PRICING DEFAULT AND FINANCIAL DISTRESS RISKS IN FOREIGN CURRENCY-DENOMINATED CORPORATE LOANS IN TURKEY

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ABSTRACT

PRICING DEFAULT AND FINANCIAL DISTRESS RISKS IN FOREIGN CURRENCY-DENOMINATED CORPORATE LOANS IN TURKEY

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The globalization leads to integration of the economies worldwide. As the firms' businesses also get integrated with each other, the financing choices of the firms diversify. Among these choices, the popularity and the share of foreign currency borrowing in total borrowing by non-financial firms increase in Turkey similar to the global developments. The main purpose of this thesis is to price the risks of default and financial distress due to foreign currency denominated loans of non-financial firms in Turkey. The valuation model of foreign currency corporate loans is established by two state variable option pricing model based on the study of Cox, Ingersoll and Ross [28]. In our model, the main risk factors are identified as the exchange rate and the interest rate, which are the state variables of the main partial differential equation whose solution gives the value of the asset. The numerical results are tested for different parameters and for different economic environments. The findings show that interest rate fluctuations are more important both for the default and financial distress option values than the fluctuations in exchange rate. However, the effect of upside movements of exchange rate on the financial distress and default values is sharper than the downside movement effect of interest rate. Furthermore, high loan-to-value (LTV) foreign currency loans result in significantly high financial distress values that cannot be disregarded and can lead to default of the firm. To the best of our knowledge, this thesis is the first study that develops a structural model to evaluate foreign currency denominated corporate loans in an option-pricing framework.

Keywords: Foreign Currency Denominated Corporate Loans, Bankruptcy, Financial Distress, Explicit Finite Difference Method, Capital Structure

ÖZ

TÜRKİYE'DE DÖVİZ CİNSİNDEN KURUMSAL KREDİLERİN İFLAS VE MALİ SIKINTI RİSKLERİNİN FİYATLANDIRILMASI

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Küreselleşme, ekonomilerin dünya genelinde bütünleşmesine yol açmaktadır. Firmaların işleri de bütünleştikçe, firmaların finansman seçenekleri çeşitlenmektedir. Bu seçenekler arasında, Türkiye'deki finansal olmayan şirketlerin toplam borçlanmaları içindeki yabancı para cinsinden borçlanmalarının payı ve popüleritesi küresel gelişmelere benzer şekilde artmaktadır. Bu tezin temel amacı Türkiye'deki finansal olmayan şirketlerin yabancı para cinsinden borçlanmalarından kaynaklanan iflas ve mali sıkıntının fiyatlandırılmasıdır. Yabancı para cinsinden kurumsal kredilerin değerleme modeli Cox, Ingersoll ve Ross'un [28] çalışması üzerine dayandırılaran iki durum değişkenli opsiyon fiyatlama modeliyle kurulmuştur. Bizim modelimizdeki temel risk faktörleri, çözümü varlık değerini veren ana kısmi diferansiyel denklemin durum değişkenleri olan döviz kuru ve faiz oranı olarak belirlenmiştir. Nümerik sonuçlar farklı parametreler ve farklı ekonomik çevreler için test edilmiştir. Bulgular faiz oranındaki dalgalanmaların, iflasın ve mali sıkıntının her ikisinin değeri için döviz kurundaki dalgalanmalardan daha önemli olduğunu göstermektedir. Ancak, döviz kurunun mali sıkıntı ve iflas değerleri üzerindeki yukarı hareketlerinin etkisi faiz oranının aşağı hareketinin etkisinden daha keskindir. Ayrıca, yüksek yabancı para cinsinden kredi/değer oranları göz

ardı edilemeyecek değerleriyle mali sıkıntıya neden olmaktadır ve firmanın iflas etmesine yol açabilir. Bildiğimiz kadarıyla, bu çalışmanın katkısı, yabancı para cinsinden kurumsal kredilerin bir modelini yapısal opsiyon fiyatlama modeli çerçevesinde geliştiren ve değerleyen ilk olması açısından ayırt edilir özelliktedir.

Anahtar Kelimeler: Döviz Cinsinden Kurumsal Krediler, İflas, Mali Sıkıntı, Açık Sonlu Farklar Yöntemi, Sermaye Yapısı

To my family,

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CHAPTER 1

INTRODUCTION

1.1 Introduction

Financing choices of the firms play an important role in their capital structure as these choices include certain risks. Types of financing that a firm can choose differ along with the risks and benefits associated with them. Traditional financing types are categorized into two groups; namely, debt financing and equity financing. Debt financing can be defined as borrowing money to pay back to the lending party. Loan origination and bond issuance are the two commonly used types of debt financing. Corporations often issue debt contracts. As the interest payment on debt is an expense to produce an income or benefit, it is tax deductible. In particular, tax-deductible business expenses are subtracted from a company's income before it is subject to taxation. By reducing taxable income, these deductions reduce a company's tax liability and thus improving its net income. Another advantage of debt financing is that the lender cannot claim an ownership unless the firm does not pay its obligations. The second traditional financing is the equity financing in which the money lent in exchange for ownership in a company. In case financing needs occur for the company, involving investors to the business may end with loss of control. Firms generally choose equity financing for their startups or when they need to raise additional equity capital to compensate the existing debt. Reports on the corporations' choices of financing sources, reveal that non-financial firms mainly use external financing sources, like loan-taking or bond issuance rather than equity financing [1].

1.2 General Overview of the Credit Market

In recent years, as the business investments grew, the corporate borrowing has expanded rapidly. Economic and financial globalization and the expansion of world trade have had effects on both the financing choice of the firms and the choice of the currency of denomination in corporate debt issues. Over the time the share of foreign debt in corporate borrowing has considerably increased. Specifically, in 1983, US firms had foreign currency denominated debt of around $1 billion which increased to $62 billion in 1998 [2]. In the European area, there is also an increasing trend in issuing foreign currency denominated credits especially for the private sector credits [42]. In addition to US and EU countries, in developing countries foreign currency bond issuance and bank borrowing by corporations rose from $81 billion in 2002 to $423 billion in 2007. Examining the external corporate borrowing in regional base, it is seen that the developing countries raised their share in corporate borrowing from external sources in all regions defined by the World Bank. According to World Bank's Global Development Finance 2009 report [3], Europe and Central Asia, including Turkey, had the largest share of the foreign currency debt in 2007. In Europe and Central Asia region corporate borrowing rose to $197 billion in 2007, from $19 billion in 2002. Three main non-financial sectors that use the biggest portion of foreign debt among the developing-country corporations are oil & gas, telecommunications, utility & energy sectors.

In align with global developments during the last decade, borrowing in foreign currency has also increased in Turkey. The share of foreign exchange loans (FX-loans) taken by corporate sector within the total cash loans was 57% in 2005 which increased to 64% in 2008. This ratio went down as a result of the exchange rate movements and the 2008 economic crisis but stayed around 60% levels. Due to the amendment to Decree No. 32 in June 2009, when firms shifted their credit utilization towards the domestic market. Over the period from 2008 to 2010, foreign currency borrowing from domestic sources continued to increase from $46 billion to $52 billion, whereas foreign borrowing from abroad decreased to $89 billion from $100 billion. Similar to the global FX-borrowing trends in sectorwise, among ten sectors listed on the Istanbul Stock Exchange (ISE), Electricity, Gas and Water Sources sub-sector use mostly FX-loans. FX-loans-to-total loans ratios for those sectors are 90.9%, 93.3% and 94.6% in 2008, 2009 and 2010, respectively [4].

The share of FX-loans of corporations in total are increasing significantly both in Turkey and in almost all economies around the world. Foreign currency debt financing affects the economic exposure of the firms. Economic exposure is defined as the long-term sensitivity of a firm's cash flows to exchange rate changes. A foreign currency transaction is exposed to the risk of fluctuations in exchange rates. A considerable amount of researches have been made to investigate the motives lying under the FX-loans issuance by non-financial firms. The results suggest that the two main reasons to issue foreign currency denominated debt are to hedge the foreign operations and to benefit from lower financing costs. In addition, transaction costs of financing, arbitrage differences in tax rates and credit-ratings of firms are the other reasons for the issuance of foreign currency denominated debt.

Although most countries have regulations for banks to limit their foreign exchange exposure, banks are still indirectly exposed to that risk due to currency mismatches on their clients' balance sheets. It is important to note, worldwide standards for the regulation of issuing foreign currency denominated debt do not exist for the non-financial corporations. However, considering the potential risks some countries put limitations on FX-loans especially for the non-financial firms. For instance, with amendment to Decree No. 32, Turkey also puts some restrictions on foreign currency borrowing both at the household and at the corporate level.

Existing academic literature investigated the main reasons for issuing FX-loans. Furthermore, the possible financial distress to firms that is created by exchange rate fluctuations is measured according to stock price performance and operating performance by statistical methods.

1.3 Motivation for the Research

For the companies that are heavily financed by FX-loans, during the life of the loan, the decrease in borrowing rates and depreciation in domestic currency are the two important risks to consider. The share of foreign currency denominated corporate loans in non-financial firms' total borrowing is increasing in recent years despite the risks created by foreign exchange exposure and financing costs. These risks are especially important because of their potential effects on capital structure of the firms. Research on credit risk valuation appears in various works in the existing literature. However, to our knowledge, there has not been a study of pricing of foreign currency loans and related default and financial risks in a contingent claims framework.

The main objective of this thesis is to develop a contingent claims approach (a structural model) to evaluate (price) both the foreign-currency risk and the interest rate risk of FX-loans at the corporate level. In other words, this thesis investigates the financial distress and default risk of non-financial corporations which are heavily financed by FX-loans. In this framework, the present study aims at setting the general characteristics of the valuation functions for the FX-loan and its related assets. Explicitly, the values of future payments, financial distress and default option, in line with the Turkish corporate credit market conditions.

Our research is developed based on these motivations and contributes to the existing literature in three ways. First of all, this study organizes the various theories into a single framework discussing foreign exchange rate exposure information at firm level, and develops a model for pricing the foreign exchange rate denominated loan pricing using option based approach. Secondly, in the extant literature two state variable contingent claims approaches in credit pricing models consider asset value of the firm and interest rate as the underlying state variables. Introducing the exchange rate as another state variable to these models requires constructing a partial differential equation (PDE) with three state variables. Making simplifying but financially reasonable assumptions, we constructed a PDE with two state variables, which are exchange rate and interest rate. Finally, in existing studies, two state variable option based loan pricing methods use CIR process to model the interest rate and Brownian motion to model the asset price. Therefore, in those methods, the main PDE, whose solution gives the asset value, is constructed with two state variables following a CIR process and a Brownian process. In contrast, in our model both underlying variables follow Cox Ingersoll Ross (CIR) process and the main PDE, whose solution gives value of asset, differs from its counterparts in the literature.

1.4 Research Content and Methodology

Various techniques are developed to measure and manage credit risk. Credit risk pricing techniques that are classified in three categories are; historical method, intensity based method, and asset based (contingent claims approach) method [32]. Historical method is mostly used by rating agencies. The default probabilities are calculated according to the firms' defaults that occurred in the past and the risk is measured by transition matrices indicating the change from one rating category to another. In intensity based approach, which is also known as reduced form model, default time is modeled with stopping time of a hazard rate process. The main shortcoming of these methods is that the models do not explain why default occurs and default is modeled as an unexpected event. In contingent claims approach, the default event is related with the capital structure of the firm and unlike other methods this method explains why default occurs. Contingent claims approach is also called as structural model in literature.

The general valuation framework used in this work is based on Cox, Ingersoll and Ross's structural model [28]. The corresponding valuation equation provides the value of the assets under study, given the terminal and boundary conditions of the problem. In the present study, a two state variable version of the model is used to cope with the two natural sources of uncertainty associated with the FX-loan, term structure risk and exchange rate risk. All changes in the term structure of interest rates are assumed to be driven by the evolution of the spot interest rate. In modeling the financial distress for the firms, the distress threshold is identified according to the financial leverage ratios of the firms where financial leverage and highly leveraged firm definitions are used following Opler and Titman's work [26].

In our calculations, we first determine the terminal and boundary conditions for each asset studied. Given these terminal and boundary conditions, it is possible to use the model to price the assets for all the other moments in time, with the exception of those when the terminal conditions do not apply. The closed form solution for the model is probably impossible to obtain. Therefore, it is necessary to use a numerical solution technique. The nature of the problem brings on the choice of an explicit finite difference method. Using this methodology, a series of simulation results are generated which identifies the sensitivity of the different assets to changes in various parameters of the model.

1.5 Organization of the Thesis

The organization of this thesis is as follows:

Chapter 1 provides an introduction to the whole work. It gives an intuitive notion of the importance of the foreign currency denominated corporate loans and credit markets, introduces the main purposes of the research and presents the methodological approach used, the content of the thesis, its structure and organization.

Chapter 2 firstly reviews the developments and growth of current foreign currency credit market, academic literature on foreign exchange exposure of firms. This chapter also presents the motives behind issuing foreign currency denominated debts, main risks associated with that type of debt financing and the contingent claims valuation techniques of debts and related products.

In Chapter 3, the contingent claims valuation model that is used to price the loan, future payments, financial distress and default option is developed. As the valuation model does not have a closed-form solution we use numerical methods to solve it.

Chapter 4 presents the transformations and the overall procedure that is required in order to reach the numerical solution of the problem.

In Chapter 5, several scenarios for the values of the key parameters, that are interest rate volatility, exchange rate volatility, correlation coefficient, loan-to-value (LTV) ratios and financial distress barrier, of the model are studied and conclusions about the values of the different assets under study are explained in both financial and mathematical perspectives.

Finally, Chapter 6 concludes the thesis identifying the most important contributions and suggesting areas for further research.

CHAPTER 2

LITERATURE REVIEW

2.1 Introduction

This Chapter summarizes the existing studies related to foreign currency borrowing by non-financial firms. the first section presents a synopsis of the foreign exchange loan market according to financial reports of the countries. the second section summarizes the main studies that explain the motivations of issuing foreign currency denominated debt. The third section provides a discussion on the risks created by foreign currency debt issuance. Finally, the last section discusses credit-pricing methods.

2.2 Foreign Exchange Loan Market

As the investments grew and businesses get integrated firms evaluate different types of financing opportunities internationally. The evolution of the shares of foreign currency denominated debt issuance in corporate borrowing is significant over the world.

The data for foreign debt contracted by developing-country corporations, taken from web-publications of The World Bank, shows that in developing countries foreign currency bond issuance and bank borrowing by corporations rose from \$81 billion in 2002 to \$423 billion in 2007. As the global financial disorder increased, it fell to \$271 billion in 2008. In 2002, 72% of the external borrowing by developing-country corporations was bank loans and this ratio has risen to 85% in 2008. Weighted portion of the total external debt consists of medium and long-term contracts. The share of the total medium and long-term external debt held by developing countries reached about 50 percent in 2008, up from 5 percent in 1989. Oil & gas, telecommunications, utility & energy sectors are the three main non-financial sectors that use the big portion of foreign debt among the developing-country corporations. Although the total borrowing is decreased from 2007 to 2008, private sector increased its share in external corporate borrowing to 75% in 2008 from 70% in 2007 [3].

In regional base, the share of foreign borrowing increases in all regions of the developing countries. Turkey is located in Europe and Central Asia region. The largest share of the foreign currency debt in 2007 is issued in Europe and Central Asia (ECA). Also in that region corporate borrowing rose to $197 billion in 2007, from $19 billion in 2002. South Asia region has realized the largest percentage increase in corporate borrowing from 2002 to 2007, which is 10 times more than its value in 2002. As compared to other regions, the rise in FX corporate borrowing in Latin America & the Caribbean and in East Asia & the Pacific was relatively humble. In 2008, foreign currency corporate borrowing dropped in all regions, except the Middle East & North Africa [3].

In Turkey borrowing in foreign currency has also increased in accordance with the foreign currency borrowing trends in the world. According to the 2009 and 2010 Financial Stability Reports of CBRT, the share of foreign currency loans within the total cash loans was 58% in 2006. This ratio was 64% in 2008 but went down as a result of the exchange rate movements and the 2008 economic crisis but stayed in 60% levels in 2009. Foreign currency borrowings of the corporate sector was $84.4 billion in 2006 which increased to $146.4 billion in 2008 decreased to $142 billion at the end of 2009 and to $141.7 billion in March 2010. Following the amendment made to Decree Number 32 on June 16, 2009, loans obtained from foreign branches and affiliates of banks established in Turkey and foreign currency loans obtained from foreign banks went down by $3.7 billion and $4 billion, respectively by March 2010. As a result of the amendment, firms have decreased the amounts of loans that they use from abroad and have chosen for domestic banks to issue FX loans. During the period 2008 to 2010, foreign currency borrowing from domestic sources continued to increase from $46 billion to $52 billion whereas foreign borrowing from abroad decreased to $89 billion from $100 billion. As a matter of fact, decrease in the total amount of FX loans mostly due to the decrease in the portion of FX loans obtained from foreign countries [4].

For the 10 sectors listed on the ISE the share of FX corporate loans in total corporate loans is shown in 2010 March Financial Stability Report of CBRT. Among these sectors Transportation, Storage and Communication and Electricity, Gas and Water Sources sectors continued to raise their shares of FX loans. And Electricity, Gas and Water Sources sector use mostly FX loans where its FX loans to Total loans ratio is 90.9%, 93.3% and 94.6% in 2008, 2009 and 2010 respectively [4].

The main terms determining the features of foreign currency contracts are maturity structure and the borrowing rate. According to May 2009 financial Stability Report of CBRT 70% of the foreign currency borrowing by corporate sector have maturities medium and long term. Electricity, Gas and Water Sources sector's medium and long term foreign borrowing to total foreign borrowing ratio is 85%. As stated previously in 2008 this ratio is 50% in developing countries. In Turkey, loan interest rates, both TL denominated and FX denominated, increased in October 2008 attributable to the global crisis. However, between November 2008 and November 2009 by the interest rate cuts made by CBRT, they fell below their September 2008 levels. During the same period interest rates of TL denominated loans almost doubled the interest rates of FX denominated loans.

2.3 Why Do Firms Issue Foreign Currency Denominated Debt?

Previous section demonstrates the growth in foreign currency denominated loan market in Turkey and in all regions of the world. As the portion of foreign borrowing by non-financial corporations in total borrowings increases, the topic attracts more attention in academic environment. In addition to risks associated with borrowing in domestic currency, foreign currency borrowing induces exchange rate risk to the firms. The foreign exchange exposure of firms and motivations of the firms to issue a debt with additional risks are examined in academic literature.

Kıymaz [5] investigates the FX exposure of firms in highly inflationary environment. In this paper, specifically the following issues are discussed: whether the foreign exchange risk is priced at the firm level, foreign exchange exposure across industries, comparison of exchange rate exposure in pre-crisis and post-crisis periods, investigation of the relation between foreign exchange exposure and magnitude of international operations. The sample of the research consists of 109 firms traded in ISE. Following the work of Dumas [36] and Adler and Dumas

[35] the exchange rate exposure is estimated by the regression coefficient of the value of the firm on the exchange rate for different states of nature. The findings show that Turkish firms are highly exposed to foreign exchange risks and their values are significantly influenced by exchange rate fluctuations. In this article it has been showed that the post-crisis foreign exchange exposures of all industries seem to be lower than those of pre-crisis. Another result of this study is that the firms with a higher degree of export and import involvement experience a greater fx-rate exposure. Solakoglu [37] examines the relationship between exchange rate exposure and firm-specific factors for Turkish firms between 2001 and 2003. Findings of this study show that firm size and foreign activities as well as being net-exporters and net importers are significant affects on foreign exchange exposure.

One of the reasons to issue foreign currency denominated debt is to hedge the foreign operations. Examining the motives behind the use of currency derivatives, Geczy et al.[13] studied 392 Fortune 500 non-financial firms in 1990. In their study, they also included the naturally hedged firms, defined as having foreign operations and foreign-denominated debt. They employ multivariate and univariate tests to identify the determinants of using currency derivatives. They find out that for naturally hedged firms R&D and short-term liquidity are not significant determinants whereas these variables are still significant determinants of derivative use for firms having foreign operations but no foreign currency denominated debt. Thus foreign currency denominated debt and currency derivatives can be used as substitutes to hedge foreign operations for this sample. Therefore firms having revenues in foreign currency may want to issue foreign currency debt in order to provide hedging of foreign exchange exposure.

Many researchers have examined the relation between the FX exposure of non-financial firms and the foreign currency debt issuance. Taek Ho Kwon and Sung C. Bae [6] study the Korean manufacturing firms between 1998 – 2005. They measure the effect of foreign currency borrowing on asymmetric foreign exchange exposure with a regression model. They find strong evidence that firm's export ratio (total export to total sales ratio) and dollar denominated debt ratio (difference between the dollar denominated liabilities and dollar denominated assets over total assets) are significantly related to the firm's asymmetric foreign exchange exposure but in the opposite direction.

Allayannis et al [7] examined the 327 of the largest East Asian non-financial firms' choice of using alternative types of loans from 1996 to 1998 as well as the effects of foreign borrowing on firms' performance. The results suggest that ability to manage the currency risk with foreign cashflows and cash reserves is a determinant to use of foreign currency debt. An unanticipated result of this study is that they find no evidence suggesting that unhedged foreign currency debt was the primary cause of the 1997 Asian crisis. This result is attributed to illiquidity of the derivatives market during the crisis.

Keloharju and Niskanen [8] also studied the determinants of the decision to raise currency debt. Their sample consists of 44 non-financial Finnish companies listing on Helsinki Stock Exchange between the years 1985 and 1991. Main drivers of foreign borrowing are tested by using probit regression framework. The findings show that firms having export to sales ratio high are most likely to raise foreign currency debt. Hence, they reach to conclusion that hedging is an important determinant of the currency of denomination decision. Kedia Mozumdar [2] investigated the same topic for large U.S. firms reported to Compustat. They also employ probit regression model to test the effects of various determinants on foreign borrowing and find that foreign borrowing relates with the foreign activity.

Whether the economic exposure of non-financial firms has effects on firms' debt financing choice is questioned by Goswami and Shrikhande [1]. In their model they used a discrete time exchange rate process to explain the data on real world. They showed that the dominant debt-financing alternative for firms faced with negative economic exposure is foreign currency debt. Also, Hekman [27], Aliber [9] described the hedging purposes of non-financial firms as the principal incentive of issuing foreign currency denominated debt.

In addition to mathematical and statistical models, survey studies show that in practice foreign currency borrowing and currency derivatives is used interchangeably. Bradley&Moles [10] conducted a survey about the techniques used to manage exchange rate risk among the sample of non-financial firms of UK. In that survey the results show that more than half of the firms issue foreign currency debt and match costs with revenues issued in the same currency to manage exchange rate exposure. Graham & Harvey [14] surveyed with 392 CFOs about the capital structure. In their survey study questions about debt, equity and foreign debt are asked. 31% of the survey respondents considered issuing foreign debt for which one of the most popular reasons is to provide a natural hedge against foreign currency devaluation.

Other than hedging purposes, low interest rates of foreign currency denominated corporate loans relative to domestic interest rates made them more preferable among other alternatives. Firms that have no cash flows in foreign currency may want to borrow in foreign currency to reduce their interest costs. Keloharju & Niskanen [8] examined the determinants of the decision to raise foreign currency denominated debt by among 44 Finnish corporations. In that study it is specified that one of the reasons for corporations to issue FX denominated debt is the fact that borrowing in foreign currency may cost less than borrowing in home currency. Allayannis et al [7] investigate the largest East Asian corporations, Kedia & Mozumdar [2] study the sample of large US firms and Aliber [9], mentioned the interest rate differential as one of the incentives for the firms to issue FX denominated debt.

Other than hedging and cost of financing advantages, higher liquidity of the debt market reduces the transaction costs of financing and affects the foreign debt issuance. Legal regimes having strong creditor rights also lower the cost of financing. Although, it creates a weak preference, investors may follow the arbitrage differences in tax rates and that affects the foreign currency debt issuance. Also, Kedia and Mozumdar [2] find that higher credit-rated firms issue significantly more foreign debt than lower credit-rated firms. Thus credit rating can also be showed as the incentive to issue more foreign currency denominated debt.

2.4 Risks Associated with Foreign Currency Debt Issuance

Risks induced by foreign borrowing can have many aspects. The previous section reveals that non-financial firms may issue foreign currency denominated debt by hedging purposes and to take advantage of lower financing costs. Related with those findings important risks will be discussed in this section.

Although most countries have regulations for banks to limit their foreign exchange exposure, banks are still indirectly exposed to that risk due to currency mismatches on their clients' balance sheets. As of March 2010, 67% percent of total loans consisted of corporate loans. In CBRT's Financial Stability Report, it is stated that the share of corporate lending used to decrease in the post-crisis period; but these loans have increased in the last quarter of 2009 [4]. To illustrate, during the 1997 Asian financial crisis banks did not exercise careful credit assessment over foreign exchange risks in their corporate customers [41]. Also, during 2001 the Brazilian currency depreciated about 40% from end of the 2000 to September 2001 causing the share of foreign currency debt in total debt to increase. Another example of sharp increase in exchange rate is Turkish lira to dollar exchange rate, which increased to 1.6 million in July 2001 from 672 thousand in 2000.

Goldstein and Turner [41] state that in a crisis, widening credit spreads and currency devaluations tend to occur at the same time. In such circumstances ”cheap” foreign currency debt can quickly become very expensive to service and refinance. The case of Korea illustrates risks of assuming cheap foreign currency financing. The won/yen exchange rate has been very stable over the past decade. Thus firms could generate large profits by borrowing in yen at low interest rates and using the proceeds to invest in higher-yielding won-denominated instruments. However, the financial crisis simultaneously cut firms’ export revenues as global demand plummeted and put the won under pressure.

On the other hand, the higher exchange rate volatility means that the hedging will be more expensive hence there will be less hedging. Therefore firms will be more exposed to exchange rate risk in such a case.

2.5 Pricing of Risky Debt

Using contingent claims approach to price risky debt is first introduced by Black and Scholes [15] in 1973 followed by Merton [16]. Credit pricing models using contingent claims approach can be classified in two categories: structural models and reduced form models. Reduced form models assume that a firm can default in any time and the event of default is independent of the firm’s capital structure. This model is generally used in cases where more data is available. Also reduced form models do not explain the reasons of the default and ignores the relation between the economics of the firm and the event of default. The study of Jarrow and Turnbull [17] is one of the first reduced form models. The structural models on the other hand define the corporate liability as a function of a firm’s capital structure and time. In that approach, the reasons behind the default are investigated and implemented to the model. As mentioned in Chapter 1 in this study structural model will be used to price the default risk of foreign currency denominated loans.

In the study of ”The Pricing of Options and Corporate Liabilities”, Black and Scholes [15] used contingent claims approach to price risky debt. They defined the value of a corporate liability as a function of the firm's value and time. This function satisfies a partial differential equation, which is also known as Black Scholes PDE. In that model asset value of the firm is assumed to follow a stochastic Brownian motion and risk free rate is assumed to be constant. Also, default of a firm is defined as a put option with strike price equals to the face value of the zero coupon debt and default occurs when the value of the assets of the firm is less than the face value of the debt.

Merton [16] defined value of the equity as a call option on the assets of a firm and derived the same solution with Black and Scholes for the debt value by subtracting the call price from the market value of the assets. Furthermore, in this study Black-Scholes method is applied to corporate debt, which is zero coupon bond and risk structure of interest rate is presented.

The first studies on the valuation of risky debt applied the contingent claims approach to zero coupon bonds, which have their only payment at the maturity of the bond. Therefore, early works on this topic assume that a firm can go bankrupt only at the maturities of the existing debts. Between the issuance date and maturity, assets of the firm can be lower than the debt obligation. Black and Cox [18] introduced a valuation model for bonds with safety covenants. The most important assumption made in this study is to assume that trading takes place continuously. In this model they defined an absorbing barrier for the value of the assets of a firm and they let default to occur prior to the maturity when the value of a firm hits this barrier. In their study default barrier is defined as a constant fraction of the present value of the promised final payment.

Vasicek [38] combined the Black Scholes model with stochastic interest rate and assumed that spot interest rate follows a mean reverting lognormal process. This model has two stochastic factors namely asset value and interest rate where the debt value satisfies a two-state PDE. Introducing stochastic interest rate to the model provide to measure the effects of changes in interest rate. However, in Vasicek's interest rate process spot rates can take negative values, which is not the case in real world. Also, Shimko, Tejima and Van Deventer [19] attempt interest rate with a stochastic process and assumed that spot interest rate follows the Ornstein-Uhlenbeck process. The important characteristic of this model is to take non-zero correlation between the asset value and the interest rate.

Longstaff and Schwartz [20] combined the Shimko, Tejima and Van Deventer's work with a constant default threshold assumption. They argue that when the firm value reaches the predefined constant threshold it is an indicator of a financial distress and financial distress triggers the default of all of the firm's debts.

Kim, Ramaswamy and Sundersan [21] considered that the firm could default due to its coupon obligations before the maturity. They also take interest rate as stochastic and used Cox Ingersoll Ross process to model the spot interest rate. Besides, they take the operating cash flows of a firm into account, instead of asset value of a firm and they state that a firm's bankruptcy is triggered when the firm's cash flows are unable to cover its interest obligations.

Ericsson and Reneby [22] and Briys and de Varenne [23] also allowed default to occur prior to maturity in their models. Briys and de Varenne used stochastic default barrier in their model, which is an extension of the Black & Cox's study where both the interest rate and the default barrier are constant. Leland [24] argued that debt values cannot be determined without knowing the firm's capital structure, which triggers the default and bankruptcy. Following this assumption Leland determined the optimal leverage ratio to declare bankruptcy. Toft & Prucyk [25] followed Leland [24] work and defined bankruptcy level in the same way.

Opler & Titman [26] studied the relation between the corporate performance and financial distress. The term financial distress is used interchangeably with being highly leveraged and they measured leverage with book values of debt divided by the book values of assets. Using book values instead of market values to identify the leverage ratios avoids the problem of including market condition related factors to the model. Their sample contains 46,799 years of data in the 1972 to 1991 period and it is divided in to two groups; firms in industries experiencing poor performance and firms in industries experiencing normal performance. Base year leverage ratios for those samples are found as 34% and 31% for firms in industries experiencing poor performance and firms in industries experiencing normal performance respectively. They also defined highly leverage firms as having leverage in deciles 8 to 10.

In option pricing methods the value of assets satisfies a partial differential equation, which does not have closed form solutions. Therefore, numerical methods are employed to reach to a solution of the PDE. The most common methods are Monte Carlo method, which is a forward pricing method and finite difference approximation to the differential equation, which is a backward pricing method. These techniques mostly used by valuing mortgage contracts in literature. Stanton [39], Kau, Keenan, Muller and Epperson [40], Azevedo-Pereira [32] used finite difference method to find the numerical solution for the value of the mortgage and its embedded options.

CHAPTER 3

CURRENCY LOAN VALUATION

3.1 Introduction

In this study, foreign currency loan and the related assets are treated as derivative assets. A derivative is an asset whose value depends on the price of some other assets, the underlyings. The underlying assets' random movements can be modeled mathematically by stochastic processes. In this chapter, properties of the stochastic processes that the underlying assets follow are discussed and the main PDE to value the derivative assets is derived. Also, payment functions of the derivative assets to be valued are covered in this chapter.

3.2 Contingent Claims Valuation Framework

3.2.1 Exchange Rate and Term Structure of Interest Rate

The pattern of interest rates over different time periods for different investment opportunities is known as the term structure of interest rates. In corporate debt valuation models, major factors affecting the value of the loan are set as the underlying state variables. Interest rate and asset price are generally determined as two major risk factors for domestic currency fixed rate loans. Examining the foreign currency denominated fixed rate loans, the interest rate and the foreign currency rate are determined as the state variables. Mean reverting square root stochastic process is used to model both of these state variables. This type of stochastic processes is used to model the commodities, which are sensitive to short-term oscillations but tends to revert back to a ”normal” long-term equilibrium level while pushing away from zero.

In this context, both interest rate and currency rate are assumed to follow mean reverting square root diffusion process defined by Cox Ingersoll and Ross, which is also known as CIR process [28]. The mean reverting square root processes for interest rate and foreign currency rate are given by the following equations respectively:

$$dr = \kappa_r(\theta_r - r)dt + \sigma_r \sqrt{r} dz_r \tag{3.1}$$

$$df = \kappa_f(\theta_f - f)dt + \sigma_f \sqrt{f} dz_f \tag{3.2}$$

Here κ is the rate of mean reversion. The greater the mean-reverting parameter value κ, the greater is the pull back to the equilibrium level. θ is the drift term brings the variable being modeled back to an equilibrium level. σ is the volatility of the process, which is proportional to the square root of the interest rate. Standardized Wiener processes for interest rate and foreign currency rate are dz_r and dz_f respectively. Relation between these two Wiener processes is given as follows:

$$dz_r dz_f = \rho dt \tag{3.3}$$

where ρ is the instantaneous correlation coefficient between two Wiener processes.

In this study, loan is considered as a derivative asset and its value is assumed to depend on the term structure of the interest rate and the foreign exchange rate. Market price of risk associated with the spot interest rate and exchange rate also affects the value of the loan. Since exchange rate is a traded asset there is no risk adjustment for that variable. Also, option pricing theory requires a risk neutral economic environment. Using spot interest rate will eliminate the market price of risk for the interest rate.

The Feyman-Kac theorem is used to derive the partial differential equation (PDE) for the value of assets $F(r, f, t)$ whose value is a function of the two state variables defined by the stochastic processes in (3.1) and (3.2). The following PDE can be derived by this method:

$$\frac{\partial F}{\partial t} + \kappa_r(\theta_r - r)\frac{\partial F}{\partial r} + \kappa_f(\theta_f - f)\frac{\partial F}{\partial f} + \rho\sigma_r\sigma_f \sqrt{r}\sqrt{f}\frac{\partial^2 F}{\partial r \partial f} + \frac{1}{2}\sigma_r^2 r \frac{\partial^2 F}{\partial r^2} + \frac{1}{2}\sigma_f^2 f \frac{\partial^2 F}{\partial f^2} - rF = 0 \tag{3.4}$$

The aim of this study is to determine the values of Financial Distress and Default prior to maturity. Therefore, backward valuation procedure is used in an iterative way. Once the value of different assets is known at maturity, it is possible to use equation (3.4) to solve for the value of these assets in previous moments in time. Small time steps backwards were used to solve the equation giving the value of the loan and loan related assets immediately after the previous payment date. According to the values obtained in previous steps, new terminal conditions are determined and this process is repeated iteratively in the life of the loan until the initial moment and the value of these assets at origination is determined.

3.3 The Foreign Currency Denominated Loan Contract

3.3.1 Notations

The study considers fixed rate foreign currency denominated corporate loans and the following notations will be used:

n = life of the loan in months

L = initial amount of loan in domestic currency terms

c = the fixed rate coupon rate

$\eta(i)$ = i^{th} payment date

f_{int} = fx rate at the origination of the loan

M = monthly payment in foreign currency

V_A = book value of the assets of a company

Th = threshold for the financial distress

$V_B(r, f, t)$ = Value at time t of the loan for borrower

$A(r, f, t)$ = Value at time t of the remaining payments of the loan

$FD(r, f, t)$ = Value at time t of the financial distress

$D(r, f, t)$ = Value at time t of the default

$F^-(r, f, t)$ = Value of the asset F immediately before the payment

$F^+(r, f, t)$ = Value of the asset F immediately after the payment

3.3.2 Valuation of Monthly Payments

The PDE (3.4) will be solved using a backward procedure. In that context, identifying the monthly payments of the fixed rate foreign currency denominated loan is necessary for the valuation procedure. At the origination of the loan, all payables are determined in foreign currency units. At payment dates, amounts in domestic currency are calculated using the relevant exchange rates.

The principal that is in foreign currency terms should be paid in full by the end of the contract. The loan under study is a fixed rate loan therefore during the life of the loan the monthly payments are same in foreign currency terms.

On the other hand, the borrower gets the loan in domestic currency at the origination date and makes the monthly payments in domestic currency. Since the exchange rate changes on payments dates monthly payments can be expressed with the following equation:

$$M = \frac{(\frac{c}{12})(1+\frac{c}{12})^n}{(1+\frac{c}{12})^n - 1} L \frac{1}{f_{int}} \tag{3.5}$$

3.3.3 Valuation of Future Payments

Valuation of future payments is similar to the valuation of foreign currency denominated bonds. Given the cash flows, its value is a function of both interest rate and exchange rate similar to other financial assets. Since the value depends on exchange rate, payments should be adjusted with the relative rate at each payment date.

By the end of the loan, value of the payment due will be the same as the last monthly payment adjusted according to the domestic currency. The terminal condition for the value of the future payments can be defined as follows:

$$A^{-}(r, f, t) = Mf \qquad \text{for } t = \eta(n) \tag{3.6}$$

After each monthly payment the present value of borrower's debt is reduced by Mf. Considering this the following terminal condition should be used for the other payment dates:

$$A^{-}(r,f,t) = A^{+}(r,f,t) + Mf \qquad \text{for } t = \eta(1),...,\eta(\text{n-1}) \tag{3.7}$$

3.3.4 Value of the Loan

The corporate loan in this study has a probability of default, which makes the value of the debt risky. The value of the risky debt is defined as the default free value of the debt minus the expected loss. Expected loss of the debt is an implicit put option on the assets of a firm [43]. Following the definition, value of the loan can be expressed as follows:

$$V_B = A - D \tag{3.8}$$

Value of future payments and value of default are affected by two state variables namely exchange rate and interest rate. Therefore, the value of the loan is also a function of these two state variables. As mentioned earlier, value of the assets of a firm is assumed to be constant and value of remaining payments depends on exchange rate and interest rate. Fluctuation in the state variables affects the value of the future payments and if the amount that the firm is obliged to pay is more than the value of its assets then the firm goes to bankruptcy. In that case lender of the loan can get as much as V_A, the value of the firm's assets. So, value of the debt can be at most V_A. At the last payment date the firm can either make the last payment or goes to bankruptcy. Hence the value at termination is given by,

$$V_B^{-}(r,f,t) = min(Mf, V_A) \qquad \text{for } t = \eta(n) \tag{3.9}$$

Following the same logic, value of the debt immediately before the other payment dates is as follows:

$$V_B^{-}(r,f,t) = min([V_B^{+}(r,f,t) + Mf], V_A) \qquad \text{for } t = \eta(1),...,\eta(\text{n-1}) \tag{3.10}$$

3.3.5 Value of the Default Option

By issuing a risky debt, borrower is obliged to pay the losses in case of default. In this case borrower offers an implicit guarantee i.e. buys an implicit put option on the assets of a firm. Therefore, value of default for a risky debt with sequential payouts can be valued as a compound put option. At each payment date, borrower has right to make the monthly payment or to leave the assets of the firm. Making the monthly payments, default option expires and the borrower issues another put option, which expires in the next payment date.

At the maturity of the loan, default option will be valueless if the value of the assets of the firm is more than the value of remaining payments of the currency loan, which is Mf. Otherwise the value of default will be the value of expected loss which is the amount left after discounting the value of the assets of the firm.

$$D^{-}(r, f, t) = max(0, Mf - V_A) \qquad \text{for } t = \eta(n) \tag{3.11}$$

For the other payment dates, terminal conditions can be defined by considering two cases namely, the case of default and no default as follows;

$$D^{-}(r, f, t) = \begin{cases} max(0, A^{-}(r, f, t) - V_A), & \text{if } A^{-}(r, f, t) \geq V_A \\ D^{+}(r, f, t), & \text{otherwise} \end{cases} \qquad \text{for } t = \eta(1), ..., \eta(\text{n-1}) \tag{3.12}$$

3.3.6 Valuation of Financial Distress

Valuing the risky debt and default with contingent claims approach received widespread attention in academic literature. In chapter 2 models for identifying the value of the default risk are discussed. It has been mentioned that Ericsson and Reneby [22] and Briys and de Varenne [23] modeled default as a barrier option. Following the reasoning behind these two theories value of the financial distress can be found by assigning a threshold, Th, for financial distress. In this study, distress barrier represents the percentage of assets of a firm when passed causing a financial distress and leading to default. In case of default firm is still in a distress situation and the value of distress is the difference between the value of remaining payments and the barrier.

A firm is assumed to be in financial distress when the value of the remaining payments of the currency denominated debt exceeds the distress barrier. Terminal conditions for the value of financial distress are similar to the conditions given for default. At termination, if the last monthly payment exceeds the barrier the firm is assumed to be in financial difficulty with the amount of the difference between these two values. Monthly payment for the current month is converted to domestic currency. Hypothetically exchange rate can go to infinity and in this case value of the option should also go to infinity. In such a case this problem is eliminated by assuming that firm's distress value is the difference between the value of the firm's assets and the threshold.

$$FD^{-}(r, f, t) = max(0, Mf - Th) \qquad \text{for } t = \eta(n) \tag{3.13}$$

At other payment dates, comparing the value of the remaining payments with the barrier distress value can be identified and the terminal conditions are given by;

$$FD^{-}(r, f, t) = \begin{cases} max(0, A^{-}(r, f, t) - Th), & \text{if } A^{-}(r, f, t) \geq Th \\ FD^{+}(r, f, t), & \text{otherwise} \end{cases} \qquad \text{for } t = \eta(1), ..., \eta(\text{n-1}) \tag{3.14}$$

CHAPTER 4

NUMERICAL SOLUTION OF A TWO-STATE VARIABLE CONTINGENT CLAIMS VALUATION MODEL USING THE EXPLICIT FINITE DIFFERENCE METHOD

4.1 Introduction

Derivative assets can be modeled by partial differential equations (PDEs) in contingent claims analysis framework. Most of the PDEs used in finance can be categorized in first order linear equations and second order linear parabolic equations that allow addopting mathematical methods to real world finance problems [30]. This chapter presents a numerical procedure for the solution of a contingent claims valuation model in order to value the assets related to foreign currency denominated loans. Considering the nature of those assets and the information available at the origination of the contract, backward valuation procedure is used. The most commonly used types of backward procedures are lattices and finite difference methods in which finite difference methods are able to compute simultaneously several starting contingent claim prices for different values of the risk parameters and can be more powerful when several option values are to be calculated simultaneously [31]. The boundary and terminal conditions necessary for the solution of the model are described according to the cash-flow structure of the asset under valuation.

4.2 The Finite Difference Methodology

In the finite difference methods the domain of the problem is divided into a grid and the PDE is discretized by replacing each partial derivative with a difference quotient. Discretizing the PDE the derivative terms are approximated by replacing them with the respective finite difference approximations that approximate their values between the nodes obtained. The finite difference equivalents of the derivative terms are derived from the Taylor series expansions.

4.2.1 Finite Difference Approximations

Let $u(x)$ be a continuous function of the single independent variable x, the x domain is discretized into a grid. The constant grid spacings in the x and t directions are given by h and k. In one-dimensional problems the function of any grid point is given by:

$$u(x_i) \equiv u(ih) \equiv u_i \text{ for } i = 0, 1, 2, ... \tag{4.1}$$

U_i^n and u_i^n are the difference and differential equations respectively at the points $x = ih$ and $t = nk$.

In two-space-dimensional problems, grid spacings for x direction are given by h and grid spacings for y direction are given by l. The function for any grid point is represented as follows:

$$u(y_j, x_i) \equiv u(jl, ih) \equiv u_{j,i} \text{ for } i = 0, 1, 2, ... \text{ and } j = 0, 1, 2, ... \tag{4.2}$$

The representation of the difference and differential equations at the point $x = ih$, $y = jl$ and $t = mk$ will be denoted by $U^n_{j,i}$ and $u^n_{j,i}$ respectively.

Finite difference methods are based on Taylor series expansions of the functions under study and their derivatives. If $u(x)$ and its derivatives are single valued, finite and continuous functions of x, Taylor series expansion for $u(x)$ can be written in two ways at the point x as:

$$u(x_i + h) = u(x_i) + hu_{x|i} + \frac{h^2}{2!}u_{xx|i} + \frac{h^3}{3!}u_{xxx|i} + ... \tag{4.3}$$

Or as:

$$u(x_i - h) = u(x_i) - hu_{x|i} + \frac{h^2}{2!}u_{xx|i} - \frac{h^3}{3!}u_{xxx|i} + ... \tag{4.4}$$

From (4.3) it is possible to obtain:

$$u_{x|i} \approx \frac{u(x_i + h) - u(x_i)}{h} \equiv \frac{u_{i+1} - u_i}{h} \approx \frac{U_{i+1} - U_i}{h} \tag{4.5}$$

This representation of differencing is in the forward t direction thus it is called forward difference approximation to a first derivative. Likewise, from (4.4) it is possible to obtain the following for the backward difference approximation to a first derivative:

$$u_{x|i} \approx \frac{u(x_i) - u(x_i - h)}{h} \equiv \frac{u_i - u_{i-1}}{h} \approx \frac{U_i - U_{i-1}}{h} \tag{4.6}$$

The approximations given above are subject to discretional truncation thus contain error which is of order h, $O(h)$.

The central difference method is obtained by adding (4.3) and (4.4) and solving for $u_{x|i}$ as follows:

$$u_{x|i} \approx \frac{U_{i+1} - U_{i-1}}{2h} \tag{4.7}$$

In this case truncation error is of order $O(h^2)$. Since the order of the error is directly proportional to the accuracy of the approximation, central difference is the most commonly used approximation method.

For the second order derivatives, adding (4.3) and (4.4) and solving for $u_{xx|i}$ will give the following central finite difference approximation:

$$u_{xx|i} \approx \frac{U_{i+1} - 2U_i + U_{i-1}}{h^2} \tag{4.8}$$

where $O(h^2)$ is the truncation error.

In order to derive the finite difference approximations to higher order derivatives the same groundwork can be followed. In this study second order derivatives are under consideration. Therefore, for the given function $u(x, y)$ the following forward finite difference approximations for the first derivative terms can be derived with truncation errors $O(h)$ and $O(l)$.

$$u_{x|j,i} = \frac{U_{j,i+1} - U_{j,i}}{h} \tag{4.9}$$

$$u_{y|j,i} = \frac{U_{j+1,i} - U_{j,i}}{l} \tag{4.10}$$

Similarly, backward finite difference approximations for the first derivative terms with the same truncation errors can be given by:

$$u_{x|j,i} = \frac{U_{j,i} - U_{j,i-1}}{h} \tag{4.11}$$

$$u_{y|j,i} = \frac{U_{j,i} - U_{j-1,i}}{l} \tag{4.12}$$

Using the same type of procedure, central difference approximations for the second order derivatives are obtained as:

$$u_{xx|j,i} = \frac{U_{j,i+1} - 2U_{j,i} + U_{j,i-1}}{h^2} \tag{4.13}$$

$$u_{yy|j,i} = \frac{U_{j+1,i} - 2U_{j,i} + U_{j-1,i}}{l^2} \tag{4.14}$$

The truncation errors have orders $O(h^2)$ and $O(l^2)$ respectively. For the mixed derivative terms the following approximation will be used:

$$u_{yx|j,i} = \frac{U_{j+1,i+1} - U_{j-1,i+1} - U_{j+1,i-1} + U_{j-1,i-1}}{4hl} \tag{4.15}$$

where the truncation error is of dimension $O(h^2) + O(l^2)$.

4.3 A Framework for the Solution of the Model Using an Explicit Finite Difference Method

4.3.1 Transformed Version of the Original PDE

In order to eliminate the possible problems due to infinite boundary problem the unbounded domain of the original PDE (3.4) is transformed on to a bounded domain. For the state variables r and f the following transformations were chosen as in Azevedo-Pereira [32] respectively:

$$y = \frac{1}{1 + \psi r} \tag{4.16}$$

$$x = \frac{1}{1 + \omega f} \tag{4.17}$$

for $\psi > 0$ and $\omega > 0$. Therefore the inverse transformations are given by:

$$r = \frac{1 - y}{\psi y} \tag{4.18}$$

$$f = \frac{1 - x}{\omega x} \tag{4.19}$$

The time variable is transformed as:

$$\tau = T - t \tag{4.20}$$

The scale factors ψ and ω affect the density of the points in the grid. Higher values of these factors results with having more number of points that correspond to small r and f. In numerical calculations these factors are chosen as the middle point of the y grid corresponds to $r = 10\%$ and the middle point of the x grid corresponds to $f = 1,5$.

The inverse transformation for the time variable is:

$$t = T - \tau \tag{4.21}$$

With these transformations the unbounded region $(0,\infty)\times(0,\infty)$ is mapped onto the region $(0,1)\times(0,1)$. Also under the transformations made the original function can be expressed in terms of new variables.

$$W(x,y,\tau) = F(r(y), f(x), t(\tau)) \tag{4.22}$$

Likewise the derivatives should be expressed in terms of new variables. First derivatives in r and f are obtained as:

$$\frac{\partial F}{\partial r} = \frac{\partial W}{\partial y}\frac{\partial y}{\partial r} \tag{4.23}$$

$$\frac{\partial F}{\partial f} = \frac{\partial W}{\partial x}\frac{\partial x}{\partial f} \tag{4.24}$$

Second derivatives are as follows:

$$\frac{\partial^2 F}{\partial r^2} = \frac{\partial^2 W}{\partial y^2}\left(\frac{\partial y}{\partial r}\right)^2 + \frac{\partial^2 y}{\partial r^2}\left(\frac{\partial W}{\partial y}\right) \tag{4.25}$$

$$\frac{\partial^2 F}{\partial f^2} = \frac{\partial^2 W}{\partial x^2}\left(\frac{\partial x}{\partial f}\right)^2 + \frac{\partial^2 x}{\partial f^2}\left(\frac{\partial W}{\partial x}\right) \tag{4.26}$$

Mixed derivative in terms of new variables is the following:

$$\frac{\partial^2 F}{\partial r \partial f} = \frac{\partial^2 W}{\partial y \partial x}\left(\frac{\partial y}{\partial r}\right)\left(\frac{\partial x}{\partial f}\right) \tag{4.27}$$

The derivatives of new variables with respect to old variables for the first derivatives are:

$$\frac{\partial y}{\partial r} = -\psi y^2 \tag{4.28}$$

$$\frac{\partial x}{\partial f} = -\omega x^2 \tag{4.29}$$

The second derivatives of new variables in terms of old variables are as follows:

$$\frac{\partial^2 y}{\partial r^2} = 2\psi^2 y^3 \tag{4.30}$$

$$\frac{\partial^2 x}{\partial f^2} = 2\omega^2 x^3 \tag{4.31}$$

After the substitutions the formulation for the transformed version of the PDE (3.4) is the following:

$$\begin{aligned} &\frac{1}{2}\sigma_f^2 f(x)\omega^2 x^4 \frac{\partial^2 W}{\partial x^2} + \rho\sigma_r\sigma_f \sqrt{r(y)}\sqrt{f(x)}\psi\omega y^2 x^2 \frac{\partial^2 W}{\partial x \partial y} \\ &+\frac{1}{2}\sigma_r^2 r(y)\psi^2 y^4 \frac{\partial^2 W}{\partial y^2} + [\sigma_f^2 f(x)\omega^2 x^3 - \omega x^2 \kappa_f(\theta_f - f(x))]\frac{\partial W}{\partial x} \\ &+[\sigma_r^2 r(y)\psi^2 y^3 - \psi y^2 \kappa_r(\theta_r - r(y))]\frac{\partial W}{\partial y} - \frac{\partial W}{\partial \tau(t)} - r(y)W = 0 \end{aligned} \tag{4.32}$$

4.3.1.1 Consistency, Stability and Convergence of the Numerical Procedure

In numerical procedures used in this study a continuous domain is discretized along the finite number of points. This procedure creates truncation errors for the problem and errors associated with the problem affects the accuracy of the solution. If the errors do not grow to be much larger, then the solution can be said to be accurate.

Following the approach used in Morton and Mayers [29] it can be said that, in order for the numerical solution of a PDE of the type under study to converge and be stable as $h, l, k \to 0$, the following condition must be satisfied:

$$\Lambda\frac{k}{h^2} + \Theta\frac{k}{l^2} \leq \frac{1}{2} \tag{4.33}$$

where;

Λ = Coefficient of the second derivative term in x in equation (4.32).
Θ = Coefficient of the second derivative term in y in equation (4.32).

h = Grid spacing in the x dimension.

l = Grid spacing in the y dimension.

k = Grid spacing in the time dimension.

The open form of the equation (4.33) is:

$$\frac{1}{2}\sigma_f^2 f(x)\omega^2 x^4 \frac{k}{h^2} + \frac{1}{2}\sigma_r^2 r(y)\psi^2 y^4 \frac{k}{l^2} \leq \frac{1}{2} \tag{4.34}$$

When $h = l$:

$$\frac{\left\{[\sigma_f^2 f(x)\omega^2 x^4] + [\sigma_r^2 r(y)\psi^2 y^4]\right\} k}{h^2} \leq 1 \tag{4.35}$$

The value of the left-hand side changes with the parameters that are included in the coefficients of the second derivative terms of (4.32). These changes will be reflected within the same lattice when the variations affect $x, H(x), y, r(y), \psi$ and ω. Otherwise, when the variations are related to the parameters that characterize the economic environment(σ_r and σ_f), the changes will be felt only between different sets of economic environment parameters. As a consequence of this phenomenon, it is necessary to define a step size for the time variable capable of providing a guarantee that the condition will be respected for each point of the grid and each set of economic environment parameters. In order to achieve this result with a reasonable margin of security, it was decided to use 66 time steps a month corresponding to a $k \approx 0.00126$. Thus grid spacing assures the satisfaction of the stability and consistency condition in all circumstances considered in the present work. The matlab code presented in Appendix A calculates the values for the equation (4.35). The parameters that underlie the matlab code are same that is used in the numerical results presented in Chapter 5. Maximum value is computed as 0.0289 which is smaller than $\frac{1}{2}$.

4.3.2 Finite Difference Representation of the PDE

4.3.2.1 Interior Points

The transformed version of the PDE (4.32) can be approximated by the following difference equation:

$$\begin{aligned}
&\frac{1}{2}\sigma_f^2 f(x)\omega^2 x^4 \frac{U^n_{j,i+1} - 2U^n_{j,i} + U^n_{j,i-1}}{h^2} \\
&+\rho\sigma_r\sigma_f \sqrt{r(y)}\sqrt{f(x)}\psi\omega y^2 x^2 \frac{U^n_{j+1,i+1} - U^n_{j-1,i+1} + U^n_{j+1,i-1} + U^n_{j-1,i-1}}{4hl} \\
&+\frac{1}{2}\sigma_r^2 r(y)\psi^2 y^4 \frac{U^n_{j+1,i} - 2U^n_{j,i} + U^n_{j-1,i}}{l^2} \\
&+[\sigma_f^2 f(x)\omega^2 x^3 - \omega x^2 \kappa_f(\theta_f - f(x))]\frac{U^n_{j,i+1} - U^n_{j,i-1}}{2h} \\
&+[\sigma_r^2 r(y)\psi^2 y^3 - \psi y^2 \kappa_r(\theta_r - r(y))]\frac{U^n_{j,i+1} - U^n_{j,i-1}}{2l} \\
&-\frac{U^{n+1}_{j,i} + U^n_{j,i}}{s} - r(y)U^n_{j,i} = 0
\end{aligned} \tag{4.36}$$

When the equation (4.36) rearranged, asset value at a certain time step can be found as a function of its own value at the previous time step recursively:

$$\begin{aligned}
U^{n+1}_{j,i} &= \left[1 - sr(y) - \sigma_r^2 r(y)\psi^2 y^4\left(\frac{s}{l^2}\right) - \sigma_f^2 f(x)\omega^2 x^4\left(\frac{s}{h^2}\right)\right]U^n_{j,i} \\
&+ \frac{1}{2}\sigma_f^2 f(x)\omega^2 x^4\left(\frac{s}{h^2}\right)(U^n_{j,i+1} + U^n_{j,i-1}) \\
&+ \frac{1}{2}\sigma_r^2 r(y)\psi^2 y^4\left(\frac{s}{l^2}\right)(U^n_{j+1,i} + U^n_{j-1,i}) \\
&+ \left[\sigma_f^2 f(x)\omega^2 x^3 - \omega x^2\kappa_f(\theta_f - f(x))\right]\left(\frac{s}{2h}\right)(U^n_{j,i+1} - U^n_{j,i-1}) \\
&+ \left[\sigma_r^2 r(y)\psi^2 y^3 - \psi y^2\kappa_r(\theta_r - r(y))\right]\left(\frac{s}{2l}\right)(U^n_{j+1,i} - U^n_{j-1,i}) \\
&+ \rho\sigma_r\sigma_f\sqrt{r(y)}\sqrt{f(x)}\psi\omega y^2 x^2\left(\frac{s}{4hl}\right)(U^n_{j+1,i+1} - U^n_{j-1,i+1} + U^n_{j+1,i-1} + U^n_{j-1,i-1})
\end{aligned} \tag{4.37}$$

Equation (4.37) can be represented as follows when the coefficients of $U^n_{j,i}$'s are perfectly isolated;

$$
\begin{aligned}
U_{j,i}^{n+1} = & \left[1 - sr(y) - \sigma_r^2 r(y)\psi^2 y^4 \left(\frac{s}{l^2}\right) - \sigma_f^2 f(x)\omega^2 x^4 \left(\frac{s}{h^2}\right)\right] U_{j,i}^n \\
& + \left[\frac{1}{2}\sigma_f^2 f(x)\omega^2 x^4 \left(\frac{s}{h^2}\right) + \left[\sigma_f^2 f(x)\omega^2 x^3 - \omega x^2 \kappa_f(\theta_f - f(x))\right]\left(\frac{s}{2h}\right)\right] U_{j,i+1}^n \\
& + \left[\frac{1}{2}\sigma_f^2 f(x)\omega^2 x^4 \left(\frac{s}{h^2}\right) + \left[\sigma_f^2 f(x)\omega^2 x^3 - \omega x^2 \kappa_f(\theta_f - f(x))\right]\left(\frac{s}{2h}\right)\right] U_{j,i-1}^n \\
& + \left[\frac{1}{2}\sigma_r^2 r(y)\psi^2 y^4 \left(\frac{s}{l^2}\right) + \left[\sigma_r^2 r(y)\psi^2 y^3 - \psi y^2 \kappa_r(\theta_r - r(y))\right]\left(\frac{s}{2l}\right)\right] U_{j+1,i}^n \\
& + \left[\frac{1}{2}\sigma_r^2 r(y)\psi^2 y^4 \left(\frac{s}{l^2}\right) + \left[\sigma_r^2 r(y)\psi^2 y^3 - \psi y^2 \kappa_r(\theta_r - r(y))\right]\left(\frac{s}{2l}\right)\right] U_{j-1,i}^n \\
& + \rho\sigma_r\sigma_f \sqrt{r(y)}\sqrt{f(x)}\psi\omega y^2 x^2 \left(\frac{s}{4hl}\right)(U_{j+1,i+1}^n - U_{j-1,i+1}^n + U_{j+1,i-1}^n + U_{j-1,i-1}^n)
\end{aligned}
\tag{4.38}
$$

The coefficients of equation (4.38) are not only the model parameters but the values are changed across the grid by the transformed variables. In order to eliminate the stability problems and keep the errors bounded the coefficients of U^n terms should be positive in equation (4.38) [29]. Since the coefficients of the second derivative terms are always positive in equation (4.32) the coefficients of the first derivative terms must be focused on. According to Morton & Mayers [29] this problem can be eliminated by using forward or backward differences for the first derivative instead of using central differences. The appropriate difference method to use can be identified according to the sign of the coefficients. In this study, forward difference approximation is used when the coefficients of the first derivative term are positive and backward difference approximation is used when the coefficient of the first derivative term is negative. Two-dimensional PDE was used to solve the asset prices therefore there are four combinations of first derivative signs.

4.3.2.2 Upper Boundary Conditions in the Transformed State Variables

The specifications that are made on the boundary of the domain to have a unique and well-behaved solution for the equation are called boundary conditions. In order to simplify the solution of the model transformed variables were used but the relevant state variables are the original ones. Therefore, the discussion about boundary conditions should be done in terms of the original state variables. The upper boundary conditions in the transformed variables correspond to the lower boundary conditions in the original PDE where, $f = r = 0$.

All the formulation related to the boundary conditions needs to consider the degenerate versions of the original PDE (3.4). Transformation of the degenerate version of the PDE to the finite difference scheme will be done by using one of the difference methods.

The Exchange Rate Dimension: When $f = 0$ obligation of the borrower will also be zero when converted to domestic currency. In such a case foreign currency borrowing has no risk in terms of exchange rate conversion. In other words exchange rate risk is eliminated. Therefore value of all the assets will be zero and the following equations hold;

$$A(r, 0) = 0 \tag{4.39}$$

$$V_B(r, 0) = 0 \tag{4.40}$$

$$D(r, 0) = 0 \tag{4.41}$$

$$FD(r, 0) = 0 \tag{4.42}$$

The Spot Interest Rate Dimension: When $r = 0$ there is no discounting but exchange rate effect on the asset values still exists. Therefore, in the valuation of assets, following degenerate form of the main PDE (3.4) will be used. It should be noted about the following equation that although the interest rate is null there exists a first derivative in r, because this element is not multiplied by the variable:

$$\frac{\partial F}{\partial t} + \kappa_r \theta_r \frac{\partial F}{\partial r} + \kappa_f (\theta_f - f) \frac{\partial F}{\partial f} + \frac{1}{2} \sigma_f^2 f \frac{\partial^2 F}{\partial f^2} = 0 \tag{4.43}$$

4.3.2.3 Lower Boundary Conditions in the Transformed State Variables

Following the same logic stated in the previous subsection lower boundary conditions in the transformed PDE correspond to upper boundary conditions of the original PDE. Therefore, in this section cases in which $f \to \infty$ and $r \to \infty$ will be analyzed.

The Exchange Rate Dimension: When $f \to \infty$, values of default and financial distress will be different than 0. But both of their values depend on the value of future payments. Thus, $\lim_{f \to \infty} F(r, f, t)$ will be determined by the following degenerate PDE:

$$\frac{\partial F}{\partial t} + \kappa_r(\theta_r - r)\frac{\partial F}{\partial r} + \kappa_f\theta_f\frac{\partial F}{\partial f} + \frac{1}{2}\sigma_r^2 r\frac{\partial^2 F}{\partial r^2} - rF = 0 \tag{4.44}$$

The Spot Interest Rate Dimension: When $r \to \infty$ any future payment does not have any value. As a result the values of all other assets are equal to zero:

$$\lim_{r\to\infty} A(r, f) = 0 \tag{4.45}$$

$$\lim_{r\to\infty} V_B(r, f) = 0 \tag{4.46}$$

$$\lim_{r\to\infty} D(r, f) = 0 \tag{4.47}$$

$$\lim_{r\to\infty} FD(r, f) = 0 \tag{4.48}$$

4.3.2.4 Corners of the Grid

Both state variables can take values in the $[0, \infty]$ interval. In order to make calculations easier those intervals are mapped to $[0, 1]$ interval and a unit grid is obtained. Therefore corners of the grid corresponds to points where both original state variables take the combination of extreme values in which $f = 0$ or $f \to \infty$ and $r = 0$ or $r \to \infty$.

Corners in the Upper Boundary of the Transformed Interest Rate Dimension of the Grid: In this case interest rate is null therefore there is no discounting and exchange rate takes the extreme values $f = 0$ or $f \to \infty$. When $r = 0$ and $f = 0$ the value of any asset can be found by the following equation:

$$F(0, 0, t) = F(\kappa_r\theta_r s, \kappa_f\theta_f s, t + s) \tag{4.49}$$

When $f \to \infty$, the value of all assets can be found by:

$$\lim_{f\to\infty} F(0, f, t) = F(\kappa_r\theta_r s, f, t + s) \tag{4.50}$$

Corners in the Lower Boundary of the Transformed Interest Rate Dimension of the Grid: Those are the points in which $r \to \infty$. The value of future payments will be 0. Thus the value of assets that depend on future payments will be also 0. Consequently, the values of assets are defined by the following equations:

$$\lim_{r \to \infty} A(r, f) = 0 \tag{4.51}$$

$$\lim_{r \to \infty} V_B(r, f) = 0 \tag{4.52}$$

$$\lim_{r \to \infty} D(r, f) = 0 \tag{4.53}$$

$$\lim_{r \to \infty} FD(r, f) = 0 \tag{4.54}$$

CHAPTER 5

ANALYSIS OF THE FOREIGN CURRENCY LOAN VALUATION MODEL RESULTS

5.1 Introduction

Securities that are traded on stock exchanges and developed over-the-counter markets are standardized products having common features. The parameters used in the valuation of such products do not vary much from one product to another. On the other hand, not only foreign currency denominated corporate loan contracts but also all types of corporate loan contracts have specifications varying according to some features of the company. Basically, the country, where the firm is established, sector of the business running, size of the firm and the purpose of the credit taken affects the contract terms [45]. Parameters affecting the valuation procedure are classified as systematic parameters and unsystematic parameters. Table 5.1 represents the parameter values used in the construction of base case economic environment (systematic parameters) and basic properties of the loan contract (unsystematic parameters). Sensitivities of the asset values to the changes in these parameter values are tested along with the analysis of financial implications of the changes.

This chapter aims to provide and discuss the numerical results provided by the model that was presented in Chapter 3. Section 5.2 presents an overview of the valuation functions for each of the variables under study and series of figures indicating the behavior of these functions for different values of the state variables at the origination of the contract given. Section 5.3 develops a mathematical analysis of the effects induced by changes in the parameters that are used to characterize the economic environment as well as the company specifications, in terms of the valuation of the assets under study. Finally section 5.4 presents the financial interpretations of the consequences induced by changes in the parameters of the contracts in terms of the valuation functions.

5.2 Overview of the Valuation Functions

We produce 3-D figures for each of the related assets (value of loan, value of future payments, value of default and value of financial distress) in order to show that the numerical solutions vary across the state space smoothly, without causing any instability and that the structure of the graphs makes sense in economic terms. Figures from 5.1 to 5.8 give a graphical image of the value of what assumed by the valuation functions associated with the contract specifications under study along the state space at the contract origination. Each presented figure is produced based on the (36×45) dimensional grid located in the economically reasonable part of the corresponding grid with dimensions (51×51). According to the (36×45) dimensional grid interest rate and exchange rate are in the intervals $[0.0020, 0.9000]$ and $[0.0305, 3.8380]$ respectively.

Figures 5.1 and 5.2 present the variable A for initial leverage ratios of 80% and 90% respectively. The resulting figure verifies the expectations for both interest rate dimension and exchange rate dimension. It is seen that value of future payments, being independent of the initial capital structure of the firm, is inversely related with the interest rate and in direct relationship with the exchange rate.

Value of the loan, V, is a function of A and D. High exchange rates triggers the event of default and increase the value of default, D. In the regions where default has positive value, V stabilizes in the value equal to total assets of the firm (Figure 5.3, Figure 5.4). Revealing the fact that in the event of default firm can pay as much as its assets independent of exchange rate increase. The effect of initial foreign currency denominated loan to total assets ratio alters the nodes where the value stabilization begins. Examining the value of the contract in terms of state variables, it is seen that until default occurs, the relations between the contract value, interest rate and exchange rate are influenced by A.

Value of the default option, D, is exhibited in Figure 5.5 and Figure 5.6. It is derived from the figures that default tends to occur in the region where interest rates are low and exchange rates are high. For the interest rate dimension, our findings are consistent with the claims on Moody's analytics report [34], which states that if the economy is in recession and the default rate is high, interest rates are often relatively low due to the traditional central bank monetary policy of lowering rates in order to awaken the economy. When the economy improves, the central bank tends to raise the rates, correspondingly.

Since an increase in interest rate (discount rate) decreases the value of both A and V, the dominant state variable affecting the value of D is exchange rate. Moreover, the default region changes according to the initial LTV ratios. With higher initial LTV ratios default occurs for the lower values of exchange rate compared to lower LTV ratios at the origination of the contract.

Finally, Figure 5.7 and 5.8 show the value of financial distress, FD, for the different values of state variables. FD has similar characteristics with the implicit put option, but it gets positive values when the value of future payments exceeds the distress threshold. Therefore, even in low exchange rates, financial distress is a problem for the firm due to foreign currency denominated loan. In the interest rate dimension, being related with A, value of FD decreases with the increase of interest rate.

5.3 Mathematical Interpretations of the Model

As noted earlier in the introduction section, the economic environment is described by the parameters given in Table 5.1. The exchange rate data required for this study is gathered from the Central Bank of the Republic of Turkey and the exchange rate parameters are estimated by employing Maximum Likelihood Estimation method. Interest rate parameters are taken from the study of Erol [44]. In this section effects of changes in the parameter values on the numeric solutions of the asset values will be analyzed.

5.3.1 Interest Rate Volatility

In Chapter 3 all the assets under study are modeled as a function of interest rate. Therefore, fluctuations in interest rate affect all the assets. Table 5.3 and 5.4 shows the sensitivities of the foreign currency denominated loan contract and related assets' values to changes in interest rate volatility, σ_r for different exchange rates.

Value of future payments, which affects the values of all the assets under study, increases as the volatility of interest rate increase. In other words, high and low levels of discount rate arising from an increment in σ_r leads to an increase in the value of A. Rising discount rates decrease the present value of expected cash-flows whereas decline in interest rates increase its value. Nevertheless, the downward changes in interest rates have more influence on the present value of expected cash-flows than upward changes in the same degree, which can be explained by the Jensen's inequality. Jensen's inequality states that expected value of a convex function of a random variable is greater than the convex function of an expectation of a random variable.

The values of default and financial distress are affected by the value of the future payments. As the volatility of interest rate and the value of future payments increase, values of D and FD also increase.

Interest rate volatility and the value of the contract have an inverse relationship. This is an unexpected result since the equation (3.8) holds and both the value of the future payments and the value of default have direct relationship with the volatility of interest rate. However, an increment in σ_r influences the value of D more than it affects the value of A.

5.3.2 Exchange Rate Volatility

Impact of changes in exchange rate volatility on the assets under study has some different characteristics with the impact of interest rate volatility. Also, exchange rate and interest rate increases and decreases affect the asset values in the model in a different way. When the interest rate increases the values of all the assets decrease whereas when exchange rate increases the value of all the assets also increase (see Table 5.5 and Table 5.6).

For the high levels of exchange rate volatility, σ_f, the value of future payments increases but for the low levels of σ_f the increase in the value of A can not be sustained. According to our numerical results, continuous increase in the value of expected payoffs occurs when the volatility of exchange rate takes values higher than 0.30.

Value of financial distress shows similar characteristic with the value of A. In chapter 3 it is stated that if the value of future payments, A, surpasses a certain predefined distress barrier then this situation induces financial distress for the firm. Therefore, value of FD having direct relationship with the value of A increases as the volatility of exchange rate increases to high levels. Although value of the default option, D, is also in direct relation with A, being in inverse relation with the value of the contract, V, keeps its value to increase continuously as σ_f increases. Value of the loan decreases as the exchange rate volatility increases due to the equation (3.8) given in chapter 3 since the changes in default value is greater than the changes in the value of future payments for the same amount of increase in σ_f.

5.3.3 Correlation Between Wiener Processes

In Table 5.7 and Table 5.8 numerical results influenced with the change in correlation coefficient between Wiener processes, ρ, are presented. Tables show that as ρ increases, the value of the assets decrease, which implies inverse relation between r and f. In other words, for the high values of r default does not tend to occur since increasing discount rate reduces the value of A. However, when r is low and f is high default is more likely to happen. As a consequence these factors creates a tendency for the value of default to evolve in inverse relationship with ρ. Change in state variables affect the value of future payments in the opposite direction and all the assets valued in this study have direct relationship with the value of A. Therefore, it is an expected result to have negative relation between the correlation coefficient and the values of the assets.

5.3.4 Initial Loan to Value Ratio

Tables 5.9 to 5.12 represents the value of assets (V, A, D, FD) for different levels of loan to value ratios and for different levels of exchange rates, $f = 1.8996$, $f = 2.2388$, $f = 2.6534$ and $f = 3.1716$. Recent developments show that Turkish lira considerably loses value, devaluates against dollar, euro and pound. According to recent exchange rate levels, $f = 1.8996$ can be thought as Turkish lira to dollar, $f = 2.2388$ and $f = 2.6534$ as Turkish lira to euro, finally $f = 3.1716$ can be thought as turkish lira to pound exchange rates. Empirical data shows that for utilities sector average FX-loan to total asset ratio was nearly 55% in

2009, when we began this study. Therefore, we tested different levels of LTV ratios between 55% – 100%.

LTV ratio is used to determine the amount of the loan. Therefore, for higher LTV ratios value of A will be higher. As LTV ratio gets closer to 1, the conditions for the value of A to pass the value of total assets of the firm is created. Following with the event of default defined in chapter 3 tends to occur. For the higher values of LTV ratios and exchange rates the value of the loan stays bounded by the value of the assets of the firm. Also for higher levels of these parameters default and financial distress have significantly higher values.

5.3.5 Financial Distress Barrier

Threshold value for foreign currency loans to total assets ratio indicates the financial distress barrier for the firm. According to model and valuation functions constructed in Chapter 3, it is expected to see the effect of changing the threshold only on the value of Financial Distress. Hence, the numeric results presented in Table 5.13 are anticipated results. Also from Table 5.14 it can be seen that the effect of increasing threshold values by 0.05 on the value of FD is in the same amount with the increment for different LTV-ratios. Those two tables are presented to show that our model works correctly.

5.4 Financial Implications of the Model

The main aim of this study is to identify the behaviors of default and financial distress in foreign currency denominated loans of non-financial firms according to their capital structure. Financial leverage is defined as the ratio of total liabilities to total assets of the firm. It is generally accepted that when this ratio gets closer to one then it is said that the firm is in financial distress, which is a signal to default for the firm. It is expected to observe the effects of foreign borrowing in firms whose borrowings are mostly in foreign currency. Most specifically, in order to identify the financial distress induced by foreign currency denominated loans clearly, that is to say to eliminate the other effects causing financial distress as much as possible, we focused on the sectors whose foreign currency denominated loan to total loans ratio is high. It is also expected that for such firms, effects of fluctuations in exchange rates on the capital structure of the firm can be seen directly. Following these arguments parameters presented in Table 5.1 are determined.

The firms in Electricity, Gas and Water Sources, or the utilities sector whose stocks are traded in ISE100 have the highest foreign currency denominated loan to total loans ratio that is 93%, among other sectors. Tourism is the second sector with the ratio of 78% and Metal Industries is the third sector with the ratio of 70% according to this listing [4]. The historical, quarterly financial statements of the firms in these sectors are obtained from Istanbul Stock Exchange website. From notes to financial statements foreign currency denominated loan composition of the firms are obtained and currency loan to total assets ratio is calculated for the period 2001 to 2009. The effect of foreign borrowing can differ according to size of the company. We classified the utilities sector firms as ”big” and ”small” considering their total asset values. Having classified the companies according to their asset values we identified distress barriers for these firms following the work of Opler & Titman [26]. The authors defined the firms having debt to asset ratio in deciles 8 to 10 as being highly leveraged. Our data shows that for the firms with higher asset values, total debt to asset ratio equals to 60% in decile 8. Therefore, it is reasonable to assume that when the foreign currency loan to total asset ratio exceeds the sector's total debt to total asset ratio in decile 8 there occurs a financial distress for the firm. Based on this assumption we set the distress barrier as 60% for companies with total asset value greater than 600M TL in energy sector. Table 5.2 shows the table of leverage ratios to relative deciles for those firms.

Firms that mostly rely on debt financing and having high volumes of foreign currency denominated loans raised their fx-borrowing to total asset ratios up to 80% between the years 2001 and 2009. In the light of these findings we took 80% as the base value for loan-to-value (LTV) ratio. However, we test the different compositions of foreign currency denominated loans to total assets ratio and identified the distress percentages for respective ratios.

Disclosed notes to financial statements show that the maturity of foreign currency loans clustered up to five years. Also, foreign borrowings are mostly denominated in dollar. Hence we choose the maturity of the loan as 60 months and initial exchange rate as 1.523 FX/TL for calculations used in MATLAB code (see Appendix B).

5.4.1 Effect of LTV Ratio

In this model LTV ratio identifies the initial loan amount as a domestic-currency loan. All asset values are in domestic currency and indicate the ratio of the valued asset to total assets of the firm. Specifically, when exchange rate is 2.6534 times the domestic currency and LTV ratio is 80%, the value of financial distress due to foreign currency denominated loan is 50% of the total assets of the firm, value of the default is 11% of the total assets of the firm, value of the remaining payments of the firm and the value of the currency loan is 110% and 99% in terms of the total assets of the firm, respectively (see Table 5.11).

In tables 5.9 to 5.12 it is seen that as the borrowing in foreign currency increases, the value of FD and D increase to levels that cannot be neglected. For lower exchange rate values, the value of financial distress reaches to 40% levels and the value of default stays in manageable levels. As exchange rate increases, the value of FD increases significantly and the value of D does not stay in manageable levels. Moreover, for higher values of exchange rates, value of FD is considerably high even for the LTV ratios around the distress barrier (see Tables 5.11 and 5.12). Default gets non-zero values for even low LTV ratios as FX-rate increases. However, for small values of D we cannot jump into conclusion that the firm experiences default. In reality, default event occurs only if corporations are borrowing very high LTV loans with considerably high FX-rates.

5.4.2 Effect of Volatilities

The valuation functions of the derivative assets are constructed considering the main risk factors. Fluctuations in exchange rate and interest rate are determined as the main factors affecting the values of the financial distress and the default. The variations of state variables are included to model with the parameters σ_f and σ_r. The analysis of sensitivities of the numerical results to change in volatility values is presented in the previous section and it is stated that both of the volatilities of the state variables are in direct relationship with the values of FD and D.

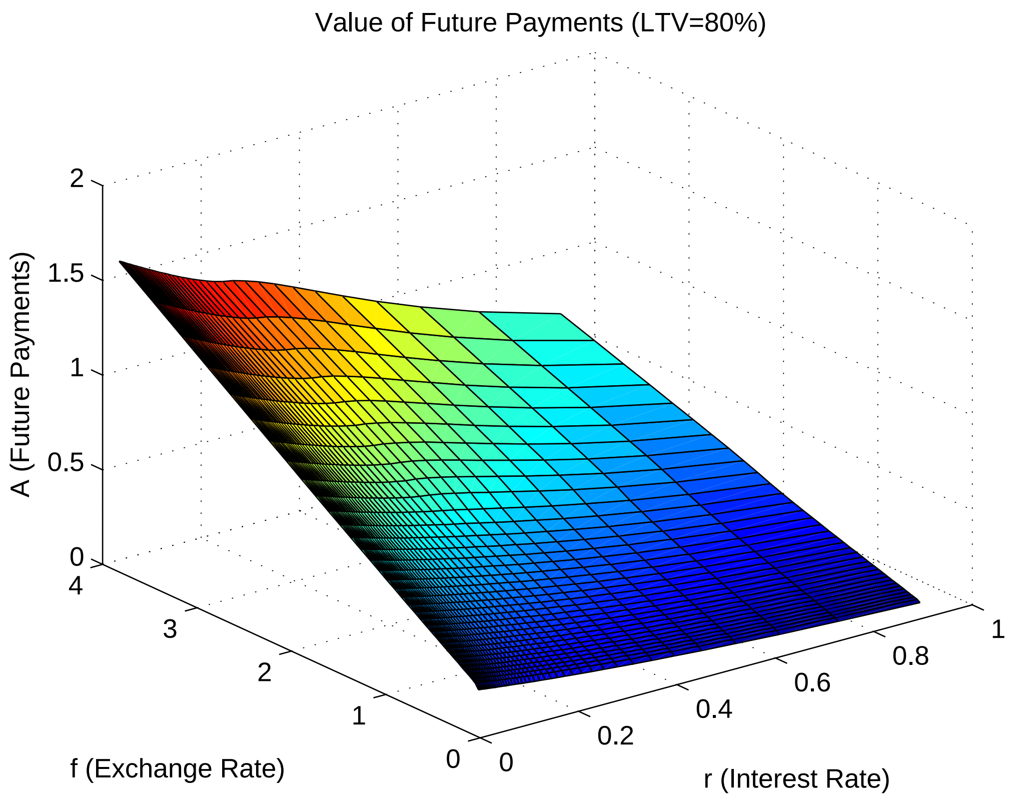
In tables 5.4 and 5.6, standalone effects of changes in the volatilities of interest rate and exchange rate on asset values are presented, respectively. Our results reveal that interest rate volatility is reasonably more effective on financial distress and default risks in comparison to exchange rate volatility. In detail, change in interest rate volatility by 0.03 changes the distress value and the default value higher compared to 0.03 change in exchange rate volatility does. On the other hand, it should be noted that the numerical results show that up to certain level of increase in σ_r the value of the assets increase. After that certain level, which is nearly 0.39 in this model, increase in interest rate volatility decreases the value of the assets sharper than while it is increasing. However, it is very unlikely for interest rate volatility to reach that levels. Furthermore, as exchange rate volatility takes high values, increase in the value of FD can be seen clearly. However, still for those levels of exchange rate volatility, the effect of σ_f on FD values is less than the effect of change in σ_r.

In summary, the effect of fluctuations of discount rate on the value of financial distress is dominant compared to the effect of variances in exchange rate. This result can be interpreted in two ways. First, since exchange rate and interest rate have inverse relation with each other when the variance for interest rate is in negative direction, the variance in exchange rate is expected to be in positive direction. Both of them are the cases that increase the value of financial distress. Moreover, increment in exchange rate increases the value of FD sharper than the decrease in r.On the other hand, for the lower values of exchange rate FD has no value. Although increase in exchange rate increases the value of FD sharper than the decrease in r, for exchange rate movements in opposite direction, for lower exchange rates FD has no value. In total, the effect of exchange rate volatility on the value of financial distress will be less compared to the effect of interest rate volatility. Second, if we consider the volatility in terms of upside risk in exchange rate dimension we see that the effect of a rise in exchange rate on the value of FD is sharper than a decrease in the value of r. In Turkey, we commonly observe upward movements in the exchange rate. Considering the exchange rate trends in Turkey and our results, which reveal the importance of the currency devaluation risk for FX-borrowing, we identify the exchange rate increase as the main risk factor for corporations that are heavily financed by FX-loans.

If volatility in interest rates leads to higher interest rates in the market, as commonly observed in Turkey, mathematically the present value of credit payments will be lower due to a higher discount factor. Correspondingly, lower payments will not result in financial distress for the corporations. Besides, higher interest rates should be explained as the higher opportunity cost of capital from financial perspective. As the cost of borrowing in domestic currency increases (high interest rates), we would expect to see lower financial distress for corporations that are heavily financed by FX-loans. Furthermore, as the cost of borrowing in foreign currency is lower than the domestic currency borrowing, in such an environment firms tend to issue more FX-loans. Turkish economy experienced that case after 2001 when the cost of borrowing in domestic currency was high firms started to issue more FX-loan, which led to higher FX-loan to total asset ratios for the firms.

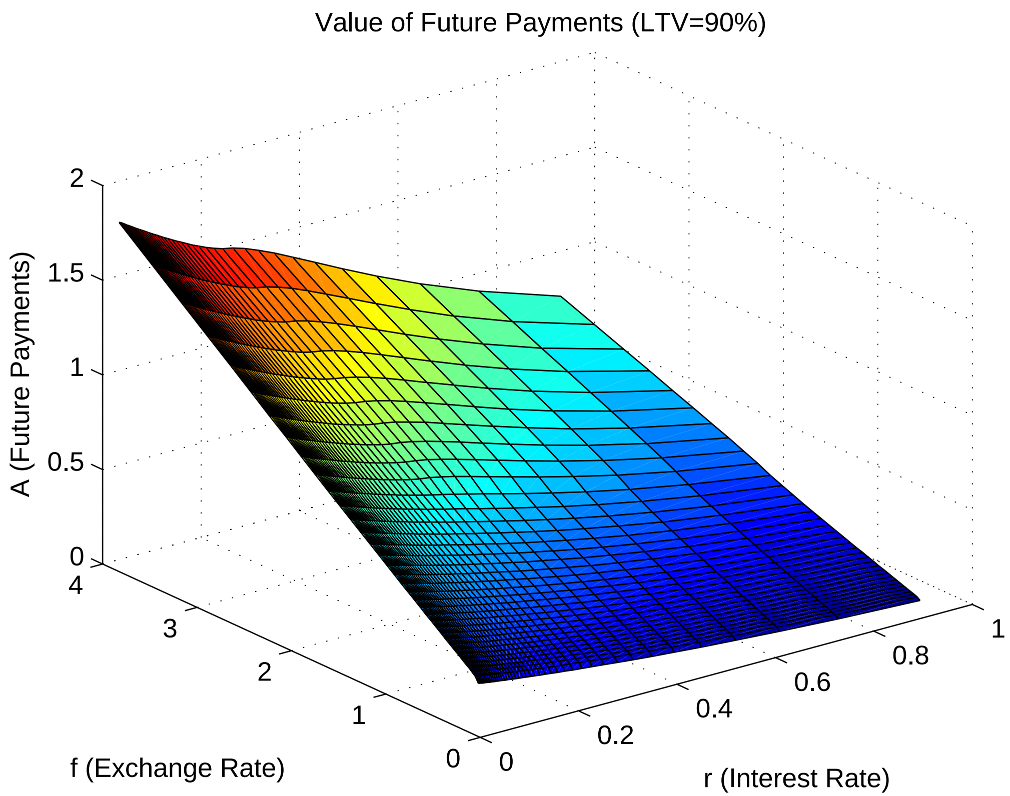
Overall, our results demonstrate that corporations that are heavily financed by FX-loans can easily experience financial distress under volatile economies like Turkey. We argue that under the volatile financial environment of Turkey, utilities sector has experienced financial distress as long as TL devaluates against US dollar and euro. In this study calculations are made based on the assumption that FX-loan positions of the firms are unhedged and asset value of the firm is constant. In that respect, the values of FD and D can be defined as the amounts of hedged positions, that firms should have in their capital structure to avoid financial distress and default, in terms of total assets value of the firms. Moreover, in financial perspective, the default event occurs only if corporations are borrowing very high LTV loans (like 95% – 100%) with considerably high exchange rates when volatility of interest rate is high. In detail, Table 5.10 shows that for the exchange rate (FX-rate) of 2.2388, 100% LTV loans result in a significant default value of 21% of the total asset value of the firm. Table 5.11 shows that for the FX-rate of 2.6534, not only 100% LTV loans but also 95% LTV loans result in default values of 38.5% and 31.6% of total asset value of the firm, respectively. Finally, the default is likely to occur with 85% – 100% LTV loans when FX-rate is at the 3.1716 level. Under these circumstances, the value of default reaches up to 36% to 60% of asset value of the firm.

Figure 5.1: Value of Future Payments (A) (LTV=0.80)



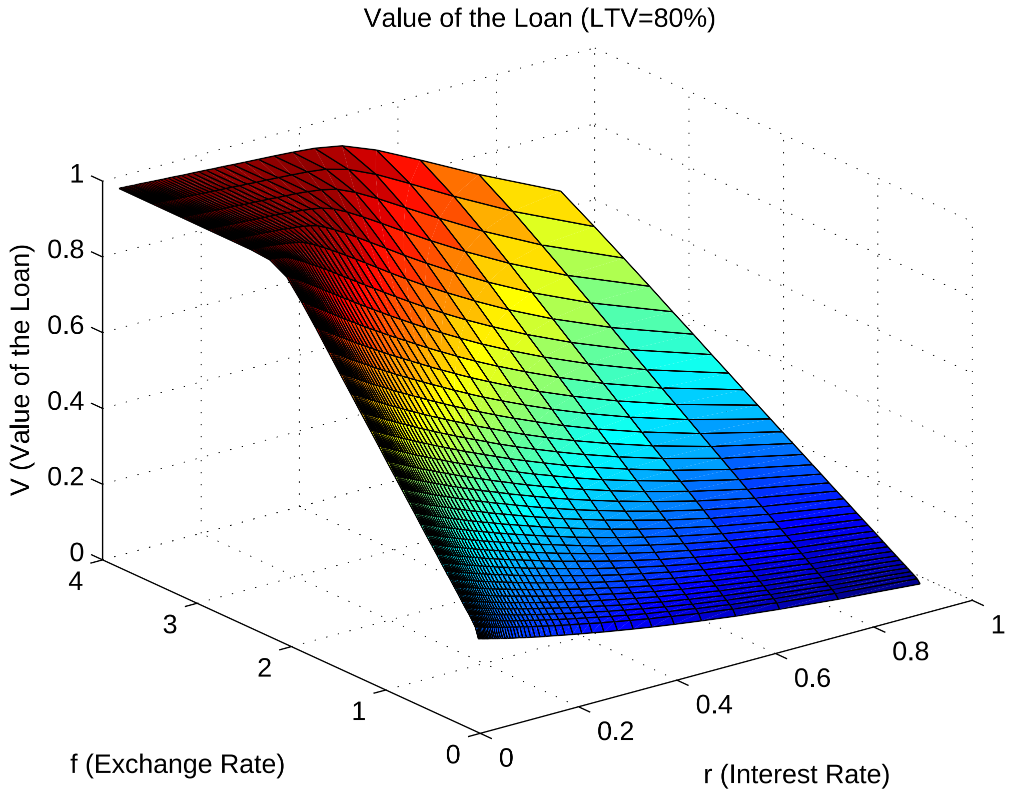
The following parameters were used in the construction of this figure: the spot interest rate r is 10%, the exchange rate f_{int} is 1.5230, long-term mean reversion rate of interest rate κ_r is 0.56, long-term mean reversion rate of exchange rate κ_f is 0.15, long term mean of interest rate θ_r is 0.24, long term mean of exchange rate θ_f is 1.86, the interest rate volatility σ_r is 0.12, the exchange rate volatility σ_f is 0.21, financial distress threshold is 0.60, LTV ratio is 0.80, the correlation coefficient ρ is 0

Figure 5.2: Value of Future Payments (A) (LTV=0.90)



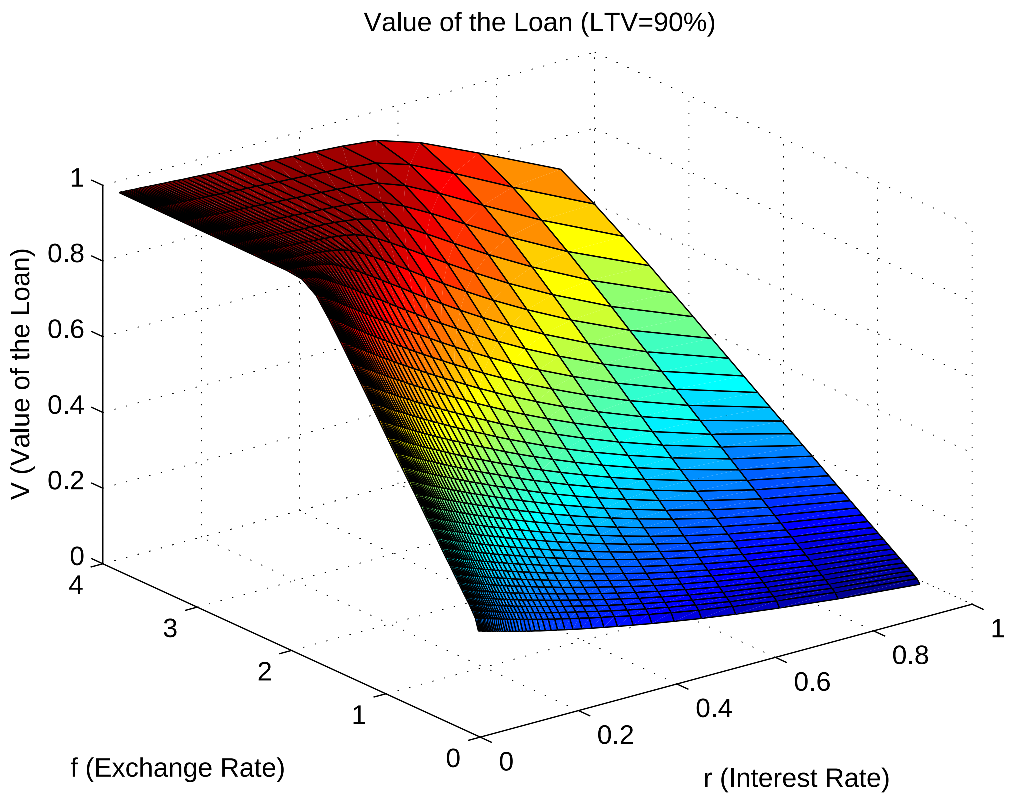
The following parameters were used in the construction of this figure: the spot interest rate r is 10%, the exchange rate f_{int} is 1.5230, long-term mean reversion rate of interest rate κ_r is 0.56, long-term mean reversion rate of exchange rate κ_f is 0.15, long term mean of interest rate θ_r is 0.24, long term mean of exchange rate θ_f is 1.86, the interest rate volatility σ_r is 0.12, the exchange rate volatility σ_f is 0.21, financial distress threshold is 0.60, LTV ratio is 0.90, the correlation coefficient ρ is 0

Figure 5.3: Value of Loan (V) (LTV=0.80)



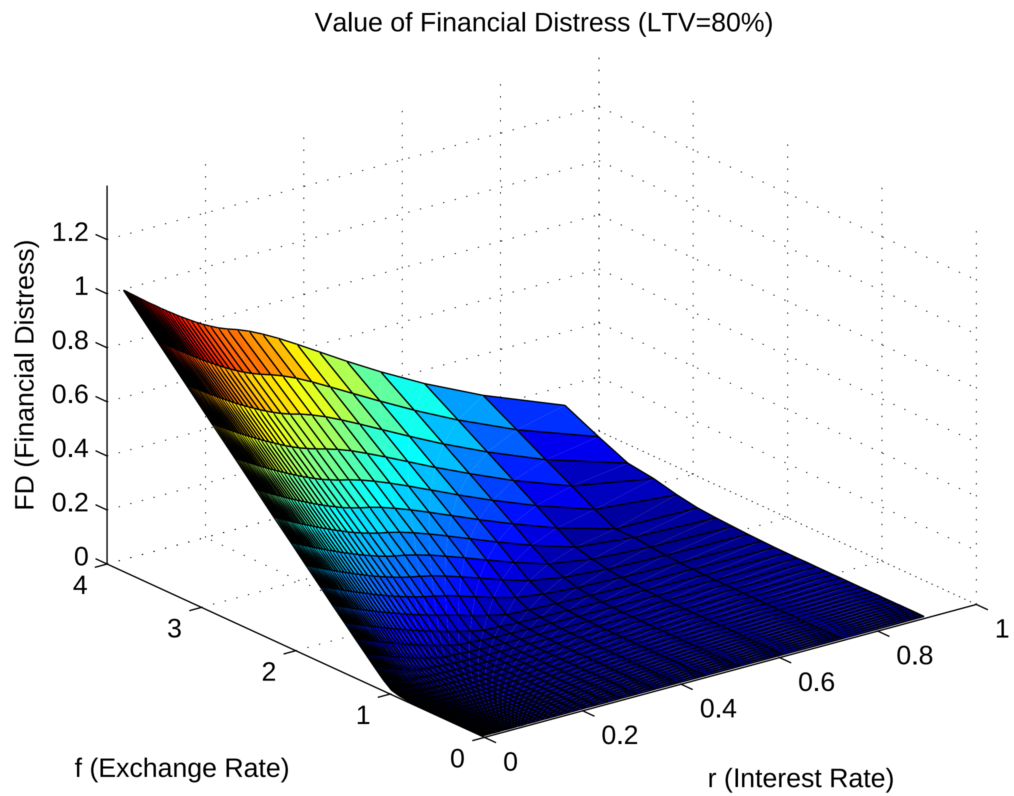
The following parameters were used in the construction of this figure: the spot interest rate r is 10%, the exchange rate f_{int} is 1.5230, long-term mean reversion rate of interest rate κ_r is 0.56, long-term mean reversion rate of exchange rate κ_f is 0.15, long term mean of interest rate θ_r is 0.24, long term mean of exchange rate θ_f is 1.86, the interest rate volatility σ_r is 0.12, the exchange rate volatility σ_f is 0.21, financial distress threshold is 0.60, LTV ratio is 0.80, the correlation coefficient ρ is 0

Figure 5.4: Value of Loan (V) (LTV=0.90)



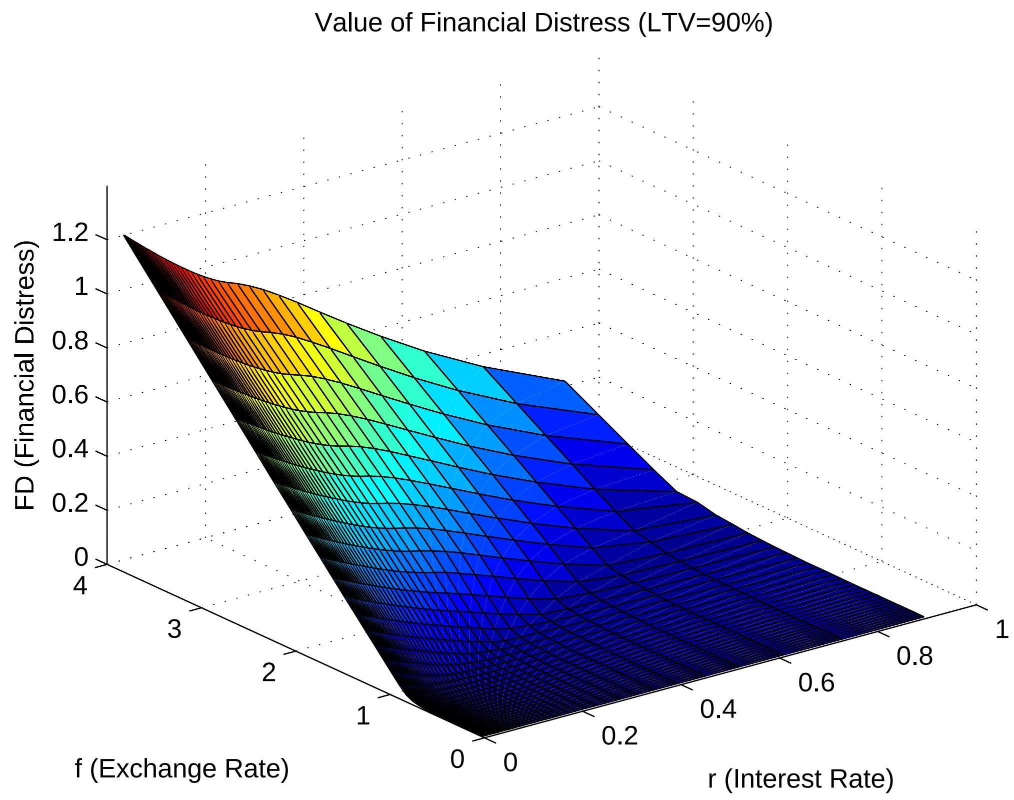
The following parameters were used in the construction of this figure: the spot interest rate r is 10%, the exchange rate f_{int} is 1.5230, long-term mean reversion rate of interest rate κ_r is 0.56, long-term mean reversion rate of exchange rate κ_f is 0.15, long term mean of interest rate θ_r is 0.24, long term mean of exchange rate θ_f is 1.86, the interest rate volatility σ_r is 0.12, the exchange rate volatility σ_f is 0.21, financial distress threshold is 0.60, LTV ratio is 0.90, the correlation coefficient ρ is 0

Figure 5.5: Value of Financial Distress (FD) (LTV=0.80)



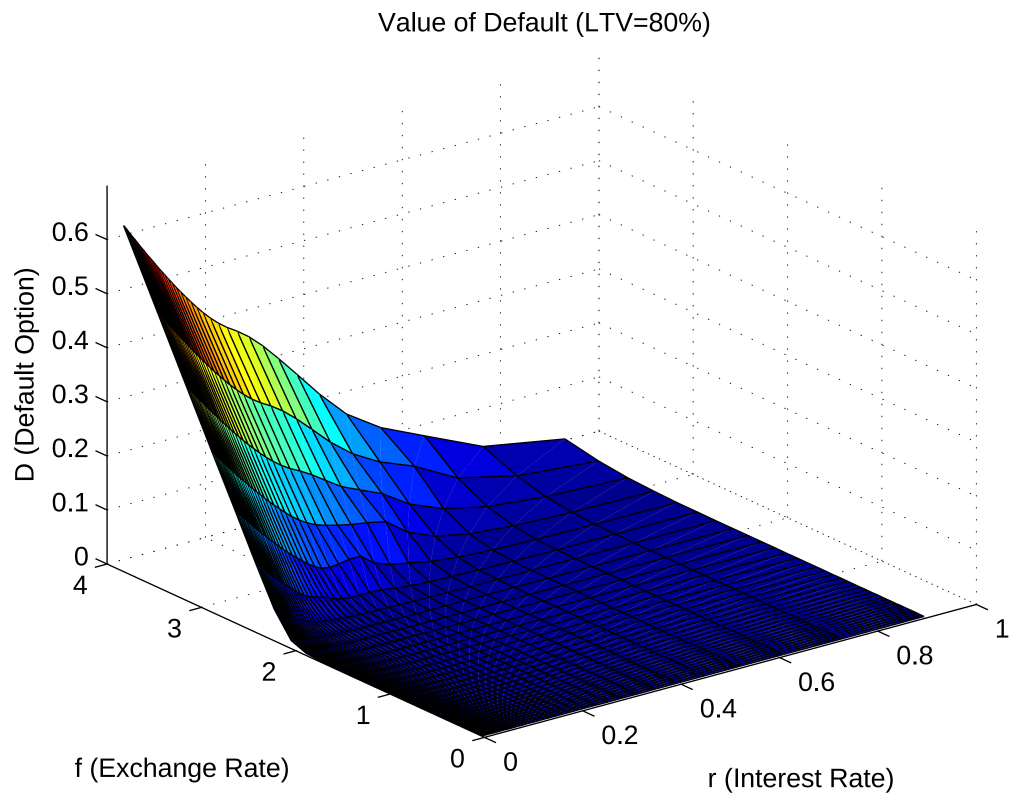
The following parameters were used in the construction of this figure: the spot interest rate r is 10%, the exchange rate f_{int} is 1.5230, long-term mean reversion rate of interest rate κ_r is 0.56, long-term mean reversion rate of exchange rate κ_f is 0.15, long term mean of interest rate θ_r is 0.24, long term mean of exchange rate θ_f is 1.86, the interest rate volatility σ_r is 0.12, the exchange rate volatility σ_f is 0.21, financial distress threshold is 0.60, LTV ratio is 0.80, the correlation coefficient ρ is 0

Figure 5.6: Value of Financial Distress (FD) (LTV=0.90)



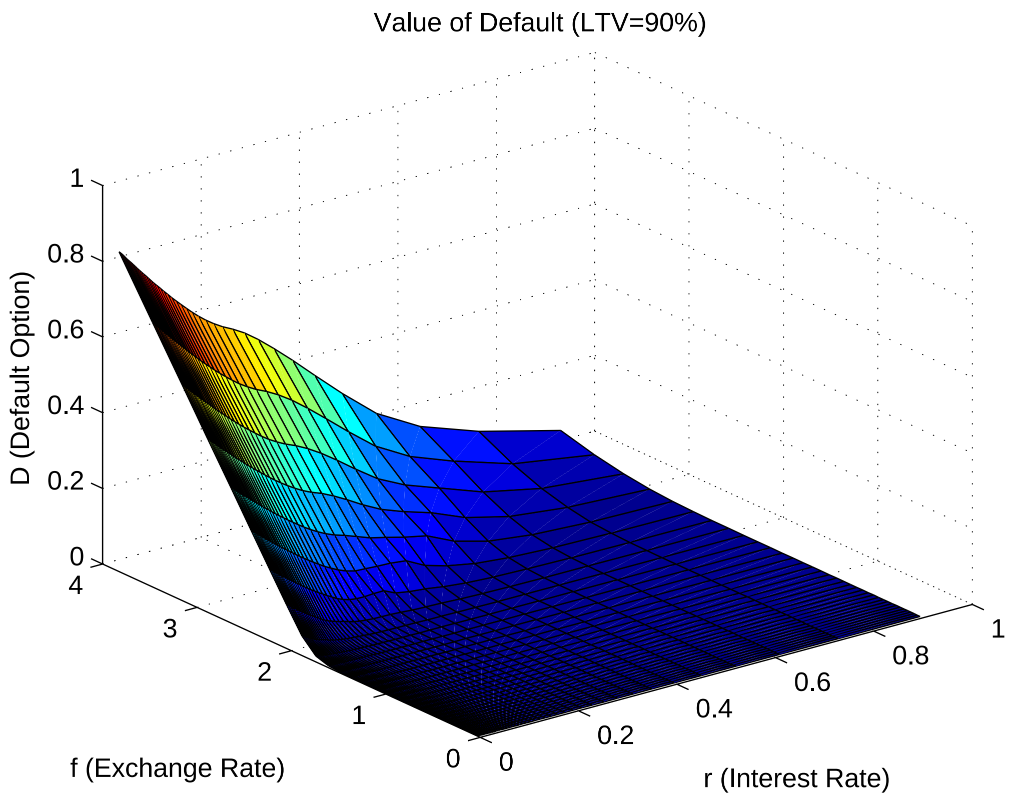
The following parameters were used in the construction of this figure: the spot interest rate r is 10%, the exchange rate f_{int} is 1.5230, long-term mean reversion rate of interest rate κ_r is 0.56, long-term mean reversion rate of exchange rate κ_f is 0.15, long term mean of interest rate θ_r is 0.24, long term mean of exchange rate θ_f is 1.86, the interest rate volatility σ_r is 0.12, the exchange rate volatility σ_f is 0.21, financial distress threshold is 0.60, LTV ratio is 0.90, the correlation coefficient ρ is 0

Figure 5.7: Value of Default (D) (LTV=0.80)



The following parameters were used in the construction of this figure: the spot interest rate r is 10%, the exchange rate f_{int} is 1.5230, long-term mean reversion rate of interest rate κ_r is 0.56, long-term mean reversion rate of exchange rate κ_f is 0.15, long term mean of interest rate θ_r is 0.24, long term mean of exchange rate θ_f is 1.86, the interest rate volatility σ_r is 0.12, the exchange rate volatility σ_f is 0.21, financial distress threshold is 0.60, LTV ratio is 0.80, the correlation coefficient ρ is 0

Figure 5.8: Value of Default (D) (LTV=0.90)



The following parameters were used in the construction of this figure: the spot interest rate r is 10%, the exchange rate f_{int} is 1.5230, long-term mean reversion rate of interest rate κ_r is 0.56, long-term mean reversion rate of exchange rate κ_f is 0.15, long term mean of interest rate θ_r is 0.24, long term mean of exchange rate θ_f is 1.86, the interest rate volatility σ_r is 0.12, the exchange rate volatility σ_f is 0.21, financial distress threshold is 0.60, LTV ratio is 0.90, the correlation coefficient ρ is 0

Table 5.1: Base Values

PARAMETERS			
Systematic Parameters		**Unsystematic Parameters**	
Spot Interest Rate, $r(0)$	10%	Maturity η	60 months
Long-term mean reversion rate of interest rate κ_r	0.56	Initial loan to value ratio	0.80
long term mean of interest rate θ_r	0.24	Financial Distress Threshold	0.60
Interest rate volatility σ_r	0.12		
Initial exchange rate f_{int}	1.5230 $\$/TL$		
long-term mean reversion rate of exchange rate κ_f	0.15		
long term mean of exchange rate θ_f	1.86		
exchange rate volatility σ_f	0.21		
Correlation coefficient ρ	0		

Table 5.2: Total Liabilities to Total Assets Ratio

Decile	$V_A \geq 600$ **M TL**	$V_A \leq 600$ **M TL**
0.1	22%	53%
0.2	27%	54%
0.3	31%	56%
0.4	35%	60%
0.5	38%	62%
0.6	42%	63%
0.7	46%	67%
0.8	60%	71%
0.9	76%	73%
1	105%	77%

Above ratios represents the leverage ratios of the firms in energy sector traded at ISE for the period between 2001 and 2009

Table 5.3: Changes in Volatility of Interest Rate (f = 1.8996)

Volatility	Financial Distress	Default	Future Payments	Loan
σ_r	FD	D	A	V
0.03	0.2426	0.0003	0.8373	0.8369
0.06	0.2432	0.0004	0.8379	0.8368
0.09	0.2479	0.0005	0.8426	0.8365
0.12	0.2518	0.0007	0.8464	0.8361
0.15	0.2562	0.0009	0.8508	0.8354
0.18	0.2699	0.0019	0.8645	0.8343
0.21	0.2867	0.0043	0.8811	0.8328
0.24	0.3050	0.0090	0.8994	0.8309
0.27	0.3235	0.0170	0.9179	0.8283
0.30	0.3413	0.0343	0.9357	0.8249
0.33	0.3569	0.0520	0.9517	0.8204
0.36	0.3797	0.0751	0.9757	0.8142
0.39	0.3829	0.0824	0.9796	0.8054

The following parameters were used in the construction of this table: the spot interest rate r is 10%, the exchange rate f is 1.8996, long-term mean reversion rate of interest rate κ_r is 0.56, long-term mean reversion rate of exchange rate κ_f is 0.15, long term mean of interest rate θ_r is 0.24, long term mean of exchange rate θ_f is 1.86, the exchange rate volatility σ_f is 0.21, financial distress threshold is 0.60, LTV ratio is 0.80, the correlation coefficient ρ is 0

Table 5.4: Changes in Volatility of Interest Rate (f = 2.2388)

Volatility	**Financial Distress**	**Default**	**Future Payments**	**Loan**
σ_r	FD	D	A	V
0.03	0.3549	0.0066	0.9496	0.9429
0.06	0.3556	0.0069	0.9502	0.9426
0.09	0.3617	0.0091	0.9563	0.9421
0.12	0.3665	0.0112	0.9611	0.9412
0.15	0.3721	0.0142	0.9667	0.9401
0.18	0.3894	0.0271	0.9839	0.9384
0.21	0.4102	0.0585	1.0047	0.9364
0.24	0.4326	0.0858	1.0270	0.9337
0.27	0.4550	0.1111	1.0493	0.9304
0.30	0.4760	0.1427	1.0705	0.9263
0.33	0.4944	0.1662	1.0892	0.9212
0.36	0.5215	0.1998	1.1175	0.9144
0.39	0.5257	0.2128	1.1224	0.9053

The following parameters were used in the construction of this table: the spot interest rate r is 10%, the exchange rate f is 2.2388, long-term mean reversion rate of interest rate κ_r is 0.56, long-term mean reversion rate of exchange rate κ_f is 0.15, long term mean of interest rate θ_r is 0.24, long term mean of exchange rate θ_f is 1.86, the exchange rate volatility σ_f is 0.21, financial distress threshold is 0.60, LTV ratio is 0.80, the correlation coefficient ρ is 0

Table 5.5: Changes in Volatility of Exchange Rate (f = 1.8996)

Volatility	Financial Distress	Default	Future Payments	Loan
σ_f	FD	D	A	V
0.03	0.2525	0	0.8472	0.8360
0.06	0.2514	0	0.8461	0.8361
0.09	0.2506	0	0.8453	0.8362
0.12	0.2502	0	0.8448	0.8363
0.15	0.2500	0.0001	0.8446	0.8363
0.18	0.2520	0.0003	0.8466	0.8362
0.21	0.2518	0.0007	0.8464	0.8361
0.24	0.2516	0.0014	0.8462	0.8357
0.27	0.2526	0.0025	0.8472	0.8349
0.30	0.2525	0.0040	0.8471	0.8339
0.33	0.2524	0.0060	0.8470	0.8324
0.36	0.2532	0.0086	0.8478	0.8305
0.39	0.2537	0.0115	0.8483	0.8283

The following parameters were used in the construction of this table: the spot interest rate r is 10%, the exchange rate f is 1.8996, long-term mean reversion rate of interest rate κ_r is 0.56, long-term mean reversion rate of exchange rate κ_f is 0.15, long term mean of interest rate θ_r is 0.24, long term mean of exchange rate θ_f is 1.86, the interest rate volatility σ_r is 0.12, financial distress threshold is 0.60, LTV ratio is 0.80, the correlation coefficient ρ is 0

Table 5.6: Changes in Volatility of Exchange Rate (f = 2.2388)

Volatility	**Financial Distress**	**Default**	**Future Payments**	**Loan**
σ_f	FD	D	A	V
0.03	0.3672	0.0003	0.9618	0.9478
0.06	0.3670	0.0009	0.9617	0.9475
0.09	0.3667	0.0021	0.9613	0.9470
0.12	0.3663	0.0036	0.9609	0.9461
0.15	0.3659	0.0057	0.9605	0.9449
0.18	0.3667	0.0083	0.9613	0.9433
0.21	0.3665	0.0112	0.9611	0.9412
0.24	0.3663	0.0146	0.9610	0.9388
0.27	0.3670	0.0185	0.9616	0.9358
0.30	0.3669	0.0226	0.9615	0.9324
0.33	0.3669	0.0271	0.9615	0.9287
0.36	0.3674	0.0320	0.9620	0.9246
0.39	0.3679	0.0371	0.9625	0.9203

The following parameters were used in the construction of this table: the spot interest rate r is 10%, the exchange rate f is 2.2388, long-term mean reversion rate of interest rate κ_r is 0.56, long-term mean reversion rate of exchange rate κ_f is 0.15, long term mean of interest rate θ_r is 0.24, long term mean of exchange rate θ_f is 1.86, the interest rate volatility σ_r is 0.12,financial distress threshold is 0.60, LTV ratio is 0.80, the correlation coefficient ρ is 0

Table 5.7: Changes in Correlation Coefficient (f = 1.8996)

Correlation	Financial Distress	Default	Future Payments	Loan
ρ	FD	D	A	V
-0.20	0.2523	0.0009	0.847	0.8376
-0.15	0.2522	0.0008	0.8468	0.8372
-0.10	0.2521	0.0008	0.8467	0.8368
-0.05	0.2519	0.0007	0.8465	0.8365
0.00	0.2518	0.0007	0.8464	0.8361
0.05	0.2516	0.0006	0.8462	0.8357
0.10	0.2515	0.0006	0.8461	0.8353
0.15	0.2513	0.0005	0.8459	0.8349
0.20	0.2512	0.0005	0.8458	0.8345

The following parameters were used in the construction of this table: the spot interest rate r is 10%, the exchange rate f is 1.8996, long-term mean reversion rate of interest rate κ_r is 0.56, long-term mean reversion rate of exchange rate κ_f is 0.15, long term mean of interest rate θ_r is 0.24, long term mean of exchange rate θ_f is 1.86, the interest rate volatility σ_r is 0.12, the exchange rate volatility σ_f is 0.21, financial distress threshold is 0.60, LTV ratio is 0.80,

Table 5.8: Changes in Correlation Coefficient (f = 2.2388)

Correlation	**Financial Distress**	**Default**	**Future Payments**	**Loan**
ρ	FD	D	A	V
-0.20	0.3675	0.0126	0.9621	0.9415
-0.15	0.3672	0.0123	0.9619	0.9414
-0.10	0.3670	0.0119	0.9616	0.9414
-0.05	0.3668	0.0116	0.9614	0.9413
0.00	0.3665	0.0112	0.9611	0.9412
0.05	0.3663	0.0109	0.9609	0.9411
0.10	0.366	0.0106	0.9606	0.9411
0.15	0.366	0.0106	0.9606	0.9411
0.20	0.3655	0.0098	0.9601	0.9409

The following parameters were used in the construction of this table: the spot interest rate r is 10%, the exchange rate f is 2.2388, long-term mean reversion rate of interest rate κ_r is 0.56, long-term mean reversion rate of exchange rate κ_f is 0.15, long term mean of interest rate θ_r is 0.24, long term mean of exchange rate θ_f is 1.86, the interest rate volatility σ_r is 0.12, the exchange rate volatility σ_f is 0.21, financial distress threshold is 0.60, LTV ratio is 0.80,

Table 5.9: Changes in LTV Ratio (f = 1.8996)

Loan-to-Value	**Financial Distress**	**Default**	**Future Payments**	**Loan**
LTV	FD	D	A	V
0.55	0.0073	0	0.5819	0.5751
0.60	0.0411	0	0.6348	0.6274
0.65	0.0931	0	0.6877	0.6796
0.70	0.1460	0	0.7406	0.7319
0.75	0.1989	0.0001	0.7935	0.7841
0.80	0.2518	0.0007	0.8464	0.8361
0.85	0.3047	0.0028	0.8993	0.8870
0.90	0.3576	0.0102	0.9522	0.9346
0.95	0.4105	0.0311	1.0051	0.9723
1	0.4634	0.0700	1.0580	0.9877

The following parameters were used in the construction of this table: the spot interest rate r is 10%, the exchange rate f is 1.8996, long-term mean reversion rate of interest rate κ_r is 0.56, long-term mean reversion rate of exchange rate κ_f is 0.15, long term mean of interest rate θ_r is 0.24, long term mean of exchange rate θ_f is 1.86, the interest rate volatility σ_r is 0.12, the exchange rate volatility σ_f is 0.21, financial distress threshold is 0.60, the correlation coefficient ρ is 0

Table 5.10: Changes in LTV Ratio (f = 2.2388)

Loan-to-Value	**Financial Distress**	**Default**	**Future Payments**	**Loan**
LTV	FD	D	A	V
0.55	0.0664	0	0.6608	0.652
0.60	0.1262	0	0.7209	0.7112
0.65	0.1863	0.0001	0.7809	0.7705
0.70	0.2464	0.0005	0.8410	0.8295
0.75	0.3064	0.0025	0.9011	0.8876
0.80	0.3665	0.0112	0.9611	0.9412
0.85	0.4266	0.0395	1.0212	0.9805
0.90	0.4867	0.0917	1.0813	0.9894
0.95	0.5467	0.1505	1.1414	0.9909
1	0.6068	0.2104	1.2014	0.9910

The following parameters were used in the construction of this table: the spot interest rate r is 10%, the exchange rate f is 2.2388, long-term mean reversion rate of interest rate κ_r is 0.56, long-term mean reversion rate of exchange rate κ_f is 0.15, long term mean of interest rate θ_r is 0.24, long term mean of exchange rate θ_f is 1.86, the interest rate volatility σ_r is 0.12, the exchange rate volatility σ_f is 0.21, financial distress threshold is 0.60, the correlation coefficient ρ is 0

Table 5.11: Changes in LTV Ratio (f = 2.6534)

Loan-to-Value	**Financial Distress**	**Default**	**Future Payments**	**Loan**
LTV	FD	D	A	V
0.55	0.1622	0.0000	0.7568	0.7464
0.60	0.2310	0.0002	0.8256	0.8141
0.65	0.2998	0.0018	0.8944	0.8810
0.70	0.3686	0.0107	0.9632	0.9429
0.75	0.4374	0.0468	1.0321	0.9840
0.80	0.5062	0.1105	1.1009	0.9903
0.85	0.5750	0.1787	1.1697	0.9910
0.90	0.6438	0.2474	1.2385	0.9910
0.95	0.7126	0.3162	1.3073	0.9910
1	0.7814	0.3850	1.3761	0.9910

The following parameters were used in the construction of this table: the spot interest rate r is 10%, the exchange rate f is 2.6534, long-term mean reversion rate of interest rate κ_r is 0.56, long-term mean reversion rate of exchange rate κ_f is 0.15, long term mean of interest rate θ_r is 0.24, long term mean of exchange rate θ_f is 1.86, the interest rate volatility σ_r is 0.12, the exchange rate volatility σ_f is 0.21, financial distress threshold is 0.60, the correlation coefficient ρ is 0

Table 5.12: Changes in LTV Ratio (f = 3.1716)

Loan-to-Value	**Financial Distress**	**Default**	**Future Payments**	**Loan**
LTV	FD	D	A	V
0.55	0.2822	0.0010	0.8768	0.8642
0.60	0.3619	0.0085	0.9565	0.9378
0.65	0.4416	0.0502	1.0363	0.9847
0.70	0.5213	0.1253	1.1160	0.9906
0.75	0.6011	0.2047	1.1957	0.9910
0.80	0.6808	0.2844	1.2754	0.9910
0.85	0.7605	0.3641	1.3551	0.9910
0.90	0.8402	0.4438	1.4348	0.9910
0.95	0.9199	0.5235	1.5145	0.9910
1	0.9996	0.6032	1.5942	0.9910

The following parameters were used in the construction of this table: the spot interest rate r is 10%, the exchange rate f is 3.1716, long-term mean reversion rate of interest rate κ_r is 0.56, long-term mean reversion rate of exchange rate κ_f is 0.15, long term mean of interest rate θ_r is 0.24, long term mean of exchange rate θ_f is 1.86, the interest rate volatility σ_r is 0.12, the exchange rate volatility σ_f is 0.21, financial distress threshold is 0.60, the correlation coefficient ρ is 0

Table 5.13: Changes in Financial Distress Barrier (f = 2.2388)

Distress Barrier	**Financial Distress**	**Default**	**Future Payments**	**Loan**
Th	FD	D	A	V
0.50	0.4656	0.0112	0.9611	0.9412
0.55	0.4161	0.0112	0.9611	0.9412
0.60	0.3665	0.0112	0.9611	0.9412
0.65	0.3170	0.0112	0.9611	0.9412
0.70	0.2674	0.0112	0.9611	0.9412
0.75	0.2179	0.0112	0.9611	0.9412
0.80	0.1683	0.0112	0.9611	0.9412

The following parameters were used in the construction of this table: the spot interest rate r is 10%, the exchange rate f is 2.2388, long-term mean reversion rate of interest rate κ_r is 0.56, long-term mean reversion rate of exchange rate κ_f is 0.15, long term mean of interest rate θ_r is 0.24, long term mean of exchange rate θ_f is 1.86, the interest rate volatility σ_r is 0.12, the exchange rate volatility σ_f is 0.21, LTV ratio is 0.80, the correlation coefficient ρ is 0

Table 5.14: Combined Effects of Th and LTV ratio on Financial Distress (f = 2.2388)

Value of Financial Distress								
Distress Barrier	**LTV Ratios**							
Th	0.60	0.65	0.70	0.75	0.80	0.85	0.90	0.95
0.50	0.2253	0.2854	0.3455	0.4055	0.4656	0.5257	0.5858	0.6458
0.55	0.1758	0.2359	0.2959	0.356	0.4161	0.4761	0.5362	0.5963
0.60	0.1262	0.1863	0.2464	0.3064	0.3665	0.4266	0.4867	0.5467
0.65	0.0769	0.1368	0.1968	0.2569	0.3170	0.377	0.4371	0.4972
0.70	0.0305	0.0874	0.1473	0.2073	0.2674	0.3275	0.3876	0.4476
0.75	0.0069	0.0404	0.09781	0.1578	0.2179	0.2779	0.338	0.3981
0.80	0.0016	0.0106	0.05	0.1084	0.1683	0.2284	0.2884	0.3485

The following parameters were used in the construction of this table: the spot interest rate r is 10%, the exchange rate f is 2.2388, long-term mean reversion rate of interest rate κ_r is 0.56, long-term mean reversion rate of exchange rate κ_f is 0.15, long term mean of interest rate θ_r is 0.24, long term mean of exchange rate θ_f is 1.86, the interest rate volatility σ_r is 0.12, the exchange rate volatility σ_f is 0.21, the correlation coefficient ρ is 0

CHAPTER 6

CONCLUSION

6.1 Summary

This thesis establishes and solves a structural model for the valuation of fixed rate foreign currency denominated corporate loans, whose conditions are identified based on non-financial corporations' foreign borrowing information in Turkey. Possible financial distress and default events caused by foreign currency denominated loan issuance at the corporate level are also priced.

First, we give an overview of the study. Foreign currency denominated corporate loan contract related concepts and market information as well as the credit pricing methodologies are introduced. Credit market in Turkey is discussed focusing on FX-loan market at the corporate level. Growing foreign currency loan shares in total corporate loans and market trends of the specifications of the contracts are discussed. Studies on foreign exchange rate borrowing and risky debt valuation techniques are reviewed. Literature on foreign borrowing by non-financial firms mostly focuses on the financing choices of the non-financial firms and motivations of issuing foreign currency denominated loan. In this study, possible risk factors are mentioned and some techniques to price credit risk in literature are summarized.

The literature review guided us in the decision of employing a contingent claims framework. We analyzed default option and financial distress in foreign currency denominated corporate loans. We develop and evaluate a model to price the FX-loan and its related assets, which are future payments, financial distress and default, in a framework based on the Cox, Ingersoll and Ross [28] equilibrium model. The main risk factors are identified as the interest rate and the exchange rate. Two-factor formulation is employed following this assumption where state variables are the main risk factors. The corresponding valuation function is a partial differential equation for which there is no closed form solution. Consequently, we employ a numerical solution technique. Valuation functions for monthly payments, borrower's debt, remaining payments, default and financial distress are developed according to their payment behaviors. Since the value of the different assets is known at termination of the contract, once the boundary conditions are identified, the model is closed and can be solved recursively using backward solution techniques.

The explicit finite difference method is used to solve the problem. The necessary steps to find a numerical solution such as transforming the original PDE and mapping its original bi-infinite domain into a unit square are also explained. The stability of the model is maintained by use of an "upwind differencing" scheme and difference equations are generated. Considering the original PDE the common boundary conditions are formulated. All the required conditions for the solution of the problem are obtained. Lastly, MATLAB code is developed to find a numerical solution.

This thesis provides a valuable study in order to evaluate foreign currency risk and interest rate risk with the corresponding financial distress and default risks of foreign currency denominated loans under volatile economic conditions. Numerical results were evaluated for different contract specifications and for different economic environments. Results for the values of foreign currency denominated corporate loan related assets are interpreted in terms of mathematical and financial perspectives. Values of default and financial distress are calculated according to the capital structure of a non-financial firm. The results are consistent with the economic knowledge and suggest that increase in exchange rate and decrease in interest rate (discount rate) cause financial distress and triggers the event of default for corporations. The findings reveal that the effect of an increment in exchange rate is sharper than the effect of a decrement in interest rate on financial distress and default values. Recently, Turkish lira has rapidly devaluated against dollar, euro and pound. Under these conditions, both financial distress and default risks for sectors that are heavily financed by FX-loans increase. Our results indicate that interest rate volatility is reasonably more effective on financial distress and default risk in comparison to exchange rate volatility. Another important result is that non-financial firms having high ratio of foreign currency loan to total assets are exposed to default risk and financial distress. Furthermore, high values of exchange rate and low values of spot interest rate trigger the events of default and financial distress to occur with values that cannot be neglected. However, even when there is an economical crisis, such default event hasn't been observed in Turkey, which can be attributable to the firms' effective management of foreign exchange risk.

After 2001 financial crisis, Turkey has experienced higher interest rates and lower exchange rates compared to the pre-2001 period. Higher interest rates can be defined as the higher opportunity cost of capital from the financial perspective. As the cost of borrowing in domestic currency increases, our results suggest that we would expect to see lower financial distress for corporations that are heavily financed by FX-loans. Firms used that opportunity by financing their operations heavily by FX-loans and in most of the sectors, especially in utilities sector, the share of FX-loans in total borrowing of the non-financial firms increased significantly. Since 2008 CBRT has decreased borrowing rates and very recently, Turkish lira has rapidly devaluated against dollar and euro. Our results show that if volatility in exchange rates leads to higher exchange rates, then the financial distress and default values for the sectors that are heavily financed by FX-loans increase in an economic environment where volatility of interest rate is also high. The values obtained for financial distress and default are also interpreted as the amounts of hedged positions, that firms should have in their capital structure to avoid financial distress and default, in terms of total assets value of the firms.

6.2 Contributions of the Research and Further Study

To the best of our knowledge, this thesis contributes to the extant literature by providing the first study that develops a structural model to evaluate foreign currency denominated corporate loans in option pricing framework. Details of the pricing model also yield some contributions. Methodological contributions can be summarized as follows. Our model assumes that the value of firm's assets is constant and uses two state variables; namely, the interest rate and exchange rate, both of which follow CIR process. In addition, the main partial differential equation used in the study differs according to the choice of stochastic processes for the state variables compared to its counterparts.

In the existing literature, pricing default risk and financial distress in FX-loans at corporate level has not been studied. This study contributes to the literature on contingent claims pricing framework for the valuation of foreign currency denominated corporate debt by measuring the corresponding default risk and financial distress values for non-financial firms. Recent global trends show that non-financial firms, especially firms in utilities sector, are heavily financed by FX-loans. Turkish firms in utilities sector also follow the global trend in using FX-loans widely. We observe that under the volatile economic environment in Turkey, these companies can still survive. This thesis demonstrates that although companies experience financial distress, especially with high LTV FX-loans where FX-rates are high they can still continue their operations. We conclude that firms may go into bankruptcy (or default) if they borrow 95% to 100% LTV loans with considerably high FX-rates ($f = 2.2388, 2.6534$), especially when interest rates considerably volatile.

The present study models exchange rate variable as a CIR process. Therefore, big discontinuities in exchange rate movements in the real world are neglected. In order to integrate such discontinuities of the exchange rate movements into the FX-loan pricing model, a jump process can be used to model the exchange rate in a further study. This study also assumes that the value of firm's assets is constant during the life of the loan. The effect of the asset value of the firm on foreign currency denominated loan can further be studied and evaluated by developing a three factor model. Finally, a finite volume model can be developed to find a corresponding numerical solution of a three factor model.

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APPENDIX A

Matlab Code for Consistency, Stability and Convergence of the Transformed PDE

```
function[CO]=Consistency(InputData)
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%'InputData' is the matrix of external parameters in the value of fix rate
%foreign currency denominated loan. The rows of the matrix are composed
%as following%

%InputData(1)=contract maturity
%InputData(2)= time step per month
%InputData(3)= long-term mean reversion rate of interest rate
%InputData(4)= long term mean of interest rate
%InputData(5)= interest rate volatility
%InputData(6)= long-term mean reversion rate of exchange rate
%InputData(7)= long term mean of exchange rate
%InputData(8)= exchange rate volatility
%InputData(9)= scale factor for interest rate transformation
%InputData(10)= scale factor for fx rate transformation
%InputData(11)= correlation coefficient
%InputData(12)= initial Loan value
%InputData(13)= coupon rate
%InputData(14)= # grid points in x direction
%InputData(15)= # grid points in y direction
%InputData(16)=Threshold
```

```
%InputData(17)=VA
%InputData(18)=initial fx rate
 %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

InputData=[ 60.0000,66.0000,0.5600,0.2400,0.1200,0.1500,1.8600,0.2100,...
10.0000,0.67, 0,0.9500,0.1000,50.0000,50.0000,0.6000,1.0000,1.5230];

%%%%%%%%%%%%%%%%%%%%%%
%Internal Parameters
%%%%%%%%%%%%%%%%%%%%%
%cn is the counter for months, cs is the counter for time steps, cx is the
%counter for fx-rate dimension, cy is the counter
%for interest rate dimension. ".....inc" represents the increment for the
%variable "....."

cnmin=1;
cnmax=InputData(1);
cninc=1;

s=1/(12*InputData(2));

csmin=1;
csmax=csmin+InputData(2);
csinc=1;

cxmin=1;
cxmax=cxmin+InputData(14);
cxinc=1;

cymin=1;
cymax=cymin+InputData(15);
cyinc=1;
```

```
% ”x” is the transformed varible in the fx-rate dimension. So the grid
% will be linear in x but not in F (fx-rate)

xmin=0;
xmax=1;
xinc=1/InputData(14);
x=xmin:xinc:xmax;

% y is the transformed varible in the interest rate dimension. So the grid
% will be linear in y but not in R (interest rate)

ymin=0;
ymax=1;
yinc=1/InputData(15);
y=ymin:yinc:ymax;

cxhalf=cxmin+round(InputData(14)/2);
cyhalf=cymin+round(InputData(15)/2);

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%Preliminary Calculations
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

%The Value of fx-rate
F(cxmax)=0;
F(cxmin)=inf;

for cx=cxmin+1:cxmax-1
   F(cx)=(1-x(cx))/(InputData(10)*x(cx));
end

%The Value of Interest Rate
R(cymax)=0;
```

```
R(cymin)=inf;

for cy=cymin+1:cymax-1
   R(cy)=(1-y(cy))/(InputData(9)*y(cy));
end


for cy=cymin+1:cymax
   for cx=cxmin+1:cxmax

      Term2=0.5*R(cy)*(InputData(5)^2)*(InputData(9)^2*...
      (y(cy)^4));
      Term3=0.5*F(cx)*(InputData(8)^2)*(InputData(10)^2*...
      (x(cx)^4));

      C(cy,cx)= Term2*(s/(xinc^2))+ Term3*(s/(yinc^2));

   end
end

%For matrices, MAX(X) is a row vector containing
%the maximum element from each column.

MaxC=max(max(C))

[CO]=[C];

end
```

APPENDIX B

Matlab Code for Loan Valuation

```
function[FR]=LoanValuation(InputData)
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%'InputData' is the matrix of external parameters in the value of fix rate
%foreign currency denominated loan.
%The rows of the matrix are composed as following%
%InputData(1)=contract maturity
%InputData(2)= time step per month
%InputData(3)= long-term mean reversion rate of interest rate
%InputData(4)= long term mean of interest rate
%InputData(5)= interest rate volatility
%InputData(6)= long-term mean reversion rate of exchange rate
%InputData(7)= long term mean of exchange rate
%InputData(8)= exchange rate volatility
%InputData(9)= scale factor for interest rate transformation
%InputData(10)= scale factor for fx rate transformation
%InputData(11)= correlation coefficient
%InputData(12)= initial Loan Value
%InputData(13)= coupon rate
%InputData(14)= # grid points in x direction
%InputData(15)= # grid points in y direction
%InputData(16)=Threshold
%InputData(17)=VA
%InputData(18)=initial fx rate
```

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

%%%%%%%%%%%%%%%%%%%%%
%Internal Parameters
%%%%%%%%%%%%%%%%%%%%%
%cn is the counter for months, cs is the counter for time steps, cx is the
%counter for fx rate dimension, cy is the counter
%for interest rate dimension. ".....inc" represents the increment for the
%variable "....."

cnmin=1;
cnmax=InputData(1);
cninc=1;

s=1/(12*InputData(2));

csmin=1;
csmax=csmin+InputData(2);
csinc=1;

cxmin=1;
cxmax=cxmin+InputData(14);
cxinc=1;

cymin=1;
cymax=cymin+InputData(15);
cyinc=1;

% "x" is the transformed varible in the fx rate dimension. So the grid
% will be linear in x but not in F (fx-rate)

xmin=0;
xmax=1;
```

```
xinc=1/InputData(14);
x=xmin:xinc:xmax;

% y is the transformed varible in the interest rate dimension. So the grid
% will be linear in y but not in R (interest rate)

ymin=0;
ymax=1;
yinc=1/InputData(15);
y=ymin:yinc:ymax;

cxhalf=cxmin+round(InputData(14)/2);
cyhalf=cymin+round(InputData(15)/2);

%%%%%%%%%%%%%%%%
%Preliminary Calculations
%%%%%%%%%%%%%%%%

%The value of fx-rate
F(cxmax)=0;
F(cxmin)=inf;

for cx=cxmin+1:cxmax-1
   F(cx)=(1-x(cx))/(InputData(10)*x(cx));
end

%the value of interest rate
R(cymax)=0;
R(cymin)=inf;

for cy=cymin+1:cymax-1
   R(cy)=(1-y(cy))/(InputData(9)*y(cy));
end
```

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%FINAL TERMINATION CONDITIONS
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

%Valuation of monthly payment

M=(((InputData(13)/12)*(1+(InputData(13)/12))^cnmax)/...
((1+(InputData(13)/12))^cnmax-1))*InputData(12)/InputData(18);

for cy=cymin:cymax

  for cx=cxmin+1:cxmax-1
    NewA(cy,cx)=M*F(cx);
    OldA(cy,cx)=NewA(cy,cx);
  end

  % F is infinite (cx=cxmin)
  NewA(cy,cxmin)=InputData(17);
  OldA(cy,cxmin)=NewA(cy,cxmin);

  % F is zero (cx=cxmax)
  NewA(cy,cxmax)=0;
  OldA(cy,cxmax)=NewA(cy,cxmax);
end

for cy=cymin:cymax
  for cx=cxmin+1:cxmax
    OldFD(cy,cx)=max(0,M*F(cx)-InputData(16));
    OldD(cy,cx)=max(0,M*F(cx)-InputData(17));
    OldV(cy,cx)=min(M*F(cx),InputData(17));
  end
```

```
    %F is infinite(cx=cxmin)
    OldFD(cy,cxmin)=InputData(17);
    OldV(cy,cxmin)=InputData(17);
    OldD(cy,cxmin)=InputData(17);
end

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%The Beginning of the Last Month
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

cn=cnmin;
for cs=csmin:csmax

    %%%BOUNDARY CONDITIONS%%%%

    %The Value of A(R,F) at the Corners

    NewA(cymax,cxmin)=OldA(cymax-1,cxmin);
    OldA(cymax,cxmin)=NewA(cymax,cxmin);

    NewA(cymax,cxmax)=OldA(cymax-1,cxmax);
    OldA(cymax,cxmax)=NewA(cymax,cxmax);

    NewA(cymin,cxmin)=0;
    OldA(cymin,cxmin)=NewA(cymin,cxmin);

    NewA(cymin,cxmax)=0;
    OldA(cymin,cxmax)=NewA(cymin,cxmax);

    %The value of A(R,F) at the Edges

    %F=0 R varies%
    for cy=cymin+1:cymax-1
```

```
   NewA(cy,cxmax)=0;
   OldA(cy,cxmax)=NewA(cy,cxmax);
end

%R=0, F varies%
for cx=cxmin+1:cxmax-1
   Term7=1-F(cx)*(InputData(8)^2)*(InputData(10)^2*...
   (x(cx)^4)*(s/xinc^2));
   Term8=0.5*F(cx)*(InputData(8)^2)*(InputData(10)^2*...
   (x(cx)^4)*(s/xinc^2));
   Term9=(F(cx)*(InputData(8)^2)*(InputData(10)^2)*...
   (x(cx)^3)-(InputData(6)*(InputData(7)-F(cx))*...
   InputData(10)*(x(cx)^2)))*(s/xinc);
   Term10=-InputData(3)*InputData(4)*InputData(9)*(y(cy)^2)*(s/yinc);

   EQ3=(F(cx)*(InputData(8)^2)*(InputData(10)^2)*...
   (x(cx)^3)-(InputData(6)*(InputData(7)-F(cx))*...
   InputData(10)*(x(cx)^2)));

   if EQ3<0
      NewA(cymax,cx)=Term7*OldA(cymax,cx)+...
      Term8*(OldA(cymax,cx+1)-OldA(cymax,cx-1))+...
      Term10*(OldA(cymax,cx)-OldA(cymax-1,cx))+...
      Term9*(OldA(cymax,cx)- OldA(cymax,cx-1));

   else
      NewA(cymax,cx)=Term7*OldA(cymax,cx)+...
      Term8*(OldA(cymax,cx+1)-OldA(cymax,cx-1))+...
      Term10*(OldA(cymax,cx)-OldA(cymax-1,cx))+...
      Term9*(OldA(cymax,cx+1)- OldA(cymax,cx));
   end
   OldA(cymax,cx)=NewA(cymax,cx);
```

```
end

%F=inf, R varies%
for cy=cymin+1:cymax-1
   Term11=1-R(cy)*(InputData(5)^2)*(InputData(9)^2*...
   (y(cy)^4)*(s/yinc^2))-R(cy)*s;
   Term12=0.5*R(cy)*(InputData(5)^2)*(InputData(9)^2*...
   (y(cy)^4)*(s/yinc^2));
   Term13=(R(cy)*(InputData(5)^2)*(InputData(9)^2)*...
   (y(cy)^3)-(InputData(3)*(InputData(4)-R(cy))*...
   InputData(9)*(y(cy)^2)))*(s/yinc);

   Term14=-(InputData(6)*InputData(7)*InputData(10)*(x(cx)^2))*(s/xinc);

   EQ4=(R(cy)*(InputData(5)^2)*(InputData(9)^2)*...
   (y(cy)^3)-(InputData(3)*(InputData(4)-R(cy))*...
   InputData(9)*(y(cy)^2)));

   if EQ4<0
      NewA(cy,cxmin)=Term11*OldA(cy,cxmin)+...
      Term12*(OldA(cy+1,cxmin)+OldA(cy-1,cxmin))+...
      Term13*(OldA(cy,cxmin)-OldA(cy-1,cxmin))+...
      Term14*(OldA(cy,cxmin)- OldA(cy,cxmin+1));
   else
      NewA(cy,cxmin)=Term11*OldA(cy,cxmin)+...
      Term12*(OldA(cy+1,cxmin)+OldA(cy-1,cxmin))+...
      Term13*(OldA(cy+1,cxmin)-OldA(cy,cxmin))+...
      Term14*(OldA(cy,cxmin)- OldA(cy,cxmin+1));
   end
   OldA(cy,cxmin)=NewA(cy,cxmin);
end
```

```
%R=inf, F varies%
for cx=cxmin+1:cxmax-1
   NewA(cymin,cx)=0;
   OldA(cymin,cx)=NewA(cymin,cx);
end

%PDE solution for A(R,F)%
for cy=cymin+1:cymax-1
   for cx=cxmin+1:cxmax-1
      Term1=1-R(cy)*(InputData(5)^2)*(InputData(9)^2*...
      (y(cy)^4)*(s/yinc^2))-F(cx)*(InputData(8)^2)*...
      (InputData(10)^2*(x(cx)^4)*(s/xinc^2))-R(cy)*s;
      Term2=0.5*R(cy)*(InputData(5)^2)*(InputData(9)^2*...
      (y(cy)^4)*(s/(yinc^2)));
      Term3=0.5*F(cx)*(InputData(8)^2)*(InputData(10)^2*...
      (x(cx)^4)*(s/(xinc^2)));

      Term4=(R(cy)*(InputData(5)^2)*(InputData(9)^2)*...
      (y(cy)^3)-(InputData(3)*(InputData(4)-R(cy))*...
      InputData(9)*(y(cy)^2)))*(s/yinc);
      Term5=(F(cx)*(InputData(8)^2)*(InputData(10)^2)*...
      (x(cx)^3)-(InputData(6)*(InputData(7)-F(cx))*...
      InputData(10)*(x(cx)^2)))*(s/xinc);
      Term6=InputData(11)*InputData(5)*InputData(8)*...
      (R(cy)^0.5)*(F(cx)^0.5)*InputData(9)*InputData(10)*...
      y(cy)^2*x(cx)^2*(s/(4*xinc*yinc));


      EQ1=(F(cx)*(InputData(8)^2)*(InputData(10)^2)*...
      (x(cx)^3)-(InputData(6)*(InputData(7)-F(cx))*...
      InputData(10)*(x(cx)^2)));

      EQ2=(R(cy)*(InputData(5)^2)*(InputData(9)^2)*...
```

```
(y(cy)^3)-(InputData(3)*(InputData(4)-R(cy))*...
InputData(9)*(y(cy)^2)));


if EQ1>=0 && EQ2>=0
    NewA(cy,cx)=Term1*OldA(cy,cx)+...
    Term2*(OldA(cy+1,cx)+OldA(cy,cx))+...
    Term3*(OldA(cy,cx+1)+OldA(cy,cx-1))+...
    Term4*(OldA(cy+1,cx)-OldA(cy,cx))+...
    Term5*(OldA(cy,cx+1)- OldA(cy,cx))+...
    Term6*(OldA(cy+1,cx+1)-OldA(cy-1,cx+1)-...
    OldA(cy+1,cx-1)+OldA(cy-1,cx-1));

elseif EQ1<0 && EQ2>=0
    NewA(cy,cx)=Term1*OldA(cy,cx)+...
    Term2*(OldA(cy+1,cx)+OldA(cy-1,cx))+...
    Term3*(OldA(cy,cx+1)+OldA(cy,cx-1))+...
    Term4*(OldA(cy+1,cx)-OldA(cy,cx))+...
    Term5*(OldA(cy,cx)-OldA(cy,cx-1))+...
    Term6*(OldA(cy+1,cx+1)-OldA(cy-1,cx+1)-...
    OldA(cy+1,cx-1)+OldA(cy-1,cx-1));

elseif EQ1>=0 && EQ2<0
    NewA(cy,cx)=Term1*OldA(cy,cx)+...
    Term2*(OldA(cy+1,cx)+OldA(cy-1,cx))+...
    Term3*(OldA(cy,cx+1)+OldA(cy,cx-1))+...
    Term4*(OldA(cy,cx)-OldA(cy-1,cx))+...
    Term5*(OldA(cy,cx+1)-OldA(cy,cx))+...
    Term6*(OldA(cy+1,cx+1)-OldA(cy-1,cx+1)-...
    OldA(cy+1,cx-1)+OldA(cy-1,cx-1));

else
    NewA(cy,cx)=Term1*OldA(cy,cx)+...
```

```
        Term2*(OldA(cy+1,cx)+OldA(cy-1,cx))+...
        Term3*(OldA(cy,cx+1)+OldA(cy,cx-1))+...
        Term4*(OldA(cy,cx)-OldA(cy-1,cx))+...
        Term5*(OldA(cy,cx)-OldA(cy,cx-1))+...
        Term6*(OldA(cy+1,cx+1)-OldA(cy-1,cx+1)-...
        OldA(cy+1,cx-1)+OldA(cy-1,cx-1));

    end
    OldA(cy,cx)=NewA(cy,cx);
end

end



%%%%Corners of the Grid %%%

%Corners of the grid in which F=0, R=0 %%%

NewV(cymax,cxmax)=OldV(cymax-1,cxmax);
NewD(cymax,cxmax)=OldD(cymax-1,cxmax);
NewFD(cymax,cxmax)=OldFD(cymax-1,cxmax);
OldV(cymax,cxmax)=NewV(cymax,cxmax);
OldD(cymax,cxmax)=NewD(cymax,cxmax);
OldFD(cymax,cxmax)=NewFD(cymax,cxmax);

%Corners of the grid in which F=inf, R=0 %%%
NewV(cymax,cxmin)=OldV(cymax-1,cxmin);
NewD(cymax,cxmin)=OldD(cymax-1,cxmin);
NewFD(cymax,cxmin)=OldFD(cymax-1,cxmin);
OldV(cymax,cxmin)=NewV(cymax,cxmin);
OldD(cymax,cxmin)=NewD(cymax,cxmin);
OldFD(cymax,cxmin)=NewFD(cymax,cxmin);
```

```
%Corners of the grid in which F=inf, R=inf %%%
NewV(cymin,cxmin)=0;
NewD(cymin,cxmin)=0;
NewFD(cymin,cxmin)=0;
OldV(cymin,cxmin)=NewV(cymin,cxmin);
OldD(cymin,cxmin)=NewD(cymin,cxmin);
OldFD(cymin,cxmin)=NewFD(cymin,cxmin);

%Corners of the grid in which F=0, R=inf %%%
NewV(cymin,cxmax)=0;
NewD(cymin,cxmax)=0;
NewFD(cymin,cxmax)=0;
OldV(cymin,cxmax)=NewV(cymin,cxmax);
OldD(cymin,cxmax)=NewD(cymin,cxmax);
OldFD(cymin,cxmax)=NewFD(cymin,cxmax);

%%%%Edges of the Grid %%%%

%F=0 R varies%
for cy=cymin+1:cymax-1

   NewD(cy,cxmax)=0;
   NewFD(cy,cxmax)=0;
   NewV(cy,cxmax)=0;
   OldD(cy,cxmax)=NewD(cy,cxmax);
   OldFD(cy,cxmax)=NewFD(cy,cxmax);
   OldV(cy,cxmax)=NewV(cy,cxmax);
end

%R=0, F varies%
for cx=cxmin+1:cxmax-1
   Term15=1-F(cx)*(InputData(8)^2)*(InputData(10)^2)*...
   (x(cx)^4)*(s/(xinc^2));
```

```
Term16=0.5*F(cx)*(InputData(8)^2)*(InputData(10)^2)*...
(x(cx)^4)*(s/(xinc^2));
Term17=(F(cx)*(InputData(8)^2)*(InputData(10)^2)*...
(x(cx)^3)-(InputData(6)*(InputData(7)-F(cx))*...
InputData(10)*(x(cx)^2)))*(s/xinc);
Term18=-InputData(3)*InputData(4)*InputData(9)*...
(y(cy)^2)*(s/yinc);

EQ5=(F(cx)*(InputData(8)^2)*(InputData(10)^2)*...
(x(cx)^3)-(InputData(6)*(InputData(7)-F(cx))*...
InputData(10)*(x(cx)^2)));
if EQ5<0
   NewV(cymax,cx)=Term15*OldV(cymax,cx)+...
   Term16*(OldV(cymax,cx+1)-OldV(cymax,cx-1))+...
   Term18*(OldV(cymax,cx)-OldV(cymax-1,cx))+...
   Term17*(OldV(cymax,cx)- OldV(cymax,cx-1));
   OldV(cymax,cx)=NewV(cymax,cx);

   NewD(cymax,cx)=Term15*OldD(cymax,cx)+...
   Term16*(OldD(cymax,cx+1)-OldD(cymax,cx-1))+...
   Term18*(OldD(cymax,cx)-OldD(cymax-1,cx))+...
   Term17*(OldD(cymax,cx)- OldD(cymax,cx-1));
   OldD(cymax,cx)=NewD(cymax,cx);

   NewFD(cymax,cx)=Term15*OldFD(cymax,cx)+...
   Term16*(OldFD(cymax,cx+1)-OldFD(cymax,cx-1))+...
   Term18*(OldFD(cymax,cx)-OldFD(cymax-1,cx))+...
   Term17*(OldFD(cymax,cx)- OldFD(cymax,cx-1));
   OldFD(cymax,cx)=NewFD(cymax,cx);

else
   NewV(cymax,cx)=Term15*OldV(cymax,cx)+...
   Term16*(OldV(cymax,cx+1)-OldV(cymax,cx-1))+...
```

```
        Term18*(OldV(cymax,cx)-OldV(cymax-1,cx))+...
        Term17*(OldV(cymax,cx+1)- OldV(cymax,cx));
        OldV(cymax,cx)=NewV(cymax,cx);

        NewD(cymax,cx)=Term15*OldD(cymax,cx)+...
        Term16*(OldD(cymax,cx+1)-OldD(cymax,cx-1))+...
        Term18*(OldD(cymax,cx)-OldD(cymax-1,cx))+...
        Term17*(OldD(cymax,cx+1)- OldD(cymax,cx));
        OldD(cymax,cx)=NewD(cymax,cx);

        NewFD(cymax,cx)=Term15*OldFD(cymax,cx)+...
        Term16*(OldFD(cymax,cx+1)-OldFD(cymax,cx-1))+...
        Term18*(OldFD(cymax,cx)-OldFD(cymax-1,cx))+...
        Term17*(OldFD(cymax,cx+1)- OldFD(cymax,cx));
        OldFD(cymax,cx)=NewFD(cymax,cx);

    end

end

%F=inf, R varies%
for cy=cymin+1:cymax-1
    Term19=1-R(cy)*(InputData(5)^2)*(InputData(9)^2*...
    (y(cy)^4)*(s/yinc^2))-R(cy)*s;
    Term20=0.5*R(cy)*(InputData(5)^2)*(InputData(9)^2*...
    (y(cy)^4)*(s/yinc^2));
    Term21=(R(cy)*(InputData(5)^2)*(InputData(9)^2)*...
    (y(cy)^3)-(InputData(3)*(InputData(4)-R(cy))*...
    InputData(9)*(y(cy)^2)))*(s/yinc);
    Term22=-(InputData(6)*InputData(7)*InputData(10)*...
    (x(cx)^2))*(s/xinc);

    EQ6=(R(cy)*(InputData(5)^2)*(InputData(9)^2)*...
```

```
(y(cy)^3)-(InputData(3)*(InputData(4)-R(cy))*...
InputData(9)*(y(cy)^2)));

if EQ6<0
   NewFD(cy,cxmin)=Term19*OldFD(cy,cxmin)+...
   Term20*(OldFD(cy+1,cxmin)+OldFD(cy-1,cxmin))+...
   Term21*(OldFD(cy,cxmin)-OldFD(cy-1,cxmin))+...
   Term22*(OldFD(cy,cxmin)- OldFD(cy,cxmin+1));

   NewD(cy,cxmin)=Term19*OldD(cy,cxmin)+...
   Term20*(OldD(cy+1,cxmin)+OldD(cy-1,cxmin))+...
   Term21*(OldD(cy,cxmin)-OldD(cy-1,cxmin))+...
   Term22*(OldD(cy,cxmin)- OldD(cy,cxmin+1));

   NewV(cy,cxmin)=Term19*OldV(cy,cxmin)+...
   Term20*(OldV(cy+1,cxmin)+OldV(cy-1,cxmin))+...
   Term21*(OldV(cy,cxmin)-OldV(cy-1,cxmin))+...
   Term22*(OldV(cy,cxmin)- OldV(cy,cxmin+1));
else
   NewFD(cy,cxmin)=Term19*OldFD(cy,cxmin)+...
   Term20*(OldFD(cy+1,cxmin)+OldFD(cy-1,cxmin))+...
   Term21*(OldFD(cy+1,cxmin)-OldFD(cy,cxmin))+...
   Term22*(OldFD(cy,cxmin)- OldFD(cy,cxmin+1));

   NewD(cy,cxmin)=Term19*OldD(cy,cxmin)+...
   Term20*(OldD(cy+1,cxmin)+OldD(cy-1,cxmin))+...
   Term21*(OldD(cy+1,cxmin)-OldD(cy,cxmin))+...
   Term22*(OldD(cy,cxmin)- OldD(cy,cxmin+1));

   NewV(cy,cxmin)=Term19*OldV(cy,cxmin)+...
   Term20*(OldV(cy+1,cxmin)+OldV(cy-1,cxmin))+...
   Term21*(OldV(cy+1,cxmin)-OldV(cy,cxmin))+...
   Term22*(OldV(cy,cxmin)- OldV(cy,cxmin+1));
```

```
    end
    OldFD(cy,cxmin)=NewFD(cy,cxmin);
    OldD(cy,cxmin)=NewD(cy,cxmin);
    OldV(cy,cxmin)=NewV(cy,cxmin);
end

%R=inf, F varies%
for cx=cxmin+1:cxmax-1
    NewD(cymin,cx)=0;
    NewFD(cymin,cx)=0;
    NewV(cymin,cx)=0;
    OldD(cymin,cx)=NewD(cymin,cx);
    OldFD(cymin,cx)=NewFD(cymin,cx);
    OldV(cymin,cx)=NewV(cymin,cx);
end

%%The Main Algorithm to Solve the Main PDE for V(F,R),FD(F,R) and D(F,R)%%

    for cy=cymin+1:cymax-1

        for cx=cxmin+1:cxmax-1
            Term23=1-R(cy)*(InputData(5)^2)*(InputData(9)^2*...
            (y(cy)^4)*(s/yinc^2))-F(cx)*(InputData(8)^2)*...
            (InputData(10)^2*(x(cx)^4)*(s/xinc^2))-R(cy)*s;
            Term26=0.5*R(cy)*(InputData(5)^2)*(InputData(9)^2*...
            (y(cy)^4)*(s/yinc^2));
            Term24=0.5*F(cx)*(InputData(8)^2)*(InputData(10)^2*...
            (x(cx)^4)*(s/xinc^2));
            Term27=(R(cy)*(InputData(5)^2)*(InputData(9)^2)*...
            (y(cy)^3)-(InputData(3)*(InputData(4)-R(cy))*...
            InputData(9)*(y(cy)^2)))*(s/yinc);
            Term25=(F(cx)*(InputData(8)^2)*(InputData(10)^2)*...
```

```
    (x(cx)^3)-(InputData(6)*(InputData(7)-F(cx))*...
    InputData(10)*(x(cx)^2)))*(s/xinc);
    Term28=InputData(11)*InputData(5)*InputData(8)*...
    (R(cy)^0.5)*(F(cx)^0.5)*InputData(9)*InputData(10)*...
y(cy)^2*x(cx)^2*(s/(4*xinc*yinc));

    EQ7=F(cx)*(InputData(8)^2)*(InputData(10)^2)*...
    (x(cx)^3)-(InputData(6)*(InputData(7)-F(cx))*...
    InputData(10)*(x(cx)^2));

    EQ8=R(cy)*(InputData(5)^2)*(InputData(9)^2)*...
    (y(cy)^3)-(InputData(3)*(InputData(4)-R(cy))*...
    InputData(9)*(y(cy)^2));


    if EQ7>=0 && EQ8>=0
        NewV(cy,cx)=Term23*OldV(cy,cx)+...
        Term24*(OldV(cy,cx+1)+OldV(cy,cx-1))+...
        Term25*(OldV(cy,cx+1)-OldV(cy,cx))+...
        Term26*(OldV(cy+1,cx)+OldV(cy-1,cx))+...
        Term27*(OldV(cy+1,cx)-OldV(cy,cx))+...
        Term28*(OldV(cy+1,cx+1)-OldV(cy+1,cx-1)-...
        OldV(cy-1,cx+1)+OldV(cy-1,cx-1));

    elseif EQ7<0 && EQ8>=0
        NewV(cy,cx)=Term23*OldV(cy,cx)+...
        Term24*(OldV(cy,cx+1)+OldV(cy,cx-1))+...
        Term25*(OldV(cy,cx)-OldV(cy,cx-1))+...
        Term26*(OldV(cy+1,cx)+OldV(cy-1,cx))+...
        Term27*(OldV(cy+1,cx)-OldV(cy,cx))+...
        Term28*(OldV(cy+1,cx+1)-OldV(cy+1,cx-1)-...
        OldV(cy-1,cx+1)+OldV(cy-1,cx-1));
```

```
    elseif EQ7>=0 && EQ8<0
        NewV(cy,cx)=Term23*OldV(cy,cx)+...
        Term24*(OldV(cy,cx+1)+OldV(cy,cx-1))+...
        Term25*(OldV(cy,cx+1)-OldV(cy,cx))+...
        Term26*(OldV(cy+1,cx)+OldV(cy-1,cx))+...
        Term27*(OldV(cy,cx)-OldV(cy-1,cx))+...
        Term28*(OldV(cy+1,cx+1)-OldV(cy+1,cx-1)-...
        OldV(cy-1,cx+1)+OldV(cy-1,cx-1));

    else
        NewV(cy,cx)=Term23*OldV(cy,cx)+...
        Term24*(OldV(cy,cx+1)+OldV(cy,cx-1))+...
        Term25*(OldV(cy,cx)-OldV(cy,cx-1))+...
        Term26*(OldV(cy+1,cx)+OldV(cy-1,cx))+...
        Term27*(OldV(cy,cx)-OldV(cy-1,cx))+...
        Term28*(OldV(cy+1,cx+1)-OldV(cy+1,cx-1)-...
    OldV(cy-1,cx+1)+OldV(cy-1,cx-1));
end


if EQ7>=0 && EQ8>=0
    NewD(cy,cx)=Term23*OldD(cy,cx)+...
    Term24*(OldD(cy,cx+1)+OldD(cy,cx-1))+...
    Term25*(OldD(cy,cx+1)-OldD(cy,cx))+...
    Term26*(OldD(cy+1,cx)+OldD(cy-1,cx))+...
    Term27*(OldD(cy+1,cx)-OldD(cy,cx))+...
    Term28*(OldD(cy+1,cx+1)-OldD(cy+1,cx-1)-...
    OldD(cy-1,cx+1)+OldD(cy-1,cx-1));

    OldD(cy,cx)=NewD(cy,cx);

elseif EQ7<0 && EQ8>=0
    NewD(cy,cx)=Term23*OldD(cy,cx)+...
```

```
    Term24*(OldD(cy,cx+1)+OldD(cy,cx-1))+...
    Term25*(OldD(cy,cx)-OldD(cy,cx-1))+...
    Term26*(OldD(cy+1,cx)+OldD(cy-1,cx))+...
    Term27*(OldD(cy+1,cx)-OldD(cy,cx))+...
    Term28*(OldD(cy+1,cx+1)-OldD(cy+1,cx-1)-...
    OldD(cy-1,cx+1)+OldD(cy-1,cx-1));

    OldD(cy,cx)=NewD(cy,cx);

elseif EQ7>=0 && EQ8<0
    NewD(cy,cx)=Term23*OldD(cy,cx)+...
    Term24*(OldD(cy,cx+1)+OldD(cy,cx-1))+...
    Term25*(OldD(cy,cx+1)-OldD(cy,cx))+...
    Term26*(OldD(cy+1,cx)+OldD(cy-1,cx))+...
    Term27*(OldD(cy,cx)-OldD(cy-1,cx))+...
    Term28*(OldD(cy+1,cx+1)-OldD(cy+1,cx-1)-...
    OldD(cy-1,cx+1)+OldD(cy-1,cx-1));

    OldD(cy,cx)=NewD(cy,cx);

else
    NewD(cy,cx)=Term23*OldD(cy,cx)+...
    Term24*(OldD(cy,cx+1)+OldD(cy,cx-1))+...
    Term25*(OldD(cy,cx)-OldD(cy,cx-1))+...
    Term26*(OldD(cy+1,cx)+OldD(cy-1,cx))+...
    Term27*(OldD(cy,cx)-OldD(cy-1,cx))+...
    Term28*(OldD(cy+1,cx+1)-OldD(cy+1,cx-1)-...
    OldD(cy-1,cx+1)+OldD(cy-1,cx-1));

    OldD(cy,cx)=NewD(cy,cx);

end
```

```
if EQ7>=0 && EQ8>=0
    NewFD(cy,cx)=Term23*OldFD(cy,cx)+...
    Term24*(OldFD(cy,cx+1)+OldFD(cy,cx-1))+...
    Term25*(OldFD(cy,cx+1)-OldFD(cy,cx))+...
    Term26*(OldFD(cy+1,cx)+OldFD(cy-1,cx))+...
    Term27*(OldFD(cy+1,cx)-OldFD(cy,cx))+...
    Term28*(OldFD(cy+1,cx+1)-OldFD(cy+1,cx-1)-...
    OldFD(cy-1,cx+1)+OldFD(cy-1,cx-1));

    OldFD(cy,cx)=NewFD(cy,cx);

elseif EQ7<0 && EQ8>=0
    NewFD(cy,cx)=Term23*OldFD(cy,cx)+...
    Term24*(OldFD(cy,cx+1)+OldFD(cy,cx-1))+...
    Term25*(OldFD(cy,cx)-OldFD(cy,cx-1))+...
    Term26*(OldFD(cy+1,cx)+OldFD(cy-1,cx))+...
    Term27*(OldFD(cy+1,cx)-OldFD(cy,cx))+...
    Term28*(OldFD(cy+1,cx+1)-OldFD(cy+1,cx-1)-...
    OldFD(cy-1,cx+1)+OldFD(cy-1,cx-1));

    OldFD(cy,cx)=NewFD(cy,cx);

    elseif EQ7>=0 && EQ8<0
    NewFD(cy,cx)=Term23*OldFD(cy,cx)+...
    Term24*(OldFD(cy,cx+1)+OldFD(cy,cx-1))+...
    Term25*(OldFD(cy,cx+1)-OldFD(cy,cx))+...
    Term26*(OldFD(cy+1,cx)+OldFD(cy-1,cx))+...
    Term27*(OldFD(cy,cx)-OldFD(cy-1,cx))+...
    Term28*(OldFD(cy+1,cx+1)-OldFD(cy+1,cx-1)-...
    OldFD(cy-1,cx+1)+OldFD(cy-1,cx-1));

    OldFD(cy,cx)=NewFD(cy,cx);
```

```
        else
            NewFD(cy,cx)=Term23*OldFD(cy,cx)+...
            Term24*(OldFD(cy,cx+1)+OldFD(cy,cx-1))+...
            Term25*(OldFD(cy,cx)-OldFD(cy,cx-1))+...
            Term26*(OldFD(cy+1,cx)+OldFD(cy-1,cx))+...
            Term27*(OldFD(cy,cx)-OldFD(cy-1,cx))+...
            Term28*(OldFD(cy+1,cx+1)-OldFD(cy+1,cx-1)-...
            OldFD(cy-1,cx+1)+OldFD(cy-1,cx-1));

            OldFD(cy,cx)=NewFD(cy,cx);

        end

        OldV(cy,cx)=NewV(cy,cx);
      end
    end

end



%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%The Remaining Months of the Contract
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
for cn=cnmin+1:cnmax

    %Terminal Conditions
    for cy=cymin:cymax
      for cx=cxmin+1:cxmax
        NewA(cy,cx)=OldA(cy,cx)+M*F(cx);
        OldA(cy,cx)=NewA(cy,cx);
      end
      NewA(cy,cxmin)=OldA(cy,cxmin);
```

```
    OldA(cy,cxmin)=NewA(cy,cxmin);
end

for cy=cymin:cymax
    for cx=cxmin+1:cxmax

        %'VBLP' represents the value of the contract just before
        %the last payment date
        VBLP(cy,cx)=OldV(cy,cx)+M*F(cx);
        NewV(cy,cx)=min(VBLP(cy,cx),InputData(17));
        OldV(cy,cx)=NewV(cy,cx);

        if OldA(cy,cx)>=InputData(17)
            NewD(cy,cx)=OldA(cy,cx)-OldV(cy,cx);
            OldD(cy,cx)=NewD(cy,cx);

        else
            NewD(cy,cx)=OldD(cy,cx);
            OldD(cy,cx)=NewD(cy,cx);
        end

        if OldA(cy,cx)>=InputData(16)
            NewFD(cy,cx)=OldA(cy,cx)-InputData(16);
            OldFD(cy,cx)=NewFD(cy,cx);

        else
            NewFD(cy,cx)=OldFD(cy,cx);
            OldFD(cy,cx)=NewFD(cy,cx);
        end

    end

    NewV(cy,cxmin)=OldV(cy,cxmin);
```

```
    OldV(cy,cxmin)=NewV(cy,cxmin);

    NewD(cy,cxmin)=OldA(cy,cxmin)-OldV(cy,cxmin);
    OldD(cy,cxmin)=NewD(cy,cx);

    NewFD(cy,cxmin)=max(0,OldA(cy,cxmin)-InputData(16));
    OldFD(cy,cxmin)=NewFD(cy,cx);

end


for cs=csmin:csmax

    %PDE solution for A(R,F)

        for cy=cymin+1:cymax-1
            for cx=cxmin+1:cxmax-1
                Term1=1-R(cy)*(InputData(5)^2)*(InputData(9)^2*...
                (y(cy)^4)*(s/yinc^2))-F(cx)*(InputData(8)^2)*...
                (InputData(10)^2*(x(cx)^4)*(s/xinc^2))-R(cy)*s;
                Term2=0.5*R(cy)*(InputData(5)^2)*(InputData(9)^2*...
                (y(cy)^4)*(s/yinc^2));
                Term3=0.5*F(cx)*(InputData(8)^2)*(InputData(10)^2*...
                (x(cx)^4)*(s/xinc^2));

                Term4=(R(cy)*(InputData(5)^2)*(InputData(9)^2)*...
                (y(cy)^3)-(InputData(3)*(InputData(4)-R(cy))*...
                InputData(9)*(y(cy)^2)))*(s/yinc);
                Term5=(F(cx)*(InputData(8)^2)*(InputData(10)^2)*...
                (x(cx)^3)-(InputData(6)*(InputData(7)-F(cx))*...
                InputData(10)*(x(cx)^2)))*(s/xinc);
                Term6=InputData(11)*InputData(5)*InputData(8)*...
                (R(cy)^0.5)*(F(cx)^0.5)*InputData(9)*InputData(10)*...
```

```
y(cy)^2*x(cx)^2*(s/(4*xinc*yinc));

EQ1=(F(cx)*(InputData(8)^2)*(InputData(10)^2)*...
(x(cx)^3)-(InputData(6)*(InputData(7)-F(cx))*...
InputData(10)*(x(cx)^2)));

EQ2=(R(cy)*(InputData(5)^2)*(InputData(9)^2)*...
(y(cy)^3)-(InputData(3)*(InputData(4)-R(cy))*...
InputData(9)*(y(cy)^2)));


if EQ1>=0 && EQ2>=0
   NewA(cy,cx)=Term1*OldA(cy,cx)+...
   Term2*(OldA(cy+1,cx)+OldA(cy,cx))+...
   Term3*(OldA(cy,cx+1)+OldA(cy,cx-1))+...
   Term4*(OldA(cy+1,cx)-OldA(cy,cx))+...
   Term5*(OldA(cy,cx+1)- OldA(cy,cx))+...
   Term6*(OldA(cy+1,cx+1)-OldA(cy-1,cx+1)-...
   OldA(cy+1,cx-1)+OldA(cy-1,cx-1));

elseif EQ1<0 && EQ2>=0
   NewA(cy,cx)=Term1*OldA(cy,cx)+...
   Term2*(OldA(cy+1,cx)+OldA(cy-1,cx))+...
   Term3*(OldA(cy,cx+1)+OldA(cy,cx-1))+...
   Term4*(OldA(cy+1,cx)-OldA(cy,cx))+...
   Term5*(OldA(cy,cx)-OldA(cy,cx-1))+...
   Term6*(OldA(cy+1,cx+1)-OldA(cy-1,cx+1)-...
   OldA(cy+1,cx-1)+OldA(cy-1,cx-1));

elseif EQ1>=0 && EQ2<0
   NewA(cy,cx)=Term1*OldA(cy,cx)+...
   Term2*(OldA(cy+1,cx)+OldA(cy-1,cx))+...
   Term3*(OldA(cy,cx+1)+OldA(cy,cx-1))+...
```

```
            Term4*(OldA(cy,cx)-OldA(cy-1,cx))+...
            Term5*(OldA(cy,cx+1)-OldA(cy,cx))+...
            Term6*(OldA(cy+1,cx+1)-OldA(cy-1,cx+1)-...
            OldA(cy+1,cx-1)+OldA(cy-1,cx-1));

        else
            NewA(cy,cx)=Term1*OldA(cy,cx)+...
            Term2*(OldA(cy+1,cx)+OldA(cy-1,cx))+...
            Term3*(OldA(cy,cx+1)+OldA(cy,cx-1))+...
            Term4*(OldA(cy,cx)-OldA(cy-1,cx))+...
            Term5*(OldA(cy,cx)-OldA(cy,cx-1))+...
            Term6*(OldA(cy+1,cx+1)-OldA(cy-1,cx+1)-...
            OldA(cy+1,cx-1)+OldA(cy-1,cx-1));

        end
        OldA(cy,cx)=NewA(cy,cx);
    end

end

%Boundary Conditions

%The value of A(R,F) at the corners
NewA(cymax,cxmin)=OldA(cymax-1,cxmin);
NewA(cymax,cxmax)=OldA(cymax-1,cxmax);
NewA(cymin,cxmin)=0;
NewA(cymin,cxmax)=0;

OldA(cymax,cxmin)=NewA(cymax,cxmin);
OldA(cymax,cxmax)=NewA(cymax,cxmax);
OldA(cymin,cxmin)=NewA(cymin,cxmin);
OldA(cymin,cxmax)=NewA(cymin,cxmax);
```

```
%The value of A(R,F) at the edges

%F=0 R varies%
for cy=cymin+1:cymax-1
   NewA(cy,cxmax)=0;
   OldA(cy,cxmax)=NewA(cy,cxmax);
end

%R=0, F varies%
for cx=cxmin+1:cxmax-1
   Term7=1-F(cx)*(InputData(8)^2)*(InputData(10)^2*...
   (x(cx)^4)*(s/xinc^2));
   Term8=0.5*F(cx)*(InputData(8)^2)*(InputData(10)^2*...
   (x(cx)^4)*(s/xinc^2));
   Term9=(F(cx)*(InputData(8)^2)*(InputData(10)^2)*...
   (x(cx)^3)-(InputData(6)*(InputData(7)-F(cx))*...
   InputData(10)*(x(cx)^2)))*(s/xinc);
   Term10=-InputData(3)*InputData(4)*InputData(9)*(y(cy)^2)*(s/yinc);

   EQ3=(F(cx)*(InputData(8)^2)*(InputData(10)^2)*...
   (x(cx)^3)-(InputData(6)*(InputData(7)-F(cx))*...
   InputData(10)*(x(cx)^2)));

   if EQ3<0
      NewA(cymax,cx)=Term7*OldA(cymax,cx)+...
      Term8*(OldA(cymax,cx+1)-OldA(cymax,cx-1))+...
      Term10*(OldA(cymax,cx)-OldA(cymax-1,cx))+...
      Term9*(OldA(cymax,cx)- OldA(cymax,cx-1));

   else
      NewA(cymax,cx)=Term7*OldA(cymax,cx)+...
      Term8*(OldA(cymax,cx+1)-OldA(cymax,cx-1))+...
      Term10*(OldA(cymax,cx)-OldA(cymax-1,cx))+...
```

```
    Term9*(OldA(cymax,cx+1)- OldA(cymax,cx));
  end

  OldA(cymax,cx)=NewA(cymax,cx);
end

%F=inf, R varies%
for cy=cymin+1:cymax-1
  Term11=1-R(cy)*(InputData(5)^2)*(InputData(9)^2*...
  (y(cy)^4)*(s/yinc^2))-R(cy)*s;
  Term12=0.5*R(cy)*(InputData(5)^2)*(InputData(9)^2*...
  (y(cy)^4)*(s/yinc^2));
  Term13=(R(cy)*(InputData(5)^2)*(InputData(9)^2)*...
  (y(cy)^3)-(InputData(3)*(InputData(4)-R(cy))*...
  InputData(9)*(y(cy)^2)))*(s/yinc);

  Term14=-(InputData(6)*InputData(7)*InputData(10)*(x(cx)^2))*(s/xinc);

  EQ4=(R(cy)*(InputData(5)^2)*(InputData(9)^2)*...
  (y(cy)^3)-(InputData(3)*(InputData(4)-R(cy))*...
  InputData(9)*(y(cy)^2)));

  if EQ4<0
    NewA(cy,cxmin)=Term11*OldA(cy,cxmin)+...
    Term12*(OldA(cy+1,cxmin)+OldA(cy-1,cxmin))+...
    Term13*(OldA(cy,cxmin)-OldA(cy-1,cxmin))+...
    Term14*(OldA(cy,cxmin)- OldA(cy,cxmin+1));
  else
    NewA(cy,cxmin)=Term11*OldA(cy,cxmin)+...
    Term12*(OldA(cy+1,cxmin)+OldA(cy-1,cxmin))+...
    Term13*(OldA(cy+1,cxmin)-OldA(cy,cxmin))+...
    Term14*(OldA(cy,cxmin)- OldA(cy,cxmin+1));
  end
```

```
    OldA(cy,cxmin)=NewA(cy,cxmin);
end



%R=inf, F varies%
for cx=cxmin+1:cxmax-1
    NewA(cymin,cx)=0;
    OldA(cymin,cx)=NewA(cymin,cx);
end

%%%Corners of the grid%%%

%Corners of the Grid in which F=0, R=0 %%%

NewV(cymax,cxmax)=OldV(cymax-1,cxmax);
NewD(cymax,cxmax)=OldD(cymax-1,cxmax);
NewFD(cymax,cxmax)=OldFD(cymax-1,cxmax);
OldV(cymax,cxmax)=NewV(cymax,cxmax);
OldD(cymax,cxmax)=NewD(cymax,cxmax);
OldFD(cymax,cxmax)=NewFD(cymax,cxmax);

%Corners of the Grid in which F=inf, R=0 %%%
NewV(cymax,cxmin)=OldV(cymax-1,cxmin);
NewD(cymax,cxmin)=OldD(cymax-1,cxmin);
NewFD(cymax,cxmin)=OldFD(cymax-1,cxmin);
OldV(cymax,cxmin)=NewV(cymax,cxmin);
OldD(cymax,cxmin)=NewD(cymax,cxmin);
OldFD(cymax,cxmin)=NewFD(cymax,cxmin);

%Corners of the Grid in which F=inf, R=inf %%%
NewV(cymin,cxmin)=0;
NewD(cymin,cxmin)=0;
NewFD(cymin,cxmin)=0;
```

```
OldV(cymin,cxmin)=NewV(cymin,cxmin);
OldD(cymin,cxmin)=NewD(cymin,cxmin);
OldFD(cymin,cxmin)=NewFD(cymin,cxmin);

%Corners of the Grid in which F=0, R=inf %%%
NewV(cymin,cxmax)=0;
NewD(cymin,cxmax)=0;
NewFD(cymin,cxmax)=0;
OldV(cymin,cxmax)=NewV(cymin,cxmax);
OldD(cymin,cxmax)=NewD(cymin,cxmax);
OldFD(cymin,cxmax)=NewFD(cymin,cxmax);



%%%%Edges of the Grid %%%

%F=0 R varies%
for cy=cymin+1:cymax-1

   NewD(cy,cxmax)=0;
   NewFD(cy,cxmax)=0;
   NewV(cy,cxmax)=0;
   OldD(cy,cxmax)=NewD(cy,cxmax);
   OldFD(cy,cxmax)=NewFD(cy,cxmax);
   OldV(cy,cxmax)=NewV(cy,cxmax);
end

%R=0, F varies%
for cx=cxmin+1:cxmax-1
   Term15=1-F(cx)*(InputData(8)^2)*(InputData(10)^2)*...
   (x(cx)^4)*(s/(xinc^2));
   Term16=0.5*F(cx)*(InputData(8)^2)*(InputData(10)^2)*...
   (x(cx)^4)*(s/(xinc^2));
```

```
Term17=(F(cx)*(InputData(8)^2)*(InputData(10)^2)*...
(x(cx)^3)-(InputData(6)*(InputData(7)-F(cx))*...
InputData(10)*(x(cx)^2)))*(s/xinc);
Term18=-InputData(3)*InputData(4)*InputData(9)*(y(cy)^2)*(s/yinc);

EQ5=(F(cx)*(InputData(8)^2)*(InputData(10)^2)*...
(x(cx)^3)-(InputData(6)*(InputData(7)-F(cx))*...
InputData(10)*(x(cx)^2)));
if EQ5<0
   NewV(cymax,cx)=Term15*OldV(cymax,cx)+...
   Term16*(OldV(cymax,cx+1)-OldV(cymax,cx-1))+...
   Term18*(OldV(cymax,cx)-OldV(cymax-1,cx))+...
   Term17*(OldV(cymax,cx)- OldV(cymax,cx-1));
   OldV(cymax,cx)=NewV(cymax,cx);

   NewD(cymax,cx)=Term15*OldD(cymax,cx)+...
   Term16*(OldD(cymax,cx+1)-OldD(cymax,cx-1))+...
   Term18*(OldD(cymax,cx)-OldD(cymax-1,cx))+...
   Term17*(OldD(cymax,cx)- OldD(cymax,cx-1));
   OldD(cymax,cx)=NewD(cymax,cx);

   NewFD(cymax,cx)=Term15*OldFD(cymax,cx)+...
   Term16*(OldFD(cymax,cx+1)-OldFD(cymax,cx-1))+...
   Term18*(OldFD(cymax,cx)-OldFD(cymax-1,cx))+...
   Term17*(OldFD(cymax,cx)- OldFD(cymax,cx-1));
   OldFD(cymax,cx)=NewFD(cymax,cx);

else
   NewV(cymax,cx)=Term15*OldV(cymax,cx)+...
   Term16*(OldV(cymax,cx+1)-OldV(cymax,cx-1))+...
   Term18*(OldV(cymax,cx)-OldV(cymax-1,cx))+...
   Term17*(OldV(cymax,cx+1)- OldV(cymax,cx));
   OldV(cymax,cx)=NewV(cymax,cx);
```

```
        NewD(cymax,cx)=Term15*OldD(cymax,cx)+...
        Term16*(OldD(cymax,cx+1)-OldD(cymax,cx-1))+...
        Term18*(OldD(cymax,cx)-OldD(cymax-1,cx))+...
        Term17*(OldD(cymax,cx+1)- OldD(cymax,cx));
        OldD(cymax,cx)=NewD(cymax,cx);

        NewFD(cymax,cx)=Term15*OldFD(cymax,cx)+...
        Term16*(OldFD(cymax,cx+1)-OldFD(cymax,cx-1))+...
        Term18*(OldFD(cymax,cx)-OldFD(cymax-1,cx))+...
        Term17*(OldFD(cymax,cx+1)- OldFD(cymax,cx));
        OldFD(cymax,cx)=NewFD(cymax,cx);

    end

end

%F=inf, R varies%

for cy=cymin+1:cymax-1
    Term19=1-R(cy)*(InputData(5)^2)*(InputData(9)^2*...
    (y(cy)^4)*(s/yinc^2))-R(cy)*s;
    Term20=0.5*R(cy)*(InputData(5)^2)*(InputData(9)^2*...
    (y(cy)^4)*(s/yinc^2));
    Term21=(R(cy)*(InputData(5)^2)*(InputData(9)^2)*...
    (y(cy)^3)-(InputData(3)*(InputData(4)-R(cy))*...
    InputData(9)*(y(cy)^2)))*(s/yinc);

    Term22=-(InputData(6)*InputData(7)*InputData(10)*(x(cx)^2))*(s/xinc);

    EQ6=(R(cy)*(InputData(5)^2)*(InputData(9)^2)*...
    (y(cy)^3)-(InputData(3)*(InputData(4)-R(cy))*...
    InputData(9)*(y(cy)^2)));
```

```
if EQ6<0
   NewFD(cy,cxmin)=Term19*OldFD(cy,cxmin)+...
   Term20*(OldFD(cy+1,cxmin)+OldFD(cy-1,cxmin))+...
   Term21*(OldFD(cy,cxmin)-OldFD(cy-1,cxmin))+...
   Term22*(OldFD(cy,cxmin)- OldFD(cy,cxmin+1));

   NewD(cy,cxmin)=Term19*OldD(cy,cxmin)+...
   Term20*(OldD(cy+1,cxmin)+OldD(cy-1,cxmin))+...
   Term21*(OldD(cy,cxmin)-OldD(cy-1,cxmin))+...
   Term22*(OldD(cy,cxmin)- OldD(cy,cxmin+1));

   NewV(cy,cxmin)=Term19*OldV(cy,cxmin)+...
   Term20*(OldV(cy+1,cxmin)+OldV(cy-1,cxmin))+...
   Term21*(OldV(cy,cxmin)-OldV(cy-1,cxmin))+...
   Term22*(OldV(cy,cxmin)- OldV(cy,cxmin+1));
else
   NewFD(cy,cxmin)=Term19*OldFD(cy,cxmin)+...
   Term20*(OldFD(cy+1,cxmin)+OldFD(cy-1,cxmin))+...
   Term21*(OldFD(cy+1,cxmin)-OldFD(cy,cxmin))+...
   Term22*(OldFD(cy,cxmin)- OldFD(cy,cxmin+1));

   NewD(cy,cxmin)=Term19*OldD(cy,cxmin)+...
   Term20*(OldD(cy+1,cxmin)+OldD(cy-1,cxmin))+...
   Term21*(OldD(cy+1,cxmin)-OldD(cy,cxmin))+...
   Term22*(OldD(cy,cxmin)- OldD(cy,cxmin+1));

   NewV(cy,cxmin)=Term19*OldV(cy,cxmin)+...
   Term20*(OldV(cy+1,cxmin)+OldV(cy-1,cxmin))+...
   Term21*(OldV(cy+1,cxmin)-OldV(cy,cxmin))+...
   Term22*(OldV(cy,cxmin)- OldV(cy,cxmin+1));
```

```
    end
    OldD(cy,cxmin)=NewD(cy,cxmin);
    OldFD(cy,cxmin)=NewFD(cy,cxmin);
    OldV(cy,cxmin)=NewV(cy,cxmin);

end

%R=inf, F varies%
for cx=cxmin+1:cxmax-1
    NewD(cymin,cx)=0;
    NewFD(cymin,cx)=0;
    NewV(cymin,cx)=0;
    OldD(cymin,cx)=NewD(cymin,cx);
    OldFD(cymin,cx)=NewFD(cymin,cx);
    OldV(cymin,cx)=NewV(cymin,cx);
end

%%The Main Algorithm to Solve the Main PDE for V(F,r),FD(F,r) and D(F,r)%%

    for cx=cxmin+1:cxmax-1

        for cy=cymin+1:cymax-1
        Term23=1-R(cy)*(InputData(5)^2)*(InputData(9)^2*...
        (y(cy)^4)*(s/yinc^2))-F(cx)*(InputData(8)^2)*...
        (InputData(10)^2*(x(cx)^4)*(s/xinc^2))-R(cy)*s;
        Term26=0.5*R(cy)*(InputData(5)^2)*(InputData(9)^2*...
        (y(cy)^4)*(s/yinc^2));
        Term24=0.5*F(cx)*(InputData(8)^2)*(InputData(10)^2*...
        (x(cx)^4)*(s/xinc^2));
        Term27=(R(cy)*(InputData(5)^2)*(InputData(9)^2)*...
        (y(cy)^3)-(InputData(3)*(InputData(4)-R(cy))*...
        InputData(9)*(y(cy)^2)))*(s/yinc);
        Term25=(F(cx)*(InputData(8)^2)*(InputData(10)^2)*...
```

```
(x(cx)^3)-(InputData(6)*(InputData(7)-F(cx))*...
InputData(10)*(x(cx)^2)))*(s/xinc);
Term28=InputData(11)*InputData(5)*InputData(8)*...
(R(cy)^0.5)*(F(cx)^0.5)*InputData(9)*InputData(10)*...
y(cy)^2*x(cx)^2*(s/(4*xinc*yinc));

EQ7=F(cx)*(InputData(8)^2)*(InputData(10)^2)*...
(x(cx)^3)-(InputData(6)*(InputData(7)-F(cx))*...
InputData(10)*(x(cx)^2));

EQ8=R(cy)*(InputData(5)^2)*(InputData(9)^2)*...
(y(cy)^3)-(InputData(3)*(InputData(4)-R(cy))*...
InputData(9)*(y(cy)^2));


if EQ7>=0 && EQ8>=0
   NewV(cy,cx)=Term23*OldV(cy,cx)+...
   Term24*(OldV(cy,cx+1)+OldV(cy,cx-1))+...
   Term25*(OldV(cy,cx+1)-OldV(cy,cx))+...
   Term26*(OldV(cy+1,cx)+OldV(cy-1,cx))+...
   Term27*(OldV(cy+1,cx)-OldV(cy,cx))+...
   Term28*(OldV(cy+1,cx+1)-OldV(cy+1,cx-1)-...
   OldV(cy-1,cx+1)+OldV(cy-1,cx-1));
elseif EQ7<0 && EQ8>=0
   NewV(cy,cx)=Term23*OldV(cy,cx)+...
   Term24*(OldV(cy,cx+1)+OldV(cy,cx-1))+...
   Term25*(OldV(cy,cx)-OldV(cy,cx-1))+...
   Term26*(OldV(cy+1,cx)+OldV(cy-1,cx))+...
   Term27*(OldV(cy+1,cx)-OldV(cy,cx))+...
   Term28*(OldV(cy+1,cx+1)-OldV(cy+1,cx-1)-...
   OldV(cy-1,cx+1)+OldV(cy-1,cx-1));
elseif EQ7>=0 && EQ8<0
   NewV(cy,cx)=Term23*OldV(cy,cx)+...
```

```
    Term24*(OldV(cy,cx+1)+OldV(cy,cx-1))+...
    Term25*(OldV(cy,cx+1)-OldV(cy,cx))+...
    Term26*(OldV(cy+1,cx)+OldV(cy-1,cx))+...
    Term27*(OldV(cy,cx)-OldV(cy-1,cx))+...
    Term28*(OldV(cy+1,cx+1)-OldV(cy+1,cx-1)-...
    OldV(cy-1,cx+1)+OldV(cy-1,cx-1));
else
    NewV(cy,cx)=Term23*OldV(cy,cx)+...
    Term24*(OldV(cy,cx+1)+OldV(cy,cx-1))+...
    Term25*(OldV(cy,cx)-OldV(cy,cx-1))+...
    Term26*(OldV(cy+1,cx)+OldV(cy-1,cx))+...
    Term27*(OldV(cy,cx)-OldV(cy-1,cx))+...
    Term28*(OldV(cy+1,cx+1)-OldV(cy+1,cx-1)-...
    OldV(cy-1,cx+1)+OldV(cy-1,cx-1));
end


if EQ7>=0 && EQ8>=0
    NewD(cy,cx)=Term23*OldD(cy,cx)+...
    Term24*(OldD(cy,cx+1)+OldD(cy,cx-1))+...
    Term25*(OldD(cy,cx+1)-OldD(cy,cx))+...
    Term26*(OldD(cy+1,cx)+OldD(cy-1,cx))+...
    Term27*(OldD(cy+1,cx)-OldD(cy,cx))+...
    Term28*(OldD(cy+1,cx+1)-OldD(cy+1,cx-1)-...
    OldD(cy-1,cx+1)+OldD(cy-1,cx-1));

    OldD(cy,cx)=NewD(cy,cx);

elseif EQ7<0 && EQ8>=0
    NewD(cy,cx)=Term23*OldD(cy,cx)+...
    Term24*(OldD(cy,cx+1)+OldD(cy,cx-1))+...
    Term25*(OldD(cy,cx)-OldD(cy,cx-1))+...
    Term26*(OldD(cy+1,cx)+OldD(cy-1,cx))+...
```

```
        Term27*(OldD(cy+1,cx)-OldD(cy,cx))+...
        Term28*(OldD(cy+1,cx+1)-OldD(cy+1,cx-1)-...
        OldD(cy-1,cx+1)+OldD(cy-1,cx-1));

        OldD(cy,cx)=NewD(cy,cx);

    elseif EQ7>=0 && EQ8<0
        NewD(cy,cx)=Term23*OldD(cy,cx)+...
        Term24*(OldD(cy,cx+1)+OldD(cy,cx-1))+...
        Term25*(OldD(cy,cx+1)-OldD(cy,cx))+...
        Term26*(OldD(cy+1,cx)+OldD(cy-1,cx))+...
        Term27*(OldD(cy,cx)-OldD(cy-1,cx))+...
        Term28*(OldD(cy+1,cx+1)-OldD(cy+1,cx-1)-...
        OldD(cy-1,cx+1)+OldD(cy-1,cx-1));

        OldD(cy,cx)=NewD(cy,cx);

    else
        NewD(cy,cx)=Term23*OldD(cy,cx)+...
        Term24*(OldD(cy,cx+1)+OldD(cy,cx-1))+...
        Term25*(OldD(cy,cx)-OldD(cy,cx-1))+...
        Term26*(OldD(cy+1,cx)+OldD(cy-1,cx))+...
        Term27*(OldD(cy,cx)-OldD(cy-1,cx))+...
        Term28*(OldD(cy+1,cx+1)-OldD(cy+1,cx-1)-...
        OldD(cy-1,cx+1)+OldD(cy-1,cx-1));

        OldD(cy,cx)=NewD(cy,cx);

    end


if EQ7>=0 && EQ8>=0
    NewFD(cy,cx)=Term23*OldFD(cy,cx)+...
```

```
    Term24*(OldFD(cy,cx+1)+OldFD(cy,cx-1))+...
    Term25*(OldFD(cy,cx+1)-OldFD(cy,cx))+...
    Term26*(OldFD(cy+1,cx)+OldFD(cy-1,cx))+...
    Term27*(OldFD(cy+1,cx)-OldFD(cy,cx))+...
    Term28*(OldFD(cy+1,cx+1)-OldFD(cy+1,cx-1)-...
    OldFD(cy-1,cx+1)+OldFD(cy-1,cx-1));

    OldFD(cy,cx)=NewFD(cy,cx);

elseif EQ7<0 && EQ8>=0
    NewFD(cy,cx)=Term23*OldFD(cy,cx)+...
    Term24*(OldFD(cy,cx+1)+OldFD(cy,cx-1))+...
    Term25*(OldFD(cy,cx)-OldFD(cy,cx-1))+...
    Term26*(OldFD(cy+1,cx)+OldFD(cy-1,cx))+...
    Term27*(OldFD(cy+1,cx)-OldFD(cy,cx))+...
    Term28*(OldFD(cy+1,cx+1)-OldFD(cy+1,cx-1)-...
    OldFD(cy-1,cx+1)+OldFD(cy-1,cx-1));

    OldFD(cy,cx)=NewFD(cy,cx);

elseif EQ7>=0 && EQ8<0
    NewFD(cy,cx)=Term23*OldFD(cy,cx)+...
    Term24*(OldFD(cy,cx+1)+OldFD(cy,cx-1))+...
    Term25*(OldFD(cy,cx+1)-OldFD(cy,cx))+...
    Term26*(OldFD(cy+1,cx)+OldFD(cy-1,cx))+...
    Term27*(OldFD(cy,cx)-OldFD(cy-1,cx))+...
    Term28*(OldFD(cy+1,cx+1)-OldFD(cy+1,cx-1)-...
    OldFD(cy-1,cx+1)+OldFD(cy-1,cx-1));

    OldFD(cy,cx)=NewFD(cy,cx);

else
    NewFD(cy,cx)=Term23*OldFD(cy,cx)+...
```

```
            Term24*(OldFD(cy,cx+1)+OldFD(cy,cx-1))+...
            Term25*(OldFD(cy,cx)-OldFD(cy,cx-1))+...
            Term26*(OldFD(cy+1,cx)+OldFD(cy-1,cx))+...
            Term27*(OldFD(cy,cx)-OldFD(cy-1,cx))+...
            Term28*(OldFD(cy+1,cx+1)-OldFD(cy+1,cx-1)-...
            OldFD(cy-1,cx+1)+OldFD(cy-1,cx-1));

            OldFD(cy,cx)=NewFD(cy,cx);

          end



          OldV(cy,cx)=NewV(cy,cx);

        end

      end
    end

  end

  end
```