ZONGULDAK BÜLENT ECEVİT ÜNİVERSİTESİ
FEN BİLİMLERİ ENSTİTÜSÜ

ULTRAHİPERBOLİK SCHRÖDINGER DENKLEMİ İÇİN BİR TERS PROBLEM

MATEMATİK ANABİLİM DALI

YÜKSEK LİSANS TEZİ

SABRİYE GÖZDE KİRLİ

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DANIŞMAN: Doç. Dr. Fikret GÖLGELEYEN

ZONGULDAK

Ağustos 2019

KABUL:

Sabriye Gözde KİRLİ tarafından hazırlanan "Ultrahiperbolik Schrödinger Denklemi İçin Bir Ters Problem" başlıklı bu çalışma jürimiz tarafından değerlendirilerek Zonguldak Bülent Ecevit Üniversitesi, Fen Bilimleri Enstitüsü, Matematik Anabilim Dalında Yüksek Lisans Tezi olarak oybirliğiyle kabul edilmiştir. 23/08/2019

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ONAY:

Yukarıdaki imzaların, adı geçen öğretim üyelerine ait olduğunu onaylarım. ..../..../2019

Prof. Dr. Ahmet ÖZARSLAN
Fen Bilimleri Enstitüsü Müdürü

“Bu tezdeki tüm bilgilerin akademik kurallara ve etik ilkelere uygun olarak elde edildiğini ve sunulduğunu; ayrıca bu kuralların ve ilkelerin gerektirdiği şekilde, bu çalışmadan kaynaklanmayan bütün atıfları yaptığımı beyan ederim.”

Sabriye Gözde KİRLİ

ÖZET

Yüksek Lisans Tezi

ULTRAHİPERBOLİK SCHRÖDINGER DENKLEMİ İÇİN BİR TERS PROBLEM

Sabriye Gözde KİRLİ

Zonguldak Bülent Ecevit Üniversitesi
Fen Bilimleri Enstitüsü
Matematik Anabilim Dalı

Tez Danışmanı: Doç. Dr. Fikret GÖLGELEYEN
Ağustos 2019, 77 sayfa

Bu tezde, bir ultrahiperbolik Schrödinger denklemi sınırsız bir bölgede Cauchy başlangıç şartları ile birlikte ele alınmıştır. Başlangıç anında çözüm hakkında verilen bir ek bilgi yardımıyla denklemin sağ tarafındaki bir bilinmeyen fonksiyonun belirlenmesi ters probleminin çözümünün tekliği araştırılmıştır. Bu kapsamda tezin birinci bölümünde, diğer bölümlerde ihtiyaç duyulan bazı temel tanım ve teoremler verilmiştir. İkinci bölümde bazı yardımcı önermeler yer almıştır. Son bölümde ise ele alınan problemin çözümünün tekliğine ilişkin bir teorem sunulmuştur.

Anahtar Kelimeler: Ultrahiperbolik Schrödinger denklemi, ters problem, çözümün tekliği.

Bilim Kodu: 403.06.00.

ABSTRACT

M. Sc. Thesis

AN INVERSE PROBLEM FOR THE ULTRAHYPERBOLIC SCHRÖDINGER EQUATION

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August 2019, 77 pages

In this thesis, an ultrahyperbolic Schrödinger equation is considered in an unbounded domain with Cauchy initial conditions. By using the given additional data at the initial time for the solution, uniqueness of solution of an inverse problem of determining a function in the right-hand side of the equation is investigated. In this context, in the first chapter, some basic definitions and theorems which are needed in the other chapters are given. In the second chapter, some auxiliary lemmata are given. In the last chapter, a theorem devoted to the uniqueness of solution of the problem is presented.

Keywords: Ultrahyperbolic Schrödinger equation, inverse problem, uniqueness of solution.

Science Code: 403.06.00.

TEŞEKKÜR

Tezimin tüm aşamalarında bilgi ve birikimi ile bana destek olan, değerli vaktini esirgemeyen, kullandığı her kelimeyle bakış açımı geliştiren, danışman hocam sayın Doç. Dr. Fikret GÖLGELEYEN'e teşekkürlerimi sunarım.

Bu süreç boyunca desteğini, ilgisini ve sabrını hiç esirgemeyen değerli aileme ve arkadaşlarıma teşekkür ederim.

İÇİNDEKİLER

SİMGELER VE KISALTMALAR DİZİNİ

SİMGELER

Q	: Verilen bir bölge
Q_γ	: Oluşturulan alt bölge
∂Q	: Q bölgesinin sınırı
$\overline{Q}$	: Q bölgesinin kapanışı
h	: Planck sabiti
$L^1(Q)$	: Q bölgesi üzerinde Lebesgue ölçülebilir ve integrallenebilir fonksiyonlar uzayı
$L^2(Q)$	: Q bölgesi üzerinde Lebesgue ölçülebilir ve karesi integrallenebilir fonksiyonlar uzayı
$C^k(Q)$	: Q bölgesinde tanımlı k. mertebeye kadar sürekli kısmi türevlere sahip fonksiyonlar uzayı
$H^k(Q)$	: Kendisi ve k. mertebeye kadar tüm genelleşmiş türevleri $L^2(Q)$ ya ait olan fonksiyonlar uzayı
λ, ν	: Büyük Parametreler
d_i	: Divergent terimler, $i = 1,2,3,4$
$\widehat{w}$	: w fonksiyonunun Fourier dönüşümü
u_t	: u fonksiyonunun t değişkenine göre kısmi türevi; $u_t = \frac{\partial u}{\partial t}$
u_{x_i}	: u fonksiyonunun x_i değişkenine göre kısmi türevi; $u_{x_i} = \frac{\partial u}{\partial x_i}$
u_{y_j}	: u fonksiyonunun y_j değişkenine göre kısmi türevi; $u_{y_j} = \frac{\partial u}{\partial y_j}$
$\nabla_x u$	: u fonksiyonunun x vektörüne göre gradienti; $\nabla_x u = (u_{x_1}, u_{x_2}, \dots, u_{x_n})$
$\nabla_y u$	: u fonksiyonunun y vektörüne göre gradienti; $\nabla_y u = (u_{y_1}, u_{y_2}, \dots, u_{y_m})$
$i = \sqrt{-1}$	: Sanal birim

BÖLÜM 1

ÖN BİLGİLER

1.1 GİRİŞ

Bu çalışmada bir

$Q=\{(x,y,t): x\in D\subset \mathbb{R}^n, y\in G\subset \mathbb{R}^m, t\in \mathbb{R}\}$

bölgesinde

$$\begin{aligned} Lu &\equiv iu_t+u_{x_1x_1}+\sum_{s=2}^{n}a_{ss}(x,y)u_{x_sx_s}-\sum_{j=1}^{m}b_{jj}(x,y)u_{y_jy_j} \\ &\quad +\sum_{r=1}^{n}a_r(x,y,t)u_{x_r}+\sum_{l=1}^{m}b_l(x,y,t)u_{y_l}+a_0(x,y,t)u \\ &= f(x,y,t)g(x,y) \end{aligned} \tag{1.1}$$

ultrahiperbolik Schrödinger denklemi

$$u(0,'x,y,t)=u_{x_1}(0,'x,y,t)=0 \tag{1.2}$$

koşulları ile birlikte ele alınacaktır. Burada $x=(x_1,x_2,...,x_n)$, $'x=(x_2,x_3,...,x_n)\in \mathbb{R}^{n-1}$, $y=(y_1,y_2,...,y_m)$, $\frac{\partial^2 u}{\partial x_1^2}=u_{x_1x_1}$, $\frac{\partial^2 u}{\partial x_s^2}=u_{x_sx_s}$, $\frac{\partial^2 u}{\partial y_j^2}=u_{y_jy_j}$, $\frac{\partial u}{\partial x_r}=u_{x_r}$, $\frac{\partial u}{\partial y_l}=u_{y_l}$ gösterimleri kullanılmıştır. Verilen bir ek bilgi yardımıyla (1.1) denkleminin sağ tarafındaki bilinmeyen fonksiyonun bulunması ters probleminin çözümünün tekliği araştırılmıştır. Bilindiği üzere kuantum mekaniğinde, bir parçacık $\psi(r,t)$ dalga fonksiyonuyla karakterize edilir. Bu fonksiyon, t anında parçacığın uzaysal konumu hakkında bilgi verir. Şimdi m kütleli ve $V(r,t)$ potansiyelinin etkisi altındaki bir parçacığı düşünelim. Bu durumda dalga fonksiyonu

$$i\hbar\frac{\partial\psi(r,t)}{\partial t}=-\frac{\hbar^2}{2m}\nabla^2\psi(r,t)+V(r,t)\psi(r,t) \tag{1.3}$$

Schrödinger denklemini sağlar. Burada $\hbar=\frac{h}{2\pi}$ olup h Planck sabitidir. Ayrıca ∇^2 Laplace operatörünü göstermektedir. Bu denklemin iki önemli özelliği vardır:

i) Schrödinger denklemi ψ ye göre lineer ve homojen bir denklemdir.

ii) Schrödinger denklemi zamana göre birinci mertebeden bir denklemdir. Bu nedenle t_0 anındaki durum bütün zamanlardaki alt durumları belirlemektedir (Peleg et al. 1998).

(1.1) denklemi genelleşmiş Schrödinger denklemi olarak da adlandırılır. (1.3) denklemi üzerine pek çok çalışma mevcut iken (1.1) denklemi ile ilgili sınırlı sayıda çalışma yapılmıştır. (1.3) denklemi için çeşitli ters problemler Baudouin ve Puel (2002), Mercado vd. (2008), Amirov ve Yamamoto (2008), Bellassoued ve Choulli (2009), Yuan ve Yamamoto (2010), Cristofol ve Soccorsi (2011), Kian vd. (2015), Triggiani ve Zhang (2015) de ele alınmıştır. Diğer yandan (1.1) denklemi için bir başlangıç değer probleminin çözülebilirliği Kenig vd. (1998, 2006) tarafından araştırılmıştır. Gölgeleyen ve Kaytmaz (2019), (1.1) denklemi için bir ters problemin çözümünün kararlılığını göstermiştir. Bu tür denklemlerin başta su dalgası problemleri olmak üzere farklı alanlarda önemli uygulamaları vardır (Davey and Stewartson 1974, Djordjevic and Redekopp 1977, Escauriaza et al. 2011, Ichinose 1990, Zakharov and Schulman 1980, Zakharov and Kuznetsov 1986). Ayrıca kuantum kinetik teorisinde de bu denklemler karşımıza çıkmaktadır.
Bu tezin birinci bölümünde temel tanım ve teoremler verilmiştir, ikinci bölümünde bazı yardımcı önermeler yer almıştır. Son bölümde ise ele alınan problemin çözümünün tekliğine ilişkin bir teorem sunulmuştur.

1.2 TEMEL TANIM VE TEOREMLER

Tanım 1.1 (Direkt Problem) *Matematiksel fizikte denklem, bölge ve koşullar verildiğinde denklemi ve koşulları sağlayan bir çözümün bulunmasına direkt problem denir (Amirov 2001).*

Tanım 1.2 (Ters Problem) *Pratikte karşılaşılan öyle problemler vardır ki, bunların çözümleri için ayrıca ek bilgiye gerek duyulur. Verilen bu ek bilgiye göre denklemin bir veya birkaç katsayısının veya sağ tarafının ya da sınır koşullarından bir veya birkaçını denklemin çözümü ile birlikte bulmak gerekir. Böyle problemlere ters problem adı verilir (Amirov 2001).*

Tanım 1.3 (Fourier Dönüşümü) $f \in L^1(\mathbb{R})$ *olmak üzere* f *fonksiyonunun Fourier dönüşümü ve eşlenik Fourier dönüşümü sırasıyla*

$$\mathcal{F}f(\xi) = \widehat{f}(\xi) = \int_{\mathbb{R}} e^{-i\xi x} f(x)dx,$$

$$\overline{\mathcal{F}}f(\xi) = \int_{\mathbb{R}} e^{i\xi x} f(x)dx$$

şeklinde tanımlanır (Gasquet and Witomski 1999). Eğer $\mathcal{F}[f] \in L^1(\mathbb{R})$ *ise* $\overline{\mathcal{F}}$ *operatörü,* $\mathcal{F}$*'nin tersi olur* ($\mathcal{F}^{-1} = \overline{\mathcal{F}}$)*. O halde* f *ve* $\widehat{f}$ *fonksiyonları* $L^1(\mathbb{R})$ *uzayına ait olduğunda, hemen hemen her yerde* $\overline{\mathcal{F}}\widehat{f}(t) = f(t)$ *yazılabilir.*

Tanım 1.4 ($C^k(Q)$ Uzayı) Q, $\mathbb{R}^n$ *uzayında bir bölge olsun. Her negatif olmayan* k *tam sayısı için* $|\alpha| \leq k$ *olmak üzere* $D^\alpha \varphi$ *kısmi türevleri* Q *bölgesinde sürekli olacak şekilde tüm* φ *fonksiyonlarının oluşturduğu vektör uzayı* $C^k(Q)$ *ile gösterilir (Adams and Fournier 2003). Bu uzayda norm aşağıdaki şekilde tanımlanır (Mikhailov 1978):*

$$\|f\|_{C^k(\overline{Q})} = \sum_{|a| \leq k} \max_{x \in \overline{Q}} |D^\alpha f(x)| .$$

Tanım 1.5 ($L^1(Q)$, $L^2(Q)$ Uzayları) $L^1(Q)$, Q *üzerinde Lebesgue ölçülebilir ve integrallenebilir tüm fonksiyonların oluşturduğu cümle;* $L^2(Q)$, Q *üzerinde Lebesgue ölçülebilir ve modülünün karesi integrallenebilir tüm fonksiyonların oluşturduğu cümledir. Bu kümelere ait bazı önemli özellikler aşağıda verilmiştir (Mikhailov 1978):*

i) $L^1(Q)$ *ve* $L^2(Q)$ *lineer uzaydır ve* Q *sınırlı bir bölge ise*

$$C(\overline{Q}) \subset L^2(Q) \subset L^1(Q),$$

ii) $L^1(Q)$, *üzerinde tanımlanan*

$$\|f\|_{L^1(Q)} = \int_Q |f(x)|\, dx$$

normuna göre bir Banach uzayıdır,

iii) $L^2(Q)$, *üzerinde tanımlanan*

$$(f_1, f_2)_{L^2(Q)} = \int_Q f_1(x)\overline{f_2(x)}dx$$

iç çarpımına göre bir Hilbert uzayıdır, bu iç çarpım ile tanımlanan norm

$$\|f\|_{L^2(Q)} = (\int_Q |f(x)|^2 \, dx)^{1/2}$$

biçimindedir.

Tanım 1.6 ($H^k(Q)$ Uzayı) *$H^k(Q)$, kendisi ve k. mertebeye kadar tüm genelleşmiş türevleri $L^2(Q)$ uzayına ait olan tüm fonksiyonların oluşturduğu cümledir. Bu cümleye ait bazı özellikler aşağıda verilmiştir:*

i) *$H^k(Q)$ lineer uzaydır ve $H^0(Q) = L^2(Q)$ olarak tanımlanır.*

ii) *$H^k(Q)$ üzerinde tanımlanan*

$$(f_1, f_2)_{H^k(Q)} = \sum_{|\alpha| \leq m} \int_Q D^\alpha f_1 D^\alpha \overline{f_2} dx$$

iç çarpımına göre bir Hilbert uzayıdır, bu iç çarpım ile tanımlanan norm

$$\|f\|_{H^k(Q)} = (\sum_{|\alpha| \leq k} \int_Q |D^\alpha f|^2 \, dx)^{1/2}$$

biçimindedir (Mikhailov 1978).

Teorem 1.1 *Eğer $f \in C^2(\mathbb{R})$ ve f, f', f'' fonksiyonları integrallenebilirse $\widehat{f}$ integrallenebilirdir (Gasquet and Witomski 1999).*

Teorem 1.2 (Plancherel) *$\mathbb{R}$'nin Lebesgue ölçümü sonsuz olduğundan $L^2(\mathbb{R})$ kümesi $L^1(\mathbb{R})$'nin alt kümesi değildir. Bu nedenle yukarıda verilen Fourier dönüşümü tanımı her $f \in L^2(\mathbb{R})$ fonksiyonu için direkt olarak uygulanabilir değildir. Ancak, eğer $f \in L^1(\mathbb{R}) \cap L^2(\mathbb{R})$ ise yukarıdaki tanımın kolayca uygulanabileceği açıktır. Ayrıca bu durumda $\widehat{f} \in L^2(\mathbb{R})$ dir. Aslında $||\widehat{f}||_{L^2} = ||f||_{L^2}$ olup bu bağıntıya Plancherel dönüşümü adı verilir. Böylece L^2 uzayında f ve $\widehat{f}$ aynı rolü oynar (Rudin 1987).*

Teorem 1.3 *Her $f \in L^2$ fonksiyonuna, aşağıdaki özellikleri sağlayan bir $\widehat{f} \in L^2$ fonksiyonu karşılık gelir:*

i) Eğer $f \in L^1 \cap L^2$ ise $\widehat{f}$ daha önce tanımlandığı şekliyle f'nin Fourier dönüşümüdür.

ii) Her $f \in L^2$ ise $||\widehat{f}||_{L^2} = ||f||_{L^2}$.

iii) $f \to \widehat{f}$ dönüşümü $L^2 \to L^2$ ye olan bir Hilbert uzayı izomorfizmidir (Rudin 1987).

BÖLÜM 2

BAZI YARDIMCI ÖNERMELER

Ele alınan ters problemin çözümünün tekliğini araştırmak için öncelikle bazı lemmalar vereceğiz. Denklemi daha uygun bir hale getirmek için yeni bir değişken dönüşümü tanımlayalım:

$$\widetilde{x}_1 = \sqrt{2x_1} - \delta_0,\ x_1 = \frac{1}{2}(\widetilde{x}_1 + \delta_0)^2,\ 2\delta_0 \leq \min\{\alpha_0, \gamma\},\ \delta_0 > 0.$$

Buna göre

$$\begin{aligned} u(x,y,t) &= u(\frac{1}{2}(\widetilde{x}_1+\delta_0)^2, x_2, ..., x_n, y, t) \equiv \widetilde{u}(\widetilde{x}_1, x_2, ..., x_n, y, t), \\ u_{x_1} &= \widetilde{u}_{\widetilde{x}_1}\frac{d\widetilde{x}_1}{dx_1} = \widetilde{u}_{\widetilde{x}_1}\frac{1}{\widetilde{x}_1+\delta_0};\ u_{x_1x_1} = \widetilde{u}_{\widetilde{x}_1\widetilde{x}_1}(\widetilde{x}_1+\delta_0)^{-2} - \widetilde{u}_{\widetilde{x}_1}(\widetilde{x}_1+\delta_0)^{-3} \end{aligned}$$

olduğu görülür. Bu durumda (1.1) denklemi aşağıdaki formlarda yazılabilir:

$$\begin{aligned} & i\widetilde{u}_t + \widetilde{u}_{\widetilde{x}_1\widetilde{x}_1}(\widetilde{x}_1+\delta_0)^{-2} - \widetilde{u}_{\widetilde{x}_1}(\widetilde{x}_1+\delta_0)^{-3} + \sum_{s=2}^{n}\widetilde{a}_{ss}\widetilde{u}_{x_sx_s} - \sum_{j=1}^{m}\widetilde{b}_{jj}\widetilde{u}_{y_jy_j} \\ & +(\widetilde{a}_1(\widetilde{x}_1+\delta_0) + (\widetilde{x}_1+\delta_0)^{-2})\widetilde{u}_{\widetilde{x}_1}\frac{1}{\widetilde{x}_1+\delta_0} + \sum_{r=2}^{n}\widetilde{a}_r\widetilde{u}_{x_r} + \sum_{l=1}^{m}\widetilde{b}_l\widetilde{u}_{y_l} + \widetilde{a}_0\widetilde{u} \\ = \ & \widetilde{f}(x,y,t)\widetilde{g}(x,y), \end{aligned}$$

$$\begin{aligned} & i\widetilde{u}_t + \widetilde{u}_{\widetilde{x}_1\widetilde{x}_1}(\widetilde{x}_1+\delta_0)^{-2} + \widetilde{u}_{\widetilde{x}_1}(\widetilde{a}_1 + (\widetilde{x}_1+\delta_0)^{-3} - (\widetilde{x}_1+\delta_0)^{-3}) + \sum_{s=2}^{n}\widetilde{a}_{ss}\widetilde{u}_{x_sx_s} \\ & -\sum_{j=1}^{m}\widetilde{b}_{jj}\widetilde{u}_{y_jy_j} + \sum_{r=2}^{n}\widetilde{a}_r\widetilde{u}_{x_r} + \sum_{l=1}^{m}\widetilde{b}_l\widetilde{u}_{y_l} + \widetilde{a}_0\widetilde{u} \\ = \ & \widetilde{f}(x,y,t)\widetilde{g}(x,y), \end{aligned}$$

$$\begin{aligned} & (\widetilde{x}_1+\delta_0)i\widetilde{u}_t + (\widetilde{x}_1+\delta_0)^{-1}\widetilde{u}_{\widetilde{x}_1\widetilde{x}_1} + (\widetilde{x}_1+\delta_0)\widetilde{a}_1\widetilde{u}_{\widetilde{x}_1} + (\widetilde{x}_1+\delta_0)\sum_{s=2}^{n}\widetilde{a}_{ss}\widetilde{u}_{x_sx_s} \\ & -(\widetilde{x}_1+\delta_0)\sum_{j=1}^{m}\widetilde{b}_{jj}\widetilde{u}_{y_jy_j} + (\widetilde{x}_1+\delta_0)\sum_{r=2}^{n}\widetilde{a}_r\widetilde{u}_{x_r} + (\widetilde{x}_1+\delta_0)\sum_{l=1}^{m}\widetilde{b}_l\widetilde{u}_{y_l} + (\widetilde{x}_1+\delta_0)\widetilde{a}_0\widetilde{u} \\ = \ & (\widetilde{x}_1+\delta_0)\widetilde{f}(x,y,t)\widetilde{g}(x,y), \end{aligned}$$

$$
\begin{aligned}
&(\widetilde{x}_1+\delta_0)^{-1}\widetilde{u}_{\widetilde{x}_1\widetilde{x}_1}+(\widetilde{x}_1+\delta_0)(\sum_{s=2}^{n}\widetilde{a}_{ss}\widetilde{u}_{x_sx_s}-\sum_{j=1}^{m}\widetilde{b}_{jj}\widetilde{u}_{y_jy_j}+i\widetilde{u}_t)\\
&+(\widetilde{x}_1+\delta_0)(\sum_{r=1}^{n}\widetilde{a}_r\widetilde{u}_{x_r}+\sum_{l=1}^{m}\widetilde{b}_l\widetilde{u}_{y_l}+\widetilde{a}_0\widetilde{u})\\
=\;&(\widetilde{x}_1+\delta_0)\widetilde{f}(x,y,t)\widetilde{g}(x,y).
\end{aligned}
$$

Burada

$$
\begin{aligned}
\widetilde{a}_{ss} &= a_{ss}(\frac{1}{2}(\widetilde{x}_1+\delta_0)^2,x_2,...,x_n,y);\;\; s=2,...,n,\\
\widetilde{a}_r &= a_r(\frac{1}{2}(\widetilde{x}_1+\delta_0)^2,'x,y,t);\;\; r=2,...,n,\\
\widetilde{a}_0 &= a_0(\frac{1}{2}(\widetilde{x}_1+\delta_0)^2,'x,y,t),\\
\widetilde{a}_1 &= a_1(\frac{1}{2}(\widetilde{x}_1+\delta_0)^2,'x,y,t)(\widetilde{x}_1+\delta_0)^{-1}-(\widetilde{x}_1+\delta_0)^{-3},\\
\widetilde{b}_{jj} &= b_{jj}(\frac{1}{2}(\widetilde{x}_1+\delta_0)^2,x_2,...,x_n,y);\;\; j=1,...,m,\\
\widetilde{b}_l &= b_l(\frac{1}{2}(\widetilde{x}_1+\delta_0)^2,'x,y,t);\;\; l=1,...,m,\\
\widetilde{f} &= f(\frac{1}{2}(\widetilde{x}_1+\delta_0)^2,'x,y,t),\\
\widetilde{g} &= g(\frac{1}{2}(\widetilde{x}_1+\delta_0)^2,'x,y)
\end{aligned}
$$

şeklinde alınmıştır. Basitlik açısından $\widetilde{x}_1$, $\widetilde{x}_s$, $\widetilde{x}_r$, $\widetilde{y}_j$, $\widetilde{y}_l$, $\widetilde{u}$, $\widetilde{a}_{ss}$, $\widetilde{a}_r$, $\widetilde{b}_{jj}$, $\widetilde{b}_l$ gösterimlerini x_1, x_s, x_r, y_j, y_l, u, a_{ss}, a_r, b_{jj}, b_l gösterimleri ile değiştirelim:

$$
\begin{aligned}
\widetilde{L}u &\equiv (x_1+\delta_0)^{-1}u_{x_1x_1}+(x_1+\delta_0)(\sum_{s=2}^{n}a_{ss}u_{x_sx_s}-\sum_{j=1}^{m}b_{jj}u_{y_jy_j}+iu_t)\\
&\quad+(x_1+\delta_0)(\sum_{r=1}^{n}a_ru_{x_r}+\sum_{l=1}^{m}b_lu_{y_l}+a_0u)\\
&= (x_1+\delta_0)f(x,y,t)g(x,y). \qquad (2.1)
\end{aligned}
$$

Şimdi yeni bir bilinmeyen fonksiyon tanımlayalım:

$w=ue^{-kt^2}$,

$k\geq\max\{d_u,d_f\}$ olup d_u, d_f sayıları üçüncü bölümde tanımlanmıştır. Buradan

$$
\begin{aligned}
u &= we^{kt^2},\;\; u_t=w_te^{kt^2}+2ktwe^{kt^2},\\
u_{x_1} &= w_{x_1}e^{kt^2},\;\; u_{x_1x_1}=w_{x_1x_1}e^{kt^2},\\
u_{x_s} &= w_{x_s}e^{kt^2},\;\; u_{x_sx_s}=w_{x_sx_s}e^{kt^2},\\
u_{y_j} &= w_{y_j}e^{kt^2},\;\; u_{y_jy_j}=w_{y_jy_j}e^{kt^2}
\end{aligned}
$$

yazılabilir. O halde (2.1) denkleminden

$$
\begin{aligned}
& (x_1+\delta_0)^{-1} w_{x_1x_1} e^{kt^2} \\
& +(x_1+\delta_0)(\sum_{s=2}^{n} a_{ss} w_{x_sx_s} e^{kt^2} - \sum_{j=1}^{m} b_{jj} w_{y_jy_j} e^{kt^2} + i(w_t e^{kt^2} + 2ktwe^{kt^2})) \\
& +(x_1+\delta_0)(\sum_{r=1}^{n} a_r w_{x_r} e^{kt^2} + \sum_{l=1}^{m} b_l w_{y_l} e^{kt^2} + a_0 w e^{kt^2}) \\
= \; & (x_1+\delta_0) f(x,y,t) g(x,y),
\end{aligned}
$$

$$
\begin{aligned}
& 2(x_1+\delta_0) iktwe^{kt^2} + (x_1+\delta_0) i w_t e^{kt^2} + (x_1+\delta_0)^{-1} w_{x_1x_1} e^{kt^2} \\
& +(x_1+\delta_0)(\sum_{s=2}^{n} a_{ss} w_{x_sx_s} e^{kt^2} - \sum_{j=1}^{m} b_{jj} w_{y_jy_j} e^{kt^2}) \\
& +(x_1+\delta_0)(\sum_{r=1}^{n} a_r w_{x_r} e^{kt^2} + \sum_{l=1}^{m} b_l w_{y_l} e^{kt^2} + a_0 w e^{kt^2}) \\
= \; & (x_1+\delta_0) f(x,y,t) g(x,y)
\end{aligned}
$$

elde edilir. Diğer bir şekilde

$$
\begin{aligned}
\widetilde{L}w + 2(x_1+\delta_0) iktw &= (x_1+\delta_0) F(x,y,t) g(x,y), && (2.2) \\
w(0,' x, y, t) &= w_{x_1}(0,' x, y, t) = w(x,y,0) = 0 && (2.3)
\end{aligned}
$$

olarak yazılabilir. Burada

$$F(x,y,t) = f(x,y,t) e^{-kt^2}$$

dir. Diğer yandan

$$
\begin{aligned}
\widetilde{L}w \; = \; & (x_1+\delta_0) i w_t + (x_1+\delta_0)^{-1} w_{x_1x_1} + (x_1+\delta_0)(\sum_{s=2}^{n} a_{ss} w_{x_sx_s} - \sum_{j=1}^{m} b_{jj} w_{y_jy_j}) \\
& +(x_1+\delta_0)(\sum_{r=1}^{n} a_r w_{x_r} + \sum_{l=1}^{m} b_l w_{y_l} + a_0 w)
\end{aligned}
$$

olduğu göz önünde bulundurularak $t = 0$ için (2.2) denkleminden

$$
\begin{aligned}
g(x,y) &= \frac{1}{f(x,y,0)} (i w_t(x,y,0)), \\
g(x,y) &= -\frac{1}{2\pi f(x,y,0)} \int_{-\infty}^{\infty} \xi \widehat{w}(x,y,\xi) d\xi && (2.4)
\end{aligned}
$$

olarak bulunur. Burada

$$2\pi f(x,y,0) = f_0, \; \mathcal{F}(w_t) = i\xi\widehat{w}, \; \mathcal{F}^{-1}\mathcal{F}(w_t) = \mathcal{F}^{-1}(i\xi\widehat{w}) = \frac{1}{2\pi} \int_{-\infty}^{\infty} i\xi\widehat{w} e^{i\xi t} d\xi$$

gösterimleri kullanılmıştır.

(2.2) denklemine ve (2.3) koşullarına t-ye göre Fourier dönüşümü uygulanır ve $i = \sqrt{-1}$ olduğu göz önünde bulundurulursa

$$\begin{aligned} & -(x_1+\delta_0)\xi\widehat{w} + (x_1+\delta_0)^{-1}\widehat{w}_{x_1x_1} + (x_1+\delta_0)(\sum_{s=2}^{n} a_{ss}\widehat{w}_{x_sx_s} - \sum_{j=1}^{m} b_{jj}\widehat{w}_{y_jy_j}) \\ & +(x_1+\delta_0)(\sum_{r=1}^{n}\widehat{a_rw_{x_r}} + \sum_{l=1}^{m}\widehat{b_lw_{y_l}} + \widehat{a_0w}) - 2(x_1+\delta_0)k\widehat{w}_\xi \\ = & -(x_1+\delta_0)\widehat{F}\frac{1}{f_0}\int_{-\infty}^{\infty}\xi\widehat{w}d\xi \end{aligned} \tag{2.5}$$

ve

$$\widehat{w}(0,'x,y,\xi) = \widehat{w}_{x_1}(0,'x,y,\xi) = 0 \tag{2.6}$$

elde edilir. Diğer taraftan $\widehat{w} = \varphi_1(x,y,\xi) + i\varphi_2(x,y,\xi)$, $\widehat{F} = f_1(x,y,\xi) + if_2(x,y,\xi)$ şeklinde yazılabildiği kabul edilerek aşağıdaki gösterimi tanımlayalım:

$$\begin{aligned} L_0\varphi \equiv & -(x_1+\delta_0)\xi\varphi + (x_1+\delta_0)^{-1}\varphi_{x_1x_1} \\ & +(x_1+\delta_0)(\sum_{s=2}^{n} a_{ss}\varphi_{x_sx_s} - \sum_{j=1}^{m} b_{jj}\varphi_{y_jy_j}) - 2(x_1+\delta_0)k\varphi_\xi \\ = & (x_1+\delta_0)^{-1}\varphi_{x_1x_1} + (x_1+\delta_0)(\sum_{s=2}^{n} a_{ss}\varphi_{x_sx_s} - \sum_{j=1}^{m} b_{jj}\varphi_{y_jy_j} - 2k\varphi_\xi - \xi\varphi). \end{aligned} \tag{2.7}$$

Burada φ, $\widehat{w}$ fonksiyonunun reel veya sanal kısmını göstermektedir. Daha açık olarak $\widehat{w} = \varphi_1(x,y,\xi) + i\varphi_2(x,y,\xi)$ olmak üzere $\varphi = \varphi_1$ veya $\varphi = \varphi_2$ dir.

Aşağıdaki gösterimleri tanımlayalım:

$$Q_\gamma = \{(x,y) : x \in \mathbb{R}^n, y \in \mathbb{R}^m, x_1 > 0, 0 < \delta x_1 + \frac{1}{2}\sum_{j=2}^{n}(x_j - x_j^0)^2 + \frac{1}{2}\sum_{s=1}^{m}(y_s - y_s^0)^2 < \gamma\},$$

$$\psi(x,y) = \delta x_1 + \frac{1}{2}\sum_{j=2}^{n}(x_j - x_j^0)^2 + \frac{1}{2}\sum_{s=1}^{m}(y_s - y_s^0)^2;$$

$\chi = e^{\lambda\psi^{-\nu}}$, $a_0 > 0$, $\gamma + a_0 = \eta < 1$, $a_0 < \psi(x,y) < \eta$.

2.1 LEMMA 2.1

Lemma 2.1 *Kabul edelim ki* $||a_{ii}||_{C^3(\overline{D\times G})} \leq M$, $2 \leq i \leq n$, $||b_{jj}||_{C^3(\overline{D\times G})} \leq M$, $1 \leq j \leq m$ *ve* $0 < \gamma < \min\{\frac{2\sqrt{2\varepsilon_0}}{3M\sqrt{n+m-1}}, 1\}$ *olsun. Burada* ε_0 *bir sabit olup* $0 < \varepsilon_0 < \frac{\alpha_1}{4}$, $0 < \varepsilon_0 < \frac{\alpha_2}{4}$ *şeklindedir. Bu durumda yeterince büyük*

$\delta_* = \delta_*(\alpha_1, \alpha_2, M, n, m, \gamma)$, $\nu_* = \nu_*(\alpha_1, \alpha_2, M, \delta, n, m, \gamma)$, $\lambda_* = \lambda_*(\alpha_1, \alpha_2, M, \nu, n, m, \gamma)$

sayıları vardır öyle ki eğer $\delta > \delta_*$, $\nu > \nu_*$ ise tüm $\varphi \in C^2(\overline{Q}_\gamma)$ ve her $\lambda > \lambda_*$ için aşağıdaki eşitsizlik geçerlidir:

$$\begin{aligned} & \psi^{\nu+1}(L_0\varphi)^2\chi^2 + 2\lambda\nu(x_1+\delta_0)\chi^2\Lambda\varphi(L_0\varphi) \\ \geq\ & 2\lambda\nu\varphi_{x_1}^2\chi^2 + \lambda\nu(x_1+\delta_0)\chi^2\sum_{i=2}^{n}\varphi_{x_i}^2 + \lambda\nu(x_1+\delta_0)\chi^2\sum_{j=1}^{m}\varphi_{y_j}^2 \\ & +4\lambda^3\nu^3\psi^{-2\nu-3}\varphi^2\chi^2 + \sum_{i=1}^{4} d_i(\varphi). \end{aligned} \tag{2.8}$$

Lemma 2.1'i ispatlamak için aşağıda Lemma 2.2 ve Lemma 2.3 verilecektir.

2.2 LEMMA 2.2

Lemma 2.2 *Lemma 2.1'in koşulları sağlansın. Eğer λ, ν ve δ parametreleri bir pozitif sabitten büyük ise bu durumda her $\varphi \in C^2(\overline{Q}_\gamma)$ aşağıdaki eşitsizliği sağlar:*

$$\begin{aligned} \psi^{\nu+1}(L_0\varphi)^2\chi^2 \geq\ & 2\lambda\nu\gamma^{-3}\delta\varphi_{x_1}^2\chi^2 + \lambda\nu(x_1+\delta_0)\delta\alpha_1\chi^2\sum_{i=2}^{n}\varphi_{x_i}^2 \\ & +\lambda\nu(x_1+\delta_0)\delta\alpha_2\chi^2\sum_{j=1}^{m}\varphi_{y_j}^2 + \lambda^3\nu^4\delta^4\psi^{-2\nu-3}\varphi^2\chi^2 \\ & +2\lambda\nu\xi(x_1+\delta_0)^2\Big(\sum_{i=2}^{n}(a_{ii}\psi_{x_i})_{x_i} - \sum_{j=1}^{m}(b_{jj}\psi_{y_j})_{y_j}\Big)\varphi^2\chi^2 + \sum_{j=1}^{3} d_j(\varphi). \end{aligned} \tag{2.9}$$

İspat. İlk olarak yeni bir bilinmeyen fonksiyon tanımlayalım:

$\vartheta = \chi\varphi.$

Bu durumda

$$\begin{aligned} \varphi &= \vartheta\chi^{-1},\ \varphi_\xi = \vartheta_\xi\chi^{-1}, \\ \varphi_{x_i} &= \chi^{-1}(\vartheta_{x_i} + \lambda\nu\psi^{-\nu-1}\psi_{x_i}\vartheta), \end{aligned}$$

$$\begin{aligned} \varphi_{x_ix_i} &= \chi^{-1}(\vartheta_{x_ix_i} + \lambda\nu\psi^{-\nu-1}\psi_{x_i}\vartheta_{x_i} + \lambda\nu\psi^{-\nu-1}\psi_{x_i}\vartheta_{x_i} \\ &\quad +(\lambda^2\nu^2\psi^{-2\nu-2}\psi_{x_i}^2 - \lambda\nu(\nu+1)\psi^{-\nu-2}\psi_{x_i}^2 + \lambda\nu\psi^{-\nu-1}\psi_{x_ix_i})\vartheta) \\ &= \chi^{-1}(\vartheta_{x_ix_i} + 2\lambda\nu\psi^{-\nu-1}\psi_{x_i}\vartheta_{x_i} + (\lambda^2\nu^2\psi^{-2\nu-2}\psi_{x_i}^2 - \lambda\nu(\nu+1)\psi^{-\nu-2}\psi_{x_i}^2 \\ &\quad +\lambda\nu\psi^{-\nu-1}\psi_{x_ix_i})\vartheta), \end{aligned}$$

$$
\begin{aligned}
\varphi_{x_1x_1} &= \chi^{-1}(\vartheta_{x_1x_1} + 2\lambda\nu\psi^{-\nu-1}\psi_{x_1}\vartheta_{x_1} + (\lambda^2\nu^2\psi^{-2\nu-2}\psi_{x_1}^2 - \lambda\nu(\nu+1)\psi^{-\nu-2}\psi_{x_1}^2 \\
&\quad +\lambda\nu\psi^{-\nu-1}\psi_{x_1x_1})\vartheta), \\
\varphi_{y_jy_j} &= \chi^{-1}(\vartheta_{y_jy_j} + 2\lambda\nu\psi^{-\nu-1}\psi_{y_j}\vartheta_{y_j} + (\lambda^2\nu^2\psi^{-2\nu-2}\psi_{y_j}^2 - \lambda\nu(\nu+1)\psi^{-\nu-2}\psi_{y_j}^2 \\
&\quad +\lambda\nu\psi^{-\nu-1}\psi_{y_jy_j})\vartheta)
\end{aligned}
$$

eşitlikleri yazılabilir. Yukarıdaki eşitlikler dikkate alınır ve gerekli hesaplamalar yapılırsa,

$$
\begin{aligned}
\psi^{\nu+1}(L_0\varphi)^2\chi^2 &= \psi^{\nu+1}((x_1+\delta_0)^{-1}\varphi_{x_1x_1} + (x_1+\delta_0)(\sum_{i=2}^{n} a_{ii}\varphi_{x_ix_i} - \sum_{j=1}^{m} b_{jj}\varphi_{y_jy_j} \\
&\quad -2k\varphi_\xi - \xi\varphi))^2\chi^2 \\
&= \chi^2\psi^{\nu+1}((x_1+\delta_0)^{-1}\chi^{-1}(\vartheta_{x_1x_1} + 2\lambda\nu\psi^{-\nu-1}\psi_{x_1}\vartheta_{x_1} \\
&\quad +(\lambda^2\nu^2\psi^{-2\nu-2}\psi_{x_1}^2 - \lambda\nu(\nu+1)\psi^{-\nu-2}\psi_{x_1}^2 + \lambda\nu\psi^{-\nu-1}\psi_{x_1x_1})\vartheta) \\
&\quad +(x_1+\delta_0)(\sum_{i=2}^{n} a_{ii}\chi^{-1}(\vartheta_{x_ix_i} + 2\lambda\nu\psi^{-\nu-1}\psi_{x_i}\vartheta_{x_i} \\
&\quad +(\lambda^2\nu^2\psi^{-2\nu-2}\psi_{x_i}^2 - \lambda\nu(\nu+1)\psi^{-\nu-2}\psi_{x_i}^2 + \lambda\nu\psi^{-\nu-1}\psi_{x_ix_i})\vartheta) \\
&\quad -\sum_{j=1}^{m} b_{jj}\chi^{-1}(\vartheta_{y_jy_j} + 2\lambda\nu\psi^{-\nu-1}\psi_{y_j}\vartheta_{y_j} \\
&\quad +(\lambda^2\nu^2\psi^{-2\nu-2}\psi_{y_j}^2 - \lambda\nu(\nu+1)\psi^{-\nu-2}\psi_{y_j}^2 + \lambda\nu\psi^{-\nu-1}\psi_{y_jy_j})\vartheta) \\
&\quad -2k\vartheta_\xi\chi^{-1} - \xi\vartheta\chi^{-1}))^2 \\
&= \psi^{\nu+1}(z_1+z_2+z_3+z_4+z_5)^2 \\
&= \psi^{\nu+1}(z_1^2 + (z_2+z_4+z_5)^2 + z_3^2 + 2z_1z_2 + 2z_1z_3 \\
&\quad +2z_1(z_4+z_5) + 2z_2z_3 + 2z_3(z_4+z_5)) \qquad (2.10)
\end{aligned}
$$

bulunur. Burada

$$
\begin{aligned}
z_1 &= 2k(x_1+\delta_0)\vartheta_\xi, \\
z_2 &= -((x_1+\delta_0)^{-1}\vartheta_{x_1x_1} + (x_1+\delta_0)(\sum_{i=2}^{n} a_{ii}\vartheta_{x_ix_i} - \sum_{j=1}^{m} b_{jj}\vartheta_{y_jy_j})), \\
z_3 &= -2\lambda\nu\psi^{-\nu-1}(\psi_{x_1}(x_1+\delta_0)^{-1}\vartheta_{x_1} + (x_1+\delta_0)(\sum_{i=2}^{n} a_{ii}\psi_{x_i}\vartheta_{x_i} - \sum_{j=1}^{m} b_{jj}\psi_{y_j}\vartheta_{y_j})), \\
z_4 &= -E_0\vartheta, \\
z_5 &= \xi(x_1+\delta_0)\vartheta,
\end{aligned}
$$

$$
\begin{aligned}
E_0 &= (x_1+\delta_0)^{-1}(\lambda^2\nu^2\psi^{-2\nu-2}\psi_{x_1}^2-\lambda\nu(\nu+1)\psi^{-\nu-2}\psi_{x_1}^2)\\
&+(x_1+\delta_0)(\sum_{i=2}^{n}a_{ii}(\lambda^2\nu^2\psi^{-2\nu-2}\psi_{x_i}^2-\lambda\nu(\nu+1)\psi^{-\nu-2}\psi_{x_i}^2+\lambda\nu\psi^{-\nu-1}\psi_{x_ix_i})\\
&-\sum_{j=1}^{m}b_{jj}(\lambda^2\nu^2\psi^{-2\nu-2}\psi_{y_j}^2-\lambda\nu(\nu+1)\psi^{-\nu-2}\psi_{y_j}^2+\lambda\nu\psi^{-\nu-1}\psi_{y_jy_j}))
\end{aligned}
$$

olarak tanımlıdır. Şimdi (2.10) eşitliğindeki terimleri tek tek değerlendirelim. Burada

$$
\begin{aligned}
\psi_{x_ix_s} &= \begin{cases}1, & i=s\\ 0, & i\neq s\end{cases} \quad (2\le i,s\le n);\ \psi_{y_jy_k}=\begin{cases}1, & j=k\\ 0, & j\neq k\end{cases} \quad (1\le j,k\le m),\\
\psi_{x_iy_j} &= 0,\ \psi_{x_1}=\delta,\ \psi_{x_1x_1}=0
\end{aligned}
$$

olduğuna dikkat edilmelidir.

İlk olarak

$$
\begin{aligned}
2\psi^{\nu+1}z_1z_2 &= 2\psi^{\nu+1}2k(x_1+\delta_0)\vartheta_\xi(-(x_1+\delta_0)^{-1}\vartheta_{x_1x_1}\\
&\quad -(x_1+\delta_0)(\sum_{i=2}^{n}a_{ii}\vartheta_{x_ix_i}-\sum_{j=1}^{m}b_{jj}\vartheta_{y_jy_j}))\\
&= -4k\psi^{\nu+1}(x_1+\delta_0)\vartheta_\xi((x_1+\delta_0)^{-1}\vartheta_{x_1x_1}\\
&\quad +(x_1+\delta_0)(\sum_{i=2}^{n}a_{ii}\vartheta_{x_ix_i}-\sum_{j=1}^{m}b_{jj}\vartheta_{y_jy_j}))
\end{aligned}
$$

$$
\begin{aligned}
&= -4k\psi^{\nu+1}\vartheta_\xi\vartheta_{x_1x_1}-4k\psi^{\nu+1}(x_1+\delta_0)^2\vartheta_\xi\sum_{i=2}^{n}a_{ii}\vartheta_{x_ix_i}\\
&\quad +4k\psi^{\nu+1}(x_1+\delta_0)^2\vartheta_\xi\sum_{j=1}^{m}b_{jj}\vartheta_{y_jy_j}\\
&= -2k(2\psi^{\nu+1}\vartheta_\xi\vartheta_{x_1x_1})-2k(x_1+\delta_0)^2\sum_{i=2}^{n}2\psi^{\nu+1}a_{ii}\vartheta_\xi\vartheta_{x_ix_i}\\
&\quad +2k(x_1+\delta_0)^2\sum_{j=1}^{m}2\psi^{\nu+1}b_{jj}\vartheta_\xi\vartheta_{y_jy_j}
\end{aligned}
$$

yazılabilir. Aşağıdaki işlemlerde ψ, a_{ii} ve b_{jj} fonksiyonlarının ξ değişkeninden bağımsız olduğu unutulmamalıdır.

$$
\begin{aligned}
2\psi^{\nu+1}\vartheta_\xi\vartheta_{x_1x_1} &= (\psi^{\nu+1}\vartheta_\xi\vartheta_{x_1})_{x_1}+(\psi^{\nu+1}\vartheta_\xi\vartheta_{x_1})_{x_1}-(\psi^{\nu+1}\vartheta_{x_1}^2)_\xi\\
&\quad -(\nu+1)\psi^\nu\psi_{x_1}\vartheta_\xi\vartheta_{x_1}-(\nu+1)\psi^\nu\psi_{x_1}\vartheta_\xi\vartheta_{x_1}+(\psi^{\nu+1})_\xi\vartheta_{x_1}^2\\
&= 2(\psi^{\nu+1}\vartheta_\xi\vartheta_{x_1})_{x_1}-2(\nu+1)\psi^\nu\psi_{x_1}\vartheta_\xi\vartheta_{x_1}-(\psi^{\nu+1}\vartheta_{x_1}^2)_\xi,
\end{aligned}
$$

$$
\begin{aligned}
2\psi^{\nu+1}a_{ii}\vartheta_\xi\vartheta_{x_ix_i} &= (\psi^{\nu+1}a_{ii}\vartheta_\xi\vartheta_{x_i})_{x_i} + (\psi^{\nu+1}a_{ii}\vartheta_\xi\vartheta_{x_i})_{x_i} - (\psi^{\nu+1}a_{ii}\vartheta_{x_i}^2)_\xi \\
&\quad -(\psi^{\nu+1}a_{ii})_{x_i}\vartheta_\xi\vartheta_{x_i} - (\psi^{\nu+1}a_{ii})_{x_i}\vartheta_\xi\vartheta_{x_i} + (\psi^{\nu+1}a_{ii})_\xi\vartheta_{x_i}^2 \\
&= 2(\psi^{\nu+1}a_{ii}\vartheta_\xi\vartheta_{x_i})_{x_i} - (\psi^{\nu+1}a_{ii}\vartheta_{x_i}^2)_\xi - 2(\psi^{\nu+1}a_{ii})_{x_i}\vartheta_\xi\vartheta_{x_i} \\
&= 2(\psi^{\nu+1}a_{ii}\vartheta_\xi\vartheta_{x_i})_{x_i} - (\psi^{\nu+1}a_{ii}\vartheta_{x_i}^2)_\xi \\
&\quad -2(\nu+1)\psi^\nu\psi_{x_i}a_{ii}\vartheta_\xi\vartheta_{x_i} - 2\psi^{\nu+1}(a_{ii})_{x_i}\vartheta_\xi\vartheta_{x_i},
\end{aligned}
$$

$$
\begin{aligned}
2\psi^{\nu+1}b_{jj}\vartheta_\xi\vartheta_{y_jy_j} &= (\psi^{\nu+1}b_{jj}\vartheta_\xi\vartheta_{y_j})_{y_j} + (\psi^{\nu+1}b_{jj}\vartheta_\xi\vartheta_{y_j})_{y_j} - (\psi^{\nu+1}b_{jj}\vartheta_{y_j}^2)_\xi \\
&\quad -(\psi^{\nu+1}b_{jj})_{y_j}\vartheta_\xi\vartheta_{y_j} - (\psi^{\nu+1}b_{jj})_{y_j}\vartheta_\xi\vartheta_{y_j} + (\psi^{\nu+1}b_{jj})_\xi\vartheta_{y_j}^2 \\
&= 2(\psi^{\nu+1}b_{jj}\vartheta_\xi\vartheta_{y_j})_{y_j} - (\psi^{\nu+1}b_{jj}\vartheta_{y_j}^2)_\xi - 2(\psi^{\nu+1}b_{jj})_{y_j}\vartheta_\xi\vartheta_{y_j} \\
&= 2(\psi^{\nu+1}b_{jj}\vartheta_\xi\vartheta_{y_j})_{y_j} - (\psi^{\nu+1}b_{jj}\vartheta_{y_j}^2)_\xi \\
&\quad -2(\nu+1)\psi^\nu\psi_{y_j}b_{jj}\vartheta_\xi\vartheta_{y_j} - 2\psi^{\nu+1}(b_{jj})_{y_j}\vartheta_\xi\vartheta_{y_j}
\end{aligned}
$$

eşitlikleri yerine yazılırsa

$$
\begin{aligned}
2\psi^{\nu+1}z_1z_2 &= -4k(\psi^{\nu+1}\vartheta_\xi\vartheta_{x_1})_{x_1} + 4k(\nu+1)\psi^\nu\psi_{x_1}\vartheta_\xi\vartheta_{x_1} + 2k(\psi^{\nu+1}\vartheta_{x_1}^2)_\xi \\
&\quad -2k(x_1+\delta_0)^2\sum_{i=2}^{n}(2(\psi^{\nu+1}a_{ii}\vartheta_\xi\vartheta_{x_i})_{x_i} - (\psi^{\nu+1}a_{ii}\vartheta_{x_i}^2)_\xi \\
&\quad -2(\nu+1)\psi^\nu\psi_{x_i}a_{ii}\vartheta_\xi\vartheta_{x_i} - 2\psi^{\nu+1}(a_{ii})_{x_i}\vartheta_\xi\vartheta_{x_i}) \\
&\quad +2k(x_1+\delta_0)^2\sum_{j=1}^{m}(2(\psi^{\nu+1}b_{jj}\vartheta_\xi\vartheta_{y_j})_{y_j} - (\psi^{\nu+1}b_{jj}\vartheta_{y_j}^2)_\xi \\
&\quad -2(\nu+1)\psi^\nu\psi_{y_j}b_{jj}\vartheta_\xi\vartheta_{y_j} - 2\psi^{\nu+1}(b_{jj})_{y_j}\vartheta_\xi\vartheta_{y_j})
\end{aligned}
$$

$$
\begin{aligned}
&= 4k(\nu+1)\psi^\nu\psi_{x_1}\vartheta_\xi\vartheta_{x_1} + 4k(\nu+1)(x_1+\delta_0)^2\psi^\nu\sum_{i=2}^{n}\psi_{x_i}a_{ii}\vartheta_\xi\vartheta_{x_i} \\
&\quad +4k(x_1+\delta_0)^2\psi^{\nu+1}\sum_{i=2}^{n}(a_{ii})_{x_i}\vartheta_\xi\vartheta_{x_i} - 4k(\nu+1)(x_1+\delta_0)^2\psi^\nu\sum_{j=1}^{m}\psi_{y_j}b_{jj}\vartheta_\xi\vartheta_{y_j} \\
&\quad -4k(x_1+\delta_0)^2\psi^{\nu+1}\sum_{j=1}^{m}(b_{jj})_{y_j}\vartheta_\xi\vartheta_{y_j} + d_2(\vartheta)
\end{aligned}
$$

$$
\begin{aligned}
&= 2\psi^{\nu+1}z_1((\nu+1)\psi^{-1}(x_1+\delta_0)^{-1}\psi_{x_1}\vartheta_{x_1} + (\nu+1)\psi^{-1}(x_1+\delta_0)\sum_{i=2}^{n}\psi_{x_i}a_{ii}\vartheta_{x_i} \\
&\quad +(x_1+\delta_0)\sum_{i=2}^{n}(a_{ii})_{x_i}\vartheta_{x_i} - (\nu+1)\psi^{-1}(x_1+\delta_0)\sum_{j=1}^{m}\psi_{y_j}b_{jj}\vartheta_{y_j} \\
&\quad -(x_1+\delta_0)\sum_{j=1}^{m}(b_{jj})_{y_j}\vartheta_{y_j}) + d_2(\vartheta) \\
&= 2\psi^{\nu+1}z_1z_6 + d_2(\vartheta)
\end{aligned}
$$

olduğu görülür. Burada

$$z_6 = (\nu+1)\psi^{-1}((x_1+\delta_0)^{-1}\psi_{x_1}\vartheta_{x_1} + (x_1+\delta_0)(\sum_{i=2}^{n} a_{ii}\psi_{x_i}\vartheta_{x_i} - \sum_{j=1}^{m} b_{jj}\psi_{y_j}\vartheta_{y_j}))$$
$$+(x_1+\delta_0)(\sum_{i=2}^{n}(a_{ii})_{x_i}\vartheta_{x_i} - \sum_{j=1}^{m}(b_{jj})_{y_j}\vartheta_{y_j})$$

dir. O halde, (2.10) eşitliğinde bulunan birinci, üçüncü, dördüncü ve beşinci terimler için $2ab \geq -a^2 - b^2$ kullanılarak

$$\begin{aligned}
& \psi^{\nu+1}(z_1^2 + z_3^2 + 2z_1z_3 + 2z_1z_2) \\
= {} & \psi^{\nu+1}(z_1^2 + z_3^2) + \psi^{\nu+1}2z_1z_3 + \psi^{\nu+1}2z_1z_2 \\
= {} & \psi^{\nu+1}(z_1^2 + z_3^2) + \psi^{\nu+1}2z_1z_3 + \psi^{\nu+1}2z_1z_6 + d_2(\vartheta) \\
= {} & \psi^{\nu+1}(z_1^2 + z_3^2 + 2z_1z_3 + 2z_1z_6) + d_2(\vartheta)
\end{aligned}$$

$$\begin{aligned}
= {} & \psi^{\nu+1}(z_1^2 + z_3^2 + 2z_1(z_3 + z_6)) + d_2(\vartheta) \\
\geq {} & \psi^{\nu+1}(z_1^2 + z_3^2 - z_1^2 - (z_3 + z_6)^2) + d_2(\vartheta) \\
= {} & \psi^{\nu+1}(z_3^2 - z_3^2 - 2z_3z_6 - z_6^2) + d_2(\vartheta) \\
= {} & \psi^{\nu+1}(-2z_3z_6 - z_6^2) + d_2(\vartheta)
\end{aligned}$$

$$\begin{aligned}
= {} & \psi^{\nu+1}(-2z_3((\nu+1)\psi^{-1}((x_1+\delta_0)^{-1}\psi_{x_1}\vartheta_{x_1} + (x_1+\delta_0)(\sum_{i=2}^{n} a_{ii}\psi_{x_i}\vartheta_{x_i} \\
& - \sum_{j=1}^{m} b_{jj}\psi_{y_j}\vartheta_{y_j})) + (x_1+\delta_0)(\sum_{i=2}^{n}(a_{ii})_{x_i}\vartheta_{x_i} - \sum_{j=1}^{m}(b_{jj})_{y_j}\vartheta_{y_j})) \\
& -((\nu+1)\psi^{-1}((x_1+\delta_0)^{-1}\psi_{x_1}\vartheta_{x_1} + (x_1+\delta_0)(\sum_{i=2}^{n} a_{ii}\psi_{x_i}\vartheta_{x_i} \\
& - \sum_{j=1}^{m} b_{jj}\psi_{y_j}\vartheta_{y_j})) + (x_1+\delta_0)(\sum_{i=2}^{n}(a_{ii})_{x_i}\vartheta_{x_i} - \sum_{j=1}^{m}(b_{jj})_{y_j}\vartheta_{y_j}))^2) + d_2(\vartheta)
\end{aligned}$$

$$
\begin{aligned}
= \;& \psi^{\nu+1}(-2z_3(\nu+1)\psi^{-1}((x_1+\delta_0)^{-1}\psi_{x_1}\vartheta_{x_1} + (x_1+\delta_0)(\sum_{i=2}^{n} a_{ii}\psi_{x_i}\vartheta_{x_i} \\
& -\sum_{j=1}^{m} b_{jj}\psi_{y_j}\vartheta_{y_j})) - 2z_3(x_1+\delta_0)(\sum_{i=2}^{n}(a_{ii})_{x_i}\vartheta_{x_i} - \sum_{j=1}^{m}(b_{jj})_{y_j}\vartheta_{y_j}) \\
& -(\nu+1)^2\psi^{-2}((x_1+\delta_0)^{-1}\psi_{x_1}\vartheta_{x_1} + (x_1+\delta_0)(\sum_{i=2}^{n} a_{ii}\psi_{x_i}\vartheta_{x_i} - \sum_{j=1}^{m} b_{jj}\psi_{y_j}\vartheta_{y_j}))^2 \\
& -2(\nu+1)\psi^{-1}((x_1+\delta_0)^{-1}\psi_{x_1}\vartheta_{x_1} + (x_1+\delta_0)(\sum_{i=2}^{n} a_{ii}\psi_{x_i}\vartheta_{x_i} - \sum_{j=1}^{m} b_{jj}\psi_{y_j}\vartheta_{y_j})) \\
& \times(x_1+\delta_0)(\sum_{i=2}^{n}(a_{ii})_{x_i}\vartheta_{x_i} - \sum_{j=1}^{m}(b_{jj})_{y_j}\vartheta_{y_j}) \\
& -(x_1+\delta_0)^2(\sum_{i=2}^{n}(a_{ii})_{x_i}\vartheta_{x_i} - \sum_{j=1}^{m}(b_{jj})_{y_j}\vartheta_{y_j})^2) + d_2(\vartheta)
\end{aligned}
$$

$$
\begin{aligned}
= \;& \psi^{\nu+1}(-2z_3(\nu+1)\psi^{-1}((x_1+\delta_0)^{-1}\psi_{x_1}\vartheta_{x_1} + (x_1+\delta_0)(\sum_{i=2}^{n} a_{ii}\psi_{x_i}\vartheta_{x_i} - \sum_{j=1}^{m} b_{jj}\psi_{y_j}\vartheta_{y_j})) \\
& -(\nu+1)^2\psi^{-2}((x_1+\delta_0)^{-1}\psi_{x_1}\vartheta_{x_1} + (x_1+\delta_0)(\sum_{i=2}^{n} a_{ii}\psi_{x_i}\vartheta_{x_i} - \sum_{j=1}^{m} b_{jj}\psi_{y_j}\vartheta_{y_j}))^2 \\
& -2z_3(x_1+\delta_0)(\sum_{i=2}^{n}(a_{ii})_{x_i}\vartheta_{x_i} - \sum_{j=1}^{m}(b_{jj})_{y_j}\vartheta_{y_j}) \\
& -2(\nu+1)\psi^{-1}((x_1+\delta_0)^{-1}\psi_{x_1}\vartheta_{x_1} + (x_1+\delta_0)(\sum_{i=2}^{n} a_{ii}\psi_{x_i}\vartheta_{x_i} - \sum_{j=1}^{m} b_{jj}\psi_{y_j}\vartheta_{y_j})) \\
& \times(x_1+\delta_0)(\sum_{i=2}^{n}(a_{ii})_{x_i}\vartheta_{x_i} - \sum_{j=1}^{m}(b_{jj})_{y_j}\vartheta_{y_j}) \\
& -(x_1+\delta_0)^2(\sum_{i=2}^{n}(a_{ii})_{x_i}\vartheta_{x_i} - \sum_{j=1}^{m}(b_{jj})_{y_j}\vartheta_{y_j})^2) + d_2(\vartheta)
\end{aligned}
$$

elde edilir. Ayrıca

$$
z_3 = -2\lambda\nu\psi^{-\nu-1}((x_1+\delta_0)^{-1}\psi_{x_1}\vartheta_{x_1} + (x_1+\delta_0)(\sum_{i=2}^{n} a_{ii}\psi_{x_i}\vartheta_{x_i} - \sum_{j=1}^{m} b_{jj}\psi_{y_j}\vartheta_{y_j}))
$$

olduğu göz önünde bulundurularak

$$
\begin{aligned}
& \psi^{\nu+1}(z_1^2 + z_3^2 + 2z_1z_3 + 2z_1z_2) \\
\geq \ & \psi^{\nu+1}(-2(-2\lambda\nu\psi^{-\nu-1}((x_1+\delta_0)^{-1}\psi_{x_1}\vartheta_{x_1} + (x_1+\delta_0)(\sum_{i=2}^{n} a_{ii}\psi_{x_i}\vartheta_{x_i} - \sum_{j=1}^{m} b_{jj}\psi_{y_j}\vartheta_{y_j}))) \\
& \times(\nu+1)\psi^{-1}((x_1+\delta_0)^{-1}\psi_{x_1}\vartheta_{x_1} + (x_1+\delta_0)(\sum_{i=2}^{n} a_{ii}\psi_{x_i}\vartheta_{x_i} - \sum_{j=1}^{m} b_{jj}\psi_{y_j}\vartheta_{y_j})) \\
& -(\nu+1)^2\psi^{-2}((x_1+\delta_0)^{-1}\psi_{x_1}\vartheta_{x_1} + (x_1+\delta_0)(\sum_{i=2}^{n} a_{ii}\psi_{x_i}\vartheta_{x_i} - \sum_{j=1}^{m} b_{jj}\psi_{y_j}\vartheta_{y_j}))^2 \\
& -2(-2\lambda\nu\psi^{-\nu-1}((x_1+\delta_0)^{-1}\psi_{x_1}\vartheta_{x_1} + (x_1+\delta_0)(\sum_{i=2}^{n} a_{ii}\psi_{x_i}\vartheta_{x_i} - \sum_{j=1}^{m} b_{jj}\psi_{y_j}\vartheta_{y_j})))
\end{aligned}
$$

$$
\begin{aligned}
& \times(x_1+\delta_0)(\sum_{i=2}^{n}(a_{ii})_{x_i}\vartheta_{x_i} - \sum_{j=1}^{m}(b_{jj})_{y_j}\vartheta_{y_j}) \\
& -2(\nu+1)\psi^{-1}((x_1+\delta_0)^{-1}\psi_{x_1}\vartheta_{x_1} + (x_1+\delta_0)(\sum_{i=2}^{n} a_{ii}\psi_{x_i}\vartheta_{x_i} - \sum_{j=1}^{m} b_{jj}\psi_{y_j}\vartheta_{y_j})) \\
& \times(x_1+\delta_0)(\sum_{i=2}^{n}(a_{ii})_{x_i}\vartheta_{x_i} - \sum_{j=1}^{m}(b_{jj})_{y_j}\vartheta_{y_j}) \\
& -(x_1+\delta_0)^2(\sum_{i=2}^{n}(a_{ii})_{x_i}\vartheta_{x_i} - \sum_{j=1}^{m}(b_{jj})_{y_j}\vartheta_{y_j})^2) + d_2(\vartheta)
\end{aligned}
$$

$$
\begin{aligned}
= \ & \psi^{\nu+1}(4\lambda\nu(\nu+1)\psi^{-\nu-2}((x_1+\delta_0)^{-1}\psi_{x_1}\vartheta_{x_1} \\
& +(x_1+\delta_0)(\sum_{i=2}^{n} a_{ii}\psi_{x_i}\vartheta_{x_i} - \sum_{j=1}^{m} b_{jj}\psi_{y_j}\vartheta_{y_j}))^2 \\
& -(\nu+1)^2\psi^{-2}((x_1+\delta_0)^{-1}\psi_{x_1}\vartheta_{x_1} + (x_1+\delta_0)(\sum_{i=2}^{n} a_{ii}\psi_{x_i}\vartheta_{x_i} - \sum_{j=1}^{m} b_{jj}\psi_{y_j}\vartheta_{y_j}))^2 \\
& +4\lambda\nu(x_1+\delta_0)\psi^{-\nu-1}((x_1+\delta_0)^{-1}\psi_{x_1}\vartheta_{x_1} \\
& +(x_1+\delta_0)(\sum_{i=2}^{n} a_{ii}\psi_{x_i}\vartheta_{x_i} - \sum_{j=1}^{m} b_{jj}\psi_{y_j}\vartheta_{y_j}))(\sum_{i=2}^{n}(a_{ii})_{x_i}\vartheta_{x_i} - \sum_{j=1}^{m}(b_{jj})_{y_j}\vartheta_{y_j})
\end{aligned}
$$

$$
\begin{aligned}
& -2(\nu+1)(x_1+\delta_0)\psi^{-1}((x_1+\delta_0)^{-1}\psi_{x_1}\vartheta_{x_1} \\
& +(x_1+\delta_0)(\sum_{i=2}^{n} a_{ii}\psi_{x_i}\vartheta_{x_i} - \sum_{j=1}^{m} b_{jj}\psi_{y_j}\vartheta_{y_j}))(\sum_{i=2}^{n}(a_{ii})_{x_i}\vartheta_{x_i} - \sum_{j=1}^{m}(b_{jj})_{y_j}\vartheta_{y_j}) \\
& -(x_1+\delta_0)^2(\sum_{i=2}^{n}(a_{ii})_{x_i}\vartheta_{x_i} - \sum_{j=1}^{m}(b_{jj})_{y_j}\vartheta_{y_j})^2) + d_2(\vartheta)
\end{aligned}
$$

$$
\begin{aligned}
= \;& \psi^{\nu+1}((4\lambda\nu(\nu+1)\psi^{-\nu-2} - (\nu+1)^2\psi^{-2}) \\
& \times((x_1+\delta_0)^{-1}\psi_{x_1}\vartheta_{x_1} + (x_1+\delta_0)(\sum_{i=2}^{n} a_{ii}\psi_{x_i}\vartheta_{x_i} - \sum_{j=1}^{m} b_{jj}\psi_{y_j}\vartheta_{y_j}))^2 \\
& +(4\lambda\nu\psi^{-\nu-1} - 2(\nu+1)\psi^{-1}) \\
& \times((x_1+\delta_0)^{-1}\psi_{x_1}\vartheta_{x_1} + (x_1+\delta_0)(\sum_{i=2}^{n} a_{ii}\psi_{x_i}\vartheta_{x_i} - \sum_{j=1}^{m} b_{jj}\psi_{y_j}\vartheta_{y_j})) \\
& \times(x_1+\delta_0)(\sum_{i=2}^{n}(a_{ii})_{x_i}\vartheta_{x_i} - \sum_{j=1}^{m}(b_{jj})_{y_j}\vartheta_{y_j}) \\
& -(x_1+\delta_0)^2(\sum_{i=2}^{n}(a_{ii})_{x_i}\vartheta_{x_i} - \sum_{j=1}^{m}(b_{jj})_{y_j}\vartheta_{y_j})^2) + d_2(\vartheta)
\end{aligned}
$$

elde edilir. Aşağıda $2ab \geq -\sigma_0 a^2 - \frac{b^2}{\sigma_0}$, $\sigma_0 = (\nu+1)\psi^{-1} > 0$ eşitsizliği kullanılırsa,

$$
\begin{aligned}
& 2((x_1+\delta_0)^{-1}\psi_{x_1}\vartheta_{x_1} + (x_1+\delta_0)(\sum_{i=2}^{n} a_{ii}\psi_{x_i}\vartheta_{x_i} - \sum_{j=1}^{m} b_{jj}\psi_{y_j}\vartheta_{y_j})) \\
& \times(x_1+\delta_0)(\sum_{i=2}^{n}(a_{ii})_{x_i}\vartheta_{x_i} - \sum_{j=1}^{m}(b_{jj})_{y_j}\vartheta_{y_j}) \\
\geq \;& -\sigma_0((x_1+\delta_0)^{-1}\psi_{x_1}\vartheta_{x_1} + (x_1+\delta_0)(\sum_{i=2}^{n} a_{ii}\psi_{x_i}\vartheta_{x_i} - \sum_{j=1}^{m} b_{jj}\psi_{y_j}\vartheta_{y_j}))^2 \\
& -\frac{1}{\sigma_0}(x_1+\delta_0)^2(\sum_{i=2}^{n}(a_{ii})_{x_i}\vartheta_{x_i} - \sum_{j=1}^{m}(b_{jj})_{y_j}\vartheta_{y_j})^2
\end{aligned}
$$

bulunur. Böylece

$$
\begin{aligned}
& \psi^{\nu+1}(z_1^2 + z_3^2 + 2z_1z_3 + 2z_1z_2) \\
\geq \;& \psi^{\nu+1}((4\lambda\nu(\nu+1)\psi^{-\nu-2} - (\nu+1)^2\psi^{-2}) \\
& \times((x_1+\delta_0)^{-1}\psi_{x_1}\vartheta_{x_1} + (x_1+\delta_0)(\sum_{i=2}^{n} a_{ii}\psi_{x_i}\vartheta_{x_i} - \sum_{j=1}^{m} b_{jj}\psi_{y_j}\vartheta_{y_j}))^2 \\
& +(2\lambda\nu\psi^{-\nu-1} - (\nu+1)\psi^{-1}) \\
& \times(-\sigma_0((x_1+\delta_0)^{-1}\psi_{x_1}\vartheta_{x_1} + (x_1+\delta_0)(\sum_{i=2}^{n} a_{ii}\psi_{x_i}\vartheta_{x_i} - \sum_{j=1}^{m} b_{jj}\psi_{y_j}\vartheta_{y_j}))^2
\end{aligned}
$$

$$
\begin{aligned}
& -\frac{1}{\sigma_0}(x_1+\delta_0)^2(\sum_{i=2}^{n}(a_{ii})_{x_i}\vartheta_{x_i} - \sum_{j=1}^{m}(b_{jj})_{y_j}\vartheta_{y_j})^2) \\
& -(x_1+\delta_0)^2(\sum_{i=2}^{n}(a_{ii})_{x_i}\vartheta_{x_i} - \sum_{j=1}^{m}(b_{jj})_{y_j}\vartheta_{y_j})^2) + d_2(\vartheta)
\end{aligned}
$$

$$
\begin{aligned}
&= \psi^{\nu+1}((4\lambda\nu(\nu+1)\psi^{-\nu-2}-(\nu+1)^2\psi^{-2}-2\lambda\nu\sigma_0\psi^{-\nu-1}+(\nu+1)\sigma_0\psi^{-1})\\
&\quad\times((x_1+\delta_0)^{-1}\psi_{x_1}\vartheta_{x_1}+(x_1+\delta_0)(\sum_{i=2}^{n}a_{ii}\psi_{x_i}\vartheta_{x_i}-\sum_{j=1}^{m}b_{jj}\psi_{y_j}\vartheta_{y_j}))^2\\
&\quad-\frac{1}{\sigma_0}(2\lambda\nu\psi^{-\nu-1}-(\nu+1)\psi^{-1})(x_1+\delta_0)^2(\sum_{i=2}^{n}(a_{ii})_{x_i}\vartheta_{x_i}-\sum_{j=1}^{m}(b_{jj})_{y_j}\vartheta_{y_j})^2\\
&\quad-(x_1+\delta_0)^2(\sum_{i=2}^{n}(a_{ii})_{x_i}\vartheta_{x_i}-\sum_{j=1}^{m}(b_{jj})_{y_j}\vartheta_{y_j})^2)+d_2(\vartheta)
\end{aligned}
$$

$$
\begin{aligned}
&= \psi^{\nu+1}(2\lambda\nu\psi^{-\nu-1}\sigma_0((x_1+\delta_0)^{-1}\psi_{x_1}\vartheta_{x_1}+(x_1+\delta_0)(\sum_{i=2}^{n}a_{ii}\psi_{x_i}\vartheta_{x_i}-\sum_{j=1}^{m}b_{jj}\psi_{y_j}\vartheta_{y_j}))^2\\
&\quad-\frac{1}{\sigma_0}2\lambda\nu\psi^{-\nu-1}(x_1+\delta_0)^2(\sum_{i=2}^{n}(a_{ii})_{x_i}\vartheta_{x_i}-\sum_{j=1}^{m}(b_{jj})_{y_j}\vartheta_{y_j})^2)+d_2(\vartheta)\\
&\geq -2\lambda\nu\psi(x_1+\delta_0)^2(\sum_{i=2}^{n}(a_{ii})_{x_i}\vartheta_{x_i}-\sum_{j=1}^{m}(b_{jj})_{y_j}\vartheta_{y_j})^2+d_2(\vartheta) \qquad (2.11)
\end{aligned}
$$

elde edilir.

İkinci olarak, (2.10) eşitliğinde altıncı terim için

$$
\begin{aligned}
2z_1(z_4+z_5)\psi^{\nu+1} &= 2(2k(x_1+\delta_0)\vartheta_\xi)(-E_0\vartheta+\xi\vartheta(x_1+\delta_0))\psi^{\nu+1}\\
&= -4k(x_1+\delta_0)E_0\vartheta\vartheta_\xi\psi^{\nu+1}+4k(x_1+\delta_0)^2\xi\vartheta\vartheta_\xi\psi^{\nu+1}\\
&= (-2k(x_1+\delta_0)\psi^{\nu+1}E_0\vartheta^2)_\xi+(2k(x_1+\delta_0)^2\xi\psi^{\nu+1}\vartheta^2)_\xi\\
&\quad -2k(x_1+\delta_0)^2\psi^{\nu+1}\vartheta^2 \qquad (2.12)
\end{aligned}
$$

yazılabilir.

Üçüncü olarak:

$$
\begin{aligned}
2z_3z_5\psi^{\nu+1} &= 2(-2\lambda\nu\psi^{-\nu-1}((x_1+\delta_0)^{-1}\psi_{x_1}\vartheta_{x_1}\\
&\quad+(x_1+\delta_0)(\sum_{i=2}^{n}a_{ii}\psi_{x_i}\vartheta_{x_i}-\sum_{j=1}^{m}b_{jj}\psi_{y_j}\vartheta_{y_j})))\xi\vartheta(x_1+\delta_0)\psi^{\nu+1}\\
&= -4\lambda\nu\xi\vartheta(x_1+\delta_0)((x_1+\delta_0)^{-1}\psi_{x_1}\vartheta_{x_1}\\
&\quad+(x_1+\delta_0)(\sum_{i=2}^{n}a_{ii}\psi_{x_i}\vartheta_{x_i}-\sum_{j=1}^{m}b_{jj}\psi_{y_j}\vartheta_{y_j}))\\
&= -4\lambda\nu\xi\psi_{x_1}\vartheta\vartheta_{x_1}-4\lambda\nu\xi\vartheta(x_1+\delta_0)^2(\sum_{i=2}^{n}a_{ii}\psi_{x_i}\vartheta_{x_i}-\sum_{j=1}^{m}b_{jj}\psi_{y_j}\vartheta_{y_j})
\end{aligned}
$$

$$
\begin{aligned}
= \quad & -2\lambda\nu(\xi\psi_{x_1}\vartheta^2)_{x_1} - 2\lambda\nu(x_1+\delta_0)^2\sum_{i=2}^{n}(a_{ii}\psi_{x_i}\xi\vartheta^2)_{x_i} \\
& +2\lambda\nu(x_1+\delta_0)^2\xi\vartheta^2\sum_{i=2}^{n}(a_{ii}\psi_{x_i})_{x_i} + 2\lambda\nu(x_1+\delta_0)^2\sum_{j=1}^{m}(b_{jj}\psi_{y_j}\xi\vartheta^2)_{y_j} \\
& -2\lambda\nu(x_1+\delta_0)^2\xi\vartheta^2\sum_{j=1}^{m}(b_{jj}\psi_{y_j})_{y_j} \qquad (2.13)
\end{aligned}
$$

olur. Burada $\psi_{x_1x_1}=0$ olduğu göz önünde bulundurulmuştur.

Dördüncü olarak, (2.10) eşitliğindeki yedinci terim için

$$
\begin{aligned}
2z_2z_3\psi^{\nu+1} \quad = \quad & 2(-(x_1+\delta_0)^{-1}\vartheta_{x_1x_1} - (x_1+\delta_0)(\sum_{i=2}^{n}a_{ii}\vartheta_{x_ix_i} - \sum_{j=1}^{m}b_{jj}\vartheta_{y_jy_j})) \\
& \times(-2\lambda\nu\psi^{-\nu-1}((x_1+\delta_0)^{-1}\psi_{x_1}\vartheta_{x_1} \\
& +(x_1+\delta_0)(\sum_{i=2}^{n}a_{ii}\psi_{x_i}\vartheta_{x_i} - \sum_{j=1}^{m}b_{jj}\psi_{y_j}\vartheta_{y_j})))\psi^{\nu+1} \\
= \quad & 4\lambda\nu((x_1+\delta_0)^{-1}\vartheta_{x_1x_1} + (x_1+\delta_0)(\sum_{i=2}^{n}a_{ii}\vartheta_{x_ix_i} - \sum_{j=1}^{m}b_{jj}\vartheta_{y_jy_j})) \\
& \times((x_1+\delta_0)^{-1}\psi_{x_1}\vartheta_{x_1} + (x_1+\delta_0)(\sum_{i=2}^{n}a_{ii}\psi_{x_i}\vartheta_{x_i} - \sum_{j=1}^{m}b_{jj}\psi_{y_j}\vartheta_{y_j}))
\end{aligned}
$$

elde edilir. Yukarıdaki eşitlikte çarpma işleminden dolayı ortaya çıkan dört terim aşağıda değerlendirilmiştir:

Birinci terim:

$$
4\lambda\nu(x_1+\delta_0)^{-2}\psi_{x_1}\vartheta_{x_1}\vartheta_{x_1x_1} = 2\lambda\nu\psi_{x_1}((x_1+\delta_0)^{-2}\vartheta_{x_1}^2)_{x_1} + 4\lambda\nu\psi_{x_1}(x_1+\delta_0)^{-3}\vartheta_{x_1}^2
$$

olarak yazılabilir.

İkinci terim için

$$
\begin{aligned}
& 4\lambda\nu\vartheta_{x_1x_1}(\sum_{i=2}^{n}a_{ii}\psi_{x_i}\vartheta_{x_i} - \sum_{j=1}^{m}b_{jj}\psi_{y_j}\vartheta_{y_j}) \\
= \quad & 2\lambda\nu\sum_{i=2}^{n}2\psi_{x_i}a_{ii}\vartheta_{x_i}\vartheta_{x_1x_1} - 2\lambda\nu\sum_{j=1}^{m}2\psi_{y_j}b_{jj}\vartheta_{y_j}\vartheta_{x_1x_1}
\end{aligned}
$$

eşitliğinde

$$
\begin{aligned}
2\psi_{x_i}a_{ii}\vartheta_{x_i}\vartheta_{x_1x_1} \quad = \quad & (\psi_{x_i}a_{ii}\vartheta_{x_i}\vartheta_{x_1})_{x_1} + (\psi_{x_i}a_{ii}\vartheta_{x_i}\vartheta_{x_1})_{x_1} - (\psi_{x_i}a_{ii}\vartheta_{x_1}^2)_{x_i} \\
& -(\psi_{x_i}a_{ii})_{x_1}\vartheta_{x_i}\vartheta_{x_1} - (\psi_{x_i}a_{ii})_{x_1}\vartheta_{x_i}\vartheta_{x_1} + (\psi_{x_i}a_{ii})_{x_i}\vartheta_{x_1}^2 \\
= \quad & 2(\psi_{x_i}a_{ii}\vartheta_{x_i}\vartheta_{x_1})_{x_1} - (\psi_{x_i}a_{ii}\vartheta_{x_1}^2)_{x_i} \\
& -2(\psi_{x_i}a_{ii})_{x_1}\vartheta_{x_i}\vartheta_{x_1} + (\psi_{x_i}a_{ii})_{x_i}\vartheta_{x_1}^2
\end{aligned}
$$

ve

$$
\begin{aligned}
2\psi_{y_j} b_{jj}\vartheta_{y_j}\vartheta_{x_1x_1} &= (\psi_{y_j} b_{jj}\vartheta_{y_j}\vartheta_{x_1})_{x_1} + (\psi_{y_j} b_{jj}\vartheta_{y_j}\vartheta_{x_1})_{x_1} - (\psi_{y_j} b_{jj}\vartheta^2_{x_1})_{y_j} \\
&\quad -(\psi_{y_j} b_{jj})_{x_1}\vartheta_{y_j}\vartheta_{x_1} - (\psi_{y_j} b_{jj})_{x_1}\vartheta_{y_j}\vartheta_{x_1} + (\psi_{y_j} b_{jj})_{y_j}\vartheta^2_{x_1} \\
&= 2(\psi_{y_j} b_{jj}\vartheta_{y_j}\vartheta_{x_1})_{x_1} - (\psi_{y_j} b_{jj}\vartheta^2_{x_1})_{y_j} \\
&\quad -2(\psi_{y_j} b_{jj})_{x_1}\vartheta_{y_j}\vartheta_{x_1} + (\psi_{y_j} b_{jj})_{y_j}\vartheta^2_{x_1}
\end{aligned}
$$

terimleri yerine yazılırsa

$$
\begin{aligned}
&4\lambda\nu\vartheta_{x_1x_1}(\sum_{i=2}^{n} a_{ii}\psi_{x_i}\vartheta_{x_i} - \sum_{j=1}^{m} b_{jj}\psi_{y_j}\vartheta_{y_j}) \\
= \;& 2\lambda\nu\sum_{i=2}^{n}(2(\psi_{x_i} a_{ii}\vartheta_{x_i}\vartheta_{x_1})_{x_1} - (\psi_{x_i} a_{ii}\vartheta^2_{x_1})_{x_i} - 2(\psi_{x_i} a_{ii})_{x_1}\vartheta_{x_i}\vartheta_{x_1} + (\psi_{x_i} a_{ii})_{x_i}\vartheta^2_{x_1}) \\
&-2\lambda\nu\sum_{j=1}^{m}(2(\psi_{y_j} b_{jj}\vartheta_{y_j}\vartheta_{x_1})_{x_1} - (\psi_{y_j} b_{jj}\vartheta^2_{x_1})_{y_j} - 2(\psi_{y_j} b_{jj})_{x_1}\vartheta_{y_j}\vartheta_{x_1} + (\psi_{y_j} b_{jj})_{y_j}\vartheta^2_{x_1})
\end{aligned}
$$

bulunur.

Üçüncü terim için

$$
\begin{aligned}
&4\lambda\nu(\sum_{i=2}^{n} a_{ii}\vartheta_{x_ix_i} - \sum_{j=1}^{m} b_{jj}\vartheta_{y_jy_j})\psi_{x_1}\vartheta_{x_1} \\
= \;& 2\lambda\nu\sum_{i=2}^{n} 2\psi_{x_1} a_{ii}\vartheta_{x_1}\vartheta_{x_ix_i} - 2\lambda\nu\sum_{j=1}^{m} 2\psi_{x_1} b_{jj}\vartheta_{x_1}\vartheta_{y_jy_j}
\end{aligned}
$$

eşitliğinde

$$
\begin{aligned}
2\psi_{x_1} a_{ii}\vartheta_{x_1}\vartheta_{x_ix_i} &= (\psi_{x_1} a_{ii}\vartheta_{x_1}\vartheta_{x_i})_{x_i} + (\psi_{x_1} a_{ii}\vartheta_{x_1}\vartheta_{x_i})_{x_i} - (\psi_{x_1} a_{ii}\vartheta^2_{x_i})_{x_1} \\
&\quad -(\psi_{x_1} a_{ii})_{x_i}\vartheta_{x_1}\vartheta_{x_i} - (\psi_{x_1} a_{ii})_{x_i}\vartheta_{x_1}\vartheta_{x_i} + (\psi_{x_1} a_{ii})_{x_1}\vartheta^2_{x_i} \\
&= 2(\psi_{x_1} a_{ii}\vartheta_{x_1}\vartheta_{x_i})_{x_i} - (\psi_{x_1} a_{ii}\vartheta^2_{x_i})_{x_1} \\
&\quad -2(\psi_{x_1} a_{ii})_{x_i}\vartheta_{x_1}\vartheta_{x_i} + (\psi_{x_1} a_{ii})_{x_1}\vartheta^2_{x_i}
\end{aligned}
$$

ve

$$
\begin{aligned}
2\psi_{x_1} b_{jj}\vartheta_{x_1}\vartheta_{y_jy_j} &= (\psi_{x_1} b_{jj}\vartheta_{x_1}\vartheta_{y_j})_{y_j} + (\psi_{x_1} b_{jj}\vartheta_{x_1}\vartheta_{y_j})_{y_j} - (\psi_{x_1} b_{jj}\vartheta^2_{y_j})_{x_1} \\
&\quad -(\psi_{x_1} b_{jj})_{y_j}\vartheta_{x_1}\vartheta_{y_j} - (\psi_{x_1} b_{jj})_{y_j}\vartheta_{x_1}\vartheta_{y_j} + (\psi_{x_1} b_{jj})_{x_1}\vartheta^2_{y_j} \\
&= 2(\psi_{x_1} b_{jj}\vartheta_{x_1}\vartheta_{y_j})_{y_j} - (\psi_{x_1} b_{jj}\vartheta^2_{y_j})_{x_1} \\
&\quad -2(\psi_{x_1} b_{jj})_{y_j}\vartheta_{x_1}\vartheta_{y_j} + (\psi_{x_1} b_{jj})_{x_1}\vartheta^2_{y_j}
\end{aligned}
$$

eşitlikleri kullanılarak

$$
\begin{aligned}
& 4\lambda\nu(\sum_{i=2}^{n} a_{ii}\vartheta_{x_ix_i} - \sum_{j=1}^{m} b_{jj}\vartheta_{y_jy_j})\psi_{x_1}\vartheta_{x_1} \\
= \; & 2\lambda\nu\sum_{i=2}^{n}(2(\psi_{x_1}a_{ii}\vartheta_{x_1}\vartheta_{x_i})_{x_i} - (\psi_{x_1}a_{ii}\vartheta_{x_i}^2)_{x_1} - 2(\psi_{x_1}a_{ii})_{x_i}\vartheta_{x_1}\vartheta_{x_i} + (\psi_{x_1}a_{ii})_{x_1}\vartheta_{x_i}^2) \\
& -2\lambda\nu\sum_{j=1}^{m}(2(\psi_{x_1}b_{jj}\vartheta_{x_1}\vartheta_{y_j})_{y_j} - (\psi_{x_1}b_{jj}\vartheta_{y_j}^2)_{x_1} - 2(\psi_{x_1}b_{jj})_{y_j}\vartheta_{x_1}\vartheta_{y_j} + (\psi_{x_1}b_{jj})_{x_1}\vartheta_{y_j}^2)
\end{aligned}
$$

elde edilir.

Dördüncü terim için

$$
\begin{aligned}
& 4\lambda\nu(x_1+\delta_0)^2(\sum_{s=2}^{n} a_{ss}\vartheta_{x_sx_s} - \sum_{k=1}^{m} b_{kk}\vartheta_{y_ky_k})(\sum_{i=2}^{n} a_{ii}\psi_{x_i}\vartheta_{x_i} - \sum_{j=1}^{m} b_{jj}\psi_{y_j}\vartheta_{y_j}) \\
= \; & 2\lambda\nu(x_1+\delta_0)^2\sum_{s,i=2}^{n} 2a_{ss}a_{ii}\psi_{x_i}\vartheta_{x_i}\vartheta_{x_sx_s} - 2\lambda\nu(x_1+\delta_0)^2\sum_{s=2}^{n}\sum_{j=1}^{m} 2a_{ss}b_{jj}\psi_{y_j}\vartheta_{y_j}\vartheta_{x_sx_s} \\
& -2\lambda\nu(x_1+\delta_0)^2\sum_{k=1}^{m}\sum_{i=2}^{n} 2b_{kk}a_{ii}\psi_{x_i}\vartheta_{x_i}\vartheta_{y_ky_k} + 2\lambda\nu(x_1+\delta_0)^2\sum_{k,j=1}^{m} 2b_{kk}b_{jj}\psi_{y_j}\vartheta_{y_j}\vartheta_{y_ky_k}
\end{aligned}
$$

eşitliğinde yer alan terimleri aşağıda değerlendirelim:

$$
\begin{aligned}
2a_{ss}a_{ii}\psi_{x_i}\vartheta_{x_i}\vartheta_{x_sx_s} \; = \; & (a_{ss}a_{ii}\psi_{x_i}\vartheta_{x_i}\vartheta_{x_s})_{x_s} + (a_{ss}a_{ii}\psi_{x_i}\vartheta_{x_i}\vartheta_{x_s})_{x_s} - (a_{ss}a_{ii}\psi_{x_i}\vartheta_{x_s}^2)_{x_i} \\
& -(a_{ss}a_{ii}\psi_{x_i})_{x_s}\vartheta_{x_i}\vartheta_{x_s} - (a_{ss}a_{ii}\psi_{x_i})_{x_s}\vartheta_{x_i}\vartheta_{x_s} + (a_{ss}a_{ii}\psi_{x_i})_{x_i}\vartheta_{x_s}^2 \\
= \; & 2(a_{ss}a_{ii}\psi_{x_i}\vartheta_{x_i}\vartheta_{x_s})_{x_s} - (a_{ss}a_{ii}\psi_{x_i}\vartheta_{x_s}^2)_{x_i} \\
& -2(a_{ss}a_{ii}\psi_{x_i})_{x_s}\vartheta_{x_i}\vartheta_{x_s} + (a_{ss}a_{ii}\psi_{x_i})_{x_i}\vartheta_{x_s}^2,
\end{aligned}
$$

$$
\begin{aligned}
2a_{ss}b_{jj}\psi_{y_j}\vartheta_{y_j}\vartheta_{x_sx_s} \; = \; & (a_{ss}b_{jj}\psi_{y_j}\vartheta_{y_j}\vartheta_{x_s})_{x_s} + (a_{ss}b_{jj}\psi_{y_j}\vartheta_{y_j}\vartheta_{x_s})_{x_s} - (a_{ss}b_{jj}\psi_{y_j}\vartheta_{x_s}^2)_{y_j} \\
& -(a_{ss}b_{jj}\psi_{y_j})_{x_s}\vartheta_{y_j}\vartheta_{x_s} - (a_{ss}b_{jj}\psi_{y_j})_{x_s}\vartheta_{y_j}\vartheta_{x_s} + (a_{ss}b_{jj}\psi_{y_j})_{y_j}\vartheta_{x_s}^2 \\
= \; & 2(a_{ss}b_{jj}\psi_{y_j}\vartheta_{y_j}\vartheta_{x_s})_{x_s} - (a_{ss}b_{jj}\psi_{y_j}\vartheta_{x_s}^2)_{y_j} \\
& -2(a_{ss}b_{jj}\psi_{y_j})_{x_s}\vartheta_{y_j}\vartheta_{x_s} + (a_{ss}b_{jj}\psi_{y_j})_{y_j}\vartheta_{x_s}^2,
\end{aligned}
$$

$$
\begin{aligned}
2b_{kk}a_{ii}\psi_{x_i}\vartheta_{x_i}\vartheta_{y_ky_k} \; = \; & (b_{kk}a_{ii}\psi_{x_i}\vartheta_{x_i}\vartheta_{y_k})_{y_k} + (b_{kk}a_{ii}\psi_{x_i}\vartheta_{x_i}\vartheta_{y_k})_{y_k} - (b_{kk}a_{ii}\psi_{x_i}\vartheta_{y_k}^2)_{x_i} \\
& -(b_{kk}a_{ii}\psi_{x_i})_{y_k}\vartheta_{x_i}\vartheta_{y_k} - (b_{kk}a_{ii}\psi_{x_i})_{y_k}\vartheta_{x_i}\vartheta_{y_k} + (b_{kk}a_{ii}\psi_{x_i})_{x_i}\vartheta_{y_k}^2 \\
= \; & 2(b_{kk}a_{ii}\psi_{x_i}\vartheta_{x_i}\vartheta_{y_k})_{y_k} - (b_{kk}a_{ii}\psi_{x_i}\vartheta_{y_k}^2)_{x_i} \\
& -2(b_{kk}a_{ii}\psi_{x_i})_{y_k}\vartheta_{x_i}\vartheta_{y_k} + (b_{kk}a_{ii}\psi_{x_i})_{x_i}\vartheta_{y_k}^2,
\end{aligned}
$$

$$\begin{aligned} 2b_{kk}b_{jj}\psi_{y_j}\vartheta_{y_j}\vartheta_{y_ky_k} &= (b_{kk}b_{jj}\psi_{y_j}\vartheta_{y_j}\vartheta_{y_k})_{y_k} + (b_{kk}b_{jj}\psi_{y_j}\vartheta_{y_j}\vartheta_{y_k})_{y_k} - (b_{kk}b_{jj}\psi_{y_j}\vartheta^2_{y_k})_{y_j} \\ &\quad -(b_{kk}b_{jj}\psi_{y_j})_{y_k}\vartheta_{y_j}\vartheta_{y_k} - (b_{kk}b_{jj}\psi_{y_j})_{y_k}\vartheta_{y_j}\vartheta_{y_k} + (b_{kk}b_{jj}\psi_{y_j})_{y_j}\vartheta^2_{y_k} \\ &= 2(b_{kk}b_{jj}\psi_{y_j}\vartheta_{y_j}\vartheta_{y_k})_{y_k} - (b_{kk}b_{jj}\psi_{y_j}\vartheta^2_{y_k})_{y_j} \\ &\quad -2(b_{kk}b_{jj}\psi_{y_j})_{y_k}\vartheta_{y_j}\vartheta_{y_k} + (b_{kk}b_{jj}\psi_{y_j})_{y_j}\vartheta^2_{y_k}. \end{aligned}$$

Buradan dördüncü terim

$$\begin{aligned} & 4\lambda\nu(x_1+\delta_0)^2(\sum_{s=2}^{n} a_{ss}\vartheta_{x_sx_s} - \sum_{k=1}^{m} b_{kk}\vartheta_{y_ky_k})(\sum_{i=2}^{n} a_{ii}\psi_{x_i}\vartheta_{x_i} - \sum_{j=1}^{m} b_{jj}\psi_{y_j}\vartheta_{y_j}) \\ = \;& 2\lambda\nu(x_1+\delta_0)^2\sum_{s,i=2}^{n}(2(a_{ss}a_{ii}\psi_{x_i}\vartheta_{x_i}\vartheta_{x_s})_{x_s} - (a_{ss}a_{ii}\psi_{x_i}\vartheta^2_{x_s})_{x_i} \\ & -2(a_{ss}a_{ii}\psi_{x_i})_{x_s}\vartheta_{x_i}\vartheta_{x_s} + (a_{ss}a_{ii}\psi_{x_i})_{x_i}\vartheta^2_{x_s}) \\ & -2\lambda\nu(x_1+\delta_0)^2\sum_{s=2}^{n}\sum_{j=1}^{m}(2(a_{ss}b_{jj}\psi_{y_j}\vartheta_{y_j}\vartheta_{x_s})_{x_s} - (a_{ss}b_{jj}\psi_{y_j}\vartheta^2_{x_s})_{y_j} \\ & -2(a_{ss}b_{jj}\psi_{y_j})_{x_s}\vartheta_{y_j}\vartheta_{x_s} + (a_{ss}b_{jj}\psi_{y_j})_{y_j}\vartheta^2_{x_s}) \end{aligned}$$

$$\begin{aligned} & -2\lambda\nu(x_1+\delta_0)^2\sum_{k=1}^{m}\sum_{i=2}^{n}(2(b_{kk}a_{ii}\psi_{x_i}\vartheta_{x_i}\vartheta_{y_k})_{y_k} - (b_{kk}a_{ii}\psi_{x_i}\vartheta^2_{y_k})_{x_i} \\ & -2(b_{kk}a_{ii}\psi_{x_i})_{y_k}\vartheta_{x_i}\vartheta_{y_k} + (b_{kk}a_{ii}\psi_{x_i})_{x_i}\vartheta^2_{y_k}) \\ & +2\lambda\nu(x_1+\delta_0)^2\sum_{k,j=1}^{m}(2(b_{kk}b_{jj}\psi_{y_j}\vartheta_{y_j}\vartheta_{y_k})_{y_k} - (b_{kk}b_{jj}\psi_{y_j}\vartheta^2_{y_k})_{y_j} \\ & -2(b_{kk}b_{jj}\psi_{y_j})_{y_k}\vartheta_{y_j}\vartheta_{y_k} + (b_{kk}b_{jj}\psi_{y_j})_{y_j}\vartheta^2_{y_k}) \end{aligned}$$

olarak bulunur. Böylece

$$\begin{aligned} & 2z_2z_3\psi^{\nu+1} \\ = \;& 2\lambda\nu\psi_{x_1}((x_1+\delta_0)^{-2}\vartheta^2_{x_1})_{x_1} + 4\lambda\nu\psi_{x_1}(x_1+\delta_0)^{-3}\vartheta^2_{x_1} \\ & +2\lambda\nu(\sum_{i=2}^{n}(2(\psi_{x_i}a_{ii}\vartheta_{x_i}\vartheta_{x_1})_{x_1} - (\psi_{x_i}a_{ii}\vartheta^2_{x_1})_{x_i} - 2(\psi_{x_i}a_{ii})_{x_1}\vartheta_{x_i}\vartheta_{x_1} + (\psi_{x_i}a_{ii})_{x_i}\vartheta^2_{x_1}) \\ & -\sum_{j=1}^{m}(2(\psi_{y_j}b_{jj}\vartheta_{y_j}\vartheta_{x_1})_{x_1} - (\psi_{y_j}b_{jj}\vartheta^2_{x_1})_{y_j} - 2(\psi_{y_j}b_{jj})_{x_1}\vartheta_{y_j}\vartheta_{x_1} + (\psi_{y_j}b_{jj})_{y_j}\vartheta^2_{x_1}) \\ & +\sum_{i=2}^{n}(2(\psi_{x_1}a_{ii}\vartheta_{x_1}\vartheta_{x_i})_{x_i} - (\psi_{x_1}a_{ii}\vartheta^2_{x_i})_{x_1} - 2(\psi_{x_1}a_{ii})_{x_i}\vartheta_{x_1}\vartheta_{x_i} + (\psi_{x_1}a_{ii})_{x_1}\vartheta^2_{x_i}) \\ & -\sum_{j=1}^{m}(2(\psi_{x_1}b_{jj}\vartheta_{x_1}\vartheta_{y_j})_{y_j} - (\psi_{x_1}b_{jj}\vartheta^2_{y_j})_{x_1} - 2(\psi_{x_1}b_{jj})_{y_j}\vartheta_{x_1}\vartheta_{y_j} + (\psi_{x_1}b_{jj})_{x_1}\vartheta^2_{y_j})) \end{aligned}$$

$$+2\lambda\nu(x_1+\delta_0)^2(\sum_{s,i=2}^{n}(2(a_{ss}a_{ii}\psi_{x_i}\vartheta_{x_i}\vartheta_{x_s})_{x_s}-(a_{ss}a_{ii}\psi_{x_i}\vartheta_{x_s}^2)_{x_i}$$
$$-2(a_{ss}a_{ii}\psi_{x_i})_{x_s}\vartheta_{x_i}\vartheta_{x_s}+(a_{ss}a_{ii}\psi_{x_i})_{x_i}\vartheta_{x_s}^2)$$
$$-\sum_{s=2}^{n}\sum_{j=1}^{m}(2(a_{ss}b_{jj}\psi_{y_j}\vartheta_{y_j}\vartheta_{x_s})_{x_s}-(a_{ss}b_{jj}\psi_{y_j}\vartheta_{x_s}^2)_{y_j}$$
$$-2(a_{ss}b_{jj}\psi_{y_j})_{x_s}\vartheta_{y_j}\vartheta_{x_s}+(a_{ss}b_{jj}\psi_{y_j})_{y_j}\vartheta_{x_s}^2)$$
$$-\sum_{k=1}^{m}\sum_{i=2}^{n}(2(b_{kk}a_{ii}\psi_{x_i}\vartheta_{x_i}\vartheta_{y_k})_{y_k}-(b_{kk}a_{ii}\psi_{x_i}\vartheta_{y_k}^2)_{x_i}$$
$$-2(b_{kk}a_{ii}\psi_{x_i})_{y_k}\vartheta_{x_i}\vartheta_{y_k}+(b_{kk}a_{ii}\psi_{x_i})_{x_i}\vartheta_{y_k}^2)$$
$$+\sum_{k,j=1}^{m}(2(b_{kk}b_{jj}\psi_{y_j}\vartheta_{y_j}\vartheta_{y_k})_{y_k}-(b_{kk}b_{jj}\psi_{y_j}\vartheta_{y_k}^2)_{y_j}$$
$$-2(b_{kk}b_{jj}\psi_{y_j})_{y_k}\vartheta_{y_j}\vartheta_{y_k}+(b_{kk}b_{jj}\psi_{y_j})_{y_j}\vartheta_{y_k}^2)) \tag{2.14}$$

elde edilir.

Beşinci olarak:

$$
\begin{aligned}
& 2z_3z_4\psi^{\nu+1} \\
= \; & 2(-2\lambda\nu\psi^{-\nu-1}((x_1+\delta_0)^{-1}\psi_{x_1}\vartheta_{x_1}+(x_1+\delta_0)(\sum_{i=2}^{n}a_{ii}\psi_{x_i}\vartheta_{x_i}-\sum_{j=1}^{m}b_{jj}\psi_{y_j}\vartheta_{y_j}))) \\
& \times(-E_0\vartheta)\psi^{\nu+1} \\
= \; & 4\lambda\nu E_0\vartheta((x_1+\delta_0)^{-1}\psi_{x_1}\vartheta_{x_1}+(x_1+\delta_0)(\sum_{i=2}^{n}a_{ii}\psi_{x_i}\vartheta_{x_i}-\sum_{j=1}^{m}b_{jj}\psi_{y_j}\vartheta_{y_j})) \\
= \; & 4\lambda\nu E_0\vartheta(x_1+\delta_0)^{-1}\psi_{x_1}\vartheta_{x_1}+4\lambda\nu E_0\vartheta(x_1+\delta_0)\sum_{i=2}^{n}a_{ii}\psi_{x_i}\vartheta_{x_i} \\
& -4\lambda\nu E_0\vartheta(x_1+\delta_0)\sum_{j=1}^{m}b_{jj}\psi_{y_j}\vartheta_{y_j} \\
= \; & 2\lambda\nu\psi_{x_1}(E_0(x_1+\delta_0)^{-1}\vartheta^2)_{x_1}-2\lambda\nu\psi_{x_1}(E_0(x_1+\delta_0)^{-1})_{x_1}\vartheta^2 \\
& +2\lambda\nu\sum_{i=2}^{n}(E_0(x_1+\delta_0)a_{ii}\psi_{x_i}\vartheta^2)_{x_i}-2\lambda\nu\sum_{i=2}^{n}(E_0(x_1+\delta_0)a_{ii}\psi_{x_i})_{x_i}\vartheta^2 \\
& -2\lambda\nu\sum_{j=1}^{m}(E_0(x_1+\delta_0)b_{jj}\psi_{y_j}\vartheta^2)_{y_j}+2\lambda\nu\sum_{j=1}^{m}(E_0(x_1+\delta_0)b_{jj}\psi_{y_j})_{y_j}\vartheta^2 \\
= \; & 2\lambda\nu\psi_{x_1}((x_1+\delta_0)^{-1}E_0\vartheta^2)_{x_1}-2\lambda\nu\psi_{x_1}((x_1+\delta_0)^{-1}E_0)_{x_1}\vartheta^2 \\
& +2\lambda\nu(x_1+\delta_0)(\sum_{i=2}^{n}((a_{ii}\psi_{x_i}E_0\vartheta^2)_{x_i}-(a_{ii}\psi_{x_i}E_0)_{x_i}\vartheta^2) \\
& -\sum_{j=1}^{m}((b_{jj}\psi_{y_j}E_0\vartheta^2)_{y_j}-(b_{jj}\psi_{y_j}E_0)_{y_j}\vartheta^2))
\end{aligned} \tag{2.15}
$$

bulunur.

(2.11)-(2.15) kullanılarak,

$$
\begin{aligned}
& \psi^{\nu+1}(z_1^2+(z_2+z_4+z_5)^2+z_3^2+2z_1z_2+2z_1z_3 \\
& +2z_1(z_4+z_5)+2z_2z_3+2z_3(z_4+z_5)) \\
\geq\ & -2\lambda\nu\psi(x_1+\delta_0)^2(\sum_{i=2}^{n}(a_{ii})_{x_i}\vartheta_{x_i}-\sum_{j=1}^{m}(b_{jj})_{y_j}\vartheta_{y_j})^2+d_2(\vartheta) \\
& -(2k(x_1+\delta_0)\psi^{\nu+1}E_0\vartheta^2)_\xi+(2k(x_1+\delta_0)^2\xi\psi^{\nu+1}\vartheta^2)_\xi-2k(x_1+\delta_0)^2\psi^{\nu+1}\vartheta^2 \\
& -2\lambda\nu(\xi\psi_{x_1}\vartheta^2)_{x_1}-2\lambda\nu(x_1+\delta_0)^2\sum_{i=2}^{n}(a_{ii}\psi_{x_i}\xi\vartheta^2)_{x_i}
\end{aligned}
$$

$$
\begin{aligned}
& +2\lambda\nu(x_1+\delta_0)^2\xi\vartheta^2\sum_{i=2}^{n}(a_{ii}\psi_{x_i})_{x_i}+2\lambda\nu(x_1+\delta_0)^2\sum_{j=1}^{m}(b_{jj}\psi_{y_j}\xi\vartheta^2)_{y_j} \\
& -2\lambda\nu(x_1+\delta_0)^2\xi\vartheta^2\sum_{j=1}^{m}(b_{jj}\psi_{y_j})_{y_j} \\
& +2\lambda\nu\psi_{x_1}((x_1+\delta_0)^{-2}\vartheta_{x_1}^2)_{x_1}+4\lambda\nu\psi_{x_1}(x_1+\delta_0)^{-3}\vartheta_{x_1}^2 \\
& +2\lambda\nu(\sum_{i=2}^{n}(2(\psi_{x_i}a_{ii}\vartheta_{x_i}\vartheta_{x_1})_{x_1}-(\psi_{x_i}a_{ii}\vartheta_{x_1}^2)_{x_i}-2(\psi_{x_i}a_{ii})_{x_1}\vartheta_{x_i}\vartheta_{x_1}+(\psi_{x_i}a_{ii})_{x_i}\vartheta_{x_1}^2) \\
& -\sum_{j=1}^{m}(2(\psi_{y_j}b_{jj}\vartheta_{y_j}\vartheta_{x_1})_{x_1}-(\psi_{y_j}b_{jj}\vartheta_{x_1}^2)_{y_j}-2(\psi_{y_j}b_{jj})_{x_1}\vartheta_{y_j}\vartheta_{x_1}+(\psi_{y_j}b_{jj})_{y_j}\vartheta_{x_1}^2)
\end{aligned}
$$

$$
\begin{aligned}
& +\sum_{i=2}^{n}(2(\psi_{x_1}a_{ii}\vartheta_{x_1}\vartheta_{x_i})_{x_i}-(\psi_{x_1}a_{ii}\vartheta_{x_i}^2)_{x_1}-2(\psi_{x_1}a_{ii})_{x_i}\vartheta_{x_1}\vartheta_{x_i}+(\psi_{x_1}a_{ii})_{x_1}\vartheta_{x_i}^2) \\
& -\sum_{j=1}^{m}(2(\psi_{x_1}b_{jj}\vartheta_{x_1}\vartheta_{y_j})_{y_j}-(\psi_{x_1}b_{jj}\vartheta_{y_j}^2)_{x_1}-2(\psi_{x_1}b_{jj})_{y_j}\vartheta_{x_1}\vartheta_{y_j}+(\psi_{x_1}b_{jj})_{x_1}\vartheta_{y_j}^2)) \\
& +2\lambda\nu(x_1+\delta_0)^2(\sum_{s,i=2}^{n}(2(a_{ss}a_{ii}\psi_{x_i}\vartheta_{x_i}\vartheta_{x_s})_{x_s}-(a_{ss}a_{ii}\psi_{x_i}\vartheta_{x_s}^2)_{x_i} \\
& -2(a_{ss}a_{ii}\psi_{x_i})_{x_s}\vartheta_{x_i}\vartheta_{x_s}+(a_{ss}a_{ii}\psi_{x_i})_{x_i}\vartheta_{x_s}^2) \\
& -\sum_{s=2}^{n}\sum_{j=1}^{m}(2(a_{ss}b_{jj}\psi_{y_j}\vartheta_{y_j}\vartheta_{x_s})_{x_s}-(a_{ss}b_{jj}\psi_{y_j}\vartheta_{x_s}^2)_{y_j}
\end{aligned}
$$

$$
\begin{aligned}
& -2(a_{ss}b_{jj}\psi_{y_j})_{x_s}\vartheta_{y_j}\vartheta_{x_s}+(a_{ss}b_{jj}\psi_{y_j})_{y_j}\vartheta_{x_s}^2) \\
& -\sum_{k=1}^{m}\sum_{i=2}^{n}(2(b_{kk}a_{ii}\psi_{x_i}\vartheta_{x_i}\vartheta_{y_k})_{y_k}-(b_{kk}a_{ii}\psi_{x_i}\vartheta_{y_k}^2)_{x_i} \\
& -2(b_{kk}a_{ii}\psi_{x_i})_{y_k}\vartheta_{x_i}\vartheta_{y_k}+(b_{kk}a_{ii}\psi_{x_i})_{x_i}\vartheta_{y_k}^2) \\
& +\sum_{k,j=1}^{m}(2(b_{kk}b_{jj}\psi_{y_j}\vartheta_{y_j}\vartheta_{y_k})_{y_k}-(b_{kk}b_{jj}\psi_{y_j}\vartheta_{y_k}^2)_{y_j} \\
& -2(b_{kk}b_{jj}\psi_{y_j})_{y_k}\vartheta_{y_j}\vartheta_{y_k}+(b_{kk}b_{jj}\psi_{y_j})_{y_j}\vartheta_{y_k}^2))
\end{aligned}
$$

$$+2\lambda\nu\psi_{x_1}((x_1+\delta_0)^{-1}E_0\vartheta^2)_{x_1} - 2\lambda\nu\psi_{x_1}((x_1+\delta_0)^{-1}E_0)_{x_1}\vartheta^2$$

$$+2\lambda\nu(x_1+\delta_0)(\sum_{i=2}^{n}((a_{ii}\psi_{x_i}E_0\vartheta^2)_{x_i} - (a_{ii}\psi_{x_i}E_0)_{x_i}\vartheta^2)$$

$$-\sum_{j=1}^{m}((b_{jj}\psi_{y_j}E_0\vartheta^2)_{y_j} - (b_{jj}\psi_{y_j}E_0)_{y_j}\vartheta^2))$$

$$= -2\lambda\nu\psi(x_1+\delta_0)^2(\sum_{i=2}^{n}(a_{ii})_{x_i}\vartheta_{x_i} - \sum_{j=1}^{m}(b_{jj})_{y_j}\vartheta_{y_j})^2 - 2k(x_1+\delta_0)^2\psi^{\nu+1}\vartheta^2$$

$$+2\lambda\nu(x_1+\delta_0)^2\xi\vartheta^2\sum_{i=2}^{n}(a_{ii}\psi_{x_i})_{x_i} - 2\lambda\nu(x_1+\delta_0)^2\xi\vartheta^2\sum_{j=1}^{m}(b_{jj}\psi_{y_j})_{y_j}$$

$$+4\lambda\nu\psi_{x_1}(x_1+\delta_0)^{-3}\vartheta_{x_1}^2$$

$$-2\lambda\nu(\sum_{i=2}^{n}(2(\psi_{x_i}a_{ii})_{x_1}\vartheta_{x_i}\vartheta_{x_1} - (\psi_{x_i}a_{ii})_{x_i}\vartheta_{x_1}^2 + 2(\psi_{x_1}a_{ii})_{x_i}\vartheta_{x_1}\vartheta_{x_i} - (\psi_{x_1}a_{ii})_{x_1}\vartheta_{x_i}^2)$$

$$-\sum_{j=1}^{m}(2(\psi_{y_j}b_{jj})_{x_1}\vartheta_{y_j}\vartheta_{x_1} - (\psi_{y_j}b_{jj})_{y_j}\vartheta_{x_1}^2 + 2(\psi_{x_1}b_{jj})_{y_j}\vartheta_{x_1}\vartheta_{y_j} - (\psi_{x_1}b_{jj})_{x_1}\vartheta_{y_j}^2))$$

$$-2\lambda\nu(x_1+\delta_0)^2(\sum_{s,i=2}^{n}(2(a_{ss}a_{ii}\psi_{x_i})_{x_s}\vartheta_{x_i}\vartheta_{x_s} - (a_{ss}a_{ii}\psi_{x_i})_{x_i}\vartheta_{x_s}^2)$$

$$-\sum_{s=2}^{n}\sum_{j=1}^{m}(2(a_{ss}b_{jj}\psi_{y_j})_{x_s}\vartheta_{y_j}\vartheta_{x_s} - (a_{ss}b_{jj}\psi_{y_j})_{y_j}\vartheta_{x_s}^2)$$

$$-\sum_{k=1}^{m}\sum_{i=2}^{n}(2(b_{kk}a_{ii}\psi_{x_i})_{y_k}\vartheta_{x_i}\vartheta_{y_k} - (b_{kk}a_{ii}\psi_{x_i})_{x_i}\vartheta_{y_k}^2)$$

$$+\sum_{k,j=1}^{m}(2(b_{kk}b_{jj}\psi_{y_j})_{y_k}\vartheta_{y_j}\vartheta_{y_k} - (b_{kk}b_{jj}\psi_{y_j})_{y_j}\vartheta_{y_k}^2))$$

$$-2\lambda\nu(\psi_{x_1}((x_1+\delta_0)^{-1}E_0)_{x_1} + (x_1+\delta_0)(\sum_{i=2}^{n}(a_{ii}\psi_{x_i}E_0)_{x_i} - \sum_{j=1}^{m}(b_{jj}\psi_{y_j}E_0)_{y_j}))\vartheta^2$$

$$+d_1(\vartheta) + d_2(\vartheta)$$

$$= 4\lambda\nu\delta(x_1+\delta_0)^{-3}\vartheta_{x_1}^2$$

$$-2\lambda\nu(\sum_{i=2}^{n}(2(\psi_{x_i}a_{ii})_{x_1}\vartheta_{x_i}\vartheta_{x_1} - (a_{ii})_{x_i}\psi_{x_i}\vartheta_{x_1}^2 + \delta(2(a_{ii})_{x_i}\vartheta_{x_1}\vartheta_{x_i} - (a_{ii})_{x_1}\vartheta_{x_i}^2))$$

$$-\sum_{j=1}^{m}(2(\psi_{y_j}b_{jj})_{x_1}\vartheta_{y_j}\vartheta_{x_1} - (b_{jj})_{y_j}\psi_{y_j}\vartheta_{x_1}^2 + \delta(2(b_{jj})_{y_j}\vartheta_{x_1}\vartheta_{y_j} - (b_{jj})_{x_1}\vartheta_{y_j}^2)))$$

$$+2\lambda\nu\sum_{i=2}^{n}a_{ii}\psi_{x_ix_i}\vartheta_{x_1}^2 - 2\lambda\nu\sum_{j=1}^{m}b_{jj}\psi_{y_jy_j}\vartheta_{x_1}^2 + I$$

$$-2(x_1+\delta_0)^2(\lambda\nu\psi(\sum_{i=2}^{n}(a_{ii})_{x_i}\vartheta_{x_i}-\sum_{j=1}^{m}(b_{jj})_{y_j}\vartheta_{y_j})^2$$
$$+(k\psi^{\nu+1}-\lambda\nu\xi(\sum_{i=2}^{n}(a_{ii}\psi_{x_i})_{x_i}-\sum_{j=1}^{m}(b_{jj}\psi_{y_j})_{y_j}))\vartheta^2)$$
$$-2\lambda\nu(\delta((x_1+\delta_0)^{-1}E_0)_{x_1}+(x_1+\delta_0)(\sum_{i=2}^{n}(a_{ii}\psi_{x_i}E_0)_{x_i}-\sum_{j=1}^{m}(b_{jj}\psi_{y_j}E_0)_{y_j}))\vartheta^2$$
$$+d_1(\vartheta)+d_2(\vartheta) \qquad (2.16)$$

elde edilir. Burada $\psi_{x_1}=\delta$ olduğu dikkate alınmıştır ve

$$\begin{aligned} I \;=\; & -2\lambda\nu(x_1+\delta_0)^2(\sum_{s,i=2}^{n}(2(a_{ss}a_{ii}\psi_{x_i})_{x_s}\vartheta_{x_i}\vartheta_{x_s}-(a_{ss}a_{ii}\psi_{x_i})_{x_i}\vartheta_{x_s}^2) \\ & -\sum_{s=2}^{n}\sum_{j=1}^{m}(2(a_{ss}b_{jj}\psi_{y_j})_{x_s}\vartheta_{y_j}\vartheta_{x_s}-(a_{ss}b_{jj}\psi_{y_j})_{y_j}\vartheta_{x_s}^2) \\ & -\sum_{k=1}^{m}\sum_{i=2}^{n}(2(b_{kk}a_{ii}\psi_{x_i})_{y_k}\vartheta_{x_i}\vartheta_{y_k}-(b_{kk}a_{ii}\psi_{x_i})_{x_i}\vartheta_{y_k}^2) \\ & +\sum_{k,j=1}^{m}(2(b_{kk}b_{jj}\psi_{y_j})_{y_k}\vartheta_{y_j}\vartheta_{y_k}-(b_{kk}b_{jj}\psi_{y_j})_{y_j}\vartheta_{y_k}^2)), \end{aligned}$$

$$\begin{aligned} d_1(\vartheta) \;=\; & -(2k(x_1+\delta_0)\psi^{\nu+1}E_0\vartheta^2)_\xi+(2k(x_1+\delta_0)^2\xi\psi^{\nu+1}\vartheta^2)_\xi \\ & -2\lambda\nu(\xi\psi_{x_1}\vartheta^2)_{x_1}-2\lambda\nu(x_1+\delta_0)^2(\sum_{i=2}^{n}(a_{ii}\psi_{x_i}\xi\vartheta^2)_{x_i}-\sum_{j=1}^{m}(b_{jj}\psi_{y_j}\xi\vartheta^2)_{y_j}) \\ & +2\lambda\nu\psi_{x_1}(((x_1+\delta_0)^{-2}\vartheta_{x_1}^2)_{x_1}+((x_1+\delta_0)^{-1}E_0\vartheta^2)_{x_1}) \end{aligned}$$

$$\begin{aligned} & +2\lambda\nu(\sum_{i=2}^{n}(2(\psi_{x_i}a_{ii}\vartheta_{x_i}\vartheta_{x_1})_{x_1}-(\psi_{x_i}a_{ii}\vartheta_{x_1}^2)_{x_i} \\ & +2(\psi_{x_1}a_{ii}\vartheta_{x_1}\vartheta_{x_i})_{x_i}-(\psi_{x_1}a_{ii}\vartheta_{x_i}^2)_{x_1}) \\ & -\sum_{j=1}^{m}(2(\psi_{y_j}b_{jj}\vartheta_{y_j}\vartheta_{x_1})_{x_1}-(\psi_{y_j}b_{jj}\vartheta_{x_1}^2)_{y_j} \\ & +2(\psi_{x_1}b_{jj}\vartheta_{x_1}\vartheta_{y_j})_{y_j}-(\psi_{x_1}b_{jj}\vartheta_{y_j}^2)_{x_1})) \end{aligned}$$

$$+2\lambda\nu(x_1+\delta_0)^2(\sum_{s,i=2}^{n}(2(a_{ss}a_{ii}\psi_{x_i}\vartheta_{x_i}\vartheta_{x_s})_{x_s}-(a_{ss}a_{ii}\psi_{x_i}\vartheta^2_{x_s})_{x_i})$$
$$-\sum_{s=2}^{n}\sum_{j=1}^{m}(2(a_{ss}b_{jj}\psi_{y_j}\vartheta_{y_j}\vartheta_{x_s})_{x_s}-(a_{ss}b_{jj}\psi_{y_j}\vartheta^2_{x_s})_{y_j})$$
$$-\sum_{k=1}^{m}\sum_{i=2}^{n}(2(b_{kk}a_{ii}\psi_{x_i}\vartheta_{x_i}\vartheta_{y_k})_{y_k}-(b_{kk}a_{ii}\psi_{x_i}\vartheta^2_{y_k})_{x_i})$$
$$+\sum_{k,j=1}^{m}(2(b_{kk}b_{jj}\psi_{y_j}\vartheta_{y_j}\vartheta_{y_k})_{y_k}-(b_{kk}b_{jj}\psi_{y_j}\vartheta^2_{y_k})_{y_j}))$$
$$+2\lambda\nu(x_1+\delta_0)(\sum_{i=2}^{n}(a_{ii}\psi_{x_i}E_0\vartheta^2)_{x_i}-\sum_{j=1}^{m}(b_{jj}\psi_{y_j}E_0\vartheta^2)_{y_j}),$$

$$\begin{aligned}
d_2(\vartheta) &= -4k(\psi^{\nu+1}\vartheta_\xi\vartheta_{x_1})_{x_1}+2k(\psi^{\nu+1}\vartheta^2_{x_1})_\xi \\
&\quad -2k(x_1+\delta_0)^2(\sum_{i=2}^{n}(2(\psi^{\nu+1}a_{ii}\vartheta_\xi\vartheta_{x_i})_{x_i}-(\psi^{\nu+1}a_{ii}\vartheta^2_{x_i})_\xi) \\
&\quad -\sum_{j=1}^{m}(2(\psi^{\nu+1}b_{jj}\vartheta_\xi\vartheta_{y_j})_{y_j}-(\psi^{\nu+1}b_{jj}\vartheta^2_{y_j})_\xi))
\end{aligned}$$

olarak tanımlıdır. Diğer taraftan $\psi_{x_i}=(x_i-x_i^0)$, $|\psi_{x_i}|=|x_i-x_i^0|\le\sqrt{2\gamma}$, $2\le i\le n$ ve $\psi_{y_j}=(y_j-y_j^0)$, $|\psi_{y_j}|=|y_j-y_j^0|\le\sqrt{2\gamma}$, $1\le j\le m$ dir. Gerçekten de

$$\begin{aligned}
\psi(x,y) &= \delta x_1+\frac{1}{2}\sum_{i=2}^{n}(x_i-x_i^0)^2+\frac{1}{2}\sum_{j=1}^{m}(y_j-y_j^0)^2<\gamma, \\
|x_i-x_i^0|^2 &< \gamma,\ |x_i-x_i^0|<\sqrt{2\gamma}, \\
|\psi_{x_i}| &= |x_i-x_i^0|\le\sqrt{2\gamma}, \\
|y_j-y_j^0|^2 &< \gamma,\ |y_j-y_j^0|\le\sqrt{2\gamma}, \\
|\psi_{y_j}| &= |y_j-y_j^0|\le\sqrt{2\gamma}
\end{aligned}$$

olduğu kolayca görülebilir. O halde yukarıdaki eşitsizlikler, $||a_{ii}||_{C^1(\overline{D\times G})}\le M(2\le i\le n)$, $||b_{jj}||_{C^1(\overline{D\times G})}\le M\ (1\le j\le m)$, $\forall\varepsilon_0>0$ için $-2ab\ge-\frac{1}{\varepsilon_0(x_1+\delta_0)}a^2-\varepsilon_0(x_1+\delta_0)b^2$ eşitsizliği ve

$$\begin{aligned}
\psi_{x_ix_s} &= \begin{cases}1, & i=s\\ 0, & i\ne s\end{cases}\ (2\le i,s\le n);\ \psi_{y_jy_k}=\begin{cases}1, & j=k\\ 0, & j\ne k\end{cases}\ (1\le j,k\le m); \\
\psi_{x_iy_j} &= 0,\ \psi_{y_jx_1}=0,\ \psi_{x_ix_1}=0
\end{aligned}$$

yardımıyla

$$\begin{aligned}
-4\lambda\nu\delta\sum_{i=2}^{n}(a_{ii})_{x_i}\vartheta_{x_1}\vartheta_{x_i} &\geq -4\lambda\nu\delta\sum_{i=2}^{n}|(a_{ii})_{x_i}||\vartheta_{x_1}\vartheta_{x_i}| \\
&= 2\lambda\nu\delta\sum_{i=2}^{n}(-2|(a_{ii})_{x_i}||\vartheta_{x_1}\vartheta_{x_i}|) \\
&\geq -2\lambda\nu\delta\sum_{i=2}^{n}((x_1+\delta_0)\varepsilon_0\vartheta_{x_i}^2 + \frac{M^2}{\varepsilon_0(x_1+\delta_0)}\vartheta_{x_1}^2) \\
&= -2\lambda\nu\delta((x_1+\delta_0)\varepsilon_0\sum_{i=2}^{n}\vartheta_{x_i}^2 + \frac{M^2}{\varepsilon_0(x_1+\delta_0)}(n-1)\vartheta_{x_1}^2), \qquad (2.17)
\end{aligned}$$

$$2\lambda\nu\delta\sum_{i=2}^{n}(a_{ii})_{x_1}\vartheta_{x_i}^2 \geq 2\lambda\nu(x_1+\delta_0)\delta\alpha_1\sum_{i=2}^{n}\vartheta_{x_i}^2, \qquad (2.18)$$

$$\begin{aligned}
4\lambda\nu\delta\sum_{j=1}^{m}(b_{jj})_{y_j}\vartheta_{x_1}\vartheta_{y_j} &\geq -4\lambda\nu\delta\sum_{j=1}^{m}|(b_{jj})_{y_j}||\vartheta_{x_1}\vartheta_{y_j}| \\
&= 2\lambda\nu\delta\sum_{j=1}^{m}(-2|(b_{jj})_{y_j}||\vartheta_{x_1}\vartheta_{y_j}|) \\
&\geq -2\lambda\nu\delta\sum_{j=1}^{m}((x_1+\delta_0)\varepsilon_0\vartheta_{y_j}^2 + \frac{M^2}{\varepsilon_0(x_1+\delta_0)}\vartheta_{x_1}^2) \\
&= -2\lambda\nu\delta((x_1+\delta_0)\varepsilon_0\sum_{j=1}^{m}\vartheta_{y_j}^2 + \frac{M^2}{\varepsilon_0(x_1+\delta_0)}m\vartheta_{x_1}^2), \qquad (2.19)
\end{aligned}$$

$$-2\lambda\nu\delta\sum_{j=1}^{m}(b_{jj})_{x_1}\vartheta_{y_j}^2 \geq 2\lambda\nu(x_1+\delta_0)\delta\alpha_2\sum_{j=1}^{m}\vartheta_{y_j}^2, \qquad (2.20)$$

$$\begin{aligned}
-4\lambda\nu\sum_{i=2}^{n}(\psi_{x_i}a_{ii})_{x_1}\vartheta_{x_i}\vartheta_{x_1} &= -4\lambda\nu\sum_{i=2}^{n}(a_{ii})_{x_1}\psi_{x_i}\vartheta_{x_i}\vartheta_{x_1} \\
&\geq -4\lambda\nu\sum_{i=2}^{n}|(a_{ii})_{x_1}||\psi_{x_i}||\vartheta_{x_i}\vartheta_{x_1}| \\
&\geq -4\lambda\nu\sqrt{2\gamma}\sum_{i=2}^{n}|(a_{ii})_{x_1}||\vartheta_{x_i}\vartheta_{x_1}| \\
&= 2\lambda\nu\sqrt{2\gamma}\sum_{i=2}^{n}(-2|(a_{ii})_{x_1}||\vartheta_{x_i}\vartheta_{x_1}|) \\
&\geq -2\lambda\nu\sqrt{2\gamma}\sum_{i=2}^{n}((x_1+\delta_0)\vartheta_{x_i}^2 + \frac{1}{(x_1+\delta_0)}|(a_{ii})_{x_1}|^2\vartheta_{x_1}^2) \\
&\geq -2\lambda\nu\sqrt{2\gamma}(x_1+\delta_0)\sum_{i=2}^{n}\vartheta_{x_i}^2 \\
&\quad -2\lambda\nu\sqrt{2\gamma}\frac{M^2}{(x_1+\delta_0)}(n-1)\vartheta_{x_1}^2, \qquad (2.21)
\end{aligned}$$

$$2\lambda\nu\sum_{i=2}^{n}(a_{ii})_{x_i}\psi_{x_i}\vartheta_{x_1}^2 \geq -2\lambda\nu M\sqrt{2\gamma}(n-1)\vartheta_{x_1}^2, \tag{2.22}$$

$$\begin{aligned}
4\lambda\nu\sum_{j=1}^{m}(\psi_{y_j}b_{jj})_{x_1}\vartheta_{y_j}\vartheta_{x_1} &= 4\lambda\nu\sum_{j=1}^{m}(b_{jj})_{x_1}\psi_{y_j}\vartheta_{y_j}\vartheta_{x_1}\\
&\geq -4\lambda\nu\sum_{j=1}^{m}|(b_{jj})_{x_1}||\psi_{y_j}||\vartheta_{y_j}\vartheta_{x_1}|\\
&\geq -4\lambda\nu\sqrt{2\gamma}\sum_{j=1}^{m}|(b_{jj})_{x_1}||\vartheta_{y_j}\vartheta_{x_1}|\\
&= 2\lambda\nu\sqrt{2\gamma}\sum_{j=1}^{m}(-2|(b_{jj})_{x_1}||\vartheta_{y_j}\vartheta_{x_1}|)\\
&\geq -2\lambda\nu\sqrt{2\gamma}\sum_{j=1}^{m}((x_1+\delta_0)\vartheta_{y_j}^2+\frac{1}{(x_1+\delta_0)}|(b_{jj})_{x_1}|^2\vartheta_{x_1}^2)\\
&\geq -2\lambda\nu\sqrt{2\gamma}(x_1+\delta_0)\sum_{j=1}^{m}\vartheta_{y_j}^2-2\lambda\nu\sqrt{2\gamma}\frac{M^2}{(x_1+\delta_0)}m\vartheta_{x_1}^2, \quad (2.23)
\end{aligned}$$

$$\begin{aligned}
-2\lambda\nu\sum_{j=1}^{m}(b_{jj})_{y_j}\psi_{y_j}\vartheta_{x_1}^2 &\geq -2\lambda\nu\sum_{j=1}^{m}|(b_{jj})_{y_j}||\psi_{y_j}||\vartheta_{x_1}^2|\\
&\geq -2\lambda\nu M\sqrt{2\gamma}m\vartheta_{x_1}^2 \quad (2.24)
\end{aligned}$$

eşitsizlikleri yazılabilir. Ayrıca I'nın terimleri için

$$\begin{aligned}
|(a_{ss}a_{ii}\psi_{x_i})_{x_s}| &= |(a_{ss})_{x_s}a_{ii}\psi_{x_i}+a_{ss}(a_{ii})_{x_s}\psi_{x_i}+a_{ss}a_{ii}\psi_{x_ix_s}|\\
&\leq M^2\sqrt{2\gamma}+M^2\sqrt{2\gamma}+M^2=M^2(2\sqrt{2\gamma}+1),
\end{aligned}$$

$$\begin{aligned}
|(a_{ss}a_{ii}\psi_{x_i})_{x_i}| &= |(a_{ss})_{x_i}a_{ii}\psi_{x_i}+a_{ss}(a_{ii})_{x_i}\psi_{x_i}+a_{ss}a_{ii}\psi_{x_ix_i}|\\
&\leq M^2\sqrt{2\gamma}+M^2\sqrt{2\gamma}+M^2=M^2(2\sqrt{2\gamma}+1),
\end{aligned}$$

$$\begin{aligned}
|(a_{ss}b_{jj}\psi_{y_j})_{x_s}| &= |(a_{ss})_{x_s}b_{jj}\psi_{y_j}+a_{ss}(b_{jj})_{x_s}\psi_{y_j}+a_{ss}b_{jj}\psi_{y_jx_s}|\\
&\leq M^2\sqrt{2\gamma}+M^2\sqrt{2\gamma}=2M^2\sqrt{2\gamma},
\end{aligned}$$

$$\begin{aligned}
|(a_{ss}b_{jj}\psi_{y_j})_{y_j}| &= |(a_{ss})_{y_j}b_{jj}\psi_{y_j}+a_{ss}(b_{jj})_{y_j}\psi_{y_j}+a_{ss}b_{jj}\psi_{y_jy_j}|\\
&\leq M^2\sqrt{2\gamma}+M^2\sqrt{2\gamma}+M^2=M^2(2\sqrt{2\gamma}+1),
\end{aligned}$$

$$\begin{aligned}
|(b_{kk}a_{ii}\psi_{x_i})_{y_k}| &= |(b_{kk})_{y_k}a_{ii}\psi_{x_i}+b_{kk}(a_{ii})_{y_k}\psi_{x_i}+b_{kk}a_{ii}\psi_{x_iy_k}|\\
&\leq M^2\sqrt{2\gamma}+M^2\sqrt{2\gamma}=2M^2\sqrt{2\gamma},
\end{aligned}$$

$$\begin{aligned}
|(b_{kk}a_{ii}\psi_{x_i})_{x_i}| &= |(b_{kk})_{x_i}a_{ii}\psi_{x_i} + b_{kk}(a_{ii})_{x_i}\psi_{x_i} + b_{kk}a_{ii}\psi_{x_ix_i}| \\
&\leq M^2\sqrt{2\gamma} + M^2\sqrt{2\gamma} + M^2 = M^2(2\sqrt{2\gamma}+1),
\end{aligned}$$

$$\begin{aligned}
|(b_{kk}b_{jj}\psi_{y_j})_{y_k}| &= |(b_{kk})_{y_k}b_{jj}\psi_{y_j} + b_{kk}(b_{jj})_{y_k}\psi_{y_j} + b_{kk}b_{jj}\psi_{y_jy_k}| \\
&\leq M^2\sqrt{2\gamma} + M^2\sqrt{2\gamma} + M^2 = M^2(2\sqrt{2\gamma}+1),
\end{aligned}$$

$$\begin{aligned}
|(b_{kk}b_{jj}\psi_{y_j})_{y_j}| &= |(b_{kk})_{y_j}b_{jj}\psi_{y_j} + b_{kk}(b_{jj})_{y_j}\psi_{y_j} + b_{kk}b_{jj}\psi_{y_jy_j}| \\
&\leq M^2\sqrt{2\gamma} + M^2\sqrt{2\gamma} + M^2 = M^2(2\sqrt{2\gamma}+1)
\end{aligned}$$

eşitsizlikleri kullanılarak

$$\begin{aligned}
&\sum_{s,i=2}^{n}(-2(a_{ss}a_{ii}\psi_{x_i})_{x_s}\vartheta_{x_i}\vartheta_{x_s}) \\
\geq\ & -2\sum_{s,i=2}^{n}|(a_{ss}a_{ii}\psi_{x_i})_{x_s}||\vartheta_{x_i}\vartheta_{x_s}| \geq M^2(2\sqrt{2\gamma}+1)\sum_{s,i=2}^{n}(-2|\vartheta_{x_i}\vartheta_{x_s}|) \\
\geq\ & -M^2(2\sqrt{2\gamma}+1)\sum_{s,i=2}^{n}(\vartheta_{x_i}^2+\vartheta_{x_s}^2) = -M^2(2\sqrt{2\gamma}+1)(\sum_{s,i=2}^{n}\vartheta_{x_i}^2 + \sum_{s,i=2}^{n}\vartheta_{x_s}^2) \\
=\ & -M^2(2\sqrt{2\gamma}+1)((n-1)\sum_{i=2}^{n}\vartheta_{x_i}^2 + (n-1)\sum_{s=2}^{n}\vartheta_{x_s}^2) \\
=\ & -2M^2(2\sqrt{2\gamma}+1)(n-1)\sum_{i=2}^{n}\vartheta_{x_i}^2,
\end{aligned}$$

$$\begin{aligned}
\sum_{s,i=2}^{n}(a_{ss}a_{ii}\psi_{x_i})_{x_i}\vartheta_{x_s}^2 &\geq -\sum_{s,i=2}^{n}|(a_{ss}a_{ii}\psi_{x_i})_{x_i}||\vartheta_{x_s}^2| \\
&\geq -M^2(2\sqrt{2\gamma}+1)\sum_{s,i=2}^{n}\vartheta_{x_s}^2 \\
&= -M^2(2\sqrt{2\gamma}+1)(n-1)\sum_{s=2}^{n}\vartheta_{x_s}^2,
\end{aligned}$$

$$\begin{aligned}
\sum_{s=2}^{n}\sum_{j=1}^{m}2(a_{ss}b_{jj}\psi_{y_j})_{x_s}\vartheta_{y_j}\vartheta_{x_s} &\geq -\sum_{s=2}^{n}\sum_{j=1}^{m}2|(a_{ss}b_{jj}\psi_{y_j})_{x_s}||\vartheta_{y_j}\vartheta_{x_s}| \\
&\geq 2M^2\sqrt{2\gamma}\sum_{s=2}^{n}\sum_{j=1}^{m}(-2|\vartheta_{y_j}\vartheta_{x_s}|) \\
&\geq -2M^2\sqrt{2\gamma}\sum_{s=2}^{n}\sum_{j=1}^{m}(\vartheta_{y_j}^2+\vartheta_{x_s}^2) \\
&= -2M^2\sqrt{2\gamma}((n-1)\sum_{j=1}^{m}\vartheta_{y_j}^2 + m\sum_{s=2}^{n}\vartheta_{x_s}^2),
\end{aligned}$$

$$
\begin{aligned}
-\sum_{s=2}^{n}\sum_{j=1}^{m}(a_{ss}b_{jj}\psi_{y_j})_{y_j}\vartheta_{x_s}^2 &\geq -\sum_{s=2}^{n}\sum_{j=1}^{m}|(a_{ss}b_{jj}\psi_{y_j})_{y_j}||\vartheta_{x_s}^2| \\
&\geq -M^2(2\sqrt{2\gamma}+1)\sum_{s=2}^{n}\sum_{j=1}^{m}\vartheta_{x_s}^2 \\
&= -M^2(2\sqrt{2\gamma}+1)m\sum_{s=2}^{n}\vartheta_{x_s}^2,
\end{aligned}
$$

$$
\begin{aligned}
\sum_{k=1}^{m}\sum_{i=2}^{n}2(b_{kk}a_{ii}\psi_{x_i})_{y_k}\vartheta_{x_i}\vartheta_{y_k} &\geq -\sum_{k=1}^{m}\sum_{i=2}^{n}2|(b_{kk}a_{ii}\psi_{x_i})_{y_k}||\vartheta_{x_i}\vartheta_{y_k}| \\
&\geq 2M^2\sqrt{2\gamma}\sum_{k=1}^{m}\sum_{i=2}^{n}(-2|\vartheta_{x_i}\vartheta_{y_k}|) \\
&\geq -2M^2\sqrt{2\gamma}\sum_{k=1}^{m}\sum_{i=2}^{n}(\vartheta_{x_i}^2+\vartheta_{y_k}^2) \\
&= -2M^2\sqrt{2\gamma}(m\sum_{i=2}^{n}\vartheta_{x_i}^2+(n-1)\sum_{k=1}^{m}\vartheta_{y_k}^2),
\end{aligned}
$$

$$
\begin{aligned}
-\sum_{k=1}^{m}\sum_{i=2}^{n}(b_{kk}a_{ii}\psi_{x_i})_{x_i}\vartheta_{y_k}^2 &\geq -\sum_{k=1}^{m}\sum_{i=2}^{n}|(b_{kk}a_{ii}\psi_{x_i})_{x_i}||\vartheta_{y_k}^2| \\
&\geq -M^2(2\sqrt{2\gamma}+1)\sum_{k=1}^{m}\sum_{i=2}^{n}\vartheta_{y_k}^2 \\
&= -M^2(2\sqrt{2\gamma}+1)(n-1)\sum_{k=1}^{m}\vartheta_{y_k}^2,
\end{aligned}
$$

$$
\begin{aligned}
\sum_{k,j=1}^{m}-2(b_{kk}b_{jj}\psi_{y_j})_{y_k}\vartheta_{y_j}\vartheta_{y_k} &\geq -2\sum_{k,j=1}^{m}|(b_{kk}b_{jj}\psi_{y_j})_{y_k}||\vartheta_{y_j}\vartheta_{y_k}| \\
&\geq M^2(2\sqrt{2\gamma}+1)\sum_{k,j=1}^{m}(-2|\vartheta_{y_j}\vartheta_{y_k}|) \\
&\geq -M^2(2\sqrt{2\gamma}+1)\sum_{k,j=1}^{m}(\vartheta_{y_j}^2+\vartheta_{y_k}^2) \\
&= -2M^2(2\sqrt{2\gamma}+1)m\sum_{j=1}^{m}\vartheta_{y_j}^2,
\end{aligned}
$$

$$
\begin{aligned}
\sum_{k,j=1}^{m}(b_{kk}b_{jj}\psi_{y_j})_{y_j}\vartheta_{y_k}^2 &\geq -\sum_{k,j=1}^{m}|(b_{kk}b_{jj}\psi_{y_j})_{y_j}||\vartheta_{y_k}^2| \\
&\geq -M^2(2\sqrt{2\gamma}+1)\sum_{k,j=1}^{m}\vartheta_{y_k}^2 \\
&= -M^2(2\sqrt{2\gamma}+1)m\sum_{k=1}^{m}\vartheta_{y_k}^2
\end{aligned}
$$

yazılabilir. Bunlara bağlı olarak

$$\begin{aligned} I \geq\ & 2\lambda\nu(x_1+\delta_0)^2(-2M^2(2\sqrt{2\gamma}+1)(n-1)\sum_{i=2}^{n}\vartheta_{x_i}^2 - M^2(2\sqrt{2\gamma}+1)(n-1)\sum_{s=2}^{n}\vartheta_{x_s}^2 \\ & -2M^2\sqrt{2\gamma}((n-1)\sum_{j=1}^{m}\vartheta_{y_j}^2 + m\sum_{s=2}^{n}\vartheta_{x_s}^2) - M^2(2\sqrt{2\gamma}+1)m\sum_{s=2}^{n}\vartheta_{x_s}^2 \\ & -2M^2\sqrt{2\gamma}(m\sum_{i=2}^{n}\vartheta_{x_i}^2 + (n-1)\sum_{k=1}^{m}\vartheta_{y_k}^2) - M^2(2\sqrt{2\gamma}+1)(n-1)\sum_{k=1}^{m}\vartheta_{y_k}^2 \\ & -2M^2(2\sqrt{2\gamma}+1)m\sum_{j=1}^{m}\vartheta_{y_j}^2 - M^2(2\sqrt{2\gamma}+1)m\sum_{k=1}^{m}\vartheta_{y_k}^2) \end{aligned}$$

$$\begin{aligned} =\ & 2\lambda\nu(x_1+\delta_0)^2((-2M^2(2\sqrt{2\gamma}+1)(n-1) - M^2(2\sqrt{2\gamma}+1)(n-1) \\ & -2M^2\sqrt{2\gamma}m - M^2(2\sqrt{2\gamma}+1)m - 2M^2\sqrt{2\gamma}m)\sum_{i=2}^{n}\vartheta_{x_i}^2 \\ & +(-2M^2\sqrt{2\gamma}(n-1) - 2M^2\sqrt{2\gamma}(n-1) - M^2(2\sqrt{2\gamma}+1)(n-1) \\ & -2M^2(2\sqrt{2\gamma}+1)m - M^2(2\sqrt{2\gamma}+1)m)\sum_{j=1}^{m}\vartheta_{y_j}^2) \end{aligned}$$

$$\begin{aligned} =\ & -2\lambda\nu(x_1+\delta_0)^2 \\ & \times((3M^2(2\sqrt{2\gamma}+1)(n-1) + 4M^2\sqrt{2\gamma}m + M^2(2\sqrt{2\gamma}+1)m)\sum_{i=2}^{n}\vartheta_{x_i}^2 \\ & +(3M^2(2\sqrt{2\gamma}+1)m + 4M^2\sqrt{2\gamma}(n-1) + M^2(2\sqrt{2\gamma}+1)(n-1))\sum_{j=1}^{m}\vartheta_{y_j}^2) \end{aligned} \quad (2.25)$$

bulunur. (2.16) eşitsizliğindeki diğer terimler için aşağıdaki değerlendirmeler yapılabilir:

$$\begin{aligned} & -(\sum_{i=2}^{n}(a_{ii})_{x_i}\vartheta_{x_i} - \sum_{j=1}^{m}(b_{jj})_{y_j}\vartheta_{y_j})^2 \\ =\ & -(\sum_{i=2}^{n}(a_{ii})_{x_i}\vartheta_{x_i})^2 + 2\sum_{i=2}^{n}\sum_{j=1}^{m}(a_{ii})_{x_i}(b_{jj})_{y_j}\vartheta_{x_i}\vartheta_{y_j} - (\sum_{j=1}^{m}(b_{jj})_{y_j}\vartheta_{y_j})^2. \end{aligned}$$

Burada $(\sum_{k=1}^{m} a_k)^2 \leq m\sum_{k=1}^{m} a_k^2$ olduğu dikkate alınarak

$$\begin{aligned} -(\sum_{i=2}^{n}(a_{ii})_{x_i}\vartheta_{x_i})^2 \ &\geq\ -(M\sum_{i=2}^{n}\vartheta_{x_i})^2 \\ &\geq\ -M^2(n-1)\sum_{i=2}^{n}\vartheta_{x_i}^2, \end{aligned}$$

$$
\begin{aligned}
2\sum_{i=2}^{n}\sum_{j=1}^{m}(a_{ii})_{x_i}(b_{jj})_{y_j}\vartheta_{x_i}\vartheta_{y_j} &\geq -2\sum_{i=2}^{n}\sum_{j=1}^{m}|(a_{ii})_{x_i}||(b_{jj})_{y_j}||\vartheta_{x_i}\vartheta_{y_j}| \\
&\geq M^2\sum_{i=2}^{n}\sum_{j=1}^{m}(-2|\vartheta_{x_i}\vartheta_{y_j}|) \\
&\geq -M^2\sum_{i=2}^{n}\sum_{j=1}^{m}(\vartheta_{x_i}^2+\vartheta_{y_j}^2) \\
&= -M^2(m\sum_{i=2}^{n}\vartheta_{x_i}^2+(n-1)\sum_{j=1}^{m}\vartheta_{y_j}^2),
\end{aligned}
$$

$$
\begin{aligned}
-(\sum_{j=1}^{m}(b_{jj})_{y_j}\vartheta_{y_j})^2 &\geq -(M\sum_{j=1}^{m}\vartheta_{y_j})^2 \\
&\geq -M^2m\sum_{j=1}^{m}\vartheta_{y_j}^2
\end{aligned}
$$

eşitsizlikleri elde edilir. Bu eşitsizlikler kullanılarak

$$
\begin{aligned}
&-(\sum_{i=2}^{n}(a_{ii})_{x_i}\vartheta_{x_i}-\sum_{j=1}^{m}(b_{jj})_{y_j}\vartheta_{y_j})^2 \\
\geq\ &-M^2(n-1)\sum_{i=2}^{n}\vartheta_{x_i}^2-M^2(m\sum_{i=2}^{n}\vartheta_{x_i}^2+(n-1)\sum_{j=1}^{m}\vartheta_{y_j}^2) \\
&-M^2m\sum_{j=1}^{m}\vartheta_{y_j}^2 \qquad (2.26)
\end{aligned}
$$

bulunur. Ek olarak

$$
2\lambda\nu\sum_{i=2}^{n}a_{ii}\psi_{x_ix_i}\vartheta_{x_1}^2 \geq -2\lambda\nu\vartheta_{x_1}^2\sum_{i=2}^{n}|a_{ii}| = -2\lambda\nu M(n-1)\vartheta_{x_1}^2, \qquad (2.27)
$$

$$
-2\lambda\nu\sum_{j=1}^{m}b_{jj}\psi_{y_jy_j}\vartheta_{x_1}^2 \geq -2\lambda\nu\vartheta_{x_1}^2\sum_{j=1}^{m}|b_{jj}| = -2\lambda\nu Mm\vartheta_{x_1}^2 \qquad (2.28)
$$

elde edilir. Böylece (2.17)-(2.28) eşitsizlikleri kullanılarak

$$
\psi^{\nu+1}(L_0\varphi)^2\chi^2 \geq E_1\vartheta_{x_1}^2+E_2\sum_{i=2}^{n}\vartheta_{x_i}^2+E_3\sum_{j=1}^{m}\vartheta_{y_j}^2+E_4\vartheta^2+d_1+d_2 \qquad (2.29)
$$

yazılabilir. Burada

$$
\begin{aligned}
E_1 &= 4\lambda\nu\delta(x_1+\delta_0)^{-3}-2\lambda\nu\delta\frac{M^2}{\varepsilon_0(x_1+\delta_0)}(n-1)-2\lambda\nu\delta\frac{M^2}{\varepsilon_0(x_1+\delta_0)}m \\
&\quad -2\lambda\nu\sqrt{2\gamma}\frac{M^2}{(x_1+\delta_0)}(n-1)-2\lambda\nu M\sqrt{2\gamma}(n-1)-2\lambda\nu\sqrt{2\gamma}\frac{M^2}{(x_1+\delta_0)}m \\
&\quad -2\lambda\nu M\sqrt{2\gamma}m-2\lambda\nu M(n-1)-2\lambda\nu Mm,
\end{aligned}
$$

$$
\begin{aligned}
E_2 &= 2\lambda\nu(x_1+\delta_0)(\delta\alpha_1 - \delta\varepsilon_0 \\
&\quad -\sqrt{2\gamma} - (x_1+\delta_0)(3M^2(2\sqrt{2\gamma}+1)(n-1) + 4M^2\sqrt{2\gamma}m + M^2(2\sqrt{2\gamma}+1)m) \\
&\quad -(x_1+\delta_0)\psi(M^2(n-1)+M^2m)),
\end{aligned}
$$

$$
\begin{aligned}
E_3 &= 2\lambda\nu(x_1+\delta_0)(\delta\alpha_2 - \delta\varepsilon_0 - \sqrt{2\gamma} \\
&\quad -(x_1+\delta_0)(3M^2(2\sqrt{2\gamma}+1)m + 4M^2\sqrt{2\gamma}(n-1) + M^2(2\sqrt{2\gamma}+1)(n-1)) \\
&\quad -(x_1+\delta_0)\psi(M^2(n-1)+M^2m)).
\end{aligned}
$$

$$
\begin{aligned}
E_4 &= -2\lambda\nu(\delta((x_1+\delta_0)^{-1}E_0)_{x_1} + (x_1+\delta_0)(\sum_{i=2}^{n}(a_{ii}\psi_{x_i}E_0)_{x_i} - \sum_{j=1}^{m}(b_{jj}\psi_{y_j}E_0)_{y_j})) \\
&\quad -2(x_1+\delta_0)^2(k\psi^{\nu+1} - \lambda\nu\xi(\sum_{i=2}^{n}(a_{ii}\psi_{x_i})_{x_i} - \sum_{j=1}^{m}(b_{jj}\psi_{y_j})_{y_j}))
\end{aligned}
$$

şeklinde tanımlıdır. Ayrıca $\delta_0 \leq \frac{1}{2}\gamma$, $\delta \geq 4$ ve $0 < \delta x_1 < \gamma$ olduğundan

$$
\begin{aligned}
x_1 &< \frac{\gamma}{\delta} \leq \frac{\gamma}{4}, \\
\delta_0 &\leq \frac{1}{2}\gamma
\end{aligned}
$$

eşitsizlikleri taraf tarafa toplanırsa $(x_1+\delta_0) < \frac{3}{4}\gamma$ olarak bulunur.

Şimdi $\vartheta_{x_1}^2$'nin katsayısı olan E_1 toplamını düzenleyelim:

$$
\begin{aligned}
E_1 &= 4\lambda\nu\delta(x_1+\delta_0)^{-3} - 2\lambda\nu\delta\frac{M^2}{\varepsilon_0(x_1+\delta_0)}(n+m-1) \\
&\quad -2\lambda\nu\sqrt{2\gamma}\frac{M^2}{(x_1+\delta_0)}(n+m-1) - 2\lambda\nu M\sqrt{2\gamma}(n+m-1) \\
&\quad -2\lambda\nu M(n+m-1)
\end{aligned}
$$

$$
\begin{aligned}
&= 2\lambda\nu(x_1+\delta_0)^{-3}(2\delta - (x_1+\delta_0)^2\delta\frac{M^2}{\varepsilon_0}(n+m-1) \\
&\quad -(x_1+\delta_0)^2\sqrt{2\gamma}M^2(n+m-1) - (x_1+\delta_0)^3 M\sqrt{2\gamma}(n+m-1) \\
&\quad -(x_1+\delta_0)^3 M(n+m-1)) \\
&= 2\lambda\nu(x_1+\delta_0)^{-3}(\delta + E_{11} - E_{12}).
\end{aligned}
$$

Burada

$$
\begin{aligned}
E_{11} &= \delta(1 - (x_1+\delta_0)^2\frac{M^2}{\varepsilon_0}(n+m-1)), \\
E_{12} &= (x_1+\delta_0)^2\sqrt{2\gamma}M^2(n+m-1) + (x_1+\delta_0)^3 M\sqrt{2\gamma}(n+m-1) \\
&\quad +(x_1+\delta_0)^3 M(n+m-1)
\end{aligned}
$$

dir. Diğer taraftan, eğer $0 < \gamma < \frac{2\sqrt{2\varepsilon_0}}{3M\sqrt{n+m-1}}$ alınırsa

$$\begin{aligned}
1-(x_1+\delta_0)^2\frac{M^2}{\varepsilon_0}(n+m-1) &> 1-(\frac{3}{4}\gamma)^2\frac{M^2}{\varepsilon_0}(n+m-1)\\
&> 1-(\frac{6\sqrt{2\varepsilon_0}}{12M\sqrt{n+m-1}})^2\frac{M^2}{\varepsilon_0}(n+m-1)\\
&= 1-\frac{72\varepsilon_0}{144M^2(n+m-1)}\frac{M^2}{\varepsilon_0}(n+m-1)\\
&= \frac{1}{2}
\end{aligned}$$

yazılır. Böylece

$$E_{11} > \delta(1-(\frac{3}{4}\gamma)^2\frac{M^2}{\varepsilon_0}(n+m-1)) > \frac{1}{2}\delta \tag{2.30}$$

bulunur.

Ayrıca $0 < \gamma < 1$ olduğu göz önünde bulundurularak E_1 içerisindeki diğer terimler için

$$\begin{aligned}
&(x_1+\delta_0)^2\sqrt{2\gamma}M^2(n+m-1)+(x_1+\delta_0)^3M\sqrt{2\gamma}(n+m-1)\\
&+(x_1+\delta_0)^3M(n+m-1)\\
<\ &(\frac{3}{4}\gamma)^2\sqrt{2\gamma}M^2(n+m-1)+(\frac{3}{4}\gamma)^3M\sqrt{2\gamma}(n+m-1)\\
&+(\frac{3}{4}\gamma)^3M(n+m-1)\\
<\ &\sqrt{2}M^2(n+m-1)+M\sqrt{2}(n+m-1)+M(n+m-1)\\
=\ &(\sqrt{2}(M+1)+1)M(n+m-1)
\end{aligned}$$

olur. Burada $E'_{12} = (\sqrt{2}(M+1)+1)M(n+m-1)$, $\delta \geq \delta_1 = 2E'_{12} \Longrightarrow \delta \geq 2E'_{12} \Longrightarrow \frac{\delta}{2} \geq E'_{12} \Longrightarrow \frac{\delta}{2} - E'_{12} \geq 0$ olarak alınırsa ve (2.30) eşitsizliği kullanılırsa

$$E_1 > 2\lambda\nu(x_1+\delta_0)^{-3}(\delta+\frac{1}{2}\delta-E'_{12}) \geq 2\lambda\nu(x_1+\delta_0)^{-3}\delta > 2\lambda\nu\gamma^{-3}\delta \tag{2.31}$$

bulunur.

Şimdi $\vartheta^2_{x_i}$'nin katsayısı olan E_2 toplamını düzenleyelim:

Bu amaçla bir ε_0 sayısını $0 < \varepsilon_0 < \frac{\alpha_1}{4}$ olacak şekilde seçelim. Bu durumda

$$\delta\alpha_1 - \delta\varepsilon_0 = \delta(\alpha_1-\varepsilon_0) > \delta(\alpha_1 - \frac{\alpha_1}{4}) = \frac{3}{4}\delta\alpha_1 \tag{2.32}$$

yazılabilir. Eğer

$$\begin{aligned}
\delta_2 = \ &\frac{4}{\alpha_1}(\sqrt{2}+3M^2(2\sqrt{2}+1)(n-1)+4M^2\sqrt{2}m\\
&+M^2(2\sqrt{2}+1)m+M^2(n-1)+M^2m)
\end{aligned}$$

olmak üzere $\delta \geq \delta_2$ alınırsa

$$
\begin{aligned}
\frac{1}{4}\alpha_1\delta &\geq \sqrt{2}+3M^2(2\sqrt{2}+1)(n-1)+4M^2\sqrt{2}m \\
&\quad +M^2(2\sqrt{2}+1)m+M^2(n-1)+M^2m, \\
-\frac{1}{4}\alpha_1\delta &\leq -(\sqrt{2}+3M^2(2\sqrt{2}+1)(n-1)+4M^2\sqrt{2}m \\
&\quad +M^2(2\sqrt{2}+1)m+M^2(n-1)+M^2m)
\end{aligned}
$$

elde edilir. O halde

$$
\begin{aligned}
E_2 &= 2\lambda\nu(x_1+\delta_0)(\delta\alpha_1-\delta\varepsilon_0 \\
&\quad -\sqrt{2\gamma}-(x_1+\delta_0)(3M^2(2\sqrt{2\gamma}+1)(n-1)+4M^2\sqrt{2\gamma}m+M^2(2\sqrt{2\gamma}+1)m) \\
&\quad -(x_1+\delta_0)\psi(M^2(n-1)+M^2m))
\end{aligned}
$$

olduğundan $0<\gamma<1$ ve $0<\psi<1$ olduğu dikkate alınırsa E_2 içerisindeki terimler aşağıdaki şekilde değerlendirilebilir:

$$
\begin{aligned}
&\sqrt{2\gamma}+(x_1+\delta_0)(3M^2(2\sqrt{2\gamma}+1)(n-1)+4M^2\sqrt{2\gamma}m+M^2(2\sqrt{2\gamma}+1)m) \\
&+(x_1+\delta_0)\psi(M^2(n-1)+M^2m) \\
<\ &\sqrt{2\gamma}+\frac{3}{4}\gamma(3M^2(2\sqrt{2\gamma}+1)(n-1)+4M^2\sqrt{2\gamma}m+M^2(2\sqrt{2\gamma}+1)m) \\
&+\frac{3}{4}\gamma\psi(M^2(n-1)+M^2m) \\
<\ &\sqrt{2}+3M^2(2\sqrt{2}+1)(n-1)+4M^2\sqrt{2}m+M^2(2\sqrt{2}+1)m+M^2(n-1)+M^2m.
\end{aligned}
$$

Sonuç olarak

$$E_2 > 2\lambda\nu(x_1+\delta_0)(\frac{3}{4}\delta\alpha_1-\frac{1}{4}\delta\alpha_1)=\lambda\nu(x_1+\delta_0)\delta\alpha_1 \tag{2.33}$$

bulunur.

Şimdi $\vartheta_{y_j}^2$'nin katsayısı olan E_3 toplamını düzenleyelim:

Bu amaçla bir ε_0 sayısını $0<\varepsilon_0<\frac{\alpha_2}{4}$ olacak şekilde seçelim. Bu durumda

$$\delta\alpha_2-\delta\varepsilon_0=\delta(\alpha_2-\varepsilon_0)>\delta(\alpha_2-\frac{\alpha_2}{4})=\frac{3}{4}\delta\alpha_2 \tag{2.34}$$

yazılabilir. Eğer

$$
\begin{aligned}
\delta_3 &= \frac{4}{\alpha_2}(\sqrt{2}+3M^2(2\sqrt{2}+1)m+4M^2\sqrt{2}(n-1) \\
&\quad +M^2(2\sqrt{2}+1)(n-1)+M^2(n-1)+M^2m)
\end{aligned}
$$

olmak üzere $\delta \geq \delta_3$ alınırsa

$$\begin{aligned}
\frac{1}{4}\alpha_2\delta &\geq \sqrt{2} + 3M^2(2\sqrt{2}+1)m + 4M^2\sqrt{2}(n-1) \\
&\quad +M^2(2\sqrt{2}+1)(n-1) + M^2(n-1) + M^2m, \\
-\frac{1}{4}\alpha_2\delta &\leq -(\sqrt{2} + 3M^2(2\sqrt{2}+1)m + 4M^2\sqrt{2}(n-1) \\
&\quad +M^2(2\sqrt{2}+1)(n-1) + M^2(n-1) + M^2m)
\end{aligned}$$

elde edilir. O halde

$$\begin{aligned}
E_3 &= 2\lambda\nu(x_1+\delta_0)(\delta\alpha_2 - \delta\varepsilon_0 - \sqrt{2\gamma} \\
&\quad -(x_1+\delta_0)(3M^2(2\sqrt{2\gamma}+1)m + 4M^2\sqrt{2\gamma}(n-1) + M^2(2\sqrt{2\gamma}+1)(n-1)) \\
&\quad -(x_1+\delta_0)\psi(M^2(n-1)+M^2m))
\end{aligned}$$

olduğundan $0 < \gamma < 1$ ve $0 < \psi < 1$ olduğu göz önünde bulundurulursa E_3 içerisindeki terimler aşağıdaki şekilde değerlendirilebilir:

$$\begin{aligned}
&\sqrt{2\gamma} + (x_1+\delta_0)(3M^2(2\sqrt{2\gamma}+1)m + 4M^2\sqrt{2\gamma}(n-1) + M^2(2\sqrt{2\gamma}+1)(n-1)) \\
&+(x_1+\delta_0)\psi(M^2(n-1)+M^2m) \\
< \; &\sqrt{2\gamma} + \frac{3}{4}\gamma(3M^2(2\sqrt{2\gamma}+1)m + 4M^2\sqrt{2\gamma}(n-1) + M^2(2\sqrt{2\gamma}+1)(n-1)) \\
&+\frac{3}{4}\gamma\psi(M^2(n-1)+M^2m) \\
< \; &\sqrt{2} + 3M^2(2\sqrt{2}+1)m + 4M^2\sqrt{2}(n-1) \\
&+M^2(2\sqrt{2}+1)(n-1) + M^2(n-1) + M^2m.
\end{aligned}$$

Sonuç olarak

$$E_3 > 2\lambda\nu(x_1+\delta_0)(\frac{3}{4}\delta\alpha_2 - \frac{1}{4}\delta\alpha_2) = \lambda\nu(x_1+\delta_0)\delta\alpha_2 \tag{2.35}$$

bulunur.

Şimdi ϑ^2'nin E_4 katsayısını ele alalım:

$$E_4 = E_{41} + E_{42} + 2(x_1+\delta_0)^2\lambda\nu\xi(\sum_{i=2}^{n}(a_{ii}\psi_{x_i})_{x_i} - \sum_{j=1}^{m}(b_{jj}\psi_{y_j})_{y_j}).$$

Burada

$$\begin{aligned}
E_{41} &= -2\lambda^3\nu^3\delta^3((x_1+\delta_0)^{-2}\psi^{-2\nu-2})_{x_1} - 2\lambda^3\nu^3\delta\sum_{i=2}^{n}(a_{ii}\psi^{-2\nu-2})_{x_1}\psi_{x_i}^2 \\
&\quad +2\lambda^3\nu^3\delta\sum_{j=1}^{m}(b_{jj}\psi^{-2\nu-2})_{x_1}\psi_{y_j}^2 - 2\lambda^3\nu^3\delta^2\sum_{i=2}^{n}(a_{ii}\psi_{x_i}\psi^{-2\nu-2})_{x_i} \\
&\quad -2\lambda^3\nu^3(x_1+\delta_0)^2\sum_{i,s=2}^{n}(a_{ii}a_{ss}\psi_{x_i}\psi_{x_s}^2\psi^{-2\nu-2})_{x_i} \\
&\quad +2\lambda^3\nu^3(x_1+\delta_0)^2\sum_{i=2}^{n}\sum_{j=1}^{m}(a_{ii}b_{jj}\psi_{x_i}\psi^{-2\nu-2})_{x_i}\psi_{y_j}^2 \\
&\quad +2\lambda^3\nu^3\delta^2\sum_{j=1}^{m}(b_{jj}\psi_{y_j}\psi^{-2\nu-2})_{y_j} + 2\lambda^3\nu^3(x_1+\delta_0)^2\sum_{j=1}^{m}\sum_{i=2}^{n}(a_{ii}b_{jj}\psi_{y_j}\psi^{-2\nu-2})_{y_j}\psi_{x_i}^2 \\
&\quad -2\lambda^3\nu^3(x_1+\delta_0)^2\sum_{j,k=1}^{m}(b_{jj}b_{kk}\psi_{y_j}\psi_{y_k}^2\psi^{-2\nu-2})_{y_j},
\end{aligned}$$

$$\begin{aligned}
E_{42} &= 2\lambda^2\nu^2(\delta((x_1+\delta_0)^{-1}E_{01})_{x_1} + (x_1+\delta_0)(\sum_{i=2}^{n}(a_{ii}\psi_{x_i}E_{01})_{x_i} - \sum_{j=1}^{m}(b_{jj}\psi_{y_j}E_{01})_{y_j})) \\
&\quad -2(x_1+\delta_0)^2k\psi^{\nu+1},
\end{aligned}$$

$$\begin{aligned}
E_{01} &= (x_1+\delta_0)^{-1}(\nu+1)\psi^{-\nu-2}\psi_{x_1}^2 \\
&\quad +(x_1+\delta_0)(\sum_{i=2}^{n}a_{ii}((\nu+1)\psi^{-\nu-2}\psi_{x_i}^2 - \psi^{-\nu-1}\psi_{x_ix_i}) \\
&\quad -\sum_{j=1}^{m}b_{jj}((\nu+1)\psi^{-\nu-2}\psi_{y_j}^2 - \psi^{-\nu-1}\psi_{y_jy_j}))
\end{aligned}$$

olarak tanımlıdır.

Şimdi E_{41} ifadesini değerlendirelim:

$\psi_{x_1} = \delta$ olduğu dikkate alınarak

$$\begin{aligned}
&-2\lambda^3\nu^3\delta^3((x_1+\delta_0)^{-2}\psi^{-2\nu-2})_{x_1} \\
&= -2\lambda^3\nu^3\delta^3(-2(x_1+\delta_0)^{-3}\psi^{-2\nu-2} - 2(\nu+1)\psi^{-2\nu-3}\psi_{x_1}(x_1+\delta_0)^{-2}) \\
&= 4\lambda^3\nu^3\delta^4(\nu+1)(x_1+\delta_0)^{-2}\psi^{-2\nu-3} + 4\lambda^3\nu^3\delta^3(x_1+\delta_0)^{-3}\psi^{-2\nu-2}, \qquad (2.36)
\end{aligned}$$

$$\begin{aligned}
&-2\lambda^3\nu^3\delta\sum_{i=2}^{n}(a_{ii}\psi^{-2\nu-2})_{x_1}\psi_{x_i}^2 \\
&= -2\lambda^3\nu^3\delta\sum_{i=2}^{n}((a_{ii})_{x_1}\psi^{-2\nu-2} - 2(\nu+1)\psi^{-2\nu-3}\psi_{x_1}a_{ii})\psi_{x_i}^2 \\
&= 4\lambda^3\nu^3\delta^2(\nu+1)\psi^{-2\nu-3}\sum_{i=2}^{n}a_{ii}\psi_{x_i}^2 - 2\lambda^3\nu^3\delta\psi^{-2\nu-2}\sum_{i=2}^{n}(a_{ii})_{x_1}\psi_{x_i}^2, \qquad (2.37)
\end{aligned}$$

$$
\begin{aligned}
& 2\lambda^3\nu^3\delta\sum_{j=1}^{m}(b_{jj}\psi^{-2\nu-2})_{x_1}\psi_{y_j}^2 \\
= \; & 2\lambda^3\nu^3\delta\sum_{j=1}^{m}((b_{jj})_{x_1}\psi^{-2\nu-2}-2(\nu+1)\psi^{-2\nu-3}\psi_{x_1}b_{jj})\psi_{y_j}^2 \\
= \; & -4\lambda^3\nu^3\delta^2(\nu+1)\psi^{-2\nu-3}\sum_{j=1}^{m}b_{jj}\psi_{y_j}^2+2\lambda^3\nu^3\delta\psi^{-2\nu-2}\sum_{j=1}^{m}(b_{jj})_{x_1}\psi_{y_j}^2, \qquad (2.38)
\end{aligned}
$$

$$
\begin{aligned}
& -2\lambda^3\nu^3\delta^2\sum_{i=2}^{n}(a_{ii}\psi_{x_i}\psi^{-2\nu-2})_{x_i} \\
= \; & -2\lambda^3\nu^3\delta^2\sum_{i=2}^{n}((a_{ii})_{x_i}\psi_{x_i}\psi^{-2\nu-2}+a_{ii}\psi_{x_ix_i}\psi^{-2\nu-2}-2(\nu+1)\psi^{-2\nu-3}\psi_{x_i}^2a_{ii}) \\
= \; & 4\lambda^3\nu^3\delta^2(\nu+1)\psi^{-2\nu-3}\sum_{i=2}^{n}a_{ii}\psi_{x_i}^2-2\lambda^3\nu^3\delta^2\psi^{-2\nu-2}\sum_{i=2}^{n}((a_{ii})_{x_i}\psi_{x_i}+a_{ii}\psi_{x_ix_i}), \qquad (2.39)
\end{aligned}
$$

$$
\begin{aligned}
& -2\lambda^3\nu^3(x_1+\delta_0)^2\sum_{i,s=2}^{n}(a_{ii}a_{ss}\psi_{x_i}\psi_{x_s}^2\psi^{-2\nu-2})_{x_i} \\
= \; & -2\lambda^3\nu^3(x_1+\delta_0)^2\sum_{i,s=2}^{n}a_{ii}a_{ss}\psi_{x_i}\psi_{x_s}^2(-2(\nu+1)\psi^{-2\nu-3}\psi_{x_i}) \\
& -2\lambda^3\nu^3(x_1+\delta_0)^2\sum_{i,s=2}^{n}(a_{ii}a_{ss}\psi_{x_i}\psi_{x_s}^2)_{x_i}\psi^{-2\nu-2} \\
= \; & 4\lambda^3\nu^3(x_1+\delta_0)^2(\nu+1)\psi^{-2\nu-3}\sum_{i,s=2}^{n}a_{ii}a_{ss}\psi_{x_i}^2\psi_{x_s}^2 \\
& -2\lambda^3\nu^3(x_1+\delta_0)^2\psi^{-2\nu-2}\sum_{i,s=2}^{n}(a_{ii}a_{ss}\psi_{x_i}\psi_{x_s}^2)_{x_i}, \qquad (2.40)
\end{aligned}
$$

$$
\begin{aligned}
& 2\lambda^3\nu^3(x_1+\delta_0)^2\sum_{i=2}^{n}\sum_{j=1}^{m}(a_{ii}b_{jj}\psi_{x_i}\psi^{-2\nu-2})_{x_i}\psi_{y_j}^2 \\
= \; & 2\lambda^3\nu^3(x_1+\delta_0)^2\sum_{i=2}^{n}\sum_{j=1}^{m}((a_{ii})_{x_i}b_{jj}\psi_{x_i}\psi^{-2\nu-2}+a_{ii}(b_{jj})_{x_i}\psi_{x_i}\psi^{-2\nu-2} \\
& +a_{ii}b_{jj}\psi_{x_ix_i}\psi^{-2\nu-2}+a_{ii}b_{jj}\psi_{x_i}(-2(\nu+1)\psi^{-2\nu-3}\psi_{x_i}))\psi_{y_j}^2 \\
= \; & -4\lambda^3\nu^3(x_1+\delta_0)^2(\nu+1)\psi^{-2\nu-3}\sum_{i=2}^{n}\sum_{j=1}^{m}a_{ii}b_{jj}\psi_{x_i}^2\psi_{y_j}^2 \\
& +2\lambda^3\nu^3(x_1+\delta_0)^2\psi^{-2\nu-2}\sum_{i=2}^{n}\sum_{j=1}^{m}((a_{ii})_{x_i}b_{jj}\psi_{x_i}+a_{ii}(b_{jj})_{x_i}\psi_{x_i}+a_{ii}b_{jj}\psi_{x_ix_i})\psi_{y_j}^2, \qquad (2.41)
\end{aligned}
$$

$$
\begin{aligned}
& 2\lambda^3\nu^3\delta^2\sum_{j=1}^{m}(b_{jj}\psi_{y_j}\psi^{-2\nu-2})_{y_j} \\
= \; & 2\lambda^3\nu^3\delta^2\sum_{j=1}^{m}((b_{jj})_{y_j}\psi_{y_j}\psi^{-2\nu-2}+b_{jj}\psi_{y_jy_j}\psi^{-2\nu-2}-2(\nu+1)\psi^{-2\nu-3}\psi_{y_j}^2 b_{jj}) \\
= \; & -4\lambda^3\nu^3\delta^2(\nu+1)\psi^{-2\nu-3}\sum_{j=1}^{m}b_{jj}\psi_{y_j}^2+2\lambda^3\nu^3\delta^2\psi^{-2\nu-2}\sum_{j=1}^{m}((b_{jj})_{y_j}\psi_{y_j}+b_{jj}\psi_{y_jy_j}), \quad (2.42)
\end{aligned}
$$

$$
\begin{aligned}
& 2\lambda^3\nu^3(x_1+\delta_0)^2\sum_{j=1}^{m}\sum_{i=2}^{n}(a_{ii}b_{jj}\psi_{y_j}\psi^{-2\nu-2})_{y_j}\psi_{x_i}^2 \\
= \; & 2\lambda^3\nu^3(x_1+\delta_0)^2\sum_{j=1}^{m}\sum_{i=2}^{n}((a_{ii})_{y_j}b_{jj}\psi_{y_j}\psi^{-2\nu-2}+a_{ii}(b_{jj})_{y_j}\psi_{y_j}\psi^{-2\nu-2} \\
& +a_{ii}b_{jj}\psi_{y_jy_j}\psi^{-2\nu-2}+a_{ii}b_{jj}\psi_{y_j}(-2(\nu+1)\psi^{-2\nu-3}\psi_{y_j}))\psi_{x_i}^2 \\
= \; & -4\lambda^3\nu^3(x_1+\delta_0)^2(\nu+1)\psi^{-2\nu-3}\sum_{j=1}^{m}\sum_{i=2}^{n}a_{ii}b_{jj}\psi_{x_i}^2\psi_{y_j}^2 \\
& +2\lambda^3\nu^3(x_1+\delta_0)^2\psi^{-2\nu-2}\sum_{j=1}^{m}\sum_{i=2}^{n}((a_{ii})_{y_j}b_{jj}\psi_{y_j}+a_{ii}(b_{jj})_{y_j}\psi_{y_j}+a_{ii}b_{jj}\psi_{y_jy_j})\psi_{x_i}^2, \quad (2.43)
\end{aligned}
$$

$$
\begin{aligned}
& -2\lambda^3\nu^3(x_1+\delta_0)^2\sum_{j,k=1}^{m}(b_{jj}b_{kk}\psi_{y_j}\psi_{y_k}^2\psi^{-2\nu-2})_{y_j} \\
= \; & -2\lambda^3\nu^3(x_1+\delta_0)^2\sum_{j,k=1}^{m}b_{jj}b_{kk}\psi_{y_j}\psi_{y_k}^2(-2(\nu+1)\psi^{-2\nu-3}\psi_{y_j}) \\
& -2\lambda^3\nu^3(x_1+\delta_0)^2\sum_{j,k=1}^{m}(b_{jj}b_{kk}\psi_{y_j}\psi_{y_k}^2)_{y_j}\psi^{-2\nu-2} \\
= \; & 4\lambda^3\nu^3(x_1+\delta_0)^2(\nu+1)\psi^{-2\nu-3}\sum_{j,k=1}^{m}b_{jj}b_{kk}\psi_{y_j}^2\psi_{y_k}^2 \\
& -2\lambda^3\nu^3(x_1+\delta_0)^2\psi^{-2\nu-2}\sum_{j,k=1}^{m}(b_{jj}b_{kk}\psi_{y_j}\psi_{y_k}^2)_{y_j} \quad (2.44)
\end{aligned}
$$

elde edilir.

Bu durumda (2.36)-(2.44) eşitliklerinden E_{41} aşağıdaki şekilde yazılabilir:

$$
\begin{aligned}
E_{41} \; = \; & 4\lambda^3\nu^3(\nu+1)\psi^{-2\nu-3}(\delta^4(x_1+\delta_0)^{-2}+\delta^2\sum_{i=2}^{n}a_{ii}\psi_{x_i}^2-\delta^2\sum_{j=1}^{m}b_{jj}\psi_{y_j}^2+\delta^2\sum_{i=2}^{n}a_{ii}\psi_{x_i}^2 \\
& +(x_1+\delta_0)^2\sum_{i,s=2}^{n}a_{ii}a_{ss}\psi_{x_i}^2\psi_{x_s}^2-(x_1+\delta_0)^2\sum_{i=2}^{n}\sum_{j=1}^{m}a_{ii}b_{jj}\psi_{x_i}^2\psi_{y_j}^2-\delta^2\sum_{j=1}^{m}b_{jj}\psi_{y_j}^2 \\
& -(x_1+\delta_0)^2\sum_{j=1}^{m}\sum_{i=2}^{n}a_{ii}b_{jj}\psi_{x_i}^2\psi_{y_j}^2+(x_1+\delta_0)^2\sum_{j,k=1}^{m}b_{jj}b_{kk}\psi_{y_j}^2\psi_{y_k}^2)
\end{aligned}
$$

$$+4\lambda^3\nu^3\psi^{-2\nu-2}(\delta^3(x_1+\delta_0)^{-3} - \frac{\delta}{2}\sum_{i=2}^{n}(a_{ii})_{x_1}\psi_{x_i}^2 + \frac{\delta}{2}\sum_{j=1}^{m}(b_{jj})_{x_1}\psi_{y_j}^2$$
$$-\frac{\delta^2}{2}\sum_{i=2}^{n}((a_{ii})_{x_i}\psi_{x_i} + a_{ii}\psi_{x_ix_i}) - \frac{(x_1+\delta_0)^2}{2}\sum_{i,s=2}^{n}(a_{ii}a_{ss}\psi_{x_i}\psi_{x_s}^2)_{x_i}$$
$$+\frac{(x_1+\delta_0)^2}{2}\sum_{i=2}^{n}\sum_{j=1}^{m}((a_{ii})_{x_i}b_{jj}\psi_{x_i} + a_{ii}(b_{jj})_{x_i}\psi_{x_i} + a_{ii}b_{jj}\psi_{x_ix_i})\psi_{y_j}^2$$
$$+\frac{\delta^2}{2}\sum_{j=1}^{m}((b_{jj})_{y_j}\psi_{y_j} + b_{jj}\psi_{y_jy_j}) - \frac{(x_1+\delta_0)^2}{2}\sum_{j,k=1}^{m}(b_{jj}b_{kk}\psi_{y_j}\psi_{y_k}^2)_{y_j}$$
$$+\frac{(x_1+\delta_0)^2}{2}\sum_{j=1}^{m}\sum_{i=2}^{n}((a_{ii})_{y_j}b_{jj}\psi_{y_j} + a_{ii}(b_{jj})_{y_j}\psi_{y_j} + a_{ii}b_{jj}\psi_{y_jy_j})\psi_{x_i}^2).$$

Yukarıdaki eşitlikte pozitif terimler atılırsa, E_4 için aşağıdaki eşitsizlik elde edilir:

$$E_4 \geq 4\lambda^3\nu^3(\nu+1)\psi^{-2\nu-3}\delta^4(x_1+\delta_0)^{-2}$$
$$+2\lambda^3\nu^3\psi^{-2\nu-2}\delta^2(2\delta(x_1+\delta_0)^{-3} - \frac{1}{\delta}\sum_{i=2}^{n}(a_{ii})_{x_1}\psi_{x_i}^2 + \frac{1}{\delta}\sum_{j=1}^{m}(b_{jj})_{x_1}\psi_{y_j}^2$$
$$-\sum_{i=2}^{n}((a_{ii})_{x_i}\psi_{x_i} + a_{ii}\psi_{x_ix_i}) - \frac{(x_1+\delta_0)^2}{\delta^2}\sum_{i,s=2}^{n}(a_{ii}a_{ss}\psi_{x_i}\psi_{x_s}^2)_{x_i}$$
$$+\frac{(x_1+\delta_0)^2}{\delta^2}\sum_{i=2}^{n}\sum_{j=1}^{m}((a_{ii})_{x_i}b_{jj}\psi_{x_i} + a_{ii}(b_{jj})_{x_i}\psi_{x_i} + a_{ii}b_{jj}\psi_{x_ix_i})\psi_{y_j}^2$$

$$+\sum_{j=1}^{m}((b_{jj})_{y_j}\psi_{y_j} + b_{jj}\psi_{y_jy_j}) - \frac{(x_1+\delta_0)^2}{\delta^2}\sum_{j,k=1}^{m}(b_{jj}b_{kk}\psi_{y_j}\psi_{y_k}^2)_{y_j}$$
$$+\frac{(x_1+\delta_0)^2}{\delta^2}\sum_{j=1}^{m}\sum_{i=2}^{n}((a_{ii})_{y_j}b_{jj}\psi_{y_j} + a_{ii}(b_{jj})_{y_j}\psi_{y_j} + a_{ii}b_{jj}\psi_{y_jy_j})\psi_{x_i}^2)$$
$$+E_{42} + 2(x_1+\delta_0)^2\lambda\nu\xi(\sum_{i=2}^{n}(a_{ii}\psi_{x_i})_{x_i} - \sum_{j=1}^{m}(b_{jj}\psi_{y_j})_{y_j}). \qquad (2.45)$$

Şimdi son eşitsizlikteki terimleri $(x_1+\delta_0) < 1$, $\delta \geq 4$, $0 < \gamma < 1$, $|\psi_{x_i}| \leq \sqrt{2\gamma}$, $|\psi_{y_j}| \leq \sqrt{2\gamma}$, $||a_{ii}||_{C^1(\overline{D\times G})} \leq M$, $||b_{jj}||_{C^1(\overline{D\times G})} \leq M$, $2 \leq i \leq n$, $1 \leq j \leq m$ eşitsizliklerini ve $\psi_{x_ix_s} = 1$ $(i = s;\ 2 \leq i, s \leq n)$, $\psi_{y_jy_k} = 1$ $(j = k;\ 1 \leq j, k \leq m)$ olduğunu göz önünde bulundurarak değerlendirelim:

$$
\begin{aligned}
&2\delta(x_1+\delta_0)^{-3} - \frac{1}{\delta}\sum_{i=2}^{n}(a_{ii})_{x_1}\psi_{x_i}^2 + \frac{1}{\delta}\sum_{j=1}^{m}(b_{jj})_{x_1}\psi_{y_j}^2 \\
&-\sum_{i=2}^{n}((a_{ii})_{x_i}\psi_{x_i} + a_{ii}\psi_{x_ix_i}) - \frac{(x_1+\delta_0)^2}{\delta^2}\sum_{i,s=2}^{n}(a_{ii}a_{ss}\psi_{x_i}\psi_{x_s}^2)_{x_i} \\
&+\frac{(x_1+\delta_0)^2}{\delta^2}\sum_{i=2}^{n}\sum_{j=1}^{m}((a_{ii})_{x_i}b_{jj}\psi_{x_i} + a_{ii}(b_{jj})_{x_i}\psi_{x_i} + a_{ii}b_{jj}\psi_{x_ix_i})\psi_{y_j}^2 \\
&+\sum_{j=1}^{m}((b_{jj})_{y_j}\psi_{y_j} + b_{jj}\psi_{y_jy_j}) - \frac{(x_1+\delta_0)^2}{\delta^2}\sum_{j,k=1}^{m}(b_{jj}b_{kk}\psi_{y_j}\psi_{y_k}^2)_{y_j} \\
&+\frac{(x_1+\delta_0)^2}{\delta^2}\sum_{j=1}^{m}\sum_{i=2}^{n}((a_{ii})_{y_j}b_{jj}\psi_{y_j} + a_{ii}(b_{jj})_{y_j}\psi_{y_j} + a_{ii}b_{jj}\psi_{y_jy_j})\psi_{x_i}^2
\end{aligned}
$$

$$
\begin{aligned}
=\;\; & 2\delta(x_1+\delta_0)^{-3} - \frac{1}{\delta}\sum_{i=2}^{n}(a_{ii})_{x_1}\psi_{x_i}^2 + \frac{1}{\delta}\sum_{j=1}^{m}(b_{jj})_{x_1}\psi_{y_j}^2 \\
& -\sum_{i=2}^{n}((a_{ii})_{x_i}\psi_{x_i} + a_{ii}\psi_{x_ix_i}) + \sum_{j=1}^{m}((b_{jj})_{y_j}\psi_{y_j} + b_{jj}\psi_{y_jy_j}) \\
& -\frac{(x_1+\delta_0)^2}{\delta^2}\sum_{i,s=2}^{n}((a_{ii})_{x_i}a_{ss}\psi_{x_i}\psi_{x_s}^2 + a_{ii}(a_{ss})_{x_i}\psi_{x_i}\psi_{x_s}^2 \\
& +a_{ii}a_{ss}\psi_{x_ix_i}\psi_{x_s}^2 + 2a_{ii}a_{ss}\psi_{x_i}\psi_{x_s}\psi_{x_sx_i})
\end{aligned}
$$

$$
\begin{aligned}
&+\frac{(x_1+\delta_0)^2}{\delta^2}\sum_{i=2}^{n}\sum_{j=1}^{m}((a_{ii})_{x_i}b_{jj}\psi_{x_i} + a_{ii}(b_{jj})_{x_i}\psi_{x_i} + a_{ii}b_{jj}\psi_{x_ix_i})\psi_{y_j}^2 \\
&+\frac{(x_1+\delta_0)^2}{\delta^2}\sum_{j=1}^{m}\sum_{i=2}^{n}((a_{ii})_{y_j}b_{jj}\psi_{y_j} + a_{ii}(b_{jj})_{y_j}\psi_{y_j} + a_{ii}b_{jj}\psi_{y_jy_j})\psi_{x_i}^2 \\
&-\frac{(x_1+\delta_0)^2}{\delta^2}\sum_{j,k=1}^{m}((b_{jj})_{y_j}b_{kk}\psi_{y_j}\psi_{y_k}^2 + b_{jj}(b_{kk})_{y_j}\psi_{y_j}\psi_{y_k}^2 \\
&+b_{jj}b_{kk}\psi_{y_jy_j}\psi_{y_k}^2 + 2b_{jj}b_{kk}\psi_{y_j}\psi_{y_k}\psi_{y_ky_j})
\end{aligned}
$$

$$
\begin{aligned}
\geq\;\; & 2\delta(x_1+\delta_0)^{-3} - \frac{1}{\delta}\sum_{i=2}^{n}|(a_{ii})_{x_1}\psi_{x_i}^2| - \frac{1}{\delta}\sum_{j=1}^{m}|(b_{jj})_{x_1}\psi_{y_j}^2| \\
& -\sum_{i=2}^{n}(|(a_{ii})_{x_i}\psi_{x_i}| + |a_{ii}\psi_{x_ix_i}|) - \sum_{j=1}^{m}(|(b_{jj})_{y_j}\psi_{y_j}| + |b_{jj}\psi_{y_jy_j}|) \\
& -\frac{(x_1+\delta_0)^2}{\delta^2}\sum_{i,s=2}^{n}(|(a_{ii})_{x_i}a_{ss}\psi_{x_i}\psi_{x_s}^2| + |a_{ii}(a_{ss})_{x_i}\psi_{x_i}\psi_{x_s}^2| \\
& +|a_{ii}a_{ss}\psi_{x_ix_i}\psi_{x_s}^2| + |2a_{ii}a_{ss}\psi_{x_i}\psi_{x_s}\psi_{x_sx_i}|)
\end{aligned}
$$

$$
\begin{aligned}
&-\frac{(x_1+\delta_0)^2}{\delta^2}\sum_{i=2}^{n}\sum_{j=1}^{m}(|(a_{ii})_{x_i}b_{jj}\psi_{x_i}\psi_{y_j}^2|+|a_{ii}(b_{jj})_{x_i}\psi_{x_i}\psi_{y_j}^2|+|a_{ii}b_{jj}\psi_{x_ix_i}\psi_{y_j}^2|)\\
&-\frac{(x_1+\delta_0)^2}{\delta^2}\sum_{j=1}^{m}\sum_{i=2}^{n}(|(a_{ii})_{y_j}b_{jj}\psi_{y_j}\psi_{x_i}^2|+|a_{ii}(b_{jj})_{y_j}\psi_{y_j}\psi_{x_i}^2|+|a_{ii}b_{jj}\psi_{y_jy_j}\psi_{x_i}^2|)\\
&-\frac{(x_1+\delta_0)^2}{\delta^2}\sum_{j,k=1}^{m}(|(b_{jj})_{y_j}b_{kk}\psi_{y_j}\psi_{y_k}^2|+|b_{jj}(b_{kk})_{y_j}\psi_{y_j}\psi_{y_k}^2|\\
&+|b_{jj}b_{kk}\psi_{y_jy_j}\psi_{y_k}^2|+|2b_{jj}b_{kk}\psi_{y_j}\psi_{y_k}\psi_{y_ky_j}|)
\end{aligned}
$$

$$
\begin{aligned}
> \quad & 2\delta-\sum_{i=2}^{n}|(a_{ii})_{x_1}\psi_{x_i}^2|-\sum_{j=1}^{m}|(b_{jj})_{x_1}\psi_{y_j}^2|\\
&-\sum_{i=2}^{n}(|(a_{ii})_{x_i}\psi_{x_i}|+|a_{ii}\psi_{x_ix_i}|)-\sum_{j=1}^{m}(|(b_{jj})_{y_j}\psi_{y_j}|+|b_{jj}\psi_{y_jy_j}|)\\
&-\sum_{i,s=2}^{n}(|(a_{ii})_{x_i}a_{ss}\psi_{x_i}\psi_{x_s}^2|+|a_{ii}(a_{ss})_{x_i}\psi_{x_i}\psi_{x_s}^2|\\
&+|a_{ii}a_{ss}\psi_{x_ix_i}\psi_{x_s}^2|+|2a_{ii}a_{ss}\psi_{x_i}\psi_{x_s}\psi_{x_sx_i}|)
\end{aligned}
$$

$$
\begin{aligned}
&-\sum_{i=2}^{n}\sum_{j=1}^{m}(|(a_{ii})_{x_i}b_{jj}\psi_{x_i}\psi_{y_j}^2|+|a_{ii}(b_{jj})_{x_i}\psi_{x_i}\psi_{y_j}^2|+|a_{ii}b_{jj}\psi_{x_ix_i}\psi_{y_j}^2|)\\
&-\sum_{j=1}^{m}\sum_{i=2}^{n}(|(a_{ii})_{y_j}b_{jj}\psi_{y_j}\psi_{x_i}^2|+|a_{ii}(b_{jj})_{y_j}\psi_{y_j}\psi_{x_i}^2|+|a_{ii}b_{jj}\psi_{y_jy_j}\psi_{x_i}^2|)\\
&-\sum_{j,k=1}^{m}(|(b_{jj})_{y_j}b_{kk}\psi_{y_j}\psi_{y_k}^2|+|b_{jj}(b_{kk})_{y_j}\psi_{y_j}\psi_{y_k}^2|\\
&+|b_{jj}b_{kk}\psi_{y_jy_j}\psi_{y_k}^2|+|2b_{jj}b_{kk}\psi_{y_j}\psi_{y_k}\psi_{y_ky_j}|)
\end{aligned}
$$

$$
\begin{aligned}
\geq \quad & 2\delta-M2\gamma(n-1)-M2\gamma m-M\sqrt{2\gamma}(n-1)-M(n-1)-M\sqrt{2\gamma}m-Mm\\
&-M^22\gamma\sqrt{2\gamma}(n-1)^2-M^22\gamma\sqrt{2\gamma}(n-1)^2-M^22\gamma(n-1)^2-M^24\gamma(n-1)\\
&-M^22\gamma\sqrt{2\gamma}(n-1)m-M^22\gamma\sqrt{2\gamma}(n-1)m-M^22\gamma(n-1)m\\
&-M^22\gamma\sqrt{2\gamma}(n-1)m-M^22\gamma\sqrt{2\gamma}(n-1)m-M^22\gamma(n-1)m-M^22\gamma\sqrt{2\gamma}m^2\\
&-M^22\gamma\sqrt{2\gamma}m^2-M^22\gamma m^2-M^24\gamma m
\end{aligned}
$$

$$
\begin{aligned}
= \quad & 2\delta-(n-1)^2(M^22\gamma\sqrt{2\gamma}+M^22\gamma\sqrt{2\gamma}+M^22\gamma)\\
&-(n-1)(M2\gamma+M\sqrt{2\gamma}+M+M^24\gamma)-m^2(M^22\gamma\sqrt{2\gamma}+M^22\gamma\sqrt{2\gamma}+M^22\gamma)\\
&-m(M2\gamma+M\sqrt{2\gamma}+M+M^24\gamma)\\
&-(n-1)m(M^22\gamma\sqrt{2\gamma}+M^22\gamma\sqrt{2\gamma}+M^22\gamma+M^22\gamma\sqrt{2\gamma}+M^22\gamma\sqrt{2\gamma}+M^22\gamma)
\end{aligned}
$$

$$\begin{aligned}
&= 2\delta - (n-1)^2(M^2 4\gamma\sqrt{2\gamma} + M^2 2\gamma) - (n-1)(M2\gamma + M\sqrt{2\gamma} + M + M^2 4\gamma) \\
&\quad -m^2(M^2 4\gamma\sqrt{2\gamma} + M^2 2\gamma) - m(M2\gamma + M\sqrt{2\gamma} + M + M^2 4\gamma) \\
&\quad -(n-1)m(M^2 8\gamma\sqrt{2\gamma} + M^2 4\gamma)
\end{aligned}$$

$$\begin{aligned}
&> 2\delta - (n-1)^2(4\sqrt{2}M^2 + 2M^2) - (n-1)(2M + \sqrt{2}M + M + 4M^2) \\
&\quad -m^2(4\sqrt{2}M^2 + 2M^2) - m(2M + \sqrt{2}M + M + 4M^2) \\
&\quad -(n-1)m(8\sqrt{2}M^2 + 4M^2)
\end{aligned}$$

$$\begin{aligned}
&= 2\delta - (4\sqrt{2}M^2 + 2M^2)((n-1)^2 + m^2 + 2(n-1)m) \\
&\quad -(2M + \sqrt{2}M + M + 4M^2)(n+m-1) \\
&= 2\delta - (4\sqrt{2}+2)M^2(n+m-1)^2 - (\sqrt{2}+3+4M)M(n+m-1).
\end{aligned}$$

Burada eğer

$$\delta \geq \delta_4 = \frac{1}{2}((4\sqrt{2}+2)M^2(n+m-1)^2 + (\sqrt{2}+3+4M)M(n+m-1))$$

alınırsa

$$\begin{aligned}
& 2\delta(x_1+\delta_0)^{-3} - \frac{1}{\delta}\sum_{i=2}^{n}(a_{ii})_{x_1}\psi_{x_i}^2 + \frac{1}{\delta}\sum_{j=1}^{m}(b_{jj})_{x_1}\psi_{y_j}^2 \\
& -\sum_{i=2}^{n}((a_{ii})_{x_i}\psi_{x_i} + a_{ii}\psi_{x_ix_i}) - \frac{(x_1+\delta_0)^2}{\delta^2}\sum_{i,s=2}^{n}(a_{ii}a_{ss}\psi_{x_i}\psi_{x_s}^2)_{x_i} \\
& +\frac{(x_1+\delta_0)^2}{\delta^2}\sum_{i=2}^{n}\sum_{j=1}^{m}((a_{ii})_{x_i}b_{jj}\psi_{x_i} + a_{ii}(b_{jj})_{x_i}\psi_{x_i} + a_{ii}b_{jj}\psi_{x_ix_i})\psi_{y_j}^2 \\
& +\sum_{j=1}^{m}((b_{jj})_{y_j}\psi_{y_j} + b_{jj}\psi_{y_jy_j}) - \frac{(x_1+\delta_0)^2}{\delta^2}\sum_{j,k=1}^{m}(b_{jj}b_{kk}\psi_{y_j}\psi_{y_k}^2)_{y_j} \\
& +\frac{(x_1+\delta_0)^2}{\delta^2}\sum_{j=1}^{m}\sum_{i=2}^{n}((a_{ii})_{y_j}b_{jj}\psi_{y_j} + a_{ii}(b_{jj})_{y_j}\psi_{y_j} + a_{ii}b_{jj}\psi_{y_jy_j})\psi_{x_i}^2 \\
> \; & 0 \qquad (2.46)
\end{aligned}$$

olur. Diğer taraftan

$$\delta^4(x_1+\delta_0)^{-2} > \delta^4 \qquad (2.47)$$

olduğundan

$$
\begin{aligned}
& 2\lambda^3\nu^3\delta^4(\nu+1)(x_1+\delta_0)^{-2}\psi^{-2\nu-3} + E_{42} \\
> \; & \lambda^2\nu^2(2\lambda\nu\delta^4(\nu+1)\psi^{-2\nu-3} \\
& +2(\delta((x_1+\delta_0)^{-1}E_{01})_{x_1} + (x_1+\delta_0)(\sum_{i=2}^{n}(a_{ii}\psi_{x_i}E_{01})_{x_i} - \sum_{j=1}^{m}(b_{jj}\psi_{y_j}E_{01})_{y_j})) \\
& -2(x_1+\delta_0)^2\frac{k}{\lambda^2\nu^2}\psi^{\nu+1}) \\
\geq \; & \lambda^2\nu^2(2\lambda\nu\delta^4(\nu+1)\psi^{-2\nu-3} \\
& -2|\delta((x_1+\delta_0)^{-1}E_{01})_{x_1} + (x_1+\delta_0)(\sum_{i=2}^{n}(a_{ii}\psi_{x_i}E_{01})_{x_i} - \sum_{j=1}^{m}(b_{jj}\psi_{y_j}E_{01})_{y_j})| \\
& -2(x_1+\delta_0)^2\frac{k}{\lambda^2\nu^2}\psi^{\nu+1}) \\
> \; & \lambda^2\nu^2(4\lambda\nu\delta^4 \\
& -2|\delta((x_1+\delta_0)^{-1}E_{01})_{x_1} + (x_1+\delta_0)(\sum_{i=2}^{n}(a_{ii}\psi_{x_i}E_{01})_{x_i} - \sum_{j=1}^{m}(b_{jj}\psi_{y_j}E_{01})_{y_j})| \\
& -2(x_1+\delta_0)^2\frac{k}{\lambda^2\nu^2}\psi^{\nu+1}) \\
\geq \; & 0,
\end{aligned}
$$

$$
2\lambda^3\nu^3\delta^4(\nu+1)(x_1+\delta_0)^{-2}\psi^{-2\nu-3} + E_{42} > 0 \tag{2.48}
$$

dır. Böylece (2.45) eşitsizliğinin, (2.46)-(2.48) eşitsizlikleri kullanılarak

$$
\begin{aligned}
E_4 \; > \; & 2\lambda^3\nu^3\delta^4(\nu+1)\psi^{-2\nu-3} + 2\lambda\nu\xi(x_1+\delta_0)^2(\sum_{i=2}^{n}(a_{ii}\psi_{x_i})_{x_i} - \sum_{j=1}^{m}(b_{jj}\psi_{y_j})_{y_j}) \\
> \; & 2\lambda^3\nu^4\delta^4\psi^{-2\nu-3} + 2\lambda\nu\xi(x_1+\delta_0)^2(\sum_{i=2}^{n}(a_{ii}\psi_{x_i})_{x_i} - \sum_{j=1}^{m}(b_{jj}\psi_{y_j})_{y_j})
\end{aligned} \tag{2.49}
$$

olduğu görülür. Sonuç olarak

$$
\begin{aligned}
\psi^{\nu+1}(L_0\varphi)^2\chi^2 \; \geq \; & 2\lambda\nu\gamma^{-3}\delta\vartheta_{x_1}^2 + \lambda\nu(x_1+\delta_0)\delta\alpha_1\sum_{i=2}^{n}\vartheta_{x_i}^2 \\
& +\lambda\nu(x_1+\delta_0)\delta\alpha_2\sum_{j=1}^{m}\vartheta_{y_j}^2 + 2\lambda^3\nu^4\delta^4\psi^{-2\nu-3}\vartheta^2 \\
& +2\lambda\nu\xi(x_1+\delta_0)^2(\sum_{i=2}^{n}(a_{ii}\psi_{x_i})_{x_i} - \sum_{j=1}^{m}(b_{jj}\psi_{y_j})_{y_j})\vartheta^2 \\
& +d_1(\vartheta) + d_2(\vartheta)
\end{aligned} \tag{2.50}
$$

değerlendirmesi bulunur.

Diğer taraftan, aşağıdaki eşitlikler kullanılarak (2.50) değerlendirmesi φ fonksiyonu için elde edilir.

$$\begin{aligned}
\chi &= e^{\lambda\psi^{-\nu}}, \chi^2 = e^{2\lambda\psi^{-\nu}}, (\chi^2)_{x_i} = -2\lambda\nu\psi^{-\nu-1}\psi_{x_i}\chi^2, \\
\vartheta &= e^{\lambda\psi^{-\nu}}\varphi, \vartheta_{x_i} = -\lambda\nu\psi^{-\nu-1}\psi_{x_i}e^{\lambda\psi^{-\nu}}\varphi + \varphi_{x_i}e^{\lambda\psi^{-\nu}} = (\varphi_{x_i} - \lambda\nu\psi^{-\nu-1}\psi_{x_i}\varphi)\chi \\
\vartheta_{x_i}^2 &= (\varphi_{x_i} - \lambda\nu\psi^{-\nu-1}\psi_{x_i}\varphi)^2\chi^2 \\
&= (\varphi_{x_i}^2 - 2\lambda\nu\psi^{-\nu-1}\psi_{x_i}\varphi\varphi_{x_i} + \lambda^2\nu^2\psi^{-2\nu-2}\psi_{x_i}^2\varphi^2)\chi^2 \\
&= \varphi_{x_i}^2\chi^2 - 2\lambda\nu\psi^{-\nu-1}\psi_{x_i}\varphi\varphi_{x_i}\chi^2 + \lambda^2\nu^2\psi^{-2\nu-2}\psi_{x_i}^2\varphi^2\chi^2.
\end{aligned}$$

$\vartheta_{x_i}^2$'nin ifadesi için

$$\begin{aligned}
-2\lambda\nu\psi^{-\nu-1}\psi_{x_i}\varphi\varphi_{x_i}\chi^2 &= -\lambda\nu\psi^{-\nu-1}\psi_{x_i}(\varphi^2)_{x_i}\chi^2, \\
\lambda\nu(\psi^{-\nu-1}\psi_{x_i}\varphi^2\chi^2)_{x_i} &= \lambda\nu(-(\nu+1)\psi^{-\nu-2}\psi_{x_i}^2\varphi^2\chi^2 + \psi^{-\nu-1}\psi_{x_ix_i}\varphi^2\chi^2 \\
&\quad +\psi^{-\nu-1}\psi_{x_i}2\varphi\varphi_{x_i}\chi^2 + \psi^{-\nu-1}\psi_{x_i}\varphi^2(-2\lambda\nu\psi^{-\nu-1}\psi_{x_i}\chi^2)) \\
&= -\lambda\nu(\nu+1)\psi^{-\nu-2}\psi_{x_i}^2\varphi^2\chi^2 + \lambda\nu\psi^{-\nu-1}\psi_{x_ix_i}\varphi^2\chi^2 \\
&\quad +\lambda\nu\psi^{-\nu-1}\psi_{x_i}2\varphi\varphi_{x_i}\chi^2 - 2\lambda^2\nu^2\psi^{-2\nu-2}\psi_{x_i}^2\varphi^2\chi^2
\end{aligned}$$

$$\begin{aligned}
-\lambda\nu\psi^{-\nu-1}\psi_{x_i}(\varphi^2)_{x_i}\chi^2 &= -\lambda\nu(\nu+1)\psi^{-\nu-2}\psi_{x_i}^2\varphi^2\chi^2 + \lambda\nu\psi^{-\nu-1}\psi_{x_ix_i}\varphi^2\chi^2 \\
&\quad -2\lambda^2\nu^2\psi^{-2\nu-2}\psi_{x_i}^2\varphi^2\chi^2 - \lambda\nu(\psi^{-\nu-1}\psi_{x_i}\varphi^2\chi^2)_{x_i}
\end{aligned}$$

eşitlikleri kullanılırsa $\psi_{x_ix_i} = 1$ $(2 \leq i \leq n)$ olduğundan

$$\begin{aligned}
\vartheta_{x_i}^2 &= \varphi_{x_i}^2\chi^2 - \lambda^2\nu^2\psi^{-2\nu-2}\psi_{x_i}^2\varphi^2\chi^2 - \lambda\nu(\nu+1)\psi^{-\nu-2}\psi_{x_i}^2\varphi^2\chi^2 \\
&\quad +\lambda\nu\psi^{-\nu-1}\varphi^2\chi^2 - \lambda\nu(\psi^{-\nu-1}\psi_{x_i}\varphi^2\chi^2)_{x_i}
\end{aligned}$$

ve buna bağlı olarak $\psi_{x_1} = \delta$, $\psi_{x_1x_1} = 0$ olduğu göz önünde bulundurularak

$$\begin{aligned}
\vartheta_{x_1}^2 &= \varphi_{x_1}^2\chi^2 - \lambda^2\nu^2\delta^2\psi^{-2\nu-2}\varphi^2\chi^2 - \lambda\nu(\nu+1)\delta^2\psi^{-\nu-2}\varphi^2\chi^2 \\
&\quad -\lambda\nu(\delta\psi^{-\nu-1}\varphi^2\chi^2)_{x_1}
\end{aligned}$$

bulunur. Benzer şekilde, $\psi_{y_jy_j} = 1$ $(1 \leq j \leq m)$ olduğu dikkate alınarak

$$\begin{aligned}
\vartheta_{y_j} &= (\varphi_{y_j} - \lambda\nu\psi^{-\nu-1}\psi_{y_j}\varphi)\chi, \\
\vartheta_{y_j}^2 &= \varphi_{y_j}^2\chi^2 - \lambda^2\nu^2\psi^{-2\nu-2}\psi_{y_j}^2\varphi^2\chi^2 - \lambda\nu(\nu+1)\psi^{-\nu-2}\psi_{y_j}^2\varphi^2\chi^2 \\
&\quad +\lambda\nu\psi^{-\nu-1}\varphi^2\chi^2 - \lambda\nu(\psi^{-\nu-1}\psi_{y_j}\varphi^2\chi^2)_{y_j}
\end{aligned}$$

elde edilir. Sonuç olarak,

$$
\begin{aligned}
& 2\lambda\nu\gamma^{-3}\delta\vartheta_{x_1}^2+\lambda\nu(x_1+\delta_0)\delta\alpha_1\sum_{i=2}^{n}\vartheta_{x_i}^2+\lambda\nu(x_1+\delta_0)\delta\alpha_2\sum_{j=1}^{m}\vartheta_{y_j}^2+2\lambda^3\nu^4\delta^4\psi^{-2\nu-3}\vartheta^2 \\
= \; & 2\lambda\nu\gamma^{-3}\delta(\varphi_{x_1}^2\chi^2-\lambda^2\nu^2\delta^2\psi^{-2\nu-2}\varphi^2\chi^2 \\
& -\lambda\nu(\nu+1)\delta^2\psi^{-\nu-2}\varphi^2\chi^2-\lambda\nu(\delta\psi^{-\nu-1}\varphi^2\chi^2)_{x_1})
\end{aligned}
$$

$$
\begin{aligned}
& +\lambda\nu(x_1+\delta_0)\delta\alpha_1\sum_{i=2}^{n}(\varphi_{x_i}^2\chi^2-\lambda^2\nu^2\psi^{-2\nu-2}\psi_{x_i}^2\varphi^2\chi^2-\lambda\nu(\nu+1)\psi^{-\nu-2}\psi_{x_i}^2\varphi^2\chi^2 \\
& +\lambda\nu\psi^{-\nu-1}\varphi^2\chi^2-\lambda\nu(\psi^{-\nu-1}\psi_{x_i}\varphi^2\chi^2)_{x_i}) \\
& +\lambda\nu(x_1+\delta_0)\delta\alpha_2\sum_{j=1}^{m}(\varphi_{y_j}^2\chi^2-\lambda^2\nu^2\psi^{-2\nu-2}\psi_{y_j}^2\varphi^2\chi^2-\lambda\nu(\nu+1)\psi^{-\nu-2}\psi_{y_j}^2\varphi^2\chi^2 \\
& +\lambda\nu\psi^{-\nu-1}\varphi^2\chi^2-\lambda\nu(\psi^{-\nu-1}\psi_{y_j}\varphi^2\chi^2)_{y_j})+2\lambda^3\nu^4\delta^4\psi^{-2\nu-3}\varphi^2\chi^2
\end{aligned}
$$

$$
\begin{aligned}
= \; & 2\lambda\nu\gamma^{-3}\delta\varphi_{x_1}^2\chi^2+\lambda\nu(x_1+\delta_0)\delta\alpha_1\chi^2\sum_{i=2}^{n}\varphi_{x_i}^2 \\
& +\lambda\nu(x_1+\delta_0)\delta\alpha_2\chi^2\sum_{j=1}^{m}\varphi_{y_j}^2+2\lambda^3\nu^4\delta^4\psi^{-2\nu-3}\varphi^2\chi^2 \\
& -2\lambda^2\nu^2\gamma^{-3}\delta(\delta\psi^{-\nu-1}\varphi^2\chi^2)_{x_1}-\lambda^2\nu^2(x_1+\delta_0)\delta\alpha_1\sum_{i=2}^{n}(\psi^{-\nu-1}\psi_{x_i}\varphi^2\chi^2)_{x_i} \\
& -\lambda^2\nu^2(x_1+\delta_0)\delta\alpha_2\sum_{j=1}^{m}(\psi^{-\nu-1}\psi_{y_j}\varphi^2\chi^2)_{y_j}
\end{aligned}
$$

$$
\begin{aligned}
& -2\lambda^3\nu^3\gamma^{-3}\delta^3\psi^{-2\nu-2}\varphi^2\chi^2-\lambda^3\nu^3(x_1+\delta_0)\delta\alpha_1\psi^{-2\nu-2}\varphi^2\chi^2\sum_{i=2}^{n}\psi_{x_i}^2 \\
& -\lambda^3\nu^3(x_1+\delta_0)\delta\alpha_2\psi^{-2\nu-2}\varphi^2\chi^2\sum_{j=1}^{m}\psi_{y_j}^2 \\
& -\lambda^2\nu^2(2\gamma^{-3}\delta^3(\nu+1)\psi^{-\nu-2}+(x_1+\delta_0)\delta\alpha_1(\nu+1)\psi^{-\nu-2}\sum_{i=2}^{n}\psi_{x_i}^2 \\
& +(x_1+\delta_0)\delta\alpha_2(\nu+1)\psi^{-\nu-2}\sum_{j=1}^{m}\psi_{y_j}^2-(x_1+\delta_0)\delta\alpha_1\psi^{-\nu-1}(n-1) \\
& -(x_1+\delta_0)\delta\alpha_2\psi^{-\nu-1}m)\varphi^2\chi^2 \qquad (2.51)
\end{aligned}
$$

bulunur. Ayrıca $(x_1+\delta_0)<\frac{3}{4}\gamma$, $\psi^{-1}>1$, $\sum_{i=2}^{n}\psi_{x_i}^2\leq 2\gamma$, $\sum_{j=1}^{m}\psi_{y_j}^2\leq 2\gamma$ olduğundan $\nu\geq\nu_1=2\gamma^{-3}\delta^{-1}+\frac{3}{2}\gamma^2\delta^{-3}(\alpha_1+\alpha_2)+1$ olmak üzere (2.51) eşitliğindeki dördüncü, sekizinci, dokuzuncu ve onuncu terimler için

$$\begin{aligned}
& \lambda^3\nu^4\delta^4\psi^{-2\nu-3}\varphi^2\chi^2 - 2\lambda^3\nu^3\gamma^{-3}\delta^3\psi^{-2\nu-2}\varphi^2\chi^2 \\
& -\lambda^3\nu^3(x_1+\delta_0)\delta\alpha_1\psi^{-2\nu-2}\varphi^2\chi^2\sum_{i=2}^{n}\psi_{x_i}^2 - \lambda^3\nu^3(x_1+\delta_0)\delta\alpha_2\psi^{-2\nu-2}\varphi^2\chi^2\sum_{j=1}^{m}\psi_{y_j}^2 \\
= \;& \varphi^2\chi^2\psi^{-2\nu-2}(\lambda^3\nu^4\delta^4\psi^{-1} - 2\lambda^3\nu^3\gamma^{-3}\delta^3 - \lambda^3\nu^3(x_1+\delta_0)\delta\alpha_1\sum_{i=2}^{n}\psi_{x_i}^2 \\
& -\lambda^3\nu^3(x_1+\delta_0)\delta\alpha_2\sum_{j=1}^{m}\psi_{y_j}^2) \\
> \;& \varphi^2\chi^2\psi^{-2\nu-2}(\lambda^3\nu^4\delta^4 - 2\lambda^3\nu^3\gamma^{-3}\delta^3 - \lambda^3\nu^3\frac{3}{4}\gamma\delta\alpha_1 2\gamma - \lambda^3\nu^3\frac{3}{4}\gamma\delta\alpha_2 2\gamma) \\
= \;& \varphi^2\chi^2\psi^{-2\nu-2}\lambda^3\nu^3\delta^4(\nu - 2\gamma^{-3}\delta^{-1} - \frac{3}{2}\gamma^2\delta^{-3}(\alpha_1+\alpha_2)) \\
\geq \;& \lambda^3\nu^3\delta^4\psi^{-2\nu-2}\varphi^2\chi^2 \qquad (2.52)
\end{aligned}$$

yazılır. Bu sonuç ve onbirinci, onikinci ve onüçüncü terimler birlikte değerlendirilirse her $\lambda \geq \lambda_1 = 2\gamma^{-3}(\nu+1) + \frac{3}{2}\gamma^2(\nu+1)(\alpha_1+\alpha_2)$, $0 < \psi < 1$ ve $\nu \geq 1$ için

$$\begin{aligned}
& \lambda^3\nu^3\delta^4\psi^{-2\nu-2}\varphi^2\chi^2 - 2\lambda^2\nu^2\gamma^{-3}\delta^3(\nu+1)\psi^{-\nu-2}\varphi^2\chi^2 \\
& -\lambda^2\nu^2(x_1+\delta_0)\delta\alpha_1(\nu+1)\psi^{-\nu-2}\varphi^2\chi^2\sum_{i=2}^{n}\psi_{x_i}^2 \\
& -\lambda^2\nu^2(x_1+\delta_0)\delta\alpha_2(\nu+1)\psi^{-\nu-2}\varphi^2\chi^2\sum_{j=1}^{m}\psi_{y_j}^2 \\
= \;& \lambda^2\nu^3\delta^4\psi^{-2\nu-2}\varphi^2\chi^2(\lambda - 2\nu^{-1}\gamma^{-3}\delta^{-1}(\nu+1)\psi^{\nu} \\
& -\nu^{-1}(x_1+\delta_0)\delta^{-3}\alpha_1(\nu+1)\psi^{\nu}\sum_{i=2}^{n}\psi_{x_i}^2 - \nu^{-1}(x_1+\delta_0)\delta^{-3}\alpha_2(\nu+1)\psi^{\nu}\sum_{j=1}^{m}\psi_{y_j}^2) \\
> \;& \lambda^2\nu^3\delta^4\psi^{-2\nu-2}\varphi^2\chi^2(\lambda - 2\gamma^{-3}(\nu+1) - \frac{3}{4}\gamma\alpha_1(\nu+1)2\gamma - \frac{3}{4}\gamma\alpha_2(\nu+1)2\gamma) \\
= \;& \lambda^2\nu^3\delta^4\psi^{-2\nu-2}\varphi^2\chi^2(\lambda - 2\gamma^{-3}(\nu+1) - \frac{3}{2}\gamma^2(\nu+1)(\alpha_1+\alpha_2)) \\
\geq \;& 0 \qquad (2.53)
\end{aligned}$$

olur. O halde yukarıdaki eşitsizliklerden (2.51) eşitliğini $\nu \geq \nu_1$ ve $\lambda \geq \lambda_1$ için

$$\begin{aligned}
& 2\lambda\nu\gamma^{-3}\delta\vartheta_{x_1}^2 + \lambda\nu(x_1+\delta_0)\delta\alpha_1\sum_{i=2}^{n}\vartheta_{x_i}^2 \\
& +\lambda\nu(x_1+\delta_0)\delta\alpha_2\sum_{j=1}^{m}\vartheta_{y_j}^2 + 2\lambda^3\nu^4\delta^4\psi^{-2\nu-3}\vartheta^2 \\
\geq \;& 2\lambda\nu\gamma^{-3}\delta\varphi_{x_1}^2\chi^2 + \lambda\nu(x_1+\delta_0)\delta\alpha_1\chi^2\sum_{i=2}^{n}\varphi_{x_i}^2 + \lambda\nu(x_1+\delta_0)\delta\alpha_2\chi^2\sum_{j=1}^{m}\varphi_{y_j}^2 \\
& +\lambda^3\nu^4\delta^4\psi^{-2\nu-3}\varphi^2\chi^2 + d_3(\varphi) \qquad (2.54)
\end{aligned}$$

olarak yazabiliriz. Burada

$$
\begin{aligned}
d_3(\varphi) &= -2\lambda^2\nu^2\gamma^{-3}\delta(\psi^{-\nu-1}\delta\varphi^2\chi^2)_{x_1} - \lambda^2\nu^2(x_1+\delta_0)\delta\alpha_1\sum_{i=2}^{n}(\psi^{-\nu-1}\psi_{x_i}\varphi^2\chi^2)_{x_i} \\
&\quad -\lambda^2\nu^2(x_1+\delta_0)\delta\alpha_2\sum_{j=1}^{m}(\psi^{-\nu-1}\psi_{y_j}\varphi^2\chi^2)_{y_j}
\end{aligned}
$$

biçiminde tanımlıdır. Böylece

$$
\begin{aligned}
\psi^{\nu+1}(L_0\varphi)^2\chi^2 &\geq 2\lambda\nu\gamma^{-3}\delta\varphi_{x_1}^2\chi^2 + \lambda\nu(x_1+\delta_0)\delta\alpha_1\chi^2\sum_{i=2}^{n}\varphi_{x_i}^2 \\
&\quad +\lambda\nu(x_1+\delta_0)\delta\alpha_2\chi^2\sum_{j=1}^{m}\varphi_{y_j}^2 + \lambda^3\nu^4\delta^4\psi^{-2\nu-3}\varphi^2\chi^2 \\
&\quad +2\lambda\nu\xi(x_1+\delta_0)^2(\sum_{i=2}^{n}(a_{ii}\psi_{x_i})_{x_i} - \sum_{j=1}^{m}(b_{jj}\psi_{y_j})_{y_j})\varphi^2\chi^2 + \sum_{j=1}^{3}d_j(\varphi)
\end{aligned}
$$

eşitsizliği bulunur ve ispat tamamlanır. ■

2.3 LEMMA 2.3

Lemma 2.3 *Aşağıdaki eşitsizlik Q_γ da sağlanır:*

$$
\begin{aligned}
&2\lambda\nu(x_1+\delta_0)\chi^2\Lambda\varphi(L_0\varphi) \\
\geq\ &-2\lambda\nu\Lambda^0\chi^2(\varphi_{x_1}^2 + M(x_1+\delta_0)^2(\sum_{i=2}^{n}\varphi_{x_i}^2 + \sum_{j=1}^{m}\varphi_{y_j}^2)) \\
&-4\lambda^3\nu^3\chi^2\psi^{-2\nu-2}(\delta^2 + M2\gamma(n+m-1))\Lambda^0\varphi^2 \\
&-C\lambda^2\nu^2\psi^{-\nu-1}\delta^2\chi^2\varphi^2 - 2\lambda\nu(x_1+\delta_0)^2\chi^2\Lambda\xi\varphi^2 + d_4(\varphi). \qquad (2.55)
\end{aligned}
$$

Burada $C > 0$, a_{ii}, b_{jj} katsayılarına ve n, m ye bağlı olup

$$
\begin{aligned}
d_4(\varphi) &= 2\lambda\nu(\chi^2\Lambda\varphi\varphi_{x_1})_{x_1} - \lambda\nu((\chi^2\Lambda)_{x_1}\varphi^2)_{x_1} \\
&\quad +2\lambda\nu(x_1+\delta_0)^2(\chi^2\Lambda\sum_{i=2}^{n}a_{ii}\varphi\varphi_{x_i})_{x_i} - \lambda\nu(x_1+\delta_0)^2((\chi^2\Lambda\sum_{i=2}^{n}a_{ii})_{x_i}\varphi^2)_{x_i} \\
&\quad -2\lambda\nu(x_1+\delta_0)^2(\chi^2\Lambda\sum_{j=1}^{m}b_{jj}\varphi\varphi_{y_j})_{y_j} + \lambda\nu(x_1+\delta_0)^2((\chi^2\Lambda\sum_{j=1}^{m}b_{jj})_{y_j}\varphi^2)_{y_j} \\
&\quad -(2\lambda\nu(x_1+\delta_0)^2\chi^2\Lambda k\varphi^2)_\xi,
\end{aligned}
$$

$$\Lambda = \sum_{i=2}^{n}(a_{ii}\psi_{x_i})_{x_i} - \sum_{j=1}^{m}(b_{jj}\psi_{y_j})_{y_j},$$

$$\Lambda^0 = \max\{(\sqrt{2\gamma}+1)M(n+m-1) + M(n-1), (\sqrt{2\gamma}+1)M(n+m-1) + Mm\}$$

şeklinde tanımlıdır.

İspat. İlk olarak aşağıdaki eşitliği ele alalım:

$$
\begin{aligned}
& 2\lambda\nu(x_1+\delta_0)\chi^2\Lambda\varphi(L_0\varphi) \\
= \; & 2\lambda\nu(x_1+\delta_0)\chi^2\Lambda\varphi((x_1+\delta_0)^{-1}\varphi_{x_1x_1} \\
& +(x_1+\delta_0)(\sum_{i=2}^{n} a_{ii}\varphi_{x_ix_i} - \sum_{j=1}^{m} b_{jj}\varphi_{y_jy_j} - 2k\varphi_\xi - \xi\varphi)) \\
= \; & 2\lambda\nu\chi^2\Lambda\varphi\varphi_{x_1x_1} + 2\lambda\nu(x_1+\delta_0)^2\chi^2\Lambda\varphi\sum_{i=2}^{n} a_{ii}\varphi_{x_ix_i} - 2\lambda\nu(x_1+\delta_0)^2\chi^2\Lambda\varphi\sum_{j=1}^{m} b_{jj}\varphi_{y_jy_j} \\
& -4\lambda\nu(x_1+\delta_0)^2\chi^2\Lambda\varphi k\varphi_\xi - 2\lambda\nu(x_1+\delta_0)^2\chi^2\Lambda\varphi\xi\varphi.
\end{aligned}
$$

Bu eşitlikteki terimler $a_{x_i}b_{x_i} = (a_{x_i}b)_{x_i} - a_{x_ix_i}b$ özdeşliğide göz önünde bulundurularak değerlendirilirse,

$$
\begin{aligned}
2\lambda\nu\chi^2\Lambda\varphi\varphi_{x_1x_1} &= 2\lambda\nu((\chi^2\Lambda\varphi\varphi_{x_1})_{x_1} - (\chi^2\Lambda)_{x_1}\varphi\varphi_{x_1} - \chi^2\Lambda\varphi^2_{x_1}) \\
&= 2\lambda\nu(\chi^2\Lambda\varphi\varphi_{x_1})_{x_1} - 2\lambda\nu(\chi^2\Lambda)_{x_1}\varphi\varphi_{x_1} - 2\lambda\nu\chi^2\Lambda\varphi^2_{x_1},
\end{aligned}
$$

$$
\begin{aligned}
2\lambda\nu(\chi^2\Lambda)_{x_1}\varphi\varphi_{x_1} &= \lambda\nu(\chi^2\Lambda)_{x_1}(\varphi^2)_{x_1} \\
&= \lambda\nu(((\chi^2\Lambda)_{x_1}\varphi^2)_{x_1} - (\chi^2\Lambda)_{x_1x_1}\varphi^2),
\end{aligned}
$$

$$
\begin{aligned}
2\lambda\nu\chi^2\Lambda\varphi\varphi_{x_1x_1} &= 2\lambda\nu(\chi^2\Lambda\varphi\varphi_{x_1})_{x_1} - \lambda\nu((\chi^2\Lambda)_{x_1}\varphi^2)_{x_1} \\
&\quad +\lambda\nu(\chi^2\Lambda)_{x_1x_1}\varphi^2 - 2\lambda\nu\chi^2\Lambda\varphi^2_{x_1},
\end{aligned} \tag{2.56}
$$

$$
\begin{aligned}
2\lambda\nu(x_1+\delta_0)^2\chi^2\Lambda\varphi\sum_{i=2}^{n} a_{ii}\varphi_{x_ix_i} &= 2\lambda\nu(x_1+\delta_0)^2\chi^2\Lambda\sum_{i=2}^{n} a_{ii}\varphi\varphi_{x_ix_i} \\
&= 2\lambda\nu(x_1+\delta_0)^2\chi^2\Lambda\sum_{i=2}^{n} a_{ii}((\varphi\varphi_{x_i})_{x_i} - \varphi^2_{x_i}) \\
&= 2\lambda\nu(x_1+\delta_0)^2\chi^2\Lambda\sum_{i=2}^{n} a_{ii}(\varphi\varphi_{x_i})_{x_i} \\
&\quad -2\lambda\nu(x_1+\delta_0)^2\chi^2\Lambda\sum_{i=2}^{n} a_{ii}\varphi^2_{x_i},
\end{aligned}
$$

$$
\begin{aligned}
& 2\lambda\nu(x_1+\delta_0)^2\chi^2\Lambda\sum_{i=2}^{n} a_{ii}(\varphi\varphi_{x_i})_{x_i} \\
= \; & 2\lambda\nu(x_1+\delta_0)^2(\chi^2\Lambda\sum_{i=2}^{n} a_{ii}\varphi\varphi_{x_i})_{x_i} - 2\lambda\nu(x_1+\delta_0)^2(\chi^2\Lambda\sum_{i=2}^{n} a_{ii})_{x_i}\varphi\varphi_{x_i} \\
= \; & 2\lambda\nu(x_1+\delta_0)^2(\chi^2\Lambda\sum_{i=2}^{n} a_{ii}\varphi\varphi_{x_i})_{x_i} - \lambda\nu(x_1+\delta_0)^2(\chi^2\Lambda\sum_{i=2}^{n} a_{ii})_{x_i}(\varphi^2)_{x_i},
\end{aligned}
$$

$$\lambda\nu(x_1+\delta_0)^2(\chi^2\Lambda\sum_{i=2}^{n}a_{ii})_{x_i}(\varphi^2)_{x_i}$$
$$= \lambda\nu(x_1+\delta_0)^2(((\chi^2\Lambda\sum_{i=2}^{n}a_{ii})_{x_i}\varphi^2)_{x_i}-(\chi^2\Lambda\sum_{i=2}^{n}a_{ii})_{x_ix_i}\varphi^2),$$

$$2\lambda\nu(x_1+\delta_0)^2\chi^2\Lambda\varphi\sum_{i=2}^{n}a_{ii}\varphi_{x_ix_i}$$
$$= 2\lambda\nu(x_1+\delta_0)^2(\chi^2\Lambda\sum_{i=2}^{n}a_{ii}\varphi\varphi_{x_i})_{x_i}-\lambda\nu(x_1+\delta_0)^2((\chi^2\Lambda\sum_{i=2}^{n}a_{ii})_{x_i}\varphi^2)_{x_i}$$
$$+\lambda\nu(x_1+\delta_0)^2(\chi^2\Lambda\sum_{i=2}^{n}a_{ii})_{x_ix_i}\varphi^2-2\lambda\nu(x_1+\delta_0)^2\chi^2\Lambda\sum_{i=2}^{n}a_{ii}\varphi_{x_i}^2, \tag{2.57}$$

$$\begin{aligned}-2\lambda\nu(x_1+\delta_0)^2\chi^2\Lambda\varphi\sum_{j=1}^{m}b_{jj}\varphi_{y_jy_j} &= -2\lambda\nu(x_1+\delta_0)^2\chi^2\Lambda\sum_{j=1}^{m}b_{jj}\varphi\varphi_{y_jy_j}\\ &= -2\lambda\nu(x_1+\delta_0)^2\chi^2\Lambda\sum_{j=1}^{m}b_{jj}((\varphi\varphi_{y_j})_{y_j}-\varphi_{y_j}^2)\\ &= -2\lambda\nu(x_1+\delta_0)^2\chi^2\Lambda\sum_{j=1}^{m}b_{jj}(\varphi\varphi_{y_j})_{y_j}\\ &\quad +2\lambda\nu(x_1+\delta_0)^2\chi^2\Lambda\sum_{j=1}^{m}b_{jj}\varphi_{y_j}^2,\end{aligned}$$

$$-2\lambda\nu(x_1+\delta_0)^2\chi^2\Lambda\sum_{j=1}^{m}b_{jj}(\varphi\varphi_{y_j})_{y_j}$$
$$= -2\lambda\nu(x_1+\delta_0)^2(\chi^2\Lambda\sum_{j=1}^{m}b_{jj}\varphi\varphi_{y_j})_{y_j}+2\lambda\nu(x_1+\delta_0)^2(\chi^2\Lambda\sum_{j=1}^{m}b_{jj})_{y_j}\varphi\varphi_{y_j}$$
$$= -2\lambda\nu(x_1+\delta_0)^2(\chi^2\Lambda\sum_{j=1}^{m}b_{jj}\varphi\varphi_{y_j})_{y_j}+\lambda\nu(x_1+\delta_0)^2(\chi^2\Lambda\sum_{j=1}^{m}b_{jj})_{y_j}(\varphi^2)_{y_j},$$

$$\lambda\nu(x_1+\delta_0)^2(\chi^2\Lambda\sum_{j=1}^{m}b_{jj})_{y_j}(\varphi^2)_{y_j}$$
$$= \lambda\nu(x_1+\delta_0)^2(((\chi^2\Lambda\sum_{j=1}^{m}b_{jj})_{y_j}\varphi^2)_{y_j}-(\chi^2\Lambda\sum_{j=1}^{m}b_{jj})_{y_jy_j}\varphi^2),$$

$$-2\lambda\nu(x_1+\delta_0)^2\chi^2\Lambda\varphi\sum_{j=1}^{m}b_{jj}\varphi_{y_jy_j}$$
$$= -2\lambda\nu(x_1+\delta_0)^2(\chi^2\Lambda\sum_{j=1}^{m}b_{jj}\varphi\varphi_{y_j})_{y_j}+\lambda\nu(x_1+\delta_0)^2((\chi^2\Lambda\sum_{j=1}^{m}b_{jj})_{y_j}\varphi^2)_{y_j}$$
$$-\lambda\nu(x_1+\delta_0)^2(\chi^2\Lambda\sum_{j=1}^{m}b_{jj})_{y_jy_j}\varphi^2+2\lambda\nu(x_1+\delta_0)^2\chi^2\Lambda\sum_{j=1}^{m}b_{jj}\varphi_{y_j}^2, \tag{2.58}$$

$$-4\lambda\nu(x_1+\delta_0)^2\chi^2\Lambda\varphi k\varphi_\xi = -(2\lambda\nu(x_1+\delta_0)^2\chi^2\Lambda k\varphi^2)_\xi, \tag{2.59}$$

$$-2\lambda\nu(x_1+\delta_0)^2\chi^2\Lambda\varphi\xi\varphi = -2\lambda\nu(x_1+\delta_0)^2\chi^2\Lambda\xi\varphi^2 \tag{2.60}$$

elde edilir. (2.56)-(2.60) eşitlikleri kullanılarak

$$\begin{aligned}
& 2\lambda\nu(x_1+\delta_0)\chi^2\Lambda\varphi(L_0\varphi) \\
= \; & 2\lambda\nu(\chi^2\Lambda\varphi\varphi_{x_1})_{x_1} - \lambda\nu((\chi^2\Lambda)_{x_1}\varphi^2)_{x_1} + \lambda\nu(\chi^2\Lambda)_{x_1x_1}\varphi^2 - 2\lambda\nu\chi^2\Lambda\varphi_{x_1}^2 \\
& +2\lambda\nu(x_1+\delta_0)^2(\chi^2\Lambda\sum_{i=2}^{n}a_{ii}\varphi\varphi_{x_i})_{x_i} - \lambda\nu(x_1+\delta_0)^2((\chi^2\Lambda\sum_{i=2}^{n}a_{ii})_{x_i}\varphi^2)_{x_i} \\
& +\lambda\nu(x_1+\delta_0)^2(\chi^2\Lambda\sum_{i=2}^{n}a_{ii})_{x_ix_i}\varphi^2 - 2\lambda\nu(x_1+\delta_0)^2\chi^2\Lambda\sum_{i=2}^{n}a_{ii}\varphi_{x_i}^2
\end{aligned}$$

$$\begin{aligned}
& -2\lambda\nu(x_1+\delta_0)^2(\chi^2\Lambda\sum_{j=1}^{m}b_{jj}\varphi\varphi_{y_j})_{y_j} + \lambda\nu(x_1+\delta_0)^2((\chi^2\Lambda\sum_{j=1}^{m}b_{jj})_{y_j}\varphi^2)_{y_j} \\
& -\lambda\nu(x_1+\delta_0)^2(\chi^2\Lambda\sum_{j=1}^{m}b_{jj})_{y_jy_j}\varphi^2 + 2\lambda\nu(x_1+\delta_0)^2\chi^2\Lambda\sum_{j=1}^{m}b_{jj}\varphi_{y_j}^2 \\
& -(2\lambda\nu(x_1+\delta_0)^2\chi^2\Lambda k\varphi^2)_\xi - 2\lambda\nu(x_1+\delta_0)^2\chi^2\Lambda\xi\varphi^2
\end{aligned}$$

$$\begin{aligned}
= \; & \lambda\nu(\chi^2\Lambda)_{x_1x_1}\varphi^2 - 2\lambda\nu\chi^2\Lambda\varphi_{x_1}^2 + \lambda\nu(x_1+\delta_0)^2(\chi^2\Lambda\sum_{i=2}^{n}a_{ii})_{x_ix_i}\varphi^2 \\
& -2\lambda\nu(x_1+\delta_0)^2\chi^2\Lambda\sum_{i=2}^{n}a_{ii}\varphi_{x_i}^2 - \lambda\nu(x_1+\delta_0)^2(\chi^2\Lambda\sum_{j=1}^{m}b_{jj})_{y_jy_j}\varphi^2 \\
& +2\lambda\nu(x_1+\delta_0)^2\chi^2\Lambda\sum_{j=1}^{m}b_{jj}\varphi_{y_j}^2 - 2\lambda\nu(x_1+\delta_0)^2\chi^2\Lambda\xi\varphi^2 + d_4(\varphi),
\end{aligned}$$

$$\begin{aligned}
& 2\lambda\nu(x_1+\delta_0)\chi^2\Lambda\varphi(L_0\varphi) \\
= \; & -2\lambda\nu\Lambda\chi^2(\varphi_{x_1}^2 + (x_1+\delta_0)^2(\sum_{i=2}^{n}a_{ii}\varphi_{x_i}^2 - \sum_{j=1}^{m}b_{jj}\varphi_{y_j}^2)) \\
& +A\varphi^2 - 2\lambda\nu(x_1+\delta_0)^2\chi^2\Lambda\xi\varphi^2 + d_4(\varphi)
\end{aligned} \tag{2.61}$$

bulunur. Burada

$$A = \lambda\nu((\chi^2\Lambda)_{x_1x_1} + (x_1+\delta_0)^2((\chi^2\Lambda\sum_{i=2}^{n}a_{ii})_{x_ix_i} - (\chi^2\Lambda\sum_{j=1}^{m}b_{jj})_{y_jy_j}))$$

dir. Diğer taraftan $|\Lambda| \leq \Lambda^0$, $|a_{ii}| \leq M$, $2 \leq i \leq n$ ve $|b_{jj}| \leq M$, $1 \leq j \leq m$ olduğundan

$$\begin{aligned}
& -2\lambda\nu\Lambda\chi^2(\varphi_{x_1}^2 + (x_1+\delta_0)^2(\sum_{i=2}^{n} a_{ii}\varphi_{x_i}^2 - \sum_{j=1}^{m} b_{jj}\varphi_{y_j}^2)) \\
\geq & -2\lambda\nu|\Lambda|\chi^2(\varphi_{x_1}^2 + (x_1+\delta_0)^2(\sum_{i=2}^{n} |a_{ii}\varphi_{x_i}^2| + \sum_{j=1}^{m} |b_{jj}\varphi_{y_j}^2|)) \\
\geq & -2\lambda\nu\Lambda^0\chi^2(\varphi_{x_1}^2 + M(x_1+\delta_0)^2(\sum_{i=2}^{n} \varphi_{x_i}^2 + \sum_{j=1}^{m} \varphi_{y_j}^2)) \qquad (2.62)
\end{aligned}$$

yazılabilir. Ayrıca A katsayısını değerlendirmek için

$$\begin{aligned}
(\chi^2\Lambda)_{x_1} &= (\chi^2)_{x_1}\Lambda + \chi^2\Lambda_{x_1}, \\
(\chi^2\Lambda)_{x_1x_1} &= ((\chi^2)_{x_1}\Lambda + \chi^2\Lambda_{x_1})_{x_1} \\
&= (\chi^2)_{x_1x_1}\Lambda + (\chi^2)_{x_1}\Lambda_{x_1} + (\chi^2)_{x_1}\Lambda_{x_1} + \chi^2\Lambda_{x_1x_1} \\
&= (\chi^2)_{x_1x_1}\Lambda + 2(\chi^2)_{x_1}\Lambda_{x_1} + \chi^2\Lambda_{x_1x_1},
\end{aligned}$$

$$\begin{aligned}
(\chi^2\Lambda a_{ii})_{x_i} &= (\chi^2)_{x_i}\Lambda a_{ii} + \chi^2\Lambda_{x_i}a_{ii} + \chi^2\Lambda(a_{ii})_{x_i}, \\
(\chi^2\Lambda a_{ii})_{x_ix_i} &= ((\chi^2)_{x_i}\Lambda a_{ii} + \chi^2\Lambda_{x_i}a_{ii} + \chi^2\Lambda(a_{ii})_{x_i})_{x_i} \\
&= (\chi^2)_{x_ix_i}\Lambda a_{ii} + (\chi^2)_{x_i}\Lambda_{x_i}a_{ii} + (\chi^2)_{x_i}\Lambda(a_{ii})_{x_i} \\
&\quad +(\chi^2)_{x_i}\Lambda_{x_i}a_{ii} + \chi^2\Lambda_{x_ix_i}a_{ii} + \chi^2\Lambda_{x_i}(a_{ii})_{x_i} \\
&\quad +(\chi^2)_{x_i}\Lambda(a_{ii})_{x_i} + \chi^2\Lambda_{x_i}(a_{ii})_{x_i} + \chi^2\Lambda(a_{ii})_{x_ix_i}
\end{aligned}$$

$$\begin{aligned}
= & \ \chi^2(\Lambda_{x_ix_i}a_{ii} + \Lambda_{x_i}(a_{ii})_{x_i} + \Lambda_{x_i}(a_{ii})_{x_i} + \Lambda(a_{ii})_{x_ix_i}) \\
& +(\chi^2)_{x_i}(\Lambda_{x_i}a_{ii} + \Lambda(a_{ii})_{x_i} + \Lambda_{x_i}a_{ii}) + (\chi^2)_{x_ix_i}\Lambda a_{ii} + (\chi^2)_{x_i}\Lambda(a_{ii})_{x_i},
\end{aligned}$$

$$\begin{aligned}
(\chi^2\Lambda a_{ii})_{x_ix_i} &= \Lambda((\chi^2)_{x_i}a_{ii})_{x_i} + (\chi^2)_{x_i}(2\Lambda_{x_i}a_{ii} + \Lambda(a_{ii})_{x_i}) \\
&\quad +\chi^2(\Lambda_{x_ix_i}a_{ii} + 2\Lambda_{x_i}(a_{ii})_{x_i} + \Lambda(a_{ii})_{x_ix_i}),
\end{aligned}$$

$$\begin{aligned}
(\chi^2\Lambda b_{jj})_{y_j} &= (\chi^2)_{y_j}\Lambda b_{jj} + \chi^2\Lambda_{y_j}b_{jj} + \chi^2\Lambda(b_{jj})_{y_j}, \\
(\chi^2\Lambda b_{jj})_{y_jy_j} &= \Lambda((\chi^2)_{y_j}b_{jj})_{y_j} + (\chi^2)_{y_j}(2\Lambda_{y_j}b_{jj} + \Lambda(b_{jj})_{y_j}) \\
&\quad +\chi^2(\Lambda_{y_jy_j}b_{jj} + 2\Lambda_{y_j}(b_{jj})_{y_j} + \Lambda(b_{jj})_{y_jy_j})
\end{aligned}$$

bağıntıları yardımıyla

$$\begin{aligned} A &= \lambda\nu((\chi^2)_{x_1x_1}\Lambda + 2(\chi^2)_{x_1}\Lambda_{x_1} + \chi^2\Lambda_{x_1x_1} \\ &+(x_1+\delta_0)^2(\sum_{i=2}^{n}(\Lambda((\chi^2)_{x_i}a_{ii})_{x_i} + (\chi^2)_{x_i}(2\Lambda_{x_i}a_{ii} + \Lambda(a_{ii})_{x_i}) \\ &+\chi^2(\Lambda_{x_ix_i}a_{ii} + 2\Lambda_{x_i}(a_{ii})_{x_i} + \Lambda(a_{ii})_{x_ix_i})) \\ &-\sum_{j=1}^{m}(\Lambda((\chi^2)_{y_j}b_{jj})_{y_j} + (\chi^2)_{y_j}(2\Lambda_{y_j}b_{jj} + \Lambda(b_{jj})_{y_j}) \\ &+\chi^2(\Lambda_{y_jy_j}b_{jj} + 2\Lambda_{y_j}(b_{jj})_{y_j} + \Lambda(b_{jj})_{y_jy_j}))) \end{aligned} \tag{2.63}$$

bulunur. Ayrıca $\psi_{x_1} = \delta$ ve $\psi_{x_1x_1} = 0$ olduğu göz önünde bulundurularak

$$\begin{aligned} \chi^2 &= e^{2\lambda\psi^{-\nu}}, \\ (\chi^2)_{x_1} &= -2\lambda\nu\psi^{-\nu-1}\psi_{x_1}e^{2\lambda\psi^{-\nu}} = -2\lambda\nu\psi^{-\nu-1}\delta\chi^2, \\ (\chi^2)_{x_1x_1} &= -2\lambda\nu(-(\nu+1)\psi^{-\nu-2}\psi_{x_1}^2\chi^2 + \psi^{-\nu-1}\psi_{x_1x_1}\chi^2 \\ &+\psi^{-\nu-1}\psi_{x_1}(-2\lambda\nu\psi^{-\nu-1}\psi_{x_1}\chi^2)) \\ &= 2\lambda\nu(\nu+1)\delta^2\psi^{-\nu-2}\chi^2 + 4\lambda^2\nu^2\delta^2\psi^{-2\nu-2}\chi^2 \\ &= 2\lambda\nu\chi^2(2\lambda\nu\delta^2\psi^{-2\nu-2} + (\nu+1)\delta^2\psi^{-\nu-2}), \end{aligned}$$

$$\begin{aligned} (\chi^2)_{x_i} &= -2\lambda\nu\psi^{-\nu-1}\psi_{x_i}e^{2\lambda\psi^{-\nu}} = -2\lambda\nu\psi^{-\nu-1}\psi_{x_i}\chi^2, \\ (\chi^2)_{x_ix_i} &= -2\lambda\nu(-(\nu+1)\psi^{-\nu-2}\psi_{x_i}^2\chi^2 + \psi^{-\nu-1}\psi_{x_ix_i}\chi^2 + \psi^{-\nu-1}\psi_{x_i}(-2\lambda\nu\psi^{-\nu-1}\psi_{x_i}\chi^2)) \\ &= 2\lambda\nu\chi^2(2\lambda\nu\psi^{-2\nu-2}\psi_{x_i}^2 + (\nu+1)\psi^{-\nu-2}\psi_{x_i}^2 - \psi^{-\nu-1}\psi_{x_ix_i}), \end{aligned}$$

$$\begin{aligned} (a_{ii}(\chi^2)_{x_i})_{x_i} &= (a_{ii})_{x_i}(\chi^2)_{x_i} + a_{ii}(\chi^2)_{x_ix_i} \\ &= (a_{ii})_{x_i}(-2\lambda\nu\psi^{-\nu-1}\psi_{x_i}\chi^2) \\ &+a_{ii}(2\lambda\nu\chi^2(2\lambda\nu\psi^{-2\nu-2}\psi_{x_i}^2 + (\nu+1)\psi^{-\nu-2}\psi_{x_i}^2 - \psi^{-\nu-1}\psi_{x_ix_i})) \\ &= 2\lambda\nu\chi^2(a_{ii}(2\lambda\nu\psi^{-2\nu-2}\psi_{x_i}^2 + (\nu+1)\psi^{-\nu-2}\psi_{x_i}^2 - \psi^{-\nu-1}\psi_{x_ix_i}) \\ &-(a_{ii})_{x_i}\psi^{-\nu-1}\psi_{x_i}), \end{aligned}$$

$$\begin{aligned} (\chi^2)_{y_j} &= -2\lambda\nu\psi^{-\nu-1}\psi_{y_j}e^{2\lambda\psi^{-\nu}} = -2\lambda\nu\psi^{-\nu-1}\psi_{y_j}\chi^2, \\ (\chi^2)_{y_jy_j} &= -2\lambda\nu(-(\nu+1)\psi^{-\nu-2}\psi_{y_j}^2\chi^2 + \psi^{-\nu-1}\psi_{y_jy_j}\chi^2 + \psi^{-\nu-1}\psi_{y_j}(-2\lambda\nu\psi^{-\nu-1}\psi_{y_j}\chi^2)) \\ &= 2\lambda\nu\chi^2(2\lambda\nu\psi^{-2\nu-2}\psi_{y_j}^2 + (\nu+1)\psi^{-\nu-2}\psi_{y_j}^2 - \psi^{-\nu-1}\psi_{y_jy_j}), \end{aligned}$$

$$
\begin{aligned}
(b_{jj}(\chi^2)_{y_j})_{y_j} &= (b_{jj})_{y_j}(\chi^2)_{y_j} + b_{jj}(\chi^2)_{y_jy_j} \\
&= (b_{jj})_{y_j}(-2\lambda\nu\psi^{-\nu-1}\psi_{y_j}\chi^2) \\
&\quad + b_{jj}(2\lambda\nu\chi^2(2\lambda\nu\psi^{-2\nu-2}\psi_{y_j}^2 + (\nu+1)\psi^{-\nu-2}\psi_{y_j}^2 - \psi^{-\nu-1}\psi_{y_jy_j})) \\
&= 2\lambda\nu\chi^2(b_{jj}(2\lambda\nu\psi^{-2\nu-2}\psi_{y_j}^2 + (\nu+1)\psi^{-\nu-2}\psi_{y_j}^2 - \psi^{-\nu-1}\psi_{y_jy_j}) \\
&\quad - (b_{jj})_{y_j}\psi^{-\nu-1}\psi_{y_j})
\end{aligned}
$$

eşitlikleri yazılabilir. Bu son eşitlikler kullanılırsa

$$
\begin{aligned}
A &= \lambda\nu(2\lambda\nu\chi^2(2\lambda\nu\delta^2\psi^{-2\nu-2} + (\nu+1)\delta^2\psi^{-\nu-2})\Lambda \\
&\quad + 2(-2\lambda\nu\delta\psi^{-\nu-1}\chi^2)\Lambda_{x_1} + \chi^2\Lambda_{x_1x_1} \\
&\quad + (x_1+\delta_0)^2(\sum_{i=2}^{n}(\Lambda 2\lambda\nu\chi^2(a_{ii}(2\lambda\nu\psi^{-2\nu-2}\psi_{x_i}^2 + (\nu+1)\psi^{-\nu-2}\psi_{x_i}^2 - \psi^{-\nu-1}\psi_{x_ix_i}) \\
&\quad - (a_{ii})_{x_i}\psi^{-\nu-1}\psi_{x_i}) - 2\lambda\nu\psi^{-\nu-1}\psi_{x_i}\chi^2(2\Lambda_{x_i}a_{ii} + \Lambda(a_{ii})_{x_i}) \\
&\quad + \chi^2(\Lambda_{x_ix_i}a_{ii} + 2\Lambda_{x_i}(a_{ii})_{x_i} + \Lambda(a_{ii})_{x_ix_i})) \\
&\quad - \sum_{j=1}^{m}(\Lambda 2\lambda\nu\chi^2(b_{jj}(2\lambda\nu\psi^{-2\nu-2}\psi_{y_j}^2 + (\nu+1)\psi^{-\nu-2}\psi_{y_j}^2 - \psi^{-\nu-1}\psi_{y_jy_j}) \\
&\quad - (b_{jj})_{y_j}\psi^{-\nu-1}\psi_{y_j}) - 2\lambda\nu\psi^{-\nu-1}\psi_{y_j}\chi^2(2\Lambda_{y_j}b_{jj} + \Lambda(b_{jj})_{y_j}) \\
&\quad + \chi^2(\Lambda_{y_jy_j}b_{jj} + 2\Lambda_{y_j}(b_{jj})_{y_j} + \Lambda(b_{jj})_{y_jy_j}))))
\end{aligned}
$$

elde edilir. Diğer taraftan $\psi_{x_ix_i} = 1$, $\psi_{y_jy_j} = 1$ ve $\psi_{x_iy_j} = 0$, $2 \le i \le n$, $1 \le j \le m$ olduğu dikkate alınarak

$$
\begin{aligned}
\Lambda &= \sum_{i=2}^{n}(a_{ii}\psi_{x_i})_{x_i} - \sum_{j=1}^{m}(b_{jj}\psi_{y_j})_{y_j} \\
&= \sum_{i=2}^{n}((a_{ii})_{x_i}\psi_{x_i} + a_{ii}\psi_{x_ix_i}) - \sum_{j=1}^{m}((b_{jj})_{y_j}\psi_{y_j} + b_{jj}\psi_{y_jy_j}) \\
&= \sum_{i=2}^{n}(a_{ii})_{x_i}\psi_{x_i} + \sum_{i=2}^{n}a_{ii} - \sum_{j=1}^{m}(b_{jj})_{y_j}\psi_{y_j} - \sum_{j=1}^{m}b_{jj},
\end{aligned}
$$

$$
\begin{aligned}
\Lambda_{x_i} &= \sum_{i=2}^{n}((a_{ii})_{x_ix_i}\psi_{x_i} + (a_{ii})_{x_i}\psi_{x_ix_i}) + \sum_{i=2}^{n}(a_{ii})_{x_i} \\
&\quad - \sum_{j=1}^{m}((b_{jj})_{y_jx_i}\psi_{y_j} + (b_{jj})_{y_j}\psi_{y_jx_i}) - \sum_{j=1}^{m}(b_{jj})_{x_i} \\
&= \sum_{i=2}^{n}(a_{ii})_{x_ix_i}\psi_{x_i} + 2\sum_{i=2}^{n}(a_{ii})_{x_i} - \sum_{j=1}^{m}(b_{jj})_{y_jx_i}\psi_{y_j} - \sum_{j=1}^{m}(b_{jj})_{x_i},
\end{aligned}
$$

$$\begin{aligned}
\Lambda_{x_ix_i} &= \sum_{i=2}^{n}((a_{ii})_{x_ix_ix_i}\psi_{x_i} + (a_{ii})_{x_ix_i}\psi_{x_ix_i}) + 2\sum_{i=2}^{n}(a_{ii})_{x_ix_i} \\
&\quad - \sum_{j=1}^{m}((b_{jj})_{y_jx_ix_i}\psi_{y_j} + (b_{jj})_{y_jx_i}\psi_{y_jx_i}) - \sum_{j=1}^{m}(b_{jj})_{x_ix_i} \\
&= \sum_{i=2}^{n}(a_{ii})_{x_ix_ix_i}\psi_{x_i} + 3\sum_{i=2}^{n}(a_{ii})_{x_ix_i} - \sum_{j=1}^{m}(b_{jj})_{y_jx_ix_i}\psi_{y_j} - \sum_{j=1}^{m}(b_{jj})_{x_ix_i},
\end{aligned}$$

$$\begin{aligned}
\Lambda_{y_j} &= \sum_{i=2}^{n}((a_{ii})_{x_iy_j}\psi_{x_i} + (a_{ii})_{x_i}\psi_{x_iy_j}) + \sum_{i=2}^{n}(a_{ii})_{y_j} \\
&\quad - \sum_{j=1}^{m}((b_{jj})_{y_jy_j}\psi_{y_j} + (b_{jj})_{y_j}\psi_{y_jy_j}) - \sum_{j=1}^{m}(b_{jj})_{y_j} \\
&= \sum_{i=2}^{n}(a_{ii})_{x_iy_j}\psi_{x_i} + \sum_{i=2}^{n}(a_{ii})_{y_j} - \sum_{j=1}^{m}(b_{jj})_{y_jy_j}\psi_{y_j} - 2\sum_{j=1}^{m}(b_{jj})_{y_j},
\end{aligned}$$

$$\begin{aligned}
\Lambda_{y_jy_j} &= \sum_{i=2}^{n}((a_{ii})_{x_iy_jy_j}\psi_{x_i} + (a_{ii})_{x_iy_j}\psi_{x_iy_j}) + \sum_{i=2}^{n}(a_{ii})_{y_jy_j} \\
&\quad - \sum_{j=1}^{m}((b_{jj})_{y_jy_jy_j}\psi_{y_j} + (b_{jj})_{y_jy_j}\psi_{y_jy_j}) - 2\sum_{j=1}^{m}(b_{jj})_{y_jy_j} \\
&= \sum_{i=2}^{n}(a_{ii})_{x_iy_jy_j}\psi_{x_i} + \sum_{i=2}^{n}(a_{ii})_{y_jy_j} - \sum_{j=1}^{m}(b_{jj})_{y_jy_jy_j}\psi_{y_j} - 3\sum_{j=1}^{m}(b_{jj})_{y_jy_j}
\end{aligned}$$

bulunur. Aşağıda $|\psi_{x_i}| \leq \sqrt{2\gamma}$, $|\psi_{y_j}| \leq \sqrt{2\gamma}$, $||a_{ii}||_{C^3(\overline{D\times G})} \leq M$ ve $||b_{jj}||_{C^3(\overline{D\times G})} \leq M$, $2 \leq i \leq n$, $1 \leq j \leq m$ eşitsizlikleri kullanılmıştır.

$$\begin{aligned}
|\Lambda| &\leq \sum_{i=2}^{n}|(a_{ii})_{x_i}\psi_{x_i}| + \sum_{i=2}^{n}|a_{ii}| + \sum_{j=1}^{m}|(b_{jj})_{y_j}\psi_{y_j}| + \sum_{j=1}^{m}|b_{jj}| \\
&\leq M\sqrt{2\gamma}(n-1) + M(n-1) + M\sqrt{2\gamma}m + Mm \\
&= M(\sqrt{2\gamma}+1)(n-1) + M(\sqrt{2\gamma}+1)m \\
&= (\sqrt{2\gamma}+1)M(n+m-1),
\end{aligned}$$

$$\begin{aligned}
|\Lambda_{x_i}| &\leq \sum_{i=2}^{n}|(a_{ii})_{x_ix_i}\psi_{x_i}| + 2\sum_{i=2}^{n}|(a_{ii})_{x_i}| + \sum_{j=1}^{m}|(b_{jj})_{y_jx_i}\psi_{y_j}| + \sum_{j=1}^{m}|(b_{jj})_{x_i}| \\
&\leq M\sqrt{2\gamma}(n-1) + 2M(n-1) + M\sqrt{2\gamma}m + Mm \\
&= (\sqrt{2\gamma}+2)M(n-1) + (\sqrt{2\gamma}+1)Mm \\
&= (\sqrt{2\gamma}+1)M(n+m-1) + M(n-1),
\end{aligned}$$

$$
\begin{aligned}
|\Lambda_{x_ix_i}| &\leq \sum_{i=2}^{n}|(a_{ii})_{x_ix_ix_i}\psi_{x_i}| + 3\sum_{i=2}^{n}|(a_{ii})_{x_ix_i}| + \sum_{j=1}^{m}|(b_{jj})_{y_jx_ix_i}\psi_{y_j}| + \sum_{j=1}^{m}|(b_{jj})_{x_ix_i}| \\
&\leq M\sqrt{2\gamma}(n-1) + 3M(n-1) + M\sqrt{2\gamma}m + Mm \\
&= M\sqrt{2\gamma}(n+m-1) + M(3(n-1)+m),
\end{aligned}
$$

$$
\begin{aligned}
|\Lambda_{y_j}| &\leq \sum_{i=2}^{n}|(a_{ii})_{x_iy_j}\psi_{x_i}| + \sum_{i=2}^{n}|(a_{ii})_{y_j}| + \sum_{j=1}^{m}|(b_{jj})_{y_jy_j}\psi_{y_j}| + 2\sum_{j=1}^{m}|(b_{jj})_{y_j}| \\
&\leq M\sqrt{2\gamma}(n-1) + M(n-1) + M\sqrt{2\gamma}m + 2Mm \\
&= M\sqrt{2\gamma}(n+m-1) + M(n+2m-1) \\
&= (\sqrt{2\gamma}+1)M(n+m-1) + Mm,
\end{aligned}
$$

$$
\begin{aligned}
|\Lambda_{y_jy_j}| &\leq \sum_{i=2}^{n}|(a_{ii})_{x_iy_jy_j}\psi_{x_i}| + \sum_{i=2}^{n}|(a_{ii})_{y_jy_j}| + \sum_{j=1}^{m}|(b_{jj})_{y_jy_jy_j}\psi_{y_j}| + 3\sum_{j=1}^{m}|(b_{jj})_{y_jy_j}| \\
&\leq M\sqrt{2\gamma}(n-1) + M(n-1) + M\sqrt{2\gamma}m + 3Mm \\
&= M\sqrt{2\gamma}(n+m-1) + M(n+3m-1)
\end{aligned}
$$

dir. Diğer taraftan

$$
\begin{aligned}
|\Lambda| &\leq \Lambda^0,\ |\Lambda_{x_i}| \leq \Lambda^0,\ |\Lambda_{y_j}| \leq \Lambda^0, \\
|\Lambda_{x_ix_i}| &\leq M\sqrt{2\gamma}(n+m-1) + M(3(n-1)+m), \\
|\Lambda_{y_jy_j}| &\leq M\sqrt{2\gamma}(n+m-1) + M(n+3m-1) \qquad (2.64)
\end{aligned}
$$

bağıntıları kullanılırsa

$$
\begin{aligned}
|A| \leq\ & \lambda\nu\Lambda^0(|4\lambda^2\nu^2\delta^2\chi^2\psi^{-2\nu-2}| + |2\lambda\nu(\nu+1)\delta^2\chi^2\psi^{-\nu-2}| + |4\lambda\nu\delta\psi^{-\nu-1}\chi^2| \\
& +(x_1+\delta_0)^2(\sum_{i=2}^{n}(|4\lambda^2\nu^2\chi^2 a_{ii}\psi^{-2\nu-2}\psi_{x_i}^2| + |2\lambda\nu(\nu+1)\chi^2 a_{ii}\psi^{-\nu-2}\psi_{x_i}^2| \\
& +|2\lambda\nu\chi^2 a_{ii}\psi^{-\nu-1}\psi_{x_ix_i}| + |2\lambda\nu\chi^2(a_{ii})_{x_i}\psi^{-\nu-1}\psi_{x_i}| + |4\lambda\nu\psi^{-\nu-1}\psi_{x_i}\chi^2 a_{ii}| \\
& +|2\lambda\nu\psi^{-\nu-1}\psi_{x_i}\chi^2(a_{ii})_{x_i}| + 2|\chi^2(a_{ii})_{x_i}| + |\chi^2(a_{ii})_{x_ix_i}|)
\end{aligned}
$$

$$
\begin{aligned}
& +\sum_{j=1}^{m}(|4\lambda^2\nu^2\chi^2 b_{jj}\psi^{-2\nu-2}\psi_{y_j}^2| + |2\lambda\nu(\nu+1)\chi^2 b_{jj}\psi^{-\nu-2}\psi_{y_j}^2| + |2\lambda\nu\chi^2 b_{jj}\psi^{-\nu-1}\psi_{y_jy_j}| \\
& +|2\lambda\nu\chi^2(b_{jj})_{y_j}\psi^{-\nu-1}\psi_{y_j}| + |4\lambda\nu\psi^{-\nu-1}\psi_{y_j}\chi^2 b_{jj}| + |2\lambda\nu\psi^{-\nu-1}\psi_{y_j}\chi^2(b_{jj})_{y_j}| \\
& +2|\chi^2(b_{jj})_{y_j}| + |\chi^2(b_{jj})_{y_jy_j}|))) \\
& +\lambda\nu(\chi^2|\Lambda_{x_1x_1}| + (x_1+\delta_0)^2(\sum_{i=2}^{n}|\chi^2 a_{ii}||\Lambda_{x_ix_i}| + \sum_{j=1}^{m}|\chi^2 b_{jj}||\Lambda_{y_jy_j}|))
\end{aligned}
$$

ve diğer bir şekilde

$$
\begin{aligned}
|A| \quad \leq \quad & 4\lambda^3\nu^3\chi^2\psi^{-2\nu-2}\Lambda^0(\delta^2 + (x_1+\delta_0)^2(\sum_{i=2}^{n}|a_{ii}\psi_{x_i}^2| + \sum_{j=1}^{m}|b_{jj}\psi_{y_j}^2|)) \\
& +\lambda^2\nu^2\psi^{-\nu-1}\delta^2\chi^2\Lambda^0(2(\nu+1)\psi^{-1} + \frac{4}{\delta} \\
& +\frac{(x_1+\delta_0)^2}{\delta^2}(\sum_{i=2}^{n}(|2a_{ii}(\nu+1)\psi^{-1}\psi_{x_i}^2| + |2a_{ii}\psi_{x_ix_i}| \\
& +|2(a_{ii})_{x_i}\psi_{x_i}| + |4\psi_{x_i}a_{ii}| + |2\psi_{x_i}(a_{ii})_{x_i}|) \\
& +\sum_{j=1}^{m}(|2(\nu+1)b_{jj}\psi^{-1}\psi_{y_j}^2| + |2b_{jj}\psi_{y_jy_j}| + |2(b_{jj})_{y_j}\psi_{y_j}| \\
& +|4\psi_{y_j}b_{jj}| + |2\psi_{y_j}(b_{jj})_{y_j}|)))
\end{aligned}
$$

$$
\begin{aligned}
& +\lambda\nu\chi^2(\sum_{i=2}^{n}(|2(a_{ii})_{x_i}|\Lambda^0 + |(a_{ii})_{x_ix_i}|\Lambda^0) + \sum_{j=1}^{m}(|2(b_{jj})_{y_j}|\Lambda^0 + |(b_{jj})_{y_jy_j}|\Lambda^0) \\
& +|\Lambda_{x_1x_1}| + (x_1+\delta_0)^2(\sum_{i=2}^{n}|a_{ii}||\Lambda_{x_ix_i}| + \sum_{j=1}^{m}|b_{jj}||\Lambda_{y_jy_j}|))
\end{aligned}
$$

elde edilir. Eğer

$$
\begin{aligned}
\lambda \quad \geq \quad & \lambda_2 = \sum_{i=2}^{n}(|2(a_{ii})_{x_i}|\Lambda^0 + |(a_{ii})_{x_ix_i}|\Lambda^0) + \sum_{j=1}^{m}(|2(b_{jj})_{y_j}|\Lambda^0 + |(b_{jj})_{y_jy_j}|\Lambda^0) \\
& +|\Lambda_{x_1x_1}| + (x_1+\delta_0)^2(\sum_{i=2}^{n}|a_{ii}||\Lambda_{x_ix_i}| + \sum_{j=1}^{m}|b_{jj}||\Lambda_{y_jy_j}|)
\end{aligned}
$$

olarak alınır ve $\nu \geq 1$ olduğu göz önünde bulundurulursa

$$
\begin{aligned}
|A| \quad \leq \quad & 4\lambda^3\nu^3\chi^2\psi^{-2\nu-2}\Lambda^0(\delta^2 + (x_1+\delta_0)^2(\sum_{i=2}^{n}|a_{ii}\psi_{x_i}^2| + \sum_{j=1}^{m}|b_{jj}\psi_{y_j}^2|)) \\
& +\lambda^2\nu^2\psi^{-\nu-1}\delta^2\chi^2\Lambda^0(2(\nu+1)\psi^{-1} + \frac{4}{\delta} \\
& +\frac{(x_1+\delta_0)^2}{\delta^2}(\sum_{i=2}^{n}(|2a_{ii}(\nu+1)\psi^{-1}\psi_{x_i}^2| + |2a_{ii}\psi_{x_ix_i}| \\
& +|2(a_{ii})_{x_i}\psi_{x_i}| + |4\psi_{x_i}a_{ii}| + |2\psi_{x_i}(a_{ii})_{x_i}|) \\
& +\sum_{j=1}^{m}(|2(\nu+1)b_{jj}\psi^{-1}\psi_{y_j}^2| + |2b_{jj}\psi_{y_jy_j}| + |2(b_{jj})_{y_j}\psi_{y_j}| \\
& +|4\psi_{y_j}b_{jj}| + |2\psi_{y_j}(b_{jj})_{y_j}|))) + \lambda^2\nu^2\chi^2
\end{aligned}
$$

$$
\begin{aligned}
&= 4\lambda^3\nu^3\chi^2\psi^{-2\nu-2}\Lambda^0(\delta^2+(x_1+\delta_0)^2(\sum_{i=2}^{n}|a_{ii}\psi_{x_i}^2|+\sum_{j=1}^{m}|b_{jj}\psi_{y_j}^2|)) \\
&+\lambda^2\nu^2\psi^{-\nu-1}\delta^2\chi^2(\Lambda^0(2(\nu+1)\psi^{-1}+\frac{4}{\delta} \\
&+\frac{(x_1+\delta_0)^2}{\delta^2}(\sum_{i=2}^{n}(|2a_{ii}(\nu+1)\psi^{-1}\psi_{x_i}^2|+|2a_{ii}\psi_{x_ix_i}| \\
&+|2(a_{ii})_{x_i}\psi_{x_i}|+|4\psi_{x_i}a_{ii}|+|2\psi_{x_i}(a_{ii})_{x_i}|) \\
&+\sum_{j=1}^{m}(|2(\nu+1)b_{jj}\psi^{-1}\psi_{y_j}^2|+|2b_{jj}\psi_{y_jy_j}|+|2(b_{jj})_{y_j}\psi_{y_j}| \\
&+|4\psi_{y_j}b_{jj}|+|2\psi_{y_j}(b_{jj})_{y_j}|)))+\frac{\psi^{\nu+1}}{\delta^2})
\end{aligned}
$$

$$
\begin{aligned}
&= 4\lambda^3\nu^3\chi^2\psi^{-2\nu-2}\Lambda^0(\delta^2+(x_1+\delta_0)^2(\sum_{i=2}^{n}|a_{ii}\psi_{x_i}^2|+\sum_{j=1}^{m}|b_{jj}\psi_{y_j}^2|)) \\
&+C\lambda^2\nu^2\psi^{-\nu-1}\delta^2\chi^2
\end{aligned}
$$

eşitsizliği elde edilir. Burada

$$
\begin{aligned}
C &= \Lambda^0(2(\nu+1)\psi^{-1}+\frac{4}{\delta}+\frac{(x_1+\delta_0)^2}{\delta^2}(\sum_{i=2}^{n}(|2a_{ii}(\nu+1)\psi^{-1}\psi_{x_i}^2|+|2a_{ii}\psi_{x_ix_i}| \\
&+|2(a_{ii})_{x_i}\psi_{x_i}|+|4\psi_{x_i}a_{ii}|+|2\psi_{x_i}(a_{ii})_{x_i}|)+\sum_{j=1}^{m}(|2(\nu+1)b_{jj}\psi^{-1}\psi_{y_j}^2| \\
&+|2b_{jj}\psi_{y_jy_j}|+|2(b_{jj})_{y_j}\psi_{y_j}|+|4\psi_{y_j}b_{jj}|+|2\psi_{y_j}(b_{jj})_{y_j}|)))+\frac{\psi^{\nu+1}}{\delta^2}
\end{aligned}
$$

dir. O halde mutlak değer tanımından ve $|a_{ii}|\leq M$, $|b_{jj}|\leq M$, $|\psi_{x_i}|^2\leq 2\gamma$, $|\psi_{y_j}|^2\leq 2\gamma$, $2\leq i\leq n$, $1\leq j\leq m$ kabullerinden

$$
\begin{aligned}
A &\geq -4\lambda^3\nu^3\chi^2\psi^{-2\nu-2}\Lambda^0(\delta^2+(x_1+\delta_0)^2(\sum_{i=2}^{n}|a_{ii}\psi_{x_i}^2|+\sum_{j=1}^{m}|b_{jj}\psi_{y_j}^2|)) \\
&-C\lambda^2\nu^2\psi^{-\nu-1}\delta^2\chi^2 \\
&\geq -4\lambda^3\nu^3\chi^2\psi^{-2\nu-2}\Lambda^0(\delta^2+M2\gamma(n-1)+M2\gamma m)-C\lambda^2\nu^2\psi^{-\nu-1}\delta^2\chi^2 \qquad (2.65)
\end{aligned}
$$

yazılabilir. (2.62) ve (2.65) eşitsizlikleri kullanılarak

$$
\begin{aligned}
&2\lambda\nu(x_1+\delta_0)\chi^2\Lambda\varphi(L_0\varphi) \\
\geq\ &-2\lambda\nu\Lambda^0\chi^2(\varphi_{x_1}^2+(x_1+\delta_0)^2M(\sum_{i=2}^{n}\varphi_{x_i}^2+\sum_{j=1}^{m}\varphi_{y_j}^2)) \\
&-4\lambda^3\nu^3\chi^2\psi^{-2\nu-2}(\delta^2+M2\gamma(n+m-1))\Lambda^0\varphi^2 \\
&-C\lambda^2\nu^2\psi^{-\nu-1}\delta^2\chi^2\varphi^2-2\lambda\nu(x_1+\delta_0)^2\chi^2\Lambda\xi\varphi^2+d_4(\varphi)
\end{aligned}
$$

ispat tamamlanır. ■

Lemma 2.1'in ispatı. (2.9) ve (2.55) eşitsizlikleri taraf tarafa toplanırsa,

$$\begin{aligned}
& \psi^{\nu+1}(L_0\varphi)^2\chi^2 + 2\lambda\nu(x_1+\delta_0)\chi^2\Lambda\varphi(L_0\varphi) \\
\geq\ & 2\lambda\nu\gamma^{-3}\delta\varphi_{x_1}^2\chi^2 + \lambda\nu(x_1+\delta_0)\delta\alpha_1\chi^2\sum_{i=2}^{n}\varphi_{x_i}^2 \\
& +\lambda\nu(x_1+\delta_0)\delta\alpha_2\chi^2\sum_{j=1}^{m}\varphi_{y_j}^2 + \lambda^3\nu^4\delta^4\psi^{-2\nu-3}\varphi^2\chi^2 \\
& +2\lambda\nu\xi(x_1+\delta_0)^2(\sum_{i=2}^{n}(a_{ii}\psi_{x_i})_{x_i} - \sum_{j=1}^{m}(b_{jj}\psi_{y_j})_{y_j})\varphi^2\chi^2 + \sum_{j=1}^{3}d_j(\varphi)
\end{aligned}$$

$$\begin{aligned}
& -2\lambda\nu\Lambda^0\chi^2(\varphi_{x_1}^2 + M(x_1+\delta_0)^2(\sum_{i=2}^{n}\varphi_{x_i}^2 + \sum_{j=1}^{m}\varphi_{y_j}^2)) \\
& -4\lambda^3\nu^3\chi^2\psi^{-2\nu-2}(\delta^2 + M2\gamma(n+m-1))\Lambda^0\varphi^2 \\
& -C\lambda^2\nu^2\psi^{-\nu-1}\delta^2\chi^2\varphi^2 - 2\lambda\nu(x_1+\delta_0)^2\chi^2\Lambda\xi\varphi^2 + d_4(\varphi)
\end{aligned}$$

$$\begin{aligned}
=\ & \sum_{i=1}^{4}d_i(\varphi) + 2\lambda\nu\varphi_{x_1}^2\chi^2(\gamma^{-3}\delta - \Lambda^0) + \lambda\nu(x_1+\delta_0)(\delta\alpha_1 - 2\Lambda^0M(x_1+\delta_0))\chi^2\sum_{i=2}^{n}\varphi_{x_i}^2 \\
& +\lambda\nu(x_1+\delta_0)(\delta\alpha_2 - 2\Lambda^0M(x_1+\delta_0))\chi^2\sum_{j=1}^{m}\varphi_{y_j}^2 \\
& +4\lambda^3\nu^3\psi^{-2\nu-3}\varphi^2\chi^2(\frac{\nu\delta^4}{4} - \psi\Lambda^0(\delta^2 + M2\gamma(n+m-1)) - \frac{C}{\lambda\nu}\delta^2\psi^{\nu+2})
\end{aligned}$$

$$\begin{aligned}
>\ & \sum_{i=1}^{4}d_i(\varphi) + 2\lambda\nu\varphi_{x_1}^2\chi^2(\gamma^{-3}\delta - \Lambda^0) + \lambda\nu(x_1+\delta_0)(\delta\alpha_1 - 2\Lambda^0M\gamma)\chi^2\sum_{i=2}^{n}\varphi_{x_i}^2 \\
& +\lambda\nu(x_1+\delta_0)(\delta\alpha_2 - 2\Lambda^0M\gamma)\chi^2\sum_{j=1}^{m}\varphi_{y_j}^2 \\
& +4\lambda^3\nu^3\psi^{-2\nu-3}\varphi^2\chi^2(\frac{\nu\delta^4}{4} - \psi\Lambda^0(\delta^2 + M2\gamma(n+m-1)) - \frac{C}{\lambda\nu}\delta^2\psi^{\nu+2}) \qquad (2.66)
\end{aligned}$$

bulunur. Burada

$\gamma^{-3}\delta - \Lambda^0 \geq 1 \Rightarrow \delta \geq (1+\Lambda^0)\gamma^3,$

$\delta\alpha_1 - 2\Lambda^0M\gamma \geq 1 \Rightarrow \delta \geq (1+2\Lambda^0M\gamma)/\alpha_1,$

$\delta\alpha_2 - 2\Lambda^0M\gamma \geq 1 \Rightarrow \delta \geq (1+2\Lambda^0M\gamma)/\alpha_2$

dir. Diğer taraftan

$$\begin{aligned} 0 &< \psi < 1,\ \lambda \geq \lambda_3 = C,\ \nu \geq 1, \\ \nu &\geq \nu_2 = \frac{4}{\delta^4}(1 + \Lambda^0(\delta^2 + M2\gamma(n+m-1)) + \delta^2) \end{aligned}$$

olduğundan

$$\begin{aligned} & \frac{\nu\delta^4}{4} - \psi\Lambda^0(\delta^2 + M2\gamma(n+m-1)) - \frac{C}{\lambda\nu}\psi^{\nu+2}\delta^2 \\ > & \frac{\nu\delta^4}{4} - \Lambda^0(\delta^2 + M2\gamma(n+m-1)) - \delta^2 \\ \geq & 1 \end{aligned}$$

olur. Burada δ, λ ve ν parametrelerini $\delta > \delta_*$, $\lambda > \lambda_*$, $\nu > \nu_*$,

$$\begin{aligned} \delta_* &= \max\{4, \delta_1, \delta_2, \delta_3, \delta_4, \delta_5\}, \\ \delta &\geq \delta_5 = \max\{(1 + \Lambda^0)\gamma^3, (1 + 2\Lambda^0 M\gamma)/\alpha_1, (1 + 2\Lambda^0 M\gamma)/\alpha_2\}, \\ \lambda_* &= \max\{\lambda_1, \lambda_2, \lambda_3\},\ \nu_* = \max\{\nu_1, \nu_2\} \end{aligned}$$

olacak şekilde seçelim. Bu durumda (2.66) eşitsizliğinden

$$\begin{aligned} & \psi^{\nu+1}(L_0\varphi)^2\chi^2 + 2\lambda\nu(x_1 + \delta_0)\chi^2\Lambda\varphi(L_0\varphi) \\ \geq & 2\lambda\nu\varphi_{x_1}^2\chi^2 + \lambda\nu(x_1 + \delta_0)\chi^2\sum_{i=2}^{n}\varphi_{x_i}^2 + \lambda\nu(x_1 + \delta_0)\chi^2\sum_{j=1}^{m}\varphi_{y_j}^2 \\ & +4\lambda^3\nu^3\psi^{-2\nu-3}\varphi^2\chi^2 + \sum_{i=1}^{4} d_i(\varphi) \end{aligned}$$

elde edilir. Böylece ispat tamamlanır.

BÖLÜM 3

ULTRAHİPERBOLİK SCHRÖDINGER DENKLEMİ İÇİN BİR TERS PROBLEM

Problem 3.1 *(1.1)-(1.2) bağıntılarından*

$$u(x,y,0) = a(x,y) \tag{3.1}$$

ek bilgisi yardımıyla (u,g) fonksiyonlar çiftinin bulunması problemini ele alalım.
Şimdi aşağıdaki kümeyi tanımlayalım:

$$\mathcal{U} = \{u(x,y,t); u \in C^3(Q), |\partial_t^\beta u|, |\partial_t^\beta u_{x_s}|, |\partial_t^\beta u_{x_s x_j}|, |\partial_t^\beta u_{y_i}|, |\partial_t^\beta u_{y_i y_l}| \leq K_u \exp(d_u t^2)\}.$$

Burada $0 \leq \beta \leq 2$, $\partial_t^\beta = \frac{\partial^\beta}{\partial t^\beta}$; $s,j = 2,...,n$; $i,l = 1,...,m$ ve K_u, $d_u > 0$ sabitleri $u(x,y,t)$ fonksiyonuna bağlıdır.

3.1 TERS PROBLEMİN ÇÖZÜMÜNÜN TEKLİĞİNİN ARAŞTIRILMASI

Teorem 3.1 *Kabul edelim ki*

$$\begin{aligned} a_{ii} &\in C^3(\overline{D \times G}),\ b_{jj} \in C^3(\overline{D \times G}), \\ \sum_{i=2}^{n} (a_{ii}(x,y))_{x_1} (\xi^i)^2 &\geq \alpha_1 |'\xi|^2,\ \ -\sum_{j=1}^{m} (b_{jj}(x,y))_{x_1} (\zeta^j)^2 \geq \alpha_2 |\zeta|^2, \end{aligned} \tag{3.2}$$

$$\sum_{i=2}^{n} a_{ii}\psi_{x_i}^2 \geq \alpha_3 \sum_{i=2}^{n} \psi_{x_i}^2,\ \ -\sum_{j=1}^{m} b_{jj}\psi_{y_j}^2 \geq \alpha_4 \sum_{j=1}^{m} \psi_{y_j}^2 \tag{3.2'}$$

olacak şekilde α_1, α_2, α_3, $\alpha_4 > 0$ sabitleri mevcut olsun.
Burada $'\xi = (\xi_2, ..., \xi_n) \in \mathbb{R}^{n-1}$ ve $\zeta = (\zeta_1, \zeta_2, ..., \zeta_m) \in \mathbb{R}^m$ olarak tanımlıdır. Ayrıca

$$f \in C(Q),\ \ |\partial_t^\beta f| \leq K_f \exp(d_f t^2),\ \ f(x,y,0) \neq 0$$

olsun. Problem 3.1 en çok bir (u,g) çözümüne sahiptir öyle ki $u \in \mathcal{U}$, $g \in C(D \times G)$.

İspat. Kabul edelim ki $a(x,y)=0$ ve (u,g) Problem 3.1'in bir çözümü olsun. (2.5) denklemi, (2.6) koşulları ve (2.7) ifadesi kullanılarak

$$L_0\varphi_s = m_s,\ s=1,2; \tag{3.3}$$

$$\varphi_s(0,' x,y,\xi) = (\varphi_s)_{x_1}(0,' x,y,\xi)=0 \tag{3.4}$$

yazılabilir. Burada

$$L_0\varphi_s=(x_1+\delta_0)^{-1}(\varphi_s)_{x_1x_1}+(x_1+\delta_0)(\sum_{i=2}^{n}a_{ii}(\varphi_s)_{x_ix_i}-\sum_{j=1}^{m}b_{jj}(\varphi_s)_{y_jy_j}-2k(\varphi_s)_\xi-\xi\varphi_s)$$

şeklinde olup m_1 ve m_2 ifadeleri için $\widehat{F}=f_1+if_2$, $\widehat{w}=\varphi_1+i\varphi_2$, $\widehat{a_rw_{x_r}}=a_{1r}+ia_{2r}$, $\widehat{a_0w}=a_{10}+ia_{20}$, $\widehat{b_lw_{y_l}}=b_{1l}+ib_{2l}$ tanımlamaları kullanılırsa

$$\begin{aligned}
-\widehat{F}\frac{1}{f_0}\int_{-\infty}^{\infty}\xi\widehat{w}d\xi &= -(f_1+if_2)\frac{1}{f_0}\int_{-\infty}^{\infty}\xi(\varphi_1+i\varphi_2)d\xi\\
&= -(f_1+if_2)\frac{1}{f_0}(\int_{-\infty}^{\infty}\xi\varphi_1d\xi+i\int_{-\infty}^{\infty}\xi\varphi_2d\xi)\\
&= -f_1\frac{1}{f_0}\int_{-\infty}^{\infty}\xi\varphi_1d\xi-if_1\frac{1}{f_0}\int_{-\infty}^{\infty}\xi\varphi_2d\xi\\
&\quad -if_2\frac{1}{f_0}\int_{-\infty}^{\infty}\xi\varphi_1d\xi+f_2\frac{1}{f_0}\int_{-\infty}^{\infty}\xi\varphi_2d\xi,
\end{aligned}$$

$$-(\sum_{r=0}^{n}\widehat{a_rw_{x_r}}+\sum_{l=1}^{m}\widehat{b_lw_{y_l}})=-(\sum_{r=0}^{n}(a_{1r}+ia_{2r})+\sum_{l=1}^{m}(b_{1l}+ib_{2l}))$$

eşitliklerinden

$$\begin{aligned}
&(x_1+\delta_0)(-\widehat{F}\frac{1}{f_0}\int_{-\infty}^{\infty}\xi\widehat{w}d\xi-\sum_{r=0}^{n}\widehat{a_rw_{x_r}}-\sum_{l=1}^{m}\widehat{b_lw_{y_l}})\\
=\ &(x_1+\delta_0)(-f_1\frac{1}{f_0}\int_{-\infty}^{\infty}\xi\varphi_1d\xi-if_1\frac{1}{f_0}\int_{-\infty}^{\infty}\xi\varphi_2d\xi\\
&-if_2\frac{1}{f_0}\int_{-\infty}^{\infty}\xi\varphi_1d\xi+f_2\frac{1}{f_0}\int_{-\infty}^{\infty}\xi\varphi_2d\xi\\
&-\sum_{r=0}^{n}(a_{1r}+ia_{2r})-\sum_{l=1}^{m}(b_{1l}+ib_{2l}))\\
=\ &m_1+m_2
\end{aligned}$$

olduğu görülür. Burada $w_{x_0}=w$ gösterimi kullanılmıştır. Böylece

$$\begin{aligned}
m_1 &= (x_1+\delta_0)(-f_1\frac{1}{f_0}\int_{-\infty}^{\infty}\xi\varphi_1d\xi+f_2\frac{1}{f_0}\int_{-\infty}^{\infty}\xi\varphi_2d\xi-\sum_{r=0}^{n}a_{1r}-\sum_{l=1}^{m}b_{1l}),\\
m_2 &= (x_1+\delta_0)(-f_1\frac{1}{f_0}\int_{-\infty}^{\infty}\xi\varphi_2d\xi-f_2\frac{1}{f_0}\int_{-\infty}^{\infty}\xi\varphi_1d\xi-\sum_{r=0}^{n}a_{2r}-\sum_{l=1}^{m}b_{2l})
\end{aligned}$$

olarak bulunur. Diğer taraftan

$$\begin{aligned}(\int_{-\infty}^{\infty}\xi\varphi_1 d\xi)^2 &= (\int_{-\infty}^{\infty}(1+\xi^2)^{-1/2}(1+\xi^2)^{1/2}\xi\varphi_1 d\xi)^2\\ &\le \int_{-\infty}^{\infty}(1+\xi^2)^{-1}d\xi\int_{-\infty}^{\infty}\xi^2(1+\xi^2)\varphi_1^2 d\xi\\ &\le C_1\int_{-\infty}^{\infty}(1+\xi^2)^2\varphi_1^2 d\xi,\end{aligned}$$

$$(\int_{-\infty}^{\infty}\xi\varphi_2 d\xi)^2 \le C_1\int_{-\infty}^{\infty}(1+\xi^2)^2\varphi_2^2 d\xi,\ C_1=\int_{-\infty}^{\infty}(1+\xi^2)^{-1}d\xi,$$

$$(\sum_{k=1}^{m}a_k)^2 \le m\sum_{k=1}^{m}a_k^2,\ (a+b+c+d)^2\le 4(a^2+b^2+c^2+d^2)$$

eşitsizlikleri kullanılarak

$$\begin{aligned}m_1^2+m_2^2 &= (x_1+\delta_0)^2(-f_1\frac{1}{f_0}\int_{-\infty}^{\infty}\xi\varphi_1 d\xi+f_2\frac{1}{f_0}\int_{-\infty}^{\infty}\xi\varphi_2 d\xi-\sum_{r=0}^{n}a_{1r}-\sum_{l=1}^{m}b_{1l})^2\\ &\quad +(x_1+\delta_0)^2(-f_1\frac{1}{f_0}\int_{-\infty}^{\infty}\xi\varphi_2 d\xi-f_2\frac{1}{f_0}\int_{-\infty}^{\infty}\xi\varphi_1 d\xi-\sum_{r=0}^{n}a_{2r}-\sum_{l=1}^{m}b_{2l})^2\end{aligned}$$

$$\begin{aligned}&\le (x_1+\delta_0)^2(4f_1^2\frac{1}{f_0^2}(\int_{-\infty}^{\infty}\xi\varphi_1 d\xi)^2+4f_2^2\frac{1}{f_0^2}(\int_{-\infty}^{\infty}\xi\varphi_2 d\xi)^2+4(\sum_{r=0}^{n}a_{1r})^2+4(\sum_{l=1}^{m}b_{1l})^2\\ &\quad +4f_1^2\frac{1}{f_0^2}(\int_{-\infty}^{\infty}\xi\varphi_2 d\xi)^2+4f_2^2\frac{1}{f_0^2}(\int_{-\infty}^{\infty}\xi\varphi_1 d\xi)^2+4(\sum_{r=0}^{n}a_{2r})^2+4(\sum_{l=1}^{m}b_{2l})^2)\\ &\le (x_1+\delta_0)^2((4f_1^2\frac{1}{f_0^2}+4f_2^2\frac{1}{f_0^2})(\int_{-\infty}^{\infty}\xi\varphi_1 d\xi)^2+(4f_2^2\frac{1}{f_0^2}+4f_1^2\frac{1}{f_0^2})(\int_{-\infty}^{\infty}\xi\varphi_2 d\xi)^2\\ &\quad +4(n+1)\sum_{r=0}^{n}(a_{1r}^2+a_{2r}^2)+4m\sum_{l=1}^{m}(b_{1l}^2+b_{2l}^2))\end{aligned}$$

$$\begin{aligned}&= (x_1+\delta_0)^2(4\frac{(f_1^2+f_2^2)}{f_0^2}((\int_{-\infty}^{\infty}\xi\varphi_1 d\xi)^2+(\int_{-\infty}^{\infty}\xi\varphi_2 d\xi)^2)\\ &\quad +4(n+1)\sum_{r=0}^{n}(a_{1r}^2+a_{2r}^2)+4m\sum_{l=1}^{m}(b_{1l}^2+b_{2l}^2))\\ &\le (x_1+\delta_0)^2(4\frac{(f_1^2+f_2^2)}{f_0^2}(C_1\int_{-\infty}^{\infty}(1+\xi^2)^2\varphi_1^2 d\xi+C_1\int_{-\infty}^{\infty}(1+\xi^2)^2\varphi_2^2 d\xi)\\ &\quad +4(n+1)\sum_{r=0}^{n}(a_{1r}^2+a_{2r}^2)+4m\sum_{l=1}^{m}(b_{1l}^2+b_{2l}^2))\end{aligned}$$

$$\begin{aligned}&= (x_1+\delta_0)^2(4\frac{C_1}{f_0^2}(f_1^2+f_2^2)\int_{-\infty}^{\infty}(1+\xi^2)^2(\varphi_1^2+\varphi_2^2)d\xi\\ &\quad +4(n+1)\sum_{r=0}^{n}(a_{1r}^2+a_{2r}^2)+4m\sum_{l=1}^{m}(b_{1l}^2+b_{2l}^2)) \qquad (3.5)\end{aligned}$$

elde edilir. Şimdi Lemma 2.1 de bulduğumuz

$$\begin{aligned} & \psi^{\nu+1}(L_0\varphi_s)^2\chi^2 + 2\lambda\nu(x_1+\delta_0)\Lambda\varphi_s(L_0\varphi_s)\chi^2 \\ \leq\ & \psi^{\nu+1}(L_0\varphi_s)^2\chi^2 + (L_0\varphi_s)^2\chi^2 + \lambda^2\nu^2(x_1+\delta_0)^2\Lambda^2\varphi_s^2\chi^2 \end{aligned}$$

eşitsizliğin sol tarafını değerlendirelim. Bu amaçla toplam sembolüde yukarıdaki eşitsizliğe dahil edilerek:

$$\begin{aligned} & \sum_{s=1}^{2}(\psi^{\nu+1}(L_0\varphi_s)^2\chi^2 + 2\lambda\nu(x_1+\delta_0)\Lambda\varphi_s(L_0\varphi_s)\chi^2) \\ \leq\ & \sum_{s=1}^{2}(\psi^{\nu+1}(L_0\varphi_s)^2\chi^2 + (L_0\varphi_s)^2\chi^2 + \lambda^2\nu^2(x_1+\delta_0)^2\Lambda^2\varphi_s^2\chi^2) \\ =\ & \sum_{s=1}^{2}((\psi^{\nu+1}+1)(L_0\varphi_s)^2\chi^2 + \lambda^2\nu^2(x_1+\delta_0)^2\Lambda^2\varphi_s^2\chi^2) \\ =\ & (\psi^{\nu+1}+1)(m_1^2+m_2^2)\chi^2 + \lambda^2\nu^2(x_1+\delta_0)^2\Lambda^2(\varphi_1^2+\varphi_2^2)\chi^2 \\ \leq\ & (\psi^{\nu+1}+1)(x_1+\delta_0)^2(4\frac{C_1}{f_0^2}(f_1^2+f_2^2)\int_{-\infty}^{\infty}(1+\xi^2)^2(\varphi_1^2+\varphi_2^2)d\xi \\ & +4(n+1)\sum_{r=0}^{n}(a_{1r}^2+a_{2r}^2) + 4m\sum_{l=1}^{m}(b_{1l}^2+b_{2l}^2))\chi^2 \\ & +\lambda^2\nu^2(x_1+\delta_0)^2\Lambda^2(\varphi_1^2+\varphi_2^2)\chi^2 \end{aligned} \quad (3.6)$$

yazılır. İkinci olarak Lemma 2.1 deki eşitsizliğin

$$\begin{aligned} & \psi^{\nu+1}(L_0\varphi_s)^2\chi^2 + 2\lambda\nu(x_1+\delta_0)\Lambda\varphi_s(L_0\varphi_s)\chi^2 \\ \geq\ & 2\lambda\nu\chi^2(\varphi_s)_{x_1}^2 + \lambda\nu(x_1+\delta_0)\chi^2\sum_{i=2}^{n}(\varphi_s)_{x_i}^2 + \lambda\nu(x_1+\delta_0)\chi^2\sum_{j=1}^{m}(\varphi_s)_{y_j}^2 \\ & +4\lambda^3\nu^3\psi^{-2\nu-3}\chi^2\varphi_s^2 + \sum_{i=1}^{4}d_i(\varphi_s) \end{aligned}$$

sağ tarafını değerlendirelim:

$$\begin{aligned} & \sum_{s=1}^{2}(2\lambda\nu\chi^2(\varphi_s)_{x_1}^2 + \lambda\nu(x_1+\delta_0)\chi^2\sum_{i=2}^{n}(\varphi_s)_{x_i}^2 + \lambda\nu(x_1+\delta_0)\chi^2\sum_{j=1}^{m}(\varphi_s)_{y_j}^2 \\ & +4\lambda^3\nu^3\psi^{-2\nu-3}\chi^2\varphi_s^2 + \sum_{i=1}^{4}d_i(\varphi_s)) \\ =\ & 2\lambda\nu\chi^2\sum_{s=1}^{2}(\varphi_s)_{x_1}^2 + \lambda\nu(x_1+\delta_0)\chi^2\sum_{s=1}^{2}\sum_{i=2}^{n}(\varphi_s)_{x_i}^2 + \lambda\nu(x_1+\delta_0)\chi^2\sum_{s=1}^{2}\sum_{j=1}^{m}(\varphi_s)_{y_j}^2 \\ & +4\lambda^3\nu^3\psi^{-2\nu-3}\chi^2\sum_{s=1}^{2}\varphi_s^2 + \sum_{s=1}^{2}\sum_{i=1}^{4}d_i(\varphi_s). \end{aligned} \quad (3.7)$$

O halde (3.6) ve (3.7) birleştirilirse

$$
\begin{aligned}
& 2\lambda\nu\chi^2 \sum_{s=1}^{2} (\varphi_s)_{x_1}^2 + \lambda\nu(x_1+\delta_0)\chi^2 \sum_{s=1}^{2}\sum_{i=2}^{n} (\varphi_s)_{x_i}^2 + \lambda\nu(x_1+\delta_0)\chi^2 \sum_{s=1}^{2}\sum_{j=1}^{m} (\varphi_s)_{y_j}^2 \\
& +4\lambda^3\nu^3\psi^{-2\nu-3}\chi^2 \sum_{s=1}^{2} \varphi_s^2 + \sum_{s=1}^{2}\sum_{i=1}^{4} d_i(\varphi_s) \\
\leq \;& (\psi^{\nu+1}+1)(x_1+\delta_0)^2 (4\frac{C_1}{f_0^2}(f_1^2+f_2^2)\int_{-\infty}^{\infty}(1+\xi^2)^2(\varphi_1^2+\varphi_2^2)d\xi \\
& +4(n+1)\sum_{r=0}^{n}(a_{1r}^2+a_{2r}^2) + 4m\sum_{l=1}^{m}(b_{1l}^2+b_{2l}^2))\chi^2 \\
& +\lambda^2\nu^2(x_1+\delta_0)^2\Lambda^2(\varphi_1^2+\varphi_2^2)\chi^2 \qquad (3.8)
\end{aligned}
$$

elde edilir. (3.8) eşitsizliğini $(1+\xi^2)^2$ ile çarpıp $(-\infty,\infty)$ aralığında integralini alalım:

$$
\begin{aligned}
& \sum_{s=1}^{2}\int_{-\infty}^{\infty}(2\lambda\nu(\varphi_s)_{x_1}^2 + \lambda\nu(x_1+\delta_0)\sum_{i=2}^{n}(\varphi_s)_{x_i}^2 + \lambda\nu(x_1+\delta_0)\sum_{j=1}^{m}(\varphi_s)_{y_j}^2 \\
& +4\lambda^3\nu^3\psi^{-2\nu-3}\varphi_s^2)\chi^2(1+\xi^2)^2 d\xi + \sum_{s=1}^{2}\sum_{i=1}^{4}\int_{-\infty}^{\infty} d_i(\varphi_s)(1+\xi^2)^2 d\xi \\
\leq \;& (\psi^{\nu+1}+1)(x_1+\delta_0)^2(4\frac{C_1}{f_0^2}\int_{-\infty}^{\infty}(f_1^2+f_2^2)(1+\xi^2)^2 d\xi \sum_{s=1}^{2}\int_{-\infty}^{\infty}(1+\xi^2)^2\varphi_s^2 d\xi \\
& +4(n+1)\int_{-\infty}^{\infty}\sum_{r=0}^{n}(a_{1r}^2+a_{2r}^2)(1+\xi^2)^2 d\xi + 4m\int_{-\infty}^{\infty}\sum_{l=1}^{m}(b_{1l}^2+b_{2l}^2)(1+\xi^2)^2 d\xi)\chi^2 \\
& +\lambda^2\nu^2(x_1+\delta_0)^2\Lambda^2\chi^2\sum_{s=1}^{2}\int_{-\infty}^{\infty}(1+\xi^2)^2\varphi_s^2 d\xi. \qquad (3.9)
\end{aligned}
$$

Burada

$$
\begin{aligned}
\overline{f_0} &= C_1 \max_{(x,y)\in\overline{D\times G}}\{\frac{1}{f_0^2}\}, \qquad (3.10) \\
C_2 &= \int_{-\infty}^{\infty}(f_1^2+f_2^2)(1+\xi^2)^2 d\xi < \infty \qquad (3.11)
\end{aligned}
$$

biçiminde tanımlıdır. (3.9) eşitsizliğinin sağ tarafındaki terimler için Parseval eşitliği kullanılır ve $|\partial_t^\beta a_r| \leq C_0$ olduğu göz önünde bulundurulursa

$$
\begin{aligned}
\int_{-\infty}^{\infty}(1+\xi^2)^2|\widehat{a_r w_{x_r}}|^2 d\xi &= \int_{-\infty}^{\infty}|\widehat{a_r w_{x_r}}|^2 d\xi + 2\int_{-\infty}^{\infty}\xi^2|\widehat{a_r w_{x_r}}|^2 d\xi + \int_{-\infty}^{\infty}\xi^4|\widehat{a_r w_{x_r}}|^2 d\xi \\
&= \int_{-\infty}^{\infty}|a_r w_{x_r}|^2 dt + 2\int_{-\infty}^{\infty}|\partial_t(a_r w_{x_r})|^2 dt + \int_{-\infty}^{\infty}|\partial_t^2(a_r w_{x_r})|^2 dt \\
&\leq 2\int_{-\infty}^{\infty}\sum_{0\leq\beta\leq 2}|\partial_t^\beta(a_r w_{x_r})|^2 dt \\
&\leq 2C_0'\int_{-\infty}^{\infty}\sum_{0\leq\beta\leq 2}|\partial_t^\beta w_{x_r}|^2 dt
\end{aligned}
$$

$$
\begin{aligned}
&= 2C_0'(\int_{-\infty}^{\infty} |w_{x_r}|^2 dt + \int_{-\infty}^{\infty} |\partial_t w_{x_r}|^2 dt + \int_{-\infty}^{\infty} |\partial_t^2 w_{x_r}|^2 dt) \\
&= 2C_0'(\int_{-\infty}^{\infty} |\widehat{w_{x_r}}|^2 dt + \int_{-\infty}^{\infty} |\widehat{\partial_t w_{x_r}}|^2 dt + \int_{-\infty}^{\infty} |\widehat{\partial_t^2 w_{x_r}}|^2 dt) \\
&= 2C_0'(\int_{-\infty}^{\infty} |\widehat{w_{x_r}}|^2 d\xi + \int_{-\infty}^{\infty} |i\xi\widehat{w_{x_r}}|^2 d\xi + \int_{-\infty}^{\infty} |(i\xi)^2\widehat{w_{x_r}}|^2 d\xi) \\
&= 2C_0' \int_{-\infty}^{\infty} (1+\xi^2+\xi^4)|\widehat{w_{x_r}}|^2 d\xi \\
&\leq 2C_0' \int_{-\infty}^{\infty} (1+\xi^2)^2|\widehat{w_{x_r}}|^2 d\xi \\
&= 2C_0' \int_{-\infty}^{\infty} (1+\xi^2)^2((\varphi_1)_{x_r}^2 + (\varphi_2)_{x_r}^2) d\xi
\end{aligned}
$$

elde edilir. Burada C_0', C_0'a bağlı bir sabittir. Ayrıca $|\widehat{a_r w_{x_r}}|^2 = a_{1r}^2 + a_{2r}^2$ olduğundan

$$
\int_{-\infty}^{\infty} (a_{1r}^2 + a_{2r}^2)(1+\xi^2)^2 d\xi \leq 2C_0' \int_{-\infty}^{\infty} (1+\xi^2)^2((\varphi_1)_{x_r}^2 + (\varphi_2)_{x_r}^2) d\xi
$$

yazılabilir. Burada $r = 0, ..., n$ için toplam alınırsa

$$
\begin{aligned}
&\int_{-\infty}^{\infty} \sum_{r=0}^{n} (a_{1r}^2 + a_{2r}^2)(1+\xi^2)^2 d\xi \\
\leq\ & 2C_0' \int_{-\infty}^{\infty} (\sum_{r=1}^{n}((\varphi_1)_{x_r}^2 + (\varphi_2)_{x_r}^2) + \varphi_1^2 + \varphi_2^2)(1+\xi^2)^2 d\xi \qquad (3.12)
\end{aligned}
$$

elde edilir. Benzer şekilde $|\partial_t^\beta b_l| \leq C_0$ olduğundan

$$
\int_{-\infty}^{\infty} (1+\xi^2)^2 |\widehat{b_l w_{y_l}}|^2 d\xi \leq 2C_0' \int_{-\infty}^{\infty} (1+\xi^2)^2((\varphi_1)_{y_l}^2 + (\varphi_2)_{y_l}^2) d\xi
$$

yazılabilir. Burada $|\widehat{b_l w_{y_l}}|^2 = b_{1l}^2 + b_{2l}^2$ yerine yazılırsa

$$
\int_{-\infty}^{\infty} (b_{1l}^2 + b_{2l}^2)(1+\xi^2)^2 d\xi \leq 2C_0' \int_{-\infty}^{\infty} (1+\xi^2)^2((\varphi_1)_{y_l}^2 + (\varphi_2)_{y_l}^2) d\xi
$$

ve $l = 1, ..., m$ için toplam alınırsa

$$
\int_{-\infty}^{\infty} \sum_{l=1}^{m} (b_{1l}^2 + b_{2l}^2)(1+\xi^2)^2 d\xi \leq 2C_0' \int_{-\infty}^{\infty} \sum_{l=1}^{m} ((\varphi_1)_{y_l}^2 + (\varphi_2)_{y_l}^2)(1+\xi^2)^2 d\xi \qquad (3.13)
$$

bulunur. O halde (3.10)-(3.11) eşitlikleri ve (3.12)-(3.13) eşitsizlikleri (3.9) eşitsizliğinde yerine yazılırsa

$$
\begin{aligned}
&\sum_{s=1}^{2} \int_{-\infty}^{\infty} (2\lambda\nu(\varphi_s)_{x_1}^2 + \lambda\nu(x_1+\delta_0) \sum_{i=2}^{n} (\varphi_s)_{x_i}^2 + \lambda\nu(x_1+\delta_0) \sum_{j=1}^{m} (\varphi_s)_{y_j}^2 \\
&+4\lambda^3\nu^3\psi^{-2\nu-3}\varphi_s^2)\chi^2(1+\xi^2)^2 d\xi + \sum_{s=1}^{2}\sum_{i=1}^{4} \int_{-\infty}^{\infty} d_i(\varphi_s)(1+\xi^2)^2 d\xi
\end{aligned}
$$

$$
\begin{aligned}
\leq\ & (\psi^{\nu+1}+1)(x_1+\delta_0)^2(4\overline{f_0}C_2\sum_{s=1}^{2}\int_{-\infty}^{\infty}(1+\xi^2)^2\varphi_s^2 d\xi \\
& +8(n+1)C_0'\int_{-\infty}^{\infty}(\sum_{r=1}^{n}((\varphi_1)_{x_r}^2+(\varphi_2)_{x_r}^2)+\varphi_1^2+\varphi_2^2)(1+\xi^2)^2 d\xi \\
& +8mC_0'\int_{-\infty}^{\infty}\sum_{l=1}^{m}((\varphi_1)_{y_l}^2+(\varphi_2)_{y_l}^2)(1+\xi^2)^2 d\xi)\chi^2 \\
& +\lambda^2\nu^2(x_1+\delta_0)^2\Lambda^2\chi^2\sum_{s=1}^{2}\int_{-\infty}^{\infty}(1+\xi^2)^2\varphi_s^2 d\xi
\end{aligned}
$$

$$
\begin{aligned}
=\ & (\psi^{\nu+1}+1)(x_1+\delta_0)^2(4\overline{f_0}C_2\sum_{s=1}^{2}\int_{-\infty}^{\infty}(1+\xi^2)^2\varphi_s^2 d\xi \\
& +8(n+1)C_0'\sum_{s=1}^{2}\int_{-\infty}^{\infty}(|\nabla_x\varphi_s|^2+\varphi_s^2)(1+\xi^2)^2 d\xi \\
& +8mC_0'\sum_{s=1}^{2}\int_{-\infty}^{\infty}|\nabla_y\varphi_s|^2(1+\xi^2)^2 d\xi)\chi^2 \\
& +\lambda^2\nu^2(x_1+\delta_0)^2\Lambda^2\chi^2\sum_{s=1}^{2}\int_{-\infty}^{\infty}(1+\xi^2)^2\varphi_s^2 d\xi \qquad (3.14)
\end{aligned}
$$

elde edilir. Böylece (3.14) eşitsizliğinden

$$
\begin{aligned}
& E_5\chi^2\sum_{s=1}^{2}\int_{-\infty}^{\infty}(\varphi_s)_{x_1}^2(1+\xi^2)^2 d\xi+E_6\chi^2\sum_{s=1}^{2}\int_{-\infty}^{\infty}\sum_{i=2}^{n}(\varphi_s)_{x_i}^2(1+\xi^2)^2 d\xi \\
& +E_7\chi^2\sum_{s=1}^{2}\int_{-\infty}^{\infty}\sum_{j=1}^{m}(\varphi_s)_{y_j}^2(1+\xi^2)^2 d\xi+E_8\chi^2\sum_{s=1}^{2}\int_{-\infty}^{\infty}\varphi_s^2(1+\xi^2)^2 d\xi \\
\leq\ & -\sum_{s=1}^{2}\sum_{i=1}^{4}\int_{-\infty}^{\infty}d_i(\varphi_s)(1+\xi^2)^2 d\xi \qquad (3.15)
\end{aligned}
$$

olduğu görülür. Burada

$$
\begin{aligned}
E_5 &= 2\lambda\nu-8(n+1)C_0'(\psi^{\nu+1}+1)(x_1+\delta_0)^2, \\
E_6 &= \lambda\nu(x_1+\delta_0)-8(n+1)C_0'(\psi^{\nu+1}+1)(x_1+\delta_0)^2, \\
E_7 &= \lambda\nu(x_1+\delta_0)-8mC_0'(\psi^{\nu+1}+1)(x_1+\delta_0)^2, \\
E_8 &= 4\lambda^3\nu^3\psi^{-2\nu-3}-4(\psi^{\nu+1}+1)(x_1+\delta_0)^2\overline{f_0}C_2 \\
& \quad -8(n+1)C_0'(\psi^{\nu+1}+1)(x_1+\delta_0)^2-\lambda^2\nu^2(x_1+\delta_0)^2\Lambda^2
\end{aligned}
$$

olarak tanımlıdır.

Eğer $\lambda > 1$, $\nu \geq \nu_3 = \frac{1}{2}+8(n+1)C_0'$, $0 < \psi < 1$ ve $(x_1+\delta_0) < 1$ eşitsizlikleri dikkate alınırsa

$$\begin{aligned}
E_5 &> 2\lambda\nu - 16(n+1)C_0' \\
&\geq 2\lambda(\frac{1}{2} + 8(n+1)C_0') - 16(n+1)C_0' \\
&= \lambda + 16\lambda(n+1)C_0' - 16(n+1)C_0' \\
&> \lambda \qquad (3.16)
\end{aligned}$$

olur.

İkinci olarak $\lambda > 1$, $\nu \geq \nu_4 = 1 + 16(n+1)C_0'$, $0 < \psi < 1$, $(x_1 + \delta_0) < 1$ eşitsizlikleri ve $(x_1 + \delta_0) \neq 0$ olduğu göz önüne alınarak

$$\begin{aligned}
E_6 &> (x_1+\delta_0)^2(\lambda\nu\frac{1}{(x_1+\delta_0)} - 16(n+1)C_0') \\
&> (x_1+\delta_0)^2(\lambda\nu - 16(n+1)C_0') \\
&\geq (x_1+\delta_0)^2(\lambda(1+16(n+1)C_0') - 16(n+1)C_0') \\
&= (x_1+\delta_0)^2(\lambda + 16\lambda(n+1)C_0' - 16(n+1)C_0') \\
&> (x_1+\delta_0)^2\lambda \qquad (3.17)
\end{aligned}$$

bulunur.

Benzer şekilde $\lambda > 1$, $\nu \geq \nu_5 = 1 + 16mC_0'$, $0 < \psi < 1$, $(x_1 + \delta_0) < 1$ seçilerek ve $(x_1 + \delta_0) \neq 0$ olduğu dikkate alınarak

$$\begin{aligned}
E_7 &> (x_1+\delta_0)^2(\lambda\nu\frac{1}{(x_1+\delta_0)} - 16mC_0') \\
&> (x_1+\delta_0)^2(\lambda\nu - 16mC_0') \\
&\geq (x_1+\delta_0)^2(\lambda(1+16mC_0') - 16mC_0') \\
&= (x_1+\delta_0)^2(\lambda + 16\lambda mC_0' - 16mC_0') \\
&> (x_1+\delta_0)^2\lambda \qquad (3.18)
\end{aligned}$$

elde edilir.

Son olarak $\lambda \geq \max\{\lambda_4, \lambda_5\}$, $\lambda_4 = (\Lambda^0)^2$, $\lambda_5 = 8\overline{f_0}C_2 + 16(n+1)C_0'$, $0 < \psi < 1$, $(x_1 + \delta_0) < 1$, $|\Lambda| \leq \Lambda^0$ ve $\nu \geq 1$ alınırsa

$$E_8 > 4\lambda^3\nu^3\psi^{-2\nu-3} - 8\overline{f_0}C_2 - 16(n+1)C_0' - \lambda^2\nu^2(\Lambda^0)^2$$

$$\begin{aligned}
\lambda^3\nu^3\psi^{-2\nu-3} - \lambda^2\nu^2(\Lambda^0)^2 &> \lambda^3\nu^3 - \lambda^2\nu^2\lambda_4 \\
&\geq \lambda^3\nu^3 - \lambda^3\nu^3 = 0,
\end{aligned}$$

$$
\begin{aligned}
\lambda^3\nu^3\psi^{-2\nu-3} - 8\overline{f_0}C_2 - 16(n+1)C_0' &> \lambda^3\nu^3 - \lambda_5 \\
&\geq \lambda^3\nu^3 - \lambda \\
&\geq \lambda^3\nu^3 - \lambda^3\nu^3 = 0,
\end{aligned}
$$

$$
E_8 > 2\lambda^3\nu^3\psi^{-2\nu-3} \tag{3.19}
$$

bulunur. Yukarıdaki (3.16)-(3.19) eşitsizlikleri (3.15) eşitsizliğinde yerine yazılırsa ve $\lambda > 1$, $\nu \geq 1$ olduğu dikkate alınırsa

$$
\begin{aligned}
&\sum_{s=1}^{2}\int_{-\infty}^{\infty}(\lambda(\varphi_s)_{x_1}^2 + \lambda(x_1+\delta_0)^2\sum_{i=2}^{n}(\varphi_s)_{x_i}^2 + \lambda(x_1+\delta_0)^2\sum_{j=1}^{m}(\varphi_s)_{y_j}^2 \\
&+2\lambda^3\nu^3\psi^{-2\nu-3}\varphi_s^2)\chi^2(1+\xi^2)^2 d\xi \leq -\sum_{s=1}^{2}\sum_{i=1}^{4}\int_{-\infty}^{\infty} d_i(\varphi_s)(1+\xi^2)^2 d\xi,
\end{aligned}
$$

$$
\sum_{s=1}^{2}\int_{-\infty}^{\infty} 2\lambda^3\nu^3\psi^{-2\nu-3}\varphi_s^2\chi^2(1+\xi^2)^2 d\xi \leq -\sum_{s=1}^{2}\sum_{i=1}^{4}\int_{-\infty}^{\infty} d_i(\varphi_s)(1+\xi^2)^2 d\xi,
$$

$$
\sum_{s=1}^{2}\int_{-\infty}^{\infty} 2\psi^{-2\nu-3}\varphi_s^2\chi^2(1+\xi^2)^2 d\xi \leq -\frac{1}{\lambda^3\nu^3}\sum_{s=1}^{2}\sum_{i=1}^{4}\int_{-\infty}^{\infty} d_i(\varphi_s)(1+\xi^2)^2 d\xi
$$

elde edilir. Son eşitsizlikte $2(1+\xi^2)^2 > 1$, $\chi^2 > 1$ alınır ve $0 < \psi < 1$ olduğu göz önünde bulundurulursa

$$
\sum_{s=1}^{2}\int_{-\infty}^{\infty} \varphi_s^2 d\xi \leq -\frac{1}{\lambda^3\nu^3}\sum_{s=1}^{2}\sum_{i=1}^{4}\int_{-\infty}^{\infty} d_i(\varphi_s)(1+\xi^2)^2 d\xi
$$

olur. Eğer $\lambda \to \infty$ için limite geçilirse $\varphi_1 = \varphi_2 = 0$ olur. Böylece $g = 0$ bulunur ve ispat tamamlanır. ■

BÖLÜM 4

SONUÇ

Bu tezde, ultrahiperbolik Schrödinger denklemi için bir ters problem ele alınmış ve başlangıç anında verilen bir ek bilgi yardımıyla çözümün tekliği araştırılmıştır. Burada kullanılan temel araç Lavrenti'ev vd. (1986) da verilmiş olan Carleman tipi bir eşitsizliktir. Bu yöntem, Klibanov ve Timonov (2004) ve Amirov ve Yamamoto (2008) tarafından parabolik, hiperbolik ve Schrödinger denklemlerine uygulanmıştır. Ayrıca tezde ele alınan problem çalışılırken Fourier analizinin kavram ve araçlarından faydalanılmıştır. Ultrahiperbolik Schrödinger denklemleri ek boyutlar içerdiğinden başta Sicim teorisi olmak üzere modern fizik kuramları açısından önem taşımaktadır.

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