuals, they prefer to work more to compensate the negative effect of increase in the tax rate. On the contrary, individuals after the 61^{st} percentile supply less labor in a week. While the effect of the change in tax scheme is both negative and positive over the population, the overall effect on the net income levels is positive for the whole population, as illustrated in Figure 4. That is, the effect of tax rates on labor supply is not reflected onto the income levels. Even though the individuals above 61^{st} percentile supply less labor compared to the case under the actual scheme, they earn more income since the decrease in the average tax rate for them outweighs the fall in labor supply. Note also that the change in weekly net income of individuals until the 52^{nd} percentile is negligible or very small, whereas after that point there is a substantial difference. Note that the individuals whose net income income exhibits a substantial increase are those who fall into the third and the fourth brackets in the current schedule, i.e. the relatively "rich" individuals.

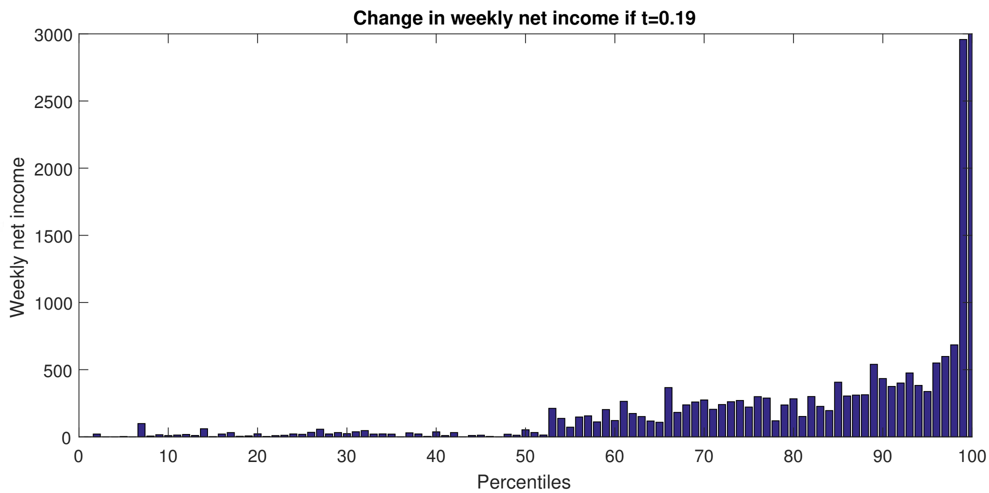

Figure 4. Change in weekly net income if $t = 0.19$

Figure 5 shows the change in utility levels over the population. Notice that Figure 5, which displays the change in utilities, is very similar to Figure 4, which displays the change in weekly net income. This similarity can be attributed to the small value of the parameter k.

The remarkable feature of the figure is that the change in the utility level is positive for each individual. The group above the median wage benefits more relative to the wage earners below the median wage. Especially, the most dramatic increase is observed for the top level income earners. The raise in the utility of the 100^{th} percentile is almost nine times of the raise in the utility of 99^{th} percentile.

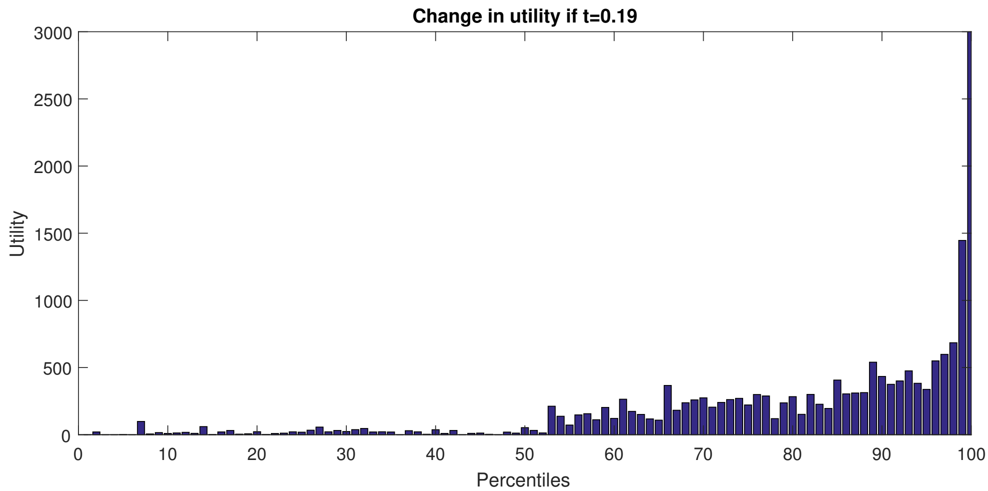

Figure 5. Change in utility levels if $t = 0.19$

4.2.2 Numerical solution for the two-bracket piecewise linear tax schedule

For the simulation, we applied grid search algorithm by increasing each of t_1 and t_2 by 0.01 in the interval [0,1], and at the same time increasing $\hat{y}$ by 500 TL in the interval [5,000 TL,150,000 TL]. The optimal levels of marginal tax rates for the first and the second brackets are t_1 = 0.21 and t_2 = 0.08, and the optimal cutoff level for the yearly income is $\hat{y}$ =106,500 TL. The revenue raised by this scheme is 800,760.64 TL.

With the optimal two-bracket schedule, only the highest wage earner (the 100^{th} percentile) pays tax at the marginal rate t_2 for some part of her income. In this case, this individual with the highest gross wage and highest gross income earns gross annual income of 234,882 TL, and she pays 22,365 TL for the 106,500 TL portion of her income, and pays 2,567 TL for the rest.

As opposed to current tax schedule, this scheme requires the burden of tax revenue to fall on the low wage levels. The top level income earner pays 74,903 TL in tax in 2014 under the actual scheme, while she pays only 24,932 TL under the hypothetical two-bracket scheme.

Compared to the revenue-equivalent simulation with one-bracket, under the two-bracket scheme the tax burden falls mainly on individuals other than the top income earners. This is due to the fact that the average marginal tax rate on the first 99 percentile increased, while that on the top income earners decrese.

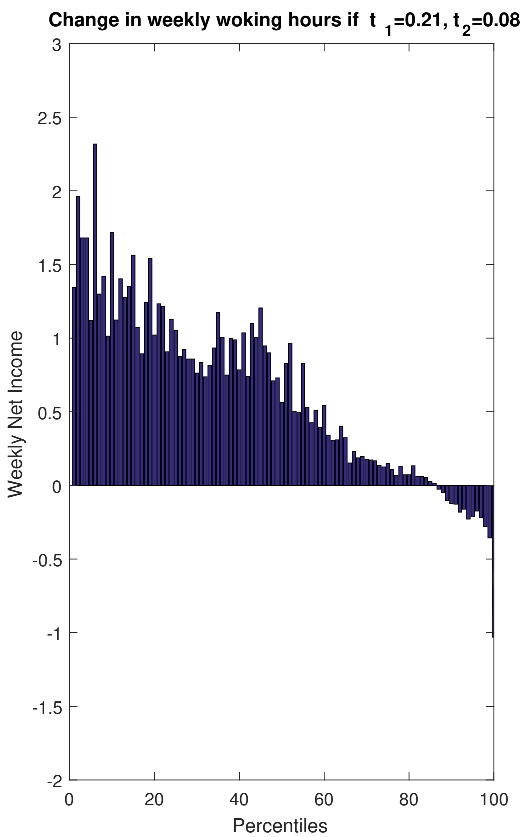

Figure 6. Change in weekly working hours if t_1=0.21 and t_2=0.08

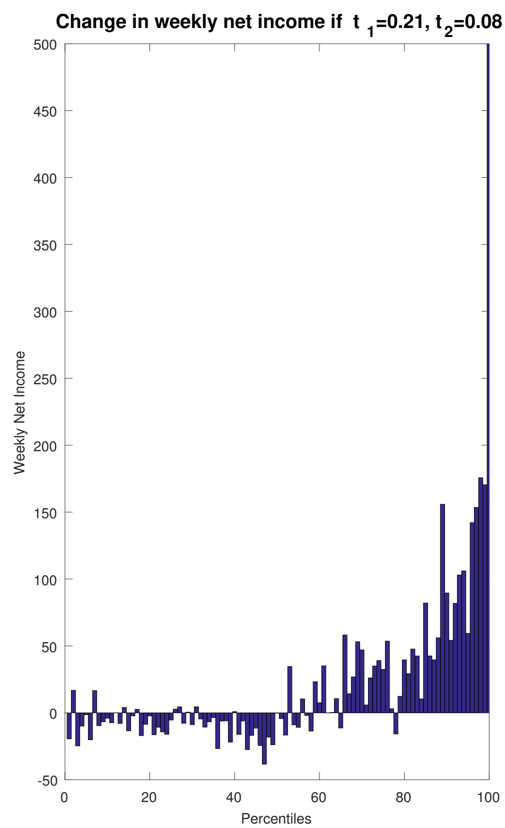

Figure 7. Change in weekly net income if t_1=0.21 and t_2=0.08

Figure 6 provides the counterfactual weekly hours of work that result from the application of the two-bracket schedule instead of the four-bracket schedule applied in 2014. We observe that the weekly working hours increase until the 83^{th} percentile, and then decrease monotonically. This is a simple consequence of the increase in average tax paid by the group before the 83^{th} percentile, and the opposite for the rest of the individuals.

Figure 7 illustrates the change in weekly net income earned. Despite the increase in their weekly working hours, the new tax schedule has negative effect on weekly net income of 'low' wage individuals. The negative effect of increased tax payment on net income overcomes the positive effect on hours of labor supplied for the individuals before the 50^{th} percentile.

A similar effect is also observed in Figure 8, which displays the change in the utility levels of individuals at each percentile. Utility diminishes for the individuals until the 50^{th} percentile, while the utility levels of individuals after the 59^{th} percentile demonstrate substantial increase.

Note that the positively affected households fall in the third and fourth brackets in the actual tax schedule, which is a consequence of the fact that average tax rate decrease substantially for these group of households. Consistent with this, the largest increase in utility is observed for the 100^{th} percentile (almost 14 times of that of the 99^{th} percentile). Since, the last two percentiles are in the highest bracket of current tax schedule, they pay marginal tax rate 35% for their income above 97,000 TL in the current schedule. However, only the highest wage earners (the top 1%) earn income falling in the the second bracket in the simulation, paying almost zero tax for income above 106,500 TL (in addition to paying substantially less tax for the income between 97,000 TL and 106,500 TL). On the other hand, the individuals at the 99^{th} percentile of the wage distribution do not experience a similar jump in their net income. The positive change in the utility of the top 1% is so much higher than the rest that the total welfare effect of the tax reform would be negative if the the top 1% is eliminated from the distribution.

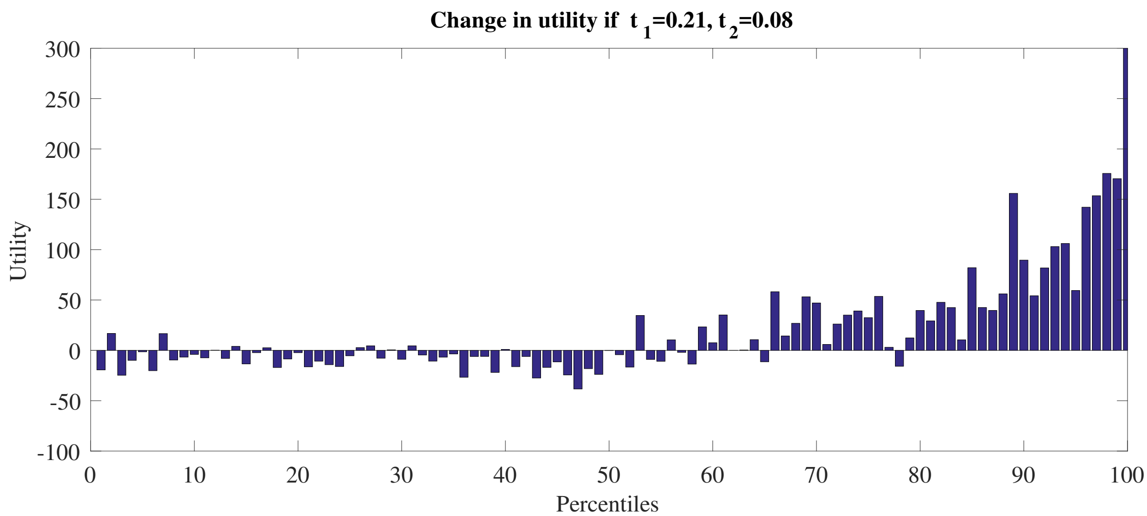

Figure 8. Change in utility levels if t_1=0.21 and t_2=0.08

4.2.3 Numerical solution for the three-bracket piecewise linear tax schedule

In this case, we applied grid search algorithm by increasing each of t_1, t_2 and t_3 by 0.01 in the interval [0,1], while increasing $\hat{y}_1$ and $\hat{y}_2$ by 5,000 TL in the interval [5,000 TL, 150,000 TL]. The optimal levels of marginal tax rates for the first, the second, and the third brackets are t_1=0.51, t_2=0.46, and t_3=0.02, while the optimal yearly cutoff income levels turn out to be $\hat{y}_1$=15,000 TL and $\hat{y}_2$ =25,000 TL. Note that $\hat{y}_1$ and $\hat{y}_2$ correspond to 10^{th} and 40^{th} percentiles in the distribution, respectively. The revenue raised by this scheme is 800,759.4 TL.

Average tax rates increase tremendously for each income group except the last two percentiles. Implications of this phenomenon are observed through changes in working hours. Figure 9 displays the change in weekly working hours. The effect on weekly working hours is almost zero at last two percentiles, whereas it increases from

3 to 22 hours for the rest of the distribution. This is consistent with what was observed in the cases of the two- and three-bracket schemes. The average tax rate paid by this group increases and a large segment of the population experiences a decrease in their income, as displayed in Figure 10. The increase in average tax rates compared to the actual four-bracket system in use in 2014 overcomes the effect of increase in hours of weekly labor supply on incomes. The change in utilities displayed in Figure 11 show a similar picture. Individuals below the 64^{th} percentile experience a decrease in their utilities as a result of drastic increase in average tax rates.

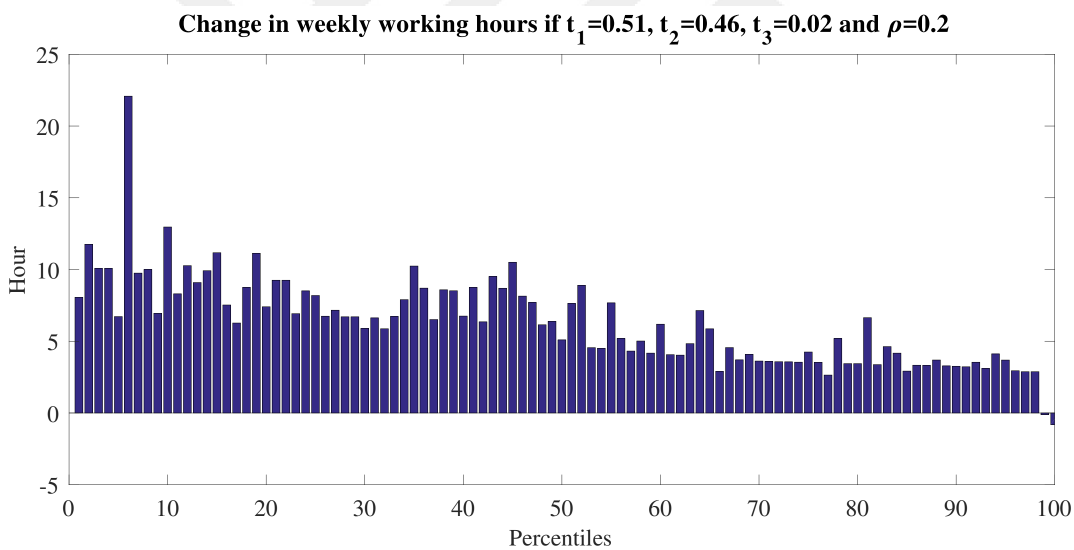

Figure 9. Change in weekly working hours if t_1=0.51, t_2=0.46 and $t_3 = 0.02$

This study reported the social welfare maximizing results in the interval of ± 2 of targeted tax revenue. For the other results for three-bracket schedule which are not reported, the marginal tax rates are never progressive, in the sense that marginal tax rate in each bracket is higher than the rate of the previous bracket for all brackets. This result implies that the model simulation on this dataset does not produce results in which the most part of the tax burden falls on high wage individuals. In other words, the results do not favor the 'low' wage individuals which is the mass group in the distribution.

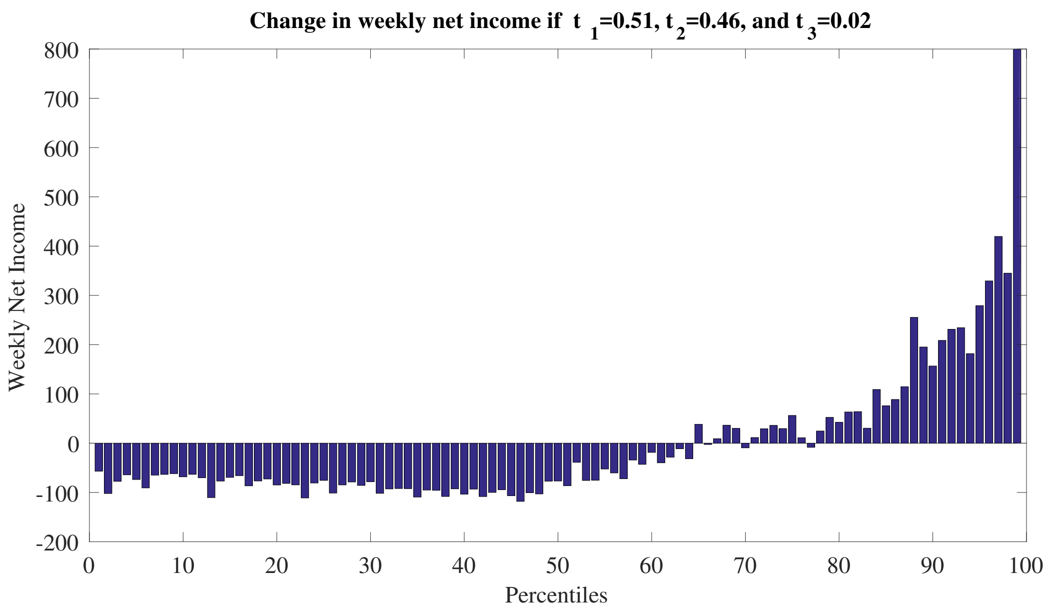

Figure 10. Change in weekly net income if t_1=0.51, t_2=0.46 and $t_3 = 0.02$

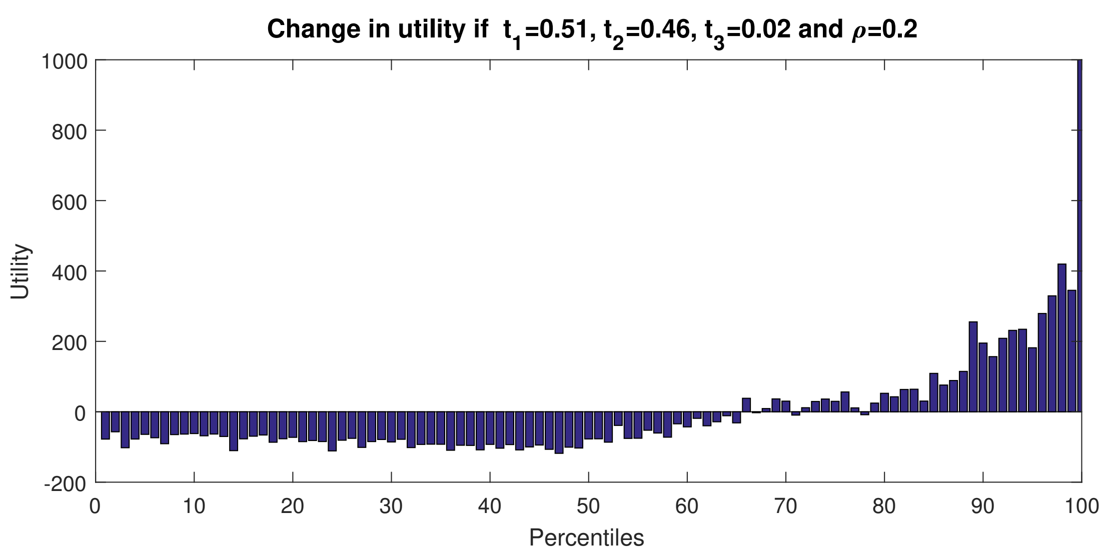

Figure 11. Change in utility levels if t_1=0.51, t_2=0.46 and $t_3 = 0.02$

4.2.3 Comparison between the two-bracket and the three-bracket tax schedules

This part aims to analyze the counterfactual changes if the scheme with three-bracket is applied instead of the two-bracket scheme. This comparison provides a look at the distributional impact of an increase in the number of brackets in revenue-equivalent piecewise linear tax schemes. Figure 12 illustrates the change in weekly working hours on individual basis when a three-bracket scheme replaces the revenue equivalent two-bracket scheme. Working hours increase substantially as a consequence of the sharp increase in average tax rates. Again, net incomes do not exhibit the same increase, as shown in Figure 13, since the rise in the tax rates outweighs the increase in labor supply until 75^{th} percentile.

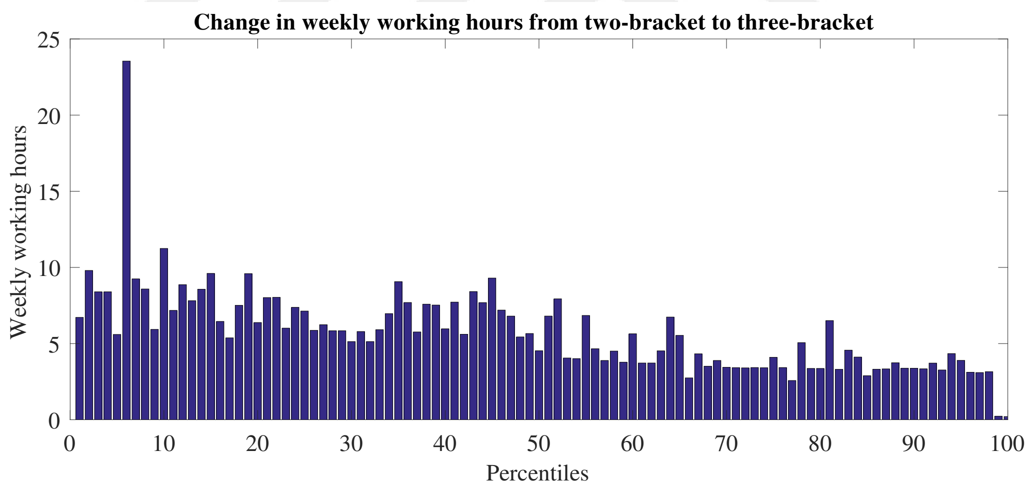

Figure 12. Change in weekly working hours: moving from the two-bracket to three-bracket tax scheme

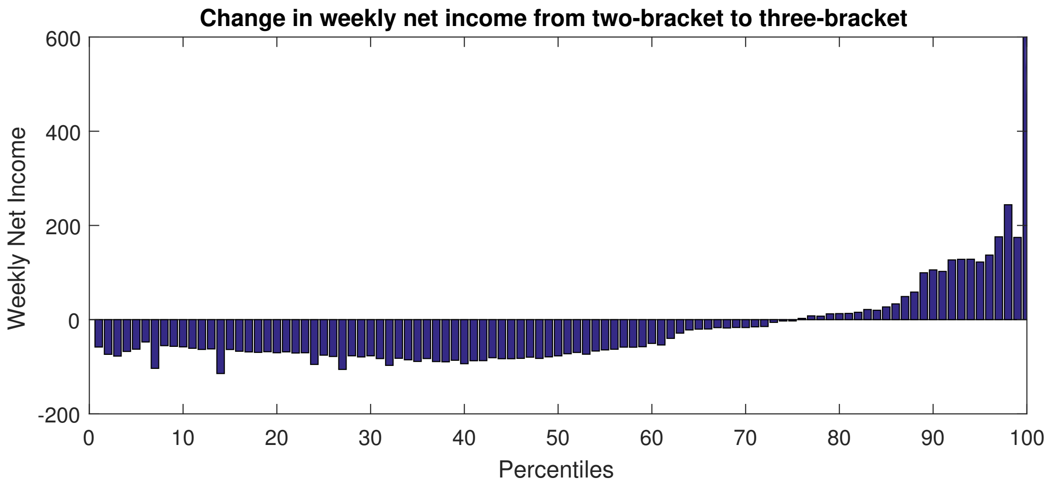

Figure 13. Change in weekly net income: moving from the two-bracket to three-bracket tax scheme

Figure 14 demonstrates the gains in benefit of individuals from increase in the number of brackets from two to three. As observed, all individuals gain from such a move, i.e. the three-bracket schedule Pareto dominates the two-bracket schedule. The top level income earners again gain the most in terms of utilities, experiencing a change in utility that is almost seven times of the previous percentile. This is a consequence of the fact that the largest proportion of her income is exposed to almost zero tax in the three-bracket scheme. In the simulations, she pays only 4,281 TL tax for 214,069 TL of her gross income (of 239,069 TL). The approximately linear shape of the curve in Figure 14 implies that an additional tax bracket in this case benefits individuals in proportional to their exogenous gross wage levels.

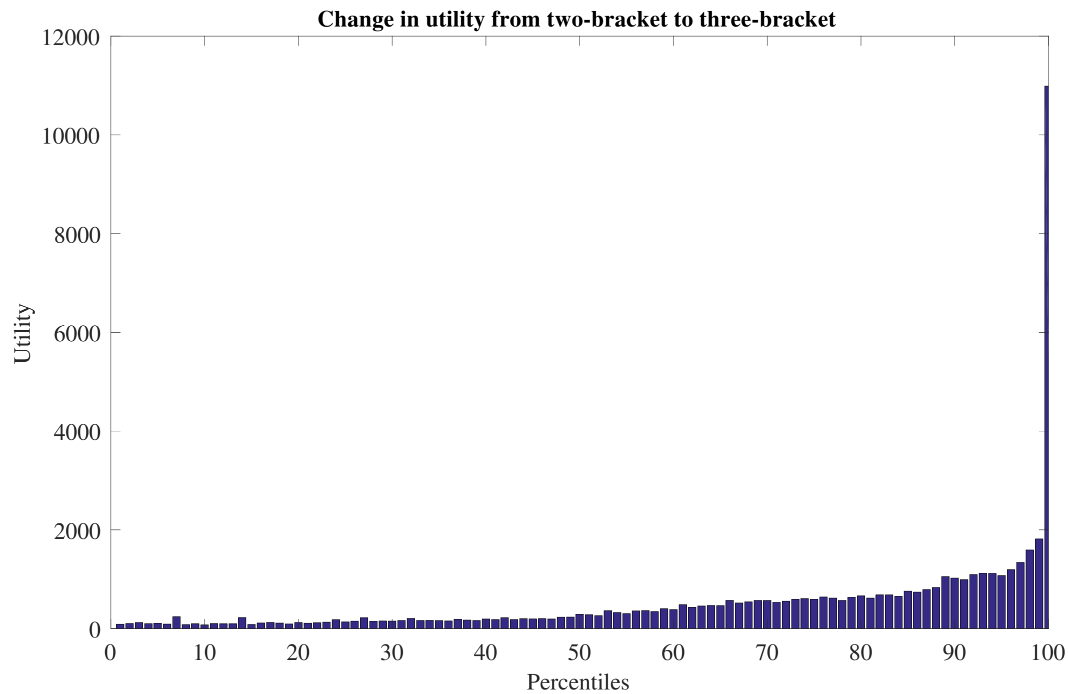

Figure 14. Change in utility: moving from the two-bracket to three-bracket tax scheme

Another feature of the curve in Figure 14 that stands out is its similarity to the reference wage distribution. The similarity is to be attributed to regressive marginal tax rates in both of the tax schemes. In the simulation with three-brackets, income levels after 40^{th} percentile are exposed to almost zero marginal tax rate, whereas they pay marginal tax rate 21% for similar gross income levels in the two-bracket scheme. Therefore, these individuals benefit from the transmission to three-bracket scheme proportional to the gross income earned, which is directly related to wage levels. The drastic benefit specific to the top level income can be attributed to two reasons. First, the marginal tax rate on top level falls from 8% to 2%. Secondly and more importantly, the portion of her income falling to the last bracket increases drastically, from 54% to 89% as the number of brackets increase from two to three. Note that the cutoff income level falls from 106,500 TL to 25,000 TL. Even though the first effect is small relative to the second, top level income benefits from increase in the number of brackets drastically as consequences of two reinforcing effects,

4.3 The effect of inequality aversion on the results

Inequality aversion parameter (ρ) represents the social planner's preferences over the relative utilities of individuals in the distribution. Until now, it was assumed as ρ=0.2 so that the results could be comparable to the Apps et al. (2013) and Adrienko et al. (2016). In this part, we elaborate on the effect of ρ on social planner's preferences over the number of brackets.

Table 4. Social Welfare Levels for Different Values of ρ

		Social Welfare Levels				
		$\rho = 0.2$	$\rho = 0.3$	$\rho = 0.5$	$\rho = 0.7$	$\rho = 0.9$
	Current Schedule	$16*10^4$	$36*10^4$	$48*10^5$	$21*10^8$	$44*10^{20}$
Simulations	1-Bracket	$17*10^4$	$38.5*10^4$	$50*10^5$	$22.4*10^8$	$46*10^{20}$
	2-Bracket	$50*10^4$	$38*10^4$	$49*10^5$	$22*10^8$	$45*10^{20}$
	3-Bracket	$88*10^4$	$35*10^4$	$44*10^5$	$19*10^8$	$39*10^{20}$

Table 4 presents the different social welfare levels for different values of ρ. At a first glance, the most apparent fact is that, for a given number of tax brackets, social welfare is not monotonic in ρ. This can easily be demonstrated theoretically as well.

Note that parameter ρ does not affect the optimal marginal tax rates and cutoff income levels for these specific simulations. For the simulation of flat tax scheme, t = 0.19 raise the exact amount of required revenue, 800,758.84 TL. Hence, it is reasonable to expect that ρ would not affect the optimal t. However, also for the two-bracket schedule, it turns out that t_1 = 0.21, t_2 = 0.08, and $\hat{y}_1$ = 106,500 TL still yield the highest social welfare among other combinations of marginal tax rates and cutoff income levels which collect revenue in ± 2 interval of 800,758.84 TL.

Moreover, for three-bracket schedule, t_1 = 0.51, t_2 = 0.46, $\hat{y}_1$ = 15,000 TL and $\hat{y}_2$ = 25,000 TL remain the same as the optimal values. The optimal parameters are not affected because the change in the values of ρ does not have an effect on ordering of social welfare levels resulting from the simulations that involve collecting of revenue in the $\pm$2 TL interval of 800,758.84 TL. It should be noticed that the ordering does not change among the simulations applied by using same ρ as well as same number of brackets. It alters the ordering of social planner's preferences among different tax schedules if number of brackets differ among the schedules.

Figure 5 previously demonstrated that the tax schedule with one-bracket Pareto dominates the current piecewise linear tax schedule. Given that ρ does not affect utility levels on individual basis, piecewise linear schedule with one-bracket always yields higher social welfare than the current piecewise linear tax schedule does. This statement holds for all values of $\rho \in [0, 1)$. Even though the scheme with two brackets does not Pareto dominate the current scheme, simulations of two-bracket scheme with each different values of ρ yield higher social welfare than the current scheme could yield. This could be attributed to that the utility of top level income increases so drastically that the decrease in the utility of first half of the distribution could be 'compensated' in total for the social planner for all value of $\rho \in [0, 1)$ [10].

The rest of the comparisons among results of simulations with different number of brackets depend on the values of ρ. For $\rho \leq 0.3$, the two-bracket schedule results in substantially higher value of social welfare than that under the one-bracket schedule. This can also be attributed to drastic increase in utility of top level income, which is the consequence of almost zero marginal tax rate for top income level.

[10]The case in which ρ=1 corresponds to Rawlsian type utility function. Hence, the highest utility would not interest the social planner.

However, the level of social welfare yielded by the one-bracket and the two-bracket schedules approach each other for values of $\rho \geq 0.3$. But for all $\rho \geq 0.3$ the one-bracket scheme slightly dominates the two-bracket scheme (that favors the top level income in expense of the rest).

While the social welfare level that results in the three-bracket scheme is above the level resulting from the two-bracket for $\rho \leq 0.3$, the situation is reversed when $\rho \geq 0.3$. This statement holds for the analysis between the current schedule and the three-bracket schedule as well. The current schedule and the two-bracket simulations are similar in terms of the following aspect. Notice that the increase in the number of brackets from the two-bracket to the three-bracket serve the benefit of higher income earners in the distribution. Likewise, compared to the current schedule, the the three-bracket scheme favors the high wage earners at the cost of others. Therefore, the increase in the social planner's preferences for the equality, i.e. an increase in ρ, leads the planner to prefer the two-bracket scheme over the three-bracket scheme as well as to prefer the two-bracket scheme over the current tax schedule.

CHAPTER 5

DISCUSSION AND CONCLUDING REMARKS

Two points in the results draw attention. First, an increase in the number of brackets does not necessarily imply increase in the social welfare for some values of inequality aversion parameter. Slemrod (1994) shows theoretically that the two-bracket scheme Pareto dominates the one-bracket scheme. On the other hand, the numerical simulations in the study states that the optimal two-bracket tax scheme does not differ markedly from the optimal flat tax scheme. This statement is compatible with our results in the case of $\rho \geq 0.3$. On the other hand, Apps et al. (2014) finds out that the piecewise linear schedule with two-bracket dominates the flat tax schedule in their numerical analysis. Even though they do not elaborate on whether the difference is significant or not, they conclude that the two-bracket is preferred over the one-bracket even for $\rho = 0.3$ which is contrary to our results. The reason for this situation could be explained by the second important point in our study.

The results of our simulations are similar to those obtained by Slemrod et al. (1994) in the sense that the marginal tax rate in the $n + 1^{st}$ bracket is lower than the marginal tax rate in n^{th} bracket. Not only the marginal tax rates but also the tax structure is regressive (in the sense that average tax rates decrease in income as described in Musgrave & Thin, 1948). An additional tax bracket allows for lower marginal tax rates on high incomes as well as for a decrease in the cut-off income level, which in turn allows individuals with high incomes benefit from almost zero marginal tax rates for a large portion of their incomes. Contrary to our results, optimal marginal tax parameters in Apps et al. (2014) is progressive. Adrienko et al. (2016) discovers similar results as well. In both of the studies, increase in the number of brackets entails higher marginal tax rates in subsequent brackets. Therefore, the schedule implicitly oblige the top incomes to contribute more for collecting targeted tax revenue. The difference between their results and the ones presented here can be attributed to different labor elasticity values assumed (positive in theirs and negative here) as well as the no non-labor In that sense, it could be argued that the optimal tax schedule in these studies helps to provide more equal income distribution. For that reason, as social planner's preferences for equality increases, she is less likely to prefer higher number of brackets in piecewise linear schedule.

The assumption of no non-labor income, including any welfare payments, is unlike most of studies in the literature. Individuals are assumed to receive zero benefit from tax revenue either directly or indirectly. This study elaborates only on how to collect the targeted amount of revenue. If some welfare payments are taken into consideration, then negative effect of the high tax rates on low income individuals could be offset partially. However it is worth to note that the inclusion of welfare payments could completely change the optimal tax structure.

Also, the problem of self-selection (Stiglitz, 1982) is not addressed in this study. Individuals with high ability, i.e. those with exogenously high wages, always have the alternative of working less, enjoying a lower level of marginal tax rates if the tax schedule is progressive in average tax rates. A desirable feature for tax structure in application is that more able individuals are given incentive to reveal their ability by earning higher incomes. However, since the optimal tax schedules found in the framework of this study are not progressive, this issue does not arise in this study.

In the simulation of the one-bracket tax schedule, the uniform tax rate of 0.19 raised the exact targeted tax revenue of 800,758.84 TL which was collected from the reference wage distribution through the piecewise linear tax schedule in 2014. The required revenue could be collected by means of both two-bracket and three-bracket schemes as well. In both cases, marginal tax rates are decreasing in the sense that the marginal tax rate in each bracket is higher than the rate in subsequent bracket. The results are in line with studies in the optimal taxation literature. Mirrless (1971) finds out similar results in his leading theoretical study, and Slemrod (1994) concludes that the optimal tax schedule performs regressive marginal tax rates in his empirical study as well.

Counterfactual analysis points out that low wage earners do not benefit from the optimal piecewise linear tax schedules with two brackets and three bracket imposed by this model, as well as the increase in number of tax brackets within the model. On the other hand, individuals with incomes in the third and fourth bracket of the actual tax scheme in use in 2014 benefited from all schemes imposed in this study. This is a consequence of the fact that marginal tax rates for 'high' income levels decrease substantially in each of the simulations.

The one-bracket tax scheme Pareto dominates the current piecewise linear schedule for the revenue equivalent tax schemes simulated in this study. Also, the schedule with two brackets yields higher social welfare than the current piecewise linear schedule does for all value of $\rho \in [0, 1)$. The comparative analysis for the three-bracket and the two-bracket as well as the comparison of these two with the current schedule depend on the value of ρ. As the inequality aversion parameter ρ increases the social planner is more likely to prefer the schemes with one-bracket and two-bracket over the scheme with three-bracket as well as the current scheme over the three-bracket scheme. These inferences are related to the regressive structure of the optimal tax parameters. Increase in the number of bracket favors the high wage earners by decreasing the marginal tax rates as well as by lowering the cutoff income levels after which households pay almost zero marginal tax.

APPENDIX

TABLES AND FIGURES

Table A1. Regression Results of Labor Supply Elasticity Estimation

Labor supply elasticity estimation

lnweek_hour	1	2	3	4	5	6
lnwage	-0.0571***	-0.0821***	-0.0674***	-0.0704***	-0.0588***	-0.0578***
		[0.0045]	[0.0048]	[0.0048]	[0.0048]	[0.0047]
age	-0.0036***				0.002	
	[0.0002]				[0.0015]	
sex	-0.0938***	-0.0917***	-0.0994***	-0.0983***	-0.0944***	-0.0942***
	[0.0046]	[0.0047]	[0.0047]	[0.0047]	[0.0046]	[0.0046]
educ	-0.0399***	-0.0339***	-0.0383***	-0.0378***	-0.0398***	-0.0399***
	[0.0015]	[0.0014]	[0.0015]	[0.0015]	[0.0015]	[0.0015]
exper			-0.0022***	0.0008		
			[0.0002]	[0.0007]		
exper_sq				-0.0001***		
				[0.0000]		
age_sq					-0.0001***	-0.0000***
					[0.0000]	[0.0000]
Constant	4.6467***	4.6439***	4.6137***	4.6113***	4.5599***	4.5908***
		[0.0264]	[0.0265]	[0.0265]	[0.0344]	[0.0263]
Observation	10207	10207	10207	10207	10207	10207
r2	0.2082	0.1898	0.1964	0.1978	0.2094	0.0263

Standard errors in brackets
*** p<0.05, ** p<0.01, ***p<0.001**

Education categories

Illiterate	0
No schooling	1
Primary school	2
Secondary school	3
General high school	4
Vocational high school	5
Higher education	6

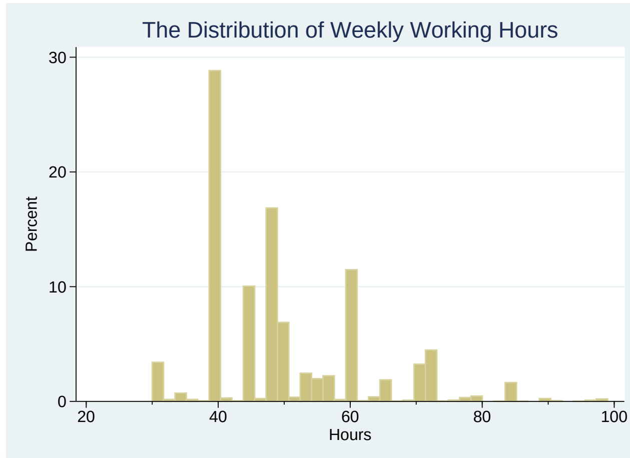

Figure A1. Distribution of weekly working hours

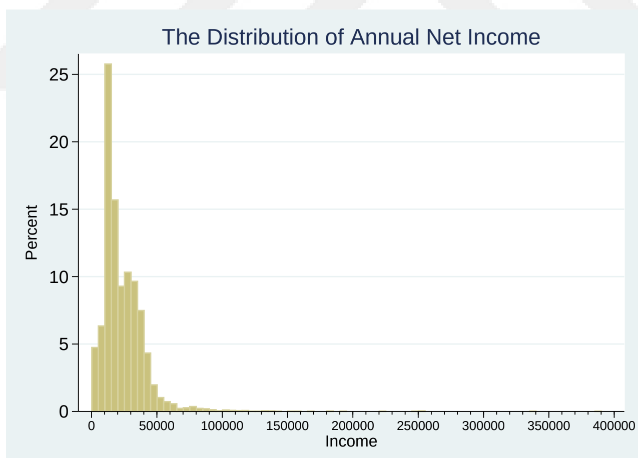

Figure A2. Distribution of annual net income

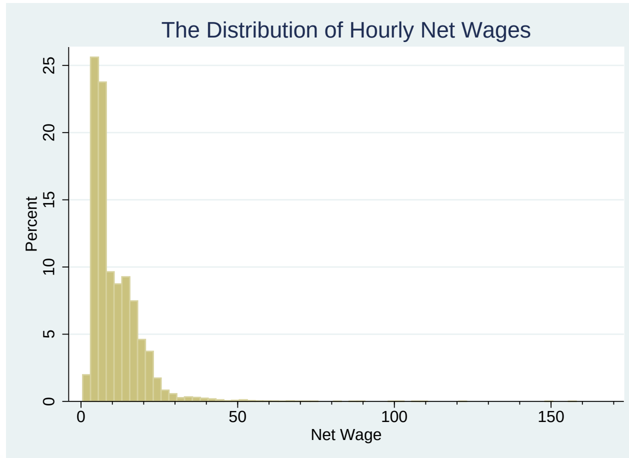

Figure A3. Distribution of hourly net wages

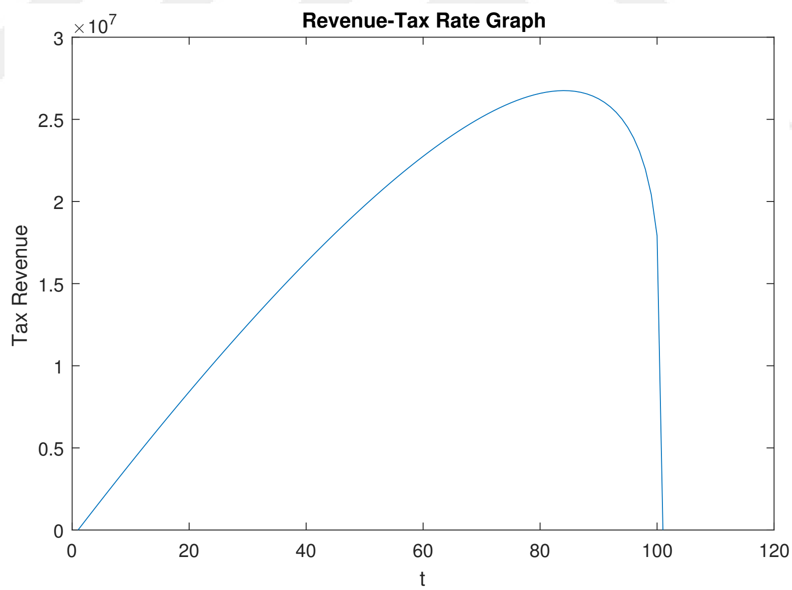

Figure A4. Laffer Curve

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