PRODUCT RECOVERY SYSTEMS: POLICY ISSUES AND DISPOSITION DECISIONS

by

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POLICY ISSUES AND DISPOSITION DECISIONS

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Tüm öğrenim hayatım boyunca olduğu gibi doktora süreci boyunca da benden sevgi ve desteklerini hiçbir zaman esirgemeyen anne ve babama...

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In the last few decades, worsening environmental problems have attracted attention to sustainable development practices. In this respect, product recovery, which aims to regain the value in end-of-life products and, thus can be regarded as an implementation towards sustainable development at the firm level, has gained importance. This dissertation addresses two interrelated decisions in the context of product recovery systems: at the strategic level, we analyze the impact of environmental policy as a coercive force on product recovery undertakings of firms; and at the tactical level, we explore the disposition decisions of a firm who is already engaged in product recovery. First, we focus on one of the main motivators of recovery practices; environmental legislation, and investigate its effectiveness in encouraging manufacturers for product recovery and redesign. We find that initial investment requirements may have a serious impact on the legislation's effectiveness. We, then, focus on two common forms of take-back legislation (tax and rate models) and compare them from the perspective of different stakeholders. We observe that in terms of the profitability of the two forms, there are some misalignments between the incentives of different stakeholders (the social planner, the manufacturers and the environment). Furthermore, we consider the decisions of a firm engaged in recovery operations and investigate the associated disposition decisions. We address this problem employing a common revenue management technique of bid price controls. As a result of our numerical experiments, we find that a dynamic approach based on bid price controls significantly outperforms a static one.

ÜRÜN GERİ KAZANIM SİSTEMLERİ:
YASAL DÜZENLEMELER VE GERİ KAZANIM ALTERNATİFLERİNİN SEÇİMİ

Öznur Özdemir

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Anahtar Kelimeler: ürün geri kazanımı, çevre ile ilgili yasal düzenlemeler, genişletilmiş üretici sorumluluğu ilkesi, geri kazanım alternatiflerinin seçimi, teklif fiyatı kontrolleri

Son yıllarda, giderek artan çevre sorunları sürdürülebilir kalkınma uygulamalarına olan ilginin artmasına neden oldu. Bu bağlamda, ömrü tükenmiş ürünlerdeki değeri geri kazanmayı amaçlayan ve sürdürülebilir kalkınmaya yönelik firma düzeyinde bir uygulama olarak görülebilecek ürün geri kazanımı önem kazandı. Bu tez çalışması, ürün geri kazanım sistemleri çerçevesinde birbiriyle ilişkili iki kararı ele almaktadır: stratejik düzeyde firmaların ürün geri kazanım üstlenimleri için zorlayıcı bir güç olan çevre politikalarının etkisi, taktik düzeyde ise halihazırda ürün geri kazanımı yapan bir firmanın, geri alınan kullanılmış ürünleri (özleri) geri kazanım seçenekleri arasında dağıtım kararları incelenmiştir. İlk olarak, geri kazanım uygulamalarının temel güdüleyicilerinden biri olan çevre ile ilgili yasal düzenlemelere odaklanılarak, bunların üreticileri ürün geri kazanım ve yeniden tasarımına yönlendirmek konusundaki etkinlikleri araştırılmıştır.

Ardından ürünlerin tüketicilerden toplanması için yaygın olarak kullanılan iki yasal düzenleme modeli ele alınmış ve bu iki model, sistemdeki farklı taraflar açısından karşılaştırılmıştır. Analizimiz sonucunda, modellerin kârlılığı bakımından farklı taraflar (sosyal düzenleyici, üreticiler ve çevre) arasında bazı uyuşmazlıklar olduğu gözlemlenmiştir. Ayrıca, halihazırda ürünlerini geri kazanan bir firmada, toplanan özler için en uygun geri kazanım seçeneklerinin belirlenmesi problemi ele alınmış, bu problem gelir yönetimi alanındaki teklif fiyatı kontrolleri yöntemiyle irdelenmiştir. Sayısal deneylerimiz, teklif fiyatı kontrolleri yöntemine dayanan dinamik bir yaklaşımın, statik bir yaklaşımdan anlamlı seviyede üstün olduğunu göstermiştir.

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TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

Chapter 1

INTRODUCTION

Over the last few decades, ever worsening environmental problems (e.g., rapid depletion of scarce natural resources and shortage of areas suitable for landfills, alarmingly increasing waste material accumulation and consequential environmental pollution) and their serious consequences for the future of humankind have increased the environmental consciousness of all social segments and attracted considerable attention to sustainable development initiatives. Government enforcements for proper waste management and recovery of the utmost value from end-of-life products have toughened and non-governmental organizations' (NGOs) emphasis on environmental friendly technologies and practices have accelerated. In the industry, this growing concern on environment materialized in the increasing emphasis put on sustainable manufacturing practices. In this sense, regaining and re-integrating end-of-life products to the industry into the different stages of the production process have become more important in the recent years. As De Brito and Dekker (2004) indicate while only the flow of products from raw material to end consumer was important twenty years ago, today firms, especially the manufacturing industry, are also really concerned with the flow of products from end customer back to producers or recovery centers. As a consequence of all these developments, product recovery systems emerged as a new field of research in addition to the traditional manufacturing systems.

1.1 What is a Product Recovery System?

In the last few years, manufacturing firms, especially the Original Equipment Manufacturers (OEMs), have begun to pay close attention to the production and distribution systems that will enable them to collect and recover used products besides manufacturing new ones. The primary drivers of this increasing emphasis on recovery systems can be sought both in the recent regulations of governments about the disposal of waste materials/used products and the increasing importance of establishing a green image in the eyes of customers as well as the possible economic gains that can be obtained from such systems. In fact, recovery systems can be considered in relation to the broad area of sustainable development. Brundland (1998) defined the sustainable development in an EU report as '...to meet the needs of the present without compromising the ability of future generations to meet their own needs'.

Hence, De Brito and Dekker (2004) argue that recovery systems can be regarded as the implementation of sustainable development at the firm level since product recovery prescribes retaining the utmost value embedded in products and, hence, avoiding any sort of waste of scarce resources.

Güngör and Gupta (1999) define product recovery as the act of minimizing the waste sent to landfills by recovering materials and parts from old or outdated products by recycling, remanufacturing and reuse.

Jayaraman et al. (1999) consider a product recovery system as a recoverable product environment, including strategies to increase product life through repair, remanufacturing, and recycling of products.

In fact, product recovery systems can be considered as part of a broader system, *closed-loop supply chain*, which combines the traditional and the reverse supply chains and, thus encompasses both manufacturing and recovery processes. Guide and Van Wassenhove (2009), in a very recent study where they provide an overview of the evolution closed-loop supply chains (integration of product recovery systems with traditional supply chain activities) research, define closed-loop supply chain management as 'the design, control, and operation of a system to maximize value creation over the entire life cycle of a product'. Although the authors adopt a strong business perspective, they still recognize the role of product recovery in the development of industrial systems that are both economically and environmentally sustainable.

In the broadest sense, a product recovery system is composed of collection of used products from the end-consumers, inspection/sorting/selection of them, implementation of the most appropriate recovery strategy (e.g., repair, refurbish, remanufacturing, and recycling), and disposal of non-recoverable waste materials/parts. After the inspection/sorting/selection stage in which the collected used products (cores) are checked for their conditions and classified according to their quality levels, they are allocated among the various recovery options.

1.2 Factors Motivating Firms for Product Recovery

There exist several factors leading OEMs or independent recovery firms to collect and recover end-of-life products, which was once considered costly and economically infeasible. We can list the primary reasons for the increasing interest towards product recovery systems in the last few decades as follows;

(i) *Rapidly depleting scarce resources and landfills and the consequent problem of environmental pollution:* One of the most important goals of product recovery systems is minimizing the amount of waste sent to landfills or disposal. Pollution arising from land filling is so serious that EU has enacted a separate directive (Council Directive 1999/31/EC of 26 April 1999 on the landfill of waste) including strict requirements on the characteristics of waste that can be land filled and how the procedure should be managed to reduce environmental impact. In this respect, Ferguson and Toktay (2006) report that in the US, the amount of Municipal Solid Waste raised to three folds of the 1960 value by 2001 and 56% of this amount is land filled. They also indicate that according to Environmental Protection Agency records, in 1999, fourteen states had no landfill capacity left and eight states had less than ten years of landfill capacity left. Similarly, according to European Environment Agency statistics, each year 1.3 billion tonnes of waste (3.5 tonnes per capita) is generated only in the European Union. Moreover, OECD estimates that the waste amount generated in Europe by 2020 will increase to 45% of the amount in 1995. For Turkey, although we do not have such exact figures, it is an undeniable fact that industrial waste accumulation and consequent environmental pollution are our ever-worsening problems mostly due to unregulated and not sufficiently controlled industrial practices. Hence, adoption and dissemination of proper waste management and product recovery activities carry even more importance for our country.

These worrying facts have led to an increase in legislative enforcement and social consciousness on environmental problems, which have ultimately motivated firms for more product recovery as discussed below.

- *Increasing environmental consciousness of society in general and consumers in particular:* Today, partly because of the worsening environmental problems like pollution, rapid depletion of natural resources, consumers begin to pay more attention to firms' concern for environmental protection. Behaving in an environmentally responsible manner improves the green image of firms and even increases the demand for firms' products. For instance, Toffel (2004:122), in reporting King and Mackkinnon's survey, states that increasing the amount of recyclable contents in products and adopting environmentally sustainable business practices were perceived to have the greatest positive impact on consumer's willingness to use a firm's products and services. The author further indicates that OEMs like Kodak, FujiFilm, HP, IBM Europe and Xerox have quickly become aware of the considerable effect of developing green brand image on firm performance and invested in product recovery activities accordingly. Similarly, Güngör and Gupta (1999) point out that in the last decades, consumers have become more sensitive to their environment and its crucial problems, which may lead to irreversible consequences if neglected. Hence, they have begun to show more interest towards environmentally friendly products that will be taken back by their manufacturers at the end of their useful lives for recovery. The authors state that this market trend is an important stimulus for the OEMs to design and market environmentally friendly products (or so called 'green products') so as to gain competitive advantage against their competitors.

- *Increasing number of environmental regulations and legislation*: In the last few decades, especially with the worsening land filling and waste disposal problems, governments' concern for proper management of end-of-life products has increased. Especially in the European countries, 'extended producer responsibility (EPR)' and 'polluter pays' principles have been widely acknowledged. As a result, a number of laws and regulations enforcing firms to undertake the responsibility of the whole life-cycle of their products have been enacted in several countries. This entire legislation bases on the principle that the responsibility of manufacturers for their products does not end with sale but extends beyond the consumer use after which products should be either recovered or properly disposed under the OEMs' control. One of the widely known legislation about extended OEM responsibility is the Waste Electronic and Electrical Equipment (WEEE) Directive of European Commission (The European Parliament and the Council of the European Union, 2003) that holds producers responsible for taking back and properly recovering their electrical and electronic equipment.

  Directive on End-of-Life Vehicles (ELV) is another common example, which aims to "make vehicle dismantling and recycling more environmental friendly, set clear and quantified targets for reuse, recycling and recovery of vehicles and their components and push producers to manufacture new vehicles also with a view to their recyclability." (European Commission, 2007).

  If we consider the specific case of our country, although all the legislation adopted from EU has not become fully active yet, for some prominent directives such as Regulation for Control of the Tyres Which have Completed Their Life-Cycles (Ömrünü Tamamlamıs Lastiklerin Kontrolü Yönetmeligi) and Recovery and Disposal of Spent Accumulators and Batteries (Atık Pil Ve Akümülatörlerin Kontrolü Yönetmeligi) binding timetables which impose specific recovery targets are already set. Hence, in a near future, establishing and conducting efficient recovery systems will become as important for Turkish manufacturers as their European counterparts. Furthermore, Turkish manufacturers, who are already selling their products in EU markets, have to abide by the current regulations that are in force in the EU member states.

(ii) *Possible economic gains in collecting, reusing or recovering used products and materials:* Expected economic gains and other benefits are the main factors that lead to voluntary and proactive involvement of manufacturers in product recovery. In contrast to what was believed in the past, today it is widely accepted that product recovery systems can contribute to firm performance a lot in economic terms (Mabee et al., 1999, Ayres et al., 1997, Şerifoğlu et al., 2006). Some of these contributions can be listed as follows;

- Raw materials, components or parts retained from returned products can be used as inputs in new production and as spare parts in after sales and repair services. These can also provide a valuable base for the parts and components supply of no longer produced models.
- Energy consumption, waste disposal costs and landfill needs can be considerably reduced.
- Capabilities gained through product recovery can be utilized in new product development/design.

To quantify these effects; Şerifoğlu et al. (2006) note that worldwide energy savings obtained by remanufacturing activities is about 120 trillion Btu/year which is equivalent to the total amount that can be produced by 8 nuclear power plants. Similarly, worldwide components/parts savings are designated as 14 million tones/year. Mabee et al. (1999), on the other hand, express that cost reductions by remanufacturing have been estimated as 30-60% of new production.

Ayres et al. (1997) coin the word 'double dividends' in order to attract attention to both increased profits and cost reductions for the firm, and the environmental improvement for the society. The authors argue that the purchased parts and materials and the waste disposal constitute a large proportion of a manufacturer's cost and these cost items can simultaneously be avoided through strategic recovery and remanufacturing systems. Xerox Corporation, Kodak, FujiFilm, Electrolux, HP, IBM, Ford Motor Company and Mercedes-Benz are just some of the examples which successfully carry out recovery operations and obtain economic gains from this business. Ferguson and Toktay (2006) and Şerifoğlu et al. (2006) report that there exist 73,000 firms engaged in product recovery in US by 1997 and an estimated $53 billion revenue has been obtained by sale of remanufactured products.

Another significant contribution of recovery activities is the improved product development and design capabilities that may be attained with the help of the experience gained in recovery operations. Knowledge about and familiarity with the most frequent part/product failures may provide manufacturers with insights and new ideas about future product designs and product characteristics besides decreasing the repair and after-sale service costs.

(iii) *Corporations' own social responsibility principles and targets:* Today, manufacturing firms' concerns are no longer limited with producing in the most efficient way and selling their goods with the highest possible profit. Partly because of the increasing consumer consciousness for the global environmental problems (e.g., pollution, depletion of natural resources, climatic changes) and partly because of the crucial effect of brand image on market demand, firms have begun to set social responsibility targets for themselves and prepare reports to present their activities in these respects. Since product recovery is one of the most effective ways of working for the social well-being and contributing to environmental protection, commitment to self set social responsibility principles is another driving factor for the adoption of product recovery systems as is the case in IBM Europe, Xerox and HP (Toffel, 2004). In this sense, Dhanda and Hill (2005) cite 'the sincere commitment to environmental issues, successfully developed and implemented ethical standards, and the existence of managers who are responsible for their operationalisation' as the primary internal drivers of product recovery systems.

1.3 Research Scope

Under the field of product recovery, one can list several problems that should be investigated. In fact, all the relevant problems and issues that have been examined for traditional manufacturing systems for years can be reconsidered for recovery systems.

In this respect, Fleischmann et al. (1997), Güngör and Gupta (1999) and Thierry et al. (1995) give the earliest review studies that summarize the main problems of the field and show new research venues. More recently, Guide and Van Wassenhove (2009) discuss the evolution of research in the broader field of closed-loop supply chains over the last fifteen years. Adopting a business perspective, they consider only the studies on value-added recovery activities (those firms are engaged in for profit purposes) and examine them under five phases. The authors argue that while the papers in the first phases have a technical engineering perspective and focus more on the individual problems (such as reverse logistics networks, production planning and inventory control systems, value of information and remanufacturing shop/line design), the studies in the later phases develop a holistic business model view. Similarly, Atasu et al. (2008) provide a review of analytic research that focus on practical problems in product recovery. They classify this research stream in four categories as: industrial engineering/operations (e.g., forecasting, inventory control and reverse logistics network design problems), design (e.g., product acquisition management, time value of product returns and product durability problems), strategy (e.g., which actor should be responsible for used product take-back and remanufacturing as a competitive weapon) and behavioral issues (e.g., customer perceptions of remanufactured goods). Finally, Sasikumar and Kannan (2008a, 2008b and 2009) provide one of the most comprehensive and the recent review of literature through a series of three papers. In the first paper, they focus on the studies on environmental regulations and inventory management while in the second one they consider the research on reverse logistics (distribution). Finally in the third paper, they adopt a broader perspective and provide both content and methodology-based classifications of all the studies in the field of reverse supply chains. Especially

Sasikumar and Kannan (2008a) emphasize the importance of environmental legislation in encouraging product recovery practices.

In this dissertation study, we focus on environmental legislation and the disposition decisions in product recovery systems. Environmental legislation is one of the most crucial coercive force leading manufacturers for product recovery. Especially in European Union, the emergence and the fast spread of product recovery practices coincide with the enactment of environmental directives such as WEEE and ELV. Hence, implications of environmental policies on product recovery decisions is a significant topic to be investigated in this context. Still there exist variations between the environmental regulations currently in force in different countries. The analysis of the possible differences between the different legislative forms can provide valuable insights both for those countries who are yet shape their own environmental policies and for future amendments and improvements in current legislation. Based on these observations, in chapters 2 and 3, we adopt a more strategic perspective and examine the implications of environmental legislation as well as the differences between the current environmental regulations originated from extended producer responsibility (EPR) principle. Once the firms start product recovery, more tactical issues such as collection of the used products, inspection of them and selection of the most appropriate or profitable recovery option come into the picture. One of the most important decisions at this stage is how to assign the available used products (or cores) between alternative recovery options, namely *disposition process*. Especially in those settings where there is high demand for the outputs of recovery activities (e.g., parts and refurbished products) and the available cores are not sufficient to meet all demand, disposition decisions of a firm play an important role to maximize her recovery earnings. In practice and in literature, disposition decisions are generally based on the quality of the available cores or the priorities of the firm (some firms a priori set the recovery alternative they will use) (see Guide and Van Wassenhove, 2003). Nevertheless, there may be problems with both of these two approaches. Hence, in chapter 4 we consider the disposition decisions of a remanufacturer.

Particularly, in chapter 2, we investigate the impact of EPR based legislation and the initial investments required to start recovery operations on the optimal product recovery decisions of manufacturers. Motivated by the fact that product redesign can increase savings from product recovery and one of the main objectives of EPR legislation is to promote redesign for recovery, we seek to understand; (1) how the redesign opportunities affect the willingness of manufacturers for product recovery and (2) under what circumstances EPR legislation can encourage manufacturers towards a product design that will facilitate recovery operations.

On the other hand, in chapter 3, we focus on the two common forms of EPR based product take-back legislation and compare the structural efficiency of tax and rate models from the perspectives of different stakeholders (i.e. manufacturers, social planner and environment). We consider both monopolistic and competitive environments and try to identify the circumstances under which the environment, the manufacturers or the consumers benefit more from one of the two models.

In chapter 4, we consider the disposition decisions (assigning an available core to a specific product recovery option) of a remanufacturer. We consider a setting where a remanufacturer has two options to recover value from an available core; refurbish and sell at a discount of a new unit's price or dismantle it and sell (or internally use) the harvested parts and where there is demand uncertainty for both options. We handle the problem from a revenue management perspective employing bid price controls.

Finally, in chapter 5 we provide the important conclusions arising from our analysis and discuss some further research venues.

Chapter 2

RECOVERY DECISIONS OF A MANUFACTURER IN A LEGISLATIVE DISPOSAL FEE ENVIRONMENT

As a result of the rapid depletion of scarce resources and landfills, and alarmingly increasing waste material accumulation; some national governments (e.g., Japan, Canada, Taiwan) and the European Union toughened their legislative enforcement and encouragement for product recovery practices in the last decades. Among the various kinds of policy instruments employed, *Extended Producer Responsibility (EPR) Principle* based policies like product take-back mandates, recycling rate targets and advance recycling fees (Walls, 2006) are the most common ones. These policies mainly aim to increase the amount and the degree of recovery and minimize the environmental impact of waste materials. They are intended to motivate firms to take into account the environmental impact of their products while planning their forward production. Another main objective of EPR legislation is to promote product design/redesign that will facilitate disassembly and recovery operations. In the European Commission Directive on WEEE and in the EPR Guidance Manual for Governments of the OECD (2001), it is clearly stated that proper legislation should encourage design and production, which take into account and facilitate dismantling and recovery, in particular the re-use and recycling of used products, their components and materials. However, environmental regulations are not always successful in attaining these objectives.

Most of the time the initial investments manufacturers should make to start recovery operations are not taken into account in these regulations. Nevertheless, this is an important concern for several manufacturers who will just start the recovery business, and may even deter manufacturers to start recovery operations. Manufacturers, who are generally reluctant to initiate product recovery voluntarily, do not want to allocate serious amount of funds for these initial costs as well.

Given all these observations, in this research we mainly seek to answer the following research questions;

- What is the impact of EPR based legislation on the optimal product recovery decisions of manufacturers?
- How do the redesign opportunities affect the willingness of manufacturers for product recovery and under what circumstances can EPR based legislation serve to encourage manufacturers towards a product design that will facilitate recovery?
- How do initial investments needed to start recovery operations affect the optimal recovery decisions?

To answer these questions we consider a legislative form in which the government imposes a disposal fee on each product sold. We assume that manufacturers are obliged to pay this fee unless they properly treat a used product for each new product they introduce to the market. This approach is inline with the EC directive on WEEE which confers the financial responsibility of recovery or disposal to producers and requires them to submit a guarantee for this purpose when placing a product on the market. This legislative form is also in the same vein with the advance recycling/recovery fees or the taxes suggested in the literature (Walls, 2006) and used by some governments (e.g., U.S., Canada).

2.1 Literature Review

The problem investigated in this chapter is related to two streams of research. The first one adopts an economic perspective and seeks to find the socially optimum amount of disposal and recycling generally through general/partial equilibrium models in a game theoretic setting. These studies compare the efficiency of various policy instruments like Pigovian taxes, disposal fees, deposit/refund systems and recycling subsidies. Some of them which are more relevant to our study, focus on product recyclability and investigate whether existing policies are efficient to attain this objective (Fullerton and Wu, 1998, Walls and Palmer, 2000 and Walls, 2000 and 2002). In this stream of research, disposal fee is considered as a downstream policy instrument in which consumers are charged for disposal of used products. Fullerton and Wu (1998), and Walls and Palmer (2000) examine the effectiveness of upstream and downstream policies[1] (e.g., Pigovian taxes, disposal fees, subsidies on recyclable design, deposit/refund systems) in achieving the socially optimum level of product recyclability. The objective is to maximize the consumer utility in Fullerton and Wu (1998) and the net social surplus (sum of the total consumer and the total producer surplus) in Walls and Palmer (2000). Similarly, Calcott and Walls (2000) and (2002) investigate whether downstream policies like deposit/refund system and Pigovian type disposal fees can encourage manufacturers for *Design for Environment* (DfE). They conclude that downstream policies are not successful in encouraging product recyclability. Palmer and Walls (1997) compare the efficiency of recycled content standards and deposit refund systems by maximizing net social surplus. Although the study includes the recycling decisions of the consumers (whether to recycle the used product instead of disposing) and the input combination decisions of the manufacturers (recycled materials vs. virgin materials), their model does not take into account product recyclability and DfE applications. Palmer et al. (1996) also do not include product recyclability decisions in their empirical comparison of deposit/refund systems, recycling subsidies and advance disposal fees. Through case studies, Palmer and Walls (1999) and Walls (2006) discuss advantages and disadvantages of different EPR policies. Palmer and Walls (1999) examine three policies: upstream combined product tax and recycling subsidy (UCTS), manufacturer take-back requirements and unit-based pricing. They conclude that UCTS is more cost effective especially in terms of transaction costs. Walls (2006) gives an extensive overview of various policies based on EPR principle and note that only limited form of DfE has been encouraged by EPR based policies.

[1]Upstream policy instruments such as taxes, subsidies or take-back rate targets, focus on producers while the downstream policy instruments such as disposal fee charged on households per unit consumption, focus on consumers.

The common features of all these studies are: (a) exogenous price and perfectly competitive manufacturers, (b) general/partial equilibrium models which include multiple decision makers, (c) focus on net social surplus or net consumer utility, (d) consumers take the responsibility for treatment of returned products.

The second stream investigates the effects and the efficiency of EPR policies from operations management perspective.

In terms of the research questions, the closest study to our research is Subramanian et al. (2008a). This study proposes a multi-period and multi-product mathematical programming model which integrates the operational decisions with the environmental considerations of the manufacturers. By a comprehensive model which takes into various operational and environmental compliance decisions like manufacturing/remanufacturing amounts, inventory levels, abatement levels, design choices, they examine the trade-offs between these decisions. In contrast to the micro level and operational approach of this paper, we consider the design for recoverability decisions from a more macro perspective to see the individual impact of such kind of decisions. Subramanian et al. (2008b) approach the design issue from a supply chain perspective and investigate the influence of both EPR legislation and supply-chain coordination on product design decisions. They discuss various contracts which can help to achieve coordination between the customer and the manufacturer and lead to more favorable product design. In a slightly different setting, Plambeck and Wang (2008) examine the impact of e-waste (electronic waste such as scrapped mobile phones, video-game consoles, televisions and computers) regulations on new product introduction frequency and the quality and durability of new products. Walther and Spengler (2005) investigate the impact of new legal requirements on e-waste (WEEE) on material flows and costs in a reverse logistics network. They suggest a linear, activity-based materials flow model and solve the model under different scenarios based on possible WEEE requirements to assess the impact of future regulations on the treatment of e-waste and to provide policy recommendations.

Atasu et al. (2009) focus on the efficiency of existing WEEE legislation. They conclude that the social planner should take into account the recycling costs and the environmental impact of different product groups to design effective legislation. By examining another policy instrument, we have similar conclusions about the importance of information about recovery costs for effective policies.

Jacobs and Subramanian (2009) approach the issue from a supply-chain perspective and examine the implications of EPR legislations on economic and environmental performance of supply chains.

In contrast to the quantitative models discussed above, Gottberg et al. (2006) provide an exploratory study to investigate the effectiveness of EPR policies on promoting eco-design (design which provides durability, energy efficiency, avoidance of toxic materials and ease of disassembly). The authors conclude that current EPR policies (i.e. charges for producer responsibility) cannot sufficiently stimulate eco-design.

Finally, in this stream, there is a group of studies which focus only on the operational issues in product design. They do not take into account legislative issues. In this sense, Debo et al. (2005) investigate the most profitable product technology for a tire manufacturing firm. Ferrer (2001) suggests the design measures of disassemablability, recyclability and reusability and develops a heuristic to determine the recovery routine of a generic widget. Mangun and Thurston (2002) provide a decision tool for manufacturers to assess whether a used product should be taken back and which parts of it should be reused, recycled or disposed under the scenarios of no market segmentation and market segmentation.

Our work differs from both of the two streams in the following respects: (a) our main contribution is taking into account the initial investments needed to start recovery and redesign operations and investigating their impact on the effectiveness of legislation; (b) we take into account product recoverability opportunities and their impact on the manufacturer's recovery decisions; (c) in our models, manufacturer bears the full responsibility for the proper treatment of their used products as required in EPR based legislations and (d) we examine the implications of EPR policies from the perspective of a manufacturer.

2.2 Models

We consider a setting where manufacturers are held responsible for each product they introduce to the market. They are required either to pay a disposal fee, d_f, or to recover a core per their unit sales. By recovering a core, we imply disassembling a used product and making use of its components/parts/materials as inputs in the production or as spare parts in maintenance and repair. Our models include a single decision maker, a manufacturer, who decides on the sales quantity and the recovery amount as a percentage of his total sales to maximize his profits. To avoid the fee, the manufacturer may also properly dispose the cores. However, since direct disposal is practically equivalent to incurring the legal fee, we assume that once the manufacturer decides to avoid the disposal fee for a core, he will prefer to disassemble it and make use of some parts or components to obtain some savings. For each core recovered, there is a unit disassembly cost, c_d, and a unit saving, s. We assume that the yield rate is hundred percent and the manufacturer can gain s from each core he disassembles. We model unit saving as a fixed proportion, χ, of the unit production cost, c. This approach is different from the current literature where savings from remanufacturing is taken into consideration by means of cost disparities between newly manufactured and remanufactured products (Savaşkan, et al., 2004; Savaşkan and Van Wassenhove, 2006; Majumder and Groenevelt, 2001). Modeling the recovery saving as a separate parameter instead of taking it into consideration via the reduced remanufacturing costs, makes our models applicable to other recovery types as well as the remanufacturing.

We consider a monopoly and assume that consumer utility (υ) from purchasing a product is uniformly distributed between 0 and 1 and the market size is normalized to 1. Hence, a consumer with utility Ω_j purchases the product if Ω_j is greater than or equal to the unit price of the product, p. Given this, we obtain a linear inverse demand function, $p = 1 - q$ where q stands for the total sales quantity of the manufacturer. To ensure that the legislation does not drive the manufacturer out of the business, we assume that the sum of the unit production cost and the unit disposal fee is smaller than 1 ($c + d_f < 1$).

Given this general setting, we examine the optimal decisions of the manufacturer under three models. In our first model (the base model), the manufacturer decides only on the recovery rate (q_r), we do not consider the design for recoverability opportunities in this model. We assume a fixed disassembly cost and do not take into account the possible cost reductions that can be obtained by improving the design of products. In the literature, it is widely acknowledged that product design is an important determinant of recovery costs and may provide substantial cost reductions in disassembly (see for instance Güngör and Gupta, 1999, Calcott and Walls, 2000, Subramanian et al., 2008(b)). However, product design is not explicitly included in the current models investigating the economic implications of product recovery, except for some recent papers (e.g., Subramanian et al., 2008 (b), Debo et al., 2005, Calcott and Walls, 2002). Also, design for recovery is an important objective of the current EPR legislations. In view of these, in our second model we assume that the manufacturer has the opportunity of increasing the recoverability of his products by product design changes. We define the appropriateness of product design for disassembly and recovery operations as *product recoverability level* (a), and let the manufacturer to decide on a so as to maximize his profits. Improving product recoverability provides a reduction in the unit disassembly cost but also leads to an increase in the unit production cost. We define the marginal cost increase as recoverability improvement cost (c_r) and the marginal cost reduction as recoverability improvement saving (σ).

The current quantitative models in the literature assume that the facilities or the capacity needed to conduct recovery operations already exist. Hence, they do not include initial investments needed to start recovery applications in the analysis. However; considering the novelty of product recovery for most of the manufacturers, we believe that initial investments for system set-up is an important concern for manufacturers and may have significant effects on their recovery decisions. To disassemble and recover a product, a manufacturer has to make an investment either to set-up his own systems or to purchase the necessary capacity from an external supplier. To improve the recoverability of his products, he also needs to invest in his production technology.

However; most of the time the manufacturers are reluctant to finance these initial investments which imply a substantial lump-sum spending for the company. They can generally allocate only a limited amount of funds which does not cover all the necessary investments. In our last model, we include these initial investments into the analysis and consider a constraint on the total amount of funds that can be allocated for this purpose. We investigate the impact of the initial investment costs on the recovery decisions of the manufacturer and the effectiveness of the legislation.

We solve all the three models under two different cost structures. First, we consider the case where both the unit disassembly cost, (c_d) and the unit recoverability improvement cost (c_r) are fixed and total cost functions are linear in quantity. Then, we consider the case where c_d is increasing in the recovery amount (qq_r) and c_r is increasing in the product recoverability (a). Table 2.1 gives notation for the parameters and the decision variables used in models.

Parameters	
d_f	unit disposal fee
c_d	unit disassembly cost
c	unit production cost
s	unit recovery saving $s = \chi c$ where χ is a fixed proportion
c_r	unit recoverability improvement cost
σ	unit saving from recoverability improvement
F	available funds for initial investments
γ	cost per unit recovery capacity
δ	cost per unit production technology improvement
Decision Variables	
q	sales quantity
q_r	recovery rate - percentage of total sales recovered $q_r \in [0, 1]$
a	product recoverability level

Table 2.1: Notation

2.2.1 Linear Cost Case

Base model

In the base model, given the unit disposal fee (d_f) and a constant unit disassembly cost (c_d), the manufacturer decides only on the sales quantity and the recovery rate to maximize his profits. The objective of the manufacturer is written as;

$$\max_{q_r, q} \quad \Pi_M = q(p-c) - q(1-q_r)d_f - qq_r c_d + qq_r s \tag{2.1}$$

$$\text{s.t.} \quad 0 \leq q_r \leq 1 \tag{2.2}$$

$$s = \chi c \text{ and } p = 1 - q \tag{2.3}$$

In the objective function, the first term ($q(p-c)$) denotes total profit from the sales, the second one ($q(1-q_r)d_f$) denotes total disposal fee paid and, finally the last two terms express total cost of disassembly and total savings from recovered cores, respectively.

Proposition 1 *The optimal decisions of the manufacturer under the base model are summarized as in Table 2.2. The manufacturer prefers to avoid the disposal fee and recover as much as his sales if $c_d - c\chi \leq d_f$. In contrast if $c_d - c\chi > d_f$, he prefers to pay the disposal fee and recovers nothing. Optimal recovery rate increases as d_f, c or χ increases but decreases as c_d increases.*

Proof. See Appendix-A.1 for all proofs. ■

	$c_d - c\chi \leq d_f$	$c_d - c\chi > d_f$
q_r	1	0
q	$\frac{1}{2}(1 - c_d - c(1-\chi))$	$\frac{1}{2}(1 - c - d_f)$
p	$\frac{1}{2}(1 + c_d + c(1-\chi))$	$\frac{1}{2}(1 + c + d_f)$
Π_M	$\frac{1}{4}(-1 - \chi c + c + c_d)^2$	$\frac{1}{4}(-1 + c + d_f)^2$

Table 2.2: Optimal solutions for the base model with linear costs given the possible parameter realizations

As Table 2.2 shows, as long as the disposal fee is equal to or just above the net cost of recovery ($c_d - \chi c$), legislation can guarantee full recovery ($q_r = 1$). In this case, the manufacturer prefers to recover as much as his sales. However, if the disposal fee is set to a lower value, the manufacturer recovers nothing and reflects the disposal fee to the sales price. In this case, legislation cannot achieve its objective in encouraging product recovery and only serves as an extra burden for both consumers and manufacturers. This result implies that the level of the disposal fee is very important to encourage product recovery via a disposal fee policy. To make the policy effective, relevant costs and savings (i.e. disassembly and production costs) specific to target industries or product groups should be carefully examined and the legal fees should be customized according to the characteristics of each product group. In this way, both high product recovery rates can be obtained and artificially high but ineffective fees can be avoided.

Model Incorporating Design for Recoverability (IDR)

In this case, we take into account the product design opportunities and allow the manufacturer to determine the recoverability of his products (a) as well as the recovery rate. When the manufacturer increases the recoverability level by a unit, the unit production cost increases by c_r but the unit disassembly cost decreases by σ. We assume that $c_r < \sigma$ since otherwise improving product recoverability would never be profitable for the manufacturer. In the linear cost setting, we assume that c_r is fixed and total recoverability improvement cost (aqc_r) is linear in a.

Given these, the objective function of the manufacturer is written as;

$$\max_{q_r,q,a} \quad \Pi_M = q(p-c) - q(1-q_r)d_f - qq_rc_d + sq_rq - c_rqa + \sigma q_rqa \tag{2.4}$$

$$\text{s.t.} \quad 0 \leq q_r \leq 1 \tag{2.5}$$

$$0 \leq a \leq 1 \tag{2.6}$$

$$s = \chi c \text{ and } p = 1 - q \tag{2.7}$$

Except for the last two terms, the objective function is the same as in the base model. The last two terms (c_rqa and σq_rqa) denote the total cost and total contribution of recoverability improvement respectively.

Proposition 2 *The optimal decisions of the manufacturer in IDR case are given as in Table 2.3. Similar to the base model, optimal solution occurs at boundaries. Recovery rate and product recoverability are mutually reinforcing each other's effect. The manufacturer prefers perfect recoverability ($a = 1$) in full recovery case and prefers zero recoverability ($a = 0$) in no recovery case. The threshold on the disposal fee for full recovery is lower due to the contribution of recoverability improvement.*

	$d_f < c_d - c\chi - \sigma + c_r$	$d_f \geq c_d - c\chi - \sigma + c_r$
a	0	1
q_r	0	1
q	$\frac{1}{2}(1 - c - d_f)$	$\frac{1}{2}(1 - c(1-\chi) - c_d - c_r + \sigma)$
p	$\frac{1}{2}(1 + c + d_f)$	$\frac{1}{2}(1 + c(1-\chi) + c_d + c_r - \sigma)$
Π_M	$\frac{1}{4}(-1 + c + d_f)^2$	$\frac{1}{4}(-1 + c(1-\chi) + c_r + c_d - \sigma)^2$

Table 2.3: Optimal solution sets for the model IDR with linear costs given the possible parameter realizations

As Table 2.3 shows, depending on the relevant costs and savings (e.g., c_d, c_r, σ, χ) and the disposal fee, the manufacturer prefers either perfect or zero recoverability. Partial product recoverability ($0 < a < 1$) is never optimal for the manufacturer. As long as the disposal fee is sufficiently high to guarantee full recovery, the manufacturer always prefers perfect recoverability.

Comparison of the base and the IDR case solutions clearly shows the effect of redesign opportunities on the recovery decisions of the manufacturer. Reflecting the contribution of recoverability improvement, the threshold on the disposal fee which ensures full recovery is lower than the base case. Since, the manufacturer can increase his recovery savings by investing in product recoverability, he prefers full recovery even at lower levels of disposal fee. Redesign opportunities increase favorableness and profitability of product recovery. This implies that to increase the effectiveness of legislation for different product groups, besides the recovery costs and savings, information about the redesign opportunities and the associated costs and savings is crucial. In this way, the policy maker can accurately estimate the disposal fee threshold that can encourage the manufacturers for full recovery and perfect recoverability and can also avoid unnecessarily increasing the sales price and the cost burden of manufacturers.

Model with Limitations on the Allocated Fund for Initial Costs (LAFIC)

In this case, we take into account the reluctance of the manufacturer to finance the initial investments. We assume that the manufacturer needs to invest γ to increase his recovery capacity by one unit. Similarly, to increase product recoverability by one unit, he needs to invest δ in his production technology. The total investment expenditure is a linear function of the recovery rate (q_r), and the recoverability level (a). However, the manufacturer is willing to allocate only a limited amount of funds (F) for these initial investments instead of completely covering these costs. Throughout this chapter for the sake of brevity, we call the amount of funds the manufacturer can allocate for these initial investments as the allocated fund.

The aim of this model is to reveal the effects of these initial expenditures and the insufficiency of the allocated funds on the recovery decisions of the manufacturer.

The objective function of the manufacturer is written as;

$$\max_{q_r,q,a} \quad \Pi_M = q(p-c) - qq_rc_d - q(1-q_r)d_f + sq_rq - c_rqa + \sigma q_rqa \tag{2.8}$$

$$\text{s.t.} \quad 0 \le q_r \le 1 \tag{2.9}$$

$$0 \le a \le 1 \tag{2.10}$$

$$a\delta + \gamma qq_r \le F \tag{2.11}$$

$$s = \chi c \text{ and } p = 1 - q \tag{2.12}$$

The objective function is the same as in the model IDR. However, with the inclusion of a nonlinear constraint, the model becomes very complex for the derivation of an analytical solution. Hence, we use numerical analysis.

Numerical Analysis: We conduct the numerical analysis over a large experimental set designed by full factorial design. We have chosen our experimental set as follows;

- F values are selected such that F is always smaller than the amount required to cover all the initial investments. For this purpose, we set F to 30% and 90% of the required amount.

- Three value levels (i.e. high, medium and low) for d_f and two value levels (i.e. high and low) for all the other parameters are used.

- Unit recoverability improvement cost (c_r) is set to be always lower than unit saving from recoverability improvement (σ) since, otherwise recoverability would never be profitable for the manufacturer and we are not interested in this case.

- Both *unit production cost is higher than unit disassembly cost* ($c > c_d$) and *unit disassembly cost is higher than unit production cost* ($c < c_d$) scenarios are covered in the experimental set. In real applications both of these cases are observable.

Given these specifications, see Appendix A.2 for the experimental set we used.

To find the global optimal solutions for our experimental set, we used a commercial global optimal solver, *Premium Solver* by *Frontline Systems*. We employed *Global Interval Search* provided by this software.

Results and Discussion

Observation 1 *Initial investment costs has a substantial influence on the recovery decisions of the manufacturer. These costs and the insufficiency of the allocated fund may completely deter the manufacturer from starting product recovery even if he would prefer full recovery and perfect recoverability* ($q_r = 1$, $a = 1$) *otherwise. This is mostly observed when* F *is relatively lower.*

This observation is not obvious at first glance. Intuitively one would expect that if recovery is more preferable for the manufacturer, then even if the allocated fund is not sufficient to cover all the initial costs, the manufacturer would still try to obtain the highest attainable recovery rate (q_r) and product recoverability (a) by entirely using his allocated fund. However, our results show that this is not always the case. Especially when the needed amount for the initial investments is substantially higher than the amount of funds the manufacturer is willing to allocate, then the manufacturer prefers not to start recovery operations. The underlying reason for this result is the interdependence between the two decision variables (a and q_r). To increase the profitability of product recoverability and to justify a positive a in the optimal solution, recovery rate should exceed a certain threshold (see the proof of Proposition 2 in Appendix A.1). On the other hand, high recovery (i.e. exceeding the necessary threshold) is only possible if product recoverability is high and recovery costs are low. However, since the manufacturer can allocate only a limited amount of funds for the initial investments, he will not be able to invest in both high product recoverability and high recovery rate. Hence, no recovery and zero recoverability solution becomes optimal given a relatively low F.

To see the impact of the constraint on optimal q_r, a, p and Π_M we compare the solutions for models IDR and LAFIC in Table 2.4. As Table 2.4 indicates when

	Model IDR Average	Model LAFIC Average	Percent Decrease
q_r	0.563	0.438	22%
a	0.563	0.271	52%
p	0.761	0.786	−3%
Π_M	0.061	0.056	9%

Table 2.4: Comparison of the model IDR and the model LAFIC solutions under linear cost structure

the amount of funds the manufacturer is willing to allocate does not cover all the initial investment costs, he substantially reduces his product recoverability (by 52%) and recovery rate (by 22%). This result also supports our finding in Observation 1. On the other hand, Π_M and p are not affected much by this constraint. Price increases slightly while the manufacturer profit decreases by only 9%. These conclusions are also verified by the statistical analysis given below.

To investigate the impact of the parameters on the optimal objective value (manufacturer profit), we use *one-at-a-time version of the global sensitivity analysis* suggested by Wagner (1995). We prefer the *least squares regression approach* to implement the method (see Wagner, 1995).

We regressed each parameter on the optimal manufacturer profit (Π_M) in simple regressions and found the corresponding adjusted R^2s. Adjusted R^2 values provide a measure of the impact of each parameter on Π_M (see Wagner, 1995). Table 2.5 shows adjusted R^2 values in order from the simple regressions with a significant relation.

Parameters	Adjusted R^2
c	0.554
d_f	0.200
c_d	0.080
F	0.008

Table 2.5: Adjusted R^2 values from simple regressions for the model LAFIC with linear costs

From Table 2.5, note that c is the most influential parameter on the optimal manufacturer profit with an adjusted R^2 of 0.554 which is considerably higher than that of the other parameters. The influence of d_f is also high while the other parameters do not have an important impact. Compared to the IDR case[2], the impact of d_f is higher and the impact of c_d is lower. This change in the relative impact of the disposal fee and the disassembly cost on the manufacturer profit can be explained by the increased preference of the manufacturer for paying the disposal fee rather than investing in recovery when the allocated fund does not cover all the initial investments.

To investigate the impact of the parameters on the decision variables (a, q_r and p), we used chi-square analysis (see Wagner, 1995). We constructed a cross-tabulation for each decision variable with respect to each parameter. In cross-tabulations, we categorized the optimal values of the decision variables into 3-tiles and grouped the parameters under the value levels used in the experimental set. Wagner (1995) suggests

[2] Although we have provided closed-form solutions for the model IDR under linear cost structure, to make comparisons with other models we have also conducted regression and chi-square analysis for this model. We run the analysis over the same experimental set used for the other models. Tables A.2 and A.3 in Appendix-A.3 summarize the results of this analysis.

using the significance probabilities (p-values) for the chi-square statistic to assess the relative influence of each parameter. However; the significance probabilities for the significantly influential parameters are all very small ($\approxeq$ 0.000) in our case and did not help much in comparing the influence of the parameters on the decision variables. Thus, we use Cramer's V coefficients to assess the parameters' relative influence.

Parameters/Decision Variables	q_r	a	p
F	0.583	0.519	0.200
d_f	0.461	0.301	0.350
c	0.158	Not Signf.	0.785
c_d	0.470	0.232	0.190
δ	Not Signf.	0.339	Not Signf.

Table 2.6: Cramer's V coefficients from chi-square analysis for the model LAFIC with linear costs

Table 2.6 summarizes the Cramer's V coefficients of the most influential parameters. The significance probabilities for all the parameters in the table (except for the *Not Signf.* cases) are 0.000, thus all the relations are highly significant.

From the table we can conclude that;

1. The three most influential parameters on the optimal recovery rate (q_r) are the allocated fund (F), the unit disassembly cost (c_d) and the unit disposal fee (d_f). Among these, F has the highest influence with a Cramer's V coefficient of 0.583.

2. The three most influential parameters on the optimal product recoverability (a) are the allocated fund, the unit production technology improvement cost (δ) and the unit disposal fee. Similar to q_r, again F has the highest influence on a.

3. The three most influential parameters on the optimal price (p) are the unit production cost (c), the unit disposal fee and the allocated fund. In contrast to q_r and a, the most influential parameter on the optimal price is the unit production cost not the allocated fund.

4. Compared to the IDR case, the influence of d_f on a and q_r decreases (see Table A.3).

Results 1, 2 and 4 support our argument that insufficiency of the allocated fund for the initial investments may have a substantial impact on the recovery decisions. F turns out to be the most important determinant of both the optimal recovery rate and the product recoverability. When the allocated fund is not sufficient to cover the initial investments, the effect of the disposal fee on the recovery variables reduces. In other words, disposal fee is less effective in boosting recovery and redesign applications when the manufacturer is reluctant to undertake all the initial costs. In contrast, optimal price is not affected by the allocated fund to the same extent. From Table 2.5, we also know that the impact of the allocated fund on Π_M is very small relative to the other parameters.

In the light of all these findings and Table 2.4, we can conclude that the reluctance of the manufacturer to undertake the initial costs and, thus the insufficiency of the allocated fund for the initial investments substantially changes the recovery choices of the manufacturer. The impact of this constraint on the recovery-related outputs which are primarily the social planner's concerns (i.e. recovery rate and recoverability) is much greater than its impact on price and total profit. In other words, insufficiency of the allocated fund affects the social planner more than the manufacturer. To design effective policies and ensure higher recovery rates, at the outset the social planner needs to consider this problem and try to find some solutions such as start-up subsidies/credits or tax cuts.

2.2.2 Nonlinear Total Cost Case

In the previous section, we solved our models given that unit disassembly and recoverability improvement costs are fixed. However, for some product groups, increasing disassembly and recoverability improvement costs may be more realistic. For instance, for some product groups unit disassembly cost depends on the condition of the core and manufacturers prefer to disassemble the cores in better condition first. Thus, unit disassembly cost increases as the manufacturer recovers more products. Similarly, in some cases, improving the recoverability of the product further becomes costlier at higher recoverability levels. Although substantial cost savings can be obtained with small modifications in the initial stages, as the recoverability level increases, getting a marginal improvement requires costlier and more complex modifications. In the light of these observations, in this section, we consider that the unit disassembly cost is increasing in the recovery amount (qq_r) and unit recoverability improvement cost is increasing in the recoverability level (a). We model unit disassembly cost, $c_d^{'}$, and unit recoverability improvement, $c_r^{'}$, as;

$$c_d^{'} = c_d + \beta_{c_d} q q_r \tag{2.13}$$

$$c_r^{'} = c_r + \beta_{c_r} a \tag{2.14}$$

β_{c_d} and β_{c_r} stands for the rate of increase in $c_d^{'}$ and $c_d^{'}$, respectively.

Given these, we investigate whether the nonlinear total cost structure alters our findings from linear cost case.

Base model

Given $c_d^{'}$ as above, the objective of the manufacturer is written as,

$$\max_{q_r, q} \quad \Pi_M = q(p-c) - qq_r c_d^{'} - q(1-q_r)d_f + sq_r q \tag{2.15}$$

$$\text{s.t.} \quad 0 \leq q_r \leq 1 \tag{2.16}$$

$$s = \chi c \text{ and } p = 1 - q \tag{2.17}$$

The objective function is the same as in the linear cost case base model. The only difference is the disassembly cost function ($c_d^{'}$) which is increasing in recovery amount in this case.

Proposition 3 *The optimal decisions of the manufacturer under the base model with increasing unit disassembly cost are summarized as in Table 2.7. In contrast to the linear cost case, a disposal fee higher than net minimum cost of disassembly ($c_d - c\chi$) is no more sufficient to guarantee full recovery. Instead of full recovery, partial recovery is preferred* *especially at a relatively lower disposal fee. As the rate of increase in disassembly cost (β_{c_d}) increases, optimal recovery rate decreases and full recovery becomes possible at a much higher disposal fee.*

	$d_f \leq c_d - c\chi$	$c_d - c\chi < d_f < \frac{c_d - c\chi + \beta_{c_d}(1-c)}{\beta_{c_d}+1}$	$\frac{c_d - c\chi + \beta_{c_d}(1-c)}{\beta_{c_d}+1} \leq d_f$
q_r	0	$\frac{c\chi + d_f - c_d}{\beta_{c_d}(1-c-d_f)}$	1
q	$\frac{(1-c-d_f)}{2}$	$\frac{(1-c-d_f)}{2}$	$\frac{(1-c-c_d+c\chi)}{2(1+\beta_{c_d})}$
p	$\frac{(1+c+d_f)}{2}$	$\frac{(1+c+d_f)}{2}$	$\frac{2\beta_{c_d}+c+c_d-c\chi+1}{2(1+\beta_{c_d})}$
Π_M	$\frac{(-1+c+d_f)^2}{4}$	$\frac{\beta_{c_d}(-1+c+d_f)^2+(-c_d+c\chi+d_f)^2}{4\beta_{c_d}}$	$\frac{(-1-\chi c+c+c_d)^2}{4(\beta_{c_d}+1)}$

Table 2.7: Optimal solutions for the base model with nonlinear total costs

A unit disassembly cost increasing in recovery amount decreases the preferability of product recovery for the manufacturer. Keeping everything else equal, the manufacturer prefers full recovery at a higher disposal fee compared to the linear case. When the disposal fee exceeds net minimum cost of disassembly ($c_d - c\chi$) but is lower than $\frac{c_d - c\chi + \beta_{c_d}(1-c)}{\beta_{c_d}+1}$, the manufacturer prefers partial recovery. Interestingly, in this case, even if the manufacturer recovers some cores and obtains some savings, he does not reflect this to his sales price. Sales price and quantity remain the same and disposal fee is reflected on consumers as in the no recovery case. Hence, setting the disposal fee higher than $\frac{c_d - c\chi + \beta_{c_d}(1-c)}{\beta_{c_d}+1}$ is important both to ensure full recovery and to minimize the effect of legislation on sales price. For this purpose, information about the cost structure of the manufacturers is essential for the policy maker.

Model incorporating design for recoverability

Given the increasing unit disassembly ($c_d^{'}$) and the increasing unit recoverability improvement costs ($c_r^{'}$), the objective of the manufacturer is written as;

$$\max_{q_r, q, a} \quad \Pi_M = q(p-c) - qq_r c_d^{'} - q(1-q_r)d_f + sq_r q - c_r^{'} qa + \sigma q_r qa \tag{2.18}$$

$$\text{s.t.} \quad 0 \leq q_r \leq 1 \tag{2.19}$$

$$0 \leq a \leq 1 \tag{2.20}$$

$$s = \chi c \text{ and } p = 1 - q \tag{2.21}$$

The objective function is same as in the linear cost case design model. However; with nonlinear total cost functions, analytical solution of the model becomes very complex. Hence, as in the LAFIC case with linear costs, we use numerical analysis. We add the rate of increase parameters (β_{c_d} and β_{c_r}) to our experimental set and conduct the analysis over this extended set. For β_{c_d} and β_{c_r} we consider two values, 0.03 and 0.09. We selected these values to ensure that even with the maximum increase in recovery costs, the total unit cost of manufacturer does not exceed 1 so that he can continue production.

Results and Discussion:

Observation 2 *Unlike the linear cost case, optimal solution does not always occur at boundaries (i.e. $q_r = 0, a = 0$ or $q_r = 1, a = 1$). Especially for high c_r and β_{c_r}, full recovery can be accompanied with partial recoverability ($0 < a < 1$) instead of perfect product recoverability ($a = 1$). Similarly, partial recovery may be accompanied by zero recoverability. Reduction in the optimal product recoverability due to the increasing unit costs is always higher than the reduction in the recovery rate and $q_r \geq a$ in the optimal solution in contrast to the linear cost case where $q_r = a$.*

Keeping everything else equal, increasing unit disassembly and unit recoverability improvement costs decrease both the optimal recovery rate and the optimal product recoverability. To see the impact of nonlinear total cost structure on q_r, a, p and Π_M we compare solutions of the model IDR with the linear and the nonlinear total costs in Table 2.8. Under nonlinear total cost structure, although the sales price and the manufacturer profit do not change to a great extent, the recovery variables substantially decrease. Supporting Observation 2, the reduction in average a is almost twice of the reduction in average q_r. This implies that to compensate for the increasing costs the manufacturer first relinquishes product recoverability improvement.

	Linear Cost Case Average	Nonlinear Total Cost Case Average	Percent Decrease
q_r	0.563	0.449	20%
a	0.563	0.271	52%
p	0.761	0.776	−2%
Π_M	0.061	0.055	10%

Table 2.8: Comparison of the model IDR solutions under linear and nonlinear total cost structures

To investigate impact of parameters on the optimal manufacturer profit and the decision variables, we use simple regressions and chi-square analysis respectively similar to the linear case. Table 2.9 gives the adjusted R^2s from the simple regressions while Table 2.10 summarizes the Cramer's V coefficients from the chi-square analysis.

Parameters	Adjusted R^2
c	0.584
d_f	0.200
c_d	0.086

Table 2.9: Adjusted R^2 values from simple regressions for the model IDR with nonlinear total costs

Table 2.9 shows that unit production cost is the most influential parameter on Π_M. Like the linear cost case, d_f and c_d also have considerable impact on Π_M. On the other hand, neither β_{c_d} nor β_{c_r} has a significant impact. Compared to the linear cost case model (see Table A.2 in Appendix-A.3), the impact of c remains almost the same. However, the impact of d_f substantially increases and the impact of c_d decreases. This can be explained by the decrease in the amount of recovery made by the manufacturer. Due to the increased recovery costs, the manufacturer prefers to pay the disposal fee rather than recovering his products. Hence, the influence of the disposal fee on his profit increases under nonlinear total cost structure. Along with the findings from Table 2.8, these results imply that the rate of increase in recovery costs does not significantly affect the manufacturer profit. However, the cost structure changes the effect of disassembly cost and disposal fee on the manufacturer profit.

Parameters/Decision Variables	q_r	a	p
d_f	0.658	0.463	0.283
c	Not Signf	Not Signf.	0.836
c_d	0.471	0.480	0.374
β_{c_r}	Not Signf.	0.518	Not Signf.

Table 2.10: Cramer's V coefficients from chi-square analysis for the model IDR with nonlinear total costs

As we can see from Table 2.10, the rate of increase in the unit recoverability improvement cost (β_{c_r}) has a significant influence on the product recoverability level (a). From Table 2.8 we have already stated that under nonlinear total cost structure the highest reduction occurs in product recoverability by 52%. Supporting this finding, β_{c_r} is the most influential parameter on the product recoverability in Table 2.10. Nonlinear total cost structure also affects the influence of the other parameters on a. Compared to the linear cost case, especially the influence of the disposal fee on a decreases. However; the same outcome is not valid for price and recovery rate. The most influential parameters and their relative impact on p and q_r do not change substantially with respect to the linear case (see Table A.3).

Overall, it seems that increasing recovery costs reduce the manufacturer's disposition to improve recoverability of his products and weaken the effectiveness of disposal fee in encouraging recoverability improvement.

Model with limitations on the allocated fund for initial costs

Similar to the linear case, the objective of the manufacturer is written as;

$$\max_{q_r,q,a} \quad \Pi_M = q(p-c) - qq_r c_d^{'} - q(1-q_r)d_f + sq_r q - c_r^{'} qa + \sigma q_r qa \tag{2.22}$$

$$\text{s.t.} \quad 0 \leq q_r \leq 1 \tag{2.23}$$

$$0 \leq a \leq 1 \tag{2.24}$$

$$a\delta + \gamma q q_r \leq F \tag{2.25}$$

$$s = \chi c \text{ and } p = 1 - q \tag{2.26}$$

Similar to the linear case, we solve the model numerically over the same experimental set. We add β_{c_d} and β_{c_r} to our experimental set and again consider two values for these parameters, 0.03 and 0.09, respectively.

Results and Discussion Table 2.11 gives a comparison of the solutions of the models IDR and LAFIC with nonlinear total costs. Similar to the linear cost case, insufficiency of the allocated fund leads to a serious reduction in q_r and a in nonlinear total costs case. Considering Table 2.11 and Table 2.4 together, the reduction due to the constraint is higher under increasing recovery costs than the reduction under fixed recovery costs. In other words, increasing disassembly and recoverability costs intensifies the impact of the constraint.

	Model IDR Average	Model LAFIC Average	Percent Decrease
q_r	0.449	0.342	24%
a	0.271	0.114	58%
p	0.776	0.790	−2%
Π_M	0.055	0.052	5%

Table 2.11: Comparison of the model IDR and the model LAFIC solutions under nonlinear total cost structure

We investigate the impact of parameters on the manufacturer profit and the decision variables through simple regressions and chi-square analysis as before. Table 2.12 shows the adjusted R^2 values from the simple regressions where Π_M is the dependent variable and a parameter is the independent variable. The table only shows the parameters with a significant relation with Π_M. From this table we see that like all the previous models, production cost is the most influential parameter and the allocated fund does not have any significant effect on the manufacturer profit. Compared to the linear cost case, the impact of c remains almost the same; however the impact of d_f

increases while the impact of c_d decreases (see Table 2.5). As we have also observed for the model IDR, increasing recovery costs intensifies the impact of disposal fee on the manufacturer profit.

Parameters	Adjusted R^2
c	0.555
d_f	0.289
c_d	0.044
χ	0.001

Table 2.12: Adjusted R^2 values from simple regressions for the model LAFIC with nonlinear total costs

Table 2.13 shows Cramer's V coefficients from chi-square analysis of each parameter and decision variables. Allocated fund is the most important determinant of optimal recovery rate and product recoverability. d_f and c_d are also influential especially on q_r. Compared to the model IDR, the impact of d_f on both q_r and a weakens. Similar to the linear cost case, when the amount of funds the manufacturer allocates is not sufficient to entirely cover the initial investments, disposal fee is less effective in encouraging product recovery and recoverability.

On the other hand, compared to the model LAFIC with linear costs, the impacts of F and δ on a are lower. Table 2.13 does not include the parameters with a Cramer's V coefficient lower than 0.2 for all decision variables. Hence, we cannot see the impact of cost parameters like c_r, β_{c_r} in the Table 2.13. However; though smaller than the parameters given in the table, these parameters also have a significant impact on a and the decrease in the Cramer's coefficients of F and δ on a can be explained by the increased impact of these parameters.

Parameters/Decision Variables	q_r	a	p
F	0.572	0.432	0.139
d_f	0.468	0.282	0.366
c	0.094	Not Signf.	0.765
c_d	0.475	0.236	0.158

Table 2.13: Cramer's V coefficients from chi-square analysis for the model LAFIC with nonlinear total costs

2.3 Conclusions

In this chapter, we investigate product recovery and recoverability decisions of a manufacturer under a legislative disposal fee. We include the impact of initial investments on the manufacturer's decisions. We examine how the recovery choices of the manufacturer change when he is reluctant to or cannot entirely cover the initial investment costs. First, we consider the case where the unit recovery costs are fixed and then we extend our models to the case where the unit disassembly and the unit recoverability improvement costs are increasing in the recovery amount and the recoverability level, respectively.

As a result of our analysis, we observe that the insufficiency of the allocated fund for the initial investments may have a serious impact on the optimal recovery decisions. If the amount of funds the manufacturer is willing to allocate is not enough for the initial investments, disposal fee policy will not be sufficient to encourage the manufacturer for more product recovery and higher product recoverability. Initial investment expenditures and the reluctance of the manufacturer to allocate sufficient funds may even completely deter the manufacturer from starting product recovery applications. Hence, in order to design effective environmental policies to encourage product recovery, these initial investment needs should be carefully taken into account and perhaps through proper subsidies their negative effects should be alleviated.

To design effective policies, information about the recovery costs and savings and the redesign opportunities for relevant product groups is also essential. To accurately estimate the disposal fee that will encourage full recovery and perfect recoverability but will not redundantly increase the cost burden of the manufacturers, the policy maker should take into account these costs/savings.

We have similar findings in both fixed and increasing recovery costs case. However; increasing recovery costs weaken the effectiveness of the disposal fee especially in encouraging redesign applications. When the cost burden increases, the manufacturer substantially reduces his investments in product recoverability irrespective of the disposal fee. Increasing recovery costs also intensify the impact of insufficiency of the fund allocated for the initial investments.

Chapter 3

AN INVESTIGATION OF THE STRUCTURAL EFFICIENCY OF EPR LEGISLATION

EPR principle based legislation involves extension of a producer's financial and/or physical responsibility to the post-consumer stage of a product's life cycle. Under EPR based legislation, manufacturers are held responsible for the collection and the proper treatment of their used products. Although the EPR based legislation is originated from Europe and The Waste Electrical and Electronic Equipment (WEEE) and End-of-Life Vehicle (ELV) directives in Europe are considered as the first examples of EPR legislation, twelve USA states (Connecticut, Maine, Maryland, Minnesota, New Jersey, North Carolina, Oklahoma, Oregon, Texas, Virginia, Washington and West Virginia) have also passed e-waste laws and a number of others are currently considering EPR legislation. However, there exist variations among the policy tools used to implement the EPR legislation. For instance, WEEE Directive in Europe holds producers physically responsible for meeting certain recycling or recovery targets for their own waste while advance recycling fees used by California charge producers on the basis of their sales. Nevertheless, manufacturers do not seem to be happy with the current legislation. They generally complain about inefficiency and ineffectiveness of the current regulations. They argue that mandatory take-back rates actually serve as hidden taxes imposed on producers and advocate that the governments or social regulators should undertake the take-back and recovery tasks (Atasu et al., 2009).

The purpose of this chapter is to determine the impact of the structural variations that exist in the current EPR legislation on the welfare of different stakeholders (e.g., consumers, producers, the environment and the social planner). For this purpose, we classify the current EPR models under two categories; *tax model* and *rate model*, and compare their efficiency from the perspectives of social planner, manufacturers, environment and consumers. In the tax model, the social planner undertakes the take-back task but requires manufacturers to pay a unit tax for their sales. On the other hand, in the rate model she determines a mandatory take-back rate and requires manufacturers to satisfy it. In other words, in the tax model manufacturers are held only financially responsible for their end-of-life products while in the rate model they are both financially and physically responsible. First, we conduct our analysis in a monopolistic environment and then extend it to a competitive market to examine the effects of competition on the relative efficiency of the two legislative forms. The model under which each stakeholder is better off changes with respect to the parameter settings. Hence, we also examine under which parameter settings the incentives of the stakeholders will be aligned in each environment.

3.1 Literature Review

The problem we focus on in this chapter is closely related to two research streams. The first stream adopts a microeconomic perspective and focuses on how the socially optimum amount of waste generation and disposal can be ensured (Palmer, Sigman and Walls, 1996, Palmer and Walls, 1997, Fullerton and Wu, 1998, Palmer and Walls, 1999, Walls and Palmer, 2000, Calcott and Walls, 2000, Calcott and Walls, 2002, Walls, 2003, Walls et al., 2003, Walls, 2006). Palmer, Sigman and Walls (1997) compare the costs of three different policies (deposit/refund system, recycling subsidies and advance disposal fees) in reducing municipal solid waste in an empirical setting and conclude that deposit/refund is the least costly policy. Similarly, Palmer and Walls (1997) compare the efficiency of deposit/refund system and recycling content standards in generating the socially optimum amount of disposal. Both of these use a partial equilibrium model with competitive markets and do not take into account product recyclability in their analysis. Fullerton and Wu (1998) and Walls and Palmer (2000) formulate models that take into account all environmental externalities throughout the whole life-cycle of a product. In this setting, they discuss the efficiency of various upstream and downstream policies (e.g., Pigovian taxes, disposal fees, subsidies on recyclable design, command and control regulatory standards and deposit and refund systems) in ensuring the socially optimum level of product recyclability. They conclude that depending on the objectives, the market failures, and the ease of implementation different policies can be useful in obtaining the social optimum. Calcott and Walls (2000 and 2002) also investigate the success of deposit/refund system and Pigovian disposal fee in encouraging DfE and product recyclability. Similar to the previous studies, they conclude that downstream policies (e.g., disposal fees and taxes imposed on products) are not useful or practical in encouraging product recyclability especially considering the lack of fully functioning recycling markets and deposit/refund type policies can be more effective at least to obtain the constrained socially optimum recyclability. However unlike Fullerton and Wu (1998), Calcott and Walls (2002) explicitly consider a recycling market and instead of simply assuming that these markets either function or not, they argue that there may be some transaction costs that may obstruct the functioning of the markets and analyze the effects of transaction costs on the efficiency of the environmental policies.

In a related stream, Palmer and Walls (1999) and Walls (2006) use case studies to discuss the pros and cons of different environmental policies. Palmer and Walls (1999) examine three specific policies (upstream combined product tax and recycling subsidy (UCTS), manufacturer take-back requirements and unit-based pricing) and conclude that UCTS, which is a special type of deposit/refund systems, is more cost effective especially in terms of transaction costs. Walls (2006) provides a more extensive overview and comparison of various policies under the EPR umbrella and presents insights from the real life applications of these policies.

The common features of all these studies, specifically the ones with mathematical models, are (i) exogenous price (ii) focus on the social optimum and to what extent it can be attained by different policies and (iii) consideration of product recyclability and DfE decisions in most of them.

Our research differs from the studies in this stream in the following aspects:(i) we assume an endogenous price; (ii) we discuss the impact of competition on the relative efficiency of the policies; (iii) we identify the possible misalignments between manufacturers and government and (iv) rather than focusing on the social optimum and the alternative policies which will ensure it, we explicitly compare two different policies used in real life to assess their relative profitability for the manufacturer(s) and the social planner.

The second stream analyzes the efficiency and effects of environmental regulations from an operations management perspective. In this respect, the most relevant paper to our research is Atasu et al. (2009) where the authors investigate the efficiency of existing WEEE legislation and conclude that to design effective legislation the recycling costs and environmental impact of different product groups as well as the competitive conditions of the market and the consumers' environmental consciousness should be carefully considered. We also focus on the efficiency of existing take-back legislations in this chapter. However unlike Atasu et al. (2009), instead of focusing on only one type of legislation we investigate the relative efficiency of different legislative forms with respect to social welfare and manufacturer profit. Similarly, Toyasaki et al. (2008) compare the favorableness of two prevalent take-back scheme for WEEE (competitive versus monopolistic) from the perspective of the manufacturers, consumers, and recyclers through two-stage sequential games between competing manufacturers and recyclers. In contrast to our problem, besides the collection they also consider the recycling stage and assume that the recycling firms undertake the collection and recycling tasks in return for a fee. Plambeck and Wang (2008) investigate the effects of e-waste regulation (specifically EPR and advance recycling fees) on new product introduction under both monopoly and duopoly settings and conclude that environmental regulation can reduce both the production and the waste amount by giving rise to less frequent new product introduction. Focusing on compliance to the directive on Restriction of Hazardous Substances (RoHS), Plambeck and Taylor (2008) investigate the choice of the regulators to rely on competitive testing (i.e. manufacturers inspect their competitors' products) or to make the inspection themselves to check whether the products comply with RoHS. They conclude that competitive testing is effective in markets dominated with a few firms but not effective in highly competitive markets with many small firms.

Jacobs and Subramanian (2009), on the other hand, take a supply chain perspective and investigate the implications of EPR regulation (specifically the take-back and recycling mandates) on both the economic and environmental performance of the supply chain and the social welfare under integrated and decentralized supply chain settings. Similar to our problem, they make use of a social welfare construct, which is the sum of manufacturer profit, environmental impact and consumer surplus. However; instead of comparing the efficiency of different legislative forms from the perspective of the stakeholders, they investigate the impact of the environmental cost sharing on the social welfare. Subramanian et al. (2008a) also adopt a supply chain perspective. They assume that the consumers and the manufacturer share the environmental costs and primarily focus on the impact of EPR on product design and the effects of supply chain coordination and different contractual arrangements that can ensure coordination in a EPR setting.

The last two studies in this stream adopt different methodological approaches from the previous studies, which mostly make use of game theoretic models. In this respect, Subramanian et al. (2008b) propose a multi-period and multi-product mathematical programming model to simultaneously examine the environmental considerations and the operational decisions of a manufacturer. By this nonlinear model they seek to determine the optimal compliance levels, pricing of new and remanufactured products and optimal design choices. Kroes and Subramanian (2006), on the other hand, present an empirical analysis on existing market based programs (in which firms are allowed to choose among different compliance strategies such as abatement, input switching, use of permits, and retirement of older dirtier facilities). With two sets of multiple regression models, they examine the relation between the compliance choices and the environmental performance and the relation between the environmental performance and the financial performance of the participant manufacturers.

3.2 Models

In this section, we propose our generic model to investigate the basic economic drivers (e.g., manufacturing, collection, recycling and environmental costs) of take-back in alternative legislative settings. We consider a market with three sets of decision makers: consumers, manufacturers and a social planner.

Our generic model considers manufacturing of a single product. The decision making process occurs in a Stackelberg setting (see Varian, 1984); first the social planner decides on the legislative structure. We assume that she makes a choice between 2 different models (see Figure 3.1):

- *Tax Model* (T): The social planner undertakes the take-back task (i.e. collection and recycling), but requires manufacturers to pay a unit tax (τ) for each product. In this case, the social planner decides both on the tax and the take-back rate (c).

- *Take-back Rate Model* (R): The social planner determines a certain take-back rate (c) and requires manufacturers to ensure this take-back rate.

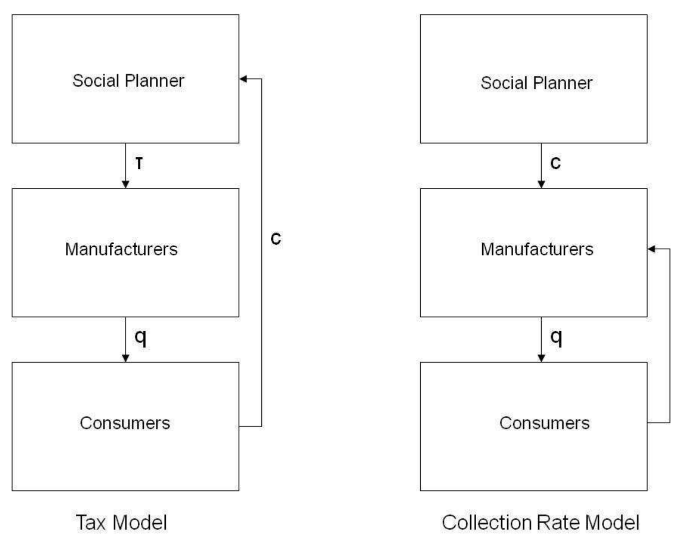

Figure 3.1: Take-back legislation models

We assume n identical manufacturers competing in a Cournot setting (see Varian, 1984). Given the choice of the social planner, each manufacturer (say i) decides on his own sales quantity (q_i) to maximize his total profit.

We assume that consumer utilities (v) from purchasing a product are uniformly distributed between 0 and 1 and the market size is normalized to 1. Hence, a consumer with utility v_j purchases the product if v_j is greater than or equal to the unit product price, p. Given this, we obtain a linear inverse demand function, $p = 1 - q$ where q stands for the total quantity sold by n manufacturers, i.e $p = 1 - \sum_{i=1}^{n} q_i$.

We assume a unit cost of μ to produce each unit, and a unit cost of χ to take back each used product from the consumers. We assume that $\chi + \mu < 1$ so that take-back legislation does not drive manufacturers out of the market.[1] We also require that the unit tax to be always nonnegative and lower than $1 - \mu$ so that there is always positive production in the market. Table 3.1 shows the notation used throughout the chapter.

Indices	
i	Manufacturer index
Decision Variables	
p	Unit price
q	Total sales quantity
q_i	Sales quantity of manufacturer i
Parameters	
μ	Unit production cost
c	Take-back rate
χ	Unit take-back cost
ϵ	Unit environmental cost for each uncollected product
τ	Unit tax imposed on manufacturers for each product they introduce to the market.
n	Number of competitors in the market
Π_E	Total environmental impact of uncollected products
Π_M	Total manufacturer profit
Π_C	Total consumer surplus

Table 3.1: Common notation

Given these costs and $k \in (T, R)$ denoting the legislation structure choice of the social planner, the manufacturer i will maximize his profit;

$$\max_{q_i} \quad \Pi_{M_i}(k) = q_i(p - \mu) - q_i \rho_k(\tau, c) \tag{3.1}$$

[1]Note that the maximum price is 1 and if the sum of the unit production cost and the unit take-back cost is higher than 1, then the manufacturers cannot survive in the market.

where,

$$\rho_k(\tau, c) = \begin{cases} \tau, & \text{if} \quad k = T; \\ c\chi, & \text{if} \quad k = R. \end{cases} \tag{3.2}$$

In the objective function given above, the first term denotes the total sales revenue and the second one denotes the total take-back cost incurred by manufacturer i in the rate model and the total tax paid by manufacturer i in the tax model. $\rho_k(\tau, c)$ stands for the unit tax imposed by the social planner in the tax model and the take-back cost per unit sales in the rate model. From the objective function of manufacturer i, one can derive the optimal q_i as;

$$q_i^* = \frac{1 - \mu - \rho_k(\tau, c)}{n+1} \tag{3.3}$$

By anticipating the manufacturer's choice, the social planner maximizes the total social welfare which consists of four main terms:

- Total Manufacturer Profit:

$$\Pi_M(k) = \sum_{i=1}^{n} \Pi_{M_i}(k) = n(q_i^* p(q_i^*) - \mu - \rho_k(\tau, c))$$

  where $p(q_i^*)$ denotes the price when the optimal quantity chosen by each manufacturer is q_i^*.

- Total Consumer Surplus:

$$\Pi_C(k) = (1 - p(q_i^*))q_i^* n/2$$

- Environmental Impact: We assume that there exists a unit environmental cost of each uncollected product denoted by ϵ (see Atasu et al., 2009 for a detailed discussion). Given this, the total environmental impact of not taking back $(1-c)$ proportion of total sales is,

$$\Pi_E(k) = -\epsilon(1-c)nq_i^*$$

Note that we define Π_E, total environmental impact, as the negative of total environmental cost, hence when we talk about high Π_E, this means low environmental cost and the more Π_E increases, the better it is for the environment. Throughout the chapter, when we actually want to imply cost, we use the term total environmental cost rather than total environmental impact.

- Depending on the particular legislative structure chosen, the social planner will either collect taxes per unit production from the manufacturers and take the responsibility of core take-back herself or impose a take-back rate on the manufacturers. Hence, given the legislative structure chosen (k), the social planner's objective includes the following cost term;

$$\phi_k(\tau, c) = \begin{cases} nq_i^*(\tau - c\chi), & \text{if} \quad k = T; \\ 0, & \text{if} \quad k = R. \end{cases} \tag{3.4}$$

Given these, the total social welfare ($SW(k)$) is written as;

$$SW(k) = \Pi_M(k) + \Pi_C(k) + \Pi_E(k) + \phi_k(\tau, c)$$

The social planner maximizes $SW(k)$ by deciding on:

- the tax (τ) and the take-back rate (c) in the tax model and
- the take-back rate (c) in the take-back rate model.

3.3 Analysis

In this section, we customize our generic model to find out the efficiency level differences between the alternative forms of legislation we consider. We first start with our base model which considers a monopoly and ignores the externality of take-back assurance (i.e. education cost). Then, we consider a competitive market and examine the impact of competition on the relative efficiency of the alternative legislation forms.

3.3.1 Monopoly Case

In this section, we consider a monopoly ($n = 1$) and compare the optimal choices of the decision makers and the resulting social welfare under the two possible legislative structures described in the previous section.

Tax model

If the social planner chooses the tax model ($k = T$), she takes the responsibility of core take-back and imposes a unit tax on the manufacturer for each product sold.

In this case the objective function of the manufacturer simplifies to;[2]

$$\max_{q} \quad \Pi_M = (p - \mu - \tau)q \tag{3.5}$$

$$\text{where} \qquad q = 1 - p \tag{3.6}$$

It is easy to see that the above objective function is concave in q and by backward induction the optimal quantity for the manufacturer, q^*, can be written as $\frac{(1-\mu-\tau)}{2}$. Given the optimal quantity, the objective of the social planner simplifies to;

$$\max_{c,\tau} \quad SW = \Pi_E + \Pi_M + \Pi_C + \tau\, q - \chi\, c\, q \tag{3.7}$$

$$s.t. \qquad 0 \leq c \leq 1 \tag{3.8}$$

$$0 \leq \tau < 1 - \mu \tag{3.9}$$

where

$$\begin{aligned}
\Pi_E &= -\epsilon(1-c)(1-\mu-\tau)/2 \\
\Pi_M &= (1-\mu-\tau)^2/4 \\
\Pi_C &= (1-\mu-\tau)^2/8
\end{aligned}$$

[2]In the rest of the chapter, when we provide the solution of each model, we skip the legislation structure identifier ($k \in T, R$) used in front of SW, Π_M, Π_C and Π_E for the sake of simplicity.

The optimal solution to the model is summarized in Proposition 4.

Proposition 4 *Under the tax model, the optimal decisions of the social planner are as given in Table 3.2. The optimal take-back rate is always chosen at its upper bound* 1 *when the take-back cost is lower than the environmental cost and at its lower bound* 0 *otherwise. In case of perfect take-back, optimal tax increases with unit take-back cost* (χ) *while it increases with the unit environmental cost* (ϵ) *in case of zero take-back.*

Proof. All proofs are provided in the Appendix-B.1. ■

	$\epsilon \leq \chi$		$\epsilon > \chi$	
	$0 < \epsilon \leq \frac{1-\mu}{2}$	$\frac{1-\mu}{2} < \epsilon < 1-\mu$	$0 < \chi \leq \frac{1-\mu}{2}$	$\frac{1-\mu}{2} < \chi < 1-\mu$
c	0	0	1	1
τ	0	$-1+2\epsilon+\mu$	0	$-1+2\chi+\mu$
q	$\frac{1-\mu}{2}$	$1-\mu-\epsilon$	$\frac{1-\mu}{2}$	$1-\mu-\chi$
Π_M	$\frac{(1-\mu)^2}{4}$	$(1-\epsilon-\mu)^2$	$\frac{(1-\mu)^2}{4}$	$(1-\chi-\mu)^2$
Π_E	$-\frac{\epsilon(1-\mu)}{2}$	$-\epsilon(1-\epsilon-\mu)$	0	0
Π_C	$\frac{(1-\mu)^2}{8}$	$\frac{(1-\epsilon-\mu)^2}{2}$	$\frac{(1-\mu)^2}{8}$	$\frac{(1-\chi-\mu)^2}{2}$
SW	$\frac{1}{8}(\mu-1)(4\epsilon+3\mu-3)$	$\frac{(1-\epsilon-\mu)^2}{2}$	$\frac{1}{8}(\mu-1)(4\chi+3\mu-3)$	$\frac{(1-\chi-\mu)^2}{2}$

Table 3.2: Optimal solutions for the tax model under all possible parameter realizations

Optimal take-back rate always occurs at boundaries. Since the social planner aims to maximize the total social welfare and our model assumes a linear cost structure, it is intuitive to obtain a perfect take-back rate when environmental cost (ϵ) is higher than take-back cost (χ) and a zero take-back rate otherwise.

Optimal tax increases with either the environmental or the take-back cost. As long as ϵ is lower than or equal to χ, the social planner prefers zero take-back rate and optimal tax increases in ϵ. In this case, if $\epsilon > \frac{1-\mu}{2}$ (i.e the marginal profit at no take-back), the social planner imposes positive tax to reduce the sales quantity and increase Π_E. When ϵ is greater than χ, on the other hand, the optimal tax increases with χ and is always smaller than the take-back cost. This implies that the take-back cost is shared by the manufacturer and the social planner in this case. In both of the cases the social planner selects the optimal tax such that she always protects the manufacturer. The optimal tax is positive only if ϵ or χ is greater than $\frac{(1-\mu)}{2}$.

Take-back rate model

In this model, which corresponds to the Waste Electric and Electronic Equipment Directive (WEEE), the social planner leaves the take-back task to the manufacturer. The social planner decides on a target take-back rate and imposes this rate on the manufacturer. Given the target rate and the take-back cost, manufacturer only decides on sales quantity to maximize his profits.
The objective of the manufacturer simplifies to;

$$\max_{q} \quad \Pi_M = (p - \mu)\, q - \chi\, q\, c \tag{3.10}$$

$$\text{where} \qquad q = 1 - p \tag{3.11}$$

It is easy to see that the manufacturer's objective is concave in q and by backward induction, optimal quantity (q^*) is $\frac{1}{2}\,(1 - c\,\chi - \mu)$.
Given the optimal quantity, the objective of the social planner is written similar to Atasu et al. (2009) as;

$$\max_{c} \quad SW = \Pi_E + \Pi_M + \Pi_C \tag{3.12}$$

$$s.t. \qquad 0 \leq c \leq 1 \tag{3.13}$$

where

$$\begin{aligned}
\Pi_E &= -\frac{1}{2}(1-c)\epsilon(1-c\chi-\mu) \\
\Pi_C &= \frac{1}{8}(1-c\chi-\mu)^2 \\
\Pi_M &= \frac{1}{4}(1-c\chi-\mu)^2
\end{aligned}$$

Proposition 5 *The optimal decision of the social planner under the take-back rate model is as summarized in Table 3.3. The optimal take-back rate is increasing in unit environmental cost, ϵ.*

Optimal take-back rate is always increasing in the environmental cost and when

	$0 < \epsilon \leq \frac{3\chi(1-\mu)}{2(\chi+1-\mu)}$	$\frac{3\chi(1-\mu)}{2(\chi+1-\mu)} < \epsilon < \frac{3\chi}{2}$	$\frac{3\chi}{2} \leq \epsilon$
c	0	$\frac{3\chi(1-\mu)-2\epsilon(1-\mu+\chi)}{\chi(3\chi-4\epsilon)}$	1
q	$\frac{(1-\mu)}{2}$	$\frac{\epsilon(1-\mu-\chi)}{4\epsilon-3\chi}$	$\frac{(1-\chi-\mu)}{2}$
Π_M	$\frac{(1-\mu)^2}{4}$	$\frac{\epsilon^2(1-\mu-\chi)^2}{(3\chi-4\epsilon)^2}$	$\frac{(1-\chi-\mu)^2}{4}$
Π_E	$-\frac{\epsilon(1-\mu)}{2}$	$\frac{\epsilon^2(2\epsilon-3\chi)(1-\chi-\mu)^2}{\chi(3\chi-4\epsilon)^2}$	0
Π_C	$\frac{(1-\mu)^2}{8}$	$\frac{\epsilon^2(1-\mu-\chi)^2}{2(3\chi-4\epsilon)^2}$	$\frac{(1-\chi-\mu)^2}{8}$
SW	$\frac{3(1-\mu)^2}{8} - \frac{\epsilon(1-\mu)}{2}$	$\frac{\epsilon^2(1-\mu-\chi)^2}{2\chi(4\epsilon-3\chi)}$	$\frac{3(1-\chi-\mu)^2}{8}$

Table 3.3: Optimal solutions for the take-back rate model under all possible parameter realizations

the environmental cost is sufficiently higher than the take-back cost, take-back rate is set to a positive value in the optimal solution. Perfect take-back is required if ϵ is greater than $\frac{3\chi}{2}$. This implies that compared to the tax model, perfect take-back (i.e. the best environmental outcome) is possible at a higher environmental cost in this model.

Comparison of monopoly models

In this section, we compare the two legislative forms with respect to the social welfare, the manufacturer profit and the environmental impact. We do not make a separate comparison for the consumer surplus since in the monopoly case, consumer surplus is always half of the manufacturer profit and comparison for the consumer surplus will be no different from the comparison for the manufacturer profit. First, we specify the optimal manufacturer profit, the environmental impact and the social welfare for all possible value intervals of environmental cost (ϵ) and take-back cost (χ). Then, for every parameter realization we evaluate the models' relative efficiency from the perspective of the social planner, the manufacturer, and the environment to see when the incentives of different market figures are aligned. To ease understanding we use the same table format as in Table 3.2.

Corollary 1 *Table 3.4 shows the dominating model with respect to social welfare under all possible realizations of unit take-back cost (χ) and unit environmental cost (ϵ). The social planner is always better-off in the tax model except for the interval where ϵ is lower than or equal to both χ and $\frac{1-\mu}{2}$. Only in this case, where the environmental cost is low enough so that zero take-back is optimal in both legislative forms, the social planner is indifferent between the tax and the take-back rate models.*

$\epsilon \leq \chi$		$\chi < \epsilon < \frac{3(1-\mu)}{2}$		$\frac{3(1-\mu)}{2} \leq \epsilon$
$0 < \epsilon \leq \frac{1-\mu}{2}$	$\frac{1-\mu}{2} < \epsilon < 1-\mu$	$0 < \chi \leq \frac{1-\mu}{2}$	$\frac{1-\mu}{2} < \chi < 1-\mu$	$0 < \chi < 1-\mu$
$SW(T) = SW(R)$	$SW(T) > SW(R)$			

Table 3.4: The dominating models for the social planner under all possible realizations of take-back cost (χ) and environmental cost (ϵ)

When the environmental cost is lower than or equal to both the take-back cost and $\frac{1-\mu}{2}$, zero take-back is optimal for both the tax and the take-back rate models. This is the case where the environmental cost is sufficiently low that social planner takes back nothing in the tax model and imposes a zero take-back rate on the manufacturer in the rate model. In fact, this case is equivalent to the no legislation case under both legislative forms and two models are equivalent with respect to the social welfare.

When ϵ exceeds $\frac{1-\mu}{2}$ or when it is higher than χ, tax model always outperforms the rate model.[3] These results imply that except for the cases when the unit environmental cost is very low, the social planner is always better-off under the tax model. That is she can increase the social welfare by undertaking the take-back responsibility and imposing a certain tax on the manufacturer.

Corollary 2 *Similar to the social welfare case, Table[4] 3.5 summarizes the comparison of the models with respect to the manufacturer profit. Tax model is either the dominating or the equally favorable legislative form for the manufacturer except for the region where*

[3]Only if the unit environmental cost is equal to the unit take-back cost in the interval of $\frac{3\chi(1-\mu)}{2(\chi-\mu+1)} \leq \epsilon < 1-\mu$, two models are equivalent with respect to the social welfare.

[4]See Proofs of Corollary 1-4 in Appendix B.1.1 for ϵ^* and χ^*

$\chi \geq \epsilon$ *and* $\frac{1-\mu}{2} < \epsilon < \frac{3(1-\mu)}{4}$. *In this interval, the take-back rate model outperforms the tax model with either zero or interior optimal take-back rate (i.e.* $0 < c < 1$*).*

$\epsilon \leq \chi$				$\chi < \epsilon < \frac{3(1-\mu)}{2}$			$\frac{3(1-\mu)}{2} \leq \epsilon$
$\epsilon \leq \frac{1-\mu}{2}$	$\frac{1-\mu}{2} < \epsilon \leq \epsilon^*$	$\epsilon^* < \epsilon < \frac{3(1-\mu)}{4}$	$\frac{3(1-\mu)}{4} < \epsilon < 1-\mu$	$\chi < \chi^*$	$\chi^* \leq \chi \leq \frac{1-\mu}{2}$	$\frac{1-\mu}{2} < \chi < 1-\mu$	$0 < \chi < 1-\mu$
$\Pi_M(T) = \Pi_M(R)$	$\Pi_M(T) < \Pi_M(R)$		$\Pi_M(T) > \Pi_M(R)$		$\Pi_M(T) = \Pi_M(R)$	$\Pi_M(T) > \Pi_M(R)$	$\Pi_M(T) > \Pi_M(R)$

Table 3.5: The dominating models for the manufacturer under all possible realizations of take-back cost (χ) and environmental cost (ϵ)

Rate model is more profitable than the tax model for the manufacturer only if the unit take-back cost is higher than or equal to the unit environmental cost and the unit environmental cost is between the best possible profit margin and $\frac{3(1-\mu)}{4}$. In this interval zero take-back with positive tax is optimal for the tax model while either zero or partial take-back is optimal in the rate model. It is intuitive that rate model with zero take-back is more profitable for the manufacturer because manufacturer is not sharing the environmental cost with no take-back and can obtain the same profit as in a no legislation environment. In the same interval, rate model with partial take-back can still outperform the tax model as long as the environmental cost is small enough ($< \frac{3(1-\mu)}{4}$). In this case, take-back rate imposed by the social planner is sufficiently small so that the total take-back cost is lower than the total tax the manufacturer has to pay in the tax model.[5]

Equality between the two models is possible when zero take-back is optimal in the rate model and zero tax is optimal in the tax model. This case is actually equivalent to the no legislation case and observed in two intervals in Table 3.5; (1) in the region of $\epsilon \leq \chi$ when the environmental cost is low enough (i.e. $\epsilon \leq \frac{1-\mu}{2}$) to ensure zero take-back and zero tax; (2) in the region of $\epsilon > \chi$ when take-back and environmental costs are not high enough (i.e. $\chi^* \leq \chi < \epsilon \leq \frac{1-\mu}{2}$) to necessitate positive tax in the tax model and positive take-back rate in the rate model. Overall, the dominance of the tax model with respect to the manufacturer profit is obvious from the comparison in Table 3.5.

[5]If the unit environmental cost is equal to the unit take-back cost or $\frac{3(1-\mu)}{4}$ in the interval of $\epsilon^* < \epsilon < (1-\mu)$, two models are equivalent with respect to manufacturer profit since the total tax the manufacturer has to pay and the take-back cost he incurs are equal in this case.

Corollary 3 *Table 3.6 summarizes the comparison of the models with respect to the environmental impact. When either perfect take-back or zero take-back (with zero tax) is optimal under both models, two models are equivalent in terms of the total environmental impact. Only in a relatively restricted interval (i.e.* $\epsilon^\dagger < \epsilon < \chi < 1 - \mu$*),*[6] *the rate model outperforms the tax model. In all other cases, the tax model is always better than the rate model.*

$\epsilon \le \chi$			$\chi < \epsilon < \frac{3(1-\mu)}{2}$				$\frac{3(1-\mu)}{2} \le \epsilon$
$\epsilon \le \frac{1-\mu}{2}$	$\frac{1-\mu}{2} < \epsilon < \epsilon^\dagger$	$\epsilon^\dagger < \epsilon < 1-\mu$	$\chi \le \frac{2\epsilon}{3}$	$\frac{2\epsilon}{3} < \chi \le \frac{1-\mu}{2}$	$\frac{1-\mu}{2} < \chi \le \frac{2\epsilon}{3}$	$\frac{2\epsilon}{3} < \chi < 1-\mu$	
$\Pi_E(T) = \Pi_E(R)$	$\Pi_E(T) > \Pi_E(R)$	$\Pi_E(T) < \Pi_E(R)$	$\Pi_E(T) = \Pi_E(R)$	$\Pi_E(T) > \Pi_E(R)$	$\Pi_E(T) = \Pi_E(R)$	$\Pi_E(T) > \Pi_E(R)$	$\Pi_E(T) = \Pi_E(R)$

Table 3.6: The dominating models for the environment under all possible realizations of take-back cost (χ) and environmental cost (ϵ).

The dominance of the tax model with respect to the environmental impact is obvious from Table 3.6. Except for the intervals where two models are equivalent and a relatively small interval $\epsilon^\dagger < \epsilon < 1 - \mu$ where the rate model is better, the tax model always outperforms the rate model. In the region where the rate model dominates, partial take-back is optimal for the rate model while zero take-back with a positive tax is optimal for the tax model. In other words, in the tax model the social planner tries to reduce the total environmental cost by reducing the production with a positive tax while in the rate model she tries to do so by imposing partial take-back on the manufacturer. In this region, requiring the manufacturer to take back at least some portion of his used products is more effective than the taxing policy since the unit environmental cost is relatively higher and taxing policy cannot be effective enough in reducing the environmental cost. In contrast, in the region of (i.e. $\frac{1-\mu}{2} < \epsilon < \epsilon^\dagger$), with the same optimal solutions, tax model dominates the rate model because the unit environmental cost is relatively lower and taxing policy can be more effective.

Two models are equivalent when either perfect or zero take-back is optimal in both models. In the first case, there is no environmental cost while in the second one

[6]See Proofs of Corollary 1-4 in Appendix-B.1.1 for $\epsilon^\dagger$.

total environmental cost is maximum for both of the models and they are equivalent with respect to the environmental impact. We observe the equivalence when unit environmental cost (ϵ) is either sufficiently smaller or higher than unit take-back cost (χ).

Corollary 4 *Table 3.7 shows the intervals where the manufacturer and/or the environment are better off under rate model in contrast to the social planner who is always better off under tax model. As the table indicates when $\chi \geq \epsilon$ and $\frac{(1-\mu)}{2} < \epsilon < (1-\mu)$, at least for one of the stakeholders the rate model outperforms the tax model in contrast to the social planner.*

$\epsilon \leq \chi$				$\chi < \epsilon < \frac{3(1-\mu)}{2}$		$\frac{3(1-\mu)}{2} \leq \epsilon$
$\epsilon \leq \frac{1-\mu}{2}$	$\frac{1-\mu}{2} < \epsilon < \epsilon^{\dagger}$	$\epsilon^{\dagger} < \epsilon < \frac{3(1-\mu)}{4}$	$\frac{3(1-\mu)}{4} < \epsilon < 1-\mu$	$\chi \leq \frac{1-\mu}{2}$	$\frac{1-\mu}{2} < \chi < 1-\mu$	$0 < \chi < 1-\mu$
NONE	MANUFACTURER	MANUFACTURER ENVIRONMENT	ENVIRONMENT	NONE		

Table 3.7: Stakeholders who are better off under the rate model under all possible realizations of take-back cost (χ) and environmental cost (ϵ)

Tax model optimizes the incentives of both the manufacturer (the consumers) and the environment if $\epsilon > \chi$. This is the interval where perfect collection is always optimal in the tax model. On the other hand, if $\epsilon \leq \chi$, all stakeholders are better off under the tax model only when environmental cost is sufficiently low (i.e. $\epsilon \leq \frac{(1-\mu)}{2}$). As ϵ increases, first the manufacturer and, then the environment switch to the rate model. For relatively low ϵ the manufacturer is worse off with the tax model while for higher ϵ environment suffers under the tax model.

3.3.2 Competition Case

In this case, we consider a competitive environment to investigate the effects of competition on the relative efficiency of the three legislative models. We assume that there are n identical manufacturers in the market and they play a Cournot game to obtain their market shares.

Tax model under competition

If the social planner chooses the tax model, she takes the responsibility of the core take-back as in the base model and imposes a unit tax on manufacturers for each unit they introduce to the market.

In this case, the objective function of each manufacturer is written as;

$$\max_{q_i} \quad \Pi_{M_i} = (p - \mu - \tau)q_i \tag{3.14}$$

$$\text{where} \qquad p = 1 - \sum_{i=1}^{n} q_i \tag{3.15}$$

It is easy to see that this objective function is concave in q_i and by the first order conditions, the optimal sales quantity for manufacturer i (q_i^*) and the optimal price (p^*) are written as;

$$q_i^* = \frac{1 - \mu - \tau}{n+1} \quad \text{and} \quad p^* = 1 - \frac{n(1 - \mu - \tau)}{n+1}$$

Given this optimal quantity, the social planner's objective is;

$$\max_{c,\tau} \quad SW = \Pi_M + \Pi_C + \Pi_E + n\, q_i\, \tau - n\, q_i\, c\, \chi \tag{3.16}$$

$$\text{s.t.} \quad 0 \leq c \leq 1 \tag{3.17}$$

$$0 \leq \tau < 1 - \mu \tag{3.18}$$

where

$$\begin{aligned} \Pi_M &= \sum_{i=1}^{n} \Pi_{M_i} = \frac{n(\mu + \tau - 1)^2}{(n+1)^2} \\ \Pi_C &= \frac{n^2(\mu + \tau - 1)^2}{2(n+1)^2} \\ \Pi_E &= -\frac{\epsilon n(1-c)(-\mu - \tau + 1)}{n+1} \end{aligned}$$

The social planner's objective is concave in the unit tax (τ) and linear in the take-back rate (c) (see the proof of proposition 6 in Appendix-B.1.2). Hence, the optimal take-back rate always occurs at the boundary values (either 0 or 1) while the optimal unit tax may take interior values. The optimal solution of the model is summarized in Proposition 6.

Proposition 6 *The optimal decisions of the social planner are given in Table 3.8. As in the monopoly case, take-back rate is always chosen at its upper bound* 1 *when the unit take-back cost is lower than the unit environmental cost and at its lower bound* 0 *otherwise. Optimal tax, on the other hand, depends on the degree of competition (n). It is increasing in n, ϵ and χ.*

	$\epsilon \leq \chi$		$\epsilon > \chi$	
	$0 < \epsilon \leq \frac{1-\mu}{n+1}$	$\frac{1-\mu}{n+1} < \epsilon < 1-\mu$	$0 < \chi \leq \frac{1-\mu}{n+1}$	$\frac{1-\mu}{n+1} < \chi < 1-\mu$
c	0	0	1	1
τ	0	$\frac{-1+(n+1)\epsilon+\mu}{n}$	0	$\frac{-1+(n+1)\chi+\mu}{n}$
q	$\frac{n(1-\mu)}{n+1}$	$1-\mu-\epsilon$	$\frac{n(1-\mu)}{n+1}$	$1-\mu-\chi$
$n\Pi_{M_i}$	$\frac{n(1-\mu)^2}{(n+1)^2}$	$\frac{(1-\epsilon-\mu)^2}{n}$	$\frac{n(1-\mu)^2}{(n+1)^2}$	$\frac{(1-\chi-\mu)^2}{n}$
Π_E	$-\frac{n\epsilon(1-\mu)}{n+1}$	$-\epsilon(1-\epsilon-\mu)$	0	0
Π_C	$\frac{n^2(1-\mu)^2}{2(n+1)^2}$	$\frac{(1-\epsilon-\mu)^2}{2}$	$\frac{n^2(1-\mu)^2}{2(n+1)^2}$	$\frac{(1-\chi-\mu)^2}{2}$
SW	$\frac{(\mu-1)n(2\epsilon(n+1)+(\mu-1)(n+2))}{2(n+1)^2}$	$\frac{(1-\epsilon-\mu)^2}{2}$	$\frac{(\mu-1)n(2\chi(n+1)+(\mu-1)(n+2))}{2(n+1)^2}$	$\frac{(1-\chi-\mu)^2}{2}$

Table 3.8: Optimal solutions for the tax model under competition given the possible parameter realizations

As in the monopoly case, optimal take-back rate (c^*) always occurs at the boundaries. It is not affected by the degree of competition (n). However, competition affects the optimal unit tax selected by the social planner. Quantity competition leads to a price decline in the market which increases the number of consumers willing to purchase the product and the total sales quantity. An increase in sales also increases the total environmental impact and reduces the total social welfare. To compensate for this adverse effect, the social planner increases the unit tax and brings down the total sales to its monopoly level when environmental cost or take-back cost is sufficiently high. On the other hand, when these costs are not sufficiently high to necessitate a positive unit tax (which is equivalent to no legislation case), total sales quantity increases and price declines with competition.

As the degree of competition (n) increases, the social planner begins to impose positive tax at even lower ϵ and χ. As a result, as Table 3.8 shows, optimal tax increases under competition.

Corollary 5 *Both the profit of each manufacturer (Π_{M_i}) and the total manufacturer profit (Π_M) decreases with the degree of competition (n) while the social welfare is not affected when tax is positive and increases with n when tax is zero.*

When the unit environmental or take-back cost is higher than the best profit margin ($\frac{1-\mu}{n+1}$), the social planner increases the unit tax to keep the sales quantity at the monopoly level, and this leads to a decline in the sales quantity of each manufacturer. Hence, sales revenue of each manufacturer decreases with the degree of competition (n). Due to higher tax, the total manufacturer profit (Π_M) also decreases with n. In contrast, social welfare remains the same because the reduction in the manufacturer profit is compensated by the increase in the tax revenue and the environmental impact remain unchanged at the monopoly level.

On the other hand, when the unit environmental or take-back cost is lower than the best profit margin (i.e. $\epsilon(\chi) < \frac{1-\mu}{n+1}$), the social planner sets the unit tax to zero and Cournot competition leads to an increase in total sales quantity and a decrease in price. Nevertheless, the sales quantity of each manufacturer still decreases with n and, the profit of each manufacturer and the total manufacturer profit reduce with competition. On contrary, the social welfare increases with n because the reduction in the manufacturer profit and the increase in the environmental cost is compensated by the increase in the consumer surplus.

Take-back rate model under competition

In this case, the objective function of manufacturer i is given as;

$$\max_{q_i} \quad \Pi_{M_i} = (p - \mu)q_i - c\chi q_i \tag{3.19}$$

$$\text{where} \qquad p = 1 - \sum_{i=1}^{n} q_i \tag{3.20}$$

It is easy to see that Π_{M_i} is concave in q_i and $q_i^* = \frac{1-c\chi-\mu}{n+1}$ by the first order conditions. Given this, the social planner's problem is written as,

$$\max_{c} \quad SW = n\Pi_{M_i} + \Pi_C + \Pi_E \tag{3.21}$$

$$\text{s.t} \qquad 0 \leq c \leq 1 \tag{3.22}$$

where

$$\begin{aligned} n\Pi_{M_i} &= \frac{n(c\chi + \mu - 1)^2}{(n+1)^2} \\ \Pi_C &= \frac{(c\chi + \mu - 1)^2 n^2}{2(n+1)^2} \\ \Pi_E &= -\frac{\epsilon(c-1)(c\chi + \mu - 1)n}{n+1} \end{aligned}$$

Depending on the relation between the unit environmental cost (ϵ) and the unit take-back cost (χ), the objective function of the social planner can be linear, concave or convex in the take-back rate c (see the proof of Proposition 7 in Appendix B.1.1). Thus, optimal take-back rate (c^*) can take both boundary (0 or 1) and interior values in this case. Proposition 7 summarizes the optimal solution set for the take-back rate model under competition.

Proposition 7 *Table 3.9 summarizes the optimal decision of the social planner for the take-back rate model under competition. The optimal take-back rate increases with the degree of the competition (n) and the unit environmental cost (ϵ).*

	$0<\epsilon\leq\frac{(1-\mu)\chi}{1-\mu+\chi}(\frac{n+2}{n+1})$	$\frac{(1-\mu)\chi}{1-\mu+\chi}(\frac{n+2}{n+1})<\epsilon<\chi(\frac{n+2}{n+1})$	$\epsilon\geq\chi(\frac{n+2}{n+1})$
c	0	$\frac{\epsilon(\mu-1)(n+1)-\chi(\epsilon(n+1)+(\mu-1)(n+2))}{\chi(\chi(n+2)-2\epsilon(n+1))}$	1
q	$\frac{(1-\mu)n}{n+1}$	$\frac{\epsilon(\chi+\mu-1)n}{\chi(n+2)-2\epsilon(n+1)}$	$\frac{(-\chi-\mu+1)n}{n+1}$
$n\Pi_{M_i}$	$\frac{n(\mu-1)^2}{(n+1)^2}$	$\frac{n\epsilon^2(\chi+\mu-1)^2}{(\chi(n+2)-2\epsilon(n+1))^2}$	$\frac{n(\chi+\mu-1)^2}{(n+1)^2}$
Π_E	$\frac{\epsilon(\mu-1)n}{n+1}$	$\frac{\epsilon^2(\chi+\mu-1)^2n(n\epsilon+\epsilon-\chi(n+2))}{\chi(\chi(n+2)-2\epsilon(n+1))^2}$	0
Π_C	$\frac{(\mu-1)^2n^2}{2(n+1)^2}$	$\frac{\epsilon^2(\chi+\mu-1)^2n^2}{2(\chi(n+2)-2\epsilon(n+1))^2}$	$\frac{(\chi+\mu-1)^2n^2}{2(n+1)^2}$
SW	$\frac{(\mu-1)n(2\epsilon(n+1)+(\mu-1)(n+2))}{2(n+1)^2}$	$-\frac{\epsilon^2(\chi+\mu-1)^2n}{2\chi(\chi(n+2)-2\epsilon(n+1))}$	$\frac{(\chi+\mu-1)^2n(n+2)}{2(n+1)^2}$

Table 3.9: Optimal solutions for the take-back rate model under competition

Quantity competition leads to a price decline which increases the number of consumers willing to purchase the product and, thus the total sales quantity increases. However, an increase in sales also increases the total environmental cost and reduces the total social welfare. To compensate for this effect, the social planner imposes a positive take-back rate at lower levels of the unit environmental cost (ϵ) in a Cournot setting. The thresholds on the unit environmental cost for partial or perfect take-back decrease as the degree of competition (n) increases. Similarly, as n increases the social planner increases the optimal take-back rate in the partial take-back case; hence total environmental cost decreases (Π_E increases) with n in this case. Overall intense competition is beneficial for the environment. However, the same effect is not valid for the manufacturers. Increase in the degree of competition reduces the total manufacturer profit.

Comparison of models under competition

In this section, we compare the two legislative forms with respect to social welfare, total manufacturer profit, and environmental impact under competition. As in the monopoly case we do not make a separate comparison for consumer surplus since consumer surplus is always equal to a constant ($\frac{n}{2}$) times the total manufacturer profit and the comparison with respect to the consumer surplus will be the same as the comparison for total manufacturer profit. We investigate how competition affects the relative efficiency of the models for each stakeholder.

Corollary 6 *Table 3.10 shows the dominating model with respect to the social welfare under competition. Competition reinforces the dominance of the tax model. As the degree of competition (n) increases, the region where tax model dominates gets larger while the region where the two models are equivalent gets smaller.*

$\epsilon \leq \chi$		$\chi < \epsilon < \frac{(n+2)(1-\mu)}{n+1}$		$\frac{(n+2)(1-\mu)}{n+1} \leq \epsilon$
$0 < \epsilon \leq \frac{1-\mu}{n+1}$	$\frac{1-\mu}{n+1} < \epsilon < 1-\mu$	$0 < \chi \leq \frac{1-\mu}{n+1}$	$\frac{1-\mu}{n+1} < \chi < 1-\mu$	$0 < \chi < 1-\mu$
$SW(T) = SW(R)$	$SW(T) > SW(R)$			

Table 3.10: The dominating models for the social planner under competition

Corollary 7 *Table[7] 3.11 shows the dominating model with respect to the total manufacturer profit under competition. Similar to the social welfare case, competition reinforces the dominance of the tax model with respect to the manufacturer profit. As the degree of competition (n) increases, the regions where tax model dominates get larger while the regions where two models are equivalent get smaller.*

$\epsilon \leq \chi$				$\chi < \epsilon < \frac{(n+2)(1-\mu)}{(n+1)}$			$\frac{(n+2)(1-\mu)}{(n+1)} \leq \epsilon$
$\epsilon \leq \frac{1-\mu}{(n+1)}$	$\frac{1-\mu}{(n+1)} < \epsilon \leq \epsilon^{**}$	$\epsilon^{**} < \epsilon < \frac{(n+2)(1-\mu)}{2(n+1)}$	$\frac{(n+2)(1-\mu)}{2(n+1)} < \epsilon < 1-\mu$	$\chi < \chi^{**}$	$\chi^{**} \leq \chi \leq \frac{1-\mu}{(n+1)}$	$\frac{1-\mu}{(n+1)} < \chi < 1-\mu$	$0 < \chi < 1-\mu$
$\Pi_M(T) = \Pi_M(R)$	$\Pi_M(T) < \Pi_M(R)$		$\Pi_M(T) > \Pi_M(R)$		$\Pi_M(T) = \Pi_M(R)$	$\Pi_M(T) > \Pi_M(R)$	$\Pi_M(T) > \Pi_M(R)$

Table 3.11: The dominating models for the manufacturer under competition

Competition strengthens the dominance of the tax model with respect to both the social welfare and the manufacturer profit. To compensate for the sales increase due to quantity competition, the social planner applies more stringent legislative tools as the degree of competition (n) increases. Under intense competition the thresholds which require positive tax and positive take-back rate decreases, hence even at relatively lower environmental cost or take-back cost, there is non-zero tax in the tax model and either partial or perfect take-back in the rate model. In other words, it becomes more difficult

[7] See the proof of corollary 7 in Appendix B.1.2 for ϵ^{**} and χ^{**}.

for manufacturers to obtain no legislation case profit in both models. Consequently, the regions where the two models are equivalent (with respect to both the social welfare and the manufacturer profit) narrow down in favor of the regions where tax model dominates.

Corollary 8 *Table[8] 3.12 summarizes the comparison of the models with respect to the environmental impact under competition. As the degree of competition (n) increases, the dominance of the tax model weakens while the dominance of the rate model for the environment strengthens.*

$\epsilon \leq \chi$			$\chi < \epsilon < \frac{(n+2)(1-\mu)}{n+1}$				$\frac{(n+2)(1-\mu)}{n+1} \leq \epsilon$
$\epsilon \leq \frac{1-\mu}{n+1}$	$\frac{1-\mu}{n+1} < \epsilon < \epsilon^{\dagger\dagger}$	$\epsilon^{\dagger\dagger} < \epsilon < 1-\mu$	$\chi \leq \frac{(n+1)\epsilon}{(n+2)}$	$\frac{(n+1)\epsilon}{(n+2)} < \chi \leq \frac{1-\mu}{(n+1)}$	$\frac{1-\mu}{(n+1)} < \chi \leq \frac{(n+1)\epsilon}{(n+2)}$	$\frac{(n+1)\epsilon}{(n+2)} < \chi < 1-\mu$	
$\Pi_E(T) = \Pi_E(R)$	$\Pi_E(T) > \Pi_E(R)$	$\Pi_E(T) < \Pi_E(R)$	$\Pi_E(T) = \Pi_E(R)$	$\Pi_E(T) > \Pi_E(R)$	$\Pi_E(T) = \Pi_E(R)$	$\Pi_E(T) > \Pi_E(R)$	$\Pi_E(T) = \Pi_E(R)$

Table 3.12: The dominating models for the environment under competition

When unit take-back cost is lower than unit environmental cost, tax model requires perfect collection and is either the equivalently favorable or the dominating model for the environment. In this case, competition only enlarges the intervals where rate model also requires perfect collection and the regions where the two models are equivalent with zero environmental impact, get larger while the regions where tax model dominates get smaller.

On the other hand, when take-back cost is higher than the environmental cost, as n increases the equivalence region gets smaller, while the region where the tax model dominates and the region where the rate model dominates get larger. Hence, the dominance of the rate model for the environment increases under intense competition. Although the tax region also gets larger in this case, overall the dominance of the tax model weakens if we also consider the previous case.

Corollary 9 *Table 3.13 shows the regions where the manufacturer and/or the environment is better off under the rate model in contrast to the social planner who is always*

[8] See the proof of corollary 8 in Appendix-B.1.2 for $\epsilon^{\dagger\dagger}$.

better off under the tax model. Competition increases the misalignment of incentives between the social planner and the other stakeholders. As the degree of competition increases, the regions where for either the manufacturer or the environment the rate model outperforms the tax model get larger.

$\epsilon \leq \chi$				$\chi < \epsilon < \frac{(n+2)(1-\mu)}{n+1}$		$\frac{(n+2)(1-\mu)}{n+1} \leq \epsilon$
$\epsilon \leq \frac{1-\mu}{n+1}$	$\frac{1-\mu}{n+1} < \epsilon < \epsilon^{\dagger\dagger}$	$\epsilon^{\dagger\dagger} < \epsilon < \frac{(n+2)(1-\mu)}{2(n+1)}$	$\frac{(n+2)(1-\mu)}{2(n+1)} < \epsilon < 1-\mu$	$\chi \leq \frac{1-\mu}{n+1}$	$\frac{1-\mu}{n+1} < \chi < 1-\mu$	$0 < \chi < 1-\mu$
NONE	MANUFACTURER	MANUFACTURER ENVIRONMENT	ENVIRONMENT	NONE		

Table 3.13: Stakeholders who are better off with the rate model under competition

In Table 3.13, since $\frac{1-\mu}{n+1}$ decreases as n increases, the regions where at least one of the stakeholders is better off with the rate model get larger under intense competition. For the same reason, the region where the tax model dominates gets smaller. Under competition, misalignment is observed between the incentives of stakeholders at relatively lower environmental cost (ϵ). In previous sections we have discussed that competition strengthens the dominance of the rate model for the environment. Accordingly, in Table 3.13, we observe that as the degree of competition increases, the region where the environment is better off with the rate model gets larger. Similarly, the dominance of the rate model for the manufacturers also increases with n. As a result, misalignment between the preferences of the social planner and the other stakeholders extends as the degree of competition increases.

3.4 Conclusions

In this chapter, we investigate the structural efficiency differences between two common forms of take-back legislation, namely tax and rate models. We consider a Stackelberg setting where first the social planner decides on the legislative structure and then, given the choice of the social planner, each manufacturer decides on his own sales quantity to maximize his total profit. We measure the efficiency of the two models from the perspectives of different stakeholders (i.e. manufacturer(s), the social planner and the environment) in monopoly and competition settings. In the competition case, we assume n identical manufacturers competing in quantity.

As a result of our analysis, we observe that tax model is either the dominating or the equally favorable model for the social planner under both monopoly and competition cases. Competition does not affect the dominance of the tax model for the social planner. In contrast, for manufacturers and consumers, rate model can outperform the tax model especially when the unit environmental cost is less than the unit take-back cost but high enough to necessitate a positive tax. Similarly, the environment is also better off with the rate model in these settings. Moreover, the settings where the environment is better off with the rate model expands, as the intensity of the competition (the number of manufacturers competing, n) rises. Nevertheless, for manufacturers, consumers, and the social planner we cannot observe the same effect; instead competition reinforces the dominance of the tax model for these stakeholders. Competition also aggravates the misalignment between the incentives of the social planner and the other stakeholders. As the degree of competition increases the parameter regions where either the manufacturer or the environment is better off under the rate model get larger.

In conclusion, our findings show that social welfare can always be maximized with the tax model. Given this result, the interesting question is why some governments, especially those in Europe, still prefer the rate model (imposing target take-back rates and assigning the take-back task to manufacturers). One possible reason behind this choice can be the reluctance of the governments to deal with the collection and recovery tasks. The set up cost of collection and recovery systems and the possible externalities (i.e. education of consumers and promotion programs to publicize take-back schemes) can also act as disincentives. Similarly, in contrast to their complaints manufacturers seem to be better off under rate model under certain parameter settings. Again externalities like transaction costs or system set-up costs or simply their reluctance to deal with take-back and recovery operations, which are not included in this analysis, may be the underlying reason for this preference of manufacturers.

Chapter 4

REFURBISH vs HARVESTING DECISIONS OF A REMANUFACTURER

In previous chapters, as a significant motivator of product recovery we focused on environmental legislation and its implications on the recovery decisions of firms. Once the firms start product recovery and can make profit by processing and making use of their used products, more tactical issues come into the picture. At this stage, an important decision for a firm is the allocation of available cores among alternative recovery options, namely the disposition decision. That is the remanufacturers should decide what product recovery options to use for the available cores. Hence, in this chapter, we examine the disposition decisions of a firm with two alternative recovery options: refurbishing the core and selling the refurbished product at some fraction of the price for the new product or dismantling it and selling the harvested parts.

In practice, remanufacturers either prioritize one option over others and try to meet the demand of that option in the first place or make their disposition decisions according to the quality of returned products. Current practice of IBM, where the primary recovery option is refurbishing, is a good example of the former approach. After replacing the worn components and reloading the necessary software, the firm tries to sell the refurbished products through its web site for about a month, and at end of this period, salvage the remaining products with almost zero profit. Only if the available cores are more than the demand estimates for refurbished products is harvesting considered (Ferguson et al., 2009). Some other remanufacturers such as Pitney Bowes and Xerox, ReCellular Inc. (Guide and Van Wassenhove, 2003), on the other hand, use quality-based decision making. Particularly, they prefer to refurbish the highest quality cores while they dismantle the lower quality ones to harvest their parts. Both of these approaches, however, may lead to loss of possible profits especially when there is demand uncertainty. Particularly, if the returned products are of comparable quality (i.e. end-of-lease cores are generally at similar quality due to the predetermined terms of use and lifetimes) or if refurbishing and harvesting are of equal value for the remanufacturer, a quality based decision making may not be optimal. Similarly when demand for either parts or refurbished products is uncertain and excessive production from one option has to be salvaged at no profit, prioritizing one recovery alternative over the others may lead to a loss of possible revenue from the ignored alternative. For instance, a static policy which requires a certain amount of refurbishing without considering the actual demand may lead to loss of possible harvesting revenue, especially when the available cores are not sufficient to meet both refurbishing and harvesting demand. Hence, there is always an opportunity for a better decision making mechanism between the possible disposition alternatives.

In this chapter, we consider this problem, which is relevant to many remanufacturers, from a revenue management perspective. We argue that a dynamic approach based on bid price controls will help remanufacturers to cope with the demand-side uncertainty. Rather than making an allocation decision at the beginning of the planning period, we recalculate the optimal decision each time after a demand is received. When a demand is received for a particular recovery alternative (for either a refurbished product or a harvested part type), first we evaluate the opportunity cost of using a core for that alternative. We allocate the core for that alternative only if its current value for the remanufacturer is higher than the opportunity cost of the core. We obtain the opportunity costs from a Linear Programming (LP) model that considers the problem over a T period planning horizon. This helps to employ a dynamic approach without losing the long-term planning perspective. The use of a deterministic LP-based algorithm for determining the bid prices requires that the problem be frequently resolved to account for the uncertainty in demand. While the need for such frequent resolving sometimes prevents the implementation of this method for large problems, we show that our problem can be solved via an efficient solution procedure based on an alternative formulation of the LP model which makes it practical to resolve the problem after every demand arrival.

4.1 Literature Review

Our study relates to two separate streams of research; disposition decisions in product recovery systems and revenue management, particularly bid price controls. The first stream can be classified under the broader topic of Close-Loop Supply Chains (CLSC) which extends the traditional supply chains to the post-consumer life of products. In addition to the traditional supply-chain problems, CLSC is concerned with the collection of used products from the end-consumers (reverse logistics), inspection and sorting of them, selection and implementation of a suitable recovery alternative (disposition decision) and the distribution/selling of recovered products. Guide and Van Wassenhove (2003) provide a review of issues related to closed-loop supply chains and discuss its similarities and differences from traditional supply-chains. Fleischmann et al. (1997) and Dekker et al. (2003) cover a number of quantitative models proposed for CLSC problems. More recently, Sasikumar and Kannan (2008a), Sasikumar and Kannan (2009), Guide and Van Wassenhove (2009), Atasu et al. (2008) and Rubio et al. (2008) review the evolution of CLCS and product recovery research over the last decades. Among these, Sasikumar and Kannan (2008a), who provide the most comprehensive and the recent review specifically on product recovery and inventory management issues in recovery systems, point out that revenue management approach for recovered products is a noteworthy subject that has not yet received sufficient attention. In this research, by making use of bid price controls, an revenue management method, in disposition decisions, we hope, at least, to contribute to fill this gap in the literature.

The disposition decision for product returns has been discussed in a number of studies. Most of these studies propose that returns should be allocated among the recovery alternatives according to their quality levels (Fleischmann et al., 1997, Guide and Van Wassenhove, 2003, Guide et al., 2005 and Mitra, 2007). Guide et al. (2005) adopt a more strategic perspective and investigate the impact of disposition process design (centralized vs. decentralized). On the other hand, Guide et al. (2008) consider a capacitated remanufacturing facility and propose a two-step disposition policy taking into account the varying processing times of the returns and the time-sensitivity of remanufactured product prices. Fleischmann et al. (2003) examine the integration of returns as a new source of spare parts with the regular supply system of the company and propose an inventory control model to address this issue.

A few studies have focused on the disposition decision; Inderfurth et al. (2001) assume stochastic returns and demand in a make-to-stock system and propose an optimal control policy for allocating scarce returned products among multiple remanufacturing options under the assumption of linear allocation of returns. Kleber et al. (2002), on the other hand, consider deterministic returns and demand, and integrate forward production with remanufacturing in an environment where all demand should be met. Similar to Inderfurth et al. (2001), they propose optimal control rules for the allocation of returns among different alternatives. In contrast to these two studies, Ferguson et al. (2009) propose a disposition strategy based on expected opportunity cost of alternatives instead of seasonal fluctuations. They handle demand uncertainty through a stochastic dynamic optimization model and show that a capacity-based Revenue Management (RM) approach (Littlewood's model) to the disposition decision can significantly increase profits. We also propose an RM approach but there are important differences between our study and this paper; (i) while refurbishing is always the more profitable but riskier alternative in Ferguson et al. (2009), we do not make such an assumption. Instead we allow the value of harvesting to vary based on the demand for parts and thus take into account the cases where harvesting can be more or less profitable than refurbishing; (ii) our implementation is an example of a make-to-order (MTO) system while Ferguson et al. (2009) consider a make-to-stock (MTS) system and (iii) to take into account demand uncertainty, Ferguson et al. (2009) propose a stochastic optimization model which decides on the optimal disposition amounts based on expected demand at the beginning of the planning horizon. Unlike their static approach, we suggest a dynamic approach using bid price controls. In other words, rather than deciding at the beginning of the planning horizon over expected demand, we reevaluate the situation each time a demand is received and choose the best option accordingly.

Bid price controls were first studied by Simpson (1989) and then by Williamson (1992). Talluri and Van Ryzin (2004, chapters 2 and 3) provide a comprehensive explanation of the method for both single resource and network capacity control problems while Talluri and Van Ryzin (1998) discuss the theoretical foundations of the method in the context of a origin-destination (OD) control problem (a network RM problem). The authors conclude that bid prices are optimal only if the opportunity cost of each itinerary (a path on the network) is equal to the sum of opportunity costs of selling each leg separately; otherwise, bid prices are only asymptotically optimal as leg capacities and sales volume get larger. Since our problem is equivalent to the single resource capacity control problem, this condition holds for our problem. Comparing the performance of a deterministic LP (DLP) based and probabilistic non-linear programming (PNLP) based bid price approach, Talluri and Van Ryzin (1998) show that DLP outperforms PNLP especially when the marginal revenue (fare) variance within each (fare) class is low or zero. This is in fact the case in our model since we assume constant part/refurbished product prices for each period. A number of studies implemented bid-price controls to hotel room allocation and airline seat inventory control problems (Higle, 2007 and Goldman et al., 2002). Finally, Klein (2007) argues that using bid prices which are computed on forecasted demand by DLP and updated only a few times during the planning horizon will lead to inferior decisions. Since we update the bid-prices each time a demand is received, our decisions are not affected by disadvantages of outdated bid-prices.

4.2 Base Model Formulation

4.2.1 Problem Definition and Basic Assumptions

In this section, we define our problem and formulate our base model. We consider a remanufacturer who receives a known amount of returns (cores) each period and plans production over a horizon of T periods. Due to the reasons stated in the introduction section of this chapter, cores do not vary significantly in terms of their quality and, thus quality ranking is not a suitable criterion for the disposition decision. The firm has two options to recover value from an available core; refurbish and sell at a discount of a new unit's price or dismantle it and sell (or internally use) the harvested parts. A core includes multiple part types in varying amounts and all of these parts can be attained with hundred percent yield rate after dismantling.

The company works on a Remanufacture-To-Order basis and cores can be kept in stock for future use, however there is no inventory of refurbished products. There is a per period unit holding cost for cores inventory. We assume that total available cores over T periods is not enough to meet all demand (refurbished product and parts demand over T periods). As is the case for some remanufacturers like IBM, we consider a setting where the total demand for refurbishing and harvesting exceed the total available cores. Still, available cores in any particular period may exceed that period's total demand. In this case, the firm may keep some portion of the excessive cores in stock for future use if it is profitable (after subtracting the holding cost). If there are still remaining cores after sparing for inventory, he may just salvage them at a zero or negligible profit. For parts, there exists a certain amount of base stock for each type and the firm can keep parts inventory only as much as the base stock amount. Inventory exceeding the base stock is just discarded at the end of the period. Also the firm has the option of outsourcing parts from an external supplier but we assume that the cost of outsourcing to the firm is always higher than the part's selling price (Ferguson et al., 2009).

We do not include unit refurbishing and harvesting costs explicitly in our model. Since we assume that the quality of cores are similar, these costs will not vary much between cores and can be ignored without loss of generality. Still, if one would like to reflect these costs in the objective, unit prices can simply be seen as unit net profits.

Demand for both the parts and the refurbished products are uncertain and forecasted over the planning horizon. We formulate our LP model using expected demands but the formulation can handle demand uncertainty through a dynamic implementation described later. In the next section, first we give our notation in Table 4.1 and formulate our LP model, which we will call as Disposition Model (DM) from now on, as follows,

Indices	
i	product type index, $i = 1, ..., N$
j	part type index, $j = 1, ..., M$
t	time period, $t = 1, ..., T$
Parameters	
D^r_{it}	demand for refurbished product type i at period t
D^p_{jt}	demand for part type j at period t
B_{it}	quantity of core type i available at period t
p^r_{it}	per unit price for refurbished product type i at period t
p^p_{jt}	per unit price for part type j at period t
h_i	per unit per period inventory holding cost of core type i
c_{jt}	per unit cost of outsourcing part type j at period t
v_{it}	unit salvage value of cores for core type i at period t
a_{ij}	number of part type j that can be harvested from one unit of core type i
O_j	base stock level for part type j
I_j	initial inventory for part type j
Decision Variables	
x_{it}	amount of core type i refurbished in period t
z_{it}	amount of core type i harvested in period t
q_{jt}	amount of part type j sold in period t
s_{it}	amount of core type i salvaged in period t
b_{it}	inventory of core type i at the end of period t
o_{jt}	amount of part type j outsourced at the end of period t
Shadow Prices	
y^c_{it}	shadow price (dual variable) associated with constraint set (4.2) for product type i in period t
y^p_{jt}	shadow price (dual variable) associated with constraint set (4.3) for part type j

Table 4.1: Notation for the disposition model (DM)

(DM)

$$\max \quad \sum_{t=1}^{T}(\sum_{i=1}^{N} p^r_{it}x_{it} + \sum_{j=1}^{M} p^p_{jt}q_{jt} + \sum_{i=1}^{N} v_{it}s_{it} - \sum_{j=1}^{M} c_{jt}o_{jt} - \sum_{i=1}^{N} h_i b_{it}) \tag{4.1}$$

$$
\begin{aligned}
s.t.\quad b_{i(t-1)} + B_{it} - (z_{it} + x_{it} + s_{it}) - b_{it} = 0 \quad & i = 1,...,N; t = 1,...,T & (4.2)\\
\textstyle\sum_{i=1}^{N} a_{ij} z_{i1} - q_{j1} + o_{j1} \geq O_j - I_j \quad & j = 1,...,M & (4.3)\\
\textstyle\sum_{i=1}^{N} a_{ij} z_{it} - q_{jt} + o_{jt} \geq 0 \quad & j = 1,...,M; t = 2,...,T & (4.4)\\
q_{jt} \leq D^p_{jt} \quad & j = 1,...,M; t = 1,...,T & (4.5)\\
x_{it} \leq D^r_{it} \quad & i = 1,...,N; t = 1,...,T & (4.6)\\
x_{it}, z_{it}, q_{jt}, s_{it}, b_{it}, o_{jt} \geq 0 \quad & i = 1,..,N; j = 1,..M; t = 1,..T & (4.7)
\end{aligned}
$$

DM is formulated for multiple product types and can be used even if there is commonality between the parts of different core types. However, in our dynamic implementation and in the alternative formulation of the disposition problem we consider only one core type ($N = 1$) and thus skip the product type index i from our variables.

The objective function maximizes the net profit of the firm over a planning horizon of T periods. It is the sum of the revenue obtained from refurbished products, parts and salvaged cores minus the cores inventory holding cost and the parts' outsourcing cost. Constraint set (4.2) represents the cores inventory balance constraint. Constraints (4.3) and (4.4) are parts inventory balance constraints for the first period and the other periods, respectively. We structure the problem such that each period starts and ends with base stock amount of parts for each type. (i.e. for the first period $I_j = O_j$). However, in the dynamic implementation parts inventory I_j may differ from the base stock, since we solve the problem at every demand realization, as will be explained in the next section. Hence, $O_j - I_j$ may not always be equal to zero for the first period and we denote this period separately. Constraint set (4.3) and (4.4) ensure that the firm starts and ends each period with at least the base-stock amount of inventory for each part type. Any amount exceeding the base stock will not be carried to the next period and discarded. Constraints (4.5) and (4.6) guarantee that the amount of parts harvested and refurbished products sold are not more than the associated demands in each period. We solve our model using a rolling horizon.

In the next section, we present two approaches to use the solutions of our disposition model to help remanufacturers in their disposition decisions. The first approach is a static implementation, where we directly implement the solution of DM while the other is a dynamic implementation based on bid-price controls which make use of the DM solution only to determine the current bid prices.

4.2.2 Model Implementations

Static implementation

In our static implementation, DM is solved based on expected demands for each period and the optimal harvesting and refurbishing amounts for each period are determined at the beginning of the planning horizon. Then, this solution is implemented throughout the planning horizon regardless of the actual demand realizations. However, when the actual demand is not equal to the expected demand, this approach will lead to loss of potential revenues for the firm. Therefore, next we also suggest a dynamic implementation where we decide each time a new demand is received instead of deciding based on expected demand at the beginning of the planning horizon.

Dynamic implementation

In our dynamic implementation, we employ a revenue management approach based on bid-price controls. As a common method in hotel room or airline seat allocation problems, a bid price control sets a threshold price which generally denotes the unit opportunity cost of remaining capacity or time, and accepts a request only if its revenue exceeds this threshold (Talluri and Van Ryzin, 2004). We still make use of the DM solution; however instead of directly implementing this solution, we use it only to determine the current bid prices. Each time a demand is received for either a refurbished product or a part type, we resolve the model with the updated parameters (demand and available cores capacity) on a rolling horizon and determine the current shadow prices (or the dual variables) associated with the cores and the parts inventory balance constraints. These dual variables give the opportunity cost of using an available core or part and are taken as the bid prices in our context.

In our dynamic implementation, we consider only one core type, hence our analysis skip the product type index i. In fact, if there is no parts commonality between core types, the analysis we provide here, can easily be extended to include multiple cores since DM can be decomposed into N single core models under this assumption.

The procedure in our dynamic implementation can be summarized as follows;

Step 1: When an order is received, update the forecasted demand values in the right hand side of constraints (4.5) and (4.6) and resolve DM with the new values.

Step 2: Check whether the order is for a refurbished product or a part.

(i) If it is for a refurbished product, compare the price of the refurbished product (p_t^r) with the current shadow price (y_t^c) of the cores inventory balance constraint. If $p_t^r \geq y_t^c$, accept the demand and refurbish one core to meet it and update the core availability in DM. Otherwise, reject the demand.

(ii) If it is for a part, first check the current inventory of that part type. If the part's current inventory (I_j) is higher than its base stock level (O_j), then without dismantling, meet the demand from stock. Otherwise calculate the revenue from dismantling one core as;[1]

$$TR_{dismantle} = \sum_{j=1}^{M} p_{jt}^p \max(\min(O_j - I_j, a_j), 0)$$

- If $TR_{dismantle} > y_t^c$, dismantle a core and update the inventory quantities for all part types.[2] Resolve DM with updated levels for parts inventory. Compare the price of the part (p_{jt}^p) with the absolute value of the new shadow price (y_{jt}^p) of the associated parts inventory balance constraint.[3] If $p_{jt}^p \geq |y_{jt}^p|$, accept the order and update the inventory of this part. Otherwise reject the order.

[1]When calculating the revenue from dismantling a core we only consider parts that can be added to the inventory up to the base stock. Revenue from any parts exceeding the base stock is ignored in this calculation since these excessive parts are considered as redundant and discarded at the end of the period.

[2]Add the pieces obtained from dismantling to the inventory of the associated part type.

[3]We take the absolute value because these dual variables are always non-positive.

- If $TR_{dismantle} \leq y_t^c$, do not dismantle.[4] Compare the current bid price with the part price. Accept if $p_{jt}^p \geq |y_{jt}^p|$, and reject otherwise. If the order is met, update the inventory of this part.

Step 3: Stop if all of the available cores (B_t) are used up or the end of the period is reached. Otherwise go to Step 1.

At the end of the period, if the parts inventory is less than the base stock for any type, it is replenished (i) first by harvesting from the remaining cores, and (ii) if remaining cores are not sufficient, by outsourcing.

4.3 An Alternative Formulation

As discussed before, the need for frequent reoptimization of the LP model is one of the main disadvantages of the bid-price approach. In our dynamic implementation, we need to resolve DM frequently after every realization of demand. To avoid this drawback, we develop an efficient solution procedure that provides an optimal solution without resorting to an optimization software. To do so, we reformulate DM in the form of a transportation model. We name this formulation as TFDM to stand for transportation formulation of the disposition model.

4.3.1 Redefinition of Variables and Parameters

In TFDM, cores available in each period become our supplies. To reformulate the disposition problem in the form of a transportation model, we exploit the fact that the alternative disposition options (i.e. refurbished products, parts and salvage) share a common source of supply and redefine the demand for harvested parts as demand for dismantling a core for different part bundles. Part bundles refer to sets of parts like $\{P_1, P_2\}$, $\{P_2, P_3\}$ or $\{P_1, P_2, P_3\}$ where P_j for $j = 1, ..., M$ denotes a part. We use these part sets to redefine the individual part demands as aggregate dismantling demands. In any period, we only consider the part bundles with a positive demand. Hence, from

[4] Note that there is no need to resolve DM since no parameters are updated in this case.

all possible bundles we determine the smallest set of bundles with a positive demand.[5] Based on this, let $\bar{z}_{kt't}$ denote the amount of part bundle k demand met in period t from the cores of period t'. We determine the smallest set of bundles with a positive demand and the associated demands for each bundle in this set using *Parameter Derivation Procedure* (PDP) given below.

In PDP, first we determine all possible part bundles and their unit revenues. Unit revenue of a part bundle is the total revenue that can be obtained from the sale of all part pieces harvested from a core. Note that by dismantling a core we can obtain multiple pieces from each part type and to find the unit revenue of a part bundle we consider the revenue from all pieces of each part type included in the bundle. Given the revenues, we determine the demand of each part bundle according to the part type whose demand can be fully met with the smallest amount of harvesting. We start from the highest revenue part bundle and continue until the demands of all part bundles are determined. To clarify consider the following example;

Example 1 *Assume that there are three part types that can be harvested from a core and for some period t their respective prices and demands are given as $p^p_{1t} = 20$, $p^p_{2t} = 15$ and $p^p_{3t} = 15$ and $D^p_{1t} = 60$, $D^p_{2t} = 40$ and $D^p_{3t} = 30$. Furthermore, $a_1 = 2$, $a_2 = 2$ and $a_3 = 3$. In this case, the set of all possible part bundles (E) is written as,*

$$E = \{b1 = \{1\}, b2 = \{2\}, b3 = \{3\}, b4 = \{1, 2\}, b5 = \{1, 3\}, b6 = \{2, 3\}, b7 = \{1, 2, 3\}\}$$

The unit revenues for these bundles are calculated as $b1 = 20, b2 = 15, b3 = 15, b4 = 70, b5 = 85, b6 = 75$ and $b7 = 115$. Part bundle 7 has the highest unit revenue and we calculate its demand as follows;

$$D_{(b7)\,t} = \min_{j \in J_{b7}} \left(\frac{D^p_{jt}}{a_j}\right) = \min(60/2, 40/2, 30/3) = 10$$

where J_{b7} is the set of parts in bundle 7. In this case, part 3 requires the smallest amount of dismantling and it determines the demand of part bundle 7.

[5]The number of all possible part bundles is equal to the number of all subsets (except the zero subset) of an M-element set ($2^M - 1$) where M is the number of part types.

In the next step, we update the set E by eliminating all the bundles with part 3; $E = \{b1 = \{1\}, b2 = \{2\}, b4 = \{1, 2\}\}$ and recalculate the demand for each part as $D^p_{1t} = D^p_{1t} - (10 \times a_1) = 60 - 20 = 40$ and $D^p_{2t} = D^p_{2t} - (10 \times a_2) = 40 - 20 = 20$. We repeat the procedure until all bundles in E are eliminated and we find all the bundles with a positive demand as; $b4$ and $D_{(b4)\,t} = 10$; and $b1$ and $D_{(b1)\,t} = 10$.

We do not need the cores inventory variables in TFDM. Instead, we explicitly include the decision variables denoting the case of meeting one period's demand from the previous period's cores as in the the transportation formulation of production planning problems. For instance, $\bar{x}_{t't}$ denotes the amount of refurbished product demand met in period t from the cores of period t'. Finally similar to DM, salvage variables are defined as $\bar{s}_{t'}$ to denote the amount of cores salvaged in period t'.

PDP formally states the derivation of the parameters in TFDM from the original parameters in DM as explained above. Table 4.2 summarizes the notation for TFDM.

Parameter Derivation Procedure (PDP):

Step 0: Let E be the set of all possible bundles of parts that can be obtained from a core and S^t be an empty set. For all bundles $k \in E$, calculate the unit revenue (p^b_{kt}) by $\sum_{j \in J_k} a_j p^p_{jt}$, where J_k denotes the set of parts in part bundle k.

Step 1: Take the highest revenue part bundle. Let this be k^* and add bundle k^* to set S^t. Determine the demand for bundle k^* as;

$$D^b_{k^*t} = \min_{j \in J_{k^*}} (D^p_{jt}/a_j)$$

Step 2: Let j^* be the part type with $\min_{j \in J_{k^*}} (D^p_{jt}/a_j)$. Eliminate all part bundles with part j^* from the set E and update the demand for each part by $D^p_{jt} = D^p_{jt} - a_j D^b_{k^*t}$.

Step 3: If set E is empty, stop. Otherwise, go to Step 1.

S^t gives the smallest set of bundles with a positive demand in period t. We repeat the procedure for all $t = 1, ..., T$ to find this set and the associated revenue of each bundle

Indices	
k	parts bundle index, $k \in S^t$ where S^t denotes the set of feasible bundles in each period t
t' and t	time period index, $t' = 1, ..., T$ and $t = t'...T$
Parameters	
D_t^r	demand for the refurbished product at period t
D_{kt}^b	demand for part bundle k at period t
B_t	quantity of available cores at period t
$p_{t't}^r$	per unit revenue from meeting a refurbished product order in period t from the cores of period t'
$p_{kt't}^b$	per unit revenue from meeting part bundle k demand in period t from the cores of period t'
$v_{t'}$	per unit revenue of salvaging in period t'
h	per unit per period inventory holding cost of a core, $h = h_1$
a_j	amount of part type j that can be obtained by dismantling a core, $a_j = a_{1j}$ for all j
Variables	
$\bar{x}_{t't}$	amount of refurbished product demand met in period t from the cores of period t'.
$\bar{z}_{kt't}$	amount of part bundle k demand met in period t from the cores of period t'.
$\bar{s}_{t'}$	amount of cores salvaged in period t'.
Shadow Prices	
u_t	shadow price (dual variable) associated with constraint set (4.9) in period t.
w_{dt}	shadow price (dual variable) associated with constraint sets (4.10-4.12) in period t.

Table 4.2: Notation for the transportation formulation of the disposition model (TFDM)

in it for each period. Given p_t^r and p_{kt}^b, $p_{t't}^r = p_t^r - h(t-t')$ and $p_{kt't}^b = p_{kt}^b - h(t-t')$.

In DM, we have written the right hand sides (RHS) of constraints (4.3) as $O_j - I_j$ stating that they are always zero since the firm starts and ends each period with the base stock for each part. However; during the dynamic implementation, within a period these RHSs may become negative or positive. In other words, the inventory of part j may be higher or lower than its base stock at some time. When this occurs, we need to make some adjustments to make these RHSs zero before we reformulate DM as a transportation model (see Appendix-C.1 for these adjustments). As Theorem 1 given below states, DM and TFDM are equivalent in terms of finding the optimal solution for the disposition problem, when $O_j - Ij = 0$. When $O_j - I_j > 0$, on the other hand, the adjustments needed to restore the right hand of $O_j - I_j$ to 0 may hurt the equivalency of the two problems. Nevertheless, TFDM can still be used to find the shadow prices for the adjusted problem and these shadow prices can be used as the bid prices in the dynamic implementation. The difference that may arise between the two dynamic implementations (using DM versus using TFDM) due to these adjustments needs to be tested experimentally which is beyond the scope of this study and will be considered in further research.

Given this setting, the reformulation of the disposition problem as a transportation model is as follows,

(TFDM)

$$\max \sum_{t'=1}^{T} \sum_{t=t'}^{T} (\bar{x}_{t't}\, p^r_{t't}) + \sum_{k \in S^t} \sum_{t'=1}^{T} \sum_{t=t'}^{T} (\bar{z}_{kt't}\, p^b_{kt't}) + \sum_{t'=1}^{T} (\bar{s}_{t'}\, v_{t'}) \tag{4.8}$$

$$s.t. \quad \textstyle\sum_{t=t'}^{T} (\bar{x}_{t't}) + \sum_{k \in S^t} \sum_{t=t'}^{T} (\bar{z}_{kt't}) + \bar{s}_{t'} = B_{t'} \qquad t' = 1, ..., T \tag{4.9}$$

$$\sum_{t'=1}^{t} \bar{x}_{t't} \le D^r_t \qquad t = 1, ..., T \tag{4.10}$$

$$\sum_{t'=1}^{t} \bar{z}_{kt't} \le D^b_{kt} \qquad t = 1, ..., T \text{ and } \forall k \in S^t \tag{4.11}$$

$$\sum_{t'=1}^{T} \bar{s}_{t'} \le \sum_{t'=1}^{T} B_{t'} \tag{4.12}$$

$$\bar{x}_{t't}, \bar{z}_{kt't}, \bar{s}_{t'} \ge 0 \qquad t = 1, ..., T \text{ and } \forall k \in S^t \tag{4.13}$$

The objective function maximizes the total revenue from the sale of refurbished products and parts, and salvaging. Constraints (4.9) are the cores supply constraints equivalent to supply constraints in a standard transportation model. Constraints (4.10), (4.11) and (4.12) are the demand constraints for refurbished products, part bundles and salvaging respectively. The firm can salvage at most as much as the available cores in each period. Hence, we take $B_{t'}$ as the right hand sides of constraint (4.12).

Theorem 1 *The optimal solution of TFDM is also optimal for DM when $O_j - I_j = 0$.*

Proof. See Appendix-C.2 for all proofs. ■

4.3.2 An Optimal Solution Procedure for TFDM

In this section, we provide an efficient procedure (TPP) to obtain an optimal solution for TFDM. First we explain the basic idea underlying our procedure and then we give the formal statement.

The firm has three options to use the available cores; refurbished product demand, part bundles demand, and salvaging. In any period t, the firm can either meet the demand of an option for that period, or for future periods (i.e. $t+1$, $t+2$,...,T) by carrying some cores from period t.

In our solution procedure, given the unit revenue of all alternative demand options and the total available cores in each period, we start from the last period. We choose the highest revenue option and meet its demand as much as we can from the available cores in that period. Then, if there are still cores left, we consider the second highest revenue option and try to meet its demand. We continue in this manner until all the available cores for the period are used. Then, we pass to the previous period and repeat the same procedure. We continue until we complete the allocation in all periods.

<u>Formal Statement of TPP:</u>

Step 0: Start from the last period (T). Set $t = T$.

Step 1: Choose the highest revenue option with a positive demand which can be met from period t. Let it be h^* and its demand is D_{h^*}. Allocate $\min(D_{h^*}, B_t)$ amount of cores to this option and update both its demand and the available cores.

Step 2: Still if there are available cores left ($B_t > 0$), go to step 1. Otherwise, set $t = t - 1$. If $t = 0$, stop, if not, go to step 1.

To clarify consider the transportation tableau[6] 4.3; to balance the total demand and the total supply (available cores) we add a dummy supply in the amount of $\sum_{t=1}^{T}(D_t^r + D_{kt}^b + B_t) - \sum_{t=1}^{T} B_t$ since we assume that the total amount of available cores is less than total demand. As in a standard transportation tableau we denote the supply (cores) for each period in rows and the demand for each option in each period in columns of the tableau. Given the transportation tableau, TPP can be restated as,

	Period 1				...	Period T					Avail. Cores
	RfPr.	B-1	...	B-M	...	RfPr.	B-1	...	B-M	Salv.	
Dummy	$\bar{x}_{d1}$	$\bar{z}_{1d1}$	$\bar{z}_{kd1}$	$\bar{z}_{Md1}$	...	$\bar{x}_{dT}$	$\bar{z}_{1dT}$	$\bar{z}_{kdT}$	$\bar{z}_{MdT}$	s_d	$\sum_{t=1}^{T}(D_t^r + D_{kt}^b)$
Period1	$\bar{x}_{11}$	$\bar{z}_{111}$	$\bar{z}_{k11}$	$\bar{z}_{M11}$	...	$\bar{x}_{1T}$	$\bar{z}_{11T}$	$\bar{z}_{k1T}$	$\bar{z}_{M1T}$	s_1	B_1
Period2	NA	NA	NA	NA	...	$\bar{x}_{2T}$	$\bar{z}_{12T}$	$\bar{z}_{k2T}$	$\bar{z}_{M2T}$	s_2	B_2
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
PeriodT	NA	NA	NA	NA	...	$\bar{x}_{TT}$	$\bar{z}_{1TT}$	$\bar{z}_{kTT}$	$\bar{z}_{MTT}$	s_T	B_T
Demand	D_1^r	D_{11}^b	D_{k1}^b	D_{M1}^b	...	D_T^r	D_{1T}^b	D_{kT}^b	D_{MT}^b	$\sum_t B_t$	

Table 4.3: Example transportation tableau

Step 0: Start from the last row (period T).

Step 1: Going across the columns that are not crossed out, choose the highest revenue cell in this row, allocate the minimum of the row and the column totals to this cell.

Step 2: Update the row and column totals accordingly and cross out either the row or the column which drops to zero. Do not cross out both if they both become zero. Stop if exactly one row or column is left uncrossed. Otherwise, continue with Step 3.

[6] RfPr. stands for refurbished product, B-M for part bundle type M and Salv. for salvaging in the table. Cells denoted by NA are not feasible.

Step 3: If a row is crossed out in Step 2, proceed to the previous row and return to Step 1. If a column is crossed out, remain in the same row and return to Step 1.

Our solution procedure generates a basic feasible solution for TFDM (see Dantzig, 1998). Given a solution by TPP, cells with an entry form the set of basic variables and the cells with no entry form the set of non-basic variables in the transportation tableau. Theorem 2 states the optimality of TPP. We prove the optimality of TPP by LP duality.

Theorem 2 *TPP provides the optimal solution of TFDM.*

Statement of the DM solution in terms of the TFDM solution

Given an optimal solution for TFDM, let us call it $S = (\bar{x}_{t't}, \bar{z}_{kt't}, \bar{s}_{t'})$, we can write the equivalent solution (S') for DM as follows;

$$
\begin{aligned}
x_t &= \sum_{t'=1}^{t} \bar{x}_{t't} && (4.14)\\
z_t &= \sum_{t'=1}^{t} \sum_{k \in S^t} \bar{z}_{kt't} && (4.15)\\
q_{jt} &= \sum_{t'=1}^{t} \sum_{k \in K_j^t} a_j \bar{z}_{kt't} \text{where } K_j^t \text{ denotes the set of feasible} && (4.16)\\
&\quad \text{bundles of period } t \text{ which include part } j \ (K_j^t \subset S^t \text{ for } \forall\ t \text{ and } j)\\
s_t &= \bar{s}_{t'} && (4.17)\\
b_t &= \sum_{t'=1}^{t} \sum_{l=t+1}^{T} \bar{x}_{t'l} + \sum_{t'=1}^{t} \sum_{l=t+1}^{T} \sum_{k \in S^l} \bar{z}_{kt'l} && (4.18)\\
o_{jt} &= 0 \text{ since we assume } c_{jt} > p_{jt}^p \text{ for all } j \text{ and } t \text{ and, we consider} && (4.19)\\
&\quad \text{the case where } O_j - I_j = 0 \text{ for all } t
\end{aligned}
$$

In the dynamic implementation we use the dual variables associated with the cores and the parts inventory balance constraints of DM; namely y_t^c and y_{jt}^p. As it will become clear in the next section, we run the dynamic implementation only for the first period, thus we need the dual variables associated with only the first period (y_1^c and y_{j1}^p). We define these dual variables directly from the solution found by TPP. In the definition of these dual variables we use the dual variable associated with the cores supply constraint of TFDM for the first period (u_1). The proof of Theorem 2 in Appendix-C.2 explains how we determine the dual variables associated with the TFDM constraints. Given u_1 and the solution S', we determine y_1^c and y_{j1}^p as follows;

(i) $y_1^c = u_1$

(ii) The value of y_{j1}^p depends on whether the constraints (4.3) are binding or not for $N = 1$,

   (a) If $a_j z_1 - q_{j1} + o_{j1} \geq O_j - I_j$ then $y_{j1}^p = 0$

   (b) If $a_j z_1 - q_{j1} + o_{j1} = O_j - I_j$ then $y_{j1}^p = \max\left(\frac{\sum_{h \neq j} a_h p_{ht}^p - u_1}{a_j}, -p_{j1}^p\right)$ where h denotes parts different from part j and for which $D_{h1}^p > q_{h1}$.

4.4 Numerical Analysis

In this section, we conduct an extensive numerical analysis to evaluate the performance of bid price approach to the disposition decisions of remanufacturers. We compare our dynamic implementation which makes use of bid price controls with our static implementation which simply implements the DM solution over an experimental set defined below. The objectives of this analysis are; (1) to assess the contribution of bid price approach in terms of handling demand uncertainty, (2) to determine the aspects (e.g., total revenue, unmet demand) the dynamic implementation significantly outperforms the static implementation and, (3) to determine under which parameter settings the discrepancy between the two approaches gets larger.

4.4.1 Experimental Design

We consider a planning horizon of 6 periods ($T = 6$) where a period is approximately equal to a month. We assume a single core type ($N = 1$) with 3 different part types ($M = 3$) that can be harvested from this core. The amount of parts that can be obtained by dismantling one core is given as $a_1 = 2$, $a_2 = 2$ and $a_3 = 3$.

For the unit price of refurbished products, we scale down the value used in Ferguson et al. (2009) by 10% and assume that $p_t^r = 100$ for all t. A constant price over the planning horizon is realistic since our planning horizon is not long enough for price changes in this industry. Similarly, we take the part prices as $p_1^p = 20$, $p_2^p = 15$ and $p_3^p = 15$. In our problem, outsourcing the parts is not preferable and always generates a loss for the firm, hence we assume that the unit costs of outsourcing parts are greater than their selling prices. We take outsourcing costs as 1.5 fold of the selling prices. Per period unit holding cost of cores is considered as 10% of the refurbished product price ($h = 10$) in our experimental set. Finally, we assume that salvage value (v_t) is 0 for all periods. For the rest of the parameters, we ran a full factorial experimental design. We consider 24 scenarios based on,

1. Refurbished product demand pattern: We consider three patterns, namely *stable, non-stationary, seasonal*, for refurbished product demand.

2. Parts demand pattern: We assume two patterns, *increasing and stationary*, for parts demand over the planning horizon.

3. The ratio of total available cores to total expected demand: We consider two cases, Total Cores (TC)/Total Expected Demand (TED)= 90% and TC/TED= 70%.

4. The ratio of total expected refurbishing demand to total expected dismantling demand: We take into account the cases of both total refurbishing demand being higher than the total dismantling demand and vice versa. In the first case, we assume that total expected refurbishing demand (TERD)/total expected dismantling (TEDD)= 6/4 and in the second case TERD/TEDD= 4/6.

We vary our scenarios mainly according to the demand patterns and the amount of available cores since we want to assess the contribution of bid price approach in handling demand uncertainty. For the expected refurbished product demand, we have scaled down the data in Denizel et al. (2009) which is very close to industrial data and represents three demand patterns as in our analysis. Table 4.4 shows the data we used for refurbished product demand.

	Stable	Non-stationary	Seasonal
Period 1	39	24	24
Period 2	38	24	28
Period 3	50	24	56
Period 4	36	34	61
Period 5	22	55	24
Period 6	31	55	23

Table 4.4: Refurbished product demand

Given the refurbishing demand, we consider two cases for dismantling demand; total expected dismantling demand is higher than total expected refurbishing demand (particularly $TEDD = TERD \times 6/4$) and total expected dismantling demand is lower than total refurbishing demand (particularly $TEDD = TERD \times 4/6$). We calculate the total demand for each part by multiplying $TEDD$ by the bill of materials values (a_j) and taking a certain percentage (100% for part 1, 85% for part 2 and 90% for part 3) of this amount to differentiate between the demand for different part types. In CLSC, parts demand is generally more predictable and less variable than the refurbished products' demand (Ferguson et al. 2009). Hence, we allocate the total parts demand either as equally among the periods (stationary pattern) or as increasing by only 10% from one period to the other (increasing pattern).

We examine the case where total available cores are not sufficient to meet all demand. Hence, in our experimental set, the amount of total available cores is always less than the total demand. We conduct our numerical analysis for the cases of both relatively abundant available cores, where $TC = 0.9 \times TED$, and relatively scarce amount of cores, where $TC = 0.7 \times TED$. For each of the scenarios we simulate the two approaches (static and dynamic policies) for the first period. A summary of scenarios is given in Table 4.5.

To run our experiments, we assume that demand arrivals for the refurbished product and the parts occur randomly according to a Poisson process with the rate of expected period demands (as given above) over the length of the simulation period. Since

	Refurbished Product Demand Pattern	Parts Demand Pattern	TC/TED	TERD/TEDD
Scenario-0	Stable	0.7	4/6	Stationary
Scenario-1	Stable	0.7	4/6	Increasing
Scenario-2	Stable	0.7	6/4	Stationary
Scenario-3	Stable	0.7	6/4	Increasing
Scenario-4	Stable	0.9	4/6	Stationary
Scenario-5	Stable	0.9	4/6	Increasing
Scenario-6	Stable	0.9	6/4	Stationary
Scenario-7	Stable	0.9	6/4	Increasing
Scenario-8	Non-stationary	0.7	4/6	Stationary
Scenario-9	Non-stationary	0.7	4/6	Increasing
Scenario-10	Non-stationary	0.7	6/4	Stationary
Scenario-11	Non-stationary	0.7	6/4	Increasing
Scenario-12	Non-stationary	0.9	4/6	Stationary
Scenario-13	Non-stationary	0.9	4/6	Increasing
Scenario-14	Non-stationary	0.9	6/4	Stationary
Scenario-15	Non-stationary	0.9	6/4	Increasing
Scenario-16	Seasonal	0.7	4/6	Stationary
Scenario-17	Seasonal	0.7	4/6	Increasing
Scenario-18	Seasonal	0.7	6/4	Stationary
Scenario-19	Seasonal	0.7	6/4	Increasing
Scenario-20	Seasonal	0.9	4/6	Stationary
Scenario-21	Seasonal	0.9	4/6	Increasing
Scenario-22	Seasonal	0.9	6/4	Stationary
Scenario-23	Seasonal	0.9	6/4	Increasing

Table 4.5: Summary of scenarios

the static implementation allocates the available cores among alternatives according to the expected demand, in this implementation we may observe excessive or insufficient refurbished products or parts when actual demand is realized. In case of insufficient production of one alternative, the firm loses the profits from the unmet orders, hence is already penalized due to her poor decision making. In case of excessive production, on the other hand, we explicitly charge a penalty cost for each refurbished product or part exceeding the actual demand (or base stock in case of parts). We set the penalty costs as 30% of the associated parts' or refurbished product's selling price. Note that in the dynamic implementation we will never observe excessive refurbished products since the firm refurbishes a core only when there is demand for it. Nevertheless, there can be excessive parts because when a core is dismantled to meet the demand of a certain part type, pieces of other part types are also obtained and these may remain unsold at the end of the period. Hence, for the sake of comparability we also charge the penalty costs on the parts' inventory over the base stock in the dynamic implementation.

Given our experimental set, we consider a simulation period of 30 days and ran a total of 350 replications for each scenario. We determined the replication number based on a sufficiently narrow 95% confidence interval (Law, 2004). We specified the replication number considering the first scenario's average net revenue and stuck to it in other scenarios. We coded DM and our implementations in C++ within the MS Visual Studio 2008 environment in connection with the CPLEX 11.2 Callable Library. We conducted all computations on a Toshiba PC with Intel Celeron 560 2.13 GHz.

Appendix-C.3 provides the results for a sample of 50 replications under scenario 0 as well as the demand arrivals and the disposition decisions under dynamic implementation for one simulation period (30 days). First, in Tables-C.1-C.4, we give the solution and the net revenue of each implementation for 50 different demand realizations occurred in a Poisson process. Then, in Tables C.5-C.13 we provide a representative simulation period and the associated disposition decisions under dynamic implementation.

4.4.2 Comparison of Two Implementations

In this section, we compare the two policies with respect to the net revenue for all scenarios. We calculate the net revenue for the static and the dynamic implimentations as follows;

- For the static implementation, first we calculate the total revenue from sales of the refurbished products and parts and salvaging. In this calculation, as the actual sales amount for either products or parts, we use the minimum of the actual demand and the planned refurbishing/harvesting at the beginning of the period. Then, we subtract the cost of outsourcing and cores inventory, and the penalty cost for excessive parts and refurbished products from the total revenue to find the net revenue.

- For the dynamic implementation, first we find the total revenue by summing the revenue from each sale during the simulation period. In this implementation, we may need to replenish deficient parts by either outsourcing or harvesting remaining cores at the end of the period. After replenishment we decide whether to salvage or keep the remaining cores for future periods. To find the net revenue at the end of the period, we subtract the outsourcing cost, penalty cost for excessive parts and the holding cost of the final cores inventory from the total sales revenue and add salvaging revenue if there is any.

In each scenario, we compare the average net revenue of the two implementations by paired t-tests with a hypothesized difference of zero. Tables 4.6-4.7 show the results of our t-tests along with the mean revenue for each implementation and the percentage difference between the mean revenues. These results indicate that in all scenarios the net revenue of the dynamic implementation is significantly higher than that of the static implementation. This result is intuitive since in the dynamic implementation we consider demand uncertainty but in the static implementation we do not.

The percentage difference between the mean revenues varies among scenarios. Figure 4.1 shows the change of the average percentage difference with respect to the refurbished product demand patterns. As the figure indicates the difference increases as the refurbished demand variability increases.

Parts demand pattern also has a similar effect on the average revenue difference between the implementations. As Figure 4.2 shows the percentage difference between the net revenues is higher under increasing parts demand pattern compared to the stationary pattern.

	Scen0		Scen1		Scen2		Scen3		Scen4		Scen5	
	Dynamic	Static	Dynamic	Static	Dynamic	Static	Dynamic	Static	Dynamic	Static	Dynamic	Static
Mean	6888	6855	6756	6716	4435	4396	4368	4343	8591	8503	7803	7541
Variance	2857	16177	5191	18398	1542	12282	2219	8906	57018	117897	332921	258488
P($T \leq t$) one-tail	0.00		0.00		0.00		0.00		0.00		0.00	
P($T \leq t$) two-tail	0.00		0.00		0.00		0.00		0.00		0.00	
Difference %	0.48		0.61		0.89		0.59		1.03		3.46	
	Scen6		Scen7		Scen8		Scen9		Scen10		Scen11	
	Dynamic	Static	Dynamic	Static	Dynamic	Static	Dynamic	Static	Dynamic	Static	Dynamic	Static
Mean	5558	5514	5359	5293	6853	6828	6307	6136	4332	4269	3946	3787
Variance	43489	70853	110795	167440	22826	29964	159212	180765	46881	76819	219072	263702
P($T \leq t$) one-tail	0.00		0.00		0.00		0.00		0.00		0.00	
P($T \leq t$) two-tail	0.00		0.00		0.00		0.00		0.00		0.00	
Difference %	0.80		1.24		0.36		2.78		1.48		4.20	

Table 4.6: Comparison of revenues for scenarios 0-11

	Scen12		Scen13		Scen14		Scen15		Scen16		Scen17	
	Dynamic	Static	Dynamic	Static	Dynamic	Static	Dynamic	Static	Dynamic	Static	Dynamic	Static
Mean	7586	7296	6278	5957	4673	4387	4119	3807	6853	6832	6295	6049
Variance	344965	186413	358423	179022	260820	119154	283918	116786	21979	29117	170012	167236
P($T \leq t$) one-tail	0.00		0.00		0.00		0.00		0.00		0.00	
P($T \leq t$) two-tail	0.00		0.00		0.00		0.00		0.00		0.00	
Difference %	3.98		5.39		6.50		8.21		0.31		4.07	
	Scen18		Scen19		Scen20		Scen21		Scen22		Scen23	
	Dynamic	Static	Dynamic	Static	Dynamic	Static	Dynamic	Static	Dynamic	Static	Dynamic	Static
Mean	4331	4289	4004	3819	7571	7309	6285	5969	4629	4378	4057	3773
Variance	43781	57400	153399	146259	327239	169615	356126	173149	267160	120732	331560	157788
P($T \leq t$) one-tail	0.00		0.00		0.00		0.00		0.00		0.00	
P($T \leq t$) two-tail	0.00		0.00		0.00		0.00		0.00		0.00	
Difference %	0.97		4.84		3.58		5.30		5.74		7.51	

Table 4.7: Comparison of revenues for scenarios 12-23

Finally, Figure 4.3 shows the change of the percentage difference between the net revenues with respect to the available cores. From the figure we observe that the percentage difference increases as the amount of available cores increases.

4.5 Conclusions

In this chapter, we examine the disposition decisions of a remanufacturer who works on a remanufacturer-to-order basis under demand-side uncertainty. We consider three disposition alternatives namely refurbishing, dismantling and salvaging and investigate how the available cores should be allocated among these alternatives when their demands are uncertain and the quality levels of the cores are similar. In contrast to

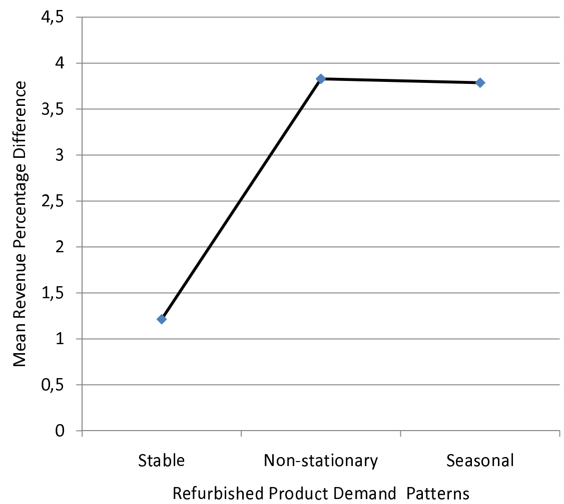

Figure 4.1: Refurbished product demand pattern vs. percentage difference between net revenues

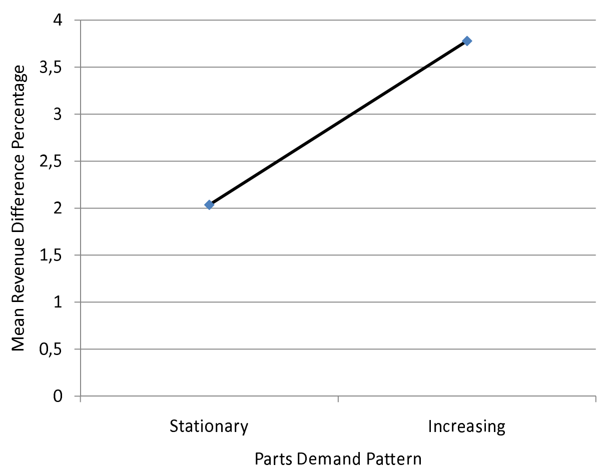

Figure 4.2: Part demand patterns vs. percentage difference between net revenues

some common approaches, which make the allocation based on expected demand at the beginning of the planning horizon, we propose a dynamic approach. To cope with demand-side uncertainty and avoid excessive or insufficient production, we employ bid price approach. To implement the approach and obtain the necessary bid prices (unit

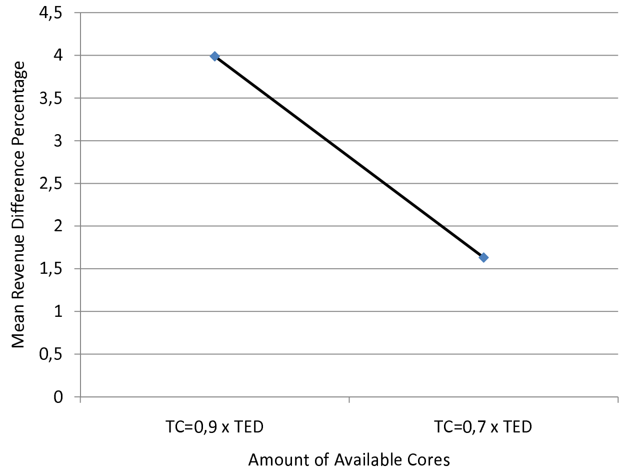

Figure 4.3: Core availability vs. percentage difference between net revenues

opportunity cost of an available core/part) we model the problem as an LP over T periods. We avoid the drawback of frequent reoptimizations required by the bid price method by reformulating the model as a transportation model and providing an efficient solution procedure for this formulation. In a large experimental set we compare our dynamic implementation with a static implementation which directly uses DM solution. As a result of our numerical analysis we observe that the dynamic implementation significantly outperforms the static one under all parameter settings and the discrepancy between the two approaches gets larger as the demand variability increases. Our results show that a dynamic approach, which can continually evaluate and balance the marginal revenue and opportunity cost of each disposition alternative, can serve remanufacturers as a better decision making mechanism than the current policies used in practice. We compare our dynamic implementation with respect to the static implementation of the DM solution which may also provide a close-to-optimal solution if demand uncertainty is not very high and still observe a significant difference. Hence, when compared to more simple approaches like priority ranking used in practice, the advantage of the dynamic implementation will become more prominent.

We consider the case of one core type. When there is no parts commonality between the core types, through a simple decomposition of the models all the analysis we have provided here can be used for multiple core types as well. In case of parts commonality, on the other hand, DM is still valid and can be used to implement the two approaches we suggest. In this case only the transportation formulation and its solution procedure will need some modifications. In the future studies, this formulation can also be extended to handle multiple core type cases.

Chapter 5

CONCLUSIONS

In this dissertation study, we focus on two interrelated decisions in the context of product recovery systems: at the strategic level the implications of the environmental legislation on the product recovery decisions of producers, and, at the tactical level the disposition decisions of a remanufacturer who is already engaged in product recovery.

In the field of environmental policy, first we address the recovery rate and the product recoverability decisions of a manufacturer under a legislative disposal fee. We take into consideration the initial investment costs for system set-up and investigate the impact of these costs on the manufacturer's decisions. Specifically, we examine how the recovery choices of the manufacturer may change when he is reluctant to or cannot entirely cover these initial expenditures. We conduct our analysis under both fixed and increasing unit disassembly and product recoverability improvement costs.

As a result of our analysis, we observe that the insufficiency of the allocated fund for the initial investments may have a significant impact on the optimal recovery decisions. Our results show that the initial expenditures and the lack of sufficient funds to cover them may even completely deter the manufacturer from starting product recovery. Hence, for effective environmental policies, these initial investment needs should be carefully taken into account and perhaps through proper subsidies their negative effects should be alleviated. The analysis in this chapter also reveals the importance of information about the recovery costs and savings and the redesign opportunities for relevant product groups. Our findings suggest that to be able to set an effective disposal fee, the policy maker should take into account these costs/savings.

In our second problem on environmental policy, we focus on the current implementations of product take-back legislation. To understand the impact of the structural variations in current legislation on the welfare of different stakeholders, we compare two common forms of product take-back legislation (tax and rate models) from the perspective of the social planner, the manufacturer(s) and the environment. We consider both monopoly and competition cases and find that regardless of the market setting, tax model is always the dominating legislative form for the social planner. Nevertheless, the same result does not seem to be true for the manufacturers or the consumers. Our results show that especially, when the unit environmental cost is lower than the unit take-back cost, rate model is more profitable for both the manufacturers and the environment. In other words, despite the complaints and the counterarguments of manufacturers, the rate model seems to serve better to their interests in certain parameter settings (i.e. when the unit environmental cost is less than the unit take-back cost but high enough to necessitate a positive tax). Competition reinforces the dominance of the tax model for all the stakeholders except for the environment. That is under intense competition rate model can serve better to reduce the environmental impact of used products than the tax model. Nevertheless, competition also aggravates the misalignment between incentives of the different stakeholders. Our results provide important implications for policy makers to assess the efficiency of each legislative form and design the most appropriate legislation structure in different settings. Still, there are a number issues like external costs (i.e. costs for promoting take-back programs, educating consumers and system set-up) which should be take into account in future research.

Finally, our last problem considers the disposition decisions of a remanufacturer with three disposition alternatives; namely refurbishing, dismantling and salvaging, and investigate how the available cores should be allocated among these alternatives under demand uncertainty. In contrast to some common approaches based on priority ranking and quality, to cope with demand-side uncertainty, we propose a dynamic approach where we employ a common revenue management method of bid price controls. We compare our dynamic implementation with a static implementation in a numerical analysis. Our findings show that the dynamic implementation significantly outperforms the static one under all parameter settings and the discrepancy between the two approaches gets larger as the demand variability increases. That is a dynamic approach, which continually evaluates and balances the marginal revenue and the opportunity cost of each disposition alternative, can generate more revenue than the current policies used in practice. In this dissertation, we consider the case of only one core type. Still, our analysis can be readily extended to handle the case of multiple core types when there is no part commonalities between the different core types. The case of part commonalities, on the other hand, can be the subject of further research. Moreover, in future research the dynamic approach we have provided here, can be compared with other approaches such as stochastic models.

Our findings provide important insights for both the policy makers and the firms who are already engaged in product recovery and can make profit from these operations. First, for policy makers or social planners, our analysis is useful in identifying the factors that may influence the effectiveness of environmental legislation under different settings as well as the efficiency implications of the current take-back legislation for various stakeholders in a product recovery system. For the manufacturers, on the other hand, our analysis in Chapter 4 suggests an efficient mechanism which can help in their disposition decisions.

Appendix A

A.1 Proofs of Chapter 2

Proof of Proposition 1. Π_M is linear in q_r, hence optimal q_r always occurs at boundaries. When we rearrange Π_M to see the coefficient of q_r;

$$\Pi_M = -d_f q + (-c - q + 1)q + (c\chi q + d_f q - c_d q)q_r$$

From this expression observe that when $c_d - c\chi \leq d_f$, Π_M is increasing in q_r, and optimal q_r (q_r^*) is 1. When $c_d - c\chi > d_f$, on the other hand, Π_M is decreasing in q_r and $q_r^* = 0$.

Π_M is concave in q and we find optimal q by the first order conditions;

$$\frac{\partial \Pi_M}{\partial q} = \chi q_r c - c - d_f - 2q - c_d q_r + d_f q_r + 1$$

$$\frac{\partial^2 \Pi_M}{\partial q^2} = -2 < 0$$

$$\frac{\partial \Pi_M}{\partial q} = 0 \quad \Leftrightarrow \quad q = \frac{1}{2}(d_f(q_r - 1) - c_d q_r + c(\chi q_r - 1) + 1)$$

We can summarize these results as below;

- $q^* = \frac{1}{2}(1 - c_d + c(\chi - 1))$ if $c_d - c\chi \leq d_f$
- $q^* = \frac{1}{2}(1 - c - d_f)$ if $c_r - c\chi > d_f$

Given these the optimal solution of the base model is given in Table 2.2.

■

Proof of Proposition 2. When the objective function (Π_M) is rearranged in terms of a, the coefficient of a is written as $q(q_r\sigma - c_r)$. Given this, when $q_r \geq \frac{c_r}{\sigma}$ ($q_r < \frac{c_r}{\sigma}$)

in the optimal solution, objective is increasing (decreasing) in a and optimal a is 1 (0). We solve the model for these two cases;

- When $a = 1$,
  - if $d_f \geq c_d - c\chi - \sigma$,

    $q_r = 1$

    $q = \frac{1}{2}(\chi c - c - c_d - c_r + \sigma + 1)$ and $p = \frac{1}{2}(-\chi c + c + c_d + c_r - \sigma + 1)$

    $\Pi_M = \frac{1}{4}(-\chi c + c + c_d + c_r - \sigma - 1)^2$

    Note that the condition of $d_f \geq c_d - c\chi - \sigma$ ensures that $\Pi_M(a = 1)$ is always increasing with q_r, hence q_r is set to its maximum value, 1. Given $a = 1$ and $q_r = 1$, we can easily find q as given above.
  - if $d_f < c_d - c\chi - \sigma$,

    $q_r = \frac{c_r}{\sigma}$

    $q = \frac{-c_d c_r + (c\chi + d_f)c_r - (c + d_f)\sigma + \sigma}{2\sigma}$ and $p = \frac{c_d c_r - (c\chi + d_f)c_r + (c + d_f + 1)\sigma}{2\sigma}$

    $\Pi_M = \frac{(c_d c_r - (c\chi + d_f)c_r + (c + d_f - 1)\sigma)^2}{4\sigma^2}$

    The condition of $d_f < c_d - c\chi - \sigma$ guarantees that $\Pi_M(a = 1)$ is always decreasing with q_r, hence q_r is set to its minimum value, $\frac{c_r}{\sigma}$, this time. Given $a = 1$ and $q_r = \frac{c_r}{\sigma}$, again we can easily find q as given above.
- When $a = 0$,
  - if $\frac{c_r(c_d - c\chi) + \sigma(c - 1)}{c_r - \sigma} > d_f \geq c_d - c\chi$,

    $q_r = \frac{c_r}{\sigma}$

    $q = \frac{-c_d c_r + (c\chi + d_f)c_r - (c + d_f)\sigma + \sigma}{2\sigma}$ and $p = \frac{c_d c_r - (c\chi + d_f)c_r + (c + d_f + 1)\sigma}{2\sigma}$

    $\Pi_M = \frac{(c_d c_r - (c\chi + d_f)c_r + (c + d_f - 1)\sigma)^2}{4\sigma^2}$

    Similar to the previous cases, the second condition on d_f ($d_f \geq c_d - c\chi$) ensures that $\Pi_M(a = 0)$ is increasing with q_r, hence q_r is set to its maximum value, which is $\frac{c_r}{\sigma}$ in this case. The first condition ($\frac{c_r(c_d - c\chi) + \sigma(c - 1)}{c_r - \sigma} > d_f$), on the other hand, requires q to be always nonnegative. Given $a = 0$ and $q_r = \frac{c_r}{\sigma}$ we find q as given above.

– if $d_f < c_d - c\chi$,

$q_r = 0$

$q = \frac{1}{2}(-c - d_f + 1)$ and $p = \frac{1+c+d_f}{2}$

$\Pi_M = \frac{1}{4}(c + d_f - 1)^2$

In the final case, the condition of $d_f < c_d - c\chi$ ensures that $\Pi_M(a = 0)$ is decreasing with q_r, hence q_r is 0. Given $a = 0$ and $q_r = 0$ we find q as given above.

Given these solution intervals, it is easy to see that when $d_f < c_d - c\chi - \sigma$, $\Pi_M(a = 0,\ q_r = 0)$ is always greater than $\Pi_M(a = 1,\ q_r = \frac{c_r}{\sigma})$. Similarly, when $d_f \geq c_d - c\chi$, solution of $\{a = 1, q_r = 1\}$ always outperforms $\{a = 0, q_r = \frac{c_r}{\sigma}\}$. This implies that interior solutions are always outperformed by the boundary solutions.
In the interval of $c_d - c\chi - \sigma \leq d_f < c_d - c\chi$, we compare the two boundary solutions;

- $a = 1$ and $q_r = 1$ is more profitable when $c_d - c\chi - \sigma + c_r \leq d_f < 2 - 2c + c\chi - c_r - c_d + \sigma$,
- $2 - 2c + c\chi - c_r - c_d + \sigma > c_d - c\chi$ and $c_d - c\chi - \sigma + c_r > c_d - c\chi - \sigma$.

Given above results we can conclude;

- When $c_d - c\chi - \sigma + c_r \leq d_f$, the optimal solution is $a = 1$, $q_m = 1$
- When $c_d - c\chi - \sigma + c_r > d_f$, the optimal solution is $a = 0$, $q_r = 0$

The optimal solution of the model is summarized in Table 2.3.

Note that since $\sigma > c_r$, $\sigma - c_r$ is always positive. Hence, in this model the threshold on the disposal fee $(c_d - c\chi - \sigma + c_r)$ for full recovery is smaller than the one in the base model. ■

Proof of Proposition 3. The lagrangian is written as;

$$L = q(p - c) - qq_r(c_d + \beta_{c_d} qq_r) - q(1 - q_r)d_f + sq_r q + \lambda(1 - q) + \mu q + \alpha(1 - q_r) + \rho q_r$$

where λ, μ, α, ρ are the lagrange multipliers. First order conditions are;

$$\frac{\partial L}{\partial q_r} = -\beta q_r q^2 + c\chi q + d_f q - (c_d + \beta qq_r)q - \alpha + \rho = 0$$

$$\frac{\partial L}{\partial q} = -\beta q q_r^2 + c\chi q_r - (c_d + \beta q q_r) q_r - c + \mu - \lambda - 2q - d_f(1 - q_r) + 1 = 0$$

Complementary slackness equations are;

$$\lambda(1-q) = 0 \qquad \mu q = 0 \qquad \alpha(1-q_r) = 0 \qquad \rho q_r = 0$$

Solving first order conditions and complementary slackness equations simultaneously and checking for dual and primal feasibility, we find the optimal solutions of the model as in Table 2.7. ■

A.2 Experimental Design for the Model LAFIC under Linear Cost Structure

We determined our parameter set through full factorial design with three values for d_f and two values for all the other parameters. To determine the values for the parameters, we considered the relations between the total cost of technology improvement and the total cost of recovery capacity investment or the total possible savings from recoverability improvement and tried to cover all possible cases.

We set values for F such that F is always smaller than the amount required to cover all the initial investments. To ensure this, we calculated the amount of funds that would be required to cover all the initial investment costs of the optimal recovery rate and the optimal product recoverability without a constraint and multiplied this amount by 0.9 and 0.3. In Table A.1, F^* denotes the amount of funds needed. Given the specifications mentioned in the chapter, our experimental set is given in Table A.1.

A.3 Regression and Chi-square Analysis for the Model IDR under Linear Cost Structure

We have given the analytical solution of the model IDR with linear costs in the chapter and used this solution for our discussions. To compare this model with the other models which we solved numerically, we also conducted a numeric study for this model. We used the same experimental set as in the previous cases. Table A.2 and A.3 summarize the results from this statistical analysis.

Parameters	Values
d_f	$0.12 - 0.25 - 0.37$
c_d	$0.22 - 0.40$
c	$0.25 - 0.45$
χ	$0.05 - 0.15$
c_r	$0.06 - 0.11$
σ	$0.12 - 0.17$
F	$F^* \times 0.3 - F^* \times 0.9$
γ	$0.07 - 0.10$
δ	$0.005 - 0.02$

Table A.1: Experimental set for the model LAFIC with linear costs

Parameters	Adjusted R^2
c	0.582
d_f	0.093
c_d	0.156

Table A.2: Adjusted R^2 values from simple regressions for the model IDR with linear costs

Parameters/Decision Variables	q_r	a	p
d_f	0.644	0.644	0.274
c	Not Signf	Not Signf.	0.789
c_d	0.462	0.462	0.451

Table A.3: Cramer's V coefficients from chi-square analysis for the model IDR with linear costs

Appendix B

B.1 Proofs of Chapter 3

B.1.1 Monopoly Case

Proof of Proposition 4. Given Π_E, Π_M and Π_C, the social planner's problem is written as,

$$\max_{\tau,c} \quad SW = \frac{3}{8}(-\mu-\tau+1)^2 - \frac{1}{2}c\chi(-\mu-\tau+1) - \frac{1}{2}(1-c)\epsilon(-\mu-\tau+1) + \frac{1}{2}\tau(-\mu-\tau+1)$$

$$\text{s.t.} \qquad 0 \leq c \leq 1$$

$$0 \leq \tau < 1-\mu$$

The social planner's objective function is concave in unit tax (τ);

$$\frac{\partial SW}{\partial \tau} = \frac{1}{4}(2c(\chi-\epsilon)+2\epsilon+\mu-\tau-1) \quad \text{and} \quad \frac{\partial^2 SW}{\partial \tau^2} = \frac{-1}{4}$$

We find optimal τ by the first order conditions as;

$$\frac{\partial SW}{\partial \tau} = 0 \quad \Leftrightarrow \quad \frac{1}{4}(2c(\chi-\epsilon)+2\epsilon+\mu-\tau-1) = 0$$

$$\Leftrightarrow \quad \tau = 2c\chi - 2c\epsilon + 2\epsilon + \mu - 1$$

On the other hand, the objective function is linear in take-back rate (c), and depending on the relation between the take-back cost (χ) and the environmental cost (ϵ), it is either monotone increasing ($\frac{\partial SW}{\partial c} > 0$) or monotone decreasing ($\frac{\partial SW}{\partial c} > 0$) in c. Hence, optimal take-back rate (c^*) always occurs on the boundaries.

$$\frac{\partial SW}{\partial c} = \frac{1}{2}(\chi-\epsilon)(\mu+\tau-1)$$

(i) $c^* = 0$ if $\chi \geq \epsilon$

(ii) $c^* = 1$ if $\chi < \epsilon$

Substituting the optimal c values in $\tau = 2c\chi - 2c\epsilon + 2\epsilon + \mu - 1$,

(i) $\tau = -1 + 2\epsilon + \mu$ when $c^* = 0$

(ii) $\tau = -1 + 2\chi + \mu$ when $c^* = 1$

We also require that unit tax to be always nonnegative and lower than $1 - \mu$ so that there is always positive production in the market. $\tau_{c=0}$ is greater than 0 if $\epsilon > \frac{1-\mu}{2}$ and lower than $1-\mu$ if $\epsilon < 1-\mu$. In this case since $\epsilon \leq \chi$, and we assume that $\chi < 1-\mu$, $\epsilon < 1 - \mu$ is already satisfied. Hence, for $c^* = 0$, optimal τ is 0 when $\epsilon \leq \frac{1-\mu}{2}$, and it is $-1 + 2\epsilon + \mu$ when $\epsilon > \frac{1-\mu}{2}$. Similarly, $\tau_{c=1}$ is greater than 0 if $\chi > \frac{1-\mu}{2}$ and lower than $1 - \mu$ if $\chi < 1 - \mu$. By assumption $\chi < 1 - \mu$ and the second condition is always satisfied. Thus, for $c^* = 1$, optimal τ is 0 when $\chi \leq \frac{1-\mu}{2}$ and, $-1+2\chi+\mu$ when $\chi > \frac{1-\mu}{2}$.

Optimal solution (c^*, τ^*) for the tax model can be summarized as;

- $c^* = 0$ and $\tau^* = 0$, if $\chi \geq \epsilon$ and $\epsilon \leq \frac{1-\mu}{2}$
- $c^* = 0$ and $\tau^* = -1 + 2\epsilon + \mu$, if $\chi \geq \epsilon$ and $\epsilon > \frac{1-\mu}{2}$
- $c^* = 1$ and $\tau^* = 0$, if $\chi < \epsilon$ and $\chi \leq \frac{1-\mu}{2}$
- $c^* = 1$ and $\tau^* = -1 + 2\chi + \mu$, if $\chi < \epsilon$ and $\chi > \frac{1-\mu}{2}$

■ **Proof of Proposition 5.** By taking the sum of Π_M, Π_E and Π_C, the social planner's objective is written as;

$$\max_{c} \quad SW = \frac{1}{8}(c\chi + \mu - 1)(3c\chi - 4c\epsilon + 4\epsilon + 3\mu - 3)$$

$$\text{s.t.} \qquad 0 \leq c \leq 1$$

Partial derivatives of the social planner's objective with respect to c are;

$$\frac{\partial SW}{\partial c} = \frac{1}{4}\left(3c\chi^2 + (-4c\epsilon + 2\epsilon + 3\mu - 3)\chi - 2\epsilon(\mu - 1)\right) \quad \text{and} \quad \frac{\partial^2 SW}{\partial c^2} = \frac{1}{4}\chi(3\chi - 4\epsilon)$$

Considering the second order partial derivatives,

- SW is concave in c when $3\chi < 4\epsilon$. In this case, by the first order conditions $c = \frac{3\chi(1-\mu)-2\epsilon(1-\mu+\chi)}{\chi(3\chi-4\epsilon)}$.
  - if $\epsilon > \frac{3\chi}{2}$, the given expression is greater than 1, thus optimal take-back rate (c^*) is 1.
  - if $\frac{3\chi(1-\mu)}{2(\chi+1-\mu)} < \epsilon < \frac{3\chi}{2}$, the given expression is between 0 and 1, and thus $c^* = \frac{3\chi(1-\mu)-2\epsilon(1-\mu+\chi)}{\chi(3\chi-4\epsilon)}$
  - if $\frac{3\chi}{4} < \epsilon \leq \frac{3\chi(1-\mu)}{2(\chi+1-\mu)}$, the expression is lower than 0 and $c^* = 0$
- SW is linear and decreasing in c when $3\chi = 4\epsilon$, and $c^* = 0$.
- SW is convex in c when $3\chi \geq 4\epsilon$, and $c^* = 0$.

Given these results the optimal decisions of the social planner in rate model can be summarized as in Table 3.3. ■

Proofs of Corollary 1-4. See the proofs for the comparison of the models under the competition case. Monopoly case is a special case of the competition case with $n = 1$. In Table 3.5, $\epsilon^* = \frac{3\chi(1-\mu)}{2(\chi-\mu+1)}$ and $\chi^* = \frac{2\epsilon\mu-2\epsilon}{2\epsilon+3\mu-3}$.
Similarly, in Table 3.6, $\epsilon^\dagger$ is the root of $-\epsilon(-\epsilon-\mu+1) = \frac{\epsilon^2(2\epsilon-3\chi)(\chi+\mu-1)^2}{\chi(3\chi-4\epsilon)^2}$. ■

B.1.2 Competition Case

Proof of Proposition 6. The social planner's problem is written as,

$$\max_{c,\tau} SW = \frac{n(\mu+\tau-1)(2c(\chi-\epsilon)(n+1)+2\epsilon(n+1)+(\mu-1)(n+2)-n\tau)}{2(n+1)^2} \tag{B.1.1}$$

$$\text{s.t.} \quad 0 \leq c \leq 1 \tag{B.1.2}$$

$$0 \leq \tau \leq 1-\mu \tag{B.1.3}$$

The social planner's objective function is concave in τ and linear in c. Hence, the optimal take-back rate always occurs at the boundaries (either 0 or 1) while the optimal tax may take interior values.

The partial derivatives of the objective function with respect to τ are;

$$\frac{\partial SW}{\partial \tau} = \frac{n(n\epsilon + \epsilon + \mu + c(\chi - \epsilon)(n+1) - n\tau - 1)}{(n+1)^2} \text{ and } \frac{\partial^2 SW}{\partial \tau^2} = -\frac{n^2}{(n+1)^2}$$

Since the objective is concave in τ we find optimal τ by the first order conditions;

$$\frac{\partial SW}{\partial \tau} = 0 \quad \Leftrightarrow \quad \tau = \frac{n\epsilon + \epsilon + \mu + c(\chi - \epsilon)(n+1) - 1}{n}$$

The objective function is linear in c;

$$\frac{\partial SW}{\partial c} = \frac{(\chi - \epsilon)n(\mu + \tau - 1)}{n+1}$$

Thus, optimal take-back rate (c^*) is either 0 or 1 depending on the relation between ϵ and χ;

(i) $c^* = 0$ if $\chi \geq \epsilon$

(ii) $c^* = 1$ if $\chi < \epsilon$

Substituting optimal c in $\tau = \frac{n\epsilon+\epsilon+\mu+c(\chi-\epsilon)(n+1)-1}{n}$,

(i) $\tau = \epsilon + \frac{\epsilon+\mu-1}{n}$ when $c^* = 0$

(ii) $\tau = \chi + \frac{\chi+\mu-1}{n}$ when $c^* = 1$

Unit tax should always be nonnegative and lower than $1-\mu$ so that there is always positive production in the market. $\tau_{c=0}$ is greater than 0 if $\epsilon > \frac{1-\mu}{n+1}$ and lower than $1-\mu$ if $\epsilon < 1-\mu$. Since $\epsilon \leq \chi$ in this case and $\chi < 1-\mu$ by assumption, $\epsilon < 1-\mu$ is always satisfied. Hence, for $c^* = 0$, optimal unit tax (τ^*) is 0 when $\epsilon \leq \frac{1-\mu}{n+1}$, and $\epsilon + \frac{\epsilon+\mu-1}{n}$ when $\epsilon > \frac{1-\mu}{n+1}$. Similarly, $\tau_{c=1}$ is greater than 0 if $\chi > \frac{1-\mu}{n+1}$ and lower than $1-\mu$ if $\chi < 1-\mu$. By assumption $\chi < 1-\mu$ and the second condition is always satisfied. Thus, for $c^* = 1$, optimal τ is 0 when $\chi \leq \frac{1-\mu}{n+1}$ and $\chi + \frac{\chi+\mu-1}{n}$ when $\chi > \frac{1-\mu}{n+1}$ as summarized in Table 3.8.

Optimal unit tax is increasing with n. The partial derivatives of the optimal unit tax with respect to n are,

when $\frac{1-\mu}{n+1} < \epsilon < 1-\mu$,

$$\frac{\partial\left(\frac{-1+(n+1)\epsilon+\mu}{n}\right)}{\partial n} = \frac{1-\mu-\epsilon}{n^2} \text{ and } \frac{1-\mu-\epsilon}{n^2} > 0 \text{ since } \epsilon < 1-\mu \quad \text{(B.1.4)}$$

when $\frac{1-\mu}{n+1} < \chi < 1-\mu$,

$$\frac{\partial\left(\frac{-1+(n+1)\chi+\mu}{n}\right)}{\partial n} = \frac{1-\mu-\chi}{n^2} \text{ and } \frac{1-\mu-\chi}{n^2} > 0 \text{ since } \chi < 1-\mu \quad \text{(B.1.5)}$$

Similarly, optimal unit tax is also increasing with ϵ when $\epsilon \leq \chi$ and it is increasing with χ when $\epsilon > \chi$. Note that it is 0 when $0 < \epsilon \leq \frac{1-\mu}{n+1}$ ($0 < \chi \leq \frac{1-\mu}{n+1}$) and rises to $\frac{-1+(n+1)\epsilon+\mu}{n}$ ($\frac{-1+(n+1)\chi+\mu}{n}$) when ϵ (χ) exceeds $\frac{1-\mu}{n+1}$. Moreover, in the interval of $\frac{1-\mu}{n+1} < \epsilon < 1-\mu$ ($\frac{1-\mu}{n+1} < \chi < 1-\mu$), it is easy to see that unit optimal tax increases with ϵ (χ). ■

Proof of Corollary 5.

We examine the behavior of the manufacturer profit and the social welfare in 4 solution intervals as in Table 3.8.

1 . ($c^* = 0$ and $\tau^* = 0$)

$$\Pi_{M_i} = \frac{(1-\mu)^2}{(n+1)^2} \text{ and } SW = \frac{(\mu-1)n(2\epsilon(n+1)+(\mu-1)(n+2))}{2(n+1)^2}$$

When we take the derivative of Π_{M_i}, $n\Pi_{M_i}$ and SW with respect to n;

$$\frac{\partial\Pi_{M_i}}{\partial n} = -\frac{2(\mu-1)^2}{(n+1)^3} \text{ and } \frac{\partial(n\Pi_{M_i})}{\partial n} = -\frac{(\mu-1)^2(n-1)}{(n+1)^3}$$

$$\frac{\partial SW}{\partial n} = \frac{(\mu-1)(n\epsilon+\epsilon+\mu-1)}{(n+1)^3}$$

$\frac{\partial\Pi_{M_i}}{\partial n}$ and $\frac{\partial(n\Pi_{M_i})}{\partial n}$ are negative given that $n > 1$. Hence, in this interval both the profit of each manufacturer (Π_{M_i}) and the total manufacturer profit ($n\Pi_{M_i}$) decreases in n. On the other hand, $\frac{\partial SW}{\partial n} > 0$ since $\epsilon \leq \frac{1-\mu}{n+1}$. Thus, SW increases in n.

2 . ($c^* = 0$ and $\tau^* = \epsilon + \frac{\epsilon+\mu-1}{n}$)

$$\Pi_{M_i} = \frac{(1-\epsilon-\mu)^2}{n^2} \text{ and } SW = \frac{1}{2}(1-\epsilon-\mu)^2$$

SW is independent of n, hence it is not affected by n. On the other hand, it is easy to see that Π_{M_i} and $n\Pi_{M_i}$ decreases in n.

3 . ($c^* = 1$ and $\tau^* = 0$)

Π_{M_i} and $n\Pi_{M_i}$ are the same as in interval 1 and the analysis for interval 1 is valid for this interval as well.

$$\begin{aligned} SW &= \frac{(\mu-1)n(2\chi(n+1)+(\mu-1)(n+2))}{2(n+1)^2} \\ \frac{\partial SW}{\partial n} &= \frac{(\mu-1)(n\chi+\chi+\mu-1)}{(n+1)^3} \end{aligned}$$

$\frac{\partial SW}{\partial n} > 0$ since $\chi \leq \frac{1-\mu}{n+1}$ in this interval. Thus, SW increases in n.

4 . ($c^* = 1$ and $\tau^* = \chi + \frac{\chi+\mu-1}{n}$)

$$\Pi_{M_i} = \frac{(1-\chi-\mu)^2}{n^2} \text{ and } SW = \frac{1}{2}(1-\chi-\mu)^2$$

SW is independent of n, hence it is not affected by n in this case. On the other hand, it is easy to see that Π_{M_i} and $n\Pi_{M_i}$ decreases in n.

Given the above analysis, we conclude that the profit of each manufacturer (Π_{M_i}) and the total manufacturer profit (Π_M) always decrease as n increases. On the other hand, when tax is positive the social welfare (SW) is not affected by n while it increases as n increases when tax is zero. ■

Proof of Proposition 7. Taking the sum of $n\Pi_{M_i}$, Π_C and Π_E, the social planner's problem is written as;

$$\max_c \quad SW = \frac{(c\chi+\mu-1)n(2\epsilon(n+1)+(\mu-1)(n+2)+c(\chi(n+2)-2\epsilon(n+1)))}{2(n+1)^2} \tag{B.1.6}$$

$$\text{s.t} \qquad 0 \leq c \leq 1 \tag{B.1.7}$$

The first and second order derivatives of the objective with respect to c are;

$$\frac{\partial SW}{\partial c} = \frac{n\left(c(n+2)\chi^2 + ((1-2c)\epsilon(n+1) + (\mu-1)(n+2))\chi - \epsilon(\mu-1)(n+1)\right)}{(n+1)^2}$$

$$\frac{\partial^2 SW}{\partial c^2} = \frac{\chi n(\chi(n+2) - 2\epsilon(n+1))}{(n+1)^2}$$

From the second order derivative, it is easy to see that;

- SW is concave in c when $\epsilon > \frac{\chi(n+2)}{2(n+1)}$. In this case, from the first order conditions, $c = \frac{\epsilon(\mu-1)(n+1)-\chi(\epsilon(n+1)+(\mu-1)(n+2))}{\chi(\chi(n+2)-2\epsilon(n+1))}$.
  - if $\epsilon \geq \chi\left(1+\frac{1}{n+1}\right)$, the expression given for c is greater than or equal to 1 and optimal c (c^*) is set to 1.
  - if $-\frac{\chi(\mu-1)(n+2)}{(\chi-\mu+1)(n+1)} < \epsilon < \chi\left(1+\frac{1}{n+1}\right)$, then the expression takes values between 0 and 1 and $c^* = \frac{\epsilon(\mu-1)(n+1)-\chi(\epsilon(n+1)+(\mu-1)(n+2))}{\chi(\chi(n+2)-2\epsilon(n+1))}$.
  - if $\frac{\chi(n+2)}{2(n+1)} < \epsilon \leq -\frac{\chi(\mu-1)(n+2)}{(\chi-\mu+1)(n+1)}$, then the expression is smaller than or equal to 0 and $c^* = 0$.
- SW is linear and decreasing in c when $\epsilon = \frac{\chi(n+2)}{2(n+1)}$ and $c^* = 0$.
- SW is convex in c when $\epsilon < \frac{\chi(n+2)}{2(n+1)}$ and $c^* = 0$.

Given these results, the optimal decisions of the social planner in the rate model under competition are as summarized in Table 3.9.

The thresholds on ϵ which determine the intervals in Table 3.9, decreases in the degree of competition (n). That is;

$$\frac{\partial(\frac{(1-\mu)\chi}{1-\mu+\chi}(\frac{n+2}{n+1}))}{\partial n} < 0 \quad \text{and} \quad \frac{\partial(\chi(\frac{n+2}{n+1}))}{\partial n} < 0$$

Moreover;

$$\frac{(\partial\frac{(1-\mu)\chi}{1-\mu+\chi}(\frac{n+2}{n+1}))}{\partial n} > \frac{\partial(\chi(\frac{n+2}{n+1}))}{\partial n}$$

In other words, as n increases, the zero or the partial take-back regions shrink while the perfect take-back interval enlarges. Hence, we can conclude that optimal take-back rate increases in the degree of competition (n). ■

Proof of Corollary 6. We can evaluate the two models in terms of social welfare under three cases; when $\epsilon \leq \chi$, $\chi < \epsilon < \frac{(n+2)(1-\mu)}{n+1}$ and $\frac{(n+2)(1-\mu)}{n+1} \leq \epsilon$.

- When $\epsilon \leq \chi$:
  - In the interval of $\epsilon \leq \frac{1-\mu}{n+1}$, optimal solution for the tax model is

    $$\left\{c = 0,\ \tau = 0,\ SW(T) = \frac{(\mu-1)n(2\epsilon(n+1)+(\mu-1)(n+2))}{2(n+1)^2}\right\}$$

    Similarly, optimal solution for the rate model is

    $$\left\{c = 0,\ SW(R) = \frac{(\mu-1)n(2\epsilon(n+1)+(\mu-1)(n+2))}{2(n+1)^2}\right\}$$

    Hence $SW(T) = SW(R)$ in this interval.
  - In the interval of $\epsilon > \frac{1-\mu}{n+1}$, optimal solution for the tax model is

    $$\left\{c = 0, \quad \tau = \epsilon + \frac{\epsilon+\mu-1}{n}\right\}$$

    In this interval there are two feasible optima for the rate model depending on the value of ϵ;

    (a) when $\epsilon \leq \frac{\chi(1-\mu)(n+2)}{(\chi-\mu+1)(n+1)}$, $c = 0$ and $SW(R) = \frac{(\mu-1)n(2\epsilon(n+1)+(\mu-1)(n+2))}{2(n+1)^2}$

    (b) when $\frac{\chi(n+2)}{n+1} > \epsilon > \frac{\chi(1-\mu)(n+2)}{(\chi-\mu+1)(n+1)}$, $c \in (0,1)$ and $SW(R) = -\frac{\epsilon^2(\chi+\mu-1)^2 n}{2\chi(\chi(n+2)-2\epsilon(n+1))}$.

    It is easy to see that $SW(T) > SW(R)$ in both of these two cases. Hence, $SW(T) > SW(R)$ in this interval.

- When $\chi < \epsilon < \frac{(n+2)(1-\mu)}{n+1}$:
  - In the interval of $0 < \chi \leq \frac{1-\mu}{n+1}$, optimal solution for tax model is

    $$\left\{c = 1,\ \tau = 0, SW(T) = \frac{(\mu-1)n(2\chi(n+1)+(\mu-1)(n+2))}{2(n+1)^2}\right\}$$

    On the other hand, there are three possible optima for the rate model depending on the value of χ;

(a) when $\chi \leq \frac{(n+1)\epsilon}{(n+2)}$, $c = 1$ and $SW(R) = \frac{(\chi+\mu-1)^2 n(n+2)}{2(n+1)^2}$

(b) when $\frac{\epsilon(1-\mu)(-n-1)}{\epsilon(n+1)-(1-\mu)(n+2)} > \chi > \frac{\epsilon(n+1)}{n+2}$, $c \in (0,1)$ and $SW(R) = -\frac{\epsilon^2(\chi+\mu-1)^2 n}{2\chi(\chi(n+2)-2\epsilon(n+1))}$

(c) when $\frac{\epsilon(1-\mu)(-n-1)}{\epsilon(n+1)-(1-\mu)(n+2)} \leq \chi$, $c = 0$ and $SW(R) = \frac{(\mu-1)n(2\epsilon(n+1)+(\mu-1)(n+2))}{2(n+1)^2}$.

It is easy to see that $SW(T) > SW(R)$ in all of these cases. Hence, $SW(T) > SW(R)$ in this interval.

– In the interval of $\frac{1-\mu}{n+1} < \chi < 1-\mu$, optimal solution for the tax model is

$$\left\{c = 1,\ \tau = \chi + \frac{\chi+\mu-1}{n},\ SW(T) = \frac{1}{2}(\chi+\mu-1)^2\right\}$$

On the other hand, depending on the value of χ there are two possible optima for the rate model;

(a) when $\chi \leq \frac{(n+1)\epsilon}{(n+2)}$, $c = 1$ and $SW(R) = \frac{(\chi+\mu-1)^2 n(n+2)}{2(n+1)^2}$

(b) when $\frac{\epsilon(1-\mu)(-n-1)}{\epsilon(n+1)-(1-\mu)(n+2)} > \chi > \frac{\epsilon(n+1)}{n+2}$, $c \in (0,1)$ and $SW(R) = -\frac{\epsilon^2(\chi+\mu-1)^2 n}{2\chi(\chi(n+2)-2\epsilon(n+1))}$

It is easy to see that $SW(T) > SW(R)$ in both of these cases. Thus, as in the previous intervals, $SW(T) > SW(R)$ in this interval.

- When $\frac{(n+2)(1-\mu)}{n+1} \leq \epsilon$:

This case is very similar to the second case. The only difference is that there exists only one possible optima for the rate model. Both in the interval of $0 < \chi \leq \frac{1-\mu}{n+1}$ and $\frac{1-\mu}{n+1} < \chi < 1-\mu$, optimal solution for the rate model is

$$\left\{c = 1,\ SW(R) = \frac{(\chi+\mu-1)^2 n(n+2)}{2(n+1)^2}\right\}$$

Similar to the previous cases, optimal social welfare for the tax model $(SW(T))$

$$\begin{aligned} &= \frac{(\mu-1)n(2\chi(n+1)+(\mu-1)(n+2))}{2(n+1)^2} && \text{when } 0 < \chi \leq \frac{1-\mu}{n+1} \\ &= \frac{1}{2}(\chi+\mu-1)^2 && \text{when } \frac{1-\mu}{n+1} < \chi \end{aligned}$$

As in the second case $SW(T) > SW(R)$ in both of these intervals.

Given this analysis, we can conclude that $SW(T) = SW(R)$ only when $\epsilon \leq \frac{(1-\mu)}{n+1}$ and $\epsilon \leq \chi$. $SW(T) > SW(R)$ otherwise as shown in Table 3.10.

As the degree of competition (n) increases, the upper threshold ($\frac{1-\mu}{n+1}$) on the equality region decreases.

$$\frac{\partial(\frac{1-\mu}{n+1})}{\partial n} < 0$$

This implies that as n increases the equality region gets smaller while the region where tax model dominates enlarges. ■

Proof of Corollary 7.

Similar to social welfare comparison, we can evaluate the two models in terms of the manufacturer profit under three cases; when $\epsilon \leq \chi$, $\chi < \epsilon < \frac{(n+2)(1-\mu)}{n+1}$ and $\frac{(n+2)(1-\mu)}{n+1} \leq \epsilon$.

- When $\epsilon \leq \chi$:
  - In the interval of $\epsilon \leq \frac{1-\mu}{n+1}$, optimal solution for tax model is

    $$\left\{c = 0,\ \tau = 0,\ \Pi_M(T) = \frac{(\mu-1)^2 n}{(n+1)^2}\right\}$$

    Similarly, optimal solution for the rate model is

    $$\left\{c = 0,\ \Pi_M(R) = \frac{(\mu-1)^2 n}{(n+1)^2}\right\}$$

    Hence $\Pi_M(T) = \Pi_M(R)$ in this interval.
  - In the interval of $\epsilon > \frac{1-\mu}{n+1}$, optimal solution for the tax model is

    $$\left\{c = 0,\ \tau = \epsilon + \frac{\epsilon+\mu-1}{n}\right\}$$

    For the rate model, on the other hand, there are two possible optima depending on the value of ϵ;

    (a) when $\epsilon \leq \epsilon^{**} = \frac{\chi(1-\mu)(n+2)}{(\chi-\mu+1)(n+1)}$, $c = 0$ and $\Pi_M(R) = \frac{(\mu-1)^2 n}{(n+1)^2}$.
    Thus $\Pi_M(R) > \Pi_M(T)$

    (b) when $\frac{\chi(n+2)}{n+1} > \epsilon > \epsilon^{**}$, optimal $c \in (0,1)$ and $\Pi_M(R) = \frac{n\epsilon^2(\chi+\mu-1)^2}{(\chi(n+2)-2\epsilon(n+1))^2}$.

In this subinterval,

$$\Pi_M(R) > \Pi_M(T) \quad \text{if} \quad \epsilon < \frac{(n+2)(1-\mu)}{(2n+2)}$$
$$\Pi_M(R) < \Pi_M(T) \quad \text{if} \quad \epsilon > \frac{(n+2)(1-\mu)}{(2n+2)}$$

- When $\chi < \epsilon < \frac{(n+2)(1-\mu)}{n+1}$:
  - In the interval of $0 < \chi \leq \frac{1-\mu}{n+1}$, optimal solution for the tax model is

    $$\left\{c = 1,\ \tau = 0,\ \Pi_M(T) = \frac{(\mu-1)^2 n}{(n+1)^2}\right\}$$

    Depending on the value of χ there are three possible optima for the rate model;

    (a) when $\chi \leq \frac{(n+1)\epsilon}{(n+2)}$, $c = 1$ and $\Pi_M(R) = \frac{n(\chi+\mu-1)^2}{(n+1)^2}$. Thus $\Pi_M(T) > \Pi_M(R)$.

    (b) when $\frac{\epsilon(n+1)}{n+2} < \chi < \chi^{**} = \frac{\epsilon(1-\mu)(-n-1)}{\epsilon(n+1)-(1-\mu)(n+2)}$, $c \in (0,1)$ and $\Pi_M(R) = \frac{n\epsilon^2(\chi+\mu-1)^2}{(\chi(n+2)-2\epsilon(n+1))^2}$. Similar to (a), $\Pi_M(T) > \Pi_M(R)$

    (c) when $\chi^{**} \leq \chi$, $c = 0$ and $\Pi_M(R) = \frac{n(\mu-1)^2}{(n+1)^2}$. Hence, $\Pi_M(T) = \Pi_M(R)$.

  - In the interval of $\frac{1-\mu}{n+1} < \chi < 1-\mu$, optimal solution for the tax model is;

    $$\left\{c = 1,\ \tau = \chi + \frac{\chi+\mu-1}{n},\ \Pi_M(T) = \frac{(\chi+\mu-1)^2}{n}\right\}$$

    Depending on the value of χ there are two possible optima for the rate model;

    (a) when $\chi \leq \frac{(n+1)\epsilon}{(n+2)}$, $c = 1$ and $\Pi_M(R) = \frac{n(\chi+\mu-1)^2}{(n+1)^2}$

    (b) when $\frac{\epsilon(n+1)}{n+2} < \chi < \frac{\epsilon(1-\mu)(-n-1)}{\epsilon(n+1)-(1-\mu)(n+2)}$, $c \in (0,1)$ and $\Pi_M(R) = \frac{n\epsilon^2(\chi+\mu-1)^2}{(\chi(n+2)-2\epsilon(n+1))^2}$

    It is easy to see that in both of these subintervals $\Pi_M(T) > \Pi_M(R)$.

- When $\frac{(n+2)(1-\mu)}{n+1} \leq \epsilon$:

  This case is very similar to the second case. The only difference is that there exists only one possible optima for the rate model.

  Both in the interval of $0 < \chi \leq \frac{1-\mu}{n+1}$ and $\frac{1-\mu}{n+1} < \chi < 1-\mu$, the optimal solution for the rate model is

$$\left\{c = 1,\ \Pi_M(R) = \frac{n(\chi+\mu-1)^2}{(n+1)^2}\right\}$$

Similar to the previous cases, the optimal manufacturer profit for the tax model is

$$\Pi_M(T) = \frac{n(\mu-1)^2}{(n+1)^2} \text{ when } 0 < \chi \leq \frac{1-\mu}{n+1}$$

$$\Pi_M(T) = \frac{(\chi+\mu-1)^2}{n} \text{ when } \frac{1-\mu}{n+1} < \chi < 1-\mu$$

In both of these intervals, $\Pi_M(T) > \Pi_M(R)$.

Given this analysis, we can conclude that the comparison of the two models with respect to manufacturer profit can be summarized as in Table 3.11.

To see how the regions in Table 3.11 are affected by the competition, we examine the behavior of thresholds as n increases.

- As n increases, the upper threshold ($\frac{1-\mu}{n+1}$) on the first equality region (see Table 3.11) decreases.

$$\frac{\partial(\frac{1-\mu}{n+1})}{\partial n} = -\frac{1-\mu}{(n+1)^2} < 0$$

This implies that as n increases this region gets smaller.

The upper threshold of the second equality region is also equal to $\frac{1-\mu}{n+1}$ and decreases in n. However; the lower threshold on this region (χ^{**}) increases in n.

$$\frac{\partial\chi^{**}}{\partial n} = \frac{\epsilon(\mu-1)^2}{(\epsilon(n+1)+(\mu-1)(n+2))^2} > 0$$

As a result of these two effects, the second equality region gets smaller as n increases.

- The lower threshold of the first tax model dominating region ($\frac{(1-\mu)(n+2)}{2(n+1)}$) decreases and the upper threshold (χ^{**}) increases as n increases.

$$\frac{\partial(\frac{(1-\mu)(n+2)}{2(n+1)})}{\partial n} = \frac{\mu - 1}{2(n+1)^2} < 0$$

Hence, this region enlarges as the degree of competition rise.

Similarly, the lower threshold of the second tax model dominating region ($\frac{1-\mu}{n+1}$) is decreasing in n and the upper threshold does not change with n. Hence, this region gets larger as the degree of competition rise too.

- Finally, note that the only rate model dominating region is expanding in n. Both the upper ($\frac{(1-\mu)(n+2)}{2(n+1)}$) and the lower ($\frac{1-\mu}{n+1}$) thresholds of this region are decreasing in n. However the decrease in the lower threshold is greater than the decrease in the upper threshold and we can say that the region expands by $\frac{1-\mu}{2(n+1)^2}$ with unit increase in n.

$$\frac{\partial(\frac{1-\mu}{n+1})}{\partial n} = -\frac{1-\mu}{(n+1)^2} < \frac{\partial(\frac{(1-\mu)(n+2)}{2(n+1)})}{\partial n} = -\frac{1-\mu}{2(n+1)^2}$$

$$-\frac{1-\mu}{2(n+1)^2} - (-\frac{1-\mu}{(n+1)^2}) = \frac{1-\mu}{2(n+1)^2}$$

Nevertheless, compared to the tax model dominating regions, this expansion is much lower, hence the dominance of tax model is reinforced when n increases as stated in the corollary.

Overall, as n increases the regions where tax model dominates gets larger while the equality regions gets smaller. ■

Proof of Corollary 8. Similar to previous comparisons, we evaluate the two models under three cases;

- When $\epsilon \leq \chi$:
  - In the interval of $\epsilon \leq \frac{1-\mu}{n+1}$, there is zero take-back in the tax model and optimal environmental impact is $-\frac{\epsilon(1-\mu)n}{n+1}$. Similarly, optimal environmental impact for the rate model is also $-\frac{\epsilon(1-\mu)n}{n+1}$. Hence, $\Pi_E(T) = \Pi_E(R)$ in this interval.

- In the interval of $\frac{1-\mu}{n+1} < \epsilon$, $\Pi_E(T) = -\epsilon(1-\epsilon-\mu)$. In this interval there are two feasible optima for the rate model;

  (a) when $\epsilon \leq \frac{\chi(1-\mu)(n+2)}{(\chi-\mu+1)(n+1)}$, zero take-back is optimal for the rate model and $\Pi_E(R) = -\frac{\epsilon(1-\mu)n}{n+1}$. Thus, $\Pi_E(T) > \Pi_E(R)$.

  (b) when $\frac{\chi(1-\mu)(n+2)}{(\chi-\mu+1)(n+1)} < \epsilon < \frac{\chi(n+2)}{n+1}$, partial take-back is optimal for the rate model and $\Pi_E(R) = \frac{\epsilon^2(\chi+\mu-1)^2 n(n\epsilon+\epsilon-\chi(n+2))}{\chi(\chi(n+2)-2\epsilon(n+1))^2}$. Given this,

$$\Pi_E(T) > \Pi_E(R) \quad \text{if} \quad \epsilon < \epsilon^{\dagger\dagger}$$
$$\Pi_E(T) < \Pi_E(R) \quad \text{if} \quad \epsilon^{\dagger\dagger} < \epsilon$$

  Here, $\epsilon^{\dagger\dagger}$ is the root of $\epsilon(\epsilon+\mu-1) = \frac{\epsilon^2(\chi+\mu-1)^2 n(n\epsilon+\epsilon-\chi(n+2))}{\chi(\chi(n+2)-2\epsilon(n+1))^2}$.

- When $\chi < \epsilon < \frac{(n+2)(1-\mu)}{n+1}$:

  - In the interval of $0 < \chi \leq \frac{1-\mu}{n+1}$, perfect take-back is optimal for the tax model and $\Pi_E(T) = 0$. For the rate model, on the other hand, there are three feasible optima depending on χ value;

    (a) when $\chi \leq \frac{(n+1)\epsilon}{(n+2)}$, perfect take-back is optimal for the rate model too and $\Pi_E(R) = 0$. Thus $\Pi_E(T) = \Pi_E(R)$.

    (b) when $\frac{\epsilon(n+1)}{n+2} < \chi < \frac{\epsilon(1-\mu)(-n-1)}{\epsilon(n+1)-(1-\mu)(n+2)}$, partial take-back is optimal and $\Pi_E(R) = \frac{\epsilon^2(\chi+\mu-1)^2 n(n\epsilon+\epsilon-\chi(n+2))}{\chi(\chi(n+2)-2\epsilon(n+1))^2}$.

    (c) When $\frac{\epsilon(1-\mu)(-n-1)}{\epsilon(n+1)-(1-\mu)(n+2)} \leq \chi$, zero-take is optimal and $\Pi_E(R) = -\frac{\epsilon(1-\mu)n}{n+1}$.

    In (b) and (c), with zero environmental impact tax model always dominates rate model.

  - In the interval of $\frac{1-\mu}{n+1} < \chi < 1-\mu$, still $\Pi_E(T) = 0$. On the other hand, there are two feasible optima for the rate model;

    (a) When $\chi \leq \frac{(n+1)\epsilon}{(n+2)}$, $\Pi_E(R) = 0$. Thus, $\Pi_E(T) = \Pi_E(R)$.

(b) When $\frac{\epsilon(n+1)}{n+2} < \chi < \frac{\epsilon(1-\mu)(-n-1)}{\epsilon(n+1)-(1-\mu)(n+2)}$, $\Pi_E(R) = \frac{\epsilon^2(\chi+\mu-1)^2 n(n\epsilon+\epsilon-\chi(n+2))}{\chi(\chi(n+2)-2\epsilon(n+1))^2}$.

It is easy to see that in this case $\Pi_E(T) > \Pi_E(R)$.

- When $\frac{(n+2)(1-\mu)}{n+1} \leq \epsilon$:

  In this case, perfect take-back is optimal both for the tax model and the rate model. Hence, $\Pi_E(T) = \Pi_E(R) = 0$ throughout the interval.

Given this analysis, the comparison of the two models with respect to the environmental impact can be summarized as in Table 3.12.

To see how the regions in Table 3.12 are affected by the competition, we examine the behavior of thresholds as n increases.

- When $\epsilon \leq \chi$;
  - As n increases, the equality region gets smaller since the upper threshold of this region ($\frac{1-\mu}{n+1}$) is decreasing in n.
  - Tax model dominating region gets larger as n increases. Note that both the upper and the lower thresholds of this region are decreasing in n. Nevertheless, the decrease in the lower threshold is greater than the decrease in the upper threshold, thus the region expands as the intensity of competition increases.
  - Rate model dominating region gets larger as n increases since the lower threshold of this region ($\epsilon^{\dagger\dagger}$) is decreasing in n.
- When $\chi < \epsilon \leq \frac{(n+2)(1-\mu)}{n+1}$;
  - Both of the two equality regions get larger as n increases. Common upper threshold of these regions is $\frac{(n+1)\epsilon}{(n+2)}$ and this threshold is increasing in n. The lower threshold of the second equality region ($\frac{1-\mu}{n+1}$) is decreasing in n. Thus, the equality regions expand as the intensity of competition increases.
  - In contrast to the equality regions, due to the same threshold behaviors tax dominating regions get smaller as n increases.

- When $\frac{(n+2)(1-\mu)}{n+1} \leq \epsilon$;

  The only region in this case is an equality region and it gets larger as n increases since the lower threshold of the region is decreasing in n.

If we consider all of these comparison regions together, it is obvious that under intense competition the regions where the rate model is at least equally favorable expand, while the regions where tax model dominates get smaller. Thus, we can conclude that as the degree of competition (n) increases, the dominance of the tax model weakens while the dominance of the rate model increases for the environment. ■

Proof of Corollary 9. Observe that the lower threshold of the misalignment region is $\frac{1-\mu}{n+1}$ and this threshold is decreasing in n. Hence, as the degree of competition increases, misalignment region gets larger. ■

Appendix C

C.1 Data Adjustments in DM Before Transportation Formulation

To reformulate DM as a transportation model we need to make the RHSs of the constraints (4.3) zero as discussed in section 4.3.1. To do this, we employ the following heuristic procedure,

(i) If $O_j - I_j$ is negative for some j, this implies that there is excessive inventory from part j and this excessive inventory can be readily used to meet some portion of the demand for part j. Hence, we deduct $\min\big(\mid O_j - I_j \mid, D^p_{j1}\big)$ from D^p_{j1} for all j such that $O_j - I_j < 0$. If $\min\big(\mid O_j - I_j \mid, D^p_{j1}\big) = D^p_{j1}$, then this implies that we will have excessive pieces for part type j at the end of the period. Since in the dynamic implementation, we charge a penalty fee for each excessive part piece at the end of the period, we need to keep record of these excessive amounts. Hence, we update e_j, the end of period excessive amount of part j over the base stock, to $e_j + \mid O_j - I_j \mid - D^p_{j1}$ in this case.

(ii) If it is positive, this implies that we have deficient inventory for part j and we need to replenish it. In this case, we can replenish the deficient parts by either (i) outsourcing or (ii) dismantling cores. If we dismantle, we sacrifice one core but we avoid outsourcing costs and can sell any part pieces exceeding the required amount if there is enough demand for that type to get some revenue. Hence, by dismantling one core, we can save;

$$S_d = \sum_{j \in P} \big(c_{j1} \min(O_j - I_j, a_j)\big) + \sum_{j \in K} \big(p^p_{j1} \min(D^p_{j1}, a_j)\big) \qquad \text{(C.1.1)}$$

where P is the set of all parts such that $O_j - I_j > 0$ and K is the set of all parts such that $O_j - I_j = 0$.

If we outsource, on the other hand, we save the core that would be dismantled otherwise, and can use this core for refurbishing if there is still demand for refurbished products. Hence, by outsourcing the deficient parts, we can save;

$$S_o = \min\big(1, D_1^r\big)p_1^r \tag{C.1.2}$$

To decide how to replenish the deficient parts,

Step 0: Determine all parts j such that $O_j - I_j > 0$ and let the set of these parts be P. Similarly, let the set of parts for which $O_j - I_j = 0$ be K.

Step 1: Check the amount of available cores (B_1).

(a) If $B_1 = 0$, then outsource all deficient parts ($o_{j1} = O_j - I_j$ for all $j \in P$). Set $O_j - I_j = 0$ for all $j \in P$ and eliminate them from P.

(b) If $B_1 > 0$, calculate S_d and S_o as defined in (C.1.1) and (C.1.2), respectively.

- If $S_d \geq S_o$, dismantle as much as $\min\Bigg(B_1, \min_{j\in P}\big(\frac{O_j - I_j}{a_j}\big)\Bigg)$. Update the inventory of each part type (I_j), the cores inventory (B_1) and the end of period amount of each part type exceeding the base stock (e_j) as follows;

$$B_1 = B_1 - \min\Bigg(B_1, \min_{j\in P}\big(\frac{O_j - I_j}{a_j}\big)\Bigg)$$

$$I_j = I_j + a_j \min\Bigg(B_1, \min_{j\in P}\big(\frac{O_j - I_j}{a_j}\big)\Bigg) \text{ for all } j \in P$$

$$e_j = e_j + a_j \min\Bigg(B_1, \min_{j\in P}\big(\frac{O_j - I_j}{a_j}\big)\Bigg) \text{ for all } j \in K$$

  Eliminate all parts j for which $O_j - I_j = 0$ from the set P and add them to the set K.

- If $S_d < S_o$, then outsource all deficient parts ($o_{j1} = O_j - I_j$ for all $j \in P$). Set $O_j - I_j = 0$ for all $j \in P$ and eliminate them from P.

Step 2: If P is empty, stop. Otherwise go to Step 1.

After these adjustments, to determine the shadow prices (bid prices) needed in the dynamic implementation we can solve TFDM using the optimal solution procedure (TPP) provided in section 4.3.2 instead of DM.

Note that when $B_1 = 0$, we do not even need to solve TFDM and determine the bid prices. In this case, we cannot meet any refurbished product order, hence we do not need the shadow prices associated with the core inventory balance constraint (y_1^c). As for the part orders, it is straightforward that the shadow prices associated with the parts inventory constraints (y_{j1}^p) for which $O_j - I_j \geq 0$, are equal to c_{j1} since the firm will have to outsource to replenish each part he sells in this case.

C.2 Proofs of Chapter 4

Proof of Theorem 1. To show that an optimal solution to TFDM is also optimal to DM, first we prove that the feasible region of TFDM ($F(TFDM)$) is a subset of the feasible region of DM ($F(DM)$) and the objective functions of the two models are equivalent over $F(TFDM)$. Then, we show that the feasible region of DM is larger than that of TFDM; however the optimal solution to DM can never be in $F(DM) \setminus F(TFDM)$.

Let $S = (\bar{x}_{t't}, \bar{z}_{kt't}, \bar{s}_{t'})$ be a solution for TFDM and let the equivalent solution for DM for $N = 1$ be S'. S' can be written as (4.14-4.19) given in Section 4.3.2. Moreover, let $f(S')$ and $\overline{f}(S)$ denote the objective function value of DM and TFDM for solutions S' and S respectively.

Part 1:

(i) To prove $F(TFDM) \subset F(DM)$, we show that $S \in F(TFDM)$ is a feasible solution to DM. To do this, we check the feasibility of S for each DM constraint.

Cores Inventory Balance Constraints: These constraints are

$$b_{(t-1)} + B_t - (z_t + x_t + s_t) - b_t = 0 \qquad t = 1, ..., T \qquad \text{(C.2.3)}$$

Substituting S from (4.14-4.19) in (C.2.3) we get,

$$\underbrace{\sum_{t'=1}^{t-1}\sum_{l=t}^{T}\bar{x}_{t'l} + \sum_{t'=1}^{t-1}\sum_{l=t}^{T}\sum_{k\in S^l}\bar{z}_{kt'l}}_{b_{(t-1)}} - \underbrace{\left(\sum_{t'=1}^{t}\sum_{l=t+1}^{T}\bar{x}_{t'l} + \sum_{t'=1}^{t}\sum_{l=t+1}^{T}\sum_{k\in S^l}\bar{z}_{kt'l}\right)}_{b_t} \quad \text{(C.2.4)}$$

$$+\underbrace{\sum_{l=t}^{T}(\bar{x}_{tl}) + \sum_{l=t}^{T}\sum_{k\in S^l}(\bar{z}_{ktl}) + \bar{s}_t}_{B_t} - \underbrace{\sum_{k\in S^t}\sum_{t'=1}^{t}\bar{z}_{kt't}}_{z_t} - \underbrace{\sum_{t'=1}^{t}\bar{x}_{t't}}_{x_t} - \bar{s}_{t'} = 0 \quad \text{(C.2.5)}$$

We know $B_t = \sum_{l=t}^{T}(\bar{x}_{tl}) + \sum_{l=t}^{T}\sum_{k\in S^l}(\bar{z}_{ktl}) + \bar{s}_t$ from TFDM by (4.9). We can write the difference $b_{(t-1)} - b_t$ as,

$$\sum_{t'=1}^{t-1}\bar{x}_{t't} - \sum_{l=t+1}^{T}\bar{x}_{tl} + \sum_{t'=1}^{t-1}\sum_{k\in S^t}\bar{z}_{kt't} - \sum_{l=t+1}^{T}\sum_{k\in S^l}\bar{z}_{ktl}$$

When we substitute $b_{(t-1)} - b_t$ in (C.2.4) and add to (C.2.5), we get;

$$\bar{x}_{tt} + \bar{z}_{ktt} - \bar{x}_{tt} - \bar{z}_{ktt} = 0$$

which shows that the solution S satisfies the cores inventory balance constraints of DM.

Parts Inventory Balance Constraints: These constraints are,

$$a_j z_t - q_{jt} + o_{jt} \geq 0 \qquad j = 1, ..., M \text{ and } t = 1, ..., T \qquad \text{(C.2.6)}$$

From (4.16) and (4.15), we know $q_{jt} = \sum_{t'=1}^{t}\sum_{k\in K_j^t} a_j\bar{z}_{kt't}$, $z_t = \sum_{t'=1}^{t}\sum_{k\in S^t}\bar{z}_{kt't}$ and $o_{jt} = 0$. When we multiply z_t with a_j,

$$a_j z_t = a_j(\sum_{t'=1}^{t}\sum_{k\in S^t}\bar{z}_{kt't}) = \sum_{t'=1}^{t}\sum_{k\in S^t}(a_j\bar{z}_{kt't})$$

K_j^t which denotes the smallest set of part bundles with a positive demand and include part j in period t, is a subset of S^t, which denotes the smallest set of all part bundles with a positive demand in period t. Hence, the summation in $a_j z_t$ is over a larger part bundle set than the summation in q_{jt} and this implies that

$a_j z_t$ is always greater than or equal to q_{jt} and the solution S also satisfies parts inventory balance constraints.

Parts Demand Constraints: These constraints are,

$$q_{jt} \leq D^p_{jt} \qquad j = 1, ..., M \text{ and } t = 1, ..., T \tag{C.2.7}$$

Substituting $q_{jt} = \sum_{t'=1}^{t} \sum_{k \in K^t_j} a_j \bar{z}_{kt't}$, we can write them as,

$$\sum_{t'=1}^{t} \sum_{k \in K^t_j} a_j \bar{z}_{kt't} \leq D^p_{jt} \tag{C.2.8}$$

From TFDM we know,

$$\sum_{t'=1}^{t} \bar{z}_{kt't} \leq D^b_{kt} \tag{C.2.9}$$

Let L^t_j be the set of part bundles with a positive demand in period t and includes part j except for the bundle k. Then, from *Parameter Derivation Procedure* we can write (C.2.9) as,

$$\sum_{t'=1}^{t} \bar{z}_{kt't} \quad \leq \quad D^b_{kt} = \min_{j \in J_k} \left(\frac{D^p_{jt} - a_j(\sum_{m \in L^t_j} D^b_{mt})}{a_j} \right) \tag{C.2.10}$$

$$\sum_{t'=1}^{t} \bar{z}_{kt't} \quad \leq \quad \frac{D^p_{jt} - a_j(\sum_{m \in L^t_j} D^b_{mt})}{a_j} \tag{C.2.11}$$

$$a_j \sum_{t'=1}^{t} \bar{z}_{kt't} + a_j \left(\sum_{m \in L^t_j} D^b_{mt} \right) \quad \leq \quad D^p_{jt} \tag{C.2.12}$$

$$a_j \sum_{t'=1}^{t} \bar{z}_{kt't} + \sum_{m \in L^t_j} a_j D^b_{mt} \quad \leq \quad D^p_{jt} \tag{C.2.13}$$

Using L_j^t we can also write (C.2.8) in a similar way,

$$\sum_{t'=1}^{t} a_j \bar{z}_{kt't} + \sum_{m \in L_j^t} \sum_{t'=1}^{t} a_j \bar{z}_{mt't} \leq D_{jt}^p \tag{C.2.14}$$

$$\sum_{t'=1}^{t} a_j \bar{z}_{kt't} + \sum_{m \in L_j^t} a_j \left(\sum_{t'=1}^{t} \bar{z}_{mt't} \right) \leq D_{jt}^p \tag{C.2.15}$$

From (C.2.9), for all part bundles m we know that,

$$\sum_{t'=1}^{t} \bar{z}_{mt't} \leq D_{mt}^b \Leftrightarrow a_j \sum_{t'=1}^{t} \bar{z}_{mt't} \leq a_j D_{mt}^b \tag{C.2.16}$$

Hence, given (C.2.13) and (C.2.16), (C.2.15) will also hold and we can conclude that part demand constraints are satisfied for the solution S.

Refurbished Product Demand Constraints: These constraints are,

$$x_t \leq D_t^r \qquad t = 1, ..., T \tag{C.2.17}$$

When we substitute $x_t = \sum_{t'=1}^{t} \bar{x}_{t't}$ from (4.14),

$$\sum_{t'=1}^{t} \bar{x}_{t't} \leq D_t^r \tag{C.2.18}$$

We know that (C.2.18) holds from TFDM. Hence, (C.2.17) also holds and we can conclude that S satisfies refurbished product demand constraints in DM.

Nonnegativity Constraints: Given our definitions of the DM variables in terms of the TFDM solution S (see equations 4.14-4.19) and the nonnegativity of the TFDM solution, S, it is obvious that these constraints are also satisfied.

(ii) $f(S') = \overline{f}(S)$ where $S \in F(TFDM)$:

$$\overline{f}(S) = \sum_{t'=1}^{T} \sum_{t=t'}^{T} (\bar{x}_{t't} p_{t't}^r) + \sum_{t'=1}^{T} \sum_{t=t'}^{T} \sum_{k \in S^t} (\bar{z}_{kt't} p_{kt't}^b) + \sum_{t'=1}^{T} (s_{t'} v_{t'}) \tag{C.2.19}$$

$$
\begin{aligned}
f(S') &= \sum_{t=1}^{T}\left(p^r_t x_t + \sum_{j=1}^{M} p^p_{jt} q_{jt} + v_t s_t - \sum_{j=1}^{M} c_{jt} o_{jt} - h b_t\right) && \text{(C.2.20)}\\
&= \sum_{t=1}^{T}\left[\left(\sum_{t'=1}^{t} \bar{x}_{t't}\right)\left(p^r_{t't} + h(t-t')\right)\right] && \text{(C.2.21)}\\
&+ \sum_{t=1}^{T}\sum_{j=1}^{M}\left[\left(\sum_{t'=1}^{t}\sum_{k\in K^t_j} a_j \bar{z}_{kt't}\right)\frac{p^b_{kt't} + h(t-t') - \sum_{\substack{h\in J_k\\ h\neq j}} a_h p^p_{ht}}{a_j}\right] && \text{(C.2.22)}\\
&+ \sum_{t'=1}^{T} v_t \bar{s}_{t'} && \text{(C.2.23)}\\
&- \sum_{t=1}^{T}\left[h\sum_{t'=1}^{t}\sum_{l=t+1}^{T} \bar{x}_{t'l} + h\sum_{t'=1}^{t}\sum_{l=t+1}^{T}\sum_{k\in S^l} \bar{z}_{kt'l}\right] && \text{(C.2.24)}
\end{aligned}
$$

In $f(S')$, (C.2.21) corresponds to $\sum_{t=1}^{T} p^r_t x_t$, namely the total revenue from the sale of refurbished products. Similarly, (C.2.22) corresponds to $\sum_{j=1}^{M} p^p_{jt} q_{jt}$, the total revenue from harvested parts. Finally, (C.2.23) and (C.2.24) correspond to total salvaging revenue and total cores inventory holding cost.

In (C.2.22), the fractional term redefines the price of part j in terms of part bundle prices ($p^b_{kt't}$). In PDP, we have defined the price of a part bundle as the sum of the prices of parts included in the bundle,

$$p^b_{kt't} = p^b_{kt} - h(t-t') \quad \text{where} \quad p^b_{kt} = \sum_{j\in J_k} a_j p^p_{jt} \tag{C.2.25}$$

When we reverse these operations, we can write the price of a part j in bundle k,

$$\sum_{j\in J_k} a_j p^p_{jt} = p^b_{kt't} + h(t-t') \tag{C.2.26}$$

$$p^p_{jt} = \frac{(p^b_{kt't} + h(t-t') - \sum_{h\in J_k h\neq j} a_h p^p_{ht})}{a_j} \tag{C.2.27}$$

where h denotes the parts in bundle k other than part j.

To show the equivalence of $f(S')$ to $\overline{f}(S)$, first we will consider the first term, (C.2.21), in $f(S')$. We can rearrange this term as,

$$\sum_{t=1}^{T}\left(\sum_{t'=1}^{t}\bar{x}_{t't}p^{r}_{t't}+\sum_{t'=1}^{t}\bar{x}_{t't}h(t-t')\right) \tag{C.2.28}$$

$$\underbrace{\sum_{t=1}^{T}\sum_{t'=1}^{t}\bar{x}_{t't}p^{r}_{t't}}_{A}+\underbrace{\sum_{t=1}^{T}\sum_{t'=1}^{t}\bar{x}_{t't}h(t-t')}_{B} \tag{C.2.29}$$

Note that B gives the total cost of meeting refurbished product demand by carrying cores from previous periods and is equal to the first term of (C.2.24) ($h\sum_{t=1}^{T}\sum_{t'=1}^{t}\sum_{l=t+1}^{T}\bar{x}_{t'l}$) which gives the total holding cost of cores used for refurbished products. Hence, these two terms cancel out each other. A, on the other hand, corresponds to the total revenue from the sale of refurbished products and is equal to the first term in $\overline{f}(S)$.

Similarly, we can rearrange (C.2.22) as,

$$\sum_{t=1}^{T}\sum_{t'=1}^{t}\sum_{j=1}^{M}\sum_{k\in K_j^t}\bar{z}_{kt't}\left[p^{b}_{kt't}+h(t-t')-\left(\sum_{\substack{h\in J_k\\ h\neq j}}a_h p^{p}_{ht}\right)\right] \tag{C.2.30}$$

Note that the expression inside the brackets gives the total revenue from all pieces of a given part type j included in the bundle k. Hence, if we calculate this revenue for each part bundle $k \in K_j^t$ and add them up (the first summation), we get the total revenue from part type j in all bundles. Similarly, in the second summation when we add these total revenues for all part types ($j = 1, ..., M$), we simply obtain the total revenue of all bundles. Hence, instead of these two summations (summation over j and $k \in K_j^t$), we can substitute the summation over all part bundles with a positive demand ($k \in S^t$) and restate (C.2.30) as;

$$= \sum_{t=1}^{T}\sum_{t'=1}^{t}\sum_{k\in S^t} \bar{z}_{kt't}p^b_{ktt} \tag{C.2.31}$$

$$= \sum_{t=1}^{T}\sum_{t'=1}^{t}\sum_{k\in S^t} \bar{z}_{kt't}\Big(p^b_{kt't} + h(t-t')\Big) \tag{C.2.32}$$

$$= \underbrace{\sum_{t=1}^{T}\sum_{t'=1}^{t}\sum_{k\in S^t} \bar{z}_{kt't}p^b_{kt't}}_{C} + \underbrace{\sum_{t=1}^{T}\sum_{t'=1}^{t}\sum_{k\in S^t} \bar{z}_{kt't}h(t-t')}_{D} \tag{C.2.33}$$

Similar to A in (C.2.29), C corresponds to the total revenue from the sales of part bundles and is equivalent to the second term in $\overline{f}(S)$. D, on the other hand, is the cost of meeting parts demand by carrying cores from previous periods and simplify with the second term in (C.2.24) ($h\sum_{t=1}^{T}\sum_{k\in S^l}\sum_{t'=1}^{t}\sum_{l=t+1}^{T}\bar{z}_{kt'l}$).

Finally, (C.2.23) denotes the total salvaging revenue and is obviously equivalent to the last term of $\overline{f}(S)$.

This analysis shows that the objectives of the two models are equivalent for $S \in F(TFDM)$.

Part 2: In this part, we show that any solution in $F(DM)\backslash F(TFDM)$ is never optimal for DM.

First, observe that given the solution $S' \in F(DM)$ and the equivalent solution S for TFDM, the demand constraints for refurbished products and salvage in TFDM are satisfied.

TFDM demand constraints for refurbished products are:

$$\sum_{t'=1}^{t} \bar{x}_{t't} \leq D^r_t \quad \text{for } t = 1, ..., T \tag{C.2.34}$$

Substituting $x_t = \sum_{t'=1}^{t} \bar{x}_{t't}$ from (4.14), we can write (C.2.34) as,

$$x_t \leq D^r_t \quad \text{for } t = 1, ..., T \tag{C.2.35}$$

which is equivalent to the refurbished product demand constraint in DM. Thus, we can conclude that S' satisfies the demand constraints for refurbished products in TFDM.

TFDM salvage demand constraint is:

$$\sum_{t'=1}^{T} \bar{s}_{t'} \leq \sum_{t'=1}^{T} B_{t'} \tag{C.2.36}$$

This constraint is also satisfied since the total amount of salvaging cannot be larger than total amount of available cores.

To show the feasibility of S' for cores supply constraint of TFDM, we need the following equalities which are derived from (4.18), (4.14) and (4.15) respectively;

$$\sum_{t=t'+1}^{T} \bar{x}_{t't} - \sum_{l=1}^{t'-1} \bar{x}_{lt'} + \sum_{k \in S^t} \sum_{t=t'+1}^{T} \bar{z}_{kt't} - \sum_{k \in S^{t'}} \sum_{l=1}^{t'-1} \bar{z}_{klt'} = b_{t'} - b_{t'-1} \tag{C.2.37}$$

$$\sum_{l=1}^{t'} \bar{x}_{lt'} = x_{t'} \tag{C.2.38}$$

$$\sum_{l=1}^{t'} \sum_{k \in S^t} \bar{z}_{klt'} = z_{t'} \tag{C.2.39}$$

Rearranging (C.2.37) we obtain,

$$\sum_{t=t'}^{T} \bar{x}_{t't} - \sum_{l=1}^{t'} \bar{x}_{lt'} + \sum_{k \in S^t} \sum_{t=t'}^{T} \bar{z}_{kt't} - \sum_{k \in S^{t'}} \sum_{l=1}^{t'} \bar{z}_{klt'} = b_{t'} - b_{t'-1} \tag{C.2.40}$$

Since S' is feasible for DM, we also know,

$$b_{t'} - b_{(t'-1)} = B_{t'} - (z_{t'} + x_{t'} + s_{t'}) \tag{C.2.41}$$

Substituting $b_{t'} - b_{(t'-1)}$ from (C.2.41) in (C.2.40) we get,

$$\sum_{t=t'}^{T} \bar{x}_{t't} - \underbrace{\sum_{l=1}^{t'} \bar{x}_{lt'}}_{x_{t'}} + \sum_{k \in S^t} \sum_{t=t'}^{T} \bar{z}_{kt't} - \underbrace{\sum_{k \in S^{t'}} \sum_{l=1}^{t'} \bar{z}_{klt'}}_{z_{t'}} = B_{t'} - (z_{t'} + x_{t'} + \underbrace{s_{t'}}_{\bar{s}_{t'}}) \quad \text{(C.2.42)}$$

$$\sum_{t=t'}^{T} \bar{x}_{t't} - x_{t'} + \sum_{k \in S^t} \sum_{t=t'}^{T} \bar{z}_{kt't} - z_{t'} = B_{t'} - (z_{t'} + x_{t'} + \bar{s}_{t'}) \quad \text{(C.2.43)}$$

$$\sum_{t=t'}^{T} \bar{x}_{t't} + \sum_{k \in S^t} \sum_{t=t'}^{T} \bar{z}_{kt't} + \bar{s}_{t'} = B_{t'} \quad \text{(C.2.44)}$$

Equation (C.2.44) is equivalent to cores supply constraint of TFDM and this shows that S' also satisfies the cores inventory constraint of TFDM.

Finally, we consider the part bundle demand constraints which are written as;

$$\sum_{t'=1}^{t} \bar{z}_{kt't} \leq D^b_{kt} \quad \text{for } t = 1, ..., T \text{ and } \forall k \in S^t \quad \text{(C.2.45)}$$

Since S' is feasible for DM, we know that;

$$q_{jt} \leq D^p_{jt} \text{ for } j = 1, ..., M \text{ and } t = 1, ..., T \quad \text{(C.2.46)}$$

Substituting $q_{jt} = \sum_{t'=1}^{t} \sum_{k \in K^t_j} a_j \bar{z}_{kt't}$ from (4.16) in (C.2.46) we get;

$$\sum_{t'=1}^{t} \sum_{k \in K^t_j} a_j \bar{z}_{kt't} \leq D^p_{jt} \quad \text{(C.2.47)}$$

Note that in parameter derivation procedure (PDP) we determine the demand for each bundle by $D^b_{kt} = \min_{j \in J_k} \left(\frac{D_{jt}}{a_j} \right)$ where J_k denotes the set of parts included in bundle k (see PDP). Let k^m be the part bundle which includes all part types, $j = 1, ..., M$ and j^m be the part type with $\min_{j \in J_{k^m}} \left(\frac{D_{jt}}{a_j} \right)$, in other words the part type whose demand can be met with the smallest amount of dismantling. Then, we can write (C.2.47) for j^m as,

$$\sum_{t'=1}^{t} a_{j^m}\bar{z}_{k^m t' t} \leq D^p_{j^m t} \text{ for } t = 1, ..., T \tag{C.2.48}$$

$$\sum_{t'=1}^{t} \bar{z}_{k^m t' t} \leq \frac{D^p_{j^m t}}{a_{j^m}} \tag{C.2.49}$$

Right hand side of (C.2.49) is equal to $D^b_{k^m t}$, hence we have shown the feasibility of demand constraint for bundle k^m which includes all part types.

Similarly, let k^s denote the $(m - s + 1)^{th}$ highest revenue bundle which includes s part types and j^s be the part type with $\min_{j \in J_{k^s}} \left(\frac{D_{jt}}{a_j}\right)$. Now we can write (C.2.47) for j^s as,

$$\sum_{k \in K^t_{j^s}} \sum_{t'=1}^{t} a_{j^s}\bar{z}_{k^s t' t} \leq D^p_{j^s t} \text{ for } t = 1, ..., T \tag{C.2.50}$$

$$\sum_{k \in K^t_{j^s}} \sum_{t'=1}^{t} a_{j^s}\bar{z}_{k^s t' t} \leq a_{j^s} \sum_{k \in K^t_{j^s}} D^b_{kt} \tag{C.2.51}$$

$$\sum_{t'=1}^{t} (\bar{z}_{k^m t' t} + \bar{z}_{k^{m-1} t' t} + ... + \bar{z}_{k^s t' t}) \leq D^b_{k^m t} + D^b_{k^{m-1} t} + ... + D^b_{k^s t} \tag{C.2.52}$$

Note that (C.2.52) is the expanded form of (C.2.51). In this inequality, we know that $\bar{z}_{k^m t' t} \leq D^b_{k^m t}$ holds. However, this does not require $\bar{z}_{k^s t' t} \leq D^b_{k^s t}$ or, in general, $\bar{z}_{k t' t} \leq D^b_{kt}$ for $\forall k \in S^t$ which we need to show to prove the feasibility of S' for the bundle demand constraints. Note that we have derived (C.2.52) from the parts demand constraints of DM and these constraints only require that the total amount of sales of a part type is smaller than its demand. In other words, for a given part type a solution where its total sales is lower than its total harvested pieces even if there is more demand for it, is feasible in DM. Given this, it is easy to see that the feasible region defined by (C.2.50) is larger than the one defined by (C.2.45).

Nevertheless, in an optimal solution to DM, all the part pieces harvested will be sold if there is demand for them. This implies that, in the corresponding TFDM solution, the demand of higher revenue part bundles will be met first and the remanufacturer will never dismantle for the part types included in lower revenue bundles before meeting all demand of a part type included in a higher revenue bundle. Hence, for all optimal solutions to DM, $\bar{z}_{kt't} \leq D^b_{kt}$ holds for $\forall k \in S^t$ given (C.2.52) and (C.2.49). In other words, all optimal solutions to DM is feasible for TFDM and $F(DM) \backslash F(TFDM)$ only consists of non-optimal solution points to DM.

Thus, along with the previous results from Part 1, we can conclude that the optimal solution of TFDM is also optimal for DM. ■

Proof of Theorem 2. We show the optimality of TPP by LP duality. In general, a basic feasible solution for a transportation problem is optimal when the objective equation coefficients (z-values) in the simplex tableau are nonnegative ($u_i+v_j-c_{ij} \geq 0$). In this inequality, u_i and v_j are simplex multipliers and denote the implicit prices (dual variables) associated with i^{th} supply constraint (or row in a transportation tableau) and j^{th} demand constraint (or column) and c_{ij} denote the objective function coefficient of the associated variable. Hence, to show the optimality of TPP, first we define the dual variables (simplex multipliers) associated with each constraint in TFDM and then show the nonnegativity of the z-values. We denote the dual variables associated with the cores supply constraints in TFDM by $u_{t'}$ and the dual variables associated with the demand constraints by w_{dt}. In our transportation formulation, we denote different demand types (refurbished product demand, part bundles demand and salvage demand) by different symbols, namely with $\bar{x}$, $\bar{z}$ and $\bar{s}$ respectively. However; for the sake of brevity, in this section, we will use a single symbol, X, and a single type of index, d, to denote all demand types. Particularly, we denote the amount of demand type d met in period t from the cores of period t' by $X_{dt't}$ and its unit revenue and demand by $P_{dt't}$ and D_{dt} respectively, where $d \in DT^t = \{rfpr, b1, ..., bM, slv\}$. In the set of demand types in period t, DT^t, $rfpr$ denotes refurbished products, b denotes part bundles with positive demand and slv denotes salvaging.

In the procedures, $Proc_u$ and $Proc_w$, described below, we use the transportation tableau 4.3 for clarification.

Determination of $u_{t'}$

Given a solution S on the transportation tableau, we find $u_{t'}$ by procedure $Proc_u$. We repeat the procedure for all $t' = 1, ..., T$.

To ease understanding, before the formal procedure, we provide the intuition behind $Proc_u$. To determine $u_{t'}$, we consider the change in the objective function value by a unit increase in the amount of available cores in period t' (RHSs of the cores supply constraints in TFDM). An additional core in period t' will be first used to meet one more unit of a not fully satisfied demand type for some period $t^* = t', ..., T$ (condition B in $Proc_u$) if there is any such demand type. If this is the case, then $u_{t'}$ will be equal to the unit revenue associated with this demand type ($P_{d^*kt^*}$ as defined in $Proc_u$). If there is no such demand type, the additional core can be used to meet one more unit of a demand type which is partially met from periods before t' (Condition A). In this case, we can avoid the associated holding cost and save one core in the period from where the core is carried. This saved core can then be used to meet another demand type which is not fully met. Therefore, in this case $u_{t'}$ will be equal to the sum of the avoided holding cost and the unit revenue associated with the demand type met by the saved core ($P_{d^*kt^*} + (t' - k)h$ in $Proc_u$).

$Proc_u$:

Step 0: Set $k = t'$.

Step 1: In row k, find d^* and t^* such that

$$\operatorname*{argmax}_{\substack{k \le t^* \le T \\ d^* \in DT^{t^*}}} \left\{ P_{d^*kt^*} | \underbrace{\sum_{l=1}^{k-1} X_{d^*lt^*} > 0}_{A} \text{ or } \underbrace{\sum_{l=1}^{t^*} X_{d^*lt^*} < D_{d^*t^*}}_{B} \right\}$$

Step 2: If B holds, then $u_{t'} = P_{d^*kt^*} + (t' - k)h$.

If A holds, then find $l' = \min_{1 \le t \le t^*} \{t | X_{d^*tt^*} > 0\}$. Set $k = l'$ and go to Step 1.

Determination of w_{dt}

Given the solution S, we find w_{dt} by procedure $Proc_w$. We repeat the procedure for all $d \in DT^t$ and $t = 1, ..., T$.

Similar to $u_{t'}$, before the formal procedure, we provide the intuition behind $Proc_w$.

To determine w_{dt}, we investigate the change in the objective function value by a unit increase in the demand of type d for period t. To meet this additional demand, we need to sacrifice a unit of demand met from period t or a period before t, and save a core in this period. To make this exchange in the most profitable way, we need to find the minimum revenue demand type. $Proc_w$ seeks this minimum revenue demand type and equates w_{dt} to the difference between the unit revenue of this demand type ($P_{\hat{d}\hat{t}\hat{l}}$ in $Proc_w$) and the unit revenue from meeting the additional unit of demand type d for period t from the cores of period $\hat{t}$.

$Proc_w$:

- If $D_{dt} > \sum_{t'=1}^{t} X_{dt't}$, then $w_{dt} = 0$.
- If not (i.e. if $D_{dt} = \sum_{t'=1}^{t} X_{dt't}$),

  Step 0: Set $k' = t$ and $d' = d$.

  Step 1: Find $\hat{t} = \min_{1 \le l \le k'} \{l | X_{d'lk'} > 0\}$. In row (period) $\hat{t}$, find $\hat{l}$ and $\hat{d}$ such that[1]

$$\underset{\substack{\hat{t} \le \hat{l} \le T \\ \hat{d} \in DT^{\hat{l}}}}{\operatorname{argmin}} \{P_{\hat{d}\hat{t}\hat{l}} | X_{\hat{d}\hat{t}\hat{l}} > 0 \text{ and } P_{\hat{d}\hat{t}\hat{l}} < P_{d'\hat{t}k'}\}$$

  Step 2: If $\hat{t} = 1$ or $\sum_{m=1}^{\hat{l}} X_{\hat{d}m\hat{l}} < D_{\hat{d}\hat{l}}$, stop and set $w_{dt} = P_{d\hat{t}t} - P_{\hat{d}\hat{t}\hat{l}}$. Otherwise, set $k' = \hat{l}$ and $d' = \hat{d}$ and go to Step 1.

Given $u_{t'}$ and w_{dt} for $t' = 1, ..., T$, $t = t', ..., T$ and $d \in DT^t$ as above, now we need to show;

$$u_{t'} + w_{dt} - P_{dt't} \ge 0 \quad \Leftrightarrow \quad \underbrace{P_{d^*kt^*} + (t'-k)h}_{u_{t'}} + \underbrace{P_{d\hat{t}t} - P_{\hat{d}\hat{t}\hat{l}}}_{w_{dt}} - P_{dt't} \ge 0 \tag{C.2.53}$$

[1]If there is no such $\hat{l}$ and $\hat{d}$, in other words if there is no cell with a positive entry and unit revenue less than $P_{d'\hat{t}k'}$, this is the degeneracy case. In this case, starting from the previous period ($\hat{t} - 1$) search each period until a cell satisfying these two conditions is found and set the period where this cell is found as $\hat{t}$.

First note that $\hat{t} \leq t$; $\hat{t} \leq \hat{l}$; $k \leq t^*$ and $k \leq t' \leq t$ by definition. For $\hat{t}$ and t', on the other hand, there are two possibilities; (i) $\hat{t} \leq t'$ or (ii) $t' \leq \hat{t}$. In Case (ii), we have $k \leq t' \leq \hat{t}$ since k is smaller than or equal to t'. In case (i), again k is the smallest one since $k \leq \hat{t}$. The intuition behind this is that k is the first period where we find the max revenue demand type which is not fully met for some period $t^* = k, ..., T$; $\hat{t}$ is the first period from where the demand of type d' for period k' is first met ($X_{d'\hat{t}k'} > 0$). To find k, we start from period (row) t' and check the demand types for all periods (e.g. in row t' we check all columns $t', ..., T$) whose demand is either not fully met or partially met from periods before t'. Hence, demand type d' for period k' is always among the columns checked since its demand is partially met from $\hat{t} \leq t'$. Starting from period t', if we find k before $\hat{t}$, this implies that in period k there exists a demand type (d^*) with a higher unit revenue than the unit revenue of type d' in that period and whose demand for period t^* is not fully met. However, this is not possible since our solution procedure meets the demand for highest revenue types first and the demand for a less revenue type will never be met before all demand of a higher revenue option is fulfilled. Hence, $k \leq \hat{t}$.

In case (i): We can write the third term in (C.2.53), $P_{d\hat{t}t}$, as $P_{dt't} - (t' - \hat{t})h$. With this substitution (C.2.53) is written as;

$$P_{d^*kt^*} + (t' - k)h + P_{dt't} - (t' - \hat{t})h - P_{\hat{d}\hat{t}\hat{l}} - P_{dt't} \geq 0$$

$$P_{d^*kt^*} + \underbrace{(t' - k)h - (t' - \hat{t})h}_{(\hat{t}-k)h} - P_{\hat{d}\hat{t}\hat{l}} \geq 0$$

$$P_{d^*kt^*} + \underbrace{(\hat{t} - k)h - P_{\hat{d}\hat{t}\hat{l}}}_{-P_{\hat{d}k\hat{l}}} \geq 0$$

$$P_{d^*kt^*} - P_{\hat{d}k\hat{l}} \geq 0$$

Since $P_{d^*kt^*}$ is the max demand type in period k (see $Proc_u$), the last inequality always holds.

In case (ii): We substitute $P_{d\hat{t}t}$ with $P_{dt't} + (\hat{t} - t')h$ and (C.2.53) is written as;

$$P_{d^*kt^*} + (t' - k)h + P_{dt't} + (\hat{t} - t')h - P_{\hat{d}\hat{t}\hat{l}} - P_{dt't} \geq 0$$

$$P_{d^*kt^*} + \underbrace{(t'-k)h + (\hat{t}-t')h}_{(\hat{t}-k)h} - P_{\hat{d}\hat{t}\hat{l}} \geq 0$$

Then similar to case (i), we can conclude that (C.2.53) holds for all $t' = 1, ..., T$, $t = t', ..., T$ and $d \in DT^t$. ■

C.3 A Sample of Numerical Results

Tables C.1-C.2 and C.3-C.4 give the results for a sample of 50 replications under scenario 0 for static and dynamic implementations, respectively. In the tables, the first four columns give the refurbished product and the parts demand received over the length of one simulation period. The fifth and the following three columns show the amount of refurbished product demand and parts' demand met, respectively. Then columns nine to eleven denote the amount of outsourcing for parts while columns twelve to fourteen give the amount of excessive parts inventory over base stock at the end of the simulation period. Finally, the last two columns show the salvaging amount and the net revenue which is calculated as described in section 4.4.2.

Tables C.5-C.13, on the other hand, show the demand arrivals and the associated disposition decisions for one simulation period (30 days) under dynamic implementation. In the tables, the first two columns denote the simulation time and the order type, respectively. Here, P1, P2, P3, and Pr stand for Part 1, Part 2, Part 3, and refurbished product, respectively. Columns three to six show the remaining demands for refurbished products and parts at each simulation time. Shadow prices associated with parts inventory constraints and cores inventory balance constraints are given in columns seven and eight, respectively. Column nine shows the decision made for the received order. Note that when a refurbished product order is received, the firm decides only whether to accept or reject it while when a part order is received first he decides whether to dismantle a core or not and then determine whether to meet the order or not. In this column, acronyms, *DisAcc*, *DisRej*, *NoDisAcc*, *NoDisRej*, *Acc*, and *Rej* stand for the decisions of accepting a part order after dismantling a core, dismantling but rejecting a part order, accepting a part order without dismantling, rejecting a part order and not dismantling, accepting a refurbished product order, and rejecting a refurbished product order, respectively. Finally, columns ten to thirteen denote the remaining inventory of cores and each part type after each decision and column fourteen gives the cumulative revenue.

Demand Data from Simulation				Static Implementation											
Refurbish product demand received	Parts' demand received			Refurbish product demand met	Parts' demand met			Outsourced			Excessive inventory over base stock at the end of the period			Salvaged	Net Revenue
	P1	P2	P3		P1	P2	P3	P1	P2	P3	P1	P2	P3		
43	104	93	119	17	92	92	119	0	0	0	0	0	18	0	6624
47	103	92	141	17	92	92	137	0	0	0	0	0	0	0	6975
36	117	96	159	17	92	92	137	0	0	0	0	0	0	0	6975
27	122	91	145	17	92	91	137	0	0	0	0	1	0	0	6956
30	94	98	145	17	92	92	137	0	0	0	0	0	0	0	6975
38	98	89	137	17	92	89	137	0	0	0	0	3	0	0	6917
54	89	84	141	17	89	84	137	0	0	0	3	8	0	0	6741
48	119	87	132	17	92	87	132	0	0	0	0	5	5	0	6780
30	115	84	139	17	92	84	137	0	0	0	0	8	0	0	6819
43	97	83	153	17	92	83	137	0	0	0	0	9	0	0	6800
41	112	83	141	17	92	83	137	0	0	0	0	9	0	0	6800
39	113	89	145	17	92	89	137	0	0	0	0	3	0	0	6917
43	99	89	141	17	92	89	137	0	0	0	0	3	0	0	6917
34	110	92	159	17	92	92	137	0	0	0	0	0	0	0	6975
32	97	91	152	17	92	91	137	0	0	0	0	1	0	0	6956
42	112	99	142	17	92	92	137	0	0	0	0	0	0	0	6975
30	129	87	156	17	92	87	137	0	0	0	0	5	0	0	6878
40	98	103	145	17	92	92	137	0	0	0	0	0	0	0	6975
38	104	83	152	17	92	83	137	0	0	0	0	9	0	0	6800
32	96	82	149	17	92	82	137	0	0	0	0	10	0	0	6780
42	105	68	139	17	92	68	137	0	0	0	0	24	0	0	6507
37	114	104	166	17	92	92	137	0	0	0	0	0	0	0	6975
35	119	102	146	17	92	92	137	0	0	0	0	0	0	0	6975
29	112	95	152	17	92	92	137	0	0	0	0	0	0	0	6975
34	112	104	172	17	92	92	137	0	0	0	0	0	0	0	6975
42	108	91	131	17	92	91	131	0	0	0	0	1	6	0	6839
33	105	82	137	17	92	82	137	0	0	0	0	10	0	0	6780
41	96	87	162	17	92	87	137	0	0	0	0	5	0	0	6878
43	124	85	135	17	92	85	135	0	0	0	0	7	2	0	6800
43	110	86	147	17	92	86	137	0	0	0	0	6	0	0	6858
29	106	102	154	17	92	92	137	0	0	0	0	0	0	0	6975
30	96	91	147	17	92	91	137	0	0	0	0	1	0	0	6956
35	99	92	137	17	92	92	137	0	0	0	0	0	0	0	6975
39	87	93	137	17	87	92	137	0	0	0	5	0	0	0	6845
39	103	84	118	17	92	84	118	0	0	0	0	8	19	0	6449
41	115	86	126	17	92	86	126	0	0	0	0	6	11	0	6644
44	117	76	136	17	92	76	136	0	0	0	0	16	1	0	6644
39	111	86	153	17	92	86	137	0	0	0	0	6	0	0	6858
32	121	79	135	17	92	79	135	0	0	0	0	13	2	0	6683
29	116	98	168	17	92	92	137	0	0	0	0	0	0	0	6975
30	125	86	160	17	92	86	137	0	0	0	0	6	0	0	6858
39	103	73	146	17	92	73	137	0	0	0	0	19	0	0	6605
34	106	85	139	17	92	85	137	0	0	0	0	7	0	0	6839
52	125	79	136	17	92	79	136	0	0	0	0	13	1	0	6702
42	104	105	153	17	92	92	137	0	0	0	0	0	0	0	6975
30	100	94	135	17	92	92	135	0	0	0	0	0	2	0	6936

Table C.1: Results of scenario 0 for static implementation

Demand Data from Simulation				Static Implementation											
Refurbish product demand received	Parts' demand received			Refurbish product demand met	Parts' demand met			Outsourced			Excessive inventory over base stock at the end of the period			Salvaged	Net Revenue
	P1	P2	P3		P1	P2	P3	P1	P2	P3	P1	P2	P3		
31	102	80	136	17	92	80	136	0	0	0	0	12	1	0	6722
41	120	85	138	17	92	85	137	0	0	0	0	7	0	0	6839
31	112	99	158	17	92	92	137	0	0	0	0	0	0	0	6975
33	90	103	137	17	90	92	137	0	0	0	2	0	0	0	6923

Table C.2: Results of scenario 0 for static implementation (continued)

Demand Data from Simulation				Dynamic Implementation											
Refurbish product demand received	Parts' demand received			Refurbish product demand met	Parts' demand met			Outsourced			Excessive inventory over base stock at the end of the period			Salvaged	Net Revenue
	P1	P2	P3		P1	P2	P3	P1	P2	P3	P1	P2	P3		
43	104	93	119	25	79	78	114	3	2	1	0	0	0	0	6803
47	103	92	141	19	89	89	132	1	1	1	0	0	0	0	6920
36	117	96	159	17	93	93	138	1	1	1	0	0	0	0	6950
27	122	91	145	15	98	91	144	2	0	1	0	5	0	0	6880
30	94	98	145	21	85	85	126	1	1	1	0	0	0	0	6890
38	98	89	137	20	87	87	129	1	1	1	0	0	0	0	6905
54	89	84	141	21	85	84	126	1	0	1	0	0	0	0	6898
48	119	87	132	19	90	87	132	2	0	1	0	1	0	0	6894
30	115	84	139	19	90	84	132	2	0	1	0	4	0	0	6840
43	97	83	153	21	85	83	126	1	0	1	0	1	0	0	6878
41	112	83	141	20	87	83	129	1	0	1	0	3	0	0	6854
39	113	89	145	21	85	85	126	1	1	1	0	0	0	0	6890
43	99	89	141	20	87	87	129	1	1	1	0	0	0	0	6905
34	110	92	159	18	91	91	135	1	1	1	0	0	0	0	6935
32	97	91	152	19	89	89	132	1	1	1	0	0	0	0	6920
42	112	99	142	18	91	91	135	1	1	1	0	0	0	0	6935
30	129	87	156	17	93	87	138	1	0	1	0	5	0	0	6860
40	98	103	145	20	87	87	129	1	1	1	0	0	0	0	6905
38	104	83	152	20	87	83	129	1	0	1	0	3	0	0	6854
32	96	82	149	22	83	82	123	1	0	1	0	0	0	0	6883
42	105	68	139	29	69	68	102	1	0	1	0	0	0	0	6778
37	114	104	166	17	93	93	138	1	1	1	0	0	0	0	6950
35	119	102	146	16	95	95	141	1	1	1	0	0	0	0	6965
29	112	95	152	15	97	95	144	1	0	1	0	1	0	0	6968
34	112	104	172	14	99	99	147	1	1	1	0	0	0	0	6995
42	108	91	131	20	87	87	129	1	1	1	0	0	0	0	6905
33	105	82	137	21	85	82	126	1	0	1	0	2	0	0	6859
41	96	87	162	20	87	87	129	1	1	1	0	0	0	0	6905
43	124	85	135	21	85	85	126	1	1	1	0	0	0	0	6890
43	110	86	147	23	81	81	120	1	1	1	0	0	0	0	6860
29	106	102	154	16	95	95	141	1	1	1	0	0	0	0	6965
30	96	91	147	19	89	89	132	1	1	1	0	0	0	0	6920
35	99	92	137	19	89	89	132	1	1	1	0	0	0	0	6920
39	87	93	137	21	85	87	126	1	3	1	0	0	0	0	6875
39	103	84	118	22	83	83	118	1	1	0	0	0	4	0	6800
41	115	86	126	24	79	79	117	1	1	1	0	0	0	0	6845
44	117	76	136	22	83	76	123	1	0	1	0	6	0	0	6766
39	111	86	153	24	79	79	117	1	1	1	0	0	0	0	6845
32	121	79	135	21	85	79	126	1	0	1	0	5	0	0	6800
29	116	98	168	14	99	98	147	1	0	1	0	0	0	0	7003
30	125	86	160	19	89	86	132	1	0	1	0	2	0	0	6889
39	103	73	146	26	75	73	111	1	0	1	0	1	0	0	6803
34	106	85	139	23	83	81	120	3	1	1	0	0	0	0	6840
52	125	79	136	23	81	79	120	1	0	1	0	1	0	0	6848
42	104	105	153	16	95	95	141	1	1	1	0	0	0	0	6965
30	100	94	135	18	95	92	135	5	2	1	0	0	0	0	6883

Table C.3: Results of scenario 0 for dynamic implementation

Demand Data from Simulation				Dynamic Implementation												
Refurbish product demand received	Parts' demand received			Refurbish product demand met	Parts' demand met			Outsourced			Excessive inventory over base stock at the end of the period			Salvaged	Net Revenue
	P1	P2	P3		P1	P2	P3	P1	P2	P3	P1	P2	P3		
31	102	80	136	22	83	80	123	1	0	1	0	2	0	0	6844
41	120	85	138	21	85	85	126	1	1	1	0	0	0	0	6890
31	112	99	158	16	95	95	141	1	1	1	0	0	0	0	6965
33	90	103	137	21	85	85	126	1	1	1	0	0	0	0	6890

Table C.4: Results of scenario 0 for dynamic implementation (continued)

Simulation	Order	Remaining Demands				Shadow Price			Remaining Inventory				Total
Time	Type	Refr.Prod.	P1	P2	P3	Parts	Cores	Decision	Cores	P1	P2	P3	Revenue
0.058	P3	38.92	107.79	91.82	145.72	-15	100	NoDisAcc	63	27	23	34	15
0.070	P1	38.91	107.75	91.78	145.66	-20	100	NoDisAcc	63	26	23	34	35
0.085	P3	38.89	107.69	91.74	145.59	-15	100	NoDisAcc	63	26	23	33	50
0.111	Pr	38.86	107.60	91.66	145.46	-	100	Acc	62	26	23	33	150
0.231	P2	38.70	107.17	91.29	144.87	-7.5	100	NoDisAcc	62	26	22	33	165
0.347	P1	38.55	106.75	90.94	144.31	-20	100	NoDisAcc	62	25	22	33	185
0.472	P1	38.39	106.30	90.55	143.70	-20	100	HarvAcc	61	26	24	36	205
0.475	P1	38.38	106.29	90.54	143.69	-20	100	NoDisAcc	61	25	24	36	225
0.542	P2	38.30	106.05	90.34	143.36	-7.5	100	NoDisAcc	61	25	23	36	240
0.600	P3	38.22	105.84	90.16	143.08	-15	100	NoDisAcc	61	25	23	35	255
0.788	Pr	37.98	105.16	89.58	142.16	-	100	Acc	60	25	23	35	355
0.792	P3	37.97	105.15	89.57	142.15	-15	100	NoDisAcc	60	25	23	34	370
0.796	P2	37.97	105.13	89.56	142.12	-7.5	100	NoDisAcc	60	25	22	34	385
0.819	P3	37.94	105.05	89.49	142.01	-15	100	NoDisAcc	60	25	22	33	400
0.895	P1	37.84	104.78	89.26	141.65	-20	100	DisAcc	59	26	24	36	420
1.116	P3	37.55	103.98	88.58	140.57	-15	100	NoDisAcc	59	26	24	35	435
1.161	Pr	37.49	103.82	88.44	140.35	-	100	Acc	58	26	24	35	535
1.182	P1	37.46	103.74	88.38	140.25	-20	100	NoDisAcc	58	25	24	35	555
1.204	P3	37.44	103.67	88.31	140.14	-15	100	NoDisAcc	58	25	24	34	570
1.212	P3	37.42	103.64	88.28	140.10	-15	100	NoDisAcc	58	25	24	33	585
1.244	P3	37.38	103.52	88.19	139.95	-15	100	NoDisAcc	58	25	24	32	600
1.425	P3	37.15	102.87	87.63	139.06	-15	100	NoDisAcc	58	25	24	31	615
1.531	P1	37.01	102.49	87.31	138.55	-20	100	NoDisAcc	58	24	24	31	635
1.570	P1	36.96	102.35	87.19	138.36	-20	100	NoDisAcc	58	23	24	31	655
1.772	P3	36.70	101.62	86.57	137.38	-15	100	NoDisAcc	58	23	24	30	670
1.825	P3	36.63	101.43	86.40	137.12	-15	100	NoDisAcc	58	23	24	29	685
2.310	P3	36.00	99.68	84.92	134.76	-15	100	NoDisAcc	58	23	24	28	700
2.489	Pr	35.77	99.04	84.37	133.89	-	100	Acc	57	23	24	28	800
2.572	P1	35.66	98.74	84.11	133.49	-20	100	NoDisAcc	57	22	24	28	820
2.582	P3	35.64	98.71	84.08	133.44	-15	100	NoDisAcc	57	22	24	27	835
2.693	P1	35.50	98.30	83.74	132.89	-20	100	NoDisAcc	57	21	24	27	855
2.746	P1	35.43	98.11	83.58	132.64	-20	100	NoDisAcc	57	20	24	27	875
2.943	P2	35.17	97.40	82.97	131.68	-7.5	100	NoDisAcc	57	20	23	27	890
3.015	Pr	35.08	97.15	82.76	131.33	-	100	Acc	56	20	23	27	990
3.271	P3	34.75	96.23	81.97	130.08	-15	100	NoDisAcc	56	20	23	26	1005
3.279	P2	34.74	96.20	81.95	130.04	-7.5	100	NoDisAcc	56	20	22	26	1020
3.427	P1	34.55	95.66	81.49	129.32	-20	100	DisAcc	55	21	24	29	1040
3.598	P3	34.32	95.05	80.97	128.49	-15	100	NoDisAcc	55	21	24	28	1055
3.608	P1	34.31	95.01	80.94	128.44	-20	100	NoDisAcc	55	20	24	28	1075
3.631	P1	34.28	94.93	80.86	128.33	-20	100	NoDisAcc	55	19	24	28	1095
3.656	P1	34.25	94.84	80.79	128.21	-20	100	NoDisAcc	55	18	24	28	1115
3.686	P1	34.21	94.73	80.70	128.06	-20	100	NoDisAcc	55	17	24	28	1135
3.722	P3	34.16	94.60	80.59	127.89	-15	100	NoDisAcc	55	17	24	27	1150
3.764	P1	34.11	94.45	80.46	127.68	-20	100	NoDisAcc	55	16	24	27	1170

Table C.5: Disposition decisions under dynamic implementation for one simulation period

Simulation	Order	Remaining Demands				Shadow Price			Remaining Inventory				Total
Time	Type	Refr.Prod.	P1	P2	P3	Parts	Cores	Decision	Cores	P1	P2	P3	Revenue
3.768	Pr	34.10	94.43	80.44	127.66	-	100	Acc	54	16	24	27	1270
3.814	P2	34.04	94.27	80.30	127.44	-7.5	100	NoDisAcc	54	16	23	27	1285
3.902	P1	33.93	93.95	80.03	127.01	-20	100	NoDisAcc	54	15	23	27	1305
3.912	P1	33.91	93.92	80	126.96	-20	100	NoDisAcc	54	14	23	27	1325
3.960	P1	33.85	93.75	79.86	126.73	-20	100	NoDisAcc	54	13	23	27	1345
4.036	P3	33.75	93.47	79.62	126.36	-15	100	NoDisAcc	54	13	23	26	1360
4.100	P3	33.67	93.24	79.43	126.05	-15	100	NoDisAcc	54	13	23	25	1375
4.125	P3	33.64	93.15	79.35	125.93	-15	100	NoDisAcc	54	13	23	24	1390
4.187	P3	33.56	92.93	79.16	125.62	-15	100	NoDisAcc	54	13	23	23	1405
4.191	P1	33.55	92.91	79.15	125.60	-20	100	NoDisAcc	54	12	23	23	1425
4.202	P3	33.54	92.87	79.11	125.55	-15	100	NoDisAcc	54	12	23	22	1440
4.256	P1	33.47	92.68	78.95	125.29	-20	100	NoDisAcc	54	11	23	22	1460
4.280	P2	33.44	92.59	78.88	125.17	-7.5	100	NoDisAcc	54	11	22	22	1475
4.409	P3	33.27	92.13	78.48	124.54	-15	100	DisAcc	53	13	24	24	1490
4.487	P3	33.17	91.85	78.24	124.16	-15	100	NoDisAcc	53	13	24	23	1505
4.610	P3	33.01	91.40	77.86	123.56	-15	100	NoDisAcc	53	13	24	22	1520
4.708	P3	32.88	91.05	77.56	123.09	-15	100	NoDisAcc	53	13	24	21	1535
4.708	P1	32.88	91.05	77.56	123.09	-20	100	NoDisAcc	53	12	24	21	1555
4.733	P3	32.85	90.96	77.48	122.96	-15	100	NoDisAcc	53	12	24	20	1570
4.879	P1	32.66	90.43	77.04	122.25	-20	100	NoDisAcc	53	11	24	20	1590
5.000	P1	32.50	90.00	76.67	121.66	-20	100	NoDisAcc	53	10	24	20	1610
5.011	P3	32.49	89.96	76.63	121.61	-15	100	NoDisAcc	53	10	24	19	1625
5.049	P2	32.44	89.83	76.52	121.43	-7.5	100	NoDisAcc	53	10	23	19	1640
5.088	P3	32.39	89.68	76.40	121.24	-15	100	NoDisAcc	53	10	23	18	1655
5.143	P1	32.31	89.49	76.23	120.97	-20	100	NoDisAcc	53	9	23	18	1675
5.184	P3	32.26	89.34	76.10	120.77	-15	100	NoDisAcc	53	9	23	17	1690
5.201	P2	32.24	89.28	76.05	120.69	-7.5	100	NoDisAcc	53	9	22	17	1705
5.208	P1	32.23	89.25	76.03	120.65	-20	100	DisAcc	52	10	24	20	1725
5.366	P2	32.03	88.68	75.55	119.89	-7.5	100	NoDisAcc	52	10	23	20	1740
5.375	P1	32.01	88.65	75.52	119.84	-20	100	NoDisAcc	52	9	23	20	1760
5.500	P3	31.85	88.20	75.13	119.23	-15	100	NoDisAcc	52	9	23	19	1775
5.655	Pr	31.65	87.64	74.66	118.48	-	100	Acc	51	9	23	19	1875
5.847	P3	31.40	86.95	74.07	117.54	-15	100	NoDisAcc	51	9	23	18	1890
5.861	P3	31.38	86.90	74.03	117.48	-15	100	NoDisAcc	51	9	23	17	1905
5.944	P1	31.27	86.60	73.77	117.07	-20	100	NoDisAcc	51	8	23	17	1925
5.954	P1	31.26	86.57	73.74	117.03	-20	100	NoDisAcc	51	7	23	17	1945
5.984	P3	31.22	86.46	73.65	116.88	-15	100	NoDisAcc	51	7	23	16	1960
6.123	P2	31.04	85.96	73.22	116.20	-7.5	100	NoDisAcc	51	7	22	16	1975
6.237	P1	30.89	85.55	72.87	115.65	-20	100	DisAcc	50	8	24	19	1995
6.250	P3	30.87	85.50	72.83	115.58	-15	100	NoDisAcc	50	8	24	18	2010
6.283	P2	30.83	85.38	72.73	115.42	-7.5	100	NoDisAcc	50	8	23	18	2025
6.517	P3	30.53	84.54	72.01	114.28	-15	100	NoDisAcc	50	8	23	17	2040
6.573	P3	30.46	84.34	71.84	114.01	-15	100	NoDisAcc	50	8	23	16	2055
6.787	P2	30.18	83.57	71.19	112.97	-7.5	100	NoDisAcc	50	8	22	16	2070

Table C.6: Disposition decisions under dynamic implementation for one simulation period (continued)

Simulation Time	Order Type	Remaining Demands				Shadow Price		Decision	Remaining Inventory				Total Revenue
		Refr.Prod.	P1	P2	P3	Parts	Cores		Cores	P1	P2	P3	
6.834	P3	30.12	83.40	71.04	112.74	-15	100	DisAcc	49	10	24	18	2085
6.888	P1	30.05	83.27	70.88	112.48	-20	100	NoDisAcc	49	9	24	18	2105
7.035	P3	29.85	82.67	70.43	111.76	-15	100	NoDisAcc	49	9	24	17	2120
7.049	P3	29.84	82.62	70.38	111.69	-15	100	NoDisAcc	49	9	24	16	2135
7.057	P3	29.83	82.59	70.36	111.65	-15	100	NoDisAcc	49	9	24	15	2150
7.179	P2	29.67	82.16	69.99	111.07	-7.5	100	NoDisAcc	49	9	23	15	2165
7.219	P2	29.62	82.01	69.86	110.87	-7.5	100	NoDisAcc	49	9	22	15	2180
7.263	P3	29.56	81.86	69.73	110.66	-15	100	DisAcc	48	11	24	17	2195
7.278	P2	29.54	81.80	69.68	110.58	-7.5	100	NoDisAcc	48	11	23	17	2210
7.450	P1	29.32	81.18	69.15	109.74	-20	100	NoDisAcc	48	10	23	17	2230
7.517	P1	29.23	80.94	68.95	109.42	-20	100	NoDisAcc	48	9	23	17	2250
7.523	P2	29.22	80.92	68.93	109.39	-7.5	100	NoDisAcc	48	9	22	17	2265
7.527	P2	29.22	80.90	68.92	109.37	-7.5	100	DisAcc	47	11	23	20	2280
7.567	P2	29.16	80.76	68.80	109.17	-7.5	100	NoDisAcc	47	11	22	20	2295
7.623	Pr	29.09	80.56	68.62	108.90	-	100	Acc	46	11	22	20	2395
7.769	P1	28.90	80.03	68.17	108.19	-20	100	DisAcc	45	12	24	23	2415
7.821	P1	28.83	79.84	68.02	107.94	-20	100	NoDisAcc	45	11	24	23	2435
7.849	P2	28.80	79.74	67.93	107.80	-7.5	100	NoDisAcc	45	11	23	23	2450
7.992	P2	28.61	79.23	67.49	107.11	-7.5	100	NoDisAcc	45	11	22	23	2465
8.040	P2	28.55	79.06	67.34	106.87	-7.5	100	DisAcc	44	13	23	26	2480
8.054	Pr	28.53	79.00	67.30	106.80	-	100	Acc	43	13	23	26	2580
8.186	P3	28.36	78.53	66.90	106.16	-15	100	NoDisAcc	43	13	23	25	2595
8.234	P1	28.30	78.36	66.75	105.93	-20	100	NoDisAcc	43	12	23	25	2615
8.286	P1	28.23	78.17	66.59	105.68	-20	100	NoDisAcc	43	11	23	25	2635
8.291	P2	28.22	78.15	66.57	105.65	-7.5	100	NoDisAcc	43	11	22	25	2650
8.318	P3	28.19	78.05	66.49	105.52	-15	100	DisAcc	42	13	24	27	2665
8.397	P3	28.08	77.77	66.25	105.13	-15	100	NoDisAcc	42	13	24	26	2680
8.431	P3	28.04	77.65	66.15	104.97	-15	100	NoDisAcc	42	13	24	25	2695
8.446	P1	28.02	77.59	66.10	104.90	-20	100	NoDisAcc	42	12	24	25	2715
8.449	P3	28.02	77.58	66.09	104.88	-15	100	NoDisAcc	42	12	24	24	2730
8.480	P2	27.98	77.47	66.00	104.73	-7.5	100	NoDisAcc	42	12	23	24	2745
8.649	P1	27.76	76.86	65.48	103.91	-20	100	NoDisAcc	42	11	23	24	2765
8.933	P1	27.39	75.84	64.61	102.53	-20	100	NoDisAcc	42	10	23	24	2785
8.950	P2	27.37	75.78	64.55	102.44	-7.5	100	NoDisAcc	42	10	22	24	2800
9.032	P2	27.26	75.49	64.30	102.05	-7.5	100	DisAcc	41	12	23	27	2815
9.055	P1	27.23	75.40	64.23	101.93	-20	100	NoDisAcc	41	11	23	27	2835
9.174	P3	27.07	74.97	63.87	101.35	-15	100	NoDisAcc	41	11	23	26	2850
9.251	P2	26.97	74.70	63.63	100.98	-7.5	100	NoDisAcc	41	11	22	26	2865
9.279	P3	26.94	74.60	63.54	100.84	-15	100	DisAcc	40	13	24	28	2880
9.387	P3	26.80	74.21	63.22	100.32	-15	100	NoDisAcc	40	13	24	27	2895
9.516	P1	26.63	73.74	62.82	99.69	-20	100	NoDisAcc	40	12	24	27	2915
9.527	P2	26.62	73.70	62.79	99.64	-7.5	100	NoDisAcc	40	12	23	27	2930
9.618	P3	26.50	73.38	62.51	99.19	-15	100	NoDisAcc	40	12	23	26	2945
9.653	P3	26.45	73.25	62.40	99.02	-15	100	NoDisAcc	40	12	23	25	2960

Table C.7: Disposition decisions under dynamic implementation for one simulation period (continued)

Simulation	Order	Remaining Demands				Shadow Price			Remaining Inventory				Total
Time	Type	Refr.Prod.	P1	P2	P3	Parts	Cores	Decision	Cores	P1	P2	P3	Revenue
9.654	P1	26.45	73.25	62.40	99.02	-20	100	NoDisAcc	40	11	23	25	2980
9.835	P3	26.22	72.59	61.84	99.14	-15	100	NoDisAcc	40	11	23	24	2995
9.861	P1	26.18	72.50	61.76	99.01	-20	100	NoDisAcc	40	10	23	24	3015
10.009	P2	25.99	71.97	61.31	97.29	-7.5	100	NoDisAcc	40	10	22	24	3030
10.126	P3	25.84	71.55	60.95	96.72	-15	100	DisAcc	39	12	24	26	3045
10.210	P2	25.73	71.24	60.69	96.31	-7.5	100	NoDisAcc	39	12	23	26	3060
10.347	P1	25.55	70.75	60.27	95.64	-20	100	NoDisAcc	39	11	23	26	3080
10.404	P2	25.48	70.55	60.09	95.37	-7.5	100	NoDisAcc	39	11	22	26	3095
10.504	P1	25.35	70.19	59.79	94.88	-20	100	DisAcc	38	12	24	29	3115
10.554	P1	25.28	70.00	59.63	94.64	-20	100	NoDisAcc	38	11	24	29	3135
10.628	P3	25.18	69.74	59.41	94.28	-15	100	NoDisAcc	38	11	24	28	3150
10.641	P3	25.17	69.69	59.37	94.21	-15	100	NoDisAcc	38	11	24	27	3165
10.731	P2	25.05	69.37	59.09	93.78	-7.5	100	NoDisAcc	38	11	23	27	3180
10.740	P1	25.04	69.34	59.06	93.73	-20	100	NoDisAcc	38	10	23	27	3200
10.836	P2	24.91	68.99	58.77	93.27	-7.5	100	NoDisAcc	38	10	22	27	3215
10.958	P1	24.76	68.55	58.40	92.67	-20	100	DisAcc	37	11	24	30	3235
11.041	P2	24.65	68.25	58.14	92.27	-7.5	100	NoDisAcc	37	11	23	30	3250
11.066	P3	24.61	68.16	58.07	92.15	-15	100	NoDisAcc	37	11	23	29	3265
11.080	P3	24.60	68.11	58.02	92.08	-15	100	NoDisAcc	37	11	23	28	3280
11.458	P1	24.11	66.75	56.86	90.24	-20	100	NoDisAcc	37	10	23	28	3300
11.553	P1	23.98	66.41	56.57	89.78	-20	100	NoDisAcc	37	9	23	28	3320
11.576	P2	23.95	66.33	56.50	89.66	-7.5	100	NoDisAcc	37	9	22	28	3335
11.598	P3	23.92	66.25	56.43	89.56	-15	100	DisAcc	36	11	24	30	3350
11.628	P2	23.88	66.14	56.34	89.41	-7.5	100	NoDisAcc	36	11	23	30	3365
11.679	P2	23.82	65.96	56.18	89.16	-7.5	100	NoDisAcc	36	11	22	30	3380
11.685	P1	23.81	65.94	56.17	89.13	-20	100	DisAcc	35	12	24	33	3400
11.744	P2	23.73	65.72	55.98	88.85	-7.5	100	NoDisAcc	35	12	23	33	3415
11.861	P3	23.58	65.30	55.63	88.27	-15	100	NoDisAcc	35	12	23	32	3430
12.001	P3	23.40	64.80	55.20	87.60	-15	100	NoDisAcc	35	12	23	31	3445
12.058	P2	23.33	64.59	55.02	87.32	-7.5	100	NoDisAcc	35	12	22	31	3460
12.237	P2	23.09	63.95	54.48	86.45	-7.5	100	DisAcc	34	14	23	34	3475
12.238	P1	23.09	63.94	54.47	86.44	-20	100	NoDisAcc	34	13	23	34	3495
12.367	P3	22.92	63.48	54.07	85.81	-15	100	NoDisAcc	34	13	23	33	3510
12.486	P3	22.77	63.05	53.71	85.24	-15	100	NoDisAcc	34	13	23	32	3525
12.528	P2	22.71	62.90	53.58	85.03	-7.5	100	NoDisAcc	34	13	22	32	3540
12.554	P1	22.68	62.80	53.50	84.90	-20	100	DisAcc	33	14	24	35	3560
12.670	P1	22.53	62.39	53.15	84.34	-20	100	NoDisAcc	33	13	24	35	3580
12.812	P2	22.34	61.88	52.71	83.65	-7.5	100	NoDisAcc	33	13	23	35	3595
12.815	P1	22.34	61.87	52.70	83.64	-20	100	NoDisAcc	33	12	23	35	3615
12.880	P2	22.26	61.63	52.50	83.32	-7.5	100	NoDisAcc	33	12	22	35	3630
12.909	Pr	22.22	61.53	52.41	83.18	-	100	Acc	32	12	22	35	3730
12.918	P2	22.21	61.50	52.39	83.13	-7.5	100	NoDisAcc	32	12	21	35	3745
12.990	P3	22.11	61.24	52.17	82.78	-15	100	NoDisAcc	32	12	21	34	3760
13.004	P3	22.10	61.19	52.12	82.71	-15	100	DisAcc	31	14	23	36	3775

Table C.8: Disposition decisions under dynamic implementation for one simulation period (continued)

Simulation	Order	Remaining Demands				Shadow Price			Remaining Inventory				Total
Time	Type	Refr.Prod.	P1	P2	P3	Parts	Cores	Decision	Cores	P1	P2	P3	Revenue
13.054	P3	22.03	61.01	51.97	82.47	-15	100	NoDisAcc	31	14	23	35	3790
13.105	P3	21.96	60.82	51.81	82.22	-15	100	NoDisAcc	31	14	23	34	3805
13.120	P1	21.94	60.77	51.77	82.15	-20	100	NoDisAcc	31	13	23	34	3825
13.162	P3	21.89	60.62	51.64	81.94	-15	100	NoDisAcc	31	13	23	33	3840
13.384	P3	21.60	59.82	50.96	80.87	-15	100	NoDisAcc	31	13	23	32	3855
13.420	Pr	21.55	59.69	50.84	80.69	-	100	Acc	30	13	23	32	3955
13.425	P1	21.55	59.67	50.83	80.66	-20	100	NoDisAcc	30	12	23	32	3975
13.481	Pr	21.48	59.47	50.66	80.39	-	100	Acc	29	12	23	32	4075
13.603	P1	21.32	59.03	50.28	79.80	-20	100	NoDisAcc	29	11	23	32	4095
13.749	Pr	21.13	58.50	49.84	79.09	-	100	Acc	28	11	23	32	4195
13.760	P3	21.11	58.47	49.80	79.04	-15	100	NoDisAcc	28	11	23	31	4210
13.794	P3	21.07	58.34	49.70	78.87	-15	100	NoDisAcc	28	11	23	30	4225
14.032	P1	20.76	57.49	48.97	77.71	-20	100	NoDisAcc	28	10	23	30	4245
14.076	P3	20.70	57.33	48.84	77.50	-15	100	NoDisAcc	28	10	23	29	4260
14.272	P1	20.45	56.62	48.23	76.55	-20	100	NoDisAcc	28	9	23	29	4280
14.298	P1	20.41	56.53	48.15	76.42	-20	100	NoDisAcc	28	8	23	29	4300
14.361	P2	20.33	56.30	47.96	76.11	-7.5	100	NoDisAcc	28	8	22	29	4315
14.417	P3	20.26	56.10	47.79	75.84	-15	100	DisAcc	27	10	24	31	4330
14.471	P2	20.19	55.91	47.62	75.58	-7.5	100	NoDisAcc	27	10	23	31	4345
14.477	P2	20.18	55.88	47.61	75.55	-7.5	100	NoDisAcc	27	10	22	31	4360
14.520	P2	20.13	55.73	47.47	75.34	-7.5	100	DisAcc	26	12	23	34	4375
14.579	P1	20.05	55.52	47.29	75.05	-20	100	NoDisAcc	26	11	23	34	4395
14.656	P3	19.95	55.24	47.06	74.67	-15	100	NoDisAcc	26	11	23	33	4410
14.669	P2	19.93	55.19	47.02	74.61	-7.5	100	NoDisAcc	26	11	22	33	4425
14.697	P2	19.89	55.09	46.93	74.48	-7.5	100	DisAcc	25	13	23	36	4440
14.697	P3	19.89	55.09	46.93	74.47	-15	100	NoDisAcc	25	13	23	35	4455
14.810	P2	19.75	54.69	46.58	73.93	-7.5	100	NoDisAcc	25	13	22	35	4470
14.825	P2	19.73	54.63	46.54	73.85	-7.5	100	NoDisAcc	25	13	21	35	4485
14.865	P3	19.68	54.49	46.41	73.66	-15	100	NoDisAcc	25	13	21	34	4500
14.881	P3	19.66	54.43	46.37	73.58	-15	100	DisAcc	24	15	23	36	4515
14.939	P2	19.58	54.22	46.19	73.30	-7.5	100	NoDisAcc	24	15	22	36	4530
14.969	P3	19.54	54.11	46.10	73.15	-15	100	NoDisAcc	24	15	22	35	4545
15.026	P1	19.47	53.91	45.92	72.87	-20	100	NoDisAcc	24	14	22	35	4565
15.133	P2	19.33	53.52	45.59	72.35	-7.5	100	NoDisAcc	24	14	21	35	4580
15.283	P3	19.13	52.98	45.13	71.62	-15	100	NoDisAcc	24	14	21	34	4595
15.367	P3	19.02	52.68	44.88	71.22	-15	100	DisAcc	23	16	23	36	4610
15.387	P1	19.00	52.61	44.81	71.12	-20	100	NoDisAcc	23	15	23	36	4630
15.412	P1	18.96	52.52	44.74	70.99	-20	100	NoDisAcc	23	14	23	36	4650
15.507	P3	18.84	52.18	44.45	70.53	-15	100	NoDisAcc	23	14	23	35	4665
15.599	P2	18.72	51.85	44.17	70.09	-7.5	100	NoDisAcc	23	14	22	35	4680
15.604	P1	18.72	51.83	44.15	70.06	-20	100	NoDisAcc	23	13	22	35	4700
15.887	P3	18.35	50.81	43.28	68.68	-15	100	NoDisAcc	23	13	22	34	4715
15.906	P2	18.32	50.74	43.22	68.59	-7.5	100	NoDisAcc	23	13	21	34	4730
16.032	P2	18.16	50.29	42.84	67.98	-7.5	100	DisAcc	22	15	22	37	4745

Table C.9: Disposition decisions under dynamic implementation for one simulation period (continued)

Simulation	Order	Remaining Demands				Shadow Price			Remaining Inventory				Total
Time	Type	Refr.Prod.	P1	P2	P3	Parts	Cores	Decision	Cores	P1	P2	P3	Revenue
16.032	P3	18.16	50.29	42.84	67.98	-15	100	NoDisAcc	22	15	22	36	4760
16.066	P3	18.11	50.16	42.73	67.81	-15	100	NoDisAcc	22	15	22	35	4775
16.135	P2	18.02	49.91	42.52	67.48	-7.5	100	NoDisAcc	22	15	21	35	4790
16.137	Pr	18.02	49.91	42.51	67.47	-	115	Rej	22	15	21	35	4790
16.221	P2	17.91	49.61	42.26	67.06	-15	115	NoDisAcc	22	15	20	35	4805
16.297	P3	17.81	49.33	42.02	66.69	-15	115	NoDisAcc	22	15	20	34	4820
16.521	P3	17.52	48.53	41.34	65.60	-15	115	NoDisAcc	22	15	20	33	4835
16.533	P2	17.51	48.48	41.30	65.54	-15	115	DisAcc	21	17	21	36	4850
16.592	P1	17.43	48.27	41.12	65.25	-20	115	NoDisAcc	21	16	21	36	4870
16.593	P3	17.43	48.27	41.12	65.25	-15	115	NoDisAcc	21	16	21	35	4885
16.637	P1	17.37	48.11	40.98	65.03	-20	115	NoDisAcc	21	15	21	35	4905
16.671	P2	17.33	47.98	40.88	64.87	-15	115	NoDisAcc	21	15	20	35	4920
16.678	P1	17.32	47.96	40.85	64.83	-20	115	NoDisAcc	21	14	20	35	4940
16.892	P2	17.04	47.19	40.20	63.79	-15	115	NoDisAcc	21	14	19	35	4955
16.917	P3	17.01	47.10	40.12	63.67	-15	115	NoDisAcc	21	14	19	34	4970
17.022	P2	16.87	46.72	39.80	63.16	-15	115	NoDisAcc	21	14	18	34	4985
17.033	P1	16.86	46.68	39.77	63.11	-20	115	NoDisAcc	21	13	18	34	5005
17.180	P1	16.67	46.15	39.32	62.39	-20	115	NoDisAcc	21	12	18	34	5025
17.198	P1	16.64	46.09	39.26	62.30	-20	115	NoDisAcc	21	11	18	34	5045
17.400	P3	16.38	45.36	38.64	61.32	-15	115	NoDisAcc	21	11	18	33	5060
17.480	P2	16.28	45.07	38.40	60.93	-15	115	DisAcc	20	13	19	36	5075
17.523	P1	16.22	44.92	38.26	60.72	-20	115	NoDisAcc	20	12	19	36	5095
17.592	P2	16.13	44.67	38.05	60.39	-15	115	NoDisAcc	20	12	18	36	5110
17.681	P1	16.02	44.35	37.78	59.95	-20	100	NoDisAcc	20	11	18	36	5130
17.771	P3	15.90	44.03	37.50	59.52	-10	100	NoDisAcc	20	11	18	35	5145
17.819	P3	15.84	43.85	37.36	59.28	-15	115	NoDisAcc	20	11	18	34	5160
17.869	P3	15.77	43.67	37.20	59.04	-15	115	NoDisAcc	20	11	18	33	5175
17.995	P1	15.61	43.22	36.82	58.43	-20	115	DisAcc	19	12	20	36	5195
18.054	P1	15.53	43.00	36.63	58.14	-20	115	NoDisAcc	19	11	20	36	5215
18.262	Pr	15.26	42.26	36.00	57.12	-	115	Rej	19	11	20	36	5215
18.269	Pr	15.25	42.23	35.98	57.09	-	115	Rej	19	11	20	36	5215
18.315	P1	15.19	42.07	35.83	56.87	-20	100	NoDisAcc	19	10	20	36	5235
18.368	P1	15.12	41.88	35.67	56.61	-20	100	NoDisAcc	19	9	20	36	5255
18.506	P2	14.94	41.38	35.25	55.94	-15	100	NoDisAcc	19	9	19	36	5270
18.515	P1	14.93	41.35	35.22	55.89	-20	100	NoDisAcc	19	8	19	36	5290
18.686	P2	14.71	40.73	34.70	55.06	-15	100	NoDisAcc	19	8	18	36	5305
18.721	P2	14.66	40.60	34.59	54.89	-15	100	NoDisAcc	19	8	17	36	5320
18.849	P3	14.50	40.14	34.20	54.27	-10	100	NoDisAcc	19	8	17	35	5335
18.972	Pr	14.34	39.70	33.82	53.67	-	100	Acc	18	8	17	35	5435
19.048	P3	14.24	39.43	33.59	53.30	-15	115	NoDisAcc	18	8	17	34	5450
19.101	P1	14.17	39.24	33.42	53.04	-20	115	NoDisAcc	18	7	17	34	5470
19.169	P1	14.08	38.99	33.22	52.71	-20	115	NoDisAcc	18	6	17	34	5490
19.177	P1	14.07	38.96	33.19	52.67	-20	115	NoDisAcc	18	5	17	34	5510
19.196	P1	14.05	38.89	33.13	52.58	-20	115	NoDisAcc	18	4	17	34	5530

Table C.10: Disposition decisions under dynamic implementation for one simulation period (continued)

Simulation	Order	Remaining Demands				Shadow Price			Remaining Inventory				Total
Time	Type	Refr.Prod.	P1	P2	P3	Parts	Cores	Decision	Cores	P1	P2	P3	Revenue
19.203	P3	14.04	38.87	33.11	52.55	-15	115	NoDisAcc	18	4	17	33	5545
19.238	P2	13.99	38.75	33.01	52.38	-15	115	DisAcc	17	6	18	36	5560
19.242	P3	13.99	38.73	32.99	52.36	-15	115	NoDisAcc	17	6	18	35	5575
19.499	P3	13.65	37.80	32.20	51.10	-15	115	NoDisAcc	17	6	18	34	5590
19.504	P2	13.65	37.79	32.19	51.08	-15	115	NoDisAcc	17	6	17	34	5605
19.541	P3	13.60	37.65	32.07	50.90	-15	115	NoDisAcc	17	6	17	33	5620
19.618	P2	13.50	37.37	31.84	50.52	-15	115	DisAcc	16	8	18	36	5635
19.620	P1	13.49	37.37	31.83	50.51	-20	115	NoDisAcc	16	7	18	36	5655
19.646	P3	13.46	37.28	31.75	50.39	-15	115	NoDisAcc	16	7	18	35	5670
19.676	P1	13.42	37.17	31.66	50.25	-20	115	NoDisAcc	16	6	18	35	5690
19.731	P3	13.35	36.97	31.49	49.98	-15	115	NoDisAcc	16	6	18	34	5705
19.755	P3	13.32	36.88	31.42	49.86	-15	115	NoDisAcc	16	6	18	33	5720
20.003	P1	13.00	35.99	30.66	48.65	-20	115	DisAcc	15	7	20	36	5740
20.065	P1	12.92	35.77	30.47	48.35	-20	115	NoDisAcc	15	6	20	36	5760
20.103	P3	12.87	35.63	30.35	48.17	-15	115	NoDisAcc	15	6	20	35	5775
20.223	P3	12.71	35.20	29.98	47.58	-15	115	NoDisAcc	15	6	20	34	5790
20.252	P1	12.67	35.09	29.90	47.44	-20	115	NoDisAcc	15	5	20	34	5810
20.325	P3	12.58	34.83	29.67	47.08	-15	115	NoDisAcc	15	5	20	33	5825
20.355	P1	12.54	34.72	29.58	46.94	-20	115	DisAcc	14	6	22	36	5845
20.407	P3	12.47	34.54	29.42	46.69	-15	115	NoDisAcc	14	6	22	35	5860
20.410	P3	12.47	34.53	29.41	46.67	-15	115	NoDisAcc	14	6	22	34	5875
20.739	P1	12.04	33.34	28.40	45.07	-20	115	NoDisAcc	14	5	22	34	5895
20.904	P3	11.83	32.75	27.90	44.27	-15	115	NoDisAcc	14	5	22	33	5910
20.923	P2	11.80	32.68	27.84	44.18	-15	115	NoDisAcc	14	5	21	33	5925
21.033	P2	11.66	32.28	27.50	43.64	-15	115	DisAcc	13	7	22	36	5940
21.158	P3	11.50	31.83	27.12	43.03	-15	115	NoDisAcc	13	7	22	35	5955
21.179	P3	11.47	31.76	27.05	42.93	-15	115	NoDisAcc	13	7	22	34	5970
21.204	P3	11.44	31.67	26.98	42.81	-15	115	NoDisAcc	13	7	22	33	5985
21.209	P2	11.43	31.65	26.96	42.78	-15	115	NoDisAcc	13	7	21	33	6000
21.342	P3	11.26	31.17	26.55	42.14	-15	115	DisAcc	12	9	23	35	6015
21.526	P3	11.02	30.51	25.99	41.24	-15	115	NoDisAcc	12	9	23	34	6030
21.612	P1	10.90	30.20	25.72	40.82	-20	115	NoDisAcc	12	8	23	34	6050
21.659	Pr	10.84	30.03	25.58	40.59	-	115	Rej	12	8	23	34	6050
21.672	P1	10.83	29.98	25.54	40.53	-20	115	NoDisAcc	12	7	23	34	6070
21.819	P1	10.64	29.45	25.09	39.81	-20	115	NoDisAcc	12	6	23	34	6090
21.914	P3	10.51	29.11	24.80	39.35	-15	115	NoDisAcc	12	6	23	33	6105
22.043	P1	10.34	28.65	24.40	38.72	-20	115	NoDisAcc	12	5	23	33	6125
22.187	P1	10.16	28.13	23.96	38.02	-20	100	NoDisAcc	12	4	23	33	6145
22.237	P3	10.09	27.95	23.81	37.78	-15	100	NoDisAcc	12	4	23	32	6160
22.288	P1	10.03	27.76	23.65	37.53	-20	100	NoDisAcc	12	3	23	32	6180
22.364	Pr	9.93	27.49	23.42	37.16	-	100	Acc	11	3	23	32	6280
22.592	P3	9.63	26.67	22.72	36.05	-15	135	NoDisAcc	11	3	23	31	6295
22.645	P3	9.56	26.48	22.56	35.80	-15	135	NoDisAcc	11	3	23	30	6310
22.650	Pr	9.56	26.46	22.54	35.77	-	135	Rej	11	3	23	30	6310

Table C.11: Disposition decisions under dynamic implementation for one simulation period (continued)

Simulation	Order	Remaining Demands				Shadow Price			Remaining Inventory				Total
Time	Type	Refr.Prod.	P1	P2	P3	Parts	Cores	Decision	Cores	P1	P2	P3	Revenue
22.719	P1	9.47	26.21	22.33	35.43	-30	135	NoDisRej	11	3	23	30	6310
22.774	P2	9.39	26.01	22.16	35.17	-15	135	NoDisAcc	11	3	22	30	6325
22.863	P1	9.28	25.70	21.89	34.74	-30	135	NoDisRej	11	3	22	30	6325
23.030	P2	9.06	25.09	21.37	33.92	-15	135	NoDisAcc	11	3	21	30	6340
23.109	P1	8.96	24.81	21.13	33.54	-30	135	NoDisRej	11	3	21	30	6340
23.130	Pr	8.93	24.73	21.07	33.43	-	135	Rej	11	3	21	30	6340
23.588	P2	8.34	23.09	19.67	31.21	0	105	DisAcc	10	5	22	33	6355
23.709	P1	8.18	22.65	19.29	30.62	-30	135	NoDisRej	10	5	22	33	6355
23.709	P3	8.18	22.65	19.29	30.61	-15	135	NoDisAcc	10	5	22	32	6370
23.792	P3	8.07	22.35	19.04	30.21	-15	135	NoDisAcc	10	5	22	31	6385
23.806	P3	8.05	22.30	19.00	30.14	-15	105	NoDisAcc	10	5	22	30	6400
23.927	P3	7.90	21.86	18.62	29.56	-15	105	NoDisAcc	10	5	22	29	6415
23.948	P3	7.87	21.79	18.56	29.45	-15	105	NoDisAcc	10	5	22	28	6430
23.949	P3	7.87	21.78	18.56	29.45	-15	105	NoDisAcc	10	5	22	27	6445
23.974	Pr	7.83	21.70	18.48	29.33	-	105	Rej	10	5	22	27	6445
24.042	P2	7.75	21.45	18.27	29.00	0	105	NoDisAcc	10	5	21	27	6460
24.051	P3	7.73	21.42	18.25	28.95	-15	135	NoDisAcc	10	5	21	26	6475
24.413	P1	7.26	20.12	17.14	27.19	-30	105	DisRej	9	7	23	29	6475
24.466	P1	7.19	19.92	16.97	26.93	-30	105	NoDisRej	9	7	23	29	6475
24.476	P1	7.18	19.89	16.94	26.88	-30	105	NoDisRej	9	7	23	29	6475
24.536	P3	7.10	19.67	16.76	26.59	-15	105	NoDisAcc	9	7	23	28	6490
24.570	P3	7.06	19.55	16.65	26.43	-15	105	NoDisAcc	9	7	23	27	6505
24.611	P3	7.01	19.40	16.53	26.23	-15	105	NoDisAcc	9	7	23	26	6520
25.003	P1	6.50	17.99	15.32	24.32	-30	105	NoDisRej	9	7	23	26	6520
25.054	P3	6.43	17.81	15.17	24.07	-15	105	NoDisAcc	9	7	23	25	6535
25.129	P1	6.33	17.54	14.94	23.70	-30	105	NoDisRej	9	7	23	25	6535
25.152	P2	6.30	17.46	14.87	23.60	0	105	NoDisAcc	9	7	22	25	6550
25.397	P1	5.98	16.57	14.12	22.40	-30	105	NoDisRej	9	7	22	25	6550
25.464	P1	5.90	16.33	13.91	22.07	-30	105	NoDisRej	9	7	22	25	6550
25.548	P3	5.79	16.03	13.65	21.67	-15	105	NoDisAcc	9	7	22	24	6565
25.997	P2	5.20	14.41	12.28	19.48	0	105	NoDisAcc	9	7	21	24	6580
26.120	P1	5.04	13.97	11.90	18.88	-30	105	DisRej	8	9	23	27	6580
26.191	P3	4.95	13.71	11.68	18.54	-15	105	NoDisAcc	8	9	23	26	6595
26.198	P2	4.94	13.69	11.66	18.50	0	105	NoDisAcc	8	9	22	26	6610
26.200	P2	4.94	13.68	11.65	18.49	0	105	NoDisAcc	8	9	21	26	6625
26.301	P2	4.81	13.32	11.34	18.00	0	105	DisAcc	7	11	22	29	6640
26.396	P3	4.69	12.97	11.05	17.54	-15	105	NoDisAcc	7	11	22	28	6655
26.450	P3	4.61	12.78	10.89	17.28	-15	105	NoDisAcc	7	11	22	27	6670
26.493	P1	4.56	12.63	10.76	17.07	-30	105	NoDisRej	7	11	22	27	6670
26.574	Pr	4.45	12.33	10.51	16.67	-	105	Rej	7	11	22	27	6670
26.636	P1	4.37	12.11	10.32	16.37	-30	105	NoDisRej	7	11	22	27	6670
26.686	P1	4.31	11.93	10.16	16.13	-30	105	NoDisRej	7	11	22	27	6670
26.819	P1	4.14	11.45	9.75	15.48	-30	105	NoDisRej	7	11	22	27	6670
27.021	P2	3.87	10.73	9.14	14.50	0	105	NoDisAcc	7	11	21	27	6685

Table C.12: Disposition decisions under dynamic implementation for one simulation period (continued)

Simulation	Order	Remaining Demands				Shadow Price			Remaining Inventory				Total
Time	Type	Refr.Prod.	P1	P2	P3	Parts	Cores	Decision	Cores	P1	P2	P3	Revenue
27.051	P1	3.83	10.62	9.04	14.35	-30	105	DisRej	6	13	23	30	6685
27.085	P3	3.79	10.50	8.94	14.19	-15	105	NoDisAcc	6	13	23	29	6700
27.116	P3	3.75	10.38	8.85	14.04	-15	105	NoDisAcc	6	13	23	28	6715
27.473	P1	3.29	9.10	7.75	12.30	-30	105	NoDisRej	6	13	23	28	6715
27.595	Pr	3.13	8.66	7.38	11.71	-	105	Rej	6	13	23	28	6715
27.668	Pr	3.03	8.39	7.15	11.35	-	105	Rej	6	13	23	28	6715
27.690	P1	3.00	8.32	7.08	11.24	-30	105	NoDisRej	6	13	23	28	6715
27.718	P1	2.97	8.22	7.00	11.11	-30	105	NoDisRej	6	13	23	28	6715
27.756	P3	2.92	8.08	6.88	10.92	-15	105	NoDisAcc	6	13	23	27	6730
27.927	P3	2.70	7.46	6.36	10.09	-15	105	NoDisAcc	6	13	23	26	6745
28.181	P3	2.37	6.55	5.58	8.85	-15	105	NoDisAcc	6	13	23	25	6760
28.184	P3	2.36	6.54	5.57	8.84	-15	105	NoDisAcc	6	13	23	24	6775
28.194	P3	2.35	6.50	5.54	8.79	-15	105	NoDisAcc	6	13	23	23	6790
28.232	P3	2.30	6.36	5.42	8.60	-15	105	NoDisAcc	6	13	23	22	6805
28.586	P2	1.84	5.09	4.34	6.88	0	105	NoDisAcc	6	13	22	22	6820
28.666	P2	1.74	4.80	4.09	6.49	0	105	NoDisAcc	6	13	21	22	6835
28.745	P2	1.63	4.52	3.85	6.11	0	105	DisAcc	5	15	22	25	6850
28.854	P3	1.49	4.13	3.52	5.58	-15	105	NoDisAcc	5	15	22	24	6865
28.953	Pr	1.36	3.77	3.21	5.10	-	105	Rej	5	15	22	24	6865
28.958	P1	1.36	3.75	3.20	5.07	-30	105	NoDisRej	5	15	22	24	6865
29.066	P3	1.21	3.36	2.86	4.55	-15	105	NoDisAcc	5	15	22	23	6880
29.113	P2	1.15	3.19	2.72	4.32	0	105	NoDisAcc	5	15	21	23	6895
29.114	P2	1.15	3.19	2.72	4.31	0	105	DisAcc	4	17	22	26	6910
29.250	P3	0.98	2.70	2.30	3.65	-15	105	NoDisAcc	4	17	22	25	6925
29.290	P1	0.92	2.56	2.18	3.46	-30	105	NoDisRej	4	17	22	25	6925
29.323	P2	0.88	2.44	2.08	3.29	0	105	NoDisAcc	4	17	21	25	6940
29.405	P3	0.77	2.14	1.83	2.90	-15	105	DisAcc	3	19	23	27	6955
29.435	P3	0.73	2.03	1.73	2.75	-15	105	NoDisAcc	3	19	23	26	6970
29.542	P1	0.60	1.65	1.40	2.23	-30	127.5	NoDisRej	3	19	23	26	6970
29.606	P1	0.51	1.42	1.21	1.92	-30	127.5	NoDisRej	3	19	23	26	6970
29.626	P3	0.49	1.35	1.15	1.82	-22.5	127.5	NoDisRej	3	19	23	26	6970
29.768	Pr	0.30	0.84	0.71	1.13	-	127.5	Rej	3	19	23	26	6970
29.937	P2	0.08	0.23	0.19	0.31	0	127.5	NoDisAcc	3	19	22	26	6985

Table C.13: Disposition decisions under dynamic implementation for one simulation period (continued)

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