THE EFFECTS OF CONTRACTUAL RELATIONSHIPS ON INVESTMENT INCENTIVES IN COMPLEMENTARY GOODS MARKET

by
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dedicated to my beloved deceased father...

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ABSTRACT

This paper tries to suggest some solutions to the debate on net neutrality where content providers and access providers in tandem produce a composite good; Internet service. Both content providers and excess providers contribute to their own component's quality and the quality that the internet users conceive depends on both components' qualities.

In our model, this debate is generalized to complementary goods market where final goods are composite goods comprising two components. The producers of the components contribute to the quality of their components where the value of the composite good is determined with a function of the qualities of components. We assumed; one firm's benefit per consumer is a exogenous constant, and the other gets benefit from pricing consumers for the final good. In the model, we investigate how introducing a price mechanism between the firms effects their investment decisions.

We found out that the optimal solution for net neutrality debate can be found by examining how the qualities of components are related to each other on determining the quality of internet service. If the qualities are more close to being complementary to each other, a positive price between the content providers and access providers is welfare improving.

TAMAMLAYICI ÜRÜNLER PİYASASINDA SÖZLEŞMELERİN YATIRIM TEŞVİKLERİNE ETKİSİ

Sinan Karadayı

Ekonomi, MA Tezi, 2008

Yar. Doç. Dr. Toker Doğanoğlu

Anahtar Kelimeler: Tamamlayıcı Ürünler, Kalite, Yatırım Teşvikleri

ÖZET

Bu tez internet servisi hakkındaki yoğun olarak tartışılmakta olan bir konuyu incelemektedir. İçerik sağlayıcıları ile erişim sağlayıcıları arasında bir fiyat varlığının topluma yararlı olup olmayacağına dair çözümler önermeye çalışmaktadır.

Kurduğumuz modelde bu tartışma daha genel bir çerçevede; tamamlayacı ürünler piyasasında incelenmiştir. Bu piyasada ortaya çıkan bileşik ürünler iki parçadan oluşmaktadır. Her parçayı üreten firma kendi parçasının kalitesine yatırım yapmaktadır ve bileşik ürünün kalitesi iki parçanın da kalitesine bağlı bir fonksiyon ile belirlenir. Modelimizde, bir firmanın her bir bileşik ürün kullanıcısından aldığı yarar sabit olarak varsayılırken, diğer firmanın ise kullanıcıları kendilerinin belirlediği bir fiyat ile ücretlendirebildiği varsayılmıştır.

Kurduğumuz modelle bulduğumuz sonuç; internet servisi hakkındaki tartışmaya çözümün parçaların kalitelerinin nasıl birbirleriyle ilişkilendirildiğinin araştırılması yolu ile bulunabileceğini öngörmektedir. Eğer parçaların kaliteleri internet servisinin kalitesini belirlemede daha çok tamamlayıcı özellik göstermekte ise, içerik sağlıyıcıları ile erişim sağlayıcıları arasında pozitif bir fiyatın olması toplumun refah seviyesini arttırıcı niteliktedir.

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Chapter 1

INTRODUCTION

1.1 The Composite Goods Markets

"Each rational person buys a product if its benefit exceeds its cost" This is the most basic teaching of economics. Demand of consumers for a product is derived by this basic idea. As the properties of a particular product are improved, the consumers get more benefit and accept to pay more. Therefore the firms make investments to increase their product's value for higher profits. In competitive markets, since profit margins are small, the incentives on investment are not very large. However if the firm is a monopolist, he does not compete with any other firms, therefore he can contribute to quality more. For example, the firm of Porsche improves its car's quality very frequently because it differentiated itself from other car brands and became a monopolist. The philosophy "Porsche Principle knows no upwards limits" explains how much they care about the quality.

Like in above example, the monopolist only takes into account the consumers and its cost function while making decision on quality investments. On the other hand, there are duopoly or oligopolistic markets where the firms have to consider other firms' decisions. Complementary goods markets are examples of this where produced components are complements for each other. Therefore producer of each component of a composite good takes into account other firms' decisions when he is investing in its quality.

There are many examples of complementary good markets having goods comprising two or more components. In some of these composite goods, the components are produced by the same firm and in some of them they are produced by different firms. A final good has a value if it has its all components, if one component does not exist, consumers will not prefer to buy the composite good. In these markets, each component has its own quality and the components' qualities determine the quality of the composite goods, therefore the firms of different components have to make strategic moves on how much to invest on the quality of its own component to maximize profits. Moreover, the incentive on investment for each firm is closely related to whether the components' contributions to the value of the final good are substitutes or complements. For example, a computer is composite good comprising hardware and software. These two components' contributions to quality can be thought as complementary to each other. We call this kind of components *quality complements*. The computer's quality is the minimum quality of hardware and software, because if the computer does not have a hardware of sufficient technology, the user can not enjoy many software applications. That is the reason why the software improvements in computer markets trigger the investments in hardware production. In other words, in a setting where components are quality complements, if the firm of one component does not invest in quality, the other firm has no incentive to increase its quality level. On the other hand, there are composite goods containing *quality substitute* components like a lecture. A lecture has both the lecturer and learning material contributing to the quality of the lecture. Both components' effect on the student's learning is almost independent of the other components's quality. Therefore in a market where the components are quality substitutes, the composite good's quality is the sum of qualities of all components, and even the firm of one component does not invest in quality, the other firm may prefer to invest to increase its profit.

The firms of the components strategically respond to the investment decisions of each others. When they integrate with each other they can internalize the external benefit of their investment decisions, therefore integrated monopolist of different component producers increases the profits and consumer surplus compared to a disintegrated monopolists, and also if they don't integrate, they provide lower quality level of the composite good in a market where components are complements. In Related Literature chapter the arguments on complementary goods markets are briefly explained.

This paper's main interest is about the price mechanism between the firms and its implications on improving the total welfare of the society, rather than the vertical integration option. The focus is on a specific type of composite good comprising two components where only one firm charges the consumer for the final good and the other firm has external benefit for each consumer using the final good. The newspaper markets have this kind of structure. A newspaper has its content about politics, magazine and daily life, besides it has commercial ads. The content and the commercial ads are two components of a newspaper. The owner of the commercial ads do not charge the newspaper readers, only the newspaper owner charges the readers, however as the number of readers increases, more people see the ads, and more costumers will buy the advertised product. So the owner of the ads externally gets benefit from newspaper readers without charging them. Furthermore, the newspaper owner charges the advertisement owners for their external benefit. In an exact manner, the markets where the interaction between the firms of components and consumers are modeled like the above example are called *Two Sided Markets*.

1.2 Two Sided Markets

Two sided markets has its own specifications and it differs from the composite goods markets in some points. Two sided markets include two groups and a platform. The groups are sellers and buyers. They interact with each other via platform. Besides, an agent's benefit for joining the platform depends on the number of agents of the other group joining to the platform. The platform can charge both sides to maximize its own profit.

Video game platforms are typical example of two sided markets, where game developers are the sellers, game console is the platform, and game players are the buyers. Consumers play video games via game console. As the number of game developers increases, the game players get more utility from the game console, and as the number of consumers increases, game developers will prefer to produce more games. So the game console producers have to attract both sides, to maximize its own profit. Therefore the platform's price mechanism is really important for the wellbeing of the society.

Each agent of a group exerts externality on the member of the other group. Therefore, platform's eagerness to attract the agents of a group depends on the size of the externality. Usually these externalities are fixed to constant parameters which means one side get a constant benefit for each agent of other group joining to the platform. The pricing mechanism for this kind of markets are analyzed by Armstrong and Roche - Tirole which enlighten our work on complementary goods markets. Their arguments are explained in Related Literature chapter extensively.

Basically, in two sided markets, the seller side and the platform are producing a composite good in tandem for the buyer; the games and game console are components of the composite good for the game players. So, the composite good's quality that the buyer side conceives is related to both the seller side's quality and the platforms's quality. In our specific model, each buyer exerts positive constant externality on the seller side while paying the platform for using the composite good.

1.3 The Debate On World Wide Web

This paper's main motivation comes from a debate on a specific two sided market where the buyer side's utility depends on the quality investment decisions of seller side and platform. These decisions together determine the quality of the final good. The specific two sided market where the platform and the seller side can be thought as firms of two components of a composite good is; *World Wide Web*.

World Wide Web changed our lives very much. We can easily acquire and offer knowledge, we can reach to many goods and services from all over the world while sitting in front of our computers. We can watch videos, download songs, communicate with our friends who are living abroad. So we can say that internet is one of the indispensable things of our daily lives. For this reason the quality of the internet is quite important for the society.

In internet there are three groups; content providers like google, skype, facebook; access providers like Turk Telecom; and the internet users. With a two sided market approach, the content providers are the sellers, the internet users are the buyers, and the access providers are the platforms. Internet users reach to the contents of google, skype and all other web sites, through the networks of access providers. Most of the content providers do not charge the internet users. They get external benefit when the users visit their web sites, because as the number of visitors increases more people see the advertisements on the web sites, and the content providers can charge the advertisement owners' more. On the other hand the access providers do not charge the content providers, but charge the internet users. As the number of internet users increase the content providers' utilities increase, also as the content variety increases the internet users gain more benefit.

Internet users' utilities depend on the quality of the internet service which comprises the network and content. For instance, when a person is watching a video on youtube; the resolution level of the content which is provided by the content provider, and the speed of download which is provided by access provider are two factors determining the conceived value of that video. Like in all composite goods, both network's and content's qualities contribute to the quality of internet service. Therefore the price mechanism between these component producers is crucial for the investment incentives on quality.

At the time being the access providers do not charge the content providers. The internet is neutral. However, net neutrality proponents and opponents are debating on whether the access providers should charge the content providers for using their network.

In Federal Trade Commission [2007], Bob Pepper, the senior managing director of Global Advanced Technology Policy at Cisco Systems says[1];

Services like web browsing e-mail, instant messaging, Voice over IP, and low-quality streaming video do not require high broadband speeds, and with a few exceptions, can actually tolerate interruptions and short delays in transmission. Dumb networks that merely send packets along and randomly drop packets during periods of congestion have been mostly sufficient to handle these types of applications. But they're not going to be sufficient if we are to realize the potential, full potential, of Web 2.0, which will focus on new applications like high quality video, user-generated content, multi-media applications.

According net neutrality opponents like Bob Pepper these new applications are going to require a wide broadband internet service which will allow the consumers to enjoy the applications without delay and interruptions. In his arguments he wants to highlight the fact that, access providers have to make big investments on their quality to keep up with the new generation contents, otherwise the internet users will not be able to enjoy these applications. Therefore the net neutrality opponents believe that the content providers should give money to the access providers to increase their incentives on investment and advance the internet's quality to a sufficient level.

On the other hand, net neutrality proponents believe that the reason why internet became so successful lies under its neutrality property. They assert that one of the most important property of internet is that the contents are reachable for every network in all around the world. If content providers have to make contracts with local access providers, the contents will be out of reach in places where the parties could not agree on the price. Also proponents assert that making contracts with each local access provider will be time consuming and costly. So they claim that, content providers should not spend time and money on the contracts, but they should spend time and money on innovation for increasing their content's quality. When they increase the content quality, they believe that more people will prefer to use internet which will indirectly increases the access providers' profits.

To sum up, net neutrality proponents argue that there should be no contracts between the content providers and access providers. They believe content providers' investments create sufficient incentives for investment in quality for access providers

[1] In [FTC, 2007] Workshop

because, as they together invest in final quality of internet service, the access providers will receive higher demands for their networks, and they can compensate their investment costs through charging more internet users with higher prices. On the other hand opponents say without access providers' investment on their networks' qualities, the users will not be able to enjoy the high quality contents, therefore access providers' investment costs must be compensated partially by the content providers, if they increase access providers' investment incentives, people's utilities from high quality contents will be higher, so more people will visit those contents and content providers' utilities will increase.

1.4 The Outline of The Thesis

There are many composite goods markets like internet comprising two components, where one firm externally benefits from the end users, while the other firm charges the end users. In following chapters, we will model this kind of composite goods market where the composite good's quality is determined via a value function of the components' qualities. We will compare the situation when there are not any contracts between the firms and when there are. First we will analyze the market when the qualities of components are perfect complements, after when the qualities of components are perfect substitutes. Under some conditions, we will show that the effects of introducing price mechanism between the firms of components depend on how the value function is set. In addition, the equilibrium contract decision between the firms and welfare maximizing price will be investigated. We will compare the findings of *quality components* case with the finding of *quality substitute* case and with the help of this comparison, we will comment on Net Neutrality Debate. Finally we will discuss how our modeling can be extended for two sided markets where there are many sellers and buyers, and more than one platform and we will finish the thesis with a conclusion part which will summarize our work briefly and our contributions to the literature.

Chapter 2

RELATED WORK

In this chapter we will try to summarize and explain some of the papers related to my thesis. The dynamics of our model, i. e. the cost structure, the settings of profit functions, utility functions, the quality arguments are determined with the help of these papers. In our work there are two relevant literature; complementary goods market and two sided markets.

2.1 Complementary goods market

The dynamics of complementary goods market are very much concern of economists. In our model, there are two complementary components and two different firms producing them. Therefore we benefited from different papers, especially for setting the utility function for consumers, including the innovation and investment arguments into my thesis.

In [Economides, 1999] the simple case where only one composite good is demanded was analyzed. They compared the quality levels when the components are owned by the same producer with quality levels when each component is produced by different producer.

In their model, they considered a market where the composite good AB comprises one unit of A and one unit of B. The quality of the composed good is the minimum of the qualities of components. They assumed the utilities of the consumers increase with quality and decrease with price. For the demand function they introduced a heterogeneity parameter, therefore the utility function is;

$$U_\Theta = \Theta q - p$$

where q stands for the quality, θ is the heterogeneity parameter which is assumed to be distributed on [0, 1] and p represents the price.

We used a similar utility function for consumers, but in our model we introduced the heterogeneity in a different way which is;

$$U_i = q - p - \xi_i.$$

Also we used a similar aspect for determining the value of the composite good.

They concluded that if the components' producers integrate and become a monopolist they will contribute to quality more than disintegrated monopolists. Also they will gain higher profits. Besides the price of the composite good will be lower in markets where the components are produced by same producers. Therefore integration is pareto improving. In disintegrated ownership, the quality improvements has higher impacts on price level, therefore, firms set lower qualities and have the same price impact.

The reason of their finding is very much related to *double marginalization.* In [Cournot, 1927] it is explained that if the quality levels are the same, the dual monopolists can not appropriate the full benefit of price decrease. However if they integrate they will internalize the external benefit of price decrease. This is the double marginalization and it leads to higher pricer.

Cournot's model was the same as the [Economides, 1999] where there is a composite good comprising two components. They analyzed the pricing strategies of two disintegrated monopolists and concluded that if they integrate they will set the price lower. And in [Sonnenschein, 1968] it is explained that that Cournot's duopoly arguments are also valid in complementary good markets.

Another paper on complementary goods is [Economides and Salop, 1992]. In their work they analyzed the composite good market, where there are multiple brands of compatible components. They compared the prices of joint ownership to independent ownership prices.

In their model each composite good comprises one unit of A and one unit of B. There are m differentiated producers of A and n differentiated producers of B. They found that if a producer of A integrates with a producer of B under some conditions prices can increase.[1] Mainly the effect of the integration depends on the sizes of the cross partials of demand. Besides they analyzed different market structures where horizontal or vertical externalities can be internalized. In the paper above, there are no quality arguments, but the analysis with increased number of brands of components and their price analysis are quite concern of our research topic.

[1]see also [Matutes and Regibeau, 1988] and [Economides, 1989]

The innovation incentives in composite goods are also investigated in another paper. In [Farrell and Katz, 2000] they analyzed the systems comprising two components, one of which is monopolized.

In their model the composite good AB comprises one unit of A and one unit of B. There is a single producer of A labeled M which may also or may not produce B. There is at least one independent producer of B. They tried to answer whether the monopoly will prefer to extract the efficiency rents in the market of component B. They concluded that M chooses to produce B component and decreases the price of it, because with this price squeeze M can gain more profits by charging consumers with higher prices for A.

In our model, like in above papers the composite goods have value if they are used together. On the other hand, there are some complementary goods where each component is valuable even though they are used without its complement. For example in their paper [Chen and Nalebuff, 2006] they analyzed the market where one good is essential for the use of other but not vice versa. In their model there are two components A and B and they produced by different monopolists. A has a value itself, but B has value only if it is used with A. Actually their main motivation comes from the debate on the policy of Microsoft. They tied their Windows operator system with Internet Explorer. In their model they found out that the optimal strategy of the monopolist producer of A is to produce a similar version of B and to give them for free or acquire firm B and set the price equal to 0.

There is another paper [Choi, 2004] where they analyzed the tying policy of Microsoft. In their paper they investigated the effects of tying decisions on R&D investment incentives. They found that if the firms ties two products, it can commit more aggressive investments. Therefore tying option can provide higher quality levels of the tied goods and can increase the wellbeing of consumers.

2.2 Two sided goods market

The two sided markets literature was helpful for determining how we are going to set the profit functions. The main papers we investigated for the dynamics of two sided markets are explained in this part.

In [Armstrong, 2005] the dynamics of two sided markets are extensively analyzed. In his work he tried to find how the platform sets profit maximizing prices. In his model, there are two groups and they interact with each other via platforms. The utility of each agent is introduced such;

$$u_j^i = \alpha_j i n^i + \zeta_j^i$$

Here n_i is the number of agents from the other side who are present on platform i, α_j^i is the benefit that agent j enjoys from interacting with each agent on the other side, and ζ_j^i is the fixed benefit the agent obtains from using that platform. In his paper he assumed α is the same for every agent of a group, and the heterogeneity is sustained via ζ. However in [Rochet and Tirole, 2003] the heterogeneity is given by α.

With above assumptions in Armstrong's work, the utilities of the groups are introduced such;

$$u_1^i = \alpha_1 n_2^i - p_1^i \qquad u_2^i = \alpha_2 n_1^i - p_2^i$$

where u_1 and u_2 are utilities of two groups and p values are the prices they give to the platforms. i stands for different platform. He assumed that the demand functions are increasing functions of these utility functions.

In his analysis he first investigated the equilibrium price when there is only one platform and concluded that the group whose elasticity is high or the external benefit that they exert on the other group is large are charged with smaller prices compared to other group.

In following chapter he increased the number of platforms and analyzed the pricing dynamics, and he concluded that the determinants of the equilibrium prices are the magnitudes of cross group externalities, the way of how fees are levied and whether the agents join one or more platform.

In our model, we used a similar utility function for the firm which externality benefits from the consumers. Therefore his analysis on prices enlightened our work very much.

Furthermore we benefited from [Rochet and Tirole,][2005] since their work provided a better understanding of two sided markets. In most of papers, two sided markets are introduced with a “you know a two sided market when you see it” flavor. However in their work the definition of two-sided market is precisely given. They say that two sided markets are the markets where not only the total of prices charged to buyers and seller matters, but the structure also matters. They gave examples of markets which can be examples of two sided markets and which can not. For instance a firm pays its labor wage and it sells products to consumers, so we can say that consumers reach to to workers via platform. However this market is not two sided, because the structure of pricing does not effect the wellbeing of society. Also they proved that Coase theorem is necessary for being two sidedness. They tried investigate effects of having a membership benefit and usage benefit at the same time with a model like this:

$$U^i = (b^i - a^i)N^j + B^i - A^i$$

where b^i is the benefit of consumer i interacting with each agent from other side, a^i is per user charge, B^i is the membership benefit and A^i is the membership cost. They figured out the optimal pricing strategies of the platform.

The multi homing option is an important factor for the pricing strategies of platforms. Multi homing is basically, if an agent of a group can join more than one platform at the same time, this means he can multi home. In [Choi, 2007] paper the tying options is analyzed in two sided markets where the agents can multi home. Their paper is also motivated from Microsoft tying policies. In their model there are two platforms A and B. Hotelling model is used to create product differentiation, there are buyers and sellers. They found out that tying can be welfare enhancing when the agents can multi home, because tying makes consumers to multi home more, which is also beneficial for agents of seller side.

To determine our model, the above papers' approaches are investigated elaborately. Therefore we can say that our model involves some of both markets' dynamics and mechanisms.

Chapter 3

MODEL

In the model there are two firms and two components; A and B. Each composite good includes one unit of A and one unit of B. The value of using the composite good AB will depend on the qualities of both A and B. Component A will represent the product of content providers and component B will be the product of access providers. So composite good AB will represent the internet service and obviously its quality depends on both the quality of the content and the quality of the network. We will denote this value with;

$$V_{AB} = f(q_A, q_B)$$

We will assume there are no cost of producing either component. The firms only incur investment costs for their component's quality. Let $k > 0$ denote the cost parameter. As k becomes larger, investment cost increases.

$$C(q_i) = \frac{kq_i^2}{4}$$

As it is mentioned before the content providers do not charge internet users and get external benefit when users visit their contents. Let $\alpha > 0$ denote the external benefit of the firm A for each user using the composite good AB. As the number of visitors increases, advertisement revenues go up for content providers. However we know that access providers charge internet user for using internet. Firm B will get profit by charging the users for buying AB with the price of P_{AB}. We investigate the effect of a contractual relationship between the firms, therefore we will assume firm B will charge firm A with a constant price for each user. In the model P_A will denote the price firm A will pay to firm B for each user. n will denote the number of consumers of composite good AB. With these notifications the firms' profits can be written as;

$$\Pi_A = (\alpha - P_A)n - \frac{kq_A^2}{4} \tag{3.1}$$

$$\Pi_B = (P_{AB} + P_A)n - \frac{kq_B^2}{4} \tag{3.2}$$

We assume there is a continuum of users with mass M where M is sufficiently large. Each user has a unit demand for AB. The consumer will purchase one unit of AB, if his utility is positive otherwise he will purchase nothing. The utility depends not only on the value of V_{AB} and the price P_{AB} but also the heterogeneity parameter ξ. Here ξ represents the affinity of a consumer using internet service. The smaller is the value of ξ the more likely it is that a consumer would like to purchase composite good AB. We assume ξ has a uniform density function distributed on the interval $[0, M]$ with a density of 1.

$$U_i = V_{AB} - P_{AB} - \xi_i \qquad \xi_i \sim U(0, M) \tag{3.3}$$

For a given V_{AB} and P_{AB} there exist a consumer i whose $\overline{\xi_i} = V_{AB} - P_{AB}$ therefore he will be indifferent between buying or not buying the composite good. Consumers whose ξs are smaller than $\overline{\xi}_i$, will have positive utilities and prefer to buy the good. Those whose $\xi > \overline{\xi}_i$ will not buy anything. The number of users is;

$$n = \int_0^{V_{AB}-P_{AB}} d\xi_i$$

$$n = V_{AB} - P_{AB}$$

As the value of the composite good increases, the number of buyers increases, and as the price of the good increases the number of buyers decreases.

We search for a subgame perfect Nash equilibrium of the model. The sequence of the events are;

1. Firms of components agree on the contract between them. i.e firm B decides on a P_A.

2. Knowing the contract decision, firms simultaneously decide on the profit maximizing quality levels.

3. Knowing the contract between the firms and the value of the composite good firm B sets the price of composite good AB that consumers will pay.

To find the equilibrium, we start with the last step where the firm of B sets the price of composite good. If we substitute the number of users into the equation 3.2 the profit function of firm B yields;

$$\Pi_B = (P_{AB} + P_A)(V_{AB} - P_{AB}) - \frac{kq_B^2}{4}$$

The profit is a concave function of P_{AB} and there is a value of P_{AB} which maximizes the profit function, therefore it can be easily verified that optimal $P_{AB}*$ and $n*$ are;

$$P_{AB}^* = \frac{V_{AB} - P_A}{2} \qquad n^* = \frac{V_{AB} + P_A}{2} \tag{3.4}$$

Substituting the values in 3.4 into the firms' profit functions 3.1 and 3.2 yields;

$$\Pi_A = (\alpha - P_A)\frac{V_{AB} + P_A}{2} - \frac{kq_A^2}{4} \tag{3.5}$$

$$\Pi_B = (\frac{V_{AB} + P_A}{2})^2 - \frac{kq_B^2}{4} \tag{3.6}$$

The consumer surplus can be computed as;

$$CS = \int_0^{V_{AB} - P_{AB}} (V_{AB} - P_{AB} - \xi_i) d\xi_i$$

$$CS = \int_0^{\frac{V_{AB} + P_A}{2}} (\frac{V_{AB} + P_A}{2} - \xi_i) d\xi_i$$

$$CS = \frac{1}{2}(\frac{V_{AB} + P_A}{2})^2 \tag{3.7}$$

The second step where the firms decide on their components' qualities is closely related to how the value function is set, because of this reason, in the following sections different kinds of value functions will be analyzed.

3.1 Perfect Complements

In this section, we analyze the value function where the qualities of the components are perfect complements. i e;

$$V_{AB} = min(aq_A, bq_B) \tag{3.8}$$

In this value function $a > 0$ and $b > 0$ are the marginal contributions of the components' qualities to the final value of the composite good. Their incentives on investment depend on the other firm's quality contribution. If one of them does not contribute to quality the other firm can not increase the value of AB. In the case of net neutrality, to increase the quality of the internet service both content provider and access provider have to increase their quality levels at the same time. The value of the composite good can be written as;

$$V_{AB} = \begin{cases} aq_A & \text{if } q_A \leq \frac{bq_B}{a} \\ bq_B & \text{if } q_B \leq \frac{aq_A}{b} \end{cases} \tag{3.9}$$

If the value function is substituted into the profit function of firm A;

$$\Pi_A = \begin{cases} (\alpha - P_A)\frac{aq_A+P_A}{2} - \frac{k(q_A)^2}{4} & if \quad q_A \leq \frac{bq_B}{a} \\ (\alpha - P_A)\frac{bq_B+P_A}{2} - \frac{k(q_A)^2}{4} & if \quad q_A > \frac{bq_B}{a} \end{cases} \tag{3.10}$$

Also the profit function of firm B becomes;

$$\Pi_B = \begin{cases} (\frac{bq_B+P_A}{2})^2 - \frac{k(q_B)^2}{4} & if \quad q_B \leq \frac{aq_A}{b} \\ (\frac{aq_A+P_A}{2})^2 - \frac{k(q_B)^2}{4} & if \quad q_B > \frac{aq_A}{b} \end{cases} \tag{3.11}$$

At the second step of the model, knowing the price between the firms, both firms simultaneously decide on their quality contribution levels. While they are making these decisions they will take into account other firm's quality decision.

Proposition 1: If $k > b^2$ holds, both profits are concave functions of their qualities. There will be multiple Nash equilibria. The equilibrium values of the quality contributions are $q_A^* = \frac{1}{a}q^*$ and $q_B^* = \frac{1}{b}q^*$ where q^* is the equilibrium value of the composite good AB;

$$V_{AB}^* = q^* \leq min[\frac{b^2 P_A}{k - b^2}, \frac{a^2(\alpha - P_A)}{k}]$$

Proof. If we look at the profit function of firm A, we can see that the first part of the profit function is concave function of q_A. Therefore we can find a q_A value which maximizes the first part.

$$\begin{aligned} \frac{\partial \Pi_A}{\partial q} &= \frac{a(\alpha - P_A)}{2} - \frac{kq}{2} = 0 \\ \widehat{q_A} &= \frac{a(\alpha - P_A)}{k} \end{aligned} \tag{3.12}$$

Firm A will contribute to quality until it does not increase the profit, therefore q_A has to be equal or less than $\widehat{q_A}$. Besides increasing q_A more than $\frac{bq_B}{a}$ will not increase the profits. Therefore firm A's best response function will be;

$$BR_A = \frac{1}{a}min[\frac{a^2(\alpha - P_A)}{k}, bq_B]$$

If we look at the profit function of B; the first part of the profit function is convex in q_B if $k < b^2$ which means higher q_B values always increase the profit of firm B, however if $k > b^2$ Π_B will be a concave function of q_B and we can find a profit maximizing q_B from the first order condition[1];

$$\frac{\partial \Pi_B}{\partial q_B} = \frac{bq_B + P_A}{2} - \frac{kq}{2} = 0$$
$$\widehat{q_B} = \frac{bP_A}{k - b^2}$$

Therefore firm B will contribute to quality until it does not increase the profit, therefore q_B has to be equal or less than $\widehat{q_B}$. Besides increasing q_B more than $\frac{aq_A}{b}$ will not increase the profits so the best response function of firm B is;

$$BR_B = \frac{1}{b} min[\frac{b^2 P_A}{k - b^2}, aq_a]$$

When we solve two best response functions together, if the following conditions hold;

$$aq_A < \frac{b^2 P_A}{k - b^2} \quad and \quad bq_B < \frac{a^2(\alpha - P_A)}{k}$$

the firms will contribute to their own components' qualities such that $aq_A = bq_B$. therefore we can define the Nash Equilibria values such;

$$q_A = \frac{q^*}{a} \;\; and \;\; q_B = \frac{q^*}{b}$$

where;

$$q^* \leq min[\frac{b^2 P_A}{k - b^2}, \frac{a^2(\alpha - P_A)}{k}]$$

□

Therefore there will be a continuum of the Nash equilibria in this model and each equilibrium values yield different profits for firms. Therefore the firms prefer one of the Nash equilibrium more than the others. The Nash Equilibrium which both firms will prefer is the Pareto efficient equilibrium of this model.

Lemma 1: The Pareto optimal equilibrium of the above Nash Equilibria is defined as;

$$q_A = \frac{q^*}{a} \;\; and \;\; q_B = \frac{q^*}{b}$$

[1]We will assume $k > b^2$ holds in the rest of our analysis since it is necessary for concavity.

where;

$$q^* = min[\frac{b^2 P_A}{k - b^2}, \frac{a^2(\alpha - P_A)}{k}]$$

Proof. If we substitute the Nash equilibrium values of q, q_A and q_B into profit functions of firms, the profit functions can be written as;

$$\Pi_A = (\alpha - P_A)\frac{q + P_A}{2} - \frac{k}{4}(\frac{q}{a})^2$$
$$\Pi_B = (\frac{q + P_A}{2})^2 - \frac{k}{4}(\frac{q}{b})^2$$

Both profit functions are concave in q, therefore we can find optimal q value for each profit function from first order conditions. For firm A the profit maximizing q is;

$$\frac{\partial \Pi_A}{\partial q} = \frac{\alpha - P_A}{2} - \frac{k}{2}(\frac{q}{a^2}) = 0$$
$$q' = \frac{a^2(\alpha - P_A)}{k}$$

However for firm B, the profit maximizing q value is;

$$\frac{\partial \Pi_B}{\partial q} = \frac{q + P_A}{2} - \frac{k}{2}(\frac{q}{b^2}) = 0$$
$$q'' = \frac{b^2 P_A}{k - b^2}$$

For any q^* smaller than both q' and q'' increasing q^* will increase both producers profits. Because of this reason they both prefer to increase their qualities at the same time to increase q^*. Therefore the Pareto optimal q* value is the minimum of q' and q'', and Pareto optimal quality level will be;

$$q_A = \frac{q^*}{a} \; and \; q_B = \frac{q^*}{b}$$

□

Case 1:In this part we will investigate the case where;

$$0 \leq P_A \leq \alpha(1 - \frac{kb^2}{kb^2 + a^2(k - b^2)})$$

Lemma 2: If the following condition holds;

$$0 \leq P_A \leq \alpha(1 - \frac{kb^2}{kb^2 + a^2(k - b^2)})$$

then the Pareto optimal value of the composite good and Pareto optimal quality contributions will be;

$$V_{AB} = \frac{b^2 P_A}{k - b^2} \tag{3.13}$$

$$q_A^* = \frac{b^2 P_A}{a(k - b^2)} \qquad q_B^* = \frac{b P_A}{(k - b^2)} \tag{3.14}$$

where both quality contribution levels are increasing in b, P_A while decreasing in k, besides as a increases optimal q_A^* decreases.

Proof. The optimal value of the q^* depends on the value of the q' and q''. When $P_A = 0$ $q'' = 0$ while $q' > 0$ and as P_A increases q'' increases and q' decreases, therefore there exist a $\overline{P_A}$ where $q' = q''$ and for P_A values smaller than $\overline{P_A}$ Pareto optimal value will be $q^* = q''$ and P_A values bigger than $\overline{P_A}$ the Pareto optimal value will be $q^* = q'$.The altering $\overline{P_A}$ is;

$$\overline{P_A} \;=\; \alpha(1 - \frac{kb^2}{kb^2 + a^2(k - b^2)}) \tag{3.15}$$

Since α , k, a, b, $k - b^2$ are positive values $\overline{P_A}$ will be between 0 and α. Then if P_A is between 0 and $\overline{P_A}$ the Pareto optimal quality contribution levels and the value of the composite good will be;

$$V = q'' \quad q_A^* = \frac{q''}{a} \quad and \quad q_B^* = \frac{q''}{b}$$

where

$$q'' = \frac{b^2 P_A}{k - b^2}$$

We investigate how the parameters effect the Pareto optimal equilibrium values of the qualities. The derivatives of the quality levels with respect to P_A gives;

$$\frac{\partial q_A}{\partial P_A} = \frac{b^2}{a(k - b^2)} \qquad \frac{\partial q_B}{\partial P_A} = \frac{b}{k - b^2}$$

Since b , a, $k - b^2$ are positive values, the above derivatives are positive which means as P_A increases both quality levels will increase.

Besides if we take the derivative of the quality levels with respect to b;

$$\frac{\partial q_A}{\partial b} = \frac{2bkP_A}{a(k - b^2)^2} \qquad \frac{\partial q_B}{\partial b} = \frac{P_A(k - b^2 + 2b)}{(k - b^2)^2}$$

Since a, k, b, $k - b^2$ are positive whenever $P_A > 0$ the above derivatives will be positive which means as b increases both qualities increase.

The derivative of the quality levels with respect to k gives;

$$\frac{\partial q_A}{\partial k} = -\frac{b^2 P_A}{a(k-b^2)^2} \quad \frac{\partial q_B}{\partial k} = -\frac{bP_A}{(k-b^2)^2}$$

Since a, b are positive, whenever $P_A > 0$ the above derivatives will be negative which means as k increases both quality contribution levels decrease.

The derivative of the quality of firm A with respect to a gives;

$$\frac{\partial q_A}{\partial k} = -\frac{b^2 P_A}{a^2(k-b^2)}$$

Since $k - b^2$ is positive, whenever $P_A > 0$ the above derivative will be negative, which means as a increases firm A's contribution level decreases. □

When P_A increases both firms will contribute to quality more, because when P_A increases firm B will gain more for each user which means the marginal returns of investing to quality increases for firm B. Therefore firm B will invest in quality more. Since firm A's best response is increasing with q_B, he will also contribute more.

Like the increase in P_A when b increases both firms will also contribute to quality more, because when B component's marginal contribution to the value of the final good increases, firm B's marginal return of investing to quality increases. Therefore firm B will invest in quality more. Since firm A's best response is increasing with q_B, he will also contribute more.

On the other hand, when k increases both firms will contribute to quality less, because when the cost parameter increases investing in quality for firm B becomes more costly which means the marginal cost of investing to quality increases for firm B. Therefore firm B will invest in quality less. Since firm A's best response is increasing with q_B, he will also invest in quality less.

Furthermore as a increases firm A decreases optimal q_A value because, firm A's best response function is decreasing with a, therefore as the marginal contribution of A increases since q_B does not change firm B will contribute to quality less.

Effects of P_A on the profits of the firms and consumer surplus when qualities are complements

If we substitute the Pareto optimal value of the composite good into equation 3.4, price of the final good and number of consumers become;

$$P_{AB} = \frac{(2b^2 - k)P_A}{2(k - b^2)} \quad and \quad n = \frac{kP_A}{2(k - b^2)}$$

Substituting these values into the profit function of the firms yields:

$$\Pi_A = (\alpha - P_A)\frac{kP_A}{2(k - b^2)} - \frac{k}{4}(\frac{b^2 P_A}{a(k - b^2)})^2 \tag{3.16}$$

$$\Pi_B = \frac{k(P_A)^2}{4(k - b^2)} \tag{3.17}$$

If we substitute the Pareto optimal equilibrium values into 3.7. Consumer surplus becomes;

$$CS = \frac{1}{2}(\frac{kP_A}{2(k - b^2)})^2 \tag{3.18}$$

When there is no contract between the producers. i. e. $P_A = 0$. Both profits and consumer surplus will be zero, because the firm B will have no incentive for investing in quality, consequently the quality of the final good, its price become 0 which makes the market collapse. Because of this reason how parties will be effected when P_A is introduced must be analyzed.

Proposition 2: When the price between the firms increased from 0 both firms' profits increase.

Proof. First we will take the derivative of value function with respect to P_A;

$$\frac{\partial V_{AB}}{\partial P_A} = \frac{2b}{k - b^2} \tag{3.19}$$

Since b and $k - b^2$ are positive, increasing P_A from 0 will increase the value of the final good.

If we take the derivative of profit function of firm A with respect to P_A yields;

$$\frac{\partial \Pi_A}{\partial P_A} = \frac{k(\alpha - 2P_A)}{2(k - b^2)} - \frac{kb^4 P_A}{2a^2(k - b^2)} \tag{3.20}$$

The above derivative is concave in P_A and at the point where $P_A = 0$ the derivative is;

$$\frac{\partial \Pi_A}{\partial P_A} |_{P_A=0} = \frac{k\alpha}{2(k-b^2)}$$

Since k, α and $k-b^2$ are positive values the above derivative will be positive which means as P_A is increased from 0, profit of firm A also increases.

If we take the derivative of profit function of firm B with respect to P_A yields;

$$\frac{\partial \Pi_B}{\partial P_A} = \frac{k(P_A)}{2(k-b^2)} \tag{3.21}$$

Since k and $k-b^2$ are positive values for any $P_A > 0$ the above derivative will be positive which means the profit of firm B is convex and increasing P_A will provide higher profits for firm of B. □

Case 2: In this part we will investigate the case where;

$$\alpha(1 - \frac{kb^2}{kb^2 + a^2(k-b^2)}) \leq P_A \leq \alpha$$

As it mention before as P_A increases the optimal q^* switches from q'' to q'. With the above assumption $q^* = q'$, therefore, the pareto optimal value of the final good and the equilibrium quality values will be;

$$V_{AB} = q^* = \frac{(\alpha - P_A)a^2}{k}$$

$$q_A^* = \frac{(\alpha - P_A)a}{k} \quad and \quad q_B^* = \frac{(\alpha - P_A)a^2}{kb}$$

Until P_A attains $\overline{P_A}$, the equilibrium value of the composite good and equilibrium qualities of the components increase, however as P_A exceeds $\overline{P_A}$ they start to decrease and when P_A reaches to α both Pareto optimal equilibrium qualities and value of the composite good becomes 0. Therefore the maximum value of these variables are attained when $P_A = \overline{P_A}$

When we substitute the Pareto equilibrium quantities into 3.5 and 3.6 the profit functions of firms turn into;

$$\Pi_A = (\alpha - P_A)\frac{(\alpha - P_A)a^2 + kP_A}{2k} - \frac{(\alpha - P_A)^2a^2}{4k} \tag{3.22}$$

$$\Pi_B = (\frac{\alpha a^2 + P_A(k-a^2)}{2k})^2 - \frac{(\alpha - P_A)^2a^4}{4kb^2} \tag{3.23}$$

To investigate how increasing P_A more than $\overline{P_A}$ effects the profit of firm A we have;

$$\frac{\partial \Pi_A}{\partial P_A} = \frac{(\alpha - P_A)(a^2 - k) - P_A k}{2k}$$

If $k - a^2 < 0$ holds for any $P_A \geq 0$ the derivative above will be negative which means increasing P_A more than $\overline{P_A}$ decreases the profit of firm A.

The contract agreement between the firms when qualities are complements

The decision for the contactual relationship between the firms is done as the first step of the sequences of the events. firm B will try to maximize his profit by choosing P_A.

Proposition 2: If $a^2 > k$ holds, firm A set a positive P_A which is in the interval $[\overline{P_A}\ ,\ \alpha]$.

Proof. As it is mentioned before for different ranges of P_A the profit function of B changes. If we look at equation 3.17 and 3.21 when $P_A < \overline{P_A}$ the profit function is convex in P_A, therefore increasing P_A increases Π_B. we can conclude that firm B will never set P_A less than $\overline{P_A}$. However when $P_A > \overline{P_A}$ with the above assumption, the profit function of firm B is concave [2] in P_A, and the derivative of Π_B with respect P_A gives;

$$\frac{\partial \Pi_B}{\partial P_A} = \frac{P_A(k - a^2)^2 + \alpha a^2(k - a^2)}{2k^2} + \frac{(\alpha - P_A)a^4}{2kb^2}$$

Since profit function of firm B is concave we can find an optimal P_A value from equating above first order condition to 0. The optimal P_A is;

$$P_A^* = \frac{\alpha a^2[a^2(b^2 - k) - kb^2]}{(k - a^2)^2 b^2 - ka^4} \tag{3.24}$$

P_A^* is bigger than $\overline{P_A}$ and smaller than α. In the appendix the necessary conditions and the details of the proof is explained. Therefore we can conclude that increasing P_A increases the profit of firm B until P_A^*. Therefore the equilibrium value of P_A will be P_A^* □

The reason why firm B chooses the above price is that as it is already mentioned, before when $P_A = 0$ firm B's profit is 0, however as P_A begins to increase in equations 3.13 and 3.14 we see that both firms gain incentives for investment and V_{AB} increases. Also in equation 3.4 the number of consumers increases as the value of the composite good increases. As a consequence, until $P_A = \overline{P_A}$ the marginal benefit of investing in quality is always higher than the marginal cost of quality investments. However when

[2] See Appendix for the proof

P_A exceeds $\overline{P_A}$ the incentives on investment changes and both firms prefer to decrease their quality contribution which leads to a lower value of the composite good and less consumer. That's why the revenue of firm B begins to decrease, but at the same time the cost of investing also decreases. The cost decrease dominates the revenue decrease until $P_A = P_A^*$ and after P_A^* the profits starts to decrease. That is why firm B will choose P_A^* as the price between the firms.

Nevertheless if a social planner decides on the price who wants to maximize welfare of the consumers, he will choose the price which maximizes the consumer surplus. For $P_A < \overline{P_A}$ consumer surplus is

$$CS = \frac{1}{2}(\frac{kP_A}{2(k-b^2)})^2$$

If we take derivative of CS with respect to P_A;

$$\frac{\partial CS}{\partial P_A} = \frac{P_A}{4}(\frac{k}{k-b^2})^2$$

The above derivative is positive for any $P_A > 0$ and the consumer surplus function is convex in P_A, which indicates that increasing price will always increase consumer surplus in the interval where $0 < P_A < \overline{P_A}$. Therefore the social planner never sets the price smaller than $\overline{P_A}$. For $P_A > \overline{P_A}$ consumer surplus is

$$CS = \frac{1}{2}(\frac{\alpha a^2 + P_A(k-a^2)}{2k})^2$$

If we take derivative of CS with respect to P_A;

$$\frac{\partial CS}{\partial P_A} = \frac{[\alpha a^2 + P_A(k-a^2)](k-a^2)}{4k^2}$$

If $k - a^2$ is negative the above derivative will be negative where P_A is in the interval $[\overline{P_A}, \alpha]$ which means increasing P_A more than $\overline{P_A}$ decreases consumer surplus. Therefore the social planner who wants to maximize wellbeing of the consumers will set the price between the firms equal to $\overline{P_A}$.

The intuition is simple. We know that until $P_A = \overline{P_A}$ the firms will have incentives for increasing their quality. Therefore as P_A increase the value of the composite good and the number of consumers increase until $\overline{P_A}$, but when P_A exceeds $\overline{P_A}$ the firms prefer to invest less in the quality which leads to lower value of the final good and less consumers. That's why as P_A increases from 0 to $\overline{P_A}$ the consumer surplus increases and it reaches its maximum at $P_A = \overline{P_A}$ and after that price level the consumer surplus decreases. Therefore consumer surplus maximizing P_A value is $\overline{P_A}$.

Finally, if a social planner who wants to maximize the total welfare of the economy, he will maximize the sum of firms profits and consumer surplus which is;

$$W = CS + \Pi_A + \Pi_B \tag{3.25}$$

Under some conditions, W function is concave in P_A and we can find a welfare maximizing P_A which is smaller than α and bigger than $\overline{P_A}$ which is;

$$P_A^w = \alpha(1 - \frac{(3b^2a^2k - k^2b^2)}{-3\,b^2a^4 + 4\,b^2a^2k + k^2b^2 + 2\,a^4k} \tag{3.26}$$

The proof of this and the welfare maximizing P_A value is given in appendix.

3.2 Quality Substitutes

In this section, we will analyze the value function where the contributions of qualities of the components to the quality of the final good are substitutes to each other. i e;

$$V_{AB} = \begin{cases} aq_A + bq_B & \text{if A and B both exist} \\ 0 & \text{otherwise} \end{cases} \tag{3.27}$$

a and b are the marginal contributions of the components' qualities to the value of the final good. As it is seen in the equation the marginal contribution of each component's quality is independent of the other components' quality. When we have this value function, we can substitute $aq_A + bq_B$ for all V_{AB} in the profit functions of firms.

$$\Pi_A = (\alpha - P_A)\frac{aq_A + bq_B + P_A}{2} - \frac{kq_A^2}{4} \tag{3.28}$$

$$\Pi_B = (\frac{aq_A + bq_B + P_A}{2})^2 - \frac{kq_B^2}{4} \tag{3.29}$$

At the second step of the setting, knowing P_A each component firms will try to maximize its profit by choosing their component's quality levels. While making their decisions, they will take into account other firm's quality decisions.

Proposition 3: If $k > b^2$ holds, both profits will be concave functions of the qualities, the equilibrium will be;

$$q_A^* = \frac{a(\alpha - P_A)}{k} \text{ and } q_B^* = \frac{a^2b\alpha + bP_A(k - a^2)}{k(k - b^2)} \tag{3.30}$$

q_A* is increasing in a and α while it is decreasing in k and P_A. q_B* is increasing in a, b and α while it is decreasing in k. Also if $a^2 > k$ q_B is decreasing in P_A.

Proof. In equation 3.31 it is seen that the firm A's marginal cost of contributing to quality is an increasing function however its marginal revenue is constant. For this reason we can find a q_A where firm A maximizes its profit. Besides in equation 3.32 firm B's both marginal revenue and marginal cost of contributing to quality is increasing function. If $k < b^2$ increasing the quality will always increase the profits, however if $k > b^2$ holds profit of B will be a concave function of q_B therefore we can find an optimum for q_B as well. With the assumption of $k > b^2$, the first order conditions are;

$$\frac{\partial \Pi_A}{\partial q_A} = \frac{a(\alpha - P_A)}{2} - \frac{kq_A}{2} = 0$$
$$\frac{\partial \Pi_B}{\partial q_B} = \frac{b(aq_A + bq_B + P_A)}{2} - \frac{kq_B}{2} = 0$$

From the first order conditions we can obtain how firms will respond to other firm's quality decisions. firm A's quality contribution decision is independent of q_B,

$$q_A = \frac{a(\alpha - P_A)}{k}$$

however firm B has a best response function that increases with q_A

$$BR_B = \frac{abq_A + bP_A}{k - b^2}$$

If we substitute the optimal q_A value into the best response function of firm B, the profit maximizing q_A* and q_B* are;

$$q_A^* = \frac{a(\alpha - P_A)}{k} \qquad q_B^* = \frac{a^2 b\alpha + bP_A(k - a^2)}{k(k - b^2)} \tag{3.31}$$

The effects of a , b , k ,P_A, α on the equilibrium quality levels can be easily derived from the above equations. If I take the derivatives of both quality level with respect to α;

$$\frac{\partial q_A^*}{\partial \alpha} = \frac{a}{k} \qquad \frac{\partial q_B^*}{\partial \alpha} = \frac{a^2 b}{k(k - b^2)}$$

Since a, b, k, $k - b^2$ are positive values the above derivatives are positive which means when α increases both q_A^* and q_B^* increase.

The derivatives of equilibrium qualities with respect to a and b yields;

$$\frac{\partial q_A^*}{\partial a} = \frac{\alpha - P_A}{k} \qquad \frac{\partial q_B^*}{\partial b} = \frac{(a^2\alpha + P_A(k-a^2))(k+b^2)}{k(k-b^2)^2}$$

Since a, b, k, $\alpha - P_A$, α are positive values the above derivatives are positive which means when a increases q_A^* increases and when b increases q_B^* increases.

The derivatives of equilibrium qualities with respect to k yields;

$$\frac{\partial q_A^*}{\partial k} = -\frac{a(\alpha - P_A)}{k^2} \qquad \frac{\partial q_B^*}{\partial k} = -\frac{b(a^2(\alpha - P_A)(2k-b^2) + k^2 P_A}{k^2(k-b^2)^2}$$

Since a, b, k, $\alpha - P_A$, α $k - b^2$ are positive values the above derivatives are negative which means as k increases q_A^* and q_B^* decrease.

Finally the derivatives of equilibrium qualities with respect to P_A yields;

$$\frac{\partial q_A^*}{\partial P_A} = -\frac{a}{k} \qquad \frac{\partial q_B^*}{\partial P_A} = \frac{b(k-a^2)}{k(k-b^2)}$$

Since a, b, k, $k - b^2$ are positive values the first derivative above is negative which means as P_A increases q_A^* decreases. Also, if $a^2 > k$ the second derivative will be negative which means as P_A increases q_B^* decreases. □

${q_B}^*$ increases with ${q_A}^*$, because when the firm A contribute to the quality, the value of the final good increases which leads to larger number of consumers and higher price firm B can charge the consumers. Therefore marginal return of contributing to quality increases for firm B, and any increase in q_A increases the incentives of firm B on investment. However a change in q_B can not effect the incentives of firm A's investment decision because, the marginal return of contributing to quality is $\frac{\alpha - P_A}{2}$. In the net neutrality case, we can say that an increase in content quality triggers access provider's investment on quality. However, if access providers contribute to quality, even though the profit of content provider increases, he sustains the same quality contribution level.

Furthermore, when α increases both qualities increase. The reason of it is when the external benefit of firm A increases, he will get higher marginal returns for each costumer, therefore he will prefer to invest in quality more, since quality contribution increases the value of the final good which leads to a higher number of users. As it is explained above, when q_A increases q_B increases as well. So whenever the external benefit of firm A increases both firms will invest in quality more. In the context of internet, if content providers can get higher external benefits for each visitor visiting their websites, both content's and network's qualities increase.

In addition, when a increases q_A* and when b increases q_B* increases, because when the marginal contributions of the qualities to the final good of the composite good increase the returns of investing to quality will be higher for both firms which will increase incentives for investment on quality. Obviously when k gets bigger, marginal cost of investment will increase which will obviously decrease the incentives of the firms for investing which leads to lower quality contributions of the firms.

When P_A increases q_A* decreases and q_B* increases. The reason of it is as the price that the firm A will pay to the firm B increases, each consumer will provide smaller revenues for firm A, so the firm A's marginal return for contributing to quality decreases which reduces the incentives for investment. Besides, if $a^2 > k$ there are two effects; first, when P_A increases, the reduction of q_A decreases the incentives of firm B. On the other hand when P_A increases, each user will provide higher revenues which increases the incentives of firm B. if $a^2 > k$ the first effect dominates the second effect of the increase in P_A which leads to lower quality levels for both firms.

Effects of P_A on the profits of the firms and consumer surplus when qualities are substitutes

If we substitute the optimal quality levels into Value function we have;

$$V_{AB} = \frac{a^2\alpha + P_A(b^2 - a^2)}{k - b^2} \tag{3.32}$$

Since price of the composite good AB and the number of consumers are functions of V_{AB}. When we substitute the above V_{AB} into P_{AB} and n we have;

$$\begin{aligned} P_{AB} &= \frac{a^2\alpha + P_A(2b^2 - a^2 - k)}{2(k - b^2)} \\ n &= \frac{a^2\alpha + P_A(k - a^2)}{2(k - b^2)} \end{aligned} \tag{3.33}$$

Knowing both optimal quality levels, equilibrium price of the composite good, and the number of consumers in terms of the parameters. The profit functions of the firms become;

$$\Pi_A = (\alpha - P_A)\frac{a^2\alpha + P_A(k - a^2)}{2k - 2b^2} - \frac{k}{4}(\frac{a(\alpha - P_A)}{k})^2 \tag{3.34}$$

$$\Pi_B = (\frac{a^2\alpha + P_A(k - a^2)}{2k - 2b^2})^2(1 - \frac{b^2}{k}) \tag{3.35}$$

If we substitute the equilibrium values of the qualities and prices into the consumer surplus to find the wellbeing of the users, we get;

$$CS = \frac{1}{2}(\frac{a^2\alpha + P_A(k-a^2)}{2k-2b^2})^2 \tag{3.36}$$

The main question of this paper is how will the existence of the contracts between the firms will effect the wellbeing of each side, so we will analyze the above profit functions and the consumer surplus when the $P_A = 0$ and when $P_A > 0$.

Proposition 4: The effect of introducing P_A depends on the values of k, a, b. When P_A is increased form 0 if $a^2 > k$ holds, the value of the composite good decreases. Also Π_A, Π_B and CS decrease.

Proof. When price between the firms increased to a positive number from 0, how the firms and consumers will be effected will depend on the below derivatives.

If we take the derivative of V_{AB} with respect to P_A;

$$\frac{\partial V_{AB}}{\partial P_A} = \frac{b^2-a^2}{k-b^2}$$

By assumption we know $k-b^2$ is positive, then if $a^2 > k$ since $k > b^2$, b will eventually be less than a. Then the above derivative will be negative which means an increase in P_A will decrease V_{AB}.

The derivative of Π_A with respect to P_A yields;

$$\frac{\partial \Pi_A}{\partial P_A}\mid_{P_A=0} = \frac{\alpha(k-2a^2)}{2k-2b^2} + \frac{a^2\alpha}{2k} = \frac{\alpha(k(k-a^2)-a^2b^2))}{2k(k-b^2)}$$

Since α, $k-b^2$ and k are positive values if $a^2 > k$, the above derivative will be positive which means an increase in P_A will decrease Π_A.

The derivative of Π_B with respect to P_A yields;

$$\frac{\partial \Pi_B}{\partial P_A}\mid_{P_A=0} = \frac{a^2\alpha(k-a^2)(1-\frac{b^2}{k})}{2(k-b^2)^2}$$

Since α and $1-\frac{b^2}{k}$ are positive values if $a^2 > k$ the above derivative will be negative which means as P_A increases, the firm B's profit decreases.

The derivative of CS with respect to P_A yields;

$$\frac{\partial\ CS}{\partial P_A}\mid_{P_A=0} = \frac{a^2\alpha(k-a^2)}{4(k-b^2)^2}$$

Since α is a positive value if $a^2 > k$ the above derivative will be negative which means as P_A increases, consumers are worse off. □

When $a^2 > k$, an increase in P_A will decrease V_{AB}, because if $a^2 > k$, increasing P_A will decrease the optimal q_A and optimal q_B. Therefore both decreases in quality contributions will lead to a lower value of the final good.

If $a^2 > k$ when P_A is increased from 0, the firm A will lose profits, because if we look at equation 3.1 we see that, when the price between the firms increases, firm A gains less for each consumer, and the value of the composite good decreases, which leads to smaller profits for firm A.

When P_A increases if $a^2 > k$, firm B will lose profits and consumers are worse off, because when we look at the equation 3.6 and 3.7, we can see that an increase in P_A has two effects; it decreases the value of the composite good which decreases the profit and consumer surplus while it directly increases the profit of firm of B and the consumer surplus, if $a^2 > k$ first effect will dominate the second one which decreases the profit of firm B and consumer surplus.

To show how each parameter changes the effect of P_A on the value of the final good, profit of the producers and consumer surplus we let one parameter change while fixing the other parameters.

In figure 3.1, the relation between a and derivatives of V ,Π_A Π_B, CS with respect to P_A is given. The other parameters are fixed; $k = 4$, $b = 1$, $\alpha = 1$. As a changes the effect of introducing a price mechanism changes. If a is small when P_A is increased from 0 all parties of the economy; the firms of components and consumers get better and also value of the composite good increases. However as a increases and exceeds a certain level, the profit of both firms and consumer surplus decreases and V_{AB} decreases if P_A is increased. Therefore as a increases all parties lose their will for the existence of price mechanism between the firms.

The reason of the above is; if a becomes larger, when P_A is introduced the decrease in q_A* becomes larger. It is seen in the equation 3.31. Also since q_A* reduction decreases q_B* at the same time, a large a can lead to a decrease in both qualities and a lower value of the final good. Also if $a^2 > k$ a lower value of the composite good decreases both the price of the composite good and the number of consumers. Therefore if the marginal contribution of the component A is sufficiently large, all firms and consumers prefer not to have a pricing mechanism. In the context of internet case, if content providers' percentage on contributing to the value of the internet is high, charging them with a positive price will lead to lower qualities of internet and hurts content providers, access providers and the consumers.

In figure 3.2, the relation between b and derivatives of V ,Π_A Π_B, CS with respect to P_A is given.The other parameters are fixed; $k = 16$, $a = 2$, $\alpha = 1$. As "b" increases all parties willingness to have a price mechanism increases, because when b is large if P_A is increased from 0, firms of the components get higher profits, consumers are better off and the value of the composite good is larger.

The reason of the above is; when b gets larger, if $P_A > 0$ is introduced, the increase in q_B gets larger, which leads to higher values of the final good. Also as b gets larger in equation 3.33 it is seen that n and P_{AB} increases. Therefore if the marginal contribution of the component B is sufficiently large, all firms and consumers prefer to have a pricing mechanism. In the context of internet case, if access providers' percentage on contributing to the value of the internet is high, charging them with a positive price will lead to higher qualities of internet service and content providers, access providers and the consumers will be better off.

In figure 3.3,the relation between k and derivatives of V ,Π_A Π_B, CS with respect to P_A is given.The other parameters are fixed; $a = 1$, $b = 2$, $\alpha = 1$. For all values of k the derivatives are positive, but as "k" increases all derivatives converge to 0 except the $\frac{\partial \Pi_A}{\partial P_A}$. Therefore as k increases, the marginal benefit of introducing a price mechanism decreases. Consumers and the firm B get smaller marginal profits. However $\frac{\partial \Pi_A}{\partial P_A}$ converges to 0.5 which means as k increases the firm of A's will also decreases but still, he will always prefer to have a positive positive P_A since in any k a positive P_A provides positive profit for him.

As k increase all the optimal quality level , price of the final good and number of consumers decrease. Therefore, the effects of introducing a price mechanism also decreases for all parties of the economy. On the other hand as α; the external benefit of firm of A increases, all optimal values increase, therefore introduction of a price mechanism positively effects the parties of the economy.

Lemma 3: When $P_A = 0$, as a, b or α increases, both firms' profits, value of the final good and consumer surplus increase. On the other side an increase in k decreases all the profits, the value of the final good and consumer surplus.

Proof. When $P_A = 0$ all the profits, consumer surplus, and value of the composite good equations become simpler. The value of the composite good becomes;

$$V_{AB} = \frac{a^2\alpha}{k - b^2} \tag{3.37}$$

Figure 3.1: "a" and $\frac{\partial V_{AB}}{\partial P_A}$, $\frac{\partial \Pi_A}{\partial P_A}$, $\frac{\partial \Pi_B}{\partial P_A}$, $\frac{\partial CS}{\partial P_A}$

Here $k = 4$, $b = 1$, $\alpha = 1$

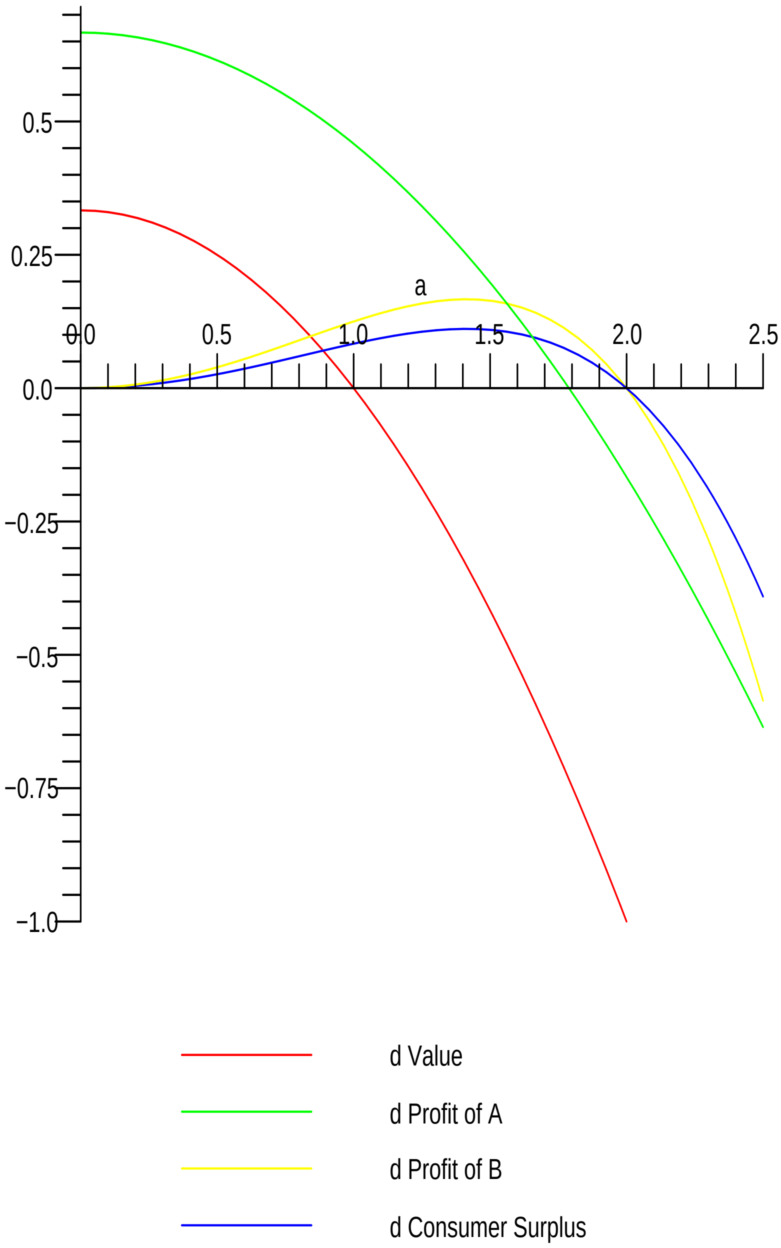


Figure 3.2: "b" and $\frac{\partial V_{AB}}{\partial P_A}$, $\frac{\partial \Pi_A}{\partial P_A}$, $\frac{\partial \Pi_B}{\partial P_A}$, $\frac{\partial CS}{\partial P_A}$

Here $k = 16$, $a = 2$, $\alpha = 1$

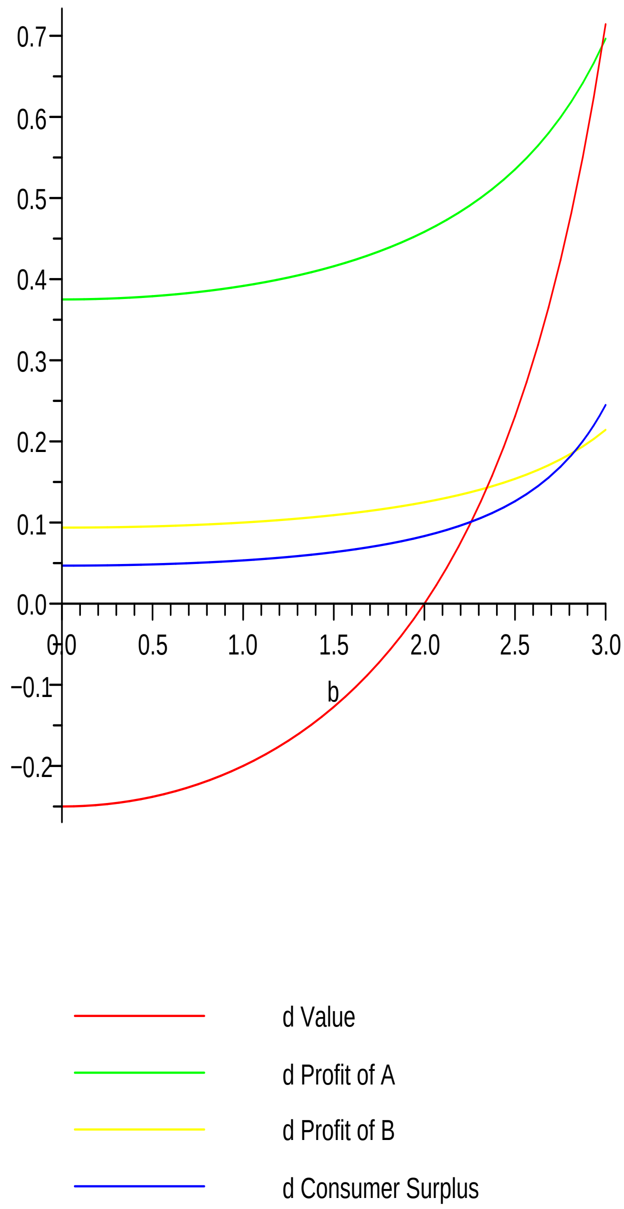
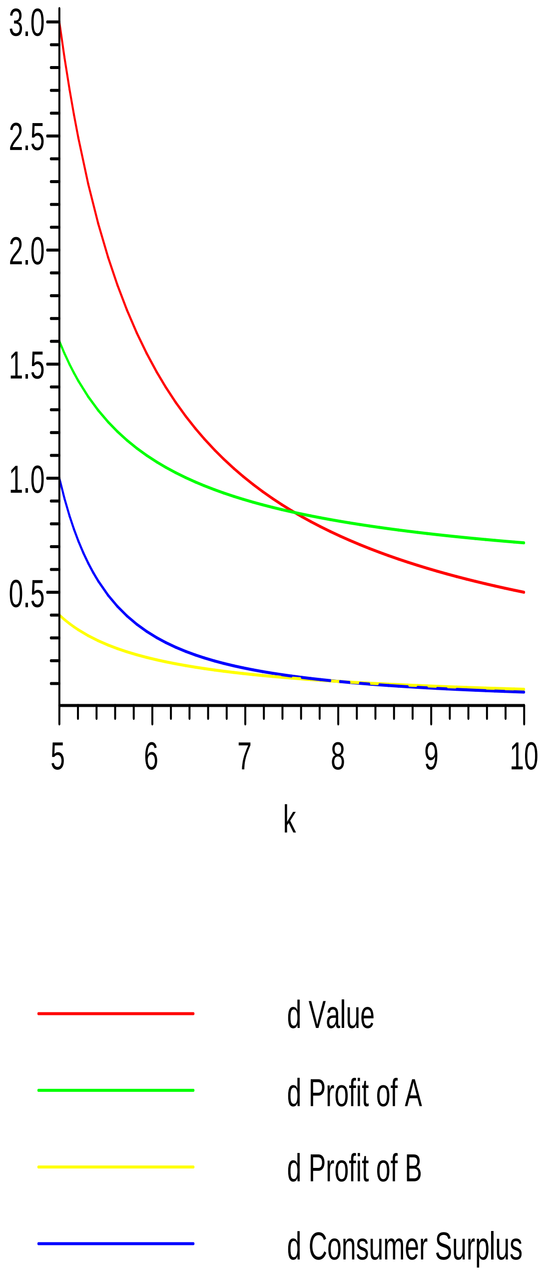


Figure 3.3: "k" and $\frac{\partial V_{AB}}{\partial P_A}$, $\frac{\partial \Pi_A}{\partial P_A}$, $\frac{\partial \Pi_B}{\partial P_A}$, $\frac{\partial CS}{\partial P_A}$

Here $a = 1$, $b = 2$, $\alpha = 1$



For how the parameters; a, b, α, k effect the value of the composite good we have;

$$\frac{\partial V_{AB}}{\partial a} = \frac{2a\alpha}{k-b^2} \qquad \frac{\partial V_{AB}}{\partial b} = \frac{(a^2\alpha)2b}{(k-b^2)^2}$$
$$\frac{\partial V_{AB}}{\partial \alpha} = \frac{a^2}{k-b^2} \qquad \frac{\partial V_{AB}}{\partial k} = -\frac{a^2\alpha}{(k-b^2)^2}$$

Since a, b, $k-b^2$ α are positive values, the first three derivatives are positive values while, the last one is negative, which means as a, b or α increases the value of final good increases.

When price between the firms set to 0, the profit of firm A becomes;

$$\Pi_A = \frac{a^2\alpha^2}{2k-2b^2} - \frac{a^2\alpha^2}{4k} \tag{3.38}$$

For how the parameters; a, b, α, k effect the profit of firm A we have;

$$\frac{\partial \Pi_A}{\partial a} = \frac{a\alpha^2}{k-b^2} - \frac{a\alpha^2}{2k} \qquad \frac{\partial \Pi_A}{\partial b} = \frac{(a^2\alpha^2)b}{(k-b^2)^2}$$
$$\frac{\partial \Pi_A}{\partial \alpha} = \frac{a^2\alpha}{k-b^2} - \frac{a^2\alpha}{2k} \qquad \frac{\partial \Pi_A}{\partial k} = -\frac{a^2\alpha^2}{2(k-b^2)^2} + \frac{a^2\alpha^2}{4k^2}$$

Since a, b α, k, $k-b^2$ are positive the first three derivatives will be positive and the last one will be negative which means as a, b α increase profit of A increases, however as k increases the profit of A decreases.

When $P_A = 0$ the profit of firm B becomes;

$$\Pi_B = \frac{a^4\alpha^2}{4k(k-b^2)} \tag{3.39}$$

If we take the derivatives of Π_B with respect to a, b α, k;

$$\frac{\partial \Pi_B}{\partial a} = \frac{a^3\alpha^2}{k(k-b^2)} \qquad \frac{\partial \Pi_B}{\partial b} = \frac{a^4\alpha^2 b}{2k(k-b^2)^2}$$
$$\frac{\partial \Pi_B}{\partial \alpha} = \frac{a^4\alpha}{2k(k-b^2)} \qquad \frac{\partial \Pi_B}{\partial k} = -\frac{a^4\alpha^2(2k-b^2)}{4k^2(k-b^2)^2}$$

Since a, b, k, α, $k-b^2$ are positive values the first three derivatives will be positive and the last one will be negative which means as a, b α increase profit of B increases, however as k increases the profit of A decreases.

If $P_A = 0$ consumer surplus is;

$$CS = \frac{1}{8}(\frac{a^2\alpha}{k-b^2})^2 \tag{3.40}$$

If we take the derivatives of consumer surplus with respect to a, b α, k;

$$\begin{aligned} \frac{\partial CS}{\partial a} &= \frac{a^3\alpha^2}{2(k-b^2)^2} \qquad & \frac{\partial CS}{\partial b} &= \frac{a^4\alpha^2 b}{2(k-b^2)^3} \\ \frac{\partial CS}{\partial \alpha} &= \frac{a^4\alpha}{2(k-b^2)^2} \qquad & \frac{\partial CS}{\partial k} &= -\frac{a^4\alpha^2(2k-b^2)}{4(k-b^2)^3} \end{aligned}$$

Since a, b, k, α, $k-b^2$ are positive values the first three derivatives will be positive and the last one will be negative which means as a, b or α increases consumer surplus increases, however as k increases consumer surplus decreases. □

The contract agreement between the firms when qualities are substitutes

The first step of the sequence of the events is deciding the price between the firms. Firm B will try to maximize his own profit by choosing P_A. Taking the derivative of his profit function with respect to P_A yields;

$$\frac{\partial \Pi_B}{\partial P_A} = \frac{[a^2(\alpha - P_A) + P_A k](k-a^2)}{2k^2(k-b^2)}$$

Since $k-b^2$, k, $\alpha - P_A$, a are positive, if $k-a^2$ is negative the above derivative will be negative for P_A in interval $[0,\alpha]$. Increasing P_A will decrease profit of firm of B. Therefore the firm B will choose the price;

$$P_A = 0$$

Therefore the profits of firms will be;

$$\Pi_A = \frac{a^2\alpha^2}{2k-2b^2} - \frac{a^2\alpha^2}{4k}$$

$$\Pi_B = \frac{a^4\alpha^2}{4k(k-b^2)}$$

The reason why firm B chooses not to charge firm A is; whenever P_A is increased from 0, as it is explained before both firms decrease their quality level, which leads smaller value of the composite good. Besides in equation 3.4 the number of consumers depends on V_{AB} and P_A. When P_A is increased the decrease in V_{AB} is larger than the increase in P_A which causes a decrease in the number of consumers. Therefore, introducing P_A is not profitable for firm B.

However if a social planner decides the price between the firms who wants to maximize wellbeing of the consumers:

$$\frac{\partial CS}{\partial P_A} = \frac{[\alpha a^2 + P_A(k - a^2)](k - a^2)}{4(k - b^2)^2}$$

If $k - a^2$ is negative, above derivative will be negative. For all values of P_A in the interval $[0\ ,\ \alpha]$ increasing P_A decreases the consumer surplus. Therefore the social planner will set the $P_A = 0$ If the price is substituted into the consumer surplus;

$$CS = \frac{1}{8}(\frac{a^2\alpha}{k - b^2})^2$$

Like in the analysis of firm B's profits, the consumer do not prefer a price between firms, because as P_A increases both the value of the composite good and the number of user decrease which decreases the wellbeing of the society.

Finally if the social planner wants to maximize the total welfare of the economy, he will maximize the sum of the profits and consumer surplus which is;

$$W = CS + \Pi_A + \Pi_B$$

We know than if $a^2 > k$, when $P_A = 0$ each derivative of consumer surplus, profit of firm A and profit of firm B with respect to P_A are negative which means increasing P_A does not increase either of the profits or consumer surplus, therefore the social planner who wants to maximize the total welfare will choose to set the price equal to 0.

Chapter 4

FINDINGS OF THE MODEL

4.1 Comparison of the Findings

In this section we will analyze the findings of the model and compare the results of quality complement case with the results of quality substitute case. While making this comparison we will assume the square of marginal contribution of firm A is larger than the cost parameter, and the square of firm B's marginal contribution is smaller than the cost parameter. i. e. $a^2 > k > b^2$. This means firm A has higher incentives for investing in quality compared to firm B. Besides we will denote the quality complement case with *QC case*, and the quality substitutes case with *QS case*.

Result 1: When $P_A = 0$, in QC case, the market collapses which results in 0 profits of both firms and 0 consumer surplus, however in QS case all firms and consumer surplus is positive.

The reason of the above result is, in QC case when $P_A = 0$ even though the firm A's quality level is sufficiently large, with the above assumption investing in the quality can not increase the profits of firm B (*equation 3.11*). Therefore firm B does not prefer to invest in quality. Since the qualities are complements, the other firm does not invest in quality either which makes the value of the composite good 0. Consequently both firms gain 0 profit and consumer surplus is 0.

However in QS case, when $P_A = 0$ even though the firm B does not invest, firm A can increase the value of the composite good. It is shown that firm A finds investing in quality profitable and the decision of investing in quality for firm A is independent from q_B. On the other hand, if we look at the equation 3.32, the profit function of firm B is increasing in q_B and an increase in q_A increases the incentives of firm B. As the number of consumers increases, firm B's marginal returns for investing in quality increases which leads to positive quality contributions and positive value of the final good. Therefore both firms gain positive profits and consumer surplus is positive.

Result 2: As P_A increase from 0, for QC case, both firms increase their investments which lead to a higher value of the final good, however in QS case, both firms decrease their investments which lead to a lower value of the final good.

The reason is; in QC case, if q_A is positive, an increase in P_A increases the incentives of firm B, and if q_B is positive firm A also prefers to invest in quality, therefore they both increase their quality levels. However, as it is already mentioned, in QS case, firm A's investment decision is independent of q_B, also his decision is decreasing with P_A. Therefore q_A decreases. Since q_B is increasing function of q_A, when q_A decreases even though each consumers provide higher for each consumer, because of the decrease in the value of the composite good and the number of consumers, firm B also loses incentives for investment and decreases his quality level. So in QS case the value of the composite good is maximum when $P_A = 0$.

Result 3: For QC case, there is a positive P_A value where the value of the composite good is maximum.

Until $P_A = \overline{P_A}$, both firms find investing in quality profitable, however when P_A exceeds $\overline{P_A}$ firm A prefers to decrease its quality level which also decreases incentives of firm B. Therefore at $P_A = \overline{P_A}$ value of the composite good is maximum in QC case.

Result 4: Consumer surplus maximizing P_A is $\overline{P_A}$ for QC case, while for QS case it is 0.

In both cases wellbeing of the consumers is totally related with value of the composite good, therefore in both cases, consumers prefer the prices where value of the composite good is maximized which is $P_A = 0$ for QS case, and $P_A = \overline{P_A}$ for QC case.

Result 5: Firm B's profit maximizing P_A choice is 0 for QS case, however for QC case, P_A^* is positive and higher than $\overline{P_A}$.

We assumed that the marginal contribution of component A is larger than both cost parameter and marginal contribution of component B, and in QS case we know that when q_A decreases that marginal return of investing in quality decreases for firm B. Also firm B's profit function is increasing in q_A. Therefore whenever P_A increases, the decrease in q_A strongly worsens all the variables of economy. For this reason, even though P_A provides revenue for firm B, because of the negative effects of the decrease in q_A, firm B prefers not to charge firm A in QS case. However in QC case increasing P_A increases both sides' incentives therefore firm B finds profitable to set a positive P_A. Nevertheless he does not prefer $\overline{P_A}$ which maximizes the value of the product but, prefers $P_A^* > \overline{P_A}$, because after $\overline{P_A}$, increasing P_A decreases the cost of investment and revenues at the same time which provides extra profit until $P_A = P_A^*$.

Result 6: If a social planner who wants to maximize the total welfare of the economy, determines the price between firms. In QS case he will set a price equal to 0, and in QC case he will chooses a positive P_A^w.

In QS case, when $P_A > 0$ is introduced all quality levels, value of the final good and the number of consumers decrease, therefore none of the parties of economy can benefit from an increase in P_A. Therefore total welfare is maximized when $P_A = 0$. However in QC case, the increase in P_A creates incentives for investment and increases the profits and consumer surplus. Therefore there is a positive P_A which a social planner would choose to maximize the total welfare of the economy.

4.2 The Implication of Findings on Net Neutrality Debate

Net neutrality debate was the main motive of this thesis, because of that, in this section, we will explain what the findings of our model can imply on the world wide web market.

As it is mentioned previously, the content providers are represented as firm A and access providers are represented as firm B, and internet service's value is determined with a function of both content's and network's qualities. Besides we analyzed the internet market assuming that content's quality's marginal contribution is higher than network's quality's.

In the analysis of the model, the effects of introducing a price between the firms depends on how qualities of components are related to each other. This paper analyzed two extreme value functions; perfect *quality substitutes* and perfect *quality complements*. Introducing P_A increases the welfare when the qualities of components are complements while when the qualities of components are substitutes increasing P_A decreases the total welfare. Therefore the relation between the components of internet service is important.

If we assume that the qualities of network and content are perfect *quality substitutes*, we would defend the neutrality of internet. We would conclude that there should be no contracts between the access provider and the content providers, since it hurts all parties in the economy. Actually if the parties allowed to have a contract between them, they will set the price equal to 0. If content providers are charged, they will prefer to invest less in quality which means they will not develop new applications or improve their softwares. This will decrease the number of internet users and value of the internet service, which will also decrease the access providers incentives on improving its network's quality.

If we assume that the qualities of network and content are perfect *quality complements*, we would be an opponent of net neutrality. We would conclude that there should be a positive price between the access providers and content providers, because when the internet is neutral, the access providers do not have incentives for investment in quality, which prevents the increase in internet's quality. Even though the content providers could find investing in quality profitable, since the access providers do not invest in quality, they also do not invest, which leads to lower quality of the internet service. Therefore the content providers should give some of their external benefit to access providers to increase the quality of internet service and the number of consumers. This policy is welfare improving for the society.

On the other hand, like in most of composite goods comprising two components, it is difficult to determine how internet service's quality is determined. The network and content are perfect complements, however they are not perfect *quality complements* because when the quality of content's increases without any improvement in network quality, the internet service's value increases, also we can not think them as perfect *quality substitutes* since their qualities' contributions on the value are not independent of each other. Because of this reason we can only conclude that if the relationship between the components of internet service is closer to being quality complements, there should be a positive price between the access providers and content providers, but if the relationship is closer to being substitutes, internet should stay neutral.

Chapter 5

CONCLUSION

In this paper we analyzed the effect of contractual relationships on the investment incentives of complementary good producers. The motivating idea was the debate on a specific two-sided market namely World Wide Web. Therefore in our analysis, we collated the dynamics of both two-sided markets and complementary goods market.

In our model, like in two-sided markets, the seller side (firm A) and the buyer side (consumers) did not interact with each others directly, the platform (firm B) interact with both of them and charged prices for each of them to maximize profits. However since there were only one firm as the producer of A and one firm as the producer of B, the model was also illustrating a market of composite goods comprising two complementary components.

We analyzed two specific value function; perfect complements and perfect substitutes. We commented on the net neutrality debate with the help of our findings. As future work we will analyze our model with different value functions such as Cobb Douglas which can represent the internet market better.

Also, our work can be extended in two-sided markets. The number of sellers and the number platforms can be increased, since there are many content providers in internet and more than one access providers.

My main finding is, in composite good markets if the components are closer to being *quality components* introducing price between the firms of components increases welfare, however if the components are closer to being *quality substitutes* there should be no price between the firms. Therefore this paper highlights the fact that the optimal solution for the debate about net neutrality can be found by determining how the qualities of content and network are related to each other.

Chapter A

APPENDIX

A.1 The P_A choice of firm A when qualities are complements

In this section we will explain the mathematical calculations of how firm B choose the optimal P_A to maximize its profit. When P_A in interval $[0, \overline{P_A}]$, the profit function is;

$$\Pi_B = \frac{k(P_A)^2}{4(k-b^2)} \tag{A.1}$$

If look at the first and second derivatives;

$$\frac{\partial \Pi_B}{\partial P_A} = \frac{k(P_A)}{2(k-b^2)} \geq 0 \quad \frac{\partial^2 \Pi_B}{\partial^2 P_A} = \frac{k}{2(k-b^2)} > 0 \tag{A.2}$$

When P_A is in interval $[0, \overline{P_A}]$ the profit function is increasing and convex in P_A. Therefore firm B will never set the price less than $\overline{P_A}$.

When P_A is in interval $[\overline{P_A}, \alpha]$ the profit function is;

$$\Pi_B = (\frac{\alpha a^2 + P_A(k-a^2)}{2k})^2 - \frac{(\alpha - P_A)^2 a^4}{4kb^2} \tag{A.3}$$

If we look at the first and second derivative of the above function;

$$\frac{\partial \Pi_B}{\partial P_A} = \frac{P_A(k-a^2)^2 + \alpha a^2(k-a^2)}{2k^2} + \frac{(\alpha - P_A)a^4}{2kb^2} \tag{A.4}$$

$$\frac{\partial^2 \Pi_B}{\partial^2 P_A} = \frac{(k-a^2)^2}{2k^2} - \frac{a^4}{2kb^2} \tag{A.5}$$

If we substitute $P_A = 0$ in the first derivative, we get;

$$\frac{\partial \Pi_B}{\partial P_A} \mid_{P_A=0} = \frac{b^2 \alpha a^2 (k - a^2)}{2k^2 b^2} + \frac{k \alpha a^4}{2k^2 b^2} \tag{A.6}$$

Since the denominators are the same and positive, if the sum of the numerators are positive it means above function is increasing in P_A which is;

$$b^2 \alpha a^2 (k - a^2) + k \alpha a^4$$

If we rearrange the equation it becomes;

$$\alpha a^4 (k - b^2) + \alpha a^2 b^2 k$$

since $k - b^2$ is positive by assumption we can conclude that the sum of numerators are positive so Π_B is increasing in P_A.

Concavity of the profit function:

As P_A increases Π_B increases, however if the profit function is concave we can find a P_A when profit of firm B has its maximum , therefore the second derivative must be negative. If we rearrange the second derivative of profit of firm B with respect to P_A

$$\frac{\partial^2 \Pi_B}{\partial^2 P_A} = \frac{(k - a^2)^2 b^2 - a^4 k}{2k^2 b^2}$$

The denominator is positive if we rearrange the numerator;

$$a^4 (b^2 - k) + b^2 k (k - 2a^2)$$

By assumption, $b^2 - k$ is negative, if the $k - 2a^2 < 0$ holds the above function will be negative, then the numerator will negative which means the second derivative is negative and profit function is concave.

Whether P_A^* is bigger than $\overline{P_A}$.

Then we found the profit maximizing P_A^* value by equating first derivative equal to 0. The optimal P_A has to be bigger than $\overline{P_A}$ because the profit function is valid for the interval $[\overline{P_A}, \alpha]$. The difference between the optimal price and $\overline{P_A}$ is;

$$P_A^* - \overline{P_A} = \frac{\alpha\, a^2 \left(-b^2 a^2 + k b^2 + a^2 k\right)}{-b^2 a^4 + 2\, b^2 a^2 k - k^2 b^2 + a^4 k} - \alpha \left(1 - \frac{k b^2}{k b^2 + a^2 \left(-b^2 + k\right)}\right) \tag{A.7}$$

$$= \frac{\alpha\, a^2 k^3 b^2}{\left(-b^2 a^4 + 2\, b^2 a^2 k - k^2 b^2 + a^4 k\right)\left(-b^2 a^2 + k b^2 + a^2 k\right)} \tag{A.8}$$

The numerator is positive, if the denominator is also positive we can conclude that P_A^* larger than $\overline{P_A}$. The first part of the denominator is:

$$\left(-b^2a^4 + 2\,b^2a^2k - k^2b^2 + a^4k\right)$$

If we rearrange it;

$$a^4\left(-b^2 + k\right) + kb^2\left(2\,a^2 - k\right)$$

Since $k - b^2$ is if $2a^2 - k$ is positive the first part of the denominator is positive. The second part of the denominator is

$$\left(-b^2a^2 + kb^2 + a^2k\right)$$

If we rearrange it;

$$a^2\left(-b^2 + k\right) + kb^2$$

Since $k - b^2$ is positive the second part is also positive then I have;

$$\frac{+}{(+)(+)}$$

Which is positive, therefore we can conclude that $P_A^* > \overline{P_A}$.

Whether P_A^* is smaller than α.

If P_A^* is smaller than α. $\alpha - P_A^*$ has to be positive;

$$\begin{aligned}\alpha - P_A^* &= \alpha - \frac{\alpha\,a^2\left(-b^2a^2 + kb^2 + a^2k\right)}{-b^2a^4 + 2\,b^2a^2k - k^2b^2 + a^4k} \\ &= \frac{\alpha\,kb^2\left(a^2 - k\right)}{-b^2a^4 + 2\,b^2a^2k - k^2b^2 + a^4k} \qquad \text{(A.9)}\end{aligned}$$

If $a^2 - k$ is positive the numerator will be positive, the denominator is also positive, therefore we can say that P_A^* is smaller than α.

The benefit of having the assumption: $a^2 > k$

If $k > a^2$, we know that the optimal price of firm B will be higher than α. If P_A is larger or equal to α, the value of the composite good and the quality contributions will be less than or equal to 0 which makes the effects of quality contributions unimportant for gaining profits. However if $k < a^2$ P_A is less than α. The value of the final good and quality contributions are positive. Besides with the assumption of $k < a^2$ the concavity of profit function and $P_A^* > \overline{P_A}$ is also satisfied.

A.2 Welfare maximizing P_A value when components are complements

Total welfare of the economy is;

$$W = CS + \Pi_A + \Pi_B; \tag{A.10}$$

If take the first and second derivatives of the function we have;

$$\begin{aligned} \frac{\partial W}{\partial P_A} &= -1/4\,\frac{3\,\alpha\,a^4b^2 - \alpha\,a^2kb^2 - 3\,b^2P_A\,a^4 + 4\,b^2P_A\,a^2k + b^2k^2P_A - 2\,k^2b^2\alpha - 2\,\alpha\,a^4k + 2\,a^4kP_A}{k^2b^2} \\ \frac{\partial^2 W}{\partial^2 P_A} &= -1/4\,\frac{-3\,b^2a^4 + 4\,b^2a^2k + k^2b^2 + 2\,a^4k}{k^2b^2} \end{aligned} \tag{A.11}$$

Necessary condition for the concavity of welfare function

If the second derivative is negative, welfare function will be concave, since denominator is positive, if

$$-3\,b^2a^4 + 4\,b^2a^2k + k^2b^2 + 2\,a^4k$$

is positive the profit function will be concave. If we rearrange the above equation

$$-2\,b^2a^4 + 2\,a^4k - \,b^2a^4 + 4\,b^2a^2k + k^2b^2$$

which is;

$$2\,a^4(k - b^2) + a^2b^2(4k - a^2) + k^2b^2$$

As the assumption we know that $k - b^2$ is positive therefore the necessary condition for concavity is;

$$4k > a^2 \tag{A.12}$$

Since welfare function is concave we can equate the first derivative to 0;

$$P_A^w = \alpha - \frac{\alpha\,kb^2\,(3\,a^2 - k)}{-3\,b^2a^4 + 4\,b^2a^2k + k^2b^2 + 2\,a^4k} \tag{A.13}$$

Since the denominator and $3a^2 > k$ are positive, we can say that P_A^w less than α.

Whether P_A^w is bigger than $\overline{P_A}$

If P_A^w is bigger than $\overline{P_A}$ the following equation has to be positive;

$$P_A^w - \overline{P_A} = -\frac{k^2b^2\alpha\,(a^4 - a^2k - 2\,kb^2)}{(-3\,b^2a^4 + 4\,b^2a^2k + k^2b^2 + 2\,a^4k)\,(-b^2a^2 + kb^2 + a^2k)}$$

We know that the first part of the denominator is positive the second part is;

$$\left(-b^2a^2 + kb^2 + a^2k\right)$$

If we rearrange it;

$$a^2(k - b^2) + kb^2$$

Since $k - b^2$ is positive the denominator will be positive. Therefore if

$$\left(a^4 - a^2k - 2\,kb^2\right)$$

is negative P_A^w will be bigger than $\overline{P_A}$.

Whether P_A^w is bigger than P_A^*

If P_A^w is smaller than P_A^*. $P_A^* - P_A^w$ has to be positive which is;

$$\frac{\alpha\,k^2b^2\left(-2\,b^2a^2k + a^6 + a^4k + 2\,k^2b^2\right)}{\left(-b^2a^4 + 2\,b^2a^2k - k^2b^2 + a^4k\right)\left(-3\,b^2a^4 + 4\,b^2a^2k + k^2b^2 + 2\,a^4k\right)}$$

We know that

$$\left(-3\,b^2a^4 + 4\,b^2a^2k + k^2b^2 + 2\,a^4k\right)$$

is positive. The other element of denominator is;

$$\left(-b^2a^4 + 2\,b^2a^2k - k^2b^2 + a^4k\right)$$

If we rearrange it;

$$a^4(k - b^2) + kb^2(2a^2 - k)$$

Since $k - b^2$ is positive if $a^2 > k$ the denominator is positive. The numerator is positive if;

$$\left(-2\,b^2a^2k + a^6 + a^4k + 2\,k^2b^2\right)$$

The necessary condition for the numerator to be positive is $a^2 > k$, because when $a^2 > k$

$$a^6 + a^4k > 2a^4k > 2a^2b^2k$$

Therefore if $a^2 > k$ the welfare optimal price between firms will be between $\overline{P_A}$ and P_A^*

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