MODELING BELIEF REVISION VIA BELIEF BASES USING SITUATION SEMANTICS

AYŞE SENA BOZDAĞ

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Ayşe Sena Bozdağ

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Modeling Belief Revision via Belief Bases Using Situation Semantics

The thesis of Ayşe Sena Bozdağ

has been approved by:

Prof. Berna Kılınç
(Thesis Advisor)

Assist. Prof. Lucas Thorpe

Assist. Prof. Alexandra Zinke
(External Member)

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ABSTRACT

Modeling Belief Revision via Belief Bases Using Situation Semantics

The belief base approach to belief representation and belief dynamics is developed as an alternative to the belief set approaches, which are pioneered by the AGM model. The belief base approach models collections of information and expectations of an agent as possibly incomplete and possibly inconsistent foundations for her beliefs. Nevertheless, the beliefs of an agent are always consistent; this is ensured by a sophisticated inference relation. Belief changes take place on the information base instead of on the belief set, providing a reasonable account of belief change, for the content of the information base is much smaller than a closed belief set, and directly accessible by the agent for its elements are characterized as explicit and non-inferential information the agent acquires, while the closed belief set represents what the agent is committed to believe. In chapter 2, I present an investigation of the belief base approach, both its statics (formation of beliefs from an information base) and its dynamics, while presenting the consequences of the approach; as well as a brief investigation of the AGM model as the representative of the belief set approaches, to make comparisons between the two approaches. In chapter 3, I offer a modal model of the statics of the belief base approach using situation semantics. The choice of semantics is primarily due to that situation semantics can model incomplete and inconsistent collections of sentences. The belief modality offered in this model is intended to capture the inference process in the belief base approach as much as possible.

ÖZET

İnanç Temeli Aracılığıyla İnanç Değişiminin Durum Semantiği ile Modellenmesi

İnanç gösterimi ve inanç dinamiğinde inanç temeli yaklaşımı, AGM modeli ile öncülenen inanç kümesi yaklaşımlarına alternatif olarak geliştirilmiştir. İnanç temeli yaklaşımı, kişilerin bilgi ve beklentilerini, inançlarının muhtemelen eksik ve tutarsız temelleri olarak modeller. Buna rağmen, kişinin inançları her zaman tutarlıdır; bu karmaşık bir çıkarım ilişkisi ile garantiye alınır. İnanç değişimleri, inanç kümesi yerine bilgi temeli üzerinde gerçekleştirilir. Bilgi temelinin içeriği inanç kümesine göre çok daha kısıtlı ve elemanları kişi tarafından doğrudan ulaşılabilir ve çıkarımsal olmayan cümleler olarak tanımlandığından, bu yaklaşım gerçekçi bir inanç dinamiği teorisi sunar. İkinci bölümde, inanç temeli yaklaşımının, hem statik (bilgi temeline dayalı inanç oluşumu) hem dinamik boyutlarının incelemesi sunuluyor. İnanç kümesi yaklaşımlarını temsilen AGM modelinin kısa bir incelemesi ile iki yaklaşım kısaca karşılaştırılıyor. Üçüncü bölümde ise, durum semantiği kullanılarak inanç temeli yaklaşımının statik boyutunun modal bir modellemesi sunuluyor. Semantik tercihi, durum semantiğinin muhtemelen eksik ve tutarsız cümle kümelerini modelleyebilmesinden kaynaklanıyor. Bu modelde sunulan inanç modalitesinin inanç temeli yaklaşımında kullanılan çıkarım ilişkisinin özelliklerini olabildiğince yansıtması hedefleniyor.

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CHAPTER 1

INTRODUCTION

The belief base approach to belief representation and belief dynamics is developed by Sven Ove Hansson in his 1999 book *A Textbook of Belief Dynamics: Theory Change and Database Updating* and by Hans Rott in his 2001 book *Change, Choice and Inference: A Study of Belief Revision and Nonmonotonic Reasoning* as an alternative to belief set approaches to belief dynamics, pioneered by the so-called AGM model developed by Carlos E. Alchourrón, Peter Gärdenfors, and David Makinson in their famous 1985 paper *On the Logic of Theory Change: Partial Meet Contraction and Revision Functions*. The theory I will sketch in Chapter 2 is based on Hans Rott's formulation of belief base and belief base dynamics.[1] Although I find this formulation more realistic, this is a normative model that specifies the pattern for what an ideally rational agent should believe. Hence, the formalism and logic.

In Chapter 3, I develop a modal representation of belief with the intention to capture the belief base approach. My primary aim is to offer a model which accurately represents the structure of an inference base, and statics of belief (the inference relation) as will be articulated in Chapter 2. The model should be able to represent an incomplete and possibly inconsistent inference base, with a belief modality which does not allow inconsistent beliefs inferred by the agent. For this purpose, I make use of situation semantics. Situation semantics is developed by Jon Barwise and John Perry in their 1983 book *Situations and Attitudes*.

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[1] While the theory I will sketch here is largely based on Rott's approach, on some issues which I find questionable in Rott's formulation, I will give my own development. Thus, in some aspects the theory I will present may not be in one-to-one correspondence with Rott's.

Situation semantics is an alternative to possible worlds semantics to model a doxastic modality. In situation semantics, situations play the same role as possible worlds play in possible worlds semantics. They may be defined as sets of propositions. Unlike the possible worlds, situations are not complete in the sense that their content is not closed under Cn, and they do not necessarily contain logical truths. Situation semantics is more suitable for modeling the belief base approach because of these features. Modal operators are defined by the relations between situations.

CHAPTER 2

THE BELIEF BASE APPROACH

2.1 Preliminaries of belief representation in the belief base approach

In Rott's theory of belief and belief dynamics, the doxastic state of an agent is represented by two sets of sentences, whose elements stand respectively for pieces of information acquired by the agent and for the expectations of the agent. The former set is called the belief base by Rott. The belief base is not a theory, but rather a possibly incomplete and possibly inconsistent set of sentences. A more accurate name for Rott's belief base would be the information base. This is because, not all pieces of information represented in the information base are accepted as beliefs by the agent. Information in this context refers to information gathered from any source, which may be subject to error, including for agents, testimonies of other agents, perception, etc. Thus, some of the information acquired by the agent may be faulty, and it is possible that elements of the information base may contradict with one another. The information base serves as a cautious foundation for the beliefs of the agent. Elements of the information base are non-inferential sentences. Which means that they have independent standings in the doxastic state of an agent, they do not depend on any other information the agent has. In this context, inferential sentences are those obtained by reflecting on this non-inferential content.

A belief set is constituted via the inferences carried on the information base. According to the belief base approach, the belief set of an agent represents what the agent is committed to believe based on the information she acquires and her expectations provided that she is rational. The belief set of the agent is always consistent and logically closed.

Strictly speaking, the consistent belief set is formed on the basis of the information base with the help of the expectation base, which serves as a background theory for the agent's inferences from the information base. Like the information base, the expectation base is a possibly incomplete and possibly inconsistent set of sentences, which stand for the agent's expectations about how things normally are. Rott does not give an account of how the expectations of an agent are established, and I will not try to give one neither. Intuitively, expectations are basic opinions the agent holds about how things are most of the time. They can be regarded as some sort of soft information; however, since they are accepted by the agent as true most of the time, they are not hold as strong as the information. In case of conflict between expectations and information, expectations are usually overridden by information. On the other hand, the expectation base is more established compared to the information base in the sense that it is more stable than the information base i.e. the agent does not acquire new expectation as she acquires new information. The formal differences between the information base and the expectation base are only the two just explained; otherwise expectations and information are treated in the same manner while forming as well as changing beliefs with an effect of only the former differentiating factor above.

Both the information base and the expectation base are further structured with a preference ordering. The preference ordering enables the agent to make rational choices between pieces of information and expectations when there are contradictions and some information and/or expectations are to be excluded in the process of forming a belief set in order to ensure consistency of beliefs. While forming the belief set, the sentences at higher orders suppress the sentences at lower orders which contradict with them or with their conclusions. Therefore, the sentences at higher orders are preferred for the inferences of the agent. Since the belief set of the agent consists of these inferences, consequently sentences at higher orders form the beliefs of the agent. Rott (2001, p. 35) defines the prioritization between clusters of propositions as follows: a prioritized information base is denoted by $H = \{H_1, H_2 \ldots H_n\}$, where higher indexed H_is are more valuable, e.g. H_2 is preferred to H_1. Consequently, $\varphi < \psi$ if and only if $\varphi \in H_i$ and $\psi \in H_j$ with $i < j$. Similarly, a prioritized expectation base is denoted by $E = \{E_1, E_2 \ldots E_m\}$. Elements of the information base are usually more preferred than the elements of the expectation base. If the two bases are combined into a set G, we have $G = \{E_1, E_2 \ldots E_m, H_1, H_2 \ldots H_n\}$(Rott, 2001). To illustrate, suppose the information base of the agent, denoted by H, consists of three propositions, such that $H = \{\neg p, q, p\}$ where $\neg p < p < q$. From now on, let us use the notation $H = \{\neg p < q < p\}$ for such prioritized bases. For the belief set which will be formed on the basis of H will be consistent, $\neg p$ will be overridden by p in the process of making inferences from the information base. The prioritization in the information base and in the expectation base significantly affects belief change strategies of agents as it affects inferences made from an information base.

2.1.1 Inference from an information base

An inference operator is defined on a prioritized set of sentences which consists of the elements of the information base and the expectation base of the agent, where the expectations are less preferred compared to the information as stated in the previous section. Let us call the set G in the previous section the inference base, defined as $G = \{E_1, E_2 \ldots E_m, H_1, H_2 \ldots H_n\} = \{G_1, G_2 \ldots G_k\}$, where k = m + n, and $G_1 = E_1$ and $G_{m+1} = H_1$. In this notation, the difference between information and expectations of an agent is simply a difference in their levels of certainty, represented by the preference ordering. Inference from an information base *H* is defined as follows: $Inf(H) = \cap\{Cn\ (G'): G' \in (G \Downarrow \bot)\}$ (Rott, 2001). According to this formulation, $(G \Downarrow \bot)$ denotes the set of all *maximally consistent best* subsets of *G*, such that a subset *G'* of *G* belongs to this set only if *G'* does not entail contradiction and *G'* is selected as among the best according to the preference ordering in the inference base. The process which forms the set $(G \Downarrow \bot)$ is explained by Rott (2001, p. 41) as follows: the set $(G \Downarrow \bot)$ is formed by maximizing the sentences accepted at each level of priority, beginning with the highest index G_i and advance step by step to G_1, with the constraint of consistency. In order to illustrate, consider an information base $H = \{s < \neg p < \neg s < r < q < p\}$ and $E = ø$. We begin with the most preferred sentence *p*, and maximizing the sentences which are picked out for $(G \Downarrow \bot)$ according to their preference ordering through the least preferred sentence *s,* $H_i = \{\neg s, r, q, p\}$, $H_j = \{s, r, q, p\}$... where $i > j$. Supposing these two are the maximally consistent best subsets of *H*, $Inf(H) = Cn(H_i) \cap Cn(H_j)$. The set formed by the inference operator Inf is consistent and closed with respect to the monotonic consequence operation Cn.[2]

The logic of the inference relation is non-monotonic. Rott's inference relation is similar to partial meet consequence relation.[3] From a set of sentences, the best

[2] Rott (2001, p.41) states in a footnote that a slightly different form of inference from a belief base is given by Sven Ove Hansson (1991, Section 2) which does not take the closure of the best subsets under classical consequence.

[3] The partial meet consequence operation, as it is defined by David Makinson (2005) is formulated as follows: "let A be a singleton premise set A={a}, K be a closed background theory, δ a selection function and (Ka) the set of all subsets of K maximally consistent with A, and CK denotes the partial meet consequence operation on K; CK(A) = ∩{Cn(K' U {a}): K' ∊ δ (Ka)}".

The AGM-style non-monotonic consequence relation in terms of Inference relation is defined by Rott (2001, p. 38) as follows: "Inf(H)= E * h, where h = ∧(IHI) i.e. conjunctions of all elements of non-prioritized belief base, and E=Cn(IEI)".

consistent subsets are picked out and then full meet consequence is applied, i.e. the intersection of the consequences of all the best subsets compose a consistent set of sentences which stand for what the rational agent is committed to believe based on the information and the expectations she holds. When new information is acquired by the agent, the consistent best subsets may change, and while the agent infers new consequences, some of her old consequences may vanish.

The background logic also belongs to a family of paraconsistent logics. One of the biggest strengths of Inf is that it can deal with inconsistent information without exploding, i.e. without inferring everything from contradiction[4]. While paraconsistent logics traditionally involve contradictory propositions in the consequence set, the inference operator employed by Rott does not. Thus, Inf is neither reflexive nor supraclassical: It is not generally the case that $H \subseteq Inf(H)$ and it follows, it is not the case that $Cn(H) \subseteq Inf(H)$ (Rott, 2001).

Rott's inference relation is what he calls a consolidation process on the inference base: $Inf(H) = Consol(G)$ (Rott, 2001). The process of consolidation simply involves eliminating inconsistencies from a set. The resulting the belief set never contains inconsistencies.

Defining the inference relation from an information base is an essential step in the belief base approach. For unlike the belief set approaches, the belief base approach does not work on a given belief set. Here, the initial belief set of the agent

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[4] A partial meet consequence operation, as it is defined by David Makinson in his 2005 book *Bridges from Classical to Nonmonotonic Logic*, works with a closed background theory and a set of propositions, which may conflict with each other, which are nonetheless consistent within themselves, unlike the information bases and expectation bases we deal with in the belief base approach. The partial meet contraction operation, upon which the partial meet consequence operation is developed, results in non-monotonic consequences by taking the intersection of *some* subsets of the belief set which does not imply the contracted proposition. While the belief base contraction operation developed via the inference operation takes some subsets of the information base which does not imply the contracted proposition and contradiction.

is constructed on the information base, and when the agent's information changes, changes in the belief set always occur as a result of the changes in the information base.

2.2 Belief revision and belief contraction in the belief base approach

The belief base approach belongs to a family of foundationalist belief change models. Changes of belief take place on the information base, and the changes in the corresponding belief set are always derived as changes resulting from the changes in the information base. Alternative to the foundationalist models are the coherentist belief change models. According to coherentist models, when new information is acquired, the process of belief change first involves making the current set of beliefs consistent with the new information by eliminating the beliefs which imply contradiction with the new information, and then adding the new information into this new belief set. On the other hand, according to foundationalist models, and in particular according to the direct mode of belief change formulated by Rott (2001), first the new information is added to the stock of information whether or not it is inconsistent with the already existing set of sentences, then the set is made consistent by applying the inference operator. The sophisticated step of belief change in the belief base approach is the inference operation, as adding new information to the information base or eliminating information from it is made easily by set-theoretical means.

The information base on which the belief changes take place is *fallible* in the sense that while the inference operation eliminates inconsistencies from the consequent belief set, inconsistent information remains in the information base, though possibly overridden by other information. A coherence inspired approach to the belief base change would eliminate the inconsistencies also from the information base as well as from the expectation base using sophisticated change operators and then forming the corresponding belief set via an ordinary closure operator[5]. This goes against one of the most important strengths of the belief base approach which allows representation of inconsistent information in the information base. The account of fallible bases also means that old information and expectations which are previously suppressed by other information or expectations become available again when the suppressing sentences are retracted in the face of new information.

While changing beliefs, a rational agent makes choices between sentences in order to ensure a consistent belief set by eliminating some of her earlier beliefs which contradict with the new information she acquired. This highlights another advantage of the belief base approach to belief change: besides the theory's ability to represent inconsistent information, the belief base theory makes the process of making choices between sentences much more realistic than the belief set approaches. It is more reasonable to suppose that the choices are made within an incomplete set of non-inferential sentences instead of within a closed set of implicitly held belief.

Let us define belief revision in the belief base approach. Let H be the information base and let K be the corresponding belief set, such that $K = Inf(H)$. Suppose we revise K by new information φ. The revision operator will be denoted by $(*)$. Accordingly, we denote the set K revised by φ as $(K * \varphi)$. Belief revision is defined as follows: $(K * \varphi) = Inf(H \, o \, \varphi)$, where o is the operation of adding φ to H, such that φ acquires *the highest* order of preference in the prioritized information base H (Rott, 2001).

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[5] The approach is called "Infallible foundations". Rott, H. (2001).

Assigning the new information the highest order of preference is due to two concerns. First, it ensures that the revision operation will be successful in the sense that the sentence by which the beliefs of the agent is revised is included in the new belief set of the agent, i.e. it will not be overridden by a sentence in the initial information base which may entail $\neg\varphi$; unless φ itself is not a contradiction, in that case φ never takes place in the new belief set due to consolidation. Second concern is due to simplicity: we avoid the complex process of assigning precise preferential values to each of the sentences in the information base and in the expectation base.[6]

To define belief contraction in the belief base approach, again let H be the initial information base and let K be the initial belief set. Let (—) denote the contraction operation, so that we denote the set K contracted with respect to φ as $(K — \varphi)$. In case of belief contraction, a set-theoretic operation of deleting φ from the initial information base would not ensure successful elimination of the sentence which is to be contracted since the contracted sentence may still be entailed by items in the information or expectation base. In order to avoid this, we define contraction with respect to a sentence as if revision by the negation of that sentence, such that $(K — \varphi) = (K * \neg\varphi^*)$. Here, $\neg\varphi^*$ stands for a phantom belief, which is used only to ensure that φ does not occur in the new belief set, but $\neg\varphi$ itself is not used for inferences and does not occur in the belief set, unless it is entailed by the other elements of the information base or by the expectation base.

In order to illustrate the process, let the prioritized information base be $H = \{p < p \vee q < \neg q\}$ and the expectation base be $E = \varnothing$. We want to contract the

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[6] Nevertheless, assigning the new information the top most priority is philosophically problematic for new information and new evidences we acquire may be uncertain, and may deserve much less priority. Models which do not assign absolute priority to the new information in revision are given by Galliers (1992) and Cross & Thomson (1992), models of Autonomous and Fact-finding belief revision respectively.

corresponding belief set *K* with respect to *p*. Note that if the contraction operation had been defined similar to the revision operation as *(K — p) = Inf(H / p)*, ¬*q* and *p v q* in the information base together would together entail *p* back into the contracted belief set. With contraction defined as (*K — p) = (K ∗ ¬p*)*, we require the agent to give up either ¬*q* or *p v q* by adding ¬*p* with the top most priority to *H*. The choice between ¬*q* and *p v q* is made according to the preference ordering in the information base. In case *p v q* < ¬*q*, the new belief set *K' = Cn{¬q}*[7]. *K'* does not involve inferences which use ¬*p.*

Before moving on to present some consequences of the belief base approach, I will briefly sketch the AGM model, the pioneer of non-monotonic belief studies, so that the novelties brought by the belief base approach will be more clear.

2.3 Belief representation and belief dynamics in the AGM model

The partial meet contraction and revision operators presented in the 1985 paper *On the logic of theory change: Partial meet contraction and revision functions* still remain as reference points for many subsequent non-monotonic belief change models, including Rott's model of contraction and revision on belief base.

The AGM model represents the doxastic state of an agent by a belief set which is closed under Cn. As opposed to the belief base approach, which presents a two layer belief change process (first from the initial information base *H* to new information base *H'*, and then from *H'* to the new belief set *K'*), in the AGM model

[7] Since the preference ordering in an information base is defined between clusters of sentences, there may be ties or incomparabilities between sentences. In such cases, the rational choice theory offers two strategies. The first one is to pick one element randomly, but in this case the choice cannot be said to be inherently rational. The second strategy is to obey the principle of indifference and look for a suitable combination of the alternatives, nevertheless a combination of the alternatives may not be as good as choosing one of them. The consequences of applying these two strategies on the example would respectively be, choosing one of ¬*q* and *p v q* for the inference base randomly, or eliminating both from the inference base, since accepting both is not an option for they together imply *p* back into the belief set.

belief contraction and belief revision take place on this given belief set. The AGM model is a coherentist belief change model: whenever the initial belief set is consistent, the new belief set obtained as the result of AGM belief change is also consistent and closed under classical consequence.

Partial meet contraction of a belief set K with respect to a sentence φ is defined as follows: $K— \varphi = \cap \delta (K \perp \varphi)$. Here, $(K \perp \varphi)$ denotes the set of all subsets of K which does not imply φ, and δ denotes a selection function which picks out some of the maximal subsets of $(K \perp \varphi)$ arbitrarily or according to a preference relation defined between the subsets of K. When δ marks of the elements of $\delta (K \perp \varphi)$ as the best elements of $(K \perp \varphi)$, we say δ is relational over a set of beliefs K. A partial meet contraction determined by such selection function is also called relational (Alchourrón, Gärdenfors, & Makinson, 1985).

Revision of a belief set is defined from contraction via the Levi identity, as follows: $K * \varphi = Cn(\cap \delta (K \perp \neg \varphi) \cup \{\varphi\})$. In the case of revision, the first step is to contract the initial belief set with respect to the negation of the sentence by which it will be revised in order to prevent inconsistency within the set when the sentence is added to it, then the sentence is simply added to the set and the resulting set is closed under Cn.

Partial meet contraction is developed in the 1985 paper as an alternative to previously employed maxi-choice contraction and full meet contraction operations. Full meet contraction and maxi-choice contraction are the limiting cases for partial meet contraction, when all such subsets are picked out by the selection function and when only one of them is picked out respectively. Rather than taking the intersection of all such subsets or piking out only one of such subsets, partial meet contraction avoids limiting the number of consequences obtained from a contracted set by taking the intersection of *some* subsets of *K* which does not imply the sentence with respect to which it is to be contracted.

2.3.1 AGM postulates for belief contraction and belief revision

The six postulates given for belief change (six for contraction and revision each) are called the AGM postulates or the Gärdenfors postulates. Alchourrón et al. (1985) defined that for every belief set K, (—) is a partial meet contraction operator if and only if it satisfies the six AGM postulates. In the 1985 paper, two additional postulates are offered for belief changes with respect to conjunctions. The eight AGM postulates for partial meet contraction are formulated as follows (Rott, 2001):

(—1) $K — \varphi = Cn(K — \varphi)$ whenever $K = Cn(K)$ (Closure)

(—2) $K — \varphi \subseteq K$ (Inclusion)

(—3) If $\varphi \notin K$, then $K \subseteq K — \varphi$ (Vacuity)

(—4) If $\varphi \in K — \varphi$, then $\varphi \in Cn(ø)$ (Success)

(—5) $K \subseteq Cn(K — \varphi) \cup \{\varphi\})$ (Recovery)

(—6) If $Cn(\varphi) = Cn(\psi)$, then $K — \varphi = K — \psi$ (Extensionality)

(—7) $(K — \varphi) \cap (K — \psi) \subseteq K — (\varphi \wedge \psi)$ (Conjunctive overlap)

(—8) If $\varphi \notin K — (\varphi \wedge \psi)$, then $K — (\varphi \wedge \psi) \subseteq (K — \varphi)$ (Conjunctive inclusion)

The first one is the closure postulate. It says that whenever the initial belief set is closed under Cn, the new set obtained by contraction is also closed.

Postulate (—2) says that in case of contraction, no new beliefs are added to the original belief set. As we shall see, this postulate is not satisfied by the belief base contraction.

Postulate (—3) says that if the sentence which will be contracted is not contained in the initial belief set, no change occurs in the belief set as a result of the contraction operation.

Postulate (—4) ensures that the operation is successful in the sense that the contracted sentence is not contained in the contracted belief set, unless it is a tautology.

The recovery postulate (—5) is the most controversial AGM postulate. It says that whenever a set is contracted with respect to a sentence and then the sentence is added to the contracted set, all the beliefs in the original set are retained. There are many counterexamples to this postulate in the literature. The postulate is not satisfied in the belief base approach.

Postulate (—6) says that logically equivalent sentences are treated alike by the contraction operation.

A contraction operation is a *transitively relational partial meet contraction operator* if it also satisfies postulates (—7) and (—8) (Alchourrón et al., 1985). The postulates concern contractions of conjunctions. Postulate (—7) says that if some belief in K survives contraction with respect to φ as well as with respect to ψ, it is also withstands contraction with their conjunction. This postulate ensures minimal loss of beliefs in case of contraction with a conjunction. While postulate (—8) says that in order to contract a belief set with respect to a conjunction, the agent must give up at least one of the conjuncts; in this case, we expect that contraction with respect to a conjunction will lead to the loss of beliefs which would have been lost in case of contraction with respect to at least one of the conjunctions.

Postulates for partial meet revision are obtained via the Levi identity, and they are formulated as follows (Rott, 2001):

(∗1) $K * \varphi = Cn(K * \varphi)$ whenever $K = Cn(K)$ (Closure)

(∗2) $\varphi \in K * \varphi$ (Success)

(∗3) $K * \varphi \subseteq Cn(K \cup \{\varphi\})$ (Expansion 1 or Inclusion)

(∗4) If $\neg\varphi \notin K$, then $Cn(K \cup \{\varphi\}) \subseteq K * \varphi$ (Expansion 2 or Vacuity)

(∗5) If $Cn(\varphi) \neq L$, then $K * \varphi \neq L$ (Consistency Preservation)

(∗6) If $Cn(\varphi) = Cn(\psi)$, then $K * \varphi = K * \psi$ (Extensionality)

(∗7) $K * (\varphi \wedge \psi) \subseteq Cn((K * \varphi) \cup \{\psi\})$

(∗8) If $\neg\psi \notin K * \varphi$, then $K * \varphi \subseteq K * (\varphi \wedge \psi)$

The first three postulates are obvious and postulate (∗6) is the same as postulate (—6). The vacuity postulate says that if the sentence by which the belief set will be revised does not contradict with the existing elements of the belief set, none of the beliefs is lost as a result of revision by that sentence.

Consistency postulate (∗5) says that the revised belief set is consistent, unless the sentence it is revised by is itself a contradiction.

Postulate (∗7) says what occurs in the belief set revised by a conjunction is also in the union set of revision by one of the conjuncts and the other conjunct. While postulate (∗8) says that if the conjuncts are consistent with each other, none of the beliefs in the belief set revised by one of the conjuncts are lost in the belief set revised by the conjunction.

We leave the discussion of postulates to the next section.

2.4 Properties of the belief base approach

In this section, I will point out some properties of representation and dynamics of belief in the belief base approach. The first property to consider is about the structure of the information base. The information base consists of pieces of information acquired by the agent, which are non-inferential sentences. These pieces of information are discursive but not necessarily in the form of atomic sentences, they may be complex. They are the foundations for other beliefs of the agent, which are inferred via the inference relation and which are not contained in the information base. In this way, the belief base approach makes a distinction between non-inferential and inferential beliefs, while the belief set approaches including the AGM model treats inferential and non-inferential beliefs equally. The distinction between non-inferential and inferential beliefs allows identifying the supporting beliefs, from which the merely derived beliefs are inferred, and merely derived beliefs. In turn, it ensures that when the supporting belief is retracted, the beliefs merely derived from it are also given up from the belief set. In other words, along with the beliefs which entail the contracted belief, the beliefs which are entailed by it are also given up, as it should be expected from a successful belief contraction. In the belief set approaches, only the beliefs which entail the contracted belief are eliminated in the new belief set, and the consequences of the contracted belief remains in the set without any support.

Another advantage of differentiating inferential and non-inferential beliefs is, that the agent chooses the sentences which will be retained as beliefs and which will be given up as the result of a belief change process within a smaller set of sentences, whose elements supposed to be explicit to the agent. According to the alternative belief set approaches, these choices are made within comparatively huge sets, a process which does not seem realistic, since in a closed belief set, some sentences may be implicit for the agent, and the agent cannot be expected to make choices among sentences which are not consciously accessible by her. In the belief base approach, expectations of an agent are implicit elements of the inference base, however the choices are not directly made on the expectation base; expectations play the role of hidden premises to make inferences from the information base.

The second property to state is the approach's ability to represent inconsistent information. Usually there are more than one information source for an agent, and even when all sources are consistent within themselves, information they provide may contradict with information from other sources.[8] A rational agent does not hold contradictory beliefs, she nevertheless may acquire contradictory information. Providing a sophisticated inference operator which applies to the information base, namely consolidation of the information base with respect to contradiction, the belief base approach allows representation of inconsistent information without risking the consistency of beliefs.

This property also adds to the explanatory power of the belief base approach. Since belief set approaches represent the doxastic state of an agent with a closed set of sentences, they cannot distinguish between two information states, both of which are inconsistent within themselves, yet which consist of distinct pieces of information. To illustrate, Let A, B be different information bases, such that $A = \{\neg q < q < p\}$ and $B = \{\neg s < s < r\}$, and suppose the expectation base is always empty. Let $A' = Cn(A)$ and $B' = Cn(B)$, and let L be the set of all sentences in the Language. Note that $A' = B' = L$. Suppose we want to contract A' and B' with respect to a sentence p. If the contraction operation operates on the belief sets, since $A' = L$, $A' — p = L — p$. Similarly, $B' — p = L — p$. Hence, contraction of p from A' and B' results in the same set of beliefs, although B does not contain p to begin with. Let us see what happens when the contraction operation takes place on the information bases. According to the belief base approach,

[8] We would like to grant the agent the choice to reject inputs coming from information sources which are self-contradictory.

A' — p = Inf(¬q, q) = Cn(q). While *B' — p = Inf(¬s, s, r) = Cn(r, s)* which is equal to *B'*.

In terms of expressive power, another advantage of the belief base approach is that the belief set is constructed on an information base taking into account the structural differences in the formulation of sentences. There are various belief bases for a fixed belief set. For instance, information bases *H = {q, p}* and *H' = {p → q, p}* have the same Cn closure and correspond to the same belief set provided that the expectation base is empty. The belief base approach captures how differences in the formulation of sentences in the information base lead to differences in change strategies. When the two agents with information bases *H* and *H'* and with *E = ø* want to contract *q* from their belief sets, they use different strategies: while the former gives up *q*, the latter gives up *p* or *p → q*. Consequently, the contracted belief sets differ from each other. Moreover, the change strategies depend on the preferential ordering of the information in the information base. In the above example, the choice of giving up *p* or *p → q* from *H'* depends on their preference ordering. Thus, the belief base approach explains how people that seem to share the same beliefs may follow different paths when incorporating new information.

2.4.1 Postulates for belief base revision and contraction

In the AGM model, the postulates for belief contraction are given first, and the postulates for belief revision follows from them via the Levi identity, as the operation of belief revision is defined by belief contraction via the Levi identity. In the belief base approach, we give primacy to belief revision, and define the belief contraction operator by the revision operator. Thus, the properties of belief base contraction depend on the properties of belief base revision. Giving primacy to belief revision is more realistic given real life belief change cases the agents face, most of which occur in the face of acquiring new information; thus most belief changes that take place are revisions by new information. The postulates for belief base revision are presented along with their counterparts as postulates of finitary inferences, as provided by Rott, to indicate how the properties of belief revision - thus properties of belief contraction - are derived from the properties of non-monotonic inference relation. Postulates for belief base revision given by Rott (2001) are as follows:

(∗1) If K = Cn(K), then K ∗ φ = Cn(K ∗ φ)

($*2^-$) If Cn(φ) ≠ L, then φ ∈ K ∗ φ

(∗3) K ∗ φ ⊆ Cn(K ∪ {φ})

($*5^+$) K ∗ φ ≠ L

(∗6) If Cn(φ) = Cn(ψ), then K ∗ φ = K ∗ ψ

(∗7) K ∗ (φ ∧ ψ) ⊆ Cn((K ∗ φ) ∪ (K ∗ ψ))

(∗8c) If ψ ∈ K ∗ φ, then K ∗ φ ⊆ K ∗ (φ ∧ ψ)

(∗8vwd) K ∗ (φ ∨ ψ) ⊆ Cn(K ∗ φ U K ∗ ψ)

The first postulate is same as the AGM postulate of closure. It follows from the Cn-closure postulate of *Inf* that *Inf(φ) = Cn(Inf(φ))*.

Belief base revision does not satisfy the AGM postulate for success (∗2). The AGM postulate says that the revised belief set always contains the sentence by which it is revised. However, success is guaranteed in belief base revisions as long as the input sentence is not a contradiction itself, for the consolidation process ensures that there cannot be any contradiction in the belief set. Thus, a weaker version of success is satisfied by belief base revision.

Postulate (∗2) corresponds to the reflexivity of the inference operation, so that; $\varphi \in Inf(\varphi)$. However, Inf fails reflexivity since if $Cn(\varphi) = L$, then $\varphi \notin Inf(\varphi)$.

Belief base revision satisfies the inclusion postulate (∗3). The corresponding postulate for Inf is a weakening of Conditionalization: $Inf(\varphi) \subseteq Cn(Inf(T) \cup \{\varphi\})$, where *Inf(T)* is a consistent theory. Rott (2001, p. 116) states that weak conditionalization does not have any motivation for non-monotonic inferences except that it corresponds to (∗3).

Belief base revision satisfies a stronger version of the consistency preservation postulate offered by the AGM model, ($*5^{+}$). The revised theory is always consistent since there is no way $\varphi \in K * \varphi$ if $Cn(\varphi) = L$. With weak success and strong consistency, the belief base approach says that rational agents refuse to accept inconsistent information (Rott, 2001). The postulate corresponds to strong consistency of *Inf*, such that; $Inf(\varphi) \neq L$.

The model satisfies extensionality postulate (∗6), which corresponds to left logical equivalence for non-monotonic inferences, such that; If $Cn(\varphi) = Cn(\psi)$, then $Inf(\varphi) = Inf(\psi)$.

Postulate (∗7) for revisions with conditionals is also satisfied. It is the counterpart of Conditionalization for non-monotonic inferences;
$Inf(\varphi \wedge \psi) \subseteq Cn(Inf(\varphi) \cup \{\psi\})$ The postulate is shown equivalent to (∗7g):
$(K * \varphi) \cap (K * \psi) \subseteq K * (\varphi \vee \psi)$, which says that whatever is in both belief sets revised with each of the conjuncts, is also in the belief set revised with their disjunction. Postulate (∗7g) corresponds to Or property for non-monotonic inferences: $Inf(\varphi) \cap Inf(\psi) \subseteq Inf(\varphi \vee \psi)$.

Belief base revision fails the AGM postulate (∗8). This means that the inference relation fails rational monotony. However, it satisfies a weakening of (∗8), (∗8c), which corresponds to cumulative monotony for the inference relation: If $\psi \in Inf(\varphi)$, then $Inf(\varphi) \subseteq Inf(\varphi \wedge \psi)$. It also satisfies (∗8vwd), which corresponds to very weak disjunctive rationality for the inference relation: $Inf(\varphi \vee \psi) \subseteq Cn(Inf(\varphi) \cup Inf(\psi))$.

Belief base revision does not satisfy postulate (∗4) for vacuity, neither it satisfies the weakening of it, the Preservation postulate: If $\neg\varphi \notin K$, then $K \subseteq (K * \varphi)$. This is because even if the input information does not contradict with the initial belief set, acquiring new information may cause the agent to reconsider her old beliefs.

Belief base contraction fails the postulate (—2) for inclusion. For eliminating a belief from the belief set may lead to reassessment of some information which are previously dominated by the eliminated belief. Belief base contraction also fails the recovery postulate (—5): when a belief set is contracted with respected to a belief and then revised by the same belief, all the beliefs in the original set may not be retained. For when a belief set is contracted with respect to some sentence, all sentences in the set which imply that sentence are also eliminated, and some of them may not recover when the contracted sentence is added again.

2.4.2 Iterated belief change

Modeling iterated belief change is only possible with binary belief revision and belief contraction operators. The AGM model works with fixed belief sets and with a family of unary change operators defined for a unique belief set and a singleton propositional input. Whereas in order to model iterated belief change, the change operators should be somehow independent from the belief set, so that they can apply to various different belief sets in the same manner. It is also important that the belief change model satisfies the principle of categorical matching, which states that an agent's doxastic state is represented in the same way prior to and after the belief change takes place. The belief base approach satisfies both conditions. First, it presents binary belief change operators, as functions from an inference base and a (singleton) propositional input to a new inference base. Second, the preference ordering is a structure inherent in the information base and in the expectation base, thus the revised or contracted bases are also structured accordingly. The AGM model fails also on this aspect. It imposes entrenchment orderings on belief sets, however those orderings are not transferred to the revised or contracted sets.

Belief revision and belief contraction strategies are further characterized by elements of the doxastic state other than the current stock of beliefs. Dispositions concerning the dynamics of belief are more stable than the beliefs themselves. This feature is usually provided by a background preference ordering, which is due to the doxastic state of the agent, but independent of the current belief set. Providing a preference ordering as such allows employing choice functions which depend neither on the current belief set nor on the new information, but which are at the same time faithful to the doxastic state of the agent. Belief change operators with such choice functions satisfy rational choice constraints which state that rational choices are independent of the choice menus.

Rott's formulation for iterated belief change is as follows (Rott, 2001):

$(K * \varphi) * \psi = K * (\varphi \wedge \psi)$ if $\{\varphi, \psi\} \nvDash \bot$,

$K * \psi$ otherwise.

2.4.3 The role of expectations

The belief base approach captures a difference between explicit and implicit elements of the doxastic state. Elements of the information base are characterized as explicit and assumed to be consciously accessible by the agents. While, elements of the expectation base function as hidden premises for making inferences. Though some expectations may take place in the inference set, they may remain implicit, such that they are not consciously accessible unless the agent reflects upon them. The idea behind defining expectations as elements of the doxastic state is that people draw conclusions which go beyond their information and their explicit beliefs.

Expectations are suppositions of agents about what normally is the case. Information which contradict with expectations are usually regarded as indicating exceptions, hence, expectations are usually overridden by information as less preferred items of the inference base. There may be exceptional cases though, where an agent prefers an expectation over some explicit information. For instance, in case the source of the information is regarded as doubtful by the agent. Although, this course of thinking requires defining a more sophisticated preference relation on the inference base instead of assigning the top most priority to the latest information.

In any case, the effect of expectations of an agent on her inferences is undeniable. Agents who have access to exactly the same pieces of information may differ in their belief states due to different expectations they hold. Thus, expectations are effective in determining how the information are interpreted. Moreover, agents may hold different expectations for different contexts, while their stock of explicit information remains unchanged. While inferences made in a friendly environment may depend on more general expectations, agents may be more cautious and even skeptical during a trial, and may employ a smaller set of expectations.

It is important to note the difference between canceled information in the information base and canceled expectations in the expectation base. When a piece of information is canceled by another information which entails its negation, due to the consolidation process and consistency of beliefs, the two pieces of information cannot take place in the same belief set. Furthermore, it is unlikely to suppose that two conflicting pieces of information may be used to make inference for the same belief set even though the inferences somehow do not conflict with each other. When a piece of information is negated by a new one, provided that the new information is accepted by the agent, the former piece of information is expected to become devoid of all its inferential force, for it should be regarded as false. The case of expectations is different. When an expectation is overridden or negated by a new piece of information, it is not regarded as false. That it is overridden merely means for the agent that there are exceptions to it, and the case which verifies this piece of information is one of such exceptions. The expectation which is overridden by a specific information can still be in use for making inferences which refer to other cases, provided those inferences do not lead to contradiction in the belief set. For instance, the agent may have heard that someone dropped a glass ball from two meters high, and the glass ball did not break. While the agent may believe in the accuracy of this information, though wondering about what may have caused such an event, she can still believe that if you drop a glass ball from two meters high it will break. Consequently, the expectation base is more stable than the information base. In order to cancel an expectation, the agent must acquire some groundbreaking information which indicates on the contrary of what the agent is used to.

Although the above discussion may indicate some conceptual difference between information and expectations, they are formulated as entities of the same kind in the model which represents doxastic states. The obvious motivation is simplicity: to enable them to be employed by the same inference operation. However, while we do not model the difference between information and expectations for the time being, there is no harm in being aware of these differences, which may be modelled by a more complicated preference structure, which is at the same time responsive to the context.

In chapter 3, I will offer a modal model of the belief base approach with situation semantics.

CHAPTER 3

MODELING THE BELIEF BASE APPROACH USING SITUATION SEMANTICS

3.1 The belief base model

The language for the belief base model has the following syntax:

AT := p, q, r…

$\varphi ::= AT \mid \neg\varphi \mid (\varphi \wedge \psi) \mid (\varphi \vee \psi) \mid (\varphi \rightarrow \psi) \mid B\varphi$

$\top$ (always true) and $\bot$ (always false)

The set of all formulas in this language is denoted by L.

Let ***F*** be a preferential frame structure; ***F*** = ⟨*S, C,* <⟩.

Here, *S* is a non-empty and finite set of situations $S = \{s_1, \ldots, s_n\}$. Situations are defined as sets of sentences. Their non-modal content consists of the atomic sentences and non-modal formulas of the language (sentences which do not contain the modal operator *B*, they may still contain other modal operators), which in the context of the belief base model represents the elements of the information base and of the expectation base. Their modal content consists of the modal formulas of the language (sentences which contain the modal operator *B*). The modal content represents the beliefs of the agent, the sentences accepted as beliefs based on the relevant inference base. Thus, a situation represents a possible inference base by its non-modal content, and the relevant belief set by its modal content. We call the union of non-modal content and modal content of a situation the content of the situation. The content of a situation is not necessarily closed under Cn.

Situations can be distinct yet compatible, and they can be parts of other larger situations. Unlike possible worlds, they are not disjoint alternatives. They can represent possible belief states which are compatible with each other.

I now define a binary partial relation C between situations. C is a parthood relation between situations, intended to represent information growth, so that for $s, x \in S$, if sCx, we say that s is a part of x, or that s is involved in x. When the valuation function is defined on the preferential frame $\boldsymbol{F}$, we will see that if sCx, then the non-modal content of s is set-theoretically contained in x, although the modal content of situations need not be persistent through C. The relation C is reflexive, anti-symmetric and transitive. C is also convergent: for all $s \in S$, there is a $s' \in S$ such that $s' \neq s$ and sCs'. As such, C is the relation of information growth between compatible belief states.

Let $<$ be an irreflexive and transitive preference relation between situations. For $s, x \in S$, we say that s is preferred to x and denote it as $x < s$. The preference relation between situations is intended to correspond to the preference relation defined by Rott between clusters of sentences, which are subsets of an inference base. The preferential structure of the situation frame plays an important role in the non-monotonic nature of belief and belief change. It enables us to define a non-monotonic consequence relation for inferring beliefs in the model, similar to the inference relation used in the belief base approach by Rott.

Let us define a preferential model structure $\boldsymbol{M}$; $\boldsymbol{M} = \langle \boldsymbol{F}, v \rangle$.

Here $\boldsymbol{F}$ is a preferential frame as defined above, and v is a mapping from atomic sentence letters to a pair (V, F) of subsets of S, v: AT → (V, F). Intuitively, the set V denotes the verifiers, and the set F denotes the falsifiers of a sentence within S. Let us denote the set of verifiers for p with $v(p)+$, and the set of falsifiers for p

with $v(p)^{-}$. If $s \in v(p)+$, we write $s \vDash p$ and say that s verifies p. If $s \in v(p)^{-}$, we write $s =| p$ and say that s falsifies p.[9]

The subsets V and F of S need not be disjoint, so that a situation s may be both in the set $v(p)+$ and in the set $v(p)^{-}$. Therefore, it is possible that $s \vDash p$ and $s =| p$ simultaneously. Moreover, it is $V \cup F$ need not be S. There may be some sentences, which are not mapped into a situation, such that it is possible that for some $s \in S$, neither $s \vDash p$ nor $s =| p$. Therefore, the content of the situations may be incomplete with respect to the language. In the context of the belief base approach, we interpret this feature as indicating that the belief state of an agent may be silent about some sentences. In other words, it may be the case that an agent has neither acquired some sentence as information nor she holds it as an expectation.

The semantics for negation, disjunction, conjunction and material implication are as follows (Fine, 2016):

$s \vDash \neg p$ if $s =| p$

$s =| \neg p$ if $s \vDash p$

$s \vDash p \wedge q$ if $s \vDash p$ and $s \vDash q$

$s =| p \wedge q$ if $s =| p$ or $s =| q$

$s \vDash p \vee q$ if $s \vDash p$ or $s \vDash q$

$s =| p \vee q$ if $s =| p$ and $s =| q$

$s \vDash p \rightarrow q$ if $s \vDash \neg p$ or $s \vDash q$

$s =| p \rightarrow q$ if $s \vDash p$ and $s =| q$

[9] This idea and notation is based on the valuation offered by Kit Fine in his 2016 paper "Truth maker semantics".

We assume that the valuation of atomic sentences is persistent through the parthood relation C: for $s, x \in S$, if $s \vDash p$ and sCx, then $x \vDash p$. With the above definitions, it follows that the non-modal content of situations is persistent through the parthood relation.

According to this model, a situation may verify a formula and its negation at the same time. In this way, inconsistent collections of information and expectations can be modelled. We need to ensure that while a situation models a possibly inconsistent inference base of an agent, it cannot contain modal formulas which contradict each other and modal formulas which quantify over contradictions. This is because the intended belief approach ensures that an agent does not accept contradictions as belief and she does not simultaneously entertain beliefs which contradict with each other. In order to ensure consistency of beliefs, we introduce an additional structure to the preferential frame given above.

Let the modalized preferential frame be a structure $\boldsymbol{F}^*=\langle S, S^*, C, <\rangle$.

Let us define S^* as a subset of S satisfying: for all $s \in S$, $s \in S^*$ if and only if it is not the case that $s \vDash \varphi$ and $s \vDash \neg\varphi$.

Truth in a model is defined via S^*: a formula φ is true in a model $\boldsymbol{M}$, i.e. $\vDash_M \varphi$ if and only if for all $s \in S^*$, $s \vDash \varphi$.

In order to use in the definition of belief, let us define a concept of "being a source for", using the two elements of the preferential frame, the parthood relation C and the set S^*. For, $s, x \in S$, we say that, s is a source for x, and denote it by sSx, if and only if sCx and $s \in S^*$ and s is maximal in the sense that it has only inconsistent proper extensions within x. Intuitively, a source for a situation can be used for inferences in a situation which it is a source for, since its content is consistent and the most informative compared to the other situations it is compatible with. In the context of the belief base approach, the source situations correspond to the maximally consistent subsets of the inference base.

In order to model a belief modality that corresponds to the inference relation employed by Rott in the belief base approach, we define a class of best situations according to the preference ordering < defined between situations. The class of best situations is a subset of the set S^*: $BEST(s) = Max_{<} \{s' \mid s' \in S^*, s'Cs\}$. So, $s' \in BEST(s)$ only if $s' \in S^*$, $s'Cs$, and there is no better situation in S^* according to the preference ordering i.e. s' is maximal in the preference ordering.

We define belief as follows: for $s \in S$, $s \vDash B\varphi$ if and only if for all $s'Ss$ such that $s' \in BEST(s)$, $s' \vDash \varphi$. The agent will be committed to believe only the sentences which are supported by all best source situations. $s =\mid B\varphi$ if and only if for some $s'Ss$ such that $s' \in BEST(s)$, $s' =\mid \varphi$.

According to a belief modality defined as such, even if the source situations for a situation are not mutually consistent, modal content of the situations is always consistent, in the sense that for all $s \in S$, it is not the case that $s \vDash B\varphi$ and $s \vDash \neg B\varphi$, and for all $s \in S$, it is not the case that $s \vDash B\varphi$ and $s \vDash B\neg\varphi$. To show the first point suppose (1) for some $s \in S$, $s \vDash B\varphi$. Then, (2) for all $s'Ss$ such that $s' \in BEST(s)$, $s' \vDash \varphi$. Suppose also (3) $s \vDash \neg B\varphi$. Then (4) for some $s'Ss$ such that $s' \in BEST(s)$, $s' =\mid \varphi$. But since $s'Ss$ only if $s \in S^*$, it is not the case that for some $s'Ss$, $s' \vDash \varphi$ and $s' =\mid \varphi$, hence, (4) contradicts with (2), therefore, it is not the case that $s \vDash B\varphi$ and $s \vDash \neg B\varphi$. To show the second point, suppose (1) and (2). Suppose also (5) $s \vDash B\neg\varphi$. Then, (6) for all $s'Ss$ such that $s' \in BEST(s)$, $s' \vDash \neg\varphi$. But since $s'Ss$ only if $s \in S^*$, it is not the case that for some $s'Ss$, $s' \vDash \varphi$ and $s' =| \varphi$, hence, (6) contradicts with (2), therefore, it is not the case that $s \vDash B\varphi$ and $s \vDash B\neg\varphi$.

Lemma: (i) if $s \in S^*$, then $BEST(s) = \{s\}$

(ii) if $s \in S^*$, then $s \vDash \varphi$ implies $s \vDash B\varphi$

Modal formulas are neither persistent through the relation C, nor are they persistent through the relation of being a source for. Let $s = \{p, \neg p\}$, $t = \{p\}$ and $u = \{\neg p\}$. Therefore, tCs and uCs, and tSs and uSs. Suppose $t, u \in BEST(s)$. Therefore, by lemma (i) $t \vDash Bp$, but it is not the case that $s \vDash Bp$ since $u \vDash \neg p$.

3.2 Basic properties of belief

In this section, I present some basic properties of belief based on the belief base model. First, let us see that the model satisfies Kripke's axiom, as well as the inference rules Necessitation and Modus Ponens.

The belief base model satisfies Kripke's axiom (known as axiom **K** in modal logic): $B(\varphi \to \psi) \to (B\varphi \to B\psi)$.

Proof: Suppose $s \in S$ and $s \vDash B(\varphi \to \psi)$. Hence, for all tSs, such that $t \in BEST(s)$, $t \vDash (\varphi \to \psi)$. Hence, for all tSs such that $t \in BEST(s)$, either $t \vDash \neg\varphi$ or $t \vDash \psi$. Suppose for some such t, $t \vDash \neg\varphi$. In this case, $s =| B\varphi$, so $s \vDash (B\varphi \to B\psi)$. If there is no such t, then for all t, $t \vDash \psi$. This implies $s \vDash B\psi$, and thus $s \vDash (B\varphi \to B\psi)$. Since s is arbitrary, for all $s \in S$, $B(\varphi \to \psi) \to (B\varphi \to B\psi)$. Hence, the axiom is satisfied.

The belief base model satisfies the rule of necessitation: $\varphi \to B\varphi$.

Proof: Suppose φ is a valid formula in $\boldsymbol{M}$, that is, $\vDash_M \varphi$. If $\vDash_M \varphi$, then for all $s \in S^*$, $s \vDash \varphi$. Also, we know that for all t and u, if tSu, then $t \in S^*$. Hence, for all

tSu such that $t \in BEST(u)$, $t \vDash \varphi$, therefore, $u \vDash B\varphi$. Since u is arbitrary, this proves, $\varphi \rightarrow B\varphi$. Hence, the rule of necessitation is valid in the model.

The belief base model satisfies Modus Ponens as well as Modal Modus Ponens. The proof for Modal Modus Ponens follows from the satisfaction of the axiom K.

The belief base model satisfies the consistency of belief (known as axiom **D** in modal logic): $\neg B(\varphi \wedge \neg\varphi)$.

Proof: Suppose for some $s \in S$, $s \vDash B(\varphi \wedge \neg\varphi)$. Hence, for all for all tSs such that $t \in BEST(s)$, $t \vDash \varphi \wedge \neg\varphi$ Hence, for all tSs such that $t \in BEST(s)$, $t \vDash \varphi$ and $t \vDash \neg\varphi$. But then, $t \notin S^*$. But since tSs, $t \in S^*$. Hence, we have a contradiction. Therefore, $s =| B(\varphi \wedge \neg\varphi)$. Since s is arbitrary, the axiom is satisfied by the model.

The belief base model satisfies the positive introspection property of belief (known as axiom **4** in modal logic): $B\varphi \rightarrow BB\varphi$

Proof: Suppose for some $s \in S$, $s \vDash B\varphi$. Hence, for all tSs such that $t \in BEST(s)$, $t \vDash \varphi$. Since $t \in S^*$ if tSs, we know that it is not the case that $t \vDash \neg\varphi$, hence, for all uSt, $u \vDash \varphi$ since if uSt, u is maximal. Therefore, $t \vDash B\varphi$. But then $s \vDash BB\varphi$. Since s is arbitrary, the axiom is satisfied by the model.

The belief base model does not satisfy the negative introspection property of belief (known as axiom **5** in modal logic): $\neg B\varphi \rightarrow B\neg B\varphi$

Proof: Suppose $s = \{\neg Bp, \neg p, p\}$. Hence, (1) for some tSs such that $t \in BEST(s), t \vDash \neg p$. Now consider (2) another situation in S, namely, $u = \{p, Bp\}$, uSs and $u \in BEST(s)$. (1) and (2) are compatible since it can be the case that $s \vDash \neg p$ and $s \vDash p$, and modal formulas need not be persistent through the source relation. But then, $s =| B\neg Bp$ since $u =| \neg Bp$ if $u \vDash Bp$. Therefore, the axiom is violated.

Consequently, the belief base model satisfies the basic properties of belief which are also satisfied by doxastic Kripke models (KD4).

According to the belief base approach we would expect the model to violate both the positive and the negative introspection. For the belief base approach models a distinction between explicit and implicit elements of the doxastic state, such that the elements of the information base are explicit while the elements of the expectation base and of the belief set may be implicit. However, the belief base model does not make this distinction. The elements of the information base, the expectation base and the belief set are represented without any distinction. That the positive introspection axiom is satisfied intuitively means that at least the beliefs of an agent are consciously accessible by the agent, directly or by reflection on the doxastic state.

However, the negative introspection property is violated. Violation of this property means that if an agent does not have a belief that φ, she may not be aware of this belief state. Intuitively, this is what we would expect to be. That an agent does not have a belief that φ may mean that the agent does not have any attitude towards φ. We can model negative belief in the form of $\neg B\varphi$ in the model, however, they do not have any intuitive meaning. Although they appear in the content of situations, they are neither elements of the inference base, nor they are elements of the belief set. Negative beliefs appear in the model only as side effects of the formulation of the belief modality.

3.2.1 Some properties of belief based on Rott's postulates for non-monotonic inferences

The belief modality in the belief base semantics is intended to model the inference relation from the base to the belief set in Rott's belief base model. In this section, we will see if it satisfies the basic postulates Rott (2001) gives for non-monotonic inferences:

(1) $Inf(\varphi) = Cn(Inf(\varphi))$

(1a) If $\psi \in Inf(\varphi)$ and $\chi \in Cn(\psi)$, then $\chi \in Inf(\varphi)$

(1b) If $\psi \in Inf(\varphi)$ and $\chi \in Inf(\varphi)$, then $\psi \wedge \chi \in Inf(\varphi)$

(2^-) $Cn(\varphi) \neq L$, then $\varphi \in Inf(\varphi)$

(3) $Inf(\varphi) \subseteq Cn(Inf(T) \cup \{\varphi\})$ where T refers to tautologies.

(5^+) $Inf(\varphi) \neq L$

(6) $Cn(\varphi) = Cn(\psi)$, then $Inf(\varphi) = Inf(\psi)$

(1a) and (1b) are obtained by splitting (1) into two conditions. While, (1) corresponds to Cn-closure for non-monotonic inferences, (1a) is the postulate for right weakening and (1b) is the postulate for And.

The postulates can be translated into the language of the belief base model as follows: *Inf(φ)* corresponds to the extension of modality *B* on a situation *s*, the sole content of which is *φ*. That is because an inference base in Rott's system corresponds to a situation in the belief base model. By the extension of modality *B* on situation *s*, I mean: $B_s = \{\varphi \in L: s \vDash B\varphi\}$.

For any $s, t \in S$, the translation of the basic postulates for belief in the belief base model is then as follows:

(B1) $s \vDash B\psi$ if and only if $\psi \in Cn(B_s)$

(B1a) If s $\vDash$ Bψ and $\psi \vdash \chi$ then s $\vDash$ Bχ

(B1b) If s $\vDash$ Bψ and s $\vDash$ Bχ then s $\vDash$ B ($\psi \wedge \chi$)

($B2^-$) If s $\in$ S* then s $\vDash$ Bψ if s $\vDash \psi$

(B3) If s $\vDash$ Bψ then Cn(T), s $\vDash$ Bψ

($B5^+$) B_s is consistent.

(B6) If s $\vDash \psi$ iff t $\vDash \psi$ then s $\vDash$ Bψ iff t $\vDash$ Bψ

Proof for (B1a): Suppose for some $s \in S$, $s \vDash B\psi$ and suppose $\psi \vdash \chi$. If $s \vDash B\psi$, then for all *tSs* such that $t \in BEST(s)$, $t \vDash \psi$. Since $\psi \vdash \chi$, then for all *tSs* such that $t \in BEST(s)$, $t \vDash \chi$. Hence, $s \vDash B\chi$.

We noted before that the content of a situation is not necessarily closed under Cn. (B1) means that, the modal extension of a situation is closed under Cn. The Cn-closure postulate of Inf means exactly that.

Proof for (B1): $\psi \in B_s$ implies $\psi \in Cn(B_s)$. Proof for the other direction follows from (B1a).

Proof for (B1b): Suppose for some $s \in S$, $s \vDash B\psi$ and $s \vDash B\chi$. If $s \vDash B\psi$, then for all *tSs* such that $t \in BEST(s)$, $t \vDash \psi$. And, if $s \vDash B\chi$, then for all *tSs* such that $t \in BEST(s)$, $t \vDash \chi$. Hence, for all *tSs* such that $t \in BEST(s)$, $t \vDash \psi$ and $t \vDash \chi$, therefore, $t \vDash \psi \wedge \chi$. Hence, $s \vDash B(\psi \wedge \chi)$.

Proof for ($B2^-$): Suppose for some $s \in S^*$, $s \vDash \psi$. That $s \in S^*$ means that if $s \vDash \psi$ then it is not the case that $s \vDash \neg\psi$. Hence, for all *tCs*, it is not the case that $t \vDash \neg\psi$. Hence, there is not a *u* such that *uSs*, and $u \vDash \neg\psi$. Also, since if *uSs, u* is maximal, $u \vDash \psi$. Therefore, $s \vDash B\psi$.

Proof for (B3): Suppose for some $s \in S$, $s \vDash B\psi$. T refers to the sentences which are true in the model, i.e. $\varphi \in Cn(T)$ if and only if for all $u \in S^*$, $u \vDash \varphi$. But then for all tSs such that $t \in BEST(s)$, $t \vDash \varphi$ since $t \in S^*$. Also, since $s \vDash B\psi$, for all tSs such that $t \in BEST(s)$, $t \vDash \psi$. Hence, $\psi, \varphi \nvdash \bot$. Therefore, if $s \vDash B\psi$ then $Cn(T)$, $s \vDash B\psi$.

Proof for ($B5^+$): B_s is consistent means that for any φ, if φ is in B_s then $\neg\varphi$ is not in B_s. The axiom-D corresponds to this property and was already proved in the previous section.

Proof for violation of (B6): Suppose $s \vDash \psi$ iff $t \vDash \psi$. Let $s = \{\neg\psi, \psi, B\psi\}$. Hence for all uSs such that $u \in BEST(s)$, $u \vDash \psi$. Let $v = \{\neg\psi\}$, and $v \in BEST(t)$ but $v \notin BEST(s)$. Hence, while $s \vDash B\psi$, it is not the case that $t \vDash B\psi$. Postulate (6) is not satisfied by the belief base model.

The postulate (B6) states that the content of the situations as far as it is captured by the monotonic background logic Cn is what really matters. This assumption is a strongly coherentist one and it is not in line with the foundationalist reasoning the model is aimed to represent. The postulate (6) is satisfied by non-monotonic inference relations, and its counterpart postulates for belief change are also satisfied by the change operations of the belief base approach while (B6) is violated by the model. This is because the postulate (6) and its counterpart postulates for belief change are formulated with singleton inputs. In translating the postulates, I supposed that the situations capture the discursive inputs such as φ in Rott's system. Which means that if the postulate (B6) is translated back in the language of sentential logic, the input for Cn or Inf are no longer singleton propositions but rather prioritized inference bases. The prioritization in the bases impacts the result of the inference operation Inf, although it does not impact the result of the monotonic consequence operation Cn. In this case the postulate would be violated also by the belief base approach. To illustrate, let H and H' be inference bases, such that $H = (\neg p < \neg q < q \vee p)$ and $H' = (\neg q < \neg p < q \vee p)$. Note that $Cn(H) = Cn(H') = $ L. Nevertheless, $p \in Inf(H)$ and $q \in Inf(H')$, while $p \notin Inf(H')$ and $q \notin Inf(H)$.

3.3 Interpretation of the model

One way to interpret the situations in the belief base model is to take them as potential sources of information for an agent. A similar interpretation is given by Marta Bilkova, Ondrej Majer, and Michal Pelis (2008, 2016) in the context of a scientific framework for an epistemic model. In their framework, different situations represent epistemic states of colleagues working in similar areas, and the contents of the situations are scientific data obtained and supplied by them. Situations which meet relevant criteria are taken as sources of knowledge by an agent, and the agent is said to know φ if φ is adequately supported by some of the source situations. The most informative (maximally consistent) source situations are interpreted as epistemic states of bosses of different scientific groups, which contain the data gathered from less informative yet compatible epistemic states.

For the belief base context, in a pointed model, the current situation represents the doxastic state of an agent, which contains the information acquired by the agent and expectations and beliefs of the agent based on them. Less informative situations, while they are alternative doxastic states themselves, are potential sources of information for our agent.

The parthood relation C in the belief base model is similar to an accessibility relation, that a situation involves another situation means that the former has access to all the information contained in the latter. Although an agent does not accept as belief all the information she has access to, nevertheless that information is acquired by the agent and thus represented in her doxastic state.

The situations which are accepted as source situations are maximally consistent. They are consistent in themselves, since the contrary reflects that some information conveyed by the situation is faulty. If the situation is inconsistent within itself, the information supplied by that situation is denied by the agent in order to avoid faulty conclusions based of faulty information. The source situations are possibly fusions of multiple less informative situations which are compatible with each other. It is supposed that an agent does not reject information which is compatible with her current stock of information. It is similar to the idea that in a scientific framework, results obtained by other colleagues can be used to form hypotheses unless they contradict with other data obtained by the agent.

Although the source situations are consistent within themselves, they may still contradict with one another. For this reason, for an agent to accept some information as belief, it is not sufficient that the information is acquired by the agent from a source situation. It is also required that the information is supported by all source situations which are selected as the best by the agent. The best source situations are interpreted as the most reliable sources of information for the agent. Since source situations are fusions of multiple situations, their preference relation is determined by the preference relation of their parts. For instance, an agent may take as the most reliable information, the results of her own experiments, or the results published by her supervisor. In this case, other most reliable sources of information may be the ones whose contents are wholly or mostly compatible with the most preferred ones. If the information is supported by all such source situations, it is accepted as belief by the agent.

CHAPTER 4

CONCLUSION

In chapter 2, I presented belief representation and belief dynamics in the belief base approach largely based on the theory of Hans Rott. The belief base approach represents the doxastic state of an agent by an information base and by an expectation base, which consist of pieces of non-inferential sentences the agent acquires as information or holds as expectations respectively. The two bases are the foundation for the beliefs of the agent, together they lead to a belief set via the inference relation. The approach allows representation of an incomplete and inconsistent stock of information the agent may acquire without risking the consistency of beliefs, since the inference relation is a consolidation process, which eliminates inconsistencies while forming the belief set. Dynamics of belief in the belief base approach differs from the belief set approaches for the belief changes take place on the information base rather than on the belief set. Although the information base may be inconsistent, the belief base approach satisfies a stronger consistency postulate than the one satisfied by the leading belief set approach, the AGM model. The approach is stronger than the alternatives in terms of expressive power as explained in section four, and it allows modeling iterated belief changes.

In chapter 3, I offered a modal model of belief which I think best captures the representation of belief in the belief base approach. For this purpose, I used situation semantics, for it allows modeling inconsistent and possibly incomplete collections of information and expectations. Developing the model, I presented some properties of belief as captured by this framework. It turned out that the situation semantics for the belief base approach satisfies the basic properties of belief which are also satisfied by doxastic Kripke models (KD4).

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