IMPACT OF CAPACITY LEVEL ON REINSURANCE AND CAT BOND MARKETS

A THESIS SUBMITTED TO
THE GRADUATE SCHOOL OF APPLIED MATHEMATICS
OF
MIDDLE EAST TECHNICAL UNIVERSITY

BY

T. TOYGAR KERMAN

IN PARTIAL FULFILLMENT OF THE REQUIREMENTS
FOR
THE DEGREE OF MASTER OF SCIENCE
IN
ACTUARIAL SCIENCES

AUGUST 2012

Approval of the thesis:

IMPACT OF CAPACITY LEVEL ON REINSURANCE AND CAT BOND MARKETS

submitted by **T. TOYGAR KERMAN** in partial fulfillment of the requirements for the degree of **Master of Science in Department of Actuarial Sciences, Middle East Technical University** by,

Prof. Dr. Bülent Karasözen ______________
Director, Graduate School of **Applied Mathematics**

Assoc. Prof. Dr. Sevtap Selçuk-Kestel ______________
Head of Department, **Actuarial Sciences**

Assoc. Prof. Dr. Sevtap Selçuk-Kestel ______________
Supervisor, **Actuarial Sciences**

Selamet Yazıcı (M.B.A), Actuary ______________
Co-supervisor, **Undersecretariat of Treasure Insurance Directorate General**

Examining Committee Members:

Prof. Dr. Gerhard Wilhelm Weber ______________
Institute of Applied Mathematics, METU

Prof. Dr. Ahmet Yakut ______________
Department of Civil Engineering, METU

Assist. Prof. Dr. Burçak Başbuğ ______________
Department of Statistics, METU

Assoc. Prof. Dr. Sevtap Selçuk-Kestel ______________
Institute of Applied Mathematics, METU

Selamet Yazıcı (M.B.A), Actuary ______________
Undersecretariat of Treasury Insurance Directorate General

Date: ______________

I hereby declare that all information in this document has been obtained and presented in accordance with academic rules and ethical conduct. I also declare that, as required by these rules and conduct, I have fully cited and referenced all material and results that are not original to this work.

Name, Last Name: T. TOYGAR KERMAN

Signature :

ABSTRACT

IMPACT OF CAPACITY LEVEL ON REINSURANCE AND CAT BOND MARKETS

Kerman, T. Toygar
M.S., Department of Actuarial Sciences
Supervisor : Assoc. Prof. Dr. Sevtap Selçuk-Kestel
Co-Supervisor : Selamet Yazıcı (M.B.A), Actuary

August 2012, 83 pages

Reinsurance is one of the most important tools to be used by insurance companies, for managing risks. This is an effective way; however, there are situations where reinsurance is insufficient, such as the occurrence of a natural hazard. When a natural hazard occurs, many insured experience loss at the same time, which drains the reinsurance market capacity. If future market capacity could be forecasted, then it would be easier for companies to decide when to include cat bonds or any other additional securities in their portfolio. In order to establish a model for market capacity, its relationship with other market parameters and the association among parameters are examined. In this study, these relationships are analyzed and used to establish an algorithm for predicting the next years reinsurance capacity. Moreover, last 10-year data for market capacity is used to establish an AR(1) model, in order to create a comparison with the algorithm. A case study of cat bonds is done, which uses the pricing load calculation of the Lane model and aims to ease the decision-making process by comparing the loads of cat bond and reinsurance pricing.

Keywords: Cat bonds, excess of loss reinsurance, natural hazards, reinsurance, reinsurance

capacity

ÖZ

KAPASITE SEVIYESININ REASÜRANS VE KATASTROF SENETI PIYASALARINA ETKISI

Kerman, T. Toygar
Yüksek Lisans, Aktüerya Bölümü
Tez Yöneticisi : Doç. Dr. Sevtap Selçuk-Kestel
Ortak Tez Yöneticisi : Selamet Yazıcı (M.B.A), Aktüer

August 2012, 83 sayfa

Sigorta şirketleri için risk yönetiminin en önemli unsurlarından biri reasüranstır. Reasürans riskin bir kısmını ya da hepsini sigorta şirketinden reasüröre devreder. Bu risk yönetimi için efektif bir yöntemdir; ancak reasüransın da hasar ödemesi yaparken yetersiz kaldığı durumlar vardır. Bu durumlardan biri doğal bir afetin meydana gelmesidir. Herhangi bir doğal afet meydana geldiğinde, birçok sigortalı aynı anda hasara maruz kalır ve bu durum market kapasitesinde azalmaya yol açar. Gelecek yılların market kapasitesi yaklaşık olarak tahmin edilebilirse, şirketlerin hangi zamanda katastrof senetlerini portföylerine katmaları gerektiğine karar vermeleri kolaylaşır. Kapasiteyi tahmin eden bir model kurabilmek için, kapasitenin kendisini etkileyen diğer piyasa değişkenlerinin kendi aralarıdaki ve kapasiteyle olan ilişkileri incelendi. Bu çalışmada, hangi zamanda katastrof seneti satın almanın mantıklı olduğunu belirlemek için değişkenler arasındaki bu ilişkiler incelenerek ve bunlardan yararlanılarak reasürans kapasitesini tahmin edebilmek için bir algoritma kuruldu. Ayrıca, algoritmayla bir kıyas oluşturabilmek için, son 10 yılın kapasite verisi kullanılarak bir AR(1) modellemesi yapıldı. Katastrof senetleri için yapılan çalışma, Lane modelindeki yük hesaplarını kullanarak katastrof senetleri için bir yük elde ederek reasürans ile kıyaslamayı hedeflemek-

tedir.

Anahtar Kelimeler: Afet tahvili, doğal afet, hasar fazlası reasüransı, reasürans, reasürör kapasitesi

To My Parents

ACKNOWLEDGMENTS

The author wishes to express his deepest gratitude to his supervisor Assoc. Prof. Dr. Sevtap Selçuk-Kestel and co-supervisor Selamet Yazıcı for their guidance, advice, criticism, encouragements and insight throughout the research.

The author would also like to thank Eureko Insurance Non-Motor Underwriting, Non Motor Claims and Reinsurance group director Süha Çele for his suggestions and comments.

The technical support of research assistant Ms. Büşra Zeynep Temoçin is gratefully acknowledged.

TABLE OF CONTENTS

LIST OF TABLES

TABLES

LIST OF FIGURES

FIGURES

CHAPTER 1

Introduction

For insurance companies, one of the most essential parts of risk management is reinsurance, which transfers some part of the risk to the reinsurer. Reinsurance, is basically insurance for insurance companies. The company who is passing the risk is called the ceding company. The purpose is to reduce the exposure to loss by passing it to a reinsurer or a group of reinsurers. This is a very effective way of managing risks; however there are situations such as natural hazards in which reinsurance is insufficient for covering all the losses. It is very difficult to deal with natural hazards because of the low frequency, high severity of catastrophic events. Between January 1989 and October 1998, the U.S. property/casualty industry incurred an inflation adjusted $98 billion in catastrophic losses, more than double the catastrophic losses experienced during the previous thirty-nine years [1].

When a natural hazard takes place, many insureds experience loss at the same time. In this case, the capital of the reinsurers may not be sufficient to cover all these losses and the insureds may not be able to collect their compensation and the risk of bankruptcy is born for reinsurance companies.

Because of the lack of capacity in the catastrophe reinsurance market, the catastrophe bond (cat bond) was introduced in the mid 90's. Though cat bonds can theoretically be issued on many hazards, they are in practice centered on natural catastrophes. Cat bonds serve an extremely useful role in their overall approach to manage catastrophe risk exposures. They allow a reinsurance company to transfer a portion of its natural catastrophe exposures to the capital markets rather than retaining the exposure on its books of business of retroceding the risk to other reinsurers. Besides these advantages, cat bonds have disadvantages such as the significantly high costs, especially when compared with the costs of buying traditional reinsurance [2].

Despite the high costs of cat bonds, they are incredibly useful in managing catastrophic risks. But not every insurance/reinsurance company is economically large enough to use cat bonds efficiently. A company must be financially powerful to take the risks of a cat bond. However, there can be situations that cat bonds are preferable to traditional insurance. Because of the low frequency of catastrophic events, we can not be sure when it will occur and we can not know the magnitude of the loss. Because of this obscurity, the reinsurance market needs other types of security other than traditional reinsurance. When too many catastrophes occur or when simply the reinsurance market goes to recession, the capacity that the reinsurance market provides diminishes. When capacity decreases, naturally the traditional reinsurance prices leap up since now every reinsurer has a more restricted quota due to the lack of capacity and only can reinsure a limited number of clients. In this kind of situations, cat bonds become more preferable and their cost is not relatively such high anymore. The literature on reinsurance, cat bonds and on the comparison of the two is very wide. One of the authors that has done many studies on reinsurance and cat bonds is J. David Cummins. In the 2002 paper, *The Global Market for Reinsurance: Consolidation, Capacity, and Efficiency* Cummins with Mary A. Weiss estimate the loss payments that would be made by reinsurers, for different sizes of losses. In the 2007 paper, *Reinsurance for Natural and Man-Made Catastrophes in the United States: Current state of the Market and Regulatory Reforms*, the response of reinsurance markets in the case of large catastrophe losses are inspected. In 2006, Darius Lakdawalla and George Zanjani published the paper *Catastrophe Bonds, Reinsurance, and the Optimal Collateralization of Risk-Transfer* in which the use of fully collateralized instruments such as cat bonds in risk exposures is analysed. In the 2008 paper, *Catastrophe Bonds and Reinsurance: The Competitive Effect of Information-Insensitive Triggers* by Silke Finken and Christian Laux, the fact of asymmetric information in the reinsurance market and how products such as cat bonds which are insensitive to the information asymmetry can be used to reach an equilibrium in the reinsurance market are analysed. Besides individual authors, also companies publish various papers on this subject. The company Lane Financial L.L.C has published many papers, one of which that focuses on the pricing on cat bonds and establishes a model for this problem. Another company, Towers Watson, has also published many papers on catastrophic risk exposure and the comparison of traditional reinsurance and cat bonds, such as the 2012 paper *Reinsurance vs. Cat bonds: Comparing and Contrasting Features*

Across the Convergence Spectrum. The contribution of this study consists of a method to forecast the reinsurance market capacity and to determine the situations where cat bonds are preferable instead of traditional reinsurance.

In this study, capacity is modeled using both regression modelling and auto-regressive modelling. The methodology used depends on the statistical analysis of the major variables and parameters in reinsurance and cat bond pricing. The variables that have an effect on capacity are determined as: Shareholders fund (SHF), premiums earned, claims paid, net investment income and combined ratio. The data for these parameters are collected from the last 10 year statistics of 23 major reinsurance companies. Due to the lack of information on total reinsurance market capacity, the historical data used for reinsurance market capacity consists of the top 40 global reinsurer's capital levels. However, it is logical to assume that the top 40 global reinsurer's shape the capacity level of the total reinsurance market. The regressions between these parameters are examined in order to determine which of the parameters can interpret capacity the best. For cat bonds, the parameters are determined as expected loss, trigger type and duration. Several statistical methods are used in the study. Descriptive anlaysis is used for a general overview and to understand the association among the variables that are examined. Among linear models, regression analysis is used to explore the impact of the key variables on determinants and autoregressive structure on time scale is analysed to determine the impact of dependency on time and historical observations. Based on the models, fitted future realizations (forecasts) are used to determine the market behaviour. Moreover an equilibrium model is used to evaluate the determinants for the preference of cat bonds to reinsurance or vice versa. The aim of this study is to establish models for forecasting reinsurance capacity using the data for the selected parameters, and since traditional reinsurance prices are in a direct link with reinsurance capacity and increase in the case of diminished capacity, to ease the decision of purchasing cat bonds by capacity evaluation. Moreover, a case study of cat bonds is done which aims to calculate a load for a specific type of cat bond in order to create a comparison with reinsurance loads.

In the next part, general definitions, components, pricing and comparison of reinsurance and cat bonds will be given. In the third part, the methodology and the models that have been used in the process will be explained. Moreover, it includes the case study of cat bonds. In the fourth part, the relationship between the data that has been collected from insurance companies are examined. According to these relations, the data is used to establish a model that aims to predict the next years capacity level. The fifth chapter will consist of conclusion and comments on further studies on the subject. The references will be given in the last part.

CHAPTER 2

Risk Transfer Mechanisms in Insurance: Reinsurance and Cat Bonds

2.1 Reinsurance

The risk spreading process starts with individual clients purchasing insurance and continues as insurance companies redistribute the risk they inccured among themselves. These companies may share individual risks in different ways or transfer the total accumulated risk to other parties. Protecting the portfolio against excessive claims is very common practice, however, there are other forms of reinsurance.

A company, which reinsures some part of its risk is called a *cedent*. The company which assumes this part is called a *reinsurer*. The negotiation between these companies are based on principles equally acceptable to both sides. These principles, theoretically, need not to be connected with payments for reinsurance. This negotiation may be direct and an agreement can be reached with a certain form of risk distribution, without paying each other for reinsurance. However, when there is a simultaneous involvement of many companies, market price mechanisms are the most realistic mechanisms of exchanging risk. In actual practice of reinsurance, there are various combinations of reinsurance, such as direct insurance, financial derivatives such as bonds, options or futures.

Reinsurance is a very important instrument that helps decreasing the risk exposure. The primary insurance company's risk and capital management is supported by reinsurance and it contributes to enhancing the size and competitiveness of insurance markets. Most common reinsurance contracts include terms such that the amount of risk transferred is limited. These kinds of contracts are called *finite reinsurance*. By finite reinsurance, additional capital and capacity is brought to insurance markets.

If the (re)insurance industry is to maintain its role in the expanding global economy, there are a number of challenges and constraints that have to be addressed. Not only the regulation on the reinsurance is increasing, but also the demand for reinsurance is increasing every year. Satisfying this demand is the role of the reinsurance industry. Because there is a lot of risk diversification involved in reinsurance, risks can be insured cheaper and with higher security, since primary insurers have less diversified portfolios. By spreading the risk to larger capital base and many other uses, reinsurance became an important tool in the global insurance market. In recent years, the complexity of reinsurance arrangements has improved, both with the alternative risk transfer methods and with the advances in the risk modelling technology. Now, much more discrete calculations of probabilities of loss, payment patterns and risk exposure quantification are available, when compared to a few years back. Nearly every reinsurance contract, whether it is traditional or non-traditional, has a provision that limits the risk assumed by the reinsurer. That is to protect the company's capital base and claims paying ability [3].

There are four defined purposes of reinsurance: financing, creating stabilization, creating capacity and catastrophe protection. Financing can be considered as limiting the liabilities for insurance companies. Purchasing reinsurance enables insurance companies to offer higher coverage amounts to its clients, so that even small insurance companies can meet higher expectations. Stabilization means that the standard deviation of the profit and loss margins are kept under a certain level so that the fluctuations in these values are small and the company's operating results are stabilized. With writing a policy comes along the expenses such as agent commissions and administrative expenses and these cause a decrease in the company's capacity. Since capacity is prudently based on the surplus on the companies, by purchasing reinsurance the company shares a portion of its expenses, more surplus becomes available and more capacity is created. Catastrophe protection is provided by reinsurance for the financial loss caused by a catastrophic event and through the use of insurance the effect of a catastrophe is reduced for companies.

The traditional ways of reinsurance basically includes *proportional reinsurance* and *excess of loss reinsurance*.

2.1.1 Facultative Reinsurance

Facultative reinsurance goes on in a couple of steps. The ceding company decides to reinsure a particular policy. The reason of reinsuring may differ. It may be the size of the policy or some circumstance such as the health of the insured. Each reinsurer reviews the underwriting material and decides whether or not to offer a reinsurance for the risk. After that, the company recieves the offers and decides which reinsurer will take the risk if the deal is made.

Facultative reinsurance is generally purchased by ceding companies for individual risks not covered by their reinsurance treaties. Underwriting expenses and in particular personnel costs are higher relative to premiums written on facultative business because each risk is individually underwritten and administered. The ability to separately evaluate each risk reinsured increases the probability that the underwriter can price the contract in order to reflect the risks involved more accurately.

2.1.2 Treaty (Automatic) Reinsurance

In this kind of reinsurance, the ceding company cannot select which policies will be reinsured. The reinsurer must reinsure all the covered policies, as long as the terms in the treaty are met. Treaty reinsurance is generally handled on excess of loss or proportinal basis.

Proportional reinsurance reimburses a fixed percentage of the incurred loss. It involves the reinsurer(s) taking a pre-determined share of every policy the insurer writes. Excess of loss reinsurance reimburses losses above a predetermined level, which is called retention. The insurance company retains the specified level and purchases reinsurer for the rest of the policy. In catastrophic reinsurance, mostly excess of loss reinsurance is preferred to proportional reinsurance.

Assume that X is the random variable for one claim and is uniformly distributed on [0,1]. This assumption can be made for sake of simplicity and does not restrict generality. If the retention level $d \leq 1$, for each claim X the amount that the ceding company covers is given as

$$X_d = \begin{cases} X, & \text{if } X \leq d, \\ d, & \text{if } X > d. \end{cases} \tag{2.1}$$

The random variable X_d is the value d with probability $1 - d$, since X is uniform on [0,1]. Moreover, for the calculation of the premium, the effect of X_d should be included in the expected loss. Therefore, the premium c that is paid by the ceding company for one claim is given as

$$c_d = (1 + \theta)E(X - X_d) \tag{2.2}$$

Here, θ is the load determined by the reinsurer and can change for different regions and for different types of catastrophes.

2.2 Cat Bonds

Catastrophe bonds were created due to the lack of capacity in the catastrophe reinsurance market. When compared to the traditional re(insurance) markets, the cat bond market is small but the continuation of its growth is expected. Actually, the growth of the cat bond market causes the growth of the traditional reinsurance market, as some hedge funds that were early cat bond investors are starting to launch their own reinsurance firms. Cat bonds became an important part of the loss-financing market and continue to supplement solutions for the reinsurance sector.

The structure of cat bonds is similar to traditional *Insurance Linked Securities* (ILS), but with an exception: if a pre-specified event (trigger) such as an earthquake or hurricane occurs before the maturity of the bonds, then the investors risk losing their accrued interest and/or the principle value of the bonds. A cat bond offering is made through an issuance vehicle that may be an insurance or reinsurance company, which is called a *Special Purpose Reinsurer* (SPR). The SPR has many duties. It provides reinsurance to a sponsoring (re)insurance company and sells notes to investors. Then the proceeds are transferred to the trustee for further reinvestment and an indemnity contract is provided to the issuing company. The investor coupon is formed by the return generated through reinvestment and the premium payment. This coupon becomes due and payable on a periodic basis. The invested proceeds that are held in the trust account are used to repay the principle of the bond at maturity. If a catastrophic event occurs and the trigger is set, the trustee holds the interest and/or principal payments for a temporary of permanent period. In this case, the principal that would be returned to the investors is used by the SPR to make the payments to the insurer. In exchange for taking catastrophe risk, the investor receives a relatively high interest rate on the bonds.

Figure 2.1 shows the structure of cat bonds:

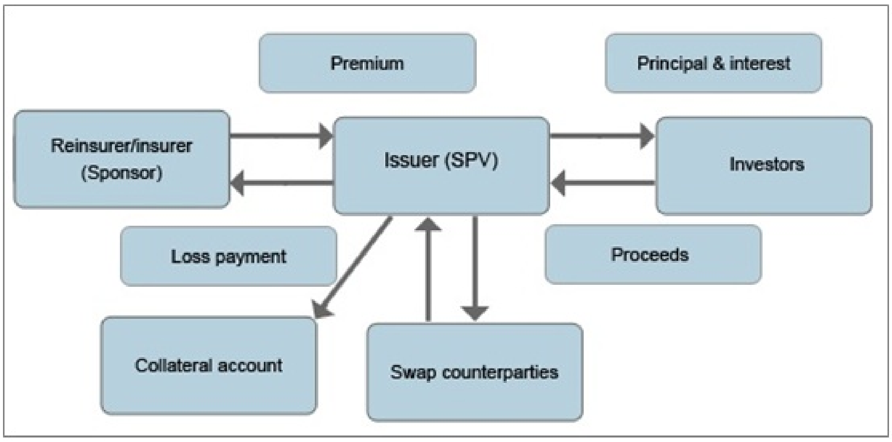


Figure 2.1: Cat Bond Structure (Source: AM Best)

2.2.1 Triggers of Cat Bonds

The trigger of a cat bond has a huge impact on its benefits, effecting both the cedant and the investor. The variables of the trigger splits into two categories. The triggers of the first category are called indemnity triggers, where payouts are based on the actual losses of the insurance company. The second category triggers are called index triggers, where the payout is linked to an index that is not directly tied to the insurers own losses. A portion of basis risk and the risk of moral hazard are included in the variables of the triggers. A tradeoff occurs between these risks when considering an optimal trigger variable.

When dealing with indemnity triggers, the payout is linked to the loss of the insurer in the specified region. If the loss of the insurer in the speficied region exceeds a pre-determined amount, the trigger would go off for a typical indemnity trigger. This causes information disclosure on the part of the insurer. For example the insurer may prefer not to share its information on portfolio composition. Also the evaluation of the losses may take some time in these cases. Besides the negatives, this kind of triggers minimizes basis risk. But there is always the moral hazard risk from the point of the investors, since the insurer might not use all its powers to get the evaluation of claims right. Indemnity triggers get less attractive to investors because of these difficulties in rating and modeling the returns.

Even though index triggers have a main concern such as the basis risk being assumed by the insurer. Index triggers gained popularity recently. Which implies that insurers can live with that kind of risk. Index triggers are preferred because of the ease of being rated by agencies and they reduce the risk of moral hazard. Index triggers can be divided into 3 categories: parametric, industry loss and modeled loss. In the first category, parametric triggers, the payout is linked to a physical event, such as the magnitude of an earthquake. Industry loss triggers are linked to the estimated losses of the catastrophic event. A modeled loss trigger simulates a model created by a modeling service, in order to calculate the estimated losses.

2.2.2 Pricing Approaches to Cat Bonds

Because of the uncertainness of cat bonds, pricing them becomes a challenge. However, they can be interpretted as a portfolio, which consists of a variable interest rate bond and an option. This way, cat bonds are evaluated by utilizing pricing models for cat options. In years many models were suggested for the pricing of cat bonds. The first model was by Cummins and Geman (1995) [4], which dealt with the pricing of catastrophe insurance futures and call spreads. The model was arbitrage based and used a Poisson jump process to play the role of the catastrophe. Brownian motion was used for the stochastic timing of claims. Lee and Yu (2002) [5] also suggested an arbitrage approach, where basis risk and the risk of moral hazard were included. However, it is not possible to derive a unique price since a unique martingale measure does not exist in the catastrophe securities market, which makes it incomplete.

In the Lane Financial (LFC) cat bond pricing model, there is a load added to expected loss in pricing cat bonds. Furthermore, it has been argued that the load should take account of the shape of the distribution and that frequency and severity of loss is a more plausible set of measures than standard deviation. So the model that LFC gives [6]

$$Load = \gamma \times PFL^{\alpha} \times CEL^{\beta}, \tag{2.3}$$

where PFL denotes the proability of first dollar loss and is calculated as $\sum p_i$, CEL is the conditional expected loss and is calculated as $[p_i/(1 - p_o)] \times L_i, \forall i \geq 0$. Here, L_i are the loss outcomes and the price estimated by this model is given by

$$Price = EL + \gamma \times PFL^{\alpha} \times CEL^{\beta}, \tag{2.4}$$

where the expected loss is the probability-weighted sum of the possible outcomes. If the bond has two outcomes which are full repayment or loss with probabilities p_0 and p_1, expected loss is calculated as $EL = 0 \times p_0 + (1) \times p_1 = p_1$. If there are many possible outcomes L_i, the expected loss becomes $EL = \sum p_i L_i$. The probability of first dollar loss is also known as attachment probability and is the inverse of frequency of loss. Conditional expected loss is used as a measure of severity of loss, i.e the size of the expected losses. Conditional expected loss can also be written as $CEL = \sum[p_i \times L_i]/(1 - p_0) = EL/PFL$. The model represents the idea that investors trade off between frequency and severity of loss. Its aim is to determine how more frequency can be traded for less severity.

2.3 Reinsurance vs. Cat Bonds

Managing catastrophe risk exposures has always been a difficult task to accomplish and cat bonds play a huge role helping this task. By the use of cat bonds, reinsurance companies can transfer a portion of their exposure to natural catastrophes to capital markets, instead of just retaining the exposure or retroceding the risks to another reinsurer. In addition, the insurance companies which are not able to obtain the necessary amount of reinsurance for the low probability high severity class of risks can benefit from the cat bonds in the same risk category. Because of all these reasons, cat bonds prevent reinsurance proces from increasing more. Moreover, cat bonds can be used as an instrument to enhance the portfolio, since they have relatively low or zero correlation with other traded assets. Cat bonds were designed for investors such as investment advisors and hedge funds and present attractive risk/return characteristics for these kind of investors.

Besides the positives, when compared to traditional reinsurance, cat bonds have significantly higher costs. One of these costs are the interest costs that insurers pay to the investors for compensation for the risk of loss of principal. Transaction costs such as underwriting fees charged by investment banks, fees charged by modeling firms to develop models to predict the frequency and severity of the event that is covered by the security, are said to be another reason for the high costs of cat bonds. Due to these costs, the companies that do purchase cat bonds, limit their cat bond investments to around 3 percent of their portfolio [7].

Even though the negatives of a cat bond seem stronger than the benefits that it can provide, they can be used to enhance the portfolio and the costs might be reduced with further analysis. In addition, they can cover a larger portion of the risk exposure than traditional reinsurance, which makes them preferable for big investors.

The price of a traditional reinsurance product depends on many aspects such as the type of the reinsurance product, the region that the product is available for or the expected loss for the specified event. All of these variables change the reinsurance price, but the major price fluctuations in the market are caused by the capacity level available in the market. Natural catastrophes have low frequency, however as historical data concludes, the frequency can increase in random years and the losses increase with it. With increased losses, less capacity becomes available to reinsurers and reinsurance price increases for compensation of losses. The price of a reinsurance treaty with criteria same as before can increase dramatically after a series of catastrophes.

After a decrease in the capacity level, the prices are expected to rise. However, reinsurance prices can rise even if sufficient capacity level is obtained. For example, in 2011, a record loss for catastrophes was recorded. The vast majority of the loss activity occured outside of the United States, Asia accounting for more than two-thirds of total insured losses. Figure 2.2 shows the cat losses recorded in 2010 and 2011,

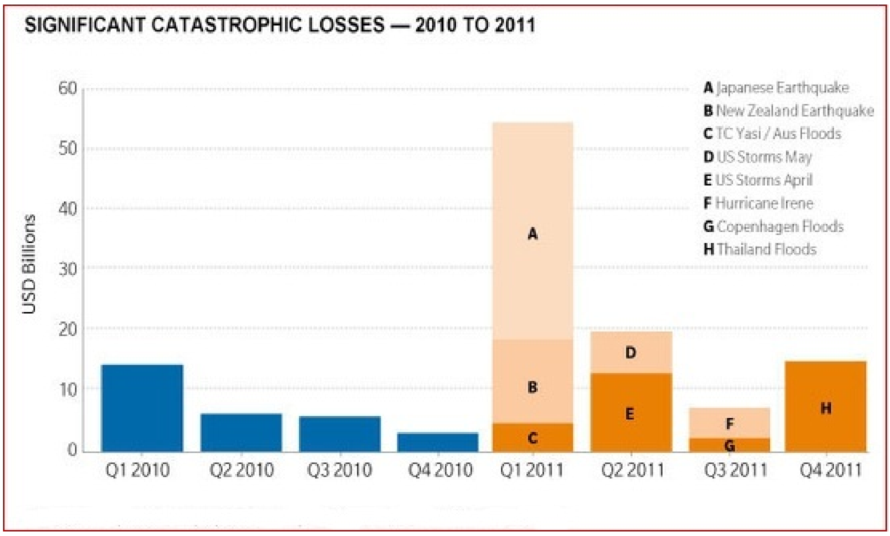


Figure 2.2: Recorded Cat Losses 2010-2011 (Source: Guy Carpenter & Company)

Even though there was enough capacity to compensate the losses, the reinsurers increased the price for traditional reinsurance contracts because of the increasing frequency of catastrophes. In Figure 2.3, it can be seen that through 2002 to 2010, the price index for earthquake excess of loss continues to decrease as the capacity index increases for Japanese non-life companies. But in the last two years, even though capacity kept rising, the reinsurance price index took a turn and increased.

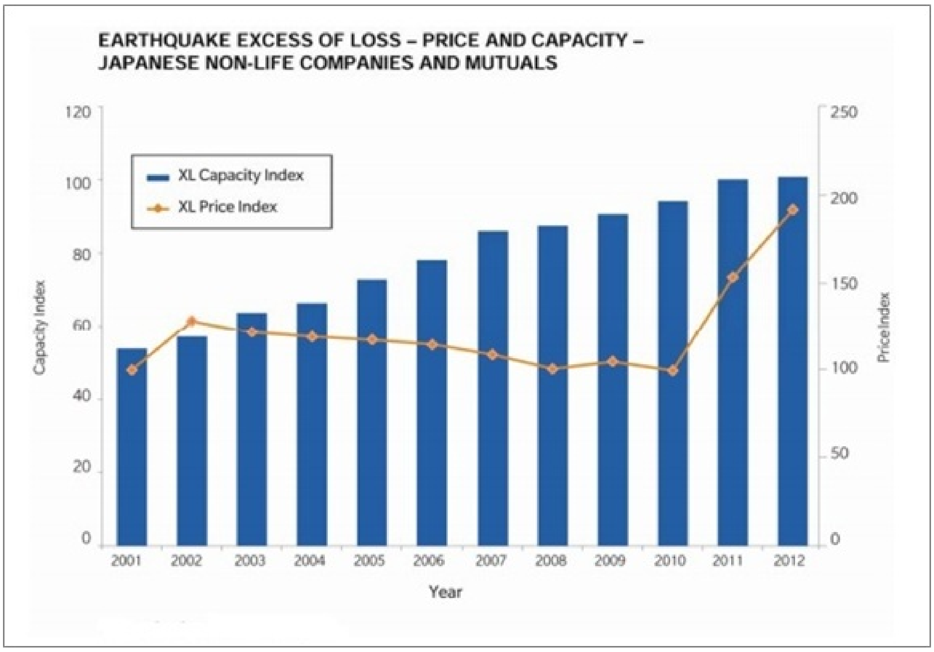


Figure 2.3: Price-Capacity Index for Japanese Non-Life Companies and Mutuals (Source: Guy Carpenter & Company)

The increase in the reinsurance price, caused by the reduction in the capacity level, makes traditional reinsurance less attractive. Other securities, such as cat bonds become more preferable, compared to traditional insurance. In the natural course of events, cat bonds tend to be relatively higher. However, in these cases they become an attractive option. To predict how the capacity level will react in the upcoming years, the approach that is taken in this study is that to relate capacity to the variables that are considered and to predict the future activity of capacity using these variables by establishing a time series.

CHAPTER 3

Methodology

In this study, the factors that affect reinsurance capacity are determined as SHF, premium income, claims paid, investment income and combined ratio. In order to model capacity with these parameters, the interactive relation among the variables should be inspected. Afterwards, to predict future capacity, the variables that can interpret capacity are forecasted. The simplest and most practical approach to be considered is linear modelling, therefore the relation among the parameters are established linearly. Moreover, linear modelling enables the forecasts to be done in a logical fashion.

Linear modelling among the variables is done by regression analysis and by this method the linear relations can be determined. The future value of the variable(s) that explain capacity with least errors are predicted and the regressions are used to specify the future value of reinsurance capacity.

3.1 Regression Analysis

Regression analysis is used to determine the systematic relation between two or more variables. If the relation between only two variables is examined, it is called a simple regression. In the case of simple linear regression, it is of the form

$$y = \beta_0 + \beta_1 x + \epsilon. \tag{3.1}$$

Here, β_0 is the constant and β_1 is the x coefficient, which represents the slope of the straight line described by the equation. ϵ is a random variable which has an expectation of 0, i.e.

$E[\epsilon] = 0$ having the assumption $\epsilon \sim N(0,\sigma^2)$ [8].

The equation generated by the regression can be used to predict new observations, as well as determining the relationship between the variables. Generally, ordinary least square method is used to derive this equation. This method minimizes the sum of squared vertical distances between the observed responses in the data and the responses predicted by the linear approximation. The sign of the coefficients in the equation specifies the direction of the relationship. The coefficients themselves, represent the mean change in the response for one unit change in the predictor, while other predictors remain constant. Each coefficient in the equation has a *p*-value, tests the null hypothesis for the coefficient, which is whether it is equal to 0 or not. In the case that it is 0, it has no effect in the model. Low *p*-values for a coefficient indicate that the predictor is consistent with the model.

3.2 Time-Series Modelling

Time series are basically recordings of processes that differ over time. This recording can either be a continuous trace or a set of discrete observations. Time series have a number of important factors. One of which is modelling that enables to develop a simple mathematical model which explains the pattern of the observed values. Another factor is forecasting, where the future value of values of the observed values can be predicted and an indication of what the uncertainty is in the prediction. Another aspect of time series is stationarity, which represents the behaviour of the series. If a series is stationary, then the values tend to vary around the same level and the variance is constant over time. In general, a time series is not stationary but most of them can be related to stationary series. An example for this case is trend models, where the series is the sum of a deterministic trend series and a stationary noise series [9].

The term ARIMA, stands for autoregressive, integrated, moving average. ARIMA models are used for forecasting time series and they are the most general class of models for this process. Each part of the model (autoregressive, integrated, moving average) represents a part of the mathematical model that is used.

When using ARIMA for forecasting, the forecasts are based on linear functions of the sample observations. The purpose of ARIMA modeling is to find the simplest model that describes the observed data, which is called parsimony. As mentioned before, ARIMA has different parts, which are the autoregressive part (AR), the integrated part (I) and the moving average part (MA). The models are written as ARIMA(p,d,q), where p represents the autoregressive part, d represents the integrated part and q represents the moving average part [10].

In ARIMA models, each observation is a function of the previous observations. The p of the autoregressive part, stands for the previous p observations that form a function for the next observation. If $p = 1$ and $d = q = 0$, then each data is a function of one previous observation, which would be represented by

$$X_t = c + \phi_1 X_{t-1} + \epsilon_t. \tag{3.2}$$

Here, X_t is the prediction at time t, X_{t-1} is the previous observation which is at time $t-1$, ϵ_t is the random error and c, ϕ_1 are constants. If $p > 1$, we get the following equation for the ARIMA model;

$$X_t = c + \phi_1 X_{t-1} + \phi_2 X_{t-2} + ... + \phi_p X_{t-p} + \epsilon_t. \tag{3.3}$$

In this equation, everything is same as above for the $p = 1$ case, except that now the prediction for the future depends on the previous p observations, that is it is a function of the previous p observations.

There are two ways in ARIMA to model the observed values. The values can be modelled directly, or the differences between consecutive observations can be modelled. If the observed values are modelled directly, then $d = 0$ for the model. The case $d = 1$ means that the modelling is done on the differences between consecutive observations. If $d = 2$, the differences of the differences are being modelled. Generally, ARIMA models are not used with $d > 2$.

The moving average part of the ARIMA model, represents with a function that how every observation is a function of the previous errors. The number of the previous errors used in the function is q, which is mentioned above as a variable of the model. The equation of the process for $q = 1$ is given as

$$X_t = c + \theta_1 \epsilon_{t-1} + \epsilon_t. \tag{3.4}$$

In this equation, as before, ϵ_t stands for the random error at time t and ϵ_{t-1} is the random error from the previous time, which is $t-1$ in this case. For $q > 1$, the above equation is extended as

$$X_t = c + \theta_1 \epsilon_{t-p} + \theta_2 \epsilon_{t-p-1} + ... + \theta_q \epsilon_{t-1} + \epsilon_t. \tag{3.5}$$

ARIMA models generally deal with nonseasonal time series, which means the differences that are used in the model are nonseasonal. However, ARIMA models designed for seasonal time series also exist. ARIMA models have a well-developed mathematical structure, which enables to calculate different model features, for example prediction intervals.

The specific ARIMA model that is used in this study is; ARIMA(1,0,0), which means that the autoregressive part of the model is $1(=p)$ and the integrated and moving average parts are not included, so they are 0. The reason for doing this is to get an AR(1) process out of ARIMA, i.e. ARIMA(1,0,0) = AR(1).

When dealing with ARIMA models and time series, the stationarity of the model must also be examined. The stationarity/non-stationarity, can influence the behaviour of the time series. Define an AR(1) series by

$$Y_t = \phi Y_{t-1} + \epsilon_t. \tag{3.6}$$

If $|\phi| = 1$, no stationary exists for the initial autoregressive equation. To test the stationarity, unit root test can be applied which inspects whether $|\phi| = 1$ or not. Unit root evolves through time and can cause problems in forecasting for stationary time series. So, for forecasting stationary time series only the case of $|\phi| < 1$ should be considered [11].

Another tool to examine the stationarity is partial autocorrelation, which measures the degree of association between Y_t and Y_{t-k} when the effects of other time lags 1, 2,..., $k-1$ are removed. The partial autocorrelation coefficient of order k (α_k) is evaluated by regressing y_t against previous observations, which gives

$$y_t = b_0 + b_1 y_{t-1} + b_2 y_{t-2} + ... + b_k y_{t-k}. \tag{3.7}$$

Stationarity implies that there are no growth or declines, that is the data fluctuates around a constant mean and the variance of the fluctuation remains constant over time.

3.3 Cat Bond Equilibrium Modelling

As mentioned in the previous chapters, the price of a traditional reinsurance contract fluctuates with the changes in the market capacity. If capacity diminishes by the increasing frequency of natural catastrophes, then the price of traditional reinsurance increases. In this case, other options for securities which are relatively more expensive, are considered more seriously, such as cat bonds. Cat bonds become preferable when capacity decreases, but this does not mean that they become cheaper. They are expensive products to purchase in any case of the market capacity, so the process of including cat bonds in the portfolio should be thorough.

Cat bonds are purchased for a variety of reasons, for example if there is a natural catastrophe expected in the upcoming months or year, the increase in the reinsurance price caused by a decrease in the market capacity or by the price cycles, or to make the portfolio more dynamic. If the market capacity for upcoming years can be estimated, it would make the decision of purchasing cat bonds much easier. However, it is extremely difficult to estimate future capacity because it depends on many variables in the global market and also on the occurence of natural catastrophes, which has a very direct relationship with capacity. Because of these facts, the level of market capacity cannot be estimated with certainty. In spite of this, capacity movement might be predicted by analysing what kind of relationship it has with the (re)insurance companies involvement in the market.

It is known that traditional reinsurance prices increase when reinsurance market capacity diminishes. In order to see how cat bond prices react with capacity decrease, the Lane model which has been explained in Subsection 2.2.2 will be used. The parameters that are used in this model are expected loss, probability of first dollar loss (attachment probability) and conditional expected loss. By using this model, cat bond prices in the years that reinsurance capacity has increased and decreased can be observed and by capacity forecast, a prediction of cat bond prices can be made.

The algorithm for capacity estimation starts with the regression between capacity and the determined market variables. After acquiring the regression between capacity and market variables, the variable(s) that interpret capacity with the most accuracy are used as response and the other variables are tested as predictors. The response with most accuracy is forecasted, and the future prediction is used in the regression equation to acquire the parameter that predicts capacity and a value for capacity is obtained.

3.3.1 Case Study: Cat Bond Pricing for California Earthquake

Due to the many parameters that are involved in cat bond pricing, the pricing process is difficult. The parameters that are taken into consideration in cat bond pricing are the trigger type, the region that the cat bond is covering, the tranche of the bond (if it is floating rate or variable rate, etc.) and the duration. All of these factors are included in the calculation of the expected loss and the attachment probability (PFL). In the Lane model for cat bond pricing, attachment probability and expected loss are considered in the calculation. Moreover, conditional expected loss (CEL) is included, which is obtained by dividing the expected loss by the attachment probability.

The pricing approach for the Lane model was given in Equation 2.4. In the equation, PFL is the inverse of frequency of loss and CEL is used as a measure of severity of loss. In this approach, non-linear regressions are made among cat bonds from the same year, in order to price a bond in the next year. For example, several bonds are chosen from the year 2001 and non-linear regressions are made between these bonds, using the equation for the Lane model. The known, issue prices in the year 2001 are used as response in the regression, and PFL and CEL are the predictors for the model. By this regression, the parameters α, β and γ are estimated. The estimated values of the parameters using cat bonds from 2001 are used in the next years pricing. Using the PFL, CEL and the estimated parameters, a price is obtained for the bond.

The cat bond data used in this thesis, consists of numerous sponsors and issuers and is available for the years 2003-2008, if the issue date is taken as a starting measure. The data is available for only 6 years, although it is sufficient for the approach taken in this study. The number of bonds in each year is different, however it does not have to be the same number to do the analysis since the regression is made among the bonds of the same year. Moreover, the number of bonds for each year is sufficient to do a regression analysis. However, the issue price for the bonds are not available in the data. So, in order to be able to do the regression, another variable risk premium (which is asked by the investors to invest in the bonds) is used instead of issue price. This replacement does not effect the consistency of the model, since issue price is a statement in terms of capacity and risk premium is in terms of risk exposure. Also, according to the research in the paper *Explaining the Spread Premiums on Catastrophe Bonds* by Debra T. Lei, Jen-Hung Wang and Larry Y. Tzeng, investors asking higher risk premium for higher catastrophic event risk is consistent with the current pricing models for cat bonds [12].

Starting from the year 2003, regression analysis is done among the cat bonds that are available for that year. Response is chosen as risk premium and the predictors are PFL and CEL. The regression estimates the parameters θ_1, θ_2 and θ_3 (which were previously denoted as α, β and γ), which can be used for the estimation of the risk premium for the next year. The estimated and actual risk premium of randomly chosen bonds are then compared in order to observe the consistency of the model. To make a comparison between cat bond prices and the reinsurance price and capacity, the following approach is considered: The average of each of the estimated parameters is taken. In order to make a logical comparison, a bond that only has California earthquake as a region and trigger is randomly chosen from each year. The average of the estimated parameters are used in the load calculation of the Lane pricing model with the statistics of the randomly chosen bonds to obtain a value for the load. Here the assumption that is made for comparing reinsurance and cat bond prices is that the expected loss is fixed for both the cat bond and the reinsurance contract. If the expected losses are the same, then the price depends on the load. The reinsurance load and cat bond load can be compared by the reinsurance purchaser, in order to benefit from the cheaper option. The averages of the parameters are for the values of the year 2003-2008, however this process can be repeated and renewed by doing the calculations with the statistics of newer bonds.

All of the regressions yield results which are logical, except for the year 2006. The stability of the variables in the regression are determined by the p-value of the predictor, which is in the interval [0,1] and should be close to zero. The p-value of the preditors in the 2006 regression were not significant, hence not resulting with an accurate model. However, the results of the 2006 regressions will also be given. The regression equation that is used is given as

$$RiskPremium = EL + \theta_3 \times PFL^{\theta_1} \times CEL^{\theta_2}. \tag{3.8}$$

The individual results and plots for each years is not presented, for the sake of simplicity. The estimated parameters and the averages of the estimated parameters are given in Table 3.1.

Table 3.1: θ_i between 2003-2008 and $\bar{\theta}_i$

Year	θ_1	θ_2	θ_3
2003	0.6235	-0.0965	0.5746
2004	0.5925	0.483	0.4852
2005	0.4212	0.3301	0.3147
2006	0.8033	1.125	1.4173
2007	0.3034	-0.2188	0.1422
2008	0.4167	0.381	0.3398
$\bar{\theta}_i$	0.5267	0.3339	0.5456

Since the *p*-values for the regression for the year 2006 did not yield accurate results, the estimated values are higher than they should be. However, after taking the mean of the estimations the error of the regression is reduced. The averages of the parameters are used for the calculation of the load with the statistics of the randomly chosen bonds. Table 3.2 shows the PFL, CEL, expected loss and the calculated load of the California EQ bonds chosen from each year and the natural logarithm of the corresponding years reinsurance capacity.

Table 3.2: Calculated Loads of Randomly Chosen Cat Bonds

Year	PFL	CEL	EL	Load	Capacity
2003	0.0159	0.805	0.0128	0.0443	12.1022
2004	0.0159	0.805	0.0128	0.0375	12.2334
2005	0.0344	0.712	0.0245	0.0680	12.3569
2006	0.0207	0.546	0.0113	0.0318	12.5955
2007	0.0097	0.701	0.0068	0.0376	12.6464
2008	0.0262	0.390	0.0102	0.0520	12.4125

The load according to the Lane model is calculated for the bonds, and these loads can be compared with the reinsurance loads by the purchaser in order to decide which catastrophe protection to buy. By including the present years cat bonds into the calculation, a purchaser can calculate the new averages of the estimated parameters and these averages can be used to calculate the load for the specified cat bond (assuming California EQ and equal expected losses with reinsurance) and compare this load with the reinsurance product of the same coverage.

In order to see the accuracy of the model is, the actual and estimated risk premium are compared. Table 3.3 shows the actual and estimated values of the risk premium for the chosen bonds with the variability

Table 3.3: Variability of Risk Premium

Year	RP_{actual}	$\widehat{RP}$	$(RP_{actual} - \widehat{RP})$
2003	0.0600	0.0571	8.044×10^{-6}
2004	0.0475	0.0503	8.190×10^{-6}
2005	0.1000	0.0925	5.557×10^{-5}
2006	0.0800	0.0431	0.0013
2007	0.0360	0.0444	7.151×10^{-5}
2008	0.0575	0.0622	2.245×10^{-5}

It can be seen that the estimates for risk premium are very close to the actual, except for year 2006. The high error in this year is caused by the inaccurate regression.

Moreover, a comparison with capacity can be done with the estimated loads for the years 2003-2008. It can be observed that the load decreases with increasing capacity and vice versa, except for the year 2006. The increase of the load in 2006 can be related to the cat bond market at the time, or can be related to the chosen cat bond. However, the model gives an idea of how the load for California EQ cat bonds react with the change in reinsurance capacity.

CHAPTER 4

Application: Case Study

By determining market parameters that have an effect on reinsurance capacity, a prediction for reinsurance capacity is made with modelling of these parameters. In order to do this, 23 (re)insurance companies which have a major effect in the reinsurance market are studied. The variables that affect capacity movement, are chosen from the last 10-year statistics of these (re)insurance companies. These variables have been determined as; shareholders fund (equity), net premiums earned, net claims paid, investment income, and combined ratio. By studying the relationships between these variables, and the relationship between capacity and these variables one can interpret capacity in terms of these parameters. These data have been collected from the annual reports of the (re)insurance companies and here it should be noted that the data available for earlier years is less than the more recent years. For some of the small companies, the annual reports or any other information on these data does not exist. So for the earlier years, the data of the larger companies is used. In the case of regression analysis that uses the sum of the variables for the specified year, an average of the available years values for the parameter is added to the sum for the companies that do not have earlier values of variables existing.

The AR(1) model that has been established to predict future capacity, uses previous data for the top 40 global reinsurance capital level, for between the years of 2000 and 2010. These level of capitals are known, so they can be fitted in the model for forecasting. While the current years market capacity is unknown, it can be interpretted by the chosen parameters. By this interpretation, the result can be used in the AR(1) model for future prediction.

4.1 Market Variables and Data

Reinsurance capacity is affected by the following variables: Shareholders fund, premium income, claims paid, investment income and combined ratio. Each of these variables have an important effect on capacity, since reinsurance capacity is mostly determined with the surplus of reinsurance companies.

However, the data for these variables were limited for the years 2000-2010. Initially, 40 (re)insurance companies were considered for the collection of the data, but because of the lack of information on these parameters for all of the companies, 40 companies reduced to 23. If data was available for earlier years than 2000 for the determined parameters, the regression and linear models that are considered in the thesis would yield better results. However, the data that is available also allows modelling and forecasting with small errors.

4.1.1 Market Capacity

Most of the reinsurance market capacity consists of the shareholders fund of the reinsurance companies, which is money given from investors and the revenue of the company. That is, capacity is the capital available for covering losses. As companies increase their capital and by this way the market capacity, they are able to use more resources and there is less risk of having insufficient funds in case of a crisis which can happen when the frequency of catastrophes increase. Figure 4.1 shows the impact of the catastrophe losses recorded in the first two quarters of 2011 to the shareholders fund of some major (re)insurance companies [13]:

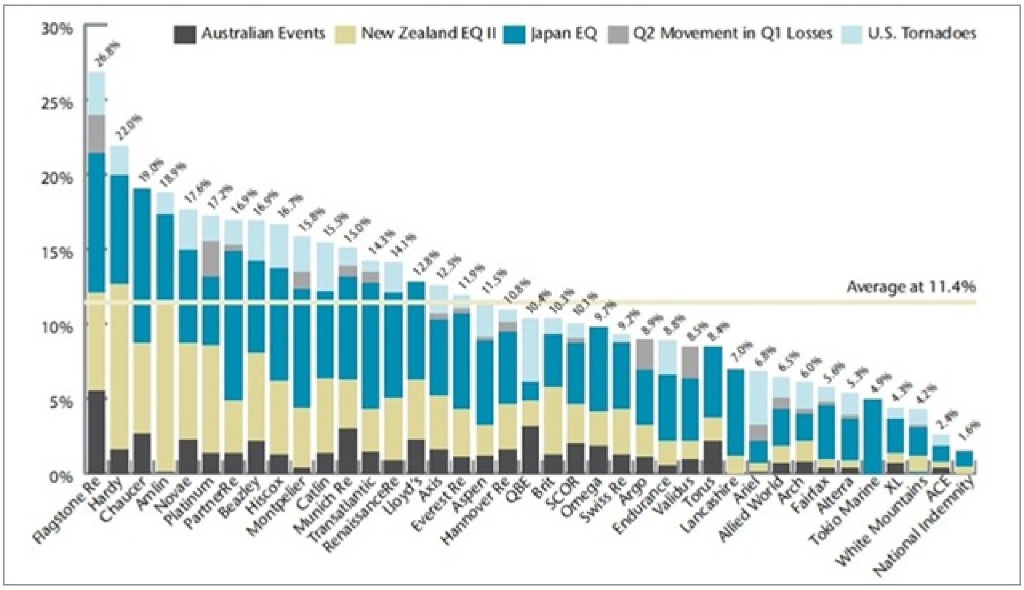


Figure 4.1: Major (Re)insurers Impact of Catastrophe Losses on Shareholders Fund in First Quarter 2011

From Figure 4.1 it can be seen how each catastrophe type that has affected the shareholders fund level of the reinsurance companies in the year 2011. In the specified period, each catastrophe has drained a percentage of shareholders fund, as the axis of Figure 4.1 represents.

The data for capacity used in this study consists of the capital levels of the top 40 global reinsurers. The capital of the top 40 global reinsurers comprises about 80% of the total market capital, so it can be assumed that these companies give shape to the course of capacity in the reinsurance market. Figure 4.2 shows the total market capital of those companies for the years 2007-2010 [14]:

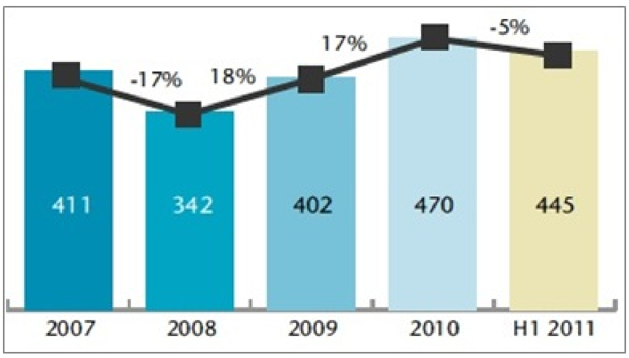


Figure 4.2: (Re)insurer Capital Change (US$billions)

In this figure, the movement of total reinsurance capacity is given for the years 2007-2011. It can be observed that, there is a significant decrease of 17% in the year 2008 and a decrease in the first quarter of 2011. Other than these years which the number of natural catastrophes were above the average, reinsurance capacity tends to increase.

Figure 4.3 shows the capital level for the top 40 global reinsurers [15]. The drop in 2008 is larger for the top 40 global reinsurers, which makes the capital level lower than 80% for that particular year. This means that the leading reinsurers were effected more with the global crisis than the other companies which are not included in this group.

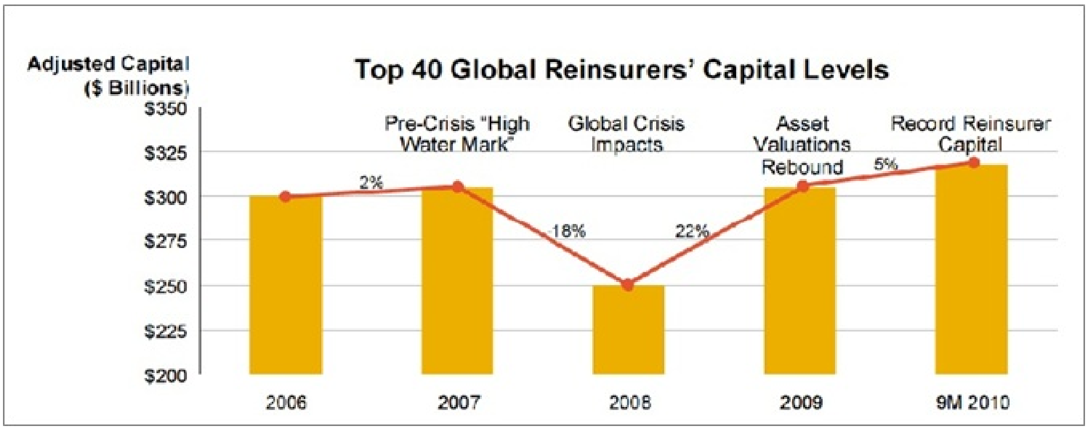


Figure 4.3: Top 40 Global Reinsurers Capital Levels

For the proposed study, the data used for capital levels includes the years 2000-2010 [16] [17], as shown in table 4.1;

Table 4.1: Reinsurance Capacity for the years 2000-2010 (US $ billions)

Year	Capacity
2000	208.5
2001	192.5
2002	150.2
2003	180.3
2004	205.5
2005	232.6
2006	295.2
2007	310.6
2008	245.8
2009	305.4
2010	315.7

Figure 4.4 shows the movement of capacity level through years 2000-2010:

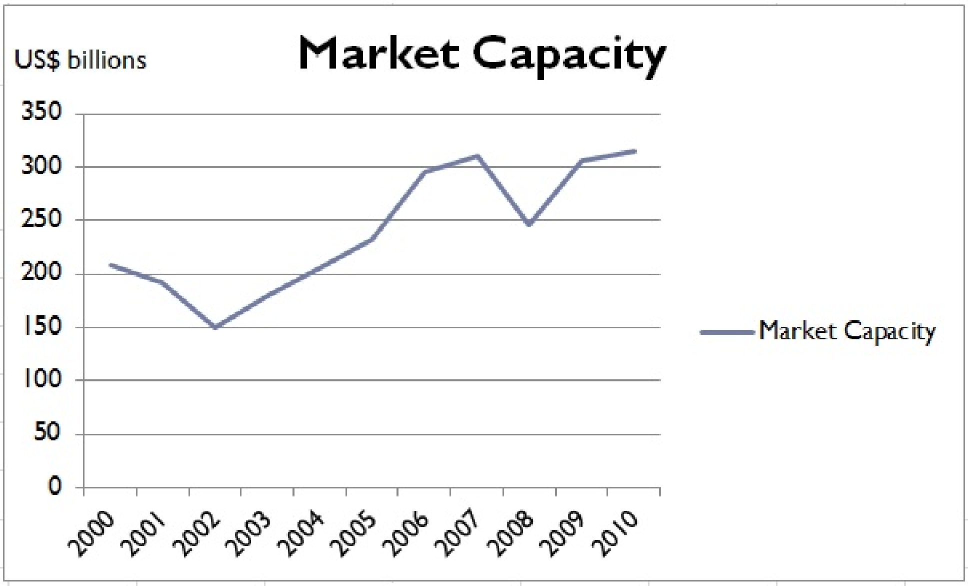


Figure 4.4: Market Capacity Level Through Years 2000-2010

Through the years 2000-2010, capacity declined significantly in 2002 and 2008. In the other years, the number of catastrophes were on the average values and the reinsurance market capacity kept increasing. In case of increasing frequence of catastrophes, market capacity declines such as in years 2002, 2007 and 2008. In 2002, catastrophes (excluding man-made catastrophes) cost over $60 billion where this number was $35 billion for 2001, most of the catastrophes being windstorms and floods in Europe, and mudslides in India and Nepal. In 2002, around 11,000 people were killed in natural catastrophes and the number of recorded natural catastrophes were about 700, which is over the average for the 90's. Many man-made catastrophes occured in 2002, however the extent of loss was not close to natural catastrophes.

Besides the global financial crisis, the hurricane season and man-made catastrophes added up to around $50 billion in 2008. 2007 was also a busy year for catastophes, however the insured losses in 2008 increased 79% from 2007. The losses of 2007 combined with the increasing losses caused by natural catastrophes caused the decline in market capacity for the year 2008. The natural catastrophes in 2008 were the second most after year 2005 (in which Hurricane Katrina occured). Although, the capital level for the top-40 global reinsurers did not suffer a significant decline, in spite of the increasing catastrophe losses. Figure 4.5 shows the total insured catastrophe losses between 1970-2007 [18],

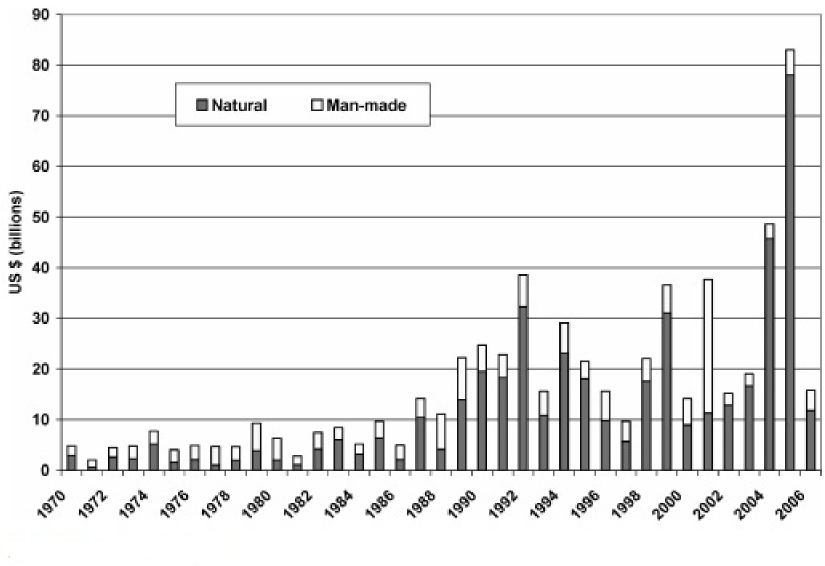


Figure 4.5: Worldwide Insured Losses Through 1970-2007 (Source: Swiss Re (2007))

From Figure 4.5, it can be observed that even though Hurricane Katrina occurred in 2005 and caused huge losses, since a big part of the losses were not insured, the insured losses for 2006 is greater.

4.1.2 Shareholders Fund (Shareholders Equity)

Shareholders fund (SHF) constitutes a significant part of reinsurer capital, thus of the market capacity. It mainly comprises of the money invested in the company and the retained earnings that are accumulated over time. It can be found as

$$Shareholders\ Fund = Share\ Capital + Retained\ Earnings - Treasury\ Shares. \quad (4.1)$$

Figure 4.6 represents the movement of shareholders fund for the chosen companies between the years 2000-2010,

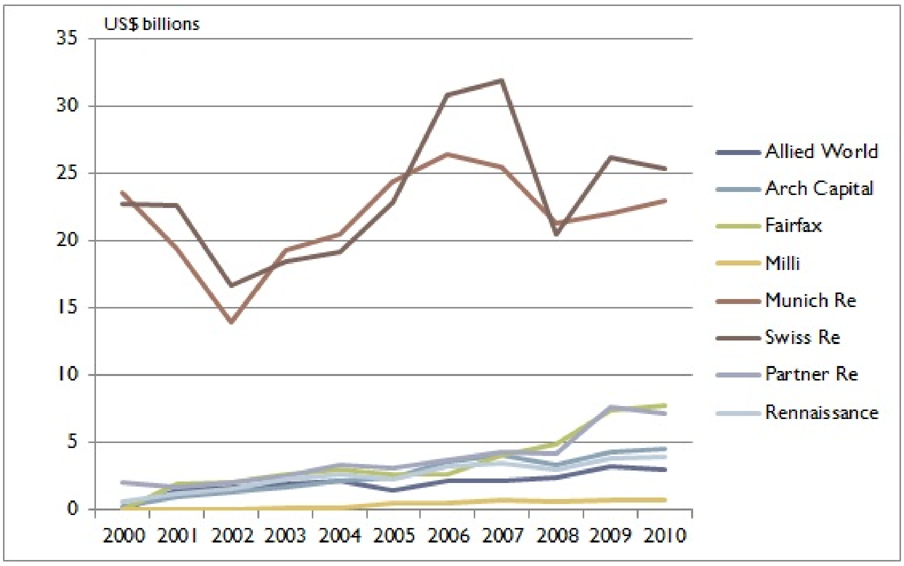


Figure 4.6: SHF Trend for 8 Chosen (Re)insurance Companies Between 2000-2010

It can be seen from the figure that the movement of shareholders fund is closely related with movement of capacity. As shown in Table 4.2 for capacity data, market capacity decreases significantly in the years 2002 and 2008. Since shareholders fund is one of the most important parts of reinsurer capital, the movements of shareholders fund and market capacity should be consistent.

4.1.3 Premiums Earned

An important aspect that effects the companies profitability is premiums earned. The difference between premium income and claims paid can be considered as a measure of profitability for a company. As collected premiums increases the profit of the company increases, hence effecting the capital level of the company which has a direct effect on the total market capital. Figure 4.7 represents the movement of premium income for the chosen companies between the years 2000-2010,

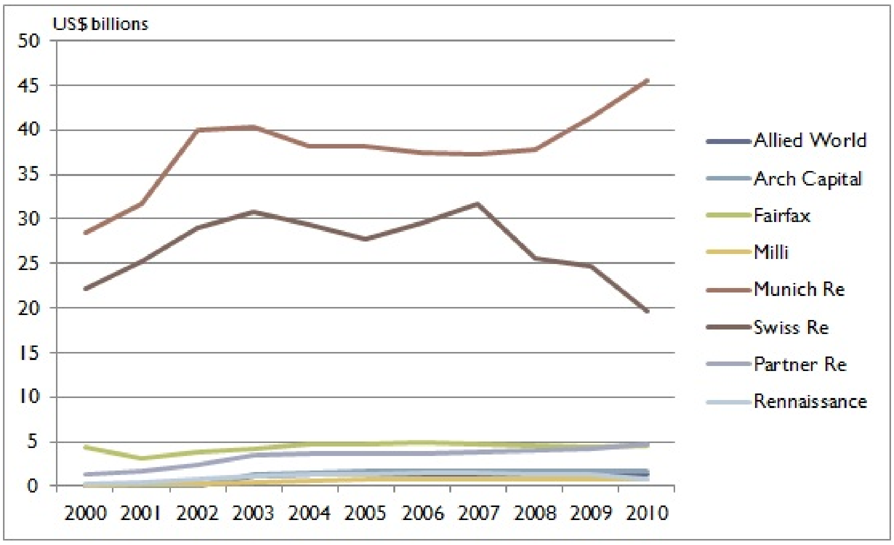


Figure 4.7: Premium Income Trend for 8 Chosen (Re)insurance Companies Between 2000-2010

Among the chosen companies, Munich Re. and Swiss Re. are significantly larger companies than the others. However, the premium income movement of these companies are close to each other, even though the amount of earned premiums are much less for the smaller companies. For the year 2008, a significant increase/decrease does not exist except for Swiss Re. The decrease of earned premiums in 2008 for Swiss Re, may mean that some customers of the company considered a change of business.

4.1.4 Claims Paid

Claims paid is also an important factor for the profitability of a company. Less claims increase the profit earned by the company and the total claims paid depends on the losses in that particular year. In the case of a catastrophe, losses increase thus reinsurer capital level diminishes. Figure 4.8 shows the movement of claims paid for the chosen companies between the years 2000-2010,

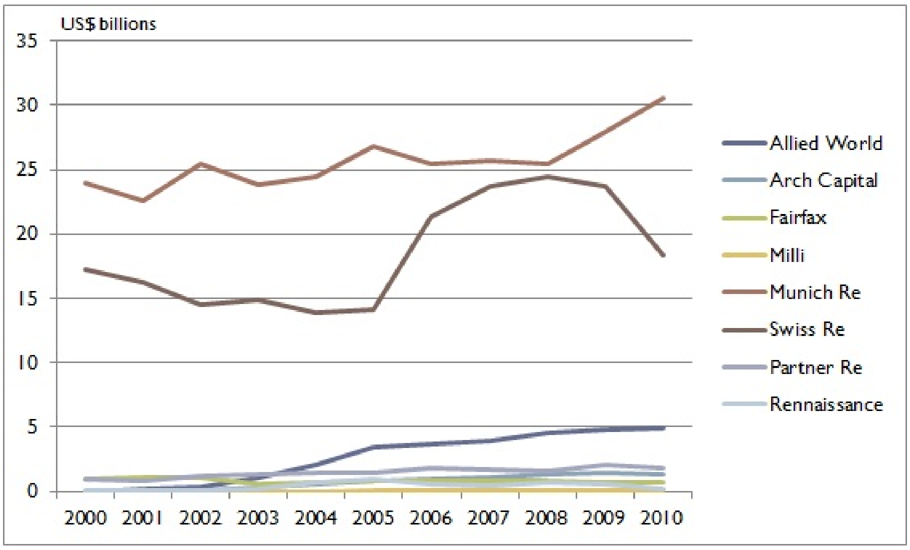


Figure 4.8: Paid Claims Trend for 8 Chosen (Re)insurance Companies Between 2000-2010

There is a general increase from the year 2005 for all of the companies. This increase is larger for some companies, since the claims paid is directly linked to the size of the company. The increase in claims paid from the year 2005 suggest that there was a slight increase in catastrophes. In the year 2008, both natural and man-made catastrophes occured above the average, causing a global crisis. However, after the year 2008 market capacity started to rise and the frequency of natural catastrophes decreased until the year 2010, causing the paid claims to diminish in general.

4.1.5 Investment Income

Besides the shareholders fund and the premiums earned, another factor that adds to the profitability of the company is investment income. The money earned from investment can be used for reinvestment, and adding more to the companies profit margin. A certain percentage of it -which depends on how much the investors contributed- is distributed among the shareholders. Figure 4.9 shows how investment income changes for the chosen companies between the years 2000-2010,

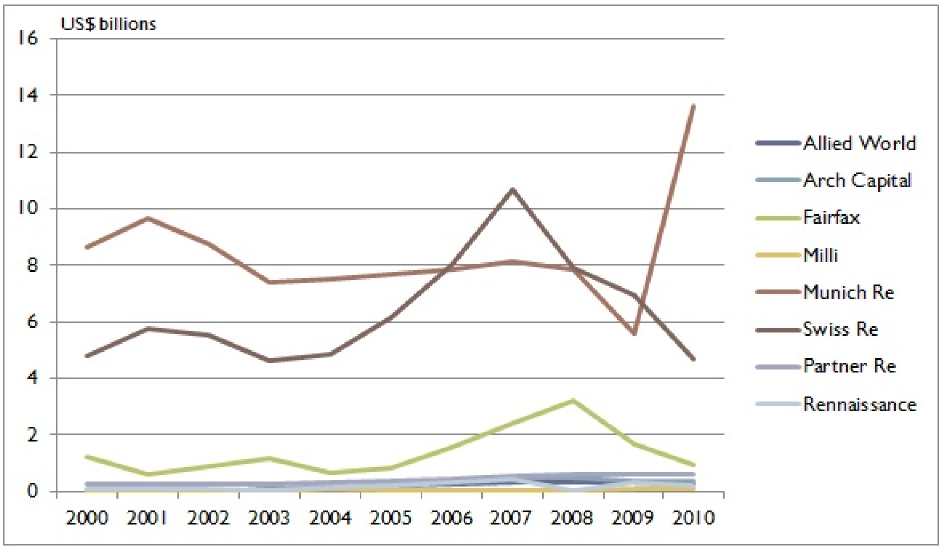


Figure 4.9: Investment Income Trend for 8 Chosen (Re)insurance Companies Between 2000-2010

Even though investment income has an effect on shareholders fund, a direct link between investment income and market capacity cannot be established. For example, shareholders fund decreased in 2008, but investment income for Fairfax, Munich Re., Partner Re. and Rennaissance did not decrease with market capacity.

4.1.6 Combined Ratio

In order to measure the profitability of the company, the combined ratio should be inspected. It gives an explanation on how the company is advancing and consists of the claims and expense ratio, which are the claims owed and operating cost as a percentage of revenue earned from collected premiums.

A combined ratio below 100% suggests that the company is making profit from underwriting, while a ratio above 100% indicates that more money is paid to cover the losses than earned premiums. However, since investment income is not included in the calculation of the combined ratio, even though the ratio is above 100% the company might be making profit, depending on the amount of investment income. The formula for combined ratio is given as

$$\textit{Combined Ratio} = \frac{\textit{Incurred Losses} + \textit{Expenses}}{\textit{Earned Premium}}. \tag{4.2}$$

Figure 4.10 represents the movement of combined ratio for the chosen companies between the years 2000-2010;

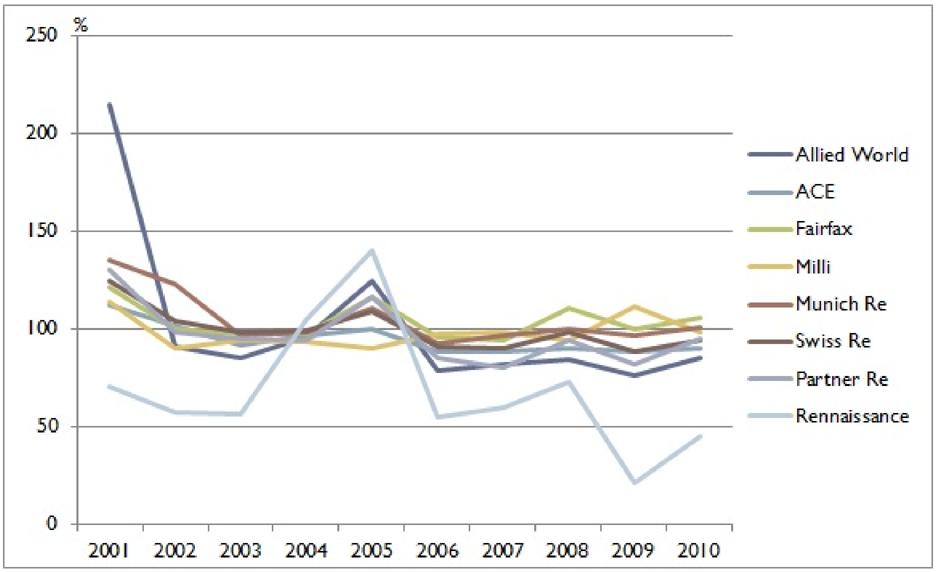


Figure 4.10: Combined Ratio Trend for 8 Chosen (Re)insurance Companies Between 2000-2010

Since premium income is in the denominator in the calculation of combined ratio, an inverse proportion exists between these two parameters. It can be seen from figure 4.10 that there is a significant increase in combined ratio for all of the companies between 2003-2005 and in the year 2008. This indicates that premium income decreased and losses increased in that particular years.

4.1.7 Categorization of (Re)insurance Companies by Region

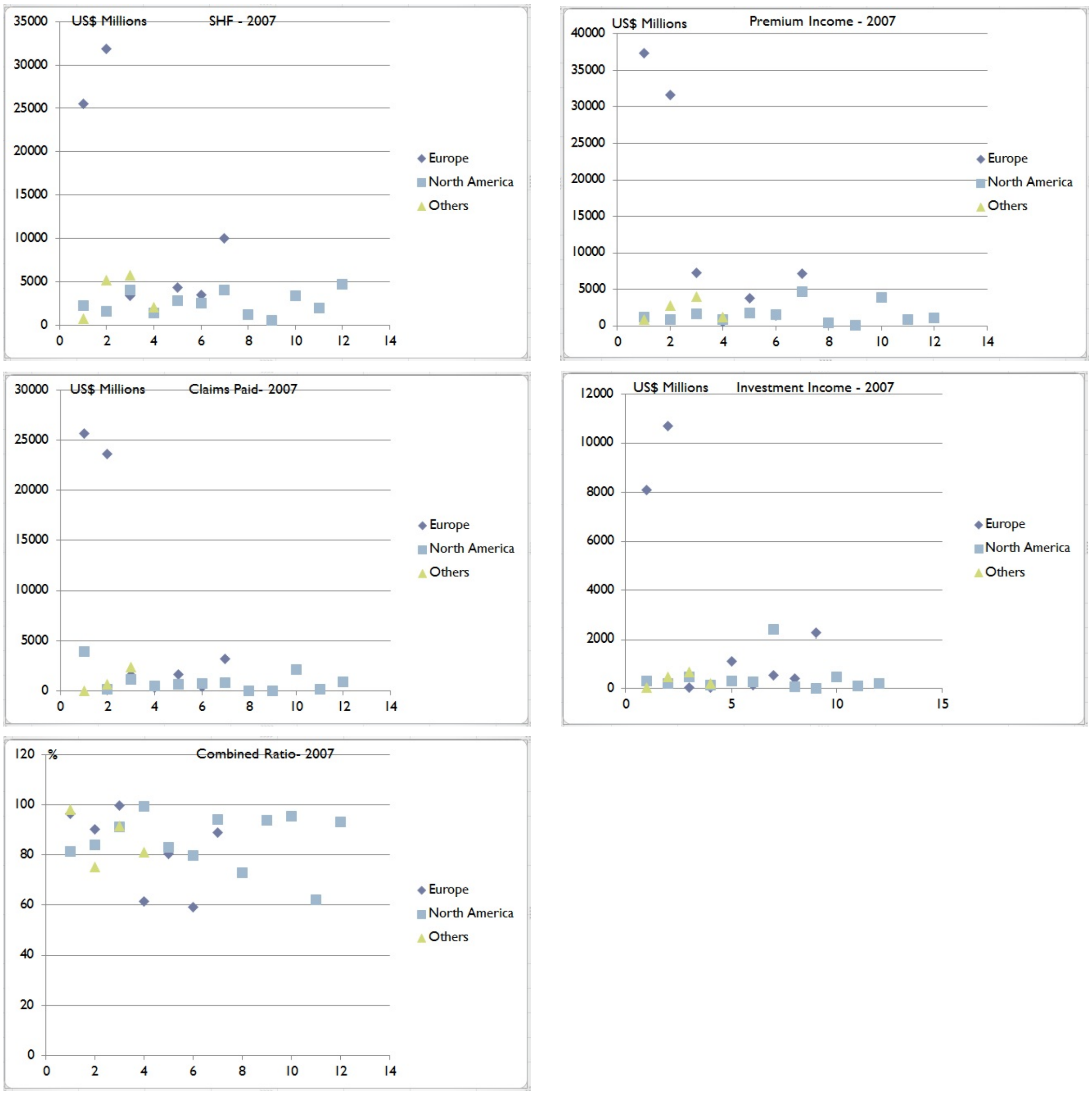
In the previous sections, the data collected for all of the 23 (re)insurance companies are summarized. In order to observe the companies statistics according to the region the company belongs, a regional categorization is done among the companies. Even though especially the larger companies have branches all around the globe, the region of a company is taken as where it is established. The regions of these companies are determined as North America, Europe and other regions, such as Bermuda or Turkey. Table 4.2 shows the regions that the companies are belonged;

Table 4.2: Regions of (Re)insurance Companies

Company	Region
Allied World	North America
Alterra	North America
Arch Capital	North America
Argo Group	North America
Aspen	North America
Axis Capital	Others
Endurance Spec.	North America
Everest Re	Others
Fairfax	North America
Flagstone Re	North America
Hannover Re	Europe
Maiden	North America
Milli Re	Others
Montpelier Re	Europe
Munich Re	Europe
Partner Re	Europe
Platinium	Others
Renaissance	Europe
Swiss Re	Europe
Transatlantic	North America
Validus	North America
White Mountains	North America
XL Group	Europe

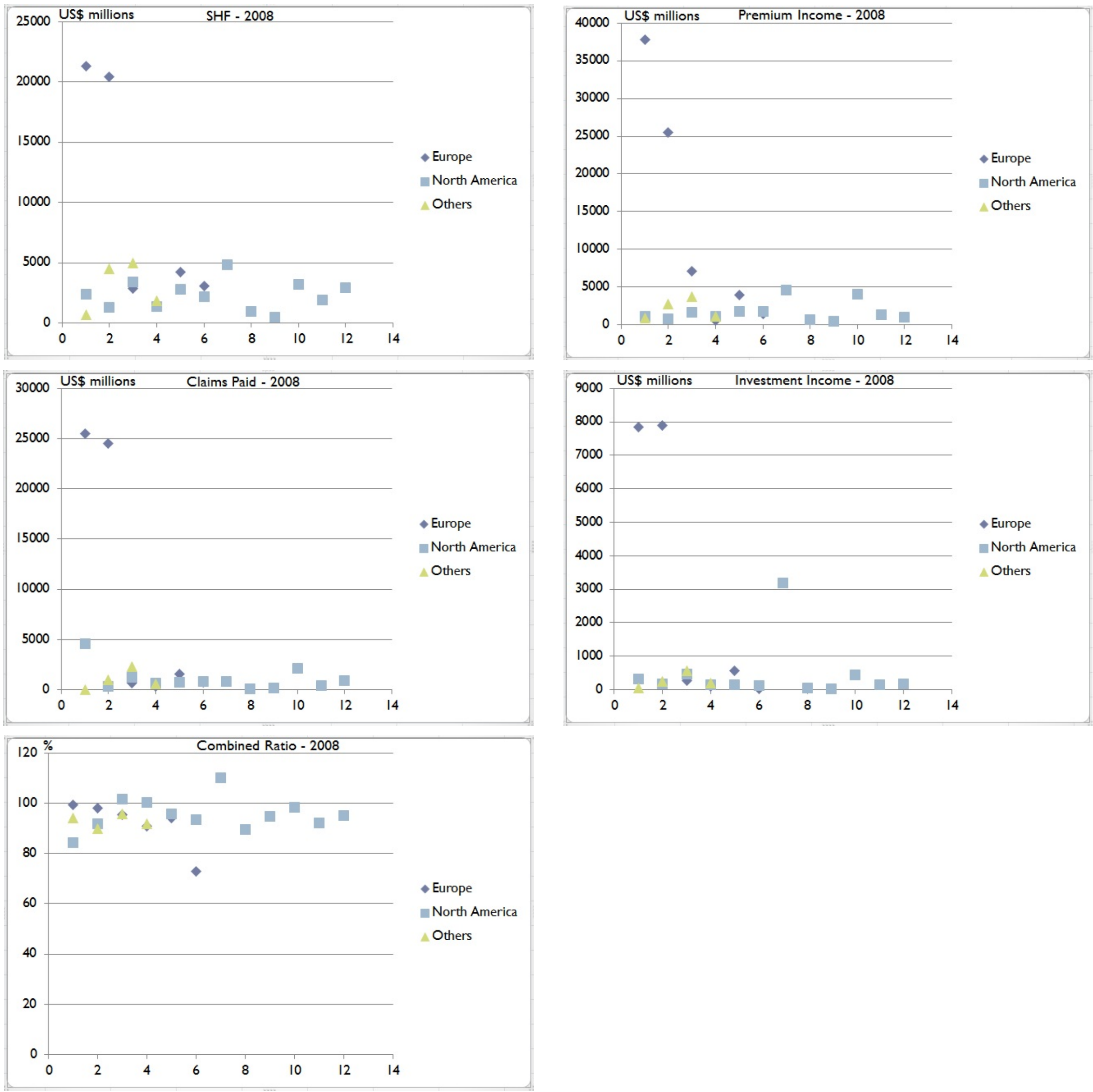
If the year 2007 is inspected, it can be seen that the European companies have larger values for the parameters except combined ratio. However, it should be noted that very large and worldwide companies such as Munich Re and Swiss Re are included in the European region. European companies have a larger premium income, claims paid, investment income and shareholders fund in average. As to combined ratios, North American companies have a larger combined ratio than the European's, which indicates the European companies made more profit than North American's and the considered other regions. This is also consistent with the fact that European companies collected more premium than North American companies. In table 4.3, the regional categorization of companies are given,

Table 4.3: Figures for Regional Categorization of Parameters for year 2007



In order to see the effect of the catastrophes in 2007 and 2008 and the reinsurance capacity decrease in 2008, the same companies are inspected for the year 2008, which yields the results in table 4.4;

Table 4.4: Figures for Regional Categorization of Parameters for year 2008



Comparison of tables 4.3 and 4.4 indicates that shareholdes fund , premium income and investment income have decreased significantly in the year 2008, while there is a slight increase in claims paid. Moreover, while most of the combined ratios in 2007 are around 80%, in 2008 this percentage increased and came closer to 100% for most of the companies.

4.1.8 A Comparison of Turkish and World Reinsurance Market

Even though the reinsurance companies in Turkey are much smaller than the global reinsurers such as Munich Re or Swiss Re, the reinsurance market in Turkey also fluctuates according to the global reinsurance market. The price fluctuations are similar, however the changes in reinsurance pricing are not exactly the same.

In order to compare the two markets, observe the reinsurance price changes in the following figures. Figure 4.11 shows the year on year rate change and cumulative rate online index from the year 1989 to 2010 for the global property catastrophe business [19],

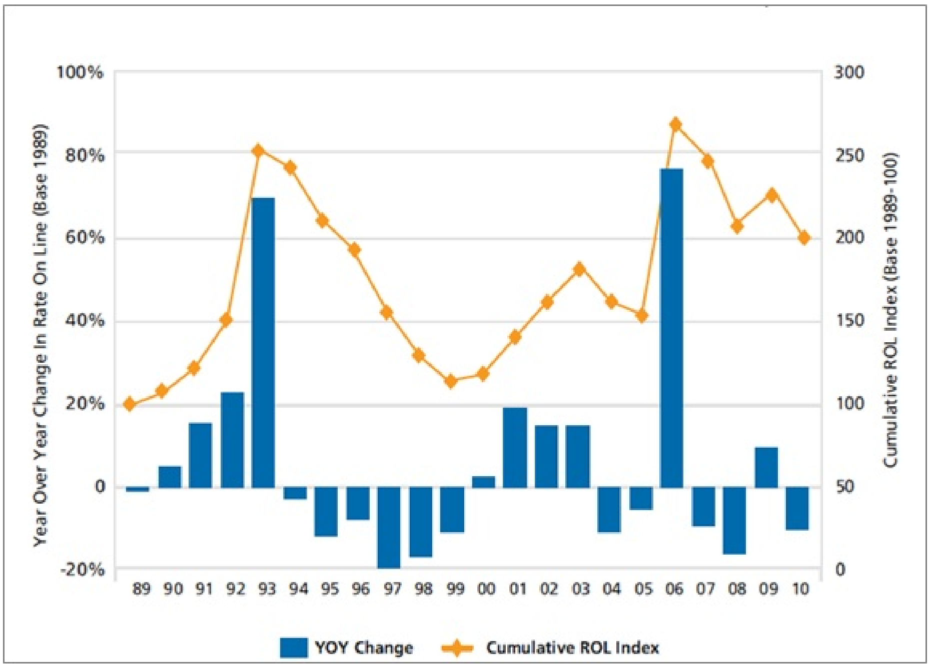


Figure 4.11: Cumulative Property Catastrophe Reinsurance Pricing between 1989-2010 (Source:Guy Carpenter, 2010)

Reinsurance prices increased in 2006, even though the reinsurance capacity did not suffer a decrease. The increase in the prices was a result of Hurricane Katrina, which caused huge losses and most of these losses were not insured. Moreover, prices increased in 2009, as a result of the diminished capacity of the previous year. From here it can be seen that reinsurance market capacity and reinsurance pricing are in a direct relationship.

Figure 4.12 represents a comparison between Turkish and world reinsurance pricing for the years 2006-2011. In catastrophe excess of loss reinsurance contracts, a standard measure that is used is taking the mid point of each layer as measured against the key zone exposure of the company or companies vs the premium which is measured as rate on line. For example, if a key zone exposure of €3 billion and a layer of €100 million in excess of €100 million and an annual premium of 1.5% rate on line is taken, the layer mid point is 50% of the limit plus the excess, which is a total of €150 million in this case. The mid point against the key zone exposure yields €150 million / €3 billion = 5%. Hence, by plotting 5% against a ROL of 1.5% produces a single point for that layer and this process is repeated until a curve is produced. In Figure 4.12 [20], the various mid points of a number of Turkish reinsurers are represented with a blue diamond, and are plotted against the market average of all catastrophic excess of loss programmes.

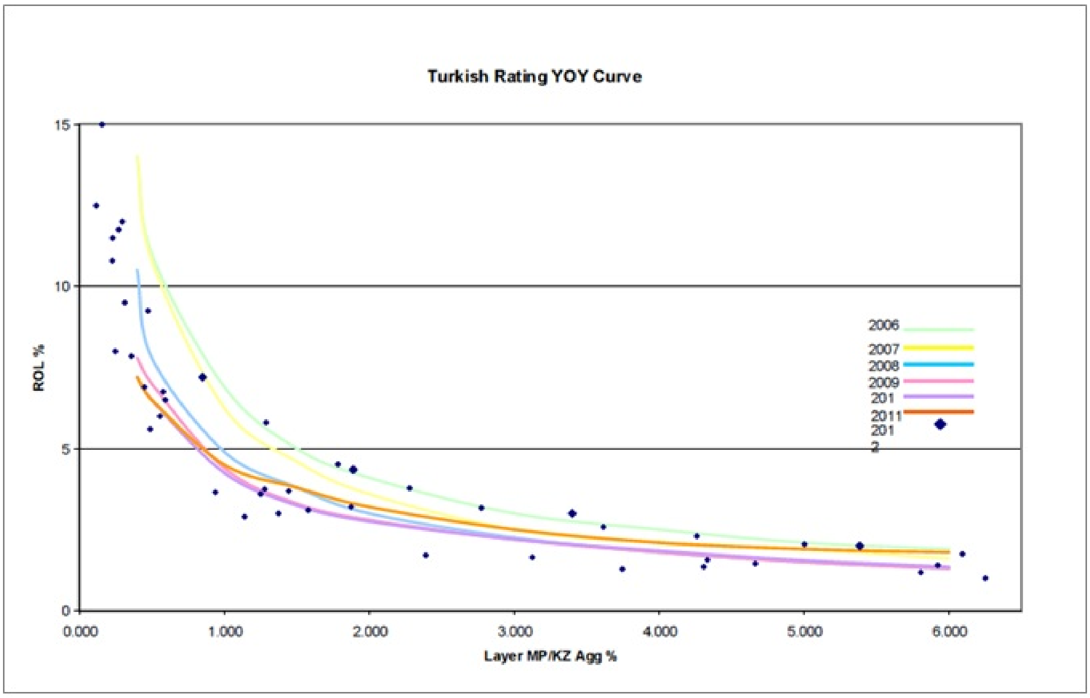


Figure 4.12: Turkish Catastrophe XL Rating Comparison (Source: Guy Carpenter)

Figure 4.12 is consistent with Figure 4.11, since the reinsurance prices are the highest in 2006 after Hurricane Katrina, slightly reduced in 2007 and continued to reduce until 2011. It can be seen that in 2011, for higher layer mp/key zone percentage, premium ROL is greater than the other years, except for 2006. The layer mid point for Turkish companies are below the market average and even though the ROL is around the curve of the global market, premiums are slightly less. This is reasonable, since the reinsurance process is done with smaller amounts when compared with the global market.

4.1.8.1 Comparison of Milli Re with Large Reinsurers

Milli Re is one of the prime reinsurers in Turkey and was set up by Türkiye İş Bankası to operate the compulsory reinsurance system and commenced operations on July 19, 1929. According to the ranking made by S&P in terms of net premiums generated in 2008, Milli Re ranked 61st among 150 largest reinsurance companies. In recent years, besides the local business Milli Re also focused on writing business from overseas markets and opened a regional branch in Singapore, which helped the development of the company. Milli Re also took on the management of Turkish Catastrophe Insurance Pool (TCIP) in the years between 2000-2005 [21].

Despite these positive facts, Milli Re is still a small company when compared with the world-wide reinsurers, such as Munich Re and Swiss Re. As Milli Re is new on writing business from foreign markets, but still the values for the considered market variables in this thesis like premiums written or shareholders fund are much lower than larger reinsurers. Figures 4.13 and 4.15 represent the shareholders fund and premium income comparison between Milli Re and 2 European companies (Munich Re and Swiss Re) and a North American company (Fairfax) for the years 2001-2010 (in $millions),

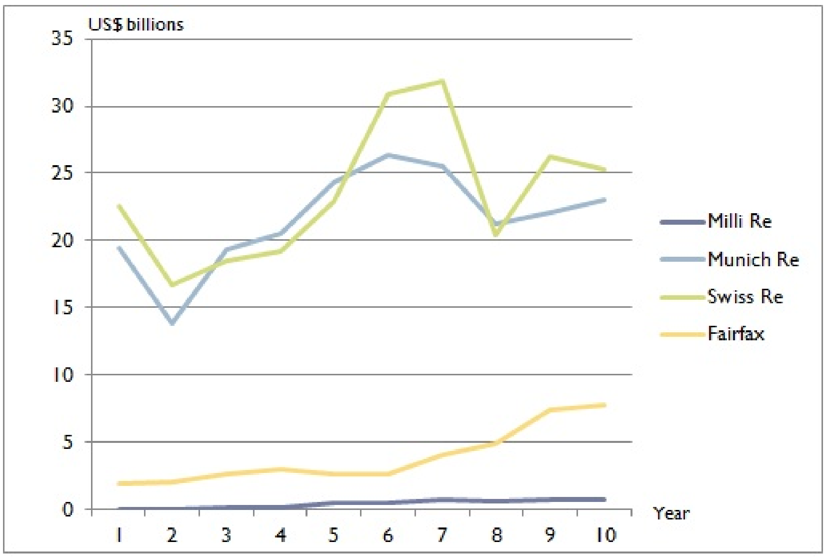


Figure 4.13: Shareholders Fund Comparison of Milli Re for years 2001-2010

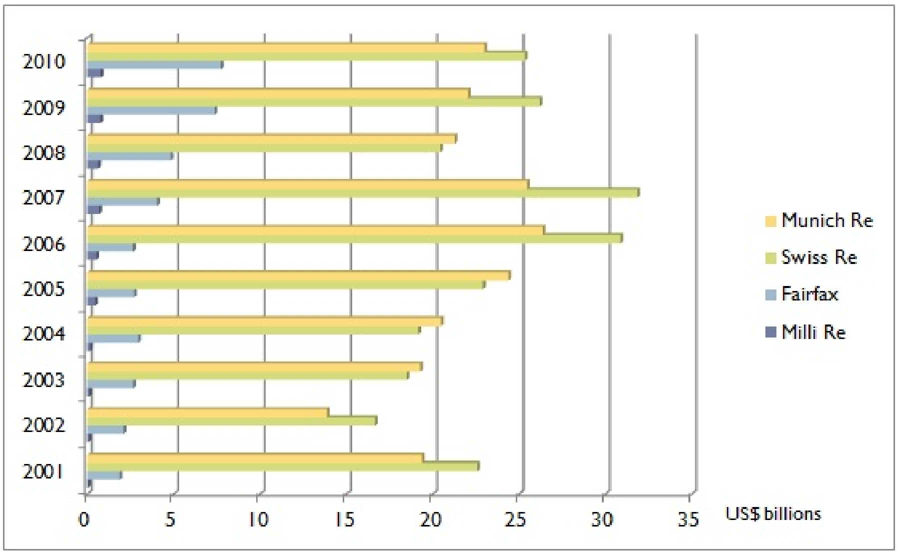


Figure 4.14: Shareholders Fund Values for years 2000-2010

From Figures 4.13 and 4.14, it can be observed that shareholders fund values are much smaller for Milli Re. Larger reinsurers such as Munich Re and Swiss Re reaches around 32 billion$ in shareholders fund, where this value is under 1 billion$ for Milli Re. However, unlike the large companies Milli Re steadily increased it shareholders fund in the last 10 years. The value for shareholders fund in 2001 was 52 million TL for Milli Re and in 2010 this value increased to 800 million TL.

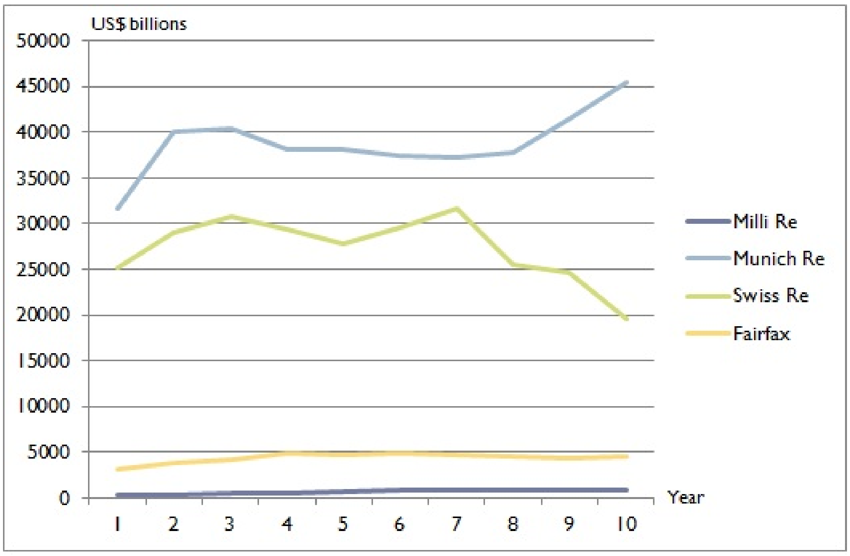


Figure 4.15: Premium Income Comparison of Milli Re for years 2001-2010

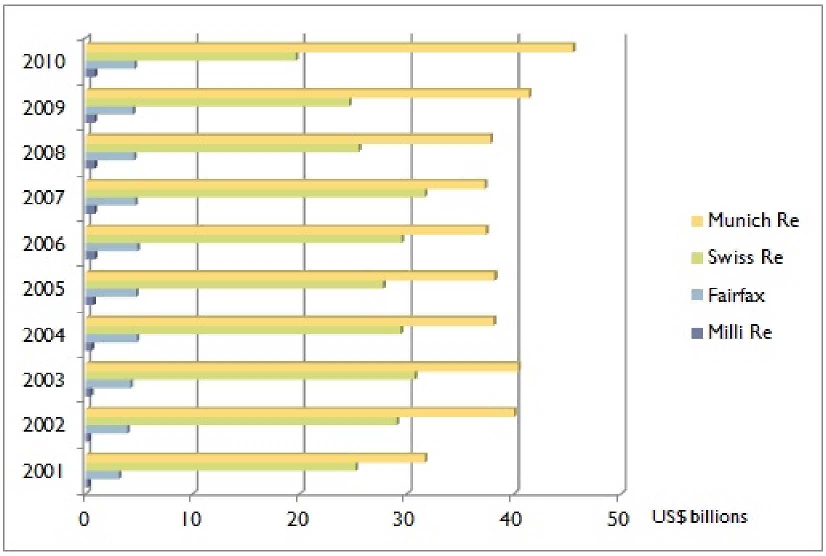


Figure 4.16: Premium Income Values for years 2000-2010

It can be observed from Figures 4.15 and 4.16 that Swiss Re had a decrease in premium income after 2007, where Munich Re had an increase after the same year. Fairfax held its premium income around 5 billion$ and Milli Re increased its premium income in the last 10 years. In 2001, Milli Re had a premium income of 257 million TL and this value increased to 855 million TL.

Figure 4.17 shows the combined ratio comparison of Milli Re with the same companies for the years 2001-2010,

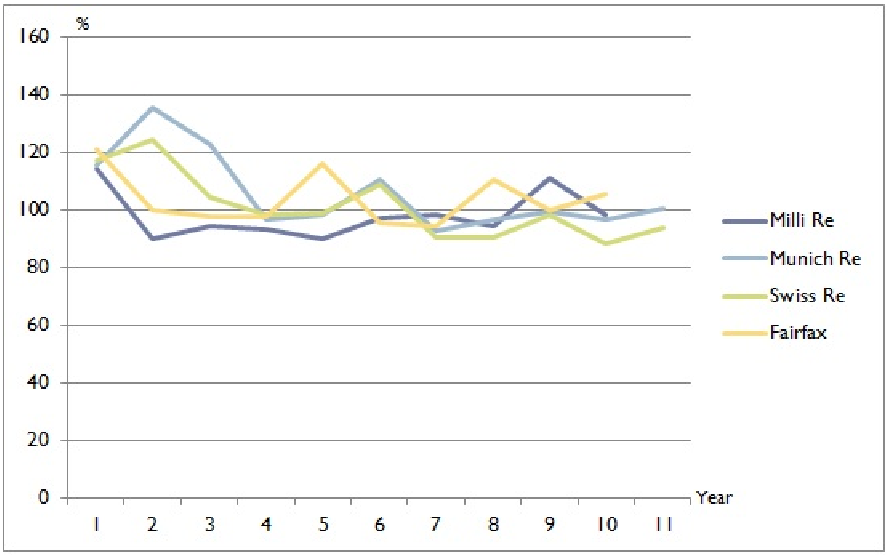


Figure 4.17: Combined Ratio Comparison of Milli Re for years 2001-2010

Even though Milli Re is a smaller company than the others, it has a better profit margin if combined ratio is taken as a measure. Up to the year 2008, Milli Re has less combined ratio than Munich Re and Swiss Re. However, it should be noted that investment income is not included in the calculation of combined ratio and the investment income of companies such as Munich Re are generally much larger than shareholders fund and premium income combined of Milli Re.

4.1.9 Impact of Reinsurance Companies on Shareholders Fund and Reinsurance Capacity

SHF is one of the most important factors that affect reinsurance market capacity. As the SHF of a company increases, it adds to the total available market capital. It is important to analyse the effects of companies on shareholders fund, since by this way the companies have the largest effect both on shareholders fund and reinsurance capacity can be observed.

The largest impact on shareholders fund by the 23 companies in the thesis are; Munich Re, Swiss Re, XL Group and Partner Re. The SHF of these companies constitute 66% of the total SHF between the years 2000-2010 and 50% of the SHF for the year 2010. In order to observe how the SHF of the top 4 companies influence reinsurance capacity, a regression analysis is done with reinsurance capacity as response and the SHF of the companies as predictors.

Table 4.5: Regression: $Capacity$ vs. SHF_{Munich}, SHF_{Swiss}, SHF_{XL}, $SHF_{Partner}$

Predictor	Coef	SE Coef	T	P
Constant	-30360	17297	-1.76	0.130
SHF_{Munich}	3.243	1.348	2.41	0.053
SHF_{Swiss}	-1.7541	0.8766	-2.00	0.102
SHF_{XL}	4.106	2.599	1.58	0.165
$SHF_{Partner}$	14.627	1.842	7.94	0.000

The R^2 for the regression is 98.8% and with an F-value of 124.44, it is a good model. Even though the p-value of XL Group is slightly above 10%, the regression analysis gives an idea of how the four companies with the most SHF affect reinsurance capacity. The mean of the residuals of the regression is close to zero, and are normal. Figure 4.18 shows the residual plots for the regression;

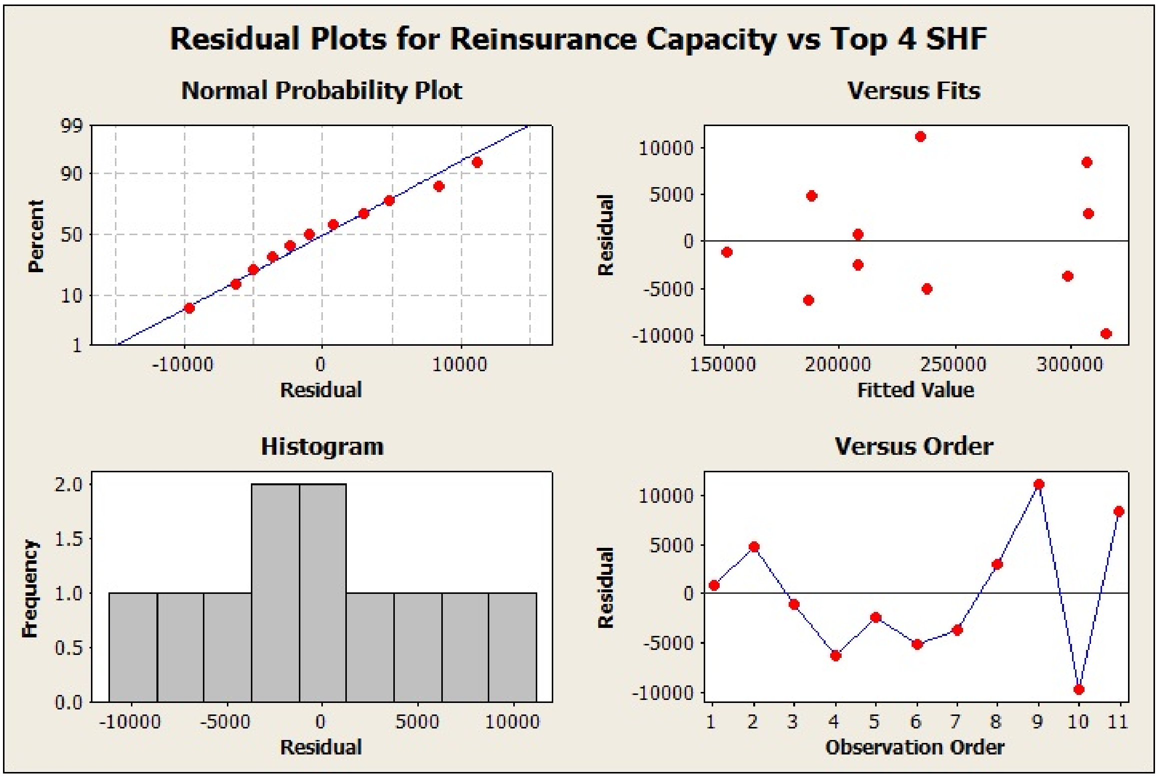


Figure 4.18: Residual Plots 1

Moreover, from the coefficients of the predictos it can be observed that the company that has the most effect on reinsurance capacity with its SHF is Munich Re, since it has the largest total SHF among the reinsurance companies.

4.2 Analysis

In order to understand the relationship between these parameters and which of the parameters interpret capacity the best, regression analysis is proposed. First, the relationships between capacity and these variables are inspected, in order to see which variable(s) interpret capacity with the most accuracy. However, to establish regressions with capacity, the market data should be annualized to obtain the same number of data with capacity. The annualization is done by summing the variables for each individidial company for the particular year.

Then, the variable that interpret capacity with the most accuracy is put in a regression as response, in order to inspect the relation with the other market parameters. The most accurate predictor in this regression is then forecasted, and the regression is used to obtain a future value for the response.

After the forecast, the future values can be fitted in the regression equation with capacity and therefore a future value for capacity can be estimated.

At this point, there are two approaches that can be considered. The first approach is to forecast the values of the variables that represent the parameter which interprets capacity the best for each individual company. Then, the forecasted values should be summed in order to obtain an annual value for the variable. After this process, the annualized variables can be put in the regression equation to estimate next years capacity. Hence, in the *Regression Analysis of Market Variables* section, firstly the regressions are not considered between the annualized values but over the original data. The second approach that can be considered is to establish the regressions between the annualized data. This method is shorter than the first approach, since it is not needed to forecast the values for each individual company. Moreover, with this number of data an accurate model cannot be established for every individual company. Therefore, regressions with the annualized data are considered and used to observe the increase/decrease in capacity later in the section.

In the *Time Series Modelling* section, the annualized values of the variables that represent the parameter which interprets capacity the best are forecasted using an auto-regressive model. With this forecasting, the value can be put in a regression to estimate a value for capacity. The aim of this estimation is not to accurately obtain a value for capacity, but to observe an increase or decrease in the value of capacity to ease the decision whether to purchase additional securities or not. Moreover, also an AR(1) model for capacity is established in order to create a comparison with the results of the first AR(1) model.

4.2.1 Descriptives and Associations

The data that has been chosen to establish a model for capacity are determined as; shareholders fund, premium income, claims paid, investment income and combined ratio. In order to understand each data, descriptive statistics are given for both the original market data and annualized market data in Tables 4.6 and 4.7 respectively. SHF stands for shareholders fund and CR for combined ratio and the annualized variables are denoted as SHF_A, $Prem_A$, $Claims_A$, Inv_A and CR_A,

Table 4.6: Descriptive Statistics of Original Market Variables

Variable	Mean	Mode	Median	St. Dev.	Min.	Max.	n
SHF	5,374	2,817, 3,349	2,817	6,933	466	31,857	173
Premium	5,877	1,633	1,676	10,171	110	45,500	173
Claims	3,593	4.1	809	7,103	2	30,600	173
Investment	1,224	81, 134.3	262	2,434	9	13,600	173
CR	92.88	96.7, 98.4	94.1	19.22	21.2	200.7	173

Table 4.7: Descriptive Statistics of Annualized Market Variables

Variable	Mean	Median	St. Dev.	Min.	Max.	n
SHF_A	107,396	105,845	32,001	64,152	147,644	11
$Prem_A$	119,250	124,638	13,103	98,532	139,463	11
$Claims_A$	61,570	62,007	9,843	48,436	74,751	11
Inv_A	22,411	21,930	5,602	16,087	33,668	11
CR_A	96.79	93	14.08	84.04	127.88	11

It can be seen from Table 4.6 that there are big differences between the minimums and the maximums of the parameters. This kind of difference is normal, since smaller companies in the earlier 2000's are considered as minimums and the latest data of the larger companies are considered as maximums, which creates the difference. The medians for premium income is 1676, that is equal number of data exists on both sides of this number. The median for claims are higher, however in the maximum value and mean for premium income are much higher than claims paid, which indicates claims are covered in overall. For the annualized market data, the standard deviations are higher than the original market data, which is caused by the increasing numbers by taking sums. Since there are only 11 data available for annualized market data, the modes do not exist.

The correlations of the parameters are measured by the Pearson's correlation coefficient, which indicates whether two variables have a linear relationship or not. The value of coefficient is between -1 and 1 and closer it gets to 1 means that the variables are linearly related and the data points fall on a line. If the value is close to 0, a linear relationship does not exist between the variables. Negative correlation is causes if on variables is increasing while the other is decreasing. However, a significant nonlinear relationship can exist when the correlation is close to 0. Table 4.8 gives the correlation matrix of the market parameters,

Table 4.8: Correlation Matrix of Market Variables

Variable	SHF	CR	Premium	Inv. Income	
CR	0.182				
Premium	0.932	0.222			
Inv.Income	0.934	0.214	0.955		
Claims	0.944	0.197	0.970	0.959	
Variable	SHF_A	CR_A	$Prem_A$	Inv_A	$Claims_A$
CR_A	-0.515				
$Prem_A$	0.887	-0.550			
Inv_A	0.884	-0.412	0.861		
$Claims_A$	0.949	-0.466	0.858	0.827	
Capacity	0.963	-0.416	0.773	0.854	0.888

As it can be seen from the first part of Table 4.8, the variables except for combined ratio have a correlation coefficient close to 1, which indicates a linear relationship. The p-values of all of the correlations are either 0 or less than 2%. In the second part of Table 4.8, correlations are similar to the first part, except they are slightly less than the first part. Correlations with capacity are also high, which makes sense since capacity relates to these parameters and changes accordingly.

4.2.2 Regression Analysis of Market Capacity

In this section, the relationship of market capacity with the other market variables are analysed according to the collected data. In order to be able to do a regression, the variables used in the regression should have the same number of data. A vast number of data is available for the market variables, however there is only 11 data available for the market capacity, which is an annual capacity data for years through 2000-2010. To have the same number of data with market capacity, other market variables are also made into an annual form. That is, the total sum of the variable for all of the companies for each year is taken in the case of shareholders fund, premiums earned, claims paid and investment income. For example, for the year 2005, the shareholders fund in 2005 of all of the companies are summed, which gives a total value of 105.8 $ billion. For combined ratio, the average of combined ratios of all companies is taken for the particular year, which is 116.9% for the year 2005. The annualized variables which are shareholders fund, premiums earned, claims paid, investment income and combined ratio are denoted as SHF_A, $Prem_A$, $Claims_A$, Inv_A and CR_A respectively.

Table 4.9 shows the result for the first regression done with capacity as the response variable in the regression. All of the market parameters are used as predictors,

Table 4.9: Regression: $Capacity$ vs. SHF_A, $Prem_A$, $Claims_A$, Inv_A, CR_A

Predictor	Coef	SE Coef	T	P
Constant	187896	99072	1.90	0.116
SHF_A	2.5370	0.5938	4.27	0.008
$Prem_A$	-1.7541	0.8766	-2.00	0.102
$Claims_A$	-1.036	1.571	-0.66	0.539
Inv_A	1.370	1.981	0.69	0.520
CR_A	229.6	411.4	0.56	0.601

The analysis of variance for this regression is given in Table 4.10.

Table 4.10: ANOVA table: $Capacity$ vs. SHF_A, $Prem_A$, $Claims_A$, Inv_A, CR_A

Source	DF	SS	MS	F	P
Regression	5	33,125,762,437	6,625,152,487	30.30	0.001
Residual Error	5	1,093,331,305	218,666,261		
Total	10	34,219,093,742			

The R^2 value of the regression is 96.8%, which indicates that the model is a good fit for the data. However, only the R^2 value is not enough to judge if the model is a good fit or not. In fact, there are a number reasons to not accept this regression as a good one. Firstly, the p-value should be observed. The p-value in the regression table is calculated from the data and represents the probability of incorrectly rejecting the null hypothesis, with a confidence level of 95%. The null hypothesis in this case is that the coefficient for the predictor is equal to 0, thus has no effect to the response with the set of predictors in the regression. In this regression, the p-values for the predictors except SHF_A are greater than 10% which is very high and means the predictor is not a good fit in the regression and has a high probability of rejecting the null hypothesis. A low p-value close to 0 indicates that the predictor is a good fit for the model.

The T-value in the regression should be greater than 2 or less than -2 for predictors with negative coefficients for a p-values less than 0.05. As it can be seen, the t values are not greater than 2 and, which suggests the predictors are not good for the regression. Although the p-value of the regression in the ANOVA table is 0.001, it does not have significance because the p-values for the predictors are high.

The F-value in the ANOVA table is calculated by dividing the factor mean square by the error mean square and when the data presents evidence against the null hypothesis the magnitude of the statistic becomes large. Thus, a higher F-value agrees with the rejection of the null hypothesis, which is good for the model. Even though the F-value for the regression is 30.30, it is not proof for a good fit since other statistics of the model do not agree.

The goodness of fit can also be seen from the graphs of residuals, since the residuals should be normally distributed or close to normal for a good fit. Figure 4.19 shows the residual plots for the regression, which can be seen that the residual are not close to normal distribution and the mean is not zero;

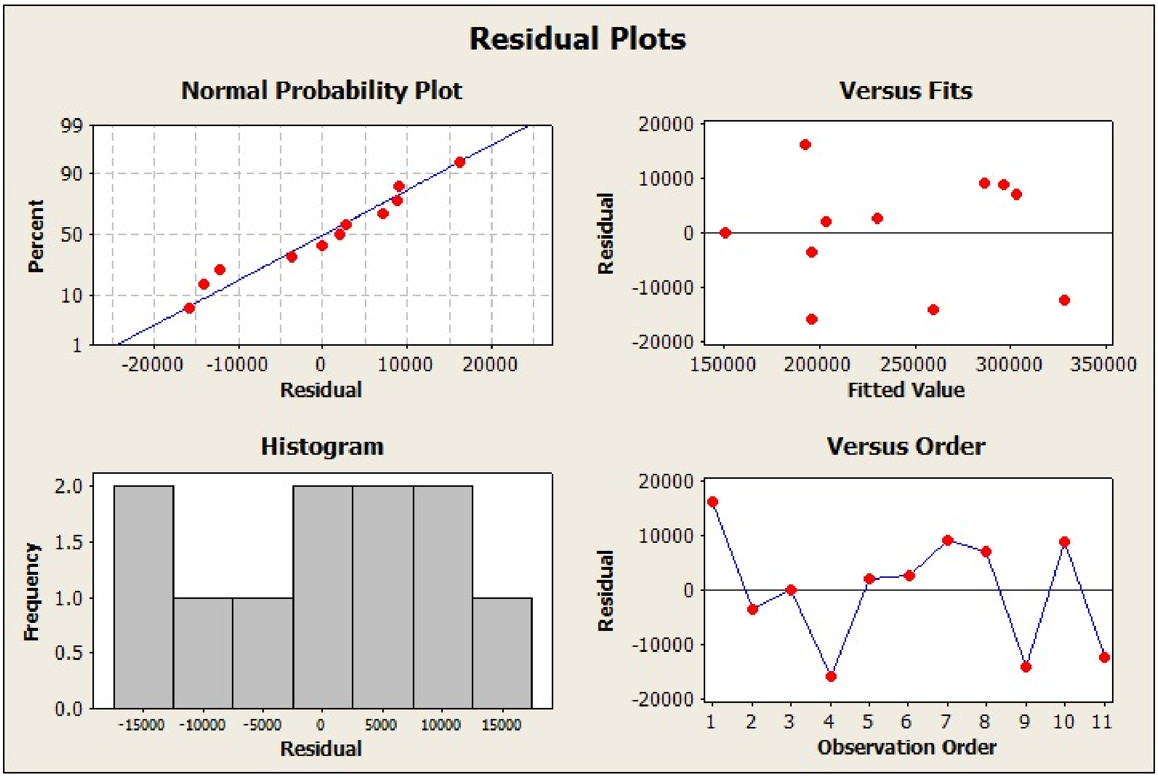


Figure 4.19: Residual Plots 2

A regression that is logical cannot be established with the data of these predictors, hence in order to make the model more consistent some of the predictors can be removed. In the next regression, the predictors Inv_A and CR_A are removed and the regression results as noted in Table 4.11.

Table 4.11: Regression: $Capacity$ vs. SHF_A, $Prem_A$, $Claims_A$

Predictor	Coef	SE Coef	T	P
Constant	216414	70871	3.05	0.018
SHF_A	2.6708	0.4888	5.46	0.001
$Prem_A$	-1.6482	0.7181	-2.30	0.055
$Claims_A$	-1.078	1.432	-0.75	0.476

The R^2 value is 96.2% for this regression. However, again it is not a good model even though the p-values for SHF_A and $Prem_A$ are low since the p-value for $Claims_A$ is significantly high. The F-value for the regression is 58.35 which is higher than the first regression and the p-value is 0.000, but since the predictors are not good fits these values do not have any importance.

Also, it can be seen from Figure 4.20 that the residuals do not have a mean close to zero and are not normally distributed,

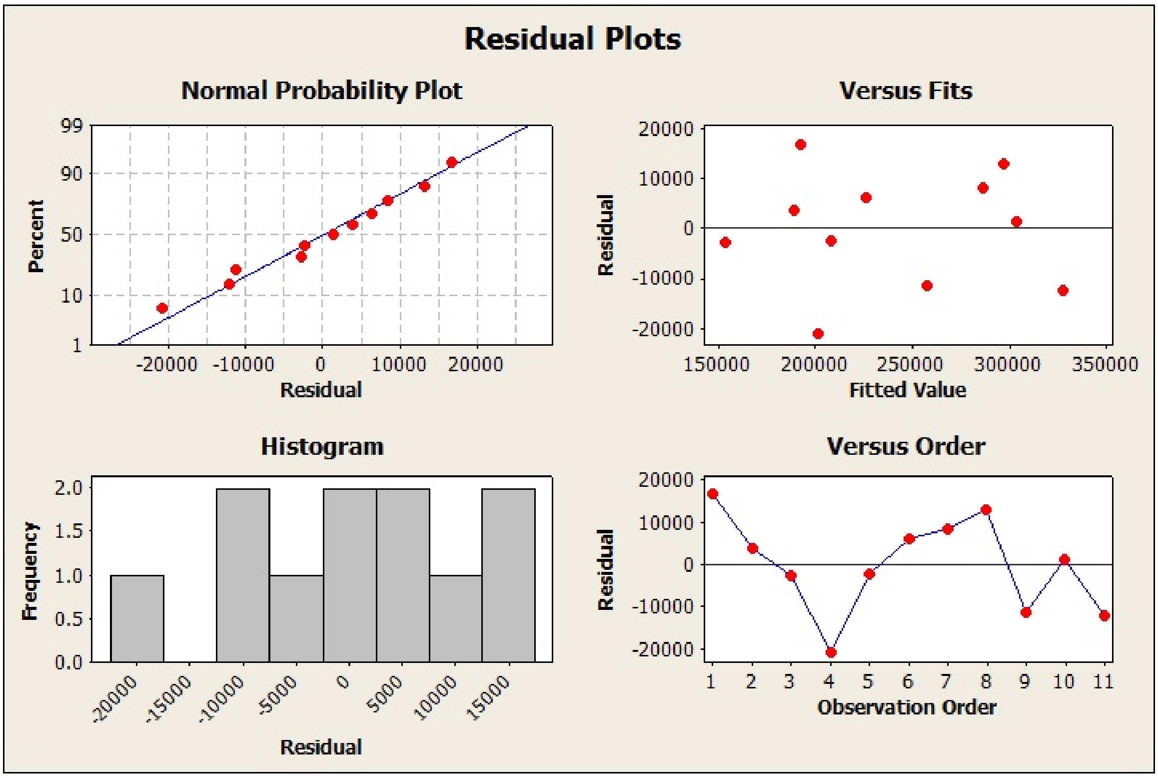


Figure 4.20: Residual Plots 3

After examining various kind of regressions with market capacity as the response, it is concluded that the best regression is fitted with the predictor SHF_A, which has an R^2 of 92.8%. Tables 4.12 and 4.13 represent the regression between capacity and SHF_A and the ANOVA table respectively.

Table 4.12: Regression: $Capacity$ vs. SHF_A

Predictor	Coef	SE Coef	T	P
Constant	51,178	18,308	2.80	0.021
SHF_{annual}	1.7605	0.1640	10.74	0.000

Table 4.13: ANOVA table: *Capacity* vs. SHF_A

Source	DF	SS	MS	F	P
Regression	1	31,740,716,398	31,740,716,398	115.26	0.000
Residual Error	9	2,478,377,344	275,375,260		
Total	10	34,219,093,742			

The regression equation is as follows,

$$Capacity = 51,178 + 1.76SHF_A. \tag{4.3}$$

It can be seen that the T-value for the constant is greater than 2 and for the predictor it is 10.74, which means the predictor is a good fit for the model. Also the p-value for the predictor is 0 and for the constant it is around 2%. The F-value in the ANOVA table is 115.26, which is greater than the previous regression and suggests that the model is consistent with a p-value of 0. The residual plots for this regression are given in Figure 4.21,

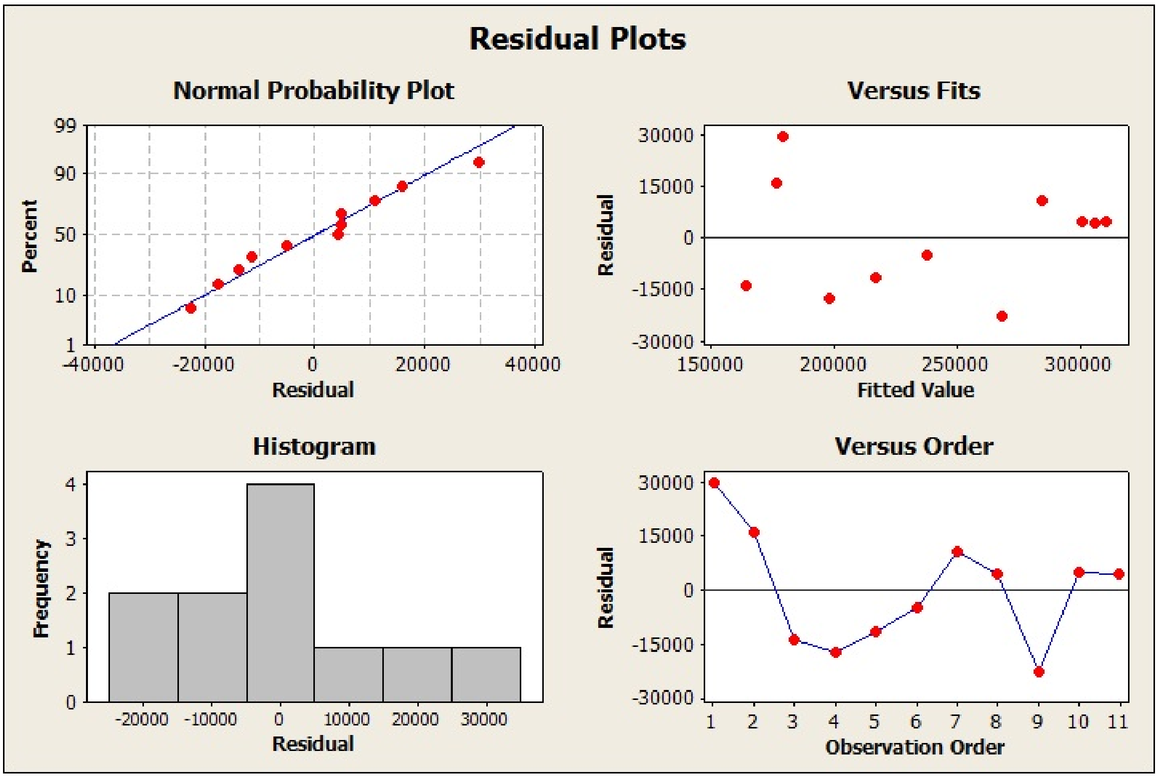


Figure 4.21: Residual Plots 4

The normal probability plot and the histogram that the residual are normally distributed and the mean is close to 0. This and the previous facts suggests that this model is significant and interprets capacity well.

In the capacity regression, the effect of the increased frequency of catastrophes should also be included. To add this effect in the regression, a dummy variable is included as trigger, where for the years that number of catastrophes increased results with one, and the years with average number of catastrophes result with 0. In the years 2000-2010, the number of catastrophes increased in years 2002 and 2008, which can also be seen from the market capacity movement. Hence, for years 2002 and 2008 the dummy variable is 1, where for other years it is equal to 0. The regression with capacity as response and SHF and trigger as predictors has an R^2 value of 95% and is the following:

$$Capacity = 62,054 + 1.70SHF_A - 23,224Trigger. \quad (4.4)$$

Tables 4.14 and 4.15 show the regression values and analysis of variance, respectively.

Table 4.14: Regression: $Capacity$ vs. $SHF_A, Trigger$

Predictor	Coef	SE Coef	T	P
Constant	62054	16657	3.73	0.006
SHF_A	1.6986	0.1445	11.75	0.000
$Trigger$	-23224	11433	-2.03	0.077

Table 4.15: ANOVA table: $Capacity$ vs. $SHF_A, Trigger$

Source	DF	SS	MS	F	P
Regression	2	32,584,007,249	16,292,003,625	79.71	0.000
Residual Error	8	1,635,086,493	204,385,812		
Total	10	34,219,093,742			

This regression has better T and p values for the predictor SHF_A and the values for the trigger are also acceptable. Even though the F-value for the regression is less than the previous regression, the effect of the increasing frequency of catastrophes is included. Also, the negative coefficient of trigger is logical, since if trigger is equal to 1, then the frequency of catastrophes have increased and capacity should be less.

Since the best predictor for capacity is the annualized SHF, the relationship of SHF with other variables should be inspected so that the variables that represent SHF can be forecasted for the next year and by using the forecasted parameters in the regression equation, next years capacity can be estimated.

4.2.3 Regression Analysis of Market Variables

In the previous section, SHF_A is determined as the best predictor for market capacity. Therefore, in the regressions in this section, firstly the original data of the market variables are used, which is all the data that has been collected from the companies and SHF as response. After SHF is used as response in the regression, SHF_A is considered as response and thus also the predictors are annualized.

4.2.3.1 Regressions with Original Market Data

In the first regression, all of the variables except for SHF are included in the predictors, which does not give a consistent model. The regression has an R^2 value of 90.2%, which is insignificant because of the values of the regression in Table 4.16.

Table 4.16: Regression: SHF vs. $Prem$, $Claims$, Inv, CR

Predictor	Coef	SE Coef	T	P
Constant	2503.1	832.4	3.01	0.003
$Prem$	0.10487	0.07346	1.43	0.155
$Claims$	0.4812	0.1097	4.39	0.000
Inv	0.9051	0.2592	3.49	0.001
CR	-6.266	8.971	-0.70	0.486

Even though this regression has an F-value of 386.54, it is not a good regression in overall because of the high p-value of combined ratio and the negative T-values. In addition, the residual plots indicate that the residuals are not normal or not close to normal in order to obtain the best fit;

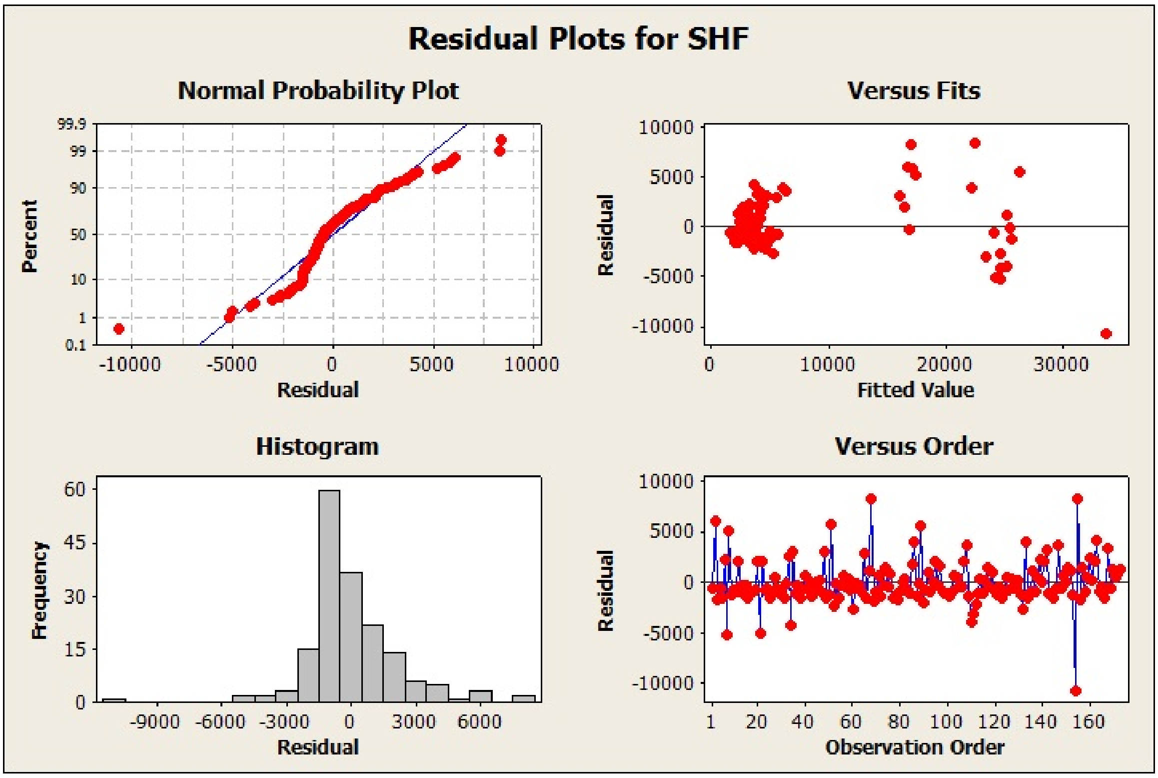


Figure 4.22: Residual Plots 5

The most accurate model is established using *Prem* and *Inv* as predictors, which is expected to be consistent since in practice shareholders fund mostly depends on the companies earnings from premiums and investment income. This regression has a R^2 value of 89%, which is significant if the other statistics are coherent. Tables 4.17 and 4.18 are the statistics for the regression and the ANOVA table,

Table 4.17: Regression: SHF vs. $Prem$, Inv

Predictor	Coef	SE Coef	T	P
Constant	1809.3	205.3	8.81	0.000
Prem	0.31209	0.05834	5.35	0.000
Inv	1.4141	0.2438	5.80	0.000

Table 4.18: ANOVA table: SHF vs. *Prem*, *Inv*

Source	DF	SS	MS	F	P
Regression	2	7,359,608,437	3,679,804,219	688.21	0.000
Residual Error	170	908,982,713	5,346,957		
Total	172	8,268,591,150			

The T-values of the regression are much greater than 2 for both the predictors and the constant, which can be taken as a good sign. All the p-values of the predictors are equal to 0, which also indicated a good model. From the ANOVA table it can be seen that the p-value of the regression is also 0 and the F-value is larger than any other previous regression. From the normal probability plot and the histogram of the residuals for this regression, we can see that the residuals are close to normal distribution (except for a few outliers) and the mean is also close to zero. Hence, this model is the most consistent among the regressions that have been tested with these variables.

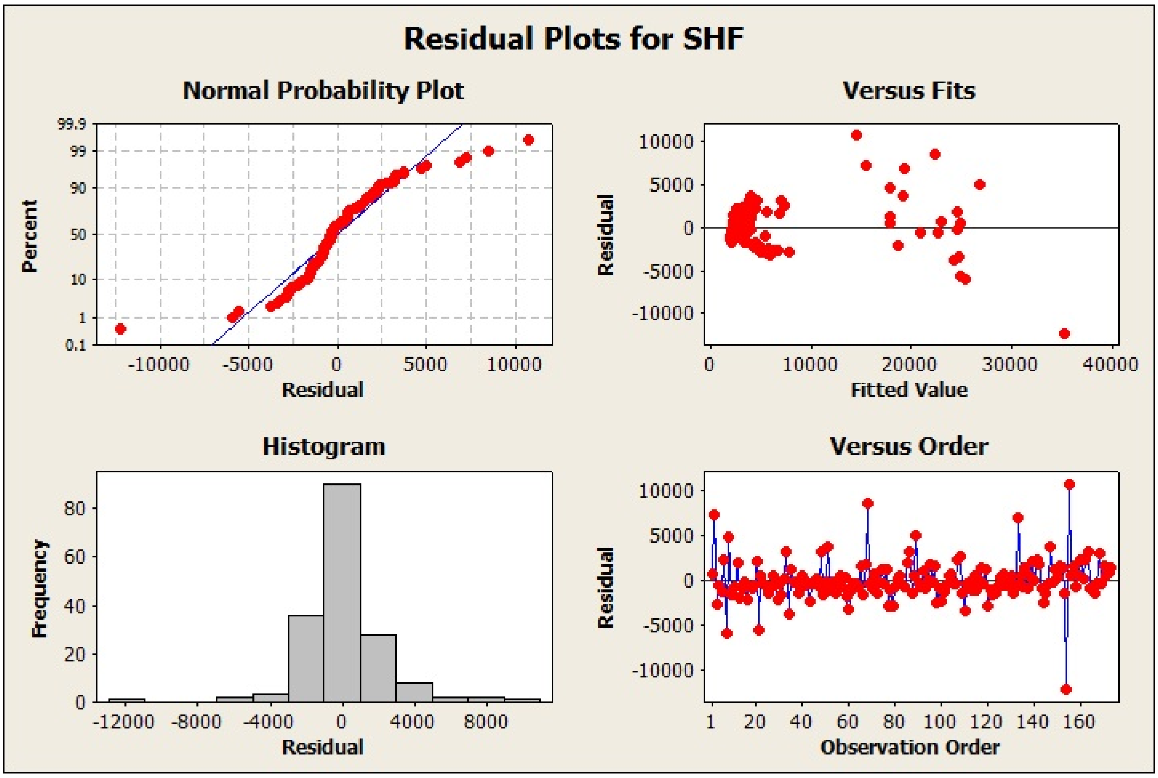


Figure 4.23: Residual Plots 6

Therefore, the regressions among the original data indicate that premium income and investment income are the best predictors for shareholders fund. In order to estimate capacity, these variables should be forecasted for each individual company and the forecasted values can be used in the regression. However, this is not the approach that is applied in this study and can be considered for further study.

4.2.3.2 Regressions with Annualized Market Data

In the first regression, SHF_A is selected as response and all other market variables except for combined ratio are chosen as predictors. Even though the regrssion has an R^2 value of 93.7%, it is not a good model since the p-values for $Prem_A$ and Inv_A are very high, which represents the probability of rejecting the null hypothesis incorrectly. Table 4.19 shows the results of the regression,

Table 4.19: Regression: SHF_A vs. $Prem_A$, $Claims_A$, Inv_A

Predictor	Coef	SE Coef	T	P
Constant	-91508	36145	-2.53	0.039
$Prem_A$	0.3351	0.05276	0.64	0.545
$Claims_A$	2.0734	0.6426	3.23	0.015
Inv_A	1.389	1.141	1.22	0.263

Here, the constant has a negative coefficient and hence has a negative T-value, but it is consistent since it is less than -2. However, the p-values are very high except for $Claims_A$, that indicates this set of predictors does not work. In addition, the residuals have a mean that is not zero or close to zero;

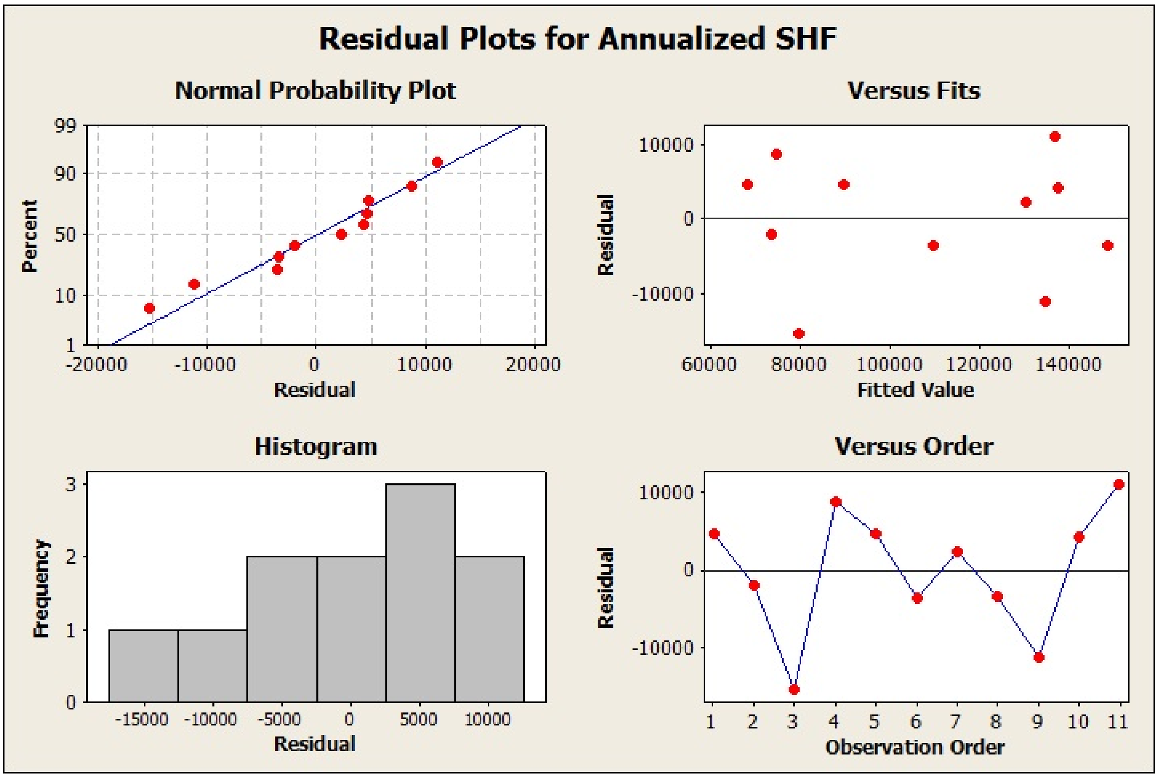


Figure 4.24: Residual Plots 7

In the next regression, $Prem_A$ and Inv_A are considered as predictors. In the regressions with the original data, $Prem$ and Inv were the best predictors for SHF, however this will not be the case for annualized data. The result of the regression is given in Table 4.20;

Table 4.20: Regression: SHF_A vs. $Prem_A$, Inv_A

Predictor	Coef	SE Coef	T	P
Constant	-93445	53314	-1.75	0.118
$Prem_A$	1.1828	0.6750	1.75	0.118
Inv_A	2.668	1.579	1.69	0.130

This predictors for this model also do not provide a good fit, as it can be seen from the p-values. Moreover, the T-values for the constant and predictors are in the interval $(-2, 2)$, which is not a good sign for the regression. From the residual plots, it is obvious that the residuals do not have a mean of 0 and are not normally distributed and decreases the quality of the model;

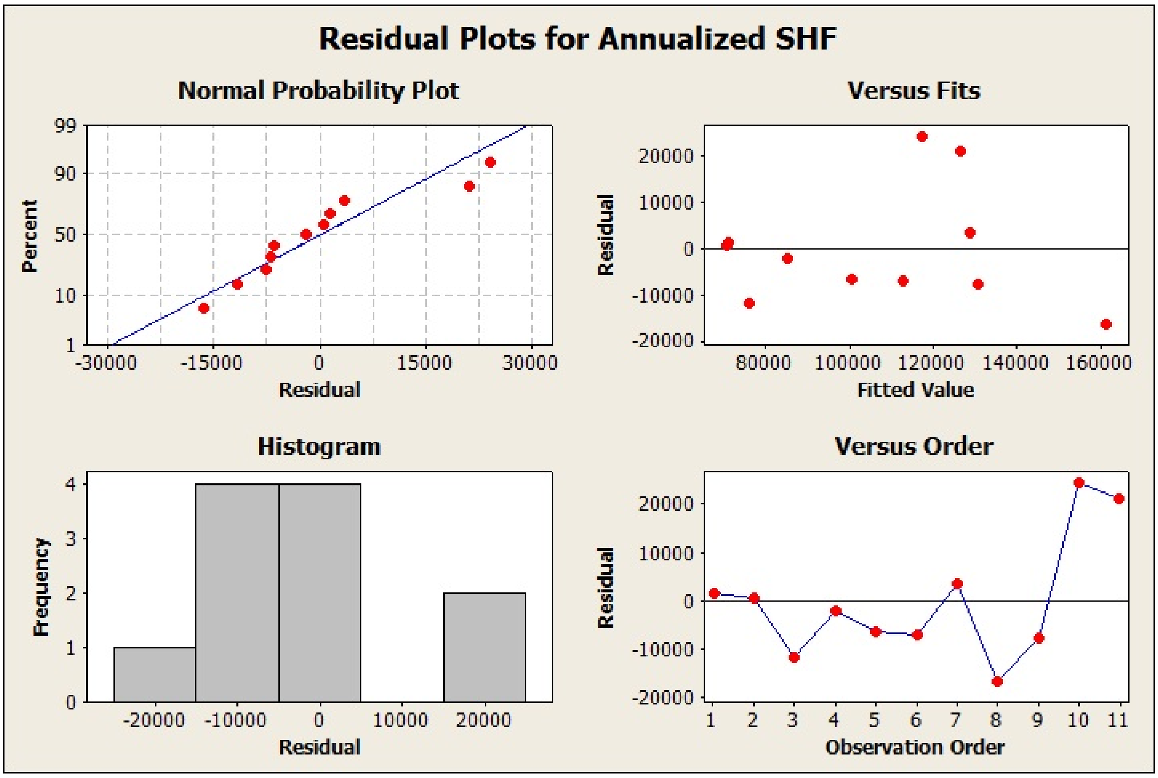


Figure 4.25: Residual Plots 8

Next, $Prem_A$ and $Claims_A$ are the predictors for SHF_A. Even though the residual plots for this regression indicate that the residuals distribution is close to normal, from the histogram it can be seen that the mean is not close to zero. The R^2 value is over 90%, but the p-value of $Prem_A$ is 18%, which is far more than it should be. Table 4.21 are the statistics and Figure 4.26 are the residual plots of the regression.

Table 4.21: Regression: SHF_A vs. $Prem_A$, $Claims_A$

Predictor	Coef	SE Coef	T	P
Constant	-117526	30014	-3.92	0.004
$Prem_A$	0.6740	0.4615	1.46	0.182
$Claims_A$	2.3451	0.6205	3.78	0.005

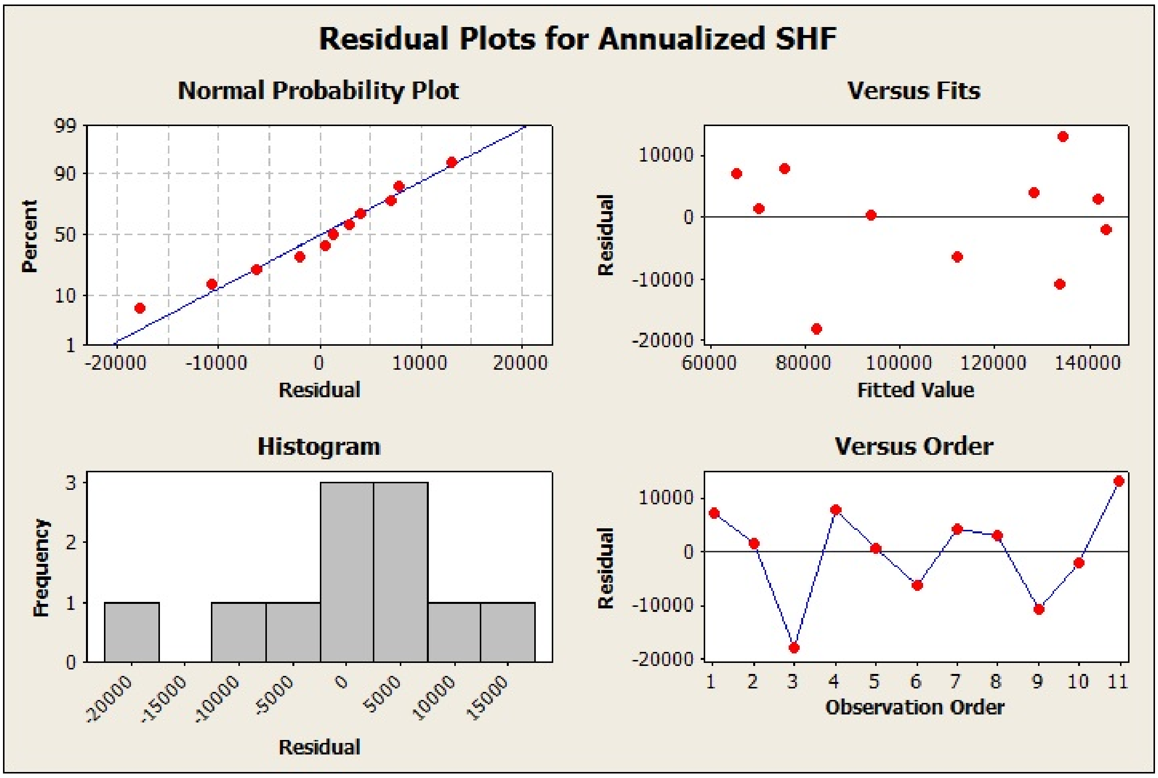


Figure 4.26: Residual Plots 9

The best regression is obtained by using a single predictor. In the following regression, first $Claims_A$ then $Prem_A$ is considered as predictor, which gives the most accurate model. Tables 4.22 and 4.23 represent the results for both of the regressions with predictor $Claims_A$ and $Prem_A$ respectively.

Table 4.22: Regression: SHF_A vs. $Claims_A$

Predictor	Coef	SE Coef	T	P
Constant	-84916	21282	-3.99	0.003
$Claims_A$	3.1200	0.3414	9.14	0.000

Table 4.23: Regression: SHF_A vs. $Prem_A$

Predictor	Coef	SE Coef	T	P
Constant	-150816	45146	-3.34	0.009
$Prem_A$	2.1653	0.3765	5.75	0.000

The T and p-values for the regression that has $Claims_A$ as predictor provides slightly better results than the latter regression. Both of the regressions residuals have a mean close to 0, however the regression with $Prem_A$ as predictor has a distribution that is closer to normal, which makes the latter the better model even though the F-value of the regression with $Claims_A$ has a greater F-value. Moreover, the AR(1) model established using $Claims_A$ has a p-value of 15% for the model, does not indicate a meaningful forecast. Figures 4.27 and 4.28 are the residual plots and Tables 4.24 and 4.25 are the ANOVA for $Claims_A$ and $Prem_A$ respectively.

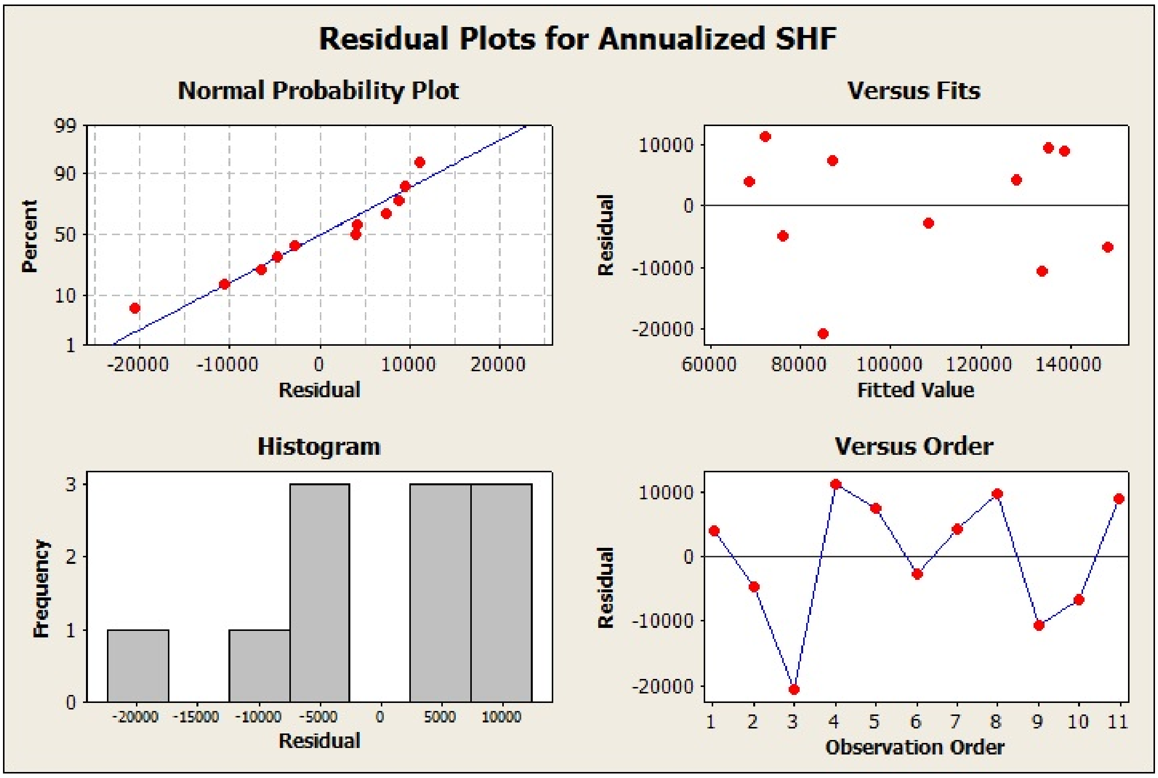


Figure 4.27: Residual Plots 10

Table 4.24: ANOVA table: SHF_A vs. $Claims_A$

Source	DF	SS	MS	F	P
Regression	1	9,244,559,827	9,244,559,827	83.51	0.000
Residual Error	9	996,252,521	110,694,725		
Total	10	10,240,812,348			

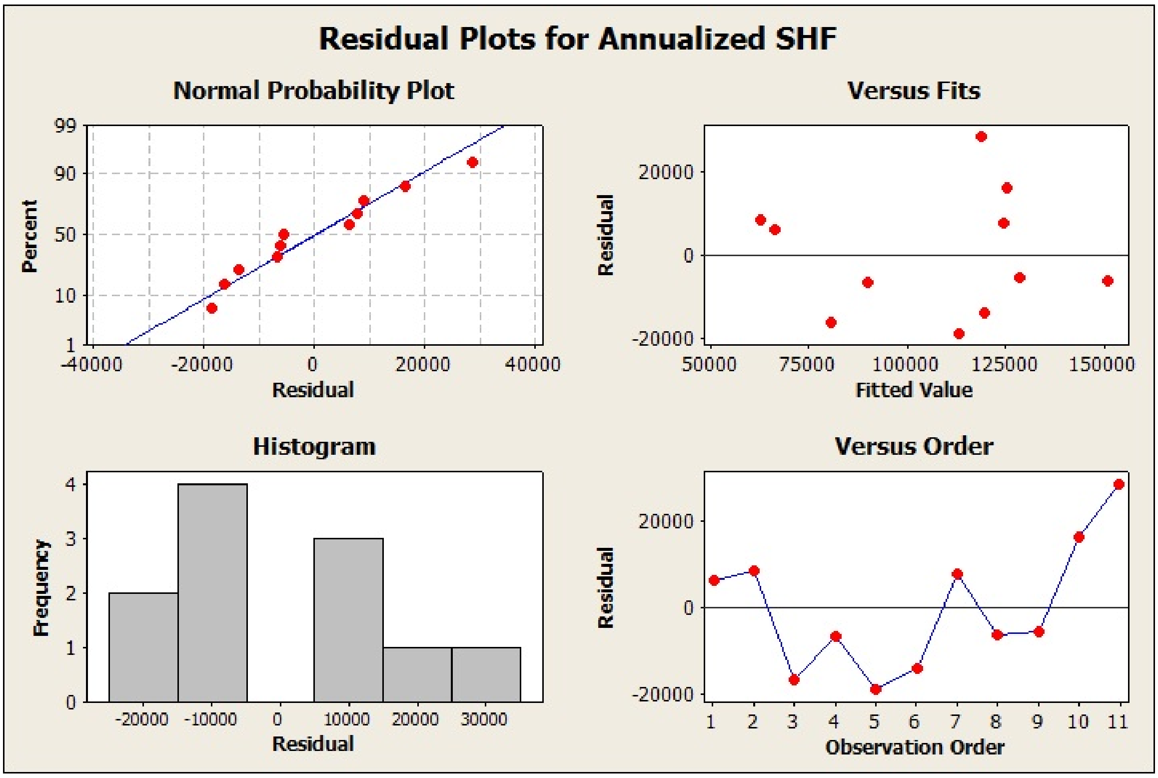


Figure 4.28: Residual Plots 11

Table 4.25: ANOVA table: SHF_A vs. $Prem_A$

Source	DF	SS	MS	F	P
Regression	1	8,050,049,731	8,050,049,731	33.07	0.000
Residual Error	9	2,190,762,616	243,418,068		
Total	10	10,240,812,348			

Observing the results of these regressions, it can be concluded that premium income is the best predictor for shareholders fund, for the annualized data. In order to increase the accuracy of the model, the same regression also is done by taking the natural logarithm of both SHF_A and $Prem_A$. The regression is given as;

Table 4.26: Regression: $ln(SHF_A)$ vs. $ln(Prem_A)$

Predictor	Coef	SE Coef	T	P
Constant	-17.716	4.660	-3.80	0.004
$ln(Prem_A)$	2.5042	0.3988	6.28	0.000

The regression done by taking the natural logarithm of both the response and the predictor yields better T-values. Moreover, the p-value of the constant is less than the previous regression. The residual plots and ANOVA table is given by Figure 4.29 and Table 4.27.

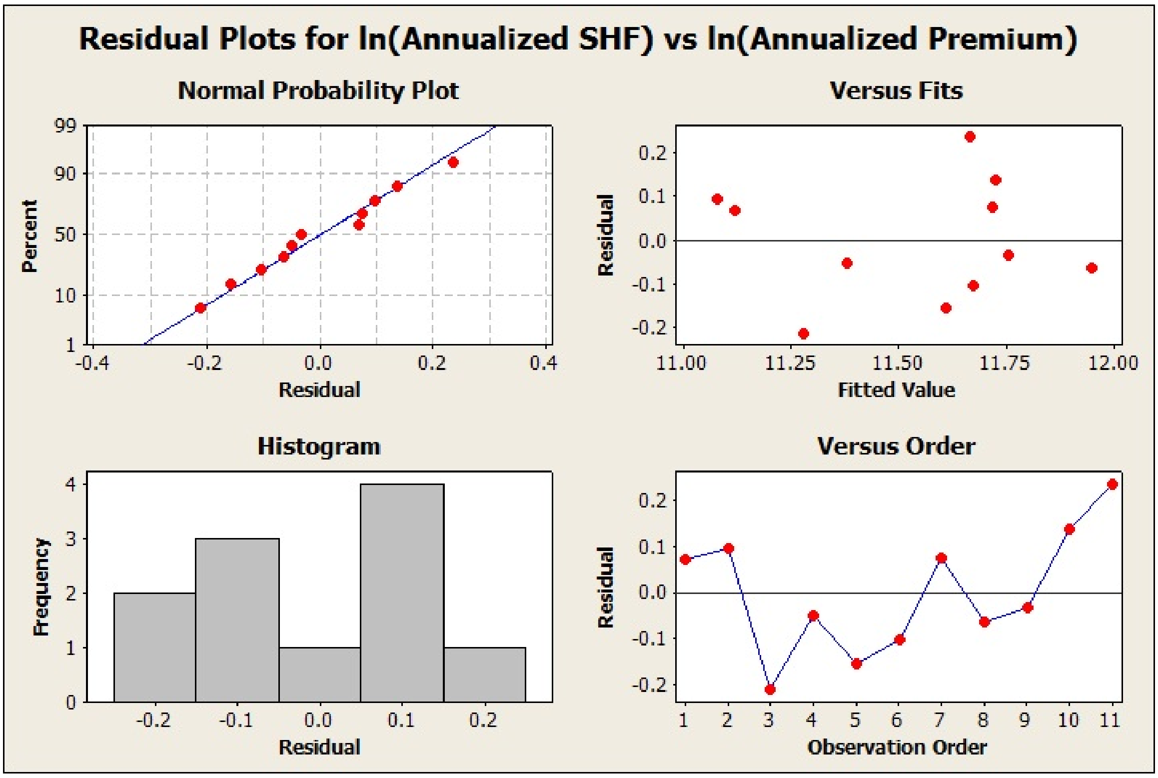


Figure 4.29: Residual Plots 12

Table 4.27: ANOVA table: $ln(SHF_A)$ vs. $ln(Prem_A)$

Source	DF	SS	MS	F	P
Regression	1	0.79868	0.79868	39.42	0.000
Residual Error	9	0.18233	0.02026		
Total	10	0.98100			

The previous F-value was 33.07, where the new F-value with the natural logarithms taken is 39.42, which also indicated that the model with natural logarithms is more accurate. Therefore, to estimate capacity, $Prem_A$ is forecasted in the next section. The forecasted value of premium income is used in the regression equation of SHF_A vs $Prem_A$. Then, the resulted SHF_A is used in the regression equation with capacity as response to estimate the level of market capacity.

4.3 Auto-regressive Modelling

In the previous section, it was concluded that $Prem_A$ is the best predictor for SHF_A among the other variables. The aim of the AR(1) modelling is to project these variables for the next year. By forecasting, these variables are then put in the regression equation to obtain a value for the next years SHF_A. By calculating the shareholders fund, a value for capacity is obtained. In addition, an AR(1) model using the capacity data is established to to compare these results. In the AR(1) models, the natural logarithm of the data is taken since the model cannot be established with large numbers. An equation of an AR(1) model is given as

$$X_t = \beta_0 + \beta_1 X_{t-1} + \epsilon_t. \tag{4.5}$$

If the AR(1) can capture the structure of dependence, there should not be any dependence in the residuals. Therefore, the residuals of the model should look random [22].

4.3.1 AR(1)/Regression Model for Annualized Premium Income

The chosen variable to be forecasted is $Prem_A$, since the AR(1) model of $Claims_A$ provides a p-value of 0.158, which indicates there is a high probability for an inaccurate forecast. The natural logarithm of the annual premium income data is taken; which is given by Table 4.28.

Table 4.28: Data: $Prem_A$

Year	$ln(Prem_A)$
2000	11.5155
2001	11.4981
2002	11.5792
2003	11.6191
2004	11.7107
2005	11.7357
2006	11.7542
2007	11.8456
2008	11.7688
2009	11.7651
2010	11.7332

The AR(1) model using annualized premium income results as stated in Table 4.29.

Table 4.29: AR(1) Model: $ln(Prem_A)$

Type	Coef	SE Coef	T	P
AR(1)	0.9670	0.1545	6.26	0.000
Constant	0.38394	0.01832	20.95	0.000
Mean	11.6382	0.5554		

As it can be seen from Table 4.29, the p-values for both the constant and the model are 0. Also, the T-values are greater than 2, which is a good sign for the model. However, the coefficient for the AR(1) model is 0.9670, which is over 95%. AR(1) models with coefficients above 95% are not accurate, and the forecast done by this model cannot be significant. Thus, $Prem_A$ can be considered as linear, and the forecast of annualized premium income can be done by regression. In order to see how premium income reacts with years and how significant this model is, regression analysis is done with $Prem_A$ as response and year as predictor. The parameter year is equal to 1 for the year 2000 and 11 for 2010. The regression analysis yields the results presented in Table 4.30.

Table 4.30: Regression: $ln(Prem_A)$ vs. $Year_t$

Predictor	Coef	SE Coef	T	P
Constant	11.5095	0.0404	285.00	0.000
$Year_t$	0.028962	0.005954	4.86	0.001

The ANOVA table is given in Table 4.31.

Table 4.31: ANOVA table: $ln(Prem_A)$ vs. $Year_t$

Source	DF	SS	MS	F	P
Regression	1	0.092265	0.092265	23.66	0.001
Residual Error	9	0.035098	0.003900		
Total	10	0.127364			

The regression has an R^2 value of 72.4%, which is significant for a regression if other important values are also significant. The T-value for the regression is 285, which is very high and good for the accuracy of the model and the p-values for both the constant and $Year_t$ are 0. From Table 4.31, it can be observed that the F-value for the regression is also significant and has a value of 23.66. The residual plots of the regression are normal, and the mean is close to zero, as presented in Figure 4.30.

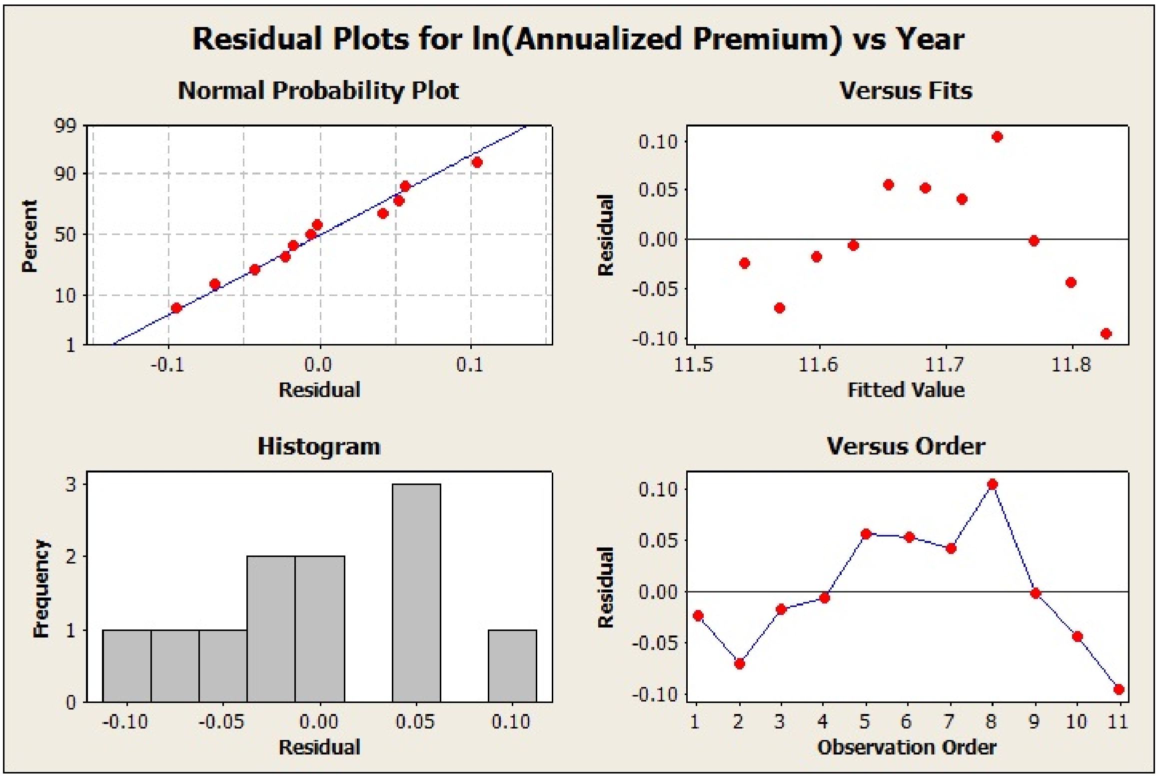


Figure 4.30: Residual Plots 13

Using this regression, the future value for annualized premium income is obtained. By putting 12 for $Year_t$ (for the year 2011), the natural logarithm of $Prem_A$ is acquired and is equal to 11.8575, which corresponds to 141,139 (US$ billions). Now, this forecasted value can be used in the regression equation with SHF_A as response to obtain a value for annualized shareholders fund. The last step is to put the obtained shareholders fund value into the regression with capacity to provide an estimate.

4.3.2 Applying the Forecast to Regression

In the previous section, the future value for annualized premium income was obtained. This value will now be put in the regression equation with annualized shareholders fund. The regression equation with SHF_A as response and $Prem_A$ as predictor is the following:

$$SHF_A = 2.17Prem_A - 150,816. \quad (4.6)$$

Hence, by inserting the forecasted annual premium income into this equation, we obtain a value of 155,455.63 (US$ billions) for the next years shareholders fund, which is an increase from the previous year. Now, by using the forecasted value in the regression equation with capacity, we can obtain an estimate for the future value of capacity. The estimation of this value is to observe the increase or decrease in the level of market capacity, not to obtain the exact accurate value. The regression equation with capacity as response was given as

$$Capacity = 62,054 + 1.70SHF_A - 23,224Trigger. \quad (4.7)$$

If trigger is equal to 0, that is the assumption that the number of catastrophes are on the average, inserting the forecast into the regression results with a vallue for reinsurance capacity of 326,328.571 (US$ billions). This is an increase from the last years reinsurance capacity, which is 315,784 (US$ billions). If the number of catastrophes are assumed to be above the average, i.e. $trigger = 1$, reinsurance capacity becomes 303,104.571 (US$ billions). The decrease from the last years capacity in this case is around 10 billions, which is consistent with the increasing number of catastrophes. The aim of obtaining the result is to observe an increase/decrease, since there is insufficient data to establish a more consistent model in order to estimate the future capacity with utmost accuracy. However, it gives an idea about the movement of capacity level, provided the data collected. In the next section, an AR(1) model using capacity data is established to compare the results of the AR(1) model and regressions and to observe if the level increases or decreases with that model.

4.3.3 AR(1) Model for Capacity

The data for previous years capacity was given in the previous sections. Similar to the AR(1) model with $Prem_A$, the natural logarithm of the data must be taken in order to establish the model. The following table shows the original data and the natural logarithms taken;

Table 4.32: AR(1) Model: *Capacity*

Year	*Capacity* (US$ billions)	*ln*(*Capacity*)
2000	208.5	12.2479
2001	192.5	12.1682
2002	150.2	11.9200
2003	180.3	12.1022
2004	205.5	12.2334
2005	232.6	12.3569
2006	295.2	12.5955
2007	310.6	12.6464
2008	245.8	12.4125
2009	305.4	12.6299
2010	315.7	12.6628

The results for the AR(1) model using capacity data are given in Table 4.33.

Table 4.33: AR(1) Model: *Capacity*

Type	Coef	SE Coef	T	P
AR(1)	0.8165	0.2381	3.43	0.008
Constant	2.27550	0.05555	40.96	0.000
Mean	12.4036	0.3028		

The T-values for both the model and the constant are greater than zero. There is a slight probability of inccorectly rejecting the null hypothesis, however it is not significant and less than 1%, whereas the p-value for the constant is zero. The equation for the AR(1) model for reinsurance capacity is given as

$$Capacity_{t+1} = 0.8165 Capacity_t + 2.275. \tag{4.8}$$

Using this equation, analysis of variance of reinsurance capacity starting from the year 2006 is done. This analysis is done by taking the square of the difference of the actual value and the predicted value for the specified year. Table 4.34 shows the actual and predicted values and the variance of reinsurance capacity starting from the year 2006.

Table 4.34: Variability of *Capacity* AR(1) Model

Year	$ln(\widehat{Capacity})$	$ln(Capacity_{actual})$	$(ln(\widehat{Capacity}) - ln(Capacity_{actual}))^2$
2006	12.3644	12.5955	0.0534
2007	12.5592	12.6464	0.0075
2008	12.6007	12.4125	0.0354
2009	12.4098	12.6299	0.0484
2010	12.5873	12.6628	0.0056

The variability is 5% for 2006, but decreases in the following years and for the last year it is under 1%.

The time series graph and the partial autocorrelation function are given by Figures 4.31 and 4.32.

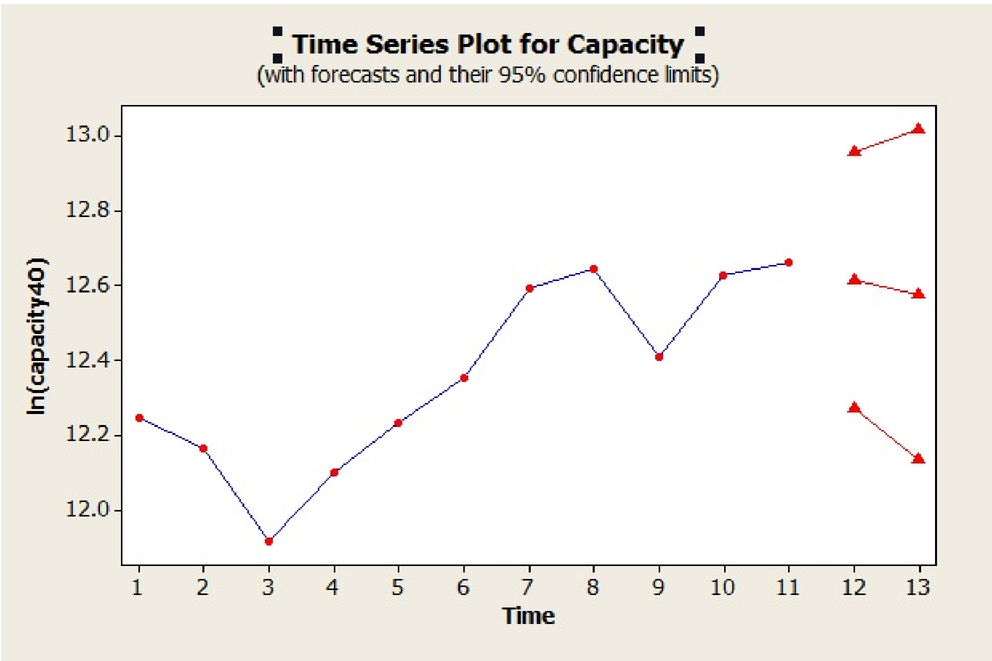


Figure 4.31: *Capacity* AR(1) Model: Time Series

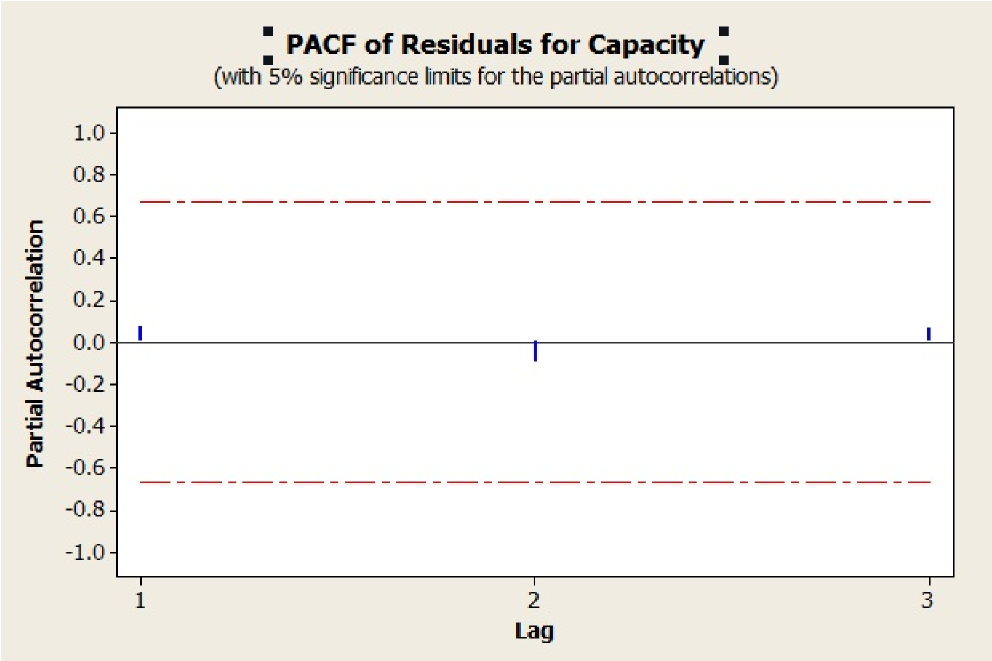


Figure 4.32: *Capacity* AR(1) Model: PACF

The movement of capacity over time can be observed from the time series graph. The PACF of the data diminishes over time, but does not become zero. However the signs of the lags interchange. The forecast of the capacity is shown in Table 4.35 with the upper and lower limits.

Table 4.35: AR(1) Model Forecast: *Capacity*

Period	Forecast	Lower	Upper
2011	12.6153	12.2745	12.9560

Here, again the exponential of the forecast must be taken to obtain the value. Hence, the forecast for the capacity of year 2011 using the AR(1) model is 301,130.7 (US$ billions). The reinsurance capacity obtained by using regressions and the forecast of annualized premium income results with 326,328.571 (US$ billions), if the trigger component in the regression is taken as 0. This means in the case of an average number of catastrophes in the next year, the reinsurance capacity will increase around $10 billion. If the trigger is taken as 1, which is the fact catastrophe frequency is above average, the value for reinsurance capacity becomes 303,104.571 (US$ billions). This forecast is very close to the prediction of the AR(1) model, established using historical data for capacity. Hence, the AR(1) model can be more accurate if the assumption of increasing frequency of catastrophes is made.

4.4 Summary of the Study

In this study, the impact of reinsurance capacity on reinsurance and cat bond prices is observed. Moreover, a model for the load of the cat bond pricing model created by Lane Financial is established. Under the assumption that the expected loss for the cat bond and the reinsurance product are the same, the model for load can be used to compare the cat bond and reinsurance loads in order to determine the cheaper option. This assumption is important, since if the expected losses are not equal, only comparing the loads would not give the cheaper option.

It is obvious that, as market capacity decreases, reinsurance prices increase. So, if capacity can be defined by some of the existing market parameters, these parameters can be forecasted and a model for predicting reinsurance capacity can be established. If future capacity is known, it would be easier to determine how the reinsurance prices will change in the upcoming year.

Reinsurance capacity is affected by many factors, but the main elements that affect it are the elements of the companies that are in the industry. The elements that are chosen for modelling are shareholders fund, premium income, claims paid, investment income and combined ratio. The data is collected from 23 (re)insurance companies for the years 2000-2010. In order to define capacity in terms of these parameters regression analysis is done among the parameters with reinsurance capacity as response. Since there is 11 annual data for market capacity, the parameters have been annualised, that is the sum of parameters of all of the companies are taken to obtain an annual value. With regression analysis, the market parameter that interpret capacity with the most accuracy is determined, which is shareholders fund. Moreover, the effect of increasing frequency of catastrophes is included in the regression. That is, for the years that the number of catastrophes are above average, the trigger is set to 1, and for the average number of catastrophes it is 0. The relationship of reinsurance capacity with shareholders fund and trigger is given as

$$Capacity = 62,054 + 1.70SHF_A - 23,224Trigger. \tag{4.9}$$

Here, capacity is defined in terms of shareholders fund and trigger. If the number of catastrophes are expected to be above average, the trigger is set to 1, which reduces the market capacity. Since if the number of catastrophes are above average, this means there is going to be more loss than average, which decreases the capital of companies. Thus capacity is decreases.

To determine capacity, shareholders fund is not directly forecasted, since the models established using the existing data are not significant, therefore not producing accurate results. Hence, again regression analysis is done, however including shareholders fund as response. The variable that defines shareholders fund then can be projected and a future value can be obtained. Regression analysis yields that premium income is the best predictor for shareholders fund. The relationship is given as

$$SHF_A = 2.17Prem_A - 150,816. \tag{4.10}$$

Therefore, by forecasting premium income, a value for shareholders fund and market capacity can be obtained. The auto-regressive model for premium income had a coefficient larger than 95%, which point to an inaccurate forecast. So, a linear regression model is applied to premium income for prediction, where year is the predictor and has a value of 1 for the year 2000, since that is the starting date for the values of parameters. The natural logarithm of premium income has been taken in order to increase the accuracy of the model. The relation is given as

$$ln(Prem) = 11.5095 + 0.028Year. \tag{4.11}$$

Using Equation 4.11, a future value for premium income is obtained, then this value is used in Equation 4.10 for the future value of shareholders fund. Finally, Equation 4.9 is used for the prediction of reinsurance capacity. If the trigger is set to 1, then a value of 303,104.571 (US$ billions) is obtained. If $trigger = 0$, then reinsurance capacity becomes 326,328.571 (US$ billions). Under the assumption of average frequency of catastrophes, reinsurance capacity increases ($315.7 billion for 2010), which is expectable since capacity has been increasing in the recent years. If the number of catastrophes is above average, then capacity decreases around $10 billion, which is similar to the capacity decreases in the recent years.

Besides the algorithm used to obtain a future value for capacity, also an AR(1) model for capacity using historical data has been established. Even though the model uses only 11 data, the results are significant. The AR(1) model predicts a future capacity value of 301,130.7 (US$ billions), which is close to the value predicted from the first model, with trigger set to 1. Hence, using existing market parameters, the capacity forecast is made.

The case study done on cat bonds, uses data for 2003-2008 for many sponsors and issuers. The Lane cat bond pricing model is used, which involves attachment probability and conditional expected loss in the calculation of the load. However, the Lane model uses issue price as response in the regression, which is not available for the existing cat bond data. Therefore, risk premium is used instead of issue price, which does not affect the consistency of the model. The formula is given as

$$RiskPremium = EL + \theta_3 \times PFL^{\theta_1} \times CEL^{\theta_2}. \tag{4.12}$$

The parameters θ_i are estimated using non-linear regression. In order to create a comparison with cat bond prices and reinsurance capacity movement, a random bond is chosen from each year with the assumption of including California earthquake, and the mean of the parameters are used in the load calculation with the statistics of the bond. These loads are compared with annual reinsurance capacity in order to see how cat bond prices react with capacity increase/decrease. In order to compare reinsurance and cat bond prices; the following assumption is made: The expected loss of the cat bond and reinsurance contract are equal. Therefore, the price is determined by the load. Using the mean of the estimated parameters, a purchaser may calculate the load of the cat bond and compare with the known load of reinsurance, hence easing the decision of catastrophe protection.

CHAPTER 5

Conclusion and Comments

Reinsurance is an industry that constantly improves. Therefore, the companies that are in the industry must adjust to these improvements. Every decade, new market instruments for catastrophe protection are created to create financial stabilization in the reinsurance market. Reinsurance prices depend on many factors, and new pricing methods are still being introduced. In order to both make profit and purchase catastrophe protection, companies have to take numerous parameters into account when establishing their portfolios.

Cat bonds are one of the securities against catastrophes and are often included in company portfolios in the recent years. Reinsurance and cat bond prices fluctuate and depend on many elements. One key element that affects reinsurance pricing is the reinsurance capacity. As market capacity diminishes, reinsurance prices increase, since there is less capital available for the reinsurance companies. At this point, cat bonds become more attractive, since they are more expensive than traditional reinsurance in normal circumstances. A company should make an analysis of cat bond and reinsurance prices according to the change in reinsurance capacity and the pricing models, in order to choose the option that is cheaper for the same circumstances. This study aims to explore the impact of reinsurance capacity to the reinsurance and cat bond prices. Furthermore, the reinsurance capacity is forecasted and a model for the load in cat bond pricing is established by regression analaysis and auto-regressive modelling, to ease the decision of catastrophe security protection.

5.1 Contribution

There are many existing studies on reinsurance and cat bond pricing and how these react with the changes in the industry. However, it is a very wide subject and some aspects that involve probability calculation lead to uncertainty. Thus, calculations of the pricing of these instruments become complicated. Predicting some factors that effect the pricing, may help the decision process.

Reinsurance capacity is the element that affects reinsurance pricing, since when capacity decreases reinsurance companies have less resources for operations, which leads to the price increase. Therefore, predicting capacity with existing values, a company might take the capacity change into consideration when deciding whether or not to purchase another security (cat bond in this case).

The case study about cat bonds is done by choosing random bonds that include California EQ from each year, for years 2003-2008. The cat bond data for these years is used in non-linear regression analysis and the parameters α, β and γ in Equation 2.3 are estimated. The mean of these variables are used on the randomly chosen cat bonds from each year and the bond prices are compared with the reinsurance capacity of the corresponding years. Moreover, the load of a cat bond can be calculated using the load from the Lane model and the mean of the estimations. Assuming that the expected loss of the considered cat bond and reinsurance contract is equal, the calculated load can be compared with the reinsurance load, since load determines price if the expected losses are the same.

The modelling and analysis done in this study aims to observe the impact of capacity on reinsurance and cat bond markets, and attempts to do a capacity forecast. Also, a model for the load of cat bonds with specific perils is established. Both of the modelling are done in order to obtain an understanding for the catastrophe protection pricing and to help with the decision process of these purchases.

5.2 Further Study

One and probably one of the most important difficulty of this study is the lack of information in the market. Initially, 40 (re)insurance companies were considered in this study, however because of insufficient information this number was reduced to 23. Moreover, the public information available for these companies start from the year 2000.

The models used in this study, yield better results when the number of data is larger. That way, a stronger model can be established, since more data points are available for the model. If the data existed for before the years 2000, and for a larger number of reinsurance companies, the models would result with more accuracy, hence yielding better predictions. Since the data is available from the year 2000, when the parameters are annualized, only 11 data points are left for establishing models and regression analysis. Increasing this number would increase the accuracy of the models with high probability. However, even with this number of data, it is possible to obtain results with accuracy.

Another difficulty is the number of parameters that reinsurance capacity depends on. The parameters that are chosen in this thesis certainly have an effect on reinsurance capacity, but capacity is a more complex concept than just being able to be defined by a few parameters. The chosen parameters might be extended or the parameters that have been left out in the regression analysis because of their insignificancy as predictors might be included by obtaining older statistics or using different modelling methods.

The case study done with cat bonds might be extended. For example, the case considered is for California cat bonds. However, cat bonds that cover larger regions and peril types may be included and reinsurance capacity comparison can be made with the results. Besides, the cat bond data used is for the years 2003-2008, which is insufficient to do a comparison with more recent years. The year interval may be extended and non-linear regression can be done for more years, which would reduce the errors in the parameter estimation.

REFERENCES

[1] Bantwal, V. and Kunreuther, H.(2000). *A Cat Bond Premium Puzzle?*. The Journal of Psychology and Financial Markets, pp.1-2.

[2] Bruggeman V. (2007). *Capital Market Instruments for Catastrophe Risk Financing*, pp.12.

[3] CEA. (2006). *Guidance Paper on Reinsurance*, pp.3-4.

[4] Cummins, J.D. and Geman, H. (1995). *Pricing Catastrophe Futures and Call Spreads: An Arbitrage Approach*.

[5] Lee, J.P. and Yu, M.T. (2002). *Pricing Default-Risky Cat Bonds with Moral Hazard and Basis Risk*.

[6] Lane, M.N. (2003). *Rationale and Results with the LFC Cat Bond Pricing Mode*, pp.2-4.

[7] Bruggeman V. (2007). *Capital Market Instruments for Catastrophe Risk Financing*, pp.13-14.

[8] Hoffmann, J.P. (2010). *Linear Regression Analysis: Applications and Assumptions*, pp.29.

[9] Ihaka, R. (2005). *Time Series Analysis*, pp.1-2.

[10] Hyndman, R. (2001). *ARIMA Processes*, pp.1-2.

[11] Ihaka, R. (2005). *Time Series Analysis*, pp.21-22.

[12] Lei, D.T., Wang, J. and Tzeng, L.Y. (2007). *Explaining the Spread Premiums on Catastrophe Bonds*, pp.8.

[13] AON Benfield. (2011). *Reinsurance Market Outlook*, pp.5.

[14] AON Benfield. (2011). *Reinsurance Market Outlook*, pp.4.

[15] Eyre, W. (2011). *U.S. Reinsurance Marketplace*, pp.3.

[16] International Association of Insurance Supervisors. (2003-2006). *Global Reinsurance Market Report*, pp.50-60.

[17] The Congress of United States, Congressional Budget Office. (2002). *Federal Reinsurance for Disasters*, pp.2-7.

[18] Cummins, J.D. (2007). *Reinsurance for Natural and Man-Made Catastrophes in the United States: Current State of the Market and Regulatory Reforms*, pp.182.

[19] Guy Carpenter. (2011). *World Catastrophe Reinsurance Market Review*, pp.20.

[20] Guy Carpenter. (2012). *Mr. Robert Easton, Personal Contact*.

[21] Milli Reasürans. (2010). *Annual Report*, pp.2-4.

[22] Root, E.D. (2010). *Time Series Analysis, Stochastic Processes*, pp.4.

APPENDIX A

Tables of Market Data

A.1 Shareholders Fund

The data is in the figure of $m for American companies, €m for European companies and TL m for Milli Re. Table A.1 shows the data collected for SHF.

Table A.1: Shareholders Fund of (Re)insurance Companies Between Years 2000-2010

Company	2000	2001	2002	2003	2004	2005	2006	2007	2008	2009	2010
Allied World	-	1,490	1,682	1,979	2,138	1,420	2,220	2,239	2,416	3,213	3,075
Alterra	-	-	-	-	-	-	1,390	1,582	1,280	1,564	2,918
Arch capital	272.2	1,020	1,411	1,710	2,241	2,480	3,590	4,035	3,432	4,323	4,513
Argo Group	-	447.5	327.7	539.2	603.4	716.1	847.7	1,384	1,352	1,614	1,626
Aspen	-	-	878.1	1,298	1,481	2,039	2,389	2,817	2,779	3,305	3,241
Axis Capital	-	-	1,961	2,817	3,238	3,512	4,412	5,158	4,461	5,500	5,625
Endurance Spec.	-	1,162	1,217	1,644	1,862	1,872	2,297	2,512	2,207	2,782	2,848
Everest Re	-	-	-	3,164	3,712	4,139	5,107	5,684	4,960	6,101	6,283
Fairfax	2,113	1,894	2,111	2,680	2,974	2,709	2,662	4,063	4,863	7,391	7,761
Flagstone	-	-	-	-	-	547.6	864.5	1,210	986	1,211	1,134
Hannover Re	-	-	-	-	-	2,601	2,897	3,349	2,830	3,711	4,456
Maiden	-	-	-	-	-	-	-	537.3	509.8	676.5	750
Milli Re	-	52.7	92.7	118.7	183.2	465.7	536.7	706.9	666.7	781	798.7
Montpelier Re	-	-	1,252	1,657	1,751	1,057	1,492	1,653	1,357	1,728	1,628
Münich Re	23.6k	19.4k	13.9k	19.3k	20.5k	24.4k	26.4k	25.5k	21.3k	22k	23k
Partner Re	2,086	1,748	2,077	2,594	3,352	3,093	3,786	4,322	4,199	7,646	7,207
Platinium	-	-	-	-	-	-	1,858	1,998	1,809	2,077	1,895
Rennaissance	700.8	1,225	1,642	2,334	2,644	2,253	3,280	3,447	3,032	3,840	3,939
Swiss Re	22.7k	22.5k	16.6k	18.5k	19.1k	22.9k	30.8k	31.8k	20.4k	26.2k	25.3k
Transatlantic	-	-	-	-	2,587	2,544	2,958	3,349	3,198	4,030	4,284
Validus	-	-	-	-	-	999.8	1,192	1,934	1,940	4,031	3,504
White Mount.	-	-	-	-	3,894	3,833	4,455	4,713	2,899	3,657	3,653
XL Group	5,573	5,437	6,570	6,937	7,739	8,472	10.1k	9,948	-	-	-

A.2 Premium Income

The data is in the figure of \$m for American companies, €m for European companies and TL m for Milli Re. The data collected is the premiums earned in one year. Table A.2 shows the data.

Table A.2: Premium Income of (Re)insurance Companies Between Years 2000-2010

Company	2000	2001	2002	2003	2004	2005	2006	2007	2008	2009	2010
Allied World	-	-	-	1,167	1,325	1,271	1,252	1,159	1,117	1,316	1,359
Alterra	-	-	-	-	899.9	1,053	665	817.9	813.5	834.4	1,172
Arch Capital	-	-	-	1,329	1,573	1,585	1,600	1,702	1,675	1,688	1,651
Argo Group	-	221.9	378.4	562.8	633.9	699	813	859.8	1,127	1,414	1,211
Aspen	-	-	120.3	812.3	1,232	1,508	1,676	1,733	1,701	1,823	1,898
Axis Capital	-	-	536.8	1,436	2,028	2,553	2,694	2,734	2,687	2,791	2,947
Endurance Spec.	-	-	369.4	1,173	1,632	1,723	1,638	1,594	1,766	1,633	1,763
Everest Re	-	-	-	-	4,425	3,963	3,853	3,997	3,694	3,894	3,934
Fairfax	4,297	3,108	3,888	4,209	4,801	4,703	4,850	4,648	4,529	4,422	4,580
Flagstone Re	-	-	-	-	-	-	192	477.1	654.1	758.4	852
Hannover Re	5,210	6,496	7,688	8,155	7,575	7,738	7,092	7,292	7,061	9,307	7,471
Maiden	-	-	-	-	-	-	-	110.1	420.1	919.9	1,171
Milli Re	-	257.4	317.5	477.4	575.9	724	853.5	838.2	849	823	855.3
Montpelier Re	-	-	329.9	705.3	788	848	583	557.2	528.5	573.2	283.5
Münich Re	28.4k	31.6k	40k	40.4k	38.1k	38.2k	37.4k	37.3k	37.8k	41.4k	45.5k
Partner Re	1,314	1,633	2,425	3,503	3,734	3,599	3,667	3,777	3,928	4,120	4,705
Platinium	-	-	-	-	-	-	1,336	1,173	1,114	937	780
Rennaissance	267.6	333	760.9	1,115	1,338	1,402	1,529	1,435	1,386	1,273	864,9
Swiss Re	22k	25.2k	29.1k	30.7k	29.4k	27.7k	29.5k	31.6k	25.5k	24.6k	19.6k
Transatlantic	-	-	-	3,171	3,661	3,384	3,604	3,902	4,067	4,039	3,858
Validus	-	-	-	-	-	-	306.5	858	1,256	1,449	1,761
White Mount.	-	-	-	-	-	-	1,241	1,146	1,000	858.8	847.9
XL Group	2,035	2,779	4,966	3,640	4,083	9,365	7,570	7,205	-	-	-

A.3 Claims Paid

The data is in the figure of \$m for American companies, €m for European companies and TL m for Milli Re. The data collected is the premiums earned in one year. Table A.3 shows the data.

Table A.3: Claims Paid of (Re)insurance Companies Between Years 2000-2010

Company	2000	2001	2002	2003	2004	2005	2006	2007	2008	2009	2010
Allied World	-	213	310.5	1,058	2,037	3,405	3,636	3,919	4,567	4,761	4,879
Alterra	136.3	136.7	106	119.6	217.6	169	361.7	184.2	368.5	574.4	-
Arch Capital	-	-	76.2	269.8	561.9	758.1	990.4	1,117	1,259	1,439	1,307
Argo Group	-	-	-	268.5	312.6	268.9	341.9	459.8	667	789	746
Aspen	-	-	3.7	53.9	164.6	551.9	469.7	695.7	739.4	808.6	666.8
Axis Capital	-	15	46.1	113.1	333.5	898.5	658.6	628.8	982.1	1,042	1,265
Endurance Spec.	-	-	-	86.8	236.4	579.9	782.9	749.5	823.2	794.6	717.2
Everest Re	-	-	-	-	1,748	2,218	2,639	2,367	2,311	2,385	2,557
Fairfax	983.9	1,072	1,082	597	707.7	862.1	748.4	786.3	835.5	729.9	736.9
Flagstone Re	-	-	-	-	-	-	4.1	32.6	105.7	136.4	184.8
Hannover Re	534.4	617.2	744.3	2,037	2,915	3,348	1,916	1,539	693	2,183	2,999
Maiden	-	-	-	-	-	-	-	26.5	159.9	209.3	365.2
Milli Re	-	-	-	-	-	2.28	4.81	4.88	6.84	3.49	5.5
Montpelier Re	-	-	4.1	53.2	150.7	275.1	40.7	361.2	317.9	213	189.3
Münich Re	23.9k	22.6k	25.4k	23.8k	24.4k	26.8k	25.4k	25.6k	25.4k	27.9k	30.6k
Partner Re	924	824	1,124	1,292	1,378	1,484	1,860	1,620	1,580	2,044	1,846
Platinium	-	-	-	-	-	-	-	-	582.3	607.2	594.2
Rennaissance	46.5	31.6	73.1	142.2	683.3	935.9	591.2	403.4	744.6	550.6	233.5
Swiss Re	17.1k	16.2k	14.4k	14.8k	13.8k	14.1k	21.2k	23.6k	24.4k	23.7k	18.3k
Transatlantic	-	-	-	1,686	1,868	2,031	2,104	2,102	2,116	2,421	2,207
Validus	-	-	-	-	-	-	13.9	156.8	406.4	507.4	673.4
White Mount.	-	-	-	-	-	-	1,070	870.2	888.8	772.9	437
XL Group	1,663	1,817	2,846	2,816	2,945	3,213	3,335	3,218	-	-	-

A.4 Investment Income

The data is in the figure of \$m for American companies, €m for European companies and TL m for Milli Re. Table A.4 gives the data.

Table A.4: Investment Income of (Re)insurance Companies Between Years 2000-2010

Company	2000	2001	2002	2003	2004	2005	2006	2007	2008	2009	2010
Allied World	-	-	-	101	129	178.6	244.4	297.9	308.8	300.7	244.1
Alterra	-	-	-	-	82.8	106.8	150	188.2	181.6	169.7	222.5
Arch Capital	-	-	76.2	81	143.7	232.9	380.2	463.1	468.1	390.1	364.8
Argo Group	-	53.6	52.9	53.6	65.1	83.9	104.5	134.3	150.2	145.5	133.6
Aspen	-	-	8.5	29.6	68.3	121.3	204.4	299	139.2	248.5	232
Axis Capital	-	-	71.2	73.9	152.1	256.7	407.1	482.3	247.2	464.4	406.9
Endurance Spec.	-	838	42.9	71	122	180.9	257.4	281.2	130.1	284	200.3
Everest Re	-	-	-	-	495.9	522.8	629.3	682.2	565.8	547.7	653.4
Fairfax	1,200	578.4	888.1	1,176	655	818.2	1,581	2,400	3,197	1,657	950.9
Flagstone Re	-	-	-	-	-	-	34.2	73.8	51.3	28.5	31.4
Hannover Re	868.6	945.7	928.4	1,071	1,116	1,123	1,181	1,121	278.5	1,120	872.2
Maiden	-	-	-	-	-	-	-	15.2	37.2	62.9	72
Milli Re	-	-	-	-	97.7	17.8	22.2	31.7	53.9	77.3	73.1
Montpelier Re	-	-	39.7	50.1	69	87	125.8	132.8	86.4	81	74.1
Münich Re	8,652	9,654	-	7,398	7,498	7,649	7,834	8,109	7,838	5,552	13.6k
Partner Re	273.6	239.6	245.2	261.7	298	365	449	523	573	596	613
Platinium	-	-	-	-	-	-	187.9	214.2	186.5	163.9	134.3
Rennaissance	77.8	75.1	104	29.6	162.7	217.2	318.2	402.4	24.2	323.9	203.9
Swiss Re	4,802	5,765	5,507	4,606	4,857	6,137	7,991	10.6k	7,881	6,935	4,684
Transatlantic	-	-	-	-	306.8	343.2	434.5	469.8	440	467	473.5
Validus	-	-	-	-	-	-	58	112.3	139.5	118.7	134.1
White Mount.	-	-	-	-	-	-	182.7	210.5	178.1	107.7	90.5
XL Group	542.5	562.6	734.5	780	995	1,475	1,978	2,249	-	-	-

A.5 Combined Ratio

The data for combined ratio is as represented in Table A.5.

Table A.5: Combined Ratio of (Re)insurance Companies Between Years 2000-2010

Company	2000	2001	2002	2003	2004	2005	2006	2007	2008	2009	2010
Allied World	-	214.5	90.8	84.9	95.9	124.4	78.8	81.3	84.2	76.1	84.9
Alterra	-	-	-	-	-	105.7	93.9	83.9	91.9	88.1	85.7
Arch Capital	-	-	-	89.2	92.4	97.5	89.9	91.2	101.7	98.1	102.3
Argo Group	-	130.3	126.6	104.2	99.8	98.7	93.8	99.4	100.5	96.9	103.2
Aspen	-	-	87	78	84	117	82.4	83	95.7	84.1	96.7
Axis Capital	-	-	70.7	73.7	84.4	101.8	77.3	75.3	89.8	79.3	88.7
Endurance Spec.	-	-	86.2	84.7	85.8	123.5	81.5	79.9	93.5	84	88.7
Everest Re	-	-	-	95.3	99	120.3	89.7	91.6	95.6	89.6	102.8
Fairfax	116.3	121	100	97.6	97.5	115.9	95.5	94	110.1	99.8	105.2
Flagstone Re	-	-	-	-	-	-	47.6	72.8	89.4	74.7	101.6
Hannover Re	108	115.7	96.3	96	97	112.8	98.4	99.7	95.4	96.6	99
Maiden	-	-	-	-	-	-	-	93.9	94.8	95.9	96.9
Milli Re	-	114	90	94	93	90	97	98	94	111	98
Monteplier Re	-	-	67.3	50.3	77.8	200.7	60.3	61.3	91	62.2	82
Münich Re	115.3	135.1	122.4	96.7	98.4	110.5	92.6	96.4	99.5	96.6	100.5
Partner Re	102.5	130.2	97.9	94.6	94.3	115.9	84.6	80.4	94.1	81.8	95
Platinium	-	-	-	-	-	-	83.6	81	91.9	76.7	86
Rennaissance	69.1	70.2	57.1	56.4	104.4	139.7	54.7	59.3	72.9	21.2	45.1
Swiss Re	117	124	104	98.4	98.9	108.7	90.4	90.2	97.9	88.3	93.9
Transatlantic	-	-	-	-	101.5	112	96	95.3	98.5	93.5	98.2
Validus	-	-	-	-	-	-	56.7	62	92.2	68.9	86.2
White Mount.	-	-	-	-	-	-	96	93	95	94	101
XL Group	106.8	140	97	91.6	96.7	137	91.7	88.8	-	-	-