ZONGULDAK BÜLENT ECEVİT ÜNİVERSİTESİ
FEN BİLİMLERİ ENSTİTÜSÜ

ULTRAHİPERBOLİK TÜRDEN KISMİ TÜREVLİ DENKLEMLER İÇİN BİR TERS PROBLEM

MATEMATİK ANABİLİM DALI

YÜKSEK LİSANS TEZİ

BÜŞRA BERİL YILDIRIM

TEMMUZ 2019

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DANIŞMAN: Doç. Dr. Fikret GÖLGELEYEN

ZONGULDAK

Temmuz 2019

KABUL:

Büşra Beril YILDIRIM tarafından hazırlanan "Ultrahiperbolik Türden Kısmi Türevli Denklemler İçin Bir Ters Problem" başlıklı bu çalışma jürimiz tarafından değerlendirilerek Zonguldak Bülent Ecevit Üniversitesi, Fen Bilimleri Enstitüsü, Matematik Anabilim Dalında Yüksek Lisans Tezi olarak oybirliğiyle kabul edilmiştir. 10/07/2019

Danışman: Doç. Dr. Fikret GÖLGELEYEN ....

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ONAY:

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"Bu tezdeki tüm bilgilerin akademik kurallara ve etik ilkelere uygun olarak elde edildiğini ve sunulduğunu; ayrıca bu kuralların ve ilkelerin gerektirdiği şekilde, bu çalışmadan kaynaklanmayan bütün atıfları yaptığımı beyan ederim."

Büşra Beril YILDIRIM

ÖZET

Yüksek Lisans Tezi

ULTRAHİPERBOLİK TÜRDEN KISMİ TÜREVLİ DENKLEMLER İÇİN BİR TERS PROBLEM

Büşra Beril YILDIRIM

Zonguldak Bülent Ecevit Üniversitesi
Fen Bilimleri Enstitüsü
Matematik Anabilim Dalı

Tez Danışmanı: Doç. Dr. Fikret GÖLGELEYEN
Temmuz 2019, 63 sayfa

Bu tezde ultrahiperbolik türden bir kısmi türevli denklem Cauchy başlangıç şartları ile birlikte ele alınmıştır. Çözüm hakkında verilen ek bilgi yardımıyla bu denklemdeki bir katsayının bulunması ters probleminin çözümünün tekliği araştırılmıştır. Bu amaçla tezin birinci bölümünde, sonraki bölümler için gerekli temel tanım ve teoremlere yer verilmiştir. İkinci bölüm bazı yardımcı önermelere ayrılmış, son bölümde ise çözümünün tekliği ile ilgili bir teorem sunulmuştur.

Anahtar Kelimeler: Ultrahiperbolik denklem, ters problem, çözümün tekliği

Bilim Kodu: 403.06.00.

ABSTRACT

M. Sc. Thesis

AN INVERSE PROBLEM FOR ULTRAHYPERBOLIC TYPE PARTIAL DIFFERENTIAL EQUATIONS

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July 2019, 63 pages

In this thesis, an ultrahyperbolic type partial differential equation is considered with some Cauchy initial data. By using the given additional data on the solution, uniqueness of solution of an inverse problem of determining one of the coefficients in the equation is investigated. For this purpose, in the first section some basic definitions and theorems which are needed in the succeeding sections are given. Section 2 is devoted to some auxiliary lemmata. In the last section, a theorem related to the uniqueness of solution of the problem is presented.

Keywords: Ultrahyperbolic equation, inverse problem, uniqueness of solution.

Science Code: 403.06.00.

TEŞEKKÜR

Tez çalışmam sırasında kıymetli bilgi, birikim ve tecrübeleri ile beni yönlendiren ve destek olan değerli danışman hocam Doç. Dr. Fikret GÖLGELEYEN'e, sonsuz teşekkür eder, saygılarımı sunarım. Çalışmalarım boyunca maddi manevi destekleriyle her zaman yanımda olan aileme de teşekkürlerimi bir borç bilirim.

İÇİNDEKİLER

SİMGELER VE KISALTMALAR DİZİNİ

SİMGELER

Ω	: Verilen bir bölge
Ω_γ	: Oluşturulan alt bölge
$\overline{\Omega}$	: Ω bölgesinin kapanışı
$\partial\Omega$	: Ω bölgesinin sınırı
$C^k(\Omega)$	: Ω bölgesinde tanımlı k. mertebeye kadar sürekli kısmi türevlere sahip fonksiyonlar uzayı
χ	: Ağırlık fonksiyonu
δ_{ij}	: Kronecker deltası
$sgn(x)$	: İşaret fonksiyonu
$\|\|u\|\|_{C^1(\overline{\Omega})}$	: $C^1(\overline{\Omega})$ uzayında norm
λ, ν	: Büyük Parametreler
d_i	: Divergent terimler, $i = 0,1,2,3,4$

BÖLÜM 1

ÖN BİLGİLER

1.1 GİRİŞ

Bu çalışmada bir

$\Omega \subset \{x, y : x = (x_1, ..., x_n) \in \mathbb{R}^n, y = (y_1, ..., y_m) \in \mathbb{R}^m, x_1 > 0\}$

bölgesinde

$$\sum_{i=1}^{n} u_{x_i x_i} - c^2 (x, y') \sum_{j=1}^{m} u_{y_j y_j} + \sum_{i=1}^{n} a_i(x) u_{x_i} + \sum_{j=1}^{m} b_j(y) u_{y_j} + a_0(x, y')u = f(x, y) \qquad (1.1)$$

ultrahiperbolik denklemi $u(0,' x, y) = u_0$, $u_{x_1}(0,' x, y) = u_1$ koşulları ile birlikte ele alınacaktır. Verilen bir ek bilgi yardımıyla (1.1) denkleminin bir katsayısının bulunması ters probleminin çözümünün tekliği araştırılmıştır.

Bilindiği üzere ikinci mertebeden iki değişken içeren bir lineer kısmi türevli denklem

$$au_{xx} + 2bu_{xy} + cu_{yy} + du_x + eu_y + fu = g \qquad (1.2)$$

formunda yazılabilir. Eğer u_{xx}, u_{xy}, u_{yy}, u_x, u_y fonksiyonları için sırasıyla $\alpha^2, \alpha\beta, \beta^2, \alpha, \beta$ gösterimleri kullanılırsa (1.2) denklemi için α ve β' ya bağlı ikinci dereceden bir polinom elde edilir:

$$P(\alpha, \beta) = a\alpha^2 + 2b\alpha\beta + c\beta^2 + d\alpha + e\beta + f.$$

(1.2) denkleminin çözümünün matematiksel özellikleri büyük oranda $P(\alpha, \beta)$ polinomunun cebirsel özellikleri tarafından belirlenir. $P(\alpha, \beta)$ ve onunla birlikte (1.2) denklemi $b^2 - ac$ diskriminantının pozitif, sıfır veya negatif olmasına göre sırasıyla hiperbolik, parabolik veya eliptik olarak sınıflandırılır. Dikkat edilirse (1.2) denkleminin tipi onun esas kısmı yani u'nun en yüksek mertebeden türevlerini içeren terimler tarafından belirlenir ve bu sınıflandırma a, b ve c katsayıları sabit olmadıkça xy düzleminde noktaya bağlı olarak değişir (Duchateau and Zachmann 1986).

Daha genel olarak ikinci mertebeden n değişkenli bir lineer kısmi türevli denklem

$$\sum_{i,j=1}^{n} a_{ij}(x)u_{x_i x_j} + \sum_{i=1}^{n} a_i(x)u_{x_i} + a(x)u = f(x) \tag{1.3}$$

şeklinde ifade edilebilir. Eğer $u_{x_i x_j} = u_{x_j x_i}$ ise (1.3) denkleminin esas kısmı $a_{ij} = a_{ji}$ olacak şekilde düzenlenebilir. Bu nedenle $A = [a_{ij}]$ matrisi simetrik olarak kabul edilebilir. Lineer cebirden bilinmektedir ki her reel simetrik $n \times n$ tipindeki matrisin n tane reel özdeğeri vardır. Bu özdeğerler, I; $n \times n$ tipinde birim matris olmak üzere $\det(A - \lambda I)$ nın yani n.dereceden bir polinomunun kökleridir. Kabul edelim ki x^0 çalışılan bölgedeki keyfi bir nokta olsun. Pozitif özdeğerlerin sayısını $n_+ = n_+(x^0)$, negatif özdeğerlerin sayısını $n_- = n_-(x^0)$ ve sıfır özdeğerlerinin sayısını $n_0 = n_0(x^0)$ ile gösterelim. Burada $n = n_+ + n_- + n_0$ olduğu açıktır.

Eğer $n_+ = n$ veya $n_- = n$ ise (1.3) denklemi x^0 noktasında eliptik tiptendir denir. $\mathbb{R}^n$ de eliptik denkleme örnek olarak

$$\Delta u = f$$

Poisson denklemi verilebilir. Burada $\Delta = \frac{\partial^2}{\partial x_1^2} + ... + \frac{\partial^2}{\partial x_n^2}$ şeklinde tanımlı olup Laplace operatörü olarak adlandırılır.

Eğer $n_+ = n - 1$ ve $n_- = 1$, veya eğer $n_+ = 1$ ve $n_- = n - 1$ ise (1.3) denklemi x^0 noktasında hiperbolik tiptendir denir. $\mathbb{R}^n$ de hiperbolik denkleme örnek olarak

$$u_{x_1x_1} + ... + u_{x_{n-1}x_{n-1}} - u_{x_nx_n} = f$$

dalga denklemi verilebilir. Daha basit olarak xt düzleminde

$$u_{tt} - u_{xx} = f$$

bir boyutlu dalga denklemi örnek olarak gösterilebilir.

Eğer $n_0 = 0$ ve $1 < n_+ < n - 1$ ise (1.3) denklemi x^0 noktasında ultrahiperbolik tiptendir denir. $\mathbb{R}^4$ de ultrahiperbolik denkleme örnek olarak

$$u_{x_1x_1} + u_{x_2x_2} - u_{x_3x_3} - u_{x_4x_4} = f(x)$$

verilebilir.

Eğer $n_0 > 0$ ise (1.3) denklemi x^0 noktasında parabolik tiptendir denir. Bütün $\mathbb{R}^n$ de parabolik olan bir denkleme örnek olarak

$$u_{x_1x_1} + ... + u_{x_{n-1}x_{n-1}} - u_{x_n} = f(x)$$

ısı denklemi verilebilir.

Açıktır ki a_{ij} katsayılarından herhangi biri sabit değil ise (1.3) denkleminin tipi noktaya bağlı olarak değişecektir (Mikhailov 1978).

Hiperbolik kısmi türevli denklemler fiziksel olarak bir zaman boyutu içermekte olup doğada cereyan eden bir çok olayın matematiksel modelinin oluşturulmasında kullanılmaktadır. Bu denklemlerin zamansal olarak çok boyut içeren bir teoriye genelleştirilmesi ultrahiperbolik denklemlerin ortaya çıkmasına sebebiyet vermiştir. Çok boyutlu zaman kavramı günümüze kadar fizikçilerin ve buna bağlı olarak matematikçilerin genel olarak uzak durduğu bir kavram olmuştur. Bunun temel nedeni çok boyutlu zaman kavramının klasik fizikteki bazı temel ilkeleri ihlal etmesidir. Buna örnek olarak belirleyicilik (determinizm) ilkesi verilebilir. Daha açık bir ifade ile zamanın çok boyutlu olması matematiksel olarak düzlemde kapalı bir eğrinin oluşmasına ve böylece zamanda ileri veya geri gidilebilmesine neden olacaktır. Bu durum ise meşhur dede paradoksunda (grandfather paradox) ifade edildiği gibi geçmiş zamanda geleceği etkileyebilecek değişiklikler yapılabilmesine yani neden sonuç zincirinin kırılmasına sebep olacaktır.

Diğer yandan, başta sicim (string) kuramı olmak üzere modern fizik kuramlarında ortaya çıkan gelişmeler klasik mekanik ve kuantum mekaniğini birleştirebilecek ve herşeyin teorisi olabilecek (theory of everything) bir teorinin oluşturulabilmesi için ek boyutlara ihtiyaç olduğunu göstermektedir. Bu ise ultrahiperbolik denklemlere olan ilgiyi arttırmıştır.

Ultrahiperbolik denklemler için bazı direkt problemler zaman ve uzay boyutunun özel durumları için Kostomarov (2002, 2006) tarafından incelenmiştir. Çözümün tekliğine ilişkin önemli sonuçlar Burskii ve Kirichenko (2008), Diaz ve Young (1971), Hörmander (1976), Owens (1947) tarafından elde edilmiştir. Son olarak, Craing ve Weinstein (2009) sınırsız bir bölgede ultrahiperbolik denklem için bir direkt problemin yerel olarak çözülebilir olduğunu göstermiştir. Ancak ters problemlerlerle ilgili sınırlı sayıda çalışma yapılmıştır. Ultrahiperbolik denklem için bazı ters problemlerin çözümlerinin tekliği ve kararlılığı Klibanov ve Bukhgeim (1981), Amirov (2001) ve Gölgeleyen ve Yamamato (2014) de araştırılmıştır.

1.2 TEMEL TANIM VE TEOREMLER

Tanım 1.1 ($C^m(\Omega)$ Uzayı) *Ω, $\mathbb{R}^n$ uzayında bir bölge olsun. Her negatif olmayan m tamsayısı için $|\alpha| \leq m$ olmak üzere $D^\alpha\varphi$ kısmi türevleri Ω bölgesinde sürekli olacak şekilde tüm φ fonksiyonlarının oluşturduğu vektör uzayı $C^m(\Omega)$ ile gösterilir (Adams and Fournier 2003, s. 10).*

Tanım 1.2 (Direkt Problem) *Matematiksel fizikte denklem, bölge ve koşullar verildiğinde denklemi ve koşulları sağlayan bir çözümün bulunmasına direkt problem denir (Amirov 2001).*

Tanım 1.3 (Ters Problem) *Pratikte karşılaşılan öyle problemler vardır ki, bunların çözümleri için ayrıca ek bilgiye gerek duyulur. Verilen bu ek bilgiye göre denklemin bir veya birkaç katsayısının veya sağ tarafının ya da sınır koşullarından bir veya birkaçını denklemin çözümü ile birlikte bulmak gerekir. Böyle problemlere ters problem adı verilir (Amirov 2001).*

Tanım 1.4 (Hadamard Anlamında İyi ve Kötü Konulmuş Problemler) *Fransız matematikçi J. S. Hadamard "İyi konulmuş problem" tanımını 20. yüzyılın başlarında aşağıdaki şekilde vermiştir.*
Kabul edelim ki U ve F metrik uzaylar, ve $A : U \to F$ bir operatör olsun

$$Au = f. \tag{1.4}$$

(1.4) denkleminin aşağıdaki koşulları sağlayan çözümünün bulunması problemine (U, F) uzay çifti için Hadamard anlamında iyi konulmuş problem adı verilir:

i) ***Varlık:*** *Her $f \in F$ için U uzayında problemin çözümü vardır.*

ii) ***Teklik:*** *Problemin çözümü U uzayında tektir.*

iii) ***Kararlılık:*** *Problemin koşulları F uzayında az değiştiğinde problemin çözümü de U uzayında az değişir (kararlılık koşulu) (Lavrent'ev et al. 1986). Bu şartlardan herhangi birinin sağlanmaması durumunda problem, (U, F) uzay çifti için Hadamard anlamında kötü konulmuş problem olarak adlandırılır. Bir (U_1, F_1) uzay çifti için iyi, başka bir (U_2, F_2) uzay çifti için kötü konulmuş probleme (U_2, F_2) uzay çifti için zayıf kötü konulmuş problem denir. Tüm uzay çiftlerinde kötü konulmuş probleme kuvvetli kötü konulmuş problem denir.*

Hadamard'a göre kötü konulmuş problemler, reel fiziksel anlamı olan pratik olayların tasvirinde kullanılamaz. Çünkü pratikte elde edilen veriler her zaman belirli bir hata payı içerir. Eğer problemin çözümü kararlı değil ise bu hatalı veriler yardımıyla bulunan çözüm, kesin çözümden çok farklı olabilir ve bu da yanlış sonuçların elde edilmesine yol açar. Bu nedenle başlangıçta birçok matematikçi sadece Hadamard anlamında iyi konulmuş problemlerle ilgilenmiştir. Ancak sonraki süreçte doğada ortaya çıkan birçok problemin kötü konulmuş problem olduğunun görülmesi matematikçilerin bu problemlerle ilgilenmesine sebep olmuştur. Bu çerçeve de Hadamard'ın kendisinin örnek olarak gösterdiği Laplace denklemi için Cauchy problemi de kötü konulmuş bir problem olup elektromanyetik teori de önemli bir model olarak ortaya çıkmaktadır (Petrovskii 1967).

Tanım 1.5 (Tikhonov Anlamında İyi ve Kötü Konulmuş Problemler)

İlk olarak Rus matematikçi A. N. Tikhonov, Hadamard anlamında kötü konulmuş problemlerin gerekliliğini ortaya koymuştur. (1.4) denkleminin aşağıdaki koşulları sağlayan çözümünün bulunması problemine Tikhonov anlamında iyi konulmuş problem adı verilir:

i) *U bir metrik uzay olmak üzere, problemin çözümü var ve belirli bir $M \subset U$ cümlesine aittir.*

ii) *Problemin çözümü M de tektir.*

iii) *Problemin çözümü M de koşullara sürekli bağımlıdır (Lavrent'ev et al. 1986).*

M cümlesine problemin doğruluk cümlesi denir ve M genellikle kompakt bir cümle olarak seçilir.
Ters ve kötü konulmuş problemler teorisi bilim ve teknolojinin birçok alanında sıklıkla ortaya çıkmaktadır. Bu alanlara örnek olarak, astronomi, kuantum mekaniği, elektrodinamik, tıbbi ve teknik tomografi, ultrason, optimal kontrol teorisi ve finans matematiği verilebilir (Kabanikhin 2008, Ramm 2010).

Uyarı 1.1 *Aşağıdaki bölümlerde*

$$\Omega_\gamma = \{(x,y) : x \in \mathbb{R}^n, y \in \mathbb{R}^m,\ x_1 > 0,\ 0 < \psi < \gamma\},$$

$$\psi(x) = \delta x_1 + \frac{1}{2}\sum_{i=2}^{n}\left(x_i - x_i^0\right)^2 + \frac{1}{2}\sum_{j=1}^{m}\left(y_j - y_j^0\right)^2 + \alpha_0;\ \chi = \exp\left(\lambda\psi^{-\nu}\right),$$

$\alpha_0 > 0$, $\gamma + \alpha_0 = n < 1$, $\alpha_0 < \psi(x) < n$ *tanımlamaları kullanılmıştır.*

BÖLÜM 2

BAZI YARDIMCI ÖNERMELER

Bu bölümde, ele alınan ters problemin çözümünün tekliğinin ispatında kullanılacak olan 3 tane lemma verilecektir. (1.1) denklemine $\tilde{x}_1 = \sqrt{2x_1} - \delta_0$ dönüşümü uygulanır ve basitlik açısından aynı gösterimler kullanılırsa denklem aşağıdaki forma indirgenebilir:

$$(x_1+\delta_0)^{-1} u_{x_1x_1} + (x_1+\delta_0)\,(\sum_{i=2}^{n} u_{x_ix_i} - c^2(x,y')\sum_{j=1}^{m} u_{y_jy_j})$$

$$+\,(x_1+\delta_0)\,(\sum_{i=1}^{n} a_i u_{x_i} + \sum_{j=1}^{m} b_j u_{y_j} + a_0(x,y')u) = f(x,y).$$

Lemma 2.1 *Kabul edelim ki* $\forall\xi \in \mathbb{R}$ *ve* $x \in \Omega$ *için*

$$-\sum_{k=1}^{m} \frac{\partial c^2}{\partial x_1}\xi_{y_k}^2 \geq \alpha_1|\xi|^2,\ \alpha_1 > 0 \tag{2.1}$$

eşitsizliği sağlansın. Eğer λ *ve* δ *parametreleri bir pozitif sabitten büyük ve* γ *sayısı*

$$0 < \gamma < \frac{4}{3}\left(Mm\varepsilon_0^{-1} + 3mM\right) < 1 \tag{2.2}$$

şartını sağlıyor ise her $\varphi(x) \in C^2(\Omega)$ *için aşağıdaki eşitsizlik yazılabilir:*

$$\begin{aligned} &\psi^{\nu+1}\left((x_1+\delta_0)^{-1}\varphi_{x_1x_1} + (x_1+\delta_0)\left(\sum_{i=2}^{n}\varphi_{x_ix_i} - c^2(x,y')\sum_{k=1}^{m}\varphi_{y_ky_k}\right)\right)^2\chi^2 \\ \geq\ & 2\lambda\nu\delta(x_1+\delta_0)^{-3}\varphi_{x_1}^2\chi^2 - 2\lambda\nu(x_1+\delta_0)^2 G_o(n,m)\sum_{i=2}^{n}\varphi_{x_i}^2\chi^2 \\ & +\lambda\nu\delta\alpha_1(x_1+\delta_0)\sum_{k=1}^{m}\varphi_{y_k}^2\chi^2 + 2\lambda^3\nu^4\delta^4(x_1+\delta_0)^{-2}\psi^{-2\nu-3}\varphi^2\chi^2 \\ & +G(\lambda,\delta)\varphi^2\chi^2 + d_0(\varphi\chi) + d_1(\varphi\chi) \end{aligned} \tag{2.3}$$

Burada $0 < \varepsilon_0 < \frac{a_1}{4m}$, $\sqrt{2\gamma} < 1$ *şeklinde tanımlıdır ve*

$$\begin{aligned} G(\lambda,\delta) &= 2\lambda\nu\delta(x_1+\delta_0)^{-3}\left(-\lambda^2\nu^2\psi_{x_1}^2\psi^{-2\nu-2} - \lambda\nu(\nu+1)\psi_{x_1}^2\psi^{-\nu-2}\right) \\ &\quad +2\lambda\nu(x_1+\delta_0)^2 G_o(n,m)\sum_{i=2}^{n}\left(\lambda^2\nu^2\psi_{x_i}^2\psi^{-2\nu-2} + \lambda\nu(\nu+1)\psi_{x_i}^2\psi^{-\nu-2}\right. \\ &\quad \left.-\lambda\nu\psi^{-\nu-1}\psi_{x_ix_i}\right) - \lambda\nu\delta\alpha_1(x_1+\delta_0)\sum_{k=1}^{m}\left(\lambda^2\nu^2\psi_{y_k}^2\psi^{-2\nu-2}\right. \\ &\quad \left.+\lambda\nu(\nu+1)\psi_{y_k}^2\psi^{-\nu-2} - \lambda\nu\psi^{-\nu-1}\psi_{y_ky_k}\right), \end{aligned}$$

$$
\begin{aligned}
& d_0(\vartheta) \\
= \ & 2\lambda\nu\delta\left((x_1+\delta_0)^{-2}\vartheta_{x_1}^2\right)_{x_1} + 4\lambda\nu\delta\sum_{i=2}^{n}(\vartheta_{x_1}\vartheta_{x_i})_{x_i} \\
& -2\lambda\nu\delta\sum_{i=2}^{n}\left(\vartheta_{x_i}^2\right)_{x_1} - 4\lambda\nu\delta\sum_{k=1}^{m}\left(c^2\vartheta_{x_1}\vartheta_{y_k}\right)_{y_k} \\
& +2\lambda\nu\delta\sum_{k=1}^{m}\left(c^2\vartheta_{y_k}^2\right)_{x_1} \\
& +2\lambda\nu\delta^3\left((x_1+\delta_0)^{-2}\left(\lambda^2\nu^2\psi^{-2\nu-2}-\lambda\nu(\nu+1)\psi^{-\nu-2}\right)\vartheta^2\right) \\
& +2\lambda\nu\delta\sum_{i=2}^{n}\left(\left(\left(\lambda^2\nu^2\psi^{-2\nu-2}\psi_{x_i}^2-\lambda\nu(\nu+1)\psi^{-\nu-2}\psi_{x_i}^2\right)+\lambda\nu\psi^{-\nu-1}\right)\vartheta^2\right)_{x_1} \\
& -2\lambda\nu\delta\sum_{k=1}^{m}\psi^{-\nu-1}\left[\left(\lambda^2\nu^2c^2\psi^{-\nu-1}\psi_{y_k}^2-\lambda\nu(\nu+1)c^2\psi^{-1}\psi_{y_k}^2+\lambda\nu mc^2\right)\vartheta^2\right]_{x_1} \\
& +4\lambda\nu\sum_{i=2}^{n}\left(\psi_{x_i}\vartheta_{x_i}\vartheta_{x_1}\right)_{x_1} - 2\lambda\nu\sum_{i=2}^{n}\left(\psi_{x_i}\vartheta_{x_1}^2\right)_{x_i} \\
& +4\lambda\nu(x_1+\delta_0)^2\sum_{i,j=2}^{n}\left(\psi_{x_i}\vartheta_{x_i}\vartheta_{x_j}\right)_{x_j} - 2\lambda\nu(x_1+\delta_0)^2\sum_{i,j=2}^{n}\left(\psi_{x_i}\vartheta_{x_j}^2\right)_{x_i} \\
& +2\lambda^2\nu^2\delta^2\sum_{i=2}^{n}\left(\left(\lambda\nu\psi^{-2\nu-2}\psi_{x_i}-(\nu+1)\psi^{-\nu-2}\psi_{x_i}\right)\vartheta^2\right)_{x_i} \\
& +2\lambda^2\nu^2(x_1+\delta_0)^2\sum_{i,,j=2}^{n}\left[\psi_{x_i}\psi^{-\nu-1}\left((n-1)-(\nu+1)\psi_{x_j}^2\psi^{-1}+\lambda\nu\psi_{x_j}^2\psi^{-\nu-1}\right)\vartheta^2\right]_{x_i} \\
& -2\lambda^2\nu^2(x_1+\delta_0)^2\sum_{i=2}^{n}\sum_{k=1}^{m}\left[c^2\psi_{x_i}\psi^{-\nu-1}\left(m-(\nu+1)\psi_{y_k}^2\psi^{-1}+\lambda\nu\psi_{y_k}^2\psi^{-\nu-1}\right)\vartheta^2\right]_{x_i} \\
& -4\lambda\nu\sum_{k=1}^{m}\left(c^2\psi_{y_k}\vartheta_{y_k}\vartheta_{x_1}\right)_{x_1} + 2\lambda\nu\sum_{k=1}^{m}\left(c^2\psi_{y_k}\vartheta_{x_1}^2\right)_{y_k} \\
& -4\lambda\nu(x_1+\delta_0)^2\sum_{i=2}^{n}\sum_{k=1}^{m}\left(c^2\psi_{y_k}\vartheta_{y_k}\vartheta_{x_i}\right)_{xi} + 2\lambda\nu(x_1+\delta_0)^2\sum_{i=2}^{n}\sum_{k=1}^{m}\left(c^2\psi_{y_k}\vartheta_{x_i}^2\right)_{y_k} \\
& +4\lambda\nu(x_1+\delta_0)^2\sum_{k,s=1}^{m}\left(c^4\psi_{y_k}\vartheta_{y_k}\vartheta_{y_s}\right)_{y_s} - 2\lambda\nu(x_1+\delta_0)^2\sum_{k,s=1}^{m}\left(c^4\psi_{y_k}\vartheta_{y_s}^2\right)_{y_k} \\
& -2\lambda^2\nu^2\sum_{i=2}^{n}\sum_{k=1}^{m}\left[c^2\psi_{y_k}\left(\lambda\nu\psi^{-2\nu-2}-(\nu+1)\psi^{-\nu-2}\right)\vartheta^2\right]_{y_k} \\
& -2\lambda^2\nu^2(x_1+\delta_0)^2\sum_{i=2}^{n}\sum_{k=1}^{m}\left[c^2\psi_{y_k}\psi^{-\nu-1}(\psi_{x_ix_i}-(\nu+1)\psi_{x_i}^2\psi^{-2}+\lambda\nu\psi_{x_i}^2\psi^{-\nu-1})\vartheta^2\right]_{y_k} \\
& +2\lambda^2\nu^2(x_1+\delta_0)^2\sum_{k,s=1}^{m}\left[c^4\psi_{y_k}\psi^{-\nu-1}\left(\psi_{y_sy_s}-(\nu+1)\psi_{y_s}^2\psi^{-1}+\lambda\nu\psi_{y_s}^2\psi^{-\nu-1}\right)\vartheta^2\right]_{y_k},
\end{aligned}
$$

$$\begin{aligned} d_1(\chi\varphi) &= -2\lambda\nu\delta(x_1+\delta_0)^{-3}\lambda\nu\left(\psi_{x_1}\psi^{-\nu-1}\varphi^2\chi^2\right)_{x_1} \\ &+2\lambda\nu(x_1+\delta_0)^2 G_o(n,m)\sum_{i=2}^{n}\lambda\nu\left(\psi_{x_i}\psi^{-\nu-1}\varphi^2\chi^2\right)_{x_i} \\ &-\lambda\nu\delta\alpha_1(x_1+\delta_0)\sum_{k=1}^{m}\lambda\nu\left(\psi_{y_k}\psi^{-\nu-1}\varphi^2\chi^2\right)_{y_k}. \end{aligned}$$

İspat. İlk olarak

$$\vartheta = \chi\varphi \tag{2.4}$$

şeklinde yeni bir bilinmeyen fonksiyon tanımlayalım. Buna göre;

$$\begin{aligned} \varphi_{x_1} &= \chi^{-1}\left(\vartheta_{x_1}+\lambda\nu\psi^{-\nu-1}\psi_{x_1}\vartheta\right), \\ \varphi_{x_1x_1} &= \chi^{-1}\left(\vartheta_{x_1x_1}+2\lambda\nu\psi^{-\nu-1}\psi_{x_1}\vartheta_{x_1}\right. \\ &\left.-\lambda\nu(\nu+1)\psi^{-\nu-2}\psi_{x_1}^2\vartheta+\lambda^2\nu^2\psi^{-2\nu-2}\psi_{x_1}^2\vartheta\right), \end{aligned} \tag{2.5}$$

$$\begin{aligned} \varphi_{x_i} &= \chi^{-1}\left(\vartheta_{x_i}+\lambda\nu\psi^{-\nu-1}\psi_{x_i}\vartheta\right), \\ \varphi_{x_ix_i} &= \chi^{-1}\left(\vartheta_{x_ix_i}+2\lambda\nu\psi^{-\nu-1}\psi_{xi}\vartheta_{xi}+\lambda\nu\psi^{-\nu-1}\psi_{x_ix_i}\vartheta\right. \\ &\left.-\lambda\nu(\nu+1)\psi^{-\nu-2}\psi_{x_i}^2\vartheta+\lambda^2\nu^2\psi^{-2\nu-2}\psi_{x_i}^2\vartheta\right), \end{aligned} \tag{2.6}$$

$$\begin{aligned} \varphi_{y_k} &= \chi^{-1}\left(\vartheta_{y_k}+\lambda\nu\psi^{-\nu-1}\psi_{y_k}\vartheta\right), \\ \varphi_{y_ky_k} &= \chi^{-1}\left(\vartheta_{y_ky_k}+2\lambda\nu\psi^{-\nu-1}\psi_{y_k}\vartheta_{y_k}+\lambda\nu\psi^{-\nu-1}\psi_{y_ky_k}\vartheta\right. \\ &\left.-\lambda\nu(\nu+1)\psi^{-\nu-2}\psi_{y_k}^2\vartheta+\lambda^2\nu^2\psi^{-2\nu-2}\psi_{y_k}^2\vartheta\right) \end{aligned} \tag{2.7}$$

yazılabilir. Burada $\varkappa = e^{\lambda\psi^{-v}}$ olarak tanımlıdır.

(2.5)-(2.7) bağıntıları kullanılarak

$$\begin{aligned} &\psi^{\nu+1}\left((x_1+\delta_0)^{-1}\varphi_{x_1x_1}+(x_1+\delta_0)\left(\sum_{i=2}^{n}\varphi_{x_ix_i}-c^2(x,y')\sum_{k=1}^{m}\varphi_{y_ky_k}\right)\right)^2\chi^2 \\ = \; &\psi^{\nu+1}\left[(x_1+\delta_0)^{-1}\left(\vartheta_{x_1x_1}+2\lambda\nu\psi^{-\nu-1}\psi_{x_1}\vartheta_{x_1}-\lambda\nu(\nu+1)\psi^{-\nu-2}\psi_{x_1}^2\vartheta\right.\right. \\ &\left.+\lambda^2\nu^2\psi^{-2\nu-2}\psi_{x_1}^2\vartheta\right) \\ &+(x_1+\delta_0)\left(\sum_{i=2}^{n}\left(\vartheta_{x_ix_i}+2\lambda\nu\psi^{-\nu-1}\psi_{xi}\vartheta_{xi}+\lambda\nu\psi^{-\nu-1}\psi_{x_ix_i}\vartheta\right.\right. \\ &\left.-\lambda\nu(\nu+1)\psi^{-\nu-2}\psi_{x_i}^2\vartheta+\lambda^2\nu^2\psi^{-2\nu-2}\psi_{x_i}^2\vartheta\right) \\ &-c^2\left(x,y^{'}\right)\sum_{k=1}^{m}(\vartheta_{y_ky_k}+2\lambda\nu\psi^{-\nu-1}\psi_{y_k}\vartheta_{y_k}+\lambda\nu\psi^{-\nu-1}\psi_{y_ky_k}\vartheta \\ &\left.\left.-\lambda\nu(\nu+1)\psi^{-\nu-2}\psi_{y_k}^2\vartheta+\lambda^2\nu^2\psi^{-2\nu-2}\psi_{y_k}^2\vartheta)\right)\right]^2 \end{aligned}$$

$$
\begin{aligned}
&= \psi^{\nu+1}\left[(x_1+\delta_0)^{-1}\left(\vartheta_{x_1x_1}-\lambda\nu(\nu+1)\psi^{-\nu-2}\psi_{x_1}^2\vartheta+\lambda^2\nu^2\psi^{-2\nu-2}\psi_{x_1}^2\vartheta\right)\right.\\
&\quad (x_1+\delta_0)\sum_{i=2}^{n}\left(\vartheta_{x_ix_i}+\lambda\nu\psi^{-\nu-1}\psi_{x_ix_i}\vartheta-\lambda\nu(\nu+1)\psi^{-\nu-2}\psi_{x_i}^2\vartheta+\lambda^2\nu^2\psi^{-2\nu-2}\psi_{x_i}^2\vartheta\right)\\
&\quad -c^2\sum_{k=1}^{m}\left(\vartheta_{y_ky_k}+\lambda\nu\psi^{-\nu-1}\psi_{y_ky_k}\vartheta-\lambda\nu(\nu+1)\psi^{-\nu-2}\psi_{y_k}^2\vartheta+\lambda^2\nu^2\psi^{-2\nu-2}\psi_{y_k}^2\vartheta\right.\\
&\quad \left.+2\lambda\nu\psi^{-\nu-1}\left((x_1+\delta_0)^{-1}\psi_{x_1}\vartheta_{x_1}+(x_1+\delta_0)\sum_{i=2}^{n}\psi_{xi}\vartheta_{xi}-c^2(x_1+\delta_0)\sum_{k=1}^{m}\psi_{y_k}\vartheta_{y_k}\right)\right]^2\\
&\geq 4\lambda\nu\left((x_1+\delta_0)^{-1}\psi_{x_1}\vartheta_{x_1}+(x_1+\delta_0)\sum_{i=2}^{n}\psi_{xi}\vartheta_{xi}-c^2(x_1+\delta_0)\sum_{k=1}^{m}\psi_{y_k}\vartheta_{y_k}\right)\\
&\quad .\left[(x_1+\delta_0)^{-1}\vartheta_{x_1x_1}+(x_1+\delta_0)\left(\sum_{i=2}^{n}\vartheta_{x_ix_i}-c^2\sum_{k=1}^{m}\vartheta_{y_ky_k}\right)\right.\\
&\quad +(x_1+\delta_0)^{-1}\left(\lambda^2\nu^2\psi^{-2\nu-2}\psi_{x_1}^2\vartheta-\lambda\nu(\nu+1)\psi^{-\nu-2}\psi_{x_1}^2\vartheta\right)\\
&\quad +(x_1+\delta_0)\left[\sum_{i=2}^{n}\left(\lambda^2\nu^2\psi^{-2\nu-2}\psi_{x_i}^2\vartheta-\lambda\nu(\nu+1)\psi^{-\nu-2}\psi_{x_i}^2\vartheta\right)+\lambda\nu\psi^{-\nu-1}\vartheta\right.\\
&\quad \left.\left.-c^2\sum_{k=1}^{m}\left(\lambda^2\nu^2\psi^{-2\nu-2}\psi_{y_k}^2\vartheta-\lambda\nu(\nu+1)\psi^{-\nu-2}\psi_{y_k}^2\vartheta\right)+\lambda\nu\psi^{-\nu-1}\vartheta\right]\right]
\end{aligned}
\tag{2.8}
$$

eşitsizliği elde edilir. Burada

$$
\psi_{x_1}=\delta,\ \psi_{x_1x_1}=0,\ \psi_{x_ix_i}=1,\ \psi_{x_iy_k}=0 \tag{2.9}
$$

olduğu göz önünde bulundurulmuştur. Şimdi son eşitsizlikteki her bir terimi aşağıdaki şekilde değerlendirelim:

Birinci terim:

$$
\begin{aligned}
&4\lambda\nu(x_1+\delta_0)^{-1}\psi_{x_1}\vartheta_{x_1}(x_1+\delta_0)^{-1}\vartheta_{x_1x_1}\\
&= 4\lambda\nu(x_1+\delta_0)^{-2}\delta\vartheta_{x_1}\vartheta_{x_1x_1}\\
&= 2\lambda\nu\delta\left(2\left((x_1+\delta_0)^{-2}\vartheta_{x_1}^2\right)_{x_1}-\left((x_1+\delta_0)^{-2}\vartheta_{x_1}^2\right)_{x_1}+\left((x_1+\delta_0)^{-2}\right)_{x_1}\vartheta_{x_1}^2\right.\\
&\quad \left.-2\left((x_1+\delta_0)^{-2}\right)_{x_1}\vartheta_{x_1}^2\right)\\
&= 2\lambda\nu\delta\left(\left((x_1+\delta_0)^{-2}\vartheta_{x_1}^2\right)_{x_1}-\left((x_1+\delta_0)^{-2}\right)_{x_1}\vartheta_{x_1}^2\right)\\
&= 2\lambda\nu\delta\left((x_1+\delta_0)^{-2}\vartheta_{x_1}^2\right)_{x_1}+4\lambda\nu\delta(x_1+\delta_0)^{-3}\vartheta_{x_1}^2\\
&= d_{01}(\vartheta)+4\lambda\nu\delta(x_1+\delta_0)^{-3}\vartheta_{x_1}^2,
\end{aligned}
\tag{2.10}
$$

ikinci terim :

$$
\begin{aligned}
& 4\lambda\nu\left(x_1+\delta_0\right)^{-1}\psi_{x_1}\vartheta_{x_1}\left(x_1+\delta_0\right)\sum_{i=2}^{n}\vartheta_{x_ix_i} \\
= \; & 2\lambda\nu\delta\sum_{i=2}^{n}2\vartheta_{x_1}\vartheta_{x_ix_i} \\
= \; & 4\lambda\nu\delta\sum_{i=2}^{n}\left(\vartheta_{x_1}\vartheta_{x_i}\right)_{x_i}-2\lambda\nu\delta\sum_{i=2}^{n}\left(\vartheta_{x_i}^2\right)_{x_1} \\
= \; & d_{02}\left(\vartheta\right), \qquad (2.11)
\end{aligned}
$$

üçüncü terim:

$$
\begin{aligned}
& -4\lambda\nu\left(x_1+\delta_0\right)^{-1}\psi_{x_1}\vartheta_{x_1}c^2\left(x_1+\delta_0\right)\sum_{k=1}^{m}\vartheta_{y_ky_k} \\
= \; & -2\lambda\nu\delta\sum_{k=1}^{m}2c^2\vartheta_{x_1}\vartheta_{y_ky_k} \\
= \; & -2\lambda\nu\delta\sum_{k=1}^{m}\left(\left(2c^2\vartheta_{x_1}\vartheta_{y_k}\right)_{y_k}-\left(c^2\vartheta_{y_k}^2\right)_{x_1}+\left(\frac{\partial c^2}{\partial x_1}\vartheta_{y_k}^2\right)-2\frac{\partial c^2}{\partial y_k}\vartheta_{x_1}\vartheta_{y_k}\right) \\
= \; & -4\lambda\nu\delta\sum_{k=1}^{m}\left(c^2\vartheta_{x_1}\vartheta_{y_k}\right)_{y_k}+2\lambda\nu\delta\sum_{k=1}^{m}\left(c^2\vartheta_{y_k}^2\right)_{x_1}-2\lambda\nu\delta\sum_{k=1}^{m}\left(\frac{\partial c^2}{\partial x_1}\vartheta_{y_k}^2\right) \\
& +4\lambda\nu\delta\sum_{k=1}^{m}\frac{\partial c^2}{\partial y_k}\vartheta_{x_1}\vartheta_{y_k} \\
= \; & d_{03}\left(\vartheta\right)-2\lambda\nu\delta\sum_{k=1}^{m}\frac{\partial c^2}{\partial x_1}\vartheta_{y_k}^2+4\lambda\nu\delta\sum_{k=1}^{m}\frac{\partial c^2}{\partial y_k}\vartheta_{x_1}\vartheta_{y_k}, \qquad (2.12)
\end{aligned}
$$

dördüncü terim:

$$
\begin{aligned}
& 4\lambda\nu\left(x_1+\delta_0\right)^{-1}\psi_{x_1}\vartheta_{x_1}\vartheta\left(x_1+\delta_0\right)^{-1}\left(\lambda^2\nu^2\psi^{-2\nu-2}\psi_{x_1}^2-\lambda\nu\left(\nu+1\right)\psi^{-\nu-2}\psi_{x_1}^2\right) \\
= \; & 2\lambda\nu\delta^3\left(x_1+\delta_0\right)^{-2}\left(\lambda^2\nu^2\psi^{-2\nu-2}-\lambda\nu\left(\nu+1\right)\psi^{-\nu-2}\right)2\vartheta\vartheta_{x_1} \\
= \; & 2\lambda\nu\delta^3\left(\left(x_1+\delta_0\right)^{-2}\left(\lambda^2\nu^2\psi^{-2\nu-2}-\lambda\nu\left(\nu+1\right)\psi^{-\nu-2}\right)\vartheta^2\right)_{x_1} \\
& +4\lambda\nu\delta^3\left(x_1+\delta_0\right)^{-3}\left(\lambda^2\nu^2\psi^{-2\nu-2}-\lambda\nu\left(\nu+1\right)\psi^{-\nu-2}\right)\vartheta^2 \\
& -2\lambda\nu\delta^3\left(x_1+\delta_0\right)^{-2}\left(-2\lambda^2\nu^2\left(\nu+1\right)\psi^{-2\nu-3}\psi_{x_1}+\lambda\nu\left(\nu+1\right)\left(\nu+2\right)\psi^{-\nu-3}\psi_{x_1}\right)\vartheta^2 \\
= \; & d_{04}\left(\vartheta\right)+4\lambda^3\nu^3\delta^3\left(x_1+\delta_0\right)^{-3}\psi^{-2\nu-2}\vartheta^2-4\lambda^2\nu^2\left(\nu+1\right)\delta^3\left(x_1+\delta_0\right)^{-3}\psi^{-\nu-2}\vartheta^2 \\
& +4\lambda^3\nu^3\delta^4\left(\nu+1\right)\left(x_1+\delta_0\right)^{-2}\psi^{-2\nu-3}\vartheta^2 \\
& -2\lambda^2\nu^2\delta^4\left(\nu+1\right)\left(\nu+2\right)\left(x_1+\delta_0\right)^{-2}\psi^{-\nu-3}\vartheta^2, \qquad (2.13)
\end{aligned}
$$

beşinci terim:

$$
\begin{aligned}
& 4\lambda\nu\left(x_1+\delta_0\right)^{-1}\psi_{x_1}\vartheta_{x_1}\left(x_1+\delta_0\right)\vartheta\left(\sum_{i=2}^{n}\left(\lambda^2\nu^2\psi^{-2\nu-2}\psi_{x_i}^2-\lambda\nu\left(\nu+1\right)\psi^{-\nu-2}\psi_{x_i}^2\right)\right. \\
& \left.+\lambda\nu\psi^{-\nu-1}\right) \\
= \; & 2\lambda\nu\delta\sum_{i=2}^{n}\left(\left(\lambda^2\nu^2\psi^{-2\nu-2}\psi_{x_i}^2-\lambda\nu\left(\nu+1\right)\psi^{-\nu-2}\psi_{x_i}^2\right)+\lambda\nu\psi^{-\nu-1}\right)2\vartheta\vartheta_{x_1} \\
= \; & 2\lambda\nu\delta\sum_{i=2}^{n}\left(\left(\left(\lambda^2\nu^2\psi^{-2\nu-2}\psi_{x_i}^2-\lambda\nu\left(\nu+1\right)\psi^{-\nu-2}\psi_{x_i}^2\right)+\lambda\nu\psi^{-\nu-1}\right)\vartheta^2\right)_{x_1} \\
& -2\lambda\nu\delta\sum_{i=2}^{n}\left(-2\lambda^2\nu^2\left(\nu+1\right)\psi^{-2\nu-3}\psi_{x_i}^2\psi_{x_1}+\lambda\nu\left(\nu+1\right)\left(\nu+2\right)\psi^{-\nu-3}\psi_{x_i}^2\psi_{x_1}\right. \\
& \left.-\lambda\nu\left(\nu+1\right)\psi^{-\nu-2}\psi_{x_1}\right)\vartheta^2 \\
= \; & d_{05}\left(\vartheta\right)+4\lambda^3\nu^3\delta^2\left(\nu+1\right)\psi^{-2\nu-3}\vartheta^2\sum_{i=2}^{n}\psi_{x_i}^2 \\
& -2\lambda^2\nu^2\delta^2\left(\nu+1\right)\left(\nu+2\right)\psi^{-\nu-3}\vartheta^2\sum_{i=2}^{n}\psi_{x_i}^2+2\lambda^2\nu^2\delta^2\left(\nu+1\right)\psi^{-\nu-2}\vartheta^2, \qquad (2.14)
\end{aligned}
$$

altıncı terim:

$$
\begin{aligned}
& -4\lambda\nu\left(x_1+\delta_0\right)^{-1}\psi_{x_1}\vartheta_{x_1}\left(x_1+\delta_0\right)c^2\vartheta\left(\sum_{k=1}^{m}\left(\lambda^2\nu^2\psi^{-2\nu-2}\psi_{y_k}^2-\lambda\nu\left(\nu+1\right)\psi^{-\nu-2}\psi_{y_k}^2\right)\right. \\
& \left.+\lambda\nu\psi^{-\nu-1}\psi_{y_ky_k}\right) \\
= \; & -2\lambda\nu\delta\sum_{k=1}^{m}\left(\left(\lambda^2\nu^2c^2\psi^{-2\nu-2}\psi_{y_k}^2-\lambda\nu\left(\nu+1\right)c^2\psi^{-\nu-2}\psi_{y_k}^2\right)+\lambda\nu\psi^{-\nu-1}\psi_{y_ky_k}c^2\right)2\vartheta\vartheta_{x_1} \\
= \; & -2\lambda\nu\delta\sum_{k=1}^{m}\left[\left(\lambda^2\nu^2c^2\psi^{-2\nu-2}\psi_{y_k}^2-\lambda\nu\left(\nu+1\right)c^2\psi^{-\nu-2}\psi_{y_k}^2+\lambda\nu\psi^{-\nu-1}\psi_{y_ky_k}c^2\right)\vartheta^2\right]_{x_1} \\
& +2\lambda\nu\delta\sum_{k=1}^{m}\left(-2\lambda^2\nu^2\left(\nu+1\right)c^2\psi^{-2\nu-3}\psi_{y_k}^2\psi_{x_1}+\lambda\nu\left(\nu+1\right)\left(\nu+2\right)c^2\psi^{-\nu-3}\psi_{y_k}^2\psi_{x_1}\right. \\
& -\lambda\nu\delta\left(\nu+1\right)c^2\psi^{-\nu-2}\psi_{y_ky_k}+\lambda^2\nu^2\frac{\partial c^2}{\partial x_1}\psi^{-2\nu-2}\psi_{y_k}^2-\lambda\nu\left(\nu+1\right)\frac{\partial c^2}{\partial x_1}\psi^{-\nu-2}\psi_{y_k}^2 \\
& \left.+\lambda\nu\frac{\partial c^2}{\partial x_1}\psi^{-\nu-1}\psi_{y_ky_k}\right)\vartheta^2 \\
= \; & d_{06}\left(\vartheta\right)-4\lambda^3\nu^3\delta^2\left(\nu+1\right)c^2\psi^{-2\nu-3}\vartheta^2\sum_{k=1}^{m}\psi_{y_k}^2 \\
& +2\lambda^2\nu^2\delta^2\left(\nu+1\right)\left(\nu+2\right)c^2\psi^{-\nu-3}\vartheta^2\sum_{k=1}^{m}\psi_{y_k}^2-2\lambda^2\nu^2\delta^2\left(\nu+1\right)mc^2\psi^{-\nu-2}\vartheta^2 \\
& +2\lambda^3\nu^3\delta\psi^{-2\nu-2}\vartheta^2\sum_{k=1}^{m}\frac{\partial c^2}{\partial x_1}\psi_{y_k}^2-2\lambda^2\nu^2\delta\left(\nu+1\right)\psi^{-\nu-2}\vartheta^2\sum_{k=1}^{m}\frac{\partial c^2}{\partial x_1}\psi_{y_k}^2 \\
& +2\lambda^2\nu^2\delta m\frac{\partial c^2}{\partial x_1}\psi^{-\nu-1}\vartheta^2, \qquad (2.15)
\end{aligned}
$$

yedinci terim:

$$
\begin{aligned}
& 4\lambda\nu\left(x_1+\delta_0\right)\sum_{i=2}^{n}\psi_{x_i}\vartheta_{x_i}\left(x_1+\delta_0\right)^{-1}\vartheta_{x_1x_1} \\
= \;& 2\lambda\nu\sum_{i=2}^{n}2\psi_{x_i}\vartheta_{x_i}\vartheta_{x_1x_1} \\
= \;& 2\lambda\nu\sum_{i=2}^{n}\Big(\left(2\psi_{x_i}\vartheta_{x_i}\vartheta_{x_1}\right)_{x_1}-\left(\psi_{x_i}\vartheta_{x_1}^2\right)_{x_i}+\left(\psi_{x_ix_i}\vartheta_{x_1}^2\right)-2\psi_{x_ix_1}\vartheta_{x_i}\vartheta_{x_1}\Big) \\
= \;& 4\lambda\nu\sum_{i=2}^{n}\left(\psi_{x_i}\vartheta_{x_i}\vartheta_{x_1}\right)_{x_1}-2\lambda\nu\sum_{i=2}^{n}\left(\psi_{x_i}\vartheta_{x_1}^2\right)_{x_i}+2\lambda\nu\sum_{i=2}^{n}\left(\psi_{x_ix_i}\vartheta_{x_1}^2\right) \\
= \;& d_{07}\left(\vartheta\right)+2\lambda\nu\left(n-1\right)\vartheta_{x_1}^2,
\end{aligned} \tag{2.16}
$$

sekizinci terim:

$$
\begin{aligned}
& 4\lambda\nu\left(x_1+\delta_0\right)\sum_{i=2}^{n}\psi_{x_i}\vartheta_{x_i}\left(x_1+\delta_0\right)\sum_{j=2}^{n}\vartheta_{x_jx_j} \\
= \;& 2\lambda\nu\left(x_1+\delta_0\right)^2\sum_{i,j=2}^{n}2\psi_{x_i}\vartheta_{x_i}\vartheta_{x_jx_j} \\
= \;& 2\lambda\nu\left(x_1+\delta_0\right)^2\sum_{i,j=2}^{n}\left(\left(2\psi_{x_i}\vartheta_{x_i}\vartheta_{x_j}\right)_{x_j}-\left(\psi_{x_i}\vartheta_{x_j}^2\right)_{x_i}+\left(\psi_{x_ix_i}\vartheta_{x_j}^2\right)-2\psi_{x_ix_j}\vartheta_{x_i}\vartheta_{x_j}\right) \\
= \;& 4\lambda\nu\left(x_1+\delta_0\right)^2\sum_{i,j=2}^{n}\left(\psi_{x_i}\vartheta_{x_i}\vartheta_{x_j}\right)_{x_j}-2\lambda\nu\left(x_1+\delta_0\right)^2\sum_{i,j=2}^{n}\left(\psi_{x_i}\vartheta_{x_j}^2\right)_{x_i} \\
& +2\lambda\nu\left(x_1+\delta_0\right)^2\sum_{i,j=2}^{n}\psi_{x_ix_i}\vartheta_{x_j}^2-4\lambda\nu\left(x_1+\delta_0\right)^2\sum_{i,j=2}^{n}\psi_{x_ix_j}\vartheta_{x_i}\vartheta_{x_j} \\
= \;& d_{08}\left(\vartheta\right)-2\lambda\nu\left(n-1\right)\left(x_1+\delta_0\right)^2\sum_{j=2}^{n}\vartheta_{x_j}^2,
\end{aligned} \tag{2.17}
$$

dokuzuncu terim:

$$
\begin{aligned}
& -4\lambda\nu\left(x_1+\delta_0\right)\sum_{i=2}^{n}\psi_{x_i}\vartheta_{x_i}c^2\left(x_1+\delta_0\right)\sum_{k=1}^{m}\vartheta_{y_ky_k} \\
= \;& -2\lambda\nu\left(x_1+\delta_0\right)^2\sum_{i=2}^{n}\sum_{k=1}^{m}2c^2\psi_{x_i}\vartheta_{x_i}\vartheta_{y_ky_k} \\
= \;& -2\lambda\nu\left(x_1+\delta_0\right)^2\sum_{i=2}^{n}\sum_{k=1}^{m}\Big(\left(2c^2\psi_{x_i}\vartheta_{x_i}\vartheta_{y_k}\right)_{y_k}-\left(c^2\psi_{x_i}\vartheta_{y_k}^2\right)_{x_i}+\left(c^2\psi_{x_i}\right)_{x_i}\vartheta_{y_k}^2 \\
& -2\left(c^2\psi_{x_i}\right)_{y_k}\vartheta_{x_i}\vartheta_{y_k}\Big) \\
= \;& d_{09}\left(\vartheta\right)-2\lambda\nu\left(x_1+\delta_0\right)^2\sum_{i=2}^{n}\sum_{k=1}^{m}\frac{\partial c^2}{\partial x_i}\psi_{x_i}\vartheta_{y_k}^2-2\lambda\nu\left(x_1+\delta_0\right)^2\sum_{i=2}^{n}\sum_{k=1}^{m}c^2\psi_{x_ix_i}\vartheta_{y_k}^2 \\
& +4\lambda\nu\left(x_1+\delta_0\right)^2\sum_{i=2}^{n}\sum_{k=1}^{m}\frac{\partial c^2}{\partial y_k}\psi_{x_i}\vartheta_{x_i}\vartheta_{y_k}+4\lambda\nu\left(x_1+\delta_0\right)^2\sum_{i=2}^{n}\sum_{k=1}^{m}c^2\psi_{x_iy_k}\vartheta_{x_i}\vartheta_{y_k},
\end{aligned} \tag{2.18}
$$

onuncu terim:

$$
\begin{aligned}
& 4\lambda\nu\left(x_1+\delta_0\right)\sum_{i=2}^{n}\psi_{x_i}\vartheta_{x_i}\vartheta\left(x_1+\delta_0\right)^{-1}\left(\lambda^2\nu^2\psi^{-2\nu-2}\psi_{x_1}^2-\lambda\nu\left(\nu+1\right)\psi^{-\nu-2}\psi_{x_1}^2\right)\\
= \; & 2\lambda\nu\delta^2\sum_{i=2}^{n}\left(\lambda^2\nu^2\psi^{-2\nu-2}\psi_{x_i}-\lambda\nu\left(\nu+1\right)\psi^{-\nu-2}\psi_{x_i}\right)2\vartheta\vartheta_{x_i}\\
= \; & 2\lambda^2\nu^2\delta^2\sum_{i=2}^{n}\left(\left(\lambda\nu\psi^{-2\nu-2}\psi_{x_i}-\left(\nu+1\right)\psi^{-\nu-2}\psi_{x_i}\right)\vartheta^2\right)_{x_i}\\
& -2\lambda^2\nu^2\delta^2\sum_{i=2}^{n}\left(-2\lambda\nu\left(\nu+1\right)\psi^{-2\nu-3}\psi_{x_i}^2+\lambda\nu\psi^{-2\nu-2}\psi_{x_ix_i}\right.\\
& \left.+\left(\nu+1\right)\left(\nu+2\right)\psi^{-\nu-3}\psi_{x_i}^2-\left(\nu+1\right)\psi^{-\nu-2}\psi_{x_ix_i}\right)\vartheta^2\\
= \; & d_{10}\left(\vartheta\right)+4\lambda^3\nu^3\left(\nu+1\right)\psi^{-2\nu-3}\delta^2\vartheta^2\sum_{i=2}^{n}\psi_{x_i}^2-2\lambda^3\nu^3\delta^2\left(n-1\right)\psi^{-2\nu-2}\vartheta^2\\
& -2\lambda^2\nu^2\delta^2\left(\nu+1\right)\left(\nu+2\right)\psi^{-\nu-3}\vartheta^2\sum_{i=2}^{n}\psi_{x_i}^2+2\lambda^2\nu^2\delta^2\left(\nu+1\right)\left(n-1\right)\psi^{-\nu-2}\vartheta^2, \quad (2.19)
\end{aligned}
$$

on birinci terim:

$$
\begin{aligned}
& 4\lambda\nu\left(x_1+\delta_0\right)\sum_{i=2}^{n}\psi_{x_i}\vartheta_{x_i}\vartheta\left(x_1+\delta_0\right)\sum_{j=2}^{n}\left(\lambda\nu\psi^{-\nu-1}\psi_{x_jx_j}-\lambda\nu\psi^{-\nu-2}\left(\nu+1\right)\psi_{x_j}^2\right.\\
& \left.+\lambda^2\nu^2\psi^{-2\nu-2}\psi_{x_j}^2\right)\\
= \; & 2\lambda^2\nu^2\left(x_1+\delta_0\right)^2\sum_{i,j=2}^{n}\left[\left(\psi^{-\nu-1}\psi_{x_i}\psi_{x_jx_j}-\left(\nu+1\right)\psi_{x_i}\psi_{x_j}^2\psi^{-\nu-2}\right.\right.\\
& \left.\left.+\lambda\nu\psi_{x_i}\psi_{x_j}^2\psi^{-2\nu-2}\right)2\vartheta\vartheta_{x_i}\right]\\
= \; & 2\lambda^2\nu^2\left(x_1+\delta_0\right)^2\sum_{i,,j=2}^{n}\left[\psi_{x_i}\left(\psi_{x_jx_j}\psi^{-\nu-1}-\left(\nu+1\right)\psi_{x_j}^2\psi^{-\nu-2}+\lambda\nu\psi_{x_j}^2\psi^{-2\nu-2}\right)\vartheta^2\right]_{x_i}\\
& -2\lambda^2\nu^2\left(x_1+\delta_0\right)^2\sum_{i,j=2}^{n}(\psi_{x_ix_i}\psi_{x_jx_j}\psi^{-\nu-1}-\left(\nu+1\right)\psi_{x_i}^2\psi_{x_jx_j}\psi^{-\nu-2}\\
& -\left(\nu+1\right)\psi_{x_ix_i}\psi_{x_j}^2\psi^{-\nu-2}+\left(\nu+1\right)\left(\nu+2\right)\psi_{x_i}^2\psi_{x_j}^2\psi^{-\nu-3}\\
& -2\left(\nu+1\right)\psi_{x_i}\psi_{x_j}\psi_{x_jx_i}\psi^{-\nu-2}+\lambda\nu\psi_{x_ix_i}\psi_{x_j}^2\psi^{-2\nu-2}\\
& -2\lambda\nu\left(\nu+1\right)\psi_{x_i}^2\psi_{x_j}^2\psi^{-2\nu-3}+2\lambda\nu\psi_{x_i}\psi_{x_j}\psi_{x_jx_i}\psi^{-2\nu-2})\vartheta^2\\
= \; & d_{11}\left(\vartheta\right)+4\lambda^3\nu^3\left(\nu+1\right)\left(x_1+\delta_0\right)^2\psi^{-2\nu-3}\vartheta^2\sum_{i,j=2}^{n}\psi_{x_i}^2\psi_{x_j}^2\\
& +4\lambda^3\nu^3\left(x_1+\delta_0\right)^2\psi^{-2\nu-2}\vartheta^2\sum_{i=2}^{n}\psi_{x_i}^2-2\lambda^3\nu^3\left(n-1\right)\left(x_1+\delta_0\right)^2\psi^{-2\nu-2}\vartheta^2\sum_{j=2}^{n}\psi_{x_j}^2\\
& -2\lambda^2\nu^2\psi^{-\nu-1}\left(n-1\right)^2\left(x_1+\delta_0\right)^2\vartheta^2+4\lambda^2\nu^2\left(\nu+1\right)\left(x_1+\delta_0\right)^2\psi^{-\nu-2}\vartheta^2\sum_{i=2}^{n}\psi_{x_i}^2
\end{aligned}
$$

$$+2\lambda^2\nu^2(\nu+1)(n-1)(x_1+\delta_0)^2\psi^{-\nu-2}\vartheta^2\sum_{i=2}^{n}\psi_{x_i}^2$$
$$+2\lambda^2\nu^2(\nu+1)(n-1)(x_1+\delta_0)^2\psi^{-\nu-2}\vartheta^2\sum_{j=2}^{n}\psi_{x_j}^2$$
$$-2\lambda^2\nu^2(\nu+1)(\nu+2)(n-1)(x_1+\delta_0)^2\psi^{-\nu-3}\vartheta^2\sum_{i,j=2}^{n}\psi_{x_i}^2\psi_{x_j}^2, \qquad (2.20)$$

on ikinci terim:

$$\begin{aligned}
& -4\lambda\nu(x_1+\delta_0)\sum_{i=2}^{n}\psi_{x_i}\vartheta_{x_i}.c^2\vartheta(x_1+\delta_0)\sum_{k=1}^{m}\left(\lambda\nu\psi_{y_ky_k}\psi^{-\nu-1}-\lambda\nu(\nu+1)\psi_{y_k}^2\psi^{-\nu-2}\right.\\
& \left.+\lambda^2\nu^2\psi_{y_k}^2\psi^{-2\nu-2}\right)\\
= \; & -2\lambda^2\nu^2(x_1+\delta_0)^2\sum_{i=2}^{n}\sum_{k=1}^{m}\left(c^2\psi_{x_i}\psi_{y_ky_k}\psi^{-\nu-1}-(\nu+1)c^2\psi_{x_i}\psi_{y_k}^2\psi^{\nu-2}\right.\\
& \left.+\lambda\nu c^2\psi_{x_i}\psi_{y_k}^2\psi^{-2\nu-2}\right)2\vartheta\vartheta_{x_i}\\
= \; & -2\lambda^2\nu^2(x_1+\delta_0)^2\sum_{i=2}^{n}\sum_{k=1}^{m}\left[c^2\psi_{x_i}\psi^{-\nu-1}\left(m-(\nu+1)\psi_{y_k}^2\psi^{-1}+\lambda\nu\psi_{y_k}^2\psi^{-\nu-1}\right)\vartheta^2\right]_{x_i}\\
& +2\lambda^2\nu^2(x_1+\delta_0)^2\sum_{i=2}^{n}\sum_{k=1}^{m}(\frac{\partial c^2}{\partial x_i}\psi_{x_i}\psi_{y_ky_k}\psi^{-\nu-1}+c^2\psi_{x_ix_i}\psi_{y_ky_k}\psi^{-\nu-1}\\
& -(\nu+1)c^2\psi_{x_i}^2\psi_{y_ky_k}\psi^{-\nu-2}-(\nu+1)\frac{\partial c^2}{\partial x_i}\psi_{x_i}\psi_{y_k}^2\psi^{-\nu-2}\\
& -(\nu+1)c^2\psi_{x_ix_i}\psi_{y_k}^2\psi^{-\nu-2}+(\nu+1)(\nu+2)c^2\psi_{x_i}^2\psi_{y_k}^2\psi^{-\nu-3}\\
& -2(\nu+1)c^2\psi_{x_i}\psi_{y_k}\psi_{y_kx_i}\psi^{-\nu-2}+\lambda\nu\frac{\partial c^2}{\partial x_i}\psi_{x_i}\psi_{y_k}^2\psi^{-2\nu-2}+\lambda\nu c^2\psi_{x_ix_i}\psi_{y_k}^2\psi^{-2\nu-2}\\
& -2\lambda\nu(\nu+1)c^2\psi_{x_i}^2\psi_{y_k}^2\psi^{-2\nu-3}+2\lambda\nu c^2\psi_{x_i}\psi_{y_k}\psi_{y_kx_i}\psi^{-2\nu-2})\vartheta^2\\
= \; & d_{12}(\vartheta)+2\lambda^2\nu^2m\psi^{-\nu-1}(x_1+\delta_0)^2\vartheta^2\sum_{i=2}^{n}\frac{\partial c^2}{\partial x_i}\psi_{x_i}\\
& +2\lambda^2\nu^2c^2m(n-1)(x_1+\delta_0)^2\psi^{-\nu-1}\vartheta^2\\
& -2\lambda^2\nu^2(\nu+1)mc^2\psi^{-\nu-2}(x_1+\delta_0)^2\vartheta^2\sum_{i=2}^{n}\psi_{x_i}^2\\
& -2\lambda^2\nu^2(\nu+1)(x_1+\delta_0)^2\psi^{-\nu-2}\vartheta^2\sum_{i=2}^{n}\sum_{k=1}^{m}\frac{\partial c^2}{\partial x_i}\psi_{x_i}\psi_{y_k}^2\\
& -2\lambda^2\nu^2(\nu+1)(n-1)(x_1+\delta_0)^2c^2\psi^{-\nu-2}\vartheta^2\sum_{k=1}^{m}\psi_{y_k}^2\\
& +2\lambda^2\nu^2(\nu+1)(\nu+2)(x_1+\delta_0)^2c^2\psi^{-\nu-3}\vartheta^2\sum_{i=2}^{n}\sum_{k=1}^{m}\psi_{x_i}^2\psi_{y_k}^2\\
& +2\lambda^3\nu^3(x_1+\delta_0)^2\psi^{-2\nu-2}\vartheta^2\sum_{i=2}^{n}\sum_{k=1}^{m}\frac{\partial c^2}{\partial x_i}\psi_{x_i}\psi_{y_k}^2
\end{aligned}$$

$$
\begin{aligned}
&+2\lambda^3\nu^3(n-1)\left(x_1+\delta_0\right)^2 c^2\psi^{-2\nu-2}\vartheta^2\sum_{k=1}^{m}\psi_{y_k}^2 \\
&-4\lambda^3\nu^3\left(\nu+1\right)\left(x_1+\delta_0\right)^2 c^2\psi^{-2\nu-3}\vartheta^2\sum_{i=2}^{n}\sum_{k=1}^{m}\psi_{x_i}^2\psi_{y_k}^2,
\end{aligned} \tag{2.21}
$$

on üçüncü terim:

$$
\begin{aligned}
&-4\lambda\nu\left(x_1+\delta_0\right)^{-1}\vartheta_{x_1x_1}c^2\left(x_1+\delta_0\right)\sum_{k=1}^{m}\psi_{y_k}\vartheta_{y_k} \\
=\;&-2\lambda\nu\sum_{k=1}^{m}2c^2\psi_{y_k}\vartheta_{y_k}\vartheta_{x_1x_1} \\
=\;&-2\lambda\nu\sum_{k=1}^{m}\left(\left(2c^2\psi_{y_k}\vartheta_{y_k}\vartheta_{x_1}\right)_{x_1}-\left(c^2\psi_{y_k}\vartheta_{x_1}^2\right)_{y_k}+\left(c^2\psi_{y_k}\right)_{y_k}\vartheta_{x_1}^2-2\left(c^2\psi_{y_k}\right)_{x_1}\vartheta_{x_1}\vartheta_{y_k}\right) \\
=\;&-4\lambda\nu\sum_{k=1}^{m}\left(c^2\psi_{y_k}\vartheta_{y_k}\vartheta_{x_1}\right)_{x_1}+2\lambda\nu\sum_{k=1}^{m}\left(c^2\psi_{y_k}\vartheta_{x_1}^2\right)_{y_k}-2\lambda\nu\sum_{k=1}^{m}\frac{\partial c^2}{\partial y_k}\psi_{y_k}\vartheta_{x_1}^2 \\
&-2\lambda\nu c^2\sum_{k=1}^{m}\psi_{y_ky_k}\vartheta_{x_1}^2+4\lambda\nu\sum_{k=1}^{m}\frac{\partial c^2}{\partial x_1}\psi_{y_k}\vartheta_{x_1}\vartheta_{y_k}+4\lambda\nu c^2\sum_{k=1}^{m}\psi_{y_kx_1}\vartheta_{x_1}\vartheta_{y_k} \\
=\;&d_{13}(\vartheta)-2\lambda\nu\sum_{k=1}^{m}\frac{\partial c^2}{\partial y_k}\psi_{y_k}\vartheta_{x_1}^2-2\lambda\nu c^2m\vartheta_{x_1}^2+4\lambda\nu\sum_{k=1}^{m}\frac{\partial c^2}{\partial x_1}\psi_{y_k}\vartheta_{x_1}\vartheta_{y_k},
\end{aligned} \tag{2.22}
$$

on dördüncü terim:

$$
\begin{aligned}
&-4\lambda\nu\left(x_1+\delta_0\right)c^2\sum_{k=1}^{m}\psi_{y_k}\vartheta_{y_k}\left(x_1+\delta_0\right)\sum_{i=2}^{n}\vartheta_{x_ix_i} \\
=\;&-2\lambda\nu\left(x_1+\delta_0\right)^2\sum_{i=2}^{n}\sum_{k=1}^{m}2c^2\psi_{y_k}\vartheta_{y_k}\vartheta_{x_ix_i} \\
=\;&-4\lambda\nu\left(x_1+\delta_0\right)^2\sum_{i=2}^{n}\sum_{k=1}^{m}\left(c^2\psi_{y_k}\vartheta_{y_k}\vartheta_{x_i}\right)_{xi}+2\lambda\nu\left(x_1+\delta_0\right)^2\sum_{i=2}^{n}\sum_{k=1}^{m}\left(c^2\psi_{y_k}\vartheta_{x_i}^2\right)_{y_k} \\
&-2\lambda\nu\left(x_1+\delta_0\right)^2\sum_{i=2}^{n}\sum_{k=1}^{m}\frac{\partial c^2}{\partial y_k}\psi_{y_k}\vartheta_{x_i}^2-2\lambda\nu c^2\left(x_1+\delta_0\right)^2\sum_{i=2}^{n}\sum_{k=1}^{m}\psi_{y_ky_k}\vartheta_{x_i}^2 \\
&+4\lambda\nu\left(x_1+\delta_0\right)^2\sum_{i=2}^{n}\sum_{k=1}^{m}\frac{\partial c^2}{\partial x_i}\psi_{y_k}\vartheta_{x_i}\vartheta_{y_k}+4\lambda\nu c^2\left(x_1+\delta_0\right)^2\sum_{i=2}^{n}\sum_{k=1}^{m}\psi_{y_kx_i}\vartheta_{x_i}\vartheta_{y_k} \\
=\;&d_{14}(\vartheta)-2\lambda\nu\left(x_1+\delta_0\right)^2\sum_{i=2}^{n}\sum_{k=1}^{m}\frac{\partial c^2}{\partial y_k}\psi_{y_k}\vartheta_{x_i}^2-2\lambda\nu c^2m\left(x_1+\delta_0\right)^2\sum_{i=2}^{n}\vartheta_{x_i}^2 \\
&+4\lambda\nu\left(x_1+\delta_0\right)^2\sum_{i=2}^{n}\sum_{k=1}^{m}\frac{\partial c^2}{\partial x_i}\psi_{y_k}\vartheta_{x_i}\vartheta_{y_k},
\end{aligned} \tag{2.23}
$$

on beşinci terim:

$$
\begin{aligned}
& 4\lambda\nu\left(x_1+\delta_0\right)c^2\sum_{k=1}^{m}\psi_{y_k}\vartheta_{y_k}c^2\left(x_1+\delta_0\right)\sum_{s=1}^{m}\vartheta_{y_sy_s} \\
= \;& 2\lambda\nu\left(x_1+\delta_0\right)^2\sum_{k,s=1}^{m}2c^4\psi_{y_k}\vartheta_{y_k}\vartheta_{y_sy_s} \\
= \;& 2\lambda\nu\left(x_1+\delta_0\right)^2\sum_{k,s=1}^{m}\Big(\left(2c^4\psi_{y_k}\vartheta_{y_k}\vartheta_{y_s}\right)_{y_s}-\left(c^4\psi_{y_k}\vartheta_{y_s}^2\right)_{y_k}+\left(c^4\psi_{y_k}\right)_{y_k}\vartheta_{y_s}^2 \\
& -2\left(c^4\psi_{y_k}\right)_{y_s}\vartheta_{y_k}\vartheta_{y_s}\Big) \\
= \;& 4\lambda\nu\left(x_1+\delta_0\right)^2\sum_{k,s=1}^{m}\left(c^4\psi_{y_k}\vartheta_{y_k}\vartheta_{y_s}\right)_{y_s}-2\lambda\nu\left(x_1+\delta_0\right)^2\sum_{k,s=1}^{m}\left(c^4\psi_{y_k}\vartheta_{y_s}^2\right)_{y_k} \\
& +2\lambda\nu\left(x_1+\delta_0\right)^2\sum_{k,s=1}^{m}\left(c^4\psi_{y_k}\right)_{y_k}\vartheta_{y_s}^2-4\lambda\nu\left(x_1+\delta_0\right)^2\sum_{k,s=1}^{m}\left(c^4\psi_{y_k}\right)_{y_s}\vartheta_{y_k}\vartheta_{y_s} \\
= \;& d_{15}\left(\vartheta\right)+2\lambda\nu\left(x_1+\delta_0\right)^2\sum_{k,s=1}^{m}\frac{\partial c^4}{\partial y_k}\psi_{y_k}\vartheta_{y_s}^2+2\lambda\nu c^4m\left(x_1+\delta_0\right)^2\sum_{s=1}^{m}\vartheta_{y_s}^2 \\
& -4\lambda\nu\left(x_1+\delta_0\right)^2\sum_{k,s=1}^{m}\frac{\partial c^4}{\partial y_s}\psi_{y_k}\vartheta_{y_k}\vartheta_{y_s}-4\lambda\nu c^4\left(x_1+\delta_0\right)^2\sum_{k=1}^{m}\vartheta_{y_k}^2, \qquad (2.24)
\end{aligned}
$$

on altıncı terim:

$$
\begin{aligned}
& -4\lambda\nu c^2\left(x_1+\delta_0\right)\sum_{k=1}^{m}\psi_{y_k}\vartheta_{y_k}\left(x_1+\delta_0\right)^{-1}(\lambda^2\nu^2\psi^{-2\nu-2}\psi_{x_1}^2-\lambda\nu\left(\nu+1\right)\psi^{-\nu-2}\psi_{x_1}^2)\vartheta \\
= \;& -2\lambda^2\nu^2\delta^2\sum_{k=1}^{m}\left[c^2\psi_{y_k}(\lambda\nu\psi^{-2\nu-2}-\left(\nu+1\right)\psi^{-\nu-2})\right]2\vartheta\vartheta_{y_k} \\
= \;& -2\lambda^2\nu^2\sum_{i=1}^{n}\sum_{k=1}^{m}\left[c^2\psi_{y_k}\left(\lambda\nu\psi^{-2\nu-2}-\left(\nu+1\right)\psi^{-\nu-2}\right)\vartheta^2\right]_{y_k} \\
& +2\lambda^2\nu^2\sum_{k=1}^{m}\Big(\lambda\nu\frac{\partial c^2}{\partial y_k}\psi_{y_k}\psi^{-2\nu-2}+\lambda\nu c^2\psi_{y_ky_k}\psi^{-2\nu-2}-2\lambda\nu(\nu+1)c^2\psi_{y_k}^2\psi^{-2\nu-3} \\
& -\left(\nu+1\right)\frac{\partial c^2}{\partial y_k}\psi_{y_k}\psi^{-\nu-2}-\left(\nu+1\right)c^2\psi_{y_ky_k}\psi^{-\nu-2}+\left(\nu+1\right)\left(\nu+2\right)c^2\psi_{y_k}^2\psi^{-\nu-3}\Big)\vartheta^2 \\
= \;& d_{16}\left(\vartheta\right)+2\lambda^3\nu^3\psi^{-2\nu-2}\sum_{k=1}^{m}\frac{\partial c^2}{\partial y_k}\psi_{y_k}\vartheta^2 \\
& +2\lambda^3\nu^3c^2\psi^{-2\nu-2}\sum_{k=1}^{m}\psi_{y_ky_k}\vartheta^2-4\lambda^3\nu^3\left(\nu+1\right)c^2\psi^{-2\nu-3}\sum_{k=1}^{m}\psi_{y_k}^2\vartheta^2 \\
& -2\lambda^2\nu^2\left(\nu+1\right)\psi^{-\nu-2}\sum_{k=1}^{m}\frac{\partial c^2}{\partial y_k}\psi_{y_k}\vartheta^2-2\lambda^2\nu^2\left(\nu+1\right)c^2\psi^{-\nu-2}\sum_{k=1}^{m}\psi_{y_ky_k}\vartheta^2 \\
& +2\lambda^2\nu^2c^2\left(\nu+1\right)\left(\nu+2\right)\psi^{-\nu-3}\sum_{k=1}^{m}\psi_{y_k}^2\vartheta^2, \qquad (2.25)
\end{aligned}
$$

on yedinci terim:

$$
\begin{aligned}
&-4\lambda\nu\left(x_1+\delta_0\right)c^2\sum_{k=1}^{m}\psi_{y_k}\vartheta_{y_k}\left(x_1+\delta_0\right)\sum_{i=2}^{n}(\lambda\nu\psi^{-\nu-1}\psi_{x_ix_i}-\lambda\nu\left(\nu+1\right)\psi^{-\nu-2}\psi_{x_i}^2\\
&+\lambda^2\nu^2\psi^{-2\nu-2}\psi_{x_i}^2)\vartheta\\
=\;&-4\lambda^2\nu^2\left(x_1+\delta_0\right)^2\sum_{i=2}^{n}\sum_{k=1}^{m}\left[c^2\psi_{y_k}\psi^{-\nu-1}(\psi_{x_ix_i}-\left(\nu+1\right)\psi^{-1}\psi_{x_i}^2+\lambda\nu\psi^{-\nu-1}\psi_{x_i}^2)\right]\vartheta\vartheta_{y_k}\\
=\;&-2\lambda^2\nu^2\left(x_1+\delta_0\right)^2\sum_{i=2}^{n}\sum_{k=1}^{m}\left[c^2\psi_{y_k}(\psi_{x_ix_i}\psi^{-\nu-1}-\left(\nu+1\right)\psi_{x_i}^2\psi^{-\nu-2}+\lambda\nu\psi_{x_i}^2\psi^{-2\nu-2})\vartheta^2\right]_{y_k}\\
&+2\lambda^2\nu^2\left(x_1+\delta_0\right)^2\sum_{i=2}^{n}\sum_{k=1}^{m}\left(\frac{\partial c^2}{\partial y_k}\psi_{x_ix_i}\psi_{y_k}\psi^{-\nu-1}+c^2\psi_{y_ky_k}\psi_{x_ix_i}\psi^{-\nu-1}\right.\\
&-\left(\nu+1\right)c^2\psi_{x_ix_i}\psi_{y_k}^2\psi^{-\nu-2}-\left(\nu+1\right)\frac{\partial c^2}{\partial y_k}\psi_{x_i}^2\psi_{y_k}\psi^{-\nu-2}-\left(\nu+1\right)c^2\psi_{x_i}^2\psi_{y_ky_k}\psi^{-\nu-2}\\
&+\left(\nu+1\right)\left(\nu+2\right)c^2\psi_{x_i}^2\psi_{y_k}^2\psi^{-\nu-3}-2c^2\left(\nu+1\right)\psi^{-\nu-2}\psi_{y_k}\psi_{x_i}\psi_{x_iy_k}\\
&+\lambda\nu\frac{\partial c^2}{\partial y_k}\psi_{x_i}^2\psi_{y_k}\psi^{-2\nu-2}+\lambda\nu c^2\psi_{y_ky_k}\psi_{x_i}^2\psi^{-2\nu-2}\\
&\left.-2\lambda\nu(\nu+1)c^2\psi_{y_k}^2\psi_{x_i}^2\psi^{-2\nu-3}+2\lambda\nu c^2\psi_{y_k}\psi_{x_i}\psi_{x_iy_k}\psi^{-2\nu-2}\right)\vartheta^2\\
=\;&d_{17}\left(\vartheta\right)+2\lambda^2\nu^2\left(n-1\right)\left(x_1+\delta_0\right)^2\psi^{-\nu-1}\vartheta^2\sum_{k=1}^{m}\frac{\partial c^2}{\partial y_k}\psi_{y_k}\\
&+2\lambda^2\nu^2c^2\left(n-1\right)m\left(x_1+\delta_0\right)^2\psi^{-\nu-1}\vartheta^2\\
&-2\lambda^2\nu^2c^2\left(\nu+1\right)\left(n-1\right)\left(x_1+\delta_0\right)^2\psi^{-\nu-2}\vartheta^2\sum_{k=1}^{m}\psi_{y_k}^2\\
&-2\lambda^2\nu^2\left(\nu+1\right)\left(x_1+\delta_0\right)^2\psi^{-\nu-2}\vartheta^2\sum_{i=2}^{n}\sum_{k=1}^{m}\frac{\partial c^2}{\partial y_k}\psi_{x_i}^2\psi_{y_k}\\
&-2\lambda^2\nu^2\left(\nu+1\right)\left(x_1+\delta_0\right)^2c^2m\psi^{-\nu-2}\vartheta^2\sum_{i=2}^{n}\psi_{x_i}^2\\
&+2\lambda^2\nu^2c^2\left(x_1+\delta_0\right)^2\left(\nu+1\right)\left(\nu+2\right)\psi^{-\nu-3}\vartheta^2\sum_{i=2}^{n}\sum_{k=1}^{m}\psi_{x_i}^2\psi_{y_k}^2\\
&+2\lambda^3\nu^3\left(x_1+\delta_0\right)^2\psi^{-2\nu-2}\vartheta^2\sum_{i=2}^{n}\sum_{k=1}^{m}\frac{\partial c^2}{\partial y_k}\psi_{x_i}^2\psi_{y_k}\\
&-4\lambda^3\nu^3(\nu+1)\left(x_1+\delta_0\right)^2c^2\psi^{-2\nu-3}\vartheta^2\sum_{i=2}^{n}\sum_{k=1}^{m}\psi_{y_k}^2\psi_{x_i}^2\\
&+2\lambda^3\nu^3\left(x_1+\delta_0\right)^2c^2m\psi^{-2\nu-2}\vartheta^2\sum_{i=2}^{n}\psi_{x_i}^2,
\end{aligned}
\tag{2.26}
$$

on sekizinci terim:

$$
\begin{aligned}
& 4\lambda\nu c^2\left(x_1+\delta_0\right)\sum_{k=1}^{m}\psi_{y_k}\vartheta_{y_k}\lambda\nu\psi^{-\nu-1}\vartheta c^2\left(x_1+\delta_0\right)\sum_{s=1}^{m}\left(\psi_{y_sy_s}-\left(\nu+1\right)\psi^{-1}\psi_{y_s}^2\right.\\
& \left.+\lambda\nu\psi^{-\nu-1}\psi_{y_s}^2\right)\\
= & 2\lambda^2\nu^2\left(x_1+\delta_0\right)^2\sum_{k,s=1}^{m}c^4\psi_{y_k}\left(\psi_{y_sy_s}\psi^{-\nu-1}-\left(\nu+1\right)\psi_{y_s}^2\psi^{-\nu-2}+\lambda\nu\psi_{y_s}^2\psi^{-2\nu-2}\right)2\vartheta\vartheta_{y_k}\\
= & 2\lambda^2\nu^2\left(x_1+\delta_0\right)^2\sum_{k,s=1}^{m}\left[c^4\psi_{y_k}\left(\psi_{y_sy_s}\psi^{-\nu-1}-\left(\nu+1\right)\psi_{y_s}^2\psi^{-\nu-2}+\lambda\nu\psi_{y_s}^2\psi^{-2\nu-2}\right)\vartheta^2\right]_{y_k}\\
& -2\lambda^2\nu^2\left(x_1+\delta_0\right)^2\sum_{k,s=1}^{m}\left(\frac{\partial c^4}{\partial y_k}\psi_{y_k}\psi_{y_sy_s}\psi^{-\nu-1}+c^4\psi_{y_ky_k}\psi_{y_sy_s}\psi^{-\nu-1}\right.\\
& -c^4(\nu+1)\psi_{y_k}^2\psi_{y_sy_s}\psi^{-\nu-2}-\left(\nu+1\right)\frac{\partial c^4}{\partial y_k}\psi_{y_k}\psi_{y_s}^2\psi^{-\nu-2}-\left(\nu+1\right)c^4\psi_{y_ky_k}\psi_{y_s}^2\psi^{-\nu-2}\\
& +\left(\nu+1\right)\left(\nu+2\right)c^4\psi_{y_k}^2\psi_{y_s}^2\psi^{-\nu-3}-\left(\nu+1\right)c^4\psi_{y_k}\frac{\partial\psi_{y_s}^2}{\partial y_k}\psi^{-\nu-2}\\
& +\lambda\nu\frac{\partial c^4}{\partial y_k}\psi_{y_k}\psi_{y_s}^2\psi^{-2\nu-2}+\lambda\nu c^4\psi_{y_ky_k}\psi_{y_s}^2\psi^{-2\nu-2}\\
& \left.-2\lambda\nu(\nu+1)c^4\psi_{y_k}^2\psi_{y_s}^2\psi^{-2\nu-3}+\lambda\nu c^4\psi_{y_k}\frac{\partial\psi_{y_s}^2}{\partial y_k}\psi^{-2\nu-2}\right)\vartheta^2\\
= & d_{18}\left(\vartheta\right)-2\lambda^2\nu^2m\psi^{-\nu-1}\vartheta^2\sum_{k=1}^{m}\frac{\partial c^4}{\partial y_k}\psi_{y_k}-2\lambda^2\nu^2c^4m^2\psi^{-\nu-1}\vartheta^2\\
& +2\lambda^2\nu^2c^4(\nu+1)m\psi^{-\nu-2}\vartheta^2\sum_{k=1}^{m}\psi_{y_k}^2+2\lambda^2\nu^2\left(\nu+1\right)\psi^{-\nu-2}\vartheta^2\sum_{k,s=1}^{m}\frac{\partial c^4}{\partial y_k}\psi_{y_k}\psi_{y_s}^2\\
& +2\lambda^2\nu^2\left(\nu+1\right)c^4m\psi^{-\nu-2}\vartheta^2\sum_{s=1}^{m}\psi_{y_s}^2-2\lambda^2\nu^2\left(\nu+1\right)\left(\nu+2\right)c^4\psi^{-\nu-3}\vartheta^2\sum_{k,s=1}^{m}\psi_{y_k}^2\psi_{y_s}^2\\
& +4\lambda^2\nu^2\left(\nu+1\right)c^4\psi^{-\nu-2}\vartheta^2\sum_{k=1}^{m}\psi_{y_k}^2-2\lambda^3\nu^3\psi^{-2\nu-2}\vartheta^2\sum_{k,s=1}^{m}\frac{\partial c^4}{\partial y_k}\psi_{y_k}\psi_{y_s}^2\\
& -4\lambda^3\nu^3c^4\psi^{-2\nu-2}\vartheta^2\sum_{k=1}^{m}\psi_{y_k}^2-2\lambda^3\nu^3c^4m\psi^{-2\nu-2}\sum_{s=1}^{m}\psi_{y_s}^2\\
& -4\lambda^3\nu^3(\nu+1)c^4\psi^{-2\nu-3}\sum_{k,s=1}^{m}\psi_{y_k}^2\psi_{y_s}^2 \qquad (2.27)
\end{aligned}
$$

şeklinde bulunur.

O halde (2.10) - (2.27) eşitliklerinden

$$\begin{aligned}
&\psi^{\nu+1}\left(\left(x_1+\delta_0\right)^{-1}\varphi_{x_1x_1}+\left(x_1+\delta_0\right)\left(\sum_{i=2}^{n}\varphi_{x_ix_i}-c^2\left(x,y'\right)\sum_{k=1}^{m}\varphi_{y_ky_k}\right)\right)^2\chi^2 \\
\geq\;& 4\lambda\nu\delta\left(x_1+\delta_0\right)^{-3}\vartheta_{x_1}^2-2\lambda\nu\delta\sum_{k=1}^{m}\frac{\partial c^2}{\partial x_1}\vartheta_{y_k}^2+4\lambda\nu\delta\sum_{k=1}^{m}\frac{\partial c^2}{\partial y_k}\vartheta_{x_1}\vartheta_{y_k} \\
&+2\lambda\nu\left(n-1\right)\vartheta_{x_1}^2-2\lambda\nu\left(n-1\right)\left(x_1+\delta_0\right)^2\sum_{j=2}^{n}\vartheta_{x_j}^2 \\
&-2\lambda\nu\left(x_1+\delta_0\right)^2\sum_{i=2}^{n}\sum_{k=1}^{m}\frac{\partial c^2}{\partial x_i}\psi_{x_i}\vartheta_{y_k}^2-2\lambda\nu c^2\left(n-1\right)\left(x_1+\delta_0\right)^2\sum_{k=1}^{m}\vartheta_{y_k}^2 \\
&+4\lambda\nu\left(x_1+\delta_0\right)^2\sum_{i=2}^{n}\sum_{k=1}^{m}\frac{\partial c^2}{\partial y_k}\psi_{x_i}\vartheta_{x_i}\vartheta_{y_k}-2\lambda\nu\sum_{k=}^{m}\frac{\partial c^2}{\partial y_k}\psi_{y_k}\vartheta_{x_1}^2-2\lambda\nu c^2m\vartheta_{x_1}^2 \\
&+4\lambda\nu\sum_{k=1}^{m}\frac{\partial c^2}{\partial x_1}\psi_{y_k}\vartheta_{x_1}\vartheta_{y_k}-2\lambda\nu\left(x_1+\delta_0\right)^2\sum_{i=2}^{n}\sum_{k=1}^{m}\frac{\partial c^2}{\partial y_k}\psi_{y_k}\vartheta_{x_i}^2 \\
&-2\lambda\nu c^2m\left(x_1+\delta_0\right)^2\sum_{i=2}^{n}\vartheta_{x_i}^2+4\lambda\nu\left(x_1+\delta_0\right)^2\sum_{i=2}^{n}\sum_{k=1}^{m}\frac{\partial c^2}{\partial x_i}\psi_{y_k}\vartheta_{x_i}\vartheta_{y_k} \\
&+2\lambda\nu\left(x_1+\delta_0\right)^2\sum_{k,s=1}^{m}\frac{\partial c^4}{\partial y_k}\psi_{y_k}\vartheta_{y_s}^2+2\lambda\nu c^4m\left(x_1+\delta_0\right)^2\sum_{s=1}^{m}\vartheta_{y_s}^2 \\
&-4\lambda\nu\left(x_1+\delta_0\right)^2\sum_{k,s=1}^{m}\frac{\partial c^4}{\partial y_s}\psi_{y_k}\vartheta_{y_k}\vartheta_{y_s}-4\lambda\nu c^4\left(x_1+\delta_0\right)^2\sum_{k=1}^{m}\vartheta_{y_k}^2 \\
&+K\left(\lambda,\nu,\psi\right)\vartheta^2+d_0\left(\vartheta\right) \qquad (2.28)
\end{aligned}$$

elde edilir.

Eğer Ω_γ alt bölgesinde $\|c^2\|_{C^1(\Omega)}\leq M$, $\left|\psi_{y_k}\right|\leq\sqrt{2\gamma}$ ve $\left|\psi_{x_i}\right|\leq\sqrt{2\gamma}$, $(1\leq i\leq n)$, $(1\leq j\leq m)$ olduğu göz önünde bulundurulursa (2.1) den

$$-2\lambda\nu\delta\sum_{k=1}^{m}\frac{\partial c^2}{\partial x_1}\vartheta_{y_k}^2\geq 2\lambda\nu\delta\alpha_1\sum_{k=1}^{m}\vartheta_{y_k}^2 \qquad (2.29)$$

elde edilir. Ayrıca

$$\begin{aligned}
4\lambda\nu\delta\sum_{k=1}^{m}\frac{\partial c^2}{\partial y_k}\vartheta_{x_1}\vartheta_{y_k} &\geq -4\lambda\nu\delta\sum_{k=1}^{m}\left|\frac{\partial c^2}{\partial y_k}\right|\left|\vartheta_{x_1}\vartheta_{y_k}\right| \\
&\geq -2\lambda\nu\delta\left(\left(x_1+\delta_0\right)\varepsilon_0\sum_{k=1}^{m}\vartheta_{y_k}^2+m\frac{M}{\left(x_1+\delta_0\right)\varepsilon_0}\vartheta_{x_1}^2\right) \\
&\geq -2\lambda\nu\delta\left(x_1+\delta_0\right)\varepsilon_0\sum_{k=1}^{m}\vartheta_{y_k}^2-2\lambda\nu\delta m\frac{M}{\left(x_1+\delta_0\right)\varepsilon_0}\vartheta_{x_1}^2, \qquad (2.30)
\end{aligned}$$

$$
\begin{aligned}
-2\lambda\nu\left(x_1+\delta_0\right)^2\sum_{i=2}^{n}\sum_{k=1}^{m}\frac{\partial c^2}{\partial x_i}\psi_{x_i}\vartheta_{y_k}^2 &\geq -2\lambda\nu\left(x_1+\delta_0\right)^2\sum_{i=2}^{n}\sum_{k=1}^{m}\left|\frac{\partial c^2}{\partial x_i}\right|\left|\psi_{x_i}\right|\left|\vartheta_{y_k}^2\right| \\
&\geq -2\lambda\nu\left(x_1+\delta_0\right)^2 M\sqrt{2\gamma}\,(n-1)\sum_{k=1}^{m}\vartheta_{y_k}^2, \qquad (2.31)
\end{aligned}
$$

$$
\begin{aligned}
-2\lambda\nu c^2(n-1)\left(x_1+\delta_0\right)^2\sum_{k=1}^{m}\vartheta_{y_k}^2 &\geq -2\lambda\nu(n-1)\left(x_1+\delta_0\right)^2\left|c^2\right|\sum_{k=1}^{m}\left|\vartheta_{y_k}^2\right| \\
&\geq -2\lambda\nu(n-1)M\left(x_1+\delta_0\right)^2\sum_{k=1}^{m}\vartheta_{y_k}^2, \qquad (2.32)
\end{aligned}
$$

$$
\begin{aligned}
4\lambda\nu\left(x_1+\delta_0\right)^2\sum_{i=2}^{n}\sum_{k=1}^{m}\frac{\partial c^2}{\partial y_k}\psi_{x_i}\vartheta_{x_i}\vartheta_{y_k} &\geq -4\lambda\nu\left(x_1+\delta_0\right)^2\sum_{i=2}^{n}\sum_{k=1}^{m}\left|\frac{\partial c^2}{\partial y_k}\right|\left|\psi_{x_i}\right|\left|\vartheta_{x_i}\vartheta_{y_k}\right| \\
&\geq -2\lambda\nu M\sqrt{2\gamma}\left(x_1+\delta_0\right)^2\sum_{i=2}^{n}\sum_{k=1}^{m}\left(\vartheta_{x_i}^2+\vartheta_{y_k}^2\right) \\
&\geq -2\lambda\nu M\sqrt{2\gamma}\left(x_1+\delta_0\right)^2 m\sum_{i=2}^{n}\vartheta_{x_i}^2 \\
&\quad -2\lambda\nu M\sqrt{2\gamma}\left(x_1+\delta_0\right)^2(n-1)\sum_{k=1}^{m}\vartheta_{y_k}^2, \qquad (2.33)
\end{aligned}
$$

$$
\begin{aligned}
-2\lambda\nu\sum_{k=1}^{m}\frac{\partial c^2}{\partial y_k}\psi_{y_k}\vartheta_{x_1}^2 &\geq -2\lambda\nu\sum_{k=1}^{m}\left|\frac{\partial c^2}{\partial y_k}\right|\left|\psi_{y_k}\right|\left|\vartheta_{x_1}^2\right| \\
&\geq -2\lambda\nu M\sqrt{2\gamma}\sum_{k=1}^{m}\vartheta_{x_1}^2, \qquad (2.34)
\end{aligned}
$$

$$
-2\lambda\nu c^2 m\vartheta_{x_1}^2 \geq -2\lambda\nu\left|c^2\right|m\vartheta_{x_1}^2 \geq -2\lambda\nu mM\vartheta_{x_1}^2, \qquad (2.35)
$$

$$
\begin{aligned}
4\lambda\nu\sum_{k=1}^{m}\frac{\partial c^2}{\partial x_1}\psi_{y_k}\vartheta_{x_1}\vartheta_{y_k} &\geq -4\lambda\nu\sum_{k=1}^{m}\left|\frac{\partial c^2}{\partial x_1}\right|\left|\psi_{y_k}\right|\left|\vartheta_{x_1}\vartheta_{y_k}\right| \\
&\geq -2\lambda\nu M\sqrt{2\gamma}\sum_{k=1}^{m}\left(\vartheta_{x_1}^2+\vartheta_{y_k}^2\right) \\
&\geq -2\lambda\nu M\sqrt{2\gamma}\sum_{k=1}^{m}\vartheta_{y_k}^2-2\lambda\nu mM\sqrt{2\gamma}\vartheta_{x_1}^2, \qquad (2.36)
\end{aligned}
$$

$$
\begin{aligned}
-2\lambda\nu\left(x_1+\delta_0\right)^2\sum_{i=2}^{n}\sum_{k=1}^{m}\frac{\partial c^2}{\partial y_k}\psi_{y_k}\vartheta_{x_i}^2 &\geq -2\lambda\nu\left(x_1+\delta_0\right)^2\sum_{i=2}^{n}\sum_{k=1}^{m}\left|\frac{\partial c^2}{\partial y_k}\right|\left|\psi_{y_k}\right|\left|\vartheta_{x_i}^2\right| \\
&\geq -2\lambda\nu\left(x_1+\delta_0\right)^2 Mm\sqrt{2\gamma}\sum_{i=2}^{n}\vartheta_{x_i}^2, \qquad (2.37)
\end{aligned}
$$

$$-2\lambda\nu c^2 m\left(x_1+\delta_0\right)^2 \sum_{i=2}^{n} \vartheta_{x_i}^2 \geq -2\lambda\nu M m\left(x_1+\delta_0\right)^2 \sum_{i=2}^{n} \vartheta_{x_i}^2, \tag{2.38}$$

$$\begin{aligned}
4\lambda\nu\left(x_1+\delta_0\right)^2 \sum_{i=2}^{n}\sum_{k=1}^{m} \frac{\partial c^2}{\partial x_i}\psi_{y_k}\vartheta_{x_i}\vartheta_{y_k} &\geq -4\lambda\nu\left(x_1+\delta_0\right)^2 \sum_{i=2}^{n}\sum_{k=1}^{m}\left|\frac{\partial c^2}{\partial x_i}\right|\left|\psi_{y_k}\right|\left|\vartheta_{x_i}\vartheta_{y_k}\right| \\
&\geq -2\lambda\nu\left(x_1+\delta_0\right)^2 M\sqrt{2\gamma}\sum_{i=2}^{n}\sum_{k=1}^{m}\left(\vartheta_{x_i}^2+\vartheta_{y_k}^2\right) \\
&\geq -2\lambda\nu\left(x_1+\delta_0\right)^2 M\sqrt{2\gamma}m\sum_{i=2}^{n}\vartheta_{x_i}^2 \\
&\quad -2\lambda\nu\left(x_1+\delta_0\right)^2 M\sqrt{2\gamma}\left(n-1\right)\sum_{k=1}^{m}\vartheta_{y_k}^2,
\end{aligned} \tag{2.39}$$

$$\begin{aligned}
2\lambda\nu\left(x_1+\delta_0\right)^2 \sum_{k,s=1}^{m} \frac{\partial c^4}{\partial y_k}\psi_{y_k}\vartheta_{y_s}^2 &\geq -2\lambda\nu\left(x_1+\delta_0\right)^2 \sum_{k,s=1}^{m}\left|\frac{\partial c^4}{\partial y_k}\right|\left|\psi_{y_k}\right|\left|\vartheta_{y_s}^2\right| \\
&\geq -4\lambda\nu\left(x_1+\delta_0\right)^2 M^2\sqrt{2\gamma}m\sum_{k=1}^{m}\vartheta_{y_k}^2,
\end{aligned} \tag{2.40}$$

$$2\lambda\nu m c^4\left(x_1+\delta_0\right)^2 \sum_{s=1}^{m} \vartheta_{y_s}^2 \geq -2\lambda\nu m M^2\left(x_1+\delta_0\right)^2 \sum_{s=1}^{m} \vartheta_{y_s}^2, \tag{2.41}$$

$$\begin{aligned}
-4\lambda\nu\left(x_1+\delta_0\right)^2 \sum_{k,s=1}^{m} \frac{\partial c^4}{\partial y_s}\psi_{y_k}\vartheta_{y_k}\vartheta_{y_s} &\geq -4\lambda\nu\left(x_1+\delta_0\right)^2 \sum_{k,s=1}^{m}\left|\frac{\partial c^4}{\partial y_s}\right|\left|\psi_{y_k}\right|\left|\vartheta_{y_k}\vartheta_{y_s}\right| \\
&\geq -2\lambda\nu\left(x_1+\delta_0\right)^2 M^2\sqrt{2\gamma}\sum_{k,s=1}^{m}\left(\vartheta_{y_k}^2+\vartheta_{y_s}^2\right) \\
&\geq -4\lambda\nu\left(x_1+\delta_0\right)^2 M^2\sqrt{2\gamma}m\sum_{k=1}^{m}\vartheta_{y_k}^2,
\end{aligned} \tag{2.42}$$

$$-4\lambda\nu c^4\left(x_1+\delta_0\right)^2 \sum_{k=1}^{m} \vartheta_{y_k}^2 \geq -4\lambda\nu M^2\left(x_1+\delta_0\right)^2 \sum_{k=1}^{m} \vartheta_{y_k}^2 \tag{2.43}$$

bağıntıları yazılabilir.

(2.28) eşitsizliğinde (2.29)-(2.43) bağıntıları kullanılarak

$$
\begin{aligned}
&\psi^{\nu+1}\left(\left(x_1+\delta_0\right)^{-1}\varphi_{x_1x_1}+\left(x_1+\delta_0\right)\left(\sum_{i=2}^{n}\varphi_{x_ix_i}-c^2\left(x,y'\right)\sum_{k=1}^{m}\varphi_{y_ky_k}\right)\right)^2\chi^2\\
\geq\ & 2\lambda\nu\left(2\delta\left(x_1+\delta_0\right)^{-3}+(n-1)-\delta Mm\left(x_1+\delta_0\right)^{-1}\varepsilon_0^{-1}-2mM\sqrt{2\gamma}-mM\right)\vartheta_{x_1}^2\\
&-2\lambda\nu\left(x_1+\delta_0\right)^2\left((n+1)+M\sqrt{2\gamma}m+Mm\sqrt{2\gamma}+Mm+M\sqrt{2\gamma}m\right)\sum_{i=2}^{n}\vartheta_{x_i}^2\\
&+2\lambda\nu\left(\delta\alpha_1-\delta\left(x_1+\delta_0\right)\varepsilon_0-\left(x_1+\delta_0\right)^2M\sqrt{2\gamma}\,(n-1)-(n-1)\,M\left(x_1+\delta_0\right)^2\right.\\
&-M\sqrt{2\gamma}\left(x_1+\delta_0\right)^2(n-1)-M\sqrt{2\gamma}-\left(x_1+\delta_0\right)^2M\sqrt{2\gamma}\,(n-1)\\
&-2\left(x_1+\delta_0\right)^2M^2\sqrt{2\gamma}m-mM^2\left(x_1+\delta_0\right)^2-2\left(x_1+\delta_0\right)^2M^2\sqrt{2\gamma}m\\
&\left.-2M^2\left(x_1+\delta_0\right)^2\right)\sum_{k=1}^{m}\vartheta_{y_k}^2+K\left(\lambda,\nu,\psi\right)\vartheta^2+d_{01}\left(\vartheta\right) \qquad (2.44)
\end{aligned}
$$

yazılabilir. Diğer taraftan $0\leq\varepsilon_0\leq\frac{\alpha_1}{4}$ olduğundan

$$
2\lambda\nu\delta\left(\alpha_1-n\varepsilon_0\right)>2\lambda\nu\delta\left(\alpha_1-\frac{\alpha_1}{4}\right)>\frac{3}{2}\lambda\nu\delta\alpha_1 \qquad (2.45)
$$

bulunur. Eğer

$$
\begin{aligned}
l_1 &= 1+3\,(n-1)\left(\frac{3}{4}\gamma\right)+4M\left(\frac{3}{4}\gamma\right)m\\
&\quad+2M\frac{3}{4}\left(\frac{\gamma}{2}\right)^{1/2}+(n-1)\,\frac{3}{4}\left(\frac{\gamma}{2}\right)^{1/2}, \qquad (2.46)
\end{aligned}
$$

$$
\delta_1=\frac{4M\sqrt{2\gamma}}{\alpha_1}l_1, \qquad (2.47)
$$

alınır ve

$$
\left(\sqrt{2\gamma}\right)^{-1}=\left(\frac{\gamma}{2}\right)^{1/2} \qquad (2.48)
$$

olduğu göz önünde bulundurulursa $\delta\geq\delta_1$ için

$$
\begin{aligned}
&2\lambda\nu\left(\delta\alpha_1-\delta\varepsilon_0-\left(x_1+\delta_0\right)^2M\sqrt{2\gamma}\,(n-1)-(n-1)\,M\left(x_1+\delta_0\right)^2\right.\\
&-M\sqrt{2\gamma}\left(x_1+\delta_0\right)^2(n-1)-M\sqrt{2\gamma}-\left(x_1+\delta_0\right)^2M\sqrt{2\gamma}\,(n-1)\\
&-2\left(x_1+\delta_0\right)^2M^2\sqrt{2\gamma}m-mM^2\left(x_1+\delta_0\right)^2-2\left(x_1+\delta_0\right)^2M^2\sqrt{2\gamma}m\\
&\left.-2M^2\left(x_1+\delta_0\right)^2\right)\sum_{k=1}^{m}\vartheta_{y_k}^2
\end{aligned}
$$

$$
\begin{aligned}
&= 2\lambda\nu\left(\delta\alpha_1-\delta\varepsilon_0-M\sqrt{2\gamma}\left(1+(x_1+\delta_0)(n-1)+(n-1)(x_1+\delta_0)\left(\sqrt{2\gamma}\right)^{-1}\right.\right.\\
&\quad\left.\left.+2(x_1+\delta_0)(n-1)+2M(x_1+\delta_0)m+2M(x_1+\delta_0)\left(\sqrt{2\gamma}\right)^{-1}\right)(x_1+\delta_0)\sum_{k=1}^{m}\vartheta_{y_k}^2\right.\\
&\geq 2\lambda\nu(x_1+\delta_0)\left(\delta\alpha_1-\delta\varepsilon_0-M\sqrt{2\gamma}l_1\right)\sum_{k=1}^{m}\vartheta_{y_k}^2\\
&\geq \left(2\lambda\nu(\delta\alpha_1-n\varepsilon_0\delta)-2\lambda\nu M\sqrt{2\gamma}l_1\right)(x_1+\delta_0)\sum_{k=1}^{m}\vartheta_{y_k}^2\\
&\geq \left(\frac{3}{2}\lambda\nu\delta\alpha_1-2\lambda\nu\delta_1\frac{\alpha_1}{4}\right)(x_1+\delta_0)\sum_{k=1}^{m}\vartheta_{y_k}^2\\
&\geq \lambda\nu\delta\alpha_1(x_1+\delta_0)\sum_{k=1}^{m}\vartheta_{y_k}^2 \qquad (2.49)
\end{aligned}
$$

elde edilir. Ayrıca kabulümüzden $\sqrt{2\gamma}<1$,

$$\gamma<\frac{4}{3}\left(Mm\varepsilon_0^{-1}+3mM\right)^{-1/2}<1$$

eşitsizliği yazılabilir. Buradan

$$\left(\frac{3}{4}\gamma\right)^2\left(Mm\varepsilon_0^{-1}+3mM\right)<1$$

ve

$$\left(\frac{3}{4}\gamma\right)^2\left(Mm\varepsilon_0^{-1}+\frac{3}{2}\gamma mM\sqrt{2\gamma}+\frac{3}{4}\gamma mM\right)<1$$

olacağından $2-\phi_0>1$ olur. O halde

$$\phi_0=\left(\left(\frac{3}{4}\gamma\right)^2Mm\varepsilon_0^{-1}+2mM\left(\frac{3}{4}\gamma\right)^3\sqrt{2\gamma}+mM\left(\frac{3}{4}\gamma\right)^3\right)<1$$

ve $x_1+\delta_0<\frac{3}{4}\gamma$ $(\delta\geq 4)$ için

$$2-\phi_1>2-\phi_0$$

olduğu görülür.

Burada

$$\phi_1=\left((x_1+\delta_0)^2Mm\varepsilon_0^{-1}+2mM(x_1+\delta_0)^3\sqrt{2\gamma}+mM(x_1+\delta_0)^3\right)<1$$

dır. Bu durumda

$$
\begin{aligned}
&2-Mm(x_1+\delta_0)^2\varepsilon_0^{-1}-2mM(x_1+\delta_0)^3\delta^{-1}\sqrt{2\gamma}-mM\delta^{-1}(x_1+\delta_0)^3\\
&> 2-\phi_1>2-\phi_0>1
\end{aligned}
$$

olduğu görülür. Eğer $\delta \geq 4$ ve

$$\gamma \leq \min\left\{\frac{1}{2}, \frac{4}{3}\left(Mm\varepsilon_0^{-1} + 3mM\right)^{-1/2}\right\}$$

olduğu dikkate alınırsa

$$\begin{aligned}
& 2\lambda\nu\left(2\delta\left(x_1+\delta_0\right)^{-3} + (n-1) - \delta Mm\left(x_1+\delta_0\right)^{-1}\varepsilon_0^{-1} - 2mM\sqrt{2\gamma} - mM\right)\vartheta_{x_1}^2 \\
= \;& 2\lambda\nu\delta\left(x_1+\delta_0\right)^{-3}\vartheta_{x_1}^2\left(2 - Mm\left(x_1+\delta_0\right)^2\varepsilon_0^{-1} - 2mM\left(x_1+\delta_0\right)^3\delta^{-1}\sqrt{2\gamma}\right. \\
& \left.-mM\delta^{-1}\left(x_1+\delta_0\right)^3\right) \\
\geq \;& 2\lambda\nu\delta\left(x_1+\delta_0\right)^{-3}\vartheta_{x_1}^2. \qquad (2.50)
\end{aligned}$$

bulunur.

Böylece

$$\begin{aligned}
& \psi^{\nu+1}\left(\left(x_1+\delta_0\right)^{-1}\varphi_{x_1x_1} + \left(x_1+\delta_0\right)\left(\sum_{i=2}^{n}\varphi_{x_ix_i} - c^2\left(x,y'\right)\sum_{k=1}^{m}\varphi_{y_ky_k}\right)\right)^2\chi^2 \\
= \;& 2\lambda\nu\left(2\delta\left(x_1+\delta_0\right)^{-3} - \delta Mm\left(x_1+\delta_0\right)^{-1}\varepsilon_0^{-1} - 2mM\sqrt{2\gamma} - mM\right)\vartheta_{x_1}^2 \\
& -2\lambda\nu\left(x_1+\delta_0\right)^2\left(n+1+M\sqrt{2\gamma}m + Mm\sqrt{2\gamma} + Mm + M\sqrt{2\gamma}m\right)\sum_{i=2}^{n}\vartheta_{x_i}^2 \\
& +2\lambda\nu\left(x_1+\delta_0\right)\left(\delta\alpha_1 - \delta\varepsilon_0 - \left(x_1+\delta_0\right)M\sqrt{2\gamma}\,(n-1) - (n-1)\,M\left(x_1+\delta_0\right)\right. \\
& -M\sqrt{2\gamma}\left(x_1+\delta_0\right)(n-1) - M\sqrt{2\gamma} - \left(x_1+\delta_0\right)M\sqrt{2\gamma}\,(n-1) \\
& -2\left(x_1+\delta_0\right)M^2\sqrt{2\gamma}m - \left(x_1+\delta_0\right)mM^2 - 2\left(x_1+\delta_0\right)M^2\sqrt{2\gamma}m \\
& \left.-2M^2\left(x_1+\delta_0\right)\right)\sum_{k=1}^{m}\vartheta_{y_k}^2 + K\left(\lambda,\nu,\psi\right)\vartheta^2 + d_0\left(\vartheta\right) \\
\geq \;& 2\lambda\nu\delta\left(x_1+\delta_0\right)^{-3}\vartheta_{x_1}^2 - 2\lambda\nu\left(1 + 2\left(x_1+\delta_0\right)^2 + 3M\sqrt{2\gamma}\left(x_1+\delta_0\right)^2 m\right. \\
& \left.+\left(x_1+\delta_0\right)^2 Mm\right)\sum_{i=2}^{n}\vartheta_{x_i}^2 + \lambda\nu\delta\alpha_1\left(x_1+\delta_0\right)\sum_{k=1}^{m}\vartheta_{y_k}^2 + K\left(\lambda,\nu,\psi\right)\vartheta^2 + d_0\left(\vartheta\right) \\
\geq \;& 2\lambda\nu\delta\left(x_1+\delta_0\right)^{-3}\vartheta_{x_1}^2 - 2\lambda\nu\left(x_1+\delta_0\right)^2 G_0(n,m)\sum_{i=2}^{n}\vartheta_{x_i}^2 \\
& +\lambda\nu\delta\alpha_1\left(x_1+\delta_0\right)\sum_{k=1}^{m}\vartheta_{y_k}^2 + K\left(\lambda,\nu,\psi\right)\vartheta^2 + d_0\left(\vartheta\right) \qquad (2.51)
\end{aligned}$$

eşitsizliği elde edilir. Burada

$$G_0(n,m) = n + 1 + 3M\sqrt{2\gamma}m + Mm \qquad (2.52)$$

dir.

Ayrıca $K(\lambda,\nu,\delta)$ fonksiyonu aşağıdaki şekilde düzenlenebilir:

$$
\begin{aligned}
K \;=\;& \lambda^3\nu^3 \Big(4\delta^3 (x_1+\delta_0)^{-3}\psi^{-2\nu-2} + 4\delta^4 (\nu+1)(x_1+\delta_0)^{-2}\psi^{-2\nu-3} \\
&+4\delta^2(\nu+1)\psi^{-2\nu-3}\sum_{i=2}^{n}\psi_{x_i}^2 - 4\delta^2(\nu+1)c^2\psi^{-2\nu-3}\sum_{k=1}^{m}\psi_{y_k}^2 \\
&+2\delta\psi^{-2\nu-2}\sum_{k=1}^{m}\frac{\partial c^2}{\partial x_1}\psi_{y_k}^2 + 4(\nu+1)\psi^{-2\nu-3}\delta^2\sum_{i=2}^{n}\psi_{x_i}^2 - 2\delta^2(n-1)\psi^{-2\nu-2} \\
&+4(\nu+1)(x_1+\delta_0)^2\psi^{-2\nu-3}\sum_{i,j=2}^{n}\psi_{x_i}^2\psi_{x_j}^2 - 4(x_1+\delta_0)^2\psi^{-2\nu-2}\sum_{i=2}^{n}\psi_{x_i}^2 \\
&-2(n-1)(x_1+\delta_0)^2\psi^{-2\nu-2}\sum_{j=2}^{n}\psi_{x_j}^2 + 2(x_1+\delta_0)^2\psi^{-2\nu-2}\sum_{i=2}^{n}\sum_{k=1}^{m}\frac{\partial c^2}{\partial x_i}\psi_{x_i}\psi_{y_k}^2 \\
&+2(n-1)(x_1+\delta_0)^2c^2\psi^{-2\nu-2}\sum_{k=1}^{m}\psi_{y_k}^2 \\
&-4(\nu+1)(x_1+\delta_0)^2c^2\psi^{-2\nu-3}\sum_{i=2}^{n}\sum_{k=1}^{m}\psi_{x_i}^2\psi_{y_k}^2 \\
&+2\psi^{-2\nu-2}\sum_{k=1}^{m}\frac{\partial c^2}{\partial y_k}\psi_{y_k} + 2c^2m\psi^{-2\nu-2} - 4(\nu+1)c^2\psi^{-2\nu-3}\sum_{k=1}^{m}\psi_{y_k}^2 \\
&+2\psi^{-2\nu-2}(x_1+\delta_0)^2\sum_{i=2}^{n}\sum_{k=1}^{m}\frac{\partial c^2}{\partial y_k}\psi_{x_i}^2\psi_{y_k} + 2(x_1+\delta_0)^2c^2m\psi^{-2\nu-2}\sum_{i=2}^{n}\psi_{x_i}^2 \\
&-4(\nu+1)(x_1+\delta_0)^2c^2\psi^{-2\nu-3}\sum_{i=2}^{n}\sum_{k=1}^{m}\psi_{y_k}^2\psi_{x_i}^2 - 2\psi^{-2\nu-2}\sum_{k,s=1}^{m}\frac{\partial c^4}{\partial y_k}\psi_{y_k}\psi_{y_s}^2 \\
&-2c^4m\psi^{-2\nu-2}\sum_{s=1}^{m}\psi_{y_s}^2 - 4(\nu+1)c^4\psi^{-2\nu-3}\sum_{k,s=1}^{m}\psi_{y_k}^2\psi_{y_s}^2 - 4c^4\psi^{-2\nu-2}\sum_{k=1}^{m}\psi_{y_k}^2\Big) \\
&+\lambda^2\nu^2\Big(-4(\nu+1)\delta^3(x_1+\delta_0)^{-3}\psi^{-\nu-2} - 2\delta^4(\nu+1)(\nu+2)(x_1+\delta_0)^{-2}\psi^{-\nu-3} \\
&-2\delta^2(\nu+1)(\nu+2)\psi^{-\nu-3}\sum_{i=2}^{n}\psi_{x_i}^2 + 2\delta^2(\nu+1)\psi^{-\nu-2} + 2\delta m\frac{\partial c^2}{\partial x_i}\psi^{-\nu-1} \\
&+2\delta^2(\nu+1)(\nu+2)c^2\psi^{-\nu-3}\sum_{k=1}^{m}\psi_{y_k}^2 - 2\delta^2(\nu+1)mc^2\psi^{-\nu-2} \\
&-2\delta(\nu+1)\psi^{-\nu-2}\sum_{k=1}^{m}\frac{\partial c^2}{\partial x_1}\psi_{y_k}^2 - 2\delta^2(\nu+1)(\nu+2)\psi^{-\nu-3}\sum_{i=2}^{n}\psi_{x_i}^2 \\
&+2\delta^2(\nu+1)(n-1)\psi^{-\nu-2} - 2\psi^{-\nu-1}(n-1)^2(x_1+\delta_0)^2 \\
&+2(\nu+1)(n-1)(x_1+\delta_0)^2\psi^{-\nu-2}\sum_{i=2}^{n}\psi_{x_i}^2 \\
&+2(\nu+1)(n-1)(x_1+\delta_0)^2\psi^{-\nu-2}\sum_{j=2}^{n}\psi_{x_j}^2 \\
&-2(\nu+1)(\nu+2)(n-1)(x_1+\delta_0)^2\psi^{-\nu-3}\sum_{i,j=2}^{n}\psi_{x_i}^2\psi_{x_j}^2
\end{aligned}
$$

$$+4\left(\nu+1\right)\left(x_1+\delta_0\right)^2\psi^{-\nu-2}\sum_{i=2}^{n}\psi_{x_i}^2+2m\psi^{-\nu-1}\left(x_1+\delta_0\right)^2\sum_{i=2}^{n}\frac{\partial c^2}{\partial x_i}\psi_{x_i}$$

$$+2c^2m(n-1)\left(x_1+\delta_0\right)^2\psi^{-\nu-1}-2\left(\nu+1\right)mc^2\psi^{-\nu-2}\left(x_1+\delta_0\right)^2\sum_{i=2}^{n}\psi_{x_i}^2$$

$$-2\left(\nu+1\right)\left(x_1+\delta_0\right)^2\psi^{-\nu-2}\sum_{i=2}^{n}\sum_{k=1}^{m}\frac{\partial c^2}{\partial x_i}\psi_{x_i}\psi_{y_k}^2$$

$$-2\left(\nu+1\right)\left(n-1\right)\left(x_1+\delta_0\right)^2c^2\psi^{-\nu-2}\sum_{k=1}^{m}\psi_{y_k}^2$$

$$+2\left(\nu+1\right)\left(\nu+2\right)\left(x_1+\delta_0\right)^2c^2\psi^{-\nu-3}\sum_{i=2}^{n}\sum_{k=1}^{m}\psi_{x_i}^2\psi_{y_k}^2$$

$$-2\left(\nu+1\right)\psi^{-\nu-2}\sum_{k=1}^{m}\frac{\partial c^2}{\partial y_k}\psi_{y_k}-2\left(\nu+1\right)mc^2\psi^{-\nu-2}$$

$$+2c^2\left(\nu+1\right)\left(\nu+2\right)\psi^{-\nu-3}\sum_{k=1}^{m}\psi_{y_k}^2+2\left(n-1\right)\left(x_1+\delta_0\right)^2\psi^{-\nu-1}\sum_{k=1}^{m}\frac{\partial c^2}{\partial y_k}\psi_{y_k}$$

$$+2c^2\left(n-1\right)\left(x_1+\delta_0\right)^2m\psi^{-\nu-1}-2c^2\left(\nu+1\right)\left(n-1\right)\left(x_1+\delta_0\right)^2\psi^{-\nu-2}\sum_{k=1}^{m}\psi_{y_k}^2$$

$$-2\left(\nu+1\right)\left(x_1+\delta_0\right)^2\psi^{-\nu-2}\sum_{i=2}^{n}\sum_{k=1}^{m}\frac{\partial c^2}{\partial y_k}\psi_{x_i}^2\psi_{y_k}$$

$$-2\left(\nu+1\right)\left(x_1+\delta_0\right)^2c^2m\psi^{-\nu-2}\sum_{i=2}^{n}\psi_{x_i}^2$$

$$+2c^2\left(\nu+1\right)\left(\nu+2\right)\left(x_1+\delta_0\right)^2\psi^{-\nu-3}\sum_{i=2}^{n}\sum_{k=1}^{m}\psi_{x_i}^2\psi_{y_k}^2$$

$$-2m\psi^{-\nu-1}\sum_{k=1}^{m}\frac{\partial c^4}{\partial y_k}\psi_{y_k}-2c^4m^2\psi^{-\nu-1}+2c^4(\nu+1)m\psi^{-\nu-2}\sum_{k=1}^{m}\psi_{y_k}^2$$

$$+2\left(\nu+1\right)\psi^{-\nu-2}\sum_{k,s=1}^{m}\frac{\partial c^4}{\partial y_k}\psi_{y_k}\psi_{y_s}^2+2\left(\nu+1\right)c^4m\psi^{-\nu-2}\sum_{s=1}^{m}\psi_{y_s}^2$$

$$-2\left(\nu+1\right)\left(\nu+2\right)c^4\psi^{-\nu-3}\sum_{k,s=1}^{m}\psi_{y_k}^2\psi_{y_s}^2$$

$$\left.+4\left(\nu+1\right)c^4\psi^{-\nu-2}\sum_{k=1}^{m}\psi_{y_k}^2\right)$$

Daha açık olarak yukarıda verilen $K\left(\lambda,\nu,\delta\right)$ fonksiyonu

$$K\left(\lambda,\nu,\delta\right)=\lambda^3\nu^3E_1+\lambda^2\nu^2E_2 \tag{2.53}$$

formunda yazılabilir.

Burada

$$
\begin{aligned}
E_1 \;=\;& 4\delta^3 (x_1+\delta_0)^{-3} \psi^{-2\nu-2} + 4\delta^4 (\nu+1)(x_1+\delta_0)^{-2} \psi^{-2\nu-3} \\
&+4\delta^2(\nu+1)\psi^{-2\nu-3}\sum_{i=2}^{n}\psi_{x_i}^2 - 4\delta^2(\nu+1)c^2\psi^{-2\nu-3}\sum_{k=1}^{m}\psi_{y_k}^2 \\
&+2\delta\psi^{-2\nu-2}\sum_{k=1}^{m}\frac{\partial c^2}{\partial x_1}\psi_{y_k}^2 + 4(\nu+1)\psi^{-2\nu-3}\delta^2\sum_{i=2}^{n}\psi_{x_i}^2 - 2\delta^2(n-1)\psi^{-2\nu-2} \\
&+4(\nu+1)(x_1+\delta_0)^2\psi^{-2\nu-3}\sum_{i,j=2}^{n}\psi_{x_i}^2\psi_{x_j}^2 - 4(x_1+\delta_0)^2\psi^{-2\nu-2}\sum_{i=2}^{n}\psi_{x_i}^2 \\
&-2(n-1)(x_1+\delta_0)^2\psi^{-2\nu-2}\sum_{j=2}^{n}\psi_{x_j}^2 + 2(x_1+\delta_0)^2\psi^{-2\nu-2}\sum_{i=2}^{n}\sum_{k=1}^{m}\frac{\partial c^2}{\partial x_i}\psi_{x_i}\psi_{y_k}^2 \\
&+2(n-1)(x_1+\delta_0)^2c^2\psi^{-2\nu-2}\sum_{k=1}^{m}\psi_{y_k}^2 \\
&-4(\nu+1)(x_1+\delta_0)^2c^2\psi^{-2\nu-3}\sum_{i=2}^{n}\sum_{k=1}^{m}\psi_{x_i}^2\psi_{y_k}^2 + 2\psi^{-2\nu-2}\sum_{k=1}^{m}\frac{\partial c^2}{\partial y_k}\psi_{y_k} \\
&+2mc^2\psi^{-2\nu-2} - 4(\nu+1)c^2\psi^{-2\nu-3}\sum_{k=1}^{m}\psi_{y_k}^2 + 2\psi^{-2\nu-2}\sum_{i=2}^{n}\sum_{k=1}^{m}\frac{\partial c^2}{\partial y_k}\psi_{x_i}^2\psi_{y_k} \\
&+2c^2m\psi^{-2\nu-2}\sum_{i=2}^{n}\psi_{x_i}^2 - 4(\nu+1)c^2\psi^{-2\nu-3}\sum_{i=2}^{n}\sum_{k=1}^{m}\psi_{y_k}^2\psi_{x_i}^2 \\
&-2\psi^{-2\nu-2}\sum_{k,s=1}^{m}\frac{\partial c^4}{\partial y_k}\psi_{y_k}\psi_{y_s}^2 - 2c^4m\psi^{-2\nu-2}\sum_{s=1}^{m}\psi_{y_s}^2 \\
&-4(\nu+1)c^4\psi^{-2\nu-3}\sum_{k,s=1}^{m}\psi_{y_k}^2\psi_{y_s}^2 - 4c^4\psi^{-2\nu-2}\sum_{k=1}^{m}\psi_{y_k}^2, \qquad (2.54)
\end{aligned}
$$

$$
\begin{aligned}
E_2 \;=\;& -4(\nu+1)\delta^3(x_1+\delta_0)^{-3}\psi^{-\nu-2} - 2\delta^4(\nu+1)(\nu+2)(x_1+\delta_0)^{-2}\psi^{-\nu-3} \\
&-2\delta^2(\nu+1)(\nu+2)\psi^{-\nu-3}\sum_{i=2}^{n}\psi_{x_i}^2 + 2\delta^2(\nu+1)\psi^{-\nu-2} \\
&+2\delta^2(\nu+1)(\nu+2)c^2\psi^{-\nu-3}\sum_{k=1}^{m}\psi_{y_k}^2 - 2\delta^2(\nu+1)mc^2\psi^{-\nu-2} \\
&-2\delta^2(\nu+1)(\nu+2)\psi^{-\nu-3}\sum_{i=2}^{n}\psi_{x_i}^2 + 2\delta^2(\nu+1)(n-1)\psi^{-\nu-2} \\
&-2\delta(\nu+1)\psi^{-\nu-2}\sum_{k=1}^{m}\frac{\partial c^2}{\partial x_1}\psi_{y_k}^2 - 2\psi^{-\nu-1}(n-1)^2(x_1+\delta_0)^2 \\
&+2\delta m\frac{\partial c^2}{\partial x_i}\psi^{-\nu-1} + 2(\nu+1)(n-1)(x_1+\delta_0)^2\psi^{-\nu-2}\sum_{i=2}^{n}\psi_{x_i}^2 \\
&+2(\nu+1)(n-1)(x_1+\delta_0)^2\psi^{-\nu-2}\sum_{j=2}^{n}\psi_{x_j}^2
\end{aligned}
$$

$$
\begin{aligned}
&-2\left(\nu+1\right)\left(\nu+2\right)\left(n-1\right)\left(x_1+\delta_0\right)^2\psi^{-\nu-3}\sum_{i,j=2}^{n}\psi_{x_i}^2\psi_{x_j}^2 \\
&+4\left(\nu+1\right)\left(x_1+\delta_0\right)^2\psi^{-\nu-2}\sum_{i=2}^{n}\psi_{x_i}^2+2m\psi^{-\nu-1}\left(x_1+\delta_0\right)^2\sum_{i=2}^{n}\frac{\partial c^2}{\partial x_i}\psi_{x_i} \\
&+2c^2m(n-1)\left(x_1+\delta_0\right)^2\psi^{-\nu-1}-2\left(\nu+1\right)mc^2\psi^{-\nu-2}\left(x_1+\delta_0\right)^2\sum_{i=2}^{n}\psi_{x_i}^2 \\
&-2\left(\nu+1\right)\left(x_1+\delta_0\right)^2\psi^{-\nu-2}\sum_{i=2}^{n}\sum_{k=1}^{m}\frac{\partial c^2}{\partial x_i}\psi_{x_i}\psi_{y_k}^2 \\
&-2\left(\nu+1\right)\left(n-1\right)\left(x_1+\delta_0\right)^2c^2\psi^{-\nu-2}\sum_{k=1}^{m}\psi_{y_k}^2 \\
&+2\left(\nu+1\right)\left(\nu+2\right)\left(x_1+\delta_0\right)^2c^2\psi^{-\nu-3}\sum_{i=2}^{n}\sum_{k=1}^{m}\psi_{x_i}^2\psi_{y_k}^2 \\
&-2\left(\nu+1\right)\psi^{-\nu-2}\sum_{k=1}^{m}\frac{\partial c^2}{\partial y_k}\psi_{y_k}-2\left(\nu+1\right)mc^2\psi^{-\nu-2} \\
&+2c^2\left(\nu+1\right)\left(\nu+2\right)\psi^{-\nu-3}\sum_{k=1}^{m}\psi_{y_k}^2+2\left(n-1\right)\left(x_1+\delta_0\right)^2\psi^{-\nu-1}\sum_{k=1}^{m}\frac{\partial c^2}{\partial y_k}\psi_{y_k} \\
&+2c^2\left(n-1\right)m\left(x_1+\delta_0\right)^2\psi^{-\nu-1}-2c^2\left(\nu+1\right)\left(n-1\right)\left(x_1+\delta_0\right)^2\psi^{-\nu-2}\sum_{k=1}^{m}\psi_{y_k}^2 \\
&-2\left(\nu+1\right)\left(x_1+\delta_0\right)^2\psi^{-\nu-2}\sum_{i=2}^{n}\sum_{k=1}^{m}\frac{\partial c^2}{\partial y_k}\psi_{x_i}^2\psi_{y_k} \\
&-2\left(x_1+\delta_0\right)^2\left(\nu+1\right)c^2m\psi^{-\nu-2}\sum_{i=2}^{n}\psi_{x_i}^2 \\
&+2c^2\left(\nu+1\right)\left(\nu+2\right)\left(x_1+\delta_0\right)^2\psi^{-\nu-3}\sum_{i=2}^{n}\sum_{k=1}^{m}\psi_{x_i}^2\psi_{y_k}^2-2m\psi^{-\nu-1}\sum_{k=1}^{m}\frac{\partial c^4}{\partial y_k}\psi_{y_k} \\
&-2c^4m^2\psi^{-\nu-1}+2c^4(\nu+1)m\psi^{-\nu-2}\sum_{k=1}^{m}\psi_{y_k}^2+2\left(\nu+1\right)\psi^{-\nu-2}\sum_{k,s=1}^{m}\frac{\partial c^4}{\partial y_k}\psi_{y_k}\psi_{y_s}^2 \\
&+2\left(\nu+1\right)c^4m\psi^{-\nu-2}\sum_{s=1}^{m}\psi_{y_s}^2-2\left(\nu+1\right)\left(\nu+2\right)c^4\psi^{-\nu-3}\sum_{k,s=1}^{m}\psi_{y_k}^2\psi_{y_s}^2 \\
&+4\left(\nu+1\right)c^4\psi^{-\nu-2}\sum_{k=1}^{m}\psi_{y_k}^2 \qquad (2.55)
\end{aligned}
$$

olarak verilmiştir.

Şimdi E_1 ve E_2 ifadelerini değerlendirelim:

İlk olarak

$$\begin{aligned} E_1 &= 4\delta^4(\nu+1)(x_1+\delta_0)^{-2}\psi^{-2\nu-3} + 4\delta^3(x_1+\delta_0)^{-3}\psi^{-2\nu-2} \\ &+4(\nu+1)(x_1+\delta_0)^2\psi^{-2\nu-3}\sum_{i,j=2}^{n}\psi_{x_i}^2\psi_{x_j}^2 + 4\delta^2(\nu+1)\psi^{-2\nu-3}\sum_{i=2}^{n}\psi_{x_i}^2 \\ &+2(n-1)(x_1+\delta_0)^2c^2\psi^{-2\nu-2}\sum_{k=1}^{m}\psi_{y_k}^2 + 2mc^2\psi^{-2\nu-2} \\ &+2c^2m\psi^{-2\nu-2}\sum_{i=2}^{n}\psi_{x_i}^2 - E_{11} \end{aligned} \tag{2.56}$$

olmak üzere

$$E_1 > 4\delta^4(\nu+1)(x_1+\delta_0)^{-2}\psi^{-2\nu-3} - E_{11} \tag{2.57}$$

yazılabilir.

$$\begin{aligned} E_{11} &= 4\delta^2(\nu+1)c^2\psi^{-2\nu-3}\sum_{i=2}^{n}\psi_{y_k}^2 - 2\delta\psi^{-2\nu-2}\sum_{i=2}^{n}\frac{\partial c^2}{\partial x_1}\psi_{y_k}^2 \\ &+2\delta^2(n-1)\psi^{-2\nu-2} + 4(x_1+\delta_0)^2\psi^{-2\nu-2}\sum_{i=2}^{n}\psi_{x_i}^2 \\ &+2(n-1)(x_1+\delta_0)^2\psi^{-2\nu-2}\sum_{j=2}^{n}\psi_{x_j}^2 - 2(x_1+\delta_0)^2\psi^{-2\nu-2}\sum_{i=2}^{n}\sum_{k=1}^{m}\frac{\partial c^2}{\partial x_i}\psi_{x_i}\psi_{y_k}^2 \\ &+4(\nu+1)(x_1+\delta_0)^2c^2\psi^{-2\nu-3}\vartheta^2\sum_{i=2}^{n}\sum_{k=1}^{m}\psi_{x_i}^2\psi_{y_k}^2 - 2\psi^{-2\nu-2}\sum_{k=1}^{m}\frac{\partial c^2}{\partial y_k}\psi_{y_k} \\ &+4(\nu+1)c^2\psi^{-2\nu-3}\sum_{k=1}^{m}\psi_{y_k}^2 - 2\psi^{-2\nu-2}\sum_{i=2}^{n}\sum_{k=1}^{m}\frac{\partial c^2}{\partial y_k}\psi_{x_i}^2\psi_{y_k} \\ &+4(\nu+1)c^2\psi^{-2\nu-3}\sum_{i=2}^{n}\sum_{k=1}^{m}\psi_{y_k}^2\psi_{x_i}^2 + 2\psi^{-2\nu-2}\sum_{k,s=1}^{m}\frac{\partial c^4}{\partial y_k}\psi_{y_k}\psi_{y_s}^2 \\ &+2c^4m\psi^{-2\nu-2}\sum_{s=1}^{m}\psi_{y_s}^2 + 4(\nu+1)c^4\psi^{-2\nu-3}\sum_{k,s=1}^{m}\psi_{y_k}^2\psi_{y_s}^2 + 4c^4\psi^{-2\nu-2}\sum_{k=1}^{m}\psi_{y_k}^2 \end{aligned} \tag{2.58}$$

olup

$$\begin{aligned} E_1 &> 4\delta^4(\nu+1)(x_1+\delta_0)^{-2}\psi^{-2\nu-3} - E_{11} \\ &= 4\delta^4(\nu+1)(x_1+\delta_0)^{-2}\psi^{-2\nu-3}\left(1-\frac{1}{\delta^2}E_{12}\right) \end{aligned} \tag{2.59}$$

bulunur. Bu eşitsizlikte

$$E_{12} = \frac{1}{4\delta^2(\nu+1)(x_1+\delta_0)^{-2}\psi^{-2\nu-3}}E_{11} \tag{2.60}$$

şeklinde tanımlıdır.

Diğer yandan c, ψ, ψ_{x_i}, ψ_{y_k} fonksiyonları $C^1\left(\overline{\Omega}\right)$ uzayında sınırlı olduğundan $x \in \overline{\Omega_\gamma}$ için E_{12} de sınırlıdır. Yani bir $M_1 > 0$ sayısı vardır öyle ki $|E_{12}| \leq M_1$. Eğer

$\delta \geq \delta_2 = \sqrt{2M_1},\ M_1 = \dfrac{\delta_2^2}{2}$

ise

$\left(1 - \dfrac{1}{\delta^2} E_{12}\right) \geq \dfrac{1}{2}$

olacağından

$$\begin{aligned} E_1 &> 4\delta^4 \left(x_1+\delta_0\right)^{-2}(\nu+1)\,\psi^{-2\nu-3} - E_{11} \\ &= 4\delta^4 \left(x_1+\delta_0\right)^{-2}(\nu+1)\,\psi^{-2\nu-3}\left(1-\frac{1}{\delta^2}E_{12}\right) \\ &\geq 2\delta^4 \left(x_1+\delta_0\right)^{-2}(\nu+1)\,\psi^{-2\nu-3} \qquad (2.61) \end{aligned}$$

elde edilir.

İkinci olarak sabitlenmiş $\delta \geq \delta_2$, ν, γ sayıları için E_2 ifadesinin de $\overline{\Omega_\gamma}$ da sınırlı olduğunu söyleyebiliriz. $\overline{\Omega_\gamma} : |E_2| \leq M_2,\ M_2 > 0$. Buna bağlı olarak
eğer $\lambda \geq \lambda_0 = M_2$ ise $\nu > 1$ için

$2\lambda^3\nu^3\delta^4 \left(x_1+\delta_0\right)^{-2}(\nu+1)\,\psi^{-2\nu-3} - \lambda^2\nu^2 M_2 \geq 0$

yazılabilir. Sonuç olarak

$$\begin{aligned} K(\lambda,\nu,\delta) &= \lambda^3\nu^3E_1 + \lambda^2\nu^2E_2 \\ &\geq 2\lambda^3\nu^3\delta^4\left(x_1+\delta_0\right)^{-2}(\nu+1)\,\psi^{-2\nu-3} + \lambda^2\nu^2E_2 \\ &= 2\lambda^3\nu^4\delta^4\left(x_1+\delta_0\right)^{-2}\psi^{-2\nu-3} + 2\lambda^3\nu^3\delta^4\left(x_1+\delta_0\right)^{-2}\psi^{-2\nu-3} + \lambda^2\nu^2E_2 \\ &\geq 2\lambda^3\nu^4\delta^4\left(x_1+\delta_0\right)^{-2}\psi^{-2\nu-3} + \left(2\lambda^3\nu^3\left(x_1+\delta_0\right)^{-2}\psi^{-2\nu-3}\delta^4 - \lambda^2\nu^2M_2\right) \\ &\geq 2\lambda^3\nu^4\delta^4\left(x_1+\delta_0\right)^{-2}\psi^{-2\nu-3} \qquad (2.62) \end{aligned}$$

olduğu görülür.

(2.44) eşitsizliği (2.49), (2.50), (2.52), (2.62) bağıntıları dikkate alınarak $\delta \geq \delta_3$,
$\lambda \geq \lambda_0$, $\nu > 1$ için

$$\begin{aligned} &\psi^{\nu+1}\left(\left(x_1+\delta_0\right)^{-1}\varphi_{x_1x_1} + \left(x_1+\delta_0\right)\left(\sum_{i=2}^{n}\varphi_{x_ix_i} - c^2\left(x,y'\right)\sum_{k=1}^{m}\varphi_{y_ky_k}\right)\right)^2\chi^2 \\ \geq\ & 2\lambda\nu\delta\left(x_1+\delta_0\right)^{-3}\vartheta_{x_1}^2 - 2\lambda\nu G_o(n,m)\sum_{i=2}^{n}\vartheta_{x_i}^2 + \lambda\nu\delta\alpha_1\left(x_1+\delta_0\right)\sum_{k=1}^{m}\vartheta_{y_k}^2 \\ & +2\lambda^3\nu^4\delta^4\left(x_1+\delta_0\right)^{-2}\psi^{-2\nu-3}\vartheta^2 + d_0\left(\vartheta\right) \qquad (2.63) \end{aligned}$$

elde edilir.

Lemma 2.1 in ispatını tamamlamak için $\varphi(x)$ fonksiyonuna geri dönmeliyiz. Bunun için

$$\begin{aligned}
\vartheta &= \chi\varphi, \\
\vartheta_{x_1} &= \left(\varphi_{x_1} - \lambda\nu\psi^{-\nu-1}\psi_{x_1}\varphi\right)\chi, \\
\vartheta_{x_1}^2 &= \varphi_{x_1}^2\chi^2 - \lambda^2\nu^2\psi_{x_1}^2\psi^{-2\nu-2}\varphi^2\chi^2 - \lambda\nu(\nu+1)\psi_{x_1}^2\psi^{-\nu-2}\varphi^2\chi^2 \\
&\quad -\lambda\nu\left(\psi_{x_1}\psi^{-\nu-1}\varphi^2\chi^2\right)_{x_1}, \\
\vartheta_{x_i} &= \left(\varphi_{x_i} - \lambda\nu\psi^{-\nu-1}\psi_{x_i}\varphi\right)\chi, \\
\vartheta_{x_i}^2 &= \varphi_{x_i}^2\chi^2 - \lambda^2\nu^2\psi_{x_i}^2\psi^{-2\nu-2}\varphi^2\chi^2 - \lambda\nu(\nu+1)\psi_{x_i}^2\psi^{-\nu-2}\varphi^2\chi^2 \\
&\quad +\lambda\nu\psi^{-\nu-1}\psi_{x_ix_i}\varphi^2\chi^2 - \lambda\nu\left(\psi_{x_i}\psi^{-\nu-1}\varphi^2\chi^2\right)_{x_i}, \\
\vartheta_{y_k} &= \left(\varphi_{y_k} - \lambda\nu\psi^{-\nu-1}\psi_{y_k}\varphi\right)\chi, \\
\vartheta_{y_k}^2 &= \varphi_{y_k}^2\chi^2 - \lambda^2\nu^2\psi_{y_k}^2\psi^{-2\nu-2}\varphi^2\chi^2 - \lambda\nu(\nu+1)\psi_{y_k}^2\psi^{-\nu-2}\varphi^2\chi^2 \\
&\quad +\lambda\nu\psi^{-\nu-1}\psi_{y_ky_k}\varphi^2\chi^2 - \lambda\nu\left(\psi_{y_k}\psi^{-\nu-1}\varphi^2\chi^2\right)_{y_k}
\end{aligned}$$

eşitlikleri (2.63) eşitsizliğinde yerine yazılırsa

$$\begin{aligned}
&\psi^{\nu+1}\left((x_1+\delta_0)^{-1}\varphi_{x_1x_1} + (x_1+\delta_0)\left(\sum_{i=2}^{n}\varphi_{x_ix_i} - c^2(x,y')\sum_{k=1}^{m}\varphi_{y_ky_k}\right)\right)^2\chi^2 \\
\geq\ & 2\lambda\nu\delta(x_1+\delta_0)^{-3}\vartheta_{x_1}^2 - 2\lambda\nu(x_1+\delta_0)^2G_o(n,m)\sum_{i=2}^{n}\vartheta_{x_i}^2 + \lambda\nu\delta\alpha_1(x_1+\delta_0)\sum_{k=1}^{m}\vartheta_{y_k}^2 \\
&+2\lambda^3\nu^4\delta^4(x_1+\delta_0)^{-2}\psi^{-2\nu-3}\vartheta^2 + d_0(\vartheta) \\
=\ & 2\lambda\nu\delta(x_1+\delta_0)^{-3}\left(\varphi_{x_1}^2\chi^2 - \lambda^2\nu^2\psi_{x_1}^2\psi^{-2\nu-2}\varphi^2\chi^2 - \lambda\nu(\nu+1)\psi_{x_1}^2\psi^{-\nu-2}\varphi^2\chi^2\right. \\
&\left.-\lambda\nu\left(\psi_{x_1}\psi^{-\nu-1}\varphi^2\chi^2\right)_{x_1}\right) - 2\lambda\nu(x_1+\delta_0)^2G_o(n,m)\sum_{i=2}^{n}\left(\varphi_{x_i}^2\chi^2 - \lambda^2\nu^2\psi_{x_i}^2\psi^{-2\nu-2}\varphi^2\chi^2\right. \\
&\left.-\lambda\nu(\nu+1)\psi_{x_i}^2\psi^{-\nu-2}\varphi^2\chi^2 + \lambda\nu\psi^{-\nu-1}\psi_{x_ix_i}\varphi^2\chi^2 - \lambda\nu\left(\psi_{x_i}\psi^{-\nu-1}\varphi^2\chi^2\right)_{x_i}\right) \\
&+\lambda\nu\delta\alpha_1(x_1+\delta_0)\sum_{k=1}^{m}\left(\varphi_{y_k}^2\chi^2 - \lambda^2\nu^2\psi_{y_k}^2\psi^{-2\nu-2}\varphi^2\chi^2 - \lambda\nu(\nu+1)\psi_{y_k}^2\psi^{-\nu-2}\varphi^2\chi^2\right. \\
&\left.+\lambda\nu\psi^{-\nu-1}\psi_{y_ky_k}\varphi^2\chi^2 - \lambda\nu\left(\psi_{y_k}\psi^{-\nu-1}\varphi^2\chi^2\right)_{y_k}\right) \\
&+2\lambda^3\nu^4\delta^4(x_1+\delta_0)^{-2}\psi^{-2\nu-3}\varphi^2\chi^2 + d_0(\varphi\chi) \\
=\ & 2\lambda\nu\delta(x_1+\delta_0)^{-3}\varphi_{x_1}^2\chi^2 - 2\lambda\nu(x_1+\delta_0)^2G_o(n,m)\sum_{i=1}^{n}\varphi_{x_i}^2\chi^2 \\
&+\lambda\nu\delta\alpha_1(x_1+\delta_0)\sum_{k=1}^{m}\varphi_{y_k}^2\chi^2 + 2\lambda^3\nu^4\delta^4(x_1+\delta_0)^{-2}\psi^{-2\nu-3}\varphi^2\chi^2 \\
&+G(\lambda,\delta)\varphi^2\chi^2 + d_0(\varphi\chi) + d_{01}(\varphi\chi) \qquad (2.64)
\end{aligned}$$

elde edilir.

Burada

$$
\begin{aligned}
& d_0\left(\chi\varphi\right) \\
= \ & 2\lambda\nu\delta\left(\left(x_1+\delta_0\right)^{-2}\vartheta_{x_1}^2\right)_{x_1}+4\lambda\nu\delta\sum_{i=2}^{n}\left(\vartheta_{x_1}\vartheta_{x_i}\right)_{x_i}-2\lambda\nu\delta\sum_{i=2}^{n}\left(\vartheta_{x_i}^2\right)_{x_1} \\
& +4\lambda\nu\delta\sum_{k=1}^{m}\left(c^2\vartheta_{x_1}\vartheta_{y_k}\right)_{y_k}+2\lambda\nu\delta\sum_{k=1}^{m}\left(c^2\vartheta_{y_k}^2\right)_{x_1} \\
& +2\lambda\nu\delta^3\left(\left(x_1+\delta_0\right)^{-2}\left(\lambda^2\nu^2\psi^{-2\nu-2}-\lambda\nu\left(\nu+1\right)\psi^{-\nu-2}\right)\vartheta^2\right)_{x_1} \\
& +2\lambda\nu\delta\sum_{i=2}^{n}\left(\left(\left(\lambda^2\nu^2\psi^{-2\nu-2}\psi_{x_i}^2-\lambda\nu\left(\nu+1\right)\psi^{-\nu-2}\psi_{x_i}^2\right)+\lambda\nu\psi^{-\nu-1}\right)\vartheta^2\right)_{x_1} \\
& -2\lambda\nu\delta\sum_{k=1}^{m}\left[\left(\lambda^2\nu^2c^2\psi^{-2\nu-2}\psi_{y_k}^2-\lambda\nu\left(\nu+1\right)c^2\psi^{-\nu-2}\psi_{y_k}^2+\lambda\nu\psi^{-\nu-1}mc^2\right)\vartheta^2\right]_{x_1} \\
& +4\lambda\nu\sum_{i=2}^{n}\left(\psi_{x_i}\vartheta_{x_i}\vartheta_{x_1}\right)_{x_1}-2\lambda\nu\sum_{i=2}^{n}\left(\psi_{x_i}\vartheta_{x_1}^2\right)_{x_i} \\
& +4\lambda\nu\left(x_1+\delta_0\right)^2\sum_{i,j=2}^{n}\left(\psi_{x_i}\vartheta_{x_i}\vartheta_{x_j}\right)_{x_j}-2\lambda\nu\left(x_1+\delta_0\right)^2\sum_{i,j=2}^{n}\left(\psi_{x_i}\vartheta_{x_j}^2\right)_{x_i} \\
& -4\lambda\nu\left(x_1+\delta_0\right)^2\sum_{i=2}^{n}\sum_{k=1}^{m}\left(c^2\psi_{x_i}\vartheta_{x_i}\vartheta_{y_k}\right)_{y_k}+2\lambda\nu\left(x_1+\delta_0\right)^2\sum_{i=2}^{n}\sum_{k=1}^{m}\left(c^2\psi_{x_i}\vartheta_{y_k}^2\right)_{x_i} \\
& +2\lambda^2\nu^2\delta^2\sum_{i=2}^{n}\left(\left(\lambda\nu\psi^{-2\nu-2}\psi_{x_i}-\left(\nu+1\right)\psi^{-\nu-2}\psi_{x_i}\right)\vartheta^2\right)_{x_i} \\
& +2\lambda^2\nu^2\left(x_1+\delta_0\right)^2\sum_{i,,j=2}^{n}\left[\psi_{x_i}\psi^{-\nu-1}\left(\left(n-1\right)\psi_{x_i}-\left(\nu+1\right)\psi_{x_j}^2\psi^{-1}+\lambda\nu\psi_{x_j}^2\psi^{-v-}\right)\vartheta^2\right]_{x_i} \\
& -2\lambda^2\nu^2\left(x_1+\delta_0\right)^2\sum_{i=2}^{n}\sum_{k=1}^{m}\left[c^2\psi_{x_i}\psi^{-\nu-1}\left(m-\left(\nu+1\right)\psi_{y_k}^2\psi^{-1}+\lambda\nu\psi_{y_k}^2\psi^{-\nu-1}\vartheta^2\right)\right]_{x_i} \\
& -4\lambda\nu\sum_{k=1}^{m}\left(c^2\psi_{y_k}\vartheta_{y_k}\vartheta_{x_1}\right)_{x_1}+2\lambda\nu\sum_{k=1}^{m}\left(c^2\psi_{y_k}\vartheta_{x_1}^2\right)_{y_k} \\
& -4\lambda\nu\left(x_1+\delta_0\right)^2\sum_{i=2}^{n}\sum_{k=1}^{m}\left(c^2\psi_{y_k}\vartheta_{y_k}\vartheta_{x_i}\right)_{xi}+2\lambda\nu\left(x_1+\delta_0\right)^2\sum_{i=2}^{n}\sum_{k=1}^{m}\left(c^2\psi_{y_k}\vartheta_{x_i}^2\right)_{y_k} \\
& +4\lambda\nu\left(x_1+\delta_0\right)^2\sum_{k,s=1}^{m}\left(c^4\psi_{y_k}\vartheta_{y_k}\vartheta_{y_s}\right)_{y_s}-2\lambda\nu\left(x_1+\delta_0\right)^2\sum_{k,s=1}^{m}\left(c^4\psi_{y_k}\vartheta_{y_s}^2\right)_{y_k} \\
& -2\lambda^2\nu^2\sum_{i=1}^{n}\sum_{k=1}^{m}\left[c^2\psi_{y_k}\left(\lambda\nu\psi^{-2\nu-2}-\left(\nu+1\right)\psi^{-\nu-2}\right)\vartheta^2\right]_{y_k} \\
& -2\lambda^2\nu^2\left(x_1+\delta_0\right)^2\sum_{i=2}^{n}\sum_{k=1}^{m}\left[c^2\psi_{y_k}\psi^{-\nu-1}((n-1)-\left(\nu+1\right)\psi_{x_i}^2\psi^{-1}+\lambda\nu\psi_{x_i}^2\psi^{-\nu-1})\vartheta^2\right]_{y_k} \\
& +2\lambda^2\nu^2\left(x_1+\delta_0\right)^2\sum_{k,s=1}^{m}\left[c^4\psi^{-\nu-1}\psi_{y_k}\left(m-\left(\nu+1\right)\psi_{y_s}^2\psi^{-1}+\lambda\nu\psi_{y_s}^2\psi^{-\nu-1}\right)\vartheta^2\right]_{y_k} \quad (2.65)
\end{aligned}
$$

$$
\begin{aligned}
G(\lambda,\nu) &= -2\lambda\nu\delta\left(x_1+\delta_0\right)^{-3}\left(\lambda^2\nu^2\psi_{x_1}^2\psi^{-2\nu-2}+\lambda\nu\left(\nu+1\right)\psi_{x_1}^2\psi^{-\nu-2}\right) \\
&\quad +2\lambda\nu\left(x_1+\delta_0\right)^2 G_o(n,m)\sum_{i=2}^{n}\left(\left(\lambda^2\nu^2\psi_{x_i}^2\psi^{-2\nu-2}+\lambda\nu\left(\nu+1\right)\psi_{x_i}^2\psi^{-\nu-2}\right)\right. \\
&\quad \left.-\lambda\nu\left(n-1\right)\psi^{-\nu-1}\right)+\lambda\nu\delta\alpha_1\left(x_1+\delta_0\right)\sum_{k=1}^{m}\left(\left(-\lambda^2\nu^2\psi_{y_k}^2\psi^{-2\nu-2}\right.\right. \\
&\quad \left.\left.-\lambda\nu\left(\nu+1\right)\psi_{y_k}^2\psi^{-\nu-2}\right)+\lambda\nu m\psi^{-\nu-1}\right), \qquad (2.66)
\end{aligned}
$$

$$
\begin{aligned}
d_1(\chi\varphi) &= -2\lambda^2\nu^2\delta\left(x_1+\delta_0\right)^{-3}\left(\left(\psi_{x_1}\psi^{-\nu-1}\varphi^2\chi^2\right)_{x_1}\right) \qquad (2.67) \\
&\quad +2\lambda^2\nu^2 G_o(n,m)\sum_{i=2}^{n}\left(\psi_{x_i}\psi^{-\nu-1}\varphi^2\chi^2\right)_{x_i} \\
&\quad -\lambda^2\nu^2\delta\alpha_1\left(x_1+\delta_0\right)\sum_{k=1}^{m}\left(\psi_{y_k}\psi^{-\nu-1}\varphi^2\chi^2\right)_{y_k}
\end{aligned}
$$

şeklinde tanımlıdır. ■

Şimdi Problem 3.1 in çözümünün tekliğini ispatlamak için gerekli olan ikinci bir lemma tanımlayalım.

Lemma 2.2 *Aşağıdaki eşitlik her $\varphi \in C^2(\Omega)$ fonksiyonu için*

$$
\begin{aligned}
&-\varphi\left(\varphi_{x_1x_1}+\left(x_1+\delta_0\right)^2\left(\sum_{i=2}^{n}\varphi_{x_ix_i}-c^2\left(x,y'\right)\sum_{k=1}^{m}\varphi_{y_ky_k}\right)\right)\chi^2 \qquad (2.68) \\
&= \varphi_{x_1}^2\chi^2+\left(x_1+\delta_0\right)^2\left(\sum_{i=2}^{n}\varphi_{x_i}^2\chi^2-c^2\sum_{k=1}^{m}\varphi_{y_k}^2\chi^2\right)+G_2(\lambda,\delta)\varphi^2\chi^2+d_{02}(\varphi)
\end{aligned}
$$

sağlanır. Burada

$$
\begin{aligned}
G_2(\lambda,\nu) &= \left(-\lambda^2\nu^2\psi^{-2\nu-2}\psi_{x_1}^2+\lambda\nu\psi^{-\nu-1}\psi_{x_1}-\lambda\nu(\nu+1)\psi^{-\nu-2}\psi_{x_1}^2\right. \\
&\quad -\left(x_1+\delta_0\right)^2\sum_{i=2}^{n}\left(\lambda\nu(\nu+1)\psi^{-\nu-2}+2\lambda^2\nu^2\psi^{-2\nu-2}\right)\psi_{x_i}^2 \\
&\quad +\lambda\nu\psi^{-\nu-1}\left(x_1+\delta_0\right)^2\left(n-1\right)+\frac{1}{2}\left(x_1+\delta_0\right)^2\sum_{k=1}^{m}\frac{\partial^2}{\partial y_k^2}c^2 \\
&\quad -\lambda\nu c^2 m\psi^{-\nu-1}\left(x_1+\delta_0\right)^2 \\
&\quad +\left(2\lambda^2\nu^2c^2\left(x_1+\delta_0\right)^2\psi^{-2\nu-2}+\lambda\nu\left(\nu+1\right)c^2\left(x_1+\delta_0\right)^2\psi^{-\nu-2}\right)\sum_{k=1}^{m}\psi_{y_k}^2 \\
&\quad \left.-2\lambda\nu\left(x_1+\delta_0\right)^2\psi^{-\nu-1}\sum_{k=1}^{m}\frac{\partial c^2}{\partial y_k}\psi_{y_k}\right)\varphi^2\chi^2,
\end{aligned}
$$

$$
\begin{aligned}
d_2(\varphi) \;=\; & -\left(\varphi\varphi_{x_1}\chi^2\right)_{x_1} - \lambda\nu\left(\psi^{-\nu-1}\psi_{x_1}\varphi^2\chi^2\right)_{x_1} + \frac{1}{2}\left(\varphi^2\chi^2\right)_{x_1} \\
& -\sum_{i=2}^{n}\left((x_1+\delta_0)^2\left(\varphi\varphi_{x_i}\chi^2 + \lambda\nu\psi^{-\nu-1}\psi_{x_i}\varphi^2\chi^2\right)\right)_{x_i} \\
& +\sum_{k=1}^{m}(x_1+\delta_0)^2\left(\chi^2\left(c^2\varphi\varphi_{y_k} - \frac{1}{2}\frac{\partial c^2}{\partial y_k}\varphi^2 + \lambda\nu c^2\psi^{-\nu-1}\psi_{y_k}\varphi^2\right)\right)_{y_k}
\end{aligned}
$$

olarak tanımlıdır.

İspat. (2.68) eşitliğinin sol tarafı için aşağıdaki eşitlikler yazılabilir.

Birinci terim:

$$
\begin{aligned}
-\varphi\varphi_{x_1x_1}\chi^2 &= -\left[\left(\varphi\varphi_{x_1}\chi^2\right)_{x_1} - \left(\varphi\chi^2\right)_{x_1}\varphi_{x_1}\right] \\
&= -\left(\varphi\varphi_{x_1}\chi^2\right)_{x_1} + \left(\varphi\chi^2\right)_{x_1}\varphi_{x_1}. \qquad (2.69)
\end{aligned}
$$

(2.69) eşitliğinde

$$
\begin{aligned}
\left(\varphi\chi^2\right)_{x_1}\varphi_{x_1} &= \varphi_{x_1}^2\chi^2 + \varphi\left(\chi^2\right)_{x_1}\varphi_{x_1} \\
&= \varphi_{x_1}^2\chi^2 - 2\lambda\nu\psi^{-\nu-1}\psi_{x_1}\varphi\varphi_{x_1}\chi^2 \\
&= \varphi_{x_1}^2\chi^2 - \lambda\nu\psi^{-\nu-1}\psi_{x_1}\left(\varphi^2\right)_{x_1}\chi^2 \qquad (2.70)
\end{aligned}
$$

dir. Ayrıca

$$
\begin{aligned}
\lambda\nu\left(\psi^{-\nu-1}\psi_{x_1}\varphi^2\chi^2\right)_{x_1} &= \lambda\nu\big[(-\nu-1)\psi^{-\nu-2}\psi_{x_1}^2\varphi^2\chi^2 \\
&\quad +\psi^{-\nu-1}\psi_{x_1}\left(\varphi^2\right)_{x_1}\chi^2 + \psi^{-\nu-1}\psi_{x_1x_1}\varphi^2\chi^2 \\
&\quad -2\lambda\nu\psi^{-2\nu-2}\psi_{x_1}^2\varphi^2\chi^2\big] \\
&= -\lambda\nu(\nu+1)\psi^{-\nu-2}\psi_{x_1}^2\varphi^2\chi^2 + \lambda\nu\psi^{-\nu-1}\psi_{x_1}\left(\varphi^2\right)_{x_1}\chi^2 \\
&\quad +\lambda\nu\psi^{-\nu-1}\psi_{x_1x_1}\varphi^2\chi^2 - 2\lambda^2\nu^2\psi^{-2\nu-2}\psi_{x_1}^2\varphi^2\chi^2 \qquad (2.71)
\end{aligned}
$$

kullanılarak

$$
\begin{aligned}
-\lambda\nu\psi^{-\nu-1}\psi_{x_1}\left(\varphi^2\right)_{x_1}\chi^2 &= -\lambda\nu\left(\psi^{-\nu-1}\psi_{x_1}\varphi^2\chi^2\right)_{x_1} - \lambda\nu(\nu+1)\psi^{-\nu-2}\psi_{x_1}^2\varphi^2\chi^2 \\
&\quad +\lambda\nu\psi^{-\nu-1}\varphi^2\chi^2 - 2\lambda^2\nu^2\psi^{-2\nu-2}\psi_{x_1}^2\varphi^2\chi^2 \qquad (2.72)
\end{aligned}
$$

ve sonuç olarak

$$
\begin{aligned}
-\varphi\varphi_{x_1x_1}\chi^2 &= -\left(\varphi\varphi_{x_1}\chi^2\right)_{x_1} - \lambda\nu\left(\psi^{-\nu-1}\psi_{x_1}\varphi^2\chi^2\right)_{x_1} \\
&\quad +\frac{1}{2}\left(\varphi^2\chi^2\right)_{x_1} + \lambda^2\nu^2\psi^{-2\nu-2}\psi_{x_1}^2\varphi^2\chi^2 + \lambda\nu\psi^{-\nu-1}\psi_{x_1}\varphi^2\chi^2 \\
&\quad +\varphi_{x_1}^2\chi^2 - \lambda\nu(\nu+1)\psi^{-\nu-2}\psi_{x_1}^2\varphi^2\chi^2 - 2\lambda^2\nu^2\psi^{-2\nu-2}\psi_{x_1}^2\varphi^2\chi^2 \qquad (2.73)
\end{aligned}
$$

bulunur.

İkinci terim:

$$\begin{aligned}
& -\sum_{i=2}^{n}\left(x_1+\delta_0\right)^2 \varphi\varphi_{x_ix_i}\chi^2 \\
= & -\sum_{i=2}^{n}\left[\left(\left(x_1+\delta_0\right)^2 \varphi\varphi_{x_i}\chi^2\right)_{x_i}-\left(\left(x_1+\delta_0\right)^2 \varphi\chi^2\right)_{x_i}\varphi_{x_i}\right] \\
= & -\sum_{i=2}^{n}\left(\left(x_1+\delta_0\right)^2 \varphi\varphi_{x_i}\chi^2\right)_{x_i}+\left(x_1+\delta_0\right)^2\sum_{i=2}^{n}\left(\varphi\chi^2\right)_{x_i}\varphi_{x_i}.
\end{aligned} \tag{2.74}$$

(2.74) eşitliğinde

$$\begin{aligned}
& \left(x_1+\delta_0\right)^2\sum_{i=2}^{n}\left(\varphi\chi^2\right)_{x_i}\varphi_{x_i} \\
= & \left(x_1+\delta_0\right)^2\sum_{i=2}^{n}\left(\varphi_{x_i}^2\chi^2+\varphi\left(\chi^2\right)_{x_i}\varphi_{x_i}\right) \\
= & \left(x_1+\delta_0\right)^2\sum_{i=2}^{n}\varphi_{x_i}^2\chi^2-\left(x_1+\delta_0\right)\sum_{i=1}^{n}2\lambda\nu\psi^{-\nu-1}\psi_{x_i}\varphi\varphi_{x_i}\chi^2 \\
= & \left(x_1+\delta_0\right)^2\sum_{i=2}^{n}\varphi_{x_i}^2\chi^2-\left(x_1+\delta_0\right)\sum_{i=1}^{n}\lambda\nu\psi^{-\nu-1}\psi_{x_i}\left(\varphi^2\right)_{x_i}\chi^2
\end{aligned} \tag{2.75}$$

dir. Ayrıca

$$\begin{aligned}
& \lambda\nu\left(x_1+\delta_0\right)^2\sum_{i=2}^{n}\left(\psi^{-\nu-1}\psi_{x_i}\varphi^2\chi^2\right)_{x_i} \\
= & \lambda\nu\left(x_1+\delta_0\right)^2\sum_{i=2}^{n}\left((-\nu-1)\psi^{-\nu-2}\psi_{x_i}^2\varphi^2\chi^2+\psi^{-\nu-1}\psi_{x_ix_i}\varphi^2\chi^2\right. \\
& \left.+\psi^{-\nu-1}\psi_{x_i}\left(\varphi^2\right)_{x_i}\chi^2-2\lambda\nu\psi^{-2\nu-2}\psi_{x_i}^2\varphi^2\chi^2\right) \\
= & \left(x_1+\delta_0\right)^2\sum_{i=2}^{n}\left(-\lambda\nu(\nu+1)\psi^{-\nu-2}\psi_{x_i}^2\varphi^2\chi^2+\lambda\nu\psi^{-\nu-1}\psi_{x_ix_i}\varphi^2\chi^2\right. \\
& \left.+\lambda\nu\psi^{-\nu-1}\psi_{x_i}\left(\varphi^2\right)_{x_i}\chi^2-2\lambda^2\nu^2\psi^{-2\nu-2}\psi_{x_i}^2\varphi^2\chi^2\right)
\end{aligned} \tag{2.76}$$

kullanılarak

$$\begin{aligned}
& -\left(x_1+\delta_0\right)^2\sum_{i=2}^{n}\lambda\nu\psi^{-\nu-1}\psi_{x_i}\left(\varphi^2\right)_{x_i}\chi^2 \\
= & \left(x_1+\delta_0\right)^2\sum_{i=2}^{n}\left(-\left(\lambda\nu\psi^{-\nu-1}\psi_{x_i}\varphi^2\chi^2\right)_{x_i}-\lambda\nu(\nu+1)\psi^{-\nu-2}\psi_{x_i}^2\varphi^2\chi^2\right. \\
& \left.+\lambda\nu\psi^{-\nu-1}\psi_{x_ix_i}\varphi^2\chi^2-2\lambda^2\nu^2\psi^{-2\nu-2}\psi_{x_i}^2\varphi^2\chi^2\right)
\end{aligned} \tag{2.77}$$

bulunur

ve ikinci terim için

$$
\begin{aligned}
& -\left(x_1+\delta_0\right)^2 \sum_{i=2}^{n} \varphi \varphi_{x_i x_i} \chi^2 \\
= & -\sum_{i=2}^{n}\left(\left(x_1+\delta_0\right)^2 \varphi \varphi_{x_i} \chi^2\right)_{x_i}+\left(x_1+\delta_0\right)^2 \sum_{i=2}^{n} \varphi_{x_i}^2 \chi^2 \\
& -\sum_{i=2}^{n}\left(\left(x_1+\delta_0\right)^2 \lambda \nu \psi^{-\nu-1} \psi_{x_i} \varphi^2 \chi^2\right)_{x_i} \\
& +\left(x_1+\delta_0\right)^2 \sum_{i=2}^{n}\left(\lambda \nu \psi^{-\nu-1} \psi_{x_i x_i}-\lambda \nu(\nu+1) \psi^{-\nu-2} \psi_{x_i}^2\right. \\
& \left.-2 \lambda^2 \nu^2 \psi^{-2 \nu-2} \psi_{x_i}^2\right) \varphi^2 \chi^2 \\
= & \left(x_1+\delta_0\right)^2 \sum_{i=2}^{n} \varphi_{x_i}^2 \chi^2 \\
& -\sum_{i=2}^{n}\left(\left(x_1+\delta_0\right)^2\left(\varphi \varphi_{x_i} \chi^2+\lambda \nu \psi^{-\nu-1} \psi_{x_i} \varphi^2 \chi^2\right)\right)_{x_i} \\
& -\left(x_1+\delta_0\right)^2 \sum_{i=2}^{n}\left(\lambda \nu(\nu+1) \psi^{-\nu-2}+2 \lambda^2 \nu^2 \psi^{-2 \nu-2}\right) \psi_{x_i}^2 \varphi^2 \chi^2 \\
& +\lambda \nu \psi^{-\nu-1}\left(x_1+\delta_0\right)^2 \sum_{i=2}^{n} \psi_{x_i x_i} \varphi^2 \chi^2
\end{aligned} \tag{2.78}
$$

elde edilir.

Üçüncü terim:

$$
\begin{aligned}
& \left(x_1+\delta_0\right)^2 \sum_{k=1}^{m} c^2 \varphi \varphi_{y_k y_k} \chi^2 \\
= & \left(x_1+\delta_0\right)^2 \sum_{k=1}^{m}\left[\left(c^2 \varphi \varphi_{y_k} \chi^2\right)_{y_k}-\left(c^2 \varphi \chi^2\right)_{y_k} \varphi_{y_k}\right] \\
= & \left(x_1+\delta_0\right)^2 \sum_{k=1}^{m}\left(c^2 \varphi \varphi_{y_k} \chi^2\right)_{y_k}-\left(x_1+\delta_0\right) \sum_{k=1}^{m}\left(c^2 \varphi \chi^2\right)_{y_k} \varphi_{y_k}
\end{aligned} \tag{2.79}
$$

(2.79) eşitliğinde

$$
\begin{aligned}
& -\sum_{k=1}^{m}\left(c^2 \varphi \chi^2\right)_{y_k} \varphi_{y_k} \\
= & -\sum_{k=1}^{m}\left[\frac{\partial}{\partial y_k} c^2 \varphi \chi^2 \varphi_{y_k}+c^2 \varphi_{y_k} \chi^2 \varphi_{y_k}-2 \lambda \nu c^2 \psi^{-\nu-1} \psi_{y_k} \varphi \varphi_{y_k} \chi^2\right] \\
= & -\sum_{k=1}^{m} c^2 \varphi_{y_k}^2 \chi^2-\sum_{k=1}^{m} \frac{\partial}{\partial y_k} c^2 \varphi \chi^2 \varphi_{y_k}+\lambda \nu \sum_{k=1}^{m} c^2 \psi^{-\nu-1} \psi_{y_k} 2 \varphi \varphi_{y_k} \chi^2
\end{aligned} \tag{2.80}
$$

dir. (2.80) eşitliğinin sağ tarafındaki ikinci terim için

$$
-\sum_{k=1}^{m} \frac{\partial}{\partial y_k} c^2 \varphi \chi^2 \varphi_{y_k}=-\frac{1}{2} \sum_{k=1}^{m} \frac{\partial}{\partial y_k} c^2\left(\varphi^2\right)_{y_k} \chi^2 \tag{2.81}
$$

olup

$$
\begin{aligned}
&\frac{1}{2}\left(\frac{\partial}{\partial y_k}c^2\varphi^2\chi^2\right)_{y_k} \\
= &\ \frac{1}{2}\left(\frac{\partial^2}{\partial y_k^2}c^2\varphi^2\chi^2+\frac{\partial}{\partial y_k}c^2\left(\varphi^2\right)_{y_k}\chi^2-2\lambda\nu\psi^{-\nu-1}\psi_{y_k}\frac{\partial}{\partial y_k}c^2\varphi^2\chi^2\right) \\
= &\ \frac{1}{2}\frac{\partial^2}{\partial y_k^2}c^2\varphi^2\chi^2+\frac{1}{2}\frac{\partial}{\partial y_k}c^2\left(\varphi^2\right)_{y_k}\chi^2-\lambda\nu\psi^{-\nu-1}\psi_{y_k}\frac{\partial}{\partial y_k}c^2\varphi^2\chi^2 \qquad (2.82)
\end{aligned}
$$

eşitliğinden

$$
\begin{aligned}
&-\frac{1}{2}\sum_{k=1}^{m}\frac{\partial}{\partial y_k}c^2\left(\varphi^2\right)_{y_k}\chi^2 \\
= &\ \frac{1}{2}\sum_{k=1}^{m}\frac{\partial^2}{\partial y_k^2}c^2\varphi^2\chi^2-\frac{1}{2}\sum_{k=1}^{m}\left(\frac{\partial}{\partial y_k}c^2\varphi^2\chi^2\right)_{y_k} \\
&-\lambda\nu\psi^{-\nu-1}\sum_{k=1}^{m}\frac{\partial}{\partial y_k}c^2\varphi^2\chi^2\psi_{y_k} \qquad (2.83)
\end{aligned}
$$

bulunur.

(2.80) eşitliğinin sağ tarafındaki üçüncü terim için

$$
\sum_{k=1}^{m}\lambda\nu c^2\psi^{-\nu-1}\psi_{y_k}2\varphi\varphi_{y_k}\chi^2=\sum_{k=1}^{m}\lambda\nu c^2\psi^{-\nu-1}\psi_{y_k}\left(\varphi^2\right)_{y_k}\chi^2 \qquad (2.84)
$$

olup

$$
\begin{aligned}
&\sum_{k=1}^{m}\left(\lambda\nu c^2\psi^{-\nu-1}\psi_{y_k}\varphi^2\chi^2\right)_{y_k} \\
= &\ \sum_{k=1}^{m}\left(\lambda\nu\frac{\partial}{\partial y_k}c^2\psi^{-\nu-1}\psi_{y_k}\varphi^2\chi^2-\lambda\nu\left(\nu+1\right)c^2\psi^{-\nu-2}\psi_{y_k}^2\varphi^2\chi^2\right. \\
&+\lambda\nu c^2\psi^{-\nu-1}\psi_{y_ky_k}\varphi^2\chi^2+\lambda\nu c^2\psi^{-\nu-1}\psi_{y_k}\left(\varphi^2\right)_{y_k}\chi^2 \\
&\left.-2\lambda^2\nu^2c^2\psi^{-2\nu-2}\psi_{y_k}^2\varphi^2\chi^2\right) \qquad (2.85)
\end{aligned}
$$

eşitliğinden

$$
\begin{aligned}
&\sum_{k=1}^{m}\lambda\nu c^2\psi^{-\nu-1}\psi_{y_k}\left(\varphi^2\right)_{y_k}\chi^2 \\
= &\ \sum_{k=1}^{m}\left(-\lambda\nu\frac{\partial}{\partial y_k}c^2\psi^{-\nu-1}\psi_{y_k}\varphi^2\chi^2\right. \\
&+\lambda\nu\left(\nu+1\right)c^2\psi^{-\nu-2}\psi_{y_k}^2\varphi^2\chi^2-\lambda\nu c^2\psi^{-\nu-1}\psi_{y_ky_k}\varphi^2\chi^2 \\
&\left.+\left(\lambda\nu c^2\psi^{-\nu-1}\psi_{y_k}\varphi^2\chi^2\right)_{y_k}+2\lambda^2\nu^2c^2\psi^{-2\nu-2}\psi_{y_k}^2\varphi^2\chi^2\right) \qquad (2.86)
\end{aligned}
$$

bulunur.

(2.83) ve (2.86) eşitlikleri yerine yazılırsa;

$$
\begin{aligned}
&(x_1+\delta_0)^2\sum_{k=1}^{m}c^2\varphi\varphi_{y_ky_k}\chi^2\\
=\;&\sum_{k=1}^{m}\left((x_1+\delta_0)^2c^2\varphi\varphi_{y_k}\chi^2\right)_{y_k}-(x_1+\delta_0)^2\sum_{k=1}^{m}c^2\varphi_{y_k}^2\chi^2\\
&+\frac{1}{2}(x_1+\delta_0)^2\sum_{k=1}^{m}\frac{\partial^2c^2}{\partial y_k^2}\varphi^2\chi^2-\frac{1}{2}(x_1+\delta_0)^2\sum_{k=1}^{m}\left(\frac{\partial c^2}{\partial y_k}\varphi^2\chi^2\right)_{y_k}\\
&-\lambda\nu\psi^{-\nu-1}(x_1+\delta_0)^2\sum_{k=1}^{m}\frac{\partial c^2}{\partial y_k}\psi_{y_k}\varphi^2\chi^2-\lambda\nu\psi^{-\nu-1}(x_1+\delta_0)^2\sum_{k=1}^{m}\frac{\partial c^2}{\partial y_k}\psi_{y_k}\varphi^2\chi^2\\
&+\lambda\nu(\nu+1)c^2\psi^{-\nu-2}(x_1+\delta_0)^2\sum_{k=1}^{m}\psi_{y_k}\varphi^2\chi^2-\lambda\nu c^2\psi^{-\nu-1}(x_1+\delta_0)^2\sum_{k=1}^{m}\psi_{y_ky_k}\varphi^2\chi^2\\
&+\sum_{k=1}^{m}\left(\lambda\nu c^2(x_1+\delta_0)^2\psi^{-\nu-1}\psi_{y_k}\varphi^2\chi^2\right)_{y_k}+2\lambda^2\nu^2c^2(x_1+\delta_0)^2\psi^{-2\nu-2}\sum_{k=1}^{m}\psi_{y_k}^2\varphi^2\chi^2\\
=\;&\sum_{k=1}^{m}(x_1+\delta_0)^2\left(\chi^2\left(c^2\varphi\varphi_{y_k}-\frac{1}{2}\frac{\partial c^2}{\partial y_k}\varphi^2+\lambda\nu c^2\psi^{-\nu-1}\psi_{y_k}\varphi^2\right)\right)_{y_k}\\
&-c^2(x_1+\delta_0)^2\sum_{k=1}^{m}\varphi_{y_k}^2\chi^2+\frac{1}{2}(x_1+\delta_0)^2\sum_{k=1}^{m}\frac{\partial^2}{\partial y_k^2}c^2\varphi^2\chi^2\\
&-\lambda\nu c^2m\psi^{-\nu-1}(x_1+\delta_0)^2\varphi^2\chi^2+2\lambda^2\nu^2c^2(x_1+\delta_0)^2\psi^{-2\nu-2}\sum_{k=1}^{m}\psi_{y_k}^2\varphi^2\chi^2\\
&+\lambda\nu(\nu+1)c^2(x_1+\delta_0)^2\psi^{-\nu-2}\sum_{k=1}^{m}\psi_{y_k}^2\varphi^2\chi^2\\
&-2\lambda\nu(x_1+\delta_0)^2\psi^{-\nu-1}\sum_{k=1}^{m}\frac{\partial c^2}{\partial y_k}\psi_{y_k}\varphi^2\chi^2
\end{aligned}
\tag{2.87}
$$

olur.

O halde (2.73), (2.78) ve (2.87) eşitlikleri kullanılarak;

$$
\begin{aligned}
&-\varphi\left(\varphi_{x_1x_1}+(x_1+\delta_0)^2\left(\sum_{i=2}^{n}\varphi_{x_ix_i}-c^2(x,y')\sum_{k=1}^{m}\varphi_{y_ky_k}\right)\right)\chi^2\\
=\;&-\left(\varphi\varphi_{x_1}\chi^2\right)_{x_1}-\lambda\nu\left(\psi^{-\nu-1}\psi_{x_1}\varphi^2\chi^2\right)_{x_1}+\frac{1}{2}\left(\varphi^2\chi^2\right)_{x_1}+\lambda^2\nu^2\psi^{-2\nu-2}\psi_{x_1}^2\varphi^2\chi^2\\
&+\lambda\nu\psi^{-\nu-1}\psi_{x_1}\varphi^2\chi^2+\varphi_{x_1}^2\chi^2-\lambda\nu(\nu+1)\psi^{-\nu-2}\psi_{x_1}^2\varphi^2\chi^2-2\lambda^2\nu^2\psi^{-2\nu-2}\psi_{x_1}^2\varphi^2\chi^2\\
&+(x_1+\delta_0)^2\sum_{i=2}^{n}\varphi_{x_i}^2\chi^2-\sum_{i=2}^{n}\left((x_1+\delta_0)^2\left(\varphi\varphi_{x_i}\chi^2+\lambda\nu\psi^{-\nu-1}\psi_{x_i}\varphi^2\chi^2\right)\right)_{x_i}\\
&-(x_1+\delta_0)^2\sum_{i=2}^{n}\left(\lambda\nu(\nu+1)\psi^{-\nu-2}+2\lambda^2\nu^2\psi^{-2\nu-2}\right)\psi_{x_i}^2\varphi^2\chi^2\\
&+\lambda\nu\psi^{-\nu-1}(x_1+\delta_0)^2\sum_{i=2}^{n}\psi_{x_ix_i}\varphi^2\chi^2
\end{aligned}
$$

$$
\begin{aligned}
& +\sum_{k=1}^{m}(x_1+\delta_0)^2\left(\chi^2\left(c^2\varphi\varphi_{y_k}-\frac{1}{2}\frac{\partial c^2}{\partial y_k}\varphi^2+\lambda\nu c^2\psi^{-\nu-1}\psi_{y_k}\varphi^2\right)\right)_{y_k} \\
& -c^2(x_1+\delta_0)^2\sum_{k=1}^{m}\varphi_{y_k}^2\chi^2+\frac{1}{2}(x_1+\delta_0)^2\sum_{k=1}^{m}\frac{\partial^2}{\partial y_k^2}c^2\varphi^2\chi^2 \\
& -\lambda\nu c^2 m\psi^{-\nu-1}(x_1+\delta_0)^2\varphi^2\chi^2+2\lambda^2\nu^2c^2(x_1+\delta_0)^2\psi^{-2\nu-2}\sum_{k=1}^{m}\psi_{y_k}^2\varphi^2\chi^2 \\
& +\lambda\nu(\nu+1)c^2(x_1+\delta_0)^2\psi^{-\nu-2}\sum_{k=1}^{m}\psi_{y_k}^2\varphi^2\chi^2-2\lambda\nu(x_1+\delta_0)^2\psi^{-\nu-1}\sum_{k=1}^{m}\frac{\partial c^2}{\partial y_k}\psi_{y_k}\varphi^2\chi^2 \\
= \; & \varphi_{x_1}^2\chi^2+(x_1+\delta_0)^2\sum_{i=2}^{n}\varphi_{x_i}^2\chi^2-c^2(x_1+\delta_0)^2\sum_{k=1}^{m}\varphi_{y_k}^2\chi^2 \\
& -\left(\varphi\varphi_{x_1}\chi^2\right)_{x_1}-\lambda\nu\left(\psi^{-\nu-1}\psi_{x_1}\varphi^2\chi^2\right)_{x_1}+\frac{1}{2}\left(\varphi^2\chi^2\right)_{x_1} \\
& -\sum_{i=2}^{n}\left((x_1+\delta_0)^2\left(\varphi\varphi_{x_i}\chi^2+\lambda\nu\psi^{-\nu-1}\psi_{x_i}\varphi^2\chi^2\right)\right)_{x_i} \\
& +\sum_{k=1}^{m}(x_1+\delta_0)^2\left(\chi^2\left(c^2\varphi\varphi_{y_k}-\frac{1}{2}\frac{\partial c^2}{\partial y_k}\varphi^2+\lambda\nu c^2\psi^{-\nu-1}\psi_{y_k}\varphi^2\right)\right)_{y_k} \\
& +\left(\lambda^2\nu^2\psi^{-2\nu-2}\psi_{x_1}^2+\lambda\nu\psi^{-\nu-1}\psi_{x_1}-\lambda\nu(\nu+1)\psi^{-\nu-2}\psi_{x_1}^2-2\lambda^2\nu^2\psi^{-2\nu-2}\psi_{x_1}^2\right. \\
& -(x_1+\delta_0)^2\sum_{i=2}^{n}\left(\lambda\nu(\nu+1)\psi^{-\nu-2}+2\lambda^2\nu^2\psi^{-2\nu-2}\right)\psi_{x_i}^2+\lambda\nu\psi^{-\nu-1}(x_1+\delta_0)^2(n-1) \\
& +\frac{1}{2}(x_1+\delta_0)^2\sum_{k=1}^{m}\frac{\partial^2}{\partial y_k^2}c^2-\lambda\nu c^2 m\psi^{-\nu-1}(x_1+\delta_0)^2 \\
& +\left(2\lambda^2\nu^2c^2(x_1+\delta_0)^2\psi^{-2\nu-2}+\lambda\nu(\nu+1)c^2(x_1+\delta_0)^2\psi^{-\nu-2}\right)\sum_{k=1}^{m}\psi_{y_k}^2 \\
& \left.-2\lambda\nu(x_1+\delta_0)^2\psi^{-\nu-1}\sum_{k=1}^{m}\frac{\partial c^2}{\partial y_k}\psi_{y_k}\right)\varphi^2\chi^2 \\
= \; & \varphi_{x_1}^2\chi^2+(x_1+\delta_0)^2\left(\sum_{i=2}^{n}\varphi_{x_i}^2\chi^2-c^2\sum_{k=1}^{m}\varphi_{y_k}^2\chi^2\right)+G_2(\lambda,\delta)\varphi^2\chi^2+d_2(\varphi) \qquad (2.88)
\end{aligned}
$$

elde edilir. Burada

$$
\begin{aligned}
d_2(\varphi) \;=\; & -\left(\varphi\varphi_{x_1}\chi^2\right)_{x_1}-\lambda\nu\left(\psi^{-\nu-1}\psi_{x_1}\varphi^2\chi^2\right)_{x_1}+\frac{1}{2}\left(\varphi^2\chi^2\right)_{x_1} \\
& -\sum_{i=2}^{n}\left((x_1+\delta_0)^2\left(\varphi\varphi_{x_i}\chi^2+\lambda\nu\psi^{-\nu-1}\psi_{x_i}\varphi^2\chi^2\right)\right)_{x_i} \\
& +\sum_{k=1}^{m}(x_1+\delta_0)^2\left(\chi^2\left(c^2\varphi\varphi_{y_k}-\frac{1}{2}\frac{\partial c^2}{\partial y_k}\varphi^2+\lambda\nu c^2\psi^{-\nu-1}\psi_{y_k}\varphi^2\right)\right)_{y_k},
\end{aligned}
$$

$$
\begin{aligned}
G_2(\lambda,\nu) \;=\;& \Big(-\lambda^2\nu^2\psi^{-2\nu-2}\psi_{x_1}^2+\lambda\nu\psi^{-\nu-1}\psi_{x_1}-\lambda\nu(\nu+1)\psi^{-\nu-2}\psi_{x_1}^2\\
&-(x_1+\delta_0)^2\sum_{i=2}^{n}\left(\lambda\nu(\nu+1)\psi^{-\nu-2}+2\lambda^2\nu^2\psi^{-2\nu-2}\right)\psi_{x_i}^2\\
&+\lambda\nu\psi^{-\nu-1}(x_1+\delta_0)^2(n-1)+\frac{1}{2}(x_1+\delta_0)^2\sum_{k=1}^{m}\frac{\partial^2}{\partial y_k^2}c^2\\
&-\lambda\nu c^2 m\psi^{-\nu-1}(x_1+\delta_0)^2\\
&+\left(2\lambda^2\nu^2c^2(x_1+\delta_0)^2\psi^{-2\nu-2}+\lambda\nu(\nu+1)c^2(x_1+\delta_0)^2\psi^{-\nu-2}\right)\sum_{k=1}^{m}\psi_{y_k}^2\\
&-2\lambda\nu(x_1+\delta_0)^2\psi^{-\nu-1}\sum_{k=1}^{m}\frac{\partial c^2}{\partial y_k}\psi_{y_k}\Big)\varphi^2\chi^2
\end{aligned}
$$

şeklinde tanımlıdır. ■

Lemma 2.3 *Aşağıdaki eşitsizlik sağlanır:*

$$
\begin{aligned}
&\psi^{\nu+1}\left((x_1+\delta_0)^{-1}\varphi_{x_1x_1}+(x_1+\delta_0)\left(\sum_{i=2}^{n}\varphi_{x_ix_i}-c^2(x,y')\sum_{k=1}^{m}\varphi_{y_ky_k}\right)\right)^2\chi^2\\
&-2\lambda\nu\beta_0\varphi\left(\varphi_{x_1x_1}+(x_1+\delta_0)^2\left(\sum_{i=2}^{n}\varphi_{x_ix_i}-c^2(x,y')\sum_{k=1}^{m}\varphi_{y_ky_k}\right)\right)\chi^2\\
\geq\;& 2\lambda\nu(x_1+\delta_0)^2\sum_{i=2}^{n}\varphi_{x_i}^2\chi^2+2\lambda\nu(x_1+\delta_0)^2\sum_{k=1}^{m}\varphi_{y_k}^2\chi^2\\
&+\lambda^3\nu^4\delta^4(x_1+\delta_0)^{-2}\psi^{-2\nu-3}\varphi^2\chi^2+d_3(\chi\varphi) \qquad (2.89)
\end{aligned}
$$

İspat. (2.68) eşitliği $2\lambda\nu\beta_0$ ile çarpılır ve (2.2) eşitsizliği ile toplanırsa

$$
\begin{aligned}
&\psi^{\nu+1}\left((x_1+\delta_0)^{-1}\varphi_{x_1x_1}+(x_1+\delta_0)\left(\sum_{i=2}^{n}\varphi_{x_ix_i}-c^2(x,y')\sum_{k=1}^{m}\varphi_{y_ky_k}\right)\right)^2\chi^2\\
&-2\lambda\nu\beta_0\varphi\left(\varphi_{x_1x_1}+(x_1+\delta_0)^2\left(\sum_{i=2}^{n}\varphi_{x_ix_i}-c^2(x,y')\sum_{k=1}^{m}\varphi_{y_ky_k}\right)\right)\chi^2\\
\geq\;& 2\lambda\nu\delta(x_1+\delta_0)^{-3}\varphi_{x_1}^2\chi^2-2\lambda\nu(x_1+\delta_0)^2G_o(n,m)\sum_{i=2}^{n}\varphi_{x_i}^2\chi^2\\
&+\lambda\nu\delta\alpha_1(x_1+\delta_0)\sum_{k=1}^{m}\varphi_{y_k}^2\chi^2+2\lambda^3\nu^4\delta^4(x_1+\delta_0)^{-2}\psi^{-2\nu-3}\varphi^2\chi^2\\
&+G(\lambda,\delta)\varphi^2\chi^2+d_0(\varphi\chi)+d_1(\varphi\chi)\\
&+2\lambda\nu\beta_0\left(\varphi_{x_1}^2\chi^2+(x_1+\delta_0)^2\left(\sum_{i=2}^{n}\varphi_{x_i}^2\chi^2-c^2\sum_{k=1}^{m}\varphi_{y_k}^2\chi^2\right)+G_2(\lambda,\delta)\varphi^2\chi^2\right)\\
&-2\lambda\nu\beta_0 d_2(\varphi)
\end{aligned}
$$

$$
\begin{aligned}
= \quad & 2\lambda\nu\delta\left(x_1+\delta_0\right)^{-3}\varphi_{x_1}^2\chi^2+2\lambda\nu\beta_0\varphi_{x_1}^2\chi^2 \\
& -2\lambda\nu\left(x_1+\delta_0\right)^2 G_o(n,m)\sum_{i=2}^{n}\varphi_{x_i}^2\chi^2+2\lambda\nu\beta_0\left(x_1+\delta_0\right)^2\sum_{i=2}^{n}\varphi_{x_i}^2\chi^2 \\
& +\lambda\nu\delta\alpha_1\left(x_1+\delta_0\right)\sum_{k=1}^{m}\varphi_{y_k}^2\chi^2-2\lambda\nu\beta_0\left(x_1+\delta_0\right)^2 c^2\sum_{k=1}^{m}\varphi_{y_k}^2\chi^2 \\
& +2\lambda^3\nu^4\delta^4\left(x_1+\delta_0\right)^{-2}\psi^{-2\nu-3}\varphi^2\chi^2+G(\lambda,\delta)\varphi^2\chi^2+2\lambda\nu\beta_0 G_2\left(\lambda,\delta\right)\varphi^2\chi^2 \\
& +d_0\left(\varphi\chi\right)+d_1\left(\varphi\chi\right)+2\lambda\nu\beta_0 d_2\left(\varphi\right)
\end{aligned}
$$

$$
\begin{aligned}
= \quad & \left(2\lambda\nu\delta\left(x_1+\delta_0\right)^{-3}+2\lambda\nu\beta_0\right)\varphi_{x_1}^2\chi^2 \\
& +\left(-2\lambda\nu\left(x_1+\delta_0\right)^2 G_o(n,m)+2\lambda\nu\beta_0\left(x_1+\delta_0\right)^2\right)\sum_{i=2}^{n}\varphi_{x_i}^2\chi^2 \\
& +\left(\lambda\nu\delta\alpha_1\left(x_1+\delta_0\right)-2\lambda\nu\beta_0\left(x_1+\delta_0\right)^2 c^2\right)\sum_{k=1}^{m}\varphi_{y_k}^2\chi^2 \\
& +\left(2\lambda^3\nu^4\delta^4\left(x_1+\delta_0\right)^{-2}\psi^{-2\nu-3}+G(\lambda,\delta)+2\lambda\nu\beta_0 G\left(\lambda,\delta\right)\right)\varphi^2\chi^2 \\
& +d_0\left(\varphi\chi\right)+d_1\left(\varphi\chi\right)-2\lambda\nu\beta_0 d_2\left(\varphi\right) \qquad (2.90)
\end{aligned}
$$

bulunur. Burada

$\beta_0 = G_0 + 1$

seçilirse;

$$
\begin{aligned}
& \left(-2\lambda\nu G_o(n,m)+2\lambda\nu\beta_0\right)\left(x_1+\delta_0\right)^2\sum_{i=2}^{n}\varphi_{x_i}^2\chi^2 \\
= \quad & \left(-2\lambda\nu G_0+2\lambda\nu\left(G_0+1\right)\right)\left(x_1+\delta_0\right)^2\sum_{i=2}^{n}\varphi_{x_i}^2\chi^2 \\
= \quad & \left(-2\lambda\nu G_0+2\lambda\nu G_0+2\lambda\nu\right)\left(x_1+\delta_0\right)^2\sum_{i=2}^{n}\varphi_{x_i}^2\chi^2 \\
= \quad & 2\lambda\nu\left(x_1+\delta_0\right)^2\sum_{i=2}^{n}\varphi_{x_i}^2\chi^2 \qquad (2.91)
\end{aligned}
$$

elde edilir. Diğer yandan

$\delta_4 = \dfrac{1}{2}\left(1+3\beta_0 M\gamma/\alpha_1\right)$

olarak alınırsa ve

$-2\lambda\nu\beta_0 c^2\sum_{k=1}^{m}\varphi_{y_k}^2\chi^2 \geq -\dfrac{3}{2}\lambda\nu\beta_0 M\gamma\sum_{k=1}^{m}\varphi_{y_k}^2\chi^2$

eşitsizliği göz önünde bulundurulursa $\delta \geq \delta_4$ için

$$\begin{aligned}
& \left(\lambda\nu\delta\left(x_1+\delta_0\right)\alpha_1 - 2\lambda\nu\beta_0\left(x_1+\delta_0\right)^2 c^2\right)\sum_{k=1}^{m}\varphi_{y_k}^2\chi^2 \\
\geq & \left(\lambda\nu\delta\alpha_1 - \frac{3}{2}\lambda\nu\beta_0 M\gamma\right)\left(x_1+\delta_0\right)\sum_{k=1}^{m}\varphi_{y_k}^2\chi^2 \\
\geq & \left(\lambda\nu\frac{1}{2}\left(1+3\beta_0 M\gamma/\alpha_1\right)\alpha_1 - 2\lambda\nu\beta_0 M^2\right)\left(x_1+\delta_0\right)\sum_{k=1}^{m}\varphi_{y_k}^2\chi^2 \\
= & \frac{1}{2}\lambda\nu\left(x_1+\delta_0\right)\sum_{k=1}^{m}\varphi_{y_k}^2\chi^2 + \left(\frac{3}{2}\lambda\nu\beta_0 M\gamma - \frac{3}{2}\lambda\nu\beta_0 M\gamma\right)\left(x_1+\delta_0\right)\sum_{k=1}^{m}\varphi_{y_k}^2\chi^2 \\
= & \frac{1}{2}\lambda\nu\alpha_1\left(x_1+\delta_0\right)\sum_{k=1}^{m}\varphi_{y_k}^2\chi^2 \qquad (2.92)
\end{aligned}$$

yazılabilir. Son olarak

$$\left[2\lambda^3\nu^4\delta^4\left(x_1+\delta_0\right)^{-2}\psi^{-2\nu-3} + G(\lambda,\delta) + 2\lambda\nu\beta_0 G_2\left(\lambda,\delta\right)\right]\varphi^2\chi^2$$

terimi için

$$\begin{aligned}
& G\left(\lambda,\nu\right) + 2\lambda\nu\beta_0 G_2(\lambda,\nu) \\
= & 2\lambda\nu\delta\left(x_1+\delta_0\right)^{-3}\left(-\lambda^2\nu^2\psi_{x_1}^2\psi^{-2\nu-2} - \lambda\nu\left(\nu+1\right)\psi_{x_1}^2\psi^{-\nu-2}\right) \\
& -2\lambda\nu G_o(n,m)\sum_{i=2}^{n}\left(-\lambda^2\nu^2\psi_{x_i}^2\psi^{-2\nu-2} - \lambda\nu\left(\nu+1\right)\psi_{x_i}^2\psi^{-\nu-2} + \lambda\nu\psi^{-\nu-1}\psi_{x_i x_i}\right) \\
& +\lambda\nu\delta\alpha_1\left(x_1+\delta_0\right)\sum_{k=1}^{m}\left(-\lambda^2\nu^2\psi_{y_k}^2\psi^{-2\nu-2} - \lambda\nu\left(\nu+1\right)\psi_{y_k}^2\psi^{-\nu-2} + \lambda\nu\psi^{-\nu-1}\psi_{y_k y_k}\right) \\
& +2\lambda\nu\beta_0\left(\lambda^2\nu^2\psi^{-2\nu-2}\psi_{x_1}^2 + \lambda\nu\psi^{-\nu-1}\psi_{x_1} - \lambda\nu(\nu+1)\psi^{-\nu-2}\psi_{x_1}^2\right. \\
& -\left(x_1+\delta_0\right)^2\sum_{i=2}^{n}\left(\lambda\nu(\nu+1)\psi^{-\nu-2} + 2\lambda^2\nu^2\psi^{-2\nu-2}\right)\psi_{x_i}^2 \\
& +\lambda\nu\psi^{-\nu-1}\left(x_1+\delta_0\right)^2\left(n-1\right) + \frac{1}{2}\left(x_1+\delta_0\right)^2\sum_{k=1}^{m}\frac{\partial^2}{\partial y_k^2}c^2 - \lambda\nu c^2 m\psi^{-\nu-1}\left(x_1+\delta_0\right)^2 \\
& +\left(2\lambda^2\nu^2 c^2\left(x_1+\delta_0\right)^2\psi^{-2\nu-2} + \lambda\nu\left(\nu+1\right)c^2\left(x_1+\delta_0\right)^2\psi^{-\nu-2}\right)\sum_{k=1}^{m}\psi_{y_k}^2 \\
& \left.-2\lambda\nu\left(x_1+\delta_0\right)^2\psi^{-\nu-1}\sum_{k=1}^{m}\frac{\partial c^2}{\partial y_k}\psi_{y_k}\right)
\end{aligned}$$

$$
\begin{aligned}
= \quad & \lambda^3\nu^3 \Bigg[-2\delta^3 (x_1+\delta_0)^{-3} \psi^{-2\nu-2} + 2G_o(n,m)\psi^{-2\nu-2}\sum_{i=2}^{n}\psi_{x_i}^2 + 2\delta^2\beta_0\psi^{-2\nu-2} \\
& -4\beta_0 (x_1+\delta_0)^2 \psi^{-2\nu-2}\sum_{i=2}^{n}\psi_{x_i}^2 + 4\beta_0 c^2 (x_1+\delta_0)^2 \psi^{-2\nu-2}\sum_{k=1}^{m}\psi_{y_k}^2 \\
& -\delta\alpha_1 (x_1+\delta_0) \psi^{-2\nu-2}\sum_{k=1}^{m}\psi_{y_k}^2 \Bigg] \\
& +\lambda^2\nu^2 \Bigg[-2(\nu+1)\delta^3 (x_1+\delta_0)^{-3}\psi^{-\nu-2} + 2G_o(n,m)(\nu+1)\psi^{-\nu-2}\sum_{i=2}^{n}\psi_{x_i}^2 \\
& -2\psi^{-\nu-1}(n-1) - 2(\nu+1)\delta\alpha_1 (x_1+\delta_0)\psi^{-\nu-2}\sum_{k=1}^{m}\psi_{y_k}^2 \\
& +2\delta\beta_0\psi^{-\nu-1} - 2\delta^2\beta_0(\nu+1)\psi^{-\nu-2} \\
& -2\beta_0(\nu+1)(x_1+\delta_0)^2\psi^{-\nu-2}\sum_{i=2}^{n}\psi_{x_i}^2 + 2\beta_0(n-1)(x_1+\delta_0)^2\psi^{-\nu-1} \\
& -2\beta_0 c^2 m (x_1+\delta_0)^2\psi^{-v-2} + 2\beta_0(\nu+1)c^2(x_1+\delta_0)^2\psi^{-\nu-2}\sum_{k=1}^{m}\psi_{y_k}^2 \\
& -4\beta_0 (x_1+\delta_0)^2\psi^{-\nu-1}\sum_{k=1}^{m}\frac{\partial c^2}{\partial y_k}\psi_{y_k} + \delta\alpha_1 m (x_1+\delta_0)\psi^{-\nu-1}\Bigg] \\
& +\lambda\nu\beta_0 (x_1+\delta_0)^2\sum_{k=1}^{m}\frac{\partial^2 c^2}{\partial y_k^2}
\end{aligned}
$$

eşitliğini yazabiliriz. Burada

$$
\begin{aligned}
E_3 \quad = \quad & -2\delta^3 (x_1+\delta_0)^{-3}\psi^{-2\nu-2} + 2\delta^2\beta_0\psi^{-2\nu-2} + 2G_o(n,m)\psi^{-2\nu-2}\sum_{i=2}^{n}\psi_{x_i}^2 \\
& -4\beta_0 (x_1+\delta_0)^2\psi^{-2\nu-2}\sum_{i=2}^{n}\psi_{x_i}^2 + 4\beta_0 c^2 (x_1+\delta_0)^2\psi^{-2\nu-2}\sum_{k=1}^{m}\psi_{y_k}^2 \\
& -\delta\alpha_1 (x_1+\delta_0)\psi^{-2\nu-2}\sum_{k=1}^{m}\psi_{y_k}^2 \Bigg], \qquad (2.93)
\end{aligned}
$$

$$
\begin{aligned}
E_4 \quad = \quad & -2(\nu+1)\delta^3 (x_1+\delta_0)^{-3}\psi^{-\nu-2} - 2\psi^{-\nu-1}(n-1) + 2\delta\beta_0\psi^{-\nu-1} \\
& -2\delta^2\beta_0(\nu+1)\psi^{-\nu-2} + 2\beta_0(n-1)(x_1+\delta_0)^2\psi^{-\nu-1} \\
& -2\beta_0 c^2 m (x_1+\delta_0)^2\psi^{-v-2} + \delta\alpha_1 m (x_1+\delta_0)\psi^{-\nu-1} \\
& +2G_o(n,m)(\nu+1)\psi^{-\nu-2}\sum_{i=2}^{n}\psi_{x_i}^2 - 2\beta_0(\nu+1)(x_1+\delta_0)^2\psi^{-\nu-2}\sum_{i=2}^{n}\psi_{x_i}^2 \\
& -2(\nu+1)\delta\alpha_1 (x_1+\delta_0)\psi^{-\nu-2}\sum_{k=1}^{m}\psi_{y_k}^2
\end{aligned}
$$

$$+2\beta_0(\nu+1)c^2(x_1+\delta_0)^2\psi^{-\nu-2}\sum_{k=1}^{m}\psi_{y_k}^2-4\beta_0(x_1+\delta_0)^2\psi^{-\nu-1}\sum_{k=1}^{m}\frac{\partial c^2}{\partial y_k}\psi_{y_k}$$
$$+\frac{1}{\lambda\nu}\beta_0(x_1+\delta_0)^2\sum_{k=1}^{m}\frac{\partial^2c^2}{\partial y_k^2} \tag{2.94}$$

dir.

Diğer bir ifade ile

$$G(\lambda,\nu)+2\lambda\nu\beta_0G_2(\lambda,\nu)=\lambda^3\nu^3E_3+\lambda^2\nu^2E_4 \tag{2.95}$$

formundadır.

c^2, ψ, ψ_{x_i}, ψ_{y_j} fonksiyonları $C\left(\overline{\Omega}_\gamma\right)$ da sınırlı olduğundan E_{31} ifadesi $x\in\overline{\Omega}_\gamma$ için sınırlı olur: $\delta\,(\delta\geq 4)$: $|E_{31}|\leq M_3$.

Burada

$$E_3=\nu\delta^2\psi^{-2\nu-3}E_{31}$$

şeklinde tanımlıdır. O halde

$$\delta\geq\delta_5=\sqrt{2M_3}$$

seçilirse;

$$\begin{aligned}
\lambda^3\nu^4\delta^4(x_1+\delta_0)^{-2}\psi^{-2\nu-3}+\lambda^3\nu^3E_3 &= \lambda^3\nu^4\delta^4(x_1+\delta_0)^{-2}\psi^{-2\nu-3}\left(1+\frac{1}{\delta^4\nu\psi^{-2\nu-3}}E_3\right)\\
&= \lambda^3\nu^4\delta^4(x_1+\delta_0)^{-2}\psi^{-2\nu-3}\left(1+\frac{1}{\delta^2}E_{31}\right)\\
&\geq \lambda^3\nu^4\delta^4(x_1+\delta_0)^{-2}\psi^{-2\nu-3}\left(1-\frac{1}{\delta^2}|E_{31}|\right)\\
&\geq \lambda^3\nu^4\delta^4(x_1+\delta_0)^{-2}\psi^{-2\nu-3}\left(1-\frac{1}{\delta^2}M_3\right)\\
&= \lambda^3\nu^4\delta^4(x_1+\delta_0)^{-2}\psi^{-2\nu-3}\left(1-\frac{1}{\delta^2}\frac{\delta_5^2}{2}\right)\\
&\geq \lambda^3\nu^4\delta^4(x_1+\delta_0)^{-2}\psi^{-2\nu-3}\left(1-\frac{1}{\delta^2}\frac{\delta^2}{2}\right)\\
&= \lambda^3\nu^4\delta^4(x_1+\delta_0)^{-2}\psi^{-2\nu-3}\left(1-\frac{1}{2}\right)\\
&= \frac{1}{2}\lambda^3\nu^4\delta^4(x_1+\delta_0)^{-2}\psi^{-2\nu-3}
\end{aligned} \tag{2.96}$$

olduğu görülür. Benzer düşünce ile sabitlenmiş $\delta\geq\delta_5$, $\nu>1$ ve γ sayıları için E_4, Ω_γ da sınırlıdır. Eğer;

$\Omega_\gamma : |E_4|\leq M_4$. $\lambda\geq\lambda_1=M_4$.

seçilirse;

$$\begin{aligned}\lambda^3\nu^4\delta^4\psi^{-2\nu-3}+\lambda^3\nu^3E_3+\lambda^2\nu^2E_4 &\geq \lambda^3\nu^4\delta^4\psi^{-2\nu-3}+\lambda^3\nu^3E_3-\lambda^2\nu^2\left|E_4\right| \\ &\geq \frac{1}{2}\lambda^3\nu^4\delta^4\psi^{-2\nu-3}-\lambda^2\nu^2\left|E_4\right| \\ &\geq \frac{1}{2}\lambda^3\nu^4\delta^4\psi^{-2\nu-3}-\lambda^2\nu^2M_4\geq 0 \qquad (2.97)\end{aligned}$$

buradan

$$\begin{aligned}&2\lambda^3\nu^4\delta^4\left(x_1+\delta_0\right)^{-2}\psi^{-2\nu-3}+G\left(\lambda,\nu\right)+2\lambda\nu\beta_0G_2(\lambda,\nu) \\ =\;&\lambda^3\nu^4\delta^4\left(x_1+\delta_0\right)^{-2}\psi^{-2\nu-3}+\lambda^3\nu^4\delta^4\left(x_1+\delta_0\right)^{-2}\psi^{-2\nu-3}+G\left(\lambda,\nu\right)+2\lambda\nu\beta_0G_2(\lambda,\nu) \\ \geq\;&\lambda^3\nu^4\delta^4\left(x_1+\delta_0\right)^{-2}\psi^{-2\nu-3} \qquad (2.98)\end{aligned}$$

eşitliği bulunur. Sonuç olarak, $\delta\geq\delta_6=\max\left\{\delta_3,\delta_4,\delta_5\right\}$ ve $\lambda\geq\lambda_2$ olmak üzere

$$\begin{aligned}&\psi^{\nu+1}\left(\left(x_1+\delta_0\right)^{-1}\varphi_{x_1x_1}+\left(x_1+\delta_0\right)\left(\sum_{i=2}^{n}\varphi_{x_ix_i}-c^2\left(x,y'\right)\sum_{k=1}^{m}\varphi_{y_ky_k}\right)\right)^2\chi^2 \\ &-2\lambda\nu\beta_0\varphi\left(\varphi_{x_1x_1}+\left(x_1+\delta_0\right)^2\left(\sum_{i=2}^{n}\varphi_{x_ix_i}-c^2\left(x,y'\right)\sum_{k=1}^{m}\varphi_{y_ky_k}\right)\right)\chi^2 \\ \geq\;&2\lambda\nu\delta\left(x_1+\delta_0\right)^{-3}\varphi_{x_1}\chi^2+2\lambda\nu\left(x_1+\delta_0\right)^2\sum_{i=2}^{n}\varphi_{x_i}^2\chi^2+\frac{1}{2}\lambda\nu\alpha_1\left(x_1+\delta_0\right)^2\sum_{k=1}^{m}\varphi_{y_k}^2\chi^2 \\ &+\lambda^3\nu^4\delta^4\left(x_1+\delta_0\right)^{-2}\psi^{-2\nu-3}\varphi^2\chi^2+d_3\left(\chi\varphi\right) \qquad (2.99)\end{aligned}$$

değerlendirmesi elde edilir. ■

BÖLÜM 3

ULTRAHİPERBOLİK DENKLEM İÇİN BİR TERS PROBLEM

Bu bölümde aşağıdaki ters problem ele alınacaktır.

Problem 3.1 *(1.1) denkleminden $a_0(x,y')$ katsayısının $u(x,y',0) = g(x,y')$ ek bilgisi yardımıyla bulunması problemi.*

Teorem 3.1 *Kabul edelim ki (1.1) eşitsizliği sağlansın. Ayrıca $g(x,y\prime) \neq 0$ ve $c(x,y')$ katsayısı y_m değişkeninden bağımsız olsun. Bu durumda Problem 1'in bir tek $a_0(x,y') \in C(\Omega)$ çözümüne sahiptir.*

$$\begin{aligned} &(x_1+\delta_0)^{-1} u_{x_1x_1} + (x_1+\delta_0)\left(\sum_{i=2}^{n} u_{x_ix_i} - c^2(x,y')\sum_{j=1}^{m} u_{y_jy_j}\right) \\ &+ (x_1+\delta_0)\sum_{i=1}^{n} a_i u_{x_i} + (x_1+\delta_0)\sum_{j=1}^{m} b_j u_{y_j} + a_0(x,y')\,u \\ = \;& f(x,y') \end{aligned}$$

denklemi

$$u(0,'x,y) = u_0('x,y) \qquad (3.1)$$

$$u_{x_1}(0,'x,y) = u_{x_1}('x,y) \qquad (3.2)$$

koşulları ve

$$u(x,y',0) = g(x,y') \qquad (3.3)$$

ek bilgisi ile birlikte ele alınacaktır. Burada (1.1) denkleminden $a_0(x,y)$ fonksiyonunu bulunması problemi ele alınacaktır.

$f_{x_{n+m}} \in C\left(\overline{\Omega}\right)$ fonksiyonuna karşılık gelen iki çözüm $u^{(1)}, u^{(2)} \in C^2\left(\overline{\Omega}\right)$ ve $a_0^{(1)}$, $a_0^{(2)}$ olsun.

Ayrıca

$\widetilde{u} = u^{(2)} - u^{(1)},$

$\widetilde{a_0} = a_0^{(2)} - a_0^{(1)}$

olsun. Bu durumda

$$\begin{aligned}
& (x_1+\delta_0)^{-1} u^{(2)}_{x_1x_1} + (x_1+\delta_0)\left(\sum_{i=2}^{n} u^{(2)}_{x_ix_i} - c^2(x,y')\sum_{j=1}^{m} u^{(2)}_{y_jy_j}\right) \\
& + (x_1+\delta_0)\sum_{i=2}^{n} a_i u^{(2)}_{x_i} + (x_1+\delta_0)\sum_{j=1}^{m} b_j u^{(2)}_{y_j} + a_0(x,y')\,u^{(2)} \\
= \; & f(x,y') \qquad (3.4)
\end{aligned}$$

ve

$$\begin{aligned}
& (x_1+\delta_0)^{-1} u^{(1)}_{x_1x_1} + (x_1+\delta_0)\left(\sum_{i=2}^{n} u^{(1)}_{x_ix_i} - c^2(x,y')\sum_{j=1}^{m} u^{(1)}_{y_jy_j}\right) \\
& + (x_1+\delta_0)\sum_{i=1}^{n} a_i u^{(1)}_{x_i} + (x_1+\delta_0)\sum_{j=1}^{m} b_j u^{(1)}_{y_j} + a_0(x,y')\,u^{(1)} \\
= \; & f(x,y'), \qquad (3.5)
\end{aligned}$$

olup

$$\begin{aligned}
& (x_1+\delta_0)^{-1}\left(u^{(2)}_{x_1x_1} - u^{(1)}_{x_1x_1}\right) + (x_1+\delta_0)\sum_{i=2}^{n}\left(u^{(2)}_{x_ix_i} - u^{(1)}_{x_ix_i}\right) \\
& -c^2(x,y')(x_1+\delta_0)\sum_{j=1}^{m}\left(u^{(2)}_{y_jy_j} - u^{(1)}_{y_jy_j}\right) + (x_1+\delta_0)\sum_{i=1}^{n} a_i\left(u^{(2)}_{x_i} - u^{(1)}_{x_i}\right) \\
& + (x_1+\delta_0)\sum_{j=1}^{m} b_j\left(u^{(2)}_{y_j} - u^{(1)}_{y_j}\right) + a_0^{(2)}u^{(2)} - a_0^{(1)}u^{(1)} \\
= \; & 0 \qquad (3.6)
\end{aligned}$$

yazılabilir. Buradan

$$\begin{aligned}
a_0^{(2)}u^{(2)} - a_0^{(1)}u^{(1)} + a_0^{(1)}u^{(2)} - a_0^{(1)}u^{(2)} &= a_0^{(1)}\left(u^{(2)} - u^{(1)}\right) + u^{(2)}\left(a_0^{(2)} - a_0^{(1)}\right) \\
&= a_0^{(1)}\widetilde{u} + u^{(2)}\widetilde{a_0} \qquad (3.7)
\end{aligned}$$

bağıntısı kullanılarak

$$\begin{aligned}
& (x_1+\delta_0)^{-1}\tilde{u}_{x_1x_1} + (x_1+\delta_0)\sum_{i=2}^{n}\tilde{u}_{x_ix_i} - c^2(x,y')(x_1+\delta_0)\sum_{j=1}^{m}\tilde{u}_{y_jy_j} \\
& + (x_1+\delta_0)\sum_{i=1}^{n} a_i\tilde{u}_{x_i} + (x_1+\delta_0)\sum_{j=1}^{m} b_j\tilde{u}_{y_j} + a_0^{(1)}\widetilde{u} \\
= \; & -\widetilde{a_0}u^{(2)} \qquad (3.8)
\end{aligned}$$

elde edilir.

Ayrıca $\widetilde{u}$ aşağıdaki koşulları sağlar:

$$\widetilde{u}\left(0,' x, y\right) = 0,\ \widetilde{u}_{x_1}\left(0,' x, y\right) = 0 \tag{3.9}$$

ve

$$\widetilde{u}\left(x, y', 0\right) = 0. \tag{3.10}$$

Şimdi Ω_γ de yeni bir bilinmeyen fonksiyon tanımlayalım:

$$\widetilde{u} = u^{(2)}w.$$

Buradan

$$\left(u^{(2)}w\right)_{x_1} = \left(u^{(2)}\right)_{x_1} w + u^{(2)}w_{x_1},$$

$$\begin{aligned}
\left(u^{(2)}w\right)_{x_1x_1} &= \left(\left(u^{(2)}\right)_{x_1} w + u^{(2)}w_{x_1}\right)_{x_1} \\
&= \left(\left(u^{(2)}\right)_{x_1x_1} w + \left(u^{(2)}\right)_{x_1} w_{x_1} + \left(u^{(2)}\right)_{x_1} w_{x_1} + u^{(2)}w_{x_1x_1}\right) \\
&= \left(u^{(2)}\right)_{x_1x_1} w + u^{(2)}w_{x_1x_1} + 2\left(u^{(2)}\right)_{x_1} w_{x_1},
\end{aligned} \tag{3.11}$$

$$\left(u^{(2)}w\right)_{x_i} = \left(u^{(2)}\right)_{x_i} w + u^{(2)}w_{x_i},$$

$$\begin{aligned}
\sum_{i=2}^{n}\left(u^{(2)}w\right)_{x_ix_i} &= \sum_{i=2}^{n}\left(\left(u^{(2)}\right)_{x_i} w + u^{(2)}w_{x_i}\right)_{x_i} \\
&= \sum_{i=2}^{n}\left(\left(u^{(2)}\right)_{x_ix_i} w + \left(u^{(2)}\right)_{x_i} w_{x_i} + \left(u^{(2)}\right)_{x_i} w_{x_i} + u^{(2)}w_{x_ix_i}\right) \\
&= \sum_{i=2}^{n}\left(u^{(2)}\right)_{x_ix_i} w + \sum_{i=2}^{n} 2\left(u^{(2)}\right)_{x_i} w_{x_i} + \sum_{i=2}^{n} u^{(2)}w_{x_ix_i},
\end{aligned} \tag{3.12}$$

$$\left(u^{(2)}w\right)_{y_j} = \left(u^{(2)}\right)_{y_j} w + u^{(2)}w_{y_j},$$

$$\begin{aligned}
\sum_{j=1}^{m}\left(u^{(2)}w\right)_{y_jy_j} &= \sum_{j=1}^{m}\left(\left(u^{(2)}\right)_{y_j} w + u^{(2)}w_{y_j}\right)_{y_j} \\
&= \sum_{j=1}^{m}\left(\left(u^{(2)}\right)_{y_jy_j} w + \left(u^{(2)}\right)_{y_j} w_{y_j} + \left(u^{(2)}\right)_{y_j} w_{y_j} + u^{(2)}w_{y_jy_j}\right) \\
&= \sum_{j=1}^{m}\left(u^{(2)}\right)_{y_jy_j} w + \sum_{j=1}^{m} 2\left(u^{(2)}\right)_{y_j} w_{y_j} + \sum_{j=1}^{m} u^{(2)}w_{y_jy_j}.
\end{aligned} \tag{3.13}$$

(3.8) denkleminde yerine yazılırsa:

$$\begin{aligned}
& (x_1+\delta_0)^{-1}\left(u^{(2)}\right)_{x_1x_1} w + (x_1+\delta_0)^{-1} u^{(2)} w_{x_1x_1} + 2(x_1+\delta_0)^{-1}\left(u^{(2)}\right)_{x_1} w_{x_1} \\
& + (x_1+\delta_0)\sum_{i=2}^{n}\left(u^{(2)}\right)_{x_ix_i} w + 2(x_1+\delta_0)\sum_{i=2}^{n}\left(u^{(2)}\right)_{x_i} w_{x_i} + (x_1+\delta_0)\sum_{i=2}^{n} u^{(2)} w_{x_ix_i} \\
& - (x_1+\delta_0)c^2(x,y')\sum_{j=1}^{m}\left(u^{(2)}\right)_{y_jy_j} w - 2(x_1+\delta_0)c^2(x,y')\sum_{j=1}^{m}\left(u^{(2)}\right)_{y_j} w_{y_j} \\
& - (x_1+\delta_0)c^2(x,y')\sum_{j=1}^{m} u^{(2)} w_{y_jy_j} + (x_1+\delta_0)\sum_{i=1}^{n} a_i\left(\left(u^{(2)}\right)_{x_i} w + u^{(2)} w_{x_i}\right) \\
& + (x_1+\delta_0)\sum_{j=1}^{m} b_j\left(\left(u^{(2)}\right)_{y_j} w + u^{(2)} w_{y_j}\right) + (x_1+\delta_0) a_0^{(1)} u^{(2)} w \\
= \; & -\widetilde{a_0} u^{(2)}
\end{aligned}$$

veya daha düzenli olarak

$$\begin{aligned}
& (x_1+\delta_0)^{-1} u^{(2)} w_{x_1x_1} + (x_1+\delta_0)\sum_{i=2}^{n} u^{(2)} w_{x_ix_i} - (x_1+\delta_0)c^2(x,y')\sum_{j=1}^{m} u^{(2)} w_{y_jy_j} \\
& +2(x_1+\delta_0)^{-1}\left(u^{(2)}\right)_{x_1} w_{x_1} + (x_1+\delta_0)\sum_{i=1}^{n}\left(a_i u^{(2)} + 2\left(u^{(2)}\right)_{x_i}\right) w_{x_i} \\
& + (x_1+\delta_0)\sum_{j=1}^{m}\left(+b_j u^{(2)} - c^2(x,y')\left(u^{(2)}\right)_{y_j}\right) w_{y_j} + (x_1+\delta_0)^{-1}\left(u^{(2)}\right)_{x_1x_1} w \\
& + (x_1+\delta_0)\left(\sum_{i=2}^{n}\left(u^{(2)}\right)_{x_ix_i} - c^2(x,y')\sum_{j=1}^{m}\left(u^{(2)}\right)_{y_jy_j} + \sum_{i=2}^{n} a_i\left(u^{(2)}\right)_{x_i}\right. \\
& \left. + \sum_{j=1}^{m} b_j\left(u^{(2)}\right)_{y_j} + a_0^{(1)} u^{(2)}\right) w \\
= \; & -(x_1+\delta_0)\widetilde{a_0} u^{(2)} \qquad (3.14)
\end{aligned}$$

bulunur.

Son eşitlikte $\left(u^{(2)}\right)^{-1}$ ile çarpılırsa;

$$\begin{aligned}
& (x_1+\delta_0)^{-1} w_{x_1x_1} + (x_1+\delta_0)\sum_{i=2}^{n} w_{x_ix_i} - (x_1+\delta_0)c^2(x,y')\sum_{j=1}^{m} w_{y_jy_j} \\
& +2(x_1+\delta_0)^{-1}\left(u^{(2)}\right)_{x_1}\left(u^{(2)}\right)^{-1} w_{x_1} + (x_1+\delta_0)\sum_{i=1}^{n}\left[\left(a_i u^{(2)} + 2\left(u^{(2)}\right)_{x_i}\right)\left(u^{(2)}\right)^{-1}\right] w_{x_i} \\
& + (x_1+\delta_0)\sum_{j=1}^{m}\left[\left(+b_j u^{(2)} - 2c^2(x,y')\left(u^{(2)}\right)_{y_j}\right)\left(u^{(2)}\right)^{-1}\right] w_{y_j}
\end{aligned}$$

$$
\begin{aligned}
&+\Bigg[(x_1+\delta_0)^{-1}\left(u^{(2)}\right)_{x_1x_1}+(x_1+\delta_0)\Bigg(\sum_{i=2}^{n}\left(u^{(2)}\right)_{x_ix_i}-c^2(x,y')\sum_{j=1}^{m}\left(u^{(2)}\right)_{y_jy_j}\\
&+\sum_{i=2}^{n}a_i\left(u^{(2)}\right)_{x_i}+\sum_{j=1}^{m}b_j\left(u^{(2)}\right)_{y_j}+a_0^{(1)}u^{(2)}\Bigg)\left(u^{(2)}\right)^{-1}\Bigg]w\\
=\ &-(x_1+\delta_0)\,\widetilde{a_0}u^{(2)}\left(u^{(2)}\right)^{-1} \qquad (3.15)
\end{aligned}
$$

elde edilir.

Burada

$$
\begin{aligned}
\overline{a_0}\ =\ &\Bigg[\Bigg((x_1+\delta_0)^{-1}\left(u^{(2)}\right)_{x_1x_1}+(x_1+\delta_0)\sum_{i=2}^{n}\left(u^{(2)}\right)_{x_ix_i}\\
&-c^2(x,y')(x_1+\delta_0)\sum_{j=1}^{m}\left(u^{(2)}\right)_{y_jy_j}+(x_1+\delta_0)\sum_{i=1}^{n}a_i\left(u^{(2)}\right)_{x_i}\\
&+(x_1+\delta_0)\sum_{j=1}^{m}b_j\left(u^{(2)}\right)_{y_j}+(x_1+\delta_0)\,a_0^{(1)}u^{(2)}\Bigg)\left(u^{(2)}\right)^{-1}\Bigg]
\end{aligned}
$$

$$\overline{a_i}=\left[\left(a_iu^{(2)}+2\left(u^{(2)}\right)_{x_i}\right)\left(u^{(2)}\right)^{-1}\right],$$

$$\overline{b_j}=\left[\left(b_ju^{(2)}-2c^2(x,y')\,(u^{(2)})_{y_j}\right)\left(u^{(2)}\right)^{-1}\right]$$

olarak alınırsa, (**??**) denklemi

$$
\begin{aligned}
&(x_1+\delta_0)^{-1}w_{x_1x_1}+(x_1+\delta_0)\sum_{i=2}^{n}w_{x_ix_i}-(x_1+\delta_0)\,c^2(x,y')\sum_{j=1}^{m}w_{y_jy_j}\\
&+2(x_1+\delta_0)^{-1}\left(u^{(2)}\right)_{x_1}\left(u^{(2)}\right)^{-1}w_{x_1}+(x_1+\delta_0)\sum_{i=1}^{n}\overline{a_i}w_{x_i}\\
&+(x_1+\delta_0)\sum_{j=1}^{m}\overline{b_j}w_{y_j}+\overline{a_0}w=-(x_1+\delta_0)\,\widetilde{a_0}(x,y') \qquad (3.16)
\end{aligned}
$$

şeklinde yazılabilir. Diğer yandan

$u^{(2)}\neq 0,$

$\widetilde{u}(0,'x,y)=0=u^{(2)}w(0,'x,y)=0\Rightarrow w(0,'x,y)=0,$

$\widetilde{u}_{x_1}(0,'x,y)=0=\left(u^{(2)}w\right)_{x_1}=\left(u^{(2)}\right)_{x_1}w+u^{(2)}w_{x_1}=0\Rightarrow w_{x_1}(0,'x,y)=0,$

$w(x,y',0)=0$

koşulları bulunur.

(3.16) denkleminin y_m değişkenine göre türevi alınır ve

$$\left(x_1+\delta_0\right)^{-1}\left(w_{x_1x_1}\right)_{y_m}+\left(x_1+\delta_0\right)\sum_{i=2}^{n}\left(w_{x_ix_i}\right)_{y_m}$$
$$-\left(x_1+\delta_0\right)c^2\left(x,y'\right)\sum_{j=1}^{m}\left(w_{y_jy_j}\right)_{y_m}$$
$$+2\left(x_1+\delta_0\right)^{-1}\left(\left(u^{(2)}\right)_{x_1}w_{x_1}\right)_{y_m}+\left(x_1+\delta_0\right)\sum_{i=1}^{n}\left(\overline{a_i}w_{x_i}\right)_{y_m}$$
$$+\left(x_1+\delta_0\right)\sum_{j=1}^{m}\left(\overline{b_j}w_{y_j}\right)_{y_m}+\left[\overline{a_0}w\right]_{y_m}=-\left(x_1+\delta_0\right)\left[\widetilde{a_0}\left(x,y'\right)\right]_{y_m}$$

sağ taraf sıfır olacağından

$$\begin{aligned}&\left(x_1+\delta_0\right)^{-1}\left(w_{x_1x_1}\right)_{y_m}+\left(x_1+\delta_0\right)\sum_{i=1}^{n}\left(w_{x_ix_i}\right)_{y_m}-c^2\left(x,y'\right)\left(x_1+\delta_0\right)\sum_{j=1}^{m}\left(w_{y_jy_j}\right)_{y_m}\\&+\left(x_1+\delta_0\right)\sum_{i=1}^{n}\overline{a_i}\left(w_{x_i}\right)_{y_m}+\left(x_1+\delta_0\right)\sum_{i=1}^{n}\frac{\partial}{\partial y_m}\overline{a_i}w_{x_i}\\&+\left(x_1+\delta_0\right)\sum_{j=1}^{m}\frac{\partial}{\partial y_m}\overline{b_j}w_{y_j}+\left(x_1+\delta_0\right)\sum_{j=1}^{m}\overline{b_j}\left(w_{y_j}\right)_{y_m}+\frac{\partial}{\partial y_m}\overline{a_0}w+\overline{a_0}w_{y_m}\\=&\ 0\end{aligned}$$

bulunur. Türevlerin yerleri değiştirilirse

$$\begin{aligned}&\left(x_1+\delta_0\right)^{-1}\left(w_{y_m}\right)_{x_1x_1}+\left(x_1+\delta_0\right)\sum_{i=1}^{n}\left(w_{y_m}\right)_{x_ix_i}-c^2\left(x,y'\right)\left(x_1+\delta_0\right)\sum_{j=1}^{m}\left(w_{y_m}\right)_{y_jy_j}\\&+\left(x_1+\delta_0\right)\sum_{i=1}^{n}\overline{a_i}\left(w_{y_m}\right)_{x_i}+\left(x_1+\delta_0\right)\sum_{i=1}^{n}\frac{\partial}{\partial y_m}\overline{a_i}w_{x_i}+\left(x_1+\delta_0\right)\sum_{j=1}^{m}\frac{\partial}{\partial y_m}\overline{b_j}w_{y_j}\\&+\left(x_1+\delta_0\right)\sum_{j=1}^{m}\overline{b_j}\left(w_{y_m}\right)_{y_j}+\frac{\partial}{\partial y_m}\overline{a_0}w+\overline{a_0}w_{y_m}\\=&\ 0\end{aligned}\tag{3.17}$$

elde edilir. Burada $z=w_{y_m}$ gösterimi kullanılırsa

$$z=w_{y_m}\Rightarrow w=sgn\left(y_m\right)\int_0^{y_m}z(x,y',\tau)d\tau,$$

$$Iz=\begin{cases}\int_0^{y_m}z(x,y',\tau)d\tau, & y_m\geq 0\\ \int_{y_m}^{0}z(x,y',\tau)d\tau, & y_m<0\end{cases},$$

$$I_{x_i}z=\begin{cases}\frac{\partial}{\partial x_i}\int_0^{y_m}z(x,y',\tau)d\tau, & y_m\geq 0\\ \frac{\partial}{\partial x_i}\int_{y_m}^{0}z(x,y',\tau)d\tau, & y_m<0\end{cases},$$

$$I_{y_j}z=\begin{cases}\frac{\partial}{\partial y_j}\int_0^{y_m} z(x,y',\tau)d\tau, & y_m\geq 0\\ \frac{\partial}{\partial y_j}\int_{y_m}^0 z(x,y',\tau)d\tau, & y_m<0\end{cases}$$

elde edilir.

(3.17) denklemininde $z=w_{y_m}$ yazılırsa

$$\begin{aligned}
&(x_1+\delta_0)^{-1} z_{x_1x_1}+(x_1+\delta_0)\sum_{i=1}^{n} z_{x_ix_i}-c^2(x,y')(x_1+\delta_0)\sum_{j=1}^{m} z_{y_jy_j}\\
&+(x_1+\delta_0)\sum_{i=1}^{n}\overline{a_i}z_{x_i}+(x_1+\delta_0)\sum_{j=1}^{m}\overline{b_j}z_{y_j}+(x_1+\delta_0)\overline{a_0}z\\
&+(x_1+\delta_0)\sum_{i=1}^{n}\frac{\partial}{\partial y_m}\overline{a_i}I_{x_i}z+(x_1+\delta_0)\sum_{j=1}^{m}\frac{\partial}{\partial y_m}\overline{b_j}I_{y_j}z\\
&+(x_1+\delta_0)\frac{\partial}{\partial y_m}\overline{a_0}Iz\\
=\;&0
\end{aligned}\tag{3.18}$$

elde edilir ve

$$z(0,'x,y)=z_{x_1}(0,'x,y)=0$$

koşulları bulunur.

Şimdi, eğer $z(x,y)$ fonksiyonu (3.18) denklemini sağlıyor ise Ω_γ da $z(x,y)=0$ olduğu gösterilecektir. Bu amaçla aşağıdaki bağıntılar kullanılır:

$$\begin{aligned}
\int_{\Omega_\gamma}(Iz)^2\chi^2 d\Omega_\gamma &\leq \gamma\int z^2\chi^2 d\Omega_\gamma,\\
\int_{\Omega_\gamma}(I_{x_i}z)^2\chi^2 d\Omega_\gamma &\leq \gamma\int z_{x_i}^2\chi^2 d\Omega_\gamma,\\
\int_{\Omega_\gamma}\left(I_{y_j}z\right)^2\chi^2 d\Omega_\gamma &\leq \gamma\int z_{y_j}^2\chi^2 d\Omega_\gamma.
\end{aligned}$$

Diğer taraftan (3.18) denkleminden

$$\begin{aligned}
&\left((x_1+\delta_0)^{-1} z_{x_1x_1}+(x_1+\delta_0)\sum_{i=2}^{n} z_{x_ix_i}-c^2(x,y')(x_1+\delta_0)\sum_{j=1}^{m} z_{y_jy_j}\right)^2\\
=\;&(x_1+\delta_0)^2\left(\sum_{i=1}^{n}(\bar{a}_iz_{x_i}+\frac{\partial\bar{a}_i}{\partial y_m}I_{x_i}z)+\sum_{j=1}^{m}(\bar{b}_jz_{y_j}+\frac{\partial}{\partial y_m}\bar{b}_jI_{y_j}z)+\bar{a}_0z+\frac{\partial\bar{a}_0}{\partial y_m}Iz\right)^2\\
\leq\;&3(x_1+\delta_0)^2\left(\left(\sum_{i=1}^{n}(\bar{a}_iz_{x_i}+\frac{\partial\bar{a}_i}{\partial y_m}I_{x_i}z)\right)^2+\left(\sum_{j=1}^{m}(\bar{b}_jz_{y_j}+\frac{\partial}{\partial y_m}\bar{b}_jI_{y_j}z)\right)^2\right.\\
&\left.+\left(\bar{a}_0z+\frac{\partial\bar{a}_0}{\partial y_m}Iz\right)^2\right.
\end{aligned}$$

$$\leq 3\left(x_1+\delta_0\right)^2\left((2n\sum_{i=1}^{n}((\bar{a}_i z_{x_i})^2+(\frac{\partial \bar{a}_i}{\partial y_m}I_{x_i}z)^2)+2m\sum_{j=1}^{m}((\bar{b}_j z_{y_j})^2+(\frac{\partial}{\partial y_m}\bar{b}_j I_{y_j}z)^2)\right.$$

$$\left.+2(\bar{a}_0 z)^2+2(\frac{\partial \bar{a}_0}{\partial y_m}Iz)^2\right)$$

$$\leq 3\left(x_1+\delta_0\right)^2\left(2n\sum_{i=1}^{n}((\bar{a}_i z_{x_i})^2+(\frac{\partial \bar{a}_i}{\partial y_m}I_{x_i}z)^2)+2m\sum_{j=1}^{m}((\bar{b}_j z_{y_j})^2+(\frac{\partial}{\partial y_m}\bar{b}_j I_{y_j}z)^2\right.$$

$$\left.+2(\bar{a}_0 z)^2+2(\frac{\partial \bar{a}_0}{\partial y_m}Iz)^2)\right)$$

$$\leq 3M_5\left(x_1+\delta_0\right)^2\left(2n\sum_{i=1}^{n}z_{x_i}^2+2n\sum_{i=1}^{n}(I_{x_i}z)^2+2m\sum_{j=1}^{m}z_{y_j}^2+2m\sum_{j=1}^{m}(I_{y_j}z)^2\right.$$

$$\left.+2z^2+2(Iz)^2\right)$$

$$\leq 6M_5\left(x_1+\delta_0\right)^2\left(n\sum_{i=1}^{n}(z_{x_i}^2+(I_{x_i}z)^2)+m\sum_{j=1}^{m}(z_{y_j}^2+(I_{y_j}z)^2)+z^2+(Iz)^2\right)$$

$$\leq 6M_5\max\{n,m\}\left(x_1+\delta_0\right)^2\left(\sum_{i=1}^{n}(z_{x_i}^2+(I_{x_i}z)^2)+\sum_{j=1}^{m}(z_{y_j}^2+(I_{y_j}z)^2)+z^2+(Iz)^2\right)$$

$$\leq 6M_5\max\{n,m\}\left(x_1+\delta_0\right)^2(1+\gamma)\left(\sum_{i=1}^{n}z_{x_i}^2+\sum_{j=1}^{m}z_{y_j}^2+z^2\right) \tag{3.19}$$

eşitsizliği yazılabilir. Ayrıca (3.18) denklemini $(x_1+\delta_0)$ ile çarpılarak, yukarıdaki benzer düşünce ile

$$\left(z_{x_1x_1}+\left(x_1+\delta_0\right)^2\sum_{i=2}^{n}z_{x_ix_i}-c^2\left(x,y'\right)\left(x_1+\delta_0\right)\sum_{j=1}^{m}z_{y_jy_j}\right)^2$$

$$=\left(x_1+\delta_0\right)^4\left(\sum_{i=1}^{n}(\bar{a}_i z_{x_i}+\frac{\partial \bar{a}_i}{\partial y_m}I_{x_i}z)+\sum_{j=1}^{m}(\bar{b}_j z_{y_j}+\frac{\partial}{\partial y_m}\bar{b}_j I_{y_j}z)+\bar{a}_0 z+\frac{\partial \bar{a}_0}{\partial y_m}Iz\right)^2$$

$$\leq 3\left(x_1+\delta_0\right)^4\left(\left(\sum_{i=1}^{n}(\bar{a}_i z_{x_i}+\frac{\partial \bar{a}_i}{\partial y_m}I_{x_i}z)\right)^2+\left(\sum_{j=1}^{m}(\bar{b}_j z_{y_j}+\frac{\partial}{\partial y_m}\bar{b}_j I_{y_j}z)\right)^2\right.$$

$$\left.+\left(\bar{a}_0 z+\frac{\partial \bar{a}_0}{\partial y_m}Iz\right)^2\right.$$

$$\leq 3\left(x_1+\delta_0\right)^4\left((2n\sum_{i=1}^{n}((\bar{a}_i z_{x_i})^2+(\frac{\partial \bar{a}_i}{\partial y_m}I_{x_i}z)^2)+2m\sum_{j=1}^{m}((\bar{b}_j z_{y_j})^2+(\frac{\partial}{\partial y_m}\bar{b}_j I_{y_j}z)^2)\right.$$

$$\left.+2(\bar{a}_0 z)^2+2(\frac{\partial \bar{a}_0}{\partial y_m}Iz)^2\right)$$

$$\leq 3\left(x_1+\delta_0\right)^4\left(2n\sum_{i=1}^{n}((\bar{a}_i z_{x_i})^2+(\frac{\partial \bar{a}_i}{\partial y_m}I_{x_i}z)^2)+2m\sum_{j=1}^{m}((\bar{b}_j z_{y_j})^2+(\frac{\partial}{\partial y_m}\bar{b}_j I_{y_j}z)^2\right.$$

$$\left.+2(\bar{a}_0 z)^2+2(\frac{\partial \bar{a}_0}{\partial y_m}Iz)^2)\right)$$

$$
\begin{aligned}
\leq\ & 3M_5\left(x_1+\delta_0\right)^4\left(2n\sum_{i=1}^{n} z_{x_i}^2+2n\sum_{i=1}^{n}(I_{x_i}z)^2+2m\sum_{j=1}^{m} z_{y_j}^2+2m\sum_{j=1}^{m}(I_{y_j}z)^2\right.\\
& \left.+2z^2+2(Iz)^2\right)\\
\leq\ & 6M_5\left(x_1+\delta_0\right)^4\left(n\sum_{i=1}^{n}(z_{x_i}^2+(I_{x_i}z)^2)+m\sum_{j=1}^{m}(z_{y_j}^2+(I_{y_j}z)^2)+z^2+(Iz)^2\right)\\
\leq\ & 6M_5\max\{n,m\}\left(x_1+\delta_0\right)^4\left(\sum_{i=1}^{n}(z_{x_i}^2+(I_{x_i}z)^2)+\sum_{j=1}^{m}(z_{y_j}^2+(I_{y_j}z)^2)+z^2+(Iz)^2\right)\\
\leq\ & 6M_5\max\{n,m\}\left(x_1+\delta_0\right)^4(1+\gamma)\left(\sum_{i=1}^{n} z_{x_i}^2+\sum_{j=1}^{m} z_{y_j}^2+z^2\right)
\end{aligned}
\tag{3.20}
$$

yazılabilir. Burada $M_5>0$ sabiti, M ve $\|u^2\|_{C^1(\overline{\Omega_\gamma})}$ büyüklüklerine bağlıdır.
Lemma 2.3 kullanılarak

$$
\begin{aligned}
& \left(\left(x_1+\delta_0\right)^{-1}\varphi_{x_1x_1}+\left(x_1+\delta_0\right)\left(\sum_{i=2}^{n}\varphi_{x_ix_i}-c^2\left(x,y'\right)\sum_{j=1}^{m}\varphi_{y_jy_j}\right)\right)^2\chi^2\\
& +\lambda^2\nu^2\beta_0^2\varphi^2\chi^2+\left(\varphi_{x_1x_1}+\left(x_1+\delta_0\right)^2\left(\sum_{i=2}^{n}\varphi_{x_ix_i}-c^2\left(x,y'\right)\sum_{j=1}^{m}\varphi_{y_jy_j}\right)\right)^2\chi^2\\
\geq\ & \psi^{\nu+1}\left(\left(x_1+\delta_0\right)^{-1}\varphi_{x_1x_1}+\left(x_1+\delta_0\right)\left(\sum_{i=2}^{n}\varphi_{x_ix_i}-c^2\left(x,y'\right)\sum_{j=1}^{m}\varphi_{y_jy_j}\right)\right)^2\chi^2\\
& -2\lambda\nu\beta_0\varphi\left(\varphi_{x_1x_1}+\left(x_1+\delta_0\right)^2\left(\sum_{i=2}^{n}\varphi_{x_ix_i}-c^2\left(x,y'\right)\sum_{j=1}^{m}\varphi_{y_jy_j}\right)\right)\chi^2\\
\geq\ & 2\lambda\nu\delta\left(x_1+\delta_0\right)^{-3}\varphi_{x_1}^2\chi^2+2\lambda\nu\left(x_1+\delta_0\right)^2\sum_{i=1}^{n}\varphi_{x_i}^2\chi^2+\frac{1}{2}\lambda\nu\alpha_1\left(x_1+\delta_0\right)\sum_{j=1}^{m}\varphi_{y_j}^2\chi^2\\
& +\lambda^3\nu^4\delta^4\psi^{-2\nu-3}\left(x_1+\delta_0\right)^{-2}\varphi^2\chi^2+d_4(\varphi)
\end{aligned}
\tag{3.21}
$$

eşitsizliği elde edilir. Son eşitsizlikte $z\equiv\varphi$ alınır ve (**??**), (**??**) bağıntıları kullanılırsa

$$
\begin{aligned}
& 6M_5\max\{n,m\}\left(x_1+\delta_0\right)^2(1+\gamma)\left(\sum_{i=1}^{n} z_{x_i}^2+\sum_{j=1}^{m} z_{y_j}^2+z^2\right)+\lambda^2\nu^2\beta_0^2\varphi^2\chi^2\\
& +6M_5\max\{n,m\}\left(x_1+\delta_0\right)^4(1+\gamma)\left(\sum_{i=1}^{n} z_{x_i}^2+\sum_{j=1}^{m} z_{y_j}^2+z^2\right)\\
=\ & 6M_5\max\{n,m\}\left(x_1+\delta_0\right)^2(1+\gamma)\left(\left(1+\left(x_1+\delta_0\right)^2\right)\left(\sum_{i=1}^{n} z_{x_i}^2+\sum_{j=1}^{m} z_{y_j}^2\right)\right.\\
& \left.+z^2\left(1+\lambda^2\nu^2\beta_0^2\left(x_1+\delta_0\right)^2\right)\right)\chi^2\\
\geq\ & 2\lambda\nu\delta\left(x_1+\delta_0\right)^{-3}z_{x_1}^2\chi^2+2\lambda\nu\left(x_1+\delta_0\right)^2\chi^2\sum_{i=2}^{n} z_{x_i}^2+\frac{1}{2}\lambda\nu\alpha_1\left(x_1+\delta_0\right)\chi^2\sum_{j=1}^{m} z_{y_j}^2\\
& +\lambda^3\nu^4\delta^4\left(x_1+\delta_0\right)^{-2}\psi^{-2\nu-3}z^2\chi^2+d_4(z)
\end{aligned}
\tag{3.22}
$$

bulunur.

O halde

$$
\begin{aligned}
&= 6M_5 \max\{n,m\}(x_1+\delta_0)^2(1+\gamma)\left(\left(1+(x_1+\delta_0)^2\right)\left(\sum_{i=1}^{n} z_{x_i}^2+\sum_{j=1}^{m} z_{y_j}^2\right)\right.\\
&\quad\left.+z^2\left(1+\lambda^2\nu^2\beta_0^2(x_1+\delta_0)^2\right)\right)\chi^2\\
&\geq 2\lambda\nu(x_1+\delta_0)^{-3} z_{x_1}^2\chi^2+2\lambda\nu(x_1+\delta_0)^2\sum_{i=2}^{n} z_{x_i}^2\chi^2+\frac{1}{2}\lambda\nu\alpha_1(x_1+\delta_0)\sum_{j=1}^{m} z_{y_j}^2\chi^2\\
&\quad+\lambda^3\nu^4\delta^4\psi^{-2\nu-3}(x_1+\delta_0)^{-2}z^2\chi^2+d_4(z) \qquad (3.23)
\end{aligned}
$$

ve buna bağlı olarak

$$
\begin{aligned}
0 &\geq 2\lambda\nu(x_1+\delta_0)^{-3} z_{x_1}^2\chi^2+2\lambda\nu(x_1+\delta_0)^2\sum_{i=2}^{n} z_{x_i}^2\chi^2+\frac{1}{2}\lambda\nu\alpha_1(x_1+\delta_0)\sum_{j=1}^{m} z_{y_j}^2\chi^2\\
&\quad+\lambda^3\nu^4\delta^4\psi^{-2\nu-3}(x_1+\delta_0)^{-2}z^2\chi^2+d_4(z)\\
&\quad-6M_5 \max\{n,m\}(x_1+\delta_0)^2(1+\gamma)\left(\left(1+(x_1+\delta_0)^2\right)\left(\sum_{i=1}^{n} z_{x_i}^2+\sum_{j=1}^{m} z_{y_j}^2\right)\right.\\
&\quad\left.+z^2\left(1+\lambda^2\nu^2\beta_0^2(x_1+\delta_0)^2\right)\right)\chi^2 \qquad (3.24)
\end{aligned}
$$

yazılabilir.

(3.24) eşitsizliğini değerlendirmek için,

$\lambda \geq \lambda_5 = \max\{\lambda_2,\lambda_3,\lambda_4\},$

$\lambda_3 = \max\left\{6M_5(1+\gamma)\max\{n,m\},\frac{1}{4}\alpha_1\right\}$

ve

$v_2 = 1+\beta_0$

alınırsa

$$
\begin{aligned}
&2\lambda\nu(x_1+\delta_0)^2\sum_{i=1}^{n} z_{x_i}^2\chi^2\\
&-6M_5\max\{n,m\}(x_1+\delta_0)^2(1+\gamma)\left(1+(x_1+\delta_0)^2\right)\sum_{i=1}^{n} z_{x_i}^2\chi^2\\
&= \left[2\lambda\nu-6M_5\max\{n,m\}(x_1+\delta_0)^2(1+\gamma)\left(1+(x_1+\delta_0)^2\right)\right](x_1+\delta_0)^2\sum_{i=1}^{n} z_{x_i}^2\chi^2\\
&\geq [2\lambda\nu-\lambda\nu](x_1+\delta_0)^2\sum_{i=1}^{n} z_{x_i}^2\chi^2\\
&= \lambda\nu(x_1+\delta_0)^2\sum_{i=1}^{n} z_{x_i}^2\chi^2,
\end{aligned}
$$

$$
\begin{aligned}
& \frac{1}{2}\lambda\nu\alpha_1 (x_1+\delta_0) \sum_{j=1}^{m} z_{y_j}^2 \chi^2 \\
& -6M_5 \max\{n,m\} (x_1+\delta_0)^2 (1+\gamma) \left(1+(x_1+\delta_0)^2\right) \sum_{i=1}^{n} z_{x_i}^2 \chi^2 \\
= & \left[\frac{1}{2}\lambda\nu\alpha_1 - 6M_5 \max\{n,m\} (x_1+\delta_0)(1+\gamma)\left(1+(x_1+\delta_0)^2\right)\right] (x_1+\delta_0) \sum_{j=1}^{m} z_{y_j}^2 \chi^2 \\
\geq & \left[\frac{1}{2}\lambda\nu\alpha_1 - \frac{1}{4}\lambda_3\nu\alpha_1\right] (x_1+\delta_0) \sum_{j=1}^{m} z_{y_j}^2 \chi^2 \\
\geq & \left[\frac{1}{2}\lambda\nu\alpha_1 - \frac{1}{4}\lambda_3\nu\alpha_1\right] (x_1+\delta_0) \sum_{j=1}^{m} z_{y_j}^2 \chi^2 \\
= & \frac{1}{4}\lambda\nu\alpha_1 (x_1+\delta_0) \sum_{j=1}^{m} z_{y_j}^2 \chi^2,
\end{aligned}
$$

ve

$$
\lambda^3\nu^3 - \lambda^3\nu^2 - \lambda\nu \geq 0
$$

olduğu dikkate alınarak

$$
\begin{aligned}
& \lambda^3\nu^4\delta^4 (x_1+\delta_0)^{-2} \psi^{-2\nu-3} z^2\chi^2 \\
& -\left(6M_5 \max\{n,m\} (x_1+\delta_0)^2 (1+\gamma)\left(1+\lambda^2\nu^2\beta_0^2 (x_1+\delta_0)^2\right)\right) z^2\chi^2 \\
\geq & \left[\lambda^3\nu^4\delta^4\psi^{-2\nu-3} - \lambda\nu - \lambda^3\nu^2 (v_2-1)^2\right] z^2\chi^2 \\
\geq & \left[\lambda^3\nu^4\delta^4\psi^{-2\nu-3} - \lambda\nu - \lambda^3\nu^2 (v-1)^2\right] z^2\chi^2 \\
\geq & \left[\lambda^3\nu^4\delta^4\psi^{-2\nu-3} - \lambda^3\nu^4\delta^4\psi^{-2\nu-3} + 2\lambda^3\nu^3 - \lambda^3\nu^2 - \lambda\nu\right] z^2\chi^2 \\
\geq & \lambda^3\nu^3 z^2\chi^2
\end{aligned}
$$

eşitsizliği elde edilir.

Bu durumda (3.24) eşitsizliği aşağıdaki şekilde bulunur:

$$
\begin{aligned}
0 \geq & 2\lambda\nu (x_1+\delta_0)^{-3} z_{x_1}^2\chi^2 + 2\lambda\nu (x_1+\delta_0)^2 \sum_{i=2}^{n} z_{x_i}^2\chi^2 + \frac{1}{2}\lambda\nu\alpha_1 (x_1+\delta_0) \sum_{j=1}^{m} z_{y_j}^2\chi^2 \\
& +\lambda^3\nu^4\delta^4\psi^{-2\nu-3} z^2\chi^2 + d_4(z) \\
& -6M_5 \max\{n,m\} (x_1+\delta_0)^2 (1+\gamma) \left(\left(1+(x_1+\delta_0)^2\right)\left(\sum_{i=1}^{n} z_{x_i}^2 + \sum_{j=1}^{m} z_{y_j}^2\right)\right. \\
& \left. +z^2\left(1+\lambda^2\nu^2\beta_0^2 (x_1+\delta_0)^2\right)\right)\chi^2 \\
\geq & \lambda\nu z_1^2\chi^2 + \lambda\nu (x_1+\delta_0)^2 \sum_{i=2}^{n} z_{x_i}^2\chi^2 + \frac{1}{4}\lambda\nu\alpha_1 (x_1+\delta_0) \sum_{j=1}^{m} z_{y_j}^2\chi^2 \\
& +\lambda^3\nu^3 z^2\chi^2 + d_4(z).
\end{aligned}
$$

Buradan

$$\lambda\nu z_1^2\chi^2 + \lambda\nu\left(x_1+\delta_0\right)^2\sum_{i=2}^{n} z_{x_i}^2\chi^2 + \frac{1}{4}\lambda\nu\alpha_1\left(x_1+\delta_0\right)\sum_{j=1}^{m} z_{y_j}^2\chi^2$$
$$+\lambda^3\nu^3 z^2\chi^2 + d_4\left(z\right) \leq 0$$

ve sonuç olarak

$$\int_{\Omega_\gamma} z^2 d\Omega_\gamma \leq -\frac{1}{\lambda^3\nu^3}\int_{\Omega_\gamma} d_4\left(z\right) d\Omega_\gamma$$

elde edilir.

Son eşitsizlikte $\lambda \to \infty$ için limit alınırsa

$$\int_{\Omega_\gamma} z^2 d\Omega_\gamma \leq 0$$

bulunur. Böylece Ω_γ da $z = 0$ olduğu görülür. İspat tamamlanmış olur.

3.1 SONUÇ

Bu tezde, ultrahiperbolik denklem için bir ters problem ele alınmış ve verilen bir ek bilgi yardımıyla çözümün tekliği araştırılmıştır. Burada kullanılan temel araç Carleman eşitsizlikleri olup Lavrenti'ev vd (1986), Amirov ve Yamamoto (2006) ve Gölgeleyen ve Yamamoto (2019) çalışmalarında verilen yöntemler kullanılmıştır. Bu denklemler fiziksel olarak çok boyutlu zaman kavramını içerdiğinden modern fizik kuramları açısından önemli bir hale gelmiştir.

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