A STRUCTURAL EQUATION MODELING STUDY: THE METACOGNITION-KNOWLEDGE MODEL FOR GEOMETRY

UTKUN AYDIN

JULY 2007

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A THESIS SUBMITTED TO
THE GRADUATE SCHOOL OF NATURAL AND APPLIED SCIENCES
OF
MIDDLE EAST TECHNICAL UNIVERSITY

BY

UTKUN AYDIN

IN PARTIAL FULFILLMENT OF THE REQUIREMENTS
FOR
THE DEGREE OF MASTER OF SCIENCE
IN
THE SECONDARY SCIENCE AND MATHEMATICS EDUCATION

JULY 2007

A STRUCTURAL EQUATION MODELING STUDY: THE METACOGNITION-KNOWLEDGE MODEL FOR GEOMETRY

submitted by **UTKUN AYDIN** in partial fulfillment of the requirements for the degree of **Master of Science in Secondary Science and Mathematics Education Department, Middle East Technical University** by,

Prof. Dr. Canan Özgen
Dean, Graduate School of **Natural and Applied Sciences** ______

Prof. Dr. Ömer Geban
Head of Department, **Secondary Science and Mathematics Education** ______

Assoc. Prof. Dr. Behiye Ubuz
Supervisor, **Secondary Science and Mathematics Education Dept., METU** ______

Examining Committee Members:

Prof. Dr. Petek Aşkar
Computer Education and Instructional Technology Dept., Hacettepe University _____

Assoc. Prof. Dr. Behiye Ubuz
Secondary Science and Mathematics Education Dept., METU _____

Assist. Prof. Dr. Yeşim Çapa Aydın
Educational Sciences Dept., METU _____

Assist. Prof. Dr. Ayhan Kürşat Erbaş
Secondary Science and Mathematics Education Dept., METU _____

Dr. Bülent Çetinkaya
Secondary Science and Mathematics Education Dept., METU _____

Date: ________________

I hereby declare that all information in this document has been obtained and presented in accordance with academic rules and ethical conduct. I also declare that, as required by these rules and conduct, I have fully cited and referenced all material and results that are not original to this work.

Name, Last name: Utkun AYDIN

Signature :

ABSTRACT

A STRUCTURAL EQUATION MODELING STUDY: THE METACOGNITION-KNOWLEDGE MODEL FOR GEOMETRY

Aydın, Utkun
M. S., Department of Secondary Science and Mathematics Education
Supervisor : Assoc. Prof. Dr. Behiye Ubuz

July 2007, 179 pages

The purpose of this study is twofold: (1) to examine the effects of knowledge on cognition and regulation of cognition on declarative knowledge, conditional knowledge, and procedural knowledge in geometry and (2) to examine the interrelationships among declarative knowledge, conditional knowledge, and procedural knowledge in geometry. The reciprocal relationships between metacognitive and knowledge factors were modeled by using data from tenth grade secondary school students.

Structural equation modeling was used to test the hypothesized relationships of two metacognitive factors (knowledge of cognition, regulation of cognition) and three knowledge factors (declarative knowledge, conditional knowledge, procedural knowledge). The observed variables representing the latent variables were determined by carrying out exploratory factor analysis and confirmatory factor analysis for the metacognitive awareness inventory and geometry knowledge test separately.

Major findings revealed: (1) Declarative knowledge significantly and positively influences conditional and procedural knowledge; (2) Procedural knowledge has a signitificant and positive direct effect on conditional

knowledge; (3) Declarative knowledge has a positive indirect effect on conditional knowledge; (4) Knowledge of cognition significantly and positively influences procedural knowledge; (5) Regulation of cognition has a significant but negative direct effect on procedural knowledge; (6) Knowledge of cognition has positive indirect effects on conditional and procedural knowledge; (7) Regulation of cognition has negative indirect effects on conditional and procedural knowledge; (8) Knowledge of cognition and regulation of cognition have non-significant direct effect on declarative and conditional knowledge.

The results showed that knowledge of cognition has the strongest direct effect on procedural knowledge and the direct effect of declarative knowledge on conditional knowledge is stronger than on procedural knowledge. In view of the findings considerable suggestions is provided for teachers, instructional designers, and mathematics education researchers.

Keywords: Structural Equation Modeling, Knowledge of Cognition, Regulation of Cognition, Declarative Knowledge, Conditional Knowledge, Procedural Knowledge, Geometry

ÖZ

BİR YAPISAL DENKLEM MODELLEME ÇALIŞMASI: GEOMETRİ İÇİN ÜSTBİLİŞ-BİLGİ MODELİ

Aydın, Utkun
Yüksek Lisans, Orta Öğretim Fen ve Matematik Alanları Eğitimi Bölümü
Tez Yöneticisi : Doç. Dr. Behiye Ubuz

Temmuz 2007, 179 sayfa

Bu çalışmanın iki amacı vardır: (1) bilişin bilgisi ve bilişin düzenlemesinin geometride ifadesel bilgi, koşullu bilgi ve işlemsel bilgiye etkilerinin incelenmesi ve (2) geometride ifadesel bilgi, koşullu bilgi ve işlemsel bilgi arasındaki karşılıklı ilişkilerin incelenmesi. Üstbilişsel ve bilgi faktörleri arasındaki ilişkiler onuncu sınıf ortaöğretim öğrencilerinin verileri kullanılarak modellenmiştir.

İki üstbilişsel (bilişin bilgisi, bilişin düzenlemesi) ve üç bilgi (ifadesel bilgi, koşullu bilgi, işlemsel bilgi) faktörü arasındaki varsayılan ilişkileri test etmek için yapısal denklem modelleme yöntemi kullanılmıştır. Örtük değişkenleri temsil eden gözlenebilen değişkenler üstbilişsel farkındalık envanteri ve geometri bilgi testlerine ayrı ayrı açıklayıcı faktör analizi ve doğrulayıcı faktör analizi uygulanarak tespit edilmiştir.

Ortaya çıkan ana bulgular: (1) İfadesel bilgi, koşullu ve işlemsel bilgiyi pozitif ve istatistiksel olarak anlamlı bir şekilde etkilemektedir; (2) İşlemsel bilgi, koşullu bilgi üzerinde pozitif ve istatistiksel olarak anlamlı direkt etkiye sahiptir; (3) İfadesel bilgi, işlemsel bilgi üzerinde pozitif ve istatistiksel olarak anlamlı indirekt etkiye sahiptir; (4) Bilişin bilgisi işlemsel bilgiyi pozitif ve istatistiksel olarak anlamlı bir şekilde etkilemektedir; (5) Bilişin düzenlemesi, işlemsel bilgi üzerinde istatistiksel olarak anlamlı fakat negatif direkt etkiye sahiptir; (6) Bilişin bilgisi, koşullu ve işlemsel bilgi üzerinde pozitif indirekt etkilere sahiptir; (7) Bilişin düzenlemesi koşullu ve işlemsel bilgi üzerinde negatif indirekt etkilere sahiptir; (8) Bilişin bilgisi ve bilişin düzenlemesi ifadesel ve işlemsel bilgi üzerinde istatistiksel olarak anlamlı bir direkt etkiye sahip değildirler.

Sonuçlar bilişsel bilginin en güçlü direkt etkisinin işlemsel bilgi üzerinde olduğunu ve ifadesel bilginin koşullu bilgi üzerindeki direkt etkisinin işlemsel bilgi üzerindeki direkt etkisinden daha güçlü olduğunu göstermiştir. Bulgular doğrultusunda öğretmenlere, program geliştiricilere, ve matematik eğitimi araştırmacılarına önemli öneriler sunulmaktadır.

Anahtar Kelimeler: Yapısal Denklem Modellemesi, Bilişin Bilgisi, Bilişin Düzenlemesi, İfadesel Bilgi, Koşullu Bilgi, İşlemsel Bilgi, Geometri

To My Parents, To My Brother and To My Grandfather…

Mukadder AYDIN, Ömer AYDIN, Orkun AYDIN and Bekir AYDIN

ACKNOWLEDGEMENTS

I would like to express my gratitude to my supervisor Assoc. Prof. Dr. Behiye UBUZ for her valuable support and gentle guidance. Her expertise and sincere encouragement provided me support throughout this study.

Thanks are owed to Prof. Dr. Giray BERBEROĞLU, Asst. Prof. Dr. Ayhan Kürşat ERBAŞ, Asst. Prof. Dr. Erdinç ÇAKIROĞLU, Inst. Ph. D. Yusuf KOÇ; their encouraging efforts, constructive criticism and suggestions during this research.

I would like to thank my committee members Prof. Dr. Petek AŞKAR, Asst. Prof. Dr. Yeşim ÇAPA AYDIN and Inst. Ph. D. Bülent ÇETİNKAYA for their critique, feedback and insight.

I am very much thankful to Erten ABDİK who helped me at the very beginning of the development of the instruments and during their administration.

I am indepted to the support of my family. I thank their far but close lovingly encouragement, moral support and appreciation which were my source of strength and inspiration in every step of the way.

I am grateful to my friends Ebru and Gökay UYAR, Gökçe and Gürdal GÖKERİ, İlham ÜÇOK, Zeynep Sonay POLAT, Ayşegül ERYILMAZ, Duygu ÖREN, Mesture and Mirkan ALTAY, Mert AYTİMUR, Koray KUTLU, Sena ERDOĞAN, Serhan MOĞOL, Çiğdem İŞ GÜZEL, Betül YAYAN, Hanife Can ŞEN, Celil EKİCİ, Semirhan GÖKÇE, Sevda YERDELEN, Gökhan SERİN, Şemsettin BEŞER, Eren CEYLAN, Demet KIRBULUT, Ebru KAYA, Oktay BEKTAŞ, Gönül KURT, Hüseyin Hüsnü YILDIRIM, Almula Elif DİLMAÇ and Özlem KORKMAZ. Thank you for sharing your love, time and interest. Your enormous moral support contributed greatly throughout the stress and difficulties.

Thank you all very much indeed.

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KNOOFCOG: Knowledge of Cognition

REGOFCOG: Regulation of Cognition

DECKNOW: Declarative Knowledge

CONKNOW: Conditional Knowledge

PROKNOW: Procedural Knowledge

JR. MAI: Junior Metacognitive Awareness Inventory

GKT: Geometry Knowledge Test

MET1-MET18: Items of Junior Metacognitive Awareness Inventory

QUES1-QUES24: Questions of Geometry Knowledge Test

SEM: Structural Equation Modeling

CHAPTER 1

INTRODUCTION

Recent research that use structural equation modeling techniques has attempted to identify, explain and understand factors effecting students' performance in mathematics such as attitudes (Ma, 1997; Ma & Xsu, 2004), anxiety (Meece, Wigfield, & Eccles, 1990), self-concept (Abu-Hilal, 2000), gender (Ethington & Wolfle, 1986; Nasser & Birenbaum, 2005), parent education (Ethington & Wolfle, 1984), motivational variables (Hammouri, 2004), interest and academic engagement (Singh, Granville, & Dika, 2002), beliefs (Papanastasiou, 2000), achievement behaviors (Ethington, 1991), home and peer environment (Reynolds & Walberg, 1992), social background factors (Rowe & Hill, 1998), school factors (Papanastasiou, 2002), metacognitive factors (Panaoura & Philippou, 2003; 2005) and institutional factors (Schreiber, 2002). Most of the proposed models highlighted the attitudinal and affective variables related to mathematics achievement in middle grades. The examination of factors affecting students' achievement in middle grades is important because it is in those years that students contemplate and negotiate their future trajectories. In the same vein, success in mathematical performance can be attributed to the complex and dynamic interaction between cognitive, affective, and motivational factors in secondary school (Volet, 1997).

Among the factors that have substantial effect on success, metacognitive factors play an important role in students' performance (Panaoura & Philippou, 2005). Metacognition refers to the ability to reflect upon one's own cognitive processes (Schraw & Dennison, 1994; Schraw, 1998) that underline the conscious use, control, and online awareness that individuals have of their own cognitive abilities and processes. It is related to learners' knowledge, awareness, and control of the processes by which they learn (Brown, 1987), and the metacognitive learner is supposed to have the sophisticated characteristics who is able to recognize, evaluate, and where needed reconstruct existing ideas (Gunstone, 1991). Initially the concept of metacognition appeared within information processing theory which shows how individuals receive and learn information. Information processing theory focuses on how people attend to environmental events, encode information to be learned, and relate it to knowledge in memory, store new knowledge in memory, and retreive it as needed (Shuell, 1986). The present study undertakes metacognition with its two essential components: Knowledge of cognition which involves knowledge about cognitive resources and regulation of cognition which involves knowledge about monitoring progress and regulating strategy usage. Researchers are convinced that successful mathematical performance depends on having not only adequate knowledge about concepts, relational rules, and procedures but also sufficient awareness of cognitive processes and control of the processes.

According to Vygotskian theory mathematics pedagogy is based on a conceptual system rather than a collection of procedures. Vygotsky (1986) underlined that effective learning depends both on the knowledge and experience already existing in the student (level of development) as well as on the student's potential to learn. While this conceptual base is important the general perception in high school teaching of mathematics in Turkey tends to be fairly procedural and that students who enter Öğrenci Seçme Sınavı (University Entrance Examination) are better equipped to deal with procedural problems rather than conceptual ones. Most students have well developed manipulation skills in mathematics but few of them expose to deeper conceptual thinking. On the other hand, several models of learning indicate that the process of procedural knowledge acquisition depends on the existing conceptual knowledge (Anderson, 1995) and further learner's proficiency in procedural knowledge is a requirement for metacognition and conceptual knowledge. Teachers often complain that students have little understanding of the basic mathematical ideas, concept definitions, facts, or relational rules and even the high achieving students are only better in a procedural way of thinking. However, according to the curriculum requirements and time constraints courses often begin with a brief review of definitions and theorems; then focus on computational applications of procedural knowledge. Unfortunately, there is evidence that in most high schools rote learning of rules and algorithms is emphasized and hence teachers devote less time and attention to conceptual knowledge (Porter, 1989). Computational proficiency is taken as the primary means to assess students' understanding of mathematics and geometry. Thus, in classes success is measured through students' ability to apply their procedural knowledge.

Knowledge is defined as the organization of information into bodies of meaningfully interconnected facts and generalizations, which serves as a vehicle for thought and problem- solving (Gagné & Briggs, 1979). It is attributed to a variety of types such as concrete and abstract knowledge, tacit and explicit knowledge, elaborated and compiled knowledge, or conceptual and procedural knowledge. Research studies emphasize that any of these types of knowledge can involve declarative, conditional, and procedural knowledge (Alexander & Judy, 1988; Alexander, Schallert, & Hare, 1991; Ryle, 1949). To distinguish between types of knowledge that can be subsumed under the heading of knowledge of mathematics, the present study subdivided this into three knowledge types. These three knowledge types correspond roughly to the knowledge construct designated as declarative, conditional, and procedural knowledge in the literature (Smith & Ragan, 1993). Drawing on this approach, it was attempted to describe relevant aspects of knowledge, which is hierarchically structured and characterized by links between concepts, principles and procedures, with reference to the context in which they function.

Most research concerning types of knowledge has tried to determine the relationship between concept and procedure learning. In general, it is agreed that the two are positively correlated and they are learned in tandem rather than independently (Hiebert & Lefevre, 1986; Rittle-Johnson & Siegler, 1998). The relationship among three different types of knowledge is explained in an iterative manner: declarative knowledge forms the ground or the base on which actions depend; conditional knowledge provides an overview that supports the connection making and assists the reconstruction of actions; procedural knowledge provides actions, changing and transforming the situations (Mason & Spence, 1999). This interconnection within the discrimination illuminates alternative ways in which useful instructional designs for geometry and as well as mathematics courses can be developed. There is consensus that a well-organized knowledge base not only facilitates the accessing of relevant information but also determines how this information is deployed in the solution process (Chinappan, 1998; Lawson & Chinnappan, 1994; Prawat, 1989). If we apply this view of organization to the domain of geometry, the way students assemble their declarative, conditional, and procedural knowledge would influence their performance. Research on geometry should therefore, direct its focus on the interrelation among knowledge types. The extent to which they utilise these knowledge during their solution process and the effectiveness with which they do so could enhance the performance in this cluster of knowledge that contains core concepts and principles, the relations between these concepts and principles, and knowledge about how to use these concepts and relations in procedures. Skemp (1976) underlined knowing what to do and why constitutes understanding in mathematics. Similarly, Schoenfeld (1988) emphasized that development of mathematical thinking and understanding requires not only mastering various facts, principles and algorithms, but also internalizing the connections among them. Thus, it is the synthesis of knowing that, knowing why, and knowing how that brings the accomplishment in different content areas of mathematics.

In this study, students' performance was measured in terms of open-ended geometry questions contextualized in declarative, conditional, and procedural knowledge. Geometry has a substantial place in mathematics curriculum which allows students to develop new insights to understand other mathematical concepts, make interrelations among other areas of mathematics such as number and algebra (Mammana & Villiani, 1998) and realize the beauty of mathematics (Serra, 1993). A considerable attention was directed to students' understanding of geometry in most mathematics curriculum documents (MEB, 2006) because they come across with the value of geometry knowledge not only inside school but also outside school while solving daily life problems (Bussi & Boero, 1998). Thus, geometry is accepted as a good starting point for teaching and learning challenging mathematical argumentation, and filling the gap between daily life experiences and mathematics. Rich store of geometry knowledge offers students a qualified scientific thinking, articulation of their ideas, and development of clearly structured arguments to support their intuition. Research literature revealed that secondary school students encounter difficulties in geometry (Usiskin, 1972). Thus, it is obvious that there is a need for new challenges that develop geometric knowledge in ways consistent with recent reccommandations for curriculum reform.

This study will provide an indepth understanding for the critical importance of metacognitive and knowledge factors affecting success in geometry by exploring the interrelations between different types of knowledge in relation to metacognitive knowledge and regulation. In view of the patterns emerged in this study considerable suggestions can be provided for teachers, instructional designers, and mathematics education researchers.

1.1 PURPOSE OF THE STUDY

The purpose of this study was twofold: (a) to examine the effects of knowledge on cognition and regulation of cognition on tenth grade students' declarative knowledge, conditional knowledge, and procedural knowledge in geometry and (b) to examine the interrelations among tenth grade students' declarative knowledge, conditional knowledge, and procedural knowledge in geometry. The problems investigated were (a) What linear structural model explains the metacognitive factors affecting tenth grade students' declarative, conditional, and procedural knowledge in geometry? and (b) What linear structural model explains the interrelationships among tenth grade students' declarative, conditional, and procedural knowledge in geometry?.

Students' answers on Junior Metacognitive Awareness Inventory which was adopted from Sperling, Howard, Miller, and Murphy (2002) and scores on Geometry Knowledge Test which was developed by the researcher were combined to run exploratory and confirmatory factor analyses. The items on each instrument were grouped to identify the latent variables. The latent variables included in the study were conceptualized as: knowledge of cognition, regulation of cognition, declarative knowledge, conditional knowledge, and procedural knowledge. Various models were tested for the hypothesis given below with respect to the framework of the model proposed in Figure 1.1.

Ho: The linear structural model between KNOOFCOG, REGOFCOG, DECKNOW, CONKNOW, and PROKNOW is not statistically significant.

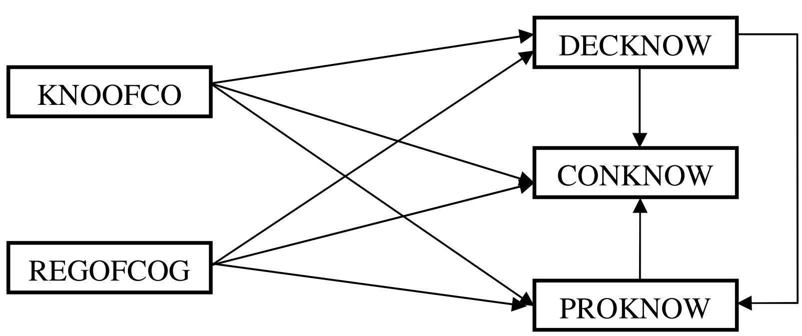


Figure 1.1 Hypothesized Metacognition-Knowledge Model

1.2 DEFINITION OF IMPORTANT TERMS

The definition of the latent variables incuded in the hypothesized model are given below:

1. Knowledge of Cognition (KNOOFCOG)

Knowledge of cognition component of metacognition refers to individual's knowledge about her/his own capabilities, beliefs, cognitive abilities and processes. It includes three subcomponents: declarative knowledge, conditional knowledge, and procedural knowledge (Sperling, et al., 2002; 2004). Declarative knowledge involves knowledge about one's self. Conditional knowledge involves individual's knowledge about when and why to allocate resources. Finally, procedural knowledge involves individual's knowledge about how to use strategies.

2. Regulation of Cognition (REGOFCOG)

Regulation of cognition component of metacognition refers to individual's knowledge about her/his own control processes during the execution of the task. It includes five subcomponents as planning, selecting, comprehension monitoring, debugging, and evaluating (Sperling, et al., 2002; 2004). These regulatory processes provide individuals better use of their cognitive resources including attention, stategy selection, and awareness of comprehension. Individual's awareness of her/his own strengths and weaknesses aids in determining which strategies to apply and the effectiveness of their applicaion.

3. Declarative Knowledge (DECKNOW)

Declarative knowledge refers to "knowing that" (Smith & Ragan, 1993). It is concerned about the facts, hypothesis, and generalizations. Learners are not required to apply the knowledge that they have acquired rather they are expected to recall, recognize, or state it in their own words. Good performance in declarative knowledge requires to know concept definitions, recall of relevant symbols and facts, and required components of a given geometrical figure.

4. Conditional Knowledge (CONKNOW)

Conditional knowledge refers to “knowing why” (Smith & Ragan, 1993). It is concerned about the relational rules, principles, propositions, and axioms. It enables individuals to predict what will happen if one of the situations or variables is changed, to explain why the condition is satisfied or not and to apply the statement to a variety of unfamiliar situations. Good performance in conditional knowledge requires to make connections among concept definitions, generate explanations regarding facts, and create meaningful links among definitions, principles and procedures.

5. Procedural Knowledge (PROKNOW)

Procedural knowledge refers to “knowing how” (Smith & Ragan, 1993). It involves procedural rules and algorithms. Good performance in procedural knowledge requires to identify the situation to which procedure applies, the correct order of algorithms, the correct completion of steps, and finally to recognize the correctly completed procedure.

1.3 THE HYPOTHESIZED METACOGNITION-KNOWLEDGE MODEL

The hypothesized Metacognition-Knowledge Model presented in Figure 1.1 was developed from the review of related literature. The first block of the model hypothesized the relationships between metacognitive and knowledge constructs. Although metacognition is a multidimensional construct, the present study focused on two components of metacognition as knowledge of cognition and regulation of cognition. Research studies emphasized that students’ awareness of their learning and knowledge of monitoring their actions contribute greatly to their success in mathematics (Artz & Armour-Thomas, 1992; Goos & Galbraith, 1996; Kramarski, Mevarech, & Lieberman, 2001; Kramarski, Mevarech, & Arami, 2002; Kramarski, 2004; Mevarech & Kramarski, 1997; Mevarech, 1999; Lucangeli & Cornoldi, 1997; Maqsud, 1997; Pugalee, 2001; 2004; Schurter, 2002; Slife, Weiss, & Bell, 1985; Sperling, et al., 2002; 2004; Stillman & Galbraith, 1998; Swanson, 1990; Tobias & Everson, 2002; Veenman, Wilhelm, & Beishuizen, 2004; Veenman, Kok, & Blöte, 2005; Wilson & Clarke, 1994). With regard to the substantial effect of metacognitive factors on students' performance (Panaoura & Philippou, 2003; 2005) success in mathematics can be recognized as a complicate interplay between metacognition and knowledge of mathematics. The bulk of the studies were investigated with respect to the components of metacognition and the context of the problems that researchers used. The review showed that students' knowledge of cognition effects their declarative knowledge (Tobias & Everson, 2002; Wilson & Clarke, 1994), conditional knowledge (Swanson, 1990; Wilson & Clarke, 1994), procedural knowledge (Maqsud, 1997; Wilson & Clarke, 1994) and students' regulation of cognition effects their declarative knowledge (Veenman, Wilhelm, & Beishuizen, 2004; Wilson & Clarke, 1994), conditional knowledge (Lucangeli & Cornoldi, 1997; Pugalee, 2001; 2004; Veenman, Wilhelm, & Beishuizen, 2004; Wilson & Clarke, 1994), and procedural knowledge (Artz & Armour-Thomas, 1992; Lucangeli & Cornoldi, 1997; Pugalee, 2001; 2004; Schurter; 2002; Stillman & Galbraith, 1998; Veenman, Wilhelm, & Beishuizen, 2004; Veenman, Kok, & Blöte, 2005; Wilson & Clarke, 1994).

The second block of the model hypothesized the interrelations among knowledge constructs. As mentioned before, the present study attempted to describe relevant aspects of knowledge, which is hierarchically structured and characterized by links between concepts, principles and procedures, with reference to the context in which they function. That is to say, declarative knowledge is the factual information base on which actions depend, procedural knowledge is the compilation of declarative knowledge into functional units that facilitate the use of appropriate strategies, and conditional knowledge is the understanding of when and where to access certain facts and procedures that provides an overview that supports connection making (Alexander & Judy, 1988; Alexander, et al., 1991). The bulk of the studies were investigated with respect to types of knowledge that the context of problems is conveyed. The review revealed that students' knowledge of concept definitions and knowledge of procedures effect their building relations among principles (Baroody & Gannon, 1984; Byrnes & Wasik, 1991; Engelbrecht, et al., 2005; Moss & Case, 1999; Rittle-Johnson & Alibali, 1999; Star, et al., 2005) and students' knowledge of concepts effects their knowledge of procedures (Byrnes & Wasik, 1991; Gelman, Meck, & Merkin, 1986; Hiebert & Wearne, 1996; Knuth, McNeil, & Alibali, 2006; Lembke & Reys, 1994; Mack, 1990; Pesek & Kirschner, 2000; Rittle-Johnson & Alibali, 1999; Rittle-Johnson, Siegler, & Alibali, 2001;).

Accordingly, this model proposed the direct and indirect relationships between latent dependent and latent independent variables. The latent depent variables were DECKNOW, CONKNOW, and PROKNOW. The latent independent variables were KNOOFCOG and REGOFCOG. KNOOFCOG is supposed to be directly linked to DECKNOW, CONKNOW, and PROKNOW. Similarly, REGOFCOG is supposed to directly effect DECKNOW, CONKNOW, and PROKNOW. CONKNOW is supposed to be influenced by both DECKNOW and PROKNOW; while PROKNOW is supposed to be influenced by only DECKNOW. Finally, KNOOFCOG and REGOFCOG are supposed to influence CONKNOW indirectly through DECKNOW and PROKNOW. Similarly, KNOOFCOG and REGOFCOG are supposed to influence PROKNOW indirectly through DECKNOW.

1.4 SIGNIFICANCE OF THE STUDY

Applications of structural equation modeling technique are fairly recent both in Turkey and other countries. Many modeling studies were conducted to investigate the attitudinal, motivational, and affective factors related to mathematics achievement. These studies are generally carried out by using data from international studies such as TIMSS and PISA. Although the focus was on learner-related factors up to now little research has been conducted to explore metacognitive factors affecting performance in geometry. The results of this study will highlight factors affecting different types of knowledge in geometry.

Quantitative and qualitative studies concerning the relationships between metacognition and performance and the interrelation among knowledge types indicated the unilateral relationships which cannot explain to what extent these constructs influence one another. The present study employs advanced statistical techniques such as structural equation modeling (SEM) in order to explain the causal bilateral relations among the constructs included in the model.

Several studies were conducted to explain the relationships among conceptual knowledge and procedural knowledge which disregard the discrimination as declarative knowledge, conditional knowledge, and procedural knowledge. These studies were limited to elementary school mathematics. There is not enough research concerning geometry performance of students in Turkey. The findings will provide an insight for the relationships among knowledge types within secondary school geometry.

Recent research tends using multiple-choice items or standardized tests as measures of performance. In the current study, performance measures are based on the scores on open-ended questions which includes an indepth understanding of students' use of heuristics.

Furthermore, research on metacognition delineates the construct within problem-solving process in the line of its regulation of cognition component; thereby excluding the affect of the other component knowledge of cognition on students' performance. The present study undertakes metacognition with both components; namely knowledge of cognition and regulation of cognition.

This study will provide an indepth understanding for the critical importance of metacognitive and knowledge factors affecting success in geometry by exploring the interrelations between different types of knowledge in relation to metacognitive knowledge and regulation. In view of the patterns emerged in this study considerable suggestions can be provided for teachers, instructional designers, and mathematics education researchers. The results show evidence for the structural relationships between metacognitive and cognitive constructs which will provide an impetus for the development of more inclusive strategies to create supportive teaching environments for teachers and effective learning environments for students. The findings will help mathematics educators and instructional designers to overview educational implications. Consequently, new ideas for the curriculum development can be emerged on the basis of the relationships in this study.

CHAPTER 2

REVIEW OF RELATED LITERATURE

This chapter involves the review of literature concerning the metacognitive and knowledge constructs together with the interrelations among them.

2.1 KNOWLEDGE OF MATHEMATICS

Knowledge is defined as the organization of information into bodies of meaningfully interconnected facts and generalizations, which serves as a vehicle for thought and problem- solving (Gagné & Briggs, 1979). Ryle (1949) and Scheffler (1965) made an initial modern distinction about knowledge of concepts and knowledge of procedures, which related, "knowing that" and "knowing how". Since then, the relationship between learners' knowledge of concepts and knowledge of procedures has gained considerable attention in cognitive science. "Knowing that" was viewed as a straightforward type of knowledge, whereas "knowing how" was introduced as the more complex type of knowledge. The "knowing that" and "knowing why and when" were attributed to mathematics education as "conceptual knowledge", which puts the focus on relationships between facts and heuristics. On the other hand, the "knowing how" was viewed as "procedural knowledge", which links the terms such as doing, process, problem solving, or strategies. In the case of mathematics, Hiebert (1986) argued that the relationship between conceptual and procedural knowledge highlights the understanding of students' tendencies to learn algorithms by rote without attempting to understand what they are doing. He underlined that knowledge of concepts and procedures are positively correlated and their interaction is critical to understand how two types of knowledge contribute to success in mathematical performance. Similarly, Anderson (1995) viewed conceptual knowledge as “knowledge about facts” and procedural knowledge as “knowledge about how to do things”. He indicated that learning begins with actions on existing conceptual knowledge and with increased practise learners begin to internalize the procedural knowledge and then leave aside the conceptual knowledge on which these procedures are based on. The acquisition of procedural knowledge relies both on knowledge of concepts and repeated use of procedures. Piaget’s (Engelbrecht, Harding, & Potgieter, 2005) learning model is in line with Anderson’s but, he further develops this process. After the learner gains proficiency with procedural knowledge, she/he begins to reflect upon the processes and subsequently gains new conceptual knowledge. Schunk (1996) emphasized the importance of the distinction among three types of knowledge in terms of their implications for teaching and learning. He defined declarative knowledge as facts, beliefs, opinions, generalizations, theories, hypotheses, and attitudes; conditional knowledge (relational rules) as a network of condition-action sequences, propositions that is consisted of principles, laws, axioms, theories, or postulates, and procedural knowledge (procedural rules) as the knowledge of how to perform cognitive activities. Descriptions of three different types of knowledge are presented in the following sections in detail.

2.1.1 DECLARATIVE KNOWLEDGE (KNOWING THAT)

Declarative knowledge refers to the knowledge that something is the case (Schunk, 1996; Smith & Ragan, 1993). In other words, declarative knowledge is the explicit knowledge that learners can report and of which they are consciously aware (Anderson, 1995). It is concerned about the facts, hypotheses, theories, generalisations, beliefs, attitudes, and opinions. The acquisition of declarative knowledge is stated to be “low-level learning” (Smith

& Ragan, 1993) and is comparable to "recall" and "understanding" levels of Bloom's taxonomy. It is the core of human learning as it helps pupils learn higher-order and complex objectives. Initially, learners should possess declarative knowledge prior to the new knowledge to be acquired and provided that more abstract level knowledge can be learned effectively. Learners are not required to apply the knowledge that they have acquired rather they are expected to recall, recognize, or state it in their own words. The objectives require a learner to recall in verbatim, paraphrased, or summarized form. Concept learning can be concerned about declarative knowledge learning that it involves recalling the critical attributes of a concept and sequentally or simultaneously comparing attributes of an instance to the attributes of the concept. The acquisition of concrete concepts (e.g. classify figures into categories by their physical characteristics and identify examples of that concept) and the acquisition of defined concepts (e.g. classify examples and nonexamples of a figure) can be gathered under this circumstance.

This type of knowledge covers three sub-types: labels and names (making mental connections), facts and lists (integrating with prior knowledge), and organized discourse (reading a text, integrating with the existing knowledge) (Smith & Ragan, 1993; Gagné & Briggs, 1979).

For declarative knowledge learning to occur, learners should engage in processes such as ***linking*** of new knowledge to the existing knowledge (Jonassen, 1991), ***organizing*** new information (Smith & Ragan, 1993), and ***elaboration*** of information (Smith & Ragan, 1993). Linking requires a meaningful presentation of new declarative knowledge (Jonassen, 1991). Organizing simplifies the acquisition of new knowledge by arranging bits of knowledge (Smith & Ragan, 1993). Elaboration makes the newly acquired knowledge more meaningful for learners while they individualize it according to their prior experiences. Meaningfulness, organization, and elaboration enhance the potential for declarative knowledge to be effectively processed and retreived. “What” and “Which” type of questions are in the context of declarative knowledge.

2.1.2 CONDITIONAL KNOWLEDGE (KNOWING WHEN AND WHY)

Conditional knowledge is concerned about rule learning and involves relational rules, which are a network of condition-action sequences (Jonassen, 1993; Schunk, 1996; Smith & Ragan, 1993). It comprises ‘if-then’ or ‘condition-action’ statements, which describe the relationships between two or more concepts in a particular domain. These relationships can also be described in ‘cause-effect’ relationship. ‘If’ refers to the cause/condition and ‘then’ refers to the effect/action statements. This type of knowledge is concerned about the propositions, principles, postulates, axioms, theorems, and laws. Smith and Ragan (1993) indicated that conditional knowledge helps students to predict, explain, or control circumstances. In other words, it enables them to predict what will happen if one of the situations or variables is changed, to explain why the condition is satisfied or not and to apply the statement to a variety of unencountered situations.

In conditional knowledge learning, identification of the known and unknown variables leads the learners to determine the effect of known variables on unknown variables. For conditional knowledge learning to occur, concepts in the situation should be determined, then which rules will be applied should be considered, and finally a conclusion about the situation will be reached (Smith and Ragan, 1993). Consequently, learners should have the ability to verbally state the relationships between the mathematical situations in order to justify given statements. When learning conditional knowledge, one should highlight both the relevant declarative and procedural knowledge.

2.1.3 PROCEDURAL KNOWLEDGE (KNOWING HOW)

Procedural knowledge involves procedural rules, which require not only recalling of knowledge but also knowing how to apply this knowledge (Smith & Ragan, 1993). It is the implicit knowledge that highlights both declarative knowledge and conditional knowledge. Procedural knowledge comprises concepts, rules, and algorithms (Schunk, 1996). This type of knowledge is concerned about action sequences such as identifying concepts, knowing how to perform cognitive activities, employing rules and algorithms, and solve problems. In other words, the acquisition of procedural knowledge requires identifying the situation to which procedure applies, the correct order of algorithms, the correct completion of steps, and finally recognizing the correctly completed procedure. It is comparable to “application”, “analysis”, “synthesis”, and “evaluation” levels of Bloom’s taxonomy. Smith and Ragan (1993) emphasized that solving mathematical problems and proving geometry problems are in the context of such processes. The steps that should be taken in order to apply procedural knowledge are: (1) determine if a situation requires doing particular cognitive tasks; (2) recall the steps in the procedure; (3) complete the steps in the procedure; and (4) analyze the completed procedures.

While solving problems learners may perform more automatically on the given task since they know more about the application of strategies. However, they must have acquired the declarative knowledge of fundamental facts and they need to select from a number of rules using their relavant conditional knowledge. Further, they should apply those rules in a logical sequence and use this unique combination in another situation. Smith and Ragan (1993) defined that process as higher order rule learning in which learners both employ conditional and procedural knowledge. This procedure also has other procedures nested in it. For example: The order of theorems to be applied, systematic presentation of rules, axioms, and algorithms as well as strategies for drawing a conclusion.

2.1.4 THE RELATIONSHIP AMONG DECLARATIVE KNOWLEDGE, CONDITIONAL KNOWLEDGE AND PROCEDURAL KNOWLEDGE

Mason and Spence (1999) explained the relationship among knowledge types in the following manner: declarative knowledge forms the ground or the base on which actions depend; conditional knowledge provides an overview that supports the connection making and assists the reconstruction of actions; procedural knowledge provides actions, changing and transforming the situations. Accordingly, declarative knowledge is the factual information, procedural knowledge is the compilation of declarative knowledge into functional units that facilitate the use of appropriate strategies, and conditional knowledge is the understanding of when and where to access certain facts and procedures (Alexander & Judy, 1988; Alexander, et al., 1991). In mathematics, students should select appropriate rules (conditional knowledge), recall declarative knowledge related to these rules, and apply algorithms (procedural knowledge). Smith and Ragan (1993) emphasized that the process of higher order rule learning or problem solving requires both the employment of conditional knowledge and procedural knowledge. While procedural knowledge is sometimes activated by automated and unconscious steps, conditional knowledge requires conscious thinking.

The relationship between students' knowledge of concepts and procedures has long been an important issue in the mathematics education particularly in the domains of counting (Gelman, Meck, & Merkin, 1986), single-digit addition (Baroody & Gannon, 1984), multi-digit addition (Fuson, 1990; Hiebert & Wearne, 1996), fractions (Byrnes & Wasik, 1991; Mack, 1990; Rittle-Johnson, Alibali, & Siegler, 2001), decimal fractions (Moss & Case, 1999; Resnick, et al., 1989), percent (Lembke & Reys, 1994); mathematical equivalence (Knuth, McNeil, & Alibali, 2006; Perry, 1991; Rittle-Johnson & Alibali, 1999), linear equations (Star, et al., 2005), calculus (Engelbrecht, Harding, & Potgieter, 2005), polygons (Pesek & Kirschner,

2000) and algebra-geometry-analytic geometry (Webb, 1979). Knowledge of concepts (declarative knowledge) were assessed through tasks that focus on the ability to make interpretations relevant to the core concepts in a variety of situations as well as the ability to make translations between verbal statements and mathematical expressions. Further, knowledge of concepts (conditional knowledge) were assessed through tasks that focus on the ability to evaluate whether the given statement is true or not and the ability to explain why. Knowledge of procedures (procedural knowledge) was assessed through tasks that focus on the ability to solve a routine problem by using appropriate formulae, procedures, and algorithms. None of these studies, however, attempted to classify conceptual knowledge as declarative and conditional knowledge; rather the particular categories introduced were conceptual knowledge and procedural knowledge. Review of the studies further revealed that the topics studied have been mainly limited to elementary school mathematics. Notably absent were studies of declarative knowledge, conditional knowledge, and procedural knowledge of geometry, algebra, and calculus.

In these studies reports on qualitative analysis displaying interview data (Hiebert & Wearne, 1996; Lembke & Reys, 1994; Mack, 1990; Moss & Case, 1999; Resnick, et al., 1989), reports on teaching experiments (Baroody & Gannon, 1984; Byrnes and Wasik, 1991; Gelman, Meck, & Merkin, 1986; Perry, 1991; Pesek & Kirschner, 2000; Rittle-Johnson & Siegler, 1999; Rittle-Johnson, Siegler, & Alibali, 2001; Rittle-Johnson & Koedinger, 2005; Star, et al., 2005), and reports on discussing quantitative analysis particularly correlational analysis (Byrnes & Wasik, 1991; Engelbrecht, et al., 2005; Knuth, et al., 2006; Rittle-Johnson & Alibali, 2001; Webb, 1979) remain the main methods. These studies tested the unilateral relationships between conceptual knowledge and procedural knowledge, disregarding the fact that a unilateral relationship cannot hold the interactional characteristics of knowledge types. Ma and Kishor (1997) suggested using advanced statistical techniques such as structural equation modeling (SEM) in order to explain the causal relations. In line with this suggestion, this study aimed to investigate the bilateral relationships between three types of knowledge and used SEM as a way of specifying their complex interaction.

Interview studies, which traced the emerging relations, revealed that conceptual knowledge enables students to integrate the most efficient procedures and modify old ones to solve a problem. Audiotaped responses of students from fifth grade to eleventh grade demonstrated that each of them had a broad general conceptual knowledge, however only a few of the students were able to explain their knowledge of procedures (Lembke & Reys, 1984). Researchers indicated that students who were successful in solving computational problems also displayed a good performance in conceptual knowledge questions. In contrast, students who did not recognize the meanings of mathematical concepts and did not have an understanding of relationships among these concepts had difficulty with applying the appropriate algorithms. Mack (1990) added more evidence to the argument in favor of teaching concepts and facts prior to procedures and algorithms. Think-aloud sessions with sixth grade students exhibited that students can construct meaningful algorithms by building upon their both informal knowledge containing knowlege of concepts. Hiebert and Wearne (1996) interviewed with elementary school students who were administered three types of tasks: understanding task (problems contextuaized in declarative knowledge), skills task (problems contextualized in procedural knowledge), and understanding of skills task (problems contextualized in conditional knowledge). Students' understanding and performance profile demonstrated that conceptual understanding enables students to construct new procedures, make sense of procedures, and deploy meaningful solution procedures. Furthermore, performance in understanding tasks and skills tasks showed close relations to understanding of skills tasks that require adjustments to relevant facts and procedures. Moss and Case (1999) conducted interviews with fourth grade students exposed to either experimental curriculum instruction in which concepts and relations among concepts were emphasized or to control curriculum instruction in which traditional view of exercises, rules for operations, and computations were emphasized. The qualitative differences in thinking patterns of students yielded similar results with previous research that when confronted with non-routine problems students fail to generate new procedures since they lack the knowledge of relevant concepts. Students in the treatment group showed some improvement on the problems that involve the application of conventional algorithms. Furthermore, they were able to reason about the relations they used in the problems, which demonstrated their sufficient knowledge of concepts. In line with previous findings, students' lack of recalling the meanings of concepts was linked to their incorrect solutions.

In common interview studies provided evidence that students' computational errors often derive from their failure to represent relations among facts, principles, and algorithms. Thus, a rich store of concepts is crucial to build upon the understanding of facts, construct connections between the facts, develop appropriate procedures and adopt acquired procedures for unfamiliar tasks. Overall responses yielded that conceptual and procedural knowledge of a particular mathematical domain helps students to make sense of their learning process and acquire the procedures with more meaning.

Experimental studies that provided evidence for the relations among knowledge types, specifically explored how instruction about concepts influences students' problem-solving procedures and how instruction about procedures influences students' understanding of concepts. Comparing students' pretest and posttest scores on written descriptions or verbal explanations traced the developmental presedence of one type of knowledge on the other. This association delineated in distinguishing between *concepts-first* and *procedures-first theories* which provide different contributions to the understanding of the contexts in which knowledge is to be conveyed. According to *concepts-first theories*, learners initially develop conceptual knowledge in a domain and then use this knowledge of concepts to generate relations and procedures. Some studies (Byrnes & Wasik, 1991; Perry, 1991; Star, et al., 2005) underlined that conceptual knowledge forms a basis for the acquisition of new procedures and fosters the learning of procedural knowledge. However, earlier work suggested that conceptual knowledge is a necessary but not sufficient condition for the acquisition of procedures. In contrast, *procedures-first theories* state that conceptual knowledge may develop after procedural knowledge. According to that, learners initially select appropriate procedures, for example by trial-and-error and then extract the relevant concepts from that repeated experience (Baroody & Gannon, 1984; Gelman, Meck, & Merkin, 1986; Hiebert & Wearne, 1996).

With regard to *concepts-first theories,* there is a consensus about the developmental presedence of conceptual knowledge over procedural knowledge that affirms procedures cannot function without first accessing concepts. However, in their second experiment Byrnes and Wasik (1991) stated that conceptual knowledge is a necessary but not sufficient condition for the acquisition of procedures. They devised three instructional interventions basically designed on to put emphasis on the relations among mathematical concepts. Fifth grade students who were placed into one of three instructional conditions were administered conceptual and procedural questions. The analysis of the knowledge interrelations yielded that conceptual knowledge forms a basis on which procedures are acquired and fosters the learning of procedural knowledge. This supported their findings in the first experiment details of which is given in the correlational studies.

With regard to *procedures-first theories,* students' computational processes were reported to be independent of their knowledge of basic facts. In their study, which encompassed three experimental settings with preschool children, Baroody and Gannon (1984) found that knowledge of algorithms might develop without or apart from knowledge of concepts. However, they underlined that children who were successful in solving problems contextualized in the knowledge of concepts were significantly more likely to perform better on solving problems that require knowledge of algorithms. In line with Piaget's model, the findings emphasized that children discover the principles of a mathematical domain within the practice process. Furthermore, the failure on mathematical tasks could be attributed to difficulties students have with conceptual and procedural competence and the utilization of the two that requires the coordination of both types of knowledge (Gelman, et al., 1986). They suggested that conceptual competence is a prerequisite for the development of the ability to assess the relations within the task and planning solution procedures, while procedural competence may lead to the acquisition of conceptual competence.

Perry (1991) made an extended synthesis of the effects of principle-based and procedure-based instruction on mathematical performance by conducting two studies. In the first study, fourth and fifth grade students who were assigned to one of two instructional conditions and administered tasks that required in-depth knowledge of principles, algorithms, and relations. The results of the post-test scores were in the favor of students who were exposed to procedure-based instruction. Overwhelmingly, students in both groups showed little level of success in transfer problems that require the knowledge of relations between concepts and algorithms. Students in the principle-based instruction were significantly more likely to demonstrate success in the understanding of relations. Thus, this implied that knowledge of principles is crucial for the acquisition of procedures and relations and further conceptual competence. In contrast, procedure-based instruction did not yield impressive results. It led students to an automatized learning in which they did not analyze why the procedure works or under what conditions it can be used. In the second study, it was investigated whether, when given both principle- and procedure-based instruction, more students take advantage of the type of being successful. Fourth and fifth grade students were exposed to one of principle-plus-procedure instruction or procedure-plus-principle instruction. Although overall students were successful on the posttest, the results were in the favor of procedure-plus-principle students. The findings supported the previous study that only a small number of students were able to solve the transfer problems correctly. Most students failed to demonstrate enough understanding of the principles that would allow successful performance on the transfer tasks. Both studies agreed that a principle is critical to produce deeper learning of transfer. It was emphasized that procedure-based instruction led to an ability to the use of appropriate procedure, whereas principle-based instruction led to a deeper conceptual understanding, which facilitates the understanding of relations. The view in this study confirmed that students should be provided with underlying mathematical concepts and principles, which would help them to figure out the procedures for solving the problems.

Despite the distinction between concepts-first and procedures-first theories, the results provide support for the view that success on mathematical tasks reflects a combination of conceptual and procedural knowledge. The findings of experimental studies support both claims that intructions, which put emphasis on the acquisition of conceptual knowledge foster learners' mathematical understanding, and intructions, which put emphasis on the acquisition of procedural knowledge, are fruitful. The results qualify the conclusions that the interaction among the knowledge of concepts, procedures, and relations leads to understanding beyond routine applications.

Correlational analyses, which were performed using the total score of tests, developed in a variety knowledge contents (e.g memorization and recognition of formulae, relational views of symbols and principles, routine application of formulae), demonstrated significant interrelations between two types of knowledge. Webb (1979) made an initial investigation to determine the relative importance of concepts and procedures in problem solving. The large overlap between tenth grade students' pretest scores and scores obtained from the think-aloud problem-solving sessions revealed that knowledge of concepts and knowledge of procedures are interrelated. The findings suggested problem-solving as a constructive act for which the interactions between concepts and procedures should be taken into consideration. In their first experiment, Byrnes and Wasik (1991) investigated the quantitave relations between conceptual knowledge scores and procedural knowledge scores of fourth and sixth grade students. Mathematical performance of fourth grade students who had been taught concepts but not computations was compared with sixth grade students who had been taught both concepts and computations. Although there was a significant correlation between conceptual and procedural knowledge scores, it appeared that conceptual knowledge is a necessary but not sufficient condition for procedural knowledge. Majority of students demonstrated adequate conceptual knowledge, which supported the developmental trend of concepts being acquired before procedures. However, it was emphasized that the impoverished conceptual knowledge base does not underlie students' computational errors. Engelbrecht, et al. (2005) encompassed three broad findings with respect to the interconnection among students' knowledge of concepts, knowledge of procedures, and their confidence levels when handling problems related to the two types of knowledge. Firstly, the comparison of college students' conceptual and procedural knowledge interestingly revealed a low correlation with a leaning towards conceptual rather than procedural. Second, students' confidence of response indices demonstrated a high correlation between their conceptual and procedural responses. In line with the first premise, students' confidence on conceptual items was significantly higher than their confidence on procedural items. This refuted the general perception that doing is easier than thinking. Finally, the results revealed significant correlations between students' confidence indices and their performance in conceptual and procedural items. When the number of overconfident students with respect to their conceptual and procedural performance was compared, results revealed similar levels. This further confirmed that no more misconceptions exist among students about their conceptual knowledge than about their procedural knowledge. A recent study by Knuth, et al. (2006) lends support to the previous contentions with respect to the students' understanding of concepts and its relation to their application of algorithms. It was emphasized that a relational view of basic concepts is necessary not only to meaningfully generate and interpret on procedures but also to meaningfully operate on procedures. The findings further highlighted that the relationship between concept understanding and procedural performance is independent of students' general mathematics ability. Even when controlling for scores on mathematics ability, the association between concept understanding and procedural performance was significant, which supported the relation is not simply because high-achievers perform better in both types of knowledge. Researchers related students' general poor performance on measures of concept understanding to the lack of explicit focus on teaching for understanding and suggested that efforts to enhance students' knowledge of concepts will pay off in increased knowledge of procedures.

Conceptual and procedural knowledge are both cause for concern because they provide support to this compromise: Greater conceptual knowledge is associated with greater knowledge of procedural knowledge or vice versa. Furthermore, research studies clarify that students have partial knowledge of concepts, relations, and procedures (Fuson, 1990) which highlights the interpretations of their later performance. The findings add more evidence to the argument in favor of the fact that two types of knowledge are learned in tandem rather than independently. Although one type of knowledge begins to emerge first, this knowledge facilitates the improvement in other types, thus leads to increasing gains among the two (Rittle-Johnson & Siegler, 1999). These results suggested an iterative model, which exhibits the bidirectional causal relations between knowledge types (Rittle-Johnson, Siegler, & Alibali, 2001). That is, conceptual knowledge influences the gains in procedural knowledge by improving problem representation, selection of correct procedures, and facilitating the deployment of new procedures. On the other hand, gains in procedural knowledge produce gains in conceptual knowledge through strengthening the use of mental resources, reflecting on why procedures work, and evaluating the correctness of possible answers to the demands of problems.

Rittle-Johnson and Alibali (1999) examined relations between students' understanding of mathematical concepts and procedures for solving problems. The instruction either focused on concepts or on procedures yielded an interconnection between two types of knowledge. The findings revealed that students' explicit or implicit understanding of the concepts and of the interrelations between concepts and principles influences their action sequences for solving problems. It was suggested that level of conceptual knowledge precedes practice skills and predicts future procedural knowledge. Students who were able to use their knowledge of concepts to constrain procedure application and adoption also have increased sense-making capabilities. However, researchers suggested that this relationship is not a unidirectional one. Indeed, it was underlined that students may learn a correct procedure and then develop an understanding of the concepts and principles underlying it. Results indicated that students, who have greater procedural knowledge, as shown by their ability to solve the problems correctly, also have greater conceptual knowledge. Interestingly, students who received procedural instruction reverted to incorrect procedures rather than attempt to adapt or generate new procedures. In contrast, students who received conceptual instruction generalized their knowledge to explain the conceptual basis of facts and procedures. Parallel results in a follow-up study (Rittle-Johnson, Siegler, & Alibali, 2001) provided causal evidence for the development of conceptual and procedural knowledge in an iterative fashion. In the experiments conducted by fifth and sixth grade students, both types of knowledge appeared to develop in a gradual process, which highlighted the iterative relations. Based on these findings, it was emphasized that exploring one type of knowledge in isolation may develop incomplete perceptions about knowledge acquisition and obscure important change processes.

Pesek and Kirschner (2000) explored the effects of this dual approach, which supported the previous views of teaching and learning. This study defined two instructional modes as relational learning and instrumental learning. Relational learning involves teaching for meaning that focus on construction of relationships among concepts, whereas instrumental learning involves teaching for recall that focus on memorization and routine application of formulas. Researchers reported finding interference effects when relational learning follows initial instrumental learning. They compared the effects of fifth grade students' receiving instrumental instruction prior to relational instruction (the I-R treatment) with the effects of relational instruction only (the R-O treatment). The posttest scores showed that students in the R-O group significantly outperformed their counterparts in the I-R group. In addition, the interview data provided further insight into effects of juxtaposing instructional modes. Two treatment groups emerged with different approaches to solving problems. The I-R group referred to formulas, operations, and routine applications as the means for problem-solving, wheras the R-O group used conceptual and challenging methods of constructing solutions. Transcriptions demonstrated that the R-O interviewees were able to make more sense of the formulas despite their lack of instructional exposure to them, whereas the I-R interviwees ungrounded use of formulas carried over into their explanations of how formulas work. Although, they relied heavily on formulas but lack the understanding of them further provided evidence for their insufficient metacognitive knowledge and regulation. Collectively, the findings indicated that initial rote learning that relies on drill-and-practice implementations could create interference to later translation into meaningful learning of concepts. In mathematics classes, most instructional time is spent on routine applications of algorithms to consolidate procedural knowledge; much less emphasis is given to students' intuition and making sense of concepts. These results suggested the incorporation of conceptual knowledge and procedural knowledge in classroom practices that would respond to the recommendations of fundamental reforms in mathematics education.

Rittle-Johnson and Koedinger (2005) further suggested designing knowledge scaffolds that elicit contextual, conceptual, and procedural knowledge to support mathematical problem-solving. It was indicated that organized knowledge requires individuals integrate their contextual, conceptual, and procedural knowledge in a particular domain. In this study, contextual knowledge (knowledge of how things work in real-world situations) was elicited by situating mathematical problems in story contexts, conceptual knowledge was viewed as the integrated knowledge of important principles and finally, procedural knowledge was considered as a direct source of problem-solving knowledge. The role of three types of knowledge was evaluated for sixth grade students' solving scaffolded problems in a design experiment. The results revealed that students were more likely to choose the correct solution on each of the scaffolded problems compared to no-scaffold problem. They accomplished better outcomes on procedural-scaffold-problem than on the contextual- or conceptual-scaffold-problems. Students' written work added additional insight into how each scaffold influenced problem-solving. The findings demonstrated that procedural scaffold greatly increased their drawing correct solutions, whereas conceptual scaffold consistently reduced their computational errors. Overwhelmingly, students were lack the ability to use alternative strategies such as visual or numeric estimation. However, the contextual, conceptual, and procedural scaffolds each helped students to reach a correct solution. There was substantial effect of the intervention, which incorporated all three scaffolds on learning that students were more successful at posttest compared to pretest. Students' improvement at posttest was due to the decrease in their errors across problems.

Taken together, the premise in the current study is that, all types of knowledge are interactive that the presence of one type of knowledge can directly or indirectly affect any other.

2.2 METACOGNITION

Recent research in cognitive psychology states the importance of metacognition in instruction. There has been a growing interest in exploring the role that aspects of metacognition play in the performance of cognitive tasks. It is convinced that metacognitive knowledge and skills are the determinants of success or failure in a variety of situations such as reading (Cross & Paris, 1988), memory (Pressley, et al., 1985), intelligence (Borkowski, 1985), aptitude (Swanson, 1990), and problem-solving (Garofalo & Lester, 1985). Initially the concept of metacognition appeared within information processing theory, which shows how individuals receive and learn information. Information processing theory focuses on how individuals attend to environmental events, encode information to be learned, and relate it to knowledge in memory, store new knowledge in memory, and retrieve it as needed (Shuell, 1986). Metacognition is also consistent with social learning theorists' notion of self-regulation as it provides the mechanism through which individuals begin to regulate one aspect of their lives and their learning (Smith, 1994; Zimmerman, 1989).

The term 'metacognition' was primarily introduced by Flavell (1971) and viewed as learners' knowledge of their own cognition. It is generally defined as 'thinking about thinking', 'knowing about knowing', or 'cognitions about cognitions'. Broadly, Brown (1987) defined metacognition as "one's knowledge and control of own cognitive system". It is also defined as "knowledge about knowledge" or "reflections about actions" (Weinert, 1987), the process by which individuals think about their own thinking to develop strategies to solve problems (O'Neil & Brown, 1998), monitoring and regulating one’s thinking (Goos, 2002), and one’s inner awareness about one’s learning process (Hennessey, 2003). In common, metacognition refers to the ability to reflect upon one’s own cognitive processes (Schraw & Dennison, 1994; Schraw, 1998) and these definitions all underline the conscious use, control, and online awareness that individuals have of their own cognitive abilities and processes. Hence, metacognition is related to learners’ knowledge, awareness, and control of the processes by which they learn (Brown, 1987), and the metacognitive learner is supposed to have the sophisticated characteristics who is able to recognize, evaluate, and where needed reconstruct existing ideas (Gunstone, 1991). Garofalo and Lester (1985) viewed that metacognition is involved in following a sequence of steps such as selecting and planning what to do and monitoring what is being done.

In general, studies that investigated metacognition used one of Flavell’s (1979) or Brown’s (1978) theories, which distinguish the components of metacognition that stem on numerous definitions of the term. Flavell defined metacognition as one’s knowledge concerning one’s own cognitive processes and products or anything related to them (Flavell, 1979). An initial framework was introduced involving “metacognitive knowledge” (what one knows about cognition) and “metacognitive experiences” (how one regulates cognition). Metacognitive knowledge includes person, task, and strategy components while metacognitive experiences include feelings of understanding and mental processes employed to strategy implementation. A complex interaction between metacognitive knowledge and metacognitive experiences affects the learning process and thus the performance. Person variables refer to the knowledge of one’s beliefs about oneself. They involve understanding one’s own memory abilities and capabilities, being aware of one’s own strengths and weaknesses and judging whether one can recall information. The task variables refer to the knowledge about the scope and the requirements of tasks in which some conditions are more difficult than others. The context of task knowledge involves both one’s beliefs about the subject itself and the nature of the tasks relevant to this subject. This kind of knowledge covers the task's context, content, and the syntax. Finally, the strategy variables refer to the knowledge of general and specific cognitive strategies (remembering, understanding, and processing information) together with the awareness of their usefulness and necessity to apply when approaching a problem situation. The context of strategy knowledge involves the knowledge of algorithms, procedures and heuristics. The awareness of such knowledge leads individuals to comprehend the problem better, organize the information given in the problem context, plan the solution steps, execute the plans and check the results. Metacognitive experiences include monitoring the knowledge while performing on a task. In terms of this regulatory aspect of metacognition, students select the appropriate strategies to understand the nature of the given task, plan the solution of the task, monitor the strategies being implemented, evaluate the strengths and weaknesses of these strategies and where needed revise information, establish new goals, or implement new strategies. When students engage in metacognitive activities these two aspects of metacognition interact. For example, students consider their skills on solving a geometry problem (person), the context and the nature of the problem (task), the strategies to be used (strategy), and if necessary, they revise and execute alternative strategies (experience).

In the present study, Brown (1978) framework of metacognition was employed because the instruments adapted and developed are based on a measure of metacognition that was developed conceptually from this framework, which provides a direct application to academic learning settings. Brown (1978) categorized metacognition in two components as "knowledge of cognition" and "regulation of cognition" which was used in later studies (Schraw & Dennison, 1994; Schraw & Moshman, 1995; Schraw, 1997). Knowledge of cognition refers to how much learners understand about their memory and learning. Regulation of cognition refers to how well learners can regulate their own memory and learning. Schraw and Dennison (1994) divided knowledge of cognition component of metacognition into three distinct areas as declarative knowledge (knowing that and about), conditional knowledge (knowing why and when), and procedural knowledge (knowing how). Regulation of cognition component of metacognition was divided into five subcomponents including planning, selecting, monitoring, evaluating, and debugging. For example, if a student knows that she/he has a difficulty in remembering the definitions of geometric figures and the properties related to these geometric figures, she/he may develop a table of definitions and properties to better facilitate memorization. The understanding of this difficulty with geometric concepts, and their understanding of when and how to use this table while solving a geometry problem are examples of knowledge of cognition. The decision to use the appropriate property or principle is an example of regulation of cognition. Therefore, students' repertoire of knowledge of cognition and regulation of cognition has a substantial effect on their knowledge acquisition in geometry.

Descriptions of two components of metacognition are presented in detail in the following sections.

2.2.1 KNOWLEDGE OF COGNITION

In general, knowledge of cognition involves knowledge about one's own capabilities, beliefs, cognitive abilities and processes. It includes at least three different kinds of metacognitive awareness: declarative knowledge, conditional knowledge, and procedural knowledge (Brown, 1987; Jacobs & Paris, 1987; Schraw & Moshman, 1995; Sperling, et al., 2002). Declarative knowledge can be defined as knowledge about one's general processing abilities (Paris, et al., 1984), self and about strategies. Individuals who know about what factors influence their own learning performance are thought to be good learners. Hence, good learners are appeared to have more knowledge about their capacity limitations, rehearsal, and distributed learning (Garner, 1987; Schneider & Pressley, 1989). Conditional knowledge was defined as the knowledge of when and why to use declarative and procedural knowledge (Garner, 1990). Good learners know when and what information to rehearse, selectively allocate their resources, and use the most effective strategies (Reynolds, 1992). Conditional knowledge also provides students to adjust to the changing situations and construct appropriate relations among the situations. Finally, procedural knowledge can be defined as knowing how to successfully solve problems (Paris, et al., 1984) and knowledge about how to use strategies. It refers to knowledge about doing things. Individuals who have high degree of procedural knowledge are more aware of how to chunk and categorize new information (Schraw, 1998), perform more automatically on the given tasks, and select the most effective strategies (Pressley, Borkowski & Schneider, 1987).

Research studies support the premise that good learners effectively possess declarative, conditional, and procedural knowledge. Such learners are able to make explicit descriptions of their own cognition, which improve their performance (Schraw & Moshman, 1995).

2.2.2 REGULATION OF COGNITION

Regulation of cognition refers to knowledge about one's own control processes during the execution of the task. It is concerned about the metacognitive activities that promote individuals' control of their own thinking and learning. Schraw and Dennison (1994) described subprocesses of regulation of cognition in five categories as (1) planning, (2) information management strategies (selecting), (3) comprehension monitoring, (4) debugging, and (5) evaluating. Planning involves goal setting and allocating resources prior to learning (Schraw & Dennison, 1994). In this category students are able to indicate the operations to be carried out and their order or the ability to recognize the correct steps necessary to perform the task

(Lucangeli & Cornoldi, 1997). Students make predictions before or after reading a problem, sequence their strategies, or allocate time or attention before beginning to solve the problem. Selecting involves skills and strategy sequences used to process information more efficiently. This process enables students to choose the most appropriate strategy. Monitoring involves the assessment of one's learning or strategy use (Schraw & Dennison, 1994). It is concerned about the on-line awareness of comprehension and task performance (Schraw & Moshman, 1995) and the ability to recognize the strategies used to solve the task (Lucangeli & Cornoldi, 1997). Monitoring ability improves as students engage in periodic self-testing while performing on a task. Debugging involves the strategies used to correct comprehension and performance errors. Students assess what is going on through the solution steps and make judgments about their computations. Finally, evaluating involves the analysis of performance and strategy effectiveness after a learning episode (Schraw & Dennison, 1994). It refers to the ability to estimate whether the accomplished results were correct or not (Lucangeli & Cornoldi, 1997). During evaluation students re-evaluate their goals and conclusions in terms of deciding whether the implemented strategy should be revised or abandoned.

Research studies affirm that regulatory processes provide students better use of their cognitive resources including attention, strategy selection, and awareness of comprehension. These processes improve performance when regulatory skills are activated by the individual and the understanding of how to use these skills are integrated into instructional settings. Besides, regulatory processes are highly automated and develop without any conscious reflection, and that they are difficult to report in many learning situations (Brown, 1987).

Several researchers as discussed above have investigated metacognition and components of metacognition. It can be concluded that metacognitive knowledge and regulation support students to become strategic and effective performers and provide them to plan, monitor, and evaluate their learning in a way that positively supports the improvement in their performance. As students become more aware of their learning and thinking, they make more deliberate thought processes in the tasks they are dealing within, make adaptive decisions and coordinate them.

2.2.3 THE RELATIONSHIP BETWEEN METACOGNITION AND KNOWLEDGE OF MATHEMATICS

A causal relationship between metacognition and mathematical performance has long been assumed to exist. That is, a more metacognitive awareness contributes to a more success in mathematics. A considerable body of research has been developed to explore the interrelation among components of metacognition and mathematical performance that relied on using particularly correlational analysis (Lucangeli & Cornoldi, 1997; Sperling, et al., 2002; 2004; Swanson, 1990; Tobias & Everson, 2002; Veenman, Wilhelm, & Beishuizen, 2004; Veenman, Kok, & Blöte, 2005), crosstab analysis (Panaoura & Philippou, 2003), latent variable modeling analysis (Panaoura & Philippou, 2005; 2007) and qualitative methods particularly interviews (Artz & Armour-Thomas, 1992; Goos & Galbraith, 1996; Maqsud, 1997; Pugalee, 2001; 2004; Stillman & Galbraith, 1998; Wilson & Clarke, 1994). Further the effect of metacognitive instruction on mathematical problem solving and reasoning has been investigated in experimental settings (Schurter, 2002; Slife, Weiss, & Bell, 1985; Kramarski, Mevarech, & Lieberman, 2001; Kramarski, Mevarech, & Arami, 2002; Kramarski, 2004; Mevarech & Kramarski, 1997; Mevarech, 1999).

Correlational studies adopted the view that knowledge of cognition or metacognitive knowledge constitutes the self-representation dimension of metacognition whereas, regulation of cognition or metacognitive skills constitute the self-regulation dimension. The findings revealed that metacognitive knowledge and skills are highly associated with mathematical performance (Lucangeli & Cornoldi, 1997; Tobias & Everson, 2002) and intellectual ability in partial (Veenman, Wilhelm, & Beishuizen, 2004; Veenman, Kok, & Blöte, 2005). One limitation of the correlation studies is that, it can not be known to what extent and with what relation components of metacognition and performance in mathematics influence each other.

Lucangeli and Cornoldi (1997) evaluated the extent to which regulatory processes related to areas of arithmetic, problem solving and geometry were able to explain the performance of third and fourth grades in each area of mathematical learning. A general view of the relation that links success in mathematics to regulatory processes demonstrated that when students are required to answer items aimed at focusing on the presence of metacognitive regulation they altered the way that they approach to a mathematical problem. Students' high scores on metacognitive questions were indicated to explain their higher mathematical competence. That is, students who performed better were supposed to have a complete knowledge of evaluating the correctness of their solution steps and final answer. The results showed evidence that regulatory processes such as planning, monitoring, and evaluation were highly associated with mathematical performance. However, the relationship was indicated to be less strong for the more automized tasks that depend on the application of a straightforward algorithm. The correlation between metacognitive knowledge and mathematical performance yielded similar results in Tobias and Everson's (2002) study. Researchers found that students who estimated that they would solve the problem correctly got high scores on the aptitude test. Studies that focused on establishing to what extent metacognitive skill development is associated with intelligence emphasize that regulatory skills develop and contribute to learning performance, partly independent of intelligence. In a study of Veenman, et al. (2005), similar to Lucangeli and Cornoldi (1997), investigated the impact of hints on the execution of metacognitive skills. Participants were administered to procedural knowledge questions with or without metacognitive hints. The analyses contrasting cues versus no-cues performance revealed a significant effect of giving metacognitive cues to students. The findings showed that metacognitive cueing yielded better learning outcomes as students are made aware of using their metacognitive skills such as designing a solution plan containing goals, keeping track on their progress, or elaborating their conclusions. Consequently, it was suggested that regulatory processes should be appropriately integrated to the learning contexts to guarantee success in performance.

Although available research has demonstrated that metacognitive processes play an essential role in solving mathematical tasks, some others indicated that this relationship is not direct (Sperling, et al., 2002; 2004; Swanson, 1990). With regard to knowledge of cognition component of metacognition, Swanson (1990) preliminarily determined whether metacognition and aptitude measures containing conditional knowledge questions were correlated. He reported that fourth and fifth grade students' knowledge of their own beliefs, strengths and weaknesses, the scope and the requirements of the given task, and general and specific cognitive strategies are not associated with their aptitude. In the same vein, further studies revealed the correlations between scores on metacognitive inventories and mathematical performance were indicated to be statistically significant but low or non-significant and negative. Although there is a general recognition that metacognitive knowledge and regulation would positively effect performance and hence facilitate learning, results illustrated that the relationship between components of metacognition and standardized achievement scores is not direct. Sperling, et al. (2002) provided further information about the correlations among Jr. MAI inventories and achievement scores. Version A which was administered to grade three to grade five students, yielded significant correlations with achievement. However, the correlations were not likely meaningful with Version B which was administered to grade six to grade nine students. In their follow-up study Sperling, et al. (2004) affirmed that metacognitive knowledge and regulation may not correspond to increased achievement. Consistent with previous findings, adult version MAI (Schraw &

Dennison, 1994) which was administered to college students, yielded a significant negative correlation between measures of mathematical achievement and metacognition. The results indicated that students' performance in mathematics is not associated with their awareness of their strengths and weaknesses, knowledge about when, why, and how to use strategies or with their knowledge about planning, implementing, monitoring, and evaluating strategy use.

Similarly, the crosstab analysis (Panaoura & Philippou, 2003) of fourth and sixth grade students' answers to metacognitive knowledge and regulation questions before-and-after solving procedural knowledge questions revealed that they do not have a rich repertoire of metacognitive knowledge and regulation. The results demonstrated that a few number of students who get the correct answer claimed that they were aware of their ability to solve it. Interestingly, only half of these students claimed that they knew their answer was correct when they finished the solution. Overall results indicated that students overestimated their abilities in understanding their solution methods, clarifying their goals or monitoring their progress toward a solution. Students who stated they could not regulate their solution strategies failed to solve the problem correctly. Further, in a follow-up study (Panaoura & Philippou, 2005) that investigated students' metacognitive knowledge and regulation in relation to their mathematical performance fourth and sixth grade students were categorized in three groups (high cognitive-high metacognitive, low cognitive-low metacognitive, low cognitive-high metacognitive) with respect to their cognitive and metacognitive processes. The findings revealed that high-achievers with a more precise relation between their cognitive and metacognitive processes were better able to evaluate the given tasks according to the strategies they can use to reach a solution. In contrast, the low-achievers were unaware of the appropriateness of the strategies necessary to draw a conclusion. Besides, the presence of such a category as low-achievers with high-metacognitive capabilities supported previous claims that some students overestimate their cognitive performance. Low-achievers were generally unaware of the ineffectiveness of the strategies they plan to use. In this sense, improvement of mathematical performance depends on promoting metacognitive awareness by building awareness among students that metacognition exists.

Research that aim to model the relations among cognitive variables (processing efficiency, working memory, and mathematical performance) and metacognitive variables (knowledge of cognition and regulation of cognition) emphasized a notable finding: no statistically significant correlations were found between metacognitive variables and mathematical performance (Panaoura & Philippou, 2007). Modeling was used to explore the interrelations among metacognitive knowledge, regulation and mathematical performance. The findings provided evidence that students' metacognitive knowledge and regulation do not directly affect their performance in mathematics.

Qualitative analyses focused both on the mathematical problem-solving processes and the underlying cognitive and metacognitive activities that directed these processes. The data collected from the think-aloud sessions were interpreted in terms of a cognitive and metacognitive framework devised for studying students' mathematical performance that comprises four categories: orientation, organization, execution, and verification (Garofalo & Lester, 1985) or for delineating the interplay between cognitive and metacognitive behaviors within problem solving processes that comprises eight categories: read, understand, analyze, explore, plan, implement, verify, and watch and listen (Artz & Armour-Thomas, 1992). In common both frameworks adopted the view that cognition is involved in doing, whereas metacognition is involved in choosing and planning what to do and monitoring what is being done.

Garofalo and Lester's (1985) framework is related to four categories that broadly defines Polya's four problem solving phases (understanding, planning, execution, review). In the orientation phase students are involved in assessing and understanding the problem. The metacognitive behaviors include comprehension strategies, analysis of information, assessment of familiarity with task, initial and subsequent representation, and assessment of problem difficulty and chances of success. Organization phase includes planning of behaviors and choice of actions. Students involve in metacognitive behaviors such as identification of problem goals, global planning, and local planning with respect to the goals identified. Execution phase encompasses regulating behaviors to conform to plans. At this phase students exhibit such behaviors as performance of local actions, monitoring of progress regarding local and global plans, and trade-off decisions. The final phase verification involves the evaluation of decisions made and of outcomes of executed plans. Students evaluate their overall processeses in orientation, organisation and execution phases. They check the adequacy and consistency of their progress. Studies that are conducted with regard to this framework showed that metacognitive actions were involved in all phases of the solution processes where more time was spent on orientation and execution activities with little time spent on organisation and verification activities. Stillman and Galbraith (1998) reported that during orientation activities students focused their attention on the problem statement in an effort to become familiar with the task. Their initial attempts to analyze the information in the problem facilitated their sorting through the information and how this information can be applied in developing a solution plan. Students' activities that demonstrated the presence of organisation processes showed that they were not aware of how they formulated or implemented the plans they have identified. The findings confirmed that students often identify their goals when the given task is more difficult and the solution needs more mathematical line of reasoning. It is emphasized that metacognition is more essential for the challenging non-routine tasks while minimal metacognitive activity is required for routine tasks that trigger an automatic response. The link between successful mathematical performance and successful deployment of metacognition is associated to students' appropriate knowledge base, which in turn related to the demands of the task (Wilson & Clarke, 1994). Interestingly, execution activities were evident while students were performing calculations and applying their knowledge about strategies, however they displayed an absence of regulating the execution activities particularly limited monitoring their progress of generating plans. Although they displayed a partial understanding of the problem context, they failed to execute a plan since they lack the ability to make a complete representation of the problem or apply formal mathematical procedures. The metacognitive behaviors associated with verification activities demonstrated that some students verify their solutions by testing alternative solution procedures. Although, they rarely attempted to evaluate their decisions and results, the frequency of these behaviors did not correspond to success in the performance. In contrast to regulatory behaviors, Maqsud (1997) indicated that high school students' metacognitive knowledge behaviors positively contribute to their mathematical performance. The interview data demonstrated that mathematics performance of the students with high general ability and high metacognition was found the highest while mathematics performance of the group with high general ability and low metacognition was lower than that of the former.

Artz and Armour-Thomas (1992) attempted to develop a framework as a synthesis of problem-solving steps identified in mathematics education research by Polya, Lester and Garofalo, and Schoenfeld, and of cognitive-metacognitive problem-solving steps identified by Flavell. The episodes were categorized as *read, understand, analyze, explore, plan, implement, verify, and watch and listen*. In reading episode students exemplify instances of doing. Students are assigned to understanding category when they make comments reflecting their attempts to clarify the meaning of the problem. In the episode of analyzing students reveal thought processes including statements about the problem and the problem-solving process. Similarly, planning episode includes student statements made about how to proceed in problem-solving process.

Exploration, implementation, and verification phases involve students' monitoring processes. The final phase watch and listen is an optional episode, which is an important dimension in the process of problem-solving for small group settings. Similar findings with previous studies that devised the former framework were emphasized. The number of cognitive and metacognitive behaviors was coded mostly as exploring followed by reading and understanding. The extent to which students exploit their metacognitive knowledge and the manner in which they monitored their progress exhibited that their failure in establishing exactly what they know, when to make use of cognitive resources runs them into difficulties on their solution attempts. Transcripts demonstrated that throughout the problem-solving session exploration accompanied by monitoring and regulating processes. Students labeled with the lowest percentage of episodes of exploration failed to solve the given task. It was concluded that the interconnection between the cognitive and metacognitive processes is complex and that the effective interaction of the two is crucial to success in analysing the given information and conditions and choosing appropriate strategies for the solution.

In the same vein, ninth grade students' written and verbal descriptions of their progress in mathematical problem-solving showed evidence of a cognitive and metacognitive framework (Pugalee, 2001; 2004). Students' writing provided information about the crucial role that initial understanding of the problem plays in facilitating students to identify the appropriate solution paths. Narrative examples that illustrate organization behaviors exhibited that students were able to develop plans to solve problems although these plans were not always the efficient ones. Furthermore, in some cases students implied their plans instead of explicitly stating it. The failure in execution of the plans was indicated to derive from their lack of the ability to completely represent the problem conditions and the skills to approach the problem solution through formal application. Consistent with the previous research, the written descriptions revealed that execution activities such as carrying out goals and making computations comprised the largest number of behaviors whereas verification activities such as checking for the final answer accounted for the smallest number of behaviors (Pugalee, 2004). Despite students' limited display of verification behaviors, the findings supported that their interaction with metacognitive behaviors along with an understanding of cognitive behaviors distinguishes them as successful problem solvers.

Research concerning the effect of metacognitive instruction undertook a comparative analysis of performance of students in cooperative learning environments with or without metacognitive intervention or questions. Mevarech and Kramarski (1997) arranged metacognitive questions that follow the four-step cognitive-metacognitive framework: orientation, organisation, execution, and evaluation (Garofalo & Lester, 1985). The use of metacognitive questions such as "What is in the problem?", "What are the differences between the problem you are working on and the previous problem you solved?", or "What is the most appropriate strategy to solve the problem?" were suggested to support students' regulatory processes during problem-solving. Results showed that metacognitive instruction produced beneficial effects on seventh grade students' mathematical performance that the ones engaged in such instruction significantly outperformed their counterparts on measures of mathematics achievement. The main differences occurred in the quality of knowledge constructed under two conditions that cognitive responses of the treatment groups often included multiple perspectives, verbal explanations enriched by evidences, and mathematical principles. In contrast, control group responses presented only the final answer without stating the relationships or justifying the elaborations. Kramarski, Mevarech, and Arami (2002) underlined that metacognitive training exerts positive effects not only on students' ability to solve routine problems but also on their ability to solve non-routine problems that require effective use of regulatory processes. Activating such processes promoted mathematical understanding and further enabled the treatment groups to understand the given task, think about appropriate strategies for solution, suggest new strategies, and compare the applied strategies. This probably led students to capture all the structural features of the problem and comprehend the problem before attempting to immediate solution procedures. Moreover, metacognitive training strengthened students' self-confidence in approaching unfamiliar tasks and supported them when coping with the complexity of such tasks. This premise was also evident in findings of Mevarech (1999) that low achievers performed best under metacognitive instruction. In common, findings of the experimental studies combine with each other that both low and high achievers benefit from metacognitive instruction. Furthermore, it was indicated that students under all conditions were able to correctly represent the routine problems. However, they encountered difficulties in representing non-routine problems. Since non-routine problems are at a higher level of cognitive complexity than routine problems, they require the activation of careful metacognitive knowledge and regulation processes. In this sense, training treatment groups to activate such metacognitive processes enabled them to organize and process the given information, justify their reasoning and hence, solve non-routine problems better than their counterparts in the control group. Follow-up studies (Kramarski, Mevarech, & Lieberman, 2001; Kramarski, 2004) which investigated the contribution of metacognitive instruction with regard to mathematical reasoning and conceptions supported previous research that when middle grade students are provided explanations to elaborate information and make connections, they become more able to reflect on the similarities and differences between previous and new problems, comprehend the problem before attempting a solution, and consider about strategies to draw a conclusion. Schurter (2002) investigated the impact of comprehension monitoring on college students' problem solving processes. The results supported that treatment groups who received increased emphasis in the use of comprehension monitoring technique performed better in problem-solving than control groups who did not receive this type of instruction. Participants were interviewed to qualitatively examine their use of metacognitive regulation and problem-solving strategies. Interestingly, no apparent increase in the conscious use of metacognitive processes among the treatment groups was reported. However, the reason for their improved performance was indicated to be attributable to their subconscious use of their metacognitive processes.

Furthermore, O'Neil and Brown (1998) suggested that open-ended and multiple-choice question formats have differential effects on metacognitive regulatory processes of students in the context of mathematics. This study employed open-ended questions in the sense of conditional knowledge questions that require students to justify their answers. Multiple-choice questions were employed in the sense of procedural knowledge questions that require students to find a final answer. The results demonstrated that open-ended questions require more cognitive strategy use and less self-checking than did multiple-choice questions. Different self-checking behaviors are utilized for varying item formats, which may be a result of the complexity of the open-ended question format that students may not know how to check their progress and solution procedures in such an unfamiliar format.

Many of the afore-cited studies provide substantial evidence in favor of the positive interrelation among metacognitive knowledge, metacognitive regulation and mathematical performance. The review of these studies revealed that context of the assessments mainly focused on elementary school mathematics and rarely on secondary school mathematics, particularly concerning procedural knowledge questions. Another limitation of several studies is that their focus is restricted to the regulation of cognition component of metacognition, thereby excluding the relation among both components of metacognition and three different types of knowledge. Besides, these studies only indicate the unilateral relationships among metacognitive components and mathematical performance, which can not explain to what extent these constructs influence one another. Veenman, Van Hout-Wolters, and Afflerbach (2006) suggested the use of PCA and LISREL analyses, which emphasize the bilateral relationships among cognitive and metacognitive constructs. The current study conceptualized that different types of knowledge, namely declarative knowledge, conditional knowledge, and procedural knowledge affected by two metacognitive constructs, namely knowledge of cognition and regulation of cognition. The structural relationships among the knowledge and metacognitive constructs were interpreted as indices of effects of one variable on the other. Thus, it was hypothesized that metacognitive constructs are positively related to three different types of knowledge in geometry.

2.3 SUMMARY OF THE LITERATURE REVIEW

Brown (1978) categorized metacognition in two components as "knowledge of cognition" and "regulation of cognition" which was used in later studies (Schraw & Dennison, 1994; Schraw & Moshman, 1995; Schraw, 1997). Metacognitive knowledge and regulation support students to become strategic and effective performers and provide them to plan, monitor, and evaluate their learning in a way that positively supports the improvement in their performance. As students become more aware of their learning and thinking, they make more deliberate thought processes in the tasks they are dealing within, make adaptive decisions and coordinate them. A considerable body of research has been developed to explore the interrelation among components of metacognition and mathematical performance that relied on using particularly correlational analysis (Lucangeli & Cornoldi, 1997; Sperling, et al., 2002; 2004; Swanson, 1990; Tobias & Everson, 2002; Veenman, Wilhelm, & Beishuizen, 2004; Veenman, Kok, & Blöte, 2005), crosstab analysis (Panaoura & Philippou, 2003), latent variable modeling analysis (Panaoura & Philippou, 2005; 2007) and qualitative methods particularly interviews (Artz & Armour-Thomas, 1992; Goos & Galbraith, 1996; Maqsud, 1997; Pugalee, 2001; 2004; Stillman & Galbraith, 1998; Wilson & Clarke, 1994). Further the effect of metacognitive instruction on mathematical problem solving and reasoning has been investigated in experimental settings (Schurter, 2002; Slife, Weiss, & Bell, 1985; Kramarski, Mevarech, & Lieberman, 2001; Kramarski, Mevarech, & Arami, 2002; Kramarski, 2004; Mevarech & Kramarski, 1997; Mevarech, 1999). These studies provide substantial evidence in favor of the positive interrelation among metacognitive knowledge, metacognitive regulation and mathematical performance. The review of these studies revealed that context of the assessments mainly focused on elementary school mathematics and rarely on secondary school mathematics, particularly concerning procedural knowledge questions. Another limitation of several studies is that their focus is restricted to the regulation of cognition component of metacognition, thereby excluding the relation among both components of metacognition and three different types of knowledge. Besides, these studies only indicate the unilateral relationships among metacognitive components and mathematical performance which can not explain to what extent these constructs influence one another. Veenman, Van Hout-Wolters, and Afflerbach (2006) suggested the use of PCA and LISREL analyses which emphasize the bilateral relationships among cognitive and metacognitive constructs.

Knowledge is defined as the organization of information into bodies of meaningfully interconnected facts and generalizations which serves as a vehicle for thought and problem- solving (Gagné & Briggs, 1979). Mason and Spence (1999) explained the relationship among knowledge types in the following manner: declarative knowledge forms the ground or the base on which actions depend; conditional knowledge provides an overview that supports the connection making and assists the reconstruction of actions; procedural knowledge provides actions, changing and transforming the situations. The relationship between students' knowledge of concepts and procedures has long been an important issue in the mathematics education particularly in the domains of counting (Gelman, Meck, & Merkin, 1986), single-digit addition (Baroody & Gannon, 1984), multi-digit addition (Fuson, 1990; Hiebert & Wearne, 1996), fractions (Byrnes & Wasik, 1991; Mack, 1990; Rittle-Johnson, Alibali, & Siegler, 2001), decimal fractions (Moss & Case, 1999; Resnick, et al., 1989), percent (Lembke & Reys, 1994); mathematical equivalence (Knuth, McNeil, & Alibali, 2006; Perry, 1991; Rittle-Johnson & Alibali, 1999), linear equations (Star, et al., 2005), calculus (Engelbrecht, Harding, & Potgieter, 2005), polygons (Pesek & Kirschner, 2000) and algebra-geometry-analytic geometry (Webb, 1979).

Knowledge of concepts (declarative knowledge) were assessed through tasks that focus on the ability to make interpretations relevant to the core concepts in a variety of situations as well as the ability to make translations between verbal statements and mathematical expressions. Further, knowledge of concepts (conditional knowledge) were assessed through tasks that focus on the ability to evaluate whether the given statement is true or not and the ability to explain why. Knowledge of procedures (procedural knowledge) was assessed through tasks that focus on the ability to solve a routine problem by using appropriate formulae, procedures, and algorithms. None of these studies, however, attempted to classify conceptual knowledge as declarative and conditional knowledge, rather the particular categories introduced were conceptual knowledge and procedural knowledge.

Review of the studies revealed that the topics studied have been mainly limited to elementary school mathematics. Notably absent were studies of declarative knowledge, conditional knowledge, and procedural knowledge of geometry, algebra, and calculus. In these studies reports on qualitative analysis displaying interview data (Hiebert & Wearne, 1996; Lembke & Reys, 1994; Mack, 1990; Moss & Case, 1999; Resnick, et al., 1989), reports on teaching experiments (Baroody & Gannon, 1984; Byrnes and Wasik, 1991; Gelman, Meck, & Merkin, 1986; Perry, 1991; Pesek & Kirschner, 2000; Rittle-Johnson & Siegler, 1999; Rittle-Johnson, Siegler, & Alibali, 2001; Rittle-Johnson & Koedinger, 2005; Star, et al., 2005), and reports on discussing quantitative analysis particularly correlational analysis (Byrnes & Wasik, 1991; Engelbrecht, et al., 2005; Knuth, et al., 2006; Rittle-Johnson & Alibali, 2001;

Webb, 1979) remain the main methods. These studies tested the unilateral relationships between conceptual knowledge and procedural knowledge, disregarding the fact that a unilateral relationship can not hold the interactional characteristics of knowledge types. Ma and Kishor (1997) suggested using advanced statistical techniques such as structural equation modeling (SEM) in order to explain the causal relations.

The afore-mentioned summary suggests that there is a need for further studies to explore the affect of metacognitive knowledge and regulation on students' different types of knowledge; in relation with the interconnection among declarative knowledge, conditional knowledge, and procedural knowledge. The effectiveness of learning mathematics stems from the ability to draw on a rich store of well-structured knowledge and heuristics as well as the ability to manage own metacognitive processes. With respect to this compromise, the present study investigates the relationships among metacognitive and cognitive constructs.

CHAPTER 3

METHODOLOGY

This chapter involves the methodology of the study comprising population and sample, the development of the instruments together with their validity and reliability, procedure, data collection and analysis including the structural equation modeling.

3.1 POPULATION AND SAMPLE

The target population of this study consists of all tenth grade secondary school students in Ankara who enrolled in geometry courses during the 2005-2006 spring semester. The accessible population is all tenth grade secondary school students in Çankaya district of Ankara. The results of the present study are generalized for this population. Total number of tenth graders in secondary schools in Çankaya district is almost 2600. The sample of this study was 297 (153 females and 144 males) tenth grade secondary students in Çankaya, Turkey. Thus, this sample size constituted at least 10% percent of the population. The ages of the selected sample ranged from 17 to 18. The selected sample was representative of diverse socioeconomic backgrounds. Prior to the study permission was taken from the Turkish Ministry of Education for 17 high schools residing in Çankaya district. However, only six (three Anatolian high schools, two private high schools and one public high school) of those 17 schools accepted to participate in the study. Anatolian high schools are the schools that take students according to the Ortaöğretim Kurumları Sınavı (OKS) conducted by Turkish Ministry of Education. This exam includes 100 multiple choice questions in four domains: Turkish Literature, Mathematics, Science, and Social Sciences. According to the total scores on this test, students are emplaced to the Anatolian high schools regarding their preferences. Students attending to private high schools have to pay a certain fee during the school year. To be accepted to public high schools students are neither required to take OKS nor pay a fee to the school administration.

3.2 INSTRUMENTS

In this study, a metacognitive awareness inventory and a geometry knowledge test were administered to the participants. Demographic data such as gender, prior math grades, and prior geometry grades were gathered from students' admission records and school records.

3.2.1 JUNIOR METACOGNITIVE AWARENESS INVENTORY (Jr. MAI Version B)

In academic settings, investigation of metacognition is generally based on two major components of metacognition as knowledge of cognition and regulation of cognition. Therefore, the current study used the Jr. MAI (see Appendix Q) that was developed by Sperling, et al. (2002). Jr. MAI was based on Schraw and Dennison's (1994) Metacognitive Awareness Inventory (MAI). The MAI includes 52 five point Likert-scale items that loaded on knowledge of cognition and regulation of cognition. MAI is considered to be appropriate for adults while Jr. MAI is presented to measure metacognition from sixth grade through ninth grade. Thus, in this study Jr. MAI was chosen to measure students' metacognitive awareness. Some items of MAI were reworded by Schraw and Dennison (1994) in order to represent a more understandable language to younger populations and they were also given more of a context to assist young learners' understanding. In this sense, both MAI and Jr. MAI were translated into Turkish and two high school teachers were consulted prior to administration. Both teachers agree that the Jr. MAI would be more answerable by their students than MAI. And they both confirmed that some items in MAI (e.g. “I set specific goals before I begin a task.”, “I use the organizational structure of the text to help me learn.”) may be difficult for students to understand. No changes were made to Jr. MAI except for the brief explanations given for items 14 and 16. The definition of “learning strategies” was given to students for clarification with regard to the advisor’s comments. They were informed that learning strategies are the methods used to accomplish a task and such strategies provide individuals effective learning. In addition, six sample learning strategies such as “After reading a problem identifying the known and unknown variables.”, “Taking notes while studying.”, and “Making connections between prior knowledge and new knowledge.” were introduced to students together with Jr. MAI.

Jr. MAI includes 18 five point Likert-scale items for use with students in grades six through nine. Items 1, 2, 3, 4, 5, 12, 13, 14, and 16 are loaded on the knowledge of cognition component of metacognition and items 6, 7, 8, 9, 10, 11, 15, 17, and 18 are loaded on the regulation of cognition component of metacognition. The Jr. MAI included eighteen items with a five choice response (never, seldom, sometimes, often, and always). This inventory was translated into Turkish by an English language teacher and then re-translated into English by another English language teacher. Deficiencies and inconsistencies were checked by the researcher and finally Turkish version of Jr. MAI was checked by a Turkish instructor PH. D. and found to be appropriate to administer, and easy to understand by the participants. All the participants who helped in the translation process were from Hacettepe University in Turkey.

3.2.2 GEOMETRY KNOWLEDGE TEST (GKT)

A knowledge test covering the content of triangles was developed to measure tenth grade students' geometrical knowledge (see Appendix R). A question pool was constructed including thirty-five questions contextualized in declarative knowledge, conditional knowledge, and procedural knowledge. In the selection of questions the researcher consulted the textbooks, test books, and a variety of geometry books. Thirty-five open-ended questions were developed in the light of the objectives of National Mathematics Curriculum (MEB, 2006) for the tenth grade geometry. The open-ended question format was used to have an insight on students' computation processes and a deeper understanding of their conceptual understanding. Questions 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 30, 31, and 32 were related to declarative knowledge, questions 11, 12, 13, 14, 15, 16, 27, 28, 34, and 35 were related to conditional knowledge, and questions 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 29, and 33 were related to procedural knowledge. Initially, the first version of the knowledge test was checked to support the appropriateness of its content to the objectives by a high school mathematics teacher for the face and content validity. Additionally, an instructor Ph. D. and an assistant professor in mathematics education from Elementary Department at Middle East Technical University, and an assistant professor and an associate professor in mathematics education from Secondary Science and Mathematics Education Department at Middle East Technical University controlled it. The test was submitted to them along with a checklist including the following categories: (i) the question is appropriate to grade level, (ii) the question contributes to the relevant knowledge type (iii) the question's wording is understandable (iv) the question's context is appropriate to geometry curriculum. They were also provided a summary description of declarative knowledge, conditional knowledge, and procedural knowledge to gather trustworthy judgments and suggestions.

Regarding the teacher's and the researchers' comments and their responses to the checklist nine of the questions were removed from the test. Those questions were the ones which were assigned to be too easy (e.g "What is the area formula of a triangle?") or too difficult (e.g "If Descartes has three sticks length of which are a, b, and c, respectively and he wants to develop a triangle, what should be the relationship between a, b, and c? Justify your answer." Some of the questions remained were checked and revised in order to make the wordings clear and suitable for the knowledge type being measured. In the final version of the knowledge test questions 1, 2, 3, 4, 5, 6, 7 were related to declarative knowledge, questions 8, 10, 12, 13, 15, 16, 18, 20, and 22 were related to conditional knowledge; and questions 9, 11, 14, 17, 19, 21, 23, and 24 were related to procedural knowledge. Question seven was developed as an adaptation from TIMSS 1999 released set for eighth grade. Question nine was taken from TIMSS 1995 released set of mathematics items for seventh and eighth grades. Question eleven was adapted from TIMSS 2003 released set of mathematics items for eighth grade. Since the same geometry content is taught in the eighth and tenth grade, all of the adapted questions were appropriate for the grade level regarding the objectives of the geometry curriculum (MEB, 2006). The draft form of the final version was resubmitted to two high school mathematics teachers and an associated professor in mathematics education from Secondary Science and Mathematics Education Department at Middle East Technical University. All the researchers and teachers were requested to comment on the clarity of the questions, their face and content validity, and the correctness of their categorizations into knowledge types. Taking into account their suggestions, no more revisions were made on the test and these twenty-four open-ended geometry questions were confirmed to be understandable and appropriate for tenth grade students and content valid to administer. The table of specifications of the questions in the GKT is presented in Table 3.1.

Table 3.1 The Table of Specifications of the Questions in GKT

Question Number	Objective
1	A) Define an equilateral triangle. B) Define an isosceles triangle. C) Define a right triangle.
2	Recall the properties of triangles.
3	Write the symbolic expressions of a triangle's side lengths and interior angles.
4	Define congruency of triangles.
5	Define similarity of triangles.
6	Given a triangle, A) Write the congruent triangles in symbolic expression. B) Write the similar triangles in symbolic expression.
7	Given a set of triangles, A) Write the congruent triangles. B) Write the similar triangles.
8	Justify the relationship between a triangle's sides and angles.
9	Apply A.S.A theorem.
10	Justify the relationship between a triangle's angles and sides.
11	Apply A.S.A theorem.
12	Justify the relationship between similar triangles.
13	Explain whether a triangle might have two right angles.
14	Apply the Fundamental Theorem of Similarity.
15	Justify the relationship between an equilateral triangle and an isosceles triangle.
16	Justify the relationship between congruent triangles.
17	Apply A.A.A theorem and Pythagoras theorem.
18	Justify the relationship between similar triangles and congruent triangles.
19	Apply Menelaus theorem for similar triangles.
20	Justify the relationship between an altitude drawn on the hypothenuse and similarity.
21	Apply the metric relations in right triangles.
22	Justify the relationship between similarity and an isosceles triangle.
23	Apply properties of an isosceles triangle.
24	Apply properties of an equilateral triangle.

In terms of covering the context of triangles regarding the objectives a hierarchical order was provided among the test items according to three types of knowledge (see Table 3.2).

Table 3.2 Specimen Questions of GKT in the Hierarchical Order

Objective: Comprehend the fundamental concepts about triangles.

Declarative Knowledge:

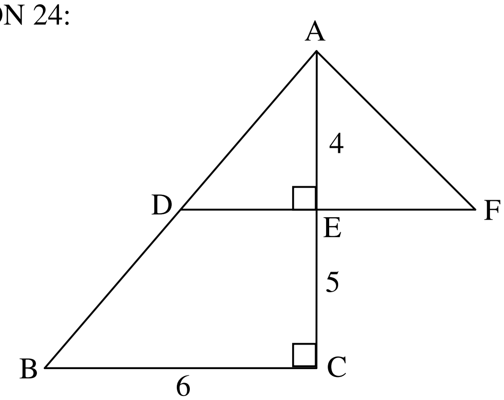
QUESTION 1b: Define what a right triangle is.

Conditional Knowledge:

QUESTION 20: 'If you draw an altitude relative to the hypotenuse in a right triangle ABC, then right triangle ABC is divided into two similar triangles.'. Is this statement true? Justify your answer.

Procedural Knowledge:

QUESTION 24:



In the figure given above, ABC, ADF, and AEF are right triangles.

$|AE| = 4$ cm, $|EC| = 5$ cm, and $|BC| = 6$ cm. What is $\frac{|EF|}{|DE|}$?

Table 3.3 shows the hierarchical distribution of the test items.

Table 3.3 Hierarchical Distributions of the Questions in GKT

Declarative Knowledge Questions	Conditional Knowledge Questions	Procedural Knowledge Questions
QUESTION 2	QUESTION 13	
QUESTION 3		
QUESTION 1a	QUESTION8	
QUESTION 1b	QUESTION 20	QUESTION 24
QUESTION 1c	QUESTION 10	QUESTION 23
	QUESTION 15	QUESTION 21
QUESTION 6	QUESTION 22	QUESTION 9
QUESTION 7		
QUESTION 4	QUESTION 16	QUESTION 11
	QUESTION 18	
QUESTION 5	QUESTION 12	QUESTION 14
		QUESTION 17
		QUESTION 19

After that the knowledge test was piloted by two tenth grade students attending to an urban high school in the spring semester of 2005-2006 academic year. The test was piloted with one of the students in order to determine the time it takes to complete. Furthermore, the other student solved each question with the researcher. Interviews were audiotaped. The aim of the pilot study was to check the clarity of the questions, the adequacy of the test duration, and the difficulty of the questions. The student who only took the test completed her solutions on the given time and confirmed that the questions were equally difficult. According to the interview data, the researcher observed that only question seven was problematic. The student declared that it would be better if any of the angle measures or side lengths were given. However, question seven was not removed from the test in order to gain an insight about students' spatial ability. Participants were allowed one class period in order to complete GKT. As mentioned above, the test contained 24 open-ended questions with increasing difficulty relatively to three different types of knowledge. In this sense, it was not expected that all students would be able to respond correctly to the more difficult items. Thus, the time limit was thought to have little effect on students' overall performance level. The reliability analyses for GKT were conducted by using SPSS 11.5 for Windows. The internal reliability of the GKT was high according to Cronbach's alpha coefficient as 0.91. Furthermore, the Cronbach's alpha reliability coefficients of the subscales were 0.67 for declarative knowledge, 0.82 for conditional knowledge, and 0.85 for procedural knowledge.

A focused holistic scoring scheme was developed reflecting the conceptual framework of declarative knowledge, conditional knowledge, and procedural knowledge with reference to Lane (1993) which served as a guide for grading the responses for each question in the GKT. For each question of the test, a five-score level (0-4) was assigned. The highest score of 4 was awarded for responses that the researchers regard as being entirely correct and satisfactory at grade ten geometry level, while the lowest score of 0 was reserved for no answer.

3.2.3 CONSTRUCT-RELATED VALIDITY, CONTENT-RELATED VALIDITY AND RELIABILITY

In order to determine the factors that would be included in the model as latent variables, several factor analyses were conducted. The items with high factor loadings were identified as latent variables. Factor analyses provided the evidence for construct-related validity for Jr. MAI and GKT. Detailed explanations of the separate factor analyses will be given in the results chapter. In addition, double translation procedure was used in order to acquire the equivalence of Jr. MAI. The evidence for content-related validity was obtained by translating the source version of the inventory into Turkish, and then translating them back to English. Furthermore, a third expert reconciled the translations. By using this procedure three people (two translators and a reconciler) recorded the discrepancies in Turkish.

After the latent variables were determined, the internal-consistency estimates of reliability were examined for each latent variable. The observed variables composing the latent variables and alpha reliability coefficients are given in Table 3.4.

Table 3.4 Observed and Latent Variables, and Reliabilities of the Latent Variables

Observed Variables	Latent Variables	Alpha Reliability Coefficients
QUES4 QUES1 QUES3 QUES5	DECKNOW	0.66
QUES15 QUES12 QUES8 QUES13	CONKNOW	0.71
QUES24 QUES21 QUES23 QUES19	PROKNOW	0.90
MET1 MET12 MET11 MET5 MET2	KNOOFCOG	0.66
MET10 MET7 MET8 MET15 MET9 MET14	REGOFCOG	0.73

Reliability analyses were conducted separately for each instrument by using SPSS 11.5. The Cronbach's alpha reliability coefficients of the instruments are given in Table 3.5.

Table 3.5 Reliability Coefficients of the Instruments

Instrument	Alpha Reliability Coefficients
Jr. MAI	0.80
Geometry Knowledge Test	0.91

The reliability coefficients of the instruments are quite high representing high reliability (Adams & Wu, 2002). On the other hand, a scoring rubric (see Appendix S) was developed by the researcher regarding the features of declarative knowledge, conditional knowledge, and procedural knowledge. In order to establish the extent of consensus on use of the scoring rubric for Geometry Knowledge Test inter-rater reliability coefficient was computed in order to make an estimation based on the correlation of scores between two raters who rate the same instrument. 37 tests were randomly selected and given to a high school mathematics teacher with three-year experienced. Intraclass correlation (ICC) was used to measure inter-rater reliability for two raters in terms of providing subjective decisions. An intraclass correlation > 0.70 is considered acceptable inter-rater reliability (Shrout & Fleiss, 1979). The ICC value of 0.83 indicated a quite high reliability and the internal consistency of the scoring rubric as used by two raters.

3.3 PROCEDURE

In the fall semester of 2006 extensive and detailed information was obtained about metacognition, achievement in geometry, and knowledge of mathematics through review of related literature.

In March 2006 Junior Metacognitive Awareness Inventory (Jr. MAI) was translated into Turkish. Prior to administration, Jr. MAI was piloted on similar-age students to assure that the items were clearly presented, answerable, and appropriately worded for Turkish students. The inventory was administered to a total of 314 sixth, seventh, and eighth grade students (122, 109 students, and 83, respectively) attending to a private school. The administration was a part of normal class procedures in all classes. Some of the classroom teachers who administered the inventory reported that they gave additional assistance to students who are in need of it. They also reported that it took students approximately 20 minutes to complete Jr. MAI. Regarding the school management’s request the researcher did not participate in the classes in order not to affect the nature of the class. Reliability analysis reported the coefficient alpha as 0.85. Thus, results of the data supported that Jr. MAI is a reliable measure of metacognition.

In April 2006 Geometry Knowledge Test (GKT) was developed covering the context of triangles by the researcher. Test design, development, and pilot study were finished by the end of April.

During May-June 2006 all the instruments were administered as part of mathematics or geometry classes. Participants were initially administered Jr. MAI. In the subsequent week they were administered GKT. For the metacognition inventory and geometry test students were allowed one class period, separately. The researcher was ready in the schools during the administration.

After the data were collected several exploratory factor analyses were conducted with the items of Jr. MAI and the questions of GKT in order to investigate the constructs. Then the related factors were selected on the basis of the related literature and the results of the obtained statistics. Subsequently, separate confirmatory factor analyses were conducted for each instrument. When the factors were decided, a detailed search was conducted for the literature review in order to base the relations among factors on a theoretical frame. Finally, the hypothesized model was tested and the modifications were done.

3.4 DATA COLLECTION

The testing was conducted in the Spring semester of 2006 during May and June. Testing of all the instruments were conducted by the mathematics or geometry teachers of the selected classes. However, the researcher was ready at each school during the administration in order to provide support and answer students' questions when needed. Data were analyzed using Statistical Package for Social Sciences 11.5 for Windows (SPSS 11.5).

3.5 DATA ANALYSES

Data gathered through the inventories, rating scale and geometry test was investigated. They were obtained in an SPSS file and variables in the file were examined. The descriptive statistics and frequency tables of Jr. MAI are presented in Appendix A and Appendix C, respectively. In addition, descriptive statistics and frequency tables of GKT are presented in Appendix B and Appendix D, respectively. After the examination of the variables, Varimax Rotated Principle Components Factor Analyses were run for each instrument by using SPSS 11.5 for Windows in order to explore the factor structures of the items. Accordingly, the observed variables with high factor loadings were selected as the latent variables. Following the exploratory factor analyses, separate confirmatory factor analyses were conducted for each instrument in order to make sure that the selected observed variables account for the latent variables. The final data set including the items that would be involved in the model testing were imported from SPSS 11.5 for Windows to PRELIS 2.3 for Windows. Furthermore, reliability analyses were conducted for each latent variable in order to obtain Cronbach alpha reliability coefficients. Finally, LISREL 8.30 for Windows (Linear Structural Relations Statistics Package Program) with SIMPLIS command language was used for estimating the models including the factors affecting different types of geometry knowledge of tenth grade students together with the interrelations among different knowledge types.

3.5.1 MISSING DATA ANALYSES

In terms of defining the items that would be analyzed, the missing percentages of the inventory and test items were one of the essential criterions. In this sense each of the items in the inventory and geometry test was analyzed in detail. The general criterion for the missing data was 10%. For Jr. MAI the values were under 10% ranging from 0.0% to 0.7%. There were no missings in GKT data. While conducting factor analyses and estimating models pairwise deletion method was used in order to handle missing data.

3.5.2 EFFECT SIZES

An effect size measure is the size of the relationship among variables. Weinfurt (1995) mentioned that the effect size can be defined as the magnitude of an independent variable's effect, usually expressed as a proportion of explained variance in the dependent variables. In other words, effect size is an indicator of the association that exists between two or more variables (Denis, 2003). Multiple regression analysis provide a better understanding for the path analysis in structural equation modeling. The multiple correlation indices are multiple correlation (R), squared multiple correlation (R^2), and adjusted squared multiple correlation ($R_{adj}{}^2$). Squared multiple correlation is a measure of the strength of the linear relationship. The measure of effect size is equivalent to the R^2 used in multiple regression. The squared multiple correlation indicates the amount of variance explained by the set of independent variables. It is used as a model fit criterion in multiple regression analysis (Schumacler & Lomax, 2004). Cohen (1988) suggested a classification of effect sizes, which were measured in terms of R^2. This classification indicated for effect sizes: 0.01 is small, 0.09 is medium and 0.25 or greater is large. In social studies, small to medium effect sizes emerge (Weinfurt, 1995).

3.5.3 STRUCTURAL EQUATION MODELING

The applications of structural equation modeling have five stages (Bollen & Long, 1993).

1. Model Specification

Model specification is the essential starting point of the modeling procedure. This model should be drawn on the basis of a literature review or a theory. The hypothesized model explains both what relationships are expected to see in the data and what relationships are not expected to emerge (Kelloway, 1998). The research literature does not contain a study that investigates a theoretical model including all the variables of the present study.

2. Identification

Model identification depends on the designation of parameters as fixed, free, or constrained (Schumacker & Lomax, 2004). Prior to the estimation of parameters, the parameters are combined to form a unique covariance matrix. Thus, the levels of model identification (underidentified, just-identified, and overidentified) depends on the covariance matrix.

3. Estimation

Estimation refers to the various estimation techniques depending on the variable scale and/or distributional property of the variable(s) used in the model. There are a variety of estimations such as maximum likelihood (ML), ordinary least squares (OLS), unweighted least squares (ULS), and generalized least squares (GLS). In LISREL program ML estimation is the default method which is consistent and asymptotically efficient in large samples. Kline (1998) indicated that ML estimation works just fine for the most types of structural equation models so long as the data have been properly screened and their distributions are reasonably normal. In this sense, maximum likelihood estimation method was used in this study.

4. Testing Fit

Testing fit involves interpreting model fit or comparing fit indices for alternative or nested models. In the literature there are a variety of fit indices which can be used according to the goodness-of-fit criteria. These indices provide a deeper understanding of what it means to say model fits the data.

5. Respecification

The purpose of the respecification is to improve the model if the fit indices indicate a poor fit. In the case of respecification, LISREL program guides for finding sources the poor fit. Accordingly, it is decided how to modify the model in terms of deleting or adding paths. When the model is modified, analyses are rerun.

In conclusion, modification suggestions were considered according to the modification indices given in the LISREL output by adding a covariance term in the syntax of the model. Subsequently, the model was retested again in terms of the goodness-of-fit criteria.

3.5.3.1 Definition of Terms for Structural Equation Modeling

The definitions of the terms used in this study are explained for the aim of clarifying the terms in order to provide a better understanding of the Path Analysis with Latent Variables.

1. Path Diagram

It is the pictorial representation of a structural equation model that indicates the relations. A path diagram in which variables are linked by unidirectional arrows or bidirectional curved arrows represents the structural relations that form the model. The unidirectional arrows represent causal relations; whereas bidirectional curved arrows represent noncausal or correlational relationships (Kelloway, 1998). In other words, the path diagram includes the indication of all parameters in a model (Hoyle, 1995).

2. Observed or Manifest Variables

Observed variables which are sometimes called indicators, are directly observable and measurable variables such as test items or questionnaire items (Schumacker & Lomax, 2004).

3. Latent or Unobserved Variables

Latent variables are the variables that are not measured directly (Kelloway, 1998). Nevertheless, they can be indirectly measured through observed variables (Schumacker & Lomax, 2004).

4. Latent Dependent Variables

Latent dependent variables are affected by other latent variables in the model and their measurement depends on the observed dependent variables.

5. Latent Independent Variables

Latent independent variables are not affected by any other latent variables in the model and their measurements depend on the observed independent variables.

6. Structural Equation Models

The factors are established as latent variables in the path models by which the structural equation models are represented. Structural equation models give the relationship between latent variables and observed variables in a theoretical perspective. There are two parts in a structural model: (i) the measurement model and (ii) the structural model.

7. The Measurement Model

It is the component of the general model in which latent variables are prescribed (Hoyle, 1995). The purpose of a measurement model is to describe how well the observed indicators serve as a measurement instrument for the latent variables (Jöreskog & Sörbom, 1993). This description is made on the basis of the confirmatory factor analyses in terms of the factor loadings. The measurement properties of the latent variables such as validity and reliability are specified in this model.

8. The Structural Model

It is the component of the general model that prescribes relations between latent variables and observed variables that are not indicators of latent variables (Hoyle, 1995). This model gives the direct and indirect relationships among latent variables that describe the amount of explained and unexplained variance. In this sense, the structural model is an indication of the extent to which hypothesized relationship is supported by the data (Schumacker & Lomax, 2004).

9. Direct Effect

The direct effect is a directional relation between two variables, that is the characterization of the relation between an independent and a dependent variable. The path coefficients, which represent the direct effects in the model, are the building blocks of the structural equation models.

10. Indirect Effect

The indirect effect is the effect of an independent variable on a dependent variable through one or more mediating variable (Hoyle, 1995).

11. Total Effect

The total effect is the sum of direct and indirect effects of an independent variable on a dependent variable.

12. LISREL 8.30 with SIMPLIS Command Language

LISREL is a computer program (Jöreskog & Sörbom, 1993) which uses the SIMPLIS command language in order to perform structural equation modeling. A more national language is used in SIMPLIS language to define LISREL models (Kelloway, 1998) in which path models are generated regarding a model formulation.

13. The Measurement Coefficients

The λ_y (lowercase lambda sub y) and λ_x (lowercase lambda sub x) values describe the relationships among the latent variables and observed variables. These values can also be defined as factor loadings, which indicate the validity coefficients.

The ε (lowercase epsilon) and δ (lowercase delta) indicate the measurement errors for the Ys and Xs, respectively. These values serve as reliability coefficients.

14. The Structure Coefficients

The β (lowercase beta) values represent the strength and direction of the relationship among the latent dependent variables.

The γ (lowercase gamma) values represent the strength and direction of the relationship between latent dependent variables and latent independent variables.

15. Factor Analysis

Factor analyses are integrated in structural equation modeling in order to create the latent variables by reducing a large number of variables to a small number of factors. For modeling purposes two types of factor analysis can be used.

15.1 Exploratory Factor Analysis (EFA)

In terms of data reduction, this technique is used to determine the factors, which are independent among each other. In EFA, how many factors there are is explored along with whether the factors are correlated, and which observed variables appear to best measure each factor (Schumacker & Lomax, 2004).

15.2 Confirmatory Factor Analysis (CFA)

Regarding a theory this technique is used to determine if the number of factors and the loadings of the observed variables on them conform to what is hypothesized. In CFA, a certain number of factors is specified along with which factors are correlated, and which observed variables measure each factor (Schumacker & Lomax, 2004).

3.5.3.2 The Goodness-of-Fit Criteria for Structural Equation Modeling

In this study LISREL 8.30 for Windows with SIMPLIS Command Language was used in formulating and estimating the models including factors affecting geometrical knowledge of tenth grade students.

These criteria are used to determine the degree to which the structural equation model fits the sample data. Interpretations of goodness-of-fit criteria are given in detail as the following:

1. Chi-Square (χ^2)

Chi-square is a measure of overall fit of the model to the data (Jöreskog & Sörbom, 1993). In this view, it indicates that there is no significant discrepancy between the covariance matrix implied by the model and the population covariance matrix. A nonsignificant chi-square implies that the model fits the data; hence the population covariance matrix can be reproduced by the model (Kelloway, 1998). The chi-square is sensitive to sample size that it decreases as the sample size increases. In other words, the chi-square criterion tends to indicate a significant probability level when the sample increases, generally above 200 (Schumacker & Lomax, 2004).

2. Goodness-of-Fit Index (GFI)

This measure does not depend on sample size explicitly. It measures how much better the model fits as compared to no model at all (Jöreskog &

Sörbom, 1993). The GFI is based on a ratio of the sum of the squared discrepancies to the observed variances. The measure of GFI ranges from 0 to 1 and the values that exceed 0.90 indicates a good fit to the data (Kelloway, 1998).

3. Adjusted Goodness-of-Fit Index (AGFI)

The AGFI refers to the adjusted GFI for the degrees of freedom of a model relative to the number of variables in the model. Similar to GFI, AGFI also ranges from 0 to 1, with the values exceeding 0.90 indicate a good fit to data (Kelloway, 1998).

4. Root-Mean-Square Residual (RMR)

This index uses the square root of the mean of the squared differences between the implied and covariance matrices. The low values of RMR whose lower bound is 0 indicate a good fit to the data (Kelloway, 1998).

5. Standardized-Root-Mean-Square Residual (S-RMR)

It is difficult to determine what a low value is for RMR so that the S-RMR is provided by LISREL. The S-RMR has a lower bound of 0 and upper bound of 1. The values less than 0.05 indicate a good fit to the data (Kelloway, 1998). However, values less than 0.10 are accepted (Kline, 1998).

6. Root-Mean-Squared Error of Approximation (RMSEA)

The RMSEA is a measure of discrepancy per degree of freedom. It is suggested that values below 0.10 indicate a moderate fit to the data, the values below 0.05 indicate a good fit to the data, and the values below 0.01 indicate an outstanding fit to the data. The RMSEA index has the advantage of going beyond point estimates to the provision of 90%. Furthermore, by testing whether the value obtained is significantly different from 0.05, LISREL provides a test of the significance of the RMSEA (Kelloway, 1998).

7. Normed Fit Index (NFI)

Bentler and Bonett (1980) suggested that the percentage improvement in fit over the baseline independence model is the basis of NFI. The NFI index ranges from 0 to 1. The values above 0.90 indicate a good fit data, which means that the model is 90% better fitting than the null model.

8. Non-normed Fit Index (NNFI)

The measure of NNFI refers to the number of degrees of freedom in the model. Unlike NFI, this index has a lower bound of 0 but an upper bound of greater than 1. The values exceeding 0.90 indicate a good fit to the data (Kelloway, 1998).

9. Comparative Fit Index (CFI)

This index is proposed on the basis of noncentral chi-square distribution. The CFI ranges from 0 to 1, with the values exceeding 0.90 indicating a good fit to the data (Kelloway, 1998).

10. Incremental Fit Index (IFI)

The IFI index takes the scaling factor as a basis (Bollen, 1989). It ranges from 0 to 1. The higher values approaching to unity indicate a good fit to the data (Kelloway, 1998).

11. Relative Fit Index (RFI)

The RFI index is based on assessing the fit of the indicator variables to the latent variables (Schumacker & Lomax, 2004). It ranges from 0 to 1. The higher values approaching to unity indicate a good fit to the data (Kelloway, 1998).

12. Relative Normed Fit Index (RNFI)

The RNFI index refers to the adjusted RFI, which provides to estimate the effects of structural model from the measurement model separately (Schumacker & Lomax, 2004).

13. Cross-Validation Index

Cudeck and Browne (1983) suggested the index of cross-validation which requires two samples as a calibration sample and a validation sample. Initially, a model was set to the calibration sample, then the discrepancy between the covariance matrix implied by the model to the covariance matrix of the validation sample. The model fits the data if this discrepancy value is small. The requirement of two samples is a practical problem for this strategy. Thus, it is suggested estimating the expected value of the cross-validation index using only data from a single sample (Kelloway, 1998).

14. Expected Value of Cross-Validation Index (ECVI)

The ECVI index refers to the estimation of the expected discrepancy over all calibration samples. It has a lower bound of zero, however it has no upper bound. The smaller values indicate a good fit to the data. Not only the point estimate of the ECVI, but also the confidence intervals for the estimate is given in LISREL output. Moreover, LISREL provides the ECVI values for the independence (null) and saturated (just-identified) models.

15. Normed Chi-Square (NC)

The NC index takes the ratio of chi-square and its degrees of freedom as a basis. The adjusted chi-square with ratios below five indicates a good fit to the data, like ratios between two and five. In addition, $\frac{\chi^2}{df}$ ratios of less than two indicate overfitting (Kelloway, 1998).

16. Parsimonious Fit Index (PFI)

James, et al. (1982) suggested the PFI as the modified NFI measure. This index deals with the number of degrees of freedom that is used to obtain a given level of fit. Fewer degrees of freedom provide a high degree of fit for parsimony (Schumacker & Lomax, 2004).

17. Parsimonious Normed Fit Index (PNFI)

The PNFI index refers to the adjusted NFI for parsimony of the model. It ranges from 0 to 1, with higher values indicating a more parsimonious fit (Kelloway, 1998).

18. Parsimonious Goodness-of-Fit Index (PGFI)

The PGFI is the adjusted GFI for the degrees of freedom. Similar to PNFI, this index ranges from 0 to 1, with higher values indicating a more parsimonious fit. There is no standard for how high either PNFI or PGFI index should be in order to indicate a parsimonious fit. It is not expected to obtain 0.90 cutoff level for both PNFI and PGFI indices (Kelloway, 1998).

Table 3.6 represents the summary of the criteria of the fit indices defined above.

Table 3.6 Criteria of the Fit Indices

Fit Index	Criterion
Chi-Square (χ^2)	Non-significant
Normed Chi-Square (NC)	NC< 5
Goodness of Fit Index (GFI)	GFI> 0.90
Adjusted Goodness of Fit Index (AGFI)	AGFI> 0.90
Root Mean Square Error of Approximation (RMSEA)	0.05 < RMSEA < 0.08 (moderate fit)
	RMSEA < 0.05 (good fit)
Root Mean Square Residual (RMR)	RMR < 0.05
Root Mean Square Residual (S-RMR)	S-RMR < 0.05
Parsimony Goodness of Fit Index (PGFI)	Higher values
Parsimony Normed Fit Index (PNFI)	Higher values
Normed Fit Index (NFI)	NFI> 0.90
Non-Normed Fit Index (NNFI)	NNFI> 0.90
Comparative Fit Index (CFI)	CFI> 0.90
Incremental Fit Index (IFI)	IFI> 0.90
Relative Fit Index (RFI)	RFI> 0.90

3.5.3.3 Fitted Residuals and Standardized Residuals

Fitted residuals depend on the unit of measurement of the observed variables while standardized residuals are independent of the units of measurement of the variables (Jöreskog & Sörbom, 1993). For each observed variable standardized residuals are calculated. Large standardized residuals that are above 2 indicate a lack of fit (Kelloway, 1998). This means that a particular covariance is not explained well by the model; hence the model should be examined to determine ways in which this particular covariance could be explained (Schumacker & Lomax, 2004). Furthermore, when the model fits the data well the fitted and standardized residuals for the model are typical and the two residual stemleaf plots look approximately normal.

CHAPTER 4

RESULTS

Results of the present study are given in two main sections as preliminary analysis and structural equation modeling. Preliminary analysis section includes the exploratory factor analyses and confirmatory factor analyses conducted for each instrument used in the study. The factors in accordance with the instruments were examined and determined. Structural equation modeling section includes the model testing for the knowledge types along with a detailed explanation of the model.

4.1 PRELIMINARY ANALYSIS

In order to reduce the number of observed variables exploratory factor analyses were conducted for the aim of grouping the variables in constructs. Separate factor analyses with the data of the main study for GKT and Jr. MAI were run to determine which sets of observed variables sharing common variance or covariance characteristics define constructs. Furthermore, factor analyses provide a deeper understanding of the relationship among these variables (Schumacker & Lomax, 2004). Then confirmatory factor analyses with the data of the main study were conducted to determine a few indicators (observed variables) of a theoretical construct (latent variables). The observed variables representing the latent variables were selected regarding the results of confirmatory factor analyses in order to be used in structural equation modeling. Finally, according to the results of the factor analyses structural equation modeling technique was used to find a metacognition-knowledge model of geometry that fit the data well.

4.1.1 RESULTS OF EXPLORATORY FACTOR ANALYSIS OF GEOMETRY KNOWLEDGE TEST

Principal components analysis was carried out for GKT by SPSS 11.5. The first exploratory factor analysis including 24 observed variables was employed to understand the underlying structure of the questions. The Kaiser-Meyer-Olkin (KMO) measure indicates whether the distribution of values is adequate to conduct analysis and varies from 0 to 1. It should be 0.60 or above in order to proceed factor analysis. On the other hand, Bartlett's Test of Sphericity measure indicates whether the factor model is inappropriate. It should be significant at $p< 0.05$. The KMO and Bartlett's Test of Sphericity values of GKT are presented in Table 4.1.

Table 4.1 KMO and Bartlett's Tests of GKT

Kaiser-Meyer-Olkin Measure of Sampling Adequacy		0.918
Bartlett's Test of Sphericity	Approx. Chi-Square	2911.392
	df	276
	Sig.	0.000

The KMO measure revealed a value greater than 0.6 and the Bartlett's Test was significant. These findings indicated that principal component analysis can be implemented to GKT. After the first analysis, a five-factor solution emerged from the data. In the five-factor solution for GKT, 57% of the sample variance was accounted for by the questions. Regarding three criteria as the scree test, interpretability of the factor solution, and the priori hypothesis that the measure was unidimensional the number of factors to rotate was determined. According to the scree plot, three factors were indicated since this test revealed that the initial hypothesis of unidimensionality was incorrect. Accordingly, a second factor analysis was conducted with the restriction of the number of factors to three using Varimax rotation. The three-factor solution demonstrated that 47.7% of the sample variance was accounted for by the questions.

After the analysis three of the twenty-four items were excluded because of their ambiguous factor loadings in the Rotated Component Matrix. Regarding that questions seven, nine and eleven were not included the next Principal Component Analysis.

Finally, the remaining twenty-one geometry questions were analyzed and this analysis yielded a three-factor solution. The observed variables representing latent variables and their factor loadings are represented below.

Table 4.2 Principle Component Factor Analysis Results of GKT

Rotated Component Matrix(a)			
	Component		
	1	**2**	**3**
QUES24	**0.852**	0.100	0.136
QUES21	**0.843**	0.151	0.132
QUES23	**0.832**	0.183	0.127
QUES19	**0.829**	0.183	0.088
QUES20	**0.667**	0.298	0.184
QUES22	**0.648**	0.366	0.090
QUES17	**0.633**	0.284	0.299
QUES18	**0.532**	0.476	0.200
QUES15	0.263	**0.710**	-0.022
QUES12	0.207	**0.652**	0.164
QUES8	0.165	**0.642**	0.166
QUES13	0.028	**0.633**	0.256
QUES10	0.184	**0.629**	0.026
QUES16	0.378	**0.521**	0.318
QUES14	0.321	**0.422**	0.257
QUES4	0.196	0.354	**0.671**
QUES1	- 0.019	0.135	**0.616**
QUES3	0.253	- 0. 016	**0.571**
QUES5	0.188	0.365	**0.567**
QUES2	0.255	- 0.054	**0.564**
QUES6	0.006	0.182	**0.512**

As it was expected GKT had three subdimensions. Subsequently, these three factors in the rotated solution were given the following names as Procedural Knowledge, Conditional Knowledge, and Declarative Knowledge respectively. They were interpreted on the basis of their context and loadings. The eigenvalues, the percentage, and the cumulative percentage of the factors were given in Table 4.3.

Table 4.3 Rotation Sums of Squared Loadings of GKT

Factors	Eigenvalue	% of Variance	Cumulative %
Procedural Knowledge	4.987	23.749	23.749
Conditional Knowledge	3.529	16.803	40.551
Declarative Knowledge	2.578	12.275	52.826

As a result of Principal Components Analysis of GKT, the observed variables were grouped under these three factors. The total variance accounted by all the factors was 52.8%. All the factors were included as latent variables for further analysis. While representing the latent variable, using all the observed variables are problematic. Therefore, in this study each latent variable was represented by four to six observed variables, which have the highest factor loadings. The selected observed variables representing the latent variable were examined regarding the eigenvalues, the percentage and cumulative percentage of the explained variance and the alpha values of reliability of the factors were represented in Table 4.4.

Table 4.4 Rotation Sums of Squared Loadings and Reliability of Factors for GKT

Observed Variables	Latent Variables	Eigenvalue	% of Var.	Cum. %	Reliability
QUES24 QUES21 QUES23 QUES19	PROKNOW	3.205	26.708	26.708	0.90
QUES15 QUES12 QUES8 QUES13	CONKNOW	2.276	18.968	45.676	0.71
QUES4 QUES1 QUES3 QUES5	DECKNOW	1.972	16.429	62.105	0.66

4.1.2 RESULTS OF EXPLORATORY FACTOR ANALYSIS OF JUNIOR METACOGNITIVE AWARENESS INVENTORY

Principal components analysis was carried out for Jr. MAI by SPSS 11.5. The exploratory factor analysis including 18 observed variables was employed to understand the underlying structure of the metacognition items. The KMO and Bartlett's Test of Sphericity values of Jr. MAI are presented in Table 4.5.

Table 4.5 KMO and Bartlett's Tests of Jr. MAI

Kaiser-Meyer-Olkin Measure of Sampling Adequacy		0.852
Bartlett's Test of Sphericity	Approx. Chi-Square	1035.986
	df	153
	Sig.	0.000

The KMO measure revealed a value greater than 0.6 and the Bartlett's Test was significant. These findings indicated that principal components analysis can be implemented to Jr. MAI. A four-factor solution emerged from the data. In the four-factor solution for Jr. MAI, 46.8% of the sample variance was accounted for by the inventory items. Regarding three criteria as the scree test, interpretability of the factor solution, and the priori hypothesis that the measure was unidimensional the number of factors to rotate was determined. According to the scree plot, three factors were indicated since this test revealed that the initial hypothesis of unidimensionality was incorrect. Accordingly, a second factor analysis was conducted with the restriction of the number of factors to three using Varimax rotation. In the three-factor solution the total variance accounted by all the factors was 46.8%. The scree plot indicated two factors. Hence, the third factor analysis was conducted by extracting the number of factors to two. Subsequently, these two components were examined and three of the 18 items were excluded from the analysis because of their ambiguous factor loadings in the Rotated Component Matrix. Regarding that items four, six, and sixteen were not included in the next Principal Component Analysis.

The factor loadings of the final two-factor solution of Jr. MAI after the items were excluded are displayed in Table 4.6.

Table 4.6 Principle Component Factor Analysis Results of Jr. MAI

Rotated Component Matrix(a)		
	Component	
	1	**2**
MET10	**0.736**	-0.109
MET7	**0.649**	0.055
MET8	**0.588**	0.204
MET15	**0.582**	0.117
MET9	**0.567**	0.260
MET14	**0.548**	0.382
MET18	**0.522**	0.290
MET17	**0.504**	-0.029
MET1	0.057	**0.642**
MET12	- 0.117	**0.629**
MET11	0.301	**0.629**
MET5	0.123	**0.604**
MET2	0.305	**0.531**
MET3	0.040	**0.474**
MET13	0.399	**0.464**

Jr. MAI had two subdimensions. Subsequently, these two factors in the rotated solution were given the following names as Regulation of Cognition and Knowledge of Cognition, respectively.

The eigenvalues, the percentage, and the cumulative percentage of the factors were given in Table 4.7.

Table 4.7 Rotation Sums of Squared Loadings of Jr. MAI

Factors	Eigenvalue	% of Variance	Cumulative %
Regulation of Cognition	3.173	21.156	21.156
Knowledge of Cognition	2.659	17.728	38.884

As a result of Principal Components Analysis of Jr. MAI, the observed variables were grouped under these two factors and included in structural equation modeling. Six observed variables and five observed variables, which have the highest factor loadings were chosen to represent the latent variables as REGOFCOG and KNOOFCOG, respectively.

The selected observed variables representing the latent variables were examined regarding the eigenvalues, the percentage, cumulative percentage of the explained variance and the alpha values of reliability of the factors were represented in Table 4.8.

Table 4.8 Rotation Sums of Squared Loadings and Reliability of Factors for Jr. MAI

Observed Variables	Latent Variable	Eigenvalue	% of Var.	Cum. %	Reliability
MET10 MET7 MET8 MET15 MET9 MET14	REGOFCOG	2.145	37.711	47.765	0.73
MET1 MET12 MET11 MET5 MET2	KNOOFCOG	2.154	23.934	23.934	0.66

4.1.3 RESULTS OF CONFIRMATORY FACTOR ANALYSIS FOR THE GEOMETRY KNOWLEDGE TEST

Confirmatory Factor Analysis was conducted to specify the observed variables that indicate the latent variables of the Geometry Knowledge Test

(GKT). Regarding the findings of the Exploratory Factor Analysis, the model was tested and ten covariance terms were added to SIMPLIS syntax in order to improve the model considering the modification indices with the highest values. The final SIMPLIS syntax for the CFA of the model is given in Appendix E. In Figure 4.1 LISREL estimates of parameters in the model in which the coefficients were in standardized values is presented. Additionally, in Figure 4.2 LISREL estimates of parameters in the model in which the coefficients were in t-values were displayed. Furthermore, LISREL estimates of parameters in GKT model with coefficients in standardized value and t-values are included in Appendix F.

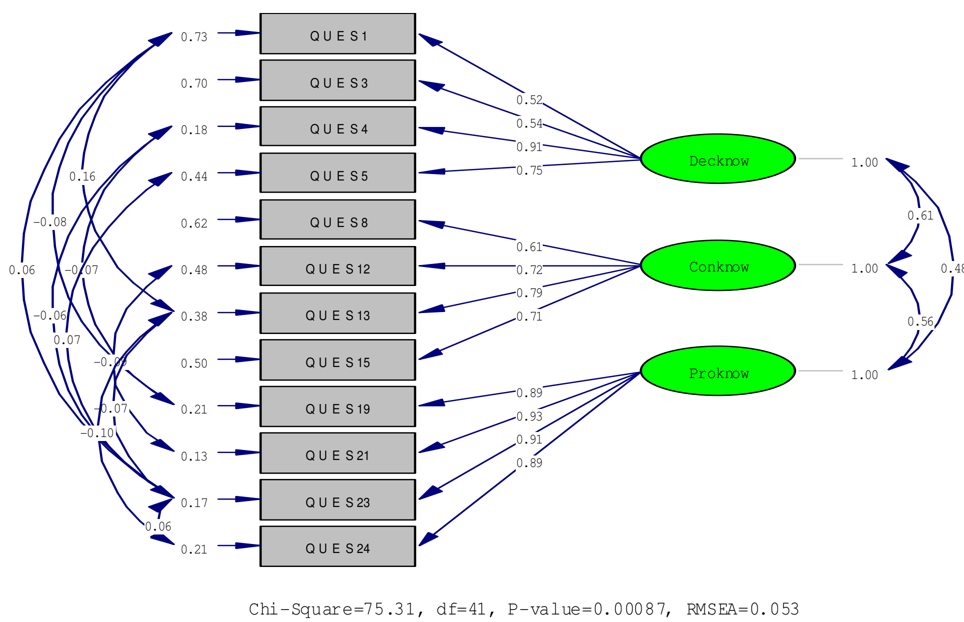


Figure 4.1 LISREL Estimates of Parameters in GKT Model with Standardized Value

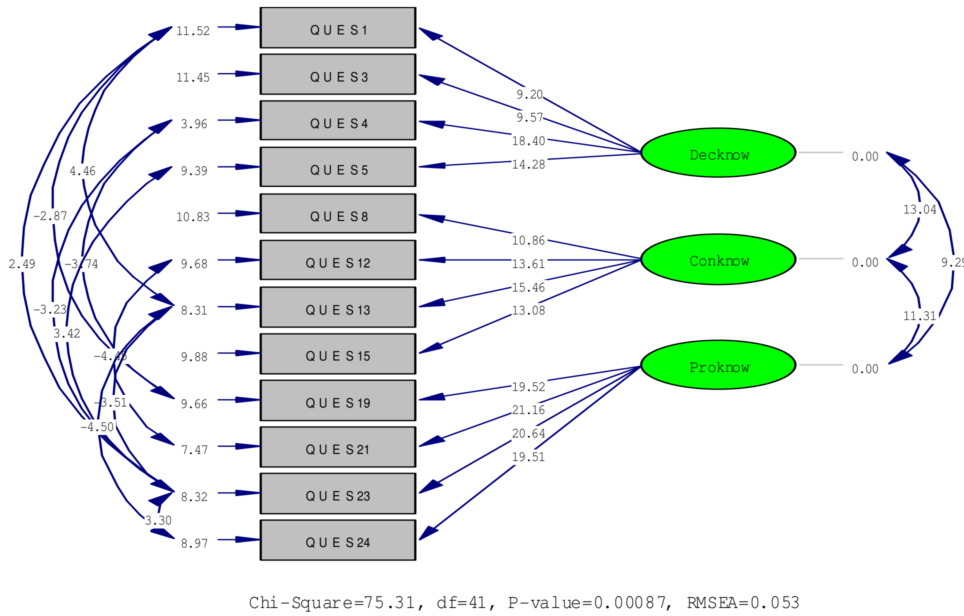


Figure 4.2 LISREL Estimates of Parameters in GKT Model with t-Values

According to the Figure 4.1 and Figure 4.2, the structural equation model was involving three latent variables and twelve observed variables. The latent variables of the model were Declarative Knowledge (DECKNOW), Conditional Knowledge (CONKNOW), and Procedural Knowledge (PROKNOW) whereas the observed variables of the model were QUES1, QUES3, QUES4, QUES5, QUES8, QUES12, QUES13, QUES15, QUES19, QUES21, QUES23, and QUES24, respectively.

Fit statistics of squared multiple correlation (R^2) was calculated for each observed variable that represent the latent variables and the results were displayed in the LISREL output. In Table 4.9 the values of R^2 that equal the proportion of explained variance was presented. These values gave the reliability of the observed variables that specify DECKNOW, CONKNOW, and PROKNOW.

Table 4.9 Squared Multiple Correlations for GKT

Latent Variables	Observed Variables	R^2
DECKNOW	QUES1	0.27
	QUES3	0.30
	QUES4	0.82
	QUES5	0.56
CONKNOW	QUES8	0.38
	QUES12	0.52
	QUES13	0.62
	QUES15	0.50
PROKNOW	QUES19	0.79
	QUES21	0.87
	QUES23	0.83
	QUES24	0.79

The values of the measurement coefficient the λ_x (lowercase lambda sub x) indicate the relationships between the latent variables and the observed variables. Furthermore, the δ (lowercase delta) is the measurement errors for the Xs, respectively. The measurement coefficients and the measurement errors of GKT was given in standardized values in Table 4.10.

Table 4.10 Measurement Coefficients of GKT

Latent Variables	Observed Variables	λ_X	δ
DECKNOW	QUES1	0.52	0.74
	QUES3	0.54	0.70
	QUES4	0.91	0.18
	QUES5	0.75	0.44
CONKNOW	QUES8	0.61	0.62
	QUES12	0.72	0.47
	QUES13	0.79	0.38
	QUES15	0.71	0.50
PROKNOW	QUES19	0.88	0.21
	QUES21	0.94	0.13
	QUES23	0.90	0.17
	QUES24	0.88	0.20

The summary statistics for fitted residuals for the model yielded the smallest fitted residual as -0.10 and largest fitted residual as 0.14. This examination indicated a good fit because each of the fitted residual values was less than 2 in absolute value (Kelloway, 1998). On the other hand, the summary statistics for standardized residuals for the model yielded the smallest standardized residual as -3.32 and the largest standardized residual was 3.44. In the output file, the fitted residuals and the standardized residuals were typical for the situation they had been calculated. In other words, the structure of both residuals displayed a similar shape. Furthermore, the stemleaf plots of both residuals were approximately normal which indicated a good fit. The stemleaf plots of the residuals are represented in Appendix G.

GKT was evaluated in terms of the goodness-of-fit-indices, which were discussed in detail in Chapter 3. The values of the goodness-of-fit criteria of the model are represented in Appendix H. The model for GKT demonstrated a significant Chi-Square value of $\chi^2 = 75.31$ with degrees of freedom, df = 41, at a significance level p = 0.000. As known, χ^2 is sensible to sample size. In this sense, this criterian indicates a significant probability level when the sample size increases, generally above 200 (Schumacker & Lomax, 2004). The sample size in this study was 297, which was large enough to make the test statistically significant. The value of the Normed Chi-Square (NC) in terms of which χ^2 / df is displayed, was 1.83 that indicated a good fit to the data with its being less than 5.

The Goodness-of-Fit Index (GFI) and the Adjusted Goodness-of-Fit Index (AGFI) of the model for GKT was 0.96 and 0.92, respectively. Both values were higher than 0.90 that indicated a good fit to the data. The Root-Mean-Square Residual (RMR) and the Standardized RMR values of the model was both equal to 0.046. Since, these values were lower than 0.05, they indicated a good fit to the data. The value of Root-Mean-Squared Error of Approximation (RMSEA) of the model was 0.053, which was between 0.05

and 0.08, indicating a good fit to the data. Additionally, RMSEA of the model was demonstrated to be in the 90 percent confidence interval for RMSEA, which was from 0.034 to 0.072.

The Normed Fit Index (NFI) and the Non-Normed Fit Index (NNFI) of the model were both equal to 0.97. These values indicated a good fit to the data, which were higher than the 0.90 cutoff level.

The values of Comparative Fit Index (CFI), Incremental Fit Index (IFI), and Relative Fit Index (RFI) were 0.98, 0.98, and 0.94, respectively. All the values were above 0.90 that indicated a good fit to the data. The Expected Cross Validation Index (ECVI) of the model was 0.50. Additionally, this criteria was among the 90 percent confidence interval for ECVI which was from 0.44 to 0.60. Since the value of ECVI was between the values of the confidence interval, it can be stated that the model fits the data. Furthermore, the value displayed for ECVI was found to be smaller than the value of ECVI for Saturated Model ($0.50 < 0.53$). This finding also supported the good fit to the data.

The Parsimony Goodness of Fit Index (PGFI) and the Parsimony Normed Fit Index (PNFI) were 0.50 and 0.60, respectively. It is stated that higher values of PGFI and PNFI indicate a more parsimonious fit. However, for these indices it is not expected to obtain 0.90 cutoff level. In this sense, the PGFI and PNFI values displayed in the output, indicate a moderate fit to the data.

The investigation of the goodness-of-fit indices of the model regarding their criteria showed that there is an overall fit between the model and the data. In this sense, it can be concluded that the model for GKT indicated a good fit to the data and in addition, a moderate parsimonious fit to the data.

Table 4.11 Goodness of Fit Indices of the Model for GKT

Fit Index	Criterion	Value
Chi-Square (χ^2)	Non-significant	75.31 (p= 0.00)
Normed Chi-Square (NC)	NC< 5	1.83
Goodness of Fit Index (GFI)	GFI> 0.90	0.96
Adjusted Goodness of Fit Index (AGFI)	AGFI> 0.90	0.92
Root Mean Square Error of Approximation (RMSEA)	0.05 < RMSEA < 0.08 (moderate fit) RMSEA < 0.05 (good fit)	0.053
Root Mean Square Residual (RMR)	RMR < 0.05	0.046
Root Mean Square Residual (S-RMR)	S-RMR < 0.05	0.046
Parsimony Goodness of Fit Index (PGFI)	Higher values	0.50
Parsimony Normed Fit Index (PNFI)	Higher values	0.60
Normed Fit Index (NFI)	NFI> 0.90	0.97
Non-Normed Fit Index (NNFI)	NNFI> 0.90	0.97
Comparative Fit Index (CFI)	CFI> 0.90	0.98
Incremental Fit Index (IFI)	IFI> 0.90	0.98
Relative Fit Index (RFI)	RFI> 0.90	0.94

4.1.4 RESULTS OF CONFIRMATORY FACTOR ANALYSIS FOR THE JUNIOR METACOGNITIVE AWARENESS INVENTORY

Confirmatory Factor Analysis was also conducted to specify the observed variables that indicate the latent variables of the Junior Metacognitive Awareness Inventory (Jr. MAI). Regarding the findings of the Exploratory Factor Analysis, the model was tested and seven covariance terms were added to SIMPLIS syntax in order to improve the model considering the modification indices with the highest values. The final SIMPLIS syntax for the CFA of the model is given in Appendix I. In Figure 4.3 LISREL estimates of parameters in the model in which the coefficients were in standardized values is presented.

Additionally, in Figure 4.4 LISREL estimates of parameters in the model in which the coefficients were in t-values were displayed. Moreover, LISREL estimates of parameters in the model with coefficients in standardized value and t-values are represented in Appendix J.

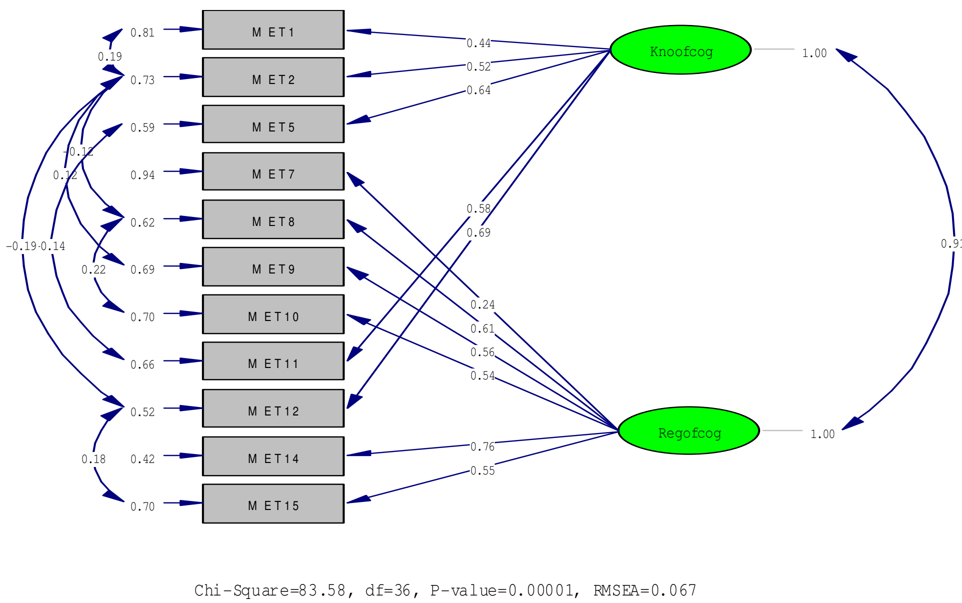


Figure 4.3 LISREL Estimates of Parameters in Jr. MAI Model with Coefficients in Standardized Value

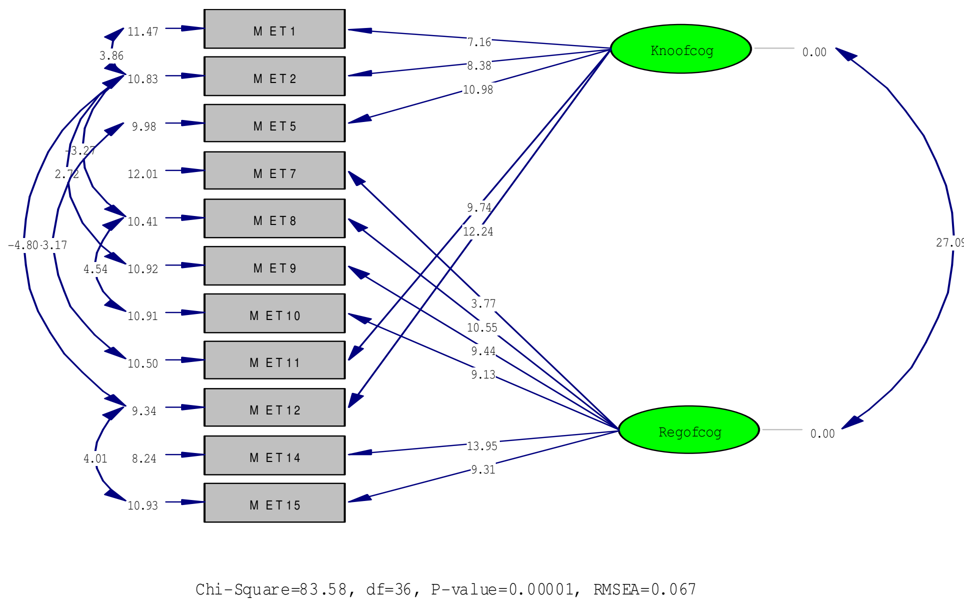


Figure 4.4 LISREL Estimates of Parameters in Jr. MAI Model with Coefficients in t-Values

According to the Figure 4.3 and Figure 4.4, the structural equation model was involving two latent variables and eleven observed variables. The latent variables of the model were Knowledge of Cognition (KNOOFCOG) and Regulation of Cognition (REGOFCOG) whereas the observed variables of the model were MET1, MET2, MET5, MET7, MET8, MET9, MET10, MET11, MET12, MET14, and MET15, respectively.

Fit statistics of squared multiple correlation (R^2) was calculated for each observed variable that represent the latent variables and the results were displayed in the LISREL output. In Table 4.12 the values of R^2 that equal the proportion of explained variance was presented. These values gave the reliability of the observed variables that specify KNOOFCOG and REGOFCOG.

Table 4.12 Squared Multiple Correlations for Jr. MAI

Latent Variables	Observed Variables	R^2
KNOOFCOG	MET1	0.19
	MET2	0.27
	MET5	0.41
	MET11	0.34
	MET12	0.48
REGOFCOG	MET7	0.06
	MET8	0.38
	MET9	0.31
	MET10	0.30
	MET14	0.58
	MET15	0.30

The values of the measurement coefficient the λ_x (lowercase lambda sub x) indicate the relationships between the latent variables and the observed variables. Furthermore, the δ (lowercase delta) is the measurement errors for the Xs, respectively. The measurement coefficients and the measurement errors of Jr. MAI were given in standardized values in Table 4.13.

Table 4.13 Measurement Coefficients of Jr. MAI

Latent Variables	Observed Variables	λ_X	δ
KNOOFCOG	MET1	0.44	0.81
	MET2	0.52	0.73
	MET5	0.64	0.59
	MET11	0.58	0.66
	MET12	0.69	0.52
REGOFCOG	MET7	0.24	0.94
	MET8	0.61	0.62
	MET9	0.56	0.69
	MET10	0.54	0.70
	MET14	0.76	0.42
	MET15	0.55	0.70

The summary statistics for fitted residuals for the model yielded the smallest fitted residual as -0.09 and largest fitted residual as 0.10. This examination indicated a good fit because each of the fitted residual values was less than 2 in absolute value (Kelloway, 1998). On the other hand, the summary statistics for standardized residuals for the model yielded the smallest standardized residual as -2.65 and the largest standardized residual was 3.14. In the output file, the fitted residuals and the standardized residuals were typical for the situation they had been calculated. In other words, the structure of both residuals displayed a similar shape. Furthermore, the stemleaf plots of both residuals were approximately normal which indicated a good fit. The stemleaf plots of the residuals are represented in Appendix K.

Jr. MAI was evaluated in terms of the goodness-of-fit-indices, which were discussed in detail in Chapter 3. The values of the goodness-of-fit criteria of the model are represented in Appendix L. The model for Jr. MAI demonstrated a significant Chi-Square value of $\chi^2 = 83.58$ with degrees of freedom, df = 36, at a significance level p = 0.000. The value of the Normed Chi-Square (NC) in terms of which χ^2 / df is displayed, was 2.32 that indicated a good fit to the data with its being less than 5.

The Goodness-of-Fit Index (GFI) and the Adjusted Goodness-of-Fit Index (AGFI) of the model for GKT was 0.95 and 0.91, respectively. Both values were higher than 0.90 that indicated a good fit to the data. The Root-Mean-Square Residual (RMR) and the Standardized RMR values of the model was both equal to 0.048. Since, these values were lower than 0.05, they indicated a good fit to the data. The value of Root-Mean-Squared Error of Approximation (RMSEA) of the model was 0.067, which was between 0.05 and 0.08, indicating a good fit to the data. Additionally, RMSEA of the model was demonstrated to be in the 90 percent confidence interval for RMSEA which was from 0.048 to 0.086.

The Normed Fit Index (NFI) and the Non-Normed Fit Index (NNFI) of the model were both equal to 0.91. These values indicated a good fit to the data which were higher than the 0.90 cutoff level.

The values of Comparative Fit Index (CFI), Incremental Fit Index (IFI), and Relative Fit Index (RFI) were 0.94, 0.94, and 0.86, respectively. Except the value of RFI, all the values were above 0.90 that indicated a good fit to the data. Since the value of RFI approaches to unity, it can be indicated that there is a good fit to the data. The Expected Cross Validation Index (ECVI) of the model was 0.49. Additionally, this criterian was among the 90 percent confidence interval for ECVI, which was from 0.41 to 0.59. Since the value of ECVI was between the values of the confidence interval, it can be stated that the model fits the data. Furthermore, the value displayed for ECVI was found to be larger than the value of ECVI for Saturated Model (0.49 > 0.45). This finding did not support the data.

The Parsimony Goodness of Fit Index (PGFI) and the Parsimony Normed Fit Index (PNFI) were 0.52 and 0.59, respectively. It is stated that higher values of PGFI and PNFI indicate a more parsimonious fit. However, for these indices it is not expected to obtain 0.90 cutoff level. In this sense, the PGFI and PNFI values displayed in the output, indicate a moderate fit to the data.

The investigation of the goodness-of-fit indices of the model regarding their criteria except RFI and the comparison of the values of ECVI and Saturated ECVI showed that there is an overall fit between the model and the data. In this sense, it can be concluded that the model for Jr. MAI indicated a good fit to the data and in addition, a moderate parsimonious fit to the data.

Table 4.14 Goodness of Fit Indices of the Model for the Jr. MAI

Fit Index	Criterion	Value
Chi-Square (χ^2)	Non-significant	83.58 (p= 0.00)
Normed Chi-Square (NC)	NC< 5	2.32
Goodness of Fit Index (GFI)	GFI> 0.90	0.95
Adjusted Goodness of Fit Index (AGFI)	AGFI> 0.90	0.91
Root Mean Square Error of Approximation (RMSEA)	0.05 < RMSEA < 0.08 (moderate fit) RMSEA < 0.05 (good fit)	0.067
Root Mean Square Residual (RMR)	RMR < 0.05	0.048
Root Mean Square Residual (S-RMR)	S-RMR < 0.05	0.048
Parsimony Goodness of Fit Index (PGFI)	Higher values	0.52
Parsimony Normed Fit Index (PNFI)	Higher values	0.59
Normed Fit Index (NFI)	NFI> 0.90	0.91
Non-Normed Fit Index (NNFI)	NNFI> 0.90	0.91
Comparative Fit Index (CFI)	CFI> 0.90	0.94
Incremental Fit Index (IFI)	IFI> 0.90	0.94
Relative Fit Index (RFI)	RFI> 0.90	0.86

4.2 SUMMARY OF EXPLORATORY FACTOR ANALYSIS AND CONFIRMATORY FACTOR ANALYSIS

The Principle Components Analysis was carried out by SPSS 11.5 and the Confirmatory Factor Analysis was carried out by LISREL 8.30. As a result of these analyses five latent variables which were indicated by four to six observed variables were selected to be included in structural equation modeling. These variables were Declarative Knowledge (DECKNOW), Conditional Knowledge (CONKNOW), Procedural Knowledge (PROKNOW), Knowledge of Cognition (KNOOFCOG), and Regulation of Cognition

(REGOFCOG). The latent variables, the observed variables, and accordingly the context of the observed variables are demonstrated in Table 4.15 and in Table 4.16, respectively.

Table 4.15 Latent and Observed Variables of Geometry Knowledge Test

Latent Variable	**Observed Variables**
Declarative Knowledge (DECKNOW)	Definition of different types of triangles (QUES1)
	Given a triangle write the sides and interior angles by symbols (QUES3)
	Definition of congruency in triangles (QUES4)
	Definition of similarity in triangles (QUES5)
Conditional Knowledge (CONKNOW)	Is the statement “If triangle ABC has three equal sides then angles A, B, C are equal to each other.” true? Justify your answer. (QUES8)
	Is the statement “If triangles ABC and DEF are similar then triangles ACB and DFE are similar.” true? Justify your answer. (QUES12)
	Is the statement “A triangle may have two right angles.” true? Justify your answer. (QUES13)
	Is the statement “Every equilateral triangle is an isosceles triangle.” true? Justify your answer. (QUES15)
Procedural Knowledge (PROKNOW)	Application of similarity in triangles (QUES19)
	Application of properties of equilateral triangle (QUES21)
	Application of properties of isosceles triangle (QUES23)
	Application of Euclid Relations and Pythagoras Theorem (QUES24)

Table 4.16 Latent and Observed Variables of Junior Metacognitive Awareness Inventory

Latent Variable	Observed Variables
Regulation of Cognition (REGOFCOG)	I ask myself questions about how well I am learning while I am learning something new. (MET10)
	I ask myself if I learned as much as I could have once I finish a task. (MET7)
	I ask myself if I have considered all options when solving a problem. (MET8)
	I ask myself periodically if I am meeting my goals. (MET15)
Knowledge of Cognition (KNOOFCOG)	I think about what I really need to learn before I begin a task. (MET9)
	I use different learning strategies depending on the situation. (MET14)
	I am a good judge of how well I understand something. (MET1)
	I can motivate myself to learn when I need to. (MET2)
	I learn best when I already know something about the topic. (MET5)
	I focus on the meaning and significance of new information. (MET11)
	I learn more when I am interested in the topic. (MET12)

After the observed variables that would be included in the structural equation modeling were determined, they were examined in terms of their frequency distributions. It was aimed to see the most and the least selected alternatives and the percentages of the responses given to the statements in Jr. MAI and of the responses given to the questions in GKT. The frequency distribution tables of each of the observed variables selected from the instruments were represented in Appendix C and Appendix D, respectively.

4.3 STRUCTURAL EQUATION MODELING

According to the results of the factor analyses the observed variables that represent the latent variables were determined and then included in the structural equation modeling. The data file containing all the variables in this study was imported into PRELIS 2.30 for Windows. The necessary steps of LISREL 8.30 for Windows with SIMPLIS command language were carried out for formulating and estimating the structural equation model of tenth grade students. In LISREL package program, SIMPLIS provides command language and PRELIS provides getting the covariance matrix. The structural equation modeling analyses were conducted by using the pairwise deletion method and the Maximum Likelihood Method of Estimation. In the analysis of this study, the significance level was taken to be 0.05.

4.3.1 THE METACOGNITION-KNOWLEDGE MODEL

Prior to the analysis a hypothetical model presented in Chapter 1, was determined regarding a theoretical base. It was hypothesized that there would be causal relationships among the variables concerning three types of geometry knowledge and two dimensions of metacognitive awareness. Then this model was tested and twenty-six covariance terms were added to SIMPLIS syntax in order to improve the model considering the modification indices with the highest values. The final SIMPLIS syntax for the Metacognition-Knowledge Model is given in Appendix M. In Figure 4.5 LISREL estimates of parameters in structural model in which the coefficients were in standardized values is presented. Additionally, in Figure 4.6 LISREL estimates of parameters in structural model in which the coefficients were in t-values were displayed. Moreover, LISREL estimates of parameters in measurement model with coefficients in standardized value and t-values are represented in Appendix N.

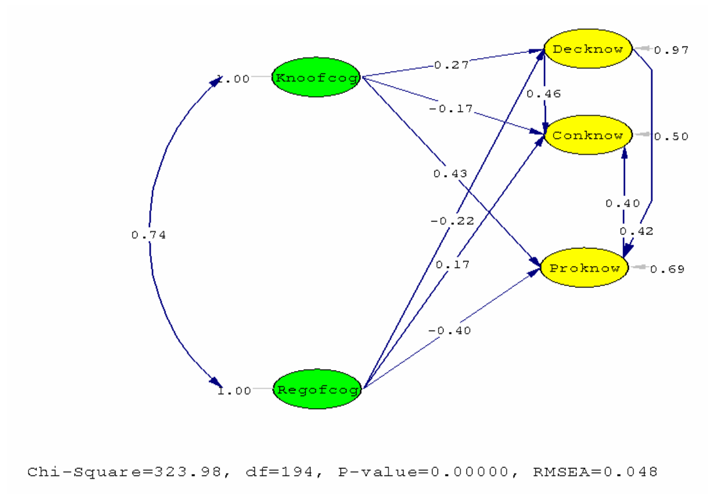


Figure 4.5 LISREL Estimates of Parameters in the Metacognition-Knowledge Model with Coefficients in Standardized Value

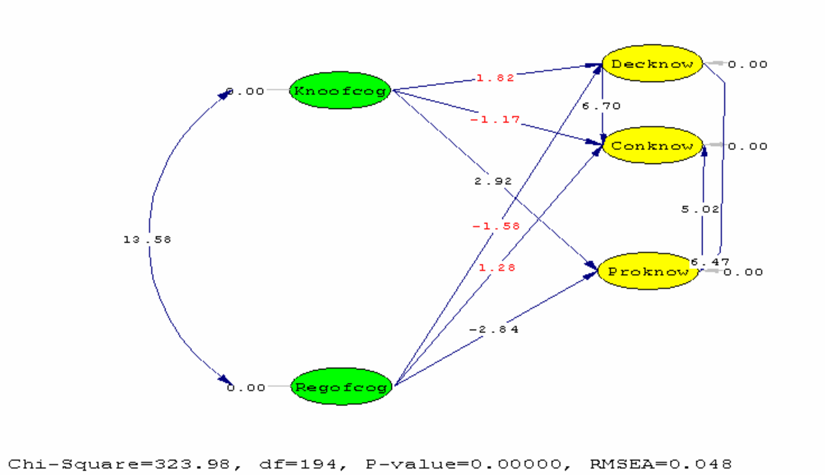


Figure 4.6 LISREL Estimates of Parameters in the Metacognition-Knowledge Model with Coefficients in t-Values

According to the Figure 4.5 and Figure 4.6, the structural equation model was involving two latent independent variables and three latent dependent variables. The latent independent variables of the model were Knowledge of Cognition (KNOOFCOG) and Regulation of Cognition (REGOFCOG) whereas the latent dependent variables were Declarative Knowledge (DECKNOW), Conditional Knowledge (CONKNOW), and Procedural Knowledge (PROKNOW).

The values of the measurement coefficients as the λ_y (lowercase lambda sub y) and the λ_x (lowercase lambda sub x) indicate the relationships between the latent variables and the observed variables. Furthermore, the ε (lowercase epsilon) and δ (lowercase delta) are the measurement errors for the Ys and Xs, respectively. The measurement coefficients of the Metacognition-Knowledge Model were given in standardized values in Table 4.17.

Besides, the values of the structure coefficient γ (lowercase gamma) indicate the strength and direction of the relationship between the latent dependent variables and the latent independent variables. Additionally, the values of the structure coefficient β (lowercase beta) indicate the strength and the direction of the relationships between latent dependent variables. The values of the structure coefficient γ (lowercase gamma) for the Metacognition-Knowledge Model were demonstrated in Table 4.18 and the values of the structure coefficient β (lowercase beta) were demonstrated in Table 4.19. Both structure coefficients for the Metacognition-Knowledge Model for geometry were displayed in standardized values.

Table 4.17 Measurement Coefficients of the Metacognition-Knowledge Model

Latent Variables	Observed Variables	λ	Measurement Error
DECKNOW	QUES1	0.57 (λ_Y)	0.68 (ε)
	QUES3	0.55 (λ_Y)	0.70 (ε)
	QUES4	0.93 (λ_Y)	0.13 (ε)
	QUES5	0.75 (λ_Y)	0.44 (ε)
CONKNOW	QUES8	0.62 (λ_Y)	0.62 (ε)
	QUES12	0.74 (λ_Y)	0.45 (ε)
	QUES13	0.79 (λ_Y)	0.38 (ε)
	QUES15	0.70 (λ_Y)	0.51 (ε)
PROKNOW	QUES19	0.90 (λ_Y)	0.20 (ε)
	QUES21	0.93 (λ_Y)	0.13 (ε)
	QUES23	0.91 (λ_Y)	0.17 (ε)
	QUES24	0.88 (λ_Y)	0.22 (ε)
KNOOFCOG	MET1	0.47 (λ_X)	0.78 (δ)
	MET2	0.58 (λ_X)	0.67 (δ)
	MET5	0.49 (λ_X)	0.76 (δ)
	MET11	0.69 (λ_X)	0.53 (δ)
	MET12	0.35 (λ_X)	0.88 (δ)
REGOFCOG	MET7	0.62 (λ_X)	0.61 (δ)
	MET8	0.65 (λ_X)	0.58 (δ)
	MET9	0.61 (λ_X)	0.62 (δ)
	MET10	0.56 (λ_X)	0.69 (δ)
	MET14	0.60 (λ_X)	0.64 (δ)
	MET15	0.50 (λ_X)	0.75 (δ)

Table 4.18 Structure Coefficients of the Metacognition-Knowledge Model

Latent Dependent Variables	Latent Independent Variables	γ
DECKNOW	KNOOFCOG	0.27
	REGOFCOG	- 0.22
CONKNOW	KNOOFCOG	- 0.17
	REGOFCOG	0.17
PROKNOW	KNOOFCOG	0.43
	REGOFCOG	-0.40

Table 4.19 Structure Coefficients of the Metacognition-Knowledge Model

Latent Dependent Variables	β	**Latent Dependent Variables**
DECKNOW	-	DECKNOW
	-	CONKNOW
	-	PROKNOW
CONKNOW	0.46	DECKNOW
	-	CONKNOW
	0.40	PROKNOW
PROKNOW	0.42	DECKNOW
	-	CONKNOW
	-	PROKNOW

In LISREL output, the squared multiple correlation (R^2) for each variable was also displayed. This measurement gives the proportion of the explained variance. For example, a value of 0.30 means that 30% of the variance of a variable is explained by another variable. In Table 4.20, the squared multiple correlations (R^2) of the observed variables are represented.

Table 4.20 Squared Multiple Correlations of the Observed Variables

Observed Variable	R^2	**Observed Variable**	R^2
QUES1	0.32	MET1	0.22
QUES3	0.30	MET2	0.33
QUES4	0.87	MET5	0.24
QUES5	0.56	MET7	0.39
QUES8	0.38	MET8	0.42
QUES12	0.55	MET9	0.38
QUES13	0.62	MET10	0.31
QUES15	0.49	MET11	0.47
QUES19	0.80	MET12	0.12
QUES21	0.87	MET14	0.36
QUES23	0.83	MET15	0.25
QUES24	0.78		

Furthermore, in LISREL for each endogenous variable in the model R^2 values are computed and accordingly, interpreted in the sense of R^2 values in regression. The effect sizes in measures of squared multiple correlation for endogenous variables were used in this study. Table 4.21 shows the index of effect sizes for the model of this study.

Table 4.21 Effect Sizes of the Model in R^2

Latent Variables	Squared Multiple Correlation
DECKNOW	0.033
CONKNOW	0.50
PROKNOW	0.31

Declarative knowledge had an effect size value of 0.033, conditional knowledge had an effect size value of 0.50, and procedural knowledge had an effect size value of 0.31. These values indicated a large effect size value because of exceeding the large index 0.25, except declarative knowledge. In the present study, the Metacognition-Knowledge Model for geometry was able to explain 3% of the variance of declarative knowledge; 50% of the variance of conditional knowledge; and 31% of the variance of procedural knowledge.

The summary statistics for fitted residuals for the model yielded the smallest fitted residual as -0.16 and largest fitted residual as 0.21. On the other hand, the summary statistics for standardized residuals for the model yielded the smallest standardized residual as -4.38 and the largest standardized residual as 4.38. In the output file, the fitted residuals and the standardized residuals were typical for the situation they had been calculated. In other words, the structure of both residuals displayed a similar shape. Furthermore, the stemleaf plots of both residuals were approximately normal which indicated a good fit. The stemleaf plots of the residuals are represented in Appendix O.

The Metacognition-Knowledge Model for geometry was evaluated in terms of the goodness-of-fit-indices, which were discussed in Chapter 3 detaily. The values of the goodness-of-fit criteria of the model are represented in Appendix P. The Metacognition-Knowledge Model for geometry demonstrated a significant Chi-Square value of $\chi^2 = 323.98$ with degrees of freedom, df = 194, at a significance level p = 0.000. As known, χ^2 is sensible to sample size. In this sense, this criterian indicates a significant probability level when the sample size increases, generally above 200 (Schumacker & Lomax, 2004). The sample size in this study was 297, which was large enough to make the test statistically significant. The value of the Normed Chi-Square (NC) in terms of which χ^2 / df is displayed, was 1.67 that indicated a good fit to the data with its being less than five.

The Goodness-of-Fit Index (GFI) and the Adjusted Goodness-of-Fit Index (AGFI) of the Metacognition-Knowledge Model for geometry was 0.91 and 0.88, respectively. These values were approaching to unity that indicated a good fit to the data.

The Root-Mean-Square Residual (RMR) and the Standardized RMR values of the model was both equal to 0.06. Since, these values were greater than 0.05, they did not indicate a good fit to the data. However, the Root-Mean-Squared Error of Approximation (RMSEA) of the model was 0.048 as a value less than 0.05, indicating a good fit to the data. Additionally, RMSEA of the model was demonstrated to be in the 90 percent confidence interval for RMSEA, which was from 0.038 to 0.057.

The Normed Fit Index (NFI) and the Non-Normed Fit Index (NNFI) of the model were 0.90 and 0.94, respectively. Both values of NFI and NNFI indicated a good fit to the data, which was higher than the 0.90 cutoff level.

The values of Comparative Fit Index (CFI), Incremental Fit Index (IFI), and Relative Fit Index (RFI) were 0.95, 0.95, and 0.87, respectively. Except RFI of the model, CFI and IFI were above 0.90 that indicated a good fit to the data. Although, the value of RFI was below 0.90 the higher values that approach to unity are also acceptable.

The Expected Cross Validation Index (ECVI) of the model was 1.65. Additionally, this criterian was among the 90 percent confidence interval for ECVI, which was from 1.49 to 1.83. Since the value of ECVI was between the values of the confidence interval, it can be stated that the model fits the data. Furthermore, the value displayed for ECVI was found to be smaller than the value of ECVI for Saturated Model (1.65 < 1.86). This finding also indicated a good fit to the data.

The Parsimony Goodness of Fit Index (PGFI) and the Parsimony Normed Fit Index (PNFI) were 0.64 and 0.69, respectively. It is stated that higher values of PGFI and PNFI indicate a more parsimonious fit. However, for these indices it is not expected to obtain 0.90 cutoff level. In this sense, the PGFI and PNFI values displayed in the output, indicate a moderate fit to the data.

Furthermore, the values of Comparative Fit Index (CFI) and Incremental Fit Index (IFI) were both 0.94, which was above 0.90 indicating a good fit to the data.

The investigation of the goodness-of-fit indices of the model regarding their criteria showed that there is an overall fit between the model and the data except the values of RMR, Standardized RMR, PGFI, and PNFI. In this sense, it can be concluded that the Knowledge Model for geometry indicated a good fit to the data and in addition, a moderate parsimonious fit to the data. Table 4.22 demonstrates the goodness of fit indices of the Metacognition-Knowledge Model

Table 4.22 Goodness of Fit Indices of the Metacognition-Knowledge Model

Fit Index	Criterion	Value
Chi-Square (χ^2)	Non-significant	323.98 (p= 0.00)
Degrees of Freedom(df)		194
Normed Chi-Square (NC)	NC< 5	1.67
Goodness of Fit Index (GFI)	GFI> 0.90	0.91
Adjusted Goodness of Fit Index (AGFI)	AGFI> 0.90	0.88
Root Mean Square Error of Approximation (RMSEA)	0.05 < RMSEA < 0.08 (moderate fit) RMSEA < 0.05 (good fit)	0.048
Root Mean Square Residual (RMR)	RMR < 0.05	0.065
Standardized Root Mean Square Residual (S-RMR)	S-RMR < 0.05	0.065
Parsimony Goodness of Fit Index (PGFI)	Higher values	0.64
Parsimony Normed Fit Index (PNFI)	Higher values	0.69
Normed Fit Index (NFI)	NFI> 0.90	0.90
Non-Normed Fit Index (NNFI)	NNFI> 0.90	0.94
Comparative Fit Index (CFI)	CFI> 0.90	0.95
Incremental Fit Index (IFI)	IFI> 0.90	0.95
Relative Fit Index (RFI)	RFI> 0.90	0.87

With respect to Figure 4.5 and according to lowercase lambda sub x values, all of the eleven metacognitive variables were significantly ($p < 0.05$) and positively loaded on KNOOFCOG and REGOFCOG, respectively. Among these variables, MET11 accounted for the greatest variance (R^2 = 0.47) of the latent variable KNOOFCOG and MET8 accounted for the greatest variance (R^2 = 0.42) of the latent variable REGOFCOG.

Similarly, the latent dependent variables DECKNOW, CONKNOW, and PROKNOW were examined and found to account for the observed variables. Regarding the lowercase lambda sub y values displayed in the

LISREL output, all the performance-based variables were found to be significantly and positively loaded on the latent dependent variables. Among these twelve variables, QUES4 with a R^2 value of 0.87; QUES13 with a R^2 value of 0.62; and QUES21 with a R^2 value of 0.87 accounted for the greatest variance of the latent dependent variables DECKNOW, CONKNOW, and PROKNOW, respectively.

LISREL Package program also demonstrates the direct and indirect effects that constitute the total effect. With respect to Figure 4.5 the direct effects of the latent independent variables on the latent dependent variables were represented in Table 4.23.

Table 4.23 Direct Effects of Latent Independent Variables on Latent Dependent Variables for the Metacognition-Knowledge Model

	KNOOFCOG	REGOFCOG
DECKNOW	0.27	- 0.22
CONKNOW	- 0.17	0.17
PROKNOW	0.43	- 0.40

The indirect effects and the total effects of latent independent variables on latent dependent variables were presented in LISREL output of the model. Table 4.24 and Table 4.25 displayed the values of the indirect effects and total effects of latent independent variables on latent dependent variables.

Table 4.24 Indirect Effects of Latent Independent Variables on Latent Dependent Variables for the Metacognition-Knowledge Model

	KNOOFCOG	REGOFCOG
DECKNOW	-	-
CONKNOW	0.34	- 0.30
PROKNOW	0.11	- 0.09

Table 4.25 Total Effects of Latent Independent Variables on Latent Dependent Variables for the Metacognition-Knowledge Model

	KNOOFCOG	REGOFCOG
DECKNOW	0.27	- 0.22
CONKNOW	0.17	- 0.13
PROKNOW	0.54	- 0.49

The results presented in Figure 4.5 showed that KNOOFCOG had a non-significant positive direct effect on DECKNOW ($\Gamma = 0.25$, $P > 0.05$) and REGOFCOG had a non-significant negative direct effect on DECKNOW ($\Gamma = - 0.22$, $P > 0.05$). KNOOFCOG had a non-significant negative direct effect on CONKNOW ($\Gamma = - 0.17$, $P > 0.05$) and REGOFCOG had a non-significant positive direct effect on CONKNOW ($\Gamma = 0.17$, $P > 0.05$). KNOOFCOG had a positive direct effect on PROKNOW ($\Gamma = 0.43$, $p < 0.05$) and REGOFCOG had a negative direct effect on PROKNOW ($\Gamma = - 0.40$, $P < 0.05$). The latent independent variables also had indirect effects on CONKNOW and PROKNOW. The latent independent variable KNOOFCOG had a positive indirect effect of 0.34 on CONKNOW and REGOFCOG had a negative indirect effect of – 0.30 on CONKNOW. On the other hand, KNOOFCOG had a positive indirect effect of 0.11 on PROKNOW, while REGOFCOG had a negative indirect effect of – 0.09 on PROKNOW. Accordingly, the total effects of the latent independent variables on latent dependent variables presented in Table 4.25 showed that KNOOFCOG had a non-significant positive total effect on DECKNOW ($\Gamma = 0.27$, $P > 0.05$) and REGOFCOG had a non-significant negative total effect on DECKNOW($\Gamma = - 0.22$, $p > 0.05$). KNOOFCOG and REGOFCOG also had non-significant total effects on CONKNOW. KNOOFCOG had a positive total effect on CONKNOW ($\Gamma =$ O.17, $P > 0.05$) and REGOFCOG had a negative total effect on CONKNOW ($\Gamma = - 0.13$, $p > 0.05$). In contrast, KNOOFCOG had significant positive total effect on PROKNOW ($\Gamma = 0.54$, $p < 0.05$) and REGOFCOG had a significant negative total effect on PROKNOW (Γ = - 0.49, P< 0.05). In this sense, among two latent independent variables KNOOFCOG had the greatest total effect on PROKNOW, while no significant total effects were indicated for DECKNOW and CONKNOW.

The results displayed in Figure 4.5 showed the direct effects of the latent dependent variables on the latent dependent variables. The values of the direct effects among the latent dependent variables were presented in Table 4.26.

Table 4.26 Direct Effects of the Latent Dependent Variables on Latent Dependent Variables for the Metacognition-Knowledge Model

	DECKNOW	CONKNOW	PROKNOW
DECKNOW	-	-	-
CONKNOW	0.46	-	0.40
PROKNOW	0.42	-	-

The indirect effects and the total effects of latent dependent variables on latent dependent variables were presented in LISREL output of the model. Table 4.27 and Table 4.28 displayed the values of indirect effects and total effects of the latent dependent variables among each other.

Table 4.27 Indirect Effects Between Latent Dependent Variables for the Metacognition-Knowledge Model

	DECKNOW	CONKNOW	PROKNOW
DECKNOW	-	-	-
CONKNOW	0.17	-	-
PROKNOW	-	-	-

Table 4.28 Total Effects between Latent Dependent Variables for the Metacognition-Knowledge Model

	DECKNOW	CONKNOW	PROKNOW
DECKNOW	-	-	-
CONKNOW	0.63	-	0.40
PROKNOW	0.42	-	-

The latent dependent variables did not have direct effects on DECKNOW. DECKNOW had a significant and positive effect on both CONKNOW and PROKNOW with Beta values of $\beta = 0.46$ and $\beta = 0.42$, respectively. DECKNOW had also a significant indirect effect of 0.17 on CONKNOW. PROKNOW had a significant and positive direct effect on CONKNOW ($\beta = 0.40$, $P < 0.05$). The total effect of latent dependent variable DECKNOW on CONKNOW indicated a significant and positive effect with a value of 0.63. Similarly, the total effect of DECKNOW on PROKNOW indicated a significant and positive effect with a value of 0.42. On the other hand, the total effect of PROKNOW on CONKNOW was 0.40, which indicated a significant and positive effect. The reciprocal relationships among the latent dependent variables showed that DECKNOW had the greatest total effect on CONKNOW with a value of 0.63. Furthermore, the results demonstrated that DECKNOW had a stronger effect on CONKNOW than PROKNOW in the Metacognition-Knowledge Model of geometry.

CHAPTER 5

DISCUSSION, CONCLUSION AND IMPLICATIONS

This chapter presents the discussion and the conclusion of the results, the interpretations of the findings, educational implications, and reccommendations for future research.

5. 1 DISCUSSION OF THE RESULTS

The review of the related literature shows that up to now very few research has been conducted including the metacognitive and knowledge factors involved in the present study. This study investigated a structural model to explain the reciprocal relationships among a set of metacognitive and knowledge variables, constituted through the use of principal component analysis and confirmatory factor analysis. The purpose of this study was twofold: (a) to examine the effects of knowledge of cognition and regulation of cognition on declarative, conditional, and procedural knowledge and (b) to examine the interrelations among declarative, conditional, and procedural knowledge.

With respect to the first block of the model interesting findings emerged in this study. The first block of the model has provided evidence about the relations and interconnections among metacognitive knowledge and regulation with respect to three different types of knowledge. The predominant role of knowledge of cognition on procedural knowledge is too important. The hypothesized relationships among knowledge of cognition, regulation of cognition, and procedural knowledge were statistically significant and were substantional in size. This supports the robustness of the theoretical model of the importance of metacognition in procedural knowledge learning. Concerning predictors of procedural knowledge, knowledge of cognition had the strongest predictive power; that as students become more aware of what they know and what they need to know, they show a higher performance in applying algorithms. This finding supports the findings of previous studies (Maqsud, 1997; Tobias & Everson, 2002; Wilson & Clarke, 1994) that students' metacognitive knowledge positively contributes to their problem-solving performance in mathematics. This provided evidence that students who have the awareness of what strategies and prior knowledge to use for what purposes are better able to integrate the most appropriate ones to their computational processes. In other words, knowledge of how to use available information to reach a correct solution leads students to make effective progress on procedural knowledge questions.

In contrast, regulation of cognition had a significant but negative effect on students' procedural knowledge. Although this result of the present study was inconsistent with previous research (Kramarski, Mevarech, & Lieberman, 2001; Kramarski, Mevarech, & Arami, 2002; Kramarski, 2004; Mevarech & Kramarski, 1997; Mevarech, 1999), it lends strong support for the claim that low-achievers who are categorized to have high-regulatory abilities tend to overestimate their mathematical knowledge (Panaoura & Philippou, 2003). This reminds that students who are aware of the demands of the task do not automatically regulate or monitor their progress while performing on procedural tasks. In some cases students subconsciously use their regulatory processes and imply their plans instead of explicitly stating them. Therefore, it might be inappropriate to assume that students who have high procedural knowledge would have few problems in regulation of cognition processes such as planning, monitoring, evaluation, and verification. The findings further support the findings of Pugalee (2001; 2004). Although, students' written and verbal descriptions of their progress in procedural knowledge questions show evidence of their regulatory processes, they might be able to organize their solution plans even they are not the efficient ones. Thus, their high procedural knowledge may be attributable to the subconscious use of their comprehension monitoring processes (Schurter, 2002). Another important explanation for this result comes from the literature that regulatory processes are highly automated and develop without any conscious reflection, and that they are difficult to report in many learning situations (Brown, 1987). The negative relationship also confirmed the results of Lucangeli and Cornoldi (1997) that metacognitive regulation processes can be used less frequently as the tasks become automized.

The effects of knowledge of cognition and regulation of cognition on declarative and conditional knowledge were practically trivial. This suggests that metacognitive knowledge and regulation do not substantially affect these types of knowledge. Although the findings provide only partial support for interpreting the positive effect of knowledge of cognition and regulation of cognition on declarative knowledge and conditional knowledge, the assertion by previous research (Panaoura & Philippou, 2003; 2007; Sperling, et al., 2002; 2004; Swanson, 1990) positing that students, even the high-achievers; are hardly aware of the knowledge about their own cognitive processes and the relative utility of these processes. Furthermore, they are unable to appraise the outcomes of their cognitive processes which would facilitate their understanding of comprehension breakdowns. The findings from this study suggest that students fail to possess their knowledge of cognition and use that knowledge to regulate their learning while performing on questions that require the recall of simple facts and the justification of the relations between facts and principles.

The discrepancy between the results of the first block of the model and those of previous studies that support the essential role of metacognitive knowledge and regulation in using effective strategies and solving complex tasks (Artz & Armour-Thomas, 1992; Lucangeli & Cornoldi, 1997; Mevarech & Kramarski, 1997) was partly attributable to the general perception in high school teaching of mathematics which tends to be fairly procedural as well as the format and context of the questions used to measure students' knowledge of mathematics. In line with the suggestions of Mevarech (1999) one would expect that complex non-routine mathematical tasks (e.g conditional knowledge questions) require more metacognitive knowledge and regulation than routine tasks at a lower level of complexity (e.g declarative and procedural knowledge questions). However, in mathematics classes students are better equipped to deal with procedural knowledge questions rather than declarative and conditional knowledge questions. It is generally accepted that students are in possession of making sense of mathematics consists in their being able to carry out routine application of procedures. Thus, the relationships mentioned above might be derived from students' unfamiliarity with making their own concept definitions and building connections between if-then statements. While, solving declarative and conditional knowledge questions they might not think about what they know about the problem or check the outcomes of the problem solution. In addition, while solving procedural knowledge questions students may not need to pay attention to important information in the problem context, plan their solution strategies, seek for an easier way to reach a correct solution, or monitor their progress as the given task requires the application of straightforward algoritms and procedures. Furthermore, the present study assessed students' knowledge of mathematics through open-ended questions that require making appropriate concept definitions, meaningful explanations for the relationships between facts and principles, and correct application of procedures. In accordance with the findings of O'Neil and Brown (1998) students might not know how to be aware of what they understand or check the solution process in such a question format. While solving open-ended questions they might focus on the indepth mathematical line of their responses rather than the utilization of metacognitive knowledge and regulation.

Nevertheless, this study's foregoing findings lend credence to the suggestions of some researchers (Kramarski, et al., 2001; Kramarski, 2004) that when students are provided explanations to elaborate information and make connections, they become more able to reflect on the similarities and differences between previous and new problems, comprehend the problem before attempting a solution, and consider about strategies to draw a conclusion. The findings of these studies emphasize the effect of regulatory processes which provide students better use of their cognitive resources including attention, strategy selection, and awareness of comprehension. However, these processes improve performance when regulatory skills are activated by the individual and the understanding of how to use these skills are integrated into instructional settings. In line with this premise, one possible reason for the nonsignificant relations between components of metacognition and declarative and conditional knowledge may be that students participated in this study were not employed to a training to activate their metacognitive knowledge and regulation which would enable them to perform better on defining concepts, recalling facts, understanding symbols, and justifying condition/action statements. Moreover, this compromises with the results of some studies (Veenman, Wilhelm, & Beishuizen, 2004; Wilson & Clarke, 2004) reviewed earlier that suggests introducing the mathematical tasks with metacognitive cues. Correct concept definitions and justifications of the relations might yield as students are made aware of using their metacognitive knowledge and regulation processes. The awareness of making predictions about their solution processes, designing their goals, keeping track on their progress, or elaborating their conclusions can be provided by metacognitive cues. Students' management of their knowledge of cognition processes can be supported by giving them metacognitive cues such as "I understand the requirements of the given task.", "What do you need to know to solve this problem?", "Do you have adequate prior knowledge to solve this problem?", "Do you think you will solve this problem correctly?", "I think about using strategies that have worked to solve a similar problem before.", "I know that I am good at solving problems like this.". Students' management of their regulation of cognition processes can be supported by giving them metacognitive cues such as "What is the best strategy to solve this problem?", "After each step, check your solution progress to prevent making computational errors.", "I need to draw a graph/table to solve this problem.", "I am on the right track.", and "Can you draw a conclusion with regard to your solution process?".

The results yielded from the second block of the model were in accordance with previous research (Pesek and Kirschner, 2000; Rittle-Johnson & Siegler, 1999; Rittle-Johnson, Siegler, & Alibali, 2001; Rittle-Johnson & Koedinger, 2005) that emphasized conceptual knowledge influences the gains in procedural knowledge, whereas procedural knowledge produce gains in conceptual knowledge. The interrelations among declarative knowledge, conditional knowledge, and procedural knowledge emerged in this study provided support in favor of the fact that knowledge of concepts and knowledge of procedures are learned in tandem rather than independently. Such an incorporation emphasizes that making sense of concept definitions can support translations among principles and routine applications of algorithms whereas drill-and-practice implementations can support later meaningful learning of principles and building mathematical relations.

Declarative knowledge influenced the gains in conditional knowledge by improving construction of relations between statements and in procedural knowledge by improving problem representation, selection of correct procedures, and facilitating the deployment of new procedures. In other words, relationships among principles and applications of procedures can not function without first accessing to definitions and facts. These findings support the findings of some previous research (Byrnes & Wasik, 1991; Mack, 1990; Moss & Case, 1999; Perry, 1991; Star, et al., 2005) that students should have a rich store of definitions and facts to explain the relations among principles and adopt adequate procedures to the solution process. When students learn to define a concept and use apropriate mathematical notations relevant to this concept, in effect, they learn to extract relations. Thus, students' justification of relations, in addition to state the definitions, includes application of procedures. Besides, procedures depend on mathematical definitions and notations. The interconnection facilitates students' understanding of what notation leads to which concept and what concept is related to which principle and procedure. Declarative knowledge of mathematics leads to an expanding conditional and procedural knowledge base for the efficient justification of relations and execution of procedures which underpins success in mathematics. It influences the way that the relationships between principles are structured and the adoption of procedures.

Gains in procedural knowledge produced gains in conditional knowledge through strengthening the use of concept-algorithm sequences, reflecting on why algorithms work, and evaluating the correctness of possible answers to the demands of problems. These findings were consistent with the findings of previous research (Baroody & Gannon, 1984; Engelbrecht, et al., 2005; Gelman, Meck, & Merkin, 1986; Hiebert & Wearne, 1996; Lembke & Reys, 1984) that indicated students who exhibit greater computational performance are likely to have a greater understanding of relations among principles. This implies that students extract relevant principles as they progress on a given task and discover the relations within the practice process. Students demonstrate the action sequences of a given problem in conjunction to the links between relational rules. Thus, causal relations are characterized by definitions, notations, and algorithms. Parallels within this synthesis are used to make the most efficient progress.

The present study specified procedural knowledge as independent of conditional knowledge and this specification was not disproved by the data. The premise is that students may not need to utilize from their conditional knowledge while performing on procedural knowledge questions. When applying straightforward algorithms, they may not tend to justify their answers or link between principles and procedures. This particular finding affirmed that knowledge of algorithms may develop without or apart from knowledge of concepts and principles. In some circumstances, this result supported a traditional view that mathematical knowledge is a set of rules of propositions. However, the interactions in the second block of the model put forward the fact that including three types of knowledge as part of a mathematics class is that it takes students' understanding to define, justify, and apply.

5. 2 CONCLUSION

The metacognitive factors affecting three different types of knowledge and the interrelation among these knowledge types are explored by using structural equation modeling. The factors included in the study are selected in accordance with the context of the measures used in order to assess students' metacognitive awareness and cognitive performance through the use of principal component analysis and confirmatory factor analysis. The Junior Metacognitive Awareness Inventory is comprised of two components of metacognition as knowledge of cognition and regulation of cognition. The Geometry Knowledge Test is comprised of open-ended questions contextualized in declarative knowledge, conditional knowledge, and procedural knowledge. The factors in the present study are knowledge of cognition, regulation of cognition, declarative knowledge, conditional knowledge, and procedural knowledge.

Although, no models exist in the literature that involve the factors of the present study, the results are generally consistent with the findings of previous research. They provide general and partial support for the relations and interconnections among cognitive and metacognitive processes. The results are summarized as follows:

1. Knowledge of cognition has a significant and positive direct effect on procedural knowledge. That is, students who know that they understand something tend to perform better on the application of procedures. Moreover, students who estimate that they will solve the problem correctly exhibits better performance in the applications of procedures.
2. Knowledge of cognition does not have a significant effect on declarative knowledge and conditional knowledge. This means that students' metacognitive knowledge does not have any impact on their knowledge of concepts, relations, and procedures. In some cases, even the high performing students are hardly aware of their ability to define a concept or to state the relationships among principles and algorithms.
3. Regulation of cognition has a significant but negative direct effect on students' procedural knowledge. This means that as students' regulatory processes increase their knowledge of procedures decrease or vice versa. Students tend to use their metacognitive regulation processes less frequently as the tasks become automized. Furthermore, regulatory processes are highly automated that may be developed subconsciously. Hence, they are difficult to report in many learning situations.
4. Regulation of cognition does not have a significant effect on declarative knowledge and conditional knowledge. This means that students' orientation, organization, evaluation and verification processes are not related with students' knowledge of concepts.
5. Knowledge of cognition has positive indirect effects on conditional knowledge and procedural knowledge. This means that knowledge of cognition not only has direct effects on conditional and procedural knowledge but it also affects these factors through influencing declarative knowldge.
6. Regulation of cognition has negative indirect effects on conditional knowledge and procedural knowledge. This means that regulation of cognition not only has direct effects on conditional and procedural

knowledge but it also affects these factors through influencing declarative knowledge.

7. Knowledge of cognition has the strongest direct effect on procedural knowledge. This means that students' awareness of what they know has a substantial effect on their computational processes.
8. The direct effect of knowledge of cognition on procedural knowledge is stronger than the direct effect of regulation of cognition on procedural knowledge. This means that students' metacognitive knowledge is associated with their procedural performance to a higher extend than their metacognitive regulation.
9. The indirect effect of knowledge of cognition on conditional knowledge is stronger than on procedural knowledge.
10. The indirect effect of regulation of cognition on procedural knowledge is stronger than on conditional knowledge.
11. Declarative knowledge has a significant and positive direct effect on conditional knowledge. This means that students' knowledge of definitions and symbols influence their ability to build relationships among concepts and principles. Thus, students who have adequate knowledge of concepts tend to make sensible and reliable justifications regarding the demands of the statements.
12. Declarative knowledge has a significant and positive direct effect on procedural knowledge. This means that students' knowledge of definitions and symbols influence their ability to apply algorithms and develop appropriate procedures. Thus, students who have adequate knowledge of concepts tend to generate new procedures and deploy these procedures in a variety of unfamiliar situations.
13. Procedural knowledge has a significant and positive direct effect on conditional knowledge. This means that students who have sufficient knowledge of procedures are better able to link between concepts and procedures. They tend to make translations among algorithms and

relational rules, and further make meaningful explanations about why these particular translations work.

14. Declarative knowledge has a positive indirect effect on conditional knowledge. This was the only indirect effect among latent dependent variables of the model.
15. Declarative knowledge has the strongest direct effect on conditional knowledge. That is, students knowledge of concept definitions improves their building relations among concepts and principles.
16. The direct effect of declarative knowledge on conditional knowledge is stronger than on procedural knowledge.
17. Declarative knowledge and to a somewhat lesser extent procedural knowledge have the strongest direct effects on conditional knowledge. This means that with enough declarative knowledge, students' computational progress improves their relational interpretations.

5. 3 IMPLICATIONS

The Metacognition-Knowledge Model possesses an accaptable fit and accordingly exhibits premise as a model for research on knowledge of mathematics and its metacognitive predictors together with the interrelations among different types of knowledge. The results provide some ideas for educational interventions such as integration of metacognitive training and instructional designs that reconceptualize knowledge of mathematics delineated within declarative, conditional, and procedural knowledge.

According to the conclusions and the literature review, the educational and pedagogical suggestions can be presented as the followings:

1. Formal instruction in the application of geometry tends to make the students' concept definitions less intuitive and more rule driven, actually narrowing rather than expanding the relational strategies and mental computations students use in solving geometry problems. Furthermore, it

severely limits students' understanding of complex problems and causes them difficulties in the long run. If students are to be able to internalize geometry, the curriculum can be designed in such a way that students recognize concept definitions and principles; that students have a clear understanding of the relationship among concepts, principles and procedures; and that students develop appropriate procedures. Instruction not only should build on the concepts and procedures that students start out with but it should move them through meaningful reasoning for making justifications in the development of those concepts and procedures. Most instruction jumps directly from the characterization of definitions to the memorization of facts and routine application of procedures without acknowleding the relationships among concepts and procedures. Moreover, majority of the students are entrenched in procedural views rather than the relational ones. Overhelmingly, many students tend to conceive geometry as a rigid subject based on rules, principles, and routine application of algorithms. There is a need to structure learning environments that reinforce the idea that geometry does not only include arbitrary rules but rather connections among these rules. Therefore, teachers, administrators and instructional designers should make clear establishments about how instruction can be sequenced to enhance the effective development of concepts, relations and procedures.

2. Competence in a subject requires all three types of knowledge. Developing students' knowledge of procedures is an important avenue for improving their knowledge of concepts and relational rules, just as developing knowledge of definitions and relational rules is essential for generation of computational algorithms and selection of appropriate procedures. This study hopes to inspire teachers to undertake fundamental classroom reform that emphasizes the relative efficiency and effectiveness of the bidirectional relations between knowledge types. Mathematics education researchers can support this instructional function by documenting different topics

contextualized in three different types of knowledge and investigating how such contexts effect students' learning and performance. The careful analysis of the hierarchical and nested relations among knowledge types and use of this analysis to inform instruction can provide different perspectives for teaching and learning.

3. With the current reform in classes educational practioners, administrators and teachers are deeply concerned with instructional methods appropriate for both low and high achieving students. The second block of the model in the present study requires a serious commitment to restructuring geometry classes that knowledge of concepts, relations, and procedures should be taught gradually to ensure success among all types of students. Distinguishing between declarative, conditional, and procedural knowledge illuminates alternative ways in which useful instructional designs for mathematics courses can be developed. The instruction might present the sequence of knowledge steps for a simplest type of knowledge (declarative knowledge), incorporate the algorithms (procedural knowledge) with links to definitions of concepts, and finally outline the relational rules (conditional knowledge) to cluster the definitions, facts, and algorithms while explaining the relationships among principles. Since the use of procedures requires students to recall the concepts relevant to the algorithms, students can be introduced definitions and facts within their solution progress. This would further enable them to build upon their prior knowledge of concepts and subsequent elaborations of procedures. Thus, it is synthesis of knowing that, knowing why, and knowing how that brings the accomplishment in mathematics. Teachers should focus on how students can do mathematics on the character of different types knowledge. Mathematics educators need to take declarative, coditional, and procedural knowledge acquisition into account when designing instructional contexts for improving relational understanding. Furthermore, they should view learning as an active process involving the interplay of metacognitive and

cognitive variables such as knowledge of cognition, regulation of cognition, declarative knowledge, conditional knowledge, and procedural knowledge.

4. Teachers can make certain changes in the context of the assessments such as shifting from classical paper-and-pencil tasks, which are generally based on the routine application of procedures to problems conceptualized in declarative, conditional, and procedural knowledge which would influence access to one or all three types of knowledge. The methods used to assess students' knowledge of mathematics are impoverished and limited that it is measured by what students perform rotely. Drawing on the questions of the knowledge test used in the present study, questions can be sequenced as follows: "What is the definition of a square?" (declarative knowledge), "If ABCD is a square then it is a rectangle. Is this statement true? Justify your answer." (conditional knowledge), "Given an ABCD square with a side length of 4 cm. Find the area of the square." (procedural knowledge). When questions are sequenced in such a manner, students would be able to capture the mathematical content as a whole.
5. In general, tests or examinations include few conditional knowledge questions that ask students to make justifications. Teachers may prepare guidelines and ask students to reason about the relations between concept definitions and theorems to acquaint students with such tasks. Furthermore, this guideline may be included metacognitive questions that activate students' metacognitive knowledge and regulation.
6. The norms for enhancing authentic geometrical tasks are not well developed in high school classrooms. One of the challenges that faces education is to develop worthwhile problems that promote students' knowledge of concepts and procedures, foster their performance in geometry, and capture students' interest. Mathematics education researchers may design open-ended question banks including tasks that

challenge students to provide meaningful concept definitions, relational explanations, and appropriate procedures in a hierarchical manner.

7. The curriculum designs and instructional methods should include activities that increase students' metacognitive knowledge and regulation processes. In the view of these activities, teachers may develop classroom discourse that emphasize metacognitive knowledge and regulation. Building an awareness among students that metacognition exists would likely affect performance. That is, promoting metacognitive knowledge and regulation would get students to become aware of their own processes. One possible implication might be introducing declarative, conditional, and procedural knowledge questions with metacognitive cues. In addition, metacognitive questions can be specified for each type of knowledge. For example; declarative knowledge questions might be introduced with questions such as "Do you have the adequate prior knowledge to state this definition?", "Do you recall the symbols relevant to the definition of this concept?"; conditional knowledge questions might be introduced with questions such as "Do you recognize the situation in which these facts and principles are related?", "Why do you use this justification that link the relations in the given statement?"; and finally, procedural knowledge questions might be introduced with questions such as "Do you think of the algorithmic sequences before attempting a solution?" and "Do you think of an easier procedure to draw a conclusion?".
8. Teachers need to be informed in the provision of support for students' metacognitive knowledge and regulation processes. Apart from the students teachers should also be introduced the effectiveness of metacognition in mathematics education. Teachers might be given instruction about what metacognition is, what role does it play in educational settings, and how its implementation contributes to success in mathematics. In the line of this instruction, they might begin to consider the implications of their tasks contextualized in declarative, conditional, and procedural knowledge from

the perspective of promoting the development of metacognitive capabilities.

9. The nonsignificant relations between metacognitive constructs and declarative and conditional knowledge remind us of the characteristics of the University Entrance Examination (ÖSS) and the placement system applying in Turkey. This exam has constraining characteristics for students that it lends students a tendency to make automized implementations without considering why their procedures work. Counsellors and teachers should work collaboratively to help students that possess commensurate abilities in geometry. Counsellors, with the help of teachers can work on students' metacognitive development. For instance, with the help of teachers, after determining the students who possess commensurate abilities in geometry courses, counsellors may guide students to use metacognitive knowledge and regulation processes specific to geometry.
10. An educational challenge may be integrating metacognitive knowledge and regulation into the mathematics study context of students. Students need to experience the fact that activities such as reflecting, planning, monitoring, and evaluating are part of the entire range of mathematical study activities. Consequently, metacognitive knowledge and regulation competencies need to be introduced into their programmes. Students should be supported to become aware that metacognitive knowledge and regulation are valuable processes for obtaining good study results. Teachers should teach students to construct knowledge about what, why and how to use learning strategies. A rich store of strategies can be used next to perform regulatory skills that enable individuals to orientate, organize, evaluate, and verify. Metacognitive knowledge and regulation can be improved through instructional practices, and that students use these new skills to improve their performance.
11. This study has put forward a model of metacognition and knowledge of geometry, that distinguishes two components of metacognition and locates

them in relation to three different types of knowledge. The classroom use of metacognition in an adapted form with teaching three different types of knowledge could stimulate students' metacognitive processes in other domains of mathematics. The promotion of metacognitive knowledge and regulation within the curriculum can start with its integration as a topic of classroom conversation. The model in this study especially provides teachers an understanding to support their conversations with students about their metacognitive processes and teaching students to monitor their thinking while solving problems.

12. Classroom environment should be transformed into a smart climate in which teachers and students reach the highest levels of awareness of cognitive, metacognitive and affective processes. There is a need to develop metacognitive instruction methods that are appropriate for each type of knowledge in specific areas of mathematics such as algebra, data analysis and probability, problem solving, and reasoning and proof. Teachers should come to terms with one another in order to attune metacognitive instructions in conjunction with teaching different types of knowledge. The issue of conditions under which each method works merits future research.

5. 4 LIMITATIONS

Although the results of this study indicated that the investigated metacognitive and knowledge factors were meaningfully related to each other, certain limitations should be kept in mind when interpreting the findings. When non-significant path cofficients from knowledge of cognition and regulation of cognition towards declarative knowledge and conditional knowledge are considered, one can conclude that these paths cannot be kept in the model. Although significant relations might have been prompted by exploring more factor loadings in Junior Metacognitive Awareness Inventory, it was preferred to sincere the Brown framework of metacognition from which the inventory was developed. On the other hand, knowledge of cognition and regulation of cognition constructs were self-reported measures of metacognition. Metacognition is a complex construct to measure with high reliability and validity. Although the measure of metacognition was fairly reliable and valid, it would have been preferable to use measures including more items to asssess the knowledge of cognition and regulation of cognition constructs to possibly strengthen them.

Another limitation of the study is that knowledge of mathematics was modeled in terms of two metacognitive constructs. There are additional attitudinal variables that have potential effect on performance in mathematics. Examples of such variables are anxiety, motivation, self-efficacy, self-concept and more. Therefore, more research is needed in order to explore the effects that these factors exert on different types of knowledge. In later studies the affects of such variables that exert on different types of knowledge also needed to be investigated next to metacognitive factors.

Geometry Knowledge Test does not include metacognitive hints. A better measure would be an instrument composed of geometry questions sequenced by metacognitive questions specific to the items in the Junior Metacognitive Awareness Inventory. Such metacognitive hints might yield significant relations between metacognitive and knowledge factors.

In Geometry Knowledge Test part of the score on each question is based on whether or not the student understood the problem. The subscores are related to the approach taken to solve a declarative, conditional and procedural question, in partial. Another means of scoring the problems such as marking right-wrong may result in different findings.

Another limitation of this research is that the findings are based on a single sample of tenth grade students. Besides, all statistical analyses were conducted by using the same data. Hence, overusing the data is another limitation issue for concern.

Despite these limitations, the fact that most of the hypothesized relationships were statistically significant and substantial in size supports the robustness of the structural model related to metacognitive and cognitive factors. There are some points of strength that lend further credibility to the results of the present study, such as the use of structural equation modeling and the specification of direct and indirect effects of the factors. Applications of LISREL are fairly recent in educational research and studies using the structural equation modeling technique are still developing. Such advanced statistical techniques that employ structural models are more robust and reflect the complexity of the relationships among various constructs by hypothesizing the direct effects.

5. 5 RECOMMENDATIONS FOR FUTURE RESEARCH

The Metacognition-Knowledge Model in this study has the potential to inform more detailed research into relationship between components of metacognition and knowledge of mathematics.

1. Future research requires cross-validation and replication with any modeling approach. Further examination of metacognitive and cognitive factors with different measures, multiple-achievement measures and larger samples of students at different grade levels is likely to provide better understanding of the role of the metacognitive factors on different types of knowledge. Better understanding of the interconnection among declarative knowledge, conditional knowledge, and procedural knowledge would lead to instructional and curricular changes and counseling interventions that would support geometry learning. Therefore, the potential of designing instructions that include the acquisition of declarative knowledge, conditional knowledge and procedural knowledge to enhance other learning outcomes merits future research.

2. More research is needed that will enhance the understanding on metacognition, components of metacognition and its affects on learning performance. Besides, if other attitudinal, motivational, and affective factors can be included in the model, percentage of variance in performormance in different types of knowledge explained by all of the latent variables increases.
3. Further research may explore the utilisation of metacognitive knowledge and regulation with the scores on other areas such as number and operations, algebra, measurement, data analysis and probability, problem solving, and reasoning and proof to explore whether the effect of metacognitive factors differ in explaining different types of knowledge in the content level.
4. The present study modeled tenth grade students metacognitive and cognitive processes. Model tests across gender would also be beneficial. These efforts would provide further insight into the process of metacognitive and cognitive development in relation to eachother.
5. Teachers may believe that metacognitive knowledge and regulation processes are valuable and they may actively reinforce these processes as an important part of the curriculum, but the result is not necessarily that students learn to be metacognitively active and further integrate these actions while they progress on different types of knowledge. More detailed research needs to be undertaken into the extent to which students act in accordance with metacognitive and cognitive processes in a way that match their teachers' instructional intentions. The focus of this study was on the relationships among students' metacognitive knowledge and regulation and knowledge of geometry. Another study could include the relationships among teachers' metacognitive knowledge and regulation and subject-matter knowledge. This concerns the arguments that teachers often lack metacognitive knowledge and regulation and that students copy their teachers' matching metacognitive behaviors. Therefore, another question

that requires further investigation is how to improve teachers' perceptions, use, and promotion of metacognition.

6. Future studies of this model and others would benefit from the use of alternative measures of the metacognitive constructs and investigation of other geometry topics.
7. It is evident that future research must continue to examine the relationships among components of metacognition and knowledge measures. An interesting research issue may be the investigation of to what extent training metacognitive knowledge and regulation processes, directed at bridging the application of nested relations among knowledge types could enhance activation of these processes.

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APPENDIX A

DESCRIPTIVE STATISTICS OF THE ITEMS OF JR. MAI

Item	Min	Max	Mean	SD	Skewness	Kurtosis
MET1	1	5	4.4074	0.8536	-1.677	2.980
MET2	1	5	3.8451	0.9530	-0.581	-0.116
MET3	1	5	3.9596	1.4698	5.931	76.074
MET4	1	5	3.6229	1.0295	-0.371	-0.494
MET5	1	5	4.4512	0.9218	-1.796	2.790
MET6	1	5	2.9461	1.2040	-0.025	-0.866
MET7	1	5	2.9024	1.3230	-0.031	-1.164
MET8	1	5	3.4209	1.1003	-0.295	-0.633
MET9	1	5	3.7811	1.0761	-0.734	-0.073
MET10	1	5	3.3434	1.1285	-0.394	-0.510
MET11	1	5	4.3872	0.8270	-1.403	1.831
MET12	1	5	4.6397	0.7632	-2.545	6.844
MET13	1	5	3.6801	0.9419	-0.542	0.150
MET14	1	5	3.6296	1.0800	-0.533	-0.213
MET15	1	5	3.4714	1.0843	-0.270	-0.591
MET16	1	5	3.0101	1.0050	-0.241	-0.105
MET17	1	5	3.2929	1.2102	-0.337	-0.801
MET18	1	5	4.0000	1.0033	1.010	0.759

APPENDIX B

DESCRIPTIVE STATISTICS OF THE QUESTIONS OF GKT

Item	Min	Max	Mean	SD	Skewness	Kurtosis
QUES1	0	4	3.656	0.633	-2.288	7.175
QUES2	0	4	3.616	0.900	-2.124	3.062
QUES3	0	4	1.899	1.175	0.876	-0.640
QUES4	0	4	2.697	1.556	-0.704	-1.090
QUES5	0	4	2.599	1.432	-0.514	-1.096
QUES6	0	4	1.787	0.932	0.535	0.452
QUES7	0	4	1.949	1.115	0.556	-0.275
QUES8	0	4	2.633	1.060	-0.387	-0.821
QUES9	0	4	3.629	0.946	-2.591	5.627
QUES10	0	4	1.986	1.117	0.509	-0.892
QUES11	0	4	3.259	1.326	-1.473	0.599
QUES12	0	4	2.289	1.344	0.106	-1.464
QUES13	0	4	3.437	1.192	-1.812	1.599
QUES14	0	4	3.505	1.087	-2.011	2.652
QUES15	0	4	2.437	1.272	-0.053	-1.285
QUES16	0	4	2.259	1.261	0.040	-1.625
QUES17	0	4	2.612	1.480	-0.464	-1.675
QUES18	0	4	2.292	1.508	-0.038	-0.526
QUES19	0	4	2.387	1.744	-0.331	-1.771
QUES20	0	4	1.626	1.212	0.600	-0.775
QUES21	0	4	2.171	1.757	-0.110	-1.771
QUES22	0	4	1.582	1.365	0.674	.0.775
QUES23	0	4	2.619	1.670	-0.643	-1.319
QUES24	0	4	2.175	1.750	.0.129	-1.747

APPENDIX C

THE FREQUENCY DISTRIBUTIONS OF THE OBSERVED VARIABLES OF JR. MAI

	Alternatives	Frequency	Percent
MET1	Never	4	1.3
	Seldom	8	4.0
	Sometimes	24	12.1
	Often	88	41.8
	Always	173	100
	Total	297	100
	Missing	0	0
	Total	297	100
MET2	Never	4	1.3
	Seldom	22	7.4
	Sometimes	71	23.9
	Often	119	40.1
	Always	81	27.3
	Total	297	100
	Missing	0	0
	Total	297	100
MET5	Never	5	1.7
	Seldom	10	3.4
	Sometimes	29	9.8
	Often	55	18.5
	Always	198	66.7
	Total	297	100
	Missing	0	0
	Total	297	100
MET7	Never	61	20.5
	Seldom	55	18.5
	Sometimes	70	23.6
	Often	74	24.9
	Always	37	12.5
	Total	297	100
	Missing	0	0
	Total	297	100
MET8	Never	14	4.7
	Seldom	48	16.2
	Sometimes	87	29.3
	Often	95	32
	Always	53	17.8
	Total	297	100
	Missing	0	0
	Total	297	100

	Alternatives	Frequency	Percent
MET9	Never	11	3.7
	Seldom	28	9.4
	Sometimes	61	20.5
	Often	112	37.7
	Always	85	28.6
	Total	297	100
	Missing	0	0
	Total	297	100
MET10	Never	23	7.7
	Seldom	41	13.8
	Sometimes	89	30
	Often	99	33.3
	Always	45	15.2
	Total	297	100
	Missing	0	0
	Total	297	100
MET11	Never	2	0.7
	Seldom	8	2.7
	Sometimes	30	10.1
	Often	90	30.3
	Always	167	56.2
	Total	297	100
	Missing	0	0
	Total	297	100
MET12	Never	3	1
	Seldom	7	2.4
	Sometimes	13	4.4
	Often	48	16.2
	Always	226	76.1
	Total	297	100
	Missing	0	0
	Total	297	100
MET14	Never	14	4.7
	Seldom	25	8.4
	Sometimes	89	30
	Often	98	33
	Always	71	23.9
	Total	297	100
	Missing	0	0
	Total	297	100
MET15	Never	12	4
	Seldom	42	14.1
	Sometimes	96	32.3
	Often	88	29.6
	Always	59	19.9
	Total	297	100
	Missing	0	0
	Total	297	100

APPENDIX D

THE FREQUENCY DISTRIBUTIONS OF THE OBSERVED VARIABLES OF GKT

	Alternatives	Frequency	Percent
QUES1	0	2	0.7
	1	0	0
	2	14	4.7
	3	66	22.2
	4	215	72.4
	Total	297	100
	Missing	0	0
	Total	297	100
QUES3	0	5	1.7
	1	147	49.5
	2	77	25.9
	3	9	3.0
	4	59	19.9
	Total	297	100
	Missing	0	0
	Total	297	100
QUES4	0	50	16.8
	1	25	8.4
	2	42	14.1
	3	28	9.4
	4	152	51.2
	Total	297	100
	Missing	0	0
	Total	297	100
QUES5	0	36	12.1
	1	35	11.8
	2	66	22.2
	3	35	11.8
	4	125	42.1
	Total	297	100
	Missing	0	0
	Total	297	100
QUES8	0	4	1.3
	1	52	17.5
	2	61	20.5
	3	112	37.7
	4	68	22.9
	Total	297	100
	Missing	0	0
	Total	297	100

	Alternatives	Frequency	Percent
QUES12	0	15	5.1
	1	100	33.7
	2	58	19.5
	3	32	10.8
	4	92	31
	Total	297	100
	Missing	0	0
	Total	297	100
QUES13	0	10	3.4
	1	36	12.1
	2	6	2
	3	7	2.4
	4	238	80.1
	Total	297	100
	Missing	0	0
	Total	297	100
QUES15	0	13	4.4
	1	69	23.2
	2	85	28.6
	3	35	11.8
	4	95	32
	Total	297	100
	Missing	0	0
	Total	297	100
QUES19	0	77	25.9
	1	35	11.8
	2	30	10.1
	3	6	2
	4	149	50.2
	Total	297	100
	Missing	0	0
	Total	297	100
QUES21	0	88	29.6
	1	43	14.5
	2	25	8.4
	3	12	4
	4	129	43.4
	Total	297	100
	Missing	0	0
	Total	297	100
QUES23	0	67	22.6
	1	20	6.7
	2	29	9.8
	3	24	8.1
	4	157	52.9
	Total	297	100
	Missing	0	0
	Total	297	100

	Alternatives	Frequency	Percent
QUES24	0	90	30.3
	1	35	11.8
	2	32	10.8
	3	13	4.4
	4	127	42.8
	Total	297	100
	Missing	0	0
	Total	297	100

APPENDIX E

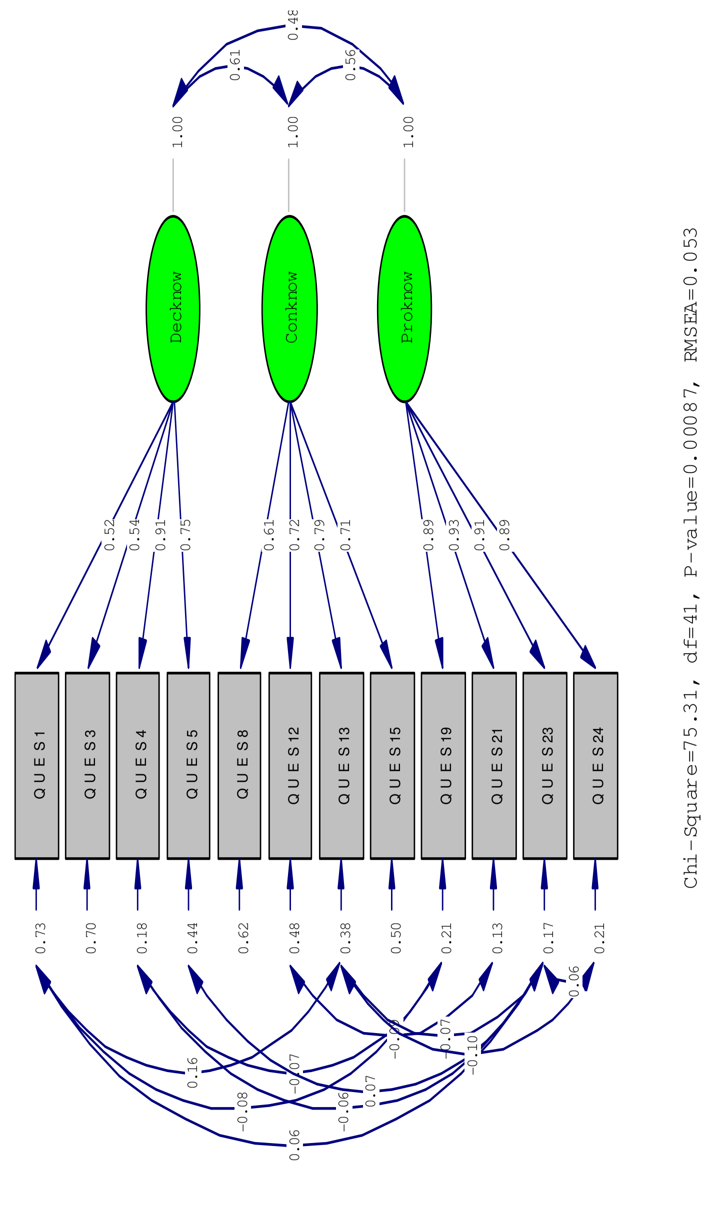
THE SIMPLIS SYNTAX FOR THE GKT MODEL

```
Real Data Set-Geometry Achievement
Observed Variables
QUES1 QUES2 QUES3 QUES4 QUES5 QUES6 QUES7 QUES8 QUES9
QUES10 QUES11 QUES12 QUES13 QUES14 QUES15 QUES16 QUES17 QUES18
QUES19 QUES20 QUES21 QUES22 QUES23 QUES24
Covariance matrix from File: geoc.cov
Sample Size = 297
Latent Variables
Decknow Conknow Proknow
Relationships

QUES1 QUES3 QUES4 QUES5 = Decknow
QUES8 QUES12 QUES13 QUES15 = Conknow
QUES19 QUES21 QUES23 QUES24 = Proknow

Set Error Covariance Between QUES21 and QUES12 Free
Set Error Covariance Between QUES23 and QUES4 Free
Set Error Covariance Between QUES13 and QUES1 Free
Set Error Covariance Between QUES24 and QUES23 Free
Set Error Covariance Between QUES19 and QUES1 Free
Set Error Covariance Between QUES19 and QUES4 Free
Set Error Covariance Between QUES23 and QUES5 Free
Set Error Covariance Between QUES23 and QUES1 Free
Set Error Covariance Between QUES24 and QUES13 Free
Set Error Covariance Between QUES23 and QUES13 Free

Path Diagram
Print Residuals
Admissibility Check = 1000
Iterations = 5000
Method of Estimation: Maximum Likelihood
Lisrel Output: EF
End of problem
```

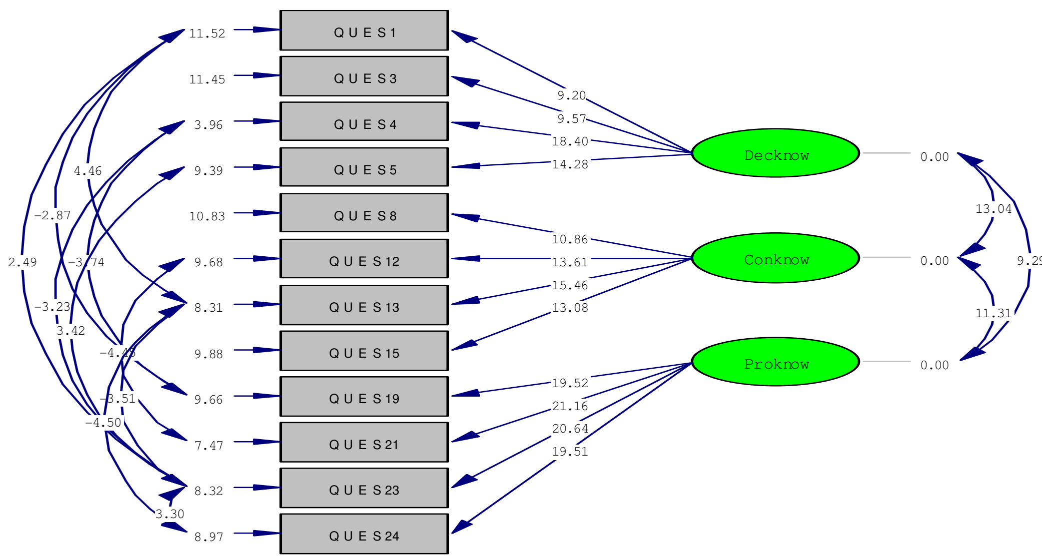
APPENDIX F

LISREL ESTIMATES OF PARAMETERS IN GKT MODEL
(COEFFICIENTS IN STANDARDIZED VALUE AND t-VALUES)

COEFFICIENTS IN STANDARDIZED VALUE



COEFFICIENTS IN t-VALUES



Chi-Square=75.31, df=41, P-value=0.00087, RMSEA=0.053

APPENDIX G

SUMMARY STATISTICS FOR RESIDUALS AND STEMLEAF PLOTS OF THE GKT MODEL

- Summary Statistics for Fitted Residuals

Smallest Fitted Residual = -0.10
Median Fitted Residual = 0.01
Largest Fitted Residual = 0.14

Stemleaf Plot

```
- 8|7
- 6|918554322
- 4|492
- 2|9676
- 0|929988742210000
  0|12345568899900001255577888
  2|2634456
  4|2
  6|264569
  8|67
 10|0
 12|3
 14|0
```

-Summary Statistics for Standardized Residuals

Smallest Standardized Residual = -3.32
Median Standardized Residual = 0.25
Largest Standardized Residual = 3.44

Stemleaf Plot

```
- 3|3
- 2|5541
- 1|887777664440
- 0|776554322100000
  0|111222333333556677899
  1|0001133355789
  2|0000111349
  3|4
```

Largest Negative Standardized Residuals
Residual for QUES24 and QUES13 -3.32

Largest Positive Standardized Residuals
Residual for QUES8 and QUES5 3.44
Residual for QUES21 and QUES3 2.93

APPENDIX H

GOODNESS-OF-FIT CRITERIA FOR THE GKT MODEL

Fit Index	***Criterion***	***Value***
Chi-Square (χ^2)	Non-significant	75.31 (p= 0.00)
Normed Chi-Square (NC)	NC< 5	1.83
Goodness of Fit Index (GFI)	GFI> 0.90	0.96
Adjusted Goodness of Fit Index (AGFI)	AGFI> 0.90	0.92
Root Mean Square Error of Approximation (RMSEA)	0.05 < RMSEA < 0.08 (moderate fit) RMSEA < 0.05 (good fit)	0.053
Root Mean Square Residual (RMR)	RMR < 0.05	0.046
Root Mean Square Residual (S-RMR)	S-RMR < 0.05	0.046
Parsimony Goodness of Fit Index (PGFI)	Higher values	0.50
Parsimony Normed Fit Index (PNFI)	Higher values	0.60
Normed Fit Index (NFI)	NFI> 0.90	0.97
Non-Normed Fit Index (NNFI)	NNFI> 0.90	0.97
Comparative Fit Index (CFI)	CFI> 0.90	0.98
Incremental Fit Index (IFI)	IFI> 0.90	0.98
Relative Fit Index (RFI)	RFI> 0.90	0.94

APPENDIX I

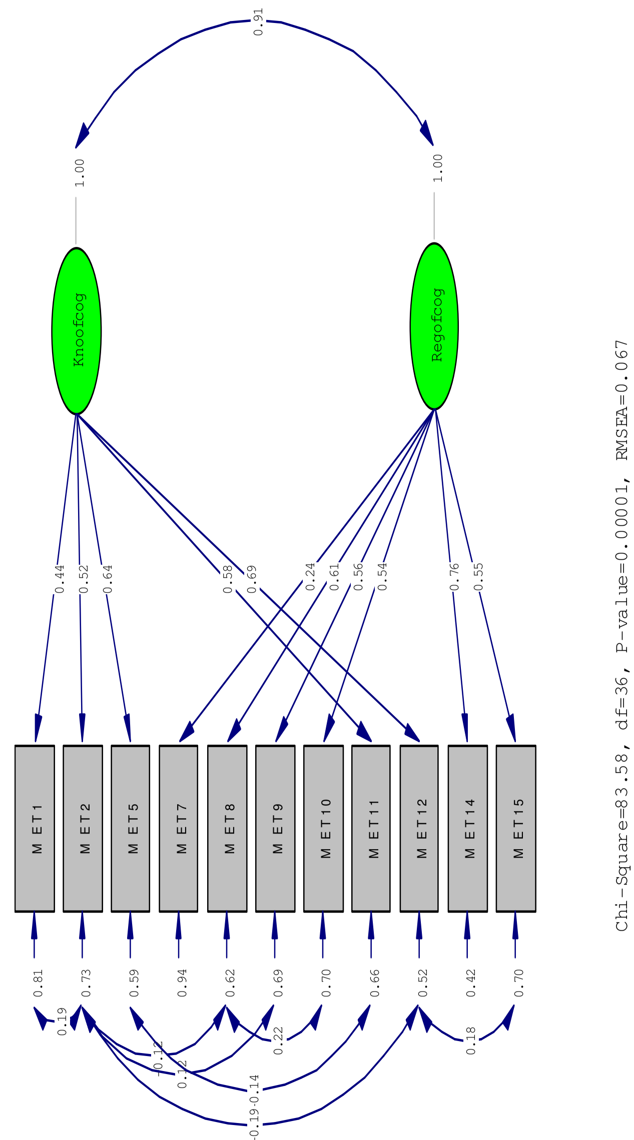
THE SIMPLIS SYNTAX FOR THE JR. MAI MODEL

```
Real Data Set-Geometry Achievement
Observed Variables
MET1 MET2 MET3 MET4 MET5 MET6 MET7 MET8 MET9
MET10 MET11 MET12 MET13 MET14 MET15
MET16 MET17 MET18
Covariance matrix from File: metc.cov
Sample Size = 297
Latent Variables
Knoofcog Regofcog
Relationships

MET1 MET2 MET5 MET11 MET12 = Knoofcog
MET7 MET8 MET9 MET10 MET14 MET15 = Regofcog

Set Error covariance Between MET10 and MET8 Free
Set Error covariance Between MET2 and MET1 Free
Set Error covariance Between MET12 and MET2 Free
Set Error covariance Between MET15 and MET12 Free
Set Error covariance Between MET8 and MET2 Free
Set Error covariance Between MET11 and MET5 Free
Set Error covariance Between MET9 and MET2 Free

Path Diagram
Print Residuals
Admissibility Check = 1000
Iterations = 5000
Method of Estimation: Maximum Likelihood
Lisrel Output: EF
End of problem
```

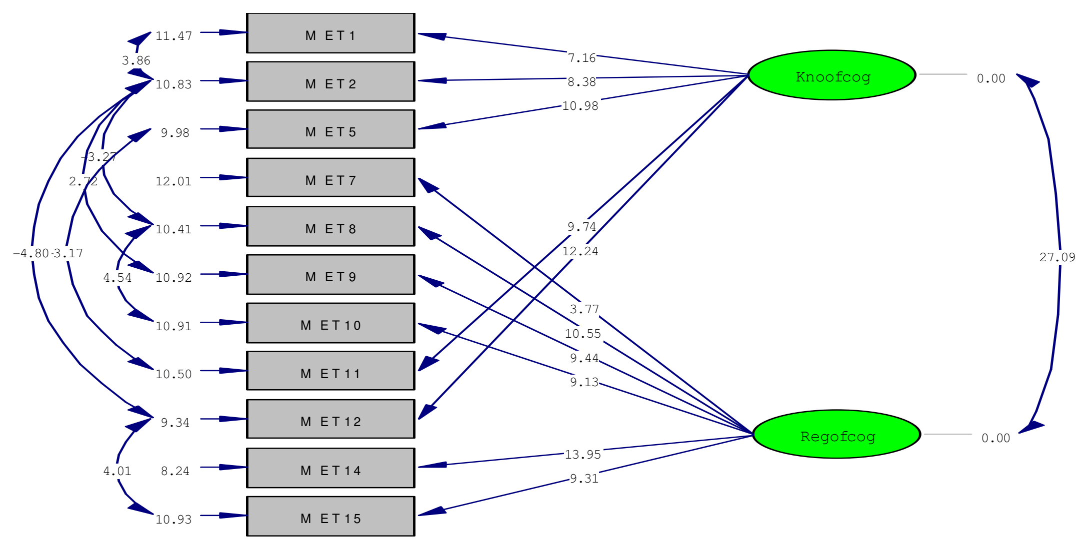
APPENDIX J

LISREL ESTIMATES OF PARAMETERS IN JR. MAI MODEL
(COEFFICIENTS IN STANDARDIZED VALUE AND t- VALUES)

COEFFICIENTS IN STANDARDIZED VALUE



COEFFICIENTS IN t-VALUES



Chi-Square=83.58, df=36, P-value=0.00001, RMSEA=0.067

APPENDIX K

SUMMARY STATISTICS FOR RESIDUALS AND STEMLEAF PLOTS OF THE JR. MAI MODEL

- Summary Statistics for Fitted Residuals

Smallest Fitted Residual = -0.09
Median Fitted Residual = 0.00
Largest Fitted Residual = 0.10

Stemleaf Plot

```
- 8|430063
- 6|83
- 4|86095400
- 2|64964
- 0|2186443200000000
  0|224456299
  2|9996
  4|347911378
  6|66822
  8|56
```

-Summary Statistics for Standardized Residuals

Smallest Standardized Residual = -2.65
Median Standardized Residual = 0.00
Largest Standardized Residual = 3.14

Stemleaf Plot

```
- 2|65522111
- 1|97666643310
- 0|988765322200000000
  0|1122344899
  1|001123334789
  2|022357
  3|1
```

Largest Negative Standardized Residuals
Residual for MET10 and MET9 -2.65

Largest Positive Standardized Residuals
Residual for MET14 and MET12 3.14
Residual for MET15 and MET10 2.68

APPENDIX L

GOODNESS-OF-FIT CRITERIA FOR THE JR. MAI MODEL

Fit Index	***Criterion***	***Value***
Chi-Square (χ^2)	Non-significant	83.58 (p= 0.00)
Normed Chi-Square (NC)	NC< 5	2.32
Goodness of Fit Index (GFI)	GFI> 0.90	0.95
Adjusted Goodness of Fit Index (AGFI)	AGFI> 0.90	0.91
Root Mean Square Error of Approximation (RMSEA)	0.05 < RMSEA < 0.08 (moderate fit) RMSEA < 0.05 (good fit)	0.067
Root Mean Square Residual (RMR)	RMR < 0.05	0.048
Root Mean Square Residual (S-RMR)	S-RMR < 0.05	0.048
Parsimony Goodness of Fit Index (PGFI)	Higher values	0.52
Parsimony Normed Fit Index (PNFI)	Higher values	0.59
Normed Fit Index (NFI)	NFI> 0.90	0.91
Non-Normed Fit Index (NNFI)	NNFI> 0.90	0.91
Comparative Fit Index (CFI)	CFI> 0.90	0.94
Incremental Fit Index (IFI)	IFI> 0.90	0.94
Relative Fit Index (RFI)	RFI> 0.90	0.86

APPENDIX M

THE SIMPLIS SYNTAX FOR THE METACOGNITION-KNOWLEDGE MODEL

```
Real Data Set-Geometry Achievement
Observed Variables
QUES1 QUES2 QUES3 QUES4 QUES5 QUES6 QUES7 QUES8 QUES9 QUES10
QUES11 QUES12 QUES13 QUES14 QUES15 QUES16 QUES17 QUES18 QUES19
QUES20 QUES21 QUES22 QUES23 QUES24 MET1 MET2 MET3 MET4 MET5 MET6
MET7 MET8 MET9 MET10 MET11 MET12 MET13 MET14 MET15 MET16 MET17
MET18
Covariance matrix from File: geoknw.cov
Sample Size = 297
Latent Variables
Decknow Conknow Proknow Knoofcog Regofcog
Relationships

QUES1 QUES3 QUES4 QUES5 = Decknow
QUES8 QUES12 QUES13 QUES15 = Conknow
QUES19 QUES21 QUES23 QUES24 = Proknow
MET1 MET2 MET5 MET11 MET12 = Knoofcog
MET7 MET8 MET9 MET10 MET14 MET15 = Regofcog

Decknow = Knoofcog Regofcog
Conknow = Decknow Proknow Knoofcog Regofcog
Proknow = Decknow Knoofcog Regofcog

Set Error Covariance Between MET14 and QUES4 Free
Set Error Covariance Between QUES21 and QUES12 Free
Set Error Covariance Between QUES13 and QUES1 Free
Set Error Covariance Between MET1 and QUES4 Free
Set Error Covariance Between QUES24 and QUES4 Free
Set Error Covariance Between QUES24 and QUES23 Free
Set Error Covariance Between MET8 and QUES23 Free
Set Error Covariance Between MET12 and QUES21 Free
Set Error Covariance Between QUES23 and QUES5 Free
Set Error Covariance Between QUES23 and QUES1 Free
Set Error Covariance Between MET2 and QUES23 Free
Set Error Covariance Between MET14 and QUES23 Free
Set Error Covariance Between MET12 and MET10 Free
Set Error Covariance Between MET12 and QUES1 Free
Set Error Covariance Between MET10 and MET1 Free
Set Error Covariance Between MET5 and QUES13 Free
Set Error Covariance Between MET1 and QUES12 Free
Set Error Covariance Between MET1 and QUES5 Free
```

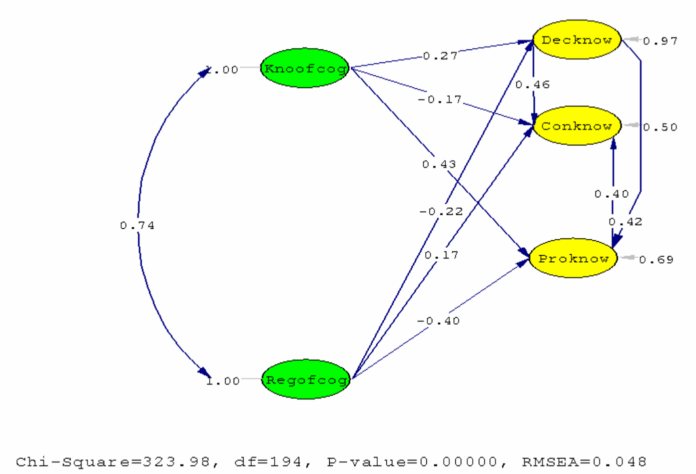
Set Error Covariance Between MET7 and QUES3 Free
Set Error Covariance Between QUES24 and QUES13 Free
Set Error Covariance Between QUES23 and QUES13 Free
Set Error Covariance Between MET12 and MET5 Free
Set Error Covariance Between MET12 and MET11 Free
Set Error Covariance Between MET11 and MET7 Free
Set Error Covariance Between MET11 and QUES1 Free
Set Error Covariance Between MET9 and MET5 Free

Path Diagram
Print Residuals
Admissibility Check = 1000
Iterations = 5000
Method of Estimation: Maximum Likelihood
Lisrel Output: EF
End of problem

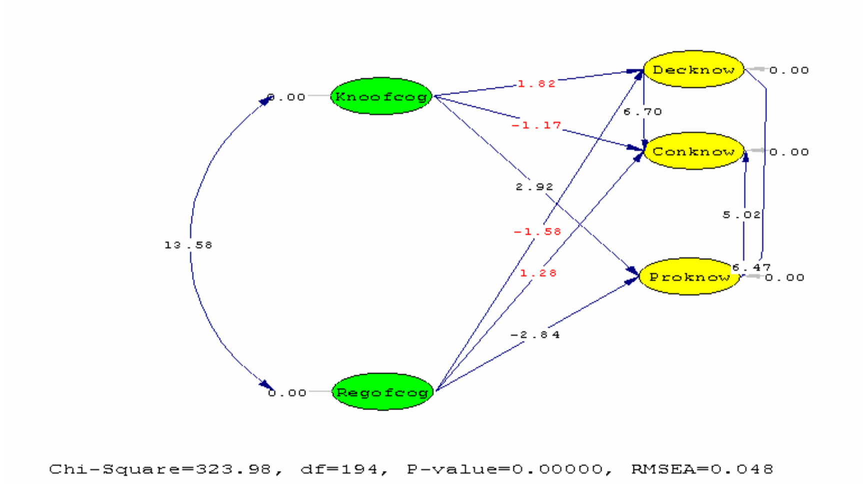
APPENDIX N

LISREL ESTIMATES OF PARAMETERS IN THE METACOGNITION-KNOWLEDGE MODEL (COEFFICIENTS IN STANDARDIZED VALUE AND t-VALUES)

COEFFICIENTS IN STANDARDIZED VALUE



COEFFICIENTS IN t-VALUES



APPENDIX O

SUMMARY STATISTICS FOR RESIDUALS AND STEMLEAF PLOTS OF THE METACOGNITION-KNOWLEDGE MODEL

- Summary Statistics for Fitted Residuals

```
Smallest Fitted Residual =   -0.16
Median Fitted Residual =    0.00
Largest Fitted Residual =    0.21
```

Stemleaf Plot

```
-16|2
-14|1860
-12|1653
-10|438543
- 8|9810876622210000
- 6|665554377654221100
- 4|66631087665300000
- 2|9988766432210888653321110
- 0|8766665433311110098777666555544333222221111110000000
  0|1223334455888899000111222555 6788
  2|011334566779990013344566666 7888
  4|00111222223346777790267778 89
  6|024779902366789
  8|068912346779
 10|0238119
 12|0579
 14|7
 16|1
 18|6844
 20|7
```

-Summary Statistics for Standardized Residuals

```
Smallest Standardized Residual =   -4.38
Median Standardized Residual =    0.00
Largest Standardized Residual =    4.38
```

Stemleaf Plot

- 4|40
- 3|886
- 3|4441111000
- 2|98877776665
- 2|4443333322000
- 1|8877666655555555
- 1|443333111000
- 0|99999888877777777666665555555
- 0|444444443333333222222222211111111111100000000000000
0|1111112222222233334444444
0|5556666666777777777777788888899999999
1|0011112222223333344
1|555556666677888888899999
2|000001111344
2|56677799
3|4
3|556668
4|024

Largest Negative Standardized Residuals				Largest Positive Standardized Residuals			
Residual for	QUES4 and	QUES1	-3.76	Residual for	QUES8 and	QUES5	2.91
Residual for	QUES4 and	QUES4	-3.78	Residual for	QUES21 and	QUES3	2.69
Residual for	QUES5 and	QUES1	-2.82	Residual for	MET1 and	QUES1	3.45
Residual for	QUES5 and	QUES3	-2.60	Residual for	MET1 and	QUES3	3.64
Residual for	QUES5 and	QUES4	-2.97	Residual for	MET1 and	QUES4	2.73
Residual for	QUES5 and	QUES5	-2.99	Residual for	MET1 and	QUES5	2.89
Residual for	QUES12 and	QUES8	-3.13	Residual for	MET1 and	QUES19	3.60
Residual for	QUES13 and	QUES4	-3.13	Residual for	MET1 and	QUES21	4.38
Residual for	QUES15 and	QUES1	-2.70	Residual for	MET1 and	QUES23	4.21
Residual for	QUES15 and	QUES4	-2.71	Residual for	MET1 and	QUES24	3.99
Residual for	QUES19 and	QUES1	-3.60	Residual for	MET1 and	MET1	3.77
Residual for	QUES19 and	QUES4	-4.05	Residual for	MET2 and	MET1	3.49
Residual for	QUES21 and	QUES13	-2.87	Residual for	MET10 and	MET10	3.53
Residual for	QUES23 and	QUES1	-2.59	Residual for	MET12 and	MET1	3.64
Residual for	QUES23 and	QUES4	-3.45	Residual for	MET14 and	MET5	2.60
Residual for	QUES24 and	QUES4	-3.12	Residual for	MET14 and	MET11	2.68
Residual for	QUES24 and	QUES13	-3.43				
Residual for	MET7 and	MET5	-3.37				
Residual for	MET7 and	MET7	-2.79				
Residual for	MET10 and	MET1	-3.07				
Residual for	MET11 and	QUES21	-4.38				
Residual for	MET12 and	QUES19	-2.69				
Residual for	MET12 and	QUES21	-3.03				
Residual for	MET12 and	MET7	-2.72				

APPENDIX P

GOODNESS-OF-FIT CRITERIA FOR THE METACOGNITION-KNOWLEDGE MODEL

Fit Index	Criterion	Value
Chi-Square (χ^2)	Non-significant	323.98 (p= 0.00)
Degrees of Freedom(df)		194
Normed Chi-Square (NC)	NC< 5	1.67
Goodness of Fit Index (GFI)	GFI> 0.90	0.91
Adjusted Goodness of Fit Index (AGFI)	AGFI> 0.90	0.88
Root Mean Square Error of Approximation (RMSEA)	0.05 < RMSEA < 0.08 (moderate fit) RMSEA < 0.05 (good fit)	0.048
Root Mean Square Residual (RMR)	RMR < 0.05	0.065
Standardized Root Mean Square Residual (S-RMR)	S-RMR < 0.05	0.065
Parsimony Goodness of Fit Index (PGFI)	Higher values	0.64
Parsimony Normed Fit Index (PNFI)	Higher values	0.69
Normed Fit Index (NFI)	NFI> 0.90	0.90
Non-Normed Fit Index (NNFI)	NNFI> 0.90	0.94
Comparative Fit Index (CFI)	CFI> 0.90	0.95
Incremental Fit Index (IFI)	IFI> 0.90	0.95
Relative Fit Index (RFI)	RFI> 0.90	0.87

APPENDIX Q

JUNIOR METACOGNITIVE AWARENESS INVENTORY

BİLİŞÜSTÜ YETİLER ANKETİ

Ad Soyad :
Sınıf :
Cinsiyet:
Geçen Yıl Aldığı Matematik Notu:

Aşağıdaki tabloda size uygun olan kutucuğun içine çarpı (X) işareti koyunuz.

	İlkokul	Ortaokul	Lise	Üniversite
Annenizin öğrenim durumu:				
Babanızın öğrenim durumu:				

Bu çalışmanın amacı sizin nasıl öğrendiğiniz ve çalıştığınız hakkında bilgi edinmektir. Doğru veya yanlış cevap yoktur. Cevaplar kendi görüşlerinizi yansıtmalıdır. Her cümleyle ilgili görüş belirtirken önce cümleyi dikkatle okuyunuz, sonra cümlede belirtilen durumun size ne derecede uygun olduğuna karar veriniz. Lütfen size en uygun olan yuvarlağın içini doldurunuz.
Teşekkürler! ☺

	Hiçbir Zaman	Nadiren	Bazen	Sık sık	Her Zaman
1. Bir şeyi anladığımı bilirim.	O	O	O	O	O
2. Gerektiğinde, öğrenmek için kendimi motive edebilirim.	O	O	O	O	O
3. Daha önce, benim için işe yaramış çalışma yollarını kullanmayı denerim.	O	O	O	O	O
4. Öğretmenin benden ne öğrenmemi beklediğini bilirim.	O	O	O	O	O
5. Konu hakkında daha önceden bilgim varsa daha iyi öğrenirim.	O	O	O	O	O
6. Öğrenirken anlamama yardımcı olacak resimler veya şemalar çizerim.	O	O	O	O	O
7. Çalışmamı bitirdiğimde kendime "Öğrenmek istediğim şeyi öğrendim mi?" diye sorarım.	O	O	O	O	O
8. Bir problemi çözmek için çeşitli çözüm yollarını denerim ve daha sonra en uygun olanını seçerim.	O	O	O	O	O
9. Çalışmaya başlamadan önce neyi öğrenmem gerektiğini düşünürüm.	O	O	O	O	O
10. Yeni bir şey öğrenirken kendime iyi gidip gitmediğime dair sorular sorarım.	O	O	O	O	O
11. Önemli bilgiye gerçekten dikkat ederim.	O	O	O	O	O
12. Konuya ilgim varsa daha çok öğrenirim.	O	O	O	O	O
13. Zihinsel açıdan güçlü olduğum noktaları, zayıf olan noktalarımı telafi etmede kullanırım.	O	O	O	O	O
14. Verilen işe bağlı olarak farklı öğrenme stratejileri* kullanırım.	O	O	O	O	O
15. Çalışmamı zamanında bitireceğimden emin olmak için ara sıra kontrol ederim.	O	O	O	O	O
16. Bazen öğrenme stratejilerini* düşünmeden kullanırım.	O	O	O	O	O
17. Bir işi bitirdikten sonra kendime "Daha kolay bir yol var mıydı?" diye sorarım.	O	O	O	O	O
18. Bir işe başlamadan önce neyi tamamlamam gerektiğine karar veririm.	O	O	O	O	O

*Öğrenme stratejileri, bir işi başarıyla tamamlamak için kullandığımız yöntemlerdir. Bu stratejiler daha iyi öğrenmemize yardımcı olur. Örneğin:

- ✓ Bir problemi okuduktan sonra bilinenleri ve bilinmeyenleri belirlemek.
- ✓ Kafamız karıştığında verilen problemi tekrar okumak ve verilenler üzerinde düşünmek.
- ✓ Bir problemi çözmek için çeşitli yaklaşımlar kullanmak.

(Daha önce öğrendiklerini düşünmek - Amacını belirlemek - Çözüm basamaklarını planlamak)

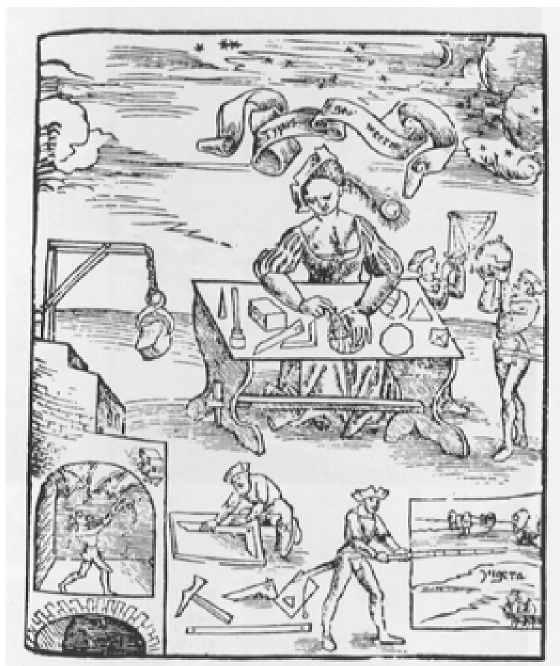
- ✓ Önemli bir bilgiyle karşılaştığımızda altını çizmek.
- ✓ Çalışırken küçük notlar almak.
- ✓ Eski bilgilerimizle yeni bilgilerimizi birleştirmek.
- ✓ Daha önce çözdüğümüz benzer örnekleri hatırlamaya çalışmak.

APPENDIX R

GEOMETRY KNOWLEDGE TEST

GEOMETRİ BİLGİ TESTİ

Ad Soyad:
Sınıf:
Geçen Dönem Aldığı Geometri Notu:



Geometri (Yunanca *γεωμετρία;geo = dünya, metria = ölçme)*

Sevgili Öğrenci;
Bu test üçgenler ünitesi ile ilgili 24 sorudan oluşmaktadır. Bazı sorular bir yada birkaç alt soru içermektedir. Bazılarında ise açıklama yapmanız istenmektedir. Sorulardaki alt sorulara verilecek cevaplara ve yapacağınız açıklamalara karşılık gelen puan değerleri bulunmaktadır. Lütfen tüm soruları cevaplamaya çalışınız.
Sınav süresi 45 dakikadır.
Başarılar…☺

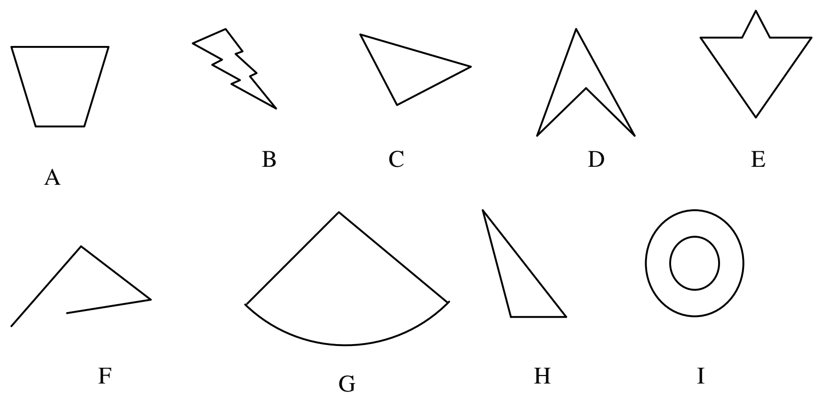
1) Aşağıda verilen üçgen çeşitlerini tanımlayınız.

A) Eşkenar üçgen:

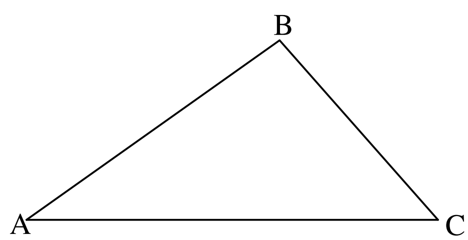
B) Dik üçgen:

C) İkizkenar üçgen:

2) Aşağıdakilerden hangisi yada hangileri üçgendir? İşaretleyiniz.



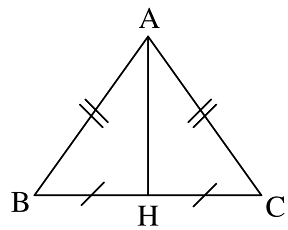
3) Aşağıda verilen ABC üçgenin kenarlarını ve iç açılarını sembolle yazarak gösteriniz.



4) "Üçgenlerin eşliği" tanımını yapınız.

5) "Üçgenlerde benzerlik" tanımını yapınız.

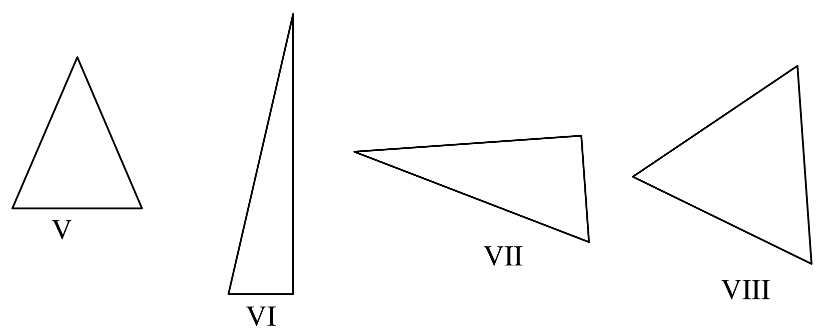
6)



Yukarıda verilen ABC üçgeninde;

A) Birbirine eş olan üçgenleri sembolle yazarak gösteriniz.
B) Birbirine benzer olan üçgenleri sembolle yazarak gösteriniz

7)

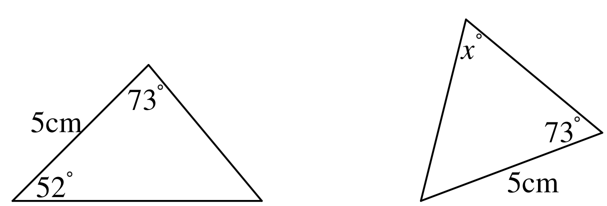


A) Yukarıda verilen üçgenlerden hangileri eştir? Numaralarını yazınız.

B) Yukarıda verilen üçgenlerden hangileri benzerdir? Numaralarını yazınız.

8) "Eğer ABC üçgeninin üç kenarı eşit uzunlukta ise A, B ve C açıları birbirine eşittir."
ifadesi doğru mudur? Cevabınızı nedenleriyle açıklayınız.

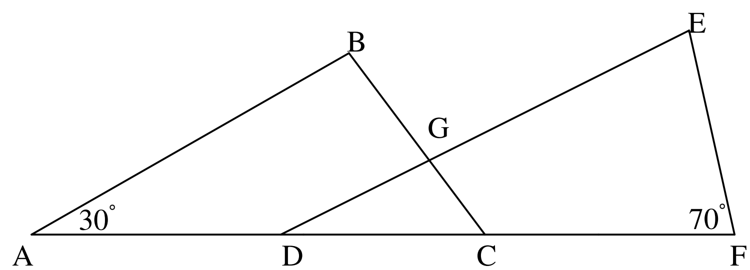
9)



Şekildeki üçgenler eştir. Bu üçgenlerin bazı kenar ve açı ölçüleri verildiğine göre x' in değeri nedir?

10) "Bir üçgende ölçüleri eşit olan açıların karşısında, uzunlukları eşit olan kenarlar vardır."
ifadesi doğru mudur? Cevabınızı nedenleriyle açıklayınız.

11)

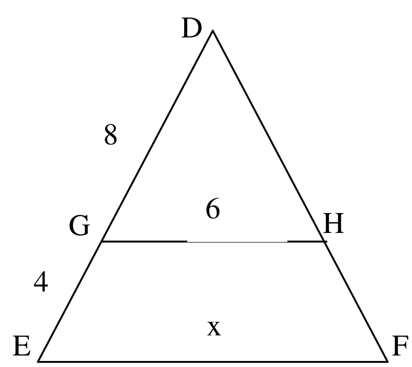


Yukarıdaki şekilde ABC ve DEF eş üçgenleri verilmiştir. $|AC| = |DF|$ dir. Buna göreDGC açısı kaç derecedir?

12) "Eğer ABC üçgeni ile DEF üçgeni benzer üçgenler ise ACB üçgeni ile DFE üçgeni benzer üçgenlerdir."
ifadesi doğru mudur? Cevabınızı nedenleriyle açıklayınız.

13) Bir üçgenin iki tane dik açısı olabilir mi? Cevabınızı nedenleriyle açıklayınız.

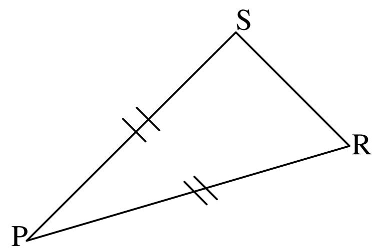
14)



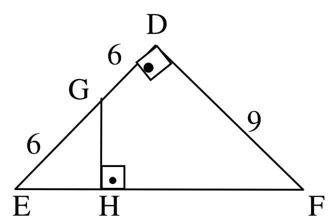
Şekildeki DEF üçgeninde $[GH]//[EF]$ dir. $|DG| = 8cm$ $|GE| = 4cm$ ve $|GH| = 6cm$ olduğuna göre $|EF| = x$ kaç cm dir?

15) “Her eşkenar üçgen bir ikizkenar üçgendir.” ifadesi doğru mudur? Cevabınızı nedenleriyle açıklayınız

16) Aşağıda verilen PRS üçgeninde $|PR| = |PS|$ dir. “Eğer SR kenarına ait bir yükseklik çizersem bu üçgeni iki eş üçgene ayırmış olurum.” ifadesi doğru mudur? Cevabınızı nedenleriyle açıklayınız.



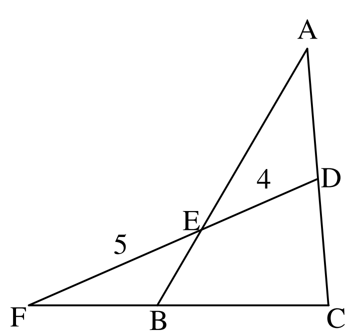
17)



DEF dik üçgeninde $[DE] \perp [DF]$, $[GH] \perp [EF]$ dir. $|EG| = |GD| = 6cm$, $|DF| = 9cm$ ise $|GH|$ kaç cm dir?

18) “Eğer iki üçgen benzer ise, bu iki üçgen aynı zamanda eştir.” ifadesi doğru mudur? Cevabınızı nedenleriyle açıklayınız.

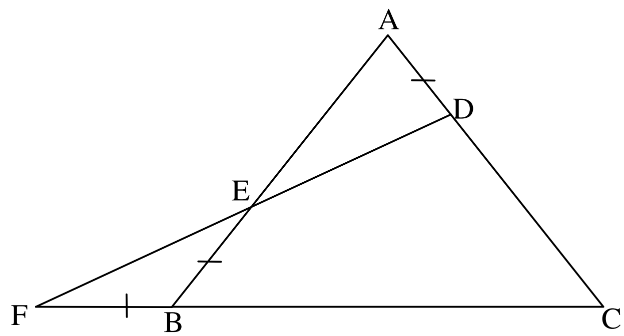
19)



Yandaki şekilde $|EF| = 5cm$, $|ED| = 4cm$, ve $|AD| = |DC|$ dir. Buna göre, $\dfrac{|FB|}{|BC|}$ kaçtır?

20) "ABC dik üçgeninde hipotenüse çizilen yükseklik, ABC üçgenini iki benzer üçgene ayırır."
ifadesi doğru mudur? Cevabınızı nedenleriyle açıklayınız.

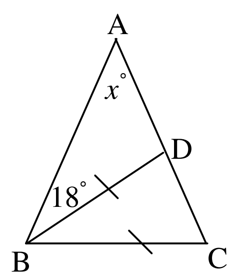
21)



Şekildeki ABC eşkenar üçgeninde $|AD| = |EB| = |BF|$ ve $|BC| = 24cm$ ise $|BF|$ kaç cm dir?

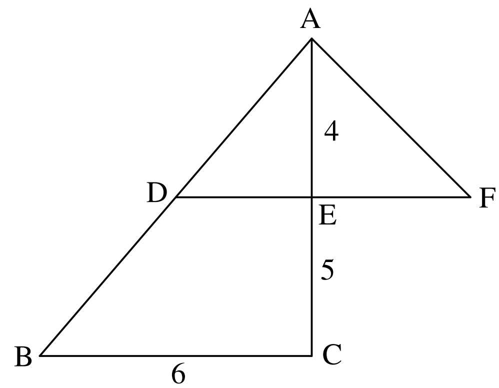
22) "Eğer $\overset{\Delta}{ABC} \cong \overset{\Delta}{ACB}$ ise ABC üçgeni ikizkenar üçgendir."
ifadesi doğru mudur? Cevabınızı nedenleriyle açıklayınız.

23)



ABC üçgeninde $|AB| = |AC|$, $|BD| = |BC|$ ve $s(\hat{ABD}) = 18^\circ$ dir. Bu verilenlere göre x açısı kaç derecedir?

24)



Yukarıdaki şekilde ABC, ADF, ve AEF dik üçgenlerdir. $|AE| = 4$ cm, $|EC| = 5$ cm, ve $|BC| = 6$ cm olduğuna göre $\frac{|EF|}{|DE|}$ kaç cm dir?

APPENDIX S

SCORING RUBRIC FOR THE GEOMETRY KNOWLEDGE TEST

<table>
<tr><th colspan="2">DECLARATIVE KNOWLEDGE QUESTIONS</th></tr>
<tr><td colspan="2">Visual Skills: recognition, observation of properties.
Verbal Skills: correct use of terminology, accurate description of the geometrical concepts.
Drawing Skills: appropriate use of symbols and notations.
Logical Skills: classification, recognition of essential properties of a geometrical concept.</td></tr>
<tr><th>Score</th><th>Description</th></tr>
<tr><td>0</td><td>❖ No answer attempted.
❖ Copies parts of the problem without attempting a solution.
❖ Uses irrelevant information.
❖ Includes declarative knowledge which completely misrepresent the problem situation.</td></tr>
<tr><td>1</td><td>❖ Shows very limited understanding and recalling of geometrical concept definitions, labels and names (symbols and notations), facts and lists.
❖ Misuse or fail to use geometrical concept definitions, labels and names (symbols and notations), facts and lists.
❖ Tries to solve the problem but includes improper definitions, and unnecessary symbols, or notations.
❖ Limitedly defines, identifies, lists, describes, and classifies the concepts.
❖ Draws a geometrical figure that is not clear or writes something that does not go with the answer.</td></tr>
<tr><td>2</td><td>❖ Shows some of the understanding and recalling of geometrical concept definitions, labels and names (symbols and notations), facts and lists.
❖ Makes significant progress towards completion of a definition but the description may be ambiguous or unclear.
❖ Includes flawed or unclear geometrical drawings representing the problem situation.</td></tr>
<tr><td>3</td><td>❖ Shows nearly complete understanding and recalling of geometrical concept definitions, labels and names (symbols and notations), facts and lists.
❖ Uses nearly correct geometrical terminology when defining a concept.
❖ Includes nearly complete and appropriate geometrical drawings (figures) representing the problem situation.</td></tr>
<tr><td>4</td><td>❖ Shows understanding and recalling of geometrical concept definitions, labels and names (symbols and notations), facts and lists.
❖ Uses appropriate geometrical terminology when defining a concept.
❖ Includes complete and appropriate geometrical drawings (figures) representing the problem situation.</td></tr>
</table>

CONDITIONAL KNOWLEDGE QUESTIONS	
Visual Skills: interpreting statements. Verbal Skills: correct use of terminology, accurate communication in describing relationships. Drawing Skills: appropriate use of symbols and notations. Logical Skills: formulating and testing hypothesis, making inferences, using counter-explanations, develop mathematical arguments about geometric relationships	
Score	**Description**
0	❖ No answer attempted. ❖ Copies parts of the problem without attempting a solution. ❖ Uses irrelevant information. ❖ Includes conditional knowledge which completely misrepresent the problem situation.
1	❖ Shows very limited explaining of the principles, theorems, relations, and statements. ❖ Fails to identify the important parts when expressing the “if-then” statements. ❖ Gives incomplete evidence of the explanation process. ❖ Places too much emphasis on unimportant relations when expressing the “if-then” statements.
2	❖ Shows some of the limited explaining of the principles, theorems, relations, and statements. ❖ Identifies some important parts when expressing the “if-then” statements. ❖ The relations expressed in the “if-then” statement is difficult to interpret and the arguments given are incomplete and logically unsound.
3	❖ Shows nearly complete explaining of the principles, theorems, relations, and statements. ❖ Identifies the most important parts when expressing the “if-then” statements. ❖ Shows general understanding of the relations in the “if-then” statements. ❖ Gives a fairly complete response with reasonably clear explanations or descriptions. ❖ Presents supporting logically sound arguments which may contain some minor gaps.
4	❖ Shows explaining of the principles, theorems, relations, and statements. ❖ Identifies all the important parts when expressing the “if-then” statements. ❖ Shows understanding of the relations in the “if-then” statements. ❖ Gives a complete response with a clear, unambiguous explanation or description. ❖ Presents strong, supporting, logically sound and complete arguments which may include counter-explanations or different aspects.

PROCEDURAL KNOWLEDGE QUESTIONS	
Visual Skills: imaging Verbal Skills: correct use of terminology Drawing Skills: appropriate use of symbols and notations, accurate application of the algorithm. Logical Skills: classification, recognition of essential properties of a geometrical concept,formulating and testing hypothesis, making inferences, using counter-explanations, appropriate use of the procedures, use visualization and spatial reasoning to solve problems.	
Score	**Description**
0	❖ No answer attempted. ❖ Copies parts of the problem without attempting a solution. ❖ Uses irrelevant information. ❖ Includes procedural knowledge which completely misrepresent the problem situation.
1	❖ Makes major computational errors when employing the algorithms and rules. ❖ Reflects an inappropriate strategy for solving the problem. ❖ Gives incomplete evidence of a solution process. ❖ The solution process is missing, difficult to identify or completely unsystemetic.
2	❖ Makes serious computational errors when employing the algorithms and rules. ❖ Gives some evidence of the solution process. ❖ The solution process is incomplete or somewhat unsystematic. ❖ Makes significant progress towards completion of the problem but the algorithm is unclear.
3	❖ Executes algorithms and rules completely. ❖ Computations are generally correct but may contain minor errors. ❖ Gives clear evidence of a solution process. ❖ The solution process is nearly complete and systematic.
4	❖ Executes algorithm and rules completely and correctly. ❖ Reflects an appropriate and systematic strategy for solving the problem. ❖ Gives evidence of a solution process. ❖ The solution process is complete and systematic.