A MODEL OF VALET PARKING

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Abstract

This thesis provides an explanation for the coexistence of regular free parking and paid valet parking in some shopping malls. The varying socio-economic levels of customers, their differing valuations of the goods sold at the mall, and an extra utility associated with valet service are the immediate reasons for the emergence of valet parking at the mall. We provide a theory based on the latter two reasons to analyze parking decisions of customers and the pricing strategy of shopping malls. Based on the sorting of different kinds of customers between valet and regular parking, various parking regimes can emerge at the mall including regular parking only, valet parking only, and mixed regimes of the two.

VALE PARK MODELLEMESİ

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Özet

Bu tezde, şehir merkezinin dışında kalan alışveriş merkezlerindeki ücretsiz otopark ve ücretli vale park uygulamasının ortaya çıkışı ve açıklamaları incelenmektedir. Müşterilerin alışveriş merkezinde satılan ürünlere farklı değer biçmeleri, farklı zenginlik seviyeleri ve vale park kullanmaktan elde edilen fayda; vale parkın ortaya çıkışı için sunulabilecek açıklamalardır. Bu saydığımız nedenlerden, modelde ilk ikisine odaklanıyoruz. Bu açıklamaları kullanarak müşterilerin park kararlarını ve alışveriş merkezlerinin fiyat stratejilerini açıklıyoruz. Modelde kullandığımız çeşitle değişkenlere (otopark masrafı, vale masrafı, vale park kullanmaktan alınan fayda) bağlı olarak farklı rejimler oluşabilir. Bunlar bütün müşterilere sadece otopark sunulan, sadece vale park hizmeti sunulan, ya da müşteriler kategorize edilerek bir kısma vale park, diğerine otopark sunulan ve hatta bazı durumlarda fiyatları iyice artırarak belli müşterilerin alışveriş merkezine gelmemesini sağlayan rejimlerdir.

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1 Introduction

We analyze the parking behavior of shopping mall customers and the pricing strategy of shopping malls by providing an extension of the base model developed by Hasker and Inci (forthcoming). A typical shopping mall allocates $4-6$ parking spaces per $1,000$ sq. ft. of gross leasable area (International Council of Shopping Centers and Urban Land Institute, 2003; henceforth, ICSC and ULI, 2003), which means that shopping malls allocate more space to parking than to stores. Considering also the facts that there are approximately $109,500$ shopping centers in the United States ranging in size from the small convenience centers to the large super-regional malls and that these centers share over half of total retail sales in the US with $\$2.26$ trillion (ICSC, 2003), we can say that shopping malls substantially contribute to parking capacity as well as the economy. Thus, an analysis of parking at shopping malls is vital. In addition, we classify shopping malls as urban and suburban. Hasker and Inci (forthcoming) define a shopping mall as suburban if its parking lot is not used as a parking garage but used by customers who visit the mall for shopping, otherwise it is urban. In urban areas, the malls may have an incentive to charge for parking in order to prevent people from using a parking lot as a free parking garage. In order to eliminate this external factor on urban malls, we focus on suburban malls where the only way to get to the mall is by car.

As one of the most important intermediate goods in the modern market economy, parking has been an increasingly significant topic in economics, especially after the early 1990s. Before that, urban planners were decisive in policy making. They concerned with setting minimum parking requirements for every land use. The Planning Advisory Service of the American Planning Association (1991) reports that it “receives hundreds of requests each year about off-street parking requirements for different land uses–in fact, we receive more requests year after year on this topic than on any other.” The aim of urban planners requiring off-street parking requirements is to overcome the peak demand for each land use. As a result, parking was free for 99% of automobile trips in the US (US Department of Transportation, 1990). While urban planners insist on minimum parking requirements, they rarely take the cost of parking into account. Shoup (1997) estimates the cost of an additional parking space in 1991 at UCLA campus as $\$23,346$. He also underlines the social cost of parking that arises from congestion and pollution externalities. In addition, excessive free parking spaces deteriorate urban design and increase automobile dependency: Shoup and Pickrell (1980) show that free parking causes more solo driving. Free parking reduces the market price of parking and so inflates demand. Together with all the side effects of minimum parking requirements, according to Shoup (1997), urban planning education does not provide any instruction for setting parking requirements. Consequently, he suggests eliminating minimum parking requirements.

Hasker and Inci (forthcoming) devise a model that justifies the common implementation of minimum parking requirements in the context of shopping mall parking. They analyze the parking behavior in shopping malls and find that the socially optimal lot size is always larger than the lot size that a profit maximizing mall charges. They also show malls largely provide free parking despite the cost of the vast amount of land devoted to parking. Moreover, they find that free parking is socially optimal in a second-best sense. In fact, the first-best social optimum is subsidizing parking, which is not implementable. In other words, the mall is willing to insure the customers who leave the mall empty-handed by embedding the cost of parking into the price of the good.

We extend the model of Hasker and Inci (forthcoming) by allowing for valet parking. So, customers are provided two types of parking: regular and valet. The benefit of valet parking to customers is that they do not need to cruise to find an empty lot and walk the distance between the parking spot and the mall because the valet point is located just at the entrance of the mall. The malls are aware that shoppers do not want to walk in a shopping mall's garage and are sensitive to the customers' need for the availability of parking. Moreover, not being able to find a close parking spot can be the cause of bad shopping experience or even driving off (Grant 2011). The shopping malls are trying new ways to assure customers close and in-easy-reach parking spaces. For instance, they deploy cell-phone applications, overhead sensors and cameras to show customers empty parking spaces. An alternative might be offering valet parking and they sometimes do so with an exclusive service of suited, presentable valet staff. The mall developer Westfield group introduced valet parking with a key fob feature, that is, to pull up your car before going to the garage. Nevertheless, to minimize the waiting time, the mall is trying to replace a new way such as text message instead of key fob. All in all, parking availability is known to be a significant factor on customer's shopping behavior and the malls are thus seeking better tools. In this sense, valet parking seems to be a good alternative in terms of close space, exclusive service, limited (or no) cruising. We incorporate these into the model by adding an extra utility associated with valet parking.

Even though customers get some utility from using valet service, there are some factors affecting the customers' decision of choosing valet service over regular parking such as difference in wealth levels, the utility from getting valet service, or the valuation of the good sold at the mall. The valuation of the good sold at the mall can be seen as the most surprising one to affect the parking behavior and we will provide the impact of it, together with the utility from using valet service, on parking behavior of customers. We will also concentrate on welfare analysis and try to determine the cases in which implementing a valet service would bring the highest welfare.

The model suggests that two types of customers exist who differ by their valuation of the good sold at the mall. Note that the customers do not differ by their valuation of the valet service or by their wealth. In our model, we focus on the effect of the valuation of the good on parking choice. All customers have primarily two choices: going to the mall or engaging in another activity. They all have the same utility from engaging in another activity. However, their utilities from going to the mall differ because they have different valuations of the good and they might prefer different kinds of parking, namely valet and regular.

Now, consider the mall's problem. It is aware that there are two types of customers. Therefore, it may offer two different types of parking to get the maximum payoff it can get if the valuations are distant enough. If it offers only regular parking with the same price to all, high types earn a higher surplus than do low types. So, when the customers have different valuations, it can be optimal for the mall to provide different kinds of parking. In the end, the mall maximizes its payoff by choosing the prices of parking and the good.

The rest of the paper is organized as follows. Section 2 reviews the literature, Section 3 presents the model and solves for equilibria, Section 4 shows comments on welfare and Section 5 concludes.

2 Literature Review

To my knowledge, this is the first attempt to analyze valet parking. Parking literature mostly focuses on traffic congestion, parking lot size, and parking fees.

Arnott and Rowse (1999) create a model of parking congestion to find an optimal parking fee. They consider a city with spatial homogeneity located on an annulus. They assume road demand to be exogenous, stochastic and time-invariant. The walking distance between the parking spot and the destination point depends on the mean density of empty parking spaces. The critical point is that people neglect the effect of their parking on the mean density of empty parking spaces. In fact, it does affect the mean density and as a consequence, the mean density of empty parking spaces becomes endogenous. Although they create a simple model due to their assumption of spatial homogeneity, they end up with complex results. They find that parking fee should be equal to the value of congestion externality imposed. However, their model generates three equilibria which can be Pareto ranked. Thus, due to multiplicity of equilibria, the social optimum is not guaranteed even with optimal pricing.

Anderson and de Palma (2004) consider parking as a common property resource and suggest pricing it in a monopolistic manner. They set up a linear city model with fixed and exogenous parking capacity at each location and without pricing, parking close to destination will be excessive according to their model. The solution they emphasize is the same as that of Knight (1924), which is private ownership of parking areas –private ownership of common property resources in Knight (1924) paper– and thus pricing them in a monopolistic manner. To better understand their contention, let me modify Pigou's (1924) "two roads" example. Consider two parking areas, one of which is curbside and close to the destination but limited in capacity and the other is a huge off-street parking thus being able to accommodate all the cars without causing anyone cruising. Suppose that we have only these two options of parking and both of them are free. In such a setting, the equilibrium will be such that the cost of getting to the destination from the off-street parking area will be equal to the cost of cruising caused by the marginal driver who preferred to take his or her chance on curb. In other words, as more drivers look for a curbside parking space, cruising for parking increases up to a point where it is equally costly to look for a curbside or off-street parking. After the equilibrium is achieved, if a few people looking for a curbside are transferred to the off-street parking, they would not incur higher cost but those who remained in curbside parking area would benefit from decreased cruising time, that is to say individual freedom does not always result in a socially optimum outcome. We can obtain the social optimum in this case by levying a tax on the curbside that makes curbside and off-street parking equally costly. Pigou thinks if the curbside is apt to private appropriation, the profit-seeking private owner would charge a rent that would prevent the market from being a social optimum. However, according to Knight (1924), the amount of charge by the owner of the road will be equal to the tax that would otherwise be imposed by the government. My model holds with Knight's contention in the sense that the mall is managed by a monopolist private owner.

Shoup (1999) discusses why minimum parking requirements are fallacious and suggests pricing curbside parking. There are two flaws of urban planners in setting minimum parking requirements according to Shoup. They neglect the price on demand for parking and the cost on its supply. What is most important in his paper is that he says minimum parking requirements bundle the cost of parking into the cost of development, and thus increase the prices of all the goods and services at the sites where free parking is offered. It is just as in our model where the price of parking is embedded in the price of the good sold at the mall.

Arbatskaya, Mukhopadhaya and Rasmusen (2007) work on parking lot size. They point that a high chance of cruising for parking at a parking lot eats up the payoff of customers from parking. The competition driving shoppers to find an empty parking space consumes their entire benefit from parking. Hence, even though parking lots are hit substantially only during Christmas, Black Friday or Super Saturday; the mall might want to provide parking lots over the average demand. For instance, occupancy of 70% may indicate too small a parking lot size given uncertainty in demand.

Russo (2013) brings an interesting approach to the topic. He analyzes the effect of institutional formation on traffic policy which is either road pricing or parking fee. He also analyzes the effect of redistribution of traffic revenues on sustainability of the policies, and the effect of financial support of central government on policy decisions of local ones. By doing so, he gives a satisfying explanation to why congestion pricing may not be feasible in some places. His model includes a central business district, a city and the hinterland with two types of traffic charges: parking fee and road toll. He also covers mode choice: individuals get a higher utility when travelling by car than public transport. His findings reveal that road pricing and parking fees can be sustainable if the city council rather than the regional government is the policy maker. This is because hinterland residents are, by and large, car-dependent. However, if there is enough government funding to compensate the local government's loss from road pricing such as public transport fare reduction, road pricing can still be feasible.

Fosgerau and de Palma (2013) suggest using a parking fee instead of congestion pricing in order to control urban congestion. Congestion pricing is an application that charges drivers on any road that is subject to congestion. It is applied in London, Stockholm, Singapore and Milan. However, its application cannot be politically feasible in some other cities. For instance, New York, Hong Kong, Manchester and Copenhagen once proposed congestion pricing and then stepped back. In this case, Fosgerau and de Palma (2013) offer using a time-varying parking fee at workplace. Considering congestion delay is strongly dependent on timing of trips and by using Vickrey's bottleneck model (1969), they show that that time-varying parking fee at workplace can reschedule commuting as congestion pricing does and as a result, it can be as efficient as congestion pricing in the places where congestion pricing is not implementable.

3 The Model

The model is based on Hasker and Inci (forthcoming) model. A risk-neutral mall tries to sell a good at price P in a unit time to risk-averse customers with a differentiable, invertible and concave utility function $u : \mathbb{R} \to \mathbb{R}^+$. It has two options of parking to offer to customers:

valet parking with a cost of c_v at a fee of t_v and regular parking with a cost of c_r at a fee of t_r. A customer visiting the mall can either purchase the good or leave the mall without shopping, and then his or her probability buying the good is denoted by ρ. If s/he decides not to visit the mall, s/he engages in another activity from which s/he earns a payoff of r with certainty.

The first difference between our model and Hasker and Inci (forthcoming)'s is that the valuations of customers for the good sold at the mall differ among customers but they are deterministic in our model. There are two types of customers in terms of valuation for the good, namely low types with low valuation and high types with high valuation. In Hasker and Inci (forthcoming)'s model, the value of the good to a customer is represented by a stochastic variable. Since there are two types in our model, the mall would want to know the proportion of high and low types, which are denoted by λ and $1 - \lambda$ respectively. The second difference is that the mall has the option of providing valet parking service along with regular parking in our model. We can really observe this new trend of offering valet service in some shopping malls such as Westfield San Francisco, Westfield Chermside, Cherry Creek, Garden City, The Florida Mall, and Dolphin Mall with a charge between 5 to 10 dollars. If there is no option of offering valet service so that the price of parking and the good is same to all, high types will definitely have a surplus due to their higher valuation of the good. In this case, the mall would seek an instrument to extract that surplus, which is valet service. We already talked about the benefits of using valet parking to customers; i.e., closer parking space and no cruising. Therefore, we say a customer gets a utility from valet parking and we denote it by x.

One problem with valet parking is that some people are concerned handing their key over to valet. In fact, they have every right to be concerned about because Gocompare.com conducted a study with 233 car insurance policies and reported that 49% of them do not cover the damage while it is in control of valet (Gocompare.com). Despite this fact, what we see in real life is that some customers continue using valet parking. In our model, we can overcome this issue by reflecting the concern of people about their car while in control of others on the utility gained by having valet service.

Different equilibrium strategies emerge based on the values of the exogenous variables such as the valuations, parking costs, proportion of high and low types. There are separating and pooling equilibria. In separating equilibria, the mall provides different parking services to different types of customers. In other words, the mall separates customers in terms of the service it offered. In pooling equilibria, the mall provides unique parking service to all customers, which means this time the mall pools the customers in terms of the service it offered.

3.1 Separating Equilibria

In separating equilibria, the mall provides different parking services to different types of customers. Not providing any service to a certain type can also be considered as a separating equilibrium. For instance, the mall may want to keep the prices of the good so high that low types do not visit the mall. This is the case in which low-type customers are not provided any parking service simply because they do not come to the mall. So, a customer is provided either valet or regular parking, or none. Keeping this in mind, we may have three different separating equilibrium strategies: 1. valet by high types, regular by low types; 2. valet by high, none by low; 3. regular by high, none by low. The proof of why a strategy in which low types use valet and high types use regular parking cannot be an equilibrium is presented in Appendix A.1. In addition, there are two other candidate strategies: valet by low types, none by high types and regular by low types and none by high types. To see why they cannot be equilibrium is trivial.

We now analyze each equilibrium configuration in turn.

3.1.1 Valet by high types, regular by low types

In this equilibrium, the best strategy for the mall is to provide both valet and regular parking and set the prices of the good and parking such that high-type customers prefer valet parking whereas low types do not. So, what leads customers' different preference of parking is the pricing strategy of the shopping mall. The mall sets the prices of the good, valet parking service and regular parking such that a seperating equilibrium is reached, in which choosing valet service makes high-types better off and choosing the regular parking makes low-types better off. Since customers differ in their preference of parking by their different valuations of the good sold at the mall, the payoff of the mall by each type of customer will be formulized differently. If a high-type customer visits the mall, s/he will shop with probability ρ, which yields an expected payoff of ρP for the mall. s/he will also pay t_v for valet service which costs c_v to the mall. Thus, the payoff of the mall from a high-type customer visiting the mall becomes $\rho P + t_v - c_v$. Now, consider a low-type customer. s/he will purchase the good with the same probability ρ but s/he will pay t_r for regular parking which costs $c_r < c_v$ to the mall. Thus, the payoff of the mall from a low-type customer visiting the mall becomes $\rho P + t_r - c_r$. Given that λ proportion of customers is high type and $1 - \lambda$ of them is low type, the overall payoff of the mall becomes

$$\Pi(P, t_v, t_r) = (\rho P + t_v - c_v)\lambda + (\rho P + t_r - c_r)(1 - \lambda) \; . \tag{1}$$

The shopping mall aims to maximize this payoff by choosing the prices in a way that both types of customers visit the mall and they do not pretend to be each other.

From the customers' point of view, there are initially two options: visiting the mall or not. We will first deal with a high-type customer who uses valet service in a separating equilibrium setting. If a high-type customer goes to the mall and purchases the good, s/he gets the utility of $u(v_H - P - t_v)$ because s/he has the valuation of v_H and s/he pays P for the good and t_v for valet parking. If s/he leaves the mall without shopping, s/he gets the utility of $u(-t_v)$. Considering that a high-type customer gets an extra utility x because s/he uses valet parking service, the expected utility of a high-type customer from visiting the mall becomes

$$E_H\left(u|\, P, t_v\right) = x + \rho u(v_H - P - t_v) + (1 - \rho)u(-t_v) \; . \tag{2}$$

Now, we can move to the low-type customer's problem. Applying the same approach, we formulize the expected utility of a low-type customer. If s/he goes to the mall and shops, his or her utility will be $u(v_L - P - t_r)$. If s/he does not shop, his or her utility will be $u(-t_r)$. So, the expected utility of a low-type customer becomes

$$E_L\left(u|\, P, t_r\right) = \rho u(v_L - P - t_r) + (1 - \rho)u(-t_r) \; . \tag{3}$$

So far, we have determined the expected utilities of each type of customers from going to the mall. If a customer does not go to the mall and engages in another activity, s/he gets a utility of $u(r)$. Since both types of customers have to visit the mall in this equilibrium, his or her expected utility from going to the mall should be greater or equal to their utility from not going to the mall. Thus, the following individual rationality constraints should be imposed in order to ensure customers visiting the mall:

$$\begin{aligned} x + \rho u(v_H - P - t_v) + (1 - \rho)u(-t_v) \quad &\geq \quad u(r) & (4) \\ \rho u(v_L - P - t_r) + (1 - \rho)u(-t_r) \quad &\geq \quad u(r) \; . & (5) \end{aligned}$$

Equation (7) is the individual rationality constraint for high-type customer, and will be represented by IRH. Equation (9) is that for a low-type customer, and will be represented by IRL. From now on, we will assume that both types of customer visit the mall since individual rationality conditions are implemented. After ensuring that all customers come to the mall, another concern is to prevent different types of customers from imitating each other. What we mean by imitation is that a high-type customer would like to use regular parking instead of valet parking or a low-type customer would like to use valet parking. In order to avoid those cases, the pricing strategy should be such that the utility of a high-type customer using valet service should be as high as his or her utility from using regular parking; and the utility of a low-type customer using regular parking should be as high as his or her utility from using valet service. We will first focus on the former and then move to the latter. We already know that the expected utility of a high-type customer using valet service is the equation (2). We need to find out what utility s/he gets if s/he mimics low types and uses regular parking. In this case, s/he will get the utility of $u(v_H - P - t_r)$ if s/he shops, and $u(-t_r)$ if s/he does not. Besides, s/he does not get the utility x because s/he uses regular parking. All in all, the expected utility of a high-type customer using regular parking becomes

$$E_H\left(u|\, P, t_r\right) = \rho u(v_H - P - t_r) + (1-\rho)u(-t_r)\,. \tag{6}$$

Then, we just need to implement the condition that the equation (2) is as high as the equation (6), which we call the incentive compatibility constraint for high types (henceforth, ICH):

$$x + \rho u(v_H - P - t_v) + (1-\rho)u(-t_v) \geq \rho u(v_H - P - t_r) + (1-\rho)u(-t_r)\,. \tag{7}$$

To sum up, a high-type customer may want to pretend to be a low type and use regular parking instead of valet service because s/he pays $t_v - t_r$ less for parking. On the other hand, s/he does not get the utility x from using valet service if s/he mimics low types. So, there is a trade-off between choosing regular or valet parking. If the effect of valet parking utility (x) dominates, s/he prefers valet service; if the effect of lower parking fee dominates, s/he prefers using regular parking. In the current setting, the pricing strategy of the mall should be such that the effect of valet parking utility surpasses the effect of less parking fee.

For the latter part, we already know the utility of a low-type customer using regular parking, which is the equation (3). However, if a low-type customer pretends to be a high type and uses valet service, his or her utility will be $u(v_L - P - t_v)$ if s/he shops or $u(-t_v)$ if s/he does not. s/he also gets the utility x from using valet service. All in all, his or her expected utility becomes:

$$E_L\left(u|\, P, t_v\right) = x + \rho u(v_L - P - t_v) + (1-\rho)u(-t_v) \tag{8}$$

Now, we can define the incentive compatibility constraint for low types (henceforth, ICL):

$$\rho u(v_L - P - t_r) + (1-\rho)u(-t_r) \geq \rho u(v_L - P - t_v) + (1-\rho)u(-t_v) + x\ . \tag{9}$$

We have defined four constraints so far, two individual rationality and two incentive compatibility constraints. Lastly, we state that price of valet parking, regular parking or of the good may not be negative:

$$P, t_v, t_r \geq 0\ . \tag{10}$$

Now, we are going to find the prices. All in all, we have one objective function, four constraints and a bunch of non-negativity conditions. Solving an optimization problem with four constraints might be troublesome. However, good news is that we can get rid of ICL and IRH and we are left with ICH and IRL only. It is because ICL and IRH do not bind whereas ICH and IRL do (The proofs are in Appendix A.2, A.3, A.4, A.5).

In what follows, we show that the mall provides the regular parking for free (See Appendix A.6). In other words, it insures the customers who use regular parking to some extent and so that the results of the equilibrium are consistent with Hasker and Inci (forthcoming).

Proposition 1 *Regular parking is used by low types and provided free; valet parking is used by high types and valet fee is always greater than zero in this equilibrium. Price of the good is below the valuation of low types.*

If t_r is equal to zero, we can find the values of P and t_v. Using the equation (5), we find P:

$$\rho u(v_L - P) + (1-\rho)u(0) = u(r) \implies P = v_L - u^{-1}[\frac{u(r) - (1-\rho)u(0)}{\rho}]\ . \tag{11}$$

Considering the fact that the equation (7) binds, we can say that t_v is always greater than t_r thus it is greater than zero. Positive t_v may imply two different meanings. The mall may be trying to attract high types in case the valet fee is lower than its cost, or the mall might be trying to make profit from valet parking in case the utility from using valet service is so high.

3.1.2 Valet by high types, none by low types

In this equilibrium, the mall does not care if low types visit the mall or not and offers only valet parking to only high types. So, it can charge a higher price for the good or valet service compared to the previous equilibrium (valet for high, free regular for low; henceforth v-h, f-l). The question is for which good does the mall charge a higher price, for the good sold at the mall or the valet? We will figure it out after solving the mall's problem. In any case, for this equilibrium to come true, it must be more profitable for the mall to increase the price of the good or valet fee in return for losing all of its low-type customers.

Consider the payoff of the mall. A high-type customer visiting the mall yields a payoff of $\rho P + t_v - c_v$ to the mall. However, low types do not visit the mall in this equilibrium, which means the mall gets the payoff from λ proportion of customers. Thus, the payoff function of the mall becomes

$$\Pi(P, t_v) = (\rho P + t_v - c_v)\lambda \ . \tag{12}$$

What remains is to implement the IRH:

$$x + \rho u(v_H - P - t_v) + (1-\rho)u(-t_v) \geq u(r) \ . \tag{13}$$

Again, the prices must be above zero:

$$P, t_v \geq 0 \ . \tag{14}$$

Proposition 2 *In this equilibrium, valet parking fee can be either positive or zero, which means the mall may be willing to cover the cost of parking. Price of the good is not related to the valuation of low types but to the valuation of high types because low types do not come to the mall. Therefore, if the mall does not care whether low types visit the mall, it charges a higher price for the good.*

In order to solve the problem, we start with finding the Lagrange function:

$$F(P, t_v, \mu) = (\rho P + t_v - c_v)\lambda + \mu[x + \rho u(v_H - P - t_v) + (1-\rho)u(-t_v) - u(r)] \ . \tag{15}$$

The FOC with respect to P is

$$\frac{\partial F(P, t_v, \mu)}{\partial P} = \rho\lambda - \mu\rho u'(v_H - P - t_v) = 0 \tag{16}$$

and by rearranging the terms, we achieve the following result:

$$u'(v_H - P - t_v) = \frac{\lambda}{\mu} \ . \tag{17}$$

The FOC with respect to t_v is

$$\frac{\partial F(P,t_v,\mu)}{\partial t_v} = \lambda - \mu\rho\underbrace{u'(v_H - P - t_v)}_{\lambda/\mu} - \mu(1-\rho)u'(-t_v) = 0 \tag{18}$$

and by rearranging the terms, we achieve the following result:

$$u'(-t_v) = \frac{\lambda}{\mu}\,. \tag{19}$$

Since $u'(v_H - P - t_v)$ and $u'(-t_v)$ both equals λ/μ, we conclude $P = v_H$. Substituting v_H with P, we find $t_v = -u^{-1}[u(r) - x]$. This can be either positive or negative depending on the values of r and x but the mall cannot charge a negative valet price. So, if the formula gives a negative result, t_v becomes zero and P is adjusted accordingly: $P = v_H - u^{-1}[\frac{u(r)-(1-\rho)u(0)-x}{\rho}]$. Note that the price is higher compared to the one in the previous equilibrium. This suggests that the mall increase the price of the good as long as it does not care about low types.

3.1.3 Regular by high types, none by low types

It might be hard to imagine how such a model exists. Assume we are in the previous equilibrium (valet-high, none-low) in which the mall provides only valet parking to only high-type customers. If providing valet parking is too costly compared to regular parking for the mall, and then the mall might increase its payoff by replacing regular parking with valet service. At this point, one may ask the question why the mall does not serve low types as well as high types in this new situation. If it does not serve low types, it means that the mall finds it more profitable to increase the price of the good and give up the payoff gained from low types. As a result, there is a chance that we can obtain this equilibrium.

Consider the payoff of the mall. A high-type customer visiting the mall yields the expected payoff of $\rho P + t_r - c_r$. As in the previous model, the mall earns this payoff from λ proportion of customers. So, the payoff function of the mall becomes

$$\Pi(P,t_r) = (\rho P + t_r - c_r)\lambda\,. \tag{20}$$

Now, let me define the IRH constraint:

$$\rho u(v_H - P - t_r) + (1-\rho)u(-t_r) \geq u(r)\,. \tag{21}$$

Finally, we implement the constraint for the prices:

$$P, t_r \geq 0\,. \tag{22}$$

Proposition 3 *Regular parking is provided free and the price of the good is below the valuation of high types in this equilibrium. This means that if the mall provides free regular parking instead of paid valet parking which is the case in the previous equilibrium, the mall charges a lower price for the good.*

In order to solve the problem, we start with finding the Lagrange function:

$$F(P, t_r, \mu) = (\rho P + t_r - c_r)\lambda + \mu[\rho u(v_H - P - t_r) + (1-\rho)u(-t_r) - u(r)]\,. \tag{23}$$

The FOC with respect to P is

$$\frac{\partial F(P, t_r, \mu)}{\partial P} = \rho\lambda - \mu\rho u'(v_H - P - t_r) = 0 \tag{24}$$

and by rearranging the terms, we achieve the following result:

$$u'(v_H - P - t_r) = \frac{\lambda}{\mu}\,. \tag{25}$$

The FOC with respect to t_r is

$$\frac{\partial F(P, t_r, \mu)}{\partial t_r} = \lambda - \mu\rho \underbrace{u'(v_H - P - t_r)}_{\lambda/\mu} - \mu(1-\rho)u'(-t_r) = 0 \tag{26}$$

and by rearranging the terms, we achieve the following result:

$$u'(-t_r) = \frac{\lambda}{\mu}\,. \tag{27}$$

Since $u'(v_H - P - t_r)$ and $u'(-t_r)$ both equals λ/μ, we conclude $P = v_H$. Plugging v_H instead of P, we find $t_r = -u^{-1}[u(r)] = -r$ which is negative. So, regular parking is provided free. Making $t_r = 0$, we recalculate the price:

$$\rho u(v_H - P) + (1-\rho)u(0) = u(r) \implies P = v_H - u^{-1}[\frac{u(r) - (1-\rho)u(0)}{\rho}]\,. \tag{28}$$

3.2 Pooling Equilibria

In pooling equilibria, the mall provides unique parking service so customers are not differentiated in terms of the parking service they choose. What distinguishes high types from low types is their valuation. Hence for pooling equilibria to come true, the valuations must be close enough to each other. If the mall observes a significant difference between the valuation of high types and low types, it might want to extract the surplus of high types by increasing the price of the good not caring about low types and as result, customers will have been separated. Of course, there are other parameters affecting the mall's pricing strategy such as parking costs, utility gained by using valet service, reservation value. Considering there are two types of parking offered, two different pooling equilibrium settings may arise.

3.2.1 Free parking for all

In this equilibrium, valet service is not provided. Therefore, both types of customers must choose regular parking. In this case, any customer visiting the mall and shopping with probability ρ yields the expected payoff of ρP to the mall. In addition, each customer pays t_r for parking which costs c_r to the mall. So, the expected payoff of the mall becomes

$$\Pi(P, t_r) = \rho P + t_r - c_r \; . \tag{29}$$

The mall wants to maximize this payoff by choosing the prices of the good and parking. However, the prices should be such that the model is sustained, which means all customers visit the mall and they use regular parking. Since all customers are provided the same kind of parking in this equilibrium, there is no need to implement the incentive compatibility constraints. We still need the individual rationality constraints because we want to make sure all customers prefer visiting the mall instead of engaging in another activity.

Consider a high-type customer first. s/he gets the utility of $u(v_H - P - t_r)$ if s/he is happy with what s/he finds out in the mall and purchases the good. Otherwise, s/he gets the utility of $u(-t_r)$. Note that one difference of his or her utility from the one in separating model is that s/he pays t_r for parking instead of t_v. Another difference is that s/he does not get the utility of x because s/he does not use valet service. So, the expected utility of a high-type customer becomes $\rho u(v_H - P - t_r) + (1 - \rho)u(-t_r)$. For a high-type customer to visit the mall, this expected utility should be as high as the utility from not going to the mall, $u(r)$.

So, the IRH constraint becomes

$$\rho u(v_H - P - t_r) + (1-\rho)u(-t_r) \geq u(r) \; . \tag{30}$$

Now, consider a low-type customer. his or her utility from visiting the mall would be $\rho u(v_L - P - t_r) + (1-\rho)u(-t_r)$. Unique difference between the utilities of high and low types is the valuations. So, the IRL constraint becomes

$$\rho u(v_L - P - t_r) + (1-\rho)u(-t_r) \geq u(r) \; . \tag{31}$$

As it is the case in all equilibria, the prices should not be less than zero because we assume the mall does not subsidize parking or shopping. Therefore, we finally implement the following conditions:

$$P, t_r \geq 0. \tag{32}$$

Proposition 4 *There can be such an equilibrium in which free parking is provided for all and this is the same strategy in the baseline model by Hasker and Inci (forthcoming). The difference is that here the price of the good is below the valuation of low types. Since the valuation is a stochastic variable in the model by Hasker and Inci (forthcoming), it does not make sense to compare between the two equilibria in terms of the prices of the good.*

The mall aims to maximize its objective function based on the two constraints and non-negativity conditions. Fortunately, one of the two constraints IRH can easily be proven not to bind whereas IRL does (See Appendix A.7). In other words, high types earn surplus because they pay lower parking fee than they would be willing to pay.

Now, reconsider the mall's problem. It deals with only one constraint and non-negativity conditions now. The Lagrange function for the problem becomes

$$F(P, t_r, \mu) = \rho P + t_r - c_r + \mu[\rho u(v_L - P - t_r) + (1-\rho)u(-t_r) - u(r)] \tag{33}$$

where μ is the Lagrange multiplier.

The FOC with respect to P is

$$\frac{\partial F(P, t_r, \mu)}{\partial P} = \rho - \mu\rho u'(v_L - P - t_r) = 0 \tag{34}$$

and by rearranging the terms, we achieve the following result:

$$u'(v_L - P - t_r) = \frac{1}{\mu}\,. \tag{35}$$

The FOC with respect to t_r is

$$\frac{\partial F(P, t_r, \mu)}{\partial t_r} = 1 - \mu\rho\underbrace{u'(v_L - P - t_r)}_{1/\mu} - \mu(1-\rho)u'(-t_r) = 0 \tag{36}$$

and by rearranging the terms, we achieve the following result:

$$u'(-t_r) = \frac{1}{\mu}\,. \tag{37}$$

From the two FOC's, we conclude $P = v_L$. By plugging in the value of v_L in the equation (31), we find $t_r = -r$. This result is exactly the same as found in Hasker and Inci (forthcoming). So, when the mall does not provide valet parking, we achieve the same results in Hasker and Inci (forthcoming) model. As stated in their work, the mall cannot subsidize customers for parking. We have non-negativity conditions to prevent subsidization. Therefore, the mall provides free parking to all customers ($t_r = 0$). Now, we go back to the equation (31) and recalculate the price:

$$\rho u(v_L - P) + (1-\rho)u(0) = u(r) \implies P = v_L - u^{-1}[\frac{u(r) - (1-\rho)u(0)}{\rho}]\,. \tag{38}$$

3.2.2 Valet parking for all

In this equilibrium, the mall provides only valet parking to all customers. The customers do not have a choice to park without paying for the valet. This means that the mall gets more payoffs by offering valet service to both low and high type customers probably in a relatively low price. Otherwise, low types would not prefer to visit the mall.

The payoff function of the mall will be similar to the one in previous equilibrium in which all customers are provided free parking. In addition, this time, the mall gets t_v from each customer for valet parking which costs c_v. So, the payoff function of the mall becomes

$$\Pi(P, t_v) = \rho P + t_v - c_v\,. \tag{39}$$

Now, we have to implement the individual rationality conditions. Consider a high-type customer first. If s/he is happy with what s/he finds out and buys the good, s/he gets the utility of $u(v_H - P - t_v)$, or else s/he gets the utility of $u(-t_v)$. Note that one difference of the utilities from those in the first pooling model is the parking fee. In the first pooling model, they pay t_n for regular parking whereas they pay t_v for valet in this equilibrium. Another difference is that s/he will get an extra utility of x because s/he uses valet service. Finally, if s/he does not go to the mall and undertake another activity, s/he gets $u(r)$. So, the IRH constraint becomes

$$x + \rho u(v_H - P - t_v) + (1 - \rho)u(-t_v) \geq u(r)\ . \tag{40}$$

Consider a low-type customer now. All variables in the expected utility function of a high-type customer above remains the same except for the valuation term. So, the IRL constraint becomes

$$x + \rho u(v_L - P - t_v) + (1 - \rho)u(-t_v) \geq u(r)\ . \tag{41}$$

In addition to the individual rationality constraints, we have to implement the condition for the prices not to be negative:

$$P, t_v \geq 0\ . \tag{42}$$

Proposition 5 *There can be such an equilibrium in which valet parking is provided for all with a positive charge or zero depending on r, x and u. In this equilibrium, price of the good is still below the valuation of low types but higher than the price in free parking for all equilibrium.*

With a similar approach as in free-for-all equilibrium, we can eliminate the IRH constraint because it does not bind (See Appendix A.8). That is why we can ignore the IRH constraint while solving the mall's problem.

Now, including the Lagrange multiplier, we obtain the function

$$F(P, t_v, \mu) = \rho P + t_v - c_v + \mu[x + \rho u(v_L - P - t_v) + (1 - \rho)u(-t_v) - u(r)]\ . \tag{43}$$

FOC with respect to P is

$$\frac{\partial F(P, t_v, \mu)}{\partial P} = \rho - \mu\rho u'(v_L - P - t_v) = 0 \tag{44}$$

and by rearranging the terms, we achieve the following result:

$$u'(v_L - P - t_v) = \frac{1}{\mu}\ . \tag{45}$$

FOC with respect to t_v is

$$\frac{\partial F(P, t_v, \mu)}{\partial t_v} = 1 - \mu\rho \underbrace{u'(v_L - P - t_v)}_{1/\mu} - \mu(1-\rho)u'(-t_v) = 0 \tag{46}$$

and what we achieve after simplification is

$$u'(-t_v) = \frac{1}{\mu} \, . \tag{47}$$

From the two FOC's, we conclude $P = v_L$ as in the previous model and the one in Hasker and Inci (forthcoming). When we plug v_L in place of P, we find $t_v = -u^{-1}[u(r) - x]$ which might be either negative or positive depending on the values of parameters r, x and the utility function u. If it is negative, t_v becomes zero and $P = v_L - u^{-1}[\frac{u(r)-(1-\rho)u(0)-x}{\rho}]$. Note that price of the good is higher than the price in the previous equilibria. This means that when the mall provides free valet parking for all, it embeds the cost of valet into the price of the good sold at the mall.

4 Comments

We have defined five different equilibria. Now, the issue is determining the circumstances under which each of the five is reached. We will now calculate the welfare as well as compare customers' and the mall's share of the cake for each equilibrium.

We generate different cases by assigning values to the exogenous variables x, ρ, v_H, v_L, λ, r, c_v, c_r and we can reach each equilibrium as shown in Table 1 and Table 2. For Table 1, we define the convex utility function of risk-averse customers as $u(m) = \sqrt{100 + m}$ where m combines the valuation of the good or the amount of money paid for the good or parking. The constant 100 is chosen to keep the inside of the square root positive. If the customer shops, his or her utility becomes $\sqrt{100 + v_{H/L} - P - t_{v/r}}$; if s/he does not, it is $\sqrt{100 - t_{v/r}}$. The former is higher than the latter in each equilibrium. Therefore, $100 - t_{v/r}$ being non-negative makes $100 + v_{H/L} - P - t_{v/r}$ positive. In a sense, the fixed amount of 100 is a maximum value a customer would spend for parking. We will henceforth call this amount the parking budget.

	$\mathbf{x}$	ρ	v_H	v_L	λ	r	c_v	c_r	P	t_v	t_r	$Eq.$	Π	U	U/Π
case 1	0.05	0.1	250	200	0.5	0.5	2	1	194.94	1.02	0	v-h,f-l	18.50	10.14	0.55
case 2	0.05	0.1	250	200	0.9	0.5	2	1	250.05	0.50	n/a	v-h,n-l	21.19	9.99	0.47
case 3	0.05	0.1	250	200	0.9	0.5	2	0.8	244.94	n/a	0	f-h,n-l	21.32	10.02	0.47
case 4	0.05	0.1	250	200	0.5	0.5	2	0.8	194.94	n/a	0	f-all	18.69	8.14	0.44
case 5	0.05	0.1	250	200	0.5	5	2	0.8	156.73	0	n/a	v-all	13.67	8.34	0.61

Table 1: Equilibrium strategies under five different cases

One striking result from the Table 1 is that the mall offers valet parking to all (henceforth; v-all) for free as shown in the last row of the Table. In fact, this result is compatible with Hasker and Inci's (forthcoming) model in which the mall wants to subsidize regular parking and embeds the price of it into the price of the good sold at the mall. Here, the mall goes beyond that and offers an additional valet service from which customers get a utility and embeds the price of valet into the good. Using the same parameter values in the last row of the Table, if the mall did not offer free valet but free regular parking to all, the price of the good would turn out to be 144.51. However, it is 156.73 in the equilibrium where the cost of valet is absorbed in the price of the good.

Let me explain how we formed the 10 cases in Tables 1 and 2:

Case 1: We started with assigning subjectively reasonable numbers to the exogenous variables of the model: $x = 0.05$, $\rho = 0.1$, $v_H = 250$, $v_L = 200$, $\lambda = 0.5$, $r = 0.5$, $c_v = 2$, and $c_r = 1$. Then we calculate the equilibrium payoff of the mall under each strategy; i.e., v-h,f-l, v-h,n-l, f-h,n-l, f-all, v-all. We compare the equilibrium payoff of the mall and see that it is largest with v-h,f-l. Finding the equilibrium prices and customer utility completes the second row of Table 1.

Case 2: Now, we can spread to other equilibria by changing the circumstances. Table 1 shows how to move from one equilibrium to another by changing only one exogenous variable. For instance, changing λ from 0.5 to 0.9 moves us to valet for high, none for low (henceforth; v-h,n-l) equilibrium from v-h,f-l. In fact, it intuitively makes sense. λ represents the proportion of high types. If it is 0.9 instead of 0.5, the mall becomes fancier in response and it does so by offering v-all.

Case 3: From that equilibrium, we can move to free for high, none for low (henceforth; f-h,n-l) by decreasing the cost of regular parking. It is reasonable because offering free parking without valet might be more profitable as the cost of regular parking decreases.

Case 4: By decreasing λ back to 0.5, we can achieve free parking for all equilibrium (henceforth, f-all). Note that if the mall has more high-type customers, it tends not to care about low types.

Case 5: Finally, by increasing the reservation value, we can switch to v-all equilibrium. Higher reservation value means better outside option. If customers have a better outside option, the mall needs to try harder to attract customers and does so by offering free valet service. Note that the mall's offering free valet parking to all is not a general condition; as it turns out that the mall charges customers for valet in the same equilibrium strategy of Table 2.

	x	ρ	v_H	v_L	λ	r	c_v	c_r	P	t_v	t_r	$Eq.$	Π	U	U/Π
case 6	0.03	0.8	50	20	0.35	0.5	1.94	0.6	19.37	1.37	0	v-h,f-l	14.91	22.56	1.51
case 7	0.03	0.8	50	15	0.35	0.5	1.94	0.6	50.07	0.79	n/a	v-h,n-l	13.62	22.36	1.64
case 8	0.02	0.8	50	15	0.35	0.5	1.94	0.6	49.37	n/a	0	f-h,n-l	13.61	22.37	1.64
case 9	0.02	0.8	40	15	0.35	0.5	1.94	0.6	14.37	n/a	0	f-all	10.90	29.07	2.67
case 10	0.02	0.8	40	15	0.35	0.5	1	0.6	15.08	0.33	n/a	v-all	11.39	29.06	2.55

Table 2: Equilibrium strategies under five more different cases

Now, look at Table 2. This time the utility function of customers is replaced with $u(m) = \sqrt{500+m}$. Again, we start with assigning values to the exogenous variables and move from one equilibrium to another by changing only one, but this time a different, variable.

Case 6: A higher parking budget relative to the valuations means that the good sold at the mall is less significant to them than it was with the utility function for Table 1. As shown in the second row of Table 2, v-h,f-l is the best strategy for the mall under the related setting.

Case 7: Consider v-h,f-l equilibrium. We had increased λ on Table 1 to switch to the next equilibrium which is v-h,n-l. Here, we apply a different strategy: decrease the valuation of low types down to a point where the mall does not care about them. Then, the mall keeps offering valet to its high-type customers but none for low types. By doing that, the mall can increase the price of the good because it does not have to sell it to low types any more. Notice that the price jumped to 50.07 (which is the valuation of high types) from 19.37 (which is close to the valuation of low types).

Case 8: Now, decrease the utility gain by having valet service from 0.03 to 0.02. Valet service is not that attractive to high-type customers now and as a result, they are offered regular parking for free.

Case 9: At this point, we had decreased λ in order to move to f-h,n-l equilibrium. There is also another way: decrease the valuation of high types down to a point where it is close enough to the valuation of low types so that low types are regarded by the mall and free regular parking is offered to all.

Case 10: Now, instead of increasing r in Table 1, decrease the cost of valet parking until the mall is willing to offer v-all.

A noticeable finding from the Tables is that valet fee is never greater than its cost, which means valet parking is offered as a loss leader in all the cases from 1 to 10. This leads us to think that the mall uses valet service not to make profit but as a tool to attract customers to the mall. However, this conclusion depends on the utility gained by using valet service. If it is high enough, the mall can increase the price of valet over its cost and make profit out of valet parking. In addition, we justify all findings of the model. First, regular parking is free under each equilibrium. Second, the price of the good is less than the valuation of high types in separating equilibria, less than the valuation of low types in pooling equilibria. Third, each equilibrium strategy can be performed depending on the exogenous variables.

policy	$\mathbf{x}$	ρ	v_H	v_L	λ	r	c_v	c_r	P	t_v	t_r	Π	U	U/Π
v-h,n-l	0.05	0.1	250	200	0.5	0.5	2	0.8	194.94	1.02	0	18.603	10.135	0.545
f-all	0.05	0.1	250	200	0.5	0.5	2	0.8	194.94	n/a	0	18.694	8.135	0.435
v-all	0.05	0.1	250	200	0.5	0.5	2	0.8	200.03	0.50	n/a	18.450	8.143	0.440

Table 3: Outcome of three different policies under a fixed setting

Look at Table 3. Notice that all the exogenous variables are controlled. Thus, we can make a ceteris-paribus comparison between the three policies. The striking result is that f-all is the least customer-friendly policy with $U/\Pi = 0.435$. Offering v-h,n-l brings higher share to customers because U/Π increases slightly. In addition, v-all and increasing the price of the good from 194.94 to 200.03 brings significantly a higher share to customers. In sum, what the Table implies is that valet parking should not be seen as a tool to exploit customers, it can be a positive solution for them in some cases.

One might question if the assigned values of the exogenous variables in Table 1 and Table 2 are realizable. ICSC reports that Canadian shopping-center sales per capita reached $\$8,631$ in 2011 (ICSC). Shoup estimated the cost per parking space at UCLA campus as $\$23,346$ in 1994 dollars. Let me assume the average spending at shopping malls is similar in the US and Canada. The price level increased with 51.75% between 1994 and 2011 so let me say the cost per regular parking space is $23,346 \times 1.5175 = 35,427.56$. It was 1 in the first two cases on Table 1. By multiplying v_H, v_L, c_v and c_r with $35,427.56$, we find P as $6,906,386.34$. Note that P is not affected by x. Since 10% of people coming to the mall shop, total spending per year is $\$6,906,386.34 \times 0.1 = \$690,638.63$ according to our model under v-h, f-l equilibrium. We will not find disproportionately different results under the other equilibria so let me apply this result to the real data and see if the assigned values are reasonable. The interpretation of our finding is that if there was only one customer who visits the mall and shops with probability 0.1 and there was only one good to be sold at the mall, then the customer, on average, would spend $\$690,638.63$ in one year. Now, let us go back to real world. Average spending is $\$8,631$. Therefore, the customer density should be $\$690,638.63/\$8,631 = 80.02$, which means there should be 80.02 customers on average in the mall at a moment. Considering that shopping malls range in size from $13,150$ to $2,574,505$ sq. ft., we say that this number can be reasonable for some small-scale malls. What about the variables in Table 2? When we follow the same process, we find the customer density 106.04, similar to the former result. What about bigger malls? Consider Mall of America which is the largest mall of the US. The mall has $135,000$ visitors per day and each of them spends $\$105$ on average (Prairie 2006). So, total spending in the mall is $\$14,175,000$ per day. We estimated regular parking cost to be $\$35,428$ per year, so $\$35,428/365 = \$97$ per day. Mall of America has $12,287$ parking lots indicating total cost of parking is $\$97 \times 12,287 = \$1,192,613$. According to the parameters that we calculated, we rearrange the variables and can obtain Table 4. What we should notice from the Table is $P = \$14,243,684$. This means that $135,000$ visitors purchase $\$14,243,684$ in a day. However, 0.1 of visitors make the purchase, so total spending per day according to our model is $\$1,424,368$. What remains is to find the number of visitors per day and compare it with the observed value of $135,000$. Mall of America reports that average spending per visitor in a month is $\$316$ (Prairie 2006). Therefore, $\$1,424,368 \times 30/\$316 = 135,225$ is the number of visitors per day according to our model. The estimated data is close to the observed one. In conclusion, we show that the model is applicable to large-scale malls such as Mall of America as the model is to small-scale malls.

x	ρ	v_H	v_L	λ	r	c_v	c_r	P	t_v	t_r
10	0.1	20000000	15000000	0.5	75000	1500000	1192613	14243684	90418	0

Table 4: Mall of America case

Before moving to Table 5, let me define welfare under each equilibrium as follows:

$$W_1 = \Pi(P, t_v, t_r) + \lambda E_H (u|\, P, t_v) + (1-\lambda) E_L (u|\, P, t_r) \quad (48)$$

$$W_2 = \Pi(P, t_v) + \lambda E_H (u|\, P, t_v) + (1-\lambda) E_L (u|\, r) \quad (49)$$

$$W_3 = \Pi(P, t_r) + \lambda E_H (u|\, P, t_r) + (1-\lambda) E_L (u|\, r) \quad (50)$$

$$W_4 = \Pi(P, t_r) + \lambda E_H (u|\, P, t_r) + (1-\lambda) E_L (u|\, P, t_r) \quad (51)$$

$$W_5 = \Pi(P, t_v) + \lambda E_H (u|\, P, t_v) + (1-\lambda) E_L (u|\, P, t_v) \quad (52)$$

where W_1 is the welfare under the first equilibrium where high types are offered valet and low types are served regular, W_2 is the welfare under the second equilibrium where high types are served valet and low types engage in another activity and so on. Π represents the profit of the mall and E_H, E_L are the expected utilities of high-type and low-type customers respectively. We will represent the total expected utility of customers by U. In short, U represents the share of customers and Π for the share of the mall from the cake.

Now, look at Table 5. It shows the welfare from each strategy under the cases defined in Table 1 and Table 2. For instance, consider case 1: the bold numbers are the welfare if the mall plays the equilibrium strategy, e.g. 28.64 is the welfare when the mall follows the equilibrium strategy which is v-h,f-l. If it offered v-h,n-l the welfare would be 21.75. Comparing 28.64 with the other values in the column, we reach the conclusion that the equilibrium strategy of the mall brings a higher welfare than does the rest of the column. However, we cannot draw such a conclusion from case 4. The equilibrium strategy of the mall which is offering f-all brings a welfare of 26.82, but we could have 28.74 if the mall offered v-h,f-l. At this point, the assumption that the mall is in a suburban area so that it is not a price taker but has a monopoly power steps in. Hence, it would set the prices itself and offer f-all which brings less welfare to society but more profit to the mall. We can observe similar situations in some other circumstances. For instance, look at the cases 4, 5, 6, 7 and 8 on Table 5. In all of them, there is another strategy which gives a higher welfare rather than the equilibrium one. Note that in general, no strategy is strictly better than another in bringing in welfare.

	case 1	case 2	case 3	case 4	case 5	case 6	case 7	case 8	case 9	case 10
v-h,f-l	**28.64**	28.73	28.75	<u>28.74</u>	<u>23.91</u>	**37.47**	33.5	33.34	33.28	33.61
v-h,n-l	21.75	**31.18**	31.13	21.75	19.56	35.98	**35.98**	35.82	33.02	33.35
f-h,n-l	21.77	31.17	**31.35**	21.87	19.57	35.99	<u>35.99</u>	**35.99**	33.19	33.19
f-all	26.63	27.92	28.12	**26.82**	21.99	<u>44</u>	<u>40.03</u>	<u>40.03</u>	**39.97**	39.97
v-all	26.64	27.93	27.93	26.64	**22.02**	<u>43.99</u>	<u>40.02</u>	<u>39.58</u>	39.52	**40.46**

Note: Bold numbers represent the equilibrium strategy of the mall, underlined ones represent the non-equilibrium strategies which bring higher welfare from the equilibrium one.

Table 5: Comparison of welfare obtained by playing five different strategies under each case

Now, consider the third column of Table 5 for case 2. We are talking about v-h,n-l equilibrium strategy. It brings the highest welfare. Hence, this strategy is a candidate for being a social optimum in a first-best sense because no free parking is provided. If a social planner also finds it as the best strategy, we can confirm it as a first-best sense social optimum with certainty. However, the social planner problem of this model is in the future agenda.

Remember that the equilibrium that Hasker and Inci (forthcoming) gives us is a second-best social optimum because the price of parking should have been negative but a negative price of parking is not implementable. However in case 2, only valet parking is offered and with a positive charge. There is one more candidate policy for being a social optimum in a first-best sense, which is v-all. This policy does not provide free regular parking either. In conclusion, there are two equilibrium strategies that are candidates for being a first-best sense social optimum: v-h,n-l; and v-all. What is common for them is that free parking is not offered.

5 Conclusion

We have created a model of valet parking by which we made the first attempt to understand the emergence of valet parking in shopping malls. We analyzed the circumstances in which offering valet parking might turn out to be the optimal solution for the mall. We reached five different equilibrium strategies: three of them are separating and two of them are pooling. In separating equilibria, the equilibrium strategies of the mall may be setting the prices of the good and the parking such that high types get valet, low types get regular; high types get valet and low types do not visit the mall; and high types get regular and low types do not visit the mall. In pooling equilibria, the equilibrium strategies of the mall may be setting the prices such that everyone gets free parking or valet parking.

In all the equilibria, one thing is common: if offered, regular parking is free. That is why we started calling it free parking instead of regular parking after Section 3. This finding is consistent with that of Hasker and Inci (forthcoming) but it seems to be contradicting to what many economists argue against minimum parking requirements that parking price should not be considered independently of parking cost and demand. However in this situation, there is a firm who is willing to cover the cost of parking. Not only shopping malls but also independent retailers on street might be willing to do that in order to attract customers. To sum up, we find an incentive to provide free parking. Note that our finding does not justify the minimum parking requirements because we do not make any analysis about parking lot size. It just shows that in some cases, free parking can be the equilibrium.

We extended the base model of Hasker and Inci (forthcoming) by adding valet parking and diversifying customers to high and low types according to their valuation of the good sold at the mall. What leads us to this extension is that shopping malls work hard to make it easier for customers to find close and in-easy-reach parking spaces. They are aware that parking experience may affect further visits of customers to the mall or bad parking experience may even be the cause of driving off. That is why they invest in technology and deploy overhead sensors, cameras or cell-phone applications to help customers find a close parking spot easily, or they offer reserved exclusive parking places by valet parking. The model is interested in explaining the increasing number of malls offering valet parking.

An important finding of the model is a candidate for being the first-best sense social optima. Hasker and Inci (forthcoming) model gives us a second-best social optimum. With the extension of valet parking and customer diversification, free parking for all may not even bring the highest welfare. There are five different equilibrium strategies the mall might want to play and two of them can be a first-best sense social optimum because no free parking is offered in them. If offered, the mall wants to cover the cost of regular parking and as a result, the price of it turns out to be negative which is not implementable. In fact, it is a similar case for valet parking. The mall is also willing to cover the cost of valet parking and it does. Notice that in Table 1 and 2, we find valet parking fee smaller than the cost of it and for those that remains positive, there is no need for non-negativity modification for sure. However, without non-negativity constraint, regular parking always turns out to be negative, that is why we modify it to zero. The equilibrium with no modification will be a candidate for being a first-best sense social optimum. For this reason, there are two candidates: valet for high, none for low; and valet for all.

We analyze the parking behavior of customers and pricing strategies of the mall through the model. For instance, we say that when there are enough of high-type visitors, we most probably face with separating equilibria in which only high types are served. This means that as there are more high types around, the mall becomes classier, increases the price of the good, draw back from offering free parking and as a result, serves only to high types. This result can be obtained also when the valuation of low types is too small compared with the valuation of high types. Then, it would be better for the mall not to consider low types and focus on high types. We show that each equilibrium can be played under different cases in Section 3 and each switch from one equilibrium to another gives us meaningful interpretations as explained in Section 4.

A Appendix

A.1 Appendix A.1

Proposition 6 *There is no equilibrium such that high types use valet parking and low types use regular parking.*

The prices set by the mall must satisfy the conditions below:

$$
\begin{aligned}
x + \rho u(v_L - P - t_v) + (1-\rho)u(-t_v) &\geq u(r) && \text{(A-1)}\\
\rho u(v_H - P - t_r) + (1-\rho)u(-t_r) &\geq u(r) && \text{(A-2)}\\
x + \rho u(v_L - P - t_v) + (1-\rho)u(-t_v) &\geq \rho u(v_L - P - t_r) + (1-\rho)u(-t_r) && \text{(A-3)}\\
\rho u(v_H - P - t_r) + (1-\rho)u(-t_r) &\geq \rho u(v_H - P - t_v) + (1-\rho)u(-t_v) + x && \text{(A-4)}
\end{aligned}
$$

They are IRL, IRH, ICL and ICH respectively. Now, think of (A-1) and (A-4). It is easy to see that right hand side of (A-4) is strictly greater than left hand side of (A-1):

$$\rho u(v_H - P - t_v) + (1-\rho)u(-t_v) + x > x + \rho u(v_L - P - t_v) + (1-\rho)u(-t_v)\ . \quad \text{(A-5)}$$

Hence left hand side of (A-4) is strictly greater than right hand side of (A-1):

$$\rho u(v_H - P - t_r) + (1-\rho)u(-t_r) > u(r)\ . \quad \text{(A-6)}$$

This means IRH does not bind.

Now, let me increase t_r by ϵ. IRL, IRH and ICH still hold after the increase, only ICL can be affected. Therefore, the mall can increase t_r until ICL binds.

Consider (A-3) and (A-4). With the summation of them, we obtain:

$$\underbrace{u(v_H - P - t_r)}_{\boxed{1}} + \underbrace{u(v_L - P - t_v)}_{\boxed{2}} \geq \underbrace{u(v_L - P - t_r)}_{\boxed{3}} + \underbrace{u(v_H - P - t_v)}_{\boxed{4}}\ . \quad \text{(A-7)}$$

Using the concavity of u and $t_v > t_r$, we say ($\boxed{3} - \boxed{2} > \boxed{1} - \boxed{4}$) $\Longrightarrow$ ($\boxed{3} + \boxed{4} > \boxed{1} + \boxed{2}$), which is a contradiction.

A.2 Appendix A.2

Proposition 7 *IRH does not bind in the setting where the equilibrium strategy is v-h,f-l.*

In order to show that IRH does not bind, consider the inequalities (7) and (5). Only difference of right-hand side of (7) ($E_H\,(u|\,P, t_r)$) from the left-hand side of (5) ($E_L\,(u|\,P, t_r)$) is that we have v_H instead of v_L. Since v_H is assumed to be strictly greater than v_L, the right-hand side of (7) is strictly greater than left-hand side of (5):

$$\underbrace{\rho u(v_H - P - t_r) + (1-\rho)u(-t_r)}_{\text{right-hand side of 7}} > \underbrace{\rho u(v_L - P - t_r) + (1-\rho)u(-t_r)}_{\text{left-hand side of 5}} . \tag{A-8}$$

From the condition above, we can conclude that left-hand side of (7) is strictly greater than right-hand side of (5):

$$x + \rho u(v_H - P - t_v) + (1-\rho)u(-t_v) > u(r) . \tag{A-9}$$

Notice that this is IRH and we have just proved that left-hand side of it is strictly greater than its right-hand side, which means that IRH does not bind, thus it can be eliminated while solving the model.

A.3 Appendix A.3

Proposition 8 *ICH binds in the setting where the equilibrium strategy is v-h,n-l.*

Now, we will show that ICH binds. In order to do that, let us increase t_v by ϵ. Since t_v does not exist in IRL, it is not affected by the increase. An increase in t_v will decrease right-hand side of ICL, so it still holds:

$$\rho u(v_L - P - t_r) + (1-\rho)u(-t_r) > \rho u(v_L - P - t_v - \epsilon) + (1-\rho)u(-t_v - \epsilon) + x \tag{A-10}$$

Lastly, we showed that IRH does not bind, so ϵ amount of increase in t_v will not affect IRH:

$$\rho u(v_H - P - t_v - \epsilon) + (1-\rho)u(-t_v - \epsilon) > u(r) \tag{A-11}$$

If IRL, ICL and IRH are not affected by the increase in t_v, what stops the mall from increasing t_v? The answer is ICH. Since ICH binds, the mall cannot increase t_v.

A.4 Appendix A.4

Proposition 9 *ICL does not bind in the setting where the equilibrium strategy is v-h,n-l.*

Given that ICH binds, we can show that ICL does not. Since ICH binds, we can rewrite

ICH as:

$$x = \rho[u(v_H - P - t_v) - u(v_H - P - t_r)] + (1-\rho)[u(-t_v) - u(-t_r)] \,. \tag{A-12}$$

If we rewrite ICL in the same way, it becomes

$$x \leq \rho[u(v_L - P - t_r) - u(v_L - P - t_v)] + (1-\rho)[u(-t_r) - u(-t_v)] \,. \tag{A-13}$$

We will find out the relation between t_v and t_r now. Adding up ICH and ICL constraint and making simplification, we obtain

$$\underbrace{u(v_H - P - t_v)}_{\boxed{1}} + \underbrace{u(v_L - P - t_r)}_{\boxed{2}} \geq \underbrace{u(v_H - P - t_r)}_{\boxed{3}} + \underbrace{u(v_L - P - t_v)}_{\boxed{4}} \,. \tag{A-14}$$

From this inequality and concavity of u, we conclude $t_v \geq t_r$. In addition, we can intuitively say that t_v is strictly greater than t_r, otherwise this equilibria in which high types prefer valet parking and low types prefer regular parking would not exist. Low types would pretend to be high type because they would pay the same or less for valet and get an extra utility of x.

Now, it is time exploit our assumptions that the utility function u of customers is monotone, increasing and concave. The last term ($(1-\rho)[u(-t_r) - u(-t_v)]$) is common in both of the equations (A-12) and (A-13). So, we can ignore that part for now. For the rest, only difference is valuations. Since v_H is greater than v_L, inputs of the utility function u are greater in ICH than those in ICL: $\boxed{3} > \boxed{2}$ & $\boxed{1} > \boxed{4}$.

Using the concavity of u and $t_v > t_r$, we say $\boxed{1} - \boxed{4} > \boxed{3} - \boxed{2}$ because an increase from v_L to v_H when the inside of utility function is with t_v has a greater effect and does it with t_r. So, the left hand side of (A-14) is strictly bigger than the right hand side, which means ICL does not bind.

A.5 Appendix A.5

Proposition 10 *IRL binds in the setting where the equilibrium strategy is v-h,n-l.*

Let me increase t_r by ϵ amount. IRH remains the same after this increase because none of the terms in IRH carries t_r. ICL is not affected by ϵ amount of increase of t_r because it does not bind:

$$\rho u(v_L - P - t_r - \epsilon) + (1-\rho)u(-t_r - \epsilon) > \rho u(v_L - P - t_v) + (1-\rho)u(-t_v) + x \tag{A-15}$$

Lastly, ICH still holds because an increase in t_r will decrease the right-hand side of it:

$$\rho u(v_H - P - t_v) + (1-\rho)u(-t_v) + x > \rho u(v_H - P - t_r - \epsilon) + (1-\rho)u(-t_r - \epsilon) \quad \text{(A-16)}$$

Therefore, IRL binds so that the mall may not increase t_r further.

A.6 Appendix A.6

Proposition 11 t$_r$= 0 *in v-h,f-l equilibrium.*

In order to prove that regular parking turns out to be free, we will show that payoff of the mall inversely related with t_r. Derivative of the payoff function of the mall is

$$\frac{\partial \Pi(P, t_v, t_r)}{\partial t_r} = \rho \frac{dP}{dt_r} + \lambda \frac{\partial t_v}{\partial t_r} + (1-\lambda)\,. \quad \text{(A-17)}$$

Consider $\frac{dP}{dt_r}$ first. We can obtain P in terms of t_r from the equation (9). When we write P in terms of t_r, we achieve the following result:

$$P = v_L - t_r - u^{-1}[\frac{u(r) - (1-\rho)u(-t_r)}{\rho}]\,. \quad \text{(A-18)}$$

Taking the derivative, we find

$$\frac{dP}{dt_r} = -1 - \frac{\frac{(1-\rho)u'(-t_r)}{\rho}}{u'\{u^{-1}[\frac{u(r)-(1-\rho)u(-t_r)}{\rho}]\}} = -1 - \frac{(1-\rho)}{\rho}\frac{u'(-t_r)}{u'(v_L - P - t_r)}\,. \quad \text{(A-19)}$$

Note that v_L must be greater than P because the minimum value t_r can get is 0 and when t_r is 0 in equation (5), v_L must be greater than P for the equation to hold. Keeping this in mind and considering that u is increasing and concave so its derivative is decreasing:

$$\frac{dP}{dt_r} < -1 - \frac{(1-\rho)}{\rho} = \frac{-1}{\rho}\,. \quad \text{(A-20)}$$

Let me go back to the equation (A-17). Plugging the value above in the equation (A-17), we find

$$\frac{d\Pi(P, t_v, t_r)}{dt_r} < \lambda(\frac{\partial t_v}{\partial t_r} - 1)\,. \quad \text{(A-21)}$$

So, if we show that $\frac{\partial t_v}{\partial t_r}$ is less than 1, payoff of the firm decreases with t_r for the values of t_r greater than zero. Then, the best strategy for the mall would be to provide free parking as in Hasker and Inci (forthcoming). Consider $\frac{\partial t_v}{\partial t_r}$ then. Let me define a new function f by putting all the terms of equation (7) on one side:

$$f(P(t_r), t_v, t_r) = x + \rho u(v_H - P - t_v) + (1-\rho)u(-t_v) - \rho u(v_H - P - t_r) - (1-\rho)u(-t_r) \; . \tag{A-22}$$

Using implicit differentiation,

$$\begin{aligned} \frac{\partial t_v}{\partial t_r} &= -\frac{\frac{\partial f(P(t_r),t_v,t_r)}{\partial t_r}}{\frac{\partial f(P(t_r),t_v,t_r)}{\partial t_v}} & \text{(A-23)} \\ &= -\frac{-\rho u'(v_H - P - t_v)\frac{dP}{dt_r} + \rho u'(v_H - P - t_r)(\frac{dP}{dt_r} + 1) + (1-\rho)u'(-t_r)}{-\rho u'(v_H - P - t_v) - (1-\rho)u'(-t_v)} & \text{(A-24)} \end{aligned}$$

We want to show $\frac{\partial t_v}{\partial t_r} < 1$. Notice that $(1-\rho)u'(-t_v) > (1-\rho)u'(-t_r)$ because t_v is greater than t_r and u is concave. So, we just need to show:

$$\rho u'(v_H - P - t_v) > -\rho u'(v_H - P - t_v)\frac{dP}{dt_r} + \rho u'(v_H - P - t_r)(\frac{dP}{dt_r} + 1) \; . \tag{A-25}$$

Rearranging the inequality, we easily achieve $u'(v_H - P - t_v) > u'(v_H - P - t_r)$ which is true.

As a result, we have showed that payoff of the mall and t_r are inversely related when t_r is greater than zero. Therefore, the mall provides free parking to low types in this equilibrium in which low types prefer regular and high types prefer valet parking.

A.7 Appendix A.7

Proposition 12 *IRH does not bind, IRL binds in the setting where the equilibrium strategy is f-all.*

Consider the inequalities (30) and (31), that is to say, IRH and IRL respectively. The sole difference between these two constraints is that we have v_H in IRH instead of v_L that exists in IRL:

$$\begin{aligned} \rho u(\underline{\mathbf{v}_H} - P - t_r) + (1-\rho)u(-t_r) &> u(r) \; . & \text{(A-26)} \\ \rho u(\underline{\mathbf{v}_L} - P - t_r) + (1-\rho)u(-t_r) &= u(r) \; . & \text{(A-27)} \end{aligned}$$

Since v_H is assumed to be greater than v_L, left-hand side of the inequality (30) is always greater than that of the inequality (31). That is why we conclude IRH does not bind whereas

IRL does.

A.8 Appendix A.8

Proposition 13 *IRH does not bind, IRL binds in the setting where the equilibrium strategy is v-all.*

Left-hand side of the inequality (40) is always greater than that of the inequality (41):

$$x + \rho u(\underline{\mathbf{v}_H} - P - t_v) + (1 - \rho)u(-t_v) \quad > \quad u(r) \; . \tag{A-28}$$

$$x + \rho u(\underline{\mathbf{v}_L} - P - t_v) + (1 - \rho)u(-t_v) \quad = \quad u(r) \; . \tag{A-29}$$

So IRH does not bind whereas IRL does. This is why we can ignore IRH while solving the mall's problem.

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