EPISTEMOLOGY OF PROBLEM SOLVING AND AN ANNOTATION FRAMEWORK FOR COLLABORATIVE PROBLEM SOLVING ENVIRONMENTS

A THESIS SUBMITTED TO
THE GRADUATE SCHOOL OF INFORMATICS
OF
MIDDLE EAST TECHNICAL UNIVERSITY

BY

TUNÇ GÜVEN KAYA

IN PARTIAL FULFILLMENT OF THE REQUIREMENTS FOR THE
DEGREE OF MASTER OF SCIENCE
IN
THE DEPARTMENT OF COGNITIVE SCIENCE

SEPTEMBER 2015

EPISTEMOLOGY OF PROBLEM SOLVING AND AN ANNOTATION FRAMEWORK FOR COLLABORATIVE PROBLEM SOLVING ENVIRONMENTS

Submitted by **Tunç Güven Kaya** in partial fulfillment of the requirements for the degree of **Master of Science in Cognitive Science, Middle East Technical University** by,

Prof. Dr. Nazife Baykal ____________________
Director, **Informatics Institute**

Prof. Dr. Cem Bozşahin ____________________
Head of Department, **Cognitive Science**

Assist. Prof. Dr. Murat Perit Çakır ____________________
Supervisor, **Cognitive Science, METU**

Assoc. Prof. Dr. Samet Bağçe ____________________
Co-Supervisor, **Department of Philosophy, METU**

Examining Committee Members

Prof. Dr. Cem Bozşahin ____________________
Head of Department, METU

Assist. Prof. Dr. Murat Perit Çakır ____________________
Cognitive Science, METU

Assoc. Prof. Dr. Samet Bağçe ____________________
Department of Philosophy, METU

Assist. Prof. Dr. Cengiz Acartürk ____________________
Cognitive Science, METU

Assist. Prof. Dr. Murat Ulubay ____________________
Department of Management, Yıldırım Beyazıt University

Date: September 10, 2015

I hereby declare that all information in this document has been obtained and presented in accordance with academic rules and ethical conduct. I also declare that, as required by these rules and conduct, I have fully cited and referenced all material and results that are not original to this wok.

Name, Last name: Tunç Güven Kaya

Signature : ____________________

ABSTRACT

EPISTEMOLOGY OF PROBLEM SOLVING AND AN ANNOTATION FRAMEWORK FOR COLLABORATIVE PROBLEM SOLVING ENVIRONMENTS

Kaya, Tunç Güven
Msc., Department of Cognitive Science
Supervisor: Asst. Prof. Dr. Murat Perit Çakır
Co-Supervisor: Assoc. Prof. Dr. Samet Bağçe

September 2015, 80 pages

Problem solving can be summarized as the activities that a subject performs when confronted with a problem. The behavioral and cognitive processes underlying these activities have been first studied by psychologists and then cognitive scientists since the beginning of the 20th century. The studies mostly focused on the psychology, philosophy and AI perspectives of problem solving activity. However, epistemological and collaborrative aspects of human problem solving have not been emphasized in related cognitive science literature. This thesis aims to investigate the underlying epistemological concepts of problem solving activity of human subjects. The formulations and definitions of problem solving related terms will be explicated and some problem solving related phenomena which were discovered by the scientists will be investigated in terms of epistemology. Moreover, the experimental environments for observing the problem solving activities of individuals and groups will be introduced to the reader along with an annotation framework which was designed for these problem solving environments. The importance of the framework comes from its event-based structure and its sensitivity for the epistemological aspects of human problem solving. In the case study, it will be demonstrated that the gist of the problem solving sessions can mostly be captured by the framework.

Keywords: Problem Solving, Epistemology, Annotation Framework, Collaborative Problem Solving Environments

ÖZ

PROBLEM ÇÖZME SÜREÇLERİNİN EPİSTEMOLOJİSİ VE GRUP PROBLEM ÇÖZÜM ORTAMLARI İÇİN BİR İŞARETLEME SİSTEMİ

Kaya, Tunç Güven
Master, Bilişsel Bilimler Bölümü
Tez Yöneticisi: Yrd. Doç. Dr. Murat Perit Çakır
Eş Tez Yöneticisi: Doç. Dr. Samet Bağçe

Eylül 2015, 80 sayfa

Problem çözme, bir öznenin problem ile karşılaştığı zaman yaptığı aktiviteler olarak kısaca tanımlanabilir. Bu aktivitelerin altında yatan davranışsal ve bilişsel süreçler 20. yüzyılın başından bu yana önce psikologlar ve ardından bilişsel bilimciler tarafından çalışılmaktadır. Bu çalışmaların çoğu psikoloji, felsefe ve yapay zeka alanlarına yoğunlaşmıştır. Ancak, epistemoloji ve grup problem çözümleri konularında yeterli çalışmanın bilişsel bilim alanında yapılmadığı görülmektedir. Bu tezin amacı insanların problem çözme süreçlerinin arkasında yatan epistemolojik nosyonları sağlamaktır. Aynı zamanda problem çözme aktivitesi ile ilgili çeşitli kavramların epistemolojik tanımları ve formülleri de okuyucuya sunulacaktır. Bununla birlikte, bireysel ve grup olarak problem çözmeyi incelemek için düzenlenmiş deneysel ortamlar tanıtılacak ve bu deneysel ortamlarda kullanılacak bir işaretleme ve analiz sistemi tanıtılacaktır. Bu açıklama sisteminin önemi etkinlik tabanlı yaklaşımından ve problem çözme etkinliğinin altında yatan epistemolojik kavramlara olan hassasiyetinden ileri gelmektedir. Örnek çalışmada, problem çözüm süreçlerinin özünün bahsi geçen işaretleme sistemi ile çoğunlukla kapsanabileceği gösterilecektir.

Anahtar Kelimeler: Problem Çözme, Epistemoloji, İşaretleme Sistemi, Grup Problem Çözüm Ortamları

To Lisa

ACKNOWLEDGMENTS

First of all I would like to thank to my supervisors Assist. Prof. Dr. Murat Perit Çakır and Assoc. Prof. Dr. Samet Bağçe. They not only helped me with writing this thesis but also expanded my horizon both in scientific and philosophical ways.

I also thank to Ece Kamer Takmaz for her helps in the design process of the framework. Without her efforts, this framework would have never existed.

Finally, I would like to thank to Prof. Dr. Cem Bozşahin, Ege Saygıner, Gökhan Gönül, Hakan Güler and Murathan Kurfalı for their priceless contributions.

TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

CHAPTER 1

INTRODUCTION

Problem solving is the study of the activities of a subject that encounters a problem. For an activity to be called problem solving there should be a goal which is not immediately available to the subject and the subject has to find her way through the obstacles to the goal. The subject can be either human or machine. In the case of a machine subject, problem solving is a research area of artificial intelligence, computer science and engineering. When the subject is human, this activity is the subject matter of psychology and philosophy. However, whether the subject is a machine or human the activity is the subject matter of cognitive science.

This thesis approaches problem solving from a cognitive science based point of view. Since cognitive science is an interdisciplinary science including philosophical, psychological, computational, and linguistic points of view, any topic studied in cognitive science should have a broad perspective that takes the nature of the problem into account.

Problem solving has been studied in terms of algorithms, heuristics, search strategies and problem space representation in the domain of artificial intelligence. These studies aided engineers in building of machines that perform both computationally and physically complex tasks. These tasks vary in a wide range from chess playing engines to autonomous space vehicles; from online shopping assistance softwares to industrial robots. Moreover, these studies shed light on scientific aspects of problem solving and led to developments theoretical domain as well (Newell & Simon, 1972).

Psychologists also have been working on problem solving since the beginning of the 20th century (Dunbar, 1998). Psychological approach to problem solving involves topics like the importance of problem representation, the effects of cultural and emotional differences among the subjects on their problem solving performances, the differences between novice and expert problem solvers, the relation of problem solving to linguistic activities and abilities, and many more.

1.1. The Aims of the Thesis

It is noteworthy to realize that the philosophical aspects of human problem solving have not been studied as much as psychological and artificial intelligence aspects. The philosophical studies on problem solving are mostly about the scientific problem solving and the problem solving abilities of scientific theories (Laudan, 1977). This thesis aims to be one of the few studies which focus on the underlying philosophical concepts of human problem solving. Also, based on our review of the related literature and to the best of our knowledge there are a limited number of studies specifically focusing on the knowledge production process and epistemological aspects of human problem solving activity (Goldman, 1983). So, the second aim of this thesis is to give epistemological formulations to scientific findings in problem solving research.

The third objective of the thesis is rather a practical one about the problem solving environments used in experimental studies. The thesis aims to provide an annotation framework which is sensitive to epistemic aspects of problem solving for both individual and collaborative problem solving environments.

1.2. The Scope of the Thesis

The scope of this thesis covers a very brief history of the studies on problem solving in cognitive science and psychology, some epistemological definitions and discussions along with problem solving related epistemological formulations and an annotation framework which is designed to assist the researchers who work with problem solving environments.

As the reader will see in the related chapters, the brief history of problem solving research will be given and some epistemological claims will be made. However, the reader should keep in mind that the main objective of this thesis is not to provide the reader an exhaustive survey about the history of problem solving related research. So, in the related sections, only the relevant studies will be mentioned briefly and the reader will be directed to more elaborate sources where she can find better accounts for the history of problem solving related research and their details. The same thing can be applied to the epistemological accounts as well. Moreover, the reader should be aware of the fact that the most of the epistemological explanations are just interpretations of what is already asserted by philosophers. The author of this thesis only collects these assertions and applies them to problem solving studies in order to give an epistemic and epistemological basis that aim to guide empirical research.

1.3. The Content of the Thesis

This thesis is comprised of four chapters: introduction, history of the studies in problem solving and epistemological aspects of problem solving, an annotation framework for individual and collaborative problem solving environments and conclusion.

In the second chapter, there are two sections. In the first section, psychology and cognitive science approaches to problem solving will be presented. Some of the concepts and phenomena regarding problem solving activity will be presented as well.

The second section of the second chapter is devoted to the epistemological aspects of problem solving activities. In that section, basic epistemological definitions will be provided in the very beginning. After that, epistemological formulations for problem solving related concepts will be given. Finally, some epistemic claims about problem solving activity of human subjects will be made.

The third chapter is about FODOR which is an annotation framework for the outputs of individual and collaborative problem solving environments. There are two sections in this chapter. In the first section, the need for such a framework and deficiencies of earlier approaches will be stated, and then the conceptual understanding of the framework and the way it works will be explained. In the second section, some case studies will be given along with the underlying assumptions of the framework.

1.4. The Notation of the Thesis

The reader should keep in mind that the term “problem solving” refers to human problem solving activity through the whole thesis, unless noted otherwise. Also, mostly in the second chapter, the terms “proposition” and “belief” are used interchangeably along with the terms “knowledge” and “judgment”. In most of the cases the term “belief” is used to emphasize the human subject’s relation with a certain proposition.

The reader should also be aware of the uses of the terms “epistemic” and “epistemological”. The former is used for the elements about knowledge while the latter is used for the elements about epistemology.

CHAPTER 2

LITERATURE REVIEW

2.1. Problem Solving in Cognitive Science and Psychology

In this section, problem solving related studies in cognitive science and (cognitive) psychology will be explained briefly along with the related concepts and phenomena. The importance of this section comes from its role as a bridge between the scientific understanding of problem solving and epistemological understanding of problem solving which will be presented in the next section. It can be assumed that the basics of the problem solving will be presented in this section, and in the following section the underlying epistemological notions of certain phenomena will be explained.

First, the basic definitions of the concepts about problem solving will be given. Then, a brief history of problem solving research will be presented and different theoretical approaches will be explained. Also, procedural (process-based) understanding of problem solving which is crucial for the next section will be introduced with the examples from Simon (1977) and Reitman (1964).

2.1.1. Basic Definitions and Concepts in Problem Solving

This subsection is devoted to the basic definitions about problem solving. The definitions differ significantly among the researchers and the traditions; however, it is plausible to think that a broad understanding for each concept can be provided. In the following of this subsection, most common understandings about the problem solving related concepts will be given and the reasons why they were chosen will be explicated when necessary.

First, we can start with a review of previously proposed definitions for the term *problem*. Throughout the history of the research on problem solving, there have been several different definitions by different researchers. One of the reasons why there is a multiplicity on the definition of problem is that each researcher tries to define the term according to her own research purposes. Some focuses on the subjective aspects of problems while others focus on the structure or even the way the problem is solved in their definitions. A good exploration about these different definitions can be found in Frensch (1995). Since, this thesis aims to capture problem solving as a whole, our definition should cover the general structure of these definitions and capture a couple of important points which were mostly clarified in (Mayer, 1999):

- Problem is a subjective term. What constitutes a problem for someone may not constitute a problem for another. It might be due to the knowledge base of the problem solver, for example "2 + 3 = ?" does not constitute a problem for a mathematics professor. It might also be due to the physical conditions of the problem solver, for instance a cow does not have to solve the problem of hunting a prey.
- The subject should also desire a solution for something to be a problem. For example, if the subject does not want to learn how the ants build their nests, then it does not constitute a problem for her. This condition can be called *directedness* or *willingness*.
- Problems also have a cognitive aspect. The subject, even if it is a machine or a person or an animal, has to make some decisions, and represent the problem in a way.
- The last point might be about the procedural nature of problems. Each problem should have at least one step between the initial state and the solution.

So, with all these conditions in our minds, we can define *problem* as: When a subject wants to reach a goal which is not immediately reachable by the subject, and has to perform some operations for reaching that goal, then the subject has a problem. Note that the definition is broad enough to cover all kinds of problems: daily problems, personal problems, scientific problems, intellectual problems, puzzles, practical problems etc.

Second, *problem solving* should be defined before investigating more complicated concepts in the body of problem solving research. Different traditions which will be presented later in this section define problem solving according to their way of studying the subject. Again, to give a more inclusionary definition for the purposes of the thesis, a broader definition is preferred here. Problem solving is the accumulation of the activities that a subject (human, computer or animal) performs when solving a problem.

So far, the most basic two definitions were provided to the reader. Now, it is time to investigate some of the most common aspects of problem solving research and give proper definitions for each of them. Two of the common concepts in problem solving are *productive* thinking and *reproductive* thinking (Mayer, 1999). Productive thinking covers the subject's way of thinking when confronted with a new problem which cannot be reduced to an older problem the subject knows about. This type of thinking takes place when the subject is presented with a novel situation. Reproductive thinking can be regarded as the opposite of productive thinking; it is the case that the subject uses her prior knowledge for the solution of a new problem. This brings us to analogical problem solving (Gick, Holyoak, 1980). Analogical problem solving is based on the subject's ability to realize the similarity between two problems. In most of the cases, the subject has a solution for one of these problems and applies this solution to the other problem. Sometimes these kinds of problems which have different cover stories and same underlying idea are called superficially dissimilar problems (Catrambone, Holyoak, 1989). The human subjects' ability to build analogies among problems is beyond the scope of this thesis, however; further information about analogy can be found in (Gentner, 1983; Gentner, Holyoak & Kokinov, 2001).

Another pair of concepts is *routine* problems and *creative* problems (Boles, 1990; Mayer, 1999). Routine problems usually cover the problems that the subject reaches a solution by mostly using reproductive thinking. In other words, routine problems are the familiar set of problems for the subject and the subject can apply what she already knows. For instance, "254 x 78 = ?" is an example of a routine problem, the solution is not immediate for an average person; however, the means for a solution (using a calculator or manipulating syntactic mathematical entities) is available to that person. Contrariwise, creative problems cover those problems that are novel to the subject. This kind of problem is mostly solved with productive thinking activities. The question "What do you do when you wake up and realize that you are the last man on earth?" can be given as an example to creative problems. Again, the important part is the subject's relation with the problem. A problem might be a creative problem for one person and a routine problem for another. The distinction between these two types of problems is important, since most of the time the researchers conduct experiments with one of these two kinds for investigating certain phenomena (Isaksen, 1985; Anderson, 1993; Hermann, 1995). For example, some researchers employing the information processing approach (explained later in the section) often use routine problems and try to show the differences between the novice and expert subjects (Chi, Feltovich & Glaser, 1981; Larkin et al. 1980; Schoenfeld & Herrmann, 1982). On the contrary, the researchers who investigate *insight* (also will be explained) mostly use creative problems to reveal underlying events that lead the subject to come up with a solution to a novel situation (Bowden & Beeman, 1998; Lockhart, Lamon & Gick, 1988). A more detailed explanation about the uses of routine and creative problems will be provided in the related subsections.

Another important concept which will be used very frequently in the rest of this section is *problem representation* (Gick & Holyoak, 1980). Problem representation can be considered as the phase before the problem solving takes place. It is more or less the process that the subject takes the problem and creates a mental representation for it. According to Richard Mayer (1995), problem representation is the combination of two components: *translating* and *integrating*. In the translation phase, the subject mentally represents everything given (propositions, figures, constraints etc.) in the problem definition and the integration phase includes the subject's efforts for building a coherent structure with the created representation. An important historical event regarding the significance of problem representation can be found in the story of Leonhard Euler (1707-1783) and the seven bridges of Königsberg problem (Alexanderson, 2006; Gribkovskaia, Halskau & Laporte, 2007). In 1735, Euler was presented with the problem of whether it was possible to cross each bridge in Königsberg only once and complete a tour or not. There were seven bridges on the river Pregel each connecting some regions of Königsberg to another. At first, Euler thought that the problem was not related to mathematics (Alexanderson, 2006, p. 568), however; then he realized that the current representation of the problem was misleading. He turned geographical representation into nodes and connectives, and understood that the problem was not about the geographical features of the city and the bridges but about the mathematical relations among the nodes and their connections. His representation of the problem as a graph paved the way for the mathematical discipline called graph theory (Biggs, Lloyd & Wilson, 1976).

2.1.2. Brief History of Problem Solving in Psychology and Cognitive Science

The very first scientific attempts to understand the underlying phenomena of human problem solving can be dated back to the works of Oswald Külpe, Karl Bühler, and Otto Selz (Dunbar, 1998). Their primary focus was on complex thinking and problem solving activities. The very first experimental studies on problem solving (mostly conceptualization tasks) also were done in the early 20th century by these researchers. Later on, their legacy was taken over by Gestalt psychologists. Gestalt psychologists also studied complexity of human thinking and problem solving activities. The problems they used in experiments were mostly creative problems in which the subjects needed to find a crucial element for the solution of the problem (Dunbar, 1998).

In 1950s, in the lead of Herbert A. Simon and Allen Newell, problem solving started to be seen as a search in a problem space. Simon and Newell not only studied human problem solving, but also tried to extend the findings of human problem solving into computer simulations that imitate human problem solving performance. This symbol-based approach of problem solving became very successful both in explaining human problem solving and simulating human performance with computers. Contemporary studies on problem solving can be seen as an extension to what Simon and Newell started.

Three different theoretical approaches can be distinguished in the course of scientific problem solving research: associationism, Gestalt tradition and information processing approach (Mayer, 1999, p. 440). Associationist theory claims that problem solving activity is mostly based on the associations between the ideas (concepts) in human mind. There is not much room for creativity in this approach because; the theory emphasizes the use of former associations in the mind.

Gestalt tradition took problem solving with a different perspective than associationists. Although they agreed with associationists that some ideas and connections between these ideas preexist in human mind, they further claimed that problem solving has a creative aspect. Their primary focus was on the creative aspects of problem solving. According to this tradition, when subjects are presented with a novel problem, first they construct their problem representation and if they fail, then they creatively reconstruct the problem representation to solve the problem. For their purposes, they mostly used problems that require the reconstruction phase. These types of problems are called *insight* problems in which the subjects need to gain some insight into the problem in order to solve it. Two classical examples of insight problems are candle and radiation problems (Duncker, 1945). These problems enabled the researchers to identify a phenomenon called *functional fixedness*. Functional fixedness can be defined as "a mental block against using an object in a new way that is required to solve a problem" with Karl Duncker's (1945) own words. In the definition, the use of objects in new ways is mentioned, however; it is not only the use of the objects in new ways that block our capacity to solve the problems but also the well-accepted methods and constructions.

There is a similar phenomenon called the *Einstellung effect* (Luchins, 1942). Einstellung means “setting” or “installation” and the phenomenon is about the human subjects’ sticking into the previous successful methods for solving a similar problem when there is a better or easier solution available. Luchins, in his famous experiment, presented several problems to the subjects (1942). The problems all had the same path for the solution and the subjects applied this solution strategy to the last problem which had an easier possible solution. The findings can be regarded as the negative effect of the previous knowledge in problem solving. These two effects (Functional fixedness and Einstellung effect) are important for the purposes of this thesis since in the following section an epistemological analysis of these effects will be given and the findings will constitute the backbone of the framework presented in the third chapter.

The last theoretical approach to problem solving is information processing approach which was introduced by Simon and Newell in 1960’s (Dunbar, 1998; Mayer, 1999). Their approach was novel in several ways: First, they did not primarily focus their research on insight but studied the underlying processes that lead the subjects to solve a problem. Second, they used verbalizations instead of introspection in their studies. Third, they also designed computer programs that would simulate human problem solving activity. And the last, they contributed to artificial intelligence domain in terms of algorithms, heuristics and strategies they introduced (Newell, Shaw & Simon, 1958; Newell & Simon, 1972; Dunbar, 1998)

According to this approach, information in human mind can be attributed as propositions and the operations on these propositions are the basics of the problem solving activity (Mayer, 1999). They defined the problem in terms of the states: the initial state in which the subject has a problem and needs to solve it, the goal state which is the solution of the problem and the intermediary states which are the states that can be reached by the subject from other states by applying certain operations to the aforementioned propositions. This representation is called problem space representation (Newell & Simon, 1972; Mayer, 1999). In the problem space representation, the problem solving activity turns into a search problem for the goal state. To solve this search problem, the subject needs to understand the structure of the problem space and have knowledge about the operations available to her. This understanding of problem solving activity has a highly formalized structure and this structure comes from the findings about the characteristics of human subjects. These characteristics are: small short-term memory with fast retrieval and fast storage, assumedly infinite long-term memory with slow storage and serial processing (Newell & Simon, 1972, p. 149).

The structure of the problem space and the way subject searches this problem space brings us to the algorithms, heuristics and search strategies. There is a huge body of research regarding these notions, but summarizing this research is beyond the scope of this thesis. Here, only the related concepts will be presented to the reader. As mentioned, Simon and Newell not only investigated human problem solving activity, but also tried to simulate it by using computer programs. The outcome of their endeavors was the General Problem Solver (Newell, Shaw & Simon, 1959). GPS was able to mimic human problem solving performance by using a strategy called *means-ends analysis*. Means-ends analysis is a strategy that recursively creates subgoals when the goal is unreachable. Once the goal or one of the subgoals is reachable, then the program acts to reach that goal and repeats these steps until the goal is achieved. The idea behind this strategy comes from the experiments on human subjects with the tasks like Towers of Hanoi problem (Newell & Simon, 1972, p. 152).

So far, the history of the scientific approaches to problem solving was briefly presented along with different theories. Also, some problem solving related concepts and phenomena were explained. This would be a rather insufficient subsection if the aim of this thesis were to provide an exhaustive survey on problem solving research, however; this is not the aim of this thesis and for the purposes of this thesis all the related content was provided to the reader. For a more elaborate comprehension of the problem solving research the reader may consult to (Dunbar, 1998; Sternberg, 1998; Runco & Pritzker, 1999).

2.1.3. Well-structured and Ill-structured problems

In the previous subsection, an important distinction in problem solving research was skipped for the sake of clarification/explicitation. That is the distinction between *well-structured* and *ill-structured* problems. These two types of problems are important due to the fact that most of the subjects' representation and solution activities are shaped according to the structure of the problem. They are also important for the purposes of this thesis since the framework which will be provided in the following chapter mostly depends on the structure of ill-structured problem solving.

Giving the definitions for these notions would be a good starting point, however; there is no simple way to define these terms only some characteristics or attributes of the terms can be provided (Simon, 1977; Newell, 1993). According to Herbert Simon (1977), well-structured problems have six criteria:

1. There should be a criterion for testing the potential solutions.
2. Initial state, goal state and the other intermediary states can be represented in at least one of the problem spaces.
3. Legal moves or operators are well-defined.
4. Any knowledge regarding the solution of the problem can be represented in the problem space.
5. Correspondence with the real world is necessary for real-world problems.
6. The solution should be able to be generated in a practical amount of search.

However, for Simon, ill-structured problem is a "residual definition" which means that the term is explained by its opposite term. Newell calls "negative definition" for the same case (Newell, 1993, p. 365). We can define or at least give some criteria for well-structured problems, but we can define ill-structured problems only by means of its deficit of these criteria.

Simon also takes one step further and claims that even chess or theorem proving are not well-structured problems unless they are presented to machinery like General Problem Solver (Newell, Shaw & Simon, 1959). According to Simon, every subgoal of these problems is well-defined; however, the problems themselves are not. This approach of seeing the problem as the collection of its components is noteworthy as will be mentioned in the following subsubsection.

2.1.3.1 Solving Ill-structured Problems

> But on this particular occasion, they didn't stop. And when we went off stage, they sang You'll Never Walk Alone to us. I'd gone to sleep thinking, "What can an audience do?" They're all crammed in there. They can stamp their feet, clap their hands and sing. So I woke up with We Will Rock You in my head… ... I came up with We Will Rock You and Freddie with We Are The Champions. His thinking was very similar.

This is taken from Days of Our Lives (2011) which is a documentary about the rock band Queen and the quote belongs to Brian May who is the lead guitarist of the band. He is talking about the underlying processes that lead them to compose two of the greatest stadium rock songs after they (Queen) decided to.

Composing a piece of music, creating a piece of art, writing a poem, designing a vehicle for the army or for a space mission, designing a building for certain purposes, sustaining a good life… These are all examples of ill-structured problems where the initial state is not clear, the goal state is obscure and there are no well-defined operators you can use at a certain state, and even if you find a solution there is no clear way to test it. However, people solve this kind of problems all the time. In this subsection, the structure of solutions for ill-structured problems will be given. The most of the following is the interpretation of the works of Allen Newell (1993), Walter Reitman (1964) and Herbert Simon (1977).
According to Simon and Reitman, there are key elements in the process of solving ill-structured problems: constraints, style and priorities. Actually, Reitman and Simon only use the term “constraints” directly, but the other concepts were present in their writings implicitly. So, it was considered to be better to include the other two terms as well. Before explaining the terms, simple characteristics of the ill-structured problem solving process should be given.

In each problem, whether it is well or ill defined, there is a goal that the subject should reach to in order to solve the problem. Moreover, the subject needs to pass through some states for reaching to that goal. So, it can be argued that if the journey of the subject from the initial state to the goal state has some coherent characteristics, then the ill-structured problems can be formalized to a certain degree. Let’s take the radiation problem (Duncker, 1945) as an example. In the problem, the subjects are supposed to save an alleged patient who has a malignant tumor in one of his internal organs. The subjects can apply ray therapy which can destroy the tumor, but the ray also kills the healthy tissue as it passes through the body of the patient. If the intensity of the ray is reduced, then it does not harm the healthy tissue but it also does not destroy the tumor. What can the subjects do in order to destroy the tumor without harming the patient?

The solution includes the use of several rays with reduced energy reaching the tumor from different directions. This way, none of the rays harm the healthy tissue because, they are not strong enough. Nevertheless, they can destroy the tumor when they focus on the same region (where the tumor is) in the body.

As one can see, both the problem and the solution are ill-structured. There are no definite or explicit terms. For example, how many low-intensity rays are needed or does the body gives the same reaction in each and every region? These can not be answered by looking at the question or the solution. However, through all these ill-structured definitions and instructions, we can clearly see that the solution requires the use of two certain strategies together: using multiple low-intensity rays and focusing the rays on the tumor. These two steps can be regarded as the subgoals of this problem or the key parts of the solution.

Defining the key parts (subgoals) for the solution is the main idea of Reitman (1964) and Simon (1977). They claimed that no matter how ill-structured the problem is, there would always be some subgoals if there is a solution. Moreover, they built an analogy between these subgoals and formal grammars. According to this understanding, problems can be represented as the accumulation of subgoals and this process can be repeated recursively for each subgoal until a well-defined problem is reached. Simon exemplifies the problem of an architect who is trying to design a house in a similar manner (1977, p. 313): “Applying the same linguistic metaphor to house design, 'house' might transform to 'general floor plan plus structure', 'structure' to 'support plus roofing plus sheathing plus utilities', 'utilities' to 'plumbing plus heating system plus electrical system', and so on.”

So, it is clear that composing a rock song can be seen as combining several different aspects together: lyrics, tempo, chord progress, intro, outro, bridge, instrumental solo, main riff etc. In a similar manner each of the problems mentioned in the beginning of this subsubsection can be crumbled into smaller pieces which can be handled more easily. The creation and identification of the subgoals for any given problem is highly related with the subject’s representation of the problem, her priorities, the constraints of the problem - and the stylistic aspects if it is a daily problem.

2.2. Epistemology of Problem Solving

This section is about the knowledge types and the knowledge production processes take part in problem solving activity. In the very beginning of the chapter, the classical definition of the term ‘knowledge’, and some basic concepts and classical discussions in epistemology will be given. Afterwards, with the notation borrowed from Goldman, the epistemological formulations of ‘problem’ and ‘problem solving’ will be presented. Additionally, Goldman’s perspective of seeing the solution of a problem as knowledge will be explained and some problem solving related phenomena which were mentioned in the previous section will be revisited in terms of epistemology.

2.2.1. Basic Concepts and Discussions in Epistemology

I would like to start by giving the classical definition of knowledge as 'justified true belief' (Ichikawa & Steup, 2012). The definition was first given in Theaetetus of Plato (369 BC) as “justified right opinions” and has been discussed since then(Lehrer & Paxson, 1969; Lehrer, 1990; Ayer & Maric, 1956). The philosophers mostly attacked the inadequacy and the vagueness of the term 'justification' in the definition. Some philosophers also advocated the idea that the classical definition of knowledge captures non-knowledge elements (Gettier, 1963). Here, however, I will not summarize these objections to the classical definition nor will I give a new definition. This definition is proper and sufficient for the objectives of this thesis. Nevertheless, the components in the definition should be clarified. According to the definition, a proposition should satisfy three conditions to be attributed as knowledge: *belief condition*, *truth condition* and *justification*. The definition can be formulated as (Truncellito, 2007; Matthias, 2005):

S knows that p if and only if,

1. S believes in p, and
2. p is true, and
3. S is justified in believing that p.

The first condition, belief condition, can be understood as the subject S's relation with the proposition p. If the subject S believes in a proposition p, then the first condition of knowledge is achieved. The condition seems very straightforward; however, there are some controversies which will be skipped here, since they are not related to the content of this thesis.

The truth condition seems less controversial, because we intuitively think that it is not wise to say that "Subject S knows p and p is wrong.", and so the truth of the proposition is indispensable for knowledge. Also, Plato in his Theaetetus in which the definition we use for knowledge was first stated made the distinction between knowledge and true beliefs (or right opinions). Nevertheless, when we look at the cumulative knowledge of humankind, we can see that the 'truths' of one era can be the 'wrongs' of another era both in the scientific and in the cultural senses. For example, in the 19th century, the scientists 'knew' that the planets were moving according to the Newtonian physics, but in the end the Newtonian physics was proven to be wrong at least in some aspects. Can we really say that those scientists did not have knowledge? Or in a similar manner, can we say that everything we have in science today might be disproven someday in the future and we do not know anything right now? A more detailed explanation on this debate can be found in the writings of several analytic philosophers like Allan Hazlett (2010), Roderick Chisholm (1966).

The last condition is justification and for the purposes of this thesis, most emphasis will be made on this component. Justification takes a belief as its subject matter and investigates if it is formed properly or improperly by the subject. This is actually the definition of *doxastic justification* (Turri, 2010). There is also the concept of *propositional justification* which is mostly mistakenly used instead of the former. Propositional justification only cares about the truth condition of the proposition without regarding how the subject came to believing in that proposition. The following example is from John Turri's paper in which Turri investigates the relation between these two types of justification (Turri,2010):

> Imagine two jurors, Miss Knowit and Miss Not, deliberating about the case of Mr. Mansour. Both jurors have paid close attention throughout the trial. As a result, both have good reason to believe that Mansour is guilty. Each juror goes on to form the belief that Mansour is guilty,

> which he in fact is. Miss Knowit believes he’s guilty because of the evidence presented during the trial. Miss Not believes he’s guilty because he looks suspicious. Miss Knowit knows that Mansour is guilty; Miss Not does not. Why the difference? Miss Knowit believes he’s guilty on the basis of the good reasons she has, whereas Miss Not, despite having good reasons at her disposal, believes based on mere suspicion.

In the excerpt both jurors have the same proposition as their beliefs. However, only one of them is said to know it. That is because the doxastic justification of the belief is present only in the case of Miss Knowit. If we were investigating propositional justification, statuses of both jurors would be the same, since they held exactly the same belief. For the rest of the thesis, the term justification will be used in the sense of doxastic justification.

2.2.2. Sources of Justification

People may have all kinds of beliefs in all kinds of fields and topics; and these beliefs might be based on intuitions, instincts, emotional conditions, wills etc. However, for a belief to become knowledge, the sources of the belief and its justification are crucial. In this section, well established sources of justification will be introduced and each of them will be explained very briefly. The sources were determined according to the writings of Steup Matthias (2014) and Robert Audi (2003). They both acknowledge that there are five legitimate sources for justification; however, they used different names for those. Audi lists the sources as: perception, memory, consciousness, reasoning and testimony. Matthias lists his five sources as: perception, memory, introspection, reason and testimony. What Audi covers with the term *consciousness* matches with the term *introspection* used by Matthias. So, *introspection* will be used for the next of the section and the thesis. The final list of sources of justification is:

a. Perception
b. Memory
c. Introspection
d. Reasoning
e. Testimony

The names for justification types are very explanatory. Nevertheless, an explanation for each type will be provided to the reader for a better understanding of the distinctions between the sources. An average person has five perceptual faculties: visual, auditory, tactile, olfactory and gustatory. Each of these faculties can play a justificatory role for the beliefs of the person. The beliefs justified by means of perceptual faculties are called perceptual knowledge. There are lots of debates going on regarding the structure and reliability of the perceptual justification processes. A detailed overview of these debates can be found in (BonJour, 2007).

The next justification tool available to us is memory. Memory can basically be defined as our ability to maintain the knowledge acquired in the past. It may be the most controversial source of justification, since we do not have any means to distinguish the false memories from the real ones. However, the authenticity of the memories is not one of the concerns of this thesis thus, related debates will be skipped. Thomas Senor's Epistemological Problems of Memory (2009) is a good source for the related discussions.

The third source of justification is introspection. Introspection can be defined as our ability to view the processes taking place in our minds. The idea is that just by "looking in" we might trace our mental states (at least some of them) and have some knowledge about our body and mind. The knowledge coming from introspection is mostly about the mental or physical states like "I am happy." or "I feel pain in my stomach.". The *foundationalists* attribute a special status to introspection. Nevertheless, there are objections to this special status and infallibility of introspective knowledge. The claims of both sides can be found in David Armstrong (1963).

The fourth source of the justification is reason or reasoning. For a belief to be justified by reasoning, the belief should not be based on perception, introspection or memory. It is also called *a priori justification* which will be introduced as a knowledge type in the following section. There are lots of discussions about the existence and the limits of a priori justification (BonJour, 1998; Christensen, Kornblith, 1997). For example, skeptics claim that there is no knowledge which is not empirical. Empiricists also claim that the boundaries of a priori justification are the boundaries of analytic knowledge which will also be mentioned in the following subsection. In a similar vein, it can also be argued that reasoning might also be used to evaluate and organize the information coming from the experiences. For example, the subject's realization that a tangram piece belongs to a certain position after unguided or random movements can be regarded as this type of justification. This argument is important, since in the following sections it will be claimed that just by exploring a well-known set of propositions some new knowledge can be gained by the subject. Laurence BonJour's In Defense of Pure Reason (1998) constitutes a good starting point for the debate.

The last source of justification is testimony which mostly comes from an authoritative figure on the subject. The figure might be anyone or anything: a politician, a scientist, an eyewitness, a good journal, an online blog etc. It is obvious that this type of justification differs from the other ones in the sense that the person does not only use his mental capacities but also relies on the knowledge of an outer source. However, most of the time it is argued that the justificatory role of the testimony depends to the subject's prior relation with the source (Graham, 2006; Faulkner, 2000). For example, if the subject hears some news from a source she does not trust, the news does not constitute justification or knowledge for her.

In the following subsections, it will be shown that each of these five types of justification is used in some aspects of problem solving activity.

2.2.3. Structure of Justification

In the previous subsection, we have seen that the subject needs at least one other justified belief to justify one of her new beliefs. For example, if the subject believes that she sees a green apple in front of her, then she should be also believing that her vision is reliable. The same current of thought is valid for the other sources of justification as well. If you remember falling from an oak tree when you were a kid, then you should also rely on the fact that your memories are accurate and so on.

If every belief we have is justified by other beliefs, then there must be some relation among our beliefs. This brings us to the investigation of the structure among beliefs, in other words structure of justification. There are two well-accepted explanations for this structure: foundationalism and coherentism. There is also another branch that claims the justification or knowledge is not possible at all (Crosby, 1988; Pratt, 2005); however, this view will be skipped since this thesis already assumes that the justification or knowledge is possible. In the rest of this subsection, first, the two kinds of possible structures will be presented and in the end their relations with problem solving activity will be discussed.

The underlying understanding for each of these approaches is that each proposition is justified by means of at least one other proposition. However, if every belief is justified by means of at least one other belief, then this justification process has to continue forever because there will always a belief left without justification. This situation is called *epistemic regress problem* or *regress argument* which can be basically formulated as (Cling, 2008):

1. The proposition p1 is justified by p2, and
2. The proposition p2 is justified by p3, and
3. The proposition p3 is justified by p4, and
4. The proposition p4 is justified by p5, and
5. ...

As can be seen in the formulation that each justificatory proposition needs another proposition to be justified and this leads to a vicious cycle. Foundationalism and coherentism are the two most important attempts to solve this cycle.

Foundationalism claims that there are *non-inferential* propositions which do not need any other propositions to be justified, in other words there are *self-justified* propositions. So, the idea is that the cycle ends when it reaches to a non-inferential proposition. But, the question is what are these self-justified or non-inferential propositions that are able to break this vicious cycle? The answers differ among the philosophers and traditions. David Armstrong (1973) claims that there is a law-like (nomological) connection between the world and the human subjects. The term he uses for this connection is "thermometer model of knowledge", he claims that the human subjects are capable of gathering some non-inferential knowledge from the world just like a *reliable* thermometer indicates the air temperature.

Alvin Goldman (1975) suggested that if there is a *causal* connection between the fact and the corresponding belief, then the belief does not need justification to become knowledge. It is slightly different from the concept of non-inferential beliefs. Goldman claims that these kinds of beliefs do not need justification and the subject just knows them without thinking any further. He supports his claim by exemplifying a chicken breeder who is capable of telling the sex of a chick without knowing how he is doing it. He is unaware of the processes taking place in his decision, but he is always right (Goldman, 1975, p. 114). The way Armstrong and Goldman illustrated how non-inferential beliefs can exist is nowadays called *reliabilism* which is an externalist view of justification. The main assertion of reliabilism can be summarized as the subject's belief that p is justified if, and only if, the subject reaches to p through reliable processes. A more detailed demonstration of reliabilism can be found in Goldman (1994), Goldman and Erik Olsson (2009).

What we have seen so far were the examples from externalist foundationalist theories, now some examples from the *internalist* approach will be presented. It can be assumed that René Descartes was one of the very first philosophers who claimed that just by introspection; we can come up with infallible knowledge (Descartes, Weissman, 1996). What he did was questioning everything and realizing that even though everything can be doubted the doubt itself is always present as well as the subject who doubts. Briefly, it can be said that just by investigating the internal states, one may come up with a belief which does not rely upon another belief for justification.

The other attempt to solve the epistemic regress problem is coherentism. Coherentist theories claim that the justificatory relation among the propositions can also be circular. What they mean is that we do not need non-inferential beliefs in order to stop the vicious cycle; a consistent set of beliefs in which each proposition is justified by means of other propositions in the set is sufficient to solve the epistemic regress problem (D. Davidson, 1984).

The possible roles of both foundationalist and coherentist theories in problem solving activity will be presented in the related subsection.

2.2.4. Knowledge Types

In the classical view, knowledge has four types: *knowledge that*, *knowledge of*, *knowledge by acquaintance* and *knowledge by description* (Truncellito, 2007; Russell, 1910). I feel the need to clarify these types of knowledge before going any further and deeper in the conceptual analysis. The first kind of knowledge, also known as propositional knowledge, is *knowledge that*. It is mostly expressed with a proposition or a declarative sentence. The main characteristics of this type of knowledge are that it can be expressed and transmitted without much effort by human subjects. "Rome is the capital city of Italy." and "Cats are mammals." are examples for this type of knowledge.

The second type is *knowledge of* which is also called *know-how* or *tacit knowledge* depending on the domain and on the context. It is the knowledge that we cannot easily express or cite like the ability to tie a knot or to ride a bike, but we use and exploit without giving any further thoughts. The primary difference between the *knowledge that* and *knowledge of* is that the former can be expressed easily and the subject is fully aware of having it; however, the latter cannot be expressed most of the time by the subject and the subject does not have to be aware of having the related knowledge.

The third and fourth types of knowledge are considered to be identical by some philosophers (Truncellito, 2007) and are called *knowledge by*. However, philosophers like Bertrand Russell (1910) make the distinction between the two types. According to Russell, knowledge by acquaintance is the knowledge of the things that are within the boundaries of our direct perception. For example, one's knowledge about the apple standing on the table in front of her is this type of knowledge. As for the other type, a good example is the knowledge of "the tallest man on earth". Even though, we have not seen him, know him in person or know anything else about him, just by the description of him, we can have an understanding of him. Also, our knowledge on historical events is of this type since we have not witnessed them nor we have the chance to witness them anymore. For each knowledge type, more detailed definitions and several more examples can be found; however, this is sufficient for the purposes of this section for now and more elaborate discussion will be made later where necessary.

Knowledge can also be categorized according to its source as *a priori* ("from the earlier" in Latin) and *a posteriori* ("from the latter" in Latin). The first type is used more or less for the knowledge that is independent from the experience. In a priori knowledge, the justification of the judgment is not based on the experiences. In Critique of Pure Reason, Immanuel Kant summarizes the a priori knowledge as "although all our cognition begins with experience, it does not follow that it arises from experience" (Kant, 1781). On the contrary, a posteriori knowledge is the knowledge that is based on the subject's experiences. To emphasize the distinction, the following two declarative sentences can be given:

a. "All bachelors are unmarried."

b. "Some bachelors want to marry as soon as possible."

If we are justified with both sentences, we can say that our justification for the first sentence is not based on experience, it is justified by definition; however, in the second sentence our justification should be based on some experience about the bachelors. So, it can be concluded that the first proposition can be known a priori while the second can be known a posteriori.

The last distinction I would like to mention is the distinction between *analytic* and *synthetic* knowledge. In the former, the justification or the truth condition of the proposition can be attained just by looking at the components of the proposition; assuming that you know the meanings of the words in the proposition. In the latter, however, to be justified or to gather the truth condition of the proposition, you need to have at least some understanding of the universe which the proposition is about. In other words, analytic propositions can be thought as the propositions in which the subject (as a concept) contains the predicate (as a concept) and synthetic propositions can be considered as the propositions in which the subject does not include the predicate. To clarify, we can look at some famous and informative examples by Kant (1781):

a. "All bodies are extended."

b. "All bodies are heavy."

It is clear that the first proposition is true by definition; however, the second proposition claims something which is not contained in the subject in the first place. So, to justify the proposition or to determine the truth condition, we need to investigate the universe.

Before Kant, it was thought that the propositions were either *analytic a priori* or *synthetic a posteriori*. The intersection of a priori and synthetic propositions was empty; as well as the intersection of a posteriori and analytic propositions. Kant argued that there is a set of propositions/judgments which are a priori in the sense that their truth does not come from the experience and synthetic in the sense that the subject does not include the predicate. “7 + 5 = 12” is an example of *synthetic a priori* judgments according to Kant because, its truth does not depend on the experience and the concept of “12” is not included in the concepts of “7”, “5” and “+”. This is brief and a rather insufficient summary of synthetic a priori judgments, however; the reader can refer to (Kant, 1781; Reichenbach, 1936; Reichenbach, 1952; Rey, 2008) in order to see a more elaborate explanation of Kant’s approach and the objections directed to this approach.

2.2.5. What Happens in Problem Solving in terms of Epistemology

In this subsection Alvin Goldman's approach to problem solving will be introduced. Also, his epistemological formulations will be borrowed for the rest of the thesis. The definitions of *problem, problem solving* and *solution* provided by Goldman will constitute the basis for the epistemological background of the framework that will be proposed in the next section for analyzing problem solving episodes.

According to Goldman, problem is a subjective issue meaning that the same thing might constitute a problem for one subject and not constitute a problem for another. It is all about the condition of the subject, and this condition includes things like familiarity with the problem, physical states, possession of a solution for the problem by the subject and the willingness of the subject to overcome the related problem. For something to constitute a problem for a subject, a couple of conditions should be met. The subject should have some goal which is not achieved immediately and the subject should have a desire for achieving that certain goal. So, problem can be defined as (Goldman, 1983, p. 23):

Person S has a problem (question) Q if, and only if

1. S wants to have an answer to Q, and
2. S believes that she doesn’t have an answer to Q.

It is also important to note that when Goldman and the author of this thesis talk about problem, it might be any kind of problem: practical, intellectual, personal etc. Replacing a light bulb or deciding what to eat for breakfast are problems in this sense. It can also be argued that each practical or personal problem has a corresponding intellectual problem (Goldman, 1983, p. 23), so we can study problems as a whole without making distinctions about their contents.

Now that we have a definition for problem, ‘solution’ can be defined in a similar manner. For a subject to have a solution to a problem means that the subject acquires some sort of knowledge that enables her to overcome or solve the problem. The definition is as follows (Goldman, 1983, p. 24):

Proposition A is a (real) solution (answer) to problem (question) if and only if,

1. A is a potential answer to Q, and

2. A is true.

As you might have noticed, there is no connection between the subject and the solution in this definition. Since, it is only the formulation of a solution, not the problem solving activity of a subject. Now, the following definition forms a connection between the subject S and the solution A:

S has a solution (answer) to Q if and only if,

There is a proposition A such that

1. S believes A,
2. S's belief in A is justified
3. S believes that A is a potential answer to Q, and
4. A is an answer to Q

Evidently, there is a similarity between the definition of knowledge in section 2.3.1. and the definition provided here for problem solving. That is because the problem solving activity of a subject can be regarded as a knowledge production process. The subject comes up with a potential solution, tries her solution for justification and if the solution is true, then the subject has a justified true solution which is a piece of knowledge by definition. Another subtle implication of this formulation is that problem can also be defined as the shortage of knowledge of the subject in terms of epistemology. This is an important point since some problem solving related phenomena like functional fixedness and Einstellung effect can be investigated in the light of this definition. The related investigations will be given towards the end of this section.

So far, the epistemological formulations for problem, solution and problem solving were given. Also, the connection between the subject and the solution was clarified along with the thesis that "solution to a problem is knowledge". In the following two subsections, the outputs of this subsection will be used to show that problem solving activity plays a justificatory role in knowledge production, and not only the propositions alone but the relations among them constitute knowledge.

2.2.5.1. The Organization of Propositions

It is obvious that some of our knowledge is in the form of propositions like "The capital city of Australia is Canberra" or "The equatorial circumference of the Moon is 10,916 km.". Even if we have never been to Australia or to the Moon, we might have these pieces of knowledge. They are true, we believe in them and we are justified in believing them by testimony, experience or any other means. It is also true that the equatorial circumference of the Earth is 40,075 km. Just by knowing these two facts about the Earth and the Moon, and by using some basic concepts of mathematics, we can also know that the Earth's equatorial circumference is greater than that of the Moon. So, not only the propositions alone, but also their relations with each other give us a piece of knowledge in some occasions. This subsection is the manifestation of the idea that when the propositions come together it might provide us new knowledge, and in some problems, mostly in insight problems, the solution comes not from the acquirement of new propositions but from the reorganization of the propositions which we already has. Now, this claim will be stated and supported with few examples, and then the underlying epistemological concepts in problem solving related to this claim will be provided.

The claim of this subsection can be summarized as: "Not only the propositions themselves, but also their relations and organization in the memory constitute knowledge." First part of the claim is straightforward, if you have a proposition p in your belief set, and p is justified and p is true, then you know p. The second part of the claim is not that straightforward; an illustration is needed here. Suppose that you are asked to evaluate or criticize the foreign policies of your country. If you do not have a ready-made answer at your disposal, you would have to think a couple of seconds or even minutes before answering. Within these couple of seconds, you probably try to probe your memory for the related propositions and arrange these propositions in a way that the organization would be consistent and provide a sufficient answer for the question you were asked. It is clear that you gain no new beliefs or pieces of knowledge from anywhere else but your own memories and among the propositions you already have. This is the case most of the time when we are required to interpret some subject which we already have some knowledge about.

In *The Sciences of the Artificial*, Herbert Simon touches upon a similar topic (Simon, 1996). He questions if the computer simulations of real world environments or objects could give us new information or not. He mentions and accepts two assertions that were made about the nature of these simulations: simulations are no better than the assumptions built into it and a computer can do only what it is programmed to do (Simon, 1996, p. 14). Even though he accepts both of these assertions, he still claims that new knowledge can be acquired by studying the simulations. The underlying idea behind his stream of thinking is " All correct reasoning is a grand system of tautologies, but only God can make direct use of that fact. The rest of us must painstakingly and fallibly tease out the consequences of our assumptions" with his own words. In a nutshell, the set of propositions is too complex for a human subject to deduct all deducible theorems in the set and the simulations give us the ability to easily manipulate the objects or environments and observe the outputs. These outputs constitute knowledge for us about the simulated phenomenon.

The relation between this process and problem solving activity is that in problem solving the subjects need to explore the environment and use the tools in order to get a comprehension about them. Even if the subject propositionally knows all the relevant knowledge about the environment and the objects in the environment, that would not be enough for her to be able to fully understand and internalize the situation she is in. By exploration the subjects gain new knowledge (mostly tacit knowledge) about the environment, and these pieces of knowledge can be added to their knowledge bases as will be seen in the following chapter.

2.2.5.2. Problem Solving as a Justificatory Process

The types and sources of justification were explained in the sections 2.3.2. and 2.3.3. respectively. We have seen that justification might have its roots in our memories or in our perceptions; it can also be depending on our ability to follow the activities in our brain (consciousness or reflection actually) or our ability to reason; and the last source of justification is the testimony which mostly comes from someone whom we see as an authority at the relevant subject. It was also concluded that justification is a necessary condition for a belief to develop into knowledge. This subsection is devoted to the idea that problem solving activity of a subject may lead her to justify her beliefs (and turn them into knowledge), make some of her knowledge get stronger and dispute some of her beliefs or knowledge pieces. First, the justificatory role of problem solving activity will be discussed and we will continue with an example designed to illustrate the changes of justificatory status of beliefs in problem solving practice. After that, related literature from Cognitive Psychology and Cognitive Science areas will be revisited in the light of the claims made in this subsection.

So far, we have seen that justification takes place when the person has relevant evidences and the proper reasoning about her beliefs. Suppose that our subject is sitting in her room reading some papers; all of a sudden she hears a strange hissing noise from outside of her room. She has couple of beliefs at the moment which are all plausible with the facts available to her. She thinks that it might be due to the wind, because she remembers leaving the window open. She also suspects that the noise might be caused by some sort of bug or even a bat or some other animal, because she lives in the countryside and it is a common thing for an animal to visit houses. To end her doubts and anxiety, she goes and checks for the source of the noise and she realizes that the noise is coming from the curtain jostling to the wall because of the wind coming from the open window. She was right in believing that the noise was due to the wind, and she got justified by directly seeing the curtain jostling the wall and hearing the noise at the same time. She goes back to her room and continues reading with the relief that she has found the source of the noise. Now, what happens here is that she has a problem of identifying the source of a strange noise, and she overcomes this problem by using her motor and perceptual abilities. At the beginning she has only some beliefs mostly based on her prior encounterings with similar situations. But, in the end, after her seeing the curtain jostling the wall and producing that certain noise, she knows the source of the noise exactly. Surely, it can be claimed that the next time she hears a similar noise she would not have to get to the other room to detect the source of the noise. That is because the solution of her former problem justifies one of her beliefs and she no longer needs to solve this problem again.

In problem solving, it can be assumed that the ends justify the means. When the person is satisfied with the result of her effort for solving a goal or a subgoal, she will have a tendency for justifying her efforts. These efforts can include problem solving strategies, algorithms, heuristics, certain uses of tools etc. The justification can be understood by using both foundationalist and coherentist approaches which were presented in the related subsection. According to the foundationalists, some beliefs or propositions do not need any further justificatory beliefs or propositions. This idea can be borrowed to explain what happens when a subject comes up with a solution to a problem. The solution of the problem (assuming that the subject has the means to evaluate correctness of the solution) does not need any further explanations or beliefs for justification; the subject *just* knows the solution when she has one. With other words, the solution to a problem can be seen as a non-inferential belief which is the starting point from which the subject begins her journey of justification. It can be formulated as:

S knows p if, and only if,

1. A is a solution for Q, and
2. S knows A, and
3. S is justified in believing that p on the basis of A, and
4. p is true.

Here, in the formulation, *p* can be regarded as a former belief of the subject which was used in the problem solving activity. Moreover, the belief p was justified on the basis of the correct solution A. For example, suppose that the road you always use for going home from your working place is blocked, and you have some reasons to believe that a certain route which you have never used may take you home. You try that new route since you do not have anything else to do, and as you expected the route takes you home. From now on, you can use this new route without any hesitations. What happens here is that you have an unjustified belief and you use that belief to solve a problem that you have never encountered before, and your unjustified belief let you solve the problem. By the solution of the problem, your unjustified belief is justified and turns into a piece of knowledge.

This attempt for providing epistemological groundings for knowledge production in problem solving activity is also consistent with the ideas of Clarence Irwing Lewis and Arhut Pap (Pap, 1946; Boyer, 1958; Stump, 2011). Lewis claimed that all the analytic rules, laws, conventions, definitions and a priori concepts has an empirical origin. In our case, this empirical origin is the problem solving activity and the solution; and the analytic rules are the strategies, constructions, heuristics which were once successful in solving a problem and transmitted to solve other problems as well. Pap also claims in a similar vein that experience based events can function an analytic role after they are encountered many times and this situation plays a crucial role in the knowledge production process of the sciences. In a nutshell, synthetic and contingent findings of problem solving sessions can play an analytic and consistent role for the subjects to support their new beliefs and justify them.

Coherentist approach for explaining the justificatory role of problem solving activity is a little bit more complicated than the foundationalist one. As mentioned, in coherentist theories, the subject has a consistent set of beliefs in which each proposition is justified by the other propositions in the set. If we want to apply this approach to problem solving activity, we need to make several assumptions: the foundationalist approach only provides us the justificatory role of the solution, there is a set of operations (propositions or beliefs) applied by the subjects in the solution and this set of operations is consistent at least to a certain degree since it enables the subject to solve the problem. An example for a set of consistent beliefs in problem solving domain might be some configurations of some tangram pieces. For instance, a large triangle can be formed by using two small triangles and the square and this little set of beliefs might act as a subsolution in some puzzles. This idea can also be generalized to problem solving routines and approaches of different subjects. Since each subject may have different set of beliefs, the way they reach to solutions might differ and sometimes their set of beliefs determines or forces the way they tend to solve the problem as will be explained in the next paragraph. In a nutshell, as you may have already anticipated, the set of operations will be our subject matter for the coherentist explanation of problem solving activity.

It can be assumed that the subject is aware of this set of operations and its conducive role in the solution. However, the subject does not know which operations are crucial and which are arbitrary for the solution. To understand status of each operation, the subject needs to solve different problems and observe the relations among these operations. This brings us to the concepts of functional fixedness and Einstellung effect.

Duncker defines functional fixedness as a mental block (Duncker, 1945). Now, it will be demonstrated that this mental block is originated from the shortage of knowledge about the use of the related objects. In the candle problem (Duncker, 1945), subjects had difficulties in finding a possible use of the given objects: a candle, a box of tacks, and matches. In the correct solution of the problem, the subjects need to use the box as a ground for holding the candle. However, the previous interactions of the subjects with the box primed them to use the box as a container not as a bed or ground for the candle. This can be seen as the subjects' inability to question their coherent set of beliefs which were useful for them before the experiment. It can be assumed that they all used the box for stocking or carrying objects in their prior encounterings. When they were presented with a problem which requires using the box as a ground, they had difficulties.

Einstellung effect (the details of this effect can be found in the previous section) is also a very similar phenomenon and can be regarded in the same way as functional fixedness (Luchins, 1942). In both phenomena, subjects lack some kind of knowledge either regarding the solution of the problem or the problem itself. However, in functional fixedness the knowledge that the subjects lack is mostly about different uses of certain objects where the lack of knowledge is due to the subjects' unawareness of their set of beliefs in Einstellung effect.

In the following chapter, an annotation framework for problem solving environments which uses the findings of this section will be presented to the reader.

CHAPTER 3

COLLABORATIVE PROBLEM SOLVING ENVIRONMENTS AND AN ANNOTATION FRAMEWORK

In this chapter individual and collaborative problem solving environments will be briefly explained along with the summary of available technologies in these environments. Afterwards, the difficulties and some problems regarding these environments will be mentioned. For a solution to those problems an annotation framework which is both consistent with the findings of the previous chapter and compatible with the technologies in use will be presented. At the end of the chapter, the outputs of a collaborative problem solving experiment will be annotated with the framework as a case study.

3.1. Individual and Collaborative Problem Solving Environments

The goal of this section is to give the reader a basic understanding of problem solving environments. At the beginning of the section the reasons why problem solving is studied in an experimental manner will be explained briefly. Then, the idea behind studying collaborative problem solving will be explicated. Also, the contemporary technologies in problem solving environments will be given along with their correspondences in the history of problem solving research. Finally, the difficulties that the researchers encounter while studying these environments will be mentioned.

The very first experimental approaches to problem solving were held by German psychologists in the early 20th century as mentioned in the previous chapter. Their primary focus was understanding the underlying cognitive (or behavioral at the very beginning) phenomena in problem solving activity of the human subjects. They mostly used what is now called creative or insightful problems for testing the abilities of the subjects and observing the processes that lead the subjects to succeed or fail in the solution to the given problem. The candle problem (Duncker, 1936), the water jar problem (Luchin, 1942) and the radiation problem (Duncker, 1936) can be regarded as the classical problems studied in that times. Later, on the problem solving research there have been different approaches mostly regarding how expertise is gained and what are the differences between expert and novice subjects, but the underlying idea for studying problem solving in an experimental manner remained the same. The researchers wanted to know what leads a human subject to the solution of a problem whether it was a creative one or a routine one.

Although the purposes of the researchers remained more or less the same, crucial changes took place in problem solving environments as the time passed and the technology evolved. At the very beginning, the researchers could only watch the sessions without recording the sessions with video recorders or similar devices. They were using introspections or verbalizations in order to understand the intentions of the subjects (Schultz & Schultz, 2007; Gilhooly, Fioratou & Henretty, 2010). Also, the problem solving environments were mostly comprised of real objects (like candles, tacks and matches in the candle problem or a piece of paper and a pen). With the introduction of personal computers into our lives, the problem solving environments changed drastically. The paper and pen were replaced with computers in most of the studies. Moreover, the data that the researchers could gather got rich with the new technologies like eye-trackers (Poole & Ball, 2006), voice recording devices and problem solving softwares. Nevertheless, some problems are still present for the researchers. One of these problems is the researcher's lack of ability to correctly identify the intentions of the subjects. The eye-tracker technology can overcome this problem to a certain degree in the context of specific experimental scenarios, however; it is still a problem to determine the exact intention of the subject in an individual problem solving in terms of her eye movement patterns in general.

Research on collaborative problem solving is another related area which focuses on how a group of individuals coordinate their activities to solve problems together. The studies shed light on previous aspects of cognitive science including shared learning, shared working, group cognition etc. There is one more purpose of these researches that interests us here which is the ability of collaborative problem solving environments to reveal the intentions of the subjects more than any tool available to the researchers. In collaborative problem solving settings, the subjects need to explain their purposes to their partners in a naturalistic way as part of coordinating their joint activity, and the researchers can identify these intentions by observing the communication which can be either written (chat sessions) or vocal among the subjects.

However, collaborative problem solving researchers need to overcome practical challenges involved with the difficulty to handle the data coming from multiple users in multiple modalities like eye-tracker data, chat session records, problem solving screen recordings. The synchronization and the organization of the data are harder than ever and some other means are needed other than data collecting technologies as eye-trackers and digital recording systems. Some frameworks (Jonassen, 1999; Avouris, Margaritis, Komis, 2004) and approaches (Avouris, Dimitracopoulou, Komis, 2003) which provide great statistical power to the researchers were offered to address these problems. However, none of them took into account the underlying epistemic phenomena and the states that the subjects need to pass in order to solve the problem.

In the following section, a framework that builds on the abovementioned frameworks and approaches will be presented. The novelty of this framework is its competence for considering both the epistemic phenomena and the procedural structure of problem solving activity.

3.2. FODOR: Framework for Data Organizing

FODOR, abbreviation for Framework of Data Organizing, is an annotation framework designed and developed for the purpose of organizing, arranging, sequencing and ordering of the data coming from eye trackers (single or multiple) and different multimodality tools like chat sessions, voice recordings, drawing tables in individual or collaborative problem solving environments. FODOR is an important tool because it gives the researcher the chance to organize the data in a way that the essence of the data is preserved while interpreting the data becomes much easier. Also, at the end, there is more statistical information available to the researcher with the use of FODOR like how many states the subject passed through or how many objects were created by which subject etc.

Interpreting multimodal data is very hard; since the data is very complex and finding a reference point to arrange the data gets more and more complicated with every new instance. FODOR has some assumptions and make some abstractions to ease the process of annotating the data. The first assumption is that the main action is held in the problem solving screen in which the subjects move the pieces or draw shapes to solve the problem and the data coming from that screen is primary, meaning that the voice recordings, chat recordings, and eye-tracker data have a supportive role. The second assumption is about the awareness of the subjects. In FODOR, it is assumed that a subject can be attributed to know some event (hints by the researcher, messages in chat session, objects in problem solving screen etc.) only if she directly looks at the area where the event is or she talks about that event. The third assumption is that the subjects are able to store the knowledge throughout the whole session or whole experiment once they get it. This is important because, we can arrange the knowledge bases of subjects only if we assume that they are able to memorize the data. Moreover, FODOR is a general framework and abstracts the reasons why the experiment is being done. Either it is a constructivist learning task (Jonassen, Rohrer-Murphy, 1999) or a shared knowledge construction research (Roschelle, Teasley, 1995), the researchers can use FODOR with the right arrangements appropriate for their research. Also, the configuration of the environment can be abstracted in the framework. In some environments two or more subjects monitor the same screen while each subject has access to different screens and manipulation tools in other environments. All these scenarios can be covered with this framework by adjusting the parameters appropriately for each subject.

In the previous chapter some of the underlying epistemic phenomena of problem solving were described along with the procedural understanding of problem solving originated from the works of Simon and Reitman. The difficulties the researchers come across while interpreting multimodal data were also stated in the beginning of this section. In the rest of this section, FODOR's ability to both consider the findings of the previous chapter and help researchers to deal with the problems involved with analyzing multimodal data will be presented starting with the key characteristics of the framework. Note that the main purpose of this framework is simplifying the researchers work while saving the essence of the problem solving activity and providing statistical data about the activity.

The framework FODOR has four key characteristics which enable the researchers to handle the data more efficiently and save the gist of the problem solving sessions:

1. FODOR is an event-based annotation tool which means that instead of using time stamps, FODOR uses events for creating a sequence. The idea is based on the work of Avouris, Dimitracopoulou and Komis (2003). In FODOR, there are three types of events: objects, communication events and conceptual events. Objects include problem related elements such as tangram pieces or geometric shapes. These events can be created during the problem solving activity in the problems like geometric construction tasks and ER drawing tasks etc. Also, these events can be present from the beginning of the session in the problems like tangram puzzle, the candle problem, and the towers of Hanoi etc. Communication events cover the elements that are used to communicate by the subjects like sentences, utterances, pieces of text etc. Conceptual events are a little bit harder to explain; these are the events that are not directly accessible by investigating the objects or communication events. This type of events mostly has derivative characteristics that can be derived from the objects and the communication events like the certain configuration of the objects or recommendations involving a solution to a subgoal. Algorithms, heuristics, strategies, problem solving constructions are all members of conceptual events set. Unlike the objects or communication events, the conceptual events tend to prevail through different problem solving sessions. For example, a certain algorithm for constructing an equilateral triangle might be discovered in the first session and reused in the following sessions by the same subjects. There are five different operations regarding the events in FODOR: recommendation, creation, modification, deletion, and objection. The details for each operation will be given under the notation of FODOR subsection.
2. FODOR is sensitive to relative perspectives of the subjects and the researchers. In problem solving environments, most of the time it is assumed that subjects are able to acquire the knowledge (or notice the changes) immediately when they are presented and sequencing the events with absolute time stamps is sufficient, however; it was noted (which can be seen in the case study) that subjects often do not realize the changes in the environment at that moment, whether it was due to the environment itself or the other subject(s). So, it is essential to generate proprietary event sequences for each of the subjects.
3. FODOR embraces a process-based approach in problem solving. The idea is that in each problem solving task, the subject(s) needs to achieve some certain subgoals and has to visit some states. The idea comes from the works of Simon (1967) and Reitman (1964) about the solution processes of ill-structured problems which were explained in the previous chapter. FODOR requires the researchers to discover these states, for each possible solution, and keep track of the state of the subject in problem solving progress. The details of this process will be provided in the case study.
4. FODOR is knowledge-sensitive. In FODOR, the researchers are expected to determine certain pieces of knowledge that the subject(s) needs to acquire in order to solve the problem. By doing this, the researchers would be able to track the progress of the subjects in terms of their knowledge bases. The underlying idea for this feature comes from the findings of the previous chapter where it was clearly shown that the knowledge related changes might play a crucial role in problem solving activity of a human subject. A more

elaborate explanation for this element will be given in the case study of FODOR.

With the given four characteristics, the researchers can combine different data sources easily and reconstruct the problem solving session from the perspectives of each subject. Moreover, the researchers are able to keep the track of knowledge related changes in problem solving activity along with the ability to follow the states of the subjects towards the solution of the problem. Furthermore, the event-based structure of FODOR enables the researchers to capture the gist of the sessions by abstracting the time-based relations among the events.

3.2.1. The Notation of FODOR

In this subsection, the notation for the framework will be presented. The reader should keep in mind that the symbolization used here is arbitrary, and each researcher is free to use her own way of symbolization for the concepts and operations as long as the symbolization is consistent.

In FODOR, as mentioned, there are three different types of events and any number of subjects and they are symbolized as the following:

Objects: $E_o^1, E_o^2, \ldots, E_o^n$

Communication events: $E_t^1, E_t^2, \ldots, E_t^n$

Conceptual events: $E_c^1, E_c^2, \ldots, E_c^n$

Subjects: $A_1, A_2, \ldots, A_n$ where ($n>0, n \in N$).

There are also operations which are like predicates or functions in first-order logic (Lindström, 1966), they take a subject and an event (only objects and conceptual events) and a time-stamp if the researcher wishes to. The researcher might identify these operations by looking at the communication among the subjects (chat sessions or voice recordings) or investigating the problem solving screen. Operations and their definitions are as follows:

Recommendation: It is the case that one of the subjects suggests to create an object or comes up with an idea for an algorithm, heuristic, construction or strategy. The symbolization is: $S(E_o^k, A_l, T_m)$ where E_o^k is the kth object in the session, A_l is the lth subject and T_m is time m. As one might notice, not each event has to have a recommendation due to the fact that the subjects might skip to creation without suggesting beforehand. Recommendations take place in communication among the subjects, and the researchers should investigate this communication to decide when and by whom the recommendation was made.

Creation: Creation is the realization of an event with the provided problem solving tools. The symbolization is: $C(E_o^k, A_l, T_m)$ Creation can be identified by the researchers by looking at the problem solving screen.

Modification: Modification is the case that a created object or conceptual event is changed by one of the subjects. The symbolization is: $M(E_o^k, A_l, T_m)$ Note that an event might be modified many times, and it should be stated by the researcher each time with the subject and event ID. Modification can be observed on the problem solving screen by the researchers.

Deletion: It is the case that a created object is deleted by one of the subjects. The same object can be created again in the session, however; it should be given a new event ID. The symbolization is: $D(E_o^k, A_l, T_m)$ The deletion can be found in the problem solving screen by the researchers.

Objection: When one of the subjects objects to an event or the subjects discuss some objects or conceptual events, it is called objection. The symbolization is: $O(E_o^k, A_l, T_m)$ Note that, the same event can be discussed by more than one subject, and each of them should be expressed separately. The researchers should investigate communication among the subjects to determine objections.

There are also states in FODOR. These states might be solutions for subgoals or certain milestones in a path that the problem solver should take in order to solve the problem. The states differ for each problem and each solution, so the researcher should keep in mind that states must be determined beforehand for each possible solution of the problem. The representations of the problem and the states are up to the researcher. Some examples for possible states might be the number of correctly placed pieces in tangram puzzle or the construction of certain shapes in geometric construction tasks. For example, in one possible solution of constructing an equilateral triangle by using a ruler and a compass, the subjects should create two intersecting congruent circles, and this might be a state for that problem. Another example might be the emptying of the tack box or attaching the box to the wall in candle problem (Duncker, 1936). The states are shown with S_n where $n > 0$ and $n \in N$. Possible states for a problem (candle problem) can be exemplified as follows:

Initial State $\rightarrow S_1 \rightarrow S_2 \rightarrow S_3 \rightarrow S_4$ (Solution)

Where S_1 is emptying of the tack box, S_2 is attaching the tack box to the wall with a tack, S_3 is melting the bottom of the candle with a burning match, and S_4 is placing the candle on the box. Another possible solution for the same problem might be:

Initial State $\rightarrow S_1 \rightarrow S_3 \rightarrow S_4 \rightarrow S_2$ (Solution)

Note that not every problem can be easily represented with states. In these cases, the researchers might use the number of steps correctly taken towards solution or left for reaching the solution, the number of correct pieces or constructions etc.

The last element of FODOR is knowledge representation of the subjects. This part requires the predetermination of the problem related and environment related knowledge pieces by the researcher. For the sake of simplicity, it is assumed that every piece of knowledge can be represented as propositional knowledge (Goldman, 1983). Nevertheless, the researcher should keep in mind that the subjects might have that certain piece of knowledge as tacit knowledge or knowledge by description etc. For problem related propositions P_n; for environment related propositions Q_n ($n > 0$ and $n \in N$) is used in the framework. The researcher should examine all the related propositions before the experiment began and keep note of these propositions. Environment related propositions may include the restrictions or allowances of the subjects in the problem solving environment as well as the manipulation of the environment or the use of the tools. In a similar vein, problem related propositions may include the pieces of knowledge that the subject(s) should acquire for solving the problem. Following is an example for a computer mediated tangram puzzle environment with two subjects where one of the subjects is able to see the goal shape but not able to manipulate the pieces and the other subject is able to see and manipulate the pieces but not allowed to see the goal shape:

Q_1: The subjects are not allowed to use any communicatory means other than provided ones (chat tool or microphones).

Q_2: The subjects are free to describe the goal shape to the other subjects.

Q_3: Double left clicking mirror the tangram pieces in Y coordinate.

Q_4: By clicking and holding, the pieces can be moved.

Q_5: By clicking on the edges of the pieces, the pieces can be rotated.

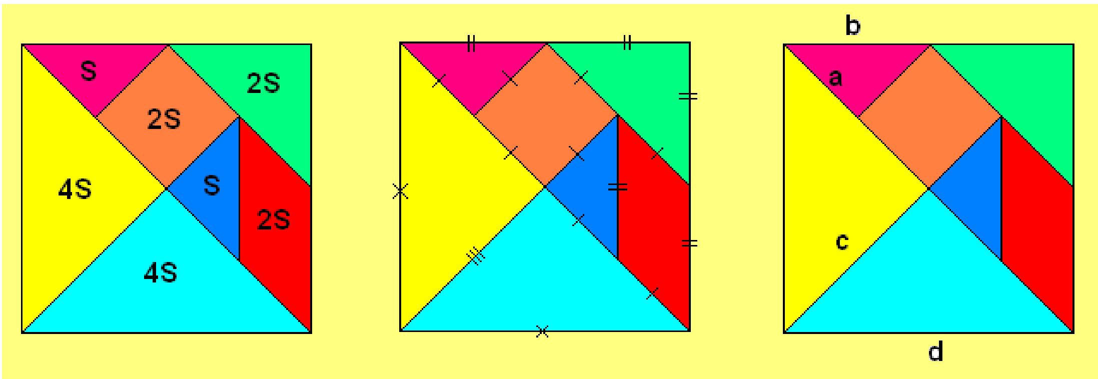

Figure 1. The relations among the tangram pieces. The leftmost shape demonstrates the areal properties of the pieces. The center shape demonstrates the equalities among the edges of the shapes. In the rightmost shape, the lengths of b,c and d are √2, 2 and 2√2 units respectively, where a is 1 unit.

P_1: There are certain proportions among the areas of the pieces. The areas of square, parallelogram, small triangle, middle-size triangle and large triangle are proportional with 2, 2, 1, 2, 4 respectively. A more detailed demonstration can be found in Figure 1.

P_2: All the triangles are right isosceles triangle.

P_3: The hypotenuses of small triangles are equal to the short edges (the ones except the hypotenuse) of middle-size triangle. The same is applicable to middle-size and large triangles as well. Also, the short edges of small triangles are equal to the edges of the square (see Figure 1).

P_4: Two small triangles can form a square, a parallelogram and another triangle with the exact sizes of the given ones (see Figure 2).

P_5: By using two small triangles and one of the square or parallelogram or middle-size triangle, you can form a large triangle (see Figure 2).

The list can go further, however; the main idea can be figured out. As you might have noticed P_3, P_4 and P_5 are combinations of a couple of propositions. The researcher might combine the propositions which she finds relevant. Nevertheless, the combination should be performed carefully because, for example, the knowledge that two small triangles can form a square does not guarantee for the subject to know the other combined propositions as well. The decision for the organization of the lists is up to the researcher's priorities. Note that not always all the related information is available to the researchers before the experiment begins. For example, the subjects might discover another way of solving a subgoal that the researcher failed to anticipate. In these cases, a new proposition can be defined for the related knowledge.

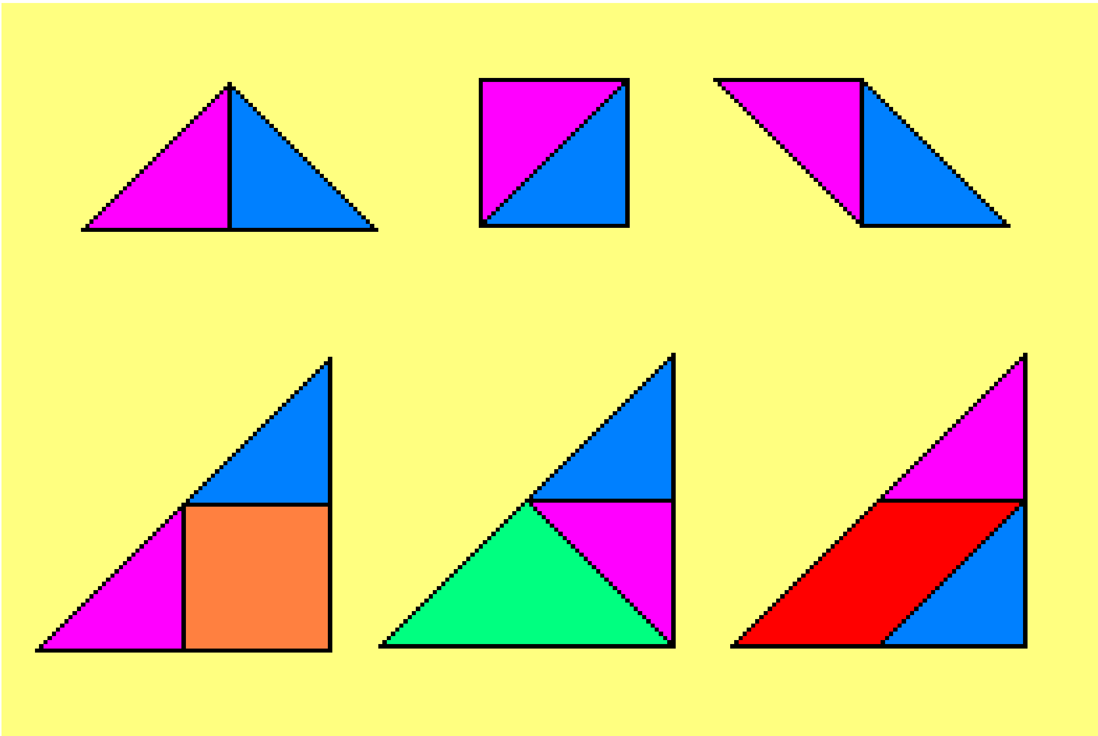

Figure 2. The combinatory properties of the tangram pieces. In the first row, different tangram pieces that can be formed by using two small triangles are shown. In the second row, different strategies for forming a large triangle are shown.

By discovering and listing the underlying propositions, the researcher is able to keep track of the knowledge changes in the knowledge bases of the subjects. It is plausible to think that even the subject fails to solve the problem, she can acquire some new knowledge about the problem solving environment and the problem itself. Moreover, the knowledge gathered in a session can be used in the following sessions. The researcher now has a sense for understanding the failures and successes of the subjects. Furthermore, the researcher might make some changes about the environment, instructions or even the problem and arrange the hints accordingly in the following experiments. Assume that the researcher finds out that environment related propositions are not well recognized by the subjects. The next time she designs an experiment, she might change the instructions in a way that the new instruction set puts away the misunderstandings.

Until now, we have seen the underlying assumptions and the way that the framework works as a concept. However, the statistical power of the framework and its performance in a practical example has not been demonstrated yet. In the following section, real outputs of a collaborative problem solving environment will be annotated with the framework; and the statistics and interpretation of the problem solving sessions will be provided to the reader.

3.2. Case Study

3.2.1 Background Information about the Study

In this section, a case study will be given to demonstrate both the use and the aforementioned qualities of the framework. The study was taken from a collaborative tangram solving experiment. In the experiment there are two subjects with different statuses. One is able to view both the solution (goal shape) and the solving process, but unable to move the pieces. The other subject is able to move the pieces, but unable to see the goal shape. In a nutshell, the subjects are presented with two different screens and the subject who is able to see the goal shape is directing the subject who is able to manipulate the pieces. In the experiment, subjects communicate through microphones connected to the computers in which the experimental software runs. There is also eye-tracker data available for both of the subjects. Each pair of subjects completes six different trials each lasting eight minutes whether the solution is found by the subjects or not. In two trials, all tangram pieces have different colors, and in the rest of the trials the pieces are all the same color. In every three minutes, a hint was provided to the subjects (see Figure 3), but only the subject who is directing the trial could see the hint. For the details about the environment and the running software, the reader can refer to (Deniz, Fal, Bozkurt & Acartürk, 2015).

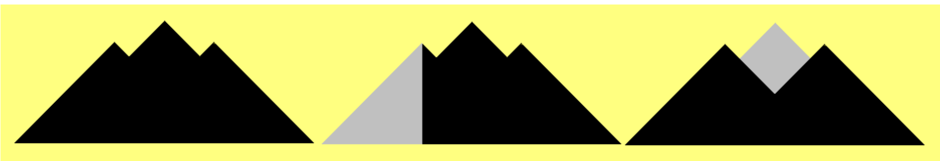

Figure 3. The providing of the hints. From left to right: the goal shape, the first hint and the second hint in the mountain puzzle trial. The light areas indicate the correct position of the related tangram piece. For example, in the second hint, the subjects are expected to place the square at the top of the "mountain".

In the case study, only the activity of one pair of subjects will be used. The subjects were presented with six tangram puzzles in the order of: vase, swan, mountain, fish, seal and chair (see Figure 4). The order of the puzzles is important because of the knowledge production process during the problem solving activity. The idea is that the subjects are able to apply successful strategies and constructions in the following trials. That is why we have to investigate the order of the puzzles and their relations first. Our aim is to find similarities between solutions of the puzzles that can help the subjects to solve the problems. Before doing that, some information about tangram puzzle and solving strategies should be provided to the reader.

3.2.2. Background Information about the Problem

When solving tangram puzzles, people use certain strategies. In this subsection, some of these strategies will be introduced. There are not any academic sources regarding tangram puzzle strategies known to the author, so the following is the outcome of the author's own tangram puzzle experiences.

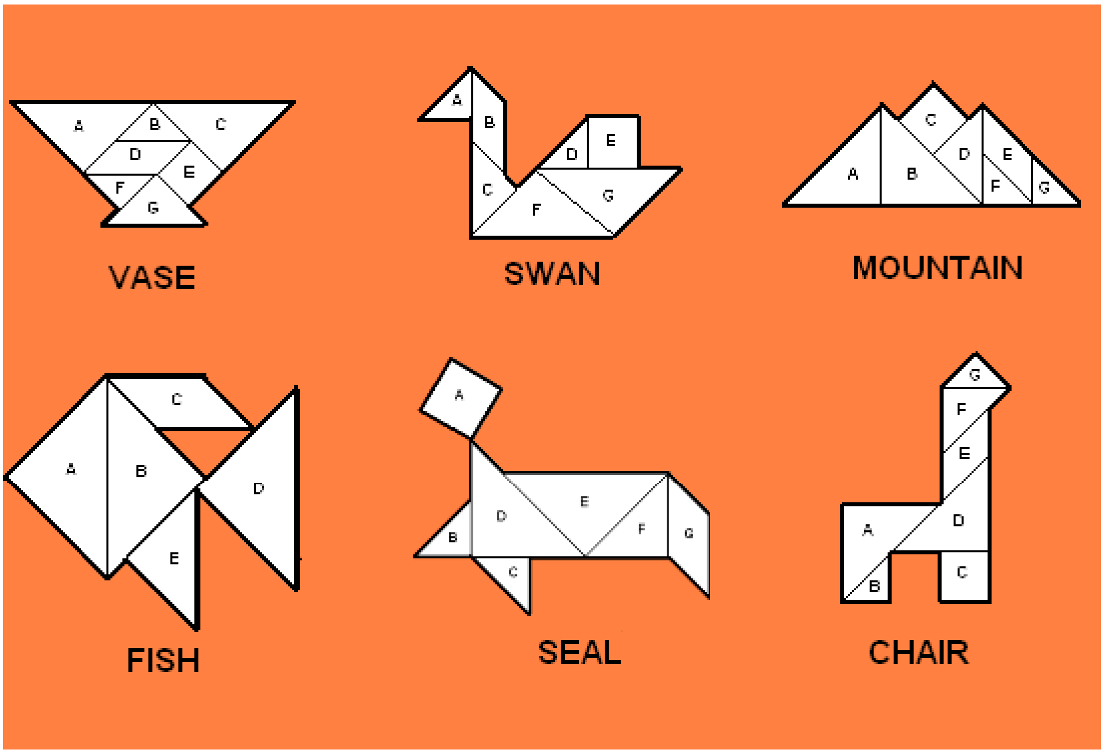

Figure 4. The tangram puzzles used in the experiment along with their solutions. From left to right, up to down: the vase, the swan, the mountain, the fish, the seal and the chair. The names are arbitrary. The letters in the puzzles show the correct places for the pieces. Note that the letters are for positions, not for pieces. For example, the square is in "A" position in the seal puzzle, but in "C" position in the chair puzzle. The solution of the fish puzzle was not completed on purpose, the idea is to show that "A", "B" and "D" positions are interchangeable and the subject needs to put two large triangles in two of them. The last position should be formed with the rest of the pieces: two small triangles and the square.

There are two basic strategies: completing isolated parts first and creating chunks. In tangram puzzles, not always all pieces are formed in a monolithic manner. In Figure 4, the fish and the seal puzzles have one isolated part each. The idea in this strategy is to find fully and partially isolated pieces and completing these pieces in the very beginning. For example, if the subject is presented with the seal puzzle, the first thing she should do is to place the square piece in "A" position. The same strategy can also be used for partially isolated pieces like "C" position in the chair puzzle and "C" and "E" positions in the fish puzzle.

The second strategy, creating chunks, is a little bit more complicated. The idea is to find familiar structures in the goal shape. In Figure 5, it can be observed that all cat puzzles share similar structures. The head is combination of two small triangles and the square in each puzzle. The subject should be aware of this kind of structures. However, the structures are not always obvious like they are in the cat examples. For example, in Figure 4, “A” and “B” positions in the fish puzzle essentially form a big square. Also, “F” and “G” positions in the swan puzzle form a big parallelogram. Actually, this parallelogram was identified by one of the subjects as we will see in the annotation part. This chunking strategy can be further enhanced by the use of the characteristics of the pieces which are demonstrated in Figure 2.

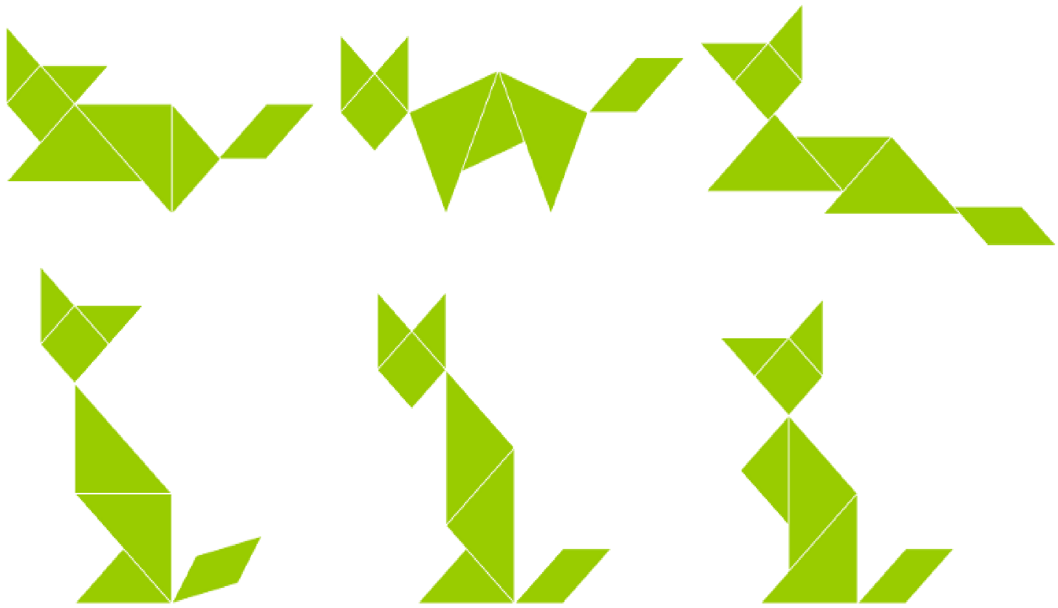

Figure 5. Retrieved from http://xiongyihui.github.io/ Representing some cat designs created by using tangram pieces.

Since the basic strategies were presented, the relations between tangram puzzles can be explored. There are several relations; however, only some of them will be provided here. The first relation is about the mountain puzzle and the following fish puzzle. In the solution of the mountain puzzle, the subjects need to form a large triangle (at the same size with the given large triangles, see Figure 2) by using smaller pieces. (“E”, “F”, “G” positions in the mountain puzzle can be investigated.) Another relation can be found between the fish puzzle and the seal puzzle. In each of these puzzles there are several fully and partially isolated pieces. Once the subjects get this idea (produce the related knowledge), they can apply this idea to other puzzles as well.

After the revealing of the relations between the puzzles; problem and problem solving environment related propositions can be listed as mentioned in the previous section. The problem related propositions are mostly about the geometric relationships among the tangram pieces (Wang & Hsuing 1942; Oflazer, 1993; Scott, 2006; Fox-Epstein & Uehara, 2014). For the sake of simplicity, these propositions are presented in Appendix A, and problem solving environment related propositions are presented in Appendix B.

As mentioned, there are objects in the framework and the objects in the tangram puzzle are the tangram pieces. For the rest of this section (and in the Appendices), the following list will be used for the objects:

E_O^1: Small Triangle 1

E_O^2: Small Triangle 2

E_O^3: Middle-size Triangle

E_O^4: Large Triangle 1

E_O^5: Large Triangle 2

E_O^6: Square

E_O^7: Parallelogram

The conceptual events in the case study were designed to be the relations between the pieces and the positions (shown in Figure 1). For example, "E_O^2 to E position" means that the second small triangle was intended to be placed in "E" position in the related puzzle. The underlying idea is that the subjects are able to recognize the structural properties of the goal shapes and they move the pieces in order to match them with the goal shape. Suppose that the subjects are trying to form the head ("A" position) in the seal puzzle. They have to put the square in "A" position in the right rotation to achieve that subgoal. Note that the rotations of the pieces are abstracted in the annotations. There are two reasons for this situation: it was assumed that the subjects are able to identify the correct rotation in most of the cases and if they cannot identify the correct rotation then the piece would cover some other regions than it is supposed to cover. This situation is indicated with "/" symbol. For instance, in the vase puzzle, if the subject tries to put one of the large triangles in "A" position with a wrong rotation (let's say hypotenuse is facing down-left), then the large triangle would occupy "D" and "F" positions as well. This situation is symbolized as "A/D/F" which means that the placed piece occupies all of these areas, and the placement is wrong.

Different formalization methods can also be used by the researchers, like the grid representation of the puzzle (Oflazer, 1993) or the exact coordinates of the pieces etc. However, this study does not aim to investigate all of the underlying concepts regarding a tangram puzzle solution session; it only tries to demonstrate the use of the framework.

There are also problem solving states in the framework as mentioned in the previous section. In tangram puzzle, the solution states can be regarded as the number of correctly placed pieces and this approach will be used for the annotation of the case trials. For example, if the subjects are in "State 4" it means that they have four pieces at their correct positions. The researchers are free to create their own state representations according to their research purposes.

3.2.3. The Annotations and Discussions

In the previous subsections, the experimental environment was introduced and problem related information was provided to the reader. In this subsection, only the basics of the annotation will be exemplified for the sake of simplicity. The basics include use of the eye-tracker data and speech recordings in the annotation process, some operations on events, and formalization phase of the events. Some screenshots will also be shared with the reader. After that, significant parts taken from the annotation will be shared with the reader. The parts are chosen in order to show the aforementioned qualities of the framework. The rest of the annotated data can be found in Appendix C. Note that voice recordings were not transcripted and eye-tracker data is not provided to the reader. That is consistent with the abstractions of the framework which were presented in the previous section. The voice recordings and eye-tracker data were only used to capture the gist of the sessions and included to the annotated data where necessary; and only the essence of these sessions will be presented here. However, the researchers can also include voice recordings, chat sessions, eye-tracker data, and any other information they have access to, if they think it is relevant to their studies.

In the annotation tables (see Appendix C) there are six columns: events, notes, time, knowledge base changes, state and storyline. Event column includes conceptual events and communication events. The objects are already created since it is a tangram puzzle solving. In the notes column, there are explanations about the events where necessary. Time column indicates the time stamps of the events and taken from the video time. Knowledge base changes are about the propositional knowledge elements that the subjects acquire or use. In the state column, the current state of the problem is given. The states are determined by the number of correctly placed pieces as mentioned earlier in the section. The last column is the storylines where the essences of the problem solving sessions are given to the reader. Since this is a case study, the notes and storylines are given a little bit more explanatory than they normally supposed to be.

In the following paragraphs, some examples taken from the annotations will be presented to the reader to demonstrate the aforementioned characteristics and the abilities of the framework. These characteristics and abilities include: first, the event-based structure which prevents the researchers from mistakes related absolute times and enables them to study the gist of the sessions. The second is the relative understanding that enables the researchers to follow the perspectives of each user. In the case study, since every subject is in a different observatory condition, the perspectives of the subjects are presented in the same table. However, in different types of problems like constructive geometry or ER (entity-relationship) diagram generating tasks, different tables for each subject can be generated. The third is the state-by-state or process-based understanding of problem solving activities. This function enables the researchers to follow the main flow of the problem solving activity by observing the states of the subjects. In the case study, there are only seven states in a linear manner, however; in different types of problems there can be more complicated state spaces. The last characteristic is the knowledge-sensitive approach of the framework. As explained in the related sections, problem can be regarded as the shortage or deficit of certain pieces of knowledge whereas the solution to the problem can be seen as the acquirement or production of these pieces of knowledge. In the framework, this understanding is used to demonstrate the knowledge bases of the subjects. The subjects can acquire knowledge and use the knowledge that was acquired before. These are all specified both in the conceptual understanding of the framework and the tables in the annotations (knowledge base changes columns in the tables). Furthermore, some statistically valuable data will be provided to the reader as the outputs of the annotation period.

The first example which emphasizes the quality of the framework is the tidiness of the outputs. As mentioned, there are multiple data sources in this environment, but the framework was able to handle these multiple sources in a neat way that the researchers can understand the gist of the sessions with a quick glimpse. This tidiness is a result of the event-based structure of the framework. The researchers can generate even better forms for visualizing the output of the framework. The details of how each line in an annotation table is generated can be found in the very beginning of Appendix C.

The explanatory power of the events and their quality might be another concern about the framework. To investigate the reliability of the events, another test was conducted. In the test, the events determined by the framework were compared with the gaze overlaps of the subjects through the trials. Although, the events were not determined directly based on the eye-tracker data, there were meaningful correspondences between the events and the gaze overlaps of the subjects. The details of the test and the related figures can be found in Appendix D; however, it would be helpful both for us and the reader to give the basics of the tests and the underlying ideas.

In the tests, a software which divides the problem solving screen into 17 areas of interest (AOI) and collects gaze information for each subject in each AOI was used. The output of the software is three rows each indicating time-based relations of the gazes. The first row shows the regions where the presenter looks at while the second row gives the same information for the operator. Also, the last row compares these two rows and presents the overlaps between them (see Figure 6).

In the test study, the outputs of the software were taken for each trial between the subjects, and they were compared with the annotated output of the framework. The comparison was based on the events identified (detected) by the framework. The assumption was that if the events were determined arbitrarily, then there would be no (or little) consistency between the gaze overlaps of the subjects and the events. However, if the events were somehow sensitive to the essence of the problem solving sessions, then there would be some consistencies between the gaze overlaps and the events. The exact nature of this relation is not known yet; however, it can be assumed that such relation exists. One sample output can be found in Figure 7 and the rest of the results of the test study can be found in Appendix D.

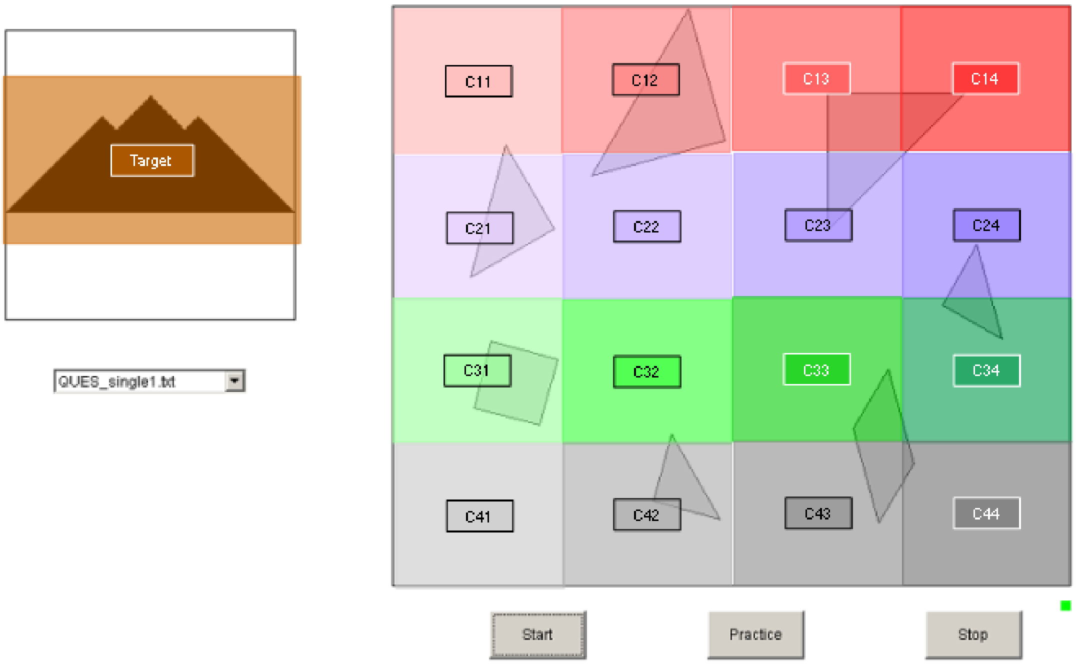

Figure 6. The areas of interest (AOIs) in the problem screen. The "target" region is only available to the presenter. The color codes are for visualization purposes which can be investigated in Figure 7.

Figure 7. The first two rows illustrate the gaze information of the subjects where the last row is their overlapping regions. All rows are time-based which means that the plots can be regarded as the changes where the subjects look at over time. The red rectangles in the last row demonstrate the events identified by the annotation framework. The name for each event is given under the corresponding rectangle. Note that some events were grouped for the sake of simplicity and better look. The idea is that the better the gaze overlaps and the event times match the more crucial information the framework can provide. As noted above, this assumption is not well-established but it is reasonable to believe in so.

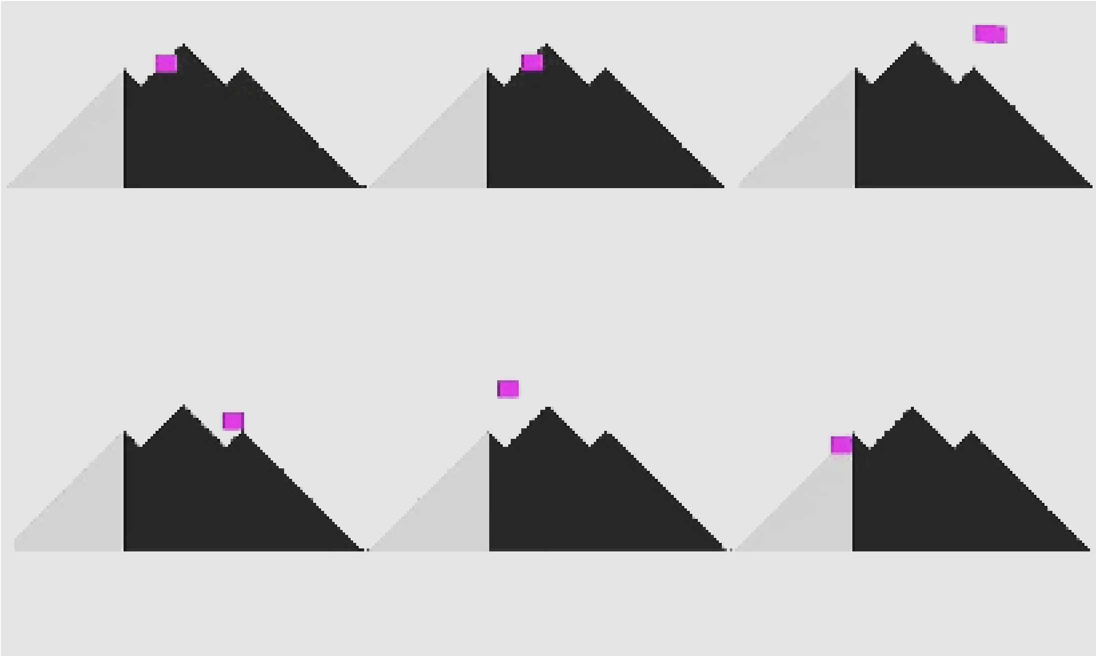

Figure 8. Subject 1 (A_1) checked the hint screen at least 5 times before she realized the hint at times: 03:18, 03:27, 04:19, 04:31, 04:38 (from left to right, top to bottom); and she finally realized the hint at 04:52.The pink square indicates the points where the subject 1 is looking at. In order to understand the nature of the hints, the reader can refer to Figure 3.

Another example can be given in order to illustrate the importance of the relative understanding. In the third trial, the first hint was provided at 3:00, however; it was not until 4:52 that the subject realized the hint and used for the solution (see Figure 8). When the eye-tracker data is analyzed it can be seen that the subject (see the pink squares in Figure 8) had checked the screen where the hints were provided at least five times before she realized the hint. If the researcher only trusted in the fact that the hint was provided to the subject or even subject checked the screen, then the researcher would be misled. The fact that she realized the hint after one minute and fifty two seconds comes from the communication events in which she declares that she noticed the hint (see Table 4).

The state-by-state representation of the problem which was introduced in 2.1.3.1 is self evident. In the annotations, the fifth column indicates the state which the subjects are in. Table 1 shows the relation between the states and the problem solving activity in terms of time. The values are given in seconds. Figure 9 demonstrates the problem solving states of the subjects for the fish puzzle. In tangram puzzle, the state spaces are not complicated as it can be in different problems like the towers of Hanoi, the candle problem etc. However, the ability to follow the states still gives valuable statistics to the researchers as can be seen in Table 1.

Table 1. The amount of time that the subjects reached to different states in each trial.

	1st Trial	2nd Trial	3rd Trial	4th Trial	5th Trial	6th Trial
1st State	86	250	318	19	17	51
2nd State	180	386	334	57	51	199
3rd State	264	458	365	164	83	238
4th State	305	-	470	223	212	246
5th State	332	-	-	260	235	286
6th State	-	-	-	265	298	407
7th State	-	-	-	269	336	441

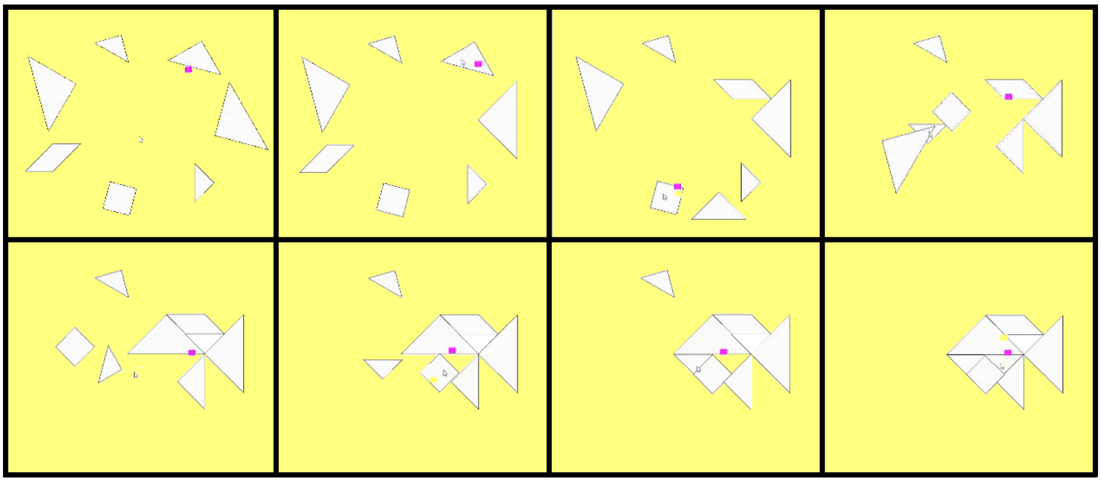

Figure 9. States for the fish puzzle. From left to right, top to bottom: Initial state to goal state. Note that the colors were manipulated for a better visual understanding.

The data provided in Table 1 not only illustrates the time durations that the subjects reach to certain states, it can also be used to examine the correlation between different states and the subjects' success or failure. For example, it is clearly seen in the table that subjects had more potential for reaching the goal if they were able to arrive at the first state earlier. Similar notions can also be speculated by observing the table. Also, problem solving characteristics and achievements of different pair of subjects can be compared by using this type of tables and data.

The last characteristic of the framework is its sensitivity about epistemic concepts like knowledge production, knowledge transmission, knowledge acquisition etc. As explained in the second chapter, our epistemological emphasis is mostly on two topics: justificatory role of a solution and the relation among the propositions (pieces of knowledge). A good example for the justificatory role of a solution might be found in the first and second trials. In the first trial, the subjects accidentally discovered the fact that they were able to form a parallelogram by using two big triangles together (P_{13}). Then by using this knowledge, they also realized that same the same configuration can be formed with the small triangles (P_8). Just by their chance, in the following trial (the second trial) the swan puzzle required the forming of a parallelogram by using two big triangles. They completed the parallelogram, but it was too late for them to solve the problem since there were twenty seconds left (see Figure 10; see Figure 12, Figure 13 and Table 4 in Appendix C). The way that the knowledge was acquired and the way it enabled the subjects to almost come up with a solution can be examined in the annotations of the first and second trials. The epistemological formulation (as provided in 2.3.5.2) of this knowledge production activity can be given as follows:

A_1 knows P_{13} if, and only if,

1. B is a solution for Q, and
2. A_1 knows B, and
3. A_1 is justified in believing that P_{13} on the basis of B, and
4. P_{13} is true.

Here, B and Q can be regarded as “F and G positions can be filled by using two big triangles” and “How F and G positions can be filled?” respectively. The situation can be summarized as: A_1 has a problem of filling the F and G positions (see Figure 4) in the swan puzzle, she uses two big triangles to fill the related regions and she realizes that the solution is correct. So, she is justified in believing that P_{13} which is the proposition that says that two big triangles can form a parallelogram and this justification is based on the correct solution. These epistemological grounds can also be given for the other propositions that were acquired in the problem solving activity.

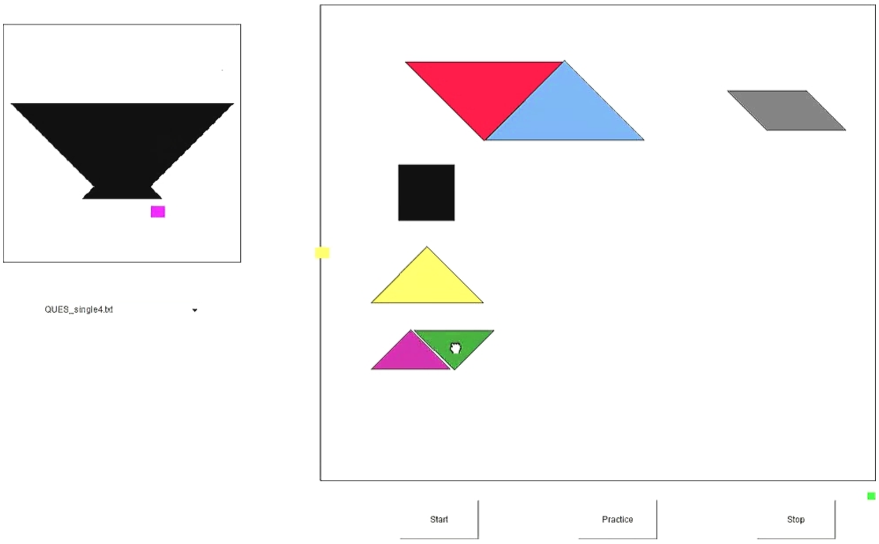

Figure 10. The realization of the propositions P_{13} and P_8. As can be seen in the screenshot, the subjects formed two more parallelograms by using the strategies provided in Figure 4. For a detailed explanation the reader may refer to Table 2.

The subject's exploration of the relations among the propositions and its relation with the knowledge production can be best observed in the cases that the operator (the subject who can manipulate the pieces without seeing the goal shape) comes up with a solution strategy which is correct. In the fish puzzle, A_1 who was the operator in that trial placed the last three pieces on her own after she randomly (random but with a purpose) moved the pieces and realized that the remaining pieces (two small triangles and the square) can form a large triangle (see Figure 2 and Figure 4) and that was exactly what they needed. The details of this process can be examined in Figure 9 and Table 5. So, it was mentioned in the previous chapter that the exploration is necessary in some cases where there are so many propositions and the relations among these propositions are not immediately available to the subject. This is also consistent with the Kantian way of thinking about *synthetic a priori* knowledge (see 2.3.4). The subjects had access to all information they needed; however, an exploration phase was necessary for them to generate new knowledge.

The last thing that the researchers are able to do by using FODOR is to collect data about the differences between subjects both in the same team and in different teams. For example, by using a quick search command (like search for "S(A_2" gives the number of events suggested by the second subject) the number of suggestions that each subject made can be gathered. In the case study, the first subject suggested 32 events and created 32 events. The other subject suggested 28 events and created 35 events. Since the trials required one of them to be active all the time, the numbers are very close to each other. In different tasks, the activity of the subjects can be roughly estimated by checking the numbers of their suggestions or creations etc. and these numbers can give valuable information about the collaborative nature of the environments.

The previous examples were just toy examples to illustrate the way FODOR works and how can it contribute to the studies of the researchers. By using aforementioned conceptual backbone (either with the same notation or with a different one), the researchers can come up with better uses of this framework suitable for their own research purposes. In the following chapter some more discussions and some possible enhancements about the framework will be presented.

CHAPTER 4

CONCLUSION

In this thesis, a brief history of problem solving research in Cognitive Science and Psychology was provided. Moreover, the basic definitions and discussions in epistemology literature were given in order to provide a ground for the epistemological explanations regarding human problem solving activities. It was demonstrated that the problem solving activities of human subjects can be investigated via knowledge related concepts in the human subjects. These concepts include knowledge representation, knowledge production, knowledge transmission and knowledge usage processes. By analyzing these processes, it was demonstrated that some specific scientific phenomena regarding problem solving such as functional fixedness, the Einstellung effect, or ill-structured problems, can be further investigated. In a nutshell, the purpose of the related chapter was to give the reader the understanding that the findings -about problem solving activity- of the sciences (Psychology, Cognitive Science, AI etc.) can also be explained from an epistemological perspective.

In the third chapter, an annotation framework was introduced. The main goal of this framework was to embrace the findings of the previous chapters and help the researchers to investigate the outputs of complex problem solving environments. The findings included the importance of the organization among the propositions, the justificatory role of problem solving activity for a subject, the steps that lead the subjects to solve the ill-structured problems and knowledge-based understanding of problem solving activity. The framework was also an event-based one which used the sequence of the events instead of absolute time stamps. Moreover, the relative perspectives of the subjects were also conserved in the framework in order to give a better gist of the problem solving activity from the eyes of each subject. After all, we can say that an annotation framework which is sensitive to knowledge-based understanding of problem solving was provided to the reader along with some different facilitator aspects in data annotation.

After the framework was introduced, a case study was also provided in order to demonstrate both the usage of the framework and to assess its qualityas an analytical tool. The case was taken from a collaborative problem solving experiment where two subjects try to solve a tangram puzzle which is essentially an ill-structured problem (see 2.1.3) together in six different trials. The annotations of these trials showed that with the use of the presented framework, the researchers are able to quantify some of the important knowledge-based changes that occur during problem solving activity, and easily track the states of the subjects in the related problem. The researchers are also able to follow the recurrent pieces of knowledge that lead the subjects to solve different problems. These pieces of knowledge can be regarded as strategies, constructions, heuristics or algorithms. The framework gives the researcher the ability to follow these notions in problem solving activities to a certain degree. For example, in the case study, it was shown that subjects discovered how they can build a parallelogram by using two right isosceles triangles; and used this finding in the following trial to solve the problem. It was also realized that the subjects explained the goal shape to each other in every trial after they learned that they were able to do so. Actually, in some cases the description of the goal shape was so successful that the operator subject who could not see the goal shape contributed to the solution by suggesting and creating some conceptual events on her own (see Table 5 and Table 6 in Appendix C). So, it can be safely argued that an important portion of problem solving activity can be followed and comprehended through knowledge related changes of the subjects and the presented framework enables the researchers to more easily track these changes.

The epistemological findings of the thesis can be generalized to other problem solving related phenomena in the future studies. For example, the collaborative aspects of problem solving activities can be linked to epistemic changes in the subjects or the performances of expert and novice subjects can be explained through different knowledge types like *knowledge that*, *knowledge of* etc. It can further be argued that formation (creation) of expertise can be regarded as the transformation of *knowledge that* into *knowledge of* and this transformation can be observed in problem solving environments.

The case study also demonstrated potential of the framework for guiding further statistical analysis of problem solving processes. With the use of the framework, the researchers can follow the states of each problem for each group of subjects (or individual subject) and compare different groups in terms of their success rates of reaching different states. The researchers are also able to evaluate the collaborative nature of the problem solving sessions to a degree by comparing the actions of each subject in the session. This can be used with subjects' gaze overlaps to investigate the role of each subject in different sessions. For example, in a constructive geometry task, one subject might be always in the role of a leader who suggests and creates most of the objects and concepts and the other subject might be behaving in a more passive way by following the leader. Another statistical use of the framework might be the evaluation of the propositional sets of different subjects or different group of subjects. This information can be used to compare different groups in terms of their knowledge production abilities and problem solving performance.

One limitation of the study is that all the annotations were made by the author and there is no way to determine that the framework can eliminate the subjective differences or not. So, the annotation framework should be subjected to an inter-rater reliability test in the future to realize its potential as an analytical tool. The current thesis has mainly focused on establishing the internal validity of the framework through philosophical analysis and empirical case studies.

Another limitation of the study is that only one type of problem is annotated for the case study. However, the problem (tangram puzzle) was chosen due to its inclusive and compulsive nature. In tangram puzzle, it is harder to determine the conceptual events than constructional geometry or ER diagram generation tasks. It is also harder to come up with an appropriate state space for the problem. Another reason why tangram puzzle solving was chosen as the case study is that the framework was designed according to constructional geometry tasks and the event-based structure was borrowed from a study that was designed for ER diagram generation task (Avouris et al., 2003). So, using a different and difficult type of problem would be a good practice both for us and the reader. Some different problems might be used for annotation in the following studies.

There can also be some engineering solutions which would ease the annotation efforts of the researchers. One of these solutions might be an automated system that would match the problem solving screen and eye-tracker data in order to give the information about the subjects' awareness about certain events. Another solution might be a computer program which would parse the output of the problem solving experiments and provide statistical data to the researchers. Speech to text softwares can also be connected to the framework both to ease the redaction process and colligate the different sources of data into a more easily comprehensible one.

In conclusion, this thesis aimed to give the researchers an annotation framework for collaborative and individual problem solving environments. The framework was meant to be designed in a way that it would be compatible with the epistemic aspects of problem solving. So far, the aim was achieved. However, there is still room for enhancements both in conceptual sense and practical sense.

REFERENCES

Alexanderson, G. (2006). About the cover: Euler and Königsberg's Bridges: A historical view. *Bulletin of the american mathematical society*, *43*(4), 567-573.

Anderson, J. R. (1993). Problem solving and learning. *American Psychologist*, *48*(1), 35.

Armstrong, D. M. (1963). Is introspective knowledge incorrigible?. *The PhilosophicalReview*, *72*, 417-432.

Armstrong, D. M. (1973). *Belief, truth and knowledge*. CUP Archive.

Audi, R. (2003). *Epistemology: A contemporary introduction to the theory of knowledge* (Vol. 2). Psychology Press.

Ayer, A. J., & Marić, S. (1956). *The problem of knowledge* (Vol. 6). London: Macmillan.

Avouris, N., Dimitracopoulou, A., & Komis, V. (2003). On analysis of collaborative problem solving: An object-oriented approach. *Computers in Human Behavior*, *19*(2), 147-167.

Avouris, N., Margaritis, M., & Komis, V. (2004, August). Modelling interaction during small-group synchronous problem-solving activities: The Synergo approach. In *Proceedings of ITS 2004 workshop on designing computational models of collaborative learning interaction* (pp. 13-18).

Biggs, N., Lloyd, E. K., & Wilson, R. J. (1976). *Graph Theory, 1736-1936*. Oxford University Press.

Boles, S. (1990). A model of routine and creative problem solving. *The Journal of Creative Behavior*, *24*(3), 171-189.

BonJour, L. (1998). *In defense of pure reason: A rationalist account of a priori justification*. Cambridge University Press.

Bowden, E. M., & Beeman, M. J. (1998). Getting the right idea: Semantic activation in the right hemisphere may help solve insight problems. *Psychological Science*, *9*(6), 435-440.

Boyer, M. W. (1958). The Kantian Element in Lewis' Theory of Knowledge. *Philosophy and Phenomenological Research*, *19*(1), 95-103.

Catrambone, R., & Holyoak, K. J. (1989). Overcoming contextual limitations on problem-solving transfer. *Journal of Experimental Psychology: Learning, Memory, and Cognition*, *15*(6), 1147.

Chi, M. T., Feltovich, P. J., & Glaser, R. (1981). Categorization and representation of physics problems by experts and novices. *Cognitive science*, *5*(2), 121-152.

Chin Jr, G., Leung, L. R., Schuchardt, K., & Gracio, D. (2002). New paradigms in problem solving environments for scientific computing. In *Proceedings of the 7th international conference on Intelligent user interfaces* (pp. 39-46). ACM.

Chisholm, R. M. (1977). *Theory of knowledge* (2nd ed.). Englewood Cliffs, NJ: Prentice-Hall.

Christensen, D., & Kornblith, H. (1997). Testimony, Memory and the Limits of the A Priori. *Philosophical Studies*, *86*(1), 1-20.

Cling, A. D. (2008). The epistemic regress problem. *Philosophical Studies*, *140*(3), 401-421.

Cox, R., & Brna, P. (1995). Supporting the use of external representations in problem solving: The need for flexible learning environments. *Journal of Artificial Intelligence in Education*, *6*, 239-302.

Crosby, D. A. (1988). *The specter of the absurd: Sources and criticisms of modern nihilism*. SUNY Press.

Davidson, D. (1984). *Inquiries into Truth and Interpretation*. Boston: Clarendon Press

Davidson, J. E. (2003). Insights about insightful problem solving. In J. E. Davidson & R. J. Sternberg (Eds.), *The psychology of problem solving*, 149-175. New York: Cambridge University Press.

Days of our lives[Motion Picture]. (2011). Universal Music.

Deniz, O., Fal, M., Bozkurt, U., & Acartürk, C. (2015). GET-Social: Group Eye Tracking environment for social gaze analysis [abstract of poster presentation]. In Ansorge, U., Ditye, T., Florack, A., & Leder, H. (Eds.), Abstracts of the 18th European Conference on Eye Movements, 2015, Vienna. *Journal of Eye Movement Research, 8*(4):1, 252.

Descartes, R., & Weissman, D. (1996). *Discourse on the Method: And, Meditations on First Philosophy*. Yale University Press.

Dunbar, K. (1998). Problem solving. In W. Bechtel & G. Graham (Eds.), *A companion to cognitive science* (pp. 289–298). London: Blackwell.

Duncker, K., & Lees, L. S. (1945). On problem-solving. *Psychological monographs, 58*(5), i.

Faulkner, P. (2000). The Social Character of Testimonial Knowledge. *The Journal of Philosophy*, 97(11), 581-601.

Frensch, P. A. (1995). *Definitions, traditions, and a general framework for understanding complex problem solving*. Universitätsbibliothek der Universität Heidelberg.

Fox-Epstein, E., & Uehara, R. (2014). The Convex Configurations of" Sei Shonagon Chie no Ita" and Other Dissection Puzzles. *arXiv preprint arXiv: 1407.1923*.

Gentner, D. (1983). Structure-mapping: A theoretical framework for analogy. *Cognitive science*, *7*(2), 155-170.

Gentner, D., Holyoak, K. J., & Kokinov, B. N. (2001). *The analogical mind: Perspectives from cognitive science*. MIT press.

Gettier, E. L. (1963). Is justified true belief knowledge? A*nalysis*, 23, 121-123.

Gick, M. L., & Holyoak, K. J. (1980). Analogical problem solving. *Cognitive psychology*, *12*(3), 306-355.

Gilhooly, K. J., Fioratou, E., & Henretty, N. (2010). Verbalization and problem solving: Insight and spatial factors. *British Journal of Psychology*, *101*(1), 81-93.

Goldman, A. I. (1975). Innate knowledge. In S. P. Stich (Ed.), *Innate ideas* (pp. 111–120). Berkeley, CA: University of California Press.

Goldman, A. I. (1983). Epistemology and the theory of problem solving. *Synthese*, *55*(1), 21-48.

Goldman, A. I. (1994). Naturalistic epistemology and reliabilism. *Midwest studies in philosophy*, *19*(1), 301-320.

Goldman, A., & Olsson, E. J. (2009). Reliabilism and the value of knowledge. In A. Haddock, A. Millar, & D. H. Pritchard (Eds.), *Epistemic value* (pp. 19–41). Oxford: Oxford University Press.

Graham, P. J. (2006). Testimonial justification: inferential or non-inferential?.*The Philosophical Quarterly*, *56*(222), 84-95.

Gribkovskaia, I., Halskau, Ø., & Laporte, G. (2007). The bridges of Königsberg—a historical perspective. *Networks*, *49*(3), 199-203.

Hazlett, A. (2010). The myth of factive verbs. *Philosophy and Phenomenological Research*, *80*(3), 497-522.

Ichikawa, J. J., & Steup, M. (2012). The analysis of knowledge. *The Stanford Encyclopedia of Philosophy (Winter 2012 Edition).* Retrieved from http://plato.stanford.edu/archives/win2012/entries/knowledge-analysis

Isaksen, S. G., & Treffinger, D. J. (1985). *Creative problem solving. The Basic Course.* New York: Bearly Limited.

Jonassen, D. H., & Rohrer-Murphy, L. (1999). Activity theory as a framework for designing constructivist learning environments. *Educational Technology Research and Development*, *47*(1), 61-79.

Kant, I. (1781). *Critique of Pure Reason. Translated by Norman Kemp Smith*. London Macmillan, 1934.

Larkin, J., McDermott, J., Simon, D. P., & Simon, H. A. (1980). Expert and novice performance in solving physics problems. *Science*, *208*(4450), 1335-1342.

Laudan, L. (1977). *Progress and its problems*. University of California Press, Berkeley.

Lehrer, K. (1990). *Theory of knowledge*. London: Routledge.

Lehrer, K., & Paxson, T. (1969). Knowledge: Undefeated justified true belief. *The Journal of Philosophy*, 66, 225-237.

Luchins, A. S. (1942). Mechanization in problem solving: the effect of Einstellung. *Psychological monographs*, *54*(6), i.

Lindström, P. (1966). First order predicate logic with generalized quantifiers. *Theoria*, *32*(3), 186-195.

Lockhart, R. S., Lamon, M., & Gick, M. L. (1988). Conceptual transfer in simple insight problems. *Memory & Cognition*, *16*(1), 36-44.

Lumsdaine, E. & Lumsdaine, M. (1995). *Creative problem solving – thinking skills for a changing world*. New York: McGraw-Hill.

Mayer, R. E. (1995). The search for insight: Grappling with Gestalt psychology's unanswered questions. In R. J. Sternberg & J. E. Davidson (Eds.), *The nature of insight* (pp. 3–32). Cambridge: MIT Press.

Mayer, R. E. (1999). Problem solving. *Encyclopedia of creativity*, *2*, 437-447. Elsevier.

Newell, A., Shaw, J. C., & Simon, H. A. (1958). Elements of a theory of human problem solving. *Psychological review*, *65*(3), 151.

Newell, A., Shaw, J. C., & Simon, H. A. (1959, January). Report on a general problem-solving program. In *IFIP Congress* (pp. 256-264).

Newell, A., & Simon, H. A. (1972). *Human problem solving.* Englewood Cliffs, NJ: Prentice-Hall.

Newell, A. (1993). Heuristic programming: ill-structured problems. In *The Soar papers (vol. 1)*, Paul S. Rosenbloom, John E. Laird, and Allen Newell (Eds.). Mit Press Series In Research In Integrated Intelligence, Vol. 1. MIT Press, Cambridge, MA, USA 3-54.

Oflazer, K. (1993). Solving tangram puzzles: A connectionist approach. *International journal of intelligent systems*, *8*, 603-616.

Palmer, D. W. (1996). Towers of Hanoi. *Journal of circuits systems and computers, 12*(2), 59-66.

Pap, A. (1946). *The a priori in physical theory.* New York: King's Cross Press.

Plato, T. (1935). *Theaetetus*. trans. FM Cornford. *The Collected Dialogues of Plato, ed. Hamilton and Cairns, 149d*, *28*, 24-25.

Poole, A., & Ball, L. J. (2006). Eye tracking in HCI and usability research. *Encyclopedia of human computer interaction*, *1*, 211-219.

Pratt, Alan. (2005). Nihilism. *The Internet Encyclopedia of Philosophy*. Retrieved from http:/ www.iep.utm.edu/n/nihilism.htm

Reichenbach, H. (1936). Logistic empiricism in Germany and the present state of its problems. *The Journal of Philosophy*, 33(6), 141-160.

Reichenbach, H. (1952). Are phenomenal reports absolutely certain?. *The Philosophical Review*, *61*(2), 147-159.

Reitman, W. R. (1964). *Heuristic Decision Procedures, Open Contraints [sic], and the Structure of Ill-defined Problems*. Graduate School of Industrial Administration, Carnegie Institute of Technology.

Rey, G. (2008). The analytic/synthetic distinction. *Stanford Encyclopedia of Philosophy*. Retrieved from http://plato.stanford.edu/entries/analytic-synthetic/

Roschelle, J., & Teasley, S. D. (1995, January). The construction of shared knowledge in collaborative problem solving. In *Computer supported collaborative learning* (pp. 69-97). Springer Berlin Heidelberg.

Runco, M. A., & Pritzker, S. R. (Eds.). (1999). *Encyclopedia of creativity* (Vol. 2). Elsevier.

Russell, B. (1910, January). Knowledge by acquaintance and knowledge by description. In *Proceedings of the Aristotelian Society* (pp. 108-128). Williams and Norgate.

Schoenfeld, A. H., & Herrmann, D. J. (1982). Problem perception and knowledge structure in expert and novice mathematical problem solvers. *Journal of Experimental Psychology: Learning, Memory, and Cognition*, *8*(5), 484.

Schultz, D., & Schultz, S. (2007). *A history of modern psychology*. Cengage Learning.

Scott, P. (2006). Convex Tangrams. *Australian Mathematics Teacher*, *62*(2), 2-5.

Senor, T. D. (2009). Epistemological problems of memory. *Stanford Encyclopedia of Philosophy*. Retrieved from http://plato.stanford.edu/entries/memory-episprob/

Simon, H. A. (1977). The structure of ill-structured problems. *Models of discovery* (pp. 304-325). Springer Netherlands.

Simon, H. A. (1996). *The sciences of the artificial*. MIT press.

Sternberg, R. J. (1998). *Thinking and problem solving* (Vol. 2). Elsevier.

Sternberg, R. J. (1999). *Handbook of creativity*. Cambridge University Press.

Steup, M. (2009). The analysis of knowledge. *Stanford encyclopedia of philosophy*. Retrieved from http://plato.stanford.edu/entries/knowledge-analysis/

Stump, D. 2011. Arthur Pap's functional theory of the a priori. *HOPOS* 1: 273–289.

ter Hark, M. (2010). The psychology of thinking before the cognitive revolution: Otto Selz on problems, schemas, and creativity. *History of psychology*, *13*(1), 2.

Truncellito, D. A. (2007). Epistemology. *Internet Encyclopedia of Philosophy*. Retrieved from http://www.iep.utm.edu/epistemo/

Turri, J. (2010). On the relationship between propositional and doxastic justification. *Philosophy and Phenomenological Research*, *80*(2), 312-326.

Wang, F. T., & Hsiung, C. C. (1942). A Theorem on the Tangram. *American Mathematical Monthly*, *49*(9), 596-599.

Wang, Y., & Chiew, V. (2010). On the cognitive process of human problem solving. *Cognitive Systems Research*, *11*(1), 81-92.

Weinstein, M. (2003). Logic in contexts. In J. A. Blair, H. V. Hansen, & C. W. Tindale (Eds.), *Informal logic @ 25, proceedings of the 2003 OSSA conference*.

APPENDICES

APPENDIX A

Considering the geometric relations among the pieces of the tangram puzzle, the following propositions can be deduced. Note that each researcher can find a different set of propositions for the purpose of her studies. The following list is generated for demonstrative purposes.

P_1: There are seven pieces: two small triangles, one middle-size triangle, two large-triangles, one square and one parallelogram. All the triangles are right triangles.

P_2: The relation among the areas of small triangles, middle-size triangle, large triangles, square and parallelogram are proportional to 1, 2, 4, 2 and 2 respectively.

P_3: The hypotenuse of the small triangles is equal in length to the short edges of the middle-size triangle.

P_4: The hypotenuse of the middle-size triangle is equal in length to the short edges of the large triangles.

P_5: The short edges of the small triangles are equal in length to the edges of the square.

P_6: The short edges of the parallelogram are equal in length to the edges of the square.

P_7: By using two small triangles, the exact same square with the given one can be formed.

P_8: By using two small triangles, the exact same parallelogram with the given one can be formed.

P_9: By using two small triangles, the exact same middle-size square with the given one can be formed.

P_{10}: By using two small triangles and the square, the exact same large triangle with the given one can be formed.

P_{11}: By using two small triangles and the parallelogram, the exact same large triangle with the given one can be formed.

P_{12}: By using two small triangles and the middle-size triangle, the exact same large triangle with the given one can be formed.

P_{13}: By using two large triangles a parallelogram can be formed.

P_{14}: All the triangles are right isoceles triangles.

P_{15}: By using two large triangles a larger right isosceles triangle can be formed.

P_{16}: Long edge of the parallelogram is equal to short edges of middle-size triangle.

APPENDIX B

Q_1: The subjects are not allowed to use any communicatory means other than provided ones (chat tool or microphones).

Q_2: The subjects are free to describe the goal shape to the other subjects.

Q_3: Double left clicking mirror the tangram pieces in Y coordinate.

Q_4: By clicking and holding, the pieces can be moved.

Q_5: By clicking on the edges of the pieces, the pieces can be rotated.

APPENDIX C

The way that the speech recordings, problem solving screen activities and eye-tracker data are used in the annotation

In the following few examples, some parts from the speech between the subjects will be presented to the reader along with corresponding screenshots in order to show the way that the conceptual events are created and the annotation tables are formed. All of the examples are taken from the first trial, and this part can be regarded as a guideline to the use of the framework. Note that only the related parts of the screen will be shared for the sake of simplicity and easier understanding. Subject 1 (A_1) is leading the session and subject 2 (A_2) is the operator. For the ease of comprehension, the excerpts will be indicated with numbers which is an arbitrary procedur.

1) A_1 at 00:38 - 00:45: Şimdi yukarda o kırmızı üçgenle birleştirmeye devam edebilirsin, yukarıda düz bir çizgi olacak. (Now you can continue to merge with that red triangle on the top, cause there will be a straight line on the top.)

A_2 at 00:46 - 00:47: Neyle birleştireceğim? (Merge with what?)

A_1 at 00:47 - 00:52: Mesela diğer mavi üçgeni de ona parallel şekilde düzelt, uzun kenarları yukarıda olsun. (For example, rotate the other blue triangle in a way that those two will be parallel to each other, the long edges will be facing up.)

This is a small excerpt from the session. A_1 suggested an event and A_2 created that. This can be regarded as a conceptual event in the framework, because the objects were already provided to the subjects and what they intended to do was to create a certain construction. The formulization of this event and the manipulations are as following: (The first two rows of Table 2 were generated according to this investigation.)

E_C^1: Put E_O^4 to A/D/F region. (Placing the red triangle.)

E_C^2: Put E_O^5 to B/D/C region. (Placing the blue triangle facing up.)

$S(A_1,E_C^1)$ & $C(A_2, E_C^1)$: A_1 suggested the conceptual event E_C^1 and A_2 created that event.

$S(A_1,E_C^2)$ & $C(A_2, E_C^2)$: A_1 suggested the conceptual event E_C^2 and A_2 created that event.

The corresponding screenshot can be observed in Figure 11.

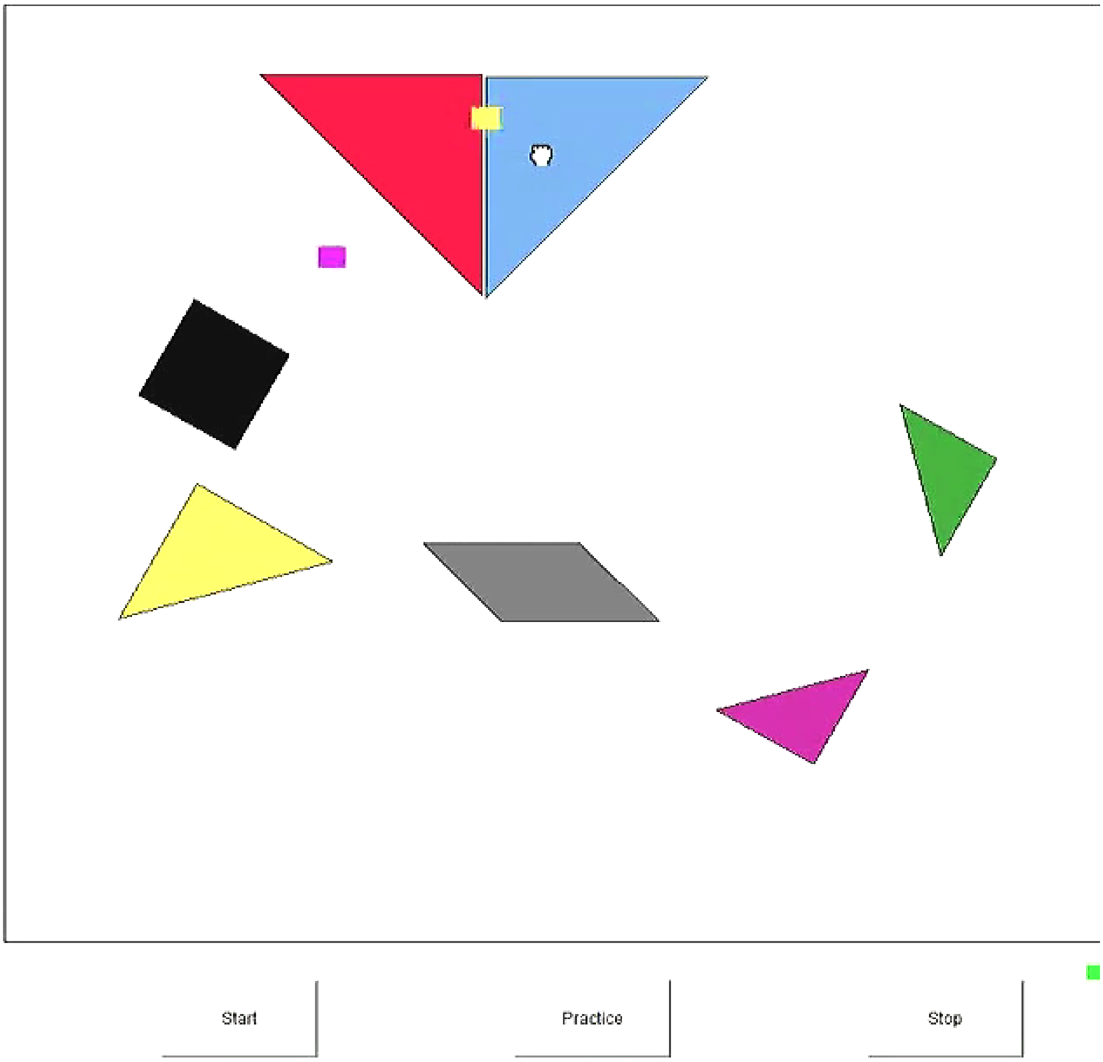

Figure 11. Creation of the first two conceptual events. The little pink and yellow squares indicate the eye-tracker data for A_1 *and* A_2 *respectively. Note that only the problem solving screen is shared here, there is also a hint screen available to the directing subject.*

2) A_1 at 01:13 - 00:52: Ya da şöyle yapalım... O üçgenlerin uzun kenarlarını yukarı getir. Hipotenüslerini üste getir ve birleştir. Evet. (Or maybe we should… Bring the long edges of these triangles to up. The hypotenuses should be facing up and merge them. OK.)

Here, A_1 suggested another conceptual event which requires the deletion of the previous ones. Actually, there are two conceptual events together; the researchers can merge them into one conceptual event if they wish to. The symbolization of the deletion of the previous conceptual events can be shown as:

$D(A_2, E_C^1)$ & $D(A_2, E_C^2)$: A_2 deleted E_C^1 and E_C^2. They gained no knowledge from these conceptual events so, the deletion was straightforward.

A_2 at 01:22 - 01:23: Şöyle mi? (Like that?) (Here, "Şöyle" (that) refers to E_C^3)

A_1 at 01:23 - 01:25: Evet, sanki… (Yes, I guess…)

A_2 at 01:26 - 01:28: Diğerini (E_O^5) de mi aynı şekilde? (Shall I put the other one (E_O^5) as this one?)

A_1 at 01:23 - 01:25: Onu (E_O^5) da öyle değil de kenarlarını birleştir. Bir daha çevir, evet. (This one (E_O^5)… Not like that. Connect the edges. Rotate once more, yes.)

A_2 at 01:43 - 01:45: Bunun (E_O^7) gibi bir şey oldu. (It looks like this one (E_O^7).) A_2 realizes that what they created was similar to the given parallelogram.

At the last four speech excerpts; two new conceptual events which include two objects were suggested by A_1 and created by A_2. The corresponding screenshot can be seen in Figure 12 and the formulations are as follows:

$S(A_1,E_C^3)$ & $C(A_2, E_C^3)$: A_1 suggested the conceptual event E_C^3 and A_2 created that event where E_C^3 is putting E_O^4 to A position. (This is correct.)

$S(A_1,E_C^4)$ & $C(A_2, E_C^4)$: A_1 suggested the conceptual event E_C^4 and A_2 created that event where E_C^4 is putting E_O^5 next to E_O^4 to form a parallelogram. This is actually one of the predetermined pieces of knowledge (P_{13}) and from the eye-tracker data (see Figure 12) it can be observed that both subjects acquired this piece of knowledge. The changes in the knowledge bases of the subjects can be followed from the 4^{th} column in the annotation tables. This situation is also consistent with the findings of the second chapter. The subjects had a belief and after the creation of the parallelogram their belief was justified and turned into a piece of knowledge.

E_C^3 is a correct move, so the state of the subjects changed accordingly. They skipped to *State 1* (S_1) which means that they had one piece at its correct location. The state changes of the subjects can be observed in the 5^{th} column of the annotation table.

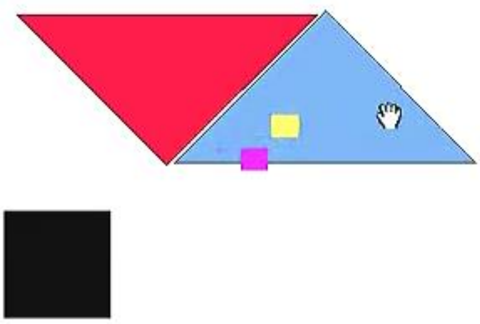

Figure 12. E_C^3 and E_C^4. Both subjects are looking at their construction, so we may safely assume that they both acquired related piece of knowledge (P_{13}).

3) A_1 at 01:54 - 01:58: O iki tane üçgeni (E_O^1 & E_O^2), yeşille pembeyi yanyana getir. (Bring these two triangles (E_O^1 & E_O^2) together, the green and the pink ones.)

A_1 suggested creating another parallelogram by using two small triangles. The formulation as following:

$S(A_1, E_C^5)$ & $C(A_2, E_C^5)$ where E_C^5 is using E_O^1 & E_O^2 to form a parallelogram.

A_2 at 02:20 - 02:21: Bu (E_C^5) aslında şununla (E_O^7) aynı zaten. (This one (E_C^5) is actually same as that one (E_O^7).)

A_2 realizes that E_C^5 is the same parallelogram with the given one (E_O^7). This is one of the predetermined pieces of knowledge (P_8, see Appendix A) and we can safely add this piece of knowledge to the subjects' knowledge bases since they both saw the similarity and talked to each other about it (see Figure 13). This can be regarded as a practical example of what was mentioned and argued in the second chapter. The subjects' acquirement of the piece of knowledge P_{13} led them to acquire another (more valuable) piece of knowledge P_8.

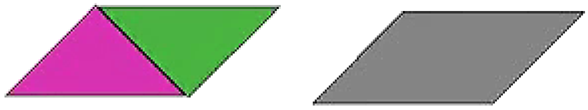

Figure 13. E_C^5. A_2 placed two shapes (E_C^5 and E_O^7) together to show their equality. This situation can be regarded as a justification process from the perspective of A_1. We can safely add P_8 to both subjects' knowledge bases. The reader can refer to Figure 2 and the related discussion for a more detailed explanation.

4) Hint 1 was given at 03:00 and A_1 realized it after a couple of seconds. Hint 1: E_O^5 to C position. After the hint was used, they had two pieces at their correct positions and they passed to S_2.

So far, how eye-tracker data can be used to determine the awareness of the subjects, how chat recordings can be used to clarify what subjects intend to do and how conceptual events are formulated and manipulated (suggestion, creation, deletion) were presented to the reader. The rest of the annotation process can be completed by applying and re-applying what were told here. The rest of the annotation results can be found in the following tables. Some important points can also be found in the third chapter.

Table 2. Annotation for the first trial. The goal shape was the vase and the subjects failed. Their knowledge bases changed as shown in the related column. They acquired the propositions P_8 and P_{13}. They had five correct pieces at most.

EVENTS	NOTES	TIME	KNOWLEDGE BASE CHANGES	STATE	STORYLINE
$S(A_1,E_C^1)$ & $C(A_2, E_C^1)$	E_C^1: Put E_O^4 to A/D/F region.	00:43	None	0	They started with a wrong strategy.
$S(A_1,E_C^2)$ & $C(A_2, E_C^2)$	E_C^2: Put E_O^5 to B/D/C region.	00:58	None	0	They continue to use the wrong strategy.
$D(A_2, E_C^1)$ & $D(A_2, E_C^2)$		01:19	None	0	The subjects realized that their construction is wrong and deleted it.
$S(A_1,E_C^3)$ & $C(A_2, E_C^3)$	E_C^3: Put E_O^4 to A position.	01:26	None	1	The subjects placed the first correct piece.
$S(A_1,E_{C4})$ & $C(A_2, E_C^4)$	E_C^4: Putting E_O^5 next to E_O^4 to form a parallelogram.	01:35	Both(P_{13})	1	Both subjects acquired the knowledge of forming a parallelogram by using two isosceles right triangles.
$S(A_1,E_C^5)$ & $C(A_2, E_C^5)$	E_C^5: Using E_O^1 & E_O^2 to form a parallelogram.. A_2 realizes that Ec5 is the same with E_O^7.	02:10	Both(P_8)	1	Subjects used the knowledge P_{13}. They also acquired the proposition P_8.
Hint 1 is given.	Hint 1: E_O^5 to C position.	03:00	None	2	The hint was used, however; the gap between

	The hint is used at 3:20.				two correctly placed pieces was not filled.
$S(A_1,E_C^6)$ & $C(A_2, E_C^6)$	E_C^6: Using E_O^1, E_O^2 & E_O^7 to form the bottom of the shape (position G).	03:44	None	2	A_1 misunderstood the bottom of the shape and they applied a wrong strategy to form the bottom which is position G in the schema.
$S(A_1, E_C^7)$ & $C(A_2, E_C^7)$	E_C^7: Integrating E_O^4 & E_O^5 with the bottom part.	04:05	None	2	The subjects decided to connect the upper half and bottom half of the shape together.
$S(A_1, E_C^8)$ & $C(A_2, E_C^8)$	E_C^8: Using E_O^6 as integrator for E_C^7. Ec6 to E position.	04:24	None	3	A_1 questions if they are ablo to finish the shape with the pieces left and she answers in negative.
Hint 2 is given.	Hint 2: E_O^7 to D position. The hint is used.	06:00	None	4	
$S(A_1, E_C^9)$ & $C(A_2, E_C^9)$	E_C^9: E_O^1 to B position.	06:32	None	5	At this moment, they have 5 pieces at their correct positions and only E_O^2 and E_O^4 left. However, they are not able to form the rest of the shape.
$D(A_2, E_C^8)$	They moved the square from its correct position.	06:37	None	4	They move the square piece from its correct position and they cannot find the correct configuration again.
$S(A_2, E_C^{10})$ & $C(A_2, E_C^{10})$	E_C^{10}: E_O^2 to E position.	06:44	None	4	
$S(A_1, E_C^{11})$ & $C(A_2, E_C^{11})$	E_C^{11}: E_O^3 to E position.	07:24	None	4	They perpetually try lefting pieces to E position until the end of the session.

Table 3. Annotation for the second trial. The goal shape was the swan and the subjects failed. Their knowledge bases changed as shown in the related column. They acquired the propositions Q_2, P_5 & P_{15}. They also used P_{13} and the use of this proposition almost enabled them to solve the problem. At the very end of the trial, A_2 understood the correct places for two more pieces, but the time ran out.

EVENTS	NOTES	TIME	KNOWLEDGE BASE CHANGES	STATE	STORYLINE
$S(A_2,E_C^1)$ & $C(A_1, E_C^1)$	E_C^1: Put E_O^3 to A/B region.	00:46	None	0	They started with a wrong strategy. Trying to form the head of the swan with a wrong triangle.
$S(A_2,E_C^2)$ & $C(A_1, E_C^2)$	E_C^2: Put E_O^4 to C/F region.	01:13	None	0	Using one of the large triangles to form the neck of the swan.
$C(A_2, E_T^1)$		01:41	Both(Q_2)	0	A_2 : Hocam mesela gördüğüm şekli söyleyebilir miyim? (Can I describe the goal shape to my partner?)
$S(A_2,E_C^3)$ & $C(A_1, E_C^3)$	E_C^3: Put E_O^6 to B position.	02:15	None	0	The subjects are trying to form the neck (B position) by using the square which is wrong.
$S(A_2,E_C^4)$ & $C(A_1, E_C^4)$	E_C^4: Putting E_O^1 next to (under) E_O^6.	02:30	Both(P_5)	0	They are still in the wrong path, however; they realized that the short edges of small triangles are compatible with the edges of the square.
$S(A_2,E_C^5)$ & $C(A_1, E_C^5)$	E_C^5: Put E_O^5 to D/F/G.	02:53	None	0	Subjects used the knowledge P_{13}. They also acquired the proposition P_8.

Hint 1 is given.	Hint 1: E_O^3 to C position. The hint was misunderstood.	03:00	None	0	The hint was misunderstood and E_O^4 was placed to C/F instead of E_O^3 to C.
$S(A_2,E_C^6)$ & $C(A_1, E_C^6)$	E_C^6: Using E_O^5 to form F.	04:10	Both(P_{15})	1	Although the construction for the head and the neck was wrong, the use of large triangle was proper. They also used two large triangles to form a larger right isosceles triangle.
$C(A_2, E_T^2)$	E_T^2: Sanırım bu yetmeyecek gibi ama. (I guess it won't be enough.)	04:13	None	1	A_2 realizes that the pieces left would not suffice for completing the shape.
$S(A_2,E_C^7)$ & $C(A_1, E_C^7)$	E_C^7: Using E_O^7 to form the tail of swan (G position).	04:56	None	1	
$S(A_1,E_C^8)$ & $C(A_1,E_C^8)$ & $D(A_1,E_C^1)$	E_C^8: Using E_O^1 instead of E_O^3. The slot was right, but the rotation was wrong.	05:25	None	1	A_1suggests to use a smaller triangle to form the head, but rejected. They rebuild E_C^1.
Hint 2 is given.	Hint 2: E_O^5 to F position. This was already achieved by the subjects.	06:00	None	1	At this moment, they have 5 pieces at their correct positions and only E_O^2 and E_O^4 left. However, they are not unable to form the rest of the shape.
$S(A_2,E_C^9)$ & $C(A_1,E_C^9)$ & $D(A_1,E_C^1)$ & $D(A_1,E_C^2)$	E_C^9: Using E_O^3 to form C position. That is actually hint 1.	06:26	None	2	They correctly replace the large triangle with middle-size triangle.
$S(A_2, E_C^{10})$ &	E_C^{10}: E_O^4 to G position.	07:38	Used(P_{13})	3	They used a strategy they learned in the

$C(A_1, E_C^{10})$					previous trial.
$S(A_2, E_C^{12})$ & $S(A_2, E_C^{13})$	E_C^{12}: E_O^1 to D position. E_C^{13}: E_O^6 to E position.	07:45	None	3	A_2 found and suggested the correct places for both E_O^1 and E_O^6, but the trial ended before A_1 was able to create.

Table 4. Annotation for the third trial. The goal shape was themountain and the subjects failed. Their knowledge bases changed as shown in the related column. They had 4 pieces at their correct positions in the end. The operator was able to put 2 pieces into their correct locations without even seeing the goal shape. So, it can be said that the use of Q_2 was very successful for the subjects.

EVENTS	NOTES	TIME	KNOWLEDGE BASE CHANGES	STATE	STORYLINE
$C(A_1, E_T^2)$	A_1 describes the shape to A_2.	00:07	Used(Q_2)	0	
$S(A_1, E_C^1)$ & $C(A_2, E_C^1)$	E_C^1: Put E_O^4 and E_O^5 to A/B and D/E/G in order to create the hills on the sides.	00:23	None	0	They started with a wrong strategy. Trying to form the hills with the large triangles both facing down.
$S(A_1, E_C^2)$ & $C(A_2, E_C^2)$	E_C^2: Put E_O^3 to B/C/D region.	01:29	None	0	The subjects are using the middle-size triangle to create the middle and higher hill.
$S(A_1, E_C^3)$ & $C(A_2, E_C^3)$	E_C^3: Put E_O^7 to B/D region.	02:25	None	0	The subjects are still following the wrong path.
Hint 1 is given.	Hint 1: E_O^4 to A region.	03:00	None	0	Hint 1 is given, but the subject did not notice the hint. They keep trying to fill the gap between the hills until the hint was noticed 1 minute 52 seconds later.
$S(A_1, E_C^4)$ & $C(A_2, E_C^4)$	E_C^4: Putting E_O^4 to A.	05:18	None	1	They were able to use the hint after two minutes it was presented.

$S(A_2,E_C^5)$ & $C(A_2, E_C^5)$	E_C^5: Put E_O^5 to E+F+G region which is a possible correct place for that piece.	05:34	None	2	Subject2, the opearator, suggests and creates ec5. The large triangles are both in possible correct locations.
Hint 2 is given.	Hint 2: E_O^6 to C position.	06:00	None	3	The hint was applied by the subjects, this time immediately.
$S(A_1,E_C^6)$ & $C(A_2, E_C^6)$	E_C^6: Using E_O^1, E_O^2, E_O^3 & E_O^7 to fill the gap between two large triangles and the square.	06:40	None	3	The subjects are trying to complete the shape by trial and error processes without minding the relations among the pieces.
$S(A_2,E_C^7)$ & $C(A_2, E_C^7)$	E_C^7: Put E_O^7 to B.	06:49	None	3	A_2, the operator, suggests and creates E_C^7. Actually, the parallelogram is in one of its correct locations but its rotation is wrong.
$S(A_1,E_C^8)$ & $C(A_2, E_C^8)$ & $D(A_2, E_C^5)$	E_C^8: Put E_O^5 to B.	07:39	None	3	The subjects moved E_O^5 to another correct position.
$S(A_2,E_C^9)$ & $C(A_2, E_C^9)$	E_C^7: Put E_O^3 to D.	07:50	None	4	A_2, the operator, suggests and creates E_C^9. It is the correct place for the middle-size triangle for this line of solution.

Table 5. Annotation for the fourth trial. The goal shape was the fish and the subjects succeeded. They acquired the proposition P_{10}.

EVENTS	NOTES	TIME	KNOWLEDGE BASE CHANGES	STATE	STORYLINE
$S(A_2,E_C^1)$ & $C(A_1, E_C^1)$	E_C^1: Put E_O^4 to D position.	00:19	None	1	They started with the true strategy for the first time. They found a correct piece for one of the isolated parts.
$S(A_2,E_C^2)$ & $C(A_1, E_C^2)$	E_C^2: Put E_O^7 to C position.	00:57	None	2	They found the correct piece for another isolated position.
$S(A_2,E_C^3)$ & $C(A_1, E_C^3)$	E_C^3: Put E_O^5 to E position.	02:04	None	2	A_2 miscalculates the area of the last isolated part and wrong piece is placed.
$C(A_1, E_T^1)$	A_2 describes the shape to A_1.	02:31	Used(Q_2)	2	
$S(A_1,E_C^4)$ & $C(A_1, E_C^4)$	E_C^4: E_O^6 to form B region.	02:44	None	3	A_1, the operator, suggests and creates E_C^4. Actually, the square is in one of its correct locations.
$D(A_1, E_C^4)$		02:48	None	2	The square was moved from its correct location.
$S(A_1,E_C^5)$ & $C(A_1, E_C^5)$	E_C^5: E_O^3 to form B region.	02:57	None	2	A_1, the operator, suggests and creates E_C^5. This is not a part of the solution since E_O^3 belongs to

					E position.
Hint 1 is given.	Hint 1: E_O^3 to E position.	03:00	None	2	A_2 understood that E_C^3 is wrong.
$S(A_2,E_C^6)$ & $C(A_1, E_C^6)$ & $D(A_1, E_C^3)$	E_C^6: E_O^3 to E position.	03:29	None	3	All three isolated positions are completed now.
$S(A_1,E_C^7)$ & $C(A_1, E_C^7)$	E_C^7: E_O^5 to form B region.	03:43	None	4	A_1, the operator, suggests and creates E_C^7. This is a part of the solution.
$C(A_2, E_T^2)$	A_2: O büyük üçgen var ya, altını da aynısı… (You should form a similar thing with that large triangle…)	04:01	Used(Q_2)	4	A_2 describes the rest of the shape to A_1. There are three pieces left (two small triangles and the square) to form a large triangle.
$S(A_1,E_C^8)$ & $C(A_1, E_C^8)$	E_C^8: E_O^6 to form A region.	04:20	None	5	A_1, the operator, suggests and creates E_C^8. This is a part of the solution.
$S(A_1,E_C^9)$ & $C(A_1, E_C^9)$	E_C^9: E_O^1 and E_O^1 to form the rest of the A region.	04:29	Both(P_{10})	7 Solved	A_1, the operator, suggests and creates E_C^9. The solution is completed.

Table 6. Annotation for the fifth trial. The goal shape was the seal and the subjects succeeded.

EVENTS	NOTES	TIME	KNOWLEDGE BASE CHANGES	STATE	STORYLINE
$S(A_1,E_C^1)$ & $C(A_2, E_C^1)$	E_C^1: Put E_O^6 to A position.	00:17	None	1	They started with a correct strategy and completed the first isolated part.
$S(A_1,E_C^2)$ & $C(A_2, E_C^2)$	E_C^2: Put E_O^3 to D position.	00:33	None	1	Wrong triangle was used.
$C(A_1, E_T^2)$	A_1: Sanki bir fok balığı da onun boynuymuş gibi. (Like it is a seal and this is the neck.)	00:35	Used(Q_2)	1	A_1 describes the shape to A_2.
$S(A_1,E_C^3)$ & $C(A_2, E_C^3)$	E_C^3: Put E_O^1 to B position.	00:51	None	2	The second isolated position was completed.
$S(A_1,E_C^4)$ & $C(A_2, E_C^4)$	E_C^4: Put E_O^2 to C position.	01:23	None	3	The last isolated position was completed.
$S(A_1,E_C^5)$ & $C(A_2, E_C^5)$	E_C^5: Filling the rest of D position with E_O^7.	01:58	None	3	
$S(A_1,E_C^6)$ & $C(A_2, E_C^6)$	E_C^6: Put E_O^4 to D position.	02:47	None	3	The piece is right, but the rotation is wrong.
Hint 1 is given.	Hint 1: E_O^4 to D position.	03:00	None	3	The hint was noticed and understood by the

					subjects.
$S(A_1,E_C^7)$ & $C(A_2, E_C^7)$ & $D(A_2, E_C^6)$	E_C^7: Hint 1.	03:32	None	4	The hint is applied. Now there are only three pieces left.
$S(A_2,E_C^8)$ & $C(A_2, E_C^8)$	E_C^8: Put E_O^5 to E position.	03:55	None	5	
$S(A_2,E_C^9)$ & $C(A_2, E_C^9)$	E_C^9: Put E_O^3 to F.	04:58	Both(P_4)	6	A_2, the operator, suggests and creates E_C^9. It is a part of the solution and one piece left.
$S(A_1,E_C^{10})$ & $C(A_2, E_C^{10})$	E_C^{10}: Put E_O^7 to G.	05:36	Both(P_{16})	7 Solved	The subjects moved E_O^5 to another correct position.

Table 7. Annotation for the sixth trial. The goal shape was the chair and the subjects succeeded. The subjects acquired P_7.

EVENTS	NOTES	TIME	KNOWLEDGE BASE CHANGES	STATE	STORYLINE
$C(A_2, E_T^1)$	A_2: Böyle zürafa gibi bir şey var. (There is something like a giraffe.)	00:04	Used(Q_2)	0	A_2 describes the shape to A_1.
$C(A_2, E_T^2)$	A_2: İki tane düz böyle fil ayağı gibi bir şey olacak. (There are two flat things like elephant legs.)	00:12	Used(Q_2)	0	A_2 describes the bottom of the shape to A_1.
$C(A_2, E_T^3)$	A_2: Sağ tarafta da uzun bir boyun ve kafası var dinozor gibi. (On the right hand side, there is a long neck ending with something like the head of a dinosaur.)	00:16	Used(Q_2)	0	A_2 describes the top of the shape to A_1.
$S(A_2,E_C^1)$ & $C(A_1, E_C^1)$	E_C^1: Put E_O^6 to C position.	00:51	None	1	The second isolated position was completed.
$S(A_1,E_C^2)$ & $C(A_1, E_C^2)$	E_C^2: Form a square by using E_O^1 and E_O^2 and put it to A/B.	01:44	Both(P_7)	1	A_2, the operator, suggests and creates E_C^2.This is not a part of the solution.
$S(A_2,E_C^3)$ & $C(A_1, E_C^3)$ & $D(A_1, E_C^3)$	E_C^3: Put E_O^4 to D/E/F position.	02:10	None	1	A_2 realizes that E_O^4 is too big to form the neck. Then removed.
Hint 1 is given.	Hint 1: E_O^4 to D position.	03:00	None	1	The hint was noticed.

$S(A_2,E_C^4)$ & $C(A_1, E_C^4)$	E_C^4: Hint 1.	03:19	None	2	The hint is applied.
$S(A_2,E_C^5)$ & $C(A_1, E_C^5)$ & $D(A_2, E_C^2)$	E_C^5: Put E_O^5 to A position.	03:58	None	3	By the use of the understanding they got from the previous piece, they identified the right place for the other large triangle.
$S(A_2,E_C^6)$ & $C(A_1, E_C^6)$	E_C^6: Put E_O^1 to B.	04:06	Used(P_7)	4	
$S(A_2,E_C^7)$ & $C(A_1, E_C^7)$	E_C^7: Put E_O^3 to E/F.	04:29	None	4	Wrong representation for the “neck”.
$S(A_2,E_C^8)$ & $C(A_1, E_C^8)$	E_C^8: Put E_O^2 to G.	04:46	None	5	Only two pieces left to the solution.
Hint 2 is given.	Hint 1: E_O^1 to B position.	06:00	None	5	The hint already has been applied by the subjects.
$S(A_2,E_C^9)$ & $C(A_1, E_C^9)$	E_C^9: Put E_O^3 to F.	06:47	None	6	A_2 realizes the formation of the middle-size triangle after struggling a while.
$S(A_1,E_C^{10})$ & $C(A_1, E_C^{10})$	E_C^{10}: Put E_O^3 to F.	07:21	None	7 Solved	A_1 realizes the certain rotation that the paralellogram can fill the gap and places it.

Appendix D

Figures (Figure 15, 16, 17, 18, 19, 20) below show the scarf plots for subject 1 (first row) and subject 2 (second row) from the tangram puzzle trials. The scarf plot shows the distribution of gaze over color-coded areas of interest on the screen in time. The third row shows the times when both participants were looking at the same section of the screen. The screen was divided into 17 areas of interest (AOI). 16 AOIs were covering the shared workspace (e.g. 11,12,…,44) as 4x4 rectangular grid (see Figure 14). The remaining AOI was the target (T), which was only available to the presenter.

The red rectangles in the third row -in the figures- indicate the correspondences between the events in the annotation results and gaze overlapping of the subjects (see Appendix C for the details of the events). As you can see, the determination of the events is consistent with the gaze occurrences of the subjects, even though the eye-tracker data was not used directly in determining the events. This can be regarded as an indicator for FODOR's ability to recognize the events in a problem solving session. A more detailed explanation about the figures are provided along with them.

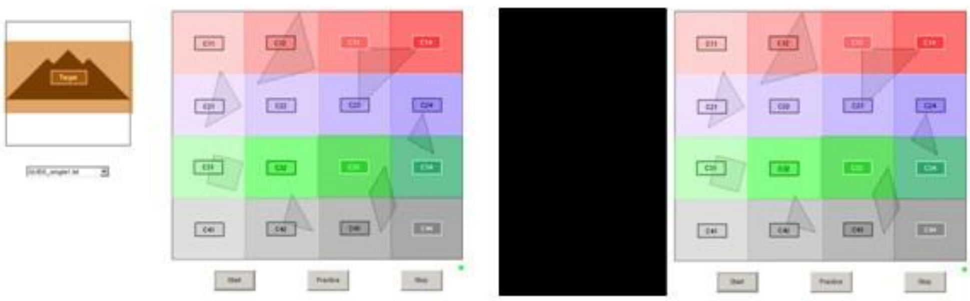

Figure 14. The structure of the AOIs.

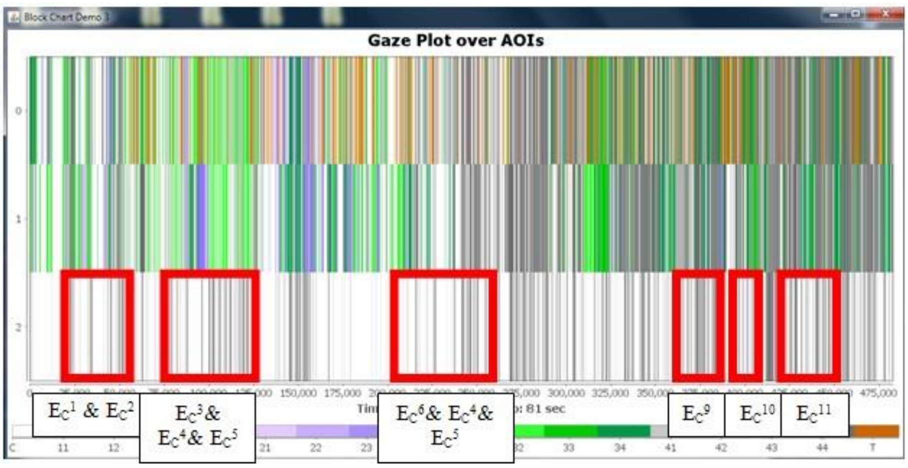

Figure 15. Gaze distributions of the subjects for the first trial. The red rectangles indicate the events in the annotation tables.

Figure 16. Gaze distributions of the subjects for the second trial. The events determined by FODOR is almost fully consistent with the gaze overlaps of the subjects.

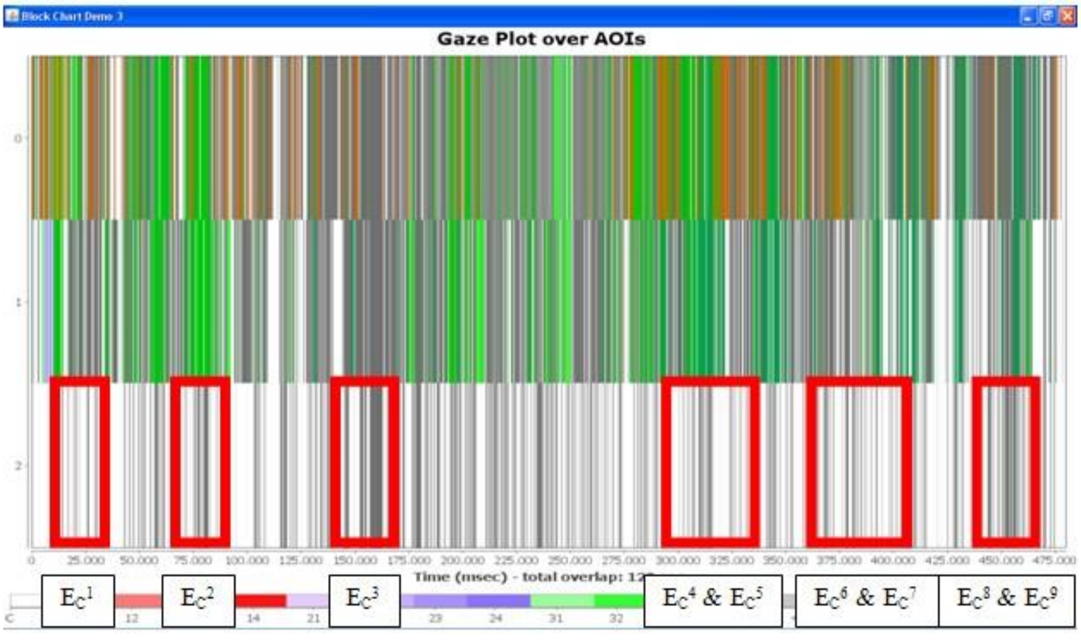

Figure 17. Gaze distributions of the subjects for the third trial. The gap in the middle of the session is due to the random moves that the subjects made which cannot be attributed as event.

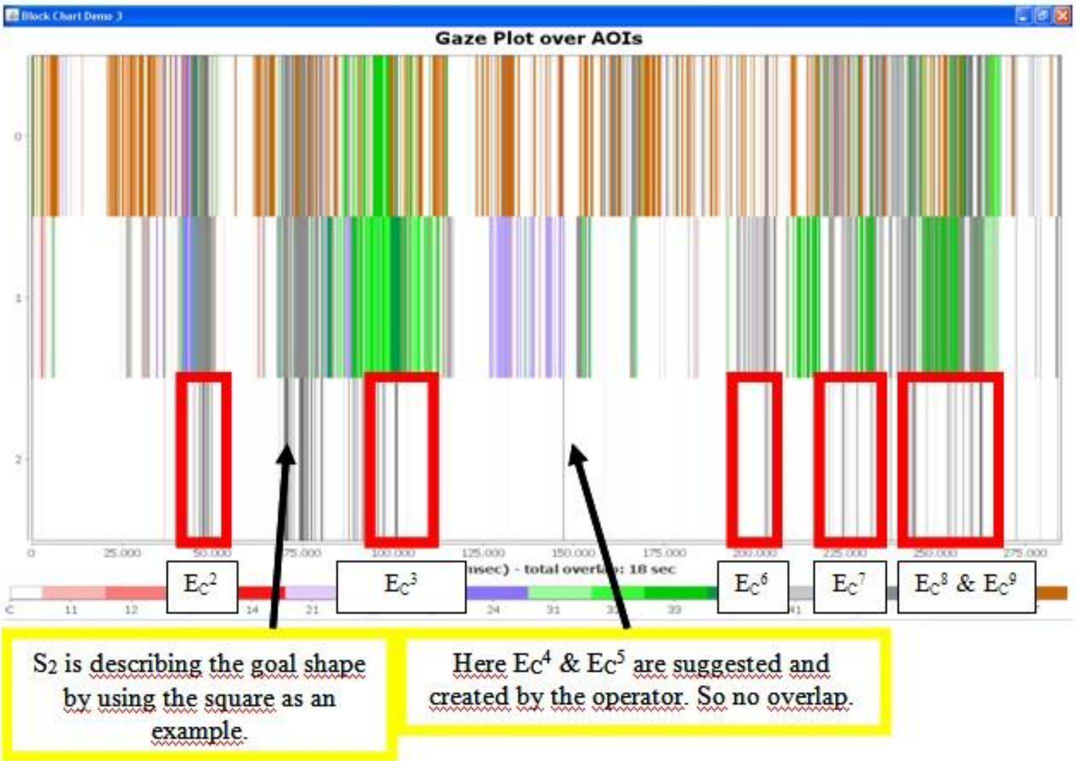

Figure 18. Gaze distributions of the subjects for the fourth trial. The gap in the middle and the absence of the events E_C^4 and E_C^5 might be explained by the fact that these events were created by the operator alone and the presenter did not take part in any of these events.

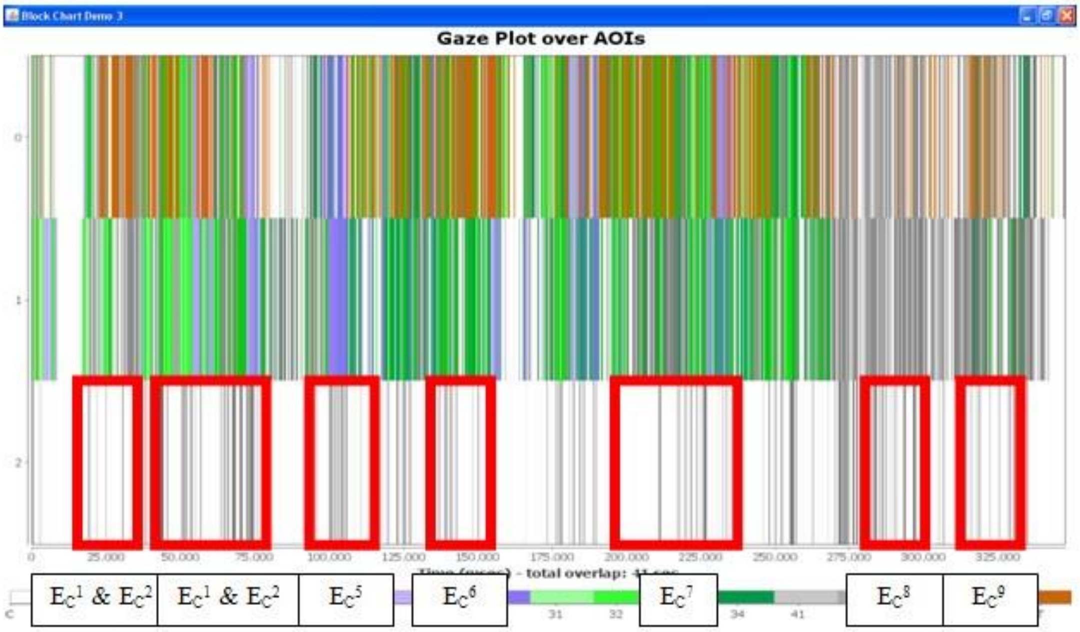

Figure 19. Gaze distributions of the subjects for the fifth trial. The events and gaze overlaps are almost identical. Keep in mind that this was one the trials that the subjects succeeded.

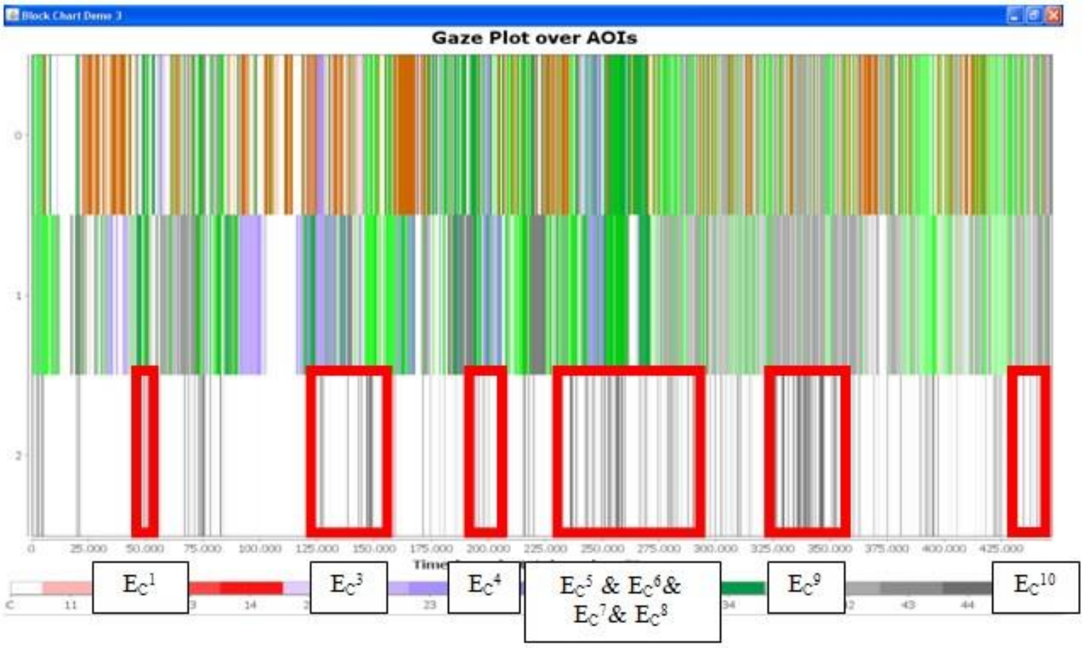

Figure 20. Gaze distributions of the subjects for the sixth trial.