A THREE PLAYER NETWORK FORMATION GAME

by

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Abstract

Efficiency and stability are the two most widely discussed issues in the networks literature. Desirable networks are such that they combine efficiency and stability. In Currarini and Morelli's (2000) non-cooperative game-theoretic model of sequential network formation, in which players propose links and demand payoffs, if the value of networks satisfy size monotonicity (i.e. the efficient networks connect all players in some way or another), then each and every equilibrium network is efficient. Our sequential game is not endogenous in terms of payoff division. The setting is such that players prefer being part of a two player network, although three player networks generate the greatest total value. However, we present our result that, the efficient complete graph is sustainable as a subgame perfect equilibrium as well as a trembling–hand perfect equilibrium. We further our analysis by examining various repeated game formulations that are most frequently used in the literature. We focus on "zero–memory" (Markov) strategies and show that our conclusion still holds under "zero–memory" (Markov) subgame perfection.

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Özet

Verimlilik ve dengenin istikrarı, şebeke literatüründe en sık tartışılan konulardır. Currarini ve Morelli 2000 tarihli makalelerinde; oyuncuların sıralı bir şekilde hem bağlantı kurmayı teklif ettiği hem de kazanç talep ettiği işbirliksiz oyun teorisi modellerinde, şebekelerin değerinin boyut monotonluğuna uyum sağlaması halinde her dengenin verimli olduğunu göstermektedir. Bizim sıralı oynanan 3 oyunculu şebeke oluşturma oyunumuzun kazanç dağılımı oyuncuların taleplerine dayanmamaktadır. Oyuncular kişisel kazançlarını göz önünde bulundurarak iki oyunculu şebekelerin parçası olmayı, üç oyunculu şebekelere tercih etmektedir. Fakat, üç oyunculu şebekeler en yüksek toplam değeri üretir, ve dolayısıyla verimlidir. Verimli ve eksiksiz olan şebekelerin alt oyun kusursuz Nash dengesi ve hatta $\varepsilon-$kusursuz denge ile elde edilebildiğini göstermekteyiz. Ardından, oyunumuzu literatürde sıkça kullanılan çeşitli tekrarlı oyun formülasyonlarına çevirmekteyiz. "Sıfır-hafıza" (Markov) stratejilere odaklanmakta ve sonucumuzun hala geçerli olduğunu göstermekteyiz.

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1 INTRODUCTION

Our social and economic lives are shaped according to different network structures we are involved in. Networks play a central role in many instances. Social networks affect which habits we do posses, how diseases spread, which products we choose to buy, how much education we obtain, which job opportunities we have etc. In other contexts network structures can affect how the information is shared among individuals in a firm and thus the firm's productivity, the trading schemes, the firm's attitudes against financial risk and contagion. These and many other examples in which the networks play significant roles, make it imperative to understand how networks emerge as well as how they affect economic interactions.

Therefore, the network literature mainly focuses on two aspects; network formation and the influences of a given network. Furthermore, the theoretical literature that is concerned with the strategic network formation is in the pursuit of two characteristics; stability and efficiency, i.e. maximizing total value the network generates.

In the article titled "Endogenous Formation of Links Between Players and Coalitions: an Application of the Shapley Value", Aumann and Myerson (1988) study a sequential network formation example with a specific allocation rule (*the Myerson value*) which results in an inefficient equilibrium network. The implication of their paper is therefore that not all fixed allocation rules are compatible with efficiency, even if the game is sequential. Later, Jackson and Wolinsky (1996) reach an axiomatic result. Their strong conclusion yields that no fixed allocation rule would ensure that at least one stable graph is efficient for every given value function. Dutta and Mutuswami (1997), on the other hand, show that mechanism design approach (where the allocation rules themselves are the mechanisms to play with) can help reconcile efficiency and stability in their paper called "Stable Networks". Specifically, they deal with the impossibility result of Jackson and Wolinsky (1996) by imposing the anonymity axiom only on the equilibrium network.

In another article titled "Network Formation with Sequential Demands" Currarini and Morelli (2000) come up with another result that if the value function satisfies size monotonicity (i.e. the efficient networks connect all players in some way or the other), then the sequential network formation process with endogenous payoff division leads all equilibria being efficient. Their setup is such that players propose links and formulate a single absolute demand, representing their final payoff demand. They also extend their result and show that it also holds when players are allowed to make link–specific demands.

The network game that we introduce also has a sequential structure. There are three players, deciding on whom to connect sequentially. The payoffs are not generated through an endogenous process: we have an allocation rule satisfying anonymity. However, our setting satisfies the size monotonicity of Currarini and Morelli (2000). Players benefit from being linked to one another. Although the highest total value is generated through networks including all players, players individually benefit most from being part of a two player network. The problem is to reach the efficient networks. We establish that, in our setting the complete and efficient network can be obtained in subgame perfect equilibrium with various dynamic network formation games: (1) We show that when the game is played only once, the above conclusion holds with subgame perfection as well as trembling–hand perfect equilibrium. Also (2) for any repeated game obtained by finite or infinite repetition of this stage game (with sequential choices) the efficient and complete network can be sustained with subgame perfection. It also needs to be mentioned that the subgame perfect equilibrium strategies employed for these results are "zero memory" (Markov), i.e. these strategies depend only on what has happened in the current stage game. We also consider another well–known formulation of the associated repeated game which involves players "updating" the current state of play individually and sequentially (the setting of Bhaskar and Vega-Redondo (2002)), and prove that the same conclusion holds with subgame perfection and "zero–memory" (Markov) strategies dependent only on the payoff relevant state of the game.

Our game consists of 3 players, proposing links sequentially: First, player 1 chooses a member of $\{2, 3, \{2, 3\}, \emptyset\}$; second, player 2 observes what player 1 has chosen and selects a member of $\{1, 3, \{1, 3\}, \emptyset\}$; finally, observing what the two players have done, player 3 chooses a member of $\{1, 2, \{1, 2\}, \emptyset\}$. A link between two players is formed only if both of them have proposed to form a link with each other. The payoff structure is as follows: if only 2 players are linked to each other, then they both get a payoff of $\frac{4}{3}$ whereas the other (isolated player) gets 0; if only one player has two links (i.e. is the central player) and the rest has only one link with the mentioned central player then the central player receives a payoff of $(1 + 2\alpha)$ and the other players get $(1 - \alpha)$ where $\alpha < \frac{1}{6}$; if, on the other hand, each player is linked to the rest of the players (i.e. the complete graph is formed) then each of the players receive a payoff of 1; and, finally, having no link yields a payoff of 0 so the empty network generates no positive payoffs for any one of the players.

The core of the NTU (non-transferrable utility cooperative) game turns out to include only the two player networks; not a surprising outcome considering the above specified payoff structure.

Although the complete network is not in the core, we show that it can be obtained under subgame perfection with strategies involving punishments of deviators from actions supporting the complete network. These punishments involve the isolation of the player who has not chosen to propose a link with both of the others. The subgame perfect equilibrium strategy is as follows: Player 1, who moves first and thus cannot condition on the past, chooses to propose to both of the others. Player 2's strategy is to propose links with both player 1 and player 3 only if player 1 has proposed links with player 2 and 3, otherwise player 2's strategy requires him to propose a link only with player 3. The same also holds for player 3 and in fact he plays an even more significant role: if only both player 1 and 2 proposes links to the two other players, player 3 chooses to propose links to player 1 and 2; however, whenever only one of the previous players proposed to form a link with both of the other players, then player 3 only responds back to that particular player; the rest of the strategy is defined formally in the associated section and at this stage it suffices to say that while honoring best responses player 3 aims to punish the player who deviated the last.

We further our analysis by letting our one–shot sequential game to become the stage game of a finitely or infinitely repeated game. That is, our context comprises of phases within periods of the repeated game. In order to avoid unnecessary complexities, the sequence of players is constant throughout the whole game. Any period starts with the first player's action and ends with that of the third. Period payoffs are generated only after the third player's move. The repetition is over the periods, namely blocks of phases in which only one player is allowed to play. This could intuitively be thought as a dynamic network formation process among three players located in three distinct meridians so that within a day (a period) the three phases can be regarded as the morning followed by the noon and finally the evening.

We show that the complete graph is sustainable under subgame perfection employing a strategy that does not depend on what has happened before today and consists of repetitions of the very same strategy described in the paragraph preceding the previous one. That is, our strategy (vector) depends only on the previous phases of the current period, and not what has happened in previous days. In other words, we show that in the finitely or infinitely repeated game the efficient complete graph is obtained under subgame perfection with the use of zero–memory (Markov) strategies.

While we were able to sustain the complete network with zero–memory subgame perfect strategies, in general memory considerations are quite important in repeated games. It is appropriate to mention that in our case we are able to get some clean cut results easily. However, in general this is not always this easy for a given arbitrary repeated game.

Indeed, repeated games are extensive form games involving finite or infinite repetitions of a given stage game. They provide players the chance to condition strategies on past behavior which enables us to sustain a wide range of equilibrium possibilities; a property which is not preferable in terms generality. The assumption that players have infinite memory is thought of being the main source of multiplicity of subgame perfect equilibrium (SPE) payoffs in repeated games. *Bounded memory* considerations (restricting players' memory, the number of consecutive periods a player can recall) have long been included in the literature in order to increase the level of realism in two aspects; avoiding the multiplicity of equilibrium problem as suggested by Aumann (1981) as well as to serve as a remedy for *complexity* concerns. There are various formulations of complexity, a particular one of which is related to the memory concerns of players, representing the imperfect and limited computational skills of human beings who can not condition their actions upon everything that has happened in the past.

However, this intuition regarding the reduction of the multiplicity of equilibrium via bounded memory was contrasted by Barlo, Carmona, and Sabourian (2009). Working with one memory, they prove that the Folk Theorem for SPE continues to hold for games in which players' action spaces are sufficiently "rich". Emphasizing the importance of the rich action spaces assumption, they additionally show that when action spaces are not "rich" it is possible that no efficient payoff vector can be supported by a one memory SPE strategy even with a discount factor near one; confirming the argument of Aumann (1981).

In a subsequent work of Barlo, Carmona, and Sabourian (2012), they produce the result that the Folk Theorem for discounted repeated game continues to hold with time-independent bounded memory pure strategies, even when the action sets are finite. Furthermore, they show that the limitation on the number of periods that players need to recall to form the result is uniform in terms of the set of individually rational payoffs, and depends solely on the desired degree of payoff approximation.

It is of some value to point out that we did not have such problems arising due to bounded memory or complexity considerations in our setting. Indeed, using the repetition of the subgame perfect strategy of the stage game, we obtained our conclusion.

However, there is another related repeated game formulation which brings about a more clear inference when someone is concerned with memory considerations. That is, the repeated game is not defined with blocks of phases (periods), but with phases that are sequentially and asynchronously updated by players. To see whether or not our conclusion can be extended to that setting as well, we present an adaptation of our model designed to accommodate the formulation of Bhaskar and Vega-Redondo (2002), and show that we are still able to sustain the complete and efficient graph in "zero–memory" (Markov) subgame perfect equilibrium, i.e. subgame perfect equilibrium in which strategies depend only on the payoff relevant state of the game.

While the bulk of literature concern repeated games obtained by the simultaneous play in the stage game, their paper "Asynchronous Choice and Markov Equilibria", Bhaskar and Vega-Redondo (2002) deal with a 2–player repeated game with asynchronous choice.[1] In their setting the total payoff depends on the discounted sum of stage-game payoffs as well as memory costs. They prove that in any asynchronously repeated game with memory costs, every subgame perfect equilibrium must only be one of the Markov equilibria; where Markov equilibria are defined as the equilibria such that players are only allowed to condition their strategies on the payoff relevant states:

> The present paper provides a theoretical foundation for Markov equilibria of repeated games with asynchronous moves that is based on memory costs. We consider two-player repeated games with discounting where players move in alternate periods. Players may condition their actions upon payoff irrelevant past events, but such conditioning is costly and their memory is finite (although arbitrarily large). Specifically, players' preferences over alternative configurations respond both to payoffs and memory requirements in the natural way: they are increasing in payoffs for identical memory requirements, but decreasing in these requirements for equal induced payoffs. This allows for either a scenario where complexity costs and payoffs are of comparable magnitudes or the commonly considered context where com-

[1] It is useful to point out that Bhaskar and Vega-Redondo (2002) emphasizes in its discussion part that this result can easily be extended to contexts with finitely many players who (almost surely) never have the chance to move simultaneously.

plexity costs are lexicographically less important than stage-game payoffs.

Markov equilibria do not possess much of a meaning under dynamic settings with simultaneous stage games, where it is only formed of repeated plays of a Nash equilibrium of that stage game. The result is solely based on repeated games with asynchronous choices. However, this paper's environment may be considered as a counterpart to those of repeated games with simultaneous move stage games. In fact, their result may be contrasted with the pioneering work of Abreu and Rubinstein (1988), who introduce complexity considerations in a repeated game context with simultaneous moves. Their focus is on the Nash equilibria, where a given strategy taken to be preferred to an alternative more complex one if both strategies yield the same payoff against the opponent's strategy on the equilibrium path. They showed that this is enough to reduce quite substantially the wide range of Nash equilibrium payoffs typically supported in standard Folk Theorems. However, Kalai and Neme (1992) later showed that just "a little perfection" is sufficient to restore the usual Folk Theorem conclusions in this concept.

The adaptation of our basic model to accommodate the setting of Bhaskar and Vega-Redondo (2002) is such that, there is a new network structure emerging after each and every players' actions at any point in time. Because we have three players, the actions played by other players in the previous two phases along with the current action specifies a new network that will determine a new payoff for players. Specifically, let the game start from any fixed network structure. Then player 1 "updates" this network structure by modifying his choice in that network by choosing a member of $\{2, 3, (2, 3), \emptyset\}$. The resulting player 1 modified network is referred to as the state of the play at the end of the first phase, and players get payoffs accordingly. In the next round, player 2 observes the state of play at the end of the first phase, and modifies this network structure by choosing a member of $\{1, 3, (1, 3), \emptyset\}$. This brings about the state of play at the end of the second phase and associated returns to players. The game continues in this fashion where the sequence of moves are always given by 1 followed by 2 and followed by 3 and followed by 1 and so and so forth.

We show that, Markov strategies are enough to deliver the desired result. Indeed, we prove that there exists an open neighborhood of parameters (discount factors and centrality measures) such that the complete network can be supported in Markov perfect equilibrium.

The Markov perfect strategies we employ have the following form: Any player conditions his actions only on the payoff relevant state. It is imperative to point out that the state of play at the beginning of the current phase (the payoff relevant state in terms of the language of Bhaskar and Vega-Redondo (2002)) can only be determined by the other players' choices in the previous two phases. The players choose to propose links to both of the players whenever the preceding player's action comprises of proposing links to both of the other players or proposing link only to the other player. Continuing in that fashion, any player chooses to propose link to the subsequent player whenever the preceding player's action is $\{\emptyset\}$ or whenever the preceding player has only proposed a link to the current player and additionally the other player has either proposed links to both of the players or only to the current player. Finally, any player proposes a link to the preceding player whenever the preceding player has proposed a link to the current player and additionally the other player has either proposed a link to the preceding player or played $\{\emptyset\}$.

To sum up, employing different repeated game formulations that are frequently used in the literature this study shows that the complete and efficient network can be sustained with "zero–memory" (Markov) perfect strategies.

In the next chapter, we introduce our model and make the necessary definitions. In Chapter 3, we provide the subgame perfect equilibria of the one-shot game and show that the complete graph is also an trembling–hand perfect equilibrium. In Chapter 4, we employ two different formulations of infinitely repeated games and present the strategies to obtain the complete and efficient network.

2 THE MODEL

The set of players is $N = \{1, 2, 3\}$ and a *graph* g is a set L of links (non-directed segments) joining pairs of players (nodes). The graph containing a link for every pair of players is called the *complete graph*, and is denoted by $g^{N=3}$. The set of all possible graphs on N is $\mathcal{G} \equiv \{g \mid g \subset g^{N=3}\}$. We denote by ij the link that joins players i and j, so that if $ij \in g$ we say that i and j are *directly connected* in the graph g. $g + ij$ denotes the graph obtained by adding the link ij to the graph g, and $g - ij$ the graph obtained by removing the link ij from g.

We let $N(g) \equiv \{i \in N \mid \exists j \in N \text{ such that } ij \in g\}$ be the set of individuals who have at least one link in network g. Let $n(g)$ be the cardinality of $N(g)$. A *path* in g connecting i_1 and i_k is a set of nodes $\{i_1, ..., i_k\} \in N(g)$ such that $i_p i_{p+1} \in g$ for all $p = 1, ..., k - 1$. We say that the *subgraph* g' of g, i.e. $g \subset g$, is a *component* of g if

- if $i \in N(g')$ and $j \in N(g')$ and $j \neq i$, then there exists a path in g' connecting i and j;
- if $i \in N(g')$ and $j \notin N(g')$, then there is no path in g connecting i and j.

So the components of a given network consist of its distinct connected subgraphs. We let the set of components of g be denoted $C(g)$, and note that $g = \cup_{g' \in C(g)} g'$.

The particular payoff structure this study considers is as follows: $u_i(\emptyset) = 0$, $u_i(ij) = \frac{4}{3}$, $u_i(ij, jk) = 1 - \alpha$, $u_i(ij, ik) = 1 + 2\alpha$, and $u_i(ij, jk, ik) = 1$; where $\alpha \in (0, 1)$. The value of a component g' is determined by $v(g') = \sum_i u_i(g')$ which results in an additive and anonymous value function $v : \mathcal{G} \to \mathbb{R}$. [1]

[1] A *value function* $v : \mathcal{G} \to \mathbb{R}$ is *anonymous* if for any permutation π, a bijection mapping N into N,

A graph $g^* \in \mathcal{G}$ is *efficient* with respect to v if $v(g^*) \geq v(g)$ for all $g \in \mathcal{G}$, and $\mathcal{G}^*(v) \subset \mathcal{G}$ denotes the set of efficient networks relative to v. In our setting, $\mathcal{G}^*(v) = \{(ij, jk), (ij, jk, ik) \mid i, j, k = 1, 2, 3, \text{ and } i \neq j \neq k\}$. Moreover, the core of this game with non-transferable utilities consists of the following three graphs, $\{\{12\}, \{13\}, \{23\}\}$. [2]

2.1 The Network Formation Game

The dynamic network formation of this study involves a repeated game under perfect information and asynchronous moves.

2.1.1 The Stage Game

Our stage game, an extensive form game with perfect information, is defined by $G = \langle N, X, \iota, (u_i)_{i \in N} \rangle$: $N = \{1, 2, 3\}$ is the set of players. X, the set of histories of the stage game, is given as follows: Let $A_i \equiv \{\{\emptyset\}, \{j\}, \{k\}, \{j, k\} \mid j \neq k \text{ and } j, k \in N \setminus \{i\}\}$ and $A \equiv \times_{i=1,2,3} A_i$ and $X \equiv \{e, a_1, (a_1, a_2), (a_1, a_2, a_3) \mid a_i \in A_i, i = 1, 2, 3\}$ where e denotes the beginning of the stage game. We let $X_1 = \{e\}$ and $X_2 = \{a_1 \mid a_1 \in A_1\}$ and $X_3 = \{(a_1, a_2) \mid a_1 \in A_1 \text{ and } a_2 \in A_2\}$. The terminal histories of the stage game are denoted by Z, and $(a_1, a_2, a_3) = Z \subset X$ is the only terminal history in X. The

and any $g \in \mathcal{G}$ it must be that $v(g^\pi) = v(g)$ where $g^\pi = \{\pi(i)\pi(j) \mid ij \in g\}$; *additive* if for any $g \in \mathcal{G}$ we have that $v(g) = \sum_{g' \in C(g)} v(g')$. The individual payoff of player i in $g \in \mathcal{G}$ is denoted by $Y_i(g, v)$ and $Y : \mathcal{G} \times V \to \mathbb{R}^N$ is referred to as the *allocation rule*. Dealing with additive value functions and allocating the value generated by any component without any waste among the individuals in that component implies a component balanced allocation rule: Y is *component balanced* if for any additive v and $g \in \mathcal{G}$ and $g' \in C(g)$ we have that $\sum_{i \in N(g')} Y_i(g', v) = v(g')$. Finally the allocation rule Y is also anonymous, satisfying $Y_{\pi(i)}(g^\pi, v^\pi) = Y_i(g, v)$ for any v,$g \in \mathcal{G}$ and permutation π.

[2] The complete graph given by $\{12, 23, 13\}$ is not in the core because $4/3 > 1$ implies that anyone of the players has a strictly profitable deviation opportunity. Moreover, clearly $\{ij, jk\}$ is also not in the core because players i and k may deviate jointly and each increase their payoffs from $1 - \alpha$ to 1; $i \neq j \neq k$ and $i, j, k = 1, 2, 3$. Next, it is also clear that the graph which does not have any links is also not in the core because all players deviating to the complete graph is a strictly profitable deviation. Finally, for any graph with $\{ij\}$, $i \neq j$ and $i, j = 1, 2, 3$, due to the fact that each player i and j obtaining a payoff of $4/3$ the only other network structure that may present a profitable deviation opportunity (for some values of α) is one given by $\{kl, lm\}$ where $k, l, m = 1, 2, 3$ and $k \neq l$ and $l \neq m$ and $k \neq m$. But this also does not work: while i may benefit (when α such that $1 + 2\alpha > 4/3$), j has to suffer (because $1 - \alpha < 4/3$).

player function $\iota : X \setminus Z \to N$ has a simple shape in order to refrain from non-fruitful technicalities: players choose sequentially and are ordered by their index, i.e. $\iota(x) = k$ whenever $x \in X_k$. For any terminal history $a = (a_1, a_2, a_3) \in Z$ with $a_i \in A_i$, the induced network is formed at the end of the stage game as follows: $a = (a_1, a_2, a_3)$ induces the graph $g(a) \equiv \{ij \in g^{N=3} \mid j \in a_i \text{ and } i \in a_j \text{ where } i \neq j \text{ and } i, j \in N\}$. Consequently, the payoffs of the stage game are defined (with a slight abuse of notation) as follows: $u_i : Z \to \mathbb{R}$ where $u_i(a) = u_i(g(a))$.

An action $a_i \in A_i$ of player i is a vector of arcs sent by i to some subset of $N \setminus \{i\}$, and a strategy of player i in the stage game is a function $\sigma_i : X_i \to A_i$.

2.1.2 The Repeated Game

The repeated game consists of repetitions of G. Time is indexed discretely: $t \in \mathbb{N}_0 \equiv \{0, 1, 2, ...\}$. We denote the action of player i in the repeated game at any date t by $a_i^t \in A_i$ and let $a^t = (a_1^t, a_2^t, a_3^t)$ be the profile of choices at t. For the purposes of providing a full specification of the details we also keep track of the time index within each period which consists of 3 phases: In phase $s \in \mathbb{N}_0$ of period $t \in \mathbb{N}_0$, player $i \in N$, such that $i = (s \mod 3) + 1$, chooses; hence, this choice happening in (t, s) equals $a^t_{(s \mod 3)+1}$. In order to provide a tangible envisagement one may contemplate time periods as days and phases as morning or noon or evening.

For any $t \geq 1$, a t–stage history is a sequence $h^t = (a^0, \ldots, a^t)$. The set of all histories are partitioned into H_1 and H_2 and H_3 as follows. In period t with a given $(t-1)$–stage history h^{t-1}, H_1 involves histories when it is the turn of player 1 (so $H_1 \equiv \{h^{t-1} : h^{t-1} \text{ is a } (t-1)\text{–stage history}\}$; H_2 when it is the turn of player 2 (thus, $H_2 \equiv \{(h^{t-1}, a_1^t) : h^{t-1} \in H_1 \text{ and } a_1^t \in A_1\}$); and finally H_3 when it is the turn of player 3 (so $H_3 \equiv \{(h^{t-1}, a_1^t, a_2^t) : (h^{t-1}, a_1^t) \in H_2 \text{ and } a_2^t \in A_2\}$).[3] We represent the initial (empty) history by h^0. Given any history $h \in H_i$ associated with time period t, a continuation (of play) w is *compatible* with h if it is given by $\left(a_i^{t+\mathbf{1}_{i=3}}, \ldots\right)$ where $\mathbf{1}_{i=3}$

[3] It is appropriate to point out that for any $h \in H$ corresponding to time period t and $(t-1)$–stage history h^{t-1}, there is $c = 1, 2, 3$ such that h equals either to $h^{t-1} \in H_1$ (the first phase of period t) or to $(h^{t-1}, a_1^t) \in H_2$ (the second phase of t) or to $(h^{t-1}, a_1^t, a_2^t) \in H_3$ (the third phase of t).

equals 1 if $i = 3$ and 0 otherwise, $t \in \mathbb{N}$. Now, combining $h \in H_i$ with a compatible continuation w delivers a history denoted by $h \cdot w$ consisting of the *concatenation of* h *followed by* w.

A *strategy* of player i is a function f_i mapping H_i into A_i; and we let F_i denote the set of all strategies of player i; and $F = F_1 \times F_2 \times F_3$ is the joint strategy space with a typical element $f = (f_1, \ldots, f_n)$. Given a strategy $f_i \in F_i$ and a history $h \in H$, we denote the *strategy induced by* f_i *at* h by $f_i|h$. Thus, for any compatible continuation w following h we have that $(f_i|h)(w) = f_i(h \cdot w)$. We will use $(f|h)$ to denote $(f_1|h, \ldots, f_n|h)$ for every $f = (f_1, \ldots, f_n) \in F$ and $h \in H$.

Any strategy profile $f \in F$ induces an *outcome path* $\pi(f) = \{\pi^0(f), \pi^1(f), \pi^2(f), \ldots\} \in \Pi$ where $\pi^0(f) = f(h^0) \in A_1$ and $\pi^s(f) = f(\pi^0(f), \ldots, \pi^{s-1}(f)) \in A_i$ where $i = (s \mod 3) + 1$ for any $s > 0$, and $\Pi \equiv \times_{k=0}^{\infty} A_{(k \mod 3)+1}$ denotes the set of all outcome paths. Notice that $(\pi^0(f), \pi^1(f), \ldots, \pi^k(f)) \in H_i$ where $i = ((k+1) \mod 3) + 1$ and involves a l–stage history with $l = \lfloor \frac{k+1}{3} \rfloor$ where for any $r \in \mathbb{R}$ the term $\lfloor r \rfloor$ denotes the floor of r; in words, $(\pi^0(f), \ldots, \pi^k(f))$ is a history happening in day $\lfloor \frac{k+1}{3} \rfloor - 1$ of the game in which it is the turn of player $((k+1) \mod 3) + 1$.

A given $(\pi^0, \ldots, \pi^k)$ with $k \geq 2$ induces $\lfloor \frac{k+1}{3} \rfloor$ many networks in every period $t \leq \lfloor \frac{k+1}{3} \rfloor - 1$: we let $\gamma_\pi^\tau = g(\pi^s, \pi^{s+1}, \pi^{s+2})$ where s is such that $(s \mod 3) + 1$ equals 1 (i.e. the first player is the one choosing at k) and $\tau = \frac{s}{3}$ and $s \leq k$.

We assume that all players discount the future payoffs by a common discount factor $\delta \in (0, 1)$. Thus, the payoff in the repeated game is given by $U_i(f, \delta) = (1 - \delta) \sum_{t=0}^{\infty} \delta^t u_i(\gamma_{\pi(f)}^t)$. For any $\pi \in \Pi$ and $t \in \mathbb{N}_0$ and $i \in N$, let $V_i^t(\pi, \delta) = (1 - \delta) \sum_{\tau=t}^{\infty} \delta^{\tau - t} u_i(\gamma_\pi^\tau)$ be the continuation payoff of player i at date t if the outcome path π is played. For simplicity, we write $V_i(\pi, \delta)$ instead of $V_i^0(\pi, \delta)$. Also, when the meaning is clear we shall not explicitly mention δ and refer to $U_i(f, \delta)$, $V_i^t(\pi, \delta)$ and $V_i(\pi, \delta)$ by $U_i(f)$, $V_i^t(\pi)$ and $V_i(\pi)$ respectively.

The repeated game described above for discount factor $\delta \in (0, 1)$ is denoted by $G^\infty(\delta)$. A strategy vector $f \in F$ is a *Nash equilibrium* of $G^\infty(\delta)$ if $U_i(f) \geq U_i(\hat{f}_i, f_{-i})$ for all $i \in N$ and $\hat{f}_i \in F_i$. Also, $f \in F$ is a *SPE* of $G^\infty(\delta)$ if $f|h$ is a Nash equilibrium for all $h \in H_1$.

For any non-empty history h associated with time period t and any integer $0 < m \leq t$, define the *m-tail of h* by

$$T^m(h) = \begin{cases} (a^{t-m}, \ldots, a^{t-1}) & \text{if } h \in H_1, \\ (a^{t-m}, \ldots, a^{t-1}, a_1^t) & \text{if } h \in H_2, \\ (a^{t-m}, \ldots, a^{t-1}, a_1^t, a_2^t) & \text{if } h \in H_3. \end{cases}$$

We also adopt the convention that

$$T^0(h) = \begin{cases} e & \text{if } h \in H_1, \\ (a_1^t) & \text{if } h \in H_2, \\ (a_1^t, a_2^t) & \text{if } h \in H_3. \end{cases}$$

For all $M \in \mathbb{N}$, we say that $f \in F$ is a *M-memory strategy* if $f(h) = f(\bar{h})$ for all $h, \bar{h} \in H$ such that $T^M(h) = T^M(\bar{h})$. It needs to be pointed out that the requirement of $T^M(h) = T^M(\bar{h})$ implies that both $h, \bar{h} \in H_c$ for some $c = 1, 2, 3$. A strategy profile f is a *M-memory SPE* if f is a M-memory strategy and a SPE.[4]

[4]The above definition of a M-memory SPE allows players to deviate to a strategy with unbounded memory. However, this definition is equivalent to the one where players are restricted to deviate to M-memory strategies. In fact, if f is a M-memory strategy, then for all $i \in N$ and $h \in H$, player i has a M-memory best-reply to $f_{-i}|h$ by Derman (1970, Theorem 1, p.23).

3 ONE–SHOT GAME

The conflict between the efficiency and stability has long been discussed in the strategic network formation literature. It is important to remind that the the paper by Currarini and Morelli (2000) deals with a sequential network formation setting, where players propose links and formulate a single absolute demand. They showed that if the value function satisfies size monotonicity (i.e. efficient networks connect players in some way or another), then the sequential network formation with endogenous payoff division leads all equilibria being efficient.

In our setting, the payoff division is not endogenous; we have have an allocation rule satisfying anonymity. However, our setting satisfies size monotonicity: The total value generated by two player networks are only $\frac{8}{3}$, whereas that of three player networks ((12 13), (12 23), (13 23), (12 23 13)) equals 3. We let $\alpha < \frac{1}{6}$, thereby players benefit most when they are part of a 2 player-linked network by getting the highest possible payoff of $\frac{4}{3}$. However, such graphs are not efficient. We show that, we are able to reach the complete graph as an equilibrium under subgame perfection and even with trembling–hand perfection.

Lemma 1 *The subgame perfect equilibrium of our one-shot game brings about four graphs; (12), (23), (13), (12 23 13).*

Proof. We analyze all possible networks that may form out of our game. There are many different combinations of strategies that yield subgame perfect equilibrium graphs but it is enough to show only one equilibrium case for each positive result. We start by giving the explicit strategy profile σ^* for the complete graph since it would also be used in the subsequent chapter:

$$\sigma_1(e) = \{2,3\}$$

$$\sigma_2(x) = \begin{cases} \{1,3\} & \text{if } a_1 = \{2,3\} \\ \{3\} & \text{if } a_1 = \{2\} \\ \{3\} & \text{if } a_1 = \{3\} \\ \{3\} & \text{if } a_1 = \{\emptyset\} \end{cases}$$

$$\sigma_3(x) = \begin{cases} \{1,2\} & \text{if } a_1 = \{2,3\} \text{ and } a_2 = \{1,3\} \\ \{2\} & \text{if } a_1 = \{3\} \text{ and } a_2 = \{1,3\} \\ \{2\} & \text{if } a_1 = \{2\} \text{ and } a_2 = \{1,3\} \\ \{2\} & \text{if } a_1 = \{\emptyset\} \text{ and } a_2 = \{1,3\} \\ \{1\} & \text{if } a_1 = \{2,3\} \text{ and } a_2 = \{3\} \\ \{2\} & \text{if } a_1 = \{3\} \text{ and } a_2 = \{3\} \\ \{2\} & \text{if } a_1 = \{2\} \text{ and } a_2 = \{3\} \\ \{2\} & \text{if } a_1 = \{\emptyset\} \text{ and } a_2 = \{3\} \\ \{1\} & \text{if } a_1 = \{2,3\} \text{ and } a_2 = \{1\} \\ \{1\} & \text{if } a_1 = \{3\} \text{ and } a_2 = \{1\} \\ \{1\} & \text{if } a_1 = \{2\} \text{ and } a_2 = \{1\} \\ \{1\} & \text{if } a_1 = \{\emptyset\} \text{ and } a_2 = \{1\} \\ \{1\} & \text{if } a_1 = \{2,3\} \text{ and } a_2 = \{\emptyset\} \\ \{1\} & \text{if } a_1 = \{3\} \text{ and } a_2 = \{\emptyset\} \\ \{1\} & \text{if } a_1 = \{2\} \text{ and } a_2 = \{\emptyset\} \\ \{1\} & \text{if } a_1 = \{\emptyset\} \text{ and } a_2 = \{\emptyset\} \end{cases}$$

Case 1: (12 23 13) In order to obtain this graph in the in the subgame perfect equilibrium, it must be the case that $\sigma_1(e) = \{2,3\}$, $\sigma_2((2,3)) = \{1,3\}$ and $\sigma_3(((2,3),(1,3))) = \{1,2\}$. We know for sure that under this specific history, player 3 has no incentive to deviate. But we have to make sure that the same arguement is valid for player 1 and 2. In order to do that, we have to fix some of the strategies. Letting $\sigma_3((2,3))) = \sigma_3((2,(1,3))) = \sigma_3((3,3)) = \sigma_3((3,(1,3))) = \sigma_3(((\emptyset),3)) = \sigma_3(((\emptyset),(1,3))) = \{2\}$, $\sigma_3((2,(\emptyset))) = \sigma_3((2,1)) = \sigma_3((3,1)) = \sigma_3((3,(\emptyset)))) = \sigma_3(((2,3),1)) = \sigma_3(((2,3),3)) = \sigma_3(((2,3),(\emptyset))) = \sigma_3(((\emptyset),1)) = \sigma_3(((\emptyset),(\emptyset))) = \{1\}$ and $\sigma_3(((2,3),(1,3))) = \{1,2\}$ allows us to further fix player 2's strategies such that player 1 will have no incentive to deviate. First of all, notice that if $\sigma_2((2)) = \{1\}$, then since player 1 has the opportunity to obtain $\frac{4}{3}$; he would choose to play $\{2\}$ instead of $\{2,3\}$. There is no other deviation possibility for either of the player 1 and 2. Therefore, $\sigma_2((2)) = \{3\}$ assures us that the complete graph, i.e. (12 23 13) the resulting subgame perfect equilibrium graph.

Case 2: (12) Let player 1 play $\{2\}$ and player 2 choose $\{1\}$; then player 3 is indifferent between choosing any of his possible actions because independent of his choice, he will end up with a payoff of 0. There is no room to deviate for either player 1 or player 2 since they end up with the highest possible payoff $\frac{4}{3}$. Regardless of player 2 and player 3's choices under other histories, this specification yields a subgame perfect equilibrium.

Case 3: (23) Let player 1 play $\{2\}$ and player 2 play $\{3\}$ under any given history. Under this kind of a history, the best response of player 3 is to play either $\{2\}$ or $\{1,2\}$; both of which yields the above mentioned graph (23). There is no incentive for player 2 to deviate because of his equilibrium payoff $\frac{4}{3}$. However, in order to guarantee that player 1 can not profitably choose another action, we need to fix player 3's strategies for other histories. Letting $\sigma_3((2,1)) = \sigma_3((2,(1,3))) = \sigma_3((2,(\emptyset))) = \sigma_3((3,3)) = \sigma_3((3,(1,3))) = \sigma_3(((2,3),3)) = \sigma_3(((\emptyset),1)) = \sigma_3(((\emptyset),3)) = \sigma_3(((\emptyset),(1,3))) = \sigma_3(((\emptyset),(\emptyset))) = \{2\}$, $\sigma_3((3,1)) = \sigma_3((3,(\emptyset))) = \sigma_3(((2,3),1)) = \sigma_3(((2,3),(\emptyset))) = \{1\}$ and $\sigma_3(((2,3),(1,3))) = \{1,2\}$ assures us with the resulting subgame perfect equilibrium graph (23).

Case 4: (13) Let player 1 play $\{3\}$ and player 3 play $\{1\}$ under any history involving $a_1 = \{3\}$. Player 1 and 3 have no incentive to deviate because they both get the highest possible payoff, i.e. $\frac{4}{3}$. Under these circumstances, player 2 is indifferent between choosing any of his actions, since his payoff is for sure 0 independent of his action choice.

Case 5: (12 23) This graph is not reached under subgame perfection. Lets see why by observing the possible deviations on the equilibrium path. In order for (12 23) to be an equilibrium outcome, player 2 should play $\{1,3\}$ on the equilibrium path. Let player 1 choose $\{2,3\}$. Then we need $\sigma_3\left(((2,3),(1,3))\right) = \{2\}$ to be the case. However, when faced with such a history, player 3 would instead choose $\{1,2\}$ since he prefers the payoff of 1 rather than $(1-\alpha)$. Now let player 1 play $\{2\}$. Then, player 3 would either choose $\{2\}$ or $\{1,2\}$ which is compatible with the desired outcome. However, this would make player 2 get $(1+2\alpha)$, hence he would profitably deviate and play $\{1\}$ and achieve $\frac{4}{3}$ for sure. Consequently, we observe that (12 23) is not reached as a subgame perfect equilibrium outcome.

Case 6: (12 13) This graph is not reached under subgame perfection. In order for (12 13) to be an equilibrium outcome, player 1 must choose $\{2,3\}$ on the equilibrium path. Let player 2 play $\{1,3\}$. For this graph to be sustainable, player 3 should choose $\{1\}$ but he would choose $\{1,2\}$ and get a payoff of 1 instead of $(1-\alpha)$. Now consider the case $\sigma_2\left((2,3)\right) = \{1\}$. Then, player 3 would either choose $\{1\}$ or $\{1,2\}$ which is compatible with the desired outcome. However, this would make player 2 get $(1-\alpha)$. Therefore he would profitably deviate and play $\{1,3\}$ and achieve $\frac{4}{3}$ since player 3's best response is $\sigma_3\left(((2,3),(1,3))\right) = \{1,2\}$.

Case 7: (13 23) This graph is not reached under subgame perfection. On the equilibrium path, it must be the case that player 3 chooses $\{1,2\}$. Let player 1 choose $\{3\}$, then we need player 2 to play either $\{3\}$ or $\{1,3\}$. However, the best response of player 3 for such a history does not include $\{1,2\}$; he would prefer to play $\{1\}$ or $\{2\}$ and receive a payoff of $\frac{4}{3}$ instead of $(1+2\alpha)$. Now suppose player 1 chooses $\{2,3\}$, then we need player 2 to choose $\{3\}$ and player 3 to play $\{1,2\}$. However, the previous arguement holds for this case as well; player 3 would prefer to play $\{1\}$ or $\{2\}$ rather than $\{1,2\}$ and receive a payoff of $\frac{4}{3}$ instead of $(1+2\alpha)$.

Case 8: $(\emptyset)$ This graph is not reached under subgame perfection. Suppose player 1 plays $\{3\}$ or $\{2,3\}$; then we know for sure that player 3's best response would not be compatible with an equilibrium payoff of 0, he always has the option to connect to player 1. Now suppose player 1 plays $\{2\}$; moving after such a history, player 2 would either respond to player 1 by playing $\{1\}$ or play $\{3\}$. In the former case the empty network is already out of consideration; whereas in the latter one, player 3's best response is either $\{1, 2\}$ or $\{2\}$ resulting in the (23) network. Finally, suppose player 1 plays $\{\emptyset\}$; then player 2 would respond by playing either $\{3\}$ or $\{1, 32\}$ both of which would induce player 3 to respond by playing either $\{2\}$ or $\{1, 2\}$ bringing about the (23) once again.

This finishes the proof. ■

It is important to notice that the complete graph is a strict equilibrium, i.e. conforming with the equilibrium continuation is strictly beneficial for every player. Therefore, it is also sustainable as a trembling–hand perfect equilibrium. In a trembling–hand perfect equilibrium, there must be arbitrarily small perturbations of all players' strategies such that every pure strategy gets strictly positive probability and each player's equilibrium strategy is still a best response to the other players' perturbed strategies. Then, the limiting strategy vector when these perturbations diminish is a trembling–hand perfect equilibrium. More specifically, consider the above introduced equilibrium strategies for the complete graph (12 23 13). Now suppose we perturb every players strategies; that is we let every player to make mistakes with a small probability of $\varepsilon_i(\sigma_i)$ where $\varepsilon_i : \Sigma_i \to (0, 1]$ satisfying $\sum_{\Sigma_i} \varepsilon_i(\sigma_i) \leq 1$. A strategy vector $\sigma^* \in \sum$, where $\sum = \Pi_{i \in N} \Delta(\Sigma_i)$ is the set of mixed strategy vectors (N is the set of players), is a *trembling–hand perfect equilibrium* if there is a sequence of trembles $\{\varepsilon^r\}$ with

$$\lim_{r \to \infty} \left(\max_{i \in N, \sigma_i \in \Sigma_i} \varepsilon_i^r(s_i) \right) = 0 \tag{3.1}$$

and a sequence $\{\sigma^r\}$, where each σ^r is an ε^r-perfect equilibrium and $\sigma^r \to \sigma^*$, where ε^r-perfect equilibrium is fixed point of the intersection of the best responses constrained to the associated $\varepsilon^{*,r}$–simplex with the convention that $\varepsilon^{*,r} \equiv \max_{i \in N, \sigma_i \in \Sigma_i} \varepsilon_i^r(\sigma_i)$. We can find such a sequence of trembles $\{\varepsilon^r\}$ with $\lim_{r \to \infty} \left(\max_{i \in N, \sigma_i \in \Sigma_i} \varepsilon_i^r(\sigma_i) \right) = 0$ assigning strictly positive probabilities $\varepsilon_i^r(\sigma_i)$ to every pure strategy off the equilibrium path (the equilibrium path we have determined above for (12 23 13)) and assigning the probability of $1 - \varepsilon_i^r(\sigma_i)$ to pure strategies on the equilibrium path for all players; such that each player's equilibrium strategy is still a best response to the other player's perturbed strategies. Thus we conclude that the SPE outcome of (12 23 13) is also a trembling–hand perfect equilibrium outcome under the above specified strategies.

4 REPEATED NETWORK FORMATION GAME

We now further our analysis by examining the repeated versions of our network formation game. Our finding is that, in both of the following repeated game settings, the complete and efficient network is supported in subgame perfection with the use of "zero–memory" (Markov) strategies.

4.1 Version One

In this section, we will go over the repeated game context, which comprises of phases within periods of the repeated game. It is useful to remind the reader that in each phase, only 1 player is allowed to play. In order to refrain from unnecessary complexities, the sequence of players is constant throughout the whole game. In each period there are three phases for the three players to move sequentially. Any period starts with first player's action and ends with that of the third player. Period payoffs are generated only after third player's move. The repetition is over the periods, namely ternary blocks of phases. As noted earlier in the introduction, this could intuitively be thought as a dynamic network formation process among three players located in three distinct meridians. Therefore, the three phases could be regarded as the morning followed by the noon and finally the evening in a given day, i.e. period.

We show that, for a repeated game, where the repetition is either finite or infinite, the complete and efficient graph can be sustained under subgame perfection employing a strategy that does not depend on what has happened before today and consists of repetitions of the very same strategy described in the Chapter 3: s^*. That is, playing the repetition of s^*, depending only on the previous phases of the current period brings about the complete and efficient graph (12 23 13) as a subgame perfect equilibrium outcome.

Lemma 2 *For any finite or infinite repetition of the extensive form stage game G, there exists a zero–memory (Markov) subgame strategy profile which induces the complete and efficient network given by (12 23 13).*

Proof. We define the strategy profile f by

$$f(h) = \sigma^* \left(T^0(h)\right), \text{ for any } h \in H.$$

That is, players condition their actions upon the previous phases of the current period and choose the corresponding action as prescribed by the strategy profile defined explicitly in the one–shot game. To be more precise, player 1 is required to play $\{2,3\}$ regardless of what has happened in the previous periods and player 2 and 3 are to choose the relevant actions from $\sigma^* (T^0(h))$ after observing (a_1^t) and (a_1^t, a_2^t) respectively. Because that σ^* is already shown to be a subgame perfect equilibrium strategy profile, it can easily be induced that regardless of the repeated game being finite or infinite, $f(h) = \sigma^* (T^0(h))$ provides the players with a zero–memory (Markov) subgame perfect equilibrium outcome of (12 23 13). ■

4.2 Version Two

Due to the fact that we are able to obtain the desired outcome, namely (12 23 13) sustainable under subgame perfection in the one–shot game, we did not have any problems arising from bounded memory or complexity considerations in the previous section. We now adapt our model to accommodate the formulation of Bhaskar and Vega-Redondo (2002), and show that we are still able to support the complete and efficient graph in zero–memory (Markov) perfect equilibrium, i.e. subgame perfect equilibrium in which strategies depend only on the payoff relevant state of the game.

Time is indexed discretely: $s \in \mathbb{N}_0 \equiv \{0, 1, 2, ...\}$, where s represents the phases in which only one of the three players is allowed to play. The game starts from a given initial state given by $(a_1, a_2, a_3) \in A_1 \times A_2 \times A_3$. This initial state of the game is not restricted to be the complete network or the empty network, yet it is not a bad idea to think of it (without loss of generality) as the complete network. The sequence of the players is fixed, that is, in the very beginning of the game player 1 starts by choosing a_1^s, where $s = 0$, followed by player 2 choosing his action in the subsequent phase; $a_2^{(s+1)}$, followed by player 3 choosing $a_3^{(s+2)}$ followed by player 1 and so on and so forth, where $a_i^{s'} \in A_i$ for all $s' = 0, 1, 2, ...$ The resulting modified network is referred to as the state of the play at the end of the phase s, after player $i = (s \mod 3) + 1$'s move: $a^s_{(s \mod 3)+1}$.

For any $s \geq 1$, an s-stage history is a sequence $\theta^s = (a_1^0, a_2^1, \ldots, a^s_{(s \mod 3)+1})$. The set of all histories are partitioned into three subsets, Θ_1 and Θ_2 and Θ_3, corresponding to the set of histories θ^s $(s \geq 1)$, where either it is the turn of player 1, i.e. when $((s+1) \mod 3) + 1 = 1$, or 2, i.e. when $((s+1) \mod 3) + 1 = 2$ or 3, i.e. when $((s+1) \mod 3) + 1 = 3$. We represent the initial (empty) history by θ^0. Given any history $\theta \in \Theta_i$ associated with time denoting phase s, a continuation (of play) η is *compatible* with θ if it is given by $\left(a^s_{(s \mod 3)+1}, \ldots\right)$. Combining $\theta \in \Theta_i$ with a compatible continuation η delivers a history denoted by $\theta \cdot \eta$ consisting of the *concatenation of* θ *followed by* η.

A *strategy* of player i is a function ϕ_i mapping Θ_i into A_i; and we let Φ_i denote the set of all strategies of player i; and $\Phi = \Phi_1 \times \Phi_2 \times \Phi_3$ is the joint strategy space with a typical element $\phi = (\phi_1, \ldots, \phi_n)$. Given a strategy $\phi_i \in \Phi_i$ and a history $\theta \in \Theta$, we denote the *strategy induced by* ϕ_i *at* θ by $\phi_i|\theta$. Thus, for any compatible continuation η following θ we have that $(\phi_i|\theta)(\eta) = \phi_i(\theta \cdot \eta)$. We will use $(\phi|\theta)$ to denote $(\phi_1|\theta, \ldots, \phi_n|\theta)$ for every $\phi = (\phi_1, \ldots, \phi_n) \in \Phi$ and $\theta \in \Theta$.

Given the initial given state of play at the beginning of the game by (a_1, a_2, a_3), any strategy profile $\phi \in \Phi$ induces an *outcome path* $\pi(\phi) = \{\pi^0(\phi), \pi^1(\phi), \pi^2(\phi), \ldots\} \in \Pi$ where $\pi^0(\phi) = (\phi_1(\theta^0), a_2, a_3)$ with $\phi_1(\theta^0) \in A_1$ and

$$\pi^s(\phi) = (\phi_i(\pi^0(\phi), \ldots, \pi^{s-1}(\phi)), \phi_j(\pi^0(\phi), \ldots, \pi^{s-2}(\phi)), \phi_k(\pi^0(\phi), \ldots, \pi^{s-3}(\phi)))$$

where $\phi_i(\pi^0(\phi), \ldots, \pi^{s-1}(\phi)) \in A_i$ and $\phi_j(\pi^0(\phi), \ldots, \pi^{s-2}(\phi)) \in A_j$ and also the term $\phi_k(\pi^0(\phi), \ldots, \pi^{s-3}(\phi))) \in A_k$ and $i = (s \mod 3) + 1$ and $j = ((s-1) \mod 3) + 1$ and $k = ((s-2) \mod 3) + 1$ for any $s > 0$; and $\Pi \equiv (A_1 \times A_2 \times A_3)^\infty$ denotes the set of all outcome paths. Notice that $(\pi^0(\phi), \pi^1(\phi), \ldots, \pi^\kappa(\phi)) \in \Theta_i$ where $i = ((\kappa+1) \mod 3) + 1$ and involves a κ–stage history; in words, $(\pi^0(\phi), \ldots, \pi^\kappa(\phi))$ is a history happening in phase κ of the game in which it is the turn of player $((\kappa+1) \mod 3) + 1$.

We assume that all players discount the future payoffs by a common discount factor $\delta \in (0,1)$. Thus, the payoff in the repeated game is given by $U_i(\phi, \delta) = (1-\delta)\sum_{t=0}^{\infty} \delta^s u_i(\pi(\phi))$. For any $\pi \in \Pi$ and $s \in \mathbb{N}_0$ and $i \in N$, let $V_i^s(\pi, \delta) = (1-\delta)\sum_{\tau=t}^{\infty} \delta^{\tau-s} u_i(\pi^\tau)$ be the continuation payoff of player i at date s if the outcome path π is played. For simplicity, we write $V_i(\pi, \delta)$ instead of $V_i^0(\pi, \delta)$. Also, when the meaning is clear we shall not explicitly mention δ and refer to $U_i(\phi, \delta)$, $V_i^s(\pi, \delta)$ and $V_i(\pi, \delta)$ by $U_i(\phi)$, $V_i^s(\pi)$ and $V_i(\pi)$ respectively.

The repeated game described above for discount factor $\delta \in (0,1)$ is denoted by $\mathbb{G}^\infty(\delta)$. A strategy vector $\phi \in \Phi$ is a *Nash equilibrium* of $\mathbb{G}^\infty(\delta)$ if $U_i(\phi) \geq U_i(\hat{\phi}_i, \phi_{-i})$ for all $i \in N$ and $\hat{\phi}_i \in \Phi_i$. Also, $\phi \in \Phi$ is a *SPE* of $\mathbb{G}^\infty(\delta)$ if $\phi|\theta$ is a Nash equilibrium for all $\theta \in \Theta$.

For any non-empty history θ associated with time period s and any integer $0 < m \leq s$, define the *m-tail of* θ by $T^m(\theta)$ which equals the last m states of play preceding phase s and equals to:

$$\Big(\Big(a^{s-m-1}_{((s-m-1) \mod 3)+1}, a^{s-m-2}_{((s-m-2) \mod 3)+1}, a^{s-m-3}_{((s-m-3) \mod 3)+1}\Big), \\ \ldots, \Big(a^{s-1}_{((s-1) \mod 3)+1}, a^{s-2}_{((s-2) \mod 3)+1}, a^{s-3}_{((s-3) \mod 3)+1}\Big)\Big).$$

Notice that Markov strategies depend only on the current payoff relevant state of play which is given by $T^0(\theta)$ for any given θ where

$$T^0(\theta) = \left(a^{s-1}_{((s-1) \mod 3)+1}, a^{s-2}_{((s-2) \mod 3)+1}, a^{s-3}_{((s-3) \mod 3)+1}\right).$$

It is useful to point that then it is the turn of player $((s-3) \mod 3)+1$ who takes the other two players' decisions $\left(a^{s-1}_{((s-1) \mod 3)+1}, a^{s-2}_{((s-2) \mod 3)+1}\right)$ as a given.

It should be noted that in our three player network formation game, the state of the play at the beginning of the current phase (the payoff relevant state in terms of the language of Bhaskar and Vega-Redondo (2002)) can only be determined by the other players' choices in the previous two phases. That is why the preceding paragraph presents the tails of histories by containing enough information to figure out the state of play in the previous phases.

For all $M \in \mathbb{N}$, we say that $\phi \in \Phi$ is a *M-memory strategy* if $\phi(\theta) = \phi(\bar{\theta})$ for all $\theta, \bar{\theta} \in \Theta$ such that $T^M(\theta) = T^M(\bar{\theta})$ with $\theta, \bar{\theta} \in \Theta_i$ for some $i = 1, 2, 3$. A strategy profile f is a *M-memory SPE* if f is a M-memory strategy and a SPE. [1]

It is useful to remind the reader of the result of Bhaskar and Vega-Redondo (2002) at this point. They have provided a theoretical foundation for the use of Markov strategies in repeated games with asynchronous moves, such that the state of the game is "updated" after each and every players' actions at any point in time:

> If admissible strategies must display finite (arbitrarily long) memory and each player incurs a "complexity cost" which depends on the memory length required by her strategy, then every *Nash equilibrium* must be in *Markovian* strategies.

Lemma 3 *The complete network can be sustained with Markov strategies for $\alpha \in \left(\frac{3}{36}, \frac{4}{36}\right)$ whenever players are sufficiently patient. Indeed, there is an open neighborhood of parameters around $\delta = 0.98$ and $\alpha = \frac{7}{72}$ such that this conclusion holds.*

[1] The above definition of a M-memory SPE allows players to deviate to a strategy with unbounded memory. However, this definition is equivalent to the one where players are restricted to deviate to M-memory strategies. In fact, if f is a M-memory strategy, then for all $i \in N$ and $h \in H$, player i has a M-memory best-reply to $f_{-i}|h$ by Derman (1970, Theorem 1, p.23).

Proof. Without loss of generality, lets fix the sequence of players as $i \to j \to k \to i \to$ .... We start by introducing the specific Markov strategies:

$$\phi_i(h) = \begin{cases} \{j,k\} & \text{if } a_j^{s-2} = \{i,k\} \text{ and } a_k^{s-1} = \{i,j\} \\ \{j,k\} & \text{if } a_j^{s-2} = \{i\} \text{ and } a_k^{s-1} = \{i,j\} \\ \{j,k\} & \text{if } a_j^{s-2} = \{k\} \text{ and } a_k^{s-1} = \{i,j\} \\ \{j,k\} & \text{if } a_j^{s-2} = \{\emptyset\} \text{ and } a_k^{s-1} = \{i,j\} \\ \{j\} & \text{if } a_j^{s-2} = \{i,k\} \text{ and } a_k^{s-1} = \{i\} \\ \{j\} & \text{if } a_j^{s-2} = \{i\} \text{ and } a_k^{s-1} = \{i\} \\ \{k\} & \text{if } a_j^{s-2} = \{k\} \text{ and } a_k^{s-1} = \{i\} \\ \{k\} & \text{if } a_j^{s-2} = \{\emptyset\} \text{ and } a_k^{s-1} = \{i\} \\ \{j,k\} & \text{if } a_j^{s-2} = \{i,k\} \text{ and } a_k^{s-1} = \{j\} \\ \{j,k\} & \text{if } a_j^{s-2} = \{i\} \text{ and } a_k^{s-1} = \{j\} \\ \{j,k\} & \text{if } a_j^{s-2} = \{k\} \text{ and } a_k^{s-1} = \{j\} \\ \{j,k\} & \text{if } a_j^{s-2} = \{\emptyset\} \text{ and } a_k^{s-1} = \{j\} \\ \{j\} & \text{if } a_j^{s-2} = \{i,k\} \text{ and } a_k^{s-1} = \{\emptyset\} \\ \{j\} & \text{if } a_j^{s-2} = \{i\} \text{ and } a_k^{s-1} = \{\emptyset\} \\ \{j\} & \text{if } a_j^{s-2} = \{k\} \text{ and } a_k^{s-1} = \{\emptyset\} \\ \{j\} & \text{if } a_j^{s-2} = \{\emptyset\} \text{ and } a_k^{s-1} = \{\emptyset\} \end{cases}$$

We have to go over 16 different cases (identified by a_j^{s-2} and a_k^{s-1}) for each player. However, since our game is symmetric, it is enough to check those different cases only for player 1 by letting $(s\text{mod}3) + 1 = 1$ and the result will hold for player 2 and player 3 as well. By the nature of the above specified strategies, each case converges to the complete graph setting $(12, 23, 13)$ at some point and consequently continues in that fashion. Therefore, it is enough to make the comparisons based on the aggregation until this particular convergence.

Case 1. $a_2^{s-2} = \{1,3\}$ and $a_3^{s-1} = \{1,2\}$.

The utility of player 1 equals 1 in each period when $a_1^s = \{2,3\}$.

When $a_1^s = \{2\}$ player 1 would not choose to deviate for any δ because of the following:

	s	$s+1$	$s+2$	$s+3$	$s+4$	$s+5$
action	$\{2\}$	$\{3\}$	$\{2\}$	$\{2,3\}$	$\{1,3\}$	$\{1,2\}$
graph	(12, 23)	(23)	(23)	(23)	(12, 23)	(12, 23, 13)
$u_1(a_1^s, \phi_{-1})$	$(1-\alpha)$	0	0	0	$(1-\alpha)$	1
$u_1(\phi)$	1	1	1	1	1	1

On the other hand, $a_1^s = \{3\}$ is also not a profitable deviation for all δ because:

	s	$s+1$	$s+2$	$s+3$
action	$\{3\}$	$\{1,3\}$	$\{1,2\}$	$\{2,3\}$
graph	(13, 23)	(13, 23)	(13, 23)	(12, 23, 13)
$u_1(a_1^s, \phi_{-1})$	$(1-\alpha)$	$(1-\alpha)$	$(1-\alpha)$	1
$u_1(\phi)$	1	1	1	1

Next a similar conclusion also holds for player 1's deviation by choosing $a_1^s = \{\emptyset\}$ regardless of the level of δ:

	s	$s+1$	$s+2$	$s+3$	$s+4$	$s+5$
action	$\{\emptyset\}$	$\{3\}$	$\{2\}$	$\{2,3\}$	$\{1,3\}$	$\{1,2\}$
graph	(23)	(23)	(23)	(23)	(12, 23)	(13, 23, 13)
$u_1(a_1^s, \phi_{-1})$	0	0	0	0	$(1-\alpha)$	1
$u_1(\phi)$	1	1	1	1	1	1

Case 2. $a_2^{s-2} = \{1\}$ and $a_3^{s-1} = \{1,2\}$.

Conforming delivers player 1 a utility of $(1-\delta)(1+2\alpha)+\delta$, because:

	s	$s+1$
action	$\{2,3\}$	$\{1,3\}$
graph	(12, 13)	(12, 23, 13)
$u_1(\phi)$	$(1+2\alpha)$	1

Considering a deviation of player 1 by choosing $a_1^s = \{2\}$ results in:

	s	$s+1$	$s+2$	$s+3$	$s+4$	$s+5$
action	$\{2\}$	$\{3\}$	$\{2\}$	$\{2,3\}$	$\{1,3\}$	$\{1,2\}$
graph	(12)	(23)	(23)	(23)	(12, 23)	(12, 23, 13)
$u_1(a_1^s, \phi_{-1})$	4/3	0	0	0	$(1-\alpha)$	1
$u_1(\phi)$	$(1+2\alpha)$	1	1	1	1	1

and it is not profitable whenever δ is sufficiently high to satisfy

$$(1-\delta)(((1+2\alpha)-4/3)+\delta+\delta^2+\delta^3+\delta^4\alpha) \geq 0. \tag{4.1}$$

We know that there exists $\delta_{(2-1)} \in (0,1)$ such that for all $\delta \geq \delta_{(2-1)}$ condition 4.1 is satisfied because the left hand side of condition 4.1 is strictly positive when δ is sufficiently close to 1.

When player 1's deviation to $a_1^s = \{3\}$ is under analysis, we obtain:

	s	$s+1$	$s+2$	$s+3$
action	$\{3\}$	$\{1,3\}$	$\{1,2\}$	$\{2,3\}$
graph	(13)	(13, 23)	(13, 23)	(13, 23, 13)
$u_1(a_1^s, \phi_{-1})$	4/3	$(1-\alpha)$	$(1-\alpha)$	1
$u_1(\phi)$	$(1+2\alpha)$	1	1	1

which implies that this deviation is not profitable when

$$(1-\delta)(((1+2\alpha)-4/3)+\delta\alpha+\delta^2\alpha) \geq 0. \tag{4.2}$$

There is $\delta_{(2-2)} \in (0,1)$ such that for all $\delta \geq \delta_{(2-2)}$ condition 4.2 holds: Consider the continuous function $b_{(2-2)} : \mathbb{R} \to \mathbb{R}$ defined by the left hand side of condition 4.2. Hence, $\frac{1}{1-\delta} b_{(2-2)}(\delta)$ is arbitrarily close to $4\alpha - \frac{1}{3}$ when δ is sufficiently close to 1. So requiring that $\alpha > \frac{3}{36}$ enables us to guarantee that $b_{(2-2)}(\delta) \geq 0$ for δ sufficiently close to 1.

Next, we observe that choosing $a_1^s = \{\emptyset\}$ is not a profitable deviation for any δ because:

	s	$s+1$	$s+2$	$s+3$	$s+4$	$s+5$
action	$\{\emptyset\}$	$\{3\}$	$\{2\}$	$\{2,3\}$	$\{1,3\}$	$\{1,2\}$
graph	$(\emptyset)$	(23)	(23)	(23)	(12, 23)	(12, 23, 13)
$u_1(a_1^s, \phi_{-1})$	0	0	0	0	$(1-\alpha)$	1
$u_1(\phi)$	$(1+2\alpha)$	1	1	1	1	1

Case 3. $a_2^{s-2} = \{3\}$ and $a_3^{s-1} = \{1,2\}$.

Conforming to ϕ_1 yields player 1 a payoff of $(1-\delta)(1-\alpha)+\delta$ because of the following:

	s	$s+1$
action	$\{2,3\}$	$\{1,3\}$
graph	(13, 23)	(12, 23, 13)
$u_1(\phi)$	$(1-\alpha)$	1

Choosing $a_1^s = \{2\}$ is not a profitable deviation for any δ because:

	s	$s+1$	$s+2$	$s+3$	$s+4$	$s+5$
action	$\{2\}$	$\{3\}$	$\{2\}$	$\{2,3\}$	$\{1,3\}$	$\{1,2\}$
graph	(23)	(23)	(23)	(23)	(12, 23)	(12, 23, 13)
$u_1(a_1^s, \phi_{-1})$	0	0	0	0	$(1-\alpha)$	1
$u_1(\phi)$	$(1-\alpha)$	1	1	1	1	1

The same conclusion also holds for $a_1^s = \{3\}$ regardless of the level of δ as a result of:

	s	$s+1$	$s+2$	$s+3$
action	$\{3\}$	$\{1,3\}$	$\{1,2\}$	$\{2,3\}$
graph	(13, 23)	(13, 23)	(13, 23)	(13, 23, 13)
$u_1(a_1^s, \phi_{-1})$	$(1-\alpha)$	$(1-\alpha)$	$(1-\alpha)$	1
$u_1(\phi)$	$(1-\alpha)$	1	1	1

Moreover, when $a_1^s = \{\emptyset\}$ clearly we obtain the desired conclusion for any δ once again:

	s	$s+1$	$s+2$	$s+3$	$s+4$	$s+5$
action	$\{\emptyset\}$	$\{3\}$	$\{2\}$	$\{2,3\}$	$\{1,3\}$	$\{1,2\}$
graph	(23)	(23)	(23)	(23)	(12, 23)	(12, 23, 13)
$u_1(a_1^s, \phi_{-1})$	0	0	0	0	$(1-\alpha)$	1
$u_1(\phi)$	$(1-\alpha)$	1	1	1	1	1

Case 4. $a_2^{s-2} = \{\emptyset\}$ and $a_3^{s-1} = \{1, 2\}$.

Conforming renders player 1 a payoff of $(1-\delta)\frac{4}{3} + \delta$ as a result of:

	s	$s+1$
action	$\{2,3\}$	$\{1,3\}$
graph	(13)	(12, 23, 13)
$u_1(\phi)$	4/3	1

Choosing $a_1^s = \{2\}$ does not result in a profitable deviation for all δ because:

	s	$s+1$	$s+2$	$s+3$	$s+4$	$s+5$
action	$\{2\}$	$\{3\}$	$\{2\}$	$\{2,3\}$	$\{1,3\}$	$\{1,2\}$
graph	$(\emptyset)$	(23)	(23)	(23)	(12, 23)	(12, 23, 13)
$u_1(a_1^s, \phi_{-1})$	0	0	0	0	$(1-\alpha)$	1
$u_1(\phi)$	4/3	1	1	1	1	1

A similar conclusion holds for $a_1^s = \{3\}$ for all δ because:

	s	$s+1$	$s+2$	$s+3$
action	$\{3\}$	$\{1,3\}$	$\{1,2\}$	$\{2,3\}$
graph	(13)	(13, 23)	(13, 23)	(12, 23, 13)
$u_1(a_1^s, \phi_{-1})$	4/3	$(1-\alpha)$	$(1-\alpha)$	1
$u_1(\phi)$	4/3	1	1	1

Next, $a_1^s = \{\emptyset\}$ is also not a profitable deviation for all δ because of the following:

	s	$s+1$	$s+2$	$s+3$	$s+4$	$s+5$
action	$\{\emptyset\}$	$\{3\}$	$\{2\}$	$\{2,3\}$	$\{1,3\}$	$\{1,2\}$
graph	$(\emptyset)$	(23)	(23)	(23)	(12, 23)	(12, 23, 13)
$u_1(a_1^s, \phi_{-1})$	0	0	0	0	$(1-\alpha)$	1
$u_1(\phi)$	4/3	1	1	1	1	1

Case 5. $a_2^{s-2} = \{1,3\}$ and $a_3^{s-1} = \{1\}$.

Conforming to ϕ_1 delivers player 1 a payoff of $(1-\delta)\left(\frac{4}{3} + \delta\frac{4}{3} + \delta^2\frac{4}{3} + \delta^3(1+2\alpha)\right) + \delta^4$ because:

	s	$s+1$	$s+2$	$s+3$	$s+4$
action	$\{2\}$	$\{1\}$	$\{1,2\}$	$\{2,3\}$	$\{1,3\}$
graph	(12)	(12)	(12)	(12, 13)	(12, 23, 13)
$u_1(\phi)$	4/3	4/3	4/3	$(1+2\alpha)$	1

Playing $a_1^s = \{2,3\}$ is not profitable for any δ whenever $\alpha \in \left(\frac{3}{36}, \frac{4}{36}\right)$ and results in:

	s	$s+1$	$s+2$	$s+3$	$s+4$
action	$\{2,3\}$	$\{1,3\}$	$\{1,2\}$	$\{2,3\}$	$\{1,3\}$
graph	(12, 13)	(12, 13)	(12, 23, 13)	(12, 23, 13)	(12, 23, 13)
$u_1(a_1^s, \phi_{-1})$	$(1+2\alpha)$	$(1+2\alpha)$	1	1	1
$u_1(\phi)$	4/3	4/3	4/3	$(1+2\alpha)$	1

Furthermore, choosing $a_1^s = \{3\}$ is also not reasonable at any level of δ due to:

	s	$s+1$	$s+2$	$s+3$	$s+4$
action	$\{3\}$	$\{1,3\}$	$\{1,2\}$	$\{2,3\}$	$\{1,3\}$
graph	(13)	(13)	(13, 23)	(12, 23, 13)	(12, 23, 13)
$u_1(a_1^s, \phi_{-1})$	4/3	4/3	$(1-\alpha)$	1	1
$u_1(\phi)$	4/3	4/3	4/3	$(1+2\alpha)$	1

Next, $a_1^s = \{\emptyset\}$ does not constitute a profitable deviation for all δ because:

	s	$s+1$	$s+2$	$s+3$	$s+4$	$s+5$
action	$\{\emptyset\}$	$\{3\}$	$\{2\}$	$\{2,3\}$	$\{1,3\}$	$\{1,2\}$
graph	$(\emptyset)$	$(\emptyset)$	(23)	(23)	(12, 23)	(12, 23, 13)
$u_1(a_1^s, \phi_{-1})$	0	0	0	0	$(1-\alpha)$	1
$u_1(\phi)$	4/3	4/3	4/3	$(1+2\alpha)$	1	1

Case 6. $a_2^{s-2} = \{1,3\}$ and $a_3^{s-1} = \{2\}$.

Conforming yields player 1 a payoff of $(1-\delta)((1-\alpha)+\delta(1-\alpha))+\delta^2$ as a result of the following

	s	$s+1$	$s+2$
action	$\{2,3\}$	$\{1,3\}$	$\{1,2\}$
graph	(12, 23)	(12, 23)	(12, 23, 13)
$u_1(\phi)$	$(1-\alpha)$	$(1-\alpha)$	1

Choosing $a_1^s = \{2\}$ does not render a profitable deviation for any δ because:

	s	$s+1$	$s+2$	$s+3$	$s+4$	$s+5$
action	$\{2\}$	$\{3\}$	$\{2\}$	$\{2,3\}$	$\{1,3\}$	$\{1,2\}$
graph	(12, 23)	(23)	(23)	(23)	(12, 23)	(12, 23, 13)
$u_1(a_1^s, \phi_{-1})$	$(1-\alpha)$	0	0	0	$(1-\alpha)$	1
$u_1(\phi)$	$(1-\alpha)$	$(1-\alpha)$	1	1	1	1

Considering $a_1^s = \{3\}$ is also not profitable regardless of the level of δ because:

	s	$s+1$	$s+2$	$s+3$
action	$\{3\}$	$\{1,3\}$	$\{1,2\}$	$\{2,3\}$
graph	(23)	(23)	(13, 23)	(12, 23, 13)
$u_1(a_1^s, \phi_{-1})$	0	0	$(1-\alpha)$	1
$u_1(\phi)$	$(1-\alpha)$	$(1-\alpha)$	1	1

Similarly, choosing to play $a_1^s = \{\emptyset\}$ is also not profitable for all δ as a result of:

	s	$s+1$	$s+2$	$s+3$	$s+4$	$s+5$
action	$\{\emptyset\}$	$\{3\}$	$\{2\}$	$\{2,3\}$	$\{1,3\}$	$\{1,2\}$
graph	(23)	(23)	(23)	(23)	(12, 23)	(12, 23, 13)
$u_1(a_1^s, \phi_{-1})$	0	0	0	0	$(1-\alpha)$	1
$u_1(\phi)$	$(1-\alpha)$	$(1-\alpha)$	1	1	1	1

Case 7. $a_2^{s-2} = \{1,3\}$ and $a_3^{s-1} = \{\emptyset\}$.

Player 1 receives a payoff of $(1-\delta)\left(\frac{4}{3} + \delta\frac{4}{3} + \delta^2\frac{4}{3} + \delta^3(1+2\alpha)\right) + \delta^4$ by conforming ϕ_1:

	s	$s+1$	$s+2$	$s+3$	$s+4$
action	$\{2\}$	$\{1\}$	$\{1,2\}$	$\{2,3\}$	$\{1,3\}$
graph	(12)	(12)	(12)	(12, 12)	(12, 23, 13)
$u_1(\phi)$	4/3	4/3	4/3	$(1+2\alpha)$	1

Considering to deviate by playing $a_1^s = \{2,3\}$ is not profitable for any δ because:

	s	$s+1$	$s+2$	$s+3$	$s+4$
action	$\{2,3\}$	$\{1,3\}$	$\{1,2\}$	$\{2,3\}$	$\{1,3\}$
graph	(12)	(12)	(12, 23, 13)	(12, 23, 13)	
$u_1(a_1^s, \phi_{-1})$	4/3	4/3	1	1	1
$u_1(\phi)$	4/3	4/3	4/3	$(1+2\alpha)$	1

It is also easy to observe that playing $a_1^s = \{3\}$ is also not a profitable option for any level of δ:

	s	$s+1$	$s+2$	$s+3$	$s+4$
action	$\{3\}$	$\{1,3\}$	$\{1,2\}$	$\{2,3\}$	$\{1,3\}$
graph	$(\emptyset)$	$(\emptyset)$	(13, 23)	(12, 23, 13)	(12, 23, 13)
$u_1(a_1^s, \phi_{-1})$	0	0	$(1-\alpha)$	1 1	
$u_1(\phi)$	4/3	4/3	4/3	$(1+2\alpha)$	1

Furthermore player 1 would not choose to play $a_1^s = \{\emptyset\}$ regardless of the level of δ because:

	s	$s+1$	$s+2$	$s+3$	$s+4$	$s+5$
action	$\{\emptyset\}$	$\{3\}$	$\{2\}$	$\{2,3\}$	$\{1,3\}$	$\{1,2\}$
graph	$(\emptyset)$	$(\emptyset)$	(23)	(23)	(12, 23)	(12, 23, 13)
$u_1(a_1^s, \phi_{-1})$	0	0	0	0	$(1-\alpha)$	1
$u_1(\phi)$	4/3	4/3	4/3	$(1+2\alpha)$	1	1

Case 8. $a_2^{s-2} = \{1\}$ and $a_3^{s-1} = \{1\}$.

Playing according to ϕ_1 results in a payoff of $(1-\delta)\left(\frac{4}{3} + \delta\frac{4}{3} + \delta^2\frac{4}{3} + \delta^3(1+2\alpha)\right) + \delta^4$ because:

	s	$s+1$	$s+2$	$s+3$	$s+4$
action	$\{2\}$	$\{1\}$	$\{1,2\}$	$\{2,3\}$	$\{1,3\}$
graph	(12)	(12)	(12)	(12, 13)	(12, 23, 13)
$u_1(\phi)$	4/3	4/3	4/3	$(1+2\alpha)$	1

Playing $a_1^s = \{2,3\}$ is not profitable for any δ whenever $\alpha \in \left(\frac{3}{36}, \frac{4}{36}\right)$ and results in:

	s	$s+1$	$s+2$	$s+3$	$s+4$
action	$\{2,3\}$	$\{1,3\}$	$\{1,2\}$	$\{2,3\}$	$\{1,3\}$
graph	(12, 13)	(12, 13)	(12, 23, 13)	(12, 23, 13)	(12, 23, 13)
$u_1(a_1^s, \phi_{-1})$	$(1+2\alpha)$	$(1+2\alpha)$	1	1	1
$u_1(\phi)$	4/3	4/3	4/3	$(1+2\alpha)$	1

Considering to play $a_1^s = \{3\}$ is also not a profitable option for any δ because:

	s	$s+1$	$s+2$	$s+3$	$s+4$
action	$\{3\}$	$\{1,3\}$	$\{1,2\}$	$\{2,3\}$	$\{1,3\}$
graph	(13)	(13)	(13, 23)	(12, 23, 13)	(12, 23, 13)
$u_1(a_1^s, \phi_{-1})$	4/3	4/3	$(1-\alpha)$	1	1
$u_1(\phi)$	4/3	4/3	4/3	$(1+2\alpha)$	1

Finally, choosing $a_1^s = \{\emptyset\}$ is clearly not a profitable deviation for all δ as a result of:

	s	$s+1$	$s+2$	$s+3$	$s+4$	$s+5$
action	$\{\emptyset\}$	$\{3\}$	$\{2\}$	$\{2,3\}$	$\{1,3\}$	$\{1,2\}$
graph	$(\emptyset)$	$(\emptyset)$	(23)	(23)	(12, 23)	(12, 23, 13)
$u_1(a_1^s, \phi_{-1})$	0	0	0	0	$(1-\alpha)$	1
$u_1(\phi)$	4/3	4/3	4/3	$(1+2\alpha)$	1	1

Case 9. $a_2^{s-2} = \{1\}$ and $a_3^{s-1} = \{2\}$.

Conforming f_1 renders player 1 a payoff of $(1-\delta)\left(\frac{4}{3} + \delta(1-\alpha)\right) + \delta^2$ as a result of:

	s	$s+1$	$s+2$
action	$\{2,3\}$	$\{1,3\}$	$\{1,2\}$
graph	(12)	(12, 23)	(23)
$u_1(\phi)$	4/3	$(1-\alpha)$	1

Player 1 would not choose to deviate by playing $a_1^s = \{2\}$ for any δ because:

	s	$s+1$	$s+2$	$s+3$	$s+4$	$s+5$
action	$\{2\}$	$\{3\}$	$\{2\}$	$\{2,3\}$	$\{1,3\}$	$\{1,2\}$
graph	(12)	(23)	(23)	(23)	(12, 23)	(12, 23, 13)
$u_1(a_1^s, \phi_{-1})$	4/3	0	0	0	$(1-\alpha)$	1
$u_1(\phi)$	4/3	$(1-\alpha)$	1	1	1	1

Next, $a_1^s = \{3\}$ is also not a profitable deviation for player 1 for all δ due to:

	s	$s+1$	$s+2$	$s+3$
action	$\{3\}$	$\{1,3\}$	$\{1,2\}$	$\{2,3\}$
graph	$(\emptyset)$	(23)	(13, 23)	(12, 23, 13)
$u_1(a_1^s, \phi_{-1})$	0	0	$(1-\alpha)$	1
$u_1(\phi)$	4/3	$(1-\alpha)$	1	1

A similar conclusion holds for $a_1^s = \{\emptyset\}$ at any level of δ because:

	s	$s+1$	$s+2$	$s+3$	$s+4$	$s+5$
action	$\{\emptyset\}$	$\{3\}$	$\{2\}$	$\{2,3\}$	$\{1,3\}$	$\{1,2\}$
graph	$(\emptyset)$	(23)	(23)	(23)	(12, 23)	(12, 23, 13)
$u_1(a_1^s, \phi_{-1})$	0	0	0	0	$(1-\alpha)$	1
$u_1(\phi)$	4/3	$(1-\alpha)$	1	1	1	1

Case 10. $a_2^{s-2} = \{1\}$ and $a_3^{s-1} = \{\emptyset\}$.

Conforming delivers player 1 a payoff of $(1-\delta)\left(\frac{4}{3} + \delta\frac{4}{3} + \delta^2\frac{4}{3} + \delta^3(1+2\alpha)\right) + \delta^4$ as a result of

	s	$s+1$	$s+2$	$s+3$	$s+4$
action	$\{2\}$	$\{1\}$	$\{1,2\}$	$\{2,3\}$	$\{1,3\}$
graph	(12)	(12)	(12)	(12,13)	(12, 23, 13)
$u_1(\phi)$	4/3	4/3	4/3	$(1+2\alpha)$	1

Considering to play $a_1^s = \{2,3\}$ is not profitable regardless of the level of δ as a result of:

	s	$s+1$	$s+2$	$s+3$	$s+4$
action	$\{2,3\}$	$\{1,3\}$	$\{1,2\}$	$\{2,3\}$	$\{1,3\}$
graph	(12)	(12)	(12, 23, 13)	(12, 23, 13)	(12, 23, 13)
$u_1(a_1^s, \phi_{-1})$	4/3	4/3	1	1	1
$u_1(\phi)$	4/3	4/3	4/3	$(1+2\alpha)$	1

Similarly, playing $a_1^s = \{3\}$ is not a rational option at any δ because:

	s	$s+1$	$s+2$	$s+3$	$s+4$
action	$\{3\}$	$\{1,3\}$	$\{1,2\}$	$\{2,3\}$	$\{1,3\}$
graph	$(\emptyset)$	$(\emptyset)$	(13, 23)	(12, 23, 13)	(12, 23, 13)
$u_1(a_1^s, \phi_{-1})$	0	0	$(1-\alpha)$	1	1
$u_1(\phi)$	4/3	4/3	4/3	$(1+2\alpha)$	1

Finally, the same result can be concluded for $a_1^s = \{\emptyset\}$ at any level of δ since:

	s	$s+1$	$s+2$	$s+3$	$s+4$	$s+5$
action	$\{\emptyset\}$	$\{3\}$	$\{2\}$	$\{2,3\}$	$\{1,3\}$	$\{1,2\}$
graph	$(\emptyset)$	$(\emptyset)$	(23)	(23)	(12, 23)	(12, 23, 13)
$u_1(a_1^s, \phi_{-1})$	0	0	0	0	$(1-\alpha)$	1
$u_1(\phi)$	4/3	4/3	4/3	$(1+2\alpha)$	1	1

Case 11. $a_2^{s-2} = \{3\}$ and $a_3^{s-1} = \{1\}$.

Conforming renders player 1 a payoff of $(1-\delta)\left(\frac{4}{3} + \delta\frac{4}{3} + \delta^2(1-\alpha)\right) + \delta^3$ because

	s	$s+1$	$s+2$	$s+3$
action	$\{3\}$	$\{1,3\}$	$\{1,2\}$	$\{2,3\}$
graph	(13)	(13)	(13, 23)	(12, 23, 13)
$u_1(\phi)$	4/3	4/3	$(1-\alpha)$	1

When player 1 chooses $a_1^s = \{2,3\}$, we obtain

	s	$s+1$	$s+2$	$s+3$
action	$\{2,3\}$	$\{1,3\}$	$\{1,2\}$	$\{2,3\}$
graph	(13)	(12, 13)	(12, 23, 13)	(12, 23, 13)
$u_1(a_1^s, \phi_{-1})$	4/3	$(1+2\alpha)$	1	1
$u_1(\phi)$	4/3	4/3	$(1-\alpha)$	1

this implies that the deviation under analysis is not profitable when

$$(1-\delta)\left(\delta\left(\frac{4}{3} - (1+2\alpha)\right) - \delta^2\alpha\right) > 0. \tag{4.3}$$

There is $\delta_{(11-1)} \in (0,1)$ such that for all $\delta \geq \delta_{(11-1)}$ condition 4.3 holds: Letting the left hand side of condition 4.3 define the continuous function $b_{(11-1)} : \mathbb{R} \to \mathbb{R}$ we observe that $\frac{1}{1-\delta} b_{(11-1)}(\delta)$ is arbitrarily close to $\frac{1}{3} - 3\alpha$ when δ is sufficiently close to 1. So requiring that $\alpha < \frac{4}{36}$ enables us to guarantee that $b_{(11-1)}(\delta) \geq 0$ for δ sufficiently close to 1.

Considering $a_1^s = \{2\}$ as player 1's deviation we obtain:

	s	$s+1$	$s+2$	$s+3$	$s+4$
action	$\{2\}$	$\{1\}$	$\{1,2\}$	$\{2,3\}$	$\{1,3\}$
graph	$(\emptyset)$	(12)	(12)	(12, 13)	(12, 23, 13)
$u_1(a_1^s, \phi_{-1})$	0	4/3	4/3	$(1+2\alpha)$	1
$u_1(\phi)$	4/3	4/3	$(1-\alpha)$	1	1

The following condition ensures that this deviation is not profitable:

$$(1-\delta)\left(\frac{4}{3} - \delta^2\left(\frac{1}{3}+\alpha\right) - \delta^3(2\alpha)\right) > 0. \tag{4.4}$$

There is $\delta_{(11-2)} \in (0,1)$ such that for all $\delta \geq \delta_{(11-2)}$ condition 4.4 holds: Letting the left hand side of condition 4.4 define the continuous function $b_{(11-2)} : \mathbb{R} \to \mathbb{R}$ we observe that $\frac{1}{1-\delta} b_{(11-2)}(\delta)$ is arbitrarily close to $1 - 3\alpha$ when δ is sufficiently close to 1. So the requirement that $\alpha < \frac{4}{36}$ also enables us to guarantee that $b_{(11-2)}(\delta) \geq 0$ for δ sufficiently close to 1.

Next, we observe that $a_1^s = \{\emptyset\}$ is not a profitable deviation for any δ because:

	s	$s+1$	$s+2$	$s+3$	$s+4$	$s+5$
action	$\{\emptyset\}$	$\{3\}$	$\{2\}$	$\{2,3\}$	$\{1,3\}$	$\{1,2\}$
graph	$(\emptyset)$	$(\emptyset)$	(23)	(23)	(12, 23)	(12, 23, 13)
$u_1(a_1^s, \phi_{-1})$	0	0	0	0	$(1-\alpha)$	1
$u_1(\phi)$	4/3	4/3	$(1-\alpha)$	1	1	1

Case 12. $a_2^{s-2} = \{3\}$ and $a_3^{s-1} = \{2\}$.

Playing according to ϕ_1 yields player 1 a payoff of $(1-\delta)(\delta(1-\alpha)) + \delta^2$ due to:

	s	$s+1$	$s+2$
action	$\{2,3\}$	$\{1,3\}$	$\{1,2\}$
graph	(23)	(12, 23)	(12, 23, 13)
$u_1(\phi)$	0	$(1-\alpha)$	1

Deviating by playing $a_1^s = \{2\}$ is not a profitable option for any δ because:

	s	$s+1$	$s+2$	$s+3$	$s+4$	$s+5$
action	$\{2\}$	$\{3\}$	$\{2\}$	$\{2,3\}$	$\{1,3\}$	$\{1,2\}$
graph	(23)	(23)	(23)	(23)	(12, 23)	(12, 23, 13)
$u_1(a_1^s, \phi_{-1})$	0	0	0	0	$(1-\alpha)$	1
$u_1(\phi)$	0	$(1-\alpha)$	1	1	1	1

Next, choosing $a_1^s = \{3\}$ is also not a profitable deviation regardless of the level of δ because:

	s	$s+1$	$s+2$	$s+3$
action	$\{3\}$	$\{1,3\}$	$\{1,2\}$	$\{2,3\}$
graph	(23)	(23)	(13, 23)	(12, 23, 13)
$u_1(a_1^s, \phi_{-1})$	0	0	$(1-\alpha)$	1
$u_1(\phi)$	0	$(1-\alpha)$	1	1

A similar conclusion also holds for $a_1^s = \{\emptyset\}$ for all δ as a result of:

	s	$s+1$	$s+2$	$s+3$	$s+4$	$s+5$
action	$\{\emptyset\}$	$\{3\}$	$\{2\}$	$\{2,3\}$	$\{1,3\}$	$\{1,2\}$
graph	(23)	(23)	(23)	(23)	(12, 23)	(12, 23, 13)
$u_1(a_1^s, \phi_{-1})$	0	0	0	0	$(1-\alpha)$	1
$u_1(\phi)$	0	$(1-\alpha)$	1	1	1	1

Case 13. $a_2^{s-2} = \{3\}$ and $a_3^{s-1} = \{\emptyset\}$.

Conforming delivers player 1 a payoff of $(1-\delta)\left(\delta\frac{4}{3}\right) + \delta^2$ as a result of:

	s	$s+1$	$s+2$	$s+3$	$s+4$
action	$\{2\}$	$\{1\}$	$\{1,2\}$	$\{2,3\}$	$\{1,3\}$
graph	$(\emptyset)$	(12)	(12)	(12, 13)	(12, 23, 13)
$u_1(\phi)$	0	4/3	4/3	$(1+2\alpha)$	1

Choosing to play $a_1^s = \{2,3\}$ is not profitable regardless of the level of δ due to:

	s	$s+1$	$s+2$ $s+3$	$s+4$	
action	$\{2,3\}$	$\{1,3\}$	$\{1,2\}$	$\{2,3\}$	$\{1,3\}$
graph	$(\emptyset)$	(12)	(12, 23, 13)	(12, 23, 13)	(12, 23, 13)
$u_1(a_1^s, \phi_{-1})$	0	4/3	1	1	1
$u_1(\phi)$	0	4/3	4/3	$(1+2\alpha)$	1

Similarly choosing $a_1^s = \{3\}$ is also not profitable at any level of δ as it can easily be observed by the following:

	s	$s+1$	$s+2$	$s+3$	$s+4$
action	$\{3\}$	$\{1,3\}$	$\{1,2\}$	$\{2,3\}$	$\{1,3\}$
graph	$(\emptyset)$	$(\emptyset)$	(13, 23)	(12, 23, 13)	(12, 23, 13)
$u_1(a_1^s, \phi_{-1})$	0	0	$(1-\alpha)$	1	1
$u_1(\phi)$	0	4/3	4/3	$(1+2\alpha)$	1

Moreover, $a_1^s = \{\emptyset\}$ is neither a profitable deviation at any δ because:

	s	$s+1$	$s+2$	$s+3$	$s+4$	$s+5$
action	$\{\emptyset\}$	$\{3\}$	$\{2\}$	$\{2,3\}$	$\{1,3\}$	$\{1,2\}$
graph	$(\emptyset)$	$(\emptyset)$	(23)	(23)	(12, 23)	(12, 23, 13)
$u_1(a_1^s, \phi_{-1})$	0	0	0	0	$(1-\alpha)$	1
$u_1(\phi)$	0	4/3	4/3	$(1+2\alpha)$	1	1

Case 14. $a_2^{s-2} = \{\emptyset\}$ and $a_3^{s-1} = \{1\}$.

Conforming result in a payoff of $(1-\delta)\left(\frac{4}{3} + \delta\frac{4}{3} + \delta^2(1-\alpha)\right) + \delta^3$ because:

	s	$s+1$	$s+2$	$s+3$
action	$\{3\}$	$\{1,3\}$	$\{1,2\}$	$\{2,3\}$
graph	(13)	(13)	(13, 23)	(12, 23, 13)
$u_1(\phi)$	4/3	4/3	$(1-\alpha)$	1

When player 1 chooses $a_1^s = \{2,3\}$, we obtain:

	s	$s+1$	$s+2$	$s+3$
action	$\{2,3\}$	$\{1,3\}$	$\{1,2\}$	$\{2,3\}$
graph	(13)	(12, 13)	(12, 23, 13)	(12, 23, 13)
$u_1(a_1^s, \phi_{-1})$	4/3	$(1+2\alpha)$	1	1
$u_1(\phi)$	4/3	4/3	$(1-\alpha)$	1

this implies that the deviation under analysis is not profitable when

$$(1-\delta)\left(\delta\left(\frac{4}{3}-(1+2\alpha)\right)-\delta^2\alpha\right) > 0. \tag{4.5}$$

There is $\delta_{(14-1)} \in (0,1)$ such that for all $\delta \geq \delta_{(14-1)}$ condition 4.5 holds: Letting the left hand side of condition 4.5 define the continuous function $b_{(14-1)} : \mathbb{R} \rightarrow \mathbb{R}$ we observe that $\frac{1}{1-\delta}b_{(14-1)}(\delta)$ is arbitrarily close to $\frac{1}{3} - 3\alpha$ when δ is sufficiently close to 1. So requiring that $\alpha < \frac{4}{36}$ enables us to guarantee that $b_{(14-1)}(\delta) \geq 0$ for δ sufficiently close to 1.

Considering $a_1^s = \{2\}$ as player 1's deviation we obtain:

	s	$s+1$	$s+2$	$s+3$	$s+4$
action	$\{2\}$	$\{1\}$	$\{1,2\}$	$\{2,3\}$	$\{1,3\}$
graph	$(\emptyset)$	(12)	(12)	(12, 13)	(12, 23, 13)
$u_1(a_1^s, \phi_{-1})$	0	4/3	4/3	$(1+2\alpha)$	1
$u_1(\phi)$	4/3	4/3	$(1-\alpha)$	1	1

The following condition ensures that this deviation is not profitable:

$$(1-\delta)\left(\frac{4}{3}+\delta^2\left((1-\alpha)-\frac{4}{3}\right)+\delta^3(1-(1+2\alpha))\right) > 0. \tag{4.6}$$

There is $\delta_{(14-2)} \in (0,1)$ such that for all $\delta \geq \delta_{(14-2)}$ condition 4.6 holds: Letting the left hand side of condition 4.6 define the continuous function $b_{(14-2)} : \mathbb{R} \rightarrow \mathbb{R}$ we observe that $\frac{1}{1-\delta}b_{(14-2)}(\delta)$ is arbitrarily close to $1 - 3\alpha$ when δ is sufficiently close to 1. So the requirement that $\alpha < \frac{4}{36}$ also enables us to guarantee that $b_{(14-2)}(\delta) \geq 0$ for δ sufficiently close to 1.

Next, playing $a_1^s = \{\emptyset\}$ is not a profitable deviation for any δ because:

	s	$s+1$	$s+2$	$s+3$	$s+4$	$s+5$
action	$\{\emptyset\}$	$\{3\}$	$\{2\}$	$\{2,3\}$	$\{1,3\}$	$\{1,2\}$
graph	$(\emptyset)$	$(\emptyset)$	(23)	(23)	(12, 23)	(12, 23, 13)
$u_1(a_1^s, \phi_{-1})$	0	0	0	0	$(1-\alpha)$	1
$u_1(\phi)$	4/3	4/3	$(1-\alpha)$	1	1	1

Case 15. $a_2^{s-2} = \{\emptyset\}$ and $a_3^{s-1} = \{2\}$.

Conforming yields player 1 a payoff of $(1-\delta)(\delta(1-\alpha)) + \delta^2$

	s	$s+1$	$s+2$
action	$\{2,3\}$	$\{1,3\}$	$\{1,2\}$
graph	$(\emptyset)$	(12, 23)	(12, 23, 13)
$u_1(\phi)$	0	$(1-\alpha)$	1

Considering $a_1^s = \{2\}$ does not bring out a profitable deviation at any δ as a result of:

	s	$s+1$	$s+2$	$s+3$	$s+4$	$s+5$
action	$\{2\}$	$\{3\}$	$\{2\}$	$\{2,3\}$	$\{1,3\}$	$\{1,2\}$
graph	$(\emptyset)$	(23)	(23)	(23)	(12, 23)	(12, 23, 13)
$u_1(a_1^s, \phi_{-1})$	0	0	0	0	$(1-\alpha)$	1
$u_1(\phi)$	0	$(1-\alpha)$	1	1	1	1

A similar conclusion holds for $a_1^s = \{3\}$ regardless of the level of δ:

	s	$s+1$	$s+2$	$s+3$
action	$\{3\}$	$\{1,3\}$	$\{1,2\}$	$\{2,3\}$
graph	$(\emptyset)$	(23)	(13, 23)	(12, 23, 13)
$u_1(a_1^s, \phi_{-1})$	0	0	$(1-\alpha)$	1
$u_1(\phi)$	0	$(1-\alpha)$	1	1

Futhermore, considering to play $a_1^s = \{\emptyset\}$ is also not a profitable deviation for any δ because:

	s	$s+1$	$s+2$	$s+3$	$s+4$	$s+5$
action	$\{\emptyset\}$	$\{3\}$	$\{2\}$	$\{2,3\}$	$\{1,3\}$	$\{1,2\}$
graph	$(\emptyset)$	(23)	(23)	(23)	(12, 23)	(12, 23, 13)
$u_1(a_1^s, \phi_{-1})$	0	0	0	0	$(1-\alpha)$	1
$u_1(\phi)$	0	$(1-\alpha)$	1	1	1	1

Case 16. $a_2^{s-2} = \{\emptyset\}$ and $a_3^{s-1} = \{\emptyset\}$.

Conforming delivers player 1 a payoff of $(1-\delta)(\delta\frac{4}{3} + \delta^2\frac{4}{3} + \delta^3(1+2\alpha)) + \delta^4$ as a result of:

	s	$s+1$	$s+2$	$s+3$	$s+4$
action	$\{2\}$	$\{1\}$	$\{1,2\}$	$\{2,3\}$	$\{1,3\}$
graph	$(\emptyset)$	(12)	(12)	(12, 13)	(12, 23, 13)
$u_1(\phi)$	0	4/3	4/3	$(1+2\alpha)$	1

Considering to play $a_1^s = \{2,3\}$ is not a profitable deviation regardless of the level of δ as it can clearly be observed by the following:

	s	$s+1$	$s+2$	$s+3$	$s+4$
action	$\{2,3\}$	$\{1,3\}$	$\{1,2\}$	$\{2,3\}$	$\{1,3\}$
graph	$(\emptyset)$	(12)	(12, 23, 13)	(12, 23, 13)	(12, 23, 13)
$u_1(a_1^s, \phi_{-1})$	0	4/3	1	1	1
$u_1(\phi)$	0	4/3	4/3	$(1+2\alpha)$	1

Similarly, $a_1^s = \{3\}$ is not a profitable deviation at any δ because:

	s	$s+1$	$s+2$	$s+3$	$s+4$
action	$\{3\}$	$\{1,3\}$	$\{1,2\}$	$\{2,3\}$	$\{1,3\}$
graph	$(\emptyset)$	$(\emptyset)$	(13, 23)	(12, 23, 13)	(12, 23, 13)
$u_1(a_1^s, \phi_{-1})$	0	0	$(1-\alpha)$	1	1
$u_1(\phi)$	0	4/3	4/3	$(1+2\alpha)$	1

Finally,choosing $a_1^s = \{\emptyset\}$ to deviate is not profitable for all δ either due to:

	s	$s+1$	$s+2$	$s+3$	$s+4$	$s+5$
action	$\{\emptyset\}$	$\{3\}$	$\{2\}$	$\{2,3\}$	$\{1,3\}$	$\{1,2\}$
graph	$(\emptyset)$	$(\emptyset)$	(23)	(23)	(12, 23)	(12, 23, 13)
$u_1(a_1^s, \phi_{-1})$	0	0	0	0	$(1-\alpha)$	1
$u_1(\phi)$	0	4/3	4/3	$(1+2\alpha)$	1	1

This finishes the proof. ■

5 CONCLUDING REMARKS

The efficiency and stability conflict is a widely discussed topic in network literature. To summarize, our result establishes that when the payoff structure is as specified our game; i.e. players prefer being part of a two player network most though three player networks generate the greatest value, in the one–shot extensive form game complete graph (12 23 13) turns out to be sustainable as subgame perfect equilibrium as well as trembling–hand perfect equilibrium. We continue our analysis by moving our setting into a various repeated game settings, where players have the chance to condition their strategies according to past behavior. The first version of repeated game is comprised of finite or infinite repetitions of our stage game, i.e. perfect information extensive form game. We have easily showed the complete and efficient graph is sustainable under subgame perfection with "zero–memory" (Markov) strategies that are repetitions of the one–shot SPE strategy. Finally we examine another repeated game formulation; version two. We adapt our model to accommodate the setting of Bhaskar and Vega-Redondo (2002), which give more clear inference when memory considerations are concerned. Our result establishes that we are still able to sustain the desired graph in "zero–memory" (Markov) perfect equilibrium.

Bibliography

Abreu, D., and A. Rubinstein (1988): "The Structure of Nash Equilibrium in Repeated Games with Finite Automata," *Econometrica: Journal of the Econometric Society*, pp. 1259–1281.

Aumann, R. (1981): "Survey of repeated games," *Essays in Game Theory and Mathematical Economics in Honor of Oskar Morgenstern*, 4, 11–42.

Aumann, R., and R. Myerson (1988): "Endogenous Formation of Links between Players and Coalitions: an application of the Shapley value," in *The Shapley Value: Essays in honor of Lloyd Shapley*, ed. by A. E. Roth, pp. 175–191. Cambridge: Cambridge University Press.

Barlo, M., G. Carmona, and H. Sabourian (2009): "Repeated Games with One-Memory," *Journal of Economic Theory*, 144(1), 312–336.

——— (2012): "The Bounded Memory Folk Theorem," Sabanci University.

Bhaskar, V., and F. Vega-Redondo (2002): "Asynchronous Choice and Markov Equilibria," *Journal of Economic Theory*, 103(2), 334–350.

Currarini, S., and M. Morelli (2000): "Network fFormation with Sequential Demands," *Review of Economic Design*, 5(3), 229–249.

Derman, C. (1970): *Finite State Markovian Decision Processes*. Academic Press, Inc.

Dutta, B., and S. Mutuswami (1997): "Stable Networks," *Journal of Economic Theory*, 76(2), 322–344.

Jackson, M. O., and A. Wolinsky (1996): "A Strategic Model of Social and Economic Networks," *Journal of Economic Theory*, 71(1), 44–74.

Kalai, E., and A. Neme (1992): "The Strength of a Little Perfection," *International Journal of Game Theory*, 20, 335–355.