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SADIK HAZER
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GEBZE
2017

GEBZE TEKNİK ÜNİVERSİTESİ

YÜKSEK LİSANS TEZİ JÜRİ ONAY SAYFASI

GTÜ Sosyal Bilimler Enstitüsü Yönetim Kurulu'nun 18.01.2017 tarih ve 2017/01 sayılı kararıyla oluşturulan jüri tarafından 24.01.2017 tarihinde tez savunma sınavı yapılan Sadık Hazer'in tez çalışması Strateji Bilimi Anabilim Dalında YÜKSEK LİSANS tezi olarak kabul edilmiştir.

JÜRİ

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(TEZ DANIŞMANI) : YRD. DOÇ. DR. Saadettin Haluk ÇİTÇİ

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ONAY

GTÜ Sosyal Bilimler Enstitüsü Yönetim Kurulu'nun ………/………/20… tarih ve ……………/………… sayılı kararı.

İMZA/MÜHÜR

ÖZET

Hem kamu hem de özel firmaları içeren karma oligopoller gelişmiş ve gelişmekte olan ülkelerde yaygındır. Birçok sektörde devlete ait kamu firmaları ve özel firmalar birbirlerine karşı rekabet eder. Karma oligopollerde, kamu firmaları hem kendi kârını hem de sosyal refahı dikkate aldığı için sadece özel işletmeleri içeren oligopollerden farklıdır. Diğer bir taraftan, karma oligopollerle ilgili tartışmanın merkezinde rekabet halindeki firmaların belirsiz ortamlardaki stratejik davranışları vardır.

Bu çalışmada, kamu ve özel sektör firmaları içeren karma oligopollerde maliyet belirsizliği karşısında bilgi üretme ve paylaşma durumlarında denge sonuçlarını analiz etmeyi amaçlamaktayız. Kamu ve özel sektörün, bilgi paylaşımında teşvikler olup olmadığı, saf stratejilerde denge sonuçlarının ortaya çıkıp çıkmadığı ve sosyal refahın firmaların bilgi paylaşım kararlarından nasıl etkilendiği cevaplamaya çalışılmıştır.

Anahtar Kelimeler: Karışık oligopol, bilgi paylaşımı, bilgi üretimi, Kamu firmaları, özel firmalar

SUMMARY

Mixed oligopolies involving both public and private firms are common in both developed and developing countries. In many sectors, state-owned public firms and private firms compete against each other. Mixed oligopolies differ from oligopolies involving only private firms in that public firms in mixed oligopolies take both its own profit and social welfare into account. Among other issues, a central debate on mixed oligopolies is the competing firms' strategic behaviors under uncertain environments.

In this study, we aim to analyze equilibrium outcomes in mixed oligopolies when public and private firms can produce information and share information on uncertain cost. We attempt to answer whether public and private firms have incentives to share information, which equilibrium outcomes emerge, and how social welfare is affected by the information sharing decisions of firms.

Key Words: Mixed oligopoly, information production, information sharing, share information, public firm, private firm

ACKNOWLEDGEMENTS

.

Would like to thank my thesis supervisor Assoc. Dr. Sadettin Haluk ÇİTÇİ for his guidance, encouragement and patiance over the last year and a half. I am always happy to be his student. I would like to thank and TUBITAK who supported me economically with the BİDEB Scholarship during my master's degree and most of all I appreciate my wife who did not spare my support during my studies.

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<u>**Abbreviations**</u>	<u>**Explanations**</u>
CS	: Consumer Surplus
PS	: Producer Surplus

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1. INTRODUCTION

Nowadays, there are common expressions that everyone has known and said: "Information is power, knowledge is power". This is certainly true in the economy as it is in many fields. Having the information, also, means having the power at the same time. However, sharing the information might be an advantage or disadvantage for the competitors. Therefore, there are many studies about the information sharing in the economy for different kind of markets. They use different assumptions and environments. However, as far as we know there is no study about information sharing in mixed oligopolies.

In developing and developed countries, mixed oligopolies become important because of the privatization and liberalization of the market by the governments. Mixed oligopolies is a kind of economy that there are at least one public firm and one private firm compete with each other. There are a lot of example in different sectors about mixed oligopolies, for example, in the network sector transportation, broadcasting, telecommunication, mail and in the energy sector gas and electricity and in the service sector insurance, banking, health care, education (Anam, Basher and Chiang, 2007). In mixed oligopolies, private firms are profit maximizer but the public firms try to maximize welfare. Therefore, in the mixed oligopoly the choices of the firms and price, demand, or produced quantity equilibriums can be different from the pure oligopolies.

Our study combines these two topics in the economy so there are two sides in the literature. One of the topics is the information sharing. In the literature, there are many papers that examine the effects of the information sharing. Generally, they are interested in the private oligopolies under cost or product uncertainty with or without product differentiation. The other topic that our study is related with is the mixed oligopolies. Mixed oligopolies are also studied in the literature under different environments with various uncertainties. The results clearly show that when a public firm plays a role in the market the equilibriums become different from the private oligopolies. However, none of the studies is about the information sharing in the mixed oligopolies."

Under which conditions, do private and public firms competing in the same market have the incentive to produce and honestly share information on their own stochastic costs? In this paper, it is presented a mixed duopoly model involving a private and a public firm with stochastic cost functions. The private firm is the profit maximizer but the public firm is the welfare maximizer because its concerns are about the utility and the social surplus. They both have two choice, one of them is to produce information about the uncertain cost and the other choice is to disclose the produced information.

As it is mentioned at the beginning “information is the power”. Therefore, generally, obtaining the information is the best interest for firms. Our results do not controvert this intuitive argument. However, previous studies about the information sharing indicate that it is the best interest for both firms to disclose their cost functions, generally. Differently from results established for information sharing in private oligopolies, under certain circumstances, both the private and welfare-maximizing public firm have incentives not to disclosure information.

2. LITERATURE

By lots of historians, the period that we lived in is described as “information age”. In the “information age”, information and information systems have been examined in the economy as well as in many fields. The term "information sharing" in the economy means that sharing and acquiring information and examination of its effects in a certain market. Xavier Vives (Vives, 1990) explains the history of information sharing as follows:

> “This literature had an early start with Ponssard (1979) and was continued by Novshek and Sonnenschein (1982), Clarke (1983), Vives (1984), and Gal-Or (1985), among others. The problem is that oligopolistic interaction seems to lead to case-by-case analysis.”

The research in the literature is mostly for pure oligopolies and there is a little research in mixed oligopolies. Vives (1984 and 1990), Shapiro (1986), Sakai and Yamato (1989), Ganuza and Jansen (2013) have been studied about the welfare analysis of information sharing in pure oligopolies. The assumptions and models that are used vary from paper to paper. A wide range of assumptions has been investigated.

The differences can be listed in the literature as follows:

- Cournot and Bertrand competition
- Number of firms
- Oligopoly and Mixed oligopolies
- State of Nature
- Private Signals
- Revelation of signals
- Cost and Demand Uncertainty
- One or two-stage game
- Etc…

In literature, research shows that even if the same kind of assumptions is used, the results might be conflicted depending on the model. All of the work on information sharing is strictly tied to the specific environment and initial assumptions.

In the following, some research examples in the literature of the information sharing will be presented and the modeling assumptions and the equilibrium results will be explained.

Clarke (1983) modeled a Cournot Oligopoly with the n number of firms on demand uncertainty. It is discussed that how market uncertainty affects incentives to collude and solution of a general Bayesian Cournot game model is introduced under imperfect information. Also, a discussion about the welfare effects of the various market outcomes is presented. The results show that universal information sharing will not take place in a general "full Bayes-Cournot" equilibrium, except some certain cases.

Vives (1984) published a paper about the information sharing in oligopolies with two private firms. The results show that in a Cournot competition, information sharing is not the optimal outcome if the products are close substitutes. However, under Bertrand competition, information sharing is the optimal outcome for both substitutes and complements.

Gal-Or (1986) argues that information sharing result in a better outcome for firms in Cournot competition. Conversely, no disclosure gives a better outcome under Bertrand competition. This result seems to conflict with established results in Vives (1984), however, there is an important difference between models in these two papers. Gal-Or (1986) derives equilibrium outcomes for competition under uncertain costs but Vives (1984) assumes demand uncertainty

Fried (1984) develops a model for an oligopoly where two private firms compete with each other in the Cournot market under cost uncertainty. His study has two objectives. One of them is how producing information affects the profit of the firms and the other one is whether produced information should be disclosed or not. Fried (1984) shows that producing information is the best response for the both firms. In addition to this result, he shows that sharing that information increases profits of the firms.

Sakai (1986) also studied Cournot and Bertrand duopoly under demand uncertainty and his results show that sharing information is the best choices for firms. Similarly, Kirby (1988) information sharing in pure oligopolies under demand uncertainty, but with assumptions of n number of firm and noisy environment. His results differ from results established in Sakai (1986) for the Cournot competition.

Ganuza and Jansen (2013) examine effects of the information sharing in oligopoly and how the welfare is affected by sharing acquired cost information. They aim to use new methodological techniques and taking the new perspective of information economics, which lets firms to extent to determine their information structures.

As it is mentioned before Vives (1984), Shapiro (1986), Sakai and Yamato (1989) have been studied the welfare analysis of information sharing in pure oligopolies. There are some studies analyzing equilibrium outcomes in mixed oligopolies under environments with uncertainty (e.g. Anam, Basher, and Chiang, 2007, Kitahara and Matsumura, 2013, Citci and Karakas, 2014) However, none of these studies focus on the analysis of information sharing in mixed oligopolies. To the best of our knowledge, this is the first paper analyzing information sharing incentives of firms and equilibrium outcomes in mixed oligopolies.

3. THE MODEL

We formulate a non-cooperative mixed duopoly model involving a public and a private firm. Both firms produce a single homogenous good in the same market. The public firm is denoted by 1 and the private firm is denoted by 2 for all the model. These two firms have the linear cost functions on uncertain marginal cost.

The cost functions are

$$Cost\ 1 = m_1 q_1 \qquad (3.1)$$

$$Cost\ 2 = m_2 q_2 \qquad (3.2)$$

Where m_1 is the marginal cost and q_1 is the amount produced of the public firm and similarly, m_2 is the marginal cost and q_2 is the amount produced of the private firm.

Demand functions are assumed to be linear.

$$P_1 = A - d(q_1 + q_2)$$

$$P_2 = A - d(q_1 + q_2)$$

Parameter A is a commonly known constant and d is normalized to 1 in the model.[1] Prices of the products are

$$P_1 = A - q_1 - q_2 \qquad (3.3)$$

$$P_2 = A - q_1 - q_2 \qquad (3.4)$$

Given demand functions, the profit function for the public firm can be represented as follows.

$$\Pi_1 = p_1 * q_1 - m_1 * q_1$$

$$\Pi_1 = (A - q_1 - q_2) * q_1 - m_1 * q_1$$

$$\Pi_1 = (A - m_2 - q_1 - q_2) * q_2$$

$$\Pi_1 = (C_1 - q_1 - q_2) * q_1 - m_1 \qquad where\ C_1 = A - m_1$$

$$\Pi_1 = C_1 q_1 - {q_1}^2 - q_1 q_2 \qquad (3.5)$$

To simplify the model, we define $C_i = A - m_i$, i={1,2} as the cost parameter. The uncertainty of cost is referred as C_1 for public firm, and as C_2 for the private firm, in the rest of the paper.

[1] The value of the constant term d does not affect qualitative results of the paper. Thus, without loss of generality, it is normalized to 1.

Given demand functions, profit function of the private firm can be represented as in (3.6).

$$\Pi_2 = p_2 * q_2 - m_2 * q_2$$

$$\Pi_2 = (A - q_1 - q_2) * q_2 - m_2 * q_2$$

$$\Pi_2 = (A - m_2 - q_1 - q_2) * q_2$$

$$\Pi_2 = (C_2 - q_1 - q_2) * q_2 \quad where\ C_2 = A - m_2$$

$$\Pi_2 = C_2 q_2 - {q_2}^2 - q_1 q_2 \tag{3.6}$$

The nature of uncertainty is formed from parameters C_1 and C_2 which are random variables. They have a known bivariate normal distribution R. Random variable C_1 has the mean $\overline{C_1}$ and variance V_1^2. Similarly, random variable C_2 has the mean $\overline{C_2}$ and variance V_2^2. It is very well known from the probability theorem but it would be nice to mention here that V_i^2 equals $E[(C_i - \overline{C_i})^2]$. The covariance V_{12} equals $r * V_1 * V_2$ where r means the coefficient of correlation. The means, variances and the covariance of cost parameters are common knowledge.

Both firms are risk neutral. The objective of the private firm is to maximize its profit and the aim of the public firm is to maximize the social welfare. Welfare function is defined by $W = CS + PS$ where *CS* is consumer surplus and *PS* is producer surplus.

Before to explain welfare function, we need to define utility function of households. The given demand functions can be derived from quadratic, strictly concave and symmetric utility functions, which is specified as the following:

$$U(q_1, q_2) = A * (q_1 + q_2) - \frac{(q_1+q_2)^2}{2} \tag{3.7}$$

Depending on this utility function, consumer surplus, CS, can be formulated as the following:

$$CS = U(q_1, q_2) - p_1 q_1 - p_2 q_2 \tag{3.8}$$

By substitution

$$CS = A * (q_1 + q_2) - \frac{(q_1+q_2)^2}{2} - p_1 q_1 - p_2 q_2 \tag{3.9}$$

Producer surplus is the summation of the profits of the firms.

$$PS = \Pi_1 + \Pi_2$$

$$PS = p_1 q_1 - m_1 q_1 + p_2 q_2 - m_2 q_2 \tag{3.10}$$

After proper substitutions, the welfare function can be formulated as the following:

$$W = CS + PS \quad (3.11)$$

$$W = A * (q_1 + q_2) - \frac{(q_1 + q_2)^2}{2} - p_1 q_1 - p_2 q_2 + p_1 q_1 - m_1 q_1 + p_2 q_2 - m_2 q_2$$

$$W = A * (q_1 + q_2) - \frac{(q_1+q_2)^2}{2} - m_1 q_1 - m_2 q_2 \quad (3.12)$$

In this environment, it is assumed that firms play the two-stage simultaneous game. Initially, each firm only knows distributions of cost functions (more specifically, the mean, variance and covariance of the marginal costs). In the first stage, each firm decides whether to produce information on its own cost and whether to share this information with the other firm. Based on information they accumulated in the first stage, firms decide the amount of quantity to produce in the second period.

There are three alternative choices for the public and private firm in the first stage of the game. Each firm may use only the information on mean values and variances of their costs, which are common knowledge, without any information production on cost functions. This choice is assumed to be the default case because in this case, each firm has no information about the exact value of its own cost and that of the other firm. The second alternative is to produce information on its own cost without disclosure of this information to the other firm. In this case, the firm has information on the exact value of its own cost, but it does not share this information with the other firm. The third and the last alternative is to produce information on its own cost and disclosure this information to the other firm.

To explain in more detail, in the complete information case, in which each firm produces information on its own cost and share this information with the other firm, firms can decide the amount of production in the second stage with knowing the exact value of the cost, the random variable C_i, for each unit of production. However, if firms produce no information on their costs in the first stage, they will have only information on distributions of costs in the second stage and they have to give production decisions based only on their estimations of cost functions.

The information set used by the firms are nominated as $I_1 = (.,.)$ for the public firm and $I_2 = (.,.)$ for the private firm. Table *3.1* shows information sets for all possible cases.

Table 3.1: Information sets for all possible cases

$Firm\ 1$ ($Public\ firm$)	$Firm\ 2$ ($Private\ firm$)		
	Do not produce information	Produce information, no disclosure	Produce and disclosure information
Do not produce information	***Case 1*** $I_1 = (\overline{C_1}, \overline{C_2})$ $I_2 = (\overline{C_1}, \overline{C_2})$	***Case 2*** $I_1 = (\overline{C_1}, \overline{C_2})$ $I_2 = (\overline{C_1}, C_2)$	***Case 3*** $I_1 = (\overline{C_1}, C_2)$ $I_2 = (\overline{C_1}, C_2)$
Produce information, no disclosure	***Case 4*** $I_1 = (C_1, \overline{C_2})$ $I_2 = (\overline{C_1}, \overline{C_2})$	***Case 5*** $I_1 = (C_1, \overline{C_2})$ $I_2 = (\overline{C_1}, C_2)$	***Case 6*** $I_1 = (C_1, C_2)$ $I_2 = (\overline{C_1}, C_2)$
Produce and disclosure information	***Case 7*** $I_1 = (C_1, \overline{C_2})$ $I_2 = (C_1, \overline{C_2})$	***Case 8*** $I_1 = (C_1, \overline{C_2})$ $I_2 = (C_1, C_2)$	***Case 9*** $I_1 = (C_1, C_2)$ $I_2 = (C_1, C_2)$

In order to guarantee for the positive amount of quantity production, we put following limitations on C_i and $\overline{C_i}$ values:

$$2\overline{C_1} \geq \overline{C_2} \geq \overline{C_1} \geq 0 \tag{3.13}$$

$$2C_1 \geq C_2 \geq C_1 \geq 0 \tag{3.14}$$

$$C_1 \geq \overline{C_2} - \overline{C_1} \geq 0 \ \ or \ \ C_1 + \overline{C_1} \geq \overline{C_2} \geq \overline{C_1} \tag{3.15}$$

$$C_2 \geq 2\overline{C_1} - \overline{C_2} \geq 0 \ or \ C_2 + \overline{C_2} \geq 2\overline{C_1} \geq \overline{C_2} \tag{3.16}$$

4. RESULTS

4.1. Naming Convention

In order to clear understanding and categorize the formulations, the nominations are made with certain names which are explained below.

"Case 1" is the main case that both firms do not produce information about themselves. In this case, they both use the mean values and variances for the calculations.

In "Case 2", the public firm does not produce information but private firm decides to produce information without disclosure. In this case, public firm and private firms use mean value and variance about public firm but private firm uses his own produced exact value in his calculations.

"Case 3" means that the public firm does not produce information but private firm produces information and shares this information with the public firm. Therefore, the public firm can use private firm's produced and shared information as well as his means and variance. The private firm uses his own produced information.

In "Case 4" firms change their roles in case 2. Private firm does not produce information but public firm decides to produce information without disclosure. In this case, public firm and private firms use mean value and variance of private firm but public firm uses his own produced exact value in his calculations.

In "Case 5" is the special case. Both firms decide to produce information, however, they do not disclose this information with his competitor. Because of that, they do not know exactly what the other's expectation. The solution of this case differs from the other but it also converges.

In "Case 6" Public firm produce information but it hides it from the private firm. Private firm produces information and share with the public one. In this case, both firms use their own exact values. The public firm uses the private firm exact value in expectations of private firm because the private firm shares it. However, private firm does not know the public firm information so it uses its expectation.

In "Case 7" firms change their roles in case 3. The private firm does not produce information but public firm produces information and shares this information with the private firm. Therefore, the private firm uses public firm's produced and shared information as well as his means and variance. Public firms uses his own produced information.

In “Case 8” firms change their roles in case 6. The public firm produces information and share with the private firm. In this case, both firms know their own exact values. The public firm uses the private firm exact value in expectations of the private firm because the private firm shares it. However, the private firm does not know the public firm information so it uses its expectation.

In “Case 9” both firms produce and disclose this produced information with their competitors. In this case, they both use their known produced values in their calculations.

4.2. Welfare and Profit Maximization

In this section, there are the calculations of the expected quantities for a certain information sets for public and private firm. To remind, the public firm wants to maximize the welfare function the private function want to maximize its profit function.

4.2.1 Welfare maximization for public firm

$$max\left[E(W|I_1) = E\left(\left(A*(q_1+q_2) - \frac{(q_1+q_2)^2}{2} - m_1q_1 - m_2q_2\right)\middle|I_1\right)\right]$$

To find maximum quantity, the derivative of the function must be equal to zero.

$$E\left(\frac{dW}{dq_1}\middle|I_1\right) = 0$$

$$E\left(\frac{d\left(A*(q_1+q_2) - \frac{(q_1+q_2)^2}{2} - m_1q_1 - m_2q_2\right)}{dq_1}\middle|I_1\right) = 0$$

$$E\left(\frac{d\left(A*q_1 + A*q_2 - \frac{{q_1}^2}{2} - q_1*q_2 - \frac{{q_2}^2}{2} - m_1q_1 - m_2q_2\right)}{dq_1}\middle|I_1\right) = 0$$

$$E(A - m_2 - q_1 - q_2|I_1) = 0$$

$$q_1 = E(A - m_2 - q_2|I_1)$$

$$q_1 = (C_1 - q_2\ |I_1) \quad where\ C_1 = A - m_1$$

$$q_1 = E(C_1\ |I_1) - E(q_2)$$

Expected quantity for public firm for a certain information set

$$q_1 = E_1(C_1) - E_1(q_2) \tag{4.1}$$

4.2.2 Profit maximization for private firm

$$max \left[E(\Pi_2|I_2) = E\left(\left((A - q_1 - q_2) * q_2 - m_2 * q_2\right)\middle|I_2\right)\right]$$

To find maximum quantity the derivative of the function must be equal to zero.

$$E\left(\frac{d\Pi_2}{dq_2}\middle|I_2\right) = 0$$

$$E\left(\frac{d\left((A - m_2 - q_1 - q_2) * q_2\right)}{dq_2}\middle|I_2\right) = 0$$

$$E\left(\frac{d(Aq_2 - m_2q_2 - {q_2}^2 - q_1q_2)}{dq_2}\middle|I_2\right) = 0$$

$$E(A - m_2 - q_1 - 2 * q_2|I_2) = 0$$

$$q_2 = E\left(\frac{A - m_2 - q_1}{2}\middle|I_2\right)$$

$$q_2 = \left(\frac{C_2 - q_1}{2}\middle|I_1\right) \quad where\ \ C_2 = A - m_2$$

$$q_2 = \frac{E(C_2\ |I_2) - E(q_1)}{2}$$

Expected quantity for private firm for a certain information set

$$q_2 = \frac{E_2(C_2\ ) - E_2(q_1)}{2} \tag{4.2}$$

4.3. Derivation of Equilibria

The best framework to achieve a converged equilibrium solution in this type of models is the simultaneous-choice model. Simultaneous-choice model is explained in detail by Cyert and De for the duopoly problems (Cyert, R. M. and M.H De Groot, 1970a). In this model, firms estimate what competitor's beliefs is and decide quantities according to this subjective probabilities. This is a kind of "I think that he thinks that I think that he thinks…" model. It seems diverging but for certain cases, this chain can be broken. Even if it is infinite, the equation converges.

$$q_1 = E_1(C_1) - E_1(q_2)$$

$$q_2 = \frac{E_2(C_2\ ) - E_2(q_1)}{2}$$

The solution of the public firm equilibrium quantity is where $E_1E_2(x)$ public firm's expectation of private firm's expectation about x. The detailed solution is given in appendix 1.

$$q_1 = E_1(C_1) - E_1(q_2)$$

$$q_1 = E_1(C_1) - E_1(\frac{E_2(C_2\ ) - E_2(q_1)}{2})$$

$$\dots$$

$$q_1 = E_1(C_1) - \frac{E_1E_2(C_2\ )}{2} + \frac{E_1E_2E_1(C_1)}{2} - \frac{E_1E_2E_1E_2(C_2\ )}{4} + \frac{E_1E_2E_1E_2E_1(C_1)}{4} - \frac{E_1E_2E_1E_2E_1E_2(C_2\ )}{8} + \frac{E_1E_2E_1E_2E_1E_2E_1(C_1)}{8} - \frac{E_1E_2E_1E_2E_1E_2E_1E_2(C_2\ )}{16}\dots$$

The solution of the private firm equilibrium quantity is below where $E_1E_2(x)$ private firm's expectation of public firm's expectation about x. The detailed solution is given in appendix 1.

$$q_2 = \frac{E_2(C_2\ ) - E_2(q_1)}{2}$$

$$q_2 = \frac{E_2(C_2\ ) - E_2(E_1(C_1) - E_1(q_2))}{2}$$

$$\dots$$

$$q_2 = \frac{E_2(C_2\ )}{2} - \frac{E_2E_1(C_1)}{2} + \frac{E_2E_1E_2(C_2\ )}{4} - \frac{E_2E_1E_2E_1(C_1)}{4} + \frac{E_2E_1E_2E_1E_2(C_2\ )}{8} - \frac{E_2E_1E_2E_1E_2E_1(C_1)}{8} + \frac{E_2E_1E_2E_1E_2E_1E_2(C_2\ )}{16} - \frac{E_2E_1E_2E_1E_2E_1E_2E_1(C_1)}{16}\dots$$

To solve these infinite equations, intermediate members are defined for all information cases except one. The case 5 ("produced information but no disclosure") cannot be solved by using these substitutions. Except for this case for all cases X_1, X_2, X_3 and X_4 can be defined as

$$X_1 = E_1(C_1\ )$$

$$X_2 = E_2E_1(C_1) = E_1E_2E_1(C_1) = E_2E_1E_2E_1(C_1) = E_1E_2E_1E_2E_1(C_1) = \cdots$$

$$X_3 = E_2(C_2\ )$$

$$X_4 = E_1E_2(C_2) = E_2E_1E_2(C_2) = E_1E_2E_1E_2(C_2) = E_2E_1E_2E_1E_2(C_2) = \cdots$$

X_2 and X_4 is expectation of the firm's about expectation of the other firms. Therefore, after a certain point all results will be equal for all cases except case 5. By substitution, equilibrium quantity of public firm can be calculated by

$$q_1 = E_1(C_1) - \frac{E_1E_2(C_2)}{2} + \frac{E_1E_2E_1(C_1)}{2} - \frac{E_1E_2E_1E_2(C_2)}{4} + \frac{E_1E_2E_1E_2E_1(C_1)}{4} - \frac{E_1E_2E_1E_2E_1E_2(C_2)}{8} + \frac{E_1E_2E_1E_2E_1E_2E_1(C_1)}{8} - \frac{E_1E_2E_1E_2E_1E_2E_1E_2(C_2)}{16} \cdots$$

$$q_1 = X_1 - \frac{X_4}{2} + \frac{X_2}{2} - \frac{X_4}{4} + \frac{X_2}{4} - \frac{X_4}{8} + \frac{X_2}{8} - \frac{X_4}{16} \cdots$$

$$q_1 = X_1 + X_2(\frac{1}{2} + \frac{1}{4} + \frac{1}{8} \cdots) - X_4(\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} \cdots)$$

$$\left(\frac{1}{2} + \frac{1}{4} + \frac{1}{8} \cdots\right) = \sum_{i=1}^{\infty} \frac{1}{2^i} = \sum_{i=0}^{\infty} \frac{1}{2^i} - \frac{1}{2^0} = \frac{1}{1 - \frac{1}{2}} - 1 = 1$$

$$q_1 = q_1^e = X_1 + X_2 - X_4 \tag{4.3}$$

Similarly, equilibrium quantity of private firm can be calculated by

$$q_2 = \frac{E_2(C_2)}{2} - \frac{E_2E_1(C_1)}{2} + \frac{E_2E_1E_2(C_2)}{4} - \frac{E_2E_1E_2E_1(C_1)}{4} + \frac{E_2E_1E_2E_1E_2(C_2)}{8} - \frac{E_2E_1E_2E_1E_2E_1(C_1)}{8} + \frac{E_2E_1E_2E_1E_2E_1E_2(C_2)}{16} - \frac{E_2E_1E_2E_1E_2E_1E_2E_1(C_1)}{16} \cdots$$

$$q_2 = \frac{X_3}{2} - X_2(\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} \cdots) + X_4(\frac{1}{4} + \frac{1}{8} + \frac{1}{16} \cdots)$$

$$\left(\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} \cdots\right) = \sum_{i=1}^{\infty} \frac{1}{2^i} = \sum_{i=0}^{\infty} \frac{1}{2^i} - \frac{1}{2^0} = \frac{1}{1 - \frac{1}{2}} - 1 = 1$$

$$\left(\frac{1}{4} + \frac{1}{8} + \frac{1}{16} \cdots\right) = \sum_{i=2}^{\infty} \frac{1}{2^i} = \sum_{i=0}^{\infty} \frac{1}{2^i} - \frac{1}{2^0} - \frac{1}{2^1} = \frac{1}{1 - \frac{1}{2}} - 1 - \frac{1}{2} = \frac{1}{2}$$

$$q_2 = q_2^e = \frac{X_3}{2} - X_2 + \frac{X_4}{2} \tag{4.4}$$

Table 34.1: X_1, X_2, X_3 and X_4 values for all possible cases

$Firm\ 1$ ($Public\ firm$)	$Firm\ 2$ ($Private\ firm$)		
	Do not produce information	Produce information, no disclosure	Produce and disclosure information
Do not produce information	***Case 1*** $X_1 = X_2 = \overline{C_1}$ $X_3 = X_4 = \overline{C_2}$	***Case 2*** $X_1 = X_2 = \overline{C_1}$ $X_3 = C_2$ $X_4 = \overline{C_2}$	***Case 3*** $X_1 = X_2$ $= \overline{C_1} + b_2 * \Delta C_2$ $X_3 = X_4 = C_2$
Produce information, no disclosure	***Case 4*** $X_1 = C_1$ $X_2 = \overline{C_1}$ $X_3 = X_4 = \overline{C_2}$	***Case 5***	***Case 6*** $X_1 = C_1$ $X_2 = \overline{C_1} + b_2 * \Delta C_2$ $X_3 = X_4 = C_2$
Produce and disclosure information	***Case 7*** $X_1 = X_2 = C_1$ $X_3 = X_4$ $= \overline{C_2} + b_1 * \Delta C_1$	***Case 8*** $X_1 = X_2 = C_1$ $X_3 = C_2$ $X_4 = \overline{C_2} + b_1 * \Delta C_1$	***Case 9*** $X_1 = X_2 = C_1$ $X_3 = X_4 = C_2$

4.4. Equilibrium Quantity Calculations

In this section, the equilibrium quantity values are derived for all cases individually. Defined X_1, X_2, X_3 and X_4 will be used except for case 5 in these derivations. For case 5, the main functions are used to find equilibrium quantities. Detailed solutions for all cases can be found in appendix 2. Results are summarized in *Table 4.2.*

Table 4.2: Table of equilibrium quantities q_1^e and q_2^e for all possible cases

$Firm\ 1$ $(Public\ firm)$	$Firm\ 2\ (Private\ firm)$		
	Do not produce information	Produce information, no disclosure	Produce and disclosure information
Do not produce information	***Case 1*** $q_1^* = 2\overline{C_1} - \overline{C_2}$ $q_2^* = \overline{C_2} - \overline{C_1}$	***Case 2*** $q_1^* = 2\overline{C_1} - \overline{C_2}$ $q_2^* = \frac{C_2}{2} - \overline{C_1} + \frac{\overline{C_2}}{2}$	***Case 3*** $q_1^* = 2\overline{C_1} + 2b_2\Delta C_2 - C_2$ $q_2^* = C_2 - \overline{C_1} - b_2\Delta C_2$
Produce information, no disclosure	***Case 4*** $q_1^* = C_1 + \overline{C_1} - \overline{C_2}$ $q_2^* = \overline{C_2} - \overline{C_1}$	***Case 5*** $q_1^* = 2\overline{C_1} - \overline{C_2} + \Delta C_1\left(\frac{2 - b_1}{2 - r^2}\right)$ $q_2^* = \overline{C_2} - \overline{C_1} + \Delta C_2\left(\frac{1 - b_2}{2 - r^2}\right)$	***Case 6*** $q_1^* = C_1 + \overline{C_1} + b_2\Delta C_2 - C_2$ $q_2^* = C_2 - \overline{C_1} - b_2\Delta C_2$
Produce and disclosure information	***Case 7*** $q_1^* = 2C_1 - \overline{C_2} - b_1\Delta C_1$ $q_2^* = \overline{C_2} - C_1 + b_1\Delta C_1$	***Case 8*** $q_1^* = 2C_1 - \overline{C_2} - b_1\Delta C_1$ $q_2^* = \frac{C_2}{2} + \frac{\overline{C_2}}{2} - C_1 + \frac{b_1 * \Delta C_1}{2}$	***Case 9*** $q_1^* = 2C_1 - C_2$ $q_2^* = C_2 - C_1$

4.5. The Excess Equilibrium Quantities' Calculations

To compare results relative to "do not produce information" case for both firms, the excess equilibrium quantities' calculations are performed. The formulation is given as

$$q_1^+ = q_1^* - \bar{q}_1$$
$$q_2^+ = q_2^* - \bar{q}_2$$

where q_1^+ is the "excess" equilibrium quantity for public firm and q_2^+ is the "excess" equilibrium quantity for private firm relative to no information case equilibrium quantity $\bar{q}_1$ and $\bar{q}_2$ respectively. Solutions are explained in detail in appendix 3.

Table 4.3: Table of equilibrium quantities q_1^+ and q_2^+ for all possible cases

$Firm\ 1$ $(Public\ firm)$	$Firm\ 2\ (Private\ firm)$		
	Do not produce information	Produce information, no disclosure	Produce and disclosure information
Do not produce information	***Case 1*** $q_1^+ = 0$ $q_2^+ = 0$	***Case 2*** $q_1^+ = 0$ $q_2^+ = \frac{\Delta C_2}{2}$	***Case 3*** $q_1^+ = (2b_2 - 1) * \Delta C_2$ $q_1^+ = (1 - b_2) * \Delta C_2$
Produce information, no disclosure	***Case 4*** $q_1^+ = \Delta C_1$ $q_2^+ = 0$	***Case 5*** $q_1^+ = \left(\frac{2 - b_1}{2 - r^2}\right)\Delta C_1$ $q_2^+ = \left(\frac{1 - b_2}{2 - r^2}\right)\Delta C_2$	***Case 6*** $q_1^+ = \Delta C_1 + (b_2 - 1)\Delta C_2$ $q_2^+ = (1 - b_2) * \Delta C_2$
Produce and disclosure information	***Case 7*** $q_1^+ = (2 - b_1) * \Delta C_1$ $q_2^+ = (b_1 - 1) * \Delta C_1$	***Case 8*** $q_1^+ = (2 - b_1) * \Delta C_1$ $q_2^+ = \frac{(\Delta C_2)}{2} + \left(\frac{b_1 - 2}{2}\right)\Delta C_1$	***Case 9*** $q_1^+ = 2 * \Delta C_1 - \Delta C_2$ $q_2^+ = \Delta C_2 - \Delta C_1$

4.6. Welfare Calculation and Profit Calculations for the Public and Private Firms.

After the equilibrium quantities are found, welfare and profit calculations are performed at equilibrium quantities for all case. Firstly, the calculations are done for the public firm to find welfare for all case. Also, excess welfare calculations are calculated to compare the results. Secondly, public firm's profits calculations are completed. Similarly, excess profits are calculated for comparison of the results by case 1 "No produce information".

4.6.1 Welfare calculation for the public firms.

Welfare calculation are performed at equilibrium quantities for "no produce information", "produce but no disclosure information" and "produce and disclosure information" cases for the public firm.

Remember the welfare function of public firm

$$W = A * (q_1 + q_2) - \frac{(q_1 + q_2)^2}{2} - m_1 q_1 - m_2 q_2$$

Expected welfare function can be calculated for an information sets $I_1 = (.,.)$ and $I_2 = (.,.)$ for equilibrium output quantities q_1^e and q_2^e which are already calculated in a certain information sets $I_1 = (.,.)$ and $I_2 = (.,.)$.

$$E(W|I_1) = E\left(\left(Aq_1^e - m_1 q_1^e + Aq_2^e - m_2 q_2^e - \frac{(q_1^e + q_2^e)^2}{2}\right) \middle| I_1\right)$$

By substitution of $C_1 = A - m_1$ and $C_2 = A - m_2$, it yields

$$E(W|I_1) = E\left(\left(C_1 q_1^e + C_2 q_2^e - \frac{(q_1^e + q_2^e)^2}{2}\right) \middle| I_1\right)$$

Table 4.4: Table of excess welfare W^+ quantities q_1^e and q_2^e for all possible cases

$Firm\ 1\ (Public\ firm)$	$Firm\ 2\ (Private\ firm)$		
	Do not produce information	Produce information, no disclosure	Produce and disclosure information
Do not produce information	***Case 1*** $W^{+} = 0$	***Case 2*** $W^{+} = \frac{3V_2^2}{8}$	***Case 3*** $W^{+} = (1-b_2)^2 V_2^2 + \frac{b_2^2 V_2^2}{2}$
Produce information, no disclosure	***Case 4*** $W^{+} = \frac{V_1^2}{2}$	***Case 5*** $W^{+} = V_2^2 \frac{3-r^2}{2(r^2-2)^2} + V_1^2 \frac{4-r^2}{2(r^2-2)^2} - 2V_{12} \frac{3-r^2}{2(r^2-2)^2}$	***Case 6*** $W^{+} = \frac{V_1^2}{2} + (1-b_2)^2 V_2^2$
Produce and disclosure information	***Case 7*** $W^{+} = \frac{V_1^2}{2} + V_1^2(1-b_1)^2$	***Case 8*** $W^{+} = \frac{V_1^2(5b_1-6)(b_1-2)}{8} + \frac{3V_2^2}{8}$	***Case 9*** $W^{+} = \frac{3V_1^2}{2} - V_2^2(1-2b_2)$

4.6.2 Profit calculation for the private firms.

Profit calculation are performed at equilibrium quantities for “no produce information”, “produce but no disclosure information” and “produce and disclosure information” cases for the private firm.

Remember the profit function of public firm;

$$\Pi_2 = (A - m_2 - q_1 - q_2) * q_2$$

Expected profit function can be calculated for an information sets set $I_1 = (.,.)$ and $I_2 = (.,.)$ for equilibrium output quantities q_1^e and q_2^e which are already calculated in a certain information sets $I_1 = (.,.)$ and $I_2 = (.,.)$.

$$E(\Pi_2|I_1) = E\left(\left((A - m_2 - q_1^e - q_2^e) * q_2^e\right)\middle|I_1\right)$$

By substitution of $C_2 = A - m_2$, it yields

$$E(\Pi_2|I_1) = E\left(\left((C_2 - q_1^e - q_2^e) * q_2^e\right)\middle|I_1\right)$$

$$E(\Pi_2|I_1) = E\left(\left(C_2 q_2^e - {q_2^e}^2 - q_1^e q_2^e\right)\middle|I_1\right)$$

Table 4.5: Table of excess profit of private firm $\prod_2^+$ at quantities q_1^e and q_2^e for all possible cases

$Firm\ 1\ (Public\ firm)$	$Firm\ 2\ (Private\ firm)$		
	Do not produce information	Produce information, no disclosure	Produce and disclosure information
Do not produce information	***Case 1*** $\Pi_2{}^+ = 0$	***Case 2*** $\Pi_2{}^+ = \frac{V_2^2}{4}$	***Case 3*** $\Pi_2{}^+ = V_2^2(1-2b_2)^2$
Produce information, no disclosure	***Case 4*** $\Pi_2{}^+ = 0$	***Case 5*** $\Pi_2{}^+ = V_2^2\frac{(1-b_2)^2}{(2-r^2)^2}$	***Case 6*** $\Pi_2{}^+ = V_2^2(1-b_2)^2$
Produce and disclosure information	***Case 7*** $\Pi_2{}^+ = V_1^2(1-b_1)^2$	***Case 8*** $\Pi_2{}^+ = V_1^2(1-b_1)^2 + \frac{V_2^2(1-r^2)}{4}$	***Case 9*** $\Pi_2{}^+ = V_2^2(1-r^2) + V_1^2(1-b_1)^2$

4.7. Results

Table 4.6: Table of excess profit of public firm W^+ and $\prod_2^+$ the profit of the public firm quantities q_1^e and q_2^e for all possible cases

$Firm\ 1$ ($Public\ firm$)	$Firm\ 2$ ($Private\ firm$)		
	Do not produce information	Produce information, no disclosure	Produce and disclosure information
Do not produce information	***Case 1*** $W^+ = 0$ $\Pi_2{}^+ = 0$	***Case 2*** $W^+ = \frac{3V_2^2}{8}$ $\Pi_2{}^+ = \frac{V_2^2}{4}$	***Case 3*** $W^+ = V_2^2 + \frac{3V_1^2 r^2}{2} - 2V_{12}$ $\Pi_2{}^+ = V_2^2 + V_1^2 r^2 - 2V_{12}$
Produce information, no disclosure	***Case 4*** $W^+ = \frac{V_1^2}{2}$ $\Pi_2{}^+ = 0$	***Case 5*** $W^+ = V_2^2 \frac{3 - r^2}{2(r^2-2)^2} + V_1^2 \frac{4 - r^2}{2(r^2-2)^2} - 2V_{12} \frac{3 - r^2}{2(r^2-2)^2}$ $\Pi_2{}^+ = \frac{V_2^2 + V_1^2 r^2 - 2V_{12}}{(2-r^2)^2}$	***Case 6*** $W^+ = V_1^2 \left(\frac{1+2r^2}{2}\right) + V_2^2 - 2V_{12}$ $\Pi_2{}^+ = V_2^2 + V_1^2 r^2 - 2V_{12}$
Produce and disclosure information	***Case 7*** $W^+ = \frac{3V_1^2}{2} - 2V_{12} + V_2^2 r^2$ $\Pi_2{}^+ = V_1^2 + V_2^2 r^2 - 2V_{12}$	***Case 8*** $W^+ = \frac{3V_1^2}{2} - 2V_{12} + V_2^2 \left(\frac{3+5r^2}{8}\right)$ $\Pi_2{}^+ = V_1^2 + V_2^2 \frac{1+3r^2}{4} - 2V_{12}$	***Case 9*** $W^+ = \frac{3V_1^2}{2} - 2V_{12} + V_2^2$ $\Pi_2{}^+ = V_2^2 - 2\,V_{12} + V_1^2$

Payoff matrix for information production-disclosure gaming decision is given in Table 4.6. Before the detail explanations of results, by the first inspection, if the correlation coefficient between the cost functions is perfect ($r = 1$ or $r \neq 1$), there are multiple equilibria in which both firms disclose their own cost functions or one of them disclose, while the other does not. If $r \neq 1$, none of the firms disclosing its cost function might also be an equilibrium. The payoff matrix of perfect correlation and perfect anti-correlation is given in appendix 6.

Lemma 1: *It is always best interest of the private firm to produce information about its own cost function. Moreover, it is best interest of the private firm to disclose its cost function in an environment where the following condition holds:*

$$r \geq \frac{3*V_2}{2*V_1} \quad or \quad r \leq \frac{V_2}{2*V_1} \qquad \textit{(4.5)}$$

Proof 1: To prove the first part of this lemma we need to show that profit of the private firm while producing information is equal or greater than its profit while not to producing information, given the public firm's all possible choices. It means that profit of the private firm in Case 2 $\geq$ Case 1 (1a), Case 5 $\geq$ Case 4 (1b) and Case 8 $\geq$ Case 7 (1c).

(1a) $$0 \leq \frac{V_2^2}{4}$$

(1b) $$0 \leq \frac{V_2^2+V_1^2r^2-2V_1V_2r}{(2-r^2)^2} = \left(\frac{V_2-V_1r}{2-r^2}\right)^2$$

(1c) $$V_1^2 + V_2^2r^2 - 2V_1V_2r \leq V_1^2 + V_2^2\frac{1+3r^2}{4} - 2V_1V_2r$$

(1a) and (1b) clearly holds. (1c) holds because correlation coefficient is equal or lower than 1. In other words;

$$V_1^2 + V_2^2r^2 - 2V_1V_2r \leq V_1^2 + V_2^2\frac{1+3r^2}{4} - 2V_1V_2r \xrightarrow{yields} r^2 \leq \frac{1+3r^2}{4}$$

And $r^2 \leq \frac{1+3r^2}{4}$ is always true where $-1 \leq r \leq 1$

To prove second part of the lemma, suppose that *(4.6)* holds. There are three conditions that need to be satisfied. These conditions showing that private firm's profit while disclosing its cost function is greater than or equal to that of while not disclosing: Case 3 $\geq$ Case 2 (1d), Case 6 $\geq$ Case 5 (1e) and Case 9 $\geq$ Case 8 (1f).

(1d) $$\frac{V_2^2}{4} \leq V_2^2 + V_1^2 r^2 - 2V_1V_2r$$

(1e) $$\frac{V_2^2+V_1^2r^2-2V_1V_2r}{(2-r^2)^2} = \left(\frac{V_2-V_1r}{2-r^2}\right)^2 \leq V_2^2 + V_1^2r^2 - 2V_1V_2r = (V_2 - V_1r)^2$$

(1f) $$V_1^2 + V_2^2\frac{1+3r^2}{4} - 2V_1V_2r \leq V_1^2 + V_2^2 - 2V_1V_2r$$

(1e) and (1f) holds because correlation coefficient is between -1 and 1($-1 \leq r \leq 1$). For (1d);

$$\frac{V_2^2}{4} \leq V_2^2 + V_1^2r^2 - 2V_1V_2r \xrightarrow{yields} 0 \leq \frac{(3V_2^2 - 2V_1r)(V_2^2 - 2V_2r)}{4}$$

Thus, (1e) is correct if *(4.7)* holds.

Lemma 2*: In an environment where correlation coefficient of cost functions is equal to or less than zero ($r \leq 0$), it is best interest of the public firm to produce information about its own cost function and to disclose it.*

Proof 2: If disclosure of information is best interest of the public firm, then following equations should be correct: Case 1 $\leq$ Case 4 $\leq$ Case 7 (2a), Case 2 $\leq$ Case 5 $\leq$ Case 8 (2b) and Case 3 $\leq$ Case 6 $\leq$ Case 9 (2c).

(2a) $$0 \leq \frac{V_1^2}{2} \leq \frac{3V_1^2}{2} - 2V_1V_2r + V_2^2r^2$$

(2b) $$\frac{3V_2^2}{8} \leq V_2^2\frac{3-r^2}{2(r^2-2)^2} + V_1^2\frac{4-r^2}{2(r^2-2)^2} - 2V_1V_2r\frac{3-r^2}{2(r^2-2)^2} \leq \frac{3V_1^2}{2} - 2V_1V_2r + V_2^2\left(\frac{3+5r^2}{8}\right)$$

(2c) $$V_2^2 + \frac{3V_1^2r^2}{2} - 2V_1V_2r \leq V_1^2\left(\frac{1+2r^2}{2}\right) + V_2^2 - 2V_1V_2r \leq \frac{3V_1^2}{2} - 2V_1V_2r + V_2^2$$

All these three conditions are true if the correlation coefficient is equal to or less than zero. (2a) is obvious because $\frac{V_1^2}{2} \leq \frac{3V_1^2}{2}$ and the terms $-2V_1V_2r$ and $V_2^2r^2$ are certainly positive. Welfare comparisons given conditions at (2b) and (2c) completes the proof.

Lemma 3: *In an environment where the public firm cannot produce information about its own cost function, the private firm is better off disclosing its own cost function if and only if the (4.5) holds.*

Proof 3: If the public firm cannot produce information, the private firm decides among actions of which payoffs are represented at cases 1, 2, and 3. Comparison of profits of the private firm at cases 1, 2 and 3 shows that its profit while disclosing its cost function (case 3) is greater than or equal than that in cases 2 and 1.

Lemma 4: *In an environment where the private firm cannot produce information about its own cost function, the public firm is better off disclosing its own cost function.*

Proof 4: If the private firm cannot produce information, the public firm decides among actions of which payoffs are represented at cases 1, 4, and 7. Comparison of welfare at cases 1, 4 and 7 shows that welfare while the public firm disclosing its cost function (case 7) is greater than or equal than that in cases 4 and 1.

Lemma 5: *If one of the firm discloses its own cost function, the other firm is also better off disclosing its own cost function.*

Proof 5: There is two part for this lemma. First, suppose that the public firm discloses its cost function. We need to show that profit of the private firm while disclosing its cost function is equal or greater than that yielding from other two actions. That means the profit of the private firm in Case 7 $\leq$ Case 9 (5a) and Case 8 $\leq$ Case 9 (5b). Second, suppose that the private firm discloses its cost function. Now, we need to show that welfare while the public firm discloses its costs function is equal or greater than that while the public firm employing other two actions. That means welfare in Case 3 $\leq$ Case 9 (5c) and Case 6 $\leq$ Case 9 (5d).

(5a) $$V_1^2 + V_2^2 r^2 - 2V_1V_2r \leq V_1^2 + V_2^2 - 2V_1V_2r$$

(5b) $$V_1^2 + V_2^2 \frac{1+3r^2}{4} - 2V_1V_2r \leq V_1^2 + V_2^2 - 2V_1V_2r$$

(5c) $$V_2^2 + \frac{3V_1^2r^2}{2} - 2V_1V_2r \leq \frac{3V_1^2}{2} - 2V_1V_2r + V_2^2$$

(5d) $$V_1^2\left(\frac{1+2r^2}{2}\right) + V_2^2 - 2V_1V_2r \leq \frac{3V_1^2}{2} - 2V_1V_2r + V_2^2$$

After simplifying inequalities; (5a) yields $r^2 \leq 1$, (5b) yields $\frac{1+3r^2}{4} \leq 1$, (5c) yields $\frac{3r^2}{2} \leq \frac{3}{2}$, (5d) yields $\left(\frac{1+2r^2}{2}\right) \leq \frac{3}{2}$. Now, it is clear that all equations hold for $(-1 \leq \mathrm{r} \leq 1)$.

Proposition 1: *Suppose that (4.5) holds. Disclosing own cost function for both firms is an equilibrium of the two-stage game. Moreover, if correlation coefficient between cost functions is imperfect (*$r \neq 1$ *or* $r \neq -1$*), this equilibrium is unique.*

Proof: We can use lemma 1 and lemma 5 to prove the proposition. Lemma 1 shows that if (4.5) holds, disclosure information is weakly dominant strategy. Moreover, if the correlation coefficient is imperfect, disclosure becomes strictly dominant strategy for the private firm. Secondly, in the proof of lemma 5, it is shown that if the private firm chooses the disclosure its cost function, disclosure becomes best response of the public firm. Therefore, the equilibrium is case 9 where both firms disclose their cost function if (4.5) holds.

Proposition 2: *No disclosure of cost functions for both firms might be an equilibrium only if the correlation coefficient is imperfect and the following condition holds:*

$$\frac{V2}{2*V1} \leq r \leq \frac{3*V2}{2*V1} \tag{4.8}$$

Proof: If (4.6) holds, no disclosure is the best for the private firm (Case 3 $\leq$ Case 2). Comparison of welfare calculation of public firms in the same interval, there are at least one region that no disclosure is, also, dominated strategy for the public firm. If (4.7) holds, no produce information is the dominant strategy for the public (Case 5 $\leq$ Case 2 and Case 8 $\leq$ Case 2).

$$\sqrt{\frac{12}{5}} < \frac{V2}{V1} < 2 \tag{4.9}$$

Therefore, if both (4.6) and (4.7) holds at the same time, case 2 becomes another equilibrium which is different from the case 9. In other words, disclosure is not the unique equilibrium anymore and no disclosure might be an equilibrium.

5. CONCLUSION

In many sectors, state-owned public firms and private firms compete against each other. Mixed oligopolies differ from oligopolies involving only private firms in that public firms in mixed oligopolies take both its own profit and social welfare into account. Therefore, strategic behaviors of the firms in the mixed oligopolies under uncertain environment differ from the pure private oligopolies. In this thesis, we examined how information sharing decisions of the public and privates firms affect the equilibrium outcomes.

Some of the results that we have found are parallel to the studies in the literature. Our results are showed that if the correlation coefficient is equal or less than zero, disclosure information for public and private firms is the best interest as the same as the previous studies in the literature about pure oligopolies.

On the other hand, there are different results for the positive correlation coefficient. In the literature, researchers show that the best interest is generally to disclose own cost function in pure oligopolies. However, we have found that there are some certain conditions that it is not true for mixed oligopolies. In other words, no disclosure of the cost functions can be an equilibrium in some certain condition. This outcome is very new in the literature which proves that equilibriums and the strategic behaviors might be changed when a public firm is includes a private oligopolies.

This study can be further extended by examining the different environments and assumptions. For example, public and privates firms' strategic behaviors can be examined under demand uncertainty or instead of Cournot competition companies' response can be examined in the Bernard competition or instead of mixed duopoly a mixed oligopoly with n number firms can be studied etc..

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BIOGRAPHY

Sadık Hazer was born in 1989 in Denizli, Turkey. He completed primary education and high school in Denizli. He graduated from Sabancı University Department of Mechatronics Engineer in 2013 (with honors). In addition to this, he started second university in Anadolu University Department of Economics in 2011. This is the other area he wanted to learn and he successfully graduated from this department in 2015. In 2014, he started his master's degree in Science of Strategy at Gebze Technical University. He is still working as research and development Engineer in SDM Sıradışı Arge and Mühendislik in Istanbul, which he has started after his graduation in 2013.

APPENDIX

APPENDIX 1. Detailed Solution of Equilibriums

For public firm's quantity

$$q_1 = E_1(C_1) - E_1(q_2)$$

$$q_1 = E_1(C_1) - E_1(\frac{E_2(C_2) - E_2(q_1)}{2})$$

$$q_1 = E_1(C_1) - \frac{E_1E_2(C_2)}{2} + \frac{E_1E_2(q_1)}{2}$$

$$q_1 = E_1(C_1) - \frac{E_1E_2(C_2)}{2} + \frac{E_1E_2(E_1(C_1) - \frac{E_1E_2(C_2)}{2} + \frac{E_1E_2(q_1)}{2})}{2}$$

$$q_1 = E_1(C_1) - \frac{E_1E_2(C_2)}{2} + \frac{E_1E_2E_1(C_1)}{2} - \frac{E_1E_2E_1E_2(C_2)}{4} + \frac{E_1E_2E_1E_2(q_1)}{4}$$

$$q_1 = E_1(C_1) - \frac{E_1E_2(C_2)}{2} + \frac{E_1E_2E_1(C_1)}{2} - \frac{E_1E_2E_1E_2(C_2)}{4} + \frac{E_1E_2E_1E_2\left(E_1(C_1) - \frac{E_1E_2(C_2)}{2} + \frac{E_1E_2(q_1)}{2}\right)}{4}$$

$$q_1 = E_1(C_1) - \frac{E_1E_2(C_2)}{2} + \frac{E_1E_2E_1(C_1)}{2} - \frac{E_1E_2E_1E_2(C_2)}{4} + \frac{E_1E_2E_1E_2E_1(C_1)}{4} - \frac{E_1E_2E_1E_2E_1E_2(C_2)}{8} + \frac{E_1E_2E_1E_2E_1E_2(q_1)}{8}$$

$$q_1 = E_1(C_1) - \frac{E_1E_2(C_2)}{2} + \frac{E_1E_2E_1(C_1)}{2} - \frac{E_1E_2E_1E_2(C_2)}{4} + \frac{E_1E_2E_1E_2E_1(C_1)}{4} - \frac{E_1E_2E_1E_2E_1E_2(C_2)}{8} + \frac{E_1E_2E_1E_2E_1E_2\left(E_1(C_1) - \frac{E_1E_2(C_2)}{2} + \frac{E_1E_2(q_1)}{2}\right)}{8}$$

$$q_1 = E_1(C_1) - \frac{E_1E_2(C_2)}{2} + \frac{E_1E_2E_1(C_1)}{2} - \frac{E_1E_2E_1E_2(C_2)}{4} + \frac{E_1E_2E_1E_2E_1(C_1)}{4} - \frac{E_1E_2E_1E_2E_1E_2(C_2)}{8} + \frac{E_1E_2E_1E_2E_1E_2E_1(C_1)}{8} - \frac{E_1E_2E_1E_2E_1E_2E_1E_2(C_2)}{16} + \frac{E_1E_2E_1E_2E_1E_2E_1E_2(q_1)}{16}$$

$$q_1 = E_1(C_1) - \frac{E_1E_2(C_2)}{2} + \frac{E_1E_2E_1(C_1)}{2} - \frac{E_1E_2E_1E_2(C_2)}{4} + \frac{E_1E_2E_1E_2E_1(C_1)}{4} - \frac{E_1E_2E_1E_2E_1E_2(C_2)}{8} + \frac{E_1E_2E_1E_2E_1E_2E_1(C_1)}{8} - \frac{E_1E_2E_1E_2E_1E_2E_1E_2(C_2)}{16} \ldots$$

For private firm's quantity

$$q_2 = \frac{E_2(C_2) - E_2(q_1)}{2}$$

$$q_2 = \frac{E_2(C_2) - E_2(E_1(C_1) - E_1(q_2))}{2}$$

$$q_2 = \frac{E_2(C_2)}{2} - \frac{E_2E_1(C_1)}{2} + \frac{E_2E_1(q_2)}{2}$$

$$q_2 = \frac{E_2(C_2)}{2} - \frac{E_2E_1(C_1)}{2} + \frac{E_2E_1(\frac{E_2(C_2)}{2} - \frac{E_2E_1(C_1)}{2} + \frac{E_2E_1(q_2)}{2})}{2}$$

$$q_2 = \frac{E_2(C_2)}{2} - \frac{E_2E_1(C_1)}{2} + \frac{E_2E_1E_2(C_2)}{4} - \frac{E_2E_1E_2E_1(C_1)}{4} + \frac{E_2E_1E_2E_1(q_2)}{4}$$

$$q_2 = \frac{E_2(C_2)}{2} - \frac{E_2E_1(C_1)}{2} + \frac{E_2E_1E_2(C_2)}{4} - \frac{E_2E_1E_2E_1(C_1)}{4} + \frac{E_2E_1E_2E_1(\frac{E_2(C_2)}{2} - \frac{E_2E_1(C_1)}{2} + \frac{E_2E_1(q_2)}{2})}{4}$$

$$q_2 = \frac{E_2(C_2)}{2} - \frac{E_2E_1(C_1)}{2} + \frac{E_2E_1E_2(C_2)}{4} - \frac{E_2E_1E_2E_1(C_1)}{4} + \frac{E_2E_1E_2E_1E_2(C_2)}{8} - \frac{E_2E_1E_2E_1E_2E_1(C_1)}{8} + \frac{E_2E_1E_2E_1E_2E_1(q_2)}{8}$$

$$q_2 = \frac{E_2(C_2)}{2} - \frac{E_2E_1(C_1)}{2} + \frac{E_2E_1E_2(C_2)}{4} - \frac{E_2E_1E_2E_1(C_1)}{4} + \frac{E_2E_1E_2E_1E_2(C_2)}{8} - \frac{E_2E_1E_2E_1E_2E_1(C_1)}{8} + \frac{E_2E_1E_2E_1E_2E_1(q_2)}{8}$$

$$q_2 = \frac{E_2(C_2)}{2} - \frac{E_2E_1(C_1)}{2} + \frac{E_2E_1E_2(C_2)}{4} - \frac{E_2E_1E_2E_1(C_1)}{4} + \frac{E_2E_1E_2E_1E_2(C_2)}{8} - \frac{E_2E_1E_2E_1E_2E_1(C_1)}{8} + \frac{E_2E_1E_2E_1E_2E_1(\frac{E_2(C_2)}{2} - \frac{E_2E_1(C_1)}{2} + \frac{E_2E_1(q_2)}{2})}{8}$$

$$q_2 = \frac{E_2(C_2\,)}{2} - \frac{E_2E_1(C_1)}{2} + \frac{E_2E_1E_2(C_2\,)}{4} - \frac{E_2E_1E_2E_1(C_1)}{4} + \frac{E_2E_1E_2E_1E_2(C_2\,)}{8}$$
$$- \frac{E_2E_1E_2E_1E_2E_1(C_1)}{8} + \frac{E_2E_1E_2E_1E_2E_1E_2(C_2\,)}{16}$$
$$- \frac{E_2E_1E_2E_1E_2E_1E_2E_1(C_1)}{16} + \frac{E_2E_1E_2E_1E_2E_1E_2E_1(q_2)}{16}$$

$$q_2 = \frac{E_2(C_2\,)}{2} - \frac{E_2E_1(C_1)}{2} + \frac{E_2E_1E_2(C_2\,)}{4} - \frac{E_2E_1E_2E_1(C_1)}{4} + \frac{E_2E_1E_2E_1E_2(C_2\,)}{8}$$
$$- \frac{E_2E_1E_2E_1E_2E_1(C_1)}{8} + \frac{E_2E_1E_2E_1E_2E_1E_2(C_2\,)}{16}$$
$$- \frac{E_2E_1E_2E_1E_2E_1E_2E_1(C_1)}{16} \cdots$$

APPENDIX 2. Detailed Solution of Equilibrium Quantity Calculations

Case 1: Public and private firms do not produce information in Cell (1, 1)

Information set for case 1

$$I_1 = (\overline{C_1}, \overline{C_2})$$
$$I_2 = (\overline{C_1}, \overline{C_2})$$

X_1, X_2, X_3 and X_4 values for case 1

$$X_1 = X_2 = \overline{C_1}$$
$$X_3 = X_4 = \overline{C_2}$$

Equilibrium solution for q_1^e

$$q_1^e = X_1 + X_2 - X_4$$
$$q_1^e = \overline{C_1} + \overline{C_1} - \overline{C_2}$$
$$q_1^e = 2 * \overline{C_1} - \overline{C_2}$$
$$q_1^e = q_1^* = \bar{q}_1 = 2 * \overline{C_1} - \overline{C_2}$$

Equilibrium solution for q_2^e

$$q_2^e = \frac{X_3}{2} - X_2 + \frac{X_4}{2}$$
$$q_2^e = \frac{\overline{C_2}}{2} - \overline{C_1} + \frac{\overline{C_2}}{2}$$
$$q_2^e = \overline{C_2} - \overline{C_1}$$
$$q_2^e = q_2^* = \bar{q}_2 = \overline{C_2} - \overline{C_1}$$

Case 2: Public firm does not produce information but private firm produces information without disclosure in Cell (1, 2)

Information set for case 2

$$I_1 = (\overline{C_1}, \overline{C_2})$$
$$I_2 = (\overline{C_1}, C_2)$$

X_1, X_2, X_3 and X_4 values for case 2

$$X_1 = X_2 = \overline{C_1}$$
$$X_3 = C_2$$
$$X_4 = \overline{C_2}$$

Equilibrium solution for q_1^e

$$q_1^e = X_1 + X_2 - X_4$$
$$q_1^e = \overline{C_1} + \overline{C_1} - \overline{C_2}$$
$$q_1^e = q_1^* = 2 * \overline{C_1} - \overline{C_2}$$

Equilibrium solution for q_2^e

$$q_2^e = \frac{X_3}{2} - X_2 + \frac{X_4}{2}$$
$$q_2^e = q_2^* = \frac{C_2}{2} - \overline{C_1} + \frac{\overline{C_2}}{2}$$

Case 3: Public firm does not produce information but private firm produces and disclosures information in Cell (1, 3)

Information set for case 3

$$I_1 = (\overline{C_1}, C_2)$$
$$I_2 = (\overline{C_1}, C_2)$$

X_1, X_2, X_3 and X_4 values for case 3

$$X_1 = X_2 = \overline{C_1} + b_2 * \Delta C_2$$
$$X_3 = X_4 = C_2$$

Equilibrium solution for q_1^e

$$q_1^e = X_1 + X_2 - X_4$$
$$q_1^e = \overline{C_1} + b_2 * \Delta C_2 + \overline{C_1} + b_2 * \Delta C_2 - C_2$$
$$q_1^e = q_1^* = 2 * \overline{C_1} + 2 * b_2 * (C_2 - \overline{C_2}) - C_2$$

Equilibrium solution for q_2^e

$$q_2^e = \frac{X_3}{2} - X_2 + \frac{X_4}{2}$$
$$q_2^e = \frac{C_2}{2} - (\overline{C_1} + b_2 * \Delta C_2) + \frac{C_2}{2}$$
$$q_2^e = q_2^* = C_2 - \overline{C_1} - b_2 * \Delta C_2$$
$$q_2^e = q_2^* = C_2 - \overline{C_1} - b_2 * (C_2 - \overline{C_2})$$

Case 4: Public firm produces information without disclosure but private firm does not produce information in Cell (2, 1)

Information set for case 4

$$I_1 = (C_1, \overline{C_2})$$

$I_2 = (\overline{C_1}, \overline{C_2})$

X1, X2, X3 and X4 values for case 4

$$X_1 = C_1$$
$$X_2 = \overline{C_1}$$
$$X_3 = X_4 = \overline{C_2}$$

Equilibrium solution for q_1^e

$$q_1^e = X_1 + X_2 - X_4$$
$$q_1^e = C_1 + \overline{C_1} - \overline{C_2}$$
$$q_1^e = q_1^* = C_1 + \overline{C_1} - \overline{C_2}$$

Equilibrium solution for q_2^e

$$q_2^e = \frac{X_3}{2} - X_2 + \frac{X_4}{2}$$
$$q_2^e = \frac{\overline{C_2}}{2} - \overline{C_1} + \frac{\overline{C_2}}{2}$$
$$q_2^e = q_2^* = \overline{C_2} - \overline{C_1}$$

Case 5: Both firms produce information without disclosure in Cell (2, 2)

Information set for case 5

$$I_1 = (C_1, \overline{C_2})$$
$$I_2 = (\overline{C_1}, C_2)$$

This case is a special case because both firm produce information but they do not disclose information. X_1, X_2, X_3 and X_4 values cannot be defined accurately different from other cases because estimation of other firm is not known exactly. In different words, the estimation of X_2 and X_4 is not possible for this case.

$$X_2 = E_2E_1(C_1) \neq E_1E_2E_1(C_1) \neq E_2E_1E_2E_1(C_1) \neq E_1E_2E_1E_2E_1(C_1) \neq \cdots$$
$$X_4 = E_1E_2(C_2) \neq E_2E_1E_2(C_2) \neq E_1E_2E_1E_2(C_2) \neq E_2E_1E_2E_1E_2(C_2) \neq \cdots$$

However, X_1, X_2, X_3 and X_4 are not needed for equilibrium solutions for this case. To calculate equilibrium solutions the very first q_1^e and q_2^e can be used. Which are

$$q_1 = E_1(C_1) - \frac{E_1E_2(C_2)}{2} + \frac{E_1E_2E_1(C_1)}{2} - \frac{E_1E_2E_1E_2(C_2)}{4} + \frac{E_1E_2E_1E_2E_1(C_1)}{4} - \frac{E_1E_2E_1E_2E_1E_2(C_2)}{8} + \frac{E_1E_2E_1E_2E_1E_2E_1(C_1)}{8} - \frac{E_1E_2E_1E_2E_1E_2E_1E_2(C_2)}{16} \cdots$$

$$q_2 = \frac{E_2(C_2\ )}{2} - \frac{E_2E_1(C_1)}{2} + \frac{E_2E_1E_2(C_2)}{4} - \frac{E_2E_1E_2E_1(C_1)}{4} + \frac{E_2E_1E_2E_1E_2(C_2)}{8} - \frac{E_2E_1E_2E_1E_2E_1(C_1)}{8} + \frac{E_2E_1E_2E_1E_2E_1E_2(C_2)}{16} - \frac{E_2E_1E_2E_1E_2E_1E_2E_1(C_1)}{16}$$

Equilibrium solution for q_1^e

$$q_1^e = E_1(C_1) - \frac{E_1E_2(C_2)}{2} + \frac{E_1E_2E_1(C_1)}{2} - \frac{E_1E_2E_1E_2(C_2)}{4} + \frac{E_1E_2E_1E_2E_1(C_1)}{4} - \frac{E_1E_2E_1E_2E_1E_2(C_2)}{8} + \frac{E_1E_2E_1E_2E_1E_2E_1(C_1)}{8} - \frac{E_1E_2E_1E_2E_1E_2E_1E_2(C_2)}{16}$$

$$E_1(C_1) = C_1$$

$$E_1E_2(C_2\ ) = \overline{C_2} + b_1\Delta C_1$$

$$E_1E_2E_1(C_1) = E_1(E_2E_1(C_1)) = E_1(\overline{C_1} - b_2 * \Delta C_2) = E_1(\overline{C_1}) - E_1(b_2\Delta C_2)$$

C_2 is not known by public firm so public firm should use its estimation. Public firm's C_2 estimation which is $E_1(C_2)$= $\overline{C_2} - b_1 * \Delta C_1$.

$$\begin{aligned} E_1E_2E_1(C_1) &= E_1(\overline{C_1}) - E_1(b_2 * \Delta C_2) = E_1(\overline{C_1}) - E_1 * b_2 * (C_2 - \overline{C_2}) \\ &= E_1(\overline{C_1}) - E_1(b_2C_2) + E_1(b_2\overline{C_2}) = E_1(\overline{C_1}) - E_1(b_2\overline{C_2}) \\ &+ E_1(b_2C_2) = E_1(\overline{C_1}) - E_1(b_2\overline{C_2}) + b_2 * (\overline{C_2} - b_1\Delta C_1) \\ &= \overline{C_1} - b_2\overline{C_2} + b_2\overline{C_2} - b_1b_2\Delta C_1 \end{aligned}$$

$$E_1E_2E_1(C_1) = \overline{C_1} + b_1b_2\Delta C_1 = \overline{C_1} + r^2\Delta C_1$$

Definition of estimation continues similarly

$$E_1E_2E_1E_2(C_2\ ) = E_1(E_2E_1E_2(C_2\ )) = \overline{C_2} + r^2b_1\Delta C_1$$

$$E_1E_2E_1E_2E_1(C_1) = E_1(E_2E_1E_2E_1(C_1)) = \overline{C_1} + r^4\Delta C_1$$

$$E_1E_2E_1E_2E_1E_2(C_2\ ) = E_1(E_1E_2E_1E_2(C_2\ )) = \overline{C_2} + r^4b_1\Delta C_1$$

$$E_1E_2E_1E_2E_1E_2E_1(C_1) = E_1(E_2E_1E_2E_1E_2E_1(C_1)) = \overline{C_1} + r^6\Delta C_1$$

$$E_1E_2E_1E_2E_1E_2E_1E_2(C_2) = E_1(E_2E_1E_2E_1E_2E_1E_2(C_2)) = \overline{C_2} + r^6b_1\Delta C_1$$

Substitution of q_1^e for given expected values

$$q_1^e = C_1 - \frac{\overline{C_2} + b_1\Delta C_1}{2} + \frac{\overline{C_1} + r^2\Delta C_1}{2} - \frac{\overline{C_2} + r^2b_1\Delta C_1}{4} + \frac{\overline{C_1} + r^4\Delta C_1}{4} - \frac{\overline{C_2} + r^4b_1\Delta C_1}{8} + \frac{\overline{C_1} + r^6\Delta C_1}{8} - \frac{\overline{C_2} + r^6b_1\Delta C_1}{16} \cdots$$

$$q_1^e = C_1 - \frac{\overline{C_2}}{2} - \frac{b_1\Delta C_1}{2} + \frac{\overline{C_1}}{2} + \frac{r^2\Delta C_1}{2} - \frac{\overline{C_2}}{4} - \frac{r^2 b_1\Delta C_1}{4} + \frac{\overline{C_1}}{4} + \frac{r^4\Delta C_1}{4} - \frac{\overline{C_2}}{8} - \frac{r^4 b_1\Delta C_1}{8} + \frac{\overline{C_1}}{8} + \frac{r^6\Delta C_1}{8} - \frac{\overline{C_2}}{16} - \frac{r^6 b_1\Delta C_1}{16} \cdots$$

$$q_1^e = \overline{C_1}\left(1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8}\cdots\right) - \overline{C_2}\left(\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16}\cdots\right) + \Delta C_1\left(1 + \frac{r^2}{2} + \frac{r^4}{4} + \frac{r^6}{8}\cdots\right) - b_1\Delta C_1\left(\frac{1}{2} + \frac{r^2}{4} + \frac{r^4}{8} + \frac{r^6}{16}\cdots\right)$$

$$\left(1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16}\cdots\right) = \sum_{i=0}^{\infty}\frac{1}{2^i} = \frac{1}{1-\frac{1}{2}} = 2$$

$$\left(\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16}\cdots\right) = \sum_{i=1}^{\infty}\frac{1}{2^i} = \sum_{i=0}^{\infty}\frac{1}{2^i} - \frac{1}{2^0} = \frac{1}{1-\frac{1}{2}} - 1 = 1$$

$$\left(1 + \frac{r^2}{2} + \frac{r^4}{4} + \frac{r^6}{8}\cdots\right) = \sum_{i=0}^{\infty}\left(\frac{r^2}{2}\right)^i = \frac{1}{1-\frac{r^2}{2}} = \frac{2}{2-r^2}$$

$$\left(\frac{1}{2} + \frac{r^2}{4} + \frac{r^4}{8} + \frac{r^6}{16}\cdots\right) = \frac{1}{2}\left(1 + \frac{r^2}{2} + \frac{r^4}{4} + \frac{r^6}{8}\cdots\right) = \frac{1}{2} * \frac{1}{1-\frac{r^2}{2}} = \frac{1}{2-r^2}$$

Substitution of q_1^e yields

$$q_1^e = 2 * \overline{C_1} - \overline{C_2} + \Delta C_1 * \frac{2}{2-r^2} - \Delta C_1 * \frac{b_1}{2-r^2}$$

$$q_1^e = 2 * \overline{C_1} - \overline{C_2} + \Delta C_1 * \left(\frac{2-b_1}{2-r^2}\right)$$

Equilibrium solution for q_2^e

$$q_2^e = \frac{E_2(C_2)}{2} - \frac{E_2E_1(C_1)}{2} + \frac{E_2E_1E_2(C_2)}{4} - \frac{E_2E_1E_2E_1(C_1)}{4} + \frac{E_2E_1E_2E_1E_2(C_2)}{8} - \frac{E_2E_1E_2E_1E_2E_1(C_1)}{8} + \frac{E_2E_1E_2E_1E_2E_1E_2(C_2)}{16} - \frac{E_2E_1E_2E_1E_2E_1E_2E_1(C_1)}{16}$$

$$E_2(C_2) = C_2$$

$$E_2E_1(C_1) = \overline{C_1} + b_2\Delta C_2$$

$$E_2E_1E_2(C_2) = E_2(E_1E_2(C_2)) = E_2(\overline{C_2} - b_1 * \Delta C_1) = E_2(\overline{C_2}) - E_2(b_1\Delta C_1)$$

C_1 is not known by private firm so private firm should use its estimation. Private firm's C_1 estimation which is $E_2(C_1)$= $\overline{C_1} - b_2 * \Delta C_2$

$$E_2E_1E_2(C_2) = E_2(\overline{C_2}) - E_2(b_1 * \Delta C_1) = E_2(\overline{C_2}) - E_2 * b_1 * (C_1 - \overline{C_1})$$
$$= E_2(\overline{C_2}) - E_2(b_1C_1) + E_2(b_1\overline{C_1}) = E_2(\overline{C_2}) - E_2(b_1\overline{C_1})$$
$$+ E_2(b_1C_1) = E_2(\overline{C_2}) - E_2(b_1\overline{C_1}) + b_1 * (\overline{C_1} - b_2\Delta C_2)$$
$$= \overline{C_2} - b_1\overline{C_1} + b_1\overline{C_1} - b_1b_2\Delta C_2$$
$$E_2E_1E_2(C_2) = \overline{C_2} + b_1b_2\Delta C_2 = \overline{C_2} + r^2\Delta C_2$$

Definition of estimation continues similarly

$$E_2E_1E_2E_1(C_1) = E_2(E_1E_2E_1(C_1)) = \overline{C_1} + r^2b_2\Delta C_2$$
$$E_2E_1E_2E_1E_2(C_2) = E_2(E_1E_2E_1E_2(C_2)) = \overline{C_2} + r^4\Delta C_2$$
$$E_2E_1E_2E_1E_2E_1(C_1) = E_2(E_1E_2E_1E_2E_1(C_1)) = \overline{C_1} + r^4b_2\Delta C_2$$
$$E_2E_1E_2E_1E_2E_1E_2(C_2) = E_2(E_1E_2E_1E_2E_1E_2(C_2)) = \overline{C_2} + r^6\Delta C_2$$
$$E_2E_1E_2E_1E_2E_1E_2E_1(C_1) = E_2(E_1E_2E_1E_2E_1E_2E_1(C_1)) = \overline{C_1} + r^6b_2\Delta C_2$$

$$q_2^e = \frac{C_2}{2} - \frac{\overline{C_1} + b_2\Delta C_2}{2} + \frac{\overline{C_2} + r^2\Delta C_2}{4} - \frac{\overline{C_1} + r^2b_2\Delta C_2}{4} + \frac{\overline{C_2} + r^4\Delta C_2}{8}$$
$$- \frac{\overline{C_1} + r^4b_2\Delta C_2}{8} + \frac{\overline{C_2} + r^6\Delta C_2}{16} - \frac{\overline{C_1} + r^6b_2\Delta C_2}{16}$$

$$q_2^e = \frac{C_2}{2} - \frac{\overline{C_1}}{2} - \frac{b_2\Delta C_2}{2} + \frac{\overline{C_2}}{4} + \frac{r^2\Delta C_2}{4} - \frac{\overline{C_1}}{4} - \frac{r^2b_2\Delta C_2}{4} + \frac{\overline{C_2}}{8} + \frac{r^4\Delta C_2}{8} - \frac{\overline{C_1}}{8}$$
$$- \frac{r^4b_2\Delta C_2}{8} + \frac{\overline{C_2}}{16} + \frac{r^6\Delta C_2}{16} - \frac{\overline{C_1}}{16} - \frac{r^6b_2\Delta C_2}{16} \cdots$$

$$q_2^e = \overline{C_2}\left(\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16}\cdots\right) - \overline{C_1}\left(\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16}\cdots\right)$$
$$- b_2\Delta C_2\left(\frac{1}{2} + \frac{r^2}{4} + \frac{r^4}{8} + \frac{r^6}{16}\cdots\right) + \Delta C_2\left(\frac{1}{2} + \frac{r^2}{4} + \frac{r^4}{8} + \frac{r^6}{16}\cdots\right)$$

$$\left(\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16}\cdots\right) = \sum_{i=1}^{\infty}\frac{1}{2^i} = \sum_{i=0}^{\infty}\frac{1}{2^i} - \frac{1}{2^0} = \frac{1}{1 - \frac{1}{2}} - 1 = 1$$

$$\left(\frac{1}{2} + \frac{r^2}{4} + \frac{r^4}{8} + \frac{r^6}{16}\cdots\right) = \frac{1}{2}\left(1 + \frac{r^2}{2} + \frac{r^4}{4} + \frac{r^6}{8}\cdots\right) = \frac{1}{2}\sum_{i=0}^{\infty}\left(\frac{r^2}{2}\right)^i = \frac{1}{2} * \frac{2}{1 - \frac{r^2}{2}}$$
$$= \frac{1}{2 - r^2}$$

$$q_2^e = \overline{C_2} - \overline{C_1} - \frac{b_2\Delta C_2}{2 - r^2} + \frac{\Delta C_2}{2 - r^2}$$
$$q_2^e = \overline{C_2} - \overline{C_1} + \Delta C_2 * \left(\frac{1 - b_2}{2 - r^2}\right)$$

Case 6: Public firm and private firms produce information but only private firm disclosures information in Cell (2, 3)

Information set for case 6

$$I_1 = (C_1, C_2)$$
$$I_2 = (\overline{C_1}, C_2)$$

X_1, X_2, X_3 and X_4 values for case 6

$$X_1 = C_1$$
$$X_2 = \overline{C_1} + b_2 * \Delta C_2$$
$$X_3 = X_4 = C_2$$

Equilibrium solution for q_1^e

$$q_1^e = X_1 + X_2 - X_4$$
$$q_1^e = C_1 + (\overline{C_1} + b_2 * \Delta C_2) - C_2$$
$$q_1^e = q_1^* = C_1 + \overline{C_1} + b_2 * (C_2 - \overline{C_2}) - C_2$$

Equilibrium solution for q_2^e

$$q_2^e = \frac{X_3}{2} - X_2 + \frac{X_4}{2}$$
$$q_2^e = \frac{C_2}{2} - (\overline{C_1} + b_2 * \Delta C_2) + \frac{C_2}{2}$$
$$q_2^e = C_2 - \overline{C_1} - b_2 * \Delta C_2$$
$$q_2^e = q_2^* = C_2 - \overline{C_1} - b_2 * (C_2 - \overline{C_2})$$

Case 7: Private firm does not produce information but public firm produces and disclosures information in Cell (3, 1)

Information set for case 7

$$I_1 = (C_1, \overline{C_2})$$
$$I_2 = (C_1, \overline{C_2})$$

X_1, X_2, X_3 and X_4 values for case 7

$$X_1 = X_2 = C_1$$
$$X_3 = X_4 = \overline{C_2} + b_1 * \Delta C_1$$

Equilibrium solution for q_1^e

$$q_1^e = X_1 + X_2 - X_4$$
$$q_1^e = C_1 + C_1 - (\overline{C_2} + b_1 * \Delta C_1)$$
$$q_1^e = C_1 + C_1 - \overline{C_2} - b_1 * \Delta C_1$$
$$q_1^e = q_1^* = 2 * C_1 - \overline{C_2} - b_1 * (C_2 - \overline{C_2})$$

Equilibrium solution for q_2^e

$$q_2^e = \frac{X_3}{2} - X_2 + \frac{X_4}{2}$$
$$q_2^e = \frac{\overline{C_2} + b_1 * \Delta C_1}{2} - C_1 + \frac{\overline{C_2} + b_1 * \Delta C_1}{2}$$
$$q_2^e = \overline{C_2} - C_1 + b_1 * \Delta C_1$$
$$q_2^e = q_2^* = \overline{C_2} - C_1 + b_1 * (C_1 - \overline{C_1})$$

Case 8: Public firms and private firms produce information but only public firm disclosures information in Cell (3, 2)

Information set for case 8

$$I_1 = (C_1, \overline{C_2})$$
$$I_2 = (C_1, C_2)$$

X_1, X_2, X_3 and X_4 values for case 8

$$X_1 = X_2 = C_1$$
$$X_3 = C_2$$
$$X_4 = \overline{C_2} + b_1 * \Delta C_1$$

Equilibrium solution for q_1^e

$$q_1^e = X_1 + X_2 - X_4$$
$$q_1^e = C_1 + C_1 - (\overline{C_2} + b_1 * \Delta C_1)$$
$$q_1^e = 2 * C_1 - \overline{C_2} - b_1 * \Delta C_1$$
$$q_1^e = q_1^* = 2 * C_1 - \overline{C_2} - b_1 * (C_1 - \overline{C_1})$$

Equilibrium solution for q_2^e

$$q_2^e = \frac{X_3}{2} - X_2 + \frac{X_4}{2}$$
$$q_2^e = \frac{C_2}{2} - C_1 + \frac{\overline{C_2} + b_1 * \Delta C_1}{2}$$
$$q_2^e = \frac{C_2}{2} + \frac{\overline{C_2}}{2} - C_1 + \frac{b_1 * \Delta C_1}{2}$$
$$q_2^e = q_2^* = \frac{C_2}{2} + \frac{\overline{C_2}}{2} + \frac{b_1 * (C_2 - \overline{C_2})}{2} - C_1$$

Case 9: Public firms and private firms produce and disclose information in Cell (3, 3)

Information set for case 9

$$I_1 = (C_1, C_2)$$
$$I_2 = (C_1, C_2)$$

X_1, X_2, X_3 and X_4 values for case 9

$$X_1 = X_2 = C_1$$
$$X_3 = X_4 = C_2$$

Equilibrium solution for q_1^e

$$q_1^e = X_1 + X_2 - X_4$$

$$q_1^e = C_1 + C_1 - C_2$$

$$q_1^e = q_1^* = 2 * C_1 - C_2$$

Equilibrium solution for q_2^e

$$q_2^e = \frac{X_3}{2} - X_2 + \frac{X_4}{2}$$

$$q_2^e = \frac{C_1}{2} - C_1 + \frac{C_2}{2}$$

$$q_2^e = q_2^* = C_2 - C_1$$

APPENDIX 3. Detailed Solution of Excess Equilibrium Quantity Calculations

Case 1: Public and private firms do not produce information in Cell (1, 1)

Excess equilibrium solution for q_1^+

$$q_1^+ = q_1^* - \bar{q}_1$$

$$q_1^+ = 2 * \overline{C_1} - \overline{C_2} - (2 * \overline{C_1} - \overline{C_2})$$

$$q_1^+ = 0$$

Excess equilibrium solution for q_2^+

$$q_2^+ = q_2^* - \bar{q}_2$$

$$q_2^+ = \overline{C_2} - \overline{C_1} - (\overline{C_2} - \overline{C_1})$$

$$q_2^+ = 0$$

Case 2: Public firm does not produce information but private firm produces information without disclosure in Cell (1, 2)

Excess equilibrium solution for q_1^+

$$q_1^+ = q_1^* - \bar{q}_1$$

$$q_1^+ = 2 * \overline{C_1} - \overline{C_2} - (2 * \overline{C_1} - \overline{C_2})$$

$$q_1^+ = 0$$

Excess equilibrium solution for q_2^+

$$q_2^+ = q_2^* - \bar{q}_2$$

$$q_2^+ = \frac{C_2}{2} - \overline{C_1} + \frac{\overline{C_2}}{2} - (\overline{C_2} - \overline{C_1})$$

$$q_2^+ = \frac{C_2}{2} - \frac{\overline{C_2}}{2}$$

$$q_2^+ = \frac{\Delta C_2}{2}$$

Case 3: Public firm does not produce information but private firm produces and disclosures information in Cell (1, 3)

Excess equilibrium solution for q_1^+

$$q_1^+ = q_1^* - \bar{q}_1$$

$$q_1^+ = 2 * \overline{C_1} + 2 * b_2 * \Delta C_2 - C_2 - (2 * \overline{C_1} - \overline{C_2})$$

$$q_1^+ = 2 * b_2 * \Delta C_2 - C_2 + \overline{C_2}$$

$$q_1^+ = (2b_2 - 1) * \Delta C_2$$

Excess equilibrium solution for q_2^+

$$q_2^+ = q_2^* - \bar{q}_2$$

$$q_2^+ = C_2 - \overline{C_1} - b_2 * \Delta C_2 - (\overline{C_2} - \overline{C_1})$$

$$q_2^+ = C_2 - \overline{C_2} - b_2 * \Delta C_2$$

$$q_1^+ = (1 - b_2) * \Delta C_2$$

Case 4: Public firm produces information without disclosure but private firm does not produce information in Cell (2, 1)

Excess equilibrium solution for q_1^+

$$q_1^+ = q_1^* - \bar{q}_1$$

$$q_1^+ = C_1 + \overline{C_1} - \overline{C_2} - (2 * \overline{C_1} - \overline{C_2})$$

$$q_1^+ = C_1 - \overline{C_1}$$

$$q_1^+ = \Delta C_1$$

Excess equilibrium solution for q_2^+

$$q_2^+ = q_2^* - \bar{q}_2$$

$$q_2^+ = \overline{C_2} - \overline{C_1} - (\overline{C_2} - \overline{C_1})$$

$$q_2^+ = 0$$

Case 5: Both firms produce information without disclosure in Cell (2, 2)

Excess equilibrium solution for q_1^+

$$q_1^+ = q_1^* - \bar{q}_1$$

$$q_1^+ = 2 * \overline{C_1} - \overline{C_2} + \Delta C_1 * \frac{2 - b_1}{2 - r^2} - (2 * \overline{C_1} - \overline{C_2})$$

$$q_1^+ = \Delta C_1 \left(\frac{2 - b_1}{2 - r^2}\right)$$

Excess equilibrium solution for q_2^+

$$q_2^+ = q_2^* - \bar{q}_2$$

$$q_2^+ = \overline{C_2} - \overline{C_1} + \Delta C_2 \left(\frac{1-b_2}{2-r^2}\right) - (\overline{C_2} - \overline{C_1})$$

$$q_2^+ = \Delta C_2 \left(\frac{1-b_2}{2-r^2}\right)$$

Case 6: Public firm and private firms produce information but only private firm disclosures information in Cell (2, 3)

Excess equilibrium solution for q_1^+

$$q_1^+ = q_1^* - \bar{q}_1$$

$$q_1^+ = C_1 + \overline{C_1} + b_2 * \Delta C_2 - C_2 - (2 * \overline{C_1} - \overline{C_2})$$

$$q_1^+ = C_1 - \overline{C_1} + b_2 * \Delta C_2 - C_2 + \overline{C_2}$$

$$q_1^+ = \Delta C_1 + (b_2 - 1) * \Delta C_2$$

Excess equilibrium solution for q_2^+

$$q_2^+ = q_2^* - \bar{q}_2$$

$$q_2^+ = C_2 - \overline{C_1} - b_2 * \Delta C_2 - (\overline{C_2} - \overline{C_1})$$

$$q_2^+ = C_2 - \overline{C_2} - b_2 * \Delta C_2 + \overline{C_1} - \overline{C_1}$$

$$q_2^+ = (1 - b_2) * \Delta C_2$$

Case 7: Private firm does not produce information but public firm produces and disclosures information in Cell (3, 1)

Excess equilibrium solution for q_1^+

$$q_1^+ = q_1^* - \bar{q}_1$$

$$q_1^+ = 2 * C_1 - \overline{C_2} - b_1 * \Delta C_1 - (2 * \overline{C_1} - \overline{C_2})$$

$$q_1^+ = 2 * C_1 - 2 * \overline{C_1} - b_1 * \Delta C_1$$

$$q_1^+ = (2 - b_1) * \Delta C_1$$

Excess equilibrium solution for q_2^+

$$q_2^+ = q_2^* - \bar{q}_2$$

$$q_2^+ = \overline{C_2} - C_1 + b_1 * \Delta C_1 - (\overline{C_2} - \overline{C_1})$$

$$q_2^+ = -C_1 + \overline{C_1} + b_1 * \Delta C_1$$

$$q_2^+ = (b_1 - 1) * \Delta C_1$$

Case 8: Public firms and private firms produce information but only public firm disclosures information in Cell (3, 2)

Excess equilibrium solution for q_1^+

$$q_1^+ = q_1^* - \bar{q}_1$$

$$q_1^+ = 2 * C_1 - \overline{C_2} - b_1 * \Delta C_1 - (2 * \overline{C_1} - \overline{C_2})$$

$$q_1^+ = 2 * C_1 - 2 * \overline{C_1} - b_1 * \Delta C_1$$

$$q_1^+ = (2 - b_1) * \Delta C_1$$

Excess equilibrium solution for q_2^+

$$q_2^+ = q_2^* - \bar{q}_2$$

$$q_2^+ = \frac{C_2}{2} + \frac{\overline{C_2}}{2} - C_1 + \frac{b_1 * \Delta C_1}{2} - (\overline{C_2} - \overline{C_1})$$

$$q_2^+ = \frac{C_2}{2} - \frac{\overline{C_2}}{2} + \frac{b_1 * \Delta C_1}{2} - C_1 + \overline{C_1}$$

$$q_2^+ = \frac{(\Delta C_2)}{2} + \Delta C_1 * \left(\frac{b_1 - 2}{2}\right)$$

Case 9: Public firms and private firms produce and disclose information in Cell (3, 3)

Excess equilibrium solution for q_1^+

$$q_1^+ = q_1^* - \bar{q}_1$$

$$q_1^+ = 2 * C_1 - C_2 - (2 * \overline{C_1} - \overline{C_2})$$

$$q_1^+ = 2 * \Delta C_1 - \Delta C_2$$

Excess equilibrium solution for q_2^+

$$q_2^+ = q_2^* - \bar{q}_2$$

$$q_2^+ = C_2 - C_1 - (\overline{C_2} - \overline{C_1})$$

$$q_2^+ = \Delta C_2 - \Delta C_1$$

APPENDIX 4. Welfare calculation for the public firms.

Welfare calculation are performed at equilibrium quantities for "no produce information", "produce but no disclosure information" and "produce and disclosure information" cases for the public firm.

Remember the welfare function of public firm

$$W = A * (q_1 + q_2) - \frac{(q_1 + q_2)^2}{2} - m_1 q_1 - m_2 q_2$$

Expected welfare function can be calculated for an information sets $I_1 = (.,.)$ and $I_2 = (.,.)$ for equilibrium output quantities q_1^e and q_2^e which are already calculated in a certain information sets $I_1 = (.,.)$ and $I_2 = (.,.)$.

$$E(W|I_1) = E\left(\left(Aq_1^e - m_1 q_1^e + Aq_2^e - m_2 q_2^e - \frac{(q_1^e + q_2^e)^2}{2}\right) \middle| I_1\right)$$

By substitution of $C_1 = A - m_1$ and $C_2 = A - m_2$, it yields

$$E(W|I_1) = E\left(\left(C_1 q_1^e + C_2 q_2^e - \frac{(q_1^e + q_2^e)^2}{2}\right)\middle| I_1\right)$$

Case 1: Public and private firms do not produce information in Cell (1, 1)

$$q_1^e = 2 * \overline{C_1} - \overline{C_2}$$

$$q_2^e = \overline{C_2} - \overline{C_1}$$

Expected welfare function for equilibrium

$$E(W|I_1) = E\left(\left(C_1 q_1^e + C_2 q_2^e - \frac{(q_1^e + q_2^e)^2}{2}\right)\middle| I_1\right)$$

$$W^* = E\left(C_1(2\overline{C_1} - \overline{C_2}) + C_2(\overline{C_2} - \overline{C_1}) - \frac{\left((2\overline{C_1} - \overline{C_2}) + (\overline{C_2} - \overline{C_1})\right)^2}{2}\right)$$

$$W^* = E\left((2C_1\overline{C_1} - C_1\overline{C_2}) + (C_2\overline{C_2} - C_2\overline{C_1}) - \frac{\overline{C_1}^2}{2}\right)$$

$$W^* = 2E(C_1)\overline{C_1} - E(C_1)\overline{C_2} + E(C_2)\overline{C_2} - E(C_2)\overline{C_1} - \frac{\overline{C_1}^2}{2}$$

After substitution of the equalities $E(C_1) = \overline{C_1}$ and $E(C_2) = \overline{C_2}$, the formulation yields

$$W^* = 2\overline{C_1 C_1} - \overline{C_1 C_2} + \overline{C_2 C_2} - \overline{C_2 C_1} - \frac{\overline{C_1}^2}{2}$$

$$W^* = \overline{W} = \frac{3\overline{C_1}^2}{2} - 2\overline{C_1 C_2} + \overline{C_2}^2$$

Similar to excess equilibrium quantity, welfare is also expressed "excess welfare" relative to "no produce information case" for both public and private firms.

Excess expected welfare function for equilibrium

$$W^+ = W^* - \overline{W}$$

$$W^+ = 0$$

Case 2: Public firm does not produce information but private firm produces information without disclosure in Cell (1, 2)

$$q_1^e = 2 * \overline{C_1} - \overline{C_2}$$

$$q_2^e = \frac{C_2}{2} - \overline{C_1} + \frac{\overline{C_2}}{2}$$

Expected welfare function for equilibrium

$$E(W|I_1) = E\left(\left(C_1 q_1^e + C_2 q_2^e - \frac{(q_1^e + q_2^e)^2}{2}\right)\Bigg| I_1\right)$$

$$W^* = E\left(C_1(2\overline{C_1} - \overline{C_2}) + C_2\left(\frac{C_2}{2} - \overline{C_1} + \frac{\overline{C_2}}{2}\right) - \frac{\left((2\overline{C_1} - \overline{C_2}) + \left(\frac{C_2}{2} - \overline{C_1} + \frac{\overline{C_2}}{2}\right)\right)^2}{2} \right)$$

$$W^* = E\left((2C_1\overline{C_1} - C_1\overline{C_2}) + \left(\frac{{C_2}^2}{2} - C_2\overline{C_1} + \frac{C_2\overline{C_2}}{2}\right) - \frac{\left(\frac{C_2}{2} + \overline{C_1} - \frac{\overline{C_2}}{2}\right)^2}{2} \right)$$

$$W^* = 2E(C_1)\overline{C_1} - E(C_1)\overline{C_2} + \frac{E\left({C_2}^2\right)}{2} - E(C_2)\overline{C_1} + \frac{E(C_2)\overline{C_2}}{2} - \frac{E\left({C_2}^2\right)}{8} - \frac{\overline{C_1}^2}{2} - \frac{\overline{C_2}^2}{8} - \frac{E(C_2)\overline{C_1}}{2} + \frac{E(C_2)\overline{C_2}}{4} + \frac{\overline{C_1 C_2}}{2}$$

After substitution of the equalities $E(C_1) = \overline{C_1}\ and\ E(C_2) = \overline{C_2}$, the formulation yields

$$W^* = 2\overline{C_1}^2 - \overline{C_1 C_2} + \frac{E\left({C_2}^2\right)}{2} - \overline{C_2 C_1} + \frac{\overline{C_2}^2}{2} - \frac{E\left({C_2}^2\right)}{8} - \frac{\overline{C_1}^2}{2} - \frac{\overline{C_2}^2}{8} - \frac{\overline{C_1 C_2}}{2} + \frac{\overline{C_2}^2}{4} + \frac{\overline{C_1 C_2}}{2}$$

$$W^* = \frac{3\overline{C_1}^2}{2} - 2\overline{C_1 C_2} + \frac{3E\left({C_2}^2\right)}{8} + \frac{5\overline{C_2}^2}{8}$$

Excess expected welfare function for equilibrium

$$W^+ = W^* - \overline{W}$$

$$W^+ = \frac{3\overline{C_1}^2}{2} - 2\overline{C_1 C_2} + \frac{3E\left({C_2}^2\right)}{8} + \frac{5\overline{C_2}^2}{8} - \left(\frac{3\overline{C_1}^2}{2} - 2\overline{C_1 C_2} + \overline{C_2}^2\right)$$

$$W^+ = \frac{3E\left({C_2}^2\right)}{8} - \frac{3\overline{C_2}^2}{8}$$

It is know that $V_1^2 = E\left({C_1}^2\right) - \overline{C_1}^2$ and $V_2^2 = E\left({C_2}^2\right) - \overline{C_2}^2$, therefore,

$$W^+ = \frac{3V_2^2}{8}$$

Case 3: Public firm does not produce information but private firm produces and disclosures information in Cell (1, 3)

$$q_1^e = 2 * \overline{C_1} + 2 * b_2 * (C_2 - \overline{C_2}) - C_2$$

$$q_2^e = C_2 - \overline{C_1} - b_2 * (C_2 - \overline{C_2})$$

Expected welfare function for equilibrium

$$E(W|I_1) = E\left(\left(C_1 q_1^e + C_2 q_2^e - \frac{(q_1^e + q_2^e)^2}{2}\right) \middle| I_1\right)$$

$$W^* = E\Bigg(C_1(2\overline{C_1} + 2b_2(C_2 - \overline{C_2}) - C_2) + C_2(C_2 - \overline{C_1} - b_2(C_2 - \overline{C_2}))$$

$$- \frac{\left((2\overline{C_1} + 2b_2(C_2 - \overline{C_2}) - C_2) + (C_2 - \overline{C_1} - b_2(C_2 - \overline{C_2}))\right)^2}{2}\Bigg)$$

The formulation is simplified by using the equations $E(C_1) = \overline{C_1}$, $E(C_2) = \overline{C_2}$, $V_1^2 = E({C_1}^2) - \overline{C_1}^2$, $V_2^2 = E({C_2}^2) - \overline{C_2}^2$ and $V_{12} = E(C_1 C_2) - \overline{C_1 C_2}$. It yields

$$W^* = E({C_2}^2) - E(C_1 C_2) - \overline{C_1 C_2} + \frac{3\overline{C_1}^2}{2} + \frac{3b_2^2 V_2^2}{2} - V_{12}$$

Excess expected welfare function for equilibrium

$$W^+ = W^* - \overline{W}$$

$$W^+ = E({C_2}^2) - E(C_1 C_2) - \overline{C_1 C_2} + \frac{3\overline{C_1}^2}{2} + \frac{3b_2^2 V_2^2}{2} - V_{12} - \left(\frac{3\overline{C_1}^2}{2} - 2\overline{C_1 C_2} + \overline{C_2}^2\right)$$

$$W^+ = \left(E({C_2}^2) - \overline{C_2}^2\right) - (E(C_1 C_2) - \overline{C_1 C_2}) + \frac{3b_2^2 V_2^2}{2} - V_{12}$$

$$W^+ = V_2^2 + \frac{3b_2^2 V_2^2}{2} - 2V_{12}$$

$$W^+ = V_2^2 + \frac{3b_2^2 V_2^2}{2} - 2(b_2 V_2^2)$$

$$W^+ = (1 - b_2)^2 V_2^2 + \frac{b_2^2 V_2^2}{2}$$

Case 4: Public firm produces information without disclosure but private firm does not produce information in Cell (2, 1)

$$q_1^e = C_1 + \overline{C_1} - \overline{C_2}$$

$$q_2^e = q_2^* = \overline{C_2} - \overline{C_1}$$

Expected welfare function for equilibrium

$$E(W|I_1) = E\left(\left(C_1 q_1^e + C_2 q_2^e - \frac{(q_1^e + q_2^e)^2}{2}\right) \middle| I_1\right)$$

$$W^* = E\left(C_1(C_1 + \overline{C_1} - \overline{C_2}) + C_2(\overline{C_2} - \overline{C_1}) - \frac{\left((C_1 + \overline{C_1} - \overline{C_2}) + (\overline{C_2} - \overline{C_1})\right)^2}{2}\right)$$

$$W^* = E\left(C_1(C_1 + \overline{C_1} - \overline{C_2}) + C_2(\overline{C_2} - \overline{C_1}) - \frac{{C_1}^2}{2}\right)$$

After substitution of the equalities $E(C_1) = \overline{C_1}\ and\ E(C_2) = \overline{C_2}$, the formulation yields

$$W^* = \frac{E({C_1}^2)}{2} + \overline{C_1}^2 - 2\overline{C_1 C_2} + \overline{C_2}^2$$

Excess expected welfare function for equilibrium

$$W^+ = W^* - \overline{W}$$

$$W^+ = \frac{E({C_1}^2)}{2} + \overline{C_1}^2 - 2\overline{C_1 C_2} + \overline{C_2}^2 \quad - \left(\frac{3\overline{C_1}^2}{2} - 2\overline{C_1 C_2} + \overline{C_2}^2\right)$$

$$W^+ = \frac{E({C_1}^2)}{2} - \frac{\overline{C_1}^2}{2}$$

It is know that $V_1^2 = E({C_1}^2) - \overline{C_1}^2$ and $V_2^2 = E({C_2}^2) - \overline{C_2}^2$, therefore,

$$W^+ = \frac{V_1^2}{2}$$

Case 5: Both firms produce information without disclosure in Cell (2, 2)

$$q_1^e = 2 * \overline{C_1} - \overline{C_2} + \Delta C_1 * \left(\frac{2 - b_1}{2 - r^2}\right)$$

$$q_2^e = \overline{C_2} - \overline{C_1} + \Delta C_2 * \left(\frac{1 - b_2}{2 - r^2}\right)$$

Expected welfare function for equilibrium

$$E(W|I_1) = E\left(\left(C_1 q_1^e + C_2 q_2^e - \frac{(q_1^e + q_2^e)^2}{2}\right) \middle| I_1\right)$$

$$W^* = E\left(C_1\left(2*\overline{C_1} - \overline{C_2} + \Delta C_1 * \left(\frac{2-b_1}{2-r^2}\right)\right) + C_2\left(\overline{C_2} - \overline{C_1} + \Delta C_2 * \left(\frac{1-b_2}{2-r^2}\right)\right) - \frac{\left(\left(2*\overline{C_1} - \overline{C_2} + \Delta C_1 * \left(\frac{2-b_1}{2-r^2}\right)\right) + \left(\overline{C_2} - \overline{C_1} + \Delta C_2 * \left(\frac{1-b_2}{2-r^2}\right)\right)\right)^2}{2} \right)$$

The formulation is simplified by using the equations $E(C_1) = \overline{C_1}$, $E(C_2) = \overline{C_2}$, $V_1^2 = E({C_1}^2) - \overline{C_1}^2$, $V_2^2 = E({C_2}^2) - \overline{C_2}^2$ and $V_{12} = E(C_1C_2) - \overline{C_1C_2}$. It yields

$$W^* = \frac{(4-4r^2)E({C_1}^2) + (8-8r^2+3r^4)\overline{C_1}^2}{2(r^2-2)^2} + \frac{(3-2r^2)E({C_2}^2) + (5-6r^2+2r^4)\overline{C_2}^2}{2(r^2-2)^2} - 4\frac{E(C_1C_2) + (3+4r^2-r^4)\overline{C_1C_2}}{2(r^2-2)^2} + \frac{\frac{3V_1^2r^2}{2} + V_2^2r^2 - 2V_{12} + 2V_{12}r^3}{2(r^2-2)^2}$$

Excess expected welfare function for equilibrium

$$W^+ = W^* - \overline{W}$$

$$W^+ = W^* - \left(\frac{3\overline{C_1}^2}{2} - 2\overline{C_1C_2} + \overline{C_2}^2\right)$$

$$W^+ = \frac{4V_1^2 + 3V_2^2 - V_1^2r^2 - V_2^2r^2 - 6V_{12} + 2V_{12}r^2}{2(r^2-2)^2}$$

$$W^+ = \frac{3V_2^2 - V_2^2r^2}{2(r^2-2)^2} + \frac{4V_1^2 - V_1^2r^2}{2(r^2-2)^2} + \frac{-6V_{12} + 2V_{12}r^2}{2(r^2-2)^2}$$

$$W^+ = V_2^2\frac{3-r^2}{2(r^2-2)^2} + V_1^2\frac{4-r^2}{2(r^2-2)^2} - 2V_{12}\frac{3-r^2}{2(r^2-2)^2}$$

Case 6: Public firm and private firms produce information but only private firm disclosures information in Cell (2, 3)

$$q_1^e = C_1 + \overline{C_1} + b_2 * (C_2 - \overline{C_2}) - C_2$$
$$q_2^e = C_2 - \overline{C_1} - b_2 * (C_2 - \overline{C_2})$$

Expected welfare function for equilibrium

$$E(W|I_1) = E\left(\left(C_1 q_1^e + C_2 q_2^e - \frac{(q_1^e + q_2^e)^2}{2}\right)\middle| I_1\right)$$

$$W^* = E\Bigg(C_1(C_1 + \overline{C_1} + b_2 * (C_2 - \overline{C_2}) - C_2) + C_2(C_2 - \overline{C_1} - b_2 * (C_2 - \overline{C_2})) - \frac{\left((C_1 + \overline{C_1} + b_2 * (C_2 - \overline{C_2}) - C_2) + (C_2 - \overline{C_1} - b_2 * (C_2 - \overline{C_2}))\right)^2}{2}\Bigg)$$

The formulation is simplified by using the equations $E(C_1) = \overline{C_1}$, $E(C_2) = \overline{C_2}$, $V_1^2 = E({C_1}^2) - \overline{C_1}^2$, $V_2^2 = E({C_2}^2) - \overline{C_2}^2$ and $V_{12} = E(C_1C_2) - \overline{C_1 C_2}$. It yields

$$W^* = \frac{E({C_1}^2)}{2} - E(C_1C_2) + E({C_2}^2) - \overline{C_1 C_2} + \overline{C_1}^2 + V_1^2 r^2 - V_{12}$$

$$W^* = \left(\frac{E({C_1}^2)}{2} - \frac{+\overline{C_1}^2}{2}\right) + \frac{3\overline{C_1}^2}{2} - (E(C_1C_2) - \overline{C_1C_2}) - \overline{C_1C_2} - \overline{C_1C_2} + \left(E({C_2}^2) - \overline{C_2}^2\right) + \overline{C_2}^2 + V_1^2 r^2 - V_{12}$$

$$W^* = \frac{V_1^2}{2} + \frac{3\overline{C_1}^2}{2} - 2V_{12} + V_2^2 + \overline{C_2}^2 + V_1^2 r^2$$

Excess expected welfare function for equilibrium

$$W^+ = W^* - \overline{W}$$

$$W^+ = W^* = \frac{V_1^2}{2} + \frac{3\overline{C_1}^2}{2} - 2V_{12} + V_2^2 + \overline{C_2}^2 + V_1^2 r^2 - \left(\frac{3\overline{C_1}^2}{2} - 2\overline{C_1C_2} + \overline{C_2}^2\right)$$

$$W^+ = \frac{V_1^2}{2} + V_2^2 + V_1^2 r^2 - 2V_{12}$$

$$W^+ = \frac{V_1^2}{2} + V_2^2 + b_2^2 V_2^2 - 2b_2 V_2^2$$

$$W^+ = \frac{V_1^2}{2} + (1 - b_2)^2 V_2^2$$

Case 7: Private firm does not produce information but public firm produces and disclosures information in Cell (3, 1)

$$q_1^e = 2 * C_1 - \overline{C_2} - b_1 * (C_2 - \overline{C_2})$$
$$q_2^e = \overline{C_2} - C_1 + b_1 * (C_1 - \overline{C_1})$$

Expected welfare function for equilibrium

$$E(W|I_1) = E\left(\left(C_1 q_1^e + C_2 q_2^e - \frac{(q_1^e + q_2^e)^2}{2}\right)\middle| I_1\right)$$

$$W^* = E\Bigg(C_1(2 * C_1 - \overline{C_2} - b_1 * (C_2 - \overline{C_2})) + C_2(\overline{C_2} - C_1 + b_1 * (C_1 - \overline{C_1}))$$

$$- \frac{\left((2 * C_1 - \overline{C_2} - b_1 * (C_2 - \overline{C_2})) + (\overline{C_2} - C_1 + b_1 * (C_1 - \overline{C_1}))\right)^2}{2}\Bigg)$$

The formulation is simplified by using the equations $E(C_1) = \overline{C_1}$, $E(C_2) = \overline{C_2}$, $V_1^2 = E\left({C_1}^2\right) - \overline{C_1}^2$, $V_2^2 = E\left({C_2}^2\right) - \overline{C_2}^2$ and $V_{12} = E(C_1 C_2) - \overline{C_1 C_2}$. It yields

$$W^* = \frac{3E\left({C_1}^2\right)}{2} - E(C_1 C_2) - \overline{C_1 C_2} + \overline{C_2}^2 + V_2^2 r^2 - V_{12}$$

$$W^* = \left(\frac{3E\left({C_1}^2\right)}{2} - \frac{3\overline{C_1}^2}{2}\right) + \frac{3\overline{C_1}^2}{2} - (E(C_1 C_2) - \overline{C_1 C_2}) - 2\overline{C_1 C_2} + \overline{C_2}^2 + V_2^2 r^2$$

$$- V_{12}$$

$$W^* = \frac{3V_1^2}{2} + \frac{3\overline{C_1}^2}{2} - 2V_{12} - 2\overline{C_1 C_2} + \overline{C_2}^2 + V_2^2 r^2$$

Excess expected welfare function for equilibrium

$$W^+ = W^* - \overline{W}$$

$$W^+ = \frac{3V_1^2}{2} + \frac{3\overline{C_1}^2}{2} - 2V_{12} - 2\overline{C_1 C_2} + \overline{C_2}^2 + V_2^2 r^2 - \left(\frac{3\overline{C_1}^2}{2} - 2\overline{C_1 C_2} + \overline{C_2}^2\right)$$

$$W^+ = \frac{3V_1^2}{2} - 2V_{12} + V_2^2 r^2$$

$$W^+ = \frac{V_1^2}{2} + V_1^2 - 2b_1 V_1^2 + b_1^2 V_1^2$$

$$W^+ = \frac{V_1^2}{2} + V_1^2(1 - b_1)^2$$

Case 8: Public firms and private firms produce information but only public firm disclosures information in Cell (3, 2)

$$q_1^e = 2 * C_1 - \overline{C_2} - b_1 * (C_1 - \overline{C_1})$$

$$q_2^e = \frac{C_2}{2} + \frac{\overline{C_2}}{2} + \frac{b_1 * (C_2 - \overline{C_2})}{2} - C_1$$

Expected welfare function for equilibrium

$$E(W|I_1) = E\left(\left(C_1 q_1^e + C_2 q_2^e - \frac{(q_1^e + q_2^e)^2}{2}\right)\middle| I_1\right)$$

$$W^*$$

$$= E\left(C_1(2 * C_1 - \overline{C_2} - b_1 * (C_1 - \overline{C_1})) + C_2\left(\frac{C_2}{2} + \frac{\overline{C_2}}{2} + \frac{b_1 * (C_2 - \overline{C_2})}{2} - C_1\right) - \frac{\left((2 * C_1 - \overline{C_2} - b_1 * (C_1 - \overline{C_1})) + \left(\frac{C_2}{2} + \frac{\overline{C_2}}{2} + \frac{b_1 * (C_2 - \overline{C_2})}{2} - C_1\right)\right)^2}{2} \right)$$

The formulation is simplified by using the equations $E(C_1) = \overline{C_1}$, $E(C_2) = \overline{C_2}$, $V_1^2 = E({C_1}^2) - \overline{C_1}^2$, $V_2^2 = E({C_2}^2) - \overline{C_2}^2$ and $V_{12} = E(C_1C_2) - \overline{C_1C_2}$. It yields

$$W^* = \frac{3E({C_1}^2)}{2} - \frac{3E(C_1C_2)}{2} + \frac{3E({C_2}^2)}{8} - \frac{\overline{C_1C_2}}{2} + \frac{5\overline{C_2}^2}{8} + \frac{5V_2^2r^2}{8} - \frac{V_{12}}{2}$$

$$W^* = \left(\frac{3E({C_1}^2)}{2} - \frac{3\overline{C_1}^2}{2}\right) + \frac{3\overline{C_1}^2}{2} - \left(\frac{3E(C_1C_2)}{2} - \frac{3\overline{C_1C_2}}{2}\right) - 2\overline{C_1C_2} + \left(\frac{3E({C_2}^2)}{8} - \frac{3\overline{C_2}^2}{8}\right) + \overline{C_2}^2 + \frac{5V_2^2r^2}{8} - \frac{V_{12}}{2}$$

$$W^* = \frac{3V_1^2}{2} + \frac{3\overline{C_1}^2}{2} - 2V_{12} - 2\overline{C_1C_2} + \frac{3V_2^2}{8} + \overline{C_2}^2 + \frac{5V_2^2r^2}{8}$$

Excess expected welfare function for equilibrium

$$W^+ = W^* - \overline{W}$$

$$W^+ = \frac{3V_1^2}{2} + \frac{3\overline{C_1}^2}{2} - 2V_{12} - 2\overline{C_1C_2} + \frac{3V_2^2}{8} + \overline{C_2}^2 + \frac{5V_2^2r^2}{8} - \left(\frac{3\overline{C_1}^2}{2} - 2\overline{C_1C_2} + \overline{C_2}^2\right)$$

$$W^+ = \frac{3V_1^2}{2} - 2V_{12} + \frac{3V_2^2}{8} + \frac{5V_2^2r^2}{8}$$

$$W^+ = \frac{3V_1^2}{2} + \frac{3V_2^2}{8} + \frac{5b_1^2V_1^2}{8} - 2b_1V_1^2$$

$$W^+ = \frac{V_1^2(5b_1 - 6)(b_1 - 2)}{8} + \frac{3V_2^2}{8}$$

Case 9: Public firms and private firms produce and disclose information in Cell (3, 3)

$$q_1^e = 2 * C_1 - C_2$$

$$q_2^e = C_2 - C_1$$

Expected welfare function for equilibrium

$$E(W|I_1) = E\left(\left(C_1 q_1^e + C_2 q_2^e - \frac{(q_1^e + q_2^e)^2}{2}\right)\Bigg| I_1\right)$$

$$W^* = E\left(C_1(2 * C_1 - C_2) + C_2(C_2 - C_1) - \frac{\left((2 * C_1 - C_2) + (C_2 - C_1)\right)^2}{2}\right)$$

The formulation is simplified by using the equations $E(C_1) = \overline{C_1}$, $E(C_2) = \overline{C_2}$, $V_1^2 = E\left({C_1}^2\right) - \overline{C_1}^2$, $V_2^2 = E\left({C_2}^2\right) - \overline{C_2}^2$ and $V_{12} = E(C_1C_2) - \overline{C_1C_2}$. It yields

$$W^* = \frac{3E\left({C_1}^2\right)}{2} - E(C_1C_2) + E\left({C_2}^2\right)$$

$$W^* = \left(\frac{3E\left({C_1}^2\right)}{2} - \frac{3\overline{C_1}^2}{2}\right) + \frac{3\overline{C_1}^2}{2} - (2E(C_1C_2) + 2\overline{C_1C_2}) - 2\overline{C_1C_2}$$

$$+ \left(E\left({C_2}^2\right) - \overline{C_2}^2\right) + \overline{C_2}^2$$

$$W^* = \frac{3V_1^2}{2} + \frac{3\overline{C_1}^2}{2} - 2V_{12} - 2\overline{C_1C_2} + V_2^2 + \overline{C_2}^2$$

Excess expected welfare function for equilibrium

$$W^+ = W^* - \overline{W}$$

$$W^+ = \frac{3V_1^2}{2} + \frac{3\overline{C_1}^2}{2} - 2V_{12} - 2\overline{C_1C_2} + V_2^2 + \overline{C_2}^2 - \left(\frac{3\overline{C_1}^2}{2} - 2\overline{C_1C_2} + \overline{C_2}^2\right)$$

$$W^+ = \frac{3V_1^2}{2} - 2V_{12} + V_2^2$$

$$W^+ = \frac{3V_1^2}{2} - V_2^2(1 - 2b_2)$$

APPENDIX 5. Profit calculation for the private firms.

Profit calculation are performed at equilibrium quantities for “no produce information”, “produce but no disclosure information” and “produce and disclosure information” cases for the private firm.

Remember the profit function of public firm;

$$\Pi_2 = (A - m_2 - q_1 - q_2) * q_2$$

Expected profit function can be calculated for an information sets set $I_1 = (.,.)$ and $I_2 = (.,.)$ for equilibrium output quantities q_1^e and q_2^e which are already calculated in a certain information sets $I_1 = (.,.)$ and $I_2 = (.,.)$.

$$E(\Pi_2|I_1) = E\left(\left((A - m_2 - q_1^e - q_2^e) * q_2^e\right)\Big|I_1\right)$$

By substitution of $C_2 = A - m_2$, it yields

$$E(\Pi_2|I_1) = E\left(\left((C_2 - q_1^e - q_2^e) * q_2^e\right)\Big|I_1\right)$$

$$E(\Pi_2|I_1) = E\left(\left(C_2 q_2^e - {q_2^e}^2 - q_1^e q_2^e\right)\Big|I_1\right)$$

Case 1: Public and private firms do not produce information in Cell (1, 1)

$$q_1^e = 2 * \overline{C_1} - \overline{C_2}$$

$$q_2^e = \overline{C_2} - \overline{C_1}$$

Expected private firms' profit function for equilibrium

$$E(\Pi_2|I_1) = E\left(\left((C_2 - q_1^e - q_2^e) * q_2^e\right)\Big|I_1\right)$$

$$\Pi_2{}^* = E\left(\left(C_2 - (2 * \overline{C_1} - \overline{C_2}) - (\overline{C_2} - \overline{C_1})\right) * (\overline{C_2} - \overline{C_1})\right)$$

$$\Pi_2{}^* = E\big((C_2 - 2 * \overline{C_1} + \overline{C_2} - \overline{C_2} + \overline{C_1}) * (\overline{C_2} - \overline{C_1})\big)$$

$$\Pi_2{}^* = E\big((C_2 - \overline{C_1}) * (\overline{C_2} - \overline{C_1})\big)$$

$$\Pi_2{}^* = E\left(C_2\overline{C_2} - C_2\overline{C_1} - \overline{C_1 C_2} + \overline{C_1}^2\right)$$

$$\Pi_2{}^* = E(C_2)\overline{C_2} - E(C_2)\overline{C_1} - \overline{C_1 C_2} + \overline{C_1}^2$$

After substitution of the equalities $E(C_2) = \overline{C_2}$, the formulation yields

$$\Pi_2{}^* = \overline{\Pi_2} = \overline{C_2}^2 - 2\overline{C_1 C_2} + \overline{C_1}^2$$

Similar to excess equilibrium quantity and welfare, private firms' profit is also expressed "excess profit" relative to "no produce information case" for both public and private firms.

Excess expected profit function for equilibrium

$$\Pi_2{}^+ = \Pi_2{}^* - \overline{\Pi_2}$$

$$\Pi_2{}^+ = 0$$

Case 2: Public firm does not produce information but private firm produces information without disclosure in Cell (1, 2)

$$q_1^e = 2 * \overline{C_1} - \overline{C_2}$$

$$q_2^e = \frac{C_2}{2} - \overline{C_1} + \frac{\overline{C_2}}{2}$$

Expected private firms' profit function for equilibrium

$$E(\Pi_2|I_1) = E\left(\left((C_2 - q_1^e - q_2^e) * q_2^e\right) \middle| I_1\right)$$

$$\Pi_2{}^* = E\left(\left(\left(C_2 - (2 * \overline{C_1} - \overline{C_2}) - \left(\frac{C_2}{2} - \overline{C_1} + \frac{\overline{C_2}}{2}\right)\right) * \left(\frac{C_2}{2} - \overline{C_1} + \frac{\overline{C_2}}{2}\right)\right)\right)$$

After substitution of the equalities $E(C_1) = \overline{C_1}\ and\ E(C_2) = \overline{C_2}$, the formulation yields

$$\Pi_2{}^* = \frac{E\left(C_2{}^2\right)}{4} - 2\overline{C_1C_2} + \overline{C_1}^2 + \frac{3\overline{C_2}^2}{4}$$

Excess expected profit function for equilibrium

$$\Pi_2{}^+ = \Pi_2{}^* - \overline{\Pi_2}$$

$$\Pi_2{}^+ = \frac{E\left(C_2{}^2\right)}{4} - 2\overline{C_1C_2} + \overline{C_1}^2 + \frac{3\overline{C_2}^2}{4} - \left(\overline{C_2}^2 - 2\overline{C_1C_2} + \overline{C_1}^2\right)$$

$$\Pi_2{}^+ = \frac{E\left(C_2{}^2\right)}{4} - \frac{\overline{C_2}^2}{4} = \frac{E\left(C_2{}^2\right) - \overline{C_2}^2}{4}$$

It is know that $V_1^2 = E\left(C_1{}^2\right) - \overline{C_1}^2$ and $V_2^2 = E\left(C_2{}^2\right) - \overline{C_2}^2$, therefore,

$$\Pi_2{}^+ = \frac{V_2^2}{4}$$

Case 3: Public firm does not produce information but private firm produces and disclosures information in Cell (1, 3)

$$q_1^e = 2 * \overline{C_1} + 2 * b_2 * (C_2 - \overline{C_2}) - C_2$$

$$q_2^e = C_2 - \overline{C_1} - b_2 * (C_2 - \overline{C_2})$$

Expected private firms' profit function for equilibrium

$$E(\Pi_2|I_1) = E\left(\left((C_2 - q_1^e - q_2^e) * q_2^e\right) \middle| I_1\right)$$

$$\Pi_2{}^* = E\Big(\Big(C_2 - (2 * \overline{C_1} + 2 * b_2 * (C_2 - \overline{C_2}) - C_2) - (C_2 - \overline{C_1} - b_2 * (C_2 - \overline{C_2}))\Big)$$
$$* (C_2 - \overline{C_1} - b_2 * (C_2 - \overline{C_2}))\Big)$$

The formulation is simplified by using the equations $E(C_1) = \overline{C_1}$, $E(C_2) = \overline{C_2}$, $V_1^2 = E\left(C_1{}^2\right) - \overline{C_1}^2$, $V_2^2 = E\left(C_2{}^2\right) - \overline{C_2}^2$ and $V_{12} = E(C_1C_2) - \overline{C_1C_2}$. It yields

$$\Pi_2{}^* = E\left(C_2{}^2\right) - 2\overline{C_1C_2} + \overline{C_1}^2 + V_1^2r^2 - 2V_{12}$$

Excess expected profit function for equilibrium

$$\Pi_2{}^+ = \Pi_2{}^* - \overline{\Pi_2}$$

$$\Pi_2{}^+ = E\left(C_2{}^2\right) - 2\overline{C_1C_2} + \overline{C_1}^2 + V_1^2r^2 - 2V_{12} - \left(\overline{C_2}^2 - 2\overline{C_1C_2} + \overline{C_1}^2\right)$$

$$\Pi_2{}^+ = E\left(C_2{}^2\right) - \overline{C_2}^2 + V_1^2r^2 - 2V_{12}$$

It is know that $V_2^2 = E\left({C_2}^2\right) - \overline{C_2}^2$, therefore,

$${\Pi_2}^+ = V_2^2 + V_1^2 r^2 - 2V_{12}$$

$${\Pi_2}^+ = V_2^2 + b_2^2 V_2^2 - 2b_2 V_2^2$$

$${\Pi_2}^+ = V_2^2(1 - 2b_2)^2$$

Case 4: Public firm produces information without disclosure but private firm does not produce information in Cell (2, 1)

$$q_1^e = C_1 + \overline{C_1} - \overline{C_2}$$

$$q_2^e = q_2^* = \overline{C_2} - \overline{C_1}$$

Expected private firms' profit function for equilibrium

$$E(\Pi_2|I_1) = E\left(\left((C_2 - q_1^e - q_2^e) * q_2^e\right)\Big|I_1\right)$$

$${\Pi_2}^* = E\left(\left(C_2 - (C_1 + \overline{C_1} - \overline{C_2}) - (\overline{C_2} - \overline{C_1})\right) * (\overline{C_2} - \overline{C_1})\right)$$

$${\Pi_2}^* = E\left((C_2 - C_1 - \overline{C_1} + \overline{C_2} - \overline{C_2} + \overline{C_1}) * (\overline{C_2} - \overline{C_1})\right)$$

$${\Pi_2}^* = E\left((C_2 - C_1) * (\overline{C_2} - \overline{C_1})\right)$$

$${\Pi_2}^* = E(C_2)\overline{C_2} - E(C_2)\,\overline{C_1} - E(C_1)\overline{C_2} + E(C_1)\overline{C_1}$$

After substitution of the equalities $E(C_1) = \overline{C_1}\ and\ E(C_2) = \overline{C_2}$, the formulation yields

$${\Pi_2}^* = \overline{C_2}^2 - 2\overline{C_1 C_2} + \overline{C_1}^2$$

Excess expected profit function for equilibrium

$${\Pi_2}^+ = {\Pi_2}^* - \overline{\Pi_2}$$

$${\Pi_2}^+ = \overline{C_2}^2 - 2\overline{C_1 C_2} + \overline{C_1}^2 - \left(\overline{C_2}^2 - 2\overline{C_1 C_2} + \overline{C_1}^2\right)$$

$${\Pi_2}^+ = 0$$

Case 5: Both firms produce information without disclosure in Cell (2, 2)

$$q_1^e = 2 * \overline{C_1} - \overline{C_2} + \Delta C_1 * \left(\frac{2 - b_1}{2 - r^2}\right)$$

$$q_2^e = \overline{C_2} - \overline{C_1} + \Delta C_2 * \left(\frac{1 - b_2}{2 - r^2}\right)$$

Expected private firms' profit function for equilibrium

$$E(\Pi_2|I_1) = E\left(\left((C_2 - q_1^e - q_2^e) * q_2^e\right)\Big|I_1\right)$$

$$\Pi_2{}^* = E\left(\left(\left(C_2 - \left(2 * \overline{C_1} - \overline{C_2} + \Delta C_1 * \left(\frac{2-b_1}{2-r^2}\right)\right)\right.\right.\right.$$

$$\left.\left.- \left(\overline{C_2} - \overline{C_1} + \Delta C_2 * \left(\frac{1-b_2}{2-r^2}\right)\right)\right) * \left(\overline{C_2} - \overline{C_1} + \Delta C_2 * \left(\frac{1-b_2}{2-r^2}\right)\right)\right)$$

The formulation is simplified by using the equations $E(C_1) = \overline{C_1}$, $E(C_2) = \overline{C_2}$, $V_1^2 = E\left(C_1{}^2\right) - \overline{C_1}^2$, $V_2^2 = E\left(C_2{}^2\right) - \overline{C_2}^2$ and $V_{12} = E(C_1C_2) - \overline{C_1C_2}$. It yields

$$\Pi_2{}^* = \frac{(1-r^2)E\left(C_2{}^2\right) + (4-3r^2+r^4)\overline{C_2}^2}{(2-r^2)^2} - \frac{2E(C_1C_2) + (-2r^4+r^2-6)\overline{C_1C_2}}{(2-r^2)^2}$$

$$+ \frac{(4+4r^2+r^4)\overline{C_1}^2}{(2-r^2)^2} + \frac{V_1^2r^2 + V_2^2r^2}{(2-r^2)^2}$$

Excess expected profit function for equilibrium

$$\Pi_2{}^+ = \Pi_2{}^* - \overline{\Pi_2}$$

$$\Pi_2{}^+ = \Pi_2{}^* - \left(\overline{C_2}^2 - 2\overline{C_1C_2} + \overline{C_1}^2\right)$$

$$\Pi_2{}^+ = \frac{V_2^2 + V_1^2r^2 - 2V_{12}}{(2-r^2)^2}$$

$$\Pi_2{}^+ = V_2^2\frac{(1-b_2)^2}{(2-r^2)^2}$$

Case 6: Public firm and private firms produce information but only private firm disclosures information in Cell (2, 3)

$$q_1^e = C_1 + \overline{C_1} + b_2 * (C_2 - \overline{C_2}) - C_2$$

$$q_2^e = C_2 - \overline{C_1} - b_2 * (C_2 - \overline{C_2})$$

Expected private firms' profit function for equilibrium

$$E(\Pi_2|I_1) = E\left(\left((C_2 - q_1^e - q_2^e) * q_2^e\right)\Big|I_1\right)$$

$$\Pi_2{}^* = E\left(\left(C_2 - (C_1 + \overline{C_1} + b_2 * (C_2 - \overline{C_2}) - C_2) - (C_2 - \overline{C_1} - b_2 * (C_2 - \overline{C_2}))\right)\right.$$

$$\left. * (C_2 - \overline{C_1} - b_2 * (C_2 - \overline{C_2}))\right)$$

The formulation is simplified by using the equations $E(C_1) = \overline{C_1}$, $E(C_2) = \overline{C_2}$, $V_1^2 = E\left(C_1{}^2\right) - \overline{C_1}^2$, $V_2^2 = E\left(C_2{}^2\right) - \overline{C_2}^2$ and $V_{12} = E(C_1C_2) - \overline{C_1C_2}$. It yields

$$\Pi_2{}^* = E\left(C_2{}^2\right) - E(C_1C_2) - \overline{C_1C_2} + \overline{C_1}^2 + V_1^2r^2 - V_{12}$$

Excess expected profit function for equilibrium

$$\Pi_2{}^+ = \Pi_2{}^* - \overline{\Pi_2}$$

$$\Pi_2{}^{+} = E\left(C_2{}^2\right) - E(C_1C_2) - \overline{C_1C_2} + \overline{C_1}^2 + V_1^2r^2 - V_{12} - \left(\overline{C_2}^2 - 2\overline{C_1C_2} + \overline{C_1}^2\right)$$

$$\Pi_2{}^{+} = E\left(C_2{}^2\right) - \overline{C_2}^2 - (E(C_1C_2) - \overline{C_1C_2}) + V_1^2r^2 - V_{12}$$

It is know that $V_2^2 = E\left(C_2{}^2\right) - \overline{C_2}^2$ and $V_{12} = E(C_1C_2) - \overline{C_1C_2}$, therefore,

$$\Pi_2{}^{+} = V_2^2 + V_1^2r^2 - 2V_{12}$$

$$\Pi_2{}^{+} = V_2^2(1 - b_2)^2$$

Case 7: Private firm does not produce information but public firm produces and disclosures information in Cell (3, 1)

$$q_1^e = 2 * C_1 - \overline{C_2} - b_1 * (C_2 - \overline{C_2})$$

$$q_2^e = \overline{C_2} - C_1 + b_1 * (C_1 - \overline{C_1})$$

Expected private firms' profit function for equilibrium

$$E(\,\Pi_2|I_1) = E\left(\left((C_2 - q_1^e - q_2^e) * q_2^e\,\right)\Big|I_1\right)$$

$$\Pi_2{}^{*} = E\left(\left(C_2 - (2 * C_1 - \overline{C_2} - b_1 * (C_2 - \overline{C_2})) - (\overline{C_2} - C_1 + b_1 * (C_1 - \overline{C_1}))\right) * (\overline{C_2} - C_1 + b_1 * (C_1 - \overline{C_1}))\,\right)$$

The formulation is simplified by using the equations $E(C_1) = \overline{C_1}$, $E(C_2) = \overline{C_2}$, $V_1^2 = E\left(C_1{}^2\,\right) - \overline{C_1}^2$, $V_2^2 = E\left(C_2{}^2\right) - \overline{C_2}^2$ and $V_{12} = E(C_1C_2) - \overline{C_1C_2}$. It yields

$$\Pi_2{}^{*} = E\left(C_1{}^2\right) - E(C_1C_2)\ - \overline{C_1C_2} + \overline{C_2}^2 + V_2^2r^2 - V_{12}$$

$$\Pi_2{}^{*} = E\left(C_1{}^2\right) - (E(C_1C_2)\ - \overline{C_1C_2}) - 2\overline{C_1C_2} + \overline{C_2}^2 + V_2^2r^2 - V_{12}$$

$$\Pi_2{}^{*} = E\left(C_1{}^2\right) - 2\overline{C_1C_2} + \overline{C_2}^2 + V_2^2r^2 - 2V_{12}$$

Excess expected profit function for equilibrium

$$\Pi_2{}^{+} = \Pi_2{}^{*} - \overline{\Pi_2}$$

$$\Pi_2{}^{+} = E\left(C_1{}^2\right) - 2\overline{C_1C_2} + \overline{C_2}^2 + V_2^2r^2 - 2V_{12} - \left(\overline{C_2}^2 - 2\overline{C_1C_2} + \overline{C_1}^2\right)$$

$$\Pi_2{}^{+} = \left(E\left(C_1{}^2\right) - \overline{C_1}^2\right) + V_2^2r^2 - 2V_{12}$$

$$\Pi_2{}^{+} = V_1^2 + V_2^2r^2 - 2V_{12}$$

$$\Pi_2{}^{+} = V_1^2 + b_1^2V_1^2 - 2b_1V_1^2$$

$$\Pi_2{}^{+} = V_1^2(1 - b_1)^2$$

Case 8: Public firms and private firms produce information but only public firm disclosures information in Cell (3, 2)

$$q_1^e = 2 * C_1 - \overline{C_2} - b_1 * (C_1 - \overline{C_1})$$

$$q_2^e = \frac{C_2}{2} + \frac{\overline{C_2}}{2} + \frac{b_1 * (C_2 - \overline{C_2})}{2} - C_1$$

Expected private firms' profit function for equilibrium

$$E(\Pi_2|I_1) = E\left(\left((C_2 - q_1^e - q_2^e) * q_2^e\right) \middle| I_1\right)$$

$$\Pi_2{}^* = E\left(\left(C_2 - (2 * C_1 - \overline{C_2} - b_1 * (C_1 - \overline{C_1}))\right.\right.$$
$$\left. - \left(\frac{C_2}{2} + \frac{\overline{C_2}}{2} + \frac{b_1 * (C_2 - \overline{C_2})}{2} - C_1\right)\right)$$
$$\left. * \left(\frac{C_2}{2} + \frac{\overline{C_2}}{2} + \frac{b_1 * (C_2 - \overline{C_2})}{2} - C_1\right)\right)$$

The formulation is simplified by using the equations $E(C_1) = \overline{C_1}$, $E(C_2) = \overline{C_2}$, $V_1^2 = E({C_1}^2) - \overline{C_1}^2$, $V_2^2 = E({C_2}^2) - \overline{C_2}^2$ and $V_{12} = E(C_1C_2) - \overline{C_1C_2}$. It yields

$$\Pi_2{}^* = E({C_1}^2) + \frac{E({C_2}^2)}{4} - E(C_1C_2) - \overline{C_1C_2} + \frac{3\overline{C_2}^2}{4} + \frac{V_2^2r^2}{4} - V_{12}$$

$$\Pi_2{}^* = E({C_1}^2) + \frac{E({C_2}^2)}{4} - 2\overline{C_1C_2} + \frac{3\overline{C_2}^2}{4} + \frac{3V_2^2r^2}{4} - 2V_{12}$$

Excess expected profit function for equilibrium

$$\Pi_2{}^+ = \Pi_2{}^* - \overline{\Pi_2}$$

$$\Pi_2{}^+ = E({C_1}^2) + \frac{E({C_2}^2)}{4} - 2\overline{C_1C_2} + \frac{3\overline{C_2}^2}{4} + \frac{3V_2^2r^2}{4} - 2V_{12}$$
$$- \left(\overline{C_2}^2 - 2\overline{C_1C_2} + \overline{C_1}^2\right)$$

$$\Pi_2{}^+ = E({C_1}^2) - \overline{C_1}^2 + \frac{E({C_2}^2) - \overline{C_2}^2}{4} + \frac{3V_2^2r^2}{4} - 2V_{12}$$

$$\Pi_2{}^+ = V_1^2 + \frac{V_2^2}{4} + \frac{3V_2^2r^2}{4} - 2V_{12}$$

$$\Pi_2{}^+ = V_1^2 - 2b_1V_1^2 + b_1^2V_1^2 + \frac{V_2^2}{4} - \frac{V_2^2r^2}{4}$$

$$\Pi_2{}^+ = V_1^2(1 - b_1)^2 + \frac{V_2^2(1 - r^2)}{4}$$

Case 9: Public firms and private firms produce and disclose information in Cell (3, 3)

$$q_1^e = 2 * C_1 - C_2$$
$$q_2^e = C_2 - C_1$$

Expected private firms' profit function for equilibrium

$$E(\Pi_2|I_1) = E\left(\left((C_2 - q_1^e - q_2^e) * q_2^e\right) \middle| I_1\right)$$
$$\Pi_2^* = E\left(\left(C_2 - (2 * C_1 - C_2) - (C_2 - C_1)\right) * (C_2 - C_1)\right)$$
$$\Pi_2^* = E\left((C_2 - C_1) * (C_2 - C_1)\right)$$
$$\Pi_2^* = E\left({C_2}^2\right) - E(C_1C_2) + E\left({C_1}^2\right)$$

Excess expected profit function for equilibrium

$$\Pi_2^+ = \Pi_2^* - \overline{\Pi_2}$$
$$\Pi_2^+ = E\left({C_2}^2\right) - E(C_1C_2) + E\left({C_1}^2\right) - \left(\overline{C_2}^2 - 2\overline{C_1C_2} + \overline{C_1}^2\right)$$
$$\Pi_2^+ = \left(E\left({C_2}^2\right) - \overline{C_2}^2\right) - \left(E(C_1C_2) - 2\overline{C_1C_2}\right) + \left(E\left({C_1}^2\right) - \overline{C_1}^2\right)$$

The formulation is simplified by using the equations $E(C_1) = \overline{C_1}$, $E(C_2) = \overline{C_2}$, $V_1^2 = E\left({C_1}^2\right) - \overline{C_1}^2$, $V_2^2 = E\left({C_2}^2\right) - \overline{C_2}^2$ and $V_{12} = E(C_1C_2) - \overline{C_1C_2}$. It yields

$$\Pi_2^+ = V_2^2 - 2\,V_{12} + V_1^2$$
$$\Pi_2^+ = V_2^2 + (V_1^2 - 2b_1V_1^2 + b_1^2V_1^2) - b_1^2V_1^2$$
$$\Pi_2^+ = V_2^2 + V_1^2(1 - b_1)^2 - b_1^2V_1^2$$
$$\Pi_2^+ = V_2^2 + V_1^2(1 - b_1)^2 - V_2^2r^2 \text{ or } \Pi_2^+ = V_2^2(1 - r^2) + V_1^2(1 - b_1)^2$$

APPENDIX 6. PERFECT CORRALATION PAYOFF TABLES

For perfect correlation r=1,

$Firm\ 1$ $(Public\ firm)$	$Firm\ 2\ (Private\ firm)$		
	Do not produce information	Produce information, no disclosure	Produce and disclosure information
Do not produce information	***Case 1*** $W^+ = 0$ $\Pi_2{}^+ = 0$	***Case 2*** $W^+ = \frac{3V_2^2}{8}$ $\Pi_2{}^+ = \frac{V_2^2}{4}$	***Case 3*** $W^+ = \frac{3V_1^2}{2} - 2V_1V_2 + V_2^2$ $\Pi_2{}^+ = V_2^2 - 2V_1V_2 + V_1^2$
Produce information, no disclosure	***Case 4*** $W^+ = \frac{V_1^2}{2}$ $\Pi_2{}^+ = 0$	***Case 5*** $W^+ = \frac{3V_1^2}{2} - 2V_1V_2 + V_2^2$ $\Pi_2{}^+ = V_2^2 - 2V_1V_2 + V_1^2$	***Case 6*** $W^+ = \frac{3V_1^2}{2} - 2V_1V_2 + V_2^2$ $\Pi_2{}^+ = V_2^2 - 2V_1V_2 + V_1^2$
Produce and disclosure	***Case 7*** $W^+ = \frac{3V_1^2}{2} - 2V_1V_2 + V_2^2$ $\Pi_2{}^+ = V_1^2 + V_2^2 - 2V_1V_2$	***Case 8*** $W^+ = \frac{3V_1^2}{2} - 2V_1V_2 + V_2^2$ $\Pi_2{}^+ = V_1^2 + V_2^2 - 2V_1V_2$	***Case 9*** $W^+ = \frac{3V_1^2}{2} - 2V_1V_2 + V_2^2$ $\Pi_2{}^+ = V_2^2 - 2V_1V_2 + V_1^2$

For perfect anti-correlation r=-1,

$Firm\ 1$ $(Public\ firm)$	$Firm\ 2\ (Private\ firm)$		
	Do not produce information	Produce information, no disclosure	Produce and disclosure information
Do not produce information	***Case 1*** $W^+ = 0$ $\Pi_2{}^+ = 0$	***Case 2*** $W^+ = \frac{3V_2^2}{8}$ $\Pi_2{}^+ = \frac{V_2^2}{4}$	***Case 3*** $W^+ = \frac{3V_1^2}{2} + 2V_1V_2 + V_2^2$ $\Pi_2{}^+ = V_2^2 + 2V_1V_2 + V_1^2$
Produce information, no disclosure	***Case 4*** $W^+ = \frac{V_1^2}{2}$ $\Pi_2{}^+ = 0$	***Case 5*** $W^+ = \frac{3V_1^2}{2} + 2V_1V_2 + V_2^2$ $\Pi_2{}^+ = V_2^2 + 2V_1V_2 + V_1^2$	***Case 6*** $W^+ = \frac{3V_1^2}{2} + 2V_1V_2 + V_2^2$ $\Pi_2{}^+ = V_2^2 + 2V_1V_2 + V_1^2$
Produce and disclosure	***Case 7*** $W^+ = \frac{3V_1^2}{2} + 2V_1V_2 + V_2^2$ $\Pi_2{}^+ = V_1^2 + V_2^2 + 2V_1V_2$	***Case 8*** $W^+ = \frac{3V_1^2}{2} + 2V_1V_2 + V_2^2$ $\Pi_2{}^+ = V_1^2 + V_2^2 + 2V_1V_2$	***Case 9*** $W^+ = \frac{3V_1^2}{2} + 2V_1V_2 + V_2^2$ $\Pi_2{}^+ = V_2^2 + 2V_1V_2 + V_1^2$