

CLOSED-LOOP SUPPLY CHAIN NETWORK DESIGN UNDER DEMAND,  
RETURN AND QUALITY UNCERTAINTY

A THESIS SUBMITTED TO  
THE GRADUATE SCHOOL OF NATURAL AND APPLIED SCIENCES  
OF  
MIDDLE EAST TECHNICAL UNIVERSITY

BY

KADİR BİÇE

IN PARTIAL FULFILLMENT OF THE REQUIREMENTS  
FOR  
THE DEGREE OF MASTER OF SCIENCE  
IN  
INDUSTRIAL ENGINEERING

MAY 2017



Approval of the thesis:

**CLOSED-LOOP SUPPLY CHAIN NETWORK DESIGN UNDER DEMAND,  
RETURN AND QUALITY UNCERTAINTY**

submitted by **KADİR BİÇE** in partial fulfillment of the requirements for the degree of  
**Master of Science in Industrial Engineering Department, Middle East Technical  
University** by,

Prof. Dr. Gülbin Dural Ünver  
Dean, Graduate School of **Natural and Applied Sciences**

\_\_\_\_\_

Prof. Dr. Murat Köksalan  
Head of Department, **Industrial Engineering**

\_\_\_\_\_

Assist. Prof. Dr. Sakine Batun  
Supervisor, **Industrial Engineering Department, METU**

\_\_\_\_\_

**Examining Committee Members:**

Assoc. Prof. Dr. Sedef Meral  
Industrial Engineering Department, METU

\_\_\_\_\_

Assist. Prof. Dr. Sakine Batun  
Industrial Engineering Department, METU

\_\_\_\_\_

Assoc. Prof. Dr. Cem İyigün  
Industrial Engineering Department, METU

\_\_\_\_\_

Assist. Prof. Dr. Melih Çelik  
Industrial Engineering Department, METU

\_\_\_\_\_

Assist. Prof. Dr. Emre Nadar  
Industrial Engineering Department, Bilkent University

\_\_\_\_\_

**Date:**

\_\_\_\_\_



**I hereby declare that all information in this document has been obtained and presented in accordance with academic rules and ethical conduct. I also declare that, as required by these rules and conduct, I have fully cited and referenced all material and results that are not original to this work.**

Name, Last Name: KADİR BİÇE

Signature :

# ABSTRACT

## CLOSED-LOOP SUPPLY CHAIN NETWORK DESIGN UNDER DEMAND, RETURN AND QUALITY UNCERTAINTY

BİÇE, KADİR

M.S., Department of Industrial Engineering

Supervisor : Assist. Prof. Dr. Sakine Batun

May 2017, 81 pages

In this study, we focus on a closed-loop supply chain (CLSC) network design problem in the presence of uncertainty in demand quantities, return rates, and quality of the returned products. We formulate the problem as a two-stage stochastic mixed-integer program that maximizes the total expected profit. The first-stage decisions in our model are facility location and capacity decisions, and the second-stage decisions are production quantities and the forward/backward flows on the network. We solve the problem by using the L-shaped method in iterative and branch-and-cut frameworks. In order to improve the computational efficiency, we consider various strategies such as adding mean-value cuts to the restricted master problem and generating multiple cuts instead of a single cut at each iteration or at each integer feasible solution. We use our numerical results to estimate the value of the stochastic solution and the expected value of perfect information in different problem settings.

Keywords: Closed-loop supply chains, network design under uncertainty, two-stage stochastic programming, L-shaped method, branch-and-cut



# ÖZ

## TALEP, GERİ DÖNÜŞ VE KALİTE BELİRSİZLİKLERİ ALTINDA KAPALI-DÖNGÜ TEDARİK ZİNCİRİ TASARIMI

BİÇE, KADİR

Yüksek Lisans, Endüstri Mühendisliği Bölümü

Tez Yöneticisi : Yrd. Doç. Dr. Sakine Batun

Mayıs 2017, 81 sayfa

Bu çalışmada talep, geri dönüş ve kalite belirsizlikleri altında bir kapalı-döngü tedarik zinciri tasarımı problemine odaklanılmıştır. Problem, beklenen kârı en iyileştirecek iki aşamalı rassal program olarak modellenmektedir. Birinci aşama kararlarını tesis açma ve kapasite belirleme kararları, ikinci aşama kararlarını ise üretim ve ağdaki ileri/geri akış kararları oluşturmaktadır. Problem, yinelemeli L-shaped ve dal ve kesik tabanlı L-shaped algoritmaları ile çözülmektedir. Sayısal performansı arttırmak için sınırlı ana probleme ortalama değer problemine dayalı eşitsizlikler eklemek ve tek kesik yerine her tekrarda veya her olurlu tamsayı çözümünde birden fazla kesik eklemek gibi geliştirmeler uygulanmaktadır. Elde edilen sayısal sonuçlar, rassal çözümün ve eksiksiz bilginin beklenen değerlerini hesaplamak için kullanılmaktadır.

Anahtar Kelimeler: Kapalı-döngü tedarik zincirleri, belirsizlik altında ağ tasarımı, iki

aşamalı rassal programlama, L-shaped algoritması, dal ve kesik algoritması







*To my family...*

## ACKNOWLEDGMENTS

First and foremost, I would like to express my gratitude to my supervisor Assist. Prof. Dr. Sakine Batun for her continuous guidance and friendly support throughout my thesis study. Apart from my thesis study, she is always there to help and inspire me about my life and my future plans therefore, she is more than a thesis advisor to me.

Besides my advisor, I would like to thank the rest of my thesis committee: Assoc. Prof. Dr. Sedef Meral, Assoc. Prof. Dr. Cem İyigün, Assist. Prof. Dr. Melih Çelik, and Assist. Prof. Dr. Emre Nadar for their insightful comments and directive questions.

Furthermore, I am also grateful to all of my friends for their endless support and motivation in every step of this process. Specially, I would like to thank Nilgün Efe for her unique support and continuous encouragement. I would also thank Bayram Gezer, Bahadır Şahin, Selin Akifođlu and Emine Gündođdu for their support and guidance in conducting necessary experiments.

Finally, I must express my very profound gratitude to my parents and my brother for their constant love and support throughout this process. It would be impossible for me to do this study without them.

## TABLE OF CONTENTS

ABSTRACT . . . . .	v
ÖZ . . . . .	vii
ACKNOWLEDGMENTS . . . . .	x
TABLE OF CONTENTS . . . . .	xi
LIST OF TABLES . . . . .	xiv
LIST OF FIGURES . . . . .	xvi
LIST OF ABBREVIATIONS . . . . .	xvii
CHAPTERS	
1 INTRODUCTION . . . . .	1
2 LITERATURE REVIEW . . . . .	5
2.1 Review of the Related Studies . . . . .	5
2.1.1 Reverse Logistics Networks . . . . .	10

2.1.1.1	Deterministic Reverse Logistics Networks . . . . .	10
2.1.1.2	Stochastic Reverse Logistics Networks	11
2.1.2	Closed-Loop Supply Chain Networks . . . . .	12
2.1.2.1	Deterministic Closed-Loop Supply Chain Networks . . . . .	12
2.1.2.2	Stochastic Closed-Loop Supply Chain Networks . . . . .	15
2.1.3	Contributions of Our Study . . . . .	23
2.2	Background Information on Stochastic Programming . . . . .	24
2.2.1	Two-Stage Stochastic Programming . . . . .	25
2.2.2	Decomposition Methods . . . . .	27
2.2.2.1	L-Shaped Method . . . . .	27
2.2.2.2	Branch-and-Cut Based L-Shaped Method	29
2.2.3	The Value of Information and the Stochastic Solution	29
3	PROBLEM DEFINITION . . . . .	31
3.1	Scope of the Problem . . . . .	31
3.2	Mathematical Formulation . . . . .	34
4	SOLUTION METHODS . . . . .	45

4.1	Multiple Cuts . . . . .	47
4.2	Mean Value Cut . . . . .	47
5	COMPUTATIONAL EXPERIMENTS . . . . .	51
5.1	Generation of Problem Instances . . . . .	51
5.2	Computational Performance of the Proposed Solution Methods	52
5.3	The Value of Stochastic Solution and the Expected Value of the Perfect Information . . . . .	62
5.4	Value of Uncertainty in Demand, Return Rate, and Quality .	64
5.5	Impact of Return and Quality Levels . . . . .	67
5.6	The Benefit of Using Closed-Loop Supply Chains . . . . .	74
6	CONCLUSION . . . . .	75

## LIST OF TABLES

### TABLES

Table 2.1 Literature Review Criteria . . . . .	7
Table 2.2 Comparison of Our Study with Existing Studies . . . . .	24
Table 5.1 Problem Classes . . . . .	51
Table 5.2 Parameters for Solution Method Comparison . . . . .	53
Table 5.3 Solution Times (in CPU seconds) for Problem Class K1 under Demand- Based Grouping . . . . .	56
Table 5.4 Solution Times (in CPU seconds) for Problem Class K1 under Demand- Rate-Based Grouping . . . . .	58
Table 5.5 Solution Times (in CPU seconds) for Problem Class K2 . . . . .	60
Table 5.6 Relative Gaps for Problem Class K3 . . . . .	61
Table 5.7 Relative Gaps for Problem Class B1 . . . . .	62
Table 5.8 VSS and EVPI values . . . . .	64
Table 5.9 Analysis of individual uncertainties for Problem Class K2 . . . . .	66
Table 5.10 Distribution of Uncertain Return and Quality Rates . . . . .	67
Table 5.11 Analysis of $\alpha_k$ (return rate) levels for Problem Class K2 . . . . .	70
Table 5.12 Analysis of $\beta_k$ (product quality) levels for Problem Class K2 . . . . .	71

Table 5.13 Analysis of  $\sigma_{jk}$  and  $\rho_{jk}$  (part quality) levels for Problem Class K2 . . . 72

Table 5.14 Analysis of  $\delta_{jk}$  (material quality) levels for Problem Class K2 . . . . 73

Table 5.15 Benefit of CLSC over Forward Supply Chain for Problem Class K2 . . . 74



# LIST OF FIGURES

## FIGURES

Figure 3.1 Network Structure . . . . . 33





## LIST OF ABBREVIATIONS

BC	Branch and cut based L-shaped method
BD	Benders' decomposition
BOM	Bill-of-materials
CLSC	Closed-loop supply chain
CPU	Central processing unit
CTR	Center
DEP	Deterministic equivalent program
EEV	Expected result of expected value solution
EF	Extensive form
EOL	End of life
EOU	End of use
EV	Expected value solution
EVPI	Expected value of perfect information
HC	Hybrid center
HSF	Hybrid sourcing facility
LP	Linear program
LS	Iterative L-shaped method
MILP	Mixed integer linear program
MINLP	Mixed integer non-linear program
OEM	Original equipment manufacturer
RHS	Right hand side
RL	Reverse logistics

RP	Recourse problem
RT	Retailer
SAA	Sample average approximation
SF	Sourcing facility
VSS	Value of the stochastic solution
WS	Wait-and-see solution



# CHAPTER 1

## INTRODUCTION

In recent decades, interest in reverse logistics (RL) and closed-loop supply chains (CLSC) grew significantly due to increasing environmental problems. With this increase, governments created legislations such as the Waste Electrical and Electronic Equipment (WEEE) for the elimination of waste (Govindan et al., 2015). But, from an environmentally friendly perspective, considering CLSCs and determination of remanufacturing is not straightforward. For instance, according to Souza (2012), in some cases, refrigerators should be recycled instead of remanufacturing because, more energy efficient refrigerators are better for the environment. Therefore, for effective recovery decisions, life cycle assessment (LCA) should also be considered carefully.

Although RL and CLSCs have emerged due to environmental concerns, they have also become revenue opportunities for decision makers in recent years (Guide and Van Wassenhove, 2009). Recovery of returned products may provide savings on raw material purchases and energy consumption of manufacturers, and manufacturers utilizing CLSCs can also profit by selling refurbished products to the secondary markets. Especially in the electronics and automotive industry, companies such as HP, Dell, Xerox and GM are already utilizing these practices (Üster et al., 2007). According to Guide et al. (2006), Hewlett-Packard have returns that cost up to 2% of their total sales, but a very small part of them are being recovered. A challenge related to LCA for product recovery would be the use of short life-cycle products. According to

Guide et al. (2006), some computer manufacturers use such products, which causes 1% value loss per week. Moreover, RL networks that operate slowly would increase time to put returned product back to market in up to 10 weeks, which leads to a loss of approximately 10% of the product's value (Guide and Van Wassenhove, 2009). Another issue with CLSCs is the fear of cannibalization, which makes refurbished products so attractive that customers tend to switch to refurbished products instead of purchasing brand new products.

CLSCs include three main types of returns: commercial returns, end-of-use (EOU) returns and end-of-life (EOL) returns (Guide and Van Wassenhove, 2009). Commercial returns refer to the returns made with consumers' consent in several days after the purchase. This type of returns is handled by small-scale processes such as repairs. EOU returns are made when products are replaced with better alternatives due to dissatisfaction of consumers even while existing products are working properly. EOU returns require more treatment such as remanufacturing to be able to rejoin the forward flow. EOL returns take place when products are no longer functional. This kind of returns typically has a single option of recovery (recycling), after which they are used as raw materials.

Regular forward supply chains include forward flow of products through suppliers, plants and distributors to customers. RL includes the reverse flow of products starting from end customers and involves activities such as collection, inspection, repairing, disassembly, disposal, recycling and remanufacturing of collected products. According to the definition by American Reverse Logistics Executive Council, RL is a process of managing the flow of raw materials and products, inventory and information from customers to the point of recovery or disposal (Rogers and Tibben-Lembke, 1998). If both forward and reverse logistics are considered at the same time, the resulting structure would be called a CLSC. In a more business oriented approach, CLSC management is defined as: "the design, control and operation of a system to maximize value creation over the entire life cycle of a product with dynamic recovery of value from different types and volumes of returns over time" (Guide and Van

Wassenhove, 2009).

As in many decision making environments, there exists three levels of problems in CLSCs: strategic, tactical and operational (Souza, 2012). Strategic problems, such as network design, affect the company in the long-run. Tactical problems can be inventory related problems and have relatively shorter-term effects on the company. Operational problems include day-to-day decisions, therefore impacts are instantaneous.

In this study, we focus on a CLSC network design problem under uncertainty. While it is a strategic problem, we consider the long-term decisions aggregated into a single decision period. In this problem, we focus on EOU returns, which have potential to be used either in remanufacturing or in recycling. On the other hand, by setting quality parameters determining the availability for remanufacturing of returned products to lower levels, we can make this model capable of handling EOL returns as well. In considering a network design problem with many factors, evaluation of uncertainty is inevitable. In this sense, we include demand, return and quality uncertainties. The problem is modeled as a two-stage stochastic program which has an objective of expected profit maximization. First-stage decisions are facility activation and capacity installation decisions. Second-stage decisions are composed of flow decisions. The problem is solved by using iterative L-shaped method and branch-and-cut based L-shaped method. In order to increase the computational efficiency, two types of modifications are used: adding multiple cuts with scenario-based grouping and adding mean-value cut. In addition to computational experiments, value of stochastic solution (VSS) and expected value of perfect information (EVPI) are reported. To obtain managerial insights, effects of uncertainty are investigated by setting uncertain parameters to various levels.

The remainder of this study is structured as follows. In Chapter 2, we summarize related studies and give background information about the solution methodology. In

Chapter 3, we define the problem and provide its mathematical formulation. In Chapter 4, we describe the implementation of the proposed solution methods and supply approaches for computational improvements. In Chapter 5, we explain the generation of problem instances, provide results about the performances of solution methods, and present VSS, EVPI, and our additional findings.



## CHAPTER 2

### LITERATURE REVIEW

#### 2.1 Review of the Related Studies

Network design in the presence of remanufacturing and recycling activities has been studied by many researchers. Different types of node-related components (e.g. customers, facilities) that have been considered in these studies can be listed as follows:

- *Suppliers*: In most cases they supply raw materials to be used in the production of products. In some cases, they are distinguished for supply of parts and by-products. In addition, recycling occasionally takes part in these facilities.
- *Plants/Manufacturing Centers/Factories*: Parts or products are manufactured at these facilities. In some cases plants are also used for remanufacturing purposes.
- *Distribution Centers/Warehouses/Depots*: In most studies, these centers are responsible for only forward flow of products. However, in some of the relevant articles, inventory holding is also possible at these centers.
- *Customers*: They create demand for brand new products (i.e., products demanded by first customers) and remanufactured products (i.e., products demanded by second customers).
- *Collection Centers*: Reverse flow of products occurs through these centers. In some cases, repairing or disassembly can also be performed at these centers.

- *Hybrid Centers*: These centers facilitate both forward and reverse flow of products with pooled or dedicated capacity.
- *Disassembly Centers/Dismantlers*: In reverse flow, product to part or material conversion is performed at these centers.
- *Disposal Centers*: Disposal of both products and parts occurs in these facilities.
- *Recycling Centers/Decomposition Centers*: These facilities recycle parts or materials either to be used in remanufacturing or to be sold.
- *Spare Part Market*: This market represents customers who have a certain demand for the obtained parts from disassembled products.
- *Repair Centers/Recyclers*: In some studies, certain amount of returns can be repaired in these facilities to re-enter the forward flow.
- *Redistributors/Resellers*: In some of the studies, second customers which are located separately from first customers require a different flow, which is directed through redistributors.
- *Retailers*: These are considered as intermediate facilities between distribution centers and final customers.

Related studies can be categorized in terms of their network structure, objectives, decisions, presence and types of uncertainty, solution methods and recovery options (Table 2.1). Mainly, two types of network structures exist: Reverse Logistics (RL) which include only the processes and decisions related to returned goods and Closed-Loop Supply Chains (CLSCs) which consist of decisions related to both forward flow of brand new products and bidirectional flow of returned/remanufactured products.



Table 2.1: Literature Review Criteria

No	Article	Network type	Objectives	Decisions	Uncertainty	Solution Method	Recovery Option
1	Alumur et al. (2012)	RL	Maximize Profit	Facility Opening, Inventory Holding	-	Exact Solution (using a solver)	Part Recovery
2	Listes and Dekker (2005)	RL	Maximize Profit	Facility Opening, Flow	Demand, Supply	Extensive Form (using a solver)	Product Recovery
3	Kara and Onut (2010b)	RL	Maximize Profit	Facility Opening, Flow	Demand, Return Rate	Extensive Form (using a solver)	Product Recovery
4	Amin and Zhang (2012)	CLSC	Maximize Profit	Facility Opening, Flow	-	Exact Solution (using a solver)	Part Recovery
5	El-Sayed et al. (2010)	CLSC	Maximize Profit	Facility Opening, Flow	-	Exact Solution (using a solver)	Part Recovery
6	Easwaran and Üster (2010)	CLSC	Minimize Cost	Facility Opening, Flow	-	Benders Decomposition	Product Recovery
7	Aravandan and Panneerselvam (2014)	CLSC	Minimize Cost	Facility Opening, Flow	-	Exact Solution (using a solver)	Part Recovery
8	Ramezani et al. (2013)	CLSC	Maximize Profit, Maximize Service Level, Minimize Defect Rate	Facility Opening, Flow	Price, Demand, Return Rate, Processing Costs	Extensive Form (using a solver)	Product Recovery

Table 2.1: Literature Review Criteria

No	Article	Network type	Objectives	Decisions	Uncertainty	Solution Method	Recovery Option
9	Zeballos et al. (2014)	CLSC	Minimize Cost	Facility Transportation Selection, Inventory Holding, Opening, Mode Flow,	Demand, Supply	Extensive Form (using a solver)	Part Recovery
10	Soleimani et al. (2016)	CLSC	Maximize Profit	Facility Transportation Activation, Inventory Holding, Opening, Link Flow,	Demand, Price, Return Rate, Maximum number of facilities	Extensive Form (using a solver)	Product Recovery
11	Listeş (2007)	CLSC	Maximize Profit	Facility Transportation Activation, Flow	Demand, Return Rate	Integer L-Shaped based decomposition	Product Recovery
12	Pishvae et al. (2009)	CLSC	Minimize Cost	Facility Opening, Flow	Demand, Return Rate, Disposal Rate, Transportation and Processing Costs	Extensive Form (using a solver)	Product Recovery
13	Lee et al. (2010)	CLSC	Minimize Cost	Facility Opening, Flow	Demand, Supply of Returned Product	Sample Average Approximation	Product Recovery
14	Jeihoonian et al. (2017)	CLSC	Maximize Expected Profit	Facility Opening, Flow	Quality Status of Returned Products	Enhanced L-Shaped Method	Part and Material Recovery
15	Üster and Hwang (2016)	CLSC	Minimize Expected Cost	Facility Opening, Capacity Expansion Flow	Demand, Return Rate	Enhanced Benders Decomposition	Product Recovery

Table 2.1: Literature Review Criteria

No	Article	Network type	Objectives	Decisions	Uncertainty	Solution Method	Recovery Option
16	Chouinard et al. (2008)	CLSC	Minimize Expected Cost	Facility Opening, Flow	Demand, Return Rate and Quality of Returned Products	SAA based Heuristics	Product Recovery

### **2.1.1 Reverse Logistics Networks**

In the literature, there exists several types studies related to RL. In this part we will present the most relevant studies to our topic by classifying them in terms of their uncertainty structure.

#### **2.1.1.1 Deterministic Reverse Logistics Networks**

In the context of RL networks, Alumur et al. (2012) studies a problem which includes customers, collection centers, inspection/disassembly centers, remanufacturing plants, external remanufacturing plants, suppliers and second markets. In addition to these echelons, external recycling nodes exist which can be located at collection, inspection and remanufacturing centers for enabling recycling processes in these facilities. In the proposed network, collected products are disassembled to their components in disassembly centers. Resulting components are either used in remanufacturing or sold to external remanufacturers or recycled/disposed. The problem is modeled as a multi-period, multi-product MILP where the objective is to maximize the profit potential for an original equipment manufacturer (OEM) by collecting returned products from customer zones to be used in remanufacturing. Revenue is generated through the sales of products to second markets, third party recycling and remanufacturing centers. Costs include the fixed costs of infrastructure and the variable costs of transportation and inventory holding. Three main sets of decision variables exist: binary facility opening, integer flow and inventory holding variables. Exact solutions for a case study with 40 collection centers are obtained by solving the related LP by using a solver (CPLEX) and the results demonstrate that using multi-period approach can lead to some gains in the profit compared to the single-period approach. In addition, this study shows that locating remanufacturing and inspection centers at the same location can lead to a significant decrease in transportation costs.

### 2.1.1.2 Stochastic Reverse Logistics Networks

Listeş and Dekker (2005) considers a RL problem where the focus is on sand recycling with sand sources, regional depots and cleaning facilities. In this problem, used sand is collected and then sorted to three categories: clean and half-clean sand which can be sold immediately and polluted sand which should be treated to be sold. In addition to the deterministic parameters, this study considers uncertain demand locations and supply level. The problem is formulated by using two-stage and three-stage stochastic programming approaches with facility opening decisions on the first and second stages and flow decisions on the third stage. The objective is to maximize the expected net profit, which is the revenue generated by selling clean or half-clean sand minus facility opening, transportation and processing costs. Decision variables include binary opening of depots and sand treatment centers and also integer flow amounts. The resulting formulation is solved in its extensive form by using a solver (CPLEX). Numerical results show that in high demand scenarios, the network is flexible in terms of demand location and improvement by stochastic approach is relatively lower. According to the interpretation, this result can be caused by the capacitated structure of the problem where high demand amounts can lead to excess investments in unused capacity. On the other hand, in low demand cases, network is highly dependent on demand locations and hence stochastic approach is more crucial. Additionally, the three-stage approach where information about material volumes are assumed to be revealed step by step and decisions are splitted over time has been seen as an effective long run strategy.

Kara and Onut (2010*b*) study a stochastic RL network design problem with a case of paper recycling including recycling centers, customers and disposal centers. In this network, the collected paper is gathered directly at recycling centers and distributed to customers after recycling or directed to the disposal center if the paper is not recyclable. In this study, uncertain parameters are return and demand levels. The problem is formulated using two-stage stochastic programming and robust programming approaches. The objective is to maximize the expected net profit which is derived from

the sales of recycled paper and reduced by the opening, processing and transportation costs. Decisions are opening of recycling centers and the flow amounts of paper. Results of two formulations obtained by solving the extensive form are compared and this comparison shows that the stochastic programming approach leads to greater amounts of demand satisfied by recycled raw materials instead of new raw materials compared to the robust formulation. An extension of this problem with the inclusion of collection centers to the considered network is studied by Kara and Onut (2010a). Their results show that a two-stage stochastic programming approach is suitable for CLSC design.

### **2.1.2 Closed-Loop Supply Chain Networks**

CLSC is more flexible and challenging than RL due to the combined structure of forward and reverse flows. In this section, we review deterministic and stochastic CLSC studies in the literature.

#### **2.1.2.1 Deterministic Closed-Loop Supply Chain Networks**

Amin and Zhang (2012) study a CLSC network design problem with a focus on product life cycles. In this problem, forward flow of products to customers is carried out by distributors and retailers. Reverse flow starts with the collection of used products at the collection sites. Commercial returns are repaired and then they reenter the forward flow. End of use returns are disassembled and used in remanufacturing as new parts. End of life returns are either recycled or disposed depending on their quality. Unlike earlier studies, this study supplies part-product conversion, therefore production decisions affect the optimal solution. The problem is modeled as a MILP where the objective is to maximize the net profit which is composed of the revenues from product sales and related processing, transportation and facility opening costs. Decision variables include binary recycling, disassembly and repair facility opening

and integer flow variables. Exact solutions to this MILP are obtained by using a solver (GAMS). Sensitivity analysis by varying disassembly capacity and total return percentage shows that increase in commercial return rate leads to greater profits due to low process and transportation costs. In addition, an extended model to analyze the problem with a secondary market for remanufactured products is also considered. Numerical results demonstrate that disassembly capacity is the most effective parameter on the objective value and the extended model performs worse than the primary model which can be due to the second customer demand satisfaction constraint of the extended model as stated in the study.

Another deterministic CLSC network design problem is studied by El-Sayed et al. (2010). This study differs from earlier studies with inclusion of suppliers and second customers as echelons. In this problem, forward flow takes place among suppliers, production facilities, distributors and customers. After being collected from the customers, products are returned to the disassembly locations and then depending on their quality they can be: repaired to be sold to second customer, sent to supplier for recycling, sent to facilities to be directly used in remanufacturing, or sent to disposal. This problem is modeled as a MILP where the objective is to maximize the total profit coming from sales minus processing, manufacturing and transportation costs. Decision variables include binary opening costs of suppliers, production facilities, distributors, disassembly locations and redistributors as well as integer flow variables. Exact solutions are obtained by using a solver (XPressSP) and effects of mean demand and return ratio are investigated. The numerical results show that the total expected profit is linearly proportional to mean demand and return ratio except for certain instances where shortage costs and capacity constraints limit the increase.

Easwaran and Üster (2010) study a CLSC network design problem with integrated forward and reverse flows. In this problem, there are only three echelons, which are hybrid sourcing facilities (HSF) for manufacturing and remanufacturing, hybrid centers (HC) for distribution and collection of products and retailers. Products are forwarded to retailers and afterwards collected from them for recovery. The problem

is modeled as a MILP where the objective is to minimize the total cost which consists of opening costs of HSFs or HCs, processing and transportation costs. Decision variables include binary facility opening and integer flow variables. This multi-product CLSC problem is solved by Benders' Decomposition (BD) approach. 12 different problem classes with number of products between 5-10, number of hybrid centers between 25-35 and number of retailers between 60-120 are considered in the numerical study. Based on these classes, comparisons of performances of traditional branch-and-cut approach and BD approach with different types of cuts proposed by authors show that proposed strengthened cuts performed better than traditional approach and multi-cut version of the proposed approach provided faster convergence. Results demonstrate that with higher return flows compared to forward flows and higher reverse channel costs, HCs are used exclusively as collection or distribution centers. Additionally, if fixed costs are dominated by transportation costs, HCs tend to be located close to HSFs and even lead to co-location. In addition, if return amounts are lower, HCs are used for both collection and distribution purposes.

Aravendan and Panneerselvam (2014) consider multi-echelon and a multi stage CLSC network that consist of manufacturers, wholesalers, retailers and first customers in forward direction and repair, collection/disassembly/refurbishing, remanufacturing, recycling, disposal centers, resellers and second customers in reverse direction. Collection of products are due to either requirement of repair or EOL status. Then, repaired products are distributed to customers and EOL products can be: remanufactured to be sold to second customer, sold to recycler, or sent to disposal depending on their quality, which is determined by fixed ratios. The problem is modeled as a mixed integer non-linear program (MINLP) where the objective is to minimize the total cost generated by facility opening, processing and transportation costs. In the considered network, collection is a push mechanism and selling to second customers is a pull mechanism. Decisions include binary facility opening decisions of manufacturers, wholesalers, retailers, repair, collection, disposal, recycling centers and resellers and continuous flow decisions. With a small sized instance which includes three manufacturers, three wholesalers, three retailers, six first customers, two repair centers,



two collectors, two resellers, two recyclers, two land-fillers and three second customers, an exact solution is obtained by using a solver (LINGO) and CPU times are considered to be compared with CPU times of future applications of meta-heuristics by future researchers.

### **2.1.2.2 Stochastic Closed-Loop Supply Chain Networks**

In addition to the network structure, uncertainty in problem parameters is also another factor to be considered while categorizing the related studies. Stochasticity, especially in demand and return rate, has been considered in recent studies.

Ramezani et al. (2013) study a stochastic multi-objective and multi-product CLSC network design problem including suppliers, plants, forward facility, hybrid facility and customers in forward channel. Distributed goods are collected in either collection centers or hybrid centers and then moved to plants for remanufacturing or to disposal depending on their quality. Recovery of collected products is performed in product level, meaning that there is no lower level than product in the recovery process. In addition, recovered products are the perfect substitutes of brand new products. In this study, uncertainty exists in product sales price, demand, processing costs and return rate at customers and handled by two-stage stochastic programming. Different from earlier studies, this study includes a multi-objective approach. Objectives include maximization of the expected profit (total revenue minus total facility opening, transportation and processing costs), maximization of service level and minimization of total number of defective raw material parts acquired from suppliers which depends on defect rates of materials. Decisions consist of first-stage binary facility opening decisions of plants, distribution, collection, hybrid and disposal centers and second-stage shipping decisions. The authors consider only one problem instance that involves six suppliers, six types of materials, five plant candidates, two products, ten customers, six recovery center(remanufacturing), three disposal center candidates, and ten scenarios. For this multi-objective problem,  $\epsilon$ -constraint method is used to obtain the

Pareto curves. Results illustrate that three objectives are conflicting in this problem. Additionally, responsiveness level is increased with a decrease in the expected profit objective or with an increase in defect rate objective. Finally, wait and see (WS), expected value (EV) and recourse problem (RP) solutions result in the same values for defect rate and responsiveness objectives. Therefore, expected value of perfect information (EVPI) is calculated for the expected profit maximization objective and leads to negative values as authors expected since it is calculated as  $EVPI = RP - WS$ . As a result, comparison between the EV approach and the RP solution (VSS) proves that the RP solution performs better than EV and hence promotes the accurateness and the use of the two-stage stochastic programming approach in this type of problem.

Zeballos et al. (2014) study a multi-period and multi-product CLSC design problem under uncertainty considering raw material suppliers, factories, warehouses, distribution centers in forward flow and dismantlers, repairing centers, disposal locations and decomposition centers in reverse flow. Flow through reverse directions is determined according to the product quality. While there is no second customer or any other market, recycled parts are considered as if new. Uncertainty in this problem consists of demand and supply parameters and is modeled with two-stage stochastic programming. The objective of this problem is to minimize the expected total cost which includes facility opening, transportation and emission costs and therefore enhance revenue gathered from recycling. To achieve this objective, binary facility opening and transportation mode selection and continuous flow and inventory holding decisions are optimized. The resulting model is solved in three forms with 81 scenarios: extensive form with full scenario tree, extensive form after applying a scenario reduction algorithm and deterministic form. In terms of CPU time, scenario reduction performs better than the full scenario form which does not converge within the considered CPU time limit. In addition, to demonstrate the importance of uncertainties, the problem is solved in two different forms: considering both uncertainties at the same time and considering each uncertain parameter separately. Besides, investigations on effects of fixed parameters show that the decrease in return rate increases

objective function value due to lower revenue, increase in collection to repair center rate improves objective value due to reduction in inventory holding costs, decrease in repair to warehouses rate decreases objective function due to increase in purchasing and inventory holding costs, increase in dismantler to decomposition rate improves objective value due to greater revenue, decrease in decomposition to supplier rate improves objective value due to reduction in purchasing costs and increase in emission costs increases objective function value due to higher total environmental cost.

Soleimani et al. (2016) study a multi-product and multi-period CLSC network consisting of suppliers, manufacturers, warehouses, distributors and customers for forward flow and disassembly centers, re-distributors, disposal centers and second customers for reverse flow. Reverse operations of the products include repair in disassembly centers, remanufacturing in manufacturers, recycling in suppliers and disposal in disposal centers. After returned products are purchased from the first customers, reverse flow of the products are determined via fixed ratios. Because of the presence of the second customers, remanufactured products are not sold as new products. While this study includes multi-products, it focuses on product recovery rather than part or material recovery because, there is a single type of part which is supplied or recycled as new part by supplier. Stochastic parameters of the problem consisted of demands and sales price of first customers, return rate, sales price of second customers, purchasing price of returned products and maximum numbers of each opened facility. Uncertainty in this problem is handled by using a scenario-based approach and solving the problem for each scenario separately. The objective is to maximize the total profit which includes the first and second product sales and facility opening, product transportation, holding and returned product purchasing costs. Decisions include binary facility opening, transportation link activation, continuous flow and inventory holding decisions for each scenario. Exact solutions are obtained for 11 different scenarios by using a solver (CPLEX). Analysis is based on three criteria: mean, standard deviation and coefficient of variation. In addition, sensitivity analysis is performed by decreasing and increasing fixed costs by 50%. The analysis proves the reliability of scenario-based approach using multiple criteria (mean, integrated mean-risk ap-

proach) decision-making procedure.

Listeş (2007) studies a generic stochastic model for CLSC network design with relatively simple network structure including echelons of only plant, market and facility. In this proposed network, forward flow is from plant to market and reverse flow is from market to facility and then to plant. In addition, each flow is restricted to have one to one relation between two sides. The network does not include any second customers and remanufactured products are considered as new ones. Uncertainty in this problem exists in demand and return rate parameters and is handled with two-stage stochastic programming approach. The objective of the proposed model is to maximize the expected profit including product sales and facility opening and transportation costs. Decisions include first-stage binary facility opening decisions for plants and facilities, transportation link activation and second-stage continuous flow decisions. To solve this problem with relatively complete recourse, the decomposition approach of L-Shaped Method is used and the impact of uncertainty is investigated. Solved sample problem includes 5 instances for each three value of the number of the markets (60, 80, 100) with 15 plant and 25 facility locations. Results show that the penalty of not collecting returned goods and savings from remanufacturing are reinforcing investments on testing processes for returned products. Another implication is that while remanufactured products are seen as the same as the new ones, total demand is satisfied with the maximum number of remanufactured products possible. Impact of uncertainty is also investigated by using different demand scenarios. Lower investments in the first stage supply savings for the low demand cases but also cause some loss of market opportunities in high demand cases. On the other hand, higher investments lead to significant loss in low demand cases due to unused capacity but provide greater amounts of captured market opportunity.

Pishvae et al. (2009) focus on a single-period, single-product, multi-stage CLSC network design with production/recovery centers, hybrid distribution-collection centers, customer zones, and disposal centers. In this study, both forward and reverse flows are passing from hybrid centers to customers and production/recovery centers respec-

tively. Reverse flow of products either to remanufacturing or disposal is determined by fixed rates. Additionally, recovered products enter the forward flow as new ones. Uncertainty in this problem exists in most of the parameters including demand, return rate, disposal fraction, transportation costs, processing costs and penalty costs for non-utilized capacity. The problem is formulated as a two-stage stochastic program. The objective is to minimize the expected total costs composed of facility opening, processing and transportation costs by optimizing first-stage binary facility opening decisions of production/recovery centers, hybrid centers and disposal centers, and second-stage continuous flow decisions. Resulting model is solved in its extensive form and solutions are analyzed. Analyses, including comparison between deterministic and stochastic solution show that non-utilized capacity costs are higher in stochastic solution due to more decentralized network structure. In addition, robustness price is investigated by decreasing fixed costs of opening which has shown that stochastic solution has a steeper decrease in objective function value with decreasing fixed costs. Further analysis demonstrates that neither stochastic nor deterministic solutions are highly sensitive to transportation costs. Moreover, total cost is more sensitive to demand compared to return ratio while both parameters are increasing the total cost.

Lee et al. (2010) study a single-product, single-period CLSC network problem. In this problem, the echelons are manufacturers, depots and customers. Depots are supposed to be used as hybrid, collection or distribution centers but processes on returned products are unclear. Uncertainty of this problem arises from demand and supply of returned products, and is handled by using two-stage stochastic programming. The objective is to minimize total cost generated by facility opening, processing and transportation costs. Decisions in this problem are first-stage binary facility opening and second-stage continuous shipping decisions. In solving this problem, integrated Sample Average Approximation method (SAA) is used to improve the solution efficiency and to reduce the variance. The problem is solved under two settings: sequential solution which considers forward decisions first and then the reverse decisions, integrated solution which considers the forward and reverse decisions simultaneously. Results

show that an integrated solution provides more cost efficient network and customer accessibility due to decentralized structure. Return rate sensitivity analysis demonstrates that, integrated method is more cost efficient with the increasing value of the return rate from 5% to 90%.

Jeihoonian et al. (2017) study a CLSC network design for durable products which consists of component and raw material suppliers, manufacturers, distribution centers and customers in forward flow of products. Reverse flow of products is among collection, disassembly, remanufacturing, bulk recycling, material recycling and disposal centers. In reverse direction, firstly, collected products are disassembled according to reverse BOMs and resulted into reusable parts, modules, materials and residues depending on random quality status of the disassembled product. Reusable parts are directly used in manufacturing of brand new products. Modules are remanufactured to be either used in manufacturing of brand new products or sold to module markets. Materials are recycled to be either used in manufacturing of brand new products or sold to recycled material markets or disposed. Residues are treated in bulk recycling centers and then can be forwarded to material recycling or disposal. Quality status of the returned product is uncertain and the problem is modeled using two-stage stochastic programming with recourse. The objective is to maximize the expected profit which is generated by the sales of products, modules and materials to relevant markets and decreased by facility opening, flow and processing costs. First-stage decisions include flow of the forward products besides the binary facility opening decisions. Second-stage decisions consist of reverse flow decisions that are made after the uncertainties on quality status are revealed. To reduce the computational complexity of the problem, a scenario reduction algorithm is used. Resulting problem is solved by using enhanced L-Shaped method with Pareto-cut selection scheme which provides improved performance by selecting deeper cuts when multiple cuts are available to be added. In the case study with durable products, 4096 scenarios are reduced using a scenario reduction algorithm to two alternatives with sizes of 500 and 1000. For each alternative, five classes with five randomly generated test instances are created. Results show that, in most of the instances, the extensive form cannot be

solved within the time limit of 7,200 seconds and lead to higher optimality gaps for larger instances. Therefore, the obtained feasible solutions are considerably far from the optimal solutions given by the enhanced L-Shaped method. In the case with 1000 scenarios, extensive solution could not be obtained for most of the instances due to time limit and memory issues. The enhanced L-Shaped method has solved almost all instances with 0.5% optimality gap.

Üster and Hwang (2016) study a CLSC network design problem which consists of sourcing facilities (SF), centers (CTR) and demand locations (RT). Sourcing facilities include suppliers/manufacturers and remanufacturers. Centers are composed of distributors and collectors. Demand locations are retailers/customers. In this network, suppliers/manufacturers supply products to customers through distribution centers and then products are returned from customers to remanufacturers through collection centers. Remanufacturers are assumed to be located only at the locations where manufacturing facilities are present and recovered products are considered to be perfect substitutes of brand new products. Additionally, co-location of distribution and collection centers is possible. Uncertain parameters include demand and return ratio of products. The problem is modeled by using two-stage stochastic programming with the objective of total expected cost minimization. Decisions of the problem consist of binary facility opening decisions and integer capacity expansion decisions in the first stage and continuous flow decisions in the second stage. The problem is solved by using enhanced Benders' Decomposition algorithm which supplies acceleration of convergence with surrogate constraints, scenario-based multiple cuts, strengthened cuts and mean-value scenario-based lower-bounding inequalities. For computational experiments, 12 problem classes with 10 test instances are considered. Results of multi-cut approach show that the best performance is achieved by using an approach including grouped cuts in time limit with 2% optimality gap. An improvement which includes two-phase method for strengthening group cuts demonstrates better performance than regular algorithm. Analysis of mean-value cut approach indicates that best performance is achieved by combining strengthened grouped cuts with mean-value cuts that are based on dual subproblem and separation schemes. In

addition, deterministic equivalent of the problem is solved by using a solver (CPLEX) with 50, 100 and 250 scenarios. In 250 scenario case, problem can not be solved in its extensive form. This result proves the usefulness of BD framework in this type of problems with higher number of scenarios. Besides computational performances, some managerial insights are obtained. Analyses on recovery location and rates show that, under high recovery rates, selecting CTRs as inspection centers is better especially if inspection costs are sensitive to location. Under low recovery rates, selecting RTs for inspection should be preferred to avoid redundant processing, transportation and capacity expansion costs by disposing insignificant returns earlier. Comparisons also show that, in high recovery case, if inspection is at SFs, all of the SFs are used as hybrid SFs for both manufacturing and remanufacturing processes. In low recovery case, if inspections are at CTRs or RTs, less number of SFs serve as hybrid SFs meaning that, inspection on earlier stages lead to a decrease in remanufacturing facility opening costs. Evaluations of relative VSS show that stochastic solution is more favorable if second-stage costs are relatively higher than first-stage costs. Finally, analysis on EV solution points out that EV solution provides some information about location decisions in the network however, it is significantly dependent on uncertainty of the parameters and hence, RP solution performs better compared to the EV solution in this type of problems.

Different from other studies, Chouinard et al. (2008) consider the bill-of-material (BOM) structure of the products which led to a more production-oriented study. In this study, the network includes user zones, service centers, processing centers, warehouses and suppliers. Service centers are used as facilities for forward and reverse flow of the products. Processing centers are distinguished as valorization centers for recovery of the products and also disposal centers. Warehouses are used to store products before their delivery to valorization or service centers. Uncertain parameters include demand, return rates and the quality of the returned product. Quality of the returned product determines the direction of the reverse flow and defined by five different product states: unknown, new, good, deteriorated, and unusable. To model this problem, two-stage stochastic programming approach is used. Objective of the



proposed model is to minimize the expected total cost which is composed of the fixed facility usage and assignment costs and the variable flow and processing costs. Decision variables are the first-stage binary facility usage and second-stage flow and processing decision variables. Heuristics based on the SAA method are used to solve the problem. The proposed methods are implemented on a case study including 13 service centers, 6 valorization centers, 2 disposal centers, 6 warehouses, 3 products, 16 part families and 62 user zones. Results demonstrate that stochastic solution tends to restrict the number of valorization centers and warehouses more strictly than deterministic solution. In addition, stochastic solution increases disassembly of products which decreases objective value by 0.35%. Besides, when demand is satisfied only with the new products, operating costs increase by 20% ignoring the recovery processes.

### **2.1.3 Contributions of Our Study**

Among the studies reviewed in Sections 2.1.1 and 2.1.2, Jeihoonian et al. (2017) and Üster and Hwang (2016) are the most relevant ones to ours. We summarize the similarities and differences in Table 2.2. Our objective is expected profit maximization. We consider uncertainty in demand, return and quality levels. We focus on recovery in lower levels such as part and material levels. As in other studies, we used L-shaped method with improvements but, we consider two types of implementations (iterative and branch-and-cut based). Similar with Üster and Hwang (2016), we introduce improvements on L-shaped method such as using scenario-based multiple cuts and mean-value cuts. In addition to this, we consider solving our problem with branch-and-cut based L-shaped method and improving grouping measure to increase performance of scenario-based multiple cuts. In Jeihoonian et al. (2017), they use Pareto-cut selection scheme to increase performance of L-shaped method. Our study differs from this study with implementation of branch-and-cut based L-shaped method, improvements on grouping strategy and consideration of mean-value cuts. In addition to the differences in the solution approach, we include part markets, disassembly processes, co-location of distribution and collection centers, refurbished

product markets and capacity expansion decisions which are included in these studies partially.

Table 2.2: Comparison of Our Study with Existing Studies

	Jeihoonian et al. (2017)	Üster and Hwang (2016)	Our study
Objective	Expected Profit Maximization	Expected Cost Minimization	Expected Profit Maximization
Uncertainty	Quality	Demand, Return	Demand, Return, Quality
Recovery	Part and Material	Product	Part and Material
Solution	Enhanced L-Shaped Method	Strengthened BD	Iterative and Branch-and-Cut based L-Shaped Method
Part Market	+	-	+
Disassembly	+	-	+
Hybrid Centers	-	+	+
Refurbished Product Market	-	+	+
Capacity Expansion	-	+	+

## 2.2 Background Information on Stochastic Programming

Stochastic programming is a type of mathematical programming where some of the parameters are uncertain. It is first introduced by Dantzig (1955) as “Linear Programming under Uncertainty”. Values of the uncertain parameters are defined explicitly for each scenario with known probabilities. The aim of this approach is to optimize given objective function with a solution that is feasible for all possible values of the uncertain parameters. Therefore, different from deterministic programs, solving a stochastic program yields an expected value of the objective function. Application of stochastic programming includes several types of problems such as inventory, assembly, portfolio selection and supply chain network design problems (Shapiro et al. (2009)).

In our study, we use two-stage stochastic programming to formulate the problem and utilize variants of the L-Shaped Method to solve instances of our problem. In this section, we briefly introduce these concepts.

### 2.2.1 Two-Stage Stochastic Programming

Two-stage stochastic programming is the simplest stochastic programming framework where decisions are grouped by dividing the problem into two stages. In the first stage, decisions which affect second-stage decisions are made. Some examples from different problem settings could be facility opening, capacity installation or any initial investment decisions which constraint the latter decisions. In the second stage, uncertainties are resolved and second-stage decisions are made for the realized scenario. A general formulation of two-stage stochastic linear program is as follows (Birge and Louveaux, 2011):

$$\begin{aligned}
 \min z = & \quad c^T x + E_{\xi}[\min q(\omega)^T y(\omega)] \\
 \text{s.t.} & \quad Ax = b, \\
 & \quad T(\omega)x + W(\omega)y(\omega) = h(\omega), \\
 & \quad x \geq 0, y(\omega) \geq 0,
 \end{aligned} \tag{2.1}$$

In this formulation, first-stage decisions are expressed by  $x$  vector, which is multiplied by first-stage cost coefficients  $c$  to generate the first-stage component of the objective function. The expectation term in the objective function represents the expected second-stage cost which depends on the second-stage cost coefficients  $q$  and second-stage decisions  $y$ , which are defined for each scenario  $\omega$ . Note that  $\xi$  represents the collection of random parameters in the problem, and hence depends on the scenario (i.e.,  $\xi$  is an abbreviated representation of  $\xi(\omega)$ ). In the constraints, apart from the first-stage constraints  $Ax = b$ , we have stochastic components for each scenario  $\omega$ .  $T(\omega)$  is the technology matrix for scenario  $\omega$  and is associated with the first-

stage decisions.  $h(\omega)$  is the stochastic RHS vector of the second-stage constraint set. And finally,  $W(\omega)$  is the recourse matrix for scenario  $\omega$ . The formulation presented in (2.1) is also known as two-stage stochastic program with recourse and sometimes shortly referred to as recourse problem.

As stated in Birge and Louveaux (2011), the deterministic equivalent program (DEP) of the problem (2.1) is given by:

$$\begin{aligned} \min z &= c^T x + \mathcal{Q}(x) \\ \text{s.t. } Ax &= b, \\ x &\geq 0 \end{aligned} \quad (2.2)$$

where

$$\mathcal{Q}(x) = E_{\xi}[Q(x, \xi(\omega))] \quad (2.3)$$

and

$$Q(x, \xi(\omega)) = \min_y \{q(\omega)^T y \mid Wy = h(\omega) - T(\omega)x, y \geq 0\} \quad (2.4)$$

First-stage decisions,  $x$ , are made when realizations of  $\xi$  are unknown. After  $x$  is determined, uncertainty is resolved and second-stage decisions,  $y$ , are made.  $\mathcal{Q}(x)$  estimates the expected value of the first-stage decisions,  $x$ .

If we explicitly represent  $Q(x, \xi(\omega))$  for each scenario, we obtain the extensive form (EF) of the presented stochastic program. As stated in Birge and Louveaux (2011), this discretized version of the DEP is as follows:

$$\begin{aligned} \min \quad & c^T x + \sum_{k=1}^K p_k q_k^T y_k \\ \text{s.t.} \quad & Ax = b \\ & T_k x + W y_k = h_k, \quad k = 1, \dots, K \\ & x \geq 0, y_k \geq 0, \quad k = 1, \dots, K \end{aligned} \quad (2.5)$$

where  $k$  represents the scenario index and  $K$  represents the total number of scenarios in the problem.

## 2.2.2 Decomposition Methods

As the number of scenarios in a recourse problem increases, the computational time required to find an optimal solution multiplies. To solve this kind of difficult problems, several decomposition methods are developed where the problem is solved by decomposing it into main and subproblems. A commonly used method is the L-Shaped Method which is a special implementation of the Benders' Decomposition (BD). In this method, different from BD, the subproblem is the second-stage of the stochastic program and is solved for each scenario separately.

### 2.2.2.1 L-Shaped Method

Main idea of the L-Shaped method is to successfully approximate the expected objective value of the second stage. In this method, problem is divided into two problems: main problem that includes the first-stage decisions and subproblem which represents the second-stage problem for a given first-stage solution. Standard L-Shaped method works on an iterative basis where, the restricted master problem (RMP) is solved to optimality in each iteration. Then, by solving subproblems, feasibility and/or optimality cuts are generated and added to the RMP. After a cut added, iteration number increases and RMP is solved again. This process continues until the point where no more cuts are needed to be added which proves the optimality. As stated in Birge and Louveaux (2011), algorithm of the L-shaped method is as follows:

*Step 0.* Set  $r = s = v = 0$

*Step 1.* Set  $v = v + 1$ . Solve the following linear program

$$\min \quad c^T x + \theta \quad (2.6)$$

$$\text{s.t.} \quad Ax = b$$

$$D_\ell x \geq d_\ell, \quad \ell = 1, \dots, r, \quad (2.7)$$

$$E_\ell x + \theta \geq e_\ell, \quad \ell = 1, \dots, s, \quad (2.8)$$

$$x \geq 0, \quad \theta \in \Re$$

where (2.7) is a feasibility cut and (2.8) is an optimality cut. In the first iteration, while there is no constraints on  $\theta$  which is the approximation of the second-stage objective value, it is equal to  $-\infty$ .

*Step 2.* Check if  $x$  is second-stage feasible. If not, add at least one feasibility cut ( $D_\ell x \geq d_\ell$ ), update  $r = r + 1$  and return to *Step 1*. If  $x$  is second-stage feasible, continue to *Step 3*. To obtain the feasibility cut, solve the following linear program for  $k = 1, \dots, K$

$$\min w' = e^T v^+ + e^T v^-$$

$$\text{s.t.} \quad Wy + Iv^+ - Iv^- = h_k - T_k x^v \quad (2.9)$$

$$y \geq 0, \quad v^+ \geq 0, v^- \geq 0,$$

where  $e^T = (1, \dots, 1)$ , for  $w' > 0$ . Let  $\zeta^v$  be the simplex multipliers and compute the following:

$$D_{r+1} = (\zeta^v)^T T_k \quad (2.10)$$

$$d_{r+1} = (\zeta^v)^T h_k \quad (2.11)$$

*Step 3.* For  $k = 1, \dots, K$ , solve the following linear program

$$\min w = q_k^T y$$

$$\text{s.t.} \quad Wy = h_k - T_k x^v \quad (2.12)$$

$$y \geq 0$$

where  $x^v$  is the first-stage decisions for iteration  $v$ . For any optimal solution to the problem for scenario  $k$ ,  $\pi_k^v$  gives the vector of simplex multipliers. To obtain the optimality cut, compute the following

$$E_{s+1} = \sum_{k=1}^K p_k (\pi_k^v)^T T_k \quad (2.13)$$

and

$$e_{s+1} = \sum_{k=1}^K p_k (\pi_k^v)^T h_k \quad (2.14)$$

Let  $w^v = e_{s+1} - E_{s+1}x^v$ . If  $\theta^v$ , which is the approximation of the second-stage objective value for iteration  $v$  is greater than  $w^v$  ( $\theta^v \geq w^v$ ) stop;  $x^v$  is the optimal solution. If not, set  $s = s + 1$ , add  $E_{s+1}x^v + \theta^v \geq e_{s+1}$  to the constraints and return to *Step 1*.

### 2.2.2.2 Branch-and-Cut Based L-Shaped Method

This method differs from the standard L-Shaped method in terms of the process of cut addition. Both problems in (2.6) and (2.12) stay the same and the cut generation process is the same with the standard L-Shaped method. In standard L-Shaped Method we add cuts when an optimal solution to the RMP is obtained however, in Branch-and-Cut based L-Shaped Method, cuts are added at each integer feasible solution of RMP. Hence, unlike the standard L-Shaped Method, RMP is solved only once which enable us to decrease computational time if RMP is difficult to solve.

### 2.2.3 The Value of Information and the Stochastic Solution

In the presence of uncertainty, there are two important measures to be considered when using stochastic programming approach, which are the expected value of perfect information (EVPI) and the value of the stochastic solution (VSS).

EVPI is the amount which the decision maker would pay in exchange of perfect future information as defined in Madansky (1960). Consider the generic form of one scenario two-stage stochastic program which can be seen below.

$$\begin{aligned} \min z(x, \xi) \quad & c^T x + \min\{q^T y \mid W(\omega)y = h(\omega) - T(\omega)x, y \geq 0\} \\ \text{s.t.} \quad & Ax = b, x \geq 0 \end{aligned} \quad (2.15)$$

Suppose that  $x'(\xi)$  is an optimal solution to above problem and hence for all scenarios  $z(x'(\xi), \xi)$  denotes objective value of the optimal solution for each scenario. As defined in Madansky (1960), expectation of  $z(x'(\xi), \xi)$  gives the wait-and-see solution

(WS) which is as follows:

$$\begin{aligned} WS &= E_{\xi}[\min_x z(x, \xi)] \\ &= E_{\xi}[z(x'(\xi), \xi)] \end{aligned} \quad (2.16)$$

Recourse problem is defined in (2.1). By solving recourse problem, we obtain the here-and-now (stochastic) solution (RP) which is the following:

$$RP = \min_x E_{\xi} z(x, \xi) \quad (2.17)$$

EVPI can be written as the difference between WS and RP as follows:

$$EVPI = RP - WS \quad (2.18)$$

VSS is the difference between the stochastic solution and the expected value of the expected value solution. Expected value problem is a single scenario problem where uncertain parameters are set to their mean values. Solution of this problem which is called as expected value solution or mean value solution (EV) is defined as follows as in Birge and Louveaux (2011):

$$EV = \min_x z(x, \bar{\xi}) \quad (2.19)$$

where  $\bar{\xi} = E(\xi)$  is the mean value of the vector of uncertain parameters. In this case, an optimal solution to (2.19) which is called as expected value solution can be denoted as  $\bar{x}(\bar{\xi})$ . Therefore, expected value of using the mean-value solution is obtained by solving second-stage problem for each scenario with given  $\bar{x}(\bar{\xi})$  which can be defined as the following:

$$EEV = E_{\xi}(z(\bar{x}(\bar{\xi}), \xi)) \quad (2.20)$$

VSS can be defined as the difference between RP and EEV which is as follows:

$$VSS = EEV - RP \quad (2.21)$$



## CHAPTER 3

### PROBLEM DEFINITION

#### 3.1 Scope of the Problem

We consider the problem of designing a single-product CLSC network under demand, return and quality uncertainty. The main components of our network are suppliers, plants, facilities (distribution/collection centers, disassembly centers, recycling centers and disposal centers) and customers (customers for brand new products, customers for refurbished products and customers in the spare part market). Distribution/collection centers (DCC) can be used only for distribution or only for collection or for both purposes at the same time. General structure of the proposed network can be seen in Figure 3.1. Plants, disassembly centers, DCCs, customers and recycling centers are used in both forward and reverse flows. Suppliers appear only in the forward flow structure and disposal centers and spare part market are included only in the reverse flow structure.

To begin with a simpler structure which would allow us to focus more on managerial insights, we consider a single-product network. This product is composed of two different parts and each part is composed of three materials. Bill-of-material (BOM) structure is kept at this basic level to acquire sufficient amount of insight about effects of multi-level BOM structure on CLSC network design without making the problem very complex. By including disassembly centers, we aim to see the effect of parameters related to multiple production and recovery levels. We include spare part markets

to explore the effect of usable part sales in a CLSC framework. Forward flow on the network starts with the raw material supply from suppliers to plants. In plants, materials are first converted to parts (manufacturing) and these parts are assembled to form new products (assembly). Products are then distributed to customers for brand new products via DCCs which serve as distribution centers in the forward flow.

After products are used until the end of their useful life for the users, with an uncertain return rate ( $\alpha_k$ ), they are collected through DCCs which are used as collection centers in reverse flow. Depending on collected product availability for recovery, which is determined by uncertain quality rate of recovery ( $\beta_k$ ), these products are sent either to the disassembly or to disposal centers. Products which are sent to disassembly centers are dismantled to their parts and if resulting parts are useful, they can be used either in remanufacturing ( $\leq \sigma_{jk}$ ) or in recycling ( $\rho_{jk}$ ) with corresponding uncertain quality rates. Alternatively, parts that are suitable for remanufacturing can be sold to the spare part markets ( $\leq \sigma_{jk}$ ). Remaining parts at the disassembly centers are sent to the disposal centers. If parts are sent to the recycling centers, they are converted to raw materials which are used in brand new product production. At this point, part to raw material conversion is dependent on its uncertain quality rate of recycling ( $\delta_{jk}$ ). If parts are sent to the plants for remanufacturing, they are utilized in production of refurbished products and these products are sold to customers as refurbished products.

We formulate the problem as a two-stage stochastic program that maximizes the expected profit. Scenario-based uncertain parameters are return rate, recoverable product rate, remanufacturable part rate, recyclable part rate, recycling rate, demand for brand new products, demand for refurbished products and demand for spare parts. Objective function consists of the revenue, the first-stage cost and the expected second-stage cost. The revenue is generated by brand new product, refurbished product and spare part sales. The first-stage cost includes fixed facility opening costs, facility selection costs and capacity expansion costs. The second-stage cost is composed of variable raw material purchasing, part and product manufacturing, remanufacturing, distribution, collection, disassembly, disposal, recycling and transportation costs.

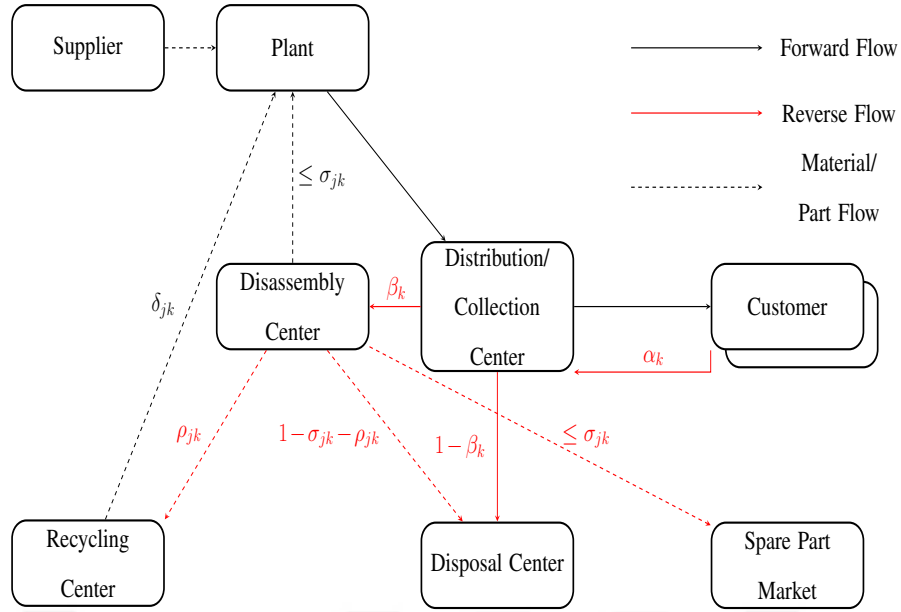


Figure 3.1: Network Structure

Under demand, return and quality uncertainty the first-stage decisions are made. The first-stage decision variables include binary facility opening decisions for plants, DCCs and disassembly centers and selection decisions of suppliers, recycling centers and disposal centers. In addition to the binary decisions, continuous capacity installation decisions for plants, DCCs and disassembly centers are also made in the first stage. Capacity of plants includes production of parts and products and remanufacturing. In DCCs, we have two types of capacity decisions which are forward and reverse capacity decisions. Sum of these dedicated capacities are constrained by total capacity of the DCC. Hence, depending on the solution, total capacity can be used either only for distribution or only for collection or for both. For suppliers, recycling centers and disposal centers, we have only maximum capacity restrictions.

When all uncertain parameters are revealed, the second-stage decisions are made. The second-stage decisions are forward and reverse flow decisions and therefore production and recovery decisions according to the described flow structure.

This problem can also be formulated as a multi-stage (e.g., three-stage) stochastic program. In that case, uncertainty in data is resolved at each stage and in the last stage, uncertain data are completely revealed. In three-stage case, first-stage decisions include facility opening and capacity decisions for forward flow. In the second stage, forward flow decisions are made, demand is observed, and facility opening and capacity decisions for reverse flow are made. In the third stage, remaining flow amounts are determined. By doing this, effects of uncertainty are captured more precisely compared to the two-stage approach. Using two-stage approach enables us to begin with a simpler structure and to obtain initial insights about the general structure and behavior of CLSCs under uncertainty.

### 3.2 Mathematical Formulation

In this section, we first introduce our notation. Then present the mathematical formulation of our problem.

#### **Indices:**

$s$ : Index for suppliers

$p$ : Index for plants

$l$ : Index for candidate DCC locations

$a$ : Index for candidate disassembly locations

$m$ : Index for customers

$n$ : Index for spare part markets

$r$ : Index for recycling centers

$w$ : Index for disposal centers

$j$ : Index for parts

$q$ : Index for materials

$k$ : Index for scenarios

**Parameters:****Fixed Costs:**

$f_s^S$ : fixed cost of selecting supplier  $s$

$f_p^P$ : fixed cost of opening plant  $p$  (i.e. opening a plant at location  $p$ )

$f_l^L$ : fixed cost of opening DCC  $l$  (i.e. opening a DCC at location  $l$ )

$f_a^A$ : fixed cost of opening disassembly center  $a$  (i.e., opening a disassembly center at location  $a$ )

$f_w^W$ : fixed cost of selecting disposal center  $w$

$f_r^R$ : fixed cost of selecting recycling center  $r$

**Variable Costs:**

$f v_p^P$ : cost of creating one unit capacity in plant  $p$

$f v_l^D$ : cost of creating one unit distribution capacity at DCC  $l$

$f v_l^C$ : cost of creating one unit collection capacity at DCC  $l$

$f v_a^A$ : cost of creating one unit capacity in disassembly center  $a$

$v_{qs}^S$ : unit purchasing cost of material  $q$  from supplier  $s$

$v_{jp}^{P1}$ : unit production cost of part  $j$  in plant  $p$

$v_p^{P2}$ : unit production cost of the product in plant  $p$

$v_p^{P3}$ : unit remanufacturing/reassembly cost of the product in plant  $p$

$v_a^A$ : unit disassembly cost of the product in disassembly center  $a$

$v_l^{L1}$ : unit distribution processing cost of the product in DCC  $l$

$v_l^{L2}$ : unit collection processing cost of the product in DCC  $l$

$v_{jr}^R$ : unit recycling cost of the part  $j$  recycling center  $r$

$v_w^{W1}$ : unit disposal cost of the product in disposal center  $w$

$v_{jw}^{W2}$ : unit disposal cost of part  $j$  in disposal center  $w$

**Transportation Costs:**

$t_{sp}^{SP}$ : unit transportation cost between supplier  $s$  and plant  $p$

$t_{pl}^{PL}$ : unit transportation cost between plant  $p$  and DCC  $l$

$t_{lm}^{LM}$ : unit transportation cost between DCC  $l$  and customer  $m$

$t_{ml}^{ML}$ : unit transportation cost between customer  $m$  and DCC  $l$

$t_{lw}^{LW}$ : unit transportation cost between DCC  $l$  and disposal center  $w$

$t_{la}^{LA}$ : unit transportation cost between DCC  $l$  and disassembly center  $a$

$t_{ap}^{AP}$ : unit transportation cost between disassembly center  $a$  and plant  $p$

$t_{ar}^{AR}$ : unit transportation cost between disassembly center  $a$  and recycling center  $r$

$t_{aw}^{AW}$ : unit transportation cost between disassembly center  $a$  and disposal center  $w$

$t_{an}^{AN}$ : unit transportation cost between disassembly center  $a$  and spare part market  $n$

$t_{rp}^{RP}$ : unit transportation cost between recycling center  $r$  and plant  $p$

**Capacities:**

$cap_s^S$ : maximum capacity of supplier  $s$

$cap_p^P$ : maximum capacity of plant  $p$

$cap_l^L$ : maximum capacity of DCC  $l$

$cap_a^A$ : maximum capacity of disassembly center  $a$

$cap_w^W$ : capacity of disposal center  $w$

$cap_r^R$ : capacity of recycling center  $r$

**Prices:**

$sp^Y$ : unit sales price of the brand new product

$sp^Z$ : unit sales price of the refurbished product

$sp_j^N$ : unit sales price of part  $j$  in the spare part market

**Rates and Other Parameters:**

$\alpha_k$ : return ratio of products in scenario  $k$

$\beta_k$ : ratio of products with value after inspection in scenario  $k$

$\sigma_{jk}$ : ratio of usable  $j$  parts in disassembly in scenario  $k$

$\rho_{jk}$ : ratio of recyclable  $j$  parts in disassembly in scenario  $k$

$\delta_{jk}$ : ratio of successful recycling of part  $j$  in scenario  $k$

$u_j^J$ : usage ratio of part  $j$  in the product

$u_{qj}^Q$ : usage ratio of material  $q$  in part  $j$

$b_q^S$ : capacity coefficient of material  $q$  in suppliers

$b_j^P$ : capacity coefficient of part  $j$  in plants

$b_j^W$ : capacity coefficient of part  $j$  in disposal centers

$b_j^R$ : capacity coefficient of part  $j$  in recycling centers

$tz_j^J$ : unit transportation coefficient of part  $j$  compared to a product

$tz_q^Q$ : unit transportation coefficient of material  $q$  compared to a product

$prob_k$ : probability of scenario  $k$

**Demands:**

$dem_{mk}^{MY}$ : demand of the brand new product at customer  $m$  in scenario  $k$

$dem_{mk}^{MZ}$ : demand of the refurbished product at customer  $m$  in scenario  $k$

$dem_{nj}^N$ : demand of spare part  $j$  in spare part market  $n$  in scenario  $k$

**Decision Variables:**

***First-Stage Decision Variables:***

$$x_s^S = \begin{cases} 1, & \text{if supplier } s \text{ is selected} \\ 0, & \text{otherwise} \end{cases}$$

$$x_p^P = \begin{cases} 1, & \text{if plant } p \text{ is opened} \\ 0, & \text{otherwise} \end{cases}$$

$$x_l^L = \begin{cases} 1, & \text{if DCC } l \text{ is opened} \\ 0, & \text{otherwise} \end{cases}$$

$$x_a^A = \begin{cases} 1, & \text{if disassembly center } a \text{ is opened} \\ 0, & \text{otherwise} \end{cases}$$

$$x_w^W = \begin{cases} 1, & \text{if disposal center } w \text{ is selected} \\ 0, & \text{otherwise} \end{cases}$$

$$x_r^R = \begin{cases} 1, & \text{if recycling center } r \text{ is selected} \\ 0, & \text{otherwise} \end{cases}$$

$g_p^P$ : amount of capacity provided to plant  $p$

$g_l^D$ : amount of distribution capacity provided to DCC  $l$

$g_l^C$ : amount of collection capacity provided to DCC  $l$

$g_a^A$ : amount of capacity provided to disassembly center  $a$

***Second-Stage Decision Variables:***

$y_{qspk}^{SP}$ : amount of material  $q$  transferred from supplier  $s$  to plant  $p$  in scenario  $k$

$y_{plk}^{PL}$ : amount of product transferred from plant  $p$  to DCC  $l$  in scenario  $k$

$y_{lmk}^{LM}$ : amount of product transferred from DCC  $l$  to customer  $m$  in scenario  $k$

$y_{mlk}^{ML}$ : amount of product transferred from customer  $m$  to DCC  $l$  in scenario  $k$

$y_{lw}^{LW}$ : amount of product transferred from DCC  $l$  to disposal center  $w$  in scenario  $k$

$y_{lak}^{LA}$ : amount of product transferred from DCC  $l$  to disassembly center  $a$  in scenario  $k$

$y_{jar}^{AR}$ : amount of part  $j$  transferred from disassembly center  $a$  to recycling center  $r$  in scenario  $k$

scenario  $k$

$y_{japk}^{AP}$ : amount of part  $j$  transferred from disassembly center  $a$  to plant  $p$  in scenario  $k$

$y_{jawk}^{AW}$ : amount of part  $j$  transferred from disassembly center  $a$  to disposal center  $w$  in scenario  $k$

$y_{jan k}^{AN}$ : amount of part  $j$  transferred from disassembly center  $a$  to spare part market  $n$  in scenario  $k$

$y_{qrp k}^{RP}$ : amount of material  $q$  transferred from recycling center  $r$  to plant  $p$  in scenario  $k$

$z_{pl k}^{PL}$ : amount of the refurbished product transferred from plant  $p$  to DCC  $l$  in scenario  $k$

$z_{lm k}^{LM}$ : amount of the refurbished product transferred from DCC  $l$  to customer  $m$  in scenario  $k$

Using the above notation, we formulate the problem as the following two-stage stochastic integer program.

$$\begin{aligned} \max z &= -(\text{First Stage Cost}) + (\text{Expected Revenue}) - (\text{Expected Second Stage Cost}) \\ &= -(\text{FC}) + (\text{ER}) - (\text{ESC}) \end{aligned} \tag{3.1}$$



where

$$\begin{aligned} \text{FC} = & \sum_s x_s^S \cdot f_s^S + \sum_p (x_p^P \cdot f_p^P + f v^P \cdot g_p^P) + \sum_l (x_l^L \cdot f_l^L + f v^D \cdot g_l^D + f v^C \cdot g_l^C) \\ & + \sum_a (x_a^A \cdot f_a^A + f v^A \cdot g_a^A) + \sum_w x_w^W \cdot f_w^W + \sum_r x_r^R \cdot f_r^R \end{aligned} \quad (3.2)$$

$$\text{ER} = \sum_k \text{prob}_k \cdot [sp^Y \cdot \sum_m \sum_l y_{lmk}^{LM} + sp^Z \cdot \sum_m \sum_l z_{lmk}^{LM} + \sum_j \sum_a \sum_n sp_j^N \cdot y_{jank}^{AN}] \quad (3.3)$$

$$\begin{aligned} \text{ESC} = & \sum_k \text{prob}_k [\sum_q \sum_s \sum_p y_{qspk}^{SP} \cdot (v_{qs}^S + t_{sp}^{SP} \cdot tz_q^Q) \\ & + \sum_p \sum_j \sum_l (u_j^J \cdot y_{plk}^{PL} + z_{plk}^{PL} \cdot u_j^J - \sum_a y_{japk}^{AP}) \cdot v_{jp}^{P1} \\ & + \sum_p \sum_l (v_p^{P2} \cdot y_{plk}^{PL} + v_p^{P3} \cdot z_{plk}^{PL}) + \sum_p \sum_l (y_{plk}^{PL} + z_{plk}^{PL}) \cdot (v_l^{L1} + t_{pl}^{PL}) \\ & + \sum_l \sum_m (y_{lmk}^{LM} + z_{lmk}^{LM}) \cdot t_{lm}^{LM} + \sum_m \sum_l y_{mlk}^{ML} \cdot (t_{ml}^{ML} + v_l^{L2}) \\ & + \sum_l \sum_a y_{lak}^{LA} \cdot (t_{la}^{LA} + v_a^A) + \sum_l \sum_w y_{lwk}^{LW} \cdot (t_{lw}^{LW} + v_w^{W1}) \\ & + \sum_j \sum_a \sum_p y_{japk}^{AP} \cdot t_{ap}^{AP} \cdot tz_j^J + \sum_j \sum_a \sum_r y_{jark}^{AR} \cdot (t_{ar}^{AR} \cdot tz_j^J + v_{jr}^R) \\ & + \sum_j \sum_a \sum_w y_{jawk}^{AW} \cdot (t_{aw}^{AW} \cdot tz_j^J + v_{jw}^{W2}) + \sum_j \sum_a \sum_n y_{jank}^{AN} \cdot t_{an}^{AN} \cdot tz_j^J \\ & + \sum_j \sum_r \sum_p y_{qrp k}^{RP} \cdot t_{rp}^{RP} \cdot tz_q^Q] \end{aligned} \quad (3.4)$$

subject to

**Flow Balance Constraints at Plants:**

$$\begin{aligned} & \sum_j \sum_l y_{plk}^{PL} \cdot u_j^J \cdot u_{qj}^Q + \sum_j [\sum_l z_{plk}^{PL} \cdot u_j^J - \sum_a y_{japk}^{AP}] \cdot u_{qj}^Q \\ & = \sum_s y_{qspk}^{SP} + \sum_r y_{qrp k}^{RP} \end{aligned} \quad \forall q, p, k \quad (3.5)$$

$$\sum_l z_{plk}^{PL} \cdot u_j^J \geq \sum_a y_{japk}^{AP} \quad \forall j, p, k \quad (3.6)$$

**Flow Balance Constraints at DCCs:**

$$\sum_p y_{plk}^{PL} = \sum_m y_{lmk}^{LM} \quad \forall l, k \quad (3.7)$$

$$\sum_p z_{plk}^{PL} = \sum_m z_{lmk}^{LM} \quad \forall l, k \quad (3.8)$$

$$\sum_m y_{mlk}^{ML} = \sum_a y_{lak}^{LA} + \sum_w y_{lwk}^{LW} \quad \forall l, k \quad (3.9)$$

$$\sum_m y_{mlk}^{ML} \cdot (1 - \beta_k) \leq \sum_w y_{lwk}^{LW} \quad \forall l, k \quad (3.10)$$

**Flow Balance Constraints at Customers:**

$$\sum_l y_{lmk}^{LM} \cdot \alpha_k \geq \sum_l y_{mlk}^{ML} \quad \forall m, k \quad (3.11)$$

**Flow Balance Constraints at Disassembly Centers:**

$$\sum_l y_{lak}^{LA} \cdot \rho_{jk} \cdot u_j^J \geq \sum_r y_{jark}^{AR} \quad \forall j, a, k \quad (3.12)$$

$$\sum_l y_{lak}^{LA} \cdot \sigma_{jk} \cdot u_j^J = \sum_p y_{japk}^{AP} + \sum_n y_{jan}^{AN} \quad \forall j, a, k \quad (3.13)$$

$$\sum_l y_{lak}^{LA} \cdot (1 - \sigma_{jk} - \rho_{jk}) \cdot u_j^J \leq \sum_w y_{jawk}^{AW} \quad \forall j, a, k \quad (3.14)$$

$$\sum_l y_{lak}^{LA} \cdot u_j^J = \sum_p y_{japk}^{AP} + \sum_n y_{jan}^{AN} + \sum_w y_{jawk}^{AW} + \sum_r y_{jark}^{AR} \quad \forall j, a, k \quad (3.15)$$

**Flow Balance Constraints at Recycling Centers:**

$$\sum_j \sum_a y_{jark}^{AR} \cdot u_{qj}^Q \cdot \delta_{jk} = \sum_p y_{qrp}^{RP} \quad \forall r, q, k \quad (3.16)$$

**Capacity Constraints:**

$$g_p^P \geq \sum_l (y_{plk}^{PL} + z_{plk}^{PL}) + \sum_j \sum_a b_j^P \cdot y_{jap}^{AP} \quad \forall p, k \quad (3.17)$$

$$g_l^D \geq \sum_p (y_{plk}^{PL} + z_{plk}^{PL}) \quad \forall l, k \quad (3.18)$$

$$g_l^C \geq \sum_m y_{mlk}^{ML} \quad \forall l, k \quad (3.19)$$

$$g_a^A \geq \sum_l y_{lak}^{LA} \quad \forall a, k \quad (3.20)$$

$$x_s^S \cdot cap_s^S \geq \sum_p \sum_q b_q^S \cdot y_{qsp}^{SP} \quad \forall s, k \quad (3.21)$$

$$x_w^W \cdot cap_w^W \geq \sum_l y_{lwk}^{LW} + \sum_a \sum_j b_j^W \cdot y_{jawk}^{AW} \quad \forall w, k \quad (3.22)$$

$$x_r^R \cdot cap_r^R \geq \sum_a \sum_j b_j^R \cdot y_{jark}^{AR} \quad \forall r, k \quad (3.23)$$

$$x_p^P \cdot cap_p^P \geq g_p^P \quad \forall p \quad (3.24)$$

$$x_l^L \cdot cap_l^L \geq g_l^D + g_l^C \quad \forall l \quad (3.25)$$

$$x_a^A \cdot cap_a^A \geq g_a^A \quad \forall a \quad (3.26)$$

**Demand Constraints:**

$$\sum_l y_{lmk}^{LM} \leq dem_{mk}^{MY} \quad \forall m, k \quad (3.27)$$

$$\sum_l z_{lmk}^{LM} \leq dem_{mk}^{MZ} \quad \forall m, k \quad (3.28)$$

$$\sum_p y_{jan k}^{AN} \leq dem_{njk}^N \quad \forall n, j, k \quad (3.29)$$

**Non-negativity and Set Constraints:**

$$x_s^S, x_p^P, x_l^L, x_a^A, x_w^W, x_r^R \in \{0, 1\} \quad \forall s, p, l, a, w, r \quad (3.30)$$

$$g_p^P, g_l^D, g_l^C, g_a^A \geq 0 \quad \forall p, l, a \quad (3.31)$$

$$y_{qspk}^{SP}, y_{plk}^{PL}, z_{plk}^{PL}, y_{lmk}^{LM}, z_{lmk}^{LM} \geq 0 \quad \forall q, s, p, l, m, k \quad (3.32)$$

$$y_{mlk}^{ML}, y_{lak}^{LA}, y_{lwk}^{LW}, y_{jap k}^{AP}, y_{jan}^{AN} \geq 0 \quad \forall p, j, m, l, a, n, w, k \quad (3.33)$$

$$y_{jar k}^{AR}, y_{jawk}^{AW}, y_{qrp k}^{RP} \geq 0 \quad \forall q, j, p, a, r, w, k \quad (3.34)$$

Expected revenue (3.3) consists of the revenue generated through the sales of brand new products, refurbished products and spare parts. First stage costs (3.2) include facility opening and capacity expansion costs of plants, DCCs and disassembly centers and facility selection costs of suppliers, disposal and recycling centers. Expected second-stage costs (3.4) are composed of product and part transportation, production, forward and reverse product processing, product disassembly, part recycling and product and part disposal costs.

Constraint (3.5) ensures that outgoing products from plants should be equal to the number of manufactured products using materials from suppliers or recycling centers. Constraint (3.6) guarantees that all of the reusable parts sent to the plant are used in remanufacturing.

Constraints (3.7) and (3.8) represent the forward flow balance at DCCs for new and remanufactured products, respectively. Constraint (3.9) ensures that collected products are sent to either disassembly or disposal centers. Constraint (3.10) guarantees that unusable products are sent to disposal, and usable ones can also be sent to disposal if needed.

Constraint (3.11) restricts the amount of returned products by the return rate.

Constraint (3.12) restricts the amount of parts sent to recycling by the amount of recyclable parts. Constraint (3.13) ensures that usable parts are either sent to the remanufacturing or to the spare part market from disassembly center. Constraint (3.14) guarantees that unusable parts are sent to disposal, and usable parts can also be sent to disposal if necessary. Constraint (3.15) represents the flow balance between returned products and outflow of parts in disassembly center.

Constraint (3.16) restricts amount of recycled parts by the success rate of the recycling process. In writing flow balance constraints we considered lowest levels (product, part, raw material) for each constraint and used conversion factors  $(u_j^J, u_{qj}^Q)$ .

Constraint (3.17) ensures that total capacities used by inflow of reusable parts and outflow of new and remanufactured products are less than the capacity of plants. Constraint (3.18) guarantees that total amounts of inflow of new and remanufactured products are less than installed distribution capacities of DCCs. Constraints (3.19)

and (3.20) make sure that amounts of returned products enter the DCCs and disassembly centers are limited to installed capacities in these facilities. Constraint (3.21) ensures that total amount of material outflow does not exceed the maximum capacities of suppliers. Constraint (3.22) guarantees that total amount of inflow of parts and products are less than maximum capacities of disposal centers. Constraint (3.23) restricts total amount of inflow of recyclable parts by maximum capacities of recycling centers. Constraints (3.24), (3.25) and (3.26) make sure that capacity installations on plants, DCCs and disassembly centers are restricted by the maximum allowed capacities of these facilities. Capacity constraints include capacity coefficients ( $b_q^S$ ,  $b_j^P$ ,  $b_j^W$ ,  $b_j^R$ ) for parts and raw materials to convert them to product scale.

Constraints (3.27), (3.28) and (3.29) represent the limitations on maximum demand level for first, second products and spare part markets, respectively. And finally, constraints (3.30)-(3.34) represent the non-negativity and set restrictions.

## CHAPTER 4

### SOLUTION METHODS

To solve the two-stage stochastic integer program presented in previous chapter, we consider three main methods: solving the extensive form (EF), iterative L-shaped method (LS) and branch-and-cut based L-shaped method (BC). Solving the extensive form, which is a single large size problem with all second-stage decision variables and constraints explicitly defined for each scenario, is computationally inefficient. To achieve computational efficiency, we use two main variants of the L-shaped method. In both implementations, the problem is decomposed into two problem groups: a restricted master problem (RMP) and a subproblem (SP) for each scenario. In our implementation, we considered the sense of the objective as minimization and treated the objective function accordingly. The initial RMP and the SP for each scenario are as follows:

$$\begin{aligned} \text{RMP: } \min z = & \quad (3.2) \\ \text{s.t. } & \quad (3.24), (3.25), (3.26), \\ & \quad (3.30), (3.31) \end{aligned} \tag{4.1}$$

SP for scenario k:

$$\begin{aligned}
\text{SP: } \min z = & -sp^Y \cdot \sum_m \sum_l y_{lmk}^{LM} - sp^Z \cdot \sum_m \sum_l z_{lmk}^{LM} - \sum_j \sum_a \sum_n sp_j^N \cdot y_{jank}^{AN} \\
& + \sum_q \sum_s \sum_p y_{qspk}^{SP} \cdot (v_{qs}^S + t_{sp}^{SP} \cdot tz_q^Q) \\
& + \sum_p \sum_j \sum_l (u_j^J \cdot y_{plk}^{PL} + z_{plk}^{PL} \cdot u_j^J - \sum_a y_{japk}^{AP}) \cdot v_{jp}^{P1} \\
& + \sum_p \sum_l (v_p^{P2} \cdot y_{plk}^{PL} + v_p^{P3} \cdot z_{plk}^{PL}) + \sum_p \sum_l (y_{plk}^{PL} + z_{plk}^{PL}) \cdot (v_l^{L1} + t_{pl}^{PL}) \\
& + \sum_l \sum_m (y_{lmk}^{LM} + z_{lmk}^{LM}) \cdot t_{lm}^{LM} + \sum_m \sum_l y_{mlk}^{ML} \cdot (t_{ml}^{ML} + v_l^{L2}) \\
& + \sum_l \sum_a y_{lak}^{LA} \cdot (t_{la}^{LA} + v_a^A) + \sum_l \sum_w y_{lwk}^{LW} \cdot (t_{lw}^{LW} + v_w^{W1}) \\
& + \sum_j \sum_a \sum_p y_{japk}^{AP} \cdot t_{ap}^{AP} \cdot tz_j^J + \sum_j \sum_a \sum_r y_{jark}^{AR} \cdot (t_{ar}^{AR} \cdot tz_j^J + v_{jr}^R) \\
& + \sum_j \sum_a \sum_w y_{jawk}^{AW} \cdot (t_{aw}^{AW} \cdot tz_j^J + v_{jw}^{W2}) + \sum_j \sum_a \sum_n y_{jank}^{AN} \cdot t_{an}^{AN} \cdot tz_j^J \\
& + \sum_j \sum_r \sum_p y_{qrpk}^{RP} \cdot t_{rp}^{RP} \cdot tz_q^Q \\
\text{s.t. } & (3.5), (3.6), (3.7), (3.8), (3.9) \\
& (3.10), (3.11), (3.12), (3.13), (3.14), \\
& (3.15), (3.16), (3.17), (3.18), (3.19), (3.20), \\
& (3.21), (3.22), (3.23), (3.27), (3.28), (3.29), \\
& (3.32), (3.33), (3.34)
\end{aligned} \tag{4.2}$$

With the addition of the first optimality cut, the auxiliary variable  $\theta$  is introduced in the RMP with coefficient of 1 in the objective function. RMP is an integer program, which is initially small and easily solvable. However, it becomes a large sized and difficult to solve problem as the number of optimality cuts increase throughout the iterations. SP is an LP, which makes it viable to use the L-shaped method which is a decomposition method with duality-based cuts. SP is feasible for each feasible first-stage solution. Therefore, our problem has relatively complete recourse, and we use only optimality cuts while implementing the L-shaped method.

In order to increase solvability, we consider adding multiple optimality cuts instead of a single cut and extending the RMP by adding a valid inequality constructed based on the mean value problem.



## 4.1 Multiple Cuts

To deliver more information from the subproblems to the master problem, multiple cuts can be added instead of aggregating all information into a single cut. We consider both a pure multi-cut approach where a cut is added for each scenario and a group-cut approach where the set of scenarios are grouped into smaller sets and an aggregate cut is added for each group. Although adding multiple cuts supplies more information to the master problem in each iteration/at each node, it also amplifies the set of constraints. Therefore, adding multiple cuts may or may not outperform the single-cut L-shaped method.

When using group-cuts, we consider two different grouping strategies: demand-based grouping and demand-rate-based grouping. Accordingly, the scenarios are sorted in a decreasing order of the considered parameter value (i.e., demand or demand-rate), and groups are constructed in a way that scenarios with highest (lowest) parameter value are in the first (last) group. In demand-based grouping, we sort scenarios according to the demand values for brand new products ( $dem_k^{MY}$ ). In demand-rate-based grouping, we also consider the effect of return and recovery rates. Therefore, we sort scenarios according to a combined measure which is the multiplication of the demand values for brand new products, return rate and product recovery rate ( $dem_k^{MY} \cdot \alpha_k \cdot \beta_k$ ).

## 4.2 Mean Value Cut

Adding a valid inequality based on the mean value problem to the RMP is another strategy that can be considered to increase the computational performance. We refer to this inequality as the mean value cut. The basic principle behind this approach is to add a constraint set to the problem to bound the expected second-stage objec-

tive value by using the second-stage objective value under the mean value scenario. For the implementation of the mean value cut approach, we add the constraint set (4.3) to the RMP. In this constraint set,  $\bar{y}$  and  $\bar{z}$  are dummy flow variable vectors,  $\bar{\alpha}$ ,  $\bar{\beta}$ ,  $\bar{\sigma}$ ,  $\bar{\rho}$ ,  $\bar{\delta}$ ,  $d\bar{e}m^{MY}$ ,  $d\bar{e}m^{MZ}$  and  $d\bar{e}m^N$  are the elements of  $\bar{\xi}$  representing the mean value scenario and  $\theta$  is the auxiliary variable representing the approximation of expected second-stage objective value. Since the mean value cut is added to the RMP,  $\theta$  becomes a part of the objective function with coefficient of 1 even before the first iteration.



$$\begin{aligned}
\theta \geq & -sp^Y \cdot \sum_m \sum_l \bar{y}_{lm}^{LM} - sp^Z \cdot \sum_m \sum_l \bar{z}_{lm}^{LM} - \sum_j \sum_a \sum_n sp_j^N \cdot \bar{y}_{jan}^{AN} \\
& + \sum_q \sum_s \sum_p \bar{y}_{qsp}^{SP} \cdot (v_{qs}^S + t_{sp}^{SP} \cdot tz_q^Q) \\
& + \sum_p \sum_j \sum_l (u_j^J \cdot \bar{y}_{pl}^{PL} + u_j^J \cdot \bar{z}_{pl}^{PL} - \sum_a \bar{y}_{jap}^{AP}) \cdot v_{jp}^{P1} \\
& + \sum_p \sum_l (v_p^{P2} \cdot \bar{y}_{pl}^{PL} + v_p^{P3} \cdot \bar{z}_{pl}^{PL}) + \sum_p \sum_l (\bar{y}_{pl}^{PL} + \bar{z}_{pl}^{PL}) \cdot (v_l^{L1} + t_{pl}^{PL}) \\
& + \sum_l \sum_m (\bar{y}_{lm}^{LM} + \bar{z}_{lm}^{LM}) \cdot t_{lm}^{LM} + \sum_m \sum_l \bar{y}_{ml}^{ML} \cdot (t_{ml}^{ML} + v_l^{L2}) \\
& + \sum_l \sum_a \bar{y}_{la}^{LA} \cdot (t_{la}^{LA} + v_a^A) + \sum_l \sum_w \bar{y}_{lw}^{LW} \cdot (t_{lw}^{LW} + v_w^{W1}) \\
& + \sum_j \sum_a \sum_p \bar{y}_{jap}^{AP} \cdot t_{ap}^{AP} \cdot tz_j^J + \sum_j \sum_a \sum_r \bar{y}_{jar}^{AR} \cdot (t_{ar}^{AR} \cdot tz_j^J + v_{jr}^R) \\
& + \sum_j \sum_a \sum_w \bar{y}_{jaw}^{AW} \cdot (t_{aw}^{AW} \cdot tz_j^J + v_{jw}^{W2}) + \sum_j \sum_a \sum_n \bar{y}_{jan}^{AN} \cdot t_{an}^{AN} \cdot tz_j^J \\
& + \sum_j \sum_r \sum_p \bar{y}_{qrp}^{RP} \cdot t_{rp}^{RP} \cdot tz_q^Q \\
& \quad \sum_j \sum_l \bar{y}_{pl}^{PL} \cdot u_j^J \cdot u_{qj}^Q + \sum_j [\sum_l \bar{z}_{pl}^{PL} \cdot u_j^J - \sum_a \bar{y}_{jap}^{AP}] \cdot u_{qj}^Q \\
& \quad = \sum_s \bar{y}_{qsp}^{SP} + \sum_r \bar{y}_{qrp}^{RP} \quad \forall q, p \\
& \quad \sum_l \bar{z}_{pl}^{PL} \cdot u_j^J \geq \sum_a \bar{y}_{jap}^{AP} \quad \forall j, p \quad (4.3) \\
& \quad \sum_p \bar{y}_{pl}^{PL} = \sum_m \bar{y}_{lm}^{LM} \quad \forall l \\
& \quad \sum_p \bar{z}_{pl}^{PL} = \sum_m \bar{z}_{lm}^{LM} \quad \forall l \\
& \quad \sum_m \bar{y}_{ml}^{ML} = \sum_a \bar{y}_{la}^{LA} + \sum_w \bar{y}_{lw}^{LW} \quad \forall l \\
& \quad \sum_m \bar{y}_{ml}^{ML} \cdot (1 - \bar{\beta}) \leq \sum_w \bar{y}_{lw}^{LW} \quad \forall l \\
& \quad \sum_l \bar{y}_{lm}^{LM} \cdot \bar{\alpha} \geq \sum_l \bar{y}_{ml}^{ML} \quad \forall m \\
& \quad \sum_l \bar{y}_{la}^{LA} \cdot \bar{\rho}_j \cdot u_j^J \geq \sum_r \bar{y}_{jar}^{AR} \quad \forall j, a \\
& \quad \sum_l \bar{y}_{la}^{LA} \cdot \bar{\sigma}_j \cdot u_j^J = \sum_p \bar{y}_{jap}^{AP} + \sum_n \bar{y}_{jan}^{AN} \quad \forall j, a \\
& \quad \sum_l \bar{y}_{la}^{LA} \cdot (1 - \bar{\sigma}_j - \bar{\rho}_j) \cdot u_j^J \leq \sum_w \bar{y}_{jaw}^{AW} \quad \forall j, a \\
& \quad \sum_l \bar{y}_{la}^{LA} \cdot u_j^J = \sum_p \bar{y}_{jap}^{AP} + \sum_n \bar{y}_{jan}^{AN} \\
& \quad \quad + \sum_w \bar{y}_{jaw}^{AW} + \sum_r \bar{y}_{jar}^{AR} \quad \forall j, a \\
& \quad \sum_j \sum_a \bar{y}_{jar}^{AR} \cdot u_{qj}^Q \cdot \bar{\delta}_j = \sum_p \bar{y}_{qrp}^{RP} \quad \forall r, q
\end{aligned}$$

$$\begin{aligned}
g_p^P &\geq \sum_l (\bar{y}_{pl}^{PL} + \bar{z}_{pl}^{PL}) && \forall p \\
&+ \sum_j \sum_a b_j^P \cdot \bar{y}_{jap}^{AP} && \\
g_l^D &\geq \sum_p (\bar{y}_{pl}^{PL} + \bar{z}_{pl}^{PL}) && \forall l \\
g_l^C &\geq \sum_m \bar{y}_{ml}^{ML} && \forall l \\
g_a^A &\geq \sum_l \bar{y}_{la}^{LA} && \forall a \\
x_s^S \cdot cap_s^S &\geq \sum_p \sum_q b_q^S \cdot \bar{y}_{qsp}^{SP} && \forall s \\
x_w^W \cdot cap_w^W &\geq \sum_l \bar{y}_{lw}^{LW} + \sum_a \sum_j b_j^W \cdot \bar{y}_{jaw}^{AW} && \forall w \\
x_r^R \cdot cap_r^R &\geq \sum_a \sum_j b_j^R \cdot \bar{y}_{jar}^{AR} && \forall r \\
\sum_l \bar{y}_{lm}^{LM} &\leq \bar{dem}_m^{MY} && \forall m \\
\sum_l \bar{z}_{lm}^{LM} &\leq \bar{dem}_m^{MZ} && \forall m \\
\sum_p \bar{y}_{jan}^{AN} &\leq \bar{dem}_{nj}^N && \forall n, j \\
\bar{y}_{qsp}^{SP}, \bar{y}_{pl}^{PL}, \bar{z}_{pl}^{PL}, \bar{y}_{lm}^{LM}, \bar{z}_{lm}^{LM} &\geq 0 && \forall q, s, p, l, m \\
\bar{y}_{ml}^{ML}, \bar{y}_{la}^{LA}, \bar{y}_{lw}^{LW}, \bar{y}_{jap}^{AP}, \bar{y}_{jan}^{AN} &\geq 0 && \forall p, j, m, l, a, n, w \\
\bar{y}_{jar}^{AR}, \bar{y}_{jaw}^{AW}, \bar{y}_{qrp}^{RP} &\geq 0 && \forall q, j, p, a, r, w
\end{aligned}$$

## CHAPTER 5

### COMPUTATIONAL EXPERIMENTS

#### 5.1 Generation of Problem Instances

We consider four problem classes with different sizes. Each class includes 10 problem instances. We use our instances to test the performance of the proposed solution methods, to estimate the value of capturing uncertainty and value of information, and to investigate the impact of problem parameters on the solution. Detailed information about the problem classes are given in Table 5.1.

Table 5.1: Problem Classes

Problem Class	# of Suppliers	# of Candidate Plant Locations	# of Candidate DCC Locations	# of Customers	# of Candidate Disassembly Center Locations	# of Recycling Centers	# of Disposal Centers	# of Spare Part Markets	# of Materials	# of Parts	# of Scenarios
K1	8	5	15	30	8	5	3	3	3	2	50
K2	8	5	15	30	8	5	3	3	3	2	250
K3	10	8	20	40	10	8	3	3	3	2	250
B1	15	10	30	60	15	10	3	3	3	2	250

Our problem instances are generated based on the instances used by Üster and Hwang (2016). The data obtained from their study includes fixed costs of opening plants and DCCs, processing costs at plants and facilities, coordinates of US cities and de-

mands of customers for new and refurbished products under each scenario. Different from their study, our objective is expected profit maximization and our problem includes disassembly centers, recycling centers, disposal centers and spare part markets. Therefore, we need additional parameters for our problem. To complete our dataset, we generate fixed and variable costs associated with disassembly centers, disposal centers and recycling centers with respect to the costs we obtained from Üster and Hwang (2016). Spare part market demands are generated as a fraction of the new product demand. Therefore, brand new product demands and spare part demands are perfectly correlated. Additionally, we create sales prices for new and refurbished products and spare parts. When generating those, we calculate the total maximum cost per product and summed with a per-unit profit margin. Candidate locations for facilities are selected randomly from the US data used in Üster and Hwang (2016). Distances are calculated as Euclidean distances between these locations. In terms of scenario-based parameters, we take demands for each scenario directly from Üster and Hwang (2016) and hence we only create return and quality parameters. These parameters are related to returned goods ( $\alpha_k$ ,  $\beta_k$ ,  $\sigma_{jk}$ ,  $\rho_{jk}$  and  $\delta_{jk}$ ) and are randomly and independently generated from uniform distribution. Detailed information about our parameters can be seen in Table 5.2.

## 5.2 Computational Performance of the Proposed Solution Methods

In this section, we compare the computational performance of the proposed solution methods. Our termination conditions are 0.1% optimality gap or 3-hour time limit. All methods are coded in JAVA using CPLEX Concert Technology 12.6 and numerical experiments are executed in a PC with Intel Core i7 3.10 GHz processor and 16 GB RAM. We solve our problem instances by using the proposed solution methods and enhancements described in Chapter 4. When implementing the branch-and-cut, our optimality cuts are added as lazy constraints.

For each grouping strategy, we consider two methods when determining the group

Table 5.2: Parameters for Solution Method Comparison

	Parameter	Value
Parameters from Üster and Hwang (2016)	$f_p^P$	[9000000,10500000]
	$f_l^L$	[7000000,7200000]
	$fv_p^P$	[40,80]
	$fv_l^D$	[40,80]
	$fv_l^C$	[80,100]
	$fv_a^A$	[40,80]
	$cap_p^P$	[10000,30000]
	$cap_l^L$	[60000,90000]
	$v_p^{P2}$	[200,300]
	$v_p^{P3}$	[120,250]
	$v_l^{L1}$	[60,100]
	$v_l^{L2}$	[60,110]
	$dem_{mk}^{MY}$	[400,4000]
	$dem_{mk}^{MZ}$	[200,2000]
	Additional parameters for our problem	$\alpha_k$
$\beta_k$		Unif[0.6,0.7]
$\sigma_{jk}, \rho_{jk}$		(1-Unif[0.6,0.8])/2, Unif[0.6,0.8]
$\delta_{jk}$		Unif[0.6,0.7]
$dem_{n,jk}^N$		$dem_{mk}^{MY} / 5$
$f_s^S$		$f_p^P / 5$
$f_a^A$		$f_l^L$
$f_w^W$		$f_l^L / 5$
$f_r^R$		$f_l^L / 5$
$v_{qs}^S$		$v_p^{P2} / 10$
$v_{jp}^{P1}$		$v_p^{P2} / 5$
$v_{jr}^R$		$v_l^{L2} / 2$
$v_a^A$		$v_l^{L2}$
$v_w^{W1}$		$v_l^{L2} / 5$
$v_w^{W2}$		$v_l^{L2} / 10$
$fv_a^A$		$fv_l^C / 2$
$cap_s^S$		$cap_p^P * 5$
$cap_a^A$		$cap_l^L / 2$
$cap_w^W$		$cap_l^L / 5$
$cap_r^R$		$cap_l^L / 5$
$tz_j^J$		0.3
$tz_j^Q$		0.1
$u_j^j$		3
$u_j^Q$		2
$b_{qj}^W$		0.5
$b_{ij}^R$		0.5
$b_{ij}^P$		0.5
$b_{ij}^S$		0.3

size. In the first one, we use constant group size which is obtained by dividing the total number of scenarios by the number of groups to be formed. In the second method, we use data-dependent group size. Given a number of groups to be formed, the break points in the scenario list are determined in a way that they correspond to the largest gaps in the considered parameter value of the consecutive scenarios. Therefore, group sizes are not constant when this method is used. To see the impact of number of groups, we consider various values in our numerical experiments.

Comparison of performances are performed by evaluating average and worst-case solution times. We report the number of unsolved instances and the related average and maximum optimality gaps as well. We also performed paired t-test to compare the computational performances. However, we were not able to show that the differences of the solution times in our results are normally distributed. Therefore, we do not report and use paired t-test results to support our conclusions about the computational performances of the proposed solution methods.

To test all alternatives, we consider the experiments related to K1 class as the pilot study. For K1, we first investigate the performance of different solution methods and enhancements under demand-based grouping strategy. The related solution times can be found in Table 5.3. By evaluating these values, our first implication is that using decomposition methods supply a significant improvement over EF solution in terms of solution times. The average solution time is 8524.17 CPU seconds for solving the EF, whereas average solution time is 1168.75 CPU seconds for LS method (achieved without adding the mean value cut and with single cut) and 52.21 CPU seconds for BC method (achieved without adding the mean value cut and with data-dependent group size where number of groups is 3).

When we compare the use of RMP without and with mean value cut, we observe that the first one performs significantly better on average and in the worst case for both solution methods. Therefore, we eliminate the use of mean value cut from further



consideration.

Analyses within each cut generation strategies show for LS that, using multiple cuts (group-cuts or pure multi-cut strategy) degrades computational performance and yield longer solution times and increased number of unsolved instances. Hence, the best performing alternative for LS is the single cut approach. Although, single cut approach improves EF solution performance, it is not better than BC performances except for the problem instance 10. Analyses within each cut generation strategy show for BC that using data-dependent groups size with 3 groups performs the best on average among all alternatives. One very close alternative is constant group size with 5 groups which also performs well. Results show that, constant group size gives the best performances for most of the instances. However, it also result in some significant worst case performances. We observe that, the benefit of using data-dependent group size is the elimination of these worst-case performances in general. Although we observe for BC that using group cuts (rather than a single-cut or pure multi-cut strategy) improves the computational performance considerably, we can not see a single dominant grouping method or group size to embrace its implementation on larger problem classes. Therefore, we continue to experiment with different groups in further experiments.

Table 5.3: Solution Times (in CPU seconds) for Problem Class K1 under Demand-Based Grouping

Instances		Iterative L-shaped method (LS)													
		Without Mean Value Cut					With Mean Value Cut								
EF		Group Cuts		Data-Dependent Group Size		Group Cuts		Data-Dependent Group Size		Group Cuts					
		Constant Group Size	Number of Groups	Constant Group Size	Number of Groups	Constant Group Size	Number of Groups	Constant Group Size	Number of Groups	Constant Group Size	Number of Groups				
Single Cut		Multi Cut		Single Cut		Multi Cut		Single Cut		Multi Cut					
		2	5	10	2	3	5	10	2	5	10	2	3	5	
1	10141.37	1332.52	10807.20	10807.21	10802.16	10808.00	10807.88	10806.53	3780.35	10802.47	10800.87	10801.97	2319.43	10510.24	3895.15
2	10799.93	<b>198.54</b>	10808.04	10808.03	10808.38	10807.20	10808.10	10806.48	10800.17	9078.37	10800.75	10802.43	2600.51	1068.75	1499.00
3	10799.92	<b>84.45</b>	10807.02	10807.28	543.69	10807.70	10803.88	10801.69	6768.53	10801.99	10801.76	10801.69	10801.45	10802.47	10800.57
4	10799.92	<b>180.24</b>	10805.96	10807.39	10808.01	10807.05	10807.06	10801.28	6740.91	10800.17	10828.64	10803.18	10800.17	10802.97	10802.63
5	3392.24	61.00	<b>20.07</b>	10808.93	10808.32	246.01	1040.99	10801.48	3103.68	3382.26	10802.25	10802.77	10801.08	10802.30	10802.77
6	10799.91	256.28	<b>183.67</b>	10801.06	10807.35	653.36	10807.50	10807.09	5686.28	10802.76	10802.80	10801.74	10802.96	4546.96	3444.80
7	3118.20	<b>50.60</b>	10807.19	10806.94	10806.97	171.70	10807.98	10807.11	2442.69	2926.71	10803.18	10803.29	10802.27	5390.50	10801.73
8	10799.91	<b>32.51</b>	4633.94	10807.92	10806.50	502.69	10801.91	10807.09	1447.15	9728.81	10803.04	10801.85	10802.25	10801.46	10801.98
9	10799.91	157.87	<b>126.35</b>	10802.82	10805.06	913.59	10800.09	10806.84	10811.20	10802.80	10802.70	10802.75	1101.56	10800.14	10802.65
10	3790.40	<b>15.31</b>	86.25	10808.83	10807.89	21.06	8670.46	10807.71	1584.10	1465.57	10801.91	10802.43	1417.91	6436.52	10800.21
Average	8524.17	<b>1168.75</b>	3118.82	10806.47	10807.20	3547.06	9615.89	10806.42	10806.01	8049.19	10804.79	10802.41	7224.96	8196.23	8445.15
Maximum	<b>10799.93</b>	10813.71	10808.15	10808.93	10808.32	10808.38	10808.00	10808.10	10811.20	10802.81	10802.80	10828.64	10803.29	10802.96	10802.77
Number of unsolved instances	1	2	10	10	3	8	8	10	10	2	5	10	6	5	7
Average Gap for unsolved instances	1%	2%	26%	34%	24%	8%	15%	46%	1%	2%	6%	10%	2%	5%	8%
Maximum Gap for unsolved instances	1%	3%	50%	67%	39%	40%	39%	136%	1%	4%	9%	21%	5%	10%	16%
Instances		Branch and cut based L-shaped method (BC)													
		Without Mean Value Cut					With Mean Value Cut								
EF		Group Cuts		Data-Dependent Group Size		Group Cuts		Data-Dependent Group Size		Group Cuts					
		Constant Group Size	Number of Groups	Constant Group Size	Number of Groups	Constant Group Size	Number of Groups	Constant Group Size	Number of Groups	Constant Group Size	Number of Groups				
Single Cut		Multi Cut		Single Cut		Multi Cut		Single Cut		Multi Cut					
		2	5	10	2	3	5	10	2	5	10	2	3	5	
1	<b>32.33</b>	47.84	42.87	205.73	70.25	37.03	33.63	75.82	560.38	406.70	284.14	384.04	210.00	220.50	585.66
2	63.51	58.43	49.20	71.75	50.48	<b>45.37</b>	60.46	109.51	162.82	131.55	159.29	147.59	122.46	113.60	120.00
3	83.03	69.68	58.54	<b>35.65</b>	89.21	54.44	73.48	730.70	266.99	312.68	445.25	498.91	736.78	272.81	261.58
4	107.78	369.95	130.10	63.04	92.75	<b>58.39</b>	69.61	120.14	231.41	273.39	288.44	154.24	235.80	744.48	193.00
5	32.79	22.38	16.86	<b>16.27</b>	31.81	19.10	27.77	31.28	193.74	125.36	237.96	311.53	66.62	89.02	100.89
6	72.57	39.06	<b>29.90</b>	51.13	47.98	67.92	107.73	420.50	198.60	276.25	275.82	148.89	252.51	891.32	252.15
7	69.85	<b>26.98</b>	36.94	66.02	30.71	55.20	39.02	104.86	260.82	552.45	167.48	139.58	124.80	362.87	189.29
8	48.50	61.54	<b>22.69</b>	42.77	51.23	111.47	56.40	64.58	128.48	444.36	83.22	135.27	376.21	194.87	335.93
9	97.55	<b>43.34</b>	108.75	192.58	44.50	51.57	57.10	257.78	142.23	326.28	176.81	166.79	168.04	116.52	127.46
10	5067.97	23.68	35.80	<b>20.87</b>	327.92	21.62	25.71	69.29	71.40	72.16	136.10	84.46	87.61	96.01	88.31
Average	567.59	76.29	53.17	76.58	83.68	<b>52.21</b>	55.09	198.45	221.69	292.12	225.45	217.13	238.08	310.20	225.82
Maximum	<b>5067.97</b>	369.95	130.10	205.73	327.92	111.47	<b>107.73</b>	730.70	560.38	552.45	445.25	498.91	736.78	891.32	585.66

We also test the performance of using group-cuts under demand-rate-based grouping strategy. The results are presented in Table 5.4. For BC method, the best average and the worst-case performances are achieved by using constant group size with 10 groups. In this strategy, we observe that using data-dependent group size is not eliminating the worst-case performances in general. Same with our previous experiment, we do not observe a single grouping method or group size which performs best for all instances. Therefore, we continue with different grouping alternatives in further experiments. For LS method, although 2 groups with data-dependent group size performs the best among grouping alternatives, using group-cuts is still outperformed by using single-cut approach. Therefore, we eliminate the use of multiple cuts for LS in our subsequent experiments.

When we compare demand-based grouping and demand-rate-based grouping (i.e., result in Table 5.3 and Table 5.4), we observe that, the best performing cases in both grouping strategies differ with changing grouping method and group size which is not an unexpected result. Comparison of average performances show that, the best performance of demand-rate based grouping is better than the case with demand-based grouping. However, on average, demand-based grouping is performing slightly better than demand-rate-based grouping.

Table 5.4: Solution Times (in CPU seconds) for Problem Class K1 under Demand-Rate-Based Grouping

Iterative L-shaped method (LS)							
Without Mean Value Cut							
Group Cuts							
Constant Group Size							
Data-Dependent Group Size							
Number of Groups							
Instances		2	5	10	2	3	5
	1	10808.66	10808.35	<b>10806.27</b>	10806.93	10808.06	10807.33
	2	<b>71.53</b>	10805.08	10807.20	200.13	10807.53	10807.26
	3	<b>133.70</b>	10807.58	10806.93	10807.26	10800.41	10807.16
	4	1261.30	10806.20	10807.07	<b>870.72</b>	10807.54	10806.89
	5	70.26	10807.67	10807.10	1039.38	<b>57.17</b>	189.54
	6	10808.31	10807.57	10807.17	10807.63	<b>10803.51</b>	10813.74
	7	10808.13	10808.01	10805.10	<b>95.41</b>	10807.65	10807.27
	8	10808.87	10806.81	10803.11	<b>729.94</b>	10807.89	10808.76
	9	10808.14	10807.51	10807.92	<b>752.67</b>	10807.92	10807.48
	10	<b>80.96</b>	10800.23	10807.84	382.09	4140.25	10807.03
Average		5565.99	10806.50	10806.57	<b>3649.22</b>	9064.79	9746.25
Maximum		10808.87	10808.35	10807.92	<b>10807.63</b>	10808.06	10813.74
Number of unsolved instances		5	10	10	<b>3</b>	8	9
Average Gap for unsolved instances		27%	16%	22%	15%	<b>14%</b>	26%
Maximum Gap for unsolved instances		42%	42%	49%	42%	<b>41%</b>	81%
Branch and Cut based L-shaped method (BC)							
Without Mean Value Cut							
Group Cuts							
Constant Group Size							
Data-Dependent Group Size							
Number of Groups							
Instances		2	5	10	2	3	5
	1	122.61	78.71	<b>72.14</b>	674.29	84.97	79.08
	2	45.20	<b>33.50</b>	50.47	66.94	95.09	56.48
	3	93.70	84.22	61.26	<b>38.97</b>	70.48	52.45
	4	65.63	46.78	<b>43.34</b>	98.22	63.31	61.00
	5	<b>15.98</b>	36.89	22.15	23.14	29.55	36.50
	6	67.86	62.00	42.43	<b>41.03</b>	73.61	99.07
	7	<b>20.93</b>	83.75	29.40	40.21	50.50	40.76
	8	102.50	<b>30.91</b>	32.66	121.23	57.98	43.74
	9	134.23	<b>43.72</b>	83.32	63.78	64.23	81.12
	10	38.07	38.11	<b>26.28</b>	28.33	42.13	32.24
Average		70.67	53.86	<b>46.34</b>	119.61	63.18	58.24
Maximum		134.23	84.22	<b>83.32</b>	674.29	95.09	99.07

The solution times of the proposed solution methods for K2 are presented in Table 5.5. We do not consider pure multi-cut approach here and in our further experiments since we do not observe significant improvements due to that approach in our earlier experiments. Results for K2 demonstrate that, although LS performs better than BC for some of the instances, it has very long solution times for difficult instances. Therefore, BC method performs significantly better on average and in the worst-case, and we do not consider using LS method in our further experiments.

When we compare performances under demand-based grouping and demand-rate-based grouping, we observe that using 10 groups with constant group size performs best on average. In some of the instances, data-dependent group size performs better, but we observe several extreme cases for these group sizes in both grouping types. When we compare average performances between two grouping strategies, we observe for K2 that, demand-based grouping performs slightly better than demand-rate-based grouping on average (679.72 for demand-based grouping, 776.09 for demand-rate-based grouping). On the other hand, when we evaluate the best performing alternatives, we see that average performance of 10 groups with constant group size is better in demand-rate-based grouping. Reason behind this measure is the extreme case in demand-based grouping with solution time 1621.85 seconds of second problem instance. When we calculate averages without worst performances we see that demand-based grouping is better on average. Considering these, we decided to consider demand-based grouping strategy in our further experiments.

Table 5.5: Solution Times (in CPU seconds) for Problem Class K2

		Branch-and-Cut based L-shaped method (BC) under demand-based grouping								
		Without Mean Value Cut								
		Single Cut	Group Cuts							
			Constant Group Size				Data-Dependent Group Size			
			Number of Groups				Number of Groups			
Instances			2	5	10	50	2	3	5	
1	<b>219.67</b>	2091.63	351.03	324.66	246.70	237.28	386.76	319.58	423.01	
2	10808.03	513.39	380.18	330.32	255.02	714.12	<b>231.16</b>	236.34	290.90	
3	572.52	608.82	1176.75	1472.83	1621.85	801.17	3046.48	<b>440.94</b>	639.09	
4	209.95	404.25	4289.53	174.68	<b>170.62</b>	477.21	193.11	459.20	275.55	
5	8123.30	197.37	120.16	193.89	<b>100.43</b>	158.27	285.18	230.05	138.58	
6	256.71	236.61	210.52	297.55	221.78	351.51	528.50	<b>153.95</b>	1002.66	
7	160.82	4342.75	173.25	136.71	<b>110.52</b>	225.37	279.52	191.35	1632.37	
8	<b>106.67</b>	325.05	246.70	704.79	199.14	296.67	312.27	267.94	3386.67	
9	1264.07	2251.81	663.58	1667.28	<b>423.39</b>	923.48	1581.71	577.09	558.56	
10	<b>53.75</b>	121.49	1597.30	470.72	97.93	198.88	1266.59	1021.23	114.29	
Average	2177.55	1109.32	920.90	577.34	<b>344.74</b>	438.40	811.13	389.77	846.17	
Maximum	10808.03	4342.75	4289.53	1667.28	1621.85	<b>923.48</b>	3046.48	1021.23	3386.67	
Average w/o	1218.61	750.05	546.61	456.24	<b>202.84</b>	384.50	562.75	319.60	563.89	
Maximum										
		Branch-and-Cut based L-shaped method (BC) under demand-rate-based grouping								
		Without Mean Value Cut								
			Group Cuts							
			Constant Group Size				Data-Dependent Group Size			
			Number of Groups				Number of Groups			
Instances			2	5	10	50	2	3	5	
1			347.17	279.33	<b>186.55</b>	332.30	268.06	2701.96	397.83	
2			334.93	<b>198.25</b>	209.63	722.52	2173.13	198.97	261.32	
3			694.80	559.43	435.33	1293.75	<b>352.37</b>	6824.33	668.05	
4			199.69	160.14	<b>125.08</b>	601.88	340.43	4765.62	211.56	
5			206.69	119.54	<b>105.58</b>	153.00	189.99	128.77	348.51	
6			237.69	230.17	<b>183.35</b>	228.66	449.56	203.84	215.11	
7			200.53	158.45	219.03	202.37	161.49	<b>152.33</b>	193.54	
8			465.72	217.61	254.37	295.12	265.92	342.80	<b>135.96</b>	
9			476.51	802.15	907.91	3061.74	<b>364.71</b>	821.96	474.56	
10			973.91	830.72	245.05	194.37	149.67	1651.28	1863.31	
Average			413.76	355.58	<b>287.19</b>	708.57	471.53	1779.19	476.97	
Maximum			973.91	<b>830.72</b>	907.91	3061.74	2173.13	6824.33	1863.31	
Average w/o			351.53	302.79	<b>218.22</b>	447.11	282.47	1218.62	322.94	
Maximum										

When solving the instances in K3 and B1 classes, we use the BC method without mean value cut. We consider the single-cut approach and group-cut approach under demand-based grouping. Since the instances in these classes are large-sized, none of them are solved within the allowed time limit. Therefore, we consider the optimality gap as our measure when evaluating the performance of the used solution methods for these classes. The results for K3 and B1 are summarized in Table 5.6 and Table 5.7, respectively. For K3, we see that almost all instances are solved with different alternatives. However, each alternative has unsolved instances. In terms of average and worst-case performance, 50 groups with constant group size performs with 0.68% average and 2.55% worst-case optimality gaps which are still greater than our target (0.1%).

Table 5.6: Relative Gaps for Problem Class K3

Branch and Cut based L-shaped method (BC) under demand-based grouping								
Without Mean Value Cut								
Group Cuts								
Single Cut								
Constant Group Size								
Data-Dependent Group Size								
Number of Groups								
Instances		2	5	10	50	2	3	5
1	426.65%	1.75%	1.46%	2.19%	<b>0.52%</b>	0.79%	2.23%	1.33%
2	400.87%	5.77%	8.75%	<b>0.10%</b>	2.47%	2.65%	0.68%	466.61%
3	2.30%	1.38%	2.58%	<b>0.10%</b>	<b>0.10%</b>	1.03%	392.74%	2.95%
4	2.01%	3.60%	1.66%	1.75%	<b>0.10%</b>	2.31%	2.29%	1.37%
5	578.41%	<b>0.10%</b>	1.91%	<b>0.10%</b>	0.16%	1.23%	7.54%	9.17%
6	2.16%	7.04%	198.05%	1.32%	0.31%	0.13%	0.14%	<b>0.10%</b>
7	1.63%	16.35%	0.09%	4.15%	0.10%	0.08%	0.09%	<b>0.07%</b>
8	1.74%	0.15%	5.20%	3.77%	0.17%	359.94%	0.16%	<b>0.10%</b>
9	2.82%	3.33%	307.22%	<b>1.66%</b>	2.55%	6.08%	201.05%	3.12%
10	3.63%	0.73%	<b>0.10%</b>	6.82%	0.30%	0.84%	1.73%	448.78%
Avg	142.22%	4.02%	52.70%	2.20%	<b>0.68%</b>	37.51%	60.87%	93.36%
Max	578.41%	16.35%	307.22%	6.82%	<b>2.55%</b>	359.94%	392.74%	466.61%

Although the instances in B1 are slightly larger than that in K3 in terms of number of decision variables, we have significantly larger gaps for these instances. As seen in Table 5.7, the smallest optimality gap value is 0.57%, which is still greater than

our optimality gap target (0.1%). This shows that, to solve such large size problems within this time limit, other solution approaches or improvements should be used.

Table 5.7: Relative Gaps for Problem Class B1

Branch and Cut based L-shaped method (BC) under demand-based grouping								
Without Mean Value Cut								
Group Cuts								
Single Cut								
Constant Group Size								
Data-Dependent Group Size								
Number of Groups								
Instances		2	5	10	50	2	3	5
1	5.33%	<b>2.81%</b>	5.37%	6.61%	3.31%	7.80%	6.27%	625.23%
2	990.38%	801.54%	735.37%	3.57%	<b>3.18%</b>	630.74%	587.50%	625.09%
3	5.63%	74.93%	514.08%	<b>3.34%</b>	17.31%	8.40%	511.97%	11.84%
4	29.43%	<b>3.33%</b>	31.34%	4.54%	3.46%	8.76%	3.87%	3.71%
5	709.73%	9.41%	<b>4.63%</b>	645.13%	5.10%	717.22%	5.88%	7.95%
6	3.02%	697.23%	3.00%	2.74%	3.80%	3.47%	3.08%	<b>2.44%</b>
7	5.37%	4.68%	14.18%	4.13%	3.91%	<b>3.80%</b>	12.66%	8.97%
8	7.30%	3.43%	4.10%	423.82%	408.66%	<b>0.57%</b>	405.17%	10.04%
9	300.32%	3.43%	<b>2.91%</b>	3.13%	3.47%	165.45%	151.77%	3.52%
10	152.70%	8.50%	2.67%	<b>2.09%</b>	3.05%	6.79%	7.35%	2.58%
Avg	220.92%	160.93%	131.76%	109.91%	<b>45.53%</b>	155.30%	169.55%	130.14%
Max	990.38%	801.54%	735.37%	645.13%	<b>408.66%</b>	717.22%	587.50%	625.23%

### 5.3 The Value of Stochastic Solution and the Expected Value of the Perfect Information

To investigate the importance of stochastic programming approach and the value of information for our problem, we calculated the VSS and EVPI measures. Our results are summarized in Table 5.8. Since we solve our instances with a gap limit of 0.1% these measures give us good bounds instead of exact values. This means that VSS values that we reported are lower bounds on VSS and EVPI values that we report are upper bounds on EVPI. In problem classes larger than K2, these bounds are more loose due to increased optimality gaps. In Table 5.8, we see that VSS values are less than 1% on average. However, we work with a strategic long term problem and the objective function takes very high values. Therefore, even low VSS values corre-



respond to a significant amount of change in the expected profit.

To have a better understanding of the issues related to low VSS values, we investigate the effect of distributions of uncertain return and quality rates. The VSS values reported in Table 5.8 belong to the instances where and quality rates are at their medium levels with a narrow range. While we have three levels for each uncertain rate between 0 and 1 to test effects of each, their ranges are quite narrow, which makes their variance quite small. Because of this, for these rates, scenarios are not significantly different from each other. To demonstrate their effect on VSS, we generate instances with uncertain rates having greater variances. To do so, we generate uncertain rates from uniform distributions whose ranges are set from lower value of low levels to higher values of high levels of each uncertain rate. By doing this we acquire the largest possible range of values for each uncertain rate. Therefore, with this setting, scenarios become more differentiated from each other. Results of this implementation show greater VSS values as expected (VSS' column in Table 5.8). In further experiments we continue with original distributions (i.e., with narrow range).

Compared to VSS, our EVPI values are higher, which shows that the expected loss due to the presence of uncertainty is quite high. Therefore, to achieve better solutions, decision maker can employ some methods to reduce uncertainty in problem parameters. By doing this, RP solution could be improved, which would results in reduced EVPI values.

Table 5.8: VSS and EVPI values

Instances	K1			K2			K3			B1	
	% EVPI	% VSS	% VSS'	% EVPI	% VSS	% VSS'	% EVPI	% VSS	% VSS'	% VSS	% VSS'
1	3.33%	0.30%	4.88%	5.05%	0.37%	3.64%	5.84%	0.08%	-8.17%	-0.42%	2.60%
2	4.55%	0.49%	4.51%	5.95%	0.42%	3.61%	5.96%	0.16%	1.64%	0.77%	1.97%
3	2.44%	0.52%	4.83%	4.16%	0.59%	1.88%	5.78%	0.44%	1.09%	2.05%	-0.38%
4	4.77%	0.65%	3.28%	5.30%	1.03%	1.74%	6.53%	0.18%	6.78%	-0.21%	1.27%
5	4.67%	0.25%	1.95%	4.18%	2.44%	0.15%	5.61%	0.17%	1.89%	0.68%	1.03%
6	4.08%	0.53%	4.66%	5.21%	1.23%	1.68%	5.64%	0.22%	0.82%	-0.18%	1.41%
7	2.53%	0.32%	6.24%	2.87%	0.50%	1.85%	4.39%	2.06%	4.13%	0.52%	-0.52%
8	3.97%	0.73%	5.17%	4.55%	0.76%	2.36%	5.46%	0.04%	2.98%	1.49%	2.09%
9	7.43%	1.05%	4.72%	5.03%	0.33%	1.80%	5.46%	0.17%	0.67%	0.48%	0.88%
10	3.34%	1.19%	4.36%	3.47%	0.49%	4.07%	7.60%	0.38%	10.73%	1.83%	5.11%
Avg	4.11%	0.60%	4.46%	4.58%	0.82%	2.28%	5.83%	0.39%	2.25%	0.70%	1.55%
Max	7.43%	1.19%	6.24%	5.95%	2.44%	4.07%	7.60%	2.06%	10.73%	2.05%	5.11%

#### 5.4 Value of Uncertainty in Demand, Return Rate, and Quality

Effects of individual uncertainties (i.e., uncertainties in demand,  $\alpha_k$ ,  $\beta_k$ ,  $\sigma_{jk}$  and  $\rho_{jk}$ , and  $\delta_{jk}$ ) are tested considering the instances in K2 by making only one parameter uncertain at a time. The rest of the parameters are kept constant at the mean value of their medium-level ranges.

We summarize our results in Table 5.9. For each instance, we report the number of suppliers worked with, the number of opened plants, the number of opened DCCs, the number of opened disassembly centers, the number of disposal centers worked with, the number of recycling centers worked with, total capacity installed on plants, total capacity of distribution, total capacity of collection, total capacity installed on disassembly centers, the number of DCCs with distribution function, the number of DCCs with collection function, % VSS and % EVPI values.

In terms of number of opened DCCs, uncertainty in demand is crucial which leads to a lower number of opened DCCs compared to the solution for other types of uncertainty. In terms of number of selected disposal and recycling centers, we observe a similar effect of demand uncertainty. When we compare installed capacities, we observe that uncertainty in demand decreases capacity installations in all facilities.

These effects show that uncertainty in demand causes decision maker to be more conservative in the first-stage decisions. By analyzing collection capacities, we see that uncertainty in  $\beta_k$  (i.e., product quality) leads to the largest collection capacity installation, which shows the criticality of quality of returned product.

Another differentiating element is the rate of co-location of distribution and collection facilities, which refers to the case where DCCs have installed capacity for both distribution and collection. We evaluate this measure to observe the differences between cases with/without co-location. From the results we evaluate this measure by comparing the number of DCCs with distribution function with the number of DCCs with collection function. Since our problem is flexible in terms of co-location, we are able to observe DCCs with pure distribution, pure collection, or combination of both. When,  $\sigma_{jk}$  and  $\rho_{jk}$  (i.e., part quality) are uncertain, co-location of facilities is almost 100%. On the other hand, when  $\alpha_k$  (i.e., return rate) is uncertain, we observe less co-location of distribution and collection facilities.

Comparison of VSS values shows that the strongest effect of uncertainty on VSS is caused by demand, which is expected due to its high impact on facility location and capacity installation decisions. Uncertainty in  $\beta_k$  and  $\sigma_{jk}$  and  $\rho_{jk}$  are also comparable. On the other hand, uncertainty in  $\alpha_k$  has the weakest effect on VSS. In other words, the uncertainty in the quality of the returned product is more critical than the uncertainty in the quantity of the returns in our problem.

Table 5.9: Analysis of individual uncertainties for Problem Class K2

	Instance No	# of Suppliers Selected	# Plants Opened	# DCCs Opened	# Disassembly Centers Opened	# Disposal Centers Selected	# Recycling Centers Selected	Total Plant Capacity	Total Distribution Capacity	Total Collection Capacity	Total Disassembly Capacity	# DCCs with Distribution	# DCCs with Collection	% YSS	% EVPI
Uncertain Demand	1	8	5	2	1	2	3	76632	69145	32267	20973	2	1	0.15%	4.81%
	2	8	5	2	1	2	4	84547	76638	33562	21802	2	2	0.19%	5.90%
	3	8	5	2	1	2	3	66751	60461	28142	18292	2	1	0.28%	4.13%
	4	8	5	1	1	1	2	65478	59661	22204	14433	1	1	0.03%	4.92%
	5	8	4	1	1	1	2	58843	55970	16342	10616	1	1	2.24%	4.04%
	6	8	5	1	1	1	3	65381	58839	24874	16168	1	1	0.93%	4.97%
	7	8	5	1	1	1	2	59507	53545	21905	14260	1	1	0.08%	2.21%
	8	8	4	1	1	1	2	62969	59555	18351	11928	1	1	0.44%	4.31%
	9	8	5	2	1	2	3	76286	68064	34508	22431	2	2	0.28%	4.77%
	10	8	4	1	1	1	1	53140	52039	9373	6073	1	1	0.42%	3.19%
	Avg	8	4.7	1.4	1	1.4	2.5	66953	61392	24153	15698	1.4	1.2	0.50%	4.33%
Max	8	5	2	1	2	4	84547	76638	34508	22431	2	2	2.24%	5.90%	
Uncertain $\alpha_k$ (return rate)	1	8	5	2	1	2	3	76632	69397	33465	21733	2	1	0.03%	0.05%
	2	8	5	2	1	2	4	84547	77193	35978	23414	2	2	0.12%	0.06%
	3	8	5	2	1	2	3	66751	60902	30132	19586	2	2	0.23%	0.08%
	4	8	5	2	1	2	3	68912	62678	31415	20408	2	1	0.13%	0.07%
	5	8	5	2	1	2	3	69697	62382	33025	21467	2	2	0.01%	0.10%
	6	8	5	2	1	2	3	67961	61489	32853	21354	2	1	0.05%	0.08%
	7	8	5	1	1	2	3	59567	54698	27073	17679	1	1	0.24%	0.08%
	8	8	5	2	1	2	4	73928	66337	36672	23794	2	1	0.15%	0.14%
	9	8	5	2	1	2	4	76286	68599	36919	23998	2	2	0.03%	0.22%
	10	8	5	2	1	2	3	62789	58118	25950	16848	2	1	0.21%	0.22%
	Avg	8	5	1.9	1	2	3.3	70707	64179	32348	21028	1.9	1.4	0.12%	0.11%
Max	8	5	2	1	2	4	84547	77193	36919	23998	2	2	0.24%	0.22%	
Uncertain $\beta_k$ (product quality)	1	8	5	2	1	2	3	76632	69408	35936	23487	2	1	0.18%	0.10%
	2	8	5	2	1	2	4	84547	77078	37460	23452	2	2	0.24%	0.14%
	3	8	5	2	1	2	3	66751	60915	32117	19587	2	2	0.49%	0.14%
	4	8	5	2	1	2	3	68912	62617	32907	20319	2	2	0.27%	0.15%
	5	8	5	2	1	2	3	69697	62319	35513	21445	2	2	0.18%	0.21%
	6	8	5	2	1	2	3	67961	61233	34255	21551	1	2	0.22%	0.12%
	7	8	5	1	1	2	3	59567	54649	27122	17565	1	1	0.38%	0.16%
	8	8	5	2	1	2	4	73928	66337	38388	23843	2	2	0.34%	0.22%
	9	8	5	2	1	2	4	76286	68645	39120	24139	2	2	0.04%	0.28%
	10	8	5	2	1	2	3	62789	58175	27201	16941	2	2	0.44%	0.28%
	Avg	8	5	1.9	1	2	3.3	70707	64138	34002	21233	1.8	1.8	0.28%	0.18%
Max	8	5	2	1	2	4	84547	77078	39120	24139	2	2	0.49%	0.28%	
Uncertain $\sigma_{j,k}$ and $\mu_{j,k}$ (part quality)	1	8	5	2	1	2	3	76632	69398	33465	21752	2	1	0.03%	0.43%
	2	8	5	2	1	2	4	84547	77907	36696	23853	2	2	0.25%	0.24%
	3	8	5	2	1	2	3	66750	61204	30359	19733	2	2	0.27%	0.34%
	4	8	5	2	1	2	3	68912	63086	31537	20499	2	2	0.15%	0.28%
	5	8	5	2	1	2	4	69697	63886	35834	23271	2	2	0.13%	0.63%
	6	8	5	2	1	2	3	67961	61220	32106	20869	2	2	0.10%	0.38%
	7	8	5	1	1	2	3	59567	54934	26837	17444	1	1	0.39%	0.33%
	8	8	5	2	1	2	4	73928	67756	38603	25291	2	2	0.32%	0.50%
	9	8	5	2	1	2	4	76287	69870	38754	25190	2	2	0.36%	0.44%
	10	8	5	2	1	2	3	62789	58930	26925	17505	2	2	0.46%	0.62%
	Avg	8	5	1.9	1	2	3.4	70707	64819	33112	21541	1.9	1.8	0.25%	0.42%
Max	8	5	2	1	2	4	84547	77907	38754	25291	2	2	0.46%	0.63%	
Uncertain $\delta_{j,k}$ (material quality)	1	8	5	2	1	2	3	76632	69775	33786	21961	2	2	0.05%	0.08%
	2	8	5	2	1	2	4	84547	77379	36255	23565	2	2	0.16%	0.08%
	3	8	5	2	1	2	3	66751	61147	30135	19601	2	1	0.27%	0.12%
	4	8	5	2	1	2	3	68912	62814	32077	20703	2	2	0.16%	0.09%
	5	8	5	2	1	2	3	69697	62664	33028	21458	2	2	0.04%	0.13%
	6	8	5	2	1	2	3	67961	61486	32194	20832	1	1	0.07%	0.10%
	7	8	5	1	1	2	3	59567	54689	27082	17604	1	1	0.29%	0.11%
	8	8	5	2	1	2	4	73928	66574	37165	24157	2	1	0.21%	0.15%
	9	8	5	2	1	2	4	76287	68811	37368	24289	2	2	0.21%	0.24%
	10	8	5	2	1	2	3	62789	58343	26294	17084	2	1	0.28%	0.23%
	Avg	8	5	1.9	1	2	3.3	70707	64368	32538	21125	1.8	1.5	0.17%	0.13%
Max	8	5	2	1	2	4	84547	77379	37368	24289	2	2	0.29%	0.24%	

## 5.5 Impact of Return and Quality Levels

To analyze the effects of different levels of uncertain parameters, we generate instances at three levels: high, medium (our original setting) and low as summarized in Table 5.10. For each instance, we reiterate our experiment by changing the level of one parameter at a time and keeping others at their medium levels to clearly observe the changes in the solutions.

Table 5.10: Distribution of Uncertain Return and Quality Rates

Parameters	Low	Medium	High
$\alpha_k$	Unif.[0.4, 0.5]	Unif.[0.6, 0.7]	Unif.[0.8, 0.9]
$\beta_k$	Unif.[0.4, 0.5]	Unif.[0.6, 0.7]	Unif.[0.8, 0.9]
$\sigma_{jk}, \rho_{jk}$	$\sigma_{jk} = (1-\text{Unif.}[0.6, 0.8])/2$ $\rho_{jk} = (1-\text{Unif.}[0.6, 0.8])/2$	$\sigma_{jk} = (1-\text{Unif.}[0.6, 0.8])/2$ $\rho_{jk} = \text{Unif.}[0.6, 0.8]$	$\sigma_{jk} = \text{Unif.}[0.6, 0.8]$ $\rho_{jk} = (1-\text{Unif.}[0.6, 0.8])/2$
$\delta_{jk}$	Unif.[0.4, 0.5]	Unif.[0.6, 0.7]	Unif.[0.8, 0.9]

We report our results in Table 5.11. When  $\alpha_k$  (return rate) is at medium and high levels, the number of facilities opened and plant, distribution, collection and disassembly capacities are significantly higher compared to the case where  $\alpha_k$  is low. Since the contribution of first-stage costs decrease when  $\alpha_k$  is low due to reduced number of facilities opened and capacities installed, the impact of the second-stage decisions value becomes higher, and hence VSS becomes higher on average for this setting. Although we observe a critical difference between medium and low  $\alpha_k$ , we do not see this difference between high and medium cases. At this point we can say that beyond some level of return rate, solutions depend on quality instead of quantity of the returned product.

Results for  $\beta_k$  (product quality) can be found in Table 5.12. When  $\beta_k$  is low, forward flow is weakened with smaller numbers of facilities opened and less capacities installed. In addition, there exists a remarkable decrease in the collection and disassembly capacities, the number of DCCs used as collection centers, the number of opened disassembly centers and the number of selected recycling centers compared to medium and high  $\beta_k$  cases. In low  $\beta_k$  cases we observe greater variation in VSS values as in low  $\alpha_k$  case. Additionally, different than  $\alpha_k$ , high  $\beta_k$  increases capacities compared to medium case which supports our insight on importance of quality over return rate.

According to the results in Table 5.13, the highest level of collection and disassembly capacities are associated with the medium part quality level which corresponds to a high value of ratio of recyclable parts. This shows that the ratio of recyclable parts ( $\rho_{jk}$ ) dominate the effect of remanufacturable parts ( $\sigma_{jk}$ ). Low level of part quality represents the case where both  $\sigma_{jk}$  and  $\rho_{jk}$  are low (i.e., the ratio of parts to be disposed is high), and the optimal network for this case is designed as a forward supply chain with no returns as expected.

We summarize the results of our analysis related to different levels of  $\delta_{jk}$  (material quality) in Table 5.14. According to these results, along with number of recycling centers, the number of facilities opened and capacities installed in both forward and reverse chain are significantly higher for high level of  $\delta_{jk}$  than the cases with medium and low  $\delta_{jk}$ . In addition to that, low  $\delta_{jk}$  leads to decreased capacities of collection and disassembly. This shows that, material quality in recycling is an important factor determining bi-directional movements of products. Similar to previous observations, in this low rate case, VSS values are higher than medium and high cases.

From these results we observe that all suppliers are selected for each case and each problem instance. This shows us that, supplier selection costs are considerably low compared to expected profit which lead to selection of as much supplier as possible.

From this, we understand that, in scope of this problem we can ignore supplier selection decisions which would make our problem easier and give us the potential to solve larger instances.



Table 5.11: Analysis of  $\alpha_k$  (return rate) levels for Problem Class K2

	Instance No	# of Suppliers Selected	# Plants Opened	# DCCs Opened	# Disassembly Centers Opened	# Disposal Centers Selected	# Recycling Centers Selected	Total Plant Capacity	Total Distribution Capacity	Total Collection Capacity	Total Disassembly Capacity	# DCCs with Distribution	# DCCs with Collection	% VSS	% EYPI
High $\alpha_k$	1	8	5	2	1	2	3	76632	69546	34022	21363	2	1	0.44%	5.10%
	2	8	4	1	0	0	0	64107	64107	0	0	1	0	0.46%	6.14%
	3	8	5	2	1	2	3	66751	60990	30121	18799	2	1	0.56%	4.42%
	4	8	4	1	1	1	1	58074	55847	13068	8359	1	1	0.29%	5.26%
	5	8	4	1	1	1	2	58843	56392	16922	10830	1	1	2.76%	3.96%
	6	8	4	1	1	1	2	58343	55988	17401	11457	1	1	1.36%	5.11%
	7	8	5	1	1	1	2	59567	53465	21483	14598	1	1	0.16%	3.20%
	8	8	4	1	1	1	2	62969	60280	19428	12434	1	1	1.07%	4.29%
	9	8	5	2	1	2	3	76287	68196	35676	22659	2	2	0.23%	5.21%
	10	8	4	1	1	1	1	53140	51984	9488	5980	1	1	0.85%	3.45%
	Avg	8	4.4	1.3	0.9	1.2	1.9	63471	59679	19760	12647	1.3	1	0.82%	4.62%
	Max	8	5	2	1	2	3	76632	69546	35676	22659	2	2	2.76%	6.14%
Medium $\alpha_k$	1	8	5	2	1	2	3	76632	69565	33929	21250	2	2	0.37%	5.05%
	2	8	4	1	0	0	0	64107	64107	0	0	1	0	0.42%	5.95%
	3	8	5	2	1	2	3	66750	61353	30207	18952	2	1	0.59%	4.16%
	4	8	4	1	1	1	1	58074	55790	13126	8432	1	1	1.03%	5.30%
	5	8	4	1	1	1	2	58843	56451	17080	11102	1	1	2.44%	4.18%
	6	8	4	1	1	1	2	58343	55972	17813	11601	1	1	1.23%	5.21%
	7	8	5	1	1	2	3	59567	55100	26469	17235	1	1	0.50%	2.87%
	8	8	4	1	1	1	2	62969	60145	19360	12456	1	1	0.76%	4.55%
	9	8	5	2	1	2	3	76287	68383	36087	22472	2	2	0.33%	5.03%
	10	8	4	1	1	1	1	53140	52102	9537	5921	1	1	0.49%	3.47%
	Avg	8	4.4	1.3	0.9	1.3	2	63471	59896	20360	12942	1.3	1.1	0.82%	4.58%
	Max	8	5	2	1	2	3	76632	69565	36087	22472	2	2	2.44%	5.95%
Low $\alpha_k$	1	8	4	1	1	1	1	63257	61269	12227	7796	1	1	1.08%	4.64%
	2	8	4	1	0	0	0	64107	64220	0	0	1	0	1.84%	5.20%
	3	8	4	1	1	1	2	57038	55166	14578	9458	1	1	1.62%	3.60%
	4	8	4	1	1	1	1	58074	55832	13256	8428	1	1	0.44%	4.41%
	5	8	4	1	1	1	2	58843	56350	17430	11187	1	1	0.40%	2.62%
	6	8	4	1	1	1	2	58343	56001	17746	11671	1	1	0.77%	4.22%
	7	8	5	1	1	1	2	59567	53595	21905	14314	1	1	0.13%	2.75%
	8	8	4	1	1	1	2	62969	60288	19540	12676	1	1	0.42%	2.75%
	9	8	4	1	1	1	2	65007	62040	18883	12671	1	1	2.11%	3.88%
	10	8	4	1	1	1	1	53140	52050	9546	5983	1	1	0.97%	3.18%
	Avg	8	4.1	1	0.9	0.9	1.5	60034	57681	14511	9418	1	0.9	0.98%	3.73%
	Max	8	5	1	1	1	2	65007	64220	21905	14314	1	1	2.11%	5.20%



Table 5.12: Analysis of  $\beta_k$  (product quality) levels for Problem Class K2

	Instance No	# of Suppliers Selected	# Plants Opened	# DCCs Opened	# Disassembly Centers Opened	# Disposal Centers Selected	# Recycling Centers Selected	Total Plant Capacity	Total Distribution Capacity	Total Collection Capacity	Total Disassembly Capacity	# DCCs with Distribution	# DCCs with Collection	% VSS	% EVPI
High $\beta_k$	1	8	5	2	1	1	3	76632	69691	25589	21288	2	1	0.36%	4.68%
	2	8	5	2	1	1	4	84547	77856	27166	23089	2	1	0.31%	5.72%
	3	8	5	1	1	1	3	66642	59025	18194	15999	1	1	0.68%	3.98%
	4	8	5	1	1	1	3	68912	61453	20412	17839	1	1	0.16%	4.54%
	5	8	4	1	1	1	2	58843	56288	13234	10917	1	1	1.87%	4.69%
	6	8	5	1	1	1	3	67961	60763	22950	19238	1	1	0.18%	4.13%
	7	8	5	1	1	1	3	59567	55263	20197	17167	1	1	0.26%	2.47%
	8	8	5	2	1	1	3	73928	65732	26428	22199	2	2	0.12%	4.85%
	9	8	5	2	1	1	3	76287	68404	27437	22759	2	2	0.24%	4.75%
	10	8	5	1	1	1	3	62789	57609	17590	15450	1	1	0.24%	3.75%
	Avg	8	4.9	1.4	1	1	3	69611	63208	21920	18594	1.4	1.2	0.44%	4.36%
	Max	8	5	2	1	1	4	84547	77856	27437	23089	2	2	1.87%	5.72%
Medium $\beta_k$	1	8	5	2	1	2	3	76632	69565	33929	21250	2	2	0.37%	5.05%
	2	8	4	1	0	0	0	64107	64107	0	0	1	0	0.42%	5.95%
	3	8	5	2	1	2	3	66750	61353	30207	18952	2	1	0.59%	4.16%
	4	8	4	1	1	1	1	58074	55790	13126	8432	1	1	1.03%	5.30%
	5	8	4	1	1	1	2	58843	56451	17080	11102	1	1	2.44%	4.18%
	6	8	4	1	1	1	2	58343	55972	17813	11601	1	1	1.23%	5.21%
	7	8	5	1	1	2	3	59567	55100	26469	17235	1	1	0.50%	2.87%
	8	8	4	1	1	1	2	62969	60145	19360	12456	1	1	0.76%	4.55%
	9	8	5	2	1	2	3	76287	68383	36087	22472	2	2	0.33%	5.03%
	10	8	4	1	1	1	1	53140	52102	9537	5921	1	1	0.49%	3.47%
	Avg	8	4.4	1.3	0.9	1.3	2	63471	59897	20361	12942	1.3	1.1	0.82%	4.58%
	Max	8	5	2	1	2	3	76632	69565	36087	22472	2	2	2.44%	5.95%
Low $\beta_k$	1	8	4	1	0	0	0	57116	57117	0	0	1	0	3.18%	3.36%
	2	8	4	1	0	0	0	64074	64075	0	0	1	0	-0.04%	3.90%
	3	8	4	1	1	1	2	57038	55206	20476	9327	1	1	2.33%	3.07%
	4	8	4	1	1	1	1	58074	55607	16842	7844	1	1	3.46%	3.60%
	5	8	4	1	1	1	2	58843	56089	22245	10677	1	1	0.15%	2.75%
	6	8	4	1	1	1	2	58343	55523	22412	10758	1	1	0.19%	3.88%
	7	8	4	1	1	1	1	50481	49022	16899	7378	1	1	1.44%	1.90%
	8	8	4	1	1	2	2	62969	59203	23716	11384	1	1	0.11%	3.07%
	9	8	4	1	1	1	2	63348	60492	20431	9837	1	1	2.08%	4.15%
	10	8	4	1	0	0	0	48936	48970	0	0	1	0	5.16%	1.71%
	Avg	8	4	1	0.7	0.8	1.2	57922	56130	14302	6720	1	0.7	1.81%	3.14%
	Max	8	4	1	1	2	2	64074	64075	23716	11384	1	1	5.16%	4.15%

Table 5.13: Analysis of  $\sigma_{jk}$  and  $\rho_{jk}$  (part quality) levels for Problem Class K2

	Instance No	# of Suppliers Selected	# Plants Opened	# DCCs Opened	# Disassembly Centers Opened	# Disposal Centers Selected	# Recycling Centers Selected	Total Plant Capacity	Total Distribution Capacity	Total Collection Capacity	Total Disassembly Capacity	# DCCs with Distribution	# DCCs with Collection	% VSS	% EYPI
High Part Quality Level	1	8	4	1	0	0	0	57117	57117	0	0	1	0	1.49%	2.00%
	2	8	4	1	0	0	0	64105	64105	0	0	1	0	0.00%	1.68%
	3	8	4	1	0	0	0	50056	50057	0	0	1	0	1.73%	1.12%
	4	8	4	1	0	0	0	51323	51323	0	0	1	0	2.39%	1.60%
	5	8	4	1	0	0	0	50182	50182	0	0	1	0	2.24%	1.47%
	6	8	4	1	0	0	0	49468	49470	0	0	1	0	1.93%	1.81%
	7	8	4	1	0	0	0	45076	45077	0	0	1	0	0.00%	0.04%
	8	8	4	1	0	0	0	52990	52990	0	0	1	0	0.56%	2.75%
	9	8	5	1	1	1	1	76287	61184	13714	9691	1	1	0.29%	3.03%
	10	8	4	1	0	0	0	48936	48936	0	0	1	0	0.00%	0.45%
	Avg	8	4.1	1	0.1	0.1	0.1	54554	53044	1371.4	969.1	1	0.1	1.06%	1.59%
	Max	8	5	1	1	1	1	76287	64105	13714	9691	1	1	2.39%	3.03%
Medium Part Quality Level	1	8	5	2	1	2	3	76632	69565	33929	21250	2	2	0.37%	5.05%
	2	8	4	1	0	0	0	64107	64107	0	0	1	0	0.42%	5.95%
	3	8	5	2	1	2	3	66750	61353	30207	18952	2	1	0.59%	4.16%
	4	8	4	1	1	1	1	58074	55790	13126	8432	1	1	1.03%	5.30%
	5	8	4	1	1	1	2	58843	56451	17080	11102	1	1	2.44%	4.18%
	6	8	4	1	1	1	2	58343	55972	17813	11601	1	1	1.23%	5.21%
	7	8	5	1	1	2	3	59567	55100	26469	17235	1	1	0.50%	2.87%
	8	8	4	1	1	1	2	62969	60145	19360	12456	1	1	0.76%	4.55%
	9	8	5	2	1	2	3	76287	68383	36087	22472	2	2	0.33%	5.03%
	10	8	4	1	1	1	1	53140	52102	9537	5921	1	1	0.49%	3.47%
	Avg	8	4.4	1.3	0.9	1.3	2	63471	59896	20360	12942	1.3	1.1	0.82%	4.58%
	Max	8	5	2	1	2	3	76632	69565	36087	22472	2	2	2.44%	5.95%
Low Part Quality Level	1	8	4	1	0	0	0	57116	57116	0	0	1	0	0.00%	1.25%
	2	8	4	1	0	0	0	64107	64107	0	0	1	0	0.00%	1.52%
	3	8	4	1	0	0	0	50056	50081	0	0	1	0	0.00%	0.74%
	4	8	4	1	0	0	0	51324	51340	0	0	1	0	0.01%	1.38%
	5	8	4	1	0	0	0	50183	50183	0	0	1	0	0.02%	0.52%
	6	8	4	1	0	0	0	49468	49519	0	0	1	0	0.00%	1.20%
	7	8	4	1	0	0	0	45076	45077	0	0	1	0	0.00%	0.04%
	8	8	4	1	0	0	0	52991	52993	0	0	1	0	0.00%	1.05%
	9	8	4	1	0	0	0	55124	55124	0	0	1	0	0.00%	1.11%
	10	8	4	1	0	0	0	48936	48936	0	0	1	0	0.00%	0.45%
	Avg	8	4	1	0	0	0	52438	52447	0	0	1	0	0.00%	0.93%
	Max	8	4	1	0	0	0	64107	64107	0	0	1	0	0.02%	1.52%

Table 5.14: Analysis of  $\delta_{jk}$  (material quality) levels for Problem Class K2

	Instance No	# of Suppliers Selected	# Plants Opened	# DCCs Opened	# Disassembly Centers Opened	# Disposal Centers Selected	# Recycling Centers Selected	Total Plant Capacity	Total Distribution Capacity	Total Collection Capacity	Total Disassembly Capacity	# DCCs with Distribution	# DCCs with Collection	% VSS	% EVPI
High $\delta_{jk}$	1	8	5	1	1	1	1	76632	64836	13965	9145	1	1	0.23%	3.18%
	2	8	5	2	1	2	3	84547	78014	32234	21050	2	2	0.45%	5.67%
	3	8	5	2	1	2	3	66751	62134	28282	18076	2	1	1.22%	4.32%
	4	8	5	1	1	1	2	67566	61218	20647	14247	1	1	0.22%	5.16%
	5	8	4	1	1	1	2	58843	56755	15754	10128	1	1	1.70%	5.23%
	6	8	5	1	1	1	2	67270	60348	23365	15438	1	1	0.33%	5.04%
	7	8	5	1	1	1	2	59567	55207	21140	14732	1	1	0.17%	2.72%
	8	8	5	2	1	2	3	73928	67615	33914	21824	2	2	0.69%	4.84%
	9	8	5	2	1	2	3	76287	70143	33911	22078	2	2	0.66%	4.78%
	10	8	5	1	1	1	2	62616	57382	17817	12103	1	1	0.79%	3.93%
	Avg	8	4.9	1.4	1	1.4	2.3	69401	63365	24103	15882	1.4	1.3	0.65%	4.49%
	Max	8	5	2	1	2	3	84547	78014	33914	22078	2	2	1.70%	5.67%
Medium $\delta_{jk}$	1	8	5	2	1	2	3	76632	69565	33929	21250	2	2	0.37%	5.05%
	2	8	4	1	0	0	0	64107	64107	0	0	1	0	0.42%	5.95%
	3	8	5	2	1	2	3	66750	61353	30207	18952	2	1	0.59%	4.16%
	4	8	4	1	1	1	1	58074	55790	13126	8432	1	1	1.03%	5.30%
	5	8	4	1	1	1	2	58843	56451	17080	11102	1	1	2.44%	4.18%
	6	8	4	1	1	1	2	58343	55972	17813	11601	1	1	1.23%	5.21%
	7	8	5	1	1	2	3	59567	55100	26469	17235	1	1	0.50%	2.87%
	8	8	4	1	1	1	2	62969	60145	19360	12456	1	1	0.76%	4.55%
	9	8	5	2	1	2	3	76287	68383	36087	22472	2	2	0.33%	5.03%
	10	8	4	1	1	1	1	53140	52102	9537	5921	1	1	0.49%	3.47%
	Avg	8	4.4	1.3	0.9	1.3	2	63471	59897	20361	12942	1.3	1.1	0.82%	4.58%
	Max	8	5	2	1	2	3	76632	69565	36087	22472	2	2	2.44%	5.95%
Low $\delta_{jk}$	1	8	4	1	0	0	0	57116	57116	0	0	1	0	0.21%	3.33%
	2	8	4	1	0	0	0	64106	64241	0	0	1	0	0.00%	3.33%
	3	8	4	1	1	1	2	57038	54558	17027	11037	1	1	2.29%	2.98%
	4	8	4	1	1	1	2	58074	55699	16706	11026	1	1	3.06%	3.70%
	5	8	4	1	1	1	2	58843	55446	19888	12831	1	1	0.17%	2.75%
	6	8	4	1	1	1	2	58343	54923	20035	13404	1	1	3.32%	3.69%
	7	8	4	1	1	1	1	50481	48310	12809	8729	1	1	0.99%	2.56%
	8	8	4	1	1	1	2	62969	59217	21908	14244	1	1	3.50%	2.90%
	9	8	4	1	1	1	2	63348	60489	18670	12696	1	1	1.53%	4.45%
	10	8	4	1	0	0	0	48977	48936	0	0	1	0	1.08%	1.15%
	Avg	8	4	1	0.7	0.7	1.3	57929	55893	12704	8397	1	0.7	1.62%	3.08%
	Max	8	4	1	1	1	2	64106	64241	21908	14244	1	1	3.50%	4.45%

## 5.6 The Benefit of Using Closed-Loop Supply Chains

In order to estimate the benefit of CLSCs over regular forward supply chains, we set the return rate ( $\alpha_k$ ) to zero to obtain forward supply chain solution, and compare this solution to the solution of our original CLSC network design model. We summarize the result in Table 5.15. Including reverse chain increases the expected profit by 5.2% on average which is a significant amount since we are dealing with long term strategic decisions.

Table 5.15: Benefit of CLSC over Forward Supply Chain for Problem Class K2

Instances	Benefit
1	3.9%
2	3.2%
3	5.5%
4	4.5%
5	5.6%
6	6.2%
7	7.1%
8	6.8%
9	6.5%
10	2.9%
Average	5.2%
Maximum	7.1%

## CHAPTER 6

### CONCLUSION

In this study we consider a single-product CLSC design problem with demand, return and quality uncertainty. The network includes suppliers, plants, DCCs, customers, disassembly centers, recycling centers, disposal centers and spare part markets. The problem is modeled as a two-stage stochastic program that maximizes the expected profit where revenue is generated by sales of brand new products, refurbished products and spare parts and the costs are associated with facility opening and selection costs, capacity expansion costs, transportation and processing costs. In the first stage, when demands, returns and qualities are uncertain, facility opening and capacity installation decisions of plants, DCCs and disassembly centers are made. In addition, suppliers, disposal centers and recycling centers are selected among candidates in the first stage. In the second stage, when uncertainties are resolved, production decisions and flow decisions of materials, parts, brand new products and refurbished products are made.

To solve this problem, we consider several alternatives for increased computational performance because solving the extensive form becomes impractical even for small size problems. Therefore, we consider two variants (iterative-based and Branch-and-Cut-based) of the L-shaped method which is a stage-wise decomposition method. To increase the computational performance of these methods, improvements such as scenario-based grouping for adding multiple cuts and adding mean value cut are implemented.

These methods are implemented in JAVA using CPLEX Concert Technology 12.6. We try to solve as large instances as possible with an optimality gap of 0.1% within 3-hours of computational time limit. Computational experiments show that LS and BC methods provide significant decrease in computational times with respect to the solution time of EF. Further experiments on computational efficiency improvements show that adding mean value cuts does not increase the computational efficiency for our problem. On the other hand, generating group cuts generally supplies better performance compared to single-cut implementations. At this point, the value of the uncertain parameters is important. When uncertain data includes significant amount of information, adding multiple cuts supplies more valuable information to the RMP and therefore improves computational performance. On the other hand, when information is not valuable enough, adding multiple cuts only increases the size of the RMP which causes computational performance to decrease. Group forming strategy is also an important element affecting computational performance. In our case, demand-based grouping performs slightly better than demand-rate-based grouping. We also estimate VSS and EVPI values by using our numerical results.

In addition to these experiments, we also test the individual effect of uncertain parameters on solutions. First, we analyze the effects of individual uncertainties in demand, return and quality by keeping the rest of uncertain parameters at their mean values. These analyses show that demand uncertainty has the greatest influence on VSS and effect of quality of return is greater than that of return rate. Comparison of EVPI values demonstrate that, if applicable, reduction of uncertainty should be applied to uncertainty in demand, which has the greatest EVPI value. We also examine the effects of levels of uncertain return and quality rates. In this analysis, we test impacts of low and high levels of each uncertain rate. These analyses show that increasing return rate beyond a certain point when qualities are unchanged does not improve solution. This shows that the effect of return rate is highly dependent on quality levels. In addition, qualities at higher levels (i.e., product rather than material) supply larger effects on solutions. Another implication is that in cases with extremely low quality,

network is designed as a forward supply chain, which avoids excess costs associated with reverse flow of products with low quality.

There are several future research directions related to this work. One extension can be improving efficiency of grouping measures (e.g., including more uncertain parameters in creating grouping measures) by considering more aspects of the network to increase effectiveness of delivery of information to the RMP. To improve the computational performance or solve larger-sized instances, heuristics can also be developed. To capture uncertainty in a more realistic setting, building multi-stage (e.g. three-stage) stochastic programming models can be an appropriate future work. In addition, considering more complex structures such as multiple products with common parts could enable exploring wider ranges of effects of product and part recovery.





## Bibliography

- Alumur, S. A., Nickel, S., da Gama, F. S. and Verter, V. (2012), 'Multi-period reverse logistics network design', *European Journal of Operational Research* **220**, 67–78.
- Amin, S. H. and Zhang, G. (2012), 'A proposed mathematical model for closed-loop network configuration based on product life cycle', *International Journal of Advanced Manufacturing Technology* **58**, 791–801.
- Aravendan, M. and Panneerselvam, R. (2014), 'An integrated multi-echelon model for a sustainable closed loop supply chain network design', *Intelligent Information Management* **6**, 257–279.
- Birge, J. R. and Louveaux, F. (2011), *Introduction to Stochastic Programming*, Springer, New York, NY.
- Chouinard, M., D'Amours, S. and Kadi, D. A. (2008), 'A stochastic programming approach for designing supply loops', *International Journal of Production Economics* **113**, 657–677.
- Dantzig, G. B. (1955), 'Linear programming under uncertainty', *Management Science* **1**(3-4), 197–206.
- Easwaran, G. and Üster, H. (2010), 'A closed-loop supply chain network design problem with integrated forward and reverse channel decisions', *IIE Transactions* **42**(11), 779–792.
- El-Sayed, M., Afia, N. and El-Kharbotly, A. (2010), 'A stochastic model for forward-reverse logistics network design under risk', *Computers and Industrial Engineering* **58**, 423–431.
- Govindan, K., Soleimani, H. and Kannan, D. (2015), 'Reverse logistics and closed-loop supply chain: A comprehensive review to explore the future', *European Journal of Operational Research* **240**, 603–626.

- Guide, V. D. R., Souza, G. C., Van Wassenhove, L. N. and D. Blackburn, J. (2006), 'Time value of commercial product returns', *Management Science* **52**, 1200–1214.
- Guide, V. D. R. and Van Wassenhove, L. N. (2009), 'The evolution of closed-loop supply chain research', *Operations Research* **57**, 10–18.
- Jeihoonian, M., Zanjani, M. K. and Gendreau, M. (2017), 'Closed-loop supply chain network design under uncertain quality status: Case of durable products', *International Journal of Production Economics* **183**, 470–486.
- Kara, S. S. and Onut, S. (2010a), 'A stochastic optimization approach for paper recycling reverse logistics network design under uncertainty', *International Journal of Environmental Science and Technology* **7**, 717–730.
- Kara, S. S. and Onut, S. (2010b), 'A two-stage stochastic and robust programming approach to strategic planning of a reverse supply network: The case of paper recycling', *Expert Systems with Applications* **37**, 6129–6137.
- Lee, D. H., Dong, M. and Bian, W. (2010), 'The design of sustainable logistics network under uncertainty', *International Journal of Production Economics* **128**, 159–166.
- Listeş, O. (2007), 'A generic stochastic model for supply-and-return network design', *Computers and Operations Research* **34**, 417–442.
- Listeş, O. and Dekker, R. (2005), 'A stochastic approach to a case study for product recovery network design', *European Journal of Operational Research* **160**, 268–287.
- Madansky, A. (1960), 'Inequalities for stochastic linear programming problems', *Management Science* **6**, 197–204.
- Pishvaei, M. S., Jolai, F. and Razmi, J. (2009), 'A stochastic optimization model for integrated forward/reverse logistics network design', *Journal of Manufacturing Systems* **28**, 107–114.

- Ramezani, M., Bashiri, M. and Tavakkoli-Moghaddam, R. (2013), 'A new multi-objective stochastic model for a forward/reverse logistic network design with responsiveness and quality level', *Applied Mathematical Modeling* **37**, 328–344.
- Rogers, D. S. and Tibben-Lembke, R. (1998), *Going backwards: Reverse logistics trends and practices*. Center for Logistics Management, University of Nevada, Reno, Reverse Logistics Executive Council.
- Shapiro, A., Dentcheva, D. and Ruszczyński, A. (2009), *Lectures on Stochastic Programming: Modeling and Theory*, MPS-SIAM, Philadelphia, PA.
- Soleimani, H., Seyyed-Esfahani, M. and Shirazi, M. A. (2016), 'A new multi-criteria scenario-based solution approach for stochastic forward/reverse supply chain network design', *Annals of Operational Research* **242**, 399–421.
- Souza, G. C. (2012), 'Closed-loop supply chains: A critical review, and future research', *Decision Sciences* **44**(1), 7–38.
- Üster, H., Easwaran, G., Akçali, E. and Çetinkaya, S. (2007), 'Benders decomposition with alternative multiple cuts for a multi-product closed-loop supply chain network design model', *Naval Research Logistics* **54**, 890–907.
- Üster, H. and Hwang, S. O. (2016), 'Closed-loop supply chain network design under demand and return uncertainty', *Transportation Science* **forthcoming**.
- Zeballos, L. J., Mendez, C. A., Barbosa-Pavoa, A. P. and Novais, A. Q. (2014), 'Multi-period design and planning of closed-loop supply chains with uncertain supply and demand', *Computers and Chemical Engineering* **66**, 151–164.