OPTIMAL PRICING AND PRODUCTION DECISIONS IN REUSABLE CONTAINER SYSTEMS

A THESIS SUBMITTED TO
THE GRADUATE SCHOOL OF NATURAL AND APPLIED SCIENCES
OF
MIDDLE EAST TECHNICAL UNIVERSITY

BY

BÜŞRA ATAMER

IN PARTIAL FULFILLMENT OF THE REQUIREMENTS
FOR
THE DEGREE OF MASTER OF SCIENCE
IN
INDUSTRIAL ENGINEERING

JULY 2010

Approval of the thesis:

OPTIMAL PRICING AND PRODUCTION DECISIONS IN REUSABLE CONTAINER SYSTEMS

submitted by **BÜŞRA ATAMER** in partial fulfillment of the requirements for the degree of **Master of Science in Industrial Engineering Department, Middle East Technical University** by,

Prof. Dr. Canan Özgen
Dean, Graduate School of **Natural and Applied Sciences** ________________

Prof. Dr. Nur Evin Özdemirel
Head of Department, **Industrial Engineering** ________________

Asst. Prof. Dr. İsmail Serdar Bakal
Supervisor, **Industrial Engineering Department, METU** ________________

Asst. Prof. Dr. Z. Pelin Bayındır
Co-supervisor, **Industrial Engineering Department, METU** ________________

Examining Committee Members:

Assoc. Prof. Dr. Yasemin Serin
Industrial Engineering, METU ________________

Asst. Prof. Dr. İsmail Serdar Bakal
Industrial Engineering, METU ________________

Asst. Prof. Dr. Z. Pelin Bayındır
Industrial Engineering, METU ________________

Assoc. Prof. Dr. Osman Alp
Industrial Engineering, Bilkent University ________________

Asst. Prof. Dr. Seçil Savaşaneril
Industrial Engineering, METU ________________

Date: ________________

I hereby declare that all information in this document has been obtained and presented in accordance with academic rules and ethical conduct. I also declare that, as required by these rules and conduct, I have fully cited and referenced all material and results that are not original to this work.

Name, Last Name: BÜŞRA ATAMER

Signature :

ABSTRACT

OPTIMAL PRICING AND PRODUCTION DECISIONS IN REUSABLE CONTAINER SYSTEMS

Atamer, Büşra
M.S., Department of Industrial Engineering
Supervisor : Asst. Prof. Dr. İsmail Serdar Bakal
Co-Supervisor : Asst. Prof. Dr. Z. Pelin Bayındır

July 2010, 94 pages

In this study, we focus on pricing and production decisions in reusable container systems with stochastic demand. We consider a producer that sells a single product to the customers in reusable containers with two supply options: (i) brand-new containers (ii) returned containers from customers. Customers purchasing the products may return the containers to the producer to receive a deposit price. The return quantity depends on both customer demand and the deposit price determined by the producer. Hence, the producer has the opportunity to manipulate the return quantity via the deposit price. The unit cost of filling brand-new containers is different than the unit cost of refilling returned containers. We also consider resource restrictions on the production operations. Our setting represents certain hybrid manufacturing / remanufacturing systems where (i) the producer collects and recovers his own products, (ii) the producer supplies both brand-new and recovered products to his customers, and (iii) the customers are indifferent between brand-new and recovered products. In this setting, we investigate the optimal pricing and production decisions in order to maximize the producer‘s profit. Our approach utilizes non-linear optimization techniques. We characterize the optimal acquisition fee and the optimal order quantity of brand-new containers analytically and investigate the effect of parameters with an extensive computational study.

Keywords: Closed-loop supply chain management, reverse logistics, acquisition management, reusable containers, deposit-refund systems

ÖZ

YENİDEN KULLANILABİLİR KONTEYNERLER İÇİN ÜCRETLENDİRME VE ÜRETİM KARARLARI

Atamer, Büşra
Yüksek Lisans, Endüstri Mühendisliği Bölümü
Tez Yöneticisi : Yard. Doç. Dr. İsmail Serdar Bakal
Ortak Tez Yöneticisi : Yard. Doç. Dr. Pelin Bayındır

Temmuz 2010, 94 sayfa

Çalışmamızın amacı, yeniden kullanılabilir konteynerlerle üretim yapan sistemlerdeki ücretlendirme ve üretim kararlarını incelemektir. Yeniden kullanılabilir konteynerlerle üretim yapan üreticilerin konteynerleri tedarik etmek için iki seçeneği vardır: (i) tedarikçiden satın alınan yeni konteynerler (ii) tüketicilerden üreticiye geri dönen konteynerler. Tüketiciler, ürünü yeniden kullanılabilir bir konteyner içinde satın alırlar ve konteyneri iade ederek depozito ücretini geri alabilirler. Tüketicilerin kullandıkları konteynerlerden üreticiye geri dönenlerin sayısı önceden kesin olarak belirlenemez; geri dönen miktar, talebe ve üreticinin belirlediği depozito ücretine bağlıdır. Böylece üretici, depozito ücretini değiştirerek geriye dönen konteyner oranını ayarlayabilir. Tedarikçiden satın alınan yeni bir konteynerle üretim yapmanın birim maliyeti ile tüketicilerden üreticiye geri dönen bir konteynerle üretim yapmanın birim maliyeti birbirinden farklıdır. Sistemdeki olası kaynak kısıtları nedeniyle ücretlendirme ve üretim kararlarının birlikte verilmesi gerekmektedir. Bu çalışmada kullandığımız yaklaşım, tedarikçilerden satın alınan yeni ürünler ile tüketicilerden geri dönen ürünlerin bir arada kullanıldığı ve şu üç temel özelliği taşıyan üretim sistemlerindeki kararlar için geçerlidir: (i) üretici sadece kendine ait ürünleri tüketicilerden geri alır ve onları tekrar üretimde kullanır, (ii) üretici hem yeni ürünler hem de geri dönmüş ürünlerle hizmet verir, (iii) tüketiciler yeni alınmış ürünler ve geri dönmüş ürünler arasında fark gözetmez. Bahsedilen kapsam çerçevesinde, üreticinin karını en iyileyecek ücretlendirme ve üretim kararları incelenmiştir. Çözüm yaklaşımımızda doğrusal olmayan en iyileme teknikleri kullanılmıştır. Depozito ücretinin ve sipariş verilmesi gereken yeni konteyner miktarının en iyi değerleri analitik olarak gösterilmiştir. Parametrelerin karar değişkenleri ve üreticinin karı üzerindeki etkileri kapsamlı bir deney çalışmasıyla incelenmiştir.

Anahtar Kelimeler: Kapalı devre tedarik zinciri yönetimi, tersine lojistik, yeniden kullanılabilir konteynerler, depozitolu sistemler

To Mom and Dad

ACKNOWLEDGMENTS

I have completed this thesis study with the support and sympathy of many people. I am now glad to have the opportunity of tendering my thanks to them.

First, I would like to thank my thesis supervisors, Dr. İsmail Serdar Bakal and Dr. Pelin Bayındır, for both their guidelines and their friendship. Their support in this thesis is only a small part of what they have given to me as an academician and as a person. Having the opportunity of working with them in the future will be a pleasure for me.

I would like to express my gratitude to the examining committee, Dr. Yasemin Serin, Dr. Osman Alp and Dr. Seçil Savaşaneril for their reviews and comments. I also would like to thank Dr. Nur Evin Özdemirel, Dr. Meral Azizoğlu, Dr. Sedef Meral, Dr. Haldun Süral and Dr. Sinan Kayalıgil for their support, not only during this thesis work, but also during my research and teaching assistance in our department. I also acknowledge Dr. Sinan Gürel for sharing his profound knowledge with me on optimization theory. I would like to thank Şule Çimen who has helped me a lot and whose smile makes me believe that every problem in this life can be solved.

I would like to thank Turkcell, Informatics Association of Turkey (TBD) and The Scientific and Technological Research Council of Turkey (TÜBİTAK), for their financial support during this thesis work.

I have spent my last two years as a teaching and research assistant at the Industrial Engineering Department of the Middle East Technical University. This department would never be a wonderful place to spend years without my other assistant friends who also have helped and supported me throughout this thesis study. I would like to thank Tülin İnkaya and Banu Lokman who have shared their friendship with me since I was a junior student. They have not only given me the answers of my questions for years, but they also taught me how to find the answers of them. Melih Çelik, my first office mate, is an irreplaceable colleague to work with; I hope we have the chance to work together again in the following years. Spending days and nights in IE 326 has never been a burden with my current office mate, Kerem Demirtaş.

Whenever I felt exhausted, Kerem's encouraging words have kept me alive. I would like to thank Özlem Karabulut and Ayşegül Demir for the friendship they have shared with me for six years. It is always good to know that Şirin Barutçuoğlu and Aras Barutçuoğlu will welcome me when I would like to drink a cup of coffee. I would like to thank Volkan Gümüşkaya and Bilge Çelik for being the most kind hearted partners in our teaching assistance experiences and apologize if I have ever broken their heart. I want to thank Erdem Çolak for all meals, drinks, songs and fun we have shared. I would like to acknowledge members of IE-AST, Sakine Batun, Mustafa Gökçe Baydoğan, Baykal Hafızoğlu and Bora Kat who made me want to be an assistant in this department.

I would like to thank two very precious friends, Sercan Oruç and Abdullah Sait Çetin, for everything they have shared with me. Without brilliant ideas and unlimited support of Sercan, this thesis would never be completed; it might even never be started. Abdullah, has not only helped me in trying to understand a simulation model or preparing a project presentation, but also helped me to find the answers of the questions I ask to myself.

During this study, I had the opportunity to share a house with the best roommates of the world: Eda Ercan and Duygu Atılgan, whose understanding and friendship I can never pay back no matter what. I would like to thank Ezgi Can Ozan who has been beside me for almost ten years, and his beautiful wife, Gülcan Özek Ozan, who has been the most hospitable friend ever.

Some friends deserve acknowledgments just because they have made me smile during this study, even they are at the other end of the world. I would like to thank Işık Karahanoğlu, Alp Oğuz and Alphan Salarvan for every single moment we shared.

I am thankful to Burçin Ak who gives me the most important reason to start every day of my life with a huge smile on my face.

Last and the most, I would like to express my gratitude to my mother, Nevin Atamer and to my father, Şafak Atamer. Every single day, they make me feel that I am the luckiest daughter in the universe. I also would like to thank my sister, Başak Atamer, for teaching me to see the beautiful details of this sacred life. Every single achievement I gain is devoted to them.

TABLE OF CONTENTS

LIST OF TABLES

TABLES

LIST OF FIGURES

FIGURES

CHAPTER 1

INTRODUCTION

Closed-loop supply chains (CLSC) focus on taking back products from customers and recovering added value by reusing the entire product, and/or some of its modules, components, and parts (Guide and Wassenhove, 2009). That is, closed-loop supply chains include both traditional forward and reverse supply chain activities and in these chains, different kinds of recovery options are utilized. Guide and Wassenhove (2003) list closed-loop supply chain activities as follows: (i) product acquisition, (ii) reverse logistics, (iii) test, sort and disposition, (iv) recovery and (v) distribution and marketing activities. Product acquisition activities are performed to collect used products from customers. Reverse logistics deal with the activities to transport products from one supply chain node to another in the reverse sequence. Testing, sorting and disposition activities are performed in order to classify used products according to their quality and choose the best reusing alternative for these products. Recovery activities are value adding recovery operations applied to the returned products, and they are performed in different levels such as reusing, repairing, remanufacturing, recycling and disposing. Distribution and marketing operations are performed in order to manage corresponding activities for refurbished products.

Product recovery activities mainly have three kinds of driving forces: economic, legislative, and social and environmental. The producers perform recovery because recovered products are economically profitable under certain conditions. In the United States, annual sales of remanufactured products are estimated to be more than $50 billion (Guide and Wassenhove, 2003). Other than economical reasons, there are governmental legislations in many countries which forces producers to engage in product recovery activities. Also, due to social and environmental responsibilies, the producers perform recovery and consolidate their company image.

Closed-loop supply chain literature includes many studies in several research areas, such as acquisition and distribution management, inventory control and production planning, supply chain management, scheduling, pricing of recoverable products and cooperation and competition among supply chain parties.

According to the classification of de Brito and Dekker (2004), two kinds of recovery operations are performed in closed-loop supply chains: process recovery and direct recovery. Process recovery operations occur at different levels such as repairing, remanufacturing, retrieval, recycling and incineration (de Brito and Dekker, 2004). Yet, if the quality of returns collected is as-good-as-new, products can be fed into market almost immediately through reuse, resale and redistribution and these operations are called direct recovery operations (de Brito and Dekker, 2004).

Reusing is a widely-used recovery option within direct recovery operation alternatives. One of the earliest practices of reuse is reuse of containers. Reusable containers are durable packages that protect the main product during its transportation through different nodes of supply chain. Typical examples of reusable containers are glass bottles of beverages and glass jars of foods. For instance, a beverage producer use glass bottles as reusable containers for packaging and sells the beverage to his customers within these bottles. After the beverage is consumed, glass bottles containing the beverage do not lose its function unless they are broken. So, many beverage producers accept returned bottles in order to reuse them in their production. Hence, reusable packaging materials can be utilized more than once. Since the producer pays for the glass bottle for once but utilize it for several times, reusing containers is usually less costly.

Our goal in this thesis is to investigate the pricing and production decisions of a production system where reusable containers are utilized. We consider a producer that sells a single product to the customers in reusable containers with two supply options: brand-new containers and returned containers from customers. The producer purchases brand-new reusable containers from an external supplier and use them in manufacturing operations; that is, he fills them. The producer also acquires returned containers from customers by paying an acquisition fee, and perform remanufacturing operations; that is, he refills them. Since customers are indifferent between buying filled or refilled products, he sells each product for the same sales price. Our setting resembles to certain hybrid manufacturing / remanufacturing systems where (i) the producer collects and recovers his own products, (ii) the producer supplies both brand-new and recovered products to his customers, and (iii) the customers are indifferent between brand-new and recovered products. Although we use a "reusable container system" framework throughout the study, the models that we construct apply to such a manufacturing / remanufacturing framework as well.

The producer wants to maximize his profit with an effective production planning. Yet, quantity of returns is generally neither constant nor deterministic; it depends on both customer demand and the deposit price determined by the producer. Hence, the producer has the opportunity to manipulate the return quantity via the acquisition fee. Since only a proportion of the containers return to the producer, he also has to decide on the order quantity of brand new reusable containers to purchase. Filling and refilling options are carried out by utilizing the same resources. Since there may be restrictions on the availability of these resources, production and pricing decisions are to be made simultaneously for a synchronized reusable container system.

In this study, we investigate the optimal pricing and production decisions in order to maximize the producer's profit. We consider two different environmental settings: (i) unrestricted resource capacity (ii) restricted resource capacity.

The rest of the study is organized as follows: In Chapter 2, we review the relevant literature and define our problem. In Chapter 3, we consider the uncapacitated environment and characterize the optimal acquisition fee and the optimal order quantity of brand-new containers; and qualify the sensitivity of these decision variables in the cost parameters. The analysis in Chapter 3 is extended to the capacitated environment in Chapter 4. Since all research questions can not be answered in the analysis in Chapter 3 and Chapter 4, a computational study is conducted and results of this study are presented is Chapter 5. The study is concluded in Chapter 6.

CHAPTER 2

LITERATURE REVIEW AND PROBLEM DEFINITION

Within Industrial Engineering / Operations Research discipline, Closed-Loop Supply Chain Management / Reverse Logistics is a rich research field with a significant number of studies on major topics such as acquisition and distribution management, supply chain network design, inventory control and production planning, lot sizing, competition and cooperation among supply chain parties, forecasting and pricing product returns. In this chapter, we summarize our review on the literature with respect to modeling returns in closed-loop supply chains.

Production systems in closed loop supply chains can be classified into two groups in terms of operations performed by the producer: pure remanufacturing systems and hybrid manufacturing / remanufacturing systems. In pure remanufacturing systems, the producer collects reusable, remanufacturable or recyclable products from the market and recovers them by performing required operations. In these systems, the producer does not supply brand-new products to the market. Whereas, in hybrid systems, the producer performs both manufacturing and remanufacturing operations and provide both recovered and brand-new products to the customers. In this thesis, our focus is on the reusable container systems where both reused and brand-new containers are supplied by the producer. In this respect, we restrict our literature review with the studies on hybrid systems.

In closed loop supply chains, two main drivers of the production are customer demand and product returns. In these systems, modeling product returns is as complicated as modeling customer demand. Returns may be deterministic or stochastic, may depend on customer demand and/or sales, may depend on acquisition prices for deposit paid to the customers for returns or the sales price of the product. The dynamics lying behind the returns are case specific and understanding these dynamics requires detailed analysis.

In the closed loop supply chain literature, there are different approaches to model product returns. One approach is to assume that quantity of customer returns is an exogenous (deterministic or random) parameter, and totally independent of other dynamics of the system. This type of scheme is most suitable for return systems where the producer collects and recovers products produced by not only himself but also other manufacturers, hence the dependency of the return stream on the producer's demand stream is weak. The focus of the studies following this approach is usually either tactical or operational; most of them are on production planning and inventory control for recovery systems.

In our study, we consider a system where the producer collects the containers supplied by only himself and our main issue is to investigate the acquisition fee and order quantity decisions; so we exclude the studies which assume that customer returns are exogenous. Hence, we present our literature review under two main headings: demand dependent returns and acquisition fee dependent returns. In Table 2.1, a summary of environmental settings of reviewed literature can be found.

2.1 Demand Dependent Returns

In modeling customer returns, one approach is to assume that quantity of customer returns depend on customer demand. In this approach, returns are generally defined as a function of demand and a fraction indicating the proportion of returns to customer demand is considered. This modeling approach is suitable for the systems where the producer collects and recovers products supplied by only himself. There are two areas of research that follows this approach. The first area focuses on production and inventory planning problems, and the second area is on forecasting studies.

Table 2.1: Summary of Reviewed Literature

Studies Reviewed	Demand Dependent Returns	Price Dependent Returns	Forecasting Returns	Returns in Inv. and Prod. Plan.	Returns as a Parameter	Returns as a Decision Variable	Deterministic Demand	Stochastic Demand
Bayindir et. al. (2003)	✓			✓		✓		✓
Dobos and Richter (2003)	✓			✓		✓	✓	
Goh and Varaprasad (1986)	✓		✓		✓			
Guide et. al. (2003)		✓		✓		✓	✓	
Hess and Mayhem (1997)		✓	✓			✓		
Kelle and Silver (1989a)	✓		✓		✓			✓
Kelle and Silver (1989b)	✓			✓	✓			✓
Kiesmüller and van der Laan (2001)	✓			✓	✓			✓
Mukhopadhyay and Setoputro (2004)		✓		✓		✓	✓	
Tang and Grubbström (2005)	✓			✓	✓		✓	
Teunter (2001)	✓			✓	✓		✓	
Teunter (2004)	✓			✓	✓		✓	

2.1.1 Forecasting Demand Dependent Returns

Kelle and Silver (1989a) introduce one of the most comprehensive studies in the literature on modeling and forecasting the returns of reusable containers. In this study, they develop four different forecasting methods to estimate returns and net demand during the lead time. These forecasting methods can be summarized as follows:

- Method 1: This simple method utilize the information on “the expected value and the variance of the demand during lead time” and “the probability of each container eventually being returned”, both of which are assumed to be known parameters. The expected return during lead time can be found by multiplying the probability that a container will ever be returned and the expected value of demand during lead time.
- Method 2: In this method, more detailed information, “the actual issues during each previous period” and “the probability of being returned in each forthcoming period for any given container”, is utilized. The total lead time return from the previous issues can be calculated with this method since the probabilities are assumed to be estimated and the previous issues are observed and known.
- Method 3: In this method, in addition to the information on “the actual issues during each previous period” and “the probability of being returned in each forthcoming period for any given container” used in Method 2, “the amount returned up to and including the present period from each previous issue” information is utilized. By considering the issues returned up to present period, conditional return probabilities are defined and updated accordingly so a more accurate method is obtained. Yet, using this method may be very expensive in practice, because it requires to track every individual container.
- Method 4: This method utilizes the information on “the actual issues during each previous period” and “the probability of being returned in each forthcoming period for any given container” as used in Method 2 and Method 3, but additionally it uses the information on “the total amount returned in each previous period”. That is, instead of using information gathered from tracking each individual container, the estimation can be done with the information of aggregate returns.

In addition to introducing these methods, Kelle and Silver (1989a) evaluate and compare their performance by a computational study. They use (i) mean positive and negative deviation to test bias, (ii) mean absolute deviation or standard deviation to measure variability and (iii) histogram of deviation as performance measures. They observe that additional information improves the forecasting performance and most of the benefit gained by utilizing the more costly method of identifying and tracking each individual container (Method 3) is achieved by utilizing only the more practical method of recording aggregate issues and aggregate returns period by period (Method 2 and Method 4).

Goh and Varaprasad (1986) make the analysis of the life-cycle of reusable containers in order to determine expected service life of reusable containers and develop an approach to analyze past data and model the return process. They approach the problem by considering the return pattern of a particular issue, which may consist of any proportion of new and used containers. The number of trips in the useful life of a container is estimated with the information regarding the probability that a container will not be returned in a given trip. This probability is calculated with the return pattern of a particular issue. They suggest that the return process can be modeled by discrete linear transfer functions with past data of containers issued.

2.1.2 Inventory and Production Planning Studies Assuming Demand Dependent Returns

The planning studies modeling returns as a function of the producer's demand can be classified into two: (i) tactical / operational level models considering returns as a parameter where the producer is passively engaged in recovery options and does not have any effort in manipulating the return stream, (ii) tactical / strategic level models where the return ratio is a decision variable and the planning problem is tactical to strategic level.

There are a number of studies that extend the traditional EOQ problem to the recovery environment where the return rate is a fixed fraction of the demand rate. Teunter (2001) and Teunter (2004) are examples of such studies. Tang and Grubbström (2005) extend these studies to the case where lead times are stochastic.

There are also a number of studies in which the demand rate is assumed to be stochastic and return rate is assumed to be a parameter depending on the demand rate.

Kelle and Silver (1989b) deal with the purchasing policies for reusable containers where demand and return rates are stochastic. They investigate the optimal purchasing policy of new reusable containers over a finite time horizon with the objective function of minimizing the total purchasing and expected holding cost under the constraint of a specified service level. They develop a stochastic model and reduce it to a deterministic, dynamic lot-sizing problem by utilizing the probability of having a negative net demand. The net demand, the consumer demand minus the number of returned containers, is critical for purchasing policy decisions; and they model the cumulative net demand by utilizing the probability of a container never being returned. The information required to find this probability is assumed to be estimated by experimentation or aggregate statistical methods. Since the number of returns in any period is assumed to be dependent upon the previous issues and recent returns, the net demand is a random variable with a time-varying distribution.

Kiesmüller and Van der Laan (2001) investigate an inventory model for a single reusable product. They assume that the random returns to the system depend on the demand stream. They introduce a constant parameter which is the probability of an item being lost because a customer has not returned it to the manufacturer after use. Additionally, they introduce another constant parameter which is the probability that a returned item being not remanufactured because of poor quality, and disposed. Hence, the number of remanufactured items that enter the serviceable inventory in a period depends on both the probability of an item returning to the producer and the probability that a returned item is remanufactured. Demands per period are assumed to be independent and follow a Poisson distribution, whereas the returns in a period are assumed to be dependent on the demand parameter with a time lag which is a given number of periods. It is shown that the number of returns entering the serviceable inventory in a period follows Poisson distribution with the parameter defined as the multiplication of expected demand rate and the probability of entering the serviceable inventory for any container. The results show that using the information about the dependence between the demand and return processes generally decreases the average relevant costs.

The second stream of research that considers demand dependent returns takes return rate as a decision variable.

Bayindir et al. (2003) investigate the benefits of utilizing returns in a hybrid manufacturing / remanufacturing system where customer demand is stochastic. In the long-run, a fraction of the end products that complete their lifetime return to the system; that is, a fraction of the customer demand is satisfied by the remanufactured items. They consider the return ratio as a decision variable. They model the production environment and useful lifetime of the product as a queuing network. For this production environment, long-run average expected cost is minimized by determining the order-up to levels for the end item and the return ratio.

Dobos and Richter (2003) investigate a production / recycling system where customer demand and return rates are deterministic and stationary. They consider the EOQ environment with recovery and define return rate as a fraction of the constant demand rate; and this fraction is called “marginal return rate”. They find the optimal levels of marginal return rate as well as marginal recycling rate, length of production and recycling cycles and production and recycling lot sizes in order to minimize EOQ-related costs.

2.2 Price Dependent Returns

Another approach used in modeling customer returns defines returns as a function of sales price, acquisition fee or refund paid to customers by the producer. Most of the studies in this group consider tactical to strategic level decision of determining the optimal acquisition fee in a single period setting.

2.2.1 Forecasting Price Dependent Returns

In order to reduce customers’ risk and to effectively compete against stores that have merchandise on display, direct marketers offer generous return policies. Hess and Mayhew (1997) highlight these policies and deal with modeling direct marketing returns. In order to accurately estimate direct marketing returns, they propose a split adjusted hazard rate model as an alternative to simple regression return modeling approaches. Both time-to-return and return rate are assumed to depend on sales prices, and hazard models are constructed accordingly. They show the robustness of split adjusted hazard models on an example data of actual returns from an apparel direct marketer.

2.2.2 Inventory and Production Planning Studies Assuming Price Dependent Returns

Guide et al. (2003) deal with product acquisition management problems in a remanufacturing environment to maximize the remanufacturer’s profit. In this environment, the profitability of remanufacturing operations depends on the quantity and quality of returned items and they assume that the quantity and quality of returns can be manipulated by quality-dependent acquisition prices. They model returns as an increasing deterministic function of acquisition fee and define customer demand as a deterministic function of selling price. They assume that return rates are independent of sales or demand rates. In order to maximize profits from remanufacturing, they determine optimal acquisition and selling prices in a single period setting.

Mukhopadhyay and Setoputro (2004) investigate optimal price and return policies for reverse logistics in e-business. Since the customers do not have the chance of physical inspection when they buy a product from e-tailers, the likelihood that customers will have some dissatisfaction with the product increases. Consequently, e-tailers offer generous return policies; and modeling demand and return functions are critical in these systems. In this study, Mukhopadhyay and Setoputro develop a model to determine optimal price and return policies to maximize e-tailer's profit. They define a simple supply chain and flow of payments as following: When customer buys a product from e-tailer, pays its selling price; if customer decides the return the product, the e-tailer gives the refund amount to the customer back. They formulate the demand for the products as a linear function of both sales price and refund paid to the customer; whereas they model the return function only dependent on refund.

2.3 Problem Definition

In this study, we consider a producer that sells a single product. The producer has two supply options: (i) Brand-new items (ii) Returns from customers. Below, we describe the problem environment in detail by focusing on a "reusable container system" where the producer supplies the product to the customers in reusable containers. Typical examples of reusable containers are glass bottles and jars used in beverage and food production. The containers can be either purchased as "brand-new" or supplied from customer returns. It should be noted that although we use a "reusable container system" framework throughout the study, the models that we construct also apply to a more general manufacturing / remanufacturing framework as well.

In reusable container systems, the producer performs both filling and refilling operations and sells his products with reusable containers. He purchases brand-new reusable containers from an external supplier and fills them; that is, he manufactures them. The producer also acquires returned containers from customers by paying acquisition fee and he refills them; that is, he remanufactures them. Customers are indifferent between buying filled or refilled products. Filling and refilling operations include different steps; for instance, returned containers require cleaning whereas brand-new containers do not. Hence, unit filling and refilling costs are non-identical. Both operations are carried out in the same facility; that is, they share same resources and consume the same capacity. Figure 2.1 depicts the flow of material and payments; the dashed lines show material flow and the solid lines show flow of payments among three entities of the supply chain.

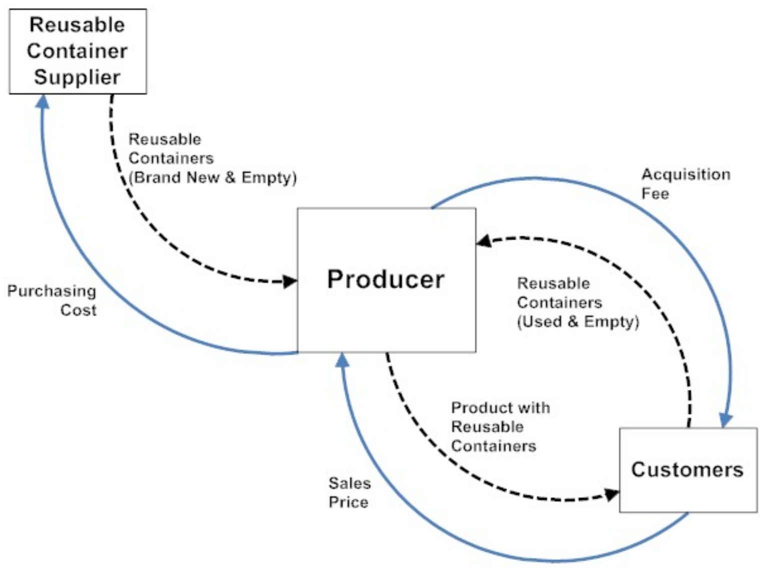

Figure 2.1: Flow of Material and Payments in Reusable Container Systems

The producer pays purchasing cost to the supplier to buy brand-new reusable containers and pays acquisition fee to the customer to take the returned reusable containers back. The producer also pays operational cost of manufacturing and remanufacturing to produce the products with reusable containers. In the revenue side of the system, the producer sells the products with reusable containers to the customers in exchange of selling price.

Our main research objective is to investigate pricing and production decisions in reusable container systems and the critical issues considered in these investigations are as follows:

- Acquisition fee dependency of the return stream
- Demand dependency of the return stream
- Common resource usage in reusable container systems

In order to address these critical issues properly, we consider a stylized model of the above environment in a single period context. Below we discuss the major assumptions of our model.

Customer demand is stochastic and the sales price is assumed to be exogenous, i.e., cannot be controlled by the producer. In Section 2.1 and 2.2, we explain that the return process is modeled in many different ways in the literature. In our study, we define returns perfectly correlated with realized demand: a fraction of realized demand returns to the system. It is expected that returns of containers are positively correlated with the demand for the products; but, perfect correlation between these two streams is our further simplification. In the literature, there are studies where returns are assumed to be a fraction of stochastic demand. Fleischmann et al. (2002) and Kiesmüller et al. (2001) assume perfect correlation between stochastic return and demand streams; and define the mean of item returns as a fraction of the mean of demand. Mostards et al. (2005, 2006) also uses perfect correlation assumption and defines the mean of net demand ("gross demand" minus "resalable returns") as a fraction of mean of gross demand.

In addition, we consider the case where all returned containers can be reused and the producer can manipulate quantity of returns by acquisition fee. Hence, by manipulating the acquisition fee, the producer can control return fraction, so the quantity of the customer returns. The producer should also determine the order quantity of brand-new reusable containers. Furthermore, we incorporate capacity restrictions on the manufacturing and remanufacturing operations. Hence, production and pricing decisions are to be made simultaneously for a synchronized reusable container system.

Within the setting explained above, our objective is to maximize the expected profit of the producer. The sequence of events in this setting are as follows and the notation used is sum-

Table 2.2: Notation Used

Decision Var.s	
Q:	Order quantity of brand-new reusable containers
f:	Unit acquisition fee of returned reusable containers
$\gamma(f)$:	Fraction of reusable containers returned for a given value of $f \geq 0$
$r(\gamma(f), D)$:	Quantity of returned containers for a given value of $D \geq 0$ and $\gamma(f) \geq 0$
Parameters	
D:	Demand realized for products with reusable containers
$g(x)$:	General probability distribution function of customer demand
$G(x)$:	General cumulative distribution function of customer demand
p:	Unit sales price of products with reusable containers
c_n:	Unit cost of purchasing brand-new reusable containers
c_r:	Unit cost of using (filling, manufacturing) brand-new reusable containers
c_f:	Unit cost of reusing (refilling, remanufacturing) returned containers
C:	Available capacity for manufacturing and remanufacturing operations

marized in Table 2.2:

- Order quantity of brand-new reusable containers, Q, and the unit acquisition fee, f, are determined. Total purchasing cost of brand-new reusable containers c_nQ is incurred.
- Demand, D, and returns, γD, are realized. Total acquisition cost for returned containers $f\gamma(f)D$ is incurred.
- The producer determines quantity of brand new containers to be filled, $M \leq Q$ and the quantity of returned containers to be refilled, $R \leq \gamma D$, to satisfy the demand without exceeding the total capacity, i.e., $M + R \leq C$. Note that since the demand has already been realized, the producer never produces more than realized demand, i.e, $M + R \leq D$.
- Total cost of filling c_rM and refilling c_fR are incurred. A total revenue of $p(M + R)$ is received where $p > c_f$ and $p > c_n + c_r$.

Note that $\gamma(f)$ is the fraction of reusable containers returned which is assumed to be an increasing, concave function of f. That is, $\frac{d\gamma(f)}{df} = \gamma'(f) \geq 0$ and $\frac{d^2\gamma(f)}{df^2} = \gamma''(f) \leq 0$. $\gamma(f)$ is equal to 0 only when f is equal to 0. When f takes a positive finite value, $0 < \gamma(f) < 1$. In the following steps of the analysis, we study with a closed form function $\gamma(f)$ and show it as γ for the sake of brevity.

In this study, the decision problem is to determine f and Q to maximize producer's expected profit within the environment setting described. In Chapter 3, the problem is investigated for an uncapacitated environment, whereas in Chapter 4, the study is extented to a capacitated setting.

Our main research questions are as follows:

- What are the optimal levels of acquisition fee and order quantity of brand-new containers in order to maximize the producer's profit?
- What are the effect of cost and demand parameters on the optimal levels of decision variables and the optimal expected profit?
- What is the effect of a restriction in production capacity on the optimal levels of decision variables and the optimal expected profit?
- How much does the producer's expected profit improve due to utilization of returns?

The optimal levels of acquisition fee and order quantity of brand-new containers are analytically investigated for the uncapacitated setting in Chapter 3 and for the capacitated setting in Chapter 4. The effect of cost parameters on optimal decisions is shown for the uncapacitated setting in Chapter 3. The effect of a restriction on production capacity on the optimal levels of the acquisition fee and order quantity of brand-new containers is investigated in Chapter 4. Yet, for the other research questions, analytical investigation is not possible, so these questions are investigated in Chapter 5 with an extensive computational study.

CHAPTER 3

ANALYSIS OF THE REUSABLE CONTAINER SYSTEMS - THE UNCAPACITATED CASE

In this chapter, we consider the reusable container systems with unlimited production capacity and investigate the following research issues analytically:

- What are the optimal levels of acquisition fee and order quantity of brand new containers to maximize the producer's expected profit?
- What are the effect of cost parameters on the optimal levels of decision variables?

The sequence of events in this setting are as follows:

- Order quantity of brand-new reusable containers, Q, and the unit acquisition fee, f, are determined. Total purchasing cost of brand-new reusable containers c_nQ is incurred.
- Demand, D, and returns, γD, are realized. Total acquisition cost for returned containers $f\gamma D$ is incurred.
- The producer determines quantity of brand new containers to be filled, $M \leq Q$ and the quantity of returned containers to be refilled, $R \leq \gamma D$, to satisfy the demand. Note that since the demand has already been realized, the producer never produces more than realized demand, i.e, $M + R \leq D$.
- Total cost of filling c_rM and refilling c_fR are incurred. A total revenue of $p(M + R)$ is received.

In order to characterize the optimal solution, we employ a two-stage modeling approach. First, we solve the second stage problem to determine the optimal production levels using new containers, M, and returned containers, R, given the first stage decisions and the demand realization. Then, we incorporate this solution to the first stage problem and solve it for optimal levels of Q and f.

The second stage problem can be stated as:

$$\text{Maximize } \pi(M,R) = p(M+R) - c_r M - c_f R \tag{3.1}$$

subject to

$$M + R \leq D \tag{3.2}$$

$$M \leq Q \tag{3.3}$$

$$R \leq \gamma D \tag{3.4}$$

$$M \geq 0 \tag{3.5}$$

$$R \geq 0 \tag{3.6}$$

Objective function (3.1) consists of total revenue gained, $p(M+R)$, total cost of using new containers, $c_r M$, and total cost of reusing returned containers, $c_f R$. Note that the total purchasing cost of new containers, $c_n Q$, and total acquisition cost of returned containers, $f\gamma D$ are not affected by the second stage decisions, hence they are excluded in the objective function. Constraint (3.2) ensures that total number of new containers filled and returned containers refilled does not exceed demand. Constraint (3.3) indicates that the level of production using new containers cannot exceed the quantity of new containers purchased. Similarly, Constraint (3.4) ensures that the level of production with returned containers are less than or equal to the quantity of returns. Constraint (3.5) and Constraint (3.6) are non-negativity constraints for M and R, respectively.

The profit function of the second stage problem is linear in decision variables M and R. Then the optimal solution is intuitive: Either M or R will have a priority over the other in satisfying the demand, depending on their corresponding profit margins, i.e., $p - c_r$ and $p - c_f$ respectively. If reusing a returned container is cheaper, that is if $c_f < c_r$, R should be set to its highest possible value, and then the remaining demand should be covered by M as much as the other constraints allow. Whereas, if using a brand new container is cheaper, that is if $c_r < c_f$, M should be set to its highest possible value, and then the remaining demand, if any, should be covered by R. An illustrative example showing the feasible region (shaded area) and the candidate solutions (black dots) of the second stage problem can be seen in Figure 3.1. In this example, demand realization is less than the total supply on hand; so, depending on the profit margins of two supply options, the producer either produces Q brand-new containers and $D - Q$ returned containers, or $D - \gamma D$ brand-new containers and γD returned containers.

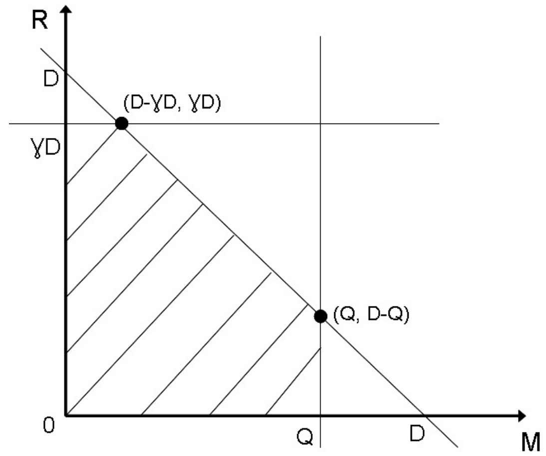

Figure 3.1: Illustrative Graphical Solution for the Second Stage Problem

Since the optimal decisions in Stage 2 depend on whether $c_r > c_f$ or not, these two cases are analyzed separately for the first stage. In Section 3.1 we analyze the case where reusing a returned container is cheaper and in Section 3.2, we analyze the other case where using a brand new container is cheaper.

3.1 P_1: Reusing a Returned Container is Cheaper ($c_f < c_r$)

In this case, for any realization of demand, returns are utilized to the fullest extent since they are the cheaper option. Proposition 3.1 characterizes the optimal production quantities given the first stage decisions, Q and f, and the demand realization, D.

Proposition 3.1 *When $c_f < c_r$, the optimal production quantities given the first stage decisions, Q and f , and the demand realization, D, the optimal production quantities are as follows:*

$$(M^*, R^*|D) = \begin{cases} (Q, \gamma D) & \textit{if } Q + \gamma D < D \\ (D - \gamma D, \gamma D) & \textit{if } Q + \gamma D > D \end{cases}$$

Proof. If $Q + \gamma D < D$, the demand can not be met in full and Constraint (3.2) is non-binding. Then, R is set to γD according to Constraint (3.4). By Constraint (3.3) M is set to Q. Hence, we have the optimal solution $(M^*, R^*) = (Q, \gamma D)$.

If $Q + \gamma D > D$, the demand can be met in full and Constraint (3.2) is binding. Then, R is set to γD according to Constraint (3.4). By Constraint (3.2) M is set to $D - \gamma D$. Hence, we have the optimal solution $(M^*, R^*) = (D - \gamma D, \gamma D)$. ■

As characterized in Proposition 3.1, the second stage optimal decisions depend on the demand realization and the values of decision variables determined in the first stage. Given the demand realization, the profit of the first stage can be expressed as follows:

$$\pi(Q, f|D) = \begin{cases} pD - c_r(D - \gamma D) - c_n Q - c_f \gamma D - f\gamma D & \text{if } Q + \gamma D > D \\ p(Q + \gamma D) - c_r Q - c_n Q - c_f \gamma D - f\gamma D & \text{if } Q + \gamma D < D \end{cases}$$

Note that regardless of the demand realization, the producer pays $c_n Q$ for purchasing brand-new containers and $f\gamma D$ for the acquisition of returns. Furthermore, since all returns are used in production, the refilling cost is $c_f \gamma D$. If $Q + \gamma D > D$ or equivalently $D < Q/(1 - \gamma)$, all demand is satisfied and the sales revenue is pD. In this case $D - \gamma D$ of the brand-new containers are used and filling cost is $c_r(D - \gamma D)$. If $Q + \gamma D < D$, or equivalently $D > Q/(1 - \gamma)$, all supply is used to satisfy the demand. Hence the sales revenue is $p(Q + \gamma D)$ and filling cost is $c_r Q$.

Based on these observations, the first stage problem can be expressed as follows:

$$\begin{aligned} [P_1] : \text{Maximize } \pi(Q, f) \quad &= \quad p \int_0^{\frac{Q}{1-\gamma}} xg(x)dx + p \int_{\frac{Q}{1-\gamma}}^{\infty} (Q + \gamma x)g(x)dx - c_r \int_0^{\frac{Q}{1-\gamma}} (1 - \gamma)xg(x)dx \\ &\quad -c_r \int_{\frac{Q}{1-\gamma}}^{\infty} Qg(x)dx - c_n Q - (f + c_f)\gamma E(X) \end{aligned}$$

subject to

$$\begin{aligned} Q &\geq 0 \\ f &\geq 0 \end{aligned}$$

Proposition 3.2 *The optimal solution to P_1 is the unique non-negative solution to $\partial\pi(Q, f)/\partial Q = 0$ and $\partial\pi(Q, f)/\partial f = 0$.*

Proof. The first and second order derivatives of $\pi(Q, f)$ with respect to f and Q are:

$$\frac{\partial\pi(Q,f)}{\partial Q} = (p - c_r)\left(1 - G\left(\frac{Q}{1-\gamma}\right)\right) - c_n \quad (3.7)$$

$$\frac{\partial\pi(Q,f)}{\partial f} = \gamma' c_r \int_0^{\frac{Q}{1-\gamma}} xg(x)dx + \gamma' p \int_{\frac{Q}{1-\gamma}}^{\infty} xg(x)dx - E(X)((c_f + f)\gamma' + \gamma) \quad (3.8)$$

$$\frac{\partial^2\pi(Q,f)}{\partial Q^2} = \frac{c_r - p}{1-\gamma} g\left(\frac{Q}{1-\gamma}\right) \quad (3.9)$$

$$\frac{\partial^2\pi(Q,f)}{\partial f^2} = \gamma'' c_r \int_0^{\frac{Q}{1-\gamma}} xg(x)dx + \gamma'' p \int_{\frac{Q}{1-\gamma}}^{\infty} xg(x)dx + \gamma'\left((c_r - p)\left(\frac{Q^2\gamma'}{(1-\gamma)^3}\right) g\left(\frac{Q}{1-\gamma}\right)\right)$$
$$-E(X)((c_f + f)\gamma'' + 2\gamma') \quad (3.10)$$

The determinant of the Hessian matrix for $\pi(Q, f)$ where $\partial\pi(Q, f)/\partial Q = 0$ and $\partial\pi(Q, f)/\partial f = 0$ is:

$$|H| = \frac{(c_r - p)E(X)(\gamma''\gamma - 2\gamma'\gamma')}{(1-\gamma)\gamma'} g\left(\frac{Q}{1-\gamma}\right)$$

The second order derivatives of $\pi(Q, f)$, Equation (3.10) and Equation (3.9) are negative on the stationary points, where the first order derivatives, Equation (3.7) and Equation (3.8) are equal to 0 for all non-negative Q and f values. Additionally, the determinant of the Hessian matrix on the stationary points of $\pi(Q, f)$ is positive for all non-negative Q and f values. Hence the expected profit function of P_1, $\pi(Q, f)$ is unimodal and has a unique maximum on $\partial\pi(Q, f)/\partial Q = 0$ and $\partial\pi(Q, f)/\partial f = 0$ (Demidenko, 2004).

When we analyze first order conditions of $\pi(Q, f)$, we see the followings:

$$\frac{\partial\pi(Q,f)}{\partial Q}|_{Q=0} = p - c_r - c_n > 0 \quad (3.11)$$

$$\frac{\partial\pi(Q,f)}{\partial Q}|_{Q\to\infty} = -c_n < 0 \quad (3.12)$$

$$\frac{\partial\pi(Q,f)}{\partial Q}|_{f=0} = \gamma' c_r \int_0^{Q} xg(x)dx + \gamma' p \int_Q^{\infty} xg(x)dx - c_f\gamma' E(X) > 0 \quad (3.13)$$

$$\frac{\partial\pi(Q,f)}{\partial Q}|_{f\to\infty} = \left(\gamma'(c_r - c_f - f) - 1\right)E(X) < 0 \quad (3.14)$$

Inequalities (3.11), (3.12), (3.13) and (3.14) show that there is a unique non-negative solution for Q and f satisfying $\partial\pi(Q, f)/\partial Q = 0$ and $\partial\pi(Q, f)/\partial f = 0$. Since the objective function of P_1 has a unique maximum, the unique non-negative solution (Q, f) to $\partial\pi(Q, f)/\partial Q = 0$ and $\partial\pi(Q, f)/\partial f = 0$ is the optimal solution to P_1. ■

3.2 P_2: Using a Brand New Container is Cheaper ($c_f > c_r$)

If $c_f > c_r$, the producer will use brand-new containers first to satisfy the demand. Using this property, Proposition 3.3 characterizes the optimal production quantities given the first stage decisions, Q and f, and the demand realization, D.

Proposition 3.3 *When $c_f > c_r$, the optimal production quantities given the first stage decisions, Q and f, and the demand realization, D, the optimal production quantities are as follows:*

$$(M^*, R^*|D) = \begin{cases} (D, 0) & \text{if } Q + \gamma D > D \text{ and } Q > D \\ (Q, D - Q) & \text{if } Q + \gamma D > D \text{ and } Q < D \\ (Q, \gamma D) & \text{if } Q + \gamma D < D \end{cases}$$

Proof. If $Q + \gamma D > D$ and $Q > D$, the demand can be met in full and Constraint (3.2) is binding. Then, M is set to D according to Constraint (3.2). Since total demand is satisfied only with new containers, R is set to 0. Hence we have the optimal solution $(M^*, R^*) = (D, 0)$.

If $Q + \gamma D > D$ and $Q < D$, the demand can be met in full and Constraint (3.2) is again binding. Then, M is set to Q according to Constraint (3.3). By Constraint (3.2), R is set to $D - Q$. Hence we have the optimal solution $(M^*, R^*) = (Q, D - Q)$.

If $Q + \gamma D < D$, the demand can not be met in full and Constraint (3.2) is non-binding. Then, M is set to Q according to Constraint (3.3). By Constraint (3.4), R is set to γD. Hence, we have the optimal solution $(M^*, R^*) = (Q, \gamma D)$. ■

As characterized in Proposition 3.3, the second stage optimal decisions depend on the demand realization and the values of decision variables determined in the first stage. Given the demand realization, the profit of the first stage can be expressed as follows:

$$\pi(Q, f|D) = \begin{cases} pD - c_r D - c_n Q - f\gamma D \text{ if } Q > D \\ pD - c_r Q - c_n Q - c_f(D - Q) - f\gamma D \text{ if } Q < D \text{ and } Q + \gamma D > D \\ p(Q + \gamma D) - c_r Q - c_n Q - c_f \gamma D - f\gamma D \text{ if } Q + \gamma D < D \end{cases}$$

Note that, regardless of the demand realization, the producer pays $c_n Q$ for purchasing brand-new containers and $f\gamma D$ for the acquisition of returns. If $D < Q$, all demand is satisfied with brand-new containers and the producer does not perform refilling. In that case, he earns a revenue of pD and pays only a filling cost of $c_r D$. If $D > Q$ and $Q + \gamma D > D$, or equivalently $Q < D < Q/(1 - \gamma)$, all demand is again satisfied and all brand-new containers on hand are utilized to the fullest extent. In that case, the producer earns a revenue of pD from sales and pays a filling cost of $c_r Q$. $D - Q$ amount of the demand is satisfied with returns, so the producer pays a refilling cost of $c_f(D - Q)$. If $Q + \gamma D < D$ or equivalently $D > Q/(1 - \gamma)$, all supply is used to satisfy the demand. Hence, the producer earns a revenue of $p(Q + \gamma D)$ from sales, pays a filling cost of $c_r Q$ from using brand new containers in production and pays a refilling cost of $c_f \gamma D$ from reusing returned containers.

Based on these observations, the first stage problem can be expressed as follows:

$$\begin{aligned} [P_2] : \text{Maximize } \pi(Q, f) \quad = \quad & p\int_0^{\frac{Q}{1-\gamma}} xg(x)dx + p\int_{\frac{Q}{1-\gamma}}^{\infty} (Q + \gamma x)g(x)dx - c_r\int_0^{Q} xg(x)dx \\ & -c_r\int_Q^{\infty} Qg(x)dx - c_f\int_Q^{\frac{Q}{1-\gamma}} (x - Q)g(x)dx - c_f\int_{\frac{Q}{1-\gamma}}^{\infty} \gamma xg(x)dx \\ & -c_n Q - f\gamma E(X) \end{aligned}$$

subject to

$$\begin{aligned} Q &\geq 0 \\ f &\geq 0 \end{aligned}$$

Proposition 3.4 *The optimal solution to P_2 is the unique non-negative solution to $\partial\pi(Q, f)/\partial Q = 0$ and $\partial\pi(Q, f)/\partial f = 0$.*

Proof. The first and second order derivatives of $\pi(Q, f)$ with respect to Q and f are:

$$\frac{\partial \pi(Q,f)}{\partial Q} = p - c_n - c_r + (c_r - c_f)G(Q) - (p - c_f)G\left(\frac{Q}{1-\gamma}\right) \tag{3.15}$$

$$\frac{\partial \pi(Q,f)}{\partial f} = (p - c_f)\gamma' \int_{\frac{Q}{1-\gamma}}^{\infty} xg(x)dx - (\gamma + f\gamma')E(X) \tag{3.16}$$

$$\frac{\partial^2 \pi(Q,f)}{\partial Q^2} = (c_r - c_f)g(Q) - \left(\frac{p-c_f}{1-\gamma}\right) g\left(\frac{Q}{1-\gamma}\right) \tag{3.17}$$

$$\frac{\partial^2 \pi(Q,f)}{\partial f^2} = (p - c_f)\gamma'' \int_{\frac{Q}{1-\gamma}}^{\infty} xg(x)dx - (p - c_f)\gamma' \left(\frac{Q^2\gamma'}{(1-\gamma)^3}\right) g\left(\frac{Q}{1-\gamma}\right) - E(X)(2\gamma' + f\gamma'') \tag{3.18}$$

The determinant of the Hessian matrix for $\pi(Q, f)$ where $\partial\pi(Q, f)/\partial Q = 0$ and $\partial\pi(Q, f)/\partial f = 0$ is:

$$|H| = \left((c_r - c_f)g(Q) - \left(\frac{p-c_f}{1-\gamma}\right) g\left(\frac{Q}{1-\gamma}\right)\right)\left(E(X)\left(\frac{\gamma'\gamma}{\gamma'} - 2\gamma'\right)\right) - (c_r - c_f)(p - c_f)\left(\frac{Q^2(\gamma')^2}{(1-\gamma)^3}\right) g(Q) g\left(\frac{Q}{1-\gamma}\right)$$

The second order derivatives of $\pi(Q, f)$, Equation (3.17) and Equation (3.18) are negative on the stationary points, where the first order derivatives, Equation (3.15) and Equation (3.16) are equal to 0 for all non-negative Q and f values. Additionally, the determinant of the Hessian matrix on the stationary points of $\pi(Q, f)$ is positive for all non-negative Q and f values. Hence the expected profit function of P_2, $\pi(Q, f)$ is unimodal and has a unique maximum on $\partial\pi(Q, f)/\partial Q = 0$ and $\partial\pi(Q, f)/\partial f = 0$ (Demidenko, 2004).

When we analyze first order conditions of $\pi(Q, f)$, we see the followings:

$$\frac{\partial \pi(Q,f)}{\partial Q}|_{Q=0} = p - c_r - c_n > 0 \tag{3.19}$$

$$\frac{\partial \pi(Q,f)}{\partial Q}|_{Q\to\infty} = -c_n < 0 \tag{3.20}$$

$$\frac{\partial \pi(Q,f)}{\partial Q}|_{f=0} = \gamma'(p - c_f) \int_{Q}^{\infty} xg(x)dx > 0 \tag{3.21}$$

$$\frac{\partial \pi(Q,f)}{\partial Q}|_{f\to\infty} = (-\gamma' f - 1)E(X) < 0 \tag{3.22}$$

Inequalities (3.19), (3.20), (3.21) and (3.22) show that there is a unique non-negative solution for Q and f satisfying $\partial\pi(Q, f)/\partial Q = 0$ and $\partial\pi(Q, f)/\partial f = 0$. Since the objective function of

P_2 has a unique maximum, the unique non-negative solution (Q, f) to $\partial\pi(Q, f)/\partial Q = 0$ and $\partial\pi(Q, f)/\partial f = 0$ is the optimal solution to P_2. ■

Propositions 3.2 and 3.4 show that the optimal acquisition fee, f^* is always non-zero under the stated assumptions, i.e, the producer always collects some used containers. In the case that the refilling option has a cost advantage over the filling option, this result is trivial. On the other hand, the fact that the producer is engaged in some refilling even if it is disadvantageous with respect to unit costs can be explained by the dependency of returns on the realized demand. Since high (low) values of demand realization indicate high (low) values of returns, refilling is used as an option to reduce the effect of uncertainty of demand in such cases. The effect of uncertainty of demand on the optimal acquisition fee is further investigated in the computational study presented in Chapter 5.

3.3 Analytical Findings for the Optimization Model

In this section, we characterize the effects of changes in cost parameters, c_f, c_n and c_r, on the optimal acquisition fee and order quantity of brand-new containers. Let Q^* and f^* denote the optimal order quantity of brand-new containers and the optimal acquisition fee, respectively.

Proposition 3.5 *Q^* is non-decreasing and f^* is non-increasing in c_f.*

Proof. We first consider the case where $c_f < c_r$. For expositional clarity, let $Q' = \partial Q^*(c_f)/\partial c_f$ and $f' = \partial f^*(c_f)/\partial c_f$.

Taking implicit derivative of $\partial\pi(Q, f)/\partial Q = 0$ with respect to c_f, we get:

$$\begin{aligned} \frac{Q'(1-\gamma) + f'\gamma' Q}{(1-\gamma)^2} g\left(\frac{Q}{1-\gamma}\right) &= 0 \\ Q'(1-\gamma) + f'\gamma' Q &= 0 \end{aligned} \tag{3.23}$$

From Equation (3.23), we observe that Q' and f' should have opposite signs since $(1-\gamma)$, Q and γ' are non-negative.

From $\partial\pi(Q, f)/\partial f = 0$, we have:

$$c_r \int_0^{\frac{Q}{1-\gamma}} x g(x) dx + p \int_{\frac{Q}{1-\gamma}}^{\infty} x g(x) dx = E(X)(c_f + f) + \frac{E(X)\gamma}{\gamma'} \tag{3.24}$$

Taking implicit derivative of $\partial\pi(Q,f)/\partial f = 0$ with respect to c_f, we get:

$$f'\gamma''c_r \int_0^{\frac{Q}{1-\gamma}} xg(x)dx + f'\gamma''p \int_{\frac{Q}{1-\gamma}}^{\infty} xg(x)dx - E(X)\left((1+2f')\gamma' + (c_f+f)f'\gamma''\right)$$
$$+\gamma'(c_r - p)\left(\frac{Q'(1-\gamma)+f'\gamma'Q}{(1-\gamma)^2}\right)\left(\frac{Q}{1-\gamma}\right)g\left(\frac{Q}{1-\gamma}\right) = 0 \tag{3.25}$$

Plugging (3.23) and (3.24) in Equation (3.25), we get:

$$\begin{aligned} f'\gamma''E(X)(c_f+f) + f'\gamma''\frac{E(X)\gamma}{\gamma'} - E(X)\left((1+2f')\gamma' + (c_f+f)f'\gamma''\right) &= 0 \\ f'\gamma''\frac{E(X)\gamma}{\gamma'} - E(X)\gamma'(2f'+1) &= 0 \quad (3.26) \end{aligned}$$

Equation (3.26) cannot hold if $f' \geq 0$, thus $f' < 0$. Since Q' and f' should have opposite signs, $Q' > 0$.

Next, we repeat our analysis for the case where $c_f > c_r$.

Taking implicit derivative of $\partial\pi(Q,f)/\partial Q = 0$ with respect to c_f, we get:

$$(c_r - c_f)g(Q)Q' - G(Q) - (p-c_f)\frac{Q'(1-\gamma)+f'\gamma'Q}{(1-\gamma)^2}g\left(\frac{Q}{1-\gamma}\right) + G\left(\frac{Q}{1-\gamma}\right) = 0 \tag{3.27}$$
$$(c_r - c_f)g(Q)Q' - (p-c_f)\frac{Q'(1-\gamma)+f'\gamma'Q}{(1-\gamma)^2}g\left(\frac{Q}{1-\gamma}\right) < 0 \tag{3.28}$$

From Equation (3.28), we observe that both Q' and f' cannot be negative since $(p - c_f)$, $(1-\gamma)$, Q and γ' are non-negative and $(c_r - c_f)$ is non-positive.

From $\partial\pi(Q,f)/\partial f = 0$ we have

$$\int_{\frac{Q}{1-\gamma}}^{\infty} xg(x)dx = \frac{E(X)(\gamma + f\gamma')}{(p-c_f)\gamma'} \tag{3.29}$$

Taking implicit derivative of $\partial\pi(Q,f)/\partial f = 0$ with respect to c_f, we get:

$$\left((p-c_f)f'\gamma'' - \gamma'\right)\int_{\frac{Q}{1-\gamma}}^{\infty} xg(x)dx - \gamma'\left((p-c_f)\left(\frac{Q'(1-\gamma)+f'\gamma'Q}{(1-\gamma)^2}\right)\left(\frac{Q}{1-\gamma}\right)g\left(\frac{Q}{1-\gamma}\right)\right)$$
$$-E(X)\left(2f'\gamma' + ff'\gamma''\right) = 0 \tag{3.30}$$

Plugging (3.29) in Equation (3.30) we get:

$$\begin{aligned} & E(X)\left(f'\left(\frac{\gamma\gamma''}{\gamma'} - 2\gamma'\right) - \frac{(\gamma + f\gamma')}{p - c_f}\right) \\ & -\gamma'\left((p - c_f)\left(\frac{Q'(1-\gamma) + f'\gamma' Q}{(1-\gamma)^2}\right)\left(\frac{Q}{1-\gamma}\right) g\left(\frac{Q}{1-\gamma}\right)\right) = 0 \end{aligned} \quad (3.31)$$

From (3.31), we observe that both Q' and f' can not be positive since $(p - c_f)$, $(1 - \gamma)$, Q and γ' are non-negative and γ'' is non-positive.

From Equation (3.27) we get:

$$(p - c_f)\frac{Q'(1-\gamma) + f'\gamma' Q}{(1-\gamma)^2} g\left(\frac{Q}{1-\gamma}\right) = (c_r - c_f) g(Q) Q' - G(Q) + G\left(\frac{Q}{1-\gamma}\right) \quad (3.32)$$

Plugging (3.32) in Equation (3.31), we get:

$$\begin{aligned} & E(X)\left(f'\left(\frac{\gamma\gamma''}{\gamma'} - 2\gamma'\right) - \frac{(\gamma + f\gamma')}{p - c_f}\right) \\ & -\gamma'\left(\frac{Q}{1-\gamma}\right)\left((c_r - c_f) g(Q) Q' - G(Q) + G\left(\frac{Q}{1-\gamma}\right)\right) = 0 \end{aligned} \quad (3.33)$$

Suppose the case where Q' is negative and f' is positive. That case contradicts with Equation (3.33) since $(p-c_f)$, $(1-\gamma)$, f, Q and γ' are non-negative, and γ'' and (c_r-c_f) are non-positive.

Due to Inequality (3.28), Equation (3.31) and the contradiction explained for Equation (3.33), Q^* is non-decreasing and f^* is non-increasing in c_f. ■

As c_f increases, unit profit margin of returns decreases. Hence, the producer decreases f^*, so the optimal expected quantity of returns, and increases Q^* in order to satisfy the demand.

Proposition 3.6 *Q^* is non-increasing and f^* is non-decreasing in c_r.*

Proof. We first consider the case where $c_f < c_r$. For expositional clarity, let $Q' = \partial Q^*(c_r)/\partial c_r$ and $f' = \partial f^*(c_r)/\partial c_r$.

Taking implicit derivative of $\partial\pi(Q, f)/\partial Q = 0$ with respect to c_r, we get:

$$\begin{aligned} -\left(1 - G\left(\frac{Q}{1-\gamma}\right)\right) - (p - c_r)\left(\frac{Q'(1-\gamma) + f'\gamma' Q}{(1-\gamma)^2}\right) g\left(\frac{Q}{1-\gamma}\right) &= 0 \\ Q'(1-\gamma) + f'\gamma' Q &< 0 \end{aligned} \quad (3.34)$$

From (3.34), we observe that both Q' and f' can not be positive, since $(1-\gamma)$, Q and γ' are non-negative.

From $\partial\pi(Q,f)/\partial f = 0$, we have:

$$c_r \int_0^{\frac{Q}{1-\gamma}} xg(x)dx + p \int_{\frac{Q}{1-\gamma}}^{\infty} xg(x)dx \quad = \quad E(X)(c_f + f) + \frac{E(X)\gamma}{\gamma'} \tag{3.35}$$

Taking implicit derivative of $\partial\pi(Q,f)/\partial f = 0$ with respect to c_r, we get:

$$\begin{aligned} & f'\gamma'' c_r \int_0^{\frac{Q}{1-\gamma}} xg(x)dx + f'\gamma'' p \int_{\frac{Q}{1-\gamma}}^{\infty} xg(x)dx - E(X)\left(2f'\gamma' + (c_f + f)f'\gamma''\right) \\ & + \gamma' \left(\int_0^{\frac{Q}{1-\gamma}} xg(x)dx + (c_r - p)\left(\frac{Q'(1-\gamma) + f'\gamma' Q}{(1-\gamma)^2}\right)\left(\frac{Q}{1-\gamma}\right) g\left(\frac{Q}{1-\gamma}\right)\right) = 0 \end{aligned} \tag{3.36}$$

Plugging (3.35) in Equation (3.36), we get:

$$\begin{aligned} & f'\gamma'' E(X)(c_f + f) + f'\gamma'' \frac{E(X)\gamma}{\gamma'} - E(X)\left(2f'\gamma' + (c_f + f)f'\gamma''\right) \\ & + \gamma' \left(\int_0^{\frac{Q}{1-\gamma}} xg(x)dx + (c_r - p)\left(\frac{Q'(1-\gamma) + f'\gamma' Q}{(1-\gamma)^2}\right)\left(\frac{Q}{1-\gamma}\right) g\left(\frac{Q}{1-\gamma}\right)\right) = 0 \end{aligned} \tag{3.37}$$

In Equation (3.37), we have:

$$\gamma' \left(\int_0^{\frac{Q}{1-\gamma}} xg(x)dx + (c_r - p)\left(\frac{Q'(1-\gamma) + f'\gamma' Q}{(1-\gamma)^2}\right)\left(\frac{Q}{1-\gamma}\right) g\left(\frac{Q}{1-\gamma}\right)\right) > 0 \tag{3.38}$$

From Equation (3.37) and Inequality (3.38), we obtain:

$$f'\gamma'' E(X)(c_f + f) + f'\gamma'' \frac{E(X)\gamma}{\gamma'} - E(X)\left(2f'\gamma' + (c_f + f)f'\gamma''\right) < 0 \tag{3.39}$$

Inequality (3.39) cannot hold if $f' \leq 0$, thus $f' > 0$. Since both Q' and f' can not be positive, $Q' < 0$.

Next, we repeat our analysis for the case where $c_f > c_r$.

Taking implicit derivative of $\partial\pi(Q,f)/\partial Q = 0$ with respect to c_r, we get:

$$-1 + G(Q) + (c_r - c_f)g(Q)Q' - (p - c_f)\frac{Q'(1-\gamma) + f'\gamma' Q}{(1-\gamma)^2} g\left(\frac{Q}{1-\gamma}\right) \quad = \quad 0 \tag{3.40}$$

$$(c_r - c_f)g(Q)Q' - (p - c_f)\frac{Q'(1-\gamma) + f'\gamma' Q}{(1-\gamma)^2} g\left(\frac{Q}{1-\gamma}\right) \quad > \quad 0 \tag{3.41}$$

From (3.41), we observe that both Q' and f' can not be positive since $(p - c_f)$, $(1 - \gamma)$, Q and γ' are non-negative, and $(c_r - c_f)$ is non-positive.

From $\partial\pi(Q, f)/\partial f = 0$ we have:

$$\int_{\frac{Q}{1-\gamma}}^{\infty} xg(x)dx = \frac{E(X)(\gamma + f\gamma')}{(p - c_f)\gamma'} \tag{3.42}$$

Taking implicit derivative of $\partial\pi(Q, f)/\partial f = 0$ with respect to c_r, we get:

$$(p - c_f)f'\gamma'' \int_{\frac{Q}{1-\gamma}}^{\infty} xg(x)dx - \gamma'(p - c_f)\left(\frac{Q'(1 - \gamma) + f'\gamma' Q}{(1 - \gamma)^2}\right)\left(\frac{Q}{1 - \gamma}\right) g\left(\frac{Q}{1 - \gamma}\right)$$
$$-E(X)\left(2f'\gamma' + ff'\gamma''\right) = 0 \tag{3.43}$$

Plugging (3.42) in Equation (3.43) we get:

$$E(X)f'\left(\frac{\gamma\gamma''}{\gamma'} - 2\gamma'\right) - \gamma'(p - c_f)\left(\frac{Q'(1 - \gamma) + f'\gamma' Q}{(1 - \gamma)^2}\right)\left(\frac{Q}{1 - \gamma}\right) g\left(\frac{Q}{1 - \gamma}\right) = 0 \tag{3.44}$$

From (3.44), we observe that both Q' and f' can not be negative since $(p - c_f)$, $(1 - \gamma)$, Q and γ' are non-negative and γ'' is non-positive.

From Equation (3.40) we get:

$$(p - c_f)\frac{Q'(1 - \gamma) + f'\gamma' Q}{(1 - \gamma)^2} g\left(\frac{Q}{1 - \gamma}\right) = (c_r - c_f)g(Q)Q' + G(Q) - 1 \tag{3.45}$$

Plugging (3.45) in Equation (3.44), we get:

$$E(X)f'\left(\frac{\gamma\gamma''}{\gamma'} - 2\gamma'\right) - \gamma'\left(\frac{Q}{1 - \gamma}\right)\Big((c_r - c_f)g(Q)Q' + G(Q) - 1\Big) = 0 \tag{3.46}$$

Consider the case where Q' is positive and f' is negative. That case contradicts with Equation (3.46).

Due to Inequality (3.41), Equation (3.44) and the contradiction in Equation (3.46), Q^* is non-increasing and f^* is non-decreasing in c_r. ■

Proposition 3.7 *Q^* is non-increasing and f^* is non-decreasing in c_n.*

Proof. We first consider the case where $c_f < c_r$. For expositional clarity, let $Q' = \partial Q^*(c_n)/\partial c_n$ and $f' = \partial f^*(c_n)/\partial c_n$.

Taking implicit derivative of $\partial \pi(Q, f)/\partial Q^* = 0$ with respect to c_n, we get:

$$\begin{aligned}
-(p - c_r)\frac{Q'(1-\gamma) + f'\gamma' Q}{(1-\gamma)^2} g\left(\frac{Q}{1-\gamma}\right) - 1 &= 0 \\
\frac{Q'(1-\gamma) + f'\gamma' Q}{(1-\gamma)^2} g\left(\frac{Q}{1-\gamma}\right) &< 0 \\
Q'(1-\gamma) + f'\gamma' Q &< 0 \qquad (3.47)
\end{aligned}$$

From (3.47), we observe that both Q' and f' can not be positive since $(1-\gamma)$, Q and γ' are non-negative.

From $\partial \pi(Q, f)/\partial f = 0$, we have

$$c_r \int_0^{\frac{Q}{1-\gamma}} x g(x) dx + p \int_{\frac{Q}{1-\gamma}}^{\infty} x g(x) dx = E(X)(c_f + f) + \frac{E(X)\gamma}{\gamma'} \qquad (3.48)$$

Taking implicit derivative of $\partial \pi(Q, f)/\partial f = 0$ with respect to c_n, we get:

$$\begin{aligned}
& f'\gamma'' c_r \int_0^{\frac{Q}{1-\gamma}} x g(x) dx + f'\gamma'' p \int_{\frac{Q}{1-\gamma}}^{\infty} x g(x) dx - E(X)\left(2f'\gamma' + (c_f + f) f'\gamma''\right) \\
& + \gamma' \left((c_r - p) \left(\frac{Q'(1-\gamma) + f'\gamma' Q}{(1-\gamma)^2} \right) \left(\frac{Q}{1-\gamma} \right) g\left(\frac{Q}{1-\gamma} \right) \right) = 0 \qquad (3.49)
\end{aligned}$$

Plugging (3.48) in Equation (3.49), we get:

$$\begin{aligned}
& f'\gamma'' E(X)(c_f + f) + f'\gamma'' \frac{E(X)\gamma}{\gamma'} + \gamma'(c_r - p)\left(\frac{Q'(1-\gamma) + f'\gamma' Q}{(1-\gamma)^2} \right) \left(\frac{Q}{1-\gamma} \right) g\left(\frac{Q}{1-\gamma} \right) \\
& - E(X)\left(2f'\gamma' + (c_f + f) f'\gamma''\right) = 0 \qquad (3.50)
\end{aligned}$$

From (3.47), we have:

$$\gamma'(c_r - p)\left(\frac{Q'(1-\gamma) + f'\gamma' Q}{(1-\gamma)^2} \right) \left(\frac{Q}{1-\gamma} \right) g\left(\frac{Q}{1-\gamma} \right) > 0 \qquad (3.51)$$

Plugging (3.51) in Equation (3.50), we get:

$$
\begin{aligned}
f'\gamma'' E(X)(c_f + f) + f'\gamma'' \frac{E(X)\gamma}{\gamma'} - E(X)\left(2f'\gamma' + (c_f + f)f'\gamma''\right) &< 0 \\
E(X)f'\left(\frac{\gamma''\gamma}{\gamma'} - 2\gamma'\right) &< 0 \qquad (3.52)
\end{aligned}
$$

Inequality (3.52) cannot hold if $f' \leq 0$, thus $f' > 0$. Since both Q' and f' can not be positive, $Q' < 0$.

Next, we repeat our analysis for the case where $c_f > c_r$.

Taking implicit derivative of $\partial\pi(Q, f)/\partial Q = 0$ with respect to c_n, we get:

$$-1 + (c_r - c_f)g(Q)Q' - (p - c_f)\frac{Q'(1-\gamma) + f'\gamma' Q}{(1-\gamma)^2} g\left(\frac{Q}{1-\gamma}\right) = 0 \qquad (3.53)$$

$$(c_r - c_f)g(Q)Q' - (p - c_f)\frac{Q'(1-\gamma) + f'\gamma' Q}{(1-\gamma)^2} g\left(\frac{Q}{1-\gamma}\right) > 0 \qquad (3.54)$$

From Inequality (3.54), we observe that both Q' and f' cannot be positive since $(p - c_f)$, $(1 - \gamma)$, Q and γ' are non-negative, and $(c_r - c_f)$ is non-positive.

From $\partial\pi(Q, f)/\partial f = 0$, we have:

$$\int_{\frac{Q}{1-\gamma}}^{\infty} xg(x)dx = \frac{E(X)(\gamma + f\gamma')}{(p - c_f)\gamma'} \qquad (3.55)$$

Taking implicit derivative of $\partial\pi(Q, f)/\partial f = 0$ with respect to c_n, we get

$$
\begin{aligned}
&(p - c_f)f'\gamma'' \int_{\frac{Q}{1-\gamma}}^{\infty} xg(x)dx - (p - c_f)\gamma' \left(\frac{Q'(1-\gamma) + f'\gamma' Q}{(1-\gamma)^2}\right)\left(\frac{Q}{1-\gamma}\right) g\left(\frac{Q}{1-\gamma}\right) \\
&-E(X)\left(2f'\gamma' + ff'\gamma''\right) = 0 \qquad (3.56)
\end{aligned}
$$

Plugging (3.55) in Equation (3.56), we get:

$$E(X)f'\left(\frac{\gamma''\gamma}{\gamma'} - 2\gamma'\right) - \gamma'\left((p - c_f)\left(\frac{Q'(1-\gamma) + f'\gamma' Q}{(1-\gamma)^2}\right)\left(\frac{Q}{1-\gamma}\right) g\left(\frac{Q}{1-\gamma}\right)\right) = 0 \qquad (3.57)$$

From the equation (3.57), we observe that both Q' and f' cannot be negative since $(p - c_f)$, $(1 - \gamma)$, Q and γ' are non-negative, and γ'' is non-positive.

From (3.53), we have:

$$(p - c_f)\left(\frac{Q'(1-\gamma) + f'\gamma' Q}{(1-\gamma)^2}\right) g\left(\frac{Q}{1-\gamma}\right) \quad = \quad -1 + (c_r - c_f)g(Q)Q' \tag{3.58}$$

Plugging (3.58) in Equation (3.57), we get:

$$E(X)f'\left(\frac{\gamma''\gamma}{\gamma'} - 2\gamma'\right) - \gamma'\left((c_r - c_f)g(Q)Q' - 1\right)\left(\frac{Q}{1-\gamma}\right) = 0 \tag{3.59}$$

Consider the case where Q' is positive and f' is negative. This case contradicts with Equation (3.59).

Due to Inequality (3.54), Equation (3.57) and the contradiction explained for Equation (3.59), Q^* is non-increasing and f^* is non-decreasing in c_n. ■

As c_r and/or c_r increases, unit profit margin of brand-new containers decreases. Hence, the producer decreases Q^* and increases f^*, so the optimal expected quantity of returns, in order to satisfy the demand.

In Proposition 3.5, Proposition 3.6 and Proposition 3.7, the effects of cost parameters on the optimal levels of decision variables are investigated analytically. The results indicate that the producer increases the acquisition fee offered to customers for returned containers, (or equivalently decreases the return quantity of brand-new containers) as

- Unit cost of refilling decreases,
- Unit cost of filling increases,
- Unit cost of purchasing brand-new containers increases.

CHAPTER 4

ANALYSIS OF THE REUSABLE CONTAINER SYSTEMS - THE CAPACITATED CASE

In this chapter, we extend the model presented in Chapter 3 to the case where the aggregate capacity of filling and refilling is limited. It is assumed that filling and refilling options are identical with respect to capacity requirements; a unit of each option requires one unit of capacity. In this chapter, we investigate following research issues analytically:

- What are the optimal levels of acquisition fee and order quantity of brand new containers in order to maximize the producer's expected profit in a capacitated environment?
- What is the effect of a restriction in production capacity on the optimal levels of decision variables?

The sequence of events in this setting are as follows:

- Order quantity of brand-new reusable containers, Q, and the unit acquisition fee, f, are determined. Total purchasing cost of brand-new reusable containers $c_n Q$ is incurred.
- Demand, D, and returns, γD, are realized. Total acquisition cost for returned containers $f\gamma D$ is incurred.
- The producer determines quantity of brand new containers to be filled, $M \leq Q$ and the quantity of returned containers to be refilled, $R \leq \gamma D$, to satisfy the demand without exceeding the total capacity C. Note that since the demand has already been realized, the producer never produces more than realized demand, i.e, $M + R \leq D$.

- Total cost of filling $c_r M$ and refilling $c_f R$ are incurred. A total revenue of $p(M+R)$ is received.

In order to characterize the optimal solution, we employ a two-stage modeling approach similar to the uncapacitated case. First, we solve the second stage problem to determine the optimal production levels for new containers, M, and returned containers, R, given the first stage decisions and the demand realization. Then, we incorporate this solution to the first stage problem and solve it for optimal levels of Q and f.

The second stage problem can be stated as:

$$\text{Maximize } \pi(Q, f) = p(M+R) - c_r M - c_f R \tag{4.1}$$

subject to

$$M + R \leq D \tag{4.2}$$

$$M + R \leq C \tag{4.3}$$

$$M \leq Q \tag{4.4}$$

$$R \leq \gamma D \tag{4.5}$$

$$M \geq 0 \tag{4.6}$$

$$R \geq 0 \tag{4.7}$$

Objective function (4.1) consists of total revenue gained, $p(M+R)$, total cost of using new containers, $c_r M$, and total cost of reusing returned containers, $c_f R$. Note that the total purchasing cost of new containers, $c_n Q$, and total acquisition cost of returned containers, $f\gamma D$ are not affected by the second stage decisions, hence they are excluded in the objective function. Constraint (4.2) ensures that total number of new containers filled and returned containers refilled does not exceed demand. Constraint (4.3) indicates that the total number of containers, filled or refilled, does not exceed the available capacity. Constraint (4.4) indicates that level of production using new containers cannot exceed the quantity of new containers purchased. Similarly, constraint (4.5) ensures that the level of production with returned containers are less than or equal to the quantity of returns. Constraint (4.6) and Constraint (4.7) are non-negativity constraints for M and R, respectively.

The profit function of the second stage problem is linear in decision variables M and R. Then the optimal solution is intuitive: Same as the uncapacitated case, either M or R will have a priority over the other in satisfying the demand, depending on their corresponding profit margins, i.e., $p-c_r$ and $p-c_f$ respectively. If reusing a returned reusable container is cheaper, that is if $c_f < c_r$, R should be set to its highest possible value, and then the remaining demand should be covered by M as much as the other constraints allow. Whereas, if using a brand new container is cheaper, that is if $c_r < c_f$, M should be set to its highest possible value, and then the remaining demand, if any, should be covered by R as much as the capacity constraint allow.

Since the optimal decisions in Stage 2 depend on whether $c_r > c_f$ or not, these two cases are analyzed separately. In Section 4.1 we analyze the case where reusing a returned reusable container is cheaper and in Section 4.2, we analyze the other case where using a brand new container is cheaper.

4.1 P_1: Reusing a Returned Container is Cheaper ($c_f < c_r$)

Proposition 4.1 characterizes the optimal production quantities given the first stage decisions, Q and f, and the demand realization, D.

Proposition 4.1 *When $c_f < c_r$, the optimal production quantities given the first stage decisions, Q and f, and the demand realization, D, are as follows:*

$$(M^*, R^*|D) = \begin{cases} (D - \gamma D, \gamma D) & \text{if } Q + \gamma D > D \text{ and } D < C \\ (Q, \gamma D) & \text{if } Q + \gamma D < D \text{ and } Q + \gamma D < C \\ (C - \gamma D, \gamma D) & \text{if } Q + \gamma D > C,\ \gamma D < C \text{ and } D > C \\ (0, C) & \text{if } \gamma D > C \end{cases}$$

Proof. If $Q + \gamma D > D$ and $D < C$, the demand can be met in full and Constraint (4.3) is non-binding; that is Constraint (4.2) is binding. By Constraint (4.5) R is set to γD. By Constraint (4.2), M is set to $D - \gamma D$. Hence, we have the optimal solution $(M^*, R^*) = (D - \gamma, \gamma D)$.

If $Q+\gamma D < D$ and $D < C$, the demand cannot be met in full and Constraint (4.3) is again non-binding. Hence both options are utilized to the fullest extent possible; that is, Constraint (4.4) and Constraint (4.5) are binding. Hence, we have the optimal solution $(M^*, R^*) = (Q, \gamma D)$.

If $Q+\gamma D > C$, $\gamma D < C$ and $D > C$, the capacity is utilized to the fullest extent and Constraint (4.2) is non-binding whereas Constraint (4.3) is binding. By Constraint (4.5), R is set to γD. By Constraint (4.3), M is set to $C - \gamma D$. Hence, we have the optimal solution $(M^*, R^*) = (C - \gamma, \gamma D)$.

If $\gamma D > C$, the capacity is utilized to the fullest extent and Constraint (4.2) is non-binding whereas Constraint (4.3) is binding. By Constraint (4.3), R is set to C. Hence, we have the optimal solution $(M^*, R^*) = (0, C)$. ■

As characterized in Proposition 4.1, the second stage optimal decisions depend on the demand realization and the values of decision variables determined in the first stage. Given the demand, the profit of the first stage can be expressed as follows:

$$\pi(Q, f|D) = \begin{cases} pD - c_r(D-\gamma D) - c_n Q - c_f\gamma D - f\gamma D & \text{if } Q+\gamma D > D \text{ and } D < C \\ p(Q+\gamma D) - c_r Q - c_n Q - c_f\gamma D - f\gamma D & \text{if } Q+\gamma D < D \text{ and } Q+\gamma D < C \\ pC - c_r(C-\gamma D) - c_n Q - c_f\gamma D - f\gamma D & \text{if } Q+\gamma D > C, \gamma D < C \text{ and } D > C \\ pC - c_n Q - c_f C - f\gamma D & \text{if } \gamma D > C \end{cases}$$

Note that regardless of the demand realization, the producer pays $c_n Q$ for purchasing brand-new containers and $f\gamma D$ for the acquisition of returns. If $Q + \gamma D > D$ and $D < C$, or equivalently $D < Q/(1-\gamma)$ and $D < C$, all demand is satisfied and the sales revenue is pD. In this case $D-\gamma D$ of the brand-new containers and all of the returned containers are utilized; $c_r(D-\gamma D)$ is incurred as total filling cost and $c_f\gamma D$ is incurred as total refilling cost. If $Q+\gamma D < D$ and $Q+\gamma D < C$, or equivalently $Q(1-\gamma) < D < (C-Q)/\gamma$, all supply is used to satisfy the demand. Hence the sales revenue is $p(Q+\gamma D)$, filling cost is $c_r Q$, and refilling cost is $c_f\gamma D$. If $Q+\gamma D > C$, $\gamma D < C$ and $D > C$, or equivalently $(C-Q)/\gamma < D < C/\gamma$ and $D > C$, the available capacity is utilized to the fullest extent and the sales revenue is pC. In that case, $C-\gamma D$ of the brand-new containers and all of the returned containers are utilized; $c_r(C-\gamma D)$ is incurred as total filling cost and $c_f\gamma D$ is incurred as total refilling cost. If $\gamma D > C$, or equivalently $D > C/\gamma$, the available capacity is again utilized fully and the sales revenue is pC. In that case, all returned containers are utilized and none of the brand-new containers are used; only $c_f C$ is incurred as refilling cost.

Note that the bounds on the demand realization in $\pi(Q, f|D)$ can be further refined, depending on whether $Q > C(1-\gamma)$ or not, which results in the following representation of $\pi(Q, f|D)$:

$$\pi(Q,f|D) = \begin{cases} \pi_i(Q,f|D) & \text{if } C \geq Q \geq C(1-\gamma) \\ \pi_{ii}(Q,f|D) & \text{if } Q \leq C(1-\gamma) \end{cases}$$

where

$$\pi_i(Q,f|D) = \begin{cases} pD - c_r(D-\gamma D) - c_n Q - c_f\gamma D - f\gamma D & \text{if } D < C \\ pC - c_r(C-\gamma D) - c_n Q - c_f\gamma D - f\gamma D & \text{if } C < D < C/\gamma \\ pC - c_n Q - c_f C - f\gamma D & \text{if } D > C/\gamma \end{cases}$$

$$\pi_{ii}(Q,f|D) = \begin{cases} pD - c_r(D-\gamma D) - c_n Q - c_f\gamma D - f\gamma D & \text{if } D < Q/(1-\gamma) \\ p(Q+\gamma D) - c_r Q - c_n Q - c_f\gamma D - f\gamma D & \text{if } Q/(1-\gamma) < D < (C-Q)/\gamma \\ pC - c_r(C-\gamma D) - c_n Q - c_f\gamma D - f\gamma D & \text{if } (C-Q)/\gamma < D < C/\gamma \\ pC - c_n Q - c_f C - f\gamma D & \text{if } D > C/\gamma \end{cases}$$

The problem of finding the Q and f values maximizing the expected profit can be formulated as follows:

$$[P_1] : \text{Maximize } \pi(Q,f) = \begin{cases} \pi_i(Q,f) & \text{if } C \geq Q \geq C(1-\gamma) \\ \pi_{ii}(Q,f) & \text{if } Q \leq C(1-\gamma) \end{cases}$$

subject to

$$\begin{aligned} Q &\geq 0 \\ f &\geq 0 \end{aligned}$$

where

$$\begin{aligned} \pi_i(Q,f) &= p\int_0^C xg(x)dx + p\int_C^\infty Cg(x)dx - c_n Q - f\gamma E(X) - c_r\int_0^C (x-\gamma x)g(x)dx \\ &- c_r\int_C^{\frac{C}{\gamma}} (C-\gamma x)g(x)dx - c_f\int_0^{\frac{C}{\gamma}} \gamma xg(x)dx - c_f\int_{\frac{C}{\gamma}}^\infty Cg(x)dx \end{aligned}$$

$$\begin{aligned}
\pi_{ii}(Q,f) &= p\int_0^{\frac{Q}{1-\gamma}} xg(x)dx + p\int_{\frac{Q}{1-\gamma}}^{\frac{C-Q}{\gamma}} (\gamma x+Q)g(x)dx + p\int_{\frac{C-Q}{\gamma}}^{\infty} Cg(x)dx - c_nQ - f\gamma E(X)\\
&- c_r\int_0^{\frac{Q}{1-\gamma}} (x-\gamma x)g(x)dx - c_r\int_{\frac{Q}{1-\gamma}}^{\frac{C-Q}{\gamma}} Qg(x)dx - c_r\int_{\frac{C-Q}{\gamma}}^{\frac{C}{\gamma}} (C-\gamma x)g(x)dx\\
&- c_f\int_0^{\frac{C}{\gamma}} \gamma xg(x)dx - c_f\int_{\frac{C}{\gamma}}^{\infty} Cg(x)dx
\end{aligned}$$

Lemma 4.1 *$\pi(Q,f)$ is a continuously differentiable function of Q and f.*

Proof.

$$\begin{aligned}
\pi_i(C(1-\gamma),f) = \pi_{ii}(C(1-\gamma),f) &= p\int_0^{C} xg(x)dx + p\int_{C}^{\infty} Cg(x)dx - c_nC(1-\gamma) - f\gamma E(X)\\
&- c_r\int_0^{C} (1-\gamma)xg(x)dx - c_r\int_{C}^{\frac{C}{\gamma}} (C-\gamma x)g(x)dx\\
&- c_f\int_0^{\frac{C}{\gamma}} \gamma xg(x)dx - c_f\int_{\frac{C}{\gamma}}^{\infty} Cg(x)dx \qquad (4.8)
\end{aligned}$$

Due to (4.8), $\pi(Q,f)$ is continuous at $Q = C(1-\gamma)$.

First order derivatives of $\pi(Q,f)$ with respect to Q and f are as follows:

$$\begin{aligned}
\frac{\partial\pi_i(Q,f)}{\partial Q} &= -c_n\\
\frac{\partial\pi_i(Q,f)}{\partial f} &= \gamma'(c_r-c_f)\int_0^{\frac{C}{\gamma}} xg(x)dx - E(X)(\gamma' f+\gamma)\\
\frac{\partial\pi_{ii}(Q,f)}{\partial Q} &= (p-c_r)\left(G\left(\frac{C-Q}{\gamma}\right) - G\left(\frac{Q}{1-\gamma}\right)\right) - c_n\\
\frac{\partial\pi_{ii}(Q,f)}{\partial f} &= \gamma'(p-c_r)\int_{\frac{Q}{1-\gamma}}^{\frac{C-Q}{\gamma}} xg(x)dx + \gamma'(c_r-c_f)\int_0^{\frac{C}{\gamma}} xg(x)dx - E(X)(\gamma' f+\gamma)
\end{aligned}$$

$$\frac{\partial \pi_i(Q,f)}{\partial Q}|_{Q=C(1-\gamma)} = \frac{\partial \pi_{ii}(Q,f)}{\partial Q}|_{Q=C(1-\gamma)} = -c_n \tag{4.9}$$

$$\begin{aligned} \frac{\partial \pi_i(Q,f)}{\partial f}|_{Q=C(1-\gamma)} = \frac{\partial \pi_{ii}(Q,f)}{\partial f}|_{Q=C(1-\gamma)} &= \gamma'(c_r - c_f)\int_0^{\frac{C}{\gamma}} xg(x)dx \\ &- E(X)(\gamma' f + \gamma) \end{aligned} \tag{4.10}$$

Equations (4.8), (4.9) and (4.10) show that $\pi(Q, f)$ is a continuously differentiable function of Q and f. ■

Proposition 4.2 *Let (Q^*, f^*) denote the optimal solution to P_1. Then $Q^* \leq C(1 - \gamma^*)$.*

Proof. Consider a feasible solution in the region $Q > C(1-\gamma)$. The expected profit function in this region is $\pi_i(Q, f)$. Since $\partial\pi_i(Q, f)/\partial Q = -c_n$, decreasing Q always improves the objective function. Hence, a feasible solution (Q, f) such that $Q > C(1 - \gamma)$ cannot be optimal. ■

Proposition 4.3 *Let (Q^*, f^*) denote the optimal solution to P_1. If there is a non-negative solution (Q^0, f^0) to $\partial\pi_{ii}(Q, f)/\partial Q = 0$ and $\partial\pi_{ii}(Q, f)/\partial f = 0$, then $Q^* = Q^0$ and $f^* = f^0$. Otherwise, $Q^* = 0$ and f^* satisfies*

$$(p - c_f)\gamma^{*\prime} \int_0^{\frac{C}{\gamma^*}} xg(x)dx - E(X)(\gamma^{*\prime} f^* + \gamma^*) = 0 \tag{4.11}$$

Proof. In Proposition 4.2, it is shown that the optimal solution satisfies $Q \leq C(1 - \gamma)$. Hence, the problem can be reformulated as

Maximize $\pi_{ii}(Q, f)$

subject to

$$\begin{aligned} Q &\leq C(1-\gamma) \\ Q &\geq 0 \\ f &\geq 0 \end{aligned}$$

The second order derivatives of $\pi_{ii}(Q, f)$ with respect to Q and f are:

$$\frac{\partial^2 \pi_{ii}(Q,f)}{\partial Q^2} = (p-c_r)\left(\left(\frac{-1}{\gamma}\right)g\left(\frac{C-Q}{\gamma}\right)-\left(\frac{1}{1-\gamma}\right)g\left(\frac{Q}{1-\gamma}\right)\right) \tag{4.12}$$

$$\begin{aligned}\frac{\partial^2 \pi_{ii}(Q,f)}{\partial f^2} &= \gamma''(p-c_r)\int_{\frac{Q}{1-\gamma}}^{\frac{C-Q}{\gamma}} xg(x)dx + \gamma''(c_r-c_f)\int_{0}^{\frac{C}{\gamma}} xg(x)dx \\ &+ \gamma'(p-c_r)\left(\left(\frac{C-Q}{\gamma}\right)\left(\frac{-(C-Q)\gamma'}{\gamma^2}\right)g\left(\frac{C-Q}{\gamma}\right)-\left(\frac{Q^2\gamma'}{(1-\gamma)^3}\right)g\left(\frac{Q}{1-\gamma}\right)\right) \\ &+ \gamma'(c_r-c_f)\left(\frac{C}{\gamma}\left(\frac{-C\gamma'}{\gamma^2}\right)g\left(\frac{C}{\gamma}\right)\right) - E(X)(\gamma'' f + 2\gamma')\end{aligned} \tag{4.13}$$

The determinant of the Hessian matrix for $\pi_{ii}(Q, f)$ where $\partial\pi_{ii}(Q, f)/\partial Q = 0$ and $\partial\pi_{ii}(Q, f)/\partial f = 0$ is:

$$\begin{aligned}|H| &= (\gamma')^2(p-c_r)^2 g\left(\frac{C-Q}{\gamma}\right)g\left(\frac{Q}{1-\gamma}\right)\left(\frac{C^2+Q^2-2QC+C^2\gamma^2-2C^2\gamma+2CQ\gamma}{(1-\gamma)^3\gamma^3}\right) \\ &+ \gamma'(p-c_r)(c_r-c_f)\left(\frac{-C^2\gamma'}{\gamma^3}\right)g\left(\frac{C}{\gamma}\right)\left(\left(\frac{-1}{\gamma}\right)g\left(\frac{C-Q}{\gamma}\right)-\frac{1}{1-\gamma}g\left(\frac{Q}{1-\gamma}\right)\right) \\ &+ E(X)(p-c_r)\left(\left(\frac{-1}{\gamma}\right)g\left(\frac{C-Q}{\gamma}\right)-\frac{1}{1-\gamma}g\left(\frac{Q}{1-\gamma}\right)\right)\left(\frac{\gamma''\gamma}{\gamma'}-2\gamma'\right)\end{aligned}$$

The second order derivatives of $\pi_{ii}(Q, f)$, Equation (4.12) and Equation (4.13) are negative on the stationary points, where the first order derivatives of $\pi_{ii}(Q, f)$ are equal to 0 for all non-negative Q and f values. Additionally, the determinant of the Hessian matrix on the stationary points of $\pi_{ii}(Q, f)$ is positive for all non-negative Q and f values. Hence the expected profit function of P_1, $\pi(Q, f)$ is unimodal and has a unique maximum on $\partial\pi_{ii}(Q, f)/\partial Q = 0$ and $\partial\pi_{ii}(Q, f)/\partial f = 0$ (Demidenko, 2004) given that there exists a non-negative (Q, f) satisfying $\partial\pi_{ii}(Q, f)/\partial Q = 0$ and $\partial\pi_{ii}(Q, f)/\partial f = 0$.

Suppose there is a non-negative solution (Q^0, f^0) to $\partial\pi_{ii}(Q, f)/\partial Q = 0$ and $\partial\pi_{ii}(Q, f)/\partial f = 0$. From $\partial\pi_{ii}(Q, f)/\partial Q = 0$, we have:

$$\begin{aligned}(p-c_r)\left(G\left(\frac{C-Q^0}{\gamma^0}\right)-G\left(\frac{Q^0}{1-\gamma^0}\right)\right)-c_n &= 0 \\ G\left(\frac{C-Q^0}{\gamma^0}\right)-G\left(\frac{Q^0}{1-\gamma^0}\right) &> 0 \\ \frac{C-Q^0}{\gamma^0} &> \frac{Q^0}{1-\gamma^0} \\ C(1-\gamma^0) &> Q^0\end{aligned}$$

Hence a non-negative solution (Q^0, f^0) satisfies the constraints and due to unimodality, it is optimal.

If there is no such non-negative (Q^0, f^0) solution, then the optimal solution lies on the boundary. We next consider possible solutions on the boundary.

Boundary Condition 1: All constraints are binding: It is not possible.

Boundary Condition 2: $Q < C(1-\gamma)$, $Q = 0$, $f = 0$: $\partial\pi_{ii}(Q, f)/\partial Q = p - c_r - c_n > 0$. Hence, the optimal solution can not be on this boundary.

Boundary Condition 3: $Q = C(1-\gamma)$, $Q > 0$, $f = 0$: $\partial\pi_{ii}(Q, f)/\partial Q = -c_n < 0$. Hence, the optimal solution can not be on this boundary.

Boundary Condition 4: $Q = C(1-\gamma)$, $Q = 0$, $f > 0$: This condition cannot be observed unless $f \to \infty$. If f approaches to infinity, then $\partial\pi_{ii}(Q, f)/\partial f = -E(X)(\gamma' f + \gamma) < 0$. Hence, the optimal solution cannot be on this boundary.

Boundary Condition 5: $Q < C(1-\gamma)$, $Q > 0$, $f = 0$: $\partial\pi_{ii}(Q, f)/\partial f = \gamma'(p - c_r)\int_Q^\infty xg(x)dx + \gamma'(c_r - c_f)\int_0^\infty xg(x)dx > 0$. Hence, the optimal solution cannot be on this boundary.

Boundary Condition 6: $Q = C(1-\gamma)$, $Q > 0$, $f > 0$: $\partial\pi_{ii}(Q, f)/\partial Q = -c_n < 0$. Hence, the optimal solution can not be on this boundary.

Boundary Condition 7: $Q < C(1-\gamma)$, $Q = 0$, $f > 0$: $\partial\pi_{ii}(Q, f)/\partial Q = (p - c_r)G(C/\gamma) - c_n$ can be negative or positive. If it is negative, the optimal solution is on the boundary $Q = 0$ and the corresponding f^* value is calculated accordingly. When Q is set to 0, the objective function can be rewritten as follows:

$$\pi_{ii}(Q, f|Q = 0) = (p - c_f)\left(\int_0^{\frac{C}{\gamma}} \gamma xg(x)dx + \int_{\frac{C}{\gamma}}^\infty Cg(x)dx\right) - f\gamma E(X)$$

The modified objective function is a function of f, and f^* can be found by the first order optimality condition of the function as below:

$$\frac{\partial\pi_{ii}(Q, f|Q = 0)}{\partial f} = \gamma^{*\prime}(p - c_f)\int_0^{\frac{C}{\gamma^*}} xg(x)dx - E(X)(\gamma^{*\prime} f^* + \gamma^*) = 0$$

■

In Proposition 4.3, the optimal solution to P_1 is characterized. In the optimal solution to P_1, f^* always takes a positive value indicating that the producer accepts returns in order to maximize his expected profit. Since $c_f < c_r$ and the producer gives priority to returned containers in P_1, this result is expected.

4.2 P_2: Using a Brand New Container is Cheaper ($c_f > c_r$)

Proposition 4.4 characterizes the optimal production quantities given the first stage decisions, Q and f, and the demand realization, D.

Proposition 4.4 *When $c_f > c_r$, the optimal production quantities given the first stage decisions, Q and f and the demand realization, D, are given as follows:*

$$(M^*, R^*|D) = \begin{cases} (D, 0) & \text{if } Q > D \text{ and } \gamma D < C \\ (Q, D-Q) & \text{if } Q + \gamma D > D,\ Q < D \text{ and } D < C \\ (Q, \gamma D) & \text{if } Q + \gamma D < D \text{ and } Q + \gamma D < C \\ (Q, C-Q) & \text{if } Q + \gamma D > C,\ Q < C \text{ and } D > C \end{cases}$$

Proof. If $Q > D$ and $D < C$, the demand can be met in full, Constraint (4.3) and Constraint (4.4) are non-binding whereas Constraint (4.2) is binding. By Constraint (4.2), M is set to D. Hence, we have the optimal solution $(M^*, R^*) = (D, 0)$.

If $Q + \gamma D > D$, $Q < D$ and $D < C$, the demand can be met in full and Constraint (4.3) is non-binding. By Constraint (4.4), M is set to Q. By Constraint (4.2) R is set to $D - Q$. Hence, we have the optimal solution $(M^*, R^*) = (D - Q, Q)$.

If $Q + \gamma D < D$ and $Q + \gamma D < C$, the demand can not be met in full and Constraint (4.3) is non-binding. Hence, both options will be utilized to the fullest extent possible; both Constraint (4.4) and Constraint (4.5) are binding. Hence, we have the optimal solution $(M^*, R^*) = (Q, \gamma D)$.

If $Q + \gamma D > C$, $Q < C$ and $D > C$, the capacity is utilized to the fullest extent and Constraint (4.2) is non-binding whereas Constraint (4.3) is binding. By Constraint (4.4), M is set to Q. By Constraint (4.3), R is set to $C - Q$. Hence, we have the optimal solution $(M^*, R^*) = (Q, C - Q)$. ■

As characterized in Proposition 4.4, the second stage optimal decisions depend on the random demand and the values of decision variables determined in the first stage. Given the demand realization and the values of decision variables determined in the first stage, the profit of the first stage can be expressed as follows:

$$\pi(Q, f|D) = \begin{cases} pD - c_r D - c_n Q - f\gamma D & \text{if } Q > D \text{ and } \gamma D < C \\ pD - c_r Q - c_n Q - c_f(D - Q) - f\gamma D & \text{if } Q + \gamma D > D,\ Q < D \text{ and } D < C \\ p(Q + \gamma D) - c_r Q - c_n Q - c_f \gamma D - f\gamma D & \text{if } Q + \gamma D < D \text{ and } Q + \gamma D < C \\ pC - c_r Q - c_n Q - c_f(C - Q) - f\gamma D & \text{if } Q + \gamma D > C,\ Q < C \text{ and } D > C \end{cases}$$

Note that regardless of the demand realization, the producer pays $c_n Q$ for purchasing brand-new containers and $f\gamma D$ for the acquisition of returns. If $Q > D$ and $\gamma D < C$, or equivalently $D < Q$ and $D < C/\gamma$, all demand is satisfied with brand-new containers and the sales revenue is pD. In this case, none of the returned containers are utilized and only $c_r D$ is incurred as total filling cost. If $Q + \gamma D > D$, $Q < D$ and $D < C$, or equivalently $Q < D < Q/(1 - \gamma)$ and $D < C$, all demand is satisfied and the sales revenue is pD. In this case, all new containers and $D - Q$ returned containers are utilized; $c_r Q$ is incurred as total filling cost and $c_f(D - Q)$ is incurred as total refilling cost. If $Q + \gamma D < D$ and $Q + \gamma D < C$, or equivalently $Q/(1 - \gamma) < D < (C - Q)/\gamma$, all supply is used to satisfy the demand. Hence the sales revenue is $p(Q+\gamma D)$, filling cost is $c_r Q$, and refilling cost is $c_f \gamma D$. If $Q+\gamma D > C$, $Q < C$ and $D > C$, or equivalently $(C - Q)/\gamma < D$ and $D > C > Q$, the available capacity is utilized to the fullest extent and the sales revenue is pC. In that case, all of the brand-new containers and $C - Q$ of the returned containers are utilized; $c_r Q$ is incurred as total filling cost and $c_f(C - Q)$ is incurred as total refilling cost.

Note that the bounds on the demand realization in $\pi(Q, f|D)$ can be further refined, depending on whether $Q > C(1 - \gamma)$ or not, which results in the following representation of $\pi(Q, f|D)$:

$$\pi(Q, f|D) = \begin{cases} \pi_i(Q, f|D) & \text{if } C \geq Q \geq C(1 - \gamma) \\ \pi_{ii}(Q, f|D) & \text{if } Q \leq C(1 - \gamma) \end{cases}$$

where

$$\pi_i(Q, f|D) = \begin{cases} pD - c_r D - c_n Q - f\gamma D & \text{if } D < Q \\ pD - c_r Q - c_n Q - c_f(D - Q) - f\gamma D & \text{if } Q < D < C \\ pC - c_r Q - c_n Q - c_f(C - Q) - f\gamma D & \text{if } D > C \end{cases}$$

$$\pi_{ii}(Q,f|D) = \begin{cases} pD - c_rD - c_nQ - f\gamma D & \text{if } D < Q \\ pD - c_rQ - c_nQ - c_f(D-Q) - f\gamma D & \text{if } Q < D < Q/(1-\gamma) \\ p(Q+\gamma D) - c_rQ - c_nQ - c_f\gamma D - f\gamma D & \text{if } Q/(1-\gamma) < D < (C-Q)/\gamma \\ pC - c_rQ - c_nQ - c_f(C-Q) - f\gamma D & \text{if } D > (C-Q)/\gamma \end{cases}$$

Then, the problem of finding the Q and f values maximizing the expected profit can be formulated as follows:

$$[P_2] : \text{Maximize } \pi(Q,f) = \begin{cases} \pi_i(Q,f) & \text{if } C \geq Q \geq C(1-\gamma) \\ \pi_{ii}(Q,f) & \text{if } Q \leq C(1-\gamma) \end{cases}$$

subject to

$$\begin{aligned} Q &\geq 0 \\ f &\geq 0 \end{aligned}$$

where

$$\begin{aligned} \pi_i(Q,f) &= p\int_0^C xg(x)dx + p\int_C^\infty Cg(x)dx - c_nQ - f\gamma E(X) - c_r\int_0^Q xg(x)dx \\ &- c_r\int_Q^\infty Qg(x)dx - c_f\int_Q^C (x-Q)g(x)dx - c_f\int_C^\infty (C-Q)g(x)dx \end{aligned}$$

$$\begin{aligned} \pi_{ii}(Q,f) &= p\int_0^{\frac{Q}{1-\gamma}} xg(x)dx + p\int_{\frac{Q}{1-\gamma}}^{\frac{C-Q}{\gamma}} (\gamma x + Q)g(x)dx + p\int_{\frac{C-Q}{\gamma}}^\infty Cg(x)dx - c_nQ \\ &- f\gamma E(X) - c_r\int_0^Q xg(x)dx - c_r\int_Q^\infty Qg(x)dx - c_f\int_Q^{\frac{Q}{1-\gamma}} (x-Q)g(x)dx \\ &- c_f\int_{\frac{Q}{1-\gamma}}^{\frac{C-Q}{\gamma}} \gamma xg(x)dx - c_f\int_{\frac{C-Q}{\gamma}}^\infty (C-Q)g(x)dx \end{aligned}$$

Lemma 4.2 *$\pi(Q,f)$ is a continuously differentiable function of Q and f.*

Proof.

$$\begin{aligned}
\pi_i(C(1-\gamma), f) = \pi_{ii}(C(1-\gamma), f) &= p\int_0^C xg(x)dx + p\int_C^\infty Cg(x)dx - c_n C(1-\gamma) - f\gamma E(X) \\
&- c_r \int_0^{C(1-\gamma)} c_r x g(x)dx - c_r \int_{C(1-\gamma)}^{\infty} C(1-\gamma)g(x)dx \\
&- c_f \int_{C(1-\gamma)}^{C} (x - C(1-\gamma))g(x)dx - c_f \int_C^\infty C\gamma g(x)dx
\end{aligned} \tag{4.14}$$

Due to (4.14), $\pi(Q, f)$ is continuous at $C(1-\gamma)$.

First order derivatives of $\pi(Q, f)$ with respect to Q and f are as follows:

$$\begin{aligned}
\frac{\partial \pi_i(Q, f)}{\partial Q} &= c_f - c_n - c_r - (c_f - c_r)G(Q) \\
\frac{\partial \pi_i(Q, f)}{\partial f} &= -E(X)(\gamma' f + \gamma) \\
\frac{\partial \pi_{ii}(Q, f)}{\partial Q} &= c_f - c_n - c_r + (p - c_f)\left(G\left(\frac{C-Q}{\gamma}\right) - G\left(\frac{Q}{1-\gamma}\right)\right) - (c_f - c_r)G(Q) \\
\frac{\partial \pi_{ii}(Q, f)}{\partial f} &= \gamma'(p - c_f)\int_{\frac{Q}{1-\gamma}}^{\frac{C-Q}{\gamma}} xg(x)dx - E(X)(\gamma' f + \gamma)
\end{aligned}$$

$$\frac{\partial \pi_i(Q, f)}{\partial Q}|_{Q=C(1-\gamma)} = \frac{\partial \pi_{ii}(Q, f)}{\partial Q}|_{Q=C(1-\gamma)} = c_f - c_n - c_r - (c_f - c_r)G(C(1-\gamma)) \tag{4.15}$$

$$\frac{\partial \pi_i(Q, f)}{\partial f}|_{Q=C(1-\gamma)} = \frac{\partial \pi_{ii}(Q, f)}{\partial f}|_{Q=C(1-\gamma)} = -E(X)(\gamma' f + \gamma) \tag{4.16}$$

Equations (4.14), (4.15) and (4.16) show that $\pi(Q, f)$ is a continuously differentiable function of Q and f. ■

Proposition 4.5 *Let (Q^*, f^*) denote the optimal solution to P_2. Then $Q^* \leq C(1-\gamma^*)$.*

Proof. Consider a feasible solution in the region $C \geq Q > C(1-\gamma)$. The expected profit function in this region is $\pi_i(Q, f)$. Since $\partial \pi_i(Q, f)/\partial f = -E(X)(\gamma' f + \gamma)$, decreasing f always improves the objective function. Hence a feasible solution (Q, f) such that $C \geq Q > C(1-\gamma)$ can not be optimal. ■

Proposition 4.6 *Let (Q^*, f^*) denote the optimal solution to P_2. If there is a non-negative solution (Q^0, f^0) to $\partial\pi_{ii}(Q, f)/\partial Q = 0$ and $\partial\pi_{ii}(Q, f)/\partial f = 0$, then $Q^* = Q^0$ and $f^* = f^0$. Otherwise, the solution is on the boundary:*

1. *If $c_f - c_n - c_r + (c_f - c_r)G(C) < 0$, then the optimal solution is $Q^* = 0$, $f^* = f$ which satisfies:*

$$(p - c_f)\gamma' \int_0^{\frac{C}{\gamma}} xg(x)dx - E(X)(\gamma' f + \gamma) \quad = \quad 0$$

2. *If $c_f - c_n - c_r + (c_f - c_r)G(C) > 0$, the optimal solution is $Q^* = C$, $f^* = 0$.*

Proof. In Proposition 4.5, it is shown that the optimal solution satisfies $Q \leq C(1 - \gamma)$. Hence the problem can be formulated as

$$\text{Maximize } \pi_{ii}(Q, f)$$

$$\text{subject to}$$

$$\begin{aligned} Q &\leq C(1-\gamma) \\ Q &\geq 0 \\ f &\geq 0 \end{aligned}$$

The second order conditions of $\pi_{ii}(Q, f)$ are:

$$\frac{\partial^2 \pi_{ii}(Q, f)}{\partial Q^2} = (p - c_f)\left(\frac{-1}{\gamma} g\left(\frac{C - Q}{\gamma}\right) - \frac{1}{1-\gamma} g\left(\frac{Q}{1-\gamma}\right)\right) - (c_f - c_r)g(Q) \tag{4.17}$$

$$\begin{aligned} \frac{\partial^2 \pi_{ii}(Q, f)}{\partial f^2} &= \gamma'(p - c_f)\left(\left(\frac{C - Q}{\gamma}\right)\left(\frac{(Q - C)\gamma'}{\gamma^2}\right) g\left(\frac{C - Q}{\gamma}\right) - \left(\frac{Q^2\gamma'}{(1-\gamma)^3}\right) g\left(\frac{Q}{1-\gamma}\right)\right) \\ &+ \gamma''(p - c_f) \int_{\frac{Q}{1-\gamma}}^{\frac{C-Q}{\gamma}} xg(x)dx - E(X)(\gamma'' f + 2\gamma') \end{aligned} \tag{4.18}$$

The determinant of the Hessian matrix for $\pi_{ii}(Q, f)$ where $\partial\pi_{ii}(Q, f)/\partial Q = 0$ and $\partial\pi_{ii}(Q, f)/\partial f = 0$ is the following:

$$\begin{aligned}
|H| &= \gamma'^2(p-c_f)^2 g\left(\frac{C-Q}{\gamma}\right) g\left(\frac{Q}{1-\gamma}\right)\left(\frac{C^2+Q^2-2QC+C^2\gamma^2-2C^2\gamma+2CQ\gamma}{(1-\gamma)^3\gamma^3}\right) \\
&+ E(X)\left((p-c_f)\left(\frac{-1}{\gamma} g\left(\frac{C-Q}{\gamma}\right) - \frac{1}{1-\gamma} g\left(\frac{Q}{1-\gamma}\right)\right) - (c_f-c_r) g\left(\frac{C}{\gamma}\right)\right)\left(\frac{\gamma''\gamma}{\gamma'} - 2\gamma'\right) \\
&+ \gamma'^2(p-c_f)(c_f-c_r) g\left(\frac{C}{\gamma}\right)\left(\left(\frac{(C-Q)^2}{\gamma^3}\right) g\left(\frac{C-Q}{\gamma}\right) - \frac{Q^2}{(1-\gamma)^3} g\left(\frac{Q}{1-\gamma}\right)\right)
\end{aligned}$$

The second order derivatives of $\pi_{ii}(Q, f)$, Equation (4.17) and Equation (4.18) are negative on the stationary points, where the first order derivatives of $\pi_{ii}(Q, f)$ with respect to Q and f are equal to 0 for all non-negative Q and f values. Additionally, the determinant of the Hessian matrix on the stationary points of $\pi_{ii}(Q, f)$ is positive for all non-negative Q and f values. Hence the expected profit function of P_2, $\pi(Q, f)$ is unimodal and has a unique maximum on $\partial\pi_{ii}(Q, f)/\partial Q = 0$ and $\partial\pi_{ii}(Q, f)/\partial f = 0$ (Demidenko, 2004) given that there exists a non-negative (Q, f) satisfying $\partial\pi_{ii}(Q, f)/\partial Q = 0$ and $\partial\pi_{ii}(Q, f)/\partial f = 0$.

Suppose there is a non-negative solution (Q^0, f^0) to $\partial\pi_{ii}(Q, f)/\partial Q = 0$ and $\partial\pi_{ii}(Q, f)/\partial f = 0$. From $\partial\pi_{ii}(Q, f)/\partial f = 0$ we have

$$\begin{aligned}
\gamma'^0(p-c_f)\int_{\frac{Q^0}{1-\gamma^0}}^{\frac{C-Q^0}{\gamma^0}} xg(x)dx - E(X)(\gamma'^0 f^0 + \gamma^0) &= 0 \\
\int_{\frac{Q^0}{1-\gamma^0}}^{\frac{C-Q^0}{\gamma^0}} xg(x)dx &> 0 \\
\frac{C-Q^0}{\gamma^0} &> \frac{Q^0}{1-\gamma^0} \\
C(1-\gamma^0) &> Q^0
\end{aligned}$$

Hence a non-negative solution (Q^0, f^0) satisfies the constraints and due to unimodality, it is optimal.

If there is no such non-negative (Q^0, f^0) solution, then the optimal solution lies on the boundary. We next consider possible solutions on the boundary.

Boundary Condition 1: All constraints are binding: It is not possible.

Boundary Condition 2: $Q < C(1-\gamma)$, $Q = 0$, $f = 0$: $\partial\pi_{ii}(Q,f)/\partial f = \gamma'(p-c_f)E(X) > 0$. Hence, the optimal solution can not be in this boundary.

Boundary Condition 3: $Q = C(1-\gamma)$, $Q = 0$, $f > 0$: $\partial\pi_{ii}(Q,f)/\partial f = -E(X)(\gamma' f + \gamma) < 0$. Hence, the optimal solution can not be in this boundary.

Boundary Condition 4: $Q < C(1-\gamma)$, $Q > 0$, $f = 0$: $\partial\pi_{ii}(Q,f)/\partial f = \gamma'(p-c_f)\int_Q^{\infty} xg(x)dx > 0$. Hence, the optimal solution can not be in this boundary.

Boundary Condition 5: $Q = C(1-\gamma)$, $Q > 0$, $f > 0$: $\partial\pi_{ii}(Q,f)/\partial f = -E(X)(\gamma' f + \gamma) < 0$. Hence, the optimal solution can not be in this boundary.

Boundary Condition 6: $Q < C(1-\gamma)$, $Q = 0$, $f > 0$: $\partial\pi_{ii}(Q,f)/\partial Q = c_f - c_n - c_r + (p - c_f)G(C/\gamma)$ can be either negative or positive. If it is negative, the optimal solution is on the boundary $Q = 0$ and the corresponding f^* value is calculated accordingly. When Q is set to 0, the objective function can be rewritten as below:

$$\pi_{ii}(Q,f|Q=0) \quad = \quad (p-c_f)\left(\int_0^{\frac{C}{\gamma}} \gamma x g(x)dx + \int_{\frac{C}{\gamma}}^{\infty} Cg(x)dx\right) - f\gamma E(X)$$

The modified objective function is a function of f and f^* is calculated with the first order optimality conditions of $\pi_{ii}(Q,f)$ as below:

$$\gamma'(p-c_f)\int_0^{\frac{C}{\gamma^*}} xg(x)dx - E(X)(\gamma^{*\prime} f^* + \gamma^*) = 0 \tag{4.19}$$

Boundary Condition 7: $Q = C(1-\gamma)$, $Q > 0$, $f = 0$: $\partial\pi_{ii}(Q,f)/\partial Q = c_f - c_n - c_r - (c_f - c_r)G(Q)$ can be either positive or negative. If it is positive, this indicates that Q^* is to be set to its upper bound and the optimal solution is $f^* = 0$ and $Q^* = C$.

The above characterization of the boundary conditions indicates that if a non-negative (Q^0, f^0) does not exist, then the optimal solution lies on the boundary. Then, if Inequality $c_f - c_n - c_r + (p - c_f)G(C\gamma) < 0$ holds, the optimal solution is $Q^* = 0$ and f^*, where f^* is the solution of the equation (4.19); or if Inequality $c_f - c_n - c_r - (c_f - c_r)G(Q) > 0$ holds, the optimal solution is $(Q^* = C, f^* = 0)$.

We show that the solution in the *Boundary Condition 7* can be valid if $\partial\pi_{ii}(Q,f)/\partial Q$ is positive under the conditions stated for the corresponding boundary. In the same way, we showed that the solution in the *Boundary Condition 6* is valid if $\partial\pi_{ii}(Q, f)/\partial Q$ is negative under the conditions stated for the corresponding boundary. We next show that only one of these boundary conditions can be valid at the same time.

Suppose the solution in the *Boundary Condition 7* is valid. Then:

$$\frac{\partial\pi_{ii}(Q, f)}{\partial Q}|_{f=0,Q=C(1-\gamma)} = c_f - c_n - c_r - (c_f - c_r)G(C) > 0 \quad (4.20)$$

For the solution in the *Boundary Condition 6* to be valid, the inequality below is to hold:

$$\frac{\partial\pi_{ii}(Q, f)}{\partial Q}|_{Q=0,Q<C(1-\gamma)} = c_f - c_n - c_r + (p - c_f)G(\frac{C}{\gamma}) < 0 \quad (4.21)$$

The inequalities (4.20) and (4.21) can not hold simultaneously. So, only one of the solutions in the boundaries can be valid for a particular problem. ■

In Proposition 4.6 the optimal solution to P_2 is characterized. We can see that the acquisition fee is positive unless the optimal solution is on the Boundary 7. Note that for the optimal solution to be on the boundary where $f^* = 0$, the unit cost of refilling should be so large to satisfy the inequality $c_f - c_n - c_r - (c_f - c_r)G(C) > 0$ and so to utilize the whole capacity by brand-new containers.

CHAPTER 5

COMPUTATIONAL STUDY

In Chapter 3 and Chapter 4, we present our analytical findings on the optimal acquisition fee and the optimal order quantity of brand-new containers. In Chapter 3, we also analytically show the effects of the cost parameters, c_f, c_n, c_r, on the optimal levels of decision variables for the uncapacitated case. In these analytical investigations, we used general, closed-form expressions for the fraction of returned containers, $\gamma(f)$ and the demand distribution, $g(x)$. Hence, we could not observe the effects of the related problem parameters on the the optimal decisions analytically.

In this chapter, we present the observations gathered from an extensive experimental analysis conducted to characterize:

- the effects of demand and return parameters (such as standard deviation of demand and sensitivity of returns to the acquisition fee) on the optimal decisions and the expected profit,
- the effects of available production capacity on the optimal decisions and the expected profit,
- the effects of cost parameters on the optimal decisions and the expected profit when production capacity is restricted,
- the effects of problem parameters on the expected profit improvement due to utilization of returns.

5.1 Computational Setting

The explicit form of expressions used in the computational study are as follows:

$\gamma(f)$ - *the fraction of returned containers:* The fraction of returned containers to the demand is assumed to be a general increasing and concave function of the acquisition fee in the analytical investigations. For the computational study, we define a complementary exponential function of acquisition fee to define the fraction of returned containers, $\gamma(f) = 1 - e^{-kf}$ where k is a constant representing the sensitivity of return fraction to the acquisition fee. Note that γ is also an increasing function of k. Recall that $\gamma(f)$ is shown by γ throughout the study for the sake of brevity.

$g(x)$ - *probability distribution function of demand for the reusable product:* Demand is assumed to follow Normal distribution with a mean of μ and a standard deviation of σ.

Note that, under the specific form of return fraction and demand, expected quantity of returns is $E(r(\gamma, D)) = \gamma E(D) = (1 - e^{-kf})\mu$.

Ranges of parameters considered in computational setting are given in Table 5.1.

Table 5.1: Ranges of Parameters Used in Computational Study

Parameter	Range
p	3.5
c_f	0.1 - 3.3
c_n	0.1 - 2.9
c_r	0.1 - 2.9
μ	2000
σ	30 - 510
k	0.15 - 2.55
C	1200 - 2800

In order to investigate the improvement in expected profit as a result of utilizing returns, we consider percent improvement, which compares the expected profit values of our optimal solution and the optimal expected profit when no returns are accepted. Percent improvement (PI) in expected profit due to utilizing returns is calculated as:

$$PI = \frac{E(\text{Optimal Profit}) - E(\text{Optimal Profit}|f = 0)}{E(\text{Optimal Profit}|f = 0)} \text{ x } 100$$

Detailed derivations of PI can be found in Appendix A.

5.2 Experimental Observations - Uncapacitated Case

In this section, we summarize our findings from computational analysis for the uncapacitated setting. We first consider the effects of problem parameters on the optimal decisions, next characterize their effects on the expected profit and comment on the benefits of utilizing returns.

5.2.1 Observations on the Optimal Decision Variables

Note that we have characterized the effects of cost parameters on the optimal acquisition fee and order quantity of new containers in Chapter 3 analytically. When there is no capacity restriction on the production operations, the optimal acquisition fee, f^*, is non-increasing and optimal order quantity of brand-new containers, Q^*, is non-decreasing in unit cost of remanufacturing, c_f; whereas f^* is non-decreasing and Q^* is non-increasing in unit cost of manufacturing, c_r, and unit cost of purchasing brand-new containers, c_n.

We next characterize the effects of demand uncertainty and returns sensitivity on the optimal decisions.

Observation 1 *The optimal acquisition fee, f^*, is non-decreasing in the standard deviation of demand, σ. The behavior of the optimal order quantity of brand-new containers, Q^*, with respect to σ depends on the unit purchasing cost of brand-new containers, c_n, and the unit cost of filling, c_r.*

The behaviors of f^* and Q^* with respect to a change in σ are illustrated for two different c_n values in Figure 5.1 and Figure 5.2, respectively. Note that returns are perfectly correlated with demand. That is, if demand turns out to be high (low), the quantity of returns will be high (low) as well. Hence, as uncertainty in demand increases, returns become the safer option. Therefore, the producer responds to an increase in uncertainty by increasing f^*. However, since returns are always insufficient to cover demand, the producer will always order new containers. The change in Q^* to a change in uncertainty depends on the relative costs of underage/overage. That is if c_n and/or c_r are large, the producer will be unwilling to have excess stock, hence Q^* decreases in σ. Otherwise, the producer will increase Q^* as σ increases to be able to satisfy the demand.

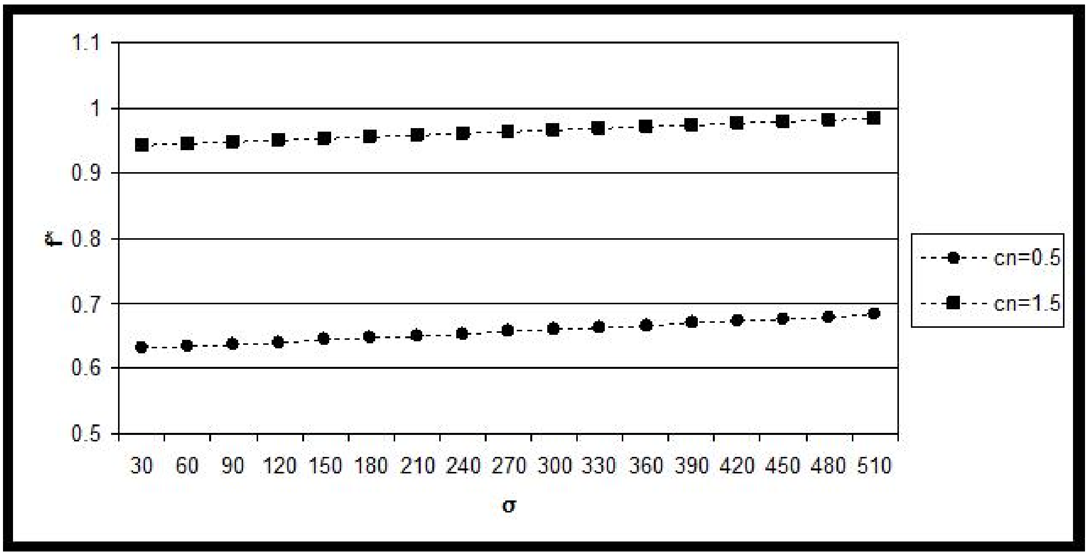

Figure 5.1: f^* versus σ (p = 3.5, c_f = 0.5, c_r = 1.5, μ = 2000, k = 1)

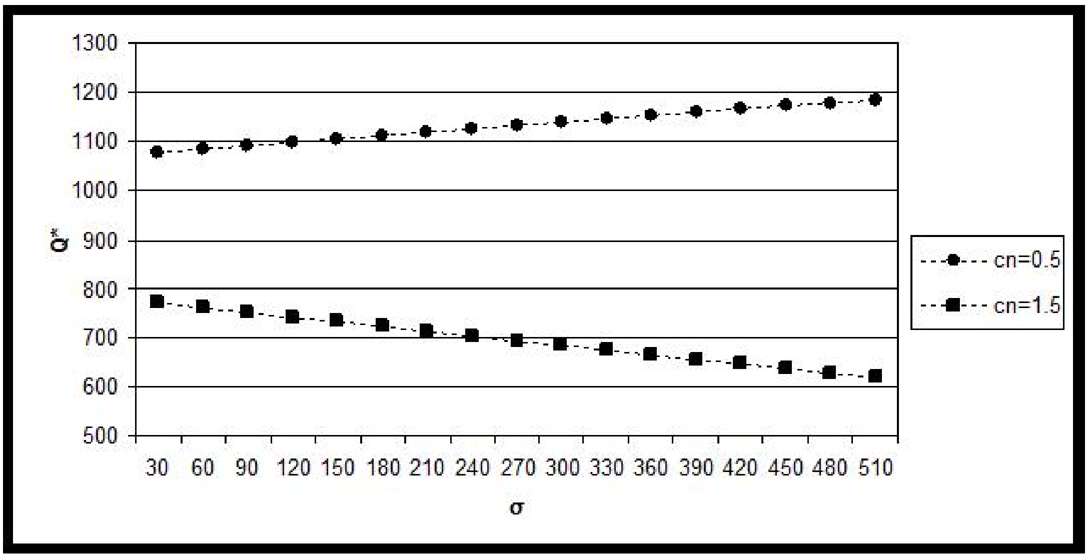

Figure 5.2: Q^* versus σ (p = 3.5, c_f = 0.5, c_r = 1.5, μ = 2000, k = 1)

In Figure 5.1 and Figure 5.2, the behaviors of f^* and Q^* are depicted for a parameter set where $c_f < c_r$. Yet, the behavior of f^* and Q^* are the same and the observation is still valid when $c_f > c_r$. As σ increases, even if $c_f > c_r$, the producer increases f^* in order to utilize more returns to decrease the effects of increasing uncertainty. The change in Q^* in σ again depends on the relative costs of underage/overage.

Observation 2 *The optimal acquisition fee,* f^**, and the optimal order quantity of brand-new containers,* Q^**, are non-increasing in the sensitivity of returns to the acquisition fee,* k*.*

The behaviors of f^*, γ^* and Q^* with respect to a change in k are depicted for two different c_n values in Figure 5.3, Figure 5.4 and Figure 5.5, respectively. Note that the return fraction is increasing in parameter k for a given f; and as k increases, the producer is not only able to collect more returns with the same acquisition fee, but he is also able to collect the equivalent quantity of returns with less acquisition fee. That is, acquiring one unit of returns becomes cheaper as k increases. In the optimal solution, the producer decreases f^* in increasing values of k down to a point where the resulting optimal return fraction is still larger. In this way, the producer utilizes the increase in k in both ways: he both collects more returns and pays less acquisition fee at the same time. That is, f^* is non-increasing and γ^*, so the optimal expected quantity of returns is non-decreasing in k. Since utilizing returns becomes cheaper and the producer collects more returns, he decreases the optimal order quantity of brand new containers and satisfies a larger portion of the demand with returns. Hence, Q^* is non-increasing in increasing values of k.

In Figure 5.3, Figure 5.4 and Figure 5.5, the optimal levels of decision variables are depicted for a parameter set where $c_f < c_r$. Yet, the behavior of f^*, γ^* and Q^* are the same and the observation is still valid when $c_f > c_r$. Even if $c_f > c_r$, utilizing one unit of returns becomes cheaper as k increases. Hence, the producer collects more returns with less acquisition fee and decreases the optimal order quantity of brand-new containers.

5.2.2 Observations on the Optimal Expected Profit and the Benefits of Utilizing the Return Option

It is obvious that the optimal expected profit would decrease in cost parameters c_f, c_n, c_r and demand uncertainty, σ; and it would increase in k. Hence, we focus more on the magnitude

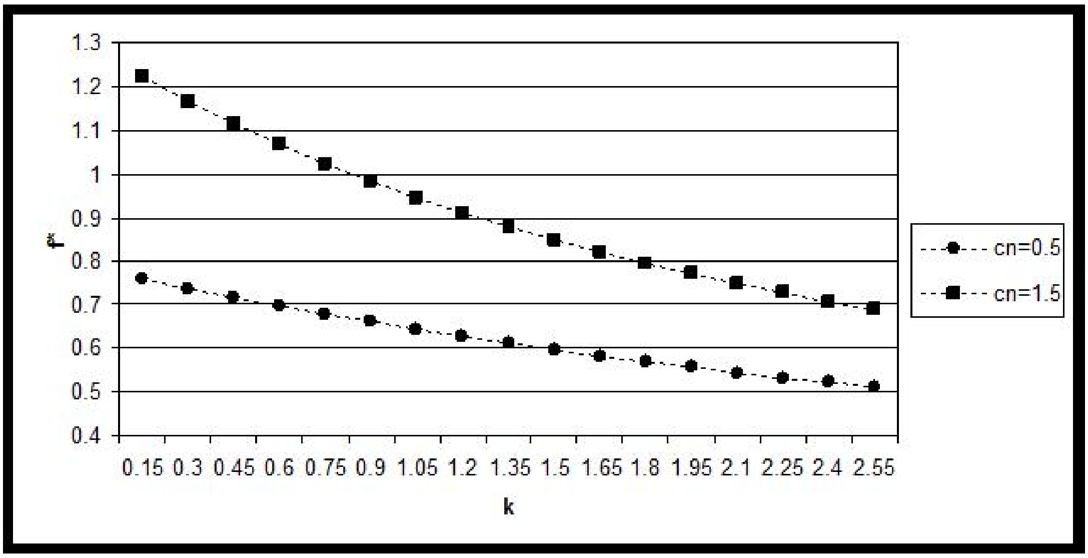

Figure 5.3: f^* versus k ($p = 3.5$, $c_f = 0.5$, $c_r = 1.5$, $\mu = 2000$, $\sigma = 200$)

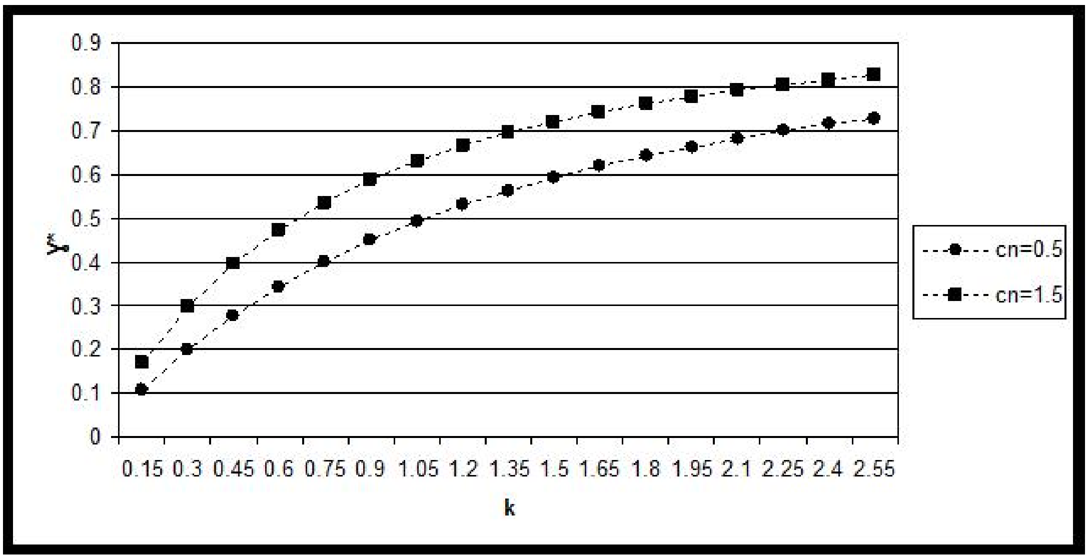

Figure 5.4: γ^* versus k ($p = 3.5$, $c_f = 0.5$, $c_r = 1.5$, $\mu = 2000$, $\sigma = 200$)

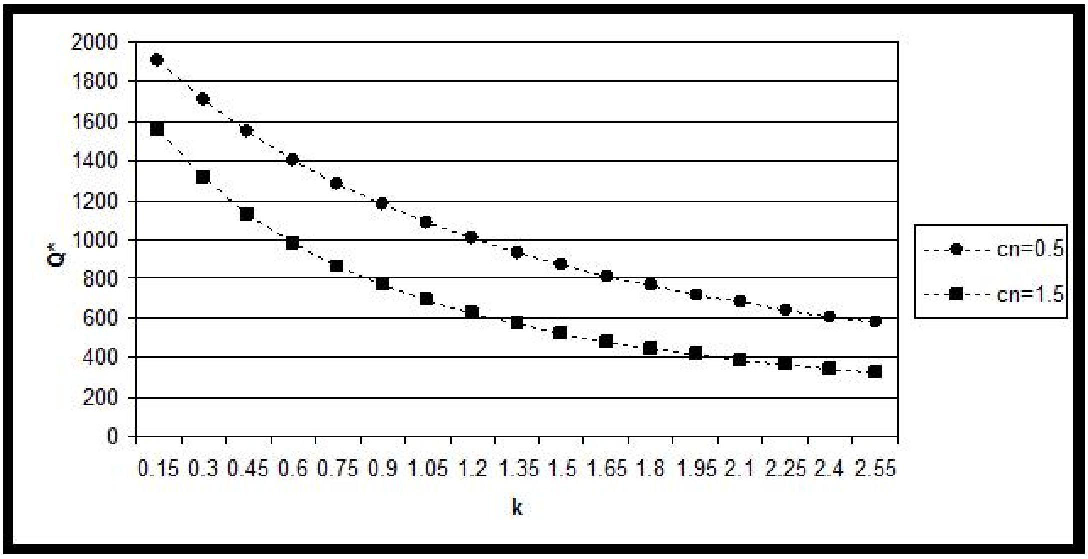

Figure 5.5: Q^* versus k (p = 3.5, c_f = 0.5, c_r = 0.6, μ = 2000, σ = 200)

of these changes.

Observation 3 *The decrease in the optimal expected profit as a result of an increase in the standard deviation of demand, σ, is more significant when the unit refilling cost, c_f, is larger.*

The behavior of the optimal expected profit is illustrated for two different c_f values in Figure 5.6. Recall that the optimal acquisition fee, so the optimal expected quantity of returns is non-increasing in c_f. Also note that the returns decrease the effect of uncertainty due to perfect correlation between the quantity of returns and the demand realization. Hence, as c_f increases and the optimal expected quantity of returns decreases, the producer becomes more prone to uncertainty and the optimal expected profit decreases more sharply in increasing values of parameter σ.

In Figure 5.6, note that we can see the behaviors of f^* and Q^* for both cases where $c_f < c_r$ and $c_f > c_r$. Independent of the relationship between c_f and c_r, the decrease in the optimal expected profit as a result of an increase σ is more significant when c_f is larger.

Observation 4 *The increase in the optimal expected profit as a result of an increase in the sensitivity of returns to the acquisition fee, k, is more significant when the unit cost of refilling, c_f, is smaller.*

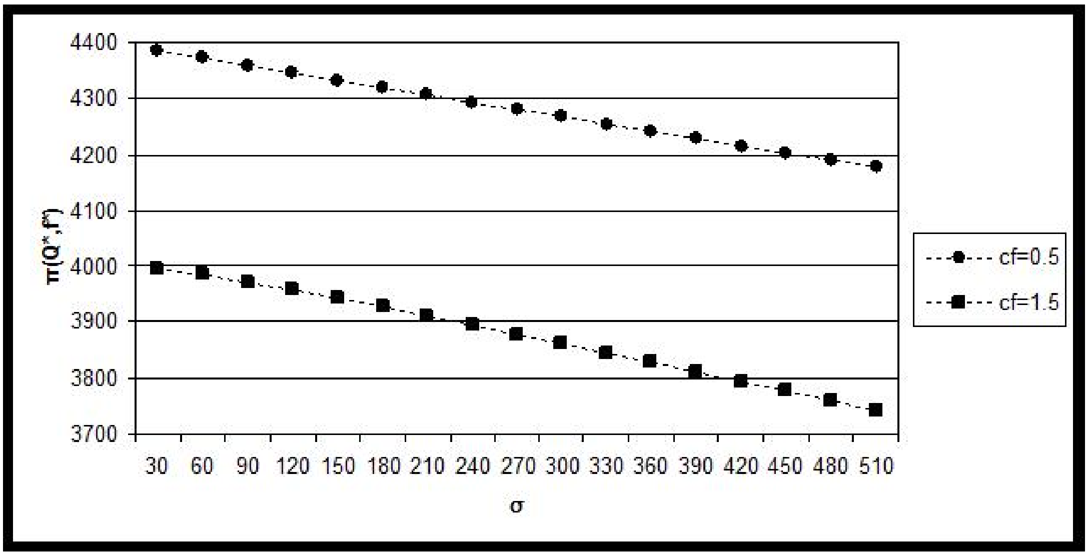

Figure 5.6: $\pi(Q^*, f^*)$ versus σ ($p = 3.5$, $c_n = 0.5$, $c_r = 1$, $\mu = 2000$, $k = 1$)

The behavior of the optimal expected profit in k is depicted for two different c_f values in Figure 5.7. With increasing values of k, the producer collects more returns by paying less acquisition fee, so the optimal expected profit increases. When c_f is smaller, returns have higher profit margins; hence, each additional return increases the optimal expected profit more. That is, when c_f is smaller the optimal expected profit increases more sharply in increasing values of parameter k.

In Figure 5.7, note that we can see the behavior of the optimal expected profit for both cases where $c_f < c_r$ and $c_f > c_r$. Independent of the relationship between c_f and c_r, the increase in the optimal expected profit as a result of an increase in k is more significant when c_f is smaller.

Observation 5 *The increase in the optimal expected profit as a result of an increase in the sensitivity of returns to the acquisition fee, k, is more significant when c_n and/or c_r is larger.*

The behavior of the optimal expected profit in k for different values of c_n and c_r is depicted in Figure 5.8 and Figure 5.9, respectively. With increasing values of k, the producer collects more returns by paying less acquisition fee and purchases less brand-new containers as explained in Observation 2; so the optimal expected profit increases. When c_n and/or c_r is larger, products with brand-new containers have lower profit margins; hence, each substitution from brand-new containers to returns increases the optimal expected profit more. That is, when c_n

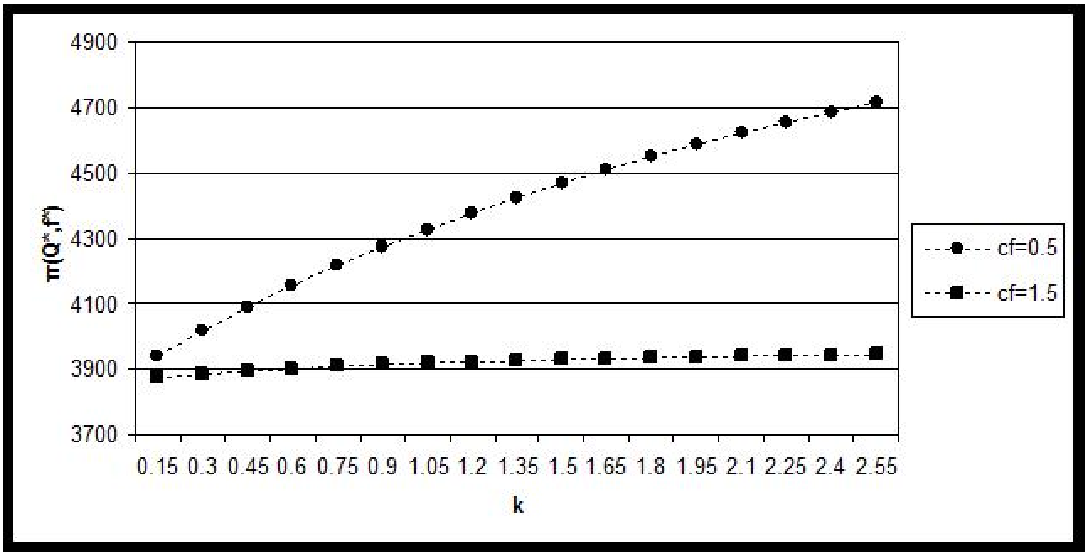

Figure 5.7: $\pi(Q^*, f^*)$ versus $k(p = 3.5, c_n = 0.5, c_r = 1, \mu = 2000, \sigma = 200)$

and/or c_r is larger, the optimal expected profit increases more sharply in increasing values of parameter k.

In Figure 5.8, we can see the behavior of the optimal expected profit for the case where $c_f < c_r$. Whereas, in Figure 5.9, the behavior of the optimal expected profit is depicted for both cases where $c_f < c_r$ and $c_f > c_r$. Since behavior of the optimal expected profit depends on the changes in profit margins of two supply options, independent of the relationship between c_f and c_r, increase in the optimal expected profit as a result of an increase in k is more significant when c_n and/or c_r is smaller.

We next consider the improvement in the optimal expected profit due to the utilization of returns in the production. It is obvious that the improvement due to utilization of returns increases in the unit purchasing cost of brand-new containers, c_n, the unit filling cost, c_r; and decreases in the unit cost of refilling, c_f. Hence, we focus on the effect of other parameters on the improvement in the optimal expected profit.

Observation 6 *The improvement due to utilization of returns is non-decreasing in standard deviation of demand, σ.*

The behavior of the improvement due to utilization of returns in σ is illustrated in Figure 5.10. Recall that f^*, so the optimal expected quantity of returns increases in σ independent

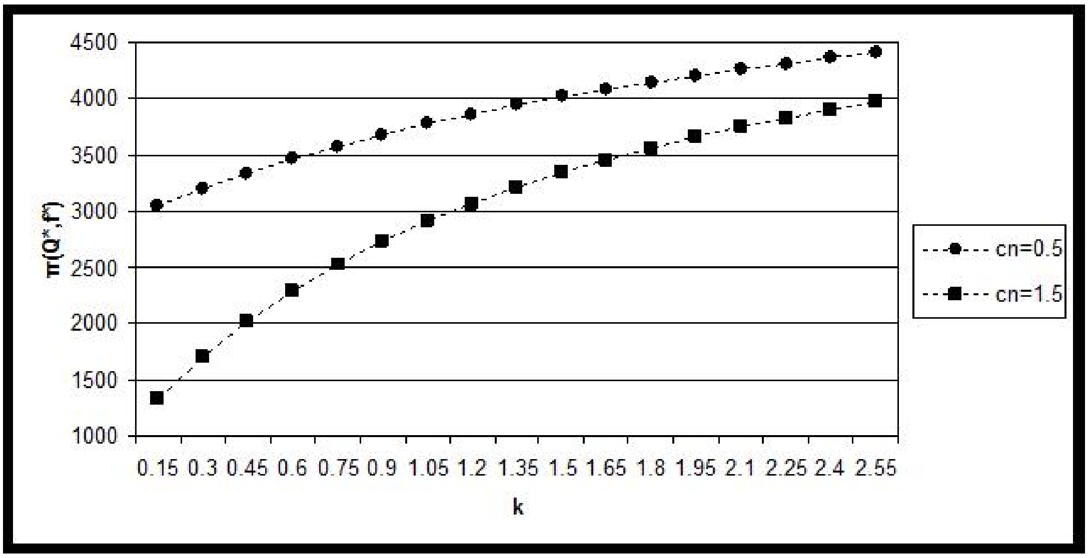

Figure 5.8: $\pi(Q^*, f^*)$ versus $k(p = 3.5, c_f = 0.5, c_r = 1.5, \mu = 2000, \sigma = 200)$

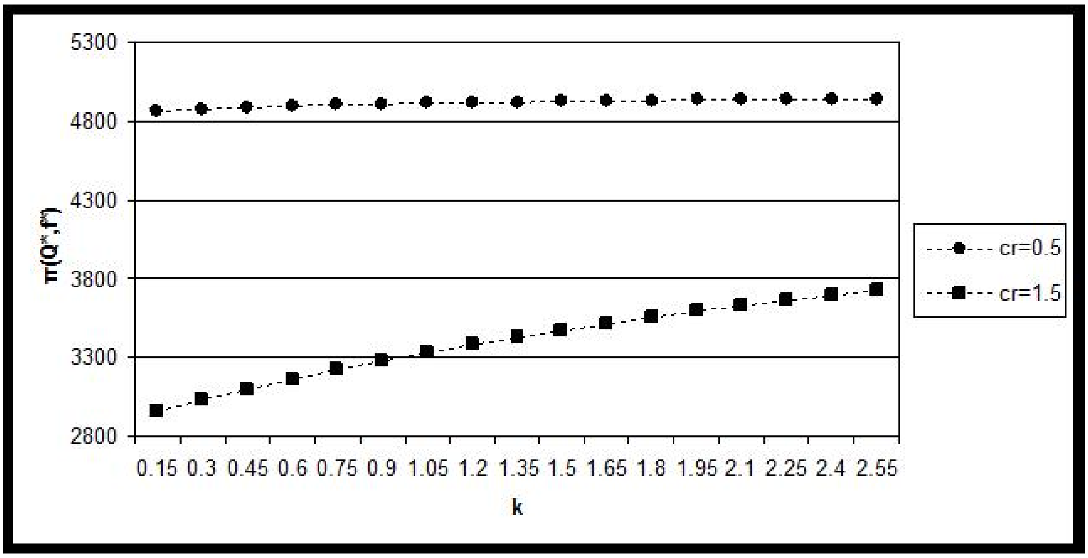

Figure 5.9: $\pi(Q^*, f^*)$ versus k $(p = 3.5, c_f = 1, c_n = 0.5, \mu = 2000, \sigma = 200)$

of the relationship between c_f and c_r. The utilization of returns becomes more critical as σ increases; since the returns do not only provide an alternative source of supply, but also decrease the effects of uncertainty due to perfect correlation between the returns and the demand realization.

Note that the percent improvement is a performance indicator to show the improvement in the expected profit due to utilization of returns, and it is calculated by comparing the cases where "returns are accepted with an acquisition fee of f^*" and "no returns are accepted by setting the acquisition fee to 0". Since accepting returns decreases the effects of uncertainty due to perfect correlation between the returns and demand realization, the percent improvement in the optimal expected profit is expected to be non-decreasing in σ.

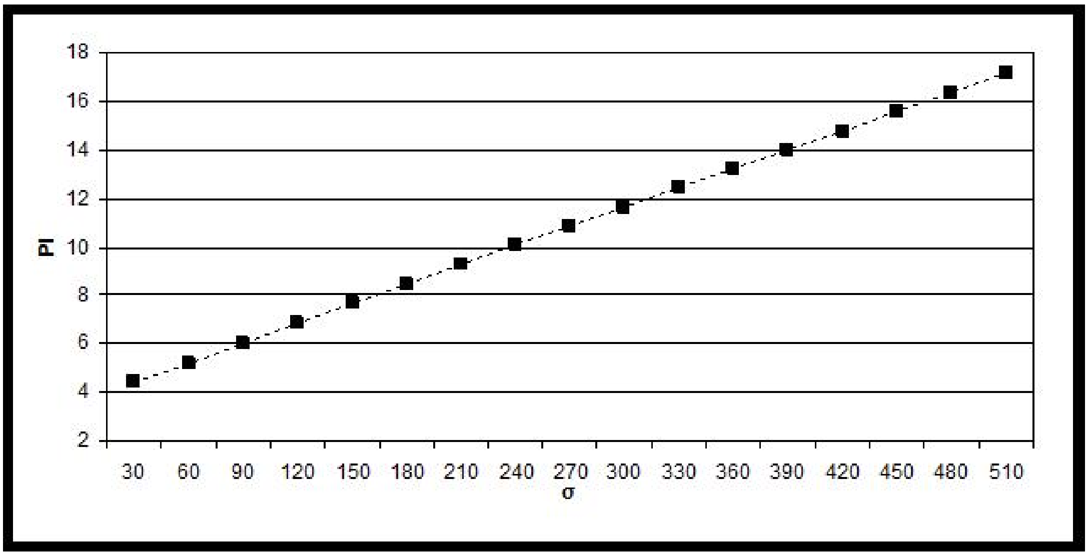

Figure 5.10: *PI* versus σ (p = 3.5, c_f = 1.5, c_n = 1.5, c_r = 0.5, μ = 2000, k = 1)

Observation 7 *The improvement due to utilization of returns is non-decreasing in the sensitivity of returns to the acquisition fee, k.*

The behavior of the improvement due to utilization of returns in k is illustrated in Figure 5.11. Recall that f^* is non-increasing and γ^*, so the optimal expected quantity of returns, is non-decreasing in k independent of the relationship between c_f and c_r. In Observation 2, we explain that, with increasing values of k, the producer collects more returns by paying less acquisition fee.

Since the producer prefers to accept more returns and to purchase less new containers as k increases; PI is expected to be non-decreasing in k since PI is calculated by comparing the cases where $\gamma = \gamma^*$ and $\gamma = 0$.

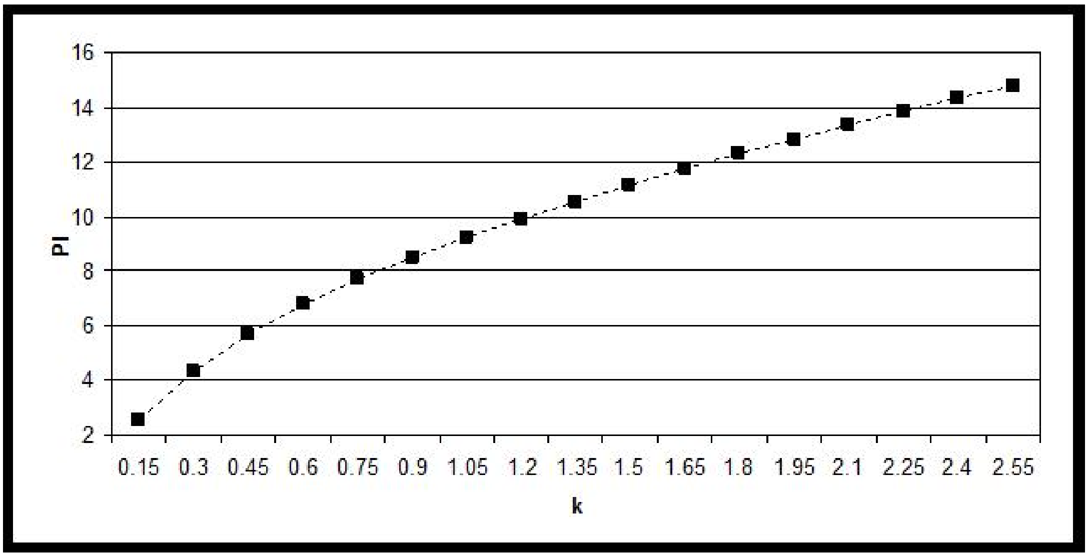

Figure 5.11: PI versus k ($p = 3.5$, $c_f = 1.5$, $c_n = 1.5$, $c_r = 0.5$, $\mu = 2000$, $std = 200$)

5.3 Experimental Observations - Capacitated Case

In this section, we summarize our findings from computational analysis for the capacitated setting. Similar to the uncapacitated setting observations, we first consider the effects of problem parameters on the optimal decisions and then we characterize their effects on the expected profit and the benefits on utilizing returns.

5.3.1 Observations on the Optimal Decision Variables

We characterize the effects of cost parameters, demand uncertainty and returns sensitivity on the optimal decisions with experimental observations.

Observation 8 *The optimal acquisition fee, f^*, is non-increasing and the optimal order quantity of brand-new containers, Q^*, is non-decreasing in the unit refilling cost, c_f.*

The behaviors of f^* and Q^* in changing c_f values for two different capacity levels are illustrated in Figure 5.12 and Figure 5.13, respectively. Recall that, in the uncapacitated case, we analytically show that the optimal acquisition fee is non-increasing and the optimal order quantity of new containers is non-decreasing in c_f. For the capacitated case, behaviors of f^* and Q^* in c_f are investigated for a tight and an ample capacity level; and it is observed that they show similar behavior to uncapacitated case as it is expected.

As c_f increases, unit profit margin of returns decreases. Hence, the producer decreases f^*, so the optimal expected quantity of returns, and increases Q^* in order to satisfy the demand. After a critical point (for the illustrative example with $C = 1500$, it is $1.9 < c_f < 2.1$ - see Figure 5.12 and Figure 5.13), he prefers to set the acquisition fee to 0 and accept no returns. Note that, in the uncapacitated case, the optimal acquisition fee is always positive; whereas in the capacitated case, f^* is set to 0 for larger values of c_f when utilizing the available capacity with brand-new containers to the fullest extent is more profitable than accepting returns. In addition, it is observed that, f^* drops to 0 more sharply when the available capacity is limited (C =1500).

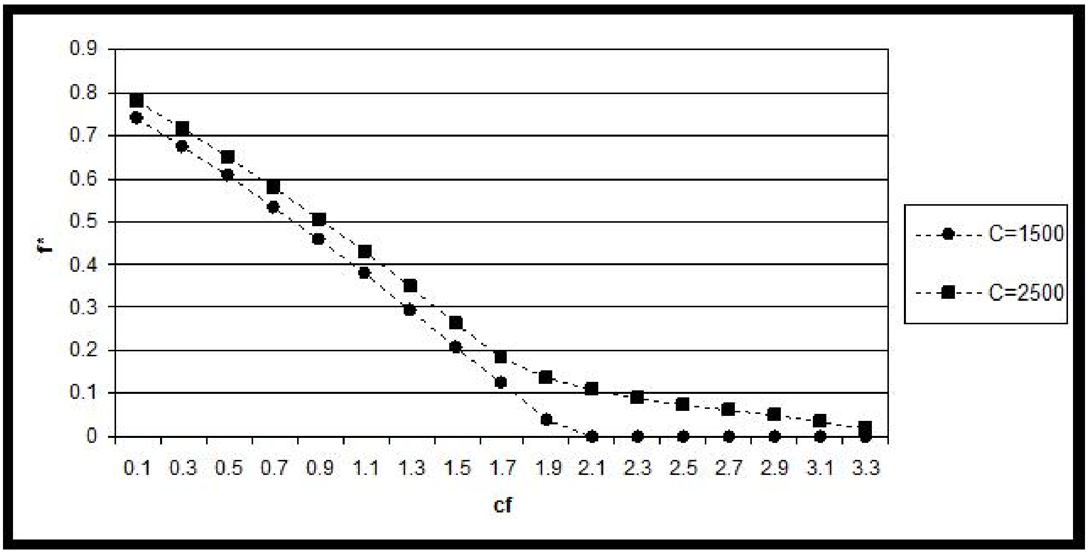

Figure 5.12: f^* versus c_f ($p = 3.5$, $c_n = 0.5$, $c_r = 1.5$, $\mu = 2000$, $\sigma = 200$, $k = 1$)

Observation 9 *The optimal acquisition fee, f^*, is non-decreasing and the optimal order quantity of brand-new containers, Q^*, is non-increasing in both the unit purchasing cost of brand-new containers, c_n, and the unit filling cost, c_r.*

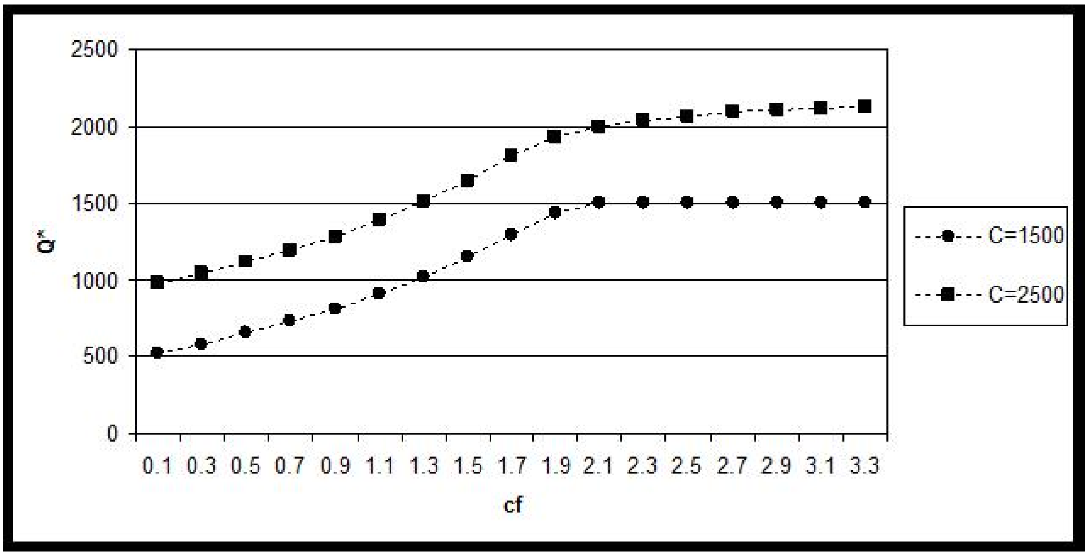

Figure 5.13: Q^* versus c_f ($p = 3.5$, $c_n = 0.5$, $c_r = 1.5$, $\mu = 2000$, $\sigma = 200$, $k = 1$)

The behaviors of f^* and Q^* in c_r for two different capacity values are depicted in Figure 5.14 and Figure 5.15, respectively. Recall that, in the uncapacitated case, we analytically show that the optimal acquisition fee is non-decreasing and the optimal order quantity of brand-new containers is non-increasing in both c_n and c_r. For the capacitated case, behaviors of f^* and Q^* in c_n and c_r are investigated for a tight and an ample capacity level; and it is observed that they show similar behavior to uncapacitated case as it is expected.

As c_n and/or c_r increases, unit profit margin of products with brand-new containers decreases. Hence, the producer decreases Q^*, and increases the optimal expected quantity of returns by increasing f^*. As seen in the illustrative examples in Figure 5.14 and Figure 5.15, the producer sets the optimal acquisition fee to 0 and accepts no returns for small values of c_r. After a critical point ($0.9 < c_r < 2.1$ for $C = 1500$), he determines a positive f^* value and begins to accept returns. Note that, in the uncapacitated case, the optimal acquisition fee is always positive; whereas in the capacitated case, f^* is set to 0 for smaller values of c_n and c_r when utilizing the available capacity with brand-new containers to the fullest extent is more profitable than accepting returns.

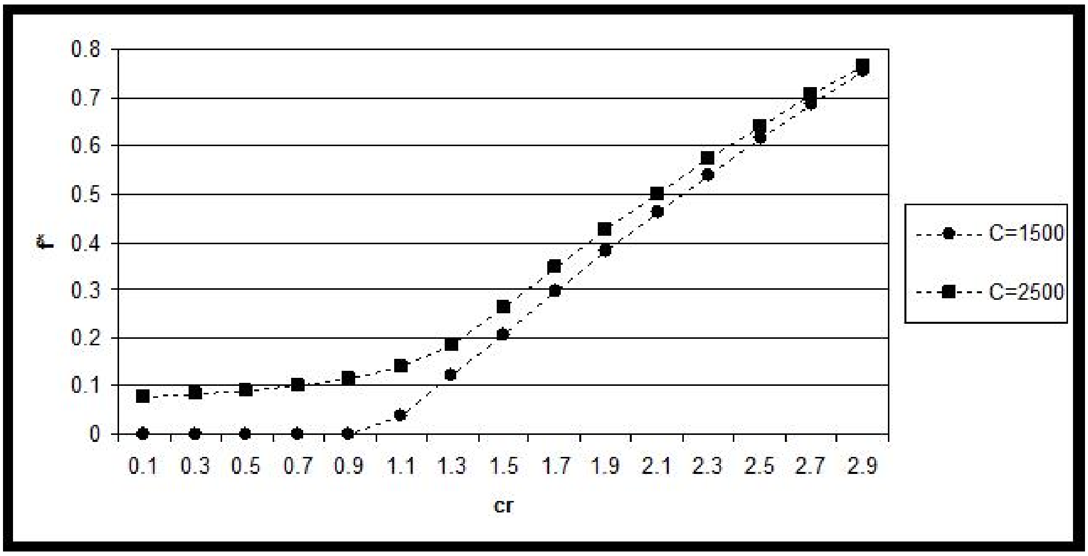

Figure 5.14: f^* versus c_n (p = 3.5, c_f = 1.5, c_n = 0.5, μ = 2000, σ = 200, k = 1)

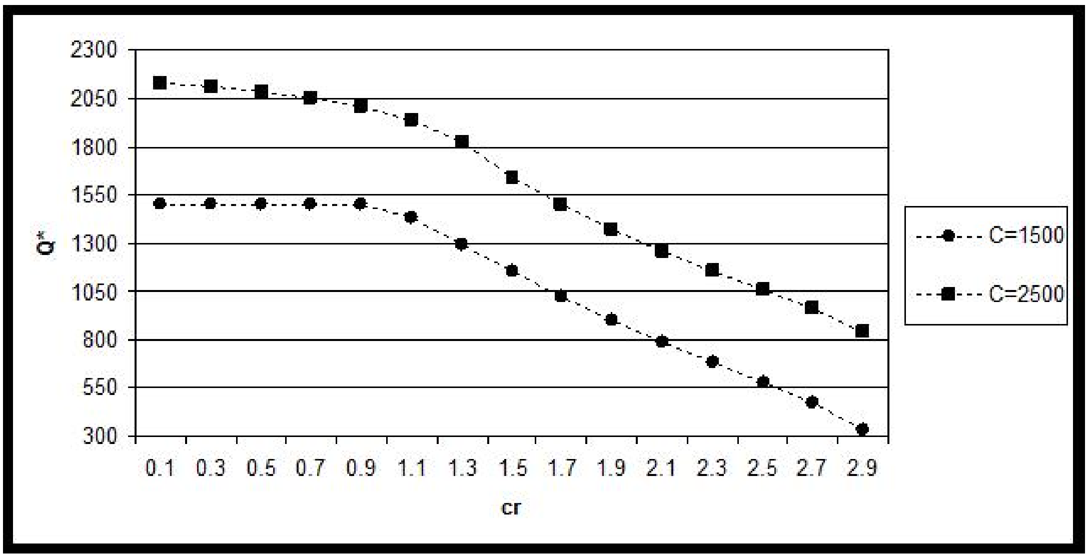

Figure 5.15: Q^* versus c_n (p = 3.5, c_f = 1.5, c_r = 0.5, μ = 2000, σ = 200, k = 1)

Observation 10 *The optimal acquisition fee, f^*, and the optimal order quantity of brand-new containers, Q^*, are non-decreasing in the available production capacity, C.*

The behavior of f^* is illustrated in Figure 5.16, and the behaviors of Q^*, $E(r(\gamma^*, D))$ and $Q^* + E(r(\gamma^*, D))$ in C are illustrated in Figure 5.17, respectively. Note that the total production level is limited with the available capacity. As capacity increases, the firm has the option to satisfy a larger portion of the demand; so both f^* and Q^* show a non-decreasing behavior.

When capacity is tight, the producer faces no demand uncertainty practically. It is almost certain that demand exceeds capacity and there is no overage. In such a case, first (when C is increased from 1200 to 1300 for the illustrative example), the producer increases total supply by increasing both f^* and Q^*; because utilizing returns is significantly cheaper and using brand-new containers is the safer supply option in the existence of return uncertainty. Then, when C is increased from 1300 to 1700, probability of underage increases due to return uncertainty. The producer chooses not to increase returns in response to an increase in capacity and just increases the quantity of brand-new containers to be purchased.

When the available capacity is around the expected value of demand, both f^* and Q^* are increasing in C. When C is around the expected value of demand the probability of having excessive stock of containers arises. In such a case, returns help decrease the uncertainty dues to its perfect correlation with demand. Hence, the producer utilizes each additional capacity with more new containers and returns, latter of which is the demand dependent supply.

When available capacity is larger, it does not constitute a binding restriction. For the results of one parameter set shown in Figure 5.16 and Figure 5.17, both decision variables are constant in increasing C values when C is increased from 2300 to 2800. The producer keeps both decision variables constant for increasing C values, because capacity is ample and the unconstrained optimal total production quantity is below the available capacity.

Observation 11 *The available capacity, C affects the behaviors of the optimal acquisition fee, f^*, and the optimal order quantity of brand-new containers, Q^*, in changing values of standard deviation of demand, σ.*

The behavior of f^* with respect to a change in σ for two different capacity values are illustrated in Figure 5.18 and Figure 5.20; and the behaviors of Q^*, $E(r(\gamma^*, D))$ and Q^* +

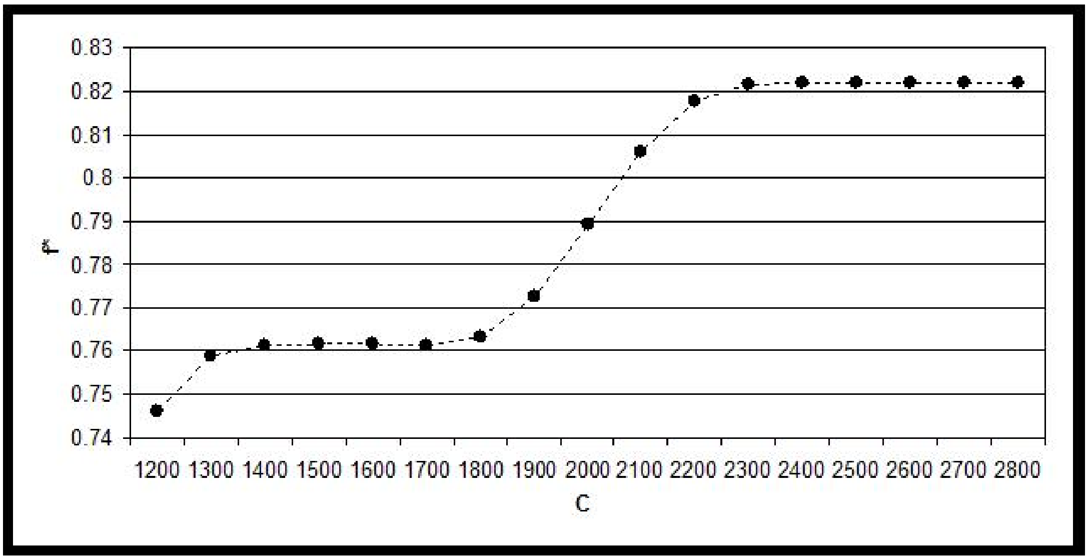

Figure 5.16: f^* versus C ($p = 3.5$, $c_f = 0.5$, $c_n = 1.5$, $c_r = 1$, $\mu = 2000$, $\sigma = 200$, $k = 1$)

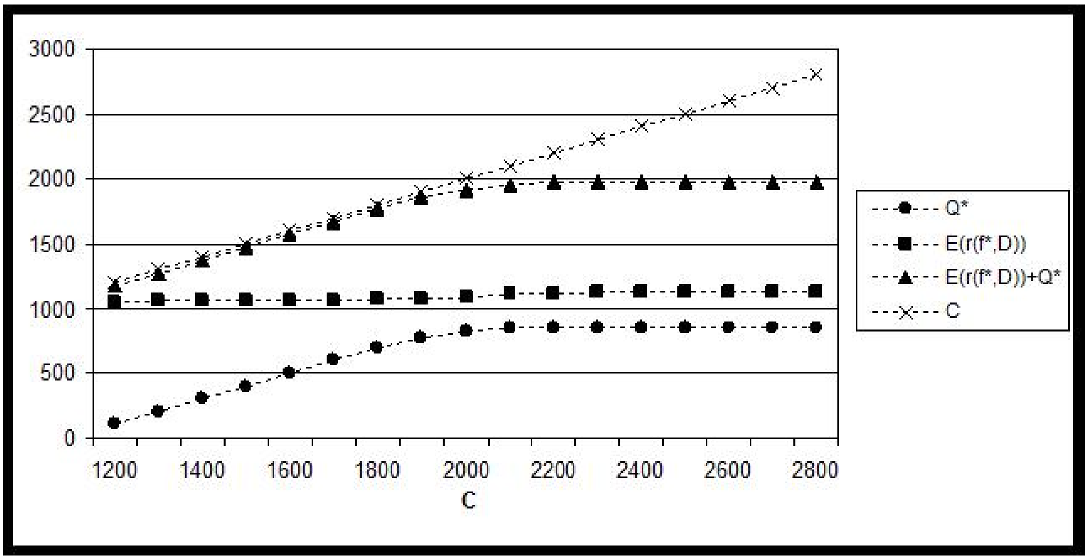

Figure 5.17: Q^*, $E(r(f^*, D))$ and $Q^* + E(r(f^*, D))$ versus C ($p = 3.5$, $c_f = 0.5$, $c_n = 1.5$, $c_r = 1$, $\mu = 2000$, $\sigma = 200$, $k = 1$)

$E(r(\gamma^*, D))$ with respect to a change in σ for two different capacity values are illustrated in Figure 5.19 and Figure 5.21, respectively.

When capacity is tight, $C = 1500$, the producer faces no demand uncertainty for the small values of σ ($\sigma < 300$) practically. It is almost certain that demand exceeds capacity and there will be no overage even though σ increases. Yet, the risk of underage due to return uncertainty exists. In such a case, the producer chooses not to increase returns in response to an increase in σ, since the return quantity is uncertain; and he chooses to increase the quantity of brand-new containers to satisfy the demand. As σ increases to higher values ($\sigma > 300$), demand uncertainty becomes as critical as return uncertainty. Hence, the producer increases f^* to decrease the effects of uncertainty. In this case, the producer can increase or decrease Q^* depending on the underage/overage costs of production alternatives. In the example shown in Figure 5.19, Q^* is decreasing in changing values of σ when $\sigma > 300$.

When $C = 2500$, the capacity does not constitute a binding restriction for the total optimal production quantity for small values of σ ($\sigma < 150$). Since there is ample capacity and the opportunity of satisfying a larger portion of the realized demand, each increase in σ increases f^*; so the producer decreases the effect of increasing uncertainty. In this case, the producer can increase or decrease Q^* depending on the underage/overage costs of production alternatives. In the example shown in Figure 5.21, Q^* is increasing in changing values of σ when $\sigma < 150$. As σ increases to higher values ($\sigma > 150$), both demand and return uncertainty affect the behaviors. Since returns are perfectly correlated with demand, the producer prefers to increase f^* more sharply to balance the demand uncertainty with return uncertainty. The behavior of Q^* again affected by the underage/overage costs, and in the example shown in Figure 5.19, Q^* is decreasing in changing values of σ when $\sigma > 150$.

Observation 12 *The optimal acquisition fee, f^*, and the optimal order quantity of brand-new containers, Q^*, are non-increasing with the sensitivity of returns to the acquisition fee, k.*

The behaviors of f^*, γ^* and Q^* in changing values of k for two different capacity levels are illustrated in Figure 5.22, Figure 5.23 and Figure 5.24 respectively. In the capacitated case, the behavior of f^*, γ^* and Q^* in k are same as in the uncapacitated case. As k increases, the producer uses the opportunity of collecting more returns by paying a lower acquisition

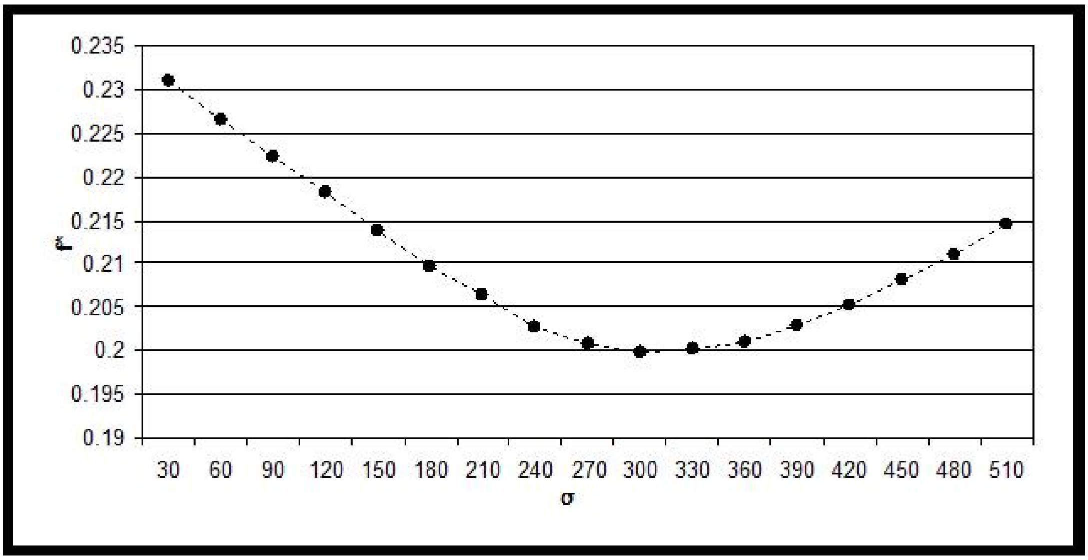

Figure 5.18: f^* versus σ (p = 3.5, c_f = 1.5, c_n = 1.5, c_r = 0.5, μ = 2000, k = 1, C = 1500)

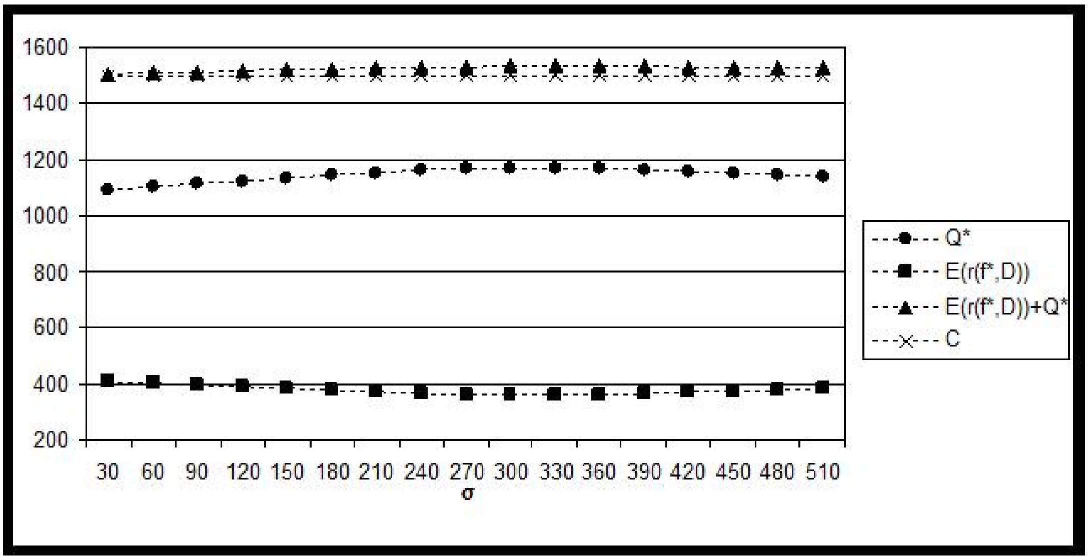

Figure 5.19: Q^*, $E(r(f^*, D))$ and $Q^* + E(r(f^*, D))$ versus σ (p = 3.5, c_f = 1.5, c_n = 1.5, c_r = 0.5, μ = 2000, k = 1, C = 1500)

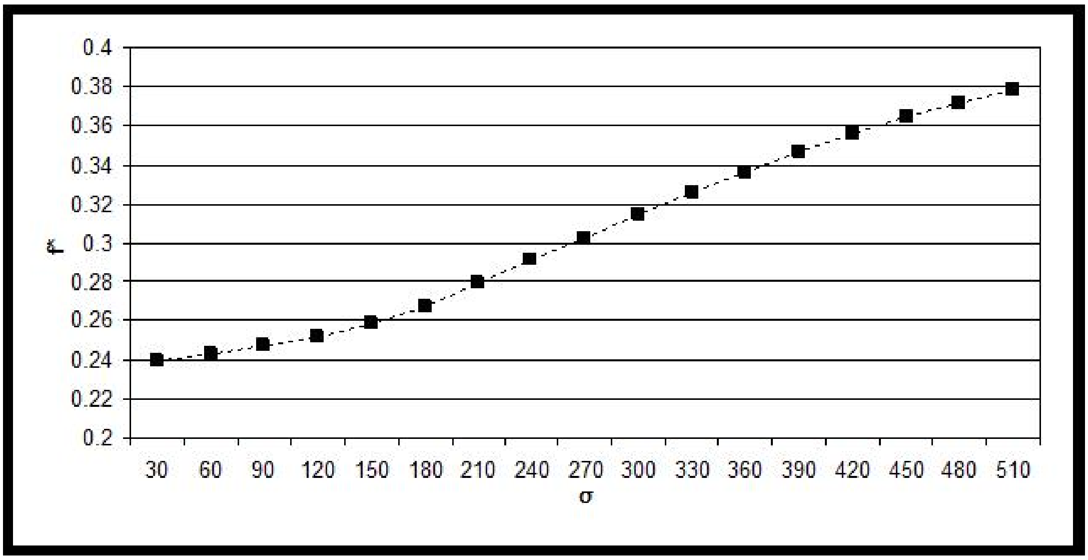

Figure 5.20: f^* versus σ (p = 3.5, c_f = 1.5, c_n = 1.5, c_r = 0.5, μ = 2000, k = 1, C = 2500)

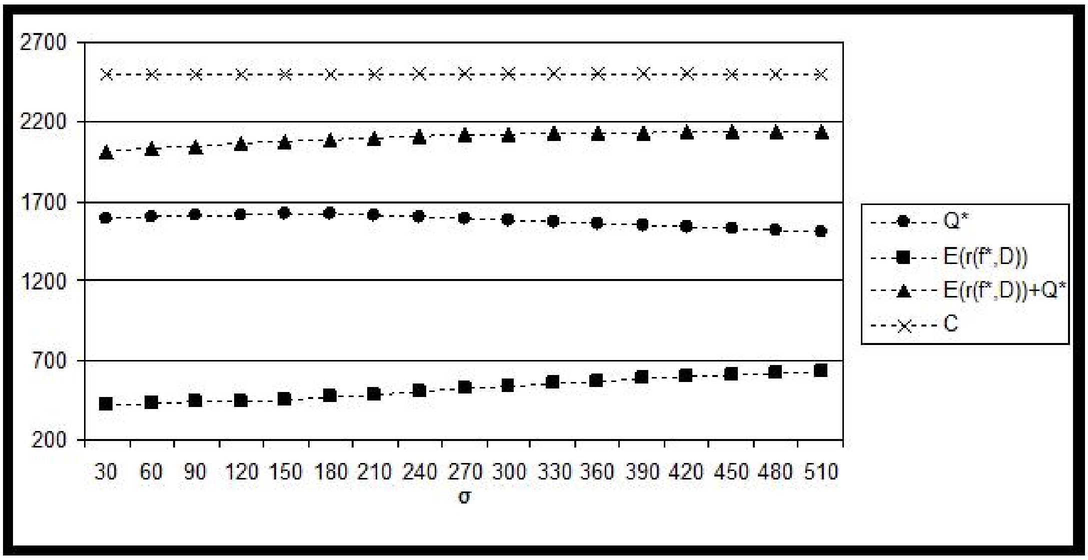

Figure 5.21: Q^*, $E(r(f^*, D))$ and $Q^* + E(r(f^*, D))$ versus σ (p = 3.5, c_f = 1.5, c_n = 1.5, c_r = 0.5, μ = 2000, k = 1, C = 2500)

fee. As larger portion of the demand can be satisfied with returns, the producer decreases the optimal order quantity of brand-new containers. Hence, f^* and Q^* are non-increasing in increasing values of parameter k, whereas γ^* and the optimal expected quantity of returns are non-decreasing.

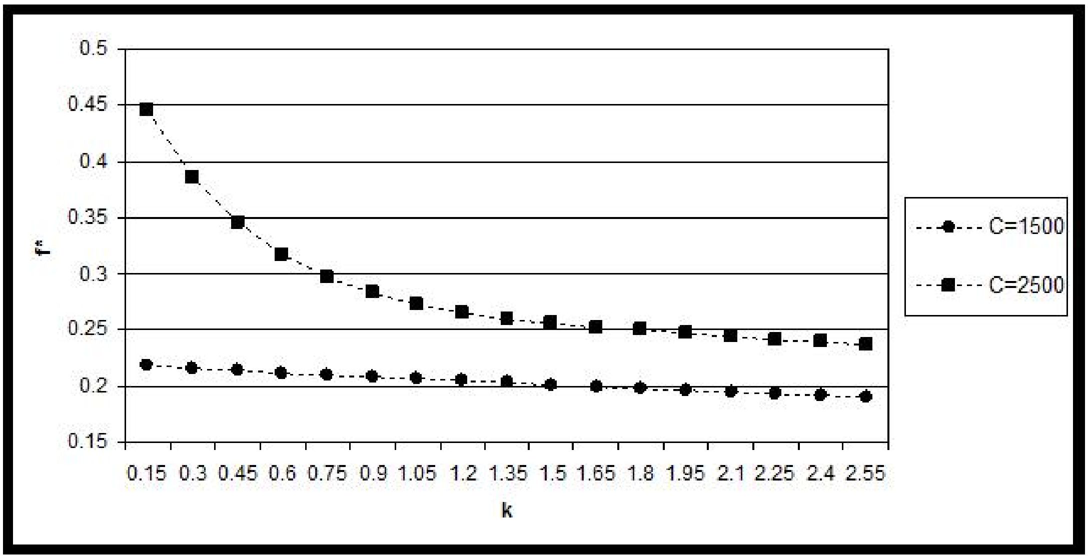

Figure 5.22: f^* versus k (p = 3.5, c_f = 1.5, c_n = 1.5, c_r = 0.5, μ = 2000, σ = 200)

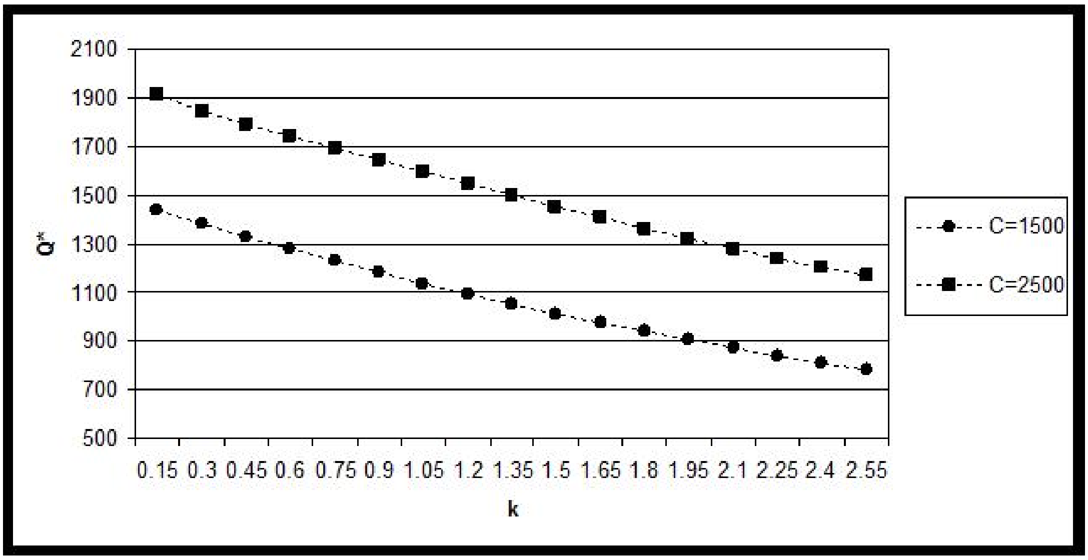

Figure 5.23: γ^* versus k (p = 3.5, c_f = 1.5, c_n = 1.5, c_r = 0.5, μ = 2000, σ = 200)

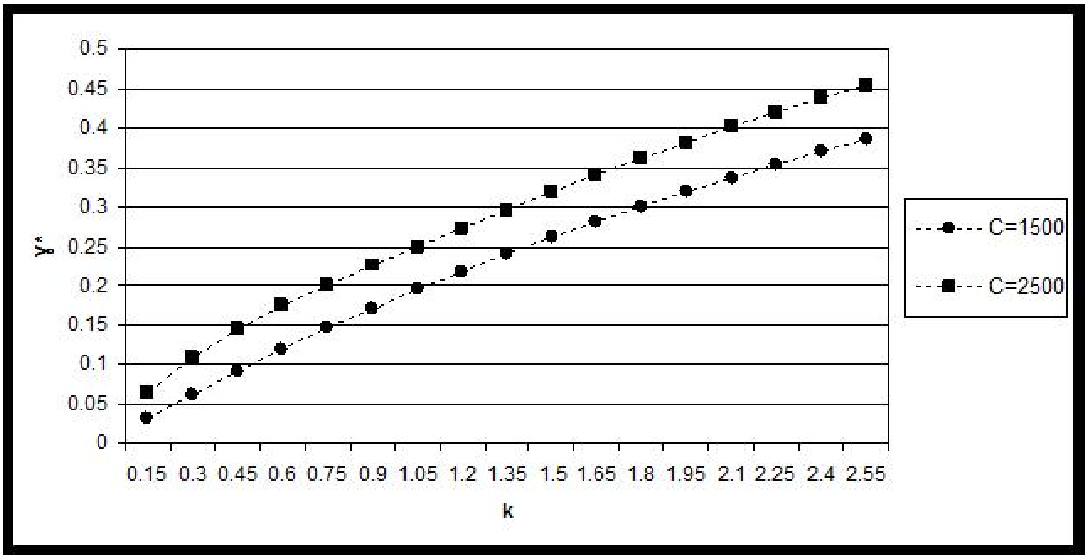

Figure 5.24: Q^* versus k (p = 3.5, c_f = 1.5, c_n = 1.5, c_r = 0.5, μ = 2000, σ = 200)

5.3.2 Observations on the Benefits of Utilizing the Return Option

It is obvious that the optimal expected profit would decrease in cost parameters c_f, c_n, c_r. As the uncertainty in demand increases with increasing values of σ, it is again obvious that the optimal expected profit would decrease. Utilizing one of the supply options, returns, becomes easier with increasing values of k, so the optimal expected profit would increase in k. In increasing values of available capacity, C, it is also obvious that the optimal expected profit shows a non-decreasing behavior. Hence, we focus more on the behavior of the improvement due to utilization of returns with respect to changes in problem parameters.

Observation 13 *The available capacity, C, affects the behavior of the improvement due to utilization of returns in standard deviation of demand,* σ.

The behavior of the improvement due to utilization of returns in σ for two different capacity values are illustrated in Figure 5.25. In Observation 12, we show that the behaviors of f^* and Q^* in σ is affected by value of C. Since the behavior of f^* and Q^* is affected by C, the behavior of the improvement due to utilization of returns for changing values of σ is also affected. The behaviors of f^* and Q^* in σ for the same parameter set in Figure 5.25 can be seen in Figure 5.18 and Figure 5.19.

When $C = 1500$ and σ is small, the producer prefers to decrease the expected quantity of returns and to increase the quantity of brand-new containers. Hence, in that case, the improvement due to utilization of returns shows a decreasing behavior in σ. For the larger values of σ, the producer prefers to decrease the effect of increasing demand uncertainty by increasing f^* and decreasing Q^*. Hence, for the case where σ is large, the improvement due to utilization of returns shows an increasing behavior in changing values of σ.

When $C = 2500$ and σ is small, the producer prefers to increase both the expected quantity of returns and the quantity of brand-new containers. In that case, depending on the cost parameters, the improvement dur to utilization of returns either increases or decreases. In the example shown in Figure 5.25, *PI* is increasing for the corresponding parameter set. When σ becomes larger, the producer prefers to decrease the effect of increasing demand uncertainty by increasing f^* and decreasing Q^*. Hence, for the case where σ is large, the improvement due to utilization of returns shows an increasing behavior in changing values of σ.

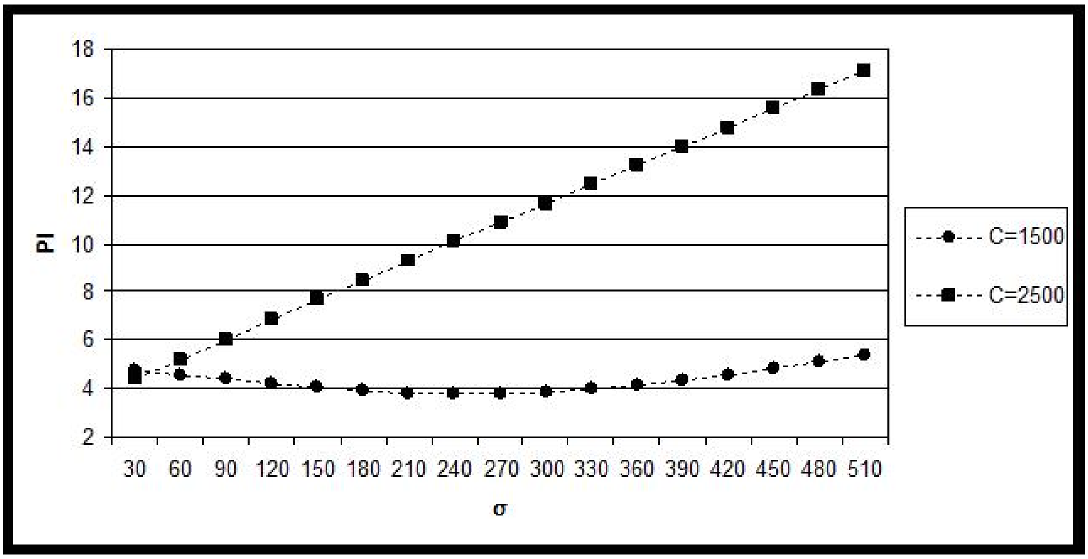

Figure 5.25: *PI* versus σ ($p = 3.5$, $c_f = 1.5$, $c_n = 1.5$, $c_r = 0.5$, $\mu = 2000$, $k = 1$)

Observation 14 *The improvement due to utilization of returns is non-decreasing in the sensitivity of returns to the acquisition fee, k.*

The behavior of percent improvement in k for two different capacity values is illustrated in Figure 5.26. Recall that f^* is non-increasing and γ^*, so the optimal expected quantity of returns, is non-decreasing in k. Note that the producer collects more returns by paying a lower acquisition fee.

Since the producer prefers to accept more returns and to purchase less new containers as k increases; *PI* is expected to be non-decreasing in k since *PI* is calculated by comparing the cases where $\gamma = \gamma^*$ and $\gamma = 0$.

5.4 Summary of Findings

With the computational study conducted in this chapter, we obtain the following results:

- In the uncapacitated setting,
  - the optimal acquisition fee is non-decreasing with standard deviation of demand, whereas the behavior of the optimal order quantity of new containers depends on other parameters.

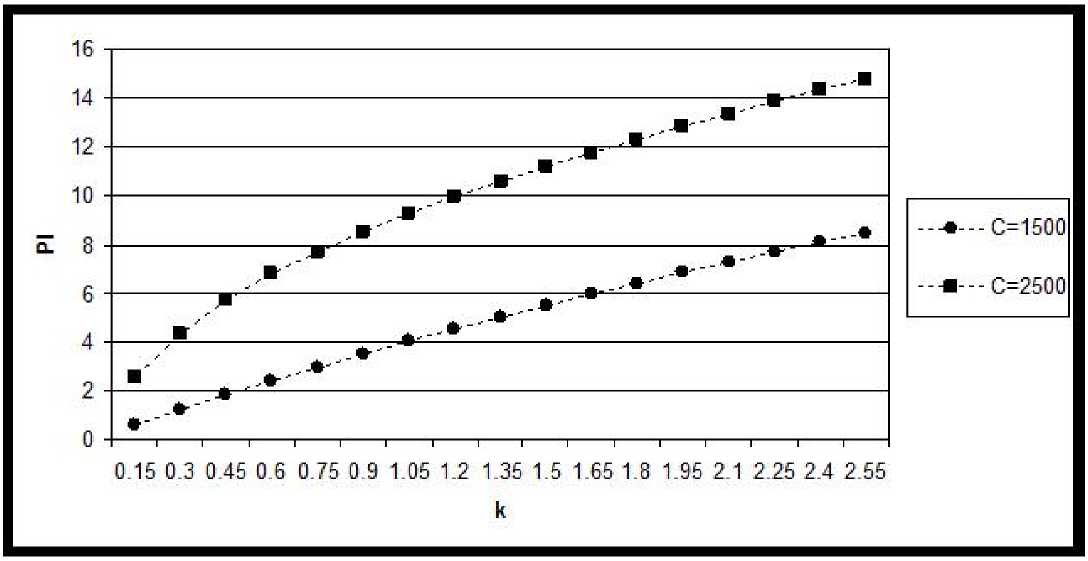

Figure 5.26: PI versus k (p = 3.5, c_f = 1.5, c_n = 1.5, c_r = 0.5, μ = 2000, σ = 200)

- the optimal acquisition fee and optimal order quantity of new containers are non-increasing with the sensitivity of return function to acquisition fee.
- the decrease in the optimal expected profit as a result of an increase in standard deviation of demand is more significant when unit cost of remanufacturing is larger.
- the increase in the optimal expected profit as a result of an increase in sensitivity of returns to acquisition fee is more significant when unit cost of remanufacturing is smaller.
- the increase in the optimal expected profit as a result of an increase in sensitivity of returns to acquisition fee is more significant when unit cost of manufacturing and/or unit cost of purchasing new containers is smaller.

- In the capacitated setting,
  - the optimal acquisition fee is non-increasing in unit cost of remanufacturing, non-decreasing in unit cost of manufacturing and unit cost of purchasing brand-new containers.
  - the optimal order quantity of brand-new containers is non-decreasing in unit cost of remanufacturing, non-increasing in unit cost of manufacturing and unit cost of purchasing brand-new containers.

  - the optimal acquisition fee and optimal order quantity of new containers are non-decreasing with the available capacity.
  - the optimal acquisition fee and the optimal order quantity of new containers are non-increasing with the sensitivity of return function to acquisition fee.

- Reusing is most profitable for the system where:
  - the unit cost of remanufacturing is low.
  - the unit cost of manufacturing is high.
  - the unit cost of purchasing brand-new containers is high.
  - the sensitivity of returns to acquisition fee is high.

CHAPTER 6

CONCLUSION

In closed-loop supply chains, reusing is one of the widely-used recovery option within direct recovery operation alternatives. Reusable containers are examples of reusing operations and in practice, many producers accept used containers from customers in order to reuse them in their production. Since reusable packaging materials are utilized for several times, accepting returned containers is profitable for the producers under certain conditions.

In the production systems where reusable containers are utilized, the producer performs both filling (manufacturing) and refilling (remanufacturing) operations. He acquires returned containers from customers in exchange of an unit acquisition fee, he reuses them in remanufacturing operations. Since only a fraction of containers returns to the producer, he also purchases brand-new reusable containers from an external container supplier and use them in manufacturing operations. In reusable container systems, customers are indifferent between buying manufactured or remanufactured products, so the manufactured and remanufactured are sold at the same sales price.

The goal of the producer is to maximize his profit with an effective production planning. Yet, such production planning decisions require the information about the return process that depends on both customer demand and the deposit price determined by the producer. Hence, the producer has the opportunity to manipulate the return quantity via the acquisition fee. In order to satisfy the demand, he also has to decide on the order quantity of brand new reusable containers. Since the producer wants to satisfy the demand in full and cannot violate restrictions on the available resources of production operations, production planning and pricing decisions are to be made simultaneously for a synchronized reusable container system.

In this thesis study, we investigate the pricing and production planning decisions of production systems where reusable containers are utilized. Since reusable container systems has the characteristics of hybrid manufacturing / remanufacuring systems, our focus is on the corresponding area of the closed-loop supply chain literature. Note that although we use a reusable container system framework throughout the study, the models that we construct apply to a more general manufacturing / remanufacturing framework as well.

The optimal pricing and production decisions to maximize the producer's profit has been investigated for two different environmental settings: (i) unrestricted resource capacity (ii) restricted resource capacity.

In Chapter 3 and Chapter 4, pricing and production decisions has been investigated analytically and optimal solutions to maximize the producer's profit are characterized. One of the major findings of these investigations is that, in a reusable container system where there is no capacity restriction on the production operations, the optimal acquisition fee is always positive and the producer should accept some returns in order to maximize his profit even though unit cost of refilling is greater than the unit cost of filling. Similarly, when there is a capacity restriction on the production operations, the optimal acquisition fee is still positive under certain conditions even though unit cost of refilling is greater than the unit cost of filling.

In Chapter 5, major findings of an extensive computational study are presented. We characterize the effects of available capacity, demand and return parameters on the optimal decisions and the expected profit. We also characterize the effects of cost parameters on the optimal decisions and the expected profit when production capacity is restricted. The improvement in the optimal expected profit due to utilization of returns are measured and the effects of problem parameters on the improvement is investigated.

There are a number of future research opportunities of our study. One of them is an extension of this work to the multi-period setting. In this study, we consider optimal pricing and production decisions in a single-period setting and do not take lead times of different production alternatives into account. The study can be extended to a multi-period setting and lead times of operations can be considered.

Another extension can be changing the simplifying assumption on the dependency of returns to the demand. We assume that returns are perfectly correlated with the demand realization. In an alternative and more realistic setting, returns can be assumed to be correlated with the sales.

In the capacitated setting, we assume that resource utilization of returned items and brand-new items are identical. In another study, resource utilization of these two options can be considered as non-identical.

Our investigation focuses on a single product with reusable containers. The existence of a product with disposable containers can be assumed as an alternative production option to satisfy customer demand. Resource utilization of reusable and disposable options can also assumed to be non-identical.

REFERENCES

[1] Bayındır, Z.P., Erkip, N. and Güllü, R. *A model to evaluate inventory costs in a remanufacturing environment*, International Journal of Production Economics **81-82** (2003), 597-607.

[2] de Brito, M. P., Dekker, R. *A framework for reverse logistics*, Reverse Logistics - Quantitative Models for Closed-Loop Supply Chains, Springer, Berlin, Germany (2004), 3-27.

[3] Demidenko, E. *Mixed Models - Theory and Applications*, Wiley Series in Probability and Statistics, New York, Wiley (2004), 657.

[4] Dobos, I. and Richter, K. *A production/recycling model with stationary demand and return rates*, Central European Journal of Operations Research **8** (2003), 35-46.

[5] Fleischmann M., Kuik R., Dekker R. *Controlling inventories with stochastic item returns: a basic model*, European Journal of Operational Research **138** (2002), 63-75.

[6] Goh, T. N. and Varaprasad, N. *A statistical methodology for the analysis of the life-cycle of reusable containers*, IIE Transactions **18-1** (1986), 42-47.

[7] Guide, V. D. R., Teunter, R. H. and Van Wassenhove, L. N. *Matching demand and supply to maximize profits from manufacturing*, Manufacturing & Service Operations Management **5-4** (2003), 303-316.

[8] Guide, V. D. R. and Van Wassenhove, L. N. *Business Aspects of Closed-loop Supply Chains*, Business Aspects of Closed-loop Supply Chains: Exploring the Issues, Carnegie Mellon University Press, Pittsburgh, PA (2003), 17-41.

[9] Guide, V. D. R. and Van Wassenhove, L. N. *The evolution of closed-loop supply chain research*, Operations Research **57-1** (2009), 10-18.

[10] Hess, J. D. and Mayhew, G. E. *Modeling Merchandise Returns in Direct Marketing*, Journal of Direct Marketing **11-2** (1997), 20-35.

[11] Kelle, P. and Silver, E. A. *Forecasting the returns of reusable containers*, Journal of Operations Management **8-1** (1989a), 17-35.

[12] Kelle, P. and Silver, E. A. *Purchasing policy of new containers considering the random returns of previously issued containers*, IIE Transactions **21-4** (1989b), 349-354.

[13] Kiesmüller, G. P. and van der Laan, E. A. *An inventory model with dependent product demands and returns*, International Journal of Production Economics **72** (2001), 73-87.

[14] Mukhopadhyay, S. K. and Setoputro, R. *Reverse logistics in e-business - optimal price and return policy*, International Journal of Physical Distribution & Logistics Management **34-1** (2004), 70-88.

[15] Mostard, J., de Koster, R., Teunter, R., *The distribution-free newsboy problem with resalable returns*, International Journal of Production Economics **97** (2005), 329-342.

[16] Mostard, J., Teunter, R., *The newsboy problem with resalable returns: a single period model and case study*, European Journal of Operations Research **169** (2006), 81-96.

[17] Tang, O. and Grubbström R. W. *Considering stochastic lead times in a manufacturing/remanufacturing system with deterministic demands and returns*, International Journal of Production Economics **93-94** (2005), 285-300.

[18] Teunter, R. H. *Economic ordering quantities for recoverable item inventory systems*, Naval Research Logistics **48** (2001), 484-495.

[19] Teunter, R. *Lot-sizing for inventory systems with product recovery*, Computers & Industrial Engineering **46** (2004), 431-441.

APPENDIX A

DERIVATIONS OF PERCENT IMPROVEMENT

Percent improvement (PI) is a performance measure in our study which is considered to investigate the improvement in expected profit as a result of utilizing returns. It compares the expected optimal profit values of our optimal solution and the solution when no returns are accepted. Percent improvement (PI) in expected profit due to utilizing returns is calculated as:

$$PI = \frac{E(\text{Optimal Profit}) - E(\text{Optimal Profit}|f = 0)}{E(\text{Optimal Profit}|f = 0)} \text{ x } 100$$

Derivations of PI for the uncapacitated case and the capacitated case are detailed below.

A.1 PI for the Uncapacitated Case

In order to find PI, we characterize the problem when no returns are accepted.

The second stage problem when no returns are accepted can be stated as:

$$\text{Maximize } \pi(Q, 0) = (p - c_r)M$$

subject to

$$\begin{aligned} M &\leq D \\ M &\leq Q \end{aligned}$$

The optimal production quantities given the first stage decision, Q, and the demand realization, D, the optimal production quantities are as follows:

$$(M^*|D) = \begin{cases} Q & \text{if } Q < D \\ D & \text{if } Q > D \end{cases}$$

Then, the first stage problem can be expresses as follows:

$$[P] : \text{Maximize } \pi(Q,0) \quad = \quad (p - c_r)\left(\int_0^Q xg(x)dx + \int_Q^\infty Qg(x)dx\right) - c_n Q$$

subject to

$$Q \quad \geq \quad 0$$

The first and second order derivatives of $\pi(Q,0)$ are the following:

$$\frac{d\pi(Q,0)}{dQ} = (p - c_r)(1 - G(Q)) - c_n$$
$$\frac{d^2\pi(Q,0)}{dQ^2} = (-p + c_r)g(Q)$$

The optimal solution to P is the unique non-negative solution to $\frac{d\pi(Q,0)}{dQ} = 0$ since $\pi(Q,0)$ is concave with respect to Q. Hence, the optimal solution to P is:

$$Q^P = G^{-1}\left(1 - \frac{c_n}{p - c_r}\right)$$

Then, PI can be derived as:

$$E(\text{Optimal Profit}|f = 0) = \pi(Q^P, 0)$$

$$PI = \frac{\pi(Q^*, f^*) - \pi(Q^P, 0)}{\pi(Q^P, 0)} \text{ x } 100 \tag{A.1}$$

A.2 PI for the Capacitated Case

In order to find PI, we characterize the problem in the capacitated setting when no returns are accepted.

The second stage problem when no returns are accepted again can be stated as:

$$\text{Maximize } \pi(Q,0) = (p - c_r)M$$

subject to

$$\begin{aligned} M &\leq C \\ M &\leq D \\ M &\leq Q \end{aligned}$$

Since Q is always set to a value smaller then C, capacity constraint is redundant and the second stage problem can be stated as:

$$\text{Maximize } \pi(Q, 0) = (p - c_r)M$$

subject to

$$\begin{aligned} M &\leq D \\ M &\leq Q \end{aligned}$$

The optimal production quantities given the first stage decision, Q, and the demand realization, D, the optimal production quantities are as follows:

$$(M^*|D) = \begin{cases} Q & \text{if } Q < D \\ D & \text{if } Q > D \end{cases}$$

Then, the first stage problem can be expresses as follows:

$$\text{[P]: Maximize } \pi(Q, 0) = (p - c_r)\left(\int_0^Q xg(x)dx + \int_Q^\infty Qg(x)dx \right) - c_n Q$$

subject to

$$\begin{aligned} Q &\leq C \\ Q &\geq 0 \end{aligned}$$

The first and second order derivatives of $\pi(Q, 0)$ are the following:

$$\begin{aligned} \frac{d\pi(Q,0)}{dQ} &= (p - c_r)(1 - G(Q)) - c_n \\ \frac{d^2\pi(Q,0)}{dQ^2} &= (-p + c_r)g(Q) \end{aligned}$$

The optimal solution to P is either the unique non-negative solution to $\frac{d\pi(Q,0)}{dQ} = 0$ or C since $\pi(Q,0)$ is concave with respect to Q but Q is restricted with the available capacity. Hence, the optimal solution to P is:

$$Q^P = min\left(C, G^{-1}\left(1 - \frac{c_n}{p - c_r}\right)\right)$$

Then, PI can be derived as:

$$E(\text{Optimal Profit}|f = 0) = \pi(Q^P, 0)$$
$$PI = \frac{\pi(Q^*, f^*) - \pi(Q^P, 0)}{\pi(Q^P, 0)} \text{ x } 100 \tag{A.2}$$

APPENDIX B

DETAILS OF OPTIMIZATION ALGORITHM

B.1 The Uncapacitated Case

Pseudocode of the optimization algorithm used in the computational study for the uncapacitated case is follows:

READ p, c_f, c_n, c_r, μ, σ and k

IF $c_r \geq c_f$

SET fUB to $p - c_f$

SET fLB to 0

SET f to $(fUB + fLB)/2$

SET QUB to $\mu + 5 \times \sigma$

SET QLB to 0

SET Q to $(QUB + OLB)/2$

CALCULATE $\partial\pi(f, Q)/\partial Q$ for P_1

WHILE $|\partial\pi(f, Q)/\partial Q| > 0.001$

IF $\partial\pi(f, Q)/\partial Q > 0.001$

SET QLB to Q

SET Q to $(QUB + OLB)/2$

CALCULATE $\partial\pi(f, Q)/\partial Q$ for P_1

ELSE

SET QUB to Q

SET Q to $(QUB + OLB)/2$

CALCULATE $\partial\pi(f, Q)/\partial Q$ for P_1

END IF

END WHILE

CALCULATE $\partial\pi(f, Q)/\partial f$ for P_1

WHILE $|\partial\pi(f, Q)/\partial f > 0.001|$

IF $\partial\pi(f, Q)/\partial f > 0.001$

SET fLB to f

SET f to $(fLB + fUB)/2$

SET QUB to $\mu + 5 \times \sigma$

SET QLB to 0

SET Q to $(QUB + OLB)/2$

CALCULATE $\partial\pi(f, Q)/\partial Q$ for P_1

WHILE $|\partial\pi(f, Q)/\partial Q| > 0.001$

IF $\partial\pi(f, Q)/\partial Q > 0.001$

SET QLB to Q

SET Q to $(QUB + OLB)/2$

CALCULATE $\partial\pi(f, Q)/\partial Q$ for P_1

ELSE

SET QUB to Q

SET Q to $(QUB + OLB)/2$

CALCULATE $\partial\pi(f, Q)/\partial Q$ for P_1

END IF

END WHILE

CALCULATE $\partial\pi(f, Q)/\partial f$ for P_1

ELSE

SET fUB to f

SET f to $(fLB + fUB)/2$

SET QUB to $\mu + 5 \times \sigma$

SET QLB to 0

SET Q to $(QUB + OLB)/2$

CALCULATE $\partial\pi(f, Q)/\partial Q$ for P_1

WHILE $|\partial\pi(f, Q)/\partial Q| > 0.001$

IF $\partial\pi(f, Q)/\partial Q > 0.001$

SET QLB to Q

```
                SET Q to (QUB + OLB)/2
                CALCULATE ∂π(f, Q)/∂Q for P1
            ELSE
                SET QUB to Q
                SET Q to (QUB + OLB)/2
                CALCULATE ∂π(f, Q)/∂Q for P1
            END IF
        END WHILE
        CALCULATE ∂π(f, Q)/∂f for P1
    END IF
END WHILE
CALCULATE π(Q, f) for P1
SET f_O to f
SET Q_O to Q
SET π(Q, f)_O to π(Q, f) for P1

ELSE
    SET fUB to p − c_f
    SET fLB to 0
    SET f to (fUB + fLB)/2
    SET QUB to μ + 5 × σ
    SET QLB to 0
    SET Q to (QUB + OLB)/2
    CALCULATE ∂π(f, Q)/∂Q for P2
    WHILE |∂π(f, Q)/∂Q| > 0.001
        IF ∂π(f, Q)/∂Q > 0.001
            SET QLB to Q
            SET Q to (QUB + OLB)/2
            CALCULATE ∂π(f, Q)/∂Q for P2
        ELSE
            SET QUB to Q
            SET Q to (QUB + OLB)/2
```

```
            CALCULATE ∂π(f, Q)/∂Q for P2
        END IF
    END WHILE
    CALCULATE ∂π(f, Q)/∂f for P2
    WHILE |∂π(f, Q)/∂f > 0.001|
        IF ∂π(f, Q)/∂f > 0.001
            SET fLB to f
            SET f to (fLB + fUB)/2
            SET QUB to μ + 5 × σ
            SET QLB to 0
            SET Q to (QUB + OLB)/2
            CALCULATE ∂π(f, Q)/∂Q for P2
            WHILE |∂π(f, Q)/∂Q| > 0.001
                IF ∂π(f, Q)/∂Q > 0.001
                    SET QLB to Q
                    SET Q to (QUB + OLB)/2
                    CALCULATE ∂π(f, Q)/∂Q for P2
                ELSE
                    SET QUB to Q
                    SET Q to (QUB + OLB)/2
                    CALCULATE ∂π(f, Q)/∂Q for P2
                END IF
            END WHILE
            CALCULATE ∂π(f, Q)/∂f for P2
        ELSE
            SET fUB to f
            SET f to (fLB + fUB)/2
            SET QUB to μ + 5 × σ
            SET QLB to 0
            SET Q to (QUB + OLB)/2
            CALCULATE ∂π(f, Q)/∂Q for P2
            WHILE |∂π(f, Q)/∂Q| > 0.001
```

IF $\partial\pi(f, Q)/\partial Q > 0.001$

SET QLB to Q

SET Q to $(QUB + OLB)/2$

CALCULATE $\partial\pi(f, Q)/\partial Q$ for P_2

ELSE

SET QUB to Q

SET Q to $(QUB + OLB)/2$

CALCULATE $\partial\pi(f, Q)/\partial Q$ for P_2

END IF

END WHILE

CALCULATE $\partial\pi(f, Q)/\partial f$ for P_2

END IF

END WHILE

CALCULATE $\pi(Q, f)$ for P_2

SET f_O to f

SET Q_O to Q

SET $\pi(Q, f)_O$ to $\pi(Q, f)$ for P_2

END IF

B.2 The Capacitated Case

Pseudocode of the optimization algorithm used in the computational study for the capacitated case is follows:

READ p, c_f, c_n, c_r, μ, σ, k and C

IF $c_r \geq c_f$

SET f to 0.1

SET Q to 1

SET $counterQ$ to 1

CALCULATE $\partial\pi(f, Q)/\partial Q$ for P_1

WHILE $|\partial\pi(f, Q)/\partial Q| > 0.001$ and $Q < C$

IF $\partial\pi(f, Q)/\partial Q > 0.001$

```
        SET Q to minimum of C and Q + C/(5 × counterQ)
        CALCULATE ∂π(f, Q)/∂Q for P1
    ELSE
        SET Q to minimum of C and Q − C/(5 × counterQ)
        CALCULATE ∂π(f, Q)/∂Q for P1
    END IF
    INCREMENT counterQ by 1
END WHILE
CALCULATE ∂π(f, Q)/∂f for P1
SET counterf to 1
WHILE |∂π(f, Q)/∂f > 0.001| and counterf < 10000
    IF ∂π(f, Q)/∂f > 0.001
        SET f to minimum of p − cf and f + (p − cf)/(5 × countf)
        SET count2 to 1
        CALCULATE ∂π(f, Q)/∂Q for P1
        WHILE |∂π(f, Q)/∂Q| > 0.001 and Q < C
            IF ∂π(f, Q)/∂Q > 0.001
                SET Q to minimum of C and Q + C/(5 × counter2)
                CALCULATE ∂π(f, Q)/∂Q for P1
            ELSE
                SET Q to minimum of C and Q − C/(5 × counter2)
                CALCULATE ∂π(f, Q)/∂Q for P1
            END IF
            INCREMENT counter2 by 1
        END WHILE
        CALCULATE ∂π(f, Q)/∂f for P1
    ELSE
        SET f to maximum of 0.001 and f − (p − cf)/(5 × countf)
        SET count2 to 1
        CALCULATE ∂π(f, Q)/∂Q for P1
        WHILE |∂π(f, Q)/∂Q| > 0.001 and Q < C
            IF ∂π(f, Q)/∂Q > 0.001
```

```
            SET Q to minimum of C and Q + C/(5 × counter2)
            CALCULATE ∂π(f, Q)/∂Q for P1
        ELSE
            SET Q to minimum of C and Q − C/(5 × counter2)
            CALCULATE ∂π(f, Q)/∂Q for P1
        END IF
        INCREMENT counter2 by 1
    END WHILE
    CALCULATE ∂π(f, Q)/∂f for P1
  END IF
  INCREMENT counterf by 1
END WHILE
CALCULATE π(Q, f) for P1
SET f_O1 to f
SET Q_O1 to Q
SET π(Q, f)_O1 to π(Q, f) for P1

SET Q to 0
SET fLB to 0
SET fUB to p
SET f to (fLB + fUB)/2
CALCULATE dπ(f, 0)/df for P1
WHILE |dπ(0, f)/df| > 0.001
  IF dπ(0, f)/df > 0.001
    SET fLB to f
    SET f to (fLB + fUB)/2
    CALCULATE dπ(Q, f)/df for P1
  ELSE
    SET fUB to f
    SET f to (fLB + fUB)/2
    CALCULATE dπ(f, 0)/df for P1
  END IF
```

```
    END WHILE
    CALCULATE ∂π(f, Q)/∂Q for P_1
    CALCULATE π(0, f)
    SET f_O2 to f
    SET Q_O2 to 0
    SET π(Q, f)_O2 to π(0, f) for P_1

    IF Q_O1 ≤ 1 AND f_O1 ≤ 0.001
    SET f_O to f_O2
    SET Q_O to Q_O2
    SET π(Q, f)_O to π(Q, f)_O2
    ELSE
    SET f_O to f_O1
    SET Q_O to Q_O1
    SET π(Q, f)_O to π(Q, f)_O1

ELSE
    SET f to 0.1
    SET Q to 1
    SET counterQ to 1
    CALCULATE ∂π(f, Q)/∂Q for P_2
    WHILE |∂π(f, Q)/∂Q| > 0.001 and Q < C
        IF ∂π(f, Q)/∂Q > 0.001
            SET Q to minimum of C and Q + C/(5 × counterQ)
            CALCULATE ∂π(f, Q)/∂Q for P_2
        ELSE
            SET Q to minimum of C and Q − C/(5 × counterQ)
            CALCULATE ∂π(f, Q)/∂Q for P_2
        END IF
        INCREMENT counterQ by 1
    END WHILE
    CALCULATE ∂π(f, Q)/∂f for P_2
```

SET $counterf$ to 1

WHILE $|\partial\pi(f,Q)/\partial f| > 0.001$ and $counterf < 10000$

    IF $\partial\pi(f,Q)/\partial f > 0.001$

        SET f to minimum of $p - cf$ and $f + (p - cf)/(5 \times countf)$

        SET $count2$ to 1

        CALCULATE $\partial\pi(f,Q)/\partial Q$ for P_2

        WHILE $|\partial\pi(f,Q)/\partial Q| > 0.001$ and $Q < C$

            IF $\partial\pi(f,Q)/\partial Q > 0.001$

                SET Q to minimum of C and $Q + C/(5 \times counter2)$

                CALCULATE $\partial\pi(f,Q)/\partial Q$ for P_2

            ELSE

                SET Q to minimum of C and $Q - C/(5 \times counter2)$

                CALCULATE $\partial\pi(f,Q)/\partial Q$ for P_2

            END IF

            INCREMENT $counter2$ by 1

        END WHILE

        CALCULATE $\partial\pi(f,Q)/\partial f$ for P_2

    ELSE

        SET f to maximum of 0.001 and $f - (p - cf)/(5 \times countf)$

        SET $count2$ to 1

        CALCULATE $\partial\pi(f,Q)/\partial Q$ for P_2

        WHILE $\partial\pi(f,Q)/\partial Q > 0.001$ and $Q < C$

            IF $\partial\pi(f,Q)/\partial Q > 0.001$

                SET Q to minimum of C and $Q + C/(5 \times counter2)$

                CALCULATE $\partial\pi(f,Q)/\partial Q$ for P_1

            ELSE

                SET Q to minimum of C and $Q - C/(5 \times counter2)$

                CALCULATE $\partial\pi(f,Q)/\partial Q$ for P_2

            END IF

            INCREMENT $counter2$ by 1

        END WHILE

        CALCULATE $\partial\pi(f,Q)/\partial f$ for P_2

```
        END IF
        INCREMENT counterf by 1
END WHILE
CALCULATE π(Q, f) for P2
SET f_O1 to f
SET Q_O1 to Q
SET π(Q, f)_O1 to π(Q, f) for P2

SET Q to 0
SET fLB to 0
SET fUB to p
SET f to (fLB + fUB)/2
CALCULATE dπ(f, 0)/df for P2
WHILE |dπ(0, f)/df| > 0.001
        IF dπ(0, f)/df > 0.001
                SET fLB to f
                SET f to (fLB + fUB)/2
                CALCULATE dπ(Q, f)/df for P2
        ELSE
                SET fUB to f
                SET f to (fLB + fUB)/2
                CALCULATE dπ(f, 0)/df for P2
        END IF
END WHILE
CALCULATE ∂π(f, Q)/∂Q for P2
CALCULATE π(0, f) for P2
SET f_O2 to f
SET Q_O2 to 0
SET π(Q, f)_O2 to π(0, f) for P2
SET ∂π(f, Q)_O2/∂Q to ∂π(f, Q)/∂Q for P2

SET Q to C
```

SET f to 0

CALCULATE $\pi(C, 0)$ for P_2

SET f_{O3} to f

SET Q_{O3} to 0

SET $\pi(Q, f)_{O3}$ to $\pi(C, 0)$ for P_2

IF $Q_{O1} > 1$ AND $f_{O1} > 0.001$

SET f_O to f_{O1}

SET Q_O to Q_{O1}

SET $\pi(Q, f)_O$ to $\pi(Q, f)_{O1}$

ELSE

  IF $\partial\pi(f, Q)_{O2}/\partial Q < 0$

    SET f_O to f_{O2}

    SET Q_O to Q_{O2}

    SET $\pi(Q, f)_O$ to $\pi(Q, f)_{O2}$

  ELSE

    SET f_O to f_{O2}

    SET Q_O to Q_{O2}

    SET $\pi(Q, f)_O$ to $\pi(Q, f)_{O2}$

  END IF

END IF

END IF