Non-cooperative Games on Dynamic Claims Problems

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Ercan Aslan

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APPROVED BY

Assoc. Prof. Dr. Özgür Kıbrıs..
(Thesis Supervisor)

Assist. Prof. Dr. Fırat İnceoğlu..

Assist. Prof. Dr. Özge Kemahlıoğlu ....

DATE OF APPROVAL: ....

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Ercan Aslan
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Abstract

In the present thesis, we analyze the Subgame Perfect Nash Equilibria (SPNE) of two different non-cooperative games. These games involve dynamic bankruptcy situations where agents have linear preferences over the set of possible allocations. We first consider a case where there are two agents and two periods (2×2) and, then, N agents and T periods ($N \times T$). For the first game (the Steel Game) we characterize the equilibria under the renowned CEA rule. For the second game (the Hospital Game), we consider a more general set of rules. Namely, we prove that a certain strategy profile is an equilibrium under the rules that satisfy bounded impact of transfers and weak (strong) claims monotonicity for 2×2 ($N \times T$) model and the payoffs of all equilibria are unique and equal to those of this profile's.

Keywords: Dynamic Claims Problems, Bankruptcy Rules, Non-cooperative Claims Game, Bounded Impact of Transfers, Weak (Strong) Claims Monotonicity.

DİNAMİK ALACAKLAR PROBLEMLERİ ÜZERİNE İŞBİRLİKÇİ OLMAYAN OYUNLAR

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Özet

Bu tezde, iki farklı işbirlikçi olmayan oyunun Alt-Oyun Yetkin Nash Dengesi'ni analiz ettik. Bu oyunlar, ajanların olası paylaşımlar üzerinde doğrusal tercihlere sahip olduğu iflas durumlarını kapsamaktadır. Öncelikle, iki ajanın ve iki dönemin varolduğu (2×2) durumu ele aldık, sonrada N ajanın ve T dönemin varolduğu durumu $(N \times T)$. İlk oyunumuzda (Çelik Oyunu) meşhur CEA kuralı altında oluşan dengeleri karakterize ettik. İkinci oyun (Hastane Oyunu) içinse daha genel bir oyunlar kümesini ele aldık. Şöyle ki, 2×2 $(N \times T)$ model için belli bir strateji profilinin, transferlerin sınırlı etkisini ve zayıf (güçlü) alacakların tekdüzeliğini sağlayan kurallar altındaki oyunlar için denge olduğunu ve bu oyunlar için denge ödüllerinin yegane ve bu profilinkine eşit olduğunu gösterdik.

Anahtar Kelimeler: Dinamik Alacaklar Problemleri, İflas Kuralları, İşbirlikçi Olmayan Alacak Oyunları, Transferlerin Sınırlı Etkisi, Zayıf (Güçlü) Alacakların Tekdüzeliği.

Contents

1 Introduction

A claims problem is a very simple allocation problem in which there is an endowment to be allocated among some agents, each characterized by a claim on the endowment. In real life, many examples of this problem exist. For instance, liquidation of a bankrupt firm among its creditors, how a state should allocate its budget based on the needs of public institutions are often times observed.

The first example is the so called bankruptcy problem. Firms raise funds from investors which can provide them with the working capital they need for their operations and let them undertake long-term investments. We care for them because many important ventures are impossible at the lack of these funds. In return, they pay the creditors the principal plus some interest. Firms depend on their cash flows to fulfill this obligation. However, there are times things go wrong and projected cash flows don't occur on time. This kind of situation may prevent the payment of the debt. Whenever a firm is insufficient to pay its creditors, there are two possible actions. It can either reorganize or go bankrupt. Reorganization, which is not the interest of the present thesis, is the act that changes the ownership structure of the firm and the maturity of the loans to let the firm stay in business and, as a result, continue to pay its debt. On the other hand, bankruptcy is the legal diagnose and declaration of a firm's insolvency. When a firm goes bankrupt, it has a liquidation value E. E is to be allocated among the creditors based on the amount due that must be paid to each creditor. Therefore, each amount due is the claim of a creditor. The problem is how to allocate this scarce value E based on the amount due of each creditor.

The second example is the so called rationing problem. It involves a central authority, most often some state department(Devlet Planlama Teşkilatı), and public institutions such as hospitals or universities. These institutions need funding from the state budget in order to finance their expenditures. The state has a pre-determined budget at each time period. However, the total amount an institution can demand depends on the proof of need. In this sense, the amount each can demand is limited. Nevertheless, each can report its claim strategically across the periods. In addition, the unpaid portion of the need can be reclaimed. Notice that the latter example includes time dimension and strategic choice of the claims reported. In this case, the allocation is repeated more than once.

The literature contains several prominent solutions to these problems. Among those, the most widely used rules are .Proportional Rule(PRO hereafter), Constrained Equal Awards(CEA hereafter), Constrained Equal Losses(CEL hereafter) and the Talmud Rule(TAL hereafter). As the name itself suggests, the PRO allocates the estate proportionally to agents' claims. For each problem, CEA comes up with a λ and offers this to each agent. The agent gets this λ if it is equal to or smaller than his claim. Otherwise, he gets his claim. In other words, CEA determines an upper bound on the payoffs and applies this upper bound anonymously. CEL works in a similar way. It uniquely determines a λ and subtracts this from each agents' claim. Each agent gets the remaining amount if it is non-negative. Otherwise, he gets zero. Finally, TAL operates in two different ways in two different situations. If the half-sum of the claims exceeds the endowment, it creates the same allocation as if CEA is applied to the half-claims. Otherwise, it works in two different steps. Firstly, everybody receives a share as much as his half-claim. Then, CEL is applied to the residual claims. These rules are discussed in detail in the following subsection.

Mainly, there are three different approaches to claims problem. : axiomatic, direct and game-theoretic approaches.(For a detailed discussion see Thomson(2003)) In the present thesis, we are interested in the game-theoretic one.

We construct a non-cooperative game in which each agent strategically allocates his claim over finite number of periods. At each period, agents move simultaneously and the game is played under complete information.

On the other hand, the current literature mainly concentrates on the characterization of the static rules. *That is,* the research is about some rules uniquely satisfying some properties or satisfying different desirable combinations of those. There isn't much discussion about the situations where the same allocation problem is repeated over time.

It is reasonable to perceive these repeated problems as a single allocation problem with time dimension if the agents subject to this problem are the same set of agents receiving shares in each period. In such a situation, all the current literature can do is to apply the same rule in each period. However, some previously-not-considered problems may arise, then. In the present thesis, we are investigating a problem of that kind. Namely, if the agents are capable of adjusting the spread of their claims strategically, then they can manipulate the payoffs using this knowledge. To investigate this, we design two distinct kinds of non-cooperative games and find out their equilibria. We are doing the same analysis both for the two agents-two periods case and for the arbitrary number of agents-arbitrary number of periods case. In the first model, the agents allot their claims to time periods. The level of the allotted claim to each period does not necessarily depend on other agents' claims. *That is,* If some agent i is playing a certain strategy, say strategy s_i, then the level of claim he uses at each period does not change with respect to other agents' claims at those periods, *i.e.*, with respect to others' strategies. In addition, at none of the periods agents can reuse the claims that they have already used at the preceding periods, regardless of the level of the shares they have received for those claims. There is an abundance of real life examples for such a situation. To illustrate, an important one is the scarce steel production in

US during WWII. In 1943, 85% of the total steel production in US was used for war effort. As a result, there was a limited supply and only a small part of it could be used for agricultural machinery and equipment production. Most of the time, agents had to trade in their worn-out machinery. However, in such situations the state can give the farmers somewhat less than what they brought in and take the whole machinery they brought for the steel needs. The details of this example can be found in the Sears Application (1942). Referring to this renowned example, we will call our first game **the Steel Game**. In the second model, agents choose how much they will claim in the first period, just like the first one. Yet, unlike the former one, in the second period their remaining claims are determined by subtracting the first period's share from the total claims available at the beginning of the game. If there is a third period, the maximum amount that an agent can claim in that period is determined by subtracting the first and the second period's shares of the agent from the total claim available at the beginning of the game and so on. Since our example regarding this model involves partitioning of a budget to hospitals, we will call it **the Hospital Game** hereafter. For the steel model, we focus on the well-known and intuitive rule CEA. As for the hospital model, we consider a broader class of rules including CEA. Namely, they are the rules satisfying bounded impact of transfers and claims monotonicity. When we extend our setting to an arbitrary number of agents and periods, we require the strong version of claims monotonicity. Fortunately, these properties are satisfied by a wide range of rules including PRO, CEA, CEL and TAL.

In both settings, our finding yield a multiplicity of equilibria but unique payoffs. In any equilibria of the steel game, based on the total claims of agents, each period has a certain parameter. If the total claim of an agent exceeds the sum of the parameters running from the first period to the last, then he claims at least as much as the relevant period's parameter at each period. Otherwise, the agent compares the parameter with his remaining claim. Then, he claims the minimum of those two. In the equilibria of the hospital game, all agents claim the maximum amount permitted by the remaining claims in hand at each period. Note that these results are due to our assumption that agents prefer the former periods to the latter ones.

1.1 Literature Review

There are very old historical examples of the claims problem. One of the earliest manuscripts where such a problem is addressed is the Babylonian Talmud. In the Talmud, there are two problems of this kind considered. The first one is called the contested garment problem. It involves two men having a conflict on how to share the worth of the garment. The second is the marriage contract problem. It involves a man and his three wives, each of which have signed a marriage contract with him. However, there isn't a general solution to such problems in Talmud. It only specifies a solution to a single problem. *That is,* for a unique set of numbers indicating the claims and the endowment. In the past, many scholars proposed allocation rules that generate the numbers in the Talmud. An allocation rule takes the claims of the agents and the endowment as input and allocates the endowment to the agents based on the claims. It is plausible to assume that if the sum of the claims doesn't exceed the endowment, then the rule gives everybody as much as his claim. The one that we use as the **Talmud Rule** in this thesis is proposed by Aumann and Maschler (1985). It is widely accepted in the literature because it is the unique rule which generates the numbers in the Talmud and at the same time satisfies some nice properties. On the other hand, this does not mean that it is the most desirable rule in each situation. For normative reasons, in many different situations many different rules are used. To illustrate, Gächter and Riedl (2006) shows us that proportional rule is considered as the fairest rule by most of the people. Their work supplements the literature by empirical evidence on three different solution concepts. Since the desirable rules proposed in the literature all rely on different properties, they claim that the attractiveness of a rule does not only depend on the theoretical aspects but also the actual perceived appeal by people once they face the problem in real life. They employ a vignette technique to observe impartial participants' perception on fairness and find out the result we mentioned above about PRO Secondly, they design a laboratory experiment where the agents with self-interests and claims bargain on allocation. They show that this game leads to an allocation similar to that of CEA's. This experiment shows us that the allocation in an equilibrium of a game might be different from normative judgements about the same situation. In order to understand this kind of actual behaviors, many authors designed different games. Garcia-Jurado et al., (2006) propose a one shot game in which each agent chooses his claim. Although claiming more generates a higher payoff in many contexts, since in their setting the agents with a lower claim has a priority over the others, each agent claims the same amount in equilibrium. Thus, the resulting allocation is the equal division. They show that all the Nash equilibria of their game yield the same payoff vector. Furthermore, one can show that in a game of that form with n agents, the strategy profile in which all agents claim $\frac{E}{n}$ is the unique Nash equilibrium. The game they formulate is a simple one in the sense that it's not sequential. Also, since an agent might lose priority and, hence, decrease his share by increasing his claim, the allocations that are proposed to different claims vectors by this game can not coincide with those of a claims monotonic rule. On the contrary, we have a sequential game and we impose claims monotonicity to the rules we use in our setting. In the seminal paper, O'Neill (1982), where the simple claims problems in the literature are first originated, a problem of n heirs and n corresponding wills is addressed.

(In other words, Rabbi Abraham Ibn Ezra's proposal about a man who dies leaving inconsistent wills to his sons) Similar to our work, the utilities of the heirs are assumed to be linear with the bequests they receive. He criticizes Ibn Ezra's premises, mainly premise 2 stating that the claims of the heirs fully overlap, and proposes and discusses alternative solutions with their pros and cons. Still, the alternatives he proposes keep the other premises. (premise 1 and premise 3) As one of the alternatives, he also proposes a non-cooperative game in which the four sons choose what part of the endowment they claim as the strategy variable. He characterizes the minimal overlap rule as the Nash equilibrium of the non-cooperative game.

Kar and Kıbrıs (2008) construct a model which involves multiple endowments. In their model, however, each agent can receive share from at most one endowment. If the preferences are single peaked and symmetric, they show that any efficient single-endowment rule can be combined by a matching rule to construct a multi-endowment efficient allocation rule. In their mechanism, firstly the matching rule assigns agents to endowments. Then, in the second stage, the single-endowment rationing rule applies to each endowment and its assigned agents. In addition, they establish two impossibility results when the domain of the single-peaked preferences is extended to asymmetric ones.

There is also a drastically growing literature on manipulation. For instance, Thomson (1984) show that given a choice correspondence and all associated manipulation games, any equilibrium allocation of such manipulation games is an equilibrium allocation of the Walrasian manipulation game. In a static bankruptcy setting, it is interesting to inquire whether a given simple claims problem embodies manipulation. As a matter of fact Ju (2003) analyzes immunity of bankruptcy rules to manipulation via splitting and merging. That work characterizes the domain of rules that satisfy equal treatment of equals, consistency, continuity and are non-manipulable via pairwise splitting and pairwise merging. (Namely, rules with superadditive and subadditive representations, respectively) Moreno-Ternero (2007) restricts attention to TAL-family of rules (for a detailed discussion on this family see Moreno-Ternero and Villar (2006)). For each member of the family, they identify on which problems it satisfies either non-manipulability via merging or non-manipulability via splitting. Moreno-Ternero (2006) provides an alternative proof to the fact that non-manipulability and PRO imply each other in an unrestricted domain. He also shows that this result continues to hold in some restricted domains.

Kıbrıs and Kıbrıs (2009) design a non-cooperative game so as to explain why proportional rule is the most widely used rule in real life bankruptcy situations. They show that the answer lies in the investment implications of the rule. Karagözoğlu (2008) supports PRO by means of a different investment-bankruptcy game

2 The 2×2 Steel Game

Let $N = \{1, 2\}$ be the set of agents and let $E^t \in \mathbb{R}_+$ be a social endowment to be allocated among members of N. For each $i \in N$, let $c_i \in \mathbb{R}_+$ be agent i's claim on the social endowment. Assume $c_1 + c_2 \geq E^t$. Let $c = (c_1, c_2)$. We call (c, E^t) a **static claims problem**. Denote the class of all static claims problems by β^{STAT}. Then $F : \beta^{STAT} \rightarrow \mathbb{R}_+^N$ is a claims rule if for each $(c, E^t) \in \beta^{STAT}$, $\sum\limits_{i \in N} F_i(c, E^t) = E^t$ and $0 \leqslant F(c, E^t) \leqslant c$. In words, given a static claims problem, F distributes the endowment among the agents.

We are interested in a framework where a group of agents have to share two social endowments that arrive in two different periods. To model this situation, denote the set of periods by $T = \{1, 2\}$. Let $E = \begin{bmatrix} E^1 \\ E^2 \end{bmatrix}$ be the vector of endowments to be divided in periods 1 and 2, respectively. We assume that each agent prefers shares from period 1 endowment over period 2 endowment. Each agent i discounts period 2 endowment with a given **discount factor** $\delta_i \in (0,1)$. Suppose that $\delta = \begin{bmatrix} \delta_1 \\ \delta_2 \end{bmatrix}$ is the vector of discount factors of agents. We represent the agent i's share by $x_i = (x_i^1, x_i^2)$, where x_i^t represent his share in period $t \in T$. We assume that the utility of agent i from x_i is of the form $u_i = x_i^1 + \delta_i x_i^2$ for $i = 1, 2$.

A **claims problem with time preferences** is a triple (c, E, δ) such that for each $t \in T$, $(c, E^t) \in ß^{STAT}$ is a static claims problem and δ is the vector that represents agents' discount factors.

We next introduce a non-cooperative game where each agent strategically decides on how to allocate his total claim c_i between the two periods. To model this, let agent i's strategy set be $S_i = [0, c_i]$. A typical strategy of i is $s_i \in S_i$ and it is interpreted as the part of i's claim used in period 1. Her remaining claim $c_i - s_i$ is used in the second period. Given a problem (c, E, δ) and a rule F, let $d = (c, E, \delta, F)$. Agent i's payoff from a strategy profile $s = (s_1, s_2)$ is then $u_i^d(s) = F_i(s_1, s_2, E^1) + \delta_i F_i(c_1 - s_1, c_2 - s_2, E^2)$. Observe that we assume that the same rule is applied in both periods.

Definition 1 *A claims game with respect to* $d = (c, E, \delta, F)$ *is*
$$G^d = \langle N, S_1, S_2, u_1^d, u_2^d \rangle \text{ satisfies } E^1, E^2 \geq 0,\ \delta \in [0,1]^2,\ \sum_{i=1}^{2} c_i \geqslant \max\left\{E^1, E^2\right\}.$$

2.1 Equilibria Under The CEA Rule

Definition 2 ***(CEA)*** *For each* $(c, E^t) \in ß^{STAT}$ *and each* $i \in N$, $CEA_i(c, E^t) \equiv \min\left\{c_i, \lambda^t\right\}$ *where* λ^t *satisfies* $\sum_{j \in N} \min\left\{c_j, \lambda^t\right\} = E^t$.

CEA advocates the idea of equal division (ED hereafter), yet respecting the differences in claims, at least in some cases. Under equal division, some agents might receive more than their claims. For this reason, ED is not an allocation rule. In contrast, under CEA each agent's claim is an upper bound for his share. Accordingly, under CEA, agents will receive the same shares as under ED as long as this amount does not exceed their claims.

Example 1 *i)* $(c_1, c_2) = (7, 9)$ *and* $E^t = 10$ *implies* $\lambda^t = 5$ *and* $CEA_1(c, E^t) = 5$, $CEA_2(c, E^t) = 5$ *and* $ED_1(c, E^t) = 5$, $ED_2(c, E^t) = 5$. *Both rules lead to the same allocation and allocate the same amount to each agent ignoring the difference in claims. ii)* $(c_1, c_2) = (3, 9)$ *and* $E^t = 10$ *implies* $\lambda^t = 7$ *and* $CEA_1(c, E^t) = 3$, $CEA_2(c, E^t) = 7$ *and* $ED_1(c, E^t) = 5$, $ED_2(c, E^t) = 5$. *CEA recognizes the difference in claims to some extend but ED not. Furthermore, ED awards agent* 1 *with a higher share than CEA.*

The allocation method used by CEA can also be explained by means of an algorithm. The algorithm works as follows:

Firstly, let everyone receive the same share, that is, start with ED. If no agent receives more than his claim, then CEA leads to ED. Otherwise, let the agents, who receive more than their claims, receive just as much as their claims and allocate the resulting surplus equally among others. After that, if no agent's share exceeds his claim, then that's the allocation. Otherwise, let the agents, who receive more than their claims, receive just as much as their claims and rearrange the resulting surplus so that the remaining agents receive equal shares from the surplus. Proceed this way until no one receives more than his claim.

Remark 1 *Let* $(c, E^t) \in ß^{STAT}$ *be given. Assume that* $c_1 + c_2 > E^t$. *If the endowment is to be distributed among the agents using CEA rule, then there exists a unique* λ *which satisfies* $\min\{\lambda, c_1\} + \min\{\lambda, c_2\} = E^t$. *Note that the previous statement remains valid if the number of agents is* $n > 2$.

For each $s \in S$, define $\lambda^1(s) \in \mathbb{R}_+$ as follows

$$\lambda^1(s) \begin{cases} \text{uniquely solves } \sum_{i=1}^{2} \min\{s_i, \lambda^1(s)\} = E^1 & \text{if } \sum_{i=1}^{N} s_i > E^1 \\ \max\{s_1, s_2\} & \text{if } \sum_{i=1}^{N} s_i \leq E^1 \end{cases}$$

and $\lambda^2(s) \in \mathbb{R}_+$ as

$$\lambda^2(s) \begin{cases} \text{uniquely solves } \sum_{i=1}^{2} \min\{s_i, \lambda^2(s)\} = E^2 & \text{if } \sum_{i=1}^{N} s_i > E^2 \\ \max\{c_1 - s_1, c_2 - s_2\} & \text{if } \sum_{i=1}^{N} s_i \leq E^2 \end{cases}$$

Consider $d = (c, E, \delta, F)$ and $G^d = \langle N, S_1, S_2, u_1^d, u_2^d \rangle$ where $c_1 + c_2 > E^1 + E^2$. In this situation, the game will lead to multiple equilibria but a unique payoff vector. This multiple equilibria will be result of redundant claims. Any agent $i \in N$ with $c_i > \frac{E^1+E^2}{2}$ is endowed with a level of claim more than the amount sufficient to obtain the same payoff. However, having a higher level of claim does not lead to a better payoff vector. Thus, we will restrict our attention to the case where $c_1 + c_2 \leq E^1 + E^2$.

Example 2 *Consider any game under CEA with $c = (160, 160)$ and $E = (100, 100)$. Let $110 \geq s_1, s_2 \geq 50$. One can see that for each player any selection of the strategy profile (s_1, s_2) constitute a Nash Equilibrium. Furthermore, the unique payoff vector is $(x_1^1, x_1^2, x_2^1, x_2^2) = (50, 50, 50, 50)$.*

Proposition 1 *Let $d = (c, E, \delta, F)$ and $G^d = \langle N, S_1, S_2, u_1^d, u_2^d \rangle$ such that $c_1 + c_2 \leq E^1 + E^2$. Then, the unique Nash Equilibrium of G^d is s^* defined as*

$$\begin{cases} s_i^* = s_j^* = \frac{E^1}{2} \text{ if } c_i, c_j > \frac{E^1}{2} \\ s_i^* = E^1 - c_j,\ s_j^* = c_j \text{ if } c_i > \frac{E^1}{2} \text{ and } c_j \leq \frac{E^1}{2} \text{ for } i, j \in N \end{cases}$$

Proof. First note that for each strategy profile $s \in S$ such that $\sum_{i=1}^{2} s_i < E^1$, there exists $i \in N$ and $s_i \in S_i$ such that $u_i^d(s_i + \epsilon, s_j) > u_i^d(s_i, s_j)$

for some $\epsilon > 0$. To see this, notice that if $\sum_{i=1}^{2} s_i < E^1$ then there exists $s_i \in S_i$ such that $s_i < c_i$. Then, there exists $\epsilon > 0$ and $(s_i + \epsilon) \in S_i$ such that $s_i + \epsilon + s_j \leq E^1$. Then, $CEA_i(s_i + \epsilon, s_j, E^1) = s_i + \epsilon$. In addition, $CEA_i(c_i - s_i, c_j - s_j, E^2) - \epsilon \leq CEA_i(c_i - s_i - \epsilon, c_j - s_j, E^2)$. Therefore, $CEA_i(s_i, s_j, E^1) + \delta_i CEA_i(c_i - s_i, c_j - s_j, E^2) \leq CEA_i(s_i + \epsilon, s_j, E^1) + \delta_i CEA_i(c_i - s_i - \epsilon, c_j - s_j, E^2)$, that is, such strategy profiles can not be a Nash Equilibrium

Case 1: $c_i \geq \frac{E^1}{2}$ and $c_j \leq \frac{E^1}{2}$. Let $s \in S$ be such that $\sum_{i=1}^{2} s_i \geq E^1$. Then, by definition, $\lambda^1(s) \geq \frac{E^1}{2}$. Thus, for each $s_i \in S_i$, $CEA_j(s_i, c_j, E^1) = c_j$, That is, Agent j can receive her full claim in the first period. Notice that his shares from $s_j^* = c_j$ are $CEA_j(s_i, c_j, E^1) = c_j$ in the first period and $CEA_j(c_i - s_i, 0, E^2) = 0$ in the second. Let $s_j' = c_j - \epsilon$ for some $\epsilon \in (0, c_j]$. Then, his shares from s_j' are $CEA_j(s_i, c_j - \epsilon, E^1) = c_j - \epsilon$ in the first period and $CEA_j(c_i - s_i, \epsilon, E^2) \leq \epsilon$ in the second. Since $\delta_j < 1$, comparing the utilities of the shares generated by s_j^* and s_j', we have $u_j^d(s_i, s_j^*) = CEA_j(s_i, c_j, E^1) + \delta_j CEA_j(c_i - s_i, 0, E^2) = c_j > c_j - \epsilon + \delta_j \epsilon \geq CEA_j(s_i, c_j - \epsilon, E^1) +$

$\delta_j CEA_j(c_i - s_i, \epsilon, E^2) = u_j^d(s_i, s_j')$ Hence $s_j^* = c_j$ is the dominant strategy for agent j.

We next claim that $s_i^* = E^1 - c_j$ is the unique best response of agent i against $s_j^* = c_j$. To see this, first note that his shares from s_i^* are $CEA_i(E^1 - c_j, c_j, E^1) = E^1 - c_j$ and $CEA_i(c_i - (E^1 - c_j), 0, E^2) = c_i - (E^1 - c_j)$ from the first and second periods respectively. Let $s_i' = s_i^* - \epsilon$ for $\epsilon \in (0, E^1 - c_j]$. Then his shares from s_i' are $CEA_i(E^1 - c_j - \epsilon, c_j, E^1) = E^1 - c_j - \epsilon$ and $CEA_i(c_i - (E^1 - c_j) + \epsilon, 0, E^2) \leq c_i - (E^1 - c_j) + \epsilon$ from the first and second periods respectively. Since $\delta_i < 1$, comparing the two we have, $u_i^d(s_i^*, s_j^*) = E^1 - c_j + \delta_i(c_i - (E^1 - c_j)) > E^1 - c_j - \epsilon + \delta_i(c_i - (E^1 - c_j) + \epsilon) \geq u_i^d(s_i', s_j^*)$. Now let $s_i' = s_i^* + \epsilon$. Then his shares are $CEA_i(E^1 - c_j + \epsilon, c_j, E^1) = E^1 - c_j$ and $CEA_i(c_i - (E^1 - c_j + \epsilon), 0, E^2) = c_i - (E^1 - c_j) - \epsilon$. We have $u_i^d(s_i^*, s_j^*) =$

$E^1 - c_j + \delta_i(c_i - (E^1 - c_j)) > E^1 - c_j + \delta_i(c_i - (E^1 - c_j) - \epsilon) = u_i^d(s_i', s_j^*)$ as desired.

Case 2: $c_i, c_j > \frac{E^1}{2}$. We have shown that strategies such that $\sum_{i=1}^{2} s_i < E^1$ can not be a Nash Equilibrium. Thus, we restrict our attention to $\sum_{i=1}^{2} s_i \geq E^1$. Then there exists $i \in N$ such that $s_i \geq \frac{E^1}{2}$. Consider $j \in N \backslash i$. Let $s_j' = \frac{E^1}{2} + \epsilon$, $\epsilon > 0$. Then, since $\sum_{i=1}^{2} c_i \leq E^1 + E^2$, we have $\sum_{i=1}^{2} c_i - s_i \leq E^2$ and $CEA_i(c_i - s_i, c_j - s_j, E^2) = c_i - s_i$ for each $i \in N$. Therefore, $u_j^d(s_i, s_j') = CEA_j(s_i, \frac{E^1}{2} + \epsilon, E^1) + \delta_j CEA_j(c_i - s_i, c_j - \frac{E^1}{2} - \epsilon, E^2) = \frac{E^1}{2} + \delta_j(c_j - \frac{E^1}{2} - \epsilon)$. On the other hand, $u_j^d(s_i, \frac{E^1}{2}) = CEA_j(s_i, \frac{E^1}{2}, E^1) + \delta_j CEA_j(c_i - s_i, c_j - \frac{E^1}{2}, E^2) = \frac{E^1}{2} + \delta_j(c_j - \frac{E^1}{2})$. Clearly, $u_j^d(s_i, \frac{E^1}{2}) > u_j^d(s_i, s_j')$. Now let $s_j' = \frac{E^1}{2} - \epsilon$. Since $\sum_{i=1}^{2} s_i \geq E^1$ we have $\lambda^1(s) \geq \frac{E^1}{2}$ and $\min\left\{s_j', \lambda^1(s)\right\} = s_j'$ Then, $u_j^d(s_i, s_j') = CEA_j(s_i, \frac{E^1}{2} - \epsilon, E^1) + \delta_j CEA_j(c_i - s_i, c_j - \frac{E^1}{2} + \epsilon, E^2) = \frac{E^1}{2} - \epsilon + \delta_j(c_j - \frac{E^1}{2}) + \delta_j \epsilon$. Hence $u_j^d(s_i, \frac{E^1}{2}) > u_j^d(s_i, s_j')$. Therefore, $s_j^* = \frac{E^1}{2}$ is the unique best response of agent j that can be in any equilibrium. Now, consider the best response of agent i against $s_j^* = \frac{E^1}{2}$ among the strategies such that $s_i \geq \frac{E^1}{2}$.. We have $u_i^d(s_i^*, \frac{E^1}{2}) = CEA_i(\frac{E^1}{2}, \frac{E^1}{2}, E^1) + \delta_i CEA_i(c_i - \frac{E^1}{2}, c_j - \frac{E^1}{2}, E^2) = \frac{E^1}{2} + \delta_i(c_i - \frac{E^1}{2})$. Let $s_i' = \frac{E^1}{2} + \epsilon$. Then, $u_i^d(\frac{E^1}{2} + \epsilon, \frac{E^1}{2}) = CEA_i(\frac{E^1}{2} + \epsilon, \frac{E^1}{2}, E^1) + \delta_i CEA_i(c_i - \frac{E^1}{2} - \epsilon, c_j - \frac{E^1}{2}, E^2) = \frac{E^1}{2} + \delta_i(c_i - \frac{E^1}{2}) - \delta_i \epsilon$. Then, $u_i^d(s_i^*, \frac{E^1}{2}) > u_i^d(\frac{E^1}{2} + \epsilon, \frac{E^1}{2})$. Hence, $s_i^* = s_j^* = \frac{E^1}{2}$ is the unique equilibrium, as desired. ■

Proposition 2 *Let $d = (c, E, \delta, F)$ and $G^d = \langle N, S_1, S_2, u_1^d, u_2^d \rangle$. Assume that $c_1 + c_2 > E^1 + E^2$. Then, the following is a Nash Equilibrium: s^* defined as*
$$\begin{cases} s_i^* = s_j^* = \frac{E^1}{2} \text{ if } c_i, c_j > \frac{E^1}{2} \\ s_i^* = E^1 - c_j,\ s_j^* = c_j \text{ if } c_i > \frac{E^1}{2} \text{ and } c_j \leq \frac{E^1}{2} \end{cases}$$
. Also, if s is a Nash Equilibrium, then it creates the same allocation as s^*, that is, $CEA(s, E^1) = CEA(s^*, E^1)$ and $CEA(c - s, E^2) = CEA(c - s^*, E^2)$.

Proof. Case 1) $c_i, c_j > \frac{E^1}{2}$. Let $s_i' = \frac{E^1}{2} + \epsilon$. Then we have $CEA_i(\frac{E^1}{2}, \frac{E^1}{2}, E^1) =$

$\frac{E^1}{2}$ and $CEA_i(\frac{E^1}{2}+\epsilon, \frac{E^1}{2}, E^1) = \frac{E^1}{2}$then $u_i^d(s_i^*, s_j^*) = CEA_i(\frac{E^1}{2}, \frac{E^1}{2}, E^1) + \delta_i CEA_i(c_i - \frac{E^1}{2}, c_j - \frac{E^1}{2}, E^2) = \frac{E^1}{2} + \delta_i CEA_i(c_i - \frac{E^1}{2}, c_j - \frac{E^1}{2}, E^2) \geq CEA_i(\frac{E^1}{2} + \epsilon, \frac{E^1}{2}, E^1) + \delta_i CEA_i(c_i - \frac{E^1}{2} - \epsilon, c_j - \frac{E^1}{2}, E^2) = u_i^d(s_i', s_j^*)$. Conversely, let $s_i' = \frac{E^1}{2} - \epsilon$. Then, $CEA_i(\frac{E^1}{2} - \epsilon, \frac{E^1}{2}, E^1) = \frac{E^1}{2} - \epsilon$. Thus, $u_i^d(s_i', s_j^*) = CEA_i(\frac{E^1}{2} - \epsilon, \frac{E^1}{2}, E^1) + \delta_i CEA_i(c_i - \frac{E^1}{2} + \epsilon, c_j - \frac{E^1}{2}, E^2) = \frac{E^1}{2} - \epsilon + \delta_i CEA_i(c_i - \frac{E^1}{2} + \epsilon, c_j - \frac{E^1}{2}, E^2) < \frac{E^1}{2} + \delta_i CEA_i(c_i - \frac{E^1}{2} + \epsilon, c_j - \frac{E^1}{2}, E^2) - \delta_i \epsilon \leq \frac{E^1}{2} + \delta_i CEA_i(c_i - \frac{E^1}{2}, c_j - \frac{E^1}{2}, E^2) = u_i^d(s_i^*, s_j^*)$. From the symmetry of claims ($c_i, c_j > \frac{E^1}{2}$) the same argument applies for agent j.

For the uniqueness part, we know that in any equilibrium $s_i \geq E^1$ for each $i \in N$. In this case, $CEA_i(s_i, s_j, E^1) = CEA_j(s_i, s_j, E^1) = \frac{E^1}{2}$. Given that $s_i \geq E^1$, since $CEA_i(c_i - s_i, c_j - s_j, E^2)$ is a non-decreasing function of s_j for all $i \in N$, for each $s_i \in S_i$ the lowest $CEA_i(c_i - s_i, c_j - s_j, E^2)$ is obtained when $s_j = \frac{E^1}{2}$. On the other hand, since $CEA_i(c_i - s_i, c_j - s_j, E^2)$ is a non-increasing function of s_i, for each $s_j \in S_j$, the highest $CEA_i(c_i - s_i, c_j - s_j, E^2)$ is obtained when $s_i = \frac{E^1}{2}$. As a result, they yield the lowest $CEA_i(c_i - s_i, c_j - s_j, E^2)$ in any equilibrium. Since the same argument holds for agent j and $CEA_i(c_i - \frac{E^1}{2}, c_j - \frac{E^1}{2}, E^2) + CEA_j(c_i - \frac{E^1}{2}, c_j - \frac{E^1}{2}, E^2) = E^2$, this lowest shares can not be increased. Thus, the payoff vector generated by s^* is unique.

Case2) $c_i > \frac{E^1}{2}$ and $c_j \leq \frac{E^1}{2}$. We are going to show that $u_j^d(s_i, c_j) > u_j^d(s_i, c_j - \epsilon)$, for $\epsilon \in (0, c_j]$ and for all $s_i \in S_i$. Then, we are going to show that $u_i^d(E^1 - c_j, c_j) \geq u_i^d(s_i, c_j)$ for all $s_i \in S_i$. First we have $u_j^d(s_i, c_j) = CEA_j(s_i, c_j, E^1) + \delta_j CEA_j(c_i - s_i, 0, E^2) = c_j > c_j - \epsilon + \delta_j \epsilon \geq CEA_j(s_i, c_j - \epsilon, E^1) + \delta_j CEA_j(c_i - s_i, \epsilon, E^2)$ for $\epsilon \in (0, c_j]$ for all $s_i \in S_i$. Namely, $s_j^* = c_j$ is the dominant strategy of agent j. Now, we will check for agent i's best response against this strategy. Playing s_i^*, his shares from the first and the second periods are $CEA_i(E^1 - c_j, c_j, E^1) = E^1 - c_j$ and $CEA_i(c_i - (E^1 - c_j), 0, E^2) = E^2$, respectively, since $c_i - (E^1 - c_j) > E^2$ by the assumption of the present proposition. Therefore, $u_i^d(E^1 - c_j, c_j) = E^1 - c_j + \delta_i E^2$. Let $s_i' = E^1 - c_j - \epsilon$

for $\epsilon \in (0, E^1 - c_j]$. Playing s_i', his shares are $CEA_i(E^1 - c_j - \epsilon, c_j, E^1) = E^1 - c_j - \epsilon$ and $CEA_i(c_i - (E^1 - c_j - \epsilon), 0, E^2) = E^2$ from the first and the second periods, respectively. Hence, $u_i^d(s_i', c_j) = E^1 - c_j - \epsilon + \delta_i E^2$. Then, we have $u_i^d(E^1 - c_j, c_j) > u_i^d(s_i', c_j)$. Conversely, let $s_i' = E^1 - c_j + \epsilon$ for $\epsilon \in (0, c_i - (E^1 - c_j)]$. Then his shares are $CEA_i(E^1 - c_j + \epsilon, c_j, E^1) = E^1 - c_j$ and $CEA_i(c_i - (E^1 - c_j + \epsilon), 0, E^2) \leq E^2$. Thus, we have $u_i^d(s_i', c_j) \leq E^1 - c_j + \delta_i E^2 = u_i^d(E^1 - c_j, c_j)$. Moreover, since s_j^* is the dominant strategy of agent j and $u_i^d(s_i, c_j) = c_j$ for all $s_i \in S_i$, then it is also true in any equilibrium. As a result, agent i can have all the remaining shares. Hence, s^* is a Nash Equilibrium and if s is a Nash Equilibrium, then it creates the same allocation as s^*. ■

3 The $N \times T$ Steel Game

Let $N = \{1, 2, ..., N\}$ be the set of agents and $T = \{1, 2, ..., T\}$ be the set of periods. For each $t \in T$, E^t is the social endowment to be allocated among the agents at period t. Let $E = \; E^1 \quad E^2 \quad . \quad E^T$ be the vector of endowments to be divided in periods $1, 2, ..., T$, respectively. For each $i \in N$, let $c_i \in \mathbb{R}_+$ be agent i's total claim to be allocated among E.[1] Denote $c = (c_1, c_2, ..., c_N)$. We assume that $\sum\limits_{i \in N} c_i \geq \max \left\{ E^1, ..., E^T \right\}$. Each agent prefers shares from E^t over shares from E^{t+k}, where $k \in \mathbb{N}_+$ and $t, t+k \in T$. That is, The agents prefer preceding periods to the succeeding ones. We denote the agent i's share by $x_i(x_i^1, ..., x_i^T)$. where x_i^t represents his share from E^t. Agents might have different discount factors from each others', however, the discount factor of an arbitrary agent for different time periods is fixed. Therefore, the utilities are of the form $u_i = \sum\limits_{t=1}^{T} \delta^{t-1} x_i^t$. We preserve the definition of a claims problem

[1] We assume that the number of elements in T is T.

with time preferences that is given in the 2×2 model. *That is,* A **claims problem with time preferences** is a triple (c, E, δ) such that for each $t \in T$, $(c, E^t) \in ß^{STAT}$ is a static claims problem and δ is the vector that represents agents' discount factors. We denote the action profile at t by s^t, agent i's strategy by s_i and the strategy profile of the whole game by s.

Steel Game: We construct a game where agents simultaneously choose how much to allocate at each period, observing which strategies are played by the players of N in the preceding periods. In this game, agent $i's$ strategy set is $S_i = \{s_i(s_i^1, s_i^2, ..., s_i^T) : 0 \leq s_i^t(s^1, s^2, ..., s^{t-1}) \leq c_i - \sum_{t=1}^{t-1} s_i^t$ for each $t \in \{2, 3, ..., T\}$ and $0 \leq s_i^1 \leq c_i$ where $\sum_{t \in T} s_i^t = c_i\}$. Once a player uses some portion of his total claim at some period, then this portion is subtracted from the total remaining claim of the agent when determining his action set for the next period. *That is,* the claims are perishable.

For each $s \in S$, define $\lambda^t(s) \in \mathbb{R}_+$ as follows

$$\lambda^t(s) \begin{cases} \text{uniquely solves } \sum_{i \in N} \min\left\{s_i^t, \lambda^t(s)\right\} = E^t & \text{if } \sum_{i \in N} s_i^t > E^t \\ \max\{s_1^t, s_2^t, ..., s_N^t\} & \text{if otherwise.} \end{cases}$$

3.1 Equilibria

Theorem 1 *Define $s_i^0 = 0$. Let $d = (c, E, \delta, F)$. Then, the following strategy profile s^* is a Subgame Perfect Nash Equilibrium of*

G^d: $s_i^{*t} = \min\left\{c_i - s_i^0 - s_i^{*1} - ... - s_i^{*t-1}, \lambda_i^{*t}\right\}$ for $t = 1, 2, ..., T-1$ and $i = 1, ..., N$. We denote $\lambda^t(c_1 - s_1^0 - s_1^1 - ... - s_1^{t-1}, ..., c_i - s_i^0 - s_i^{*1} - ... - s_i^{*t-1}, ..., c_N - s_N^0 - s_N^1 - ... - s_N^{t-1})$ by λ_i^{*t}. Moreover, the payoffs generated by this profile is unique for all SPNE and if $\sum_{i=1}^{N} c_i \leq \sum_{t=1}^{T} E^t$, then s^* is the unique Subgame Perfect Nash Equilibrium of G^d.

Proof. We first show that λ^t is non-increasing in claims for $\sum_{i \in N} c_i \geq E^t$. Let

$\sum\limits_{i\in N} c_i \geq E^t$ and let $c=(c_1,...,c_N)$, $c'=(c_1,...,c_i+\epsilon,...,c_N)$. Assume that $\lambda^t(c') > \lambda^t(c)$. We have $c'_i \geq c_i$ for all $i \in N$ and, hence, $\sum\limits_{i\in N} c_i \geq E^t$ implies $\sum\limits_{i\in N} c'_i > E^t$. Then, there exists $k \in N$ such that $c_k > \lambda^t(c')$ because, otherwise, $E^t = \sum\limits_{i\in N} F_i(c', E^t) = \sum\limits_{i\in N} c'_i$, which is not the case. Also, there exists j such that $c_j \geq \lambda^t(c)$ by definition of $\lambda^t(.)$. Since $c'_i \geq c_i$ for all $i \in N$ and $\lambda^t(c') > \lambda^t(c)$, we have $\min\left\{c'_i, \lambda^t(c')\right\} \geq \min\left\{c_i, \lambda^t(c)\right\}$ for all $i \in N$. If $c_j > \lambda^t(c)$, then $\min\left\{c_j, \lambda^t(c)\right\} = \lambda^t(c) < \min\left\{c'_j, \lambda^t(c)\right\}$. Hence, $\sum\limits_{k\in N} \min\left\{c'_k, \lambda^t(c')\right\} > \sum\limits_{k\in N} \min\left\{c_k, \lambda^t(c)\right\} = E^t$. Contradiction. If $c_j = \lambda^t(c)$ and there does not exist any $l \in N$ such that $c_l > \lambda^t(c)$, then $\sum\limits_{i\in N} c_i = E^t$. This implies that $F_i(c, E^t) = c_i$ for all $i \in N$. Then, $F_i(c', E^t) = \min\left\{c_i+\epsilon, \lambda^t(c')\right\} > c_i = F_i(c, E^t)$ for that particular $i \in N$. Then, $\sum\limits_{i\in N} F_i(c', E^t) = \sum\limits_{i\in N} \min\left\{c'_i, \lambda^t(c)\right\} > \sum\limits_{i\in N} \min\left\{c_i, \lambda^t(c)\right\} = E^t$. Contradiction. Hence, $\lambda^t(c') \leq \lambda^t(c)$. As a result, given any s^t_{-i}, $\lambda^{*t}_i = \min\limits_{s^t_i} \lambda^t(s^t_i, s^t_{-i})$. Then,

$\min\left\{\min\left\{c_i - s^0_i - s^{*1}_i - ... - s^{*t-1}_i, \lambda^{*t}_i\right\}, \lambda^{*t}_i\right\} =$

$\min\left\{c_i - s^0_i - s^{*1}_i - ... - s^{*t-1}_i, \lambda^{*t}_i\right\} = s^{*t}_i$. *That is,* . $F_i(s^{*t}_i, s_{-i}, E^t) = s^{*t}_i$ (1)

By a similar argument, we have $F_i(s^t_i, s_{-i}, E^t) = s^t_i$ where $s^t_i = s^{*t}_i - \epsilon$, for some $\epsilon > 0$. Since CEA satisfies BIT, which is discussed in the next section, $CEA_i(c_i+\epsilon, c_j-\epsilon, c_{-\{i,j\}}, E^t) - CEA_i(c, E^t) \leq \epsilon$. Since CEA satisfies strong claims monotonicity because of non-increasing $\lambda^t(.)$, we have $CEA_i(c_i+\epsilon, c_j, c_{-\{i,j\}}, E^t) \leq CEA_i(c_i+\epsilon, c_j-\epsilon, c_{-\{i,j\}}, E^t)$. Hence, $CEA_i(c_i+\epsilon, c_j, c_{-\{i,j\}}, E^t) - CEA_i(c, E^t) \leq \epsilon$ (2)

>From (1) and (2) we have $\sum\limits_{t\in T} F^t_i(s^*_i, s_{-i}, E^t) > \sum\limits_{t\in T} F^t_i(s_i, s_{-i}, E^t)$ for all $s_{-i} \in S_{-i}$ where s^*_i is the strategy consisting of $s^{*t}_i, t \in T$, s_i includes s^t_i and $F^t_i(.)$ is agent $i's$ period t share. Therefore, any $s^t_i < s^{*t}_i$ is strictly dominated.

Considering sums over $t \in \{T-K, ..., T\}$, one can see that such strategies are strictly dominated in any subgame consisting of the last K periods of the game. Once the strictly dominated strategies are eliminated at each subgame, the remaining strategies s_i^t are such that for all $i \in N$ $s_i^t \geq s_i^{*t}$, for all $t \in T$ and for each $(s_{-i}^1, ..., s_{-i}^{t-1})$. $s_i^t > s_i^{*t}$ implies $\lambda^t(s_i^{*t}, s_{-i}) < s_i^t$ and $\lambda^t(s_i^{*t}, s_{-i}) < c_i - s_i^1 - ... - s_i^{t-1}$ That is, $s_i^{*t} = \lambda^t(s_i^{*t}, s_{-i})$. Since $\lambda^t(s^t) = \lambda^t(s^{*t})$ for $s^t \geq s^{*t}$, we have $F^t(s^t, E^t) = \min\left\{s^t, \lambda^t(s^t)\right\} = \lambda^t(s^t) = \lambda^t(s^{*t}) = s_i^{*t} = F_i^t(s^{*t}, E^t)$. Therefore, any such s_i^t yields the same shares as s_i^{*t}. Then, s^* is a SPNE. Moreover, if $\sum\limits_{i \in N} c_i \leq \sum\limits_{t \in T} E^t$, the unique such s_i^t is s_i^{*t} for all $t \in T$. Hence s^* is the unique SPNE for that case. The proof is complete. ■

4 The Hospital Game

Hospital Game: Similarly, the agents simultaneously decide on how much to allocate at each period, observing which strategies are played by the players of N in the preceding periods. Yet, unlike the steel game, the claims are not always perishable. In the hospital game, to determine the action set of an agent at some $t \in T$, we subtract the shares he received in the preceding periods from his total claim c_i, instead of subtracting the claims he used in the previous periods. Therefore, the agent i's strategy set is $S_i = \{s_i(s_i^1, s_i^2, ..., s_i^T) : 0 \leq s_i^t(s^1, s^2, ..., s^{t-1}) \leq c_i - \sum\limits_{t=1}^{t-1} F_i(s^t, E^t)$ for each $t \in \{2, ..., T\}$ and $0 \leq s_i^1 \leq c_i$ where $\sum\limits_{t \in T} s_i^t = c_i\}$.

We adapt the following definition from Thomson (2007):

Definition 3 *"**Bounded Impact of Transfers**" : A rule $F(.)$ satisfies "Bounded Impact of Transfers" (BIT hereafter) if for each $(c, E^t) \in$ß^{STAT} and each pair $\{i, j\} \subseteq N$, each $c_i' > c_i$ and each $c_j' < c_j$, if $c_i + c_j = c_i' + c_j'$, then $F_i(c_i', c_j', c_{N/\{i,j\}}, E^t) - F_i(c, E^t) \leq c_i' - c_i$ and $F_j(c, E^t) - F_j(c_i', c_j', c_{N/\{i,j\}}, E^t) \leq$*

$c_j - c'_j$

By BIT, we consider a static situation where an agent transfers some part of his claim to some other agent. Under a rule satisfying BIT, the change in agents' after-transfer-shares must be less than the transferred amount of claim. Note that in this definition, checking for BIT involves the transfer between two agents. However, by lemma 1, we will show that if BIT holds for a rule, then so does an extended version of it. In other words, even if an agent transfers some part of his claim to more than one agent, the change in each agents share will be less than the change in his claim.

Remark 2 *We don't impose BIT as a normative criteria. That is to say, we don't claim that the rules satisfying BIT have a superiority or an inferiority over the other rules. On the other hand, BIT is satisfied by a wide range of rules. We want to show that the strategy profile we defined in theorem 3, is an equilibrium for a large number of rules. Since requiring BIT and strong claims monotonicity is enough to prove that theorem and most of the renowned rules satisfy them, we are able to obtain our results.*

Remark 3 *It is known that the class of rules that satisfy claims monotonicity is very large, including PRO, CEA, CEL and TAL. Moreover, the class of rules that satisfy BIT is also large and contains PRO, CEA, CEL and TAL as well.*

Proposition 3 *PRO, CEA, CEL and TAL satisfy BIT*

Proof. A somewhat weaker statement of BIT implies the original version of it. However, it is easier to use the weaker version in some proofs. Then, we start our proof by showing this relation.

Assume $\sum_{k\in N} c_k \leq E^t$. Then, $c_i^o + c_j^0 + \sum_{k\in N\backslash\{i,j\}} c_k = \sum_{k\in N} c_k \leq E^t$ where $c_i^0 = c_i - \epsilon$ and $c_j^0 = c_j + \epsilon$. Then, any $F : \beta^{STAT} \rightarrow \mathbb{R}_+^N$ with an enlarged domain to pairs $(c, E^t) \notin \beta^{STAT}$ satisfies BIT and the relation that we show below for any pairs $(c, E^t), (c^o, E^t) \notin \beta^{STAT}$. Thus, we show the relation when $(c, E^t), (c^o, E^t) \in \beta^{STAT}$.

Let $(c, E^t), (c^o, E^t) \in \beta^{STAT}$ such that $c_i^0 = c_i - \epsilon$ and $c_j^0 = c_j + \epsilon$ for some $i, j \in N$ and $c_k^o = c_k$ for all $k \in N\backslash\{i,j\}$, for some $\epsilon > 0$. We want to show that $F_j(c_i^0, c_j^0, E^t) - F_j(c_i, c_j, E^t) \leq \epsilon$ for all such $(c, E^t), (c^o, E^t) \iff F_i(c_i, c_j, E^t) - F_i(c_i^0, c_j^0, E^t) \leq \epsilon$ for all such $(c, E^t), (c^o, E^t)$.

($\Longrightarrow$) Assume $F_j(c_i^0, c_j^0, E^t) - F_j(c_i, c_j, E^t) \leq \epsilon$ for all such $(c, E^t), (c^o, E^t)$. Let $c_i' = c_i^o + \epsilon$ and $c_j' = c_j^o - \epsilon$ for some $i, j \in N$ and $c_k' = c_k^0$ for all $k \in N\backslash\{i,j\}$, for some $\epsilon > 0$. (That is, $(c, E^t) = (c', E^t)$) Then, we have $F_i(c_i', c_j', E^t) - F_i(c_i^0, c_j^0, E^t) \leq \epsilon$ by our assumption. Since, $c_i' = c_i$ and $c_j' = c_j$ we have $F_i(c_i, c_j, E^t) - F_i(c_i^0, c_j^0, E^t) \leq \epsilon$ as desired.

Conversely, assume $F_i(c_i, c_j, E^t) - F_i(c_i^0, c_j^0, E^t) \leq \epsilon$ for all such $(c, E^t), (c^o, E^t)$. Let $c_i' = c_i^o + \epsilon$ and $c_j' = c_j^o - \epsilon$ for some $i, j \in N$ and $c_k' = c_k^0$ for all $k \in N\backslash\{i,j\}$, for some $\epsilon > 0$. Then, we have $F_j(c_i^0, c_j^0, E^t) - F_j(c_i', c_j', E^t) \leq \epsilon$. *That is,* $F_j(c_i^0, c_j^0, E^t) - F_j(c_i, c_j, E^t) \leq \epsilon$ for all such $(c, E^t), (c^o, E^t)$. As a result, it is enough to show that one side of this relation is satisfied by a rule F to show that it satisfies BIT as well.

Let $(c, E^t), (c^o, E^t) \in \beta^{STAT}$ such that $c_i^0 = c_i - \epsilon$ and $c_j^0 = c_j + \epsilon$ for some $i, j \in N$ and $c_k^o = c_k$ for all $k \in N\backslash\{i,j\}$, for some $\epsilon > 0$, for the following cases:

PRO: Since $\sum_{k\in N} c_k = \sum_{k\in N} c_k^o$, $F_j(c^o, E^t) - F_j(c, E^t) = \frac{E^t c_j^o}{\sum_{k\in N} c_k^o} - \frac{E^t c_j}{\sum_{k\in N} c_k} = \frac{E^t}{\sum_{k\in N} c_k}(c_j^o - c_j) = \frac{E^t}{\sum_{k\in N} c_k}\epsilon \leq \epsilon$ since $\frac{E^t}{\sum_{k\in N} c_k} \leq 1$. Hence, PRO satisfies BIT.

CEA: Let $\lambda^t = \lambda^t(c, E^t)$ and $\lambda^{ot} = \lambda^t(c^o, E^t)$. There are 2 possible cases:

Case i) $\lambda^{ot} \geq \lambda^t$. $F_i(c^o, E^t) = \min\left\{c_i^0, \lambda^{ot}\right\} = \min\left\{c_i - \epsilon, \lambda^{ot}\right\} \geq$

$\min\left\{c_i-\epsilon,\lambda^t-\epsilon\right\} = F_i(c,E^t)-\epsilon$ That is, Along with the relation we showed above, CEA satisfies BIT in this case.

Case ii) $\lambda^{ot} < \lambda^t$. Since we are checking for the case $\sum\limits_{k\in N} c_k > E^t$, there exists at least one $z \in N$ such that $c_z > \lambda^t$. We might have either $c_i \leq \lambda^t$ or $c_i > \lambda^t$. Assume $c_i \leq \lambda^t$. Then, $z \neq i$. $\min\left\{c_z,\lambda^{ot}\right\} = \lambda^{ot} < \lambda^t = \min\left\{c_z,\lambda^t\right\}$. Agent j's share can increase at most ϵ. That is, $F_j(c^o,E^t) = \min\left\{c^0_j,\lambda^{ot}\right\} = \min\left\{c_j+\epsilon,\lambda^{ot}\right\} \leq \min\left\{c_j+\epsilon,\lambda^t\right\} \leq \min\left\{c_j,\lambda^t\right\}+\epsilon = F_j(c,E^t)+\epsilon$. Hence, since $F_i(c^o,E^t) = \min\left\{c^0_i,\lambda^{ot}\right\} = \min\left\{c_i-\epsilon,\lambda^{ot}\right\} \leq \min\left\{c_i-\epsilon,\lambda^t-\epsilon\right\} = \min\left\{c_i,\lambda^t\right\}-\epsilon = F_i(c,E^t)-\epsilon$, we have $\sum\limits_{k\in N} F_k(c^o,E^t) = F_i(c^o,E^t)+F_j(c^o,E^t)+\lambda^{ot}+\sum\limits_{k\in N\setminus\{i,j,z\}} F_k(c^o,E^t) < F_i(c,E^t)-\epsilon+F_j(c,E^t)+\epsilon+\lambda^t+\sum\limits_{k\in N\setminus\{i,j,z\}} F_k(c,E^t) = E^t$ *That is,* $c_i \leq \lambda^t$ violates efficiency. Contradiction.

Thus, we must have $c_i > \lambda^t$. We might have either $\lambda^t-\lambda^{ot} \leq \epsilon$ or $\lambda^t-\lambda^{ot} > \epsilon$. Assume $\lambda^t-\lambda^{ot} > \epsilon$. $F_i(c^o,E^t) = \min\left\{c^0_i,\lambda^{ot}\right\} = \lambda^{ot} < \lambda^t-\epsilon = \min\left\{c_i,\lambda^t\right\}-\epsilon = F_i(c,E^t)-\epsilon$. Moreover, as we showed above, $F_j(c^o,E^t) \leq F_j(c,E^t)+\epsilon$. Since $\sum\limits_{k\in N\setminus\{i,j\}} F_k(c^o,E^t) \leq \sum\limits_{k\in N\setminus\{i,j\}} F_k(c,E^t)$, we have $\sum\limits_{k\in N} F_k(c^o,E^t) = F_i(c^o,E^t)+F_j(c^o,E^t)+\sum\limits_{k\in N\setminus\{i,j\}} F_k(c^o,E^t) < F_i(c,E^t)-\epsilon+F_j(c,E^t)+\epsilon+\sum\limits_{k\in N\setminus\{i,j\}} F_k(c,E^t) = \sum\limits_{k\in N} F_k(c,E^t) = E^t$ That is, $\lambda^t-\lambda^{ot} > \epsilon$ violates efficiency. Hence, we have $\lambda^t-\lambda^{ot} \leq \epsilon$. Then, $F_i(c^o,E^t) = \min\left\{c^0_i,\lambda^{ot}\right\} \geq \min\left\{c_i-\epsilon,\lambda^t-\epsilon\right\} = F_i(c,E^t)-\epsilon$ as desired. Combining **Case i)** and **Case ii),** we have $F_i(c,E^t)-F_i(c^o,E^t) \leq \epsilon$ for all such $(c,E^t),(c^o,E^t)$. Hence, by using the relation we found above, CEA satisfies BIT.

CEL: There are 2 possible cases.

Case i) $\lambda^{ot} \geq \lambda^t$. $F_j(c,E^t)+\epsilon = \max\left\{c_j-\lambda^t,0\right\}+\epsilon = \max\left\{c_j-\lambda^t+\epsilon,\epsilon\right\} \geq \max\left\{c_j-\lambda^t+\epsilon,0\right\} \geq \max\left\{c^o_j-\lambda^{ot},0\right\} = F_j(c^o,E^t)$.

Case ii) $\lambda^{ot} < \lambda.^t$ $F_i(c,E^t)-\epsilon = \max\left\{c_i-\lambda^t,0\right\}-\epsilon =$

$\max\left\{c_i - \lambda^t - \epsilon, -\epsilon\right\} \leq \max\left\{c_i^o - \lambda^{ot}, 0\right\} = F_i(c^o, E^t)$. $\sum\limits_{k \in N \setminus \{i,j\}} F_k(c, E^t) = \sum\limits_{k \in N \setminus \{i,j\}} \max\left\{c_k - \lambda^t, 0\right\} \leq \sum\limits_{k \in N \setminus \{i,j\}} \max\left\{c_k^o - \lambda^{ot}, 0\right\}$ (1)

Since $\sum\limits_{k \in N} F_k(c, E^t) = \sum\limits_{k \in N \setminus \{i,j\}} \max\left\{c_k - \lambda^t, 0\right\} + \max\left\{c_i - \lambda^t, 0\right\} + \max\left\{c_j - \lambda^t, 0\right\} =$
$\sum\limits_{k \in N \setminus \{i,j\}} \max\left\{c_k^o - \lambda^{ot}, 0\right\} + \max\left\{c_i^o - \lambda^{ot}, 0\right\} + \max\left\{c_j^o - \lambda^{ot}, 0\right\} =$
$\sum\limits_{k \in N} F_k(c^o, E^t)$. Together with (1), we have $\max\left\{c_i - \lambda^t, 0\right\} + \max\left\{c_j - \lambda^t, 0\right\} \geq \max\left\{c_i^o - \lambda^{ot}, 0\right\} + \max\left\{c_j^o - \lambda^{ot}, 0\right\}$ *That is,* $\max\left\{c_i - \lambda^t, 0\right\} - \max\left\{c_i^o - \lambda^{ot}, 0\right\} \geq \max\left\{c_j^o - \lambda^{ot}, 0\right\} - \max\left\{c_j - \lambda^t, 0\right\}$ (2)

$\max\left\{c_i - \lambda^t, 0\right\} - \epsilon = \max\left\{c_i - \lambda^t - \epsilon, -\epsilon\right\} \leq \max\left\{c_i^o - \lambda^{ot}, 0\right\}$ *That is,* $\epsilon \geq \max\left\{c_i - \lambda^t, 0\right\} - \max\left\{c_i^o - \lambda^{ot}, 0\right\} \geq \max\left\{c_j^o - \lambda^{ot}, 0\right\} - \max\left\{c_j - \lambda^t, 0\right\} = F_j(c^o, E^t) - F_j(c, E^t)$ as desired.

Combining **Case i)** and **Case ii),** we have $F_j(c^o, E^t) - F_j(c, E^t) \leq \epsilon$ for all such $(c, E^t), (c^o, E^t)$. Thus, CEL satisfies BIT.

TAL: Since $\sum\limits_{k \in N} c_k = \sum\limits_{k \in N} c_k^o$, which part of TAL will be applied before and after the transfer is fixed. Assume $\sum\limits_{k \in N} \frac{c_k}{2} \geq E^t$. Then, $F_k(c, E^t) = \min\left\{\frac{c_k}{2}, \lambda^t\right\} = CEA_k(\frac{c}{2}, E^t)$ for all $k \in N$ and $F_k(c^o, E^t) = \min\left\{\frac{c_k^o}{2}, \lambda^t\right\} = CEA_k(\frac{c^o}{2}, E^t)$ for all $k \in N$. Hence, $F_j(c^o, E^t) - F_j(c, E^t) = CEA_k(\frac{c^o}{2}, E^t) - CEA_k(\frac{c}{2}, E^t) \leq \frac{\epsilon}{2}$ since CEA satisfies BIT.

Let $\sum\limits_{k \in N} \frac{c_k}{2} < E^t$. Then, $F_j(c, E^t) = c_j - \min\left\{\frac{c_j}{2}, \lambda^t\right\} = \max\left\{\frac{c_j}{2}, c_j - \lambda^t\right\} = \frac{c_j}{2} + \max\left\{0, \frac{c_j}{2} - \lambda^t\right\}$. We also have $\frac{c_j^o}{2} = \frac{c_j + \epsilon}{2}$. Since we are checking for the case $\sum\limits_{k \in N} c_k > E^t$, we have $\sum\limits_{k \in N} F_k(c, E^t) = E^t$. Then, $\sum\limits_{k \in N} (\frac{c_k}{2} + \max\left\{0, \frac{c_k}{2} - \lambda^t\right\}) = E^t$, $\sum\limits_{k \in N} \max\left\{0, \frac{c_k}{2} - \lambda^t\right\} = E^t - \sum\limits_{k \in N} \frac{c_k}{2}$. Hence, $\frac{c_j}{2} + \max\left\{0, \frac{c_j}{2} - \lambda^t\right\} = \frac{c_j}{2} + CEL_j(\frac{c}{2}, E^t - \sum\limits_{k \in N} \frac{c_k}{2})$, $F_j(c^o, E^t) - F_j(c, E^t) = \frac{c_j^o}{2} - \frac{c_j}{2} + CEL_j(\frac{c^o}{2}, E^t - \sum\limits_{k \in N} \frac{c_k^o}{2}) - CEL_j(\frac{c}{2}, E^t - \sum\limits_{k \in N} \frac{c_k}{2}) \leq \frac{\epsilon}{2} + \frac{\epsilon}{2} = \epsilon$, since CEL satisfies BIT. Since in both cases of TAL, we showed that the required condition for agent j is satisfied for all such $(c, E^t), (c^o, E^t)$, the required condition for agent i is also satisfied for all such $(c, E^t), (c^o, E^t)$. Hence, TAL satisfies BIT. ■

The following definitions are from Thomson, (2003):

Definition 4 ***(Weak Claims Monotonicity):*** *A rule $F(.)$ satisfies weak claims monotonicity if for each $(c, E^t) \in ß^{STAT}$, each $i \in N$, and each $c_i' > c_i$, we have $F_i(c_i', c_{-i}, E^t) \geq F_i(c, E^t)$. A rule $F(.)$ is said to be* ***strongly claims monotonic*** *if it is weakly claims monotonic and for each $(c, E^t) \in ß^{STAT}$, each $i \in N$, and each $c_i' > c_i$, we have $F_{-i}(c_i', c_{-i}, E^t) \leq F_{-i}(c, E^t)$ for each $-i \in N \backslash \{i\}$.*

Next, we will present an example to show that we can not make a conclusion using only claims monotonicity, but not BIT.

Example 3 *Consider the rule F defined by the following algorithm: Let $K_1 = \{k \in N \text{ such that } c_k = \max\{c_1, c_2, ..., c_N\}\}$. If $\sum_{k \in K_1} c_k \geq E^t$, then $F_k(c, E^t) = \frac{E^t}{K_1}$[2]. If $\sum_{k \in K_1} c_k < E^t$, then $F_k(c, E^t) = c_k$ and for*

$K_2 = \{z \in N \backslash K_1 \text{ such that } c_z = \max N \backslash K_1\}$ if $\sum_{z \in N \backslash K_1} c_z \geq E^t$, $F_z(c, E^t) = \frac{E^t - \sum_{k \in K_1} F_k(c, E^t)}{K_2}$ and if otherwise, F proceeds the same way until the entire endowment is allocated or there is no claim left. Since nobody can lose his priority ranking, (i.e., to which K_t he belongs) by increasing his claim, F is strongly claims monotonic. Nevertheless, it is easy to check that F does not satisfy BIT by considering the following case: $c_1 = 90$, $c_2 = 80$, $c_1^o = 80$, $c_2^o = 90$, $E^t = 100$. One can see that our equilibrium profile is not an equilibrium for F under at least some parameters by checking the following example: Let $c_1 = 90$, $c_2 = 80$, $\delta_1 = 0.9$, $E^1 = 55$, $E^2 = 60$, $E^3 = 55$. Assume agent 2 plays 80 at

[2]We assume that number of elements in K_t is K_t

$t = 1$ and then, his entire remaining claim at each period. Against this strategy, if agent 1 plays 90 at $t = 1$ and then, his entire remaining claim at each period, he gets: $u_1(90, .) = 83.35$. Instead, if he plays 80 at $t = 1$ and then, his entire remaining claim at each period, he gets: $u_1(80, .) = 83.525 > 83.35$. Therefore, the strategy profile we specified as equilibrium is no more so in this example.

Theorem 2 *Let G^d be a 2×2 hospital game. Assume $c_1 + c_2 \leq E^1 + E^2$. Then, if F is claims monotonic, then the profile s^* such that $s_i = c_i$ for all $i \in N$ is an equilibrium of G^d and, moreover, if s is an equilibrium of G^d then it generates the same payoffs as s^*. Assume $c_1 + c_2 > E^1 + E^2$. Then, if F satisfies claims monotonicity and BIT, then the profile s^* such that $s_i = c_i$ for all $i \in N$ is an equilibrium of G^d and, moreover, if s is an equilibrium of G^d then it generates the same payoffs as s^*.*

Proof. We denote $x_i^1 = F_i(s_i, s_j, E^1), x_i^{o1} = F_i(s_i^o, s_j, E^1), x_i^{\Delta 1} = F_i(s_i^{\Delta}, s_j, E^1)$, $x_i^2 = F_i(c_i - F_i(s_i, s_j, E^1), c_j - F_j(s_i, s_j, E^1), E^2)$ and so on. We first show that ,given $c_1 + c_2 > E^1 + E^2$, at equilibrium $s_1 + s_2 < E^1$ is not possible. The same argument also applies when $c_1 + c_2 \leq E^1 + E^2$, however, we don't use that in our proof. Given $s_1 + s_2 < E^1$ there exists an agent i with $s_i < c_i$. Otherwise, we would have $c_i + c_j < E^1$ which is contradictory to our assumption. With the same s_j, consider a profile such that $s_i^o + s_j = E^1$ or ($s_i^o + s_j < E^1$ and $s_i^o = c_i$). If the latter is true, then since it's the highest possible utility with parameter c_i, $s_i^o = c_i$ is a best response of agent i. Assume $s_i^o + s_j = E^1$. Let $s_i^o - s_i = \epsilon$. Then, $s_i^o = x_i^{o1}$ and $s_i = x_i^1$, hence $(c_i - x_i^1) - (c_i - x_i^{o1}) = \epsilon$. If $(c_j - x_j^1) - (c_j - x_j^{\Delta 1}) = -\epsilon$ then by BIT, $x_i^2 - x_i^{\Delta 2} \leq \epsilon$. Furthermore, we have $(c_j - x_j^1) = (c_j - x_j^{o1})$. By claims monotonicity, we have $x_i^{o2} \geq x_i^{\Delta 2}$. Hence, $x_i^2 - x_i^{o2} \leq \epsilon$.. We also have $x_i^{o1} - x_i^1 = \epsilon$. Therefore, $x_i^{o1} + \delta_i x_i^{o2} > x_i^1 + \delta_i x_i^2$ That is, agent i has an incentive to deviate. The same argument applies for agent j

when $s_i = c_i$ and $s_i + s_j < E^1$. Hence, at equilibrium, we have $s_1 + s_2 \geq E^1$.

Case i) $c_1 + c_2 \leq E^1 + E^2$. By our assumption on F, $x_i^1 + x_i^2 \leq c_i$ for all $s \in S$. Consider $s_i = c_i$ and some $s_j \in S_j$. If $c_i + s_j \leq E^1$, then $x_i^1 = c_i$, which gives the highest possible utility with parameter c_i. If $s_i + s_j > E^1$ then, $x_i^1 + x_j^1 = E^1$. Together with $c_i + c_j \leq E^1 + E^2$, we have $(c_i - x_i^1) + (c_j - x_j^1) \leq E^2$. Thus, $x_i^2 = (c_i - x_i^1)$ and $x_j^2 = (c_j - x_j^1)$ That is, $x_i^1 + x_i^2 = c_i$ for $i \in N$. Consider $s_i^o < s_i$. By claims monotonicity, $x_i^{o1} \leq x_i^1$. If $x_i^{o1} = x_i^1$, then $u_i^o = u_i$. Assume $x_i^{o1} < x_i^1$ and $x_i^1 - x_i^{o1} = \epsilon$. Since $x_i^{o1} + x_i^{o2} \leq c_i$, we have $x_i^{o2} - x_i^2 \leq \epsilon$. Then, $u_i - u_i^o = (x_i^1 - x_i^{o1}) + \delta_i(x_i^2 - x_i^{o2}) \geq \epsilon - \delta_i\epsilon > 0$. Hence, $s_i = c_i$ is the weakly dominant strategy for both players. Thus, s^* such that $s_i = c_i$ for all $i \in N$ is an equilibrium and at any equilibrium s we have the same payoffs as that of $s^{*\prime}s$.

Case ii) Again, consider $s_i = c_i$ and some $s_j \in S_j$. If $c_i + s_j \leq E^1$, then $x_i^1 = c_i$, which gives the highest possible utility with parameter c_i. Let $s_i + s_j > E^1$. Then, we have $x_i^1 + x_j^1 = E^1$. Consider any $s_i^o \in S_i$ such that $s_i^o < s_i = c_i$. By claims monotonicity, $x_i^{o1} \leq x_i^1$. If $x_i^{o1} = x_i^1$, then $u_i^o = u_i$. Assume $x_i^{o1} < x_i^1$ and $x_i^1 - x_i^{o1} = \epsilon$. We have already shown that if $s_i^o + s_j < E^1$ then agent i has an incentive to deviate and increase s_i^o until $s_i^o + s_j = E^1$ is satisfied. Let $s_i^o + s_j \geq E^1$. Again, $x_i^{o1} + x_j^1 = E^1$. Therefore, $x_j^{o1} - x_j^1 = \epsilon$. Then, $(c_i - x_i^1) - (c_i - x_i^{o1}) = -\epsilon$ and $(c_j - x_j^1) - (c_j - x_j^{o1}) = \epsilon$. By BIT, $x_i^{o2} - x_i^2 \leq \epsilon$. $u_i - u_i^o = (x_i^1 - x_i^{o1}) + \delta_i(x_i^2 - x_i^{o2}) \geq \epsilon - \delta_i\epsilon > 0$ since $\delta_i < 1$. Hence, $s_i = c_i$ is the weakly dominant strategy of agent i. Since the same argument applies for agent j, the payoffs of s^* is unique at any equilibrium s. Thus, the proof is complete. ■

Theorem 3 *($N \times T$ **Hospital Model**) Let G^d be an $N \times T$ hospital game. Then, if F is strongly claims monotonic and satisfies BIT, the strategy profile s^* such that $s_k^1 = c_k$, $s_k^2 = c_k - F_k(c, E^1)$, $s_k^3 = c_k - F_k(c, E^1) - F_k(c - F_k(c, E^1), E^2)$,..., $s_k^N = c_k - F_k(c, E^1) - ... - F_k(c - F(c, E^1) - ... - F(c -$*

$F(c, E^1) - ... - F(...), E^N)$. *(That is, each agent $k \in N$ uses his entire remaining claim at each of the periods) is a SPNE and any SPNE s generates the same payoffs as s^*.*

We first prove that BIT implies an extended version of itself by the following lemma.

Lemma 1 *Let $(c, E^t), (c^o, E^t) \in ß^{STAT}$. BIT implies that the following statement is true: Let $c_i^0 = c_i - \epsilon$, $c_k^o = c_k + \epsilon_k$ for $k \in N \setminus \{i\}$ where $\sum\limits_{k \in N \setminus \{i\}} \epsilon_k = \epsilon$. Then, we have $F_i(c, E^t) - F_i(c^o, E^t) \leq \epsilon$ and $F_k(c^o, E^t) - F_k(c, E^t) \leq \epsilon_k$ for all $k \in N \setminus \{i\}$.*

Proof. Define $\epsilon_0 = 0$. We have $F_i(c_1 + \epsilon_{1,} c_2 + \epsilon_2, ..., c_k + \epsilon_k, ..., c_i - \sum\limits_{j=0}^{k} \epsilon_j, ..., c_N) - F_i(c_1 + \epsilon_{1,}..., c_{k+1} + \epsilon_{k+1}, ..., c_i - \sum\limits_{j=0}^{k+1} \epsilon_j, ..., c_N) \leq \epsilon_{k+1}$ for $k = \{0, 1, ..., N-1\}$ by BIT. Then we have, $\sum\limits_{k=0}^{N-1} F_i(c_1 + \epsilon_{1,} c_2 + \epsilon_2, ..., c_k + \epsilon_k, ..., c_i - \sum\limits_{j=0}^{k} \epsilon_j, ..., c_N) - F_i(c_1 + \epsilon_{1,}..., c_{k+1} + \epsilon_{k+1}, ..., c_i - \sum\limits_{j=0}^{k+1} \epsilon_j, ..., c_N) =$

$$= F_i(c, E^t) - F_i(c_1 + \epsilon_{1,}..., c_i - \sum_{j=1}^{N} \epsilon_j, ..., c_N + \epsilon_N) \leq \sum_{j=1}^{N} \epsilon_j = \epsilon \text{ as desired. } \blacksquare$$

Now, we are checking for the first group of subgames of the hospital game, namely for the last two periods.

Lemma 2 ($N \times 2$ Equilibrium) *For a $N \times 2$ hospital game, the strategy profile s^* such that $s_k = c_k$ for each $k \in N$ is a Nash equilibrium and any Nash equilibrium s yields the same payoffs as s^*.*

Proof. By claims monotonicity, all the remaining claims are used in the last period. As a consequence, we can write the strategies as a function of only the first period's claims so that we don't use superscripts for notational simplicity.

Consider any $F_k(c_k, s_{-k}, E^1) - F_k(s_k, s_{-k}, E^1) = \epsilon > 0$ for some $s_{-k} \in S_{-k}$. $\sum\limits_{-k \in N\setminus\{k\}} F_{-k}(s_k, s_{-k}, E^1) - F_{-k}(c_k, s_{-k}, E^1) = \sum\limits_{-k \in N\setminus\{k\}} \epsilon_{-k} = \epsilon$. Hence, $(c_k - F_k(s_k, s_{-k}, E^1)) - (c_k - F_k(c_k, s_{-k}, E^1)) = \epsilon$ and $(c_{-k} - F_{-k}(c_k, s_{-k}, E^1)) - (c_{-k} - F_{-k}(s_k, s_{-k}, E^1)) = \epsilon_{-k}$ for all $-k \in N\setminus\{k\}$. From BIT we have $F_k((c_k - F_k(s_k, s_{-k}, E^1)), (c_{-k} - F_{-k}(s_k, s_{-k}, E^1)), E^2) - F_k((c_k - F_k(c_k, s_{-k}, E^1)), (c_{-k} - F_{-k}(c_k, s_{-k}, E^1)), E^2) \leq \epsilon$

Hence, $F_k(c_k, s_{-k}, E^1) + \delta_k F_k((c_k - F_k(c_k, s_{-k}, E^1)), (c_{-k} - F_{-k}(c_k, s_{-k}, E^1)), E^2)$
$> F_k(s_k, s_{-k}, E^1) + \delta_k F_k((c_k - F_k(s_k, s_{-k}, E^1)), (c_{-k} - F_{-k}(s_k, s_{-k}, E^1)), E^2)$
for all $s_{-k} \in S_{-k}$. Hence, $s_k = c_k$ is a weakly dominant strategy for agent k. Since this is true for all $k \in N$, we have the desired result. ■

Proof. *(of the theorem 3)* We have proved our claimed for $T = 2$ by the above lemma. Now, we are going to prove it for arbitrary T, by assuming it for $T - 1$. Notice that by this assumption, we can write the strategies as a function of only the actions in the first period. We denote $F_k(s_k, s_{-k}, E^t) = x_k^t(s_k, s_{-k})$ and $F_k(c_k, s_{-k}, E^t) = x_k^t(c_k, s_{-k})$ for $k \in N$ and $t \in T$. Let $s_k < c_k$ be such that $x_k^1(c_k, s_{-k}) - x_k^1(s_k, s_{-k}) = \epsilon > 0$ for some $s_{-k} \in S_{-k}$ and for some $k \in N$. By strong claims monotonicity we have $x_{-k}^1(s_k, s_{-k}) - x_{-k}^1(c_k, s_{-k}) = \epsilon_{-k}$ for each $-k \in N\setminus\{k\}$ where $\sum\limits_{-k \in N\setminus\{k\}} \epsilon_{-k} = \epsilon$. Here, by $s_k^t(s_k, s_{-k})$ for $t = \{2, ..., T\}$ we denote the SPNE of the subgames where each agent uses his entire remaining claim at each of the remaining periods after playing (s_k, s_{-k}) at $t = 1$.

$s_{-k}^2(c_k, s_{-k}) - s_{-k}^2(s_k, s_{-k}) = \epsilon_{-k}$ and $s_k^2(s_k, s_{-k}) - s_k^2(c_k, s_{-k}) = \epsilon$. Denote $x_k^2(s_k, s_{-k}) - x_k^2(c_k, s_{-k}) = \psi_k^2$ and $x_{-k}^2(c_k, s_{-k}) - x_{-k}^2(s_k, s_{-k}) = \psi_{-k}^2$. [3]From BIT, we have $\psi_{-k}^2 \leq \epsilon_{-k}$ and $\psi_k^2 \leq \epsilon$. From efficiency and claims boundedness, $\sum\limits_{-k \in N\setminus\{k\}} x_{-k}^2(c_k, s_{-k}) + x_k^2(c_k, s_{-k}) = \sum\limits_{-k \in N\setminus\{k\}} x_{-k}^2(s_k, s_{-k}) + x_k^2(s_k, s_{-k})$ *That is,* $\sum\limits_{-k \in N\setminus\{k\}} \psi_{-k}^2 = \psi_k^2$

[3] ψ_{-k}^2 and ψ_k^2 are functions of (s_k, s_{-k}). However, since we fixed a strategy profile begining with (s_k, s_{-k}), we won't write this each time for simplicity.

We then have $s^3_{-k}(c_k, s_{-k}) - s^3_{-k}(s_k, s_{-k}) = \epsilon_{-k} - \psi^2_{-k}$ and $s^3_k(s_k, s_{-k}) - s^3_k(c_k, s_{-k}) = \epsilon - \psi^2_k$.

$\psi^3_{-k} = x^3_{-k}(c_k, s_{-k}) - x^3_{-k}(s_k, s_{-k}) \leq \epsilon_{-k} - \psi^2_{-k}$, $\psi^3_k = x^3_k(s_k, s_{-k}) - x^3_k(c_k, s_{-k}) \leq \epsilon - \psi^2_k$ and

$s^4_{-k}(c_k, s_{-k}) - s^4_{-k}(s_k, s_{-k}) = \epsilon_{-k} - \psi^2_{-k} - \psi^3_{-k}$ and $s^4_k(s_k, s_{-k}) - s^4_k(c_k, s_{-k}) = \epsilon - \psi^2_k - \psi^3_k$, proceeding the same way we have, $\psi^T_k = x^T_k(s_k, s_{-k}) - x^T_k(c_k, s_{-k})$ and $\psi^T_{-k} = x^T_{-k}(c_k, s_{-k}) - x^T_{-k}(s_k, s_{-k})$, then $\psi^T_k \leq \epsilon - \psi^2_k - \psi^3_k - ... - \psi^{T-1}_k$ That is, $\epsilon \geq \sum_{t=2}^{T} \psi^t_k$. Therefore, since $\delta_k < 1$, $\sum_{t=1}^{T} x^t_k(c_k, s_{-k}) \geq \sum_{t=1}^{T} x^t_k(s_k, s_{-k}) > \sum_{t=1}^{T} \delta^{t-1}_k x^t_k(s_k, s_{-k})$ as desired. As a result, using the entire available claim at each period of each subgame is a weakly dominant strategy. Therefore, everyone playing s^* is a SPNE and every SPNE s must generate the same payoffs. Thus, the proof is complete. ■

5 Conclusion

Both for the steel and the hospital games, we have showed that if the agents are impatient, *that is,* ., they care more for the previous periods, then they tend to claim higher at those periods as long as claiming higher generates returns. This is also a consequence of the fact that the extra returns and the extra perished claims are at the same amount in our models. We found to what extend claiming higher generates returns in the steel game. For the hospital game, we have showed that, since the agents don't lose the claims for which they don't receive shares in return, each uses his entire (remaining) claim at each period. Note that this is also due to the fact that decreasing the share at some period, don't yield a higher share in the succeeding periods. (BIT) Fortunately, the class of rules that satisfy strong claims monotonicity and BIT is very large and our analysis includes them.

One can further inquire whether there is any possibility for cooperation. First note that there is a basis for cooperation only if the profile of utilities at the cooperation strategies satisfies some two conditions. The first one requires the existence of an agent who benefits from this cooperation. The second condition says that no player in the cooperation should end up with a utility less than the utility he receives if he doesn't cooperate. Notice that these two conditions together form the weakest requirement for such a cooperation. If one of these conditions does not hold, then either one agent gets worse-off or each cooperating agent gets the same utility he gets in the non-cooperating equilibrium. As a result, there is no cooperation at the absence of a such a utility profile. For a dynamic situation with two agents and two periods, Turhan (2009) has showed that given that the discount factors of the agents are not equal to each other and at each period each agent has a claim less or equal to the endowment of that period, an allocation is Pareto optimal if and only if the agent with the lower discount factor receives his entire claim at the first period and the other agent receives his at the second period. This is, however, not the case in many equilibria of our games. For instance consider the 2×2 Hospital Game under CEA where $(c_1, c_2, E^1, E^2) = (100, 100, 100, 100)$. In any equilibrium of the game, we have $(x_1^1, x_1^2, x_2^1, x_2^2) = (50, 50, 50, 50)$. Given that $\delta_1 = 0.8$ and $\delta_2 = 0.4$, the allocation $(70, 20, 30, 80)$ Pareto dominates the allocation $(50, 50, 50, 50)$. In addition, we know that in the equilibria of the games we constructed, the payoffs are unique. Therefore, at the absence of binding agreements, any cooperation is not sustainable. On the other hand, if there are binding agreements to sustain such a cooperation, then the strategy profiles they involve are not SPNE.

Secondly, it is natural to ask whether a central authority, say government, can achieve a Pareto optimal outcome as an equilibrium of the Hospital Game, if he is able to change the allocation rule at each period. In such a setting, the agents either know which rule will be used at each period certainly before the game starts or the government chooses the rule at each period without informing the agents. As a third alternative, at each period the government might announce only that period's rule before the agents choose their strategies. If the government doesn't inform the agents before the game starts, at each period he can either choose the rule with observing the agents' claims for that period or without observing them. However, in each of these settings, if we assume that the government chooses the rule among a set of rules satisfying BIT and strong claims monotonicity, then our results are still valid. That is, the government can not create a Pareto dominating outcome. As a matter of fact, most of the rules satisfying some "desirable" properties also satisfy BIT and strong claims monotonicity. As a consequence, the government's choices won't change the agents' behavior in equilibrium.

Finally, another open question is how the government would behave if he could determine the endowment at each period given the total endowment to be allocated among the periods. For a government caring for the agents' utilities, we expect him to allocate the entire endowment to the first period because we assume that all agents prefer the first period over the others.

There are instances where $\sum_{i \in N} c_i \leq \sum_{t=1}^{T-1} E^t$. In those cases some of the periods at the end are idle. In other words, it is obvious that in any equilibrium there won't be any claim in those periods. Our analysis is valid for such games as well.

As a future research question, one can extend the preferences to single-peaked ones which are also very common in the literature. Another interesting open question involves the equilibrium if there are multi-issue endowments at the periods. That is, agents might receive shares from different endowments at the same period. It's interesting to ask under what conditions agents tend to claim from the same endowment and under what conditions they rather prefer different endowments

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