BALLOONING MODES IN MULTIPLE-MIRROR MAGNETIC FIELD AT THE RESEARCH OF THERMONUCLEAR FUSION

A THESIS SUBMITTED TO
THE GRADUATE SCHOOL OF NATURAL AND APPLIED SCIENCES
OF
THE MIDDLE EAST TECHNICAL UNIVERSITY

BY

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IN PARTIAL FULFILLMENT OF THE REQUIRMENTS FOR

THE DEGREE OF

MASTER OF SCIENCE

IN

THE DEPARTMENT OF PHYSICS

JULY 1996

Approval of the Graduate School of Natural and Applied Sciences

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ABSTRACT

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June 1996, 66 pages.

The problem of stability in the multiple-mirror plasma confinement is investigated through the energy principle. As a strategy, we assumed some functions simulating the profile of the confinement shells, and then maximum β value, corresponding to the most stable configuration, is searched with different mathematical methods in the assistance of the packet programe Mathematica. An upper β limit is reached as a result of magnetic field optimization.

Keywords : Multiple-Mirror, Plasma Confinement, Energy Principle, Confinement Shell, β value.

ÖZ

TERMONÜKLEER FÜZYON ARAŞTIRMALARINDA ÇOKLU AYNA MANYETİK ALANININ BALONSU MODLARI

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Haziran 1996, 66 sayfa.

Çoklu ayna plazma sıkıştırmalarındaki kararlılık problemi Enerji prensibinin verileri ile incelendi. Sıkıştırma hücreleri için benzetim fonksiyonları varsayılarak en kararlı durum için maksimum β değeri araştırıldı. Çeşitli matematiksel metodların kullanımında Matematika paket programından yararlanıldı ve β değeri için, manyetik alan düzenlemesi ile, üst limite ulaşıldı.

Anahtar Kelimeler : Çoklu ayna, Plazma sıkıştırması, Enerji prensibi, Sıkıştırma hücresi, β değeri.

ACKNOWLEDGEMENTS

I express sincere appreciation to my supervisor Assoc. Prof. Dr. Serhat Çakır for his valuable guidance and his continued encouragement throughout all stages of my Ms. study.

I am also deeply thankful to my co-supervisor Prof. Dr. V.V. Mirnov for his continuos interest and concern, he not only taught some physics but also he showed me how to participate in scientific researches.

Special thanks go to my friend Mr. Cem Özdoğan for his technical assistance in the computer duties and to Mr. Abdullah Göksu for his valuable advises.

TABLE OF CONTENTS

LIST OF TABLES

TABLE

LIST OF FIGURES

FIGURES

CHAPTER 1

INTRODUCTION

Because all conventional energy sources are decaying, one of the most important problem of our civilization is to find an alternative energy source. Not only petroleum and coal but also uranium will end in future. What is plenty in our planet is water, namely hydrogen and oxygen. Hydrogen is sufficient to perform the fusion reaction in available circumstances. The fusion reaction produces approximately a thousand times more enery than that of the fission. And the fuel of the fusion is plenty unlike the other one, that needs uranium. In addition to the advantage of abundant fuel and efficient output, the fusion reaction is safer, because at the end of the reaction there is no many radioactive material. Namely the end products of the reaction are much less dangerous in comparision to fission. Unfortunately, although it is the safest the most efficient and the most approptiate way of energy production. there is a big problem, prohibiting the fusion reaction to be maintained in lab conditions, so that it will be used in energy production. This reaction can be maintained at very high temperatures of the order of 100 million degree Celcious, and this much temperature is very destructive for any material imagined, except for a different form of matter that is plasma. Even in plasma, there are quite a lot of problems. The plasma must be stable for a certain time interval, during which sufficient energy must be produced to heat the plasma, overcome the energy losses and supply heat to the power station. At present two main approaches are being investigated to achive this aim, magnetic confinement and inertial confinement.In this thesis multiple-mirror confinement, one of the basic version of magnetic confinement will be reviewed and our study that is the simulation of the ballooning modes in multiple-mirror magnetic field will be presented.

1.1 Physics Requirements of Fusion

The basis of energy production by nuclear fusion is that the binding energy per nucleon varies from nucleus to nucleus and is particularly high for moderately light nuclei, like e.g. He^4. Examples for energy producing fusion reactions of practical interest are

$$D + T \rightarrow (He^4 + 3.5MeV) + (n + 14.1MeV) \tag{1.1}$$

$$D + D \rightarrow (He^3 + 1MeV) + (n + 3MeV) \tag{1.2}$$
$$\rightarrow (T + 0.8MeV) + (p + 2.5MeV)$$

There are two conditions to be fulfilled for being able to generate fusion reaction at a useful rate. The first is that the typical energy of the nuclei to be fused is high enough to overcome the Coulomb barrier between them. This energy could be interpreted as temperature. And for example for DT reaction the temperature required is $10keV \approx 10^8\ ^0K$. The second condition expresses the fact that fusion energy production is of interest if it is, at least, larger than the energy needed to make the reactions possible. This implies

$$P_{fus}\tau_E > 3nT \tag{1.3}$$

where n is the plasma density, τ_E the energy confinement time of the plasma, and P_{fus} the fusion power density which for a 1:1 mixture of D and T is

$$P_{fus} = (n^2/4) < \sigma v_r >_{DT} \mathcal{E}_{DT} \tag{1.4}$$

with $\mathcal{E}_{DT}$ = 17.6 MeV the energy produced per reaction. For T=10 keV where $< \sigma v_r >= 10^{-16} cm^3/s$, Eq.(1.3) yields the so-called break-even criterion

$$n\tau_E > 0.7\text{x}10^{14} cm^{-3} s \quad (1.5)$$

It implies a requirement on the quality of plasma confinement. The ignition condition which ensures that a confined plasma is kept at thermonuclear temperatures by self-heating through the charged particles generated in fusion reactions, has an analogous structure, but is more stringent. For a DT plasma, it is obtained when $\mathcal{E}_{DT}$ in Eq.(1.4) is replaced by the α particle energy $\mathcal{E}_\alpha$=3.5 MeV, yielding

$$n\tau_E > 3\text{x}10^{14} cm^{-3} s \quad (1.6)$$

This is known as the Lawson criterion which requests that the fusion power output has to exceed the bremsstrahlung losses, the one caused by the collisions of particles, at least.

There are two possibilities from Eq.(1.6) to generate limitted fusion power in a limitted volume. Either to increase the particle density so far that inertia is sufficient for a small enough volume. This is the concept of inertial confinement which produces fusion power in a series of microexplosions. The other method consists of a drastic reduction of the free expansion velocity and the particle density selected such that the plasma pressure becomes acceptable. This is the concept of magnetic confinement which has the potential of steady state operation.

1.2 Magnetic Confinement

The magnetic confinement aims at steady state operation under reactor conditions, Eq.(1.6) requires confinement times at the order of a second at the pressures which can be confined by the magnetic fields. if free streaming is allowed between the ends of a device, having the length of the device at the order of 100 m, a reasonable one, then the improvement factor reads a quite large value as,

$$\frac{\tau_{conf}}{L} V_i \approx 10^4 \tag{1.7}$$

There exists some other limitations such as, forces limit practical fields to values 5 to 6 T. Moreover in order to have some reasonable β the operating density is supposed to be around $2 \times 10^{20} m^{-3}$and if we determine the supposed characteristics from all these limitations they read, **optimum temperature $T \approx 12 - 14 keV$, the density $n \approx 2 \times 10^{20} m^{-3}$ and the confinement time $\tau \approx 1s$**

There are some differnt topological mirror configurations in order to satisfy the stability, but this is not satisfied yet. There are still sensitive microinstabilities arising from particle interactions with plasma waves and leading to the low energy particles lost faster than the high energy ones. The loss cone thus formed results in an inversion of population and in the generation of the drift cyclotron loss cone instability. Recovery of the energy of the particles leaving the system through the ends would be essential for arriving at a power balance positive enough.

The conceptual basis for analyzing the new configurations relies on distinguishing the problems of equilibrium, stability, and transport. By equilibrium, we mean the confinement of a finite pressure assembly of particles for many gyro-periods in a state of macroscopic balance, possibly with a non-Maxwellian distribution of velocity. This is necessary condition for a confinement geometry, but it leads to a further requirement: perturbations to this equilibrium state should either damp away or saturate, ending in a new equilibrium configuration. (while most theoretical analysis of stability assume infinetesimal disturbances, stability to finite perturbations is the physically relevant test.) Conditions for equilibrium and stability have been most significant for establishing the criteria for confinement configuration. If these criteria can be met, then the success of the approach depends on the rate of loss of confined plasma to the external world on a slower, transport, timescale. The equilibrium, stability and transport timescales are quite small. Also it is the time to introduce a parameter known as fusion factor β about which we will interest in

$$\beta = \frac{8\pi P}{B_{min}^2} \tag{1.8}$$

Having clarified the theory of the physics requirements of fusion [1] it seems good to indicate the aim of the thesis in a way that we are interested in the maximum β allowable for the stable confinement. In this thesis different values of β are obtained for different profiles determining the confinement cells. From these numerical results of β one can decide on the parameters of a reasonable configuration. In next sections, technical informations will be given about Mirror Confinement, then our study and its sensible consequence on the current situation will be clarified.

CHAPTER 2

MIRROR DEVICES

'Mirror Machines' rely on the reflection of charged particles by regions of strong magnetic field. To a good approximation, a charged particle in a strong magnetic field behaves such that its magnetic moment, μ, is a constant. The constancy of this quantity, which is proportional to the ratio of perpendicular kinetic energy to magnetic field strength, means that as a particle moves into a region of stronger magnetic field its perpendicular kinetic energy increases at the expense of its parallel energy. For sufficiently strong magnetic field regions this can lead to reflection and hence by a suitable distribution of field to confinement.

Early experiments with radioactive sources in such arrangements of magnetic fields showed that individual particles could readily be confined for a million transits. Unfortunately the initial enthusiasm based on such experiments was lost when it was found that collective effects, the so-called flute or interchange instabilities, could act to drive plasma across magnetic field lines and hence limit confinement. These macroscopic instabilities were dramatically removed by the introduction of minimum β field configurations. However this improved state of affairs only brought attention to collective effects on a microscopic scale. Although these themselves did not lead to dramatic escape of plasma they did enhance the diffusion losses. Calculations have shown that even without the enhanced diffusion, classical diffusion due to Coulomb interactions was such as to make 'Mirror Machines' only marginally able to reach reactor conditions. This added diffusion almost served as a death blow to such machines. In the last few years it has been realized that these microscopic instabilities can to some extent be controlled, for example by 'dribbling' into the machine cold plasma. Theoretical calculations suggest that a Mirror Fusion device of reasonable size is possible. These recent results together with the possibility of curing the basic mirror problems by tying mirror machines such as in the 'Tandem Mirror' concept has led to renewed enthusiasm in the mirror programme.

One of the main advantages of mirror machines over the more popular 'Tokamak' systems is that the maximum β value (that is the ratio of plasma pressure to magnetic field pressure) obtainable, is at least an order of magnitude greater. This of course has important consequences when one considers the overall efficiency of a fusion device.

2.1 Mirror Confinement

The magnetic moment, μ, of a charged particle in a strong magnetic field, defined such that

$$\mu = \frac{mv_\perp^2}{2|B|} \tag{2.1}$$

is an adiabatic invariant. That is, to a good approximation this quantity may be considered as constant on the same footing as a true constant of motion namely the kinetic energy, $E = m(v_\perp^2 + v_\parallel^2)/2$. Here $v_\perp$ is the component of the particle velocity perpendicular to the magnetic field B and $v_\parallel$ is the parallel component. Consider the motion of particle moving along a field line into a region of increasing field strength. The quantities $v_\perp$ and $v_\parallel$ may be considered as functions of the distance, l say, along the field line and from the constancy of μ and E one finds

$$v_\parallel^2(l) = v_\parallel^2(0) - v_\perp^2(0)[\frac{|B(l)|}{|B(0)|} - 1] \tag{2.2}$$

where l=0 is taken as origin. From this relation it is seen that as the particle moves such that $|B(l)|$ increases, its velocity along the field line decreases, therefore the possibility exists that for some l, $v_{\|}^2(l) = 0$. At this particular point on the field line the particle is reflected, and the condition for reflection may be written in the form

$$sin^2(\theta_0) > l/\epsilon(l) \tag{2.3}$$

where $v_{\perp}(0)/v_{\|}(0) = tan\theta_0$, and $|B(l)|/|B(0)| = \epsilon(l)$, the mirror ratio, where $B(l) = B_{max}$ and $B(0) = B_{min}$. From this one sees that particles with a sufficiently large perpendicular velocity component($\theta_0 \rightarrow \frac{\pi}{2}$) will be reflected. This is the basic concept underlying a 'Mirror Machine'. Two coaxial Helmholtz coils, with currents flowing in the same direction, serve as a confinement device for charged particles for sufficiently large ratio of perpendicular to parallel velocity. If ϵ_m is the maximum value of $\epsilon(l)$ along the field line then the critical angle, defined such that $sin^2(\theta_c) = 1/\epsilon_m$, is called the loss-cone angle. A significant feature of any device based on the above principle is that even assuming perfect confinement of the reflected particles the distribution of particles in velocity space will inherently have a void, namely all that part of velocity space where $\theta < \theta_c$.

Laboratory experiments have shown that particles could be confined in Mirror Machines for millions of transits, but perhaps the most spectacular demonstration of this effect concerns the Van-Allen radiation belts. The earth's magnetic field acts essentially as a gigantic mirror machine with particles being reflected as they move along field lines towards the magnetic poles. Charged particles tend to be trapped in this field configuration and it is the particles which give rise to the radiation belts around the earth. They have been artificially filled up and subsequently found to empty on a time scale of the order of a month, in which time a typical particle will circumnavigate the earth many times.

Unfortunately, the perfect confinement of these particles as demonstrated above does not happen in practice. Various physical effects, ignored in the above treatment, lead to a finite confinement time for particles. Obviously those effects which lead to the smallest time are the most important ones. And the confinement time must be increased to a reasonable value so that the confinement will be applicable to reactor designs and ultimately to the production of energy. In next chepter those effects leading to smallest time will be studied in the name of Mirror Instabilities.

2.1.1 Mirror Instabilities and β Limit

In the mirror confinement charged particles interact with one another and if their density is sufficiently high such collective interactions can lead to large scale instabilities. The instability most relevant to mirror machines is the so called Flute or Interchange instability. Under certain circumstances the interchange in position of a volume of magnetic flux with a volume of plasma is energetically possible. This can lead to flutes of plasma moving across magnetic field lines and this leads to a loss of plasma from the confinement region. The condition that such an interchange is energetically unfavourable, that is that the plasma is stable, is that the difference in the value of quantity

$$\delta \int dl/|B| < 0. \tag{2.4}$$

Here dl is an element of length along the field line. For simple mirror arrangements the condition can be recast into the form

$$\int \frac{dl}{Rr|B|} < 0. \tag{2.5}$$

where R is the radius of curvature of the field line and r its distance from the axis. In the simplest mirror arrangement of two coils the radius of curvature is positive in the centre region between the coils where the field is smallest and this bad region outweighs the good regions near the coils themselves. Thus the simplest mirror configuration is unfortunately unstable.

A minimum β configuration has the right stability properties. It is readily shown, by considering the field lines in the vicinity of a point where $|B|$ has an absolute minimum, that the above inequality is satisfied for a whole range of field lines that pass through the minimum region and further these individual field lines have mirror reflecting properties. Perhaps the simplest way in which to produce a minimum B configuration is to take two coils of elliptic, rather than circular, cross-section and place them axially but with their long axes at right angles, that is like two deformed Helmholtz coils. The desire to maximize the region of minimum B, that is the volume in which the plasma is stable to flute instabilities, has led to the idea of fields produced by coils shaped in the form of a tennis ball that is known as Yin-Yang coils. The introduction of such coils led to a spectacular increase of at least an order of magnitude in the confinement time of plasma in mirror machines. Although from time to time various people have speculated about other ways of eliminating the flute instabilities, such as line tying. It is universally accepted that a mirror reactor system must be based on a minimum β configuration. The main disadvantage of such a system is a loss of simplicity in the magnetic coil design and hence a considerable increase in the associated engineering problems, such as cooling and structural stability. In short the flute instability can be removed but at the expense of simplicity in magnetic field configuration.

2.1.2 Micro Instabilities

Microscopic instabilities in general cause to fluctuations in the electric and magnetic fields. These fluctuating fields do not led to a direct loss of plasma but they do lead to an enhancement of the appropriate diffusion constants and this can lead to a further loss. This diffusion is usually referred to as 'anomalous' diffusion to distinguish it from 'classical' diffusion which is associated with particle-particle interactions rather than the collective interactions responsible for microinstabilities. Any confinement device based on the mirror concept must have an associated loss-cone, the velocity space distribution is non-Maxwellian. Further there must exist magnetic and plasma density gradients perpendicular to the major axis of the fields. These two inherent features of mirror confinement devices are the basic source of instabilities that exist in mirror machines. The lineer theory of such instabilities has receieved considerable attention over a period of years.

It is found that the presence of the loss-cone drives an unstable mode at the ion-cyclotron frequency which is convective in nature. This means that the disturbance with maximum growth rate is convective along a field line with a finite velocity. Thus by imposing the condition that the disturbance must not grow more than a factor of order 10 above thermal noise one obtains a limit on the axial length of the system, L, of the form

$$L/\rho_i \approx \sqrt{m_i/m_e} \tag{2.6}$$

where ρ_i is the ion Larmor radius and m_e and m_i the electron and ion masses respectively.

The spatial inhomogeneities give rise to the so-called drift waves and these can couple to the free energy available in the loss-cone distribution to give rise to the drift loss-cone instabilities. Not suprisingly an analysis of these modes led to a limit on the radius of the plasma R_p, such that R_p/ρ_i must be geater than a number of order 50.

Finally, a limit on the density of plasma that can be confined is obtained in the form of a limit on β, the ratio of plasma pressure to magnetic pressure. This again arises from the inherent non-Maxwellian nature of the plasma velocity distribution in that the effective temperature in the axial direction is different from that in the perpendicular direction.

The associated modes are called mirror modes. The limits on β obviously depend on the ratio of effective temperatures and hence on ϵ the mirror ratio. It suffices to say here that the critical values of β are significantly higher than the few per cent that both experiment and theory suggest for Tokamak systems.

In summary it should be stated that the study of instabilities in the linear regime in mirror machines is extremely sophisticated and the results obtained have significant experimental backing. They can be used with a good degree of confidence in design studies. Actually it is becoming more and more apparent, particularly from experiment, that these instabilities saturate due to nonlinear effects, and that the level of enhanced fluctuations that they give rise to is much smaller than once thought. The extra anomalous diffusion that these fluctuations lead to may be easier to accommodate by say increasing the size of the machine than trying to design the machine such that no instabilities occur in the first place.

Diffusion in velocity space leads to particles entering the loss-cone and hence being lost out of the machine. Since the driving force behind these instabilities is essentially due to presence of the loss-cone, it seems obvious that any method that can be used to fill it would be advantageous. In machines in which plasma is produced by neutral injection, normal to the main magnetic field, the resulting ion distribution is highly unisotropic and then the stabilization by transit ions can be significant. A further stabilizing influence is the presence of a cold plasma component which tends to fill the loss-cone and significantly decrease the strength of the instability. Somewhat suprisingly it has been found that just a few per cent of cold plasma can lead to effective stabilization. The fluctuations, and hence the diffusion constant, grow to such a value as to maintain marginal stability. Under these conditions the ion energy confinement time is essentially the electron-drag time which itself depends on the inverse of the difference between the electron and ion temperature. In the form of mirror machines this difference is high, due to the method of injection, and hence the energy confinement time is relatively low.

In summary it may be stated that the linear theory of microscopic instabilities in mirror configurations seems to be reasonably well understood and that the theory can be used to give relatively good design criterion. The nonlinear theory is not in such a good shape but recent experiments are suggesting that the strength of the nonlinearity is not as large as once feared and a reanalysis of already existing nonlinear theories may be sufficient to obtain useful results from the point of view of design parameters.

2.1.3 Classical Diffusion Losses

Before the discussion of the loss of particles in a mirror machine due to particle particle collisions, it is necessary to consider in a little more detail some of the basic features of the ion and electron energy distributions in such machines.

Although the basic concept of a mirror machine is the reflection of particles from regions of high fields this is only really appropriate for the ions. The electrons have such a large collision frequency that they are readily scattered out through the loss-cone, and in fact are held in the machine by a positive electrostatic potential, the ambipolar potential, which is set up by this preferential loss of electrons. The electron loss rate is then reduced to the ion loss rate. This mode of confinement results in the electrons having a Maxwellian velocity distribution with a cut-off at an energy proportional to this cut-off potential.

It now seems most probable that mirror-reactor systems will be fuelled by the injection of energetic neutrals. Subsequent ionization leads to a plasma composed of cold electrons and hot ions, with the ratio of energies less than a factor 0.1. Under these conditions the main effect of ion-electron interaction is for the electronsto drag the ions down in energ. This itself does not lead to a loss of ions but to a loss of energy as the energetic electrons escape over the ambipolar potential. It is then the subsequent ion-ion collisions which lead to ions entering loss-cone and hence giving rise to a loss of ions.

2.1.4 Tandem Mirror

A number of proposals have been made which aim to increase the obtainable value of Q that is the efficiency factor of the reactor, and hence remove the major difficulty associated with a mirror reactor system. The latest, and most probably the most interesting one, is the so-called Tandem

Mirror.

One of the main contributors to the low energy confinement time is the electron drag. This becomes a dominant effect because the electron temperature tends to be of order 1/10 that of the ions. Contributing factors to this are the mode of injection and also the fact that the electrons are held in by the ambipolar potential which is energetically selective such that the more energetic electrons are lost. A number of modifications have been suggested which aim to increase the electron temperature and thus improve the confinement time. One of the more recent suggestions has been the Tandem Mirror.

The Tandem Mirror consists essentially of a long selonoid stopper by conventional minimum β mirror machines at each end. One of the main advantages of this system is that the ambipolar potential is much reduced in the selonoidal region though high in mirrors. In fact it is the electrostatic potential which is used to contain the ions in the selonoidal region. The electrons move readily through the device and as usual are contained by the overall end to end ambipolar potential. A consequence of this is that the electron temperature is more or less constant throughout the device, whilst the ion temperature is high in the mirror plugs whilst relatively low in the selenoid. The net result is that the ion and electron temperatures in the selenoidal region are much more comparable than in conventional mirror systems and the electron drag effects are dramatically reduced. It is hoped, by proper design of the injection scheme for the ions, to control separately the ion plasma parameters in the selenoid and the mirrors such as to give an overall energy confinement time in the selenoidal region of order n_p/n_c times the ion-ion collision time. Here n_p is the ion density in the mirrors whilst n_c is the value in the selenoidal region. The value of n_c is dictated by the demand of sufficient fusion power release in the selenoidal region. The stoppering mirrors must be such to have a value of n_p significantly greater than n_c to produce a large ambipolar potential but also the ion temperature must be sufficiently high to give an electron temperature in the selenoid comparable to ion fusion temperatures in that region.

Calculations based on the results, from both experimental and theoretical studies suggest that with stoppering mirrors based on such a device connected to a long selenoid could give overall Q values in the range 2 to 3.

2.2 The Possibility of a Mirror Reactor

Our essential and ultimate interest is the Q value that is the ratio of fusion energy produced to the energy supplied by the injection process. This may be expressed in the form

$$Q = \frac{n\tau}{4}(\sigma v)\frac{E_n}{E_0} \tag{2.7}$$

where (σv) is the average rate of fusion reaction, E_n the nergy release in such an interaction, E_0 the injection energy, n the ion density and τ their lifetime or confinement time.

It is apparent as the years progress that one is developing a better understanding of both the linear and nonlinear phases of the microscopic instabilities and that theory and experiment are much in accord. On this basis a set of design parameters has been proposed by different groups. The major disadvantage of such a device, based on a simple mirror concept, is that the Q value will not be significantly different from unity. Present thoughts indicate that such a machine will only be economically viable if the energy lost by particles escaping is efficiently used to produce electricity by some method of direct conversion.

As we said before the main advantage of a mirror reactor, over ones based on the Tokamak system, comes from the fact that in the latter devices the maximum attainable value for β is lower than the values considered feasible for mirrors. A low value of β means that most of the energy in the system is in the form of magnetic energy and this energy has to be supplied externally. If a highly efficient conversion from particle kinetic energy to electricity can be made feasible then the disadvantage of ions escaping from the loss-cone could be turned into an advantage for mirror reactors. From a more engineering point of view the design of magnetic coils for toroidal devices is far more complicated than that for simple mirrors. However the imposition of minimum β requirements tends to minimize this advantage. Further, as has been seen the principal loss of particles is due to velocity space diffusion into the loss-cone and thus there is no direct advantage gained in this respect from large physical dimensions. Possible mirror reactors are significantly smaller and hence for example less costly in magnetic field demands, than those based on other confinement concepts. Finally the plague of Tokamak systems, namely the presence of heavy ion species due to plasma bombardment of the vacuum chamber walls, is considerably less important in mirror machines. This is because ion loss through the loss-cone does not allow time for build up of significant densities and, further, heavy ions have a preferential loss.

However one always has to return to the basic restriction namely the low value of Q. Hence the importance of new concepts such as the tandem mirror with estimated Q value of order 2 or 3. If the initial optimism for such devices turns out to have a solid foundation then mirror machines could well be the basis for the first fusion reactor systems. [2]

CHAPTER 3

HISTORY

It seems good to look for the history of the multiple-mirror confinement with the physical problems and aspects of each advance In the mirrors it means the transition to the dense plasma confinement. In fact, since mirror's confinement time is very sensitive to the presence of high frequency microinstabilities one should has the open systems whose length exceeds the mean free path (m.f.p.) of the charge particles. Under this condition the velocity space distribution function is very close to the Maxwellian one which therefore eliminates the thermodynamic drive for a wide class of microinstabilities.

If one simply fills the "classical" mirror machine with a dense plasma it will certainly lead to the extremely high longitudinal plasma losses. One of the possible way to reduce the losses is so called gas dynamic trap regime.

One other possibility consists in corrugating the magnetic field. Configurations of this type have been called "corrugated open traps" or "multiple mirrors". A multiple mirror trap consists of a set of mirrors which are connected to each other at their ends and have full length L which exceeds the m.f.p. of the particles λ . If, at the same time, the corrugation period $l < \lambda$ then transiting ion will be trapped in one of the mirrors due to the Coulomb collisions before it leaves the plasma. When after a few oscillations it again becomes transiting, the direction of its longitudinal velocity changes with respect to its initial velocity in a random way. The resulting motion of ions is therefore a random walk with characteristic space step λ and time interval λ/v_{Ti}, where v_{Ti} is the velocity of ions. As a result the free inertial plasma flow in a uniform magnetic field turns into slow diffusional expansion along the corrugated magnetic field, which is described by the usual diffusive law:

$$z^2 \sim Dt \sim (\lambda v_{Ti})t \tag{3.1}$$

The effect of the reduction of the plasma expansion rate can be interpreted in another way as the result of the friction of transiting particles colliding with the trapped particles. The latter ones give their momentum to the magnetic field. One should emphasizes that in a straight magnetic field the diffusive process is not possible because the collision of a pair of ions does not change their total momentum; in a corrugated field such scattering results in trapping in mirrors with high probability that corresponds to the loss of the total momentum with corresponding decrease of the plasma expansion velocity.

The mechanism that is to some extend similar to the mechanism of MMC (Multible Mirror Confinement) was discussed by J.Tuck [10], who analyzed the slowing down of longitudinal plasma motion in dense pinch plasma by use of rough walls. The boundary between plasma with $\beta = 1$ and magnetic field (i.e. no field inside the plasma) was considered as such a wall. Particle motion in transverse direction was limited by reflection from the corrugated magnetic surface that at the same time provided the diffusional motion in longitudinal direction.

The statement of the problem that is very close to the MMC was considered by R.Post [11]. The approach taken was based on the Monte-Carlo calculations of the particle behavior in the corrugated field with fixed scattering centers. As to the results of this paper because the plasma temperature and density were chosen such that $\lambda \gg L$ the wrong conclusion was made that there is a linear dependence of lifetime on the number of mirrors.

This was pointed out by G. Logan, A. Lichtenberg and M. Lieberman from Berkeley [12] who used the same fixed center model (FCM) in the proper m.f.p. regime, so that the right square dependence of lifetime on number of mirrors was found. In connection with the use of FCM one should note that this model cannot, in principle, be used for providing the existance of diffusive MMC. In fact, under the assumption of fixed unmovable centers the particle lose momentum even in a straight magnetic field, so the diffusive character of plasma expansion in a corrugated field cannot be confirmed by the FCM calculation. Although the result of [12] was not rigorous it encouraged the authors to pursue a self consistent treatment of MMC [13].

Independently from the work been carried out in Berkeley and at approximately the same time a self consistent analysis of MMC was performed at Novosibirsk Institute of Nuclear Physics, where the effect of MMC was found and presented in paper [14]. It contained a precise description of MMC based on the diffusive equation for plasma density that was derived from the full set of kinetic equations with taking into account actual ion-ion, electron-electron and electron-ion collisions.

In summary, the concept of MMC was independently invented and developed in Novosibirsk and Berkeley from the beginning of 70's. From that time until the present it has been extensively studied from both theoretical and experimental points view. The bibliography of multiple mirror systems now numbers more than 100 papers which cover a wide area of the problems.

Although the results of the basic confinement studies at Novosibirsk and Berkeley gave the similar results, two quite different reactor embodiments were suggested.

The Berkeley group considered a steady state reactor with $\beta < 1$. The superconducting magnetic fields were used to have rather high density ($n \simeq 10^{16} cm^{-3}$) to keep the length short. For this concept the MHD stability is of a major concern. On this basis a set of design parametrs has been proposed by Livermore (Berkeley) group as $\beta \approx 0.7$, $R_P/\rho_i \geq 40$, $L/\rho_i \leq 60$, with ion energy of order 200 keV and an electron energy of 1/10 of this.

In contrast, the Novosibirsk group considered a dense plasma ($10^{17} \div 10^{18} cm^{-3}$) pulsed reactor concept heated by means of relativistic electron beams. Since at high density and thermonuclear temperature, megagaus fields are required, the "wall confinement" ($\beta \gg 1$) was considered, for which the magnetic field suppresses only the transverse thermal conductivity, but equilibrium in this direction is provided by the rigid camera walls.

To analyze what effect of corrugation can have for the fusion reactor we restrict our attention to a pulsed reactor and first consider a simpler system consisting of a magnetic field without corrugation. Let us assume that in a long solenoid with a straight magnetic field a relatively short bunch of hot ($T \simeq 10KeV$) dense plasma is prepared at $t = 0$. This bunch then freely expands along a magnetic field and energy containment time is determined by the time of longitudinal expansion $t_{exp} = L/v_{Ti}$. Thus in terms of limitation on the length L Lawson criteria means

$$nL > 3 \times 10^{22} cm^{-2} \quad (3.2)$$

It shows that even if the density is high enough $n \simeq 10^{17} cm^{-3}$, the length of reactor should be not less than 3000 m.

If one takes now the time t_{dif} of plasma diffusion in the case of small scale strong corrugation and substituting it in Lawson criteria for the temperature of 10 KeV, the following limitation for the length of the system is obtained

$$nL > 2 \times 10^{21}/k \quad cm^{-2} \tag{3.3}$$

Comparing these two we see that with the same densities the corrugation allows a reduction of the length of approximately a factor of $10 \cdot k$

A more accurate analysis, based on diffusional equation presented above was treated numerically for the case of pure D-T plasma by B.Knyazev and P.Chebotaev [16]. Under the approach taken the initial space density distribution, temperature T_0 and energy W_0 per unit of transverse plasma cross-section were fixed. At $t > 0$ density and temperature evolution due to end losses, bremsstrahlung radiation and α-particle heating were calculated. Dividing fusion energy output by W_0 one finds the reactor efficiency Q .

The study of the dependence $Q(T_0)$ with a given W_0 showed that a maximum value of this function is achieved with $T_0 = 4 - 5KeV$, which is therefore, the optimal range of values from the point of view of the axial confinement. This optimization allows to find optimal values of T_0 and W_0 for given Q that determines the product n_0L, where n_0 is a characteristic value of plasma density. By means of choosing higher values of density the length of the system can be made shorter. The main limitation on n_0 is concerned with the limitation on the pressure which the containing walls can support. The results of the analysis are given in the Table 3.1.

Table 3.1 Pulsed reactor axial confinement optimization ($\epsilon = 2$)

	$n = 8\times10^{16} cm^{-3}$ $p_0 = 1000 bar$	$n = 2\times10^{17} cm^{-3}$ $p_0 = 3000 bar$
$Q = 0.4$ $W_0 = 2.5 MJ/cm^2$	$170m$	$60m$
$Q = 1$ $W_0 = 5 MJ/cm^2$	$340m$	$120m$
$Q = 4$ $W_0 = 10 MJ/cm^2$	$700m$	$230m$

The further improvement of the reactor characteristics can be achieved by inserting a small amount of heavy impurities with $Z \leq 10$ [17]. It allows to increase the hydrogen ion trapping frequency without a remarkable rise in the bremsstrahlung losses. To find an optimal amount and density profile of the impurities the numerical analysis of a self consistent motion of plasma with impurities was carried out. It was shown that by means of impurities the reactor efficiency can be increased 3-4 times in comparison with the case of a pure hydrogen plasma.

In summarizing the axial and radial wall confinement optimization there are the following parameters of the pulsed MM reactor with Q=1:

$$L = 60 \text{ m}, \qquad R = 4 \text{ cm}, \qquad B_{vmin} = 7.5 \text{ T}, \qquad \epsilon_v = 2,$$

$$n = 6\times 10^{17} cm^{-3}, \qquad T = 5 \text{ KeV}, \qquad W_0 = 120 \text{ MJ}$$

In comparison, there are the results of optimization of steady state MM reactor with Q=1:

$$L = 312 \text{ m}, \qquad R = 3 \text{ cm}, \qquad B_{vmin} = 9.3 \text{ T}, \qquad \epsilon_v = 1.75$$

$$n = 5.5\times 10^{16} cm^{-3}, \qquad T = 4.6 \text{ KeV}, \qquad \beta = 0.8$$

The results given for the parameters of pulsed and steady state reactor are quite feasible. Since the pulsed reactor operates with $\beta \gg 1$ on axis, the problems of plasma distortion of the magnetic field becomes critical. This question has been considered in detail, and the results appear to be favorable for reactor operation. An important advantage of the pulsed concept is that the "wall confinement" provides stability against MHD modes in an axisymmetric magnetic field.

A main consideration in a magnetically confined steady state reactor is MHD stability. It requires the use of linked quadrupoles which give distorted magnetic surfaces and a limited range of stabilization. In addition, since the resulting field is only ave-min-B it may be subject to ballooning modes for $\beta \simeq 1$. Because of the short connection length between good and bad curvature regions, ballooning modes are found still to be stable within the range $\beta = 0.8$. The radial diffusion, which is enhanced because of the fanned magnetic field, is also found to be acceptable at this value of β.

The basic axial confinement theory has been checked, experimentally in most of the confinement regimes discussed. Because $\lambda \propto T^2/n$ it is quite easy to scale the m.f.p. to laboratory plasmas that operate over the range of values of λ/l. This was initially done in steady state Alkali plasmas with simple mirrors, in which the plasma was stabilized by line-tieing to the ionizing source [21,22]. Reasonable agreement between theory and experiment was obtained, over the complete range $L \geq \lambda \geq l$ that corresponds to the "small scale" regime. Later, the regime of large scale corrugation $\lambda \leq l$ was studied in Nagoya experiment [23] that confirmed the theory predictions for these range of parameters.

Important "ideal" regime $l_m < \lambda < l$ was investigated with a 10 meter device, giving good agreement with theory [24]. In this device a pulsed hydrogen source was used to create the plasma, which had to be stabilized with quadrupoles. The stabilization was achieved in agreement with the theoretical predictions of ave-min-B stability. The plasma was also stabilized with respect to ballooning modes up to the highest $\beta \sim 0.25$ obtainable in the device.

In summary, the axial part of MM concept looks quite reliable from both theoretical and experimental points of view while the physics of "wall confinement" regime requires a serious experimental verification. It could be done, in principle, at present GOL-3 installation at BINP if the magnetic field of the device is reduced up to the value that provides REB heated plasma with $\beta \geq 1$. The current plasma parameters at GOL-3 $n \simeq 10^{16} cm^{-3}$, $T \simeq 100 eV$ corresponds to the range of $\lambda \simeq 5 cm$. In case of transition from straight magnetic field to the multiple mirror one the "large scale corrugation" regime will be inevitably realized in the device that is not effective from MMC point of view. It could be reasonable if the temperature increases at $3 - 4$ times. However, as it shown in the next chapter, there is a strong β limitation caused by ballooning instability that can prevent from the achievement of high β values in the machine for line-tying stabilized axisymmetrical multiple mirrors.

CHAPTER 4

BALLOONING MODES IN MULTIPLE MIRROR MAGNETIC FIELD

We had interested in Multible mirrors in Chapter II, for the MHD (Magnetohydrodynamics) consideration, let us assume that, the trap consists of N mirror cells which form multiple mirror field configuration. There is a cylindrical symetry through z axis, that is why cylindrical coordinates (r, θ, z) are considered and the profile is determined by r(z) along z. For odd number of cells it is convenient to place origin to the middle of the central cell where, l, is the length of each cell and, L, is the length of the N cells.

Magnetic system of a multiple mirror consists of some mirror cells having strong magnetig field B_{max} at the ends of each cell and relatively weak magnetic field B_{min} at the middles of each cell, the trap of the field line is visualized as shown in Figure 4.1.

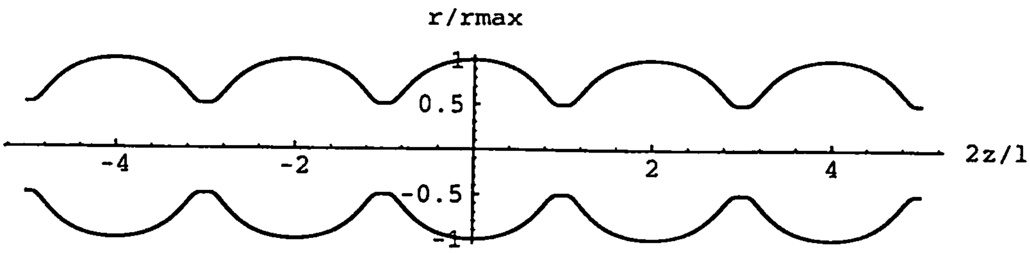

Figure 4.1. Trap of multiple mirror.

In order to analyze ballooning stability we will make use of energy principle [25] from which we have

$$\delta W = const \int_{-L}^{L} dz[h(\psi,z)(\frac{d\xi(z)}{dz})^2 + g(\psi,z)\xi(z)^2] \qquad (4.1)$$

where $h(\psi,z)$ is

$$h(\psi,z) = (1 + 4\pi\frac{P_\perp - P_\parallel}{B^2})\frac{1}{Br^2} \qquad (4.2)$$

and in our case pressure is isotropic, so $P_\perp = P_\parallel$ then Eq.(4.2) becomes

$$h(\psi,z) = \frac{1}{Br^2} = \frac{1}{\psi} \qquad (4.3)$$

and $g(\psi,z)$ is

$$g(\psi,z) = -\frac{4\pi}{B^2 r}\frac{d^2 r}{dz^2}\frac{\partial}{\partial\psi}(P_\parallel + P_\perp) \qquad (4.4)$$

for $P_\perp = P_\parallel \doteq P$ it becomes

$$g(\psi, z) = -\frac{8\pi}{B^2 r}\frac{d^2 r}{dz^2}\frac{\partial P}{\partial \psi} \tag{4.5}$$

if P is introduced as

$$P = P_0(1 - \frac{\psi}{\psi_0}) \tag{4.6}$$

then its first derivative is

$$\frac{\partial P}{\partial \psi} = -\frac{P_0}{\psi_0} \tag{4.7}$$

if Eq.(4.7) is substituted into Eq.(4.5) then

$$g(\psi, z) = \frac{8\pi}{B^2 r}\frac{d^2 r}{dz^2}\frac{P_0}{\psi_0} \tag{4.8}$$

Right after the substitution of Eq.(4.8) and Eq.(4.3) into Eq.(4.1) we get

$$\delta W = \frac{const}{\psi}\int_{-L}^{L} dz[(\frac{d\xi(z)}{dz})^2 - \frac{8\pi P_0}{B^2 r}\frac{d^2 r}{dz^2}\frac{\psi}{\psi_0}\xi(z)^2\,] \tag{4.9}$$

Here Eq.(4.9), the integral evaluated over the whole trap can be negative (unstable). It results from the second, pressure driven term. The negative contribution is caused by the region of unfavorable curvature, $d^2r/dz^2 < 0$, localized inside the transition area. It is known to drive interchange (flute) instability, for which $\xi(z) = const$. It is good to note that for interchange displacement $\xi(z) = const$, the integral of energy

Eq.(4.1) is always negative (unstable) independently from the form of r(z). However, as shown in [26], among all allowable profiles r(z) there exists the optimal one, minimizing negative integral contribution. Noting $\psi/\psi_0 \approx 1$ and $\beta = 8\pi P_0/B_{min}^2$ Eq.(4.9) becomes

$$\delta W = \frac{const}{\psi} \int_{-L}^{L} dz[(\frac{d\xi(z)}{dz}^2) - \frac{\beta}{r}\frac{d^2r}{dz^2}\xi(z)^2] \tag{4.10}$$

We are considering axisymmetrical magnetic field. In this case we suppose that interchange modes are stabilized by line tying stabilization effect.

4.1 Estimation

Before solving this equation we make a rough estimation in a way that, for a single cell we have

$$\begin{aligned} \frac{d\xi}{dz} &= \frac{\xi_o}{l/2} = 2\frac{\xi_0}{l} \\ (\frac{d\xi}{dz})^2 &= 4\frac{\xi_0^2}{l^2} \\ \frac{dr}{dz} &= \frac{r_0}{l/2} = 2\frac{r_0}{l} \\ \frac{d^2r}{dz^2} &= 4\frac{r_0}{l^2} \end{aligned} \tag{4.11}$$

Once the equation set of Eq.(4.11) is substituted into Eq.(4.10) we get

$$\delta W = \frac{const}{\psi} \int_{-L}^{L} dz[4\frac{\xi_0^2}{l^2} - \beta 4\frac{\xi_0^2}{l^2}] \tag{4.12}$$

$$\delta W = \frac{const}{\psi} 4\frac{\xi_0^2}{l^2} \int_{-L}^{L} dz[1-\beta] \tag{4.13}$$

if $\delta W = 0$ then $\beta = 1$

if $\delta W > 0$ then $\beta < 1$ stability condition

if $\delta W < 0$ then $\beta > 1$ unstability

If we generalize this estimation for the whole picture of N cells with the length of l for each single cell and with the length of L for the whole picture we have

$$\begin{aligned} \frac{d\xi}{dz} &= \frac{\xi_0}{L/2} = 2\frac{\xi_0}{L} \\ (\frac{d\xi}{dz})^2 &= 4\frac{\xi_0}{L^2} \\ \frac{dr}{dz} &= \frac{r_0}{l/2} = 2r_0 l \\ \frac{d^2}{dz^2} &= 4\frac{r_0}{l^2} \end{aligned} \tag{4.14}$$

Again if the set of Eq.(4.14) is substituted into Eq.(4.10) we get

$$\delta W = \frac{const}{\psi} \int_{-L}^{L} dz[4\frac{\xi_0^2}{L^2} - \beta 4\frac{\xi_0^2}{l^2}] \tag{4.15}$$

$$\delta W = \frac{const}{\psi} 4\xi_0^2 \int_{-L}^{L} (\frac{1}{L^2} - \frac{\beta}{l^2}) \tag{4.16}$$

if $\delta W = 0$ then $\beta = l^2/L^2 = (l/L)^2 = 1/N^2$

if $\delta W > 0$ then $\beta < 1/N^2$ stability condition

if $\delta W < 0$ then $\beta > 1/N^2$ unstability

4.2 Trials of probe functions

After this rough estimation we analyzed Eq(4.10) through some mathematical methods by the help of a software, Mathematica. Our study is simply based on assuming some probe functions determining the profile, and then searching for the most appropriate one for stability in terms of the β value that profile gives. The maximum β value describes the most stable situation.

Initially a little more modification is carried out in our equation in a way that, we assume a plasma displacement of the form $\xi = \xi_0(1 - z^2/L^2)$ that yields

$$(\frac{d\xi}{dz})^2 = \frac{4\xi_0^2 z^2}{L^4} \tag{4.17}$$

and normalized magnetic field is introduced as $b = B/B_{min}$ that yields

$$B^2 = B_{min}^2 b^2 \tag{4.18}$$

if we make use of Eq.(4.17) and Eq.(4.18) in Eq.(4.9) then we obtain

$$\delta W = \frac{const}{\psi} \int_{-L}^{L} dz[(\frac{d\xi}{dz})^2 - \frac{8\pi P_0}{B_{min}^2 r} \frac{1}{b^2} \frac{d^2 r}{dz^2} \xi^2] \tag{4.19}$$

also we have

$$Br^2 = B_{min} r_{max}^2$$

$$\frac{B}{B_{min}} = b = (\frac{r_{max}}{r})^2 = \frac{1}{(r/r_{max})^2}$$

so we get

$$b^2 = \frac{1}{(r/r_{max})^4} \tag{4.20}$$

After this preparation we started to assume some probe functions simulating the profiles, our method is simply based on the asumption of a function then, the boundaries were arranged by playing with the constants of this function.

4.2.1 Trial of a sinusoidal profile of field line

As a first trial, a sinusoidal function shown in Figure 4.2. was used for the simulation of the confinement shells.

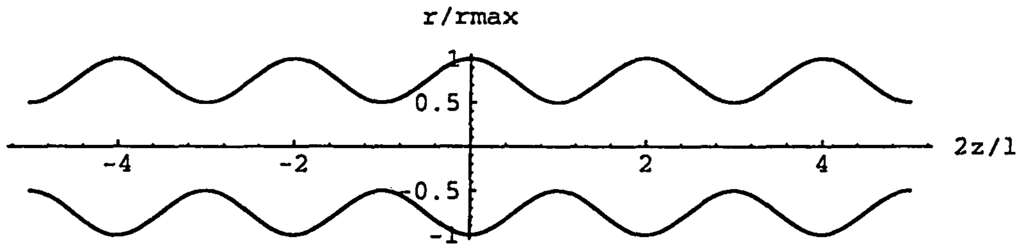

Figure 4.2. Sinusoidal profile of field line.

where the function is constructed as follows

$$r = \frac{r_{max} - r_{min}}{2} cos(\frac{z}{l} 2\pi) + \frac{r_{max} + r_{min}}{2} \tag{4.21}$$

and we have mirror ratio $\epsilon = r_{min}/r_{max}$

$$\frac{r}{r_{max}} = \frac{1-\epsilon}{2} cos(\frac{z}{l} 2\pi) + \frac{1+\epsilon}{2} \tag{4.22}$$

$$\frac{d^2}{dz^2}(\frac{r}{r_{max}}) = -(\frac{1-\epsilon}{2}) cos(\frac{z}{l} 2\pi)(\frac{2\pi}{l}^2) \tag{4.23}$$

Eq.(4.19) is modified by making use of Eq.(4.17), Eq.(4.20) and introducing new radius r/r_{max}.Then W becomes

$$\delta W = \frac{const}{\psi} \int_{-L}^{L} dz [\frac{4\xi_0^2 z^2}{L^4} - \frac{\beta}{r/r_{max}(r_{max}/r)^4} \frac{d^2(r/r_{max})}{dz^2} \xi_0^2 \frac{(L^2 - z^2)^2}{L^4}] \tag{4.24}$$

Eq.(4.22), Eq.(4.23) are used in Eq.(4.24) with the simplifying parameters $L/l = N$ and $z/l = x$ so that it becomes

$$\delta W = \frac{const}{\psi}\frac{L}{L^2}\int_0^1 dx[4\xi_0^2 x^2$$
$$-\beta[(\frac{1+\epsilon}{2})(\frac{1-\epsilon}{1+\epsilon}cos(xN\pi)+1)]^3$$
$$\frac{\epsilon-1}{2}cos(xN\pi)(N\pi)^2(1-x^2)^2] \qquad (4.25)$$

if we simplify the coefficients it becomes

$$\delta W = const[\int_0^1 4x^2dx -$$
$$\int_0^1 \beta N^2\pi^2[A(Bcos(xN\pi)+1]^3$$
$$Ccos(xN\pi)(1-x^2)^2dx] \qquad (4.26)$$

where

$$A = \frac{1+\epsilon}{2}$$
$$B = \frac{1-\epsilon}{1+\epsilon}$$
$$C = \frac{\epsilon-1}{2}$$

Now the Eq.(4.26) is ready for numerical evaluation by computer for different values of A,B and C that are dependent of the values of ϵ . Table 4.1. shows the different values of these coefficients for different values of mirror ratio ϵ.

Table 4.1. Coefficients of Eq.(4.26) for different values of ϵ.

ϵ	A	B	C
1/2	3/4	1/3	-1/4
1/5	6/10	4/6	-4/10
1/100	101/200	99/101	-99/200

Eq.(4.26) is analyzed for different values of Number of cells and Mirror ratios by making use of the numerical integration packages of Mathematica, Table 4.2. shows the numerical results of our computer analyses for β values in percentage.

Table 4.2. β values for different N and ϵ by direct integration.

N	ϵ=1/2	ϵ=1/5	ϵ=1/100
5	18.68	10.55	8.71
10	4.67	2.64	2.18
100	0.046	0.026	0.021

It is seen that maximum value of % β is 18.68 for mimimum N and minimum ϵ. Namely the smaller Mirror ratio and smaller Number of cells corresponds to a more stable situation.

Another method to analyze Eq.(4.9) is to transform it into a differential equation and then studying this differential equation by shooting some different values for physical parameters of this equation.

we first consider with Eq.(4.1) that is

$$\delta W = const \int_{-L}^{L} dz[h\psi, z(\frac{d\xi(z)}{dz})^2 + g(\psi, z)\xi z^2] \tag{4.27}$$

we assume a small perturbation in plasma displacement as

$$\xi = \xi_0 + \delta\xi \tag{4.28}$$

$$\frac{\delta\xi}{dz} = \frac{d\xi_0}{dz} + \frac{d\delta\xi}{dz} = \xi_0\prime + \delta\xi\prime \tag{4.29}$$

Then we make use of Eq.(4.28) and Eq.(4.29) in Eq.(4.27), neglecting the nonlinear terms $\delta\xi\prime^2$ and $\delta\xi^2$, δW becomes:

$$\delta W = const \int_{-L}^{L} [h(\psi, z)(\xi_0\prime^2 + 2\xi_0\prime\delta\xi\prime) + g(\psi, z)(\xi_0^2 + 2\xi_0\delta\xi)]dz \tag{4.30}$$

and the non-perturbed energy variation is

$$\delta W_0 = const \int_{-L}^{L} [h(\psi, z)\xi_0\prime^2 + g(\psi, z)\xi_0^2]dz \tag{4.31}$$

if we calculate the variation in the variation of energy then

$$\Delta\delta W = \delta W - \delta W_o$$

$$= 2\ const \int_{-L}^{L} [h(\psi, z)\xi_0\prime\delta\xi\prime + g(\psi, z)\xi_0\delta\xi]dz$$

$$= const \int_{-L}^{L} [h(\psi, z)\frac{d\xi_0}{dz}\frac{d\delta\xi}{dz} + g(\psi, z)\xi_0\delta\xi]dz$$

$$= const \int_{-L}^{L} [h(\psi, z)\frac{d^2\xi_0}{dz^2} + g(\psi, z)\xi_0]dz\delta\xi \qquad (4.32)$$

Having substituted $h(\psi, z)$ and $g(\psi, z)$ for radius r/r_{max} of Eq.(4.22) we get our differential equation as follows

$$\frac{d^2\xi(z)}{dz^2} - (N\pi)^2\beta[A(1 + Bcos(zN\pi)]^3Ccos(zN\pi)\xi(z) = 0 \qquad (4.33)$$

Notice A,B and C was depending upon mirror ratio ϵ. The analyses of this differential equation for boundary conditions, $\xi(0) = 1$ and $\xi\prime(0) = 0$, is carried out by Mathematica in a way that, Mathematica draws the solution of differential equation for different values of N and ϵ. The best profile was searched from these graphs, actually the result of this analyses is exactly confirming our previous approach. For example for N=5 and $\epsilon = 1/2$ we tried some β values. At 0.18 the profile became the best fitting one with our expectations as shown in Figure 4.3.

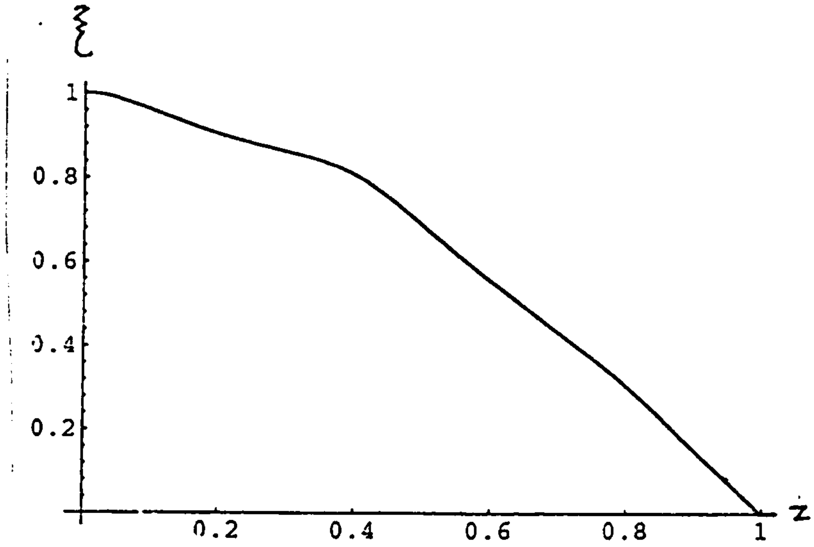

Figure 4.3. Best fitting sinusoidal profile at $\beta = 0.18$.

Through the same method we investigated β values for different ϵ as shown in below table It is apparent that the results in Table 4.3. that is inferred by differential equation is quite close to those of direct integration in Table 4.2.

Table 4.3 β values for different ϵ at N=5 by differential equation.

N	ϵ=1/2	ϵ=1/5	ϵ=1/100
5	18.52	10.35	8.51

Indeed these two approaches were giving the same results at the same numbers through different methods. Actually simulation of the profiles of the mirror cells by such a sinusoidal function is not perfect. There is a better profile satisfied by the square-root function.

4.2.2 Trial of a square-root profile of field line

Square-root function is the best choice for simulating the confinement cells, however it has a problem as well. The second derivative of this function is not zero at the contact points of the cells as seen in Figure 4.4. That is why it does not satisfy the continuity of the simulation for more than one cells.

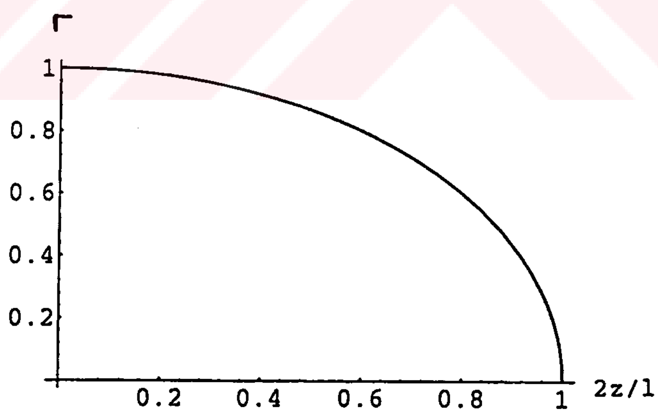

Figure 4.4. Square-root profile of field line.

We modified this profile by introducing some parameters to arrange the second derivative at the boundary and at the same time satisfying the boundary conditions as well. So the radius becomes

$$r = \epsilon + (1-\epsilon)e^{\mu+(\mu/x-1)}\sqrt{\frac{1-(\frac{\nu+x^2}{1+\nu})^\sigma}{1-(\frac{\nu}{1+\nu})^\sigma}} \qquad (4.34)$$

where $\epsilon = r_{min}/r_{max}$ is mirror ratio, $x = z/L$, μ is the local scale of magnetic field variation at point $z = 0$, ν is the local scale of magnetic field variation at point $z = L$ and σ is the variation of parameter for the width of curve. We tried different values for these parameters to find the best simulation profile. Table 4.4. shows the corresponding numerical values of below integral.

$$I = \int_0^1 r^3 \frac{d^2r}{dz^2} dz \qquad (4.35)$$

Table 4.4. I values at $\epsilon = 1/1000$ and $\nu = 0.01$ for different μ and σ values.

σ	$\mu = 0.001$	$\mu = 0.0001$
0.3	0.7845	0.7834
0.4	0.7734	0.7726
0.5	0.7816	0.7811

The integral I is the coefficient of β, that is why the minimum value of integral corresponds to the maximum stability as it can be seen from Eq.(4.24). Notice r/r_{max} refers to r here. The profile for the most stable simulation is shown in Figure 4.5.

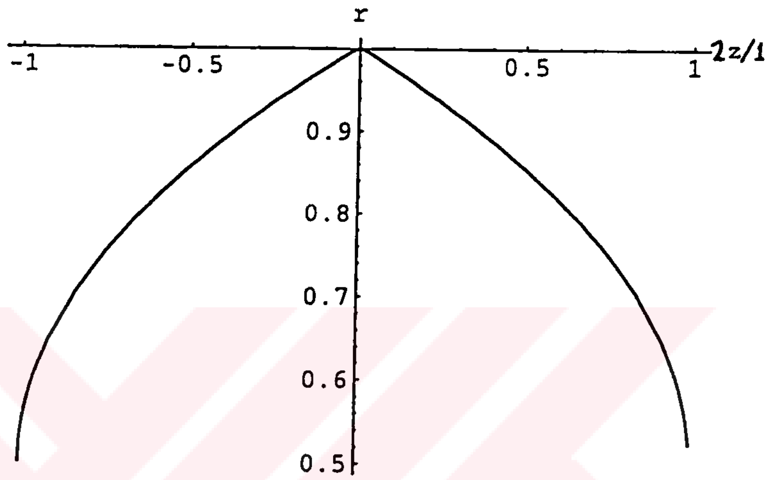

Figure 4.5. Modified square-root profile of field line for the most stable situation.

If we focus on the vicinity of $z = L$, the second derivative is zero as seen in Figure 4.6. so that the continuity of the cells would be satisfied.

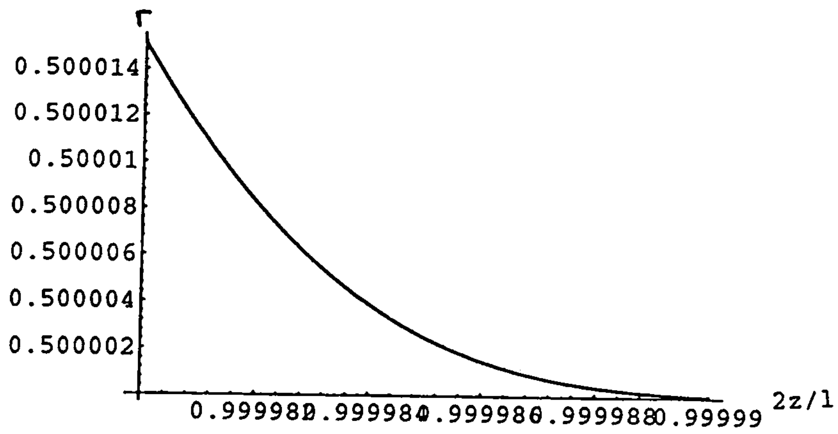

Figure 4.6. Modified square-root profile of field line for the maximum stable situation zoomed at z=L.

Table 4.4. showed that smaller μ values give less I, so in order to be more precise we decreased μ and ν values as much as possible. Effect of mirror ratio is also studied for $\epsilon = 1/10$ and $e = 1/1000$. Table 4.5. and Table 4.6. shows I values for these ϵ values respectively.

Table 4.5. I values at $\epsilon = 1/10$ and $\nu = \mu = 0.0001$ for different σ values.

σ	I
0.2	1.04809
0.3	0.77335
0.4	0.77334
0.5	0.78001

Table 4.6. I values at $\epsilon = 1/1000$ and $\nu = \mu = 0.0001$ for different σ values.

σ	I
0.4	0.7750
0.5	0.7532
0.6	0.7749

As it is seen from these tables, mirror ratio has a little effect. For larger values of ϵ we get less I values. And at $\sigma = 0.5$ we get our minimum I value as 0.7532 as in consistency with previous studies that means our simulation is appropriate for ballooning modes.

4.3 β analysis with average equation

As a last approach we have investigated the profile function in another way that we derived another integral playing role in the value of β. We multiply Eq.(4.32) by $1/l$ and equate it to zero so that we obtain β_0 (critical β)

$$\frac{1}{l}\int_{-l/2}^{l/2} dz\frac{d^2\xi}{dz^2} = \int_{-l/2}^{l/2} \frac{1}{r\beta}\frac{d^2r}{dz^2}\frac{\partial P}{\partial\psi}\xi(z)^2\frac{dz}{l} \tag{4.36}$$

left hand side is the definition of derivation so

$$\frac{d\xi(l/2)/dz - d\xi(-l/2)/dz}{l} = \frac{d^2\xi}{dz^2} \tag{4.37}$$

By introducing r/r_{max} as r and determining β_0 as $8\pi P_0/B_{min}^2$ Eq.(4.36) becomes

$$\frac{d^2\xi}{dz^2} + \xi(z)^2\int_{-l/2}^{l/2} r^3\frac{d^2r}{dz^2}dz(\beta_0 n) = 0 \tag{4.38}$$

second term is evaluated by parts yielding

$$\frac{d^2\xi}{dz^2} + \frac{3l}{r_{max}^4}\int_{-l/2}^{l/2} r(z)^2(\frac{dr}{dz})^2 dz(\beta_0 n) = 0 \tag{4.39}$$

From the Eq.(4.39) we determine the critical value of β as follows

$$\beta_0^{(S),(E)} \leq \pi^2 q^{(S),(E)}/nN^2 I' \tag{4.40}$$

where $q^{(S)} = 1$, $q^{(E)} = e$, n is the configuration factor and I' is

$$I' = \frac{3l}{r_{max}^4}\int_{-l/2}^{l/2} dz r(z)^2(\frac{dr}{dz})^2 \tag{4.41}$$

We numerically calculated I' and then from this $\beta_0\%$ for different ϵ as follows

Table 4.7. β_0 values for different ϵ at N=5 by differential equation.

ϵ	$\beta_0\%$ for Sq.-root	$\beta_0\%$ for Sinusoidal
1/2	23.16	18.44
1/3	15.48	12.68
1/100	11.76	8.60
1/1000	11.68	8.52

As it is seen in Table 4.7. square-root function gives the maximum $\%\beta$ value while the sinusoidal one is in consistancy with what we found out in our other approaches.

The better simulation of the square-root over the sinusoidal one is shown in Figure 4.7.

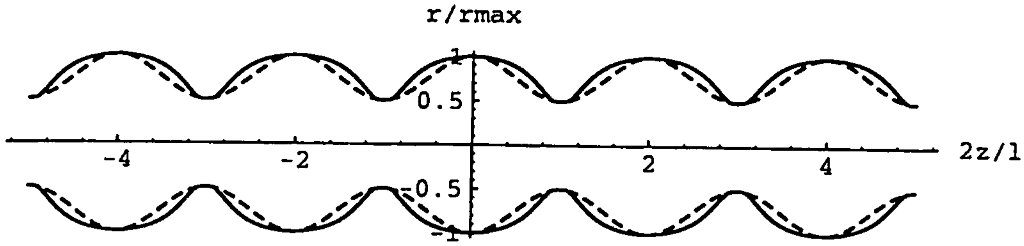

Figure 4.7. Multible-mirror field lines (dashed curves refers to sinusoidal profile and full curves to optimal square-root profile).

If we increase the number of cells we face with a less stable situation as we predicted. Table 4.7. and Table 4.8. demonstrate this in a way that for a larger value of N we get β values less, indicating a less stable situation.

Table 4.8 β_0 values for different ϵ at N=10 by differential equation.

ϵ	β_0% for Sq.-root	β_0% for Sinusoidal
1/2	5.79	4.61
1/3	3.87	3.17
1/100	2.94	2.15
1/1000	2.92	2.13

CHAPTER 5

CONCLUSION

In this thesis, the profile of the confinement shells in the multiple-mirror trap was simulated by using the theory of energy principle. **An important result was obtained that there is a very strong limit on β value in axisymetric magnetic field.** In previous experiments either low temperature Q-machine plasma was studied or plasma has been confined in min β multiple mirror configuration. As a further step in experimental activity a high β programe is planned to be developed at Novosibirsk. It is based on axisymetric multiple-mirror configuration. The results obtained shows that ballooning instability can prevent from reaching high β values in these future experiments. **Through our study an upper limit was established for β, in a way that, for large number of cells N, a very low ballooning margin was predicted in line-tying stabilized multiple mirror trap($\beta_0 \approx 5\%$ for $N = 10$).** This limit is valid even if the mirror ratio is not too high. This interesting result is supposed to be directly concerned in the future experiments of this kind.

Actually the best simulating function was determined with all its parameters for the profile, among many of the other trials. The theory of energy principle and the trial function determining the boundaries of the confinement were used to establish the most stable configuration which is predicted by the β value that configuration gives. Greater β value corresponds to a more stable situation. Indeed the most succesfull simulation was by the modified square-root function that is presented one more here

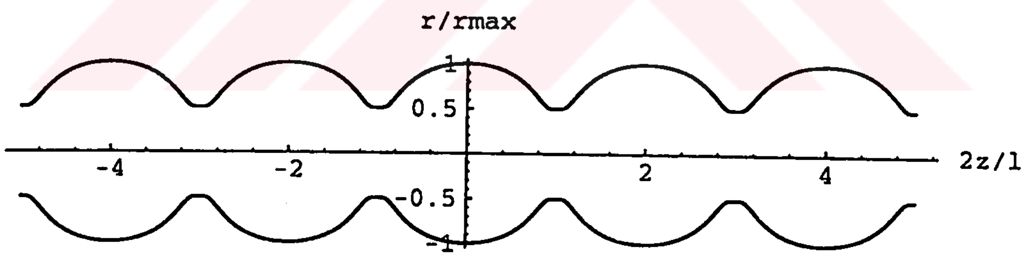

Figure 5.1. Profile of the modified square root function.

Throughout the study a contemporary method was developed not only for theoratical estimates of complicated configurations but also for the arrangements of the physical parameters of the experimental installations. Because of the high expenses of these experiments it is crucial to make a trustable simulation before experiments, that is why the development of such studies are supposed to be encouraged by those interested in the confinement.

It seems good to indicate that even if we theoritically satisfy the stability, our technological limits are not permiting to practise this theory at this stage. But the researchs are advancing the situation and one day the stability is supposed to be satisfied for maintaining fusion reaction in lab conditions. This is the common expectation of all experts of this research.

REFERENCES

[1] Folker Engelmann, **Plasma Physics and Controlled Fusion, 2**, Nato ASI Series, (1984) p.15

[2] G. Rowlands, **Plasma Physics and Nuclear Research, 17** (1981) p.401

[3] Francis F. Chen, **Introduction to Plasma Physics and Controlled Fusion**, Plenum Press, (1984)

[4] George Schmidth, **Physics of High Temperature Plasmas**, Acedemic Press, (1966)

[5] Stephen Wolfram, **Mathematica Reference Guide**, Adison-Wesley, (1992)

[6] V. Adamchik, J. Adams, A. Bocharov, et al., **Guide to Standard Mathematica Packages**, Wolfram Research Inc. , (1993)

[7] Ali B. Canbel, **Plasma Physics and Magnetofluid-Mechanics**, Mc. Graw-Hill, (1963)

[8] T.J.M. Boyd, J.J. Sanderson, **Plasma Dynamics**, Noble Inc., (1969)

[9] William B. Pardo, Hary S. Robertson, **Plasma Instabilities and Anomalous Transport**, Univ. of Miami Press, (1966)

[10] J.L.Tuck, **Rep. CN-24/K-5 on 3rd IAEA Int. Conf. on Plasma Physics and Controlled Thermonuclear Fusion**, Novosibirsk, v.2 (1968) p.595

[11] R.F.Post, **Physics Review Letters, 18** (1967) p.232

[12] B.G.Logan, A.J.Lichtenberg, M.A.Lieberman, **Physics Review Letters, 28** (1972) p.144

[13] A.Makhijani, A.J.Lichtenberg, M.A.Lieberman, B.G.Logan, **Physics of Fluids, 17**(1974)

[14] G.I.Budker, V.V.Mirnov, D.D.Ryutov, **JETP Letters, 14** (1971) p.320

[15] V.V.Mirnov, D.D.Ryutov, **Nuclear Fusion, 12** (1972) p.627

[16] B.A.Knyazev, P.Z.Chebotaev, **Nuclear Fusion, 24** (1984) p.555

[17] V.V.Mirnov, D.D.Ryutov, **Proc. 7th Europ. Conf. on Plasma Physics and Controlled Thermonuclear Fusion**, Lausanne (1975) P.143

[18] G.E.Vekstein, D.D.Ryutov, M.D.Spector, P.Z.Chebotaev, **J.of Applied Mechanics and Technical Physics, 15** (1974) p.731

[19] G.E.Vekstein, **Reviews of Plasma Physics**, Consultants Bureau, N.Y. v.15 (1990) pp.1-57

[20] P.Z.Chebotaev, B.A.Knyazev, V.V.Mirnov, G.E.Vekstein, **Proc. 10th Europ. Conf. on Plasma Physics and Controlled Fusion**, Moscow, v.1 (1981) C-6

[21] B.G.Logan, I.G.Brown, M.A.Lieberman, A.J.Lichtenberg, **Physics Review Letters, 29** (1972) p.1435

[22] G.I.Budker, V.V.Danilov, E.P.Kruglyakov, D.D.Ryutov, E.V.Shun'ko, **JETP Letters, 17** (1973) p.117

[23] A.Komori, M.Inutake, R.Hatakeyama, N.Sato, **Physics Letters, 78A** (1980) p.143

[24] M.Tuszewski, D.Price, M.A.Lieberman, **Nuclear Fusion, 19** (1979) p.1244

[25] W.A. Newcomb, **Journal of Plasma Physics 26** (1981) p. 529

[26] O.A. Bushkova, V.V. Mirnov, **Termoyadernij Sintez, v.5** (1986) p. 19 (in Russian)