

THE EFFECTS OF PHYSICAL MANIPULATIVE WITH OR WITHOUT  
SELF-METACOGNITIVE QUESTIONING ON SIXTH GRADE STUDENTS'  
KNOWLEDGE ACQUISITION IN POLYGONS

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DECEMBER 2007

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## **ABSTRACT**

### **THE EFFECTS OF PHYSICAL MANIPULATIVE WITH OR WITHOUT SELF-METACOGNITIVE QUESTIONING ON SIXTH GRADE STUDENTS' KNOWLEDGE ACQUISITION IN POLYGONS**

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This study compared the effect of the use of physical manipulative with self-metacognitive questioning versus manipulative without self-metacognitive questioning on the knowledge acquisition in polygons. Participants were 220 sixth grade students. A pretest, treatment and posttest two-group design was used. There were two treatment groups: manipulative with self-metacognitive questioning (MAN+META) and manipulative without self-metacognitive questioning (MAN). Three distinct knowledge tests were designed by the researcher: Declarative, conditional and procedural. Declarative knowledge test consisted of 18 multiple-choice questions. The conditional and procedural knowledge tests consisted of six and ten open-ended questions respectively. Mixed design analysis of variance results revealed that there is a significant effect for time but no group-by-time interaction effect suggesting that both groups responded equally well to treatment in the amount of change in their scores on the two outcome measures: pretests and posttests. A follow up analysis (paired t-test) was conducted to evaluate the impact of time on students' pretest and posttest scores. The large effect size indicated that there was a statistically significant increase in scores of all three tests.

Keywords: Manipulative, declarative knowledge, conditional knowledge, procedural knowledge, polygon, elementary school mathematics.

## ÖZ

### BİLİŞÜSTÜ YETİ SORULARI İÇEREN VEYA İÇERMEYEN SOMUT MATERYEL KULLANIMININ 6. SINIF ÖĞRENCİLERİNİN ÇOKGEN BİLGİLERİNE ETKİSİ

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Bu çalışmanın amacı, geometride bilişüstü yeti ve somut materyal kullanımının veya sadece somut materyal kullanımının 6.sınıf öğrencilerinin çokgen bilgilerine etkisini araştırmaktır. Bu çalışmaya 220 altıncı sınıf öğrencisi katıldı. Çalışma ön test, öğretim metodu ve son test içeren deneysel bir çalışmadır. Bu çalışmada iki farklı öğretim metodu kullanılmıştır; bunlar sadece somut materyalle öğretim ve somut materyal ve bilişüstü yeti soruları kullanılan öğretimdir. Bu çalışmada ölçüm araçları olarak İfade Bilgi Testi, Koşullu Bilgi Testi ve İşlemsel Bilgi Testleri ön test ve son test olarak kullanıldı ve bu testler araştırmacı tarafından geliştirilmiştir. İfade Bilgiyi ölçen test, 18 çoktan seçmeli test sorusundan, işlemsel bilgiyi ölçen test, 6 tane açık uçlu sorudan ve işlemsel bilgiyi ölçen test 10 tane açık uçlu sorudan oluşmaktadır. İstatistiksel analiz sonuçlarına göre, iki gurubun da ön test ve son test değerlendirmelerinin eşit miktarda değişim göstermiş olması, zamana bağlı anlamlı bir fark bulunduğunu, ancak iki grup arasında fark bulunmadığını ortaya koymuştur. Zamanın öğrencilerin ön test ve son test puanlarına etkisini değerlendirmek için ikili t-test uygulanmıştır. Geniş etki büyüklüğü üç testin puanlarında istatistiksel olarak anlamlı bir artış olduğunu göstermiştir.

Anahtar Kelimeler: Somut materyal, ifadesel bilgi, koşullu bilgi, işlemsel bilgi, ilköğretim matematik, çokgenler.



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## LIST OF ABBREVIATIONS

### ABBREVIATIONS

PreDecKT: Students' Declarative Knowledge Pretest Scores

PreConKT: Students' Conditional Knowledge Pretest Scores

PreProKT: Students' Procedural Knowledge Pretest Scores

PosDecKT: Students' Declarative Knowledge Posttest Scores

PosConKT: Students' Conditional Knowledge Posttest Scores

PosProKT: Students' Declarative Knowledge Posttest Scores

MAN: Students who exposed to manipulative instruction

MAN+META: Students who exposed to manipulative and self-metacognitive questioning instruction.

MANOVA: Multivariate Analysis of Variance

ANOVA: Analysis of Variance

df: Degree of Freedom

N: Sample Size

$\alpha$ : Significance Level

MEB: Turkish Ministry of National Education

TTKB: Board of Education

**IMPROVE:** **I**ntroducing the new concepts, **M**etacognitive questioning, **P**racticing, **R**eviewing and reducing difficulties, **O**btaining mastery, **V**erification and **E**nrichment.



## **CHAPTER I**

### **INTRODUCTION**

In American Heritage Dictionary of the English Language, 4th Edition, manipulative are “any of various objects designed to be moved or arranged by hands as a means of developing motor skills or understanding abstractions, especially in mathematics” (as cited in Branch, 2006). The different versions of manipulative definitions were stated by researchers and educators (Branch, 2006; Denman, 1984; Denu, 1992; Moch, 2001; Young, 1983). The common aspect of all these definitions is the active involvement of children in the learning process by touching and removing the objects. These mostly defined physical (concrete) manipulative whereas with the advances in technology a new kind, the virtual manipulative, was introduced. Reimer and Moyer (2005) defined the virtual manipulative as “essentially replicas of physical manipulative placed on the World Wide Web in the computer applets with additional advantageous features” (p.159).

Teachers and educators mostly prefer using manipulative in teaching mathematics at the elementary level, particularly fractions, place value and decimal numbers (e.g. Krech, 2000; Wearne & Hiebert, 1988). Manipulative, however, could be more effective on geometry than arithmetic due to the nature of the geometry. The geometric tasks often require visualization, spatial orientation or concrete whereas arithmetic tasks usually deal with numbers that are abstract for the students (Martin, Lukong, & Reavas, 2007; Soylu, 2005).

The studies on using manipulative in geometry can be grouped into three: (1) physical manipulative, (2) virtual manipulative, (3) physical versus virtual. Most of these studies, however, assessed the academic achievement by using procedural knowledge and/or conceptual knowledge (mostly declarative knowledge and rarely

conditional knowledge). This reveals that recent research has unanswered the effects of manipulative in teaching geometry on three different types of knowledge as defined by cognitive psychologists. Cognitive psychologists (e.g. Smith & Ragan, 1993) defined three distinct knowledge types: Declarative, conditional and procedural. Declarative knowledge is “knowing that” something is the case. It is often described as the "what" type of information. Procedural knowledge is "how" type of information that tells us the rules to follow to accomplish a task. Conditional knowledge relates to contexts and circumstances of using specific procedures, addressing "when," "where" and "why" information. Conditional knowledge consists of if-then or condition-action statements (Smith & Ragan, 1993).

Researchers mostly agree that using manipulative can be an effective way to teach mathematics, but the problem is “In which conditions manipulative should be implemented to be effective?” Just using manipulative is not a guaranteed success (Baroody, 1989; NCTM, 2000). In other words, manipulative should be used with other teaching methods to provide benefits to the students (Heddens, 1997; Suydam & Higgins, 1976).

Self-metacognitive questioning as suggested by Maverech and Kramarski (1997) is a kind of strategy for helping learners to reflect on their problem solving processing. When students used self-metacognitive questioning, they could focus on the important parts of the problems, analyze the problems and they could gain the ability to relate new knowledge to prior knowledge (Palinscar & Brown, 1984). Self-metacognitive questioning has been used in mathematics since 1970’s.

Based on the findings of previous studies summarized above, the aim of this study is to investigate the effect of manipulative with or without metacognitive questioning on students’ declarative, conditional and procedural knowledge. Due to available accommodation of the school where the study was carried out, physical manipulative were used. Although several studies have been conducted to investigate the use of manipulative in teaching geometry, the use of physical manipulative

combined with self metacognitive questioning and assessing the performance using three distinct knowledge types, declarative, conditional and procedural, is novel.

## **1.1 THE RESEARCH QUESTIONS**

The study sought to address the following research questions

1. Do the students' declarative, conditional and procedural knowledge on polygons improve in both environments: manipulative without self-metacognitive (MAN) instruction and manipulative with self-metacognitive (MAN+META) instruction?

2. What are the students' views related to the effects of use of physical manipulative with or without self-metacognitive questioning on students' acquisition of declarative knowledge, conditional knowledge and procedural knowledge on geometry including polygons?

## **1.2 HYPOTHESIS**

In order to answer the first research question the following hypothesis was used:

There will be no significant mean difference between the pretest and the posttest scores of sixth grade students' on declarative, conditional, and procedural knowledge in both two teaching environments: manipulative without self-metacognitive questioning and manipulative with self-metacognitive questioning.

## **1.3 DEFINITION OF IMPORTANT TERMS**

Physical manipulative: Physical manipulative is multisensory tools that help students learn more by experiencing hands-on situations: building and creating, taking apart, combining shapes, sorting and classifying.

Declarative Knowledge: Declarative knowledge refers to “knowing that” something is the case (Smith & Ragan, 1993). It involves about the facts, hypothesis, and generalizations. The key words for declarative knowledge are “explain”, “describe”, “summarize” and “list.

Conditional Knowledge: Conditional knowledge refers to knowing when and why to use declarative and procedural knowledge (Garner, 1990). Conditional knowledge involves rule learning in the form of ‘if-then’ or ‘action-condition’ statements (Smith & Ragan, 1993).

Procedural Knowledge: “Knowledge needed to put what the students declaratively into practice” (Hall, 1998). Procedural knowledge involves “how” type of information that tells us the rules to follow to accomplish a task and refers to knowledge about doing things (Schraw, 1998; Smith & Ragan, 1993).

Self-metacognitive Questioning: It is a kind of self-metacognitive training containing four type questions: comprehension, connection, strategic and reflection. These questions derived from the literature (Maverech & Kramarski, 1997).

#### **1.4 THE SIGNIFICANCE OF THE STUDY**

Elementary mathematics curriculum guides for grades 1-8 recommend the use of physical manipulative in all elementary classrooms (MEB, 2004). However, there is not much evidence on not only the effectiveness of use of physical manipulative in teaching geometry but also the possible effects of use of physical manipulative on students’ geometry knowledge. A few studies were conducted in this area in Turkey (e.g. Bayram, 2004), but these studies reported the effectiveness of physical manipulative by comparing with the traditional methods. They did not take into account the combined effect of physical manipulative used together with other teaching methods, as suggested by the literature (e.g. Heddens, 1997; Suydam & Higgs, 1976). This study will examine the results of implementing physical manipulative together with self-metacognitive questioning as a teaching method.

Besides, the worldwide studies on the use of physical manipulative reported the students' academic achievement generally based on procedural and declarative knowledge (e.g. Cramer, Post & delMas, 2002; Fuson & Briars, 1990) and rarely on conditional knowledge (e.g. Garrity, 1998), but not on three types. This study reports students' geometry knowledge as declarative, conditional and procedural. Although several studies have been conducted to investigate the use of manipulative in teaching geometry, the use of physical manipulative combined with self metacognitive questioning and assessing the knowledge acquisition using three distinct knowledge types, declarative, conditional and procedural, is novel.

## **CHAPTER II**

### **REVIEW OF RELATED LITERATURE**

This chapter presents a review of literature relevant to research with regard to the use of physical manipulative in learning and teaching mathematics.

#### **2.1. THEORETICAL BACKGROUND**

Comenius who is theoretical educator initiated the studies about using physical manipulative (Szendre, 1996). His famous book, the *Orbis Pictus* has become an important device in schools. Comenius' principle was that students should learn to use the reality of the senses and just not words. He suggested using the tools of real life or at least their pictures in the classroom (Szendre, 1996). Comenius proposed a curriculum for elementary education that is used in the primary grades today. He emphasized the fact that learning comes through the senses. Whenever possible the teacher must be concrete, permit the child to observe for himself/herself, and arrange for the child to have direct experience in learning by doing.

The father of the use of concrete materials (physical manipulative) is Pestalozzi. He asserted that the observation and senses are the first steps in any learning process. He invented tables to teach arithmetic.

Pestalozzi's theory of education is based on the importance of a pedagogical method that corresponds to the natural order of individual development and of concrete experiences. To Pestalozzi the individuality of each child is paramount; it is something that has to be cultivated actively through education. He opposed to the prevailing system of memorization

learning and strict discipline and sought to replace it with a system based on love and an understanding of the child's world. His belief that education should be based on concrete experience led him to pioneer in the use of tactile objects, such as plants and mineral specimens, in the teaching of natural science to youngsters (Columbia Encyclopedia, 2001-07, Pestalozzi, Johann Heinrich section).

Instead of dealing with words, he argued, children should learn through activity and through things. Students should be free to pursue their own interests and draw their own conclusions. Children should not be given ready-made answers instead should find answers themselves. To do this their own powers of seeing, judging and reasoning should be cultivated; their self-activity should be encouraged (Szendre, 1996).

However, the major theoretical base for using manipulative in mathematics teaching comes from the studies of Piaget, Brunner, and Dienes. Each of them represents the cognitive view of learning (Post, 1981)

Piaget defined the four stages of intellectual development: sensory motor stage, birth-to-two years; preoperational stage, two years-to-seven; concrete operational stage, seven-to-eleven years; and formal operation stage, eleven-to-up years. Each stage has its own characteristic involving major tasks to be accomplished. In the sensory motor stage, the mental structures are mainly concerned with mastery by concrete objects. In the preoperational stage, the mastery of symbols takes place. In the concrete operation stage, children learn mastery of classes, relations and numbers. The last stage deals with the mastery of thoughts (Evans, 1973).

Based on Piaget's studies and characteristics of preoperational and concrete operational stages, educators suggested using physical manipulative for teaching mathematics in these two stages. It is concluded that children especially in the elementary level, learn best using concrete objects. The concrete operational level is important mathematically, because of the nature of the operations in mathematics.

These operations include classifications and ordering, which are gained in the concrete operational stage (Copeland, 1970). As a conclusion, gaining such kind of abilities is possible by using physical manipulative.

Piaget also defined how physical and logical mathematical knowledge is gained by abstractions. He believed that human knowledge is active, rather than passive copy. “Knowing an object doesn’t mean copying. It means acting on it.” (Corry, 1996) Physical knowledge, the knowledge based on the experience in general, is concrete. On the other hand, logical mathematical knowledge is gained from the “action” on the objects, not from the object acted on. For example, a child under seven was given a set of pebbles and asked to place them in a row. He is asked to count them in one direction and then in other. He then tries another arrangement. He discovers that the sum always the same independent from the ordering. This is the commutative property. He figures out the property from the action on the pebbles not from the pebbles themselves (Piaget, 1968). Therefore, finding the mathematical properties by using physical manipulative is possible by actively involving cognitively and this marks the mathematical deduction.

Piaget divided abstraction, which is very important issue in mathematics education into two parts: Simple abstraction and reflective abstraction. Simple abstraction involves individual actions such as throwing, rubbing, pushing, touching. On the other hand, in the reflective abstraction coordinated action is important. These coordinated actions are the roots of logical thoughts. Concurrent with this theory, Young (1983) defined manipulative material as objects, which represent mathematical ideas that can be abstracted through physical enrolment with the objects. It is important to note that mathematical ideas are abstract, but nature of human growth and development demands pedagogical approaches that involve representation of these ideas. (Beattie, 1986)

In summary, “Perhaps the most important single proposition that the educator can derive from Piaget’s work and its use in the classroom, is that children,



especially young ones, learn best from concrete activities” (as cited in Post, 1981, Herbert & Opper, 1969, p.221).

One of the most important researchers that suggest using physical manipulative in mathematics teaching is Zoltan P. Dienes. Zoltan is influenced by Piaget’s studies but unlike Piaget, he was exclusively interested in mathematics teaching. He has also invented *Dienes Blocks* (a kind of physical manipulative). He explained six stages on learning mathematics: Free exploration, playing the rules, comparison, representation, symbolization and formalization. Stage 1 and 2 are the basis of using physical manipulative. These stages describe the general characteristics of a sequence of experiences that result in the appropriate development and subsequent abstraction of a given concept (Dienes & Goldin, 1971).

Bruner (1966) suggested that a mathematical concept could be represented in three ways: Enactively (by physical representation), iconically (through pictorial representation), and symbolically (in written symbols). Enactive representation was defined as where the things get “represented in muscles” (e.g. motor skills, rolling). In this stage, he recommended that using physical manipulative. Behr, Lesh, Post, & Silver, 1983, as cited in Cathcart, Pothier, Vance & Bezuk (2003) expended Bruner’s three modes of representation and suggested that these three modes could be extended to five: 1) real world situations, 2) manipulative, 3) pictures, 4) spoken symbols, and 5) written symbols. For example, the concept of five might be represented with five-finger (real world situation), with the unifix cube (physical manipulative model), with a picture of five flowers (pictures), by saying the word five (oral language) and writing the word five or the symbol 5 (written symbol). Using such kind of representation enhances students’ understanding.

The Van Hiele is also another important theorist who is interested in teaching geometry. He was influenced by Piaget. He and his wife Dina developed five levels of geometric thinking: (1) visualization: students recognize figures as total entities (triangles, squares), but do not recognize properties of these figures (“a rectangle is like door”). (2) Analysis: Students analyze component parts of the figures (opposite

angles of parallelograms are congruent), but interrelationships between figures and properties cannot be explained. (3) Informal deduction: Students can establish interrelationships of properties within figures (in a quadrilateral, opposite sides being parallel necessitates opposite angles being congruent) and among figures (a square is a rectangle because it has all the properties of a rectangle). Informal proofs can be followed but students do not see how the logical order could be altered nor do they see how to construct a proof starting from different or unfamiliar premises. (4) Deduction: At this level, the significance of deduction as a way of establishing geometric theory within an axiom system is understood. The interrelationship and role of undefined terms, axioms, definitions, theorems and formal proof is seen. The possibility of developing a proof in more than one way is seen. (4) Rigor: Students at this level can compare different axiom systems. (Anne; 1991)

Van Hiele (1999) also claimed that students generally cannot reach the level informal deduction and developing this kind of thinking, and instruction is more important than age or biological maturation. Instruction can foster development from one level to another. He suggested using physical manipulative to develop geometric thinking. “Rich and stimulating instruction in geometry can be provided through playful activities with mosaics, such as pattern blocks or design tiles, with puzzles like tangrams, or with the special seven piece mosaic”(p.310). Using such kind of activities can enrich student’s visual structures and develop knowledge of shapes and their properties.

## **2.2 WHAT IS PHYSICAL MANIPULATIVE?**

Physical manipulative are concrete objects that students are able to grasp with their hands. Students have a chance to manipulate them. These physical manipulative are also categorized in two ways: Commercial and teacher or student-made manipulative. Commercial manipulative come in many shapes and forms and they vary in price and complexity. Calculators, cards, rulers, protractors, dices, graph paper, measuring cups, spinners, thermometers, pattern blocks, dominoes, Cuisenaire rods or strips, geo-boards, tangrams and pentominoes are the examples of

commercial physical manipulative. Many teachers prefer using this kind of manipulative because their colors and shapes are attractive and they are easy to store. Moreover, using the commercialized manipulative, teachers save valuable preparation and instruction time since they do not have to create or locate materials (Tankersley, 1993). Commercial manipulative, however, could be expensive. Student or teacher made manipulative have a larger range of materials. Most of these are easily handled objects such as coins, beans, toothpicks etc. On the other hand, student or teacher made physical manipulative are not as good as commercial counterparts in terms of their re-usability and storage.

### **2.3 HOW PHYSICAL MANIPULATIVE USED IN THE CLASSROOM**

Educators agree that using manipulative is effective way to teach mathematics, but the problem is “In what conditions manipulative should be implemented to be effective?” Educators and teachers have been working in this issue, and they suggested different guidelines. (Burns, 1996; Denu, 1992; Heddens, 1997; Heuser, 2000; Stein & Bovalino, 2001)

One of the most important aspects of using physical manipulative in the classroom is to select the materials. Heddens (1997) suggested that manipulative materials should relate to the students' real world. For example, the use of an abacus is not something that is used in Malawian daily life. Instead of abacus stones, eating utensils, tins, beans, apples, peanuts, sticks, etc. would be more appropriate. Also manipulative materials must be selected that are appropriate for the concept being developed and appropriate for the developmental level of the students. For example, one stick may be placed on a place value chart in the ones place; however one stick should not be placed in the tens place. Another selection criteria for selecting the manipulative is to being easily manipulated or simplest possible materials (Suydam and Higgins, 1976)

Another aspect of using physical manipulative is to introduction to the students. Each student needs materials to manipulate independently, students allowed

enough time to explore manipulative their own manner (Denu, 1992; Moyer, 1986; Zoltan & Goldin, 1971). On the contrary, if teachers give students too much time in unsystematic and non-productive exploration this will make the manipulative less effective.

Presenting the manipulative to the classroom is an another important aspect. After giving enough time to exploration, clear expectations should be established for both goals and how students may use materials. Teachers must be able to articulate their purposes using manipulative. If they do not tell the students why manipulative are so important for their lesson, they are not unable to have help their students made the connections from models to an internalized ideas. Students also need simple guidelines for it was acceptable and what was not acceptable. (Burns, 1996; Moyer, 1986;), but, teachers do not tell or show students how to work systematically. Such a kind of use of manipulative can cause student thinking and reasoning become routine and mechanical (Stein & Bovalino, 2001). Demonstrations by the teacher or by one student are not sufficient. “When children are encouraged to follow their own interests while manipulating objects, they learn more than when the teacher directs each movement” (Heuser, 2000). Since manipulative can play a role in students’ construction of meaningful ideas, they should be used before formal instruction, such as teaching algorithms. However, teachers and students should avoid using manipulative as an end without careful thought rather than as a means to that end (Clements & Battista, 1986).

Research indicates that simply, using manipulative is not sufficient. Manipulative should be used with the other teaching practices, including pictures, diagrams, textbooks, films, and similar materials to provide benefits to the students. (Suydam & Higgins, 1976).And also cooperative grouping that works well with the manipulative. Working in pairs or slightly larger groups provides students in all subject areas to learn good team skills while also learning the material (Branch, 2006).

Another effective strategy to use along with manipulative is good questioning skills. When used in a classroom, good questioning allows the teacher to guide the students to attain the information and skills. In the same way, it is important to “ask probing questions to focus children’s thinking when using manipulative,” as this allows the teacher to get the students where he wants them to be without directly telling them what to do (Waite-Stupiansky and Stupiansky, 1998). Olkun & Toluk (2004) proposed that appropriate use of manipulative could be made by using the teacher questioning on aspects of geometric shapes. By a way students’ relational understanding of plane geometric shapes can be utilized in “hands-on mind-on environment” (p.9). Asking traditional questions which focus on calculating correct answer should be placed by asking why, how questions.

#### **2.4 USE OF PHYSICAL MANIPULATIVE IN LEARNING MATHEMATICS**

Research in mathematics education has mixed results about the use of physical manipulative in the learning process of mathematics. How manipulative is supposed to help, students learn mathematical concepts and skills remain unclear. Thompson (1992) explained this situation by using the word “equivocacy” (p. 123). It means while several studies have proved that using manipulative promotes achievement, several studies have found no difference observed on achievement. Despite these mixed results, National Council of Teacher of Mathematics has encouraged the use of manipulative at all grade levels since 1940 (Hartshorn & Robert-Boren, 1990) since using manipulative can facilitate student’s understanding and learning of mathematical ideas.

Researchers investigated the effects of physical manipulative on various measures of mathematics and geometry by comparing to traditional teaching: achievement on fractions including procedural and conceptual knowledge (declarative and conditional) (Cramer, Post & delMas, 2002), operations on whole numbers including procedural knowledge (Cotter, 2000), achievement on operations on whole numbers including procedural knowledge (Fuson & Briars, 1990); achievement on area of the polygons and solids including procedural knowledge

(Bayram, 2004), achievement on lines and angles including declarative, conditional and procedural knowledge (Garrity, 1998), achievement on word problems, place value and decimals, area and perimeter of polygons including declarative and procedural knowledge (Kjos & Long, 1994). In all these studies only Garrity (1998) has attempted to assess effects of physical manipulative on conditional and declarative knowledge by asking questions in the form of true-false and fill in the blanks questions. She did not measure these knowledge types in separately. The other studies assessed the academic achievement by using procedural knowledge and/or conceptual knowledge tests. All these studies reported that the students who exposed to instruction, which utilized physical manipulative significantly, outperformed than the students who exposed traditional teaching. This significant effect was associated with the active involvement of the students by constructing their own knowledge. They claim that physical manipulative enhance students' abilities to explain and represent their thinking using visual models. Moreover, physical manipulative helped students' transition from concrete to abstract symbolic level.

On the contrary, some researchers proved that physical manipulative did not demonstrate positive outcomes on academic achievement involving procedural knowledge, conceptual knowledge and problem solving in arithmetic and geometry (e.g. Baker & Beisel, 2001; Pesek & Kirshner, 2000). Boulton-Lewis et al. (1997) explained one of the reasons why using physical manipulative was not effective as "increased processing load caused by concrete representation" (p.379). Additional processing load occurred when the function of the physical manipulative was not clear for students, or the teachers were not aware of this cognitive load. Another reason could be the link between the symbolic mathematical representation and physical manipulative were not clear. The connection between the mathematical representations and physical manipulative should be well established; otherwise, students refused using physical manipulative (Clements & McMillen, 1996; Boulton-Lewis et al., 1997). And the third reason was physical manipulative could produce students mental actions different from the teachers wished, such as when adding 5

and 4 on the number line students first found 5 and then by counting get 9. This procedure only consisted counting strategies whereas students should have been understood the algorithm of the addition (Clements & McMillen, 1996).

The studies investigated the effects of physical manipulative on learning geometry by interviewing and/or observing students (Battista & Clements, 1996, 1998; Bishop, 1997; Missetra, 2000; Owens & Clements, 1998). In these studies, they reported that the physical manipulative was a kind of vehicle to encourage problem solving, knowledge construction and to help students gain the ability to relate new knowledge with the prior knowledge. Furthermore, students could develop an appreciation of the meaning of the mathematical concepts by experience exploring the relationship with the physical manipulative. Use of physical manipulative provided an environment that promotes deeper discussion. Outhred & Mitchelmore (2000) opposed and claimed that physical manipulative might not be effective for two reasons. Firstly, students' attention could not be drawn to understand the structure while having fun. The other reason was that physical manipulative may "conceal the very relations they are intended to illustrate" (p.146).

The effect of use of physical manipulative on students' spatial ability has also been investigated by the researchers. Battista & Clements (1996, 1998) studied the physical manipulative to investigate both 2-D and 3-D enumeration of cubes, due to the fact the enumeration develops spatial structuring of students. Their study revealed that students could construct spatial structuring by reflecting their actions such as moving physical manipulative on their perceptual and motor actions. Furthermore, "the perceptual and physical actions students performed during counting became inputs for the structuring process" (Battista & Clements, 1996; p.20) They do not "read off" these structures from objects, but instead, employ a process of "constructive structurization" that enriches objects with non-perceptual content. Ben-Chaim (1988) also investigated the spatial ability on middle school students. The results of the study revealed that prior to study there was a gender difference of the spatial ability in favor of boys, after the instruction physical manipulative eliminated the gender effect.

There are also some studies that compare the effects of physical manipulative with the virtual manipulative. Suh & Moyer (2007) proposed that both virtual and physical manipulative had differential effects on elementary students' achievements. Physical manipulative students showed "more tactile features had more opportunities for invented strategies and more mental mathematics" (p. 164). The reason behind these improvement students had the chance the opportunity to manipulate objects more freely than the virtual group and this manipulation help students develop more mental mathematics and allowed them process numerical relations. On the other hand, virtual manipulative had the features that "explicit linking of visual and symbolic modes, guided step by step support in algorithmic process and immediate feedback system" (p.165). Because of these features, students carried out procedures accurately and developed self-checking system to correct their answers. Berlin & White (1986) also compared the effects of physical manipulative and virtual manipulative on students' spatial ability at the elementary level. They were also consistent with the Suh & Moyer that both physical and virtual manipulative had differential effects on students' gender, socioeconomic level and age.

## **2.5 SELF-METACOGNITIVE QUESTIONING AND PROBLEM SOLVING IN MATHEMATICS**

Self-questioning was described as a metacognitive activity, since it enabled learners how to test themselves, and how to comprehend successfully (King, 1992; Palinscar & Brown, 1984; Williamson, 1996). King studied (1989, 1990, 1992) effects of self-questioning on students' learning from readings and students' comprehension of reading. Her studies revealed that asking and answering thought self-questions helped students elicit more explanations. Also, self-questioning also provided active processing for students, helped them focusing on comprehension and monitoring their activities.

Gourgey (1998) claimed that the findings in self- questioning in reading text are the same as metacognition in problem solving in mathematics. He explained this claim as



once students have acquired the basics (computation in mathematics as compared with decoding in reading), their ability to think in the domain is based on clarifying goals, understanding important concepts, monitoring understanding, clarifying confusion, predicting appropriate directions, and choosing appropriate actions (p.86).

The researchers investigated the effects of self-metacognitive questioning on various measures of mathematics: Mathematical achievement by focusing problem solving (Gourgey, 1998; Kramarski, Maverech & Arami, 2004; Lester, Garofalo & Kroll, 1989; Maverech, 1999; Swanson, 1990), mathematical knowledge (Maverech & Fridkin, 2006), mathematical e-learning environments (Maverech & Gutman, 2006), mathematical conceptions and alternative conceptions (Kramarski, 2004). The focus of these studies was to teach students how to reason mathematically by asking and answering the self-metacognitive questioning. In all these studies, the researchers reported that the students using self-metacognitive questioning outperformed than those who used self-metacognitive questioning.

The reasons of these positive effects of using self-metacognitive questions have been explained as follows: Self-metacognitive questioning, using comprehension questions (e.g. what is the problem about? What is the meaning of...) students become sensitive to the relevant parts of the related tasks and look for the all the information about the tasks (Kramarski, 2004; Kramarski & Maverech, 2003).

Self-metacognitive questioning, using connection questions (e.g. “How are this problem/ task different/similar from what you have already solved?”) might guide students how to integrate and generate the knowledge that have already learned. (Gourgey; 1989, Kramarski, Maverech & Arami, 2004),

Self-metacognitive questioning, using strategic questions (e.g., what strategy, tactic, or principle can be used to solve the problem or complete the task? Why are this strategy, tactic, or principle and the most appropriate for this problem or task?) students might focus on things to think, this might make students use reflections to construct mathematical knowledge (Gourgey; 1989, Maverech & Gutman, 2006),

Self-metacognitive questioning, using reflection questions (e.g. Does the solution make sense? What am I wrong?) might lead students to pay attention to the given information and this might make students understand the problem better. Moreover, Kramarski & Maverech (2003) proposed that when these four kinds of questions were used together, students had developed more mental explanations on mathematical reasoning.

In all these studies, except one (Maverech & Fridkin, 2006), researchers also indicated that the positive effects of asking self-metacognitive questions in small groups were more significant than the working with individuals. They proposed that asking these kinds of questions more suitable in small groups than individuals since working with small group gave students a chance to discuss the tasks and help each other to understand missing points.

Another important aspect of the self-metacognitive questioning is frequency of usage. It has been found that self-metacognitive questioning was more effective when it was practiced over prolonged period with the day-to-day exercises rather than integrated in a unit (Lester, Garofalo & Kroll, 1989).

## **2.6 TYPES OF KNOWLEDGE: DECLARATIVE, CONDITIONAL AND PROCEDURAL**

There are three types of knowledge of knowledge of cognition: Declarative, conditional and procedural. This type of classification has been made since 1990 is to enhance problem solving in general (Schraw, 1998; Smith & Ragan 1993), however several researchers classified the mathematical knowledge as conceptual and procedural (Rittle-Johnson & Alibali, 1999; Rittle-Johnson, Siegler & Alibali, 2001,; Webb, 1979). In general, they defined conceptual knowledge as facts, concepts, principals and algorithms that were needed to solve problems and procedural knowledge as mastery of computational skills, algorithms and procedures that were needed to solve problems.

### **2.6.1 DECLARATIVE KNOWLEDGE**

Smith & Ragan (2005) defined declarative knowledge as “knowing that” something is the case. Declarative knowledge involves verbal information and the key words for declarative knowledge are “explain”, “describe”, “summarize” and “list”. Declarative knowledge involves memorization, but it is different from the “rote” memorization (Smilkstein, 1993; Smith & Ragan 1993). Unlike the rote memorization, declarative is knowledge acquired within meaningful structures. For example, declarative knowledge involves knowing what the book or the teacher says. It involves the facts, hypotheses, generalizations, theories, generalizations, beliefs, attitudes, and opinions. It is descriptive and constructed of propositions (Hall, 1998).

This type of knowledge includes three subtypes. (1) Labels and names: requires making links between two elements. Learning foreign language vocabulary is an example of this kind of knowledge. (2) Facts and lists: These types of knowledge require meaningful integration in to prior knowledge. (3) Organized discourse: This type of knowledge requires comprehension while reading a text and integrating with existing knowledge is also important (Gagne & Briggs 1979; Smith & Ragan, 1993).

It is important for students to acquire the declarative knowledge correctly because incorrect initial declarative knowledge will prevent students proceed with problem. Yet, there are several ways to teach declarative knowledge correctly to the students, the teacher can ask to students repeat what was said, to write it down, to read it out, to paraphrase it (Hall, 1998; Smilkstein, 1993).

### **2.6.2 PROCEDURAL KNOWLEDGE**

Hall (1998) defined procedural knowledge as “knowledge needed to put what the students declaratively into practice” (p.37). Procedural knowledge involves “how” type of information that tells us the rules to follow to accomplish a task and refers to knowledge about doing things (Schraw, 1998; Smith & Ragan, 1993).

Finding the area of a polygon with the given sides is an example of procedural knowledge. Much of this knowledge is represented as rules, concepts, algorithms, strategies, tactics, heuristics, and plans. Unlike declarative knowledge, it is prescriptive. The key word for procedural knowledge is “after”. In other words, the procedure follows the word “after” (Hiebert & Lefevre, 1986, p.8).

Teaching procedural knowledge requires different techniques depending on its complexity. If it is a simple procedural knowledge, it can be thought straightforwardly with a step presented, demonstrated, and then practiced. If it is complex procedural knowledge, it must initially be simplified, later elaborated into form that is more complex. In general, there are four steps that should be taken in order to apply procedural knowledge: (1) determining if a situation requires doing a particular cognitive task; (2) recalling the steps in the procedure (declarative knowledge); (3) completing the steps in the procedures; and (4) analyzing the completed procedures (Smith & Ragan, 1993).

### **2.6.3 CONDITONAL KNOWLEDGE**

Conditional knowledge refers to knowing when and why to use declarative and procedural knowledge (Garner, 1990). Conditional knowledge involves rule learning in the form of ‘if-then’ or ‘action-condition’ statements. Moreover, this type of knowledge is concerned about the propositions, principals, postulates, axioms, theorems, and laws (Smith & Ragan, 1993). ‘If all the lengths of sides of two polygons are equal then, they are equal’ is an example of conditional knowledge. When students adequately learn the conditional knowledge, they become successful practitioners in scientific methods and assertions (Ward, Overton & Byners, 1990). Conditional knowledge enables students to adjust to the changing situational demands of each learning task.

Teaching conditional knowledge requires determination of the concepts, consideration of rules and reaching the conclusion about the concept. Students

should have the ability to verbally state the relationship with the given mathematics situations (Smith & Ragan, 1993).

## **2.7 SUMMARY OF THE LITERATURE REVIEW**

Physical manipulative are concrete objects that students are able to grasp with their hands and there is a strong theoretical base for the use physical manipulative to teach mathematics (Bruner, 1962; Dienes & Goldin, 1971; Piaget, 1968; Van Hiele, 1999), but the implementation of the manipulative is very important. There are too many aspects to be considered. These are explained in detailed in the literature as follows: One of the most important aspects of using physical manipulative in the classroom is to select the materials (Heddens, 1997). Each student needs material to manipulate independently, students allowed enough time to explore physical manipulative their own manner (Moyer, 1986). Presenting the physical manipulative to the classroom is also important. After giving enough time to exploration, clear expectations should be established for both goals and how students may use materials (Burns, 1996; Moyer, 1986). Physical Manipulative should be used with the other teaching practices, including pictures, diagrams, textbooks, films, and similar materials to provide benefits to the students (Suydam & Higgins, 1976). While implementing physical manipulative teachers should use good questioning skills (Olkun & Toluk, 2004; Waite-Stupiansky and Stupiansky, 1998). Research in mathematics education has mixed results about the use of physical manipulative in the learning process of mathematics. How manipulative is supposed to help, students learn mathematical concepts and skills remain unclear.

Researchers investigated the effects of physical manipulative on various measures of mathematics and geometry (Battista & Clements, 1996, 1998; Battista & Mc Millen, 1996; Bayram, 2004; Beisel & Baker, 2001; Bishop, 1997; Boulton-Lewis, 1997; Cramer, Post & delMas, 2002; Fuson & Briars, 1990; Garrity, 1998; Kjos & Long, 1994; Missetra, 2000; Owens & Clements 1993; Pesek & Kirshner, 2000; Thompson, 1992). Most of these studies, however, assessed the academic achievement by using procedural knowledge and/or conceptual knowledge (mostly

declarative knowledge and rarely conditional knowledge). Some of them reported that effect of physical manipulative were significant on students' mathematics and geometry achievement (e.g. Bayram, 2004; Cramer, Post & delMas, 2002; Fuson & Briars, 1990; Garrity, 1998). However, some them reported that effect of physical manipulative were not significant on students' mathematics and geometry achievement (Pesek & Kirshner, 2000; Thompson, 1992). Despite these mixed results, National Council of Mathematics strongly advised the use of physical manipulative (NCTM, 2000). Most of these studies, however, assessed the academic achievement by using procedural knowledge and/or conceptual knowledge (mostly declarative knowledge and rarely conditional knowledge). This reveals that recent research has unanswered the effects of manipulative in teaching geometry on three different types of knowledge as defined by cognitive psychologists. Cognitive psychologists (e.g. Smith & Ragan, 1993) defined three distinct knowledge types: Declarative, conditional and procedural

On the other hand, self-metacognitive questioning as suggested by Kramarski and Maverech (2003) and Maverech and Kramarski (1997) are a kind of strategy for helping learners to reflect on their problem solving processing consisting of four kinds of questions: Comprehension, connection, strategic and reflections. Research indicates that simply, using manipulative is not sufficient, for this reason in this study self-metacognitive, questioning and use of physical manipulative were combined.

Although several studies have been conducted to investigate the use of manipulative in teaching geometry, the use of physical manipulative combined with self metacognitive questioning and assessing the performance using three distinct knowledge types as declarative, conditional and procedural, is novel.

## **CHAPTER III**

### **METHOD**

The aim of this chapter is to report participants, variables, the instruments, treatments, data collection and methods used to analyze data will be explained briefly.

#### **3.1 PARTICIPANTS**

Participants were 220 sixth-grade students in a public elementary school in Sincan, Ankara. 111 (approximately 50%) students were male, 109 (approximately 50%) were female. The students were in the range of 12-13 years of age enrolled in five classes each including approximately 44 students. Classes were randomly assigned to two groups: using manipulative with or without metacognitive questioning. Three classrooms (N=129) received instruction based on MAN+META, two classrooms (N=91) received instruction based on MAN. Classes were randomly assigned to the method of instruction.

#### **3.2 VARIABLES**

There are three dependents and two independent variables of this study. Dependent variables were students' posttest scores on three kind's geometry knowledge tests: Declarative Knowledge Test, Conditional Knowledge Test, and Procedural Knowledge Tests on polygons. The independent variables were the teaching methodology (MAN and MAN+META) and time.

### **3.3 INSTRUMENTS**

In this study, three measuring tools were used: Declarative Knowledge Test (DecKT), Conditional Knowledge Test (ConKT) and Procedural Knowledge Test (ProKT)

#### **3.3.1 DECLARATIVE KNOWLEDGE TEST**

To assess students' declarative knowledge related to polygon unit a test Declarative Knowledge Test (see Appendix A) was developed by the researcher considering the general and specific objectives of polygons unit in sixth grade elementary mathematics curriculum (Board of Education, 2004).

Multiple-choice Declarative Knowledge Test (DecKT) included 18-items was based on polygon unit (polygons, similarity and congruency of polygons, classification of the triangles according to their sides and angles, properties of square and rectangles, perimeter of polygons, area of rectangle, square, triangle and mixed shapes) covered in the sixth grade mathematics course. Questions were adapted from textbooks (Fraser, 1999; Kaya & Salman, 1997; MEB, 2006; Özer, Budak, Altınordu & Çatal, 1999).

DecKT had questions on identifying polygons, similar and congruent polygons, and regular polygons as well as knowing the definitions and the properties of polygons, similar and congruent polygons, regular polygons, triangles, rectangle, and perimeter and area of polygons. Eight of the 18 questions were related with identifying and the rests were on knowing the definitions and properties. The possible score on DecKT was ranged from 0 to 18. DecKT questions were scored "0" for incorrect answer and "1" for correct answer.

#### **3.3.2 CONDITIONAL KNOWLEDGE TEST**

To assess students' conditional knowledge related to polygon unit a test Conditional Knowledge Test (see Appendix A) was developed by the researcher



considering the general and specific objectives of polygons unit in sixth grade elementary mathematics curriculum (Board of Education, 2004)., 2004). The test composed of six open-ended questions. The questions were based on polygon unit (polygons, similarity and equality of polygons, classification of the triangles according to their sides and angles, properties of square and rectangles, perimeter of polygons, area of rectangle, square, triangle and mixed shapes) covered in the sixth grade mathematics course. Questions also were adapted from textbooks (Fraser, 1999; Kaya & Salman, 1997; MEB, 2006; Özer, Budak, Altınordu & Çatal, 1999).

ConKT questions focused on understanding a network of ‘if-then’ statements, which describe the relationships between two concepts; congruent triangles and isosceles triangle, congruent and similar polygons, scalene, and right triangle, sides and angles relations of polygons, square and rectangle, and the relationship between area and perimeters of polygons.

ConKT questions were scored based on the rubric developed by Lane (1993) see Appendix B. For each question of the test, the researcher assigned a five-score level (0-4). The highest score of four was awarded for responses that the researchers regard as being entirely correct and satisfactory at grade sixth geometry level, while the lowest score of zero was reserved for no answer. The possible scores on ConKT ranged from 0 to 40.

### **3.3.3 PROCEDURAL KNOWLEDGE TEST**

To assess students’ procedural knowledge related to polygon unit a test Procedural Knowledge test (see Appendix A) was developed by the researcher considering the general and specific objectives of polygons unit in sixth grade elementary mathematics curriculum (Board of Education, 2004)., 2004).

The test composed of ten open-ended questions and based on polygon unit (polygons, similarity and equality of polygons, properties of square and rectangles, perimeter of polygons, area of rectangle, square, triangle and mixed shapes) covered

in the sixth grade mathematics course. Questions also were adapted from textbooks (Fraser, 1999; Kaya & Salman, 1997; MEB, 2006; Özer, Budak, Altınordu & Çatal, 1999).

Procedural knowledge questions focused on finding the perimeter of a square, equilateral triangle, and mixed shapes as well as finding the area of mixed shapes, rectangle and square, and finding the sides of square and equilateral triangle. Four of the ten questions were related with finding perimeter of the given shape. Two of the ten questions were related with finding area. Two of the ten questions were related with finding the sides of square and equilateral. Two of the ten questions were related with relationship perimeter and area.

ProKT questions were scored based on the rubric developed by Lane (1993) see Appendix B. For each question of the test, the researcher assigned a five-score level (0-4). The highest score of four was awarded for responses that the researchers regard as being entirely correct and satisfactory at grade sixth geometry level, while the lowest score of zero was reserved for no answer. The possible scores on ConKT ranged from zero to fourty.

### **3.4 VALIDITY AND RELIABILITY OF INSTRUMENTS**

Validity and reliability was studied on Declarative, Procedural and Conditional Tests.

#### **3.4.1 VALIDITY AND RELIABILITY OF DECLARATIVE KNOWLEDGE TEST**

To check the face validity, DeckKT was submitted to a three-member validation panel composed of one subject-area expert (a university professor) and two mathematics teachers at the elementary school level. Their judgments regarding the extent to which the items or questions were spread to cover the topics in polygons, language level, and the cognitive level measured were used to form the

final version of the tests. The table of specifications of the questions in the DecKT was presented in Table 3.1.

Table 3.1 Table of Specification of the Questions in DecKT

Question	Objective
1	Identify polygons
2	Identify non-polygons
3	Define polygons
4	Know the properties of similar polygons
5	Identify similar polygons
6	Define regular polygons
7	Identify congruent polygons
8	Identify interior and exterior regions of a polygon
9	Identify regular polygons
10	Recall the properties of isosceles triangle
11	Recall the properties of scalene triangle
12	Recall the properties of classification of triangles
13	Identify the rectangle
14	Recall the properties of the square
15	Recall the properties of rectangle
16	Know the properties of polygons
17	Know the definition of perimeters of polygon
18	Define area of a polygon

For the main study, the KR-21 internal consistency reliability was obtained as 0.64. In the case of main study, 88 percent of the students lied in the first standard deviation for the PosDecKT. This also might lowered the reliability.

### **3.4.2 VALIDITY AND RELIABILITY OF CONDITIONAL KNOWLEDGE TEST**

To check the face validity, ConKT was submitted to a three-member validation panel composed of one subject-area expert (a university professor) and two mathematics teachers at the elementary school level. Their judgments regarding the extent to which the items or questions were spread to cover the topics in

polygons, language level, and the cognitive level measured were used to form the final version of the tests. The table of specifications of the questions in the ConKT was presented in Table 3.2.

Table 3.2 Table of Specification of the Questions in ConKT

Question	Objective
1	Justify the relationship between similar and congruent polygons
2	Justify the relationship between a scalene triangle's side and angles
3	Justify the relationship between equilateral and isosceles triangles
4	Justify the relationship between a polygon's sides and angles
5	Justify the relationship between a rectangle and a square
6	Justify the relationship between perimeter and area of a polygon

Conditional Knowledge Test was submitted to 70 on sixth grade students in two different schools in the 2005-2006 academic year. The reliability coefficient for pilot study, the Croanbach alpha of ConKT was obtained as 0.81.

On the other hand, a scoring rubric (see Appendix B) was developed by the researcher based on Lane (1993). In order to establish the extent of consensus on use of the scoring rubric for the ConKT inter-rater reliability coefficient was computed. The researcher and a four-year-experienced elementary school mathematics teacher scored randomly selected 40 tests from each one. Intraclass correlation (ICC) was used to measure inter-rater reliability in terms of providing subjective decisions. The ICC value of 0.85 indicated a quite high reliability and the internal consistency of the scoring rubric as used by two raters. After finding the intraclass correlation coefficient, the consensus was reached by discussing with the teacher.

### **3.4.3 VALIDITY AND RELIABILITY OF PROCEDURAL KNOWLEDGE TEST**

To check the face validity, ProKT was submitted to a three-member validation panel composed of one subject-area expert (a university professor) and

two mathematics teachers at the elementary school level. Their judgments regarding the extent to which the items or questions were spread to cover the topics in polygons, language level, and the cognitive level measured were used to form the final version of the tests. The table of specifications of the questions in the ProKT was presented in Table 3.3.

Table 3.3 Table of Specification of the Questions in ProKT

Question	Objective
1	Find the perimeter of mixed shapes consisting of square and equilateral triangle
2	Find the perimeter of mixed shapes
3	Find the perimeter of equilateral triangle
4	Find the sides of the rectangle with the given perimeter
5	Find the sides of the square with the given area
6	Find the area of the mixed shapes
7	Find the area of the mixed shapes consisting of rectangle and triangle
8	Find the area of the mixed shapes consisting of similar triangles
9	Find the area and the perimeter of a square and rectangle and compare its magnitudes
10	Find relationship between perimeter and area mixed shapes

On the other hand, a scoring rubric (see Appendix B) was developed by the researcher based on Lane (1993). In order to establish the extent of consensus on use of the scoring rubric for the ProKT inter-rater reliability coefficient was computed. The researcher and a four-year-experienced elementary school mathematics teacher scored randomly selected 40 tests from each one. Intraclass correlation (ICC) was used to measure inter-rater reliability in terms of providing subjective decisions. The ICC value of 0.82 indicated a quite high reliability and the internal consistency of the scoring rubric as used by two raters. After finding the intraclass correlation coefficient, the consensus was reached by discussing with the teacher.

For the main study, the Croanbach alpha coefficient was obtained as 0.85. The reliability coefficient of the instrument is quite high representing high reliability (Adams & Wu, 2002).

### 3.5 TREATMENTS

All classes studied the Polygon unit. In all classes mathematics was taught four times a week according to the mathematics curriculum (M.E.B, 2004) adopted by the Turkish Ministry of Education. Three teachers were involved in the study. All teachers had more than five year experience in teaching mathematics. Teachers were exposed to training about manipulative instruction at the beginning of the 2006-2007 academic years. Training was given by inspectors of Turkish Ministry of the Education during three days and six hours per day. In addition, teachers were given metacognitive instruction by the researcher during two days and two hours per day. Two of the teachers had taught two classes: MAN+META and MAN groups. The other teacher had one class with MAN+META instruction.

The MAN+META lessons consisted of three interdependent components, using physical manipulative, group work, metacognitive questioning, and MAN class lessons consisted of two interdependent component; using physical manipulative and group work. Before the study started, the MAN+META group was given two hours lesson on how to deal with and respond to the metacognitive questions. Students in both groups used the same physical manipulative with the same problems/tasks, and used the same textbook (MEB, 2006). Only difference between two groups was that metacognitive questions were provided in the MAN+META groups' worksheets worked on after introducing each concept. The worksheets worked in the MAN, however, did not include any metacognitive questions.

Each treatment consisted of two parts: Introduction lesson and worksheet study lesson. The introduction lesson in each group included three sub-parts: Teachers' introduction to the whole class and free play with manipulative (about 5 min), group seatwork study (about 25 min) and teacher review with the whole class (about 10 min). In the worksheet study lesson worksheets including problems and exercises were distributed to each student and then students tried to solve them by themselves. Following this, students discussed their answers within groups and then some groups shared their answers with the class. In MAN + META groups,

metacognitive questions were provided in each worksheet but not in MAN groups' worksheets.

Based on students' academic achievement, groups were formed as low, medium and high achievers. There were six students in each group including two low achievers, two medium achievers and two high achievers. An English teacher, mathematics teacher, Turkish Teacher, Science Teacher and Social Science Teacher were consulted while forming the groups. All teachers gave their opinion for each student's academic success in their classes. The result of these consultations revealed that each teacher's opinions for each student were quite close to each other.

### **3.6 MANIPULATIVE INSTRUCTION**

The aim of the activities with the use of physical manipulative in the lesson plans was to develop declarative, conditional, and procedural knowledge. For example, seven pieces of mosaics were used to form different geometric figures and then it was discussed with the students what the common properties of the figures formed to be able to state the definition of polygon. By folding, a rectangular paper into equal small pieces of rectangles students had the chance to see the relationship between the similar rectangles by comparing the small pieces with whole piece. Informally (students used measure norms such as the length of math book to find the perimeter of surface of their desks) and formally (students used a ruler to find the perimeter of surface of their desks) measuring the side lengths of geometric shapes formed on the geoboards and the objects in their class students figured out the formula of perimeter.

As explained above three kinds of manipulative were used in this study: Seven pieces mosaics, geoboards and origami. Manipulative materials were selected considering the criteria suggested by the literature (Heddens, 1997; Suydam & Higgins, 1976). One of the most important selecting criteria was appropriateness for the concept being developed and appropriateness for the developmental level of students. Second criterion was that manipulative were easily manipulated and

simplest materials. In addition, the third criterion was that all students studied with the same manipulative and each student had the manipulative individually.

Each introduction lesson started with a teacher's short presentation of the new materials to the whole class as such "Today we will use seven pieces mosaics consisting seven different geometric shapes that you have learned before and let's start to investigate them". Following the teachers' introduction, the free play stage started. In this stage, students were allowed enough time to explore the manipulative by their own manner. Then, students worked within groups. Students tried to discover definitions, properties or rules of the related concept. For example, with the seven piece mosaics, students tried to make different polygons by using two pieces, three pieces and they shared their findings with the group. Then, class discussion was started with each group presenting their findings with the whole class. The teacher wrote the groups findings to the blackboard. Based on each group's findings the definition, properties or rules of a concept were established by discussing within whole class. Lastly, the teacher summarized the whole things discussed in the lesson.

### **3.7 SELF-METACOGNITIVE INSTRUCTION**

Self-metacognitive questioning instruction based on the studies of Mevarech and Kramarski (1997) called IMPROVE. Mevarech and Kramarski (1999) recommended that full set of self-addressed questions were more effective than asking each kind of question by itself. For that reason while implementing the self-metacognitive questioning four full set of questions were used: Comprehension, connection, strategic and reflection.

Comprehension questions: The aim of the comprehension questions was to make students analyze specific points of the problem. "what is the problem about", "What is the meaning of mathematical concepts" and regarding the polygon unit, "what is the problem about ?", "what does the polygon mean?", " What does closed curve represent?", "What does the height of a triangle represent?" were the examples of the comprehension questions.



Connection questions: The aim of the connection questions was to gain ability to students to make connections between previous and new knowledge. The type of the connection questions was “what are the similarities/differences between the problem at hand and the problems you have solved in the past?”

Strategy questions: The aim of the strategy questions was to make students to find out which strategy was appropriate for solving the problem and for what reasons. (e.g., “what are the strategies/tactics/principles appropriate for solving the problem and why?”)

Reflection questions: The aim of the reflection questions was to reflect on students’ understanding and feelings while solving problem. (e.g., “what did I do wrong here?” “Does the solution make sense?”).

Students practiced the questions written on their worksheets in individualized settings and the teacher provided assistance as needed. At the end of the lesson, the teacher reviewed the solution of the mathematical problems by modeling the meta-cognitive questioning.

### **3.8 DATA COLLECTION AND ANALYSIS**

In this study, quasi-experimental design was used to find the effects of the manipulative instruction with or without metacognition questioning on the students’ declarative, conditional and procedural knowledge. There were two groups. One group received manipulative instruction without metacognitive questioning while other group received the manipulative instruction with metacognitive questioning. The outline of the procedure represented in table 3.4.

Table 3.4 Outline of the Procedure

	MAN Group	MAN+META Group	Time Schedule
Pretests	DecKT	DecKT	26 March 2007
	ConKT	ConKT	26 March 2007
	ProKT	ProKT	26 March 2007
Treatment	Manipulative Instruction	Manipulative Instruction and Self-Metacognitive Questioning	2 April–11 May 2007
Posttests	DecKT	DecKT	14 May 2007
	ConKT	ConKT	26 May 2007
	ProKT	ProKT	26 May 2007

The researcher developed eight lesson plans (see Appendix C) and worksheets (see Appendix D). All lesson plans were piloted on sixth grade students in a public elementary school other than the one used in the main study. This pilot study was conducted to check whether the lesson plans could be applied in classroom settings, how the classroom settings should be arranged, whether directions given were clear, how the classroom management could be accomplished, and whether the objectives could be achieved. The pilot study also provided the researcher to gain experience about the lesson plans and how to use them in the classroom effectively. After the piloting, some revisions were made. For example, in the first lesson plan: group works added for item 4 and the first question were changed in the first worksheet.

Twelve students were interviewed individually after the instruction and audio taped. Six of the students were from MAN group including two high, two medium, and two low achievers. Other six students were MAN+META group including two high, two medium, and two low achievers. These interviews took about approximately 20 minutes for each student. These interviews were transcribed and coded using narrative analysis procedure (Tesch, 1990 as cited Creswell, 1994). The researchers did not have any specific in mind during the initial reading of the data. Themes evolved during the coding process. Common ideas were coded and

translated into few generalized themes. After the themes were chosen and coded, the researcher reread data and listed some specific examples from the interviews that matched each theme to demonstrate how the data collected in the interviews supported the theme. There were two themes: use of the manipulative on geometry and the role of the self-metacognitive questioning.

The study was conducted during six weeks in the second semester of the academic year 2006-2007. At the beginning of the study, groups were asked to respond to Geometry Knowledge Tests including Declarative, Conditional and Procedural Knowledge Tests as pre-tests. Subsequently, each teacher began teaching the Polygon unit according to the instructional method to which he/she was assigned, using the materials specially designed for that method of the study. Geometry Knowledge Tests were re-administered as post-tests.

In the pilot study 25 minutes, 35 min and 40 min were given for Declarative Knowledge Test, Conditional Knowledge Test and Procedural Knowledge Tests respectively. Any problem was not encountered for the timing. After the data collection, reliability analysis and factor analysis for DecKT and exploratory factor analysis for ConKT and ProKT were conducted to investigate content validity of the instruments.

In order to be able to combine MAN classes as one group independent t-tests were conducted on pre and post Declarative, Conditional and Procedural Knowledge Test Scores. In order to be able to combine MAN+META classes as a group independent one-way ANOVA were conducted on Pre and Post Declarative, Conditional and Procedural Knowledge Test Scores.

A mixed design analysis of variance was conducted to assess if there was a significant mean difference between the pretest and the posttest scores of sixth grade students' on declarative, conditional, and procedural knowledge in both two teaching environments: MAN and MAN+META. If a significant result was observed for any of the variables, a follow up analysis was conducted for that significant variable.

As we could not control the independency in situations other than the classrooms, we took the more radical approach by setting the level of significance to 0.01 as suggested by Hair, Black, Babin, Anderson, and Tatham (1995) for whole analysis.

## CHAPTER IV

### RESULTS

This chapter is divided into four sections. First section presents preliminary analysis of the data: independent t-tests and one way ANOVA, second section deals with the descriptive statistics, the third section gives the inferential statistics, and the fourth section gives qualitative results.

#### 4.1 PRELIMINARY ANALYSIS

##### 4.1.1 INDEPENDENT T-TEST FOR MAN CLASSES

An independent t-test was conducted to investigate whether two classes (MAN group) were equal or not according to their pre-tests scores and post-tests scores for three knowledge tests. Table 4.1 presents independent t-test results for PreDecKT, PreConKT, PreProKT, PosDecKT, PosConKT and PosProKT.

The results showed that there was no significant difference in scores for one of the MAN group class ( $M=6.39$ ,  $SD=0.96$ ) and other class of MAN group ( $M=6.60$ ,  $SD=1.98$ ) for PreDecKT  $t(89) = -2.88$ ,  $p=0.05$ . There was no significant difference in scores for one of the MAN group class ( $M=0.72$ ,  $SD=1.03$ ) and other class of MAN group ( $M=0.78$ ,  $SD=1.08$ ) for PreConKT  $t(89) = -1.41$ ,  $p=0.16$ . There was no significant difference in scores for one of the MAN group class ( $M=6.10$ ,  $SD=4.31$ ) and other class of MAN group ( $M=6.60$ ,  $SD=4.32$ ) for PreProKT  $t(89)=-1.21$ ,  $p=0.23$ . There was no significant difference in scores for one of the MAN group class ( $M=9.16$ ,  $SD=2.93$ ) and other class of MAN group ( $M=9.27$ ,  $SD=3.24$ ) for PosDecKT  $t(89) = -0.17$ ,  $p=0.87$ . There was no significant difference in scores for one of the MAN group class ( $M=8.21$ ,  $SD=5.12$ ) and other class of MAN group

( $M=8.90$   $SD=4.51$ ) for PosConKT  $t(89) = -1.76, p=0.82$ . There was no significant difference in scores for one of the MAN group class ( $M=15.56, SD=8.5$ ) and other class of MAN group ( $M=16.20, SD=8.65$ ) for PosProKT  $t(89) = -0.36, p=0.72$ . Thus, it is concluded that two of the MAN group' classes were equal according to their declarative, conditional and procedural knowledge. As a conclusion, the scores of pretest and posttest of Declarative, Conditional and Procedural Tests was able to be combined for main analysis.

Table 4.1 T-test Results for MAN Group Classes

		Levene's Test for Equality of Variances		t-test for Equality of Means				
		F	Sig.	t	df (2-tailed)	Sig.	95% Confidence Interval of the Difference	
Equal variances							Lower	Upper
PreDecKT	assumed	.588	.445	-2.875	89.000	.05	-2.217	-.405
	not assumed			-2.875	86.546	.05	-2.218	-.405
PreConKT	assumed	1.239	.269	-1.411	89.000	.162	-.749	.127
	not assumed			-1.411	85.746	.162	-.749	.127
PreProKT	assumed	6.729	.011	-1.210	89.000	.229	-3.640	.884
	not assumed			-1.210	71.296	.230	-3.647	.892
PosDecKT	assumed	.921	.340	-.170	89.000	.865	-1.408	1.185
	not assumed			-.170	87.157	.865	-1.408	1.186
PosConKT	assumed	.514	.475	-1.759	89.000	.082	-3.454	.210
	not assumed			-1.759	87.936	.082	-3.454	.210
PosProKT	assumed	.059	.808	-.357	89.000	.722	-4.236	2.947
	not assumed			-.357	87.973	.722	-4.236	2.947

#### 4.1.2 ONE-WAY INDEPENDENT ANALYSIS OF VARIANCE FOR MAN+META CLASSES

A one-way independent analysis of variance (ANOVA) was conducted to investigate whether three classes (MAN+META group) were equal or not according to their pre-test scores and post-tests scores for three knowledge tests. Table 4.2 presents one-way ANOVA results for PreDecKT, PreConKT, PreProKT, PosDecKT, PosConKT and PosProKT.

Table 4.2 Independent One-way ANOVA for MAN+META Classes

		Sum of Squares	df	Mean Square	F	Sig.
PreDecKT	Between Groups	8.985	2	4.492	.939	.394
	Within Groups	602.705	126	4.783		
	Total	611.690	128			
PreConKT	Between Groups	6.675	2	3.337	2.131	.123
	Within Groups	197.279	126	1.566		
	Total	203.953	128			
PreProKT	Between Groups	9.831	2	4.916	.208	.813
	Within Groups	2983.750	126	23.681		
	Total	2993.581	128			
PosDecKT	Between Groups	20.493	2	10.246	.829	.439
	Within Groups	1557.383	126	12.360		
	Total	1577.876	128			
PosConKT	Between Groups	342.824	2	171.12	6.929	.028
	Within Groups	3117.145	126	24.739		
	Total	3459.69	128			
PosProKT	Between Groups	609.052	2	304.526	4.387	.144
	Within Groups	8747.103	126	69.421		
	Total	9356.155	128			

The results showed, as seen Table 4.2, that there was not significant difference between three classes of MAN+META groups at the level  $p>0.01$  for PreDecKT  $F(2,216)=0.94$ ,  $p=0.39$ . There was not significant difference between three classes of MAN+META groups at the level  $p>0.01$  for PreConKT  $F(2,216)=2.13$ ,  $p=0.12$ . There was not significant difference between three classes of

MAN+META groups at the level  $p>0.01$  for PreProKT  $F(2,216)=0.21$ ,  $p=0.81$ . There was not significant difference between three classes of MAN+META groups at the level  $p>0.01$  for PosDecKT  $F(2,216)=0.83$ ,  $p=0.44$ . There was not significant difference between three classes of MAN+META groups at the level  $p>0.01$  for PosConKT  $F(2,216)=6.93$ ,  $p=0.28$ . There was not significant difference between three classes of MAN+META groups at the level  $p>0.01$  for PosProKT  $F(2,216)=4.39$ ,  $p=0.14$ . As a conclusion, the scores of pretest and posttest of Declarative, Conditional and Procedural Tests was able to be combined for main analysis.

#### 4.1.3 INDEPENDENT T-TEST

An independent t-test was conducted to compare the pre-tests and post-tests scores of DeckKT, ConKT and ProKT for MAN and MAN+META groups. Table 4.3 presents independent t-test results for PreDecKT, PreConKT and PreProKT.

Table 4.3 Independent T-test

		Levene's Test for Equality of Variances		t-test for Equality of Means				
		F	Sig.	t	df	Sig. (2-tailed)	95% Confidence Int. of the Diff.	
							Lower	Upper
PreDecKT	Equal variances assumed	.088	.767	.199	218.000	.842	-.537	.658
	not assumed			.198	190.253	.843	-.541	.661
PreConKT	assumed	.530	.467	.435	218.000	.664	-.248	.388
	not assumed			.450	212.139	.654	-.238	.378
PreProKT	assumed	1.165	.282	1.864	218.000	.064	-.074	2.672
	not assumed			1.828	179.475	.069	-.103	2.701



The results showed that there was no significant difference in scores for MAN group ( $M=6.49$ ,  $SD=2.52$ ) and MAN+META group ( $M=6.43$ ,  $SD=2.19$ ) for PreDecKT  $t(218)=0.2$ ,  $p=0.84$ . There was no significant difference in scores for MAN group ( $M=0.79$ ,  $SD=1.04$ ) and MAN+META group ( $M=0.72$ ,  $SD=1.26$ ) for PreConKT  $t(218) =0.45$ ,  $p=0.63$ . There was no significant difference in scores for MAN group ( $M=6.46$ ,  $SD=5.42$ ) and MAN+META group ( $M=5.16$ ,  $SD=4.84$ ) for PreProKT  $t(219)=1.8$ ,  $p=0.67$ . It was concluded that two groups are equal according to their declarative, conditional and procedural knowledge.

## 4.2 DESCRIPTIVE STATISTICS

Descriptive statistics related to students' geometry knowledge pretest scores PreDecKT, PreConKT and PreProKT, students' geometry knowledge posttest scores PosDecKT, PosConKT, PosProKT for MAN and MAN+META groups are given in Table 4.4.

As shown in Table 4.4, the MAN group showed a mean increase of 2.70 from PreDecKT to PosDecKT. On the other hand, the mean of the MAN+META group increased 3.55 from PreDecKT to PosDecKT. It can be seen that MAN+META group gained declarative knowledge slightly better than the MAN group.

Similarly, as shown in Table 4.4, the MAN group showed a mean increase of 7.70 from PreConKT to PosConKT. On the other hand, the mean of the MAN+META group increased 8.20 from PreConKT to PosConKT. It can be seen that MAN+META group gained conditional knowledge slightly better than the MAN group.

Similarly, as shown in Table 4.4, the MAN group showed a mean increase of 9.33 from PreProKT to PosConKT. On the other hand, the mean of the MAN+META group increased 10.98 from PreConKT to PosProKT. It can be seen that MAN+META group gained conditional knowledge slightly better than the MAN group.

Table 4.4 Basic Descriptive Statistics Related to the Declarative Knowledge Test Scores, Conditional Knowledge Test Scores and Procedural Knowledge Test Scores

	MAN Group		MAN+META Group	
	Pretest	Posttest	Pretest	Posttest
<b>Scores on DecKT</b>				
N	91	91	129	129
Mean	6.49	9.19	6.43	9.98
Standard Deviation	2.523	3.069	2.186	3.550
Skewness	0.791	0.298	0.564	0.032
Kurtosis	1.271	-0.283	0.209	-0.763
Range	13	15	11	17
<b>Scores on ConKT</b>				
N	91	91	129	129
Mean	0.79	8.46	0.72	8.92
Standard Deviation	1.049	4.470	1.262	5.217
Skewness	1.022	0.555	4.662	0.558
Kurtosis	-0.32	0.214	3.916	-0.413
Range	3	22	22	11
<b>Scores on ProKT</b>				
N	91	91	129	129
Mean	6.46	15.79	5.16	16.14
Standard Deviation	5.427	8.524	4.836	8.598
Skewness	1.466	0.638	1.936	0.531
Kurtosis	2.187	-0.1	5.76	-0.286
Range	24	36	29	37

### 4.3 INFERENCE STATISTICS

A mixed design analysis of variance was conducted to test the hypothesis: There will be no significant mean difference between the pretest and the posttest scores of sixth grade students' on declarative, conditional, and procedural knowledge in both two teaching environments: manipulative without self-metacognitive questioning and manipulative with self-metacognitive questioning.

### 4.3.1 ASSUMPTIONS OF MIXED DESIGN OF MANOVA

Linearity of relations among dependent variables, multivariate normality, equality of variances, homogeneity of variance-covariance matrices between groups assumptions (Leech, Barret, & Morgan, 2005) of mixed design of MANOVA were checked.

For checking the linearity of relations among dependent variable assumption, scatter plots was generated between each pair of variables.

Figures 4.1, 4.2, 4.3, 4.4, 4.5, 4.6 show no evidence of non-linearity.

Multivariate normality was checked by looking the skewness and the kurtosis values of PosDecKT, PosConKT and PosProPKT as seen in Table 4.4 that all values were approximately acceptable range in order to verify the univariate normality in the score distribution.



Figure 4.1 Scatter Plots of PosDecKT and PosConKT for the MAN GROUP



Figure 4.2 Scatter Plots of PosDecKT and PosConKT for the MAN+META GROUP

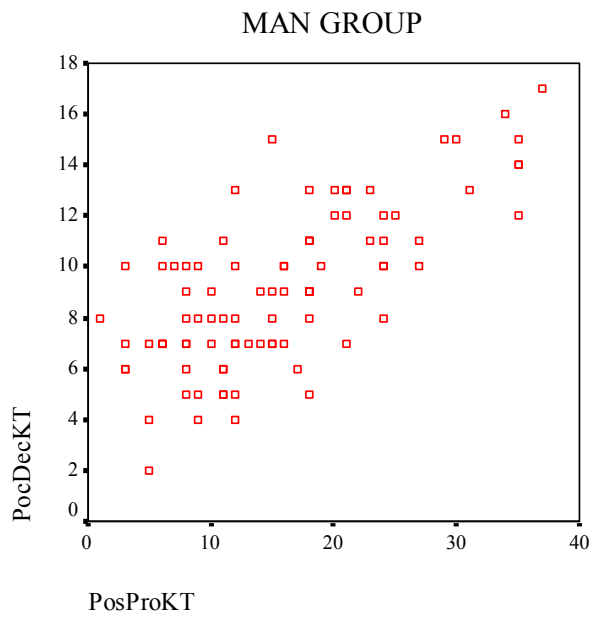


Figure 4.3 Scatter Plots of PosDecKT and PosProKT for the MAN GROUP

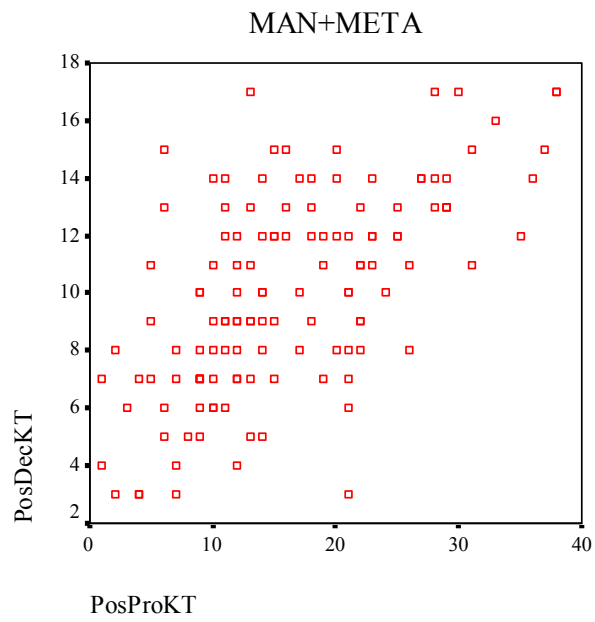


Figure 4.4 Scatter Plots of PosDecKT and ProConKT for the MAN+META GROUP

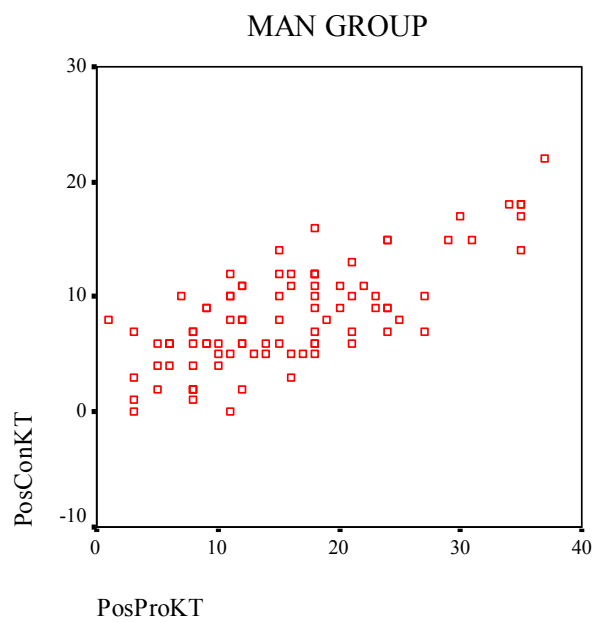


Figure 4.5 Scatter Plots of PosConKT and PosProKT for the MAN GROUP

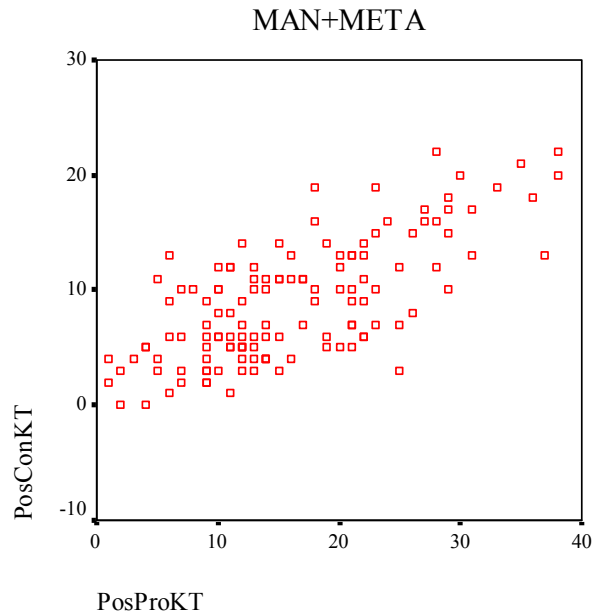


Figure 4.6 Scatter Plots of PosConKT and PosProKT for the MAN+META GROUP

Homogeneity of variance covariance matrices, that is, the variance covariance matrices are equal across groups. The statistical procedure that used to examine this assumption was Box's Test of Equality of Covariance matrices in SPSS. Box's Test of Equality of Covariance Matrices revealed as seen in Table 4.5 that the assumption of homogeneity of variance-covariance matrices was violated. The value  $p= 0.000$  indicated the significant result but, the larger sample size produces larger variance and covariance, since the more conservative  $\alpha$  level .01 was selected as suggested (Hair, Anderson, Tatham, & Black, 1995) the violation of assumption was eliminated. Therefore, this significant result will not cause any problem and it was assumed that this assumption was met.

Table 4.5 Box's Test of Equality of Covariance Matrices (a)

Box's M	53.469
F	2.468
df1	21
df2	137935.732
Sig.	0.000

Equality of variances was tested by using The Levene's Test of Equality of Error Variances. As seen in Table 4.6 this assumption was violated for PosConKT that  $p=0.041$ , it should be greater than 0.05, but this significant result, will not cause any problem since the proportion between the numbers of students of groups is less than 1.5 (Hair, Anderson, Tatham, & Black, 1995). Therefore, it was assumed that equality of variances assumption was met.

Table 4.6 Levene's Test of Equality of Error Variances (a)

	F	df1	df2	Sig.
DKTTOTAL	.088	1	218	.767
POSDKTOT	3.505	1	218	.063
CKTTOTAL	.530	1	218	.467
POSKTOT	4.228	1	218	.041
PKTTOTAL	1.165	1	218	.282
POSPKTOT	.097	1	218	.755

#### 4.3.2 MIXED DESIGN ANALYSIS OF VARIANCE RESULTS

As shown in Table 4.7 mixed design multivariate analysis of variance indicated a significant main effect for time  $F(3, 216) = 200.56, p=0.00$ ) but no group-by-time interaction effect  $F(3, 216) = 1.55, p=0.20$ ) and also no group main effect  $F(3, 216) = 1.7, p=0.168$  suggesting that both groups responded equally well to treatment in the amount of change in their scores on the two outcome measures: Pretests and Posttests.

Table 4.7 Multivariate Test Results

Effect	Wilks' Lambda	F	Hypothesis df	Error df	Sig.	Partial Eta Squared	Observed Power(a)
Intercept	.073	916.842	3.000	216.000	.000	.927	1.000
GROUP	.977	1.700	3.000	216.000	.168	.023	.441
TIME	.264	200.560	3.000	216.000	.000	.736	1.000
TIME * GROUP	.979	1.550	3.000	216.000	.203	.021	.405

#### 4.3.4. FOLLOW UP ANALYSIS: PAIRED T-TESTS

Since there was a significant main effect found for time effect as reported above. A paired t-test was conducted to evaluate the impact of time on students' pretest and posttest scores on the DecKT, ConKT and ProKT as follow up analysis.

Table 4.8 Paired T-test

		Paired Differences					t	df	Sig. (2-tailed)
		Mean	Std. Dev.	99% Confidence Interval of the Difference					
				Std. Mean	Lower				
Pair 1	PreDecKT PosDecKT	-3.22	3.443	.232	-3.83	-2.62	-13.883	219	.000
Pair 2	PreConKT PosConKT	-8.04	4.941	.333	-8.90	-7.17	-24.126	219	.000
Pair 3	PreProKT PosProKT	-10.33	7.854	.530	-11.70	-8.95	-19.502	219	.000



Table 4.9 Descriptive Statistics for Paired T-test

		Mean	N	Std. Deviation	Std. Error Mean
Pair 1	PreDecKT	6.46	220	2.209	.149
	PosDecKT	9.68	220	3.354	.226
Pair 2	PreConKT	.75	220	1.177	.079
	PosConKT	8.79	220	4.908	.331
Pair 3	PreProKT	5.70	220	5.117	.345
	PosProKT	16.03	220	8.522	.575

Table 4.8 and Table 4.9 showed that there was a statistically significant increase in DecKT scores from PreDecKT ( $M=6.46$ ,  $SD=2.21$ ) to PosDecKT ( $M=9.68$ ,  $SD=3.53$ ),  $t(219)=-13.88$ ,  $p=0$ . Since all the values of level of significance, values were equal to zero. The adjustment of level of significance was not conducted. The eta-squared statistic calculated as 0.46 and indicated large effect size (Cohen, 1988 as cited in Pallant, 2003).

Similarly, there was a statistically significant increase in ConKT scores from PreConKT ( $M=0.75$ ,  $SD=1.17$ ) to PosConKT ( $M=8.80$ ,  $SD=4.9$ ),  $t(219)=-24.13$ ,  $p=0$ . The eta-squared statistic calculated as 1.46 and indicated large effect size (Cohen, 1988 as cited in Pallant, 2003).

Similarly, there was a statistically significant increase in ProKT scores from PreProKT ( $M=5.70$ ,  $SD=5.12$ ) to PosProKT ( $M=16.03$ ,  $SD=8.5$ ),  $t(219)=-19.50$ ,  $p=0$ . The eta-squared statistic calculated as 0.63 and indicated large effect size (Cohen, 1988 as cited in Pallant, 2003).

According to results of repeated measures of multivariate analysis of variance as explained above the null hypothesis was fail to reject.

#### 4.4 QUALITATIVE RESULTS

A detailed analysis of the MAN and MAN+META students' responses as elicited in the interview provided some noteworthy findings as summarized below.

#### 4.4.1 USE OF THE PHYSICAL MANIPULATIVE

Manipulative appealed to student's senses: they were touched, handled, moved, and observed. This active involvement enhanced students' learning. In addition, they provided a means for external representation, which involved the student in learning, thereby improved his/her knowledge. Students continually explored by cutting, folding, forming, building, drawing, and discussing various "challenge situations". Some excerpts from students' responses are below:

*When you see with your eyes, it is more effective. Since we saw what we have done in the classroom, we learned better. (Student# 1)*

*When you touch and feel it is easy to understand what we are learning. (Student# 4)*

*When doing myself, I realize what the important point is, I understand better what I have learned or not. (Student# 8)*

*Since we tried to find truth ourselves, we can keep it in our mind... we comprehended better and they will stay in our mind. (Student# 5)*

*We learned by seeing, it was far more just memorizing. (Student# 6)*

*When I was at exam, I picture the model in my head to solve the problem and I continued to think about the model to help me with geometry problems. (Student# 1)*

Students stated the definitions and properties of the polygons by themselves by moving physical manipulative and they had chance to compare properties of different polygons. They explained this as follows:

*By using seven piece mosaics I learned that a polygon could be formed using another polygon. For example, a trapezoid could be formed with two triangles. (Student# 7)*

*When we constructed the shapes using seven piece mosaics, we figured out the definition of polygon ourselves. (Student# 11)*

*Using protractor I saw that all the angles of a rectangle is  $90^0$ . (Student# 11)*

*Using geoboards, I realized the difference between the rectangle and square. I see and touch the corners of the rectangle and made the diagonals. I compare the diagonals by using ruler. (Student# 3)*

*I learned perimeter by measuring side by side. I learned the meaning of area by the square units on the geoboard. All of them could stay in my mind. (Student# 6)*

*On the geoboard I made an equilaterals triangle near it, I made isosceles triangle and measure all the angles and sides by ruler than I saw that an equilateral triangle was an isosceles triangle. (Student# 5)*

*By cutting, folding papers; I made similar and equal polygons, put them together, comparing the similar and equal polygons, I understand why equal polygons similar.*

*When I measured the sides of the rectangle by using ruler on the geoboard, I easily compute the perimeter of the rectangle. (Student# 3)*

*First, I made mixed shapes whatever I want, than I calculate the area by counting unit squares easily. (Student# 1)*

The excitement the students felt during touching and moving the physical manipulative affected their learning. Further, students were not forced to memorize the facts. Some examples of the students' comments about having fun are as follows:

*Absolutely, it was more fun. For example, constructing geometric shapes by our hands using geoboards, using colorful rubbers, cutting or folding papers were too much enjoyable. We did not understand how time passed. The rings bell in a shorter time. When it is enjoyable, we understand better. (Student# 2)*

*When doing my self, I enjoyed too much. (Student# 11)*

*When we used seven piece mosaics, we did different shapes such as a robot or sailing boat it was enjoyable. (Student# 12)*

*Constructing different shapes by using colorful rubber bands on geoboards were interesting. (Student# 9)*

*Cutting or folding papers were enjoyable I felt that we were in the art lesson instead of math lesson. It was not boring. (Student# 10)*

Teachers posed different kinds of questions such as “what .....”, “why....”, “How did you calculate.....”, “How are ..... and similar”, “Explain why it is wrong/true” while students were working with physical manipulative. These questions forced them to think and took their attention toward exploration. Some examples of the student’s comments are as follows:

*When the teacher asked, “Explain how you found answer” I had to think all the steps and I had to express my thoughts. (Student# 6)*

*When the teacher asked, “What would be the area if the length of sides of a square doubled” I had to think different situations. (Student# 4)*

As students studying on physical manipulative, they worked with groups. Working with groups affected their learning positively. Group works facilitated them to learn the responsibility, provided motivation to learn and enabled them to acquire knowledge by seeing others’ behaviors, receiving different ideas, understanding others points of view. The social interaction among the students assisted the construction of knowledge. They helped to each other, by this way learned from each other.

*Everybody was helping each other. I taught my friend something and they taught something to me, too. We transferred knowledge to each other... You [we]*

*were also observing the others while you [we] were doing in the lesson. By this way, we have learned. (Student# 5)*

*We competed with other groups and this forced us to think different solutions. (Student# 2)*

#### **4.4.2 SELF-METACOGNITIVE QUESTIONING**

These questions made students to think in a systematic way and realize which subject they understood or did not understand. They focused important part of the problems and analyzed the steps. Students' comments on self-metacognitive questioning are as follows:

*Asking my self these questions and then answering them made me already solve the problem. (Student# 3)*

*When I asked my self these (self-metacognitive) questions, I found out that I could not understand for example the classification; I went back and studied (Student# 2).*

*When I asked my self these (self-metacognitive) questions, I had to think systematically. (Student# 3)*

Comprehension questions made students investigate relevant and irrelevant information to find what they need to solve the problem and forced to students to make revision of the previous subjects. Connection questions took the attention of the students to the structure of the given problem. Using connection questions also helped students to develop conditional knowledge. Answering the connection questions gave opportunity to students to integrate existing knowledge to the prior knowledge. Strategic questions forced students made plan to solve the problem and helped them to elaborate the information in the given tasks. Moreover, reflection questions paid their attention to check whether their solutions make sense or not. Some examples of the students comments as follows:

*Asking my self what is the problem about, I could concentrate on what I needed to know to solve the problem. If I do not understand the problem what was about, I reread the problem. (Student# 1)*

*In a problem, it was related about width and length of a rectangle. First, I could not solve. I could solve the answering “What was the length and width represent”. Because I realized that, I did not know the meaning of length and width of a rectangle. (Student# 3)*

*For example giving the answer of the question “what is the similarities and differences between the problem at hand and the problem solved” I could explain the question “ If a triangle is equal than it is also similar. Explain, give examples and draw their shapes” I thought the subject sets and I remember “If two sets are equal, than they are congruent” From this point of view, I could give the reasons of triangle questions and examples and draw their shapes. It was easy. (Student# 1)*

*After I read the problem, I describe the problem with my words and this made me aware of what I have to solve. (Student# 4)*

*I have never told before, the result had a meaning. I was not used to thinking of my solution. Before these questioning, after I got solution it was over. After I have learned these questioning, I learned checking the result whether it did make sense. For example, if I got negative value for the length of a rectangle after solving an equation, when I checked the result I thought that it must be wrong because length could not be negative. (Student# 1)*

#### **4.5. SUMMARY OF THE RESULTS**

The independent t-test analysis for MAN group classes revealed that MAN group classes were equal according to their declarative, conditional and procedural knowledge before the treatment.

The one-way ANOVA for MAN+META classes revealed that MAN+META classes were equal according to their declarative, conditional and procedural knowledge before the treatment.

The independent t-test analysis for MAN and MAN+META group revealed that MAN and MAN+META groups are equal according to their declarative, conditional and procedural knowledge before the treatment.

The mixed design analysis of variance indicated that there is not a significant difference between MAN and MAN+META. This means that both groups responded equally well to treatment for change in their scores on the two outcome measures: Pretests and Posttests.

Qualitative analysis results revealed that there were two themes: use of the manipulative on polygons and the role of the self-metacognitive questioning. Students in both treatment group students' opinions related with the effect of physical manipulative instruction were very positive. Use of the physical manipulative affected their learning positively since they were easier, more logical, interesting, and concrete for them. Students also stated that use of physical manipulative provided active involvement and this active involvement made students learn definitions polygons and properties of polygons. Students also mentioned that working as groups affected their learning. Group works facilitated them to learn the responsibility, provided motivation to learn and enabled them to acquire knowledge by seeing others' behaviors, receiving different ideas, understanding others' points of view. Students emphasized that the excitement they felt during the activities has also affected their learning. Exciting and interesting classroom environment took their attention and provided them learn better.

Students in MAN+META group reported that asking self-metacognitive questions made students solve problems in systematically. Using self-metacognitive questioning made students investigate relevant and irrelevant information to find what they need to solve the problem and forced students make revision of the previous subjects. Students emphasized that self meta-cognitive questions took their

attention to the structure of the given problem and helped them to find the solution of the problem. Moreover, self meta-cognitive questions paid their attention check whether their solutions make sense or not. Students also stated that self-metacognitive questions gave them a chance to check the solution of the problem.



## **CHAPTER V**

### **DISCUSSIONS AND IMPLICATIONS**

#### **5.1 DISCUSSIONS**

The main objective of this study was to investigate the effect of physical manipulative with and without self-metacognitive instruction on students' gaining declarative, conditional and procedural knowledge on geometry. When we compare the means of the scores of the posttests of the three knowledge tests, we can see that MAN+META group facilitated slightly better than the MAN group, however, the difference between two groups was not statistically significant.

The reasons for not being able to observe a significant difference between MAN and MAN+META group can be listed as follows:

(1) Teacher questioning: Olkun & Toluk (2005) studied the effect of teacher questioning on use of manipulative in geometric shapes. They proposed that questions such as "How did you find? Why..., explain..." asked by the teachers increase effect of use of physical manipulative. By this way students' relational understanding of plane geometric shapes can be utilized in "hands-on mind-on environment" (p. 9). Similarly, Waite-Stupiansky and Stupiansky (1998) and Heddens (1997) suggest teachers to ask probing questioning to focus on children's way of thinking rather than asking the correct answer. While using physical manipulative, asking traditional questions which focus on calculating correct answer should be replaced by asking why, how questions. Although the teachers asked these kinds of questions, this may also help students develop ability to ask similar questions themselves, which resembles those employed in self-metacognitive questioning. In my study, the teachers were instructed to follow teacher-questioning methods in physical manipulative as explained above. This may cause MAN group

students to develop a kind of self-metacognitive questioning them, thus, decreasing the difference between two groups in this sense.

(2) Individual versus group settings: In literature, it was found that asking self-metacognitive questions in small groups was more effective than asking in individual (Kramarski, Maverech & Arami, 2004; Maverech, 1999; Maverech & Kramarski, 2004). They found that asking this kind of questions was more suitable in small groups since this gave students a chance to discuss the tasks and help each other to understand missing points. In this study, self-metacognitive questioning was practiced in individual settings due to physical limitations. If this technique had been practiced in small groups, there could have been a significant difference between the MAN and MAN+META groups.

(3) Application period: Lester, Garofalo & Kroll (1989) found that using self-metacognitive questioning was more effective when it was practiced over prolonged period with the day-to-day regular exercises rather than integrated in a unit. In this study, self-metacognitive questioning was practiced for six weeks. Its effects could have been more significant if it was practiced over a longer period.

The slight improvement in knowledge acquisition in MAN+META group compared to the MAN group can be explained by the ability of systematic thinking that is introduced through four kinds of self-metacognitive questions:

(i) *Comprehension questions* can be of the form “what is the meaning of ...?”, “what does ... represent?” etc. Asking this kind of questions helped students understand what the problem was and encouraged them to think what was needed to be able to solve the problem. This forced the students review the definitions, properties, facts etc. thus enhance their declarative knowledge. Similarly, Kramarski (2004) and Kramarski & Maverech (2003) found that asking and answering comprehension questions made students focus on relevant and irrelevant parts of the problem by reflecting on problem solving processing.

(ii) *Connection questions* such as “How is this problem/task different/similar from what you have already solved?” provide students a chance to relate previous knowledge with the existing one. Gourgey (1989) and Kramarski, Maverech & Arami (2004) found that using connection questions made students become sensitive to relevant parts of the related tasks and look for all the information about the tasks. These finding related to the gaining declarative knowledge.

(iii) & (iv) Both *strategic questions* and *reflection questions* helped students improve their procedural knowledge. Strategic questions such as ““what strategy/tactic/principle can be used in order to solve the problem/task?”” leads students to make a plan and think each step of the solution. Reflection questions such as “does the solution make sense” may force students go over the solution steps, i.e. the whole procedure when a result that does not make sense is obtained.

Although there was not a significant difference between MAN and MAN+META groups, the findings from this study indicate that use of physical manipulative improved students’ declarative, conditional and procedural knowledge. This effect can be explained by two factors: active involvement and working in small groups.

Active involvement gives students a chance to discover the definitions of the related unit themselves by touching, removing and feeling the physical manipulative. For example forming the seven piece mosaics in different shapes may make students discover the definition of polygons. Comparison of physical manipulative may make the students to realize the relationship between geometric tasks. Students realized the relationship between similar and equal polygons by comparing the folded papers. The students may also develop computational skills (i.e. procedural knowledge) through active involvement, such as first measuring the length of sides of the polygons and their perimeters by a ruler themselves, and then recording these in their notebooks and finally making calculations. In general, active involvement might play a crucial role both physically and cognitively. These findings are consistent with those of Bayram (2004), Garrity (1998), Bishop (1997), Battista & Clements (1996,

1998), Missetra (2000), and Owens & Clements (1992b, 1993) all of whom proposed that using physical manipulative help students constructing declarative and procedural knowledge.

The improvement in gaining declarative, conditional and procedural knowledge using physical manipulative can also be attributed to working in small groups, which is stated as the second factor above. Working with group may especially help acquirement of conditional knowledge. When we look at the mean scores of the PreConKT for both MAN and MAN + META group, they were 0.79 and 0.72 out of 24, respectively. These results show that students almost had no conditional knowledge prior to instruction. After the instruction, the mean scores were increased to 8.46 and 9.02 for MAN and MAN+META groups, respectively. Although the posttest results were still way below the full score, an improvement in conditional knowledge was observed in their explanations that they provided within their solutions. Working with groups may have an important role in this improvement since it facilitated a discussion environment and the students expressed themselves better in such a setting. They also helped each other and they stated that their complementary knowledge had an important impact.

## **5.2 INTERNAL VALIDITY**

The internal validity refers to the degree to which observed differences on the dependent variable are directly related to the independent variable, not to some other (extraneous) variable (Fraenkel & Wallen, 2003). Possible threats internal validity and the methods used to cope with them discussed in this section.

In this study, quasi-experimental design is used. The groups were randomly assigned to treatment. Quasi-experimental designs control the following threats: Subject characteristics, mortality, instrument decay, testing, history, maturation and regression. On the other hand, leaves location, data collector characteristics, data collector bias, attitudinal, and implementation threats to be controlled (Fraenkel & Wallen, 2003).

Data collector characteristics and data collector bias are assumed to be controlled by training teachers to ensure standard procedures under which data collected. The same curriculum, same materials, same tests are used for both MAN and MAN+META groups to cope with the attitudinal threat. Usage of the pre-tests for the tests assisted to verify that for the two groups had the same characteristics. In order to prevent Hawthorn effect, the self-metacognitive questions are written under the worksheets of MAN+META groups. Other group has taken the same worksheet without self-metacognitive questions. To control implementer effect, the teachers are trained by the researcher to standardize the conditions under which the treatments are implemented and also the researcher has made observations through out the study. Finally, to ensure the confidentiality, names of the students, teachers and the school are not stated at any part of the study.

### **5.3. EXTERNAL VALIDITY**

The external validity is the extent to which results of the study can be generalized (Fraenkel & Wallen, 2003). There are two kinds of external validity: Population generalizability and ecological generalizability. Population generalizability refers to the degree which a sample represents population of interest. Ecological generalizability refers to the degree, which the results of the study can be extended to other settings and conditions (Fraenkel & Wallen, 2003).

220 sixth grade students were chosen as a sample of convenience. This kind of non-random sample of convenience limits the generalizability of the study. Application of the testing procedure was conducted in ordinary classrooms for both pilot and main study groups during the regular class time, there possibly no remarkable differences among environmental conditions. Therefore, it assumed that the external effects were sufficiently controlled by the setting used in this study.

## 5.4 IMPLICATIONS OF THE STUDY

This study holds the following implications for educational practice:

The significant performance of the physical manipulative instruction on both MAN and MAN+META groups revealed that geometry topic of polygons can be taught effectively and efficiently in the specified period given in the curriculum by carefully developed physical manipulative instruction. In addition, the slightly significant better performance of the MAN+META group on the given instruments suggests that physical manipulative can be used with self-metacognitive questioning on problem solving. This kind of instruction can be developed in other topics of geometry and mathematics on different levels.

Curriculum developers should take the implementation of physical manipulative instruction into consideration during curriculum development process. They could involve physical manipulative instruction with self-metacognitive questioning as a teaching method in new curricula.

Authors of mathematics education books should consider physical manipulative with self-metacognitive instruction method as an effective teaching method in mathematics education and give example lesson plans in their books.

Pre-service teacher training programs should involve a course to inform prospective teacher about the benefits of physical manipulative with self-metacognitive instruction and assist them to gain knowledge and skills about preparation of lesson plans and implementation of lessons.

School administrators should help teachers on implementing physical manipulative with self-metacognitive lesson plans like providing physical manipulative sets, classes with more spaces to facilitate lessons. School administrators could prepare workshops about how to put into practice physical manipulative into considerations.

## **5.5 RECOMMENDATIONS FOR FURTHER RESEARCH**

Based on the results of this study, the following recommendations are made for further research:

In this study, the effect physical manipulative with and without self-metacognitive instruction assessed on students' declarative, conditional and procedural knowledge. Replication of the study can be studied on different topics of mathematics and different variables. Further research is recommended with the use of physical manipulative and physical manipulative combining with different teaching methods. Complete randomization if provided in a replication of this study would allow researcher to generalize over a wider population.

## REFERENCES

- Adams, R., & Wu, M. (2002). PISA 2000 Technical Report. OECD, Paris. Retrieved December, 6, 2007 from [www.pisa.oecd.org/dataoecd/53/19/33688233.pdf](http://www.pisa.oecd.org/dataoecd/53/19/33688233.pdf).
- Alexander, P. A., & Judy, J. E. (1988). The interaction of domain-specific and strategic knowledge in academic performance. *Review of Educational Research*, 58(4), 375-404.
- Baker, J. D., & Beisel, R. W. (2001). An experiment in three approaches to teaching average to elementary school children. *School Science and Mathematics*, 101(1), 23-31.
- Ball, S. (1988). Computers, concrete materials and teaching fractions. *School Science and Mathematics*, 88(6), 470-475.
- Baroody, A. J. (1989). Manipulative don't come with guarantees. *Arithmetic Teacher*, 37 (2), 4-5.
- Battista, M. T. (1999). Fifth graders' enumeration of cubes in 3D arrays: conceptual progress in an inquiry-based classroom. *Journal of Research in Mathematics Education*, 40(4), 417-448.
- Battista, M. T. & Clements, D. H. (1996). Students' understanding of three dimensional rectangular arrays of cubes. *Journal of Research in Mathematics Education*, 27, 258-292.
- Battista, M. T., Clements, D. H., Arnoff, J., Battista, K., & Van Auken Borrow, C. (1998). Students' spatial structuring of 2D arrays of squares. *Journal for Research in Mathematics Education*, 29(5), 503-532.
- Bayram, S. (2004). The effect of concrete models on eighth grade students' achievement and attitude toward geometry. Master's Thesis, Middle East Technical University, Ankara, Turkey.



- Beattie, I. A. (1986). Modeling operations and algorithms. *Arithmetic Teacher*, 33 (6), 23-28.
- Ben-Chaim, D., Lappan, G., & R. T. Houang. (1988). The effect of instruction on spatial visualization skills of middle school boys and girls. *American Educational Research Journal*, 25(1), 51-71.
- Berlin, D., & White, A. (1986). Computer simulations and the transition from concrete manipulation of objects to abstract thinking in elementary school Mathematics. *School Science and Mathematics*, 86, 468-479.
- Bishop, J. W. (1997). Understanding of mathematical patterns and their symbolic representations. Paper presented at the Annual Meeting of the American Educational Research Association, Chicago. ERIC Document. ED 410 107.
- Board of Education (2004). Talim Terbiye Kurulu Başkanlığı. 1-8 sınıflar matematik dersi eğitim programları. Retrieved December, 6, 2007 from <http://ttkb.meb.gov.tr>.
- Boulton-Lewis, G., Cooper, T., Atweh, B., Pillay, H., Wills, L. & Mutch, S. (1997). Processing load and the use of concrete representations and strategies for solving linear equations. *Journal of Mathematical Behavior*, 16 (4), 379-397.
- Branch, M. (2006). Making math manipulative: The implementation and best practice for integrating manipulative into the mathematics curriculum. Retrieved December, 6, 2007, from <http://www.wm.edu/education/599/06projects/branch.pdf>.
- Brown, A. L. (1987). Metacognition, executive control, self-regulation and other more mysterious mechanisms. In F. E. Weinert and R. H. Kluwe (eds.), *Metacognition, Motivation, and Understanding*, chapter 3, (pp.65–116). Lawrence Erlbaum Associates, Hillsdale, New Jersey.
- Bruner, J. S. (1966). *Studies in cognitive growth: collaboration at the center for cognitive studies*. New York: Wiley.
- Burns, M. (1985). The role of the questioning. *Arithmetic Teacher*, 32, 14-16

- Burns, M. (1996). How to make the most of math manipulative. *Instructor*, 105(7), 45-51.
- Cain-Caston, M. (1996). Manipulative queen. *Journal of Instructional Psychology*, 23, 270-274.
- Catcher, G. W., Pothier Y. M., Vance, J. M. & Bezuk, N. S. (2000). *Learning mathematics in elementary and middle school: Learning and teaching mathematics* (3rd ed.). Upper Saddle River: Pearson Education Inc.
- Clements, D. C., & Battista, M. (1986). Geometry and geometric measurement. *Arithmetic Teacher*, 33(6), 29-32.
- Clements, D. H. & Mcmillen, S. (1996). Rethinking concrete manipulative. *Teaching Children Mathematics*, 2, 270-279.
- Clements, D. C. (1997) (Mis?) Constructing constructivism. *Teaching Children Mathematics*, 4, 198-200.
- Cohen, J. (1988). *Statistical power analysis for the behavioral sciences* (2nd Ed.). Hillsdale, NJ: Lawrence Earlbaum Associates.
- Columbia Encyclopedia. (2001-07). Pestalozzi, Johann Heinrich (6th Ed.). New York: Columbia University Press. Retrieved December, 6, 2007 from <http://www.bartleby.com/65/pe/Pestaloz.html>
- Copeland R.W. (1970). *How children learn mathematics: Teaching implications Piaget's research* (3rd ed.). New York: Mcmillan.
- Corry, M. (1996). Jean Piaget's Genetic Epistemology. Retrieved December, 6, 2007 from <http://home.gwu.edu/~mccorry/corry2.htm>
- Cotter, J. A. (2000). Using language and visualization to teach place value. *Teaching Children Mathematics*, 7(2), 108-114.

- Cramer, A. K., Post, T. R. & delMas, R. C. (2002). Initial fractions by fourth and fifth grade students: A comparison of the effect of using commercial curricula with effects of using rational number project. *Journal for Research in Mathematics Education*, 33(2), 111-144
- Creswell, J. W. (1994). *Research design: qualitative and quantitative approaches*. California: Sage Publications.
- Denman, T. (1984). Great kits you can buy. *Instructor*, 44(2), 59.
- Denu, B.G. (1992). The marvels of manipulative. *Instructor*, 10(8), 44-45.
- Dienes, Z. P. And E. W. Goldin (1971). *Approach to modern mathematics*. New York: Herder and Herder.
- Evans, R. (1973). *Jean Piaget: The Man and his ideas*. New York: E. P. Dutton & Co., Inc.
- Flavell, J. H. (1979). Metacognition and cognitive monitoring: A new area of cognitive developmental inquiry. *American Psychologist*, 34(10), 906–911.
- Fraenkel, J. & Wallen, N. (2003). *How to design and evaluate research in education*. (5th ed.). New York: Mc-Graw Hill.
- Fraser, P. (1993). *New Mathematics. Book I*. Ankara: TED Ankara College Foundation
- Fuson K. C. & Briars, D. J. (1990). Using base-ten blocks learning/teaching approach for first and second grade place value and multidigit addition and subtraction. *Journal for Research in Mathematics Education*, 21(3), 180-206
- Gagné, R. M., & Briggs, L. J. (1979). *Principles of instructional design*. New York: Holt, Rinehart and Winston.

- Garner, R. (1990). When children and adults do not use learning strategies: Toward a theory of settings. *Review of Educational Research*, 60, 517-529.
- Garofalo, J. & Lester, K. F. (1985) Metacognition, cognitive monitoring, and mathematical performance. *Journal for Research in Mathematics Education*, 16(1), 163-176.
- Garrity, C. (1998). Does the use of hands-on-learning, with manipulative, improves the scores of secondary school education geometry students: an action research projects submitted to Saint Xavier University (Chicago, Illinois). Available ERIC Document. ED 422 179.
- Gourgey, A. F. (1998). Metacognition in basic skills instruction. *Journal not defined*, 26: 81 Netherlands: Kluwer Academic Publishers.
- Hair, J. F., Anderson, R. E., Tatham, R.L., & William, C. B. (1995). *Multivariate analysis with readings* (4th ed.) London: Collier McMillen.
- Hall, N. (1998). Concrete representations and the procedural analogy theory. *Journal of Mathematical Behavior*, 17(1), 33-51.
- Harston, R. & Boren, S. (1990). Experiential learning: Using manipulative. Available ERIC Document. ED321967.
- Heddens, J. W. (1997). Improving mathematics by using manipulative. Retrieved December, 6, 2007, from <http://www.fed.chuck.edu.hk/~flee/mathfor/edumath/9706/13heddens.html>.
- Heuser, D. (2000). Mathematics class becomes learner centered. *Teaching Children Mathematics*, 6(5), 288-95.
- Hiebert, J., & Lefevre, P. (1986). *Conceptual and procedural knowledge in mathematics: An introductory analysis*. In J. Hiebert (Ed.). Hillsdale, NJ: Erlbaum.

- Joyner, M. J. (1990). Using manipulative successfully. *Arithmetic Teacher*, October, 6-7
- Kaya, R. A., & Salman M. (1997). *Ortaokullar İçin Matematik 1*. İstanbul: Taş Kitapçılık & Yayıncılık.
- King, A. (1989). Effects of self-questioning training on college students' comprehension of lectures. *Contemporary Educational Psychology*, 14, 366-381.
- King, A. (1990). Enhancing peer interaction and learning in the classroom through reciprocal questioning. *American Educational Research Journal*, 27(4), 664-687.
- King, A. (1992). Comparison of self-questioning, summarizing, and note taking-review as strategies for learning from lectures. *American Educational Research Journal*, 29(2), 303-323.
- Kjos, R. & Long, K. (1994). Improving critical thinking and problem solving in fifth grade mathematics. An action research project submitted at Saint Xavier University. Available ERIC Document ED 383 525.
- Kramarski, B. (2004). Making sense of graphs: Does metacognitive instruction make a difference on students' mathematical conceptions and alternative conceptions? *Learning and Instruction*, 14, 593-619.
- Kramarski, B., & Mevarech, Z. R. (2003). Enhancing mathematical reasoning in the classroom: The effects of cooperative learning and metacognitive training. *American Educational Research Journal*, 40(1), 281-310.
- Kramarski, B., Mevarech, Z. R. & Arami, M. (2004). The effects of metacognitive instruction on authentic task. *Educational Studies*, 49, 225-250.
- Krech, B. (2000). Model with manipulative. *Instructor*, 109 (8), 6

- Lane, S. (1993). The conceptual framework for the development of a mathematics performance assessment instrument. *Educational Measurement: Issues and Practice, Summer*, 16-23.
- Leech, L. N., Barret C.K., & Morgan, G. A. (2005). *SPSS for intermediate statistics: Use and interpretation*. (2nd Ed.). Mahmah: New Jersey.
- Lester, F. K., Garofalo, J. & Kroll, D. L. (1989). The role of metaconition in problem solving. Final report. ERIC Document. ED 314 255.
- Martin, T., Lukong, A., & Reaves (2007). The role of manipulative in arithmetic and geometry tasks. *Journal of Education and Human Development*, 1(1), 1-14.
- MEB. (2006). *İlköğretim Matematik 6*. Ankara: Devlet Ders Kitapları Dizisi.
- Mevarech, Z. R. & Kramarski, B. (1997). IMPROVE: A multidimensional method for teaching mathematics in heterogenuous classrooms. *American Educational Research Journal*, 34, 365-394.
- Mevarech, Z. R. (1999). Effects of metacognitive training embedded in cooperative settings on mathematical problem solving. *The Journal of Educational Research*, 92(4), 195-205.
- Mevarech, Z. R. & Gutman, M. (2006). How can self-regulated learning be supported in mathematical e-learning environments? *Journal of Computer Assisted Learning*, 22, 24-33
- Mevarech, Z. R. & Fridkin, S. (2006). The effects of IMPROVE on mathematical knowledge, mathematical reasoning and meta-cognition. *Metacognition Learning*, 1, 85-97.
- Moch, P.L. (2001). Manipulative work. *The Educational Forum*, 66(1), 81-87.
- Moyer, P. S., Bolyard, J. J., & Spikell, M. A. (2002). What Are Virtual Manipulative? *Teaching Children of Mathematics*, 8(6), 372-377.

- National Council of Teachers of Mathematics. (2000). Principles and standards for school mathematics. Reston, VA: NCTM.
- Olkun, S. & Toluk, Z. (2004). Teacher questioning with an appropriate manipulative may make a big difference. *IUMPST: The Journal*, 2, 1-11. Retrieved December, 6, 2007 from [www.k-12prep.math.ttu.edu](http://www.k-12prep.math.ttu.edu).
- Outhred, L. N. & Mitchelmore, M. C. (2000) Young childrens' intuitive understanding rectangular area measurement. *Journal For Research in Mathematics Education*, 31(2), 144-167
- Owens K. D., (Ken) Clements M. A. (1998) Representation in spatial problem solving in the classroom. *Journal of Mathematical Behavior*, 17(2), 197-218.
- Özer, H., Budak, M., Altnordu, R., & Çatal, Z. (2001). *İlköğretim Matematik Kitabı. Öğretmen Kılavuzu*. İstanbul: Özer Yayıncılık.
- Palinscar, A. S. & Brown, A. L. (1984) Reciprocal teaching of comprehension fostering and monitoring activities. *Cognition and Instruction*, 1(2), 117-175.
- Pallant, J. (2001). SPSS Survival Manual. Philadelphia: Open University Press.
- Pesek D. D. & Kirshner D. (2000). Interference of instrumental instruction in subsequent relational learning. *Journal for Research in Mathematics Education*, 31(5), 524-540.
- Piaget, J. (1968). *Genetic Epistemolog*, a series of lectures delivered by Piaget at Columbia University. Retrieved December, 6, 2007 from <http://www.marxists.org/reference/subject/philosophy/works/fr/piaget.htm>
- Post, T. R. (1981). The role of manipulative materials in the learning of mathematical concepts. Retrieved December, 6, 2007 from [http://cehd.umn.edu/rationalnumberproject/81\\_4.html](http://cehd.umn.edu/rationalnumberproject/81_4.html)
- Reimer, K., Moyer Patrica S. (2005). Third grades learn about fractions using virtual manipulative: A classroom study. *Journa. Of Computers in Mathematics and Science Teaching*, 24(1), 5-25.

- Rittle-Johnson, B., & Alibali, M. W. (1999). Conceptual and procedural knowledge of mathematics: Does one lead to the other? *Journal of Educational Psychology, 91*(1), 175-189.
- Rittle-Johnson, B., Siegler, R. S., & Alibali, M. W. (2001). Developing conceptual understanding and procedural mathematics: An iterative process. *Journal of Educational Psychology, 93*(2), 346-362.
- Rust, A. (1999). A study of the benefits of math manipulative versus standard curriculum in the comprehension of mathematical concepts. Dissertation Paper. ERIC document. ED 436 395.
- Schraw, G. (1998). Promoting general metacognitive awareness. *Instructional Science, 26*(1-2), 113-125.
- Smilkstein, R. (1993). Acquiring knowledge and using it. Dissertation paper. ERIC Document. ED 382 238.
- Smith, P. L., & Ragan, T. J. (1993). *Instructional Design*. New York: Macmillan.
- Soylu, Y. (2007) The role of the geometric models in the explanation of determinant and the properties of a determinant. *Turkish Online Journal of Distance Education, 8*(1), 12-22.
- Sowell, J. E. (1989). Effects of manipulative materials in mathematics instruction. *Journal for Research in Mathematics Education, 20*(5), 498-505.
- Stein, M. K., & Bovalino, J. W. (2001). Manipulative: One piece of the puzzle. *Mathematics Teaching in the Middle School, 6*(6), 356-60.
- Suh, J. & Moyer, P. S. (2007). Developing students' representational fluency using virtual and physical algebra balances *Journal of Computers in Mathematics and Science Teaching, 26*(2), 155-173



- Suydam, M. N., & Higgins J. L. (1976). Review and synthesis of studies activity-based approach to mathematics teaching. Final Report, NIE Contract No.400-75-0063.
- Swanson, H. L. (1990). Influence of metacognitive knowledge and aptitude on problem solving. *Journal of Educational Psychology*, 82(2), 306-314.
- Szendrei, J. (1996). Concrete materials in the classroom. *International Handbook of Mathematics Education*, (pp.411-434). Netherlands: Kluwer Academic Publisher.
- Tankersley, K. (1993). Teaching math their way. *Educational Leadership*, 50, 12-13.
- Teppo, A. R. (1991). Van Hiele level of geometric thoughts revisited. *Mathematics Teacher*, 84(3), 210-221.
- Thompson, W. P. (1992). Notations, conventions, constrains: Contributions to effective uses of concrete materials in elementary mathematics. *Journal for Research in Mathematics Education*, 23(2), 123-147.
- Van Hiele P. M. (1999). Developing geometric thinking through activities that begin with play. *Teaching Children Mathematics*, 5(6), 310-316
- Waite-Stupiansky, S. & Stupiansky, N. G. (1998). Math in action: Hands-on, minds-on math. *Instructor*, 108(3), 85.
- Ward, S. L., Byners J. P., & Overton, W. F. (1990). Organization of knowledge and conditional reasoning. *Journal of Educational Psychology*, 82(4), 832-837.
- Webb, N. L. (1979). Processes, conceptual knowledge, and mathematical problem-solving ability. *Journal for Research in Mathematics Education*, 10(2), 83-93.
- Wearne D. & Hiebert J. (1988). A cognitive approach to meaningful mathematical instruction: Testing a local theory using decimal numbers. *Journal for Research in Mathematics Education*, 19(5), 371-384.

Williamson, R. A. (1996). Self-questioning: An aid to metacognition. *Reading Horizons, 37*, 30-47

Young, L. S. (1983). How Teacher educators use manipulative materials with preservice teachers. *Teaching Children Mathematics, 11*, 12-13.

## APPENDIX A

### KNOWLEDGE TESTS

#### DECLARATIVE KNOWLEDGE TEST

İsim:

Soyisim:

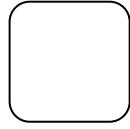
Sınıf:

No:

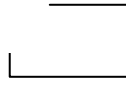
Bu testte 18 tane çoktan seçmeli soru vardır.

1. Aşağıdakilerden hangisi çokgendir?

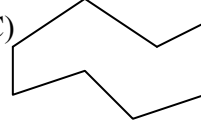
A)



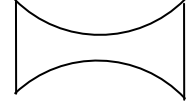
B)



C)

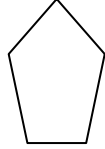


D)



2. Aşağıdakilerden hangisi çokgen değildir?

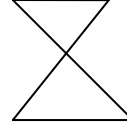
A)



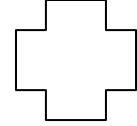
B)



C)



D)



3. Aşağıdakilerden hangisi daima doğrudur?

A) Bütün kapalı şekiller çokgendir.

B) Köşeleri olan bütün geometrik şekiller çokgendir.

C) Üç veya daha fazla doğrunun kesişmesiyle oluşan kapalı şekiller çokgendir.

D) İki veya daha fazla doğrunun kesişmesiyle oluşan şekiller çokgendir.

4. Benzer üçgenler \_\_\_\_\_ açılara \_\_\_\_\_ kenarlara sahiptir.cümlesinde boşluklara gelmesi gereken kelimeler aşağıdakilerden hangisidir?

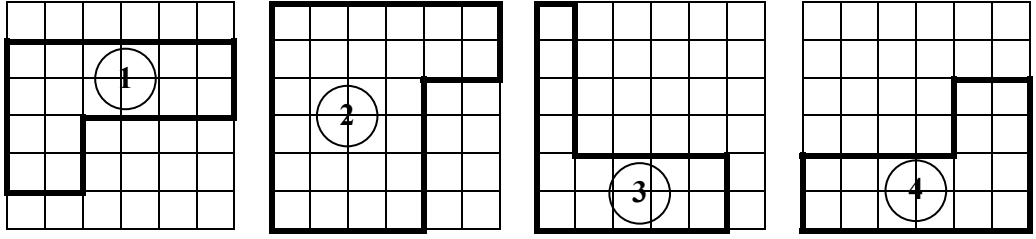
A) eşit, eşit

B) eşit, orantılı

C) orantılı, eşit

D) orantılı, orantılı

5. Aşağıdaki çokgenlerden hangileri benzedir?



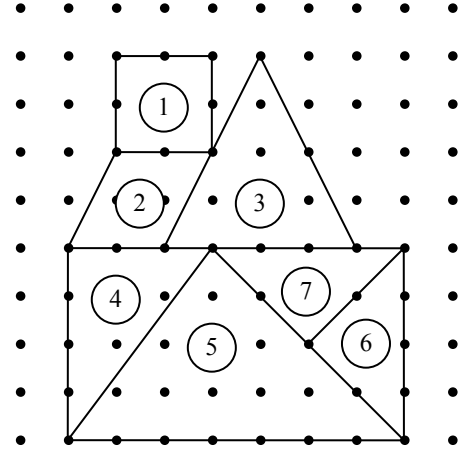
- A) 1 ve 2    B) 1 ve 3    C) 1 ve 4    D) 3 ve 4

6. Düzgün çokgenler \_\_\_\_\_ açılara \_\_\_\_\_ kenarlara sahiptir cümlesinde boşluklara gelmesi gereken kelimeler aşağıdakilerden hangisidir?

- A) eşit, eşit    B) eşit, orantılı    C) orantılı, eşit    D) orantılı, orantılı

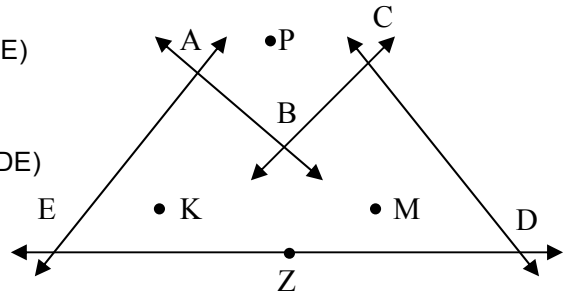
7. Yandaki şekilde eş çokgenler hangileridir?

- A) 1 ve 2    B) 3 ve 5  
C) 4 ve 7    D) 6 ve 7

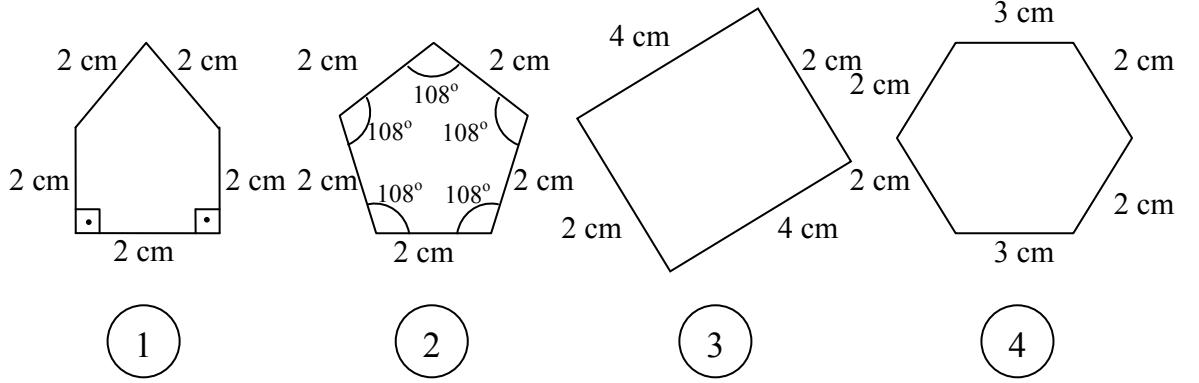


8. Yandaki şekile göre aşağıdakilerden hangisi doğrudur?

- A)  $p \in \text{iç}(ABCDE)$     B)  $k \notin \text{iç}(ABCDE)$   
C)  $z \in \text{dış}(ABCDE)$     D)  $m \in \text{iç}(ABCDE)$



9. Aşağıdaki şekillerden hangisi düzgün çokgendir?



- A) 1 B) 2 C) 3 D) 4

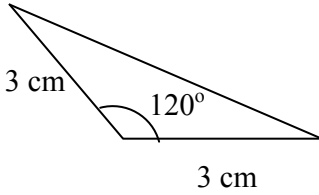
10. Bir üçgenin iki iç açısının ölçüsü  $45^\circ$  ise, bu üçgen aşağıdakilerden hangisidir?

- A) İkizkenar B) Çeşitkenar C) Geniş açılı D) Dar açılı

11. Açılarının ölçüleri 48, 62 ve 70 olan üçgen aşağıdakilerden hangisidir?

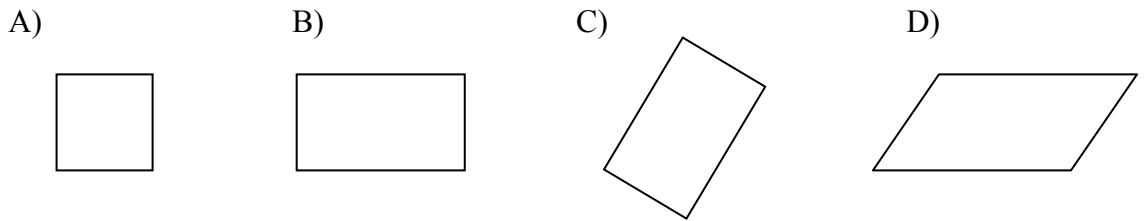
- A) Geniş Açılı B) Dar Açılı C) İkizkenar D) Eşkenar

12. Yandaki üçgen için en uygun sınıflandırma aşağıdakilerden hangisidir?



- A) Geniş açılı, çeşitkenar üçgen  
B) Geniş açılı, ikizkenar üçgen  
C) Dar açılı, çeşitkenar üçgen  
D) Dar açılı, ikizkenar üçgen

13. Aşağıdakilerden hangisi dikdörtgen değildir?



14. Aşağıdakilerden hangisi Karenin özelliklerinden biri değildir?

- A) Dört kenarı eşittir. B) Dört açısının ölçüsü eşittir  
C) Köşegenleri dik açı ile kesişir. D) Köşegenlerinin uzunlukları eşit değildir.

15. Aşağıdakilerden hangisi dikdörtgenin özelliklerinden biridir?
- A) Dört kenarı eşittir.                      B) Köşegenleri 90°'lik açı ile kesişirler.  
C) Köşegenler birbirini ortalar.        D) Köşegenlerin uzunlukları eşit değildir.
16. Bir çokgeni tanımlamak için en az kaç kenara ihtiyaç vardır?
- A) 2                                      B) 3                                      C) 4                                      D) 5
17. Bir çokgenin çevresini hesaplariken;
- A) çokgenin en dış kısmını oluşturan kenarların uzunluklarını toplarız.  
B) çokgenin iç açıları toplanır.  
C) çokgenin köşe sayıları toplanır.  
D) çokgenin içinde veya dışındaki bütün kenar uzunlukları toplanır.
18. Çokgenlerin alanları ile ilgili olarak aşağıdakilerden hangisi yanlıştır?
- A) Bir çokgenin alanı o çokgenin yüzeyini kaplayan birim karelerin sayısıdır.  
B) Bir çokgenin alanı kenar sayısı arttıkça artar.  
C) Bir çokgen birden fazla çokgenin birleşiminden oluşuyorsa, alanı kendisini oluşturan çokgenlerin alanları toplamına eşittir.  
D) Çokgenlerin kenar uzunlukları değiştikçe alanları değişir.

## CONDITIONAL KNOWLEDGE TEST

İsim:

Soyisim:

Sınıf:

No:

1. “Eş çokgenler aynı zamanda benzerdir.” ifadesi doğru mu, yanlış mıdır? Doğru ise neden doğru olduğunu, yanlış ise neden yanlış olduğunu açıklayınız.
2. Bir çeşitkenar üçgen aynı zamanda dik açılı üçgen olabilir mi? Olabilirse neden olabilir? Olamazsa neden olamaz? Açıklayınız.
3. “Bir eşkenar üçgen aynı zamanda ikizkenar üçgendir” ifadesi doğru mu, yanlış mıdır? Doğru ise neden doğru olduğunu, yanlış ise neden yanlış olduğunu açıklayınız.
4. “Bir çokgende kenar uzunlukları eşit ise, açıları da eşittir” ifadesi doğru mu, yanlış mıdır? Doğru ise neden doğru olduğunu, yanlış ise neden yanlış olduğunu açıklayınız.
5. “Kare, dört kenarı eşit, bir dikdörtgendir” ifadesi doğru mu, yanlış mıdır? Doğru ise neden doğru olduğunu, yanlış ise neden yanlış olduğunu açıklayınız.
6. “Birim karelerle oluşturulan bir çokgenin alanı  $n$  birim kare ise, bu şeklin olası en büyük çevre uzunluğu  $2n+2$ ” ifadesi doğru mudur? Yanlış mıdır? Doğruluğunu veya yanlışlığını bir örnek üzerinde gösteriniz.

## PROCEDURAL KNOWLEDGE TEST

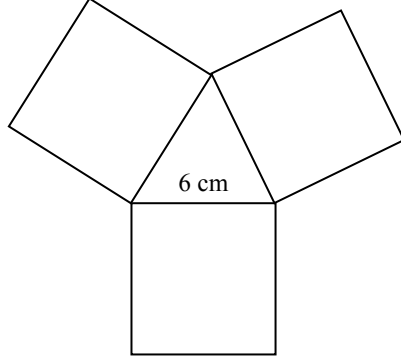
İsim:

Soyisim:

Sınıf:

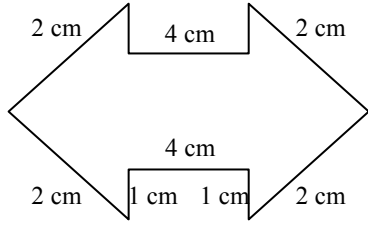
No:

1.



Yandaki şekilde bir eşkenar üçgenin üç kenarına kareler çizilmiştir. Eşkenar üçgenin bir kenarı 6 cm dir. Oluşan şeklin çevresi kaç cm dir? Açıklayarak yapınız.

2.



Yandaki şeklin çevresi kaç cm dir? Açıklayarak yapınız.

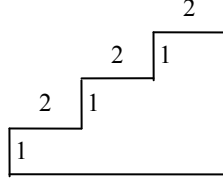
3. Çevre uzunluğu 44cm olan bir kare ile aynı kenar uzunluğuna sahip bir eşkenar üçgenin çevresi kaç cm dir? Açıklayarak yapınız

4. Dikdörtgen biçimindeki bir bahçenin çevresinin uzunluğu 260 m dir. Boyu eninin 2 katından 20 cm eksik ise bahçenin eni ve boyu cm dir? Açıklayarak yapınız

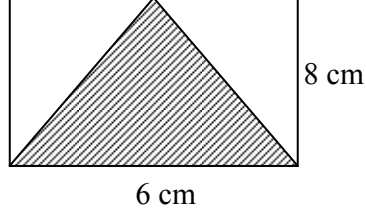
5. Alanı  $49 \text{ m}^2$  olan karenin alanının 4 katı alana sahip karenin kenar uzunluğu kaç kaç cm dir? Açıklayarak yapınız



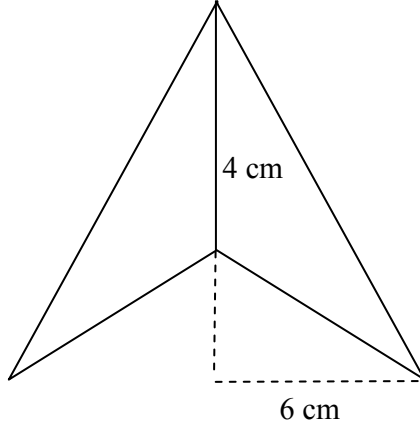
6. Yandaki şeklin alanı kaç  $\text{cm}^2$  dir? Açıklayarak yapınız



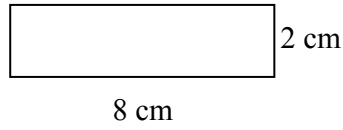
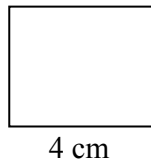
7. Yandaki taralı şeklin alanı kaç  $\text{cm}^2$  dir?



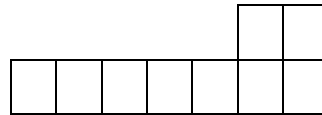
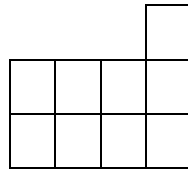
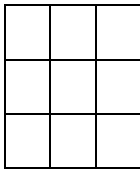
8. Yandaki şekildeki üçgenler eş üçgenlerdir. Buna göre şeklin alanı kaç  $\text{cm}^2$  dir? Açıklayarak yapınız



9. Aşağıdaki kare ve dikdörtgenin çevre ve alanları için neler söylenebilir? Açıklayarak yapınız



- 10.



Alanları eşit olarak verilen şekillerin çevre uzunluklarını bulunuz. Aynı alana sahip olası en büyük çevre uzunluğunu veren cebirsel ifadeyi bularak uygun şekli çiziniz.

## APPENDIX B

### SCORING RUBRIC FOR THE GEOMETRY KNOWLEDGE TEST

#### CONDITIONAL KNOWLEDGE QUESTIONS

Visual Skills: interpreting statements.

Verbal Skills: correct use of terminology, accurate communication in describing relationships.

Drawing Skills: appropriate use of symbols and notations.

Logical Skills: formulating and testing hypothesis, making inferences, using counter-explanations, develop mathematical arguments about geometric relationships

#### Score Description

##### 0

\_ No answer attempted.

\_ Copies parts of the problem without attempting a solution.

\_ Uses irrelevant information.

\_ Includes conditional knowledge which completely misrepresent the problem situation.

##### 1

\_ Shows very limited explaining of the principles, theorems, relations, and statements.

\_ Fails to identify the important parts when expressing the “if-then” statements.

\_ Gives incomplete evidence of the explanation process.

\_ Places too much emphasis on unimportant relations when expressing the “if-then” statements.

## **2**

\_ Shows some of the limited explaining of the principles, theorems, relations, and statements.

\_ Identifies some important parts when expressing the “if-then” statements.

\_ The relations expressed in the “if-then” statement is difficult to interpret and the arguments given are incomplete and logically unsound.

## **3**

\_ Shows nearly complete explaining of the principles, theorems, relations, and statements.

\_ Identifies the most important parts when expressing the “if-then” statements.

\_ Shows general understanding of the relations in the “if-then” statements.

\_ Gives a fairly complete response with reasonably clear explanations or descriptions.

\_ Presents supporting logically sound arguments which may contain some minor gaps.

## **4**

\_ Shows explaining of the principles, theorems, relations, and statements.

\_ Identifies all the important parts when expressing the “if-then” statements.

\_ Shows understanding of the relations in the “if-then” statements.

\_ Gives a complete response with a clear, unambiguous explanation or description.

\_ Presents strong, supporting, logically sound and complete arguments which may include counter-explanations or different aspects.

## PROCEDURAL KNOWLEDGE QUESTIONS

Visual Skills: imaging

Verbal Skills: correct use of terminology

Drawing Skills: appropriate use of symbols and notations, accurate application of the algorithm.

Logical Skills: classification, recognition of essential properties of a geometrical concept, formulating and testing hypothesis, making inferences, using counter-explanations, appropriate use of the procedures, use visualization and spatial reasoning to solve problems.

### Score Description

**0**

- No answer attempted.
- Copies parts of the problem without attempting a solution.
- Uses irrelevant information.
- Includes procedural knowledge which completely misrepresent the problem situation.

**1**

- Makes major computational errors when employing the algorithms and rules.
- Reflects an inappropriate strategy for solving the problem.
- Gives incomplete evidence of a solution process.
- The solution process is missing, difficult to identify or completely unsystematic.

## **2**

- \_ Makes serious computational errors when employing the algorithms and rules.
- \_ Gives some evidence of the solution process.
- \_ The solution process is incomplete or somewhat unsystematic.
- \_ Makes significant progress towards completion of the problem but the algorithm is unclear.

## **3**

- \_ Executes algorithms and rules completely.
- \_ Computations are generally correct but may contain minor errors.
- \_ Gives clear evidence of a solution process.
- \_ The solution process is nearly complete and systematic.

## **4**

- \_ Executes algorithm and rules completely and correctly.
- \_ Reflects an appropriate and systematic strategy for solving the problem.
- \_ Gives evidence of a solution process.
- \_ The solution process is complete and systematic.

## APPENDIX C

### LESSON PLANS

#### DERS PLANI 1

**Ders:** Matematik

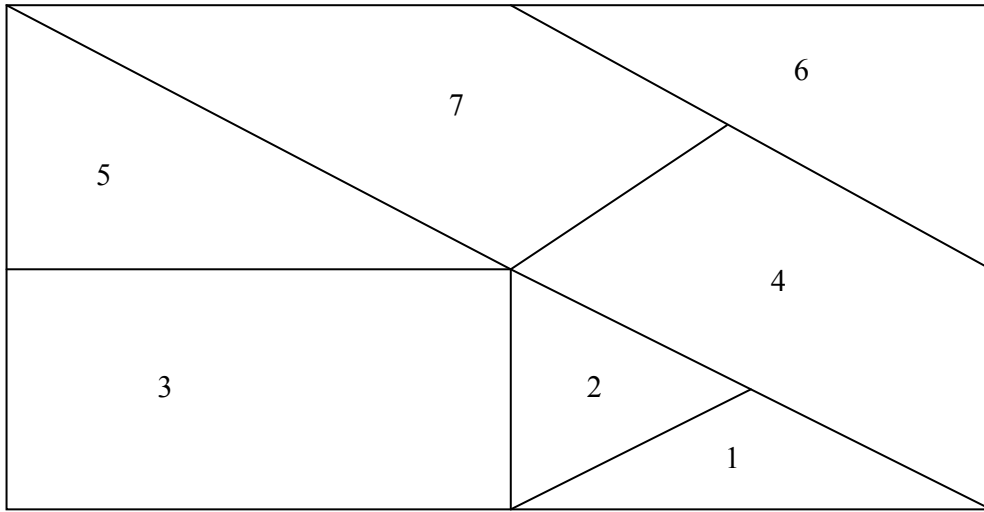
**Süre:** 2 ders saati

**Kazanımlar:**

1. Çokgenler tanımını yapar.

**Alt Öğrenme Alanı:**

Çokgenler



**Materyal:**

7 parça mozaik seti: 1 tane ikizkenar üçgen, 1 tane eşkenar üçgen, 2 tane dik üçgen, 1 tane dikdörtgen, 1 tane yamuk, 1 tane paralel kenar., 1 tane ikizkenar yamuk tan oluşur.

**Giriş:**

Her öğrenciye bir tane mozaik seti verilir. Materyali tanımları için 5 dk verilir. Materyaldeki parçaları kullanarak istedikleri şekilleri yapabilecekleri söylenir. Mesela ev veya adam gibi.

**Gelişme:**

1. Öğrencilerden 2 parça kullanarak, materyaldeki başka bir parçayı elde etmeleri istenir. Mesela 5 veya 6'yı kullanarak 3 yapabilecekleri söylenir.
2. Elde ettikleri parçayı başka parçalar kullanarak yapmaları istenir
3. Daha sonra üç parça kullanarak, materyaldeki başka bir parçayı elde etmeleri istenir. Öğretmen, grupların arasında dolaşarak gözlem yapar.
4. Öğrencilerden istedikleri parçaları kullanarak bir geometrik şekil elde etmeleri istenir. Gruplarındaki arkadaşlarıyla bu şekilleri paylaşmaları istenir. Her grubun sözcüsü gruplarında ne yaptıklarını sınıfa anlatır.
5. Defterlerine elde ettikleri geometric şekillerin etrafından çizerek. Hangi parçalardan elde ettiklerini tablo yaparak yazmaları istenir. Öğretmen her grup sözcüsünün söylediklerini tahtaya yazar.
6. Tahtaya yazılanlara göre çokgen tanımı öğrencilere yaptırılır. Öğretmen öğrencilerden fikirleri tahtaya maddeler halinde yazar. Sınıfla birlikte tartışma ortamı yaratır. Bu tartışmanın sonucunda; Kapalı şekillerin çokgen oldukları, çokgende kenarların kesişmemesi gerektiği ve kenarların doğruların kesişmesiyle oluştuğu vurgulanır. Çokgenlerin kenarlarına göre isimlendirildiği de öğrencilere buldurulur (Üçgen, dörtgen, beşgen ...).
7. Bir çokgende her bir köşenin ve açının ikişer doğrunun kesişmesiyle oluştuğunu vurgulamak için D.K sayfa 85 deki örnek sınıfta uygulanır.
8. Öğrencilere derste yapmaları için Çalışma Kağıdı 1 verilir.

**Ödev:**

Dergi, gazete, vb. yayınlarda gördüğünüz çokgen modellerini kesip bir kağıda yapıştırınız.



## **DERS PLANI 2**

**Ders:** Matematik

**Süre:** 2 ders saati

### **Kazanımlar:**

1. Düzgün çokgenin tanımını yapar. Düzgün çokgen olanlarla, düzgün çokgen olmayanları ayırır.

### **Alt Öğrenme Alanı:**

Çokgenler

### **Materyal:**

7 parça mozaik seti

### **Giriş:**

Çokgen tanımı ile ilgili sorular sorarak tekrar yapılır.

### **Gelişme:**

1. Öğrencilerden materyaldeki iki veya daha fazla şekli kullanarak önce bir dikdörtgen, sonra bir kare elde etmeleri istenir.

2. Kare ve dikdörtgenin benzer özellikleri öğrencilere sorulur. Her grubun aralarında tartışıp sonucu sınıfla paylaşması istenir.

3. Kare ve dikdörtgenin farklı özellikleri sorulur. Her grubun aralarında tartışıp sonucu sınıfla paylaşması istenir.

4. Kare ve dikdörtgen gibi benzer veya farklı özellikleri olan başka iki geometrik şekiller bulup bulamayacağımız öğrencilere sorulur. (5 kenarı eşit, 5 açısı eşit olan bir beşgenle, kenarları ve açıları farklı bir beşgen gibi örnekler öğrencilere bulurtulur)

5. Kare ve dikdörtgen karşılaştırılarak düzgün çokgen ve düzgün olmayan çokgenin tanımları öğrencilere yaptırılır. Düzgün çokgen ve düzgün olmayan çokgen arasındaki fark vurgulanır.

6. Öğrencilere ders kitaplarındaki sayfa 87 deki alıştırmayı yaptırılır

7. Öğrencilere Çalışma Kağıdı 2 dağıtılır.

### **Ödev:**

Dergi, gazete, vb. yayınlarda gördüğünüz düzgün ve düzgün olmayan çokgen modellerini kesip bir kağıda yapıştırınız.

### **DERS PLANI 3**

**Ders:** Matematik

**Süre:** 2 ders saati

#### **Kazanımlar:**

1. Eşlik ve benzerlik arasındaki ilişkiyi açıklar.
2. Eş ve benzer çokgenlerin kenar ve açı özelliklerini açıklar.

#### **Alt Öğrenme Alanı:**

Çokgenler

#### **Materyal:**

Öğrencilerin getirdikleri yapraklar, makas, A4 kağıdı.

#### **Giriş:**

1. Öğrencilere eşlik ve benzerlik hakkında ne bildikleri sorulur.
2. Çevrede bulunan bir ağaçtan farklı yapraklar sınıfa getirilerek, karşılaştırılır. Bu yaprakların birbirine benzeyip benzemediği sorularak öğrencilerin görüşü alınır.
3. Ders kitabındaki örnekler inceleyerek öğrencilerin benzerlik ve eşlik konusundaki bilgileri alınır.günlük hayattan eş ve benzer şekillere örnek vermeleri istenir. (Fotokopi makinesi ve fotoğrafların büyütülmesi ve küçültülmesi örnek verilebilir.)
4. Eşlik ve benzerliğin tanımını öğrencilerden kendi cümlelerini kullanarak yapmaları istenir.

#### **Gelişme:**

1. A4 kağıdını düzgün bir şekilde kendi üzerinde 3 kez katladıktan sonra düzgün bir çokgen çiziniz.Çizdiğiniz çokgeni kenarlarından keserek kağıttan ayırınız.Oluşan çokgensel bölgeleri karşılaştırınız. Değişik şekilde çokgen çizen öğrencilerin yaptıkları sınıfta örnek gösterilir.

Elde edilen çokgensel bölgeler arasındaki ilişkiyi açıklayınız.

2. A4 kağıdını düzgün bir şekilde kendi üzerinde 3 kez katladıktan sonra açınız. Kat çizgilerinden oluşan dikdörtgensel bölgelere numara veriniz. Numara verdiğiniz dikdörtgensel bölgelerle A4 kağıdı arasında ne gibi bir ilişki var?
3. Bu iki etkinlikle öğrencilere eş ve benzer çokgenler arasındaki fark sorulur? Eş çokgenlerin karşılıklı açı ve kenarlarının eş olduğu, benzer çokgenlerin ise karşılıklı açıların eş, ancak kenar uzunluklarının farklı olduğu vurgulanır.
4. Eş şekillerin benzer, ama benzer şekillerin eş olamacağı vurgulanır.
5. Ders kitabı sayfa 89'daki örnek yaptırılır.
6. Çalışma Kağıdı 3 öğrencilere dağıtılır.

**Ödev:** Günlük hayattaki kıyafetlerimizde eş veya benzer çokgenler kullanıyor muyuz? Modelleme yaparak getiriniz.

## **DERS PLANI 4**

**Ders:** Matematik

**Süre:** 4 ders saati

### **Kazanımlar:**

1. Üçgenleri açılarına ve kenarlarına göre sınıflandırır.

### **Alt Öğrenme Alanı:**

Çokgenler

### **Materyal:**

Geometri tahtası, renkli lastikler.Geometri tahtası, Resim-iş derslerinde öğrencilere kare şeklinde tahtanın üzerine çivi çaktırlarak 10\*10 boyutlarında yapılır.

### **Giriş:**

1. Öğrencilerin açı çeşitleri ile ilgili bilgileri sorularak, dar açı, geniş açı , dik açı ölçüleri hatırlatılır.
2. Öğrencilere ders kitaplarındaki fotoğraflardaki üçgen modelleri inceletilerek, günlük hayatta gördükleri üçgenlerden örnek vermeleri istenir.

### **Gelişme:**

1. Her öğrenciye bir geometri tahtası verilir.Geometri tahtasında istedikleri şekilleri yapmaları için 5 dk. verilir
2. Öğrencilerden geometri tahtasını ve lastikleri kullanarak aşağıdaki üçgenleri elde etmeleri istenir.

Üç tane dar açısı olan bir üçgen

Bir tane dik açısı olan bir üçgen

Bir tane geniş açısı olan bir üçgen

İki tane dik açısı olan üçgen

Bir tane dik açısı, bir tane açısı geniş açısı olan üçgen

İki tane açısı geniş açı olan üçgen

3. Oluşturduğunuz üçgenleri noktalı kağıda çiziniz.
4. Yukarıda istenilen üçgenlerden hangisini veya hangilerini oluşturamadınız.Neden? Düşüncelerinizi önce grubunuzla sonra, sınıfla tartışınız.
5. Öğrencilere üçgenleri açılarına göre nasıl sınıflandırabileceğimizi sorarız.
6. Öğrencilerden geometri tahtasını ve lastikleri kullanarak aşağıdaki üçgenleri elde etmeleri istenir.

Üç kenarının uzunlukları eşit olan bir üçgen

İki kenarının uzunlukları eşit olan bir üçgen

Üç kenarının uzunluđu farklı bir üçgen

7. Üçgenleri kenarlarına göre nasıl sınıflandırabiliriz.Önce grubunuzla, sonra sınıfla tartışınız.
8. Ders sonunda öğrencilere üçgenlerin sınıflandırılması ile ilgili kavram haritası yaptırılır.
9. Çalışma Kağıdı 4 dağıtılır.

**Ödev:** Günlük hayatta üçgenleri nasıl sınıflandırıyoruz. Bir paragraf yazınız.

## DERS PANI 5

**Ders:** Matematik

**Süre:** 4 ders saati

### **Kazanımlar:**

1. Kare ve dikdörtgenin açıları, kenarları ve köşegenleri arasındaki ilişkiyi açıklar.

### **Alt Öğrenme Alanı:**

Çokgenler

### **Materyal:**

Geometri tahtası, renkli lastikler

### **Giriş:**

1. Kare ve dikdörtgenin tanımları sorularak öğrencilerin ön bilgileri alınır.

### **Gelişme:**

1. Öğrencilerden geometri tahtasında istedikleri uzunluklara sahip bir dikdörtgen yapmaları istenir. Kenar uzunluklarını cetvelle ölçmeleri istenir. Ölçtüğü uzunlukları aşağıdaki tabloya yazmaları istenir.

Bir kenarının uzunluğu (kısa)	
Diğer kenarının uzunluğu (uzun)	
Bir köşegenin uzunluğu	
Diğer köşegen uzunluğu	
Köşegenlerin kesim nok. Diğer köşelere olan uzaklığı (4 parça)	
Bir köşegenin diğer köşelerle oluşturduğu açılarının ölçüsü	

2. Öğrencilere köşegenin tanımı sorulur. Köşegenin tanımı yapıldıktan sonra, daha önce yaptıkları dikdörtgenin köşegenini lastikle yapmaları ve uzunluğunu cetvelle ölçmeleri istenir.

3. Aynı işlemleri diğer köşegen için yapmaları istenir.

4. Başka köşegen çizilip çizilemeyeceği sorulur? Neden çizilemeyeceği tartışılır.

5. Köşegenlerin kesim noktasının, ayrı ayrı dört köşesine olan uzunlukları cetvelle ölçülür.

6. Ölçme sonuçlarına göre dikdörtgenin

a) Karşılıklı kenarları paralel ve aynı uzunluktadır.

b) Komşu kenarları birbirine diktir.

c) Köşegen uzunlukları birbirine eşittir. Sembolle gösterir.

d) Köşegenler birbirini ortalar.

Özellikleri vurgulanır.

7. Öğrencilerden geometri tahtasında istedikleri uzunluklara sahip bir kare yapmaları istenir kenar uzunluklarını cetvelle ölçmeleri istenir. Ölçtükları uzunlukları aşağıdaki tabloya yazmaları istenir.

Bir kenarının uzunluğu (kısa)	
Diğer kenarının uzunluğu (uzun)	
Bir köşegenin uzunluğu	
Diğer köşegen uzunluğu	
Köşegenlerin kesim nok. Diğer köşelere olan uzaklığı (4 parça)	
Bir köşegenin diğer köşelerle oluşturduğu açılar ölçüsü	

8. Yaptıkları karenin köşegenini lastikle yapmaları ve uzunluğunu cetvelle ölçmeleri istenir.

9. Aynı işlemleri diğer köşegen için yapmaları istenir.

10. Başka köşegen çizilip çizilemeyeceği sorulur? Neden çizilemeyeceği tartışılır.

11. Köşegenlerin kesim noktasının, ayrı ayrı dört köşesine olan uzunlukları cetvelle ölçülür.

12. Köşegenlerin, kenarlarla oluşturduğu açıları iletke kullanarak ölçmeleri istenir. Bu ölçümleri kayıt etmeleri istenir.

13. Ölçme sonuçlarına göre Kare'nin;

a) Bütün kenarları birbirine eşittir.

b) Komşu kenarları birbirine eşittir. Sembolle gösterir.

c) Köşegenler birbirini dik keserek ortalar.

d) Köşelerde oluşan komşu tümler açıları eşittir

Özellikleri vurgulanır.

14. Çalışma Kağıdı 5 dağıtılır.

**Ödev:** Günlük hayatta dikdörtgen ve karenin özelliklerini nerelerde kullanıyoruz. Bir kağıda yapıstırıp getiriniz.

## DERS PLANI 6

**Ders:** Matematik

**Süre:** 2 ders saati

### **Kazanımlar:**

1. Çokgenlerin çevresini hesaplar.
2. Çokgenlerin çevresi ile ilgili problem kurar ve çözer.

### **Alt Öğrenme Alanı:**

Çokgenler

### **Materyal:**

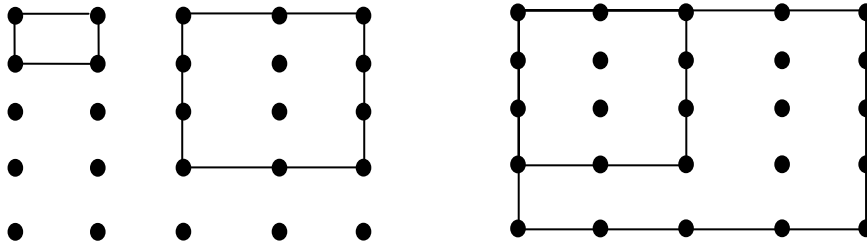
Geometri tahtası, renkli lastikler.

### **Giriş:**

1. Öğrencilere çevre kelimesinden ne anladıkları sorulur.
2. Öğrencilere günlük hayatta çevre hesaplamalarını nerede kullandığımız sorulur.

### **Gelişme:**

1. Geometri tahtasında istedikleri şekilleri yapmaları için 2-3 dk zaman verilir.
2. Sıralarının çevresini standart olmayan ölçme birimi kullanarak hesaplamaları istenir. (silgi, defter gibi).
3. Öğrencilerden geometri tahtasına 4 farklı dikdörtgen yapmaları her birinin çevresini, birim kareleri kullanarak ve cetvelle ölçerek çevrelerini hesaplamaları istenir.
4. Öğrencilere dikdörtgenin çevresini hesaplamak için nasıl bir formül kullanmamız gerektiği sorulur? Neden formüle ihtiyaç duyduğumuz sorulur?
5. Öğrencilerden aşağıdaki şekilleri geometri tahtasını kullanarak yapmalarını ve her bir şeklin çevresini hesaplamaları istenir.



6. Öğrencilere Çalışma Kağıdı 6 dağıtılır.



## DERS PLANI 7

**Ders:** Matematik

**Süre:** ders saati

### Kazanımlar:

1. Kare, dikdörtgen ve üçgenin alanını hesaplar.

### Alt Öğrenme Alanı:

Çokgenler

### Materyal:

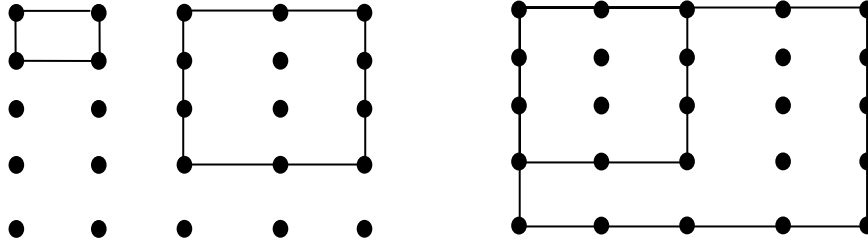
Geometri tahtası, renkli lastikler.

### Giriş:

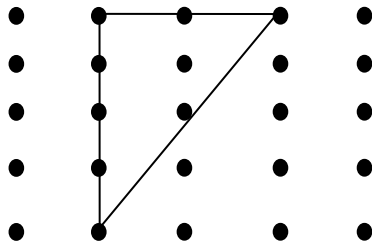
1. Öğrencilere alan kelimesinden ne anladıkları sorulur.
2. Öğrencilere günlük hayatta alan hesaplamalarını nerede kullandığımız sorulur.
3. Öğrencilerden defterlerinin yüzeyini kendi seçtikleri bir ölçü birimi (sözlük, silgi v.b) ile ölçmeleri istenir. Öğrenciler bulduklarını sınıfla paylaşırlar.

### Gelişme:

1. Geometri tahtasında istedikleri şekilleri yapmaları için 2-3 dk zaman verilir.
2. Öğrencilerden aşağıdaki şekilleri yaparak bu şekillerin alanını birim kare cinsinden bulmaları istenir.



3. Karenin alan formülü öğrencilere buldurtulur.
4. Öğrencilerden geometri tahtasında 2'ye 2 birimlik bir kare oluşturmaları istenir ve bu karenin alanını bulmaları istenir. Aynı kare üzerinde ikinci bir lastik kullanarak karenin alanının yarısına sahip bir dikdörtgen yapmaları istenir. Yine aynı kare üzerinde karenin alanının dörtte birine sahip bir kare yapmaları istenir.



5. Öğrencilerden yandaki şekli oluşturmaları istenir.İkinci bir lastikle bu şekili dikdörtgene tamamlamaları istenir. Dikdörtgenin ve üçgenin alanlarını bulmaları istenir. Üçgenin alanını bulmak için nasıl bir yol izledikleri sorulur.
6. Aynı örneği kareye tamamlanan bir üçgen çizdirilerek öğrencilere yaptırılır.
7. Dik üçgenlerin alan formülü öğrencilere buldurtulur.
8. Paralelkenarın da iki eş üçgene bölünebileceği öğrencilere gösterilerek üçgenler için genel alan formülü öğrencilere buldurtulur.
9. Öğrencilere Çalışma Kağıdı 7 dağıtılır.

**Ödev:** Günlük hayatta çokgenlerin alan hesaplamalarını nerelerde kullanıyoruz. Bu konuyla ilgili bir paragraph yazınız.

## DERS PLANI 8

**Ders:** Matematik

**Süre:** 2 ders saati

### Kazanımlar:

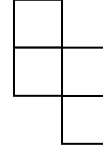
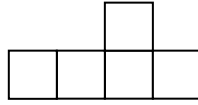
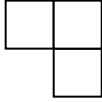
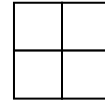
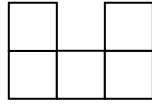
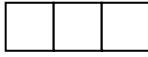
1. Alan ve çevre arasındaki ilişkiyi kurar.
2. Dikdörtgenel ve karesel bölgelerin alanlarını hesaplar.

### Giriş:

1. Alan ve çevrenin birbiriyle bağıntıları olup olmadığı sorulur.

### Gelişme:

1. Öğrencilerden kenar uzunluğu 1 br olan bir kare yapmaları istenir. Bu karenin çevresini bulup not etmeleri istenir. Benzer şekilde 2 kareden oluşan bir şekil çizmeleri istenir. Bu şeklin de çevre uzunluğunu hesaplamaları istenir. Aynı işlemi üç, dört, beş kare için yapılır ve öğrencilerden aşağıdaki soruları cevaplamaları istenir.



- Her bir şekil için olası en büyük çevre uzunluğuna sahip şekil hangisidir. Çevre uzunlukları kaç br dir?
  - Her bir şeklin olası en büyük çevre uzunluğunu ve alanını gösteren bir tablo oluşturunuz?
  - Tablodaki veriler arasında bir örüntü var mı?
  - Aynı alana sahip şekillerin olası en büyük çevre uzunluğunu veren cebirsel ifadeyi bulunuz?
2. Çalışma Kağıdı 8 öğrencilere dağıtılır.

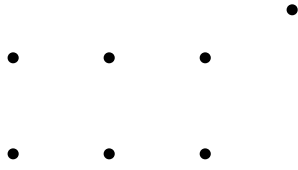
**Ödev:** Günlük hayatta çokgenlerin alan ve çevre arasındaki ilişkiyi hesaplamalarını nerelerde kullanıyoruz? Bu konuyla ilgili bir paragraf yazınız.

## APPENDIX D

### WORKSHEETS

#### ÇALIŞMA KAĞIDI 1

1. Aşağıdaki noktaları çokgen oluşturacak şekilde birleştiriniz.



A) Oluşturduğunuz çokgeni isimlendiriniz.

B) Çokgen oluşturmayacak şekilde birleştiriniz. Neden çokgen olmadığını açıklayınız.

2. Aşağıdaki ifadelerin başına doru ise “D” yanlışsa “Y” harfi koyunuz.

( ) Bütün çokgenlerin köşesi vardır.

( ) Her tip çokgende açı ve kenar bulunur.

( ) Bütün çokgenler kapalı şekillerdir.

( ) Bir çokgende tamamlayan bir kenar olabilir.

( ) Bir geometrik şeklin çokgen olabilmesi için 2 veya daha fazla kenarı olması gerekir.

( ) Bir çokgenin dış bölgesi, üzerinde bulunduğu düzlemin; çokgenin kendisi ile dış bölgesi dışında kalan kısmıdır.

#### Sorular:

1. Soru ne hakkında? Ne soruluyor? Bu soru hangi konu ile ilgili?

2. Çokgen neyi ifade eder? Çokgenin tanımını kendi cümlelerinizle yapınız?

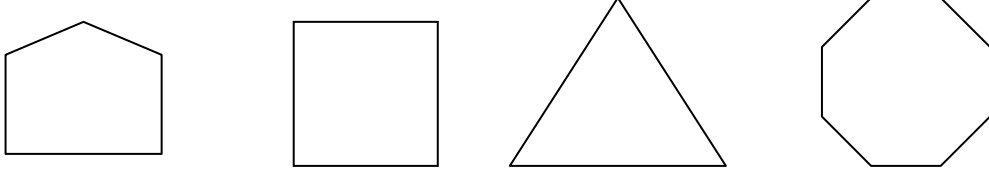
3. Bu soruların daha önce öğrendiğiniz konularla ilişkisi var mı? Neden? Açıklayınız. Benzer yada farklı özellikleri var mı?

4. Bu soruları çözmek için nasıl bir yol izlemeliyiz?

5. Bulduğum sonuç anlamlı mı? veya Nerede yanlış yaptım?

## ÇALIŞMA KAĞIDI 2

1. Aşağıdaki çokgenlerin kenarlarını ve açılarını inceleyerek düzgün çokgen olup olmadıklarını belirleyiniz. Nedenini açıklayınız.



2. Aşağıdaki ifadelerde doğru olan ifadenin başına “D” harfi, yanlış olan ifadenin başına “Y” harfi koyunuz.

- ( ) Eşkenar üçgen düzgün çokgendir.
- ( ) Dikdörtgen düzgün çokgendir.
- ( ) Kare düzgün çokgendir
- ( ) Bir düzgün beşgenin kenar uzunlukları eşit, açılarının ölçüleri farklıdır.
- ( ) Bir düzgün altıgenin kenar uzunlukları eşit, açılarının ölçüleri eşittir.

4. Kenar uzunlukları birbirine eşit ancak düzgün olmayan bir çokgen çiziniz. Neden düzgün olmadığını açıklayınız?

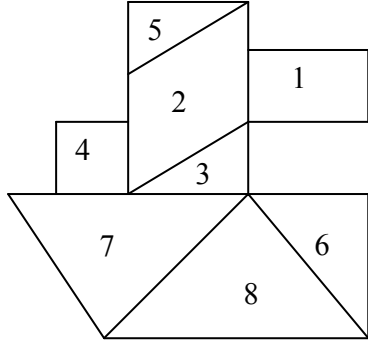
5. Günlük hayatta kullandığımız düzgün çokgenlere örnek veriniz.

### Sorular:

1. Soru ne hakkında? Ne soruluyor? Bu soru hangi konu ile ilgili?
2. Düzgün çokgen neyi ifade eder? Düzgün çokgenin tanımını kendi cümlelerinizle yapınız? Düzgün olmayan çokgen neyi ifade eder?
3. Bu sorunun daha önce öğrendiğiniz konularla ilişkisi var mı? Neden? Açıklayınız. Benzer yada farklı özellikleri var mı?
4. Bu soruyu cevaplamak için nasıl bir yol izlemeliyiz.
5. Cevabım anlamlı mı? veya Nerede yanlış yaptım?

### ÇALIŞMA KAĞIDI 3

1.



Yandaki şekildeki eş ve benzer çokgenleri bulunuz.?

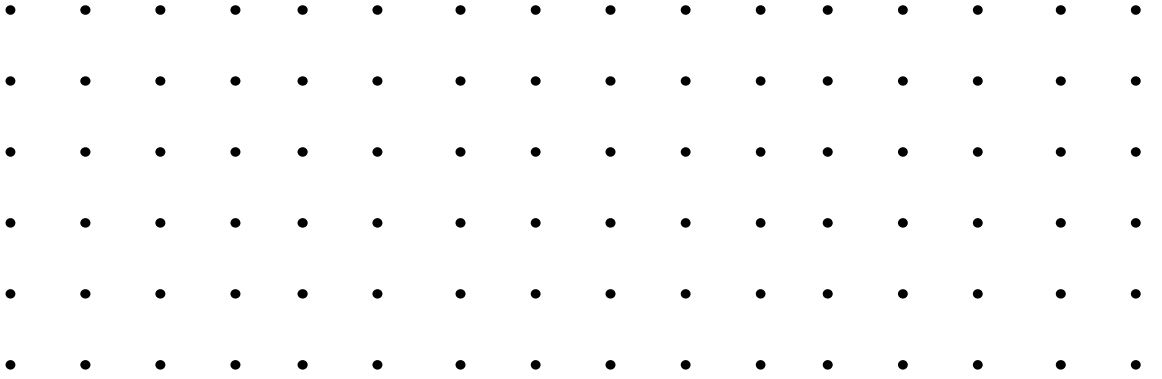
2.

Aşağıdaki cümlelerin başına doğru ise D yanlışsa Y harfi koyunuz.

- ( ) İki geometrik şeklin eş olması için büyüklüklerinin aynı olması yeterlidir.
- ( ) Açılı ölçüleri ve kenar uzunlukları birbirine eşit olan iki kare eşittir.
- ( ) İki üçgen benzer ise karşılıklı açıların ölçülerinin uzunlukları eşit, kenar uzunlukları eşittir.
- ( ) Benzer çokgenler her zaman eşittir.

3.

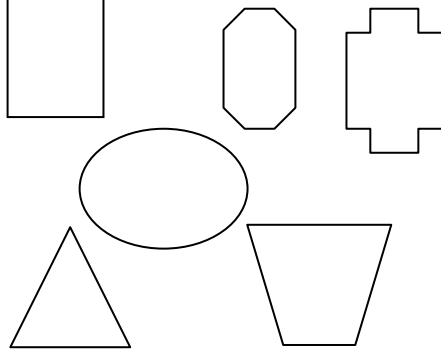
Aşağıdaki noktalı kağıda 4 tane eş fakat farklı duruşlarda 4 tane üçgen çiziniz.



4.

6-D sınıfının camı kırıldı. Yerine yeni cam takıldı .Yeni cam ile eski cam benzer midir? Eş midir?Açıklayınız.

5. Aşağıdaki şekillere benzer şekiller çiziniz.

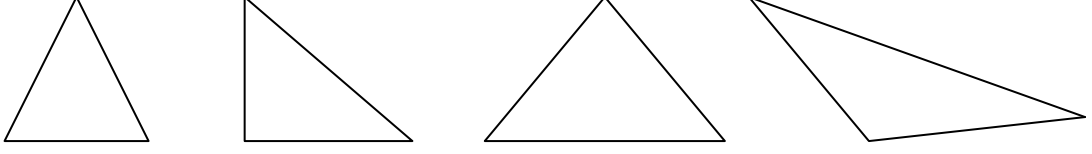


**Sorular:**

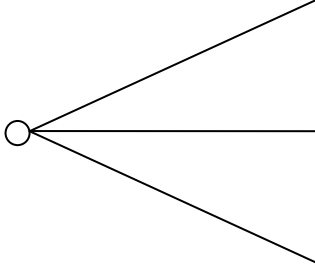
1. Soru ne hakkında? Ne soruluyor? Bu soru hangi konu ile ilgili?
2. Eş çokgenlerin özellikleri nedir? Benzer çokgenlerin özellikleri nedir? Benzer ve eş çokgenin tanımını kendi cümlelerinizle yapınız?
3. Bu sorunun daha önce öğrendiğiniz konularla ilişkisi var mı? Neden? Açıklayınız. Benzer yada farklı özellikleri var mı?
4. Bu soruyu cevaplamak için nasıl bir yol izlemeliyiz.
5. Verdiğim cevap anlamlı mı? veya Nerede yanlış yaptım?

#### ÇALIŞMA KAĞIDI 4

1. Cetvel ve açı ölçer kullanarak aşağıda verilen üçgenlerin kenar ve açı ölçülerini bulunuz. Her bir üçgeni açılara ve kenarlarına göre sınıflandırınız.



2. Hentbolda penaltı atışı 7 m'den yapılmaktadır. Penaltı atışı yapan bir kişinin penaltı atma anı modellenmiştir. Modelde oluşan üçgenleri açılara ve kenarlarına göre sınıflandırınız.



3. Geniş açılı bir üçgen aynı zamanda eşkenar üçgen olabilir mi? Neden? Açıklayınız.

4. Üçgenlerin sınıflandırması ile ilgili kavram haritası çiziniz.

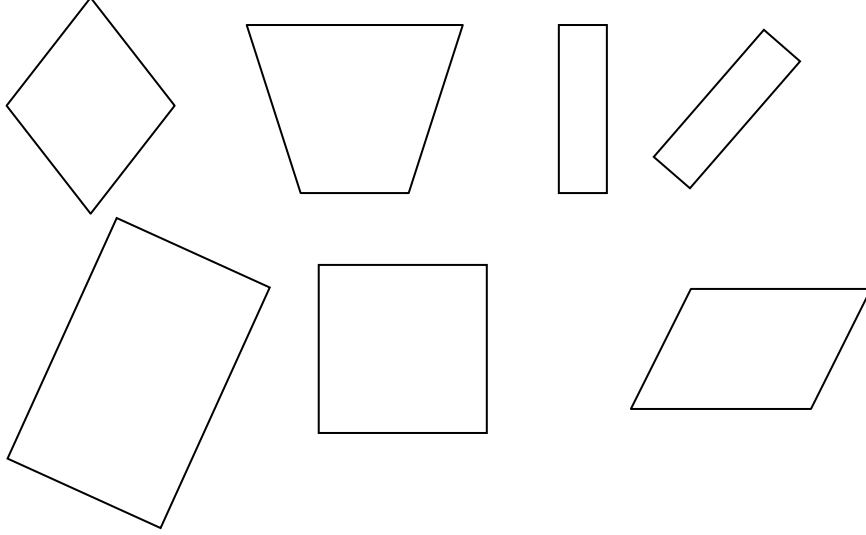
#### Sorular:

1. Soru ne hakkında? Ne soruluyor? Bu soru hangi konu ile ilgili?
2. Üçgenleri açılara göre sınıflandırmak ne demek? Üçgenleri kenarlarına göre sınıflandırmak ne demek? Geniş açılı üçgenin tanımı nedir? Eşkenar üçgenin tanımı nedir?
3. Bu sorunun daha önce öğrendiğiniz konularla ilişkisi var mı? Neden? Açıklayınız. Benzer yada farklı özellikleri var mı?
4. Bu soruyu cevaplamak için nasıl bir yol izlemeliyiz.
5. Verdiğim cevap anlamlı mı? veya Nerede yanlış yaptım?

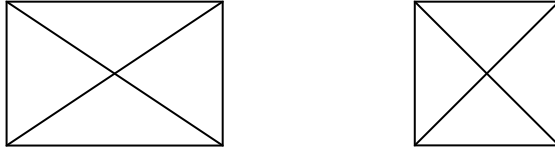


## ÇALIŞMA KAĞIDI 5

1. Aşağıda verilen dörtgenlerden kare veya dikdörtgen olanları belirleyiniz. Nasıl belirlediğinizi açıklayınız.



2. Aşağıdaki şekillerde eş açıları ve kenarları bulunuz.



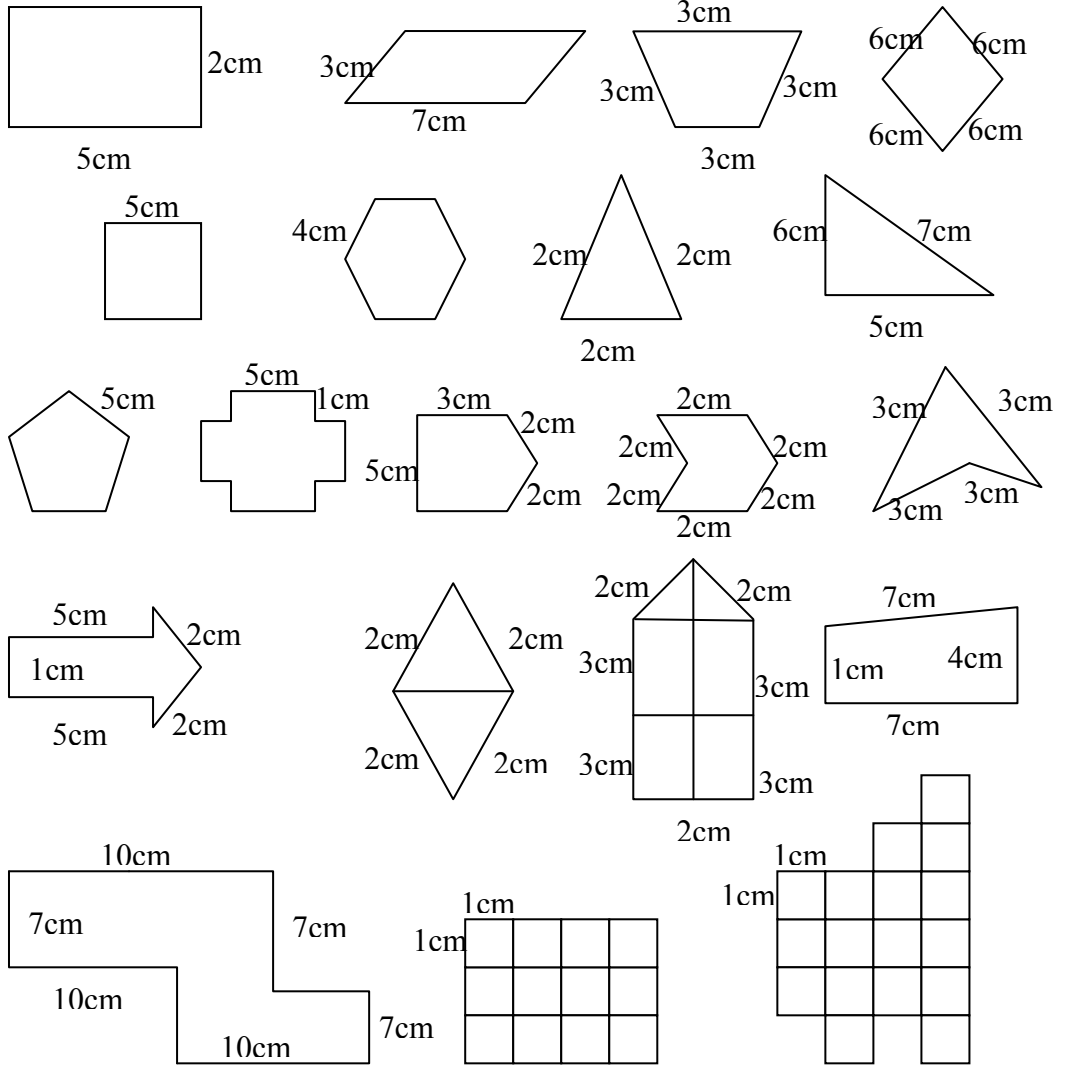
3. Kare ve dikdörtgenin ortak özellikleri nedir? Cümle ile ifade ediniz.

### Sorular:

1. Soru ne hakkında? Ne soruluyor? Bu soru hangi konu ile ilgili?
2. Kare neyi ifade eder? Karenin tanımını kendi cümlelerinizle yapınız?  
Dikdörtgen neyi ifade eder? Dikdörtgenin tanımını kendi cümlelerinizle yapınız?
3. Bu sorunun daha önce öğrendiğiniz konularla ilişkisi var mı? Neden? Açıklayınız. Benzer yada farklı özellikleri var mı?
4. Bu soruyu cevaplamak için nasıl bir yol izlemeliyiz.
5. Verdiğim cevap anlamlı mı? veya Nerede yanlış yaptım?

## ÇALIŞMA KAĞIDI 6

1. Aşağıdaki geometrik şekillerin çevresini hesaplayınız.



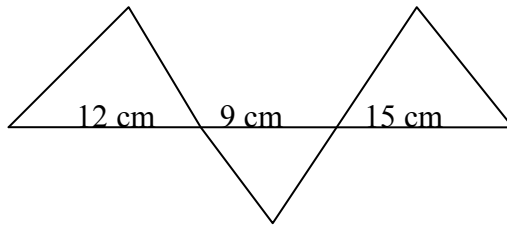
2. Bir basketbol sahasının kenarlarına çizgi çizilecektir. Basketbol sahasının eni 14m, boyu 26m dir. Buna göre basket bol sahasının kenarlarını belirlemek için kaç m çizgi çizmek gerekir.

3. Çevre uzunluğu 35cm olan bir bir beşgen, bir yedigen ve bir dikdörtgen çiziniz.

4. Bir dikdörtgenin bir kenarının uzunluğu, diğer kenarının uzunluğundan 2 m fazladır. Dikdörtgenin çevresi 16m ise bu kenar uzunlukları nedir?

5. Çevre uzunlukları 40cm olan karelerden 5 tanesini kullanarak farklı çokgenler elde ediniz. Elde ettiğiniz çokgenlerin çevrelerini bulunuz.

6. Bir kenar uzunluğu 3,5 m olan bir eşkenar üçgenin çevresinin uzunluğu kaç m dir?



7. Yandaki şekilde üçgenlerin her biri eşkenar üçgendir. Üçgenlerin çevreleri toplamı kaç cm dir.

8. Uzunluğu 54cm olan bir telde ardışık üç çift sayı olan bir üçgen yapılıyor. Üçgenin en büyük kenarının uzunluğu kaç cm dir?

### Sorular:

1. Problem ne hakkında? Problem ne ile ilgili? Problem hangi konu ile ilgili? Bu problemde ne soruluyor?

2. Çevre neyi ifade eder? Bir çokgenin çevresini nasıl hesaplarız? Alan neyi ifade eder? Alanı nasıl hesaplarız?

3. Bu problemin daha önce öğrendiğiniz konularla ilişkisi var mı? Neden? Açıklayınız. Benzer yada farklı özellikleri var mı?

4. Bu problemi çözmek için nasıl bir yol izlemeliyiz.

5. Bulduğum sonuç anlamlı mı? veya Nerede yanlış yaptım?

## ÇALIŞMA KAĞIDI 7

1. Aşağıdaki şekillerin alanlarını hesaplayınız.



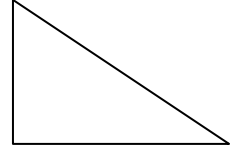
6 cm



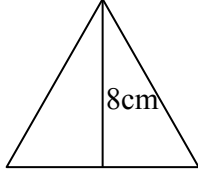
4cm

3cm

4cm



4cm



10cm

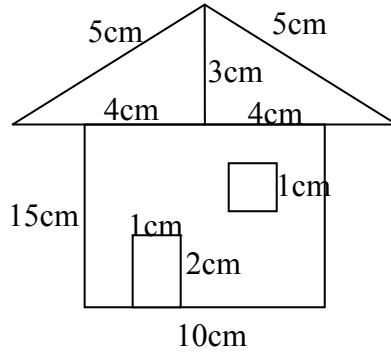
8cm



10cm

6cm

- 2.

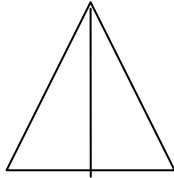


Yandaki ev krokisinde kapı ve pencerenin dışında kalan alan nedir?

3. Bir bahçıvan, eni 40m, boyu 60m olan bahçesinde kenarı en büyük olacak şekilde kare biçiminde bir yer ayırarak, domates dikmek istiyor. Domates dikilecek yerin çevresinin uzunluğu kaç m olur?

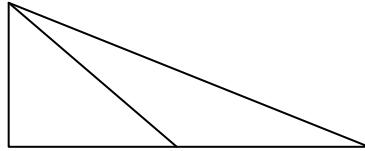
4. Çevresinin uzunluğu 120m olan bir karenin alanı kaç  $m^2$  dir.

5. Aşağıdaki üçgenlerin yüksekliğini bulunuz.



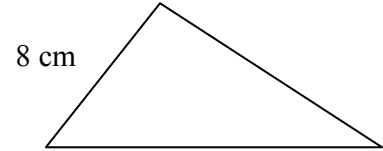
2 cm

Alanı  
 $10cm^2$



4 cm

Alanı  
 $20cm^2$



8 cm

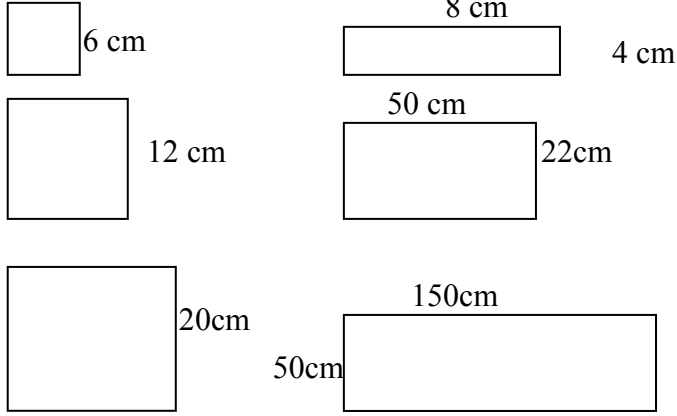
Alanı  
 $40cm^2$

**Sorular:**

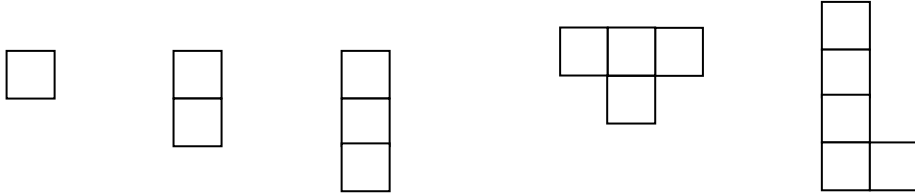
1. Problem ne hakkında? Problem ne ile ilgili? Problem hangi konu ile ilgili? Bu problemde ne soruluyor?
2. Çevre neyi ifade eder? Bir çokgenin çevresini nasıl hesaplarız? Alan neyi ifade eder? Alanı nasıl hesaplarız?
3. Bu problemin daha önce öğrendiğiniz konularla ilişkisi var mı? Neden? Açıklayınız. Benzer yada farklı özellikleri var mı?
4. Bu problemi çözmek için nasıl bir yol izlemeliyiz.
5. Bulduğum sonuç anlamlı mı? veya Nerede yanlış yaptım?

## ÇALIŞMA KAĞIDI 8

1. Aşağıda kenar uzunlukları verilen kare ve dikdörtgenin çevreleri ve alanları için ne söyleyebiliriz?

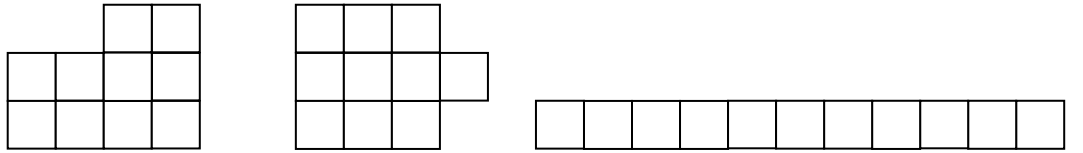


2. Aşağıdaki şekillerin alan ve çevreleri için ne söylenebilir.



3. Ahmet'in köyünde mart ayında tarlalara ağaç dikilir. Ahmet alanları eşit olan üç tarlasından çevresine 2 m aralıklarla ağaç dikecektir.

- Çevresine en fazla ağaç dikilebilecek tarla hangisidir?
- Bu tarlaya kaç tane ağaç dikilebilir ?



Her birim karenin kenarı 20 m dir.

### Sorular:

- Problem ne hakkında? Problem ne ile ilgili? Problem hangi konu ile ilgili? Bu problemde ne soruluyor?
- Çevre neyi ifade eder? Bir çokgenin çevresini nasıl hesaplarız? Alan neyi ifade eder? Alanı nasıl hesaplarız? Alan ve çevre arasında nasıl bir ilişki vardır?
- Bu problemin daha önce öğrendiğiniz konularla ilişkisi var mı? Neden? Açıklayınız. Benzer yada farklı özellikleri var mı?
- Bu problemi çözmek için nasıl bir yol izlemeliyiz.
- Bulduğum sonuç anlamlı mı? veya Nerede yanlış yaptım?

## **APPENDIX E**

### **INTERVIEW QUESTIONS**

The interviewed students were posed the following questions: For MAN and MAN+META groups.

Has the use of physical manipulative affected your learning? How?

Could you explain the effects of physical manipulative on your learning? For tangrams, geoboards and origami separately.

How has the use of physical manipulative affected your learning on definitions and properties of polygons?

How has the use of physical manipulative affected your learning on finding the relations between the properties of polygons?

How has the use of physical manipulative affected your learning on computational skills and algorithms of the problems about the polygons?

In addition to these questions, interviewed students were posed the following questions: For MAN+META groups.

What was the effect of questions in the worksheets on your acquisition of facts, definitions and the properties of the polygons?

What was the effect of questions in the worksheets in understanding the relations between the polygons?

What was the effect of questions in the worksheets in finding the solutions of the problems related to polygons?