AN APPROACH FOR QUALITY CONTROL CHART APPROPRIATENESS EVALUATION BASED ON DESIRABILITY FUNCTIONS

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ABSTRACT

AN APPROACH FOR QUALITY CONTROL CHART APPROPRIATENESS EVALUATION BASED ON DESIRABILITY FUNCTIONS

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Quality control charts are among the oldest and most powerful tools in statistical process control. Several control charts have been developed for specific needs and characteristics of processes. However, their proper implementation requires expert knowledge about statistics and properties of these charts. In this study, an effective approach is developed to evaluate appropriateness of control charts for the process to be monitored and the process owner. This approach can be used to recommend the most appropriate control chart to a novice process owner. The chart evaluation problem is formulated as a multi-criteria decision making problem, and desirability functions are utilized for its solution. The evaluation criteria are identified and the desirability functions are constructed based on knowledge from literature and experts. The approach is developed and tested for commonly used $\bar{X}$-R, $\bar{X}$-S, I-MR, I-MR-R and Exponentially Weighted Moving Average (EWMA) variable control charts; and its parameters are fine-tuned by statistical experimentation and optimization.

Keywords: Quality Control Chart, Expert System, Recommendation System, Statistical Process Control, Desirability Function

ÖZ

KALİTE KONTROL ŞEMALARININ UYGUNLUK DEĞERLENDİRİLMESİ İÇİN ÇEKİCİLİK FONKSİYONUNA DAYALI BİR YAKLAŞIM

Tunç, Sıdıka

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Kalite kontrol şemaları, istatistiksel süreç kontrol çalışmalarında kullanılan en güçlü araçlardan biri olduğu için, literatürde farklı süreç ve veri yapılarına uygun birçok kontrol şeması bulunmaktadır. Fakat kontrol şemalarının doğru şekilde kullanılması, istatistik ve kontrol şemalarının özellikleri hakkında bilgi birikimi gerektirmektedir. Bu çalışmada, kontrol şemalarının izlenecek süreç ve sahibi için ne kadar uygun olduğunu değerlendirecek etkili bir yaklaşım geliştirilmiştir. Bu yaklaşım ilgili konuda yeterince bilgi sahibi olmayan bir karar vericiye en uygun şemayı önerecek bir öneri sisteminde kullanılabilir. Şemaların uygunluk değerlendirmesi problemi, çok amaçlı karar verme problemi olarak formüle edilmiş ve çözüm yöntemi olarak çekicilik fonksiyonları kullanılmıştır. Değerlendirme kriterleri ve çekicilik fonksiyonları literatürdeki bilgilere göre ve uzman görüşünden yararlanılarak oluşturulmuştur. Yaklaşım sıkça kullanılan $\bar{X}$-R, $\bar{X}$-S, I-MR, I-MR-R ve EWMA şemaları için geliştirilmiş ve test edilmiştir; yaklaşımda kullanılan parametrelerin istatistiksel deney tasarımı ve optimizasyon yoluyla ince ayarı yapılmıştır.

Anahtar Kelimeler: Kontrol Şeması, Uzman Sistem, Tavsiye Sistemi, İstatistiksel Süreç Kontrolü, Çekicilik Fonksiyonu

To my mother,

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CHAPTER 1

INTRODUCTION

Control charts are effective tools of statistical process control (SPC), which help industrial organizations to detect special causes that disturb stability of their processes and correct for them. Control charts are applied in the following steps: control chart selection, construction, analysis of patterns, and elimination of special causes, if exist. Choosing a proper control chart has utmost importance for the remainder of the steps, since control charts have some assumptions and properties that considerably affect their performance in process control. Although control chart construction and pattern analysis are supported by many statistical software products, many industrial organizations, especially small or medium sized ones lack expertise required to select the most appropriate charts for their processes. Hence, they may not use these tools in process control correctly and effectively. Statistical software products might need an approach that evaluates appropriateness of control charts to fully support the statistical process control.

For example, in-control average run length (in-control ARL) and out-of-control ARL can be considered as one of the performance measures to evaluate the suitability of a control chart for the data. In-control ARL can be defined as the average number of points plotted before an out-of-control point is plotted even if the process is in control. It is natural to observe an out-of-control point in a control chart even if process is stable because of the common causes of variation in process. Out-of-control ARL can be defined as the average number of points plotted to observe an out-of-control point after an assignable cause is observed in process. In other words, it is the time to detect the change in the process. For a properly selected control chart, in-control ARL should be large and out-of-control ARL should be small. That is, control chart should not give too many false alarms when process is in-control and it should detect the change in process as early as possible. To illustrate, if data obtained from a process is distributed with Gamma with parameters 4 and 1, in-control ARLs for Exponentially Weighted Moving Average (EWMA) and I chart are 259 and 97, respectively, while out-of-control ARLs for 1 standard deviation shift for EWMA and I chart are 9.6 and 15, respectively (Borror et al., 1999). This example shows that distribution of data is not suitable to use I chart if maximum information about process from a control chart is desired.

Results of literature survey show that there exist studies about recommendation systems for control chart selection; however, rule-based approach used in these studies should be improved so that decision process of experts in this problem is better represented. Moreover, it is not easy to answer the questions asked by these recommendation systems if the user does not have enough knowledge about statistics.

In this study, a method is developed which can be used to recommend the most appropriate variable control chart from a set of commonly used ones to a user according to assumptions and properties of the control charts, characteristics of the process, and measurements. It evaluates each control chart in the set for its appropriateness level, and formulates this evaluation problem as a multi-criteria decision making problem. The method has the advantages that information about the process can easily be obtained even from a novice user, and knowledge about control charts and opinions of experts are well represented, even if some are contradicting.

Proposed approach is designed for only $\bar{X}$-R, $\bar{X}$-S, I-MR, I-MR-R and EWMA variables control charts because rule-based approach is suitable for the selection process of attributes control charts. If data obtained from observed process are attribute type, proper control chart can be easily selected by considering data type, which can be nonconformities/defects or nonconforming units/defectives; and sample size used in data, which can be variable or constant. Moreover, it is assumed that data are not auto-correlated, since most of the charts are designed for data in which there is no auto-correlation. Because of this assumption, it is also assumed that there does not exist an engineering process control in the processes observed. Furthermore, this approach is designed for ordinary manufacturing processes. It is also assumed that the user cannot provide the true mean and standard deviation of the quality characteristic; therefore, these are estimated in phase 1.

The remainder of the thesis is organized as follows: in Chapter 2, literature review and background information about control charts, decision support systems, expert systems, recommender systems, classification problems and desirability functions are presented. In Chapter 3, scope of the approach, evaluation method, tests and optimization are explained. In Chapter 4, an illustrative example is provided. In Chapter 5, concluding remarks are given.

CHAPTER 2

LITERATURE REVIEW AND BACKGROUND

For this study, literature is investigated for control charts, decision support systems, expert systems, recommender systems, classification problems and desirability functions.

2.1 Control Charts

Control charts are effective tools of statistical process control (SPC), which help industrial organizations to detect special causes that disturb stability of their processes and correct for them. Montgomery (2009) defined statistical process control as a set of problem-solving tools used to achieve process stability and improve process capability. Among these tools, which are histogram or steam-and-leaf plot, check sheet, Pareto chart, cause and effect diagram, defect concentration diagram, scatter diagram and control chart; control charts can be considered as the most technically developed and informative ones. The first control chart was developed by Walter A. Shewhart in the 1920s.

Regardless of how well a process is designed and operating, there exists a natural variability, which is considered as unavoidable. If only natural variability is observed in a process; in other words, if a process is operating with only chance causes of variation, it is said to be in statistical control.

Sources of variability, which are not part of chance causes, are called as assignable causes of variation. If there exits any assignable causes of variation in a process, it is said to be an out-of-control process. Main purpose of using control charts is to identify whether a process is in control or out-of-control by evaluating signals and patterns observed in a chart. Ensuring that process is in statistical control is important for two main reasons. The first reason is to prevent producing too many defective products or products with many defects. The second reason is that if a process is in statistical control, mean and variance of important quality characteristics can be estimated and one can make prediction about future of the process.

The control chart can be defined as a graphical representation of the quality characteristics measured (Montgomery, 2009). A typical control chart includes a center line (CL), upper control limit (UCL) and lower control limit (LCL), which are calculated using the mean and variance of a quality characteristic. A typical control chart can be seen Figure 1.

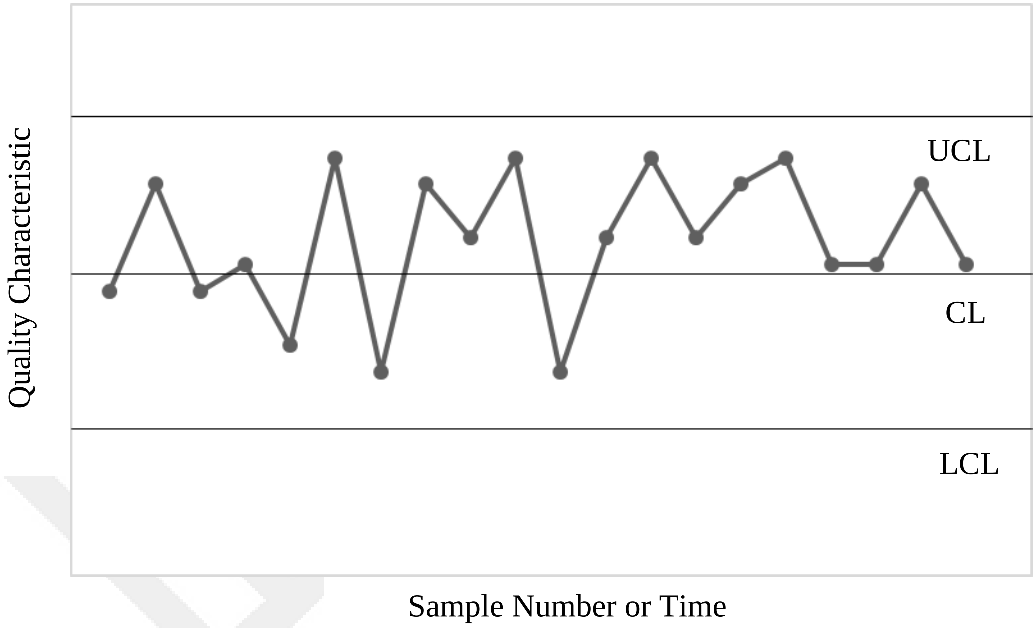

Figure 1 A Typical Control Chart

Control charts are used to understand whether a process is in control or out-of-control. If a control chart gives a signal, which means if a point is plotted beyond UCL or LCL; or if points plotted represent a systematic or nonrandom pattern, it can be concluded that there exist assignable causes of variation. If this is the case, possible causes should be investigated and corrected to maintain a stable process.

Control charts can be classified under two groups, variables and attributes control charts. Variables control charts can be used to monitor quality characteristics, which constitute variable data such as length and volume, while attributes control charts can be used for attribute data such as number of defects and fraction of non-conforming units.

Control charts can be also grouped as univariate and multivariate control charts according to number of quality characteristics observed. Although preliminary control charts are univariate, multivariate control charts started to be developed in 1940s since there emerged a need for simultaneous monitoring of more than one quality characteristics (Bersimis et al., 2005).

On the other hand, there exists a continuous improvement in statistical process control and new charting techniques are being developed. Tsung and Wang (2010) explained the need for adaptive charting techniques and make a review of recent developments in these techniques.

First of all, most of the charts perform well in detecting a specific magnitude of shift. To illustrate, I chart, $\bar{X}$-R chart, and multivariate Hotelling's T^2 chart are good at detecting large shift, whereas EWMA and multivariate Cumulative Sum (CUSUM) perform well in detecting small shifts. Some researchers claim that changing design parameters of conventional charts, it is possible to obtain a control chart which can detect unknown or mixed-range shifts.

Secondly, monitoring more than one quality characteristics has become a more important challenge in statistical process control as industries become more complex. In addition to multivariate Hotelling's T^2, multivariate CUSUM and EWMA, principal component analysis (PCA), partial least square (PLS) and independent component analysis (ICA) based control charts are developed.

Thirdly, mean of the quality characteristics of in control processes is assumed to be constant for most of the control charts. However, drifting trends in mean can be natural because of the properties of a process such as aging of a tool. Adaptive charting techniques can be useful in such situations as well.

Lastly, most of the control charts assume that data obtained from process are not auto-correlated. Furthermore, they assume that there does not exist a feedback controller, which is common in engineering process control. In these cases, dynamic shifts can be observed, which are more difficult to detect compared to sustained shift. To handle dynamic shifts, adaptive charts such as backward CUSUM chart and Dynamic T^2 chart are developed.

In addition to adaptive charting techniques, there are also studies in the literature to deal with the problem of data which consist of linguistic terms. First control chart which uses fuzzy numbers was developed by Alipour and Noorossana (2010), who developed a fuzzy multivariate EWMA chart. Moreover, Gildeh and Shafiee (2015) advanced I-MR chart to cope with auto-correlated fuzzy observations.

Despite the significant improvements in control charts, Shewhart control charts are the keystone of these improvements and they are still commonly used in industries. Shewhart control charts include $\bar{X}$-R chart, $\bar{X}$-S chart and I-MR chart, which are for variable data; and *p* chart, *np* chart, *c* chart and *u* chart, which are for attribute data.

$\bar{X}$-R chart and $\bar{X}$-S chart can be used if subgroup size used in data is greater than 1 while I-MR can be used if subgroup size is equal to 1. If the quality characteristic observed in a process is fraction of nonconforming/defective units or number of nonconforming/defective units, *p* chart or *np* chart can be used, respectively. Furthermore, c chart or u chart can be practiced if the quality characteristic is number of nonconformities/defects in a unit or average number of nonconformities/defects per unit, respectively. Detailed properties and assumptions are given in Chapter 3.

2.2 Decision Support Systems, Expert Systems, Recommender Systems, Classification Problems and Their Use in Control Chart Applications

A decision support system can be defined as information system designed to support the decision maker (Donovan and Jacoby, 1977). An expert system, on the other hand, can be defined as a computer system that uses human expertise in an area in order to perform similar functions performed by a human expert in that area (Malak, 1999). Both of these systems can help to increase effectiveness of decisions made by a user in a specific problem situation or area by the help of information and decision mechanisms they contain.

Recommendation or recommender system can be defined as applications or techniques that make recommendations about users' needs. Recommendation systems first collect data about users' preferences and then use them to make personalized predictions about users' need (Kim and Chen, 2015). Recommendation systems are classified according to the approaches they utilize and there exist three main categories, which are content-based recommendations, collaborative recommendations and hybrid approaches. Content-based recommendations are made based on users' past preferences while collaborative recommendations are made according to what people with similar tastes and preferences liked in the past. Hybrid approaches combine them (Adomavicius and Tuzhilin, 2005).

Many decision support, expert and recommendation systems either classify alternatives or select the best alternative (choice problems). Classification/sorting problems are well stated in the literature by multi-criteria decision making researchers. Both classifications and sorting problems deal with assigning alternatives to predefined groups. While groups are defined in a nominal way in classification, they are defined in an ordinal way in sorting (Zopounidis and Doumpos, 2002).

In the literature, there exist studies about decision support systems or expert systems developed for issues of statistical process control such as control chart selection, implementation and interpretation. The most recent expert system for control chart selection was proposed by Chang and Lee (2013). It consists of two different parts for the users who have different expertise levels. They construct a rule-based approach for novice users and scenario-based approach for experts. Before 1999, there exist many studies (Alexandar and Jagannathan, 1986; Dağlı and Stacey, 1988; Hosni and Elshennaway, 1988; Lall and Stanisloa, 1992; Masud and Thenappan, 1993) which include limited number of control charts and criteria for selection. Malak (1999) examined these studies and developed an improved expert system that covers five alternative variables control charts. As in other studies, Malak (1999) also used a rule-based approach for selecting the most appropriate chart considering nine criteria such as distribution of data, inspection properties, and process characteristics.

Malak (1999) showed that current decision support systems or expert systems can be advanced by introducing new alternative control charts and/or criteria for this selection problem. However, it is also possible to improve decision mechanism that these systems contain.

2.3 An Example to Illustrate Common Structure Used in the Literature

Assume that there exists a decision support system in which there are two alternative charts, $\bar{X}$-R and I-MR, and alternatives are evaluated based on two criteria, namely distribution of data and production size.

The rule-based approach, which is common approach in studies in the literature, consists of following two rules.

1. If data are normally distributed, both charts can be used. If not, $\bar{X}$-R chart can be used but I-MR chart cannot be used.

2. If number of units per production run is greater than equal to 100, both charts can be used. If not, I-MR chart can be used but $\bar{X}$-R chart cannot be used.

These rules constitute the decision tree in Figure 2, which is used in selection process.

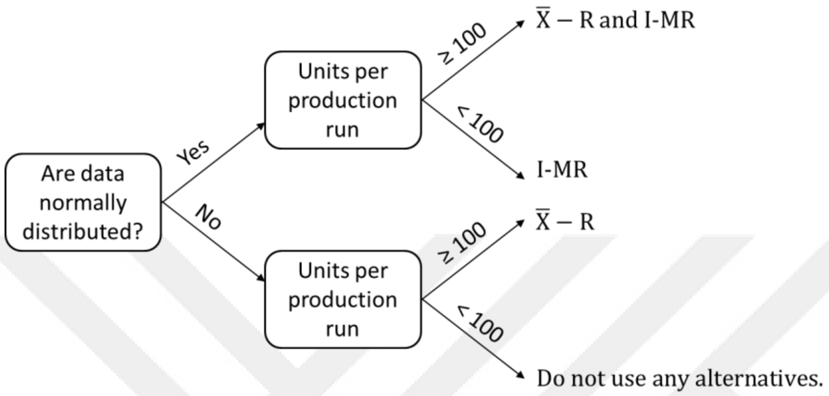

Figure 2 Decision Tree Used in the Selection Process

If there exists a process from which data collected are not normally distributed and in which number of units per production run is 100, $\bar{X}$-R chart can be suggested by the decision support system described. Figure 3 shows the decision tree corresponding to explained scenario.

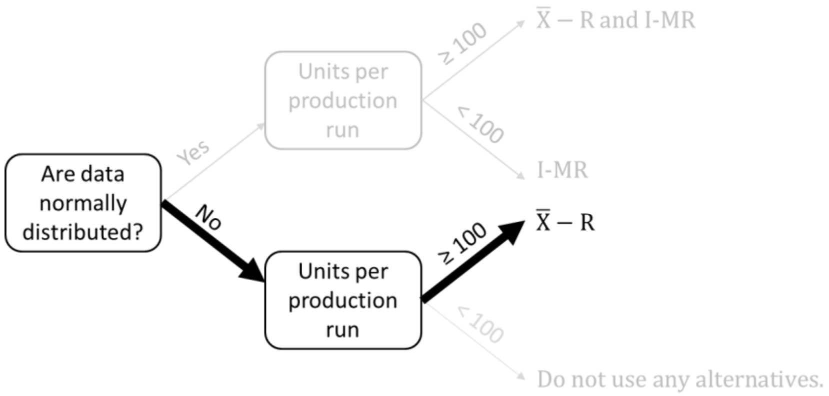

Figure 3 Decision Tree for Scenario 1

If number of units per production run is 99 in this process, this approach cannot suggest any alternatives. Figure 4 shows the decision tree corresponding to this scenario.

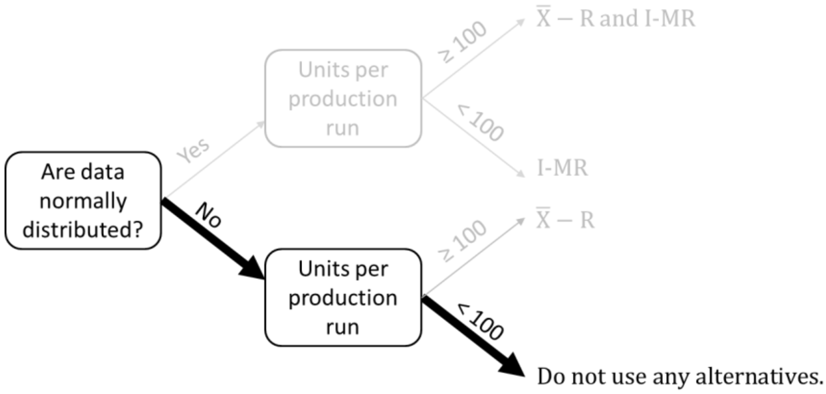

Figure 4 Decision Tree for Scenario 2

These two example scenarios show that small changes in properties of the observed process can cause a significant change in the suggestion of such systems. The aim of the expert systems is somehow representing the thinking process of the experts in the related area. If one unit decrease in the number of units per production does not change the suggestion of expert in real life, it can be concluded that a rule-based structure is not appropriate for the decision support systems for control chart selection, which includes some ambiguous rules.

2.4 Desirability Functions

Desirability functions are commonly used for multi-objective decision making problems since they can easily convert the multi-objective problem to a problem with only one objective. There exist different desirability functions in the literature; however, Derringer and Suich's (1980) desirability functions are common. The main idea behind the desirability function approach is choosing the alternative that yields the maximum overall desirability. An example desirability function is given in Equation (1).

$$d(x) = \begin{Bmatrix} 0 & \text{if } x \le l \\ \left(\dfrac{x-l}{u-l}\right)^r & \text{if } l \le x \le u \\ 1 & \text{if } x > u \end{Bmatrix} \tag{1}$$

where d is the desirability value, l and u are the maximum and the minimum values that x can take, respectively and r is the shape parameter.

Overall desirability, i.e. desirability index, can be calculated in different ways. The commonly used ones are given in Equations (2), (3), and (4) (Trautmann and Mehnen, 2008).

$$D_j = \prod_i \left(d_{ij}{}^{w_i} \right)^{\frac{1}{\sum_i w_i}} \tag{2}$$

$$D_j = \min_i \left\{ d_{ij} \right\} \tag{3}$$

$$D_j = \frac{1}{\sum_i w_i} \sum_i \left(w_i d_{ij} \right) \tag{4}$$

where D_j is the overall desirability of chart j, d_{ij} is the desirability of chart j in criterion i, and w_i is the weight of criterion i in the final selection for the control chart evaluation problem.

In control chart evaluation problem, desirability value of a chart in each criterion is critical, so calculation of overall desirability value as in Equation (3) is not suitable. Calculation in Equation (2) is more convenient than in Equation (4), in which effect of a criterion can be ignored in some special circumstances. For instance, if a control chart is not desirable according to a criterion, i.e. d_{ij} equals to 0, this chart should not be used because all assumptions and requirements of a chart should be satisfied so that it is recommended. However, if Equation (4) is used to calculate overall desirability value, D_j can still be greater than some threshold value, which means that it is recommended. As a result, for each alternative, the overall desirability is found as the geometric mean of individual desirability values of the alternative, calculated under all the decision criteria separately. The fact that d_{ij} equals to zero means that chart j is not desirable according to criterion i, and the fact that d_{ij} equals to 1 means that chart j is totally desirable according to criterion i.

CHAPTER 3

CONTROL CHART APPROPRIATENESS EVALUATION METHOD

In this study, the main problem can be considered as a classification problem for a given case (decision maker with preferences and facts about process and quality characteristics). Desirability function approach is found appropriate to solve this classification problem and to decrease the sharpness of the rule-based approach. According to this approach, first, alternative charts and chart evaluation criteria are determined. Then, for each and every criterion, an appropriate individual desirability function is defined. In order to determine parameters of these desirability functions, a set of process control scenarios are generated using statistical design of experiments. For each scenario, opinions of a panel of experts are obtained in terms of appropriateness of the alternative control charts. Then, parameters of the desirability functions are found as a solution of an optimization problem that aims to minimize differences between choices of the experts and those indicated by overall desirability values of the charts over the scenarios. The parameters are tested with new scenarios, and further fine-tuned by optimization until satisfactory results are obtained. Details of the approach are presented in the following subsections.

3.1 Alternative Control Charts

In this study, $\bar{X}$-R, $\bar{X}$-S, I-MR, I-MR-R, and EWMA charts are considered as the alternatives. They are commonly used in practice for continuous type of data, and supported by many statistical software products such as Minitab. There is a vast amount of literature about these charts providing detailed information summarized by studies such as Montgomery (2009) and Devor et al. (1992). Some important characteristics of these charts can be listed as follows:

1. $\bar{X}$-R charts are easy to implement, and they do not have demanding assumptions.
2. $\bar{X}$-S charts outperform $\bar{X}$-R charts when subgroup sample size is relatively large.
3. I-MR charts do not require a lot of data.
4. I-MR-R charts outperform the others when there exists variation both within and between subgroups.

5. EWMA chart can be implemented for many types of distributions, they can detect small shifts in the process mean sooner than the others, and they are suitable for short run processes.

3.2 Criteria for Selection

Criteria for selection of the most appropriate chart are stated as the following by the literature including Burr (1976), Grant and Leavenworth (1980), Hradeskey (1995), and Malak (1999): Distribution of data, user skill, production size, inspection properties, and shift size to be detected. In addition to these criteria, Montgomery (2009) suggests that the reason for improvement, other process characteristics and aim of controlling quality should also be considered in selecting the most appropriate control chart. Moreover, available time for chart construction should be taken into account, since some charts require longer time due to the amount of data required. In addition, type of variation in data, which can be between or within subgroups or both, has an impact on the selection.

Therefore, following criteria are chosen to evaluate the control charts, $\bar{X}$-R, $\bar{X}$-S, I-MR, I-MR-R and EWMA.

1. Distribution of data
2. Reason for improvement in process
3. Type of variability in data
4. Aim of quality control
5. Shift size to be monitored in quality characteristic
6. Production size
7. Inspection properties
8. Available time for chart construction

3.2.1 Distribution of Data

Distribution of data has an important impact on the appropriateness of a control chart for use. For this criterion, the following facts are taken into account.

- Non-normality of data is not a serious problem if sample size is greater than 1 and $\bar{X}$ chart is used to monitor the mean (Borror et al., 1999).

- Burr (1967) stated that the common normal theory control limit constants are robust to the distribution of data and only severe deviation from normal distribution may affect them. Shilling and Nelson (1976), Chan, Hapuarachchi and Macpherson (1988)'s studies show that if sample size is 4 or 5, $\bar{X}$ chart is robust to normality assumption (as cited by Montgomery, 2009).
- It can be also concluded that I and MR charts in I-MR-R chart are robust to normality assumption since I chart consists of the average values of the samples like $\bar{X}$ chart and MR chart consists of the differences of I values, which can be considered as normally distributed since they are the averages.
- Montgomery (2009) claimed that if the sampling distribution is not symmetric, using three-sigma control limits on the R chart will not produce an α-risk of 0.0027 even when the sampling distribution is normal. Therefore, R chart is more sensitive to departures from normality.
- Similar to R, departure from normality and symmetric distribution affect the S chart (He and Grigoryan, 2003).
- Borror et al. (1999) showed that if data is not normally distributed, control limits of I chart become very tight and in-control ARL decreases significantly.
- They also showed that EWMA chart is robust to normality assumption if proper parameters, which are $\lambda = 0.05$ and $L = 2.492$, are chosen.

To sum up, I-MR chart requires normality of data, whereas EWMA chart is robust to non-normal data. Moreover, significant departure from normality should be cared when $\bar{X}$-R, $\bar{X}$-S and I-MR-R charts are used.

Alternative control charts can be directly evaluated if the user knows whether data obtained from process are normally distributed or not. However, if not, normality test can be performed and p-value of the data, which is the smallest level of significance that would lead to rejection of the null hypothesis (Montgomery, 2009), can be analyzed to evaluate alternatives. In normality tests, null hypothesis is that data are sampled from a population which follows normal distribution. Minitab suggests that using value of 0.05 for α is common; however; some decision makers choose to use value of 0.10 or 0.15 to increase confidence. Therefore, following functions are chosen to evaluate alternative charts.

If a user gives an information that data are normally distributed, then x becomes “1”. Otherwise, x becomes “0” and desirability values are calculated as shown in Equation 1.1, 1.2 and 1.3.

i : distribution of data
j : alternative control charts

$$d_{ij}(x) = \begin{cases} 1 & \text{if } x = 1 \\ 0.70 & \text{if } x = 0 \end{cases} \text{ for } j\text{: } \bar{X}\text{-R, } \bar{X}\text{-S and I-MR-R} \quad (1.1)$$

$$d_{ij}(x) = \begin{cases} 1 & \text{if } x = 1 \\ 0.10 & \text{if } x = 0 \end{cases} \text{ for } j\text{: I-MR} \quad (1.2)$$

$$d_{ij}(x) = 1 \text{ for } j\text{: EWMA} \quad (1.3)$$

If the data are normally distributed, none of the alternative control charts will be penalized in this criterion. Otherwise, I-MR charts are penalized at most by taking a desirability value of 0.1 in this criterion and X̄-R, X̄-S and I-MR-R charts will take a desirability value of 0.7.

If a user provides a p-value, then it will be used to evaluate the d_{ij} values using Equations 1.4, 1.5 and 1.6.

$$d_{ij}(y) = \frac{1}{\left(1 + 0.3 \times e^{-50y}\right)^{\frac{1}{0.03}}} \text{ for } j\text{: } \bar{X}\text{-R, } \bar{X}\text{-S and I-MR-R} \quad (1.4)$$

$$d_{ij}(y) = \frac{1}{\left(1 + 0.3 \times e^{-50y}\right)^{\frac{1}{0.01}}} \text{ for } j\text{: I-MR} \quad (1.5)$$

where y is the p-value.

$$d_{ij}(y) = 1 \text{ for } j\text{: EWMA} \quad (1.6)$$

To evaluate the p-value for the desirability value in this criterion, Generalized Logistic Function is chosen since effect of increase in p-value between 0.05 and 0.1 is more significant than between 0.1 and 1 on desirability value in this criterion. Therefore, initial points in the function are used to estimate the parameters of the function.

- For X̄-R, X̄-S and I-MR-R charts, $d_{ij}(0.05) = 0.50$, $d_{ij}(0.10) = 0.90$ and $d_{ij}(0.15) = 0.99$ are used.

- For I-MR chart, $d_{1j}(0.05) = 0.10$, $d_{1j}(0.10) = 0.70$ and $d_{1j}(0.15) = 0.99$ are used.

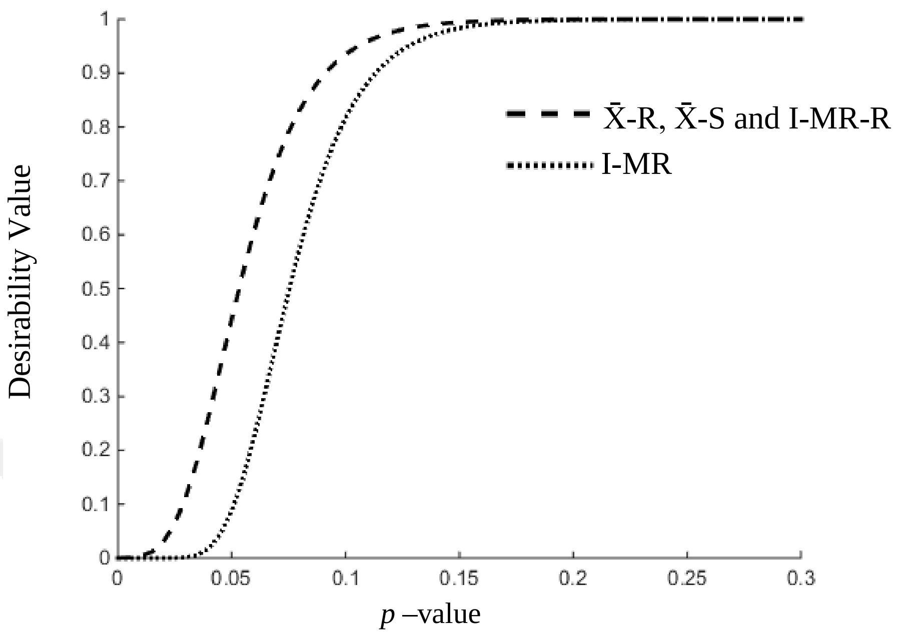

Figure 5 Desirability Functions for Distribution of Data Based on the p-value

3.2.2 Reason for Improvement in Process

Montgomery (2009) described the following situations, in which using X̄-R and X̄-S charts could be more informative than other control charts.

- There exist some problems in an existing process, in which specified tolerances cannot be held.
- The process is analyzed according to number of defects and/or defective rate; however, it is statistically out of control or the yield is unacceptable.
- There exist very narrow specifications in the process.
- The operator should decide whether or not to make adjustments in the process.
- Product specification is desired to change.

Montgomery (2009) also stated that if a new process is observed, using X̄-R and X̄-S charts provides more information. Therefore, it is asked that how these situations are valid for the process observed. The following options are presented and desirability functions for this criterion are determined accordingly using in Equation 2.1 and 2.2.

- All/most of them are valid.

- Some of them are valid.
- None of them is valid.
- It is a new process.

i : reason for improvement in process
j : alternative control charts

$$d_{ij}\left(x\right)=\begin{cases}1 & \text{if } x=1\\ 1 & \text{if } x=0\\ 0.99 & \text{if } x=0.50\\ 1 & \text{if } x=0.25\end{cases} \text{ for } j\text{: X̄-R and X̄-S} \tag{2.1}$$

$$d_{ij}\left(x\right)=\begin{cases}0.90 & \text{if } x=1\\ 0.95 & \text{if } x=0\\ 0.99 & \text{if } x=0.50\\ 0.90 & \text{if } x=0.25\end{cases} \text{ for } j\text{: I-MR, I-MR-R and EWMA} \tag{2.2}$$

$$\text{where } x=\begin{cases}1 & \text{if All/most of them are valid}\\ 0 & \text{if Some of them are valid}\\ 0.5 & \text{if None of them is valid}\\ 0.25 & \text{if It is a new process.}\end{cases}$$

By selecting the desirability functions above, X̄-R and X̄-S charts are favored more than I-MR, I-MR-R and EWMA charts, as Montgomery (2009) suggested.

3.2.3 Type of Variability in Data

In X̄-R and X̄-S charts, there should exist only common causes of variation within subgroups (Devor et al. 1992). Therefore, it is not required to observe variation within subgroups, since it is natural. On the other hand, in some manufacturing processes, assignable causes of variation can occur within subgroups and these causes should be also analyzed to obtain a statistically in-control process. To illustrate, imagine a drilling machine that has 5 heads. In this process, one of the important quality characteristics can be radius of holes in a product. If a subgroup consists of 5 measurements obtained from 5 holes in a product, two measurements from one product can differ because of some assignable causes such as wearing out in one head of the drilling machine.

Among the alternative control charts, only I-MR-R chart can handle such situations; so; whether or not assignable causes of variation within subgroup exist is asked to user and desirability functions shown in Equation 3.1, 3.2 and 3.3 are chosen for this criterion.

i : type of variability in data
j : alternative control charts

$$d_{ij}(x) = \begin{cases} 0.2 & \text{if } x = 1 \\ 1 & \text{if } x = 0 \end{cases} \quad \text{for } j\text{: } \bar{X}\text{-R and } \bar{X}\text{-S} \tag{3.1}$$

$$d_{ij}(x) = \begin{cases} 0.5 & \text{if } x = 1 \\ 1 & \text{if } x = 0 \end{cases} \quad \text{for } j\text{: EWMA and I-MR} \tag{3.2}$$

$$d_{ij}(x) = \begin{cases} 0 & \text{if } x = 1 \\ 1 & \text{if } x = 0 \end{cases} \quad \text{for } j\text{: I-MR-R} \tag{3.3}$$

where $x = \begin{cases} 1 & \text{if variation exists both within and between subgroups} \\ 0 & \text{if variation exists only between subgroups} \end{cases}$

3.2.4 Aim of Quality Control

Montgomery (2009) stated that the purpose of using control charts can also affect which control chart to be used. If inspection is to be minimized when process is in-control or process stability and capability should be continually validated, Montgomery (2009) suggested to use X̄-R and X̄-S charts. Thus, whether or not the user's aim is related to them is asked and the answer is evaluated in desirability functions shown in Equation 4.1 and 4.2.

i : aim of quality control
j : alternative control charts

$$d_{ij}(x) = \begin{cases} 1 & \text{if } x = 1 \\ 1 & \text{if } x = 0 \\ 0.99 & \text{if } x = 0.5 \end{cases} \quad \text{for } j\text{: } \bar{X}\text{-R and } \bar{X}\text{-S} \tag{4.1}$$

$$d_{ij}(x) = \begin{Bmatrix} 0.90 & \text{if } x = 1 \\ 0.95 & \text{if } x = 0 \\ 0.99 & \text{if } x = 0.5 \end{Bmatrix} \text{ for } j\text{: EWMA,I-MR, and I-MR-R} \tag{4.2}$$

$$\text{where } x = \begin{Bmatrix} 1 & \text{if two purposes are valid} \\ 0 & \text{if one of them is valid} \\ 0.5 & \text{if none of them is valid} \end{Bmatrix}$$

3.2.5 Shift Size to Be Monitored in the Quality Characteristic

$\bar{X}$-R, $\bar{X}$-S, I-MR and I-MR-R charts are good at detecting large shifts, whereas EWMA is good at detecting both small and large shifts according to out-of-control average run length (ARL) performance measure. Borror et al. (1999) compared the performance of EWMA and I chart with various shift sizes and distributions by analyzing out-of-control ARLs, which are given in Table 1. Montgomery (2009) suggested a rule of thumb which states that it is better to use smaller values of λ to detect smaller shifts; moreover, Table 1 shows that EWMA chart with $\lambda = 0.05$ and $L = 2.492$ can quickly detect small shifts, which are less than or equal to 1.5σ. Table 1 also shows that EWMA chart performs well in detecting the large shifts like I chart.

Table 1 Out-of-Control ARL's for Various Shifts and Distributions (Borror et al., 1999)

Control Charts	**Distribution**	**Shift (Number of Standard Deviations)**					
		0.5	**1.0**	**1.5**	**2.0**	**2.5**	**3.0**
EWMA $\lambda = 0.05$ $L = 2.492$	Normal	26.5	10.8	6.8	5.0	4.0	3.4
	GAM (4,1)	26.4	11.0	6.9	5.1	4.1	3.4
	GAM (3,1)	26.4	11.0	7.0	5.1	4.1	3.5
	GAM (2,1)	26.4	11.1	7.0	5.2	4.1	3.5
	GAM (1,1)	26.4	11.2	7.1	5.3	4.2	3.5
	GAM (0.5,1)	26.6	11.4	7.3	5.4	4.3	3.6
	t_{50}	26.5	10.8	6.8	5.0	4.0	3.4
	t_{40}	26.0	11.0	6.7	5.0	4.0	3.3
	t_{30}	26.0	11.0	6.7	5.0	4.0	3.3
	t_{20}	26.0	11.0	6.7	5.0	4.0	3.3
	t_{15}	26.0	11.0	6.7	5.0	4.0	3.3
	t_{10}	26.0	11.0	6.7	5.0	4.0	3.3
	t_8	25.0	11.0	6.7	5.0	4.0	3.3
	t_6	25.0	11.0	6.7	5.0	4.0	3.3
	t_4	25.0	11.0	6.7	5.0	4.0	3.3
I	Normal	155.2	44.0	15.0	6.3	3.0	2.0
	GAM (4,1)	34.2	15.0	7.7	4.5	3.0	2.2
	GAM (3,1)	31.0	14.0	7.4	4.5	3.0	2.2
	GAM (2,1)	27.0	12.6	7.0	4.4	3.0	2.2
	GAM (1,1)	21.7	11.0	6.4	4.2	3.0	2.3
	GAM (0.5,1)	18.3	9.7	6.0	4.1	3.0	2.4
	t_{50}	137.0	43.0	15.0	6.4	3.3	2.0
	t_{40}	133.0	43.0	15.0	6.4	3.3	2.0
	t_{30}	126.0	42.0	15.0	6.5	3.3	2.0
	t_{20}	115.0	41.0	15.0	6.6	3.3	2.0
	t_{15}	106.0	41.0	16.0	6.7	3.3	2.0
	t_{10}	92.0	40.0	16.0	6.9	3.4	2.0
	t_8	83.0	39.0	16.0	7.1	3.4	2.0
	t_6	73.0	38.0	17.0	7.5	3.6	2.0
	t_4	63.0	38.0	19.0	9.0	4.0	2.0

For normally distributed data, change in out-of-control ARL for I chart can be seen in Figure 6. If the desirability value of I charts for this criterion is considered as 1 when 3σ shift is to be observed, the desirability values should be decreased when smaller shifts are to be observed according to the out-of-control ARL.

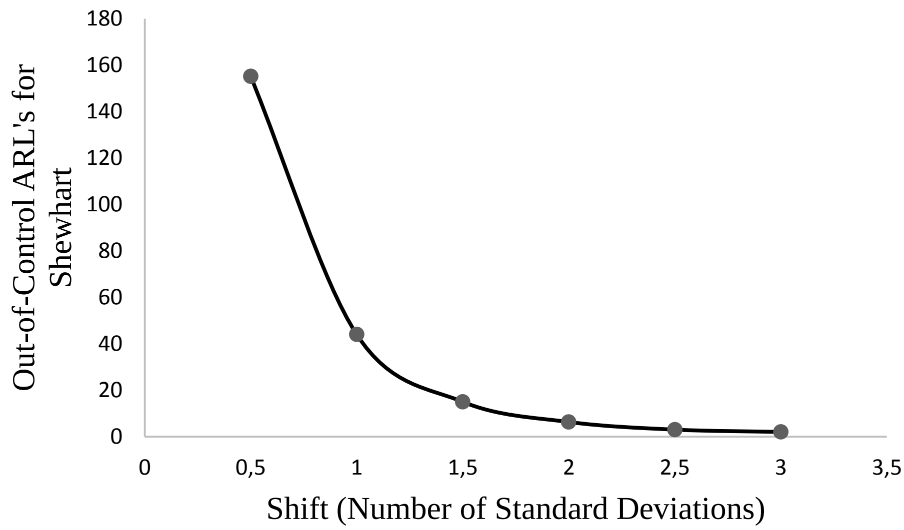

Figure 6 Out-of-Control ARLs for Different Shift Sizes

In addition to ARL, Woodall and Mahmoud (2005) recommended using signal resistance (*SR*), the largest standard deviation of the sample mean from the target value not resulting in an immediate out of control signal, as performance measure to compare control charts in order to take into account inertia, which is defined as a measure of the resistance of a control chart to signaling a particular process shift.

$SR = L\sigma$ when the control limits of $\bar{X}$ chart are $\pm L$

$SR = L\sqrt{\frac{(2-\lambda)}{\lambda}}\sigma$ when the control limits of EWMA chart are $\pm L\sqrt{\frac{\lambda}{(2-\lambda)}}$

Therefore, the signal resistance of I chart with 3-sigma control limits is 3σ and the signal resistance of EWMA chart with parameters $\lambda = 0.05$ and $L = 2.492$ is about 15σ. Since the signal resistance of EWMA chart is larger than that of I chart, it can be concluded that I chart outperform EWMA chart in detecting large shifts although their out-of-control ARL's are close. Since ARLs of $\bar{X}$ chart for various sample sizes follow the similar pattern, desirability functions given in Equations 5.1 and 5.2 for this criterion are developed by using these information.

i : shift size to be monitered
j : alternative control charts

$$x = \frac{k}{\left(\frac{max - min}{4}\right)}$$

$$d_{ij}(x) = \frac{1}{\left(1 + 700 \times e^{-5x}\right)^{\frac{1}{1.2}}} \quad \text{for } j\text{: } \bar{X}\text{-R, } \bar{X}\text{-S , I-MR and I-MR-R} \tag{5.1}$$

$$d_{ij}(x) = \frac{1}{\left(1 + 0.000002 \times e^{10x}\right)^{\frac{1}{5}}} \quad \text{for } j\text{: EWMA} \tag{5.2}$$

where *max* is the maximum measurement, *min* is the minimum measurement, *k* is the minimum amount of deviation to be monitored and *x* is the shift size.

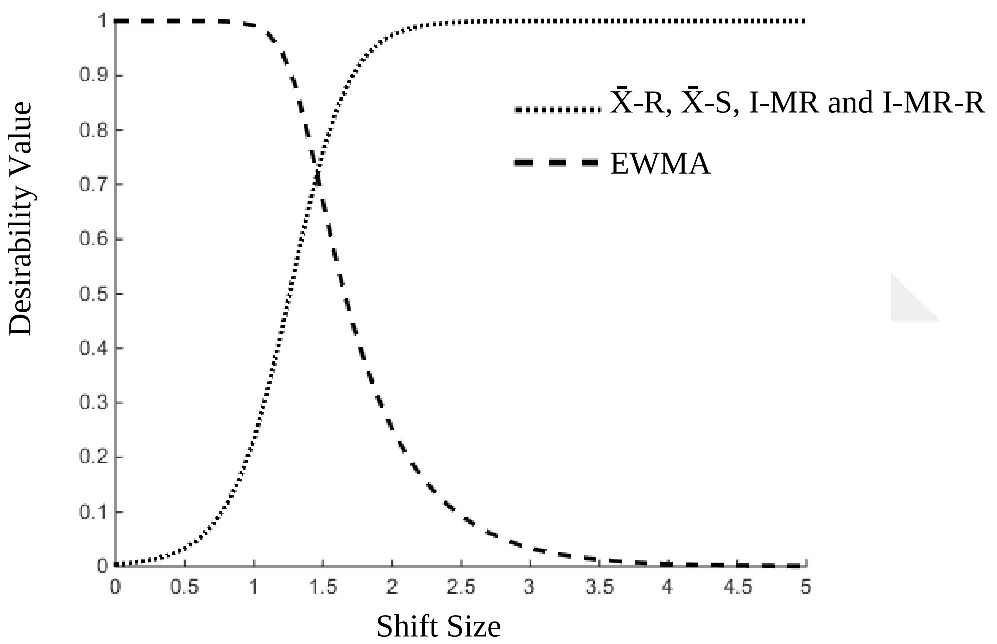

Figure 7 Desirability Functions for Shift Size to be Monitored

3.2.6 Production Size

X̄-R, X̄-S and I-MR-R charts require relatively more data than I-MR and EWMA charts since subgroup size is greater than or equal to 2 in the former group. In other words, X̄-R, X̄-S and I-MR-R charts generally cannot be used for short run processes while I-MR and EWMA charts can be used. It is not easy to determine a threshold value above which X̄-R, X̄-S and I-MR-R charts can be used. Malak (1999) uses threshold value of 100; however, this value may not be appropriate for some cases. For example, if the user wants to observe a process in which 90 data can be collected and subgroup size of 3 is used, $\bar{X}$-R chart can still be suggested for this process because 60 (3*20) data can be used to estimate the mean and standard deviation and remaining 30 data can be used to analyze the process. Therefore, decision tree in Figure 8 and desirability function in Equations 6.1, 6.2, and 6.3 are used to calculate the desirability values in this criterion.

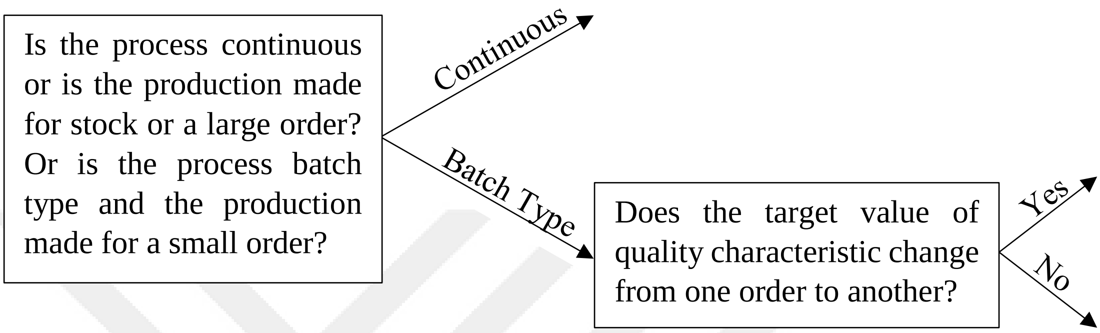

Figure 8 Decision Tree for Production Size

i : production size
j : alternative control charts

$$d_{ij}(x) = \begin{cases} 1 & \text{if } x = 1 \\ 0.2 & \text{if } x = 0 \end{cases} \text{ for } j\text{: } \bar{X}\text{-R, } \bar{X}\text{-S and I-MR-R} \tag{6.1}$$

$$d_{ij}(x) = \begin{cases} 1 & \text{if } x = 1 \\ 0.7 & \text{if } x = 0 \end{cases} \text{ for } j\text{: I-MR} \tag{6.2}$$

$$d_{ij}(x) = 1 \text{ for } j\text{: EWMA} \tag{6.3}$$

where

$$x = \begin{cases} 1 \text{ if the process is continuous or the production is made for stock or a larger order} \\ 1 \text{ if the process is batch type but target value of quality characteristic does not} \\ \quad \text{change from one order to another} \\ 0 \text{ if the process is batch type and target value of quality characteristic changes} \\ \quad \text{from one order to another} \end{cases}$$

3.2.7 Inspection Properties

Inspection properties is an important criterion since it is directly related to the cost of using control charts to improve processes. Since one of the purposes of control charts is to decrease cost of production by decreasing the number of products which do not have required properties, gain obtained from a control chart should be greater than cost of control chart so that statistical process control via control charts is preferred.

There exist some studies in the literature that suggest methods to calculate cost of using a control chart. For example, Dağlı and Stacey (1988) used Duncan's (1956) single assignable cause model to calculate expected loss per hour in a process. However, these methods require lots of information such as time required to find the assignable cause and cost of taking the sample and providing such information can be very difficult for novice users.

That is why, to get information about inspection properties from the user, 3 situations are created and detailed explanation for these situations are provided. The desirability functions, which evaluate these situations, are given in Equations 7.1, 7.2, 7.3, and 7.4.

i : inspection properties
j : alternative control charts

$$d_{ij}(x) = \begin{Bmatrix} 1 & \text{if } x = 1 \\ 0.4 & \text{if } x = 0 \\ 0.1 & \text{if } x = 0.5 \end{Bmatrix} \text{ for } j\text{: } \bar{X}\text{-R and } \bar{X}\text{-S} \qquad (7.1)$$

$$d_{ij}(x) = \begin{Bmatrix} 1 & \text{if } x = 1 \\ 0.8 & \text{if } x = 0 \\ 0.2 & \text{if } x = 0.5 \end{Bmatrix} \text{ for } j\text{: I-MR} \qquad (7.2)$$

$$d_{ij}(x) = \begin{Bmatrix} 1 & \text{if } x = 1 \\ 0.5 & \text{if } x = 0 \\ 0.1 & \text{if } x = 0.5 \end{Bmatrix} \text{ for } j\text{: I-MR-R} \qquad (7.3)$$

$$d_{ij}(x) = \begin{Bmatrix} 1 & \text{if } x = 1 \\ 0.8 & \text{if } x = 0 \\ 0.6 & \text{if } x = 0.5 \end{Bmatrix} \text{ for } j\text{: EWMA} \qquad (7.4)$$

where

$$x = \begin{cases} 1 \text{ if the inspection process is effortless and collecting data is not a problem} \\ 0 \text{ if the inspection process is not easy and collecting data frequently may} \\ \quad \text{cause some problems.} \\ 0.5 \text{ if the inspection process is challenging and collecting data frequently} \\ \quad \text{is not possible.} \end{cases}$$

3.2.8 Available Time for Chart Construction

Before observing the process, mean and standard deviation of the quality characteristic should be estimated in phase 1 for $\bar{X}$-R, $\bar{X}$-S, I-MR and I-MR-R charts since it is assumed that the user cannot provide them directly. On the other hand, the user can start observing the process immediately if the EWMA chart is used. Amount of time required to estimate the mean and standard deviation and start observing depends on how fast data are collected and how many data points are needed. By using these two, it is possible to estimate the chart construction time.

Moreover, the user can have a time constraint about construction. For example, the user may want to start process monitoring in 2 days but he can wait until 4 days. In this situation, if it takes 5 days to construct $\bar{X}$-R chart and 1 day to construct the I-MR chart, the latter can be preferred.

To evaluate the alternative charts in desirability function given in Equation 8.5 according to the required time to construct, which is calculated according to Equation 8.1, 8.2, 8.3, and 8.4, the following questions are asked to the user.

- How much time is required to measure “k” units?
- What is the targeted time for chart construction?
- What is the maximum time for chart construction?

i : available time for chart construction
j : alternative control charts

$x_j = a \times 2 \times 20$ for j: $\bar{X}$-R and $\bar{X}$-S (8.1)

$x_j = a \times 20$ for j: I-MR (8.2)

$$x_j = \frac{a \times 20 \times \max\{k,2\}}{k} \quad \text{for } j\text{: I-MR-R} \tag{8.3}$$

$$x_j = a \quad \text{for } j\text{: EWMA} \tag{8.4}$$

$$d_{ij}\left(x_j\right) = \begin{cases} 1 & \text{if } x_j \le t \\ 0.8 + (1-0.8)\left(\dfrac{x_j - m}{t-m}\right)^{0.25} & \text{if } t < x_j < m \\ (1-0.8)\left(\dfrac{x_j - 1.5m}{m - 1.5m}\right)^{4} & \text{if } x_j \ge m \end{cases} \quad \text{for all } j \tag{8.5}$$

where a is the time required to measure k units, t is the targeted time for chart construction, m is the maximum time for chart construction and x_j is the estimated minimum time for chart *j*.

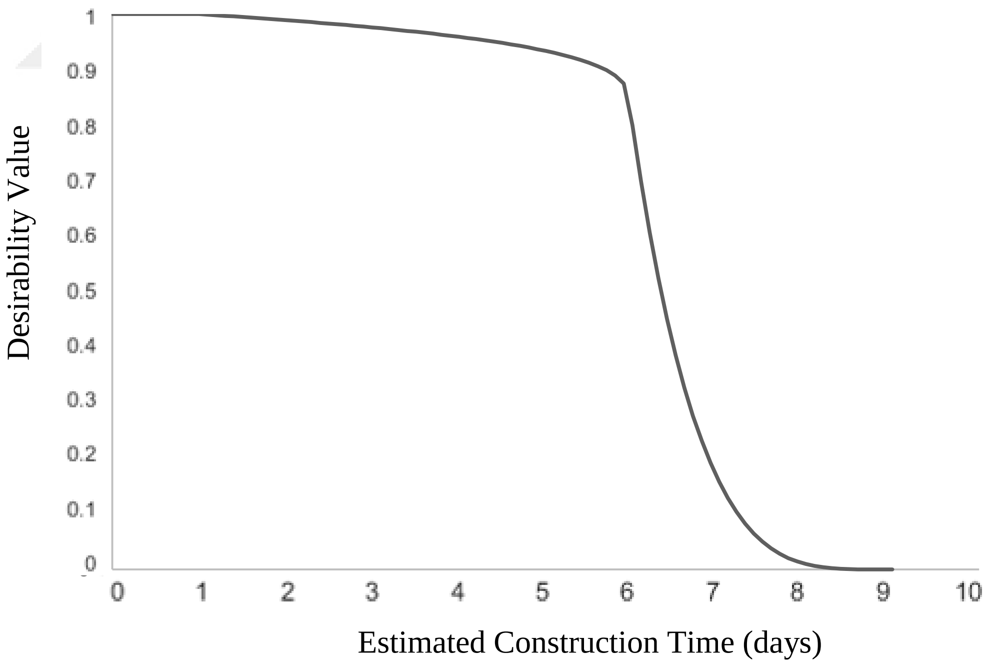

Figure 9 An Example Desirability Function for Available Time for Chart Construction (t= 6 days and m= 9 days)

3.3 Determination of Weights

To calculate the overall desirability value of each alternative, importance of each criterion should be known. Weight of each criterion is determined by Analytical Hierarchy Process (AHP) since criteria are independent from each other and application of AHP is straightforward (Saaty, 2008). In pairwise comparisons, a scale from 1 to 9 is used (given in Table 2) and decisions of three experts in statistical process control are evaluated (given in Table 3).

Table 2 The Fundamental Scale of Absolute Numbers

Intensity of Importance	Definition	Explanation
1	Equal importance	Two criteria contribute equally to the overall desirability
2	Weak or slight	
3	Moderate importance	Experience and judgement slightly favour one criterion over another
4	Moderate plus	
5	Strong importance	Experience and judgement strongly favour one criterion over another
6	Strong plus	
7	Very strong or demonstrated importance	An criterion is favoured very strongly over another
8	Very, very strong	
9	Extreme importance	The evidence favouring one criterion is of the highest possible order of affirmation

Table 3 Pairwise Comparison of Criteria

	1	2	3	4	5	6	7	9
Distribution of data	1	9	1	9	5	7	7	7
Reason for improvement in process	1/9	1	1/7	1	1/5	1/3	1	1/3
Type of variability in data	1	7	1	7	7	5	7	7
Aim of quality control (4)	1/9	1	1/7	1	1/5	1/3	1/5	1/5
Shift size to be monitored	1/5	5	1/7	5	1	3	3	3
Production size	1/7	3	1/5	3	1/3	1	1	1

Inspection properties	1/7	1	1/7	5	1/3	1	1	1
Available time for chart construction	1/7	3	1/7	5	1/3	1	1	1

Each entry in a column is divided by the sum of all rows in corresponding column and then weight of each criterion is obtained by calculating average of all entries in a row, given in Table 4.

Table 4 Weights of Criteria

	1	2	3	4	5	6	7	8
Weight	0.3297	0.0305	0.3183	0.0250	0.1178	0.0593	0.0555	0.0638

3.4 Design of Tests

The chart appropriateness evaluation system utilizes facts from the literature, but also relies on expert knowledge and judgement, which are prone to errors. To verify the approach and correct for possible deficiencies, a number of process control scenarios is generated using statistical design of experiments. Initially, two or three answers are determined for each criterion as factor levels. Fractional factorial designs are preferred for scenario generation, since the total number of scenarios would be very large (4374) if the full factorial design were used. According to the number of factors and their levels, L18 and L36 Orthogonal Arrays (Cavazzuti, 2013) are chosen, which provide 54 scenarios in total, given in Appendix A.

Table 5 Criteria and Their Levels Used in Designs

Criteria	Level 1	Level 2	Level 3
Distribution of data	Normal	Not normal	*p*-value=0.15
Reason for improvement in process	Many of them selected from the list	Some of them selected from the list	None of them selected from the list
Type of variability in data	Within/between variation	Only between variation	-
Aim of quality control	Many of them selected from the list	Some of them selected from the list	None of them selected from the list
Shift size to be monitored in quality characteristic	3σ	2σ	1σ
Production size	Continuous or the production is made for stock or a larger order	Batch type but the target value of quality characteristic does not change	Batch type and the target value of quality characteristic changes
Inspection properties	Effortless and collecting data is not a problem	Not easy and collecting data frequently may cause some problems	Challenging and collecting data frequently is not possible
Available time for chart construction	$t = 40$ $m = 41$ $x_j = 40$ for j : $\overline{\text{X}}$-R, $\overline{\text{X}}$-S and I-MR-R $x_j = 20$ for j : I-MR $x_j = 1$ for j : EWMA	$t = 10$ $m = 30$ $x_j = 40$ for j : $\overline{\text{X}}$-R and $\overline{\text{X}}$-S $x_j = 20$ for j : I-MR and I-MR-R $x_j = 1$ for j : EWMA	$t = 10$ $m = 50$ $x_j = 40$ for j : $\overline{\text{X}}$-R and $\overline{\text{X}}$-S $x_j = 20$ for j : I-MR and I-MR-R $x_j = 1$ for j : EWMA

The experts are asked to evaluate each alternative control chart as a panel for each and every scenario of the L18 design. In the evaluation process, the experts have categorized each control chart as “strongly recommended”, “recommended”, “recommended with reservations”, “not recommended”, and “absolutely not recommended” for each scenario described by the design. In addition, overall desirability of each control chart is calculated for each scenario. To compare the experts’ evaluations and the results of the desirability approach, classifications are also represented numerically. Initial bounds of these classes are also determined by the panel of experts as 1, 0.95, 0.80, 0.60, 0.50 and 0. For instance, if a chart is evaluated as strongly recommended, its overall desirability is expected to be between 1.00 and 0.95 to conclude that the two evaluations agree with each other. A chart under a scenario may fall into a recommendation class but its overall desirability value calculated based on the initial parameters may not agree with the corresponding recommendation.

Table 6 Numerical Equivalent of Categories

Desirability Values	**Categories**
$0.95 < D_j \leq 1$	Strongly recommended
$0.80 < D_j \leq 0.95$	Recommended
$0.60 < D_j \leq 0.80$	Recommended with reservations
$0.50 < D_j \leq 0.60$	Not recommended
$0 \leq D_j \leq 0.50$	Absolutely not recommended

Some disagreements are observed, and the experts are asked to reconsider their evaluations about them. They have changed some of their evaluations. For the others, they have insisted on their answers. Then, the experts are asked to evaluate the second set of scenarios defined by the L36 design in a similar manner. It is observed that there exist 17, 13, 4 and 9 disagreements in 54 scenarios for $\bar{X}$-R / $\bar{X}$-S, I-MR, I-MR-R and EWMA charts, respectively, 5 of them being very large in terms of the amount of deviation.

It is considered that the panel's evaluations can be used to adjust the parameters of the desirability functions by a mathematical model. The idea is to minimize the total amount of deviation of the overall desirability values from upper and lower limits of the corresponding classes by adjusting (or fine-tuning) the function parameters and the weights as well as the lower and upper limits. For this purpose, the following mathematical model is constructed.

3.5 Mathematical Model

Sets

i: criteria (1: Distribution of data, 2: Reason for improvement in process, 3: Type of variability in data, 4: Aim of quality control, 5: Shift size to be monitored, 6: Production size, 7: Inspection properties, 8: Available time for chart construction)

j: alternative control charts $\left(1: \overline{\text{X}}\text{-R}/\overline{\text{X}}\text{-S},\ 2: \text{I-MR},\ 3: \text{I-MR-R},\ 4: \text{EWMA}\right)$

k: scenarios $(1, \ldots, n)$

where n is total number of scenarios considered.

m: classes $\left(1: \text{ absolutely not recommended}, \ldots, 5: \text{strongly recommended}\right)$

Parameters

E_{jk}: experts′ opinion for chart j in scenario k

P_{ik}: value or level of criterion i in scenario k

Decision Variables

UL_m: upper limit of class m

LL_m: lower limit of class m

w_i: weight of criterion i

D_{jk}: overall desirability value of chart j in scenario k

d_{ijk}: desirability value of chart j in criterion i and scenario k

x_{jk}: deviation from upper limit for chart j in scenario k

y_{jk}: deviation from lower limit for chart j in scenario k

- Distribution of data

$b1_j$: desirability value of chart j if data is not normally distributed

$q1_j$: dimensionless parameter for chart j

$\beta 1_j$: growth rate parameter for chart j

$v1_j$: parameter that affects near which asymptote maximum growth occurs for chart j

- Reason for improvement in process

$a2_j$:desirability value of chart j if all/most of the situations are valid

$b2_j$:desirability value of chart j if some of the situations are valid

$c2_j$:desirability value of chart j if none of the situations is valid

$e2_j$:desirability value of chart j if the process is new

- Type of variability in data

$a3_j$:desirability value of charty j if variation exists both within and between subgroups

- Aim of quality control

$a4_j$:desirability value of chart j if two purposes are valid
$b4_j$:desirability value of chart j if one of them is valid
$c4_j$:desirability value of chart j if none of them is valid

- Shift size to be monitored

$q5_j$: dimensionless parameter for chart j
$\beta 5_j$: growth rate parameter for chart j
$v5_j$: parameter that affects near which asymptote maximum growth occurs for chart j

- Production size

$b6_j$:desirability value of chart j if the process is short run and target value of the quality characteristic changes from one order to another

- Inspection properties

$b7_j$:desirability value of chart j if the inspection process is not easy and collecting data frequently may cause some problems
$c7_j$:desirability value of chart j if the inspection process is challenging and collecting data frequently is not possible

- Available time for chart construction

$r81_j$: exponent parameter for chart j if the target time is less than maximum time
$r82_j$: exponent parameter for chart j if the target time is greater than maximum time
$n8_j$: desirability values of chart j if estimated time for chart j is greater than maximum time

<u>Objective function</u>

$$\text{Min } Z = \sum_{j=1}^{4} \sum_{k=1}^{n} \left(x_{jk} + y_{jk} \right)$$

Subject to;

1. $D_{jk} = \prod_{i=1}^{8} d_{ijk}^{\,w_i}$ for $\forall j$ and $\forall k$

First constraint calculates the overall desirability value of each control chart in each scenario according to the desirability values in criteria and weights of criteria.

2. If $E_{jk} = m$ then,

   $D_{jk} \le UL_m + x_{jk}$ for $\forall j$ and $\forall k$

   $D_{jk} \ge LL_m - y_{jk}$ for $\forall j$ and $\forall k$

Second constraint guarantees that if chart *j* is assigned to class *m* in scenario *k* by the experts, its overall desirability values in corresponding scenario should be between upper and lower limit of that class. To guarantee feasibility of this constraint, some deviations from limits are allowed, but the sum of these deviations is minimized in the objective function.

3. $LL_m = UL_{m-1}$ for $m = 2, 3, 4, 5$
4. $LL_1 = 0$
5. $UL_5 = 1$

Constraint 3, 4, and 5 simply state that lower limit of a class is the upper limit of the previous class and lower limit of first class and upper limit of fifth class are 0 and 1, correspondingly.

6. $\sum_{i=1}^{8} w_i = 1$

6^{th} constraint suggests that sum of the weights should be 1 as in AHP.

7. If $P_{1k} = 1$ then,

$d_{1jk} = 1$ for $\forall j$

If $P_{1k} = 2$ then,

$d_{1jk} = b1_j$ for $j = 1, 2, 3$ and $d_{14k} = 1$

If $P_{1k} = 3$ then,

$$d_{1jk} = \frac{1}{\left(1 + q1_j e^{-0.15\beta 1_j}\right)^{1/v1_j}} \text{ for } j = 1, 2, 3 \text{ and } d_{14k} = 1$$

7th constraint calculates the desirability value of each chart in distribution of data criterion according to the levels in this criterion.

8. $b1_1 = b1_3$

$b1_2 < b1_3$

$q1_1 = q1_2 = q1_3$

$\beta 1_1 = \beta 1_2 = \beta 1_3$

$v1_1 = v1_3$

$v1_2 < v1_3$

$$\frac{1}{\left(1 + q1_j e^{-0 \times \beta 1_j}\right)^{1/v1_j}} = b1_j \text{ for } j = 1, 2, 3$$

Set of constraints in 8 guarantees logical relations between parameters in distribution of data criterion. It suggests that;

- If data are not normally distributed, desirability value of X̄-R / X̄-S and I-MR-R should be equal and they are greater than that of I-MR.
- Dimensionless and growth parameters for X̄-R / X̄-S, I-MR and I-MR-R should be equal but parameter that affects near which asymptote maximum growth occurs for X̄-R / X̄-S and I-MR-R is greater than that of I-MR.
- Desirability value of a chart when data are not normally distributed is equal to desirability value of a chart when *p*-value is 0.

9. If $P_{2k} = 1$ then,

$d_{2jk} = a2_j \ \text{ for} \forall j$

If $P_{2k} = 2$ then,

$d_{2jk} = e2_j \ \text{ for} \forall j$

If $P_{2k} = 3$ then,

$d_{2jk} = b2_j \ \text{ for} \forall j$

9th constraint calculates the desirability value of each chart in reason for improvement in process criterion according to the levels in this criterion.

10. $a2_1 = 1$

$b2_1 = 1$

$e2_1 = 1$

$c2_1 = c2_2 = c2_3 = c2_4$

$a2_2 = a2_3 = a2_4$

$b2_2 = b2_3 = b2_4$

$e2_2 = e2_3 = e2_4$

Set of constraints in 10 suggests that other alternatives than X̄-R / X̄-S charts should be penalized if some of the reasons is selected from the list provided. Otherwise, all alternatives' desirability value should be equal.

11. If $P_{3k} = 1$ then,

$d_{3jk} = a3_j \ \text{ for} \forall j$

If $P_{3k} = 2$ then,

$d_{3jk} = 1$ for $j = 1$, 2, 4 and $d_{33k} = 0$

11th constraint calculates the desirability value of each chart in type of variability in data criterion according to the levels in this criterion.

12. $a3_2 = a3_4$

$a3_3 = 1$

Set of constraints in 12 implies that desirability value of I-MR and EWMA should be equal if variation exists both within and between subgroups and desirability value of I-MR-R should be 1 in such situation.

13. If $P_{4k} = 1$ then,

$$d_{4jk} = a4_j \text{ for } \forall j$$

If $P_{4k} = 2$ then,

$$d_{4jk} = b4_j \text{ for } \forall j$$

If $P_{4k} = 3$ then,

$$d_{4jk} = c4_j \text{ for } \forall j$$

13th constraint calculates the desirability value of each chart in aim of quality control criterion according to the levels in this criterion.

14. $a4_1 = 1$

$$b4_1 = 1$$
$$a4_2 = a4_3 = a4_4$$
$$b4_2 = b4_3 = b4_4$$
$$c4_1 = c4_2 = c4_3 = c4_4$$
$$c4_2 \geq b4_2 \geq a4_2$$

Set of constraints in 14 suggests that other alternatives than X̄-R / X̄-S charts should be penalized if one or two are selected from the list and it can be effected by number of reasons selected. If none of them is selected, desirability value of all alternatives should be equal.

15. If $P_{5k} = 1$ then,

$$d_{5jk} = \frac{1}{\left(1 + q5_j e^{3\beta 5_j}\right)^{1/v5_j}} \text{ for } \forall j$$

If $P_{5k} = 2$ then,

$$d_{5jk} = \frac{1}{\left(1 + q5_j e^{2\beta 5_j}\right)^{1/v5_j}} \text{ for } \forall j$$

If $P_{5k} = 3$ then,

$$d_{5jk} = \frac{1}{\left(1 + q5_j e^{\beta 5_j}\right)^{1/v5_j}} \text{ for } \forall j$$

15th constraint calculates the desirability value of each chart in shift size to be monitored criterion according to the levels in this criterion.

$$q5_1 = q5_2 = q5_3$$

16. $\beta 5_1 = \beta 5_2 = \beta 5_3$

$$v5_1 = v5_2 = v5_3$$

Set of constraints in 16 guarantees that desirability function for X̄-R / X̄-S, I-MR and I-MR-R is the same.

17. If $P_{6k} = 1$ or $P_{6k} = 2$ then,

$$d_{6jk} = 1 \text{ for } \forall j$$

If $P_{6k} = 3$ then,

$$d_{6jk} = b6_j \text{ for } \forall j$$

$$b6_1 = b6_3$$

17th constraint calculates the desirability value of each chart in production size criterion according to the levels in this criterion.

18. If $P_{7k} = 1$ then,

$$d_{7jk} = 1 \text{ for } \forall j$$

If $P_{7k} = 2$ then,

$$d_{7jk} = b7_j \text{ for } \forall j$$

If $P_{7k} = 3$ then,

$$d_{7jk} = c7_j \text{ for } \forall j$$

18th constraint calculates the desirability value of each chart in inspection properties criterion according to the levels in this criterion.

19. $b7_2 \geq b7_1 + 0.1$

$$b7_2 \geq b7_3 + 0.1$$

$$c7_2 \geq c7_1 + 0.1$$

$$c7_2 \geq c7_3 + 0.1$$

Set of constraints in 19 ensures that as the inspection becomes harder desirability value of alternative charts should be penalized more than 0.1.

20. If $P_{8k} = 1$ then,

$d_{8jk} = 1 \text{ for } \forall j$

If $P_{8k} = 2$ then,

$$d_{81k} = (1-n8_1)\left(\frac{40-1.5\times 30}{30-1.5\times 30}\right)^{r82_1},\ d_{8jk} = n8_j + (1-n8_j)\left(\frac{20-40}{10-40}\right)^{r81_j} \text{ for } j = 2,3$$

$and\ d_{84k} = 1$

If $P_{8k} = 3$ then,

$$d_{81k} = n8_1 + (1-n8_1)\left(\frac{40-50}{10-50}\right)^{r81_1},\ d_{8jk} = n8_j + (1-n8_j)\left(\frac{20-50}{10-50}\right)^{r81_j} \text{ for } j = 2,3$$

$and\ d_{84k} = 1$

20th constraint calculates the desirability value of each chart in available time for chart construction criterion according to the levels in this criterion.

$$r81_1 = r81_2 = r81_3 = r81_4$$

21. $$r82_1 = r82_2 = r82_3 = r82_4$$

$$n8_1 = n8_2 = n8_3 = n8_4$$

Set of constraints in 21 guarantees that shape of the desirability function in this criterion is the same for all charts.

In addition the constraints described, initial values, upper and lower limits for all function parameters, weights and limits are provided.

3.6 Results of Optimization

To test and, if necessary, improve the mathematical model results, a cross validation approach is used. The data set consisting of experts' evaluations in scenarios are divided into two, training and test scenarios. Evaluations in training scenarios are used as an input to the mathematical model to find the optimal values of the parameters used in the approach. Then the optimal parameters are tested using the test scenarios. If the results are satisfactory, i.e. experts' evaluations and the results of the approach are small enough, both training and test scenarios are used as an input to the mathematical model to find updated optimal parameter values. Then additional test scenarios are generated and the updated optimal parameters are tested at these new scenarios. If the new test results are also satisfactory, it is concluded that the final results of the mathematical model can be used in the approach. On the other hand, if the optimal parameters found using the initial training scenarios do not perform satisfactorily at the initial test scenarios, these training and test scenarios are taken as a second training set, new test scenarios are generated, and the validation procedure is repeated. This continues until obtaining satisfactory test results.

According to this procedure, scenarios in L36 and in L18 are used as the initial training and test sets, respectively. The model that consists of 2245 constraints and 1683 decision variables is solved by GAMS/IPOPT. As it can be seen from Table 3, not only is the number of disagreements at these 36 scenarios decreased to a great extent by optimization, but also most of the inconsistencies at the scenarios generated by L18 is removed.

Table 7 Comparison of Initial and Optimal Parameters Based on 36 Training Scenarios

Control Chart	**Number of Disagreements**					
	with initial parameters			**with optimal parameters (when L36 is used)**		
	L36	**L18**	**Total**	**L36**	**L18**	**Total**
$\bar{X}$-R/$\bar{X}$-S	12	5	17	0	2	2
I-MR	8	5	13	1	0	1
I-MR-R	2	2	4	0	2	2
EWMA	6	3	9	0	1	1
Total deviation	1.700	0.718	2.418	0.005	0.100	0.105

It is concluded that using scenarios in L18 and L36 together as an input to the model can improve the parameter values further. Therefore, the updated model that includes 3334 constraints and 2475 decision variables is solved by GAMS/IPOPT. As a result, the total deviation at the combined 54 scenarios is further decreased as given in Table Table 8.

Table 8 Comparison of Optimal Parameters Based on 54 Training Scenarios

Control Chart	**Number of Disagreements**					
	with optimal parameters (when L36 is used)			**with optimal parameters (when L36 and L18 are used)**		
	L36	**L18**	**Total**	**L36**	**L18**	**Total**
$\bar{X}$-R/$\bar{X}$-S	0	2	2	1	2	3
I-MR	1	0	1	1	2	3
I-MR-R	0	2	2	1	0	1
EWMA	0	1	1	0	0	0
Total deviation	0.005	0.100	0.105	0.019	0.025	0.044

To test the final parameters, extra scenarios are generated by using criteria levels in Table 9 and L8 design.

Table 9 Criteria Levels Used in Test Scenarios

Criteria	Level 1	Level 2
Distribution of data	*p*-value=0.10	*p*-value=0.03
Reason for improvement in process	None of them selected from the list	-
Type of variability in data	Only between variation	Within/between variation
Aim of quality control	None of them selected from the list	-
Shift size to be monitored in quality characteristic	2.5σ	1σ
Production size	Continuous or the production is made for stock or a larger order	Batch type and the target value of quality characteristic changes
Inspection properties	Effortless and collecting data is not a problem	Challenging and collecting data frequently is not possible
Available time for chart construction	$t = 40$ $m = 41$ $x_j = 40$ for $j:\overline{\mathrm{X}}$-R, $\overline{\mathrm{X}}$-S and I-MR-R $x_j = 20$ for $j:$ I-MR $x_j = 1$ for $j:$ EWMA	$t = 10$ $m = 30$ $x_j = 40$ for $j:\overline{\mathrm{X}}$-R and $\overline{\mathrm{X}}$-S $x_j = 20$ for $j:$ I-MR and I-MR-R $x_j = 1$ for $j:$ EWMA

When the experts' evaluations and the method's results are compared, it is observed that there is only one disagreement for $\overline{\mathrm{X}}$-R with deviation of 0.013 and one disagreement for I-MR-R with deviation of 0.003. Therefore, optimal parameters are reasonable.

3.7 Analysis of Optimal Function Parameters and Weights

To improve the approach, it is possible to generate more scenarios and calibrate the approach further with these scenarios. However, when the result of two models is compared, it is observed that convergence of function parameters and weight of criteria are at acceptable levels.

To understand whether the convergence is at acceptable level or not, difference between effects of function parameters found by two models is compared instead of comparing change in each individual function parameters. The reason is that important changes in a function parameter can be negligible due to the structure of the decision mechanism. For instance, let a_{jk} be a function parameter which determines the desirability value of chart *j* in criterion *k*, whose weight is 0.01. If the value of a_{jk} is 0.2 in one solution and 0.4 in other solution, it can be concluded that 100% increase in a function parameter is significant and this parameter is not converged. On the other hand, if the effect of this parameter on overall desirability value $(d_k^{w_k})$ is analyzed by considering the weight of corresponding criterion, it is observed that effect of 100% increase in this parameter corresponds to 1% increase in $d_k^{w_k}$.

Among 48 function parameters that directly affect the overall desirability value as explained in above example, change in the effect of 46 function parameters is less than 2% when results of two models are compared. On the other hand, change in the effect of other two parameters is about 30%.

For the distribution of data, shift size to be monitored and available time for chart construction, whose desirability functions are continues, graphs in Figure 9, 10 and 11 are analyzed.

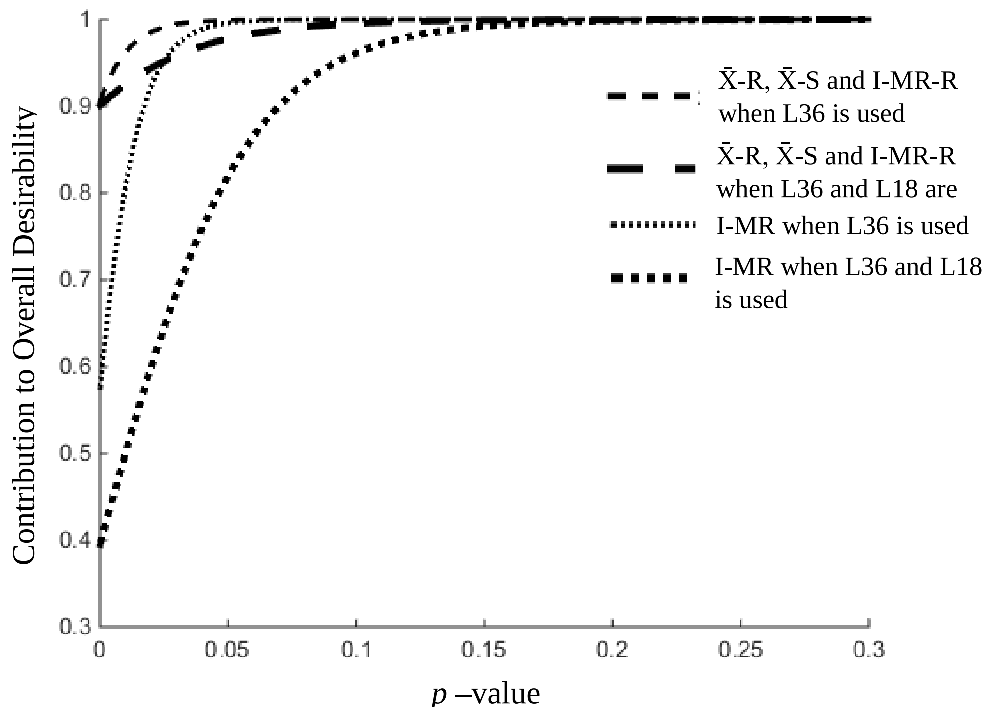

Figure 10 Change in the Effect of Functions Used in Distribution of Data

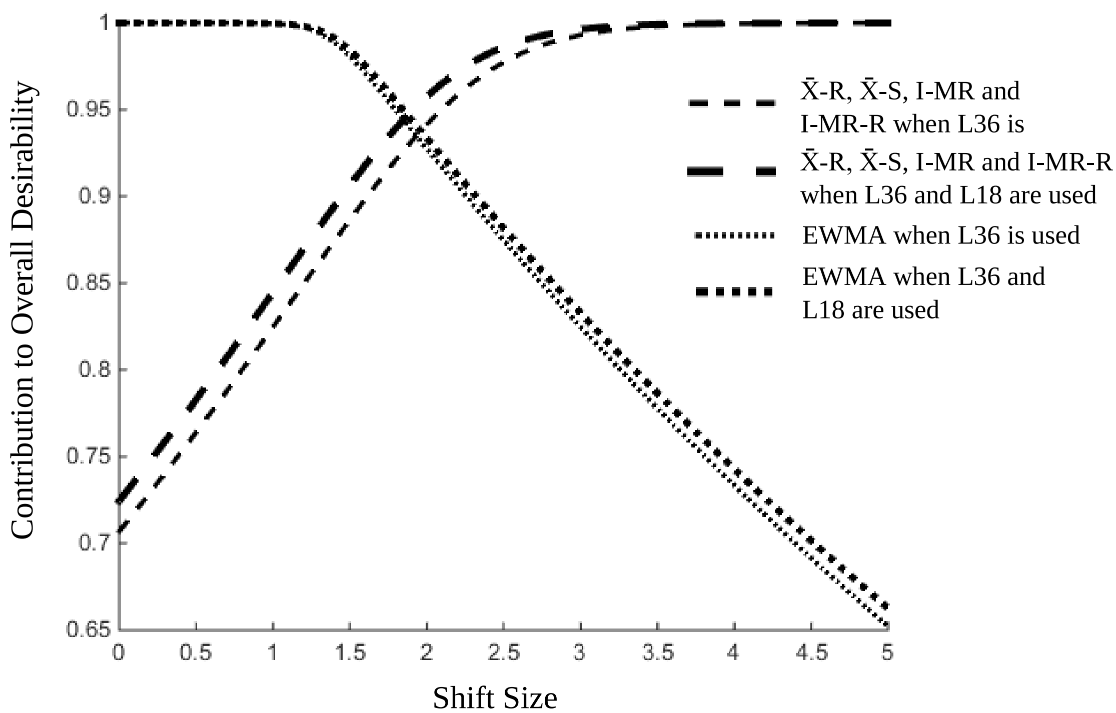

Figure 11 Change in the Effect of Functions Used in Shift Size to be Monitored

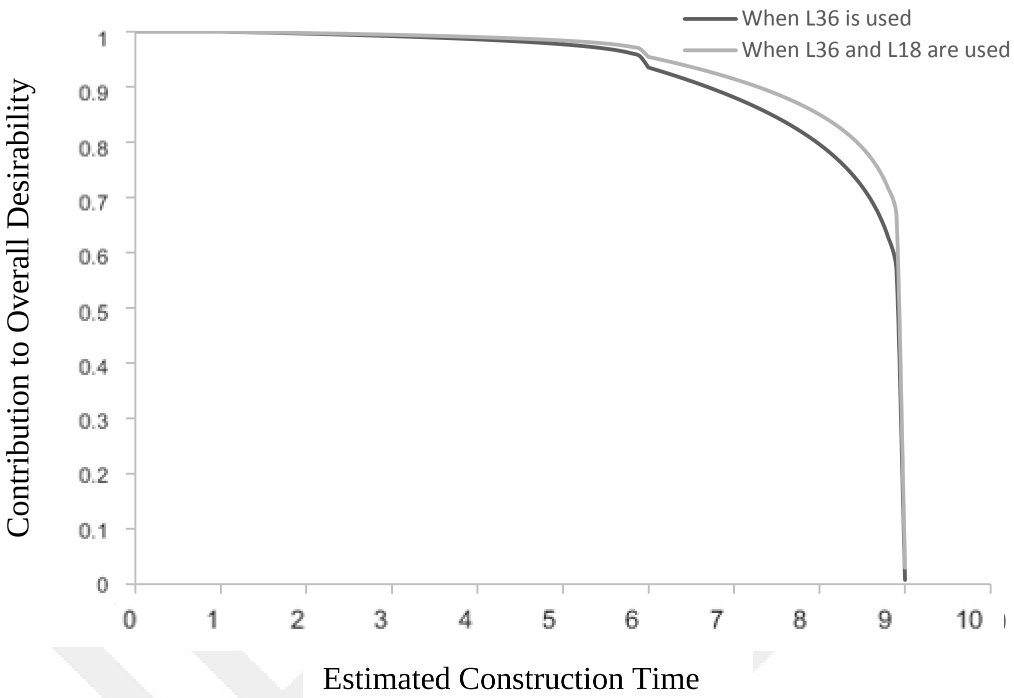

Figure 12 Change in the Effect of Functions Used in Available Time for Chart Construction

As it can be seen from Figure 10 and 11, it can be concluded that convergence is at acceptable level in these two criteria when the parameters found in two models are compared. On the other hand, effect of functions used in distribution of data seems different in these two solutions. Since the functions found by the model, in which 54 scenarios are used, are more conservative and this change does not cause many inconsistencies in scenarios in L36, it is decided to use the last solution in the system.

As in stated mathematical model, weights of criteria are also decision variable. When the weights found by AHP and the model are compared, it is observed that two most important criteria are distribution of data and type of variability in data, which constitute about 65% of total weight both in results of AHP and the model.

To conclude, although using different scenario evaluation in the model as a parameter may slightly change decision mechanism, scenarios in L36 and L18 are sufficient to argue that final parameters can be used to select the most appropriate control chart.

CHAPTER 4

AN ILLUSTRATIVE EXAMPLE

Imagine a dive watch producer. For the divers, who are the main consumers of this product, an important quality characteristic is maximum amount of time at a specific depth that the watch can function. To test the watches produced, the company uses 20 meters depth pool and measures the time until a watch fails. Although they can produce 200 watches per day with the new assembly line they have designed, they can test only one watch per day because it is very costly, and it requires about 2 hours to complete the test. Because of the same reasons, the company wants to minimize the inspection after process stability is ensured in the new assembly line. A mechanical engineer, who is the responsible person of the production process, wants to start monitoring the process according to this quality characteristic using an appropriate quality control chart within 30 days, because he needs to report the process quality to the management as soon as possible. His aim is to complete chart construction in 10 days, and if he observes any problems, he can have time to make the required adjustments until report submission. He has collected some sample data so far, and he has an access to a statistical software package that can perform the normality test. However, he cannot decide which control chart to use for his process. Furthermore, he has observed that value of maximum and minimum measurements are 100 minutes and 60 minutes, respectively. The company aims to produce watches that can resist about 80 minutes in 20 meters depth, therefore he expects to get a signal from the control chart if 10 minutes of deviation occurs.

In the situation explained above, the approach can be utilized effectively by using the questions and the answers given in Table 10 where possible answers are also given according to situation described.

Table 10 Possible Questions and Answers for the Example Problem

Are your data normally distributed? • Yes • No • **I do not know**
What is the p-value of your sample data? ***p*-value = 0.10 (obtained by a normality test)**

Table 11 Possible Questions and Answers for the Example Problem (continued)

Which one/s of the following best explain the reason for improvement in your process? • There exist some problems in an existing process, in which specified tolerances cannot be held. • The process is analyzed according to defect and/or defective rate; however, it is statistically out of control or the yield is unacceptable. • There exist very narrow specifications in the process. • The operator should decide whether or not to make adjustments in the process. • Product specification is desired to change. • **It is a new process.**
Can any assignable cause of variation exist within subgroups? • Yes • **No**
Which one/s of the following best explain the purpose of using control charts? • **Inspection is to be minimized when process is in-control or process stability.** • Capability should be continually validated.
Provide the following information that you observed in your sample data? • **Maximum measurement: 100 minutes** • **Minimum measurement: 60 minutes** • **Minimum amount of deviation that you want to detect: 10 minutes**
Is the process continuous, or is the production made for stock or a large order? Or is the process batch type or the production is made for a small order? • **Continuous** • Batch type
Which one of the following best explain the inspection properties of your process? • Inspection process is effortless and collecting data is not a problem. • Inspection process is not easy and collecting data frequently may cause some problems. • **Inspection process is challenging and collecting data frequently is not possible.**
Answer the following questions about time constraint for chart construction. • **How much time is required to measure “k” units? 1 day for k=1** • **What is the targeted time for chart construction? 10 days** • **What is the maximum time for chart construction? 30 days**

These answers are evaluated in the corresponding desirability functions. The overall desirability value of each alternative control chart is calculated and the corresponding class of that chart is determined according to the class bounds given in Table 12. The results are summarized in Table 11.

Table 12 Results for the Illustrative Example

Weights	**Criteria**	**X̄-R/X̄-S**	**I-MR**	**I-MR-R**	**EWMA**
0.31	Data distribution	0.99	0.88	0.99	1.00
0.03	Reason for improvement in process	1	0.48	0.48	0.48
0.35	Type of variability in data	1	1	0	1
0.03	Aim of quality control	0.50	0.50	0.50	0.50
0.09	Shift size to be monitored in quality characteristic	0.17	0.17	0.17	1
0.07	Production size	1	1	1	1
0.07	Inspection properties	0.05	0.38	0.28	1
0.05	Available time for chart construction	0.06	0.87	0.06	1
Overall Desirability		0.59	0.73	0.00	0.96
Recommendation		Not recommended	Recommended with reservations	Absolutely not recommended	Strongly recommended

Table 13 Optimal Class Bounds

Desirability Values	**Categories**
$0.90 < D_j \leq 1$	Strongly recommended
$0.80 < D_j \leq 0.90$	Recommended
$0.67 < D_j \leq 0.80$	Recommended with reservations
$0.38 < D_j \leq 0.67$	Not recommended
$0 \leq D_j \leq 0.38$	Absolutely not recommended

CHAPTER 5

CONCLUSION

The quality control chart evaluation approach developed in this study is based on a novel and effective approach. It can directly be used in an expert or recommender system that obtains information about characteristics of the process and the measurement system used from a novice process owner (user or decision maker) and recommends charts from its archive to the user with explanations about how strongly each chart can or cannot be recommended and why. The method has the advantages that information about the process can easily be obtained from the user, and knowledge about the control charts and opinions of SPC experts are well represented, even if some are conflicting.

It can be concluded that the desirability function approach is more powerful than the rule-based approaches in control chart evaluation and selection in terms of representation of expert knowledge. Moreover, with desirability functions, it is easier to handle ambiguous situations. However, it is still possible to encounter complicated cases where none of the charts in the achieve are appropriate, in which case the users need to directly consult with the experts to have special chart solutions designed for them.

The chart evaluation and recommendation system can be revised to handle more charts and other process conditions using a similar approach proposed in this study. The approach proposed and implemented in this study can also be adapted for development of other evaluation and recommendation systems by customizing the desirability functions, scenario generation and optimization according to particular needs of the case under consideration.

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APPENDICES

APPENDIX A Scenarios

Table 14 Scenarios

Scenarios /criteria	1	2	3	4	5	6	7	8
1	1	1	1	1	1	1	1	1
2	1	2	1	2	2	2	2	2
3	1	3	1	3	3	3	3	3
4	2	1	1	1	2	2	3	3
5	2	2	1	2	3	3	1	1
6	2	3	1	3	1	1	2	2
7	3	1	1	2	1	3	2	3
8	3	2	1	3	2	1	3	1
9	3	3	1	1	3	2	1	2
10	1	1	2	3	3	2	2	1
11	1	2	2	1	1	3	3	2
12	1	3	2	2	2	1	1	3
13	2	1	2	2	3	1	3	2
14	2	2	2	3	1	2	1	3
15	2	3	2	1	2	3	2	1
16	3	1	2	3	2	3	1	2
17	3	2	2	1	3	1	2	3
18	3	3	2	2	1	2	3	1
19	1	1	1	1	1	1	1	1
20	2	2	1	2	2	2	2	2
21	3	3	1	3	3	3	3	3
22	1	1	1	1	1	2	2	2
23	2	2	1	2	2	3	3	3
24	3	3	1	3	3	1	1	1
25	1	1	1	2	3	1	2	3
26	2	2	1	3	1	2	3	1
27	3	3	1	1	2	3	1	2
28	1	1	1	3	2	1	3	2
29	2	2	1	1	3	2	1	3
30	3	3	1	2	1	3	2	1
31	1	2	1	3	1	3	2	1
32	2	3	1	1	2	1	3	2
33	3	1	1	2	3	2	1	3
34	1	2	1	3	2	1	1	3
35	2	3	1	1	3	2	2	1
36	3	1	1	2	1	3	3	2
37	1	2	2	1	3	3	3	1
38	2	3	2	2	1	1	1	2
39	3	1	2	3	2	2	2	3

Table 15 Scenarios (continued)

Scenarios /criteria	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**
40	1	2	2	2	3	3	1	2
41	2	3	2	3	1	1	2	3
42	3	1	2	1	2	2	3	1
43	1	3	2	2	1	2	3	3
44	2	1	2	3	2	3	1	1
45	3	2	2	1	3	1	2	2
46	1	3	2	2	2	2	1	1
47	2	1	2	3	3	3	2	2
48	3	2	2	1	1	1	3	3
49	1	3	2	3	3	2	3	2
50	2	1	2	1	1	3	1	3
51	3	2	2	2	2	1	2	1
52	1	3	2	1	2	3	2	3
53	2	1	2	2	3	1	3	1
54	3	2	2	3	1	2	1	2

APPENDIX B Experts' Opinions for Each Chart in Each Scenario

Table 16 Experts' Opinions for Each Chart in Each Scenario

j/k	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**
1	2	1	1	1	1	1	1	1	2
2	3	2	2	1	1	1	2	2	3
3	5	4	3	3	3	4	3	3	5
4	3	3	3	3	3	3	3	3	3
j/k	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**
1	4	2	3	2	4	2	2	5	3
2	5	4	3	1	1	1	3	5	4
3	1	1	1	1	1	1	1	1	1
4	4	3	5	4	3	5	5	4	3
	19	20	21	22	23	24	25	26	27
1	2	1	1	1	1	2	2	1	1
2	3	1	2	3	1	3	3	1	2
3	5	3	3	5	2	5	5	3	2
4	3	3	3	3	3	3	3	3	3
j/k	**28**	**29**	**30**	**31**	**32**	**33**	**34**	**35**	**36**
1	1	1	1	1	1	2	1	1	1
2	2	1	2	2	1	3	2	1	2
3	3	4	3	3	3	5	4	4	3
4	3	3	3	3	3	3	3	3	3
j/k	**37**	**38**	**39**	**40**	**41**	**42**	**43**	**44**	**45**
1	3	3	2	3	4	3	3	2	4
2	4	1	3	4	1	3	4	1	5
3	1	1	1	1	1	1	1	1	1
4	4	3	5	4	3	5	3	5	4
j/k	**46**	**47**	**48**	**49**	**50**	**51**	**52**	**53**	**54**
1	3	2	4	2	2	2	2	2	3
2	3	1	4	4	1	3	3	1	5
3	1	1	1	1	1	1	1	1	1
4	5	4	3	4	3	5	5	4	3

APPENDIX C Value of Each Criterion in Each Scenario

Table 17 Value of Each Criterion in Each Scenario

i/k	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**
1	1	1	1	2	2	2	3	3	3
2	1	2	3	1	2	3	1	2	3
3	1	1	1	1	1	1	1	1	1
4	1	2	3	1	2	3	2	3	1
5	1	2	3	2	3	1	1	2	3
6	1	2	3	2	3	1	3	1	2
7	1	2	3	3	1	2	2	3	1
8	1	2	3	3	1	2	3	1	2
i/k	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**
1	1	1	1	2	2	2	3	3	3
2	1	2	3	1	2	3	1	2	3
3	2	2	2	2	2	2	2	2	2
4	3	1	2	2	3	1	3	1	2
5	3	1	2	3	1	2	2	3	1
6	2	3	1	1	2	3	3	1	2
7	2	3	1	3	1	2	1	2	3
8	1	2	3	2	3	1	2	3	1
i/k	**19**	**20**	**21**	**22**	**23**	**24**	**25**	**26**	**27**
1	1	2	3	1	2	3	1	2	3
2	1	2	3	1	2	3	1	2	3
3	1	1	1	1	1	1	1	1	1
4	1	2	3	1	2	3	2	3	1
5	1	2	3	1	2	3	3	1	2
6	1	2	3	2	3	1	1	2	3
7	1	2	3	2	3	1	2	3	1
8	1	2	3	2	3	1	3	1	2
i/k	**28**	**29**	**30**	**31**	**32**	**33**	**34**	**35**	**36**
1	1	2	3	1	2	3	1	2	3
2	1	2	3	2	3	1	2	3	1
3	1	1	1	1	1	1	1	1	1
4	3	1	2	3	1	2	3	1	2
5	2	3	1	1	2	3	2	3	1
6	1	2	3	3	1	2	1	2	3
7	3	1	2	2	3	1	1	2	3
8	2	3	1	1	2	3	3	1	2
i/k	**37**	**38**	**39**	**40**	**41**	**42**	**43**	**44**	**45**
1	1	2	3	1	2	3	1	2	3
2	2	3	1	2	3	1	3	1	2
3	2	2	2	2	2	2	2	2	2
4	1	2	3	2	3	1	2	3	1
5	3	1	2	3	1	2	1	2	3
6	3	1	2	3	1	2	2	3	1

Table 18 Value of Each Criterion in Each Scenario (continued)

i/k	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**
7	3	1	2	1	2	3	3	1	2
8	1	2	3	2	3	1	3	1	2
i/k	**46**	**47**	**48**	**49**	**50**	**51**	**52**	**53**	**54**
1	1	2	3	1	2	3	1	2	3
2	3	1	2	3	1	2	3	1	2
3	2	2	2	2	2	2	2	2	2
4	2	3	1	3	1	2	1	2	3
5	2	3	1	3	1	2	2	3	1
6	2	3	1	2	3	1	3	1	2
7	1	2	3	3	1	2	2	3	1
8	1	2	3	2	3	1	3	1	2