Essays on the Economics of Parking and Mixed Oligopoly

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ABSTRACT

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Keywords: parking; congestion pricing; value of time; mixed duopoly; endogenous timing; information acquisition.

This dissertation comprises of three chapters. In the first chapter, we investigate the existence of self-financing and Pareto-optimal parking pricing scheme by developing a simple static model in which travelers, differing in terms of the value of time, optimally choose between the car and public transit as well as trip duration. To curb congestion, public authorities charge parking fee per unit of time. We derive condition(s) which may ensure the pricing scheme to be both Pareto-optimal and self-financing in the sense that no external funds are required. Numerical results, when the value of time follows some specific rational functional form have also been reported that guarantees the existence of Pareto-optimal and self-financing price scheme. In the second chapter, we contribute to the literature on endogenous timing in a mixed duopoly, where a public firm is competing against a domestic private firm, by exploring the role of information advantage by a firm to act as a market leader in a quantity setting game. We find that under asymmetric information, both type of Stackelberg equilibria with either firm acting as a leader coexist only for the low variance of the demand shock. However, under high variance, only one of the firms acquires costly information which helps it to endogenously act as a market leader. In the third chapter, we allow the public firm to have a foreign-owned private competitor. We find that when the foreign-owned private firm is informed, multiple equilibria co-exist and under high uncertainty only the public firm acquires costly information and endogenously acts as a market leader.

ÖZET

Parklanmanın Ekonomisi ve Karma Oligopol Piyasalar Hakkında Makaleler

Tariq Sultan

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Tez Danışmanı: Doç. Dr. Eren İnci

Anahtar Kelimeler: parklanma; sıkışıklık fiyatlandırması; zaman değeri; karma oligopol; içsel zamanlama; bilgi toplama.

Bu tez üç bölümden oluşmaktadır. İlk bölümde, farklı zaman değerlerine sahip seyahat eden kişilerin bulunduğu ve bu kişilerin araba ve toplu taşıma arasında, bununla birlikte ulaştırma süresi konusunda seçim yaptığı basit bir statik model geliştirerek, kendi kendini finansmanın varlığını ve Pareto-optimal park ücretlendirilmesinin taslağını araştırıyoruz. Modelde, trafik yoğunluğunu azaltmak adına, kamu tarafından zamanla orantılı olarak artan bir park ücreti tahsil edilmektedir. Parklandırma taslağının Pareto-optimal olması ve aynı zamanda kendi kendine finansmanın, dışardan bir finansmana gerek duymadan, var olması için gerekli şartları bu modelden elde ediyoruz. Zaman değerinin spesifik fonksiyonel formları için Pareto-optimal ve kendi kendine finansmanı sağlayan sayısal analiz sonuçları da rapor edilmiştir. İkinci bölümde, bir yerel özel firmaya karşı bir kamu firmasının rekabet ettiği, bir üretim miktarı belirleme oyunundaki firmanın bilgi avantajının bu firmanın pazar lideri olmasındaki rolünü araştırarak, karma duopolideki firmaların içsel zamanlama kararları literatürüne katkı yapıyoruz. Asimetrik bilgi altında, iki firmadan birinin Stackelberg lideri olarak rol alabildiği ardışık hamle dengelerinin ancak küçük derecede talep şoku durumunda var olduğunu görüyoruz. Diğer yandan, talep şokunda yüksek varyans olduğunda, firmalardan sadece birinin maliyetli bilgi elde ettiğini ve bu bilginin o firmanin içsel bir şekilde pazar lideri olarak rol almasına yardım ettiğini görüyoruz. Üçüncü bölümde, kamu firmasının rakibi olan özel firmanın yabancılara ait olmasına izin veriyoruz. Yabancılara ait özel firmanın kamu firmasına kıyasla bilgi avantajı olduğunda, birden fazla denge durumu var olmaktadır, ve yüksek belirsizlik altında, sadece kamu firmasının maliyetli bilgi elde ederek içsel bir şekilde pazar lideri olarak rol aldığını buluyoruz.

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CHAPTER 1

Trip duration, mode choice, and existence of Pareto-optimal and self-financing parking prices

1.1 Introduction

The modern day lifestyle has boosted car ownership rate among the societies. Every car is required to be parked somewhere, at home, at the office, at the shopping mall, or at restaurants. In his landmark book "The High cost of free parking", Shoup (2005) writes that if the whole world achieve the car ownership rate of what the United States had in 2000, there would be 4.7 billion cars, which would require the whole world to provide parking area equivalent to the whole area of England or Greece.[1] In many parts of the world, we hardly pay for parking spaces. Parking is provided freely by the employer if we go for the job, it is provided free when we visit restaurants, and shopping malls also provide free parking (with few exceptions of course) when we go for shopping.

Free provision of parking has resulted in extensive use of automobiles and every vehicle on road contribute to the negative congestion externality on others. With the increase in the ownership of vehicles, lots of problems have emerged in the form of road congestion, pollution, and energy scarcity etc. According to some reasonable estimates, OECD (2014) reports that road transport has contributed nearly $1 trillion in the year 2010 towards the cost of air pollution to OECD member states.[2] There have been many suggestions to get rid of these sort of problems to be incorporated into transport and environmental policies. Out of these many

[1] If we assume that each car requires four parking spaces, then Shoup (2005) claims that it would require the whole world to provide parking space equivalent to that of France or Spain.

[2] Total cost of air pollution was estimated to be $1.7 trillion including both deaths and health effects. Air pollution costed Turkey $38,725 million in the year 2005 and $58,548 million in the year 2010 and caused 28045 and 28924 deaths in years 2005 and 2010, respectively (OECD, 2014).

suggestions, relevant to this study is to devise policies that should curb auto use and to make a significant investment in the public transportation system (e.g, OECD, 2014).

In order to get a remedy to these problems, a significant amount of research has been produced. It cannot be underestimated that how free parking aggravates these problems, in addition, however, it creates other problems like biasedness towards the choice of a car against public transportation.[3] Istanbul is ranked sixth in world cities in terms of worst traffic jams and congestion.[4] Overall almost 49 percent of free flow travel time (uncongested situation) is wasted on roads due to congestion. While extra 63 and 91 percent of free flow travel time is required to travel during the morning peak and evening peak rush hours respectively and much extra time is wasted on the road due to traffic congestion. In principle, politicians and public consider the problem of congestion to be very serious that needs a prompt solution. However, less than the marginal social cost of road pricing has led to inefficient use of different transport modes thus contributing to the problem than remedy.

Parking policy has been used as a demand management policy to combat with problem of congestion and to deal with the unnecessary delays. There has been issues regarding the acceptability of parking policy among public and political elite. Results from several survey studies in the literature across different parts of world show that commuters or road users may accept the congestion pricing in the form of a road or parking toll, if they can be sure about the redistribution of the toll revenue towards improving the road infrastructure, better public transportation and reductions in fuel and car ownership taxes.

With the growth in urbanized population, the demand for public transportation has increased tremendously thus leading to crowdedness in these public services. Contribution towards crowdedness when one gets into the public transit, has the property of reciprocal negative externality, in the same way as in road congestion, therefore it should be priced efficiently. Using a survey data on Paris subway, Prud'homme et al. (2012) estimated that eight percent increase in the passengers per square meters during the period from 2002 to 2007 imposed a welfare loss of at least euro 75 million in the year 2007.[5] Another study based on survey data on Paris subway, by Haywood and Koning (2013) estimated the welfare cost of a passenger that has a seat in the transit. Their finding reveals that under the highly congested situations this cost increase from euro 2.42 to euro 3.69 for such passengers.

But the issue of crowdedness in public services has not gained as much attention

[3]Gillen (1977) shows how parking cost affects the mode choice.

[4]Source: https://www.tomtom.com/en_gb/trafficindex/

[5]In case of Melbourne, Austrailia, Veitch, Partridge and Walker (2013) estimated that in the year 2011, crowding in the city trains has imposed a yearly cost of about 208 million euros.

as the road congestion. While considering the issue of crowding in public transit, de Palma et al. (2015) study the impact of time-varying transit fare to resolve this issue. In their model, commuters have the flexibility to delay or schedule their trips early to avoid crowding discomfort but face a scheduling cost. With this tradeoff, they analyze three fare regimes, no fare, a uniform fare and a train dependent (time-varying) fare and show that crowdedness can never be vanished completely even if fully flexible fare regime is implemented since all transit trains are assumed to have some degree of crowdedness. They find that when crowding cost function is convex and train capacity to carry passengers is fixed, welfare gains from implementing time-varying fare decreases with the increase in the number of commuters. By endogenizing the number and capacity of trains, they find that optimal time-varying fares may result in higher numbers and capacity of trains compared to the uniform form fares. They conclude that existing transit services can be better utilized with congestion pricing. They calibrated their model for Paris RER A line and estimated a welfare gain of euro 0.27 and euro 0.45 per user with implementing optimal uniform fare and optimal time-varying fare, respectively.

Generally, it is thought that congestion pricing, whether in the form of road usage fee or parking fee benefits the richer segment of the society, hence raising questions from an equity point of view. As far as equity is concerned, there is a difference of opinion among economists and planning experts because they divide different groups on the basis of a different set of indicators. Economists, for example, take income as the basis, on the other hand, planning literature defines groups on a broader base such as identifying a disadvantaged group as far the availability of public services is concerned. While formulating a congestion policy, care should be taken to properly address different groups vulnerable to that scheme and local conditions and which aspect of equity is under consideration, may also be kept in mind.[6]

The literature encompasses four type of equity: horizontal equity, vertical equity, the cost principle and the benefit principle. Horizontal equity states that individuals of same classes may be treated alike in terms of tax payments and vertical equity, broadly speaking states that individuals of different groups of the society may be treated differently which provides, among others, one justification for progressive taxes. According to the cost principle of equity, the ones who contribute more to the social cost should bear it and the benefit principle states that who enjoys the social benefit, should pay for it.

In this paper, we study a simple and static bi-modal, single origin-destination model similar to one in Liu et al. (2009) and Nie and Liu (2010). In our model, we allow travelers to optimally choose trip length/duration to maximize their utility

[6] For a detailed discussion on the issue of equity of congestion pricing and to see the main messages of the literature, see for example Ecola and Light (2009).

and to deal with the issue of congestion, local authorities impose a per unit of time parking charge. We believe that charging per unit of time parking fee may provide the authorities with more flexibility to collect and use of revenue and may face less opposition both from the public and political circles in terms of its acceptance. Normally it is hard for the public to accept a congestion charge on the existing lanes than on the new ones. The issue of equity is addressed by redistributing the revenue collected from parking to the ones who have been affected by the imposition of parking charges. We define the system performance in terms of total system travel time and derive the general conditions under which a per unit of time parking prices are both Pareto-improving, in the sense that everyone is better off in terms of travel time and self-financing (no external funds are required). From Pareto-improving or Pareto-optimality, we mean that no single traveler is worse-off in terms of travel time and from self-financing we mean that in order to implement Pareto-optimal parking prices, the govt. subsidizes the public transit in a way that no external funds are required.[7] We further discuss what happens to the existence of Pareto-improving and self-financing per unit of time parking pricing scheme, when the distribution of the value of time takes different functional forms (i.e., concave, linear, rational etc.). We undertake a numerical exercise when the value of time distribution takes the form of a rational function of the first order of different shape and report that such scheme exists.

The organization of this paper is as follows. We review literature in Section 1.2 and discuss model formulation in Section 1.3 and derive the main results in section 1.4. Section 1.5 discusses the numerical exercise and reports the results and Section 1.6 concludes.

1.2 Literature Review

The economic literature on road congestion pricing is not scant but the literature on parking pricing to curb congestion has not gained the deserved attention. To have a glimpse of the literature on the economics of parking see Inci (2015). In this section, we briefly review studies on congestion pricing and their acceptability among politicians and public. Giuliano (1992) highlights the importance of the issue of the traffic congestion and how the U.S transportation policy has incorporated the urgency of the congestion issue. The author identifies the winners and losers segments of the population from the congestion pricing and argues that the list of

[7] Since govt. needs funds to compensate the group of travelers who have been shifted from driving cars to use public transit.

loser group can be minimized or completely eliminated if the toll is applied on the newly added facility rather than applying it to the existing facilities.[8] He goes on further to discuss the potential ways for successful implementation of pricing policy and concludes that priority may be given to raising the public acceptance even if a compromise to some extent over the economic efficiency of the system has to be made. Giuliano (1994) also recognizes the fact that it is difficult to address all the equity concerns of congestion pricing because it is affected by numerous factors. A policy may fulfill the criteria of equity and fairness from one's perspective but it may not be of such standards from the point of view of direct affectees of the pricing scheme.

Using survey data, Ubbels and Verhoef (2006) empirically investigated the hurdles in the acceptance of road pricing in Dutch road users who are usually exposed to congestion. They observed a very low level of acceptance of road pricing at its own but however, acceptance level of pricing policy may rise if it is accompanied by the use of revenue to remove or reduce car ownership tax, gasoline taxes and to improve road networks. The commuters with higher education level and higher value of time found pricing policy to be more justified since this subpopulation receives the highest benefits in terms of time savings. And perception about how a policy can reduce the congestion may affect the acceptance.

While considering heterogeneity in terms of the value of time among road users to evaluate different congestion pricing policies, Small and Yan (2001) show that with the moderate level of heterogeneity, second best pricing policy achieves only 16 to 33 percent (approximately) of welfare level of what is being achieved through first-best pricing scheme. Interestingly, a robust result with respect to the level of heterogeneity reveals that revenue-maximizing produces an outcome in the form of higher prices but achieves lower welfare level compared with what is being attained by second best pricing policy. The authors recognize the importance of heterogeneity of value of time to improve the effectiveness of partial pricing scheme through product differentiation in the periods of high demand and congestion.

Many studies have ignored the important issue of heterogeneity of value of time among the commuters. For example, Adler and Cetin (2001) developed a model with a single origin-destination pair connected via two routes with homogeneous commuters having a constant value of time. They analyzed the congestion pricing schemes on a desirable road(congested) during peak rush hours to divert the commuters to the less desirable road(relatively less congested and have ample capacity). The revenue generated from the tolled road is directly redistributed to the users of the less desirable road which may help eliminate equity concerns over congestion

[8] This is being said keeping in view of the policy failures of 1-10 (Santa Monica Freeway) "Diamond Lane" in Los Angeles and Boston's Southeast Expressway HOV lane.

pricing among the public. In this way, they show that travel costs for everyone declines and the system can get rid of waiting time in the lines on roads.

To deal with the road congestion, Glazer and Niskanen (1992) analyze the parking prices, when the road usage is underpriced or not priced at all. When roads are not properly priced, they argue that a lump sum parking fee may improve welfare while hourly or per unit of time parking fee can not. This is because, in their model, the increase in per unit of time parking fee induces parkers to park for shorter duration and leave the parking spaces more frequently thus inviting others to park and hence contributing towards the higher level of congestion. When roads are efficiently priced, they show that marginal cost parking price (per unit of time) is optimal and their models yield no lump sum parking fee. Their model ignores the heterogeneity among consumers in terms of the value of time which is very important when evaluating the parking prices from an equity perspective. Since the value of time-saving from reduced congestion is of more worth to rich than the less privileged segment of society, so any policy aimed at reducing congestion favors those who have the high value of time. Therefore the distribution of proceeds from parking charges or road tolls has significant importance as suggested in many studies, some have been cited above. De Borger and Russo (2017) show that how local retailers lobby the city governments to influence the parking pricing policies to remain below the efficient pricing.

Arnott et al. (1994) argue that congestion (in the form of queuing) on roads can be eliminated if a proper toll scheme depending on time is implemented. They also shed light on the importance of the distribution of the toll revenues and they claim that if toll revenue is not redistributed then its benefits are regressive that benefits only rich segment of the society.

Nourinejad and Roorda (2017) argue that hourly parking fee does not always increase demand, in fact, it can decrease or increase the parking demand depending upon the dwell time elasticity with respect to parking price per unit of time. Demand increases when such elasticity is high, it may increase or decrease when dwell time is inelastic. The authors recommend that a wise parking policy should incorporate dwell time elasticity since the nature of parking pricing is of fundamentally different to the road pricing because the later always dampens traffic demand when it is raised.

The self-financing principle of Mohring and Harwitz (1962) has a great importance in the literature of transportation economics. Using several assumptions[9], they show that road capacity costs can be just recovered from toll charges (i.e. self-

[9] They use a single origin-destination pair linked with one route, road users are homogeneous in terms of the value of time, toll charges and capacity are choice variables, capacity cost function as linear and their congestion function possess homogeneous of degree zero property.

financing roads) and it can be welfare improving. To see the robustness of their results by relaxing different assumptions implied in Mohring and Harwitz (1962), a lot of literature on self-financing emerged after.[10] Verhoef and Mohring (2009) review the follow-up literature on self-financing roads that investigate the relationship between proceeds from road toll and costs associated with roads. They also provide guidance for the social planner when considering the self-financing principle of Mohring and Harwitz (1962) to device toll policy. On the basis of numerical results reported in the paper, they argue that while interpreting this principle, mixing up of capital costs with investment costs and imposing balanced budget restrictions when networks are operating under second-best conditions, may result in welfare losses.

Our study is closely related to Liu et al. (2009) and Nie and Liu (2010). Liu et al. (2009), while considering two modes (car and public transit) and a single origin and destination pair derived the conditions for the existence of Pareto-improving and revenue-neutral congestion pricing schemes. Revenue neutral in the sense that tolling authority uses the proceeds from the tolls on roads collected as a fixed charge per user to subsidize the public transit users as a lump sum while keeping in view the objective of reducing congestion in an equitable way. While deriving the above said conditions, they took into account the general distribution of the value of time among the travelers' population. But for a uniform distribution of the value of the time, they show that Pareto-improving and revenue-neutral congestion pricing schemes always exists for any target level of road users that improves the system performance in terms of total system travel time reduction. Since the value of time of the person indifferent between the two modes is critical, they show that a higher value of time for this commuter, representing higher inequality, is useful to resolve the issue of inequity. On the other hand, Nie and Liu (2010) by using the same model settings as in Liu et al. (2009), derived the conditions for the existence of self-financing and Pareto-improving congestion tolling scheme while abandoning the requirement of revenue neutrality. They show that Liu et al. (2009) result is highly dependent on the shape of the distribution of the value of time and with a general type of distribution it may just not be possible for a pricing scheme to be Pareto-optimal without external funds requirement. They revealed that Liu et al. condition is satisfied as long as the distribution is concave in nature. In this paper, the existence conditions are also derived when the value of time distribution

[10] For example, Strotz (1964) relaxed the homogeniety of the road users in terms of value of time. Arnott and Kraus (1995) and Yang and Meng (2002) extended the basic set up of single link of Mohring and Harwitz (1962) to multiple links. Oum and Zhang (1990) uses the more realistic nature of road capacity being discrete and Small (1999) departed from perfect competition environment. Kraus (1981) studied if the assumptions of linearity of capacity costs and the homogeniety of degree zero of congestion fuctions are relaxed and Mouche et al.(2007) prvodied a well defined mathematical proof of the self-financing rule, are the few but not all papers to study self-financing principle.

is of first-order rational function type. They conclude that a toll scheme is Pareto-improving only if it is revenue neutral or it is revenue maximizing. Here in this paper, we derive the same existence condition as they do but we differ in two aspects. In our analysis, toll authority charges a parking fee per unit of time to suppress car use and ultimately to curb congestion and second, we allow the travelers to choose the optimal trip duration. This is important because charging per unit of time parking fee may force individuals to reduce their trip durations and hence we need to see whether Pareto-optimal and self-financing scheme still exists when by taking this into account.

1.3 The Model

We provide a simple model in which travelers make a trip to downtown from a fixed origin located in suburbs.[11] The number of trips (total demand) are represented by D, which we assume to be fixed exogenously. Individuals optimally choose trip length (duration) and the mode of transportation between the car and public transit (train or buses specified to run on separate lanes). An individual going to his destination by car maximizes the following net utility function:

$$S(l) = V(l) - \theta(N)\left[t_c(N_c) + c(l) + l\right] - F_c \tag{1}$$

We now discuss the elements of the $S(l)$ above one by one. $V(l)$ is the gross utility obtained from spending l (trip length of duration) units of time on shopping or recreation and we assume that $V(l)$ exhibit standard properties. It is strictly concave and increasing in its argument l such that there exists a unique solution to the above problem. $\theta(N)$ is value of time distribution function which we assume is a continuous function and is different for different individuals. Let $G(\theta)$ be the distribution of θ among individuals. $G(\theta_k)$ is defined as number of individuals for which value of time $\theta \geq \theta_k$ which ultimately leads to $G(\theta_{\min}) = D$ and $G(\theta_{\max}) = 0$. $t_c(N_c)$ is the travel time on road when an individual decides to go by car. It is a strictly increasing and strictly convex function of number of car drivers on the road. $c(l)$ is the time spent on searching for a parking lot at the destination which we assume is a positive function of trip length. $\theta(N)(t_c(N_c) + c(l))$ represents the value of in-vehicle time and we also include the term l to represent the opportunity

[11] Since our focus in this research is to address the social inequity issues and we ignore the spatial inequity issues.

cost or time cost of the trip.[12] Finally the term F_c represents the operating cost of vehicle/car which may include for example fuel cost, insurance cost and wear and tear cost. The first-order condition of equation (1) with respect to l gives its unique value at which objective function of car drivers is maximized and we represent it by $l^*_{c/nt}$, where the subscripts c and nt represents car users and no toll (without any parking charges) respectively. $V\left(l^*_{c/nt}\right)$ is large enough such that the optimal $S\left(l^*_{c/nt}\right) \geq 0$, otherwise individual prefers outside option of not to make the trip at all. Hence, we have:

$$S\left(l^*_{c/nt}\right) = V\left(l^*_{c/nt}\right) - \theta\left(N\right)\left[t_c\left(N_c\right) + c\left(l^*_{c/nt}\right) + l^*_{c/nt}\right] - F_c \geq 0 \tag{2}$$

In the same fashion, an individual going to his destination by using public transit maximizes the following net utility function $S(l)$:

$$S(l) = V\left(l\right) - \theta\left(N\right)\left[t_b + l\right] - F_b \tag{3}$$

where, $V\left(l\right), \theta\left(N\right)$, and l are the same as discussed above and t_b is the time spent in public transit service en route to the destination. We assume that t_b is fixed exogenously and transit service has enough capacity without creating potential congestion[13] and is operating at constant returns to scale technology. Further, it is assumed that $t_b > t_c\left(0\right)$ which means transit time is strictly longer than free flow travel time by car. F_b represents fixed cost associated with travel by public transit which potentially may include the bus fare.

Let's represents the solution to the first-order condition of (3) with respect to l as $l^*_{b/nt}$, where the subscripts 'b' and 'nt' represents transit(bus) users and no toll respectively. And accordingly, we have the following optimized version of the objective function $S\left(l^*_{b/nt}\right) \geq 0$:

$$S\left(l^*_{b/nt}\right) = V\left(l^*_{b/nt}\right) - \theta\left(N\right)\left[t_b + l^*_{b/nt}\right] - F_b \geq 0 \tag{4}$$

By equating equations (2) and (4), we get the individual indifferent between using either of the mode.

$$V\left(l^*_{b/nt}\right) - V\left(l^*_{c/nt}\right) - \theta_e\left[l^*_{b/nt} - l^*_{c/nt}\right] + \Delta F = \theta_e\left[t_b - t_c\left(N_e\right) - c\left(l^*_{c/nt}\right)\right] \tag{5}$$

[12] $\theta\left(N\right)l$ represents the opportunity cost. For justification of this term see for example Arnott and Inci (2006).

[13] To have a glimpse of the literature on cost of crowding in public transit and its remedies see, for example, Prud'homme et al. (2012) and de Palma et al. (2015)

where N_e represents the number of car users in the absence of any toll scheme, $\theta_e = \theta(N_e)$ is the value of time of the indifferent user and $\Delta F = F_c - F_b > 0$ shows the difference between the operating costs of two modes which is assumed to be positive because owning a car is expensive. The above condition is just like Wardrop's first principle, which states that travelers continue to make choices between car and the public transit until neither travel mode becomes strictly better than the other.

Now, we derive the indifference condition when a public authority imposes a parking charge per unit of time and charges/subsides the public transit. In the similar fashion, we write the optimized objective function for the car drivers as follows:

$$S\left(l^{\circ}_{c/t}\right) = V\left(l^{\circ}_{c/t}\right) - \theta(N)\left[t_c(N_c) + c\left(l^{\circ}_{c/t}\right) + l^{\circ}_{c/t}\right] - f_c\left(l^{\circ}_{c/t}\right) - F_c \geq 0, \quad (6)$$

where $l^{\circ}_{c/t}$ is the optimal level of trip length chosen via optimization, where the subscripts ct and t represents car users and toll respectively and $f_c > 0$ is the parking fee (toll) per unit of time. Similarly the optimized objective function for transit (bus) riders after imposition of a toll scheme is:

$$S\left(l^{\circ}_{b/t}\right) = V\left(l^{\circ}_{b/t}\right) - \theta(N)\left[t_b + l^{\circ}_{b/t}\right] - f_b - F_b \geq 0, \quad (7)$$

where $l^{\circ}_{b/t}$ is the optimal level of trip length chosen via optimization process. Since we are interested in Pareto-optimal toll scheme, f_b may potentially be the transit subsidy (i.e. $f_b \leq 0$). Now equating equations (6) and (7) yields the indifference equilibrium condition once after toll scheme is imposed:

$$V\left(l^{\circ}_{b/t}\right) - V\left(l^{\circ}_{c/t}\right) - \theta_p\left(l^{\circ}_{b/t} - l^{\circ}_{c/t}\right) + f_c\left(l^{\circ}_{c/t}\right) - f_b + \Delta F = \theta_p\left[t_b - t_c(N_p) - c\left(l^{\circ}_{c/t}\right)\right] \quad (8)$$

where N_p is the number of car users after a parking charge (toll) is imposed and $\theta_p = \theta(N_p)$ is the value of time of the $N_p th$ individual.

Since our focus is to evaluate the toll scheme from equity point of view keeping in mind the heterogeneity of the individuals in terms of value of time, there are three groups of individuals after the imposition of toll.

1.3.1 Transit users

Those who use public transit(bus) before and after the toll, their optimal utility changes by only f_b. Pareto-optimality requires $f_b \leq 0$. Thus revenue neutrality or self-financing requires $f_c \geq 0$.

$$f_b \leq 0 \quad \& \quad f_c \geq 0 \tag{9}$$

1.3.2 Tolled off from car

For this group of individuals who have been tolled off the car(road) ($N_p \leq N \leq N_e$), the change in their utility $\Delta V(N)$ is as follows:

$$\Delta V(N) = V\left(l^\circ_{b/t}\right) - V\left(l^*_{c/nt}\right) - \theta(N)\left[t_b - t_c(N_e) - c\left(l^*_{c/nt}\right) - l^*_{c/nt} + l^\circ_{b/t}\right] - f_b + \Delta F \tag{10}$$

Among this group, individual with value of time θ_p, who is indifferent between using car and transit after the toll is being imposed suffers the most. So if this person receives enough subsidy such that $\Delta V(N_p) \geq 0$, then all of this group may be better off. By using equation (5) and with some algebraic manipulation in the above equation, we get:

$$f_b \leq (V l^\circ_{b/t}) - V(l^*_{b/nt}) + (\theta_e - \theta_p)[t_b - t_c(N_e) - c(l^*_{c/nt})] - \theta_p(l^\circ_{b/t} - l^*_{c/nt}) + \theta_e(l^*_{b/nt} - l^*_{c/nt}) < 0 \tag{11}$$

1.3.3 Car users

This group of individuals($N \leq N_p$) remain as car users before and after the imposition of toll, the change in their utility $\Delta V(N)$ can be written as:

$$\Delta V(N) = V(l^\circ_{c/t}) - V(l^*_{c/nt}) + \theta_p[t_c(N_e) - t_c(N_p) + c(l^*_{c/nt}) - c(l^\circ_{c/t}) + l^*_{c/nt} - l^\circ_{c/t}] - f_c(l^\circ_{c/t}) \tag{12}$$

Among this group, individual with value of time θ_p, receives the least benefit and Pareto-optimality require that $\Delta V(N_p) \geq 0$, hence we have :

$$f_c \overset{\circ}{l}_{c/t} \leq V\left(l^\circ_{c/t}\right) - V\left(l^*_{c/nt}\right) + \theta_p\left[t_c(N_e) - t_c(N_p) + c\left(l^*_{c/nt}\right) - c\left(l^\circ_{c/t}\right) + l^*_{c/nt} - l^\circ_{c/t}\right] \tag{13}$$

Proposition 1 *A parking price scheme makes everyone better off iff followings are met:*

1. $f_b \leq V(l^\circ_{b/t}) - V(l^*_{b/nt}) - (\theta_e - \theta_p)[t_b - t_c(N_e) - c\left(l^*_{c/nt}\right)] - \theta_p(l^\circ_{b/t} - l^*_{c/nt}) + \theta_e(l^*_{b/nt} - l^*_{c/nt})$

2. $f_c \overset{\circ}{l}_{c/t} \leq V(l^{\circ}_{c/t}) - V(l^{*}_{c/nt}) + \theta_p[t_c(N_e) - t_c(N_p) + c(l^{*}_{c/nt}) - c(l^{\circ}_{c/t}) + l^{*}_{c/nt} - l^{\circ}_{c/t}]$

1.4 Revenue-neutral or self-financing Toll System

Now define the maximum revenue $I_{\max}$ that can be generated from the toll scheme as follows:

$$I_{\max} = f_c\left(l^{\circ}_{c/t}\right) N_p + f_b\left(D - N_p\right)$$

where N_p is number of car users and $D - N_p$ is number of transit users after the toll is being imposed. Using equations (9),(11) and (13), $I_{\max}$ can be written as:

$$\begin{aligned} I_{\max} = {} & N_p[V\left(l^{\circ}_{c/t}\right) - V\left(l^{*}_{c/nt}\right) + \theta_p(t_c(N_e) - t_c(N_p) + c(l^{*}_{c/nt}) - c(l^{\circ}_{c/t}) + l^{*}_{c/nt} - l^{\circ}_{c/t})] \\ & + (D - N_p)[V(l^{\circ}_{b/t}) - V(l^{*}_{b/nt}) + (\theta_e - \theta_p)(t_b - t_c(N_e) - c(l^{*}_{c/nt}))] \\ & + (D - N_p)[-\theta_p(l^{\circ}_{b/t} - l^{*}_{c/nt}) + \theta_e(l^{*}_{b/nt} - l^{*}_{c/nt})] \end{aligned} \tag{14}$$

Definition 1 (Pareto-optimal parking prices) *A parking price scheme is Pareto-optimal, or alternatively, it is Pareto-improving if all of the travelers are better-off and no single traveler is worse-off in terms of travel time.*

Definition 2 (self-financing parking prices) *A parking price scheme is self-financing in a sense that the govt. does not require any external funds in order to implement Pareto-optimal parking prices.*

As prevalent in the transportation literature, we define the system performance in terms of total travel time, as follows:

$$T_{nt} = N_e\left(t_c\left(N_e\right) + c\left(l^{*}_{c/nt}\right)\right) + \left(D - N_e\right) t_b, \tag{15}$$

$$T_t = N_p\left(t_c\left(N_p\right) + c\left(l^{\circ}_{c/t}\right)\right) + \left(D - N_p\right) t_b, \tag{16}$$

where T_{nt} and T_t represent the total system travel time without toll and toll respectively and the above two equations are self explanatory. Using information in equations (5), (9), (11), and (13) into (14) and using the above two equations (15) and (16) along with some algebraic manipulation, we get the following equation for maximum revenue:

$$
\begin{aligned}
I_{\max} = &-\theta_p T_t + \theta_p T_{nt} + \underline{N_p(V(l^{\circ}_{c/t}) - V(l^{*}_{c/nt})) - \theta_p N_p(l^{\circ}_{c/t} - l^{*}_{c/nt})} \\
&+ \underline{\left((D - N_p) - \frac{\theta_p (D - N_e)}{\theta_e}\right)(V(l^{\circ}_{b/t}) - V(l^{*}_{c/nt})) - \theta_p(N_e - N_p)(l^{\circ}_{b/t} - l^{*}_{c/nt})} \\
&+ \underline{\frac{\theta_p (D - N_e)}{\theta_e}(V(l^{\circ}_{b/t}) - V(l^{*}_{b/nt})) - \theta_p(D - N_e)(l^{\circ}_{b/t} - l^{*}_{b/nt})} \\
&+ \left((D - N_p) - \frac{\theta_p (D - N_e)}{\theta_e}\right)\Delta F
\end{aligned} \tag{17}
$$

The first underlined term in the above equation represents the gross utility gain/loss or alternatively recreational gain/loss of car users adjusted with opportunity cost without taking into account the time saving due to imposition of parking charges as it is already captured by the term T_t. The second underlined term shows the gross utility gain/loss of those who have been tolled off from using car adjusted with opportunity cost and the third underlined term represents the utility gain/loss of transit users adjusted with their respective opportunity cost. since self-financing requires $I_{\max} \geq 0$, so we have the following condition:

$$
\begin{aligned}
T_t \leq\ & T_{nt} + \Delta F\left(\frac{D - N_p}{\theta_p} - \frac{(D - N_e)}{\theta_e}\right) \\
&+ \frac{N_p}{\theta_p}\left(V\left(l^{\circ}_{c/t}\right) - V\left(l^{*}_{c/nt}\right)\right) - N_p\left(l^{\circ}_{c/t} - l^{*}_{c/nt}\right) \\
&+ \left(\frac{D - N_p}{\theta_p} - \frac{D - N_e}{\theta_e}\right)\left(V\left(l^{\circ}_{b/t}\right) - V\left(l^{*}_{c/nt}\right)\right) - (N_e - N_p)\left(l^{\circ}_{b/t} - l^{*}_{c/nt}\right) \\
&+ \frac{(D - N_e)}{\theta_e}\left(V\left(l^{\circ}_{b/t}\right) - V\left(l^{*}_{b/nt}\right)\right) - (D - N_e)\left(l^{\circ}_{b/t} - l^{*}_{b/nt}\right)
\end{aligned} \tag{18}
$$

In order to economize on notations, we will just write third term on the right hand side of the above inequality as $\Delta V_{(\text{car users})}$(utility gain/loss of car users adjusted with trip length) and forth term as $\Delta V_{(\text{tolled off from car})}$ and the last term as $\Delta V_{(\text{bus users})}$. Hence we, record above discussion as the following Proposition.

Proposition 2 (Self-finance and Pareto-optimal parking prices) *For a given targeted number of car users N_p(after imposing parking charge), parking pricing scheme always be both self-financing and Pareto-optimal if the following condition is satisfied:*

$$
T_t \leq T_{nt} + \Delta F\left(\frac{D - N_p}{\theta_p} - \frac{(D - N_e)}{\theta_e}\right) + \Delta V_{(car\ users)} + \Delta V_{(tolled\ off\ from\ car)} + \Delta V_{(bus\ users)}
$$

The above condition is similar to one derived in Nie and Liu (2010) but it has three additional terms since we allow travelers to decide on the duration of their trips as well. To see the signs of the additional terms on the right-hand side for when $F(\theta)$ is a general concave function needs a rigorous proof, which is beyond the scope of the present study. We may differ with Nie and Liu (2010) if the sign comes out to be negative which may be interesting and may indicate that a Pareto-optimal and self-financing parking pricing scheme may not exist without external funds or subsidy. However, we provide some numerical results when the value of time distribution $F(\theta)$ follows a first order rational function and our results ensure the existence of Pareto-improving and self-financing parking pricing scheme and our results are in line with the one reported in Nie and Liu (2010). Since we have allowed travelers to optimally decide on trip duration, we expect that charging parking fee per unit of time may be more efficient and can resolve the equity issues more effectively if not completely. It may also provide the authorities with more flexibility to collect and use of revenue and may face less opposition both from the public and political parties in terms of its acceptance.

1.5 Numerical Exercise

In this section, we report some results from numerical exercise to see the existence of a Pareto-optimal and self-financing parking pricing scheme when value of time distribution follows a first order rational function of the form $F(\theta) = D(\theta_{\max} - \theta)/\sigma\theta + \theta_{\max}$ as used in Nie and Liu (2010), where θ is value of time, $\theta_{\max}$ is the maximum value it can take, D is total demand and σ is a parameter. We adopted some values from Nie and Liu (2010) such as $\Delta F = 4$, $D = 1000$, $\theta_{\max} = 40$, $t_b = 1.5$,$t_c(N_c) = 0.75\left(1 + 0.15\,(N_c/500)^5\right)$ to make the results comparable. We employ utility function as $V(l) = 50\ln(1+l)$, where number 50 is used to make sure that benefits are larger than costs, cruising for parking function as $c(l) = l/12$.[14] In the absence of any parking charges, the car user and bus users optimize the equations (19) and (20) respectively, by choosing on trip length l.

$$(50)\ln(1+l) - \theta\left(t_c(N_c) + \left(\frac{1}{12}\right)l + l\right) - F_c \tag{19}$$

$$(50)\ln(1+l) - \theta(t_b + l) - F_b \tag{20}$$

And in the presence of parking charges, the car users and transit users optimize the following functions as given in equations (21) and (22) respectively by choosing on trip length l. We assume parking fee to be $f_c = \theta/5$ to make the numerical results

[14] We use cruising function to be linear just for tractability and convenience.

tractable.

$$(50)\ln(1+l)-\theta\left(t_c(N_c)+\left(\frac{1}{12}\right)l+l\right)-l.f_c-F_c \tag{21}$$

$$(50)\ln(1+l)-\theta(t_b+l)-F_b-f_b \tag{22}$$

With $\rho = 1.5$ and using all the above information into equation (5), we get the value of time of indifferent user to be $\theta_e = 11.218$ and accordingly $N_e = 506.48$ are the car users in the absence of any parking charges, so individuals with value of time $\theta \geq 11.218$ will use cars. The system's total travel time as defined above in equation (15) when there are 506.48 numbers of car users, is calculated to be $T_{nt} = 1312.4$. Then we find the range of the road users in which $T_t < T_{nt}$ as $(281.57, 540.68)$ and their corresponding range of value of time as $(10.145, 20.204)$. And lastly, we see whether in this range, the condition reported in Proposition 2 is fulfilled or not to ensure the existence of self-financing and Pareto-optimal pricing scheme.

The table (1) below presents the values of different terms on right hand side of the condition stated in Proposition 2 corresponding to different values of θ in the range, where $T_t < T_{nt}$, $(10.145, 20.204)$ or accordingly the range of car users $(281.57, 540.68)$. By comparing the values in columns (3) and (4), we can observe that for values of θ in the range[15] $(13-14, 20.204)$ or for the range of number of car users between $(281.57, 453.78-426.23)$, Proposition 2 is satisfied which guarantees that for this user flow (number of car users) Pareto-optimal and self-financing parking pricing scheme exists. Similarly the results for the rational function with values of $\rho = 3.1$ and $\rho = 0.8$ are reported below in tables (2) and (3) respectively. For rational function with $\rho = 3.1$, the value of time of indifferent user is found to be $\theta_e = 10.159$ and accordingly $N_e = 417.40$ are the car users in absence of parking charges, so individual with value of time $\theta \geq 10.159$ will use cars. And by looking at table (2), we can see that for values of θ in the range $(12-13, 15.629)$ or for the range of number of car users between $(275.53, 362.69-336.24)$, Proposition 2 is satisfied which guarantees that for this user flow (number of car or road users) Pareto-optimal and self-financing parking pricing scheme exists.[16] For rational function with $\rho \geq 3.1$, Nie and Liu (2010) concluded that Pareto-optimal and self-financing scheme does not exists when toll authority charges a lump-sum congestion charge. However our numerical results show that such scheme exists for $\rho = 3.1$ when parking charges are imposed per unit of time as a means to curb congestion. For rational function with $\rho = 0.8$, the value of time of indifferent user is found to be $\theta_e = 12.38$ and accordingly $N_e = 553.46$ are the car users in absence

[15] Exact number for lower limit needs to find by using some sophisticated software like Matlab.

[16] Exact number for lower limit needs to find by using some sophisticated software like Matlab.

of parking charges, so individual with value of time $\theta \geq 12.38$ will use cars. When we examine table (3), we observe that for values of θ in the range, where $T_t < T_{nt}$ is $(11.56, 24.120)$ or for the range of number of car users between $(267.81, 577.28)$, Proposition 2 is satisfied but our relevant feasible range is for values of $\theta \geq 12.38$. Hence we can conclude that for values of θ in the range $(12.38, 24.120)$ and accordingly the number of car users between $(267.81, 553.46)$ existence of Pareto-optimal and self-financing parking pricing scheme is guaranteed.

Table 1: $\rho = 1.5$

(1) value of time (θ)	(2) user flow (N_p)	(3) T_t	(4) RHS (5+6+7+8+9)	(5) T_{nt}	(6) $\Delta F()$	(7) ΔV (car users)	(8) ΔV (tolled off from car)	(9) ΔV (transit users)
10.15	540.68	1312.4	1310	1312.4	5.13	−35.48	39.42	−11.45
11	513.27	1289.6	1261.5	1312.4	1.02	−55.70	4.20	−0.43
12	482.76	1273.9	1244.6	1312.4	−3.56	−57.53	−1.81	−4.88
12.73	461.49	1267.9	1251.1	1312.4	−6.75	-50.02	12.29	−16.84
13	453.78	1266.6	1256.5	1312.4	−7.91	-46.01	20.79	−22.77
14	426.23	1265.2	1287.3	1312.4	−12.04	-27.68	64.86	−50.20
15	400	1268	1330.6	1312.4	−15.79	-6.52	125.18	−84.46
16	375	1273.6	1382	1312.4	−19.72	15.07	197.88	−123.61
17	351.15	1281.2	1411	1312.4	−23.30	35.65	279.99	−166.24
18	328.36	1290.1	1498	1312.4	−26.72	54.40	369.26	−211.24
19	306.57	1299.9	1559.1	1312.4	−30	70.89	463.92	−211.33
20	285.71	1310.2	1620.8	1312.4	−33.12	84.90	562.60	−258.11
20.20	281.57	1312.4	1633.3	1312.4	−33.74	87.44	583.12	−306

Table 2: $\rho = 3.1$

(1) value of time (θ)	(2) user flow (N_p)	(3) T_t	(4) RHS (5+6+7+8+9)	(5) T_{nt}	(6) ΔF ()	(7) ΔV (car users)	(8) ΔV (tolled off from car)	(9) ΔV (transit users)
8.41	478.19	1329.2	1454.9	1329.2	18.85	12.82	148.73	−54.69
9	456.55	1316.8	1364.9	1329.2	12.14	−24.70	70.17	−21.91
10	422.54	1305.6	1283.4	1329.2	1.59	−51.20	4.25	−0.36
10.16	417.40	1304.6	1276.7	1329.2	0.007	−52.51	0.000002	0.0
11	391.36	1302.3	1261.2	1329.2	−8.07	51.02	−0.11	−8.83
11.84	367.21	1303.5	1271.6	1329.2	−15.56	−40.28	30.18	−31.86
12	362.69	1304.1	1276.1	1329.2	−16.96	−37.52	38.93	−37.65
13	336.24	1309.0	1314.5	1329.2	−25.16	−18.19	108.94	−80.43
14	311.75	1315.8	1367.8	1329.2	−32.75	3.06	201.18	−132.88
15	289.02	1323.8	1422.5	1329.2	−39.80	23.71	309.36	−199.99
15.63	275.53	1329.2	1473.0	1329.2	−42.98	35.87	383.46	−231.62

Table 3: $\rho = 0.8$

(1) value of time (θ)	(2) user flow (N_p)	(3) T_t	(4) RHS (5+6+7+8+9)	(5) T_{nt}	(6) ΔF ()	(7) ΔV (car users)	(8) ΔV (tolled off from car)	(9) ΔV (transit users)
11.57	577.28	945.79	1283.9	1314.2	1.90	−45.55	17.68	−4.26
12	564.52	953.22	1266.8	1314.2	0.88	−53.44	6.07	−0.86
13	535.71	973.12	1249.9	1314.2	−1.42	−58.11	−2.65	−2.12
14	507.81	995.82	1256.2	1314.2	−3.65	−50.11	9.11	−13.09
15	480.77	1020.3	1278.5	1314.2	−5.82	−35.54	36.83	−31.20
16	454.55	1045.8	1311.5	1314.2	−7.91	−17.26	77.11	−54.57
17	429.1	1071.8	1352.0	1314.2	−9.95	2.25	127.34	−81.82
18	404.41	1097.7	1397.4	1314.2	−11.92	21.55	185.5	−111.94
19	380.43	1123.3	1445.9	1314.2	−13.84	39.73	250	−144.16
20	357.14	1148.4	1496.3	1314.2	−15.71	56.21	319.55	−177.91
21	334.51	1172.8	1547.8	1314.2	−17.52	70.66	393.15	−212.74
22	312.5	1196.4	1599.5	1314.2	−19.28	82.93	469.95	−248.32
23	291.10	1219.2	1651.	1314.2	−20.99	92.94	549.28	−284.36
24	270.27	1241.1	1702.1	1314.2	−22.66	100.72	630.58	−320.67
24.12	267.81	1243.7	1708.3	1314.2	−22.85	101.5	640.45	−325.03

1.6 Conclusion

In this chapter, we investigate the existence of self-financing and Pareto-optimal parking pricing scheme by developing a simple static model. We consider an environment in which heterogenous travelers, in terms of the value of time, optimally choose the trip duration and simultaneously choose between different modes of transportation such as car and public transit in order to maximize their respective utilities. Public authorities charge parking fee per unit of time to curb congestion. While taking into account the issue of equity, we derive condition(s) which may ensure the pricing scheme to be both Pareto-optimal (no one is worse off in terms of travel time) and self-financing in the sense that no external funds are required. Then, we see what happens when the value of time distribution takes different functional form.

We numerically check the existence of such pricing scheme when the value of time distribution follows some specific rational functional forms. Nie and Liu (2010) show that for some specific functional forms such scheme does not exist without external subsidy but for the same functional form, we show that such scheme may exist. In this paper, we assume that all travelers start their journey from the same origin, relaxing this assumption and taking the spatial inequity issues into account may reveal some interesting results into the analysis which are beyond the scope of the current study. Equity issues cannot be solved completely if some travelers are residing in an area where they are lacking in availability of public transit services. We also assume here that transit services are not going to get crowded but incorporating crowding cost similar to one in de Palma et al. (2015), into the objective function of transit users may give some more insight to the present analysis. And we expect relaxing the assumption of fixed demand may challenge the results presented here.

CHAPTER 2

Information acquisition and endogenous timing in a mixed duopoly under uncertainty

2.1 Introduction

In many countries, the presence of publicly owned firms while competing against private firms can be observed in many industries like for example health, education, telecommunication, insurance, banking, postal services and transport among others. The literature recognizes this kind of market structure as the mixed oligopoly. The research in mixed oligopoly gained momentum in the past decade, although the literature on the subject is not new.[17] Historically public firms have enjoyed a monopoly in certain sectors in many countries but with the passage of time competition has increased with the participation in the form domestic private firms and foreign-owned private firms. At the initial stages of market entry by private firms,

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[17]Merrill and Schneider (1966) highlight the importance of ownership structure and conclude that the presence of publicly owned firms in an oligopolistic market structure is useful in improving the performance of the market in terms of lower prices and higher production levels. Anderson et al. (1997) analyze the implication of privatization and they found that privatization of public firm leads to higher prices in the short run thus harming the consumers. However, it will benefit consumers via increased varieties introduced by new entrants in the market because privatizing public firm is like removing the barrier to the market entry. Cremer, Marchand, and Thisse (1989) study how the presence of public firm while competing against private firms affects the performance of the oligopolistic market. They addressed the questions like whether total surplus increases more when one of the existsing private firms is converted to public or it increases more when a new public enterprise is created. They show that when the public firm pays a small premium to its workers over the workers in private firms, converting one private firm to public firm in the oligopolistic market improves total welfare in a second-best way. However, if there are already many public firms in the industry, then privatizing all but one, can be best in terms of improving total welfare. But when the public firm pays a large premium to its workers per unit of output, total surplus increases by converting all private firms to the public firms. Further, they claim that changing the status of existing private firms to a public firm is always superior to creating a new public enterprise.

public firms have owned a larger market share and thus enjoyed the first mover advantage (market leadership) and assuming exogenously the role of public firms as Stackelberg leader was reasonable. But now since public firms face significant competitive pressure from private or foreign-owned private firms, the order of firms moves is more of an endogenous in nature.[18]

The endogenous determination of simultaneous versus sequential moves in oligopolistic market structure got popularity since the seminal work of Hamilton and Slutsky (1990). The endogenous order of moves in mixed oligopoly was first studied by Pal (1998). In an environment, where uncertainty looms regarding market demand, firms choices to become market leader or follower in making strategic decisions are endogenous in nature. And further, firm's motivation to acquire costly information regarding market demand conditions largely depends upon whether a firm is enjoying a leadership position in a market or it is acting as a follower. For example, Raju and Roy (2000) found that information has a great value under high uncertainty and in more competitive industry. Moreover information is of great benefit to the firm acting as a market leader and competing in a Stackelberg fashion than competing in a Bertrand way, however, they assume firm's strategic position as market leader to be exogenous.

In this paper, we contribute to the literature on endogenous timing decisions of firms in mixed duopoly by exploring the role of information advantage by a firm (an early information of uncertain market demand) to endogenously become a market leader in a quantity setting game.[19] We consider a market of homogeneous goods, where a publicly owned firm is competing against a purely private firm. The public firm maximizes the social welfare (the sum of producer surplus and consumer surplus) by optimally choosing its output which can be made in one of the two periods early or late. While private firm faces the same problem of setting its optimal quantity in one of the two periods early or late by maximizing its own profit. Firms face a linear inverse demand function and produce with quadratic cost functions. The market demand is stochastic and if firms make their output decisions in the first period without having any information about market demand, they, being risk-averse maximize the expected value of their respective objective functions. Exact market demand is revealed to both firms before the start of the second period thus firms may have a perishable information advantage. We allow firms to have information asymmetry about market demand. In order to see how a firm endogenously chooses

[18] Fjell and Heywood (2002) discuss the state of competition in the telecommunication industry of Norway. Since January 1998 when the barriers to market entry were removed, the state-owned Telenor faced significant competition from the firms owned by different countries, for example, United States, France, the Netherlands, Sweden, and Ireland etc.

[19] In a two-stage model where firms choose capacities in the first stage and prices in the second stage, Kreps and Scheinkman (1983) argue that these results coincide with the Cournot outcome.

to become a leader in the market, we employ the framework of extended games with observable delay developed by Hamilton and Slutsky (1990). In observable delay games, firms first decide on the timings of strategic decisions and commit to it. If both of the firms opt to produce early in period one or both delay their output decisions to the second period, they will compete in Cournot fashion. But if one firm commits to produce early while other firm delays its output, the first firm will act as a Stackelberg leader and other will be the follower. In this case, the firm acting as Stackelberg follower will observe the actual quantity produced by the leading firm in period one and it will set its output accordingly. We also consider firms to endogenously acquire costly information about the market demand by adding an extra stage to the game as in Gilpatric and Li (2015) and derive the conditions under which it is optimal for firms to acquire costly information about market demand.

Given that both firms have acquired information, we find that two types of Stackelberg equilibria with either firm acting as leader coexist. This case is same as if there is no uncertainty regarding demand and we just added an additional term to the demand intercept and thus we get same results as in Pal (1998). The case where it is given that no firm has acquired information, two Stackelberg equilibria in pure strategies with either firm acting as leader coexist under mild degree of variance of demand. However, for higher uncertainty of demand reflected by variance of demand shock, firms endogenously decide to produce in the second period thus competing in Cournot fashion. This case has been studied in Anam et al. (2007). Under the case of asymmetric information where only public firm have acquired costly information about market demand, both types of Stackelberg equilibria with either firm acting as market leader exists only for small degree of demand uncertainty. However, for large demand uncertainty represented by the variance of the stochastic intercept term, only one Stackelberg equilibrium exists in which public firm, while being informed about market demand, acts as a market leader. Under asymmetric information situations, it is not a strictly dominant strategy for information advantaged firm to move early as oppose to the case of private duopolies.[20] A similar result emerges when the private firm is assumed to have acquired costly information. For a smaller degree of demand uncertainty, both Stackelberg equilibria with either firm acting as a market leader co-exist. However under higher demand volatility as reflected by the variance of stochastic intercept, only one Stackelberg equilibrium with private firm acting as leader exists. Unless both firms have not acquired information, firms in mixed duopoly considered in this paper always move in a sequential way and don't choose the same period to compete in a Cournot fashion.

[20] Gilpatric and Li (2015) conclude that in private duopoly, it a strictly dominant strategy for information advantaged firm to move early, while for other information disadvantaged firm it depends upon the uncertainty of market demand.

The results of the information acquisition stage show that it is not optimal for both firms to acquire costly information. So, there is no equilibrium where both firms acquire information. This is in contrast to the private duopoly case, as Gilpatric and Li (2015) show that in private duopoly, there is an equilibrium in which both firms acquire information and play Cournot game in the early period. However, under highly uncertain demand conditions, we find that only one firm acquires the costly information and becomes the leader of the market. So in the presence of high uncertainty, early information of market demand helps the firm to endogenously act as a market leader. Under low variance of the demand shock, no firm acquires information and two types of sequential equilibria exist with either firm acting as a leader with some parameter restrictions. An equilibrium outcome with certain parametric restrictions under mild variance of demand shock also emerges where no firm acquires information and then they choose quantities in the second period while competing in Cournot fashion.

The organization of this paper is as follows. We briefly review literature in Section 2.2 and discuss model formulation in Section 2.3. In the Sections 2.4 and 2.5, we derive sub-game perfect Nash equilibria of the timing game under no information and full information (symmetric information) cases respectively. The results under the cases of asymmetric information are discussed in Sections 2.6 and 2.7. We discuss information acquisition stage in Section 2.8 and Section 2.9 concludes.

2.2 Literature Review

There are two streams of literature which are relevant to the present context. One stream of literature is related to the endogenous sequence of moves by firms in oligopolistic markets. The second line of research is related to the incentives of firms to acquire market information. In the private duopoly where firms are playing a quantity setting game, the profit of a Stackelberg leader always exceeds than that of Cournot profits when firms are facing linear demands and constant marginal costs and Cournot profits are higher than that of a Stackelberg follower. Many studies have taken the order of firm's move in an exogenous way. The endogenous determination of simultaneous versus sequential moves by firms in oligopoly got popularity since the seminal work of Hamilton and Slutsky (1990).[21] Amir and Grilo (1999)

[21] Hamilton and Slutsky (1990) propose two ways to endogenize the timing of moves by firms in an oligopoly model; extended games with action commitment and extended games with observable delays. In the observable delay games, firms pre-commit to the periods in which to produce in pre-stage and then they play the actual game in one of the two periods by optimally choosing the quantities in the periods committed earlier

derive the conditions on demand and cost functions and find that log-concavity of inverse demand function is sufficient to derive firms to play endogenously Cournot in timing game. This holds irrespective of the shape of the cost function. On the other hand, if inverse demand function is log-convex in nature, then firms reaction functions are increasing and they behave in a Stackelberg fashion taking the role of both leader and follower endogenously. However, it requires firms to produce their goods free of cost. Dowrick (1986) finds that the slope of the reaction functions of the profit-maximizing firms plays the key role in agreeing over the assigned roles as a leader or follower in the Stackelberg model. He shows that when reaction functions are negatively sloped both of the firms prefers to take the role of Stackelberg leader. If the firms' reaction functions are positively sloped, then if the role of leadership is preferred by one firm then the other firm prefers to be a follower unless they face similar cost and demand structures in that case both of the firms prefer being a follower to acting as a leader.

Spencer and Brander (1992) study the endogenous moves (early commitment versus flexible delay) in a private duopoly where firms face uncertain market demand. They show that in equilibrium, firms compete in a Cournot game in period one (early commitment) under low uncertainty and they compete again in a Cournot fashion in period two when demand uncertainty is quite high. In a setting, where one firm has exogenously given the choice to become Stackelberg leader, they find that it prefers to retain the status of leadership just under low uncertainty while under high uncertainty, it prefers to compete in a Cournot fashion when uncertainty is resolved.[22] However, they show that the firm having a better information of market demand shock acts as a Stackelberg leader. In a pure duopoly, Liu (2005) compares the strategic advantage of being a Stackelberg leader (early commitment) versus the benefits of being fully informed while acting as Stackelberg follower (retaining flexibility). In his model firms face an uncertain demand and their roles as leader and follower are assigned exogenously. The leader makes his output choice on the expected value of demand (not knowing the actual value) but follower makes his output choice having the true value of market demand. He shows that the benefit of remaining flexible outweighs the strategic advantage of moving early if the true value of demand is quite high or quite low from its expected value.[23] In this case, Stackelberg follower earns a higher payoff than Stackelberg leader. For a large range of parameters in his model, Stackelberg leadership strategy is preferred by the firm over playing Cournot.

In a signaling game, Mailath (1993) analyzes the implication of asymmetric in-

[22] In their model, if the firm prefers to become Stackelberg leader and moves early then it is perfectly informed about the timing of the move of its opponent.

[23] A very low realized value of demand may lead to negative profit for the Stackelberg leader.

formation regarding quantity decisions of duopolist firms. He considers an environment where market demand is uncertain and potentially can take three values high, medium and low. One of the firms is exogenously informed about market demand and have the option of moving early than the uninformed firm or it can delay its output and can set simultaneous with the uninformed firm. He shows that in a stable equilibrium the informed firm chooses to move early and become Stackelberg leader irrespective of its private information. Although informed firm could get an ex-ante higher payoff while choosing its quantity along with uninformed firm simultaneously but because of the stability requirements, only Stackelberg equilibrium emerges where the informed firm acts as a market leader.

In a similar framework, Normann (1997) finds that another Stackelberg equilibrium exists in which uninformed firm acts as a leader, while all informed firms follow when uninformed firms have the opportunity to move. In the framework of extended games with observable delay, where firms first decide on the periods in which they will take actions and they are committed to it, Normann (2002) studies the endogenous timing decisions of the firms in a model formulation similar to the one in Mailath (1993).[24] The results show that in addition to the Stackelberg outcomes with either firm acting as a market leader, the Cournot outcome also emerges which is supported by a wide range of parameters. In all of these papers, firms are exogenously allowed to have information asymmetry.

In a pure duopoly, van Damme and Hurkens (1999) study the endogenous timing decisions of firms while playing extended games with action commitment as introduced by Hamilton and Slutsky (1990). Firms face linear demand and produce with constant marginal costs, however one firm has lower marginal cost thus being more efficient than the other firm. They show that each of the Stackelberg outcomes is an equilibrium of this game. But when the criterion of risk dominance of Harsanyi and Selton (1988) is applied they show that there is a unique Stackelberg equilibrium in which low-cost firm acts as a leader because it is most costly for the high-cost firm to commit early. Shi (2015) develop a pure duopoly model where firms face an uncertain demand and have options to produce in period one or costlessly wait and produce in period two without having information about market demand. Firms also have an option to do costly market research to know about market demand. Shi (2015) find that if market research is too costly or too cheap, then firms play in the same period and Cournot outcome emerges in this game. However, for intermediate cost range, market leadership arises endogenously. To see more research on the role of information and incentives for firms to share information see for example

[24] Normann (1997) builds on the framework of extended games with action commitment introduced by Hamilton and Slutsky (1990).

Raju and Roy (2000) and Yan et al. (2012).[25]

Daughety and Reinganum (1994) analyze the endogenous sequencing decisions of the firms in a signaling game, where the slope of the market demand for homogenous goods can take possibly two values. They allow ex-ante symmetric firms to acquire information and then decide to set their quantities in one of two periods. The results show that asymmetry arises in the equilibrium in the sense that only one firm acquires information and the informed firm acts as a Stackelberg leader. Both firms don't acquire information in the equilibrium unless it is free. Since the true market demand is never revealed to the follower, firms play a signaling game. In a pure duopoly model with horizontally differentiated goods, where firms face cost uncertainty, Albaek (1990) finds that a Natural Stackelberg Situation emerges in quantity competition.[26] And firm with higher cost variance acts as a Stackelberg leader in the equilibrium.

The literature on mixed oligopoly has proliferated recently within the last decade. The issue of simultaneous versus sequential moves regarding quantity setting in mixed oligopoly was first studied by Pal (1998). He shows that two sub-game perfect Nash equilibria endogenously emerge and can coexist; the one in which public firm acts as a leader and the other in which private firm acts as a leader. When there are more than two periods in which firms can produce, Pal (1998) shows that all private firms choose to produce in period one while public firm produces afterward.[27] However, social welfare is higher when public firm acts as a follower rather than when it acts as a leader. He considers constant marginal costs of production and assumes that public firm produces at a higher cost thus being less efficient. However, Matsumura (2003a) finds a Stackelberg outcome where the public firm acts as a leader when competing against a foreign-owned private firm. And this outcome is

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[25] Raju and Roy (2000) studied the value of information and they found that it is of great value under high demand uncertainty and in more competitive industry represented by high product substitutability. Moreover, in private duopoly, information benefits more to the firm acting as a market leader and competing in a Stackelberg fashion than competing in a Bertrand way. However, they assume firm's strategic position as market leader to be exogenous. In a game theoretic model, Yan et al. (2012) study the incentives of an upstream manufacturer to share information with downstream retailers differing in their market shares or positions. They find that it is optimal for the manufacturer to share information with one small and less dominant retailer.

[26] In his model, both firms know the distribution of costs but they do not know the actual values of their own as well as their opponent's costs. A Natural Stackelberg Situation is defined as the situation, where both firms agree on assigning the role of leadership to one firm and the role of following to the other firm and further, both firms prefer this situation over Nash.

[27] Jacques (2004) claims that this result is sensitive to the number of private firms. specifically, he shows that this result holds only when the number of private firms ıs greater than or equal to two. When there is singly private firms competing against the public firm, there is another equilibrium in which public firm chooses period one and private firm follows. Lu (2007) highlights the another sub-game perfect Nash equilibrium in which all private firm make their quantity choices in any period except the last one while public firm acts as a follower.

socially efficient as opposed to the Pal (1998).

By allowing two periods of production, Matsumura (2003b) investigates the endogenous order of moves of firms in a mixed duopoly. He finds that multiple equilibria exist including Cournot type and Stackelberg type equilibrium with private firm acting as a leader. However, he shows that no Stackelberg type equilibrium with public firm acting as a leader exists. By adding small inventory cost into the model, he claims that unique Stackelberg type equilibrium with public firm acting as a follower exists. In these papers market demand functions are deterministic thus firms face no uncertainties regarding demand. Anam et al. (2007) investigated the endogenous timing decisions of the firms in mixed duopoly when firms face uncertainty regarding market demand. By using the framework developed by Hamilton and Slutsky (1990), they find that multiple equilibria in the quantity-setting game exist in a mixed duopoly. Specifically, they show that along with two Stackelberg outcomes with either public and private firm acting as a leader, Cournot outcome also appears, where both firms produce in period two when uncertainty is resolved. However social welfare is higher when the private firm takes the role of leader and public firm follows thus confirming the result in Pal (1998). Moreover, when the public firm is competing against a foreign private firm, they show that under moderate uncertainty, public firm act as a leader and this outcome is socially efficient as well in line with Matsumura (2003a). However in their model, there is no role of information acquisition, firms are homogeneous in terms of the level of information about market demand.

Our study is related to Gilpatric and Li (2015) who develop a model, where two profit-maximizing firms facing uncertain market demand, decide on whether to produce in period one or two. They show that under asymmetric information, it is a strictly dominant strategy for an information advantaged firm to produce in period one. In their model, if a firm chooses to produce in period two, it becomes fully informed about market demand before the start of the period two. However, the timing decision of less informed or information disadvantaged firm hinges on the variance of demand shock since it faces a trade-off between the strategic advantage of moving early versus the value of being fully informed about market demand while producing in period two. They show that under high variance of the demand shock, the less informed firm chooses to capitalize on being fully informed and acts as a follower. Hence information asymmetry endogenously leads to a Stackelberg market structure. However, under the low variance, a standard Cournot outcome appears and both firms produce in period one. In the information acquisition stage which appears before the timing decision stage, they find that when the variance of demand shock is high, both firms acquire information and play Cournot in period one in the subsequent game. Neither of the firms acquires information when the variance is low and for intermediate ranges of the variance of the demand shock, only one firm acquires costly information. However, they show that endogenous leadership only arises when there is a significant difference between the fixed costs of acquiring information between the firms. We apply their model set up to the mixed duopoly market structure.[28]

In the context of mixed duopoly, Tomaru and Kiyono (2010) investigate the endogenous timing decisions of the firms, facing increasing marginal costs in a quantity setting game. They show that two types of sequential equilibrium coexist with either firm acting as Stackelberg leader thus their results conform to the findings of Pal (1998) even when firms face increasing marginal costs.[29] While Lu and Poddar (2009) find that simultaneous move cannot be sustained as a subgame perfect Nash equilibrium in mixed duopoly when firms are endogenously deciding on capacity then quantity. Their results are also in line with the findings on endogenous timing in mixed duopoly cited earlier that multiple equilibria can exist where either type of firm acts as a leader. Naya (2015) studies the endogenous timing decisions of a partially privatized firm in a quantity setting while competing against a private domestic firm in a differentiated goods market. The results reveal that under a lower degree of privatization, both types of sequential equilibria exist with either firm acting as Stackelberg leader. Under medium level of privatization, only one equilibrium exists with private firm acting as a leader while under the higher level of privatization firms compete in Cournot fashion endogenously.

However, more recently, Matsumura and Ogawa (2017a) overturned this standard result of endogenous timing game in mixed duopoly as in Pal (1998) and Matsumura (2003a) among others. While investigating the endogenous timing decisions of firms in a mixed duopoly, they find that in the presence of a significant negative production externality, firms endogenously choose to compete in Cournot fashion. Under negative externality, they show that firms make sequential moves when competing in prices, again in contrast to the standard findings in the literature on mixed duopolies. So they conclude that in the presence of a significant negative externality, mixed duopolies behave in the same way as private duopolies in the

[28]It is well-known result in the mixed oligopoly that the public firm will monopolize over all the production if the public firm is equally efficient as the private firm and produces with constant marginal cost (see for example Bárcena-Ruiz (2012)). In order to avoid this problem, we assume that both firms produce by using quadratic cost functions while Gilpatric and Li (2015) allow firms to produce with the constant marginal costs since they deal with only profit-maximizing firms. Pal (1998) deals with this problem by assuming that public firm produces with a positive constant marginal cost while the private firm has zero marginal cost. In Anam et al. (2007) both firms produce with quadratic costs.

[29]In their model, inverse demand function is deterministic and more general rather than linear and firms face similar convex cost functions. While in Pal (1998) and Matsumura (2003a) firms face linear inverse demand functions and have constant marginal cost.

endogenous timing game. While introducing product differentiation into the model as in Dixit (1979), Matsumura and Ogawa(2017b) study the endogenous timing decisions of firms in a mixed duopoly in quantity-setting game. They found that two Stackelberg equilibria with either firm acting as leader exist. However, from the social welfare perspective, it is desirable when public firm acts as a follower. They also show that the equilibrium with public firm acting as the leader is risk-dominant and robust under the high degree of horizontal product differentiation. While competing with foreign-owned private firm, they show that two sequential move equilibria; public leadership and foreign-owned private firm leadership exists. But social welfare is higher under the leadership of public firm and it is risk-dominant and thus a robust equilibrium.[30]

In a homogenous goods market, Ogawa and Kazuhiko (2006) study the price setting behavior of firms in a mixed duopoly. Firms are exogenously assigned the roles of leader and follower and they also allow firms to set prices simultaneously. They show that private firm while taking the role of leadership, sets higher price than the price set under the leadership of public firm and under some parametric restrictions this price is also higher than the Nash price set simultaneously. Public firms while taking care of the consumer surplus as well as the profit of private firms sets the same price as set by the private firm irrespective of the role it enjoys. Bárcena-Ruiz (2007) considers the mixed duopoly and differentiated goods market to analyze the endogenous moves of firms in the pricing game. He shows that firms in mixed duopoly set their prices simultaneously as opposed to the private duopoly where firms set their prices sequentially. However in quantity-setting game firms move sequentially in the mixed duopoly.

In another paper, Gilpatric and Li (2016) while considering the endogenous role of firms as a leader or follower in a differentiated Bertrand duopoly facing uncertain demand, find that there is always an equilibrium, in which information advantaged firm acts as a leader. Since in their model both firms know the actual market demand in the second period, the less informed firm capitalizes on this and behaves as a follower to perfectly known with market demand. They show that there is no equilibrium where both firms choose to buy the information before deciding on timings to move.

[30]In a different context, Zhang and Li (2013) analyze the timing of location decisions of a public and a private firm in a Hotelling-type model, with firms facing the uncertainty regarding the locations of the consumer. They show that when the degree of uncertainty is too high, both firm delay their entrance to the market while they find that there is no equilibrium accompanied with the small degree of uncertainty.

2.3 Model

We consider a mixed duopoly model, where firm 1 is publicly owned, while firm 2 is a pure domestic private firm. Both firms are making strategic decisions on quantities while facing an uncertain demand. Firms are selling homogenous goods and face a linear inverse demand function of the following form:

$$p = A + \psi - q_i - q_j \tag{23}$$

which has a stochastic intercept term ψ. We assume ψ is a random variable having a continuous c.d.f $\psi \sim F(.)$ with $E[\psi] = 0$ and $var[\psi] = \sigma^2 > 0$. Following Hamilton and Slutsky (1990), firms play an extended game with observable delay where firms have to decide on the timing of the their actions (commitment stage) as well as on the actual actions (action stage). In the extended games, firms pre-commit to the periods in which to produce in pre-stage and then they play the actual game in two periods and optimally produce the quantities in the periods committed. If both firms have committed to produce earlier, they play a standard Cournot game in period one but if they both have committed to delay their output, they play Cournot game in period two. But if one firm has committed to play earlier, while other commits to late, then they compete in a Stackelberg fashion in output game. We allow firms to have information asymmetry in the sense that if a firm has acquired costly information then it knows the exact realization of the demand shock before the start of period 1 and thus it is informed. While, to the uninformed firm, demand uncertainty is resolved before the start of the period 2. So the informed firm has a perishable informational advantage over the other firm because true demand is also revealed to the other firm before the start of the second period. Following Gilpatric and Li (2015), we assume that firms acquire information about market demand through a costly market research at a fixed cost $F > 0$ and it also increases the marginal cost of the firm by $k > 0$. Both firms produce by using a convex cost function of the following form:

$$C_i(q_i) = I_i k q_i + \frac{q_i^2}{2}, \tag{24}$$

where

$$I_i = \begin{cases} 1, & \text{if firm } i \text{ acquires information} \\ 0, & \text{otherwise} \end{cases} \tag{25}$$

The profit function of firm i will be as follows:

$$\Pi_i = \left(A + \psi - q_i - q_j^{'}\right) q_i - C_i\left(q_i\right), \tag{26}$$

where,

$$q_j^{'} = \begin{cases} q_j\left(q_i\right), & \text{if firm } j \text{ moves after firm } i \\ q_j, & \text{otherwise} \end{cases} \tag{27}$$

The objective of the firm 1, being a public firm, is to maximize social welfare which is the sum of profits of both firms and consumer surplus.

$$SW = \Pi_1 + \Pi_2 + CS \tag{28}$$

Following the literature, we use the expected value of consumer surplus as a measure of consumer welfare irrespective of whether the firm is informed about market demand or otherwise.[31] Specifically, consumer surplus is written as:

$$E\left[CS\right] = E\left(A(q_1 + q_2) - \frac{(q_1 + q_2)^2}{2} - p_1 q_1 - p_2 q_2\right) \tag{29}$$

using inverse demand function and after simplification, the expected value of consumer surplus reduces to:

$$E\left[CS\right] = (q_1 + q_2)^2/2 \tag{30}$$

The figure 1 below presents the sequence of events which is as follows. In the first stage, firms simultaneously and non-cooperatively decide on to acquire costly information about market demand and at the end of this stage their choices become common knowledge. In order to endogenize the timing decisions of the firms, we follow the framework of observable delay games of Hamilton and Slutsky (1990). In the second stage, firms simultaneously make choices about the timing of their production decisions and strictly commit to it. At the end of this stage, choices of firms regarding timings become common knowledge. After that, firms make their production decisions according to timing choices made earlier. If both firms have opted to produce early in period one, then production takes place only in the period one of production stage. But if firms have opted to produce in different periods, the firm who opted to produce early becomes the leader and produces in the period one

[31] Anam et al. (2007) use expected consumer surplus as a measure of consumer welfare. If consumers are risk-neutral and face a demand whose income elasticity is zero, then Stennek (1999) claim that expected consumer surplus is an appropriate measure for the welfare of consumers in uncertain environments. However, these conditions do not hold empirically all the time. Schlee (2008) also supports this idea that expected consumer surplus is a fairly good measure of consumer welfare under uncertainties.

of production stage as shown in figure 1. While the follower produces in the period two of the production stage. After that consumers make their purchases.

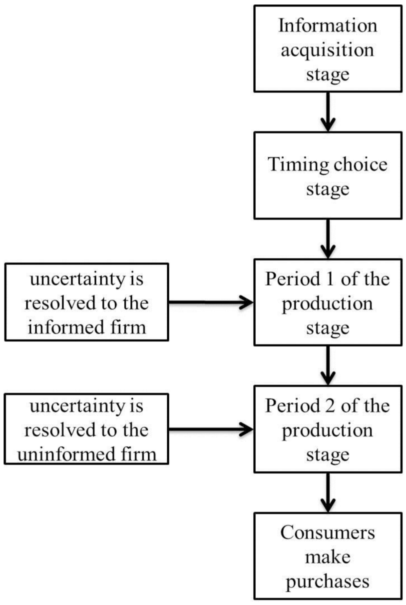

Figure 1: Sequence of events

In the section 2.4, we exogenously assume that both of the firms have not acquired information (thus both uninformed), while in section 2.5, we exogenously assume that both of the firms have acquired information (thus both informed) and analyze their timing decisions. While in the sections 2.6 and 2.7, we exogenously hold that only the public firm and only the private firm have acquired costly information respectively and we study the endogenous timing choices of the firms. We solve the model by using backward induction.

2.4 No firm has acquired information

In this section, we assume that both firms have not acquired costly information however, they will learn about the realization of market demand shock before the start of period two. In this case, both firms will optimally choose their respective quantities by maximizing the expected values of the following pay off functions, if they both decided to produce in period one.

$$E\left(SW\right)=\int\left(A+\psi-q_1-q_2\right)\left(q_1+q_2\right)dF\left(\psi\right)-C_1\left(q_1\right)-C_2\left(q_2\right)+E\left(CS\right), \tag{31}$$

$$E\left(\Pi_2\right)=\int\left(A+\psi-q_1-q_2\right)q_2dF\left(\psi\right)-C_2\left(q_2\right). \tag{32}$$

Since both firms have not acquired information, $I_i = 0$ holds for both firms and so there are no k terms in the above functions. If both the firms commit to produce early in period one (Early, Early), they will play compete in a standard Cournot fashion. Taking FOCs of the above payoff functions and simultaneously solving them, we get following optimal Nash quantities and accordingly their respective ex-ante expected payoffs:

$$q_1^*=\frac{2}{5}A,\qquad q_2^*=\frac{1}{5}A, \tag{33}$$

$$E\left(SW^*\right)=\frac{8}{25}A^2,\; E\left(\Pi_2^*\right)=\frac{3}{50}A^2. \tag{34}$$

Now suppose that both firms have committed to compete in Cournot fashion in period two (Late, Late), they will learn the actual value of market demand shock (for example as ψ_0) before the start of period two. Given ψ_0, firms will maximize the actual payoff functions (not in expected terms). Taking FOCs and simultaneously solving them, we get the following optimal quantities and payoffs in expected terms:

$$q_1^*=\frac{2}{5}(A+\psi_0),\quad q_2^*=\frac{1}{5}(A+\psi_0), \tag{35}$$

$$E\left(SW^*\right)=\frac{8}{25}A^2+\frac{8}{25}\sigma^2, \tag{36}$$

$$E\left(\Pi_2^*\right)=\frac{3}{50}A^2+\frac{3}{50}\sigma^2. \tag{37}$$

Now suppose the public firm has committed to produce early in period one, while private firm commits to produce late in period two, they compete in a Stackelberg fashion in which public firm acts as a leader while private firm acts as a follower (Early, Late). Solving from backward induction, we get following optimal quantities under public firm leadership:

$$q_1^l=\frac{5}{14}A,\quad q_2^f=\frac{9}{42}A+\frac{1}{3}\psi_0. \tag{38}$$

In this case, their corresponding ex-ante expected payoffs are:

$$E\left(SW^{*}\right)=\frac{9}{28}A^{2}+\frac{2}{9}\sigma^{2}, \tag{39}$$

$$E\left(\Pi_{2}^{*}\right)=\frac{81}{1176}A^{2}+\frac{196}{1176}\sigma^{2}. \tag{40}$$

If the private firm has opted to produce early in period one and public firm commits to produce in period two, they compete in a Stackelberg fashion with private firm acting as a leader, while public firm behaves as a follower (Late, Early). Their optimal quantities and payoffs in expected terms are:

$$q_{1}^{f}=\frac{1}{16}(5A+4\psi_{0}),\quad q_{2}^{l}=\frac{1}{4}A, \tag{41}$$

$$E\left(SW^{*}\right)=\frac{21}{64}A^{2}+\frac{16}{64}\sigma^{2}, \tag{42}$$

$$E\left(\Pi_{2}^{*}\right)=\frac{1}{16}A^{2}. \tag{43}$$

The table below summarizes the ex-ante expected payoff to the players. Based on these payoffs and by straightforward calculations, we find Nash equilibria of endogenous timing game in pure strategies and results are recorded in the following proposition.

		Firm 2	
		Early	Late
Firm 1	Early	$\frac{8}{25}A^{2}, \frac{3}{50}A^{2}$	$\frac{9}{28}A^{2}+\frac{2}{9}\sigma^{2}, \frac{81}{1176}A^{2}+\frac{196}{1176}\sigma^{2}$
	Late	$\frac{21}{64}A^{2}+\frac{16}{64}\sigma^{2}, \frac{1}{16}A^{2}$	$\frac{8}{25}A^{2}+\frac{8}{25}\sigma^{2}, \frac{3}{50}A^{2}+\frac{3}{50}\sigma^{2}$

Table 4: Payoff matrix when both firms are uninformed

Proposition 3 (both firms are uninformed) *Given that both of the firms have not acquired information and they will learn the realization of market demand before the start of period 2, then: i) there is no equilibrium in which firms produce in the period 1. ii) there is a pure strategy Stackelberg equilibrium with public firm leadership iff* $0\leq\sigma^{2}\leq 9A^{2}/616$. *iii) there is a pure strategy Stackelberg equilibrium with private firm acting as a leader iff* $0\leq\sigma^{2}\leq A^{2}/24$. *iv) there is a pure strategy equilibrium with both firms producing in period 2 iff* $\sigma^{2}>A^{2}/24$.

Proof. For proof see Proposition 1 in Anam et al. (2007). ■

2.5 Both firms have acquired information

In this section, we assume that both public as well as the private firm have acquired information ($I_i = 1 \; \forall i = 1, 2$) and are thus informed about market demand realizations before the start of period 1. By knowing exactly, the realization of random intercept of demand as ψ_0, the objective functions of both firms are:[32]

$$SW = (A + \psi_0 - q_1 - q_2)(q_1 + q_2) - C_1(q_1) - C_2(q_2) + E(CS), \tag{44}$$
$$\Pi_2 = (A + \psi_0 - q_1 - q_2) q_2 - C_2(q_2). \tag{45}$$

Given that both of the firms have acquired information and opted to produce either in period 1 (Early, Early) or in period 2 (Late, Late), they will play a standard Cournot game. FOCs give the following reaction functions which are negatively sloped reflecting that quantities are strategic substitutes:

$$q_1(q_2) = \frac{1}{2}(A - k + \psi_0) - \frac{1}{2}q, \tag{46}$$
$$q_2(q_1) = \frac{1}{3}(A - k + \psi_0) - \frac{1}{3}q_1, \tag{47}$$

Simultaneously solving the above reaction functions, we get the following optimal quantities:

$$q_1^* = \frac{2}{5}(A - k + \psi_0), \; q_2^* = \frac{1}{5}(A - k + \psi_0), \tag{48}$$

Their corresponding profits in expected terms are as follows:

$$E(SW^*) = \frac{8}{25}(A - k)^2 + \frac{8}{25}\sigma^2, \tag{49}$$
$$E(\Pi_2^*) = \frac{3}{50}(A - k)^2 + \frac{3}{50}\sigma^2. \tag{50}$$

If the public firm has committed to produce early in period 1, while private firm commits to produce late in period 2, they compete in a Stackelberg fashion in which public firm acts as a leader while private firm acts as a follower. Using backward induction, we solve first solve the follower's problem which gives the reaction function of the private firm as given by equation (47). Public firm by taking into account the reaction function of firm 2, maximizes its objective function as specified in equation (44). Optimal quantities of the leader and follower are:

$$q_1^l = \frac{5}{14}(A - k + \psi_0), \; q_2^f = \frac{3}{14}(A - k + \psi_0), \tag{51}$$

[32] Since the fixed cost of acquiring information F is a sunk cost, so it is excluded from optimization.

Their respective ex-ante expected payoffs are as follows:

$$E(SW^*) = \frac{9}{28}(A-k)^2 + \frac{9}{28}\sigma^2, \quad (52)$$
$$E(\Pi_2^*) = \frac{27}{392}(A-k)^2 + \frac{27}{392}\sigma^2. \quad (53)$$

If private firm commits to produce early in period 1 and public firm produces in period 2, they compete in a Stackelberg fashion in which private firm acts as a leader while public firm becomes the follower. Again by using backward induction, we find the following optimal quantities :

$$q_1^f = \frac{3}{8}(A-k+\psi_0), \quad q_2^l = \frac{1}{4}(A-k+\psi_0), \quad (54)$$

In private firm leadership case, their corresponding payoffs in expected terms are:

$$E(SW^*) = \frac{21}{64}(A-k)^2 + \frac{21}{64}\sigma^2, \quad (55)$$
$$E(\Pi_2^*) = \frac{1}{16}(A-k)^2 + \frac{1}{16}\sigma^2. \quad (56)$$

The table below summarizes the payoff to the players in expected terms.

		Firm 2 Early	Firm 2 Late
Firm 1	Early	$\frac{8(A-k)^2}{25} + \frac{8\sigma^2}{25}, \frac{3(A-k)^2}{50} + \frac{3\sigma^2}{50}$	$\frac{9(A-k)^2}{28} + \frac{9\sigma^2}{28}, \frac{27(A-k)^2}{392} + \frac{27\sigma^2}{392}$
	Late	$\frac{21(A-k)^2}{64} + \frac{21\sigma^2}{64}, \frac{(A-k)^2}{16} + \frac{\sigma^2}{16}$	$\frac{8(A-k)^2}{25} + \frac{8\sigma^2}{25}, \frac{3(A-k)^2}{50} + \frac{3\sigma^2}{50}$

Table 5: Payoff matrix when both firm are informed

The following proposition presents the main result of this section regarding Nash equilibria of endogenous timing game in pure strategies.

Proposition 4 (both firms have acquired information) *Given that both firms have acquired information and are informed about demand realization in period 1, then: i) there is no equilibrium in which firms produce in the same period in the extended game. ii) there are two sequential move equilibria with either public or private firm acting as a leader.*

Proof. i) (Early, Early) is not an equilibrium because in this case public firm can get higher payoff by deviating to late and its incremental payoff is $SW^*(Late, Early) - SW^*(Early, Early) = 13((A-k)^2 + \sigma^2)/1600 > 0$. Private firm also benefits by deviating to late and its incremental payoff is $\Pi_2^*(Early, Late) - \Pi_2^*(Early, Early) = 87((A-k)^2 + \sigma^2)/9800 > 0$. Similarly (Late, Late) is not an equilibrium because both firms have incentives to deviate. Incremental payoff to firm 1 is $SW^*(Early, Late) - SW^*(Late, Late) = ((A-k)^2 + \sigma^2)/700 > 0$ and incremental payoff to firm 2 while deviating to early is $\Pi_2^*(Late, Early) - \Pi_2^*(Late, Late) = ((A-k)^2 + \sigma^2)/400 > 0$.

ii) Straightforward calculations reveal that (Early, Late) is indeed an equilibrium since no firm has the incentive to deviate. Deviation payoff to firm 1 is $SW^*(Late, Late) -$ $SW^*(Early, Late) = -((A-k)^2 + \sigma^2)/700 < 0$ and deviation payoff to firm 2 is $\Pi_2^*(Early, Early) - \Pi_2^*(Early, Late) = -87((A-k)^2 + \sigma^2)/9800 < 0$. Playing (Late,Early) is another equilibrium since deviation does not benefit either of the firm. Deviation payoff to firm 1 is $SW^*(Early, Early) - SW^*(Late, Early) = -13((A-k)^2+\sigma^2)/1600 < 0$ and deviation payoff to firm 2 is $\Pi_2^*(Late, Late) - \Pi_2^*(Late, Early) = -((A-k)^2 + \sigma^2)/400 < 0$. ■

2.6 Only public firm has acquired information

In this section, we assume that only public firm has acquired information ($I_1 = 1$) and thus it knows the specific value of market demand shock as ψ_0 before the start of period one. While, the private firm having information disadvantage ($I_2 = 0$) over the public firm, will learn the market demand realization before the start of period two. It will maximize the expected profit if it chooses to produce in period one. Objective functions of the firms will look like:

$$SW = (A + \psi_0 - q_1 - q_2)(q_1 + q_2) + E(CS) - C_1(q_1) - C_2(q_2), \tag{57}$$

$$E(\Pi_2) = \int (A + \psi - q_1 - q_2) q_2 dF(\psi) - C_2(q_2). \tag{58}$$

Suppose both firms have committed to produce early in period one, they will compete in a Cournot fashion (Early, Early). Given that only public firm has acquired information, their optimal quantities and payoffs in expected terms are as follows:

$$q_1^* = \frac{1}{10}(4A - 6k + 5\psi_0), \quad q_2^* = \frac{1}{5}(A+k), \tag{59}$$

$$E(SW^*) = \frac{1}{100}\left(32A^2 - 36Ak + 32k^2\right) + \frac{1}{4}\sigma^2, \tag{60}$$

$$E(\Pi_2^*) = \frac{3}{50}(A+k)^2. \tag{61}$$

Now suppose that both firms commit to play Cournot game in period two (Late, Late), then at the start of period two, the private firm will also learn the realized value of market demand shock. Cournot quantities and firm's corresponding payoffs in expected terms are as follows:

$$q_1^* = \frac{1}{5}(2A - 3k + 2\psi_0), \quad q_2^* = \frac{1}{5}(A + k + \psi_0), \tag{62}$$

$$E(SW^*) = \frac{1}{25}\left(8A^2 - 9Ak + 8k^2\right) + \frac{8}{25}\sigma^2, \tag{63}$$

$$E(\Pi_2^*) = \frac{3}{50}(A+k)^2 + \frac{3}{50}\sigma^2. \tag{64}$$

If the public firm has opted to produce early in period one, while private firm chooses to produce late in period two, they compete in a Stackelberg fashion in which public firm acts as a leader while private firm follows. Optimal quantities and corresponding ex-ante expected payoffs under public firm leadership are:

$$q_1^l = \frac{1}{14}(5A - 9k + 5\psi_0), \; q_2^f = \frac{3}{14}(A + k + \psi_0) \tag{65}$$

$$E(SW^*) = \frac{1}{28}\left(9A^2 - 10Ak + 9k^2\right) + \frac{9}{28}\sigma^2 \tag{66}$$

$$E(\Pi_2^*) = \frac{27}{392}(A+k)^2 + \frac{27}{392}\sigma^2 \tag{67}$$

Now suppose that private firm has committed to produce early in period one and public firm produces late in period two, they compete in a Stackelberg fashion in which private firm acts as a leader while public firm becomes a follower. Here, the private firm will maximize its expected profit by taking into account the reaction function of the public firm. In this case of private firm leadership, their optimal quantities and expected payoffs are:

$$q_1^f = \frac{1}{8}(3A - 5k + 4\psi_0), \quad q_2^l = \frac{1}{4}(A + k), \tag{68}$$

$$E(SW^*) = \frac{1}{64}\left(21A^2 - 22Ak + 21k^2\right) + \frac{1}{4}\sigma^2, \tag{69}$$

$$E(\Pi_2^*) = \frac{1}{16}(A+k)^2. \tag{70}$$

The ex-ante expected payoffs to the players found in this section are recorded in the table below. Then, we find pure strategy Nash equilibria of the endogenous timing game and summarize the discussion of this section in the proposition below.

Proposition 5 (only the public firm has acquired information) *Following that only public firm has acquired information and is thus informed about demand realization in period 1, then: i) there is no equilibrium in which firms produce in the same period. ii) there is a pure strategy Stackelberg equilibrium with public firm acting as a leader iff* $\sigma^2 \geq 0$. *iii) there is another pure strategy Stackelberg equilibrium with*

Firm 2

		Early	Late
Firm 1	Early	$\frac{32A^2-36Ak+32k^2}{100}+\frac{\sigma^2}{4}, \frac{3(A+k)^2}{50}$	$\frac{9A^2-10Ak+9k^2}{28}+\frac{9\sigma^2}{28}, \frac{27(A+k)^2}{392}+\frac{27\sigma^2}{392}$
	Late	$\frac{21A^2-22Ak+21k^2}{64}+\frac{\sigma^2}{4}, \frac{(A+k)^2}{16}$	$\frac{8A^2-9Ak+8k^2}{25}+\frac{8\sigma^2}{25}, \frac{3(A+k)^2}{50}+\frac{3\sigma^2}{50}$

Table 6: Payoff matrix when only the public firm is informed

private firm acting as a leader iff $0 \leq \sigma^2 \leq \left(A^2+2Ak+k^2\right)/24$.

Proof. i) It is clear that (Early, Early) is not an equilibrium because both firms have incentives to deviate and can get higher payoffs. For example public firm can get a higher payoff by deviating to late and its incremental payoff is $SW^*\left(Late, Early\right) - SW^*\left(Early, Early\right) = 13\left((A+k)^2+\sigma^2\right)/1600 > 0$. And private firm also benefits by deviating to late and its incremental payoff is $\Pi_2^*\left(Early, Late\right) - \Pi_2^*\left(Early, Early\right) = 3\left(29(A+k)^2+225\sigma^2\right)/9800 > 0$. Similarly (Late, Late) is not an equilibrium because both firms have incentives to deviate.Incremental payoff to firm 1, in this case, is $SW^*\left(Early, Late\right) - SW^*\left(Late, Late\right) = \left((A+k)^2+\sigma^2\right)/700 > 0$ and incremental payoff to firm 2 while deviating to early is $\Pi_2^*\left(Late, Early\right) - \Pi_2^*\left(Late, Late\right) = \left((A+k)^2-24\sigma^2\right)/400$ which is > 0 as long as $\sigma^2 < (A+k)^2/24$. Since the public firm has a clear incentive to deviate so, (Late, Late) is not an equilibrium irrespective of the level of σ^2.

ii) (Early, Late) is an equilibrium since no firm has the incentive to deviate. Deviation payoff to firm 1 is $SW^*(Late, Late) - SW^*(Early, Late) = -((A+k)^2+\sigma^2)/700 < 0$ and deviation payoff to firm 2 is $\Pi_2^*(Early, Early) - \Pi_2^*(Early, Late) = -(29(A+k)^2+225\sigma^2)/9800 < 0$. For (Late,Early) to be an equilibrium we require that both firms have no incentive to deviate unilaterally. Deviation does not benefit to the public firm because its deviation payoff is negative $SW^*(Early, Early) - SW^*(Late, Early) = -13(A-k)^2/1600 < 0$ and deviation payoff to firm 2 is $\Pi_2^*\left(Late, Late\right) - \Pi_2^*\left(Late, Early\right) = (24\sigma^2-(A+k)^2)/400$ which is negative or equal to zero whenever $0 \leq \sigma^2 \leq (A+k)^2/24$. ■

2.7 Only private firm has acquired information

In this section, we exogenously allow the only private firm to acquire costly information ($I_2 = 1$) and it knows exact realization of market demand shock (for example as ψ_0). While public firm having information disadvantage ($I_1 = 0$), will learn the realized value of demand shock at the start of period two if it opts to defer its production to period two. Their objective functions, in this case, can be written as:

$$E\left(SW\right)=\int\left(A+\psi-q_1-q_2\right)\left(q_1+q_2\right)dF\left(\psi\right)+E\left(CS\right)-C_1\left(q_1\right)-C_2\left(q_2\right), \tag{71}$$

$$\Pi_2=\left(A+\psi_0-q_1-q_2\right)q_2-C_2\left(q_2\right). \tag{72}$$

Suppose both firms opted to produce early in period one and play Cournot game, the public firm will maximize its payoff in expected terms and the private firm will maximize its payoff given the realized value of market demand shock as ψ_0. Followings are the optimal quantities and corresponding payoffs in expected terms if they both have opted to produce early in period one:

$$q_1^*=\frac{1}{5}\left(2A+k\right),\quad q_2^*=\frac{1}{15}\left(3A-6k+5\psi_0\right), \tag{73}$$

$$E\left(SW^*\right)=\frac{8A^2-7Ak+7k^2}{25}+\frac{2\sigma^2}{9}, \tag{74}$$

$$E\left(\Pi_2^*\right)=\frac{(3A-6k)^2}{150}+\frac{25\sigma^2}{150}. \tag{75}$$

Now suppose both firms simultaneous produce in period two and play Cournot game, the public firm will also learn the realized value of demand shock before the start of period two. Optimization of the payoff functions given ψ_0, yields the following optimal quantities and corresponding ex-ante expected payoffs:

$$q_1^*=\frac{1}{5}\left(2A+k+2\psi_0\right),\ q_2^*=\frac{1}{5}\left(A-2k+\psi_0\right), \tag{76}$$

$$E\left(SW^*\right)=\frac{8A^2-7Ak+7k^2}{25}+\frac{8\sigma^2}{25}, \tag{77}$$

$$E\left(\Pi_2^*\right)=\frac{3(A-2k)^2}{50}+\frac{3\sigma^2}{50}. \tag{78}$$

If the public firm has committed to produce early in period one and private firm opts to produce in period two, the public firm acts as a Stackelberg leader, while firm acts as Stackelberg follower. The public firm maximizes the expected value of social surplus while taking into account the optimal response function of the private firm. Solving the problem from backward induction, we find following optimal quantities and expected payoffs under public firm leadership:

$$q_1^l = \frac{1}{14}(5A + 4k), \quad q_2^f = \frac{3}{14}(A - 2k) + \frac{1}{3}\psi_0, \tag{79}$$

$$E(SW^*) = \frac{81A^2 - 72Ak + 72k^2}{252} + \frac{56\sigma^2}{252}, \tag{80}$$

$$E(\Pi_2^*) = \frac{(9A - 18k)^2}{1176} + \frac{196\sigma^2}{1176}. \tag{81}$$

The case, where private firm opts to produce early in period one while acting as a Stackelberg leader and public firm acting as a Stackelberg follower will learn the realized value of market demand shock before the start of period two. Solving in the same way as above, we get optimal quantities and expected payoffs under private firm leadership as:

$$q_1^f = \frac{1}{8}(3A + 2k + 3\psi_0), \quad q_2^l = \frac{1}{4}(A - 2k + \psi_0), \tag{82}$$

$$E(SW^*) = \frac{1}{25}\left(21A^2 - 20Ak + 20k^2\right) + \frac{2}{9}\sigma^2, \tag{83}$$

$$E(\Pi_2^*) = \frac{1}{16}(A - 2k)^2 + \frac{1}{16}\sigma^2. \tag{84}$$

The following table summarizes the above discussion and presents the expected payoff to the players. Nash equilibria in this case of information asymmetry are recorded in the following proposition.

		Firm 2: Early	Firm 2: Late
Firm 1	Early	$\frac{8A^2-7Ak+7k^2}{25} + \frac{2\sigma^2}{9}$, $\frac{(3A-6k)^2}{150} + \frac{25\sigma^2}{150}$	$\frac{81A^2-72Ak+72k^2}{252} + \frac{56\sigma^2}{252}$, $\frac{(9A-18k)^2}{1176} + \frac{196\sigma^2}{1176}$
	Late	$\frac{21A^2-20Ak+20k^2}{64} + \frac{21\sigma^2}{64}$, $\frac{(A-2k)^2}{16} + \frac{\sigma^2}{16}$	$\frac{8A^2-7Ak+7k^2}{25} + \frac{8\sigma^2}{25}$, $\frac{3(A-2k)^2}{50} + \frac{3\sigma^2}{50}$

Table 7: Payoff matrix when only private firm is informed

Proposition 6 (only the private firm has acquired information) *Given that only private firm has acquired information and is thus informed about demand realization in period one, then: i) there is no equilibrium in which firms produce in the same period. ii) there is a pure strategy Stackelberg equilibrium with public firm acting as a leader iff* $\sigma^2 \leq 9(A^2 - 4Ak + 4k^2)/616$. *iii) there is another pure strategy Stackelberg equilibrium where the private firm acts as leader iff* $\sigma^2 \geq -A^2 + 4Ak - 4k^2$.

Proof. i) (Early, Early) is not an equilibrium because the public firm can get a higher payoff by deviating to late and its incremental payoff is $SW^*(Late, Early) - SW^*(Early, Early) = (A^2 + 1525\sigma^2 - 468Ak + 2727k^2)/14400 > 0$. And private firm also benefits by deviating to late and its incermental payoff is $\Pi_2^*(Early, Late) - \Pi_2^*(Early, Early) = 87(A - 2k)^2/9800 > 0$. Similarly (Late, Late) is not an equilibrium because both firms have incentives to deviate. Incremental payoff to public firm is $SW^*(Early, Late) - SW^*(Late, Late) = (9(A-2k)^2 - 616\sigma^2)/6300 > 0$ and incremental payoff to private firm, while deviating to early is, $\Pi_2^*(Late, Early) - \Pi_2^*(Late, Late) = ((A - 2k)^2 + \sigma^2)/400 > 0$.

ii) (Early, Late) is indeed an equilibrium since no firm has the incentive to deviate. Deviation payoff to firm 1 is $SW^*(Late, Late) - SW^*(Early, Late) = (616\sigma^2 - 9(A-2k)^2)/6300 < 0$ and deviation payoff to firm 2 is $\Pi_2^*(Early, Early) - \Pi_2^*(Early, Late) = -87(A - 2k)^2/9800 < 0$. Playing (Late,Early) is another equilibrium since deviation does not benefit either of the firms. Deviation payoff to public firm is $SW^*(Early, Early) - SW^*(Late, Early) = -(117(A-2k)^2 + 1525\sigma^2)/14400 < 0$ and deviation payoff to private firm is $\Pi_2^*(Late, Late) - \Pi_2^*(Late, Early) = -((A - 2k)^2 + \sigma^2)/400 < 0$. ■

2.8 Information acquisition

In this section, we derive equilibria of the costly information acquisition stage which appears before the timing stage. We divide the variance of the demand shock into five regions. In the first region, where firms face high uncertainty, specifically when variance of the demand shock $\sigma^2 \geq (A^2 + 2Ak + k^2)/24$, following both firms have acquired information, there are two Stackelberg equilibria (Early, Late) and (Late, Early) with either firm acting as the leader. While, following that neither of the firms acquires information, they play Cournot in period two (Late, Late) in this range and following that only public firm has acquired information, (Early, Late) exists and given that only private firm has acquired information, only (Late, Early) exists in this range. In the second region, $A^2/24 \leq \sigma^2 < (A^2 + 2Ak + k^2)/24$, two Stackelberg equilibria (Early, Late) and (Late, Early) exists in cases considered in sections 2.5 and 2.6, while one Stackelberg equilibrium (Late, Early) exists in case of section 2.7 and one Cournot equilibrium (Late, Late) exists in case discussed in section 2.4.

In the third region, where variance $9A^2/616 \leq \sigma^2 < A^2/24$, both types of Stackelberg equilibria (Early, Late) and (Late, Early) exists in cases discussed in sections 2.5 and 2.6, while one Stackelberg equilibrium (Late, Early) exists under cases considered in sections 2.4 and 2.7. In the fourth region where variance $(9A^2 - 36Ak + 36k^2)/616 \leq \ \sigma^2 < 9A^2/616$, both types of Stackelberg equilibria (Early, Late) and (Late, Early) exist in all the cases except for the case where the only private firm has acquired information. Given that only private firm has acquired information, only one sequential equilibrium (Late, Early) exists in this range. Last region, where $0 \leq \ \sigma^2 < (9A^2 - 36Ak + 36k^2)/616$, there exists both types of Stackelberg equilibria (Early, Late) and (Late, Early) exist in all four cases discusses in the previous sections. With working on all the corresponding possible payoff tables in all the regions and checking all the possibilities of equilibria, we derive the following result regarding Nash equilibria in pure strategies in the information acquisition stage.

Proposition 7 (endogenous information acquisition) *The results of the overall game are: i) there is no pure strategy equilibrium where both firms acquire information. ii) there is a pure strategy equilibrium where only public firm acquire information and play sequentially in the subsequent game with public firm acting as leader iff* $\sigma^2 \geq max\{-A^2 + 700F + 250Ak - 225k^2, (3A^2 + 448F + 160Ak - 144k^2)/32, 9(28F + 10Ak - 9k^2)/25\}$ *iii) there is a pure strategy equilibrium where private firm acquire information while public firm does not and play sequentially in the subsequent game with private firm acting as leader iff* $\sigma^2 \geq max\{-A^2 + 400F + 100Ak - 100k^2, 4(4F + Ak - k^2), 9A^2/616\}$ *iv(a)) there is a pure strategy equilibrium where neither firm acquire information and play cournot in the second period (late, late) iff* $max\{(13A^2 - 1600F - 550Ak + 525k^2)/112, A^2/24\} \leq \sigma^2 < -A^2 + 400F + 100Ak - 100k^2$. *iv(b)) there is a pure strategy equilibrium where neither firm acquire information and play sequentially in which public firm acts a leader iff* $0 \leq \sigma^2 < min\{9(-3A^2 + 448F + 154Ak - 147k^2)/112, 9(28F + 10Ak - 9k^2)/25, 9A^2/616\}$. *iv(c)) there is a pure strategy equilibrium where neither of the firms acquires information and play sequentially in which private firm acts a leader iff* $0 \leq \sigma^2 \leq min\{3(-5A^2 + 748F + 216Ak - 216k^2)/392, (3A^2 + 448F + 160Ak - 144k^2)/32, 16F + 4Ak - 4k^2, A^2/24\}$.

2.9 Conclusion

In this chapter, we study the endogenous timing decisions of firms in a mixed duopoly and examine whether an information advantaged firm has incentives to become the market leader in a quantity-setting game. We consider a market for homogeneous goods, where a publicly owned firm is competing against a domestically owned private firm while facing uncertain demand. The objective of the public firm is to maximize the social welfare, while private firm maximizes its own profit. Firms decide to set their quantities in one of the two periods. The market demand is stochastic in the sense that if firms make their output decisions in the first period without having any information about market demand, they, being risk-neutral maximize the expected values of their corresponding objective functions. We allow firms to have information asymmetry. Exact market demand is revealed before the start of the second period thus one firm may have a perishable information advantage over the other.

By employing the framework of extended games with observable delay developed by Hamilton and Slutsky (1990), we summarize the results of endogenous timing game as follows. Given that both firms know the exact realization of market demand by acquiring information, two type of Stackelberg equilibria with either firm acting as a leader coexist. The case where no firm has acquired information, we find two Stackelberg type equilibria in pure strategies with either firm acting as leader coexist under the mild degree of variance of the demand shock. However, under a high degree of uncertainty of demand, firms endogenously decide to produce in the second period thus competing in Cournot fashion. In the case of asymmetric information where only the public firm has acquired costly information about market demand, both type of Stackelberg equilibria coexist only for the small degree of demand uncertainty. But, under high variance of the demand shock, only one Stackelberg equilibrium exists in which the public firm acts as a market leader. Under asymmetric information situations, it is not a strictly dominant strategy for information advantaged firm to move early as opposed to the case of private duopolies. A similar result emerges when the private firm is assumed to have acquired costly information. For a smaller degree of demand uncertainty, both type of Stackelberg equilibria coexist. However, under high demand volatility, only private firm leadership Stackelberg equilibrium exists. Unless both firms have not acquired information, we find that firms in mixed duopoly always move in a sequential way and don't choose the same period to compete in Cournot fashion.

We also consider firms to endogenously acquire costly information about the market demand by adding an extra stage to the game. The results of the information acquisition stage show that it is not optimal for both firms to acquire costly information. This is in contrast to the profit maximizing firms case, where an equilibrium in which both firms acquire information and play Cournot game in the period one exists as shown by Gilpatric and Li (2015). However, under highly uncertain demand conditions, we find that only one firm acquires the costly information and becomes the leader in the market. So in the presence of high uncertainty, early information of market demand helps the firm to endogenously act as a market leader. Under low variance of the demand shock, no firm acquires information and two types of

Stackelberg equilibria exist with either firm acting as a leader with some parameter restrictions. An equilibrium outcome with certain parametric restrictions under mild variance of demand shock also emerges where no firm acquires information and then they choose quantities in the second period while competing in a Cournot fashion.

In this chapter, we allow one private firm to compete against the public firm, the model can be extended by adding more private firms. We work out with linear demand function, however, it remains to see whether our results hold or otherwise by using a more general demand function. Adding foreign private firms into the model while competing against the public firm, is another possible extension. Another way to extend our model is to introduce partial privatization of the public firm and to see whether endogenous sequencing or the incentives for acquiring information change or not. In the present model, firms are producing homogeneous products, what happens when they are competing in a differentiated goods market is another question to explore.

2.A Appendix: Proof of Proposition 7 (endogenous information acquisition)

We briefly sketch the proof of this proposition.

Case 1 ($\sigma^2 \geq (A^2 + 2Ak + k^2)/24$): As described in the main text, when the variance of demand shock $\sigma^2 \geq (A^2 + 2Ak + k^2)/24$, there are two Stackelberg equilibria (Early, Late) and (Late, Early) in this range given that both firms have acquired information. While, following that neither of the firms has acquired information, they play Cournot in period two (Late, Late) in this range and following that only the public firm has acquired information, only one Stackelberg equilibrium (Early, Late) exists. Given that only private firm has acquired information, only one Stackelberg equilibrium (Late, Early) exists in this range. Then, we have following payoff tables:

		Firm 2: acquire	Firm 2: don't acquire
Firm 1	acquire	$\frac{9(A-k)^2}{28}+\frac{9\sigma^2}{28}-2F$, $\frac{27(A-k)^2}{392}+\frac{27\sigma^2}{392}-F$	$\frac{9A^2-10Ak+9k^2}{28}+\frac{9\sigma^2}{28}-F$, $\frac{27(A+k)^2}{392}+\frac{27\sigma^2}{392}$
	don't acquire	$\frac{21A^2-20Ak+20k^2}{64}+\frac{21\sigma^2}{64}-F$, $\frac{(A-2k)^2}{16}+\frac{\sigma^2}{16}-F$	$\frac{8A^2}{25}+\frac{8\sigma^2}{25}$, $\frac{3A^2}{50}+\frac{3\sigma^2}{50}$

		Firm 2: acquire	Firm 2: don't acquire
Firm 1	acquire	$\frac{21(A-k)^2}{64}+\frac{21\sigma^2}{64}-2F$, $\frac{(A-k)^2}{16}+\frac{\sigma^2}{16}-F$	$\frac{9A^2-10Ak+9k^2}{28}+\frac{9\sigma^2}{28}-F$, $\frac{27(A+k)^2}{392}+\frac{27\sigma^2}{392}$
	don't acquire	$\frac{21A^2-20Ak+20k^2}{64}+\frac{21\sigma^2}{64}-F$, $\frac{(A-2k)^2}{16}+\frac{\sigma^2}{16}-F$	$\frac{8A^2}{25}+\frac{8\sigma^2}{25}$, $\frac{3A^2}{50}+\frac{3\sigma^2}{50}$

The difference between above two tables originates from the payoffs following (acquire, acquire). In the first table, payoffs correspond to the (Early, Late) while in table two, payoffs correspond to (Late, Early) equilibrium. Working with these payoffs tables, we find that there is no pure strategy equilibrium where both of the firms acquire information. There is a pure strategy equilibrium where only public firm acquires information and play sequentially in the subsequent game with public firm acting as a leader iff $\sigma^2 \geq \max\{-A^2 + 700F + 250Ak - 225k^2,$ $(A^2 + 2Ak + k^2)/24$. There is a pure strategy equilibrium where only private firm acquires information and then play (Late, Early) iff $\sigma^2 \geq \max\{-A^2+400F+100Ak-100k^2, (A^2 + 2Ak + k^2)/24\}$. There is a pure strategy equilibrium where neither of the firms acquires information and play Cournot in the second period (late, late) iff $(A^2 + 2Ak + k^2)/24 \leq \sigma^2 < \min\{-A^2 + 400F + 100Ak - 100k^2, -A^2 + 700F + 250Ak - 225k^2\}$.

Case 2 ($A^2/24 \leq \sigma^2 < (A^2 + 2Ak + k^2)/24$): In this range of variance of demand shock, ($A^2/24 \leq \sigma^2 < (A^2 + 2Ak + k^2)/24$), two Stackelberg equilibria (Early, Late) and (Late, Early) exist in cases considered in sections 2.5 and 2.6, while one Stackelberg equilibrium (Late, Early) exists in case discussed in section 2.7 and one Cournot equilibrium (Late, Late) exists in the case discussed in section 2.4. So in this range of variance, in addition to the two payoff tables above, we have following two tables as well.

		Firm 2	
		acquire	don't acquire
Firm 1	acquire	$\frac{9(A-k)^2}{28}+\frac{9\sigma^2}{28}-2F$, $\frac{27(A-k)^2}{392}+\frac{27\sigma^2}{392}-F$	$\frac{21A^2-22Ak+21k^2}{64}+\frac{\sigma^2}{4}-F$, $\frac{(A+k)^2}{16}$
	don't acquire	$\frac{21A^2-20Ak+20k^2}{64}+\frac{21\sigma^2}{64}-F$, $\frac{(A-2k)^2}{16}+\frac{\sigma^2}{16}-F$	$\frac{8A^2}{25}+\frac{8\sigma^2}{25}$, $\frac{3A^2}{50}+\frac{3\sigma^2}{50}$

		Firm 2	
		acquire	don't acquire
Firm 1	acquire	$\frac{21(A-k)^2}{64}+\frac{21\sigma^2}{64}-2F$,, $\frac{(A-k)^2}{16}+\frac{\sigma^2}{16}-F$	$\frac{21A^2-22Ak+21k^2}{64}+\frac{\sigma^2}{4}-F$, $\frac{(A+k)^2}{16}$
	don't acquire	$\frac{21A^2-20Ak+20k^2}{64}+\frac{21\sigma^2}{64}-F$, $\frac{(A-2k)^2}{16}+\frac{\sigma^2}{16}-F$	$\frac{8A^2}{25}+\frac{8\sigma^2}{25}$, $\frac{3A^2}{50}+\frac{3\sigma^2}{50}$

Working with these four payoff tables, we find that there is no pure strategy equilibrium where both firms acquire information. There is a pure strategy equilibrium where public firm acquires information and play (Early, Late) iff $\max\{-A^2+700F+250Ak-225k^2, A^2/24\} \leq \sigma^2 < (A^2+2Ak+k^2)/24$ and there is another pure strategy equilibrium where private firm acquire information and subsequently acts as a leader iff $\max\{-A^2+400F+100Ak-100k^2, (-3A^2-448F-148Ak+4k^2)/3, A^2/24\} \leq \sigma^2 < (A^2+2Ak+k^2)/24$. There is a pure strategy equilibrium where neither of the firms acquires information and play Cournot in the second period (Late, Late) iff $\{(13A^2-1600F-550Ak+525k^2)/112, A^2/24\} \leq \sigma^2 < \min\{-A^2+700F+250Ak-225k^2, (A^2+2Ak+k^2)/24, -A^2+400F+100Ak-100k^2\}$. Proceeding in the same way, we can derive conditions for other regions of variances for different type of equilibria as discussed in the main text under section 2.8, and combining all the conditions, we get the Proposition 7.

CHAPTER 3

Information acquisition and endogenous sequencing in mixed duopoly with a foreign competitor

3.1 Introduction

In different parts of the world, the presence of publicly owned firms while competing against foreign-owned private firms is evident in many industries like for example health, education, telecommunication, insurance, banking, postal services and transport among others. In the United States, Packing and over-night delivery industry is an example where we can observe public and private firms compete together. Similarly, in the Norwegian oil industry, the publicly-owned Statoil faces significant competition from two foreign-owned private firms Esso Norge and Norske Shell and in the telecom sector of the country, the state-owned Telenor has many competing firms owned by different countries, Like, United States, France, the Netherlands, Sweden, and Ireland etc.[33] The market structure where public firm competes against private firms is known as mixed oligopoly. The research on mixed oligopoly gained momentum in the past decade or so, although the literature on the subject is not new (see for example Merrill and Schneider (1966), Anderson et al. (1997), and Cremer, Marchand, and Thisse (1989) among others).

Historically public firms have enjoyed monopoly in certain sectors in many countries but with the passage of time competition has increased with the participation in the form domestic private firms and foreign-owned private firms. Many studies have discussed the consequences of privatizing the public firm while competing in the product market. And many of these studies exogenously assumed the order of firm's moves. Since assuming the different order of moves produces significantly different results, it is important to analyze the incentives of firms to endogenously

[33] See Fjell and Heywood (2002) for details.

choose the order of moves. The endogenous determination of simultaneous versus sequential moves in oligopolistic market structure got popularity since the seminal work of Hamilton and Slutsky (1990).[34] In case of mixed oligopoly with one public firm competing against many domestic private firms, the endogenous order of moves was first studied by Pal (1998). He shows that different order of moves carries significant different welfare implications. And in mixed oligopoly when a public firm has foreign-owned private competitors, to the best of our knowledge, Matsumura (2003a) paper is the first to investigate the endogenous timing decisions of firms. In an environment, where uncertainty looms regarding market demand, firms choices to become the market leader or follower in making strategic decisions are endogenous in nature. And further, firm's motivation to acquire costly information regarding market demand conditions largely depends upon whether a firm is enjoying a leadership position in a market or it is acting as a follower. For example, Raju and Roy (2000) found that information has a great value under high uncertainty and in more competitive industry. Moreover information is of great benefit to the firm acting as a market leader and competing in a Stackelberg fashion than competing in a Bertrand way, however, they assume firm's strategic position as market leader to be exogenous.

In this paper, we contribute to the literature on endogenous sequencing of moves by firms in mixed duopoly, where a publicly-owned firm is competing against a foreign-owned private firm, by exploring the role of information advantage by a firm (an early information of uncertain market demand) to endogenously become a market leader in a quantity setting game. We consider a market of homogeneous goods, where a publicly owned firm is competing against a purely private firm. The public firm maximizes the social welfare[35] (the sum of its own profit and consumer surplus) by optimally choosing its output which can be made in one of the two periods early or late. While foreign-owned firm maximizing its own profit by optimally setting its quantity in one of the two periods early or late. Firms face a linear and stochastic inverse market demand function and if firms make their output decisions in the first period without having any information about actual market demand, they, being risk-neutral maximize the expected value of their respective objective functions. We assume that firms produce with quadratic cost functions. Exact market demand is revealed to both firms before the start of the second period thus firms may have

[34] Hamilton and Slutsky (1990) propose two ways to endogenize the timing of moves by firms in an oligopoly model; extended games with action commitment and extended games with observable delays. In the observable delay games, firms pre-commit to the periods in which to produce in pre-stage and then they play the actual game in one of the two periods by optimally choosing the quantities in the periods committed earlier

[35] Since the foreign-owned private firm is assumed to remit all of its profit back to its home country of origin, its profits are excluded from the objective function of the public firm.

a perishable information advantage. We allow firms to have information asymmetry about market demand. In order to endogenize the firm's sequencing of moves, we employ the framework of extended games with observable delay developed by Hamilton and Slutsky (1990). In this framework, firms first decide on the timings of their moves and then commit to it in the action game played later. If both of the firms commit to produce early in period one or both delay their output decisions to the second period, they will compete in a standard Cournot fashion. But if one of the firms commits to produce early while other delays its output, they will compete in a Stackelberg fashion with firm producing early will act as a leader. In this case, the firm acting as Stackelberg follower will observe the actual quantity produced by the leading firm and it will set its output accordingly. We also consider firms to endogenously acquire costly information about the market demand by adding an extra stage to the game as in Gilpatric and Li (2015) and derive the conditions under which it is optimal for firms to acquire costly information about market demand.

When both firms are exogenously assumed to have acquired costly information, we show that both ((Early, Late) & (Late, Early)) Stackelberg equilibria with either firm acting as a leader coexist. Given that no firm has acquired information, a Stackelberg equilibrium in pure strategies with public firm acting as a leader exists under a mild degree of variance of demand. In this case, there is another Stackelberg equilibrium with private firm leadership but it exists only when there is no uncertainty regarding demand. However high uncertainty of demand is accompanied by firms to endogenously produce in the second period thus competing in a Cournot fashion.[36] In case of information asymmetry, when the only public firm is assumed to have acquired costly information about market demand, a Stackelberg equilibrium with public firm acting as a market leader always exists.[37] However, Stackelberg equilibrium with foreign-owned private firm leadership exists only when there is no uncertainty regarding market demand.[38] Under asymmetric information situations, it is not a strictly dominant strategy for information advantaged firm to move early as opposed to the case of private duopolies.[39] When the private firm is assumed to have acquired costly information, both Stackelberg equilibria ((Early, Late) & (Late, Early)) co-exist only for a smaller degree of demand uncertainty. However, in this case, under high variance of the demand shock, Stackelberg equilibrium with

[36] This case has been considered and studied by Anam et al. (2007).

[37] This type of Stackelberg equilibrium always exists irrespective of the level of uncertainty of market demand reflected by the variance of the stochastic intercept term.

[38] Specifically, this type of Stackelberg equilibrium exists only when the variance of the stochastic intercept term is zero.

[39] Gilpatric and Li (2015) conclude that in private duopoly, it a strictly dominant strategy for information advantaged firm to move early, while for other information disadvantaged firm it depends upon the uncertainty of market demand.

foreign-owned private firm acting as a leader coexists with a Cournot equilibrium in period two (Late, Late).

The results of the information acquisition stage reveal that it is not optimal for both firms to acquire costly information. So, both firms acquiring information (acquire, acquire) cannot be sustained as an equilibrium. This is in contrast to the profit-maximizing duopoly case.[40] However, under high uncertainty, we find that only one public firm acquires costly information and becomes the leader of the market. So in the presence of high uncertainty, an early signal of market demand helps the public firm to endogenously act as a market leader. We show that in the information acquisition stage there is no equilibrium where only foreign-owned private firm acquires costly information. However, under low variance of the demand shock, no firm acquires information and subsequently, Stackelberg equilibrium with public firm leadership emerges. There is another equilibrium where no one acquires information and then compete in a Stackelberg fashion with foreign-owned private firm acting as a leader but only when there is no uncertainty regarding market demand.

The organization of this paper is as follows. We briefly review literature in Section 3.2 and discuss model formulation in Section 3.3. In the Sections 3.4 and 3.5, we derive Nash equilibria of the timing game under no information and full information (symmetric information) cases respectively. The results under the cases of asymmetric information are discussed in Sections 3.6 and 3.7. We discuss information acquisition stage in Section 3.8 and Section 3.9 concludes.

3.2 Literature Review

Firm's choices regarding endogenous timing have been widely studied in the industrial organization literature. There are two strands of literature which are relevant to us. One stream of literature is related to the endogenous timing decisions by firms in oligopolistic markets. The second line of research is related to the incentives of firms to acquire costly market information. Many papers have considered the moves by firms in an exogenous way. Since the different order of moves produces significantly different results, it is important to study the order of firm's move endogenously. The endogenous determination of simultaneous versus sequential moves by firms in oligopoly got popularity since the seminal work of Hamilton and Slutsky

[40] Gilpatric and Li (2015) show that in private duopoly where firm's sole objective is to maximize their own profits, there is an equilibrium in which both firms acquire information and subsequently compete in Cournot fashion early in period one.

(1990). They propose two ways to endogenize the timing of moves by firms in an oligopoly model; extended games with action commitment and extended games with observable delays. In the observable delay games, firms pre-commit to the periods in which to produce in pre-stage and then they play the actual game in one of the two periods by optimally choosing the quantities in the periods committed earlier.[41]

First, we review some studies related to the endogenous timing of firms in pure oligopolies. In a pure duopoly, Liu (2005) analyzes the strategic advantage of early commitment while acting as a Stackelberg leader versus the benefits of flexibility and being fully informed while acting as a Stackelberg follower. While facing uncertain market demand, firms are exogenously assigned the roles of leader and follower. When the true value of demand is quite high or quite low from its expected value, he shows that firm prefers to remain flexible while acting as a Stackelberg follower and it earns a higher profit than being acting as a Stackelberg leader.[42] However, the firm prefers to play Stackelberg leadership strategy than playing Cournot strategy supported by a large range of parameters.

In a private duopoly, Spencer and Brander (1992) analyze the endogenous timing decisions of firms (early commitment versus flexible delay) in the presence of uncertainty regarding market demand. Under low uncertainty, their results show that firms prefer to compete in Cournot fashion in period one (early commitment). They compete again in a Cournot fashion in period two when demand uncertainty is quite high. They show that when only one of the firms is exogenously allowed to become Stackelberg leader, it will do so when there is low uncertainty but it prefers to compete in a Cournot fashion under high uncertainty.[43]

Mailath (1993) studies the role of asymmetric information in the quantity decisions of profit-maximizing duopolist firms. In his model market demand is uncertain and potentially, it can take three values high, medium and low. One of the firms is exogenously allowed to have an information advantage over the other firm. The

[41] In pure oligopolies, Amir and Grilo (1999) derive the conditions on demand and cost functions and find that log-concavity of inverse demand function is sufficient to derive firms to play endogenously Cournot in timing game. This holds irrespective of the shape of the cost function. On the other hand, if inverse demand function is log-convex in nature, then firms reaction functions are increasing and they behave in a Stackelberg fashion taking the role of both leader and follower endogenously. However, it requires firms to produce their goods free of cost. Dowrick (1986) finds that the slope of the reaction functions of the profit-maximizing firms plays the key role in agreeing over the assigned roles as a leader or follower in the Stackelberg model. He shows that when reaction functions are negatively sloped both of the firms prefers to take the role of Stackelberg leader. If the firms' reaction functions are positively sloped, then if the role of leadership is preferred by one firm then the other firm prefers to be a follower unless they face similar cost and demand structures in that case both of the firms prefer being a follower to acting as a leader.

[42] A very low realized value of demand may lead to negative profit for the Stackelberg leader.

[43] In their model, if the firm prefers to become Stackelberg leader and moves early then it is perfectly informed about the timing of the move of its opponent.

informed firm has the choice to move early than the uninformed firm or it can set its quantity simultaneously. In a stable equilibrium, the information advantaged firm optimally chooses to become Stackelberg leader. He shows that, although the informed firm could get an ex-ante higher profit while playing Cournot strategy, but because of the stability requirements, the informed firm play Stackelberg strategy and acts as a market leader. While allowing an uninformed firm to move, in a similar model, Normann (1997) shows that another Stackelberg equilibrium under the leadership of the uninformed firm exists. In the framework of extended games with observable delay, Normann (2002) shows that in addition to the Stackelberg outcomes with either firm acting as a market leader, the Cournot outcome supported with a large range of parameters also emerges. In all of these papers, firms are exogenously allowed to have information asymmetry.[44]

van Damme and Hurkens (1999) study the endogenous timing decisions of firms in a pure duopoly while applying the framework of extended games with action commitment of Hamilton and Slutsky (1990). They consider firms to face linear demand and produce with constant marginal costs, however one firm has lower marginal cost than the other firm. They show that both types of the Stackelberg equilibria exist but Stackelberg equilibrium under the leadership of low-cost firm only survives when the criterion of risk dominance of Harsanyi and Selton (1988) is applied because early commitment is more costly for the high-cost firm. Shi (2015) considers an environment, where, pure duopolist firms face an uncertain demand. Firms have three options; to produce early in period one or costlessly wait and produce in period two without having information about market demand or to do costly market research. He shows that if market research is too costly or alternatively too cheap, firms choose their quantities simultaneously. However, market leadership endogenously emerges for the intermediate values of cost.

While exploring the role of information, Daughety and Reinganum (1994) analyze the endogenous sequencing decisions of the firms in a signaling game, where the slope of the market demand for homogenous goods can take possibly two values. They allow ex-ante symmetric firms to acquire information and then decide to set their quantities in one of two periods. The results show that asymmetry arises in the equilibrium in the sense that only one firm acquires information and the informed firm acts as a Stackelberg leader. Both firms don't acquire information in the equilibrium unless it is free. Since the true market demand is never revealed to the follower, firms play a signaling game. In a pure duopoly model with horizontally differentiated goods, where firms face cost uncertainty, Albaek (1990) found that a

[44] Normann (1997) builds on the framework of extended games with action commitment while Normann (2002) applies the framework of extended games with observable delay introduced by Hamilton and Slutsky (1990).

Natural Stackelberg Situation emerges in quantity competition and firm with higher cost variance acts as a Stackelberg leader in the equilibrium.[45] To see more research on the role of information and incentives for firms to share information see for example Raju and Roy (2000) and Yan et al. (2012).[46]

Pal (1998) is the first author to investigate the issue of simultaneous versus sequential moves in a quantity setting game by firms in a mixed oligopoly. In his model, one public firm has many domestic private competitors and they produce homogenous goods with constant marginal costs. The public firm produces at a higher cost thus being less efficient. He shows that two types of Stackelberg equilibria coexist; the one in which public firm acts as a leader and the other in which private firm acts as a leader. When more than two periods are added to the model, he shows that all private firms decide to produce early in period one while public firm produces afterward.[47] But, social welfare is higher in the Stackelberg equilibrium where the public firm acts as a follower. However, Matsumura (2003a) found a Stackelberg outcome where the public firm acts as a leader when competing against a foreign-owned private firm. This outcome is socially efficient as opposed to the Pal (1998). By allowing two periods of production, Matsumura (2003b) investigated the endogenous order of moves of firms in a mixed duopoly. He finds that multiple equilibria exist including Cournot type and Stackelberg type equilibrium with private firm acting as a leader. However, he shows that no Stackelberg type equilibrium with public firm acting as a leader exists. By adding small inventory cost into the model, he claims that unique Stackelberg type equilibrium with public firm acting as a follower exists. In these papers market demand is deterministic thus firms face no uncertainties.

By using the framework of extended games with observable delay, Lu (2006) ana-

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[45] In his model, both firms know the distribution of costs but they do not know the actual values of their own as well as their opponent's costs. A Natural Stackelberg Situation is defined as the situation, where both firms agree on assigning the role of leadership to one firm and the role of following to the other firm and further, both firms prefer this situation over Nash.

[46] Raju and Roy (2000) studied the value of information and they found that it is of great value under high demand uncertainty and in more competitive industry represented by high product substitutability. Moreover, in private duopoly, information benefits more to the firm acting as a market leader and competing in a Stackelberg fashion than competing in a Bertrand way. However, they assume firm's strategic position as market leader to be exogenous. In a game theoretic model, Yan et al. (2012) study the incentives of an upstream manufacturer to share information with downstream retailers differing in their market shares or positions. They find that it is optimal for the manufacturer to share information with one small and less dominant retailer.

[47] Jacques (2004) claims that this result is sensitive to the number of private firms. specifically, he shows that this result holds only when the number of private firms ıs greater than or equal to two. When there is singly private firms competing against the public firm, there is another equilibrium in which public firm chooses period one and private firm follows. Lu (2007) highlights the another sub-game perfect Nash equilibrium in which all private firm make their quantity choices in any period except the last one while public firm acts as a follower.

lyzes the endogenous timing decisions of firms in a quantity-setting mixed oligopoly. He considers a homogenous goods market where one public firm is competing against many domestic private firms and many foreign-owned private firms. His results show that public firm optimally decides not to become the leader of the all foreign-owned private firms while it chooses to be the follower of all the domestic private firms. The number of domestic private firms and the number of foreign-owned private firms is important for the existence of multiple equilibria in his model. Bárcena-Ruiz and Garzón (2010) study the endogenous timing decisions of firms in a mixed duopoly where one semipublic firm is competing against many private firms in a quantity setting game. They consider firms to be equally efficient and produce homogenous goods with constant marginal cost. Firms decide on the timings of their moves in observable delay games of Hamilton and Slutsky (1990). They show that Cournot outcome emerges as the Subgame Perfect Nash Equilibrium in this environment which is in contrast to the result in Pal (1998).

Lu (2011) analyzes the endogenous timing of moves by firms in mixed oligopoly when the objective of private firms is to maximize the relative profits rather than the absolute profits. He considers all firms to produce homogenous good with constant marginal cost while facing linear demand function and the public firm is less efficient than private firms in his model. His results reveal that simultaneous move cannot be sustained as an equilibrium and two type of Stackelberg equilibria emerge; the one in which public firm acts as a leader and in the other equilibrium it acts as a follower. He shows that in subgame perfect Nash equilibrium with public firm acting as a leader, social welfare increase when private firms maximize relative profits but relative profit maximizing behavior of private firms has no effect on social welfare when in the equilibrium, the public firm acts as a follower.

Anam et al. (2007) investigate the endogenous timing decisions of the firms in mixed duopoly when firms face uncertainty regarding market demand. By using the framework developed by Hamilton and Slutsky (1990), they find that multiple equilibria in the quantity-setting game exist in a mixed duopoly. Specifically, they show that along with two Stackelberg outcomes with either public and private firm acting as a leader, Cournot outcome also appears, where both firms produce in period two when uncertainty is resolved. However social welfare is higher when the private firm takes the role of leader and public firm follows thus confirming the result in Pal (1998). Moreover, when the public firm is competing against a foreign private firm, they show that under moderate uncertainty, public firm act as a leader and this outcome is socially efficient as well in line with Matsumura (2003a). However in their model, there is no role of information, firms are homogeneous in terms of the level of information about market demand.

We build our model on Gilpatric and Li (2015). In their model, two profit-maximizing firms, while, facing uncertain market demand, decide on the timing to produce and if a firm chooses to produce in period two, it becomes fully informed about market demand before the start of the period two. They allow firms to have information asymmetry and that it is a strictly dominant strategy for an information advantaged firm to move early. However, the timing decision of information disadvantaged firm depends on the variance of the demand shock. They show that when the variance of the demand shock is high, the informed disadvantaged firm acts as a follower and becomes fully informed so information asymmetry endogenously leads to a Stackelberg market structure. However, a standard Cournot outcome in period one appears when the variance is low. They also allow firms to endogenously acquire information and they find that when the variance of demand shock is high, both firms acquire information and play Cournot in period one. For the medium range of the variance, they show that only one of the firms acquires costly information, however, endogenous leadership only arises when there is a significant difference between the fixed costs of acquiring information between the firms. We apply their model set up to the mixed duopoly market structure where a public firm competes with a foreign-owned private firm.[48]

In a differentiated duopoly, where firms face uncertain demand, Gilpatric and Li (2016) analyze the endogenous order of moves in the price-setting game. They show that in the equilibrium, information advantaged firm always acts as a leader and the less informed firm behaves as a follower to perfectly known with market demand since both firms know the actual market demand in the second period. While endogenizing the information acquisition decisions, they show that both firms acquiring information cannot be sustained as an equilibrium.

In the context of mixed duopoly, Tomaru and Kiyono (2010) investigated the endogenous timing decisions of the firms, facing increasing marginal costs in a quantity setting game. They show that two types of sequential equilibrium coexist with either firm acting as Stackelberg leader thus their results conform to the findings of Pal (1998) even when firms face increasing marginal costs.[49] While Lu and Poddar (2009) find that simultaneous move cannot be sustained as a subgame perfect Nash

[48]It is well-known result in the mixed oligopoly that the public firm will monopolize over all the production if the public firm is equally efficient as the private firm and produces with constant marginal cost (see for example Bárcena-Ruiz (2012)). In order to avoid this problem, we assume that both firms produce by using quadratic cost functions while Gilpatric and Li (2015) allow firms to produce with the constant marginal costs since they deal with only profit-maximizing firms. Pal (1998) deals with this problem by assuming that public firm produces with a positive constant marginal cost while the private firm has zero marginal cost. In Anam et al. (2007) both firms produce with quadratic costs.

[49]In their model, inverse demand function is deterministic and more general rather than linear and firms face similar convex cost functions. While in Pal (1998) and Matsumura (2003a) firms face linear inverse demand functions and have constant marginal cost.

equilibrium in mixed duopoly when firms are endogenously deciding on capacity then quantity. Their results are also in line with the findings on endogenous timing in mixed duopoly cited earlier that multiple equilibria can exist where either type of firm acts as a leader. Naya (2015) studied the endogenous timing decisions of a partially privatized firm in a quantity setting while competing against a private domestic firm in a differentiated goods market. The results reveal that under a lower degree of privatization, both types of sequential equilibria exist with either firm acting as Stackelberg leader. Under medium level of privatization, only one equilibrium exists with private firm acting as a leader while under the higher level of privatization firms compete in Cournot fashion endogenously.

While investigating the endogenous timing decisions of firms in a mixed duopoly, Matsumura and Ogawa (2017a) find that in the presence of a significant negative production externality, firms endogenously choose to compete in Cournot fashion as opposed to Pal (1998) and Matsumura (2003a) among others. Under negative externality, they show that firms make sequential moves when competing in prices, again in contrast to the standard findings in the literature on mixed oligopoly. They conclude that in the presence of a significant negative externality, mixed duopolies behave in the same way as private duopolies in the endogenous timing game. While introducing product differentiation into the model as in Dixit (1979), Matsumura and Ogawa(2017b) studied the endogenous timing decisions of firms in a mixed duopoly in quantity-setting game. They found that two Stackelberg equilibria with either firm acting as a leader exist. However, from the social welfare perspective, it is desirable when public firm acts as a follower. They also show that the equilibrium with public firm acting as the leader is risk-dominant and robust under the high degree of horizontal product differentiation. While competing with foreign-owned private firm, they show that two sequential move equilibria; public leadership and foreign-owned private firm leadership exists. But social welfare is higher under the leadership of public firm and it is risk-dominant and thus a robust equilibrium.[50] In all of the papers cited above, no one study the role of information advantage on the endogenous timing decisions of the firms in the mixed oligopoly. In this chapter, we fill this gap in a mixed duopoly where a public has a foreign-owned private competitor.

In a mixed duopoly, Ogawa and Kazuhiko (2006) study the price setting behavior of firms when they are competing in a homogeneous goods market. Firms are exogenously assigned the roles of leader and the follower. They show that price is

[50] In a different context, Zhang and Li (2013) analyze the timing of location decisions of a public and a private firm in a Hotelling-type model, with firms facing the uncertainty regarding the locations of the consumer. They show that when the degree of uncertainty is too high, both firm delay their entrance to the market while they find that there is no equilibrium accompanied with the small degree of uncertainty.

higher under the leadership of private firm than under the leadership of the public firm. With some parametric restrictions, this price is even higher than the Nash price set simultaneously. However, the public firm sets the same price as set by the private firm irrespective of the role it enjoys. Bárcena-Ruiz (2007) considers the mixed duopoly and differentiated goods market to analyze the endogenous moves of firms in the pricing game. He shows that firms in mixed duopoly set their prices simultaneously as opposed to the private duopoly where firms set their prices sequentially.

3.3 Model

We study a simple mixed duopoly model, where a publicly owned firm (firm 1), has a foreign-owned private competitor. Firms are competing in a homogenous goods market and face a stochastic linear inverse demand function of the following form:

$$p = A + \psi - q_i - q_j, \tag{85}$$

In the equation (85) above, ψ is a stochastic term in nature. Specifically, ψ is a random variable having a continuous c.d.f $\psi \sim F(.)$ with the properties $E[\psi] = 0$ and $var[\psi] = \sigma^2 > 0$.

Both firms are making decisions on the timing of production which can be made in period one (Early) or in period two (Late). In order to analyze the endogenous sequence of moves, we apply the framework of extended games with observable delay developed by Hamilton and Slutsky (1990). In the extended game with observable delay, firms simultaneous decide on the timing of their moves in the pre-stage and then make their production decisions according to the timing committed earlier. A standard Cournot outcome (Early, Early) or (Late, Late) will emerge if both of the firms have opted to produce in period or in period two in the pre-stage. While they will compete in Stackelberg fashion in the output game if they both have committed to produce in different periods.

Since market demand is uncertain, we allow firms to acquire information about market demand through a costly market research as in Gilpatric and Li (2015). If a firm has acquired information, it knows the exact realized value ψ before the start of production period one. Following the literature, we assume that, to the uninformed firm, demand uncertainty is resolved before the start of the period two. So the informed firm has a perishable informational advantage over the uninformed firm. Firms acquire information at a fixed cost $F > 0$ and it also increases the marginal cost of the firm by $k > 0$. Both firms produce their products with quadratic cost function which takes the following form:

$$C_j(q_j) = I_j k q_j + \frac{q_j^2}{2}, \tag{86}$$

where,

$$I_j = \begin{cases} 1, & \text{if firm } j \text{ acquires information} \\ 0, & \text{otherwise} \end{cases} \tag{87}$$

The profit function of firm j is as follows:

$$\Pi_j = \left(A + \psi - q_j - q_i^{'}\right) q_j - C_j(q_j), \tag{88}$$

where,

$$q_i^{'} = \begin{cases} q_i(q_j), & \text{if firm } i \text{ moves after firm } j \\ q_i, & \text{otherwise} \end{cases} \tag{89}$$

Since firm 1 is publically owned, its objective is to maximize social welfare which is the sum of its own profit and consumer surplus. The profit of firm 2 is not included in the social welfare calculations because it is a foreign-owned private firm and it remits all of its profit back to its home country. Hence social welfare is:

$$SW = \Pi_1 + CS \tag{90}$$

We use the expected value of consumer surplus as a measure of consumer welfare irrespective of whether the firms have acquired information about market demand or otherwise.[51] Hence, consumer surplus is written as:

$$E[CS] = E\left(A(q_1 + q_2) - \frac{(q_1 + q_2)^2}{2} - p_1 q_1 - p_2 q_2\right) \tag{91}$$

Using inverse demand function and after simplification, we have:

$$E[CS] = (q_1 + q_2)^2/2 \tag{92}$$

[51] Anam et al. (2007) use expected consumer surplus as a measure of consumer welfare. If consumers are risk-neutral and face a demand whose income elasticity is zero, then Stennek (1999) claim that expected consumer surplus is an appropriate measure for the welfare of consumers in uncertain environments. However, these conditions do not hold empirically all the time. Schlee (2008) also supports this idea that expected consumer surplus is a fairly good measure of consumer welfare under uncertainties.

The figure 2 below explains how the game is being played. In the first stage, which we call as information acquisition stage, firms simultaneously and endogenously decide on to acquire costly information about market demand in a non-cooperative way. At the end of this stage, firms decisions are announced and become common knowledge. As described before, to study the endogenous timing decisions, firms play observable delay games of Hamilton and Slutsky (1990). After the information acquisition stage, firms simultaneously and non cooperatively decide on the timing of their production decisions and strictly commit to it. At the end of this stage (we call it as timing choice or commitment stage), firm's choices on the timings of their production are announced and become common knowledge. In the next stage (production or action stage), firms make their production decisions according to timing choices made earlier. If in the timing choice stage, both of the firms decided to produce early in period one, production takes place only in the period one of the production stage and after that consumers make their purchases. But if they both have opted to produce in a different period, the leader makes his production decisions in the period one and follower produces in the period two of the production stage. In the last stage, consumers make their purchase decisions and the market clears.

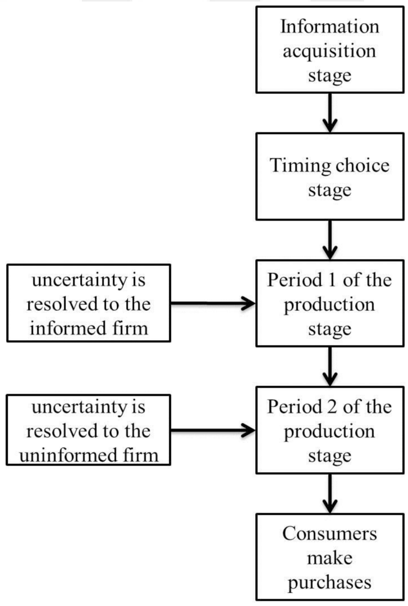

Figure 2: Sequence of events

In the section 3.4 below, we exogenously maintain the assumption that neither of the firms has acquired information (thus both uninformed), while in section 3.5, we exogenously assume that both of the firms have acquired information (thus both informed) and study their endogenous order of moves. In the section 3.6 below, we exogenously assume that only the public firm has acquired costly information and while in the section 3.7, we assume that only the private firm has acquired costly information and we examine the firm's endogenous choices regarding the timings of their moves. We solve the model by using backward induction. In the section 3.8, we study the firm's choices on acquiring costly information.

3.4 No firm has acquired information

In this section, we assume that both firms have not acquired costly information but however, they will learn about the realized value of market demand before the start of the period. In this case, both firms will optimally choose their respective quantities by simultaneously maximizing the expected values of the following pay off functions, if they both decided to produce in period one.

$$E\left(SW\right) = \int \left(A + \psi - q_1 - q_2\right) q_1 dF\left(\psi\right) + E\left(CS\right) - C_1\left(q_1\right), \tag{93}$$

$$E\left(\Pi_2\right) = \int \left(A + \psi - q_1 - q_2\right) q_2 dF\left(\psi\right) - C_2\left(q_2\right), \tag{94}$$

Since both firms have not acquired information, $I_i = 0$ holds and so there are no k terms in the above functions. Taking FOC and simultaneously solving them, we get following optimal quantities, if they both firms have committed to produce in period one:

$$q_1^* = \frac{1}{2}A, \quad q_2^* = \frac{1}{6}A, \tag{95}$$

Their respective ex-ante expected payoffs are:

$$E\left(SW^*\right) = \frac{19}{72}A^2, \tag{96}$$

$$E\left(\Pi_2^*\right) = \frac{1}{24}A^2. \tag{97}$$

If both the firms choose to compete in Cournot fashion in period two, they will learn the actual realization of market demand shock before the start of period two. Simultaneous maximization of above payoff functions by taking into account the realized value of demand shock as ψ_0, we get following optimal quantities and ex-ante expected payoffs:

$$q_1^* = \frac{1}{2}(A + \psi_0), \quad q_2^* = \frac{1}{6}(A + \psi_0), \tag{98}$$
$$E(SW^*) = \frac{19}{72}A^2 + \frac{19}{72}\sigma^2, \tag{99}$$
$$E(\Pi_2^*) = \frac{1}{24}A^2 + \frac{1}{24}\sigma^2. \tag{100}$$

Now suppose that the public firm is committed to produce in period one, while firm 2 produces in period two, they compete in a Stackelberg fashion in which the public firm acts as a leader while firm 2 acts as a follower. Solving from backward induction, we get following optimal quantities:

$$q_1^l = \frac{8}{17}A, \quad q_2^f = \frac{9}{51}A + \frac{1}{3}\psi_0, \tag{101}$$

In this case, their corresponding payoffs in expected terms are as follows:

$$E(SW^*) = \frac{9}{34}A^2 + \frac{1}{18}\sigma^2, \tag{102}$$
$$E(\Pi_2^*) = \frac{27}{578}A^2 + \frac{1}{6}\sigma^2. \tag{103}$$

If the foreign private firm has opted to produce in period one and public firm produces in period two, they compete in a Stackelberg fashion in which private firm acts as a leader, while public firm behaves as a follower. Their optimal quantities and payoffs in expected terms are:

$$q_1^f = \frac{1}{2}(A + \psi_0), \quad q_2^l = \frac{1}{6}A, \tag{104}$$
$$E(SW^*) = \frac{19}{72}A^2 + \frac{1}{4}\sigma^2, \tag{105}$$
$$E(\Pi_2^*) = \frac{1}{24}A^2. \tag{106}$$

The table below summarizes the ex-ante expected payoff to the players.

		Firm 2: Early	Firm 2: Late
Firm 1	Early	$\frac{19A^2}{72}, \frac{A^2}{24}$	$\frac{9A^2}{34} + \frac{\sigma^2}{18}, \frac{27A^2}{578} + \frac{\sigma^2}{6}$
	Late	$\frac{19A^2}{72} + \frac{\sigma^2}{4}, \frac{A^2}{24}$	$\frac{19A^2}{72} + \frac{19\sigma^2}{72}, \frac{A^2}{24} + \frac{\sigma^2}{24}$

Table 8: Payoff matrix when both firms are uninformed

By straightforward calculations based on above payoff table, we find Nash equilibria of endogenous timing game in pure strategies and results are recorded in the following proposition.

Proposition 8 (both firms are not informed) *Given that no firm has acquired costly information, the results of endogenous timing game are : i) there is no equilibrium in which firms produce in the period 1 ii) public firm leadership (Early, Late) appears as a pure strategy equilibrium iff* $0 \leq \sigma^2 \leq A^2/255$*. iii) private firm leadership (Late, Early) emerges as a pure strategy equilibrium iff* $\sigma^2 = 0$*. iv) there is a pure strategy equilibrium with both firms producing in period 2 (Cournot) iff* $\sigma^2 \geq A^2/255$.

Proof. For proof see Proposition 2 in Anam et al. (2007). ■

3.5 Both firms have acquired information

In this section, we assume that both firms have exogenously acquired information ($I_i = 1$ holds for both firms) and are thus informed about the actual realization of market demand before the start of period 1. Hence by knowing exactly, the realization of random intercept of demand as ψ_0, the objective functions of both firms are:[52]

$$SW = (A + \psi_0 - q_1 - q_2)\, q_1 - C_1(q_1) + E(CS), \tag{107}$$

$$\Pi_2 = (A + \psi_0 - q_1 - q_2)\, q_2 - C_2(q_2). \tag{108}$$

Taking FOCs and simultaneously solving them yields the following optimal quantities, if both firms have opted to produce either in period 1 or in period 2:

$$q_1^* = \frac{1}{2}(A - k + \psi_0),\ q_2^* = \frac{1}{6}(A - k + \psi_0), \tag{109}$$

In this case, their respective ex-ante expected payoffs are:

$$E(SW^*) = \frac{19}{72}(A-k)^2 + \frac{19}{72}\sigma^2, \tag{110}$$

$$E(\Pi_2^*) = \frac{1}{24}(A-k)^2 + \frac{1}{24}\sigma^2. \tag{111}$$

If firm 1 (public firm) has opted to produce in period one, while firm 2 committs to produce in period 2, they compete in a Stackelberg fashion in which public firm acts as a leader while foreign-owned private firm acts as a follower. We solve the model

[52]Since the fixed cost of acquiring information F is a sunk cost, so it is excluded from optimization.

by using backward induction and first solve the follower's problem which gives the reaction function of firm 2 as $q_2(q_1) = (A - k + \psi_0 - q_1)/3$. Firm 1 by taking into account the reaction function of firm 2, maximizes its objective function as specified in equation (107). Optimal quantities of the leader and the follower are:

$$q_1^l = \frac{8}{17}(A - k + \psi_0), \; q_2^f = \frac{3}{17}(A - k + \psi_0), \tag{112}$$

Corresponding expected payoffs are as follows:

$$E\left(SW^*\right) = \frac{9}{34}(A-k)^2 + \frac{9}{34}\sigma^2, \tag{113}$$

$$E\left(\Pi_2^*\right) = \frac{27}{578}(A-k)^2 + \frac{27}{578}\sigma^2. \tag{114}$$

If firm 2 has opted to produce in period one and firm 1 chooses to produce in period two, they compete in a Stackelberg fashion with foreign-owned private firm acting as a leader while public firm as the follower. Again by using backward induction, optimal quantities and their respective payoffs in expected terms are as follows: :

$$q_1^f = \frac{1}{2}(A - k + \psi_0), \;\; q_2^l = \frac{1}{6}(A - k + \psi_0), \tag{115}$$

$$E\left(SW^*\right) = \frac{19}{72}(A-k)^2 + \frac{19}{72}\sigma^2, \tag{116}$$

$$E\left(\Pi_2^*\right) = \frac{1}{24}(A-k)^2 + \frac{1}{24}\sigma^2. \tag{117}$$

The table below summarizes the payoff to the players in expected terms.

		Firm 2: Early	Firm 2: Late
Firm 1	Early	$\frac{19(A-k)^2}{72} + \frac{19\sigma^2}{72}, \frac{(A-k)^2}{24} + \frac{\sigma^2}{24}$	$\frac{9(A-k)^2}{34} + \frac{9\sigma^2}{34}, \frac{27(A-k)^2}{578} + \frac{27\sigma^2}{578}$
	Late	$\frac{19(A-k)^2}{72} + \frac{19\sigma^2}{72}, \frac{(A-k)^2}{24} + \frac{\sigma^2}{24}$	$\frac{19(A-k)^2}{72} + \frac{19\sigma^2}{72}, \frac{(A-k)^2}{24} + \frac{\sigma^2}{24}$

Table 9: Payoff matrix when both firms are informed

The following proposition presents the main result of this section about Nash equilibria in pure strategies in endogenous timing game.

Proposition 9 (both firms have acquired information) *Given that both firms have acquired information, then the results of endogenous timing game are: i) simultaneous moves cannot be sustained as an equilibrium ii) there are two Stackelberg equilibria with either firm acting as a leader iff* $\sigma^2 \geq 0$.

Proof. i) (Early, Early) can not be an equilibrium because the foreign-owned private firm gets higher payoff by deviating to late and its incermental payoff is $\Pi_2^*(Early, Late) - \Pi_2^*(Early, Early) = 35((A-k)^2 + \sigma^2)/6936 > 0$ and in this case public firm is indifferent between playing Early and Late given that foreign-owned private firm is playing Early since $SW^*(Early, Early) = SW^*(Late, Early)$. Similarly (Late, Late) is not an equilibrium because public firm has incentive to deviate and its incremental payoff is $SW^*(Early, Late) - SW^*(Late, Late) = ((A-k)^2 + \sigma^2)/1224 > 0$ and foreign-owned private firm remains indifferent between playing Late and Early since $\Pi_2^*(Late, Late) = \Pi_2^*(Late, Early)$.

ii) Straightforward calculations reveal that (Early, Late) is indeed an equilibrium since no firm has the incentive to deviate. Deviation payoff to firm 1 is $SW^*(Late, Late) -$ $SW^*(Early, Late) = -((A-k)^2 + \sigma^2)/1224 < 0$ and deviation payoff to firm 2 is $\Pi_2^*(Early, Early) - \Pi_2^*(Early, Late) = -(35((A-k)^2 + \sigma^2)/6936) < 0$. Playing (Late,Early) is another equilibrium since deviation does not strictly benefit either of the firms because $SW^*(Early, Early) = SW^*(Late, Early)$ and $\Pi_2^*(Late, Late) = \Pi_2^*(Late, Early)$. ■

3.6 Only public firm has acquired information

In this section, we assume that only public firm has acquired information ($I_1 = 1$) and is thus informed about market demand realizations before the start of period one and firm 2 having information disadvantage ($I_2 = 0$) over the public firm, will maximize the expected profit if it chooses to produce in period one. Objective functions of the firms will be as follows:

$$SW = (A + \psi_0 - q_1 - q_2)\, q_1 - C_1(q_1) + E(CS), \tag{118}$$

$$E(\Pi_2) = \int (A + \psi - q_1 - q_2)\, q_2 dF(\psi) - C_2(q_2). \tag{119}$$

Given that only public firm has acquired information, their optimal quantities and payoffs in expected terms are as follows, if both the firms have opted to produce early in period one:

$$q_1^* = \frac{1}{2}(A - k + \psi_0), \quad q_2^* = \frac{1}{6}(A + k), \tag{120}$$

$$E(SW^*) = \frac{1}{72}(19A^2 - 34Ak + 19k^2) + \frac{18}{72}\sigma^2, \tag{121}$$

$$E(\Pi_2^*) = \frac{1}{24}(A + k)^2. \tag{122}$$

Since at the start of period two, firm 2 will also learn the realized value of market demand shock, they will choose following optimal quantities while competing in Cournot fashion in period two.

$$q_1^* = \frac{1}{2}\left(A - k + \psi_0\right), \quad q_2^* = \frac{1}{6}\left(A + k + \psi_0\right), \tag{123}$$

Their corresponding payoffs in expected terms are as follows:

$$E\left(SW^*\right) = \frac{1}{72}(19A^2 - 34Ak + 19k^2) + \frac{19}{72}\sigma^2, \tag{124}$$

$$E\left(\Pi_2^*\right) = \frac{1}{24}(A + k)^2 + \frac{1}{24}\sigma^2. \tag{125}$$

If the public firm is committed to produce early in period one and firm 2 has opted to produce late in period two, they compete in a Stackelberg fashion in which public firm acts as a leader while firm 2 follows. Their optimal quantities are:

$$q_1^l = \frac{1}{17}\left(8A - 9k + 8\psi_0\right), \; q_2^f = \frac{3}{17}\left(A + k + \psi_0\right). \tag{126}$$

In this case, their respective ex-ante expected payoffs are written as:

$$E\left(SW^*\right) = \frac{1}{34}(9A^2 - 16Ak + 9k^2) + \frac{9}{34}\sigma^2, \tag{127}$$

$$E\left(\Pi_2^*\right) = \frac{27}{578}(A + k)^2 + \frac{27}{578}\sigma^2. \tag{128}$$

Now if the private firm has opted to produce early in period one and public firm commits to produce late in period two, they compete in a Stackelberg fashion in which private firm acts as a leader while public firm behaves as a follower. Here, the private firm will maximize its expected profit by taking into account the reaction function of the public firm. The optimal quantities and expected payoffs are:

$$q_1^f = \frac{1}{2}\left(A - k + \psi_0\right), \quad q_2^l = \frac{1}{6}\left(A + k\right), \tag{129}$$

$$E\left(SW^*\right) = \frac{1}{72}\left(19A^2 - 34Ak + 19k^2\right) + \frac{18}{72}\sigma^2, \tag{130}$$

$$E\left(\Pi_2^*\right) = \frac{1}{24}(A + k)^2. \tag{131}$$

The table below shows the ex-ante expected payoffs to the players.

We find pure strategy Nash equilibria of the endogenous timing game and summarize the discussion of this section in the proposition below.

Proposition 10 (only public firm has acquired information) *Given that only public firm has acquired costly information, then in the endogenous timing game i)*

		Firm 2	
		Early	Late
Firm 1	Early	$\frac{(19A^2-34Ak+19k^2)}{72}+\frac{18\sigma^2}{72}$, $\frac{(A+k)^2}{24}$	$\frac{(9A^2-16Ak+9k^2)}{34}+\frac{9\sigma^2}{34}$, $\frac{27(A+k)^2}{578}+\frac{27\sigma^2}{578}$
	Late	$\frac{(19A^2-34Ak+19k^2)}{72}+\frac{18\sigma^2}{72}$, $\frac{(A+k)^2}{24}$	$\frac{(19A^2-34Ak+19k^2)}{72}+\frac{19\sigma^2}{72}$, $\frac{(A+k)^2}{24}+\frac{\sigma^2}{24}$

Table 10: Payoff matrix when only the public firm is informed

simultaneous move cannot be sustained as an equilibrium outcome. ii) there is a pure strategy Stackelberg equilibrium in which the public firm acts as a leader iff $\sigma^2 \geq 0$. *iii) there is another pure strategy Stackelberg equilibrium with private firm acting as a leader iff* $\sigma^2 = 0$.

Proof. i) (Early, Early) is not an equilibrium since the foreign-owned private firm has the incentive to deviate and can get higher payoffs. Its incremental payoff is $\Pi_2^*(Early, Late) - \Pi_2^*(Early, Early) = (35(A+k)^2 + 324\sigma^2)/6936 > 0$. However, the public firm is indifferent between playing Early and Late since $SW^*(Early, Early)$ $= SW^*(Late, Early)$. (Late, Late) is not an equilibrium since public firm has incentive to deviate and incremental payoff to public firm is $SW^*(Early, Late) - SW^*(Late, Late) = ((A+k)^2 + \sigma^2)/1224 > 0$. While in this case, foreign-owned private firm has no incentive to deviate because $\Pi_2^*(Late, Late) > \Pi_2^*(Late, Early)$.

ii) (Early, Late) is an equilibrium since no firm has the incentive to deviate. Deviation payoff to public firm is $SW^*(Late, Late) -$ $SW^*(Early, Late) = -((A+k)^2 + \sigma^2)/1224 < 0$ and deviation payoff to firm 2 is $\Pi_2^*(Early, Early) - \Pi_2^*(Early, Late) = -(35(A+k)^2 + 324\sigma^2)/6936 < 0$.

iii) For (Late, Early) to be an equilibrium, we require that both firms have no incentives to deviate unilaterally. Since $SW^*(Late, Early) = SW^*(Early, Early)$, deviation does not strictly benefit to the public firm because it is indifferent between playing Late and Early given that foreign-owned private firm is playing Early. And foreign firm does not prefer to deviate as long as $\sigma^2 = 0$ because $\Pi_2^*(Late, Late) - \Pi_2^*(Late, Early) = \sigma^2/4$. ■

3.7 Only foreign-owned private firm has acquired information

In this section, we assume that only private firm has acquired costly information ($I_2 = 1$) and it knows the exact realization of the market demand shock. While public firm having information disadvantage ($I_1 = 0$), will learn the realized value of demand shock before the start of period two if it opts to defer its production to period two. If both the firms have committed to produce early in period one, their objective functions, in this case, can be written as:

$$E(SW) = \int (A + \psi - q_1 - q_2) q_1 dF(\psi) - C_1(q_1) + E(CS), \tag{132}$$

$$\Pi_2 = (A + \psi_0 - q_1 - q_2) q_2 - C_2(q_2). \tag{133}$$

Since firms are playing Cournot game (Early, Early), the public firm will maximize its payoff in expected terms while the foreign-owned private firm will maximize its payoff in actual terms since it knows the exact market demand shock. Following are the optimal quantities and their corresponding ex-ante expected payoffs, if both of the firms have opted to produce early in period one:

$$q_1^* = \frac{1}{2}A, \quad q_2^* = \frac{1}{6}(A - 2k + 2\psi_0), \tag{134}$$

$$E(SW^*) = \frac{1}{72}(19A^2 - 4Ak + 4k^2) + \frac{4}{72}\sigma^2, \tag{135}$$

$$E(\Pi_2^*) = \frac{1}{24}(A - 2k)^2 + \frac{4}{24}\sigma^2. \tag{136}$$

But if the firms simultaneously produce in period two and play Cournot game (Late, Late), the public firm will also learn the realized value of demand shock as ψ_0 before the start of period two. Simultaneous maximization yields the following optimal quantities and corresponding payoffs in expected terms:

$$q_1^* = \frac{1}{2}(A + \psi_0), \; q_2^* = \frac{1}{6}(A - 2k + \psi_0), \tag{137}$$

$$E(SW^*) = \frac{1}{72}(19A^2 - 4Ak + 4k^2) + \frac{19}{72}\sigma^2, \tag{138}$$

$$E(\Pi_2^*) = \frac{1}{24}(A - 2k)^2 + \frac{1}{24}\sigma^2. \tag{139}$$

If the public firm has committed to produce early in period one, while foreign-owned private firm produces in period two, the public firm acts as a Stackelberg leader, while firm acts as Stackelberg follower. The public firm maximizes the expected value of social surplus while taking into account the optimal response function of the foreign private firm. solving the problem from backward induction, we find following optimal quantities and expected payoffs.

$$q_1^l = \frac{1}{17}\left(8A + k\right), \quad q_2^f = \frac{3}{17}\left(A - 2k\right) + \frac{1}{3}\psi_0, \tag{140}$$

$$E\left(SW^*\right) = \frac{1}{306}(18A^2 - 18Ak + 18k^2) + \frac{17}{306}\sigma^2, \tag{141}$$

$$E\left(\Pi_2^*\right) = \frac{1}{1734}(9A - 18k)^2 + \frac{289}{1734}\sigma^2. \tag{142}$$

The case, where foreign-owned private firm commits to produce early in period one while acting as a Stackelberg leader and public firm acting as a Stackelberg follower we get optimal leader-follower quantities and expected payoff as:

$$q_1^f = \frac{1}{2}\left(A + \psi_0\right), \quad q_2^l = \frac{1}{6}(A - 2k + \psi_0), \tag{143}$$

$$E\left(SW^*\right) = \frac{1}{72}\left(19A^2 - 4Ak + 4k^2\right) + \frac{19}{72}\sigma^2, \tag{144}$$

$$E\left(\Pi_2^*\right) = \frac{1}{24}(A - 2k)^2 + \frac{1}{24}\sigma^2. \tag{145}$$

Since public firm will learn the realized value of market demand shock before the start of period two, its optimal quantity contains ψ_0. Following table summarizes the discussion of this section and presents the expected payoffs to the players. Nash equilibria of endogenous timing game in this case are recorded in the following proposition.

		Firm 2: Early	Firm 2: Late
Firm 1	Early	$\frac{(19A^2-4Ak+4k^2)}{72} + \frac{4\sigma^2}{72}$, $\frac{(A-2k)^2}{24} + \frac{4\sigma^2}{24}$	$\frac{(18A^2-18Ak+18k^2)}{306} + \frac{17\sigma^2}{306}$, $\frac{(9A-18k)^2}{1734} + \frac{289\sigma^2}{1734}$
	Late	$\frac{\left(19A^2-4Ak+4k^2\right)}{72} + \frac{19\sigma^2}{72}$, $\frac{(A-2k)^2}{24} + \frac{\sigma^2}{24}$	$\frac{(19A^2-4Ak+4k^2)}{72} + \frac{19\sigma^2}{72}$, $\frac{(A-2k)^2}{24} + \frac{\sigma^2}{24}$

Table 11: Payoff matrix when only the foreign-owned private firm is informed

Proposition 11 (only foreign-owned private firm has acquired information) *Assuming that only foreign-owned private firm has acquired costly information, then the results of the endogenous timing game are: i) there is no equilibrium in which firms produce simultaneously in period 1 (Early, Early). ii) there is a pure strategy Stackelberg equilibrium in which public firm acts as a leader iff* $\sigma^2 \leq (A - 2k)^2/255$. *iii) there is a pure strategy Stackelberg equilibrium with private firm acting as a leader iff* $\sigma^2 \geq 0$. *iv) there is another equilibrium in pure strategies where firms compete in Cournot fashion in period 2 (Late, Late) iff* $\sigma^2 \geq (A - 2k)^2/255$.

Proof. i) Playing Cournot in period 1 (Early, Early) is not an equilibrium because both firms have incentives to deviate and can get higher payoffs. The incremental payoff to the public firm is $SW^* \left(Late, Early\right) - SW^* \left(Early, Early\right) = 5\sigma^2/24 > 0$. By deviating to late foreign-owned private firm's incremental payoff is $\Pi_2^* \left(Early, Late\right) - \Pi_2^* \left(Early, Early\right) = \left(35(A-2k)^2\right)/6936 > 0$.

ii) For (Early, Late) to be an equilibrium we require that both firms have no incentives to deviate. Firm 2 has no incentive to deviate because its incremental payoff is $\Pi_2^* \left(Early, Early\right) - \Pi_2^* \left(Early, Late\right) = -\left(35(A-2k)^2\right)/6963 < 0$. And by deviating, firm 1 receives $SW^*(Late, Late) -$ $SW^*(Early, Late) = (225\sigma^2 - (A - 2k)^2)/1224$ which is ≤ 0, iff $\sigma^2 \leq (A-2k)^2/255$.

iii) Playing (Late,Early) is indeed an equilibrium since deviation does not benefit either of the firms. By deviating firm 1 receives the incremental payoff of $SW^*(Early, Early) -$ $SW^*(Late, Early) = -5\sigma^2/24 < 0$. While the foreign-owned private firm is indifferent between playing Late and Early because $\Pi_2^*(Late, Late) = \Pi_2^*(Late, Early) = ((A-2k)^2 + \sigma^2)/24$.

iv) Playing Cournot in period 2 (Late, Late) to be an equilibrium, we require that both firms do not deviate (unilaterally). Incremental payoff to firm 1 is $SW^*(Early, Late) - SW^*(Late, Late) = (A-2k)^2/1224 - 5\sigma^2/24$ which is ≤ 0 iff $\sigma^2 \geq (A-2k)^2/255$ and given that public firm plays Late, foreign-owned private firm is indifferent between playing Late and Early because $\Pi_2^*(Late, Early) = \Pi_2^*(Late, Late) = ((A-2k)^2 + \sigma^2)/24$, and thus it has no incentive to deviate. ■

3.8 Information acquisition

In this section, we endogenize the decisions of the firms to acquire costly information and discuss the incentives of the firms to engage in acquiring such information. Following Gilpatric and Li (2015), we add an additional stage to the model that appears before the stage in which firms decide on the timing of the move. In order to derive the results of the endogenous information acquisition stage, we divide the variance of demand shock into three regions. First region where $0 \leq \sigma^2 < (A-2k)^2/255$, both types of Stackelberg equilibria (Early, Late) and (Late, Early) exists in all of the four cases discussed in the previous sections. In the second region, where variance lies between $(A-2k)^2/255$ and $A^2/255$ ($(A-2k^2)/255 \leq \sigma^2 < A^2/255$), both Stackelberg equilibria (Early, Late) and (Late, Early) exists following both firms have acquired information, (Early, Late) exists in the cases following neither of the firms has acquired information and only public firm has acquired information. While in this region (Late, Early) and

(Late, Late) exists in the case where the only private firm has acquired information.

In the last region, where firms face high uncertainty, specifically when the variance of demand shock $\sigma^2 \geq A^2/255$, two Stackelberg equilibria (Early, Late) and (Late, Early) with either firm acting as leader exists following both firms have acquired information. While, following that neither of the firms acquires information, they play Cournot in period two (Late, Late) in this range and following that only public firm has acquired information, (Early, Late) exists and given that only private firm has acquired information, (Late, Early) and (Late, Late) exists in this range. Checking all the possibilities, we derive the following result regarding Nash equilibria in pure strategies in the information acquisition stage.

Proposition 12 (endogenous information acquisition) *The results of the overall game are i) there is no pure strategy equilibrium where both firms acquire information. ii) there is a pure strategy equilibrium where only public firm acquires information (acquire, don't acquire) and subsequently playing in a Stackelberg fashion with public firm acting as a leader (Early, Late) iff* $\sigma^2 \geq max\{-A^2 + 1224F + 576Ak - 324k^2, 9\left(34F + 16Ak - 9k^2\right)/64, \left(-A^2 + 1224F + 576Ak - 324k^2\right)/18\}$ *iii) there is no pure strategy equilibrium where only foreign-owned private firm acquires information. So (don't acquire, acquire) cannot be sustained as an equilibrium. iv) there is a pure strategy equilibrium where neither of the firms acquires information (don't acquire, don't acquire) and (a) play Cournot in the second period (Late, Late) iff* $A^2/255 \leq \sigma^2 < -A^2 + 1224F + 576Ak - 324k^2$*. (b) play sequentially in which public firm acts a leader (Early, Late) iff* $0 \leq \sigma^2 < min\left\{\left(A^2 + 1224F + 578Ak - 323k^2\right)/238, 9\left(34F + 16Ak - 9k^2\right)/64, A^2/255\right\}$*. (c) play sequentially in which private firm acts a leader (Late, Early) iff* $\sigma^2 \leq min\{0, 4\left(6F + Ak - k^2\right), (-35A^2 + 6936F + 1296Ak - 1296k^2)/1156, (-A^2 + 1224F + 576Ak - 324k^2)/18\}$*.*

3.9 Conclusion

In this paper, we study the role of information regarding uncertain market demand in the mixed duopoly, where information advantaged firm may have incentives to become the market leader in the quantity-setting game. We consider a mixed duopoly market structure where a publicly-owned firm is competing against a foreign-owned private firm. The objective of the public firm is to maximize the social welfare, while foreign-owned private firm maximizes its own profit. Firms produce homogeneous goods by using quadratic cost function. The market demand is stochastic and if firms make their output decisions in the first period without having any information about market demand, they, being risk-neutral maximize the expected values of their respective objective functions. Following the literature, we assume that exact market demand is revealed to both firms before the start of the second period thus one firm may have a perishable information advantage over the other. In order to study the endogenous sequence of moves, we apply the framework of extended games with observable delay developed by Hamilton and Slutsky (1990).

We show that when both firms are exogenously assumed to have acquired costly information, two types of Stackelberg equilibria with either firm acting as a market leader coexist. Given that no firm has acquired information, an equilibrium with public firm leadership exists under a mild degree of variance of demand. However, under high uncertainty of demand, firms endogenously choose to produce in the second period while competing in a Cournot fashion. In this case, there is another Stackelberg equilibrium with private firm leadership but it exists only when there is no uncertainty regarding demand. In case of information asymmetry, when only the public firm is assumed to have acquired costly information, a Stackelberg equilibrium with public firm acting as a market leader always exists but equilibrium with foreign-owned private firm leadership exists only when there is no uncertainty regarding market demand. Under asymmetric information situations, it is not a strictly dominant strategy for information advantaged firm to move early as opposed to the case of private duopolies as shown by Gilpatric and Li (2015). When only the private firm is assumed to have acquired information, both Stackelberg equilibria co-exist only for a smaller degree of demand uncertainty. However, in this case, under high variance of the demand shock, Stackelberg equilibrium with foreign-owned private firm leadership coexists with a Cournot equilibrium in period two.

The results of the overall game by adding information acquisition stage reveal that it is not optimal for both firms to acquire costly information which is in contrast to the profit-maximizing duopoly case. We find that under high uncertainty, only the public firm acquires information and becomes the leader of the market. So in the presence of high uncertainty, an early signal of market demand helps the public firm to endogenously act as a market leader. However, under low variance of the demand shock, no firm acquires information and subsequently, Stackelberg equilibrium with public firm leadership emerges. There is another equilibrium where no firm acquires information and then compete in a Stackelberg fashion with foreign-owned private firm acting as a leader but only when there is no uncertainty regarding market demand. We show that there is no equilibrium where only the foreign-owned private firm acquires information.

In this paper we allow one foreign-owned private firm to compete against the public firm, and our model can be extended by adding a domestic private firm. We work out with linear demand function, however, it remains to see whether our results hold or otherwise by using a more general demand function. Another way to extend our model is to introduce partial privatization of the public firm and to see whether endogenous sequencing or the incentives for acquiring information change or not. In the present model, firms are producing homogeneous products, what happens when they are competing in a differentiated goods market is another question to explore. In this paper, firms produce by using quadratic cost functions and are perfectly informed about their own as well as their rival's cost, adding uncertainty regarding own cost and the rival's cost as in Albaek (1990) and studying the timing decisions of firms is an important to explore which is left for future research.

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3.A Appendix: Proof of Proposition 12 (endogenous information acquisition)

Here, we sketch the proof of this proposition.

Case 1 ($\sigma^2 \geq A^2/255$): As described in the main text, when variance of demand shock is $\sigma^2 \geq A^2/255$, following that both firms have acquired information, both Stackelberg equilibria (Early, Late) and (Late, Early) exist. While, following that neither of the firms has acquired information, they play cournot in period two (Late, Late) in this range and given that only the public firm has acquired information, only one Stackelberg equilibrium (Early, Late) exists in this range. Following that only private firm have acquired information, two equilibria (Late, Early) and (Late, Late) exist in this range. Then we have following four payoff tables:

Firm 1 \ Firm 2	acquire	don't acquire
acquire	$\frac{9(A-k)^2}{34} + \frac{9\sigma^2}{34} - F,\ \frac{27(A-k)^2}{578} + \frac{27\sigma^2}{578} - F$	$\frac{(9A^2-16Ak+9k^2)}{34} + \frac{9\sigma^2}{34} - F,\ \frac{27(A+k)^2}{578} + \frac{27\sigma^2}{578}$
don't acquire	$\frac{(19A^2-4Ak+4k^2)}{72} + \frac{19\sigma^2}{72},\ \frac{(A-2k)^2}{24} + \frac{\sigma^2}{24} - F$	$\frac{19A^2}{72} + \frac{19\sigma^2}{72},\ \frac{A^2}{24} + \frac{\sigma^2}{24}$

Firm 1 \ Firm 2	acquire	don't acquire
acquire	$\frac{9(A-k)^2}{34} + \frac{9\sigma^2}{34} - F,\ \frac{27(A-k)^2}{578} + \frac{27\sigma^2}{578} - F$	$\frac{(9A^2-16Ak+9k^2)}{34} + \frac{9\sigma^2}{34} - F,\ \frac{27(A+k)^2}{578} + \frac{27\sigma^2}{578}$
don't acquire	$\frac{(19A^2-4Ak+4k^2)}{72} + \frac{19\sigma^2}{72},\ \frac{(A-2k)^2}{24} + \frac{\sigma^2}{24} - F$	$\frac{19A^2}{72} + \frac{19\sigma^2}{72},\ \frac{A^2}{24} + \frac{\sigma^2}{24}$

Firm 1 \ Firm 2	acquire	don't acquire
acquire	$\frac{19(A-k)^2}{72} + \frac{19\sigma^2}{72} - F,\ \frac{(A-k)^2}{24} + \frac{\sigma^2}{24} - F$	$\frac{(9A^2-16Ak+9k^2)}{34} + \frac{9\sigma^2}{34} - F,\ \frac{27(A+k)^2}{578} + \frac{27\sigma^2}{578}$
don't acquire	$\frac{(19A^2-4Ak+4k^2)}{72} + \frac{19\sigma^2}{72},\ \frac{(A-2k)^2}{24} + \frac{\sigma^2}{24} - F$	$\frac{19A^2}{72} + \frac{19\sigma^2}{72},\ \frac{A^2}{24} + \frac{\sigma^2}{24}$

Firm 1 \ Firm 2	acquire	don't acquire
acquire	$\frac{19(A-k)^2}{72} + \frac{19\sigma^2}{72} - F,\ \frac{(A-k)^2}{24} + \frac{\sigma^2}{24} - F$	$\frac{(9A^2-16Ak+9k^2)}{34} + \frac{9\sigma^2}{34} - F,\ \frac{27(A+k)^2}{578} + \frac{27\sigma^2}{578}$
don't acquire	$\frac{(19A^2-4Ak+4k^2)}{72} + \frac{19\sigma^2}{72},\ \frac{(A-2k)^2}{24} + \frac{\sigma^2}{24} - F$	$\frac{19A^2}{72} + \frac{19\sigma^2}{72},\ \frac{A^2}{24} + \frac{\sigma^2}{24}$

The difference between above two tables originates from the payoffs following (acquire, acquire) and following (don't acquire, acquire). In the first two table, payoffs correspond to the (Early, Late) and in the last two tables, it corresponds to (Late, Early) equilibrium following (acquire, acquire). While, following (don't acquire, acquire), in the first and third tables payoffs corresponds to (Late, Early) equilibrium and in the second and fourth tables payoffs corresponds to (Late, Late) equilibrium. Working with these payoffs tables, we find that there is no pure strategy equilibrium where both firms acquire information. There is a pure strategy equilibrium where only the public firm acquires information and subsequently, play (Early, Late) (public firm acting as a leader) iff $\sigma^2 \geq \max\{-A^2 + 1224F + 576Ak - 324k^2, A^2/255\}$. There is no pure strategy equilibrium where only the foreign-owned private firm acquires information in this range. There is a pure strategy equilibrium where neither of the firms acquires information and play Cournot in the second period (Late , Late) iff $A^2/255 \leq \sigma^2 \leq -A^2 + 1224F + 576Ak - 324k^2$.

Case 2 ($(A-2k)^2/255 \leq \sigma^2 < A^2/255$): In this case, following that both of the firms have not acquired information, they play (Early, Late) instead of (Late, Late). That is the only difference between this case and the case 1 discussed above. So will have following four payoffs matrices.

		Firm 2	
		acquire	don't acquire
Firm 1	acquire	$\frac{9(A-k)^2}{34} + \frac{9\sigma^2}{34} - F,\ \frac{27(A-k)^2}{578} + \frac{27\sigma^2}{578} - F$	$\frac{(9A^2-16Ak+9k^2)}{34} + \frac{9\sigma^2}{34} - F,\ \frac{27(A+k)^2}{578} + \frac{27\sigma^2}{578}$
	don't acquire	$\frac{(19A^2-4Ak+4k^2)}{72} + \frac{19\sigma^2}{72},\ \frac{(A-2k)^2}{24} + \frac{\sigma^2}{24} - F$	$\frac{9A^2}{34} + \frac{\sigma^2}{18},\ \frac{27A^2}{578} + \frac{\sigma^2}{6}$

		Firm 2	
		acquire	don't acquire
Firm 1	acquire	$\frac{9(A-k)^2}{34} + \frac{9\sigma^2}{34} - F,\ \frac{27(A-k)^2}{578} + \frac{27\sigma^2}{578} - F$	$\frac{(9A^2-16Ak+9k^2)}{34} + \frac{9\sigma^2}{34} - F,\ \frac{27(A+k)^2}{578} + \frac{27\sigma^2}{578}$
	don't acquire	$\frac{(19A^2-4Ak+4k^2)}{72} + \frac{19\sigma^2}{72},\ \frac{(A-2k)^2}{24} + \frac{\sigma^2}{24} - F$	$\frac{9A^2}{34} + \frac{\sigma^2}{18},\ \frac{27A^2}{578} + \frac{\sigma^2}{6}$

		Firm 2	
		acquire	don't acquire
Firm 1	acquire	$\frac{19(A-k)^2}{72} + \frac{19\sigma^2}{72} - F,\ \frac{(A-k)^2}{24} + \frac{\sigma^2}{24} - F$	$\frac{(9A^2-16Ak+9k^2)}{34} + \frac{9\sigma^2}{34} - F,\ \frac{27(A+k)^2}{578} + \frac{27\sigma^2}{578}$
	don't acquire	$\frac{(19A^2-4Ak+4k^2)}{72} + \frac{19\sigma^2}{72},\ \frac{(A-2k)^2}{24} + \frac{\sigma^2}{24} - F$	$\frac{9A^2}{34} + \frac{\sigma^2}{18},\ \frac{27A^2}{578} + \frac{\sigma^2}{6}$

		Firm 2	
		acquire	don't acquire
Firm 1	acquire	$\frac{19(A-k)^2}{72} + \frac{19\sigma^2}{72} - F,\ \frac{(A-k)^2}{24} + \frac{\sigma^2}{24} - F$	$\frac{(9A^2-16Ak+9k^2)}{34} + \frac{9\sigma^2}{34} - F,\ \frac{27(A+k)^2}{578} + \frac{27\sigma^2}{578}$
	don't acquire	$\frac{(19A^2-4Ak+4k^2)}{72} + \frac{19\sigma^2}{72},\ \frac{(A-2k)^2}{24} + \frac{\sigma^2}{24} - F$	$\frac{9A^2}{34} + \frac{\sigma^2}{18},\ \frac{27A^2}{578} + \frac{\sigma^2}{6}$

Working with these four payoff tables we find that again, there is no pure strategy equilibrium where both firms acquire information. There is a pure strategy equilibrium where only the public firm acquires information and then firms play (Early, Late) iff $\max\{9(34F + 16Ak - 9k^2)/64, (A-2k)^2/255\} \leq \sigma^2 < A^2/255$. In this range of variance, there is no pure strategy equilibrium where only the foreign-owned private firm acquires information. there is a pure strategy equilibrium where neither of the firms acquires information and play (Early, Late) iff $(A-2k)^2/255 \leq \sigma^2 \leq \min\{9(34F + 16Ak - 9k^2)/64, A^2/255\}$. Similarly, we can derive conditions for the last region of variance $(0 \leq \sigma^2 < (A-2k)^2/255)$ and combining all the conditions, we get the Proposition 12.