THE DEVELOPMENT OF VAN HIELE GEOMETRIC THINKING LEVELS IN A COMPUTER-SUPPORTED COLLABORATIVE LEARNING ENVIRONMENT

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DECLARATION OF ORIGINALITY

I, Emre Özkan, certify that

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ABSTRACT

The Development of Van Hiele Geometric Thinking Levels

in a Computer-Supported Collaborative Learning Environment

This mixed methods study investigated the use of a well-designed computer-supported collaborative learning environment, namely Virtual Math Teams, to develop Turkish middle school students' van Hiele geometric thinking levels and attitudes towards mathematics and technology. The study also qualitatively looked into students' Virtual Math Teams discourse to better understand factors leading to higher geometric thinking scores. The sample of the study consisted of 5th and 7th grade students who are at the visualization level according to the van Hiele geometric thinking test from a public and a private schools in İstanbul (*n*=24). For treatment, students are presented with instruction, which was developed based on Hiele's phased-based instruction strategy, within the Virtual Math Teams environment on quadrilaterals (trapezoid, rhombus, parallelogram, rectangle, and square). The data were collected by using van Hiele geometric test and mathematics and technology scale for the quantitative part, and involved the Virtual Math Teams chat logs for qualitative part. The quantitative results showed a statistically significant development on the students' van Hiele geometric thinking. Furthermore, the students' attitudes towards mathematics and technology did not change except a subscale that is attitudes to learning mathematics with technology. There was a statistically significant increase in the students' attitudes to learning mathematics with technology after the treatment. Qualitative results pointed out that collaboration among students could be an essential factor to develop students' geometric thinking levels within the Virtual Math Teams environment.

ÖZET

Bilgisayar Destekli İşbirliğiyle Öğrenme Ortamında Öğrencilerin van Hiele Geometrik Düşünme Seviyelerinin Gelişimi

Bu karma yöntem çalışması, iyi tasarlanmış bir bilgisayar destekli işbirliğiyle öğrenme ortamı olan Sanal Matematik Takımları ortamının Türkiye'deki ortaokul öğrencilerinin van Hiele geometrik düşünme düzeylerinin ve matematik ve teknolojiye yönelik tutumlarının gelişimini incelemiştir. Ayrıca, öğrencilerin daha yüksek geometrik düşünme puanlarına yol açan faktörleri daha iyi anlamak için, öğrencilerin Sanal Matematik Takımları ortamındaki söylemine nitel olarak bakılmıştır. Araştırmanın örneklemi, İstanbul'da bir devlet okulu ve bir özel okulda okuyan ve van Hiele geometrik düşünme testine göre seviyesi görsel düzey olan beşinci ve yedinci sınıf öğrencilerden oluşmuştur (*n*=24). Uygulama olarak, öğrencilere, van Hiele'nin aşama temelli öğretim stratejisine dayalı olarak geliştirilen dörtgenler konusu (yamuk, eşkenar dörtgen, paralelkenar, dikdörtgen ve kare) Sanal Matematik Takımları ortamında sunulmuştur. Veriler, nicel kısım için van Hiele geometrik düşünce seviye testi ve matematik ve teknoloji ölçeği ile toplanmış, nitel kısım ise Sanal Matematik Takımları sohbet kayıtlarını içermiştir. Nicel sonuçlar, öğrencilerin van Hiele geometrik düşünce seviyelerinin istatistiksel anlamlı olarak geliştirdiğini göstermiştir. Bunun yanında, öğrencilerin matematik ve teknolojiye yönelik tutumları, teknolojiyle matematik öğrenmeye yönelik tutum olan bir alt ölçek dışında değişmemiştir. Öğrencilerin matematik ile teknolojiyi öğrenme tutumları ise istatistiksel anlamlı olarak artmıştır. Nitel sonuçlar, öğrenciler arasındaki işbirliğinin, öğrencilerin Sanal Matematik Takımları ortamı içinde geometrik düşünce seviyesini geliştirmede önemli bir faktör olabileceğine işaret etmektedir.

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ABBREVIATIONS

CDASSG: Cognitive Development and Achievement in Secondary School Geometry

CPS: Collaborative Problem-solving

CSCL: Computer-supported Collaborative Learning

DGS: Dynamic Geometry Software

MTAS: Mathematics and Technology Attitudes Scale

OECD: Organisation for Economic Co-operation and Development

PISA: Programme for International Student Assessment

TIMMS: Trends in International Mathematics and Science Study

VMT: Virtual Math Teams

VHGT: Van Hiele Geometric Thinking Test

CHAPTER 1

INTRODUCTION

Geometry is a crucial discipline in the field of mathematics. However, international studies such as Trends in International Mathematics and Science Study (TIMMS) and Programme for International Student Assessment (PISA) have shown that the geometry level of students in Turkey has been below the international average. According to TIMMS data in 2011, for example, the level of students at the 4^{th} grade in Turkey was below the international average and this difference was found to be statistically significant (Oral & McGivney, 2013). The geometry level of students at the 8^{th} grade in Turkey is also below the international average based on the TIMSS data in 2011. Oral and McGivney (2013) claimed that this results pointed to important problems in the quality of geometry education in Turkey. It is argued that the geometry curricula of elementary and middle schools in Turkey misguided students by leading them to memorize definitions and properties of geometric shapes (Olkun, Sinoplu, & Deryakulu, 2009). Students are not expected to make reasoning about geometrical shapes and their features. Furthermore, the use of learning materials in geometry lessons can be another factor. Olkun et al. (2009) stated that teachers in Turkey lack technological learning materials such as dynamic geometry software and knowledge about how to use those materials.

One of the most popular studies about students' geometry achievement and teaching geometry was conducted by two Dutch mathematics educators, Dina van Hiele-Geldof and Pierre Marie van Hiele in the late of 1950s. They studied students' level of understanding geometry and how geometry levels developed. They established a theory that described students' geometric thinking levels as-visualization, analysis, informal deduction, formal deduction and rigor. Furthermore, they proposed an instructional model to move students from one level to the next. This model consists of five learning phases which are inquiry, direct orientation, explication, free orientation and integration. Several studies conducted to test the validity of van Hiele's theory of understanding geometry (e.g., Usiskin, 1982; Burger & Shaughnessy, 1986; Senk, 1989). These studies helped to confirm the validity of the van Hiele theory and showed that this theory can be used to document students' development of geometrical thinking.

Several studies were conducted to determine how to increase students' level of geometry understanding (e.g., Abdullah, Surif, Tahir, Ibrahim and Zakaria, 2015; Abdullah & Zakaria, 2013; Halat, 2006; Duatepe-Paksu & Ubuz, 2009; 2013; Karakuş & Peker, 2015; Kutluca, 2013). The researchers examined the effect of different instructional methods (e.g., drama-based instruction, instruction using a dynamic geometry software). In these studies, not only the instructional method but also the effect of social interaction among students and students' group work became prominent in increasing students' understanding of geometry. Duatepe-Paksu and Ubuz (2009) stated that working as a group in a drama-based instruction enabled students to discuss their ideas with their teammates and had a positive effect on students' engagement, attitudes and geometry achievement. Furthermore, Kutluca (2013) stated that providing students a learning environment with dynamic geometry software (DGS) in which they can share and discuss their ideas comfortably can help increasing students' geometry achievement. Similarly, Karakuş and Peker (2015) conducted a study to examine the effect of DGS on students' geometry achievement and emphasized the role of students' collaborative work on the development of students' geometric thinking. However, to the best knowledge of the author, there are not any studies that investigated the students' van Hiele geometrical thinking levels in a Computer-Supported Collaborative Learning (CSCL) environment.

CSCL is one of the popular areas of learning sciences. According to Stahl, Koschmann and Suthers (2006), "CSCL is an emerging branch of the learning sciences concerned with studying how people can learn together with the help of computers." (p. 409). Virtual Math Teams (VMT) is one of the well-designed CSCL environments in which mathematical ideas are discussed online. VMT is defined as "an open-source, virtual, collaborative learning environment that affords synchronous text-based interaction (chat) with an embedded multi-user dynamic geometry application, GeoGebra" (Oner, 2016, p. 60). VMT enables students to work collaboratively proving multi-user version of GeoGebra. In the VMT environment, students from all over the world come together and solve mathematics problems by discussing and working collaboratively.

The collaboration is a complex mechanism to assess. In PISA 2015, collaborative problem solving framework was presented. In this document, students' collaborative problem solving skills were intended to be assessed by crossing four individual problem solving components with the three main collaboration components. In the current study, along with investigating the role of VMT on improving geometric thinking levels, we also intended to understand how collaboration among students affected this development. Since, the tasks in the present study were not designed as problem solving activities, only the collaboration components of the PISA 2015 were taken into consideration.

Several studies were conducted to examine the effects of computer- assisted instruction on students' attitudes towards technology and mathematics (e.g., Akgül, 2014; Pilli, 2008). Students' attitudes should be taken into consideration since the effectiveness of technological tools is limited by negative attitudes towards mathematics (Pierce& Stacey, 2004).

In the study of Pilli (2008), the effects of a computer software, namely Frizbi Mathematics 4 on students' towards technology and mathematics were examined. Pilli (2008) found that the students who were taught by the computer software had significantly higher scores than students who were taught traditionally. In the study by Akgül (2014), the effects of a DGS-based instruction on students' towards technology and mathematics were examined. Akgül (2014) did not find any significant difference between the scores of the students who taught by DGS-based instruction and the students who taught by traditional instruction. However, Akgül (2014) pointed out that the students who took DGS-based instruction expressed verbally their positive opinions about using the DGS tool. When all these are considered, working with CSCL can also positively affect the student's attitudes towards technology and mathematics.

1.1 Purpose of the study

The purpose of the current study is to examine the role of working within a CSCL environment (VMT) on middle school students' geometric thinking levels and their attitudes towards technology and mathematics, and to understand how collaboration among students influenced their geometry learning while working within the VMT environment.

1.2 Research questions

This study was designed to answer the following main research questions:

1. Is there a statistically significant difference between the pretest and posttest scores on VHGT (Usiskin, 1982), after working within VMT?
2. Is there a statistically significant difference between the pretest and posttest scores on MTAS (Barkatsas et al., 2007), after working within VMT?
3. How did the collaboration among participants, as defined as the PISA framework, influence their geometry learning while working within the VMT environment?

CHAPTER 2

LITERATURE REVIEW

In this chapter, the literature related to the study will be presented in the eight parts. Firstly, the geometry achievement of the Turkish students in several international studies will be discussed. Next, the van Hiele's theory of geometrical thinking levels and its validity in terms of determining the students' geometry understanding will be presented. Then, the strategies to develop van Hiele geometrical thinking levels will be mentioned. After that, the studies that investigated the use of dynamic geometry software to improve students' van Hiele levels will be reviewed. Later on, Virtual Math Teams (VMT) will be mentioned. Next, PISA 2015 collaborative problem solving framework will be presented. In the following part, students' attitudes towards technology and mathematics will be mentioned. In the last part, the summary of this chapter will be given.

2.1 Geometry understanding level of students in Turkey

Tutak and Birgin (2008) stated that student geometry achievement was assessed in several international studies such as TIMSS and PISA. Köseleci-Blanchy and Şaşmaz (2011) stated that the average score of students in Turkey is one of the worst in participating Organisation for Economic Co-operation and Development (OECD) countries in PISA tests that took place in 2006 and 2009. The geometry achievement of students in Turkey does not increase over the years. The average score of students in Turkey is below 46 points the average of OECD countries in PISA test that took place in 2012 and 70 points below the average of OECD countries in PISA test that took place in 2015 (MEB, PISA 2015 Ulusal Raporu, 2016). When the data from the

TIMSS 2011 were analyzed in detail, one can see that Turkey is below the TIMSS scale average in all content domains at 4th grade level, however the geometry scores of Turkish students appear to be the lowest score in all domains (Oral & McGivney, 2013). Oral and McGivney (2013) pointed out that Turkey is 22 points below the scale average in the geometry domain at the 4th grade and this difference was found to be statistically significant. When the data for the 8th grade are considered, the results are not very different from the 4th grade. The average mathematics score of students in Turkey (452) is 48 points below the TIMSS scale average in general. In geometry domain, the average score of the students in Turkey (454) is also below the TIMSS scale average. When the last TIMSS 2015 data are considered, the results are not different from the past results. The average score of the students at the 4th grade in Turkey is 483 and 17 points below the TIMSS scale average (MEB, 2016). According to the result of 4th grade students, Turkey took 36th place out of 49 countries. TIMMS 2015 result also showed that the average score of the students at the 8th grade in Turkey is 458 and 42 points below the TIMSS scale average (MEB, 2016). According to the result of 8th grade students, Turkey took 24th place out of 39 countries (MEB, 2016).

Cansız-Aktaş and Aktaş (2012) claimed that students' geometry achievement is low in international scores since students' geometry understanding levels are not taken into consideration in instruction. Hence, one of the main issues in increasing the level of geometry understanding can be related to the students' geometric thinking levels. Another crucial issue is about the learning activities used in geometry instruction. Students are mostly required to memorize the geometric concepts rather than understanding the relationships among geometrical concepts. Fidan and Türnüklü (2010) claimed that learning activities that require students to construct their own knowledge can be more effective compared to presenting knowledge directly to them. In this respect, developing learning activities based on the constructivist approach and considering students' level of understanding geometry can increase students' geometry achievement.

2.2 The van Hiele Theory

Geometry is one of the main subjects of the school mathematics. However, most students have not been very successful in school geometry (Köseleci-Blanchy & Şaşmaz, 2011). One of the explanations to this problem can be students' difficulty with the higher order cognitive processes required to be successful in geometry (Usiskin, 1982). Two Dutch mathematics educators, Dina van Hiele-Geldof and Pierre Marie van Hiele, worked on the problems in teaching school geometry and developed a theory explaining the levels of geometrical thinking in the late 1950s. The van Hiele Level Theory focused on describing students' cognitive development regarding geometry and suggested teaching strategies to support this development.

2.2.1 The aspects of the van Hiele Theory

There are three main aspects of the van Hiele Level Theory : the existence of levels, properties of the levels, and the movement from one level to the next (Usiskin, 1982).

2.2.1.1 The existence of levels

Van Hieles described five levels of understanding in geometry which are numbered from 0 to 4. These levels were originally named by Dina van Hiele-Geldof as the basic level, the aspect of geometry, the essence of geometry, insight into the theory of geometry, and scientific insight into geometry. However, several studies have used different schemes for these levels and numbered them from 1 to 5 (e.g. Duatepe, 2000; Halat, 2006; Tutak &Birgin, 2008; Karakuş &Peker, 2015). This arrangement helps researchers to use pre-cognition level for students who cannot achieve the requirements of the first van Hiele level which is called as the visual level (Karakuş & Peker, 2015). Hence, the new version of the van Hiele level scheme was used in this study.

Level 1 (Visualization). The visual level is the lowest of the van Hiele geometric thinking levels. At the visual level, nonverbal thinking becomes prominent and shapes are judged according to their appearance (van Hiele, 1999). The features of the figures are not important for the children at this level. For example, students at this level might say that “It is a rectangle because it looks like a box” (van Hiele, 1999).

Level 2 (Analysis). At this level, the properties of a figure become more important than their appearance and children can talk about the features of shapes. Burger and Schaughnessy (1986) stated that children at this level can establish necessary properties of geometrical concepts. The children judges figures by considering their properties rather than what they look like. Van Hiele (1999) described children’s thinking at this level by giving an example: “ an equilateral triangle has such properties as three sides; all sides equal ; three equal angles; and symmetry, both about a line and rotational.” However, children cannot logically order the properties of the shapes at this level (van Hiele, 1999).

Level 3 (Informal deduction). At informal deduction level, the properties of the concepts can be logically ordered by students (van Hiele, 1999). Furthermore, students can differentiate the necessary and sufficient properties of a concept (Burger

& Schaughnessy, 1986). Students use definitions of the geometrical shapes in expressing the relationships. Van Hiele (1999) stated that students at this level can explain why all squares are rectangles by using properties of squares and rectangle. However, this is not like a formal proof. Usiskin (1982) stated that students can make simple deduction but cannot understand mathematical proof. At this level, students cannot understand the intrinsic meaning of deduction such as axioms, postulates and theorems (van Hiele, 1999). In this respect, students at this level have difficulty in understanding Euclidean geometry which includes formal deductions such as axioms, postulates, theorems (Van Hiele, 1999).

Level 4 (Deduction). At the formal deduction level, students can understand the intrinsic meaning of deduction. Students at this level can understand the importance of deduction and the roles of axioms, postulates and proofs in making formal deduction (Usiskin, 1982). Therefore, it is stated that the context of a mathematical system can be reasoned formally by students at this level (Burger & Schaughnessy, 1986).

Level 5 (Rigor). The last level of van Hiele levels of understanding geometry is rigor. At the rigor level, students do not need concrete models to study in different geometries (Burger & Schaughnessy, 1986). Usiskin (1982) stated that students at this level can go beyond the Euclidean geometry and understand non-Euclidean geometries.

2.2.1.2 Properties of the levels

In understanding geometry, the van Hiele theory claims that there are basic properties about all levels (Usiskin, 1982). Firstly, students must follow an order between levels. In other words, levels are sequential. Hence, a student in Level 1 cannot reach Level 3 without passing through Level 2. Secondly, the level of thought which is intrinsic in current level becomes extrinsic for the next level. Thirdly, each level contains its own terminology, its own linguistic symbols and network of relationship related to these symbols. Lastly, the students who are at different levels cannot communicate efficiently.

2.2.1.3 The movement from one level to the next

Usiskin (1982) stated that student's geometrical thinking levels can be developed by effective instruction based on the van Hiele theory. If teachers can prepare a proper instruction according to the students' geometry understanding level, students can pass through from one level to the next. In this respect, Usiskin (1982) stated that there are five learning phases which was suggested by the van Hiele theory for supporting students to pass from one level to the next. These learning phases are inquiry (information), direct orientation, explication, free orientation and integration. If students are provided with instruction in which these phases are embedded, they can move from one level of van Hiele geometry understanding level to the next.

Phase 1: Information (Inquiry). The first of the phases of learning based on van Hiele theory is inquiry (information). Crowley (1987) stated that student and teacher are in a conversation in this phase. At this initial phase, the role of the teacher is to understand students' prior knowledge about the geometric content and to observe students' language to express their thoughts about geometric contents. In this respect, the teacher asks questions to understand students' prior knowledge and prepare them for further activities (Crowley, 1987). Van Hiele (1999) claimed that children should be provided materials which encourage them to discover certain structures. In this phase, students are prepared to learn the characteristics of geometric concepts.

Phase 2: Direct Orientation. In the direct orientation phase, tasks should be presented to students in a way that students can gradually realize the characteristic structure of the task (van Hiele, 1999). In this phase, students are expected to become familiar with the characteristics of given geometric shapes and concepts. Students are given the opportunity to change the shapes of given geometric object in order to explore their features. Furthermore, in this phase, tasks should be short and students' responses should be specific.

Phase 3: Explication. In this phase, students are expected to discuss the problems and express their opinions in their own words which are discovered in previous phases. Teacher, at this stage, introduces the relevant mathematical terminology to support students' understanding.

Phase 4: Free Orientation. In the fourth phase, free orientation, students are expected to solve the task with multiple steps (Crowley, 1987).That is students are expected to expand their learning. Hence, they can be more proficient with what they have already known.

Phase 5: Integration. In the last phase, integration, students are led to summarize and relate what they learned (Crowley, 1987). Van Hiele (1999) stated that teachers should plan the tasks, lead students to use the terminology in their discussion, and encourage them to explain their ideas and problem solving strategies.

2.3 Validity of the van Hiele Theory

Several studies were conducted to investigate the usefulness of the can Hiele theory in school geometry (e.g., Usiskin, 1982; Burger & Shaughnessy, 1986; Senk, 1989).

In these studies, the characteristics of van Hiele levels and the assessment of students thinking processes concerning levels were discussed.

One of the projects about van Hiele Theory of levels was conducted by Usiskin and Senk (1982). The name of the project was Cognitive Development and Achievement in Secondary School Geometry (CDASSG) and it mainly focused on the relationship between van Hiele Theory of levels and students' achievement at secondary school geometry. For the project, 2699 students from 13 public geometry schools participated.

The study of Usiskin (1982) focused on the extent van Hiele levels can be indicator of students' success in secondary school geometry tasks. Furthermore, Usiskin developed a test to show whether van Hiele levels of students predict students' achievement at school geometry. The test includes 25 multiple-test questions intended to measure students van Hiele levels. The sample of the study consisted of 2700 students at the10th grade. Students were given two tests before and after the geometry course. One of the tests intended to measure van Hiele thinking levels of students and the other one is a standardized geometry test. The scores of students at the beginning of the course and at the end of the course were analyzed. The results of the study showed that there was a positive correlation between the van Hiele levels test and standardized geometry test on both cases. The correlation between the van Hiele test scores and standardized geometry test scores was .52 at the beginning of the course and .67 at the end of the course. Hence, Usiskin concluded that van Hiele Theory levels can be used as an indicator of student geometry achievement.

Senk (1989) conducted another study based on the CDASSG project. In this study, Senk (1989) focused on questioning the relationship between van Hiele levels, geometry proof writing achievement and achievement on objective tests of standard geometry content not involving proof. The sample of this study was selected from the pool of CDASSG project. In this study, 241 students from 11 state high schools was selected. The students were given the CDASSG proof test, the van Hiele test, and tests for knowledge and standard content. All these tests were developed in the CDASSG project. A positive relationship between achievement in writing geometry proofs and van Hiele levels of geometric thinking was determined. Furthermore, this study also showed a positive relationship between achievement in writing geometry proofs and achievement on standard non proof geometry content. As another important finding of this study, Senk (1989) stated that the teacher and curriculum has an important effect on students' achievement in writing geometry proofs.

The study of Burger and Shaughnessy (1986) focused on developing an alternative test apart from Usiskin's paper and pencil multiple choice test. That is, van Hiele levels were examined by using clinical interview tasks about triangles and quadrilaterals. The sample of this study were selected from kindergarten students to college students (n=45). Three main research questions were investigated in the study. The first question focused on assessing van Hiele levels in determining students thinking process on geometry tasks. The second question investigated whether the behaviors of the students on the task indicated their geometrical thinking levels. The last question was about the design of the interview and its effectiveness on students' reasoning on geometry tasks. As a result of this study, van Hiele levels were found to be useful indicators of students' reasoning on geometry tasks. Furthermore, the levels can be characterized operationally by the behaviors of students. Hence, it can be concluded that van Hiele levels can be used in school geometry as an indicator of student's thinking processes in geometry tasks.

2.4 Strategies on to develop van Hiele geometrical thinking levels

After van Hiele Theory of levels was accepted as a useful indicator of students' understanding of school geometry, several studies were conducted to investigate the factors affecting the van Hiele levels of understanding geometry (e.g. Halat, 2006; Duatepe-Paksu & Ubuz, 2009; Siew, Chong, & Abdullah, 2013)

One of the studies about van Hiele levels of understanding geometry conducted by Halat (2006) and focused on the effect of gender differences on the acquisition of van Hiele levels and students' motivation. Specifically the study questioned the effect of gender differences on the acquisition of van Hiele levels and the effect of van Hiele based curricula on the motivation of boys and girls. The sample of the study consisted of 150 students at 6^{th} grade in a public middle school in Florida, USA. Quasi-experimental design was used and the students were assigned to two groups considering gender. The students were taught the polygons and tessellations topics in five weeks of instruction. The results of the study indicated that there is not any statistically significant differences between boys and girls in terms of the acquisition of the van Hiele levels of understanding geometry. Furthermore, although boys' mean score of motivation scores was higher that girls' mean score, the analysis of covariance (ANCOVA) revealed that there was not a statistically significant effect of gender on students' motivation in learning geometry. Halat (2006) emphasized that the results of the study showed a considerable decrease on the gender gap in terms of mathematics areas.

Siew, Chong, and Abdullah (2013) conducted a study to examine the effect of van Hiele phases of learning by using tangrams on students' geometric thinking at visual and analysis level. The sample of the study consisted of 221 students at 3rd grade. Pretest and posttest single group design was used in the study. The students were taught two-dimensional geometry by using tangram activities and they were given a van Hiele geometrical thinking test before and after the intervention. The pretest and posttest scores were analyzed for three groups of students which consisted of high, moderate and low ability students. The results showed that there was a statistically significant difference between the pretest and posttest scores at both visual and analysis level for all high, moderate and low ability students. The results also showed that the instruction has the greatest impact on the scores of low ability students.

Duatepe-Paksu and Ubuz (2009) conducted a study to question the effect of drama-based instruction on geometry achievement, retention of geometric topics, geometric thinking levels and attitudes toward mathematics. The sample of this study consisted of 7th grade students from a public school (n=102). There were three classes which included 34 students for each. The findings of the study revealed that drama-based instruction had statistically significant effect on geometry achievement, retention of geometry topics, attitudes toward geometry. Duatepe-Paksu and Ubuz (2009) emphasized the importance of working as a group in lessons. In the drama-based instruction, students worked as a group, had an opportunity to have social interaction in to express, discuss and justify their ideas. Duatepe-Paksu and Ubuz (2009) stated that social interaction in drama-based instruction had a positive effect on students' engagement, motivation and achievement. However, the analysis of the data of this study failed to show a statistically significant effect of drama-based instruction on students' van Hiele levels and attitudes towards mathematics. Duatepe-Paksu and Ubuz (2009) claimed that a longer period of time is needed for improving van Hiele levels.

2.5 Use of dynamic geometry software to improve students' van Hiele levels

Dynamic geometry software (DGS) was defined as the computer software that enables students to explore geometric relationships and make conjectures by manipulating geometrical objects on the computer screen (Güven& Kosa, 2008). In DGS environment, learning the features of geometrical objects and their relationships becomes easier for learners.

Stahl (2015) stated that geometry is a discipline in which dependencies are constructed with geometric figures and provable relationships are discovered. In a well-designed DGS environment, students have opportunities to develop their geometric thinking skills by moving forward from a visual solution to constructing dependencies (Oner, 2016). Stahl (2015) advocated that the construction of dependencies are made clear in dynamic geometry environments such as GeoGebra. One of the important features of DGSs is dragging. In DGS environments, if a figure is constructed properly, the theoretical relationships of the figure remain the same even under dragging (Oner, 2016). Hence students are provided an opportunity to construct geometric dependencies with geometric figures and test whether the construction is proper or not by using the drag test. There are many studies that investigated the effect of instruction with DGS on students' geometric thinking levels. These findings showed that using DGS enabled students to develop their geometric thinking levels (e.g.; Kutluca, 2013; Abdullah & Zakaria, 2013; Abdullah, Surif, Tahir, Ibrahim and Zakaria, 2015; Karakuş & Peker, 2015).

The study that conducted by Akgül (2014) investigated the effect of DGS-based instruction on students' geometry achievement, students' van Hiele levels of geometric thinking and their attitudes toward mathematics and technology. Experimental research study design was used in the study. The participants of the study selected from a private school in Ankara in Turkey. Two intact classes were selected from the school and one of the classes was randomly assigned as control group and the other one as experimental group. Each group consisted of 17 students at the 8th grade. The subject of transformation geometry (fractals, translation, reflection and rotation) was taught by using DGS-based instruction (GeoGebra) for experimental group. Control group was taught the same content by using the traditional method. The results indicated that DGS-based instruction had a statistically significant effect on students' geometry achievement and geometric thinking. However, the results also indicated that there was no significant effect of DGS-based instruction on students' attitudes towards mathematics and technology.

Another study based on DGS and van Hiele theory was conducted by Abdullah and Zakaria (2013). In the study, the effectiveness of van Hiele phase-based instruction was investigated. Quasi-experimental design was implemented in this study. While the experimental group had an intervention with instruction based on dynamic geometry software (Geometer's Sketchpad), the control group had an instruction with the traditional approach. The concepts of transformation and quadrilaterals were presented to both groups by following the van Hiele's phases of learning which are inquiry, direct orientation, explanation, free orientation and integration. It was found that both control and treatment groups had an increase on their van Hiele geometry levels. Furthermore, the treatment group who studied with Geometer's Sketchpad activities had statistically significant higher scores than the control group.

Another study that investigated the effect of dynamic geometry software on the van Hiele geometry understanding level was conducted by Karakuş and Peker (2015). In the study, the effects of DGS- based instruction and concrete materials based instruction was compared in terms of van Hiele levels and spatial ability for pre-service teachers. The participants were 61 pre-service primary teachers in the second year undergraduate program in the Department of Elementary Education at Afyon Kocatepe University. While the control group (n=29) were implemented DGS-based instruction, the experimental group (n=32) used concrete materials in the instruction. The results showed that the both spatial ability scores and van Hiele geometry understanding levels increased for both groups. Furthermore, the study showed that there was no correlation between spatial ability and van Hiele levels. Therefore, Karakuş and Peker (2015) claimed that if relevant content were implemented in either with DGS or physical manipulatives, the van Hiele geometry understanding level of students and their spatial ability could enhance. In this study, the researchers let participants to discuss the given activities among them for both control and experimental groups. Karakuş and Peker (2015) emphasized that the role of the teacher was to assist students to express their opinions and argue their ideas rather than becoming a source of knowledge. The teacher did not give direct knowledge to the students. He only guided the instruction process and managed the classroom discussion. This is crucial since the interaction and discussion among students can be a crucial factor in increasing their geometry level. The researchers not only gave instructional materials to the students but also created a classroom atmosphere in which students discussed their opinions and learned from each other. Hence, collaboration among students might be another factor that increases the van Hiele geometry understanding of students.

Abdullah, Surif, Tahir, Ibrahim and Zakkaria (2015) conducted a study to investigate the effectiveness of Geometer's Sketchpad learning activities on students' van Hiele levels. Quasi-experimental design was applied in this study. The participants were randomly selected from a secondary school in Malaysia (n=94). The participants were randomly distributed into the control group (n= 47) and the treatment group (n=47). The participants were given the van Hiele Geometry Test before and after the course. Two groups were thought the transformations during the 6 weeks period. While the topic was presented to control group by using traditional methods, Geometer's Sketchpad learning activities which were prepared considering van Hiele's phases of learning geometry were presented to the treatment group. The data from the two groups were analyzed quantitatively and qualitatively. In quantitative analysis, it was found that there was a significant difference between the control and the treatment group in their levels of geometrical thinking levels. While, all the students in both groups were able to increase their level of thinking from Level 1 (visualization) to Level 2 (analysis), some students in treatment group were able to increase their level of thinking to Level 3 (informal deduction) according to the results of Wilcoxon signed-rank test. Abdullah et al. (2015) mentioned that the findings from interviews support the results of quantitative analysis. Furthermore, they stated that the findings revealed the effectiveness of DGS-based learning activities concerning the van Hiele's phases of learning including information, guided orientation, explication, free orientation, and integration. The roles of students and teacher were reported as other crucial aspects in this study. Abdullah et al. (2015) claimed that the constructivist approach and van Hiele's phase of learning model can be a major factor to increase the level of students' geometrical thinking by considering their perspectives to the teacher and students' role in learning environment.

The effect of dynamic geometry software (DGS) on students' van Hiele levels of understanding geometry was examined by Kutluca (2013). The sample of students was selected from students at the 11th grade (n=42) and quasi-experimental method was used in this study. Traditional teaching strategy was used for the control group, consisting of 18 students. Twenty four students in the control group used GeoGebra (a kind of DGS) as a computer-supported instruction. In this study, “Van Hiele Level of Geometric Understanding Test” developed by Usiskin (1982) and translated into Turkish by Duatepe (2000) was used to collect the data. In the analysis of the study, Wilcoxon signed-rank test was used to compare the pre-test and post-test of both control group and experimental group. The analysis showed that there was a significant increase in the van Hiele geometry understanding of students in experimental group. On the other hand, it was found that there was no significant difference between pre-test and post-test of students in control group. The results showed that using GeoGebra as an instructional tool had a positive effect on increasing the van Hiele geometry understanding level of students. Kutluca (2013) claimed that instruction with GeoGebra enabled students to construct their own knowledge by drawing and dragging their own shapes and discovering the features of these shapes. He also emphasized the role of the learning environment in which students can comfortably share and discuss their ideas with each other and actively participated in learning activity (Kutluca, 2013). Thus, this study also highlighted collaboration as an important factor for increasing the van Hiele geometry understanding level of students.

To sum up, several studies showed that students’ geometric thinking levels can be increased with the DGS-based instruction (e.g., Kutluca, 2013; Karakuş & Peker, 2015; Abdullah & Zakaria, 2013; Akgül, 2014). Some of these studies emphasized the role of collaborative learning environment in DGS-based instruction as a crucial factor that affected the quality of instruction (e.g., Kutluca, 2013;

Karakuş & Peker, 2015). In this respect, Virtual Math Team (VMT) Project, which is one example of well-designed computer-supported collaborative learning (CSCL) environments, will be presented in the next section.

2.6 Virtual math teams (VMT)

Virtual Math Teams (VMT) is a well-known computer-supported collaborative learning (CSCL) tool. CSCL has become a popular area in education in the last decades. Although collaborative learning promotes learning socially, academically and psychologically, the effects of collaborative learning has increased with the help of technology. Wei and Ismail (2010) asserted that collaborative learning enables mathematicians to produce and discuss strategies to solve mathematics problems. When the importance of increasing students' problem solving abilities is concerned, CSCL tools provide affordances to support problem solving.

In VMT, middle school and high school students have an opportunity to work collaboratively with an integrated platform which is constituted by technological, pedagogical and analytic components (Khoo & Stahl, 2015). VMT is defined as "an open-source, virtual, collaborative learning environment that affords synchronous text-based interaction (chat) with an embedded multi-user dynamic geometry application, GeoGebra" (Oner, 2016, p. 60). The project brings students from all over the world and creates a virtual community of mathematics in which mathematical ideas are presented and argued. VMT provides students with online chat and GeoGebra (a kind of dynamic geometry software) and offer them an interactive learning environment. Stahl (2013) claimed that after the cooperation with GeoGebra, VMT provide students with an online platform in which a chat window, a virtual white board and other interactive dynamic-geometry tools and help them to learn about geometry construction dynamically as a group rather than individually.

2.7 PISA 2015 collaborative problem-solving framework

Collaborative problem solving has been specified as a crucial and essential skill in education and also in workforce (OECD, 2017). Collaborative problem solving competency has been defined as a "capacity of an individual to effectively engage in a process whereby two or more agents attempt to solve a problem by sharing the understanding and effort required to come to a solution and pooling their knowledge, skills and efforts to reach that solution" (OECD, 2017, p. 6).Thus, the success of the groups depends on the collaboration among the group members.

In PISA 2015, a collaborative problem-solving framework document, the assessment of collaborative problem solving (CPS) skills were presented. In this document, definition, organization of the domain and assessment of CPS competences were broadly explained.

Collaborative problem solving is a complex mechanism that consists of the incorporation of individual problem-solving skills and collaboration process (OECD, 2017).In the assessment of PISA (2012), the cognitive skills of individual problem solving has been identified as exploring and understanding; representing and formulating; planning and executing; and monitoring and reflecting (OECD, 2010, p. 20-21). However, in collaborative work, team members also needs additional cognitive and social skills including shared understanding, knowledge and information flow, to create and understand an appropriate team organization, and to perform coordinated actions to solve the problem (OECD, 2017). Thus, in the PISA 2015 Collaborative Problem-Solving Framework, three main components of successful collaboration among group members were presented as establishing and maintaining shared understanding, taking appropriate action to solve the problem, and establishing and maintaining group organization. These components of CPS originated from the combination of individual problem solving and collaboration process. In PISA 2015, the components of individual problem solving were crossed with the major components of successful collaboration (see Table 1).

Table 1. Matrix of Collaborative Problem-solving Skills for PISA 2015

	Establishing and maintaining shared understanding	Taking appropriate action to solve the problem	Establishing and maintaining group organization
Exploring and understanding	Discovering perspectives and abilities of team members	Discovering the type of collaborative interaction to solve the problem, along with goals	Understanding roles to solve the problem
Representing and formulating	Building shared representation and negotiating the meaining of the problem	Identifying and describing tasks to be completed	Following rules of engagement, (e.g. prompting other team members to perform their tasks)
Planning and formulating	Communicating with team members about the actions to be/being performed	Enacting plans	Following rules of engagement, (e.g. prompting other team members to perform the tasks
Monitoring and reflecting	Monitoring and repairing the shared understanding	Monitoring result of actions and evaluating success in solving the problem	Monitoring, providing feedback and adapting the team organization and roles

Establishing and maintaining shared understanding. The first component of the successful collaboration is establishing common ground among group members. Students need to build to a shared understanding about the task for communicating successfully. Shared understanding is about how students' abilities, knowledge, perspectives interact with those of other members (OECD, 2017). In order to build and maintain shared understanding among group members, they needs to create an information flow among themselves by communicating the right information at the right time, and to attempt to overcome the deficiencies in shared knowledge. (OECD, 2017, p. 12-13).

Taking appropriate action to solve the problem. The second component of successful collaboration is taking appropriate action to solve the problem. Students needs to make an effort on solving the problem by understanding the problem tasks and constraints, creating team goals and taking appropriate communication acts such as explaining, justifying, negotiating and debating (OECD, 2017, p. 13).

Establishing and maintaining group organization. The last component of the successful collaboration is the team organization. A group cannot be successful without establishing and maintaining group organization (OECD, 2017). In order to collaboratively work on the problem task, group organization must be established and maintained. Thus, students must know their role in the group, fulfill the requirements of their role, check whether their teammates performing their roles appropriately, and handle with the communication problems (OECD, 2017, p. 13). The authority of the group is also important. Group organization may be established by a strong group leader or more democratically based on the type of the problem (OECD, 2017, p. 13).

In the PISA 2015, certain factors that affect the collaborative problem solving competency were identified. These factors are core skills (collaborative skills and problem-solving skills) and additional factors which are student background (prior knowledge and characteristics), and context (task characteristics, problem scenario, medium, team composition).

In PISA 2015, collaborative problem solving competences of students in collaborative problem solving contexts have been assessed individually. Computer-based agents have been used in order to operationalize the assessment. In this assessment, what was measured was the extent students reflect their collaborative problem on their solving skills in a collaborative problem solving context rather than their level of collaborative problem solving skills (OECD, 2017). Student performance in a collaborative problem solving context has been measured with three different ways which are performing certain actions (i.e. moving a cursor on a display that the other participants can see, editing a joint document), communicating as a group, and the output of products.

Although in PISA 2015, students' collaborative problem solving skills have been assessed individually, in the present study, we did not focus the assessment at the individual level. We concentrated on the issue to understand how the group interaction occurred among group members by analyzing the VMT text chats. Therefore, we used the PISA Framework to understand what components of the model were evident in the data. In the present study, the collaboration components of the model which are *establishing and maintaining shared understanding*, *taking appropriate action to solve the problem*, and *establishing and maintaining group organization* were used. Since we did not focus to assess students individually and the tasks were not problem-solving tasks, individual problem solving components of the model were not taken into consideration in the present study.

2.8 Students' attitudes towards technology and mathematics

With technology integrated into the mathematics instruction, it is easier to assist students' mathematical problem solving and explore mathematical concepts (Pierce, Stacey, & Barkatsas, 2007). However, the potential benefits of technology are not always actualized in mathematics instruction (Artigue, 2012). Students' attitudes towards mathematics and technological learning tools used in the instruction may influence their learning behaviors such as confidence, motivation and engagement and learning outcomes (Reed, Drijvers, & Kirshner, 2010). Reed et al. (2010) stated that the learning behaviors of students in mathematics instruction assisted with computer tools can be affected by their attitudes towards mathematics and mathematical computer tools. In this regard, students who have positive attitudes towards mathematics and the mathematical computer tools might be more likely to learn mathematics than students who have negative attitudes.

Pierce and Stacey (2004) conducted four case studies in order to clarify the variables that influenced the effective use of technological tools in mathematics leaning. In this study, it was found that both technical competence and personal attitudes are crucial for students' development. Pierce and Stacey (2004) also emphasized that students' negative attitudes towards mathematics tools rather than technological issues, limit the effectiveness of instruction that is assisted with technological tools.

Pilli (2008) conducted an experimental study to examine the effect of the computer software Frizbi Mathematics 4 on students' mathematics achievement, retention and their attitudes towards mathematics and computer assisted learning. The sample of this study consisted of 26 students for control group and 29 students for experimental group. The participants were selected from 4th grade students in a primary school in Gazimagusa, North Cyprus. While students in control group was taught three units which are multiplication of natural numbers, division of natural numbers, and fractions with traditional instruction, the students in experimental group were taught the same content with the computer software Frizbi Mathematics 4. The results showed that the experimental group had better scores than control group in achievement tests and attitude scales. The difference between two groups was statistically significant.

The results of the study by Akgül (2014), contradicted the results of Pilli (2008). While Pilli (2008) found that the instruction with a computer tool increased students' attitudes towards mathematics and technology, Akgül (2014) did not find any significant differences in terms of students' attitudes towards mathematics and technology between students in the control group taught by traditional instruction and the experimental group taught by DGS-based instruction. Akgül (2014) pointed out that although the students in experimental group expressed their feelings verbally that they like learning mathematics with new technological tools and thought that GeoGebra was an effective tool to learn mathematics, the results in terms of students' attitudes towards mathematics and technology showed that the difference between groups was not statistically significant.

2.9 Summary

When the results of PISA and TIMSS are considered, it can be said that students at the middle school in Turkey have room to improve their success on international geometry achievement tests. One of the main reasons of this issue is related to the instructional strategies used in classrooms, in which the geometric thinking levels of students are not often taken into consideration (Cansız-Aktaş & Aktaş, 2012).

Another problem is about the lack of effective learning activities that are prepared based on the constructivist approach that also consider students geometric thinking levels (Fidan & Türnüklü, 2010).

The geometric thinking levels of students were well described by van Hiele's theory of understanding geometry (Usiskin, 1982). The van Hiele theory described five sequential geometric thinking levels that are visualization level, analysis level, informal deduction level, deduction level and rigor level. Students can pass through from the current level to the next level if effective instruction based on van Hiele theory was provided (Usiskin, 1982). Several studies approved that van Hiele geometric thinking levels scheme was valid indicator of the achievement in school geometry (e.g. Usiskin, 1982; Burger & Shaughnessy, 1986; Senk, 1989).The van Hiele phased based instruction consists of five phases which are inquiry (information), direct orientation, explication, free orientation and integration.

Several studies were conducted to investigate the variables to increase students' van Hiele geometric thinking levels (e.g. Halat, 2006; Duatepe-Paksu & Ubuz, 2009; Siew, Chong, & Abdullah, 2013). The result of the study by Halat (2006) showed that gender did not have a statistically significant effect on students' geometric thinking levels. The study by Duatepe and Ubuz (2009) showed that students' geometric thinking levels can be increased by using drama-based geometry instruction. The study by Siew, Chong and Abdullah (2013) showed that using tangrams can increase the students' geometric thinking levels especially for low ability students.

Researchers also investigated the effect of DGS-based on van Hiele geometric thinking levels (e.g.; Kutluca, 2013; Abdullah & Zakaria, 2013; Abdullah, Surif, Tahir, Ibrahim and Zakaria, 2015; Karakuş & Peker, 2015). DGS-based instruction was found effective increasing students' van Hiele geometric thinking levels. Some researchers emphasized that learning environments in which students can collaboratively work on geometry problems, share and discuss their ideas with other students were very influential besides the use of DGS-based materials in instruction (e.g., Kutluca, 2013; Karakuş & Peker, 2015). However there is no study in the literature that investigated the development of students' van Hiele geometric thinking levels in the context of a CSCL environment.

As all these reasons were taken into consideration, the current study aimed to investigate the role of a well-designed CSCL environment namely the VMT environment with multi-user GeoGebra on the development of students' van Hiele geometric thinking. It also looked into students' attitudes towards mathematics and technology as a results of working within the VMT environment. In addition, this study explored how collaboration, as defined by PISA framework, influenced geometric thinking development. Thus, the following main research questions guided the current study:

1. Is there a statistically significant difference between the pretest and posttest scores on VGHT (Usiskin, 1982), after working within VMT?
2. Is there a statistically significant difference between the pretest and posttest scores on MTAS (Barkatsas et al., 2007), after working within VMT?
3. How did the collaboration among participants, as defined by the PISA framework, influence their geometry learning while working within the VMT environment?

CHAPTER 3

METHODOLOGY

In this chapter, the following sections (1) research design, (2) population and sample, (3) treatment, (4) instruments, and (5) data collection procedure and (6) data analysis will be presented.

3.1 Research design

In the present study, embedded mixed methods design was used. Mixed methods research consists both quantitative and qualitative data in a single study (Creswell, 2014). Researchers uses both quantitative and qualitative data in order to understand the research problem or question more deeply. Unquestionably, both quantitative and qualitative researches have some strengths and weaknesses. While quantitative research provides conclusions for large numbers of people, the conclusions of quantitative research has limited generalizability. On the other hand, while qualitative research provides detailed perspectives of few people, quantitative research provides limited understanding of the perspectives of participants (Creswell, 2015). In this respect, mixing or blending of both quantitative and qualitative data can provide better understanding of the research problem than either each of them separately (Creswell, 2014).

There are many types of mixed methods study design. Researchers selects the types of mixed methods according to certain strategies. In the literature, there are several typologies for deciding the type of mixed methods design (e.g., Creswell, 2014; Teddlie & Tashakkori, 2009). Researchers decide the type of mixed methods design that is suitable for their own study by using these strategies. In order to decide the type of mixed methods design for the present study, the typology suggested by Creswell (2014) was used.

Creswell (2014) stated six types of mixed methods which are:

a) Convergent parallel mixed methods design
b) Explanatory sequential mixed methods design
c) Exploratory sequential mixed methods design
d) Embedded mixed methods design
e) Transformative mixed methods design
f) Multiphase mixed methods design

In order to decide the type of mixed methods design, Creswell (2014) first suggests considering the timing of the data collection. The quantitative and qualitative data can be collected either at the same time or sequentially. While in the convergent parallel mixed methods design the data collected concurrently, in the explanatory sequential and exploratory sequential designs data collected in sequence. In the present study the quantitative and qualitative data were collected concurrently. The other factor is on the emphasis placed on each database. While there can be an equal emphasis on both qualitative and quantitative databases in some mixed methods studies, other studies can show priority for one of the databases. In the present study, while the quantitative data played the primary role, the qualitative data played a supplementary role. Furthermore, since the students talks in the chat were recorded in the VMT environment, the qualitative data were collected during the intervention. When all the factors were considered, it was logical to decide the type of design in the present study as an embedded mixed methods design. Embedded design was called as “intervention design” in the subsequent publication of the same author (see Creswell, 2015).

The aim of current study was to examine the role of working within a CSCL environment (VMT) on middle school students' geometric thinking levels and their attitudes towards technology and mathematics, and to understand how did collaboration among participants, as defined by the PISA framework influence their geometry learning while working within the VMT environment. In the present study, while the quantitative data were collected with the quantitative data collection instruments (i.e., pretests and posttests) at the beginning and end of intervention the qualitative data were collected with the qualitative data collection instrument (i.e., transcripts of students' chat logs) during the intervention. These two databases were used to answer three research questions of the study. The quantitative data were used to answer the first and second research questions and the qualitative data were used to answer the third research question. In the Figure 1, the research design of the present study was illustrated. Since the quantitative data played the primary role in the study, "QUAN" was written by using upper cases. On the other hand, lower cases were used in the writing of the "qual", since qualitative data played the supplementary role in the study.

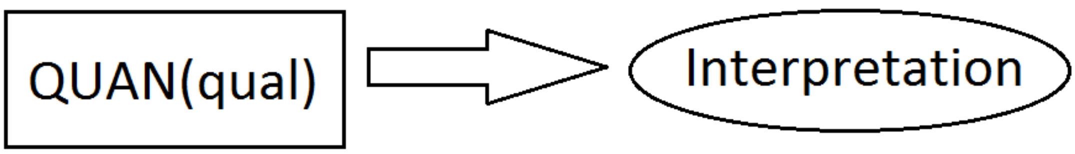


Figure 1. The embedded design

Regarding the quantitative part, the independent variable of the present study is the instruction presented within the context of VMT. Table 2 shows the dependent variables of the study which are students' pretests and posttest scores on the Van

Hiele Geometric Thinking Test (VHGT) and Mathematics and Technology Attitudes Scale (MTAS). The VHGT and MTAS were presented in Appendix A and Appendix B, respectively.

Table 2. Classification of the Variables

Name	Dependent/independent	Type of variable
Instruction in the context of VMT	Independent	Categorical
Posttest scores on the Van Hiele Geometric Thinking Test	Dependent	Continuous
Posttest scores on the Mathematics and Technology Attitudes Scale	Dependent	Continuous

3.2 Population and sample

The target population of the study was middle school students in public schools in Turkey. In this study, the purposive sampling method was used for quantitative part; participants needed to satisfy three criteria: be a middle school student, have a personal computer and Internet access, and be at the visualization level (Level 1) according to the van Hiele geometric thinking level test. After the schools were selected, among the ones willing to cooperate in the study, the students in these middle schools were given the van Hiele geometrical thinking levels test and the mathematics and technology attitude scale. Later on, the students who are at visualization level (Level 1) according to the van Hiele geometrical thinking levels test were determined as the study participants. Data were collected from twenty-four 5^{th} and 7^{th} grade students in a public and private middle schools (Table 3).

For the qualitative part of the study, purposeful sampling method was used. There are many approaches of purposeful sampling according to the needs of studies (e.g. maximal variation sampling, homogenous sampling and extreme case sampling). In the present study, maximal variation sampling was used. In maximal variation sampling, the participants who are different in terms of some factors such as gender, race, mathematics achievements are selected in order to reflect the these differences of participants at the end of the study (Creswell & Plano Clark,2007). In this respect, two groups were selected by using maximal variation sampling method and their chat logs were analyzed qualitatively. The groups were selected according to their scores on VGHT tests. Group 1 was selected as the successful group because two participants out of three in this group have increased their van Hiele Geometric Thinking from Level 1 to Level 2. Group 2 was selected as the unsuccessful group because none of the participants in this group has increased their van Hiele Geometric Thinking Level (see Table 4). The level of both students stayed the same in this group. All the participants of the qualitative part were fifth grade students.

In this study, a set of VMT activities with DGS (explained in detail below) were designed to help students who are at van Hiele Level 1 (visualization level) to reach up Level 2 (analysis level). Şener-Akbay (2012) found that 65% of the Turkish 7th-8th grade students are at Level 1. Thus, this study aimed to support the majority of the middle school students' learning geometry. The five VMT activities were designed based on the van Hiele's five learning phases, which included inquiry, direct orientation, explication, free orientation and integration.

Table 3. Participants of the Study for Quantitative Part

School	5th Graders		7th Graders		Total	
	Female	Male	Female	Male	Female	Male
Public	3	3	4	3	7	6
Private	6	5	0	0	6	5
Total	9	8	4	3	13	11

Table 4. Participants of the Study for the Qualitative Part

Student	Group	VGHT Pretest	VGHT Posttest
Emir	1	Level 1	Level 2
Sude	1	Level 1	Level 2
Lara	1	Level 1	Level 1
Öykü	2	Level 1	Level 1
Naz	2	Level 1	Level 1
Note:Names are pseudonyms			

3.3 Treatment

The content that addressed in these activities includes the properties of five quadrilaterals (trapezoid, parallelogram, rhombus, rectangle and square) and their relationships with each other. In the 6-8th grade curriculum, the geometry content area in general focused on the features of geometric shapes and their relationships. In this respect, students are expected to recognize the features of geometric shapes and discover the relationships between them. Students are expected to figure out that square is a special kind of rectangle and parallelogram (MEB, 2018). Also they are expected to discover that rectangle and parallelogram is special kind of trapezoid (MEB, 2018).

3.3.1 Designed VMT activities

Activity 1 (Inquiry): The first activity addresses the inquiry learning phase in which students are expected to recognize the types of five quadrilaterals (rectangle, square, parallelogram, rhombus, and trapezoid) and express their opinions by using their present geometric vocabulary.

Choi-Koh (2000) conducted a study in which van Hiele's phase-based learning strategy was used to increase students' geometry understanding. In this study, Choi-Koh devised activities with Geometer's Sketchpad by considering van

Hiele's phase-based learning strategy. As the initial step, students were expected to use their geometric vocabulary and recognize the geometric shapes. In the inquiry phase, students were presented three different types of triangles which are isosceles triangles, equilateral triangles and right triangles. Students were expected to identify these triangles from given pictures. Similarly, students were expected to differentiate among rectangles, squares, parallelograms, rhombuses and kites in given drawings in the inquiry phase.

In this activity, groups of students are expected to explore the pre-made GeoGebra sketch (see Figure 2) and the types of quadrilaterals by considering their properties. During the activity, students are encouraged to drag the vertices of the five quadrilaterals and discuss about them in the chat with each other. At the end of the activity, students are expected to differentiate the types and write their names on the first activity (Activity 1) of their worksheet.

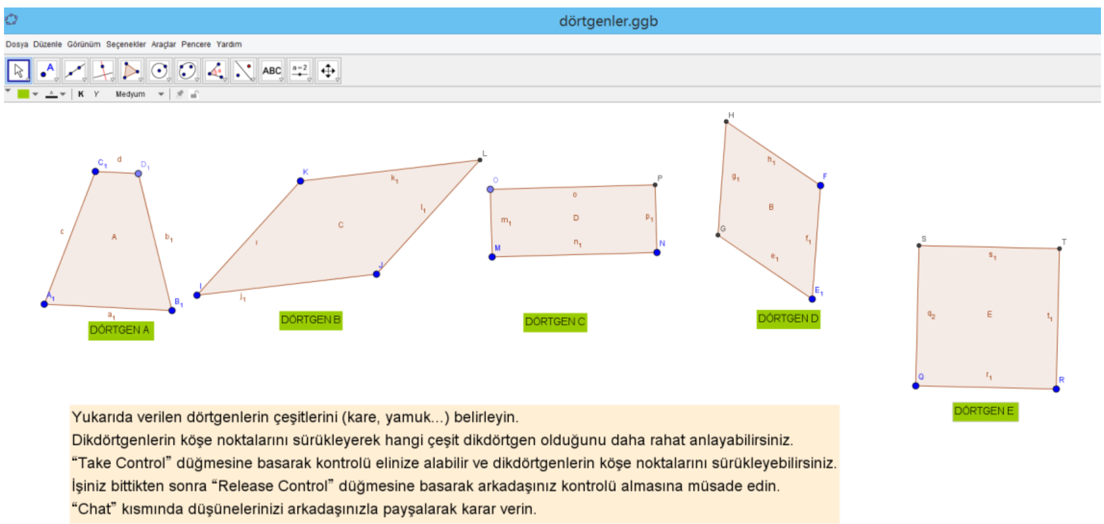


Figure 2. Activity 1 screen

Activity 2 (Direct orientation): The second activity that consists of two tasks addresses the direct orientation phase. As the first task, students are expected to explore the properties of types of quadrilaterals in terms of side lengths and diagonal lengths. As the second task, they are expected to explore the properties quadrilaterals in terms of angle measures.

Choi-Koh (2000) used a similar activity for the direct orientation phase. In this study, students were expected to explore the characteristics of equilateral triangle, isosceles triangle, and right triangle by examining the sketch created with the Geometer's Sketchpad software (GSP). In the study by Abdullah and Zakaria (2013), students were expected to explore the properties of given quadrilaterals by using GSP. At this stage, dragging capability of GSP software helped students to change shapes of any quadrilateral and manipulate them easily.

In Activity 2, groups work within the VMT environment, explore the pre-made GeoGebra sketch (see Figure 3, Figure 4, Figure 5, Figure 6, and Figure 7) and explore the properties of types of quadrilaterals in terms of side and diagonal lengths and angle measures. In order to achieve this, they are expected to drag the points of quadrilaterals and observe the changes in the measures of side lengths, diagonal lengths and angles. Furthermore, they are expected to share their observations and opinions with their teammates and have a discussion as a group using the chat tool. Finally they are expected to answer the questions in the worksheets as a group.

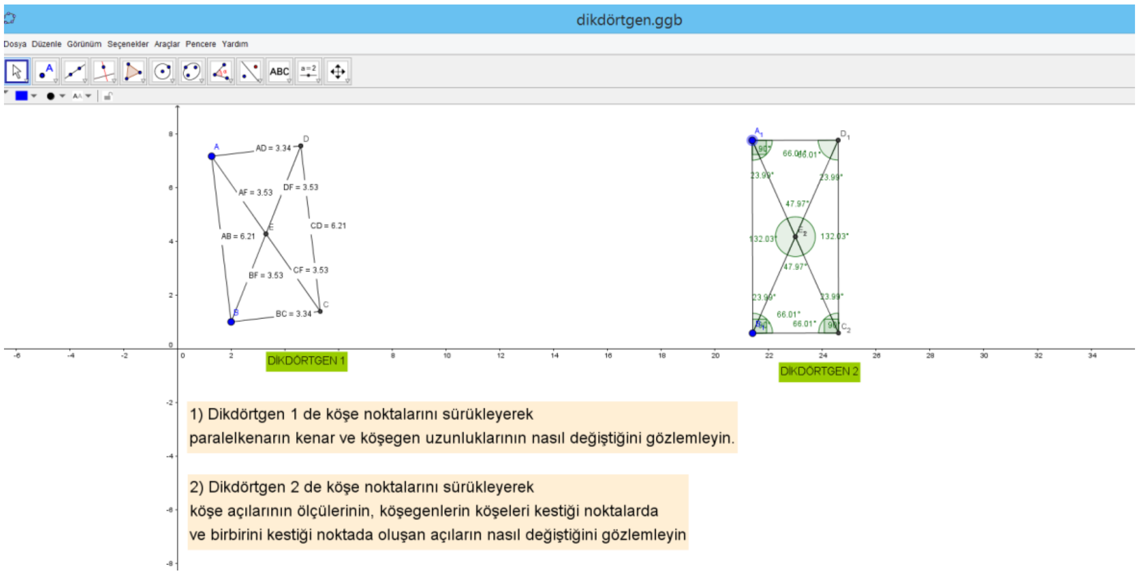


Figure 3. Activity 2 rectangle screen

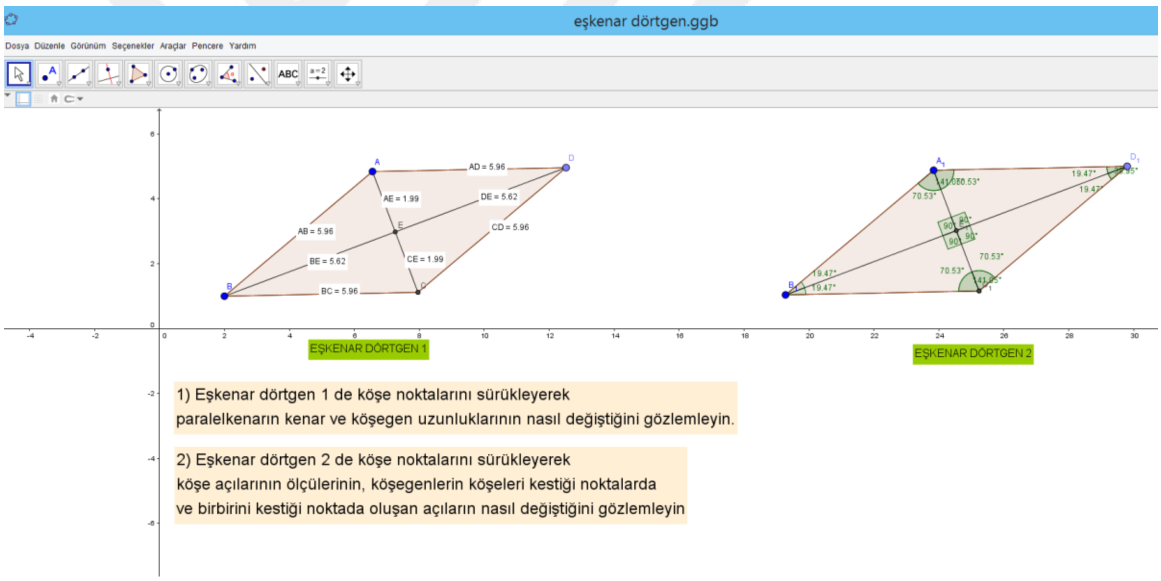


Figure 4. Activity 2 rhombus screen

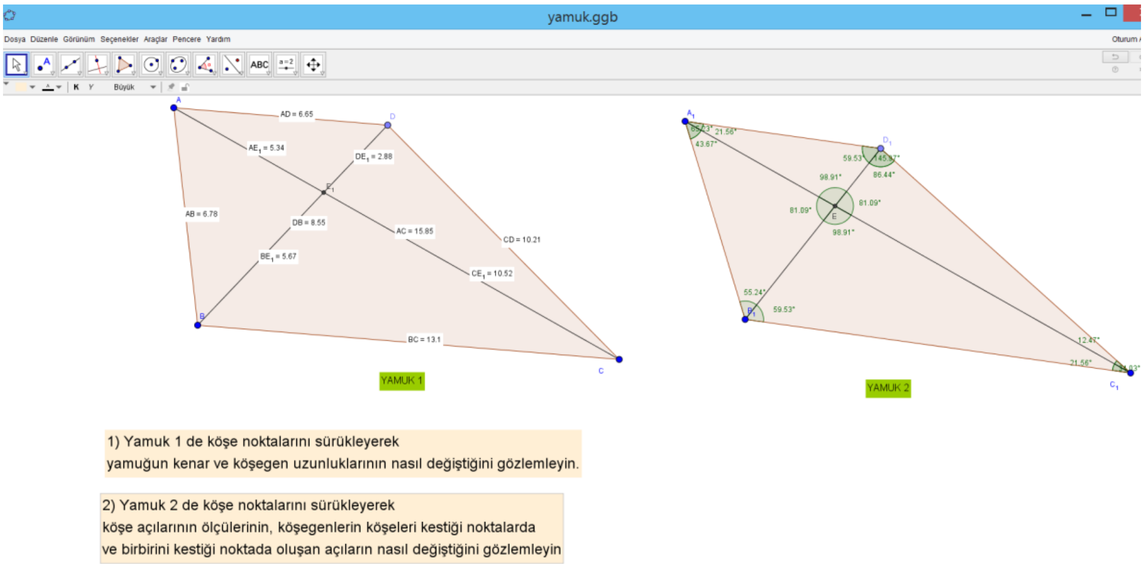


Figure 5. Activity 2 trapezoid screen

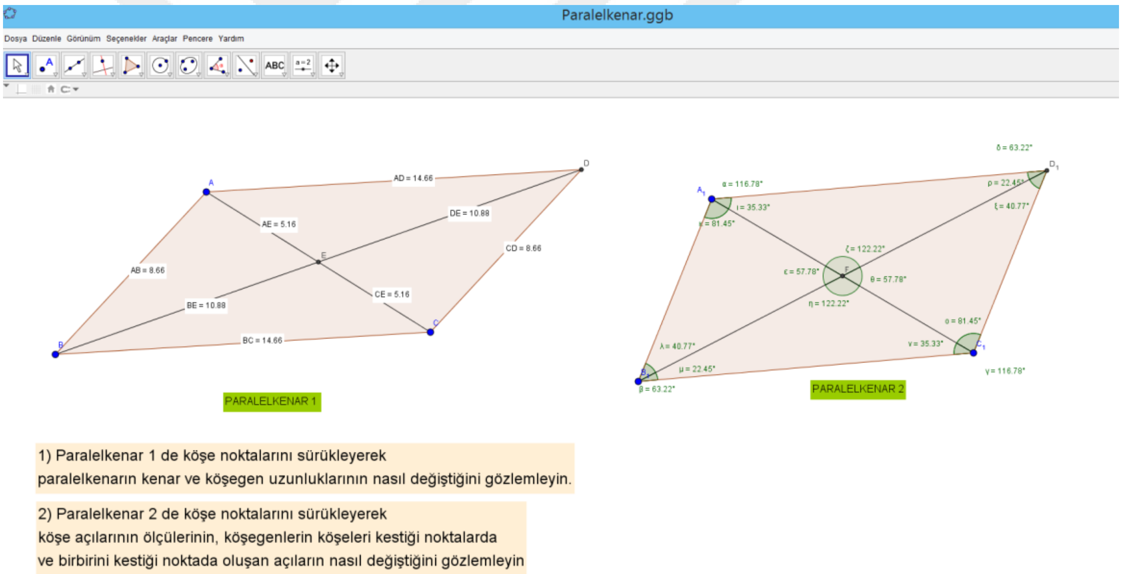


Figure 6. Activity 2 parallelogram screen

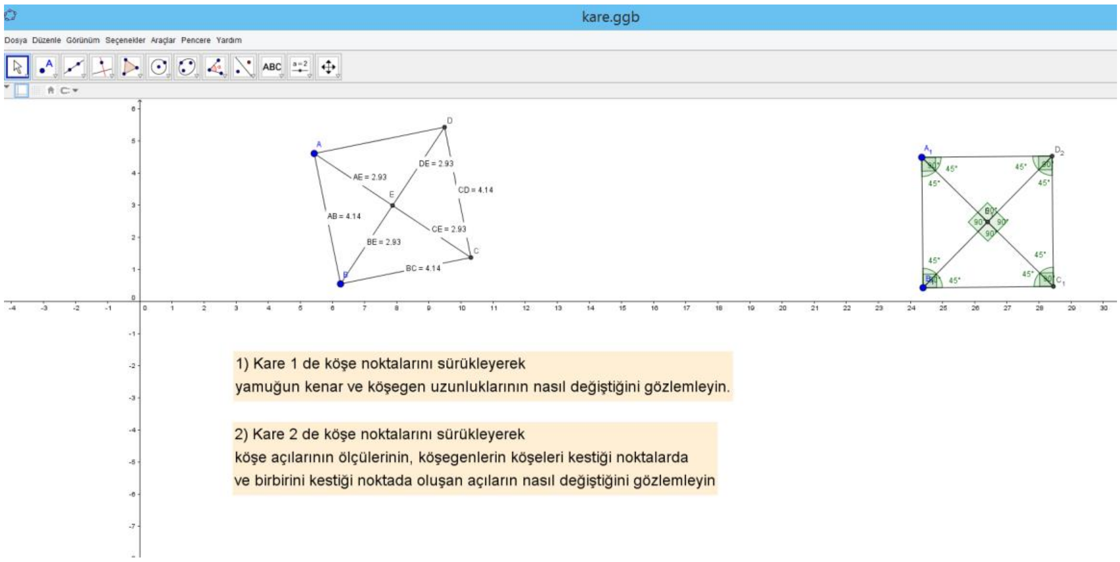


Figure 7. Activity 2 square screen

Activity 3 (explication): The third activity addresses the explication phase. In this activity students are expected to learn the formal mathematical terminology about sides, diagonals and angles of the five quadrilaterals.

In the study by Choi-Koh (2000), the students discussed the properties of equilateral triangle, isosceles triangle, and right triangle and the teacher participated in the discussion to give the relevant mathematical terminology about the properties of each triangle.

In the Activity 3, students are expected to use the relevant terminology (e.g. right angle, opposite sides) expressing the properties of each type of quadrilaterals about their side lengths, diagonal lengths and angles, such as “each angle of square is a right angle”, “opposite sides of parallelogram are parallel and equal” and “diagonals of rhombus are perpendicular to each other”. Google Drive folder is created in which the relevant geometrical terminology with definitions are provided. Students are expected to discuss the properties of types of quadrilaterals using the new terminology the teacher participates in the chat as facilitator.

Activity 4 (Free Orientation): The fourth activity addresses the free orientation phase. In this activity, students are expected to generate the types of quadrilaterals (trapezoid, parallelogram, rhombus, rectangle and square) changing measures of side lengths and angles of given quadrilaterals by using the slider tool provided on the VMT screen.

In the study by Choi-Koh (2000), for example, students were given two different vertices and expected to find the third vertex to get an equilateral triangle, an isosceles triangle, and a right triangle. In this activity, students were expected to explore several ways to solve these problems.

As the free orientation activity, groups work within the VMT environment, explore the pre-made GeoGebra sketch (see Figure 8) and generate a square, a rectangle, a parallelogram, a rhombus, and a trapezoid changing measures of side lengths and angles of given quadrilateral by using sliders. There are several ways to generate a square, a rectangle, a parallelogram, a trapezoid and a rhombus. Hence, each student can construct a square, parallelogram, trapezoid, rectangle and rhombus by considering their theoretical properties. In this activity, each member of the groups are expected to construct each type of quadrilaterals. Students can help each other by using the chat tool. Later on, they are expected to complete the Activity 4 in the worksheet.

Activity 5 (Integration): The last activity addresses the integration phase. In this activity students are expected to review and summarize the properties of types of quadrilaterals.

In the study by Choi-Koh (2000), students, at the final phase, were expected to review all their leaning about the definition of each triangle, properties of each triangle, classification of triangles, formulate the area of a triangle and sort out that the areas of triangles having the same base and the same altitude are equal. The role of teacher at this stage to help students to overview the field of the study and integrate them into what they explored in the activities. At this phase students are encouraged to use the language of the analysis level.

As the final activity, students are expected to fill the given table which is about the properties of types of quadrilaterals. In this step, students are expected to discuss their opinions before to fill each box in the table. The table is the final product of the group.

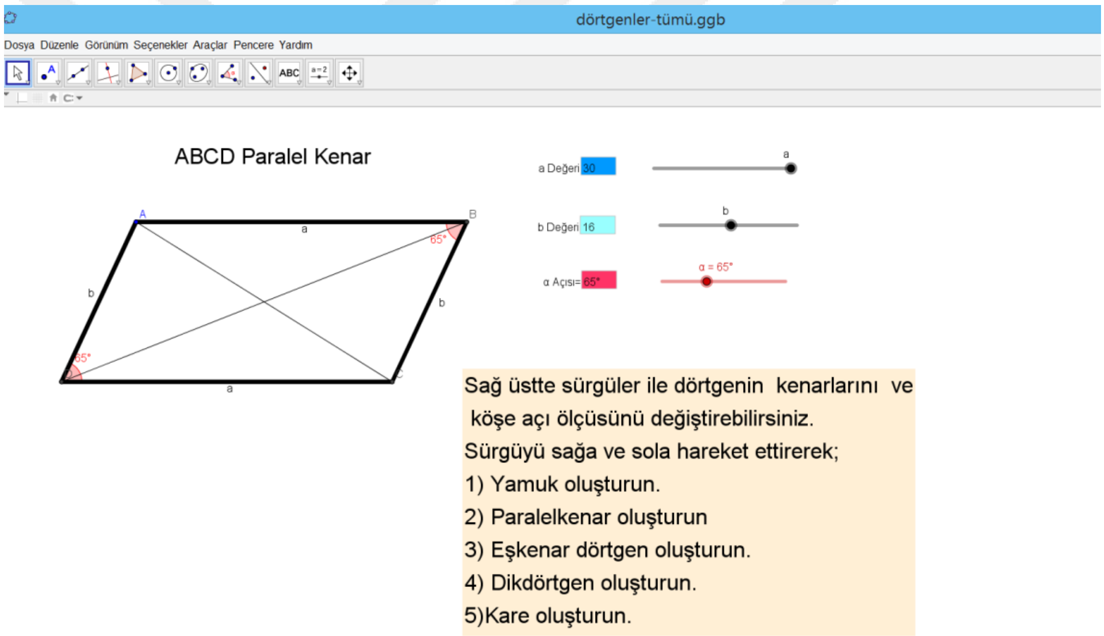


Figure 8. Activity 4 constructing types of quadrilaterals

3.4 Instruments

The instruments of the current study were the van Hiele geometry test (VHGT) and mathematics and technology Scale (MTAS).

3.4.1 The van Hiele geometry test

In this study, the van Hiele geometry test (VHGT) was used in order to evaluate students' geometric thinking levels. The test which includes 25 multiple choice questions and developed by Usiskin (1982). The first five questions address Level 1, second five questions address Level 2, and third five questions represents Level 3, and so on for Level 4 and Level 5. Duatepe (2000) translated the test into Turkish and found the Cronbach Alpha reliability measures as 82, .51, .70, .72, and .59 for each level of the test respectively. In this study, the first 15 items of the VHGT were used since middle school students can only reach up Level 3 (van Hiele, 1986). Furthermore, Şener-Akbay (2012) found that none of the 434 middle school students in Turkey could achieve Level 4.

The grading system by Usiskin (1982) was used to determine the van Hiele levels of students. Usiskin (1982) stated that a student needs to give at least three correct answers to be successful at a certain level. For example, if a student gets at least three correct answers on items 1-5 (Level 1) but doesn't get at least three answers on 6-10 items (Level 2), the van Hiele geometric thinking level of the student is decided as Level 1. The scores of van Hiele Geometric Thinking Level Tests were used to examine the students' geometric thinking levels. Students get 1 point for each correct answer and 0 point for each incorrect answer. Since only 15 questions were considered in the study, the range of scores for VHGT are between 0 and 15.

3.4.2 Mathematics and technology attitude scale (MTAS)

The Mathematics and Technology Attitudes Scale (MTAS) which is used to determine the students' attitudes towards mathematics and technology, is originally developed by Barkatsas et al. (2007). The scale was translated into Turkish by Boyraz (2007). There are 20 items in the scale. It consists of five subscales which are mathematical confidence [MC], confidence with technology [TC], attitude to learning mathematics with technology [MT], affective engagement [AE] and behavioral engagement [BE]. A five point scale from strongly agree to strongly disagree (scored from 5 to 1) is used to determine students agreement with each statement.

3.5 Data collection procedure

Before the present study was conducted, the ethical approval for the study was obtained from the Committee on Human Subjects Research of Boğaziçi University (İNAREK). After that, the permission that was required to conduct the study were presented to the principals of the participating schools. Then, the parents of participants were informed and asked their permission by sending a letter. The study was conducted in two different schools. The first school was a public middle school in Sarıyer/İstanbul. The study was conducted in this school during the spring semester of the 2016-2017 academic year. The second school was a private middle school in Kağıthane/İstanbul. The study was conducted in this school during the fall semester of 2017-2018 academic year. There were three phases of the current study including pretest, treatment and posttest.

First of all, the van Hiele Geometric Thinking Test (VHGT) and The Mathematics and Technology Attitudes Scale (MTAS) were given as pretests. These tests took part approximately 40 minutes. The study participants were selected from the students who are in level 1 (Visualization) based on VGHT pretest scores. As stated before, a student needs to answer at least three questions correctly to reach Level 1 on item 1-5 in the VHGT pretest.

After determining participants, they were informed about the procedure of the study by the researcher. All participants were provided with a Google Drive account and VMT account by the researcher. Google Drive enables users to share folders with other users. Also, by using Google docs, a document can be edited by multi-users. Since the students were required to get activity worksheets and work on worksheets all together as a group, Google Drive was useful in the study. Before the treatment the students were informed by the researcher about how they login in VMT and how to work with it. They were also informed about how to use Google Drive. The mathematics teachers of the schools were also available for help if students had any misunderstandings or problems with technical issues. The participants completed the tasks/ activities as groups of two or three.

The treatment consisted of five tasks/activities (as explained above) which were created within VMT environment. The students participated in the treatment by using their own personal computers either in the school or at home. Since there was no internet access in the public school, the students in this school participated in the study in their home. The treatment took place according to students' available time. Thus, each group participated in the treatment at different times. The time schedule was created for each group separately. The students were required to be online at the same time, which were decided before. The students who violated the group work or having technological problems were removed from the study.

In the private middle school, on the other hand, the treatment was implemented in the school. The school principal arranged eight class hours in two different days for the treatment. The treatment were implemented to all the groups at the same time. All the other procedures the same as described above.

The five different tasks related to five phases of van Hiele Theory were in the treatment. Each of the activities were designed to complete in 40 minutes. However students were welcome to complete the task according to their pace. The researcher was also online as a facilitator while groups were working.

After the treatment, the participants who completed the tasks were given the same tests (VHGT and MTAS) as posttest. These tests also took approximately 40 minutes.

3.6 Data analysis

In order to answer the first two research questions for the quantitative part, quantitative data analyses were used to analyze the data acquired from the van Hiele Geometric Thinking Test (VHGT) and The Mathematics and Technology Attitudes Scale (MTAS). The IBM SPSS statistical software (Version 22) were used for the quantitative data analysis of the study. The data were analyzed, using both descriptive and inferential statistics analyses.

For the first question, (Is there a statistically significant difference between the pretest and posttest scores on VGHT, after working within VMT?), the mean, median, standard deviation, skewness and kurtosis values of pretest and posttest scores of students on the VHGT test were calculated. In order to show the general characteristics of the sample, box plots were also used for both pre and posttests. In inferential statistics, the paired sample t-test (as the data were normal) used to compare the students' geometric thinking scores before and after the intervention.

The significance level of .05 was used in the analysis. In order to use the paired-samples t-test, there are three assumptions were checked.

Assumption 1. In order to use the paired-samples t-test, one has to have one dependent variable which is continuous. In the present study there is only one dependent variable (scores on VHGT tests) that is continuous. Thus, this assumptions was checked.

Assumption 2. Also, one has to have one independent variable that consists of two categorical, related groups. Related groups means that the groups are dependent to each other. There are the same participants in each group. For example, you can have results of one group which is measured on two different time on the same dependent variable. In the present study, the participants' scores have been measured before and after the treatment. The participants before the intervention consists of a group, and the same participants after the participants consists the other group. Thus, this assumption was also checked.

Assumption 3. The last assumption of the paired-samples t-test is about the distribution of the scores. There should be no significant outliers and the distribution of differences between two related groups should be normally distributed. Since there were no significant outliers and the scores were normally distributed (explained in the result part), the last assumption of the paired-samples t-test was also checked.

For the second question, (Is there a statistically significant difference between the pretest and posttest scores on MTAS, after working within VMT?), the mean, median, standard deviation, skewness and kurtosis values of pretest and posttest scores of students on the MTAS were calculated. In order to show the general characteristics of the sample, box plots were also used in the descriptive analysis. In inferential statistics, the paired sample t-test and Wilcoxon signed-rank test were used to investigate the difference between the students' pretest scores and posttest scores on the MTAS and its subscales.

Before the paired sample t-test was used, its assumptions were checked. The current study consisted of only one dependent variable (scores on math attitude scale) that is measured at continuous level and two related groups (the participants before the treatment and after the treatment). These two assumptions were satisfied. However, the last assumption (distribution of the data) was not satisfied for some subscales. The paired sample t-test were used for the MTAS, MT and TC scores since they were normally distributed. Since the AE, BE and MC scores were not normally distributed, Wilcoxon signed-rank test was used for these scores. It is a nonparametric test equivalent to the paired sample t-test.

For the third question, (How did the collaboration among participants, as defined by the PISA framework, influence their geometry learning while working within the VMT environment?), the VMT chat logs of the two groups were qualitatively analyzed. These two groups were selected by using purposeful maximal variation method. That is, while the VHGT pretest scores of the students in both groups were very close to each other, their VHGT posttest scores were differed.

The chat logs of two groups were qualitatively analyzed by using directed content analysis approach. In directed approach content analysis, predetermined codes which are derived from either theories or relevant research findings guides the analysis (Hsieh & Shannon, 2005).

In the current study, the codes were derived from PISA CPS framework. In this framework, the components of the collaboration were presented as (1) Establishing and maintaining shared understanding, (2) Taking appropriate action to solve the problem, (3) Establishing and maintaining group organization. Since the activities that were used in the present study were not typical problem solving activities, we changed “Taking appropriate action to solve the problem” to “Taking appropriate action to complete the tasks”. Three components of the collaboration, their definitions and the proficient behaviors of these components were stated in Table 5.

Table 5. Collaboration components, their definitions, and proficient behaviors

Collaboration components	Brief explanation of the collaboration process	Proficient behavior
A. Establishing and maintaining shared understanding	Creating an information flow among themselves by communicating the right information at the right time, and to attempt to overcome the deficiencies in shared knowledge.	A1. Discovers others' abilities and shares information about own ability A2. Discusses the tasks - asks questions, responds to others' questions A3. Communicates during monitoring and resolution of group work
B. Taking appropriate action to complete the tasks	Making an effort on completing the task by understanding the task assignments properly.	B1. Understands the type of interaction needed, makes sure to know who does what B2. Describes and discusses tasks and task assignment B3. Enacts plans together with others and performs the actions of the assigned role B4. Monitors and evaluates others' work
C. Establishing and maintaining team organization	Being aware of their role in the group, fulfill the requirements of their role, check whether their teammates performing their roles appropriately, and handle with the communication problems	C1. Acknowledges and enquires about roles C2. Follows rules of engagement - complies with plan, ensures others comply with the plan C3. Monitors team organization - notices issues, suggests ways to fix them

The unit of analysis was identified as sections in VMT chat logs in which participants talked about a single issue. Each of these sections were coded by the researcher in terms of the components of the CPS framework. The frequency tables of the codes presented and the collaboration of the groups were compared by using excerpts which derived from the codes.

To establish the reliability of the coding, another researcher, who has been trained in the coding scheme, independently coded 25 % of the data. The agreement between the two coders is found to be 90 %.

The chat postings of the students were translated into English by the researcher and the original chat logs were given in Appendix C.

CHAPTER 4

RESULTS

The aim of the present study was to examine the role of working within a CSCL environment (VMT) on middle school students' geometric thinking levels and their attitudes towards technology and mathematics, and to understand how did the collaboration among participants, as defined by the PISA framework, influence their geometry learning while working within the VMT environment?. In this chapter, the results of the data analyses based on the research questions of the present study are provided. Firstly, the descriptive statistics of the participants' scores on VGHT test and MTAS were examined. Then, the specific findings based on each research question were presented separately.

4.1 The van Hiele geometric thinking level development

Research question 1: Is there a statistically significant difference between the pretest and posttest scores on VGHT (Usiskin, 1982), after working within VMT?

The descriptive statistics related to the pretest and posttest on van Hiele Geometric Thinking (VHGT) Test were given in Table 6. The participants performed better after the intervention (*M* = 7.83, *SD* = 1.34) as opposed to before the intervention (*M* = 6.67, *SD* = 1.31)

Table 6. Descriptive statistics Related to VHGT Pretest and Posttest Scores

	N	Skewness	Kurtosis	Mean	St. Dev.	Std. Error
PreVHGT	24	-.035	1.002	6.667	1.308	.267
PostVHGT	24	.589	-.115	7.833	1.341	.273

Furthermore, Table 7 shows that the van Hiele geometric thinking level of eleven students has increased from visualization (Level 1) to analysis (Level 2). The level of one student has increased from visualization (Level 1) to informal deduction (Level 3), the level of one student has decreased to Level 0. On the other hand, the van Hiele geometric thinking level of eleven students stayed the same.

Table 7. Students' Van Hiele Geometric Thinking Levels

	Before the treatment	After the treatment
Level 0	0	1
Level 1	24	11
Level 2	0	11
Level 3	0	1

As can be seen from the box plot below, there is only one outlier in the scores (see Figure 9). In order to deal with the outliers, you can exclude them or keep them if they are not extremely different from the other scores. Since the value of the outlier in this analysis did not display to be extreme, this outlier was not excluded from the analysis.

In order to check the distribution of differences between two related groups, the normality test was used. Shapiro-Wilk's test is used for the sample with smaller size. The distribution is accepted as normally distributed when the significant level is greater than .05. Table 8 shows that the difference scores before the intervention and after the intervention were normally distributed, as assessed by Shapiro-Wilk's test (p = .068).

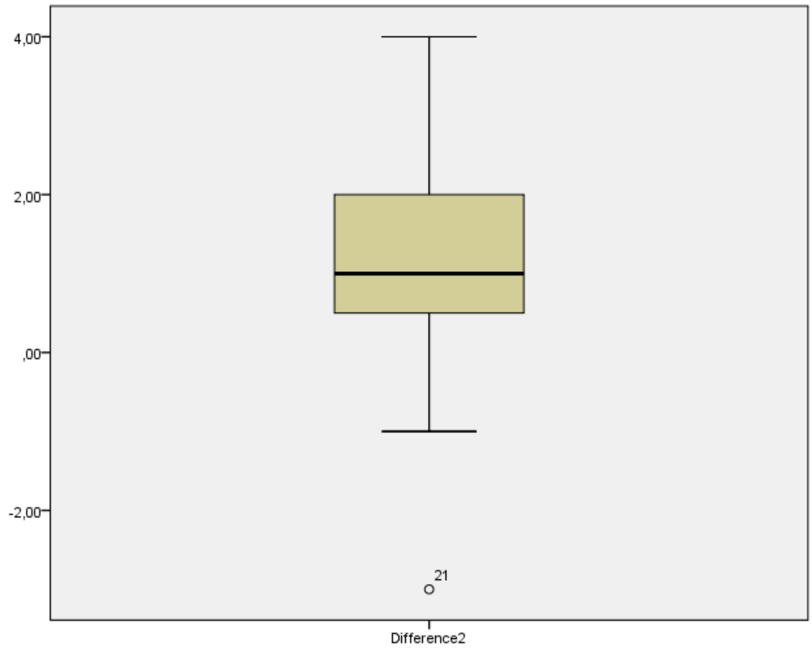


Figure 9. Boxplot of the pretest and posttest scores on VHGT

Table 8. Normality Test Related to VHGT Scores

	Kolmogorov-Smirnov			Shapiro-Wilk		
	Statistic	df	Sig.	Statistic	df	Sig.
differenceVHGT	.206	24	.010	.923	24	.068

The paired-samples t-test was used to examine the mean difference between the students' scores on VGHT test before and after treatment. Table 9 shows that the participants elicited a statistically significant increase in the geometric thinking after the intervention compared to before the intervention, t (23) = 3.83, $p < .05$, $d = .78$. The effect size ($d = .78$) for this analysis indicated a larger effect compared to the effect size (.40) that was found in the study by Akgül.

Table 9. Paired-samples t-Test Related to VHGT Pretest and Posttest Scores

	t	df	Sig. (2- tailed	Cohen's d
PreVHGT- PostVHGT	3.826	23	.001	.78

4.2 Attitudes towards mathematics and technology

Research Question 2: Is there a statistically significant difference between the pretest and posttest scores on MTAS (Barkatsas et al., 2007), after working within VMT?

Table 10 shows that the mean scores of the participants on the attitude scale before the treatment (*M* = 82.54, *SD* = 9.97) and after the treatment (*M* = 82.71, *SD* = 11.97) were very close to each other. Besides, the mean scores of the participants on the confidence with technology and affective engagement subscores before the treatment (*M* = 15.54, *SD* = 3.02; *M* = 16.71, *SD* = 3.29) and after the treatment (*M* =15.88, *SD* = 3.23; *M* =16.92, *SD* = 3.02) were very close to each other. There were a slightly decrease in the mean scores of the participants on the behavioral engagement and mathematical confidence subscore after the treatment (*M* = 16.21, *SD* = 3.22; *M* = 17.25, *SD* = 3.10) compared to before the treatment (*M* = 17.42, *SD* = 2.90; *M* = 18.08, *SD* = 2.89).On the other hand, the mean score of the participants the attitude to learning mathematics with technology subscore after the treatment (*M* = 16.46, *SD* = 3.65) increased compared to before the treatment (*M* = 14.79, *SD* = 3.09).

Table 10. Descriptive Statistics Related to Pretest and Posttest of MTAS

	N	Skewness	Kurtosis	Mean	St. Dev.	Std. Error
MathAttitudePost	24	-.843	.183	82.71	11.97	2.442
MathAttitudePre	24	-.933	1.227	82.54	9.97	2.036
BEPost	24	-.693	.018	16.21	3.22	.657
BEPre	24	-1.454	2.268	17.42	2.90	.593
TCPost	24	-.222	-1.228	15.88	3.23	.660
TCPre	24	-.230	-1.237	15.54	3.02	.617
AEPost	24	-.770	-.624	16.92	3.02	.617
AEPre	24	-1.692	3.862	16.71	3.29	.671
MTPost	24	-1.113	.787	16.46	3.65	.744
MTPre	24	-.076	-.217	14.79	3.09	.631
MCPost	24	-1.109	.608	17.25	3.10	.632
MCPre	24	-1.768	3.094	18.08	2.89	.590

As shown in Figure 10, two significant outliers in the BE, three outliers in MC and one significant outlier in AE were detected in the data. Hence, the Wilcoxon signed-rank test was used for these scores. On the other hand, there are no outliers in MTAS, TC, and MT scores. Also, Table 11 shows that the differences between the scores of the participants for MTAS, TC and MT before and after the treatment were normally distributed, as assessed by Shapiro-Wilk's test (p = .896) for MTAS, (p = .382) for MT subscale, (p = .386) for MT subscale.

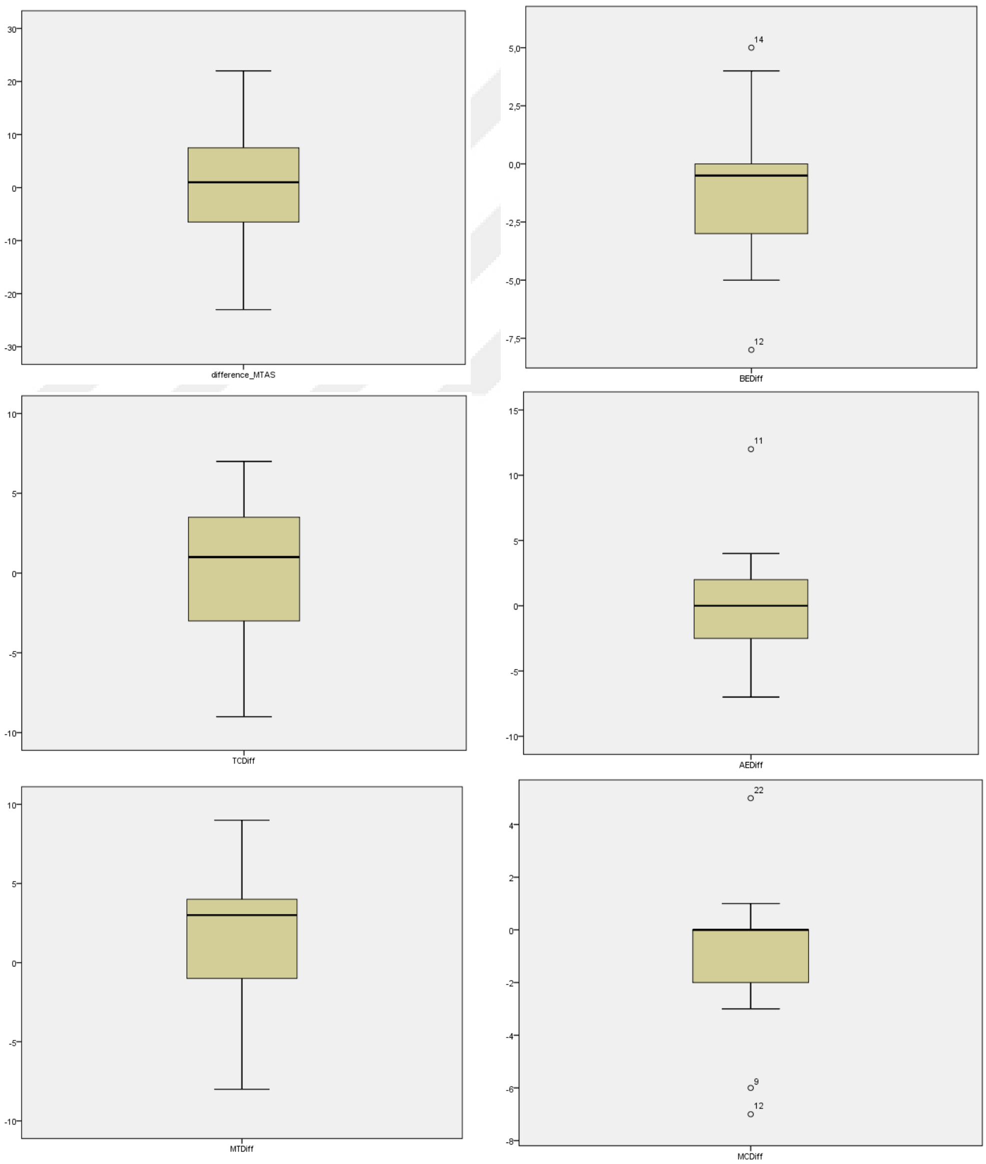


Figure 10. Boxplot of the pretest and posttest scores on MTAS and its subscales

Table 11. Normality Test Related to MT Scores

	Kolmogorov-Smirnov			Shapiro-Wilk		
	Statistic	df	Sig.	Statistic	df	Sig.
differenceMTAS	.112	24	.200	.980	24	.896
TCDiff	.148	24	.190	.957	24	.382
MTDiff	.177	24	.050	.957	24	.386

The paired-sample t-test shows that there was a statistically significant increase in the mean scores on MT subscale, *t* (23) = 2.119, *p* = .045, *d* = .45. (Table 12). The effect size (*d* = .45) for this analysis indicated a medium effect (Cohen, 1988). On the other hand, there were no statistically significant differences between the MTAS scores before and after treatment, *t* (23) = .076, *p* = .940, *d* = .02. Furthermore, there were no statistically significant differences between the TC scores before and after treatment, *t* (23) = .396, *p* = .696, *d* = .08.

Table 12. Paired-samples t-Test Related to MTAS Pretest and Posttest Scores

	t	df	Sig. (2- tailed)	Cohen's d
MTASPre- MTASPost	.076	23	.940	.02
TCPre-TCPost	.396	23	.696	.08
MTPre-MTPost	2.119	23	.045	.45

Wilcoxon signed-rank test revealed that there were no statistically significant differences between the scores on MC, p = .058, r = .39 (see Table 13). Furthermore, there were no statistically significant differences between the scores on BE, *p* = .060, *r* = .38. The test also revealed that there were no statistically significant differences between the scores on AE, *p* = .965, *r* = .01.

The observed power was .05 for MTAS, .48 for BE, .07 for TC, .06 for AE and .38 for MC. It means that the sample size of this study might be small to detect statistically significant differences between pretest and posttest scores.

Table 13. Wilcoxon signed-rank test results

	MCPost - MCPre	BEPost - BEPre	AEPost - AEPre
Z	-1.894[b]	-1.877[b]	-.044[b]
Asymp. Sig. (2-tailed)	.058	.060	.965
a. Wilcoxon Signed-rank Test			
b. Based on positive ranks.			

4.3 Collaboration in groups based on qualitative results

Research Question 3: How did the collaboration among participants, as defined by the PISA framework, influence their geometry learning while working within the VMT environment?

Although there was a statistically significant increase in terms of the VHGT scores, not all students were able to improve their geometric thinking levels. Before the intervention, all students were at the visualization level (Level 1) according to VHGT. However, after they worked within VMT environment, their van Hiele geometric thinking developments differed. What could account for this difference? To provide an answer to this question, we investigated two groups of students in terms of their collaboration.

The groups were selected based on their success in improving their van Hiele geometric thinking levels within VMT environment. The Group 1 consisted of three fifth grade students (Emir, Sude, and Lara) who are at Level 1 before the treatment. They are in the same class in the private middle school. After the treatment, two students (Emir and Sude) increased their van Hiele geometric thinking level from visualization (Level 1) to analysis (Level 2). On the other hand, the level of Lara stayed the same. The Group 2 consisted of two fifth grade students (Öykü, Naz) who were at visualization level (Level 1) before the treatment. They are in the same class in the public middle school. After the treatment, their van Hiele geometric thinking levels stayed the same.

Table 14 shows the frequencies of the codes of two groups in terms of collaboration components. According to this result, there were more evidence of shared understanding and group organization in VMT chat logs of Group 1 compared to Group 2. On the other hand, the group talk was not different in terms of taking appropriate action in both groups.

Table 14. Frequencies of the codes

Codes	Sub-codes	Group 1	Group 2
A. Shared understanding	A1	0	0
	A2	12	5
	A3	6	3
	Total	18	8
B. Taking appropriate action	B1	0	0
	B2	1	2
	B3	5	6
	B4	4	4
	Total	10	12
C. Group organization	C1	6	4
	C2	5	3
	C3	4	1
	Total	15	8

4.3.1 Establishing and maintaining shared understanding

Group 1 and 2 were first qualitatively analyzed according to shared understanding, the first component of the PISA 2015 collaboration framework. In this part, we tried to understand how the groups established and maintained shared understanding by making reference to appropriate examples from the chat logs. We firstly analyzed the students’ discourse qualitatively about the shared understanding in each of group, separately. After that, the two groups were compared based on shared understanding.

4.3.1.1 Group 1 on establishing and maintaining shared understanding

When Group 1 was analyzed qualitatively according to shared understanding, it can be said that the students' in Group 1 showed the proficient behaviors A2 and A3 mostly. That is, they discussed their opinions about the properties of quadrilaterals, asking questions and responding others' questions (A2); and keep the communication to maintain group work (A3). However, we did not find any evidence that shows they discovered others' abilities and inform others about their own ability (A1).

In the study, the students were expected to learn the properties of each type of quadrilaterals in a computer-supported collaborative learning environment. Hence, it was very crucial to learn from each other. When the chat logs of Group 1 were analyzed, it can be seen that they keep the communication to maintain group work, discussed their opinions with teammates, expressed what they explored, asked questions to learn from the others and respond the others' questions. Some representative examples from Group 1's chat log were discussed below.

In Activity 1 (Inquiry), they showed an example how they communicated to establish a shared understanding by discussing in order to determine the types of quadrilaterals (see Excerpt 1). Emir and Sude firstly expressed their opinions about the given shapes (see lines 21, 22, 27, 30, 31, 33, 34, and 35). While they have the same idea about trapezoid and square, they have different idea about which shape belongs to parallelogram family. While Sude claimed that Quadrilateral D is parallelogram (see line 30), Emir claimed that Quadrilateral C is parallelogram. After that, Sude tried to convince Emir by saying that "I have already tried it" (see line36).

Excerpt 1.

line	participant	chat posting
21	Emir	It looks like trapezoid.
22	Sude	I think quadrilateral A is a trapezoid.
23	Sude	A trapezoid.
25	Sude	I think quadrilateral A is a trapezoid.
27	Emir	Quadrilateral C is a parallelogram.
28	Emir	Do not break the shape.
30	Sude	I think Quadrilateral D is a parallelogram.
31	Emir	I think Quadrilateral C is a parallelogram.
33	Sude	Quadrilateral D is a parallelogram.
34	Emir	I think Quadrilateral B is a square.
35	Sude	Yes but Quadrilateral D is a parallelogram.
36	Sude	I have already tried it.
37	Sude	Fix it.

In Activity 3 (Discussion), after Lara asked others about their opinions, they had a discussion on the properties of the types of quadrilaterals (see Excerpt 2). They asked questions about the properties of the given quadrilateral (see line 131). Furthermore, they stated if they agreed with the opinion of others (see lines 121,127 and 128). When they disagreed about an issue, they tried to convince each other. For example, when Emir and Sude had different opinions about a question, Lara supported Sude’s opinion (see lines 122,123 and 127). After that, Sude mounted an argument to support their opinion (see 132 and 133).

There was another example of how the students in Group 1 came up with a shared understanding (see Excerpt 3). There was a discussion about the properties about the rectangle (Activity 3). Sude claimed that the lengths of all sides of rectangle are not equal (see line 252). On the other hand, Emir claimed that the lengths of all sides of rectangle are equal (see line 253). Firstly, Lara agreed with Emir (see line 256). However, after Sude explained her opinion, Lara changed her mind and accepted that the lengths of opposite sides of rectangle are equal (see lines 258, and 259).

Excerpt 2.

line	Participant	chat posting
115	Sude	I think that the answer of second question is “No”.
116	Lara	I think it is “Yes”.
117	Emir	I think that the first question is “False”
118	Sude	Or the second question is “True”
119	Sude	I think the second question is “False” and the first question is “True”
120	Emir	I think the second question is “True”
121	Sude	I agree.
122	Emir	I think the third question is “False”.
123	Sude	I think the third question is “True”.
127	Lara	I agree Sude.
128	Lara	It is OK, Emir.
129	Sude	Emir, the third one is “True”.
130	Lara	I think it was very easy.
131	Lara	Are the lengths of all sides equal?
132	Sude	The teacher said that the lengths of opposite sides are equal.
133	Sude	That is to say, I and Lara were right.

Excerpt 3.

line	participant	chat posting
252	Sude	The lengths of all sides of rectangle are not equal.
253	Emir	The lengths of all sides of rectangle are equal.
254	Sude	How they can be equal?
255	Sude	They cannot.
256	Lara	They can be. Why not?
257	Sude	One of the side lengths must be smaller and the other one must be larger.
258	Lara	It is sensible.
259	Lara	It is so sensible.
260	Emir	The lengths of all sides of a rectangle are equal.
261	Sude	The lengths of opposite sides are equal.
262	Sude	No.
263	Lara	Decide properly.
264	Sude	The lengths of all sides are not equal.
266	Lara	Ok. The lengths of opposite sides are equal.

The participants of the study were the 5th grade students, so it was not easy to stay on the task. In Excerpt 4, we can see that how the group work was broken and how they maintained group work. They were bored with the activity and teasing the others (see lines 91, 92, 93, 94, 95, and 96). After that, Lara ended the unnecessary debate by asking a question (see line 98). After that, the group work maintained again and they continued to discuss the properties of quadrilaterals (see lines 102, and 103).

Excerpt 4

line	participant	chat posting
91	Sude	Cry Emir (she is teasing)
92	Emir	It is none of your business.
93	Emir	I said to you Sude.
94	Emir	Cry baby Sude (he is teasing)
95	Lara	That is exactly you.
96	Lara	Emir.
97	Sude	Come on Emir.
98	Lara	Do you have the control Sude?
99	Lara	?????
100	Sude	Yes.
101	Lara	Thanks.
102	Sude	I think that the first one is “No”.
103	Lara	I think the first one is not equal.

In excerpt 5, we can see how the Group 1 was maintaining group work again. Lara did a very good job to communicate with others and have them keep working on the task. Lara made a great effort to motivate the others to discuss (see lines 239, 243, 247, 248, 249, and 250). Then, Sude supported her (see line 251) and the group started to discuss the task again (see lines 252, and 253).

Excerpt 5

line	participant	chat posting
238	Lara	Finally.
239	Lara	Emir, lets discuss with you.
240	Lara	Then…
241	Lara	Until Sude finish their work.
242	Lara	Come on!
243	Lara	Emir! That is your turn.
244	Lara	Sweety…
245	Lara	Do not be offline…
246	Lara	Come on!
247	Lara	Come on Emir!
248	Lara	You can do that.
249	Lara	Come on Sude!
250	Lara	Let's discuss.
251	Sude	We start for the shapes on the left.
252	Sude	The lengths of all sides are not equal.
253	Emir	The lengths of all sides are equal.

4.3.1.2 Group 2 on establishing and maintaining shared understanding

When Group 2 was analyzed qualitatively according to shared understanding, they had some problems about shared understanding. Although they tried to discuss the properties of the quadrilaterals at the beginning of the study, they could not communicate properly with each other to establish and maintain the shared understanding. In this respect, we can say that Group 2's collaboration was not successful when shared understanding, which is a crucial component of collaboration was considered. Here, we discuss how they failed to show proficient behavior in terms of "A2" and "A3". On the other hand, similar to Group 1, we did not find any evidence that shows they discovered others' abilities and inform others' about their own ability (A1).

In Excerpt 6, we can see that the group members tried to establish a shared understanding in Activity 3 (Discussion). Naz expressed her idea about what she explored about the properties of a type of quadrilateral (see lines 54 and 55) and asked a question (see lines 55 and 59). On the other hand, Öykü also expressed her opinions about what she explored (see line 60) and proposed a solution for better understanding (see lines 60, and 61). In this respect, it can be seen that they did a good job for shared understanding. However, we can see that they were paying attention on different tasks and not following each other (see lines 74 and 76).

Excerpt 6.

line	participant	chat posting
54	Naz	I think the answer of first question is “not equal”.
55	Naz	The lengths of opposite sides are equal.
56	Naz	How we would do the answer?
57	Öykü	Did you change the shape?
58	Öykü	The corner points stayed the same.
59	Naz	No, I mean that do you think they are equal?
60	Öykü	The lengths of opposite sides are equal.
61	Öykü	Drag the corner points.
62	Öykü	And look the shape.
64	Öykü	Naz, look at!
65	Öykü	It can be like that.
66	Naz	I think so. However in the first question is “Are the lengths of all sides equal?” What you said is the answer of your question.
67	Naz	I think so.
68	Öykü	I think the first one is parallelogram.
69	Naz	But it was asked that “are they equal?”
70	Öykü	I think the lengths of opposite sides are equal.
71	Naz	Öykü, are you in the parallelogram part?
72	Öykü	I think the lengths of all sides are equal for Quadrilateral C and D.
73	Naz	Is it “Yes”?
74	Naz	We are not in there, we are in trapezoid part.
75	Öykü	The lengths of opposite sides are equal for Quadrilateral A.
76	Naz	We did not complete Activity 1

As it can be seen in Excerpt 7, the problem on establishing a shared understanding about the properties of quadrilaterals (Activity 3) continued. Actually, both students tried to discuss what they explored and thought about the properties of quadrilaterals (see lines 96, 100 and 104). Furthermore, they asked questions to each other (see line 100, 104 and 105). However, they did not focus on the same task.

Hence, they did not understand what the other was talking about (107, and 113). This situation prevented them to establish a shared understanding.

Excerpt 7.

line	participant	chat posting
96	Öykü	The lengths of opposite sides are equal and it is a trapezoid.
97	Öykü	Naz, do you see?
98	Naz	Öykü, is this you?
99	Öykü	Yes.
100	Naz	What do you say? I said that the first question is “not equal”. What do you think?
101	Öykü	Naz?
102	Öykü	We dragged the corner points.
103	Öykü	Did you realize?
104	Öykü	The lengths of opposite sides are equal.
105	Naz	What do you think about the answer?
106	Naz	The answer?
107	Öykü	What for do you say “they are not equal”?
108	Öykü	The lengths of opposite sides are equal.
109	Naz	For the first question.
110	Naz	What about you?
111	Naz	What?
112	Öykü	Do you realize what I did, Naz?
113	Naz	What are you talking about?

As can be seen from the Excerpt 8 (Activity 3), the discussion between group members continued. However, they did not express what they explored about the properties of given quadrilaterals and explained their ideas. There is not any important evidence in chat logs that shows the discussion on the properties of given quadrilaterals. Thus, they did not find an opportunity to check whether their ideas about the properties were correct or not. They only focused on changing the position of the given quadrilaterals and writing missing values about quadrilaterals (side lengths, angles etc.) on the worksheet. As it can be easily seen from the Excerpt 8, Naz started to do a task (see line 209) and finished it (see line 210). After that, Öykü started to do another task (see line 211), and finished it (see line 213). However, they only focused on completing the worksheet without any need to explain what they did, thus failed to establish a shared understanding about the properties of given quadrilaterals.

Excerpt 8.

line	participant	chat posting
195	Naz	So, we are in the rhombus task.
196	Öykü	Start!
197	Naz	OK!
198	Naz	Finished
199	Naz	That is your turn
200	Öykü	I am starting
201	Naz	The last 3 activities
202	Naz	Start!
203	Naz	Your playing is needless, it is not changing.
204	Öykü	It is changing.
205	Naz	During that time, I will deal with the shape that near to your work.
206	Öykü	But, change it
207	Öykü	But, change it
208	Naz	Ok!
209	Naz	I am starting.
210	Naz	Finished!
211	Öykü	I starting to rhmbos-2 part.
212	Naz	Start!
213	Öykü	Finished!
214	Naz	OK, I am starting
215	Öykü	Ok.
216	Öykü	Ok, I am starting, too
217	Naz	Öykü, release the control.
218	Öykü	Ok, I did it.

In Activity 5 (Integration), Group 2 was required to determine the properties of each type of quadrilaterals. If the given property was correct for the given quadrilateral, they were required to indicate it by putting “check mark” on. However, the main expectation was to discuss and summarize what they learned from the previous activities about the properties of quadrilaterals. The students in this group, however, only put check marks on the worksheet without any discussion and displaying shared understanding (see Excerpt 9).

Excerpt 9.

line	participant	chat posting
284	Öykü	I am writing on the table.
285	Naz	Ok.
286	Öykü	Ok.
287	Naz	Öykü, you did wrong. You need to leave blank if it is not.
288	Öykü	I filled the table for the rectangle
290	Öykü	I put “-”.
291	Naz	Don put “-”. You need to put “+”.
292	Naz	Do not put “+” for all of them, they are different.
293	Öykü	Ok.
294	Öykü	I put “+” for the true and “-” for the false.
295	Öykü	Finished!

In Activity 2 (Directed Orientation), one can see that sometimes they were able to talk about the same thing. (see Excerpt 10; lines 260, 261, 263,264, and 273). However, this communication between group members has to do with division of labor or cooperation. That is, they expressed which part of the tasks was finished and which part of the task they would start. However, this communication was not similar to that of the Group 1. In Group 1, the communication between the group members led them to focus on what they explored. In this respect, the communication in Group 2 enabled students to complete the tasks quickly but did not help them to learn from each other.

Excerpt 10.

line	participant	chat posting
260	Öykü	There are squares left.
261	Öykü	We are in Square 1.
263	Öykü	Ok, finished!
264	Öykü	We are in Square 2.
271	Naz	Finished!
272	Öykü	Yes.
273	Öykü	The last activity…

When two groups are compared specifically in terms of shared understanding, there were crucial differences between Group 1 and Group 2. While the work of students in Group 1 could be named as collaboration, the group work in Group 2 was closer to cooperation rather than collaboration. That is, while the Group 1 worked on the tasks as synchronized and coordinated, Group 2 divided the tasks between the participants and dealt with their parts individually. Even if it was more difficult to discuss the ideas, the students in Group 1 motivated themselves to share their ideas that comes from their explorations, ask questions, respond to others' questions, explain their ideas and come up with a shared understanding (see Excerpt 1, Excerpt 2, and Excerpt 3). Furthermore they communicated successfully to each other to maintain shared understanding (see Excerpt 4 and Excerpt 5). On the other hand, Group 2 had problems in establishing and maintaining a shared understanding (see Excerpt 6 and Excerpt 7). After they had problems about talking about the same things, they shared parts of the task and only completed their own parts (see Excerpt 8, Excerpt 9, and Excerpt 10). They did not express their ideas and discuss them. There was not very much evidence to show that they contributed each other's learning by asking questions, expressing what they explored and discussing ideas. Thus, in Group 2, one can conclude that students were not able to successfully collaborate considering their level of shared understanding. On the other hand, when establishing and maintaining shared understanding criterion is considered, it can be said that students in Group 1 effectively collaborated and learned from each other compared to students in Group 2.

4.3.2 Taking appropriate action to complete the task

Group 1 and 2 were qualitatively analyzed according to taking appropriate action to complete the tasks. In this part, we tried to understand how the groups took action to complete the task assignments by making references to appropriate examples (excerpts) from the chat logs. We firstly analyzed the students' discourse qualitatively about the shared understanding for each group, separately. After that the two groups were compared based on this component of the collaboration.

4.3.2.1 Group 1 on taking appropriate action to complete the task

When the chat logs of Group 1 was qualitatively analyzed based on taking appropriate action to complete the tasks, it can be seen that, the group members made the necessary plan to complete the tasks as a group and displayed an effort to do their own part (B3); and evaluated others' work and asked for explanations if necessary (B4).

In activity 0 (training), Sude evaluated the work done by Emir and notified him that he did not complete his own part appropriately (see line 18). Then, she asked him to reconstruct the roof of the house (see line 19). After that, she recognized that Emir had a problem to do his part, and asked him to give her the control (see line 20). Here we can see that the group members monitored and evaluated each other's' work.

Excerpt 11.

line	participant	chat posting
18	Sude	Emir, you could not construct the body.
19	Sude	Reconstruct the roof.
20	Sude	Emir, give me the control.

Here, in Excerpt 12, Emir expressed that Lara was doing her part very slowly (see line 84). Then Lara apologized (see line 89). From this example, we can understand that Emir was monitoring Lara's work. Furthermore, after Emir's caution, Lara took control again to complete her part in the task.

Excerpt 12.

line	participant	chat posting
84	Emir	Lara is so slow.
85	Lara	Emir, I have a problem with my computer.
86	Lara	I am sorry.
87	Lara	Really…
88	Lara	Sude, did you complete?
89	Lara	I am taking the control.

Group 1 decided together how they would proceed with the task. Here (Excerpt 13), Emir wanted to continue with the other part of the task (see line 190). However, Sude objected to Emir's idea by saying that "we have not discussed, yet" (see line 191). Lara also supported Sude's idea (see line 191). After the discussion part was completed, they continued with the next part (see line 195). From this example, we can understand that they enacted the plans together as a group.

Excerpt 13.

line	participant	chat posting
190	Emir	That is enough. Let's pass another task.
191	Sude	Emir, we have not discussed, yet.
192	Lara	No, we have not
193	Lara	!!!!!!!!!!!!!!
195	Sude	Now, we can pass to the shape on the left.

4.3.2.2 Group 2 on taking appropriate action to complete the task

When the chat logs of Group 2 was qualitatively analyzed based on taking appropriate action to complete the tasks, there is some evidence that shows that the students in Group 2 sometimes explained the tasks to each other (B2), they made the necessary plan to complete the tasks as a group and had an effort to do their own part (B3); and evaluate the others' work and warn him/her if necessary (B4). The explanations based on taking appropriate action to complete tasks for Group 2 are presented below by giving excerpts.

Here, in Excerpt 14, we see how the group members planned to complete the task (Activity 0). Öykü asked Naz to draw the window of the house (see lines 16, and 17). Then, Naz wanted Öykü to release control in order to draw it (see lines 18 and 19).

Excerpt 14.

line	participant	chat posting
16	Öykü	Take the control if you want.
17	Öykü	Construct the window.
18	Naz	I will construct KLMJ.
19	Naz	Release the control.

We can see from the Excerpt 15 that Öykü monitored Naz's moves and stated her observations about Naz's work (see lines 57, and 58).

Excerpt 15.

line	participant	chat posting
57	Öykü	Did you change the shape?
58	Öykü	The corner points stayed the same.

In Excerpt 16, we can see another example of group members' plan to complete the task assignments (Activity 2). Firstly, Naz stated which part of the task she wants to complete (see line 205). Then, Öykü stated that she would start to the rhombus part of the task (see line 211). Meanwhile, they approved each other's plan (see lines 206, and 212).

Excerpt 16.

line	participant	chat posting
205	Naz	During that time, I will deal with the shape that near to your work.
206	Öykü	But, change it.
208	Naz	Ok.
209	Naz	I am starting.
210	Naz	Finished.
211	Öykü	I starting to rhmbos-2 part.
212	Naz	Start.

In Excerpt 17, we can see how the members planned to do the task assignments in Activity 4 (Free Orientation) and fulfilled their responsibility in the group work. Öykü and Naz enacted the plan together to complete the task assignments (see lines 279, 282, and 284) and informed each other that they followed the plan (see lines 280, 283, and 284).

To sum up, according to the qualitative results based on taking appropriate action to complete the tasks, it can be seen that the students in both Group 1 and Group 2 often enacted the plans together with others and fulfilled their own responsibilities (B3). We can understand this from Excerpt 12 and 13 for Group 1; and Excerpt 14, Excerpt 16, and Excerpt 17 for Group 2. Furthermore the students in both groups monitored and evaluated others' work in their respective groups (B4). We can see the examples of this in Excerpt 11 and Excerpt 12 for Group 1 and Excerpt 15 for Group 2. Considering the criterion "taking appropriate action to complete the tasks", one can conclude that there was not any noteworthy differences between the two groups collaboration.

Except 17.

line	participant	chat posting
279	Öykü	Let's construct a square right now.
280	Naz	Ok, it is your turn.
281	Öykü	I have constructed a rectangle.
282	Naz	Ok. Start! That is your turn.
283	Öykü	Ok.
284	Öykü	I am writing on the table right now.
285	Naz	Ok.

4.3.3 Establishing and maintaining group organization

Group 1 and 2 chat logs were also qualitatively analyzed according to group organization. In this part, we tried to understand how group organization was established and maintained by giving appropriate examples (Excerpts) from the chat logs. We firstly analyzed the students' discourse qualitatively about the shared understanding for each group, separately. After that, the two groups were compared based on this component of the collaboration.

4.3.3.1 Group 1 on establishing and maintaining group organization

When the chat logs of Group 1 was qualitatively analyzed based on group organization, there is some evidence that shows that the students in Group 1 tried to fulfill their responsibilities about their role in the group work (C1); engaged in the group work, stuck the group plan and ensures that others follow the plan (C2) and keep eye on group organization and proposed a way to fix any problem in group organization (C3).

We can see from Excerpt 18 how team organization in Group 1 was established before starting Activity 2 (Free Orientation). The teams were required to explore the characteristic features of the quadrilaterals by dragging the corner points of the quadrilaterals and change their shapes in three different positions. Here, they needed to have a plan to complete the activity successfully. In this task, Sude determined in which order the activity will be done (see lines 45, 46, and 47). She also determined how much time each should have control and complete the task (see line 48). The other students agreed with Sude (see lines 49, and 51) and accepted their roles in the group work establishing group organization

Excerpt 18.

line	participant	chat posting
45	Sude	Emir is starting first.
46	Sude	Then, me.
47	Sude	Then, Lara.
48	Sude	Two minutes for each of us…
49	Lara	I agree.
51	Emir	I am the first.

In Excerpt 19, Emir expressed that there was a problem in Lara's work in Activity 2. (see line 79). Sude was monitoring the talk and urged Lara to take the control and complete her part (see line 82). She wanted to ensure that group follows the plan.

Excerpt 19.

line	participant	chat posting
79	Emir	What should I do?
80	Emir	Lara has a problem.
81	Lara	I am taking the control.
82	Sude	Take control, come on!

In Excerpt 20, we see that the students in Group 1 was monitoring each other's work and ensure that group organization was maintained (Activity 3). Lara warned Emir not to talk about irrelevant things (see lines 110,111,112) and Sude stated that "Let's talk about the tasks." (see line 113). Thus, the group was able to stay on task.

Excerpt 20.

line	participant	chat posting
110	Lara	No unnecessary talk!
111	Lara	Emir!
112	Lara	Come on Emir!
113	Sude	Let's talk about the tasks.

4.3.3.2 Group 2 on establishing and maintaining group organization

When the chat logs of Group 2 were qualitatively analyzed based on group organization, we can say that the students in Group 2 had some problems in establishing and maintaining group organization.

As can be seen in Excerpt 21, Naz and Öykü tried to establish the team organization more democratically (Activity 3). Here, Naz kindly looked for the Öykü's approval regarding their roles in completing the tasks (see lines 37, 38 and 39).

Excerpt 21.

line	participant	chat posting
37	Naz	Öykü, I want to answer the first question.
38	Naz	Ok?
39	Naz	If you want to, of course.

In Excerpt 22 (in Activity 2), Naz and Öykü wanted to check each other’s work (see lines 122 and 125), and they did (see lines 123, and 127). Thus, we can understand that they were engaged in the task and ensured that the other complied with the plan.

Excerpt 22.

line	participant	chat posting
122	Naz	Öykü, check what I wrote.
123	Öykü	I checked.
124	Naz	Dou you think that it is correct?
125	Öykü	Check what I wrote.
126	Öykü	It is ok.
127	Naz	I think so.

In Excerpt 23, we see another example of how the team members in Group 2 checked each other to ensure the plan was followed. Öykü stated that they were in the parallelogram part and wanted Naz to do her own part (see lines 172 and 173). After that, Naz realized that she focused on the wrong task and explained her situation (see lines 175 and 177).

Excerpt 23.

line	participant	chat posting
172	Öykü	We are in the parallelogram task.
173	Öykü	I did. That is your turn.
174	Öykü	If you want to.
175	Naz	Ok, I was confused. It is written “Location 1” and I thought that we need to do something.
176	Öykü	Ok.
177	Naz	I am doing.

Meanwhile, Group 2 had some problems with the group organization in Activity 2 (see Excerpt 24). For instance, while Öykü was working on Activity 1 (see line 76), Naz was dealing with the Activity 2 (see line 74), focusing on different tasks at the same time. Although a long time have passed, they were not able to solve the problem about the group organization (see Excerpt 25). Öykü asked Naz to see what she did (see line 112). However Naz did not understand what Öykü talked about (see line 113).

Excerpt 24.

line	participant	chat posting
68	Öykü	I think the first one is paralleogram.
69	Naz	But it was asked that “are they equal?”.
70	Öykü	I think the lenghts of opposite sides are equal.
71	Naz	Öykü, are you in the paralleogram part?
72	Öykü	I think the lenghts of all sides are equal for Quadrilateral C and D.
73	Naz	Is it “Yes” ?
74	Naz	We are not in there, we are in trapezoid part.
75	Öykü	The lenghts of opposite sides are equal for Quadrilateral A.
76	Öykü	We did not complete the Activity 1.

Excerpt 25.

line	participant	chat posting
112	Öykü	Naz, did you see what I have done?
113	Naz	What are you talking about?
114	Öykü	The things what I wrote on the table…
115	Naz	No, I cannot see what you did.

Unquestionably, the group organization is crucial in order to maintain an effective collaboration in the group work. The group members should engage in group work, follow the group plan and check others’ engagement. Furthermore, if the group organization is somehow broken, the members should be able to repair it. In this respect, the qualitative results show that Group 1 did a good job in establishing and maintaining group organization. They were mostly engaged in the tasks as a group, complied with the group plan and checked if everyone was complying with the plan (see Excerpt 18 and Excerpt 19). Besides, they were able to fix group organization when there was a problem about it (see Excerpt 20). On the other hand, the students in Group 2 sometimes struggled with establishing and maintaining group organization (see Excerpt 24 and Excerpt 25).

To sum up, the result of qualitative analysis showed that the members of Group 1 collaborated much more effectively compared to the members of Group 2. The main aim of collaborative learning is to learn together. In an effective collaborative learning environment, group members are responsible for learning of each other’s. The VMT tasks asked students to study the characteristic features of the types of quadrilaterals by collaborating in small groups.

The main differences between the two groups were most notably identified in terms of the two components of PISA CPS framework: “shared understanding” and “group organization.” When the groups were compared in terms of these components, one can conclude that Group 1 members were more successful in group organization and establishing a shared understanding. That is, Group 1 they engaged in group work, followed the plan and checked each other’s work and solved the problems about group organization when there was a problem. Furthermore, they discussed their opinions based on their explorations from the activities and tried to maintain a shared understanding. On the other hand, Group 2 members were not able to successfully deal with the problems in group organization, preferred to divide the tasks between them and completed the tasks individually without sharing their ideas and discussing about the task. These aspects of their collaboration might have prevented them to learn from each other.

CHAPTER 5

DISCUSSION

The current mixed methods study investigated the role of working within a CSCL environment (VMT) on middle school students' geometric thinking levels and their attitudes towards technology and mathematics, and understand how the collaboration among participants, as defined by the PISA framework, influence their geometry learning while working within the VMT environment. Three main research questions (1) "Is there a statistically significant difference between the pretest and posttest scores on VGHT (Usiskin, 1982), after working within VMT?", (2) "Is there a statistically significant difference between the pretest and posttest scores on MTAS (Barkatsas et al., 2007), after working within VMT?", (3) "How did the collaboration among participants, as defined by the PISA framework, influence their geometry learning while working within the VMT environment?" guided this study. In order to answer these research questions, VMT based activities, designed based on the van Hiele phased based instruction, were used with 24 (5^{th} and 7^{th} grade) students who were all at Level 1. The results of the present study were discussed below considering previous literature.

5.1 Development of students' van Hiele geometric thinking

The results of the present study showed that students' geometric thinking levels can be developed in a well-designed CSCL environment even when the treatment is not long. In the previous literature, there were several studies that found the dynamic geometry learning environment to be effective for increasing students' van Hiele geometric thinking (Kutluca, 2013; Abdullah & Zakaria, 2013; Abdullah, Surif, Tahir, Ibrahim and Zakaria, 2015; Karakuş & Peker, 2015). The findings in the current study supported the findings from the previous literature.

Researchers observed that freely sharing and discussing their ideas in a learning environment affected students' learning positively (e.g., Kutluca, 2013; Karakuş & Peker, 2015). Actually, they implied the importance of the collaboration among students. However, there were no study that examined students' geometric thinking in a well-designed CSCL environment. In this respect, the results of the present study expanded the previous literature by finding that van Hiele geometric thinking levels can also be developed in a well- designed CSCL environment, where collaboration among students has been the guiding design element of the learning environment.

The results of the current study also supported the previous literature about the effectiveness of van Hiele phased-based instruction. For example, Siew, Chong, and Abdullah (2013), examined the effectiveness of van Hiele phases of learning by using tangrams and the results showed that students' geometric thinking increased after the instruction. In the present study, tasks were designed in VMT environment by considering van Hiele phased-based learning strategy and student' geometric thinking also increased. Thus, we can say that the current study corroborated the effectiveness of van Hiele phased-based instruction in developing students' geometric thinking levels.

5.2 Attitudes towards mathematics and technology

In the current study, we also investigated students' attitudes towards mathematics and technology. The inferential result of MTAS revealed that the students' scores on MTAS and its four subscales (mathematical confidence, confidence with technology, behavioral engagement, and affective engagement) were not different between two data points (pre and post-test). However, the statistical analysis revealed that students elicited a statistically significant increase in the attitudes towards learning mathematics with technology subscale.

The results based on students' attitudes towards mathematics and technology is consisted with the study by Akgül (2014). In the study by Akgül (2014), the students in the experimental group were taught with GeoGebra in three weeks and results showed that there were no statistically significant effect of computer-assisted instruction on students' mathematics and technology attitudes. On the other hand, the results of the study by Pilli (2008) revealed that computer-assisted instruction had a statistically significant effect on students' attitudes towards mathematics and technology. In this study, the participants were the 4^{th} grade students and the study was conducted in a 15 weeks.

One of the reason for not detecting change in students' attitudes towards mathematics and technology might be the duration of the study. The students in the current study completed the VMT activities in a week. This might be a short time to change the attitude. In the study by Pilli (2008), for example, the students were instructed during all the semester (15 weeks) and their attitudes changed positively. On the other hand, in the study by Akgül (2014), the students were instructed in three weeks and there were no significant difference on their attitudes. The other reason might be the age of the students. It becomes more difficult for students to change their attitudes as they grow older (Pilli, 2008). It might be easier to change students' attitudes when they are at primary school.

The interesting finding of the study based on students' attitudes towards learning mathematics with technology is the increase in their attitudes towards learning mathematics and technology subscale. The students in Turkey were mostly familiar with the traditional instruction. Teacher gives lecture and students learn by listening. However, in the current study, they were expected to learn the characteristic features of types of quadrilaterals by collaborating each other. They found an opportunity to construct the types of quadrilaterals by using dynamic geometry software and learn their features by exploring and discussing with their friends. During the study, the researcher observed that the students verbally stated that they were very excited to work within VMT environment as a group. Actually, in today’s world, students meet technology in the earlier ages and spend lots of time with technological devices such as smart phones and tablets and personal computers. However, they have less opportunities to learn with these tools, especially in geometry classrooms.

The use of technology in mathematics instruction might increase expectations about learning outcomes. However, technology-assisted instruction cannot meet the expectations about learning outcomes every time (Artigue, 2012). If students have negative attitudes towards mathematics tools, the potential benefit of technological tool might be limited (Pierce & Stacey, 2004).In this respect, the finding of the current study about students’ attitudes towards learning mathematics with technology might be crucial. Students can regard technological devices such as personal computers and tablets for playing games not for learning. However, if they feel that they can learn mathematics with the technological devices, their engagement in technology-assisted learning activities might increase. This might increase their mathematics learning.

5.3 Collaboration in groups

Apart from attitudes, one other important factor that might affected students' geometric thinking development in the VMT environment might be the quality of collaboration. Thus, we investigated the VMT chat logs of two groups (Group1 and Group 2) who were both at Level 1 initially but showed different improvements after the study, in terms of the PISA 2015 CPS framework components. Although, the students in both Group 1 and Group 2 were at the visualization level (Level 1) before the intervention, their levels were differentiated from each other after the intervention. Two students in Group 1 has increased their level from Visualization (Level 1) to Analysis (Level 2) and the level of the other student stayed the same. However, the level of the students in Group 2 did not change and their level stayed the same. Collaboration among group members plays a central role in the success of groups (PISA, 2015, p.4). The result of the qualitative part of the study showed that Group 1 members much more effectively collaborated compared to Group 2. This result brought a perspective that the effective collaboration among the students can be another important factor that supports geometric thinking development in a CSCL environment.

In order to compare the collaboration of groups, three components of the collaboration (shared understanding, taking appropriate action and group organization) that were presented in PISA 2015 Collaborative Problem-solving Framework were used in the qualitative part of the current study. The VMT chat logs of two groups were compared based on these components of collaboration. The qualitative results showed that the members of Group 1 effectively collaborated based on the analysis on "shared understanding" and "group organization" components compared to the members of Group 2. There were not any crucial difference between two groups in terms of “taking appropriate action” component.

Team members need to establish and maintain shared understanding in order to collaborate effectively. In order to establish and maintain shared understanding, team members should communicate with each other about the task, negotiate the meaning of the task and monitor each other’s understanding (OECD, 2015). According to qualitative analysis based on shared understanding, the members of Group 1 expressed and discussed their ideas about the characteristic features of quadrilaterals and met on common ground.

On the other hand, the members of Group 2, were not that successful in establishing and maintaining shared understanding. Group 2 members did not share much information about what they explored. They could not dealt with the deficiencies in the shared understanding. Hence they were not able to realize each other’s misunderstandings and help each other. This might be an important reason for not being able to increase their geometric thinking levels.

Another crucial component of collaboration is group organization. Team members should establish and maintain group organization in order to collaborate effectively (OECD, 2015). When two groups were qualitatively analyzed, the results based on the group organization showed that the members of Group 1 were more successful than the members of Group 2. In order to establish and maintain group organization, students should accomplish their responsibility in the group work and monitor to each other to ensure the maintaining of the group organization (OECD, 2015). The qualitative analysis showed that, the members of Group 1 planned how to complete the activities and executed the plan during the team work. They followed each other’s work and fixed the problems about group organization. However, the members of Group 2 were not able to fix the problem about group organization. The problems about group organization might have affected their learning.

In the current study, the researcher helped students to understand the tasks/activities and to solve the possible technological problems. On the other hand, the researcher did not interfere in the discussion of the students. If the researcher had helped the Group 2 members to solve the problem in their group organization, they could have collaborate more effectively. In this respect, the teacher can play an essential role in facilitating students to collaborate. In such a collaborative learning environment (online or not), teacher should follow the discussion among students and help them if they have difficulty in collaborating.

The findings of this study suggest several ideas about the design of geometry learning environments. Firstly, mathematics teachers should integrate computer-supported collaborative learning environments such as VMT into geometry instruction. Secondly, teachers should design activities to increase students' collaboration skills. In a well-designed computer-supported collaborative learning environment, students can take an opportunity to discover the features of geometric shapes and the relationships between them. If students can collaborate each other effectively, they can learn geometrical ideas by discussing. Thus, they can learn geometric concepts and relationships between them without memorizing.

5.4 Recommendations and implications for further research

The current study contributes to mathematics education research in the following ways. First of all, the present study presented designed activities based on van Hiele phase-based instruction to be used on the VMT environment on quadrilaterals for middle school. The students' in Turkey do not perform well at the mathematics domain in the international studies such as PISA and TIMSS. Olkun et al. (2009) stated that there was a need for technology-integrated instruction to provide students an effective geometry learning. Thus, teachers can integrate VMT-based activities designed in this study into their classes to teach geometry more effectively.

Furthermore, in the current study, the instruction was designed for the students at visualization level (Level 1). The researchers can conduct research studies by designing instruction for students at other levels. Since the participants of the present study were middle school students, in further research, participants may be selected from primary or high school students.

Secondly, the qualitative results of the current study showed that students who collaborate effectively are more likely to increase their geometric thinking. In the current education system, students are not taught how to collaborate and there is not a lesson in the current curriculum to improve their collaboration skills. Collaboration is one of the crucial skill for 21st century for success of groups, families, corporations, public institutions, organization and government agencies (OECD, 2015). Hence, curriculum developers may develop a curriculum by considering students' collaboration skills. In the current study, the groups were not selected by considering students' collaboration skills. However, the present study brought a new perspective for further researches about collaboration. Thus, an experimental design might be used in further research in order to investigate cause and effect relationship.

Lastly, in the current study, students' attitudes towards learning mathematics with technology has increased. Hence, mathematics teachers can use technological tools more frequently in their lessons. Most students find mathematics difficult to learn. However by using technological tool such as dynamic geometry software, learning and teaching mathematics can be more effective.

5.5 Limitations of the study

One of the limitations of the current study about the sampling methodology. In the current study, purposeful sampling was used to select participants. That is, the participants of the study were selected from the students who have personal computers and internet access. Thus, this limits the generalization of the findings of this study.

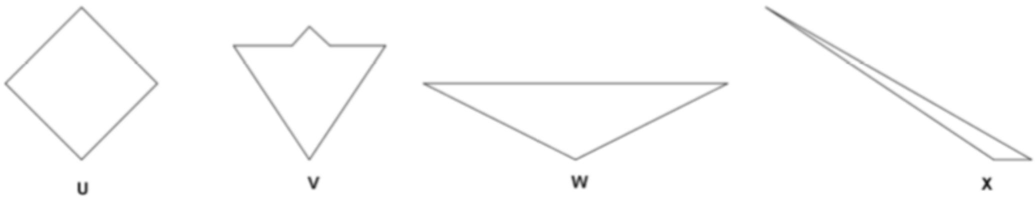
Another limitation is about the design of the study. In the quantitative part of the current study, there was no control group. Only one group was used in the quantitative part. This design is called as pre-experimental design and mostly used to measure a new program or service. Since there is no control group, this design failed to show a cause and effect relationship. However, it can provide guidance for further research.

APPENDIX A

VAN HIELE GEOEMTRIC THINKING LEVEL TEST

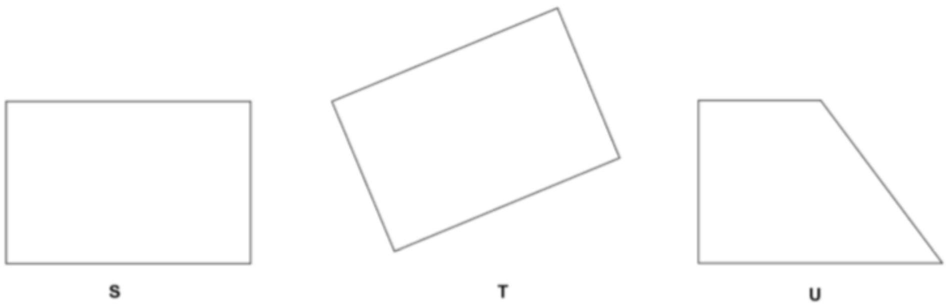
1. Which of these are squares?
   (A) K only
   (B) L only
   (C) M only
   (D) L and M only
   (E) All are squares

2. Which of these are triangles?



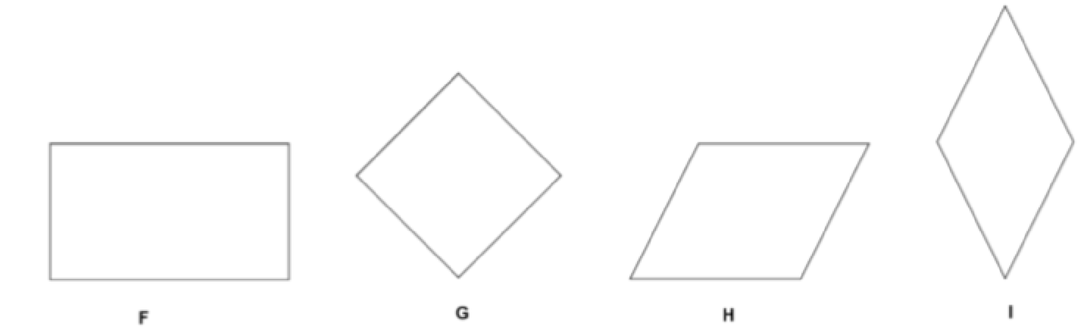
   (A) None of these are triangles.
   (B) V only
   (C) W only
   (D) W and X only
   (E) V and M only

3. Which of these are rectangles?



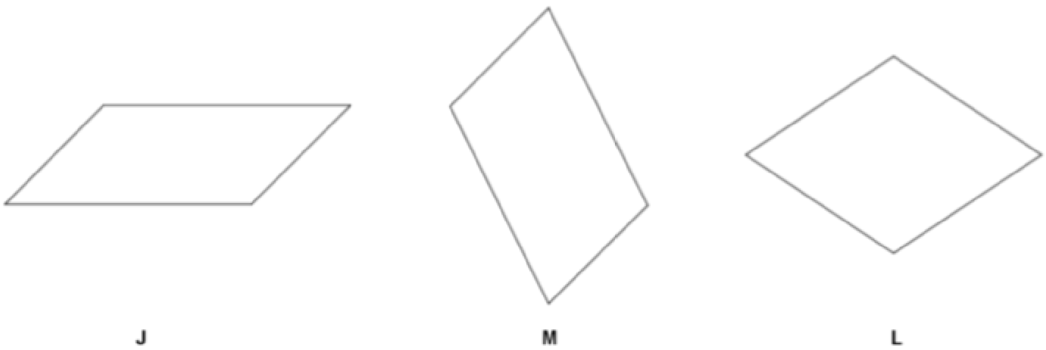
   (A) S only
   (B) T only
   (C) S and T only
   (D) S and U only
   (E) All are rectangles

4. Which of these are squares?



(A) None of these are squares.
(B) G only
(C) F and G only
(D) G and I only
(E) All are squares

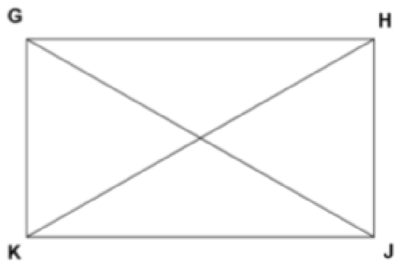
5. Which of these are parallelograms?



(A) J only
(B) L only
(C) J and M only
(D) None of these are parallelograms.
(E) All are parallelograms.

6. PQRS is a square.
Which relationship is true in all squares?
(A) **$\overline{PR}$ and $\overline{RS}$ have the same length.**
(B) **$\overline{QS}$ and $\overline{PR}$ are perpendicular.**
(C) **$\overline{PS}$ and $\overline{QR}$ are perpendicular.**
(D) **$\overline{PS}$ and $\overline{QS}$ have the same length.**
(E) **Angle Q is larger than angle R.**

7. In a rectangle GHJK, $\overline{GJ}$ **and** $\overline{HK}$ are the diagonals.



Which of (A)-(D) is not true in every rectangle?

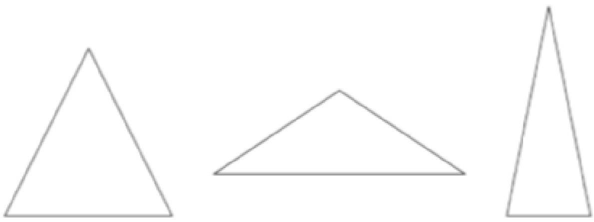
(A) There are four right angles.
(B) There are four sides.
(C) The diagonals have the same length.
(D) The opposite sides have the same length.
(E) All of (A)-(D) are true in every rectangle.

8. A rhombus is a 4-sided figure with all sides of the same length.
Here are three examples

Which of (A)-(D) is not true in every rhombus?

(A) The two diagonals have the same length.
(B) Each diagonal bisects two angles of the rhombus.
(C) The two diagonals are perpendicular.
(D) The opposite angles have the same measure.
(E) All of (A)-(D) are true in every rhombus.

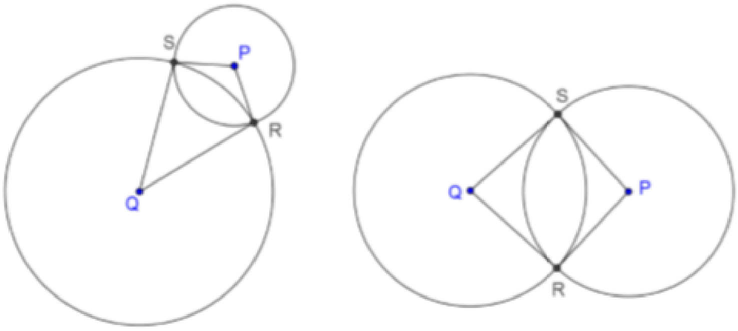
9. An isosceles triangle is a triangle with two sides of equal length.
Here are three examples.



Which of (A)-(D) is true in every isosceles triangle?

(A) The three sides must have the same length.
(B) One side must have twice the length of another side.
(C) There must be at least two angles with the same measure.
(D) The three angles must have the same measure.
(E) None of (A)-(D) is true in every isosceles triangle.

10. Two circles with centers P and Q intersect at R and S to form a 4-sided figure PRQS. Here are two examples.



Which of (A)-(D) is <u>not</u> always true?

(A) PRQS will have two pairs of sides of equal length.
(B) PRQS will have at least two angles of equal measure.
(C) The lines $\overline{\mathbf{PQ}}$ **and** $\overline{\mathbf{RS}}$ will be perpendicular
(D) Angles P and Q will have the same measure.
(E) All of (A)-(D) are true.

11. Here are two statements.

Statement 1: *Figure F is a rectangle.*
Statement 2: *Figure F is a triangle.*

Which is correct?

(A) If 1 is true, then 2 is true.
(B) If 1 is false, then 2 is true.
(C) 1 and 2 cannot both be true.
(D) 1 and 2 cannot both be false.
(E) None of (A)-(D) is correct.

12. Here are two statements.

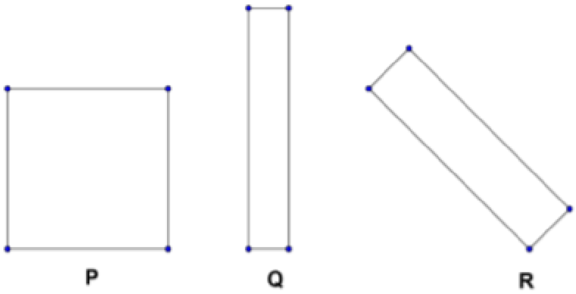
Statement S: *ΔABC has three sides of the same length.*
Statement T: *In ΔABC, $\angle B$ and $\angle C$ have the same measure.*

Which is correct?

(A) Statements S and T cannot both be true.
(B) If S is true, then T is true.
(C) If T is true, then S is true.
(D) If S is false, then T is false.
(E) None of (A)-(D) is correct.

13. Which of these can be called rectangles?

(A) All can.
(B) Q only
(C) R only
(D) P and Q only
(E) Q and R only



14. Which is true?

(A) All properties of rectangles are properties of all squares.
(B) All properties of squares are properties of all rectangles.
(C) All properties of rectangles are properties of all parallelograms.
(D) All properties of squares are properties of all parallelograms.
(E) None of (A)-(D) is true.

15. What do all rectangles have that some parallelograms do not have?

(A) Opposite sides equal
(B) Diagonals equal
(C) Opposite sides parallel
(D) Opposite angles equal
(E) None of (A)-(D)

APPENDIX B

MATHEMATICS AND TECHNOLOGY SCALE

FIVE SUBSCALES:
Mathematics Confidence [MC],
Confidence with Technology [TC)],
Attitude to learning mathematics with technology [MT],
Affective Engagement [AE] *and*
Behavioural Engagement [BE]. (MTg = MT - graphing calculators)
To tailor MT items to a particular class, change the words "graphics calculators" to the technology used by that class (e.g. computers, graphics calculators, computer algebra systems). Do not change TC items.

		Hardly Ever	**Occas-ionally**	**About Half the time**	**Usually**	**Nearly Always**
1.	I concentrate hard in mathematics [BE]	HE	Oc	Ha	U	NA
2.	I try to answer questions the teacher asks [BE]	HE	Oc	Ha	U	NA
3.	If I make mistakes, I work until I have corrected them. [BE]	HE	Oc	Ha	U	NA
4.	If I can't do a problem, I keep trying different ideas. [BE]	HE	Oc	Ha	U	NA
5.	I am good at using computers [TC]	SD	D	NS	A	SA
6.	I am good at using things like VCRs, DVDs, MP3s and mobile phones [TC]	SD	D	NS	A	SA
7.	I can fix a lot of computer problems [TC]	SD	D	NS	A	SA
8.	I am quick to learn new computer software needed for school [TC]	SD	D	NS	A	SA
9.	I have a mathematical mind [MC]	SD	D	NS	A	SA
10.	I can get good results in mathematics [MC]	SD	D	NS	A	SA
11.	I know I can handle difficulties in mathematics [MC]	SD	D	NS	A	SA
12.	I am confident with mathematics [MC]	SD	D	NS	A	SA
13.	I am interested to learn new things in mathematics [AE]	SD	D	NS	A	SA
14.	In mathematics you get rewards for your effort [AE]	SD	D	NS	A	SA
15.	Learning mathematics is enjoyable [AE]	SD	D	NS	A	SA
16.	I get a sense of satisfaction when I solve mathematics problems [AE]	SD	D	NS	A	SA
17.	I like using graphics calculators for mathematics [MTg]	SD	D	NS	A	SA
18.	Using graphics calculators in mathematics is worth the extra effort [MTg]	SD	D	NS	A	SA
19.	Mathematics is more interesting when using graphics calculators. [MTg]	SD	D	NS	A	SA
20.	Graphics calculators help me learn mathematics better [MTg]	SD	D	NS	A	SA

APPENDIX C

VMT CHAT LOGS (ORIGINAL FORM)

Excerpt 1.

line	participant	chat posting	
21	Emir	yamuğa benziyor	*It looks like a trapezoid.*
22	Sude	bence dörtgen a yamuk	*I think quadrilateral A is a trapezoid.*
23	Sude	yamuk	*A trapezoid.*
25	Sude	bence dörtgen a yamuk	*I think quadrilateral A is a trapezoid.*
27	Emir	c paralel kenar	*Quadrilateral C is a parallelogram.*
28	Emir	bozmayın	*Do not break the shape.*
30	Sude	bence dörtgen d paralel	*I think Quadrilateral D is a parallelogram.*
31	Emir	bence dörtgen c paralel kenar	*I think Quadrilateral C is a parallelogram.*
33	Sude	paralel d	*Quadrilateral D is a parallelogram.*
34	Emir	dörtgen b kare bence	*I think Quadrilateral B is a square.*
35	Sude	evet ama dörtgen d paralel	*Yes but Quadrilateral D is a parallelogram.*
36	Sude	ben denedim	*I have already tried it.*
37	Sude	onu düzeltin	*Fix it.*

Excerpt 2.

line	participant	chat posting	
115	Sude	2. soru hayır bence	*I think that the answer of second question is "No".*
116	Lara	bence evet	*I think it is "Yes".*
117	Emir	1 inci soru yanlış bence	*I think that the first question is "False"*
118	Sude	yada doğru 2. soru	*Or the second question is "True"*
119	Sude	2. soru yalnış 1. soru ise doğru bence	*I think the second question is "False" and the first question is "True"*
120	Emir	2 inci soru doğru bence	*I think the second question is "True"*
121	Sude	aynen	*I agree.*
122	Emir	3 üncü soru yanlış bence	*I think the third question is "False".*
123	Sude	3. soru doğru bence	*I think the third question is "True".*
127	Lara	aynensude	*I agree Sude.*
128	Lara	Tamam Emir	*It is OK, Emir.*
129	Sude	3. doğru Emir	*Emir, the third one is "True".*
130	Lara	bence çok kolaydı	*I think it was very easy.*
131	Lara	Bütün kenarları eşit mi?	*Are the lengths of all sides equal?*
132	Sude	karşılıklı birbirine eşit dedi hoca	*The teacher said that the lengths of opposite sides are equal.*
133	Sude	yani Larayla dediğimiz doğru çıktı	*That is to say, I and Lara were right.*

Excerpt 3.

line	participant	chat posting	
252	Sude	dikdörtgenin bütün kenarları eşit değildi	*The lengths of all sides of rectangle are not equal.*
253	Emir	dikdörtgenin bütun kenarları eşittir	*The lengths of all sides of rectangle are equal.*
254	Sude	nasıl eşit olsun ki	*How they can be equal?*
255	Sude	olamaz	*They cannot.*
256	Lara	olur niye olmasın ki	*They can be. Why not?*
257	Sude	ya birisi küçük ya birisi büyük olmak zorunda	*One of the side lengths must be smaller and the other one must be larger.*
258	Lara	yani mantıklı	*It is sensible.*
259	Lara	hem de çok mantıklı	*It is so sensible.*
260	Emir	dikdörtgenin bütün kenarları eşittir	*The lengths of all sides of a rectangle are equal.*
261	Sude	karşılıklı kenarları eşittir	*The lengths of opposite sides are equal.*
262	Sude	hAYIR	*No.*
263	Lara	düzgün karar verin	*Decide properly.*
264	Sude	bütün kenarları eşit değildir	*The lengths of all sides are not equal.*
266	Lara	tmm karşılıklı kenarlar eşit	*Ok. The lengths of opposite sides are equal.*

Excerpt 4

line	participant	chat posting	
91	Sude	AĞLA EMIR	*Cry Emir (she is teasing)*
92	Emir	sanane salyangoz	*It is none of your business.*
93	Emir	sude sana dedim	*I said to you Sude.*
94	Emir	sude AĞLAAĞLA AĞLA AĞLA	*Cry baby Sude (he is teasing)*
95	Lara	sensin o	*That is exactly you.*
96	Lara	Emir	*Emir.*
97	Sude	emir hadi	*Come on Emir.*
98	Lara	sude kontrol Emir'te mi	*Do you have the control Sude?*
99	Lara	?????	*?????*
100	Sude	yes	*Yes.*
101	Lara	thanks	*Thanks.*
102	Sude	birinci hayır bence	*I think that the first one is "No".*
103	Lara	bence 1. eşit değil	*I think the first one is not equal.*

Excerpt 5.

line	participant	chat posting	
238	Lara	sonunda be	*Finally.*
239	Lara	Emir seninle tartışalım	*Emir, lets discuss with you.*
240	Lara	bari	*Then…*
241	Lara	sude yapana kadar	*Until Sude finish their work.*
242	Lara	hayde	*Come on!*
243	Lara	sıra Emir sende	*Emir! That is your turn.*
244	Lara	GENCOKiM	*Sweety…*
245	Lara	gitme gitme	*Do not be offline…*
246	Lara	yaaaaaaaaaaaaaaaaaaaaaa	*Come on!*
247	Lara	hadi Emir	*Come on Emir!*
248	Lara	yaparsın	*You can do that.*
249	Lara	hadisude	*Come on Sude!*
250	Lara	tartışalım	*Let's discuss.*
251	Sude	soldaki şekiller için başlıyoruz	*We start for the shapes on the left.*
252	Sude	dikdörtgenin bütün kenarları eşit değildi	*The lengths of all sides are not equal.*
253	Emir	dikdörtgenin bütun kenarları eşittir	*The lengths of all sides are equal.*

Excerpt 6.

line	participant	chat posting	
54	Naz	birin cevapı bence eşit değil	*I think the answer of first question is "not equal".*
55	Naz	karşılıklı kenarları birbirine eşit	*The lengths of opposite sides are equal.*
56	Naz	bide cevapları nasıl yapıcaz	*How we would do the answer?*
57	Öykü	şekli değiştirdin mi	*Did you change the shape?*
58	Öykü	köşe noktalar aynı duruyor	*The corner points stayed the same.*
59	Naz	yok sence eşitmi	*No, I mean that do you think they are equal?*
60	Öykü	karşı kenarlar birbirine eşit	*The lengths of opposite sides are equal.*
61	Öykü	ama biraz noktalardan sürükle	*Drag the corner points.*
62	Öykü	şekline bakalım	*And look the shape.*
64	Öykü	naz bak	*Naz, look at!*
65	Öykü	böyle de oluyor	*It can be like that.*
66	Naz	bencede ama 1. soruda bütün kenarları birbirineb eşit midir diyor senin dediğin kendi sorunun cevanbı bence	*I think so. However in the first question is "Are the lengths of all sides equal?" What you said is the answer of your question.*
67	Naz	bence öyle	*I think so.*
68	Öykü	bence bir paralel kenar	*I think the first one is parallelogram.*
69	Naz	ama eşit midir diyor	*But it was asked that "are they equal?"*
70	Öykü	karşılıklı iki kenar eşit bence	*I think the lengths of opposite sides are equal.*
71	Naz	c ve e de bütün kenarlar eşit bence	*Öykü, are you in the parallelogram part?*
72	Öykü	c ve e de bütün kenarlar eşit bence	*I think the lengths of all sides are equal for Quadrilateral C and D.*
73	Naz	evetmi	*Is it "Yes"?*
74	Naz	biz orda değiliz yamuktayız	*We are not in there, we are in trapezoid part.*
75	Öykü	dörtgen a da karşılıklı iki kenar birbirine eşit bence	*The lengths of opposite sides are equal for Quadrilateral A.*
76	Öykü	act 1 i yapmadık ki	*We did not complete Activity 1*

Excerpt 7.

line	participant	chat posting	
96	Öykü	iikarşılıklı kenarları birbirine eşit yamuk bir de	*The lengths of opposite sides are equal and it is a trapezoid.*
97	Öykü	naz görüyormusun	*Naz, do you see?*
98	Naz	öykü sen misin	*Öykü, is this you?*
99	Öykü	evet	*Yes.*
100	Naz	neyi gördünmü ben ilk soruya eşit değil diyorum sen	*What do you say? I said that the first question is "not equal". What do you think?*
101	Öykü	naz	*Naz?*
102	Öykü	kenarları oynattık	*We dragged the corner points.*
103	Öykü	gördün mü	*Did you realize?*
104	Öykü	karşılıklı kenarları ve köşe uzunluklrı birbirine eşit	*The lengths of opposite sides are equal.*
105	Naz	sence cevao	*What do you think about the answer?*
106	Naz	cevap	*The answer?*
107	Öykü	neye eşit değil diyosun	*What for do you say "they are not equal"?*
108	Öykü	karşılıklı kenarlar eşit	*The lengths of opposite sides are equal.*
109	Naz	1. soruya	*For the first question.*
110	Naz	sen	*What about you?*
111	Naz	nr	*What?*
112	Öykü	naz yaptıklarımı görüyomusun	*Do you realize what I did, Naz?*
113	Naz	ne diyorsun	*What are you talking about?*

Excerpt 8.

line	participant	chat posting	
195	Naz	veeeeeeee eşkenar dörtgendeyizzzzzzzzzzzzz	*So, we are in the rhombus task.*
196	Öykü	başlaaa	*Start!*
197	Naz	okeyyyyyy	*OK!*
198	Naz	bittiiiiii	*Finished*
199	Naz	sıra sendeeee	*That is your turn*
200	Öykü	başlıyorum	*I am starting*
201	Naz	son 3 aktivite	*The last 3 activities*
202	Naz	başla	*Start!*
203	Naz	öykü boşuna oynama değişmiyor	*Your playing is needless, it is not changing.*
204	Öykü	değişiyo	*It is changing.*
205	Naz	ben o sırada sennin yaptığının yanındakini yapıyım	*During that time, I will deal with the shape that near to your work.*
206	Öykü	ama değiştir	*But, change it*
207	Öykü	ama değiştir,	*But, change it*
208	Naz	tamam	*Ok!*
209	Naz	başlıyorum	*I am starting.*
210	Naz	bitti	*Finished!*
211	Öykü	2.eşkenar dörtgene başlıyorum	*I starting to rhmbos-2 part.*
212	Naz	başla yumoşum	*Start!*
213	Öykü	bitti	*Finished!*
214	Naz	tamam başlıyorumm	*OK, I am starting*
215	Öykü	tmm	*Ok.*
216	Öykü	tmm ben başlıyorum	*Ok, I am starting, too*
217	Naz	öykü bırak	*Öykü, release the control.*
218	Öykü	tamam ben yptım	*Ok, I did it.*

Excerpt 9.

line	participant	chat posting	
284	Öykü	yazıyorum şimdi tabloya	*I am writing on the table.*
285	Naz	taamam	*Ok.*
286	Öykü	tmm	*Ok.*
287	Naz	öykü yanlış yaptın olmayanları boş bırakacaksın	*Öykü, you did wrong. You need to leave blank if it is not.*
288	Öykü	ben dikdörtgeni doldurdum	*I filled the table for the rectangle*
290	Öykü	bende - koydum	*I put "-".*
291	Naz	işte koymak yok artı koyucaksın	*Don put "-". You need to put "+".*
292	Naz	artı alt alta değil onların hepsi farklı	*Do not put "+" for all of them, they are different.*
293	Öykü	tmm	*Ok.*
294	Öykü	doğrulara artı yanlışlara eksi koydum	*I put "+" for the true and "-" for the false.*
295	Öykü	bitti	*Finished!*

Excerpt 10.

line	participant	chat posting	
260	Öykü	kareler var ya	*There are squares left.*
261	Öykü	kare 1 deyiz	*We are in Square 1.*
263	Öykü	tmm bitti	*Ok, finished!*
264	Öykü	kare 2 deyiz	*We are in Square 2.*
271	Naz	bitti	*Finished!*
272	Öykü	evet	*Yes.*
273	Öykü	son etkinlik	*The last activity…*

Excerpt 11.

line	participant	chat posting	
18	Sude	emir gövdeyi yapamadın ki	*Emir, you could not construct the body.*
19	Sude	çatıyı tekrar yapın	*Reconstruct the roof.*
20	Sude	emir kontrolü bana ver	*Emir, give me the control.*

Excerpt 12.

line	participant	chat posting	
84	Emir	lara çok yavaş	*Lara is so slow.*
85	Lara	ya Emir benim bilgisayarda sıkıntı çıktı	*Emir, I have a problem with my computer.*
86	Lara	özür dilerim	*I am sorry.*
87	Lara	cidden	*Really…*
88	Lara	yaptın mısude?	*Sude, did you complete?*
89	Lara	alıyorum kontrolü	*I am taking the control.*

Excerpt 13.

line	participant	chat posting	
190	Emir	yeter başka yere geçelim	*That is enough. Let's pass another task.*
191	Sude	bilim 4 daha tartışmadık	*Emir, we have not discussed, yet.*
192	Lara	aynen tartışmadık	*No, we have not*
193	Lara	!!!!!!!!!!!!!!!!!!!!!!!!!	*!!!!!!!!!!!!!*
195	Sude	artık soldaki şekle geçebiliriz	*Now, we can pass to the shape on the left.*

Excerpt 14.

line	participant	chat posting	
16	Öykü	istersen kontrolü al	*Take the control if you want.*
17	Öykü	camı yap	*Construct the window.*
18	Naz	klmj yi yapıyım	*I will construct KLMJ.*
19	Naz	sen kontrolü bırak	*Release the control.*

Excerpt 15.

line	participant	chat posting	
57	Öykü	şekli değiştirdin mi	*Did you change the shape?*
58	Öykü	köşe noktalar aynı duruyor	*The corner points stayed the same.*

Excerpt 16.

line	participant	chat posting	
205	Naz	ben o sırada sennin yaptığının yanındakini yapıyım	*During that time, I will deal with the shape that near to your work.*
206	Öykü	ama değiştir	*But, change it.*
208	Naz	tamam	*Ok.*
209	Naz	başlıyorum	*I am starting.*
210	Naz	bitti	*Finished.*
211	Öykü	2.eşkenar dörtgene başlıyorum	*I starting to rhmbos-2 part.*
212	Naz	-> başla yumoşum	*Start.*

Except 17.

line	participant	chat posting	
279	Öykü	şimdi bir kare oluşturalım	*Let's construct a square right now.*
280	Naz	tamam sıra sende	*Ok, it is your turn.*
281	Öykü	dikdörgen oluşturdum	*I have constructed a rectangle.*
282	Naz	tamam başla sende sıra	*Ok. Start! That is your turn.*
283	Öykü	tmm	*Ok.*
284	Öykü	yazıyorum şimdi tabloya	*I am writing on the table right now.*
285	Naz	taamam	*Ok.*

Excerpt 18.

line	participant	chat posting	
45	Sude	sırailk emirde	*Emir is starting first.*
46	Sude	sonra ben	*Then, me.*
47	Sude	sonra lara	*Then, Lara.*
48	Sude	2 dk süre var	*Two minutes for each of us…*
49	Lara	aynen	*I agree.*
51	Emir	ben birim	*I am the first.*

Excerpt 19.

line	participant	chat posting	
79	Emir	ben ne yapayım	*What should I do?*
80	Emir	lara patladı	*Lara has a problem.*
81	Lara	alıyorum açıldı	*I am taking the control.*
82	Sude	take control hadiiii çabuk olllllll	*Take control, come on!*

Excerpt 20.

line	participant	chat posting	
110	Lara	Geyik yok	*No unnecessary talk!*
111	Lara	Emir	*Emir!*
112	Lara	Hadi Emir	*Come on Emir!*
113	Sude	Düzgün dersle alakalı konuşalım	*Let's talk about the tasks.*

Excerpt 21.

line	participant	chat posting	
37	Naz	öykü ilk soruyu ben cevaplayım	*Öykü, I want to answer the first question.*
38	Naz	olurmu	*Ok?*
39	Naz	tabi sende istersen	*If you want to, of course.*

Excerpt 22.

line	participant	chat posting	
122	Naz	öykü benim yazdıklarıma bir bak	*Öykü, check what I wrote.*
123	Öykü	baktım	*I checked.*
124	Naz	sence öylemi	*Dou you think that it is correct?*
125	Öykü	sende benimkilere bak	*Check what I wrote.*
126	Öykü	öyle	*It is ok.*
127	Naz	bencede	*I think so.*

Excerpt 23.

line	participant	chat posting	
172	Öykü	parelelkenar 2 deyiz	*We are in the parallelogram task.*
173	Öykü	yaptım sende	*I did. That is your turn.*
174	Öykü	yani yapmak istersen	*If you want to.*
175	Naz	tamam ben karıştırmışım üsteki konum 1 felan yazıyo ya ondada birşey yapıcaz zannettim	*Ok, I was confused. It is written "Location 1" and I thought that we need to do something.*
176	Öykü	tamam	*Ok.*
177	Naz	yapıyorum	*I am doing.*

Excerpt 24.

line	participant	chat posting	
68	Öykü	bence bir paralel kenar	*I think the first one is paralleogram.*
69	Naz	ama eşit midir diyor	*But it was asked that "are they equal?".*
70	Öykü	karşılıklı iki kenar eşit bence	*I think the lenghts of opposite sides are equal.*
71	Naz	öykü sen paralel kenar damısın	*Öykü, are you in the paralleogram part?*
72	Öykü	c ve e de bütün kenarlar eşit bence	*I think the lenghts of all sides are equal for Quadrilateral C and D.*
73	Naz	evetmi	*Is it "Yes" ?*
74	Naz	biz orda değiliz yamuktayız	*We are not in there, we are in trapezoid part.*
75	Öykü	dörtgen a da karşılıklı iki kenar birbirine eşit bence	*The lenghts of opposite sides are equal for Quadrilateral A.*
76	Öykü	act 1 i yapmadık ki	*We did not complete the Activity 1.*

Excerpt 25.

line	participant	chat posting	
112	Öykü	naz yaptıklarımı görüyomusun	*Naz, did you see what I have done?*
113	Naz	ne diyorsun	*What are you talking about?*
114	Öykü	tabloya yazdıklarımı	*The things what I wrote on the table…*
115	Naz	hayır ne yaptın bende gözükmüyor	*No, I cannot see what you did.*

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