ESSAYS IN OPTIMAL TAXATION

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ABSTRACT

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Keywords: optimal capital taxation, entrepreneurial investment,wealth creation, productive efficiency

The mismatch between wealth level and entrepreneurial ability creates productive inefficiency in the market and under financial frictions, the credit market may not be able to solve this problem. In such a world, decreasing wealth inequality with capital taxation might be helpful in bringing the economy to its production possibilities frontier but capital taxation is costly as it distorts wealth creation. I derive optimal capital tax formulas that reflect this trade-off between productive efficiency benefit and wealth accumulation cost of capital taxes. With numerical analysis, I show that when the mismatch is high, government tends to intervene in the economy more and optimal capital taxes are higher. Also, having a lower elasticity of labor with respect to wage for agents leads to higher optimal taxation rate.

ÖZET

OPTIMAL VERGİLENDİRME ALANINDA MAKALELER

MEHMET TAYYİP DEMİR

EKONOMİ YÜKSEK LİSANS TEZİ, HAZİRAN 2019

Tez Danışmanı: Doç. Dr. HAKKI YAZICI

Anahtar Kelimeler: optimal sermaye vergilendirmesi, girişimci yatırımı, varlık oluşumu, üretim etkinliği

Varlık seviyesi ve girişimci yeteneğindeki uyumsuzluk, üretimsel etkinsizliğe yol açmaktadır. Finansal sürtüşmeler altında, kredi piyasası bu problemi çözemeyebilir. Bu durumda sermaye vergilendirmesi yoluyla varlık eşitsizliğini azaltmak, ekonomiyi üretim olanakları eğrisine ulaşmaya yardımcı olabilir. Ancak sermaye vergilendirmesi aynı zamanda varlık birikiminde bozulmalara yol açmaktadır. Üretimsel etkinlik-varlık birikimi bozulması arasındaki bu ödünleşimi gösteren optimal sermaye vergilendirilmesi bu çalışmada elde edilmiştir. Sayısal analizler ışığında, uyumsuzluk arttıkça devlet ekonomiye daha fazla müdahale etmektedir ve optimal sermaye vergisi artmaktadır. Ayrıca kişilerin ücrete göre çalışma esnekliği azaldığı durumda optimal vergilendirme artış göstermektedir.

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To My Little Brother

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1. INTRODUCTION

The role of governments has become even more significant due to the rise in economic inequalities all around the world. And taxation is a natural policy outlet for governments who are concerned about inequality. Many policy makers and governments are willing to deal with this problem. However, attempting to lower inequality through taxes entails productive inefficiency, which gives rise to the well-known equity-efficiency trade-off. Economists have worked on various aspects of this trade-off[1]. In this paper, we aim to focus on a relatively new role of taxation.

The contribution of entrepreneurs is inevitably vital for economic growth. As Kitao (2008) expresses, entrepreneurs have 40% of the wealth in the economy and produce 50% of the total output. They start businesses if the resources are adequate to create. For those who suffer from the lack of resources, cannot start their own businesses without the credit markets' support. However, when the financial markets do not work properly, entrepreneurs cannot get loans from financial intermediaries which limits the entrepreneurial investment. As De Nardi, Doctor, and Krane (2007) suggest, market frictions create economically inefficient investment levels. Hence, solving this inefficiency due to the imperfect correlation of entrepreneurial ability and wealth is a prominent issue for governments. With the presence of taxation, this problem can be achieved. In such a world, decreasing wealth inequality by means of taxation is a helpful way of bringing the economy to its production possibilities frontier. On the other hand, taxing capital is of course also costly as it distorts the wealth creation of agents. The optimal capital tax rate in my model is the one that optimally trades off the efficiency benefit and wealth creation distortions of capital taxation. I derive optimal tax formulas that summarize this trade-off.

[1] The prominent papers that analyse this trade-off are Mirrlees (1971), Diamond (1998) and Saez (2001).

In this set-up, the agents live two periods and have different abilities to generate wealth and entrepreneurial ability. The first period is the time when individuals create their wealth and in the second period, agents make an investment decision among capital goods, risk-free assets and entrepreneurial ability. As a real-world implication, the first period can be seen as the time after individuals graduate and start to work in order to create wealth. Or, it can be considered as the time when the old generation works and bequeaths it to his/her child who will invest the capital goods later on. However, agents who have ideas may not have enough wealth level which leads to investing inadequately as there are financial frictions. The government enters the story here. As financial frictions are presented, with the transfers to individuals, the binding agents can be better-off. The crucial point is the timing of the transfers. In order for transfers to be effectively used, they are distributed before the investment decision is made. In that way, poor agents with ideas can use them on entrepreneurial activity. Chang, Chang, and Kim (2018) indicate that taxing wealth, capital or both of them is widely used by governments so that the redistribution, to decrease inequality, is achieved. In this paper, I focus on capital taxation to finance lump-sum transfers.

My results suggest that there is no trade-off between productive efficiency and wealth creation distortion in case of exogenous wealth. As there are no distortions from taxing agents, government imposes taxes until it reaches the policy where every individual becomes unconstrained. Also, using the concepts of elasticity of wealth and entrepreneurial activity with respect to after-tax rates of each capital, optimal marginal taxation for entrepreneurial investment and risk-free investment are derived. The behavioural and mechanical responses of marginal taxation is shown in detail.

My main results are as follows. The optimal tax formulas indicate that when the elasticity of entrepreneurial investment with respect to after-tax rate of entrepreneurial investment is high, marginal tax on entrepreneurial investment is lower while the tax is higher when the elasticity of entrepreneurial investment with respect to after-tax rate of risk-free assets is high. Lastly, with quantitative analysis, I demonstrate that in the economy where the correlation between initial productivity to create wealth and entrepreneurial ability is high, government tends to intervene in the economy less than the economy in which the distribution of initial productivity and ability is independent. In high correlation economy, the transfers to individuals and taxation are quantitatively lower than the low correlation economy implying that as the mismatch is getting lower, laissez-faire economy becomes efficient and no government intervention is needed. In other words, when the mismatch between wealth level and entrepreneurial ability in the economy is high, optimal marginal taxation and transfers are also high. Moreover, as the elasticity of labor decreases, the distortion of wealth creation is alleviated implying higher taxes on entrepreneurial production and risk-free assets can be imposed by government.

The motivation of this paper resembles real-life implementations because the subsidization of entrepreneurs is one of the prominent policies for governments. In the U.S., the entrepreneurs can borrow from Small Business Administration (SBA), U.S. Department of Agriculture (USDA), Small Business Lending Fund (SBLF) and Gov-Loans[2]. The aim of these programs is to give financial support to agents who could not take any loans from the financial market such as banks. The U.S. Government gives guaranty to the banks who are the members of these programs and in case of default, the government pays the agents' part of the debt. In this way, the risk of failure to pay is lowered. Also, the government offers grants for the non-commercial institution for research and development purposes. In Turkey, Tübitak [3] and KOSGEB[4] are the main foundations to assist entrepreneurs. The support mainly consists of innovation, R& D, capital support, domestication support, capacity improvement, SME finance support and enterprise development, growth and internationalization support which offer both grants and loans. The most similar support to our model offers a hundred thousand lira reimbursable payment for entrepreneurs which is lent by the financial institutions that have agreement with KOSGEB. Nonetheless, without the presence of capital , no agents can open a business or start-ups. In my setting, this problem can be achieved by giving enough transfers and taxing them on their capital.

[2]For further information https://www.usa.gov/funding-options

[3]For more information https://www.kosgeb.gov.tr/site/tr/genel/destekler/3/destekler

[4]For details see http://www.tubitak.gov.tr/tr/destekler/girisimcilik/ulusal-destek-programlari

Related Literature

This paper is related to the recent literature on optimal capital taxation. The closest paper, written by Yazici (2016), expresses a model where individuals are heterogeneous with respect to their wealth level and entrepreneurial ability. However, the ability is a dummy variable whether agents have ideas or not which is different from my model. Also, individuals have private information about their types, allocations and returns. He finds that poor agents who have ideas ought to be subsidized and in order to eliminate the probability of subsidizing people who pretend to have ideas, timing and amount of the transfers should be meticulously determined. Knowles and Boar (2017) also create a similar model where risk-free projects and risky projects can be invested by entrepreneurs. The workers supply labor and live hand-to-mouth but do not own any wealth. In order to have a balanced budget, government imposes taxes on consumption and labor for workers and wealth, capital and consumption for entrepreneurs. The main concern of government in this framework is to redistribute from entrepreneurs to poor workers so that inequality can be lowered. Under the equity-efficiency trade-off, they find that optimal consumption and wealth taxation is positive and higher whereas optimal capital taxation is negative. This is because capital taxation negatively influences poor entrepreneurs to invest their risky-project and causes inefficient investment level. Also, high taxation on capital lowers all the entrepreneurs saving level.

Saez and Stantcheva (2018) develop a model to derive optimal linear and non-linear capital taxation formula by using elasticities which highlights equality-efficiency trade-off. The model consists of linear utility for consumption and concave utility for wealth which generates immediate adjustment of capital in response to taxes. In this structure, government imposes taxes on labor income and capital income. Shourideh et al. (2012) studies optimal capital taxation in order to achieve efficiency and redistribution for agents with idiosyncratic entrepreneurial risk and private information in a multi-period setting. In the model, trade-off between risk and return is presented and three main capital goods are taxed, namely; risk-free assets, entrepreneurial ability and bequests. It can be seen as an attempt on how wealthy agents should be taxed. Likewise, Albanesi (2006) proposed a model implementing Mirrlees model with private information in which hidden actions affects stochastic return for entrepreneurial capital. She demonstrates that negative wedge on observable risky capital and positive wedge on risk-free asset are likely to be seen. The characterization of optimal taxation for different market structure under the constrained-efficient allocation is the main contribution of that paper.

Kitao (2008) constructs a dynamic general equilibrium model where individuals decide whether to work in corporate sector or non-corporate sector and they have heterogeneous entrepreneurial ability. He shows that lowering risk-free asset rises aggregate investment and output whereas entrepreneurial investment is negatively affected. Likewise, Panousi (2010) studies the impact of changes in the capital income tax rate on entrepreneurial investment Agents confront with idiosyncratic investment risk which leads to creating trade-off between provision of insurance for income risk and distorting investment. He found that capital taxation stimulates capital accumulation due to the general-equilibrium effects of the insurance. Quantitatively, when capital taxation is 25%, output per worker is 2.2% higher than zero taxation which is not consistent with complete market and Bewley model. In this paper, I focus relatively new capital taxation on two distinct goods which creates a trade-off between wealth accumulation and entrepreneurial productivity .

The paper is organized as follows. Section 2 presents the environment and objectives of the society. Section 3 shows the optimal tax formulas. Section 4 demonstrates the numerical analysis and Section 5 concludes. All proofs can be found in Appendix A and the MATLAB algorithm can be found in Appendix B.

2. ENVIRONMENT

The economy consists of a unit continuum of agents, each denoted by w and θ where w and θ are jointly distributed by $F(w,\theta)$. Agents live two periods. In the beginning, they work l hours and generate wealth $k = wl$ where w represents the ability to generate wealth. Individuals choose their working hours in consideration of utility of the second period. In the second period, people divide their wealth between entrepreneurial investment, denoted e and investment in a risk-free technology, s, which has a fixed return r. Entrepreneurial investment returns θe^{α}, where θ is a known return. Agents are heterogeneous with respect to their wealth level, k and entrepreneurial ability, θ. Initial wealth level is endogenous that makes taxing capital costly. People cannot borrow from financial intermediaries and lending-borrowing activities between individuals are not allowed, that is $s \geq 0$. This assumption is going to imply financial system cannot solve the mismatch problem. Before the investment decision, government distribute transfers L to individuals which implies $e \leq k+L$. Also, the government has no redistributive motive across agents. The resources of government comes from linear capital taxes on entrepreneurial activity (τ_e) and risk-free bonds (τ_s). Consequently, an agent's problem for a given government policy $P = (\tau_e, \tau_s, L)$ is:

$$\max_{k} U(V(k;P), \frac{k}{w}) = V(k;P) - h(l)$$

where $h(l) = \frac{l^{1+\gamma}}{1+\gamma}$, a strictly increasing and convex function and $\gamma > 1$, representing reciprocal of the frisch elasticity of labor supply indicating the response of labor with the change of w because the utility function is seperable.

In the second period, each individual maximizes

$$\max_{e} V(k;P) = (1-\tau_s)r(k+L-e) + (1-\tau_e)\theta e^{\alpha}.$$

Government maximizes a social welfare function

$$W = \max_{\tau_s,\tau_e,L} \int_{\theta}\int_{w} U(\theta,w)d\theta dw = \int_{\theta}\int_{w} \Big[(1-\tau_s)r(k+L-e) + (1-\tau_e)\theta e^{\alpha} - h(k/w)\Big] F(w,\theta)dwd\theta.$$

The government maximizes W subject to budget constraint and incentive compatibility constraint. The budget constraint assures that current value of transfers are less than total earnings

$$rL \leq \tau_s r E(k+L-e) + \tau_e E(\theta e^{\alpha})$$

where expectation is taken with respect to θ and w.

Other constraint, incentive compatibility, ensures that every individual (w,θ) solves his maximization problem.

3. OPTIMAL TAX FORMULAS

In this section, we aim to obtain optimal tax formulas. In order to do that, government problem is maximized subject to budget constraint and incentive compatibility. Yet, before that it is useful to study the following benchmark.

Proposition 1 *Suppose $h(l) = 0$ and all agents work $l = 1$ hour. Then in the optimal solution:*
i- $\tau_e^* = \tau_s^* = \tau^*$
ii- $\tau^* = 0.5$
iii- *If* $E(k(w,\theta) - e^*(w,\theta)) + E(\theta e(w,\theta)^{*\alpha}\frac{1}{r}) + k(w',\theta') \geq e(w',\theta')^{FB}$ *holds* $\forall$ *individual* (w',θ') *first best-solution is reached.*

Proof 1 *Relegated to Appendix A.*

Having exogenous initial wealth level makes our problems unchallenging and not interesting. Since changing taxes does not create distortions, government collect all the resources in the economy in order to give enough transfers to productive agents. The only thing that matters is whether the transfers, L, is high enough to produce first-best level of entrepreneurial investment for all individuals. Therefore, there is no trade-off between wealth accumulation and productive efficiency. Note that, in this set-up, if there is a financial intermediary that allows agents to borrow and lend at r, first-best level can also be achieved.

Now, I will analyse the behavioural responses of agents which is crucial in order to derive optimal tax formulas.

Lemma 1 *In the problem where initial wealth is endogenous, for binding individuals, we have* $\frac{\partial e}{\partial L} = \frac{\partial k}{\partial L} + 1$, $-1 < \frac{\partial k}{\partial L} < 0$, $\frac{\partial k}{\partial \tau_s} = \frac{\partial e}{\partial \tau_s} = 0$, *and* $\frac{\partial k}{\partial \tau_e} = \frac{\partial e}{\partial \tau_e} < 0$. *For non-binding agents, we have* $\frac{\partial e}{\partial L} = \frac{\partial k}{\partial L} = 0$, $\frac{\partial k}{\partial \tau_s} < 0$, $\frac{\partial k}{\partial \tau_e} = 0$, $\frac{\partial e}{\partial \tau_s} > 0$ *and* $\frac{\partial e}{\partial \tau_e} < 0$.

Proof 2 *Relegated to Appendix A.*

Intuitively, increasing transfers causes binding individuals to lower their initial wealth level but increases entrepreneurial investment. Individuals do not directly increase their investment by L since in the first period, agents start to work less to decrease their dis-utility. For non-binding individuals, there is no wealth effect since agents have already chosen their optimal wealth level and entrepreneurial activity.

Changing marginal tax on risk-free bonds, τ_s, does not influence binding agents' decisions of entrepreneurial investment and initial wealth. Since they use all their resources to entrepreneurial activity, no behavioural response is observed. On the other hand, initial wealth of non-binding individuals is distorted due to the increase in tax rate. As their gains through risk-free bonds decline, they begin to work less in the first period. Also, these agents investment decision shifts to the entrepreneurial activity as a substitution.

As a response of increasing marginal tax on entrepreneurial investment, τ_e, binding individuals reduce both their entrepreneurial activity and initial wealth by the same amount. Since the return of entrepreneurial investment declines, agents would decide to work less. For non-binding agents, there is no modification due to the no wealth effect. However, they decrease their entrepreneurial investment as marginal return falls.

Proposition 2 *Optimal marginal taxation on risk-free bonds satisfies:*

$$(\lambda - 1)E(rs) = E(\lambda \frac{\tau_s r}{1-\tau_s} \epsilon_{ks} k) + \frac{1}{1-\tau_s} E([-\lambda \tau_s r + \lambda \tau_e \theta \alpha e^{\alpha-1}] \epsilon_s e). \tag{3.1}$$

where ϵ_{ks} represents the elasticity of wealth with respect to after tax rate of risk-free bonds while ϵ_s refers to elasticity of entrepreneurial investment after tax rate of risk-free bonds.

Proof 3 *Relegated to Appendix A.*

As an interpretation, the term on the left hand side is analogous to the redistributive (covariance) term in the optimal labor tax problem. Like in the labor tax case, the negative term represents the average cost to agents of the mechanical decline in their welfare coming from the unit rise in τ_s, which decreases each agent's consumption by $rs(w, \theta)$. The positive on the left hand side is the social benefit associated with distributing $rs(w, \theta)$ back to people through the lump-sum transfer. In the labor tax case, this simply increases average utility by a multiple of $E(rs)$ (when there are no income effects of L on labor supply). Here, the story is different. The main benefit of distributing $E(rs)$ via lump-sum transfers is the relaxation of the borrowing constraints of constrained entrepreneurs. (Increasing L has behavioral costs too in our case.)

The terms on the right hand side are associated with various costs of increasing τ_s. The first term equals to the behavioural cost through the effect of changing τ_s on initial wealth (sum of a government budget cost). The second term reflects the behavioural cost associated with people's reactions in terms of entrepreneurial investment due to change in government budget.

Proposition 3 *Optimal marginal taxation on entrepreneurial activity satisfies*

$$(\lambda - 1)E(\theta e^{\alpha}) = \frac{1}{1-\tau_e}E(\lambda\tau_s r\epsilon_{ke}k) + \frac{1}{1-\tau_e}E([-\lambda\tau_s r + \lambda\tau_e\theta\alpha e^{\alpha-1}]\epsilon_e e). \tag{3.2}$$

in which ϵ_{ke} refers to the elasticity of wealth with respect to after entrepreneurial investment tax rate whereas ϵ_e refers to elasticity of entrepreneurial activity after tax rate of entrepreneurial investment.

Proof 4 *Relegated to Appendix A.*

Interpretation of optimal marginal taxation of entrepreneurial activity is that the left hand side shows the benefit of increasing τ_e by one unit while the right hand side demonstrates the total costs of changing τ_e. The term on the the negative term of the left hand side represents the average cost to agents of the mechanical decrease in their welfare coming from the unit rise in τ_e, which decreases each agent's consumption by $\theta e(w,\theta)^{\alpha}$. The positive on the left hand side is the social benefit associated with distributing $\theta e(w,\theta)^{\alpha}$ back to people through the lump-sum transfer. Likewise, the main benefit of distributing $E(\theta e^{\alpha})$ via lump-sum transfers is the relaxation of the borrowing constraints of constrained entrepreneurs.

The terms on the right hand side are associated with costs of increasing τ_e. The first term is equal to the behavioural cost through the effect of changing τ_e on initial wealth (sum of a government budget cost). The second term reflects the behavioural cost associated with people's reactions in terms of entrepreneurial investment on government budget. Note that behavioural response related to entrepreneurial investment consists of both benefit and costs because due to the increase in τ_e, households' decision on entrepreneurial activity ($e(w,\theta)$) decline while risk-free bond investment $(s(w,\theta))$ rises. Changing from entrepreneurial investment to risk-free assets leads to gain by the first term through government's budget whereas the cost arises by second term through government's budget.

Having $\lambda > 1$ is crucial for productive efficiency. As it is mentioned that the left hand side of optimal tax formulas show the benefit of imposing taxation, the term becomes positive. This implies that government's redistribution through transfers contributes agents to relax their constraints. In Proposition 4, I show that $\lambda > 1$ implying an improvement in productive efficiency.

Proposition 4 *In the optimal tax problem, λ is strictly greater than* 1.

Proof 5 *Relegated to Appendix A.*

4. NUMERICAL ANALYSIS

In this section, I aim to show how the transfers distributed by the government will improve productive efficiency quantitatively. I worked on two different economies where Economy I has no correlation between the agents' entrepreneurial ability and initial productivity to generate wealth. In Economy II, the correlation between entrepreneurial ability and initial productivity is positive. The common parameters and probabilities are shown in Table 4.1. Interest rate is normalized to 1 in the model. The elasticity of labor with respect to wages, the Frisch elasticity of labor supply, is assigned as 1. Initial productivity to create wealth and ability level have five types where the probability of initial productivity is equal for both economies.

Table 4.1 Common Parameters for Economies

Parameters	Values
r	1
α	0.5
γ	1
w	$[0.5, 1, 1.8, 3, 5]$
θ	$[1, 2, 3, 4, 8]$
$P(w)$	$[0.2, 0.3, 0.35, 0.1, 0.05]$

In order to differentiate the economies in terms of correlation, two conditional probabilities are presented, as shown in Table 4.2 so that Economy I has independently distributed by initial productivity and ability and Economy II has positively correlated. As five different values for initial productivity and ability are presented, in both economies, 25 types of individuals exist in the society. Please note that before the taxation, Economy I consists of 39.9% binding agents while Economy II is composed of 38.7%

Table 4.2 Conditional Probabilities for Economies

Economies	$P(\theta\|w)$
Economy I	$\begin{bmatrix} 0.225 & 0.225 & 0.225 & 0.225 & 0.1 \\ 0.225 & 0.225 & 0.225 & 0.225 & 0.1 \\ 0.225 & 0.225 & 0.225 & 0.225 & 0.1 \\ 0.225 & 0.225 & 0.225 & 0.225 & 0.1 \\ 0.225 & 0.225 & 0.225 & 0.225 & 0.1 \end{bmatrix}$
Economy II	$\begin{bmatrix} 0.25 & 0.25 & 0.25 & 0.25 & 0 \\ 0.2375 & 0.2375 & 0.2375 & 0.2375 & 0.05 \\ 0.225 & 0.225 & 0.225 & 0.225 & 0.1 \\ 0.2125 & 0.2125 & 0.2125 & 0.2125 & 0.15 \\ 0.2 & 0.2 & 0.2 & 0.2 & 0.2 \end{bmatrix}$

The optimal policy for government in each economy can be found in Table 4.3. After the taxation, the percentage of binding people decreases to %20.75 for Economy I while it is lower for Economy II, 18.63%. For both economies, utilitarian social welfare function increases implying there exists an improvement in productive efficiency.

Table 4.3 Optimal Policy

Policy	Economy I	Economy II
τ_s^*	22%	16%
τ_e^*	30%	23%
L^*	2.21	1.54

In Economy II, the government tends to intervene in the economy less than Economy I implying that the transfers to individuals and taxation are quantitatively lower. Furhermore, as the correlation goes to 1, laissez-faire economy becomes efficient. Also, in Economy II, with lower taxation, the population of binding individuals is lowered nearly the same as Economy I implying that agents with more entrepreneurial ability become unconstrained in Economy II.

Sensitivity to Labor Supply Elasticity

In order to explore the effect of labor supply elasticity, I define Economy III which has the same properties with Economy II except Frish elasticity of supply is 0.5 ($\gamma = 2$). The results can be found in Table 4.4.

Table 4.4 Sensitivity Analysis

Policy	Economy II	Economy III
τ_s^*	16%	39%
τ_e^*	23%	41%
L^*	1.54	3.22

Notice that optimal marginal taxation on both entrepreneurial production and risk-free assets is higher in Economy III. As Frisch elasticity of labor supply decrease, individuals' response to labor becomes less sensitive. Therefore, distortion of wealth creation due to taxation lowers which helps government to increase tax rates. In Economy III, thanks to transfers to agents, the population of binding people decreases from 39.6%.to 7.5% which is much less than other economies. This shows how important the agents' elasticity of labor with respect to wage while determining the tax policies.

5. CONCLUSION

This paper gives an insight to policy-makers in order to achieve productive efficiency in the market. By deriving optimal taxation formulas, mechanical response and behavioral response are explained with costs and benefits of taxation. With the help of taxation, poor agents with ideas become less constrained and contribute to the output level of economy. This is a fundamental analysis to cope with the mismatch in terms of initial wealth and entrepreneurial ability. While imposing policy, individuals may distort their wealth level which engenders the trade-off between wealth accumulation and productive efficiency. Numerical analysis stresses that correlation between wage and ability, and sensitivity of labor with respect to wage are vital for taxation policy.

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APPENDIX A: Proofs

Please note that i represents individual type which refers to the type of individual (w,θ).

Proof of Proposition 1:

i- Before taxation, household's first-best optimal solution $\forall i \notin B$ is

$$r = \alpha\theta_i e_i^{\alpha-1}$$

and $\forall i \in B$ is

$$r + \mu_i = \alpha\theta_i e_i^{\alpha-1}$$

Therefore, the aim is to reduce μ_i meaning relaxation of borrowing constraint so much that marginal returns are equal. After taxation, optimal formula for non-binding individuals becomes

$$(1-\tau_s)r = (1-\tau_e)\alpha\theta_i e_i^{\alpha-1}$$

while for binding individuals:

$$(1-\tau_s)r + \mu_i = (1-\tau_e)\alpha\theta_i e_i^{\alpha-1}.$$

Notice that τ_e^*, τ_s^* and L^* are the solution of optimal tax problem. Suppose $\tau_e^* \neq \tau_s^*$. Without loss of generality, let $\tau_e^* > \tau_s^*$.

Consider new policy:

1-) $\tau_e^{**} = \tau_e^* - \epsilon$ where $\epsilon > 0$

2-) $\tau_s^{**} = \tau_s^* + \delta$ where δ is a real number such that L^* is unchanged.

If $\delta > -\epsilon$ that implies the policy $P = (\tau_s^*, \tau_e^*, L^*)$ is pareto dominated by $P^* = (\tau_s^{**}, \tau_e^{**}, L^*)$, proof is done.

For the policy P^* individuals' optimal investment level is $e_i^* = [\frac{(1-\tau_e^*)\alpha\theta_i}{(1-\tau_s^*)r+\mu_i}]^{\frac{1}{1-\alpha}}$ and for P^{**} optimal level is $e_i^{**} = [\frac{(1-\tau_e^{**})\alpha\theta_i}{(1-\tau_s^{**})r+\mu_i}]^{\frac{1}{1-\alpha}}$. Decreasing τ_e^* will not influence constrained individuals since they uses all their transfers and wealth level as an entrepreneurial activity. Hence, the optimal level for non-binding people boils down as follows:

$$e_i^* = \left[\frac{(1-\tau_e^*)\alpha\theta_i}{(1-\tau_s^*)r}\right]^{\frac{1}{1-\alpha}}$$

and

$$e_i^{**} = \left[\frac{(1-\tau_e^{**})\alpha\theta_i}{(1-\tau_s^{**})r}\right]^{\frac{1}{1-\alpha}}.$$

For the policy P, government's budget constraint is:

$$\tau_s^* r \int_i (k_i + L - e_i^*) di + \tau_e^* \int_i \theta_i e_i^{*\alpha} di = rL^* \tag{A.1}$$

and for the policy P^*, government's budget constraint is:

$$\tau_s^{**} r \int_i (k_i + L - e_i^{**}) di + \tau_e^{**} \int_i \theta_i e_i^{**\alpha} di = rL^* \tag{A.2}$$

Substracting (A.1) from (A.2) gives us the following:

$$\begin{aligned} \tau_s^* r \int_{i \notin B} (e_i^* - e_i^{**}) di + \delta r \int_i (k_i + L - e_i^{**}) di + \tau_e^* \int_{i \notin B} \theta_i e_i^{**\alpha} di - \tau_e^* \int_{i \notin B} \theta_i e_i^{*\alpha} di \\ - \epsilon \int_i \theta_i e_i^{**\alpha} di = 0 \end{aligned} \tag{A.3}$$

Suppose $\delta = -\epsilon$. We know that $e_i^{**} = e_i^*$ so equation(A.3) boils down as follows:

$$-\epsilon r \int_i (k_i + L - e_i^*) di - \epsilon \int_i \theta_i e_i^{*\alpha} di = 0 \tag{A.4}$$

which cannot hold as LHS is smaller than zero. As an intiution the same L^* cannot be achieved due to the decline in government revenue.

Suppose $\delta < -\epsilon$ which implies that $e_i^{**} < e_i^*$. Notice that since L^* does not change where $L = \frac{Y}{r}$, then no variation is observed in output level.It is known that total output level is $Y = r\int_i (k_i - e_i) di + \int_i \theta_i e_i^{\alpha} di$ and if one channel increases, the other channel must be decrease at the same level. Therefore equation(A.2) comparing with equation(A.1) can be written as follows:

$$\underbrace{\tau_s^{**}}_{\text{decrease by } \delta} \overline{r} \underbrace{\int_i (\overline{k_i} + \overline{L} - e_i^{**}) di}_{\text{increase by } X \text{ unit}} + \overbrace{\tau_e^{**}}^{\text{decrease by } \epsilon} \overbrace{\int_i \theta_i e_i^{**\alpha} di}^{\text{decrease by } X \text{ unit}} = \overline{rL^*}. \tag{A.5}$$

In a formal way,

$$\Delta X[\tau_s^* - \delta - \tau_e^* + \epsilon] = 0$$

Here, LHS is negative while RHS is zero which contradicts with our supposition. Then, $\tau_e = \tau_s = \tau$. As an inference, decreasing τ_e brings e_i^* closer to e^{FB} for non-binding people. For binding individuals, they will not be worse-off. Hence P is pareto dominated by the new policy, P^*.

ii- For the second part of the proof we know that:

$$rL = \tau r(\int_i (k_i + L - e_i)di) + \tau \int_i \theta_i e_i^\alpha di.$$

Some manipulation on above equation gives

$$rL(1-\tau) = \tau \left[r(\int_i (k_i - e_i)di) + \int_i \theta_i e_i^\alpha di \right] \tag{A.6}$$

Notice that for the maximum level of subsidy, $L = \frac{Y}{r}$ must hold, where Y is total production in the economy and $Y = r\int_i (k_i - e_i)di + \int_i \theta_i e_i^\alpha di$. Then,

$$rL(1-\tau) = rL\tau$$

which gives

$$\tau = 0.5.$$

iii- Intuitively, individuals who are productive but not wealthy are subsidized by government and, give individuals opportunity to choose first-best level of investment if the maximum level of transfers ,that is $L = \frac{r\int_i (k_i - e_i)di + \int_i \theta_i e_i^\alpha di}{r}$ causes to inequality holds. If it does not hold, then whatever the level of L, first best solution cannot be achieved.||

Proof of Lemma 1:
Remember the problem;

$$\max_{y_i} U(V(k_i;P), \frac{y_i}{w_i}) = V(k_i;P) - h(l_i)$$

where $h(l_i) = \frac{l_i^{1+\gamma}}{1+\gamma}$, a strictly increasing and convex function.

Under policy P,

If $(w_i, \theta_i) \mid i \in B$, then

$$e_i^*(w_i, \theta_i; P) = k_i + L$$
$$V(k_i; P) = (1-\tau_e)\theta_i(k_i+L)^\alpha.$$

Taking derivative of e_i with respect to L is as follows:

$$\frac{\partial e_i}{\partial L} = \frac{\partial k_i}{\partial L} + 1.$$

Increasing transfers help people to relax their constraint contributing to increase their optimal level of entrepreneurial investment.

Our maximization problem is:

$$\max_{k_i}(1-\tau_e)\theta_i(k_i+L)^\alpha - \frac{(\frac{k_i}{w_i})^{1+\gamma}}{1+\gamma}$$

First order condition of the problem boils down as below:

$$(1-\tau_e)\alpha\theta_i(k_i+L)^{\alpha-1} - \frac{1}{w_i}(\frac{k_i}{w_i})^\gamma = 0 \tag{A.7}$$

Let

$$H(\theta_i, w_i, \tau_e, L, k_i) = (1-\tau_e)\alpha\theta_i(k_i+L)^{\alpha-1} - \frac{1}{w_i}(\frac{k_i}{w_i})^\gamma = 0. \tag{A.8}$$

The function H has continuous partial derivatives $H_{k_i}, H_{\theta_i}, H_{w_i}, H_{\tau_e}, H_L$ where $H(..) = 0$ and H_{k_i} is nonzero, then there exists an implicit function satisfies $k_i = h(\theta_i, w_i, (1-\tau_e), L)$. The implicit function h is continuous and has continuous partial derivatives. Moreover, $H_{k_i}dk_i + H_{\theta_i}d\theta_i + H_{w_i}dw_i + H_{\tau_e}d\tau_e + H_L dL = 0$ which help us write the following formula:

$$\frac{\partial k_i}{\partial L} = -\frac{H_L}{H_{k_i}}.$$

Therefore $\forall i \in B$,

$$\frac{\partial k_i}{\partial L} = -\frac{\alpha(\alpha-1)(1-\tau_e)\theta_i(k_i+L)^{\alpha-2}}{\alpha(\alpha-1)(1-\tau_e)\theta_i(k_i+L)^{\alpha-2} - \gamma\frac{1}{w_i^2}(\frac{k_i}{w_i})^{\gamma-1}}.$$

Numerator and denominator of partial derivative is negative ($\alpha - 1 < 0$), so we have $-1 < \frac{\partial k_i}{\partial L} < 0$.
Change in τ_s and τ_e on k_i are derived by the same way.

$$\frac{\partial k_i}{\partial \tau_s} = -\frac{H_{\tau_s}}{H_{k_i}} = 0.$$

$$\frac{\partial k_i}{\partial \tau_e} = -\frac{-\alpha\theta_i(k_i+L)^{\alpha-1}}{\alpha(\alpha-1)(1-\tau_e)\theta_i(k_i+L)^{\alpha-2} - \gamma\frac{1}{w_i^2}(\frac{k_i}{w_i})^{\gamma-1}} < 0.$$

Note that as $e_i = k_i + L$,$\forall i \in B$ we have $\frac{\partial k_i}{\partial \tau_e} = \frac{\partial e_i}{\partial \tau_e}$ and $\frac{\partial k_i}{\partial \tau_s} = \frac{\partial e_i}{\partial \tau_s}$. Thence:

$$\frac{\partial k_i}{\partial \tau_e} = \frac{\partial e_i}{\partial \tau_e} < 0$$

$$\frac{\partial k_i}{\partial \tau_s} = \frac{\partial e_i}{\partial \tau_s} = 0$$

If $(w_i, \theta_i) \mid i \notin B$, then

$$e_i^*(w_i, \theta_i; P) = \left[\frac{(1-\tau_e)\theta_i\alpha}{(1-\tau_s)r}\right]^{\frac{1}{1-\alpha}}$$

$$V(k_i; P) = (1-\tau_s)r(k_i + L - e_i^*(w_i, \theta_i; P) + (1-\tau_e)\theta_i e_i^{*\alpha}(w_i, \theta_i; P).$$

Since e_i does not depend on L partial derivative of e_i with respect to L is zero, $\frac{\partial e_i}{\partial L} = 0$. As an intuition, increasing transfers will not affect optimal solution because they have enough resources to choose investment level.

$\forall i \notin B$, our maximization problem is:

$$\max_{k_i}(1-\tau_s)r(k_i + L - e_i^*(w_i, \theta_i; P)) + (1-\tau_e)\theta_i e_i^{*\alpha}(w_i, \theta_i; P) - \frac{(\frac{k_i}{w_i})^{1+\gamma}}{1+\gamma}$$

First order condition pops up as below:

$$(1-\tau_s)r - \frac{1}{w_i}(\frac{k_i}{w_i})^{\gamma} = 0. \tag{A.9}$$

Let

$$F(\theta_i, w_i, \tau_s, k_i) = (1-\tau_s)r - \frac{1}{w_i}(\frac{k_i}{w_i})^{\gamma} = 0. \tag{A.10}$$

The function F has continuous partial derivatives $F_{k_i}, F_{\theta_i}, F_{w_i}, F_{\tau_s}, F_L$ where $F(..) = 0$ and F_{k_i} is nonzero, then there exists an implicit function satisfies $k_i =$

$f(\theta_i, w_i, \tau_s, L)$. The implicit function h is continuous and has continuous partial derivatives. Furthermore, $F_{k_i} dk_i + F_{\theta_i} d\theta_i + F_{w_i} dw_i + F_{\tau_s} d\tau_s + F_L dL = 0$ which help us write the following formula: As function F has continuous partial derivatives, by implicit function theorem, we can write the following formula:

$$\frac{\partial k_i}{\partial L} = -\frac{F_L}{F_{k_i}}.$$

Hence $\forall i \notin B$,

$$\frac{\partial k_i}{\partial L} = 0.$$

$\forall i \notin B$, $\frac{\partial k_i}{\partial \tau_s}$ can be derived by

$$\frac{\partial k_i}{\partial \tau_s} = -\frac{F_{\tau_s}}{F_{k_i}} = -\frac{-r}{-\gamma \frac{1}{w_i^2} (\frac{k_i}{w_i})^{\gamma - 1}} < 0.$$

$\forall i \notin B$, $\frac{\partial k_i}{\partial \tau_e}$ reflects as

$$\frac{\partial k_i}{\partial \tau_e} = -\frac{F_{\tau_e}}{F_{k_i}} = 0.$$

In order to find $\frac{\partial e_i}{\partial \tau_e}$ and $\frac{\partial e_i}{\partial \tau_s}$, we will use the first order condition of second period's problem, $G(\tau_s, \tau_e, \theta_i, e_i, r) = -(1-\tau_s)r + (1-\tau_e)\theta_i \alpha e_i^{\alpha - 1} = 0$. The function G has continuous partial derivatives $G_{e_i}, G_{\theta_i}, G_{\tau_s}, G_{\tau_e}, G_r$ where $G(..) = 0$ and G_{e_i} is nonzero, then there exists an implicit function satisfies $e_i = g(\theta_i, \tau_e, \tau_s, r)$. The implicit function g is continuous and has continuous partial derivatives. Also, $G_{e_i} de_i + G_{\theta_i} d\theta_i + G_r dr + G_{\tau_e} d\tau_e + G_{\tau_s} d\tau_s = 0$ which help us write the following formula:

$$\frac{\partial e_i}{\partial \tau_s} = -\frac{G_{\tau_s}}{G_{e_i}} = \frac{e_i}{(1-\alpha)(1-\tau_e)} > 0$$

$$\frac{\partial e_i}{\partial \tau_e} = \frac{G_{\tau_e}}{G_{e_i}} - \frac{e_i}{(1-\alpha)(1-\tau_e)} < 0$$

||

Proof of Proposition 2

Taking derivative of government's problem with respect to τ_s gives:

$$\tau_s : \int_i [-rs_i + [-(1-\tau_s)r + (1-\tau_e)\theta_i \alpha e_i^{\alpha-1}] \frac{\partial e_i}{\partial \tau_s} + [(1-\tau_s)r - \frac{k_i^\gamma}{w_i^{1+\gamma}}] \frac{\partial k_i}{\partial \tau_s}] di$$

$$+ \lambda [r \int_i s_i di + r\tau_s \int_i (\frac{\partial k_i}{\partial \tau_s} - \frac{\partial e_i}{\partial \tau_s}) di + \tau_e \alpha \int_i \theta_i e_i^{\alpha-1} \frac{\partial e_i}{\partial \tau_s} di] = 0. \tag{A.11}$$

Using first order condition of household shows up the following equation:

(A.12)

$$\int_i -rs_i di+\int_i[\mu_i\frac{\partial e_i}{\partial\tau_s}+[(1-\tau_s)r-\frac{k_i^\gamma}{w_i^{1+\gamma}}]\frac{\partial k_i}{\partial\tau_s}]di+\lambda[r\int_i s_i di+r\tau_s\int_i\frac{\partial s_i}{\partial\tau_s}di+\tau_e\alpha\int_i\theta_i e_i^{\alpha-1}\frac{\partial e_i}{\partial\tau_s}di]=0.$$

Another form of (A.12) can be written as follows:

$$\int_i\Big[-rs_i+[(1-\tau_s)r-\frac{k_i^\gamma}{w_i^{1+\gamma}}]\frac{\partial k_i}{\partial\tau_s}\Big]di+\int_{i\in B}\Big[\mu_i\frac{\partial e_i}{\partial\tau_s}\Big]di+\lambda\Big[r\int_i s_i di+r\tau_s\int_i\frac{\partial k_i}{\partial\tau_s}di$$
$$-\int_i r\tau_s\frac{\partial e_i}{\partial\tau_s}di+\tau_e\alpha\int_i\theta_i e_i^{\alpha-1}\frac{\partial e_i}{\partial\tau_s}di\Big]=0. \quad \text{(A.13)}$$

Notice that $\frac{\partial x}{\partial y}=-\frac{\partial x}{\partial 1-y}$.Multiplying equation with $\frac{k_i(1-\tau_s)}{k_i(1-\tau_s)}$ and $\frac{e_i(1-\tau_s)}{e_i(1-\tau_s)}$ respectively gives us the following equation:

$$(\lambda-1)\int_i rs_i di=\int_i(r-\frac{k_i^\gamma}{(1-\tau_s)w_i^{1+\gamma}}+\lambda\frac{\tau_s r}{1-\tau_s})\epsilon_{ks}k_i di+\frac{1}{1-\tau_s}\int_{i\in B}\mu_i\epsilon_s e_i di$$
$$+\frac{1}{1-\tau_s}\int_i(-\lambda\tau_s r+\lambda\tau_e\theta_i\alpha e_i^{\alpha-1})\epsilon_s e_i di. \quad \text{(A.14)}$$

Note that for individuals $i\in B$ we know that $\frac{\partial e_i}{\partial\tau_s}=\frac{\partial k_i}{\partial\tau_s}=0$. Also, from the first order condition for $i\notin B$, $(1-\tau_s)r=\frac{k_i^\gamma}{w_i^{1+\gamma}}$. Therefore, (A.14) becomes:

(A.15)

$$(\lambda-1)\int_i rs_i di=\int_{i\notin B}\lambda\frac{\tau_s r}{1-\tau_s}\epsilon_{ks}k_i di+\frac{1}{1-\tau_s}\int_{i\notin B}(-\lambda\tau_s r+\lambda\tau_e\theta_i\alpha e_i^{\alpha-1})\epsilon_s e_i di.$$

||

Proof of Proposition 3

Taking derivative of government's problem with respect to τ_e:

$$\tau_e:\int_i[-\theta_i e_i^\alpha+[-(1-\tau_s)r+(1-\tau_e)\theta_i\alpha e_i^{\alpha-1}]\frac{\partial e_i}{\partial\tau_e}+[(1-\tau_s)r-\frac{k_i^\gamma}{w_i^{1+\gamma}}]\frac{\partial k_i}{\partial\tau_e}]di$$
$$+\lambda[\int_i\theta_i e_i^\alpha di+r\tau_s\int_i(\frac{\partial k_i}{\partial\tau_e}-\frac{\partial e_i}{\partial\tau_e})di+\tau_e\alpha\int_i\theta_i e_i^{\alpha-1}\frac{\partial e_i}{\partial\tau_e}di]=0 \quad \text{(A.16)}$$

Using first order condition of household and $\frac{\partial s_i}{\partial\tau_s}=\frac{\partial(k_i+L-e_i)}{\partial\tau_e}$:

(A.17)

$$\int_i[-\theta_i e_i^\alpha+\mu_i\frac{\partial e_i}{\partial\tau_e}+[(1-\tau_s)r-\frac{k_i^\gamma}{w_i^{1+\gamma}}]\frac{\partial k_i}{\partial\tau_e}]di+\lambda[\int_i\theta_i e_i^\alpha di+r\tau_s\int_i\frac{\partial s_i}{\partial\tau_e}di+\tau_e\alpha\int_i\theta_i e_i^{\alpha-1}\frac{\partial e_i}{\partial\tau_e}di]=0.$$

Another form of (A.17) can be written as following:

$$\int_i [-\theta_i e_i^\alpha + [(1-\tau_s)r - \frac{k_i^\gamma}{w_i^{1+\gamma}}]\frac{\partial k_i}{\partial \tau_e}]di + \int_{i\in B}\mu_i \frac{\partial e_i}{\partial \tau_e}di + \lambda[\int_i \theta_i e_i^\alpha di + r\tau_s \int_i \frac{\partial k_i}{\partial \tau_e}di + \int_i (\tau_e \alpha\theta_i e_i^{\alpha-1} - \tau_s r)\frac{\partial e_i}{\partial \tau_e}di] = 0. \tag{A.18}$$

Note that $\frac{\partial x}{\partial y} = -\frac{\partial x}{\partial 1-y}$. Multiplying equation with $\frac{k_i(1-\tau_e)}{k_i(1-\tau_e)}$ and $\frac{e_i(1-\tau_e)}{e_i(1-\tau_e)}$ respectively gives us the following equation:

$$(\lambda-1)\int_i \theta_i e_i^\alpha di = \frac{1}{1-\tau_e}\int_i ((1-\tau_s)r - \frac{k_i^\gamma}{w_i^{1+\gamma}} + \lambda\tau_s r)\epsilon_{ke}k_i di + \frac{1}{1-\tau_e}\int_{i\in B}\mu_i \epsilon_e e_i di + \frac{1}{1-\tau_e}\int_i [-\lambda\tau_s r + \lambda\tau_e\theta_i\alpha e_i^{\alpha-1}]\epsilon_e e_i di. \tag{A.19}$$

Note that for individuals $i \in B$ we know that $\frac{\partial e_i}{\partial \tau_e} = \frac{\partial k_i}{\partial \tau_e}$ and $\frac{\partial k_i}{\partial \tau_e} = 0\ \forall i \notin B$. Therefore, (A.19) becomes:

$$(\lambda-1)\int_i \theta_i e_i^\alpha di = \frac{1}{1-\tau_e}\int_{i\in B}((1-\tau_s)r - \frac{k_i^\gamma}{w_i^{1+\gamma}} + \lambda\tau_s r)\epsilon_{ke}k_i di + \frac{1}{1-\tau_e}\int_{i\in B}\mu_i \epsilon_{ke} k_i di + \frac{1}{1-\tau_e}\int_i [-\lambda\tau_s r + \lambda\tau_e\theta_i\alpha e_i^{\alpha-1}]\epsilon_e e_i di. \tag{A.20}$$

From the FOC of binding individuals, $(1-\tau_e)\alpha\theta_i(k_i+L)^{\alpha-1} = \frac{k_i^\gamma}{w_i^{1+\gamma}}$ and $(1-\tau_s)r + \mu_i = (1-\tau_e)\alpha\theta_i(k_i+L)^{\alpha-1}$. Hence, (A.20) becomes:

$$(\lambda-1)\int_i \theta_i e_i^\alpha di = \frac{1}{1-\tau_e}\int_{i\in B}\lambda\tau_s r\epsilon_{ke}k_i di + \frac{1}{1-\tau_e}\int_i [-\lambda\tau_s r + \lambda\tau_e\theta_i\alpha e_i^{\alpha-1}]\epsilon_e e_i di. \tag{A.21}$$

||

Proof of Proposition 4

CASE I: ($\lambda > 1$ when $\tau_e \geq \tau_s$)

$$(\lambda-1)\int_i \theta_i e_i^\alpha di = \frac{1}{1-\tau_e}\int_{i\in B}\lambda\tau_s r\epsilon_{ke}k_i di + \frac{1}{1-\tau_e}\int_i [-\lambda\tau_s r + \lambda\tau_e\theta_i\alpha e_i^{\alpha-1}]\epsilon_e e_i di. \tag{A.22}$$

Note that $\epsilon_{ke} > 0$ suggesting first term of RHS is positive. For binding individuals $(1-\tau_s)r + \mu_i = (1-\tau_e)\alpha\theta_i e_i^{\alpha-1}$ implying that $(1-\tau_e)\alpha\theta_i e_i^{\alpha-1} > (1-\tau_s)r$. If $\tau_e \geq \tau_s$ then $\theta_i e_i^{\alpha-1} > r$. For non-binding individuals $\theta_i e_i^{\alpha-1} \geq r$ which makes the parenthesis of second term positive. As we know that $\epsilon_e > 0$ for all i, last term is positive. Therefore, $\lambda > 1$ must hold.

CASE II: ($\lambda > 1$ when $\tau_e < \tau_s$)

Using (A.15) that is:

$$(\lambda - 1)\int_i rs_i di = \int_{i \notin B} \lambda \frac{\tau_s r}{1-\tau_s}\epsilon_{ks} k_i di + \frac{1}{1-\tau_s}\int_{i \notin B}(-\lambda\tau_s r + \lambda\tau_e\theta_i\alpha e_i^{\alpha-1})\epsilon_s e_i di. \tag{A.23}$$

Notice that $\epsilon_{ks} > 0$ which makes the first term of RHS positive. For non-binding households, we know that $(1-\tau_s)r = (1-\tau_e)\alpha\theta_i e_i^{\alpha-1}$.Using our supposition, $\tau_s > \tau_e$, implies $r > \alpha\theta_i e_i^{\alpha-1}$. Therefore paranthesis of second term is negative. As we also know that $\epsilon_s < 0$, this suggests that second term of RHS is positive. Thus, LHS must be also positive.||

APPENDIX B: MATLAB Algorithm

Step I Define parameters, wealth vector, probabilities, initial transfers and tolerance level.

Step II Obtain the population matrix

Step III Given tax policy, for each type of theta and the value of wealth, determine whether it is binding or not (according to individual constraint)

Step IV Calculate the second period value functions for different theta and wealth levels

Step V For each type of wage, theta and wealth levels, find the life-time value function

Step VI Maximize the value function by choosing wealth level

Step VII Given optimal wealth level, find the optimal level of entrepreneurial investment and life-time value function

Step VIII Define the government's budget and find the optimal transfer level until government's budget converges to tolerance level.

Step IX Calculate the utilitarian social welfare function function by using population matrix

Step X Repeat these steps for all marginal tax schemes

Step XI The policy that gives maximum SWF is the optimal solution