STOCHASTIC MODELING OF ELECTRICITY MARKETS

A THESIS SUBMITTED TO
THE GRADUATE SCHOOL OF APPLIED MATHEMATICS
OF
MIDDLE EAST TECHNICAL UNIVERSITY

BY

İREM TALASLI

IN PARTIAL FULFILLMENT OF THE REQUIREMENTS
FOR
THE DEGREE OF PHILOSOPHY OF DOCTORATE
IN
FINANCIAL MATHEMATICS

JANUARY 2012

Approval of the thesis:

STOCHASTIC MODELING OF ELECTRICITY MARKETS

submitted by **İREM TALASLI** in partial fulfillment of the requirements for the degree of **Philosophy of Doctorate in Department of Financial Mathematics, Middle East Technical University** by,

Prof. Dr. Ersan Akyıldız ______________
Director, Graduate School of **Applied Mathematics**

Assoc. Prof. Dr. Ömür Uğur ______________
Head of Department, **Financial Mathematics**

Assoc. Prof. Dr. Azize Hayfavi ______________
Supervisor, **Department of Financial Mathematics**

Examining Committee Members:

Prof. Dr. Gerhard Wilhelm Weber ______________
Institute of Applied Mathematics, METU

Prof. Dr. Gül Ergün ______________
Department of Statistics, Hacettepe University

Assoc. Prof. Dr. Azize Hayfavi ______________
Institute of Applied Mathematics, METU

Assoc. Prof. Dr. Ömür Uğur ______________
Institute of Applied Mathematics, METU

Assist. Prof. Dr. Yeliz Yolcu Okur ______________
Institute of Applied Mathematics, METU

Date: ______________

* Write the country name for the foreign committee member.

I hereby declare that all information in this document has been obtained and presented in accordance with academic rules and ethical conduct. I also declare that, as required by these rules and conduct, I have fully cited and referenced all material and results that are not original to this work.

Name, Last Name: İREM TALASLI

Signature :

ABSTRACT

STOCHASTIC MODELING OF ELECTRICITY MARKETS

Talaslı, İrem
Ph.D., Department of Financial Mathematics
Supervisor : Assoc. Prof. Dr. Azize Hayfavi

January 2012, 88 pages

Day-ahead spot electricity markets are the most transparent spot markets where one can find integrated supply and demand curves of the market players for each settlement period. Since it is an indicator for the market players and regulators, in this thesis we model the spot electricity prices. Logarithmic daily average spot electricity prices are modeled as a summation of a deterministic function and multi-factor stochastic process. Randomness in the spot prices is assumed to be governed by three jump processes and a Brownian motion where two of the jump processes are mean reverting. While the Brownian motion captures daily regular price movements, the pure jump process models price shocks which have long term effects and two Ornstein Uhlenbeck type jump processes with different mean reversion speeds capturing the price shocks that affect the price level for relatively shorter time periods. After removing the seasonality which is modeled as a deterministic function from price observations, an iterative threshold function is used to filter the jumps. The threshold function is constructed on volatility estimation generated by a GARCH(1,1) model. Not only the jumps but also the mean reverting returns following the jumps are filtered. Both of the filtered jump processes and residual Brownian components are estimated separately. The model is applied to Austrian, Italian, Spanish and Turkish electricity markets data and it is found that the weekly forecasts, which are generated by the estimated parameters, turn out to be able to capture the characteristics of the observations.

After examining the future contracts written on electricity, we also suggest a decision technique which is built on risk premium theory. With the help of this methodology derivative market players can decide on taking whether a long or a short position for a given contract. After testing our technique, we conclude that the decision rule is promising but needs more empirical research.

Keywords: Electricity spot price, stochastic multi-factor model, jump process, GARCH (1,1), risk premium

ÖZ

ELEKTRİK PİYASALARININ STOKASTİK MODELLEMESİ

Talaslı, İrem
Doktora, Finansal Matematik Bölümü
Tez Yöneticisi : Doç. Dr. Azize Hayfavi

Ocak 2012, 88 sayfa

Gün öncesi elektrik piyasaları, piyasa oyuncularının her uzlaşma dönemine ait toplam arz ve talep eğrilerini içeren en şeffaf spot elektrik piyasasıdır. Bu tezde hem piyasa oyuncuları hem de düzenleme kurumları açısından gösterge niteliği taşıyan gün öncesi elektrik fiyatları modellenmektedir. Logaritması alınan günlük ortalama spot elektrik fiyatları deterministik bir fonsiyon ve çok faktörlü stokastik süreçler ile modellenmiştir. Spot fiyalardaki rassallık iki tanesi ortalamaya dönen olmak üzere üç adet sıçrama süreci ve bir Brown hareketi tarafından yönetilmektedir. Brown hareketi günlük olağan fiyat hareketlerini yakalarken, sıçrama süreçlerinden ilki uzun dönemde etkili olan fiyat şoklarının, farklı ortalamaya dönüş hızlarına sahip olan iki tane Ornstein Uhlenbeck tipi sıçrama süreci ise fiyat seviyesi üzerinde görece daha kısa süre etkili olan fiyat şoklarının açıklanmasında kullanılmaktadır. Deterministik bir fonksiyon ile modellenen mevsimsellik etkisinin gözlenen fiyatlardan ayrıştırlmasından sonra, iteratif eşik fonksiyonu kullanılarak fiyatlardaki sıçramalar filtrelenmektedir. Söz konusu eşik fonksiyonunda GARCH(1,1) kullanılarak heasplanan oynaklık tahminleri kullanılmaktadır. Sadece sıçramalar değil, onları takip eden ortalamaya dönüş getirileri de filtre yardımı ile ayrılmaktadır. Filtrelenen sıçrama süreçleri ve bu süreçten geriye kalan Brown hareketine ait parametreler ayrı ayrı hesaplanmaktadır. Model Avusturya, İtalya, İspanya ve Türkiye'deki elektrik piyasa verilerine uygulanmış ve elde edilen parametereler kullanılarak yapılan haftalık fiyat tahminlerinin fiyat gözlemlerine yakınsadığı görülmüştür.

Elektrik üzerine yazılan vadeli sözleşmeler incelenerek, risk primi teorisine dayalı bir yöntem önerilmiştir. Bu yöntem piyasa oyuncularına, herhangi bir vadeli elektirk sözleşmesinde kısa ya da uzun pozisyon alınması doğrultusunda bilgi vermektedir. Yapılan testler sonucunda söz konusu yöntemin uygulabilir olduğu ancak daha fazla ampirik çalışmayla desteklenmesi gerektiği sonucuna varılmıştır.

Anahtar Kelimeler: Spot elektrik fiyatı, stokastik faktör modelleri, sıçrama süreçleri, GARCH (1,1), risk primi

In the memory of Prof. Dr. Hayri Körezlioğlu

ACKNOWLEDGMENTS

I would like to express my sincere thanks to everyone who contributed to my research and have helped me in the formation of this thesis.

I would like to express my gratitude to my advisor Assoc. Prof. Dr. Azize Hayfavi for her guidance and encouragement throughout the whole process.

I am grateful to Assoc. Prof. Dr. Ömür Uğur, Assist. Prof. Dr. Yeliz Yolcu Okur, Prof. Dr. Gerhard Wilhelm Weber and Assist. Prof. Dr. Kasırga Yıldırak for their guidance.

I would like to express my special thanks to my parents, Azra and Aykut Yıldırım and to my husband Anıl Talaslı for their endless support and patience.

I am thankful to Zehra Ekşi Altay, Aslı Kunt and all of my friends in IAM for their valuable help.

Finally I would like to thank all of my colleagues in the Central Bank of Turkey, for the patience and tolerance they have shown to me. I would like to express my speacial thanks to Dr. Emrah Ekşi, Dr. Eşref Mehmet Göksu, Bengi Deliorman, Özlem Kumbaracı, Metin Kavut, Özgür Ergül, Kevser Öztürk, Nihan Güvendik, Füsun Esen, Arzu Ağcaer, Bülent Yaman, Bülent Usta and Suat Aydın.

TABLE OF CONTENTS

ABSTRACT iv

ÖZ vi

DEDICATION viii

ACKNOWLEDGMENTS ix

TABLE OF CONTENTS x

LIST OF TABLES xii

LIST OF FIGURES xiii

CHAPTERS

1 INTRODUCTION 1

2 SPOT ELECTRICITY MARKETS 4

2.1 Characteristics of Electricity as a Commodity 6

2.2 The Motivation in Modeling Spot Electricity Prices 8

2.3 Spot Electricity Models in Literature 10

3 THE SPOT PRICE MODEL 20

3.1 Deterministic Part 22

3.2 Jump Detection with Threshold Method 23

3.3 Determining The Threshold 27

3.4 Decomposition of the Stochastic Processes 31

4 ESTIMATION OF SPOT ELECTRICITY PRICE PROCESS AND FORECATING SPOT ELECTRICITY PRICES 37

4.1 Model Estimation 38

4.2 Forecasting 45

5 ELECTRICITY FUTURES MARKETS 48

5.1 Stochastic Futures Pricing Models in Literature 50

5.1.1 Spot price approach 51

5.1.2 Instantaneous delivery models 53

5.1.3 Direct modeling of the observed future contracts 56

5.2 Risk Premium Approach 58

6 CONCLUSION AND OUTLOOK 64

APPENDICES

A Preliminaries 66

A.1 Definitions 66

A.2 Heath Jarrow Merton Framework for the Stochastic Modeling of Interest Rate Dynamics 66

A.3 Risk Premium 68

B Matlab Codes 70

REFERENCES 83

VITA 88

LIST OF TABLES

TABLES

Table 4.1 Day of the week parameters 40

Table 4.2 Deterministic function parameters 40

Table 4.3 Number of jumps filtered by the iterative GARCH(1,1) volatility threshold $r(h)$ 41

Table 4.4 Jump intensities=number of filtered jumps/total number of returns 43

Table 4.5 Discrete time mean reversion rates 43

Table 4.6 Jump distributions 44

Table 5.1 Suggested future contract position according to the risk premium 63

LIST OF FIGURES

FIGURES

Figure 2.1 Demand and supply curves from Italian spot electricity market 7

Figure 2.2 Optional caption for list of figures 7

Figure 4.1 Daily average price series 38

Figure 4.2 Estimated weekly cycle for Italian electricity market 39

Figure 4.3 Decomposition of deterministic and stochastic components 41

Figure 4.4 Separated jumps and mean reverting returns 42

Figure 4.5 Autocorrelation functions of the original return series and the filtered returns after the jump series are extracted 44

Figure 4.6 QQ-plots of the original return series and the filtered returns 45

Figure 4.7 Observed and forecasted price series starting from 2011 46

Figure 4.8 Observed and forecasted price series through the forecast period 46

Figure 4.9 MWE of each week in the forecast period and their average 47

Figure 5.1 TURDEX-December 2011 contract 61

Figure 5.2 Optional caption for list of figures 62

Figure 5.3 Suggested future positions' pay offs 63

CHAPTER 1

INTRODUCTION

In the past two decades, electricity industries in many countries which were initially designed as vertically integrated national or state dominated monopolies have been experiencing a deregulation process. Throughout this process, generation, transmission, distribution and marketing activities have been seperated and opened to competition wherever it is possible and profitable. The rise in the number of market players together with the competition and development of relatively more liberal electricity markets caused spot prices to be determined by supply and demand. This development increased the importance of accurate electricity price forecasting and therefore electricity price modeling. As a commodity, electricity has different characteristics than other commodities as well as financial assets due to its non-storability, its demand inelasticity and significant seasonality of its consumption and production. Electricity consumption is mainly influenced by industrial production, business activities and weather conditions, therefore electricity prices show intra daily, weekly and annual seasonal behaviour. Electricity prices are also mean reverting processes with high volatility and sharp price spikes. Non-storability of electrical energy and necessity of continuous supply and demand balance in the transmission mechanism cause direct reflection of supply and demand shocks on electricity prices. As soon as these extreme conditions disappear, their effects on price levels also fade away. Additional to bilateral electricity trade contracts, whose conditions are predetermined, a significant amount of electricity trade takes place in the spot markets. The spot electricity markets are completed with electricity derivatives trading taking place both in over the counter markets and derivative exchanges. Because of the non-storability of electricity, it is not possible to use main stream derivatives valuation models based on no arbitrage theory. However, due to their high informative content about market players' expectations or hedging behaviour of the agents, derivative accurate modeling of electricity derivatives is a necessity.

We model the daily series of equally weighted average of system marginal prices for each day's balancing intervals (hour / half an hour) formed in the day-ahead markets. Due to mean reversion property of electricity prices, modeling with Ornstein Uhlenbeck type processes are very common in literature. The need for a better fit of model forecasts with observations leads to more complicated models with latent variables and multiple regimes. However, with the increasing number of parameters, the estimation process also gets complicated. Filtering techniques are one of the widely used methods in parameter estimation of the jump processes. In our spot electricity price model, three jump processes without predetermined jump size distributions and a Brownian motion are combined with a deterministic seasonality term. While the Brownian motion captures the daily regular price movements, the pure jump process models the price shocks which have long term effects and two Ornstein Uhlenbeck type processes with different mean reversion speeds that capture the price shocks that have short term effects. One of the mean reverting processes is assumed to model price shocks that revert back in the next observation (price spikes) and the other is assumed to model price shocks that take a few days to fade away (semi-spikes). An iterative threshold derived by using estimated volatility with GARCH(1,1) is used to filter the price jumps. We construct an algorithm for step by step parameter estimation of a multi-factor price model.

One of the main goals of this thesis is to propose spot and future contract price models which can be used in recently established liberal Turkish electricity spot market and electricity future contracts that are traded in national derivatives exchange. However, since we do not prefer to be restrained by the small sample size, we test our model with relatively more mature markets' spot electricity data. We apply our spot price model to four different countries; Austria, Italy, Spain and Turkey all of which heavily depend on thermal sources for electricity generation. The hydro sources in these countries generally follow thermal sources in terms of electricity generation. In all of the four countries, a higher proportion of the generated electricity is consumed by the industry. Austria, Spain and Italy have mature markets relative to Turkish day-ahead spot electricity market which has been taken into full operation in December 2009 and is still evolving through its final market design. For all of the examined countries, the separation results are found to be in accordance with our initial expectations.

In the analysis of the electricity futures pricing, we summarize three main approaches used for electricity forward and future contracts modeling in the literature. We cannot propose a contract price model due to data shortage. Instead of proposing a future contracts price model, we offer a decision technique where the given contract prices are used. With this technique which is built on the risk premium theory, derivative market players can decide taking whether a long or a short position. After testing our technique, we conclude that the decision rule is promising but needs more empirical research.

In this context, spot electricity price processes are discussed in detail in Chapter 2. Our multi-factor spot electricity price model is summarized in Chapter 3. Chapter is devoted to the estimation of model parameters and Monte Carlo simulations. Chapter 5 is dedicated to electricity futures.

CHAPTER 2

SPOT ELECTRICITY MARKETS

Spot electricity, referring to electrical power produced for current consumption, is provided by a transformation industry which can be split into three processes.

- Electricity is produced by generators, who burn fuels such as coal, natural gas or nuclear fuel in power plants or use the gravitational energy of water or the wind force. Generated energy is then injected into a high voltage network.
- Network operator (an independent system operator) is responsible for the transmission of electricity and protection of the global balance between electricity given into and taken out of the transmission system in order to prevent a possible collapse. It operates a software system that allows the agents to exchange electricity on the high voltage network, which is the natural place for wholesale trading.
- Marketing companies and distributors get the electricity from the high voltage network, cascade it down to network distributions with lower voltage and sell it to industrial or residential consumers and handle the metering and billing.

A few decades ago, in most of the countries worldwide, electricity sector was a vertically integrated industry, where generation transmission and distribution is done by a single entity. In this setting, prices were determined by regulators to reflect the cost of generation, transmission and distribution and these prices were used to change in a deterministic manner. However, over the last twenty years, electricity markets in many countries are experiencing a deregulation process, aiming to introduce competition in generation, supply activities and distribution. The first step in this deregulation process is unboundling the activities conducted by the national monopolies. Unboundling the vertically integrated utilities means identifying and separating the different tasks attached to a single entity in the traditional organization. Then these tasks are opened to competition wherever it is possible and profitable. Contrary to generation and distribution, which are appropriate to adapt a competitive market structure, transmission is generally classified as a natural monopoly that should be operated by a single system operator. Managing the grid system requires continuously keeping the entire transmission system at equilibrium. Since it is easier to manage the whole system from a single control point, transmission has generally left out from the liberalization and privatization processes.

The upsurge in the number of market players together with the competition and development of relatively more liberal electricity markets caused electricity to become a commodity whose price is determined by supply and demand. In this market design, long-term bilateral contracts between generators and wholesales (and retailers) dominate the most of the electricity trading. However, if approximately 80% of the trade takes place according to the predetermined conditions of these bilateral contracts, the remaining 20% takes place in the spot electricity markets. The spot electricity market refers to the electricity trading that takes place in a day-ahead market, which is managed by the system operator. For their residual generation capacity that is not bounded by bilateral agreements or for their extra electricity demand, market players send their bids to the system operator in terms of prices and quantities for each settlement period (generally settlement periods are hours or half hours) of the following day. The system operator collect these bids, ranks them by merit order from the least expensive to the next least expensive and so forth, then builds the supply and demand functions. Two of the possible designs that the system operator can manage are given below.

- It is only the suppliers that make bids and the system operator is responsible for computing the expected demand for each settlement period of the following day; intersecting this demand with the supply function provides the system marginal price.
- It is both buyers and sellers who make bids to the pool and then the system operator has to build an analogous demand function, which is a quasivertical line since electricity demand is fairly inelastic to price changes. The marginal price is again defined by the intersection of the curves.

Contrary to bilateral contracts whose conditions are not publicly announced, system clearing prices published every day by the operator. The latter is assumed to be a good indicator for the spot electricity markets. On the other hand, the ultimate role of a spot market is to ensure that total generation meets total demand. Although most of the energy trading is scheduled in advance (from years to day ahead), some imbalances unavoidably occur in real time because of various generation, load, and transmission factors. Generation factors include plant outages, generators not following their schedules accurately, the use of some generation to provide ancillary services, and the intermittent nature of some generation sources (like wind). Load factors include sudden changes in weather forecasting errors, and intrahour load changes. Transmission factors include local and regional congestion, unscheduled flows, and forced outages. All these factors, separetly and in combination, require the system operator to have access to generation output so that it can move up or down from interval to interval. This real time balancing is provided by the reserve capacity mechanism where market players are obliged to hold a reserve capacity, which can be taken into operation in a few minutes if the system operator needs it.

2.1 Characteristics of Electricity as a Commodity

As a commodity, electricity has different characteristics than other commodities and financial assets due to its non-storability, its demand inelasticity and significant seasonality effect in its consumption and production. Unlike the financial assets traded for investment purposes, electricity is traded in order to be consumed. This close link with the real economy and daily life causes electricity price to exhibit a different behaviour than those of the financial assets.

As a secondary energy source, created by the conversion of the other energy resources, electricity is difficult and expensive to store or transmit between different regions. Therefore, the spot price of the electricity is set by the short term supply demand equilibrium, and as already stressed, this equilibrium should be maintained at all times for the safety and the continuity of the whole regional transmission system. Theoretical supply and demand curves of an hour, in a day-ahead market is given in Figure 2.1. This figure is taken from the Italian system operator GME's web site [46] and it shows the supply and demand curves of 12th hour of 30 June 2011, which are constructed according to market players' bids. In Figure 2.2(a) the hourly equilibrium prices for a random week are illustrated. And a longer series of daily prices, computed from the daily averages of hourly prices, are shown in Figure 2.2(b).

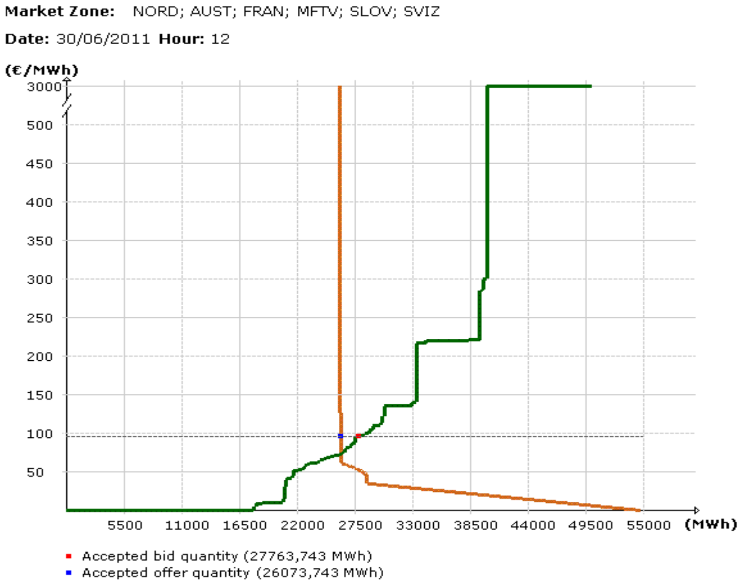
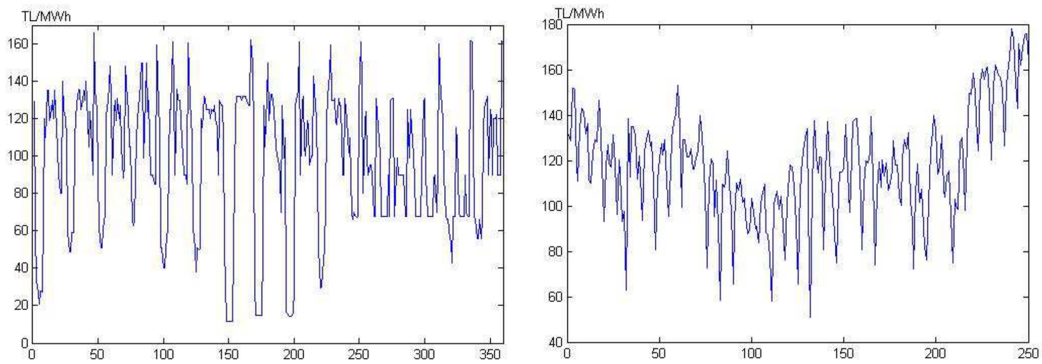


Figure 2.1: Demand and supply curves from Italian spot electricity market

These representative series can help us identify some of the outstanding features of spot prices for electricity.



(a) Hourly electricity prices in Turkish spot electricity market between 1-15 March 2010

(b) Daily electricity prices in Turkish spot electricity market between 1 Dec 09 - 30 Sep 2010

Figure 2.2: Spot electricity prices

1. **Seasonal Behavior:** Electricity consumption is mainly influenced by economic cycles, industrial production, business activities and weather conditions. Therefore electricity prices show intra daily, weekly and annual seasonal behaviour. In accordance with the literature, we assume that seasonal behaviour is governed by a deterministic function. Since we will work with daily average prices, only weekly and yearly seasonality will

be considered.

2. **Mean Reversion:** Contrary to stock prices which can be evolved freely in any direction, electricity prices are known for their anti-persistent nature against supply and demand shocks. Electricity prices generally gravitate around the cost of production. Abnormal market conditions may lead to price spreads in the short run, but in the long run supply will be adjusted and prices will return to the level dictated by the cost of production. Moreover, one of the main factors determining electricity consumption is the temperature which is also a mean reverting process. As a result, spot electricity prices show strong mean reverting behaviour.

3. **High Volatility and Sharp Spikes:** Non-storability of electrical energy and necessity of continuous supply and demand balance in the transmission mechanism cause direct reflection of supply and demand shocks on electricity prices. As it can be seen from Figure 2.1 electricity demand is highly inelastic. The characteristic of the supply stack can also contribute to the price volatility. For low levels of demand, generators supply electricity by using base load units with low marginal costs; as higher quantities needed, generation plants with high marginal costs enter into the system. Therefore higher demand levels may lead to jumps in the prices observed. Moreover, in case of a plant failure, high cost generation plants have to supply energy to the system. However, when the conditions causing the price shocks disappear, prices revert back to their long term equilibrium levels.

2.2 The Motivation in Modeling Spot Electricity Prices

Market agents usually incorporate three instruments for electricity trading: the pool (spot market operated by the system operator), bilateral contracts and the derivative securities. In the pool, agents submit bids, consisting of a set of quantities at certain prices for the following day. And the system operator clears the market and announces the set of clearing prices for the next day. These daily bids should include the price forecasts as well as production and consumption plans of the agents. Successful forecasts of the following day's price can help producers develop revenue maximizing strategies, or maximization of consumers' utility as it is also stated in Contreras et al. (2003) [30].

Moreover, agents need medium term quantity and price projections in order to determine bilateral contract terms. With the help of reliable daily price forecasts, producers or consumers can price these bilateral contracts more efficiently. Also for market players who want to be hedged against the price volatility, expected future spot prices are essential in valuation of derivative instruments.

We will model the daily spot prices (equally weighted average of the hourly equilibrium prices of each day), instead of the hourly system balancing prices. Mainstay of our approach is the observation done by Meyer-Brandis et al. (2007) in [57], which states that hourly / half hourly quoted electricity prices cannot be seen as a time series, since in most of the electricity markets, the delivery prices for all 24 hours of a given day are pre-determined by the system operator simultaneously on the previous day (day-ahead prices). After this determination, day-ahead market closes and a limited number of transactions take place randomly in real time balancing market. Thus, there is no causality relationship between different hourly prices on the same day, and hourly electricity data should rather be seen as 24 dependent daily series than a single series of hourly prices. With this line of reasoning, Meyer-Brandis et al. (2007) [57] suggest the following model for hourly electricity prices:

$$X_t^h = X_t^h f(t,h) + \epsilon_t^h \tag{2.1}$$

where

- X_t^h, $h\,\{\in 1,\ldots,24\}$ is the electricity price on day t and hour h
- X_t^h is the common factor for day t
- $f(t,h)$ is a slowly varying intradaily pattern depending both on the day t and the hour h
- ϵ_t^h is a white noise process

In this setting, most of the variability observed in electricity price, as well as all interesting statistical features, e.g. mean reversion, are assumed to be contained in the average daily price series.

2.3 Spot Electricity Models in Literature

In general, electricity price models can be grouped under three general topics. The first approach is based on econometric time series models. Apart from the basic autoregressive (AR) and autoregressive moving average (ARMA) specifications, a wide range of alternative models have been proposed. This long list of models includes: autoregressive integrated moving average (ARIMA) and seasonal ARIMA models (Zhou et al. (2004) [80]), autoregressions with heteroscedasticity (Garcia et al. (2005) [41]) or with heavy tailed (Weron (2005) [77]) innovations, AR models with exogenous variables (ARX models), AR and ARX models with thresholds (Misiorek et al. (2006) [58]), regime switching regressions with fundamental variables (Weron (2006) [78]) and mean reverting jump diffusions (Knittel et al. (2005)[50]).

The second group of models consists of fundamental / forward based models, the futures prices and their relation with the spot electricity prices are the main focus of the study, and the dynamics of the whole futures price curve is modeled by using Heath-Jarrow-Morton (HJM) framework. See for instance, [3], [17], [27], [51]. A general discussion of HJM-type models in the context of power future is given in Benth et al. (2008) [10]. They dedicate a substantial part of their analysis to the relation of spot, forward and swap price dynamics. However, since non-storability of the electricity causes the break up of no arbitrage condition between spot and future markets, futures prices do not reveal any information about price dynamics on a daily timescale, but they can provide only a poor approximation to the complex structure of spot market prices.

The classical starting point for the commodity price modeling is the Schwartz one-factor model [70] , which is an extension of geometric Brownian motion allowing for mean reversion:

$$S_t = S_0 \exp(X_t), \tag{2.2}$$

$$dX(t) = \alpha(\mu - X(t))dt + \sigma dB(t), \tag{2.3}$$

where B_t is a standard Brownian motion, σ is the volatility of the process and α is the speed of the process reverting to its long term mean μ. Many electricity price models use this process or its variants as building blocks. For instance, spot price models proposed in Lucia et al.

(2002) [53] are given (2.4) and (2.5) by

$$S_t = h(t) + X_t, \tag{2.4}$$

$$S_t = \exp(h(t) + X_t), \tag{2.5}$$

where S_t is the spot price, X_t is an Ornstein-Uhlenbeck process and $h(t)$ is a deterministic component, which is used to account for the seasonal effects. Weekly seasonality due to higher weekday prices than weekend prices, and spot prices' cyclical behaviour throughout the year are all included in spot price dynamics by the term $h(t)$. Benth et al. (2008) [10] defined (2.4) type models as arithmetic models and models with the form given in (2.5) as geometric models in which logarithmic prices can be characterized by an Ornstein-Uhlenbeck process.

Spot price models can be examined under two subgroups: single factor and multi-factor models. In a single factor model the spot price itself is a Markov process while in a multi-factor model the spot price $S_t = g(X_1^t, ..., X_k^t)$ is a function of a multi-dimensional Markov process. Here $g : \mathbb{R}^k \to \mathbb{R}^+$ and since g is not one-to-one, these models have both unknown and hidden components. Not only Lucia and Schwartz (2002) [53], but also Cartea et al. (2005) [26], Barlow (2002) [4], Geman et al.(2005) [42] proposed single factor models. Many of these models, unlike Lucia and Schwartz's, take price spikes into account too. With an additional jump term included in an Ornstein-Uhlenbeck process, Cartea et al. (2005) [26], improved their models to cover also the price spikes:

$$\log S_t = h(t) + Y_t, \tag{2.6}$$

$$dY(t) = -\alpha Y_t dt + \sigma dB_t + J_t dN_t, \tag{2.7}$$

where B_t is a Brownian motion, $h(t)$ is assumed to capture the seasonal patterns of the spot price and the term $J_t dN_t$ enables the process to have discrete random spikes which are combination of a Poisson process N_t and a jump size distribution J_t. In (2.7) the process dN_t is approximated by a Bernoulli process with parameter ldt and J_t is assumed to be lognormally distributed. Cartea et al. (2005) [26] apply this one factor jump diffusion model spot electricity markets in England and Wales. Unfortunately, unless the price data series is not long enough, few price spike observations lead to difficulties in estimation of the parameters. Moreover, one factor models including a jump process are expected to have a high speed of mean reversion. Otherwise, jumps can have permanent effects on the price levels.

Barlow (2002) [4] introduced a nonlinear Ornstein-Uhlenbeck model for spot power prices. In this model, the price is obtained by equalization of the demand level with a deterministic non-linear supply function to take price spikes into account. He used the inverse function of the Box-Cox transformation:

$$S_t = \begin{cases} f_\alpha(X_t), & \text{if } 1 + \alpha X_t > \epsilon_0, \\ \epsilon_0^{1/\alpha}, & \text{if } + \alpha X_t \leq \epsilon_0, \end{cases}$$

$$dX_t = -\lambda(X_t - \alpha)dt + \sigma dB(t), \tag{2.8}$$

where $f_\alpha(x) = (1 + \alpha x)^{1/\alpha}$, $\alpha \neq 0$, $\epsilon_0^{1/\alpha}$ is assumed to be the maximum price level taken as a constant and $f_0(x) = e^x$.

When $\alpha = 0$, an exponential Ornstein-Uhlenbeck process is retrieved for S_t. The case $\alpha = 1$ yields a regular Ornstein-Uhlenbeck process. The model is fitted to Alberta and California markets using the maximum likelihood estimation.

Geman et al. (2006) [43] also proposed a mean reverting jump process, where the long term mean level is assumed to represents the marginal cost of electricity production, which can be a constant, a periodic function or a periodic function with a trend. Random moves around the average trend represent the temporary supply demand imbalances in the network. The model assumes that the natural logarithm of electricity price is described by a stochastic differential equation of the form

$$dE(t) = [h(t) + \Theta(\mu(t) - E(t^-))]\,dt + \sigma dW(t) + f(E(t^-))dJ(t); \tag{2.9}$$

where $h(t)$ is a deterministic seasonality function, Θ is a positive parameter representing the average variation of the price per unit of shift away from the trend, mean reversion level. The process reverts back to a deterministic mean level rather than the stochastic pre-spike value. σ is the volatility attached to the Brownian shocks. The last term in the equation represents the discontinuous part of the model featuring price spikes. This effect is characterized by three quantities: occurrence, direction and size of jumps. The function f assumes ± 1 depending on the level of the spot prices:

$$f(E(t^-)) = \begin{cases} +1, & \text{if} E(t) < \tau(t), \\ -1, & \text{if} E(t) \geq \tau(t). \end{cases}$$

Steps followed by the authors in the estimation of model (2.9) are summarized below.

1. The first step is the detection of jumps in the raw market data. Authors observed that log returns tend to cluster close to either their average mean or to the largest observed values. In other words, data suggest that either there is a jump, in which case variation due to the continuous part is negligible; or there is no jump and variations in the price level are due to the continuous part of the process. It is concluded that by using a price change threshold T, main driver of the changes observed in price levels can be identified.

2. An affine function and two cosine functions with 12 and 6 month periods are used to estimate the deterministic dynamics of the jump free price series:

$$\mu(t; \alpha, \beta, \gamma, \epsilon, \delta, \zeta) = \alpha + \beta t + \gamma \cos(\epsilon + 2\pi t) + \delta \cos(\zeta + 4\pi t). \tag{2.10}$$

   The first term represents the fixed cost linked to the production of power while second one drives the long run linear trend in the total production cost. The overall effect of the third and the fourth terms is a periodic path displaying two maxima per year.

3. The third step is the determination of the jump intensity function λ: Let

$$\lambda(t) = \left(\frac{2}{1 + |\sin[\pi(t - \tau)/k]|} - 1 \right)^d. \tag{2.11}$$

   With this model, jump occurrence exhibits peaking levels at multiples of k years beginning at time τ. The power d allows us to adjust the dispersion of jumps around peaking times and it is included among parameters to be estimated.

4. The probability distribution of the jump sizes is assumed to be a truncated version of the exponential distribution with parameter θ:

$$p(x; \theta, \varphi) = \frac{\theta \exp(-\theta x)}{1 - \exp(-\theta \varphi)}. \tag{2.12}$$

   The model parameter θ and φ is estimated by using the log likelihood function.

5. The estimation of the constant Brownian volatility over observation dates is as follows;

$$\sigma = \sqrt{\sum_{i=0}^{n-1} \left(\Delta \bar{E}(t_i) \right)^2}, \tag{2.13}$$

where $\left(\Delta\bar{E}(t_i)\right)^2$ represents the square of the continuous parts of observed logarithmic price variations between consecutive days.

The last two models neither include the convenience yield as a factor, nor considers the valuation of the futures contracts (or any other kind of derivative). The single factor models are quite tractable and their parameters are relatively easy to estimate. However, they have a serious constraints: they cannot explain the relation between the spot and the futures prices well enough. Spot price models depending on more than one factor help to develop the relation between spot and future commodity prices.

For instance, Brennan et al. (1985) [19] assumed that the spot copper price follows a geometric Brownian motion and incorporated a convenience yield to their model that is proportional to the spot price:

$$dS_t = \mu S_t dt + \sigma S_t dz, \tag{2.14}$$

$$C(S,t) = cS. \tag{2.15}$$

The idea of a constant convenience yield holds only under restrictive assumptions, since the theory of storage is rooted in an inverse relationship between the convenience yield and level of inventories. Gibson et al. (1990) [44] took an important step to a more realistic model of economy by introducing a stochastic convenience yield rate. The spot price S_t of the commodity is described by a geometrical Brownian motion and the convenience yield rate δ_t is described by an Ornstein-Uhlenbeck process with equilibrium level α and the rate of mean reversion κ:

$$dS_t = (\mu - \delta_t)S_t dt + \sigma_1 S_t dz_1, \tag{2.16}$$

$$d\delta_t = \kappa(\alpha - \delta_t)dt + \sigma_2 dz_2, \tag{2.17}$$

$$dz_1 dz_2 = \rho dt. \tag{2.18}$$

Significant contributions to this kind of models have been made by Schwartz (1997) [70]. He reviewed one and two factor models and developed a three factor model under stochastic convenience yield and interest rates. Inclusion of the interest rate as a third factor makes forward and futures prices different:

$$dS_t = (r_t - \delta_t)S_t dt + \sigma_1 S_t dz_1, \tag{2.19}$$

$$d\delta_t = \kappa(\alpha - \delta_t)dt + \sigma_2 dz_2, \tag{2.20}$$

$$dr_t = a(m - r_t)dt + \sigma_3 dz_3, \tag{2.21}$$

$$dz_1 dz_2 = \rho_1 dt, \ dz_2 dz_3 = \rho_2 dt, \ \ dz_1 dz_3 = \rho_3 dt. \tag{2.22}$$

This model was originally developed for copper and oil market. Kalman filter algorithm was used to estimate the parameters in the models.

In [53], Lucia and Schwartz (2002) analyzed the Nordic power market and model the spot price as

$$S_t = h(t) + X_t + Y_t, \tag{2.23}$$

$$dX_t = -\lambda X_t dt + \sigma_X dW_X, \tag{2.24}$$

$$dY_t = -\mu_t dt + \sigma_Y dW_Y, \tag{2.25}$$

$$dW_X dW_Y = \rho dt. \tag{2.26}$$

The function $h(t)$ is deterministic, and it is intended to capture the predictable component in the spot price, i.e, seasonal effects. This function distinguishes between weekdays and includes a monthly seasonal component employing dummy variables. The idea of this model is to have a non-stationary process for the long term equilibrium price level Y and short term mean reverting component X. They estimated all the parameters simultaneously by nonlinear least squares method.

The multi-factor models described so far do not capture one of the most characteristic features of the electricity prices, jumps or spikes. Several authors, such as Deng (2000) [33] and Villaplana (2004) [76] extend these models with both diffusion and jumps. In the work of Villaplana, power prices are modeled according to non-observable state variables that account for the short term movements and long term trends in electricity prices:

$$\ln S_t = h(t) + X_t + Y_t, \tag{2.27}$$

$$dX_t = -\kappa_X X_t dt + \sigma_X dW_1 + J_u dN(\lambda_u) + J_d dN(\lambda_d), \tag{2.28}$$

$$dY_t = -\kappa_Y(\mu - Y_t)dt + \sigma_Y dW_2, \tag{2.29}$$

$$dW_1 dW_2 = \rho dt. \tag{2.30}$$

The jump components are characterized by $N(\lambda_u)$ and $N(\lambda_d)$:, Poisson processes with intensities λ_u and λ_d respectively and by random jumps of sizes J_u and J_d with a specified distribution like Gaussian or exponential.

Deng (2000) [33] and Villaplana (2004) [76] set their models in affine jump diffusion framework which enabled them to use transformed results of Duffie et al. (2000) [37] to derive tractable closed form solutions for a variety of contracts. Deng proposed more sophisticated mean reverting jump diffusion models with deterministic / stochastic volatility and regime switching. This seems a good way of dealing with the dramatic changes in the electricity prices. However, both of the authors concluded that the trajectories produced by their models are fairly different from the ones observed in the market.

Cartea et al. (2005) [26] built a model for wholesale power prices defined by two state variables (demand and capacity) and calculate the forward premium: D_t and C_t representing demand and capacity:

$$D_t = f_D(t) + X_t^D, \tag{2.31}$$

$$C_t = f_C(t) + X_t^C, \tag{2.32}$$

where f_D, f_C are deterministic functions and X_t^D, X_t^C are independent Ornstein-Uhlenbeck processes. They constructed the spot price process as

$$S_t = \beta \exp(\alpha) D_t + \gamma C_t. \tag{2.33}$$

Models discussed in Benth et al. (2008) [11] constitute the starting point of this thesis. A brief summary of their theoretical framework is given below.

- The spot markets of electricity quote prices on an hourly or half hourly basis. Thus, it will not make sense to talk about spot price of electricity at any time t. On the other hand, if there exists an electricity futures market, electricity contracts (settled according to the hourly prices) are traded in a continuous market in the sense that the actors can buy or sell at any time as long as they find a counterpart in the market. Hence, contrary to most other commodity markets where there is a liquid trading in both spot and future / forwards, we face the situation of a discrete spot and a continuous time futures market.

- A continuous time stochastic process $\widetilde{S}(t)$, represents the unobserved instantaneous spot price of electricity at time t with delivery time $[t, t + dt)$. The process $\widetilde{S}(t)$ can be regarded as the price market participants would pay if they could buy electricity at time t with infinitesimal delivery time. In the market, we observe the price of electricity with delivery over a specified hour. Following some simple calculations, authors reached the conclusion that:

$$S_i^d = \widetilde{S}(t_i^d), \tag{2.34}$$

where S_i^d is the price of electricity for the ith hour of day d and $\widetilde{S}(t_i^d)$ is the instantaneous price at the beginning of the ith hour of day d. Since we have the spot price at time moments t_i^d, we have the observations of an underlying continuous time spot price process of electricity.

- Representing the logarithmic prices or the prices itself by a series of Ornstein-Uhlenbeck processes allows us to model different speeds of mean reversion and to incorporate a mixture of jump and diffusional behavior of the prices. Price spikes can be modeled by an Ornstein-Uhlenbeck process having a low frequency of big jumps with fast mean reversion, while more normal price variations are represented by a slower mean-reverting process driven by a Brownian motion.

- Seasonality in jumps is captured by using an independent increment process in the jump model.

Geometric models are formalized as:

$$\ln S(t) = \ln \Delta(t) + \sum_{i=1}^{m} X_i(t) + \sum_{j=1}^{n} Y_i(t), \tag{2.35}$$

$$dX_i(t) = (\mu_i(t) - \alpha_i(t) X_i(t))dt + \sum_{k=1}^{p} \sigma_{ik}(t) dB_k(t), \tag{2.36}$$

$$dY_j(t) = (\delta_j(t) - \beta_j(t) Y_j(t))dt + \nu_j(t) dI_j(t), \tag{2.37}$$

where the deterministic seasonal price level is modeled by the function $\Delta(t)$, which is assumed to be continuously differentiable. An additional drift term can be imposed by the jump components, since they are not assumed to be martingales. For instance, the occurrence of price spikes should add an amount to the overall expected spot price in excess of the seasonal function.

Arithmetic models are given as

$$S(t) = \Delta(t) + \sum_{i=1}^{m} X_i(t) + \sum_{j=1}^{n} Y_i(t). \tag{2.38}$$

Independent increment processes are also assumed to be independent. The arithmetic models can lead to negative prices, although this kind of price movements can be observed: they are rare events. A class of arithmetic models having zero probability of negative prices can be constructed by supposing $m = 0$ and the seasonality function $\Delta(t)$ is a floor towards which the processes Y_j revert. Moreover, it is assumed that the probability of negative jumps is zero. Then, under the assumption that $\Delta(t)$ is positive, the spot price model produces only positive prices. Hence, there are negative and positive price fluctuations arising from a combination of downward mean reversion and upward jumps. If the sum of the jumps over the increment is stronger than total contribution by mean reversion, we observe a random price increase. A price decay is observed otherwise.

Authors provide closed form solutions for forwards and options on forwards. This model coupled with a good description of seasonality provides a precise characterization of electricity spot price behavior. Although the model seems to capture the stylized facts of the spot price market such as mean reversion, seasonality and price spikes, there are no precise statistical analysis about the quality of the model. However, they suggested the particle filter as a possible solution for estimation of the parameters in the model. As a result, parameter estimation for the model appears to be a significant challenge.

The need for a better fit of model forecasts with observations leads to more complicated models with hidden variables and multiple regimes. However, with an increasing number of parameters, the estimation process also gets complicated. Filtering techniques are one of the widely used methods in parameter estimation of the jump processes. Pirino et al. (2010) [65] used an iterative threshold filtering in identification of spikes in their univariate jump model and use the seperated processes for parameter estimation. Their model is nonparametric in the sense that it is free from parametric model assumptions and flexible in capturing the dynamics of the data. The estimation is performed in two steps. In the first step, spikes are identified by means of an iterative filtering technique. Then, series of spikes are used to estimate a seasonal jump intensity function.

This work proposes a combination of Benth et al. (2008) [11] multi-factor models and Pirino et al. (2010) [65] threshold spike detection idea. In our model, three jump processes without predetermined jump size distributions and a Brownian motion are combined with a deterministic seasonality term where two of the jump processes are mean reverting. While the Brownian motion captures the daily regular price movements, pure jump process models price shocks which have long term effect, and further the two Ornstein-Uhlenbeck type processes with different mean reversion speeds capture price shocks that have short term effects. One of the mean reverting processes is assumed to model price shocks that revert back in the next observation (price spikes) and the other is assumed to model price shocks that take a few days to fade away (semi-spikes). An iterative threshold derived by using estimated volatility with GARCH(1,1) is used to filter the price jumps. Although in Mancini (2009) [54], a threshold in order to separate jumps form Brownian motion is proposed and in Mancini et al. (2010) [55] GARCH(1,1) volatility estimation is suggested to be used in calculation of threshold, none of the models investigated in these researches include as many factors as our model have. On the other hand, instead of Pirino et al. (2010) [65] which use a kernel based estimator for threshold, we use GARCH(1,1).

CHAPTER 3

THE SPOT PRICE MODEL

Given the filtered probability space $(\Omega, F, \{F_t\}_{t\in[0,T]}, \mathbb{P})$ where the filtration $\{F_t\}_{t\in[0,T]}$ satisfies the usual conditions i.e., the filtration $\{F_t\}$ is right continuous and F_0 contains all P-null sets. Moreover, W_t is a standard Brownian motion and $J_t^{(1)}$, $J_t^{(2)}$ and $J_t^{(3)}$ are finite activity pure jump processes. A process is said to have finite activity if almost all paths of the process have only a finite number of jumps along finite time intervals. For the formal definition of the finite activity jump processes, see Appendix A.1. The simplest examples include the are Poisson and compound Poisson processes. Lévy measure $\kappa^{(i)}$ for $i = 1, 2, 3$ satisfies $\kappa^{(i)}(dt, dx) = \lambda^{(i)} dt F^{(i)}(dx)$, where $\lambda^{(i)}$ represents the expected number of jumps in the unit time interval, $F(dx)$ represents the jump size distribution. Following Benth et al. (2008) [11], we assume that there exits a continuous electricity price process, S_t governed by the exponential price equation given in (3.1).

$$S_t = e^{\Lambda_t + P_t}, \tag{3.1}$$

where Λ_t is the deterministic seasonality function and P_t, governs deseasonalized logarithmic prices process. Although observing negative prices is possible in spot electricity markets, we rule out this possibility by using an exponential price model, since it is a rare event. There will be three jump factors and a diffusion term in the model. The dynamics of P_t is:

$$dP_t = \sigma_t dW_t + dJ_t^{(1)} + dY_t + dZ_t, \tag{3.2}$$

$$dY_t = -\nu Y_t dt + dJ_t^{(2)}, \tag{3.3}$$

$$dZ_t = -\beta Z_t dt + dJ_t^{(3)}, \tag{3.4}$$

where $J_t^{(1)}$, $J_t^{(2)}$ and $J_t^{(3)}$ are compound Poisson processes with the corresponding jump intensities $\lambda_t^{(1)}$, $\lambda_t^{(2)}$ and $\lambda_t^{(3)}$, $N_t^{(1)}$, $N_t^{(2)}$ and $N_t^{(3)}$ are the respective jump counting Poisson processes.

We do not make any initial assumptions about the distribution of the jump processes. However, it is assumed that W_t, $J_t^{(1)}$, $J_t^{(2)}$ and $J_t^{(3)}$ are mutually independent with $J_0^{(i)} = 0$ for $i = 1, 2, 3$ and stochastic volatility σ_t is a progressively measurable processes.

Therefore, stochastic process P_t is composed of a stochastic volatility Brownian motion, assumed to capture regular daily movements of spot prices. A pure jump process $J_t^{(1)}$ representing structural changes that have long term effects on electricity prices like privatization, technological advances etc. the two non-Gaussian Ornstein-Uhlenbeck processes with jump processes $J_t^{(2)}$ and $J_t^{(3)}$ have different dissipation rates for price shocks created by the jumps. In this setting Y_t, spike process, represents any abrupt change in price level which is reversed quickly, like a price shock affecting the price levels only a couple of hours rather than the whole day. On the other hand, Z_t, semi-spike process, having a lower mean reversion period, assumed to capture factors affecting price level in spot market longer than spike process, like extreme weather conditions which can be effective for a few days; or supply chain problems, system failures that take a few days to resolve.

Solving (3.2) yields

$$\begin{aligned} P_t = \int_0^t \sigma_s dW_s + \int_0^t dJ_s^{(1)} + Y_0 e^{-\nu t} + \int_0^t e^{-\nu(t-s)} dJ_s^{(2)} + Z_0 e^{-\beta t} \\ + \int_0^t e^{-\beta(t-s)} dJ_s^{(3)}, \end{aligned} \tag{3.5}$$

where we assume $P_0 = Y_0 + Z_0$. If we express jump components by summation:

$$\begin{aligned} P_t = \int_0^t \sigma_s dW_s + \sum_{k=1}^{N_t^{(1)}} \Delta_k J^{(1)} + Y_0 e^{-\nu t} + \sum_{k=1}^{N_t^{(2)}} e^{-\nu(t-\tau_k^{(2)})} \Delta_k J^{(2)} + Z_0 e^{-\beta t} \\ + \sum_{k=1}^{N_t^{(3)}} e^{-\beta(t-\tau_k^{(3)})} \Delta_k J^{(3)}, \end{aligned} \tag{3.6}$$

where $\tau_k^{(i)}$ is the random time of the kth jump of the ith jump process. Here $\Delta_k J^{(i)} := J^{(i)}(\tau_k^{(i)}) - J^{(i)}(\tau_k^{-(i)})$ becomes the amount of the kth jump of the ith jump process. On the other hand, $\Delta J_{t_n}^{(i)} := J_{t_n}^{(i)} - J_{t_{n-1}}^{(i)}$. The latter difference formula holds for all of the processes in (3.2), (3.3) and (3.4). Moreover, for simplification, it is assumed that there can only be one jump in each observation interval.

Returning back to the process $\ln S_t$

$$\ln S_t = \Lambda_t + \int_0^t \sigma_s dW_s + \sum_{k=1}^{N_t^{(1)}} \Delta_k J^{(1)} + Y_0 e^{-\nu t} + \sum_{k=1}^{N_t^{(2)}} e^{-\nu(t-\tau_k^{(2)})} \Delta_k J^{(2)} + Z_0 e^{-\beta t}$$
$$+ \sum_{k=1}^{N_t^{(3)}} e^{-\beta(t-\tau_k^{(3)})} \Delta_k J^{(3)}. \tag{3.7}$$

It is well known that both Brownian motion and the Lévy processes are semimartingales. Moreover according to Protter (2004) [66] (Chapter 5, Theorem 19) the stochastic integral process $\int_0^t H_s dX_s$ is a semi-martingale if H is a left continuous right limited (cáglád) process and X is a semi-martingale. Then assuming that σ_t is a cáglád process each of the terms in P_t are semi-martingales. Again referring to Protter (2004) [66](Chapter 2, Theorem 1); the set of semi-martingales is a vector space for the given probability space. Therefore P_t is also a semi-martingale. In the following sections we discuss the modules of the estimation procedure and our spot price model.

3.1 Deterministic Part

It is assumed that logarithmic spot prices are sum of two independent components, a predictable deterministic component Λ_t and a stochastic component P_t. As already discussed, electricity prices, heavily affected by weather conditions, exhibit a cyclical behavior through the year. Not only the demand side but also the supply side may show seasonal variations, like the hydro units heavily dependent on precipitation and snow melting. Besides the annual seasonality, like it is shown in Figure 2.2 there is a significant day of the week effect in the spot prices, due to varying electricity demand.

Typically we do not observe S_t continuous in time t, but in the form of discrete observations. Working with discrete observation series, total observation period $[0, T]$ is divided into N equal intervals with length $h = T/N$. Thus, $S_{t_n} = S_{nh}$ for $n = 1, ..., N$.

Although our intention at the beginning was modeling Λ_t, the deterministic function as a summation of a constant, a linear trend (aiming to capture the inflationary pressures on the electricity prices), and sinusoidal weekly and annual functions, but it is observed that weekly sinusoidal cycles are insufficient in the elimination of weekly autocorrelation pattern, therefore we divide the procedure into two: inspired by Weron (2006) [78] and Mayer et al. (2011)

[56], with the procedure summarized below, persistent weekly autocorrelation in the logarithmic price series is eliminated.

Procedure for elimination of persistent weekly autocorrelation

1. Data is smoothed using a moving average filter:

$$m(t_n) = \frac{1}{7}(\ln S_{t_{n-3}} + \ldots + \ln S_{t_{n+3}}).$$

2. Each days deviation from the corresponding moving average is calculated. Then for each day of the week average of that days deviation is computed. For instance, if the first day of your observation series is Monday the deviation is calculated as follows:

$$w(monday) = \frac{1}{k}\sum_{j=0}^{k-1}[\ln(S_{(7j+1)h}) - m((7j+1)h)],$$

   which is the corresponding weekly seasonality term, where k is the number of Mondays in the observation set.

3. Summation of daily seasonalities is normalized to ensure that they add up to zero for each week.

After subtracting the estimated daily effects from logarithmic price series, coefficients of annual seasonality function with linear trend $\alpha_0 + \alpha_1 t_n + \alpha_2 \cos((2\pi(t_n - \alpha_3))/365)$ are estimated, where $n = 1, ..., N$. Therefore, the deterministic seasonality function becomes:

$$\Lambda_{t_n} = w(t_n) + \alpha_0 + \alpha_1 t_n + \alpha_2 \cos\left(\frac{2\pi\,(t_n - \alpha_3)}{365}\right). \tag{3.8}$$

3.2 Jump Detection with Threshold Method

From now on, we are assuming that deterministic seasonal part is removed and we are working with deseasonalized logarithmic price process P_t. In the related literature it is not possible to find a common definition for a price spike. In a broader sense, it is widely accepted that price jumps are movements in the price level that surpass a threshold and price spikes are composed of price observations that exceeded threshold level for a short period of time. The key variable that should be found is the level of this threshold. The existing literature provides some answers to this question like detecting jumps using boundaries implied by normal distribution (Borovkova et al., 2004 [16]), filtering raw data using different deterministic thresholds and selecting the threshold level which gives the best calibrated model in terms of approaching to the kurtosis of the original daily price variations (Geman, 2006 [43]), or using wavelet techniques (Stevenson, 2001 [71]). Our filtering technique which is based mainly on the work of Mancini (2009) [54] is summarized below.

In a continuous time setting, considering the asymptotic theory, for sufficiently small intervals, jumps can be detected as the increments of the process which are numerically too large to come from a continuous variation. In order to test this statement, it is necessary to determine how much a continuous process can move on a specified time interval or to determine the distribution of the largest increments generated by continuous component of the process. The stochastic continuous component in our price process is the stochastic volatility Brownian motion. The first step is to find an upper limit on these continuous movements, in other words how much a Brownian motion can move on a given time interval h. Paul Levy's law for the modulus of continuity of Brownian motion paths implies that

$$\lim_{h\to 0} \sup_{n\in(1,\ldots,N)} \frac{|\Delta W_{t_n}|}{\sqrt{2h\log\frac{1}{h}}} \le 1 \ \ almost \ \ surely \ \ (a.s.)$$

When we adopt Theorem 1 of Mancini (2009) [54] to our stochastic process P_t, we get the following result.

Theorem 3.2.1 ***Identification of the intervals where no jumps occurred:*** *Consider the system given in (3.2), (3.3) and (3.4) where all the jump processes, $J^{(1)}$, $J^{(2)}$ and $J^{(3)}$ are mutually independent, finite activity jump processes such that $for\ all\ t_n \in [0,T]$, $P\left(\sum_{i=1}^{3}\Delta N_{t_n}^{(i)} \neq 0, \sum_{i=1}^{3}\Delta J_{t_n}^{(i)} = 0\right) = 0$. Suppose also that*

1. $\limsup_{h\to 0} \frac{\sup_{n\in(1,\ldots,N)} |\int_{t_{n-1}}^{t_n} \sigma_s^2 ds|}{h} \le M(\omega) < \infty \quad a.s.$,

2. *The threshold level $r(h)$ is a deterministic function of lag h between the observations, such that $\lim_{h\to 0} r(h) = 0$ and $\lim_{h\to 0} \frac{h\log\frac{1}{h}}{r(h)} = 0$.*

Then for $P-almost\ all\ \omega$, there is $\tilde{h}(\omega) > 0$ such that $for\ every\ h \le \tilde{h}(\omega)\ n = 1,\ldots,N$:

$$I_{\{(\Delta P_{t_n})^2 \le r(h)\}}(\omega) = I_{\left\{\sum_{i=1}^{3}\Delta N_{t_n}^{(i)}=0\right\}}(\omega). \tag{3.9}$$

Proof: Like Mancini (2009) [54], the proof of the theorem is divided in two steps. Firstly, we show that , $I_{\left\{\sum_{i=1}^{3}\Delta N_{t_n}^{(i)}=0\right\}}(\omega) \leq I_{\{(\Delta P_{t_n})^2 \leq r(h)\}}(\omega)$ $a.s.$ for small h. In the second step we prove that $I_{\left\{\sum_{i=1}^{3}\Delta N_{t_n}^{(i)}=0\right\}}(\omega) \geq I_{\{(\Delta P_{t_n})^2 \leq r(h)\}}(\omega)$ $a.s.$ for small h.

Before starting; we prove that stochastic integral $\int_0^t \sigma_s dW_s$ is a bounded process. Assuming that σ_t is a continuous process; $\int_0^t \sigma_s dW_s$ is a time changed Brownian motion. Dambis (1965) [32] and Dubins and Schwartz (1965) [36] showed that "any continuous martingale is a time changed Brownian motion". In 1978, Monroe [59] extended this result to a more general setting that "any semi-martingale is a time changed Brownian motion". More precisely there is a Brownian motion W with respect to the filtration $\{F_t\}_{t\geq 0}$ such that for each $t \geq 0$, $\omega(to)[X]_t(\omega)$ is stopping time and $X_t = W_{[X]_t}$, where X_t is a local martingale. In particular, if W is a Brownian motion, and σ is square integrable then we let $X_t = \int_0^t \sigma_s dW_s$. Since $[X]_t = \int_0^t \sigma_s^2 ds$ we have $\int_0^t \sigma_s dW_s =^d W_{\int_0^t \sigma_s^2 ds}$. Considering

$$\sup_{n\in(1,\ldots,N)} \frac{\left|\int_{t_{n-1}}^{t_n} \sigma_s dW_s\right|}{\sqrt{2h\log\frac{1}{h}}} \leq \sup_{n\in(1,\ldots,N)} \frac{\left|W_{\int_0^t \sigma_s^2 ds} - W_{\int_0^{t_{n-1}} \sigma_s^2 ds}\right|}{\sqrt{2\int_{t_{n-1}}^t \sigma_s^2 ds \log\frac{1}{\int_{t_{n-1}}^t \sigma_s^2 ds}}} \sup_{n\in(1,\ldots,N)} \frac{\sqrt{2Mh\log\frac{1}{Mh}}}{\sqrt{2h\log\frac{1}{h}}}$$
$$\sup_{n\in(1,\ldots,N)} \frac{\sqrt{2\int_{t_{n-1}}^t \sigma_s^2 ds \log\frac{1}{\int_{t_{n-1}}^{t_n} \sigma_s^2 ds}}}{\sqrt{2Mh\log\frac{1}{Mh}}}. \tag{3.10}$$

When we take the limit of the above process as $h \to 0$ the first argument is smaller than or equal to 1 due to modulus of continuity as the time index is the stochastic integral instead of h. Due to assumption 1 and monotonicity of the function $x\log(1/x)$ in the neighborhood of 0 second argument is also bounded and therefore (3.10) is bounded as $h \to 0$

Now we may carry out the steps to prove the theorem:

1. For each ω, $J_{0,h} = \left\{n \in \{1, ..., N\} : \sum_{i=1}^{3} \Delta N_{t_n}^{(i)} = 0\right\}$. To show that for small h, $I_{\left\{\sum_{i=1}^{3}\Delta N_{t_n}^{(i)}=0\right\}}(\omega) \leq I_{\{(\Delta P_{t_n})^2 \leq r(h)\}}(\omega)$, it is sufficient to prove that for small h, $\sup_{J_{0,h}}(\Delta P_{t_n})^2 \leq r(h)$ holds $a.s.$. On the set $J_{0,h}$ for $i = 1, 2, 3$, we have $N_{t_n}^{(i)} = N_{t_{n-1}}^{(i)}$,

$$\Delta P_{t_n} = \underbrace{\int_{t_{n-1}}^{t_n} \sigma_s dW_s}_{A_n} + \underbrace{Y_0 e^{-\nu t}(1-e^{\nu h}) + \sum_{k=1}^{N^{(2)}_{t_{n-1}}} e^{-\nu(t_n-\tau^{(2)}_k)}(1-e^{\nu h})\Delta_k J^{(2)}}_{B_n}$$

$$+ \underbrace{Z_0 e^{-\beta t}(1-e^{\beta h}) + \sum_{k=1}^{N^{(3)}_{t_{n-1}}} e^{-\beta(t_n-\tau^{(3)}_k)}(1-e^{\beta h})\Delta_k J^{(3)}}_{C_n}. \tag{3.11}$$

The square of ΔP_{t_n} is summation of six arguments. Now we check each of these arguments relative to the threshold function $r(h)$ as $h \to 0$:

$$\sup_{t_n \in J_{0,h}} (\Delta P_{t_n})^2 \leq \sup_{t_n \in J_{0,h}} |A_n|^2 + \sup_{t_n \in J_{0,h}} |B_n|^2 + \sup_{t_n \in J_{0,h}} |C_n|^2 + \sup_{t_n \in J_{0,h}} (2|A_n||B_n|).$$
$$+ \sup_{t_n \in J_{0,h}} (2|A_n||C_n|) + \sup_{t_n \in J_{0,h}} (2|B_n||C_n|)$$

Considering $|B_n||C_n|$ we obtain that

$$\lim_{h\to 0} \sup_{J_{0,h}} \frac{|Y_0 e^{-\nu t}\left(1-e^{\nu h}\right) + \sum_{k=1}^{N^{(2)}(t_{n-1})} e^{-\nu(t_n-\tau^{(2)}_k)}(1-e^{\nu h})\Delta_k J^{(2)}|}{\sqrt{h \log \frac{1}{h}}}$$
$$\frac{|Z_0 e^{-\beta t}(1-e^{\beta h}) + \sum_{k=1}^{N^{(3)}(t_{n-1})} e^{-\beta(t_n-\tau^{(3)}_k)}(1-e^{\beta h})\Delta_k J^{(3)}|}{\sqrt{h \log \frac{1}{h}}} \frac{h \log \frac{1}{h}}{r(h)}$$

is equal to 0. $|B_n|^2$ and $|C_n|^2$ follow the same line of reasoning and goes to 0 as $h \to 0$. On the other hand we have already showed that $\sup_{t_n \in J_{0,h}} |A_n|/\sqrt{2h \log \frac{1}{h}}$ as $h \to 0$ is bounded. Therefore $\sup_{t_n \in J_{0,h}} |A_n^2|/\sqrt{2h \log \frac{1}{h}}$ is also bounded. The $|A_n||B_n|$ yields;

$$\lim_{h\to 0} \sup_{J_{0,h}} \frac{|\int_{t_{n-1}}^{t_n} \sigma_s dW_s|}{\sqrt{2h \log \frac{1}{h}}} \times \frac{|Y_0 e^{-\nu t}(1-e^{\nu h}) + \sum_{k=1}^{N^{(2)}(t_{n-1})} e^{-\nu(t_n-\tau^{(2)}_k)}(1-e^{\nu h})\Delta_k J^{(2)}|}{\sqrt{2h \log \frac{1}{h}}} \times \frac{2h \log \frac{1}{h}}{r(h)}. \tag{3.12}$$

We know that the first factor is bounded and the third factor goes to 0. Second factor also goes to 0 by L'Hospital rule. Therefore, cross terms $|A_n||B_n|$ and $|A_n||C_n|$ relative to the threshold, go to zero in the limit. As a result, we conclude that if $n \in J_{0,h}$ as $h \to 0$ $\sup_{J_{0,h}} (\Delta P_{t_n})^2 \leq r(h)$.

2. In order to establish the second inequality, it is assumed that for any ω, $J_{1,h} = \{n \in (1, ..., N) : \sum_{i=1}^{3} \Delta N_{t_n}^{(i)} \neq 0$. It is sufficient to show that for small h, $\inf_{J_{1,h}} (\Delta P_{t_n})^2 \geq r(h)$ $a.s.$ to prove that $for\ every\ n$, $I_{\left\{\sum_{i=1}^{3} \Delta N_{t_n}^{(i)}=0\right\}}(\omega) \geq I_{\{(\Delta P_{t_n})^2 \leq r(h)\}}(\omega)$ holds almost surely.

   Working in the subset $J_{1,h}$ in addition to the terms in the previous step there are also jump terms; $\Delta_{N^{(1)}(t_n)} J^{(1)} + e^{-\nu(t_n - \tau_{N^{(2)}(t_n)})} \Delta_{N^{(2)}(t_n)} J^{(2)} + e^{-\beta(t_n - \tau_{N^{(3)}(t_n)})} \Delta_{N^{(3)}(t_n)} J^{(3)}$. Consider that $\tau_{N(t_n)} \in (t_{n-1}, t_n]$ as $h \to 0$, $\tau_{N(t_n)} \to t_n$ and $\tau_{N(t_n^-)} \to t_{n-1}$ therefore discounting terms for the mean reverting jump processes collapse to 1 as $h \to 0$. Moreover, with the assumption that there can only be one jump on each time interval, we get

$$\liminf_{h\to 0}\inf_{J_{1,h}} \frac{(|\Delta_{N^{(1)}(t_n)} J^{(1)} + e^{-\nu(t_n - \tau_{N^{(2)}(t_n)})} \Delta_{N^{(2)}(t_n)} J^{(2)}}{r(h)} \\ \frac{+e^{-\beta(t_n - \tau_{N^{(3)}(t_n)})} \Delta_{N^{(3)}(t_n)} J^{(3)}|)^2}{r(h)} = \infty, \tag{3.13}$$

   Since the jump processes we consider are finite processes, the numerator is a fixed amount while the denominator goes to 0 as $h \to 0$. Let us look to the cross terms

$$\lim_{h\to 0}\inf_{J_{1,h}} \frac{(|\Delta J^{(1)}(\tau_{N^{(1)}(t_n)}) + e^{-\nu(t_n - \tau_{N^{(2)}(t_n)})} \Delta_{N^{(2)}(t_n)} J^{(2)}}{r(h)} \\ \frac{+e^{-\beta(t_n - \tau_{N^{(3)}(t_n)})} \Delta_{N^{(3)}(t_n)} J^{(3)}|)}{r(h)} \left[\frac{|A_n|}{\sqrt{r(h)}} + \frac{|B_n|}{\sqrt{r(h)}} + \frac{|C_n|}{\sqrt{r(h)}} \right]. \tag{3.14}$$

   The summation in the brackets is bounded at the limit and the first term goes to 1 as $h \to 0$. As a result,

$$\lim_{h\to 0}\inf_{J_{1,h}} \frac{(\Delta P_{t_n})^2}{r(h)} = \infty. \tag{3.15}$$

   this completes the proof.

■

3.3 Determining The Threshold

In Mancini et al. (2010) [55] h^α, $\alpha \in (0, 1)$, is used as the deterministic, constant threshold function. In their model for the interest rates Mancini and Reno stated that Theorem 3.2.1

holds even if the threshold function $r(h)$ varies with time according to

$$r_t(h) = c_t \tilde{r}(h), \tag{3.16}$$

where $\tilde{r}(h)$ satisfies the threshold conditions and c_t is an a.s. bounded stochastic process which is also bounded away from 0. Following Mancini and Reno; Pirino et al. (2010) [65] adapted the following threshold in their discrete time electricity model, iterated on the integer Z;

$$\theta_{t_i}^Z = c_v^2 \frac{\sum_{j=-L, j\neq -1,0,1}^{L} K(\frac{i}{L}) \Delta_{i+j} P^2 I_{\left\{\Delta_{i+j} P^2 \leq \theta_{t_{i+j}}^{Z-1}\right\}}}{\sum_{j=-L, j\neq -1,0,1}^{L} K(\frac{i}{L}) \Delta_{i+j} I_{\left\{\Delta_{i+j} P^2 \leq \theta_{t_{i+j}}^{Z-1}\right\}}} \tag{3.17}$$

where the parameter c_v sets the number of the standard deviations after which an observation is considered to be above the threshold. The value of the c_v potentially determines the number of detected jumps, however it is denoted that since in the case of electricity price, jumps are too large hence the choice of this value is mostly uninfluential.

Corsi et al. (2009) [31] state that, in applications, it is natural to scale threshold function with respect to local spot return variance $r_t = c^2 \hat{\sigma}_t$, where $\hat{\sigma}_t$ is an auxiliary estimator of σ_t and c is a positive constant.

We use the multiple of estimated stochastic volatility as a threshold function. Although the proposed model for the electricity prices is in continuous time, since our price observations are discrete, we will use a discrete GARCH(1,1) process for the stochastic volatility modeling. This is not an arbitrary choice: in Nelson (1992) [62], it is stated that ARCH type models are remarkably robust to certain types of misspecification such that as long as the process is well approximated by a diffusion, broad classes of ARCH models provide consistent estimates of the conditional variance. Nelson (1992) [62] especially stresses that in this context the term *'estimate'* corresponds to its use in the filtering literature rather than the statistics literature, i.e., the GARCH(1,1) model with fixed parameters produces estimates of the true variance vector at each time point in the same sense that a Kalman filter produces estimate of unobserved state variables in a linear system. Let us assume that there are no jumps at time t, then our instantaneous deseasonalized logarithmic price process is sum of a deterministic mean reversion part of the previously observed jumps and initial values of process Y_t and Z_t described by μ_t plus a diffusion component by standard Brownian motion B_t:

$$dP_t = d\mu_t + \sigma_t dB_t. \tag{3.18}$$

Using a sequence of discrete time observations, with a fixed observation interval h, we approach to continuous time stochastic volatility process with a GARCH process. Let $(\epsilon_{nh})_{n\in N_0}$ be an i.i.d. sequence of standard normal random variables, let ξ_h, $\delta_h \geq 0$ and $\eta_h > 0$ and let (P_{0h}, σ^2_{0h}) be starting random variables independent of $(\epsilon_{nh})_{n\in N_0}$. Then $(P_{nh} - P_{(n-1)h}, \sigma^2_{nh})_{n\in N_0}$ defined recursively by

$$P_{nh} = P_{(n-1)h} + \Delta_n\mu_h + h^{1/2}\sigma_{nh}\epsilon_{nh}, \quad (n \in N), \tag{3.19}$$

$$\sigma^2_{nh} = \xi_h + (\eta_h\epsilon^2_{(n-1)h} + \delta_h)\sigma^2_{(n-1)h}, \; (n \in N). \tag{3.20}$$

Equations above define a GARCH(1,1) process. Here $\Delta_n\mu_h = B_n + C_n$, where B_n and C_n are given in (3.11). Note that, $(P_{nh}, \sigma^2_{nh})_{n\in\mathbb{N}_0}$ is embedded into a continuous time process $(P_{t,h}, \sigma^2_{t,h})_{t\geq 0}$ by defining

$$P_{t,h} := P_{nh}, \quad \sigma^2_{t,h} := \sigma^2_{nh}, \quad nh \leq t < (n+1)h. \tag{3.21}$$

Nelson (1990) [61] gives the conditions for $(P_{t,h}, \sigma^2_{t,h})_{t\geq 0}$ to converge weakly to some process $(P_t, \sigma^2_t)_{t\geq 0}$ as $h \to 0$. Suppose that there are constants $\xi \geq 0$, $\theta \in \mathbb{R}$ and $\eta > 0$ as well as the starting random variables (P_0, σ^2_0) such that (P_{0h}, σ^2_{0h}) converges weakly to (P_0, σ^2_0) as $h \to 0$, $\mathbb{P}(\sigma^2_0 > 0) = 1$. Suppose further that

$$\lim_{h\to 0} h^{-1}\xi_h = \xi, \;\; \lim_{h\to 0} h^{-1}(1 - \delta_h - \eta_h) = 0, \;\; \lim_{h\to 0} 2h^{-1}\eta^2_h = \eta^2. \tag{3.22}$$

hold, then $(P_{t,h}, \sigma^2_{t,h})_{t\geq 0}$ converges weakly as $h \to 0$ to the unique solution $(P_t, \sigma^2_t)_{t\geq 0}$ of the diffusion equation

$$dP_t = d\mu_t + \sigma_t dW_t, \quad t > 0, \tag{3.23}$$

$$d\sigma^2_t = (\xi - \theta\sigma^2_t)dt + \eta\sigma^2_t dB_t, \;\; t > 0, \tag{3.24}$$

where B_t and W_t are independent Brownian motions.

An example of possible choices satisfying the necessary limit conditions (3.22); $\xi_h = \xi h$, $\delta_h = 1 - \eta\sqrt{h/2} - \theta h$ and $\eta_h = \eta\sqrt{h/2}$. Using these parameters if we rewrite the equation

(3.20):

$$\sigma^2_{nh} = \xi h + \eta\sqrt{h/2}\frac{(\Delta P_{t_n} - \Delta_n \mu_h)^2}{h} + (1 - \eta\sqrt{h/2} - \theta h)\sigma^2_{(n-1)h}. \tag{3.25}$$

When we rewrite the recursive relation for the term σ, we obtain

$$\begin{aligned}\sigma^2_{nh} &= \xi h + \frac{\eta}{\sqrt{2h}}(\Delta P_{t_{n-1}} - \Delta_{n-1}\mu_h)^2 + (1 - \eta\sqrt{h/2} - \theta h)(\xi h + \frac{\eta}{\sqrt{2h}}(\Delta P_{t_{n-2}} - \Delta_{n-2}\mu_h)^2. \\ &+ (1 - \eta\sqrt{h/2} - \theta h)(\xi h + \frac{\eta}{\sqrt{2h}}(\Delta P_{t_{n-3}} - \Delta_{n-3}\mu_h)^2 + \\ &+ (1 - \eta\sqrt{h/2} - \theta h)\sigma^2_h)\cdots)\end{aligned} \tag{3.26}$$

Rearranging the above equation yields

$$\begin{aligned}\sigma^2_{nh} &= \xi h(1 + (1 - \eta\sqrt{h/2} + \theta h) + \cdots + (1 - \eta\sqrt{h/2} + \theta h)^{i-2}) \\ &+ \frac{\eta}{\sqrt{2h}}\sum_{j=1}^{n-1}(1 - \eta\sqrt{h/2} + \theta h)^{i-1-j}(\Delta P_{t_j} - \Delta_j\mu_h)^2 + (1 - \eta\sqrt{h/2} + \theta h)^{n-1}\sigma^2_h.\end{aligned} \tag{3.27}$$

Now we define the iterative threshold function

$$r^I_n(h) := h^\alpha c^2(\hat{\sigma}^I_{nh})^2, \tag{3.28}$$

where $I \in N$ represents the number of iterations, $c \in R^+$ and $\alpha \in (\frac{1}{2}, 1)$. c can be regarded as a constant that determines after the number of standard deviations a price movement is classified as a jump. Mancini et al. (2010) [**?**] omit the dependence of threshold r on h in their implementation of a similar threshold estimator, since intervals between consecutive observations of spot market prices are fixed.

$$\begin{aligned}(\hat{\sigma}^I_{nh})^2 &= \xi^I h(1 + (1 - \eta^I\sqrt{h/2} + \theta^I h) + \cdots + (1 - \eta^I\sqrt{h/2} + \theta^I h)^{n-2}) \\ &+ \frac{\eta^I}{\sqrt{2h}}\sum_{j=1}^{n-1}(1 - \eta^I\sqrt{h/2} + \theta^I h)^{n-1-j}(\Delta P_{t_j} - \Delta\mu_h(t_n))^2 \\ &\mathbb{I}_{\left\{(\Delta P_{t_j} - \Delta\mu_h(t_j))^2 \le r^{I-1}_j\right\}} + (1 - \eta^I\sqrt{h/2} + \theta^I h)^{n-1}(\hat{\sigma}^I_h)^2.\end{aligned} \tag{3.29}$$

By using this filter, we replace the observations with stochastic residual higher than the threshold with the expected value of the Brownian motion, 0. Threshold function satisfies two necessary asymptotic conditions:

1. As $h \to 0$, it is obvious that $r^I(h)$ goes to 0 for all $I \in N$.

2. On the other hand, $\lim_{h\to 0} \frac{h\log(1/h)}{r^I(h)} = 0$ for all $I \in N$ if

$$
\begin{aligned}
&\frac{h^{\alpha}\xi h(1+(1-\eta\sqrt{h/2}-\theta h))+...+(1-\eta\sqrt{h/2}-\theta h)^{n-2}+h^{\alpha}\frac{\eta}{\sqrt{2h}}}{h\log(1/h)}+\\
&+\frac{(\Delta P_{t_n}-\Delta\mu_h(t_n))^2+...+h^{\alpha}\frac{\eta}{\sqrt{2h}}(1-\eta\sqrt{h/2}-\theta h)^{n-2}(\Delta P_{t_n}-\Delta\mu_h(t_n))^2}{h\log(1/h)}+\\
&+\frac{h^{\alpha}(1-\eta\sqrt{h/2}-\theta h)^{n-1}\sigma_1^2}{h\log(1/h)}
\end{aligned}
\tag{3.30}
$$

as $h \to 0$. above equation goes to ∞. Therefore it can be stated that the threshold function satisfies the second limit assumption of Theorem 3.2.1.

3.4 Decomposition of the Stochastic Processes

After determining the structural form of the threshold function, the next step is to construct an algorithm to separate jumps from continuous part. Since the price observations and threshold function are discrete time processes, the algorithm will be defined on discrete observations. Moreover this algorithm must both detect the jumps and determine the type of jump process (whether the jump is a pure jump, spike or semi-spike). Volatility of the diffusion part, jump processes' frequency, jump size distributions and mean reversion rates will be estimated using the results of this separation.

First of all, definitions which are necessary for this separation process will be given. The following framework is taken from the continuous time autoregression (CAR) processes of Brockwell et al. (2007) [22].

Definition 3.4.1 *A CAR(1) process Y_t, driven by the Lévy process $\left\{J_t^{(2)}, t \geq 0\right\}$, with parameter $\nu \in R$ is defined to be a strictly stationary solution of the stochastic differential equation*

$$dY_t = -\nu Y_t dt + dJ_t^{(2)}. \tag{3.31}$$

Since $J_t^{(2)}$ has bounded variation on compact intervals and process Y_t is Markov due to independent increment Lévy process, we have for all $t > s \geq 0$ that:

$$Y_t = e^{-\nu t} Y_0 + \int_s^t e^{-\nu(t-u)} dJ^{(2)}(u). \qquad (3.32)$$

The necessary and sufficient conditions for the stationarity of the process Y_t are given in [21] and [22]. According to these papers, processes Y_t and Z_t, given in (3.5), are strictly stationary solutions of (3.3) and (3.4) if and only if ν and β are greater than 0, and Y_0 and Z_0 have distributions $\int_0^\infty e^{-\nu u} dJ^{(2)}(u)$, $\int_0^\infty e^{-\beta u} dJ^{(3)}(u)$ respectively, given that Y_0 and Z_0 are independent of $\left\{J_t^{(2)}, t \geq 0\right\}$, $\left\{J_t^{(3)}, t \geq 0\right\}$ respectively and $E((J_1^{(2)})^2) < \infty$, $E((J_1^{(3)})^2) < \infty$. According to this statement, given that Y_t and Z_t are stationary processes, we conclude that ν and β are greater than 0.

Discrete analogous of spike process $\{Y_{nh}, n = 0, ..N\}$ is represented as the autoregressive process (AR(1) process) as follows

$$Y_{nh} = \phi Y_{(n-1)h} + \Delta J_{t_n}^{(2)}, \qquad (3.33)$$

where $\phi = e^{-\nu h}$. Since $\nu > 0$, $\phi \in (0, 1)$. From (3.33), we can derive,

$$\Delta Y_{hn} = (\phi - 1) Y_{(n-1)h} + \Delta J_{t_n}^{(2)}. \qquad (3.34)$$

The steps followed in parameter estimation are listed below. Also application of these steps are discussed in the following chapter.

1. **Initialization:** Using deseasonalized logarithmic price returns we estimate parameters of GARCH(1,1) process.

2. **Detection and Separation of Jump Processes:** For the given observation series, it is assumed that the initial values of processes Y_t and Z_t are 0, i.e., $Y_0 = 0$ and $Z_0 = 0$. With the estimated GARCH volatility we compute the threshold value. Since jumps are assumed to be rare events and mean reversion of both spike and semi-spike process are high, when the nth squared return is higher than threshold level, whole return ΔP_{t_n} is attributed to a jump. Due to fast mean reversion of the jump processes, the cumulative

effect of previous jumps are assumed to be contained in the boundaries formed by using Brownian motion. With a simple calculation for $i = 1, 2, 3$;

$$
\begin{aligned}
\Delta P_{t_n} &= \Delta J_{t_n}^{(i)}, \\
\Delta P_{t_{n+1}} &= (\rho^{(i)} - 1)\Delta P_{t_n} + \Delta W_{t_{n+1}}, \\
&\vdots \\
\Delta P_{t_{n+6}} &= (\rho^{(i)})^5(\rho^{(i)} - 1)\Delta P_{t_n} + \Delta W_{t_{n+6}},
\end{aligned}
\tag{3.35}
$$

where $\rho^{(i)}$ is the corresponding mean reversion rate which is 0 for $J^{(1)}$.

(a) We take the return higher than the threshold as a jump and replace it with the expected value of the Brownian component, namely 0.

(b) We form the boundary conditions in order to test whether this jump comes from a mean reverting process or not. For instance, if there is a positive jump $\Delta J_{t_n}^{(i)}$ for $i = \{1, 2, 3\}$ at nth time interval , and if we assume that $\rho^{(i)} \in (0, 1)$, for $i = \{1, 2\}$, boundary condition for the first successive return is defined as

$$
-1 \times \Delta_i P - c\hat{\sigma}_{i+1} < \Delta_{i+1} P < c\hat{\sigma}_{i+1}. \tag{3.36}
$$

where c is a positive constant. If this condition is satisfied we save this data point, replace with 0 and recalculate the estimated standard deviation for $(n+1)$st return. Implied interval for the mean reversion rate is then calculated as

$$
\left(\frac{\Delta P_{t_{n+1}} - c\hat{\sigma}_{n+1}}{\Delta P_{t_n}} + 1, \frac{\Delta P_{t_{n+1}} + c\hat{\sigma}_{n+1}}{\Delta P_{t_n}} + 1 \right) \tag{3.37}
$$

Using this interval and (3.36), corresponding boundary conditions for each return is derived.

(c) We examine six returns following each jump point, since most of the jumps are found to fade away completely until the sixth time interval following the jump. Successive steps satisfying the boundary conditions are replaced with 0 and then the next return is tested. If the return fails to satisfy the boundary conditions, algorithm stops and returns to the first step. At every iteration, mean reversion components for all jumps are retested and reassigned due to changing volatility estimates. Separated returns are assumed to be the summation of jumps, mean reversion contributions of former jumps and Brownian increments.

3. **Classification of Jump Processes**The iteration procedure stops when GARCH filter detects no more jumps. Then, in order to estimate mean reversion rates we classify collected jump points as pure jumps, spikes and semi-spikes as follows:

   - If a jump point is not followed by mean reverting steps, it is classified as a pure jump.
   - If the points separated as mean reversion components jointly satisfies for $k = n, \ldots, n+6$; $\| \sum_{j=n}^{k} \Delta P_{t_j} \| - c\hat{\sigma}_{k+1} \leq |\Delta_{k+1} P|$, these points are classified as a realization of spike process. Then the others are classified as semi-spikes. By this separation, we take jumps with successive observations all of which are in c standard deviations neighborhood as spikes.

   At end of this step, there will be four separate vectors: one vector of pure jumps including price steps classified as pure jumps and zeros for the other entries, one vector for spike process composed of jumps and successive mean reverting returns and zeros; one vector of semi-spike process observations and one vector of the filtered returns in which jumps and mean reverting observations are replaced with zeros while other entries are equal to returns of deseasonalized price series.

4. **Estimation of Mean Reversion Rates:** Since we do not have full spike and semi-spike processes, in order to estimate the mean reversion rates, artificial price series are created by summing seperated jumps and following mean reverting returns and leaving zeros unchanged. For illustration we take the spike process Y, assuming that $\tau_{N^{(2)}(t_n)} = t_n$:

$$\begin{aligned} P^Y_{\tau_{N^{(2)}(t_n)}} &= P^Y_{t_n} := \Delta P_{t_n} = \Delta_{N^{(2)}(t_n)} J^{(2)} \\ P^Y_{t_{n+1}} &:= \Delta P_{t_n} + \Delta P_{t_{n+1}} = \phi P^Y_{\tau_{N^{(2)}(t_n)}} + \int_{t_n}^{t_{n+1}} \sigma_s dW_s \\ P^Y_{t_{n+2}} &:= \Delta P_{t_n} + \Delta P_{t_{n+1}} + \Delta P_{t_{n+2}} = \phi^2 P^Y_{\tau_{N^{(2)}(t_n)}} + \int_{t_{n+1}}^{t_{n+2}} \sigma_s dW_s \end{aligned} \tag{3.38}$$

$$\begin{aligned} Corr(P^Y_{\tau_{N^{(2)}(t_n)}}, P^Y_{\tau_{N^{(2)}(t_n)}+1}) &= \frac{E(P^Y_{t_n} P^Y_{t_{n+1}}) - E(P^Y_{t_n})E(P^Y_{t_{n+1}})}{\sqrt{Var(P^Y_{t_n})}\sqrt{Var(P^Y t_{n+1})}} \\ &= \frac{\phi \sqrt{Var(P^Y_{t_n})}}{\sqrt{Var(P^Y_{t_{n+1}})}} \\ Corr(P^Y_{\tau_{N^{(2)}(t_n)}}, P^Y_{\tau_{N^{(2)}(t_n)}+2}) &= \frac{\phi^2 \sqrt{Var(P^Y t_n)}}{\sqrt{Var(P^Y t_{n+2})}} \end{aligned} \tag{3.39}$$

In (3.39), conditional probability of observing another jump in the mean reverting part is taken to be 0. In addition, the effect of previous jumps are assumed to be indistinguishable from the Brownian component. Correlation coefficient estimator, in case of missing observations proposed by Takeuchi (1995) is

$$\widehat{Corr}(l) = \frac{\sum_{n=1}^{N-1} P^Y t_n P^Y t_{n+l}}{\sqrt{\sum_{n=1}^{N-1} a(n+l)(P^Y t_n)^2}\sqrt{\sum_{n=1}^{N-1} a(n)(P^Y t_{n+l})^2}}, \tag{3.40}$$

where $a(n) = 1$ if the nth observation exists, and 0 otherwise. Using (3.40), correlation of price jumps starting from the first to the sixth successive generated prices are estimated and putting sample variances in (3.39) ϕ is calculated. After determining mean reversion rates, we refine price process by subtracting each jumps mean reversion effect from the rest of the series.

5. **Determination and Estimation of Jump Size Distributions:** Separated jump observations in all three jump processes are settled at the tail of the empirical distribution with a gap around 0. In theory, each tail can be modeled separately. However, since jumps are rare events, with three different jump processes, it is difficult to find sufficient amount of observations to fit 6 tail distributions. Therefore, absolute value of the jump point observations are used in specification of the jump size distributions. Among the long list of jump distributions, the highlighted ones in literature are exponential, normal, lognormal and inverse Gaussian. Except normal distribution, all of the distributions have positive support. Like Ane et al. (2010), we also add Burr distribution to this list and its cumulative distribution function is

$$F(x) = 1 - \left(1 + \left(\frac{x-\mu}{\beta}\right)^{\alpha}\right)^{-\theta}, \tag{3.41}$$

where α and θ are positive shape parameters, β is positive scale parameter and $\mu \in R$ is the location parameter. By taking the absolute value of negative jumps, it is assumed that both tails are of the same distribution. Empirical weight of each sign among all jumps is taken as the probability of jumps direction. For each of three jump processes we estimate the jump distributions. Anderson Darling, Kolmogorov-Smirnov and Chi-square statistics are used in determination of the jump size distribution. The jump frequencies are also approximated by the ratio of the number of jump observations of each process to the total number of observations.

6. **Estimation of Continuous Process' Volatility:** The last term is the stochastic volatility of Brownian motion. Although stochastic volatility models, or GARCH type models can be fitted to the volatility of the filtered price series acquired from Step 4, volatility is taken constant, not to over parametrize the model.

CHAPTER 4

ESTIMATION OF SPOT ELECTRICITY PRICE PROCESS AND FORECATING SPOT ELECTRICITY PRICES

Parameter estimation of the multi-factor spot electricity price model developed in the previous chapter will now be carried out in this chapter. Moreover, by using the estimated parameters day ahead price forecasts are generated and the evaluation of the model is made. For this procedure we use four daily average spot price series from Austrian, Italian, Spanish and Turkish markets. First three countries have relatively mature spot electricity markets, while liberal Turkish spot electricity market was founded in August 2006 and taken into operation in December 2009. Austrian market data is taken from EXAA (Energy Exchange Austria), Austrian energy and environmental exchange founded on 8 June 2001 and started spot market trading in electric power on 21 March 2002. Italian market data is collected from GME, Italian Power Exchange has been functioning since April 2004. Spanish data is taken from OMEL's daily market. For the listed markets equally weighted average of 24 hourly equilibrium prices constitute the daily observations. Lengths of the observation series are 3399 (from March 2002 to July 2011) for Austria, 2617 (form April 2004 to May 2011) for Italy, 4139 (from January 2000 to April 2011) for Spain and 577 (from December 2009 to July 2011) for Turkey. In jump process modeling the importance of a long observation series is generally accepted. Since jumps are assumed to be rare events, probability of acquiring enough number of jump observations for parameter estimation increases with the length of the series. An infant market like Turkey may not provide enough data for estimation of our parameter intensive model. Therefore in order to test the applicability and the validity of our model we use not only the series in Turkey but also other three series shown in Figure 4.1.

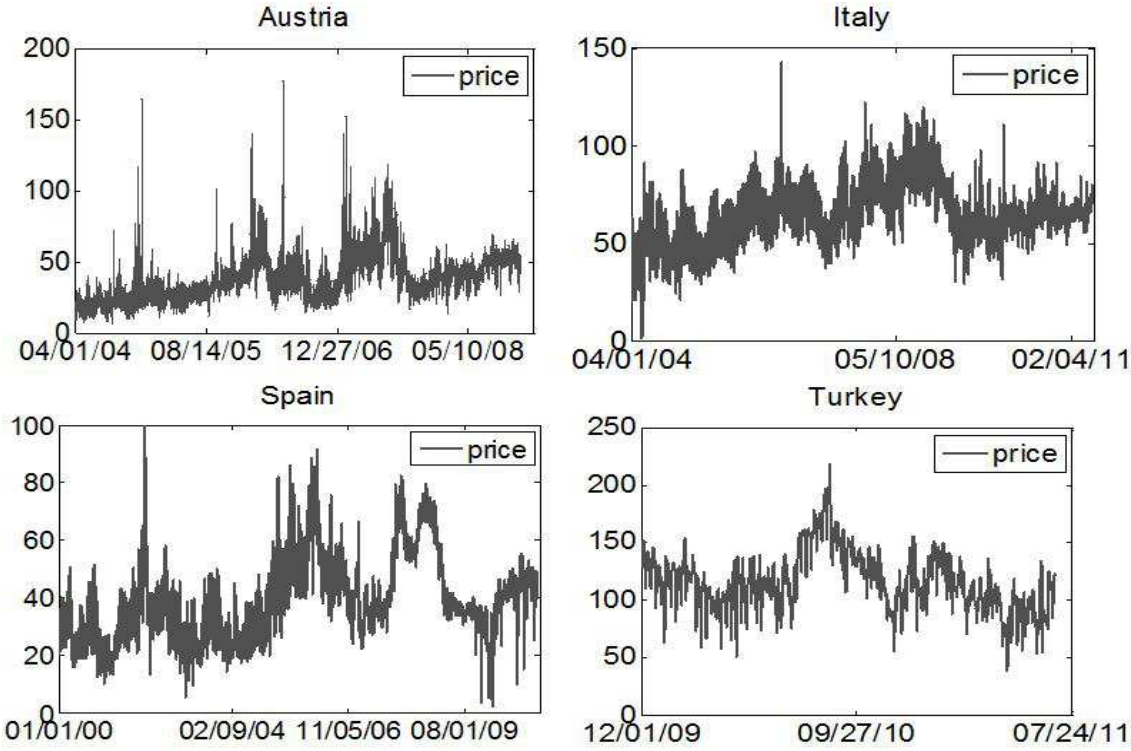


Figure 4.1: Daily average price series

4.1 Model Estimation

Firstly, the weekly pattern in logarithmic prices is extracted according to the procedure described in Section 3.1. For all of the price series excluding Turkey, it is observed that this procedure achieves minor recovery in the weekly autocorrelation. Concluding that the main problem is due to the chancing magnitude of the weekly cycles through time, we approximate weekly pattern for each year separately, which helps us overcome the weekly autocorrelation problem. Due to the relative shortness of the Turkish data, we conclude that one weekly pattern is sufficient to reflect the overall weekly seasonality. Weekly pattern estimated for Italy is given below. In Figure 4.2 we can see the convergence between the average daily prices through time. This fact may be explained by developing technology smoothing production and consumption. After subtracting the weekly cycles, deterministic annual seasonality and trend function is fitted to the residual, using robust nonlinear fitting algorithm nlinfit, a built in function in optimization toolbox of MATLAB.

Contrary to some of the similar researches, eliminating jump points before fitting determinis-

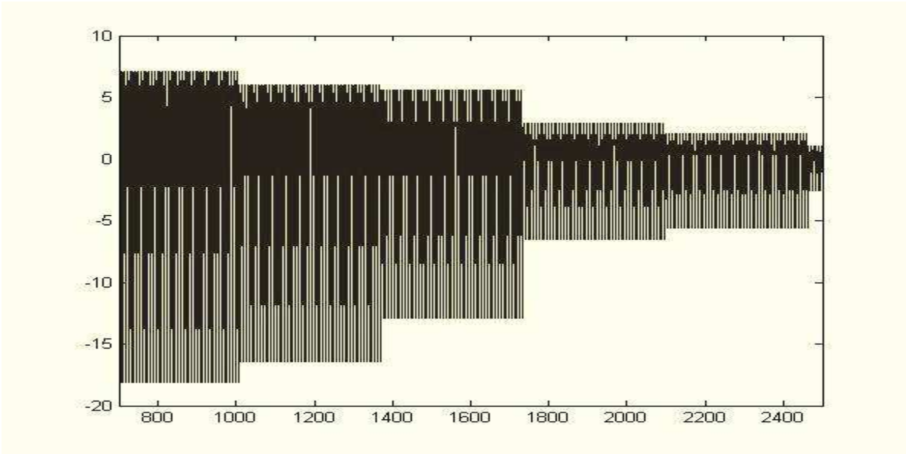


Figure 4.2: Estimated weekly cycle for Italian electricity market

tic function to avoid bias in estimated parameters, we conclude that results acquired by robust non linear fitting which uses iteratively reweighted least squares with a bisquare weighting function are as good as the former method. Coefficients of the deterministic function is given in Table 4.1 and Table 4.2. For Turkey, the coefficient of the trend parameter α_1 has a minus sign, contrary to an expected positive time effect on prices. Since the day-ahead spot electricity market started full functioning recently in Turkey, this negative trend may be caused by falling prices due to increasing efficiency and rising competition with increasing number of players in the market or falling input prices and demand because of the global financial crisis affecting the world since 2007.

Applying the threshold function on the residual returns, we acquire the jump points and consecutive mean reverting returns following the jumps. The parameter c in 3.28 is taken as 2.5 for Austria, 2.5 for Italy, 2.6 for Spain and 2.3 for Turkey and h is assumed to be 1. According to this filtration the number of jumps are given in Table 4.3. By dividing the number of jumps to the total number of observations, the jump intensities are calculated and presented in Table 4.4.

Using Takeschi formula in (3.40), correlation coefficients for 6 lags are computed. Then using (3.39) mean reversion rates of the filtered spike and semi-spike processes are computed. See Table 4.5.

Given these autocorrelation coefficients, approximated ν and β values for Austrian, Italian, Spanish and Turkish price series are equal to 2.03, 2.41, 1.61, 1.78 ($\hat{\nu}$ values), 0.40, 0.40,

	Mon.	Tues.	Wed.	Thurs.	Fri.	Sat.	Sun.
Austria							
2002	0.11	0.18	0.18	0.14	0.11	-0.19	-0.53
2003	0.14	0.21	0.20	0.15	0.08	-0.23	-0.54
2004	0.08	0.16	0.13	0.10	0.07	-0.15	-0.39
2005	0.09	0.13	0.14	0.10	0.05	-0.16	-0.34
2006	0.09	0.16	0.17	0.15	0.08	-0.18	-0.47
2007	0.11	0.18	0.16	0.16	0.07	-0.21	-0.47
2008	0.07	0.14	0.13	0.10	0.07	-0.14	-0.38
2009	0.04	0.12	0.11	0.10	0.05	-0.10	-0.32
2010	0.06	0.11	0.07	0.07	0.04	-0.09	-0.26
2011	0.07	0.09	0.08	0.05	0.03	-0.11	-0.21
Italy							
2004	0.07	0.14	0.13	0.180	0.15	-0.17	-0.52
2005	0.05	0.12	0.13	0.11	0.11	-0.16	-0.35
2006	0.04	0.08	0.10	0.10	0.07	-0.12	-0.27
2007	0.04	0.09	0.08	0.07	0.07	-0.09	-0.26
2008	0.07	0.07	0.07	0.05	0.02	-0.04	-0.16
2009	0.02	0.02	0.05	0.03	0.02	-0.02	-0.11
2010	0.01	0.02	0.02	0.03	0.01	0.01	-0.08
2011	0.01	0.02	0.01	0.01	0.01	0.01	-0.03
Spain							
2000	0.05	0.08	0.10	0.09	0.11	-0.12	-0.32
2001	0.07	0.05	0.08	0.09	0.03	-0.06	-0.25
2002	0.07	0.08	0.06	0.10	0.08	-0.12	-0.27
2003	0.08	0.06	0.06	0.08	0.09	-0.08	-0.29
2004	0.04	0.04	0.06	0.050	0.05	-0.05	-0.19
2005	0.04	0.05	0.08	0.10	0.03	-0.08	-0.22
2006	0.04	0.05	0.04	0.05	0.02	-0.05	-0.15
2007	0.04	0.04	0.02	0.03	0.02	-0.03	-0.11
2008	0.02	0.02	0.01	0.003	0.003	-0.01	-0.04
2009	0.02	0.01	0.01	0.01	0.02	-0.01	-0.05
2010	0.04	0.04	0.04	0.04	-0.01	-0.06	-0.10
2011	0.03	0.03	0.01	-0.001	-0.03	0.01	-0.05
Turkey	0.02	0.07	0.07	0.06	0.04	-0.04	-0.21

Table 4.1: Day of the week parameters

$$\Lambda_{t_n} = w(t_n) + \alpha_0 + \alpha_1 t_n + \alpha_2 \cos \frac{2\pi(t_n - \alpha_3)}{365}$$

	α_0	α_1	α_2	α_3
Austria	3.26	0.0002	-0.089	-50.1
Italy	4.08	0.0001	0.046	-212.9
Spain	3.37	0.0001	-0.079	-74.5
Turkey	4.86	-0.0004	-0.17	254.62

Table 4.2: Deterministic function parameters

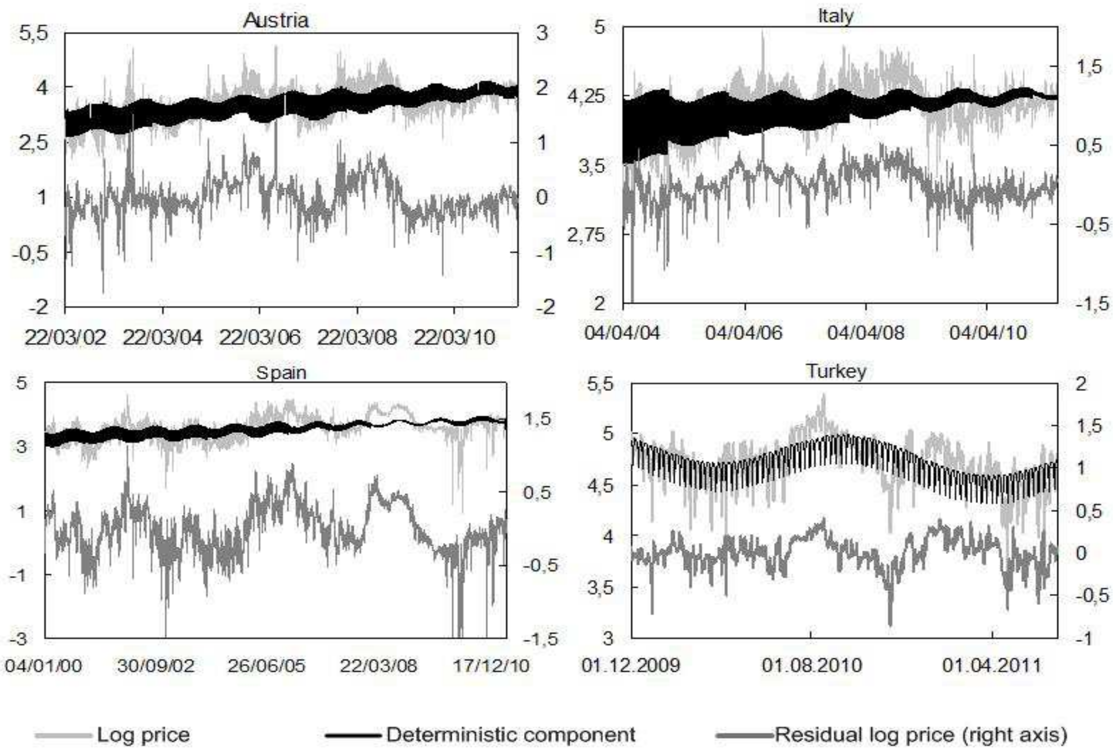


Figure 4.3: Decomposition of deterministic and stochastic components

	Pure Jumps	Spike Jumps	Semi-spike Jumps
Austria	38	100	62
Italy	17	98	39
Spain	31	87	53
Turkey	13	23	12

Table 4.3: Number of jumps filtered by the iterative GARCH(1,1) volatility threshold $r(h)$

0.92, 0.43 ($\hat{\beta}$ values) respectively. Finding the mean reversion rates for the spike and semi-spike process, whole series is refined by subtracting the lagged effects of the previous jumps from the following observations. And it is observed that this refinement cause minor changes in the jump sizes as expected. Since spike and semi-spike processes have high mean reversion rates and jumps are rare enough, effect of the price jumps almost fades away till the occurrence of the next jump.

After determination of the jump sizes, nognormal, inverse Gaussian, gamma, exponential, Levy and Burr distributions are fitted to the absolute jump sizes for each of the processes. Then, by dividing the number of observed positive jumps to the total number of jumps, the

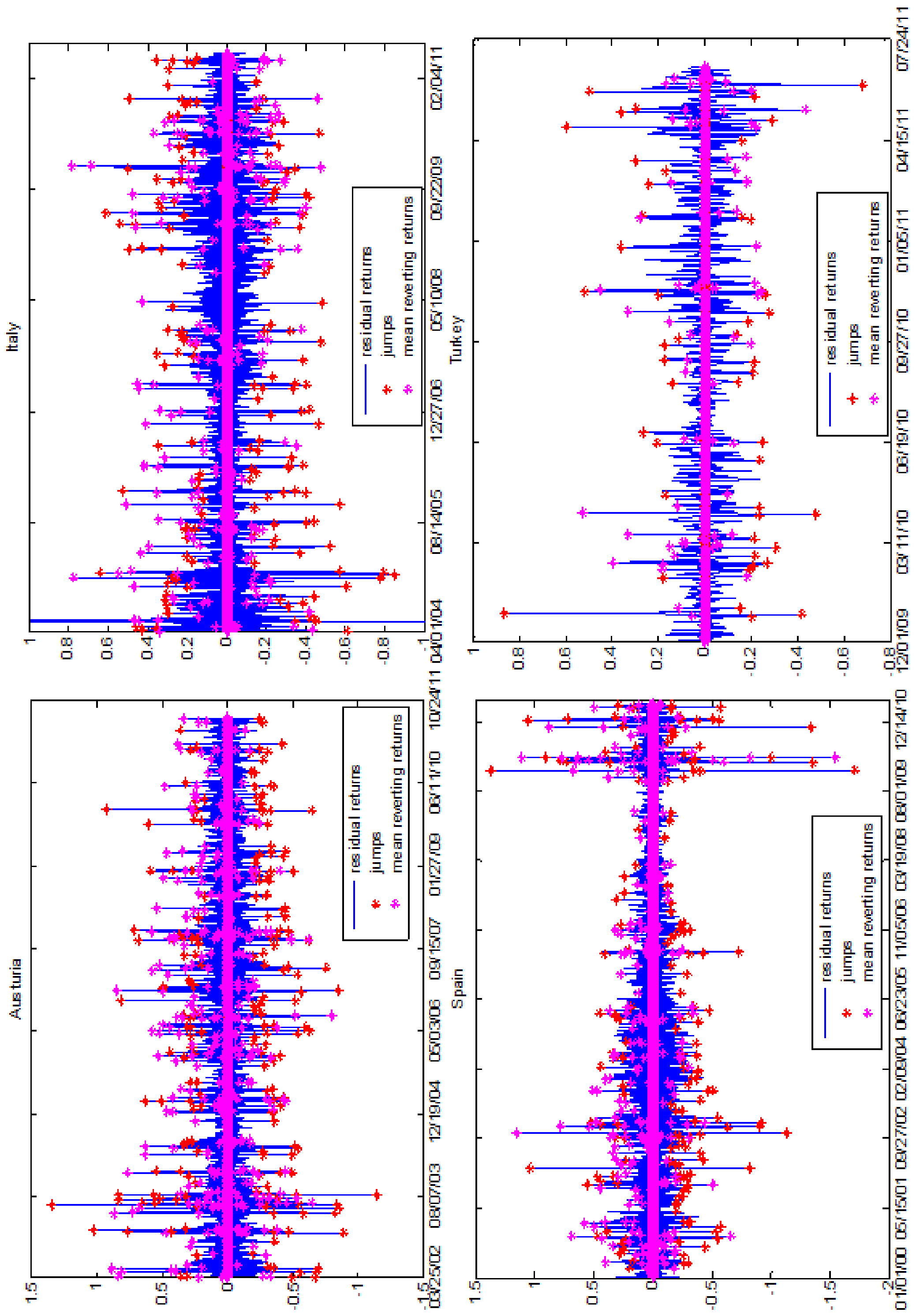


Figure 4.4: Separated jumps and mean reverting returns

	Pure Jumps	Spike Jumps	Semi-spike Jumps
Austria	0.011	0.030	0.018
Italy	0.007	0.038	0.015
Spain	0.008	0.021	0.013
Turkey	0.023	0.039	0.021

Table 4.4: Jump intensities=number of filtered jumps/total number of returns

	Austria	Italy	Spain	Turkey
Spike process	0.10	0.09	0.20	0.17
Semi-spike process	0.67	0.67	0.40	0.65

Table 4.5: Discrete time mean reversion rates

probability of observing a positive jump is derived. For each country we assume that all of the jumps are coming from the same type of distribution, but only parameters of the distribution and probability of the jump direction differs. Such a choice is made in order to decrease the complexity of the estimation and forecasting procedure.

In Table 4.6 estimated parameter values for lognormal and Burr distributions which fit best to the jumps are given where the lognormal distribution is

$$F(x) = \Phi\left(\frac{ln(x-\gamma)-\mu}{\sigma}\right)$$

and the Burr distribution is

$$F(x) = 1 - \left(1 + \left(\frac{x-\gamma}{\beta}\right)^{\alpha}\right)^{-k}.$$

After finding the best fitting distributions to the jumps, we are left with the filtered price series, assumed to represent the Brownian motion part of the price process. Autocorrelation functions and the quantile plot of these filtered price returns are depicted in Figure 4.5 and 4.6. Original series is also presented in order to see the results of our procedure. Although some of the series like Austria suggest usage of stochastic volatility models in order not to increase the parametrization, volatility is assumed to be constant. Therefore estimated standard deviations for the filtered return series are 0.10 for Austria, 0.087 for Italy, 0.11 for Spain and 0.09 for Turkey.

	Dist	P1	P2	P3	$prob(\Delta J > 0)$
Austria	Lognormal	σ	μ	γ	
PJ		1.17	-1.84	0.19	0.47
SP		0.67	-1.71	0.14	0.29
SS		1.00	-2.07	0.18	0.52
Italy	Burr	k	α	β	
PJ		0.61	4.09	0.22	0.53
SP		0.52	4.63	0.22	0.41
SS		0.45	6.02	0.20	0.59
Spain	Burr	k	α	β	
PJ		0.52	3.85	0.27	0.39
SP		0.84	4.21	0.25	0.24
SS		0.36	5.36	0.20	0.42
Turkey	Burr	k	α	β	
PJ		0.35	7.18	0.18	0.31
SP		0.58	7.75	0.18	0.35
SS		0.29	7.59	0.22	0.67

Table 4.6: Jump distributions

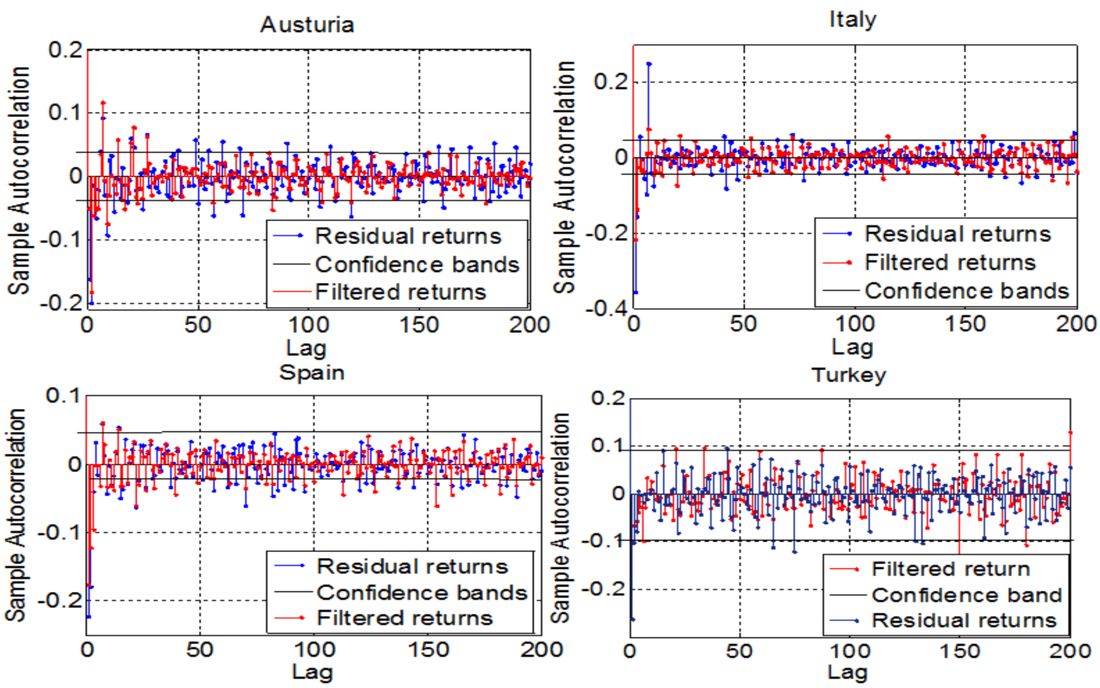


Figure 4.5: Autocorrelation functions of the original return series and the filtered returns after the jump series are extracted

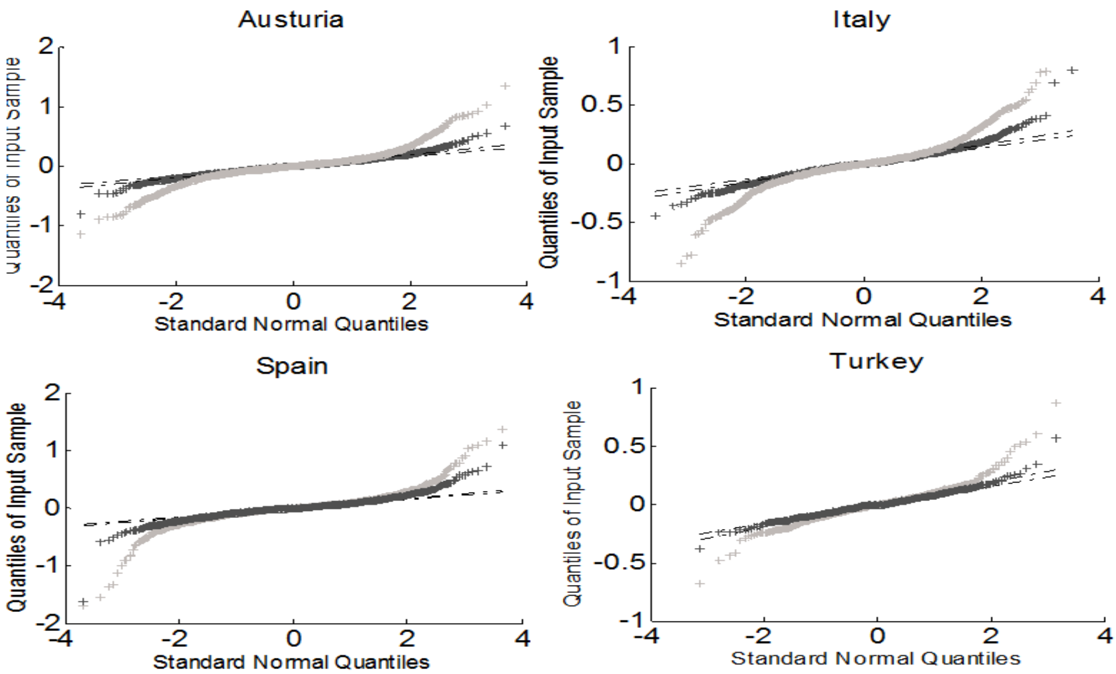


Figure 4.6: QQ-plots of the original return series and the filtered returns

4.2 Forecasting

For each of the price series, using the estimated model parameters given in the previous section, daily average spot prices are forecasted for the following days. Forecasts are based on the paths created by Monte Carlo simulation. Forecast periods are 91 days for Austria (from July 11 to October 9), 119 days for Italy (from May 29 to September 24), 154 days for Spain (from April 26 to September 26) and 91 days for Turkey (from July 1 to September 29). The forecasts are made weekly using the available data and model parameters, 100000 paths are generated for the following seven days. Mean value of the generated daily values are taken as forecasted price of the corresponding day. Then the realized price for that week is added to the observation series and the whole procedure is repeated beginning from the jump detection step. By using the newly created jump series, forecasts for the following week are generated. Original price series starting from January 1st, 2011 to the end of the forecasting period and zoomed the realizations and forecasted prices in the corresponding forecast period are given in Figure 4.7 and Figure 4.8.

In order to check forecast accuracy, daily analougus of linear Mean Weekly Error (MWE)

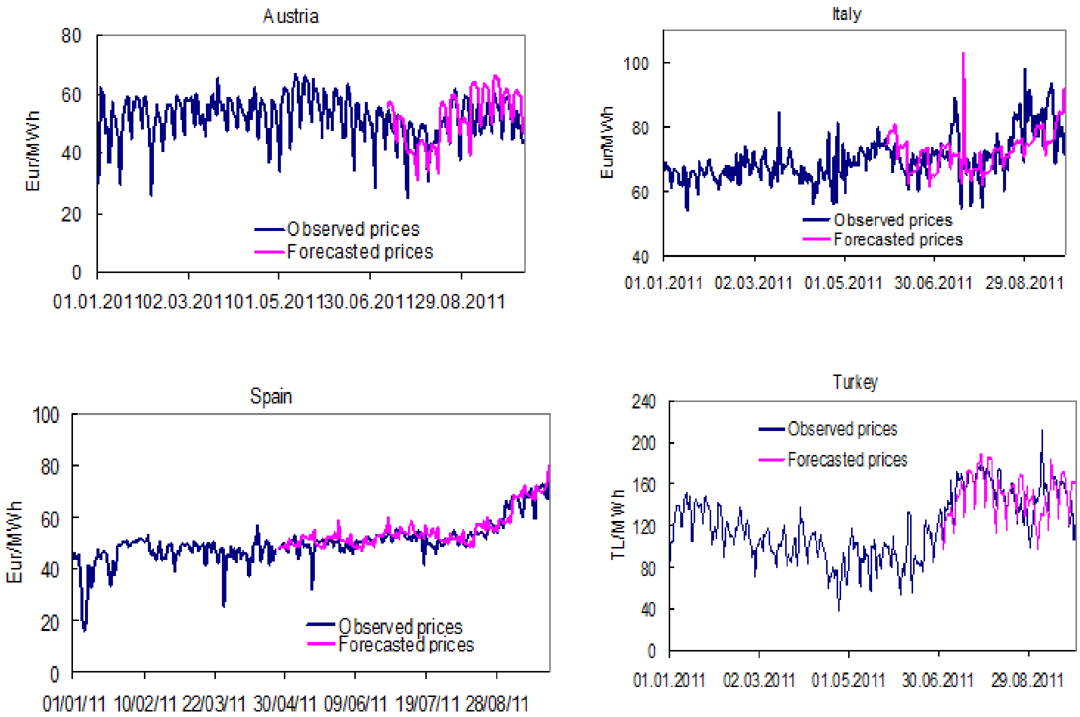


Figure 4.7: Observed and forecasted price series starting from 2011

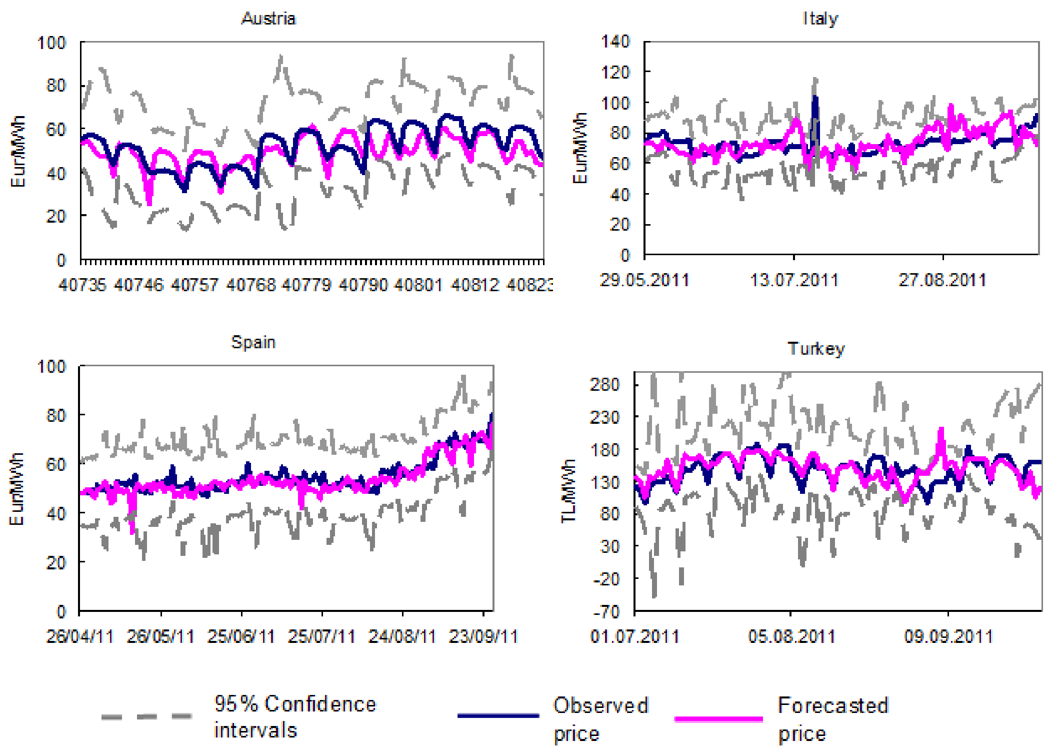


Figure 4.8: Observed and forecasted price series through the forecast period

given in [78] for hourly data is computed for each week:

$$MWE = \frac{1}{7}\sum_{d=1}^{7}\frac{\left|S_d - \hat{S}_d\right|}{\bar{S}_7},$$

where $\bar{S}_7$ is the mean observed price for the given week. MWE values for each week and their mean value are given in Figure 4.9.

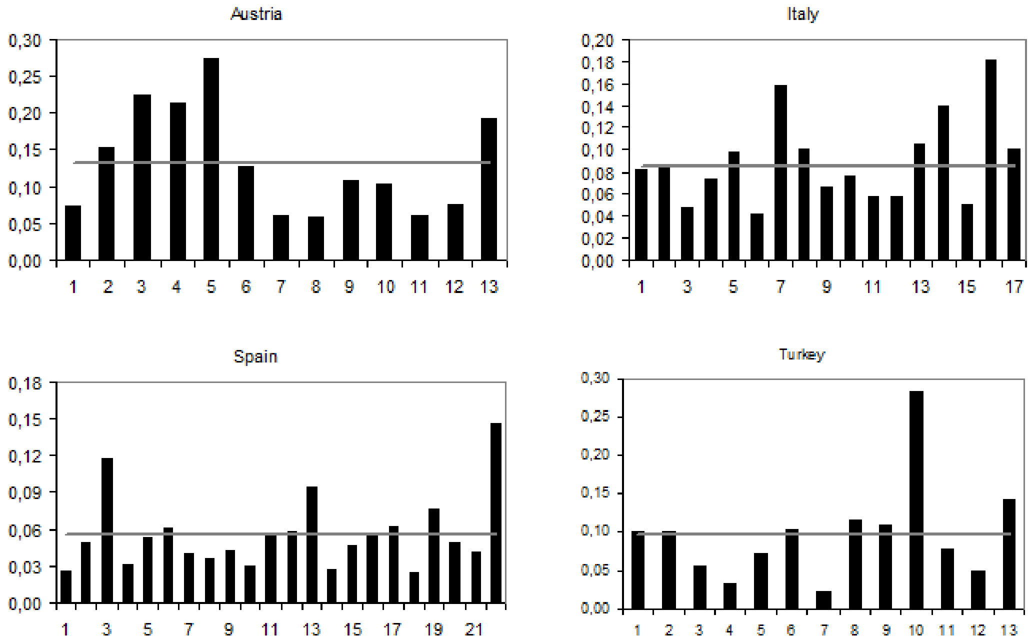


Figure 4.9: MWE of each week in the forecast period and their average

According to MWE's of each week and their corresponding mean levels, it can be concluded that estimated model shows the best performance for the Spanish data then Italy, Turkey and Austria follow in order.

CHAPTER 5

ELECTRICITY FUTURES MARKETS

In deregulated electricity markets, it is generally observed that increasing competition in retail electricity markets results in greater price volatility as the industry encourages market driven prices by moving away from administratively determined, cost-based rates and encourages market-driven prices. Price volatility introduces new risks for generators, consumers, and marketers. In a competitive environment, some generators will sell their power in potentially volatile spot markets and will be at risk if spot prices are insufficient to cover generation costs. Consumers will face greater seasonal, daily, and hourly price variability and, for commercial businesses, this uncertainty could make it more difficult to assess their long-term financial position. Finally, power marketers sell electricity to both wholesale and retail consumers, often at fixed prices. Marketers who buy on the spot market will face the risk that the spot market price can substantially exceed fixed prices specified in contracts.

Electricity futures and other derivatives can help each of these market participants to manage, or hedge, price risks in a competitive electricity market. Futures contracts are legally binding and negotiable contracts that call for the future delivery of a commodity. In most cases, physical delivery does not take place, and the futures contract is closed by buying or selling a futures contract on or near the delivery date, or by design, the contracts are subject to financial settlement. While the futures contracts are traded in organized exchanges, forwards and various types of options are traded over the counter (OTC). The first exchange to introduce electricity futures was the Scandinavian power exchange Nordpool, where monthly, quarterly and annual futures contracts are traded. Besides the New York Mercantile Exchange, London International Petroleum Exchange, UK Power Exchange, European Power Exchange, Powernext, APX UK are all well known exchanges where electricity futures are traded. Most of the national deregulated spot electricity markets around the world are completed with derivative exchanges where trading of standardized electricity contracts trading take place.

On the other hand, a large variety of electricity derivatives are traded among market participants in the OTC markets, including forward contracts, swaps, plain vanilla options and exotic options like spark spread options, swing options, swaptions and so on. In this chapter, only the standardized futures contracts will be discussed. A detailed discussion of other instruments can be found in [33]. The main reason of concentration on the futures markets is to gain an insight about the monthly electricity futures contracts which have been traded since September 2011 in the Turkish Derivatives Exchange (TurkDEX). Moreover, it is widely accepted that futures being traded on organized exchanges reflect a higher market consensus and transparency than other OTC traded products. In addition, credit risk and monitoring costs constitute a lower part of the futures prices than the other products since exchanges implement strict margin requirements to ensure financial performance of all trading parties.

A typical futures contract is a standardized, transferable and obligatory contract to buy or sell a specified quantity of the underlying asset at a particular future point in time (maturity) for a specified price contracted today (futures price). The seller of the contract is also obliged to sell the underlying asset, in our case the electricity. The maturity, quality and quantity of the underlying asset are all standardized. The only negotiable aspect of the contract is the fixed price paid for the underlying asset at maturity; the futures price. There are no initial costs of entering a futures contract. Due to changing market conditions and price expectations, the value of a particular futures contract, however, does change over time. The value of each futures contract is marked to market according to the calculated market value for that trading day. This means that financial positions are valued based on the current fair market price. Differences between previous day's value and the current market price are settled immediately and the gain/loss of a position is added/withdrawn from the margin account of the position holder. Since the risk for both parties is unlimited theoretically, exchanges use these margin accounts to guarantee that the contract obligations would be fulfilled.

Different from other future contracts, electricity futures have a delivery period instead of a fixed delivery time. Generally contracts with weekly, monthly, quarterly and annual delivery periods are traded. The regulations, number of traded contracts or trading and delivery periods varies across the countries. For instance, in Turkey only the monthly base load contracts are traded at the moment. The reference price for TurkDEX base load electricity futures contracts are the average of the day-ahead hourly prices of the corresponding delivery month. Contract size is equal to the number of hours in the contract month times 0,01 MWh. The quoted futures price is the price of 1 MWh. Contracts for the current month and the following three months are traded simultaneously. Contracts are traded until the last trading day of the delivery month. Daily settlement price is the weighted average price of all transactions performed within the last 10 minutes before the closing of the trading session.

There is a growing literature on pricing of electricity derivatives. In the following section, future contract pricing models proposed in literature will be examined. However, since almost all of these models need a long stream of observations for parameter estimation, in Section 5.2 an alternative methodology which can be used in an infant derivative market like Turkey's is discussed. Although most of the models described in this chapter are inapplicable for the market in Turkey due to the lack of historical data, they are discussed in detail since the development and calibration of such a stochastic model will be one of the further research topics.

5.1 Stochastic Futures Pricing Models in Literature

Parallel to the increasing importance of derivative contracts in the power markets, researches aiming to model these contracts have also been increasing. However, since the electricity is not a tradable asset in the classical sense due to its non-storability, models used in financial markets need some modifications. In the literature, electricity futures price process modeling has been done by following either the spot price approach or the futures price approach. In the spot price approach, firstly electricity spot price process is accurately modeled and then by using additional conditions which are related to spot and futures prices, futures price process is derived. In the futures price approach, instead of modeling the spot price and deriving futures prices, the futures prices are directly modeled. This approach is based on Heath-Jarrow-Merton (HJM) framework which is developed for fixed income markets. Moreover, there exists two types of applications for the HJM framework in electricity futures pricing. The first approach initially models the futures contracts with fixed delivery and then corresponding dynamics for futures, delivering electricity during a period are derived. The second approach is directly modeling the futures price with a delivery period (For a summary of HJM framework Appendix A.2). These three approaches (spot price approach, futures price approach based on fixed delivery contracts, futures price approach based on futures contracts with delivery periods) and related literature are summarized in the following sub-sections.

In this context, as a result of a general literature review, widely accepted features of the future contract prices are listed below. Any futures contract model is expected to contain below mentioned features:

- Future contracts show lower price volatility than the spot asset. Short term changes or price shocks have less effect on the futures prices, since futures prices depend on the average spot price observed during the delivery period and most of the spot price shocks are not persistent as it is shown in the previous chapter.

- Length of the delivery period affects the futures price volatility. The longer the delivery period, the lower the futures price volatility.

- Time left to delivery is also proved to be effective on the futures price volatility. This effect is known as the Samuelson effect, as the maturity approaches volatility observed in the futures price also rises.

- Futures prices are also found to exhibit a seasonal pattern like the spot electricity prices. For instance, if the spot electricity prices are higher in the winter months, futures contracts with winter delivery are also expected to have higher prices.

5.1.1 Spot price approach

Main stream spot electricity price models used in the literature have already been discussed in the previous chapter. In this context, Lucia et al. (2002) calibrate their one and two factor spot price models to derivative contracts. Pilipovic (1998) uses a two factor spot electricity model, which leads to a complicated closed form expression for fixed delivery futures. Benth et al. (2008b) calculate the fixed delivery forward contract prices by using their multi-factor exponential spot electricity price models. However, as it is stated in Benth et al. (2009a) all of these models exhibit market incompleteness due to jump processes in the models and/or mutlidimesional Brownian motions driving the spot price dynamics. Moreover, the non-storability of the underlying commodity rules out the possibility to hedge the derivatives by trading the underlying. In this context any probability measure Q equivalent to the objective probability measure P is risk neutral and due to the absence of perfect hedge, market price of risk has to be estimated.

For a given spot price which is represented with semimartingale S_t and constant risk free interest rate r, under the risk neutral probability measure Q discounted expected payoff of the futures contact must be equal to its price today. Since it is costless to enter these contracts

$$e^{-r(T-t)}E_Q(S_T - f(t,T)|F_t) = 0, \tag{5.1}$$

where $f(t,T)$ is the price of futures contracted started on day t with delivery on day T, $t \leq T$. Assuming that $S_T \in L^1(Q)$, the space of integrable random variables with respect to Q and $f(\cdot,T)$ is an adapted process

$$f(t,T) = E_Q(S_T|F_t). \tag{5.2}$$

With the same line of reasoning and the assumption that the settlement takes place continuously on the delivery period, for the futures contract with delivery period $[T_1,T_2]$, we obtain

$$E_Q\left(\int_{T_1}^{T_2} e^{-r(u-t)}(S_u - F(t,T_1,T_2))du|F_t\right) = 0, \tag{5.3}$$

$$F(t,T_1,T_2) = E_Q\left(\int_{T_1}^{T_2} \frac{re^{-ru}}{e^{-rT_1} - e^{-rT_2}} S_u du|F_t\right). \tag{5.4}$$

Using the Fubini theorem, we get

$$F(t,T_1,T_2) = \int_{T_1}^{T_2} \frac{re^{-ru}}{e^{-rT_1} - e^{-rT_2}} f(t,u)du. \tag{5.5}$$

If the settlement takes place at the end of the delivery period, T_2,

$$F(t,T_1,T_2) = \int_{T_1}^{T_2} \frac{1}{T_2 - T_1} f(t,u)du. \tag{5.6}$$

As it is said at the beginning, determination of the risk free probability measure is the crucial step in futures contract pricing, using the spot price approach. For instance in Benth et al. (2008a) the Esscher transform is used in order to restrict the possible set of equivalent martingale measures. However, in most cases it is not possible to find an analytic solution for $F(t,T_1,T_2)$ processes. In general, multi-factor spot electricity models aiming a detailed description of the spot prices lead to complicated future price dynamics. As a result, not only the calibration of the pricing measure on the market data becomes a very challenging task in the electricity markets, but also the flexibility of the spot models are rarely sufficient to generate future curves that are consistent with the observed curves.

5.1.2 Instantaneous delivery models

In fixed income markets, instead of modeling the prices, the forward rates are directly modeled using the HJM approach. In analogy to HJM approach for the forward interest rates, many authors have proposed a HJM type model for electricity forward and future curves. For instance, Koekebakker et al. (2005), construct a financial market where the uncertainty is described by a K dimensional Brownian motion $(W_1, ..., W_k)$, which is defined on the probability space (Ω, F, Q) with the filtration F satisfying the usual conditions. The probability measure Q represents the equivalent martingale measure and risk free rate, r, which is assumed to be constant. Assuming that the futures market is represented by a continuous futures price function $f(t, T)$, where the futures price processes are martingales under Q by construction. Two types of models are proposed in Koekebakker et al. (2005);

- *Futures price process which is independent of the futures price level:* The dynamics are given by

$$df(t,T) = \sum_{i=1}^{K} \sigma_i^A(t,T) dW_i(t), \tag{5.7}$$

  where $(W_1, ..., W_k)$ are independent Brownian motions and $\sigma_i^A(t,T)$ are time dependent deterministic volatility functions. The solution of the (5.7) and the distribution of the futures prices are

$$f(t,T) = f(0,T) + \sum_{i=1}^{K} \int_0^t \sigma_i^A(s,T) dW_i(s), \tag{5.8}$$

$$f(t,T) \sim N\left(f(0,T), \quad \sum_{i=1}^{K} \int_0^t (\sigma_i^A(s,T))^2 ds \right). \tag{5.9}$$

- *Futures price process which is proportional to the futures price level:* The dynamics of the futures price are given by

$$\frac{df(t,T)}{f(t,T)} = \sum_{i=1}^{K} \sigma_i^B(t,T)dW_i(t), \tag{5.10}$$

the solution of (5.10) and the distribution of the futures prices are

$$f(t,T) = f(0,T)exp\left(-\frac{1}{2}\sum_{i=1}^{K}\int_0^t (\sigma_i^B(s,T))^2 ds + \sum_{i=1}^{K}\int_0^t \sigma_i^B(s,T)dW_i(s)\right), \tag{5.11}$$

$$\ln f(t,T) \sim N(f(0,T) - \frac{1}{2}\sum_{i=1}^{K}\int_0^t (\sigma_i^B(s,T))^2 ds, \sum_{i=1}^{K}\int_0^t .(\sigma_i^B(s,T))^2 ds) \tag{5.12}$$

Since the HJM framework does not imply a specific model for the spot electricity prices and the volatility functions used in the above equations are flexible enough, a wide variety of future price dynamics can be constructed from the above models. For instance, one factor spot electricity price model proposed by Luica et al. (2002) is consistent with the futures price model in (5.8) taking $\sigma^A(t,T) = \sigma e^{-\kappa(T-t)}$ where σ and κ are positive constants. Bjerksund et al. (2000) propose two different models for fixed delivery futures price modeling. Their one factor model has the volatility function $\sigma^B(t,T) = \frac{a}{T-t+b} + c$, where a, b and c are positive constants. For both of the Lucia et al. (2002) and Bjerksund et al. (2000) volatility functions decrease with maturity and approach to 0 as $T \to \infty$. Moreover Bjerksund et al. (2000) propose a three factor model with volatilities $\sigma_1^B(t,T) = \frac{a}{T-t+b}$, $\sigma_2^B(t,T) = (\frac{2ac}{T-t+b})^{\frac{1}{2}}$ and $\sigma_2^B(t,T) = c$ where all parameters are assumed to be positive. In their paper authors argue that the one factor model is adequate for contingent claim pricing, while three factor model has better performance in risk management. Then $F(t,T_1,T_2)$ being today's contract price with delivery period $[T_1,T_2]$ where $t \leq T_1 < T_2$, and assuming that the contract price is paid as a constant cash flow during the delivery period; the price of this contract is given as

$$F(t,T_1,T_2) = \int_{T_1}^{T_2} w(r,u)f(t,u)du, \tag{5.13}$$

where

$$w(r,u) = \frac{e^{-r(u-t)}}{\int_{T_1}^{T_2} e^{-r(u-t)}du}. \tag{5.14}$$

Although by using (5.13) and (5.14), dynamics of the actually traded futures can be captured, as Benth et al. (2008b) argue that with this method, the implied dynamics of the futures contract price with delivery period $[T_1,T_2]$ can become very complicated. In their referenced paper, dynamics are examined by using the fixed time delivery futures contract following the stochastic differential equation presented in (5.15):

$$df(t,T) = \sigma(t,T)f(t,T)dW(t). \tag{5.15}$$

Then the implied dynamics for the futures contract with delivery period $[T_1, T_2]$ is given as;

$$dF(t,T_1,T_2) = \Sigma(t,T_1,T_2)dW(t), \tag{5.16}$$

where the volatility dynamics of the futures contract with delivery period $[T_1, T_2]$ is

$$\Sigma(t,T_1,T_2) = \int_{T_1}^{T_2} \hat{w}(u,T_1,T_2)\sigma(t,u)f(t,u)du \tag{5.17}$$

and

$$\hat{w}(t,T_1,T_2) := \frac{e^{(-rt)}}{\int_{T_1}^{T_2} e^{(-ru)}du} \tag{5.18}$$

for the given constant risk free interest rate r. After integration by parts we see

$$\Sigma(t,T_1,T_2) = \sigma(t,T_2)F(t,T_1,T_2) - \int_{T_1}^{T_2} \delta_2\sigma(t,u)\int_{T_1}^{u} \hat{w}(\tau,T_1,T_2)f(t,\tau)d\tau du. \tag{5.19}$$

In (5.19) δ_2 denotes partial differentiation with respect to the second variable of the respective function. Since $\hat{w}(\tau,T_1,T_2)/\hat{w}(\tau,T_1,u)$ is independent of τ,

$$\Sigma(t,T_1,T_2) = \sigma(t,T_2)F(t,T_1,T_2) - \int_{T_1}^{T_2} \delta_2\sigma(t,u)\frac{\hat{w}(\tau,T_1,T_2)}{\hat{w}(\tau,T_1,u)}F(t,T_1,u)du \tag{5.20}$$

Then using (5.16) and (5.20) yield

$$dF(t,T_1,T_2) = \sigma(t,T_2)F(t,T_1,T_2)dW(t) - \int_{T_2}^{T_1} \delta_2\sigma(t,u)\frac{\hat{w}(\tau,T_1,T_2)}{\hat{w}(\tau,T_1,u)}F(t,T_1,u)dudW(t). \tag{5.21}$$

In Benth et al. (2008b) it is shown that when the volatility function σ is not a function of the expiration date of the contract, i.e., $\delta_2\sigma(t,u) = 0$, $F(t,T_1,T_2)$ has lognormal dynamics. However, it is emphasized that in realistic models the volatility depends strongly on the time of delivery of the contract. Moreover, if we start with a model for future contracts with fixed delivery, we need to estimate the parameters using the market traded future contracts with delivery over a period. Due to the possibility of ending with complicated dynamics for the futures price process, it may not always be possible to integrate $F(\cdot, T_1, T_2)$ in order to estimate the model parameters.

Alternatively, instead of fitting the parameters of the observed futures price process $F(\cdot, T_1, T_2)$, $f(\cdot, T)$ prices can be extracted from the observed futures contract prices. This can be done by using the smoothing technique proposed by Benth et al. (2007b). Then, the theoretical $f(\cdot, T)$ processes constructed under the risk neutral probability Q are transformed according to the objective probability measure P and the parameters are estimated.

5.1.3 Direct modeling of the observed future contracts

Since the instantaneous delivery future contracts are not actually traded in the electricity markets, in order to avoid additional data formation problem mentioned in the previous subsection and model complications that may arise, Benth et al. (2008b) propose direct modeling of the future contracts with the delivery period $[T_1, T_2]$, which are the contracts traded in organized exchanges. In this context, authors adapted the HJM framework and they specify arbitrage free dynamics for the process $F(\cdot, T_1, T_2)$, which are valid for all delivery periods within a predetermined time horizon. However, the authors concluded that this goal is hard to achieve while preserving the flexibility of the models which can be easily adapted to option pricing and risk management models. Moreover, it is shown that a lognormal model for $F(\cdot, T_1, T_2)$ dynamics cannot satisfy the no arbitrage condition and also has a volatility process depending on the delivery period of the contract at the same time. The solution to this problem is given as modeling not all of the available contracts simultaneously but modeling the building blocks of the futures market. This means modeling the contracts that cannot be decomposed into other traded contracts. For instance, if 1 year contract and 12 months contracts for each month of the following year are traded in the market simultaneously, $F(\cdot, T_1, T_2)$ process only models the monthly contracts and ignores the yearly contract. In this framework, a lognormal one factor future price dynamics under the equivalent risk neutral martingale measure Q is given by

$$dF(t, T_1, T_2) = \Sigma(t, T_1, T_2)F(t, T_1, T_2)dW(t), \tag{5.22}$$

where $\Sigma(t, T_1, T_2)$ is a continuously differentiable and positive function representing the volatility and W(t) is a Wiener process under the risk neutral measure. When it is assumed that the settlement of the contract takes place at the maturity T_2, market futures volatility $\Sigma(t, T_1, T_2)$ can be associated to the instantenous delivery futures volatility $\sigma(t, T)$ as follows:

$$\Sigma(t, T_1, T_2) = \frac{1}{T_2 - T_1} \int_{T_1}^{T_2} \sigma(t, u) du. \tag{5.23}$$

Considering the (5.23), Benth et al. (2008b) examine six different volatility functions, which are suggested and used in the commodity market researches. And they test their performances in modeling futures price process, $F(\cdot, T_1, T_2)$. First of the examined volatility function belongs to the Schwartz's (1997) one factor oil price model; $\sigma(t, u) = ae^{-b(t-u)}$, where a,b ≥ 0. By the (5.23) corresponding volatility is found as $\Sigma(t, T_1, T_2) = a\phi(T_1, T_2)$, where $\phi(T_1, T_2) = \frac{e^{-b(T_1-t)} - e^{-b(T_2-t)}}{b(T_1-T_2)}$. The basic constant volatility model appears if $b = 0$. While the Schwartz model reflects only the maturity effect and ignores the seasonality, the instantenous delivery futures volatility $\sigma(t, u) = a(t)e^{-b(u-t)}$ picks up also the seasonality effect with the term $a(t)$, where $a(t) = a + \sum_{j=1}^{J}(d_j \sin(2\pi jt) - f_j \cos(2\pi jt))$. In this model t is given in years, d_j and f_j are real constants and $a, b \geq 0$. Then $\Sigma(t, T_1, T_2) = a\phi(T_1, T_2)$. The fourth instantenous volatility is $\sigma(t, u) = a((1 - c)e^{-b(t-u)} + c)$, where $a, b \geq 0$ and $0 \leq c \leq 1$. The associated volatility for $F(t, T_1, T_2)$ is $\Sigma(t, T_1, T_2) = a((1 - c)\phi(T_1, T_2) + c)$, where $\phi(T_1, T_2) = \frac{e^{-b(T_1-t)} - e^{-b(T_2-t)}}{b(T_1-T_2)}$ again. When this model is combined with seasonal spot volatility, $\sigma(t, u) = a(t)((1-c)e^{-b(t-u)}+c)$ then futures volatility is $\Sigma(t, T_1, T_2) = a(t)((1-c)\phi(T_1, T_2)+c)$, where $a, b \geq 0$, $0 \leq c \leq 1$ d_j and f_j are constants. The last model is $\Sigma(t, T_1, T_2) = a(t) + c\phi(T_1, T_2)$ with $\sigma(t, u) = ce^{-b(t-u)} + a(t)$, where $a(t)$ modeling the seasonality as it is given before. According to this model the spot price volatility ($\sigma(t, t)$) is $a(t) + c$ and long-run volatility ($u \to \infty$) is governed by the seasonality term $a(t)$. Authors note that with these properties, last model has a clear separation of maturity and seasonal effects.

It is assumed that under the physical probability measure P, dynamics of financial electricity contracts traded at exchanges can be described as

$$dF(t, T_1, T_2) = \lambda\Theta(t, T_1, T_2)F(t, T_1, T_2)dt + \Theta(t, T_1, T_2)F(t, T_1, T_2)dW(t), \tag{5.24}$$

where $B(t)$ is a Brownian motion under P, λ represents the market price of risk (which is assumed to be constant) and the function $\Theta(t, T_1, T_2)$ is deterministic. Then the logarithmic returns of the futures contract ($r(t, T_1, T_2)$) is equal to;

$$\ln\left(\frac{F(t+\Delta t, T_1, T_2)}{F(t, T_1, T_2)}\right) = \int_t^{t+\Delta t} (\lambda\Theta(s, T_1, T_2) - \frac{1}{2}\Theta^2(s, T_1, T_2))ds$$
$$\int_t^{t+\Delta t} \Theta(s, T_1, T_2)dB(s). \tag{5.25}$$

Therefore logarithmic returns $r(t, T_1, T_2)$ are normally distributed with mean $m(t, T_1, T_2) = \int_t^{t+\Delta t}(\lambda\Theta(s, T_1, T_2) - \frac{1}{2}\Theta^2(s, T_1, T_2))ds$ and variance $v(t, T_1, T_2) = \int_t^{t+\Delta t} \Theta^2(s, T_1, T_2)ds$. Then the return of the ith contract is

$$r^i(t, T_1, T_2) = m^i(t, T_1, T_2) + v^i(t, T_1, T_2)\epsilon^i(t), \tag{5.26}$$

where $\epsilon(t) \sim N(0, 1)$. In Benth et al. (2008b) parameters are estimated using log-likelihood functions and it is concluded that maturity effect is very significant and modeling this effect with a simple exponential function is insufficient. Estimation results also verify the existence of seasonal volatility, and an additive specification is able capture the maturity and seasonality effects in the volatility.

5.2 Risk Premium Approach

Direct modeling of futures price processes approach, discussed in the previous section, uses future contracts price observations for the parameter estimation. However, in Turkish electricity markets, due to the lack of historical futures market price data, we have to build our own methodology on the spot electricity prices. In Subsection 5.1.1, we see that modeling the electricity futures price process by using the spot price process requires identification of risk neutral probability measure Q. Researchers usually connect the risk neutral probability to the concept of market price of risk. The market price of risk is the difference between the drift in the original probability measure P and the drift in the risk neutral measure Q in the stochastic differential equation governing the spot price dynamics and it is assumed to reflect how investors are compensated for bearing risk by holding the asset.

Another important and widely referred quantity relating futures and expected spot prices is the market risk premium $\pi(t, (T_2 - T_1))$. This premium is defined as the difference of the electricity futures price $F(t, T_1, T_2)$ and the conditional expectation of the average day-ahead price of electricity during the future delivery period $[T_1, T_2]$, with respect to the objective probability measure P. It is assumed to depend on time to maturity and length of the delivery period:

$$F(t, T_1, T_2) = E_P\left[\frac{1}{T_2 - T_1}\int_{T_1}^{T_2} S_u du | F_t\right] + \pi(t, (T_2 - T_1)). \tag{5.27}$$

According to the Keynesian theory, market risk premium depends on the risk preference of the hedgers and speculators. Therefore, the future curves observed in the market reflect not only the forecasts of the commodity spot price in the future but also the dominant hedging tendency in the market. Assuming that the main motivation for the agents to engage in the future contracts is risk diversification, producers, who have made substantial investments, have incentive to reduce variability in their profits by trading these instruments. Similarly, consumers (can be either intermediaries or use electricity in their production process) also have incentive to hedge their positions in the market and to diversify the market price risk.

The hedging and risk diversification preferences of producers and consumers generally cover different time horizons. For instance, while a producer is exposed to market uncertainty for a longer period of time, determined by the remaining life of its assets, a consumer has to make decisions for shorter periods. In other words, gains in terms of risk diversification for consumers and producers vary across time. These differences in the hedging preferences and imbalances between buyers and sellers of the future contracts are assumed to create the market risk premium. In Benth et al. (2006) it is stated that the further out one looks, consumers will have less incentives to buy future contracts where producers' desire to hedge does not diminish as quickly as consumers'. Situations where $\pi(t, (T_2 - T_1) > 0$ (positive market risk premium) are associated with higher consumer demand to cover their positions than producers. Conversely, when the producers' desire to hedge outweighs consumers', $\pi(t, (T_2 - T_1)) < 0$ (negative market risk premium).

After calibration of the spot electricity model, according to (5.27), risk premium must be added in order to price the derivative instrument. In estimation of the risk premium, ex-ante and ex-post estimation methods are used in the literature. Ex-ante or expected risk premium method uses the original definition of the risk premium given in (5.27). In this methodology the choice of an appropriate spot price model is essential for the derivation of a consistent risk premium. Claiming that by using ex-ante method it is difficult to reach consistent and robust results, some of the researchers propose an ex-post approach. The ex-post risk premium is given as

$$F(t, T_1, T_2) = \int_{T_1}^{T_2} S_u du + \breve{\pi}(t, (T_2 - T_1)). \tag{5.28}$$

In this approach, instead of the expected average day ahead prices, the average of the observed market prices through the delivery period $[T_1, T_2]$ are used. Moreover, subtracting $\int_{T_1}^{T_2} S_u du$ from both sides of (5.27) gives

$$F(t, T_1, T_2) - \int_{T_1}^{T_2} S_u du = E_P\left[\frac{1}{T_2 - T_1}\int_{T_1}^{T_2} S_u du | F_t\right] - \int_{T_1}^{T_2} S_u du + \pi(t, (T_2 - T_1)), \tag{5.29}$$

$$F(t, T_1, T_2) - \int_{T_1}^{T_2} S_u du = \pi(t, (T_2 - T_1)) + \epsilon_t. \tag{5.30}$$

Therefore, under the assumption that market participants form their forecasts based on rational expectations the ex-post risk premium equals the ex-ante risk premium plus a noise term with a mean equals to 0. By subtracting average realized spot prices during the delivery periods from historically observed futures prices and assuming that trader expectations are unbiased in the long run, risk premiums are studied empirically. This approach is applied by Wilkens et al. (2007) who examine futures prices on the German EEX market. They find positive but highly volatile risk premiums for futures contracts with times to maturity up to six months. In the Nordic electricity markets, Botterud et al. (2002) identify positive risk premiums for futures contracts with a time to maturity up to one year. On the other hand, many researchers have modeled the extracted risk premium series. Bessembinder et al. (2002) study the electricity forwards and concluded that the forward risk premium is negatively related to the variance and positively related to the skewness of expected electricity spot prices. Douglas et al. (2008) relate observed risk premiums to indirect storability by showing that higher natural gas inventory levels reduce the forward risk premium in the PJM market, especially during extremely warm and cold periods.

As it is already mentioned, for now it is not possible to discover the appropriate dynamics governing neither for the futures price nor for the related risk premium in Turkish electricity futures market. Although it is possible to determine the necessary building blocks, without calibration and testing the significance of the theoretical factors, applicability of any model will be limited. However, we can still propose an elementary approach, in order to decide whether or not to buy/sell a given futures contact as long as we have a sound model for the spot electricity model by using (5.27). For a given future contract's price, $F(t, T_1, T_2)$, first of all the expected average spot price for period $[T_1, T_2]$, $E_P\left[\frac{1}{T_2-T_1}\int_{T_1}^{T_2} S_u du | F_t\right]$ is found using a Monte Carlo simulation. Future contract prices higher than the expected average spot price ($F(t, T_1, T_2) > E_P\left[\frac{1}{T_2-T_1}\int_{T_1}^{T_2} S_u du | F_t\right]$), offers a positive risk premium and can be used by the

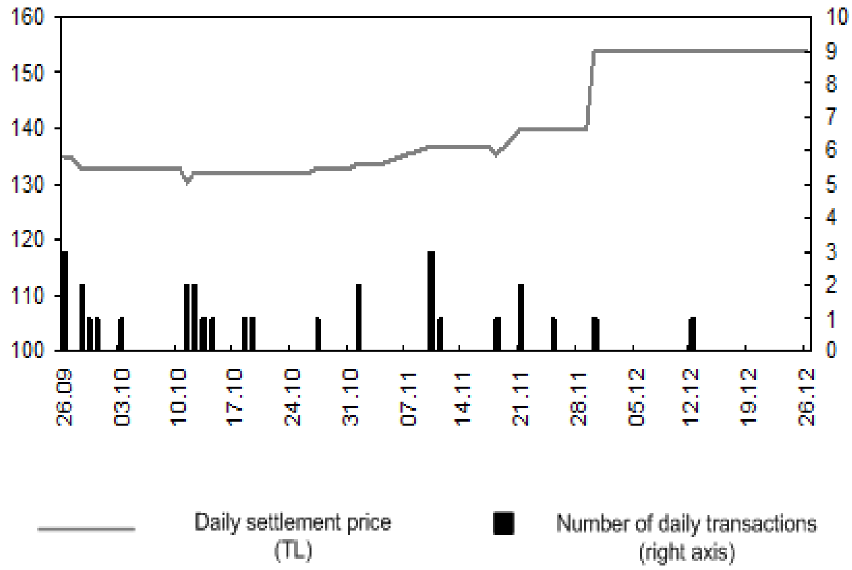


Figure 5.1: TURDEX-December 2011 contract

producers or any investor who wants to take a short position on these contracts. Contrarily, when $F(t, T_1, T_2) < E_P\left[\frac{1}{T_2-T_1}\int_{T_1}^{T_2} S_u du|F_t\right]$, future contract has a negative risk premium and is appropriate for taking a long position. As a result, 0 risk premium appears as a natural boundary for a risk neutral investor.

In order to test our simple approach, we will use the only contract that has been traded in TURDEX since 26 September 2011. This contract is a base load contract with delivery through December 2011. It is obvious that only one contract is not sufficient to reach robust conclusions. However, considering that this illiquid and thin market conditions will continue for a while, we can still see whether our approach leads to positive pay off or not. Development of the daily settlement price and number of transactions for December 2011 base load contract is shown in Figure 5.1.

Starting with the first trading day of the futures contract (26 September 2011), for every Monday from 26 September to 26 December we form monthly average expected spot electricity price for December, by using our model given in chapter 3 (5000 Monte Carlo simulations). Taking the future contract price at that day as given, we calculate the risk premium for the contract as equal to $F(\cdot, T_1, T_2) - E_P[\frac{1}{T_2-T_1}\int_{T_1}^{T_2} S_u du|F_{\cdot}]$. Theoretically if this premium is different than zero, either long or short position holders are expected to gain a positive risk premium. Therefore, if the mean value of the simulated risk premium is higher than zero, we conclude that taking a short position can yield a positive return. If the risk premium is negative, we conclude that taking a long position is appropriate. And we compute the revenue of holding the suggested position in the futures contact, due to the daily mark to market. First of all, simulated risk premium distributions for four days of each month are given in Figure 5.2.

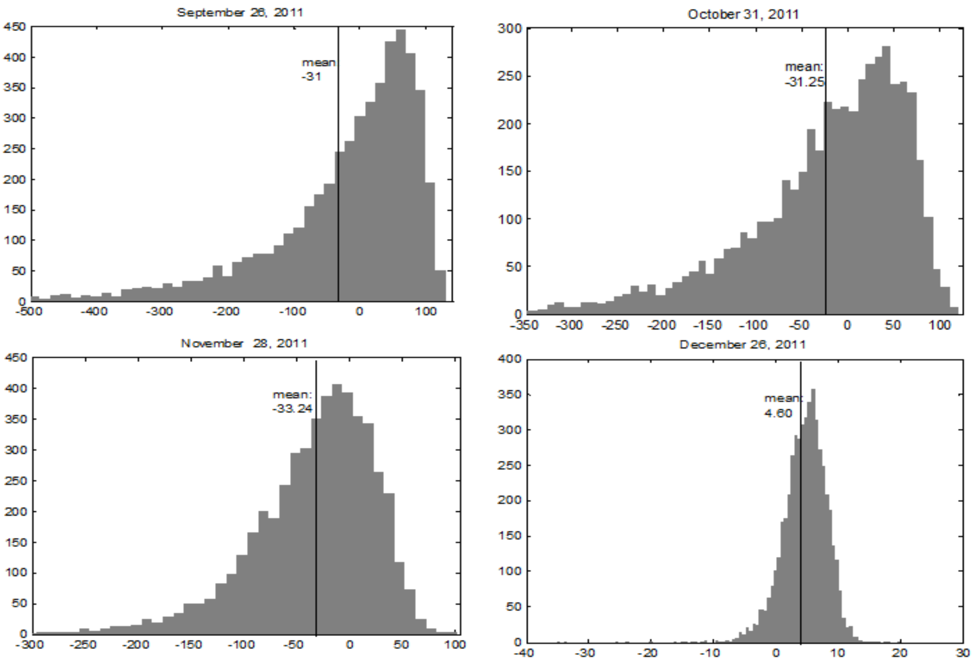


Figure 5.2: Spot electricity prices

From the risk premiums in the Figure 5.2, except 26 December, our naive approach leads us to long positions. The dates, direction of the estimated risk premium and the suggested positions are given in Table 5.1).

Lastly as of January 1st, 2012, calculated daily pay offs of the suggested positions for the given dates which are shown in Figure 5.3. Considering each December contract is written on 744 MWh electricity, 11 out of 13 positions taken according to our approach ended the contract with a positive payoff. It is far out of reach to conclude that, suggested elementary approach is an alternative for the market agents. However, as a promising method, we continue to analyze and test this method.

Date	Risk Premium Sign	Suggested Position
26 September	-	long
3 October	-	long
10 October	-	long
17 October	+	short
24 October	-	long
31 October	-	long
14 November	-	long
21 November	-	long
28 November	-	long
5 December	-	long
12 December	+	short
19 December	+	short
26 December	+	short

Table 5.1: Suggested future contract position according to the risk premium

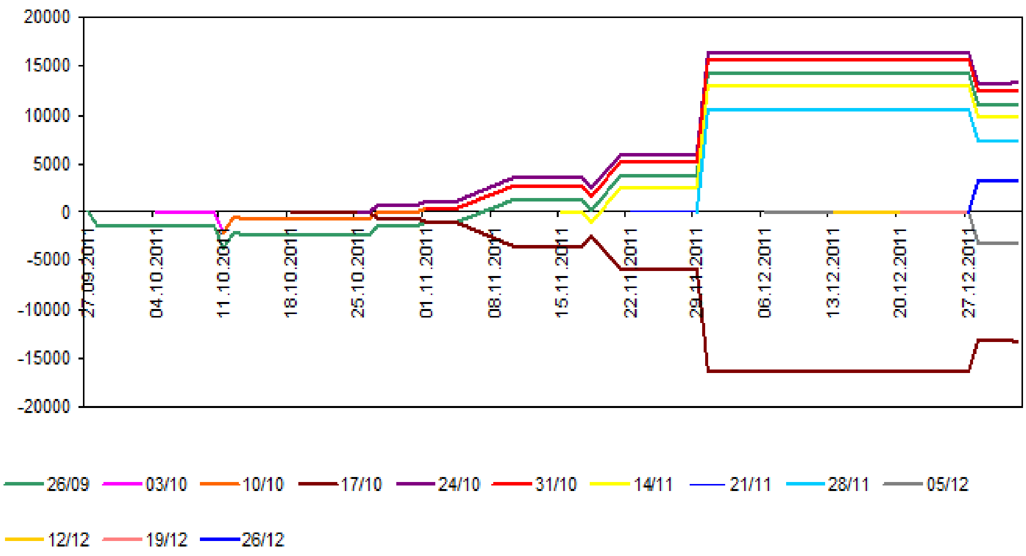


Figure 5.3: Suggested future positions' pay offs

CHAPTER 6

CONCLUSION AND OUTLOOK

In the past two decades, electricity industries in many countries which were initially designed as vertically integrated national or state dominated monopolies have been experiencing a deregulation process. The increase in the number of market players together with the competition and development of relatively more liberal electricity markets caused electricity to become a commodity whose price is determined by supply and demand. Day-ahead spot electricity markets, are the most transparent spot markets where one can find integrated supply and demand curves of the market players for each settlement period. We model spot electricity prices, since it is an indicator for the market players and regulators. Logarithmic daily average spot electricity prices are modeled as a summation of a deterministic function and multi-factor stochastic process. Randomness in the spot prices is assumed to be governed by pure jumps and mean reverting jump processes additional to a Brownian motion.

In order to estimate the model parameters, following Mancini's (2009) [54] approach, jump processes are seperated by using a parametric threshold function which is composed of a multiple of stochastic volatility estimate generated by GARCH(1,1) model. Although the idea of using a threshold function for the separation of jumps is not original, using a GARCH type threshold in electricity price modeling is uncommon. By including two mean reverting processes instead of one, we can separate price jumps being effective only for one day which are mostly due to hourly jumps in any given day from the jumps that affect the price level for more than a day. One of the main goals of this thesis is to propose spot and future contract price models which can be used in recently established liberal Turkish electricity spot market and electricity future contracts that are traded in the national derivatives exchange. However, since we do not prefer to be restrained by the small sample size, we test our model with relatively more mature markets' spot electricity data. For all of the examined countries, the separation results are found to be in accordance with our initial expectations. The occurrence of pure jumps and semi-spikes are less than price spikes as expected, and jumps are rare events as it is confirmed by their low jump intensity estimations. Burr distribution is also found to be good at capturing the distributional properties of the electricity price jumps besides the widely excepted jump distributions. Moreover, the week ahead forecast performance of the model shows that GARCH threshold multi-factor jump model can also be a useful alternative for the market practitioners.

In the derivatives front, although we summarize three main approaches used for electricity forward and future contracts modeling in the literature, we cannot propose a contract price model due to data shoratge. Instead of proposing a future contracts price model, we offer a decision technique where the given contract prices are used. With this technique which is built on the risk premium theory, derivative market players can decide whether to take a long or a short position. After testing our technique, we conclude that the decision rule is promising but needs more empirical research.

By taking this thesis as a starting point in electricity market modeling, further research can develop hourly spot electricity model and use this model in pricing future contract valuation by defining a new risk measure which can be applied to electricity portfolios. Since hourly day-ahead prices are more likely to be 24 different series than a single hourly price series, panal data techniques are assumed to be employed.

APPENDIX A

Preliminaries

A.1 Definitions

Definition A.1.1 *A cadlag, adapted, real valued stochastic process* $L = (L_t)_{0 \leq t \leq T}$ *with* $L_0 = 0$ *a.s. is called a Levy process if the following conditions are satisfied:*

1. *L has independent increments, i.e.,* $L_t - L_s$ *is independent of* F_s *for any* $0 \leq s < t \leq T$.

2. *L has stationary increments, i.e., for any* $0 \leq s < t \leq T$ *the distribution of* $L_{t+s} - L_t$ *does not depend on t.*

3. *L is stochastically continuous, i.e., for every* $0 \leq t \leq T$ *and* $\epsilon > 0$, $\lim_{s \to t} P(|L_t - L_s| > \epsilon) = 0$.

A Lévy process X_t which has a characteristic triplet (γ, A, ν) and therefore a characteristic function

$$E[e_{iuX_t}] = \exp\left(i\nu u - \frac{1}{2}u^2 A + \int_{-\infty}^{\infty} (e^{iux} - 1 - iuI_{|x|\leq 1})\nu(dx)\right) \tag{A.1}$$

is a finite activity jump process if $\nu(R) < \infty$, i.e., almost all paths of X_t have finite number of jumps on every compact interval.

A.2 Heath Jarrow Merton Framework for the Stochastic Modeling of Interest Rate Dynamics

Related concepts are listed below

- The short rate, $r(t)$, is the annualized interest rate at which an entity can borrow money for an infinitesimally short period of time from time .

- Instantaneous forward rate, $f(t,T)$ is the annualized interest rate, contracted at t, over the infinitesimal small interval $(T, T + \Delta t)$.

- A zero-coupon bond with maturity date T is a contract that guarantees 1 unit of payment on the date T. The price at time t of a zero-coupon bond with maturity T is denoted by $P(t,T)$.

Then

$$f(t,T) := -\frac{d \log P(t,T)}{dt}, \tag{A.2}$$

$$r(t) = f(t,t), \tag{A.3}$$

and

$$P(t,T) = \exp\left(-\int_t^T f(t,s)ds\right). \tag{A.4}$$

In the HJM framework, it is assumed that under the equivalent martingale measure Q

$$df(t,T) = \alpha(t,T)dt + \sigma(t,T)dW(t), \tag{A.5}$$

$$f(0,T) = f^*(0,T),$$

where W is a d-dimensional Brownian motion under Q, α represents the drift of the forward rates, σ represents the volatility of the forward rates and $f * (0,T)$ is the observed initial forward curve.

Since the forward rate dynamics are modeled directly under the martingale measure Q, prices are arbitrage free. Then market satisfies the following equations:

$$P(0,T) = \exp\left(-\int_0^T f(0,s)ds\right), \tag{A.6}$$

$$P(0,T) = E^Q\left[\exp\left(-\int_0^T r(s)ds\right)\right], \tag{A.7}$$

if the HJM drift condition holds.

HJM drift condition: Assume that the family of forward rates is given by Eq. (A.5) and that the induced bond market is arbitrage free. Then there exists a d-dimensional vector process $\lambda(t) = [\lambda_1(t), ..., \lambda_d(t)]$ with the property that for all $T \geq 0$ and for all $t \leq T$, we have

$$\alpha(t,T) = \sigma(t,T)\int_t^T \sigma'(t,s)ds - \sigma(t,T)\lambda(t), \tag{A.8}$$

where ’ denotes transpose.

Schematically, the use of the HJM model can now be written as follows:

1. Specification of the volatilities $\sigma(t,T)$
2. The drift parameters of the forward rates are then uniquely determined by Eq. (A.8).
3. Integrate the forward rate dynamics to get the forward rates as

$$f(t,T) = f^*(t,T) + \int_0^t \alpha(s,T)ds + \int_0^t \sigma(s,T)dW(s). \tag{A.9}$$

A.3 Risk Premium

Lets assume that $\theta(t)$ is a 4 dimesional vector of real valued constants; $\theta(t) = (\hat{\theta}, \tilde{\theta}^{(1)}, \tilde{\theta}^{(2)}, \tilde{\theta}^{(3)})$

$$\hat{Z}^{\theta}(t) = \exp\left(\int_0^t \hat{\theta}dB(u) - \frac{1}{2}\int_0^t \hat{\theta}^2 du\right) \tag{A.10}$$

and for i=1,2,3

$$\tilde{Z}^{\theta}(t) = \exp\left(\int_0^t \tilde{\theta}dJ^{(i)}(u) - \phi^{(i)}(t,s,-i\tilde{\theta})\right), \tag{A.11}$$

where $\phi^{(i)}$ is the corresponding log-moment generating function.

Lets define an equivalent probability measure Q^{θ} such that $Z^{\theta}(t)$ is the density process of the Radon-Nikodym derivative dQ^{θ}/dP. Then with respect to probability measure Q^{θ} the processes

$$B^{\theta}(t) = B(t) - \hat{\theta}t \tag{A.12}$$

are Brownian motions. And the characteristic function of $J^{(i)}$ for $i = 1,2,3$, is

$$E^{\theta}[e^{iuJ^{(i)}}|F_t] = \exp(iu\int_0^t\int_{|z|<1} z(e^{\tilde{\theta}z} - 1)l(du,dz) + \int_0^t\int_R (e^{iuz} - 1 - iuzI_{|z|<1})e^{\tilde{\theta}z}l(dz,du)) \tag{A.13}$$

Therefore, according to the definition of the forward risk premium,

$$R(t,\tau) := E^{Q^{\theta}}[S(\tau)|F_t] - E[S(\tau)|F_t], \tag{A.14}$$

$$R(t,\tau) = \Lambda(t)\ \ (\Theta(t,\tau,\theta) - \Theta(t,\tau,0))\ \ \exp(\frac{1}{2}\int_t^\tau \sigma^2(u)du)\ \ exp(\int_t^\tau \sigma(u)dB(u))$$

$$exp(\int_t^\tau dJ^{(1)}(u))\ \ \exp(e^{-\nu t}Y(t))\ \ \exp(e^{-\beta t}Z(t)), \tag{A.15}$$

where

$$ln\Theta(t,\tau,\theta) = \sum_{i=1}^{3}\phi^{(i)}(t,\tau,e^{-df(i)t}+\tilde{\theta}) - \phi^{(i)}(t,\tau,\tilde{\theta}) + \int_t^\tau \sigma(u)\hat{\theta}du, \tag{A.16}$$

$df(i)$ representing the corresponding discount factor, 0 for $i = 1$, ν for $i = 2$ and β for $i = 3$.

APPENDIX B

Matlab Codes

In this appendix, Matlab codes used for jump detection and separation of jump processes from Brownian motion observations and classification of jumps are given. These three procedures are original contributions of this research. Therefore leaving commonly used seasonality function fitting or Monte Carlo simulation procedures to the reader only these jump process detection and classification codes are given here.

First of all as it is alraedy stated deterministic seasonality function is fitted to logarithmic price series. Then residual series is separated to its stochastic components. This separation procedure starts with iterative filterin of mean reverting jump processes. Corresponding matlab codes used in this filterin is as follows.

```
function [Jumpsmr, Jumps, Filteredretf, Volf, GARCHP]=jumpfilter1(diffRes,cons)
%diffRes: first difference of deseasonalized logarithmic prices
%cons: mutiplier of estimated stochastic volatility, generally in the neighborhood of 3
Filteredret=zeros(length(diffRes),601);
Filteredret(:,1)=diffRes;
Vol=zeros(length(diffRes),600);
Retfiltered=zeros(length(diffRes),600);
for k=1:600
Jumps(:,k) Retfiltered(:,k) Vol(:,k) GARCHP(:,k)
=jump(Filteredret(:,k),cons);
Jumpsmr(:,1:k) Filteredret(:,k+1) Vol(:,k)
=jumpsmeanrev(Jumps(:,1:k),cons,diffRes,Vol(:,k),Retfiltered(:,k),GARCHP(:,k));
if Filteredret(:,k+1)-Filteredret(:,k)==zeros(length(Filteredret(:,k)),1);
Filteredretf=Filteredret(:,k+1);
```

Volf=Vol(:,k);

GARCHP=GARCHP(:,k);

break

end

end

%%

function [Jumps, Retfiltered, Vol,GARCHP]=jump(diffRes,cons)

Coeff
=garchfit(diffRes);

Retfiltered=diffRes;

Jumps=zeros(length(diffRes),1);

c=Coeff.K;

a=Coeff.ARCH;

b=Coeff.GARCH;

Vol=zeros(length(diffRes),1);

Vol(1)=c+a*(mean(diffRes))$^2 + b * (std(diffRes))^2$;

$fori = 2 : length(diffRes)$

$Vol(i) = c + a * (Retfiltered(i - 1))^2 + b * Vol(i - 1)$;

$if(diffRes(i))^2 >= cons^2 * Vol(i)$;

$Retfiltered(i) = 0$;

$Jumps(i) = diffRes(i)$;

break

end

end

Coeff
$= garchfit(Retfiltered)$;

$cf = Coeff.K$;

$af = Coeff.ARCH$;

$bf = Coeff.GARCH$;

$GARCHP = [cf, af, bf]'$;

%%

$function[Jumpsmr, Filteredretf, Vol] = jumpsmeanrev2(Jumpsf, cons, diffRes, Vol,$
$Filteredretf, GARCHP)$

```
x = size(Jumpsf);
row = x(1);
col = x(2);
cf = GARCHP(1);
af = GARCHP(2);
bf = GARCHP(3);
alljumps = sum(Jumpsf, 2);
Jumpsf1 = zeros(row, col);
rank1 = zeros(row, 1);
forj = 1 : col
fori = 1 : row
ifJumpsf(i, j)  = 0
rank1(i) = j;
break
end
end
end
nr = 0;
rank2 = zeros(row, 1);
fori = 1 : row;
ifalljumps(i)  = 0;
nr = nr + 1;
rank2(i) = nr;
end
end
fori = 1 : row
ifalljumps(i)  = 0;
Jumpsf1(:, rank2(i)) = Jumpsf(:, rank1(i));
end
end
Jumpsmr = zeros(row, col);
forj = 1 : col;
fori = 1 : row − 7;
```

$ifJumpsf1(i, j) \ = 0sum(Jumpsmr(i, :)) == 0;$

$Jumpsmr(i, j) = diffRes(i);$

$ifmin(-diffRes(i)-cons*sqrt(Vol(i+1)), -cons*sqrt(Vol(i+1))) < diffRes(i+1)diffRes(i+1) < max(-diffRes(i) + cons * sqrt(Vol(i + 1)), cons * sqrt(Vol(i + 1)));$

$Jumpsmr(i + 1, j) = diffRes(i + 1);$

$Filteredretf(i + 1) = 0;$

$Vol(i + 2) = cf + af * (Filteredretf(i + 1))^2 + bf * Vol(i + 1);$

$ifdiffRes(i) > 0sum(diffRes(i : i + 1)) < 0 + cons * sqrt(Vol(i + 2))$

$break$

$elseifdiffRes(i) < 0sum(diffRes(i : i + 1)) > 0 - cons * sqrt(Vol(i + 2))$

$break$

$elseif$

$min([((diffRes(i + 1) - cons * sqrt(Vol(i + 1)))/diffRes(i)) * (((diffRes(i + 1) - cons * sqrt(Vol(i + 1)))/diffRes(i)) + 1) * diffRes(i), ((diffRes(i + 1) + cons * sqrt(Vol(i + 1)))/diffRes(i))*(((diffRes(i+1)+cons*sqrt(Vol(i+1)))/diffRes(i))+1)*diffRes(i), ((diffRes(i + 1) - cons * sqrt(Vol(i + 1)))/diffRes(i)) * (((diffRes(i + 1) + cons * sqrt(Vol(i + 1)))/diffRes(i)) + 1) * diffRes(i), ((diffRes(i + 1) + cons * sqrt(Vol(i + 1)))/diffRes(i)) * (((diffRes(i+1)-cons*sqrt(Vol(i+1)))/diffRes(i))+1)*diffRes(i)]) - cons*sqrt(Vol(i+2)) < diffRes(i+2)diffRes(i+2) < max([((diffRes(i+1)-cons*sqrt(Vol(i+1)))/diffRes(i)) (((diffRes(i + 1) - cons * sqrt(Vol(i + 1)))/diffRes(i)) + 1) * diffRes(i), ((diffRes(i + 1) + cons*sqrt(Vol(i+1)))/diffRes(i))*(((diffRes(i+1)+cons*sqrt(Vol(i+1)))/diffRes(i))+1) * diffRes(i), ((diffRes(i + 1) - cons * sqrt(Vol(i + 1)))/diffRes(i)) * (((diffRes(i + 1) + cons * sqrt(Vol(i + 1)))/diffRes(i)) + 1) * diffRes(i), ((diffRes(i + 1) + cons * sqrt(Vol(i + 1)))/diffRes(i))*(((diffRes(i+1)-cons*sqrt(Vol(i+1)))/diffRes(i))+1)*diffRes(i)]) + cons * sqrt(Vol(i + 2));$

$Jumpsmr(i + 2, j) = diffRes(i + 2);$

$Filteredretf(i + 2) = 0;$

$Vol(i + 3) = cf + af * (Filteredretf(i + 2))^2 + bf * Vol(i + 2);$

$ifdiffRes(i) > 0sum(diffRes(i : i + 2)) < 0 + cons * sqrt(Vol(i + 3))$

$break$

$elseifdiffRes(i) < 0sum(diffRes(i : i + 2)) > 0 - cons * sqrt(Vol(i + 3))$

$break$

$elseif$

$min([((diffRes(i+1) - cons * sqrt(Vol(i+1)))/diffRes(i)) * (((diffRes(i+1) - cons *$
$sqrt(Vol(i+1)))/diffRes(i)) + 1)^2 * diffRes(i), ((diffRes(i+1) + cons * sqrt(Vol(i+1)))/$
$diffRes(i)) * (((diffRes(i+1) + cons * sqrt(Vol(i+1)))/diffRes(i)) + 1)^2 * diffRes(i), ((diffRes$
$(i+1) - cons * sqrt(Vol(i+1)))/diffRes(i)) * (((diffRes(i+1) + cons * sqrt(Vol(i+$
$1)))/diffRes(i)) + 1)^2 * diffRes(i), ((diffRes(i+1) + cons * sqrt(Vol(i+1)))/diffRes(i)) *$
$(((diffRes(i+1) - cons * sqrt(Vol(i+1)))/diffRes(i)) + 1)^2 * diffRes(i)]) - cons * sqrt(Vol(i+$
$3)) < diffRes(i+3)diffRes(i+3) < max([((diffRes(i+1) - cons * sqrt(Vol(i+1)))/diffRes(i))$
$(((diffRes(i+1) - cons * sqrt(Vol(i+1)))/diffRes(i)) + 1)^2 * diffRes(i), ((diffRes(i+1) +$
$cons * sqrt(Vol(i+1)))/diffRes(i)) * (((diffRes(i+1) + cons * sqrt(Vol(i+1)))/diffRes(i)) +$
$1)^2 * diffRes(i), ((diffRes(i+1) - cons * sqrt(Vol(i+1)))/diffRes(i)) * (((diffRes(i+1) +$
$cons * sqrt(Vol(i+1)))/diffRes(i)) + 1)^2 * diffRes(i), ((diffRes(i+1) + cons * sqrt(Vol(i+$
$1)))/diffRes(i)) * (((diffRes(i+1) - cons * sqrt(Vol(i+1)))/diffRes(i)) + 1)^2 * diffRes(i)]) +$
$cons * sqrt(Vol(i+3));$

$Jumpsmr(i+3, j) = diffRes(i+3);$

$Filteredretf(i+3) = 0;$

$Vol(i+4) = cf + af * (Filteredretf(i+3))^2 + bf * Vol(i+3);$

$ifdiffRes(i) > 0sum(diffRes(i : i+3)) < 0 + cons * sqrt(Vol(i+4))$

$break$

$elseifdiffRes(i) < 0sum(diffRes(i : i+3)) > 0 - cons * sqrt(Vol(i+4))$

$break$

$elseif$

$min([((diffRes(i+1) - cons * sqrt(Vol(i+1)))/diffRes(i)) * (((diffRes(i+1) - cons *$
$sqrt(Vol(i+1)))/diffRes(i)) + 1)^3 * diffRes(i), ((diffRes(i+1) + cons * sqrt(Vol(i+1)))/$
$diffRes(i)) * (((diffRes(i+1) + cons * sqrt(Vol(i+1)))/diffRes(i)) + 1)^3 * diffRes(i), ((diffRes$
$(i+1) - cons * sqrt(Vol(i+1)))/diffRes(i)) * (((diffRes(i+1) + cons * sqrt(Vol(i+$
$1)))/diffRes(i)) + 1)^3 * diffRes(i), ((diffRes(i+1) + cons * sqrt(Vol(i+1)))/diffRes(i)) *$
$(((diffRes(i+1) - cons * sqrt(Vol(i+1)))/diffRes(i)) + 1)^3 * diffRes(i)]) - cons * sqrt(Vol(i+$
$4)) < diffRes(i+4)...diffRes(i+4) < max([((diffRes(i+1) - cons * sqrt(Vol(i+1)))/diffRes(i))$
$(((diffRes(i+1) - cons * sqrt(Vol(i+1)))/diffRes(i)) + 1)^3 * diffRes(i), ((diffRes(i+1) +$
$cons * sqrt(Vol(i+1)))/diffRes(i)) * (((diffRes(i+1) + cons * sqrt(Vol(i+1)))/diffRes(i)) +$
$1)^3 * diffRes(i), ((diffRes(i+1) - cons * sqrt(Vol(i+1)))/diffRes(i)) * (((diffRes(i+1) +$
$cons * sqrt(Vol(i+1)))/diffRes(i)) + 1)^3 * diffRes(i), ((diffRes(i+1) + cons * sqrt(Vol(i+$
$1)))/diffRes(i)) * (((diffRes(i+1) - cons * sqrt(Vol(i+1)))/diffRes(i)) + 1)^3 * diffRes(i)]) +$

$cons * sqrt(Vol(i+4));$

$Jumpsmr(i+4, j) = diffRes(i+4);$

$Filteredretf(i+4) = 0;$

$Vol(i+5) = cf + af * (Filteredretf(i+4))^2 + bf * Vol(i+4);$

$ifdiffRes(i) > 0sum(diffRes(i : i+4)) < 0 + cons * sqrt(Vol(i+5))$

$break$

$elseifdiffRes(i) < 0sum(diffRes(i : i+4)) > 0 - cons * sqrt(Vol(i+5))$

$break$

$elseif$

$min([((diffRes(i+1) - cons * sqrt(Vol(i+1)))/diffRes(i)) * (((diffRes(i+1) - cons * sqrt(Vol(i+1)))/diffRes(i)) + 1)^4 * diffRes(i), ((diffRes(i+1) + cons * sqrt(Vol(i+1)))/ diffRes(i)) * (((diffRes(i+1) + cons * sqrt(Vol(i+1)))/diffRes(i)) + 1)^4 * diffRes(i), ((diffRes (i+1) - cons * sqrt(Vol(i+1)))/diffRes(i)) * (((diffRes(i+1) + cons * sqrt(Vol(i+1)))/diffRes(i)) + 1)^4 * diffRes(i), ((diffRes(i+1) + cons * sqrt(Vol(i+1)))/diffRes(i)) * (((diffRes(i+1) - cons * sqrt(Vol(i+1)))/diffRes(i)) + 1)^4 * diffRes(i)]) - cons * sqrt(Vol(i+5)) < diffRes(i+5) ... diffRes(i+5) < max([((diffRes(i+1) - cons * sqrt(Vol(i+1)))/diffRes(i)) (((diffRes(i+1) - cons * sqrt(Vol(i+1)))/diffRes(i)) + 1)^4 * diffRes(i), ((diffRes(i+1) + cons * sqrt(Vol(i+1)))/diffRes(i)) * (((diffRes(i+1) + cons * sqrt(Vol(i+1)))/diffRes(i)) + 1)^4 * diffRes(i), ((diffRes(i+1) - cons * sqrt(Vol(i+1)))/diffRes(i)) * (((diffRes(i+1) + cons * sqrt(Vol(i+1)))/diffRes(i)) + 1)^4 * diffRes(i), ((diffRes(i+1) + cons * sqrt(Vol(i+1)))/diffRes(i)) * (((diffRes(i+1) - cons * sqrt(Vol(i+1)))/diffRes(i)) + 1)^4 * diffRes(i)]) + cons * sqrt(Vol(i+5));$

$Jumpsmr(i+5, j) = diffRes(i+5);$

$Filteredretf(i+5) = 0;$

$Vol(i+6) = cf + af * (Filteredretf(i+5))^2 + bf * Vol(i+5);$

$ifdiffRes(i) > 0sum(diffRes(i : i+5)) < 0 + cons * sqrt(Vol(i+6))$

$break$

$elseifdiffRes(i) < 0sum(diffRes(i : i+5)) > 0 - cons * sqrt(Vol(i+6))$

$break$

$elseif$

$min([((diffRes(i+1) - cons * sqrt(Vol(i+1)))/diffRes(i)) * (((diffRes(i+1) - cons * sqrt(Vol(i+1)))/diffRes(i)) + 1)^5 * diffRes(i), ((diffRes(i+1) + cons * sqrt(Vol(i+1)))/ diffRes(i)) * (((diffRes(i+1) + cons * sqrt(Vol(i+1)))/diffRes(i)) + 1)^5 * diffRes(i), ((diffRes$

$(i+1) - cons * sqrt(Vol(i+1)))/diffRes(i)) * (((diffRes(i+1) + cons * sqrt(Vol(i+1)))/diffRes(i)) + 1)^5 * diffRes(i), ((diffRes(i+1) + cons * sqrt(Vol(i+1)))/diffRes(i)) * (((diffRes(i+1) - cons * sqrt(Vol(i+1)))/diffRes(i)) + 1)^5 * diffRes(i)]) - cons * sqrt(Vol(i+6)) < diffRes(i+6)...diffRes(i+6) < max([((diffRes(i+1) - cons * sqrt(Vol(i+1)))/diffRes(i)) (((diffRes(i+1) - cons * sqrt(Vol(i+1)))/diffRes(i)) + 1)^5 * diffRes(i), ((diffRes(i+1) + cons * sqrt(Vol(i+1)))/diffRes(i)) * (((diffRes(i+1) + cons * sqrt(Vol(i+1)))/diffRes(i)) + 1)^5 * diffRes(i), ((diffRes(i+1) - cons * sqrt(Vol(i+1)))/diffRes(i)) * (((diffRes(i+1) + cons * sqrt(Vol(i+1)))/diffRes(i)) + 1)^5 * diffRes(i), ((diffRes(i+1) + cons * sqrt(Vol(i+1)))/diffRes(i)) * (((diffRes(i+1) - cons * sqrt(Vol(i+1)))/diffRes(i)) + 1)^5 * diffRes(i)]) + cons * sqrt(Vol(i+6));$

$Jumpsmr(i+6, j) = diffRes(i+6);$

$Filteredretf(i+6) = 0;$

$Vol(i+7) = cf + af * (Filteredretf(i+6))^2 + bf * Vol(i+6);$

break

else

break

end

else

break

end

else

break

end

else

break

end

else

break

end

else

break

end

end

end

$fori = 2 : row - 7$

$ifJumpsmr(i, j) \;\; = 0Jumpsmr(i - 1, j) == 0;$

$ifabs(diffRes(i)) < abs(sum(Jumpsmr(i : i + 6, j)))$

$Jumpsmr(i + 6, j) = 0;$

$Filteredretf(i + 6) = diffRes(i + 6);$

$ifabs(diffRes(i)) < abs(sum(Jumpsmr(i : i + 5, j)))$

$Jumpsmr(i + 5, j) = 0;$

$Filteredretf(i + 5) = diffRes(i + 5);$

$ifabs(diffRes(i)) < abs(sum(Jumpsmr(i : i + 4, j)))$

$Jumpsmr(i + 4, j) = 0;$

$Filteredretf(i + 4) = diffRes(i + 4);$

$ifabs(diffRes(i)) < abs(sum(Jumpsmr(i : i + 3, j)))$

$Jumpsmr(i + 3, j) = 0;$

$Filteredretf(i + 3) = diffRes(i + 3);$

$ifabs(diffRes(i)) < abs(sum(Jumpsmr(i : i + 2, j)))$

$Jumpsmr(i + 2, j) = 0;$

$Filteredretf(i + 2) = diffRes(i + 2);$

$ifabs(diffRes(i)) < abs(sum(Jumpsmr(i : i + 1, j)))$

$Jumpsmr(i + 1, j) = 0;$

$Filteredretf(i + 1) = diffRes(i + 1);$

end

end

end

end

end

end

end

end

$fori = 2 : row - 7$

$ifJumpsmr(i, j) \;\; = 0Jumpsmr(i - 1, j) == 0;$

$if0.06 * abs(diffRes(i)) > abs(sum(Jumpsmr(i + 1 : i + 6, j)))$

$Jumpsmr(i + 1, j) = 0;$

$Filteredretf(i+1) = diffRes(i+1);$

$Jumpsmr(i+2, j) = 0;$

$Filteredretf(i+2) = diffRes(i+2);$

$Jumpsmr(i+3, j) = 0;$

$Filteredretf(i+3) = diffRes(i+3);$

$Jumpsmr(i+4, j) = 0;$

$Filteredretf(i+4) = diffRes(i+4);$

$Jumpsmr(i+5, j) = 0;$

$Filteredretf(i+5) = diffRes(i+5);$

$Jumpsmr(i+6, j) = 0;$

$Filteredretf(i+6) = diffRes(i+6);$

end

end

end

$Vol(i+2) = cf + af * (Filteredretf(i+1))^2 + bf * Vol(i+1);$

$Vol(i+3) = cf + af * (Filteredretf(i+2))^2 + bf * Vol(i+2);$

$Vol(i+4) = cf + af * (Filteredretf(i+3))^2 + bf * Vol(i+3);$

$Vol(i+5) = cf + af * (Filteredretf(i+4))^2 + bf * Vol(i+4);$

$Vol(i+6) = cf + af * (Filteredretf(i+5))^2 + bf * Vol(i+5);$

$Vol(i+7) = cf + af * (Filteredretf(i+6))^2 + bf * Vol(i+6);$

end

$Filteredretf = diffRes - sum(Jumpsmr, 2);$

%%

$Classification \quad of \quad the \quad detected \quad jump \quad points.$

function [pj,sp,ss, noj, sp0, ss0]=jumpclass(diffRes, Jumpsmr, Vol,cons)

%pj: vector of pure jump points

%sp: vector of detected spike processes

%ss: vector of detected semi spike processes

%noj: number of jumps

%sp0: only the spike jump points

%ss0: only the semi-spike jump points

x=size(Jumpsmr);

```
row=x(1);
col=x(2);
pj=zeros(row,1);
sp=zeros(row,1);
ss=zeros(row,1);
sp0=zeros(row,1);
ss0=zeros(row,1);
noj=zeros(1,5);
for j=1:col
for i=2:row
if Jumpsmr(i,j) =0  Jumpsmr(i+1,j)==0  Jumpsmr(i-1,j)==0;
pj(i)=diffRes(i);
break
elseif Jumpsmr(i,j) =0  Jumpsmr(i+1,j) =0  sign(Jumpsmr(i,j))==-sign(Jumpsmr(i+1,j))  sign(Jumpsmr(i-
1,j))==0;
if abs(Jumpsmr(i,j))-cons*sqrt(Vol(i+1))¡=abs(Jumpsmr(i+1,j));
if Jumpsmr(i+2,j)==0
sp(i)=diffRes(i);
sp0(i)=diffRes(i);
sp(i+1)=diffRes(i+1);
break
elseif sign(Jumpsmr(i,j))==-sign(Jumpsmr(i+2,j))  Jumpsmr(i+2,j) =0  abs(sum(Jumpsmr(i:i+1,j)))-
cons*sqrt(Vol(i+2))¡=abs(Jumpsmr(i+2,j));
if Jumpsmr(i+3,j)==0
sp(i)=diffRes(i);
sp0(i)=diffRes(i);
sp(i+1)=diffRes(i+1);
sp(i+2)=diffRes(i+2);
break
elseif sign(Jumpsmr(i,j))==-sign(Jumpsmr(i+3,j))  Jumpsmr(i+3,j) =0  abs(sum(Jumpsmr(i:i+2,j)))-
cons*sqrt(Vol(i+3))¡=abs(Jumpsmr(i+3,j));
if Jumpsmr(i+4,j)==0
sp(i)=diffRes(i);
```

```
sp0(i)=diffRes(i);
sp(i+1)=diffRes(i+1);
sp(i+2)=diffRes(i+2);
sp(i+3)=diffRes(i+3);
break
elseif sign(Jumpsmr(i,j))==-sign(Jumpsmr(i+4,j))  Jumpsmr(i+4,j) =0
abs(sum(Jumpsmr(i:i+3,j)))-cons*sqrt(Vol(i+4))¡=abs(Jumpsmr(i+4,j));
if Jumpsmr(i+5,j)==0
sp(i)=diffRes(i);
sp0(i)=diffRes(i);
sp(i+1)=diffRes(i+1);
sp(i+2)=diffRes(i+2);
sp(i+3)=diffRes(i+3);
sp(i+4)=diffRes(i+4);
break
elseif sign(Jumpsmr(i,j))==-sign(Jumpsmr(i+5,j))  Jumpsmr(i+5,j) =0  abs(sum(Jumpsmr(i:i+4,j)))-
cons*sqrt(Vol(i+5))¡=abs(Jumpsmr(i+5,j));
if Jumpsmr(i+6,j)==0
sp(i)=diffRes(i);
sp0(i)=diffRes(i);
sp(i+1)=diffRes(i+1);
sp(i+2)=diffRes(i+2);
sp(i+3)=diffRes(i+3);
sp(i+4)=diffRes(i+4);
sp(i+5)=diffRes(i+5);
break
elseif sign(Jumpsmr(i,j))==-sign(Jumpsmr(i+6,j))  Jumpsmr(i+6,j) =0  abs(sum(Jumpsmr(i:i+5,j)))-
cons*sqrt(Vol(i+6))¡=abs(Jumpsmr(i+6,j));
sp(i)=diffRes(i);
sp0(i)=diffRes(i);
sp(i+1)=diffRes(i+1);
sp(i+2)=diffRes(i+2);
sp(i+3)=diffRes(i+3);
```

```
sp(i+4)=diffRes(i+4);
sp(i+5)=diffRes(i+5);
sp(i+6)=diffRes(i+6);
break
end
end
end
end
end
end
end
end
end
ss=sum(Jumpsmr,2)-pj-sp;
for i=1:row
if ss(i) =0  ss(i-1)==0
ss0(i)=diffRes(i);
end
end
for i=1:row
if pj(i) =0
noj(1)=noj(1)+1;
end
end
for i=1:row
if sp0(i) =0
noj(2)=noj(2)+1;
end
end
for i=1:row
if ss0(i) =0
noj(3)=noj(3)+1;
end
```

```
end
for i=1:row
if sp(i) =0
noj(4)=noj(4)+1;
end
end
for i=1:row
if ss(i) =0
noj(5)=noj(5)+1;
end
end
```

REFERENCES

[1] Ane, T., Ureche-Rangau, L., Gambet, J.B., Bouverot, J., 2008. Robust outlier detection for Asia- Pacific stock index returns. Journal of International Financial Institutions Money 18, 326-343.

[2] Ane, T., Métais, C., 2010. Jump distribution characteristics: evidence from European stock markets. International Journal of Business and Economics 9, 1-22.

[3] Audet, N., Heiskanen, P., Keppo, J., Vehvilainen, I.,2004. Modeling electricity forward price dynamics in Nordic market. Modeling Prices in Competitive Electricity Markets, chapter 12, Wiley.

[4] Barlow, M.T., 2002. A diffusion model for electricity prices. Mathematical Finance 12, 287-298.

[5] Bagdadioglu, N., Odyakmaz, N., 2008. Turkish electricity reform. Utilities Policy xxx, 1-9. doi:10.1016/j.jup.2008.02.001.

[6] Benth, F.E., Cartea, A., Kiesel, R., 2006. Pricing forward contracts in power markets by the certainty equivalence principle: explaining the sign of the market risk premium. http://ssrn.com/abstract=941117.

[7] Benth, F.E., Saltyté-Benth, J., 2005. Stochastic temperature modeling and weather derivatives. Applied Mathematical Finance 12, 53-85.

[8] Benth, F.E., Kallsen, J., Meyer-Brandis, T., 2007a. A non-Gaussian Ornstein-Uhlenbeck process for electricity spot price modeling and derivatives pricing. Applied Mathematical Finance 14(2), 153-169.

[9] Benth, F.E., Koekebakker, S., Ollmar, F., 2007b. Extracting and applying smooth forward curves from average-based commodity contracts with seasonal variation. Journal of Derivatives, Fall 15(1), 52-66.

[10] Benth, F.E., Saltyté-Benth, J., Koekebakker, S., 2008a. Stochastic Modeling of Electricity and Related Markets. World Scientific, Singapore.

[11] Benth, F.E., Koekebakker, S., 2008b. Stochastic modeling of financial electricity contracts. Energy Economics 30, 1116-1157.

[12] Benth, F.E., Sgarra, C., 2009a. The risk premium and Esscher transform in power markets. Working paper, http://ssrn.com/abstract=1507627.

[13] Benth, F.E., Saltyté-Benth, J., 2009b. Dynamic pricing of wind futures. Energy Economics 31, 16-24.

[14] Bjerksund, P., Rasmussen, H., Stensland, G., 2000. Valuation and risk management in the Nordic electricity market. Working paper, Institute of Finance and Management Science, Norwegian School of Economics and Business Administration.

[15] Bessembinder, H., Lemmon, M.L., 2002. Equilibrium pricing and optimal hedging in electricity forward markets. Journal of Finance, 57(3), 13471382.

[16] Borovkova, S., Permana, F., 2004. Modeling electricity prices by the potential jump diffusion. Proceedings of the Stochastic Finance 2004 Conference, Lisbon, Portugal.

[17] Borovkova, S., Geman, H., 2006. Analysis and modeling of electricity futures prices. Studies in Nonlinear Dynamics Econometrics, 10 (3), article 6.

[18] Botterud, A., Bhattacharyya, A.K., Ilic M.D., 2002. Futures and spot prices- an analysis of the Scandinavian electricity market. Proceedings of the 34th Annual North American Power Symposium.

[19] Brennan, M. J., Schwartz, E. S., 1985. Evaluating natural resource investments. Journal of Business, 58(2), 135-157.

[20] Brockwell, P.J., 2001. Lévy-driven CARMA processes. Ann. Inst. Statist. Math. 53(1), 113-124.

[21] Brockwell, P.J., Davis, R.A., 2007. Continuous-time Gaussian autoregression. Statistica Sinica 17, 63-80.

[22] Brockwell, P.J., Davis, R.A., Yang, Y., 2007. Estimation for non-negative Lévy-driven Ornstein-Uhlenbeck processes. Journal of Applied Probability 44, 977-989.

[23] Bunn, D.W., Karakatsani, N., 2003. Forecasting electricity prices. London Business School, London.

[24] Burger, M., Klar, B., Müller, A., Schindlmayr, G., 2004. A spot market model for pricing derivatives in electricity markets. Quantitative Finance 4, 109-122.

[25] Camadan, E., Erten, I.E., 2010. An evaluation of the transitional Turkish electricity balancing and settlement market: lessons for the future. Renewable and Sustainable Energy Reviews 15, 1325-1334.

[26] Cartea, A., Figueroa, M.G., 2005. Pricing in electricity markets: a mean reverting jump diffusion model with seasonality. Applied Mathematical Finance 12, 313-335.

[27] Clewlow, L., Strickland, C., 2000. Energy Derivatives: Pricing and Risk Management. Lacima Publications.

[28] Cont, R., Mancini, C., 2007. Nonparametric tests for analyzing the fine structure of price fluctuations. Financial Engineering Report No 2007-13, Center for Financial Engineering, Columbia University.

[29] Cont, R., Tankov, P., 2004. Financial Modeling with Jump Processes. Chapman Hall/CRC, Florida.

[30] Contreras, J., Espinola, R., Nogales, F.J., Conejo, A.J., 2003. ARIMA models to predict next-day electricity prices. Power Systems, IEEE Transactions on, 18(3), 1014-1020.

[31] Corsi, F., Pirino, D., Reno, R., 2009. Threshold bipower variation and the impact of jumps on volatility forecasting. Laboratory of Economics and Management (LEM) Papers Series 2010/11, Sant'Anna School of Advanced Studies, Pisa.

[32] Dambis, K.E., 1965. On the decomposition of continuous martingales. Theory of Probability and Its Application 10, 410-410.

[33] Deng, S., 2000. Stochastic Models of Energy Commodity Prices and Their Applications: Mean-reversion with Jumps and Spikes. University of California Energy Institute.

[34] Doornik, J.A., Ooms, M., 2005. Outlier Detection in GARCH Models. Tinbergen Institute Discussion Paper, TI 2005-092/4.

[35] Douglas, S., Popova, J., 2008. Storage and the electricity forward premium. Energy Economics, 30(4), 17121727.

[36] Dubins, L., Schwarz, G., 1965. On continuous martingales. Proceedings of the National Academy of Sciences USA 53,913-916.

[37] Duffie, D., Pan, J., Singleton, K., 2000. Transform analysis and asset pricing for affine jump-diffusions. Econometrica, 68(6), 13431376.

[38] Erlwein, C., Benth, F.E., Mamon, R., 2010. HMM filtering and parameter estimation of an electricity spot price model. Energy Economics 32, 1034-1043.

[39] Escribano, A., Pena, J.I., Villaplana, P., 2002. Modeling electricity prices: International evidence. Working paper, Universidad Carlos III de Madrid.

[40] Frikha, N., Lemaire, V., 2009. Joint Modeling of Gas and Electricity Prices. hal-00421289.

[41] Garcia, R.C., Contreras, J., van Akkeren, M., Garcia, J.B.C., 2005. A GARCH forecasting model to predict day-ahead electricity prices. IEEE Transaction on Power Systems, 20(2), 867-874.

[42] Geman, H., 2005. Commodities and Commodity Derivatives. Wiley-Finance, John Wiley and Sons, Chichester.

[43] Geman, H., Roncoroni, A., 2006. Understanding the fine structure of electricity prices. Journal of Business 79(3), 1225-1261.

[44] Gibson, R., Schwartz, E.S., 1990. Stochastic convenience yield and the pricing of oil contingent claims. The Journal of Finance, 45(3), 959-976.

[45] Glasserman, P., 2002. Monte Carlo Methods in Financial Engineering. Springer, New York.

[46] GME official web site, http://www.mercatoelettrico.org/En/Default.aspx.

[47] Hamada, M., 2004. Fair pricing of energy derivatives: A comparative study. Proceeding: World Energy Congress, Sdney, Australia.

[48] Hambly, B., Howison S., Kluge, T., 2009. Modeling spikes and pricing swing options in electricity markets. Quantitative Finance 9, 937949.

[49] Kara cor, Z., Güvenek, B., 2010. Enerji Piyasası Reformlarının Elektrik Enerjisi Piyasasına Etkisi. Yönetim ve Ekonomi 17(1), 147-166. Y

[50] Knittel, C.R., Roberts, M.R., 2005. An emprical examination of restructured electricity prices. Energy Economics, 27(5), 791-817.

[51] Koekebakker, S., Ollmar, F., 2005. Forward curve dynamics in the Nordic electricity market. Managerial Finance 31(6), 74-95.

[52] Kluge, T. Pricing swing options and other electricity derivatives. PhD thesis, University of Oxford; 2006.

[53] Lucia, J.J., Schwartz, E.S., 2002. Electricity prices and power derivatives: Evidence from the Nordic Power Exchange. Review of Derivatives Research 5(1), 5-50.

[54] Mancini, C., 2009. Non-parametric threshold estimation for models with stochastic diffusion coefficient and jumps. Scandinavian Journal of Statistics 36, 270-296.

[55] Mancini, C., Reno, R., 2010. Threshold estimation of Markov models with jumps and interest rate modeling. Journal of Econometrics, doi:10.1016/j.jeconom.2010.03.019.

[56] Mayer, K., Schmid, T., Weber, F., 2011. Modeling electricity spot prices- Combining mean reversion, spikes and stochastic volatility. Working paper, Technische Universitat München.

[57] Meyer-Brandis, T., Tankov, P., 2007. Multi factor jump-diffusion models of electricity prices. Working paper, Université Paris VII, France.

[58] Misiorek, A., Trueck, S., Weron, R., 2006. Point and interval forecasting of spot electricity prices: Linear vs. non-linear time series models. Studies in Nonlinear Dynamics Econometrics 10(3), article 2.

[59] Monroe, I., 1978. Processes that can be imbedded in Brownian motion. Annals of Probability, 6(1), 42-56.

[60] Morters, P., Peres, Y., 2010. Brownian Motion. Cambridge University Press, Cambridge.

[61] Nelson, D.B., 1990. ARCH models as diffusion approximations. Journal of Econometrics 45, 7-39.

[62] Nelson, D.B., 1992. Filtering and forecasting with misspecified ARCH models I. Journal of Econometrics 52, 61-90.

[63] Nelson, D.B., Foster, D.P., 1994. Asymptotic filtering theory for univariate Arch models. Econometrica 62(1), 1-41.

[64] Pilipovic, D., 1998. Energy Risk: Valuing and Managing Energy Derivatives. Mc Graw-Hill, New York.

[65] Pirino, D., Reno, R., 2010. Electricity prices: A nonparametric approach. International Journal of Theoretical and Applied Finance 13(2), 285-299.

[66] Protter, P.E., 2004. Stochastic Integration and Differential Equations. Springer-Verlag Berlin, Heidelberg.

[67] Ralph, D., 2006. Complementarity models and the architecture of energy markets. International Symposium on Mathematical Programming (ISMP), 19th, 30 July - 4 Aug 2006, Rio de Janeiro, Brazil.

[68] Reno, R., 2006. Nonparametric estimation of stochastic volatility models. Economic Letters 90(3), 390-395.

[69] Revuz, D., Yor, M. 1991. Continuous Martingales and Brownian Motion. Springer-Verlag Berlin, Heidelberg.

[70] Schwartz, E.S., 1997. The stochastic behaviour of commodity prices: Implications for valuation and hedging. Journal of Finance LII(3), 923-973.

[71] Stevenson, M., 2001. Filtering and forecasting spot electricity prices in the increasingly deregulated Australian electricity market. QFRC Research Paper 63, University of Technology, Sydney.

[72] Takeuchi, K., 1995. A comment on Recent Development of Economic Data Analysisat the 63rd Annual Meeting of Japan Statistical Society.

[73] Tashpulatov, S.N., 2011. Estimating the volatility of electricity prices: The case of the England and Wales wholesale electricity market. Working paper CERGE-EI No 439.

[74] Vazquez, M., Barquín, J., Batlle, C., 2006. Electricity forward and volatility curves computation based on Monte Carlo simulation. Proceeding: 9th International Conference on Probabilistic Methods Applied to Power Systems KTH, Stockholm, Sweden

[75] Verhoeven, P., McAleer, M., 2000. Modelling outliers and extreme observations for ARMA-GARCH processes. Econometric Society World Congress 2000 Contributed Papers, no:1922.

[76] Villaplana, P., 2004. A two-state variables model for electricity prices. Presentation: Third World Congress of the Bachelier Finance Society, Chicago.

[77] Weron, R., 2005. Heavy tails and electricity prices. The Deutsche Bundesbank's 2005 Annual Fall Conference, Eltville.

[78] Weron, R., 2006. Modeling and Forecasting Electricity Loads and Prices: A Statistical Approach. Wiley, Chichester.

[79] Wilkens, S., Wimschulte, J., 2007. The pricing of electricity futures: evidence from the European energy exchange. Journal of Futures Markets, 27(4), 387410.

[80] Zhou, M., Yan, Z., Ni, Y., Li, 2004. An ARIMA aproach to forecasting electricity price with accuracy improvement by predicted errors. Proceedings of the IEEE Power Engineering Society General Meeting, 233-238.

VITA

PERSONAL INFORMATION

Surname, Name:	Talaslı, İrem
Nationality:	Turkish (TC)
Date and Place of Birth:	15 August 1981, Ankara
email:	irem.talasli@gmail.com

EDUCATION

Ph.D. Department of Financial Mathematics, 2012, January
Institute of Applied Mathematics
Middle East Technical University, Ankara
Supervisor: Assoc. Prof. Dr. Azize Hayfavi
Thesis Title: Stochastic Modeling of Electricity Markets

M.Sc. Department of Financial Mathematics, 2005, July
Institute of Applied Mathematics
Middle East Technical University, Ankara
Supervisor: Prof. Dr. Hayri Körezlioğlu
Thesis Title: Coherent Risk Measures

B.Sc. Department of Economics, 2003, June
Middle East Technical University, Ankara

WORK EXPERIENCE TRAINING

July 2006-Present Central Bank of the Republic of Turkey, Markets Division, Ankara
Feb - April 2006 PricewaterhouseCoopers, Financial Risk Dept, İstanbul - Intern
Aug 2004 İstanbul Stock Exchange, Research Dept., İstanbul - Intern