MATHEMATICAL MODELING AND SOLUTION APPROACHES FOR BALANCING TURKISH ELECTRICITY DAY AHEAD MARKET

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ABSTRACT

MATHEMATICAL MODELING AND SOLUTION APPROACHES FOR BALANCING TURKISH ELECTRICITY DAY AHEAD MARKET

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In the Turkish Electricity Market, electricity trade is carried out largely through Bilateral Agreements and the emerging short term imbalances are settled in the Balancing Power Market, particularly the Day Ahead Market. In the Day Ahead Market, the participants submit their bids for each hour of the next day in the form of price-quantity pairs and the Market Operator evaluates those bids using an optimization tool. After the evaluation of the bids, a Market Clearing Price at every hour of the next day and the accepted bids are announced. In this thesis, a mathematical model for balancing a day in the Turkish Electricity Day Ahead Market is proposed. All types of bids, including hourly, block and flexible bids, are included in the model. As the objective function of the model, "total economic welfare", which is the sum of consumer surplus and producer surplus, is used. In the model, "paradoxically rejected block orders" are also taken into consideration and a bi-criteria solution approach is proposed for this purpose. An extension of this solution method is also applied as a second approach. Since the proposed model is a mixed integer non-linear programming model, a linear approximation to the objective function is proposed in order to overcome the possible problems at the solution phase. Both models are tested by using several different, generated data sets, and applying the proposed bi-criteria solution approaches. Both solution approaches include total economic welfare and the number of paradoxically rejected block orders as the two criteria. The results and performance of the proposed methods and models are discussed at the end of the study.

Keywords: Electricity Sector, Day Ahead Market, Mixed Integer Non-linear Programming, Paradoxically Rejected Orders, Balancing/Clearing

ÖZ

TÜRKİYE GÜN ÖNCESİ ELEKTRİK PİYASASI DENGELEMESİ İÇİN MATEMATİKSEL MODELLEME VE ÇÖZÜM YAKLAŞIMLARI

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Türkiye Elektrik Piyasası'nda, elektrik ticaretinin büyük bir kısmı uzun dönemli İkili Anlaşmalar vasıtası ile yapılmakta, bu anlaşmalar sonrasında oluşan kısa süreli dengesizlikler ise başta Gün Öncesi Piyasası olmak üzere, Dengeleme Güç Piyasası'nda giderilmektedir. Gün Öncesi Piyasası'nda, katılımcıların bir sonraki günün her bir saati için ayrı ayrı fiyat-miktar ikilileri şeklinde verdiği teklifler, Piyasa İşletmecisi tarafından bir optimizasyon aracı kullanılarak değerlendirilir. Bu değerlendirme sonucunda, ertesi günün her bir saati için bir Piyasa Takas Fiyatı ve bu fiyata bağlı olarak kabul edilen teklif miktarları açıklanır. Bu çalışmada Türkiye Gün Öncesi Piyasası'nın bir günlük dengelenmesi için matematiksel bir model önerilmiştir. Bu modelde piyasadaki tüm teklif tipleri (saatlik, blok ve esnek teklifler) dikkate alınmaktadır. Modelin amaç fonksiyonu olarak, tüketici fazlası ve üretici fazlasının toplamı olan "toplam ekonomik refah" kullanılmıştır. Modelin çözümünde 'paradoksal reddedilen blok teklifler'i de göz önüne alınmış ve bu amaçla çift kriterli bir çözüm yaklaşımı oluşturulmuştur. İkinci bir çözüm yaklaşımı olarak, ilk yöntemin bir uzantısı kullanılmıştır. Önerilen matematiksel modelin karışık tamsayılı doğrusal olmayan bir model olmasından kaynaklanan çözüm aşamasındaki olası güçlükleri gidermek için amaç fonksiyonunun doğrusal yaklaştırması da önerilmiş ve bu modeller oluşturulan farklı veri setleri üzerinden, önerilen çift kriterli çözüm yaklaşımları kullanılarak test edilmiştir. Çözüm yaklaşımlarındaki iki kriter olarak, toplam ekonomik refah ve paradoksal reddedilen blok teklif sayısı belirlenmiştir. Çalışmanın sonunda, önerilen yöntemlerin sonuçları ve performansı tartışılmıştır.

Anahtar Kelimeler: Elektrik Piyasası, Gün Öncesi Piyasası, Karışık Tamsayılı Doğrusal Olmayan Programlama, Paradoksal Reddedilen Teklifler, Dengeleme

To my family

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LIST OF ABBREVIATIONS

ANN	Artificial Neural Network
APX	Amsterdam Power Exchange
ARIMA	Autoregressive Integrated Moving Average
ATC	Available Transmission Capacity
Belpex	Belgium Power Exchange
BSR	Balancing and Settlement Regulation *(Dengeleme ve Uzlaştırma Yönetmeliği)*
CWE	Central Western Europe
DAM	Day Ahead Market
EDCO	Electricity Distribution Company
EMCC	European Market Coupling Company
EML	Electricity Market Law *(Elektrik Piyasası Kanunu)*
EMRA	Energy Market Regulatory Authority *(Enerji Piyasası Düzenleme Kurumu)*
ENDEX	European Energy Derivatives Exchange
EPEX	European Power Exchange
EPIAS	Energy Markets Operation Company *(Enerji Piyasaları İşletme Anonim Şirketi)*
EUAS	Turkish Electricity Generation Company *(Türkiye Elektrik Üretim Anonim Şirketi)*
FMCP	Final Market Clearing Price
GARCH	Generalized Autoregressive Conditional Heteroscedasticity
IPP	Independent Power Producer
KKT	Karush-Kuhn-Tucker
MCP	Market Clearing Price
MENR	Ministry of Energy and Natural Resources *(Enerji ve Tabii Kaynaklar Bakanlığı)*
MIP	Mixed Integer Programming
MINLP	Mixed Integer Non-Linear Programming
MWh	Megawatt-hours (equivalent to 1.000 kilowatt-hours)
OIB	Privatization Administration (of Turkey) *(Özelleştirme İdaresi Başkanlığı)*
OIZ	Organize Industrial Zone *(Organize Sanayi Bölgesi)*
PMUM	Market Financial Settlement Center *(Piyasa Mali Uzlaştırma Merkezi)*
PPA	Power Purchase Agreement

PRB	Paradoxically Rejected Block (order)
PTDF	Power Transfer Distribution Factor
PX	Power Exchange
RL	Retail Licensee
SOS	Special Ordered Set
TEAS	Turkish Electricity Generation and Transmission Company *(Türkiye Elektrik Üretim İletim Anonim Şirketi)*
TEDAS	Turkish Electricity Distribution Company *(Türkiye Elektrik Dağıtım Anonim Şirketi)*
TEIAS	Turkish Electricity Transmission Company *(Türkiye Elektrik İletim Anonim Şirketi)*
TEK	Turkish Electricity Institution *(Türkiye Elektrik Kurumu)*
TETAS	Turkish Electricity Trading and Contracting Company *(Türkiye Elektrik Ticaret ve Taahhüt Anonim Şirketi)*
UMCP	Unconstrained Market Clearing Price
WTC	Wholesale Trading Company

CHAPTER 1

INTRODUCTION

The Turkish Electricity Market is a place where electricity is generated, transmitted, distributed and traded among the market players. Until recently, the market has been dominated by completely state-owned companies and it was highly regulated. Throughout the history, the electricity sector was a completely government-run, disorganized sector until the establishment of the state-owned *Turkish Electricity Institution* (TEK) in 1970, that controlled all activities related to generation, transmission, distribution and sales of electricity (Privatization Administration, OIB, 2010). Based on the envisioned privatization policies and efficiency targets of the government, the initial unbundling of TEK took place by the establishment of the public companies *Turkish Electricity Generation and Transmission Company* (TEAS) and *Turkish Electricity Distribution Company* (TEDAS) in 1994. With the enactment of the Electricity Market Law (EML) no. 4628 in 2001, which was replaced by the new EML no. 6446 in March 2013, the liberalization of the market that is financially strong, stable, transparent and competitive is aimed to be achieved (Energy Market Regulatory Authority, EMRA, EML, 2013b).

Supplement to the EMLs, the liberalization of the electricity sector has been initiated with the publication of Electricity Sector Reform and Privatization Strategy Paper in 2004. Based on the Strategy Paper (EMRA, 2004), the electricity distribution network has been divided into 21 regions and each region has been privatized independently. With the enactment of the new EML (EMRA, 2013b), the privatized distribution companies have further unbundled into two separate and independent bodies as distribution companies and incumbent retailers, which has the responsibility to serve the customers who do not or cannot choose their own supplier. The Strategy Paper also envisages the privatization of hydro power plants in Turkey, with the exception of some large ones which will remain publicly owned. During this process, several thermal power plants also have been and are being privatized. Another goal of the Paper was to reduce the electricity consumption limit for freely choosing a supplier to 0, which has not been fully achieved as of 2015.

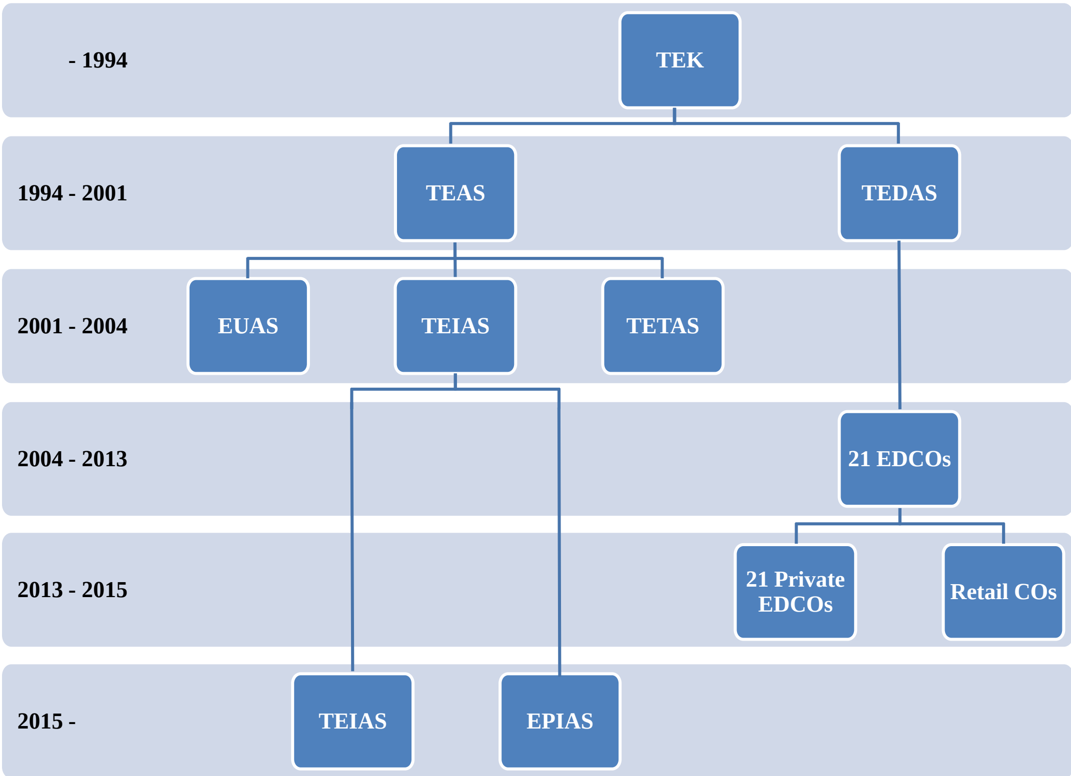

Figure 1.1: Historical Evolution of the Institutions in Turkish Electricity Market

Based on these laws and the Strategy Paper, the market was divided, first (in 2001) into 4 sectors, and later on into 6 separate and independent sectors in 2013, which are *generation*, *transmission*, *wholesale*, *distribution*, *retail (supply)* and *market operation*. In Figure 1.1, the historical evolution of the Turkish electricity market is shown based on institutions (OIB, 2010; TEDAS web site, 2014; TEIAS web site, 2014)[1]. In addition to the institutions and companies in separated sectors, EMRA is an autonomous state body regulating the whole market.

[1] The information related to the market development is retrieved from TEDAS and TEIAS websites, http://www.tedas.gov.tr/en/Pages/AboutUs.aspx and http://www.teias.gov.tr/Eng/CompanyBrief.aspx

The entities that are somehow involved in the process of electricity trading (from its generation to its -wholesale or retail- sales and consumption) can be defined as the “market players”. Players in most of the aforementioned sectors are either fully or partially private or in the process of privatization although some sectors are still completely state controlled. For instance, *Turkish Electricity Transmission Company* (TEIAS) is the sole responsible public company responsible for the electricity transmission and therefore called the *(Transmission) System Operator*. Furthermore, *Turkish Electricity Trading and Contracting Company* (TETAS) is the state organization that purchases electricity from the generators and sells it to the distribution companies, and *Turkish Electricity Generation Company* (EUAS) is still the dominating power in the generation market while most of the power plants in Turkey are planned to be privatized by the end of 2015. The major players in the aforementioned segments of the Turkish Electricity Market are given below.

- **Transmission:** TEIAS.
- **Distribution:** 21 private Electricity Distribution Companies (EDCOs), some Organized Industrial Zones (OIZs).
- **Generation:** State-owned EUAS power plants, Independent Power Producers (IPPs), Auto-producers, Micro Generation Units, some OIZs.
- **Wholesale:** TETAS, private *Wholesale Trading Companies* (WTCs).
- **Retail:** Incumbent Retailers (separated from privatized EDCOs) and other standalone *Retail Licensees* (RLs).
- **Buyers:** TETAS, WTCs, RLs, Eligible Customers (those having the option of choosing their own supplier).

Among most of the above players, the electricity trade is carried out; (1) through long-term bilateral contracts between buyers and wholesalers, (2) through bilateral contracts between retailers and the eligible customers, and (3) through the Balancing and Settlement market, also called “spot market”. The large part of the electricity purchase and sale is done via *Power Purchase Agreement*s (PPA), involving the parties such as TETAS, WTCs, Incumbent Retailers (of EDCOs), private and public (EUAS) generators, within the framework of yearly or multi-year bilateral contracts. The general structure of the long-term electricity trade through bilateral contracts is given in Figure 1.2 (EMRA, EML, 2013b).

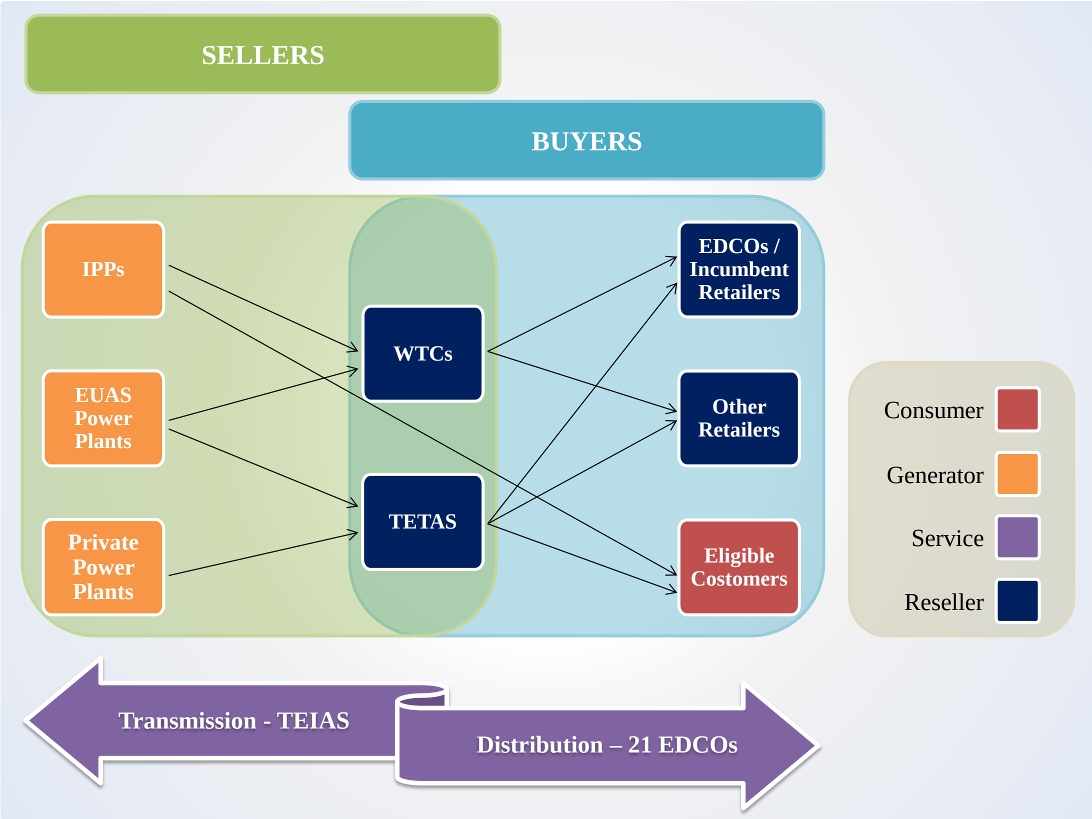

Figure 1.2: Electricity Trade between Main Players in the Turkish Electricity Market

According to the existing law and other legislation, all the "unbalanced" amount of power remaining after the execution of the bilateral contracts is settled in the spot market that is called the Balancing and Settlement Market, operated by *Market Financial Settlement Center* (PMUM). Since the end of 2011, one of the main stages of the settlement mechanism is the Day Ahead Market (DAM). In a general sense, the purpose of the DAM is to balance the next day's hourly supply and demand in such a way that the balancing costs are minimized while satisfying the operation safety and integrity constraints in accordance with the supply security and supply quality criteria. DAM is operated in order to provide the market players with the opportunity to purchase and sell electricity for the next day so as to balance their activities on top of their contractual obligations. In addition, DAM enables the system operator (TEIAS) to balance the whole system by determining a reference value for the hourly electricity price. DAM is operated by PMUM, soon to become (by the end of 2015) *Energy Markets Operation Company* (EPIAS), that is referred to as the *Market Operator*.

The players involved in the day ahead market are the licensed generators, autoproducers, large consumers –who can adjust their consumption, wholesale companies, incumbent retailers (of EDCOs) and the power plants that can load and de-load based on the market balance. All of these aforementioned entities that take part and operate in the DAM are in general referred to as "market participants". The inclusion of the large consumers in the market makes the demand more price elastic since they can influence the prices by lowering/increasing their demand at high/low price periods. In addition, market participants are allowed to form portfolios of different technologies or combine demand and supply bids to reduce their risks and have more flexibility in their operations. For instance, a portfolio bid can be a combination of wind and hydro or of wind and natural gas units, reducing the risk of supply shortage when relying on only wind. The offered consumption and generation amounts can be adjusted by the bidders from both sides according to the price levels. It should be noted that reactive (electrical) power is not involved in the spot market trading process; only active power is traded in Turkish DAM[2]. According to the official data published on PMUM website (2015)[3], trade volume of DAM was about 32% of the total electricity market in 2014.

Up to now, information on the Turkish Electricity Market in general is given with a brief introduction to the DAM. As the main focus of this thesis is the DAM and specifically its balancing, the DAM will be discussed and analyzed in more detail from Chapter 2 on. The remainder of this thesis is organized as follows: In Chapter 2, detailed information about the Turkish Electricity DAM is given with relevant definitions, players, examples and description of its current settlement mechanism. Types of bids valid in the DAM, how they are evaluated and how the hourly prices are determined are also described in this chapter. The literature on the DAM implementations around the world and specifically balancing of spot markets are reviewed

[2] There are two kinds of electrical energy carried in the network. Active power is the electrical energy that can be converted into useful work, therefore it is commonly used and traded in the whole market. On the other hand, reactive power is the ancillary energy that arises from the generation of active power and it is mostly useful for adjusting the voltage profile of the network (for a detailed description of active power, see, for instance, Chakrabarti, 2003).

[3] https://rapor.pmum.gov.tr/rapor/xhtml/piyasaHacimFiziksel.xhtml

in Chapter 3. The most common areas related to DAMs are the day ahead price forecasting, electricity generator portfolio and bidding strategy optimization and market clearing/balancing. In Chapter 4, the mathematical model proposed to solve the balancing problem for the Turkish Electricity DAM, involving hourly, block and flexible bids, is presented. In the mathematical model, the aim is to maximize total welfare. For this purpose, a non-linear objective function and its linear approximation is used to represent the total welfare. In order to achieve feasibility, paradoxically rejected block orders are allowed in the model. A bi-criteria solution approach is proposed, where a trade-off between the total welfare value and the number of paradoxically rejected orders is presented. An extension of this, where only the minimum number of paradoxically rejected orders is found by a two-step method, is also proposed as another solution approach. The numerical results of the mathematical model based on different objective functions and using the proposed solution approaches are discussed in Chapter 5. Finally, with comments and suggestions for further studies on Turkish electricity spot market clearing, the thesis ends in Chapter 6.

CHAPTER 2

THE TURKISH ELECTRICITY DAY AHEAD MARKET

2.1 The Operation of the Day Ahead Market

The DAM is operated daily on an hourly basis (from 00:00 to 23:00) and the participation of all market players is not compulsory. The ones that do not participate in the DAM can either trade electricity only by bilateral contracts or by directly bidding at the Balancing Power Market that runs in real time after the DAM is closed. Participants of the DAM are obliged to provide a collateral payment for the fulfillment of their obligations on settlement of payments under the DAM. The amount of such collateral payments will be just enough to cover any risks likely to arise from any and all market activities conducted by each relevant market player. In the DAM, the participants can submit their bids for either only the next day or up to the next 5 days. The average Day Ahead Price (Market Clearing Price, MCP) and the balancing amount for each hour of the day are determined separately by the Market Operator on the previous day. MCP is the calculated DAM Price that is determined by drawing global supply and demand curves of all the bids submitted to the system. The gives the MCP and the decision of whether a bid is accepted or rejected is made based on this price. The debts and receivables between the participants in the DAM are settled on the following day after the completion of all the electricity transactions.

For the sake of understanding, the term ‘bid’ is used to describe the submissions of the participants of the DAM whereas the term ‘order’ refers to the translation of the bids into the structure we use in our mathematical model. How bids in the DAM are converted into the form that are orders in our model is described in Chapter 4 in detail.

The usual operation scheme of the DAM, given in BSR in effect from January 2013 onwards (EMRA, BSR, 2013a) and in the DAM User Manual published by PMUM (2013), is summarized below.

- **00:00 – 09:30:** The system operator (TEIAS) determines the hourly transmission capacity between different regions for the next day and reports these capacities to the Market Operator so that they are announced to the market participants.
- **00:00 – 16:00:** Bilateral agreements of the market participants are submitted to the DAM portal.
- **00:00 – 11:30:** All the DAM participants submit their hourly bids for the next day to the Market Operator.
- **11:30 – 12:00:** The collateral payments of the participants are checked so that their eligibility to place bids in the DAM is ensured.
- **11:30 – 12:00:** Participants' bids are confirmed and verified by the Market Operator.
- **12:00 – 13:00:** All the verified bids are evaluated by the optimization tool, and the MCP and the Market Clearing Amounts for each hour of the next day are determined.
- **13:00 – 13:30:** The approved purchase/sale amounts are reported to the market participants. They have the right to object to the reports if there is a mistake.
- **14:00:** After the objections are evaluated, the final and ultimate MCP and the couplings for the 24 hours of the next day are announced.

2.2 Bidding Types in the Day Ahead Market

In addition to their bilateral agreements, the market players are allowed to submit hourly, block and/or flexible bids within the scope of DAM. All bids include price and quantity pair(s) and all prices have a sensitivity of 1%. The price can be in Turkish

Lira (TL), Euro or US Dollars per MWh while the quantity of electricity to be sold/ purchased is stated in terms of “lots”, where each lot corresponds to 10 MWh. The sign of the bid quantity determines the direction of the bid; that is, a positive quantity means a purchase bid and a negative lot means a sales bid. The highest bid price can be 2,000 TL and the largest bidding quantity can be 100,000 Lots (or 10,000 MWh) for both positive (purchase) and negative (sales) bids.

There are four types of bids that can be submitted to the DAM, described in the following paragraphs (EMRA, BSR, 2013a; PMUM, 2013). Tables 2.1, 2.2 and 2.3 show an example for an hourly purchase (demand) bid, an hourly sales (supply) bid and a block bid, respectively.

- **Hourly Bids**: Hourly bids are the main type of bids in the DAM, consisting of price-quantity pair(s) at a single hour of the day for which the bid is made. They can be both in sales and purchase directions. There can be at most 64 price-quantity pairs (32 in each direction) within an hourly bid and the same price level cannot be used for sales and purchase directions in the same bid. The prices of the hourly bids must be in increasing order and the empty/unused values between two consecutive price levels are filled out by the Market Operator using linear interpolation when drawing the respective supply and demand curves. The quantities must be submitted in a non-increasing order for hourly purchase bids and non-decreasing order for hourly sales bids.

Table 2.1: An Example Hourly Purchase Bid

	Price (TL)	0	91	110	120	130	140	2,000
Hour 7–8	Number of Lots	46,700	44,580	41,680	40,280	38,780	38,180	38,180
	MWh	4,670	4,458	4,168	4,028	3,878	3,818	3,818

Table 2.2: An Example Hourly Sales Bid

	Price (TL)	0	75	100	120	130	2,000
Hour 7–8	Number of Lots	$-15,100$	$-25,100$	$-35,750$	$-39,650$	$-40,650$	$-40,650$
	MWh	$-1,510$	$-2,510$	$-3,575$	$-3,965$	$-4,650$	$-4,650$

- **Block Bids:** Block bids cover between 4 to 24 consecutive and complete hours which cannot be processed separately. A block bid is a price-quantity pair for the time slot (at least 4 hours) it spans. The number of block bids a participant can submit is limited to 50 and each of them is either accepted fully or rejected as a whole (no partial acceptance is allowed). Up to 3 block bids submitted for the same bidding area[1] in the same direction (sales or purchase) can be linked. That is, when a block bid is linked to another one, it can only be accepted if the other one is accepted. When a third bid is involved, i.e., the second bid is linked to third one, then the linked bid is processed only if the other two are accepted. For instance, the block bid shown in Table 2.3 is linked to another block bids; namely, bid $no.\ 577$. Assuming that bid $no.\ 577$ is linked to a third bid $no.\ 578$, then block bid in Table 2.3 can be accepted only if the bid $no.\ 577$ and $no.\ 578$ are both accepted because accepting bid $no.\ 577$ requires the acceptance of bid $no.\ 578$. Otherwise, the example block bid is rejected. More than one linked (block) bids can be submitted by the same participant.

Table 2.3: An Example (Sales) Block Bid

Starting Period	Number of Hours	Price (TL)	Number of Lots	MWh	Link
7	4	100	$-1,750$	-175	Ord. $no.\ 577$

- **Flexible Bids:** Flexible bids are the hourly bids that are not made at a specific hour. In other words, they are special block bids that are submitted for a single, unspecified hour. This means that they do not have a time component and only include a price and a *negative* quantity (flexible bids are only sales bids). Similar to block bids, flexible orders cannot be executed partially. Each market participant can submit up to 10 flexible bids. The purpose of including such flexible bids is to reduce the MCP. Since this type of bids is not attached to a specific hour, they can be processed at any hour of the day. With this flexibility, they are accepted in such a way that the MCP is lower and overall benefit of the market players is higher. Current practice of the Market Operator is that the flexible bids are accepted at the hour with the highest MCP of the day. The

[1] At the time of the writing of this thesis, there is no zonal pricing in Turkey. The DAM consists of one single bidding area.

numerical examples provided below will clarify how flexible bids are evaluated in this manner.

- **Bilateral Agreements:** Bilateral agreements cover all 24 hours of the day and they are predetermined through the long-term contracts made between the market players. One party of the agreement submit positive (purchase) quantities whereas the other one submits negative (sales) of the same amounts. If these bids of the two parties match, then both bids will be accepted by the Market Operator. Note that although bilateral contracts are scheduled in the DAM, they are not counted in the stack of bids in the determination of DAM clearing prices and quantities.

2.3 Current Settlement Mechanism

Recall that the Market Operator uses an optimization tool to evaluate and match (couple) the bids and to calculate the MCP with respect to the accepted bids. In principle, there are ultimately two different prices in the DAM, which are calculated and determined for each hour by the Market Operator (EMRA, BSR, 2013a). The first price is the *Unconstrained Market Clearing Price (UMCP)*, which is, as the name implies, calculated as a common single price for each hour of the day, without considering the capacity constraints between the bidding areas. Therefore, the separation between bidding areas is ignored. UMCP is calculated by assuming there are no constraints on transmission capacity and it is based on the coupling of all bids coming from all bidding areas. The second price realized in the DAM is the *Final Market Clearing Price (FMCP)*. The FMCP is determined separately for each hour and for each bidding area by considering the constraints on transmission capacity between the bidding areas. It is calculated by making the necessary adjustments on the UMCP in order to satisfy the capacity constraints.

The current DAM clearing mechanism works in four steps. The first three steps are followed to calculate the UMCP while the last step is the determination of the FMCP. These steps are described below and summarized in Figure 2.1.

Step 1. The UMCP and the corresponding clearing amount are determined temporar-

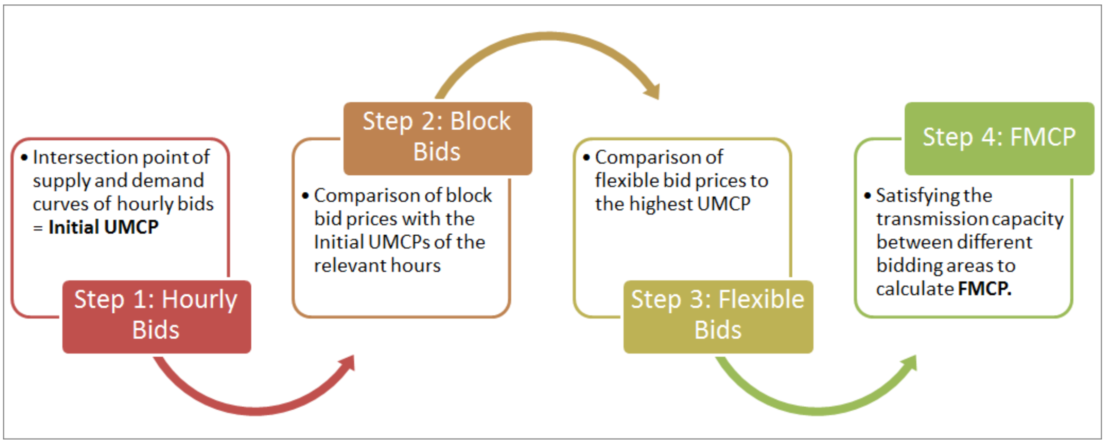

Figure 2.1: Current DAM Settlement Mechanism Applied by PMUM

ily by first drawing the supply and demand curves of the market based only on hourly bids. All of the hourly purchase (sales) bids are arranged in a descending (ascending) order so that they look like a single purchase (sales) bid. While drawing the curves, two consecutive price-quantity pairs are connected using linear interpolation. The point where the supply and demand curves intersect defines the initial UMCP and the corresponding clearing amount. In Figure 2.2, the individual demand and supply curves of the example hourly bids given earlier in Tables 2.1 and 2.2 are shown. These bids will be the basis of the initial (temporary) UMCP calculation. At this stage, only hourly bids are considered and the UMCP is 120.75 TL as seen in Figure 2.3. In this case, the unconstrained equilibrium quantity is $4,016.68$ MWh.

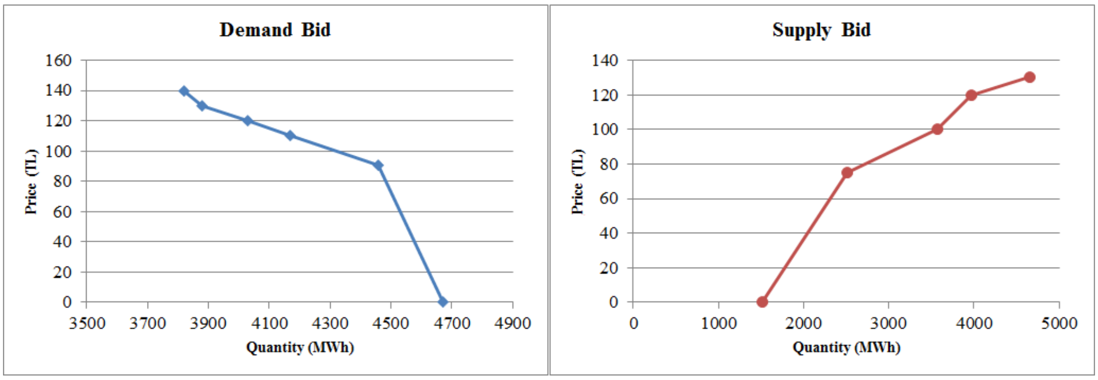

Figure 2.2: Supply and Demand Curves of Example Hourly Bids

Step 2. Block bids are compared to the previously found market clearing conditions. The ones that lower the overall costs of the day (increase the welfare), that

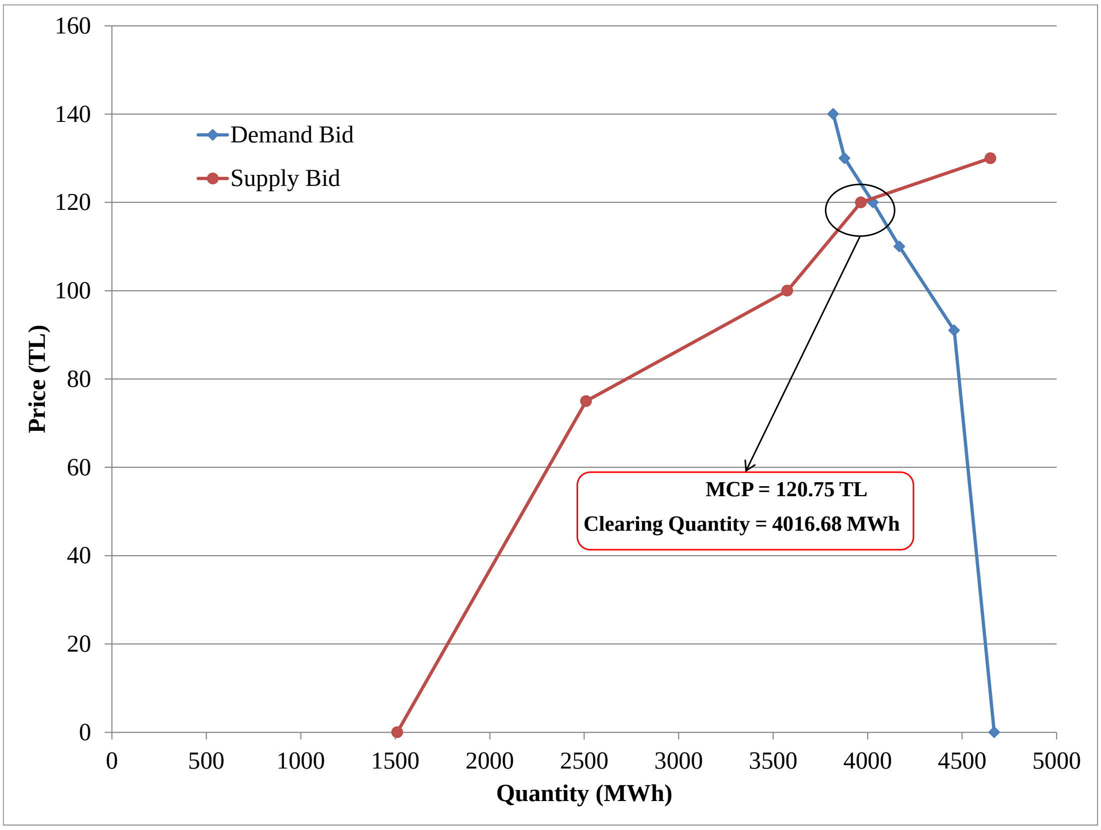

Figure 2.3: Calculation of MCP based on Example Hourly Bids

is, creating as low MCPs as possible, are accepted and incorporated into the new supply and demand curves. To obtain lower daily costs, purchase block orders, whose prices are higher and sales block orders whose prices are lower than the weighted average UMCPs calculated for the all relevant hours, are accepted. For instance, the sales block bid given in Table 2.3 is accepted because its price is 100 TL and it is lower than the incumbent UMCP, 120.75 TL.

Step 3. The flexible bids are taken into consideration in a similar manner and since they can be only in the sales direction, the ones having a lower price than the UMCP are accepted at the hour where the highest UMCP is prevailing. After the evaluation of the flexible bids, the UMCP and temporary clearing amounts for each hour of the next day are determined.

Step 4. When the UMCP is calculated, the supply and demand amounts at that price level are also automatically determined. At this point, the Market Operator calculates the energy flow between the bidding areas. If the energy flow between all bidding areas is less than or equal to the transmission capacity allocated and announced by the System Operator, then the UMCP is designated as the FMCP. However, if there exists at least one bidding area having the trade flow larger than the capacity, the UMCP is decreased in the energy-surplus-area and increased in the energy-deficit-area such that the transmission capacity is not exceeded.

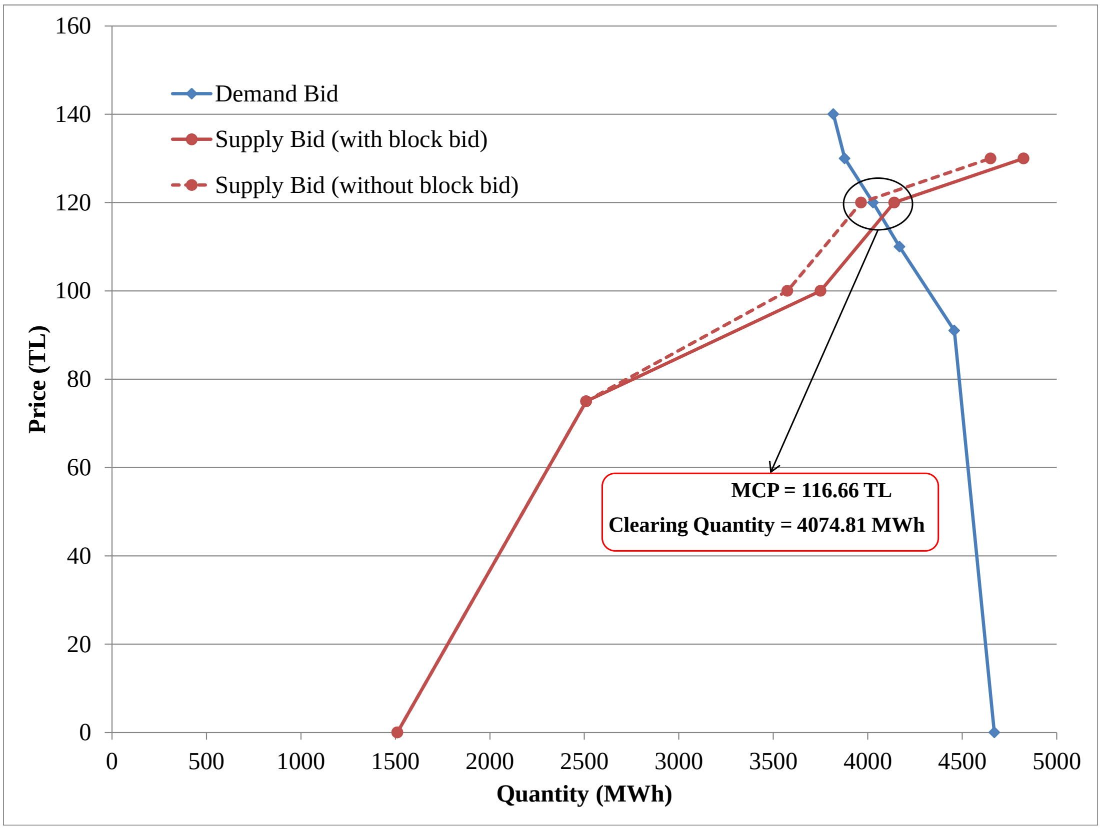

Figure 2.4: Calculation of MCP based on Example Hourly and Block Bids

The process described in the fourth step is applied to all bidding areas that require excess capacity at the calculated UMCP. This adjustment is done so as to ensure the maximum flow of energy from the lower-price areas to the higher-price areas at the lowest possible price in all bidding areas. The calculated new price levels are assigned as the FMCPs for the relevant bidding areas. While the found MCPs have of 6 decimal digits of precision, they are rounded to 2 decimal digits (Kuruş) for financial convenience. Note that the current practice in Turkey is to define the whole network as a single bidding area and there is no transmission capacity constraint except the network's overall technical capacity. Therefore, the UMCP calculated after the evaluation of all hourly, block and flexible bids (in Step 3) is also the FMCP, which will simply be referred to as MCP in the following chapters of this thesis. In the case of our example bids given above, two hourly bids given in Tables 2.1 and 2.2 are both partially accepted. Assuming it is submitted for only one hour, the sales block bid shown in Table 2.3 is also accepted because its price (100 TL) is lower than the equilibrium price of the hourly bids (see Figure 2.3 for the equilibrium price). In the end, the MCP is calculated as 116.66 TL and the corresponding clearing amount is $4,074.81$ MWh since the inclusion of the supply block bid has slightly shifted the supply curve to the right, as demonstrated in Figure 2.4.

CHAPTER 3

LITERATURE REVIEW

In this chapter, the literature and practices on electricity DAMs and specifically their balancing and clearing are reviewed. In the literature, there are three main categories of electricity DAM studies. The first one is the largely studied "forecasting of MCP"; that is, the market participants, especially electricity generators, aim to forecast the hourly day ahead prices, or as called in general "spot prices". The second category of DAM studies is the "optimization of portfolio and bidding strategy", again from the generators' point of view. The final category of DAM studies is the "balancing (clearing) of the DAM", which looks at the problem from the market operator's point of view and is also the focus of this thesis. Throughout the chapter, the studies in these three main categories are reviewed briefly, with an emphasis on the last category.

The studies in the aforementioned first two categories are reviewed in Section 3.1. These are all related to the operation of electricity DAM but not relevant to our purpose, which is to find the balance of supply and demand, and the MCP. Section 3.2 gives a detailed summary of the most commonly known studies about the clearing of DAM.

3.1 Studies on Forecasting of MCP and Optimization of Portfolio and Bidding Strategy

One of the most commonly covered areas related to DAM is the forecasting of MCP. There are numerous studies and methods in the literature; a number of them are mentioned in this paragraph with a brief summary. For instance, Amjady and Keynia (2008) propose a hybrid method to forecast day ahead prices that uses wavelet transform and a cascade of neural network and evolutionary algorithms. Another hybrid method, composed of wavelet transform, autoregressive integrated moving average (ARIMA) and least squares support vector machine, is proposed by Zhang et al. (2012). Vilar et al. (2012) provide a nonparametric functional regression model and a semi-functional partial linear model to forecast day ahead electricity demand and prices. Vahidinasab et al. (2008) use an Artificial Neural Network (ANN) with Levenberg-Marquardt learning algorithm. In this study, sensitivity analysis is used to determine optimal input combination and fuzzy c-mean algorithm is implemented to cluster daily load patterns and to forecast day ahead electricity prices. Hong and Wu (2012) present a hybrid principle component analysis method with a multi-layer feed-forward neural network to forecast marginal prices in a DAM. Another hybrid method is proposed by Voronin and Partanen (2012) for the forecasting of so-called normal range prices in the Finnish spot market. The method includes ARIMA-based models used for linear relationship in the price series and a generalized autoregressive conditional heteroscedasticity (GARCH) model for non-homogeneous variance of residual terms. A neural network is used to combine the previous two predictions. For the price spikes, i.e., prices that are above some specified threshold, they propose k-nearest neighbor model (for the spike values) and a Gaussian mixture model (for their probabilities). Garcia et al. (2005) also apply GARCH to forecast day ahead prices in Spanish and California markets, for which they propose a flowchart to obtain the model.

The second main category of DAM studies is the strategy optimization. In most of the cases, the solution approach is from the generators' point of view. For example, Yücekaya (2013) develops a methodology for price taking generation units to determine the best bidding strategy under stochastic price scenarios. The model makes use of Monte Carlo simulation and maximizes the expected profit. Bajpai and Singh (2007) use a fuzzy adaptive particle swarm optimization, which aims to find the optimal bidding strategy of a thermal generating unit by modeling the production cost as a sinusoidal nonlinear function and the start-up cost as an exponential function. The other generators' behaviors are also included as probability distribution functions. In the mathematical model, the profit of a generator is maximized. Wong et al. (2009) present a simulation model, where each generator analyzes the historical data, forecasts the demand and prices, and develops and adjusts its bidding strategy based on its deductions. In the model, risk assessment is included as a stage in the decision making process of a generator and objective of each participant is to maximize its profit. However, the simulation model proposed in this paper does not aim to find the optimal strategy but to determine the most suitable bidding quantity and price for the risk preference of individual generators under expected scenario outcomes. Francisco and Nerves (2010) develop a two-step approach to model the bidding strategy of Independent Power Producers (IPPs). In the first step, a security and capacity constrained economic dispatch model to clear energy and reserve markets is solved to determine expected spot prices (MCPs). The model minimizes the total supply, demand and reserve cost plus a constraint violation penalty. In the second step, based on the IPPs marginal cost and the calculated marginal price, the optimal bid price of the IPP is obtained. This is done by maximizing the probability of gain, i.e., probability of acceptance and obtaining a positive revenue. In another study, Foroud et al. (2011) propose an optimal bidding strategy for generation and distribution companies, which takes into account the other participants' bidding and operating conditions. Their methodology consists of two levels; The upper level is a multi-objective payoff maximization problem, where each market participant maximizes all market players' profits simultaneously. The lower subproblem is a security constrained cost minimization market clearing problem for the system operator. Genetic Algorithm and fuzzy satisfying method is used to solve the proposed multi-objective model.

An extension for optimal generator behavior is the strategy optimization in the combination of long-term contracts and short-term bidding markets, which is usually referred to as "portfolio optimization". Ramos et al. (2010) propose a model for generation companies to find the optimal balance between bilateral contracts and spot market that maximizes profit and minimizes the price and volume risks. To forecast the spot prices, a multivariate linear regression model is used. The efficient frontier showing the combination set of expected return and risk is presented based on the probability distribution of annual profit calculated for different generation quantities. Feng et al. (2008) present a different, Genetic Algorithm based Stochastic Programming model for the same portfolio optimization problem. In the model, expected utility function, defined by the difference between revenue and costs of the generation company plus its initial wealth, is maximized. Alternative portfolios make up the genes in the algorithm and Monte Carlo simulation is used to calculate their fitness values. The model by Yin et al. (2008) considers long-term contracts and spot market, as well, but the long-term decisions are represented in the form of forward contracts, which are agreements between two parties to buy or sell an asset on a future date for a specified price (Benhamou, 2007). In their solution approach, first the prices in the spot market are forecast using a time varying volatility model. Next, long-term portfolio selection model, comprised of the estimation of mean and variance of forward contract and spot market returns, is established. Finally, the optimal portfolio of forward contract and spot market bids is found by Differential Evolution algorithm.

3.2 Studies on Electricity Day Ahead Market Clearing

There are only a limited number of detailed studies in the literature about the balancing (clearing) problem of electricity DAMs, especially concerning Turkish case. Even fewer studies are publicly available as this subject is mostly a commercial issue and there is a limited opportunity to obtain real life data. In the following paragraphs, studies from both academic resources and commercial applications are discussed in detail.

Weidlich and Veit (2008) build a simulation model of the German electricity wholesale market in three steps, one of which is for the DAM. In their DAM model, the demand side is assumed to be a fixed load that does not depend on the market price. So, they only simulate the supply side with the goal of electricity generators being to maximize their profit subject to capacity constraints. In the study by Güler et al. (2010), the effect of DAM on the real time balancing market and the system security are investigated. The DAM is not cleared by a model but impact of its clearing on the consumer and producer surpluses is assessed. Vlachos and Biskas (2011) aim to clear the multi-zone, single period (1 hour) European spot market, where assuming different clearing prices for demand and supply in the same zone is allowed and the flow between bidding zones is constrained. Both supply and demand bids submitted in each zone are represented as aggregated single bids and the model is formulated as a Mixed Complementarity Problem, where the demand prices are dependent on a function of explicit supply prices in different zones. A similar paper by Farahmand et al. (2012) looks at the integration of Northern European markets, based on different generation facilities such as thermal, hydro and wind. The objective function is the minimization of total cost, consisting of fuel costs, start-up costs and reservoir usage costs for hydro units. In the constraints, the start-up of the generation units, their minimum and maximum generation capacities, and transmission capacity between regions are taken into consideration. The balance equation is constructed not at the supply-demand level but the balance of physical power exchange at each bus[1] is ensured. Muñoz-Álvarez et al. (2012) again look at the clearing problem from the generators' point of view, by categorizing them as large, medium, small and micro generators. For the clearing mechanism, three equilibrium models are defined; (1) between large generators and discretized partial residual demand, (2) between large generators and discretized residual demand, and (3) between all generators, including large, medium, small and micro, and discretized partial residual demand. In all three cases, the objective function is defined as the minimization of total expected generation costs, energy shortage and excess costs, and capacity reserve and addition costs, under different scenarios. Both supply and demand are stochastic, thus their expectations are formulated. Using a cost of flexibility rights, which is the right to deviate from scheduled generation plan, the results of three equilibria are consolidated. This study aims to clear single hour market, based on generation costs and physical balances rather than economical indicators and parameters. In addition, the first step of the study by Francisco and Nerves (2010) and the lower subproblem in the paper of Foroud et al. (2011), which are described in Section 3.1 also approach to the market clearing problem from a narrower point of view.

To the best of our knowledge, there is only one study directly related to clearing of Turkish Electricity DAM. The study was recently published; which is another indicator that this is a relatively new subject in the Turkish literature. In his paper, Derinkuyu (2015) proposes a mathematical model to solve the DAM clearing problem in Turkey. He formulates the problem as a Mixed Integer Programming (MIP)

[1] Bus is a common point in the power network, where several generators operating in parallel are connected, to provide the energy needed (Saadat, 1999).

model that finds an exact solution. In his model, he introduces all relevant types of bids in the current Turkish practice; hourly bids or as Derinkuyu puts it, single bids, block bids including linked blocks, and flexible bids. He defines hourly bids as SOS2 (Special Ordered Set of type 2) variables and interpolates on them to find the MCP. He proposes minimizing the sum of the hourly MCPs of a given day as the objective function. This objective is used due to the political stress on the Market Operator and because direct welfare maximization is not possible with real instances (we propose new approaches to overcome this issue). In order to decrease the processing time of the model, he develops a solution methodology that is comprised of the reduction of the problem size and calculation of an initial solution. To reduce the problem size, all hourly bids submitted at each hour are aggregated and a total of 24 hourly bids are obtained (one for each hour of the day). For variable elimination, lower and upper bounds for the MCP of each hour are calculated by assuming all supply and demand block and flexible bids are accepted, respectively, and then the bids that are not satisfying those bounds are removed iteratively. We also introduce lower and upper bounds for MCP in our model in Chapter 4 in the same way. As for the initial solution, the clearing problem is solved by applying the current hierarchical heuristic method applied by the market operator (EMRA, BSR, 2013 and PMUM, 2013). That is, hourly bids are cleared first and then block and flexible bids are introduced. Based on this initial solution, variable elimination method that he suggested is applied by introducing a perturbation parameter to define upper and lower bounds on the found MCP. The reduced MIP is solved with the updated price bounds. This procedure is repeated until no improvement is observed (optimal solution is found). In the end, computational experiments using real data are provided and optimal solution is found within 1 hour in almost all instances while most of the cases are solved less than only a minute.

One of the most commonly known, publicly available commercial study about DAM clearing is the COSMOS coupling system by Djabali et al. (2011), which is also our main source of inspiration when building our model presented in Chapter 4. COSMOS includes a mathematical model and a heuristic algorithm that uses branch-and-bound to find a feasible, close-to-optimal solution to the Central Western European (CWE) market coupling problem. CWE region combines APX (Amsterdam

Power Exchange), ENDEX (European Energy Derivatives Exchange), Belpex (Belgium Power Exchange) and EPEX (European Power Exchange) spot markets. As it involves more than one region, it includes both market operation and network constraints where the objective function is maximization of social welfare. Social welfare is calculated by summing the difference between consumer surplus and producer surplus, and the total congestion revenue due to electricity flow between regions. Network constraints include the available transmission capacity constraints that limit the cross border trade, and balance of energy flow between regions based on accepted bids. In the market constraints, only hourly bids and block bids are included. It is claimed in the description document that the COSMOS algorithm can be easily modified to facilitate the evaluation of linked block bids and flexible bids, as well as integration of other constraints for possible extensions to the neighboring exchange markets, the latter of which is not relevant for the Turkish case. Hourly bids are handled in COSMOS model and algorithm in such a way that the ones that are "in the money", as it is put in the document, are fully accepted, while the ones that are "at the money" are partially accepted, meaning the MCP is between the price limits of the bid (details of price limits are discussed in Chapter 4). The ones that are "out of the money" are always rejected. Definitions for the block bids are the same as their Turkish counterparts, and they are similarly constrained by the "fill-or-kill" constraints. However, this property works in one direction only; a block bid is rejected when it is out of the money, i.e., when the weighted (over the quantity offered at every hour the block bid is attached to) average of the demand (supply) bid price is lower (higher) than the weighted average of the MCPs of the relevant hours. The fact that accepting the block bids that are in the money is not forced necessitates the introduction of the concept of "paradoxically rejected block bids". In this case, a block bid can be rejected even if it satisfies the weighted average price criteria, hence it becomes paradoxically rejected. Although it is not explicitly stated in the COSMOS document, this concept is considered and included in the model as it is necessary to obtain feasibility. Therefore, we also introduce paradoxically rejected bids into our model in Chapter 4; not only for block bids but also for flexible bids, by a different approach from the model in COSMOS.

The COSMOS algorithm works as follows: First, the linear relaxation of the initial clearing problem is solved. That is, partial acceptance of block bids is allowed. After finding a solution to the relaxed problem, branch-and-bound algorithm is executed by branching on partially accepted block bids. Decision variables used for those block bids are gradually forced to take the values of 0 or 1 at each step. This way, a feasible or, if possible, an optimal solution that satisfies all market and network constraints, including the fill-or-kill property, is aimed to be found within the time limit (10 minutes in the COSMOS case). It is stated that an initial feasible solution is found within less than 30 seconds at all instances and the optimal solution is found in most of the cases before reaching the time limit.

As stated earlier, the mathematical model proposed in this thesis is mainly based on the COSMOS model by Djabali et al. (2011). One main difference is that, in COSMOS, a common notation is used to denote the price and quantity parameters of supply and demand bids. They are distinguished by the sign of the quantity parameter. Although we also use different signs for quantities in different directions, we differentiate between supply and demand bids with independent representation of variables and parameters. The way we handle paradoxically rejected block –and flexible– bids is also new compared to COSMOS, for which we introduce a new set of binary variables to handle those bids. However, the main difference of our model is that we directly find the optimal solution while COSMOS works with a heuristic algorithm. All differences of our model from COSMOS, as well as our contributions, are discussed in Chapter 4 in detail. The mathematical formulation of the COSMOS model (with common notation to denote the price and quantity parameters of supply and demand bids) is provided in Appendix A. The related references are given also in Chapter 4.

Martin et al. (2014) solves the European DAM clearing problem by decomposing it into a quadratic master problem and a linear pricing problem for the congestion rent. In the mathematical model, total economic surplus is maximized, as in the case of COSMOS (Djabali et al., 2011) and the model we propose in Chapter 4. The bid prices are defined as the market participants' marginal willingness to pay. Therefore, the producer and consumer surpluses of hourly bids are calculated by taking the integral of a participant's marginal willingness to pay curve. Similarly, block bid surpluses are the difference between total willingness to pay over all relevant hours and the multiplication of MCP and the bid quantities at the relevant hours. Surplus of flexible bids is very similar to those of block bids, only flexible bids are executed at most one hour as per their definition. In this study, paradoxically rejected bids are defined as the bids that does not result in a ''surplus maximizing solution" from the bidder's point of view.

The constraints of the model proposed by the authors are very similar to those used in COSMOS. Constraints regarding energy flow balance, available transmission capacity, acceptance of block (and flexible) bids are parallel in both studies. Here, just like in COSMOS, rejection of block and flexible bids is not forced in order to take care of the paradoxically rejected bids. On the other hand, maximization of congestion rent between bidding areas are represented as constraints to the welfare maximization model, using dual variables and Karush-Kuhn-Tucker (KKT) optimality conditions. A similar approach is adopted also for the acceptance of hourly bids. The available IBM CPLEX solvers is not able to solve the described model in 30 minutes, nor NLP solvers solve before 10 minutes. Thus, a heuristic method and an exact solution algorithm is proposed. First, the price conditions of block and flexible bids are relaxed and the model is solved. The optimal solution to this relaxed model satisfies the hourly bid price conditions but may not satisfy the block and flexible bid price conditions. In this case, a ''bid cut" is added to the relaxed model to discard this infeasible (to the original model) solution. This process is repeated until a feasible solution is found. Feasibility of a given solution is tested by an LP that minimizes the loss incurred by the executed block and flexible bids when their price conditions are relaxed. As for the bid cuts, two alternatives are used. In the first one, *at least* one of the loss incurring block or flexible bids is forced to be rejected. In the second bid cut, *exactly* one is prohibited. The first cut is said to possibly lead to suboptimal solutions but is a fast heuristic while the second one is slower but converges to global optimal solution. Both methods are tested using real data and they are compared to the performance of the optimizer currently used by the European Market Coupling Company (EMCC). Maximum solution time of the heuristic bid cut method is reported as 1.1 minutes and the average is only 4 seconds. The second method is slower but improves 4% of the solutions found by the first method. 38% of the cases end with an optimal solution and the remaining ones reach the time limit before reducing the absolute gap and proving optimality.

A very similar mechanism to that of Turkey is the Romanian DAM trading system called SAPRI (n.d). In the SAPRI system, three types of bids are allowed, namely hourly bids, block bids and flexible bids. Their definitions are exactly the same as the ones in Turkish DAM given in Chapter 2. The evaluation mechanism of SAPRI is as follows: First, the aggregated supply and demand curves of only hourly bids are drawn, just like in the Turkish case (see Section 2.3 in Chapter 2). Later, block bids are integrated iteratively in two steps. In the first step, after calculation of MCPs considering all block bids are accepted, block bids that do not satisfy the price criteria are excluded one by one, and a new MCP is calculated. When there does not remain any block bid violating the price criteria, re-inclusion of the favorable-priced bids is checked. The ones that remain rejected although they satisfy the price criteria (the ones that cannot be accepted because they affect the acceptance decision of previously accepted bids) are called "paradoxically rejected bids", as in the case of COSMOS (Djabali et al., 2011). Flexible bids are evaluated iteratively as the Turkish Market Operator considers them. That is, the ones having lower price than the highest prevailing MCP calculated are accepted at the hour having the highest MCP, as long as they do not disrupt the decision of previously accepted hourly and block bids.

Biskas et al. (2014) formulate the European DAM models in their two-part study with a very similar approach to the ones just described. They formulate the clearing problem of European DAM as a Power Pool where bids in the market are represented at the participant level (e.g., generation units), and as a Power Exchange (PX) where all bid types are included in the model. The power pool is represented by a unit commitment model since the cost of participants includes the start-up, shut-down and energy reserve expenses. Only power balance and available transmission capacity are considered in this model. However, in the PX model, hourly, block, linked block, flexible, and convertible block bids are included (convertible blocks are bids that can be converted to hourly bids when certain conditions hold). The objective function is the minimization of total cost incurred minus the total utility obtained by each market participant or each unit of each market participant, depending on the structure of the bidding areas. In the objective function, the cost of sellers/importers and the utility of buyers/exporters are defined by the multiplication of price and quantity of the corresponding bids whereas cost and utility of supply and demand bids are also considered as the multiplication of price, quantity and acceptance level of the bids. Power balance and available transmission capacity between bidding areas are added to PX model as constraints, as well, as in the case of power pool model. Later on, the two models (power pool and PX models) are combined as a single European market clearing model. In this MILP model, the import and export agents are excluded. MCP in this combined model is calculated using the Lagrange multiplier of the power balance equation and the power transfer distribution factors (PTDFs) of bidding areas, based on a reference area that needs to be defined by the market operator in order to compute PTDFs. Although the clearing conditions on hourly, flexible and linked block bids are considered, there is no comparison of any bid type to MCP. Instead, the authors suggest an algorithm in the second part of their study to deal with the prices. The suggested iterative algorithm is devised for the integrated model of power pools and power exchanges, and it works as follows. First, the MILP model described in the first part is solved. Later, the prices of block, linked block, convertible block and flexible bids are compared to the calculated MCP to determine the paradoxically rejected/accepted bids. The values of the decision variables representing those paradoxically rejected bids are fixed to 0 and eligible convertible block bids are converted to hourly bids. Then, the model is solved again.

Related to DAM clearing, there are also some studies suggesting models and algorithms for simultaneous clearing of active and reactive energy markets. To our knowledge, the operation of active and reactive power markets is decoupled and clearing is separate or at least sequential (see, for instance, Singh et al., 2011, and El-Samahy et al., 2006, for recent implementations on reactive power markets in different countries). Rabiee et al. (2009) look at the coupled active and reactive power clearing problem by considering system security constraints and minimizing the bidding cost of generators for active power and total payment for reactive power including the lost opportunity cost. In another study, Aghaei et al. (2009a) suggest a two-stage solution method where first random scenarios for generation behavior of the units are generated by Monte Carlo Simulation and then they are inserted into a series of optimization problems, objectives of which are minimization of expected generation, capacity reserve, lost opportunity and interruption cost. The same authors also formulate the joint energy and reactive power DAM clearing problem as a multi-objective mathematical program where minimization of generation cost and optimization of system security (minimization of voltage drop and line overload) are the competing objectives (Aghaei et al., 2009b). The multi-objective mathematical problem is solved using ϵ-constraint method. Reddy et al. (2011), too, propose a multi-objective approach with several objectives such as augmented payment function minimization, loss minimization, maximization of load served and minimization of Voltage Stability Enhancement Index. In this study, Strength Pareto Evolutionary Algorithm 2+ approach is used.

The next chapter demonstrates the mathematical model built for clearing the Turkish Electricity DAM, which is a modified and developed version of COSMOS (Djabali et al., 2011). Proposed solution approaches are also provided in Chapter 4.

CHAPTER 4

THE PROPOSED MATHEMATICAL MODEL AND SOLUTION APPROACHES

Recall from the previous chapters that there are only a limited number of studies on the balancing/clearing of Electricity DAMs. The model proposed on the COSMOS CWE Market Coupling Algorithm (Djabali et al., 2011) cited and discussed in Chapter 3 focuses on the DAM clearing problem and suggests a solution with an economical approach. In their model, social welfare is maximized subject to price conditions of hourly and block bids. Derinkuyu (2015) solves the DAM clearing problem in Turkish case by minimizing the daily sum of the hourly MCPs, while satisfying the price and balancing criteria. The algorithm by Biskas et al. (2014) approaches the problem from a flow-based, inter-zonal power exchange and market splitting point of view while minimizing the cost of generating, importing and transmitting energy, rather than optimizing some economic or social term with respect to price-quantity pairs of the market orders. Martin et al. (2014) optimizes the welfare maximization problem in European DAM by making use of an bid cutting algorithm. In this thesis, we use an economic term called *total welfare* as the objective of our mathematical model, as in COSMOS and Martin et al. (2014).

In this chapter, the proposed mathematical model to solve the balancing problem for Day Ahead clearing in the Turkish Electricity Market is discussed. The (hourly) clearing of the Turkish Electricity DAM requires the simultaneous balancing of hourly bids, block bids that span at least 4 hours, and the flexible bids that can be processed during any hour of a day. The current market operator approaches the problem as a separated yet connected multi-stage problem, where the decisions on accepting or rejecting the hourly bids are made first while processing of the block bids and the flexible bids comes second and third, respectively, based on the clearing of the hourly bids. This stepwise approach makes the solution to the Turkish Electricity DAM clearing problem somewhat a sub-optimal solution and gives room for improvement. In this thesis, we propose a mathematical model (based on COSMOS Model of Djabali et al., 2011) to solve the market clearing problem, processing the hourly, block and flexible bids all at once. The model proposed in this thesis is solved using 15-day Turkish market data generated as described in Chapter 5. In the model, total economic welfare (in very broad definition, the sum of consumer surplus and the producer surplus) is aimed to be maximized while satisfying the price-quantity matching, bid coupling and market clearing constraints. In order to properly run the model, hourly bids submitted in the Turkish DAM need to be converted to "hourly orders", which are the specific entities evaluated in our model. This is done by taking the two of (usually) several price levels of an hourly *bid* and denoting them as upper and lower price limits to accept and reject an hourly *order*. The difference between the two *bid* quantities corresponding to the two price levels makes up the *order* quantity. The method to transform hourly bids into orders, how the price limits and quantity parameters of an hourly supply and demand orders are calculated are elaborated in Section 4.2. Note that block and flexible bids do not require any conversion process and are directly used as orders in the model.

Before continuing with the model description in Section 4.2, some definitions and descriptions are given first in Section 4.1 to clarify the objective function, i.e., total welfare, and the parts of the objective function that constitute the general welfare term.

4.1 Economic Welfare

As stated several times earlier, the objective function of the model proposed in this thesis is maximizing the total (economic) welfare. In a very general sense, welfare can be defined as the overall well-being of the whole society as the term implies while, economically, it refers to a different yet somewhat related thing: it is the sum of consumer surplus and the producer surplus, as defined, for example, by Perloff (2008).

That is, if we can somehow separate the society into two groups of entities, producers and consumers, and measure (in numbers) those groups' individual well-being, then the sum of their well-being "values" will give the overall welfare of that society. However, different economists and social scientists have defined and tried to explain (total) welfare in different ways as exemplified in the next paragraph.

The term, as "economic surplus", was first brought up by the economist Baran (1957) although its components had been defined much earlier by Alfred Marshall (1920). Baran (1957) defines the economic surplus very broadly as "the difference between society's actual current output and its actual current consumption". As stated above, it is divided into two related quantities; namely, the consumer surplus and the producer surplus. Consumer surplus is the monetary difference between the amount a customer is willing to pay and the amount he/she actually pays (see Perloff, 2008, for a similar definition). Marshall (1920) defines this value as the total excess of the price a consumer would pay for a good over the real price of that good. Similarly, producer surplus is the monetary gain a supplier earns by selling at a price higher than the price he/she is willing to sell for (Perloff, 2008). Marshall (1920) has divided the producer's surplus into further two categories: worker's surplus, which is the rate earned by the worker in addition to the pleasure he/she gets from the work, and saver's surplus, which is the extra earning an owner receives in excess of the amount for which he/she is compelled to make the saving. According to Samuelson and Nordhaus (1992), efficient markets operate at the maximum economic surplus. In other words, when the overall satisfaction or the utility of the market participants are at its highest, the market is in equilibrium and the economic surplus, or total welfare as referred to throughout this thesis, is maximum.

Each individual producer or consumer has his/her own preferences and price decisions; so, basically every point of the supply or demand curve refers to a distinct choice of a producer or a customer, respectively. When all of these individual preferences are aggregated, the overall supply and demand curves are obtained. The area that falls between these aggregate curves and the price and quantity axes makes up the total economic surplus value, i.e., total welfare. Figure 4.1 shows an example breakdown of the total welfare into consumer surplus and producer surplus using the example hourly bids given in Chapter 2. As Perloff (2008) describes, the area above the supply curve and below the market price (the pink region) gives the producer surplus. Similarly, the area below the demand curve and over the market price (the blue region) is the consumer surplus. Total welfare, defined as “the area between supply and demand curves and the two axes”, can be clearly seen in the graph. As

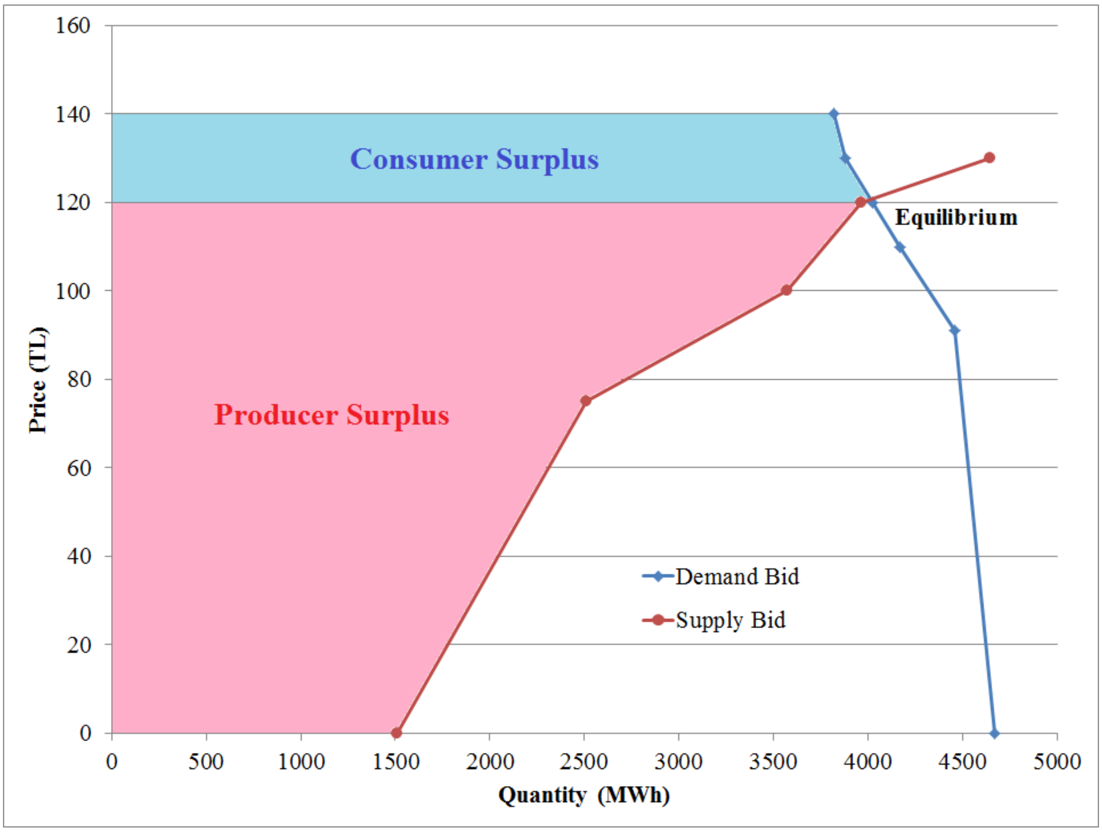

Figure 4.1: Consumer and Producer Surplus based on the Supply and Demand Curves of Example Hourly Bids

it is demonstrated in Figure 4.1, when there are only two hourly bids submitted by two players for a single hour, the problem of electricity DAM clearing can be solved graphically and using simple calculations by hand. In fact, even if there are more than one hourly bids, they can be aggregated and treated as a single hourly bid, as Derinkuyu (2015) does (see Chapter 3). However, balancing (clearing) of Turkish Electricity DAM is done every day for the next 24 hours and it involves not only many hourly purchase and sales bids but also block bids spanning at least four hours and flexible bids that can be executed during any hour of the day. Therefore, a more complex tool is needed to solve this problem, for which we propose a mathematical model.

4.2 The Mathematical Model

In this section, the mathematical model, due to the CWE Market Coupling Algorithm by Djabali et al. (2011) called COSMOS, is adapted, modified and extended for balancing (clearing) the Turkish Electricity DAM. The differences between the model we present for Turkey and the COSMOS model are itemized below.

- We introduce linked block bids and flexible bids into the COSMOS model. For some cases, working with linked block bids and flexible bids is the practice in the Turkish DAM although not very common. The model we propose includes all three types of bids applicable in the Turkish DAM, namely hourly bids, flexible bids, and block bids, including linked block bids.
  As defined earlier in Section 2.2 in Chapter 2, the linked block bids are two or three block bids that are in the same direction (either sales or purchase) and a linked block bid can only be accepted if the other (linked) block bid(s) are accepted. The flexible bids are the sales bids that can be undertaken at any hour of the day.

- The block bids in Turkey consist of a single price-quantity pair whereas distinct amounts of electrical energy can be submitted for each hour of a block bids in the CWE case.

- There is no zonal pricing and bidding applications in Turkey at the time of this thesis unlike the case in the CWE region where the COSMOS algorithm is used for clearing of different bidding areas (mostly countries). The network constraints in COSMOS can be seen in Appendix A.

- In our model, we define different sets of parameters and variables for supply and demand orders, for both hourly and block orders. (See definitions in Section 4.2.1.) In COSMOS, a common notation is used for both supply and demand orders and they are only differentiated by the sign of the quantity parameters. Note that the Turkish DAM and COSMOS have opposite signs for supply and demand quantities (See definitions in Appendix A).

- The constraints regarding the block bids in the model in COSMOS include only the condition to accept a block bid. That is, if a block bid is accepted, then it must be 'in the money', meaning the accepted block bid must satisfy the price criteria. The other way around is not included in COSMOS, i.e., a block bid that is in the money does not have to be accepted. Recall from Chapter 3 (e.g., Biskas et al., 2014, and SAPRI, n.d.) that the block bids –and for some cases, flexible bids– that are rejected even though they satisfy the price criteria are called 'paradoxically rejected' (block/flexible) bids. COSMOS model deals with this kind of bids by not forcing (with a constraint) a block bid that is in- or out of the money to be accepted or rejected, respectively. However, we consider additional constraints in our model for both cases; "to accept a block bid if it satisfies price criteria (and if the trade-off is resolved without paradoxically rejecting the bid)" and "to reject a block bid if it does not satisfy the price criteria". We include an auxiliary binary variable to allow and keep track of paradoxically rejected block and flexible bids. Details of how we do that are given in Section 4.2.2.

In the Turkish DAM's bidding structure, the hourly bids include more than one price-quantity pair in each bid, as demonstrated in Section 2.2 in Chapter 2. However, as stated in the beginning of this chapter, we use a different structure in our mathematical model, which involves hourly *orders*. Each distinct quantity component of an hourly bid is treated as a single hourly order and this quantity is determined by the difference between the two (consecutive) quantities having two consecutive price levels. The two prices corresponding to the quantities that give the aforementioned difference are called the initial and final price of hourly orders and their definitions depend on the direction of the order.

Recall from Section 2.2 in Chapter 2 that the price levels of hourly bids are submitted in an increasing order. Suppose we take two consecutive price-quantity pairs of an hourly bid; refer to the lower price as p_{low} and the corresponding quantity as q_{low}. Similarly, the higher price level and the quantity attached to that price will be denoted by p_{high} and q_{high}, respectively.

In order to go from an *hourly purchase bid* to an *hourly demand order*, the following steps are followed: (1) define the initial price of the demand order as equal to p_{high},

(2) define the final price of the demand order as equal to p_{low}, (3) determine the order quantity by the difference $q_{low} - q_{high}$. Since the quantities submitted in hourly purchase (demand) *bids* are put in a decreasing order ($q_{low} \geq q_{high}$), the calculated quantity of the hourly demand *order* is positive. Note that an hourly purchase *bid* having the price level of $2,000$ TL is always executed. Therefore, we need to define, for each hourly purchase *bid*, an additional hourly demand *order* that has $2,000$ TL as its both initial and final prices. The *order* quantity is equal to the *bid* quantity at the price level of $2,000$ TL.

Table 4.1: Conversion from Actual Purchase Bid to Demand Orders

Order no.	Order Type	Initial Price (p_{high})	Final Price (p_{low})	q_{low}	q_{high}	Quantity ($q_{low} - q_{high}$)
$i = 1$	Demand	91	0	$4,670$	$4,458$	212
$i = 2$	Demand	110	91	$4,458$	$4,168$	290
$i = 3$	Demand	120	110	$4,168$	$4,028$	140
$i = 4$	Demand	130	120	$4,028$	$3,878$	150
$i = 5$	Demand	140	130	$3,878$	$3,818$	60
$i = 6$	Demand	$2,000$	140	$3,818$	$3,818$	0
$i = 7$	Demand	$2,000$	$2,000$	$3,818$	0	$3,818$

The transition from an *hourly sales bid* to an *hourly supply order* is almost the same, except for the definitions of initial and final prices: (1) define the initial price of the supply order as equal to p_{low}, (2) define the final price of the supply order as equal to p_{high}, (3) determine the order quantity by the difference $q_{low} - q_{high}$. Since the quantities submitted in hourly sales (supply) *bids* are put in an increasing order in absolute value ($q_{low} \leq q_{high}$), the calculated quantity of the hourly supply *order* is negative. Note that, similar to the purchase bids, an hourly sales bid having the price level of 0 TL is always executed. Thus, for each hourly sales bid, an additional hourly supply *order* that has 0 TL as its both initial and final prices needs to be included in the model to make sure that it is processed. The order quantity is equal to the bid quantity at the price level of 0 TL.

In Tables 4.1 and 4.2, the initial price, the final price, q_{low}, q_{high}, and the calculated order quantities of hourly purchase and sales bids, respectively, are shown. Those de-

Table 4.2: Conversion from Actual Sales Bid to Supply Orders

Order no.	Order Type	Initial Price (p_{low})	Final Price (p_{high})	q_{low}	q_{high}	Quantity ($q_{low} - q_{high}$)
$j = 1$	Supply	0	0	0	$1,510$	$-1,510$
$j = 2$	Supply	0	75	$1,510$	$2,510$	$-1,000$
$j = 3$	Supply	75	100	$2,510$	$3,575$	$-1,065$
$j = 4$	Supply	100	120	$3,575$	$3,965$	-390
$j = 5$	Supply	120	130	$3,965$	$4,650$	-685
$j = 6$	Supply	130	$2,000$	$4,650$	$4,650$	0

mand and supply orders are derived from the example hourly purchase and sales bids given in Tables 2.1 and 2.2 in Chapter 2, respectively. The graphical representation of the derived hourly orders can also be seen in Figure 4.2. Each line segment of the supply (demand) bid curves in the figure corresponds to one hourly supply (demand) *order*. The parametric representation of prices and quantities of hourly orders and how the initial and final prices are utilized in making the acceptance decision of the orders are described in Section 4.2.1. Note that the block and flexible orders in our mathematical model have gone through no conversion process and they are exactly the same as the block and flexible bids in the Turkish DAM.

4.2.1 Parameters and Decision Variables

Let $Accept_{hr}^{D}(i)$ and $Accept_{hr}^{S}(j)$ denote the decision variables for rate of acceptance of hourly demand order i and hourly supply order j. Similarly, $Accept_{bl}^{D}(b)$ and $Accept_{bl}^{S}(c)$ denote the decision variables for acceptance of demand block order b and supply block order c, respectively. Finally, the decision variable for accepting flexible order f at hour h is denoted by $Accept_{fl}(f, h)$. Recall from Chapter 2 that hourly orders can be fully or partially accepted while flexible and block orders are either fully accepted or fully rejected. Also, flexible orders are not attached to any specific time slot and can be processed at any hour of the day. Thus, they are denoted using an additional index h that represents the hour of the day. When a flexible order f is accepted at hour h, the variable $Accept_{fl}(f, h)$ takes the value of 1 and otherwise it becomes 0. In the model, $Accept_{hr}^{D}(i)$ and $Accept_{hr}^{S}(j)$ are defined as nonnegative

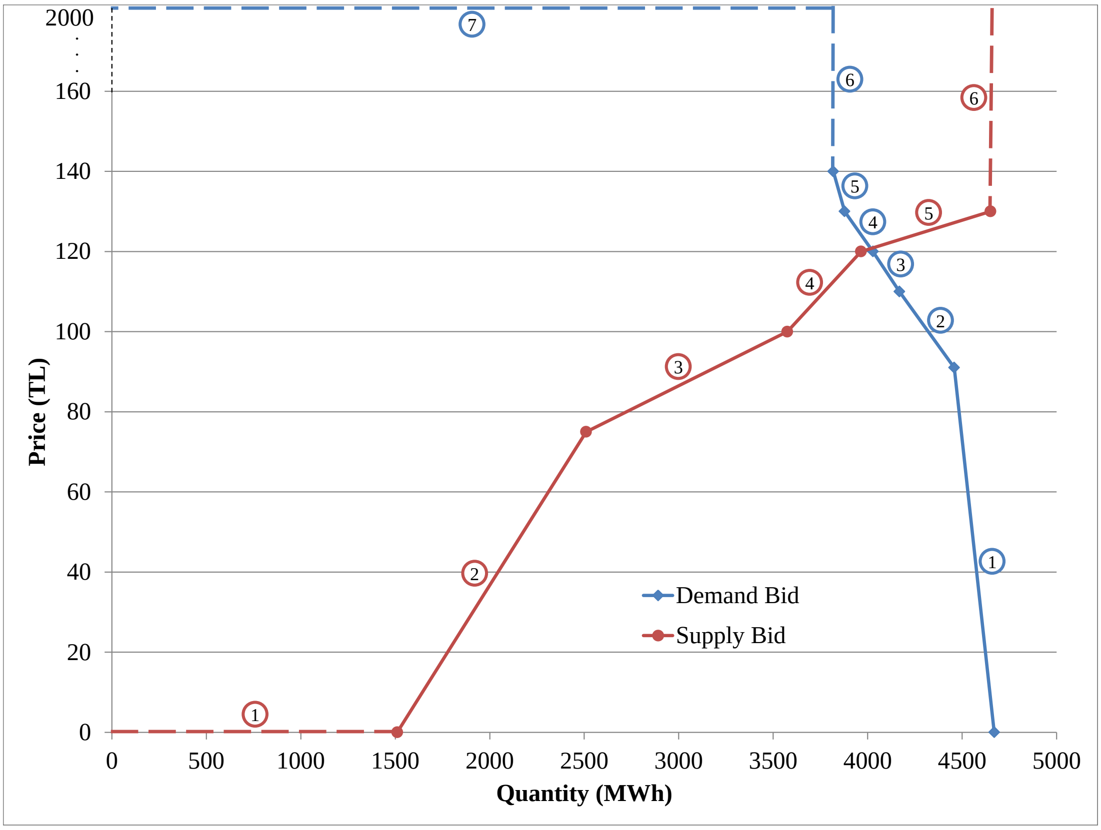

Figure 4.2: Transition from Hourly *Bids* in DAM to Hourly *Orders* in the Mathematical Model

decision variables that can take any value in the closed interval $[0, 1]$. $Accept_{bl}^{D}(b)$, $Accept_{bl}^{S}(c)$ and $Accept_{fl}(f, h)$ are binary decision variables taking a value of 0 (full rejection) or 1 (full acceptance). That is,

$$0 \leq Accept_{hr}^{D}(i) \leq 1 \qquad \text{for } i = 1, ..., I, \tag{4.1}$$

$$0 \leq Accept_{hr}^{S}(j) \leq 1 \qquad \text{for } j = 1, ..., J, \tag{4.2}$$

$$Accept_{bl}^{D}(b) \in \{0, 1\} \qquad \text{for } b = 1, ..., B, \tag{4.3}$$

$$Accept_{bl}^{S}(c) \in \{0, 1\} \qquad \text{for } c = 1, ..., C, \tag{4.4}$$

$$Accept_{fl}(f, h) \in \{0, 1\} \qquad \text{for } f = 1, ..., F \text{ and } h = 1, ..., 24. \tag{4.5}$$

Also, let the nonnegative decision variable $MCP(h)$ be the (final) MCP that is effective at hour h. Note that $MCP(h)$ is calculated based on all three types of orders in the model and it is based on their price levels and proportion of acceptance:

$$0 \leq MCP(h) \leq 2000 \qquad \text{for } h = 1, ..., 24. \tag{4.6}$$

All types of orders have different price structures, and therefore different price parameters. For instance, demand and supply block orders b and c have a single price, denoted by $p^D_{bl}(b)$ and $p^S_{bl}(c)$, which are the prices applicable for each hour spanned by the block order. Similarly, the single price parameter $p_{fl}(f)$ is used for flexible orders, which can only be submitted in the supply direction. On the other hand, the hourly orders present a different picture as they have two separate price levels. The first price level of the hourly orders can be briefly referred to as the initial price; the price at which an hourly order starts to be (partially) accepted. It is denoted in the model by $p^D_{hr}(i,0)$ and $p^S_{hr}(j,0)$, respectively, for hourly demand and hourly supply orders. The second price can be considered as the final price; it is the price level at which an hourly order is fully accepted. An hourly demand or an hourly supply order is fully accepted at hour h if a certain (final) price level, denoted by $p^D_{hr}(i,1)$ and $p^S_{hr}(j,1)$, respectively, is reached by the $MCP(h)$. For hourly supply orders, the starting (initial) price for accepting the order is always less than or equal to the (final) price beyond which the order is completely accepted: $p^S_{hr}(j,0) \leq p^S_{hr}(j,1)$. For hourly demand orders, the opposite is true: the initial price is at least as large as the final price: $p^D_{hr}(i,0) \geq p^D_{hr}(i,1)$.

$$0 \leq p^D_{hr}(i,0) \leq 2000 \quad \text{and} \quad 0 \leq p^D_{hr}(i,1) \leq 2000 \qquad \text{for } i = 1, ..., I, \quad (4.7)$$

$$p^D_{hr}(i,0) \geq p^D_{hr}(i,1) \qquad \text{for } i = 1, ..., I, \quad (4.8)$$

$$0 \leq p^S_{hr}(j,0) \leq 2000 \quad \text{and} \quad 0 \leq p^S_{hr}(j,1) \leq 2000 \qquad \text{for } j = 1, ..., J, \quad (4.9)$$

$$p^S_{hr}(j,1) \geq p^S_{hr}(j,0) \qquad \text{for } j = 1, ..., J. \quad (4.10)$$

Now, as expected, the meaning of the initial and final prices depends on the direction of the hourly order; that is, whether it is a supply order or a demand order. Basically, $p^S_{hr}(j,0)$ of an hourly *supply order* is the minimum price at which that order starts to be partially accepted. If $MCP(h)$ is lower than this price, then the respective *supply order* will be rejected. Similarly, $p^S_{hr}(j,1)$ of an hourly *supply order* is the minimum price at which that order can be fully accepted. If $MCP(h)$ is higher than or equal to this price, then the relevant order will be fully accepted.

The explanation of the price levels is almost the same for demand direction except that the minimum price levels of supply orders become maximum levels in demand cases. To clarify, $p^D_{hr}(i,0)$ of an hourly *demand order* is the maximum price to start

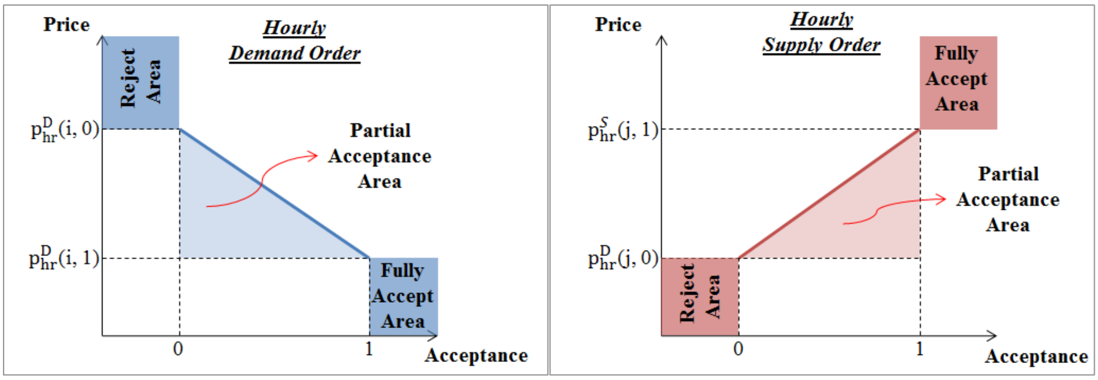

Figure 4.3: Initial and Final Prices $\big(p.(\cdot, 0)$ and $p.(\cdot, 1)\big)$ of Hourly Demand and Supply Orders

accepting the order (minimum price at which the order is rejected) and $p^D_{hr}(i, 1)$ of an hourly *demand order* is the maximum price at which the relevant demand order can be fully accepted. If $MCP(h)$ is lower than $p^D_{hr}(i, 1)$, then the hourly demand order will be fully accepted.

The relationship between the two price levels and their effect on the rate of acceptance of the orders –based on the level of $MCP(h)$– are illustrated in Figure 4.3. For both supply and demand orders, any clearing price $MCP(h)$ that is between the initial and final prices indicates that the order is accepted only partially.

Similar to the price structures, each order type has a unique quantity component that is attached to the relevant price level(s). As stated earlier, the block bids in Turkish Electricity DAM (block orders in our model) have single quantity that is associated with the price components $p^D_{bl}(b)$ and $p^S_{bl}(c)$, and the given price-quantity pairs are effective for every hour that a block order spans. In our mathematical model, the quantities submitted by block orders b and c are denoted by the parameters $q^D_{bl}(b)$ and $q^S_{bl}(c)$, respectively. The quantity parameter is positive for demand block orders, $q^D_{bl}(b) \geq 0$, and negative for supply block orders, $q^S_{bl}(c) \leq 0$. The hourly demand and supply orders have a very similar annotation and the quantities are represented by the parameters $q^D_{hr}(i)$ and $q^S_{hr}(j)$, respectively. The quantities of hourly orders are applicable at the single hour they are submitted for and they are again positive for demand orders and negative for supply orders. Recall that these parameters are different than the bid quantities submitted in the DAM and they are calculated based on those bid quantities. As for the flexible orders that are submitted only in the supply direction, the quantity is represented by the negative parameter $q_{fl}(f)$.

As stated earlier in Section 4.2, orders from $i = 1$ to $i = 7$ in Table 4.1 are actually converted from the single purchase bid given in Table 2.1 in Chapter 2 and orders from $j = 1$ to $j = 6$ in Table 4.2 are derived from the single sales bid given in Table 2.2. Based on the prevailing MCP calculated at the hour they are submitted (which is the 8th hour of the day), it is determined which portions of the single bids shown in Chapter 2 (in other words, which individual orders given in Tables 4.1 and 4.2) will be accepted. Consider the case where $MCP(8) = 120.75$ TL, which is actually the equilibrium price for the two bids as shown in Figure 2.3. In this case, the maximum prices to fully accept the *demand* orders 7, 6 (which is a redundant order as the quantity is 0) and 5 are higher than the MCP, which means that these orders are fully accepted:

$$p^D_{hr}(7,1) \geq MCP(8) \;\;\Rightarrow\;\; Accept^D_{hr}(7) = 1,$$
$$p^D_{hr}(6,1) \geq MCP(8) \;\;\Rightarrow\;\; Accept^D_{hr}(6) = 1, \text{ and}$$
$$p^D_{hr}(5,1) \geq MCP(8) \;\;\Rightarrow\;\; Accept^D_{hr}(5) = 1.$$

Moreover, demand order 4 is partially accepted since the MCP is lower than the price at which the order starts to be accepted while it is higher than the maximum price at which that *demand* order is fully accepted:

$$p^D_{hr}(4,1) < MCP(8) < p^D_{hr}(4,0) \;\;\Rightarrow\;\; 0 < Accept^D_{hr}(4) < 1.$$

Similarly, the supply orders 1, 2, 3 and 4 are all fully accepted since the minimum prices to fully accept those *supply* orders are lower than the MCP:

$$p^S_{hr}(1,1) \leq MCP(8) \;\;\Rightarrow\;\; Accept^S_{hr}(1) = 1,$$
$$p^S_{hr}(2,1) \leq MCP(8) \;\;\Rightarrow\;\; Accept^S_{hr}(2) = 1,$$
$$p^S_{hr}(3,1) \leq MCP(8) \;\;\Rightarrow\;\; Accept^S_{hr}(3) = 1, \text{ and}$$
$$p^S_{hr}(4,1) \leq MCP(8) \;\;\Rightarrow\;\; Accept^S_{hr}(4) = 1.$$

In addition, as the MCP is lower than the minimum fully-accept-price of the supply order 5 and higher than the initial price at which the order starts to be accepted, the supply order 5 is partially accepted, i.e.,

$$p^S_{hr}(5,0) < MCP(8) < p^S_{hr}(5,1) \;\;\Rightarrow\;\; 0 < Accept^S_{hr}(5) < 1.$$

When all of the above calculations and decisions are translated for the Turkish Electricity DAM, we can say that both of the purchase and sales bids given in Tables 2.1 and 2.2 in Chapter 2 are partially accepted when the MCP is 120.75 TL.

As for determining the accepted hourly supply and demand quantities, we can sum the $q^D_{hr}(i)$ and $q^S_{hr}(j)$ amounts of each of the fully accepted orders and add, on top of that amount, the amount found by interpolating the quantity of the partially accepted order using the price limits $p^D_{hr}(i,0)$, $p^D_{hr}(i,1)$, $p^S_{hr}(j,0)$ and $p^S_{hr}(j,1)$, and MCP. For instance, since the *demand* orders 7, 6 and 5 are all fully accepted we sum the quantities attached to those orders and obtain $3,878$ MWh. For the 4th order which is partially accepted, we interpolate between 120 and 130 for 120.75 to find how much of the 150 MWh offered is executed. That is,

$$Accept^D_{hr}(4) = \frac{(130 - 120.75) \times 150}{(130 - 120)} = \frac{9.25 \times 150}{10} = 0.925$$

and the corresponding accepted amount is

$$Accept^D_{hr}(4) \times q^D_{hr}(4) = 0.925 \times 150 = 138.68 \text{ MWh.}$$

In a similar manner, we can calculate the total accepted quantity supplied by summing up the submitted quantities of the fully accepted *supply* orders 1, 2, 3 and 4, and add the accepted portion of -685 MWh submitted in supply order 5, which is calculated again by interpolation as follows:

$$Accept^S_{hr}(5) = \frac{(120.75 - 120)}{(130 - 120)} = \frac{0.75}{10} = 0.075$$

and the corresponding accepted amount is

$$Accept^S_{hr}(5) \times q^S_{hr}(5) = 0.075 \times (-685) = -51.68 \text{ MWh.}$$

In the end, we find the equilibrium quantity of the purchase and sales bids given in Tables 2.1 and 2.2 (in other words, the orders in Tables 4.1 and 4.2) as $4,016.68$ MWh as shown in Figure 2.3. That is,

$$\begin{aligned} & q^D_{hr}(7) + q^D_{hr}(5) + q^D_{hr}(5) + Accept^D_{hr}(4) \times q^D_{hr}(4) \\ & = 3,878 + 138.68 \\ & = 4,016.68 \text{ MWh.} \end{aligned}$$

Alternatively,

$$\begin{aligned} & q^S_{hr}(1) + q^S_{hr}(2) + q^S_{hr}(3) + q^S_{hr}(4) + Accept^S_{hr}(5) \times q^S_{hr}(5) \\ & = (-3965) + (-51.68) \\ & = -4,016.68 \text{ MWh.} \end{aligned}$$

The price and quantity parameters described above do not have a time component h. To compare the prices to the MCP of a specific hour, we need additional parameters to associate hourly and block orders with the hours they are attached to. (Recall that flexible orders can be processed at any hour; so, there will be no need to define a time parameter for flexible orders). These parameters are $hour^D(i,h)$ and $hour^S(j,h)$ for hourly demand and supply orders; and $hours^D(b,h)$ and $hours^S(c,h)$ for demand and supply block orders, respectively. These parameters are defined as follows:

$$hour^D(i,h) = \begin{cases} 1 & \text{if hourly demand order } i \text{ is submitted for hour } h, \\ 0 & \text{otherwise, for } i = 1, ..., I \text{ and } h = 1, ..., 24, \end{cases} \tag{4.11}$$

$$hour^S(j,h) = \begin{cases} 1 & \text{if hourly supply order } j \text{ is submitted for hour } h, \\ 0 & \text{otherwise, for } j = 1, ..., J \text{ and } h = 1, ..., 24, \end{cases} \tag{4.12}$$

$$hours^D(b,h) = \begin{cases} 1 & \text{if span of demand block order } b \text{ includes hour } h, \\ 0 & \text{otherwise, for } b = 1, ..., B \text{ and } h = 1, ..., 24, \end{cases} \tag{4.13}$$

$$hours^S(c,h) = \begin{cases} 1 & \text{if span of supply block order } c \text{ includes hour } h, \\ 0 & \text{otherwise, for } c = 1, ..., C \text{ and } h = 1, ..., 24. \end{cases} \tag{4.14}$$

Using the above four parameters, the relationship between the orders, their time slot (a single hour or a set of hours) and $MCP(h)$ can be set up in the model. This way, the decision on whether or not accepting an order is made based on their (initial and final) prices, $p^D_{hr}(i,0)$, $p^S_{hr}(j,0)$ and $p^D_{hr}(i,1)$, $p^S_{hr}(j,1)$, and the MCP of the hour(s) that the order is valid for. In Section 4.2.2 below, the constraints of the mathematical model that are formulated for the purpose of determining the MCP and the execution of all types of orders are described.

4.2.2 Constraints

Recall that our model is based on COSMOS Model by Djabali et al. (2011) with additions of linked block orders, flexible orders, and a new method to deal with paradoxically rejected block and flexible orders. Furthermore, in Turkish DAM, block orders do not have separate quantities for each hour of the order, and there is no zonal pricing in Turkey. Finally, we use distinct notation for demand and supply orders to make it easier to distinguish different directions of orders, as demonstrated above in Section 4.2.1.

To begin with, the values that the decision variables can take should be specified in the set constraints. Recall (4.1) through (4.5): $Accept_{hr}^{D}(i)$ and $Accept_{hr}^{S}(j)$ can take any fractional value between and including 0 and 1 whereas $Accept_{bl}^{D}(b)$, $Accept_{bl}^{S}(c)$ are integer variables that can only take the values 0 or 1 in the model. The other decision variable, $MCP(h)$, gives the final MCP and can take any nonnegative value: $MCP(h) \geq 0$ for $h = 1, ..., 24$, although in practice the upper level is 2,000 TL (see (4.6) on page 37).

4.2.2.1 Constraints for Hourly Orders

The constraints of the mathematical model, which make sure that the fully accepted and partially accepted hourly orders satisfy the initial and final price limits as demonstrated in Section 4.2.1 are described in this section.

The first set of structural constraints is about the criterion to accept an hourly order. With the constraints (4.15) and (4.16) below, it is stated that an hourly demand order i and an hourly supply order j can be accepted only if it benefits from the price difference. That is, an hourly demand (supply) order can be –at least partially– accepted if its initial price is higher (lower) than the MCP. Since the price at which an hourly demand order starts to be accepted must be higher than the MCP, the left hand side of the inequality (4.15) is defined as $p_{hr}^{D}(i,0) - MCP(h)$, whereas it is $MCP(h) - p_{hr}^{S}(j,0)$ for the hourly supply order in (4.16). With these two constraints, we only set values of two binary variables $y_1^D(i)$ and $y_1^S(j)$, and how they work to determine the value of acceptance variables is described below.

For $i = 1, ..., I$,

$$p_{hr}^{D}(i,0) - \sum_{h=1}^{24} MCP(h) \times hour^{D}(i,h) \quad \leq \quad M \times \left(1 - y_1^{D}(i)\right). \qquad (4.15)$$

For $j = 1, ..., J$,

$$\sum_{h=1}^{24} MCP(h) \times hour^S(j,h) - p^S_{hr}(j,0) \quad \leq \quad M \times \big(1 - y^S_1(j)\big). \tag{4.16}$$

where M is a sufficiently large number and $y^D_1(i)$ and $y^S_1(j)$ are binary decision variables; $y^D_1(i) \in \{0,1\}$ for $i = 1, ..., I$ such that

$$y^D_1(i) = \begin{cases} 1 & \text{if } MCP(h) \geq p^D(i,0) \text{ (hourly demand order } i \text{ is rejected),} \\ 0 & \text{if } MCP(h) \leq p^D(i,0). \end{cases}$$

$y^S_1(j) \in \{0,1\}$ and for $j = 1, ..., J$ such that

$$y^S_1(j) = \begin{cases} 1 & \text{if } MCP(h) \leq p^S(j,0) \text{ (hourly supply order } j \text{ is rejected),} \\ 0 & \text{if } MCP(h) \geq p^S(j,0). \end{cases}$$

When the above two constraints are evaluated together with rejection inequalities (4.17) through (4.20) and partial acceptance constraints (4.27) through (4.30), it is made sure that the binary variables $y^D_1(i)$ and $y^S_1(j)$ take the correct values so that the decision variables $Accept^D_{hr}(i)$ and $Accept^S_{hr}(j)$ also take the correct value representing the acceptance decisions. These constraints are explained next.

The following two couple of constraints make sure that when the criterion of the MCP being larger (smaller) than the price at which an hourly supply (demand) order starts to be accepted is not satisfied, then the order in question is rejected.

For $i = 1, ..., I$,

$$\sum_{h=1}^{24} MCP(h) \times hour^D(i,h) - p^D_{hr}(i,0) \quad \leq \quad M \times y^D_1(i), \tag{4.17}$$

$$Accept^D_{hr}(i) \quad \leq \quad 1 - y^D_1(i). \tag{4.18}$$

For $j = 1, ..., J$,

$$p^S_{hr}(j,0) - \sum_{h=1}^{24} MCP(h) \times hour^S(j,h) \quad \leq \quad M \times y^S_1(j), \tag{4.19}$$

$$Accept^S_{hr}(j) \quad \leq \quad 1 - y^S_1(j). \tag{4.20}$$

In the constraints (4.17) through (4.20) above, it is ensured that when the initial price of an hourly demand (supply) order is lower (higher) than the MCP of the hour that the order is placed for, then the corresponding order is rejected due to (4.18) and (4.20). The constraints (4.15) and (4.16) are included in the model so that when the aforementioned price conditions do not hold, $y_1^D(i) = 0 \ \big(y_1^S(j) = 0\big)$ and the corresponding hourly demand order i (supply order j) is not forced to be rejected.

In order to fully accept an hourly order, $p_{hr}^D(i,1)$ must be larger than the MCP for demand orders and $p_{hr}^S(j,1)$ must be smaller than the MCP for supply orders as stated in Section 4.2.1. The following constraints are included in the mathematical model for this purpose.

For $i = 1, ..., I$,

$$p_{hr}^D(i,1) - \sum_{h=1}^{24} MCP(h) \times hour^D(i,h) \ \leq \ M \times \big(1 - y_2^D(i)\big), \qquad (4.21)$$

$$\sum_{h=1}^{24} MCP(h) \times hour^D(i,h) - p_{hr}^D(i,1) \ \leq \ M \times y_2^D(i), \qquad (4.22)$$

$$Accept_{hr}^D(i) - 1 \ \geq \ -y_2^D(i), \qquad (4.23)$$

For $j = 1, ..., J$,

$$\sum_{h=1}^{24} MCP(h) \times hour^S(j,h) - p_{hr}^S(j,1) \ \leq \ M \times \big(1 - y_2^S(j)\big), \qquad (4.24)$$

$$p_{hr}^S(j,1) - \sum_{h=1}^{24} MCP(h) \times hour^S(j,h) \ \leq \ M \times y_2^S(j), \qquad (4.25)$$

$$Accept_{hr}^S(j) - 1 \ \geq \ -y_2^S(j), \qquad (4.26)$$

where $y_2^D(i)$ and $y_2^S(j)$ are binary decision variables; $y_2^D(i) \in \{0,1\}$ for $i = 1, ..., I$ such that

$$y_2^D(i) = \begin{cases} 0 & \text{if } MCP(h) \leq p^D(i,1) \text{ (hourly demand order } i \text{ is fully accepted)}, \\ 1 & MCP(h) \geq p^D(i,1). \end{cases}$$

$y_2^S(j) \in \{0,1\}$ for $j = 1, ..., J$ such that

$$y_2^S(j) = \begin{cases} 0 & \text{if } MCP(h) \geq p^S(j,1) \text{ (hourly supply order } j \text{ is fully accepted)}, \\ 1 & MCP(h) \leq p^S(j,1). \end{cases}$$

The way the above set of constraints works is as follows. When an hourly demand order i or an hourly supply order j is fully accepted, i.e., $y_2^D(i) = 0$ or $y_2^S(j) = 0$, the constraints (4.23) and (4.26) indicate that $Accept^D_{hr}(i) \geq 1$ or $Accept^S_{hr}(j) \geq 1$, respectively. In addition to this condition, $Accept^D_{hr}(i) \leq 1$ and $Accept^S_{hr}(j) \leq 1$ due to (4.1) and (4.2) on page 37. This makes the value of the $Accept^{\cdot}_{hr}(\cdot)$ variables equal to 1, meaning that the hourly order i or j is fully accepted when $y^D_2(i) = 0$ or $y^S_2(j) = 0$, respectively.

The final set of structural constraints related to the hourly orders in the Turkish DAM are about the price conditions that an order must satisfy in order to be *partially* accepted.

For $i = 1, ..., I$,

$$\sum_{h=1}^{24} MCP(h) \times hour^D(i,h) - \Big(p^D_{hr}(i,0) - Accept^D_{hr}(i) \times \big(p^D_{hr}(i,0) - p^D_{hr}(i,1)\big)\Big) \leq M \times y^D_1(i) + M \times \big(1 - y^D_2(i)\big), \tag{4.27}$$

$$\sum_{h=1}^{24} MCP(h) \times hour^D(i,h) - \Big(p^D_{hr}(i,0) - Accept^D_{hr}(i) \times \big(p^D_{hr}(i,0) - p^D_{hr}(i,1)\big)\Big) \geq -M \times y^D_1(i) - M \times \big(1 - y^D_2(i)\big). \tag{4.28}$$

For $j = 1, ..., J$,

$$\sum_{h=1}^{24} MCP(h) \times hour^S(j,h) - \Big(p^S_{hr}(j,0) + Accept^S_{hr}(j) \times \big(p^S_{hr}(j,1) - p^S_{hr}(j,0)\big)\Big) \leq M \times y^S_1(j) + M \times \big(1 - y^S_2(j)\big), \tag{4.29}$$

$$\sum_{h=1}^{24} MCP(h) \times hour^S(j,h) - \Big(p^S_{hr}(j,0) + Accept^S_{hr}(j) \times \big(p^S_{hr}(j,1) - p^S_{hr}(j,0)\big)\Big) \geq -M \times y^S_1(j) - M \times \big(1 - y^S_2(j)\big). \tag{4.30}$$

Constraints (4.27) and (4.28) are redundant for all combinations of $\big(y^D_1(i), y^D_2(i)\big)$ except for the case this pair is equal to $(0, 1)$, which makes the right-hand side of both inequalities 0. In other words, constraints (4.27) and (4.28) are given for the case $p^D_{hr}(i,1) \leq MCP(h) \leq p^D_{hr}(i,0)$ to determine the corresponding $Accept^D_{hr}(i)$ making use of the equality $MCP(h) = p^D_{hr}(i,0) - Accept^D_{hr}(i) \times \big(p^D_{hr}(i,0) - p^D_{hr}(i,1)\big)$. Almost the same conditions apply for the hourly supply orders. In that case, constraints (4.29) and (4.30) are redundant for all combinations of $\big(y^S_1(j), y^S_2(j)\big)$ except when this pair is equal to $(0,1)$. In other words, (4.29) and (4.30) are given for the case $p^S_{hr}(j,0) \leq MCP(h) \leq p^S_{hr}(i,1)$ to determine the corresponding $Accept^S_{hr}(j)$ using the equality $MCP(h) = p^S_{hr}(j,0) + Accept^S_{hr}(j) \times \big(p^S_{hr}(j,1) - p^S_{hr}(j,0)\big)$. In short, the interpolation of MCP between the price limits of the hourly order i or j determines how much of that order will be accepted.

There are three outcomes of the model that can be observed related to the hourly orders. An hourly order can either be (1) fully rejected, (2) fully accepted, or (3) partially accepted. This decision is made based on the comparison of MCP at the hour the order is submitted for, to the initial and final prices of the order, $p^{\cdot}_{hr}(\cdot,0)$ and $p^{\cdot}_{hr}(\cdot,1)$.

For a given hourly demand order i such that $hour^D(i,h) = 1$, the following analysis can be made based on the comparison between the MCP and the initial price of the order.

Case I-1. When $MCP(h) > p^D_{hr}(i,0)$, we have
$y^D_1(i) = 1$ due to (4.17),
$Accept^D_{hr}(i) = 0$ due to (4.18), meaning hourly demand order i is rejected.

Case I-2. When $MCP(h) < p^D_{hr}(i,0)$, we have
$y^D_1(i) = 0$ due to (4.15). The hourly demand order i is either partially or fully accepted, depending on the value of $y^D_2(i) = 0$, which is based on the comparison of MCP and the final price.

Case I-3. When $MCP(h) = p^D_{hr}(i,0)$, we have

Subcase I-3.1. $y^D_1(i) = 1$,
$Accept^D_{hr}(i) = 0$; hourly demand order i is rejected (see Case I-1 above).

Subcase I-3.2. $y_1^D(i) = 0$,

$Accept_{hr}^D(i) = 0$; hourly demand order i is rejected due to (4.27) and (4.28). Also, $y_2^D(i) = 1$ due to (4.22) as $MCP \geq p_{hr}^D(i, 1)$.

A similar analysis, which makes use of the comparison between the MCP and the final price of a given hourly demand order i can be summarized as follows.

Case F-1. When $MCP(h) < p_{hr}^D(i, 1)$, we have

$y_2^D(i) = 0$ due to (4.21),

$Accept_{hr}^D(i) = 1$ due to (4.23), meaning hourly demand order i is fully accepted.

Case F-2. When $MCP(h) > p_{hr}^D(i, 1)$, we have

$y_2^D(i) = 0$ due to (4.22). The hourly demand order i is either partially accepted or rejected, depending on the comparison of MCP and the initial price.

Case F-3. When $MCP(h) = p_{hr}^D(i, 1)$, we have

Subcase F-3.1. $y_2^D(i) = 0$,

$Accept_{hr}^D(i) = 1$; hourly demand order i is fully accepted (see Case F-1 above).

Subcase F-3.2. $y_2^D(i) = 1$,

$Accept_{hr}^D(i) = 1$, hourly demand order i is fully accepted due to (4.27) and (4.28). Also, $y_1^D(i) = 0$ due to (4.15) as $MCP \leq p_{hr}^D(i, 0)$.

Based on the analyses above, we can demonstrate how the mathematical model behaves by the changing combination of the pair $\left(y_1^D, y_2^D\right)$.

(1) $\left(y_1^D(i), y_2^D(i)\right) = (1, 0)$

In this case, $y_1^D(i = 1$ implies $MCP(h) \geq p^D(i, 0)$ and $y_2^D(i) = 0$ implies $MCP(h) < p^D(i, 1)$, which is *not possible* because $p^D(i, 0) > p^D(i, 1)$ for demand orders.

(2) $\big(y_1^D(i), y_2^D(i)\big) = (1, 1)$

This pair implies $MCP(h) \geq p^D(i, 0)$ and $MCP(h) > p^D(i, 1)$, which is possible depending on the values of $p^D(i, 0)$ and $p^D(i, 1)$. Also, $y_1^D(i) = 1$ means that $Accept_{hr}^D(i) = 0$, and $y_2^D(i) = 1$ does not violate this outcome in any of the constraints.

(3) $\big(y_1^D(i), y_2^D(i)\big) = (0, 0)$

$y_2^D(i) = 0$ implies $MCP(h) \leq p^D(i, 1)$, which also means that hourly demand order i is fully accepted. Having $y_1^D(i) = 0$ does not violate this outcome in any of the constraints.

(4) $\big(y_1^D(i), y_2^D(i)\big) = (0, 1)$

This pair implies $p^D(i, 1) \leq MCP(h) \leq p^D(i, 0)$, which can result in either full acceptance, partial acceptance or full rejection of order i. This is determined by the constraints (4.27) and (4.28).

Note that almost the same analysis can be done for the hourly supply orders, for which (4.29) and (4.30) are the partially acceptance constraints.

4.2.2.2 Constraints for Block Orders

In this section, constraints for the block orders are given. Recall from Section 4.2 that block bids submitted in the Turkish DAM are directly incorporated into our model as block orders without going through a conversion process. Since block orders can only be accepted or rejected fully and cannot be partially executed, there is only one main set of constraints (about when to accept those orders and when to reject them) for each direction. The constraints below are very similar to their counterparts for hourly orders. The decision of whether accepting or rejecting a block order is based upon the total price difference over all hours at which a block order is placed. This summation is equivalent to the sum of individual price differences at every hour the block order placed at. This equivalence relationship is given below, showing that the sum of price differences is equal to the difference between total prices (difference between MCP and the block price).

$$p_{bl}^D(b) \times \sum_{h=1}^{24} hours^D(b,h) - \sum_{h=1}^{24} MCP(h) \times hours^D(b,h)$$
$$= \sum_{h=1}^{24} hours^D(b,h) \times \big(p_{bl}^D(b) - MCP(h)\big)$$

The constraints (4.31) and (4.33) specify the acceptance conditions. In the constraints (4.32) and (4.34), the rejection of block orders are specified.

For $b = 1, ..., B$,

$$p_{bl}^D(b) \sum_{h=1}^{24} hours^D(b,h) - \sum_{h=1}^{24} MCP(h) \times hours^D(b,h) \leq M \times Accept_{bl}^D(b) + M \times \big(1 - y_3^D(b)\big), \tag{4.31}$$

$$\sum_{h=1}^{24} MCP(h) \times hours^D(b,h) - p_{bl}^D(b) \sum_{h=1}^{24} hours^D(b,h) \leq M \times \big(1 - Accept_{bl}^D(b)\big), \tag{4.32}$$

For $c = 1, ..., C$,

$$\sum_{h=1}^{24} MCP(h) \times hours^S(c,h) - p_{bl}^S(c) \sum_{h=1}^{24} hours^S(c,h) \leq M \times Accept_{bl}^S(c) + M \times \big(1 - y_3^S(c)\big), \tag{4.33}$$

$$p_{bl}^S(c) \sum_{h=1}^{24} hours^S(c,h) - \sum_{h=1}^{24} MCP(h) \times hours^S(c,h) \leq M \times \big(1 - Accept_{bl}^S(c)\big), \tag{4.34}$$

When the summation of prices of a demand (supply) block order is lower (higher) than the total MCPs at the hours the block order is submitted, then the constraints (4.32) and (4.34) imply that the demand (supply) block order is rejected. However, the opposite is not always imposed. As stated in Section 4.2, some block orders can be rejected even if they satisfy the price criteria, and these orders are called "paradoxically rejected block (PRB) orders". Therefore, unlike the constraints regarding the acceptance of hourly orders or the rejection of block orders, we have an additional term $M \times \big(1 - y_3^{\cdot}(\cdot)\big)$ in the block order acceptance constraints. How this term helps allowing and tracking PRB orders is as follows. When a demand (supply) block order is paradoxically rejected, then $y_3^D(b) = 0$ $\big(y_3^S(c) = 0\big)$, which makes the constraint (4.31) $\big($(4.33)$\big)$ redundant. This way, these constraints do not force the mentioned block order to be accepted although that order satisfies the price criteria. Thus, it becomes a PRB order.

$$y_3^D(b) = \begin{cases} 0 & \text{if demand block order } b \text{ is paradoxically rejected,} \\ 1 & \text{if demand block order } b \text{ is either accepted or fairly rejected,} \end{cases}$$

and

$$y_3^S(c) = \begin{cases} 0 & \text{if supply block order } c \text{ is paradoxically rejected,} \\ 1 & \text{if supply block order } c \text{ is either accepted or fairly rejected.} \end{cases}$$

In the constraint (4.35) $\big($(4.36)$\big)$ below, we make sure that when a demand block order b (supply block order c) is accepted, the corresponding $y_3^D(b)$ is 1 $\big(y_3^S(c)$ is $1\big)$ and the acceptance constraint (4.31) $\big($(4.33)$\big)$ is active. When demand block order b (supply block order c) is paradoxically rejected, i.e., $y_3^D(b) = 0$ $\big(y_3^S(c) = 0\big)$, this is imposed by again (4.35) $\big($(4.36)$\big)$. On the other hand, we need constraint (4.37) $\big($(4.38)$\big)$ to ensure that the acceptance constraint (4.31) $\big($(4.33)$\big)$ is not allowed to be redundant due to $y_3^D(b)$ ($y_3^S(c)$) when demand block order b (supply block order c) is *fairly* rejected, i.e., the order is rejected because it does not satisfy the price criteria. That is, we force $y_3^D(b)$ ($y_3^S(c)$) to take the value of 1 in case of fair rejection, which is in accordance with the definition of $y_3^D(b)$ ($y_3^S(c)$).

For $b = 1, ..., B$ and $c = 1, ..., C$,

$$Accept_{bl}^D(b) \leq y_3^D(b), \tag{4.35}$$

$$Accept_{bl}^S(c) \leq y_3^S(c). \tag{4.36}$$

$$\sum_{h=1}^{24} MCP(h) \times hours^D(b,h) - p_{bl}^D(b) \sum_{h=1}^{24} hours^D(b,h) \leq M \times y_3^D(b), \tag{4.37}$$

$$p_{bl}^S(c) \sum_{h=1}^{24} hours^S(c,h) - \sum_{h=1}^{24} MCP(h) \times hours^S(c,h) \leq M \times y_3^S(c), \tag{4.38}$$

Recall from Chapter 2 that some block orders are linked to other block orders, which means that a linked block order can only be accepted when the block order(s) it is linked to is (are) accepted. For that purpose, the constraints below are added to the model. Note that the linked block orders must be submitted in the same direction and at most three block orders can be linked.

For the case of two linked block orders,

$$Accept^{D}_{bl}(b') \leq Accept^{D}_{bl}(b), \quad \text{if demand block order } b' \text{ is linked to order } b, \tag{4.39}$$
$$Accept^{S}_{bl}(c') \leq Accept^{S}_{bl}(c), \quad \text{if supply block order } c' \text{ is linked to order } c. \tag{4.40}$$

For the case of three linked block orders, the acceptance of demand block order b'' (supply block order c'') is dependent on the acceptance of two other block orders b and b' (c and c'). That is, the block order b (c) can be accepted at any case whereas block order b' (c') can only be executed if block order b (c) is accepted (see (4.39) and (4.40) above), and block order b'' (c'') can be accepted only if block orders b and b' (c and c') are accepted (EMRA, BSR, 2013a). The opposite is not enforced, i.e., the block order b' (c') does not have to be accepted when block order b (c) is accepted.

$$Accept^{D}_{bl}(b'') \leq Accept^{D}_{bl}(b'), \text{if demand order } b'' \text{ is linked to orders } b \text{ and } b', \tag{4.41}$$
$$Accept^{S}_{bl}(c'') \leq Accept^{S}_{bl}(c'), \text{if supply order } c'' \text{ is linked to orders } c \text{ and } c'. \tag{4.42}$$

In the constraints (4.41) and (4.42) above, linked block orders b'' and c'' are associated with only block orders b' and c', respectively. However, with the constraints (4.39) and (4.40), block orders b' and c' are linked to block orders b and c. This way, linking block orders b'' and c'' to block orders b and c, respectively, is also ensured.

4.2.2.3 Constraints for Flexible Orders

One can expect that the constraints regarding the decisions of accepting and rejecting the flexible orders will be almost the same as those defined for block orders. In fact, flexible orders are some special kind of block orders, placed to be executed at a single, unspecified hour. However, although the two types are very similar, there are significant differences between the constraints written for the two types of orders. Since flexible orders are not submitted for a specific hour, they can be processed at any hour of the day. This characteristic of the flexible orders requires that price, $p_{fl}(f)$, of each order f is compared to the MCP at every hour of the day. In addition, as the flexible orders can be placed only in the supply direction, there is no need for adding a superscript to denote the direction. Based on these inferences, the conditions for which a flexible order f can be accepted at an hour h are given below.

$$\sum_{h=1}^{24} Accept_{fl}(f,h) \quad \leq \quad 1 \qquad\qquad \text{for } f = 1, ..., F. \tag{4.43}$$

With the above constraint, we ensure that each flexible order is executed only at a single hour or not executed at all.

For $f = 1, ..., F$ and $h = 1, ..., 24$,

$$MCP(h) - p_{fl}(f) \quad \leq \quad M \times Accept_{fl}(f,h) + M \times \big(1 - y_4(f,h)\big). \tag{4.44}$$

where

$$y_4(f,h) = \begin{cases} 0 & \text{if flexible order } f \text{ is paradoxically rejected at hour } h, \\ 1 & \text{if flexible order } f \text{ is either accepted or fairly rejected at hour } h. \end{cases}$$

Constraint (4.44) above is a modified version of (4.33), where the only difference from the supply block orders is that price comparison is done for each hour h, instead of comparing the total price differences. Just like block orders, flexible orders can also be paradoxically rejected; that's why we work with the term $M \times \big(1 - y_4(f,h)\big)$ on the right hand side of (4.44). When a flexible order f is rejected at hour h even if its price is lower than $MCP(h)$ (paradoxically rejected), we have $y_4(f,h) = 0$, making (4.44) redundant. Two constraints below are very similar to their counterparts given for block orders; making sure of the correct functioning of $y_4(f,h)$ for all cases a flexible order f is accepted and *paradoxically* or *fairly* rejected at hour h.

For $f = 1, ..., F$ and $h = 1, ..., 24$,

$$p_{fl}(f,h) - MCP(h) \leq M \times y_4(f,h), \tag{4.45}$$

$$Accept_{fl}(f,h) \quad \leq \quad y_4(f,h). \tag{4.46}$$

The rejection inequality of flexible orders is similar to the constraint (4.34) written for supply block orders as seen below.

For $f = 1, ..., F$ and $h = 1, ..., 24$,

$$p_{fl}(f) - MCP(h) \quad \leq \quad M \times \big(1 - Accept_{fl}(f,h)\big). \tag{4.47}$$

4.2.2.4 Constraint for Hourly Market Balance

Up to this point, the conditions that define and limit the price ranges for which hourly, block and flexible orders are partially and fully accepted or rejected are discussed. However, the decision of whether or not accepting an order, or how much of it to accept, depends not only on the prices corresponding to that order but also the decision made about other orders submitted in the market. In order for the Turkish Electricity DAM, in fact for any competitive market, to be feasible, the total supplied amount must be equal to the total quantity demanded. Therefore, we write the following constraint about the feasibility of the order couplings so that the supply at every hour is not short of the demand.

$$\sum_{i=1}^{I} hour^D(i,h) \times q_{hr}^D(i) \times Accept_{hr}^D(i) + \sum_{j=1}^{J} hour^S(j,h) \times q_{hr}^S(j) \times Accept_{hr}^S(j)$$
$$+ \sum_{b=1}^{B} hours^D(b,h) \times q_{bl}^D(b) \times Accept_{bl}^D(b) + \sum_{c=1}^{C} hours^S(c,h) \times q_{bl}^S(c) \times Accept_{bl}^S(c)$$
$$+ \sum_{f=1}^{F} q_{fl}(f) \times Accept_{fl}(f,h) = 0 \qquad \text{for } h = 1, ..., 24. \tag{4.48}$$

In the constraint (4.48) above, the quantity of orders of any type that is accepted at a certain hour h is summed up. Note that the quantities are included in the inequality together with their sign without their absolute value taken. Since demand order quantities have positive sign and supply order quantities have negative sign, the total on the left hand side must be equal to zero to have a balance of supply and demand.

4.2.3 The Objective Function

All of the constraints given in Section 4.2.2 serve the purpose of determining which orders to accept and which ones to reject so as to balance or clear the Turkish Electricity DAM. In this section, we formulate the objective function of our mathematical model, which is “Total Economic Welfare”, and clarify how it is constructed and how it works.

As stated several times, the objective function of the mathematical model proposed in this thesis is the total economic welfare of the Turkish Electricity DAM. The objective function consists of three distinct expressions; each of them is included for one type of order in the DAM, namely hourly orders, block orders and flexible orders. First, define the objective function, $TotWel$, as the sum of surpluses of three types of orders:

$$
\begin{aligned}
TotWel = \sum_{i=1}^{I} q_{hr}^{D}(i) \times Accept_{hr}^{D}(i) \times \Bigg(& \frac{p_{hr}^{D}(i,0) + p_{hr}^{D}(i,1)}{2} \\
& + \frac{p_{hr}^{D}(i,0) - p_{hr}^{D}(i,1)}{2} \times \left(1 - Accept_{hr}^{D}(i)\right) \Bigg) \\
& + \sum_{j=1}^{J} q_{hr}^{S}(j) \times Accept_{hr}^{S}(j) \times \Bigg(\frac{p_{hr}^{S}(j,0) + p_{hr}^{S}(j,1)}{2} \\
& + \frac{p_{hr}^{S}(j,0) - p_{hr}^{S}(j,1)}{2} \times \left(1 - Accept_{hr}^{S}(j)\right) \Bigg) \\
& + \sum_{b=1}^{B} \sum_{h=1}^{24} q_{bl}^{D}(b) \times Accept_{bl}^{D}(b) \times p_{bl}^{D}(b) \times hours^{D}(b,h) \\
& + \sum_{c=1}^{C} \sum_{h=1}^{24} q_{bl}^{S}(c) \times Accept_{bl}^{S}(c) \times p_{bl}^{S}(c) \times hours^{S}(c,h) \\
& + \sum_{f=1}^{F} \sum_{h=1}^{24} q_{fl}(f) \times Accept_{fl}(f,h) \times p_{fl}(f)
\end{aligned}
\tag{4.49}
$$

In (4.49), the first two lines give the sum of consumer surplus for the hourly demand orders and the next two lines give the producer surplus of the hourly supply orders. Similarly, the fifth and sixth lines of (4.49) give the consumer surplus and producer surplus of block orders, respectively. The (producer) surplus arising from the flexible orders is calculated by the term in the last line. Next, it is clarified how these formulations give the consumer or producer surplus based on the example hourly orders. The three cases for which the economic surplus or welfare is calculated are described next.

1. **When an hourly order is rejected**, i.e., the MCP is higher than the price $p_{hr}^{D}(i,0)$ at which an hourly demand order i starts to be accepted or lower than the price $p_{hr}^{S}(j,0)$ at which an hourly supply order j starts to be accepted.
   In this case, $MCP(h)$ falls outside the interval between two prices. Therefore, the hourly order is rejected and $Accept_{hr}^{\cdot}(\cdot) = 0$, meaning the whole term in $TotWel$ corresponding to the order under consideration becomes 0, i.e., the

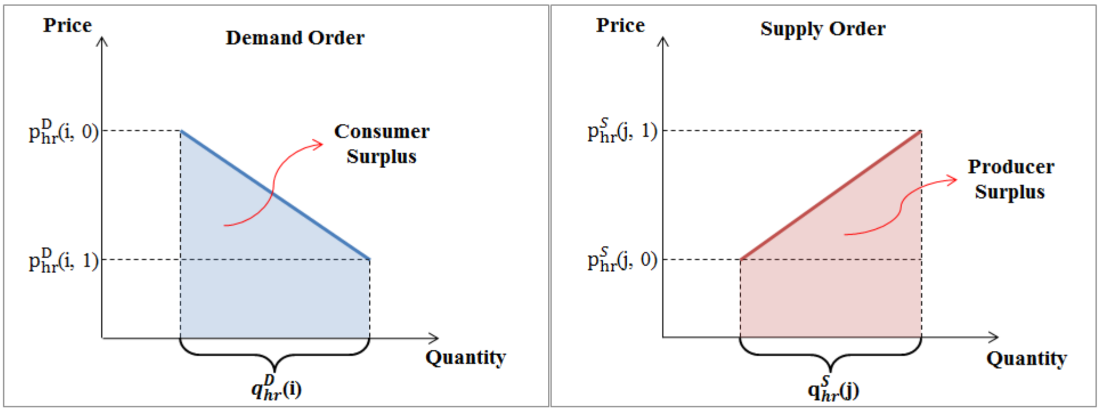

Figure 4.4: The Consumer and Producer Surplus When an Hourly Order is Fully Accepted

surplus is 0.

2. **When an hourly order is fully accepted**, i.e., the MCP is lower than the maximum price $p^D_{hr}(i, 1)$ for which an hourly demand order i is fully accepted or the MCP is higher than the minimum price $p^S_{hr}(j, 1)$ for which an hourly supply order j is fully accepted.
   This case is the opposite of the first item above: $Accept^{\cdot}_{hr}(\cdot) = 1$ and the area below the line that is connecting the two price limits is calculated as the surplus as shown in Figure 4.4 for both demand and supply orders. The formula for the areas of the shaded (blue and red) trapezoids is
$$q^{\cdot}_{hr}(\cdot) \times \frac{p^{\cdot}_{hr}(\cdot, 0) + p^{\cdot}_{hr}(\cdot, 1)}{2}$$
   since the term
$$\frac{p^{\cdot}_{hr}(\cdot, 0) - p^{\cdot}_{hr}(\cdot, 1)}{2} \times \left(1 - Accept^{\cdot}_{hr}(\cdot)\right)$$
   is cancelled out due to $Accept^{\cdot}_{hr}(\cdot) = 1$.

3. **When an hourly order is partially accepted**, i.e., the MCP is between the two price limits, $p^{\cdot}_{hr}(\cdot, 0)$ and $p^{\cdot}_{hr}(\cdot, 1)$.
   When this is the case, whole of the objective function formula in (4.49) that is related to the hourly orders remains valid. Value of the surplus for a given partially accepted hourly order i or j is calculated as follows.
$$Surplus = \left(\frac{p^{\cdot}_{hr}(\cdot, 0) + MCP(h)}{2}\right) \times q^{\cdot}_{hr}(\cdot) \times Accept^{\cdot}_{hr}(\cdot),$$

where

$$MCP(h) = p^{\cdot}_{hr}(\cdot,0) + \Big(p^{\cdot}_{hr}(\cdot,1) - p^{\cdot}_{hr}(\cdot,0)\Big) \times Accept^{\cdot}_{hr}(\cdot)$$

due to (4.27) through (4.30). Therefore,

$$\begin{aligned}
Surplus = &\left(\frac{p^{\cdot}_{hr}(\cdot,0) + p^{\cdot}_{hr}(\cdot,0) + \Big(p^{\cdot}_{hr}(\cdot,1) - p^{\cdot}_{hr}(\cdot,0)\Big) \times Accept^{\cdot}_{hr}(\cdot)}{2}\right) \\
&\times q^{\cdot}_{hr}(\cdot) \times Accept^{\cdot}_{hr}(\cdot) \\
= &\left(\frac{p^{\cdot}_{hr}(\cdot,0) + p^{\cdot}_{hr}(\cdot,0)}{2} + \left(\frac{p^{\cdot}_{hr}(\cdot,0) - p^{\cdot}_{hr}(\cdot,1)}{2}\right) \times \big(1 - Accept^{\cdot}_{hr}(\cdot)\big)\right. \\
&\left. - \frac{\Big(p^{\cdot}_{hr}(\cdot,0) - p^{\cdot}_{hr}(\cdot,1)\Big)}{2}\right) \times q^{\cdot}_{hr}(\cdot) \times Accept^{\cdot}_{hr}(\cdot) \\
= &\left(\frac{p^{\cdot}_{hr}(\cdot,0) + p^{\cdot}_{hr}(\cdot,1)}{2} + \left(\frac{p^{\cdot}_{hr}(\cdot,0) - p^{\cdot}_{hr}(\cdot,1)}{2}\right) \times \big(1 - Accept^{\cdot}_{hr}(\cdot)\big)\right) \\
&\times q^{\cdot}_{hr}(\cdot) \times Accept^{\cdot}_{hr}(\cdot). \qquad (4.50)
\end{aligned}$$

The region that constitutes the consumer and producer surpluses are now smaller shaded (blue and red) trapezoids (respectively) shown in Figure 4.5. The areas are calculated as in (4.50). Since the amount supplied is actually designated with a negative quantity, the producer surplus is mathematically a negative term. Therefore, the total welfare is in fact calculated by subtracting the producer surplus from the consumer surplus (in absolute terms), represented by the shaded triangular area in Figure 4.5.

The calculation of block and flexible order surpluses is rather simple as they only involve the multiplication of the price and the quantity parameters for the accepted orders, $Accept^{\cdot}_{bl}(\cdot) = 1$ or $Accept_{fl}(f,h) = 1$, and it is equal to zero when the block or flexible order in question is rejected, $Accept^{\cdot}_{bl}(\cdot) = 0$ or $Accept_{fl}(f,h) = 0$. The difference between two types is that since block orders are placed for at least four hours, the calculation of surplus includes the multiplication of the term $hours^{\cdot}(\cdot,h)$. For all three types of orders, the quantity parameter is included in the formula with its sign. That is, the calculation is actually "{the total of consumer surplus} − {the total of producer surplus}" as the quantity is positive for demand orders and negative for supply orders. In other words, total welfare is the "{the blue area} − {the red area}"

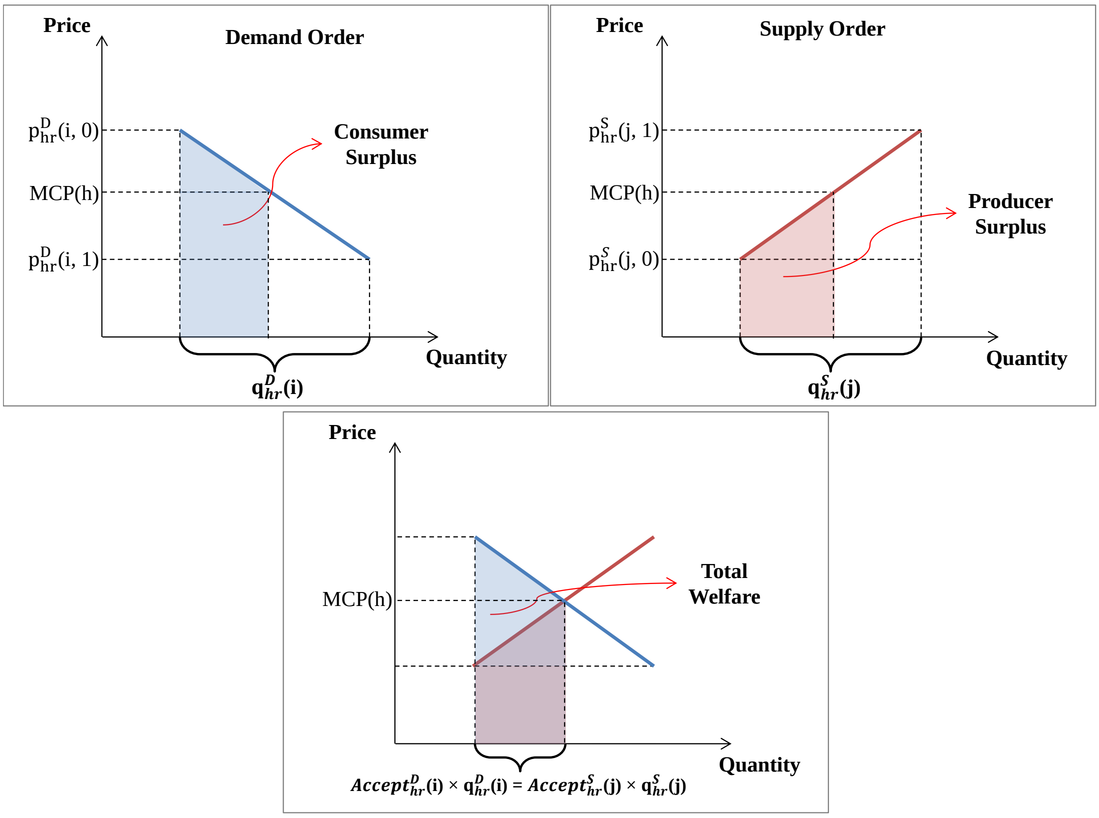

Figure 4.5: The Consumer and Producer Surplus When an Hourly Order is Partially Accepted

as illustrated in Figure 4.5.

4.2.4 Linear Approximation

In the objective function given in Section 4.2.3, the hourly surplus term involves the square of the acceptance decision variables, $\left(Accept^D_{hr}(i)\right)^2$ and $\left(Accept^S_{hr}(j)\right)^2$, which makes the total welfare a quadratic function. In addition, in general our mathematical model includes several binary decision variables, which also affects the solution time of the model. The binary decision variables are inevitably used due to the characteristic of the Turkish Electricity DAM but non-linearity can be avoided by using a slightly different expression in the objective function. For that purpose, we propose a linear approximation to the quadratic calculation of "Total Welfare" function, which is based on the study due to Hazell and Norton (1986).

$$
\begin{aligned}
TotWel' = &\sum_{i=1}^{I} q_{hr}^{D}(i) \times Accept_{hr}^{D}(i) \times \frac{p_{hr}^{D}(i,0) + p_{hr}^{D}(i,1)}{2} \\
&+ \sum_{j=1}^{J} q_{hr}^{S}(j) \times Accept_{hr}^{S}(j) \times \frac{p_{hr}^{S}(j,0) + p_{hr}^{S}(j,1)}{2} \\
&+ \sum_{b=1}^{B} \sum_{h=1}^{24} q_{bl}^{D}(b) \times Accept_{bl}^{D}(b) \times p_{bl}^{D}(b) \times hours^{D}(b,h) \\
&+ \sum_{c=1}^{C} \sum_{h=1}^{24} q_{bl}^{S}(c) \times Accept_{bl}^{S}(c) \times p_{bl}^{S}(c) \times hours^{S}(c,h) \\
&+ \sum_{f=1}^{F} \sum_{h=1}^{24} q_{fl}(f) \times Accept_{fl}(f,h) \times p_{fl}(f) \qquad (4.51)
\end{aligned}
$$

In the above equation, the hourly order surplus terms are linearized while the way block and flexible order surpluses are calculated is not changed, as compared to the original objective function in (4.49).

Recall from Figure 4.5 in Section 4.2.3 that when an hourly order is partially accepted, only a part of the area under the supply or demand curve is factored in the calculated of the objective function. The fact that hourly orders are allowed to be partially accepted makes the objective function non-linear since the rate of acceptance depends directly on the interpolation of MCP between two price limits. How this interpolation affects the calculation of hourly order surplus and makes it non-linear can be seen in (4.50) in Section 4.2.3.

What we do to find a linear approximation to the non-linear objective function is that we simply remove the interpolation term from the equation. Since some hourly orders will be partially accepted with an acceptance rate close to 1 and some will be partially accepted with an acceptance rate close to 0, the approximation would result in losing some and winning some of the original objective function value, depending on the direction of the hourly order. Therefore, the linearized objective would be (hopefully) a good approximation to the actual total welfare. In Figure 4.6, how the total welfare is approximated is shown, based on the example hourly bids given in Tables 2.1 and 2.2 in Chapter 2. It can be observed from the graphic that the quadratic welfare curve is represented by a piecewise linear function, where the welfare value is estimated at distinct price levels and connected at each of two adjacent points with a linear line.

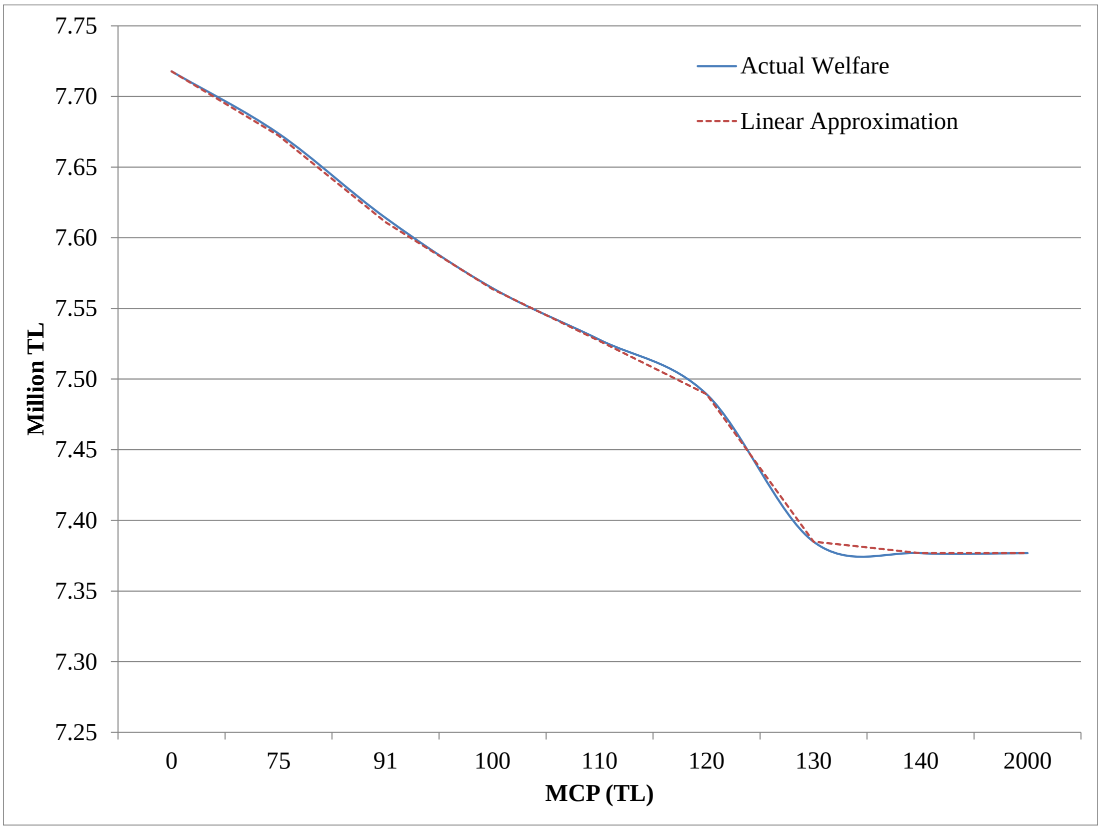

Figure 4.6: Actual Welfare vs. Linear Approximation of Total Welfare at Different MCPs for the Example Hourly Bids

The solution performance of the model that has linearized welfare as its objective value is tested and discussed in Chapter 5.

4.3 Alternative Solution Approaches

In this section, alternative approaches are proposed to solve the clearing problem of Turkish Electricity DAM using the mathematical model provided in Section 4.2.

In most of the real life cases, the DAM clearing problem is infeasible without the presence of paradoxically rejected orders. This is because there does not exist a feasible set of linear prices that maximizes the total welfare due to non-convexity of the welfare function (Derinkuyu, 2015). Therefore, we need to introduce paradoxically rejected orders into our model to reach at least feasibility. Although the concept of paradoxically accepted orders can also be included, we only consider the paradoxical rejection case in our model. In fact, in all of 15 instances, our model did not give any feasible solution when we do not allow paradoxically rejecting block and flexible orders. Only solutions of 3 cases turned out to have no *block* order that is paradoxically rejected (see Table 5.3 in Chapter 5). However, when the concept of paradoxically rejected orders comes into picture, the question of how and to what extent they will be allowed arises. For that purpose, we propose two different solution approaches, which are summarized below.

1. **Iteratively solve the original problem, where the total number of paradoxically rejected block orders allowed is updated at every iteration.** In this bi-criteria approach, we obtain several results to the DAM clearing problem, using the mathematical model given in Section 4.2. To limit the number of paradoxically rejected block orders, we introduce a new constraint: the sum of binary variables $\big(1 - y_3^{\cdot}(\cdot)\big)$ is limited by a certain number, n. $\big($Recall that $y_3^{\cdot}(\cdot)$ makes the acceptance constraints redundant when a block order is a PRB order. Also, recall constraints (4.35) through (4.38) for the use of $y_3^{\cdot}(\cdot)\big)$.

$$\sum_{b=1}^{B} \big(1 - y_3^D(b)\big) + \sum_{c=1}^{C} \big(1 - y_3^S(c)\big) \quad \leq \quad n. \tag{4.52}$$

   Note that we propose this solution approach to deal with the case regarding paradoxically rejected *block* orders only, as flexible orders are submitted for any hour of the day and they are executed at most at one hour. In this case, for instance, accepting a flexible order f at an hour h may cause the same flexible order to be paradoxically rejected at the other 23 hours of the day. Hence, it is difficult to limit the number of paradoxically rejected flexible orders.

   By systematically changing the value of n in (4.52), we allow different number of paradoxically rejected block orders in the problem solution. Each solution delivers a different set of accepted and rejected orders, different MCPs and different objective function values, which is total welfare. It is up to the Market Operator to choose between a solution that maximizes the total welfare while allowing a certain number of PRB orders, a solution that has the minimum possible number of PRB orders, or a solution that is neither of the two extremes. The efficient frontier will provide all the possible alternative solutions to the Market Operator for resolving the trade-off between the number of PRB orders and the total welfare.

The solution algorithm proposed for this approach is given below.

Initialization;
$iter = 0,$ iteration counter;
Let $n = \infty$ and solve the mathematical model for maximizing $TotWel$;
Let $TotWel^{*}$ be the optimal objective value of the problem;
Set $Welfare(iter) = TotWel^{*}$;
Let $n^{*} = \sum_{b=1}^{B} \left(1 - y_3^D(b)\right) + \sum_{c=1}^{C} \left(1 - y_3^S(c)\right)$, number of PRB orders in the optimal solution;
Set $n = n^{*} - 1$;
Introduce upper bound for Total Welfare; $TotWel \leq Welfare(iter)$;
while $n \neq 0$ *and the solution is not infeasible* **do**
  $iter = iter + 1$;
  Solve the mathematical model for maximizing $TotWel$;
  $Welfare(iter) = TotWel^{*}$;
  Calculate $n^{*} = \sum_{b=1}^{B} \left(1 - y_3^D(b)\right) + \sum_{c=1}^{C} \left(1 - y_3^S(c)\right)$;
  Update $n = n^{*} - 1$;
  $TotWel \leq Welfare(iter)$;
end

Algorithm 1: Solution algorithm

We first solve the model without limiting the number of PRB orders allowed, and we obtain the maximum number of PRB orders that maximizes the total welfare. Next, we start the algorithm by making n equal to obtained number of PRB orders in the solved model, minus one. At each step of the solution algorithm, we solve the model and decrease the limit on the allowed number of PRB orders, until we reach 0 PRB orders or get an infeasible solution.

2. **First minimize the number of paradoxically rejected orders, then maximize the total welfare.** With this two-step solution approach, the model would try to find the solutions with *the minimum number of paradoxically rejected*

block orders.

At the first step, the original problem with all the constraints given in Section 4.2 is solved. Only difference is that the objective function is not the maximization of total welfare but the minimization of total number of PRB orders. For this purpose, the left-hand side of (4.52) is defined as the objective function equation. Again, as in the first solution approach, we do not calculate or optimize the number of paradoxically rejected *flexible* orders.

After finding the minimum number of PRB orders, the outcome of the optimal objective value in the first step is input to the model provided in the previous solution approach. The model, which includes (4.49) as the objective function (Max. Total Welfare) is solved while the number of PRB orders is limited by (4.52). In this constraint, the value of n on the right-hand side is equal to the optimal objective value of the first step in this solution approach.

The solution method described in the first bi-criteria approach is applied and numerical results for 15 days of data are provided in Chapter 5. The algorithm is tested using both the original non-linear model and the linear approximation of the objective function. The second two-step approach is also applied to the same data set, the solution times and the model outcomes are compared to the first approach.

CHAPTER 5

COMPUTATIONAL RESULTS

In this chapter, the results of the numerical experiments performed to test the mathematical model and the solution algorithm given in Chapter 4 are given. First, the procedure we followed to generate the date set is described. Later in Section 5.2, experimental results of our solution approaches, using the data generated are provided.

In order to perform numerical experiments to test our mathematical model and the proposed solution methods, we need market data for the orders placed in the Turkish DAM user portal. However, as this kind of information is classified as a trade secret, it is not publicly or exclusively shared by TEIAS. Therefore, we need to generate data set of our own. For that purpose, we generate a 15-day sample data set for hourly, block and flexible bids. We base our data generation method on probability distributions when producing bid prices and quantities. Regarding the number of bids submitted to the DAM, we make use of the information provided by Derinkuyu (2015) who somehow managed to obtain and use real market data. In his paper, the average number of hourly, block and flexible bids submitted in a day as well as the number of price levels used in the hourly bids are provided. We also try to bear in mind the publicly available information about the past electricity prices on the PMUM website (2015).

On the average, each 24-hour real data consists of around 7,500 hourly bids having approximately 30,000 price levels (Derinkuyu, 2015). Recall from Chapter 4 that we convert hourly bids submitted to the Turkish DAM to hourly orders in such a way that each price level of each bid defines an hourly order. In other words, 7,500 hourly bids with a total of 30,000 price levels mean 30,000 hourly orders to be included in our model. However, we assume between 6,000 and 7,000 of them are "dummy" orders, i.e. orders with 0 MWh of quantity. This assumption stems from our estimation that a proportion of the price levels in the hourly bids will not correspond to a quantity change. As for the block bids, which are directly incorporated into our model as block orders, there are on average 150 of them, including a few (up to 14) linked block orders. Flexible orders (bids) are much more rarely observed in the Turkish DAM. Hence, we have one or two flexible orders on each day of our data set.

The next section is devoted to the data generation scheme we develop and implement to generate bidding data. In Section 5.2, the numerical results of every instance, including the equilibrium quantity and MCP at each hour, and the resulting welfare value as well as the number of paradoxically rejected orders (if any) are provided for both solution approaches proposed in Section 4.3. Optimality and the processing times of the model are also discussed in that section.

5.1 Data Generation Scheme

As stated, we need to generate sample market data in order to test our model's performance due to the confidentiality of the PMUM data and strict secrecy of TEIAS. We develop a generic procedure to generate hourly, block and flexible bids. The parameters used in the generation procedure can be different for all three types of bids, yet the method would be essentially the same. Below is the summary of our method, first for hourly bids to demonstrate the way hourly bid data is generated. Next, the process of generating block bid data is described, in representation of both block and flexible bids.

The steps below are followed when hourly bids are generated.

1. **Generate unique bid prices**

    i. *Determine the probability distribution of hourly bid prices to be generated.*

    ii. *Generate a certain number of prices.*

    iii. *Round all values to the nearest integer.*

The price levels used in the hourly bids are mostly integer. Exceptional cases will be taken into consideration in the coming steps.

iv. *Remove duplicate price values.*
When the prices are rounded, it is very likely that we observe the same price a number of times. With this step, we obtain unique prices.

v. *Sort the prices in ascending order.*
At this point, we have unique integer prices that will be assigned to hourly bids as bid price levels.

2. **Generate constant hourly bids**

i. *Determine the number and probability distribution of constant hourly bid quantities.*
These bids consist of only two price levels, 0 and $2,000$ TL, which are respectively the lower and upper limits for hourly bid prices in the Turkish DAM. These kind of bids are largely observed in practice to the extent we know and they are always processed independent of the MCP.

3. **Generate regular hourly bids**
These bids have more than two price levels and at least two distinct quantity values are attached to different price levels.

i. *Determine the number and probability distribution of the maximum quantity of regular hourly bids.*
This is the maximum amount of electricity that is estimated to be demanded or supplied in an hourly bid.

ii. *With a certain probability, assign a non-zero quantity at the initial price level of the bid; that price level is 0 TL for supply bids and 2,000 TL for demand bids.*
In most cases, an hourly demand bid has 0 MWh at the highest price level, 2,000 TL, and a supply bid has 0 MWh at the lowest price level, 0 TL. A small number of them will have a non-zero quantity, meaning that a certain amount of electricity will be demanded or supplied no matter what

the MCP is.

iii. *Assign the number of discrete quantity jumps in a bid.*
There should be a discrete probability distribution to represent the number of distinct quantities/price levels in a bid.

iv. *Randomly assign price levels to the distinct quantity levels.*
For each quantity jump, we assign a different price level. The prices will be selected and assigned in a descending order for demand bids and an ascending order for supply bids.

v. *With a certain probability, generate an additional price level, which is* 1 *Kuruş (*0.01 *TL) higher or lower than the previous one.*
Generally, the quantity of an hourly bid changes at a price level that is just 0.01 TL higher or lower than the previous price level. The quantity jump will be at this level. For instance, a demand bid quantity is 100 MWh at 100 TL, and it decreases to 90 MWh at 100.01 TL.

vi. *Randomly assign discrete quantity values to the distinct price levels.*
Starting with the maximum bid quantity (generated at Step 2-b-i) at the initial price level, we decrease the bid quantity by a random percentage of the quantity at the next price level. This is done up to the price level 2,000 TL for demand orders, and down to 0 TL for supply orders, where the quantity is either 0 or non-zero, depending on the outcome of Step 3-ii.

The procedure to generate block bids is summarized next.

1. *Determine the number of supply and demand block bids*

2. *Randomly assign the length of the period each block bid spans, which must be between 4 and 24 hours*

3. *Determine the starting period of the block bid based on the length of its validity time*

4. *Determine the probability distribution of price parameter of the block bids*

5. *Determine the probability distribution of the block bid quantity.*

6. *Determine the number of linked block bids and match the bids.*

As the flexible bids are special types of block bids, a similar method to that of block bids can be followed in order to generate flexible bids. Since they are submitted for a single unspecified hour, there is no need for creating the length of period and a starting point. Therefore, only 4^{th} and 5^{th} steps above are applied for each flexible bid.

Using the above procedure, a data set can be generated, based on the sectoral experience, assumptions and preferences/choices of any user who cannot possibly reach real data, just like we did. To use in the numerical experiments to test our mathematical model, we generated a data set of 15 days, using the assumptions listed below:

- The number of bid prices to be generated is 500.
- The bide prices are generated from Normal distribution with $mean = 200$ and $standard\ deviation = 50$.
- The number of constant hourly supply bids and constant hourly demand bids is 75 each at every hour.
- The number of regular hourly supply bids is 100 and the number of regular hourly demand bids is 75 at every hour.
- Constant hourly bid quantities and maximum quantities of regular hourly bids are generated using Normal distribution with $mean = 500$ and $standard\ deviation = 150$.
- The number of price level changes in each hourly bid is determined using the following probability distribution:

$$P(X = x) = \begin{cases} 0.50, & x = 1 \\ 0.20, & x = 2 \\ 0.15, & x = 3 \\ 0.10, & x = 4 \\ 0.05, & x = 5 \end{cases} \tag{5.1}$$

where $P(X = x)$: probability that the number of price level changes will be equal to x,

- With probability $^2/_3$, an additional price level, which is 0.01 TL lower than the next price level, is generated.
- With probability $^1/_3$, an hourly bid is exactly the same as another hourly bid submitted at the previous hour. We expect that the characteristics of bids submitted at consecutive hours to be considerably the same.
- With probability 0.1, the minimum quantity of a regular hourly bid is made non-zero.
- The number of supply and demand block bids is 75 each in five cases, 100 in five cases and 50 in five cases.
- The duration of a block bid is uniformly distributed between 4 and 24.
- The quantity of a block bid is uniformly distributed between 0 and 1000 MWh
- For the sake of achieving feasibility in the experiments, the price of a block bid is generated with the help of mean and standard deviation of the *generated* hourly bids. They are used as the mean and standard deviation of the Normal distribution function used to create block bid prices.
- The number of flexible bids is 1 or 2 in each case, determined randomly.
- The price and quantity of a flexible bid is generated using the same probability distributions as the price and quantity of block bids.

So as to incorporate the generated bid data into our model, we apply the conversion procedure discussed in Section 4.2, to transform the generated hourly bids into hourly orders. We then solve the model using the converted hourly orders, block orders and flexible orders, where the last two types do not need any conversion and such orders can be interchangeably called bids. The results are provided in the next section.

5.2 Experimental Results

In this section, we provide the numerical results of the model obtained by using the generated data set. The outcomes of the first solution approach are provided in Section 5.2.1. Later in Section 5.2.2, the solutions obtained by applying the second proposed solution method are presented. All instances are solved using a 2.7 GHz laptop that has 12 GB RAM.

In Table 5.1 below, detailed breakdown of each daily data set, showing the number of hourly, block, linked block and flexible bids and orders is provided. The number of bids are based on the information provided by Derinkuyu (2015) while the rest of the specifications of the data depend on the assumptions made in Section 5.1. To test the sensitivity of the model and the solution approaches to the size of block bid pool, we increase and decrease the number of block bids/orders in 5 trials for each case.

Table 5.1: Data Used in the Experiments

Trial no.	Number of Hourly Demand Price Levels	Number of Hourly Supply Price Levels	Number of Price Levels with $q_{hr}(\cdot) = 0$	Number of Hourly Demand Orders	Number of Hourly Supply Orders	Number of Demand Block Bids/ Orders	Number of Supply Block Bids/ Orders	Number of Linked Block Pairs	Number of Flexible Bids/ Orders
1	10,181	12,759	6,315	5,469	6,493	75	75	10	2
2	10,101	12,559	6,353	5,432	6,453	75	75	4	2
3	10,437	12,397	6,288	5,544	6,411	75	75	9	2
4	10,408	12,683	6,313	5,505	6,368	75	75	5	2
5	10,409	12,783	6,361	5,541	6,458	75	75	6	2
6	10,266	12,785	6,298	5,513	6,521	100	100	4	2
7	10,267	12,762	6,283	5,569	6,466	100	100	14	2
8	10,425	12,669	6,323	5,587	6,420	100	100	8	2
9	10,211	12,778	6,430	5,459	6,442	100	100	4	2
10	10,291	12,331	6,399	5,513	6,316	100	100	8	2
11	10,360	12,794	6,259	5,533	6,397	50	50	3	2
12	10,272	12,792	6,369	5,494	6,521	50	50	5	1
13	10,196	12,498	6,378	5,470	6,405	50	50	5	2
14	10,306	12,287	6,249	5,449	6,362	50	50	5	2
15	10,278	12,689	6,305	5,520	6,521	50	50	8	2
Avg.	10,294	12,638	6,328	5,507	6,437	75	75	6.53	1.93

We have, on the average, 29,260 hourly *bids* on a one-day data set, where 6,328 of them are ‘zero bids’. This translates into 11,943 hourly *orders*, 5,507 of which are demand orders and 6,437 are supply orders. There are 150 block orders on each day, of which 13.1 are linked block orders (6.5 linked pairs). On the average, each day includes 1.93 flexible orders.

5.2.1 Results of the Bi-criteria Solution Approach

In the experiments presented in this section, the solution algorithm described in Section 4.3 is applied. In each trial, an efficient frontier of solutions, changing in terms of welfare value and the number of PRB orders, is obtained. The Market Operator is free to choose any of the solutions on the frontier. Graphical representation of an efficient frontier is exemplified in Figure 5.1[1]. At every iteration, the number of PRB orders allowed is updated based on the outcome of the previous iteration. It may be observed that the number of PRB orders is reduced by more than one. This points between these numbers are the points where the total welfare does not change while the number of PRB orders decreases one by one. Therefore, we do not need to solve for those points.

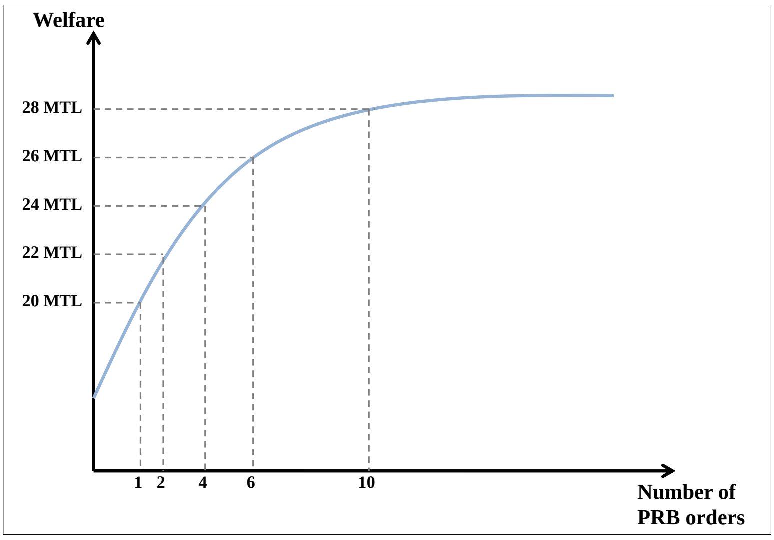

Figure 5.1: Efficient Frontier for Total Welfare vs. Number of Paradoxically Rejected Block Orders

In all of the 15 instances of 24-hour bidding data, the model is able to find the efficient frontier with respect to total welfare and the number of paradoxically rejected

[1] Note that the number of PRB orders shown in the figure may seem unrealistically high; such high volume of PRB orders would not be both beneficial in terms of Total Welfare and fair in the eyes of market participants.

block (PRB) orders within less than one hour, which is actually the time limit of the Market Operator in Turkey. However, since we cannot get hold of real market data from TEIAS or PMUM (EPIAS), we had to make assumptions on probability distributions and amount of bids when generating the data. Although we try to keep the assumptions as simple as possible, it is not likely that the data we generated completely represent the real cases in the Turkish DAM. We expect that the performance of our model and the solution approach will be much better when experimented with real data.

Table 5.2: CPU Times to Solve the Non-linear Problem (seconds)

Trial no.	Iter. no.1	Iter. no.2	Average	Total Time
1	59.77	1.27	30.52	61.04
2	15.48	0.87	8.17	16.35
3	9.57	0.96	5.26	10.53
4	90.97	0.93	45.95	91.91
5	112.77	0.84	56.80	113.61
6	126.48	0.89	63.68	127.37
7	2,803.40	0.97	1,402.18	2,804.36
8	943.86	0.86	472.36	944.71
9	332.75	1.71	167.23	334.46
10	214.85	0.98	107.91	215.83
11	2.90		2.90	2.90
12	6.35	0.92	3.63	7.26
13	5.51	0.96	3.23	6.47
14	2.57		2.57	2.57
15	4.80		4.80	4.80
Avg.	315.47	1.01	158.48	316.28

On the average, it takes the model around 316 seconds (a little over 5 minutes) to find every point on the efficient frontier for a given day. The efficient frontier is derived within the range that is based on the maximum and minimum possible number of PRB orders. In 3 out of 15 instances, the model is able to find a solution that does not involve any paradoxically rejected block orders and all 3 of them are the days having fewer (100 in total) block orders. On the remaining 12 days, we obtain a solution after 2 iterations. On the days having 75 supply and 75 demand block orders, the solution procedure ends in 59 seconds on average, whereas it is only less than 5 seconds for the 50 supply-50 demand case and close to 885 seconds (around 15 minutes) for the 100 supply-100 demand days.

On the other hand, every subproblem (iteration), that is, each individual model solution to find the total welfare *given a certain number of paradoxically rejected block orders is allowed*, lasts a little over 2.5 minutes. The CPU times of each iteration, as well as average and total time to solve the problem is presented in Table 5.2. In 7 instances, the efficient frontier is derived in less than a minute while 1 instance lasts exactly 1 minute and another 1.5 minutes.

Table 5.3: Total Welfare (in thousand TL) and the Number of Paradoxically Rejected Block Orders at Each Iteration of Every Instance

Trial no.	Iter. no.1		Iter. no.2	
	Welfare	PRB	Welfare	PRB
1	109,199.0	6	inf.	5
2	100,209.2	1	inf.	0
3	110,249.8	2	inf.	1
4	111,877.8	2	inf.	1
5	96,865.2	1	inf.	0
6	116,866.7	3	inf.	2
7	103,306.9	8	inf.	7
8	104,700.7	5	inf.	4
9	106,292.5	1	inf.	0
10	108,775.2	2	inf.	1
11	98,118.3	0		
12	111,831.6	1	inf.	0
13	97,616.8	1	inf.	0
14	101,121.3	0		
15	92,584.9	0		
Avg.	104,647.72	2.20		

*inf. : infeasible

The results of the experiments using the aforementioned data sets are given in Table 5.3. Average welfare value is 104.648 million TL and the average number of PRB orders a day is 2.20. In 8 out of 15 cases, we obtain no or only one PRB orders where the maximum number of PRB orders observed in a solution is 8. On 3 days, there are no PRB orders. The detailed list of the outcomes for the number of PRB orders and the Total Welfare values are shown in Table 5.3.

In addition to the original non-linear total welfare model, we also experimented using the linear approximation of the objective function, which is given in Section 4.2.4. The solution times of every iteration are shown in Table 5.4. In 11 out of 15 days, the linear model has reached the time limit of 1 hour before completing the efficient frontier. 3 of those 11 cases have ended without providing any solution at the last iteration. The final iteration of the remaining 8 instances is terminated with at least a feasible solution. The average time to determine all points on the efficient frontier is 2,400 seconds, i.e., 40 minutes. When there are 50 supply and 50 demand block orders, the solution time is 15.5 minutes whereas the 200-block order case are all lasted one hour, terminating with a solution in 3 instances and without one in 2 instances at the last step. The outcome of the linear model on the average-day (150 block orders) trials present a very similar picture where only one instance completely derives the efficient frontier in less than an hour (42.5 minutes). As these results and the ones given in Table 5.2 show, the non-linear model outperforms its linear approximation in terms of solution time in all cases.

Table 5.4: CPU Times to Solve the Linear Approximation Model (seconds)

Trial no.	Iter. no.1	Iter. no.2	Iter. no.3	Iter. no.4	Average	Total Time
1	136.39	(> 1 hr)			1,800.00	3,600.00
2	835.13	(> 1 hr)			1,800.00	3,600.00
3	202.04	(> 1 hr)			1,800.00	3,600.00
4	354.68	(> 1 hr)			-	(> 1 hr)
5	729.59	1203.88	610.32	1.82	636.40	2,545.61
6	195.10	(> 1 hr)			1800.00	3600.00
7	1,526.85	(> 1 hr)			-	(> 1 hr)
8	146.27	(> 1 hr)			1,800.00	3,600.00
9	1,643.17	(> 1 hr)			1,800.00	3,600.00
10	11.75	1,183.66	(> 1 hr)		-	(> 1 hr)
11	179.42	95.47	1.25		92.05	276.14
12	113.39	1.09			57.24	114.48
13	258.60	(> 1 hr)			1,800.00	3,600.00
14	22.97	346.61	31.75	1.93	100.82	403.26
15	20.01	2,586.18	(> 1 hr)		1,200.00	3,600.00
Avg.	425.51	1,988.23	409.28	1.87	1,096.03	2,400.45

*(> 1 hr): reached time limit of 1 hour before completion

For only one case the linear model is able to find a solution with no PRB orders, which overlaps with the non-linear model's results. In total, 8 data sets give the same PRB order outcome in both linear and non-linear models, 5 of them shows exactly the same welfare values for both models at the last step. The differences in the optimal welfare values of three data sets can be ignored as they give no higher than 0.01% improvement in terms of total welfare when the non-linear objective is considered. Moreover, the average welfare of *all* iterations of the linear approximation model is only less than 0.1% smaller compared to the average optimal welfare value of non-linear model. In addition to this slight difference in terms of total welfare, it takes non-linear model to find the optimal efficient frontier 7.5 times less than the linear model. The detailed results of the linear approximation are shown in Table 5.5. It should also be noted that the linear approximation of the total welfare value is on average 13.2% less than the calculated actual welfare.

Table 5.5: Total Welfare (in thousand TL) and the Number of Paradoxically Rejected Block Orders at Each Iteration of Every Instance Using Linear Approximation

Trial no.	Iter. no.1		Iter. no.2		Iter. no.3		Iter. no.4	
	Welfare	PRB	Welfare	PRB	Welfare	PRB	Welfare	PRB
1	109,184.4	7	109,197.0.	6				
2	100,125.5	5	100,130.2	4				
3	110,236.8	4	110,242.8	3				
4	111,619.5	8	(> 1 hr)	7				
5	96,800.5	3	96,858.2	2	96,865.2	1	inf.	0
6	116,833.4	4	116,864.0	3				
7	103,231.6	14	(> 1 hr)	13				
8	104,698.9	5	inf.	4				
9	106,287.8	2	106,292.5	1				
10	108,761.3	3	108,775.2	2	inf.	1		
11	98,064.1	2	98,074.4	1	inf.	0		
12	111,831.6	1	inf.	0				
13	97,343.9	6	97,377.8	5				
14	100,066.3	4	100,078.1	2	100,085.4	1	100,121.3	0
15	92,392.3	8	92,394.7	7	92,435.5	6		
Avg.	104,500.8	5.13	103,316.7	3.43	96,902.9	2.00	101,121.3	0.00

*inf. : infeasible

The resulting equilibrium quantities in *MWh* and the average MCPs in *Turkish Liras* at every hour, denoted by $EqQuant(h)$ and $MCP(h)$, respectively, are presented as the average of 15 non-linear model runs in Table 5.6.

MCP is within the range of 53–167 TL in all cases and the equilibrium quantity is be-

Table 5.6: Average Outcomes of Equilibrium Quantities ($(EqQuant(h))$ in MWh and MCPs $\big(MCP(h)\big)$ in TL

Hour h	1	2	3	4	5	6	7	8	9	10	11	12
$MCP(h)$	147.74	136.65	131.80	128.80	119.92	115.84	106.34	103.32	104.78	101.99	102.45	98.59
$EqQuant(h)$	61,901	63,521	64,023	65,864	67,235	67,281	68,080	68,693	68,946	69,202	69,117	69,088

Hour h	13	14	15	16	17	18	19	20	21	22	23	24
$MCP(h)$	103.27	105.93	105.92	109.86	107.79	110.74	117.97	123.45	126.16	131.22	139.68	143.39
$EqQuant(h)$	68,634	68,415	68,175	68,204	67,966	67,898	67,639	64,941	64,787	64,196	63,926	61,860

tween 57.5 and 73.6 GWh. As the results show, the average MCP (of all hours of all runs) is slightly lower than the observed clearing prices in Turkey (117.65 TL compared to 167 TL, the average in 2014, published on PMUM website[2], 2015). This may be an indicator that we used lower bid prices. On one hand, the objective of the market operator is to keep the prices at minimum (Derinkuyu, 2015) whereas we approach the problem from a welfare maximizing perspective. Although the Turkish Market Operator prefers to seek minimum MCPs using a heuristic solution method, we believe that the allocation of bids to have the maximum total welfare is economically more meaningful. Furthermore, it is expected that solving the welfare maximization problem will be computationally easier than the minimum-price model as the balance of supply and demand naturally gives, more or less, the maximum welfare solution (see Figure 4.1 in Chapter 4.1). In that sense, we present a set of alternative solutions to the existing ones, providing an efficient frontier of solutions with respect to total welfare and the number of paradoxically rejected block orders allowed, which is presented in Table 5.3. On the other hand, we may obtain higher prices if we modify the bid prices generated using the data generation scheme presented in Section 5.1. Nevertheless, we aim to represent the real life instances (in terms of MCPs and the number of bids submitted) as much as possible, with the simplest assumptions that need to be made. In all cases, we obtain the optimal solution to the market clearing problem of Turkish Electricity DAM within the time limit of 1 hour.

5.2.2 Results of Minimum PRB Solution Approach

In this section, we present and discuss the outcome of the second solution approach given in Section 4.3. In this method, we first solve the MIP whose objective function

[2] https://rapor.pmum.gov.tr/rapor/xhtml/ptfSmfListeleme.xhtml

is the minimization of total number of PRB orders where constraints are the constraints to the original model described in Section 4.2.2. Later, we restrict the number of PRB orders in the welfare maximization model, for which we use the optimal objective value of the first subproblem. A summary of the results obtained by applying the second solution approach are provided in Table 5.7.

Table 5.7: CPU Times to Solve the Minimum PRB Order Problem and the Solution Status

Trial no.	Time To Solve the MIP (sec)	Time to Solve the MINLP (sec)	Total Time (sec)	Solution Found
1	3,606.66	8.62	3,615.28	Optimal
2	2,092.80	15.93	2,108.72	Optimal
3	3,601.54	7.94	3,609.48	Optimal
4	3,602.01	98.62	3,700.63	Optimal
5	134.31	165.47	299.78	Optimal
6	-	-	-	no solution
7	3,605.67	1,053.56	4,659.24	Optimal
8	3,601.68	32.68	3,634.35	Optimal
9	3,603.02	386.46	3,989.48	Optimal
10	3,619.91	38.67	3,658.57	Optimal
11	15.76	2.5	18.26	Optimal
12	66.74	6.34	73.08	Optimal
13	700.36	3.95	704.32	Optimal
14	21.31	2.09	23.4	Optimal
15	45.80	2.12	47.92	Optimal
Avg.	2,022.68	130.35	2,153.04	

In 14 out of 15 instances, this solution approach is able to find the optimal solution, which is also obtained by the first solution approach as presented in Table 5.3. In one of the 200-block order instances (6^{th} instance), the model could not return any feasible solution to the first MIP. Hence, the MINLP in the second step is skipped and the outcome of the solution approach for this instance is labeled as ''no solution".

The average time to find the optimal solution by solving the two subproblems is close to 36 minutes. Note that only the original, non-linear objective function is used in this approach, as it is observed in Section 5.2.1 that the linear approximation model takes much longer in terms of CPU time of both individual iterations and complete derivation of the efficient frontier. In fact, the average solution time of the MINLP step of this approach is a little longer than 2 minutes, which is slightly lower than the average solution time of the non-linear model iterations presented in Table 5.2 (2.5 minutes). However, the real bottleneck of this solution approach is the initial MIP step, where it takes 34 minutes on the average. In 7 instances, the model terminates with only a feasible, non-optimal solution to the MIP after running for an hour. Nevertheless, the optimal welfare maximizing solution with the minimum number of PRB orders is still found within reasonable times. This is because our data set gives the minimum-PRB solution even if no bound is provided for the number of PRB orders (see Table 5.3 in Section 5.2.1). Although this observation may be due to the characteristic of the data we generated, the argument that it takes too long to find the optimal solution to the MIP will still be valid when working with real data.

As all of the above model results obtained by applying both proposed solution approaches show, the non-linear model, which iteratively maximizes welfare given a number of PRB orders is allowed, performs much better in terms of CPU times and optimal welfare values compared to its linear approximation and the second solution approach. Even working with days having higher or lower number of block orders does not close this performance gap between models and solution approaches. Plus, it allows the Market Operator to choose between several efficient solutions with respect to total welfare and the number of PRB orders. Thus, we can say that it is a fast and efficient method to find a set of alternative solutions to the DAM balancing problem, which the Market Operator needs to solve everyday.

CHAPTER 6

CONCLUSION

In this study, a mathematical model and two solution approaches are developed to solve the clearing problem in Turkish Electricity DAM. The aim of the study is to provide an alternative method to the existing market settlement mechanism applied by the Market Operator, PMUM (EPIAS). Our model and the method is the first in the literature to find the optimal solution to the Turkish case, using maximization of total welfare as the objective function. Furthermore, we develop a solution approach to deal with the paradoxically rejected block (PRB) orders, which is unique and has not been studied in this way in any other work that we know of. In fact, our approach is the first in literature to solve the maximum-welfare DAM clearing problem to optimality with PRB orders besides Martin et al. (2014). Other studies mostly find a solution by using heuristic methods.

In the mathematical model, total welfare is calculated as the sum of consumer and producer surplus of all types of bids, namely the hourly, block and flexible bids. In order to numerically work with the model we propose, we first generate sample market data to test the performance of our model and the proposed solution approach. The data generated represent the “bids” placed in the DAM, and we apply a conversion operation so as to include them in our mathematical model as “orders”. Each order has a non-zero quantity and a single order price, with the exception of hourly orders. They have two price levels, which are the prices the hourly order starts to be accepted or rejected.

Since the welfare function is non-linear, one may expect that the problem would be hard, even impossible, to solve, or it would take a long time to solve it. Considering this, we propose a modified, linearized version of the objective function. The linear approximation of the welfare formula is in fact the piecewise linear representation of the quadratic function. However, the performance of the model turns out to be the opposite of this expectation, as the results of our solution approaches show.

As a solution method, we first propose a bi-criteria algorithm that iteratively reduces the number of PRB orders allowed in the solution. By implementing the algorithm, we obtain a set of solutions having different welfare values and different number of PRB orders, which constitutes an efficient frontier of possible solutions that the Market Operator can choose from. With this method, our model is able to produce the set of optimal solutions to the clearing problem within a very short time, 5 minutes on the average. The optimal objective value of the linear approximation model deviates from the actual non-linear welfare value by over 13% on the average and it takes the linear model 7.5 times longer to completely determine the solutions on the efficient frontier, when compared to the performance of the non-linear model.

In addition, another solution approach consisting of two subproblems is proposed, where the minimization of the number of PRB orders is done at the first step. Next, the optimal objective value of this problem is input to the welfare maximization model as the upper limit of the number of PRB orders allowed. This way, we aim to directly find the optimal solution (in terms of total welfare) with the smallest number of PRB orders possible. However, the CPU times of especially the MIP in the first step is too long, even past 1 hour in half of the instances. Moreover, the solutions with minimum number of PRB orders can be found by the first approach in much shorter times. Therefore, this solution method can be discarded as long as there is no significant improvement in solution times of the first step.

Regarding the future work in the area, the solution approach to find the minimum number of PRB orders could be improved to get a result within more reasonable time. One modification could be that the PRB orders are penalized by a sufficiently large coefficient in the objective function. This will result in a feasible solution with the minimum number of PRB orders, and the solution having the maximum number of PRB orders can be obtained by not penalizing the PRB orders at all. This will be a marginal improvement to our second solution approach, as the solution steps will be reduced from two to one, which will have a slightly more complex objective function then our original MINLP. In another method, paradoxically accepted block orders can also be taken into consideration in further studies, to handle the “unfair” outcomes in both acceptance and rejection directions. Furthermore to these approaches, paradoxically rejecting/accepting an order may not be allowed when the difference between the order’s price and the MCP is larger than a certain amount or a certain percentage of MCP.

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APPENDIX A

COSMOS MODEL

A.1 Variables

- Acceptance of an hourly (supply and demand) order:

$$0 \leq Acc_{hr}(i) \leq 1 \qquad \text{for } i = 1, ..., I. \tag{A.1}$$

- Acceptance of a block (supply and block) order:

$$Acc_{bl}(b) \in \{0, 1\} \qquad \text{for } b = 1, ..., B. \tag{A.2}$$

- Energy flow on line l at hour h:

$$Flow(l, h) \geq 0 \qquad \text{for } l = 1, ...L \text{ and } h = 1, ..., 24. \tag{A.3}$$

- Congestion price of capacity of line l at hour h:

$$Price_{ATC}(l, h) \geq 0 \qquad \text{for } l = 1, ..., L \text{ and } h = 1, ..., 24. \tag{A.4}$$

- Market Clearing Price in bidding area (market) m at hour h:

$$MCP(m, h) \geq 0 \qquad \text{for } m = 1, ..., M, \text{ and } h = 1, ..., 24. \tag{A.5}$$

A.2 Parameters

- Quantity of hourly order i:

$$Q_{hr}(i) \leq 0 \text{ for demand,} \quad \text{for } i = 1, ..., I, \tag{A.6}$$

$$Q_{hr}(i) \geq 0 \text{ for supply,} \quad \text{for } i = 1, ..., I. \tag{A.7}$$

- Initial price of hourly order i (price at which hourly order i starts to be accepted): $P_{hr}(i,0)$.

- Final price of hourly order i (price at which hourly order i is fully accepted): $P_{hr}(i,1)$.

$$P_{hr}(i,0) \geq P_{hr}(i,1) \quad \text{for demand,} \qquad \text{for } i = 1, ..., I, \tag{A.8}$$

$$P_{hr}(i,0) \leq P_{hr}(i,1) \quad \text{for supply,} \qquad \text{for } i = 1, ..., I. \tag{A.9}$$

- The hour at which hourly order i is submitted:

$$1 \leq hour(i) \leq 24 \qquad \text{for } i = 1, ..., I. \tag{A.10}$$

- The bidding area at which hourly order i is submitted:

$$area_{hr}(i) \qquad \text{for } i = 1, ..., I. \tag{A.11}$$

- Quantity of block order b at hour h:

$$Q_{bl}(b,h) \leq 0 \text{ for demand,} \qquad \text{for } b = 1, ..., B \text{ and } h = 1, ...24, \tag{A.12}$$

$$Q_{bl}(b,h) \geq 0 \text{ for supply,} \qquad \text{for } b = 1, ..., B \text{ and } h = 1, ...24. \tag{A.13}$$

- Price of block order b:

$$P_{bl}(b) \qquad \text{for } b = 1, ..., B. \tag{A.14}$$

- The hours that block order b spans:

$$hours(b) \qquad \text{for } b = 1, ..., B. \tag{A.15}$$

- The bidding area at which block order b is submitted:

$$area_{bl}(b) \qquad \text{for } B = 1, ..., B. \tag{A.16}$$

- The capacity of ATC (Available Transmission Capacity) line l at hour h:

$$Cap_{ATC}(l,h) \geq 0 \qquad \text{for } l = 1, ..., L \text{ and } h = 1, ..., 24. \tag{A.17}$$

- The bidding area line l originates from

$$from(l) \qquad \text{for } l = 1, ..., L. \tag{A.18}$$

- The bidding area line l leads to

$$to(l) \qquad \text{for } l = 1, ..., L. \tag{A.19}$$

A.3 Market Constraints

- Hourly order i can be accepted only if it is in- or at the money:
  for $i = 1, ..., I$,

$$\begin{aligned} Acc_{hr}(i) > 0 \\ \Rightarrow Q_{hr}(i) \times \Big(MCP\big(area_{hr}(i), hour(i)\big) - P_{hr}(i,0)\Big) \geq 0. \end{aligned} \tag{A.20}$$

- Hourly order i must be rejected only if it is out of the money:
  for $i = 1, ..., I$,

$$\begin{aligned} Q_{hr}(i) \times \Big(P_{hr}(i,0) - MCP\big(area_{hr}(i), hour(i)\big)\Big) > 0 \\ \Rightarrow Acc_{hr}(i) = 0. \end{aligned} \tag{A.21}$$

- Hourly order i is partially accepted only if it is at the money (MCP is between inital and final prices):
  for $i = 1, ..., I$,

$$\begin{aligned} 0 < Acc_{hr}(i) < 1 \\ \Rightarrow MCP\big(area_{hr}(i), hour(i)\big) \\ = P_{hr}(i,0) + \big(P_{hr}(i,1) - P_{hr}(i,0)\big) \times Acc_{hr}(i). \end{aligned} \tag{A.22}$$

- Hourly order i must be fully accepted if it is in the money:
  for $i = 1, ..., I$,

$$\begin{aligned} Q_{hr}(i) \times \Big(P_{hr}(i,1) - MCP\big(area_{hr}(i), hour(i)\big)\Big) < 0 \\ \Rightarrow Acc_{hr}(i) = 1. \end{aligned} \tag{A.23}$$

- Block order b can be accepted only if it is in the money:
  for $b = 1, ..., B$,

$$
\begin{aligned}
& Acc_{bl}(b) = 1 \\
\Rightarrow & \sum_{h \in hours(b)} Q_{bl}(b,h) \times \Big(MCP\big(area_{bl}(b),h\big) - P_{bl}(b)\Big) \geq 0.
\end{aligned}
\tag{A.24}
$$

A.4 Network Constraints

- The energy flow in bidding area m must be in balance at hour h:

$$
\begin{aligned}
& \sum_{\substack{i=1 \\ h \in hour(i) \\ area_{hr}(i)=m}}^{I} Acc_{hr}(i) \times Q_{hr}(i) + \sum_{\substack{b=1 \\ h \in hours(b) \\ area_{bl}(b)=m}}^{B} Acc_{bl}(b) \times Q_{bl}(b,h) \\
& \qquad = \sum_{\substack{l=1 \\ from(l)=m}}^{L} Flow(l,h) - \sum_{\substack{l=1 \\ to(l)=m}}^{L} Flow(l,h).
\end{aligned}
\tag{A.25}
$$

- The energy flow on line l cannot exceed the capacity:

$$
Flow(l,h) \leq Cap_{ATC}(l,h). \tag{A.26}
$$

- Congestion price of an ATC line can be positive only if the line is congested (capacity is fully utilized):

$$
Price_{ATC}(l,h) > 0 \quad \Rightarrow \quad Flow(l,h) = Cap_{ATC}(l,h). \tag{A.27}
$$

- The positive congestion price of an ATC line is equal to the difference between two bidding areas that the line l is connecting:

$$
Price_{ATC}(l,h) = MCP\big(to(l),h\big) - MCP\big(from(l),h\big). \tag{A.28}
$$

A.5 Objective Function

The objective function is the maximization of total welfare, defined as the sum of consumer and producer surplus, plus the congestion revenue.

$$
\begin{aligned}
TotWel = \sum_{i=1}^{I} Q_{hr}(i) \times Acc_{hr}(i) \times \Bigg(& \frac{P_{hr}(i,0) + P_{hr}(i,1)}{2} \\
& + \frac{P_{hr}(i,0) - P_{hr}(i,1)}{2} \times \big(1 - Acc_{hr}(i)\big) \Bigg) \\
& + \sum_{b=1}^{B} \sum_{\substack{h=1 \\ h \in hours(b)}}^{24} Q_{bl}(b,h) \times Acc_{bl}(b) \times P_{bl}(b)
\end{aligned}
\tag{A.29}
$$