# MEAN REVERSION IN INTERNATIONAL EQUITY MARKETS

## AND TIME-VARYING RISK PREMIUM

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## MEAN REVERSION IN INTERNATIONAL EQUITY MARKETS AND TIME-VARYING RISK PREMIUM

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Mean Reversion in International Equity Markets and Time-Varying Risk Premium

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## DECLARATION OF ORIGINALITY

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#### ABSTRACT

## Mean Reversion in International Equity

## Markets and Time-Varying Risk Premium

Mean reversion is a phenomenon that has been consistently observed and refuted in several studies over the last decades. This study first aims at shedding further light on the issue by assessing mean reversion on recent data in a broad range of international equity markets including developed and emerging markets and international indices provided by MSCI. Variance ratio computations and a novel distribution-free statistical test based on randomization are used on dollar denominated nominal, real and excess returns of these equity markets. The results indicate that mean reversion exists in both developed and emerging countries, albeit its statistical significance is occasionally dubitable. Moreover, firm size and return type exhibit significant effects on the degree of mean reversion.

As Turkish market displays a strong mean reversion in the empirical tests, the second part of the thesis aims at identifying the cause of this apparent anomaly. Equity risk premium estimations generated via two-pass cross-sectional regressions reveal that the mean reversion is due to dynamic nature of equity risk-premium. The results indicate that the mean reversion in Turkish equity market is rather a result of time-varying behavior of rational investors than market inefficiency.

### ÖZET

Uluslararası Sermaye Piyasalarında Ortalamaya Dönme Eğilimi ve Zamanla Değişen Risk Primleri

Ortalamaya dönme eğilimi, geçtiğimiz kırk yılda birçok çalışma tarafından sürekli olarak gözlemlenmiş, birçok çalışma tarafından da varlığı reddedilmiş bir olgudur. Bu tezin ilk amacı güncel bir veri seti kullanarak gelişmiş ve gelişmekte olan pazarlardan ve MSCI tarafından sağlanan uluslararası endekslerden oluşan geniş bir yelpazede ortalamaya dönme eğilimini araştırarak bu konunun aydınlatılmasına katkıda bulunmaktır. Bu doğrultuda, bu uluslararası sermaye endekslerinin ve bahsi geçen pazarların sermaye piyasalarının dolar bazındaki nominal, reel ve fazla getirileri üzerinde varyans oranı hesaplamaları yapılmış ve rasgeleleştirmeye dayanan dağılımdan bağımsız bir istatistiksel test uygulanmıştır. Bazı durumlarda istatistiksel önem şüphelere yol açsa da, sonuçlar hem gelişmiş, hem de gelişmekte olan ülkelerde ortalamaya dönme eğiliminin var olduğunu göstermektedir. Bununla beraber firma büyüklüğü ve getiri tipinin ortalamaya dönme eğiliminin derecesi üzerinde önemli etkileri olduğu gözlemlenmiştir.

Türkiye pazarı ampirik testlerde güçlü bir ortalamaya dönme eğilimi gösterdiğinden, tezin ikinci kısmı görülen bu anomalinin nedenlerini tespit etmeyi amaçlamaktadır. İki geçişli kesitsel regresyonlarla üretilen sermaye risk primleri, ortalamaya dönme eğiliminin sermaye risk primlerinin dinamik doğasından ileri geldiğini ortaya koymaktadır. Sonuçlara göre Türkiye sermaye piyasasındaki ortalamaya dönme eğilimi pazarın etkin olmamasından değil, rasyonel yatırımcıların davranışlarının zamanla değişmesinden kaynaklanmaktadır.

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## ABBREVIATIONS

ACWI	All Country World Index
ADF	Augmented Dickey-Fuller
AMEX	American Stock Exchange
AR	Auto-Regressive
BIST	Borsa İstanbul
CAPM	Capital Asset Pricing Model
EM	Emerging Markets
ERP	Equity Risk Premium
FM	Frontier Markets
TKYD	Turkish Institutional Investment Managers' Association
MSCI	Morgan Stanley Capital International
NASDAQ	National Association of Securities Dealers Automated
	Quotation
NYSE	New York Stock Exchange
PP	Phillips-Perron
SUR	Seemingly Unrelated Regressions
USD	United States Dollar
WRDS	Wharton Research Data Services

#### CHAPTER 1

### MEAN REVERSION IN INTERNATIONAL EQUITY MARKETS

### 1.1 Introduction

Movements of stock prices have been a crucial part of the finance literature for decades and probably will continue to be so for decades to come. Practitioners and academics alike have been trying to understand how stock prices move since the early days of modern financial markets in the eighteenth century. There are numerous theories and published papers trying to explain how stock prices change over time. What makes it such a hot topic is that it has the possibility of opening doors to endless economic gains. If one can understand how stock prices move, he can amass profits by trading on that information. For example, if we can prove that a certain stock's price has a cyclical behavior (a.k.a. mean reversion) and if we can identify certain properties of that behavior such as the half-life, we can buy that stock when it's at its lowest level and sell it when it's at its highest. There is certainly some degree of randomness in stock prices, therefore we might not be as successful as we'd like in our predictions. Yet it is undeniable that understanding the patterns of stock prices presents us with an incomparable opportunity for profits.

The most dominant theory regarding stock returns is the random walk theory. This theory maintains that holding period returns of a stock are independent and identically distributed (i.i.d.). This idea renders technical analysis completely useless since it is impossible to predict future stock prices with past prices. According to this theory, stock prices have no memory; therefore historical prices have no practical use to us. So if we assume stock prices follow a random walk with no drift, naive forecast is our best option; which means the forecast for the next period's price is the current price. If there is a drift, we just add the stock's expected return to the forecast.

Advocates of the random walk theory believe it is not possible to outperform the market without bearing any additional risk. Burton Malkiel (1973), who is credited with popularizing the idea, claims: "a blindfolded monkey throwing darts at a newspaper's financial pages could select a portfolio that would do just as well as one carefully selected by the experts" (p.24). Assuming he was talking about the risk adjusted returns, what he said would be true if prices were following a random walk.

The popularity of the random walk theory had a dramatic increase with the introduction of the Efficient Markets Hypothesis (hereafter EMH) in the 1960s. In general terms, EMH holds that in an efficient market, "prices fully reflect available information" (Fama, 1970, p.384). Although they are not exactly the same, it is clear that EMH makes a strong case for the random walk theory.

Let's assume that prices do not follow a random walk and holding period returns display some serial correlation. We could make reliable forecasts on future prices based on this information and potentially realize risk-free returns using those forecasts. However, in an efficient market historical prices would be available to all investors and they too would make the same forecasts as we did, hence they would make the same trades as we did. Consequently, the forces of supply and demand would force the prices to come to a level that would eliminate any riskless profit opportunity coming from our analysis. Therefore, EMH and random walk theory seem to go hand in hand for the most part.

If prices do not follow a random walk and serial correlations between holding-period returns are not zero, it means there is a certain degree of predictability

in stock prices. Positive serial correlations point towards a trend in stock prices, whether it be increasing or decreasing. This means the stock price in the next period will likely move in the same direction it moved in the last period. This is called momentum and investment strategies based on this idea are called momentum strategies. Momentum strategies typically involve buying stocks that performed well in the past and short selling stocks that performed poorly in the past.

In contrast, if serial correlations are negative; stock prices tend to fluctuate around a certain mean or trend. This is called mean reversion. Investment strategies based on this phenomenon are called contrarian strategies. Contrarian strategies look to buy stocks that have performed poorly in the past and short sell stocks that have performed well in the past. Both of these investment strategies aim to outperform the market by following two diametrically opposite routes.

Coexistence of the two opposing views can be explained by the fact that serial correlations can have different signs for different holding periods or different lags. There might be momentum in the short-term and mean reversion in the longterm or vice versa. In this case, a more sophisticated investment strategy involving both momentum and contrarian perspectives may be adopted.

If EMH and random walk theory are considered the traditional paradigm in explaining stock returns, financial market anomalies are the empirical patterns that are in violation of these central ideas. There are numerous empirical anomalies documented over the decades capturing both cross-sectional and time-series patterns in returns of securities. Mean reversion and aversion occupy important places on the list of anomalies violating EMH, even though there are some differences of opinion in the literature.

Fama and French (1988) argue that serial correlations may be the result of "time varying equilibrium expected returns generated by rational investor behavior" (p.266). Moreover, Conrad and Kaul (1988) find evidence in favor of a stationary expected return process, which substantiates the earlier statement. Since there is no consensus as to even their existence, we deem it is worthy of further study to explore these phenomena.

Therefore, this study investigates if international equity indices, both developed and emerging, show any signs of these anomalies on recent data using a robust and novel methodology.

#### 1.2 Literature review

Mean reversion, especially in finance, is a relatively new concept. Vasicek (1977) proposed a mean-reverting Ornstein-Uhlenbeck process for spot interest rates. He assumed spot rates had an instantaneous drift,  $\alpha(\gamma - r)$ , which pulls the spot rate back to its long-term average  $\gamma$  whenever it deviates from it. By definition, the force that pulls the process to its mean is proportional to the magnitude of the deviation. This extension of continuous time models that have been popularized in the 70s found a natural application in interest rates, as it was long perceived that interest rates exhibit mean reversion in empirical observations.

Mean reversion in stock prices, however, was first investigated by DeBondt and Thaler (1985) under the name of price reversals. They studied the price reversals of U.S. stocks for the period between 1926 and 1982. They formed winner and loser portfolios by ranking all the stocks in the New York Stock Exchange with respect to their returns for every 3-year period in their data and picked the top 35 stocks as winners and bottom 35 stocks as losers for each period. Thus, they obtained a different winner and loser portfolio for every 3-year formation period. Then they calculated the returns of these portfolios for the formation periods as well as the 3-year test periods that come after it. Finally, they calculated the average returns for all winner and loser portfolios for the entire horizon. They repeated this process for different formation periods and portfolio sizes as well.

Their findings were remarkable. They found that on average, the loser portfolios earned 24.6% more than the winner portfolios in the subsequent test period. They also discovered that as the returns in the formation period grow larger in absolute terms, so do the following price reversals. To show that the difference in returns cannot be explained solely by the difference in risk, they also calculated CAPM betas for winner and loser portfolios for each of the formation period. Surprisingly, not only loser portfolios were outperforming winner portfolios, they were also significantly less risky. They interpreted these findings as the result of overreaction of investors and concluded that this is a violation of weak-form market efficiency. This study paved the way for other researchers to explore this new phenomenon and gain more insight on how stock prices move.

Chan (1988) challenged DeBondt and Thaler's (1985) results by claiming that their method of measuring the betas of winner and loser portfolios was biased. He argued that DeBondt and Thaler (1985) underestimated the risk of loser portfolios and overestimated the risk of winner portfolios by calculating their betas with the formation period's data because the market value of these companies changed substantially during that period. Assuming market value has a crucial impact on a company's risk, loser stocks should become significantly more risky at the end of the formation period since they lose a big portion of their market value and winner

stocks should become significantly less risky due to the large increase in their market value. However, DeBondt and Thaler (1985) disregarded this effect by assuming a constant beta throughout the formation and test periods.

Chan (1988) suggested an alternative method for measuring risk which involves calculating different betas for the formation and test periods. He repeated the same procedure as DeBondt and Thaler (1985) with the new betas and he found only weak evidence of price reversals. According to his results, abnormal returns to the contrarian strategy were very small, and probably economically insignificant, considering transaction costs and various other factors that can erase that small profit margin.

French and Roll (1986), while investigating the difference in the volatility of stock prices between trading and non-trading hours, reported negative serial correlations in daily returns in all lags up to 13 except lag 1. Although they used these results for a different purpose, these negative auto correlations can still be regarded as significant evidence in favor of short-term mean reversion. They also calculated actual to implied variance ratios for several return horizons and reported ratios smaller than 1 in all of them. More importantly, the variance ratios they calculated got smaller as the return horizon got longer. They reported variance ratios of 0.894 and 0.883 for three month and six month periods respectively, which point towards mean reversion in the long run. The way they calculated these variance ratios is very similar to that of Cochrane (1988), whose methodology was taken as the basis by the likes of Poterba and Summers (1989) and Lo and MacKinlay (1988). Even though they were not concerned with mean reversion at all, they provided substantial evidence for future researchers to move forward with, nonetheless. In fact, this simple idea provided the foundation to test random walk and efficient

market hypothesis without being tied to an asset pricing model, a problem which led the academics to question the aforementioned studies.

Lo and MacKinlay (1988) used variance ratios to explore whether or not stock prices follow random walk. They also developed a statistical testing method for variance ratios that relies on asymptotic approximations. They used weekly returns on both equal and value-weighted NYSE-AMEX indices between 1926 and 1985. They found significant evidence of positive autocorrelations in the weekly data and therefore rejected the random walk hypothesis. Their rejection of the random walk hypothesis was much stronger for the equal-weighted index since the variance ratios for that index were much larger than those of the value-weighted index.

Furthermore, they formed three different decile portfolios from the companies in NYSE-AMEX index with respect to size; as small, medium and large and tested those portfolios separately. They discovered that positive autocorrelations get larger, thus the rejection of random walk stronger, as the firm size decreases. This consolidates the finding of Lo and MacKinlay (1988) above since small firms' weights are relatively larger in the equal-weighted index.

Lastly, they repeated the same process for every 625 individual security in the index and found out that they couldn't reject the random walk hypothesis for individual securities. They attributed this result to individual returns containing "much company-specific, or idiosyncratic noise that makes it difficult to detect the presence of predictable components" (p.56).

Fama and French (1988) adopted a different approach to discover if there is any predictability in stock prices. They assumed stock prices consist of two separate AR (1) processes, a permanent component which follows a random walk and a

transitory component which is mean-reverting. What differentiates them is the coefficient of lagged term  $x_{t-1}$ , which is equal to 1 for the permanent component and smaller than 1 for the transitory component. This means shocks to the permanent component, as the name suggests, are permanent while shocks to the transitory component fade away slowly.

This model allowed them to predict the proportion of the variance in stock returns explained by the mean-reverting component with the regression slopes coming from regressing consecutive holding period returns to one another. The way the model works is that, if there is a transitory component in the stock price; regression slopes, which they use as a proxy for the autocorrelation coefficients, should have a U-shaped pattern. In other words, the regression coefficient should be close to zero for short holding periods, decrease as the holding period increases and then start to move back towards zero after a certain point.

Fama and French (1988) put their model to test by using real monthly returns for the 1926-1985 period. They formed 10 equal-weighted decile portfolios on the basis of market value as well as 17 equal-weighted industry portfolios from all the stocks in New York Stock Exchange. They performed regressions for all of these portfolios in yearly return horizons from 1 to 10. While they did not observe any apparent pattern in industry portfolios, the decile portfolios demonstrated clear Ushaped patterns that suggest mean-reverting components in prices. The effect of the mean-reverting component diminished however, as the firm size increased. Even though a direct comparison would not be appropriate since their time horizon was a lot shorter, this is consistent with the findings of Lo and MacKinlay (1988), whose rejection of the random walk hypothesis was much stronger for small companies.

Moreover, they divided their data set into two and conducted the same analysis for two sub-periods: 1926-1940 and 1941-1985. By doing so, they discovered that mean reversion was less pronounced in the latter period and strong negative autocorrelations in returns were largely due to first 15 years of the data covering the Great Depression years.

Poterba and Summers (1989) compared the methods of Fama and French (1988) and Lo and MacKinlay (1988) with respect to their power in detecting mean reversion and concluded that variance ratio method used by Lo and MacKinlay (1988), although not nearly powerful enough, is much more powerful than the method of Fama and French (1988). Therefore, they used variance ratio tests to investigate if there was long-term mean reversion in stock prices.

They used monthly returns of NYSE stocks for the 1926-1985 period. They reported variance ratios separately for nominal, real and excess returns for the valueweighted as well as the equal-weighted indices. Their results suggested positive serial correlation in stock returns in horizons shorter than one year and negative serial correlation in horizons longer than one year. This, together with the results of Lo and MacKinlay (1988), makes a strong case for momentum strategies in the short-term, while advocating for contrarian strategies for the longer horizons.

Their rejection of random walk in favor of mean reversion was stronger for the equal-weighted index compared to the value-weighted index and for excess and nominal returns compared to real returns. They also discovered that post-war period exhibited mean reversion in a less pronounced manner than the pre-war period. Most of these results are consistent with the findings of Fama and French (1988) and Lo and MacKinlay (1988).

Poterba and Summers (1989) also looked at some equity markets outside the United States as well. They included a total of 17 countries, ranging from developed markets like United Kingdom to emerging markets such as South Africa. Almost all of these countries displayed mean aversion in the short-term and mean reversion in the long-term, just like the U.S. market. There were some exceptions however, most notably Spain, whose variance ratios were very high for longer horizons. The mean 96-month variance ratio off all countries was 0.653 when Spain is excluded, which is well below unity.

Though not as strong as the indices, according to Poterba and Summers (1989), individual firms also exhibit some long-term mean reversion. Average 96-month variance ratio of 82 firms calculated with nominal returns was 0.678.

Kim, Nelson and Startz (1991) criticized the findings of Poterba and Summers (1989) and Fama and French (1988) on mean-reverting behavior of stock prices. They used both the variance ratio method of Poterba and Summers (1989) and regression method of Fama and French (1988) to investigate mean reversion in USA for the 1926-1986 period. They calculated monthly excess and real returns and formed value-weighted and equal-weighted portfolios from all the stocks in NYSE.

They divided their data set into two sub-periods: 1926-1946 and 1947-1986. The reasoning behind this was to discover if there were any differences between the pre-war and post-war periods in terms of stock price behavior. Their results for the 1926-1986 period resemble those of Poterba and Summers' (1989). Variance ratios for both value-weighted and equal-weighted portfolios were declining well below unity as the holding period was increasing. However, the sub-period results indicated a behavior shift in stock prices after the war. While the pre-war period displayed a

great degree of mean reversion for both indices, it was not the case for the post-war period. In fact, the value-weighted portfolio seemed to be mean-averting in the postwar period. They obtained similar results with the regression method of Fama and French (1988) as well.

In order to test their results, they created empirical distributions of variance ratios and regression coefficients by randomization. To put it simply, they shuffled their data 1000 times and calculated the same statistic for every shuffle to come up with an empirical distribution for their test statistic. The merits of this approach comes from the fact that it requires no assumptions about the distribution of the test statistic. This separates them from other researchers who use asymptotic approximations.

After testing their results, Kim et al. (1991) concluded that mean reversion was "primarily a phenomenon of the 1926-1946 period which includes the Great Depression and World War II when the stock market was highly volatile" (p.526). Moreover, they claimed that the evidence for mean aversion in the post-war period was as strong as the evidence for mean reversion over the entire 1926-1986 period.

McQueen (1992) also investigated long-term mean reversion in the U.S. market by making a few changes to the methodologies of Fama and French (1988) and Kim et al. (1991). They used generalized least squares estimators instead of ordinary least squares estimators to avoid the homoscedasticity assumption. They also looked at the differences in regression slopes of pre-war and post-war data to test if there was any change in the behavior of stock prices.

They reported results of both GLS and OLS regressions. While the OLS method provided significant evidence for mean reversion, they could not reject the

random walk hypothesis with the GLS method. Furthermore, sub-period results showed that mean reversion was only apparent in the pre-war data. They concluded that after correcting for heteroscedasticity and small sample sizes and recognizing the extraordinary nature of the 1927-1946 period, there wasn't any convincing evidence against random walk.

Richards (1997) studied winner-loser reversals just like DeBondt and Thaler (1985) with a few key differences: First; instead of working with individual stocks in a single market, he treated 16 different markets as individual assets and formed winner and loser portfolios among those assets. Second, his data set was from the period 1969-1995, which was more recent compared to that of DeBondt and Thaler (1985). Finally, to test the return on the contrarian strategy, he used bootstrapping and resampling to obtain simulated critical values rather than theoretically derived ones. His contrarian strategy involved going long on the loser portfolio and going short on the winner portfolio.

His results were consistent with the earlier literature. He found a momentum effect in horizons shorter than one year where winner portfolios continued to outperform loser portfolios in the short-term. In longer horizons, however, this effect turned in the opposite direction and loser portfolios started to outperform winner portfolios. Returns to the contrarian strategy reached their highest level at 3 and 4-year horizons with average annual returns of 6.4 and 5.8 percent respectively. In addition, he found no evidence that loser portfolios were riskier than winner portfolios in the test periods. On the other hand, he mentioned a *small country effect*, which comes from the fact that winner-loser reversals are stronger in smaller markets. These findings can be especially important for international funds that invest in various financial markets.

Balvers, Wu and Gilliland (2000) used an interesting approach to investigate mean reversion. They built a parametric model in which price of a certain index is determined by its deviations from a reference index. With this model, not only they could test the hypothesis of mean reversion, they were also able to find the half-life of mean reversion, if there was any.

Their focus was on international markets, just like Richards (1997). They used yearly returns of 18 developed market indices as well as the world index from 1969 to 1996. They used both the World and U.S. indices as the reference index in their model. They applied standard Augmented Dickey-Fuller unit root test to their data to test the random walk hypothesis.

Initially, when they tested all the countries separately; they could not reject the random walk hypothesis for most of the countries. However, when they pooled the data for all 18 countries to gain more statistical power; they were able to reject the random walk hypothesis at the 1 percent significance level. They found a half-life of 3.5 years when they used the World index as reference and 3.1 years when they used the U.S. index as reference. These results demonstrate how much power one can gain by using a panel approach like Balvers et al. (2000).

Having established mean reversion in international markets, they developed a parametric investment strategy to take advantage of the predictability in stock prices. With a rolling regression model, they forecasted the next period's return with past returns for every period and calculated the returns of their strategy, which was to buy the index with the highest expected return for the next period and sell the index with the lowest expected return for the next period. With this strategy, they outperformed both buy-and-hold and random walk based trading strategies as well as the contrarian

strategy of DeBondt and Thaler (1985). Their zero net investment portfolio produced a 9% annual excess return on average.

Chaudhuri and Wu (2003) also reported some interesting results. They tested the null hypothesis of random walk for 17 emerging markets using Augmented Dickey-Fuller (ADF) and Phillips and Perron (PP) tests. Without pooling the data, they rejected the random walk hypothesis for 5 countries at the 5% significance level with the ADF test and 10% significance level with the PP test. When they performed panel-based tests like Balvers et al. (2000) did, they were able to reject the random walk hypothesis for all countries at conventional significance levels. They found the half-life of the mean reversion to be between 31 and 36 months. Finally, they used Seemingly Unrelated Regression (SUR) technique instead of OLS to increase the power even more and reject the null hypothesis once again. However, they estimated shorter half-lives with SUR; between 25 and 33 months, which suggested a stronger mean reversion than what they found with OLS.

Gropp (2004) looked for mean reversion in industry stock returns in the USA. He formed portfolios for all the industry groups in Fama and French (1988) except *other*. In order to do that, he used stocks that trade in AMEX and NASDAQ exchanges, as well as the NYSE. Using the methodology of Balvers et al. (2000), he pooled all the industry data together and carried out a panel-based test. Just like Balvers et al. (2000) and Chaudhuri and Wu (2003), he rejected the null hypothesis of random walk in favor of mean reversion for the NYSE data with an implied halflife of approximately seven years. NASDAQ and AMEX results were more surprising however, considering both of these exchanges were founded after World War II. Conflicting with the earlier findings of Kim et al. (1991) and McQueen (1992), Gropp (2004) claimed to reject the random walk hypothesis comfortably in

both exchanges. Estimated half-lives for these exchanges were shorter than NYSE, 5.5 years on average.

Given the earlier evidence supporting both return continuation and mean reversion, Balvers and Wu (2006) developed a trading model that combined momentum and contrarian strategies. They believed that although the mean reversion effect seemed stronger than momentum effect, a single asset could demonstrate both at different holding periods.

Their joint strategy outperformed separate momentum and contrarian strategies as well as a pure random walk strategy. They found that transaction costs erased less than half of the excess returns yet still left a sizable profit to the investor. They noted that "even if transaction costs preclude one from actually undertaking a momentum (or contrarian) strategy profitably, they do not imply that momentum (or mean reversion) disappears; it is still an anomalous feature of financial markets" (p.44).

Mukherji (2011) revisited the issue of mean reversion with a more recent data. He used the monthly returns of small and large-cap U.S. stock indices for the period 1926-2007. In order to surmount the small sample barrier, he utilized bootstrapping which involved pulling 10-year samples from the original data set 1000 times with replacement.

He used both the variance ratio method of Lo and MacKinlay (1988) and the regression method of Fama and French (1988) to investigate mean reversion. As predicted, he observed a greater tendency for mean reversion in small company stocks. He concluded that although it had weakened in the last decades, mean reversion was still present in the U.S. data; especially for small company stocks.

Spierdijk, Bikker and Hoak (2012) tested mean reversion across 18 OECD countries with an unusually large data set, covering the 1900-2009 period. With a data set longer than a century, they were able to reject the null hypothesis of random walk in favor of mean reversion for only 8 countries out of 18. By combining all the data together with a panel approach and assuming a constant speed of mean reversion across all countries, they found an average half-life of 18.5 years.

They also conducted a rolling-window test with 27 year-long windows, in which they allowed the speed of mean reversion to be different in each window. This time they found half-lives ranging from 2.0 years to 22.6 years. According to their results, speed of mean reversion tends to fluctuate a lot over time and it is usually higher in periods of economic instability such as Great Depression, World War II and the Oil Crisis of 1973. This study shows how much the results of such an analysis depend upon the choice of data sample.

Jegadeesh (1990) explored the possibility of seasonality in the predictability of stock prices. Initially, he found negative first and second order autocorrelation and positive higher order autocorrelation in monthly stock returns. However, when he separated the month January from the rest, a different pattern emerged. This time all the autocorrelation estimates up to lag 11 turned out to be negative, indicative of a seasonality effect.

Jegadeesh (1991) investigated this phenomenon further. He found evidence of mean reversion in the equally weighted index of U.S. stocks over the period 1926-1988 but discovered that the month January was solely responsible for this result.

He also examined the post-war sub-period alone and found no evidence of mean reversion after World War II. However, even for that period, he reported some degree of price reversals in January.

The findings of Jegadeesh (1990 and 1991) casted a shadow upon the results of Poterba and Summers (1989) and many others and called most of the evidence provided in favor of mean reversion into question.

There is also a large body of literature on momentum strategy as well. Although the main focus of this paper is mean reversion, adverting some of the articles on momentum would be helpful in presenting a more comprehensive literature review.

Jegadeesh and Titman (1993) tested different momentum-based strategies for the U.S. market over the 1965-1989 period. Their trading strategy, which was buying stocks that had a good performance in the past and selling stocks that had a poor performance in the past, generated positive returns for holding periods between 3 and 12 months. For example, the specific strategy of selecting stocks to buy and sell based on the performance in the past 6 months and holding that portfolio for 6 months into the future yielded a yearly excess return of 12.01%. However, these returns started to dissipate as the holding period increased beyond 1 year. These results contributed to the earlier evidence in favor of the general rule of *momentum in the short-term/mean reversion in the long-term*. Jegadeesh and Titman (1993) also argued that these abnormal returns cannot be attributed to risk since the average beta of the zero net cost portfolio of the above strategy was negative.

Carhart (1997) claimed "buying last year's top-decile mutual funds and selling last year's bottom-decile mutual funds yields a return of 8 percent per year" (p.79).

Rouwenhorst (1998) looked at 12 European countries over the period 1980-1995 to see if these markets displayed return continuation like the USA. He found that "an internationally diversified portfolio of past winners outperformed a portfolio of past losers by about 1 percent per month" (p.283)

Chan, Hameed and Tong (2000) implemented momentum strategies on international stock markets and found statistically and economically significant returns. They also claimed that return continuation was stronger if it followed an increase in trading volume.

Jegadeesh and Titman (2001) asserted that momentum effect continued to persist in U.S. market in the 1990s, more specifically in the eight years subsequent to Jegadeesh and Titman (1993).

Lewellen (2002) provided further evidence on momentum by investigating the role of industry, size and book-to-market factors. He showed that even the welldiversified size and book-to-market portfolios exhibited a considerable degree of momentum.

Patro and Wu (2004) tried to shed further light on momentum and examined 18 developed markets for the period 1979-1998. They rejected the random walk hypothesis with daily and weekly data for most of the countries. They also noted that these equity indices displayed significant return continuation in the short-term.

By analyzing 38 country indices, Bhojraj and Swaminathan (2006) inferred that after the portfolio formation, winners outperformed losers in the first 3 to 12 months, but underperformed losers in the subsequent 2 years.

#### 1.3 Data and methodology

In order to assess mean reversion, I follow the methodology of Poterba and Summers (1989) which relies upon variance ratios. This is mainly because it is a reliable method which was tested many times over the course of last 3 decades. It is also very intuitive, easy to understand and easy to interpret.

If the return series of a stock follows random walk, the variance of its kperiod return must be k times the variance of its 1-period return, assuming we use continuously compounded returns.

(1) 
$$R_k = r_1 + r_2 + \dots + r_k$$

Here,  $R_k$  is the k-period return and returns on the right hand side are 1-period returns. If we want to get the variance of  $R_k$ :

(2) 
$$Var(R_k) = \sum_{i=1}^k \sum_{j=1}^k Cov(r_i, r_j)$$

If the series follows a random walk, returns must be independent. In this case, the equation reduces to:

$$Var(R_k) = k \times \sigma^2$$

This proves that under the strict assumptions of random walk, the variance of holding period returns is proportional to the length of the holding period itself.

The variance ratio statistic is defined as:

(4) 
$$VR(k) = \frac{Var(r_t^k)}{Var(r_t^1)*k}$$

where  $r_t^k$  and  $r_t^1$  are k-period and 1-period returns respectively. From Equation (3), we can see that this statistic has to be in unity for a random walk. Poterba and Summers (1989) used a variation of this statistic in their analysis which is:

(5) 
$$VR(k) = \frac{Var(r_t^k)/k}{Var(r_t^{12})/12}$$

In other words, they took 12 months as the base period instead of 1 month. This method draws a clear line between short-term (less than 1 year) and long-term (more than 1 year) and makes it easier to make separate inferences about both.

Cochrane (1988) showed that variance ratios can also be expressed as a linear combination of sample autocorrelations:

(6) 
$$VR(k) \cong 1 + 2\sum_{j=1}^{k-1} \frac{(k-j)}{k} \hat{\rho}(j)$$

where  $\hat{\rho}(j)$  is sample autocorrelation at lag j. From this equation we can see that for k > 1, positive autocorrelations lead to a variance ratio bigger than 1 and negative autocorrelations lead to a variance ratio smaller than 1. If autocorrelations at all lags are 0, which is the case for a perfect random walk, the variance ratio has to be at unity. We can also see that as we go up to higher lags, weights of the autocorrelations decrease, which means lower lag autocorrelations have a larger impact on the variance ratio.

Looking at the variance ratios, we can have an idea about the overall behavior of our time series. If the variance ratios are significantly smaller than 1, that will lead us to infer that the time-series in question is mean-reverting. On the contrary; if variance ratios are larger than 1, it is implied that the series is a mean-averting one.

When Poterba and Summers (1989) applied Cochrane's (1988) results to their version of the variance ratio formula, they reached the formulation:

(7) 
$$VR(k) \cong 1 + 2\sum_{j=1}^{11} j\left(\frac{k-12}{12k}\right) \hat{\rho}(j) + 2\sum_{j=12}^{k-1} \frac{k-j}{k} \hat{\rho}(j)$$

The most important practical difference between (6) and (7) is that in the latter; for k < 12, variance ratios smaller than 1 imply positive autocorrelation and variance ratios larger than 1 imply negative autocorrelation, whereas it is the opposite for (6). However, for k > 12, it is the same for both formulas. In this version, absolute weights of the autocorrelations increase up to lag 11 and start to decrease after lag 13, forming an inverted V shape.

Kendall and Stuart (1976) showed that under the null hypothesis of serial independence;

(8) 
$$E[\hat{\rho}(j)] = -1/(T-j)$$

where  $\hat{\rho}(j)$  is the sample autocorrelation at lag *j* and *T* is the sample size. This creates a downward bias in variance ratios, pushing them below unity. To avoid this, Poterba and Summers (1989) made a bias correction by calculating the expected value of the variance ratio under the null hypothesis of serial independence and dividing the variance ratios estimated from the sample by this value.

(9) 
$$E[VR(k)] = \frac{12+5k}{6k} + \frac{2}{k} \sum_{j=1}^{k-1} \frac{T-k}{T-j} - \frac{1}{6} \sum_{j=1}^{11} \frac{T-12}{T-j}$$

Variance ratios reported in the results section has been corrected accordingly.

Although variance ratios convey very useful information, the null hypothesis of random walk should be statistically tested in order to reach a conclusive result. There are different ways of testing variance ratios. However, most of these methods rely heavily upon several assumptions made about the distribution of stock returns and variance ratios, which may or may not hold in real life. Hence, I use a more robust testing method proposed by Kim et al. (1991) which does not make any assumptions about the underlying distribution.

Kim et al. (1991) utilize a method called randomization which involves creating an empirical distribution of variance ratios by shuffling the data set 1000 times and calculating the variance ratios for each shuffle. By changing the order of returns, shuffling removes any autocorrelation present in the data set, making it as close to random walk as possible. This allows the null hypothesis of random walk or the null hypothesis that the variance ratio equals to 1, to be tested by comparing the actual variance ratio to the empirical distribution of variance ratios obtained with randomization. If the variance ratio lies below or above a certain percentile (which also serves as the significance level) of the empirical distribution, the null hypothesis can be rejected. If not, it means there is no statistical proof of mean reversion or aversion in the data set.

The data set consists of 16 MSCI (Morgan Stanley Capital International) value-weighted equity indices. Among these 16 indices, 6 of them are developed (USA, UK, France, Germany, Japan and Australia) and 6 of them are emerging (Brazil, Mexico, Turkey, South Africa, China and India) market indices. Obtaining a well-diversified set which includes major developed and emerging markets was the primary aim when choosing the countries.

Remaining 4 are World, Emerging Markets, ACWI (All Country World Index) and Frontier Markets indices. World index consists of 23 developed markets and Emerging Markets (hereafter EM) index consists of 24 emerging markets. ACWI index brings together the World and EM indices and covers a total of 47 countries. Lastly, Frontier Markets (hereafter FM) index is composed of 29 frontier markets. Table 1 shows the list of countries covered by each international index.

The available data set covers different time periods for different indices. For World, EM and ACWI indices, the data extends from 1988 to 2017. However, the range of the FM index is much shorter and it only covers the period 2002-2017.

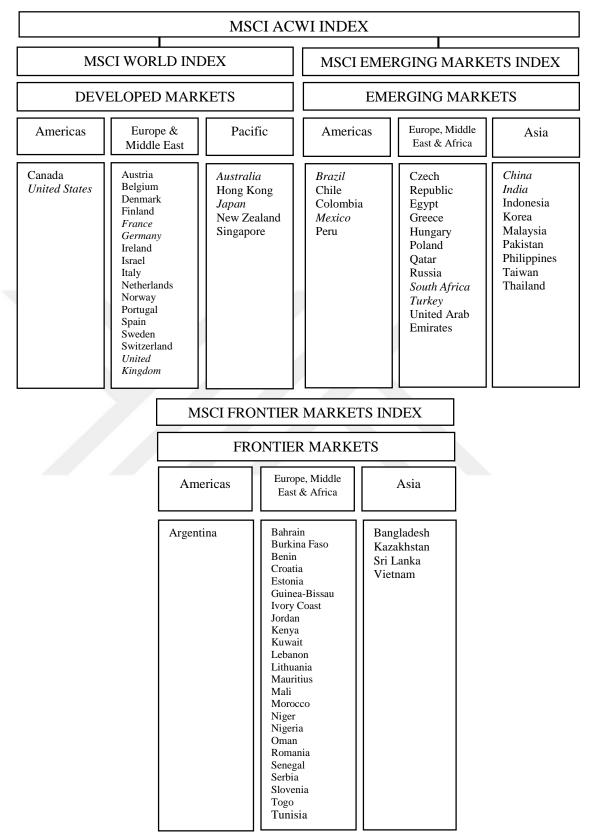
Developed market indices cover between 1970 and 2017. Brazil, Mexico and Turkey cover the period 1988-2017 and South Africa, China and India cover the period 1993-2017.

For the purpose of a detailed analysis; I calculate nominal, real and excess monthly returns on both total return (dividends reinvested) and price indices. All of the indices are denominated in U.S. dollars, rather than local currencies because it is very difficult to find reliable inflation and risk-free interest rate data for all of the listed countries. Furthermore, our preliminary tests on nominal returns denominated in local currencies yielded extreme results in some emerging markets, which we attributed to high levels of inflation in those countries.

The CPI data of U.S. Bureau of Labor Statistics and 1-month Treasury bill rates from WRDS (Wharton Research Data Services) have been used to calculate real and excess returns.

Furthermore, small-cap, mid-cap and large-cap World, EM, ACWI indices have also been extracted to investigate any possible size effect. These indices however, cover only between 1994 and 2017. FM index is not included here because MSCI does not offer large and mid-cap versions of this index.

Using MSCI data makes the entire analysis more reliable, standardized and consistent, since the same methodology has been used to calculate all of the indices. This brings all of the countries to even ground and makes them more comparable to each other.



#### Table 1. Breakdown of the MSCI Indices

This table reports the breakdown and composition of the MSCI international equity indices used in the study. Individual countries written with italic font are the ones included in the analysis.

## 1.4 Results

Table 2 reports the summary statistics of the data set. According to Table 2, emerging markets' average return of 0.68% is significantly higher than the developed markets' average return of 0.46%, which is to be expected since their volatility is also considerably higher with 6.68% standard deviation compared to the 4.25% of developed markets. We can see that the ACWI index is dominated by developed markets as its average return and standard deviation is almost identical to those of the World index. Surprisingly, frontier markets sit between developed and emerging markets in terms of both metrics. In fact, the average return and standard deviation of frontier markets are very close to their developed counterparts.

Index	Mean (% per month)	Standard Deviation (% per month)
World	0.46%	4.25%
EM	0.68%	6.68%
ACWI	0.45%	4.33%
Frontier	0.48%	5.32%
USA	0.56%	4.37%
UK	0.44%	6.05%
France	0.51%	6.43%
Germany	0.55%	6.31%
Japan	0.61%	5.93%
Australia	0.37%	7.11%
Brazil	0.84%	14.43%
Mexico	1.10%	8.66%
Turkey	0.40%	14.95%
South Africa	0.60%	7.74%
China	-0.04%	9.51%
India	0.60%	8.34%

Table 2	Summarv	Statistics
I a D C Z.	Summary	Statistics

This table reports summary statistics of the monthly nominal log-returns of all MSCI equity indices.

We can see the differences between developed and emerging markets in more detail in the second panel of Table 2. None of the emerging markets has a lower volatility than any of the developed markets, as expected. However, this trend does not fully extend to average returns. Some of the emerging markets have lower average returns than developed markets, such as Turkey with 0.4% average return and China with -0.04% average return. China's results are particularly interesting since it does not provide a positive return for a substantial amount of risk. Moreover, Australia has the lowest average monthly return among all the developed markets with 0.37%, despite being the most volatile with 7.11% standard deviation.

On the other hand, these results are obtained with returns that are denominated in USD rather than local currencies and standard deviation is not the only measure of risk, nor the most accurate one. Therefore, Table 1 only gives us a rough idea about the characteristics of our data set and serves as a starting point for our analysis.

# 1.4.1 International results

Figures 1, 2 and 3 show the calculated variance ratios for World, EM, ACWI and FM indices for nominal, real and excess monthly log-returns, respectively. There aren't any drastic differences between the three plots and all four indices exhibit some degree of mean reversion in all of them. World and ACWI indices seem to be going hand in hand, which further proves that companies in the former dominate the ACWI index.

Interestingly, World index shows stronger mean reversion than the EM index in all return types for holding periods longer than 5 years. However, for holding periods shorter than 5 years, EM index is below the others in terms of variance ratios, by a big margin.

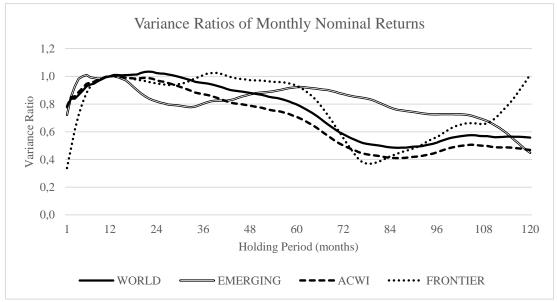


Figure 1. Variance ratios of the monthly nominal log-returns of MSCI World, EM, ACWI and FM indices from 1 month to 120 months

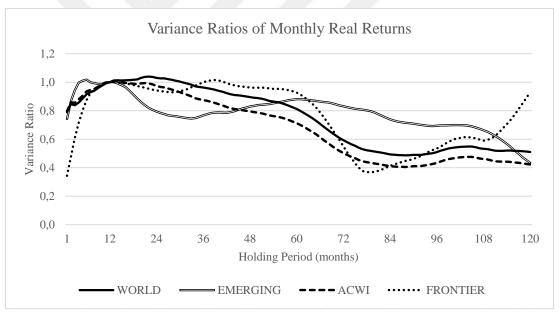


Figure 2. Variance ratios of the monthly real log-returns of MSCI World, EM, ACWI and FM indices from 1 month to 120 months

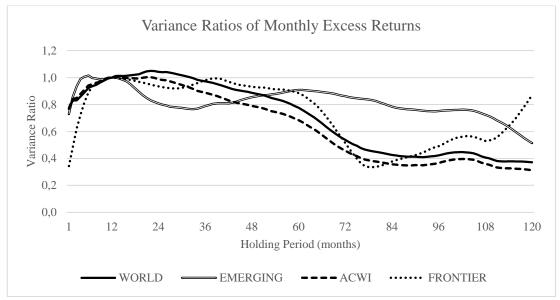


Figure 3. Variance ratios of the monthly excess log-returns of MSCI World, EM, ACWI and FM indices from 1 month to 120 months

FM index stays close to World and ACWI indices until around 80 months and then jumps ahead and starts to approach unity which is the hallmark of random walk. Moreover, there are signs of mean aversion, or momentum, in holding periods up to 1 year in all four indices, with FM index being the strongest.

All in all, there doesn't seem to be any significant difference in variance ratio patterns between different return types and further investigation is needed to gain more insight into this matter.

Table 3 reports the actual variance ratios for several holding periods, as well as the p-values obtained with randomization for all return types and all four indices. Since the p-values are obtained from an empirical distribution via randomization, it is free from the shortcomings of assuming a standard distribution like normal.

		1 month	6 months	24 months	36 months	48 months	72 months	96 months	120 months
PANEL A NOMINA	A: AL RETURNS								
WORLD	Variance Ratio	0.777	0.928	1.024	0.953	0.881	0.582	0.522	0.558
	P-value	0.088	0.223	0.572	0.466	0.415	0.213	0.250	0.367
EM	Variance Ratio	0.722	1.010	0.819	0.803	0.870	0.873	0.726	0.449
	P-value	0.038	0.487	0.168	0.278	0.437	0.529	0.464	0.265
ACWI	Variance Ratio	0.782	0.946	0.971	0.871	0.788	0.502	0.451	0.469
	P-value	0.090	0.289	0.455	0.354	0.325	0.144	0.168	0.266
FM	Variance Ratio	0.338	0.880	0.949	1.008	0.973	0.555	0.562	1.013
	P-value	0.000	0.173	0.489	0.588	0.605	0.355	0.456	0.673
PANEL E REAL RE									
WORLD	Variance Ratio	0.789	0.916	1.030	0.962	0.895	0.593	0.509	0.510
	P-value	0.129	0.232	0.602	0.546	0.520	0.330	0.345	0.421
EM	Variance Ratio	0.742	1.018	0.794	0.767	0.829	0.831	0.694	0.434
	P-value	0.060	0.528	0.113	0.204	0.373	0.481	0.426	0.248
ACWI	Variance Ratio	0.796	0.935	0.974	0.874	0.794	0.503	0.433	0.422
	P-value	0.114	0.243	0.479	0.358	0.328	0.162	0.188	0.239
FM	Variance Ratio	0.343	0.873	0.942	0.998	0.962	0.550	0.533	0.931
	P-value	0.000	0.084	0.388	0.539	0.525	0.182	0.254	0.637
PANEL C EXCESS	C: RETURNS								
WORLD	Variance Ratio	0.767	0.923	1.043	0.972	0.887	0.538	0.422	0.371
	P-value	0.076	0.198	0.633	0.513	0.447	0.173	0.143	0.141
EM	Variance Ratio	0.728	1.016	0.808	0.789	0.855	0.862	0.752	0.514
	P-value	0.038	0.540	0.128	0.230	0.402	0.496	0.455	0.317
ACWI	Variance Ratio	0.773	0.941	0.988	0.888	0.791	0.457	0.366	0.312
	P-value	0.075	0.269	0.487	0.367	0.323	0.107	0.100	0.105
FM	Variance Ratio	0.343	0.887	0.935	0.980	0.932	0.515	0.491	0.870
	P-value	0.000	0.163	0.487	0.635	0.621	0.362	0.367	0.567

Table 3. Randomization Results of the International Indices

This table reports variance ratios and their respective p-values (obtained through randomization) of monthly nominal, real and excess log-returns of MSCI World, EM, ACWI and FM equity indices for several holding periods.

According to Table 3, although there was an obvious trend of mean reversion in Figures 1, 2 and 3; we cannot statistically reject the null hypothesis of random walk with conventional significance levels for holding periods larger than 1 year. This is true for all indices and for all return types. However, we can reject the null hypothesis of random walk for EM and FM indices for the 1-month holding period with a significance level of 10% on all return types. Same is true for World and ACWI indices too, except for the real returns. Sheer size of the p-values show that mean aversion is the strongest in frontier markets and weakest in developed markets.

In summary, we observe statistically significant results signaling strong momentum effect in emerging and frontier markets and moderate momentum effect in developed markets in very short holding periods. But we cannot find any substantial statistical evidence of mean reversion for any holding period in any of the indices.

In order to see if the results are swayed by dividend payments in equity markets, we conduct tests with index values that include only capital gains against those with total returns including dividend payments. Figure 4 and Table 4 compare the variance ratios of returns with and without dividend adjustments. This comparison is made for all four indices and with nominal, real and excess monthly log-returns; but only the first one is reported as the differences in results were negligible. Therefore, Panel A of Table 3 and Panel A of Table 4 show the same exact numbers.

Figure 4 shows almost identical results for price and total return indices. Only the FM index displays a visible, though not significant enough, difference between

the two. Table 4 consolidates this result with p-values that vary slightly between the price and total return indices within the same category.

Hence, it can be argued that dividend payments do not constitute a significant difference for the purpose of this analysis.

In order to test the impact of size on mean reversion, we conduct tests with size based indices. Figure 5 and Table 5 compare the variance ratios of large-cap mid-cap and small-cap versions of World, EM, and ACWI indices. FM index has been excluded from this part of the analysis because it only had the small-cap index. Again, this analysis is carried out for nominal, real and excess monthly log-returns separately. Since the results are very similar, only the results of nominal returns are reported here for brevity.

Figure 5 shows that World and ACWI indices are still very similar in terms of size-based comparison. Between 12 and 60 months, small-cap index shows the strongest mean reversion, followed by mid-cap and large-cap indices respectively. Small and mid-cap indices are very close to each other, especially for ACWI, whereas the large-cap index is further separated from the other two, even going beyond unity at certain holding periods. However, second halves of these plots are much more complex and hard to interpret.

EM indices seem to be mean-reverting until around 30 months, after which variance ratios start to increase. In terms of the degree of mean reversion, large-cap index is in the lead while the other two are closer to unity, or random walk. Variance ratios start to decrease again after 8 years, which is a very long holding period.

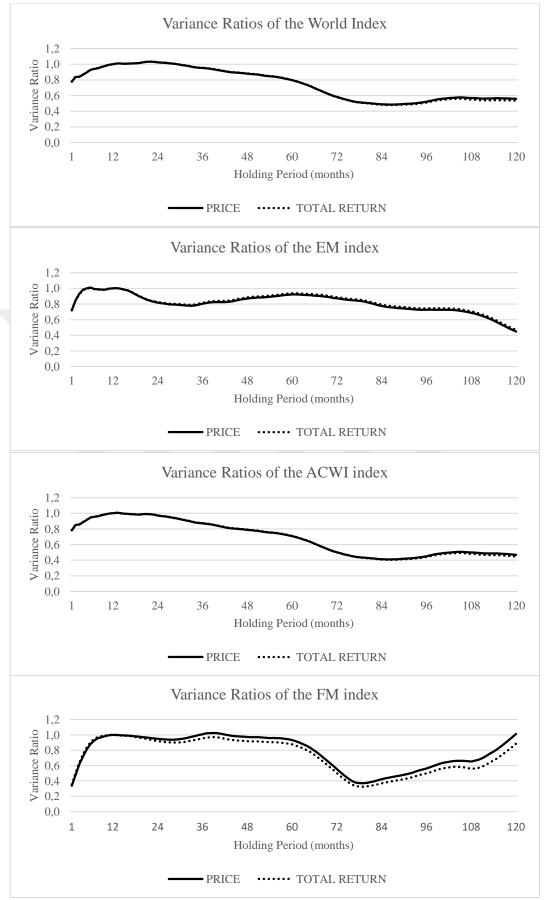


Figure 4. Variance ratios of the monthly nominal log-returns of both the price and total return indices of MSCI World, EM, ACWI and FM indices from 1 month to 120 months

		1 month	6 months	24 months	36 months	48 months	72 months	96 months	120 months
PANEL A PRICE IN									
WORLD	Variance Ratio	0.777	0.928	1.024	0.953	0.881	0.582	0.522	0.558
	P-value	0.088	0.223	0.572	0.466	0.415	0.213	0.250	0.367
EM	Variance Ratio	0.722	1.010	0.819	0.803	0.870	0.873	0.726	0.449
	P-value	0.038	0.487	0.168	0.278	0.437	0.529	0.464	0.265
ACWI	Variance Ratio	0.782	0.946	0.971	0.871	0.788	0.502	0.451	0.469
ACWI	P-value	0.782	0.289	0.455	0.354	0.325	0.302	0.451	0.409
				0.040	1				
FM	Variance Ratio	0.338	0.880	0.949	1.008	0.973	0.555	0.562	1.013
_	P-value	0.000	0.173	0.489	0.588	0.605	0.355	0.456	0.673
PANEL B									
TOTAL R	ETURN INDEX								
WORLD	Variance Ratio	0.780	0.931	1.025	0.956	0.884	0.581	0.511	0.534
	P-value	0.088	0.222	0.606	0.489	0.444	0.224	0.255	0.335
EM	Variance Ratio	0.718	1.005	0.826	0.816	0.885	0.889	0.745	0.470
LIVI	P-value	0.041	0.467	0.320	0.290	0.885	0.533	0.467	0.470
ACWI	Variance Ratio	0.785	0.948	0.972	0.875	0.792	0.501	0.441	0.448
	P-value	0.095	0.305	0.445	0.332	0.306	0.154	0.179	0.254
FM	Variance Ratio	0.353	0.904	0.920	0.954	0.918	0.503	0.498	0.884

#### Table 4. Index-Based Comparison of the Randomization Results

This table reports variance ratios and their respective p-values (obtained through randomization) of the monthly nominal log-returns of both the price and total return indices of MSCI World, EM, ACWI and FM equity indices. Price index includes only capital gains whereas total return index takes dividends into account as well.

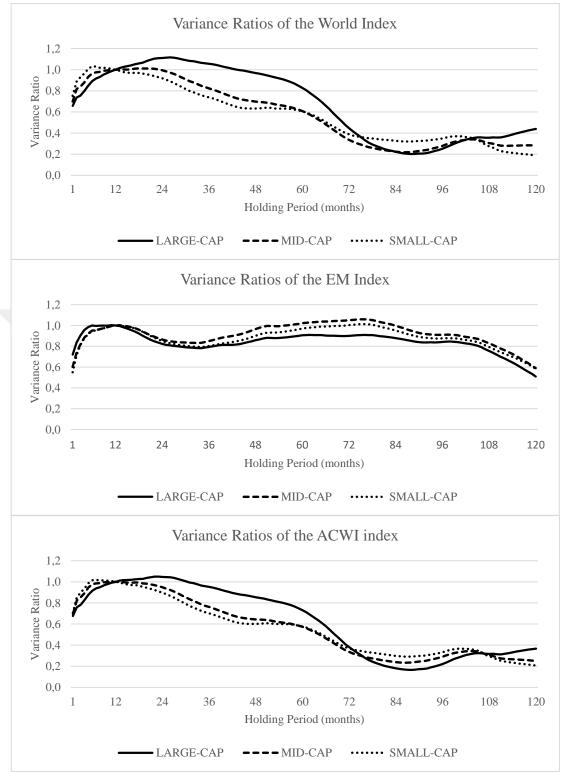


Figure 5. Variance ratios of the monthly nominal log-returns of large-cap, mid-cap and small-cap versions of MSCI World, EM and ACWI indices from 1 month to 120 months

		1 month	6 months	24 months	36 months	48 months	72 months	96 months	120 months
PANEL A: LARGE-C	AP INDEX								
WORLD	Variance Ratio	0.658	0.892	1.112	1.057	0.969	0.446	0.251	0.439
	P-value	0.014	0.134	0.779	0.645	0.561	0.150	0.054	0.317
EM	Variance Ratio	0.720	1.000	0.823	0.794	0.858	0.901	0.841	0.510
	P-value	0.067	0.488	0.199	0.302	0.447	0.548	0.556	0.377
ACWI	Variance Ratio	0.676	0.916	1.046	0.953	0.854	0.380	0.221	0.366
	P-value	0.031	0.199	0.648	0.514	0.458	0.101	0.033	0.219
PANEL B: MID-CAP									
WORLD	Variance Ratio	0.698	0.965	0.994	0.824	0.699	0.336	0.278	0.283
	P-value	0.044	0.350	0.545	0.324	0.259	0.063	0.061	0.118
EM	Variance Ratio	0.602	0.952	0.866	0.849	0.965	1.051	0.910	0.589
	P-value	0.008	0.358	0.260	0.368	0.554	0.653	0.623	0.457
ACWI	Variance Ratio	0.698	0.977	0.948	0.761	0.644	0.335	0.290	0.250
	P-value	0.049	0.395	0.435	0.243	0.204	0.055	0.058	0.065
PANEL C: SMALL-C	AP INDEX								
WORLD	Variance Ratio	0.753	1.033	0.920	0.739	0.633	0.388	0.349	0.189
	P-value	0.083	0.601	0.367	0.213	0.188	0.083	0.125	0.028
EM	Variance Ratio	0.549	0.947	0.850	0.802	0.901	1.003	0.875	0.584
	P-value	0.002	0.289	0.237	0.326	0.493	0.645	0.613	0.463
ACWI	Variance Ratio	0.706	1.015	0.897	0.701	0.601	0.368	0.332	0.206
	P-value	0.043	0.504	0.338	0.195	0.176	0.090	0.124	0.038

Table 5. Size-Based Comparison of the Randomization Results

This table reports variance ratios and their respective p-values (obtained through randomization) of monthly nominal log-returns of the large-cap, mid-cap and small-cap versions of the World, EM and ACWI indices for several holding periods.

In holding periods shorter than 1 year, all the indices exhibit mean aversion, or momentum; but the sorting is different compared to long-term. For World and ACWI; large-cap index has the strongest momentum, followed by mid-cap and small-cap indices; which means size has an opposite effect here. In the long-term, as the firm size gets bigger; return series approaches random walk whereas in the shortterm larger size leads to stronger momentum.

On the other hand, large-cap index of the emerging markets shows the weakest momentum in the short-term compared to mid and small-cap indices which have almost identical variance ratios.

In summary, in the long-term, smaller size seems to be resulting in stronger mean reversion for developed markets and somewhat weaker mean reversion for emerging markets. On the contary, in the short-term, smaller size leads to weaker momentum for developed markets and stronger momentum for emerging markets.

Table 5 confirms these results with numerical p-values although they do not allow for rejection of the random walk hypothesis except for very short holding periods.

# 1.4.2 National results

Since, international indices did not yield conclusive statistical results on the existence of mean reversion, we carried on testing the individual countries to assess if less diversified single country equity indices display any significant violation of random walk. Figure 6 plots the variance ratios of the monthly nominal log-returns of all 12 national indices. According to Figure 6, the selection of countries seems to

be well-diversified, ranging from mean-averting countries to countries that are close to random walk and to mean-reverting countries.

Among the developed markets, Japan stands out as the only country which shows mean-averting behavior, especially for holding periods more than 6 years. Its variance ratios even go beyond 1.6 at the higher end. USA's variance ratios fluctuate around unity, which is an indication that it follows more or less a random walk. The rest; namely UK, France, Germany and Australia display various degrees of mean reversion. Australia seems to be the strongest in this regard, followed by Germany. Variance ratios of UK and France are very close and they show weaker tendencies for mean reversion compared to Australia and Germany.

Emerging markets are more dispersed compared to developed markets, with a wider range of variance ratios and more complicated trends. Turkey is undoubtedly the most mean-reverting country here, with a sizable difference in variance ratios between her and others. Its variance ratios go even below 0.2 at certain holding periods. Although not as dramatic as Turkey, India also exhibits mean-reverting behavior, especially between 12 and 48 months.

Brazil displays varying behavior throughout the range of holding periods. Its variance ratios are on a downward slope between 12 and 24 months, which is a sign of mean reversion, but they start to increase after that point, reaching almost unity at 60 months. Then they start to decrease again and continue to do so until the end of the range. South Africa seems to follow a random walk in holding periods between 12 and 60 months but its variance ratios start to decline after that point, pushing it towards the mean reversion zone.

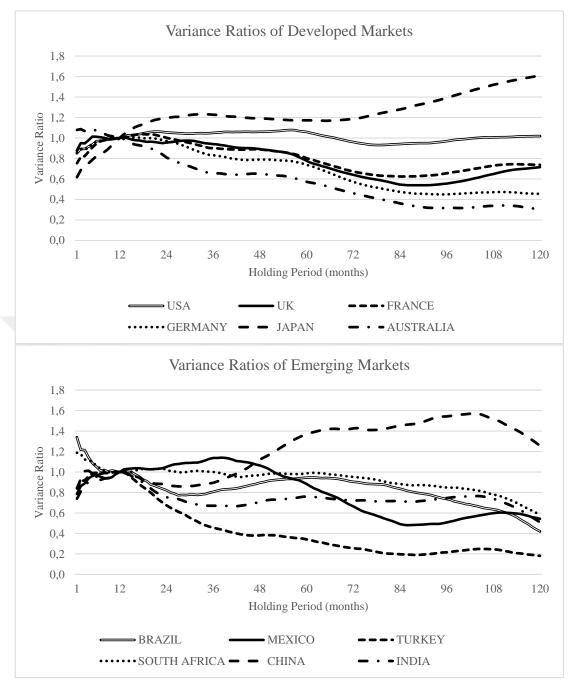


Figure 6. Variance ratios of the monthly nominal log-returns of 6 MSCI developed market indices (USA, UK, France, Germany, Japan and Australia) and 6 MSCI emerging market indices (Brazil, Mexico, Turkey, South Africa, China and India) from 1 month to 120 months

Mexico and China have the most interesting results among the emerging markets. Mexico seems to be mean-averting between 12 and 48 months and meanreverting for longer holding periods, forming an S shape. China on the other hand, starts out as a mean-reverting country after 12 months but surpasses unity at around 40 months and becomes a mean-averting country. A quite strong one as well, with variance ratios reaching almost 1.6.

In the short-term, Brazil show a quite strong tendency for mean reversion and South Africa follows it with a slightly weaker one, while the rest seem to exhibit mean-averting behavior.

Furthermore, all of the developed markets show mean aversion in the shortterm except Australia, which is slightly mean-reverting. Notably, Japan's variance ratios are quite smaller than the others.

Table 6 displays the actual variance ratios and their p-values of the time series in Figure 6 for certain holding periods. When the statistical evidence is taken into consideration, there is almost no significant statistical proof to most of the inferences made from Figure 6. However, there are some exceptions to this: Turkey and Australia have substantial proof of mean reversion in several holding periods. Turkey's p-values are especially small, proving its mean-reverting behavior beyond any reasonable doubt. Japan's mean aversion also has some merit, since the null hypothesis of random walk can be rejected in holding periods up to 24 months.

Aside from that, there is also proof that France violates random walk in very short holding periods but that is as far as it goes with a significance level of 5%. If a significance level of 10% is used, this list can be extended to a few more countries and holding periods.

Figure 7 and Table 7 show the results of the same analysis done with real returns. For the most part, they are very similar to the results shown in Figure 6 and Table 6. USA constitutes the biggest difference between the two parts. It has gone from being the most prominent random walk among all 12 markets to having the highest variance ratios among developed markets. However, it is still a random walk from a purely statistical perspective since none of its p-values exceed 5.

Moreover, Australia's mean reversion is toned down a little bit in real returns as its variance ratios and p-values are higher. In result, null hypothesis of random walk can be rejected for fewer holding periods. In addition to that, Japan's mean aversion is also weaker compared to nominal returns but we couldn't reject the null hypothesis to begin with, so it is not a major concern. The impact of inflation is visible in certain countries, yet not to the extent to change the statistical significance of the results.

Lastly, Figure 8 and Table 8 show the results of the same analysis done with excess returns. When excess returns are used, variance ratios of USA decline a significant amount and approach to their initial levels. But it is still a contender for the most mean-averting developed market.

Japan's mean aversion is further weakened and its variance ratios go below unity in holding periods between 5 and 8 years. It now seems like a market which is very close to random walk and whose variance ratios fluctuate around unity.

UK has become much more mean-reverting and it has the lowest variance ratios among all developed markets when excess returns are used. This is reflected in its p-values which allow us to reject the null hypothesis of random walk in holding periods longer 5 years.

		1 month	6 months	24 months	36 months	48 months	72 months	96 months	120 months
PANEL A: DEVELOPED	MARKETS								
USA	Variance Ratio	0.850	0.969	1.055	1.051	1.060	0.961	0.968	1.019
	P-value	0.123	0.340	0.708	0.666	0.657	0.541	0.561	0.614
UK	Variance Ratio	0.878	1.013	0.954	0.941	0.892	0.640	0.555	0.715
	P-value	0.177	0.523	0.402	0.447	0.409	0.185	0.170	0.390
FRANCE	Variance Ratio	0.754	0.942	1.002	0.901	0.887	0.674	0.655	0.738
	P-value	0.029	0.217	0.534	0.344	0.383	0.193	0.250	0.385
GERMANY	Variance Ratio	0.860	0.962	0.970	0.831	0.790	0.572	0.449	0.454
	P-value	0.154	0.310	0.437	0.220	0.249	0.104	0.076	0.132
JAPAN	Variance Ratio	0.618	0.820	1.196	1.228	1.192	1.186	1.391	1.610
	P-value	0.001	0.002	0.955	0.891	0.799	0.749	0.839	0.897
AUSTRALIA	Variance Ratio	1.079	1.073	0.810	0.654	0.651	0.460	0.316	0.305
	P-value	0.662	0.779	0.065	0.031	0.088	0.049	0.019	0.034
PANEL B: EMERGING N	<b>MARKETS</b>								
BRAZIL	Variance Ratio	1.340	1.051	0.822	0.806	0.895	0.904	0.737	0.420
	P-value	0.912	0.680	0.150	0.256	0.453	0.549	0.465	0.217
MEXICO	Variance Ratio	0.785	0.971	1.049	1.136	1.063	0.664	0.507	0.542
	P-value	0.092	0.384	0.630	0.713	0.614	0.290	0.232	0.348
TURKEY	Variance Ratio	0.840	0.992	0.675	0.456	0.381	0.256	0.216	0.182
	P-value	0.155	0.445	0.021	0.007	0.009	0.007	0.007	0.007
SOUTH	Variance Ratio	1.188	1.059	1.025	1.001	0.969	0.952	0.848	0.581
AFRICA	P-value	0.737	0.657	0.593	0.558	0.551	0.607	0.586	0.415
CHINA	Variance Ratio	0.840	0.951	0.876	0.893	1.120	1.425	1.543	1.257
	P-value	0.184	0.321	0.284	0.423	0.698	0.842	0.873	0.815
INDIA	Variance Ratio	0.739	0.954	0.753	0.671	0.707	0.720	0.745	0.510
	P-value	0.069	0.338	0.103	0.143	0.277	0.411	0.516	0.383

Table 6. Randomization Results of the National Indices (Nominal Returns)

This table reports variance ratios and their respective p-values (obtained through randomization) of monthly nominal log-returns of 12 MSCI equity indices for several holding periods. Panel A reports the results for developed market indices and Panel B reports the results for emerging market indices.

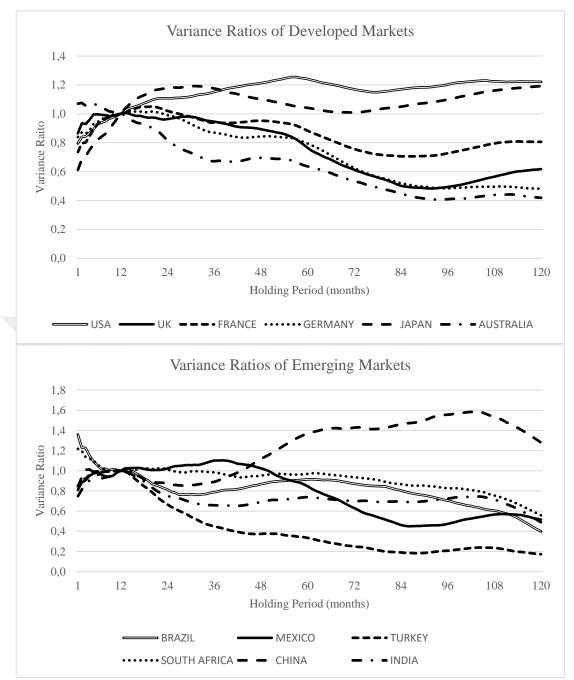


Figure 7. Variance ratios of the monthly real log-returns of 6 MSCI developed market indices (USA, UK, France, Germany, Japan and Australia) and 6 MSCI emerging market indices (Brazil, Mexico, Turkey, South Africa, China and India) from 1 month to 120 months

		1 month	6 months	24 months	36 months	48 months	72 months	96 months	120 months
PANEL A: DEVELOPED	MARKETS								
USA	Variance Ratio	0.795	0.929	1.107	1.152	1.212	1.170	1.202	1.221
	P-value	0.063	0.190	0.804	0.778	0.797	0.706	0.722	0.742
UK	Variance Ratio	0.865	0.997	0.963	0.945	0.894	0.613	0.493	0.618
	P-value	0.158	0.451	0.407	0.450	0.401	0.142	0.105	0.267
FRANCE	Variance Ratio	0.735	0.926	1.021	0.939	0.952	0.757	0.726	0.806
	P-value	0.017	0.152	0.578	0.416	0.483	0.284	0.325	0.442
GERMANY	Variance Ratio	0.839	0.943	0.991	0.869	0.843	0.625	0.483	0.481
	P-value	0.118	0.233	0.510	0.289	0.326	0.173	0.108	0.163
JAPAN	Variance Ratio	0.613	0.819	1.178	1.176	1.102	1.010	1.099	1.191
JAFAN	P-value	0.013	0.019	0.933	0.814	0.692	0.587	0.676	0.727
		1.050	1.050	0.015	0.671	0.005	0.500	0.400	0.410
AUSTRALIA	Variance Ratio P-value	1.070 0.661	1.059 0.738	0.815 0.076	0.671 0.049	0.695 0.130	0.533 0.086	0.408 0.057	0.419 0.101
DANEL D.									
PANEL B: EMERGING N	MARKETS								
BRAZIL	Variance Ratio	1.359	1.055	0.814	0.788	0.868	0.871	0.705	0.399
	P-value	0.916	0.648	0.128	0.251	0.424	0.505	0.418	0.200
MEXICO	Variance Ratio	0.809	0.981	1.023	1.101	1.025	0.630	0.474	0.516
	P-value	0.124	0.437	0.562	0.682	0.601	0.248	0.191	0.306
TURKEY	Variance Ratio	0.847	0.995	0.666	0.449	0.374	0.249	0.209	0.173
IUKKEI	P-value	0.847	0.995	0.000	0.449	0.374	0.249	0.209	0.173
COLUMN	W'D'	1 010	1.0.02	1.015	0.004	0.051	0.026	0.020	0.550
SOUTH AFRICA	Variance Ratio P-value	1.218 0.769	1.062 0.664	1.015 0.572	0.984 0.532	0.951 0.547	0.936 0.596	0.828 0.571	0.559 0.422
		6 o :=	0.051	0.055	0.055	=			
CHINA	Variance Ratio P-value	0.847 0.234	0.951 0.326	0.873 0.239	0.890 0.384	1.117 0.683	1.427 0.827	1.555 0.852	1.280 0.789
		-			-				
INDIA	Variance Ratio P-value	0.748 0.067	0.954 0.308	0.748 0.098	0.659 0.125	0.689 0.266	0.699 0.388	0.725 0.495	0.490 0.353
TT1 : / 1 1	orts variance ratio								

Table 7. Randomization Results of the National Indices (Real Returns)

This table reports variance ratios and their respective p-values (obtained through randomization) of monthly real log-returns of 12 MSCI equity indices for several holding periods. Panel A reports the results for developed market indices and Panel B reports the results for emerging market indices.

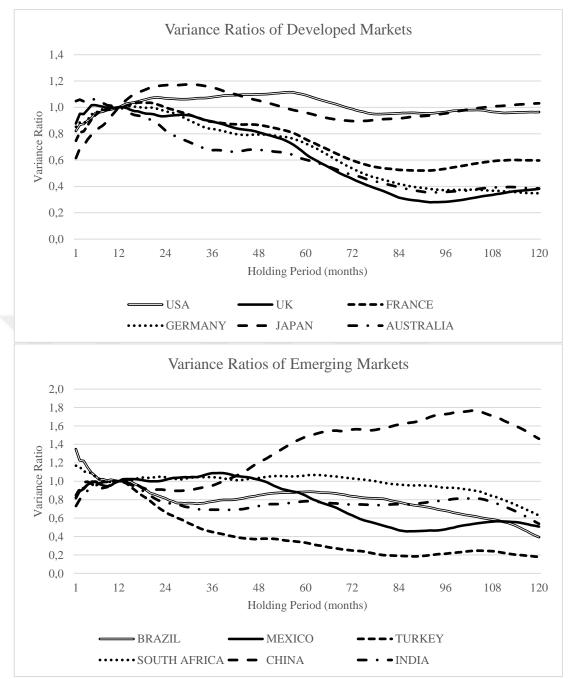


Figure 8. Variance ratios of the monthly excess log-returns of 6 MSCI developed market indices (USA, UK, France, Germany, Japan and Australia) and 6 MSCI emerging market indices (Brazil, Mexico, Turkey, South Africa, China and India) from 1 month to 120 months

		1 month	6 months	24 months	36 months	48 months	72 months	96 months	120 months
PANEL A: DEVELOPED	MARKETS								
USA	Variance Ratio	0.821	0.957	1.069	1.078	1.099	0.988	0.965	0.965
	P-value	0.107	0.302	0.714	0.686	0.686	0.556	0.556	0.582
UK	Variance Ratio	0.878	1.017	0.933	0.893	0.810	0.457	0.283	0.381
	P-value	0.183	0.565	0.328	0.345	0.278	0.041	0.011	0.087
FRANCE	Variance Ratio	0.746	0.942	1.000	0.891	0.866	0.599	0.534	0.597
	P-value	0.016	0.193	0.540	0.348	0.368	0.133	0.151	0.243
GERMANY	Variance Ratio	0.852	0.960	0.971	0.836	0.794	0.536	0.371	0.347
<b>GERMAN</b> I	P-value	0.832	0.900	0.971	0.830	0.794	0.086	0.035	0.052
JAPAN	Variance Ratio	0.615	0.824	1.169	1.151	1.052	0.896	0.955	1.031
	P-value	0.002	0.005	0.922	0.792	0.621	0.456	0.552	0.620
AUSTRALIA	Variance Ratio	1.048	1.057	0.825	0.674	0.679	0.488	0.358	0.385
	P-value	0.574	0.709	0.087	0.052	0.109	0.047	0.029	0.078
PANEL B:									
EMERGING N	MARKETS								
BRAZIL	Variance Ratio	1.349	1.056	0.815	0.778	0.847	0.837	0.680	0.393
	P-value	0.913	0.681	0.113	0.198	0.364	0.437	0.380	0.175
MEXICO	Variance Ratio	0.815	0.992	1.014	1.088	1.016	0.629	0.479	0.508
MLACO	P-value	0.144	0.462	0.579	0.661	0.574	0.257	0.205	0.303
TURKEY	Variance Ratio	0.846	0.997	0.665	0.449	0.374	0.248	0.214	0.180
	P-value	0.169	0.443	0.018	0.005	0.010	0.009	0.008	0.012
SOUTH	Variance Ratio	1.168	1.049	1.043	1.044	1.035	1.030	0.929	0.629
AFRICA	P-value	0.716	0.630	0.648	0.634	0.630	0.641	0.626	0.458
CHINA	Variance Ratio	0.822	0.940	0.905	0.952	1.203	1.560	1.728	1.460
	P-value	0.162	0.292	0.322	0.477	0.746	0.872	0.899	0.856
	Variana D. (	0.721	0.040	0.765	0.001	0.720	0.744	0.705	0.520
INDIA	Variance Ratio P-value	0.731 0.058	0.949 0.301	0.765 0.106	0.691 0.158	0.730 0.308	0.744 0.444	0.795 0.565	0.539 0.385
This table rer	orts variance rat								

Table 8. Randomization Results of the National Indices (Excess Returns)

This table reports variance ratios and their respective p-values (obtained through randomization) of monthly excess log-returns of 12 MSCI equity indices for several holding periods. Panel A reports the results for developed market indices and Panel B reports the results for emerging market indices.

A comparison among nominal, real and excess return results yields that developed markets are visibly more affected by the choice than the emerging markets. This can be seen in Table 9 which reports bigger absolute average percent changes in every return type for the developed markets.

A reasonable explanation for this might be the fact that we are using dollar denominated prices for all markets. Inflation and interest rates are usually much higher in emerging countries and when dollar denominated prices are used instead of prices denominated in local currencies, this might cause the effects of inflation and interest rates to be understated.

Another thing to note is the sign differences. Changing the return type has inverse effects for developed and emerging markets no matter which return types are used. For example, switching from nominal returns to real returns causes the variance ratios to increase for developed markets while it causes the variance ratios to decrease for emerging markets. Deciding on which return type to use for this type of analysis requires a more detailed investigation and it partly depends on investor profile and preference. It could be an excellent focal point for a future research paper on this topic.

	REAL-NOMINAL Avg. % Change	EXCESS-NOMINAL Avg. % Change	EXCESS-REAL Avg. % Change
Developed Markets			
USA	14.63%	0.69%	-11.55%
UK	-4.43%	-22.60%	-19.86%
FRANCE	7.23%	-8.42%	-14.18%
GERMANY	5.47%	-7.06%	-11.68%
JAPAN	-11.59%	-17.55%	-7.26%
AUSTRALIA	13.78%	6.93%	-5.51%
Average	4.18%	-8.00%	-11.68%
Emerging Markets	-		
BRAZIL	-2.78%	-4.96%	-2.26%
MEXICO	-3.78%	-4.02%	-0.24%
TURKEY	-2.24%	-1.72%	0.55%
SOUTH AFRICA	-1.65%	5.87%	3.32%
CHINA	-1.65%	5.87%	3.32%
INDIA	-2.12%	3.33%	5.60%
Average	-2.37%	0.73%	1.71%
Total Average	0.90%	-3.64%	-4.98%

Table 9. Comparison of the Return Types

This table reports the average percent differences in variance ratios (1 month through 120 months) between different return types for all 12 countries in the data set.

## 1.5 Conclusion

This study was performed to shed some light on the times series behavior of international equity indices and to see if they show any signs of anomalies such as mean reversion or aversion. While all of the indices appear to have some degree of one or the other, most of them fail to show strong statistical significance to reject random walk hypothesis. This could be related to power of the test, which can be improved with more data; or simply the choice of data sample, as pointed out by Spierdijk et al. (2012).

There was no evidence against the random walk for the international indices in the long-term, but there was evidence that EM and FM indices exhibit momentum in the short-term. Among the national indices, Turkey and Australia were proven to have mean reversion in the long-term while Japan was proven to have mean aversion in the short-term, although it is somewhat weakened when real or excess returns are used. Australia's mean reversion was also much stronger when nominal returns were used.

Furthermore, there was evidence of mean aversion for France in the shortterm and mean reversion for UK in the long-term, although the latter was only in excess returns.

National indices yielded more extreme p-values that allowed us to reject the random walk hypothesis for more indices and at more holding periods. Since international indices consist of companies from different countries, they contain less country-specific risk than national indices. This aggregate nature of the international indices pushes them more towards random walk and makes it harder to detect predictability in their return series.

The effect of dividends on the variance ratios was deemed negligible by the price-total return index comparison.

On the other hand, size seems to be an important factor as there were significant differences in variance ratios between large, mid and small-cap equity indices. However, its impact varies quite a bit with respect to holding period and the market type. In the long-term, smaller size seems to push the variance ratios downwards for developed markets and upwards for emerging markets whereas in the short-term, it weakens the effect of momentum for developed markets and amplifies it for emerging markets.

Finally, changing the return type has a greater impact on developed markets than it has on emerging markets. This could be due to the high levels of inflation and

interest rates in the emerging markets. Aside from the magnitude, the outcome of changing the return type is also different for developed and emerging markets. Going from one return type to the other, variance ratios always go opposite ways; upwards for developed markets and downwards for emerging markets or vice versa.

Aside from the return predictability, variance ratios also convey important information about the riskiness of stock markets in the long run. Having variance ratios smaller than 1 means the stock market in question is less volatile in the long run because long-term variances are smaller than short-term variances on a per-year basis. On the contrary, variance ratios larger than 1 imply more volatility in the longterm.

Hence, it can be said that mean-reverting countries in this analysis such as Turkey and Australia are less risky in the long-term whereas mean-averting countries such as Japan are less risky in the short-term.

The implications of these results on investing decisions are just as important as the implications of price-predictability.

Overall, the results of this study were complicated, yet intriguing. It is clear that, different parts of the world behave differently in almost every aspect and different countries within those parts are no different. The only thing certain is that further research on this topic is needed to gain more knowledge on the behaviors of equity indices and make more accurate deductions about them.

#### CHAPTER 2

# TIME-VARYING RISK PREMIUM

#### 2.1 Introduction

Establishing mean reversion or aversion in a time series is an important step on its own in examining dynamics of a time series. A natural progression would be delving deeper to understand the underlying reasons behind the behavior of the time series. In understanding the dynamics of security prices, the question boils down to the fundamental issue of market efficiency. Do these results mean markets are inefficient or the prices actually reflect rational behavior of the investors? In other words, is mean reversion/aversion an anomaly or not? This issue remains an open question in academic literature to this day with fervent supporters on both sides.

Fama and French (1986) assert that negative serial correlation in returns could be due to market inefficiency or it might be the result of time varying expected returns generated by rational investor behavior. They call this a "critical but unresolvable issue" (p.3).

Lo and MacKinlay (1988) argue that rejection of the random walk hypothesis does not mean there is an inefficiency in stock-price formation. Poterba and Summers (1989) lean more towards the inefficiency argument by saying noise trading provides a plausible explanation for the predictability in stock prices.

Ball and Kothari (1989) stand on the opposite side of the argument by claiming that negative serial correlation in returns are mostly caused by changing relative risks and thus expected returns.

Conrad and Kaul's (1988) assertion that variation in expected returns constitute a large portion of return variances also supports the proposition that return predictability of stocks does not contradict the EMH.

Furthermore, Ferson and Harvey (1991) conclude that time variation in expected risk premiums is mostly responsible for the predictability of equity returns and their findings "strengthen the evidence that the predictability of returns is attributable to time-varying, rationally expected returns" (p.412).

In light of all of these conflicting arguments, I will investigate if predictable variation in stock returns can be linked to the variation in expected returns by computing the equity risk premium in a dynamic manner. The calculations are based on rolling two-pass cross sectional regressions of the Turkish stock market data. In the country results, Turkey showed a strong tendency for mean reversion, therefore it is one of the most suitable candidates for this type of analysis. More specifically, I will try to discover if the changes in the expected equity risk premium are responsible for the predictable variation in returns of the Turkish stock market. Figure 9 shows monthly returns of MSCI Turkey index and Borsa Istanbul's BIST 100 index. These are arguably the most prominent and representative indices for the Turkish equity market and as can be deduced from the figure as well as statistically proven in the previous chapter, they both exhibit mean reversion to a great degree.

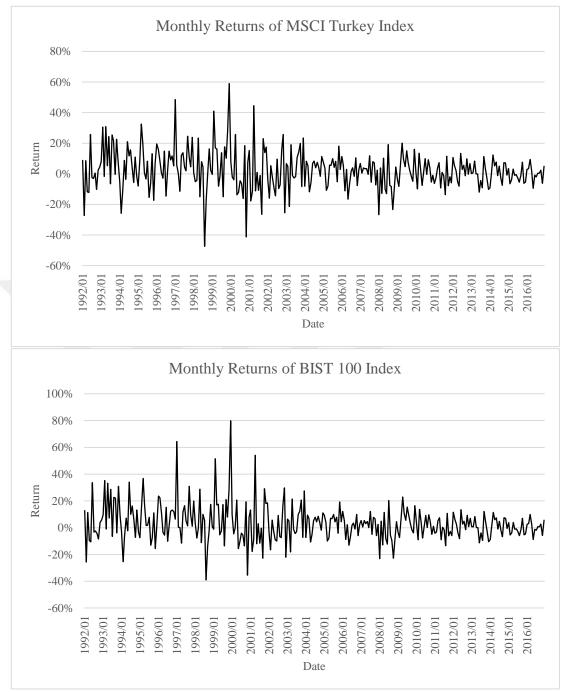


Figure 9. Monthly nominal returns of MSCI Turkey and BIST 100 indices

### 2.2 Data and methodology

The data for this study come from a proprietary source<sup>1</sup>, personally collected and adjusted so as to be more complete and reliable than any other source available for Turkish financial markets. The stock data are monthly returns of all common stocks (548 in total) that are traded on Borsa Istanbul during the period January 1992 through December 2016. These returns are adjusted to reflect dividends, capital changes and any other corporate action like splits, spin-offs, mergers, delisting and bankruptcies. The market return is the value-weighted average of available stocks for each period, which is a more appropriate measure of market return than the often used free-float weighted market indices. The risk-free rate is usually a problematic piece of data for Turkey as short-term government bonds do not exist for extensive periods. This problem is resolved by computing monthly returns from an index of overnight interest rates provided by Turkish Institutional Investment Managers' Association (TKYD) and Borsa İstanbul (BIST) under the name of BIST-KYD. All the data is denominated in the local currency.

In order to capture the time variation in equity risk premiums, one has to use a dynamic asset pricing model. I choose to use Capital Asset pricing Model (CAPM) by Sharpe (1964) and Lintner (1965) in a simple dynamic setting. However, I do not use a conditional version of the model as is the standard in the literature. Instead, I use a direct estimation of conditional betas and risk premiums using rolling window

<sup>&</sup>lt;sup>1</sup> I am immensely grateful to Ali Nezih Akyol, a PhD candidate at Boğaziçi University Department of Management, for kindly providing the data set, which is more comprehensive and reliable than any other data source available on Turkish financial markets. Mr. Akyol collected raw stock price data and corporate actions from multiple sources including Borsa Istanbul and data vendors like Thomson Reuters and Finnet for completeness. He meticulously adjusted them for dividends and other corporate actions. He computed the value-weighted market returns to be used in lieu of Borsa Istanbul Indices that are free-float weighted. He also computed monthly risk-free returns based on over-night interest rates to provide a key piece of data that is unfortunately not available for Turkish market in any other form.

regressions, a method similar to the one employed by Lewellen and Nagel (2006). The advantage of this method is its simplicity and the fact that one does not have to identify a set of state variables for conditioning information, which are usually unknown to the investors.

Beta values of individual stocks are calculated through a first-pass of standard time-series regression for CAPM,

(10) 
$$r_{it} = \alpha_{it} + \beta_i \times r_{mt} + \varepsilon_{it}$$

where  $r_{it}$  is the excess return of stock *i* at time *t*,  $\alpha_{it}$  is the abnormal returns of stock *i* at time *t*,  $\beta_i$  is the stock's beta which indicates how closely it follows the market,  $r_{mt}$  is the excess return of the market portfolio at time *t* (i.e. equity risk premium) and  $\varepsilon_{it}$  is the error term.

Each month from January 1997 through December 2016, betas of the individual stocks are estimated using the past 60 months' data, by regressing the monthly excess returns against the monthly equity risk premiums. These equity risk premiums are ex post and calculated by subtracting the risk-free interest rate from the monthly market return. As a requirement, each stock has to have at least 30 months of uninterrupted data prior to the month beta estimation is made. Figure 10 tracks how many stocks meet this data requirement and are eventually used in the analysis each month.

After estimating the betas of the stocks that meet the data requirement; in every month, a cross-sectional OLS regression is performed which regresses the monthly excess returns of the stocks against their betas, where the model estimates the regression coefficient  $\lambda$ , equity risk premium in the stock market.

(11) 
$$r_i = \gamma + \lambda \times \beta_i + \zeta_i$$

One of the most important decisions that needs to be made when performing such an analysis is whether to use individual stocks or to form portfolios. While the likes of Black, Jensen and Scholes (1972), Fama and Macbeth (1973) and Ferson and Harvey (1991) form portfolios to perform cross sectional regressions; there are others such as Ang, Liu and Schwarz (2008) and Chordia, Goyal and Shanken (2015) who use individual stocks.

Fama and Macbeth (1973) say more precise beta estimates can be made when portfolios are used instead of individual stocks. On the other hand, Ang, et al. (2008) argue that more precise estimates of beta do not lead to better estimates of the risk premia. They claim that variance of the risk premia estimates decreases when individual stocks are used as opposed to portfolios. The results below support this claim. Nevertheless, both approaches have been adopted in this analysis for the sake of robustness.

Using the individual stocks as observations, a monthly equity risk premium estimation is obtained for each of the 240 months between January 1997 and December 2016. This enables us to see the time variation in expected risk premia, priced by the classic CAPM model.

In order to test the robustness of risk premia estimation to the selection between individual stocks and portfolios, a methodology very similar to that of Fama and Macbeth (1973) is also used. Each month, 20 equal-weighted portfolios are formed using the already estimated betas of individual stocks.

Let *N* be the total number of stocks whose betas are estimated and let int(N/20) be the largest integer equal to or less than *N*/20. After sorting these stocks in an increasing order with respect to beta, int(N/20) stocks are allocated to every

portfolio. If *N* is even, first (lowest beta) and last (highest beta) portfolios each get additional  $\frac{1}{2} [N - 20 \operatorname{int}(N/20)]$  stocks. If *N* is odd, one more stock is put into the last portfolio.

Blume (1970) showed that for any portfolio *p* composed of *N* Stocks, with weights  $x_i$ , *i*=1,2,...,*N*;

(12) 
$$\hat{\beta}_p = \sum_{i=1}^N x_i \times \hat{\beta}_i$$

For equal-weighted portfolios, this becomes a simple average of betas. Returns obviously behave in the same manner as well. Therefore; each month, betas and returns of the portfolios are calculated by averaging the betas and returns of the included stocks. I choose to compute portfolio betas as a simple average of stock betas for simplicity and not to limit the dataset further by performing additional regressions for portfolio betas like Fama and Macbeth (1973).

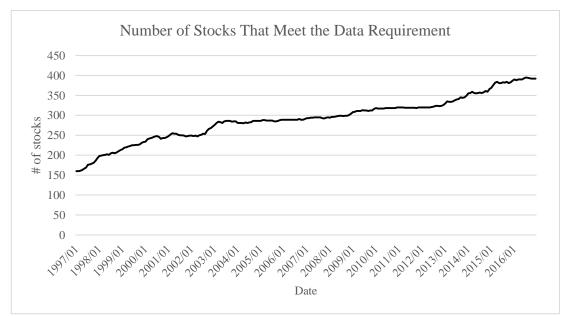


Figure 10. Number of stocks that meet the data requirement, from 1997 to 2016

# 2.3 Results

The summary of descriptive statistics are provided in Table 10. As expected, the volatility of portfolio returns increase along with beta values. Largest gaps between portfolio betas are between Portfolio 1-2 and Portfolio 19-20, with 0.25 and 0.31 differences respectively. This implies first and last portfolios predominantly consist of outlier firms.

The average portfolio returns, although not monotone, show an increasing trend along with risk levels measured by betas and volatilities. Interestingly, average return of the market portfolio is larger than all portfolios. This is a surprising, yet perfectly possible result. Due to data requirements, these 20 portfolios do not cover the entire market and the composition of the portfolios change every month. On top of that, portfolios are equal-weighted while the market return is value-weighted. Therefore, it is not possible to form a direct connection between the portfolio returns and the market return.

The time series of monthly risk premia, as estimated by a cross-sectional regression of individual stocks as well as portfolios, are shown in Figure 11 for a period of 20 years between January 1997 and December 2016. Both graphs visually suggest equity risk premium to be a mean-reverting process.

Portfolio	Average Beta	Average Monthly Return	Standard Deviation of Monthly Returns
Portfolio 1	0.20	3.02%	10.65%
Portfolio 2	0.45	3.23%	13.03%
Portfolio 3	0.54	3.04%	11.64%
Portfolio 4	0.61	2.74%	11.79%
Portfolio 5	0.66	3.81%	12.83%
Portfolio 6	0.71	3.18%	12.33%
Portfolio 7	0.75	2.76%	12.10%
Portfolio 8	0.78	3.28%	13.48%
Portfolio 9	0.82	3.03%	13.12%
Portfolio 10	0.85	2.64%	13.06%
Portfolio 11	0.89	3.79%	13.92%
Portfolio 12	0.92	3.49%	13.69%
Portfolio 13	0.95	3.90%	16.84%
Portfolio 14	0.99	3.23%	15.57%
Portfolio 15	1.02	3.14%	13.71%
Portfolio 16	1.06	3.23%	13.64%
Portfolio 17	1.11	3.30%	14.19%
Portfolio 18	1.16	2.98%	14.15%
Portfolio 19	1.23	3.31%	14.51%
Portfolio 20	1.54	3.44%	14.67%
Market Portfolio	1.00	4.31%	13.51%

Table 10. Summary Statistics

This table reports the summary statistics of the value-weighted market portfolio and 20 portfolios that are formed during the analysis, for the period 1997-2016. Regression intercepts are not forced to 0.

Moreover, the level of expected ERP in both scenarios usually hovers around 0 and does not change much over the years. However, the volatility of the ERP seems to increase in times of economic distress and decrease in times of economic stability and comfort. Risk premia changes drastically from one month to another during the 1999-2003 period which includes the 2001 financial crisis (which was national, not global) and also during the 2008-2010 period which includes the 2008 global financial crisis and stays fairly stable during other times.

Two plots are very similar except a few minor differences: The time series generated with individual stocks is slightly less volatile and its confidence intervals are a little bit tighter than the time series generated with portfolios.

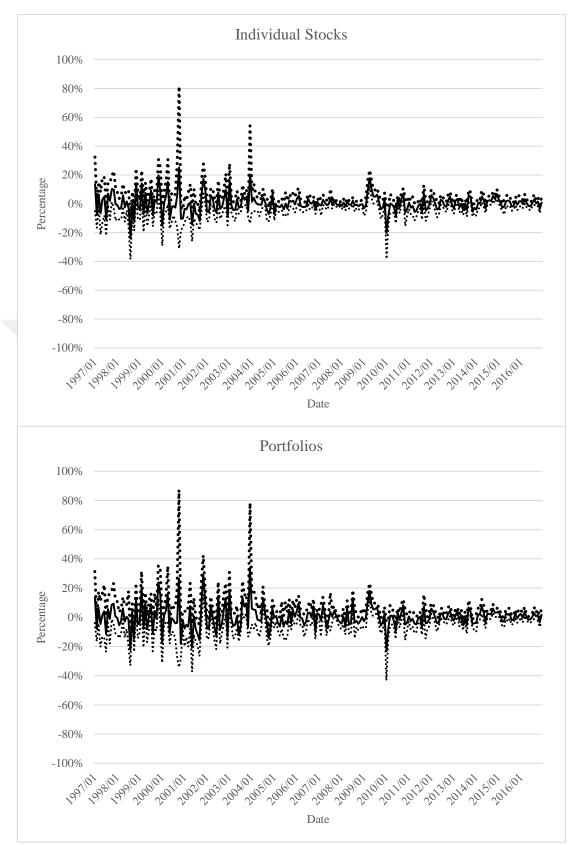


Figure 11. Time series of estimated monthly risk premiums, generated with individual stocks and portfolios, from 1997 to 2016

Figure 12 displays equity risk premiums computed by forcing the regression alphas to be zero. Axis scales have been kept constant in all graphs for a better comparison.

The time series of equity risk premiums exhibit similar properties to the case where the intercept is not suppressed. A key difference between the two cases is that the confidence intervals get tighter, but the risk premium itself becomes more volatile in the latter case. This means more precise, but somewhat less accurate estimates of the equity risk-premium can be made when intercepts are forced to 0. These results can also be observed in Table 12, which reports summary statistics for each case.

Figure 13 and 14 plot the expected ERP series and market return on top of each other for every scenario. This makes it easier to see how closely they track each other and to what extent market return is affected by the risk-premium.

Even though market return and estimated risk premia seem slightly more overlapping when intercepts are forced to 0, they still go very much hand in hand in the alternative scenario. This finding gives significant support to the argument that mean-reverting prices actually reflect rational investor behavior via temporal variation in required equity risk premium.

In order to assess the dynamics of the time series of estimated monthly equity risk premium, the same variance ratio test which was applied to several national equity indices before has also been applied to the estimated ERP series. The previous tests revealed that the Turkish equity market returns showed significant mean reversion. The current test would reveal if the mean reversion observed in market returns can be attributed to a similar property in the equity risk premium.

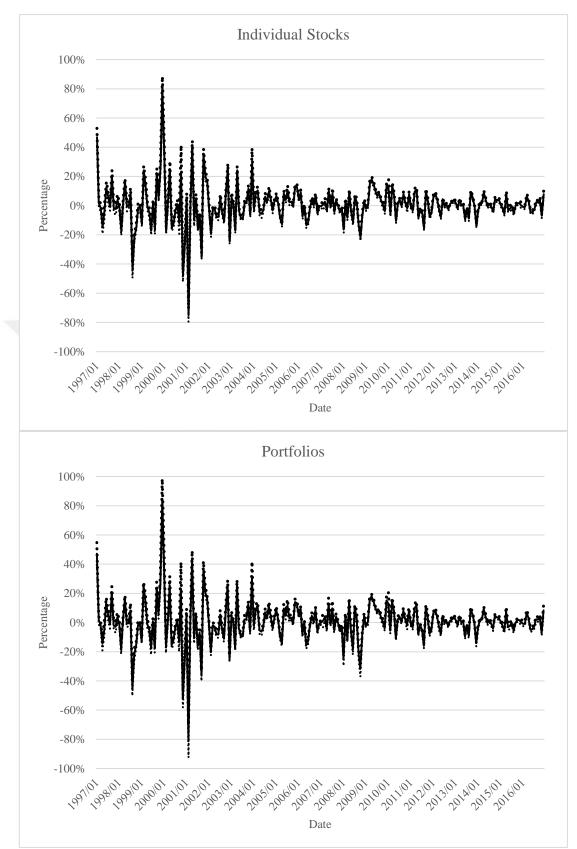


Figure 12. Time series of estimated monthly risk premiums, generated with individual stocks and portfolios and by forcing regression intercepts to 0, from 1997 to 2016

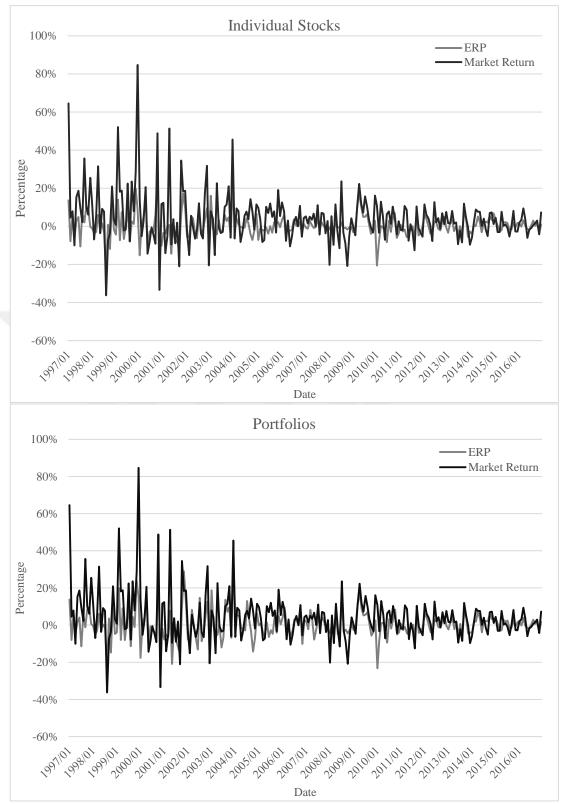


Figure 13. Monthly returns of the market portfolio and estimated monthly risk premiums from 1997 to 2016, generated with individual stocks and portfolios

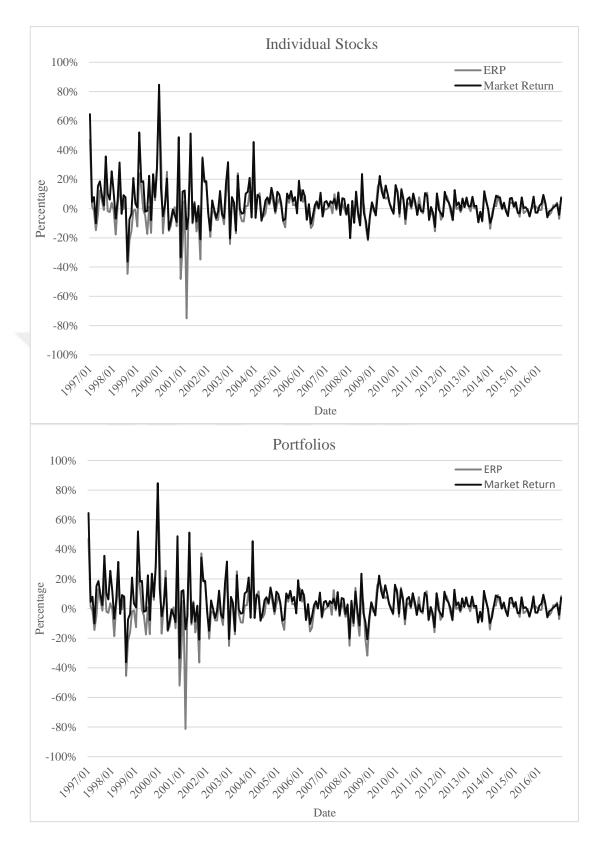


Figure 14. Monthly returns of the market portfolio and estimated monthly risk premiums from 1997 to 2016, generated with individual stocks and portfolios and by forcing regression intercepts to 0

The time series of equity risk premium exhibits mean-reverting properties in the long-term, as can be seen in Figure 15 and Table 11. Variance ratios start to decline rapidly after 12 months and stay below 0.4 after 60 months in all scenarios. When intercepts are forced to zero, the series becomes more mean-reverting in holding periods 1 to 8 years. The null hypothesis of random walk is rejected for periods longer than 1 year in all scenarios with a significance level of 5%.

In the short-term (in holding periods shorter than 1 year), the series is meanreverting if intercepts are not forced to zero and mean-averting if intercepts are forced to zero. Short-term results are inconclusive based on the discrepancy in results as well as statistical significance levels. To recall, MSCI Turkey index was also mean-averting in the short-term.

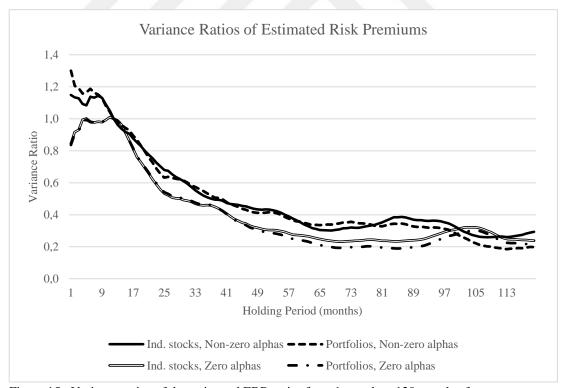


Figure 15. Variance ratios of the estimated ERP series from 1 month to 120 months, for every scenario

## Table 11. Randomization Results

		1 month	6 months	24 months	36 months	48 months	72 months	96 months	120 months
Individual Stocks,	VR	1.149	1.137	0.702	0.509	0.438	0.317	0.359	0.293
Non-zero Alphas	P-value	0.672	0.779	0.070	0.049	0.073	0.066	0.190	0.130
Portfolios,	VR	1.300	1.188	0.656	0.534	0.415	0.352	0.317	0.198
Non-zero Alphas	P-value	0.825	0.867	0.054	0.080	0.061	0.124	0.145	0.030
Individual Stocks,	VR	0.844	0.982	0.557	0.460	0.324	0.234	0.284	0.238
Zero Alphas	P-value	0.202	0.420	0.012	0.033	0.015	0.023	0.109	0.069
Portfolios, Zero Alphas	VR	0.836	0.980	0.566	0.465	0.313	0.196	0.248	0.212
	P-value	0.194	0.409	0.007	0.030	0.012	0.006	0.063	0.048

This table reports variance ratios and their respective p-values (obtained through randomization) of the expected monthly ERP series in 4 different scenarios for several holding periods.

An overview of the statistical properties of the equity risk premium time series are presented in Table 12 with an overall comparison between the four different calculation methods. Monthly average ERP is slightly higher when intercepts are forced to 0 whereas the standard deviation doubles. On the contrary, 95% confidence intervals are a lot tighter when intercepts are forced to 0. There is also a small increase in standard deviation and confidence interval range going from individual stocks to portfolios.

R-squared statistics rise significantly when intercepts are forced to 0. There is a 16% difference when individual stocks are used and almost 40% difference when portfolios are used. Using portfolios rather than individual stocks yields stronger Rsquared numbers as well. This means the cross-sectional variance of the betas explain the cross-sectional variance of the monthly asset returns much better when intercepts are forced to 0 and portfolios are used instead of individual stocks.

On the other hand, the time series of expected monthly ERP's is more meanreverting when intercepts are forced to 0. The null hypothesis of random walk can be rejected in more holding periods (almost 7 times as much for individual stocks and 6 times as much for portfolios) when intercepts are forced to 0. The magnitude of mean reversion seems to get larger also when portfolios are used rather than individual stocks.

The correlation between the market return and expected equity risk-premium is very important for this analysis as it measures the relationship between two variables and to what extent the movements in one track the movements in the other. Correlation between the two also rise when intercepts are forced to 0. In this regard, there isn't any significant difference between individual stocks and portfolios.

Evaluating the results altogether, forcing intercepts (i.e. alphas or abnormal returns) to 0 does seem to improve the explanatory power of regressions, but at a cost of increased volatility in the estimated parameter that is time-varying equity risk premium. Same pattern is observed for using portfolios over individual stocks, which is in line with the findings of Ang et al. (2008).

## Table 12. Comparison of the Different Methods

	NON-ZERO	ALPHAS	ZERO ALPHAS			
	Individual Stocks	Portfolios	Individual Stocks	Portfolios		
Average monthly ERP	0.26%	0.27%	0.70%	0.60%		
Standard Deviation of ERP	6.08%	7.77%	13.57%	14.46%		
Average width of the 95% confidence intervals	12.23%	14.66%	4.58%	5.19%		
Average R-squared	0.98%	14.04%	16.82%	53.14%		
Minimum VR	0.26	0.18	0.23	0.19		
# of months that reject random walk, with $\alpha$ =0.05	10	15	67	82		
Correlation with the market portfolio	0.59	0.65	0.86	0.86		

This table reports several statistics and compares the different calculation methods.

## 2.4 Conclusion

Since the estimated risk premia comes from Sharpe-Lintner CAPM model, it represents the risk premium rational investors expect from an efficient equity market. Therefore; if the market return and the expected equity risk premium go hand in hand, in other words if the trends in the expected returns match the trends in the actual returns well enough, it points towards an efficient market. This is a case where an apparent anomaly can be rationally explained in a dynamic setting, where the market efficiency, in fact, is not violated.

According to these results, this is the case for the Turkish equity market. Time series of expected ERP's track the market return very closely and the risk premium seems just as mean-reverting as the market itself. Hence, the empirically observed mean reversion in the Turkish equity market can be attributed to the time varying nature of equity risk premium demanded by the investors. Taken to its natural conclusion, Turkish equity market can still be considered efficient when the parameters of our model are allowed to reflect the dynamic nature of the market itself.



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