

IMPROVED VIEWSHED ANALYSIS ALGORITHMS FOR AVIONICS
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ABSTRACT

IMPROVED VIEWSHED ANALYSIS ALGORITHMS FOR AVIONICS APPLICATIONS

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Viewshed analysis is a common GIS capability used in various domains with various requirements. In avionics, viewshed analysis is a part of accuracy critical applications and the real time operating systems in embedded devices use preemptive scheduling algorithms to satisfy performance requirements. Therefore, to effectively benefit from the viewshed analysis, a method should be both fast and accurate.

Although R3 algorithm is accepted as an accuracy benchmark, R2 algorithm with lower accuracy is preferred in many cases due to its better execution time performance. This thesis prioritizes accuracy and presents an alternative approach to improve execution time performance of the R3 algorithm. Considering different execution environments, improved versions of R3 are implemented for CPU and GPU. The experiment results show that CPU implementation of improved algorithms achieve 1.23x to 13.51x speedup depending on the observer altitude, range and topology of the terrain. In GPU implementation experiments up to 2.27x speedup is recorded. In addition to execution time performance improvements, the analysis results prove that proposed algorithms are capable of providing higher accuracy like R3.

Keywords: geographic information systems, avionic applications, viewshed analysis, line of sight analysis, parallel programming

ÖZ

AVİYONİK UYGULAMALAR İÇİN GELİŞTİRİLMİŞ GÖRÜNÜRLÜK ANALİZİ ALGORİTMALARI

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Görünürlük analizi, farklı gereksinimlerle farklı çalışma alanlarında kullanılan bir Coğrafi Bilgi Sistemleri yeteneğidir. Aviyonik sistemlerde, görünürlük analizi doğruluk kritik uygulamaların bir parçasıdır ve gömülü cihazlardaki gerçek zamanlı işletim sistemleri, performans gereksinimlerini sağlamak için kesintili zamanlama algoritmaları kullanmaktadır. Bu nedenle, görünürlük analizinden etkin bir şekilde yararlanmak için kullanılan yöntemler hem hızlı hem de doğru olmalıdır.

R3 algoritması analiz sonuçlarının doğruluğu konusunda bir ölçü olarak kabul edilse de daha iyi işleme zamanı performansı nedeniyle daha düşük doğruluğa sahip R2 algoritması tercih edilmektedir. Bu tez analiz sonuçlarının doğruluğunu ön planda tutarak R3 algoritmasının işleme zamanı performansını geliştirmeye yönelik alternatif bir yaklaşım sunmaktadır. Farklı çalışma ortamları gözetilerek R3 algoritmasının geliştirilmiş versiyonları merkezi işlem birimi ve grafik işlem birimi için kodlanmıştır. Merkezi işlem birimi ile yapılan deneylerde; analiz irtifasına, menziline ve topolojiye bağlı olarak standart R3 algoritmasına göre 1.23 ile 13.51 kat arasında hızlanma görülmüştür. Grafik işlem biriminde yapılan deneylerde ise tavsiye edilen algoritmalarda standart R3'e göre 2.27 kata kadar hızlanma kaydedilmiştir. İşleme zamanı performansındaki bu artışların yanı sıra, analiz sonuçları önerilen algoritmaların R3 gibi yüksek doğruluk değerleri sunabilme yeteneğini göstermiştir.

Anahtar Sözcükler: coğrafi bilgi sistemleri, aviyonik uygulamalar, görünürlük analizi, görüş hattı analizi, paralel programlama



Dedicated to the orphans of war

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LIST OF ABBREVIATIONS

GIS	Geographic Information Systems
CPU	Central Processing Unit
GPU	Graphical Processing Unit
GPGPU	General Purpose Graphical Processing Unit
RTOS	Real Time Operating System
MSA	Minimum Sector Altitude
X2	2 x 2 Granularity R3 CPU Algorithm
X4	4 x 4 Granularity R3 CPU Algorithm
X8	8 x 8 Granularity R3 CPU Algorithm
X16	16 x 16 Granularity R3 CPU Algorithm
DGR3	Dynamic Granularity R3 CPU Algorithm
DGRHYB	Dynamic Granularity R3 Hybrid Algorithm
DGRGPU	Dynamic Granularity R3 GPU Algorithm
R3GPU	R3 GPU Algorithm
R2GPU	R2 GPU Algorithm
X2GPU	2 x 2 Granularity R3 GPU Algorithm
X4GPU	4 x 4 Granularity R3 GPU Algorithm
X8GPU	8 x 8 Granularity R3 GPU Algorithm
X16GPU	16 x 16 Granularity R3 GPU Algorithm
TIN	Triangulated irregular network
DEM	Digital elevation model
Min	Minimum value
Max	Maximum value
Avg	Average value

CHAPTER 1

INTRODUCTION

Viewshed analysis indicates visible regions on a terrain to an observer located at a certain altitude and position. The analysis environment involves an elevation model for the terrain and an observer with a given range of vision. The output is generally an image, as seen in Figure 1, depicting visible and invisible areas on the terrain as a guide for decision support. The red and green pixels on the output image represent invisible and visible points, respectively.



Figure 1: Viewshed analysis output

If the observer is moving or changing the altitude, the whole analysis is revisited since the output will be different. This is because view angles to all points from previous analysis change and new elevation points will be added to range of vision while some points are disposed due to changing observer position.

Viewshed analysis is a very common practice and depending on what application domain requires, the analysis algorithm and elevation model may vary because different quality parameters are adopted in different application domains like architecture, archeology, game development, physics and avionics.

1.1. Motivation and Research Question

Viewshed analysis applications offer different results in terms of accuracy, execution time and resource allocation depending on chosen algorithm and elevation model. The purpose of application may prioritize one or more of these attributes. Since the output of viewshed analysis is a guide depicting visible regions, the accuracy is an important metric to evaluate different algorithms. For the cases observer position on terrain changing continuously, execution time is a concern. Additionally, memory efficiency and other resource requirements are effective to evaluate the practical application.

There are two fundamental viewshed analysis algorithms, R2 and R3 (Mehta, Ray, & Franklin, 1994), the researchers study to improve for raster type elevation models. These studies focus on different application methods of the algorithms like using external memory (Andrade, Magalhães, Magalhães, Franklin, & Cutler, 2011) or derivation of new approaches (Haverkort, Toma, & Wei, 2013). This is because standard R2 algorithm is a fast solution but its accuracy is not sufficient for some applications, whereas R3 is the benchmark for the accuracy (Zalik & Kaučič, 2002) and the slowest algorithm.

In avionics applications, R2 algorithm may be preferred because of execution time concern despite the low level accuracy. This is because real time operating systems with preemptive scheduling algorithms (Vestal, 2007) impose constraints for task completion time and R3 is the slowest method. The problem is that these algorithms approach the visibility problem from two opposite points of view. As an alternative Van Kreveld's algorithm (Van Kreveld, 1996) tries to offer a balanced solution between R3 and R2, however, the amount of temporary storage required by the algorithm is a significant problem and false negative visibility decision count of Van Kreveld's is worse than R2 (Yılmaz, 2017).

This thesis approaches the problem from avionics perspective and as the literature survey shows the previous works do not offer a convenient and feasible solution for avionic applications concerning both accuracy and execution time. Hence our research question is: what should be a practical and high performance solution for avionics applications? The solution should present fast execution time, high accuracy and moderate resource allocation besides using raster type digital elevation model which is the common data type used in avionics systems (Bailey, Parrish, Kramer, Harrah, & Arthur).

1.2. Objective and Contribution

There are different avionics applications utilizing viewshed analysis to ensure flight safety. These capabilities provide users with visible regions on the terrain to a threat or ally. In challenging environmental conditions, as seen in Figure 2, the accuracy of such capabilities is a very critical issue. Hence, our objective is to present an alternative algorithm that is faster than R3 and provides higher accuracy compared to R2.

In this research we propose a family of algorithms, derived from R3, named granularity algorithms. Each one of these solutions offers improvement in execution time while almost matching the exact accuracy of R3. For altitudes below MSA (minimum sector altitude) (Federal Aviation Administration, n.d.) or close to the ground, the suggested solutions reach the best execution time which means they can be used especially for low level flights. Additionally, the design approach of proposed algorithms brings a solution to scalability problem of large elevation data models. The effect of increasing resolution and extending analysis area to execution time is compensated by the granularity algorithms since they use processed elevation models. Therefore, in expense of moderate increase in memory and disk usage, elevation models with higher resolution than DTED-2 (NGA.mil, 2015) can be used in avionics applications.

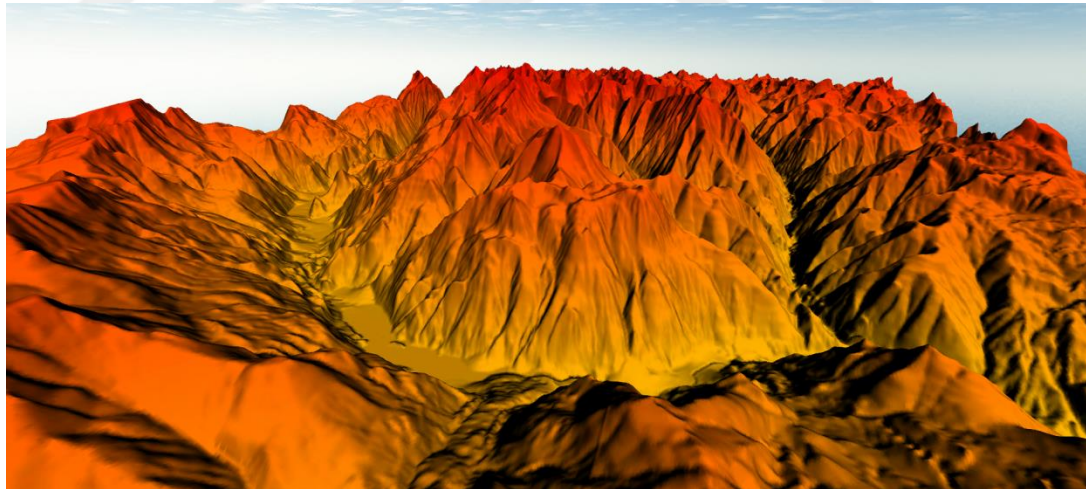


Figure 2: Terrain model visualization using Global Mapper GIS Software

The proposed algorithms are implemented for CPU and GPU using raster type digital elevation model. Depending on the range of analysis, processing unit or design restrictions, one of the suggested solutions can be preferred using results of this research.

1.3. Thesis Organization

The organization of thesis document is as follows:

Chapter 1 introduces viewshed analysis and presents the motivation and objective of the thesis,

Chapter 2 provides background information about viewshed analysis and related work,

Chapter 3 introduces the proposed algorithms, alternative implementations and their rationale,

Chapter 4 presents experiments conducted to validate the proposed algorithms and discusses the results,

Chapter 5 presents concluding remarks and suggests directions for future work,

Appendix A includes the pseudo code for the proposed algorithms,

Appendix B presents experiment tile statistics, execution time and accuracy charts for the evaluation of proposed algorithms.

CHAPTER 2

BACKGROUND INFORMATION AND PREVIOUS WORK

In this chapter components of viewshed analysis are defined and an overview of algorithms that researchers study is presented.

2.1 Elevation Model

There are two very common data types for digital elevation models which are TIN and raster type DEM (Trautwein, et al., 2016).

In 2D grid structure of raster type DEM, terrain is divided into equal sized rectangular geographic regions. Each cell in the grid represents the height of a rectangular region by an elevation value as illustrated in Figure 3. This structure is simple and easy to work with while calculating the index of a geographic region on the grid and fetching the elevation value (Trautwein, et al., 2016). Therefore, raster type DEM is preferred for studies in avionics applications (Bailey, Parrish, Kramer, Harrah, & Arthur).

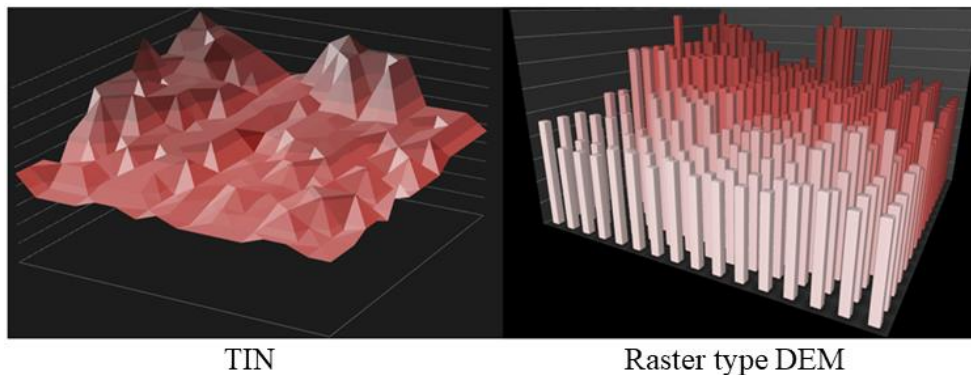


Figure 3: TIN and raster type DEM

As an alternative of raster type DEM, TIN consists of triangulated data points (See Figure 3). The number of points in a TIN and their distribution depends on the shape and complexity of the terrain. Therefore, shape and size of each triangle in TIN can be different

and this structure provides size reduction capability. However, use of TIN requires more effort to process for analyses (Trautwein, et al., 2016).

2.2. Line of Sight

Suppose an observer with a predefined range of analysis is placed above the terrain at a certain altitude. When we examine visibility of a point on the terrain, we need to define the line of sight between the observer and the target point. Since we use the digital model of the terrain, the line between observer and target point should be rasterized. In this research we use Bresenham's line drawing algorithm (Bresenham, 1965) to calculate the list of points constituting the line of sight between the observer and target point as illustrated in Figure 4.

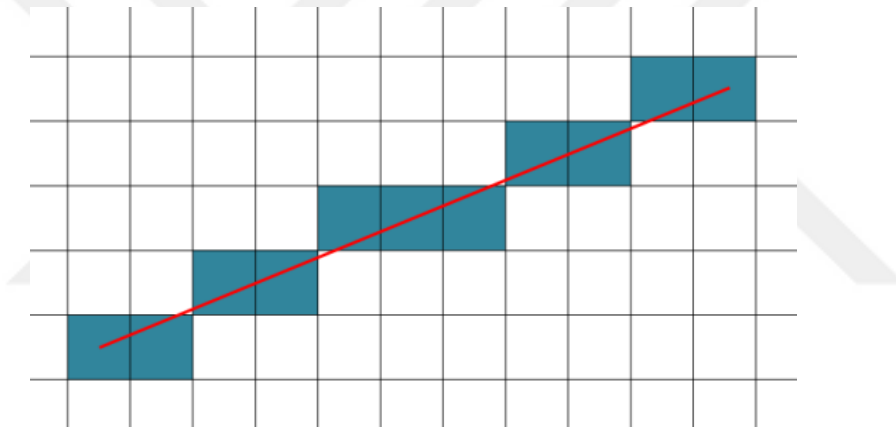


Figure 4: Line drawn by Bresenham's algorithm

To determine visibility of a target point, slope value between the observer and the target point should be compared with all slope values between the observer and other intermediary points in the line of sight. Calculation of slope value of a point with respect to observer is given in Equation 1.

Equation 1: Slope calculation of a point with respect to observer

$$\frac{(\text{elevation of target point on terrain}) - (\text{altitude of observer})}{(\text{distance between observer and target point})}$$

In Figure 5, red dashed line represents line of sight between two points P1 and P2. Suppose that observer is at P1 and the target point to check visibility is P2. H is an intermediary point on the line of sight between P1 and P2.

If the slope value between P2 and P1 is smaller than the slope value between H and P1, P2 is marked as invisible. Otherwise, we can say that target point in P2 is visible.

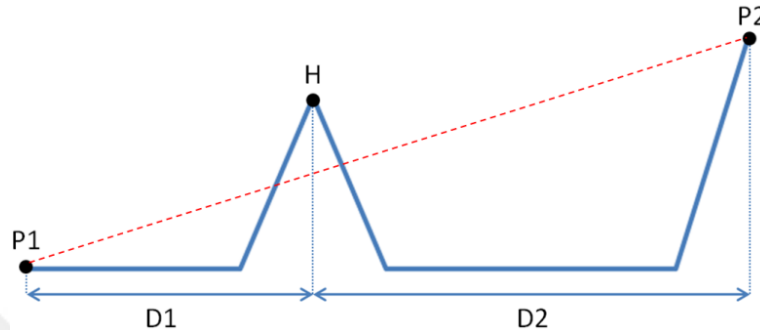


Figure 5: Visibility on a line of sight

The visibility calculation for this example is given in Equation 2.

Equation 2: Visibility calculation on a line of sight

$$\begin{aligned}
 &H_{\text{Height}} : \text{Elevation of intermediary point H} \\
 &P1_{\text{Height}} : \text{Observer altitude} \\
 &P2_{\text{Height}} : \text{Target point altitude} \\
 &D1 : \text{Distance between observer and intermediary point H} \\
 &D2 : \text{Distance between intermediary point H and P2} \\
 &\text{If } \frac{H_{\text{Height}} - P1_{\text{Height}}}{D1} \leq \frac{P2_{\text{Height}} - P1_{\text{Height}}}{D1 + D2}, \text{ P2 point is visible} \\
 &\text{If } \frac{H_{\text{Height}} - P1_{\text{Height}}}{D1} > \frac{P2_{\text{Height}} - P1_{\text{Height}}}{D1 + D2}, \text{ P2 point is not visible}
 \end{aligned}$$

2.3. Viewshed Analysis Algorithms

Viewshed analysis investigates the visibility of all points on the terrain within the range of vision. In line of sight based ray-casting viewshed analysis algorithms, visibility of each target point in area is determined by examining the points in the drawn line of sight (Mehta, Ray, & Franklin, 1994). These algorithms are also considered as ray-casting methods (Carver & Washtell, 2012). If the line of sight from observer to the target point is obstructed by another point on the terrain, the target point is marked as invisible. In Figure 6 green points on the terrain refer to visible points. The red points on the terrain refer to invisible points since their visibility is obstructed by terrain.

Blelloch, Van Kreveld, R2 and R3 presented by Franklin, Ray and Mehta are well known examples of such ray casting algorithms (Mehta, Ray, & Franklin, 1994) computing visibility by sending a ray from observer to the target point. Using the same approach

researchers also offered some line of sight scanning methods (Ying, Li, Mei, & Gao, 2008) to reduce the weight of computation.

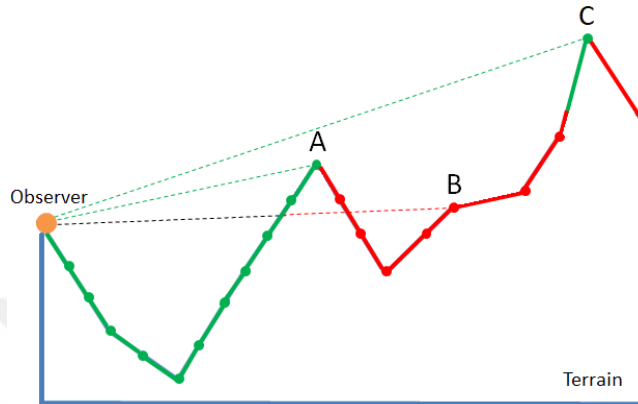


Figure 6: Line of Sight

Additionally, Mehta, Ray and Franklin defined a wave front approach, named as Xdraw, in which dataset is processed as layers unlike line of sight approach. Visibility is determined starting from the nearest points to the observer position and calculated layer by layer until the last layer is analyzed. Visibility of each layer is determined only using the information from the previous layer unlike ray-casting methods.

2.3.1. R3

R3 is a line of sight based algorithm accepted as the accuracy benchmark. The algorithm simply computes the visibility of each target point separately, in other words, a separate line of sight is drawn from observer to each target point to determine visibility. This is because R3 is not an approximate method making estimation or reusing information regarding visibility of previously analyzed target points (Zalik & Kaučič, 2002).

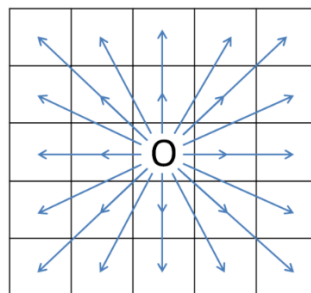


Figure 7: R3 Algorithm

Assume that the elevation grid is given in Figure 7 for viewshed analysis and O is the position of observer. Computing visibility of each cell in this brute force fashion makes R3 the slowest algorithm with $O(n^3)$ complexity for $(n \times n)$ analysis area. Therefore,

researchers try to speed up the algorithm using new technologies like GPGPU (Axell & Fridén, 2015) or works to avoid unnecessary computations (Shrestha & Panday, 2018) with different methods for a better execution time.

2.3.2. R2

R2 algorithm uses a similar approach to R3 but instead of solving the problem with brute force fashion it draws lines of sight only to boundary cells of the analysis area (See Figure 8). While moving on a line of sight to determine visibility of a boundary target point, R2 computes visibility of intermediary points too. The algorithm compares slope value of an intermediary point with its predecessors' slope values. If current point's slope value is smaller than any slope value belongs to previous points on the same line of sight, current point is marked as invisible.

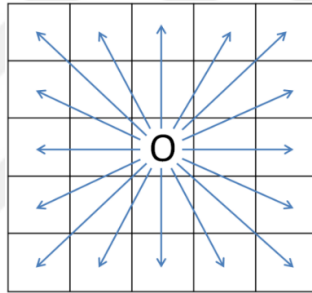


Figure 8: R2 Algorithm

Since different line of sights can intersect at some intermediary points, we can say that visibility of such intersection point is computed multiple times and visibility decision of these cells can be controversial.

T1	T2	T3	T4	T5	T6	T7	T8	T1	T2	T3	T4	T5	T6	T7	T8
T9	T10	T11	T12	T13	T14	T15	T16	T9	T10	T11	T12	T13	T14	T15	T16
T17	T18	T19	T20	T21	T22	T23	T24	T17	T18	T19	T20	T21	T22	T23	T24
T25	T26	T27	T28	T29	T30	T31	T32	T25	T26	T27	T28	T29	T30	T31	T32
T33	T34	T35	T36	T37	T38	T39	T40	T33	T34	T35	T36	T37	T38	T39	T40
T41	T42	T43	T44	T45	T46	T47	T48	T41	T42	T43	T44	T45	T46	T47	T48
T49	T50	T51	T52	T53	T54	T55	T56	T49	T50	T51	T52	T53	T54	T55	T56
O	T57	T58	T59	T60	T61	T62	T63	O	T57	T58	T59	T60	T61	T62	T63

Figure 9: Bresenham lines for R2 Algorithm

This is because different line of sights may draw different paths to these intersection points. Please check Figure 9 in which lines of sight are drawn for T4 and T5 target points

starting from observer position represented with O. These two lines of sight have a common intersection point T42 and paths to T42 are different. The line of sight from O to T4 uses slope value of T49, whereas, the line of sight from O to T5 uses slope of T50 to determine visibility of T42. For this reason, visibility of T42 will be determined by the last visiting line of sight and the order of visibility calculation affects the accuracy of R2. However, the algorithm offers better execution time with complexity $O(n^2)$ compared to R3.

2.3.3. Blleloch

To compute viewsheds for an area, each line of sight can be processed in parallel by R2 or R3 algorithms. Additionally, Blleloch parallelizes the execution of a single line of sight using maximum prefix scan method (Blleloch, 1997).

Maximum prefix scan finds the maximum element in a given list of numbers starting from the 0^{th} index to the given i^{th} index. The input for the algorithm is the slope values of cells in a line of sight. Blleloch does maximum prefix scan in parallel with binary comparison and it requires $n/2$ threads for n slope values to be compared. To find the maximum slope for a line of sight with n elements, this procedure should be repeated $\log(n)$ times. Algorithm 1 explains how Blleloch determines viewsheds using maximum prefix scan on a line of sight.

Algorithm 1: Application of maximum prefix scan algorithm for visibility

```
SlopeList: List of target slope values along a line of sight
MaxPrefixScanOutput: Output of maximum prefix scan algorithm

INPUT: SlopeList

SlopeList = {3, 1, 5, 0, 7, 6}
n: Last item index in SlopeList which is 5
n ≥ i ≥ 0, MaxPrefixScanOutput[i] is the maximum of SlopeList[0...i]
MaxPrefixScanOutput = {3, 3, 5, 5, 7, 7}

IF MaxPrefixScanOutput[i] > SlopeList[i]
    Point in index i is invisible
ELSE
    Point in index i is visible
```

In Figure 10 you can see input data to be processed by maximum prefix scan algorithm and in Figure 11 you can see the output of maximum prefix scan algorithm.

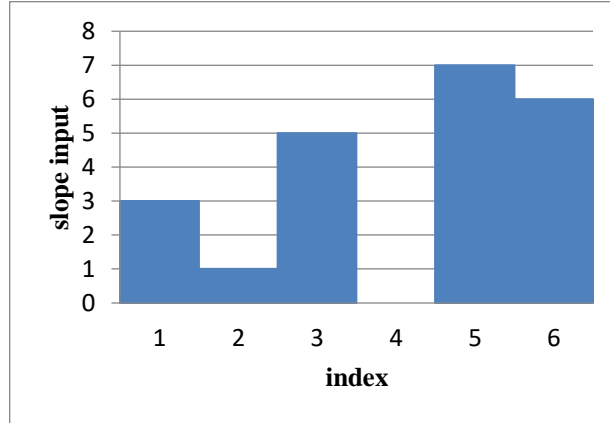


Figure 10: Input slope data for maximum prefix scan algorithm

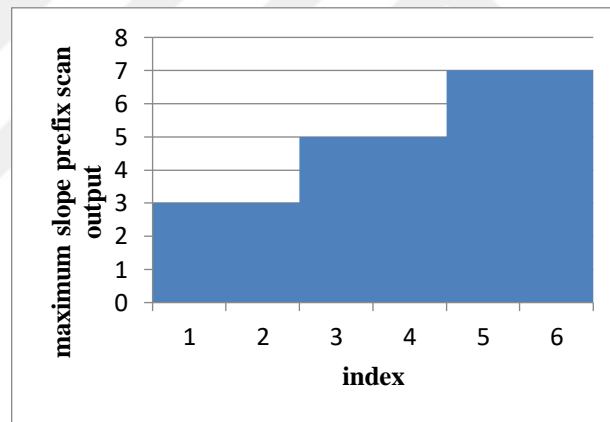


Figure 11: Output of maximum prefix scan algorithm

Blelloch's approach improves R2 by focusing on the bottleneck of the algorithm which is the execution time spent for a line of sight. Complexity of the algorithm is $O(n/\text{number of processors} + \log n)$ for $(n \times n)$ analysis area. However, Blelloch's method is not useful considering scalability due to required number of threads.

2.3.4. Van Kreveld's Algorithm

Van Kreveld's algorithm has a different approach to solve visibility problem (Van Kreveld, 1996). Van Kreveld's method draws a reference line from observer point to the border of the analysis area and algorithm sweeps all cells, holding elevation value, in the terrain grid. There are three types of events or states for each cell in the elevation grid which are enter, center and exit (See Figure 12).

- If sweeping reference line enters the cell, it is marked as **enter** event for the cell.

- If sweeping reference line passes from the center of the cell, it is marked as **center** event for the cell.
- If sweeping reference line is leaving the cell, it is marked as **exit** event for the cell.

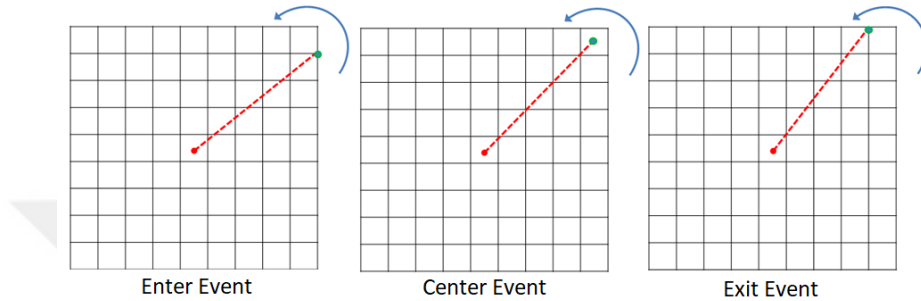


Figure 12: Events for cell in Van Kreveld's algorithm

For all cells, azimuth angle of these three events are calculated and collected in a to-do list. The list is sorted with respect to value of azimuth angles to define an order for execution of events. As the reference line sweeps, the events in the to-do list are visited and depending on the type of events different actions are taken by the algorithm using a balanced tree called **active**.

- If an **enter** event is seen, the cell is pushed into **active**.
- If a **center** event is seen, visibility of related cell should be determined using **active**.
- If an **exit** event is seen, related cell should be removed from the **active**.

Enter and center events of cells in the active balanced tree are placed according to their distance to the observer as seen in Figure 13.

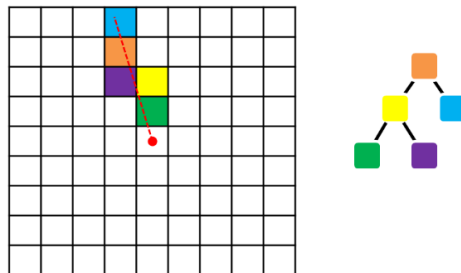


Figure 13: Active balanced tree

While iterating list of events if a center event for a cell is found, active balanced tree is traversed to find the maximum slope value (Van Kreveld, 1996). The execution time spent for this operation takes $\log(n)$ for n elements by the help of balanced tree structure of

active list. If the maximum slope value is greater than the slope value of the cell with center event, the cell is marked as invisible. In other words; to mark a cell as visible, the slope value of the cell in center state should be larger than all slope values of cells with enter and center state in active tree.

Van Kreveld's algorithm is considered as a nice option between R3 and R2 due to high accuracy and better execution time with $O(n^2 \log n)$ complexity. There are also improved alternatives of the method offering better execution time like HLD (Nguyen, Duy, & Duong, 2018) and parallel sweep line algorithm implemented for GPU (Ferreira, Andrade, Franklin, Magalhães, & Pena, 2013), however, the improvement is only in execution time. Moreover, studies show that the algorithm tends to mark visible points as invisible (Yilmaz, 2017). As an example; use of this algorithm to calculate the coverage of a threat may be misleading for the user who wants to avoid visible regions to the threat. Besides, implementation of the algorithm requires extra memory for three types of events with float type azimuth angle for a single elevation cell in the terrain grid.

2.3.5. Xdraw

Xdraw (Mehta, Ray, & Franklin, 1994) is a wave front algorithm processing the terrain data layer by layer and compute visibility from the inside out as seen in Figure 14. There are many variations developed from Xdraw since it is an approximate approach offering different estimation methods (Larsen, 2015). Mainly algorithm defines a set of predecessor points and using their elevation values computes a horizon to address the visibility of target point. Each target point's visibility depends on relative points in the previous layer. While computing visibility of the target point; minimum, maximum, mean elevation values of relative points can be used or elevation interpolation methods can be applied.

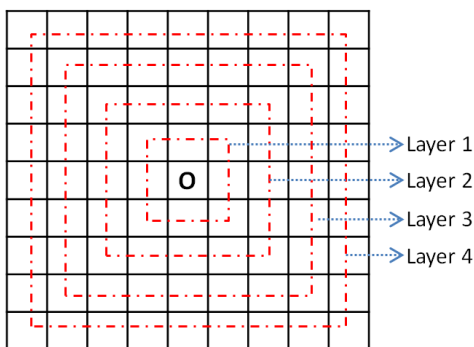


Figure 14: Xdraw algorithm

Xdraw is a faster alternative to R2 but its accuracy is worse. By the help of different estimation methods, algorithm accuracy can be increased (Larsen, 2015) but it is still not enough to use Xdraw for avionic applications with accuracy concern.

2.3.6 Comparison of Viewshed Analysis Algorithms

The comparison of algorithms summarized in previous sections is given in Table 1. Execution time and accuracy comparison of algorithms are provided for both CPU and GPU implementation. Blelloch parallel prefix scan algorithm has no CPU version.

Table 1: Comparison of common algorithms

Viewshed analysis algorithms in CPU	Execution Time	Accuracy	Extra Required Memory
R2	better than Van Kreveld's	worse than R3 better than Xdraw	-
R3	slowest algorithm	best accuracy	-
Van Kreveld's Algorithm	better than R3, slower than R2	better than R2 worse than R3	3 FLOAT type lists for events and a binary search tree
Xdraw	better than R2 in some cases	worse than R2	-
Viewshed analysis algorithms in GPU	Execution Time	Accuracy	Extra Required Memory
R2	better than Van Kreveld's	worse than R3 better than Xdraw	-
R3	slowest algorithm	best accuracy	-
Blelloch Parallel Prefix Scan	better than R2	same with R2	copy of line of sight data for each target point
Van Kreveld's Algorithm	better than R3	better than R2 worse than R3	3 FLOAT type lists for events and a binary search tree
Xdraw	better than R2	worse than R2	-

CHAPTER 3

PROPOSED SOLUTION DESIGN AND IMPLEMENTATIONS

In this chapter, proposed viewshed analysis algorithms and corresponding CPU/GPU implementations are introduced. Additionally, implementation details of R2 and R3 algorithms are given in Section 3.3 for comparison.

3.1. Proposed Solution and Design Rationale

The proposed approach of this study is based on standard R3. To reach the maximum accuracy level, we define **granularity family algorithms** which are derived from R3 by making improvements to reduce the execution time. All algorithms in this family use ray casting approach and Bresenham's line drawing method (Bresenham, 1965). Names of these granularity algorithms are **X2, X4, X8, X16, DGRHYB** and **DGR3**.

The complexity of proposed algorithms is $O(n^3)$. For the execution time improvement, preprocessed elevation models are used by granularity algorithms besides the original digital elevation model. In other words, the total amount of data used by the analysis is increased but total amount of computation in runtime is decreased by the help of processed elevation models while preserving the accuracy.

The implementation of granularity algorithms is done for both CPU and GPU to see whether parallelism is a utility to be exploited by the proposed solutions. Recently GPUs are used in avionics for such purposes due to their computing capability and limited execution times assigned for such tasks. Besides, DGRHYB algorithm presents a different approach by running different parts of analysis in CPU and GPU. This method might be helpful to divide problem instead of pushing the workload to a single processing unit for cases in which there are also other tasks.

The proposed analysis model consists of 3 phases as summarized in Figure 15. Since preparation of processed elevation models is not done in analysis runtime, it is named as phase-0. As these elevation models are prepared, execution of granularity algorithms starts

with phase-1 and then continues with phase-2. If observer altitude or position changes phase-1 and phase-2 are executed again to update the analysis result.

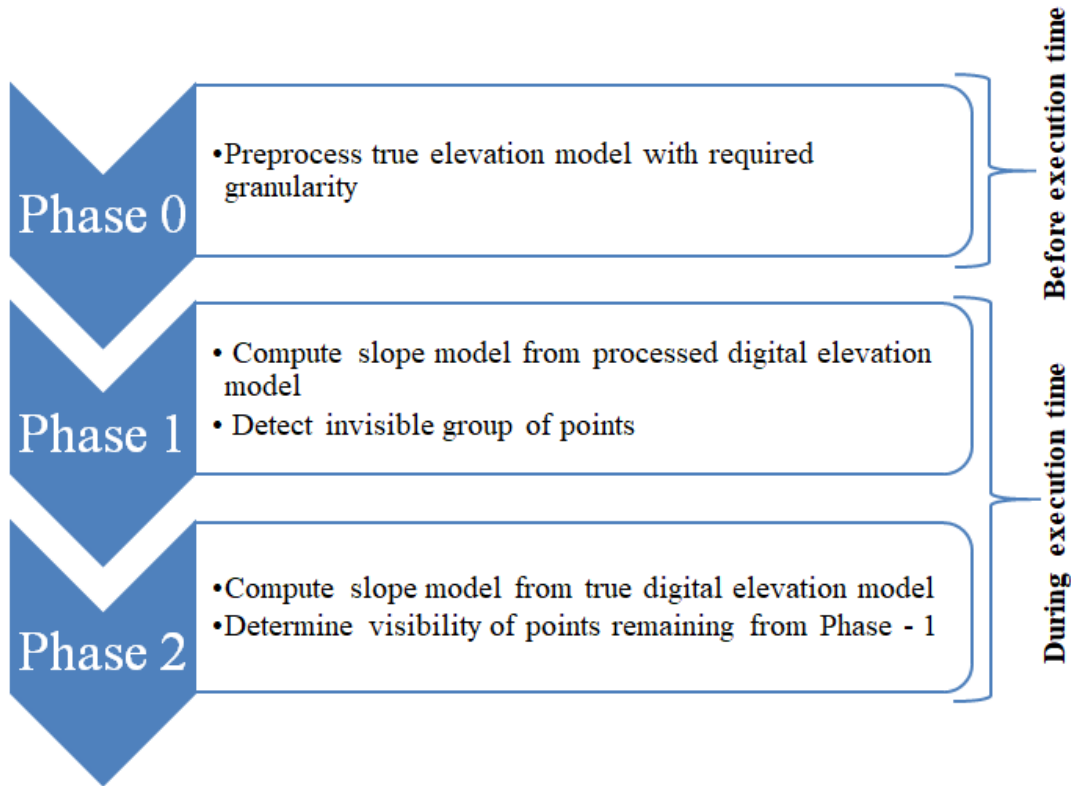


Figure 15: Proposed approach

3.1.1. Phase-0: Prepare Processed Elevation Models

Since the size of digital elevation model is the most effective factor in execution time, suggested approach focus on redesigning the elevation model to gain speedup. In this phase, true digital elevation model with real size is processed to create smaller size models without losing fundamental information. To retrieve the smaller version of digital elevation model, spatially adjacent elevation values are grouped and represented as a tile by saving the information of minimum and maximum values within the group.

The illustration of how true elevation model is processed to create smaller size model is given in Figure 16. Each red square in figure represents a group on elevation model. When elevation values are grouped by 2×2 tiles, we retrieve an elevation model whose size is $\frac{1}{4}$ of the original. Each element in processed elevation model only holds the minimum and maximum within the group of four spatially nearby elevation values.

The procedure shown in Figure 16 is applied to original elevation model with 2×2, 4×4, 8×8 and 16×16 tiles to be used by proposed algorithms.

851.38	857.337	863.997	874.323	887.249	900.235	914.554	930.376
861.523	863.952	869.127	878.777	890.81	904.083	920.59	941.307
871.856	872.813	876.64	884.678	897.618	911.792	929.704	955.812
881.208	883.516	886.53	893.234	903.304	914.403	936.461	962.321
892.042	895.357	898.723	904.836	912.234	925.406	946.504	972.197
905.827	910.211	913.687	919.902	928.09	941.036	958.72	982.315
917.267	921.971	927.35	935.42	947.064	960.047	975.434	994.003
925.286	929.591	937.169	948.787	963.115	976.249	990.621	1005.079

Group by 2x2

Min: 851.38 Max: 863.952	Min: 863.997 Max: 878.777	Min: 887.249 Max: 904.083	Min: 914.554 Max: 941.307
Min: 871.856 Max: 883.516	Min: 876.64 Max: 893.234	Min: 897.618 Max: 914.403	Min: 929.704 Max: 962.321
Min: 892.042 Max: 910.211	Min: 898.723 Max: 919.902	Min: 912.234 Max: 941.036	Min: 946.504 Max: 982.315
Min: 917.267 Max: 929.591	Min: 927.35 Max: 948.787	Min: 947.064 Max: 976.249	Min: 975.434 Max: 1005.079

Figure 16: Forming down scaled elevation model with group by 2×2

Please check the notations given in Table 2 which will be useful to define the proposed algorithms in the rest of the document. The names of granularity algorithms are relevant to processed elevation models they use. DGR3 (dynamic granularity R3) algorithm is

named considering that it uses multiple processed elevation models with different granularities.

Table 2: Notations for algorithm descriptions

Notation	Content
n	Dimension of $n \times n$ analysis area
E_Model	Original Elevation model
E2_Model	Granularity elevation model consisting of elements with minimum and maximum elevation values for each 2×2 tile in E_Model
E4_Model	Granularity elevation model consisting of elements with minimum and maximum elevation values for each 4×4 tile in E_Model
E8_Model	Granularity elevation model consisting of elements with minimum and maximum elevation values for each 8×8 tile in E_Model
E16_Model	Granularity elevation model consisting of elements with minimum and maximum elevation values for each 16×16 tile in E_Model
G	Granularity dimension for grouping $G \times G$ points
EG_Model	Refers to E2_Model, E4_Model, E8_Model, E16_Model depending on value of G.
S_Model	Slope values of target points with respect to observer.
S2_Model	Minimum and maximum slope values of target points with 2×2 granularity
S4_Model	Minimum and maximum slope values of target points with 4×4 granularity
S8_Model	Minimum and maximum slope values of target points with 8×8 granularity
S16_Model	Minimum and maximum slope values of target points with 16×16 granularity
SG_Model	Refers to S2_Model, S4_Model, S8_Model, S16_Model depending on value of G.
V_Table	Visibility table representing viewshed analysis result with same size of E

In Table 3, the size of each elevation model is given and algorithms using them marked with a cross.

Table 3: Processed elevation models used by algorithms

	X2	X4	X8	X16	DGR3	Elevation Model Size		
E_Model	X	X	X	X	X	512×512	1024×1024	2048×2048
E2_Model	X					256×256	512×512	1024×1024
E4_Model		X			X	128×128	256×256	512×512
E8_Model			X		X	64×64	128×128	256×256
E16_Model				X	X	32×32	64×64	128×128

The dimensions of processed elevation models are calculated according to three predefined sizes of original elevation model which are 512×512 , 1024×1024 and

2048×2048. The total size of the digital elevation models required by the proposed algorithms is larger than standard R3. The dimensions of processed elevation model with G×G granularity is 1/(G×G) of the original model but we double the size of each element in model since we need both minimum and maximum elevation values of G×G tile. The maximum data size for proposed solutions is given in Equation 3.

Equation 3: Calculation of size increase for proposed algorithms

Data size of E2_Model is 1/2 of E_Model
 Data size of E4_Model is 1/8 of E_Model
 Data size of E8_Model is 1/32 of E_Model
 Data size of E16_Model is 1/128 of E_Model
 The maximum data size increase is $1/2 + 1/8 + 1/32 + 1/128 < 0.67$

3.1.2. Phase-1: Detection of Invisible Groups

The first phase of actual analysis execution is detection of invisible groups. To find the invisible group of points, slope model is calculated from the processed elevation model. Reminding that each element in processed elevation models has a minimum and maximum elevation value, the calculated slope models also consist of tiles with minimum and maximum slope values. To generate the slope model, Equation 4 is applied for each cell in processed elevation data.

Equation 4: Slope calculation with granularity elevation model in phase-1

ObserverX: Observer position along X axis in Cartesian grid
 ObserverY: Observer position along Y axis in Cartesian grid
 TargetX: Target cell position along X axis in Cartesian grid
 TargetY: Target cell position along Y axis in Cartesian grid
 ObserverAltitude: Altitude of observer
 Distance: Euclidean distance between observer and the target cell
 TargetMinElev: Minimum elevation value of target cell
 TargetMaxElev: Maximum elevation value of target cell
 TargetMinSlope: Minimum slope value of target cell
 TargetMaxSlope: Maximum elevation value of target cell

$$\text{Distance} = \sqrt{(\text{TargetX} - \text{ObserverX})^2 + (\text{TargetY} - \text{ObserverY})^2}$$

$$\text{TargetMinSlope} = (\text{TargetMinElev} - \text{ObserverAltitude}) / \text{Distance}$$

$$\text{TargetMaxSlope} = (\text{TargetMaxElev} - \text{ObserverAltitude}) / \text{Distance}$$

The result of slope calculation on elevation model is one of the slope models given in Table 4.

Table 4: Dimensions of slope models retrieved from processed elevation models

	Slope Model Size		
S2_Model	256×256	512×512	1024×024
S4_Model	128×128	256×256	512×512
S8_Model	64×64	128×128	256×256
S16_Model	32x32	64×64	128×128

As slope model is available we draw separate line of sight for each cell in granularity slope model as we do in R3 algorithm. We cannot determine visible points using any granularity slope models, however, the information of minimum and maximum slope values helps us to find invisible group of points.





Min Slope: 21.38 Max Slope: 63.952 Target_Cell 	Min Slope: 63.997 Max Slope: 87.777	Min Slope: 87.249 Max Slope: 90.083	Min Slope: 14.554 Max Slope: 41.307
Min Slope: 71.856 Max Slope: 83.516	Min Slope: 16.64 Max Slope: 23.4 	Min Slope: 89.618 Max Slope: 91.403	Min Slope: 29.704 Max Slope: 62.321
Min Slope: 92.042 Max Slope: 10.211	Min Slope: 88.723 Max Slope: 99.902	Min Slope: 92.238 Max Slope: 141.036 	Min Slope: 46.504 Max Slope: 82.305
Min Slope: 17.267 Max Slope: 99.51	Min Slope: 27.35 Max Slope: 48.87	Min Slope: 47.064 Max Slope: 76.249	Min Slope: 5.434 Max Slope: 10.079 Source_Cell 

Figure 17: Visibility on slope model with 16×16 granularity

In Figure 17 we see an example of S16_Model (See Table 2) and there is a line of sight drawn from Source_Cell to Target_Cell marked with orange color. Other intermediary

points are marked with blue color. Since the maximum slope of Target_Cell is smaller than the minimum slope value of an intermediary point, Target_Cell will be marked as invisible. Since granularity of slope model is 16×16 , we can mark $16 \times 16 = 256$ points as invisible in V_Table (See Table 2).

3.1.3. Phase-2: Determine Visibility for Remaining Points

Suppose that X4 algorithm is run and phase-1 is just completed. In Figure 18, we see that observer is represented with O and red cells are marked as invisible by using S4_Model (See Table 2). This example illustrates the recent state of viewshed analysis result just before phase-2 starts.

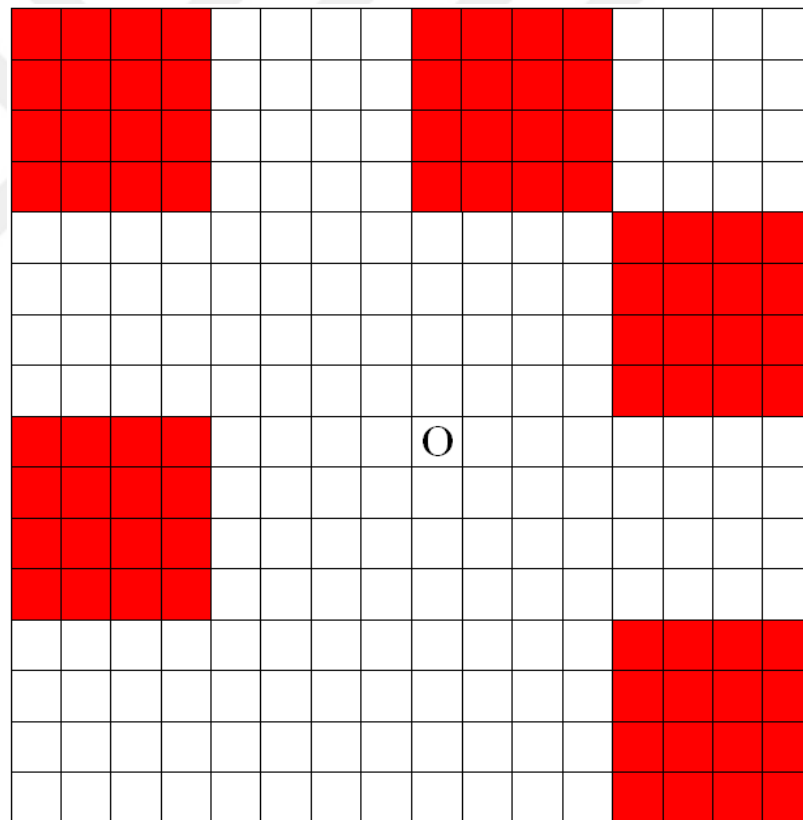


Figure 18: V_Table after phase-1

The visibilities of unmarked cells will be resolved in phase-2, therefore, slope values of all target points are calculated by applying Equation 5 using the original elevation model.

Equation 5: Slope calculation with original elevation model in phase-2

ObserverX: Observer position along X axis in Cartesian grid
ObserverY: Observer position along Y axis in Cartesian grid
TargetX: Target cell position along X axis in Cartesian grid
TargetY: Target cell position along Y axis in Cartesian grid
ObserverAltitude: Altitude of observer
Distance: Euclidean distance between observer and the target cell
TargetElev: Elevation value of target cell
TargetSlope: Slope value of target cell

$$\text{Distance} = \sqrt{(\text{TargetX} - \text{ObserverX})^2 + (\text{TargetY} - \text{ObserverY})^2}$$
$$\text{TargetSlope} = (\text{TargetElev} - \text{ObserverAltitude}) / \text{Distance}$$

After S_Model (See Table 2) is prepared, visibility computation for unmarked points starts. Phase 2 works like standard R3 but it aims to reduce the amount of computation by reusing visibility information of the previously analyzed points. Therefore, visibility calculation of remaining points is done from the inside out. To decide visibility of each target point, a separate line of sight is drawn and algorithm checks whether it is possible to avoid slope comparison operations along that line of sight.

Since we draw the line of sight using Bresenham's algorithm, we can find the position of previous point of target cell. If the slope value of previous point on the line of sight is larger than the target point's slope value, we can mark the target cell as invisible. Algorithm 2 explains how predecessor position is calculated and visibility of target point is decided.

Algorithm 2: Detection of invisible point by checking the slope of predecessor

ObserverX: Observer position along X axis in Cartesian grid
ObserverY: Observer position along Y axis in Cartesian grid
TargetX: Target cell position along X axis in Cartesian grid
TargetY: Target cell position along Y axis in Cartesian grid
DeltaX: Absolute value of distance between observer and target along X axis
DeltaY: Absolute value of distance between observer and target along Y axis
PrevPointX: Position of target point's predecessor along X axis
PrevPointY: Position of target point's predecessor along Y axis
SlopeModel: 2D data holding target point slope values calculated using original elevation model.

INPUT: ObserverX, ObserverY, TargetX, TargetY, SlopeModel

$$\text{DeltaX} = |\text{TargetX} - \text{ObserverX}|$$
$$\text{DeltaY} = |\text{TargetY} - \text{ObserverY}|$$

Algorithm 2 (continued)

```

IF (TargetX - ObserverX) > 0 THEN IteratorX = 1
IF (TargetX - ObserverX) < 0 THEN IteratorX = -1
IF (TargetY - ObserverY) > 0 THEN IteratorX = 1
IF (TargetY - ObserverY) < 0 THEN IteratorX = -1

IF (DeltaX - DeltaY) > DeltaY
    PrevPointX = TargetX - IteratorX
    PrevPointY = TargetY
ELSE IF (DeltaY - DeltaX) > DeltaX
    PrevPointX = TargetX
    PrevPointY = TargetY - IteratorY
ELSE
    PrevPointX = TargetX - IteratorX
    PrevPointY = TargetY - IteratorY

IF SlopeModel[PrevPointX][PrevPointY] > SlopeModel[TargetX][TargetY]
    Mark target point as invisible

```

If the slope of predecessor is not greater than slope of the target cell, we can determine visibility by analyzing two relative points of the target.

1	2	3	4	T3
6	7	8	T1	T2
11	12	13	14	15
16	17	18	19	20
21	22	23	24	25
26	27	28	29	30
31	32	33	34	35
36	37	38	39	40
41	42	43	44	45
46	47	48	49	50
O	52	53	54	55

1	2	3	4	T3
6	7	8	T1	T2
11	12	13	14	15
16	17	18	19	20
21	22	23	24	25
26	27	28	29	30
31	32	33	34	35
36	37	38	39	40
41	42	43	44	45
46	47	48	49	50
O	52	53	54	55

1	2	3	4	T3
6	7	8	T1	T2
11	12	13	14	15
16	17	18	19	20
21	22	23	24	25
26	27	28	29	30
31	32	33	34	35
36	37	38	39	40
41	42	43	44	45
46	47	48	49	50
O	52	53	54	55

Line of sight to T1 Line of sight to T2 Line of sight to T3

Line of sight to T1 = {O, 46, 42, 37, 32, 28, 23, 18, 14, T1}
 Line of sight to T2 = {O, 46, 42, 37, 33, 28, 24, 19, 15, T2}
 Line of sight to T3 = {O, 46, 42, 37, 33, 28, 23, 19, 14, T2, T3}

Figure 19: Reuse of visibility information in phase 2

In Figure 19, three different lines of sight and list of points in them presented for three target cells T1, T2 and T3. Comparing point list of T3 with other point lists shows that we cannot use visibility information of T2 or T1 due to different intermediary points written

in red color. However, union of T1 and T2 lists covers the line of sight points assigned for T3 and we may determine visibility of target by analyzing T1 and T2 in two specific cases where both of the points are visible or invisible.

If T1 and T2 point are visible and their slope values are smaller than the slope of T3, we can mark T3 as invisible without examining the whole line of sight. Moreover, if T1 and T2 are invisible and their slope values are greater than slope of T3, T3 must be invisible. For other cases, slope of target point is compared with slope values of all points on the line of sight as we see in standard R3. Algorithm 3 explains how position of relative points found.

Algorithm 3: Determine visibility using relative points' lines of sight

```
ObserverX: Observer position along X axis in Cartesian grid
ObserverY: Observer position along Y axis in Cartesian grid
TargetX: Target cell position along X axis in Cartesian grid
TargetY: Target cell position along Y axis in Cartesian grid
DeltaX: Absolute value of distance between observer and target along X
axis
DeltaY: Absolute value of distance between observer and target along Y
axis
RelP1X: Position of first relative point along X axis
RelP1Y: Position of first relative point along Y axis
RelP2X: Position of second relative point along X axis
RelP2Y: Position of second relative point along Y axis

INPUT: ObserverX, ObserverY, TargetX, TargetY

DeltaX = |TargetX - ObserverX|
DeltaY = |TargetY - ObserverY|

IF (TargetX - ObserverX) > 0 THEN IteratorX = 1
IF (TargetX - ObserverX) < 0 THEN IteratorX = -1
IF (TargetY - ObserverY) > 0 THEN IteratorY = 1
IF (TargetY - ObserverY) < 0 THEN IteratorY = -1

IF (DeltaX > DeltaY)
    RelP1X = TargetX - IteratorX
    RelP1Y = TargetY - IteratorY
    RelP2X = TargetX - IteratorX
    RelP2Y = TargetY

ELSE
    RelP1X = TargetX - IteratorX
    RelP1Y = TargetY - IteratorY
    RelP2X = TargetX
    RelP2Y = TargetY - IteratorY
```

3.2. Proposed Granularity Algorithm Implementations

In this section we define the implementation details of proposed granularity algorithms and elevation model processing. Besides CPU and GPU implementation of each algorithm, hybrid approach of DGRHYB is also explained.

The implementation steps are executed in the given order. To understand the notations used for data models please check Table 2.

3.2.1. Elevation Model Processing

Since the suggested solutions use processed elevation models, we should start with elevation model processing. These implementation steps are run before the analysis runtime and depending on value of G ; $E2_Model$, $E4_Model$, $E8_Model$ or $E16_Model$ is created from E_Model .

- 1) Create empty EG_Model with $(n/G) \times (n/G)$ dimensions, consisting of elements with minimum and maximum elevation attributes
- 2) For each $G \times G$ group in E_Model find minimum and maximum elevation values
- 3) Place minimum and maximum elevation values of each $G \times G$ group to respective location in EG_Model

3.2.2. Granularity Algorithms in CPU

At the end of elevation model processing EG_Model is obtained for the granularity algorithms. The following implementation steps are executed in analysis runtime and they are same for X2, X4, X8, X16 and DGR3 algorithms. Firstly phase-1 steps are run to detect invisible group of points using EG_Model .

1. Calculate SG_Model from EG_Model with respect to observer altitude and position using Euclidean distance
2. Draw separate line of sight for each target point with $G \times G$ granularity whose visibility is not determined yet
3. Iterate on each drawn line of sight to target groups in SG_Model
 - a. If minimum slope value of target point is larger than maximum slope values of points along the line of sight, mark $G \times G$ points in target as invisible using V_Table .
4. Prepare list of remaining points according to V_Table

At the end of phase-1 some invisible regions may be found or not. Visibility of remaining points is resolved by phase-2 with following steps.

5. Calculate S_Model from E_Model with respect to observer altitude and position using Euclidean distance
6. Draw separate line of sight to each target point remaining from phase-1
7. Iterate on each drawn line of sight to target points in S_Model
 - a. If slope value of target point is smaller than the slope value of predecessor, mark target point as invisible using V_Table
 - b. Else if slope value of target point is smaller than the slope values of two invisible relative points, mark target point as invisible using V_Table
 - c. Else if slope value of target point is larger than the slope values of two visible relative points, mark target point as visible using V_Table
 - d. Else if slope value of target point is larger than slope values of all points along the line of sight, mark target point as visible using V_Table
 - e. Else if slope value of the target point is smaller than slope value of any point along the line of sight, mark target point as invisible using V_Table

DGR3 algorithm uses three types of elevation models besides the original elevation model. Therefore, elevation model processing is applied for 16×16, 8×8 and 4×4 granularities. In analysis runtime algorithm starts phase-1 with E16_Model and repeats the same implementation steps using E8_Model and E4_Model in order. For the remaining points phase-2 is run once like other granularity algorithms starting from step 5.

3.2.3. Granularity Algorithms in GPU

GPU implementations of proposed algorithms are done using CUDA API developed by NVIDIA. (NVIDIA Corporation, n.d.). In CUDA, we define kernels which are functions executed by assigned threads in parallel with single instruction multiple data approach. To run a kernel we firstly decide launch parameters which are the grid and block structure for the threads (NVIDIA Corporation, n.d.).

In each phase of granularity algorithms, two kernels are defined to compute slope values and determine visibility of points. Although same kernels are run in proposed algorithms, calculation of launch parameters is different for DGR3 in phase-1. For the other proposed algorithms (X2, X4, X8, X16) the equation to define grid and block structure does not change (See Equation 6).

Equation 6: Launch parameters of both kernels in phase-1 for proposed algorithms except DGR3

```
N: Dimension of analysis area which can be 512, 1024 or 2048
G: Granularity dimension of algorithm
NG: Dimension of analysis area with granularity G

NG = (N / G)
BlockDim = 16
GridDim = NG / BlockDim

2D Block structure = (BlockDim, BlockDim)
2D Grid structure = (GridDim, GridDim)
```

Since each thread works for only one target point, same launch parameters are used for slope calculation and invisible group detection tasks in phase-1 of X2, X4, X8 and X16 algorithms. In Equation 6, you can see grid calculation where the block dimension is assigned as 16 for both kernels of phase-1.

DGR3 repeats phase-1 three times with different granularities. For slope calculation kernel in phase-1, the algorithm uses same launch parameters given in Equation 6. However, in detection of invisible groups, grid and block structure depends on the count of remaining points which are not marked as invisible. For each run of phase-1, launch parameters are calculated again as explained in Equation 6 and Equation 7.

Equation 7: DGR3 kernel launch parameters calculation in phase-1

```
G: Granularity dimension of algorithm
Rem: Remaining point count whose visibility is not determined in
phase-1
RemG: Remaining point count whose visibility is not determined yet
with granularity G

RemG = Rem / G
BlockDim = 16
GridDim =  $\sqrt{\text{RemG} / (\text{BlockDim} \times \text{BlockDim})}$ 

2D Block structure = (BlockDim, BlockDim)
2D Grid structure = (GridDim, GridDim)
```

In phase-2 launch parameter equations are same for all proposed algorithms. The grid and block structure for slope calculation of phase-2 is given in Table 5.

Table 5: Phase-2 slope computation launch parameters for proposed algorithms

Analysis Dimension	Grid Structure	Block Structure
512×512	(32,32)	(16,16)
1024×1024	(64,64)	(16,16)
2048×2048	(128,128)	(16,16)

To determine visibility of remaining points in phase-2, 1D grid and 2D block structure is used as shown in Equation 8 because using 2D grid may cause unnecessary occupancy of thread blocks.

Equation 8: Phase-2 visibility computation kernel launch parameters

```

Rem: Remaining point count whose visibility is not determined in
phase-1

BlockDim = 16
GridDim = (Rem+(BlockDim × BlockDim)-1) / (BlockDim×BlockDim)

2D Block structure = (BlockDim, BlockDim)
1D Grid structure = (GridDim)

```

To process data in GPU, we need to make copy operation from host memory to device memory. The device operations are executed in parallel using kernel launch parameters and then result data is copied back to host. Therefore, the implementation steps begin with data copy to execute phase-1.

In Host:

- 1) Copy EG_Model to device memory

In Device:

- 2) Calculate SG_Model from EG_Model with respect to observer altitude and position using Euclidean distance
- 3) Draw separate line of sight for each target point with G×G granularity whose visibility is not determined yet
- 4) Iterate on each drawn line of sight to target groups in SG_Model
 - a. If minimum slope value of target point is larger than maximum slope values of points with along the line of sight, mark G×G points in target as invisible using V_Table.

In Host:

- 5) Copy V_Table to host memory
- 6) Prepare list of remaining points according to V_Table

At the end of phase-1 the visibility results are copied to host memory. In phase-2 remaining point list is prepared and copied to device memory to determine visibility of points.

- 7) Copy E_Model and list of remaining points to device memory

In Device:

- 8) Calculate S_Model from E_Model with respect to observer altitude and position using Euclidean distance
- 9) Draw separate line of sight to each target point remaining from phase-1
- 10) Iterate on each drawn line of sight to target point
 - a. If slope value of target point is smaller than the slope value of predecessor, mark target point as invisible using V_Table
 - b. Else if slope value of target point is smaller than the slope values of two invisible relative points, mark target point as invisible using V_Table
 - c. Else if slope value of target point is larger than the slope values of two visible relative points, mark target point as visible using V_Table
 - d. Else if slope value of target point is larger than slope values of all points along the line of sight, mark target point as visible using V_Table
 - e. Else if slope value of the target point is smaller than slope value of any point along the line of sight, mark target point as invisible using V_Table

In Host:

- 11) Copy V_Table to host memory

The implementation steps above are executed once by X2, X4, X8 and X16 algorithms. DGR3 repeats steps 1-6 for E16_Model, E8_Model and E4_Model with given order and then executes phase-2 starting from step 7.

3.2.4. Hybrid Granularity Algorithm

DGRHYB algorithm executes phase-1 in CPU and then switches to GPU to execute phase-2. In phase-1, DGRHYB repeats steps 1-4 in CPU three times as DGR3. After

phase-1 is completed, the algorithm continues phase-2 in GPU by executing the steps 5-10. For slope calculation DGRHYB uses launch parameters given in Table 5 and then visibility computation of remaining points is done with grid and block structure calculated with Equation 8.

3.3. R3 and R2 Algorithm Implementations

3.3.1. R3 in CPU

The implementation steps for standard R3 is given below:

- 1) Calculate S_Model from E_Model with respect to observer altitude and position using Euclidean distance
- 1) Draw separate line of sight for each target point in S_Model
- 2) Iterate on each drawn line of sight to target points
 - a. If slope value of the target point is greater than slope values of all points along the line of sight, mark target point in V_Table as visible.
 - b. If slope value of the target point is less than slope value of any point along the line of sight, mark target point in V_Table as invisible.

3.3.2. R2 in CPU

The implementation steps for standard R2 is given below:

Calculate S_Model from E_Model with respect to observer altitude and position using Euclidean distance

Draw separate line of sight only for target points on the edges of S_Model

Iterate on each drawn line of sight to determined target points

- a. If slope value of the target point is greater than slope values of all points along the line of sight, mark target point in V_Table as visible.
- b. If slope value of the target point is less than slope value of any point along the line of sight, mark target point in V_Table as invisible.

3.3.3. R3 in GPU

R3 algorithm runs a thread for each target point in parallel using 2D indexing since the elevation data and viewshed output is 2D. The block size is defined as 16×16 and the grid dimension varies depending on the size of analysis area.

For slope calculation and visibility computation kernels, same launch parameters are used shown in Table 6.

Table 6: R3 kernel launch parameters

Analysis Dimension	R3 Grid Structure	R3 Block Structure
512×512	(32,32)	(16,16)
1024×1024	(64,64)	(16,16)
2048×2048	(128,128)	(16,16)

GPU implementation of algorithm follows same steps with CPU implementation. Only E_Model is copied to device memory in the beginning and visibility result data V_Table is copied to host memory at the end of the analysis.

3.3.4. R2 in GPU

R2 algorithm computes slope values in the analysis area using the same launch parameters with R3 as given in Table 6. However, visibility computation kernel uses 1D grid and blocks.

Table 7: R2 kernel launch parameters

Analysis Area	R2 Grid Structure	R2 Block Structure
512×512	8	256
1024×1024	16	256
2048×2048	32	256

This is because R2 algorithm draws line of sight only for points on the edges of analysis area. The block size is arranged as 256 which is equal to 16×16 which is used for 2D block structure of compared algorithms. Table 7 shows grid and block dimensions for three different analysis dimensions. GPU implementation of algorithm follows same steps with CPU implementation. Only E_Model is copied to device memory in the beginning and visibility result data V_Table is copied to host memory at the end of the analysis.



CHAPTER 4

EVALUATION

In this chapter we will see the comparison of implemented proposed algorithms with R3 and R2 in terms of accuracy, execution time and resource allocation. R2 and R3 implementations used in evaluations are given in Section 3.3. Digital elevation models used in analysis experiments are retrieved from ASTER (NASA, n.d.). Experiment tiles are chosen in three sizes which are 512, 1024 and 2048. For each size, 25 different tiles are used to assess results in different terrain topologies. In Appendix B experiment tile statistics are provided. Since the elevation models are extracted with Cartesian coordinates, Euclidian distance is used in slope calculations.

For each experimented tile, the viewshed analyses are carried out for seven different observer altitudes and the observer is located at the center of the tile in the experiments. The altitudes are determined according to terrain elevation value at the observer's position and standard deviation of elevation levels in the tile. Hence, sensible changes in viewshed can be examined, as the altitude is increased. The observer altitudes are calculated as follows:

$$A_0 = E_o + 2 \text{ Meters}$$

and

$$A_c = E_o + c * \sigma \quad \text{for } c = 1,2,3,4,5$$

Where, A_c for $c=0..5$ is the altitude of the observer, E_o is elevation at the observer's position, σ is the standard deviation of all elevations in the tile. Another observer altitude level to be tested is MSA (minimum sector altitude). In aviation, MSA (Federal Aviation Administration, n.d.) is used as a common reference to provide minimum clearance of 300 meters above the terrain (Federal Aviation Administration, n.d.).

$$A_{msa} = E_o + 300 \text{ Meters}$$

Each algorithm is run on each experiment tile with 7 different altitudes, in other words, 525 experiments are run per algorithm in this research. In each experiment, speedup ratios for the proposed algorithms with respect to R3 are calculated to make comparisons with R3 algorithm's execution time. CPU versions of algorithms are implemented with single thread. For GPU implementation CUDA parallel programming platform and API is used (NVIDIA Corporation, n.d.). Elevation data, slope data and distance calculations use FLOAT data type for maximum accuracy.

Hardware and software specifications of experimental setup are listed below.

OS: Windows 10

IDE: Visual Studio 2010 with C++

CUDA Version: v9.2

Architecture: x64

CPU: Intel(R) Core(TM) i7-4700QM CPU @ 2.40GHz 4 Core

GPU: NVIDIA Kepler Compute Capability 3.0

RAM: 16 GB

4.1. Experiments with CPU Implementation

4.1.1. Execution Time

In this section, execution time performances of the proposed algorithms are compared to execution times of R3 algorithm. The speedup parameters changing by the altitude are depicted with figures. Analyses are performed on three different tile sizes and the data points in the charts correspond to the speedup value obtained.

According to the results presented in Figure 20, Figure 21, Figure 22 and Figure 23, speedup per altitude graph shows that the fastest algorithms are DGR3, X4 and X8 for all altitude levels. Execution time of X2 and X16 algorithms proves that very small and very large granularities reduce the execution time improvement provided by phase-1.

X2 is the algorithm spending most time for phase-1 among other granularity algorithms because E2_Model is the largest processed elevation model. On the other hand, X16 is the algorithm using smallest input data E16_Model but increasing granularity decreases the possibility to find invisible group of points. In this case phase-1 becomes less effective for X16 and algorithm spends so much time for phase-2.

For analysis on 512×512 tiles, X4 and DGR3 algorithms offer best execution time with altitude levels lower than A_3 (See Figure 20). DGR3 algorithm takes advantage of using processed elevation models with large granularities at lowest altitude level. In contrast, increasing observer altitude decreases wide invisible regions and causes DGR3 to lose time in phase-1. X8 algorithm provides the best execution time for altitude levels higher than A_3 because contribution of phase-1 dramatically reduces for all granularity algorithms

and X8 completes phase-1 faster than DGR3 and X4. Although X8 spends more time for phase-2 compared to X4 and DGR3, it becomes the fastest algorithm due to small analysis dimension (See Figure 20).

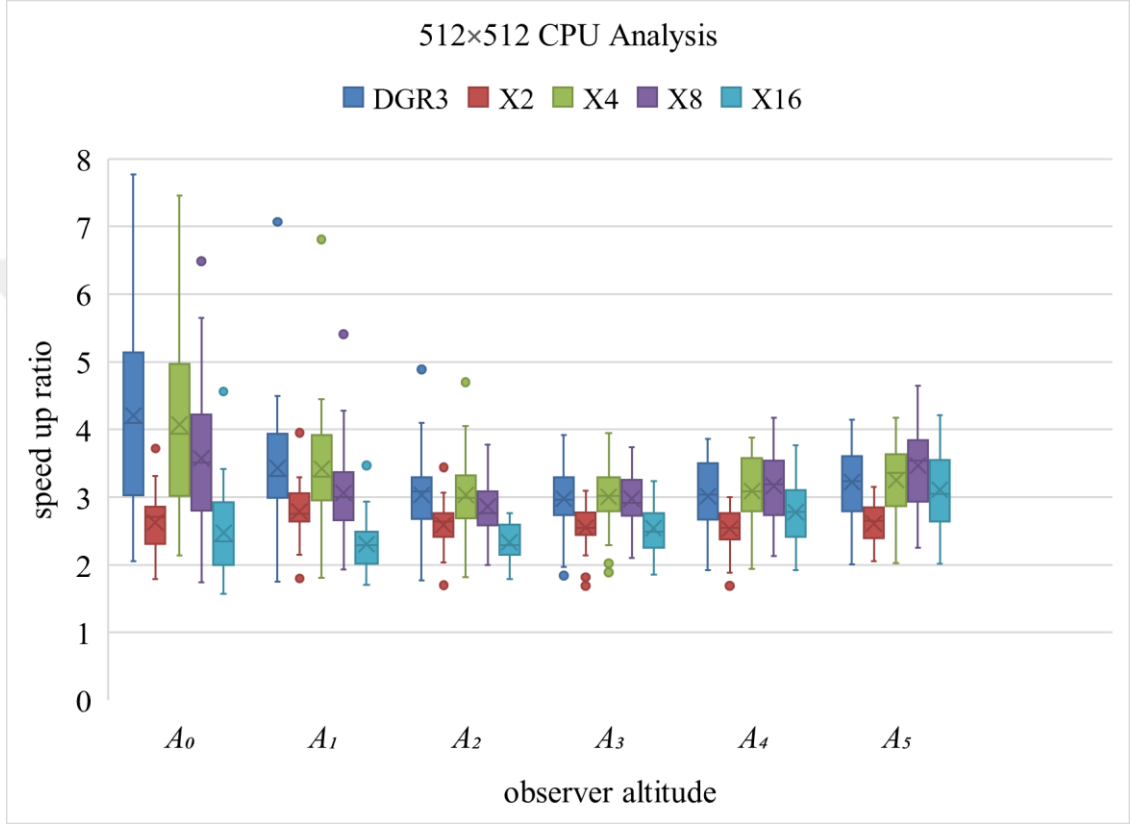


Figure 20: CPU algorithm execution time comparison on 512×512 tiles

The execution time statistics given in Table 8 for 512×512 tiles prove that DGR3 can reach 7.77x speedup ratio. Although, X8 is the fastest algorithm at high altitude levels, DGR3 and X4 are the best choices according to average speedup values.

Table 8: Execution time statistics of CPU algorithms on 512×512 tiles

	DGR3	X2	X4	X8	X16
min	1.75	1.69	1.81	1.74	1.57
max	7.77	3.95	7.46	6.49	4.58
avg	3.30	2.62	3.30	3.21	2.65

When analysis tile size is increased to 1024×1024, DGR3 and X4 benefits from phase-1 more by the help of 4×4 granularity and X8 performs slower due to spending more time for phase-2 by the increased analysis area dimension (See Figure 21). On the ground and low altitude levels DGR3 is slightly faster than X4 since it uses 8×8 and 16×16 granularities besides 4×4 to determine invisible groups more quickly. For the maximum

altitude level, gap of speedup ratio between X4, DGR3 and X8 disappears as it can be seen from Figure 21.

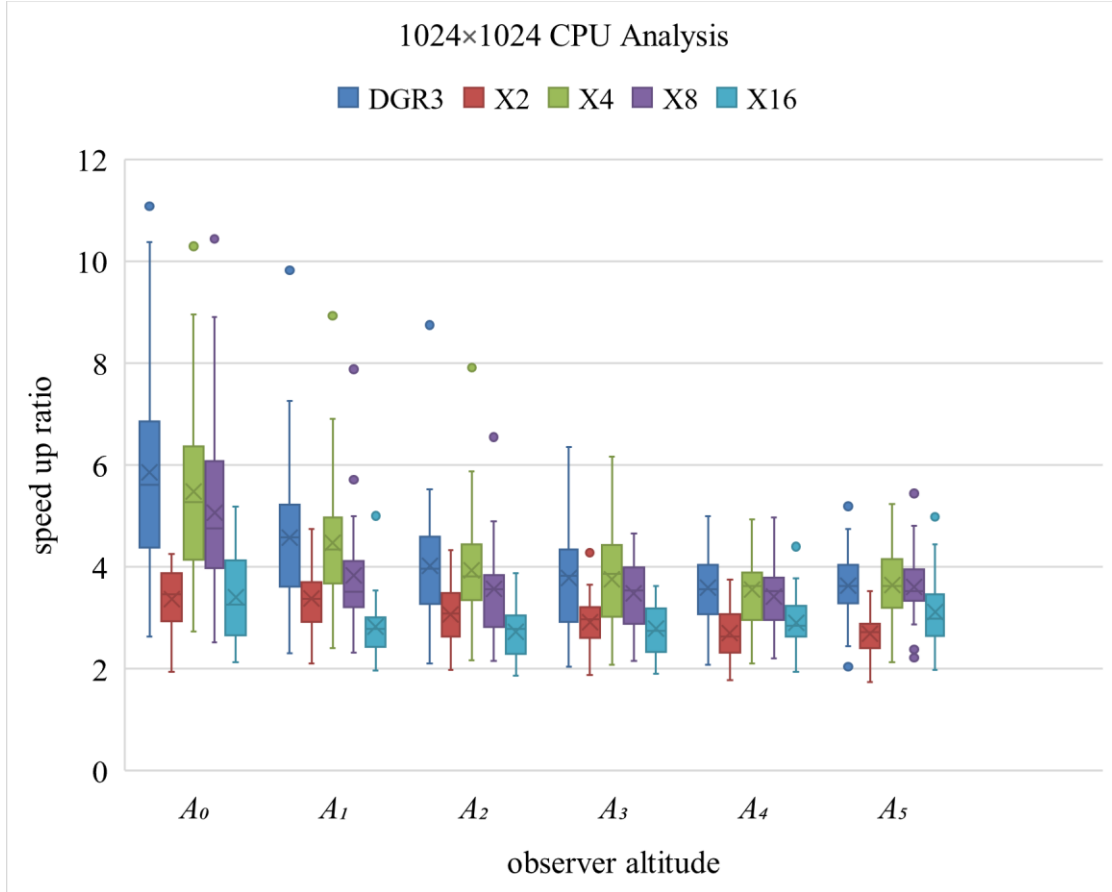


Figure 21: CPU algorithm execution time comparison on 1024x1024 tiles

According to Table 9, all algorithms have better speedup statistics compared to Table 8. Hence, it can be said that 1024x1024 is the ideal size for proposed algorithms. Average speedup values of DGR3 and X4 exceed 4.10x and starts to increase the gap with X8.

Table 9: Execution time statistics of CPU algorithms on 1024x1024 tiles

	DGR3	X2	X4	X8	X16
min	2.04	1.74	2.08	2.15	1.87
max	11.08	4.74	10.29	10.44	5.18
avg	4.18	2.99	4.10	3.82	2.99

As it can be seen from Figure 22, for 2048x2048 tiles, X4 offers the best execution time with 4x4 granularity. Execution time results at low altitude show that X4 benefits from phase-1 more than other algorithms. DGR3 is apparently slower than X4 due to overhead caused by repeating phase-1 for 8x8 and 16x16 granularities. In smaller analysis areas

overhead was not considerable but as we increase analysis area, it started to decelerate DGR3.

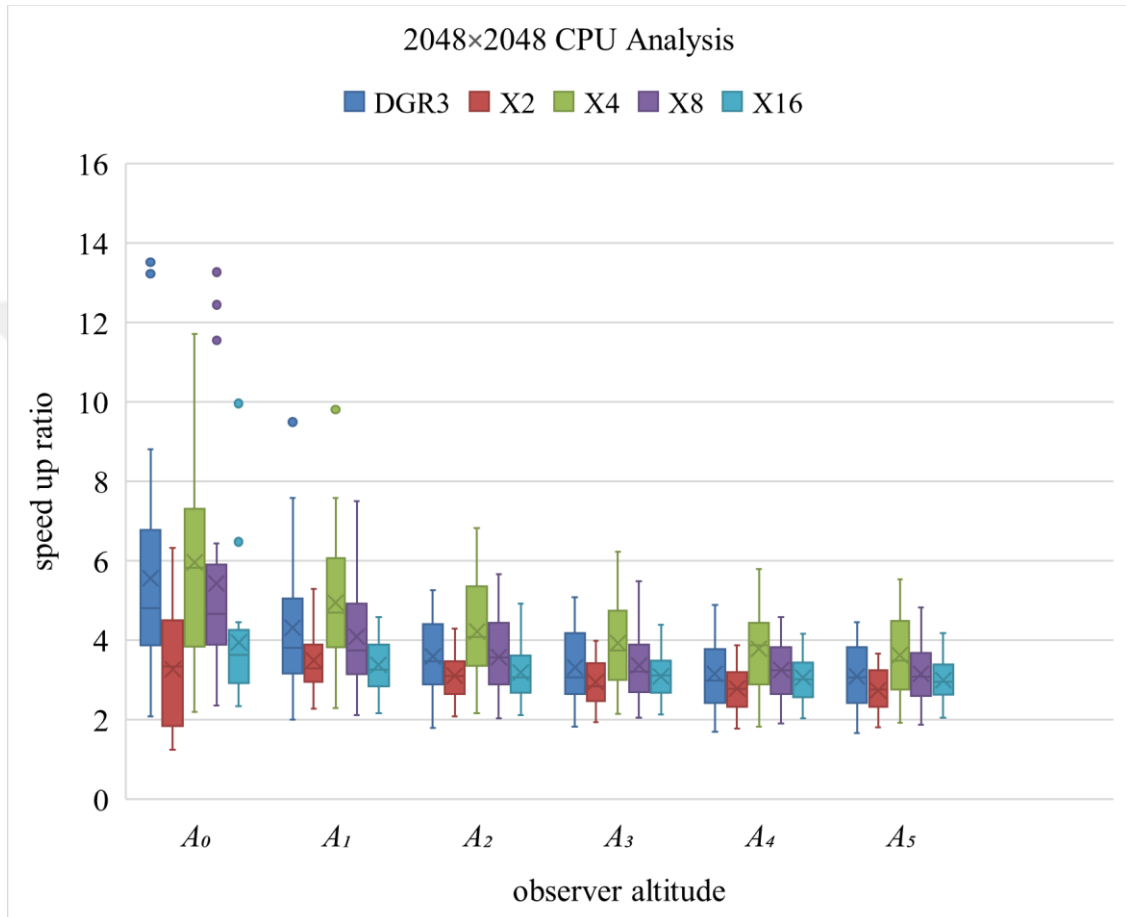


Figure 22: CPU algorithm execution time comparison on 2048x2048 tiles

In Table 10, lower minimum and higher maximum speedup ratios are seen because of the increased analysis area.

Table 10: Execution time statistics of CPU algorithms on 2048x2048 tiles

	DGR3	X2	X4	X8	X16
min	1.61	1.23	1.82	1.87	2.02
max	13.51	6.32	11.70	13.26	9.95
avg	3.72	2.99	4.29	3.72	3.24

However, average speedup ratios start to decrease compared to Table 9 which proves that 1024x1024 tile size is optimal for proposed algorithms. Results of MSA tests shown in Figure 23 also confirm that 1024x1024 is the optimal size for the proposed algorithms. For smaller analysis areas X8, X4 and DGR3 performs similar in terms of computation

time but as we extend the range of vision, the graphs indicate that X4 is the best overall choice.

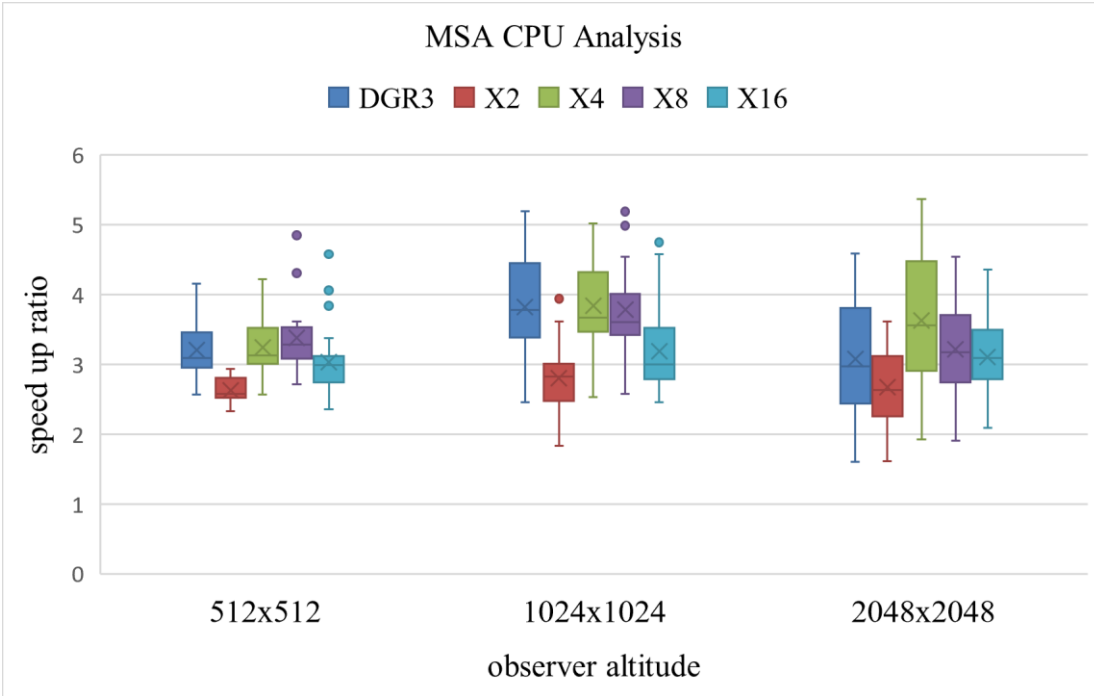


Figure 23: CPU algorithm execution time comparison for MSA

To sum up, experiment results prove that using large point tiles complicates detection of invisible groups while small point tiles cause extra overhead due to processing large data sets. Proposed algorithms perform much better at low altitude levels because phase-1 offer greater speedup compared to phase-2. On the other hand, phase-2 is effective in cases altitude level is increased. The best practice for CPU will be using DGR3 for observers on ground or with low level altitudes. As observer altitude approaches to A_3 or MSA X4 algorithm should be applied.

4.1.2. Accuracy

Approximate methods to compute viewsheds reuse information or makes estimation to reduce the execution time. On the other hand, R3 draws a separate line of sight to each target point and it is accepted as an exact method by researchers (Zalik & Kaučič, 2002). Therefore, the performance comparisons in this study are made with respect to R3.

Table 11 shows the discrepancies (i.e., non-matching point count) between the output of each algorithm and the output obtained by R3 algorithm. According to minimum non-matching point counts, there are cases for proposed algorithms providing exactly same accuracy with R3. Additionally, proposed algorithms avoid any false positive decisions (mistakenly marked as visible) as given in Table 12 thanks to Phase-2. The false negative

decisions (mistakenly marked as invisible) are done in Phase-1 because in slope calculation same distance value is assigned to a tile of points which may lead to wrong minimum and maximum slope value calculation. Therefore, the largest average non-matching point count belongs to X2 which uses the largest processed elevation model.

Table 11: Accuracy of CPU algorithms

	non-matching point count								
	512×512=262144			1024×1024=1048576			2048×2048=4194304		
	min	max	avg	min	max	avg	min	max	Avg
R2	135	11028	5702.14	68	38090	20110.32	875	173843	78584.72
DGR3	0	637	15.55	0	329	27.13	0	883	113.71
X2	1	437	52.99	1	1130	185.68	6	3255	874.55
X4	0	249	8.02	0	329	22.42	0	578	86.55
X8	0	590	6.84	0	91	4.06	0	485	30.71
X16	0	176	1.72	0	67	1.16	0	497	10.37

DGR3 uses 3 types of processed elevation models and accumulates wrong decisions made by Phase-1 of X16, X8 and X4 algorithms. However, average non-matching point count of R2 is still 8.95x of maximum non-matching point count of DGR3 on the smallest analysis area. Besides, DGR3 offers average 15.55 non-matching points compared to 5702.14 of R2 which is a significant difference. As the size of analysis area increases, the distinction between granularity algorithms and R2 becomes larger.

Table 12: Accuracy statistics of algorithms (FN: False Negative, FP: False Positive)

	non-matching point count								
	512×512			1024×1024			2048×2048		
	min	max	avg	min	max	avg	min	max	avg
FN R2	55	5080	2340.78	19	17816	8921.07	416	81436	35096.59
FP R2	80	5948	3361.36	49	21233	11189.25	459	92407	43488.13
FP DGR3	0	0	0.00	0	0	0.00	0	0	0.00
FN DGR3	0	637	15.55	0	329	27.13	0	883	113.71
FP X2	0	0	0.00	0	0	0.00	0	0	0.00
FN X2	1	437	52.99	1	1130	185.68	6	3255	874.55
FP X4	0	0	0.00	0	0	0.00	0	0	0.00
FN X4	0	249	8.02	0	329	22.42	0	578	86.55
FP X8	0	0	0.00	0	0	0.00	0	0	0.00
FN X8	0	590	6.84	0	91	4.06	0	485	30.71
FP X16	0	0	0.00	0	0	0.00	0	0	0.00
FN X16	0	176	1.72	0	67	1.16	0	497	10.37

In Figure 24, we see visual comparison of false positive and false negative decisions of R2 and DGR3 with respect to output of R3. Orange points represent false negative decisions (mistakenly marked as invisible) and blue points represent false positive decisions (mistakenly marked as visible).

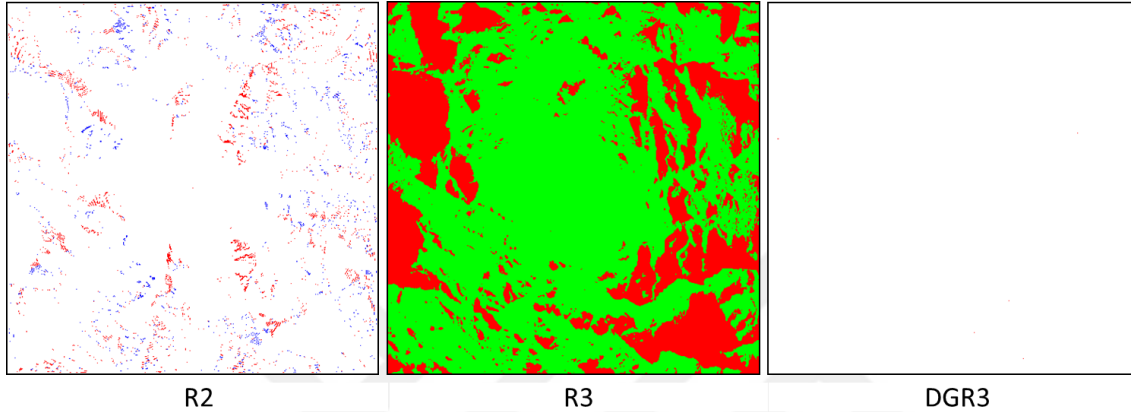


Figure 24: Visual comparison of non-matching points of R2 and DGR3 with R3

Total number of false negative and false positive decisions of R2 for this experiment is 5678 (See Table 13) which is lower than the average value according to Table 11 but the difference in viewshed output looks quite misleading and this accuracy may not be satisfactory for some applications.

Table 13: Accuracy statistics of experiment in Figure 24

	False Negative	False Positive
R2	3674	2004
DGR3	6	0

Proposed algorithms draw a line of sight for group of points and iterate on them. In other words, they apply R3 with larger resolution and all points in the same target group assumed to be on the same line of sight and distance to the observer. Therefore, detection of invisible groups in phase-1 may cause wrong visibility decisions for some of the points within the group. When we increase the number of groups by decreasing the granularity of down scaled elevation models, we see more mistaken visibility decisions. This fact explains average non-matching point count relation between X2, X4, X8 and X16. On the other hand, increasing granularity may cause a wrong decision affecting many points in a larger group and cause wrong decisions to accumulate in a specific region which is an important problem. In Figure 25, we see original output of R3 and wrong visibility decisions of X2 and DGR3. Orange points represent false negative decisions (mistakenly marked as invisible) in output of X2 and DGR3. When we analyze the outputs of algorithms using same approach with different granularities we see wrong decisions of X2 is dispersed unlike DGR3.

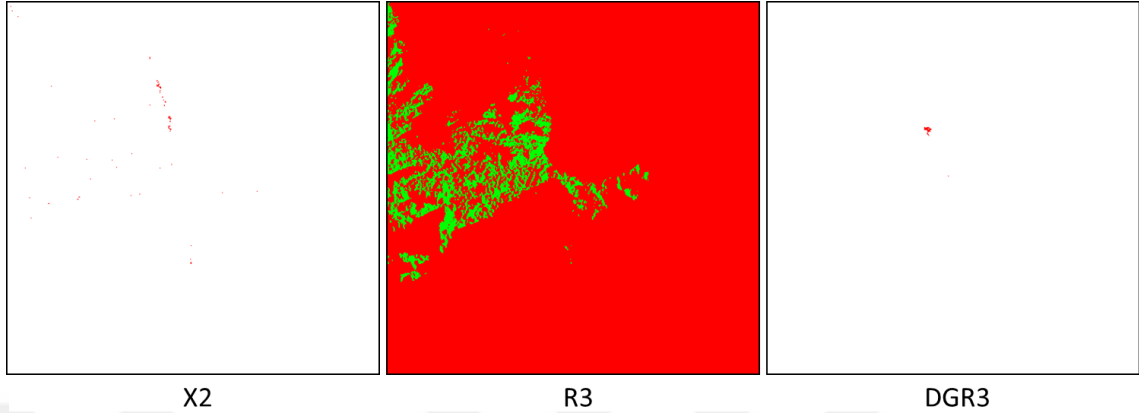


Figure 25: Visual comparison of non-matching points of X2 and DGR3 with R3

Van Kreveld's algorithm is seen as considerable alternative to R3 due to accuracy. The study of Johansson and Lundberg (Johansson & Lundberg, 2016) presents accuracy results of Van Kreveld's algorithm on 2001×2001 tiles. The experiment results show that there are 50346 non-matching points, meanwhile, X2 algorithm offers the worst accuracy with 3255 unmatched points on 2048×2048 tiles (See Table 11). In other words, granularity algorithms offer significant accuracy results compared to Van Kreveld's algorithm.

4.2. Experiments with GPU Implementation

4.2.1. Execution Time

GPU execution time performances of the proposed algorithms are compared with respect to R3. Parallelization of proposed algorithms sketch different execution time graphs depending on the size of the analysis area. Since all threads run in parallel, the amount of serial computations in an algorithm is very effective on the execution time. In this case, execution time of R3 is determined by the visible point which is the most distant one from the observer position. Considering that decreasing observer altitude extends the invisible regions and cause threads to finish visibility computation quickly, R3 provides the best execution time for the minimum altitude level. Therefore, proposed algorithms are slower than R3 at low altitude levels as seen in Figure 26, Figure 27 and Figure 28.

The execution time results of granularity algorithms indicate sequential execution of Phase-1 and Phase-2 cause overhead compared to parallel implementation of R3. Especially on smallest analysis area proposed algorithms are significantly slower than R3 as shown in Figure 26.

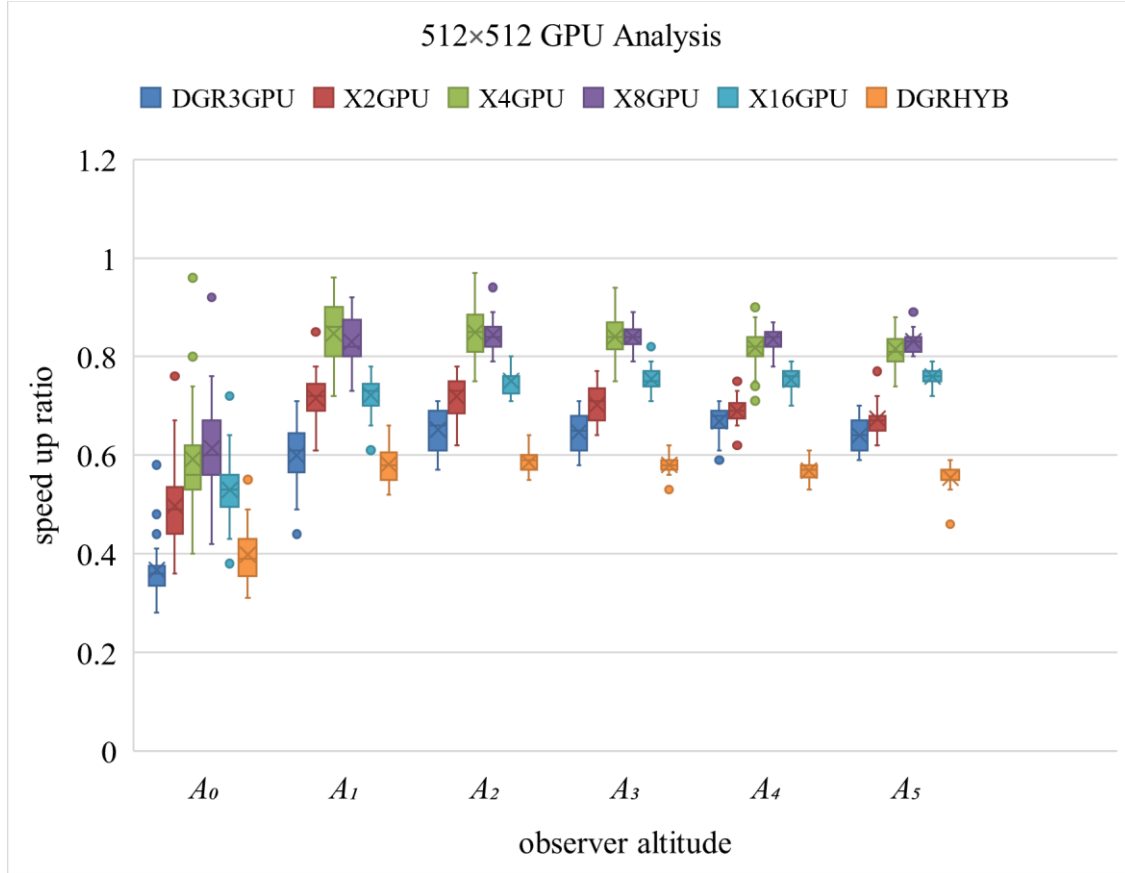


Figure 26: GPU algorithm execution time comparison on 512x512 tiles

Maximum speed up values given in Table 14 reveals that granularity algorithms failed to achieve any execution time improvement. DGRHYB and DGR3GPU are the slowest granularity algorithms because of sequentially repeating Phase-1 besides the execution of Phase-2.

Table 14: Execution time statistics of GPU algorithms on 512x512 tiles

	DGR3GPU	X2GPU	X4GPU	X8GPU	X16GPU	DGRHYB
min	0.28	0.36	0.40	0.42	0.38	0.31
max	0.71	0.85	0.97	0.94	0.82	0.66
avg	0.60	0.67	0.80	0.80	0.72	0.55

According to Figure 27 and Table 15, on 1024x1024 analysis dimensions we see X8 and X4 achieve speedup (See Figure 27) by the increasing workload for a single thread. This is because execution of phase-1 helps X4 and X8 to be affected less from the growing analysis area compared to R3. However, R3 is still faster on the ground and increasing altitude decelerate execution time performance of X4 and X8.

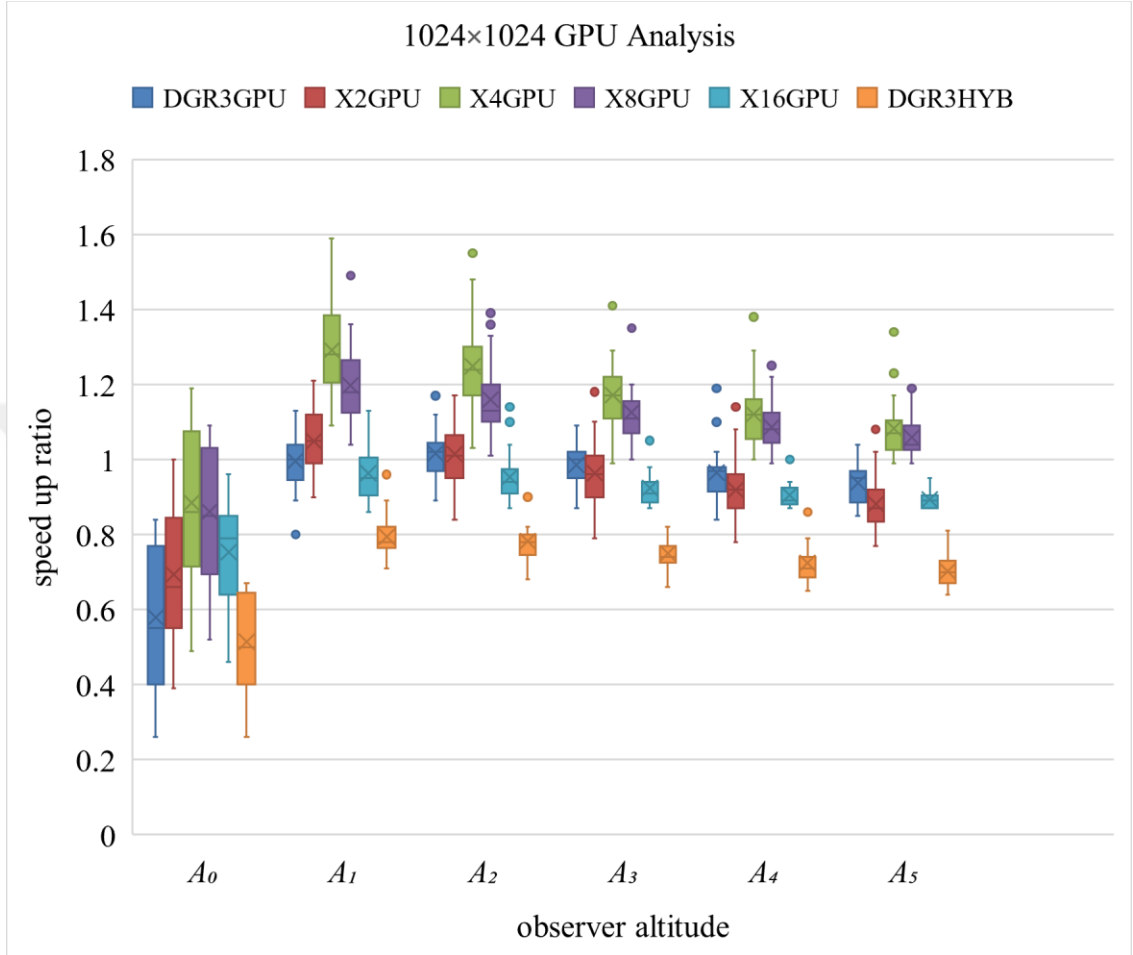


Figure 27: GPU algorithm execution time comparison on 1024×1024 tiles

Moreover, comparison of analysis results in Figure 26 and Figure 27 indicates that DGR3GPU shows improvement by the increased analysis area while DGRHYB is still slowest since it executes phase-1 in CPU.

Table 15: Execution time statistics of GPU algorithms on 1024×1024 tiles

	DGR3GPU	X2GPU	X4GPU	X8GPU	X16GPU	DGRHYB
min	0.26	0.39	0.49	0.52	0.46	0.26
max	1.19	1.21	1.59	1.49	1.14	0.96
avg	0.92	0.91	1.13	1.08	0.90	0.71

In case of 2048×2048 tile size, DGR3GPU benefits more from invisible phase-1 and becomes one of the top three algorithms. We also notice the execution time difference between algorithms is more obvious.

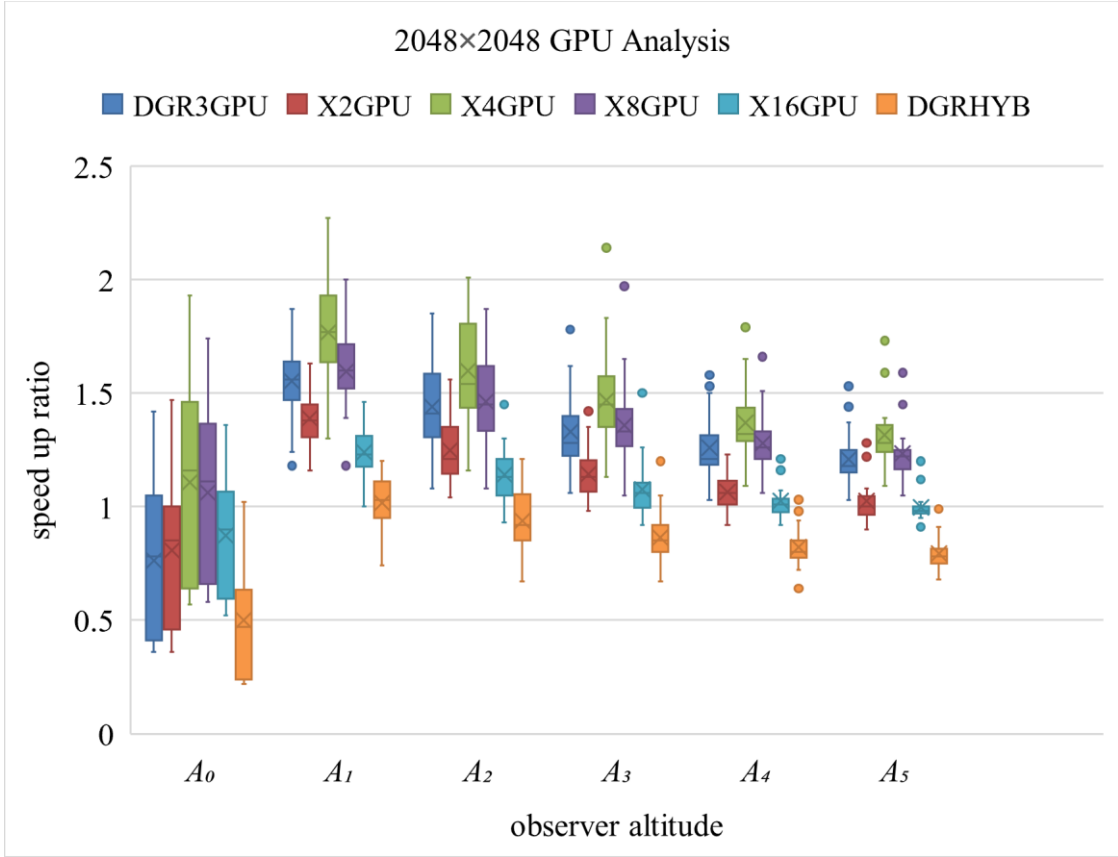


Figure 28: GPU algorithm execution time comparison on 2048×2048 tiles

The execution time graph in Figure 28 shows that granularity algorithms provide improvement only on large analysis areas and X4 is the best algorithm with 1.41x average speed up to 2.27x maximum (See Table 16). The second fastest algorithm is X8 with 1.31x average speed up to 2.0x maximum thanks to 8×8 granularity and DGR3 GPU is the third because of repeating phase-1.

Table 16: Execution time statistics of GPU algorithms on 2048×2048 tiles

	DGR3GPU	X2GPU	X4GPU	X8GPU	X16GPU	DGRHYB
min	0.36	0.36	0.57	0.58	0.52	0.22
max	1.87	1.63	2.27	2.00	1.50	1.21
avg	1.25	1.09	1.41	1.31	1.05	0.81

We see MSA results also support (See Figure 29) that X4GPU, X8GPU and DGR3GPU offer execution time improvement only for 2048×2048 analysis area.

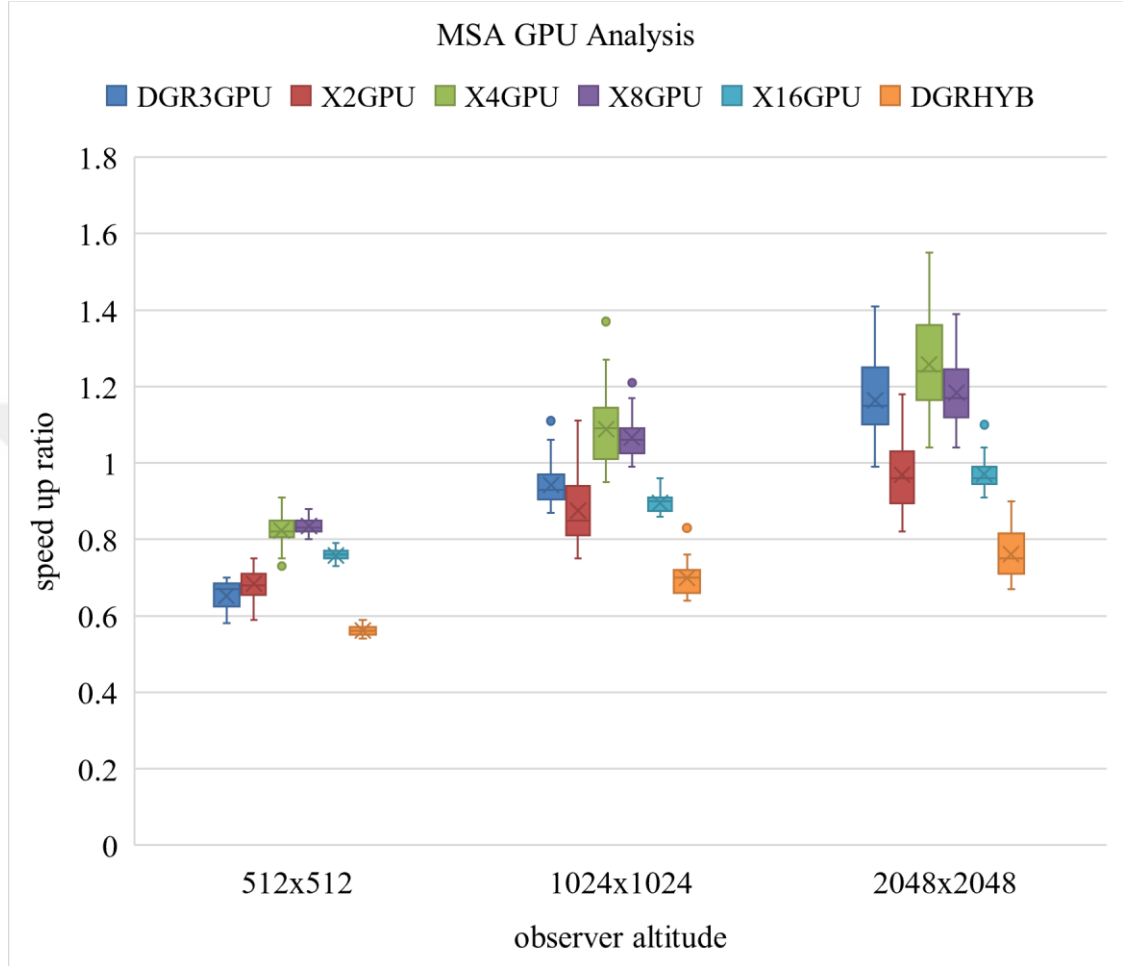


Figure 29: GPU algorithm execution time comparison for MSA

The reason for DGRHYB to be the slowest algorithm is the execution time overhead caused by running phase-1 in CPU. If we compare DGRHYB with CPU version of R3, we see amount of speedup given in Figure 30. The rationale to use hybrid approach can be reducing the workload in CPU while using both processing units in a system.

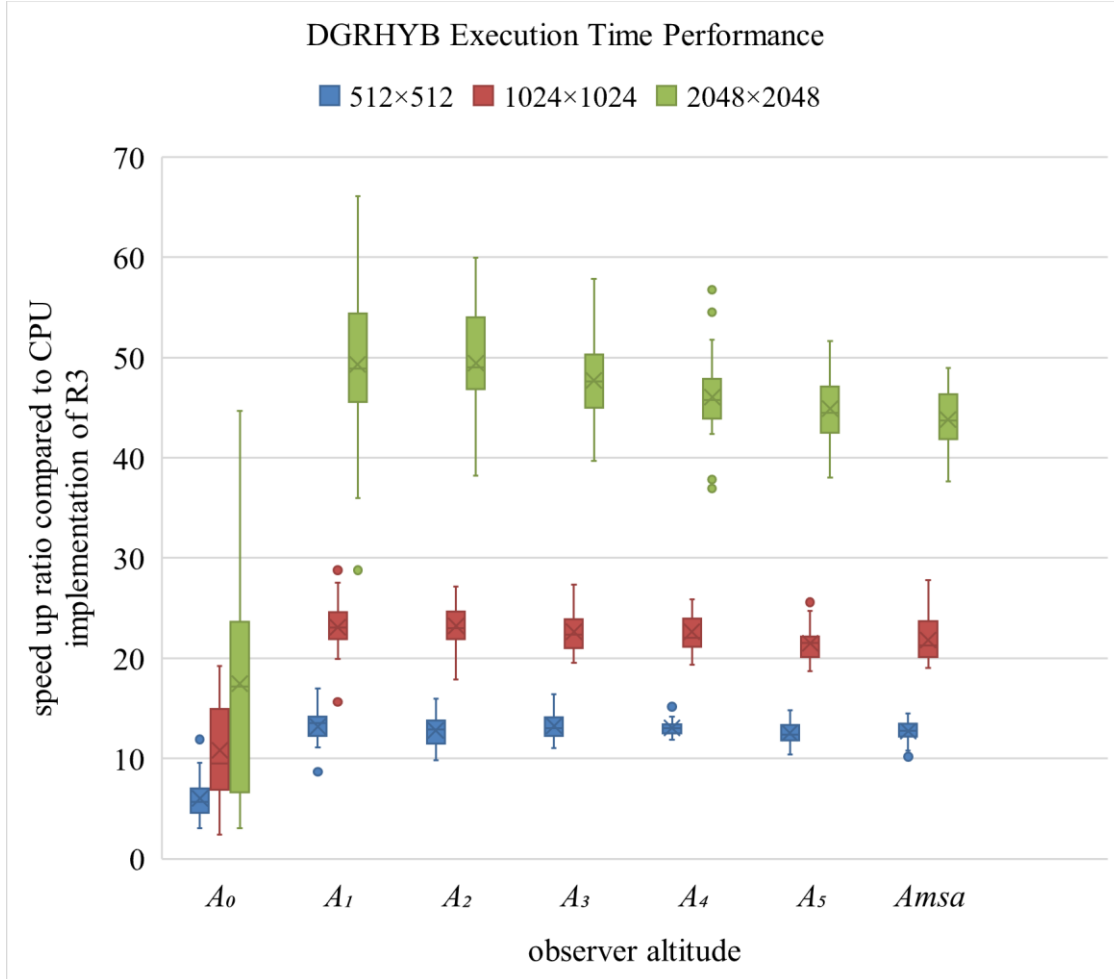


Figure 30: Hybrid algorithm speedup with respect to R3 CPU implementation

Combining all analysis results, we can infer that GPU implementations of granularity algorithms achieve speedup only at altitude levels higher than A_0 for large analysis areas. Although X4 comes forward with best execution time results, X8 is also a nice option for high altitude levels.

4.2.2. Accuracy

GPU implementation provides the same accuracy values since the proposed algorithms are deterministic solutions. However, R2 algorithm acts differently as the visibilities of some points are calculated multiple times. GPU implementation of R2 algorithm is not deterministic in terms of visibility because GPU threads processing different line of sights have intersection points and the path to these intersection points is also different. When we ran R2 algorithm in GPU on the same analysis tile twice, we can see different statistics (See Table 17).

Table 17: Changing statistics of R2 algorithm for same analysis

non-matching point count of R2 GPU implementation		
	False Negative	False Positive
Test 1	1611	2386
Test 2	1589	2213

In CPU implementation this is not a problem since processing is sequential and always in the same order. However, GPU threads running parallel reach the same intersection point in an arbitrary order and the visibility will be determined by the last thread visiting the point. Intersecting line of sights may find different visibility results for intersection points and in real time applications this may cause flickering pixels because of GPU threads racing to overwrite visibility.

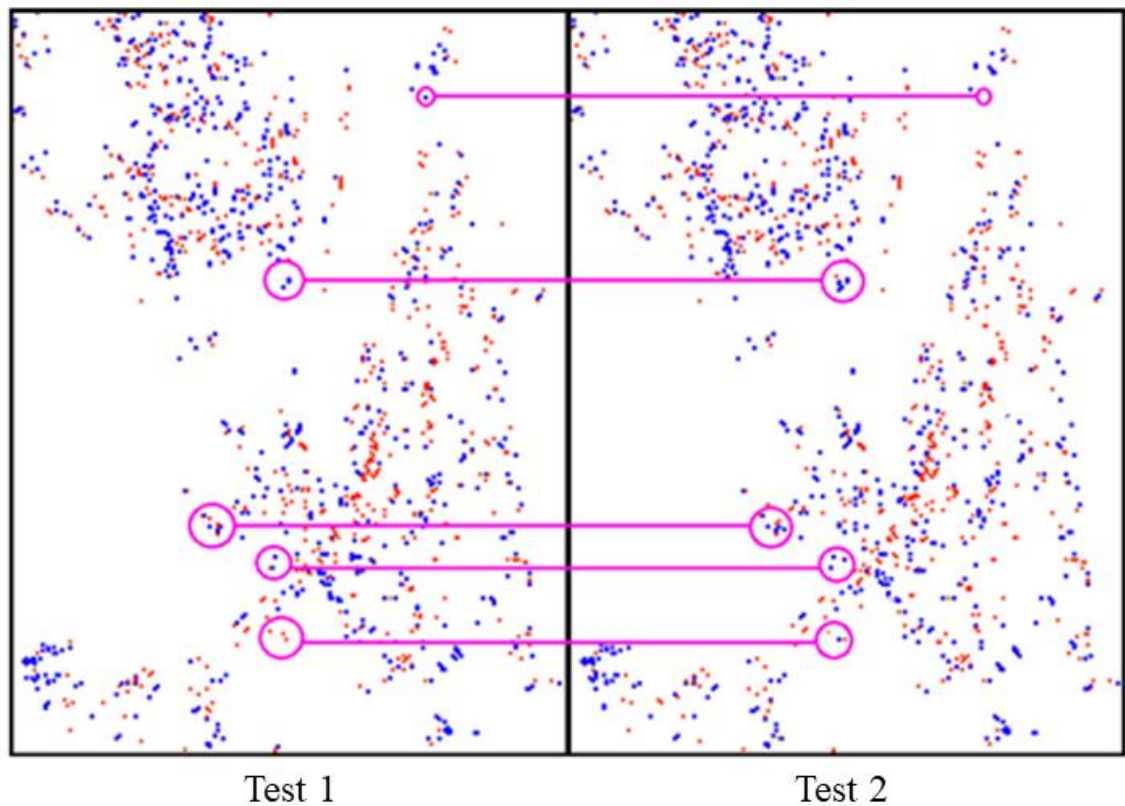


Figure 31: Changing pixels due to racing threads of R2 algorithm in GPU

In Figure 31 wrong visibility decisions of two analysis outputs of R2 are shown. Although we used same elevation data for two analyses, we see different wrong visibility decisions. Orange points in figure represent false negative points (mistakenly marked as invisible) and blue points represent (mistakenly marked as visible) false positive decisions.

4.3. Resource Usage

In Table 18 the resource requirements of each algorithm are given because it is also an important criterion in practical settings. The numbers in red color represents the external requirement to implement the algorithms (See Table 18). Proposed algorithms require maximum 0.67x more space in disk and 1.0x more memory.

For GPU implementation amount of data given in Table 18 is copied to device memory. Therefore, GPU memory requirements are exactly same with CPU implementation. In order to decrease the disk and memory requirements, the algorithm can also be implemented with INTEGER data type.

Apart from R2, Van Kreveld’s algorithm, which is considered as a fast alternative to R3, does not require extra disk space for implementation. However, the algorithm needs extra temporary storage for the runtime. The event lists (enter, center and exit) and active balanced tree require more than 3x extra memory in total (Johansson & Lundberg, 2016). Therefore, proposed granularity methods seem more feasible compared to Van Kreveld’s, if disk space is not a constraint.

Table 18: Resource usage of algorithms

Algorithms on different analysis dimensions	Disk Usage	Memory Usage in KB		
	Elevation (FLOAT,FLOAT)	Slope FLOAT	Remaining Point List (SHORT,SHORT)	Visibility BOOL
X2 512×512	512 + 1024	1024	1024	256
X2 1024×1024	2048 + 4096	4096	4096	1024
X2 2048×2048	8192 + 16384	16384	16384	4096
X4 512×512	128 + 1024	1024	1024	256
X4 1024×1024	512 + 4096	4096	4096	1024
X4 2048×2048	2048 + 16384	16384	16384	4096
X8 512×512	32 + 1024	1024	1024	256
X8 1024×1024	128 + 4096	4096	4096	1024
X8 2048×2048	512 + 16384	16384	16384	4096
X16 512×512	8 + 1024	1024	1024	256
X16 1024×1024	32 + 4096	4096	4096	1024
X16 2048×2048	128 + 16384	16384	16384	4096
DGR3 512×512	168 + 1024	1024	1024	256
DGR3 1024×1024	672 + 4096	4096	4096	1024
DGR3 2048×2048	2688 + 16384	16384	16384	4096
DGRHYB 512×512	168 + 1024	1024	1024	256
DGRHYB 1024×1024	672 + 4096	4096	4096	1024
DGRHYB 2048×2048	2688 + 16384	16384	16384	4096

CHAPTER 5

CONCLUSION AND FUTURE WORK

In this research, accuracy is considered as the primary concern in viewshed analysis. Accordingly, a set of algorithms, which are mainly derived from R3 algorithm, are proposed to improve the processing time performance of R3 algorithm while preserving its accuracy. The proposed algorithms are implemented for both CPU and GPU.

Findings of the experiments confirm that CPU versions of DGR3 and X4 are considerable options to R3 in any conditions. Besides the execution time improvement, proposed algorithms never display an invisible point as visible unlike R2 and Van Kreveld's algorithm. The significant decrease in number of false negative decisions compared to other approximate algorithms can also be a motivation to choose granularity algorithms. On the other hand, GPU application of the suggested approach does not offer as much speedup as CPU but on large analysis dimensions DGR3GPU, X4GPU, X8GPU can offer speedup above 2x with respect to R3GPU. Moreover, R2GPU algorithm is not a reliable option in terms of accuracy due to flicker effect caused by the racing threads.

Resource requirements of an algorithm are also an important issue for practical applications. DGR3, which requires the maximum resource among proposed solutions, can be considered as moderate compared to Van Kreveld's and it can be revised to change the granularity level in runtime with respect to MSA to achieve the best execution time.

To sum up DGR3, X4 and X8 are nice options to R3, if a CPU is assigned for the task. Depending on design constraints and execution time requirements these algorithms can be useful alternatives to R2.

To solve the execution time issues of viewshed analysis, algorithms like Xdraw and R2 are proposed. In this study, the motivation is to improve R3 while keeping the accuracy as high as possible. However, none of the algorithms proposed is as fast as R2. In contrast to the approach proposed in this thesis, future work may be on improvement of R2 algorithm's accuracy while preserving execution time performance.

For a specific range of analysis, wrong decisions of R2 can be anticipated. This is because the procedure to draw the line of sight and compute visibility is certain. Since there is no estimation involved in viewshed computation, the pattern of errors can be analyzed and wrong decision points can be found to develop a method minimizing false visibility decisions. Nevertheless, in order to preserve the execution time performance of R2, error correction method should not change the complexity of the algorithm.

Moreover, since different algorithms offer advantages on different terrain topologies, a method can be developed to switch between algorithms on the fly. Implementation of such an adaptive solution requires real time analysis of the terrain. Depending on changing parameters like roughness and standard deviation of elevation data, we can run the optimal algorithm to provide high accuracy and fast execution time. Additionally, making such an analysis of the terrain can help to find points whose visibility does not change by the vertical or horizontal movement of observer to avoid redundant visibility computations. In conclusion, execution time requirement of real time terrain analysis should be studied and solutions using this method should be compared with R2, R3 and granularity algorithms in terms of performance and practicality.

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APPENDICES

APPENDIX A

R3 Pseudocode

$O_{x,y}$: Position of observer in cartesian coordinates
 A_0 : Altitude of observer
 i : x component of point cartesian coordinate
 j : y component of point cartesian coordinate
 n : Dimension of analysis area
 $Distance_{i,j}$: Euclidean distance between observer and point with indices i and j
 E : Elevation model data
 S : Slope model data
 V : Visibility result data
 $E_{i,j}$: Elevation value of point with indices i and j
 $S_{i,j}$: Slope value from observer to point with indices i and j
 $T_{i,j}$: Target point to determine visibility with indices i and j
 $V_{i,j}$: Visibility result of target point with indices i and j
BresenhamPoint: Position of any point on Bresenham line between observer and $T_{i,j}$

Assign $O_{x,y}$ as $(n/2, n/2)$

// Compute Slope

FOR $\langle i,j \rangle$: $\langle 0\dots n, 0\dots n \rangle$

$S_{i,j} = (E_{i,j} - A_0) / Distance_{i,j}$

END FOR

// Compute Visibility

FOR $\langle i,j \rangle$: $\langle 0\dots n, 0\dots n \rangle$

 IsTargetInvisible = FALSE

 WHILE IsTargetInvisible is FALSE

 Iterate on Bresenham line points from $O_{x,y}$ to $T_{i,j}$

 IF $S_{i,j} < S_{BresenhamPoint}$

$V_{i,j} = Invisible$

 IsTargetInvisible = TRUE

```

        END IF
    END WHILE
    IF IsTargetInvisible is FALSE
         $V_{i,j} = \text{Visible}$ 
    END IF
END FOR

```

R2 Pseudocode

$O_{x,y}$: Position of observer in cartesian coordinates
 A_0 : Altitude of observer
 i : x component of point cartesian coordinate
 j : y component of point cartesian coordinate
 n : Dimension of analysis area
 $\text{Distance}_{i,j}$: Euclidean distance between observer and point with indices i and j
 E : Elevation model data
 S : Slope model data
 V : Visibility result data
 $E_{i,j}$: Elevation value of point with indices i and j
 $S_{i,j}$: Slope value from observer to point with indices i and j
 $T_{i,j}$: Target point to determine visibility with indices i and j
 $V_{i,j}$: Visibility result of target point with indices i and j
 BresenhamPoint: Position of any point on Bresenham line between observer and $T_{i,j}$

Assign $O_{x,y}$ as $(n/2, n/2)$

// Compute Slope

```

FOR <i,j>: <0...n, 0...n>
     $S_{i,j} = (E_{i,j} - A_0) / \text{Distance}_{i,j}$ 
END FOR

```

// Compute Visibility

R2RangeIndices <i,j> = <0, 0...n> U <n, 0...n> U <0...n, 0> U <0...n, n>

```

FOR <i,j>: <R2RangeIndices>
    MaxSlope = Minimum float value
    IsTargetInvisible = FALSE
    WHILE IsTargetInvisible is FALSE
        Iterate on Bresenham line points from  $O_{x,y}$  to  $T_{i,j}$ 
        IF  $S_{\text{BresenhamPoint}} \geq \text{MaxSlope}$ 
             $V_{\text{BresenhamPoint}} = \text{Visible}$ 

```

```

                MaxSlope = SBresenhamPoint
            ELSE
                VBresenhamPoint = Invisible
            END IF
        END WHILE
    END FOR

```

Elevation Data Preprocessing Pseudocode

A_0 : Altitude of observer
i: x component of point cartesian coordinate
j: y component of point cartesian coordinate
n: Dimension of analysis area
G: Dimension of group size
E: Elevation model data
 $E_{i,j}$: Elevation value of point with indices *i* and *j*.
 GroupMax: Temporary minimum elevation found
 GroupMin: Temporary maximum elevation found
 EGMin: *G*×*G* minimum elevation value model
 EGMax: *G*×*G* maximum elevation value model
 Index: Temporary iteration index

```

/* Phase 0 - Prepare Processed Elevation Models */
Index = 0
FOR <i,j>: <0...n, 0...n>
    GroupMax = Ei,j
    GroupMin = Ei,j

    FOR <p,r>: <0...G, 0...G>
        IF Ei+p,j+r > GroupMax
            GroupMax = Ei+p,j+r
        END IF
        IF Ei+p,j+r < GroupMin
            GroupMin = Ei+p,j+r
        END IF
    END FOR

    EGMinIndex = GroupMin
    EGMaxIndex = GroupMax

    Index = Index + 1
    i = i + G
    j = j + G

```

END FOR

Family of Granularity Algorithms Pseudocode

$O_{x,y}$: Position of observer in cartesian coordinates
 A_0 : Altitude of observer
 i : x component of point cartesian coordinate
 j : y component of point cartesian coordinate
 n : Dimension of analysis area
 G : Dimension of group size
 $Distance_{i,j}$: Euclidean distance between observer and point with indices i and j
 $EGMin_{i,j}$: Minimum elevation value in $G \times G$ group with indices i and j
 $EGMax_{i,j}$: Maximum elevation value in $G \times G$ group with indices i and j
 E : Elevation model data
 S : Slope model data
 $E_{i,j}$: Elevation value of point with indices i and j
 $SGMin_{i,j}$: Minimum slope value in $G \times G$ group with indices i and j
 $SGMax_{i,j}$: Maximum slope value in $G \times G$ group with indices i and j
 $S_{i,j}$: Slope value from observer to point with indices i and j
 V : Visibility result data
 $V_{i,j}$: Visibility result of target point with indices i and j
 $T_{i,j}$: Target point or group of points to determine visibility with indices i and j
 D_x : Absolute value of distance between observer and target point along x axis
 D_y : Absolute value of distance between observer and target point along y axis
 R : Points not marked as invisible according to $V_{i,j}$
 xIt : Iterator attribute for x axis
 yIt : Iterator attribute for y axis
BresenhamPoint: Position of any point on Bresenham line between observer and $T_{i,j}$

/* Phase 1 Determine visibility of Remaining Points */

// Compute Slope
Assign $O_{x,y}$ as $(n/(2*G), n/(2*G))$
FOR $\langle i,j \rangle$: $\langle 0 \dots n/G, 0 \dots n/G \rangle$
 $SGMin_{i,j} = (EGMin_{i,j} - A_0) / Distance_{i,j}$
 $SGMax_{i,j} = (EGMax_{i,j} - A_0) / Distance_{i,j}$
END FOR


```

// Compute Visibility
FOR <i,j>: <0...G, 0...G>
    IsTargetInvisible = FALSE
    WHILE IsTargetInvisible is FALSE
        Iterate on Bresenham line points from  $O_{x,y}$  to  $T_{i,j}$ 
        IF  $SG_{Min_{i,j}} < SG_{Max_{BresenhamPoint}}$ 
            FOR <p,r>: <0...G, 0...G>
                 $V_{i*G+p,j*G+r} = Invisible$ 
            ENDFOR
            IsTargetInvisible = TRUE
        END IF
    END WHILE
END FOR

/* Phase 2 Determine visibility of Remaining Points */

// Compute Slope
Prepare R according to V
Assign observer position as (n/2, n/2)
FOR <i,j>: <0...n, 0...n>
     $S_{i,j} = (E_{i,j} - A_0) / Distance_{i,j}$ 
END FOR

// Compute Visibility
xIt = 1
yIt = 1

IF( $O_x > i$ )
    xIt = -1
END IF

IF( $O_y > j$ )
    yIt = -1
END IF

FOR each point  $T_{i,j}$  in R
    IF ( $Dx - Dy > Dy$  AND  $S_{i,j} < S_{i-xIt,j}$ )
         $V_{i,j} = Invisible$ 
    ELSE IF ( $Dy - Dx > Dx$  AND  $S_{i,j} < S_{i-xIt,j}$ )
         $V_{i,j} = Invisible$ 
    ELSE IF  $Dx > Dy$  AND
        (( $V_{i-xIt,j-yIt}$  is Visible AND  $S_{i-xIt,j-yIt} \geq S_{i,j}$ ) OR
        ( $V_{i-xIt,j}$  is Visible AND  $S_{i-xIt,j} \geq S_{i,j}$ ))
         $V_{i,j} = Visible$ 
    ELSE IF  $Dy > Dx$  AND

```

```

        ((Vi-xIt,j-yIt is Visible AND Si-xIt,j-yIt ≥ Si,j) OR
        (Vi,j-yIt is Visible AND Si-xIt,j ≥ Si,j))
            Vi,j = Visible
    ELSE
        IsTargetInvisible = FALSE
        WHILE IsTargetInvisible is FALSE
            Iterate on Bresenham line points from Ox,y to Ti,j
                IF Si,j < SBresenhamPoint
                    Vi,j = Invisible
                    IsTargetInvisible = TRUE
                END IF
            END WHILE
        IF IsTargetInvisible is FALSE
            Vi,j = Visible
        END IF
    END IF
END FOR

```

R3GPU Pseudocode

O_{x,y}: Position of observer in cartesian coordinates
A₀: Altitude of observer
i: x component of point cartesian coordinate
j: y component of point cartesian coordinate
n: Dimension of analysis area
Distance_{i,j}: Euclidean distance between observer and point with indices i and j
E: Elevation model data
S: Slope model data
E_{i,j}: Elevation value of point with indices i and j
S_{i,j}: Slope value from observer to point with indices i and j
T_{i,j}: Target point to determine visibility with indices i and j
V: Visibility result data
V_{i,j}: Visibility result of target point with indices i and j
BresenhamPoint: Position of any point on Bresenham line between observer and T_{i,j}

COPY E to device memory
Assign O_{x,y} as (n/2, n/2)

```

// Compute Slope
SLOPE_CALC_KERNEL
    i = (blockDim.x * blockIdx.x) + threadIdx.x

```

```

        j = (blockDim.y * blockIdx.y) + threadIdx.y
        Si,j = (Ei,j - A0) / Distancei,j
    END SLOPE_CALC_KERNEL

// Compute Visibility
R3_VISIBILITY_KERNEL
    i = (blockDim.x * blockIdx.x) + threadIdx.x
    j = (blockDim.y * blockIdx.y) + threadIdx.y
    IsTargetInvisible = FALSE
    WHILE IsTargetInvisible is FALSE
        Iterate on Bresenham line points from Ox,y to Ti,j
        IF Si,j < SBresenhamPoint
            Vi,j = Invisible
            IsTargetInvisible = TRUE
        END IF
    END WHILE
    IF IsTargetInvisible is FALSE
        Vi,j = Visible
    END IF
END R3_VISIBILITY_KERNEL

COPY V to device memory

```

R2GPU Pseudocode

$O_{x,y}$: Position of observer in cartesian coordinates
 A_0 : Altitude of observer
 i : x component of point cartesian coordinate
 j : y component of point cartesian coordinate
 n : Dimension of analysis area
 $Distance_{i,j}$: Euclidean distance between observer and point with indices i and j
 E : Elevation model data
 S : Slope model data
 $E_{i,j}$: Elevation value of point with indices i and j
 $S_{i,j}$: Slope value from observer to point with indices i and j
 $T_{i,j}$: Target point to determine visibility with indices i and j
 V : Visibility result data
 $V_{i,j}$: Visibility result of target point with indices i and j
 $BresenhamPoint$: Position of any point on Bresenham line between observer and $T_{i,j}$

```

COPY E to device memory
Assign  $O_{x,y}$  as (n/2, n/2)

// Compute Slope
SLOPE_CALC_KERNEL
    i = (blockDim.x * blockIdx.x) + threadIdx.x
    j = (blockDim.y * blockIdx.y) + threadIdx.y
     $S_{i,j} = (E_{i,j} - A_0) / \text{Distance}_{i,j}$ 
END SLOPE_CALC_KERNEL

// Compute Visibility
R2_VISIBILITY_KERNEL
    IF blockIdx.x < 2
        i = (blockDim.x * blockIdx.x) + threadIdx.x
        j = 0
    ELSE IF blockIdx.x > 1 AND blockIdx.x < 4
        i = (blockDim.x * (blockIdx.x - 2)) + threadIdx.x
        j = n-1
    ELSE IF blockIdx.x > 3 AND blockIdx.x < 6
        i = 0
        j = (blockDim.x * (blockIdx.x - 4)) + threadIdx.x
    ELSE IF blockIdx.x > 5 AND blockIdx.x < 8
        i = n - 1
        j = (blockDim.x * (blockIdx.x - 6)) + threadIdx.x
    END IF

    MaxSlope = Minimum float value
    IsTargetInvisible = FALSE
    WHILE IsTargetInvisible is FALSE
        Iterate on Bresenham line points from  $O_{x,y}$  to  $T_{i,j}$ 
        IF  $S_{\text{BresenhamPoint}} < \text{MaxSlope}$ 
             $V_{\text{BresenhamPoint}} = \text{Invisible}$ 
        ELSE
             $V_{\text{BresenhamPoint}} = \text{Visible}$ 
             $\text{MaxSlope} = S_{\text{BresenhamPoint}}$ 
        END IF
    END WHILE
END R2_VISIBILITY_KERNEL

Copy V to host memory

```

Family of Granularity Algorithms GPU Pseudocode

$O_{x,y}$: Position of observer in cartesian coordinates

A_0 : Altitude of observer
 i : x component of point cartesian coordinate
 j : y component of point cartesian coordinate
 n : Dimension of analysis area
 G : Dimension of group size (granularity dimension)
 $Distance_{i,j}$: Euclidean distance between observer and point with indices i and j
 $ExMin_{i,j}$: Minimum elevation value in group with indices i and j
 $ExMax_{i,j}$: Maximum elevation value in group with indices i and j
 E : Elevation model data
 S : Slope model data
 $E_{i,j}$: Elevation value of point with indices i and j
 $SGMin_{i,j}$: Minimum slope value in $G \times G$ group with indices i and j
 $SGMax_{i,j}$: Maximum slope value in $G \times G$ group with indices i and j
 $S_{i,j}$: Slope value from observer to point with indices i and j
 $T_{i,j}$: Target point to determine visibility with indices i and j
 V : Visibility result data
 $V_{i,j}$: Visibility result of target point with indices i and j
 Dx : Absolute value of distance between observer and target point along x axis
 Dy : Absolute value of distance between observer and target point along y axis
 R : Points not marked as invisible according to $V_{i,j}$
 xIt : Iterator attribute for x axis
 yIt : Iterator attribute for y axis
 $BresenhamPoint$: Position of any point on Bresenham line between observer and $T_{i,j}$

```
// Phase 1 - Detection of Invisible Groups
```

```
// Compute Slope
```

```
Prepare R according to V
```

```
Assign  $O_{x,y}$  as  $(n/(2*G), n/(2*G))$ 
```

```
Copy E to device memory
```

```
SLOPE_G_CALC_KERNEL
```

```
     $i = (\text{blockDim}.x * \text{blockIdx}.x) + \text{threadIdx}.x$ 
```

```
     $j = (\text{blockDim}.y * \text{blockIdx}.y) + \text{threadIdx}.y$ 
```

```
     $SGMin_{i,j} = (EGMin_{i,j} - A_0) / Distance_{i,j}$ 
```

```
     $SGMax_{i,j} = (EGMax_{i,j} - A_0) / Distance_{i,j}$ 
```

```
END SLOPE_G_CALC_KERNEL
```

```
// Compute Visibility
```

```
VISIBILITY_G_KERNEL
```

```

    Ti,j = R[(blockIdx.y * gridDim.x * blockDim.x * blockDim.y) +
(blockIdx.x * blockDim.x * blockDim.y) + (threadIdx.y * blockDim.x) +
threadIdx.x
    IsTargetInvisible = FALSE
    WHILE IsTargetInvisible is FALSE
    Iterate on Bresenham line points from Ox,y to Ti,j
        IF SGMini,j < SGMaxBresenhamPoint
            FOR <p,r>: <0...G, 0...G>
                Vi*G+p,j*G+r = Invisible
            ENDFOR
            IsTargetInvisible = TRUE
        END IF
    END FOR
END VISIBILITY_G_KERNEL
Copy V to host memory
Prepare R according to V
Copy R to device memory

// Phase 2 Determine visibility of Remaining Points

// Compute Slope
Assign Ox,y as (n/2, n/2)
SLOPE_CALC_KERNEL
    i = blockDim.x * blockIdx.x + threadIdx.x
    j = blockDim.y * blockIdx.y + threadIdx.y
    Si,j = (Ei,j - A0) / Distancei,j
END SLOPE_CALC_KERNEL

// Compute Visibility
IMPR3_VISIBILITY_KERNEL
    Ti,j = R[blockIdx.x * blockDim.x * blockDim.y) + (blockDim.x *
threadIdx.y) + threadIdx.x]

    xIt = 1
    yIt = 1

    IF(Ox > i)
        xIt = -1
    END IF

    IF(Oy > j)
        yIt = -1
    END IF

    IF (Dx - Dy) > Dy AND Si,j < Si-xIt,j
        Vi,j = Invisible

```

```

ELSE IF (Dy - Dx) > Dx AND  $S_{i,j} < S_{i-xIt,j}$ 
     $V_{i,j} = \text{Invisible}$ 
ELSE IF  $Dx > Dy$  AND
    (( $V_{i-xIt,j-yIt}$  is Visible AND  $S_{i-xIt,j-yIt} \geq S_{i,j}$ ) OR
    ( $V_{i-xIt,j}$  is Visible AND  $S_{i-xIt,j} \geq S_{i,j}$ ))
     $V_{i,j} = \text{Visible}$ 
ELSE IF  $Dy > Dx$  AND
    (( $V_{i-xIt,j-yIt}$  is Visible AND  $S_{i-xIt,j-yIt} \geq S_{i,j}$ ) OR
    ( $V_{i,j-yIt}$  is Visible AND  $S_{i-xIt,j} \geq S_{i,j}$ ))
     $V_{i,j} = \text{Visible}$ 
ELSE
    IsTargetInvisible = FALSE
    WHILE IsTargetInvisible is FALSE
        Iterate on Bresenham line points from  $O_{x,y}$  to  $T_{i,j}$ 
            IF  $S_{i,j} < S_{\text{BresenhamPoint}}$ 
                 $V_{i,j} = \text{Invisible}$ 
                IsTargetInvisible = TRUE
            END IF
        END WHILE
        IF IsTargetInvisible is FALSE
             $V_{i,j} = \text{Visible}$ 
        END IF
    END IF
END IMPR3_ VISIBILITY_KERNEL

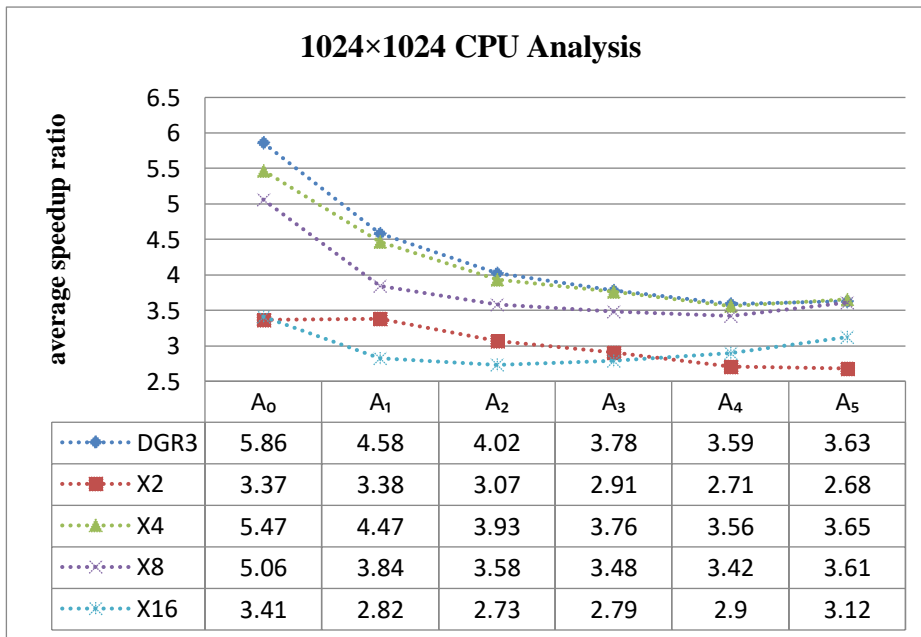
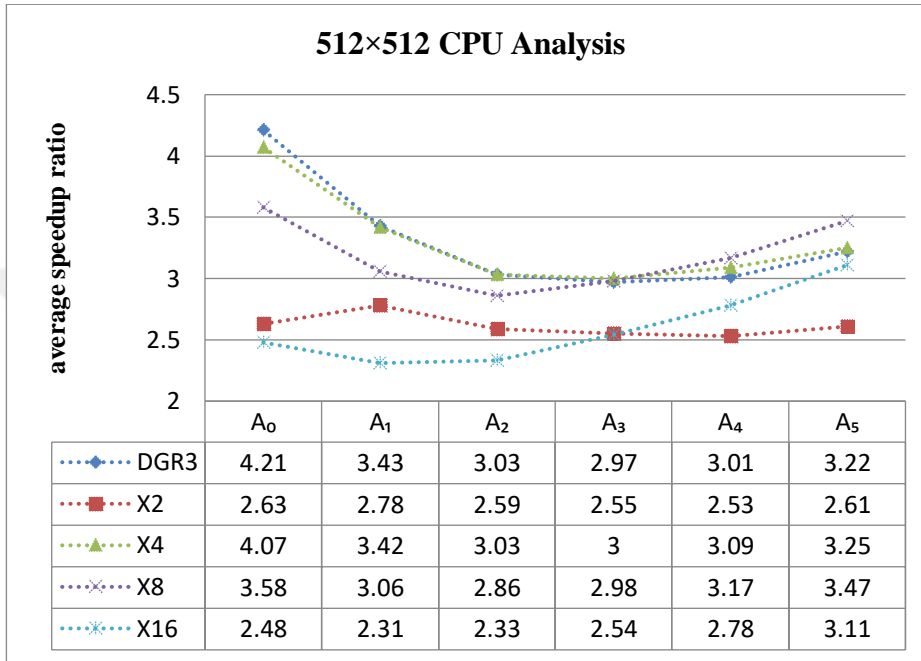
```

APPENDIX B

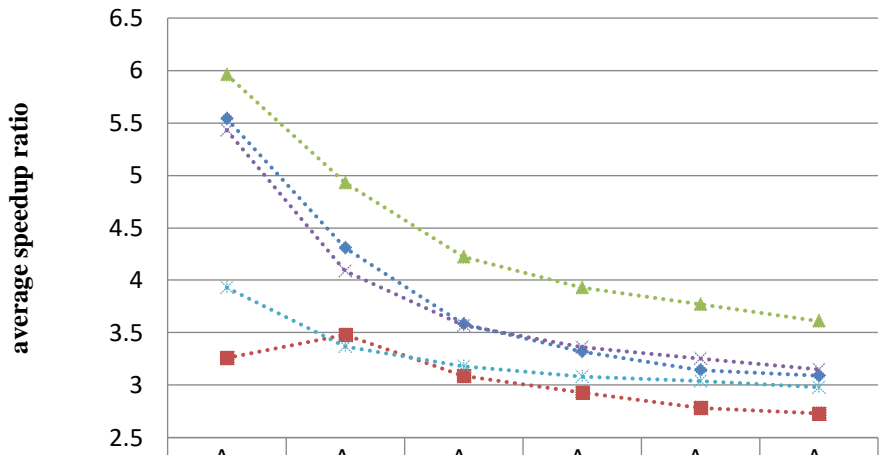
Experiment Tile Statistics

Tile No	512×512		1024×1024		2048×2048	
	Std Dev	Mean	Std Dev	Mean	Std Dev	Mean
1	41.96	824.93	285.81	1564.45	234.67	338.74
2	69.79	893.41	242.15	1187.25	290.41	569.89
3	93.7	853.14	66.87	733.29	274.89	971.56
4	101.04	965.59	57.86	710.04	252.82	1150.64
5	112.98	1096.93	54.49	788.86	112.71	1146.36
6	113.55	1045.2	83.02	823.19	219.12	1189.95
7	130.51	996.93	249.11	946.62	170.55	987.83
8	135.15	809.05	419.54	1327.08	132.66	1007.45
9	140.81	897.45	388.4	1440.12	107.44	1048.94
10	147.39	801.64	282.47	1702.37	175.95	1067.02
11	167.35	770.5	328.26	1310.15	124.2	1122.88
12	169.07	495.57	168.29	985.29	423.26	511.63
13	178.26	1122.35	173.07	1209.07	288.97	296.58
14	179.56	486.8	149.06	891.12	406.72	483.39
15	181.69	721.93	149.03	970.35	222.63	646.1
16	183.23	372.34	253.39	1243.74	217.05	935.28
17	183.42	630.06	299.39	1455.55	242.66	1302.41
18	184.03	940.64	304.54	1006.82	263.52	1207.56
19	209.6	373.34	397.88	1273.84	216.5	1126.66
20	215.02	1073.03	408.17	1103.28	81.04	1050.72
21	217.82	719.3	241.62	1331.93	63.9	991.4
22	226.67	457.35	194.86	923.9	155.8	1044.38
23	234.58	414.42	311.51	1432.01	182.15	1247.32
24	309.56	645.85	226.03	1169.96	223.89	944.14
25	325.14	670.35	289.64	1208.36	186.32	150.18

Experiments with CPU Implementation

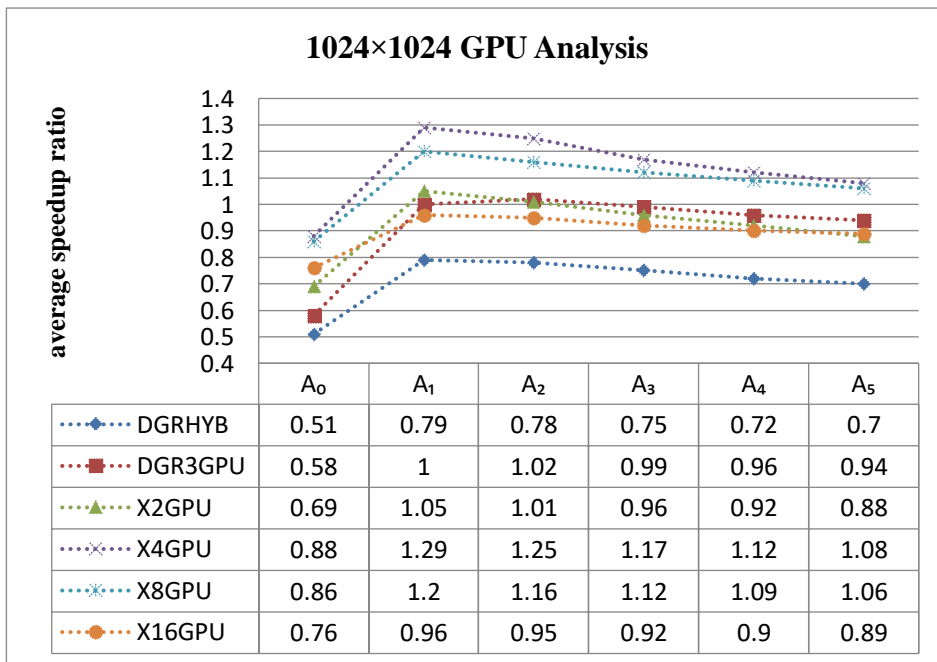
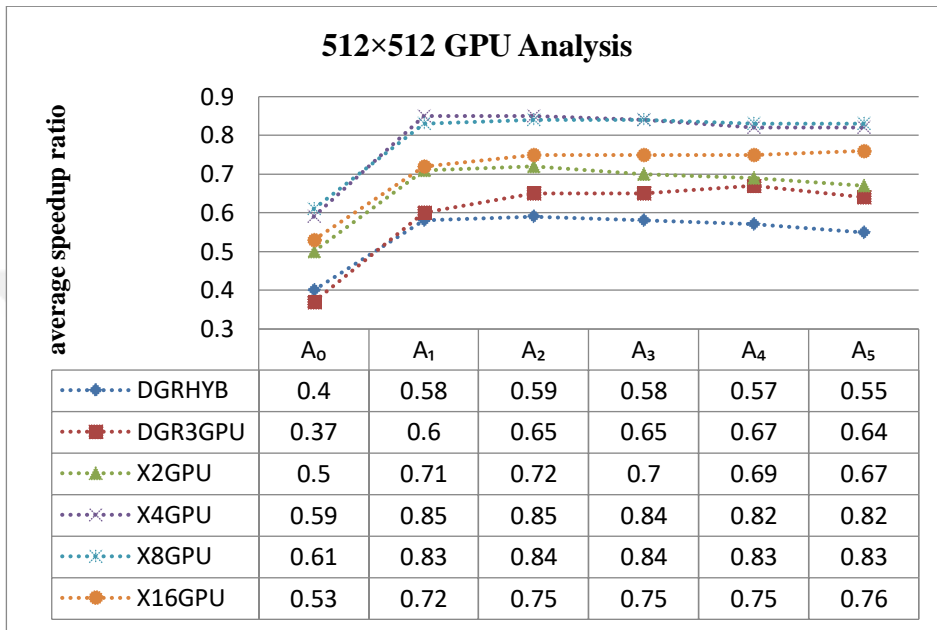


2048×2048 CPU Analysis

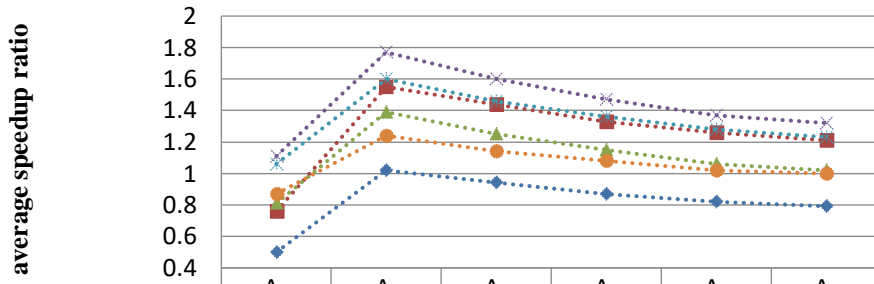


	A ₀	A ₁	A ₂	A ₃	A ₄	A ₅
◆ DGR3	5.54	4.31	3.58	3.32	3.14	3.09
■ X2	3.26	3.48	3.09	2.93	2.78	2.73
▲ X4	5.96	4.93	4.22	3.93	3.77	3.61
× X8	5.43	4.09	3.57	3.36	3.25	3.15
* X16	3.93	3.37	3.18	3.08	3.04	2.98

Experiments with GPU Implementation



2048×2048 GPU Analysis



	A ₀	A ₁	A ₂	A ₃	A ₄	A ₅
◆ DGRHYB	0.5	1.02	0.94	0.87	0.82	0.79
■ DGR3GPU	0.76	1.55	1.44	1.33	1.26	1.21
▲ X2GPU	0.81	1.39	1.25	1.15	1.06	1.02
× X4GPU	1.11	1.77	1.6	1.47	1.37	1.32
* X8GPU	1.06	1.6	1.46	1.36	1.28	1.23
● X16GPU	0.87	1.24	1.14	1.08	1.02	1

Accuracy Results on 512×512 Tiles

512×512 tile no and observer altitude	non-matching point count with respect to R3						
	R2	DGR3	X2	X4	X8	X16	R2GPU
TILE_5_1082.01	135	3	10	3	0	0	127
TILE_5_1192.99	4283	43	71	43	0	0	4127
TILE_5_1305.97	5859	12	41	4	0	8	5764
TILE_5_1418.95	6266	7	68	7	0	0	6178
TILE_5_1531.93	6732	3	63	3	0	0	6676
TILE_5_1644.92	6883	8	54	8	0	0	6828
TILE_5_1720.33	6636	9	48	8	5	0	6592
TILE_18_1026.45	896	107	1	0	74	40	781
TILE_18_1208.48	6154	17	83	17	0	0	6005
TILE_18_1392.51	8575	2	51	2	0	0	8420
TILE_18_1576.54	8572	5	55	5	0	0	8447
TILE_18_1760.58	7610	4	32	4	0	0	7560
TILE_18_1944.61	6822	2	25	2	0	0	6776
TILE_18_1798.13	7424	5	31	5	0	0	7382
TILE_3_772.619	860	0	111	0	0	0	852
TILE_3_864.315	9802	4	112	4	0	0	9682
TILE_3_958.011	10223	0	101	0	0	0	10093
TILE_3_1051.71	9428	0	71	0	0	0	9404
TILE_3_1145.4	8421	2	58	2	0	0	8388
TILE_3_1239.1	7327	3	64	3	0	0	7312
TILE_3_1541.09	4708	1	21	1	0	0	4677
TILE_4_1128.24	474	1	9	1	0	0	412
TILE_4_1227.28	4363	2	48	1	1	0	4285
TILE_4_1328.32	6725	11	77	11	0	0	6601
TILE_4_1429.36	7995	9	89	9	0	0	7926
TILE_4_1530.4	8660	9	69	9	0	0	8583
TILE_4_1631.44	8667	2	43	2	0	0	8592
TILE_4_1597.96	8652	3	43	3	0	0	8595
TILE_22_255.678	2803	14	168	13	1	0	2709
TILE_22_480.349	5883	24	88	16	8	0	5791
TILE_22_707.021	6306	13	46	9	1	3	6269
TILE_22_933.692	6302	12	52	12	0	0	6289
TILE_22_1160.36	5990	7	57	7	0	0	5951

TILE_22_1387.03	5678	6	43	6	0	0	5649
TILE_22_1306.86	5811	5	43	5	0	0	5767
TILE_25_653.042	1900	0	10	0	0	0	1815
TILE_25_976.186	5651	13	34	13	0	0	5570
TILE_25_1301.33	6151	4	46	4	0	0	6121
TILE_25_1626.48	6121	4	27	4	0	0	6126
TILE_25_1951.62	5840	2	28	2	0	0	5812
TILE_25_2276.76	5168	2	18	2	0	0	5132
TILE_25_1657.39	6086	4	34	4	0	0	6088
TILE_20_1213.58	830	3	9	3	0	0	743
TILE_20_1426.6	3842	0	38	0	0	0	3719
TILE_20_1641.62	5865	2	40	2	0	0	5778
TILE_20_1856.63	6158	1	42	1	0	0	6156
TILE_20_2071.65	5955	4	41	4	0	0	5964
TILE_20_2286.67	5626	3	25	3	0	0	5633
TILE_20_1730.12	6152	1	42	1	0	0	6136
TILE_6_1218.55	996	26	9	27	1	0	965
TILE_6_1330.09	5504	17	54	13	4	0	5318
TILE_6_1443.64	6896	7	59	7	0	0	6739
TILE_6_1557.19	7338	3	43	3	0	0	7231
TILE_6_1670.73	7197	2	43	2	0	0	7127
TILE_6_1784.28	6849	2	48	2	0	0	6828
TILE_6_1671.32	7188	2	47	2	0	0	7127
TILE_21_817.056	1003	30	39	18	16	0	924
TILE_21_1032.87	4886	49	59	17	35	0	4765
TILE_21_1250.69	6676	5	63	4	1	0	6556
TILE_21_1468.51	7028	3	57	1	2	0	6965
TILE_21_1686.33	6546	0	36	0	0	0	6550
TILE_21_1904.14	5810	1	24	1	0	0	5824
TILE_21_1541.14	6936	7	49	7	0	0	6869
TILE_2_846.845	1920	50	94	50	0	0	1815
TILE_2_914.637	6735	3	79	3	0	0	6555
TILE_2_984.428	7892	7	91	7	0	0	7763
TILE_2_1054.22	8283	4	94	4	0	0	8216
TILE_2_1124.01	8354	5	68	5	0	0	8290
TILE_2_1193.8	8311	5	69	5	0	0	8295
TILE_2_1451.31	6137	1	36	1	0	0	6142
TILE_1_806.641	521	0	11	0	0	0	472
TILE_1_846.606	8713	4	116	4	0	0	8527
TILE_1_888.571	10665	7	125	4	3	0	10481

TILE_1_930.536	11028	1	100	1	0	0	10924
TILE_1_972.501	10766	0	106	0	0	0	10661
TILE_1_1014.47	10069	1	76	1	0	0	10009
TILE_1_1440.38	3905	0	16	0	0	0	3925
TILE_15_713.188	469	0	3	0	0	0	377
TILE_15_892.874	4633	28	37	15	16	0	4482
TILE_15_1074.56	5249	28	35	4	24	0	5109
TILE_15_1256.25	5540	8	41	8	0	0	5465
TILE_15_1437.93	5870	3	31	0	3	0	5784
TILE_15_1619.62	5811	1	27	1	0	0	5782
TILE_15_1572.78	5826	3	37	3	0	0	5769
TILE_23_285.586	1056	0	94	0	0	0	984
TILE_23_518.168	4536	6	31	6	0	0	4515
TILE_23_752.749	5734	45	78	32	13	0	5732
TILE_23_987.331	5875	52	50	7	11	39	5839
TILE_23_1221.91	5492	3	42	2	0	1	5446
TILE_23_1456.49	5236	0	38	0	0	0	5185
TILE_23_1475.98	5217	0	24	0	0	0	5173
TILE_16_139.641	1506	11	18	10	1	0	1427
TILE_16_320.876	5090	4	39	3	1	0	4978
TILE_16_504.111	5742	6	46	5	1	0	5726
TILE_16_687.345	5498	5	44	5	0	0	5505
TILE_16_870.58	5168	0	41	0	0	0	5176
TILE_16_1053.81	5072	5	41	5	0	0	5085
TILE_16_1115.18	5177	3	34	3	0	0	5181
TILE_24_653.796	906	6	77	0	6	0	840
TILE_24_961.357	5296	9	55	9	1	0	5235
TILE_24_1270.92	7025	9	47	6	3	0	6953
TILE_24_1580.48	6725	0	29	0	0	0	6673
TILE_24_1890.04	5699	2	29	0	2	0	5643
TILE_24_2199.6	4758	1	23	1	0	0	4738
TILE_24_1558.51	6801	0	39	0	0	0	6739
TILE_11_673.651	957	1	14	1	0	0	888
TILE_11_838.996	2806	0	17	0	0	0	2666
TILE_11_1006.34	3927	7	27	7	1	0	3846
TILE_11_1173.69	5812	9	37	9	0	0	5780
TILE_11_1341.03	6959	7	48	7	0	0	6940
TILE_11_1508.38	6843	4	58	4	0	0	6816
TILE_11_1460.27	6918	14	50	8	6	0	6907
TILE_17_766.002	798	0	22	0	0	0	717

TILE_17_947.423	2905	225	57	49	0	176	2789
TILE_17_1130.84	5900	56	65	20	26	10	5792
TILE_17_1314.26	7517	10	52	9	1	0	7411
TILE_17_1497.68	7506	7	49	7	0	0	7420
TILE_17_1681.11	6541	4	43	4	0	0	6495
TILE_17_1405.77	7556	7	45	6	1	0	7472
TILE_13_1247.4	792	27	17	2	1	24	684
TILE_13_1423.66	4627	7	60	7	0	0	4531
TILE_13_1601.92	6662	4	61	4	0	0	6597
TILE_13_1780.17	7165	6	53	6	0	0	7165
TILE_13_1958.43	7520	2	60	2	0	0	7497
TILE_13_2136.69	7109	0	43	0	0	0	7104
TILE_13_1817.21	7346	7	55	6	1	0	7336
TILE_10_951.737	1291	531	141	249	321	0	1249
TILE_10_1097.13	5232	17	93	16	1	0	5084
TILE_10_1244.51	7753	7	66	6	1	0	7701
TILE_10_1391.9	7523	3	64	3	0	0	7492
TILE_10_1539.29	7018	5	50	5	0	0	6991
TILE_10_1686.68	6334	1	43	1	0	0	6307
TILE_10_1583.18	6838	0	39	0	0	0	6821
TILE_14_540.374	558	6	37	5	1	0	532
TILE_14_717.938	7545	5	103	4	1	0	7354
TILE_14_897.502	8257	2	70	2	0	0	8117
TILE_14_1077.07	7379	7	57	7	0	0	7291
TILE_14_1256.63	6749	1	38	1	0	0	6703
TILE_14_1436.19	5971	3	41	3	0	0	5968
TILE_14_1214.91	6862	6	41	6	1	0	6823
TILE_12_474.915	1263	9	157	9	0	0	1218
TILE_12_641.989	4842	14	42	14	0	0	4726
TILE_12_811.063	6858	14	65	5	9	0	6820
TILE_12_980.137	7557	5	62	5	0	0	7553
TILE_12_1149.21	7132	4	44	4	0	0	7097
TILE_12_1318.29	6127	6	43	6	0	0	6122
TILE_12_1227.18	6736	8	49	8	0	0	6720
TILE_7_1196.47	1079	4	154	4	0	0	1040
TILE_7_1324.98	4256	5	35	5	0	0	4183
TILE_7_1455.48	5735	0	52	0	0	0	5641
TILE_7_1585.99	6184	7	34	7	0	0	6074
TILE_7_1716.5	5965	1	29	1	0	0	5888
TILE_7_1847.01	5247	8	35	8	0	0	5192

TILE_7_1681.34	6088	2	38	2	1	0	5986
TILE_8_681.975	1240	1	6	1	0	0	1126
TILE_8_815.124	5208	4	42	4	0	0	5111
TILE_8_950.273	5934	2	53	2	0	0	5840
TILE_8_1085.42	6441	3	41	3	0	0	6407
TILE_8_1220.57	6589	4	50	4	0	0	6591
TILE_8_1355.72	6071	5	41	5	0	0	6079
TILE_8_1492.57	5520	4	32	4	0	0	5532
TILE_19_671.188	645	637	437	156	590	0	628
TILE_19_878.786	6202	32	97	32	0	0	6106
TILE_19_1088.39	6877	6	56	6	0	0	6833
TILE_19_1297.98	6781	17	50	17	0	0	6768
TILE_19_1507.58	6258	2	29	2	0	0	6200
TILE_19_1717.18	5392	0	21	0	0	0	5359
TILE_19_1151.38	6923	3	59	3	0	0	6874
TILE_9_745.292	851	0	14	0	0	0	717
TILE_9_884.102	3180	7	17	7	0	0	2968
TILE_9_1024.91	5586	3	45	3	0	0	5411
TILE_9_1165.72	6365	8	57	8	0	0	6189
TILE_9_1306.53	6964	9	59	9	0	0	6858
TILE_9_1447.34	7271	5	53	5	0	0	7164
TILE_9_1643.28	6603	6	39	6	0	0	6601

Accuracy Results on 1024×1024 Tiles

1024×1024 tile no and observer altitude	non-matching point count with respect to R3						
	R2	DGR3	X2	X4	X8	X16	R2GPU
TILE_5_755.081	6735	3	135	3	0	0	6552
TILE_5_807.567	21642	3	329	3	0	0	20988
TILE_5_862.052	30587	10	410	6	4	0	29961
TILE_5_916.538	35201	11	412	11	0	0	34733
TILE_5_971.024	37372	15	365	14	1	0	36890
TILE_5_1025.51	38090	14	345	10	4	0	37653
TILE_5_1297.05	32799	7	239	5	2	0	32767
TILE_18_869.188	564	0	81	0	0	0	523
TILE_18_1171.73	35982	20	402	18	2	0	35511
TILE_18_1476.27	35715	15	307	14	1	0	35456
TILE_18_1780.81	31049	20	224	20	0	0	30926
TILE_18_2085.35	26026	18	162	17	1	0	26035
TILE_18_2389.89	21988	19	99	19	0	0	21996
TILE_18_2860.57	17167	2	98	2	0	0	17125
TILE_3_693.999	1405	0	30	0	0	0	1184
TILE_3_758.865	9486	13	105	13	0	0	8973
TILE_3_825.731	18076	5	193	5	0	0	17615
TILE_3_892.597	24454	10	293	10	0	0	24121
TILE_3_959.464	28990	9	291	9	0	0	28592
TILE_3_1026.33	32137	21	308	20	1	0	31725
TILE_3_1540.75	30481	9	149	9	0	0	30435
TILE_4_667.636	912	1	5	1	0	0	775
TILE_4_723.497	8492	236	166	236	0	0	8131
TILE_4_781.359	21125	13	250	13	0	0	20583
TILE_4_839.22	28008	9	298	8	1	0	27417
TILE_4_897.081	31962	11	353	11	0	0	31454
TILE_4_954.943	34151	27	319	26	1	0	33667
TILE_4_1244.2	34519	10	261	10	0	0	34270
TILE_22_902.835	867	0	15	0	0	0	808
TILE_22_1095.69	17212	13	287	10	3	0	16989
TILE_22_1290.55	30506	27	403	27	0	0	29943
TILE_22_1485.41	31991	25	325	25	0	0	31602
TILE_22_1680.27	30828	9	302	9	0	0	30573

TILE_22_1875.12	28823	16	267	16	0	0	28635
TILE_22_1935.44	28229	23	231	23	0	0	28085
TILE_25_946.191	1029	3	8	3	0	0	867
TILE_25_1233.83	9586	27	128	21	6	0	9130
TILE_25_1523.46	14627	41	176	37	6	0	14233
TILE_25_1813.1	19611	45	248	36	9	0	19332
TILE_25_2102.74	22993	27	259	25	3	0	22737
TILE_25_2392.37	23929	26	236	26	0	0	23699
TILE_25_2215.07	23844	31	224	31	0	0	23567
TILE_20_1022.51	7668	87	109	83	4	0	7516
TILE_20_1428.68	24879	39	253	38	1	1	24654
TILE_20_1836.85	26663	18	180	16	2	0	26420
TILE_20_2245.02	22594	13	108	9	4	0	22354
TILE_20_2653.19	19334	15	101	14	1	0	19224
TILE_20_3061.35	16816	3	89	3	0	0	16678
TILE_20_2832.31	18149	8	84	8	0	0	17975
TILE_6_818.615	3476	3	151	3	0	0	3438
TILE_6_899.639	23137	75	276	73	2	0	22396
TILE_6_982.662	30132	14	307	14	0	0	29499
TILE_6_1065.69	32044	15	256	14	1	0	31519
TILE_6_1148.71	31966	34	235	26	10	0	31587
TILE_6_1231.73	30611	16	233	16	1	0	30338
TILE_6_1556.39	23987	11	162	6	5	0	23855
TILE_21_1647.85	747	2	11	2	0	0	677
TILE_21_1887.47	19583	25	167	19	6	0	19162
TILE_21_2129.08	22697	13	173	13	0	0	22211
TILE_21_2370.7	22912	19	163	15	4	0	22536
TILE_21_2612.31	23127	13	160	13	0	0	22833
TILE_21_2853.93	22176	10	164	10	0	0	21995
TILE_21_2436.32	23033	12	164	11	1	0	22657
TILE_2_1355.85	68	0	1	0	0	0	64
TILE_2_1596	19116	65	235	45	21	0	18500
TILE_2_1838.16	26789	44	294	24	21	0	26266
TILE_2_2080.31	29203	23	262	22	1	0	28928
TILE_2_2322.47	30004	20	219	19	1	0	29881
TILE_2_2564.62	29213	24	210	19	5	0	29090
TILE_2_2703.94	28446	15	170	14	1	0	28394
TILE_1_1706.36	3064	23	86	23	0	0	2954
TILE_1_1990.17	15697	22	155	21	1	0	15240
TILE_1_2275.98	22130	14	196	13	2	0	21517

TILE_1_2561.79	24977	23	217	16	7	1	24457
TILE_1_2847.6	25736	47	218	45	4	0	25421
TILE_1_3133.41	25370	33	183	32	3	0	25172
TILE_1_2777.12	25529	30	258	29	1	1	25193
TILE_15_785	2120	3	16	3	0	0	1957
TILE_15_932.031	12129	12	93	9	3	3	11849
TILE_15_1081.06	16244	37	195	36	4	0	15857
TILE_15_1230.09	18652	36	197	24	12	0	18332
TILE_15_1379.13	20614	30	176	18	14	0	20250
TILE_15_1528.16	23384	31	235	23	8	0	23024
TILE_15_2083.11	26485	8	198	8	1	0	26355
TILE_23_1239.48	8268	27	1130	22	5	0	8237
TILE_23_1548.99	24421	35	245	31	4	0	24082
TILE_23_1860.5	24397	33	204	27	6	0	24162
TILE_23_2172.01	23950	33	217	31	2	0	23879
TILE_23_2483.52	23220	21	192	18	3	0	23169
TILE_23_2795.04	22055	27	148	27	0	0	22043
TILE_23_2485.08	23203	22	192	19	3	0	23119
TILE_16_1395.93	4282	10	21	6	4	0	3963
TILE_16_1647.32	12300	45	122	39	6	0	11767
TILE_16_1900.71	19959	47	215	45	4	0	19572
TILE_16_2154.1	24719	30	271	29	1	0	24308
TILE_16_2407.5	26546	42	233	32	10	0	26198
TILE_16_2660.89	26607	31	230	31	0	0	26306
TILE_16_2391.8	26531	30	252	24	8	0	26153
TILE_24_830	6231	8	68	6	2	0	5793
TILE_24_1054.03	12571	14	105	14	0	0	12272
TILE_24_1280.06	16246	29	159	23	6	0	16072
TILE_24_1506.09	21367	25	180	25	0	0	21183
TILE_24_1732.12	24063	37	204	28	9	0	23911
TILE_24_1958.15	24636	32	222	22	10	0	24436
TILE_24_2311.12	22960	18	145	18	0	0	22820
TILE_11_1222.62	2430	4	17	4	0	0	2200
TILE_11_1548.87	12892	83	215	60	11	12	12632
TILE_11_1877.13	17204	29	159	14	15	0	16890
TILE_11_2205.38	19171	18	158	12	7	5	18864
TILE_11_2533.64	20011	18	119	17	1	0	19728
TILE_11_2861.89	20006	27	133	26	1	0	19737
TILE_11_2801.98	20089	28	158	26	2	0	19828
TILE_17_1917.87	1315	0	4	0	0	0	1201

TILE_17_2215.25	7119	101	117	24	91	0	6759
TILE_17_2514.64	15197	130	172	50	40	42	14840
TILE_17_2814.02	20483	49	189	39	10	1	20099
TILE_17_3113.41	22690	13	191	12	1	0	22470
TILE_17_3412.8	23821	28	155	28	2	0	23693
TILE_17_2642.31	17450	49	151	34	11	8	17082
TILE_13_1148.56	2753	3	624	3	0	0	2682
TILE_13_1319.63	15742	10	110	7	3	0	15021
TILE_13_1492.7	17813	20	141	20	2	0	17481
TILE_13_1665.76	19961	17	162	17	0	0	19809
TILE_13_1838.83	20877	28	175	21	7	0	20766
TILE_13_2011.9	21173	20	166	19	1	0	21146
TILE_13_2174.49	21364	24	153	16	8	0	21314
TILE_10_1586.7	2577	7	13	7	0	0	2272
TILE_10_1867.16	5236	7	35	7	0	0	4668
TILE_10_2149.63	7478	13	70	13	2	0	7071
TILE_10_2432.1	11650	30	88	27	4	0	11315
TILE_10_2714.56	16432	16	148	15	1	0	16083
TILE_10_2997.03	19388	33	200	32	2	0	19133
TILE_10_2864.56	18189	25	187	25	1	0	17871
TILE_14_812.509	5957	8	62	6	3	0	5750
TILE_14_959.573	16230	46	180	34	14	0	15930
TILE_14_1108.64	19815	31	209	30	2	1	19709
TILE_14_1257.7	23071	42	239	36	6	0	22909
TILE_14_1406.76	24574	85	226	34	3	48	24380
TILE_14_1555.83	24176	90	199	21	2	67	24000
TILE_14_1879.59	21368	19	130	19	0	0	21210
TILE_12_925.406	3961	4	28	4	0	0	3467
TILE_12_1091.7	15003	31	181	21	1	9	14651
TILE_12_1259.99	23913	45	245	22	23	0	23578
TILE_12_1428.28	24731	36	193	32	4	0	24348
TILE_12_1596.57	24058	14	189	13	1	0	23834
TILE_12_1764.86	23172	14	136	13	1	0	22911
TILE_12_2015.92	21693	4	96	4	0	0	21542
TILE_7_746.002	4399	1	55	1	0	0	4241
TILE_7_993.115	12264	64	192	58	6	0	11818
TILE_7_1242.23	16671	38	165	27	11	0	16388
TILE_7_1491.34	20908	35	181	28	11	0	20551
TILE_7_1740.45	22288	34	173	26	7	1	21906
TILE_7_1989.57	23581	30	193	30	0	0	23343

TILE_7_2276.83	23582	26	164	25	1	0	23464
TILE_8_1911.89	2136	0	7	0	0	0	1942
TILE_8_2329.43	15726	39	147	38	1	0	15439
TILE_8_2748.98	25161	35	210	23	13	0	24819
TILE_8_3168.52	28609	42	203	28	15	0	28253
TILE_8_3588.06	29824	22	169	16	6	0	29672
TILE_8_4007.6	29615	17	141	15	2	0	29524
TILE_8_2680.37	23939	29	208	24	11	0	23663
TILE_19_1339.41	9040	6	21	6	0	0	8624
TILE_19_1735.29	16100	18	102	12	3	3	15561
TILE_19_2133.17	20880	21	129	17	4	0	20425
TILE_19_2531.05	23389	51	148	22	29	0	23098
TILE_19_2928.94	24460	20	162	14	6	0	24282
TILE_19_3326.82	24388	21	135	20	2	0	24318
TILE_19_2736.71	24004	22	157	21	1	0	23745
TILE_9_1893.12	4598	329	82	329	6	0	4529
TILE_9_2279.53	19544	19	153	15	4	0	19080
TILE_9_2667.93	24545	26	180	26	0	0	24168
TILE_9_3056.33	27223	21	159	19	4	0	26826
TILE_9_3444.74	27718	25	175	22	3	0	27495
TILE_9_3833.14	27470	11	147	11	0	0	27417
TILE_9_2903.67	26588	21	160	19	2	0	26200

Accuracy Results on 2048×2048 Tiles

2048×2048 tile no and observer altitude	non-matching point count with respect to R3						
	R2	DGR3	X2	X4	X8	X16	R2GPU
TILE_5_1041.02	12531	23	197	23	0	0	11965
TILE_5_1151.73	47849	82	595	49	35	49	45489
TILE_5_1264.44	65824	117	764	114	3	0	63072
TILE_5_1377.15	79604	302	902	192	132	8	76970
TILE_5_1489.86	89714	199	1015	142	54	13	87176
TILE_5_1602.57	98036	101	977	96	6	0	95915
TILE_5_2021.96	111727	99	1039	83	16	0	110314
TILE_18_1078.79	16045	12	149	5	7	0	14799
TILE_18_1340.31	64106	151	698	104	40	12	61590
TILE_18_1603.83	71263	263	692	138	155	497	69523
TILE_18_1867.36	76486	216	841	92	115	25	75430
TILE_18_2130.88	82233	154	881	106	28	73	81563
TILE_18_2394.41	90375	107	884	95	15	2	89845
TILE_18_2898.04	97385	112	860	107	6	21	96925
TILE_3_1629.43	875	588	557	578	351	0	773
TILE_3_1902.32	71470	134	771	111	22	1	70025
TILE_3_2177.22	86772	149	783	127	24	0	84608
TILE_3_2452.11	96922	120	810	99	21	0	94612
TILE_3_2727	102203	114	843	98	17	0	100022
TILE_3_3001.89	105885	105	857	97	13	1	103991
TILE_3_2383.85	94677	132	796	114	18	0	92338
TILE_4_1293.92	3816	93	174	80	13	0	3693
TILE_4_1544.74	65681	80	649	70	10	0	63301
TILE_4_1797.56	76917	83	748	76	7	0	74879
TILE_4_2050.38	84612	81	844	68	16	0	82927
TILE_4_2303.2	90586	102	885	83	19	0	88905
TILE_4_2556.02	94020	95	838	70	26	0	92577
TILE_4_2601.62	94707	97	822	87	11	0	93339
TILE_22_903.02	4324	6	312	6	0	2	4185
TILE_22_1056.82	90585	48	1481	45	3	0	87533
TILE_22_1212.63	124067	59	1815	55	4	1	121068
TILE_22_1368.43	142732	55	1931	46	9	0	140088
TILE_22_1524.24	153312	49	1924	36	13	1	151212

TILE_22_1680.04	156665	38	1827	35	4	0	154983
TILE_22_2437.29	138585	18	1137	17	1	0	138255
TILE_25_11	3129	43	47	14	4	28	2648
TILE_25_195.315	14206	306	473	277	33	0	13063
TILE_25_381.631	25105	205	495	163	55	0	23793
TILE_25_567.946	39557	383	604	215	185	7	38120
TILE_25_754.261	55595	296	768	243	55	0	54059
TILE_25_940.577	71093	225	842	204	24	0	69371
TILE_25_1423.12	77166	128	771	101	32	10	75610
TILE_20_998.58	2832	12	185	7	5	0	2779
TILE_20_1077.63	60305	27	929	27	0	0	57576
TILE_20_1158.67	96473	29	1424	25	5	5	92882
TILE_20_1239.71	119535	33	1598	33	0	0	115886
TILE_20_1320.76	135395	34	1828	29	5	0	132267
TILE_20_1401.8	146510	43	1925	32	12	2	143479
TILE_20_1943.44	156895	35	1419	35	1	4	155679
TILE_6_1359.1	4155	2	117	2	0	0	3953
TILE_6_1576.23	53746	149	779	113	41	47	51946
TILE_6_1795.35	78813	123	1010	103	22	0	76836
TILE_6_2014.48	94775	88	1067	73	16	17	92787
TILE_6_2233.6	106570	92	1120	87	12	1	104712
TILE_6_2452.73	115225	125	1156	110	15	0	113481
TILE_6_2589.82	118821	99	1113	80	20	1	117293
TILE_21_945.968	6588	4	124	4	0	0	6029
TILE_21_1007.87	53602	220	1024	220	0	0	51021
TILE_21_1071.78	87912	14	1493	14	0	0	84830
TILE_21_1135.68	112625	27	1855	27	0	0	109510
TILE_21_1199.58	130483	29	1967	29	0	0	127358
TILE_21_1263.49	144051	30	2113	24	7	0	141152
TILE_21_1548.13	173843	25	2018	25	0	0	171737
TILE_2_770.449	2565	4	23	4	0	0	2037
TILE_2_1058.86	26279	193	372	159	37	0	24912
TILE_2_1349.26	52772	217	672	198	19	0	51210
TILE_2_1639.67	74930	251	847	175	92	4	73201
TILE_2_1930.08	89835	185	879	153	32	12	88160
TILE_2_2220.48	98115	162	986	142	23	0	96563
TILE_2_2064.68	94135	159	947	138	22	0	92598
TILE_1_97.061	17958	494	659	491	3	0	17862
TILE_1_329.736	48230	173	501	109	87	46	46534
TILE_1_564.411	54859	144	626	107	37	9	53484

TILE_1_799.085	62563	218	656	168	51	3	61179
TILE_1_1033.76	69239	161	649	115	46	18	67822
TILE_1_1268.43	76459	168	779	118	54	0	74898
TILE_1_1610.27	88030	202	833	136	78	3	86284
TILE_15_707.713	5017	9	78	8	1	6	4504
TILE_15_928.339	40785	81	415	66	8	13	38297
TILE_15_1150.96	54737	92	558	63	29	4	52342
TILE_15_1373.59	68689	109	710	82	27	0	66599
TILE_15_1596.22	79256	122	855	105	18	0	77338
TILE_15_1818.84	88645	112	990	99	11	2	87100
TILE_15_2188.91	97547	122	998	113	10	0	96333
TILE_23_1256.17	4026	12	88	12	0	0	3747
TILE_23_1436.32	62601	156	763	106	49	4	59932
TILE_23_1618.47	92974	334	1116	156	197	32	90555
TILE_23_1800.62	109102	190	1201	131	61	4	107116
TILE_23_1982.77	115053	148	1236	114	37	0	113547
TILE_23_2164.92	116648	139	1173	102	35	4	115484
TILE_23_2430.68	113424	186	1001	112	55	20	112775
TILE_16_919.464	1953	7	44	0	7	0	1929
TILE_16_1134.51	96004	71	1055	67	4	0	93229
TILE_16_1351.56	113848	82	1069	72	11	0	111536
TILE_16_1568.61	115981	90	990	81	12	0	114306
TILE_16_1785.66	112371	116	934	88	29	2	111367
TILE_16_2002.71	106952	94	834	89	7	4	106242
TILE_16_2226.52	101142	82	818	71	13	0	100703
TILE_24_919.464	1933	7	44	0	7	0	1898
TILE_24_1141.36	96667	72	1048	67	6	1	93876
TILE_24_1365.25	113589	84	1054	73	12	0	111380
TILE_24_1589.14	114901	76	999	71	7	0	113326
TILE_24_1813.03	110581	114	887	95	21	0	109508
TILE_24_2036.92	104689	84	841	83	3	0	103990
TILE_24_2226.52	99663	78	804	67	13	0	99214
TILE_11_1173.48	1468	4	3255	4	0	0	1434
TILE_11_1295.68	55054	240	794	115	129	16	52491
TILE_11_1419.87	80413	104	1056	51	50	3	78168
TILE_11_1544.07	101863	72	1323	52	20	0	100048
TILE_11_1668.26	120497	88	1390	66	23	0	118534
TILE_11_1792.46	130667	75	1480	41	34	5	128782
TILE_11_1974.72	137515	58	1440	55	6	0	135833
TILE_17_992.225	10681	10	89	10	0	0	10111

TILE_17_1232.89	49791	57	496	53	6	0	47412
TILE_17_1475.55	57700	63	548	57	8	0	55658
TILE_17_1718.22	61329	114	545	67	15	37	59713
TILE_17_1960.88	63947	110	602	90	22	0	62688
TILE_17_2203.55	67506	70	621	62	10	1	66593
TILE_17_2751.21	72447	92	643	81	12	0	71855
TILE_13_919.943	18961	62	177	60	6	14	18503
TILE_13_1206.91	50046	600	676	320	465	2	47833
TILE_13_1495.89	61207	883	941	509	485	289	59258
TILE_13_1784.86	74636	667	989	302	258	157	72903
TILE_13_2073.83	76548	224	627	155	36	38	75257
TILE_13_2362.8	74428	115	590	91	25	0	73395
TILE_13_1808.63	75181	627	949	403	175	106	73484
TILE_10_867	8534	6	277	2	4	0	8099
TILE_10_1040.95	49080	41	596	30	11	3	46499
TILE_10_1216.91	64298	66	856	45	22	0	61805
TILE_10_1392.86	74600	78	880	49	29	0	72534
TILE_10_1568.82	82332	55	982	53	2	0	80454
TILE_10_1744.77	95227	133	1147	119	14	0	93736
TILE_10_2094.32	111508	49	1131	39	11	0	110289
TILE_14_69.286	1197	0	11	0	0	0	1140
TILE_14_474.005	77839	62	753	51	11	0	76569
TILE_14_880.723	75189	82	629	75	9	54	74607
TILE_14_1287.44	77229	92	692	77	18	3	76856
TILE_14_1694.16	80682	133	692	120	15	1	80287
TILE_14_2100.88	82201	98	659	71	27	3	81833
TILE_14_2436.65	82695	84	611	78	7	0	82325
TILE_12_69.286	1250	0	11	0	0	0	1193
TILE_12_490.549	77340	65	692	51	14	0	76078
TILE_12_913.813	73370	72	610	56	17	0	72929
TILE_12_1337.08	74679	96	615	70	27	9	74316
TILE_12_1760.34	77100	79	678	66	15	15	76680
TILE_12_2183.6	77932	103	631	82	21	1	77547
TILE_12_2436.65	78652	83	584	77	7	0	78310
TILE_7_921.229	15993	13	289	12	5	0	15536
TILE_7_1089.78	100983	76	1176	64	13	0	98190
TILE_7_1260.33	121405	88	1323	80	10	0	119390
TILE_7_1430.88	126922	63	1222	52	11	0	125469
TILE_7_1601.44	126117	76	1176	70	7	3	125353
TILE_7_1771.99	122539	70	1082	67	3	0	121998

TILE_7_2343.16	105119	31	743	25	7	0	104972
TILE_8_920.73	18866	17	299	14	3	0	18064
TILE_8_1051.39	40399	37	523	36	3	0	38297
TILE_8_1184.06	68465	55	884	40	15	0	66205
TILE_8_1316.72	95422	59	1220	57	2	0	93133
TILE_8_1449.39	114795	57	1513	57	1	1	112538
TILE_8_1582.05	127303	49	1623	49	0	0	125412
TILE_8_1709.08	136895	59	1678	55	5	0	135075
TILE_19_963	1908	0	6	0	0	0	1853
TILE_19_1177.5	85021	65	931	45	21	14	82089
TILE_19_1394	94540	99	899	64	37	15	92318
TILE_19_1610.5	96085	76	806	59	18	0	94376
TILE_19_1827	95533	96	780	67	31	2	94158
TILE_19_2043.5	95498	70	700	49	21	1	94380
TILE_19_2571.84	92009	73	576	55	19	0	91406
TILE_9_1042.97	11285	3	113	3	0	0	10394
TILE_9_1148.41	46395	13	523	13	0	0	44137
TILE_9_1255.85	84789	30	1008	22	8	0	81965
TILE_9_1363.29	105244	46	1223	38	10	0	102776
TILE_9_1470.72	119073	45	1409	45	0	0	116653
TILE_9_1578.16	130535	69	1504	53	17	0	128256
TILE_9_2039.94	146026	43	1256	35	7	2	144568