A LOCAL PARAMETER ESTIMATOR BASED ON ROBUST LAV ESTIMATION

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VOLKAN ÖZDEMİR

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Approval of the thesis:

A LOCAL PARAMETER ESTIMATOR BASED ON ROBUST LAV ESTIMATION

submitted by VOLKAN ÖZDEMİR in partial fulfillment of the requirements for the degree of Master of Science in Electrical and Electronics Engineering Department, Middle East Technical University by,

Prof. Dr. Gülbin Dural Ünver Dean, Graduate School of Natural and Applied Sciences Prof. Dr. Gönül Turhan Sayan Head of Department, Electrical and Electronics Eng. Assist. Prof. Dr. Murat Göl Supervisor, Electrical and Electronics Eng.Dept.,METU **Examining Committee Members:** Prof. Dr. Ali Nezih Güven Electrical and Electronics Engineering Dept., METU Assist. Prof. Dr. Murat Göl Electrical and Electronics Engineering Dept., METU Prof. Dr. Muammer Ermiş Electrical and Electronics Engineering Dept., METU Prof. Dr. Işık Çadırcı Electrical and Electronics Engineering Dept., Hacettepe.U. Assoc. Prof. Dr. Mehmet Timur Aydemir Electrical and Electronics Engineering Dept., Gazi U. Date: _____

I hereby declare that all information in this document has been obtained and presented in accordance with academic rules and ethical conduct. I also declare that, as required by these rules and conduct, I have fully cited and referenced all material and results that are not original to this work.

Name, Last Name: VOLKAN ÖZDEMİR

Signature :

ABSTRACT

A LOCAL PARAMETER ESTIMATOR BASED ON ROBUST LAV ESTIMATION

Özdemir, Volkan M.S., Department of Electrical and Electronics Engineering Supervisor : Assist. Prof. Dr. Murat Göl

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There are parameter errors in power system models due to the change of weather conditions, such as temperature and humidity changes, miscommunication between the control center and the transducers of circuit breakers and tap changers, etc. Because of the incorrect parameters, the state estimator may provide biased state estimates which may lead to many serious economic and operational results. In order to prevent that, one must identify and correct those parameter errors. This work proposes a local parameter estimator based on the robust Least Absolute Value (LAV) estimator. Considering the increasing number of Phasor Measurement Units (PMUs), their fast refreshing rate and high accuracy, the proposed method will employ PMU measurements in local parameter estimation which will provide a more reliable system model.

In general, a PMU measures the current phasor flowing through a branch, and the voltage phasor of the sending bus of the considered branch. However, it is known that those two measurements are not enough to estimate the parameters of that branch. Therefore, multiple measurements taken in different time instants will be used in the parameter estimation process for measurement redundancy, assuming that the state estimates are also available.

It is known that the LAV estimator is a computationally expensive despite being robust in the presence of enough measurement redundancy. Note that, the parameter estimation problem is a non-linear problem, which increases the computational burden; since the vector to be estimated consists of not only the parameters of the considered branch, but also the bus voltages of the sending and the receiving ends of the considered branch. This deficiency will be eliminated by performing local parameter estimation, which is a very small sized problem compared to the state estimation problem's size

Keywords: Phasor Measurement Units, Least Absolute Value, Robust Parameter Estimation, Local Parameter Estimation

GÜRBÜZ LAV YÖNTEMİNE DAYANAN YEREL PARAMETRE KESTİRİMCİSİ

Özdemir, Volkan Yüksek Lisans, Elektrik ve Elektronik Mühendisliği Bölümü Tez Yöneticisi – : Yrd. Doç. Dr. Murat Göl

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Güç sistemi modellerinde, sıcaklık ve nem değişiklikleri gibi hava şartları nedeniyle bazı parametre hataları oluşmaktadır. Aynı zamanda, kontrol merkezi, devre kesiciler ve kademe değiştiricileri arasındaki iletişimsizlik de güç sistemi parametre hatalarına sebep olabilmektedir. Yanlış parametrelerden dolayı, durum kestirimcisi, yanlı tahminlerde bulunabilir ve bu yüzden ciddi ekonomik ve operasyonel sonuçlarla karşılaşılabilir. Bu durumun önüne geçebilmek için, bu parametre hatalarının tespit edilmesi ve düzeltilmesi gerekmektedir. Fazör Ölçüm Cihazlarının (Phasor Measurement Unit - PMU) gün geçtikçe yaygınlaştığı ve hızlı tepki süreleri göz önüne alınarak, bu tezde En Az Mutlak Değerli (Least Absolute Value - LAV) gürbüz tahmincisi yöntemine dayanan bir yerel parametre kestirimcisi önerilmektedir. Geliştirilmiş olan metot daha güvenilir bir sistem modeli oluşturacaktır.

Genel olarak bir PMU, bir hattan akan Faz Akımını ve ilgili hattın gönderen

tarafındaki Fazör Gerilimini ölçer. Bununla birlikte, bu iki ölçümün, hat parametrelerin tahmini için yeterli olmadığı bilinmektedir. Bu nedenle, sistemde halihazırda bir durum kestirimcinin de varolduğu kabul edilerek, parametre tahmini için ölçüm artıklığını arttırmak amacıyla farklı anlarda çoklu ölçümler kullanılacaktır.

LAV kestirimcisinin gürbüz olmasına rağmen işlemsel olarak çok masraflı olduğu bilinmektedir. Parametre kestiriminin linear olmaması ve kestirilen vektörde ilgili hatların parametre verilerinin yanında gönderen ve alan bara gerilimlerinin değerlerinin de tahmin edilecek olması kestirim probleminin hesapsal yükünü arttırmaktadır. Kestirilen parametrelerin yerel olması, problemin boyutunu küçültür ve bu kusuru yok eder.

Anahtar Kelimeler: Faz Ölçüm Cihazı, Mutlak En Küçük Değer, Gürbüz Parametre Kestirimi, Yerel Parametre Kestirimi To my grandmother

Refika Babür Köker. . .

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LIST OF ABBREVIATIONS

PMU	Phasor Measurement Unit
SE	State Estimation
WLS	Weighted Least Square
LAV	Least Absolute Value
LS	Least Square
EMS	Energy Management System
GPS	Global Positioning System
LP	Linear Programing
IEEE	The Institute of Electrical and Electronics Engineers
NHOT	Neglect the Higher Order Terms
MSE	Mean Squared Error

CHAPTER 1

INTRODUCTION

1.1 Problem Definition

State estimation (SE) in power systems is one of the most essential functions of Energy Management Systems (EMS) for the reliability of the whole system operation [1]. SE assumes a true model of the power system [1–4], and hence knowledge of system topology and true values of the line and transformer parameters are extremely important for the accuracy of SE [1–7]. Although SE assumes perfect knowledge of the system, it is affected by three types of malefactions, [8] which are:

- Bad data on measurements
- Topology errors
- Parameter errors [8]

Bad data and topology error has serious effects on the results of SE, and hence there are various methods developed on the literature to overcome those issues. This work focuses on the parameter errors. When parameters have errors, the state estimator may generate biased state estimates which will conceal the actual states and lead unreliable information about the system. Biased state estimates may cause catastrophic events during the operation because EMS applications and decision routes rely on the estimates generated by the estimator. Performance of the programs that run on the EMS depends on the parameters strongly. Topological errors can be identified easily in SE, which may be treated as parameter errors, while the small errors in the parameters such as branch impedance etc. will create reliability problems [9].

1.2 Estimation Algorithms

Well known parameter estimation techniques can be sorted via two sections, which are off-line methods [5–13] and on-line methods [14–18]. Those techniques are employs Least Squares (LS) estimator [1]. Since LS is a not a robust estimator, even in the presence of single bad data, the estimation results will be biased. Relationship between residuals and parameter errors constitute the main basis of off-line and on-line methods [19].

In order to guarantee unbiased parameter estimates, one must employ normalized residuals test [20], which has a significant computational cost due to the mandatory matrix inversion. In this thesis LAV algorithm will be proposed against the LS for obtaining a robust yet computationally competitive estimator [21, 22].

LS estimator is computationally faster compared to the LAV estimator, if no bad data exists in the measurement set. However, robustness of the LAV estimator makes it more desirable for the proposed parameter estimation method defined in this thesis.

The robust LAV estimator, which is an L-1 estimator, has an iterative solution scheme for non-linear problems [23]. Although the iterative solution means longer solution time, when the estimation problem is formulated as a local parameter estimation problem, which is a very small sized problem in comparison of centralized problems, like state estimation etc, the deficiency will be compensated immediately. The robustness thanks to the LAV estimator and computational efficiency thanks to the local problem formulation makes the proposed parameter estimation method more advantageous compared to the existing methods.

1.3 Phasor Measurements Units

A Phasor Measurement Unit (PMU) is a digital device which can read analog signal voltage with the help of analog to digital converters and produces voltage and current phasor measurements which are called synchrophasors [9, 24]. Synchronized Phasor Measurement Units (PMUs) are the newest and the most reliable trend in the power system world so the number and importance of the PMUs in power grids increase day by day. The SCADA systems provide asynchronized and low resolution measurements, which reduce the reliability of the estimations based on those measurements. On the other hand, the fast refresh rate and high accuracy of PMU measurements enables use of more reliable, computationally efficient and robust estimation methods.

It is proposed to use of the PMU measurements in local parameter estimation which will be the next step of the power system parameter estimation in this work. It is a fact that PMUs take synchronized bus voltage phasor and line current phasor measurements 30 times a second with respect to the Global Positioning System (GPS) [25]. Main two reason of using GPS in PMUs are;

- Ability of determining real coordinates which tells the exact location of the device,
- Having an access to an accurate global clock,

It is a known fact that, the system states and PMU measurements are linearly related, which makes PMU measurements desirable in SE. However, once the parameter estimation is considered, the estimation problem becomes non-linear due to the relation between the system parameters, system states and measurement set. Therefore, although the SE problem has single step solution for systems measured solely by PMUs, the parameter estimation problem will have a iterative solution.

1.4 WLS Estimator

This section presents the WLS estimation which is originally a Gaussian concept of estimation by least squares that was used in astronomical calculations in 18^{th} century [26]. From its invention till the Kalman's studies in 20^{th} century, it nurtured so many estimation theories. Although it was used in order to understand the motions of the planets by Gauss, today it is used widely in power system in many aspects mainly in power system state estimation.

In a power system with;

- *m* measurements
- *n* system states
- Measurement vector, z is non-linearly related to the system states as shown below

$$z = h(x) + e \tag{1.1}$$

x is the system state vector, h(x) is the non-linear function that relates z and x, and e is the measurement error vector.

$$z^T = \begin{bmatrix} z_1 & z_2 & z_3 & \dots & z_m \end{bmatrix}$$
(1.2)

$$h(x) = \begin{bmatrix} h_1(x_1, x_2, x_3...x_n) \\ h_2(x_1, x_2, x_3...x_n) \\ h_3(x_1, x_2, x_3...x_n) \\ \vdots \\ \vdots \\ \vdots \\ \vdots \\ h_m(x_1, x_2, x_3...x_n) \end{bmatrix}$$
(1.3)

$$e^T = \begin{bmatrix} e_1 & e_2 & e_3 & \dots & e_m \end{bmatrix}$$
(1.4)

 $E(e_k)$ is assumed to be equal to 0 and $E[e_ke_l] = 0$. The main aim of a WLS estimator is minimizing the objective function which is:

$$J(x) = \sum_{k=1}^{m} R_{kk}^{-1} * (z_k - h_k(x))^2$$
(1.5)

$$J(x) = (z - h(x))^T * R^{-1} * (z - h(x))$$
(1.6)

To have the minimum, below condition must be satisfied:

$$\frac{\partial J(x)}{\partial x} = 0 \tag{1.7}$$

Calling $\frac{\partial J(x)}{\partial x}$ as g(x) and by expanding g(x) into its Taylor series around the x^k :

$$g(x) = g(x^{k}) + G(x^{k})(x - x^{k}) + NHOT$$
(1.8)

By assuming;

•

- k as the iteration index
- x^k as the vector at k

•
$$G(x) = \frac{\partial g(x^k)}{\partial x}$$

The estimation problem can be solved iteratively by Newton-Raphson method;

$$x^{k+1} = x^k - G(x^k)^{-1} * g(x^k)$$
(1.9)

Where G(x) is called the Gain Matrix which is large but sparse and symmetric

1.5 Contribution

In this thesis, a new local parameter estimation method is proposed which uses the information that is obtained from PMU measurements. Least Absolute Value (LAV) algorithm is used for estimating line parameters through the thesis without needing a bad data process while it is assumed that there exists a state estimator which may or may not be robust. In state estimation, problem is linear whenever the considered system is measured by PMU's, but in parameter estimation case, measurements and line parameters are not linearly related. Therefore, in order to comprehend an iterative solution must be employed, which in general is computationally expensive. the proposed method employs local formulation of the parameter estimation problem, which reduces the size of the estimation problem significantly. The proposed method is evaluated by simulations under different conditions.

1.6 Outline of the Thesis

This thesis is organized as follows:

- Chapter 2 shows LAV estimator in general and how it is applicable to parameter estimation in power systems.
- Problem formulation of the proposed solution is restated in Chapter 3. This chapter also gives the information about system model and how it is implemented for both algorithms.
- The performance comparison between LS and LAV is revealed in Chapter 4. It is proven that LAV is better than the LS in some aspects while showing LAVs shining properties. This chapter also contains simulations and the results of the comparison between LAV and LS estimators in terms of accuracy and computational performance.
- Chapter 5 contains the simulation results of the proposed method under different real life scenarios and shows the power of the power system parameter estimation by LAV.

• Finally, The thesis is concluded in Chapter 6 and new ideas for possible future work is given.

CHAPTER 2

LAV ESTIMATOR

This chapter firstly presents the Least Absolute Value (LAV) estimation of an unknown vector in a linear regression. After that a brief review of linear programming is given, followed by the simplex solution method of linear programming problems.

2.1 Linear Regression

Firstly consider the regression model given below:

$$z_i = A_i^T x + e_i \tag{2.1}$$

where;

- z_i is a set of m observations $i = 1, 2, 3, \dots, m$
- A_i is a set of m vectors $A_i \epsilon R^n, i = 1, 2, 3, \dots, m$
- x is an unknown vector $x \in \mathbb{R}^n, i = 1, 2, 3, \dots, m$
- e_i the random error for each observation

For understanding the linear regression, one should know the basic minimization problem formulation as follows:

• A is a matrix of mxn and A_i^T is the *i*th row of A,

- c is a vector while all of its elements equals to 1's and $c\epsilon R^m$,
- r is the vector of observation residuals where $r \epsilon R^m$.

With all the above information for finding the least absolute value estimate \hat{x} of the unknown vector x, one should solve the following:

minimize
$$c^T |r|$$
 (2.2)

subject to
$$z - Ax = r$$
 (2.3)

2.2 Simplex Solution Method

Objective function of LAV estimator is defined as below for a system with m measurements and n states [1]:

minimize
$$\sum_{i=1}^{n} |r_i|$$
 (2.4)

subject to
$$z_i = h_i(x) + r_i$$
, while $1 < i < m$ (2.5)

In (2.4) and (2.5):

- r_i is the residual value of i^{th} ,
- z_i is simply the i^{th} measurement,
- Finally h_i is a nonlinear function which gives the relation between state vector x and the measurement vector, z_i

LAV optimization problem can be expressed and solved as an equivalent linear programming (LP) problem if one re-arranges the equations and defines new variables which are all non-negative [1]. If x^0 is assumed as an initial solution for the state and one gets the first order approximation of $h_i(x)$ around the x^0 , one gets the objective function as:

$$J(x^k) = \sum_{i=1}^{m} (p_i^k + l_i^k)$$
(2.6)

By (2.6) we can write the measurement residual vector at the kth state estimation iteration as below:

$$r^k = p^k + l^k \tag{2.7}$$

$$r^{k} = z - h(x^{k}) - H(x^{k}) * \Delta x$$
 (2.8)

$$r^{k} = \Delta z^{k} - H(x^{k}) * \Delta x^{k}$$
(2.9)

Since what we are doing is showing the roots of linear programing by LAV, we can easily get rid of the superscript k for simplicity. Than for a closer look of the problem which is solved at iteration k can be written easily as:

Minimize
$$\sum_{i=1}^{m} (p_i + l_i)$$
 (2.10)

Subject to
$$H * \Delta x_p - H * \Delta x_l + p - l = \Delta z$$
 (2.11)

$$\Delta x_p, \Delta x_l, p, l \ge 0$$
 where $\Delta x = \Delta x_p - \Delta x_l$ (2.12)

For writing simpler like general LP problem, it can be summarized as below:

$$Minimize \ c^T * Y \tag{2.13}$$

Subject to
$$M * Y = b$$
 while $Y \ge 0$ (2.14)

Detailed explanation of the (2.13) and (2.14) can be found in below:

$$c^T = [0_m, 1_n] \tag{2.15}$$

$$0_n = [0, 0...0, 0]_{nx1} \tag{2.16}$$

$$1_m = [1, 1....1, 1]_{mx1} \tag{2.17}$$

$$b = \Delta z \tag{2.18}$$

$$Y = [\Delta x_p^T, \Delta x_l^T, p^T, l^T]$$
(2.19)

$$M = [H, -H, I_m, -I_m]$$
(2.20)

$$I_m = eye(m) \tag{2.21}$$

In (2.15), Z_n is the 1x2N vector consisting of zeros and O_m is the 1x2m vector consisting of ones. Δx_p and Δx_l are 1xN, and p and l are 1xm vectors listed in (2.12);

For the iterative solution, one must follow the algorithm shown in below:

- Initialize x_0 as a flat start,
- Solve the linear programing problem given in (2.15) using a linear programing solver,
- Check if $\Delta x_k < \epsilon$
- If 'yes' stop
- $\bullet\,$ If 'no' update the Y and M and go to second step

CHAPTER 3

PROPOSED PARAMETER ESTIMATION METHOD

In Chapter 1 and 2, review of the parameter estimation is given. In this chapter, the new way of parameter estimation method is implemented.

State estimation takes a crucial part in EMS for system security and reliability, in order to build a proper system model one needs proper system data namely parameter values since the data given from the factory are not always reliable EMS needs to have a parameter estimation tool for a secure system. The proposed parameter estimation method employs LAV estimator in an iterative manner. This chapter introduces the proposed parameter estimation method.

EMS may estimates all parameters of a given power system, as well as the subset of the system parameter, like local parameter estimation [1]. If the parameters used in the state estimation tools are accurate then the accuracy of the estimates will be much higher. EMS builds a system model according to those parameters and system topology. The most reliable way of the representing the system model is basic two port π model. While the EMS deals with the whole system for estimating states of the buses, it consumes a lot of CPU power during the process and CPU loading problem is same as if one tries to estimates of the every parameters of the whole system. Since there is a high computational loading while estimating the all parameters, local parameter estimation is preferred in this thesis. Starting from this chapter a local parameter estimation method will be built. For building the local parameter estimation model, such that parameters of each line are estimated separately as a single estimation problem, consider the two-bus system given in Fig.3.1 by two port π model, where;

- g_{12} is the conductance between bus-1 and bus-2,
- b_{12} is the susceptance between bus-1 and bus-2,
- b_{11} is the charging-susceptance of the transmission line.



Figure 3.1: 2-Bus Sample System

The proposed method employs PMU measurements, as they are time stamped and fast refreshed measurements. The only PMU located in Fig.3.1 is st the BUS-1 due to system simplicity and cost issues. It will generate the voltage phasor measurements of BUS-1 and current phasor measurements between BUS-1 and BUS-2. The relation between the system parameters and the PMU measurements that can be taken is expressed as below.

$$I_{ij}^{meas} = Re(I_{ij}^{meas}) + Im(I_{ij}^{meas})$$

$$(3.1)$$

$$Re(I_{ij}^{meas}) = g_{ij}(Re(V_i) - Re(V_j)) - (b_{ij} + b_{ii})(Im(V_i)) + b_{ij}Im(V_j)$$
(3.2)

$$Im(I_{ij}^{meas}) = g_{ij}(Im(V_i) - Im(V_j)) - (b_{ij} + b_{ii})(Re(V_i)) - b_{ij}Re(V_j)$$
(3.3)

In (3.1), (3.2) and (3.3):

• $Re(I_{ij}^{meas})$ is the real part of the current phasor measurement of the PMU located in BUS-i,

- $Im(I_{ij}^{meas})$ is the imaginary part of the current phasor measurement of the PMU located in BUS-i,
- $Re(V_k)$ is the real part of BUS-k voltage phasor
- $Im(V_k)$ is the imaginary part of BUS-k voltage phasor

In the power system given in Fig.3.1, because of the economic constraints, there will be a single PMU located either at Bus-1 or Bus-2. A single scan of a PMU (Voltage and Current phasor measurements) satisfies observability for state estimation in the system given in Fig.3.1. However, considering the additional states (line parameters) of the parameter estimation problem, a single scan will cause un-observability. Therefore, at least three measurement scans are required for the observability of the parameter estimation problem. Considering the fast refresh rate of PMU measurements [25], this thesis proposes to use multiple PMU scans taken from the same measurement unit at consecutive time instants to solve parameter estimation problem.

Using only the voltage and current phasor measurements obtained by a PMU makes the parameter estimation vulnerable to measurements error associated with that PMU. In order to improve the robustness of the parameter estimation, the state estimates of the system from the EMS state estimator are also employed as measurements.

3.1 Building the Jacobian Matrix for Local Parameter Estimation

This thesis employs non-linear parameter estimation formulation, which is stated below:

$$z = h(x) + e \tag{3.4}$$

In (3.4) measurement vector z with size of 8nx1, is defined as below where measurements are taken at n different time instants;

$$Z^{T} = \begin{bmatrix} V^{m,r} & V^{m,i} & V^{e,r} & V^{e,i} & I^{m,r} & I^{m,i} \end{bmatrix}$$
(3.5)

In (3.5):

- $V^{m,r}$ is the vector of real parts of the voltage phasor measurements taken at the sending end of the branch (1xn),
- $V^{m,i}$ is the vector of imaginary parts of the voltage phasor measurements taken at the sending end of the branch (1xn),
- $V^{e,r}$ is the vector of real parts of the voltage phasor estimates at the sending and receiving ends of the branch (1x2n),
- $V^{e,i}$ is the vector of imaginary parts of the voltage phasor estimates at the sending and receiving ends of the branch (1x2n),
- $I^{m,r}$ is the vector of real parts of the current phasor measurements from the sending end to the receiving end of the branch (1xn),
- $I^{m,i}$ is the vector of imaginary parts of the current phasor measurements from the sending end to the receiving end of the branch (1xn)

The state vector x with size (4n+3)x1 is defined as follows;

$$x^{T} = \begin{bmatrix} V^{r} & V^{i} & g_{ij} & b_{ij} & b_{ii} \end{bmatrix}$$
(3.6)

In (3.6):

- V^r is the vector of real parts of the voltage phasors of the sending and receiving ends of the branch (1xn),
- V^i is the vector of imaginary parts of the voltage phasors of the sending and receiving ends of the branch (1x2n),
- g_{ij} is the series conductance of the branch (1x1),
- b_{ij} is the series susceptance of the branch (1x1),

• b_{ii} is the charging susceptance of the branch (1x1).

It is clear that the PMU measurements are non-linearly related to the state vector x defined in (3.6). The non-linear relations of the PMU measurements and the state vectors are mapped in to the h(x) matrix function with size (8n)x1 as below:

$$h(x)^{T} = \begin{bmatrix} V_{i}^{m,r} & V_{i}^{m,i} & I_{ij}^{m,r} & I_{ij}^{m,i} & V_{i}^{r} & V_{j}^{i} & V_{j}^{r} \end{bmatrix}$$
(3.7)

To obtain the Jacobian Matrix of the h(x), partial derivatives of the function h(x) must be written;

$$\frac{\partial h(x)}{\partial x} = H \tag{3.8}$$

In (3.8), H is the Jacobian matrix of the size (8n)x(4n+3). n is assumed as 1, than H is formed for the system given in 3.1. The H matrix is given below:

$$H = \begin{bmatrix} \frac{dV_{i}^{m,r}}{dV_{i}^{r}} & \frac{dV_{i}^{m,r}}{dV_{j}^{r}} & \frac{dV_{i}^{m,r}}{dV_{i}^{i}} & \frac{dV_{i}^{m,r}}{dV_{j}^{i}} & \frac{dV_{i}^{m,r}}{dV_{j}^{i}} & \frac{dV_{i}^{m,r}}{dU_{j}^{i}} & \frac{dV_{i}^{m,r}}{dU_{j}^{i}} & \frac{dV_{i}^{m,r}}{dU_{j}^{i}} & \frac{dV_{i}^{m,r}}{dU_{j}^{i}} & \frac{dV_{i}^{m,r}}{dU_{i}^{i}} & \frac{dV_{i}^{m,r}}{dU_{i}^{i}} & \frac{dV_{i}^{m,r}}{dU_{i}^{i}} & \frac{dV_{i}^{m,r}}{dU_{j}^{i}} & \frac{dV_{i}^{m,r}}{dU_{j}^{i}} & \frac{dV_{i}^{m,r}}{dU_{j}^{i}} & \frac{dV_{i}^{m,r}}{dU_{j}^{i}} & \frac{dV_{i}^{m,r}}{dU_{j}^{i}} & \frac{dV_{i}^{m,r}}{dU_{j}^{i}} & \frac{dV_{i}^{m,r}}{dU_{j}^{i}} & \frac{dV_{i}^{m,r}}{dU_{j}^{i}} & \frac{dV_{i}^{m,r}}{dU_{j}^{i}} & \frac{dV_{i}^{m,r}}{dU_{j}^{i}} & \frac{dV_{i}^{e,r}}{dU_{j}^{i}} & \frac{dV_{i}^{i}}{dU_{j}^{i}} & \frac{dV_{i}^{e,r}}{dU_{j}^{i}} & \frac{dV_{i}^{i}}{dU_{j}^{i}} & \frac{$$

Elements of the Jacobian Matrix, H is given below one by one:

$$\frac{dV_i^{m,r}}{dV_i^r} = 1 \tag{3.10}$$

$$\frac{dV_i^{m,r}}{dV_i^i} = \frac{dV_i^{m,r}}{dV_j^r} = \frac{dV_i^{m,r}}{dV_j^i} = 0$$
(3.11)

$$\frac{dV_i^{m,r}}{dg_{ij}} = \frac{dV_i^{m,r}}{db_{ij}} = \frac{dV_i^{m,r}}{db_{ii}} = 0$$
(3.12)

$$\frac{dV_i^{m,i}}{dV_i^i} = 1 \tag{3.13}$$

$$\frac{dV_i^{m,i}}{dV_i^r} = \frac{dV_i^{m,i}}{dV_j^r} = \frac{dV_i^{m,i}}{dV_j^i} = 0$$
(3.14)

$$\frac{dV_i^{m,i}}{dg_{ij}} = \frac{dV_i^{m,i}}{db_{ij}} = \frac{dV_i^{m,i}}{db_{ii}} = 0$$
(3.15)

$$\frac{dI_{ij}^{m,r}}{dV_i^r} = g_{ij} \tag{3.16}$$

$$\frac{dI_{ij}^{m,r}}{dV_i^i} = -b_{ij} - b_{ii}$$
(3.17)

$$\frac{dI_{ij}^{m,r}}{dV_j^r} = -g_{ij} \tag{3.18}$$

$$\frac{dI_{ij}^{m,r}}{dV_j^i} = b_{ij} \tag{3.19}$$

$$\frac{dI_{ij}^{m,r}}{dg_{ij}} = Re(V_i) - Re(V_j) \tag{3.20}$$

$$\frac{dI_{ij}^{m,r}}{db_{ij}} = -Im(V_i) + Im(V_j)$$
(3.21)

$$\frac{dI_{ij}^{m,r}}{db_{ii}} = -Im(V_i) \tag{3.22}$$

$$\frac{dI_{ij}^{m,i}}{dV_i^r} = b_{ij} + b_{ii} \tag{3.23}$$

$$\frac{dI_{ij}^{m,i}}{dV_i^i} = g_{ij} \tag{3.24}$$

$$\frac{dI_{ij}^{m,i}}{dV_j^r} = -b_{ij} \tag{3.25}$$

$$\frac{dI_{ij}^{m,i}}{dV_j^i} = -g_{ij} \tag{3.26}$$

$$\frac{dI_{ij}^{m,i}}{dg_{ij}} = Im(V_i) - Im(V_j)$$
(3.27)

$$\frac{dI_{ij}^{m,i}}{db_{ij}} = Re(V_i) - Re(V_j)$$
(3.28)

$$\frac{dI_{ij}^{m,i}}{db_{ii}} = Re(V_i) \tag{3.29}$$

$$\frac{dV_i^{e,r}}{dV_i^r} = 1 \tag{3.30}$$

$$\frac{dV_i^{e,r}}{dV_i^i} = \frac{dV_i^{e,r}}{dV_j^r} = \frac{dV_i^{e,r}}{dV_j^i} = \frac{dV_i^{e,r}}{dg_{ij}} = \frac{dV_i^{e,r}}{db_{ij}} = \frac{dV_i^{e,r}}{db_{ii}} = 0$$
(3.31)

$$\frac{dV_i^{e,i}}{dV_i^i} = 1 \tag{3.32}$$

$$\frac{dV_i^{e,i}}{dV_i^r} = \frac{dV_i^{e,i}}{dV_j^r} = \frac{dV_i^{e,i}}{dV_j^i} = \frac{dV_i^{e,i}}{dg_{ij}} = \frac{dV_i^{e,i}}{db_{ij}} = \frac{dV_i^{e,i}}{db_{ii}} = 0$$
(3.33)

$$\frac{dV_j^{e,r}}{dV_j^r} = 1 \tag{3.34}$$

$$\frac{dV_j^{e,r}}{dV_i^i} = \frac{dV_j^{e,r}}{dV_j^r} = \frac{dV_j^{e,r}}{dV_j^i} = \frac{dV_j^{e,r}}{dg_{ij}} = \frac{dV_j^{e,r}}{db_{ij}} = \frac{dV_j^{e,r}}{db_{ii}} = 0$$
(3.35)

$$\frac{dV_j^{e,i}}{dV_j^i} = 1 \tag{3.36}$$

$$\frac{dV_j^{e,i}}{dV_i^r} = \frac{dV_i^{e,i}}{dV_i^i} = \frac{dV_j^{e,i}}{dV_j^r} = \frac{dV_j^{e,i}}{dg_{ij}} = \frac{dV_j^{e,i}}{db_{ij}} = \frac{dV_j^{e,i}}{db_{ii}} = 0$$
(3.37)

To sum up the matrix representation, equations between (3.10) and (3.37) are written into H in (3.38):

$$H = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ g_{ij} & -g_{ij} & -(b_{ij} + b_{ii}) & b_{ij} & V_i^{Re} - V_j^{Re} & V_j^{Im} - V_i^{Im} \\ (b_{ij} + b_{ii}) & -b_{ij} & g_{ij} & -g_{ij} & V_i^{Im} - V_j^{Im} & V_j^{Re} - V_i^{Re} & V_i^{Re} \end{bmatrix}$$
(3.38)

It is clear that even though H is a massive matrix, a large part of the elements are zeros which will cause a drop in the processor power necessity during the local parameter estimation.

As seen in (3.38), H is a rank-deficient matrix if n is less than 3. As mentioned before, this thesis proposes to use multiple time scans to perform parameter estimation. "3" is the minimum number of scans that should be taken to obtain observability. However, in order to have a robust estimator, measurement redundancy is required. In order to have a robust estimator, each state should have 4 redundant measurements [23]. Therefore, this work proposes the use of at least 6 measurement scans for single bad data robustness. Thanks to the fast refresh rate of PMUs (30 times/second) and small size of the parameter estimation problem (48x27 for n = 6), the computational time and burden of the proposed method is very small.

CHAPTER 4

COMPARISON BETWEEN WLS AND LAV ESTIMATORS

Chapter 1 and 2 reveals the basic history of the parameter estimation while in Chapter 3, the new way of parameter estimation method is implemented. Chapter 4 shows the comparison between WLS and LAV Estimators.

WLS is the mostly employed estimator to solve the estimation problems, due to its simplicity and fast solution. However in this work, it is proposed to employ LAV estimator to solve the parameter estimation problem. This chapter provides a comparison between the WLS and LAV estimators, to validate the choice of the estimator for robust parameter estimation problem.

The 2-bus system given in Fig.3.1 is employed for the simulation purpose. True values of the line parameters selected for simulation purposes are given in the IEEE 30-Bus sample system and the values are used in this chapter are stated in Table 4.1. Note that the actual values of the line are not known for sure since these values can easily be affected by different conditions [27].

In this work 3 different scenarios were employed to validate the proposed method in MATLAB environment using a Windows Operating System. In the 1st scenario, parameter estimation with no bad data case is run for both LS and LAV estimators. In the 2nd scenario, single bad parameter data was introduced and the simulation results were compared to LAV estimator. Finally a bad measurement was introduced to the measurement set and the estimation results of LS and LAV estimators are compared. In all simulations, the measurement set is in the form of (3.5) and the state vector is in the form of (3.6). Measurement set consists of 6 time scans as indicated in the previous chapter.

The performance of the both estimators are compared by the mean squared errors (MSE) which is the average of the squares of errors which is the difference between actual values and what is estimated by the estimators [28]. Calculation of the MSE is shown below:

$$MSE = \frac{1}{n} * \sum_{k=1}^{n} (X_k^e - X_k)^2$$
(4.1)

where;

- *n* is the number of predictions
- X^e is the estimated value
- X is the actual value

Table 4.1: Transmission Line Parameters

Transmission Line Parameters	
g_{12}	5.2246 pu
b_{12}	-15.646 pu
b_{11}	0.0528 pu

4.1 Scenario-1

In this scenario, no bad data were introduced to the measurement set. However, to make the simulation more realistic, Gaussian error was added to all measurements. This scenario was run 100 times and the results are presented in Table 4.2.

As seen in Table 4.2, both estimators are converged to the true values in comparable durations. Note that no special effort is spent for estimator optimization.

Mean	Squared Errors	and Mean Time
	MSE of g_{12}	2.27e-011
WLS	MSE of b_{12}	2.87e-014
	MSE of b_{11}	2.71e-13
	mean time	$0.028 {\rm seconds}$
	MSE of g_{12}	9.23e-10
LAV	MSE of b_{12}	1.006e-10
	MSE of b_{11}	4.23e-12
	mean time	$0.059 {\rm seconds}$

 Table 4.2: Simulation Results for Scenario 1

4.2 Scenario-2

In this scenario, it is assumed that the given parameter information is incorrect, such that the series susceptance was assumed to be 3 times larger than the true value. In this scenario, a Gaussian error was added to the measurement set and the simulations were conducted 100 times. Simulation results are presented in Table 4.3 and Fig.2.

As seen in Table 4.3 and Fig.4.1, the proposed method converged to the true values in acceptable duration with acceptable accuracy. Note that, the increase in simulation duration and decrease in accuracy are caused by the incorrect initial values of the parameter estimation problem.



Figure 4.1: Parameter Estimations of LAV Scenario-2

Table 4.3: Simulation Results for Scenario 2

Mean Squared Errors and Mean Time		
	MSE of g_{12}	0.0984
LAV	MSE of b_{12}	0.0958
	MSE of b_{11}	0.0063
	mean time	0.15 seconds

4.3 Scenario-3

In this scenario, it is assumed that the measurement set includes a bad measurement. Gaussian error was added to the measurement set and the simulations were conducted 100 times. In each simulation, a measurement is selected as bad randomly. Simulation results for LS and LAV estimators are presented in Table 4.3.

Mean Squared Errors and Mean Time		
	MSE of g_{12}	7.62
WLS	MSE of b_{12}	6.86
	MSE of b_{11}	0.067
	mean time	$0.05 \ { m seconds}$
	MSE of g_{12}	6.64e-8
LAV	MSE of b_{12}	7.39e-8
	MSE of b_{11}	4.66e-9
	mean time	$0.03 {\rm seconds}$

Table 4.4: Simulation Results for Scenario 3

As seen in Table 4.3, the proposed method converged to true values in similar duration with Scenario 1. On the other hand, LS had a high mean squared error, which indicates that it converged to incorrect parameters. In order to obtain unbiased estimates with LS, one needs to perform bad data detection and identification process as well, which requires extra computational burden.

CHAPTER 5

SIMULATIONS AND NUMERICAL RESULTS

5.1 Line Parameter Estimation with non-Robust State Estimator

Chapter 1 and 2 contains review of the parameter estimation and Chapter 3 shows the new way of parameter estimation method while Chapter 4 shows the comparison between WLS and LAV Estimators. Finally this chapter presents the numerical results obtained via simulations to validate the proposed method. In different case studies, different amount of bad data is considered for different measurement types.

The 2-bus system given in Fig.3.1 is again employed for simulation purpose but this time the state estimator which provides the state estimates used in the parameter estimation is a non robust estimator so it will generate biased and wrong state variables.True values of the line parameters selected for simulation purposes are stated in Table 4.1. Note that the actual values of the line parameters are not known for sure since these values can easily be affected by different conditions.

5.1.1 Parameter Estimation with non-Robust State Estimator with Errors in V_1^e

In this section, 4 different scenarios were employed to validate the proposed method in MATLAB environment using a Windows Operating System. All of the scenarios are run for 100 times using Monte Carlo simulations. In the 1st scenario, parameter estimation with only one bad data in V_1^e during the 7 time scan is run for LAV. In the 2nd scenario, parameter estimation with 2 bad data in V_1^e during 7 time scan is run for LAV. In the 3rd scenario, parameter estimation with 3 bad data in V_1^e during 7 time scan is run for LAV. In the 4th scenario, parameter estimation with 6 bad data in V_1^e during 7 time scan is run for LAV.

5.1.1.1 Scenario-1: Single Error in V_1^e

In this scenario it is assumed that the given parameter information is correct but the state estimator used in the system gives one bad data for V_1^{e,t_1} . For example, at time t_1 the state estimator gives $V_1^{e,t_1} = 0$. The multiple bad data in the observation set is eliminated successfully by the proposed LAV based estimator, and unbiased estimates are obtained. Simulation results are presented in Table 5.1.

Table 5.1: Simulation Results for 6.1.1.1. Scenario-1

Mean Squared Errors and Mean Time		
	MSE of g_{12}	0.1361
LAV	MSE of b_{12}	0.1368
	MSE of b_{11}	0.0077
	mean time	$0.6064 {\rm seconds}$

5.1.1.2 Scenario-2: Two Errors in V_1^e

In this scenario it is assumed that the given parameter information is correct but the state estimator used in the system gives 2 bad data for V_1^{e,t_1} . For example, at time t_1 the state estimator gives $V_1^{e,t_1} = 0$ and at time t_2 the state estimator gives $V_1^{e,t_2} = 0$. The multiple bad data in the observation set is eliminated successfully by the proposed LAV based estimator, and unbiased estimates are obtained. Simulation results are presented in Table 5.2.

-	Mean Squared Errors and Mean Time		
		MSE of g_{12}	0.1566
	LAV	MSE of b_{12}	0.1447
		MSE of b_{11}	0.0078
		mean time	0.60062 seconds

Table 5.2: Simulation Results for 6.1.1.2. Scenario-2

5.1.1.3 Scenario-3: Three Errors in V_1^e

In this scenario it is assumed that the given parameter information is correct but the state estimator used in the system gives 3 bad data for V_1^{e,t_1} . For example, at time t_1 the state estimator gives $V_1^{e,t_1} = 0$, at time t_2 the state estimator gives $V_1^{e,t_2} = 0$. and at time t_3 the state estimator gives $V_1^{e,t_3} = 0$. The multiple bad data in the observation set is eliminated successfully by the proposed LAV based estimator, and unbiased estimates are obtained.Simulation results are presented in Table 5.3.

Table 5.3: Simulation Results for 6.1.1.3. Scenario-3

Mean Squared Errors and Mean Time		
	MSE of g_{12}	0.1673
LAV	MSE of b_{12}	0.1568
	MSE of b_{11}	0.0090
	mean time	$0.60778 { m seconds}$

5.1.1.4 Scenario-4: Four Errors in V_1^e

In this scenario it is assumed that the given parameter information is correct but the state estimator used in the system gives 4 bad data for V_1^{e,t_1} . For example, at time t_1, t_2, t_3 and t_4 the state estimator gives $V_1^{e,t_1} = V_1^{e,t_2} = V_1^{e,t_3} = V_1^{e,t_4} = 0$. The multiple bad data in the observation set is eliminated successfully by the proposed LAV based estimator, and unbiased estimates are obtained.Simulation results are presented in Table 5.4.

Mean Squared Errors and Mean Time		
	MSE of g_{12}	0.1685
LAV	MSE of b_{12}	0.1658
	MSE of b_{11}	0.0095
	mean time	$0.60595 \ { m seconds}$

Table 5.4: Simulation Results for 6.1.1.4. Scenario-4

5.1.1.5 Scenario-5: Five Errors in V_1^e

In this scenario it is assumed that the given parameter information is correct but the state estimator used in the system gives 5 bad data for V_1^{e,t_1} . For example, at time t_1, t_2, t_3, t_4 and t_5 the state estimator gives $V_1^{e,t_1} = V_1^{e,t_2} =$ $V_1^{e,t_3} = V_1^{e,t_4} = V_1^{e,t_5}0$. The multiple bad data in the observation set is eliminated successfully by the proposed LAV based estimator, and unbiased estimates are obtained.Simulation results are presented in Table 5.5.

Table 5.5: Simulation Results for 6.1.1.5. Scenario-5

Mean Squared Errors and Mean Time		
	MSE of g_{12}	0.1698
LAV	MSE of b_{12}	0.16858
	MSE of b_{11}	0.0099
	mean time	0.61527 seconds

5.1.1.6 Scenario-6: Six Errors in V_1^e

In this scenario it is assumed that the given parameter information is correct but the state estimator used in the system gives 6 bad data for V_1^e . Since there are 7 time instants and nearly all of the estimator voltage values are biased, parameter estimation is not successful as the previous scenarios. Simulation results are presented in Table 5.6. In Fig.5.1 one can see the performance of the parameter estimator by itself. The critical issue about this scenario is, even though all of the inputs are biased, parameter estimator sometimes estimates the true values as well during the 100 times repetitive scenario run. If one expands the time instants of the parameter estimator which can be easily done by using 8 time instants instead of using 6 time instants, parameter estimator will be again successful and the time consumption of the estimation process is nearly the same. Simulation results are presented in Table 5.7 and Fig.5.2. In Fig.5.2, *iter* means the total number of the iterations in order to estimate the parameters.

Table 5.6: Simulation Results for 6.1.1.6. Scenario-6

Mean Squared Errors and Mean Time		
	MSE of g_{12}	44.4461
LAV	MSE of b_{12}	116.2870
	MSE of b_{11}	112.4563
	mean time	0.54831 seconds



Figure 5.1: Parameter Estimations Results for 6.1.1.4. Scenario-4

Mean Squared Errors and Mean Time		
	MSE of g_{12}	0.7507
LAV	MSE of b_{12}	0.7197
	MSE of b_{11}	0.0613
	mean time	0.46776 seconds

Table 5.7: Simulation Results for 6.1.1.4. Scenario-4 with 8 Time Instants



Figure 5.2: Parameter Estimations Results for 6.1.1.4. Scenario-4 with 7 Time Instants

5.1.2 Parameter Estimation with non-Robust State Estimator with Errors in V_2^e

The 2-bus system given in Fig.3.1 is again employed for simulation purposes but this time the state estimator which gives the used input in the parameter estimation is a non robust estimator so it will generate biased and wrong state variables at time instants. True values of the line parameters selected for simulation purposes are stated in Table 4.1. Note that the actual values of the line are not known for sure since these values can easily be affected by different conditions. In this section, 2 different scenarios were employed to validate the proposed method in MATLAB environment using a Windows Operating System. All of the scenarios are run for 100 times for getting unbiased observations for the operator.

In the 1st scenario, parameter estimation with only one bad data in and in 7 time scan is run for LAV. In the 2st scenario, parameter estimation with 2 bad data in and in 7 time scan is run for LAV.

5.1.2.1 Scenario-1: Single Error in V_2^e

In this scenario it is assumed that the given parameter information is correct but the state estimator used in the system gives one bad data for V_2^e . For example, at time t_1 the state estimator gives $V_2^{e,t_1} = 0$. The multiple bad data in the observation set is eliminated successfully by the proposed LAV based estimator, and unbiased estimates are obtained. Simulation results are presented in Table 5.8

Table 5.8: Simulation Results for 6.1.2.1. Scenario-1

Mean Squared Errors and Mean Time		
	MSE of g_{12}	0.0542
LAV	MSE of b_{12}	0.01223
	MSE of b_{11}	0.00508
	mean time	$0.56871 \ { m seconds}$

5.1.2.2 Scenario-2: Double Error in V_2^e

In this scenario it is assumed that the given parameter information is correct but the state estimator used in the system gives 2 bad data for V_2^e . Although there are 7 time instants and all of the estimator voltage values are biased, parameter estimation is not successful as the previous scenarios. Simulation results are presented in Table 5.9. In Fig.5.3 one can see the performance of the parameter estimator by itself. The critical issue about this scenario is, even though results generally bad, parameter estimator sometimes estimates the true values as well during the 100 times repetitive scenario run. If one expands the time instants of the parameter estimator which can be easily done by using 8 time instants instead of using 7 time instants, parameter estimator will be again successful and the time consumption of the estimation process is nearly the same. Simulation results are presented in Table 5.10.

Table 5.9: Simulation Results for 6.1.2.2. Scenario-2

Mean Squared Errors and Mean Time		
	MSE of g_{12}	37.985
LAV	MSE of b_{12}	196.548
	MSE of b_{11}	116.858
	mean time	0.52894 seconds



Figure 5.3: Parameter Estimations Results for 6.1.2.2. Scenario-2

Mean Squared Errors and Mean Time		
	MSE of g_{12}	0.6507
LAV	MSE of b_{12}	0.5856
	MSE of b_{11}	0.0552
	mean time	0.4789 seconds

Table 5.10: Simulation Results for 6.1.2.2. Scenario-2 with 8 Time Instants

5.1.3 Parameter Estimation with non-Robust State Estimator with Errors in V_1^e and V_2^e

In this section, 4 different scenarios were employed to validate the proposed method in MATLAB environment using a Windows Operating System. All of the scenarios are run for 100 times for getting unbiased observations for the operator.

In the 1st scenario, parameter estimation with only one bad data in each of V_1^e and V_2^e in 7 time scan is run. In the 2nd scenario, parameter estimation with 2 bad data in each of V_1^e and V_2^e in 7 time scan is run. In the 3rd scenario, parameter estimation with 3 bad data in each of V_1^e and V_2^e in 7 time scan is run and finally In the 4th scenario, parameter estimation with 4 bad data in each of V_1^e and V_2^e in 7 time scan is run.

5.1.3.1 Scenario-1: Single Error in V_1^e and V_2^e

In this scenario it is assumed that the given parameter information is correct but the state estimator used in the system gives one bad data for V_1^e and V_2^e . For example, at time t_1 the state estimator gives $V_1^{e,t_1} = V_2^{e,t_1} = 0$. The multiple bad data in the observation set is eliminated successfully by the proposed LAV based estimator, and unbiased estimates are obtained. Simulation results are presented in in Table 5.11.

Mean Squared Errors and Mean Time		
	MSE of g_{12}	0.1080
LAV	MSE of b_{12}	0.1230
	MSE of b_{11}	0.0061
	mean time	0.3540 seconds

Table 5.11: Simulation Results for 6.1.3.1. Scenario-1

5.1.3.2 Scenario-2: Double Error in V_1^e and V_2^e

In this scenario it is assumed that the given parameter information is correct but the state estimator used in the system gives 2 bad dataa for V_1^e and V_2^e . For example, at time t_1 the state estimator gives $V_1^{e,t_1} = V_2^{e,t_1} = 0$, n time t_2 the state estimator gives $V_1^{e,t_1} = V_2^{e,t_1} = V_1^{e,t_2} = V_2^{e,t_2} = 0$. The multiple bad data in the observation set is eliminated successfully by the proposed LAV based estimator, and unbiased estimates are obtained. Simulation results are presented in in Table 5.12.

Table 5.12: Simulation Results for 6.1.3.2. Scenario-2

Mean Squared Errors and Mean Time		
	MSE of g_{12}	9.0618
LAV	MSE of b_{12}	1.5132
	MSE of b_{11}	0.1896
	mean time	$0.3540 {\rm seconds}$

5.1.3.3 Scenario-3: Three Errors in V_1^e and V_2^e

In this scenario it is assumed that the given parameter information is correct but the state estimator used in the system gives 3 bad data for V_1^e and V_2^e . For example, at time t_1 the state estimator gives $V_1^{e,t_1} = V_2^{e,t_1} = 0$, at time t_2 the state estimator gives $V_1^{e,t_1} = V_2^{e,t_1} = V_1^{e,t_2} = V_2^{e,t_2} = 0$ and at time t_3 the state estimator gives $V_1^{e,t_1} = V_2^{e,t_1} = V_1^{e,t_2} = V_2^{e,t_2} = V_1^{e,t_3} = V_2^{e,t_3} = 0$. The multiple bad data in the observation set is eliminated successfully by the proposed LAV based estimator, and unbiased estimates are obtained. Simulation results are presented in in Table 5.13 .

Mean Squared Errors and Mean Time		
	MSE of g_{12}	0.1527
LAV	MSE of b_{12}	0.1144
	MSE of b_{11}	0.0067
	mean time	0.3822 seconds

Table 5.13: Simulation Results for 6.1.3.3. Scenario-3

5.1.3.4 Scenario-4: Four Errors in V_1^e and V_2^e

In this scenario it is assumed that the given parameter information is correct but the state estimator used in the system gives 4 bad data for V_1^e and V_2^e . Although there are 7 time instants and all of the estimator voltage values are biased, parameter estimation is not successful as the previous scenarios. Simulation results are presented in Table 5.14. In Fig.5.4 one can see the performance of the parameter estimator by itself. The critical issue about this scenario is, even though results generally bad, parameter estimator sometimes estimates the true values as well during the 100 times repetitive scenario run. If one expands the time instants of the parameter estimator which can be easily done by using 8 time instants instead of using 7 time instants, parameter estimator will be again successful and the time consumption of the estimation process is nearly the same. Simulation results are presented in Table 5.15.

Table 5.14: Simulation Results for 6.1.3.4. Scenario-4

Mean Squared Errors and Mean Time		
	MSE of g_{12}	35.9401
LAV	MSE of b_{12}	15.2967
	MSE of b_{11}	0.7106
	mean time	$0,52698 { m seconds}$



Figure 5.4: Parameter Estimations Results for 6.1.3.3. Scenario-4

Table 5.15: Simulation Results for 6.1.3.4. Scenario-4 with 8 Time Instants

Mean Squared Errors and Mean Time		
	MSE of g_{12}	0.1582
LAV	MSE of b_{12}	0.1367
	MSE of b_{11}	0.0069
	mean time	0.3125 seconds

5.2 Line Parameter Estimation with Robust State Estimator with Errors in PMU

The 2-bus system given in Fig.3.1 is again employed for simulation purpose but this time the state estimator which gives the used input in the parameter estimation is robust but the PMU devices located on the bus-1 is not calibrated properly, so it will generate biased and wrong measurement states at time instants.True values of the line parameters selected for simulation purposes are stated in Table 4.1. Note that the actual values of the line are not known for sure since these values can easily be affected by different conditions.

5.2.1 Parameter Estimation with Robust State Estimator with Errors in V_1^m

In this section, 6 different scenarios were employed to validate the proposed method in MATLAB environment using a Windows Operating System. All of the scenarios are run for 100 times for getting unbiased observations for the operator.

In the 1st scenario, parameter estimation with only one bad data in each of V_1^m in 7 time scan is run. In the 2nd scenario, parameter estimation with 2 bad data in each of V_1^m in 7 time scan is run. In the 3rd scenario, parameter estimation with 3 bad data in each of V_1^m in 7 time scan is run. In the 4th scenario, parameter estimation with 4 bad data in each of V_1^m in 7 time scan is run. In the 5th scenario, parameter estimation with 5 bad data in each of V_1^m in 7 time scan is run. In the 5th scenario, parameter estimation with 5 bad data in each of V_1^m in 7 time scan is run and finally in the 6th scenario, parameter estimation with 3 bad data in each of V_1^m in 7 time scan is run.

5.2.1.1 Scenario-1: Single Error in V_1^m

In this scenario it is assumed that the given parameter information is correct but the state estimator used in the system gives one bad data for V_1^m . For example, at time t_1 the state estimator gives $V_1^{m,t_1} = 0$. The multiple bad data in the observation set is eliminated successfully by the proposed LAV based estimator, and unbiased estimates are obtained. Simulation results are presented in Table 5.16.

Mean Squared Errors and Mean Time		
	MSE of g_{12}	0.0963
LAV	MSE of b_{12}	0.0989
	MSE of b_{11}	0.0062
	mean time	0.3452 seconds

Table 5.16: Simulation Results for 6.2.1.1. Scenario-1

5.2.1.2 Scenario-2: Double Error in V_1^m

In this scenario it is assumed that the given parameter information is correct but the PMU used in the system gives 2 bad data for V_1^m . For example, at time t_1 the state estimator gives $V_1^{m,t_1} = 0$ and at time t_2 the state estimator gives $V_1^{m,t_2} = 0$. The multiple bad data in the observation set is eliminated successfully by the proposed LAV based estimator, and unbiased estimates are obtained. Simulation results are presented in Table 5.17.

Table 5.17: Simulation Results for 6.2.1.2. Scenario-2

Mean Squared Errors and Mean Time		
	MSE of g_{12}	0.1404
LAV	MSE of b_{12}	0.1772
	MSE of b_{11}	0.0085
	mean time	0.3337 seconds

5.2.1.3 Scenario-3: Three Errors in V_1^m

In this scenario it is assumed that the given parameter information is correct but the PMU used in the system gives 3 bad data for V_1^m . For example, at time t_1 PMU gives $V_1^{m,t_1} = 0$, at time t_2 PMU gives $V_1^{m,t_2} = 0$ and at time t_3 PMU gives $V_1^{m,t_3} = 0$. The multiple bad data in the observation set is eliminated successfully by the proposed LAV based estimator, and unbiased estimates are obtained. Simulation results are presented in Table 5.18.

Mean Squared Errors and Mean Time		
	MSE of g_{12}	0.1412
LAV	MSE of b_{12}	0.1782
	MSE of b_{11}	0.0079
	mean time	0.3258 seconds

Table 5.18: Simulation Results for 6.2.1.3. Scenario-3

5.2.1.4 Scenario-4: Four Errors in V_1^m

In this scenario it is assumed that the given parameter information is correct but PMU used in the system gives 4 bad data for V_1^m . For example, at time t_1 PMU gives $V_1^{m,t_1} = 0$, at time t_2 PMU gives $V_1^{m,t_2} = 0$, at time t_3 PMU gives $V_1^{m,t_3} = 0$ and at time t_4 PMU gives $V_1^{m,t_4} = 0$. The multiple bad data in the observation set is eliminated successfully by the proposed LAV based estimator, and unbiased estimates are obtained. Simulation results are presented in Table 5.19.

Table 5.19: Simulation Results for 6.2.1.4. Scenario-4

Mean Squared Errors and Mean Time		
	MSE of g_{12}	0.1412
LAV	MSE of b_{12}	0.1782
	MSE of b_{11}	0.0079
	mean time	0.3258 seconds

5.2.1.5 Scenario-5: Five Errors in V_1^m

In this scenario it is assumed that the given parameter information is correct but PMU used in the system gives 5 bad data for V_1^m . For example, at time t_1 the PMU gives $V_1^{m,t_1} = 0$, at time t_2 PMU gives $V_1^{m,t_2} = 0$, at time t_3 PMU gives $V_1^{m,t_3} = 0$ and at time t_4 PMU gives $V_1^{m,t_4} = 0$. The multiple bad data in the observation set is eliminated successfully by the proposed LAV based estimator, and unbiased estimates are obtained. Simulation results are presented in Table

Table 5.20: Simulation Results for 6.2.1.5. Scenario-5

Mean Squared Errors and Mean Time		
	MSE of g_{12}	0.1412
LAV	MSE of b_{12}	0.1782
	MSE of b_{11}	0.0079
	mean time	$0.3258 {\rm seconds}$

5.2.1.6 Scenario-6: Six Errors in V_1^m

In this scenario it is assumed that the given parameter information is correct but the PMU used in the system gives 6 bad data for V_1^m . Although there are 7 time instants and 3 of the PMU values are biased, parameter estimation is not successful as the previous scenarios. Simulation results are presented in Table 5.21. In Fig.5.5 one can see the performance of the parameter estimator by itself. The critical issue about this scenario is, even though results generally bad, parameter estimator sometimes estimates the true values as well during the 100 times repetitive scenario run. If one expands the time instants of the parameter estimator which can be easily done by using 8 time instants instead of using 7 time instants, parameter estimator will be again successful and the time consumption of the estimation process is nearly the same. Simulation results are presented in Table 5.22.

Table 5.21: Simulation Results for 6.2.1.6. Scenario-6

Mean Squared Errors and Mean Time				
	MSE of g_{12}	8.6097		
LAV	MSE of b_{12}	1.9281		
	MSE of b_{11}	2.1579		
	mean time	0.39654 seconds		

5.20.



Figure 5.5: Parameter Estimations Results for 6.2.1.6. Scenario-6

Tab	le 5.22:	Simul	lation	Results	for	6.2.1.6.	Scenario-6	with 8	Time	Instants
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Mean Squared Errors and Mean Time				
	MSE of g_{12}	0.1232		
LAV	MSE of b_{12}	0.1134		
	MSE of b_{11}	0.0052		
	mean time	0.3127 seconds		

5.2.2 Parameter Estimation with Robust State Estimator with Errors in I_{12}^m

In this section, 2 different scenarios were employed to validate the proposed method in MATLAB environment using a Windows Operating System. All of the scenarios are run for 100 times for getting unbiased observations for the operator.

In the 1^{st} scenario, parameter estimation with only one bad data in I_{12}^m in 7 time

scan is run. In the 2^{nd} scenario, parameter estimation with 2 bad data in I_{12}^m in 7 time scan is run.

5.2.2.1 Scenario-1: Single Error in I_{12}^m

In this scenario it is assumed that the given parameter information is correct but PMU used in the system gives 1 bad data for I_{12}^m . For example, at time t_1 the PMU gives $I_{12}^{m,t_1} = 0$. The multiple bad data in the observation set is eliminated successfully by the proposed LAV based estimator, and unbiased estimates are obtained. Simulation results are presented in Table 5.23.

Table 5.23: Simulation Results for 6.2.2.1. Scenario-1

Mean Squared Errors and Mean Time			
	MSE of g_{12}	0.1334	
LAV	MSE of b_{12}	0.1599	
	MSE of b_{11}	0.0075	
	mean time	0.39443 seconds	

5.2.2.2 Scenario-2: Double Error in I_{12}^m

In this scenario it is assumed that the given parameter information is correct but the PMU used in the system gives 2 bad data for I_{12}^m . Although there are 7 time instants and 2 of the PMU values are biased, parameter estimation is not successful as the previous scenarios. Simulation results are presented in Table 5.24. In Fig.5.6 one can see the performance of the parameter estimator by itself. The critical issue about this scenario is, even though results generally bad, parameter estimator sometimes estimates the true values as well during the 100 times repetitive scenario run. If one expands the time instants of the parameter estimator which can be easily done by using 8 time instants instead of using 7 time instants, parameter estimator will be again successful and the time consumption of the estimation process is nearly the same. Simulation results are presented in Table 5.25.

Mean Squared Errors and Mean Time				
	MSE of g_{12}	6.9578		
LAV	MSE of b_{12}	11.1321		
	MSE of b_{11}	0.3784		
	mean time	$0.37397 \ seconds$		

Table 5.24: Simulation Results for 6.2.2.2. Scenario-2



Figure 5.6: Parameter Estimations Results for 6.2.2.2. Scenario-2

Table 5.25: Simulation Results for 6.2.2.2. Scenario-2 with 8 Time Instants

Mean Squared Errors and Mean Time				
	MSE of g_{12}	0.1175		
LAV	MSE of b_{12}	0.1383		
	MSE of b_{11}	0.0068		
	mean time	$0.39015 \ { m seconds}$		

5.2.3 Parameter Estimation with Robust State Estimator with Errors in V_1^m and I_{12}^m

In this section, 2 different scenarios were employed to validate the proposed method in MATLAB environment using a Windows Operating System. All of the scenarios are run for 100 times for getting unbiased observations for the operator.

In the 1st scenario, parameter estimation with only one bad data in each of I_{12}^m and V_1^m in 7 time scan is run. In the 2nd scenario, parameter estimation with 2 bad data in each of I_{12}^m and V_1^m in 7 time scan is run.

5.2.3.1 Scenario-1: Single Error in V_1^m and I_{12}^m

In this scenario it is assumed that the given parameter information is correct but PMU used in the system gives 1 bad data for each of I_{12}^m and V_1^m . For example, at time t_1 the PMU gives $I_{12}^{m,t_1} = V_1^m = 0$. The multiple bad data in the observation set is eliminated successfully by the proposed LAV based estimator, and unbiased estimates are obtained.. Simulation results are presented in Table 5.26.

Table 5.26: Simulation Results for 6.2.3.1. Scenario-1

Mean Squared Errors and Mean Time				
	MSE of g_{12}	0.1355		
LAV	MSE of b_{12}	0.1549		
	MSE of b_{11}	0.0080		
	mean time	$0.39972 \ seconds$		

5.2.3.2 Scenario-2: Double Error in V_1^m and I_{12}^m

In this scenario it is assumed that the given parameter information is correct but the PMU used in the system gives 2 bad data for each of I_{12}^m and V_1^m . Although there are 7 time instants and 2 of the PMU values are biased, parameter estima-
tion is not successful as the previous scenarios. Simulation results are presented in Table 5.27. In Fig.5.7 one can see the performance of the parameter estimator by itself. The critical issue about this scenario is, even though results generally bad, parameter estimator sometimes estimates the true values as well during the 100 times repetitive scenario run. If one expands the time instants of the parameter estimator which can be easily done by using 8 time instants instead of using 7 time instants, parameter estimator will be again successful and the time consumption of the estimation process is nearly the same. Simulation results are presented in Table 5.28.

Table 5.27: Simulation Results for 6.2.3.2. Scenario-2

Mean Squared Errors and Mean Time		
	MSE of g_{12}	3.0210
LAV	MSE of b_{12}	8.3504
	MSE of b_{11}	0.3097
	mean time	$0.36985 \ seconds$



Figure 5.7: Parameter Estimations Results for 6.2.3.2. Scenario-2

Mean Squared Errors and Mean Time		
	MSE of g_{12}	0.1341
LAV	MSE of b_{12}	0.1571
	MSE of b_{11}	0.0088
	mean time	0.3984 seconds

Table 5.28: Simulation Results for 6.2.3.2. Scenario-2 with 8 Time Instants

5.3 Transformer Parameter Estimation with non-Robust State Estimator



Figure 5.8: 2-bus sample system with a Simple Transformers

The 2-bus system with transformers given in Fig.5.8 is employed for simulation purpose. True values of the line parameters selected for simulation purposes are stated in Table 5.29. Note that the actual values of the line are not known for sure since these values can easily be affected by different conditions. Pi model is considered during the scenarios. Measurement set consists of 7 time scans as well.

 Table 5.29:
 Transmission Line Parameters

Transmission Line Parameters		
Y_{12}	-1.786 pu	
b_{12}	-0.969 pu	

5.3.1 Transformer Parameter Estimation with Robust State Estimator with Error in Parameter a

In this scenario it is assumed that the given parameter information is incorrect, such that the transformers tap ratio to be 3 times larger than the true value. In this scenario Gaussian error was added to the measurement set and the simulations were conducted 100 times. As seen in Table 5.30, the proposed method converged to the true values in acceptable duration with acceptable accuracy.

Table 5.30: Simulation Results for Scenario 6.2.3.1.

Mean Squared Errors and Mean Time		
	MSE of g_{12}	0.1291
LAV	MSE of b_{12}	0.0249
	MSE of b_{11}	9.0920e-4
	MSE of a	6.1042e-4
	mean time	$0.3240 {\rm seconds}$

5.3.2 Transformer Parameter Estimation with non-Robust State Estimator with Errors in V_1^e

In this section, 6 different scenarios were employed to validate the proposed method in MATLAB environment using a Windows Operating System. All of the scenarios are run for 100 times for getting unbiased observations for the operator.

In the 1st scenario, parameter estimation with only one bad data in V_1^e in 7 time scan is run. In the 2nd scenario, parameter estimation with 2 bad data in V_1^e in 7 time scan is run. In the 3rd scenario, parameter estimation with 3 bad data in V_1^e in 7 time scan is run. In the 4th scenario, parameter estimation with 4 bad data in V_1^e in 7 time scan is run.In the 5th scenario, parameter estimation with5 bad data in V_1^e in 7 time scan is run.and finally in the 6th scenario, parameter estimation with 6 bad data in V_1^e in 7 time scan is run.

5.3.2.1 Scenario-1: Single Error in V_1^e

In this scenario it is assumed that the given parameter information is correct but state estimator that is used in the system gives 1 bad data for V_1^e . For example, at time t_1 the state estimator gives $V_1^e = 0$. The multiple bad data in the observation set is eliminated successfully by the proposed LAV based estimator, and unbiased estimates are obtained. Simulation results are presented in Table 5.31.

Table 5.31: Simulation Results for 6.3.2.1. Scenario-1

Mean Squared Errors and Mean Time		
	MSE of g_{12}	1.0699e-010
LAV	MSE of b_{12}	2.3544e-009
	MSE of b_{11}	1.0090e-010
	MSE of a	2.5103e-010
	mean time	0.3874 seconds

5.3.2.2 Scenario-2: Double Error in V_1^e

In this scenario it is assumed that the given parameter information is correct but state estimator that is used in the system gives 2 bad data for V_1^e . For example, at time t_1 the state estimator gives $V_1^{e,t_1} = 0$ and at time t_2 the state estimator gives $V_1^{e,t_2} = 0$. The multiple bad data in the observation set is eliminated successfully by the proposed LAV based estimator, and unbiased estimates are obtained. Simulation results are presented in Table 5.32.

Table 5.32: Simulation Results for 6.3.2.2. Scenario-2

Mean Squared Errors and Mean Time		
	MSE of g_{12}	1.0787e-010
LAV	MSE of b_{12}	2.1224e-008
	MSE of b_{11}	1.0950e-010
	MSE of a	2.4121e-010
	mean time	0.3815 seconds

5.3.2.3 Scenario-3: Three Errors in V_1^e

In this scenario it is assumed that the given parameter information is correct but state estimator that is used in the system gives 3 bad data for V_1^e . For example, at time t_1 the state estimator gives $V_1^{e,t_1} = 0$, at time t_2 the state estimator gives $V_1^{e,t_2} = 0$ and at time t_3 the state estimator gives $V_1^{e,t_3} = 0$. The multiple bad data in the observation set is eliminated successfully by the proposed LAV based estimator, and unbiased estimates are obtained. Simulation results are presented in Table 5.33.

Mean Squared Errors and Mean Time		
	MSE of g_{12}	2.0799e-008
LAV	MSE of b_{12}	3.3566e-008
	MSE of b_{11}	1.5090e-009
	MSE of a	8.5204 e-010
	mean time	$0.3985 \ seconds$

Table 5.33: Simulation Results for 6.3.2.3. Scenario-3

5.3.2.4 Scenario-4: Four Errors in V_1^e

In this scenario it is assumed that the given parameter information is correct but state estimator that is used in the system gives 4 bad data for V_1^e . For example, at time t_1 the state estimator gives $V_1^{e,t_1} = 0$, at time t_2 the state estimator gives $V_1^{e,t_2} = 0$, at time t_3 the state estimator gives $V_1^{e,t_3} = 0$ and at time t_4 the state estimator gives $V_1^{e,t_4} = 0$. The multiple bad data in the observation set is eliminated successfully by the proposed LAV based estimator, and unbiased estimates are obtained. Simulation results are presented in Table 5.34.

Mean Squared Errors and Mean Time		
	MSE of g_{12}	2.0799e-007
LAV	MSE of b_{12}	3.2111e-007
	MSE of b_{11}	1.1589e-008
	MSE of a	8.8954 e - 009
	mean time	$0.3815 \ { m seconds}$

Table 5.34: Simulation Results for 6.3.2.4. Scenario-4

5.3.2.5 Scenario-5: Five Errors in V_1^e

In this scenario it is assumed that the given parameter information is correct but state estimator that is used in the system gives 5 bad data for V_1^e . For example, at time t_1 the state estimator gives $V_1^{e,t_1} = 0$, at time t_2 the state estimator gives $V_1^{e,t_2} = 0$, at time t_3 the state estimator gives $V_1^{e,t_3} = 0$, at time t_4 the state estimator gives $V_1^{e,t_4} = 0$ and at time t_5 the state estimator gives $V_1^{e,t_5} = 0$. The multiple bad data in the observation set is eliminated successfully by the proposed LAV based estimator, and unbiased estimates are obtained. Simulation results are presented in Table 5.35.

Table 5.35: Simulation Results for 6.3.2.5. Scenario-5

Mean Squared Errors and Mean Time		
	MSE of g_{12}	2.0524e-006
LAV	MSE of b_{12}	3.1598e-006
	MSE of b_{11}	1.1452e-007
	MSE of a	8.7496e-008
	mean time	0.37614 seconds

5.3.2.6 Scenario-6: Six Errors in V_1^e

In this scenario it is assumed that the given parameter information is correct but the state estimator used in the system gives 6 bad data for V_1^e . Although there are 7 time instants and 6 of the PMU values are biased, parameter estimation is not successful as the previous scenarios. Simulation results are presented in Table 5.36. In Fig.5.9 and in Fig.5.10 one can see the performance of the parameter estimator by itself. The critical issue about this scenario is, even though results generally bad, parameter estimator sometimes estimates the true values as well during the 100 times repetitive scenario run. If one expands the time instants of the parameter estimator which can be easily done by using 8 time instants instead of using 7 time instants, parameter estimator will be again successful and the time consumption of the estimation process is nearly the same. Simulation results are presented in Table 5.37 in Fig.5.11 and in Fig.5.12

Mean Squared Errors and Mean Time		
	MSE of g_{12}	1.3283
LAV	MSE of b_{12}	1.9771
	MSE of b_{11}	0.2037
	MSE of a	0.3173
	mean time	$0.36985 \ { m seconds}$

Table 5.36: Simulation Results for 6.3.2.6. Scenario-6

Table 5.37: Simulation Results for 6.3.2.6. Scenario-6 with 8 Time Instants

Mean Squared Errors and Mean Time		
	MSE of g_{12}	0.1341
LAV	MSE of b_{12}	0.1571
	MSE of b_{11}	0.0088
	MSE of a	0.3097
	mean time	$0.3984 \ seconds$



Figure 5.9: Parameter Estimations Results for 6.3.2.6. Scenario-6



Figure 5.10: Tap Ratio Estimation Results for 6.3.3.6. Scenario-6



Figure 5.11: Parameter Estimations Results for 6.3.2.6. Scenario-6



Figure 5.12: Tap Ratio Estimation Results for 6.3.3.6. Scenario-6

5.3.3 Transformer Parameter Estimation with non-Robust State Estimator with Errors in V_2^e

In this section, 2 different scenarios were employed to validate the proposed method in MATLAB environment using a Windows Operating System. All of the scenarios are run for 100 times for getting unbiased observations for the operator.

In the 1st scenario, parameter estimation with only one bad data in V_2^e in 7 time scan is run. In the 2nd scenario, parameter estimation with 2 bad data in V_2^e in 7 time scan is run.

5.3.3.1 Scenario-1: Single Error in V_2^e

In this scenario it is assumed that the given parameter information is correct but state estimator that is used in the system gives 1 bad data for V_2^e . For example, at time t_1 the state estimator gives $V_2^{e,t_1} = 0$. The multiple bad data in the observation set is eliminated successfully by the proposed LAV based estimator, and unbiased estimates are obtained. Simulation results are presented in Table 5.38.

Table 5.38: Simulation Results for 6.3.3.1. Scenario-1

Mean Squared Errors and Mean Time		
	MSE of g_{12}	1.0699e-010
LAV	MSE of b_{12}	2.3544e-009
	MSE of b_{11}	1.0090e-010
	MSE of a	2.5103e-010
	mean time	0.3874 seconds

5.3.3.2 Scenario-2: Double Error in V_2^e

In this scenario it is assumed that the given parameter information is correct but the state estimator used in the system gives 2 bad data for V_2^e . Although there are 7 time instants and 6 of the PMU values are biased, parameter estimation is not successful as the previous scenarios. Simulation results are presented in Table 5.39. In Fig.5.13 and in Fig.5.14 one can see the performance of the parameter estimator by itself. The critical issue about this scenario is, even though results generally bad, parameter estimator sometimes estimates the true values as well during the 100 times repetitive scenario run. If one expands the time instants of the parameter estimator which can be easily done by using 8 time instants instead of using 7 time instants, parameter estimator will be again successful and the time consumption of the estimation process is nearly the same. Simulation results are presented in Table 5.40 and in Fig.5.15

Mean	Mean Squared Errors and Mean Time		
	MSE of g_{12}	19.6872	
LAV	MSE of b_{12}	31.3483	
	MSE of b_{11}	1.5684	
	MSE of a	0.1245	
	mean time	$0.36145 \ \mathrm{seconds}$	

Table 5.39: Simulation Results for 6.3.3.2. Scenario-2

Table 5.40: Simulation Results for 6.3.3.2. Scenario-6 with 8 Time Instants

Mean Squared Errors and Mean Time		
	MSE of g_{12}	1.4607 e-07
LAV	MSE of b_{12}	5.3091 e-08
	MSE of b_{11}	4.8862e-09
	MSE of a	2.3890e-09
	mean time	$0.3984 \ seconds$



Figure 5.13: Parameter Estimations Results for 6.3.3.2. Scenario-2



Figure 5.14: Tap Ratio Estimation Results for 6.3.3.2. Scenario-2



Figure 5.15: Tap Ratio Estimation Results for 6.3.3.2. Scenario-2 with 8 Time Instants

5.3.4 Transformer Parameter Estimation with non-Robust State Estimator with Errors in V_1^e and V_2^e

In this section, 2 different scenarios were employed to validate the proposed method in MATLAB environment using a Windows Operating System. All of the scenarios are run for 100 times for getting unbiased observations for the operator.

In the 1^{st} scenario, parameter estimation with only one bad data for each V_1^e and V_2^e in 7 time scan is run. In the 2^{nd} scenario, parameter estimation with 2 bad data for each V_1^e and V_2^e in 7 time scan is run.

5.3.4.1 Scenario-1: Single Error in V_1^e and V_2^e

In this scenario it is assumed that the given parameter information is correct but state estimator that is used in the system gives 1 bad data for V_1^e and V_2^e . For example, at time t_1 the state estimator gives $V_1^{e,t_1} = V_2^{e,t_1} = 0$. The multiple bad data in the observation set is eliminated successfully by the proposed LAV based estimator, and unbiased estimates are obtained. Simulation results are presented in Table 5.41.

Mean Squared Errors and Mean Time		
	MSE of g_{12}	7.8436e-008
LAV	MSE of b_{12}	5.0663 e-008
	MSE of b_{11}	4.2440e-009
	MSE of a	2.4319e-009
	mean time	0.37562 seconds

Table 5.41: Simulation Results for 6.3.4.1. Scenario-1

5.3.4.2 Scenario-2: Double Error in V_1^e and V_2^e

In this scenario it is assumed that the given parameter information is correct but state estimator that is used in the system gives 2 bad data for V_1 and V_2^e . For example, at time t_1 the state estimator gives $V_1^{e,t_1} = V_2^{e,t_1} = 0$ and at time t_2 the state estimator gives $V_1^{e,t_2} = V_2^{e,t_2} = 0$. The multiple bad data in the observation set is eliminated successfully by the proposed LAV based estimator, and unbiased estimates are obtained. Simulation results are presented in Table 5.42.

Mean Squared Errors and Mean Time		
	MSE of g_{12}	0.0453
LAV	MSE of b_{12}	0.1427
	MSE of b_{11}	0.0051
	MSE of a	0.0053
	mean time	0.37125 seconds

Table 5.42: Simulation Results for 6.3.4.2. Scenario-2

5.3.4.3 Scenario-3: Three Errors in V_1^e and V_2^e

In this scenario it is assumed that the given parameter information is correct but the state estimator used in the system gives 3 bad data for V_1^e and V_2^e . Although there are 7 time instants and 2 of the PMU values are biased, parameter estimation is not successful as the previous scenarios. Simulation results are presented in Table 5.43. In Fig.5.16 and in Fig.5.17 one can see the performance of the parameter estimator by itself. The critical issue about this scenario is, even though results generally bad, parameter estimator sometimes estimates the true values as well during the 100 times repetitive scenario run. If one expands the time instants of the parameter estimator which can be easily done by using 8 time instants instead of using 7 time instants, parameter estimator will be again successful and the time consumption of the estimation process is nearly the same. Simulation results are presented in Table 5.44 and in Fig.5.18

Table 5.43: Simulation Results for 6.3.4.3. Scenario-3

Mean Squared Errors and Mean Time		
	MSE of g_{12}	19.6872
LAV	MSE of b_{12}	31.3483
	MSE of b_{11}	1.5684
	MSE of a	0.1245
	mean time	0.36145 seconds

Mean Squared Errors and Mean Time		
	MSE of g_{12}	2.1394e-07
LAV	MSE of b_{12}	2.1420e-07
	MSE of b_{11}	1.2245e-08
	MSE of a	6.5588e-09
	mean time	0.3911 seconds

Table 5.44: Simulation Results for 6.3.4.3. Scenario-3 with 8 Time Instants



Figure 5.16: Parameter Estimations Results for 6.3.4.2. Scenario-2



Figure 5.17: Tap Ratio Estimation Results for 6.3.4.3. Scenario-3



Figure 5.18: Tap Ratio Estimation Results for 6.3.4.3. Scenario-3 with 8 Time Instants

5.4 Transformer Parameter Estimation with Robust State Estimator with Errors in PMU

The 2-bus system given in Fig.5.8 is again employed for simulation purpose but this time the state estimator which gives the used input in the parameter estimation is robust but the PMU devices located on the bus-1 is not calibrated properly, so it will generate biased and wrong measurement states at time instants.True values of the line parameters selected for simulation purposes are stated in Table 5.29. Note that the actual values of the line are not known for sure since these values can easily be affected by different conditions.

5.4.1 Transformer Parameter Estimation with Robust State Estimator with Errors in V_1^m

In this section, 6 different scenarios were employed to validate the proposed method in MATLAB environment using a Windows Operating System. All of the scenarios are run for 100 times for getting unbiased observations for the operator.

In the 1st scenario, parameter estimation with only one bad data in each of V_1^m in 7 time scan is run. In the 2nd scenario, parameter estimation with 2 bad data in each of V_1^m in 7 time scan is run. In the 3rd scenario, parameter estimation with 3 bad data in each of V_1^m in 7 time scan is run. In the 4th scenario, parameter estimation with 4 bad data in each of V_1^m in 7 time scan is run. In the 5th scenario, parameter estimation with 5 bad data in each of V_1^m 7 time scan is run and finally in the 6th scenario, parameter estimation with 3 bad data in each of V_1^m in 7 time scan is run.

5.4.1.1 Scenario-1: Single Error in V_1^m

In this scenario it is assumed that the given parameter information is correct but state estimator that is used in the system gives 1 bad data for V_1^m . For example, at time t_1 the state estimator gives $V_1^{m,t_1} = 0$. The multiple bad data in the observation set is eliminated successfully by the proposed LAV based estimator, and unbiased estimates are obtained. Simulation results are presented in Table 5.45.

Mean Squared Errors and Mean Time		
	MSE of g_{12}	2.0799e-008
LAV	MSE of b_{12}	3.3566e-008
	MSE of b_{11}	1.5090e-009
	MSE of a	8.5204 e-010
	mean time	0.34122 seconds

Table 5.45: Simulation Results for 6.4.1.1. Scenario-1

5.4.1.2 Scenario-2: Double Error in V_1^m

In this scenario it is assumed that the given parameter information is correct but state estimator that is used in the system gives 2 bad data for V_1^m . For example, at time t_1 and t_1 the state estimator gives $V_1^{m,t_1} = V_1^{m,t_2}0$. The multiple bad data in the observation set is eliminated successfully by the proposed LAV based estimator, and unbiased estimates are obtained. Simulation results are presented in Table 5.46.

Table 5.46: Simulation Results for 6.4.1.2. Scenario-2

Mean Squared Errors and Mean Time		
	MSE of g_{12}	1.3681 e-08
LAV	MSE of b_{12}	1.1986e-08
	MSE of b_{11}	9.4490e-10
	MSE of a	4.8059e-10
	mean time	$0.341369 { m seconds}$

5.4.1.3 Scenario-3: Three Errors in V_1^m

In this scenario it is assumed that the given parameter information is correct but state estimator that is used in the system gives 3 bad data for V_1^m . For example, at time t_1 , t_1 and t_3 the state estimator gives $V_1^{m,t_1} = V_1^{m,t_2} = V_1^{m,t_3}0$. The multiple bad data in the observation set is eliminated successfully by the proposed LAV based estimator, and unbiased estimates are obtained. Simulation results are presented in Table 5.47.

Table 5.47: Simulation Results for 6.4.1.3. Scenario-3

Mean Squared Errors and Mean Time		
	MSE of g_{12}	2.6864 e-08
LAV	MSE of b_{12}	1.4287e-08
	MSE of b_{11}	1.1380e-09
	MSE of a	6.7359e-10
	mean time	0.34587 seconds

5.4.1.4 Scenario-4: Four Errors in V_1^m

In this scenario it is assumed that the given parameter information is correct but state estimator that is used in the system gives 4 bad data for V_1^m . For example, at time t_1 , t_1 , t_3 and t_4 the state estimator gives $V_1^{m,t_1} = V_1^{m,t_2} = V_1^{m,t_3} = V_1^{m,t_4}0$. The multiple bad data in the observation set is eliminated successfully by the proposed LAV based estimator, and unbiased estimates are obtained. Simulation results are presented in Table 5.48.

Table 5.48: Simulation Results for 6.4.1.4. Scenario-4

Mean Squared Errors and Mean Time		
	MSE of g_{12}	1.3568e-08
LAV	MSE of b_{12}	1.9069e-08
	MSE of b_{11}	1.6117e-09
	MSE of a	7.2774e-10
	mean time	0.35268 seconds

5.4.1.5 Scenario-5: Five Errors in V_1^m

In this scenario it is assumed that the given parameter information is correct but state estimator that is used in the system gives 5 bad data for V_1^m . For example, at time t_1 , t_1 , t_3 , t_4 and t_5 the state estimator gives $V_1^{m,t_1} = V_1^{m,t_2} = V_1^{m,t_3} =$ $V_1^{m,t_4}0 = V_1^{m,t_5}0$. The multiple bad data in the observation set is eliminated successfully by the proposed LAV based estimator, and unbiased estimates are obtained. Simulation results are presented in Table 5.49.

Table 5.49: Simulation Results for 6.4.1.5. Scenario-5

Mean Squared Errors and Mean Time		
	MSE of g_{12}	4.9655e-08
LAV	MSE of b_{12}	4.0168e-08
	MSE of b_{11}	5.1770e-09
	MSE of a	2.5772e-09
	mean time	0.332548 seconds

5.4.1.6 Scenario-6: Six Errors in V_1^m

In this scenario it is assumed that the given parameter information is correct but the PMU used in the system gives 6 bad data for V_1^m . Although there are 7 time instants and 6 of the PMU values are biased, parameter estimation is not successful as the previous scenarios. Simulation results are presented in Table 5.50. In Fig.5.19 and in Fig.5.20 one can see the performance of the parameter estimator by itself. The critical issue about this scenario is, even though results generally bad, parameter estimator sometimes estimates the true values as well during the 100 times repetitive scenario run. If one expands the time instants of the parameter estimator which can be easily done by using 8 time instants instead of using 7 time instants, parameter estimator will be again successful and the time consumption of the estimation process is nearly the same. Simulation results are presented in Table 5.51 and in Fig.5.21



Figure 5.19: Parameter Estimations Results for 6.4.4.6. Scenario-6



Figure 5.20: Tap Ratio Estimation Results for 6.4.1.6. Scenario-6



Figure 5.21: Tap Ratio Estimation Results for 6.4.4.6. Scenario-6 with 8 Time Instants

Table 5.50: Simulation Results for 6.4.1.6. Scenario-6

Mean Squared Errors and Mean Time		
	MSE of g_{12}	0.2582
LAV	MSE of b_{12}	0.1220
	MSE of b_{11}	0.0284
	MSE of a	0.0223
	mean time	0.34129 seconds

Mean Squared Errors and Mean Time		
	MSE of g_{12}	1.4607 e-07
LAV	MSE of b_{12}	5.3091 e-08
	MSE of b_{11}	4.8862e-09
	MSE of a	2.3890e-09
	mean time	0.3984 seconds

Table 5.51: Simulation Results for 6.4.1.6. Scenario-6 with 8 Time Instants

5.4.2 Transformer Parameter Estimation with Robust State Estimator with Errors in I_{12}^m

In this section, 2 different scenarios were employed to validate the proposed method in MATLAB environment using a Windows Operating System. All of the scenarios are run for 100 times for getting unbiased observations for the operator.

In the 1st scenario, parameter estimation with only one bad data in I_{12}^m in 7 time scan is run. In the 2nd scenario, parameter estimation with 2 bad data in I_{12}^m in 7 time scan is run.

5.4.2.1 Scenario-1: Single Error in I_{12}^m

In this scenario it is assumed that the given parameter information is correct but state estimator that is used in the system gives 1 bad data for I_{12}^m . For example, at time t_1 the state estimator gives $I_{12}^{m,t_1} = 0$. The multiple bad data in the observation set is eliminated successfully by the proposed LAV based estimator, and unbiased estimates are obtained. Simulation results are presented in Table 5.52.

Mean Squared Errors and Mean Time		
	MSE of g_{12}	1.7192e-008
LAV	MSE of b_{12}	8.5635e-009
	MSE of b_{11}	1.0324 e-009
	MSE of a	5.0888e-010
	mean time	0.36248 seconds

Table 5.52: Simulation Results for 6.4.2.1. Scenario-1

5.4.2.2 Scenario-2: Double Error in I_{12}^m

In this scenario it is assumed that the given parameter information is correct but the PMU used in the system gives 2 bad data for I_{12}^m . Although there are 7 time instants and only 2 of the PMU values are biased, parameter estimation is not successful as the previous scenarios. Simulation results are presented in Table 5.53. In Fig.5.22 and in Fig.5.23 one can see the performance of the parameter estimator by itself. The critical issue about this scenario is, even though results generally bad, parameter estimator sometimes estimates the true values as well during the 100 times repetitive scenario run. If one expands the time instants of the parameter estimator which can be easily done by using 8 time instants instead of using 7 time instants, parameter estimator will be again successful and the time consumption of the estimation process is nearly the same. Simulation results are presented in Table 5.54 and in Fig.5.24

Table 5.53: Simulation Results for 6.4.2.2. Scenario-2

Mean Squared Errors and Mean Time		
	MSE of g_{12}	1.3992
LAV	MSE of b_{12}	2.9840
	MSE of b_{11}	0.0654
	MSE of a	0.1511
	mean time	0.34784 seconds

Mean Squared Errors and Mean Time		
	MSE of g_{12}	1.2058e-07
LAV	MSE of b_{12}	8.5493 e-08
	MSE of b_{11}	3.7169e-09
	MSE of a	2.0987e-09
	mean time	$0.36897 \ seconds$

Table 5.54: Simulation Results for 6.4.2.2. Scenario-2 with 8 Time Instants



Figure 5.22: Parameter Estimations Results for 6.4.2.2. Scenario-2



Figure 5.23: Tap Ratio Estimation Results for 6.4.2.2. Scenario-2



Figure 5.24: Tap Ratio Estimation Results for 6.4.2.2. Scenario-2 with 8 Time Instants

5.4.3 Transformer Parameter Estimation with Robust State Estimator with Errors in V_1^m and I_{12}^m

In this section, 2 different scenarios were employed to validate the proposed method in MATLAB environment using a Windows Operating System. All of the scenarios are run for 100 times for getting unbiased observations for the operator.

In the 1st scenario, parameter estimation with only one bad data in each of I_{12}^m and V_1^m in 7 time scan is run. In the 2nd scenario, parameter estimation with 2 bad data in each of I_{12}^m and V_1^m in 7 time scan is run.

5.4.3.1 Scenario-1: Single Error in V_1^m and I_{12}^m

In this scenario it is assumed that the given parameter information is correct but state estimator that is used in the system gives 1 bad data for V_1^m and I_{12}^m . For example, at time t_1 the state estimator gives $I_{12}^{m,t_1} = V_1^{m,t_1} = 0$. The multiple bad data in the observation set is eliminated successfully by the proposed LAV based estimator, and unbiased estimates are obtained. Simulation results are presented in Table 5.55.

Mean Squared Errors and Mean Time		
	MSE of g_{12}	1.56978e-008
LAV	MSE of b_{12}	8.45891e-009
	MSE of b_{11}	2.0124e-008
	MSE of a	4.015e-009
	mean time	0.347815 seconds

Table 5.55: Simulation Results for 6.4.3.1. Scenario-1

5.4.3.2 Scenario-2: Double Error in V_1^m and I_{12}^m

In this scenario it is assumed that the given parameter information is correct but the PMU used in the system gives 2 bad data for V_1^m and I_{12}^m . Although there are 7 time instants and only 2 of the PMU values are biased, parameter estimation is not successful as the previous scenarios. Simulation results are presented in Table 5.56. In Fig.5.25 and in Fig.5.26 one can see the performance of the parameter estimator by itself. The critical issue about this scenario is, even though results generally bad, parameter estimator sometimes estimates the true values as well during the 100 times repetitive scenario run. If one expands the time instants of the parameter estimator which can be easily done by using 8 time instants instead of using 7 time instants, parameter estimator will be again successful and the time consumption of the estimation process is nearly the same. Simulation results are presented in Table 5.57 and in Fig.5.27

Mean Squared Errors and Mean Time		
	MSE of g_{12}	1.3992
LAV	MSE of b_{12}	2.9840
	MSE of b_{11}	0.0654
	MSE of a	0.1435
	mean time	0.34112

Table 5.56: Simulation Results for 6.4.3.2. Scenario-2

Table 5.57: Simulation Results for 6.4.3.2. Scenario-2 with 8 Time Instants

Mean Squared Errors and Mean Time		
	MSE of g_{12}	1.2489e-07
LAV	MSE of b_{12}	6.9826e-08
	MSE of b_{11}	2.954e-09
	a	1.0658e-08
	mean time	$0.36215 \ \mathrm{seconds}$



Figure 5.25: Parameter Estimations Results for 6.4.3.2. Scenario-2



Figure 5.26: Tap Ratio Estimation Results for 6.4.3.2. Scenario-2



Figure 5.27: Tap Ratio Estimation Results for 6.4.2.2. Scenario-2 with 8 Time Instants

CHAPTER 6

CONCLUSION

This thesis introduces a robust parameter estimator against bad measurements, which employs LAV estimator. In order to increase the computational performance (decrease the computational burden), the estimation problem is developed locally, for a single line measured by a PMU. The required measurement redundancy is maintained using multiple PMU scans and state estimates. The proposed method is validated with simulations.

The contribution of the proposed work can be listed as follows:

- The proposed parameter estimation method is robust against bad measurements.
- Increasing the PMU measurement snapshots will also improve the robustness of the estimator. Note that compared to WLS the proposed LAV estimation is slightly constitutionally expensive. However, considering the performance of the LAV estimator under bad data and incorrect parameter conditions, the case studies simulated the superiority of the proposed method over WLS estimator.
- Thanks to the proposed local estimation approach, the size of the estimation problem is very small. Therefore, the computational burden of the parameter estimation problem is small, even the number of measurement snapshots increases, which enables removal of bad data with long duration.
- Thanks to the fast and reliable solution, the parameter estimation can be performed multiple times during a day or week, according to the change

of environmental conditions, that cause change in line and transformer parameters.

- The developed method can be used at the control center for each branch or transformer separately. Note that parameter estimation is not required to be performed as frequent as state estimation. Therefore, for computational ease, parameter estimation of each branch and transformer can be performed one at a time.
- Parameter errors generally flagged as bad measurements in state estimation. Using a reliable parameter estimator will increase the trust to the measurements and enable a more reliable system operation.
- Numerical studies showed that even the state estimates are unbiased, i.e. a reliable state estimator is available in EMS or the state estimates are biased, parameters can be estimated correctly.

The major drawback of the proposed method is the need for a PMU located at the line with the parameters to be estimated. Note that this necessity applies to all parameter estimation techniques, and hence it does not constitute a disadvantage to the proposed method over the methods available in the literature. As a future work, PMU designs can be improved by designing a unique FPGA code that can solve the parameter estimation problem locally for the PMUs.

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APPENDIX A

MATLAB CODE OF PARAMETER ESTIMATION WITH ROBUST STATE ESTIMATOR

```
tic
clc
close all
clear all
k=1;
g=0;
b=0;
bs=0;
k=1;
tts=100;
Xs=zeros(3,tts);
i s matrixi=zeros(tts,1);
test no=1;
for dongu=1:tts
```

os = 7;

```
zdatameas=xlsread ( random.xlsx , meas );
 z e s t = [];
 x flat = z eros ((os 4), 1);
 for k = 1:2:(os 4)
      x flat (k) = 1 + 0.1 rand;
      x flat (k+1) = 0.1 rand;
 \operatorname{end}
 x est = [x flat; 5.2246 ; 15.646 ; 0.0528];
 gbbsest = [
                  x e s t ((os 4+1))
                (x est((os 4+2)) x est((os 4+3)))
                  x est((os 4+1))
                  x est((os 4+2));
                 (x est((os 4+2)) + x est((os 4+3)))
                  x est((os 4+1)) = x est((os 4+2))
                  x est((os 4+1))];
 Hgbbsye = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix};
      0 \ 1 \ 0 \ 0;
      gbbsest;];
 H sol est = zeros (size (Hgbbsye, 1) os
                                    , size (Hgbbsye, 2) os);
 H sol est = DiagK (Hgbbsye, os);
```

```
H est = H sol est;
z est = H est x flat;
z e s t = z e s t + 0.001 rand ((4 o s), 1);
measnum est = length(zest);
R est = 0.0001. eye (meas num est,
                                          measnum est);
G est = H est inv(Rest) Hest;
x est=inv(Gest) Hest inv(Rest) z est;
x = [x f l a t; 5.2246; (15.6463); 0.0528];
xK = [x flat; 5.2246; 15.646; 0.0528];
\mathbf{z=}\left[ \;\right];
for k=1:os
    V1mr = z est (4 (k 1) + 1);
```

V1er = x est (4 (k 1) + 1);

V1mi = z e s t (4 (k 1) + 2);

```
V1ei=x est (4 (k1)+2);
V2er=x est (4 (k1)+3);
V2ei=x est (4 (k1)+4);
I12mr=z est (4 (k1)+3);
I12mi=z est (4 (k1)+4);
z block=[V1mr; V1mi; I12mr;
I12mi; V1er; V1ei; V2er; V2ei];
z=[z;z block];
```

 end

epsilon = 5;

h = []; hbk=ones(8,1);k=0;

for k=0:(os 1) hbk(1)=x((4 k+1)); hbk(2)=x((4 k+2));hbk(3)=x((os 4+1))

$$\begin{aligned} hbk(4) &= & x((os \ 4+1)) \\ & (x((4 \ k+2)) \ x((4 \ k+4))) \\ & +x((os \ 4+2)) \\ & (x((4 \ k+1)) \ x((4 \ k+3))) \\ & +x((os \ 4+3)) \ x((4 \ k+1)) \ ; \end{aligned}$$

$$\begin{array}{ll} \mbox{hbk}(5) = \ x \ ((4 \ k+1)); \\ \mbox{hbk}(6) = \ x \ ((4 \ k+2)); \\ \mbox{hbk}(7) = \ x \ ((4 \ k+3)); \\ \mbox{hbk}(8) = \ x \ ((4 \ k+4)); \end{array}$$

h = [h; hbk];

 end

x e s k i = x;z=z h;

 $\label{eq:tr} \begin{array}{l} {\rm tr=0;} \\ {\rm while} & (\,{\rm epsilon}\;\; 0\,.\,0\,5\,) \end{array}$

$$\begin{array}{l} \text{g b } \text{bs} = & \left[\text{x} \left(\left(\text{ os } 4+1 \right) \right) & \left(\begin{array}{c} \text{x} \left(\left(\text{ os } 4+2 \right) \right) \\ \text{x} \left(\left(\text{ os } 4+3 \right) \right) \right) & \text{x} \left(\left(\text{ os } 4+1 \right) \right) & \text{x} \left(\left(\text{ os } 4+2 \right) \right) \end{array} \right) ; \\ & \left(\text{x} \left(\left(\text{ os } 4+2 \right) \right) + \text{x} \left(\left(\text{ os } 4+3 \right) \right) \right) & \text{x} \left(\left(\text{ os } 4+1 \right) \right) \\ & \text{x} \left(\left(\text{ os } 4+2 \right) \right) & \text{x} \left(\left(\text{ os } 4+1 \right) \right) \end{array} \right]; \\ \end{array}$$

```
H g b b s y a z i l i = \begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix};
     0 \ 1 \ 0 \ 0;
     gbbs;
     1 \ 0 \ 0 \ 0;
     0 \ 1 \ 0 \ 0;
     0 \ 0 \ 1 \ 0;
    0 \ 0 \ 0 \ 1];
H sol=zeros (size (H g b bs yazili ,1)
                      os, size (Hgbbs yazili, 2) os);
H sol=DiagK(H g b b s y a z i l i , os);
HVASIL = [];
for k = 1:os
     Vlikisim = \begin{bmatrix} 0 & 0 & 0 \end{bmatrix};
                       0 \ 0 \ 0;
                     (x((4(k1)+1))x((4(k1)+3)))
                       (x((4(k1)+2))x((4(k1)+4)))
                   x((4(k1)+2));
                     (x((4(k1)+2))x((4(k1)+4)))
                      (x((4(k1)+1))x((4(k1)+3)))
                       x((4(k1)+1));
          0 \ 0 \ 0 \ ;
          0 \ 0 \ 0 ;
          0 0 0 ;
          0 \ 0 \ 0 \ ];
```

```
H V A SIL = [H V A SIL; Vlikisim];
```

 end

```
H = [H \text{ sol } H V A SIL];
  R = 0.000001. eye (8 os , 8 os ); 16,16
  m = size(H, 1);
  n=size(H,2);
cLAV = [zeros(1,n) zeros(1,n)]
                      100 \text{ ones}(1,m) \quad 100 \text{ ones}(1,m);
cLAV (2 n + [3, 4, 11, 12, 19, 20, 27, 28,
                                35, 36, 43, 44, 51, 52]) = 1;
cLAV (2 n+m+[3,4,11,12,19,20,27,28
                                ,35,36,43,44,51,52) = 1;
  Al = [H \ H \ sparse(eye(m)) \ sparse(eye(m))];
  Yl = linprog(cLAV, [], [], ]
            Al, z, zeros (2 (m+n), 1),
                      100 \text{ ones} (2 (m+n), 1));
  xLAV=Yl(1:n) Yl(n+1:n+n);
  z=Yl(2 n+1:2 n+m) Yl(2 n+m+1:end);
  x e s k i = x e s k i + x L AV;
  epsilon = max(abs(xLAV));
```

 $x{=}x \ e \ s \ k \ i \ ;$

tr = tr + 1;

 end

```
i s matrixi(test no)=tr;
test no=test no+1;
Xs(1,dongu)=x((os 4)+1);
Xs(2,dongu)=x((os 4)+2);
Xs(3,dongu)=x((os 4)+3);
```

end

 \mathbf{x} ;

```
\begin{array}{l} A \ G = x K \left( \left( \ os \ 4 \right) + 1 \right); \\ A \ B = x K \left( \left( \ os \ 4 \right) + 2 \right); \\ A \ B \ S = x K \left( \left( \ os \ 4 \right) + 3 \right); \\ \\ A \ G \ M \ A \ T = \ A \ G \ . \ on \ es \left( 1 \ , \ dong u \right); \\ A \ B \ M \ A \ T = \ A \ B \ . \ on \ es \left( 1 \ , \ dong u \right); \\ A \ B \ S \ M \ A \ T = \ A \ B \ S \ . \ on \ es \left( 1 \ , \ dong u \right); \\ \\ A \ B \ S \ M \ A \ T = \ A \ B \ S \ . \ on \ es \left( 1 \ , \ dong u \right); \\ \\ A \ B \ S \ M \ A \ T = \ A \ B \ S \ . \ on \ es \left( 1 \ , \ dong u \right); \\ \\ t \ g \ f = \ 0; \\ t \ b \ f = \ 0; \\ \\ t \ b \ s \ f = \ 0; \\ \\ t \ g \ f = \ t \ g \ f + \left( X \ s \ ( 1 \ , k \ ) \ \ A \ G \ ) \ 2; \\ \\ t \ g \ f = \ t \ g \ f + \left( X \ s \ ( 1 \ , k \ ) \ \ A \ B \ ) \ 2; \\ \end{array}
```

t b s f = t b s f + (Xs (3, k) A B S) 2;

 end

```
tgMSE=sqrt(tgf)
tbMSE=sqrt(tbf)
tbsMSE=sqrt(tbsf)
plot(Xs(1,:),g);
hold on;
plot(Xs(2,:),b);
plot(Xs(2,:),b);
plot(Xs(3,:),r);
plot(Xs(3,:),r);
plot(AGMAT,k.);
plot(ABMAT,k.);
plot(ABMAT,k.);
plot(ABSMAT,k.);
legend(g12,b12,b11,iter);
xlabel(times);
ylabel(pu);
title(g12 b12 b11 parameters);
```

```
alitsum=0;
for k=1:tts;
```

alitsum=alitsum+i s matrixi(k);

 end

```
o it er = a litsum / tts
```

toc