T.C.

GEBZE TEKNİK ÜNİVERSİTESİ

SOSYAL BİLİMLER ENSTİTÜSÜ

INFORMATION TRANSMISSION IN MIXED OLIGOPOLY

Erdem ÇELİK

YÜKSEK LİSANS TEZİ

İKTİSAT ANABİLİM DALI

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Tez Danışmanı

DOÇ. DR. Sadettin Haluk Çitçi

GEBZE

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GEBZE TEKNİK ÜNİVERSİTESİ	**YÜKSEK LİSANS JÜRİ ONAY FORMU**

GTÜ ..Sosyal Bilimler..Enstitüsü Yönetim Kurulu'nun ..../......./........ tarih ve ..../.......... sayılı kararıyla oluşturulan jüri tarafından ..../......./......... tarihinde tez savunma sınavı yapılan ....'ın tez çalışması ....İKTİSAT.....................Anabilim Dalında YÜKSEK LİSANS tezi olarak kabul edilmiştir.

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ÖZET

Birçok ülkede kamu firmaları ve özel firmalar birbirleri ile rekabet halindedir. Hem kamu hem de özel firmaları içeren karma oligopoller, sadece özel firmaları içeren oligopollerden farklılık gösterir. Kamu firmaları hem firmaların karını hem de sosyal refahı arttırmak isterken, özel firmalar kendi karlarını arttırmak isterler. Bu yüzden karma oligopollerin rekabet ettiği durumlarda çıkan sonuçlar ile sadece özel firmaların rekabet ettiği durumlar arasında çıkan sonuçlar farklılık gösterebilir.

Literatürde yapılmış çalışmaların çoğunda firmaların talebi tam olarak bildiği varsayılırken, gerçek hayatta bu varsayımın tutarlı olmadığını biliyoruz. Talep belirsizliği, firmaların ürettiği ürünlere yönelik tüketici talebini doğru bir şekilde tahmin edemediği durumlarda ortaya çıkar. Bu çalışmada, kamu ve özel firmaların rekabet ettiği karma oligopollerde firmaların talep belirsizliği durumunda ne yapacağını inceliyoruz. Firmaların talep hakkında topladığı bilgileri rekabet ettiği firma ile paylaşıp paylaşmadığını, paylaşır ise ne kadarını paylaştığını analiz ediyoruz. Sonuçlara baktığımızda, kamu firması rekabet ettiği özel firma ile her zaman bilgi paylaşımında bulunurken, özel firmanın bilgi paylaşımı kararı ürünlerin türüne göre değişiklik gösteriyor. Firmaların ürettiği ürünler ikame edilebilir mallar ise, özel firma rekabet ettiği kamu firması ile bilgi paylaşımında bulunmuyor. Ürünler tamamlayıcı mallar olduğunda ise, özel firma kamu firması ile bilgi paylaşımında bulunuyor.

Anahtar Kelimeler: Karma oligopol, talep belirsizliği, bilgi paylaşımı, kamu firmaları, özel firmalar

SUMMARY

In many countries, public and private firms are in the same environment with a strong competition. Mixed oligopolies which include both public and private firms differ from pure oligopolies which include only private firms. Public firms seek to maximize firms profit and social welfare while private firms seek to maximize only its own profit. For this reason, the consequences of mixed oligopoly competition may differ from the consequences of pure oligopoly.

In the literature, most of the studies assume that demand is fully known. We know that this assumption does not hold in real life. Demand uncertainty occurs when firms can not accurately predict consumer demand for products they produce. In this study, we research the consequences of the competition under demand uncertainty in mixed oligopolies. When companies collect information about demand, we analyze whether they share this information with a competitor, and if so, how much they share. The results show that public firm always shares information with private firm while private firm's decision on information sharing depends on the type of the goods. If the products of the firms are substitutable, private firm does not share information with public firm. If the products are complements, private firm shares information with public firm.

Key Words: Mixed oligopoly, demand uncertainty, information sharing, public firm, private firm

TEŞEKKÜR

Bu çalışmanın en başından en sonuna kadar gerçekleşen süreçlerin tamamında desteğini her zaman gösteren sayın hocam Doç. Dr. Sadettin Haluk ÇİTÇİ 'ye çok teşekkür ederim.

LIST OF CONTENTS

LIST OF ABBREVIATIONS

Abbreviations	**Explanations**
CS	: Consumer Surplus
PS	: Producer Surplus
NP	: No Pooling
CP	: Complete Pooling

LIST OF FIGURES

Table No: **Page**

LIST OF TABLES

Table No: **Page**

1. INTRODUCTION

Information plays an important role for firms no matter what business they are in. Accurate and reliable information helps companies improve their decision making. Also, information sharing among competitors may lead companies to get a better knowledge about the market they are competing in. One of the benefits of information sharing is, companies understand the market better which enables companies to produce more efficiently. Companies may share every type of information with each other, for example, information about production cost, sales data, customers demand and so on. On the other side, there are competitional disadvantages of sharing information with competitors. It will be easier for companies to monitor each other when they have more detailed information about their competitors. Increased transparency in the market can allow collusion among competitors that may lead to higher pricing.

In a mixed oligopolistic market, there is at least one public and one private firm that compete with each other. In the last three decades, there has been a worldwide movement towards (at least partial) privatization of public firms. In many countries, several sectors of activity are characterized by the presence of both public and private firms. Some of the examples about mixed oligopolies are network sector (broadcasting, mail, transportation and telecommunication), service sector (banking, insurance, healthcare and education) and energy sector (gas and electricity) (Donder and Roemer, 2009). Public firms are considered to behave different than private firms where the objective of public firm is maximizing total welfare which is equal to the sum of consumer and producer surplus. To give an example from Turkey, Ziraat Bank (banking sector), TOKİ (construction sector) and TRT (media sector) are some of the public firms that are in competition with private firms. The objective of these firms is not only maximizing their profits but also maximizing social welfare. When there is an economic slowdown, government may ask banks to decrease the interest rate. As a result of this demand, public-owned banks may offer a loan with low interest rate to increase economic activities. Therefore, competition in mixed oligopoly may leads to quite different conclusions than those found in the pure oligopoly.

Most of the studies in the literature focused on pure oligopolies under cost or demand uncertainty with homogenous or differentiated product. Differently, this study examines the effect of information sharing in mixed oligopolies under demand uncertainty. In the model, firms compete in quantities (a la cournot competition), and there is uncertainty on the intercept of each firm's demand function. Each firm receives a private signal that provides an estimation about the common price intercept of the demand functions. Firms may give authority to the agency to reveal their private signal and make it available for other firms. If a firm decides not to reveal its information, none of its private information will be put in a common pool. This case represents no pooling (NP). If a firm decides to partially reveal its information, part of its private information will be put in a common pool. This case represents partial pooling. If a firm decides to reveal all of its information, all of its private information will be put in a common pool. This case represents complete pooling (CP). The game has two stages in our model. At the first stage, firms inform the agency about how much of their private information to put in a common pool. At the second stage, each firm chooses its quantity of output.

The result obtained in our study indicate that the game has a unique subgame perfect equilibrium at the first stage. Public firm always put all of its private information in a common pool (Complete Pooling). Private firm's incentive for information sharing depends on the type of the goods (substitutes or complements). If the goods are substitutes, private firm does not put any of its private information in a common pool (No pooling). If the goods are complements, private firm put all of its private information in a common pool (Complete Pooling).

2. LITERATURE

Since the late 1970’s, there has been extensive theoretical research about information sharing in oligopoly. Early contributions to the literature were made by Ponssard (1979), Novshek and Sonnenschein (1982), Clarke (1983), Fried (1984), Vives (1984) and Gal-Or (1985). In the following years, many other studies have been done about information sharing in oligopoly.

Most of the studies have similar basic structure but assumptions and models may vary from paper to paper. Some of the differences in the literature are as follows:

- Cournot or Bertrand Competition
- Cost or Demand Uncertainty
- Product Differentiation
- Number of Firms
- State of Nature
- Noisy or Perfect Signals
- Revelation of Signals
- One or Two Stage Game

Each of the above assumptions may make a significant change about the results of information sharing in oligopoly. After researching the literature, we see the results are very sensitive to the assumptions and even making similar but only slightly different assumptions may cause completely different equilibrium outcomes.

The results of the some researches in the literature on information sharing in oligopoly are as follows.

Fried (1984) examines a duopoly model about information sharing under cost uncertainty. The focus of the study is information producing and information sharing. The results show that producing information is always optimal and information sharing is generally beneficial for both firms.

Vives (1984) develops a duopoly model about information sharing under demand uncertainty. In cournot competition, complete sharing is a dominant strategy if the goods are substitutes and no sharing is a dominant strategy if the goods are complements. In Bertrand competition, the results are reversed.

Gal-Or (1986) considers a duopolistic market where uncertainty is about unknown private costs. The result about information sharing depends nature of the competition (Cournot or Bertrand). Sharing is a dominant strategy with Cournot competition and no sharing is a dominant strategy with Bertrand competition.

Medin, Rodriguez, & Rodriguez (2003) analyze the information sharing in oligopoly where firms receive private information about random demand. The study has two scenarios and each one examines different unknown parameter. The parameters are either an unknown intercept or an unknown slope of the random demand. It is shown that if the private signals are accurate enough, information sharing is profitable among firms for both of the scenarios.

Our study is also related to the literature analyzing mixed oligopolies. Study of mixed oligopolies has become significantly popular after privatization of public firms has spread in many economies around the world. Haraguchi and Matsumura (2015) compare Cournot and Bertrand competition in a mixed oligopoly. They find that price competition is better for public firms. For private firms, it depends on the number of private firms in oligopoly. If the number of private firms are at least five, quantity competition is more profitable for private firms. Çitçi and Karakaş (2014) analyze mixed oligopoly where firms choose the capacities and prices sequentially with differentiated products. If the realized demand is higher than expected demand, both firms hold under capacity. If the realized demand is lower than expected demand, both firms hold excess capacity. If the realized demand is medium, firms hold under or excess capacity according to whether the products are complements or substitutes. When the products are substitutes, private firm holds under capacity but public firm holds excess capacity. When the products are complements, both firms hold under capacity.

Closed to our study, Çitçi and Hazer (2016) examine the incentives to produce information and to share information about uncertain cost in mixed oligopoly. They showed that if the correlation coefficient is equal or less than zero, both public and private firm share information with each other. This study focuses on information sharing under cost uncertainty with homogenous products while our study focuses on information sharing under demand uncertainty with differentiated products.

In the literature, most of the studies on information sharing only include private firms. Vives (1984), Shapiro (1986), and Sakai and Yamato (1989) have welfare analysis in their model, however, they also do not include public firms in their studies.

To the best of our knowledge, this is the first study to analyze the information sharing between public and private firms under demand uncertainty. Our findings indicate that information sharing outcomes are much different under mixed oligopoly than that of established under private oligopolies.

3. MODEL

We have a non-cooperative mixed duopoly model that includes one public and one private firm. Each firm producing a differentiated good in the same market. From now on, the public firm is denoted by 1 and the private firm is denoted by 2.

Demand functions are assumed to be linear.

$$p_1 = \alpha - \beta q_1 - \gamma q_2$$

$$p_2 = \alpha - \gamma q_1 - \beta q_2$$

where $\alpha > 0,\ \beta > |\gamma| \geq 0$. The goods are substitutes, complements or independent depending on the value of γ. If $\gamma > 0$, the goods are substitutes. If $\gamma < 0$, the goods are complements. If $\gamma = 0$, the goods are independent. If $\beta = \gamma$, the goods are perfect substitutes. If $\beta = -\gamma$, the goods are perfect complements. α is the demand intercept and γ/β varies from 1 to -1. Also, p_1 is the price and q_1 is the quantity of the goods produced by public firm and p_2 is the price and q_2 is the quantity of the goods produced by private firm.

Each firm's marginal costs are equal and constant regardless of the units of the goods they produce. We assume that prices of the goods are calculated after marginal costs have been deducted. Firms compete in the quantity of output they produce.

Given demand functions, the profit function for the public firm can be formulated as follows:

$$\Pi_1 = p_1 q_1$$

$$\Pi_1 = (\alpha - \beta q_1 - \gamma q_2) * q_1$$

$$\Pi_1 = \alpha q_1 - \beta {q_1}^2 - \gamma q_1 q_2$$

Also, given demand functions, profit function of the private firm can be formulated as follows:

$$\Pi_2 = p_2 q_2$$

$$\Pi_2 = (\alpha - \gamma q_1 - \beta q_2) * q_2$$

$$\Pi_2 = \alpha q_2 - \gamma q_1 q_2 - \beta {q_2}^2$$

Both firms are risk neutral. The objective of the public firm is to maximize the social welfare and the objective of the private firm is to maximize its profit. Welfare function is described as $W = CS + PS$, where CS stands for consumer surplus and PS stands for producer surplus. So, welfare is the sum of consumer surplus and producer surplus.

Before finding out the objective function of the public firm, we need to explain consumer surplus and producer surplus.

Consumer surplus can be formulated as follows:

$$CS = U(q_1, q_2) - \sum_{i=1}^{2} p_i q_i$$

$$CS = U(q_1, q_2) - p_1 q_1 - p_2 q_2$$

Utility function of the consumer is assumed as the following:

$$U(q_1, q_2) = \alpha * (q_1 + q_2) - \frac{\left(\beta {q_1}^2 + 2\gamma q_1 q_2 + \beta {q_2}^2\right)}{2}$$

Utility function is supposed to be quadratic, strictly concave and symmetric in the quantity of the goods produced by public and private firm.

Producer surplus is the profit that firms get from involvement in the market.

$$PS = \Pi_1 + \Pi_2$$

$$PS = p_1 q_1 + p_2 q_2$$

Now, we are ready to explain the objective functions of the firms.

Objective function of public firm is (social welfare):

$$W = CS + PS$$

$$W = \alpha * (q_1 + q_2) - \frac{\left(\beta {q_1}^2 + 2\gamma q_1 q_2 + \beta {q_2}^2\right)}{2} - p_1 q_1 - p_2 q_2 + p_1 q_1 + p_2 q_2$$

$$W = \alpha * (q_1 + q_2) - \frac{\left(\beta {q_1}^2 + 2\gamma q_1 q_2 + \beta {q_2}^2\right)}{2}$$

Objective function of private firm is (profit function of private firm):

$$\Pi_2 = (\alpha - \gamma q_1 - \beta q_2) * q_2$$

We also assume that demand intercept, α, is a random variable in the model. Following to Vives (1984) we further assume that random variable α is normally distributed with mean $\bar{\alpha}$ and variance $V(\alpha)$. Each firm observes a private noisy signal for the random variable α. These signals involve demand intercept and noise. So, the equation of signals can be shown as $s_i = \alpha + \varepsilon_i$, $i = 1{,}2$ where s_1 is the signal that public firm observes and s_2 is the signal that private firm observes and $\varepsilon_1$and ε_2 are the error terms of the signals. We have bivariate, normally distributed error terms in the model. Error terms are independent and uncorrelated with α. Their mean is zero and variances of error terms are equal to or greater than their covariance ($v_i \geq \sigma_{12} \geq 0, i = 1,2$). All of these are common knowledge.

By having this information, we have the following equations.

$E(\alpha|s_i) = (1 - t_i)\bar{\alpha} + t_i s_i$ and $E(s_j|s_i) = (1 - d_i)\bar{\alpha} + d_i s_i$, with $t_i = V(\alpha)/(V(\alpha) + v_i)$ and $d_i = (V(\alpha) + \sigma_{12}/(V(\alpha) + v_i)$, $i = 1,2, i \neq j$. It can be seen that $1 \geq d_i \geq t_i \geq 0$ since $v_i \geq \sigma_{12} \geq 0$.

Signals give more precise information about the demand intercept when the variance decreases. The conditional expectation formula is as the following:

$$E(\alpha|s_i) = (1 - t_i)\bar{\alpha} + t_i s_i$$

If the precision of the signals increase, t_i increases because when t_i increases $E(\alpha|s_i)$ gets closer to s_i than $\bar{\alpha}$. Also, t_i increases as v_i decreases because $t_i = V(\alpha)/(V(\alpha) + v_i)$. While the signal goes from being perfectly precise to being completely imprecise, v_i goes from 0 to ∞ and t_i goes from 1 to 0. When the signals are perfectly precise, $E(\alpha|s_i) = s_i$, $v_i = 0$ and $t_i = 1$. When the signals are completely imprecise, $E(\alpha|s_i) = \bar{\alpha}$, $v_i = \infty$ and $t_i = 0$.

To model information sharing process, we assume that there is an independent trade agency that collects the observation samples. The trade agency receives an n observation sample $(t_{i1}, t_{i2}, t_{i3}, \dots, t_{in})$, where $t_{ik} = a + u_{ik}$ and u_{ik}'s are independent and identically distributed random variables. Their mean is zero, variance σ_u^2 and independent with a. Firm 1 (public firm) receives n_1 observation sample and allows the trade agency to reveal $\lambda_1 n_1$ observation where $0 \leq \lambda_1 \leq 1$. Also, firm 2 (private firm) receives n_2 observation sample and allows the trade agency to reveal $\lambda_2 n_2$ observation where $0 \leq \lambda_2 \leq 1$. There are $\lambda_1 n_1 + \lambda_2 n_2$ observation sample in the common pool that available for both public and private firm, public firm has $n_1 - \lambda_1 n_1$ private observation sample and private firm has $n_2 - \lambda_2 n_2$ private observation. The signals firms receive are the best estimation of a that depend on their own observation sample plus observation sample put by other firm in the common pool. The signal public firm receives, s_1, is based on $n_1 + \lambda_2 n_2$ and the signal private firm receives, s_2, is based on $n_2 + \lambda_1 n_1$. $s_1 = \alpha + \left(1/(n_1 + \lambda_2 n_2)\right)\left(\sum_{k-1}^{n_1} u_{1k} + \sum_{k=1}^{\lambda_2 n_2} u_{2k}\right), s_2 = \alpha + \left(1/(n_2 + \lambda_1 n_1)\right)\left(\sum_{k-1}^{n_2} u_{2k} + \sum_{k=1}^{\lambda_1 n_1} u_{1k}\right)$. We have bivariate, normally distributed error terms with zero means in the model where, $v_1 = \sigma_u^2/(n_1 + \lambda_2 n_2\,), v_2 =$

$\sigma_u^2/(n_2 + \lambda_2 n_2\,)$ and $\sigma_{12} = \big((\lambda_1 n_1 + \lambda_2 n_2)/(n_1 + \lambda_2 n_2)(n_2 + \lambda_1 n_1)\big)\sigma_u^2$. Remember that $v_i \geq \sigma_{12} \geq 0, i = 1{,}2$.

We can analyze the effect of information sharing. When firms do not share their information with each other ($\lambda_1 = \lambda_2 = 0$), $v_1 = \sigma_u^2/n_1$, $v_2 = \sigma_u^2/n_2$ and $\sigma_{12} = 0$. When firms share all of their information with each other ($\lambda_1 = \lambda_2 = 1$), $v_1 = v_2 = \sigma_{12} = \sigma_u^2/(n_1 + n_2)$. As it is seen, if firms share their private information, variance of the error terms decrease and the correlation of the error terms increase.

Public and private firms play two-stage game. There are simultaneous moves within each stage but previous stage observed before the next stage begins. Timing of the game is as follows: in the first stage, both firms receive private noisy signal about the uncertain demand parameter. Each firm decides the amount of information to share with its competitor. In the second stage, based on their collected and received information about uncertain demand, each firm decides how much to produce. At the end of the second stage, the game ends.

4. RESULTS

4.1. Expected Welfare and Profit Maximization

In this section, calculations will be made to find out the expected production quantities of public and private firm. As known, the objective of public firm is to maximize the expected social welfare and the objective of private firm is to maximize its own expected profit.

4.1.1. Welfare maximization for public firm

To find the expected production quantity of public firm that maximizes the value of the expected welfare, the first derivative of expected welfare with respect to q_1 should be equal to zero. Also, to distinguish the maximum points from minimum points, the second derivative of expected welfare with respect to q_1 should be negative.

$$E\left(\frac{dW}{dq_1}\middle|s_1\right) = 0 \; and \; E\left(\frac{d^2W}{dq_1{}^2}\middle|s_1\right) < 0$$

$$\max\left[E(W|s_1) = E\left(\alpha * (q_1 + q_2) - \frac{(\beta q_1{}^2 + 2\gamma q_1 q_2 + \beta q_2{}^2)}{2}\middle|s_1\right)\right]$$

$$E\left(\frac{d\left(\alpha * (q_1 + q_2) - \frac{(\beta q_1{}^2 + 2\gamma q_1 q_2 + \beta q_2{}^2)}{2}\right)}{dq_1} \mid s_1\right)$$

$$E\left(\frac{d\left(\alpha q_1 + \alpha q_2 - \frac{\beta q_1{}^2}{2} - \gamma q_1 q_2 - \frac{\beta q_2{}^2}{2}\right)}{dq_1} \mid s_1\right)$$

$$E\left(\frac{dW}{dq_1}\middle|s_1\right) = E((\alpha - \beta q_1 - \gamma q_2)|s_1) = 0$$

$$E\left(\frac{d^2W}{dq_1{}^2}\middle|s_1\right) = -\beta$$

As known $\beta > 0$, so the second derivative of expected welfare with respect to q_1 is negative and it shows that the first derivative of expected welfare with respect to q_1 is the maximum point of the function.

$$E((\alpha - \beta q_1 - \gamma q_2)|s_1) = 0$$

$$\beta q_1 = E(\alpha - \gamma q_2)$$

The amount of the goods produced by public firm that maximizes the value of the expected welfare is:

$$q_1{}^*(s_1) = \frac{1}{\beta} E(\alpha - \gamma q_2|s_1)$$

$$q_1{}^*(s_1) = \frac{1}{\beta} E\big((\alpha|s_1) - \gamma E(q_2{}^*(s_2)|s_1)\big)$$

4.1.2. Profit maximization for private firm

To find the expected production quantity of private firm that maximizes the value of the expected profit of private firm, the first derivative of expected profit of private firm with respect to q_1 should be equal to zero. Also, to distinguish the maximum points from minimum points, the second derivative of expected profit of private firm with respect to q_1 should be negative.

$$E\left(\frac{d\Pi_2}{dq_2}\bigg|s_2\right) = 0 \; and \; E\left(\frac{d^2\Pi_2}{dq_2{}^2}\bigg|s_2\right) < 0$$

$$max\big[E(\,\Pi_2|s_2) = E\big(((\alpha - \gamma q_1 - \beta q_2) * q_2)\big)|s_2\,\big]$$

$$E\left(\frac{d((\alpha - \gamma q_1 - \beta q_2) * q_2)}{dq_2}\bigg|s_2\right) = 0$$

$$E\left(\frac{d(\alpha q_2 - \gamma q_1 q_2 - \beta q_2{}^2)}{dq_2}\bigg|s_2\right) = 0$$

$$E\left(\frac{d\Pi_2}{dq_2}\bigg|s_2\right) = E((\alpha - \gamma q_1 - 2\beta q_2)|s_2) = 0$$

$$E\left(\frac{d^2\Pi_2}{dq_2{}^2}\bigg|s_2\right) = -\beta$$

As known $\beta > 0$, so the second derivative of expected profit of private firm with respect to q_1 is negative and it shows that the first derivative of expected profit of private firm with respect to q_1 is the maximum point of the function.

$$E\Big(\big((\alpha - \gamma q_1 - 2\beta q_2)\big|s_2\big)\Big) = 0$$

$$2\beta q_2 = E(\alpha - \gamma q_1)$$

The amount of the goods produced by private firm that maximizes the value of the expected profit of private firm is:

$$q_2{}^*(s_2) = \frac{1}{2\beta} E\big((\alpha - \gamma q_1)\big|s_2\big)$$

$$q_2{}^*(s_2) = \frac{1}{2\beta} E\big((\alpha|s_2) - \gamma E(q_1{}^*(s_1)|s_2)\big)$$

Bayesian equilibrium of the model is $q_1{}^*(s_1)$ and $q_2{}^*(s_2)$. Expected welfare is maximized at $q_1{}^*(s_1)$ if the private firm produce $q_2{}^*(s_2)$ and profit of private firm is maximized at $q_2{}^*(s_2)$ if the public firm produce $q_1{}^*(s_1)$.

4.2. Equilibrium Output Strategies

We have the value of $q_1{}^*(s_1)$ in terms of $q_2{}^*(s_2)$ and we have the value of $q_2{}^*(s_2)$ in terms of $q_1{}^*(s_1)$.

$$q_1{}^*(s_1) = \frac{1}{\beta}\big(E_1(a) - \gamma E_1(q_2)\big)$$

and

$$q_2{}^*(s_2) = \frac{1}{2\beta}\big(E_2(a) - \gamma E_2(q_1)\big)$$

$$q_1{}^*(s_1) = \frac{1}{\beta}\left(E_1(a) - \gamma E_1\left(\frac{1}{2\beta}\big(E_2(a) - \gamma E_2(q_1)\big)\right)\right)$$

$$q_1{}^*(s_1) = \frac{1}{\beta}\left(E_1(a) - \gamma E_1\left(\frac{1}{2\beta}\left(E_2(a) - \gamma E_2\left(\frac{1}{\beta}\big(E_1(a) - \gamma E_1(q_2)\big)\right)\right)\right)\right)$$

$$\ldots$$

The above equations can be extended infinitely. This is called “I think that he thinks that I think..." model. The equation seems like diverging to infinity but this chain can be broken. As long as firms know $E(\alpha|s_1)$, $E(\alpha|s_2)$, $E(s_1|s_2)$ and $E(s_2|s_1)$, the infinite chain will converge and solution will be found.

4.2.1. Equilibrium output strategy of public firm

The following calculations should be performed to obtain the equilibrium strategy of public firm. The detailed solution of equilibrium output strategy of public firm is given in appendix 1.

$$q_1{}^*(s_1) = \frac{1}{\beta} E\big((\alpha|s_1) - \gamma E(q_2{}^*(s_2)|s_1)\big)$$

$$q_2{}^*(s_2) = \frac{1}{2\beta} E\big((\alpha|s_2) - \gamma E(q_1{}^*(s_1)|s_2)\big)$$

To make the calculations easier, we will use some notations:

$$E_1(q_2) = E(q_2{}^*(s_2)|s_1)$$

$$E_2(q_1) = E(q_1{}^*(s_1)|s_2)$$

$$E_1(a) = \mathrm{E}(\alpha|s_1)$$

$$E_2(a) = \mathrm{E}(\alpha|s_2)$$

$$E_1E_2(a) = E\big((\alpha|s_2)|s_1\big)$$

$$E_2E_1(a) = E\big((\alpha|s_1)|s_2\big)$$

$$q_1{}^*(s_1) = \frac{1}{\beta}E_1(a) - \frac{\gamma}{2\beta^2}E_1E_2(a) + \frac{\gamma^2}{2\beta^3}E_1E_2E_1(a) - \frac{\gamma^3}{2^2\beta^4}E_1E_2E_1E_2(a) + \frac{\gamma^4}{2^2\beta^5}E_1E_2E_1E_2E_1(a) - \frac{\gamma^5}{2^3\beta^6}E_1E_2E_1E_2E_1E_2(a) \ldots$$

$$q_1{}^*(s_1) = \frac{1}{\beta}\bar{\alpha}\left(1 + \frac{\gamma^2}{2\beta^2} + \left(\frac{\gamma^2}{2\beta^2}\right)^2 + \left(\frac{\gamma^2}{2\beta^2}\right)^4 \ldots\right) - \frac{\gamma}{2\beta^2}\bar{\alpha}\left(1 + \frac{\gamma^2}{2\beta^2} + \left(\frac{\gamma^2}{2\beta^2}\right)^2 \ldots\right) + \frac{t_1(s_1 - \bar{\alpha})}{\beta}\left(1 + \frac{\gamma^2 d_1 d_2}{2\beta^2} + \left(\frac{\gamma^2 d_1 d_2}{2\beta^2}\right)^2 + \left(\frac{\gamma^2 d_1 d_2}{2\beta^2}\right)^4 \ldots\right) - \frac{\gamma d_1 t_2(s_1 - \bar{\alpha})}{2\beta^2}\left(1 + \frac{\gamma^2 d_1 d_2}{2\beta^2} + \left(\frac{\gamma^2 d_1 d_2}{2\beta^2}\right)^2 + \left(\frac{\gamma^2 d_1 d_2}{2\beta^2}\right)^4 \ldots\right)$$

$$q_1{}^*(s_1) = \frac{1}{\beta}\bar{\alpha}\left(\frac{2\beta^2}{2\beta^2 - \gamma^2}\right) - \frac{\gamma}{2\beta^2}\bar{\alpha}\left(\frac{2\beta^2}{2\beta^2 - \gamma^2}\right) + \frac{t_1(s_1 - \bar{\alpha})}{\beta}\left(\frac{2\beta^2}{2\beta^2 - \gamma^2 d_1 d_2}\right) - \frac{\gamma d_1 t_2(s_1 - \bar{\alpha})}{2\beta^2}\left(\frac{2\beta^2}{2\beta^2 - \gamma^2 d_1 d_2}\right)$$

$$q_1{}^*(s_1) = \frac{\bar{\alpha}(2\beta - \gamma)}{2\beta^2 - \gamma^2} + \frac{2\beta - \gamma d_2}{2\beta^2 - \gamma^2 d_1 d_2} t_1(s_1 - \bar{\alpha})$$

$$A_1 = \frac{\bar{\alpha}(2\beta - \gamma)}{2\beta^2 - \gamma^2}$$

$$B_1 = \frac{2\beta - \gamma d_2}{2\beta^2 - \gamma^2 d_1 d_2}$$

$$q_1{}^*(s_1) = A_1 + B_1 t_1(s_1 - \bar{\alpha})$$

4.2.2. Equilibrium output strategy of private firm

The following calculations should be performed to obtain the equilibrium strategy of private firm. The detailed solution of equilibrium output strategy of private firm is given in appendix 2.

$$q_1{}^*(s_1) = \frac{1}{\beta}\left(E_1(a) - \gamma E_1(q_2)\right)$$

$$q_2{}^*(s_2) = \frac{1}{2\beta}\left(E_2(a) - \gamma E_2(q_1)\right)$$

$$q_2{}^*(s_2) = \frac{1}{2\beta}E_2(a) - \frac{\gamma}{2\beta^2}E_2E_1(a) + \frac{\gamma^2}{2^2\beta^3}E_2E_1E_2(a) - \frac{\gamma^3}{2^2\beta^4}E_2E_1E_2E_1(a) + \frac{\gamma^4}{2^3\beta^5}E_2E_1E_2E_1E_2(a) - \frac{\gamma^5}{2^3\beta^6}E_2E_1E_2E_1E_2E_1(a) \ldots$$

$$q_2{}^*(s_2) = \frac{1}{2\beta}\bar{\alpha}\left(1 + \frac{\gamma^2}{2\beta^2} + \left(\frac{\gamma^2}{2\beta^2}\right)^2 + \left(\frac{\gamma^2}{2\beta^2}\right)^4 \ldots\right) - \frac{\gamma}{2\beta^2}\bar{\alpha}\left(1 + \frac{\gamma^2}{2\beta^2} + \left(\frac{\gamma^2}{2\beta^2}\right)^2 \ldots\right) + \frac{t_2(s_2-\bar{\alpha})}{2\beta}\left(1 + \frac{\gamma^2 d_1 d_2}{2\beta^2} + \left(\frac{\gamma^2 d_1 d_2}{2\beta^2}\right)^2 + \left(\frac{\gamma^2 d_1 d_2}{2\beta^2}\right)^4 \ldots\right) - \frac{\gamma d_2 t_1(s_2-\bar{\alpha})}{2\beta^2}\left(1 + \frac{\gamma^2 d_1 d_2}{2\beta^2} + \left(\frac{\gamma^2 d_1 d_2}{2\beta^2}\right)^2 + \left(\frac{\gamma^2 d_1 d_2}{2\beta^2}\right)^4 \ldots\right)$$

$$q_2{}^*(s_2) = \frac{1}{2\beta}\bar{\alpha}\left(\frac{2\beta^2}{2\beta^2-\gamma^2}\right) - \frac{\gamma}{2\beta^2}\bar{\alpha}\left(\frac{2\beta^2}{2\beta^2-\gamma^2}\right) + \frac{t_2(s_2-\bar{\alpha})}{2\beta}\left(\frac{2\beta^2}{2\beta^2-\gamma^2 d_1 d_2}\right) - \frac{\gamma d_2 t_1(s_2-\bar{\alpha})}{2\beta^2}\left(\frac{2\beta^2}{2\beta^2-\gamma^2 d_1 d_2}\right)$$

$$q_2{}^*(s_2) = \frac{\bar{\alpha}(\beta-\gamma)}{2\beta^2-\gamma^2} + \frac{\beta-\gamma d_1}{2\beta^2-\gamma^2 d_1 d_2}t_2(s_2-\bar{\alpha})$$

$$A_2 = \frac{\bar{\alpha}(\beta-\gamma)}{2\beta^2-\gamma^2}$$

$$B_2 = \frac{\beta-\gamma d_1}{2\beta^2-\gamma^2 d_1 d_2}$$

$$q_2{}^*(s_2) = A_2 + B_2 t_2(s_2-\bar{\alpha})$$

The unique Bayesian equilibrium of the game is $q_1{}^*(s_1)$ and $q_2{}^*(s_2)$.

$$q_1{}^*(s_1) = A_1 + B_1 t_1(s_1-\bar{\alpha}), where\ A_1 = \frac{\bar{\alpha}(2\beta-\gamma)}{2\beta^2-\gamma^2}\ and\ B_1 = \frac{2\beta-\gamma d_2}{2\beta^2-\gamma^2 d_1 d_2}$$

$$q_2{}^*(s_2) = A_2 + B_2 t_2(s_2 - \bar{\alpha}), where\ A_2 = \frac{\bar{\alpha}(\beta - \gamma)}{2\beta^2 - \gamma^2}\ and\ B_2 = \frac{\beta - \gamma d_1}{2\beta^2 - \gamma^2 d_1 d_2}$$

If public firm produce $A_1 + B_1 t_1(s_1 - \bar{\alpha})$, the best reply of private firm is to produce $A_2 + B_2 t_2(s_2 - \bar{\alpha})$. If private firm produce $A_2 + B_2 t_2(s_2 - \bar{\alpha})$, the best reply of public firm is to produce $A_1 + B_1 t_1(s_1 - \bar{\alpha})$.

4.3. Expected Welfare and Expected Profit Calculation

Equilibrium output strategies of public and private firm are as follows:

$$q_1{}^*(s_1) = A_1 + B_1 t_1(s_1 - \bar{\alpha}), A_1 = \frac{\bar{\alpha}(2\beta - \gamma)}{2\beta^2 - \gamma^2}, B_1 = \frac{2\beta - \gamma d_2}{2\beta^2 - \gamma^2 d_1 d_2}$$

$$q_2{}^*(s_2) = A_2 + B_2 t_2(s_2 - \bar{\alpha}), A_2 = \frac{\bar{\alpha}(\beta - \gamma)}{2\beta^2 - \gamma^2}, B_2 = \frac{\beta - \gamma d_1}{2\beta^2 - \gamma^2 d_1 d_2}$$

Expected welfare function can be calculated with equilibrium output strategies $q_1{}^*$ and $q_2{}^*$ which are already found in “Equilibrium output strategies”.

4.3.1. Expected Welfare Calculation for public firm

The following calculations should be performed to obtain the objective function of public firm (social welfare). The detailed solution of expected welfare is given in appendix 3.

$$E(W|s_1) = E\left(\alpha * (q_1 + q_2) - \frac{\left(\beta q_1{}^2 + 2\gamma q_1 q_2 + \beta q_2{}^2\right)}{2}\middle| s_1\right)$$

$$E(W|s_1) = E\left(\alpha q_1 + \alpha q_2 - \frac{\left(\beta q_1{}^2 + 2\gamma q_1 q_2 + \beta q_2{}^2\right)}{2}\middle| s_1\right)$$

$$E(W|s_1) = E\left(\alpha(s_1) q_1{}^*(s_1) + \alpha(s_1) q_2{}^*(s_2)|s_1 - \frac{\beta\left(q_1{}^*(s_1)\right)^2}{2}\right.$$
$$\left. - \gamma q_1{}^*(s_1) q_2{}^*(s_2)|s_1 - \frac{\beta(q_2{}^*(s_2)|s_1)^2}{2}\right)$$

$$E(W|s_1) = E\big(\alpha(s_1) q_1{}^*(s_1)\big) + E\big(\alpha(s_1) q_2{}^*(s_2)|s_1\big) - \frac{\beta}{2} E\left(\left(q_1{}^*(s_1)\right)^2\right)$$
$$- \gamma E(q_1{}^*(s_1) q_2{}^*(s_2)|s_1) - \frac{\beta}{2} E((q_2{}^*(s_2)|s_1)^2)$$

To calculate expected welfare, we first need to solve the equations,

$E\big(\alpha(s_1) q_1{}^*(s_1)\big), E\big(\alpha(s_1) q_2{}^*(s_2)|s_1\big), E\left(\left(q_1{}^*(s_1)\right)^2\right), E((q_2{}^*(s_2)|s_1)^2), E(q_1{}^*(s_1) q_2{}^*(s_2)|s_1)$

$$E\big(\alpha(s_1)q_1{}^*(s_1)\big) = \bar{\alpha}A_1 + 2B_1t_1V(\alpha)$$
$$E\big(\alpha(s_1)q_2{}^*(s_2)|s_1\big) = \bar{\alpha}A_2 + 2B_2t_2d_1V(\alpha)$$
$$E\big(q_1{}^*(s_1)\big)^2 = A_1{}^2 + B_1{}^2t_1V(\alpha)$$
$$E(q_2{}^*(s_2)|s_1)^2 = A_2{}^2 + B_2{}^2t_2{}^2d_1{}^2(V(\alpha) + v_1)$$
$$E(q_1{}^*(s_1)q_2{}^*(s_2)|s_1) = A_1A_2 + B_1B_2t_2d_1V(\alpha)$$
$$E(W|s_1) = \bar{\alpha}A_1 + 2B_1t_1V(\alpha) + \bar{\alpha}A_2 + 2B_2t_2d_1V(\alpha) - \frac{\beta}{2}\big(A_1{}^2 + B_1{}^2t_1V(\alpha)\big)$$
$$- \gamma\big(A_1A_2 + B_1B_2t_2d_1V(\alpha)\big) - \frac{\beta}{2}\big(A_2{}^2 + B_2{}^2t_2{}^2d_1{}^2(V(\alpha) + v_1)\big)$$
$$E(W|s_1) = \frac{\bar{\alpha}^2 * (7\beta^3 - 6\beta^2y - 2\beta y^2 + 2y^3)}{2 * (2\beta^2 - y^2)^2} + 2B_1t_1V(\alpha) + 2B_2t_2d_1V(\alpha)$$
$$- \frac{\beta}{2}B_1{}^2t_1V(\alpha) - \gamma B_1B_2t_2d_1V(\alpha) - \frac{\beta}{2}\big(B_2{}^2t_2{}^2d_1{}^2(V(\alpha) + v_1)\big)$$

4.3.2. Expected Profit Calculation for private firm

Expected profit of private firm function can be also calculated with equilibrium output strategies $q_1{}^*$ and $q_2{}^*$ which are already found in “Equilibrium output strategies”.

The following calculations should be performed to obtain the objective function of private firm (profit of private firm). The detailed solution of expected profit of private firm is given in appendix 4.

$$\Pi_2 = p_2 * q_2$$
$$E(\Pi_2|s_2) = E\left(\big((\alpha - \gamma q_1 - \beta q_2) * q_2\big)\Big|s_2\right)$$
$$E(\Pi_2|s_2) = E\big((\alpha|s_2 - \gamma q_1{}^*(s_1)|s_2 - \beta q_2{}^*(s_2)|s_2) * q_2{}^*(s_2)\big)$$

$E(\alpha - \gamma q_1{}^*(s_1)|s_2) = 2\beta q_2{}^*(s_2)$ according to the first order conditions, so

$$E(\Pi_2|s_2) = \beta\big(q_2{}^*(s_2)\big)^2$$
$$E(\Pi_2|s_2) = \beta E\big(A_2 + B_2t_2(s_2 - \bar{\alpha})\big)^2$$
$$E(s_2 - \bar{\alpha})^2 = V(\alpha) + v_2$$
$$E(\Pi_2|s_2) = \beta\big(A_2{}^2 + B_2{}^2t_2{}^2V(\alpha) + v_2\big)$$
$$E(\Pi_2|s_2) = \beta\big(A_2{}^2 + B_2{}^2t_2V(\alpha)\big)$$

4.4. Results

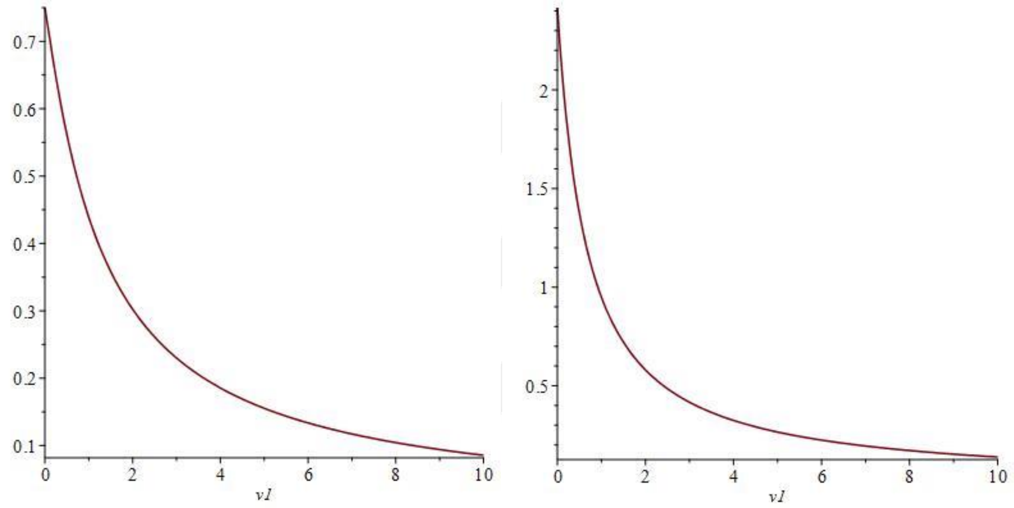
Proposition 1a. In equilibrium, expected welfare

a) increases when the precision of public firm's information increases.
b) increases when the precision of private firm's information increases.
c) increases when the correlation of the signals increase.

Proof.

We know from previous section that when the signal gives more precise information to the public firm, v_1 decreases.

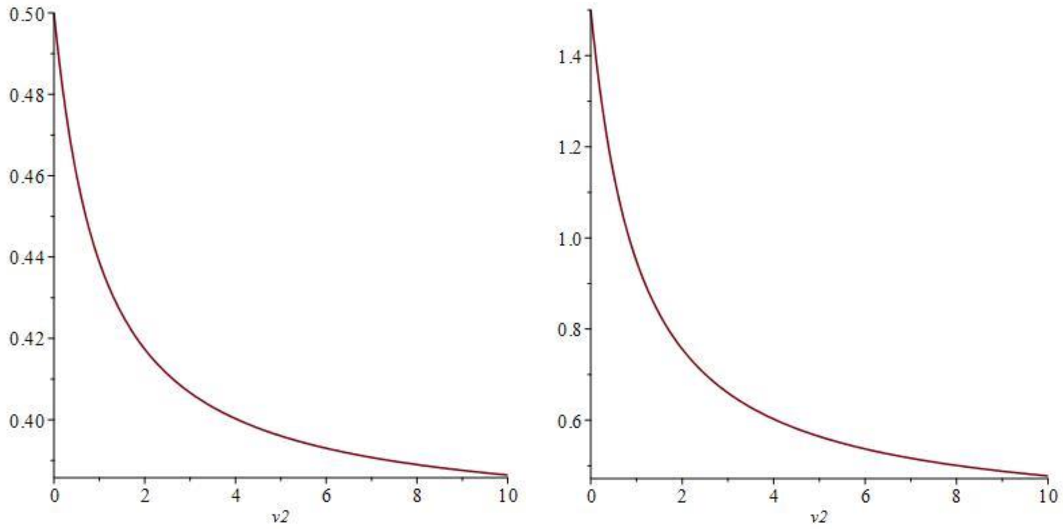
Figure 4. 1: Effect of v_1 on Expected Welfare



The graph on the left side shows the effect of v_1 on expected welfare when the goods are substitutes ($\gamma > 0$), the graph on the right side shows the effect of v_1 on expected welfare when the goods are complements ($\gamma < 0$). To make the calculations easier, we assume $v_2 = 1$, $V(\alpha) = 1$, $\sigma_{12} = 1$ and $\beta = 2$. γ is assumed 1 if the goods are substitutes and γ is assumed -1 if the goods are complements. As it is seen, expected Welfare increases as v_1 decreases.

We know from previous section that when the signal gives more precise information to the private firm, v_2 decreases.

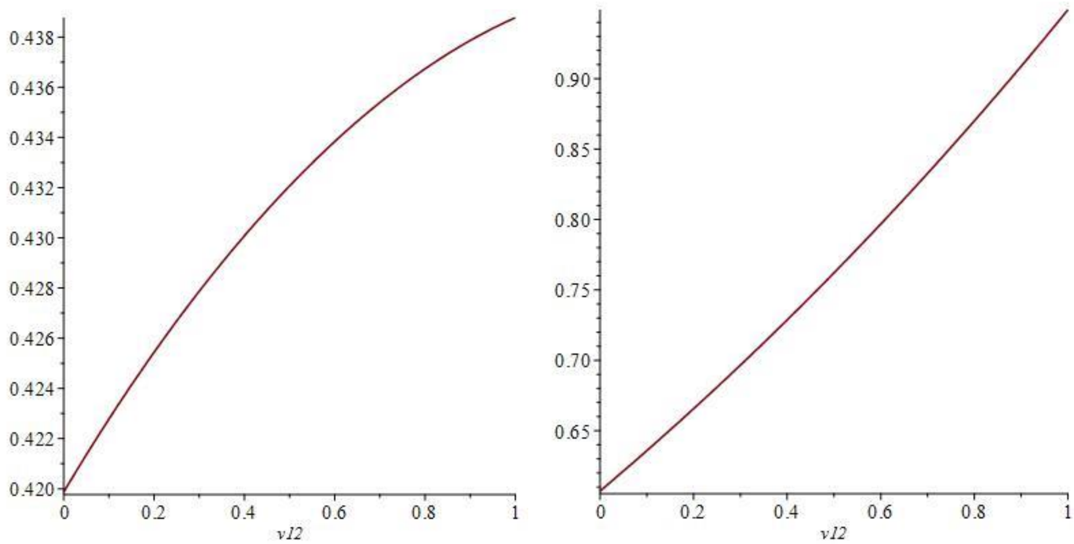
Figure 4. 2: Effect of v_2 on Expected Welfare



The graph on the left side shows the effect of v_2 on expected welfare when the goods are substitutes ($\gamma > 0$), the graph on the right side shows the effect of v_2 on expected welfare when the goods are complements ($\gamma < 0$). To make the calculations easier, we assume $v_1 = 1$, $V(\alpha) = 1$, $\sigma_{12} = 1$ and $\beta = 2$. γ is assumed 1 if the goods are substitutes and γ is assumed -1 if the goods are complements. As it is seen, expected Welfare increases as v_2 decreases.

Lastly, effect of correlation of the signals on expected welfare is as the following.

Figure 4. 3: Effect of σ_{12} on Expected Welfare



The graph on the left side shows the effect of σ_{12} on expected welfare when the goods are substitutes ($\gamma > 0$), the graph on the right side shows the effect of v_2 on expected welfare when the goods are complements ($\gamma < 0$). To make the calculations easier, we assume $v_1 = 1$, $v_2 = 1$, $V(\alpha) = 1$ and $\beta = 2$. γ is assumed 1 if the goods are substitutes and γ is assumed -1 if the goods are complements. As it is seen, expected Welfare increases as σ_{12} increases.

Proposition 1b. In equilibrium, expected profit of private firm

a) increases when the precision of private firm's information increases.
b) increases, decreases or remains the same when the precision of public firm's information increases depending on the type of the goods (substitutes, complements or independent).
c) increases, decreases or remains the same when the correlation of the signals increase depending on the type of the goods (complements, substitutes or independent).

Proof. Recall that

$E(\Pi_2|s_2) = \beta\left(A_2{}^2 + B_2{}^2 t_2 V(\alpha)\right)$,

The slope in question is:

$B_2 t_2 = \left(\frac{\beta - \gamma d_1}{2\beta^2 - \gamma^2 d_1 d_2}\right) * t_2$

Remember that $d_i = (V(\alpha) + \sigma_{12}/(V(\alpha) + v_i)$ and $t_i = V(\alpha)/(V(\alpha) + v_i)$

a)

$$B_2 t_2 = \left(\frac{\beta - \gamma d_1}{2\beta^2 - \gamma^2 d_1 d_2}\right) * \frac{V(\alpha)}{(V(\alpha) + v_2)}$$

$$B_2 = \frac{\beta - \gamma d_1}{2\beta^2 - \gamma^2 d_1 d_2}$$

We know from previous section that when the signal gives more precise information to the private firm, v_2 decreases.

If v_2 decreases d_2 increases, if d_2 increases denominator of the B_2 decreases, if denominator of the B_2 decreases B_2 increases.

$$t_2 = \frac{V(\alpha)}{(V(\alpha) + v_2)}$$

If v_2 decreases denominator of the t_2 decreases, if denominator of the t_2 decreases t_2 increases.

So, both B_2 and t_2 increases when v_2 decreases.

b)

$$\frac{dB_2}{dv_1} = \frac{\beta(2\beta - \gamma d_2)(V(\alpha) + \sigma_{12})\gamma}{4\left(\beta^2(V(\alpha) + v_1) - \frac{(V(\alpha) + \sigma_{12})d_2\gamma^2}{2}\right)^2}$$

We know from previous section that when the signal gives more precise information to the public firm, v_1 decreases.

When $dB_2/dv_1 > 0$, the slope of the function is positive. Positive slope tells us that as v_1 increases, B_2 increases or as v_1 decreases, B_2 decreases.

When $dB_2/dv_1 < 0$, the slope of the function is negative. Negative slope tells us that as v_1 increases, B_2 decreases or as v_1 decreases, B_2 increases.

Denominator of the dB_2/dv_1 is always positive because the square of every real number is positive.

β is always positive because $\beta > 0$.

$2\beta - \gamma d_2$ is always positive because $\beta \geq \gamma$ and $1 \geq d_2$.

$V(\alpha) + \sigma_{12}$ is always positive because $v_i \geq \sigma_{12} \geq 0$ and $V(\alpha) \geq 0$.

γ can be positive, 0 or negative.

Thus,

If γ is positive then dB_2/dv_1 will be positive. In other words, when the goods are substitutes ($\gamma > 0$), expected profit of private firm increases as the precision of public firm's information increases.

If γ is 0 then dB_2/dv_1 will be 0. In other words, when the goods are independent ($\gamma = 0$), expected profit of private firm remains the same as the precision of public firm's information increases.

If γ is negative then dB_2/dv_1 will be negative. In other words, when the goods are complements ($\gamma < 0$), expected profit of private firm decreases as the precision of public firm's information increases.

$$sign \frac{dB_2}{dv_1} = sign\, \gamma$$

c)

$$\frac{dB_2}{d\sigma_{12}} = -\frac{\gamma(V(\alpha) + v_2)\left(\left(\beta^2 - \beta\gamma + \frac{\gamma^2}{2}\right)\left(V(\alpha)\right)^2 + \left((v_1 + v_2)\beta^2 - \gamma(v_1 + \sigma_{12})\beta + \sigma_{12}\gamma^2\right)V(\alpha) + \beta^2 v_1 v_2 - \beta v_1 \sigma_{12}\gamma + \frac{(\sigma_{12})^2\gamma^2}{2}\right)}{2\left(\left(\beta^2 - \frac{\gamma^2}{2}\right) - \left(V(\alpha)\right)^2 + \left((v_1 + v_2)\beta^2 - \sigma_{12}\gamma^2\right)V(\alpha) + \beta^2 v_1 v_2 - \frac{(\sigma_{12})^2\gamma^2}{2}\right)^2}$$

Denominator of the $dB_2/d\sigma_{12}$ is always positive because the square of every real number is positive.

$(V(\alpha) + v_2)$ is always positive because $v_i \geq \sigma_{12} \geq 0$ and $V(\alpha) \geq 0$.

$\beta^2 - \beta\gamma + \frac{\gamma^2}{2}$ is always positive because $\beta \geq \gamma$.

$(v_1 + v_2)\beta^2 - \gamma(v_1 + \sigma_{12})\beta + \sigma_{12}\gamma^2$ is always positive because $\beta \geq \gamma$ and $v_i \geq \sigma_{12} \geq 0$.

$\beta^2 v_1 v_2 - \beta v_1 \sigma_{12}\gamma$ is always positive $\beta \geq \gamma$ and $v_i \geq \sigma_{12} \geq 0$.

γ can be positive, 0 or negative.

Thus,

If γ is positive then $dB_2/d\sigma_{12}$ will be negative. In other words, when the goods are complements ($\gamma < 0$), expected profit of private firm increases as the correlation of the signals increases.

If γ is 0 then $dB_2/d\sigma_{12}$ will be 0. In other words, when the goods are independent ($\gamma = 0$), expected profit of private firm remains the same as the correlation of the signals increases.

If γ is negative then $dB_2/d\sigma_{12}$ will be positive. In other words, when the goods are substitutes ($\gamma > 0$), expected profit of private firm decreases as the correlation of the signals increases.

$$sign\frac{dB_2}{d\sigma_{12}} = -sign\,\gamma$$

Lemma 1.

a) v_i decreases when λ_j increases and is independent of $\lambda_i, i = 1,2,\ j \neq i$.

b) σ_{12} increases when λ_i increases if $\lambda_j < 1,\ i = 1,2,\ j \neq i$. Or else σ_{12} is independent of λ_i.

Proof.

a)

$$v_i = \frac{\sigma_u^2}{(n_i + \lambda_j n_j)}$$

If λ_j increases, denominator of v_i increases, if denominator of v_i increases v_i decreases.

Since there is no λ_i in the equation, v_i is independent of λ_i.

b)

$$\sigma_{12} = \big((\lambda_1 n_1 + \lambda_2 n_2)/(n_1 + \lambda_2 n_2)(n_2 + \lambda_1 n_1)\big)\sigma_u^2$$

$$\frac{d\sigma_{12}}{d\lambda_1} = \left(-\frac{(\lambda_2 - 1)n_1 n_2}{(\lambda_2 n_2 + n_1)(\lambda_1 n_1 + n_2)^2}\right)\sigma_u^2$$

$d\sigma_{12}/d\lambda_1$ is positive when $\lambda_2 < 1$.

$d\sigma_{12}/d\lambda_1$ is 0 when $\lambda_2 = 1$.

$$\frac{d\sigma_{12}}{d\lambda_2} = \left(-\frac{(\lambda_1 - 1)n_1 n_2}{(\lambda_2 n_2 + n_1)^2(\lambda_1 n_1 + n_2)}\right)\sigma_u^2$$

$d\sigma_{12}/d\lambda_2$ is positive when $\lambda_1 < 1$.

$d\sigma_{12}/d\lambda_2$ is 0 when $\lambda_1 = 1$.

Lemma 2a. Expected welfare increases when λ_1 and λ_2 increase.

Proof.

Table 4. 1: Effect of λ_1 and λ_2 on Expected Welfare

	Public Firm (Firm 1)	
	Substitutes ($\gamma > 0$)	Complements ($\gamma < 0$)
$\lambda_1 \uparrow$	$v_2 \downarrow W \uparrow$ $\sigma_{12} \uparrow W \uparrow$	$v_2 \downarrow W \uparrow$ $\sigma_{12} \uparrow W \uparrow$
$\lambda_2 \uparrow$	$v_1 \downarrow W \uparrow$ $\sigma_{12} \uparrow W \uparrow$	$v_1 \downarrow W \uparrow$ $\sigma_{12} \uparrow W \uparrow$

According to Lemma 1, increase in λ_1, decreases the variance of the error term of the firm 2 and increases the correlation of the signals (If $\lambda_2 < 1$). According to proposition 1a, both effects ($v_2 \downarrow and\ \sigma_{12} \uparrow$) increase the expected welfare. According to Lemma 1, increase in λ_2, decreases the variance of the error term of the firm 1 and increases the correlation of the signals (If $\lambda_1 < 1$). According to proposition 1a, both effects ($v_1 \downarrow and\ \sigma_{12} \uparrow$) increase the expected welfare.

Lemma 2b. Expected profit of firm 2 decreases with λ_2 if the goods are substitutes. Expected profit of firm 2 increases with λ_2 and with λ_1 if the goods are complements.

Proof.

Table 4. 2: Effect of λ_1 and λ_2 on Expected Profit of Private Firm

	Private Firm (Firm 2)	
	Substitutes ($\gamma > 0$)	Complements ($\gamma < 0$)
$\lambda_2 \uparrow$	$v_1 \downarrow \Pi_2 \downarrow$ $\sigma_{12} \uparrow \Pi_2 \downarrow$	$v_1 \downarrow \Pi_2 \uparrow$ $\sigma_{12} \uparrow \Pi_2 \uparrow$
$\lambda_1 \uparrow$	$v_2 \downarrow \Pi_2 \uparrow$ $\sigma_{12} \uparrow \Pi_2 \downarrow$	$v_2 \downarrow \Pi_2 \uparrow$ $\sigma_{12} \uparrow \Pi_2 \uparrow$

According to Lemma 1, increase in λ_2, decreases the variance of the error term of the firm 1 and increases the correlation of the signals (If $\lambda_2 < 1$). According to proposition 1a, both effects ($v_1 \downarrow and\ \sigma_{12} \uparrow$) increase the expected profit of firm 2 if the goods are complements. If the goods are substitutes, both effects ($v_1 \downarrow and\ \sigma_{12} \uparrow$) decrease the expected profit of firm 2. According to Lemma 1, increase in λ_1, decreases the variance of the error term of the firm 2 and increases the correlation of the signals (If $\lambda_1 < 1$). According to proposition 1a, both effects ($v_2 \downarrow and\ \sigma_{12} \uparrow$) increase the expected profit of firm 2 if the goods are complements. If the goods are substitutes, the first effect ($v_2 \downarrow$) increases the expected profit of firm 2 but second effect ($\sigma_{12} \uparrow$) decreases the expected profit of firm 2. So, the net effect of λ_1 on expected profit of private firm is not certain if the goods are substitutes.

Proposition 2. The two-stage game has a unique subgame perfect equilibrium in dominant strategies at the first stage if the goods are not independent. If the goods are substitutes, public firm chooses complete pooling and private firm chooses no pooling. If the goods are complements, both firms choose complete pooling. If the goods are independent, any $\lambda_1\ and\ \lambda_2$ is an equilibrium.

Proof.

Table 4. 3: Sub-game perfect equilibrium

	Substitutes ($\gamma > 0$)	Complements ($\gamma < 0$)
Public Firm	Complete Pooling	Complete Pooling
Private Firm	No Pooling	Complete Pooling

In the first stage of the game, each firm decides the amount of information to share with its competitor. According to Lemma 2a, complete pooling ($\lambda_1 = 1$) is a dominant strategy for the public firm since expected welfare increases when λ_1 increases regardless of the value of λ_2. According to Lemma 2b, no pooling is a dominant strategy ($\lambda_2 = 0$) for the private firm since expected profit of firm 2 decreases when λ_2 increases regardless of the value of λ_1 if the goods are substitutes and complete pooling is a dominant strategy ($\lambda_2 = 1$) for the private firm since expected profit of firm 2 increases when λ_2 increases regardless of the value of λ_1 if the goods are complements.

5. CONCLUSION

We analyzed a model of mixed duopoly including one public firm and one private firm. Most of the studies have focused on the pure oligopolies where only private firms compete with each other. Conflicting results may occur between pure and mixed oligopoly because objective functions of the public and private firm are different. Private firms are considered pure profit maximizers while public firms aim to maximize social welfare and take consumer surplus into account. In our study, we examined the incentives for public and private firm to share their private information on uncertain demand when they are competing with each other in the same environment.

If public and private firms play two-stage simultaneous game under demand uncertainty, our results show that both firms reveal their private information to their competitor in most cases. As public firm always share its private information of demand intercept with its competitor, private firm also shares its information if the goods are complements.

Some of the results that we find is in line to the results of earlier studies in the literature. The conclusion in the literature is that generally it may be beneficial for firms to share their private information about uncertain demand with each other if they compete in quantities and if the goods they produce are complements. However, many of the studies in the literature also show that information sharing is not optimal for private firms when the goods they produce are substitutes. In this study, we establish that when a private and a public firm compete in quantities and when each receives noisy signals about uncertain demand, they completely share their private signals with each other if the goods are complements. However, private firm does not share any information with the public firm if the goods are substitutes. Yet, the public firm continues to share its private information with the private firm even though it does not share. Although, it is possible for firms to share their private information partially with each other, this case never arises in equilibrium. In other words, the private firm chooses to share completely or not to share at all its private information with the public firm depending on whether goods are complements or substitutes. The public firm always completely share its private information on uncertain demand with the private firm even in cases that its competitor private firm does share none of its private information with the public firm. It shows that the social welfare advantage of reducing demand uncertainty is larger than the benefit of competitive advantage of the private information.

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ÖZGEÇMİŞ

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6. APPENDIX

APPENDIX 1. Detailed Solution of equilibrium output strategy of the public firm

$$q_1^*(s_1) = \frac{1}{\beta} E\big((\alpha|s_1) - \gamma E(q_2^*(s_2)|s_1)\big)$$
$$q_2^*(s_2) = \frac{1}{2\beta} E\big((\alpha|s_2) - \gamma E(q_1^*(s_1)|s_2)\big)$$

To make the calculations easier, we will use some notations;

$$E_1(q_2) = E(q_2^*(s_2)|s_1)$$
$$E_2(q_1) = E(q_1^*(s_1)|s_2)$$
$$E_1(a) = \mathrm{E}(\alpha|s_1)$$
$$E_2(a) = \mathrm{E}(\alpha|s_2)$$
$$E_1E_2(a) = E\big((\alpha|s_2)|s_1\big)$$
$$E_2E_1(a) = E\big((\alpha|s_1)|s_2\big)$$

Now, we can rewrite the $q_1^*(s_1)$ and $q_2^*(s_2)$

$$q_1^*(s_1) = \frac{1}{\beta}\big(E_1(a) - \gamma E_1(q_2)\big)$$

$$q_2^*(s_2) = \frac{1}{2\beta}\big(E_2(a) - \gamma E_2(q_1)\big)$$

$$q_1^*(s_1) = \frac{1}{\beta}\left(E_1(a) - \gamma E_1\left(\frac{1}{2\beta}\big(E_2(a) - \gamma E_2(q_1)\big)\right)\right)$$

$$q_1^*(s_1) = \frac{1}{\beta}\left(E_1(a) - \gamma E_1\left(\frac{1}{2\beta}\left(E_2(a) - \gamma E_2\left(\frac{1}{\beta}\big(E_1(a) - \gamma E_1(q_2)\big)\right)\right)\right)\right)$$

$$\cdots$$

$$q_1^*(s_1) = \frac{1}{\beta}E_1(a) - \frac{\gamma}{2\beta^2}E_1E_2(a) + \frac{\gamma^2}{2\beta^3}E_1E_2E_1(a) - \frac{\gamma^3}{2^2\beta^4}E_1E_2E_1E_2(a)$$
$$+ \frac{\gamma^4}{2^2\beta^5}E_1E_2E_1E_2E_1(a) - \frac{\gamma^5}{2^3\beta^6}E_1E_2E_1E_2E_1E_2(a) \ldots$$

$$E_1(a) = \bar{\alpha} + t_1(s_1 - \bar{\alpha})$$

$$E_1E_2(a) = \bar{\alpha} + t_2\big((1-d_1)\bar{\alpha} + d_1s_1 - \bar{\alpha}\big) = \bar{\alpha} + t_2(\bar{\alpha} - d_1\bar{\alpha} + d_1s_1 - \bar{\alpha})$$
$$= \bar{\alpha} + t_2(-d_1\bar{\alpha} + d_1s_1) = \bar{\alpha} + t_2\big(d_1(s_1 - \bar{\alpha})\big) = \bar{\alpha} + d_1t_2(s_1 - \bar{\alpha})$$

$$E_1E_2E_1(a) = \bar{\alpha} + d_2t_1\big((1-d_1)\bar{\alpha} + d_1s_1 - \bar{\alpha}\big) = \bar{\alpha} + d_2t_1(\bar{\alpha} - d_1\bar{\alpha} + d_1s_1 - \bar{\alpha})$$

$$= \bar{\alpha} + d_2t_1(-d_1\bar{\alpha} + d_1s_1) = \bar{\alpha} + d_2t_1\big(d_1(s_1 - \bar{\alpha})\big)$$

$$= \bar{\alpha} + d_1d_2t_1(s_1 - \bar{\alpha})$$

$$E_1E_2E_1E_2(a) = \bar{\alpha} + d_2d_1t_2\big((1-d_1)\bar{\alpha} + d_1s_1 - \bar{\alpha}\big)$$

$$= \bar{\alpha} + d_2d_1t_2(\bar{\alpha} - d_1\bar{\alpha} + d_1s_1 - \bar{\alpha}) = \bar{\alpha} + d_2d_1t_2(-d_1\bar{\alpha} + d_1s_1)$$

$$= \bar{\alpha} + d_2d_1t_2\big(d_1(s_1 - \bar{\alpha})\big) = \bar{\alpha} + {d_1}^2d_2t_2(s_1 - \bar{\alpha})$$

$$E_1E_2E_1E_2E_1(a) = \bar{\alpha} + {d_2}^2d_1t_1\big((1-d_1)\bar{\alpha} + d_1s_1 - \bar{\alpha}\big)$$

$$= \bar{\alpha} + {d_2}^2d_1t_1(\bar{\alpha} - d_1\bar{\alpha} + d_1s_1 - \bar{\alpha}) = \bar{\alpha} + {d_2}^2d_1t_1(-d_1\bar{\alpha} + d_1s_1)$$

$$= \bar{\alpha} + {d_2}^2d_1t_1\big(d_1(s_1 - \bar{\alpha})\big) = \bar{\alpha} + {d_1}^2{d_2}^2t_1(s_1 - \bar{\alpha})$$

$$E_1E_2E_1E_2E_1E_2(a) = \bar{\alpha} + {d_2}^2{d_1}^2t_2\big((1-d_1)\bar{\alpha} + d_1s_1 - \bar{\alpha}\big)$$

$$= \bar{\alpha} + {d_2}^2{d_1}^2t_2(\bar{\alpha} - d_1\bar{\alpha} + d_1s_1 - \bar{\alpha})$$

$$= \bar{\alpha} + {d_2}^2{d_1}^2t_2(-d_1\bar{\alpha} + d_1s_1) = \bar{\alpha} + {d_2}^2{d_1}^2t_2\big(d_1(s_1 - \bar{\alpha})\big)$$

$$= \bar{\alpha} + {d_1}^3{d_2}^2t_2(s_1 - \bar{\alpha})$$

$$\ldots$$

$$q_1{}^*(s_1) = \frac{1}{\beta}\big(\bar{\alpha} + t_1(s_1 - \bar{\alpha})\big) - \frac{\gamma}{2\beta^2}\big(\bar{\alpha} + d_1t_2(s_1 - \bar{\alpha})\big)$$

$$+ \frac{\gamma^2}{2\beta^3}\big(\bar{\alpha} + d_1d_2t_1(s_1 - \bar{\alpha})\big) - \frac{\gamma^3}{2^2\beta^4}\Big(\bar{\alpha} + {d_1}^2d_2t_2(s_1 - \bar{\alpha})\Big)$$

$$+ \frac{\gamma^4}{2^2\beta^5}\Big(\bar{\alpha} + {d_1}^2{d_2}^2t_1(s_1 - \bar{\alpha})\Big) - \frac{\gamma^5}{2^3\beta^6}\Big(\bar{\alpha} + {d_1}^3{d_2}^2t_2(s_1 - \bar{\alpha})\Big)\ldots$$

$$q_1{}^*(s_1) = \frac{1}{\beta}\bar{\alpha} + \frac{t_1(s_1 - \bar{\alpha})}{\beta} - \frac{\gamma}{2\beta^2}\bar{\alpha} - \frac{\gamma d_1t_2(s_1 - \bar{\alpha})}{2\beta^2} + \frac{\gamma^2}{2\beta^3}\bar{\alpha} + \frac{\gamma^2d_1d_2t_1(s_1 - \bar{\alpha})}{2\beta^3}$$

$$- \frac{\gamma^3}{2^2\beta^4}\bar{\alpha} - \frac{\gamma^3{d_1}^2d_2t_2(s_1 - \bar{\alpha})}{2^2\beta^4} + \frac{\gamma^4}{2^2\beta^5}\bar{\alpha} + \frac{\gamma^4{d_1}^2{d_2}^2t_1(s_1 - \bar{\alpha})}{2^2\beta^5}$$

$$- \frac{\gamma^5}{2^3\beta^6}\bar{\alpha} - \frac{\gamma^5{d_1}^3{d_2}^2t_2(s_1 - \bar{\alpha})}{2^3\beta^6}\ldots$$

$$q_1{}^*(s_1) = \frac{1}{\beta}\bar{\alpha}\left(1 + \frac{\gamma^2}{2\beta^2} + \left(\frac{\gamma^2}{2\beta^2}\right)^2 + \left(\frac{\gamma^2}{2\beta^2}\right)^4 \dots\right)$$

$$- \frac{\gamma}{2\beta^2}\bar{\alpha}\left(1 + \frac{\gamma^2}{2\beta^2} + \left(\frac{\gamma^2}{2\beta^2}\right)^2 \dots\right)$$

$$+ \frac{t_1(s_1 - \bar{\alpha})}{\beta}\left(1 + \frac{\gamma^2 d_1 d_2}{2\beta^2} + \left(\frac{\gamma^2 d_1 d_2}{2\beta^2}\right)^2 + \left(\frac{\gamma^2 d_1 d_2}{2\beta^2}\right)^4 \dots\right)$$

$$- \frac{\gamma d_1 t_2(s_1 - \bar{\alpha})}{2\beta^2}\left(1 + \frac{\gamma^2 d_1 d_2}{2\beta^2} + \left(\frac{\gamma^2 d_1 d_2}{2\beta^2}\right)^2 + \left(\frac{\gamma^2 d_1 d_2}{2\beta^2}\right)^4 \dots\right)$$

We know that we can determine the value of the convergent series,

$$1 + \frac{\gamma^2}{2\beta^2} + \left(\frac{\gamma^2}{2\beta^2}\right)^2 + \left(\frac{\gamma^2}{2\beta^2}\right)^4 \dots = \frac{1}{1 - \frac{\gamma^2}{2\beta^2}} = \frac{1}{\frac{2\beta^2 - \gamma^2}{2\beta^2}} = \frac{2\beta^2}{2\beta^2 - \gamma^2}$$

$$1 + \frac{\gamma^2 d_1 d_2}{2\beta^2} + \left(\frac{\gamma^2 d_1 d_2}{2\beta^2}\right)^2 + \left(\frac{\gamma^2 d_1 d_2}{2\beta^2}\right)^4 \dots = \frac{1}{1 - \frac{\gamma^2 d_1 d_2}{2\beta^2}} = \frac{1}{\frac{2\beta^2 - \gamma^2 d_1 d_2}{2\beta^2}}$$

$$= \frac{2\beta^2}{2\beta^2 - \gamma^2 d_1 d_2}$$

Now, we can solve the equation of $q_1{}^*(s_1)$,

$$q_1{}^*(s_1) = \frac{1}{\beta}\bar{\alpha}\left(\frac{2\beta^2}{2\beta^2 - \gamma^2}\right) - \frac{\gamma}{2\beta^2}\bar{\alpha}\left(\frac{2\beta^2}{2\beta^2 - \gamma^2}\right) + \frac{t_1(s_1 - \bar{\alpha})}{\beta}\left(\frac{2\beta^2}{2\beta^2 - \gamma^2 d_1 d_2}\right)$$

$$- \frac{\gamma d_1 t_2(s_1 - \bar{\alpha})}{2\beta^2}\left(\frac{2\beta^2}{2\beta^2 - \gamma^2 d_1 d_2}\right)$$

$$q_1{}^*(s_1) = \frac{2\beta\bar{\alpha}}{2\beta^2 - \gamma^2} - \frac{\gamma\bar{\alpha}}{2\beta^2 - \gamma^2} + \frac{2\beta t_1(s_1 - \bar{\alpha})}{2\beta^2 - \gamma^2 d_1 d_2} - \frac{\gamma d_1 t_2(s_1 - \bar{\alpha})}{2\beta^2 - \gamma^2 d_1 d_2}$$

$$q_1{}^*(s_1) = \frac{2\beta\bar{\alpha} - \gamma\bar{\alpha}}{2\beta^2 - \gamma^2} + \frac{2\beta t_1(s_1 - \bar{\alpha}) - \left(\gamma d_1 t_2(s_1 - \bar{\alpha})\right)}{2\beta^2 - \gamma^2 d_1 d_2}$$

$$d_1 t_2 = d_2 t_1$$

$$q_1{}^*(s_1) = \frac{2\beta\bar{\alpha} - \gamma\bar{\alpha}}{2\beta^2 - \gamma^2} + \frac{t_1(s_1 - \bar{\alpha}) * (2\beta - \gamma d_2)}{2\beta^2 - \gamma^2 d_1 d_2}$$

$$q_1{}^*(s_1) = \frac{\bar{\alpha}(2\beta - \gamma)}{2\beta^2 - \gamma^2} + \frac{t_1(s_1 - \bar{\alpha}) * (2\beta - \gamma d_2)}{2\beta^2 - \gamma^2 d_1 d_2}$$

$$q_1{}^*(s_1) = \frac{\bar{\alpha}(2\beta - \gamma)}{2\beta^2 - \gamma^2} + \frac{2\beta - \gamma d_2}{2\beta^2 - \gamma^2 d_1 d_2} t_1(s_1 - \bar{\alpha})$$

$$A_1 = \frac{\bar{\alpha}(2\beta - \gamma)}{2\beta^2 - \gamma^2}$$

$$B_1 = \frac{2\beta - \gamma d_2}{2\beta^2 - \gamma^2 d_1 d_2}$$

$$q_1{}^*(s_1) = A_1 + B_1 t_1 (s_1 - \bar{\alpha})$$

APPENDIX 2. Detailed Solution of equilibrium output strategy of the private firm

$$q_1{}^*(s_1) = \frac{1}{\beta}\big(E_1(a) - \gamma E_1(q_2)\big)$$

$$q_2{}^*(s_2) = \frac{1}{2\beta}\big(E_2(a) - \gamma E_2(q_1)\big)$$

$$q_2{}^*(s_2) = \frac{1}{2\beta}\left(E_2(a) - \gamma E_2\left(\frac{1}{\beta}\big(E_1(a) - \gamma E_1(q_2)\big)\right)\right)$$

$$q_2{}^*(s_2) = \frac{1}{2\beta}\left(E_2(a) - \gamma E_2\left(\frac{1}{\beta}\left(E_1(a) - \gamma E_1\left(\frac{1}{2\beta}\big(E_2(a) - \gamma E_2(q_1)\big)\right)\right)\right)\right)$$

$$\dots$$

$$q_2{}^*(s_2) = \frac{1}{2\beta}E_2(a) - \frac{\gamma}{2\beta^2}E_2E_1(a) + \frac{\gamma^2}{2^2\beta^3}E_2E_1E_2(a) - \frac{\gamma^3}{2^2\beta^4}E_2E_1E_2E_1(a)$$
$$+ \frac{\gamma^4}{2^3\beta^5}E_2E_1E_2E_1E_2(a) - \frac{\gamma^5}{2^3\beta^6}E_2E_1E_2E_1E_2E_1(a) \dots$$

$$E_2(a) = \bar{\alpha} + t_2(s_2 - \bar{\alpha})$$

$$E_2E_1(a) = \bar{\alpha} + t_1\big((1-d_2)\bar{\alpha} + d_2 s_2 - \bar{\alpha}\big) = \bar{\alpha} + t_1(\bar{\alpha} - d_2\bar{\alpha} + d_2 s_2 - \bar{\alpha})$$
$$= \bar{\alpha} + t_1(-d_2\bar{\alpha} + d_1 s_1) = \bar{\alpha} + t_1\big(d_2(s_2 - \bar{\alpha})\big) = \bar{\alpha} + d_2 t_1(s_2 - \bar{\alpha})$$

$$E_2E_1E_2(a) = \bar{\alpha} + d_1 t_2\big((1-d_2)\bar{\alpha} + d_2 s_2 - \bar{\alpha}\big) = \bar{\alpha} + d_1 t_2(\bar{\alpha} - d_2\bar{\alpha} + d_2 s_2 - \bar{\alpha})$$
$$= \bar{\alpha} + d_1 t_2(-d_2\bar{\alpha} + d_2 s_2) = \bar{\alpha} + d_1 t_2\big(d_2(s_2 - \bar{\alpha})\big)$$
$$= \bar{\alpha} + d_2 d_1 t_2(s_2 - \bar{\alpha})$$

$$E_2E_1E_2E_1(a) = \bar{\alpha} + d_1 d_2 t_1\big((1-d_2)\bar{\alpha} + d_2 s_2 - \bar{\alpha}\big)$$
$$= \bar{\alpha} + d_1 d_2 t_1(\bar{\alpha} - d_2\bar{\alpha} + d_2 s_2 - \bar{\alpha}) = \bar{\alpha} + d_1 d_2 t_1(-d_2\bar{\alpha} + d_2 s_2)$$
$$= \bar{\alpha} + d_1 d_2 t_1\big(d_2(s_2 - \bar{\alpha})\big) = \bar{\alpha} + d_2{}^2 d_1 t_1(s_2 - \bar{\alpha})$$

$$E_2E_1E_2E_1E_2(a) = \bar{\alpha} + d_1{}^2 d_2 t_2\big((1-d_2)\bar{\alpha} + d_2 s_2 - \bar{\alpha}\big)$$
$$= \bar{\alpha} + d_1{}^2 d_2 t_2(\bar{\alpha} - d_2\bar{\alpha} + d_2 s_2 - \bar{\alpha}) = \bar{\alpha} + d_1{}^2 d_2 t_2(-d_2\bar{\alpha} + d_2 s_2)$$
$$= \bar{\alpha} + d_1{}^2 d_2 t_2\big(d_2(s_2 - \bar{\alpha})\big) = \bar{\alpha} + d_2{}^2 d_1{}^2 t_2(s_2 - \bar{\alpha})$$

$$E_2E_1E_2E_1E_2E_1(a) = \bar{\alpha} + {d_1}^2{d_2}^2 t_1\big((1-d_2)\bar{\alpha} + d_2 s_2 - \bar{\alpha}\big)$$
$$= \bar{\alpha} + {d_1}^2{d_2}^2 t_1(\bar{\alpha} - d_2\bar{\alpha} + d_2 s_2 - \bar{\alpha})$$
$$= \bar{\alpha} + {d_1}^2{d_2}^2 t_1(-d_2\bar{\alpha} + d_2 s_2) = \bar{\alpha} + {d_1}^2{d_2}^2 t_1\big(d_2(s_2 - \bar{\alpha})\big)$$
$$= \bar{\alpha} + {d_2}^3{d_1}^2 t_1(s_2 - \bar{\alpha})$$
$$\ldots$$

$$q_2{}^*(s_2) = \frac{1}{2\beta}\big(\bar{\alpha} + t_2(s_2 - \bar{\alpha})\big) - \frac{\gamma}{2\beta^2}\big(\bar{\alpha} + d_2 t_1(s_2 - \bar{\alpha})\big)$$
$$+ \frac{\gamma^2}{2^2\beta^3}\big(\bar{\alpha} + d_2 d_1 t_2(s_2 - \bar{\alpha})\big) - \frac{\gamma^3}{2^2\beta^4}\big(\bar{\alpha} + {d_2}^2 d_1 t_1(s_2 - \bar{\alpha})\big)$$
$$+ \frac{\gamma^4}{2^3\beta^5}\big(\bar{\alpha} + {d_2}^2{d_1}^2 t_2(s_2 - \bar{\alpha})\big) - \frac{\gamma^5}{2^3\beta^6}\big(\bar{\alpha} + {d_2}^3{d_1}^2 t_1(s_2 - \bar{\alpha})\big)\ldots$$

$$q_2{}^*(s_2) = \frac{1}{2\beta}\bar{\alpha} + \frac{t_2(s_2 - \bar{\alpha})}{2\beta} - \frac{\gamma}{2\beta^2}\bar{\alpha} - \frac{\gamma d_2 t_1(s_2 - \bar{\alpha})}{2\beta^2} + \frac{\gamma^2}{2^2\beta^3}\bar{\alpha}$$
$$+ \frac{\gamma^2 d_2 d_1 t_2(s_2 - \bar{\alpha})}{2^2\beta^3} - \frac{\gamma^3}{2^2\beta^4}\bar{\alpha} - \frac{\gamma^3 {d_2}^2 d_1 t_1(s_2 - \bar{\alpha})}{2^2\beta^4} + \frac{\gamma^4}{2^3\beta^5}\bar{\alpha}$$
$$+ \frac{\gamma^4 {d_2}^2{d_1}^2 t_2(s_2 - \bar{\alpha})}{2^3\beta^5} - \frac{\gamma^5}{2^3\beta^6}\bar{\alpha} - \frac{\gamma^5 {d_2}^3{d_1}^2 t_1(s_2 - \bar{\alpha})}{2^3\beta^6}\ldots$$

$$q_2{}^*(s_2) = \frac{1}{2\beta}\bar{\alpha}\left(1 + \frac{\gamma^2}{2\beta^2} + \left(\frac{\gamma^2}{2\beta^2}\right)^2 + \left(\frac{\gamma^2}{2\beta^2}\right)^4 \ldots\right)$$
$$- \frac{\gamma}{2\beta^2}\bar{\alpha}\left(1 + \frac{\gamma^2}{2\beta^2} + \left(\frac{\gamma^2}{2\beta^2}\right)^2 \ldots\right)$$
$$+ \frac{t_2(s_2 - \bar{\alpha})}{2\beta}\left(1 + \frac{\gamma^2 d_1 d_2}{2\beta^2} + \left(\frac{\gamma^2 d_1 d_2}{2\beta^2}\right)^2 + \left(\frac{\gamma^2 d_1 d_2}{2\beta^2}\right)^4 \ldots\right)$$
$$- \frac{\gamma d_2 t_1(s_2 - \bar{\alpha})}{2\beta^2}\left(1 + \frac{\gamma^2 d_1 d_2}{2\beta^2} + \left(\frac{\gamma^2 d_1 d_2}{2\beta^2}\right)^2 + \left(\frac{\gamma^2 d_1 d_2}{2\beta^2}\right)^4 \ldots\right)$$

We know that we can determine the value of the convergent series,

$$1 + \frac{\gamma^2}{2\beta^2} + \left(\frac{\gamma^2}{2\beta^2}\right)^2 + \left(\frac{\gamma^2}{2\beta^2}\right)^4 \ldots = \frac{1}{1 - \frac{\gamma^2}{2\beta^2}} = \frac{1}{\frac{2\beta^2 - \gamma^2}{2\beta^2}} = \frac{2\beta^2}{2\beta^2 - \gamma^2}$$

$$1 + \frac{\gamma^2 d_1 d_2}{2\beta^2} + \left(\frac{\gamma^2 d_1 d_2}{2\beta^2}\right)^2 + \left(\frac{\gamma^2 d_1 d_2}{2\beta^2}\right)^4 \ldots = \frac{1}{1 - \frac{\gamma^2 d_1 d_2}{2\beta^2}} = \frac{1}{\frac{2\beta^2 - \gamma^2 d_1 d_2}{2\beta^2}}$$
$$= \frac{2\beta^2}{2\beta^2 - \gamma^2 d_1 d_2}$$

Now, we can solve the equation of ${q_2}^*(s_2)$,

$$
{q_2}^*(s_2) = \frac{1}{2\beta}\bar{\alpha}\left(\frac{2\beta^2}{2\beta^2-\gamma^2}\right) - \frac{\gamma}{2\beta^2}\bar{\alpha}\left(\frac{2\beta^2}{2\beta^2-\gamma^2}\right) + \frac{t_2(s_2-\bar{\alpha})}{2\beta}\left(\frac{2\beta^2}{2\beta^2-\gamma^2 d_1 d_2}\right)
$$

$$
- \frac{\gamma d_2 t_1(s_2-\bar{\alpha})}{2\beta^2}\left(\frac{2\beta^2}{2\beta^2-\gamma^2 d_1 d_2}\right)
$$

$$
{q_2}^*(s_2) = \frac{\beta\bar{\alpha}}{2\beta^2-\gamma^2} - \frac{\gamma\bar{\alpha}}{2\beta^2-\gamma^2} + \frac{\beta t_2(s_2-\bar{\alpha})}{2\beta^2-\gamma^2 d_1 d_2} - \frac{\gamma d_2 t_1(s_2-\bar{\alpha})}{2\beta^2-\gamma^2 d_1 d_2}
$$

$$
d_1 t_2 = d_2 t_1
$$

$$
{q_2}^*(s_2) = \frac{\beta\bar{\alpha}-\gamma\bar{\alpha}}{2\beta^2-\gamma^2} + \frac{\beta t_2(s_2-\bar{\alpha}) - \gamma d_1 t_2(s_2-\bar{\alpha})}{2\beta^2-\gamma^2 d_1 d_2}
$$

$$
{q_2}^*(s_2) = \frac{\bar{\alpha}(\beta-\gamma)}{2\beta^2-\gamma^2} + \frac{\beta-\gamma d_1}{2\beta^2-\gamma^2 d_1 d_2} t_2(s_2-\bar{\alpha})
$$

$$
A_2 = \frac{\bar{\alpha}(\beta-\gamma)}{2\beta^2-\gamma^2}
$$

$$
B_2 = \frac{\beta-\gamma d_1}{2\beta^2-\gamma^2 d_1 d_2}
$$

$$
{q_2}^*(s_2) = A_2 + B_2 t_2(s_2-\bar{\alpha})
$$

APPENDIX 3. Detailed Solution of Expected Welfare

$$
E(W|s_1) = E\left(\alpha * (q_1+q_2) - \frac{\left(\beta {q_1}^2 + 2\gamma q_1 q_2 + \beta {q_2}^2\right)}{2}\middle| s_1\right)
$$

$$
E(W|s_1) = E\left(\alpha q_1 + \alpha q_2 - \frac{\left(\beta {q_1}^2 + 2\gamma q_1 q_2 + \beta {q_2}^2\right)}{2}\middle| s_1\right)
$$

$$
E(W|s_1) = E\left(\alpha(s_1){q_1}^*(s_1) + \alpha(s_1){q_2}^*(s_2)|s_1 - \frac{\beta\left({q_1}^*(s_1)\right)^2}{2}\right.
$$

$$
\left. - \gamma {q_1}^*(s_1){q_2}^*(s_2)|s_1 - \frac{\beta({q_2}^*(s_2)|s_1)^2}{2}\right)
$$

$$
E(W|s_1) = E\left(\alpha(s_1){q_1}^*(s_1)\right) + E\left(\alpha(s_1){q_2}^*(s_2)|s_1\right) - \frac{\beta}{2}E\left(\left({q_1}^*(s_1)\right)^2\right)
$$

$$
- \gamma E({q_1}^*(s_1){q_2}^*(s_2)|s_1) - \frac{\beta}{2}E(({q_2}^*(s_2)|s_1)^2)
$$

To calculate expected welfare, we first need to solve the equations,

$$
E\left(\alpha(s_1){q_1}^*(s_1)\right), E\left(\alpha(s_1){q_2}^*(s_2)|s_1\right), E\left(\left({q_1}^*(s_1)\right)^2\right), E(({q_2}^*(s_2)|s_1)^2), E({q_1}^*(s_1){q_2}^*(s_2)|s_1)
$$

$$
E\left(\alpha(s_1){q_1}^*(s_1)\right) = E\left(\left((1-t_1)\bar{\alpha} + t_1 s_1\right)\left(A_1 + B_1 t_1(s_1-\bar{\alpha})\right)\right)
$$

$$E\big(\alpha(s_1)q_1{}^*(s_1)\big)$$
$$= E\left(A_1\big((1-t_1)\bar{\alpha}+t_1 s_1\big)\right)$$
$$+ E\left(\big(B_1 t_1(s_1-\bar{\alpha})\big)\big((1-t_1)\bar{\alpha}+t_1 s_1\big)\right)$$

$$E\big(\alpha(s_1)q_1{}^*(s_1)\big) = A_1 E\big((1-t_1)\bar{\alpha}+t_1 s_1\big) + E\left(\big(B_1 t_1(s_1-\bar{\alpha})\big)\big(\bar{\alpha}+t_1(s_1-\bar{\alpha})\big)\right)$$

$$E\big(\alpha(s_1)q_1{}^*(s_1)\big)$$
$$= A_1 E(\bar{\alpha}-t_1\bar{\alpha}+t_1 s_1) + E\left(\big(B_1 t_1(s_1-\bar{\alpha})\big)\bar{\alpha}\right)$$
$$+ E\left(\big(B_1 t_1(s_1-\bar{\alpha})\big)\big(t_1(s_1-\bar{\alpha})\big)\right)$$

$$E\big(\alpha(s_1)q_1{}^*(s_1)\big)$$
$$= A_1\big(E(\bar{\alpha})-t_1 E(\bar{\alpha})+t_1 E(s_1)\big) + B_1 t_1 E\big(\bar{\alpha}(s_1-\bar{\alpha})\big)$$
$$+ B_1 t_1{}^2 E((s_1-\bar{\alpha})^2)$$

To calculate $E\big(\alpha(s_1)q_1{}^*(s_1)\big)$, we first need to solve $E(\bar{\alpha})-t_1 E(\bar{\alpha})+t_1 E(s_1)$, $E\big(\bar{\alpha}(s_1-\bar{\alpha})\big)$, $E((s_1-\bar{\alpha})^2)$

$$E(\bar{\alpha}) = E(s_1) = \bar{\alpha}$$
$$E(\bar{\alpha})-t_1 E(\bar{\alpha})+t_1 E(s_1) = \bar{\alpha}-t_1\bar{\alpha}+t_1\bar{\alpha}$$
$$E(\bar{\alpha})-t_1 E(\bar{\alpha})+t_1 E(s_1) = \bar{\alpha}$$
$$E\big(\bar{\alpha}(s_1-\bar{\alpha})\big) = E(s_1\bar{\alpha}-\bar{\alpha}^2)$$
$$E(\alpha) = E(s_1)$$
$$E\big(\bar{\alpha}(s_1-\bar{\alpha})\big) = E(\alpha^2)-\bar{\alpha}^2$$
$$E\big(\bar{\alpha}(s_1-\bar{\alpha})\big) = V(\alpha)$$
$$E(s_1-\bar{\alpha})^2 = E(\alpha+\varepsilon_1-\bar{\alpha})^2$$
$$E(s_1-\bar{\alpha})^2 = E\big((\alpha-\bar{\alpha})+(\varepsilon_1-0)\big)^2$$
$$E(s_1-\bar{\alpha})^2 = E\left(\big(\alpha-E(\alpha)\big)+\big(\varepsilon_1-E(\varepsilon_1)\big)\right)^2$$
$$E(s_1-\bar{\alpha})^2 = V(\alpha)+v_1$$

Now, we can solve $E\big(\alpha(s_1)q_1{}^*(s_1)\big)$,

$$E\big(\alpha(s_1)q_1{}^*(s_1)\big) = \bar{\alpha}A_1 + B_1 t_1 V(\alpha) + B_1 t_1{}^2(V(\alpha)+v_1)$$
$$t_1 = \frac{V(\alpha)}{(V(\alpha)+v_1)}$$
$$E\big(\alpha(s_1)q_1{}^*(s_1)\big) = \bar{\alpha}A_1 + B_1 t_1 V(\alpha) + B_1\left(\frac{V(\alpha)}{(V(\alpha)+v_1)}\right)^2 (V(\alpha)+v_1)$$

$$E\big(\alpha(s_1)q_1{}^*(s_1)\big) = \bar{\alpha}A_1 + B_1t_1V(\alpha) + B_1\frac{V(\alpha)^2}{(\,V(\alpha)+v_1)}$$

$$E\big(\alpha(s_1)q_1{}^*(s_1)\big) = \bar{\alpha}A_1 + B_1t_1V(\alpha) + B_1t_1V(\alpha)$$

$$E\big(\alpha(s_1)q_1{}^*(s_1)\big) = \bar{\alpha}A_1 + 2B_1t_1V(\alpha)$$

$$E\big(\alpha(s_1)q_2{}^*(s_2)|s_1\big)$$
$$= E\left(\big((1-t_1)\bar{\alpha} + t_1s_1\big)\left(A_2 + B_2t_2\big((1-d_1)\bar{\alpha} + d_1s_1 - \bar{\alpha}\big)\right)\right)$$

$$E\big(\alpha(s_1)q_2{}^*(s_2)|s_1\big)$$
$$= E\left(A_2\big((1-t_1)\bar{\alpha} + t_1s_1\big)\right)$$
$$+ E\left(\left(B_2t_2\big((1-d_1)\bar{\alpha} + d_1s_1 - \bar{\alpha}\big)\right)\big((1-t_1)\bar{\alpha} + t_1s_1\big)\right)$$

$$E\big(\alpha(s_1)q_2{}^*(s_2)|s_1\big)$$
$$= A_2E\big((1-t_1)\bar{\alpha} + t_1s_1\big)$$
$$+ E\left(\left(B_2t_2\big(d_1(s_1-\bar{\alpha})\big)\right)\big(\bar{\alpha} + t_1(s_1-\bar{\alpha})\big)\right)$$

$$E\big(\alpha(s_1)q_2{}^*(s_2)|s_1\big)$$
$$= A_2E(\bar{\alpha} - t_1\bar{\alpha} + t_1s_1) + E\big(B_2t_2\big(d_1(s_1-\bar{\alpha})\big)\bar{\alpha}\big)$$
$$+ E\left(B_2t_2\big(d_1(s_1-\bar{\alpha})\big)t_1(s_1-\bar{\alpha})\right)$$

$$E\big(\alpha(s_1)q_2{}^*(s_2)|s_1\big)$$
$$= A_2\big(E(\bar{\alpha}) - t_1E(\bar{\alpha}) + t_1E(s_1)\big) + B_2t_2d_1E\big(\bar{\alpha}(s_1-\bar{\alpha})\big)$$
$$+ B_2t_2t_1d_1E((s_1-\bar{\alpha})^2)$$

Remember that

$$E(\bar{\alpha}) - t_1E(\bar{\alpha}) + t_1E(s_1) = \bar{\alpha}$$

$$E\big(\bar{\alpha}(s_1-\bar{\alpha})\big) = V(\alpha)$$

$$E(s_1-\bar{\alpha})^2 = V(\alpha) + v_1$$

Now, we can solve $E\big(\alpha(s_1)q_2{}^*(s_2)|s_1\big)$,

$$E\big(\alpha(s_1)q_2{}^*(s_2)|s_1\big) = \bar{\alpha}A_2 + B_2t_2d_1V(\alpha) + B_2t_2t_1d_1(V(\alpha)+v_1)$$

$$t_1 = \frac{V(\alpha)}{(\,V(\alpha)+v_1)}$$

$$E\big(\alpha(s_1)q_2{}^*(s_2)|s_1\big) = \bar{\alpha}A_2 + B_2t_2d_1V(\alpha) + B_2t_2\frac{V(\alpha)}{(\,V(\alpha)+v_1)}d_1(V(\alpha)+v_1)$$

$$E\big(\alpha(s_1)q_2{}^*(s_2)|s_1\big) = \bar{\alpha}A_2 + B_2t_2d_1V(\alpha) + B_2t_2d_1V(\alpha)$$

$$E\big(\alpha(s_1)q_2{}^*(s_2)|s_1\big) = \bar{\alpha}A_2 + 2B_2t_2d_1V(\alpha)$$

$$E\left(q_1{}^*(s_1)\right)^2 = E\left(A_1 + B_1 t_1 (s_1 - \bar{\alpha})\right)^2$$

$$E\left(q_1{}^*(s_1)\right)^2 = E\left(A_1{}^2 + B_1{}^2 t_1{}^2 (s_1 - \bar{\alpha})^2 + 2A_1 B_1 t_1 (s_1 - \bar{\alpha})\right)$$

$$E\left(2A_1 B_1 t_1 (s_1 - \bar{\alpha})\right) = 2A_1 B_1 t_1 E(s_1 - \bar{\alpha})$$

Since $Es_1 = \bar{\alpha}$,

$$E\left(2A_1 B_1 t_1 (s_1 - \bar{\alpha})\right) = 0$$

$$E\left(q_1{}^*(s_1)\right)^2 = E\left(A_1{}^2 + B_1{}^2 t_1{}^2 (s_1 - \bar{\alpha})^2\right)$$

$$E\left(q_1{}^*(s_1)\right)^2 = \left(A_1{}^2 + B_1{}^2 t_1{}^2 E(s_1 - \bar{\alpha})^2\right)$$

$$E\left(q_1{}^*(s_1)\right)^2 = \left(A_1{}^2 + B_1{}^2 t_1{}^2 E((s_1 - \bar{\alpha})^2)\right)$$

Remember that

$$E(s_1 - \bar{\alpha})^2 = V(\alpha) + v_1$$

$$E\left(q_1{}^*(s_1)\right)^2 = A_1{}^2 + B_1{}^2 t_1{}^2 V(\alpha) + v_1$$

$$t_1 = \frac{V(\alpha)}{(V(\alpha) + v_1)}$$

$$E\left(q_1{}^*(s_1)\right)^2 = A_1{}^2 + B_1{}^2 \left(\frac{V(\alpha)}{(V(\alpha) + v_1)}\right)^2 V(\alpha) + v_1$$

$$E\left(q_1{}^*(s_1)\right)^2 = A_1{}^2 + B_1{}^2 \left(\frac{V(\alpha)}{(V(\alpha) + v_1)}\right)^2 V(\alpha) + v_1$$

$$E\left(q_1{}^*(s_1)\right)^2 = A_1{}^2 + B_1{}^2 \frac{V(\alpha)^2}{(V(\alpha) + v_1)}$$

$$E\left(q_1{}^*(s_1)\right)^2 = A_1{}^2 + B_1{}^2 t_1 V(\alpha)$$

$$E(q_1{}^*(s_1) q_2{}^*(s_2)|s_1)$$
$$= E\left(\left(A_1 + B_1 t_1 (s_1 - \bar{\alpha})\right)\left(A_2 + B_2 t_2\left((1 - d_1)\bar{\alpha} + d_1 s_1 - \bar{\alpha}\right)\right)\right)$$

$$E(q_1{}^*(s_1) q_2{}^*(s_2)|s_1)$$
$$= E(A_1 A_2) + E\left(A_1\left(B_2 t_2 d_1 (s_1 - \bar{\alpha})\right)\right) + E\left(\left(B_1 t_1 (s_1 - \bar{\alpha})\right)A_2\right)$$
$$+ E\left(\left(B_1 t_1 (s_1 - \bar{\alpha})\right)\left(B_2 t_2 d_1 (s_1 - \bar{\alpha})\right)\right)$$

$$E\left(A_1\left(B_2 t_2 d_1 (s_1 - \bar{\alpha})\right)\right) = A_1\left(B_2 t_2 d_1 E(s_1 - \bar{\alpha})\right)$$

Since $Es_1 = \bar{\alpha}$,

$$E\left(A_1\left(B_2 t_2 d_1 (s_1 - \bar{\alpha})\right)\right) = 0$$

$$E\left(\left(B_1 t_1 (s_1 - \bar{\alpha})\right)A_2\right) = \left(A_2 B_1 t_1 E(s_1 - \bar{\alpha})\right)$$

Since $Es_1 = \bar{\alpha}$,

$$E\left(\left(B_1 t_1 (s_1 - \bar{\alpha})\right) A_2\right) = 0$$

$$E(q_1{}^*(s_1) q_2{}^*(s_2) | s_1) = E(A_1 A_2) + E\left(\left(B_1 t_1 (s_1 - \bar{\alpha})\right)\left(B_2 t_2 d_1 (s_1 - \bar{\alpha})\right)\right)$$

$$E(q_1{}^*(s_1) q_2{}^*(s_2) | s_1) = A_1 A_2 + B_1 t_1 B_2 t_2 d_1 E(s_1 - \bar{\alpha})^2$$

Remember that

$$E(s_1 - \bar{\alpha})^2 = V(\alpha) + v_1$$

$$E(q_1{}^*(s_1) q_2{}^*(s_2) | s_1) = A_1 A_2 + B_1 t_1 B_2 t_2 d_1 (V(\alpha) + v_1)$$

$$t_1 = \frac{V(\alpha)}{(V(\alpha) + v_1)}$$

$$E(q_1{}^*(s_1) q_2{}^*(s_2) | s_1) = A_1 A_2 + B_1 \frac{V(\alpha)}{(V(\alpha) + v_1)} B_2 t_2 d_1 (V(\alpha) + v_1)$$

$$E(q_1{}^*(s_1) q_2{}^*(s_2) | s_1) = A_1 A_2 + B_1 B_2 t_2 d_1 V(\alpha)$$

$$E(q_2{}^*(s_2) | s_1)^2 = E\left(A_2 + B_2 t_2 \left((1 - d_1)\bar{\alpha} + d_1 s_1 - \bar{\alpha}\right)\right)^2$$

$$E(q_2{}^*(s_2) | s_1)^2 = E\left(A_2 + B_2 t_2 (\bar{\alpha} - d_1 \bar{\alpha} + d_1 s_1 - \bar{\alpha})\right)^2$$

$$E(q_2{}^*(s_2) | s_1)^2 = E\left(A_2 + B_2 t_2 (-d_1 \bar{\alpha} + d_1 s_1)\right)^2$$

$$E(q_2{}^*(s_2) | s_1)^2 = E\left(A_2 + B_2 t_2 \left(d_1 (s_1 - \bar{\alpha})\right)\right)^2$$

$$E(q_2{}^*(s_2) | s_1)^2 = E\left(A_2{}^2 + B_2{}^2 t_2{}^2 \left(d_1 (s_1 - \bar{\alpha})\right)^2 + 2 A_2 B_2 t_2 d_1 (s_1 - \bar{\alpha})\right)$$

$$E\left(2 A_2 B_2 t_2 d_1 (s_1 - \bar{\alpha})\right) = 2 A_2 B_2 t_2 d_1 E(s_1 - \bar{\alpha})$$

Since $Es_1 = \bar{\alpha}$,

$$E\left(2 A_2 B_2 t_2 d_1 (s_1 - \bar{\alpha})\right) = 0$$

$$E(q_2{}^*(s_2) | s_1)^2 = E\left(A_2{}^2 + B_2{}^2 t_2{}^2 \left(d_1 (s_1 - \bar{\alpha})\right)^2\right)$$

$$E(q_2{}^*(s_2) | s_1)^2 = A_2{}^2 + B_2{}^2 t_2{}^2 d_1{}^2 E(s_1 - \bar{\alpha})^2$$

Remember that

$$E(s_1 - \bar{\alpha})^2 = V(\alpha) + v_1$$

$$E(q_2{}^*(s_2) | s_1)^2 = A_2{}^2 + B_2{}^2 t_2{}^2 d_1{}^2 (V(\alpha) + v_1)$$

$$E(W | s_1) = \bar{\alpha} A_1 + 2 B_1 t_1 V(\alpha) + \bar{\alpha} A_2 + 2 B_2 t_2 d_1 V(\alpha) - \frac{\beta}{2}\left(A_1{}^2 + B_1{}^2 t_1 V(\alpha)\right)$$
$$- \gamma\left(A_1 A_2 + B_1 B_2 t_2 d_1 V(\alpha)\right) - \frac{\beta}{2}\left(A_2{}^2 + B_2{}^2 t_2{}^2 d_1{}^2 (V(\alpha) + v_1)\right)$$

$$E(W|s_1) = \bar{\alpha}A_1 + \bar{\alpha}A_2 - \frac{\beta}{2}{A_1}^2 - \frac{\beta}{2}{A_2}^2 - \gamma A_1 A_2 + 2B_1 t_1 V(\alpha) + 2B_2 t_2 d_1 V(\alpha)$$
$$- \frac{\beta}{2}{B_1}^2 t_1 V(\alpha) - \gamma B_1 B_2 t_2 d_1 V(\alpha) - \frac{\beta}{2}\left({B_2}^2 {t_2}^2 {d_1}^2 (V(\alpha) + v_1)\right)$$

$$E(W|s_1) = \frac{\bar{\alpha}^2 * (7\beta^3 - 6\beta^2 y - 2\beta y^2 + 2y^3)}{2 * (2\beta^2 - y^2)^2} + 2B_1 t_1 V(\alpha) + 2B_2 t_2 d_1 V(\alpha)$$
$$- \frac{\beta}{2}{B_1}^2 t_1 V(\alpha) - \gamma B_1 B_2 t_2 d_1 V(\alpha) - \frac{\beta}{2}\left({B_2}^2 {t_2}^2 {d_1}^2 (V(\alpha) + v_1)\right)$$

APPENDIX 4. Detailed Solution of Expected Profit of the private firm

$$\Pi_2 = p_2 * q_2$$
$$E(\,\Pi_2|s_2) = E\left(\left((\alpha - \gamma q_1 - \beta q_2) * q_2\right)\Big|s_2\right)$$
$$E(\,\Pi_2|s_2) = E\left((\alpha|s_2 - \gamma {q_1}^*(s_1)|s_2 - \beta {q_2}^*(s_2)|s_2) * {q_2}^*(s_2)\right)$$

$E(\alpha - \gamma {q_1}^*(s_1)|s_2) = 2\beta {q_2}^*(s_2)$ according to the first order conditions, therefore

$$E(\,\Pi_2|s_2) = E\left(\left(2\beta {q_2}^*(s_2) - \beta {q_2}^*(s_2)\right) * {q_2}^*(s_2)\right)$$
$$E(\,\Pi_2|s_2) = \beta\left({q_2}^*(s_2)\right)^2$$
$$E(\,\Pi_2|s_2) = \beta E\left(A_2 + B_2 t_2 (s_2 - \bar{\alpha})\right)^2$$
$$E(\,\Pi_2|s_2) = \beta E\left({A_2}^2 + {B_2}^2 {t_2}^2 (s_2 - \bar{\alpha})^2 + 2A_2 B_2 t_2 (s_2 - \bar{\alpha})\right)$$
$$E\left(2A_2 B_2 t_2 (s_2 - \bar{\alpha})\right) = 2A_2 B_2 t_2 E(s_2 - \bar{\alpha})$$

Since $Es_2 = \bar{\alpha}$,

$$E\left(2A_2 B_2 t_2 (s_2 - \bar{\alpha})\right) = 0$$
$$E(\,\Pi_2|s_2) = \beta E\left({A_2}^2 + {B_2}^2 {t_2}^2 (s_2 - \bar{\alpha})^2\right)$$
$$E(\,\Pi_2|s_2) = \beta\left({A_2}^2 + {B_2}^2 {t_2}^2 E(s_2 - \bar{\alpha})^2\right)$$
$$t_2 = \frac{V(\alpha)}{(\,V(\alpha) + v_2)}$$
$$E(\,\Pi_2|s_2) = \beta\left({A_2}^2 + {B_2}^2 {t_2}^2 E((s_2 - \bar{\alpha})^2)\right)$$
$$E(s_2 - \bar{\alpha})^2 = E(\alpha + \varepsilon_2 - \bar{\alpha})^2 = E\left((\alpha - \bar{\alpha}) + (\varepsilon_2 - 0)\right)^2$$
$$E\left(\left(\alpha - E(\alpha)\right) + \left(\varepsilon_2 - E(\varepsilon_2)\right)\right)^2 = V(\alpha) + v_2$$
$$E(\,\Pi_2|s_2) = \beta\left({A_2}^2 + {B_2}^2 {t_2}^2 V(\alpha) + v_2\right)$$
$$t_2 = \frac{V(\alpha)}{(\,V(\alpha) + v_2)}$$

$$E(\,\Pi_2|s_2) = \beta\left({A_2}^2 + {B_2}^2\left(\frac{V(\alpha)}{(\,V(\alpha)+v_2)}\right)^2 V(\alpha)+v_2\right)$$

$$E\,\Pi_2 = \beta\left({A_2}^2 + {B_2}^2\frac{(V(\alpha))^2}{(V(\alpha)+v_2)}\right)$$

So,

$$E(\,\Pi_2|s_2) = \beta\left({A_2}^2 + {B_2}^2 t_2 V(\alpha)\right)$$