SUBSTITUTION ELASTICITIES IN AN ENERGY-AUGMENTED CES PRODUCTION FUNCTION: AN EMPIRICAL ANALYSIS FOR TURKEY

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An Empirical Analysis for Turkey

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DECLARATION OF ORIGINALITY

I, Merve Küçük, certify that

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- this thesis contains no material that has been submitted or accepted for a degree or diploma in any other educational institution;
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ABSTRACT

Substitution Elasticities in an Energy-Augmented CES Production Function:

An Empirical Analysis for Turkey

Energy is an undebatable key element in today's modern world. Whether it is the residential sector, services sector or the industry, the role of energy in our lives is highly important. This importance gets only magnified if energy is not produced domestically in the required amounts and therefore needs to be imported at highly set prices, as it is the case for Turkey. All these factors together bring about the question on how dependent on energy the production process for the Turkish economy is. A key parameter for the assessment of this dependency, moreover for the interpretation of the role of energy in the production process, is the elasticity of substitution.

The aim of this thesis is to estimate a production function for Turkey, which takes capital, labor and energy as input factors. This production function with its parameters will give insights about the elasticity of substitution of capital, labor and energy. The estimations are carried out on a dataset for the entire Turkish economy covering a time period of 27 years. Due to the differences in technological efficiencies and production structures between countries, there is a need for the adaptation of global estimations on country levels, and the results provide strong evidence supporting the need for country-specific estimations of production functions. Estimated values present relatively higher elasticity of substitution values for Turkey, when compared with values from studies performed on a group of countries.

ÖZET

Enerji Eklemeli Sabit Esneklik Üretim Fonksiyonunun İkame Esneklikleri:

Türkiye Üzerine Deneysel Çalışma

Günümüz dünyasında enerji tartışılmaz temel bir öğedir. Konut sektörü, hizmet sektörü ve sanayi fark etmeksizin, enerjinin hayatlarımızdaki rolü çok önemlidir. Yurt içi enerji üretiminin oluşan talebi karşılayamaması ve enerjini yüksek fiyatlarda ithal edilmesi durumunda, Türkiye için söz konusu olduğu gibi, bu önem daha da artmaktadır. Tüm bu etkenler, Türkiye ekonomisinin üretimde enerjiye ne derecede bağlı olduğu sorusunu gündeme getirmektedir. Bu bağlılığın ölçülmesi, daha doğrusu üretim sürecinde enerjinin rolünün anlaşılması için, anahtar gösterge ikame esnekliğidir.

Bu tezin amacı Türkiye için sermaye, işgücü ve enerjiyi girdi olarak alan bir üretim fonksiyonunun hesaplanmasıdır. Bu üretim fonksiyonu içinde barındırdığı parametreler ile beraber sermaye, işgücü ve enerjinin elastikiyetlerine ilişkin bilgi verecektir. Hesaplamalar Türkiye ekonomisi üzerine 27 yılı kapsayan bir veri seti ile gerçekleştirilmiştir. Ülkeler arasında teknolojik verim ve üretim süreleri anlamında farklılıklar olabileceğinden ötürü, global düzeyde yapılan geniş çaplı hesaplamaların ve araştırmaların ülke seviyelerine uyarlanmaları gerekmektedir. Bu tez kapsamında elde edilen sonuçlar bu görüşü destekleyici deliller ortaya koymaktadır. Türkiye için elde edilen ikame elastikiyeti değerleri, birkaç ülkeden oluşturulmuş gruplar üzerinde yapılan araştırmalarda elde edilen değerlere kıyasla daha yüksek çıkmaktadır.

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I dedicate this thesis to my parents, Ayşe and Seyit, who made it possible for me financially and who encouraged me to undertake this master's degree.

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CHAPTER 1

INTRODUCTION

The role of energy in the production process is highly important for countries which have high budget deficits arising among others from energy imports (IEA, 2016). Unfortunately, Turkey is one of these countries. The latest International Energy Agency (IEA) country review on Turkey states that Turkey is highly dependent on oil and gas imports as only 24.8% of energy supply is met by domestic production. Total supply of energy that is consumed domestically (TPES) is the lowest for Turkey among all IEA members with 1.7 tons of oil-equivalent per capita in 2015, in comparison to the IEA average of 4.5 tons of oil-equivalent per capita (IEA, 2016). With a highly import dependent situation in terms of energy trade dynamics, the assessment of the role of energy in the production process becomes essential for potential thorough policy analyses and projections.

While neo-classical capital–labor aggregate production functions do not take energy as an input factor, due to the view of energy as an intermediate product, energy crises throughout history have emphasized the role of energy in economic growth. This led to the inclusion of energy into the production function by some researchers (Brockway, Heun, Santos, Barrett, 2017). Indeed, Brockway et al. (2017) point out that the view of energy as output of capital and labor can be weakened through the claim that capital cannot be made without labor either, but still it is not regarded as an intermediate product. Therefore, nowadays, besides labor and capital, energy constitutes an important input factor in the production process.

With the increased amount of research in the field of energy and trade, many countries have developed their own energy trade modelling programs. The Energy Modelling Forum, for instance, under the body of the Stanford University has been among the institutions contributing to the development of the grounds for energy-economy research since its establishment in 1976. Alan S. Manne has developed in 1977 the model summarized in the paper “ETA-MACRO: A model of energy-economy interactions” where energy and economy interactions are linked (Manne, 1978). Since then, various models have been developed and are still being developed to analyze the energy trade in terms of raw material to end-product (Reuter, Kuehner, & Wohlgemuth, 1996). In general, Process Engineering (PE) Models, Computable General Equilibrium (CGE) Models, Macroeconomic Growth (MG) Models and Aggregate Optimization (AO) Models are at the heart of the methodology of analyses focusing on energy trade. CGE models are commonly used in studies trying to asses impacts of various policies, particularly focusing on the energy-economy linkage (Bergman, 2005). While on the trade side, these models are tools providing information about the implications of trade dynamics, the measure in the production process revealing information about the relationship between energy and non-energy inputs is the elasticity of substitution, which is a key parameter for economic and policy analysis. Specifically, elasticity of substitution shows to what degree two inputs can be substitutes for one another (Brockway et al., 2017). The calculation of this parameter is done from the production function under focus. As it is identified by Koesler and Schymura (2015), any policy-oriented numerical model must pay attention to the elasticities, because they are the key parameters determining comparative static behavior.

While inputs and outputs are similar for some countries, technological development levels, capital value shares and energy efficiencies can cause wide variations between some others. For example, while the adjusted labor share for selected G20 countries is at approximately 0.55, as it will be further elaborated on in this study, this value is different for Turkey (ILO, & OECD, 2015). This fact together with other country-specific differences create a need for the estimation of customized substitution elasticities obtained based on country specific data.

In the literature, there are very few studies concentrating on Turkey. The most recent and relevant study is by Andic (2016), where a normalized constant elasticity of substitution (CES) form production function is estimated for Turkey. Andic takes solely capital and labor as inputs and does not include energy. This makes the results obtained in that study inapplicable to energy oriented policy analyses and studies. Besides this recent research on Turkey, there is no literature on the estimation of a production function with capital, labor and energy as input factors.

At this junction, this thesis estimates the customized substitution elasticities for Turkey using a production function in the CES form with Hicks-neutral technology and constant returns to scale using data from 1988 to 2014. The choice of production function being of the CES form has been made because CES functions are a more generalized type of production function and do not come with assumptions unlike the Cobb-Douglas and Leontief functions (Besanko and Braeutigam, 2005). Of course, the inclusion of energy into the production function can be done in different ways. For example, while Bosetti, Carraro, Galeotti, Massetti, and Tavoni prefer a (KL)E nesting structure (Bosetti, Carraro, Galeotti, Massetti, & Tavoni, 2006), where capital (K) and labor (L) are combined first and the composite is thereon combined with energy (E), Burniaux, Martin, Nicoletti, and

Oliveira-Martins use a (KE)L nesting structure (Burniaux, Martin, Nicoletti, and Oliveira-Martins, 1992). On the other hand, Shen and Whalley decide to adopt a (EL)K nesting structure in their study on China (Shen & Whalley, 2013). This thesis includes energy into the production function through widely used (KL)E nesting structure.

Moreover, there are numerous studies with a sectoral focus which aim to reveal elasticity parameters for those specific sectors as well as a general overview for the economy. While van der Werf (2008) uses the OECD International Sectoral Database, which has a detailed industry breakdown based on the ISIC Review categorization, for his study on elasticities, some other papers in the literature have a broader approach with fewer sectors, as in Blitzer, Cetin, and Manne (1970), with the title “A Dynamic Five-sector Model for Turkey, 1967-82”. The study by Kemfert and Welsch (2000) on Germany on the other hand uses both approaches, where two different data sets are considered: Aggregate time series data for the entire German industry and disaggregated time series data for seven different sectors. This study, however, avoids a sectoral approach and uses aggregated data for the entire Turkish economy.

To conclude, this study contributes to the literature by presenting a production function for the entire Turkish economy with capital, labor and energy as input. The thesis is structured as follows. After this introduction, Chapter 2 presents the existing literature on this field to better situate the importance of the analysis conducted for Turkey. Chapter 3 then explains the methodology adopted throughout this study and Chapter 4 introduces the data used for the case of Turkey. The empirical results are presented in Chapter 5, followed by a discussion and conclusion in Chapter 6.

CHAPTER 2

LITERATURE REVIEW

The study of production functions with three inputs dates back to the 1950s with Robert Merton Solow (Solow, 1956). The first applications of inserting energy as input factor into the production function follows within the following decade. A comprehensive study on the incorporation of energy into the production function was undertaken by Berndt and Wood (1975) who were the first to undertake an empirical study on estimating the elasticities of substitution between energy and non-energy inputs. The motivation in their study was to put together a research, which would give an understanding of consequences of higher priced energy inputs.

The establishment of the Energy Modelling Forum (EMF) in 1976 at the Stanford University was aimed at concentrating completely on the topic of “Energy and the Economy”, which was the name of their first study, and has been carrying out research on this field ever since (EMF, 1977). Manne, Mendelsohn and Richels (1995) contribute to the literature through their study, where they take capital (K), labor (L) and energy as input, yet additionally also separate electric (E) from non-electric (N) energy. They apply a (KL)(EN) nesting structure, where the elasticity of substitution between the two input factor bundles is taken to be constant (Manne, Mendelsohn, & Richels, 1995). In their study, the elasticity of substitution between the (KL) and (EN) composites is taken as 0.4 on the basis of a “back casting” experiment for the USA, and this reference value is then maintained throughout their study for the USA and OECD countries. Gerlagh and Van der Zwan (2003), on the other hand, takes the same nesting structure but chooses to separate energy based on its type into fossil (F) and non-fossil (N) fuels and uses a production function of the (KL)(FN) nesting structure.

An important model on energy and the economy is, among others, the Emissions Prediction and Policy Analysis (EPPA) Model by the Massachusetts Institute of Technology (MIT). The EPPA model has at its heart a production functions of the CES form, where two functions presented, one for the agricultural sector and one for the services, industrial transportation, energy intensive and other industries. What is common though in both functions is that capital and labor are nested first, and other inputs are then nested with this composite (Paltsev et al., 2005). In this study, as in its prior versions, the estimations and calculations are done for the entire world economy with data from the Global Trade Analysis Project (GTAP) dataset developed by the Purdue University.

A country-specific study for the elasticity of substitution parameters in the production function is undertaken by Kemfert and Welsch (2000) for Germany. To estimate the substitution elasticities in the German industry, they develop two approaches, one with aggregate time series data for the entire German industry and one with disaggregated time series data for the chemical, stone and earth, non-ferrous metal, vehicles, food, and paper industries. They start with three different nesting structures (KE)L, (KL)E and (EL)K, and conclude that while for some sectors the (KL)E nest is more appropriate, for the entire German industry the (KE)L nest is the most useful nesting structure in contrast to the widely spread view (Kemfert & Welsch, 2000).

The approach regarding the estimation method has evolved over the time as well. Kmenta (1967) uses the Taylor expansion formula for the estimation of the production function. He obtains his approximation formula through taking the logarithm of the CES function and accordingly applying a first-order Taylor series expansion to the logarithmized CES function. This approach has a generalized solution method for the CES function under different circumstances. It transforms the non-linear functional form of the CES function to a linear form and makes the use of simple least squares estimation possible. Following Kmenta (1967), Van der Werf (2008) tries to discover through a thorough study the optimal nesting structure given the three input factors capital, labor and energy. Unlike Kmenta (1967), his study uses a cost function based approach. Van der Werf finds out that based on industry level data on 12 OECD countries, the nesting structure where capital and labor are combined first, fits the data best, but at the same time, the nest where all three inputs are combined simultaneously cannot be rejected for most countries and industries.

The research mentioned so far on the estimation of production functions with more than two inputs have one thing in common: They use comprehensive price data, which is in some cases difficult to obtain. While obtaining data on sector prices can be a problem in the case of sector specific analysis, in the case of macro analyses, the aggregation of factors creates a need for a price index to be calculated, bringing with it the problem of choosing the most appropriate method out of a wide sea of indexation ways. Henningsen and Henningsen (2011) as well as Koesler and Schymura (2000) try to get around this problem by developing a non-linear least squares estimation method. Neither one of these two studies require extensive price data to be at hand. The method developed by Henningsen and Henningsen makes the estimation through the R package called micEconCES which they developed themselves. This R package contains various estimation methods including Kmenta’s Taylor series expansion for an appropriate type of function and others methods such as the Levenberg-Marquardt for non-linear estimation cases (Henningsen and Henningsen, 2011). An important feature to this research is that it re-estimates the Kemfert and Welsch study using the Kmenta (1967) and various other approximations with the exact same data provided in the annex of Kemfert and Welsch (2000). The re-estimation results are in large deviations from the original results no matter which estimation and optimization method is used and no reasonable explanation could be found for this divergence. In this context, Henningsen and Henningsen (2011) conclude that linear approaches using the Kmenta approximation are not proper approaches for CES function estimation.

Following Henningsen and Henningsen (2011), Koesler and Schymura contribute to the literature by applying the methods developed by Henningsen and Henningsen to the data retrieved from the World Input-Output Database (WIOD) with the goal of obtaining elasticities for the (((KL)E)M) nesting structure. Here, E stands for energy and M represents intermediate inputs, which can be used by researchers during their studies of various topics (Koesler & Schymura, 2015). Their data set covers 40 countries and 35 industries with detailed information on primary, secondary as well as tertiary sectors. Their analysis reveals that Cobb-Douglas and Leontief production functions should be rejected for the majority of sectors, just as Van der Werf (2008) found out, and provides a detailed set of substitution elasticities covering a wide sectoral breakdown.

All in all, even though there are some, such as Koesler and Schymura (2015) who claim that there are no substantial variations in substitution elasticities between regions, country specific research on various countries reveal that significant results for production functions based on country specific data can in some cases only be obtained through certain nesting ways. Su, Zhou, Nakagami, Ren, and Mu (2012) start by estimating all three nesting forms of a capital, labor and energy composed CES function with the extension to the existing literature in the form of a relatively larger dataset. Since they focus on China, they approach their estimation with two subdivided periods, before and after China's reform, more specifically, from 1953 to 1978 and then from 1979 to 2006. Su et al. (2012) use the estimation method applied by Mishra (2006), which shows that for the loss function minimization the Differential Evolution (DE) and Repulsive Particle Swarm (RPS) methods outperform the other methods. As a result, Su et al. (2012) indicate that while all nesting structures are insignificant the only economically meaningful result can be obtained for the (KE)L nesting structure, where E represents energy. Shen and Whalley (2013) contribute to the literature through their working paper at the National Bureau of Economic Research (NBER) by taking the research by Su et al. (2012) and extending it to the extent that they use normalized CES production functions and perform grid search based optimization methods. Their results therefore turn out to have lower standard errors with statistically significant results for the (EL)K nesting structure.

A study on Turkey has been undertaken by Andic (2016) where the estimation of a normalized CES production function for Turkey is set as goal. Andic takes just capital and labor as inputs and does not include energy, at which point it diverges from the so far mentioned references as well as the research goal of this study. She employs a system approach and determines the elasticity of substitution and the total factor productivity. Besides this recent research on Turkey, there is no literature on the estimation of a production function with capital, labor and energy as input factors based on Turkish data. This study tries to fill this gap.

CHAPTER 3

METHODOLOGY

Production functions in general are categorized according to three criteria: technology, elasticity of substitution and returns to scale (Besanko and Braeutigam, 2005). Technology can be incorporated into a production function in three different way: The Hicks-neutral technology, Harrod-neutral technology and Solow-neutral technology, which can also be referred to as factor augmenting, labor augmenting and capital augmenting respectively. Functions can have either constant or variable elasticity of substitution. And returns to scale can be decreasing, constant or increasing. The CES function can be regarded as a generalization for a production and does not make certain assumptions regarding the nature of the function, such as Cobb-Douglass and Leontief functions, which turn out to be not very appropriate production functions for many sectors as the study by Koesler and Schymura demonstrates (Koesler & Schymura, 2015).

The production function estimated in this study is of the CES form with Hicks-neutral technology and constant returns to scale and is denoted as follows,

$$Y = A\left[\alpha\left(K^{KPVS}L^{1-KPVS}\right)^{\frac{\sigma-1}{\sigma}} + (1-\alpha)E^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}}$$

The dependent variable Y denotes output, whereas the independent variables K, L and E represent respectively capital, labor and energy. The parameters A, KPVS and σ stand for total factor productivity, capital value share and elasticity of (technical) substitution between the capital-labor bundle and energy inputs respectively.

KPVS is calculated following the paper by Atiyas and Bakis (2013). While Atiyas and Bakis concentrate on the labor share (LS) and apply their notation accordingly, our focus is on the capital value share. Hence, we apply the notation $LS = 1 - KPVS$ to their approach. The numerical value for KPVS is obtained using the below formula,

$$1 - KPVS = \frac{W}{(Y - T)} \frac{1}{(1 - z)}$$

W denotes the compensation of employees, Y denotes GDP, T stands for net indirect taxes and z is an adjustment factor representing the share of self-employment in the labor. The adjustment created by multiplying with $\frac{1}{1-z}$ brings the assumption that the wage earned by self-employed people is equal to the wage earned by employees.

The method to solve the problem of estimating the production function at focus is the nonlinear least squares (NLS) regression. The non-linear solution to the estimation problem of the parameters, will come through the minimization of the following sum of squares, $S(A, \alpha, \sigma)$.

$$S(A, \alpha, \sigma) = \frac{1}{2} \sum_{t=1988}^{2014} e_t^2$$

$$S(A, \alpha, \sigma) = \frac{1}{2} \sum_{t=1988}^{2014} \left[Y_t - A \left[\alpha (K_t^{KPVS} L_t^{1-KPVS})^{\frac{\sigma-1}{\sigma}} + (1 - \alpha) E_t^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}} \right]$$

Minimizing $S(A, \alpha, \sigma)$ means choosing the parameters A, α and σ in such a way that the sum of squares of the error terms, e_t, i.e. difference between the Y_t and the value for $A * \left[\alpha (K_t^{KPVS} L_t^{1-KPVS})^{\frac{\sigma-1}{\sigma}} + (1 - \alpha) E_t^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}}$ calculated with the iterated values for the parameters plugged in, will be minimized. For the case of this study, this maximization will be through the derivatives of $S(A, \alpha, \sigma)$ with respect to the parameters to be estimated, which are as mentioned before A, α and σ.

$$\frac{dS(A, \alpha, \sigma)}{dA} = 0$$

$$\frac{dS(A, \alpha, \sigma)}{d\alpha} = 0$$

$$\frac{dS(A, \alpha, \sigma)}{d\sigma} = 0$$

The equations obtained from these derivatives do not have explicit solutions. Following the book “Econometric Analysis – 7th Edition” by William H. Greene, we get to the definition of a nonlinear regression model which is as follows: “A nonlinear regression model is one for which the first-order conditions for least squares estimation of the parameters are nonlinear functions of the parameters.” (Greene, 2011, p. 186).

At this point, it is important to point out that the solution to the above equations can only be found, if at all, given that the number of observations, *t*, is greater than the number of parameters, *n*, to be estimated. In the case of this study, we have $t = 27 > n = 3$ and can conclude that this condition is satisfied. For situations where the solution to the first order derivatives cannot be calculated analytically, numerical methods must be applied. These numerical methods consist of iterative algorithms which require starting values for the parameters to be estimated. The iterative process takes the starting values and tries to reach an optimum through certain rules for repeatedly making the same calculations with the next available values for the parameters. These rules are defined as optimization methods (Kuan, 2004).

Chung-Min Kuan (2004) describes in his book that for the minimization of $S(A, \alpha, \sigma)$, we start by summarizing the parameter vector as $\beta = (A, \alpha, \sigma)$. An algorithm for the parameter vector β can be expressed as presented below where the superscript *i* denotes the result from the i^{th} iteration.

$$\beta^{i+1} = \beta^i + s^i d^i$$

We can see that an algorithm used the i^{th} iteration result for calculating the $i+1^{th}$ outcome by adjusting for an amount of $s^i d^i$, where d is the direction of change and s controls for its amount. The optimum result will be obtained for a point where the gradient vector $g(\beta)$ for the first order Taylor expansion of $S(A, \alpha, \sigma)$ will be equal to zero.

Taylor expansion of $S(A, \alpha, \sigma) = S(\beta)$ around β^*:

$$S(\beta) \approx S(\beta^*) + g(\beta^*)'(\beta - \beta^*)$$

Replacing β with β^{i+1} and $\beta^*$with β^i:

$$S\left(\beta^{i+1}\right) \approx S\left(\beta^i\right) + g(\beta^i)'(s^i d^i)$$

If $d^i = -g(\beta^i)$ then we have the following:

$$S\left(\beta^{i+1}\right) \approx S\left(\beta^i\right) - s^i g(\beta^i)' g(\beta^i)$$

Since $g\left(\beta^i\right)' g\left(\beta^i\right) \geq 0$, this means that a value small enough for s can be found making $S\left(\beta^{i+1}\right)$ decreasing, whereas for a minimum β the gradient vector will be already equal to zero making a further adjustment impossible. This explains the basic method of an optimization algorithm.

Numerous optimization algorithms exist in the theory. Henningsen and Henningsen (2011) use several optimization algorithms for their nonlinear least squares estimation, besides also applying the Kmenta approximation. They make use of the Levenberg-Marquart algorithm, which is the most commonly used optimization algorithm and is also set as default algorithm in numerous statistical softwares (Henningsen and Henningsen, 2011). Additionally, they also use the Conjugate Gradients method (Nocedal & Wright, 2006), Newton method (Schnabel, Koontz, & Weiss, 1985), Broyden-Fletcher-Goldfarb-Shanno algorithm (Broyden, 1970, Fletcher, 1970, Goldfarb, 1970, Shanno, 1970), Nelder-Mead algorithm

(Nelder & Mead 1965), Simulated Annealing algorithm (Belisle, 1992), Differential Evolution algorithm (Mullen, Ardia, Gil, Windover, & Cline, 2011) and numerous other algorithms, which additionally impose a parameter constraint (Henningsen and Henningsen, 2011). Koesler and Schymura (2015) on the other hand go with the make their estimations based on the commonly used Levenberg-Marquart algorithm.

In this study, the default NLS method of the statistical software EViews was used, which is the Gauss-Newton optimization method with the Marquart step method. The Gauss-Newton method is based on a linear Taylor series approximation to the nonlinear regression function, which is in our case the production function under focus. The iterative estimator is calculated through the transformation of the optimization to a series of linear least squares regressions (Greene, 2011). If we rewrite our production function as below,

$$Y = A\left[\alpha(K^{KPVS}L^{1-KPVS})^{\frac{\sigma-1}{\sigma}} + (1-\alpha)E^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}}$$

$$y = h(x,\beta) + e$$

then the Gauss-Newton method will make a linear estimation to $h(x,\beta)$ at a particular value for the parameter vector β^0. As it is described by Greene (2011), the estimation will look as mentioned below.

$$h(x,\beta) \approx h(x,\beta^0) + \sum_{t=1988}^{2014} \frac{dh(x,\beta^0)}{d\beta_t^0}(\beta_t - \beta_t^0)$$

$$h(x,\beta) \approx \left[h(x,\beta^0) - \sum_{t=1988}^{2014} \beta_t^0 \frac{dh(x,\beta^0)}{d\beta_t^0}\right] + \sum_{t=1988}^{2014} \beta_t \frac{dh(x,\beta^0)}{d\beta_t^0}$$

Setting the notation to be so that $x_t^0 = \frac{dh(x,\beta^0)}{d\beta_t^0}$ we will have for a given value of β^0, x_t^0 to be a function of data only. Then the above estimation equation can be rewritten as follows.

$$h(x,\beta) \approx \left[h(x,\beta^0) - \sum_{t=1988}^{2014} x_t^0 \beta_t^0\right] + \sum_{t=1988}^{2014} x_t^0 \beta_t$$

$$h(x,\beta) \approx h(x,\beta^0) - x^{0'}\beta^0 + x^{0'}\beta$$

This implies,

$$y \approx h^0 - x^{0'}\beta^0 + x^{0'}\beta + e$$

By rearranging this equation, we can obtain a linear equation.

$$y^0 = y - h^0 - x^{0'}\beta^0 = x^{0'}\beta + e^0$$

where

$$e^0 = e + \left[h(x,\beta) - \left\{h^0 - \sum_{t=1988}^{2014} x_t^0 \beta_t^0 + \sum_{t=1988}^{2014} x_t^0 \beta_t\right\}\right]$$

Since in the equation of y^0 all errors are included and accounted for, this equation can be written as an equality instead of an estimation. This estimation is then estimated through linear least squares. The step method has also been taken as the default step method which is the Marquardt method. As it has been described on the EViews webpage, the Marquardt algorithm serves as a modifier to the Gauss-Newton method, by adding a correction matrix to the Hessian of the production function. Thereby, the obtained parameter estimations are brought closer towards the gradient vector improving the result. To conclude, the estimations in this study are achieved without any complications and divergence through the most commonly used nonlinear estimation methods.

CHAPTER 4

DATA

This section will introduce the data used in this thesis together with the calculation of some intermediary parameters. Section 1 of Chapter 4 presents the approach applied and data used for the capital value share calculation. Section 2 of Chapter 4 concentrates solely on the data used for the main nonlinear regression estimation for the production function. All data used were either directly in terms of real values or were converted to their real equivalents with reference base year 2011. Both sections include detailed definitions of the data used together with its sources.

4. 1 Data for KPVS calculation

The data, which is used for the calculation of the KPVS, is obtained from the Turkish Statistical Institute (TUIK), the Turkish Ministry of Finance (MUHASEBAT) database and the OECD Stats. As recommended by Atiyas and Baris (2013), real GDP values obtained from the income approach are used. The value for the adjustment factor, z, is obtained from OECD Stats. The data obtained from TUIK and MUHASEBAT was in nominal terms. Therefore, the output, compensation of employees and net indirect taxes were turned into their real values with base year 2011 through the necessary adjustments with the consumer price index for Turkey retrieved from OECD Stats. While the data used for the calculation of KPVS is provided in Appendix A - Data for KPVS Calculation, key measures and sources are summarized in Table 1.

Table 1. Data Definitions and Sources for KPVS Calculation

Abbreviation	Variable	Definition	Data Source
Y	Gross domestic product	Real gross domestic product at constant prices (base year 2011)	TUIK, OECD Stats
W	Compensation of employees	Real total compensation of employees at constant prices (base year 2011)	TUIK, OECD Stats
T	Net indirect taxes	Real taxes - subsidies on production and imports (base year 2011)	TUIK, MUHASEBAT, OECD Stats
Z	Share of self-employment	Employment of employers, workers who work for themselves, members of producers' co-operatives, and unpaid family workers	OECD Stats

This calculation was undertaken for data on the years 2009 to 2015 and yearly values for KPVS for this period were obtained as a result. Since the base year of this study is 2012 the value of KPVS for that particular year, which equals $1 - KPVS = 0.50$, is used throughout this study.

4.2 Data and descriptive statistics for production function

The estimation of the production function requires data on the variables output (Y), capital stock (K), labor (L) and energy (E). For output, real GDP data taken from the Penn World Tables is used. While for the KPVS calculation GDP calculated through income method is taken, for the production function GPD calculated through expenditure method is used. This is partially due to the fact that investment series, which are used for the capital stock calculation, are obtained from the GDP calculated through expenditure method. Data on employment for the labor (L) input factor was taken from TUIK and covers all working women and men above the age of fifteen. Data on energy has been retrieved from OECD Stats as primary energy supply in tonnes of oil equivalent. This data is prepared and published by the IEA on a yearly basis. The Turkish Ministry of Energy and Natural Resources also regularly publishes this data, but for the sole purpose of consistency of data sources, the data from OECD Stats is used. While there are numerous studies which incorporate energy with data in terms of energy units, there are also just as many studies which use energy data in terms of energy cost. The EMF (1977) outlines clearly in the report "Energy and the economy" how energy can be taken as energy cost and present various aggregation and indexation methods for prices. Among these methods some studies, including studies with limited access to energy price data, use the method of taking energy supply in terms of tonnes of oil equivalent and multiplying with the real crude oil import prices. One such study is the paper by Edwin Van der Werf. With reference to Edwin Van der Werf's research, the same approach was adopted and energy data has been taken as energy cost incurred to the Turkish economy (Van der Werf, 2008). Therefore, primary energy supply in tonnes of oil equivalent is multiplied with Turkey's real crude oil import prices in USD per barrel of oil, obtained from OECD Stats, with base year 2011 just as capital stock and real GDP data used in the production function estimation. The barrel prices are converted to tonnes prices using the OPEC conversion table taken from the annual statistical bulletin (OPEC, 2017). At this point it is worth mentioning that a weighted approach for the calculation of energy cost according to energy source was evaluated and acknowledged as well. A weighted cost could be calculated based on the source breakdown of energy supply. Yet this would require data on prices for Turkey for each one of these sources which were not obtainable for the earlier periods analyzed in this thesis. Moreover, detailed price data for each particular energy source for Turkey is only available for the more recent years and some particular years for the earlier periods. Therefore, the adaptation of this method would have restricted the number of observation years for the data.

The Turkish government does not publish data on capital stock. Therefore, this data series had to be obtained from other sources. There are already existing studies on capital stock in Turkey by Bulutay et al. (1974) and more recently by Saygili et al. (2005). These studies cover respectively the periods 1923 - 1948 and 1972 - 2005. These data series were not used, as they do not cover the most recent period and their extrapolations through mathematical methods would not be robust given data availability problems. While there are several methods for the calculation of capital stock, one of these methods is the perpetual inventory methods. Starting from 1988 onwards, the capital stock was calculated through the perpetual inventory method, following the equations below. According to this method capital depreciates over one period at the depreciation rate of δ.

$$K_{t+1} = (1 - \delta)K_t$$

This creates the need for the definition of the initial capital stock value for the time period considered, i.e. for the year 1988. To identify K_0 the following method is applied where it is assumed that the economy is close to a steady state (Atiyas and Bakis, 2013).

$$\frac{K_{t+1}}{K_t} - 1 = g_t = -\delta + \frac{I_t}{K_t}$$

If we assume that we are at a steady state at time $t = 0$, i.e. in the year 1988, then we can find K_0 from $K_0 = \frac{I_0}{\bar{g}+\delta}$ where $\bar{g}$ can be taken as the average GDP growth rate for ten years starting from t_0 onwards. Based on educated opinions and some other models such as the MARKAL model, the depreciation rate has been taken as 5% (Manne and Wene, 1992). Yet, the assumption of the economy being at a steady state in 1988 is a controversial topic, where no certain objective decision can be made for the case of Turkey as a developing economy. Therefore, throughout this study the data for capital stock is taken from the Penn World Tables, which is calculated through the economic definition of capital accumulation through the following formula, where I_t denotes the amount of investment in that particular year.

$$K_{t+1} = (1 - \delta)K_t + I_t$$

Given this formula, capital stock series can be constructed without the assumption of a certain economic growth rate. For example, for a depreciation rate assumption of $\delta = 5\%$, a lifetime of capital means 20 years. Hence for calculating K_{t+20} there will only be need for the investment data of the past 20 years and no need for K_t. The capital stock data published by the Penn World Table database applies exactly this method and therefore can be appropriately used for this study on Turkey.

Table 2. Data Definitions and Sources for the Production Function

Abbreviation	Variable	Definition	Data Source
Y	Gross domestic product	Real GDP at constant national prices (in million 2011 USD)	Penn World Table
K	Capital stock	Capital stock at constant national prices (in million 2011 USD)	Penn World Table
L	Labor	Employed women and men above the age of fifteen	TUIK
E	Energy	Primary energy supply (toe) Crude oil import prices (USD with base year 2011 per barrel of oil)	OECD Stats

An overview of the data for the production function together with the definitions and sources is presented in Table 2. The dataset used for the production function estimation is provided in Appendix B.

Table 3. Descriptive Statistics for Variables used in Production Function

	GDP	Capital Stock	Labor	Energy
Mean	905,780,784,722	2,101,556,937,500	21,129,407	33,876,671,837
Median	828,538,250,000	1,946,603,500,000	21,194,000	16,035,357,373
Maximum	1,442,669,875,000	3,707,828,500,000	25,932,000	94,831,506,872
Minimum	527,702,937,500	967,656,937,500	17,754,000	8,701,621,343
Std. Dev.	278,244,583,207	803,749,412,438	2,054,661	29,236,221,969
Skewness	0.433674	0.428693	0.686242	0.990473
Kurtosis	1.940591	2.058360	3.110704	2.429338

A fundamental step in economics analysis is the analysis of the data itself. In this regard, a summary on the descriptive statistics is presented. Table 3 presents the key factors of descriptive statistics for the variables used in the production function. Note that while the GDP, capital stock and energy variables are in real USD with base year 2011, labor is given as a plain number.

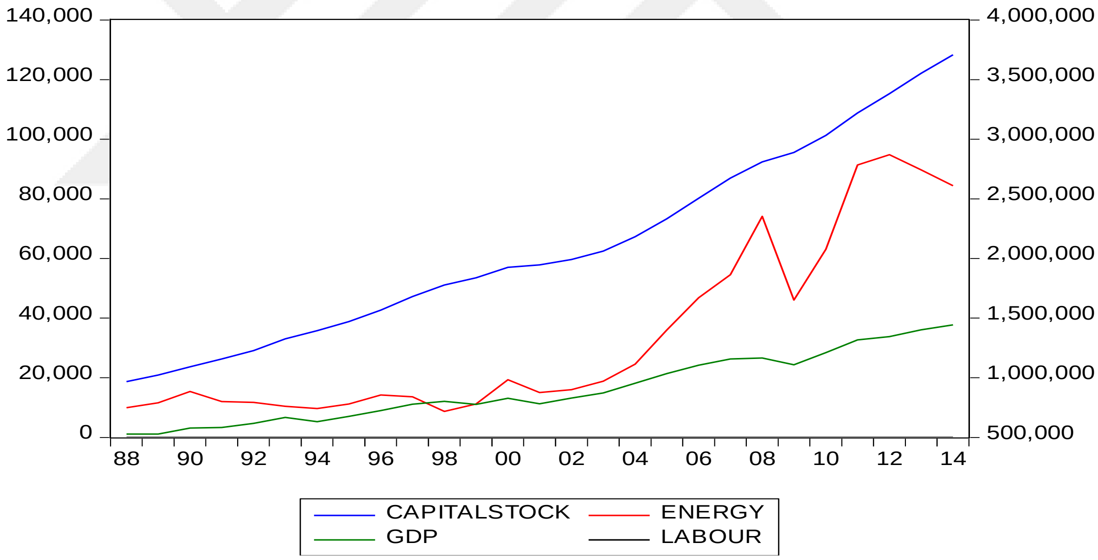


Figure 1. Graph of raw data used in the production function

The relationship between the independent and dependent variables becomes more evident once the data series are plotted. In Figure 1 the relationships are visualized. Note that the axis labels are presented in millions. All three independent variables show an increasing trend during the time period considered. Among the independent variables, while capital stock and labor series show a steady increasing trend energy costs displays higher fluctuations. When energy cost is compared to energy unit data we see that energy itself does not show this kind of fluctuations and instead shows a steadily increasing trend. This points towards the importance in the utilization of energy cost given the objective of an economic analysis. Energy itself would fail to capture and present these dynamics which have their reflections on the dependent variable output.

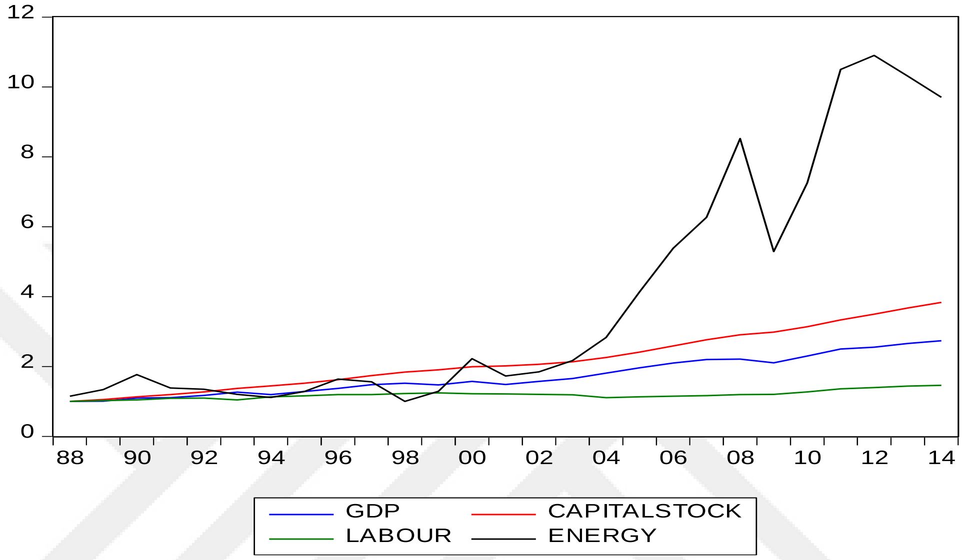


Figure 2. Graph of scaled data used in the production function

The value for labor is low compared to the values of the other variables, therefore it is not visible in the graph when the data it plotted with its real values. With this regard, in order to completely visualize the dynamics in the variables during the time period considered it is worth plotting the scaled data, with the scaling method being the division by the minimum value observed throughout the period considered, which is done in Figure 2. Just as it is visible in Figure 1, the changes in the dynamics of the energy cost are too extreme to be neglected.

Table 4. Augmented Dickey-Fuller Test Statistic Values

	Raw Data		Logarithmized Data		Scaled Data	
Variable	Level	Difference	Level	Difference	Level	Difference
GDP	0.941713	-4.901975	-0.237377	-5.875371	0.944847	-4.891251
Capital Stock	1.691296	-1.817959	-0.991070	-2.667983	1.696743	-1.833093
Labor	0.082672	-4.026030	-0.343214	-4.324500	0.086345	-4.018494
Energy	-0.188680	-5.243892	-0.351973	-4.791576	-0.188775	-5.244297

The visualization of the data series reveals clearly that there are trends in the data. In order to test this hypothesis unit root tests have been conducted on each data series and their correlograms have been analyzed. These analyses have been performed on the raw data, the logarithmized data and the scaled data. As a result, these tests revealed that for each type of data the data series has a unit root. The t-statistic values from the Augmented Dickey-Fuller tests are summarized below in Table 4. For the raw data, the logarithmized data and the scaled data all variables had a unit root in their level data, no matter which confidence interval, 1%, 5% or 10%, was looked at. These unit roots did not persist if the test were performed on the first level differences of these variables with the sole exception to capital stock. For the capital stock series only the second level difference did not have a unit root.

The stationarity of the data series could have been obtained through taking the respective number of differences or detrending the data. But both of these options were tried and had their drawbacks. Taking the first level or second level differences as well as detrending the data creates a data series with negative values. Leaving the meaning of the estimation results aside, solely from a technical perspective this was not possible since in this case negative values were tried to be raised to non-integer powers, which is not possible in the real mathematical environment. From the interpretation side, even if taking the differences or detrending the data would have given a logical value for the parameter estimates, from the economical perspective the obtained estimates would not have represented the definitions which were tried to be obtained. This idea is supported by the existing literature on elasticity of substitution estimations. To present some evidence, it can be mentioned that neither of the estimations conducted by Su et al. (2012), Kemfert and Welsch (2000) or Van der Werf (2008) mention any detrending or differentiating performed on the data, even though they present extensively the data they have used. Therefore, the same approach was followed and the raw data was used throughout the estimations.

The estimations are performed on two datasets: scaled and not-scaled data. While several normalization methods for data exists, the method adopted in this study, after a long period of search for the optimal scaling method, is the division by the minimum method. In this method each data series, capital, labor, and energy, is divided respectively by its minimum value for time period considered. This method is used in order to avoid divergences caused by taking the power of values less than 1. Several other scaling methods are tried out as well, such as using the logarithmical values, yet neither method has given significant estimation results or has given as significant results as the division by minimum method. Therefore, the estimations have been performed for the data, as it is, and for the data scaled according to the division by its minimum values. Not surprisingly, these two estimations result in the same estimation outputs as it will be presented in Chapter 5.

CHAPTER 5

EMPIRICAL FINDINGS

The estimations in this study are made with the use of the statistical software EViews, using its built in nonlinear least square estimation tool. Due to the nature of nonlinear least squares estimation, certain input values for the parameters are required. The starting value for the parameter σ is chosen according to existing literature on a similar estimation and hence has not been changed, while the starting values for A is chosen according to the scaling method. Results are presented for a certain starting value of α but grid search has been performed on this parameter, which are not presented in order to avoid redundancies. The results together with the starting values will be presented in two sections: for the data, as it is, and for the scaled data.

5.1 Not scaled data estimation results

The estimations presented in this section are carried out without any scaling performed on the data. The units are changed appropriately in order to establish equal orders of magnitude. The regression output “regr” is presented in Table 3 with the coefficients M(1), M(2) and M(3), where M(1), M(2) and M(3) are respectively the estimates for the parameters A, α and σ. The starting values for the parameters were set as M(1)=25, M(2)=0.4 and M(3)=0.3. The starting value for σ was chosen to be 0.3 based on the study by Kumbaroglu, Karali and Arikan (Kumbaroglu et al., 2008), while the other values are set as they are based on expert guess. Even though a grid search for numerous other starting parameter values is performed, and the results lead to the same output. It is important to point out that even though no restrictions are manually imposed on the parameters, the estimated values are within the meaningful intervals supporting the significance of the estimation results.

Table 5. Estimation Output with Not Scaled Data

Dependent Variable: GDP
Method: Least Squares (Gauss-Newton / Marquardt steps)
Date: 12/01/17 Time: 18:24
Sample: 1988 2014
Included observations: 27
Convergence achieved after 27 iterations
Coefficient covariance computed using outer product of gradients
GDP=M(1)*(M(2)*KLCOMPOSITE^((M(3)-1)/M(3))+(1-M(2))
*ENERGY^((M(3)-1)/M(3)))^(M(3)/(M(3)-1))

	Coefficient	Std. Error	t-Statistic	Prob.
M(1)	110.1588	7.800216	14.12253	0.0000
M(2)	0.779849	0.131559	5.927748	0.0000
M(3)	0.644032	0.225783	2.852443	0.0088

R-squared	0.983722	Mean dependent var	9.06E+08
Adjusted R-squared	0.982366	S.D. dependent var	2.84E+08
S.E. of regression	37652860	Akaike info criterion	37.83016
Sum squared resid	3.40E+16	Schwarz criterion	37.97414
Log likelihood	-507.7071	Hannan-Quinn criter.	37.87297
Durbin-Watson stat	0.860725		

The results show that convergence was achieved after 27 iterations and each one of the estimated values for the parameters is statistically significant. No matter whether 10%, 5% or even 1% confidence interval selection, the coefficient estimates are significant as it is observable from their probabilities. The predictive power of the model is strong with a 98% adjusted R^2 value. The Durbin-Watson statistics indicates that there might be serial correlation in the residuals. Since the data is not scaled and the data series consists of high number entries the value obtained for the sum squared residuals is high. Yet, this does not mean that the model is not valid, since the other indicators point towards significance of the model. Still, the high value of the sum squared residuals is among the motivators to undertake the estimation on scaled data. Even though the scaled data output is taken as final estimation output of this study, for the sake of presenting the consistency and similarity between the scaled and not scaled data estimation output, the output of the not scaled data is presented as well.

Table 6. Confidence Intervals for Estimation Output with Not Scaled Data

Coefficient Confidence Intervals
Date: 12/01/17 Time: 18:27
Sample: 1988 2014
Included observations: 27

Variable	Coefficient	90% CI Low	90% CI High	95% CI Low	95% CI High	99% CI Low	99% CI High
M(1)	110.1588	96.81356	123.5041	94.05995	126.2577	88.34208	131.9755
M(2)	0.779849	0.554767	1.004931	0.508324	1.051373	0.411886	1.147811
M(3)	0.644032	0.257745	1.030320	0.178040	1.110025	0.012532	1.275532

The confidence intervals for the coefficients are presented in Table 6. With the obtained standard deviation for each coefficient estimation, the confidence intervals are given for each significance level. It is clearly concludable that the estimated values remain within the borders of the confidence intervals no matter which significance level is chosen.

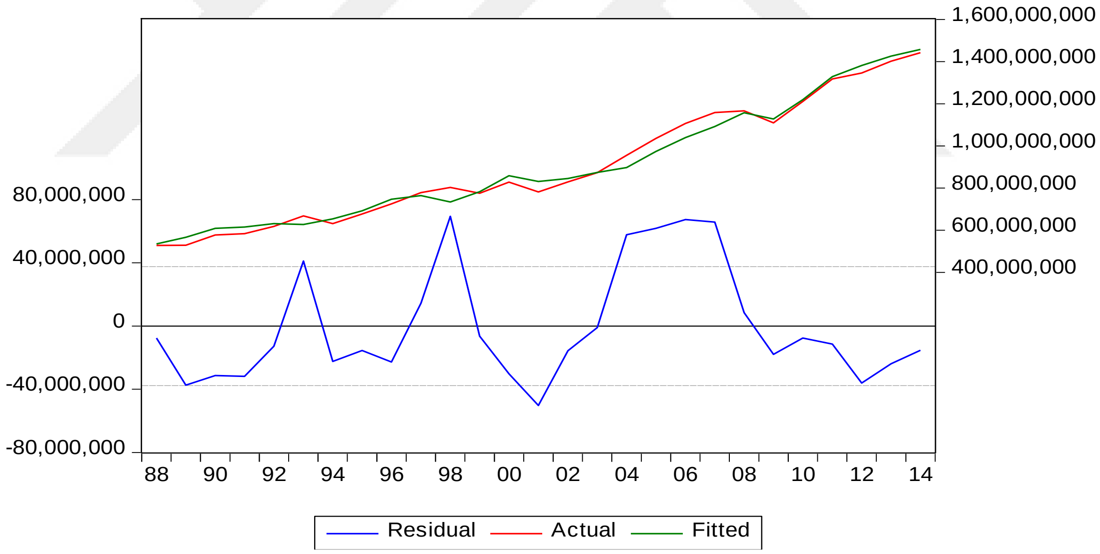


Figure 3. Actual, fitted, residuals graph for estimation output with not scaled data

When the actual and predicted values for output are plotted, presented in Figure 3, a close proximity is clearly observable as it is presented in the below figure. Particularly, over the more recent years of the considered period the fitted results are closer to the actual results than in previous periods. This may be due to the fact that the study took for several parameters and data real value 2011 as the base year.

5.2 Scaled data estimation results

Scaling applied to the raw data in this section is to achieve a normalization of the data. For this purpose, each data entry of each of the three independent variables—capital, labor and energy—is divided by the minimum value of that particular series over the time period considered (1988-2014). Not surprisingly, the minimum values are the values for the starting year 1988. The regression output is named “regr” and the coefficients are M(1), M(2) and M(3), where M(1), M(2) and M(3) are respectively the estimated values for the parameters A, α and σ, as it is the case in section 5.1. The starting values for the parameters are similarly to the ones in previous section, with the sole difference on the starting value for A, which is set to be equal to 1 due to the scaling applied. Overall, the starting values for the presented output are M(1)=1, M(2)=0.4 and M(3)=0.3. When compared to the estimation output using the raw data, the significance of the coefficient estimations is higher in the scaled case. The estimation results are presented in Table 7.

Table 7. Estimation Output with Scaled Data

Dependent Variable: GDP
Method: Least Squares (Gauss-Newton / Marquardt steps)
Date: 12/01/17 Time: 18:30
Sample: 1988 2014
Included observations: 27
Convergence achieved after 18 iterations
Coefficient covariance computed using outer product of gradients
GDP=M(1)*(M(2)*KLCOMPOSITE^((M(3)-1)/M(3))+(1-M(2))
*ENERGY^((M(3)-1)/M(3)))^(M(3)/(M(3)-1))

	Coefficient	Std. Error	t-Statistic	Prob.
M(1)	0.992745	0.013665	72.64781	0.0000
M(2)	0.842243	0.049231	17.10794	0.0000
M(3)	0.645513	0.225444	2.863299	0.0086

R-squared	0.983872	Mean dependent var	1.716667
Adjusted R-squared	0.982528	S.D. dependent var	0.537265
S.E. of regression	0.071017	Akaike info criterion	-2.347351
Sum squared resid	0.121042	Schwarz criterion	-2.203369
Log likelihood	34.68924	Hannan-Quinn criter.	-2.304538
Durbin-Watson stat	0.863832		

Table 8. Confidence Intervals for Estimation Output with Scaled Data

Coefficient Confidence Intervals
Date: 12/01/17 Time: 18:33
Sample: 1988 2014
Included observations: 27

		90% CI		95% CI		99% CI	
Variable	Coefficient	Low	High	Low	High	Low	High
M(1)	0.992745	0.969365	1.016124	0.964541	1.020948	0.954524	1.030965
M(2)	0.842243	0.758015	0.926472	0.740635	0.943852	0.704547	0.979940
M(3)	0.645513	0.259805	1.031221	0.180220	1.110806	0.014960	1.276066

The regression output shows that the model has a high adjusted R^2 value, which means that we can predict the 98% of the dependent variable output, i.e. GDP, with our production function as we have defined it. Comparing the Akaike Schwarz criteria of the two models, the results indicate that the scaled output results are more favorable and should be preferred to the not-scaled case. Indifferent of the choice of a 10%, 5% or 1% confidence interval, the parameter estimates remain certainly within the confidence intervals. The confidence intervals are presented in Table 8.

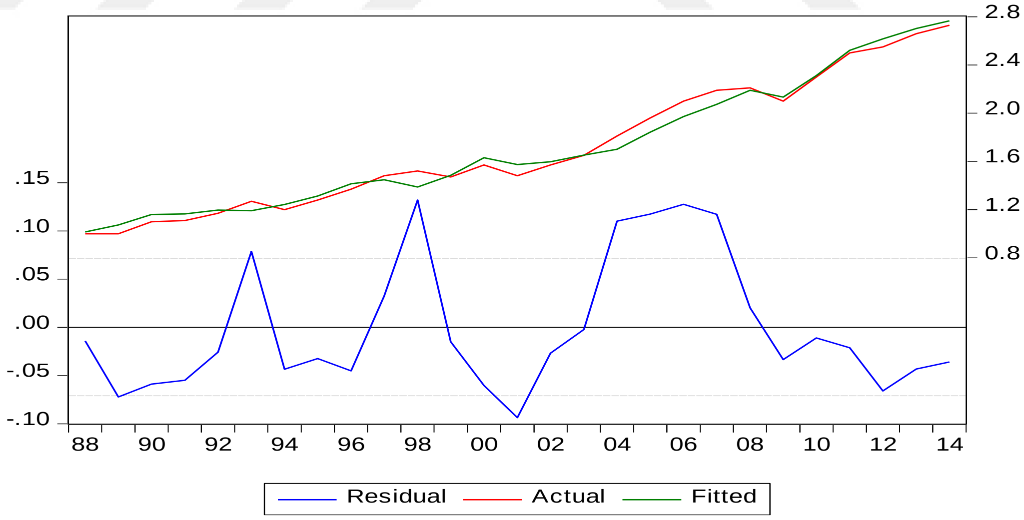


Figure 4. Actual, fitted, residuals graph for estimation output with scaled data

The actual, fitted and residuals graph shows that the fitted values are in line with the actual values throughout the entire time period considered and no significant deviation from the actual values is observable for any particular year. The residuals oscillate around the zero line. While the table for the actual and fitted values together with the residuals is presented in Appendix C, the graph is presented in Figure 4.

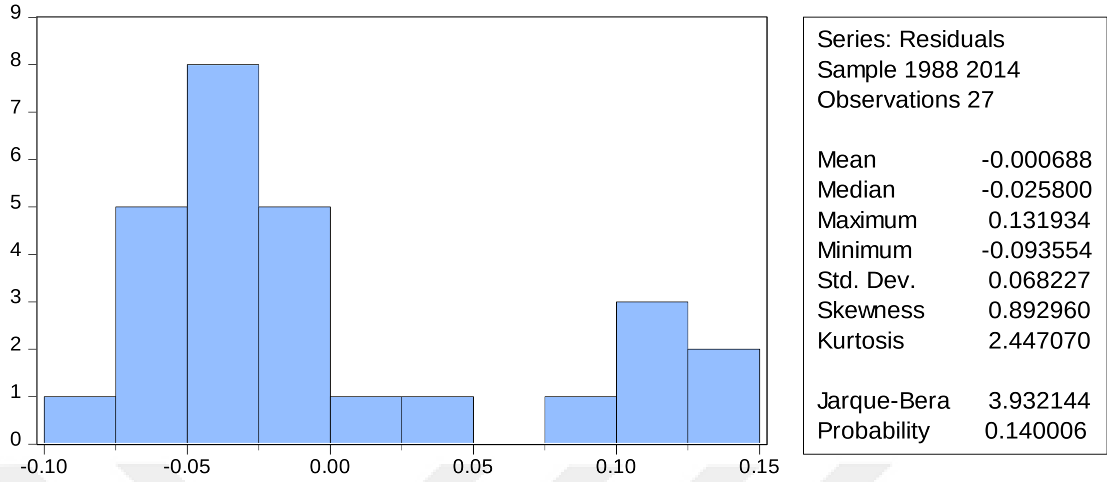


Figure 5. Normalization test for residuals of estimation output with scaled data

Figure 5 presents the normalization test results for the residuals. The mean and median of the residuals is in close proximity to zero, providing supporting evidence for the robustness of this model. In Figure 5, it is observable that even though the residuals are slightly skewed to the right, when plotted they still appear normally distributed. The Jarque-Bera statistic supports this argument, when we compare its value to the chi-square critical value for our degrees of freedom. For each of significance levels 10%, 5% and 1%, the test statistic indicates that the null hypothesis cannot be rejected, and hence our residuals are normally distributed.

To conclude, the findings obtained from the estimations suggest that we can indeed formulate a production function for Turkey which is of the CES form and has capital, labor and energy as inputs entering the function in (KL)E nesting structure. The parameter estimates give for the production function the values $A = 0.992$, $\alpha = 0.842$ and $\sigma = 0.645$. Therefore, the function can be rewritten as follows.

$$Y = A * \left[\alpha * (K^{KPVS} * L^{1-KPVS})^{\frac{\sigma-1}{\sigma}} + (1-\alpha) * E^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}}$$

$$Y = 0.992 * \left[0.842 * (K^{0.5} * L^{0.5})^{\frac{0.645-1}{0.645}} + (1 - 0.842) * E^{\frac{0.645-1}{0.645}}\right]^{\frac{0.645}{0.645-1}}$$

$$Y = 0.992 * [0.842 * (K^{0.5} * L^{0.5})^{-0.550} + (0.158) * E^{-0.550}]^{-1.818}$$

Among the parameter estimates, the economic interpretation for $\sigma = 0.645$ can be made in the way that the elasticity of technical substitution for the capital-labor bundle and energy input to be equal to 0.645. Moreover, taken into consideration the Turkish economy, the capital-labor bundle and energy inputs can be technically substituted for each other a rate of 0.645.

CHAPTER 6

DISCUSSION AND CONCLUSION

The result of this study provides supporting evidence for the necessity of country specific estimations. The initial motivation for undertaking this study consisted of the aim to derive parameter values for a nested CES function with capital, labor and energy as input factors for Turkey. Existing literature already provides some estimations for elasticity of substitutions. While Bosetti et al. (2006) had found the substitution elasticity between the capital-labor bundle and energy to be $\sigma = 0.5$, Gerlagh and Van der Zwaan (2003) and Manne et al. (1995) find this value to be $\sigma = 0.4$. While Paltsev et al. (2005) finds the same outcome as Bosetti et al. (2006), there are numerous research with close but slightly different results. This study reveals that with an estimation performed on the entire Turkish economy, the elasticity of substitution between the capital-labor bundle and energy is $\sigma = 0.645$. The interpretation of this value should not be made in the direction that the estimated value is completely different from what existing literature has obtained so far. The estimated elasticity of substitution value is the estimate for a particular point in time. But according to its confidence intervals with respect to the 1%, 5% and 10% confidence level, the values obtained from previous studies are in close proximity to the estimated value in this thesis. Even though the result is not drastically different from the existing estimates, given the importance of this value, especially when its application will be on a macro level, digit level differences become important.

No doubt, there are many modifications, which can be performed on any research trying to estimate these parameters. Estimations can be done for data on a particular country as well as on aggregated data for a number of countries. Similarly, while the elasticity can be estimated for a particular industry or a group of several industries, it can also be estimated for an entire economy, as it has been done in this study. Even if two different studies have the same nesting and functional structure, they can diverge from each other based on the data used. While labor data comes as numbers and capital stock mostly in monetary terms, energy data can enter the function estimation in many forms. While some studies use energy consumption, other research uses primary energy supply. Similarly, energy can be taken in terms of joule or other power units, or energy can be taken in the form of energy cost. Both types of data for energy in the production function are equally common, and this thesis follows amongst other the energy data approach applied by Van der Werf (2008) and takes energy cost. All these modifications can cause variations in the estimated parameter values and therefore create a need for aim-specific estimations.

In this study, even if not presented in detail, several data series have been tested. Data on capital stock and labor are not changed throughout the different estimations, yet energy data has undergone some changes. Estimations with energy in terms of peta joule and tonnes of oil equivalent have under neither scaling method led to significant conclusions for the substitution elasticity. Energy, in terms of energy cost, however led directly to significant results for data on Turkey satisfying all convergence criteria of nonlinear least squares estimation. This brings with itself the implication that even though energy consumption does increase over the time in relation with output, it does not have a nonlinear relation as it is indicated in the production function. On the other hand, total energy cost, together with capital and labor, does indeed have a nonlinear link to the economic output as it is also supported with economic theory. This draws attention to the fact that while energy consumption increases, the increase in energy consumption due to increase in output, is more closely linked to the increase in energy prices, which are omitted when solely energy in power units is taken as an input. But it is also worth pointing out that there are studies which take energy data as energy itself in terms of joule and conclude with significant results for other countries and industries, such as in Koesler and Schymura (2015).

This study imposed the (KL)E nesting structure to the production function, but there are also studies, such as the paper by Sue et al. (2012), which investigate numerous different nesting forms and try to observe the most significant structure for an economy as well as for some particular sectors. It is worth mentioning that, while this study has estimated the production function for the entire Turkish economy, the nesting structure might be different from the estimated function for some particular industries, with a more industry specific input factor structure. Particularly some industries, such as the cement industry for example, are more energy intensive than others and do require more energy and capital inputs than compared to labor input. For sector specific analyses, it is therefore of use to estimate beforehand sector specific production functions and its parameters before making conclusions.

The production function structure is not derived based on trials on different versions, but is imposed according to the research aim of this study. The derived parameter estimates are obtained through estimations performed on data for the entire Turkish economy. The main goal was to find the elasticity of substitution for the capital-labor bundle and energy. The thesis presents results for substitution elasticities which are above the values applied and discovered in prior research (Bosetti et al.,2006, Manne et al., 1995 and Paltsev et al., 2005), which can be interpreted as the elasticity of capital-labor and energy is higher in Turkey than the average value. Hence, this means that if the price of the capital-labor bundle increases, it can be relatively easily substituted for with energy. Another way of interpreting these results is that the high share of capital and labor in the production can be the result of energy prices in Turkey being relatively high due to the high share of imported energy used to meet the domestic energy demand. Hence, a reduction in energy prices can lead to a less capital-labor intensive economy for the case of Turkey.

A further point of investigation can be the application of this approach to the various sectors of the Turkish economy. Moreover, a production function estimation can be performed for the driving industries of the Turkish economy where the results can shed light on policy making fostering these industries. In this case, industry specific price can be obtained for the input factors, which could bring along besides the elasticity of technical substitution also the elasticity of substitution based on changes in prices.

Another potential topic for further research could be estimation of different types of production functions for Turkey. For instance, instead of assuming directly a Cobb Douglas form, capital and labor can be taken to be of a CES form. At the same time, based on the research question under focus, the energy input factor can be disaggregated based on electric and non-electric energy. These possible topics and numerous other ones remain potential questions for further research.

APPENDIX A

DATA FOR KPVS CALCULATION

Table A1. Data for KPVS Calculation

Sign	Y	W	T	z	1-z	1-KPVS	KPVS
Year	GDP (mil. TL)	Compensation of Employees (mil. TL)	Net Indirect Taxes (mil. TL)	Self-Employment Share	Adjustment Factor	Adjusted Labor Share	Capital Value Share
2009	1,154,993	310,939	116,634	0.40	0.60	0.50	0.50
2010	1,235,088	334,568	142,775	0.39	0.61	0.50	0.50
2011	1,394,477	371,489	160,329	0.38	0.62	0.49	0.51
2012	1,441,500	402,766	160,240	0.37	0.63	0.50	0.50
2013	1,546,090	432,597	181,262	0.36	0.64	0.49	0.51
2014	1,604,569	463,491	168,777	0.34	0.66	0.49	0.51
2015	1,703,874	498,384	184,873	0.33	0.67	0.49	0.51

APPENDIX B

DATA FOR PRODUCTION FUNCTION

Table B1. Data for Production Function

Sign	Y	K	L	E		
Year	GDP	Capital Stock	Labour	Energy		
Unit	Real GDP at constant national prices (mil. USD)	Real capital stock at constant national prices (mil. USD)	All men and women above age 15 employed (thousand)	Tonne of oil equivalent (thousand toe)	Real crude oil import prices (USD/barrel)	Real cost of primary energy supply (mil. USD)
1988	527,703	967,657	17,754	47,290	28.80	9,982
1989	529,031	1,022,162	18,222	49,100	32.30	11,624
1990	577,994	1,091,569	18,539	52,720	39.79	15,375
1991	583,350	1,157,767	19,288	51,980	31.56	12,026
1992	618,259	1,228,304	19,459	53,630	29.85	11,734
1993	667,979	1,327,095	18,499	56,890	24.99	10,422
1994	631,537	1,395,754	20,006	56,210	23.55	9,704
1995	676,952	1,471,528	20,586	61,570	24.77	11,179
1996	724,374	1,566,721	21,194	66,920	29.04	14,245
1997	778,911	1,680,505	21,204	70,410	26.33	13,590
1998	802,994	1,778,473	21,778	71,750	16.55	8,702
1999	775,970	1,838,701	22,048	70,450	21.70	11,206
2000	828,538	1,925,322	21,581	75,960	34.76	19,354
2001	781,332	1,946,604	21,524	70,240	29.19	15,030
2002	829,493	1,992,272	21,354	74,220	29.47	16,035
2003	873,168	2,062,372	21,147	77,880	33.08	18,882
2004	954,921	2,182,020	19,632	80,730	41.56	24,594
2005	1,035,149	2,332,883	20,066	84,210	58.34	36,010
2006	1,106,507	2,505,195	20,423	93,150	68.60	46,840
2007	1,158,165	2,673,324	20,738	100,000	74.41	54,543
2008	1,165,796	2,810,613	21,194	98,710	102.46	74,134
2009	1,109,536	2,888,749	21,277	97,790	64.24	46,048
2010	1,211,136	3,033,170	22,594	106,660	80.73	63,117
2011	1,317,386	3,220,938	24,110	113,510	109.81	91,365
2012	1,345,412	3,382,558	24,821	118,220	109.44	94,832
2013	1,401,819	3,551,956	25,524	116,940	104.64	89,694
2014	1,442,670	3,707,829	25,932	121,540	94.74	84,404

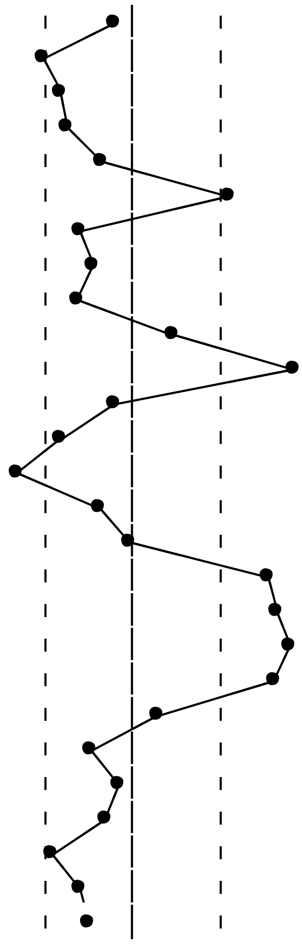
APPENDIX C

ACTUAL, FITTED, RESIDUALS

Table C1. Actual, Fitted, Residuals Table

obs	Actual	Fitted	Residual
1988	1	1.01416...	-0.0141...
1989	1	1.07213...	-0.0721...
1990	1.1	1.15871...	-0.0587...
1991	1.11	1.16484...	-0.0548...
1992	1.17	1.19580...	-0.0258...
1993	1.27	1.19129...	0.07870...
1994	1.2	1.24340...	-0.0434...
1995	1.28	1.31232...	-0.0323...
1996	1.37	1.41500...	-0.0450...
1997	1.48	1.44738...	0.03261...
1998	1.52	1.38806...	0.13193...
1999	1.47	1.48495...	-0.0149...
2000	1.57	1.63022...	-0.0602...
2001	1.48	1.57355...	-0.0935...
2002	1.57	1.59668...	-0.0266...
2003	1.65	1.65222...	-0.0022...
2004	1.81	1.69991...	0.11008...
2005	1.96	1.84254...	0.11745...
2006	2.1	1.97223...	0.12776...
2007	2.19	2.07280...	0.11719...
2008	2.21	2.18967...	0.02032...
2009	2.1	2.13347...	-0.0334...
2010	2.3	2.31096...	-0.0109...
2011	2.5	2.52113...	-0.0211...
2012	2.55	2.61592...	-0.0659...
2013	2.66	2.70323...	-0.0432...
2014	2.73	2.76588...	-0.0358...

Residual Plot



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