# TEMPERATURE IN TURKEY AND TURKISH DAY AHEAD ELECTRICITY MARKET PRICES: MODELING AND FORECASTING

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submitted by **KAMİL DEMİRBERK ÜNLÜ** in partial fulfillment of the requirements for the degree of **Master of Science in Department of Financial Mathematics, Middle East Technical University** by,

Prof. Dr. Bülent Karasözen
Director, Graduate School of **Applied Mathematics** ________________

Assoc. Prof. Dr. Ömür Uğur
Head of Department, **Financial Mathematics** ________________

Assoc. Prof. Dr. Ali Devin Sezer
Supervisor, **Department of Financial Mathematics** ________________

**Examining Committee Members:**

Prof. Dr. Gerhard-Wilhelm Weber
Institute of Applied Mathematics, METU ________________

Assoc. Prof. Dr. Ali Devin Sezer
Institute of Applied Mathematics, METU ________________

Prof. Dr. Sibel Tarı
Department of Computer Engineering, METU ________________

Assist. Prof. Dr. Özlem İlk
Department of Statistic, METU ________________

Assist. Prof. Dr. Başak Tanyeri
Faculty of Management, Bilkent University ________________

**Date:** ________________

**I hereby declare that all information in this document has been obtained and presented in accordance with academic rules and ethical conduct. I also declare that, as required by these rules and conduct, I have fully cited and referenced all material and results that are not original to this work.**

Name, Last Name: KAMİL DEMİRBERK ÜNLÜ

Signature :

# ABSTRACT

TEMPERATURE IN TURKEY AND TURKISH DAY AHEAD ELECTRICITY MARKET PRICES: MODELING AND FORECASTING

Ünlü, Kamil Demirberk
M.S., Department of Financial Mathematics
Supervisor : Assoc. Prof. Dr. Ali Devin Sezer

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One of the key steps of the liberalization of the Turkish electricity market has been the establishment of PMUM (Turkish day ahead electricity market). The aim of this study is to explore the dynamics of electricity prices observed in this market and their relation with temperature observed in Turkey. The electricity price process is studied as a univariate process and the same process is studied along with temperature together as a two-dimensional process. We give a fairly complete model of temperature. We observe that the electricity prices in Turkey exhibit many of the features that similar prices exhibit in other world markets. In particular, Turkish day ahead prices are seasonal; every year the price seems to follow a path similar to the one years preceding it. To simplify our analysis we focus our study to a 35 day period where every year the prices show a relatively simple behavior. We study the effects of the fluctuations in temperature in this period on the fluctutations in the day ahead electricity price.

Keywords: Turkish day ahead electricity market prices, temperature, autoregression, forecast-

ing, modeling

# ÖZ

TÜRKİYE'DE HAVA SICAKLIĞI VE TÜRKİYE GÜN ÖNCESİ ELEKTRİK PİYASASI FİYATLARI : MODELLEME VE TAHMİN

Ünlü, Kamil Demirberk
Yüksek Lisans, Finansal Matematik Bölümü
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Türkiye elektrik marketinin özeleştirilmesindeki en önemli adımlardan biri 2009'da elektrik borsasının (PMUM) çalışmaya başlamasıdır. Bu çalışmanın amacı bu borsada oluşan elektrik fiyatının dinamiklerini ve bu dinamiklerin sıcaklık ile ilişkisini "time series" modelleri kullanarak incelemektir. Türkiye'deki elektrik fiyatları diğer elektrik piyasalarındaki bütün özellikleri göstermektedir. Bunlardan en önemlisi fiyatların mevsimsel yani yıldan yıla benzer şekillerde hareket etmesidir. Analizleri basitleştirmek için hemen hemen aynı özelliklere sahip 35 günlük bir dönem üzerinde çalıştık ve bu dönemde sıcaklıklardaki dalgalanmaların fiyattaki dalgalanmalar üzerindeki etkisini inceledik.

Anahtar Kelimeler: Türkiye gün öncesi elektrik piyasası fiyatları, hava sıcaklığı, otoregresif, tahmin, modelleme

*To my family*

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# CHAPTER 1

# INTRODUCTION

The liberalization movement of electricity market started in 1982 in Chile. Similar developments in continental Europe and the U.S. followed soon afterward. Transition to the free market model of the Turkish electricity market started in 2001 with the Electricity Market Law no. 4628. Generation, wholesale, transmission and distribution activities were separated and some of them were privatized. This transformed the Turkish electricity market into a competitive one and with the increasing competition, on December 1, 2009 the Turkish day ahead electricity market started its operations. Almost 20% of the electricity trade in Turkey takes place in this market while the remaining is realized through bilateral contracts. Establishment of the Turkish day ahead electricity market is a significant development because it allows the forces of supply and demand to establish a fair and public price for electricity. Establishment of this price is essential for the actors in the energy sector to make efficient decisions. One of the key elements in the use of a price is to understand its dynamics. Because the market is so new there are only a very small number of works that analyze the prices that are realized in the Turkish day ahead electricity market. Currently we are only aware of [35], which uses a continuous time framework.

In this thesis our aim is to explore the dynamics of the prices observed in the Turkish day ahead electricity market. There are two levels of our work: 1) study the price process as a univariate process, 2) study the same process and temperature as a two dimensional process. The goal of the latter part is to understand to what extent temperature influences the day ahead prices. Our approach to this modeling problem is explained in the following paragraph. We have not been able to find publicly available models of temperature in Turkey. For this reason, we have built a preliminary model of temperature dynamics in Turkey. Our approach to this is also explained in the next paragraph.

We are interested in two processes: temperature and electricity prices established at the day ahead market in Ankara. Most of the temperature models that we have reviewed make use of periodic functions to capture the cyclical behavior of temperature (see Chapter 4 for examples). It appears to us that such an approach often leads to complicated models with many parameters. In this thesis, we propose a simple approach based on the following observation. The temperature process alternates between two phases: a heating phase and a cooling phase. We approximate both of these phases by a model of the form $\Lambda_t + X_t$, where $\Lambda_t$ is an affine function of time and $X_t$ is a stationary process (see for definition of stationary [10, p.207-208]). Conditioning on the heating and cooling phases, we see that very simple time series models capture the random stationary part very well. These ideas are developed and tested in detail in Chapter 4. Our analysis suggests that the length of the heating and cooling phases are *random*. Therefore, a full model for temperature dynamics that is based on the idea above would have to include random variables that represent these lengths. In this thesis, however, we confine ourselves to the dynamics of the temperature process conditioned on a fixed phase and leave the modeling of these lengths to future work.

We take a similar approach in modeling electricity prices. Upon examining the graphs of the price process in Chapter 5, one immediately sees that the electricity prices also exhibit cyclical behavior. However, this behavior is much more complicated than the dynamics of the temperature process. After listing some very salient features of the prices process, we have focused on a particular period (the time interval between February 2 and March 22) where clearly the prices seem to follow a model of the form $\Psi_t + Y_t$, where $\Psi$ is, again, an affine function of time and $Y$ a stationary process. Once again, the length of this period and its starting point seems to be random [1] and a full model of prices would have to include random variables to model these "global" features. However, for the purposes of this thesis, we limit ourselves to an analysis that conditions on this period where the price movement is particularly simple. Once again, upon conditioning on this period, we see in Chapter 5, that simple time-series models capture the dynamics of $Y_t$ remarkably well.

A primary aim of this thesis is to begin an analysis of the effect of temperature on electricity prices. In our approach this effect can be studied in two levels: globally and locally. The global variables in our temperature model are: slopes of the cooling and heating trends, the

[1] More precisely: the interval (Feb 2 ; March 22) is not random; but the lengths and the starting point of the periods in each year that exhibit the aforementioned trends are random

length of the cooling and heating periods. The local variables are the fluctuations in the temperature conditioned on the global variables. The global variables of our (incomplete) price model are: the starting point of the price-decrease period (the period that includes the February 2-March 22 interval specified above) and its length. A global analysis of the effect of temperature on prices would look at the dependence between the global variables. This is not simple and may require more data than what is currently available. Instead, in this thesis, we limit ourselves to a local analysis. That is, we look at the following question: conditioning on the global variables (i.e., fixing the values of the global variables) what is the relation between fluctuations in temperature and the fluctuations in price? To condition on the global variables we limit our analysis to the February-March period referred to above where all of the global variables described above seem to be fixed. The answer to the question we have just stated, according to our analysis, is that there is little relation between price and temperature fluctuations, see Chapter 5.

The next chapter is about electricity markets. The first section of this chapter is about the characteristics of an electricity market and electricity as a commodity. The second and the third sections summarize the liberalization movements of electricity markets in the world and the development of the Turkish electricity market. The last section gives information about how day ahead electricity prices are formed in Turkey.

Chapter 3 is devoted to a review of the literature on modeling of energy prices. The first section reviews AR and ARMA models. The second section summarizes ARIMA models in the literature. The last section presents studies of ARIMA and ARIMAX.

Chapter 4 contains our analysis and modeling of temperature in Turkey. Its second section presents several alternative models and ideas we have initially considered. The third section presents the ideas based on conditioning that we have found most effective.

Chapter 5 is devoted to the modeling of the Turkish day ahead electricity market. In the second section of this chapter we provide information about the price dynamics of the Turkish electricity market. The last section presents the model and the ideas outlined above.

Chapter 6 includes conclusions and provides ideas and questions for future work.

# CHAPTER 2

# SPOT ELECTRICITY MARKETS

## 2.1 Electricity as a Commodity

Electricity, without doubt, is one of the most important building blocks of modern human life. "It is easy to control, non-polluting at the location of its usage and convenient; it is used in the application of heat, light and power "[42, p.23]. It is a form of energy which is generated from other energy sources such as coal, natural gas, oil, nuclear power etc. Electricity is a fungible good which means that there is no difference in a unit of electricity, whether it is generated in a hydroelectric power plant in Turkey or in a nuclear power plant in France. In this respect, it is highly suitable for trading. Conduction of electricity (the ability to transfer electricity through a medium) is bounded so it is very difficult and prohibitively expensive to sell electricity physically, for example, to Germany which is generated in the U.S. This makes a global market for electricity impractical (currently there is no such market) but for example traders in the U.S can buy or sell financial derivatives of electricity from the German market for speculation or hedging purposes. Lack of storability will be discussed in the next section. Further characteristics of electricity as a commodity are summarized in Figure 2.1 which is taken from [42, p.27].

### 2.1.1 Structure of Electricity Markets

Operations in the electricity markets can be divided into four main categories.

**Generation** Electricity is produced by the generation companies, it can be generated in two different ways. First, by fuel fired power plants which burn coal, natural gas, oil and nuclear

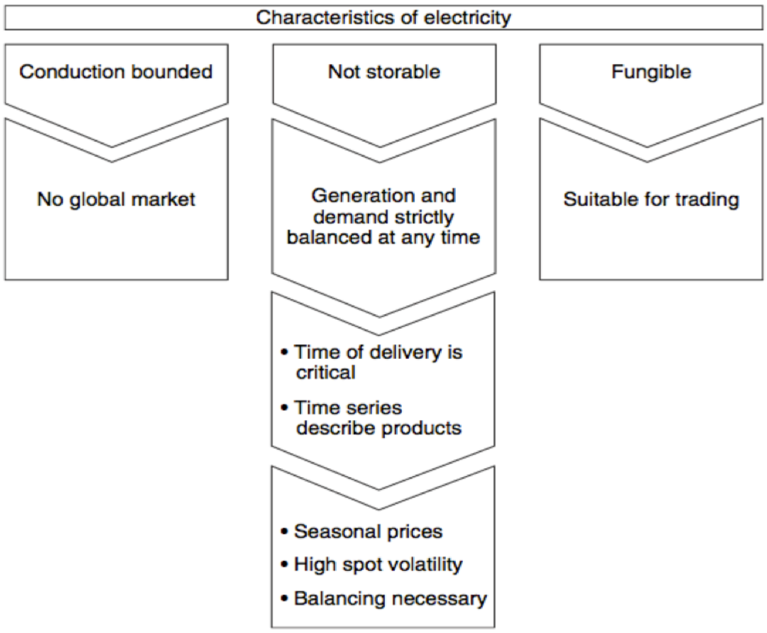


Figure 2.1: Characteristics of electricity.

fuel. Second, by using renewable energy sources such as wind force, geothermal energy, solar energy and gravitational force of water.

**Wholesale** Wholesale is the direct sale of electricity to eligible customers who are allowed to make bilateral contracts with the suppliers. There can be another consumer type in the market which is non-eligible consumers. These are not allowed to make bilateral contracts. Also exportation and importation of electricity occurs in this stage.

**Transmission** Transmission is the process of transportation of electricity from power plants to substations, through high-voltage electricity lines.

**Distribution and Retail** Retail and distribution is the sale of electricity and distribution of electricity to end users.

Electricity markets can be structured in two different ways. The first one is "vertically integrated" electricity markets in which generation, transmission and distribution are controlled by the same state-owned company. The second one is deregulated electricity markets or liberalized electricity markets. In this type of markets, generation, distribution, and wholesale are managed by different private companies but transmission is usually performed by the state-owned company.

### 2.1.2 Characteristics of Electricity Markets

**High volatility** Price volatility is the variance of its returns per unit time. More volatile a price, more fluctuations we expect it to exhibit in a short period of time. The most idiosyncratic feature of electricity markets is the high volatility of prices, which mainly stems from the non-storability of electricity. Non-storability implies that demand and supply need to match instantaneously. The ratio of percentage change in quantity demanded [supplied] to the percentage change in price is called demand [supply] elasticity or elasticity in demand [supply] [7, p.27]. It is defined as $(\Delta Q/Q)/(\Delta P/P)$. If this ratio is less than 1, demand [supply] is said to be inelastic. Also when demand inelasticity is "combined with inelastic supply, small changes in either the supply or demand for electricity can have huge effects on prices and this is probably the main cause of a highly volatile market "[12]. In the next example, we compare the volatility of electricity prices with other commodities and financial products, the difference seems to be clear:

- treasury bills and notes have volatility of less than 0.5%,
- stock indices have a moderate volatility of about 1-1.5%,
- commodities like crude oil or natural gas have volatility of 1.5-4%,
- very volatile stocks have volatility not exceeding 4%,
- electricity exhibits extreme volatility - up to 50%. [60, p.26]

**Price spikes and mean reversion** Prices spikes and mean reversion are the other features of electricity markets. An unanticipated large and sudden change is called a *price spike*, reversion of the price from this high level to its prior value is called *mean reversion*. Price spikes are usually caused by demand shocks. For example, when quantity demanded increases suddenly, power plants with high marginal costs such as oil-fired power plants will probably enter into the system. Since the price of oil is higher than the other primary energy sources, this will lead to a price jump. After the disappearance of the event that caused the price shock the price is likely to move back to its normal level, that is, it means reverts.

A long term mean reversion also seem to be present in the electricity markets. The electricity prices from one year to the next seem to be comparable (see Chapter 6 for an example of this in the Turkish market). We think that a full explanation of this is still an area that requires further work. However, we will not be addressing it in this thesis, for now let us mention the following perspectives:

"These prices are mean-reverting because weather is a dominant factor which influences equilibrium prices through changes in demand "[1]. Also M. Čulík and J. Valecký define that: "weather is cyclical factor and mean reversion process with the tendency to revert to mean level (which can change in time), this affects electricity demand and therefore equilibrium long-term price"[43].

**Seasonal behavior** The last feature is seasonality, that is, electricity prices behave periodically in parallel with the seasons. Demand is usually influenced by climate conditions like temperature and amount of daylight. In hot summer days, people starts to run air conditions or in winter number of daylight hours are declined so electricity is used for lighting applications. This factors increase the price of electricity in summer and winter but in spring and fall the demand will be lower so the prices will be lower. Also it is well known that demand for electricity is affected by the industrial activities, on holidays factories, banks, government offices and firms do not work and this causes decrease in demand so the prices will be lower. On the other hand, on weekdays electricity consumption is high so the prices will be higher. One of the primary goals of this thesis is to study the relation between prices and climate at a local level; see Chapter 6.

## 2.2 Liberalization of Electricity Markets

Liberalization is the process of splitting the vertically integrated monopoly structure in which generation, transmission and distribution of electricity is managed by the same government agency. The main motivation behind this idea is to introduce competition and transparency into the electricity sector. The hope is that increased competition and transparency (if successfully realized) would lead to improvement in both generation and transmission technologies. After liberalization, prices would be determined by supply and demand rather than government and this may cause more efficient prices for all participants in the market [60, p.1].

The timeline below shows the evolution of liberalization of electricity markets.

- 1982: The first country to try to liberalize the electricity sector is Chile. In 1982, Chile started to split generation and distribution companies. Before this, prices were determined according to a cost base formula. After the liberalization effort, the prices were set according to a marginal cost system. Electricity trading started in order to meet customer needs. A complete transformation to a deregulated market started with a large scale privatization in 1986 [47].

- 1989: British electricity sector reform started in 1989 by the Electricity Act which divided “The Central Electricity Generation Board”, the regulatory board which owned all the generation and transmission in the whole of England and Wales, into four public limited companies in 1990. In 1995, competition was introduced by the foundation of the “Electricity Pool,” a wholesale market for electricity. The wholesale market for England and Wales operated until 2005, and later Scotland joined as well [14] [60, p.1].

- 1991: British market reform was followed by Nordic countries, between 1991 and 2000. The Nordic market integrates electricity markets Denmark, Finland, Norway and Sweden into a single market [23].

- 1992: After the Nordic market, the U.S. electricity reform started in 1992 with the “Energy Policy Act”, and continued with the idea of splitting transmission and generation systems in California in 1994. By 2003 almost 45% of electricity generation plants were owned by private investors [40, p.38].

- 1996: New Zealand reformed its electricity sector and launched a deregulated electricity market, “The New Zealand Electricity Market (NZEM),” in 1996 [60, p.22].

- 1998:

  - Australian National Electricity Market (NEM) was founded in 1998 and at the end of 2002 all customers were able to choose their own retail suppliers [16].

- Electricity Act 1998 became valid in Netherlands which was designed to increase competition in its electricity market while maintaining efficiency and security on supply and demand side. In 1999, the “Amsterdam Power Exchange” (APX), the day ahead spot market, was founded. “European Energy Derivatives Exchange” (Endex) founded in 2003 is an OTC-market for trading standardized contracts of different maturities. ENDEX merged with APX in 2009 and the liberalization was completed [8] [25].
- The liberalization of German electricity market, largest electricity producer in Europe, started in 1998. One year later customers had the right to choose their own retailer. This ended the local monopoly supply which lasted more than 100 years and German wholesale prices fell by 60% [36].
- Spanish electricity reform was started in 1998. The main stated goal was to increase liquidity and trust on wholesale electricity market which would increase the efficiency of prices. The Spanish market is organized as: the day ahead market, several intraday market and ancillary market [11].

## 2.3 Historical Development of Turkish Electricity Market

### 2.3.1 Era of Private Companies

The first power plant was established in London in 1882 and “the first power plant in Turkey was put into operation in Tarsus in 1902 with 2 kW installed power capacity which was a dynamo connected to the water mill, after eleven years later the first big scaled power plant was established in İstanbul in 1913 ”[46] [62]. In her work Zeytinli says that at the time of the foundation of the Republic of Turkey in 1923, there were 38 power plants and all of them were own by private corporations. Between 1923-1930, Turkey tried to liberalize the economy where it allowed foreign joint stock companies operations [22]. In 1926, “the first Turkish electricity company, Kayseri ve Civari Elektrik Turk Elektrik, Inc., was established “[3]. Until 1935 electricity market in Turkey was formed by private investors. According to the TEİAŞ report, Etibank, the Mineral Research and Exploration Institution (MTA) and Electrical Works Survey Administration (EİEİ) were established in 1935 and later the Bank of Provinces and State Hydraulic Works (DSİ) were established [62]. These developments in state institutions of Turkey, accelerated the state investments on power plants. In 1948, some power plants ere put into service by government. By the begging of 1930s there was a common belief in all over the world that ownership of electricity sector by government would be beneficial [19]. Following this idea, nationalization of Turkish electricity industry was completed by 1944 [19]. When we come to early 1950s, we see government and private sector entities together involved in the construction and operation of power plants. Entities such as Çukurova Electric Co. and Kepez Electric Co. were founded in these years. They were founded to solve the electricity needs of Adana and Antalya provinces [62]. The Ministry of Energy and Natural Resources (MENR) was established in 1963 [19] to regulate the Turkish electricity policy, The period between the years 1913 and 1970 is the Era of Private Sector.

### 2.3.2 The TEK Era

The period between 1970 and 1984 can be referred to as "the TEK era." Turkish Electricity Authority (TEK) was established in 1970 as a state-owned company which controls the country's electricity industry. Hepbasli states [3]:

> "All generation assets were passed to TEK except the ones that belong to Cukurova Elektrik T.A.S. and Kepez ve Antalya Havalisi Elektrik Santralleri T.A.S... The transmission and distribution business, which was managed by the municipals, were left to the local governments."

An important event during the TEK era was the instability in the electricity system caused by the worldwide energy crisis in the 1970s. Large foreign dependence on the primary resources used by thermal power plants in Turkey had a major role in this problem [62].

In 1982 by Law No. 2705 transmission and distribution facilities' ownership were also transferred to TEK [52]. Until 1984, private entities did not activity involve in this electricity market.

### 2.3.3 Era of TEK and Private Companies

"In 1984 Law no. 3096, named as Respecting Authorization to Institutions other than the TEK for Generation, Transmission, Distribution and Trade of Electricity "is issued which allowed the private sectors to build and operate the electricity generation, transmission and distribution systems for the first time in the history of Turkish Republic"[2]. After this law private sector entities again started to involve in electricity sector and monopoly of TEK started to disappear. Private participants involved in the electricity market by the following financing technics: Build Operate and Transfer (BOT), Transfer of Operating Rights (TOR) and Build Operate Own (BOO) [19]. In a BOT system private companies build and operate a power plant for a specific period of time, for that time government give guarantee to purchase a certain amount electricity. At the end of the specified time ownership of the plant transfers to government without any cost. Main difference in BOO from BOT is that at the end of contract ownership of the power plant remains at the private investor. In the TOR system a private investor retains all rights to operate in a specific area. Government transfer the right to operate in a given sector to a private sector for a certain period of time and at the end of the contract again ownership is transferred to government agency. Actually it is a kind of leasing. BOT and BOO are used to ensure the establishment of new power plants by the private sector in Turkey and TOR is used to attract private investors to involve in the distribution of electricity sector [32]. Until the end of 1999 twenty two power plants were established by BOT [49]. Almost ten private entities were participated in the generation, transmission and distribution of electricity in Turkey between the years 1988 and 1992 [62]. In 1994, TEK, public-owned and vertically integrated monopoly, had come to end of his life by the Law of Council of Ministers [62] [49]. TEK was "split into two separate state owned enterprises as Turkish Electricity Generation Transmission Company (TEAŞ) and Turkish Electricity Distribution Company (TEDAŞ) "[49]. As it can be understood from the name of the companies, TEAŞ engaged in the production of electricity and TEDAŞ engaged in the transmission of electricity where they are bought reported to MENR. The period between the years 1984 and 2000 is the Era of TEK and Private Sector.

### 2.3.4 Liberalization Era

The period since 2001 can be referred to as the "Liberalization". In 2001 Energy Market Regulatory Authority (EMRA) was established. EMRA is structured as an independent regulatory entity which is "responsible for preparing and implementing secondary legislation, authorizing market participants, approving and publishing tariffs, monitoring and supervising market participants, conducting technical, legal and financial audits, settling disputes, approving, amending and enforcing performance standards, and, where necessary, applying sanctions "[48]. Corporations need to obtain a license for distribution, transmission, generation and wholesale or retail, to take place in the electricity market. Privatization has gained momentum after 2001 with Electricity Market Law no. 4628 which split TEAŞ into three public owned entities: Turkish Electricity Transmission Company (TEIAŞ), Turkish Electricity Generation Company (EÜAŞ) and Turkish Electricity Trading and Contracting Company (TETAŞ). These are responsible for the following respectively: transmission and system operation, generation, wholesale trading and contracting. Lastly TEDAŞ split into twenty one regional distribution companies.

There are 289 private companies and 6 public institutions in the production sector, 112 private companies and 1 public institution in the wholesale sector, 12 private companies and 9 public institutions in the distribution and retail sector, 108 private autoproducer and 1 public autoproducer, 1 private autoproducer group according to their licenses as of June 2012. Table 2.1 shows the development of Turkey's installed capacity where thermal represents the capacity produced in thermal power plants, hydroelectric represents the capacity produced in the hydroelectric power plants and geothermal and wind represents the capacity produced by the wind power and geothermal power. Units are in MW [62]. Installed capacity is the maximum electricity production at an instant in time [17, p.82].

## 2.4 The Turkish Electricity Market

Turkey produces electricity from the following primary energy sources: natural gas, coal, hydraulic, liquid fuels and other renewable energy sources. Table 2.2 shows the percentage Turkey electricity generation by primary sources in 2011 [21, p.22]. Table 2.2 shows that almost half of the Turkish electricity production comes from natural gas; coal comes in the second place. 73 % of electricity production is made by these two energy sources and the rest is produced from renewable energy sources and others. An obvious future study is the impact of the prices of these commodities on electricity prices in Turkey. Wind and other renewable energy sources may have small percentage in Turkish electricity productions but in the near future their shares may increase with new investments and incentives on electricity generation using these methods.

Table 2.1: Development of Turkey's installed capacity.

| Year | Thermal | Hydroelectric | Geothermal and Wind | Total |
|---|---|---|---|---|
| 1913 | 17.2 | 0.1 | | 17.3 |
| 1923 | 32.7 | 01 | | 32.8 |
| 1930 | 74.8 | 3.2 | | 78 |
| 1940 | 209.2 | 7.8 | | 217 |
| 1950 | 389.9 | 17.9 | | 407.8 |
| 1960 | 860.5 | 411.9 | | 1272.4 |
| 1970 | 1509.5 | 725.4 | | 2234.9 |
| 1980 | 2987.9 | 2130.8 | | 5118.7 |
| 1990 | 9535.8 | 6764.3 | 17.5 | 16317.6 |
| 2000 | 16052.5 | 111175.2 | 36.4 | 27264.1 |
| 2001 | 16623.1 | 11672.9 | 36.4 | 28332.4 |
| 2002 | 19568.5 | 12240.9 | 36.4 | 31845.8 |
| 2003 | 22974.4 | 12645.4 | 33.9 | 25587 |
| 2004 | 24144.7 | 12645.4 | 33.9 | 36824 |
| 2005 | 25902.3 | 12906.1 | 35.1 | 38843.5 |
| 2006 | 27420.2 | 13062.7 | 81.9 | 40564.8 |
| 2007 | 27271.6 | 13394.9 | 169.2 | 40835.7 |
| 2008 | 27595 | 13828.7 | 393.5 | 41817.2 |
| 2009 | 29339.1 | 14553.3 | 868.8 | 41817.2 |
| 2010 | 32278.5 | 15831.2 | 1414.4 | 49524.1 |

Table 2.2: Turkey electricity generation by primary sources.

| Primary Sources | Percentage |
|---|---|
| Natural Gas | 44.71% |
| Hydraulic | 22.80% |
| Coal | 28.26% |
| Wind | 2.07% |
| Liquid Fuels | 1.67% |
| Geothermal | 0.29% |
| Waste and Other | 0.20% |
| Total | 100% |

The next table shows the percentage share of the producers in Turkey's electricity generation [21, p.21]. Most of the electricity is produced by EÜAŞ but this is expected to decrease due the privatization in the future. The second biggest producers are the private generation companies and TOR (see subsection 2.3.3) with 29%, auto producers, entities that produce electricity for their own usages, have the least share with 5.13%.

Table 2.3: Share of the producers in Turkey's electricity generation.

| Producers | Percentage |
|---|---|
| EÜAŞ | 40.42 % |
| Private generation companies & TOR | 29.17% |
| Build own operate | 19.67% |
| Build operate transfer | 5.61% |
| Auto producers | 5.13% |
| Total | 100% |

### 2.4.1 Turkish Day Ahead Electricity Market

The market operator of Turkish electricity market is the "Electricity Market Services and Financial Settlement Department" (PMUM) which operates as a part of TEIAŞ. PMUM is responsible for day ahead planning, the day ahead market operation, settlement and data publishing activities. The system operator of Turkish electricity market is the "National Load Dispatch Center (MYTM)". MYTM is responsible for forecasting day ahead electricity demand and maintaining a balance between supply and demand. Almost 80% of electricity agreements are made via bilateral contracts between producers, wholesalers and customers, the rest takes place in the spot market operated by PMUM. The main function of the the spot market is to balance the excess or deficit that occur in the bilateral agreements. The balancing mechanism is divided into two parts as day ahead balancing and real-time balancing. Day ahead market consists of activities to balance next day's production and/or consumption. Real-time balancing is used to balance the supply and demand in real time. This balancing system provides the backup capacity that can be activated within a maximum of 15 minutes system by the system operator. All licensed producers, wholesalers, autoproducers, retailers and eligible consumers participate in the day ahead and real-time market. Eligible consumers that consume 25mW or above per year, are freely choose their suppliers.

#### 2.4.1.1 The Price Formation of Day Ahead Electricity Market

Most of what follows is from [20], which is a document in Turkish. Here we explain its contents in English. Day ahead electricity prices (SGÖF) are determined as follows: in the first step direction of the system is determined by comparing total daily production schedule and consumption forecasts. If the total consumption is greater than total production, there will be an energy gap or deficit in the system; if the total consumption is less than total production then there will be energy surplus in the system. In the case of an energy gap, offers in the direction of sale to the system is accepted and in the case of energy surplus offers in the direction of purchase from the system is accepted. If the total consumption is equal to the total production then the system will be in equilibrium so there won't be any sale or purchase. An example of this procedure is given in Table 2.4 [20].

Table 2.4: Determining the direction of the system.

| Hour | Total Production Schedule (mWh) | Total Consumption Forecast (mWh) | Difference | System Direction | Accepted Offer Direction |
|---|---|---|---|---|---|
| 8 | 15000 | 16000 | -1000 | Energy Gap | Sale to the system |
| 9 | 17000 | 15500 | 1500 | Energy Surplus | Purchase form the system |
| ... | ... | ... | ... | .... | ... |
| ... | ... | ... | ... | .... | ... |
| 11 | 18000 | 18000 | 0 | Equilibrium | Equilibrium |
| 12 | 14000 | 17000 | -3000 | Energy Gap | Sale to the system |
| 13 | 13000 | 14500 | -1500 | Energy Gap | Sale to the system |
| 14 | 14000 | 12000 | 2000 | Energy Surplus | Purchase form the system |
| ... | ... | ... | ... | .... | ... |
| ... | ... | ... | ... | .... | ... |

In the second step, for each hour the offers/prices that the participants propose to the system are sorted. Sale offers are ranked in descending order and purchase offers are ranked in ascending order. An example of this procedure is given in Table 2.5 [20].

The table on the left below shows how prices are ordered if there is an energy gap or sale to the system and the table on the right shows how prices are ordered if there is an energy surplus or purchase from the system. This procedure is done for all hours in a day.

Table 2.5: Ranking the system offers.

| The Amount of Offer (mWh) | Offer Price (TL/mWh) |
|---|---|
| 100 | 70 |
| 100 | 71 |
| 200 | 72 |
| 200 | 73 |
| 100 | 73 |
| 150 | 74 |
| 200 | 75 |
| 175 | 76 |
| 100 | 80 |
| 200 | 82 |

| The Amount of Offer (mWh) | Offer Price (TL/mWh) |
|---|---|
| 100 | 75 |
| 90 | 73 |
| 50 | 50 |
| 67 | 47 |
| 63 | 45 |
| 5 | 43 |
| 88 | 42 |
| 120 | 41 |
| 56 | 30 |
| 130 | 30 |

In the third step, hourly offers are evaluated to compensate the difference between the total production schedule and the estimated consumption for each hour. In the case of energy gap, offers are accepted from the lowest price to the highest price until the gap is closed and in the case of energy surplus offers are accepted from the highest price to the lowest price until the surplus is eliminated. The last accepted offer that compensates the difference is called SGÖF. In the evaluation process it is assumed that all offers can be fulfilled partially. Figure 2.2 shows how this procedure is performed [20].

In this example we assume that there is an energy gap at ten o'clock and the gap is 1000 mWh. In order to close this gap enough amount of offers are accepted and the last accepted offer's price 98TL will be SGÖF for ten o'clock. In this case we assume that there is no block or/and flexible offer. (See the following paragraphs for the definition of these terms.)

| Sale to the system offers for ten o'clock | |
|---|---|
| Amount of Offer(mWh) | Offer Price(TL/mWh) |
| 100 | 70 |
| 100 | 70 |
| 150 | 72 |
| 100 | 75 |
| 150 | 80 |
| 100 | 85 |
| 200 | 90 |
| 100 | 98 |
| 200 | 100 |
| 100 | 100 |

Acceptance of offers are stoped at this point.

Last accepted offer is 100 mWh and with this offer 1000 mWh gap is closed. The offer prices = 98TL/mWh will be the SGÖF for 10 o'clock.

Figure 2.2: Formation of SGÖF with hourly offers.

Also there can be block offers in the system. Block offers are given at least for four hours and it can be given five times a day. If block offers are accepted it must be used for consecutive hours. Block offers may decrease SGÖF for some hours and it may increase SGÖF for another hour so to accept the block offers net effect for 24 hours must be examined. Block offers are usually used by the producers in order to increase the efficiency of power plant. Figure 2.3 shows how the SGÖF is effected if there is a suitable block offer [20].

In this example we assume that there is an energy gap again at ten o'clock and the gap is 1000 mWh and we have a block offer of 300 mWh with the price 83 TL/mWh. In this situation block offer decreases the SGÖF so it is accepted and the new SGÖF for ten o'clock is determined to be 85 TL/mWh. Also it is assumed that there is no flexible offer.

| Block offer of sale to the system | |
|---|---|
| Amount of Offer(mWh) | Offer Price(TL/mWh) |
| 300 | 83 |

| Sale to the system offers for ten o'clock | |
|---|---|
| Amount of Offer(mWh) | Offer Price(TL/mWh) |
| 100 | 70 |
| 100 | 70 |
| 150 | 72 |
| 100 | 75 |
| 150 | 80 |
| 100 | 85 |
| 200 | 90 |
| 100 | 98 |
| 200 | 100 |
| 100 | 100 |

Acceptance of offers are stoped at this point.

Last accepted hourly offer is 100 mWh with 85 TL/mWh
by this offer 700 mWh of gap is closed and the rest is closed
by the block offer of 300 mWh.
The offer prices = 85TL/mWh will be the SGÖF for 10 o'clock.

Figure 2.3: Formation of SGÖF with hourly and block offers.

In addition to block offers there can be flexible offers in the system. A flexible offer is an hourly offer in which the hour is not stated, it can be used in any hour of a day. If the flexible offer decreases the SGÖF it should be accepted. Figure 2.4 shows how the SGÖF is effected if there is a suitable block offer [20].

Let's assume that we have a flexible offer of 100 mWh with 79 TL/mWh and energy gap is 1000 mWh. This offer decreases the SGÖF so it is accepted and the new SGÖF for ten o'clock will be 80 TL/mWh.

| Flexible offer of sale to the system | |
|---|---|
| Amount of Offer(mWh) | Offer Price(TL/mWh) |
| 100 | 79 |

| Block offer of sale to the system | |
|---|---|
| Amount of Offer(mWh) | Offer Price(TL/mWh) |
| 300 | 83 |

| Sale to the system offers for ten o'clock | |
|---|---|
| Amount of Offer(mWh) | Offer Price(TL/mWh) |
| 100 | 70 |
| 100 | 70 |
| 150 | 72 |
| 100 | 75 |
| 150 | 80 |
| 100 | 85 |
| 200 | 90 |
| 100 | 98 |
| 200 | 100 |
| 100 | 100 |

Acceptance of offers are stoped at this point.

Last accepted hourly offer is 150 mWh with 80 TL/mWh
by this offer and block offer 900 mWh of gap is closed and the rest is closed by the flexible offer of 100 mWh.
The offer prices = 80TL/mWh will be the SGÖF for 10 o'clock.

Figure 2.4: Formation of SGÖF with hourly, block and flexible offers.

# CHAPTER 3

# LITERATURE REVIEW

In a competitive electricity market, price modeling and forecasting allow producers and costumers to take optimal actions. Producers can develop investment plans while costumers can build up strategies in long term contracts with these forecasted prices. There are many studies in the literature about electricity demand modeling, spot electricity price forecasting and electricity derivatives pricing. In the current literature, spot electricity prices are typically modeled by stochastic differential equations (SDE) [6] [34], artificial neural networks (ANN) [33], regime switching models [41] [58], dynamic regression [26], generalized autoregressive conditional heteroscedasticity (GARCH) [53] [65] and autoregressive moving average (ARMA) type models. The present thesis mainly employes ARMA type models. Therefore, in this literature review we have mostly confined ourselves to an overview of these type of models.

## 3.1 Forecasting with AR and ARMA

Unless otherwise noted all of the displays in this section are taken from [15], [38], [61], [63]. Authors in [15] discussed the forecasting power of ARMA, ARMA combination with GARCH, gaussian mixture and switching regime approach. In order to provide a basic knowledge about econometric models used to estimate spot and future electricity prices, all models used in the paper [15] will be summarized below.

Models are characterized by parameter $Q$ which is estimated by maximizing the log likelihood function $L(Q)$ given by:

$$maximize_{\{\Theta\}} L(Q) = \sum_{t=1}^{T} \ln\left(f[\epsilon_t|\psi_{t-1}; Q]\right). \tag{3.1}$$

(This is given as display 1 in [63])

$\psi_{t-1}$ is the sigma algebra generated by the random variables up to time $t-1$.

Autoregressive moving average: ARMA($p$,$q$) is defined as follows:

$$\begin{aligned} \hat{y}_t &= y_t + \epsilon_t \\ &= \sum_{z=1}^{p} \alpha_z y_{t-z} + \sum_{z=1}^{q} \epsilon_{t-z} + \epsilon_t, \end{aligned} \tag{3.2}$$

where $y_t$ is the observed value while $\hat{y}_t$ represents the predicted value and $\epsilon_t$ indicates the error, i.e., $\epsilon_t = \hat{y}_t - y_t$. The error is assumed to be normally distributed with the probability distribution:

$$f[\epsilon_t|\psi_{t-1}] = \frac{1}{\sqrt{2\pi}\sigma} exp\left(-\left(\frac{\epsilon_t - \mu}{\sqrt{2}\sigma}\right)^2\right).$$

The authors estimate $\mu$ and $\sigma$ via maximum likelihood. Authors than suggests the use of ARMAX (Autoregressive moving average model with exogenous inputs) model to take into account influences of exogenous variable. Their work in this direction is summarized in Subsection 3.3 .

General autoregressive conditional heteroscedasticity: In this case again the error is distributed normal but its variance with respect to time, that is $\epsilon_t \sim$ i.i.d $N(\mu, {\sigma_t}^2)$ with the probability distribution:

$$f[\epsilon_t|\psi_{t-1}] = \frac{1}{\sqrt{2\pi}\sigma_t} exp\left(-\left(\frac{\epsilon_t - \mu}{\sqrt{2}\sigma_t}\right)^2\right).$$

GARCH(p,q) is defined as:

$${\sigma_t}^2 = \omega + \sum_{z=1}^{p} \alpha_z {\sigma_{t-z}}^2 + \sum_{z=1}^{q} \beta_z {\epsilon_{t-z}}^2, \tag{3.3}$$

where $\omega > 0$, $\alpha_z \geq 0$ and $\beta_z \geq 0$ [63]. These are the sufficient conditions for positive variance and the parameters can be found by log likelihood function in 3.1 [39].

Gaussian mixture: Authors state that if the fat tail occurs from the deviations of the normality hypothesis it is useful to use Gaussian mixture rather than GARCH and it is defined as follows:

$$f[\epsilon_t|\psi_{t-1}] = \sum_{j=1}^{m} p[S_t = j] f[\epsilon_t|S_t = j, \psi_{t-1}], \tag{3.4}$$

(This is given as display 7 in [63])

and the probability distribution of the state variable $S_t$ is given in (3.5) which can be estimated by (3.1).

$$f[\epsilon_t|S_t = j, \psi_{t-1}] = \frac{1}{\sqrt{2\pi}\sigma_j} exp\left(-\left(\frac{\epsilon_t - \mu_j}{\sqrt{2}\sigma_j}\right)^2\right). \tag{3.5}$$

Switching regime: The last model used in this paper is switching regime in which the probability occurrence of state variable is time variant that is $P[S_t = j|\psi_{t-1}]$. It is called state dependent switching regime model:

$$f[\epsilon_t|\psi_{t-1}] = \sum_{j=1}^{m} P[S_t = j|\psi_{t-1}] f[\epsilon_t|S_t = j, \psi_{t-1}].$$

and the probabilities can be calculated by (3.6).

$$P[S_t = j|\psi_{t-1}] = \sum_{i=1}^{2} P[S_t = j|S_{t-1} = i] P[S_{t-1} = i|\psi_{t-1}], \tag{3.6}$$

where two state, first order Markow switching is used , $S_t$ depends on the state of the last time step $t-1$ and transition matrix T represents the probability of changing or remaining in the respective regime.

$$T = \begin{bmatrix} P[S_t = 1|S_{t-1} = 1] & P[S_t = 1|S_{t-1} = 2] \\ P[S_t = 2|S_{t-1} = 1] & P[S_t = 2|S_{t-1} = 2] \end{bmatrix}$$

$$= \begin{bmatrix} \rho_{11} & 1-\rho_{22} \\ 1-\rho_{11} & \rho_{22} \end{bmatrix}.$$

(This is given as display 10 in [63])

Lastly the probabilities of the two price regimes at $t = 0$ is defined as:

$$P[S_0 = 1|\psi_0] = \frac{1-\rho_{22}}{2-\rho 22-\rho_{11}},$$
$$P[S_0 = 2|\psi_0] = \frac{1-\rho_{11}}{2-\rho 22-\rho_{11}},$$

and for any time $t > 0$ the probabilities are calculated by:

$$P[S_t = j|\psi_t] = \frac{P[S_t = j|\psi_{t-1}]f[\epsilon_t|S_t = j, \psi_{t-1}]}{f[\epsilon_t|\psi_{t-1}]}.$$

The parameters $\mu_j$, $\sigma_j$, $\rho_{11}$ and $\rho_{22}$ can be estimated by (3.1).

These explained models are used to extend ARIMA model and they are used to predict the spot electricity prices of the European Energy Exchange AG (EEX) which represent the German electricity market. Historical electricity prices of the spot and two reserve market are used as data. The accuracy of the models are checked by the mean absolute error (MAE), fraction of variance ($R^2$), the mean absolute percentage error (MAPE), the value of the log likelihood function (LLF) and the Schwarz Bayes Information criterion (SBC). "Low values in MAE and MAPE and high values in $R^2$ indicate a good prediction accuracy while high value of LLF and low value in SBC shows a good representation of the price distribution."[63]

ARMA model is compared with the extended ARMA models and results show that additional price information of the reserve market as an exogenous variable of ARMAX doesn't improve the representation of the prices of spot market and the prediction power. ARMA with GARCH improve the representation of price distributions while doesn't improve the prediction power. ARMA with GM increases the representation of the price distribution while decreases the forecasting power. ARMA with SR improves the representation of distributions. When we look at the reserve market, results shows that all the discussed extended models except GARCH improve the representation of the price distribution and prediction power. In the case of GARCH extension it increases the representation of prices while no improvements in prediction power.

The forecasting power of univariate time series models are tested in [38] for the Leipzig Power Exchange. The Leipzig Power Exchange was the electricity market for Germany before it was merged with EEX in 2002. The data set contains the period between 16 June, 2000 and 15 October, 2001. All models in this study used to estimate complete data set as a single time series and 24 time series for each hours in order to see the dynamic behavior of each hours. Models used in this study are: AR(1) process, AR(1) process with time varying intercept, ARMA process with time varying intercept, crossed ARMA process with time varying intercept, ARMA processes with jumps and unobserved components model. In order to test the forecasting power root mean square error (RMSE), MAE and Diebold Mariano (DM) [27] are used. DM is used to determine which model has the best forecasting power. The results shows that forecasting hour by hour strategy increases the forecasting power of the univariate time series.

The last study for this part is [61] which uses 12 time series methods to estimate hourly spot prices of California and Nord Pool market. The datasets used in this study include data from the years 1999 to 2000 for California and 1998 to 1999, 2003 to 2004 for Nord Pool electricity market. Load data is used as an exogenous variable in California market and air temperature is used as an exogenous variable in the Nordic market.

The first model considered in this study is autoregressive (AR) models given in (3.7). The logarithms of price and load data are used to reduce variance. Moreover the mean of price data and the median of load data was removed to center the data around zero.

$$p_t = \phi_1 p_{t-24} + \phi_2 p_{t-48} + \phi_3 p_{t-168} + \phi_4 mp_t + \psi_1 z_t + d_1 D_{Mon} + d_2 D_{Sat} + d_3 D_{Sun} + \epsilon_t. \quad (3.7)$$

(This is given as display 1 in [61])

To cope with weekly seasonal behavior autoregressive structure $p_{t-24}$, $p_{t-48}$ , $p_{t-168}$ and three dummy variables $D_{Mon}$, $D_{Sat}$, $D_{Sun}$ are used. The variable $z_t$ denotes the log load forecast for the California market and actual temperature for the Nord Pool. The minimum of the previous day's 24 hourly log price is $mp_t$ which is used to link previous day's price signals to today's bidding behavior. If the parameter $\psi_1$ equals to zero, the model will be AR otherwise it is ARX. It is noted that "parameters can be estimated by minimizing the Final Prediction Error (FPE) criterion "[61].

The next model is spike preprocessed model. This model is almost same as described above, the main difference is: the price spikes are replaced with a less extreme value. This is done by the technique called damping scheme. In this technique"an upper limit $T$ is set on the price which is equal to the mean plus three standard deviation of the price in the calibration period ". Then all the prices greater than $T$ are transformed to $P_t = T + T \log 10\left(\frac{P_T}{T}\right)$ and the spike preprocessed model is denoted by p-ARX and p-AR.

Also regime switching model is discussed in this paper. In order to model with spikes in the data Threshold Autoregressive (TAR) is used.

Instead of time series models, a continuous time stochastic differential equation is used to estimate spot electricity prices. Mean reverting jump diffusions model is defined in (3.8).

$$dp_t = (\alpha - \beta p_t) + \sigma dW_t + J dq_t, \tag{3.8}$$

(This is given as display 3 in [61])

where $W_t$ represents the Brownian motion and responsible for small fluctuations around the mean. The independent compound Poisson process denoted by $q_t$ which produces jumps with the size $J$ and frequency $\lambda$. $J$ is a Gaussian distribution with mean $\mu$ and variance $\delta^2$. Seasonal behavior is captured by the $\alpha$ which is a deterministic function of time. Since the data is discrete and the model is continuous authors apply transformations to the model to get a discrete time model which is given in (3.9).

$$p_t = \phi_1 p_{t-24} + \psi_1 z_t + d_1 D_{Mon} + d_2 D_{Sat} + d_3 D_{Sun} + \epsilon_{t,i}, \tag{3.9}$$

(This is given as display 4 in [61])

where

$$i = \begin{cases} 1, & \text{if no jump occurred in this time period.} \\ 2, & \text{if there was a jump.} \end{cases}$$

and $\epsilon_{t,1} \sim N(0, \sigma^2)$, $\epsilon_{t,2} \sim N(\mu, \sigma^2 + \delta^2)$.

Lastly, we observe that semiparametric models are also used in the modeling of electricity prices. The main idea behind semiparametric models according to authors is that “a nonparametric kernel density estimator shows a better fit to the data. If this is the case perhaps the time series models would give a more accurate result.”. To test this situation four semiparametric models are used with two different estimators. The nonparametric estimators for autoregressive models are: iterated Hsieh Manski estimator (IHM) and the smoothed nonparametric ML estimator.

The results of point forecasts show that models with exogenous variable perform better then the rest for the California market but the situation is not same for the Nordic market. Analysis for interval forecasts show that two semiparametric models are superior than the rest. Overall performance of semiparametric models are again better than the other discussed models for California electricity market and Nord Pool.

## 3.2 Forecasting with ARIMA

Authors in [37] try to forecast the day ahead electricity prices of Spain and California by using autoregressive integrated moving average (ARIMA) models. Their aim is to use ARIMA model today in order to predict tomorrow's hourly electricity prices. For the Spanish electricity market models is as follows:

$$\begin{gathered}(1-\phi_1 B^1-\phi_2 B^2-\phi_3 B^3-\phi_4 B^4-\phi_5 B^5)\\(1-\phi_{23} B^{23}-\phi_{24} B^{24}-\phi_{47} B^{47}-\phi_{48} B^{48}-\phi_{72} B^{72}-\phi_{96} B^{96}-\phi_{120} B^{120}-\phi_{144} B^{144})\\(1-\phi_{168} B^{168}-1-\phi_{336} B^{336}-1-\phi_{504} B^{504})\log p_t\\=c+(1-\theta_1 B^1-\theta_2 B^2)(1-\theta_{24} B^{24})(1-\theta_{168} B^{168}-\theta_{336} B^{336}-\theta_{504} B^{504})\epsilon_t.\end{gathered} \tag{3.10}$$

(This is given as display 3 in [37])

This model doesn't use differentiation and also needs five previous hours to predict the next hour. Three weeks have been used to predict and check the performance of the model. The first one is an usual demand weak, the second is a low demand week and the last one is a high demand week. Daily mean errors are around 5% for the first weak, 8% for the second weak and 7% for the last weak. Also different statistical tools are used to check the accuracy of the model like: the average prediction error, Mean Weak Error (MWE) and Forecast Mean Square Error (FMSE). Authors conclude that average prediction error is around 10% with and without explanatory variables.

Model for the Californian market is

$$\begin{gathered}(1-\phi_1 B^1-\phi_2 B^2)\\(1-\phi_{23} B^{23}-\phi_{24} B^{24}-\phi_{47} B^{47}-\phi_{48} B^{48}-\phi_{72} B^{72}-\phi_{96} B^{96}-\phi_{120} B^{120}-\phi_{144} B^{144})\\(1-\phi_{167} B^{167}-\phi_{168} B^{168}-\phi_{169} B^{169}-\phi_{192} B^{192})\\(1-B)(1-B^{24})(1-B^{168})\log p_t\\=c+(1-\theta_1 B^1-\theta_2 B^2)(1-\theta_{24} B^{24}-\theta_{48} B^{48}-\theta_{72} B^{72}-\theta_{96} B^{96})\\(1-\theta_{144} B^{144})\\(1-\theta_{168} B^{168}-\theta_{336} B^{336}-\theta_{504} B^{504})\epsilon_t.\end{gathered} \tag{3.11}$$

(This is given as display 4 in [37])

The models need the previous two hours to predict the next and also uses hourly, daily and weekly differentiation. One week is used to test the power of the model which is the prior to the begging of price volatility and electricity crisis. After computations 5% of mean error is founded, also accuracy of the model is checked again by statistical tools same in the Spanish case. In this case average prediction error is around 5% in the stable period and around 11% in the volatility period. It is concluded that accuracy of this model is better than the previous model before the electricity crisis.

Extended ARIMA models are worked in [44] [45] for Californian electricity market. The authors in [44] forecasts daily electricity prices then modify the predicted error to improve the accuracy. The procedure of forecasting and modification is summarized below.

The time domain for the prices are denoted by $t \in \{D^- + D^+\}$, where $D^-$ represents the historical time domain and $D^+$ refers to the forecasting domain. $P^-(t)$ is the historical prices, by using this forecasting price model $M$ is set up. Error series $\{E^-, t \in D^-\}$, which is also a time series, are obtained by comparing the real historical prices and the model $M$. This error series are used to set up error forecasting model which is denoted by $M_{E1}$. After $M$ and $M_{E1}$ are established, prices are forecasted ($\tilde{P}^-(t)$ is the forecasted price) by the model $M$ at time $t \in D^-$ and the model errors $\tilde{E}^-(t)$ are forecasted by $M_{E1}$ at time $t \in D^-$. The equation 3.12 is used to modify the forecasted prices and residual error is calculated by the equation 3.13.

$$\tilde{P}^*(t) = \tilde{P}^-(t) + \tilde{E}^-(t), \qquad t \in D^-. \tag{3.12}$$

$$E_1(t) = P^-(t) - \tilde{P}^*(t), \qquad t \in D^-. \tag{3.13}$$

(These are given as display 9 and10 in [44])

If the accuracy of the model is not at the desired level authors continue to modify the forecasted prices one more time and then accuracy of the model is checked again by the residual error. This procedure goes on until the desired accuracy level is reached. Model $M$ that is set up in $t \in D^-$ used to forecast prices $\tilde{P}^+(t)$ in $t \in D^+$ and in the same manner $M_{Ei}$ is used to forecast errors $\tilde{E}^+$ in $t \in D^+$. Forecasted prices are adjusted by (3.14).

$$\tilde{P}^{*+}(t) = \tilde{P}^+(t) + \sum_i \tilde{E}^+(t), \qquad t \in D^+. \tag{3.14}$$

(This is given as display 11 in [44])

The historical prices of California Power Market is used as an example and the model for historical prices is defined in 3.15 and historical prices of the years 1999 and 2000 is used as an numerical example. In both cases 50 historical prices are used to forecast next 10 days prices.

$$(1 - \phi_1 B - \phi_2 B^2)(1 - B)(1 - B^7)P_t = c + \epsilon_t. \tag{3.15}$$

(This is given as display 12 in [44])

Model needs previous two days prices to forecast next day and first order differentiation is used to eliminate linear trend while seventh order is used to eliminate the weekly trend. The parameters of 3.15 is determined by regressive analysis which will be the model $M$. ARIMA M with the same order but different parameters are used to forecast prices for both years but the error forecasting models $M_{Ei}$ are differs in both cases. This is because the prices of 1999 is more stationary than 2000 according to authors. Error forecasting models $M_{E1}$ and $M_{E2}$ for 1999 are AR(3) and AR(2), respectively, while error forecasting model for 2000 are ARIMA(3,1,1) and AR(3), respectively. It is concluded that extended ARIMA model increases the accuracy level while it provides an easy way of modeling.

Electricity price forecasting by Wavelet-ARIMA technique is used in [4]. Wavelet transform is used to divide price series in to three to six pieces which are expected to present a better behavior. In this study twenty four hours of electricity prices of Spanish market is forecasted. Forecasting method consist of three parts. In the first part historical prices are decomposed in to four series by the wavelet transform. In the second step appropriate ARIMA model is used to forecast prices in each series. In last part inverse wavelet transform is used to reconstruct the estimate series in order to forecast the prices of the target day. Four weeks of four seasons of 2002 is used to compare the wavelet-ARIMA model with classical ARIMA model. Weekly error of the wavelet-ARIMA and ARIMA is below 4.8% and 6.3%, respectively, for winter. The authors claim: "The performance of wavelet-ARIMA for the spring week is more accurate than winter with a weekly error below 5.7% while the weekly error for ARIMA model is 6.4%. For the summer weekly error of wavelet-ARIMA is 10.7% while weekly error of ARIMA is 13.4%. Fall season is the worst case for the both techniques weekly error of wavelet-ARIMA and ARIMA is 13.8% and 11.3%, respectively. As a result of this study it is founded that; wavelet-ARIMA technique is superior than ARIMA technique for the Spanish electricity market "[4].

In [5] time series analysis, neural networks and wavelets techniques, to predict 24 hours electricity prices for PJM, are compared. Compared time series techniques are ARIMA, dynamic regression and transfer function. The results of this study as follows: the dynamic regression and transfer function models are more accurate than ARIMA models and wavelet models behave similarly to ARIMA models while neural network techniques do not show good performance (see p.1039 [5] for results)

## 3.3 Forecasting with ARMAX and ARIMAX

In this section two studies used ARIMAX model to forecast spot and futures electricity prices, and one study that uses ARMAX to forecast electricity prices are summarized. In the first study weekly futures prices at Nord Pool are compared with the forecasted spot prices by using a time series model with external variables ARIMAX in [31]. It is thought that future prices can be seen as good indicator of spot prices. Temperature, precipitation, reservoir levels, power load and basis are used as external variables. The period between January 1, 1998 and September 30, 2007, which contains 509 weekly prices, is used for analysis. Spot reference is the for each week is the average of the 168 system prices of the same week (24hours $\times$ 7days).

Properties of external variables and their transformations are as follows:

- Reservoir levels ($R_t$): Reservoir levels are the percentage of the total hydropower capacity in the Nord Pool area.
- Weather variables: Temperature index (NTI) and the precipitation index (NPI) are used in the study. The precipitation index is transformed to weekly frequency by (3.16) and by using NTI the variable heating degrees week (HDW) is defined in (3.17) which is the difference between 18 degrees celsius and the NTI of each day.

$$P_t = \sum_{i=1}^{7} NPI_{t,i}. \tag{3.16}$$

$$HDW_t = \sum_{i=1}^{7} (18 - NTI_{t,i})^+. \tag{3.17}$$

- Basis: Basis is the difference between the future prices and the spot prices. It is expressed by $B(t, T_i) = F(t, T_i) - S(t), i = 1, 2, 3, 4$. $F(t, T_i)$ is the futures prices on week

$t$ to deliver in week $T_i$ and $T_i = t + i$. Four weekly contracts nearest to the delivery are used.

The following model is used to estimate external variables $R_t$, $HDW_t$ and precipitin ($P_t$).

$$X_t = \alpha + \beta_t + \sum_{i=1}^{p} (\gamma_i \sin \frac{2\pi}{52} it + \delta_i \cos \frac{2\pi}{52} it) + \sum_{j=1}^{k} \theta_j X_{t-j} + u_t, \tag{3.18}$$

where $X_t$ represents $R_t$, $HDW_t$ and $P_t$.

(This is given as display 1 in [31])

Estimation of the basis is done by (3.19).

$$B(t, T_i) = \alpha + \sum_{i=1}^{p} (\gamma_i \sin \frac{2\pi}{52} it + \delta_i \cos \frac{2\pi}{52} it) + \sum_{j=1}^{k} \sum_{i=1}^{4} \gamma_{ij} B(t - j, T_i) + u_t, \tag{3.19}$$

for $i = 1, 2, 3, 4$.

(This is given as display 2 in [31])

These estimates in (3.19) and (3.18) will be used in the exante approach. Lastly the spot prices model is defined in (3.20).

$$(1 - L)S(t) = c + \alpha_1 \sin \frac{2\pi}{52} + \alpha_2 \cos \frac{2\pi}{52} + \beta R_t + \gamma P_t + \delta HDW_t + \phi B(t - 1, T_1) + \mu_t, \tag{3.20}$$
$$\mu_t = \psi_1 \mu_{t-1} + \psi_2 \mu_{t-2} + \psi_3 \mu_{t-3} + \psi_4 \mu_{t-4} + \epsilon_t.$$

$L$ is the lag operator, $\mu_t$ is the residual and $\epsilon_t$ is white noise ($N(0, \sigma^2)$).

(This is given as display 3 in [31])

Forecasting is done by using two approaches: expost approach which is done by using real observed external values and in an exante approach each external variable are forecasted by with electricity prices. The whole period is split into two subperiods. The ex post subperiod contains data between the 1st week of 1998 and the 39th week of 2003 which has 300 observations and the exante subperiod starts from the 40th week of 2003 and finishes at the 39th week of 2007 which contains 209 observations. To estimate the forecasting accuracy of ARIMAX model two different forecasting methods are considered. First method is the myopic method which uses the present spot price as a forecasted settlement price at the expiration week of each futures. Second method is the futures method that uses the present futures prices as the forecasted settlement price in the corresponding forecasting week.

Results of this study shows that ARIMAX model has lower mean squared error (MSE) than myopic and futures methods in almost all cases. According to the authors "ARIMAX model is superior than the other compared model in the Nord Pool market"[31].

Another study on electricity futures is [55]. The aim of this study is to develop a model for long term electricity prices of EEX which represents the features of German electricity market. The formula used in this study to calculate the price of standard futures contract is

$$F_{t,T} = S_{t,T}(1 + r - \lambda)^{T-t}. \tag{3.21}$$

$F_{t,T}$ represents the price of the futures contract, $S_{t,T}$ denotes the spot price, $T - t$ is the time to maturity and the term $(1 + r - \lambda)^{T-t}$ shows the risk premium. Authors changed the equation in 3.21 because "this model implies no direct link between the spot and futures prices."In order to solve this problem expected spot price $E(S_{t,T})$ is used instead of spot prices which is stated in (3.22). In order to obtain a linear equation (3.22) is transformed into (3.23), a logarithmic form.

$$F_{t,T} = E(S_{t,T})(1 + r - \lambda)^{T-t}, \tag{3.22}$$

$$\log F_{t,T} = \log E(S_{t,T} + (T - t) \log (1 + r - \lambda). \tag{3.23}$$

(These are given as display 2 and 3 in [55])

It is stated that "the term $\log (1 + r - \lambda)$ will stay stable as long as it remains far from the maturity so the main factor that determine the future price is expected spot price"[55]. Expected future spot price is mostly influenced by the supply and demand of electricity. To estimate futures prices, variables that directly influence electricity supply and demand are used in this study. These variables are used as external variables of an ARIMAX model. External variables are categorized in three groups. The first group is futures on prices of oil, natural gas and coal. The second group contains emission allowances and the last group variables reflect financial market conditions, which include EUR/USD exchange rate, spread[1] and the Prime Utilities Index (UTIL). Phelix Base Futures with next year's delivery is used as a depended variable and data contains variables between the period of 2006 and June 2009.

[1] Spread is the difference between 10 years and 1 year government bonds in Germany and UTIL contains stocks of companies that involves in the energy sector.

ARIMAX model that used in this study define in (3.24).

$$Y_t = \alpha + \sum_{i=1}^{p} \beta_i Y_{t-i} + \epsilon_t + \sum_{j=1}^{q} \theta_j \epsilon_{t-j} + \sum_{k=1}^{b} \gamma_k X_{t-k}. \tag{3.24}$$

(This is given as display 5 in [55])

The model is estimated by ordinary least squares (OLS). Results of the study shows that all external variables have significant explanatory power on electricity future prices and the relations between external variables.

ARIMA and ARIMAX models are used in [59] to predict the spot electricity prices of California market. The exogenous variable of ARMAX is the system loads. The data from the period July 5, 199 to April 2, 2000 is used to calibrate the models and the period between April 3, 2000 and December 3, 2000 is used to test the models. In this study 24 hours of electricity spot prices are tried to forecast. The logarithmic transformation is applied to data in order to obtain a more stable variance, also the mean was removed to center the data. General ARMAX model used in this study is give in (3.25). (Unless otherwise noted all of the displays in this section are taken from [59])

$$A(p)P_t = C(r,k)Z_t + B(q)\epsilon_t, \tag{3.25}$$

where

$$C(r,k)Z_t = Z_{t-k} + c_1 Z_{t-k-1} + ... + c_r Z_{t-k-6},$$

$Z_t$ is the value of exogenous variable at time $t$. Authors found that moving average part ($B(q)\epsilon_t$) decreases the performance so only the ARX model is used, that is $B(q)\epsilon_t = \epsilon_t$. Optimal model is of the form

$$A(p)p_t = p_t - a_1 p_{t-24} - a_2 p_{t-48} - a_3 p_{t-168} - a_4 mp_t,$$

where $mp_t$ is a function of all prices on the previous day. Three dummy variable are injected to the model in order to cope with the weekly seasonality and the new model is as follows:

$$p_t - a_1 p_{t-24} - a_3 p_{t-168} - a_4 mp_t = c_1 z_t + d_{Mon} + d_{Sat} + d_{Sun} + \epsilon_t,$$

where $d_{Mon}, d_{Sat}, d_{Sun}$ are the coefficients of the dummies.

Accuracy of the model is checked by mean daily error and mean weekly error. Results shows that AR model is almost good as ARX but the ARIMA and ARIMAX models are not as successful as ARX.

# CHAPTER 4

# MODELING TEMPERATURE

## 4.1 Introduction

To model temperature dynamics we have considered several alternatives. The approach that we found most effective is presented in Section 5.3. Earlier alternatives and considerations that led to this approach are given in Section 5.2. The main idea of Section 5.3 is the following: temperature dynamics consist of two alternating phases: cooling and heating. Conditioning on these phases significantly simplify analysis and obviate the need to use more complicated models, such as those reviewed and discussed in Section 5.2.

### 4.1.1 Data

We have average daily temperature data of İzmir, İstanbul and Ankara. The data measured in degree Celsius and it consists of the period between December 1, 2009 and May 31, 2012. The data set was obtained from the Turkish State Meteorological Service (MGM). In the set we have some missing data and we assign values to the missing ones by using linear interpolation. The next table shows the descriptive statistics of the daily temperature data for each cities while Figure 4.1 shows the daily average daily temperature for each cities.

Table 4.1: Descriptive statistics of the average daily temperature for each cities.

| City | N | Min | Median | Max | Mean | Std. Deviation | Variance |
|---|---|---|---|---|---|---|---|
| Ankara | 913 | -9.9 | 11.2 | 32.4 | 12.02 | 9.1 | 82.84 |
| İzmir | 913 | 1.2 | 16.7 | 33.5 | 17.63 | 7.66 | 58.71 |
| İstanbul | 913 | -2.5 | 13.7 | 29.9 | 14.4 | 7.85 | 61.66 |

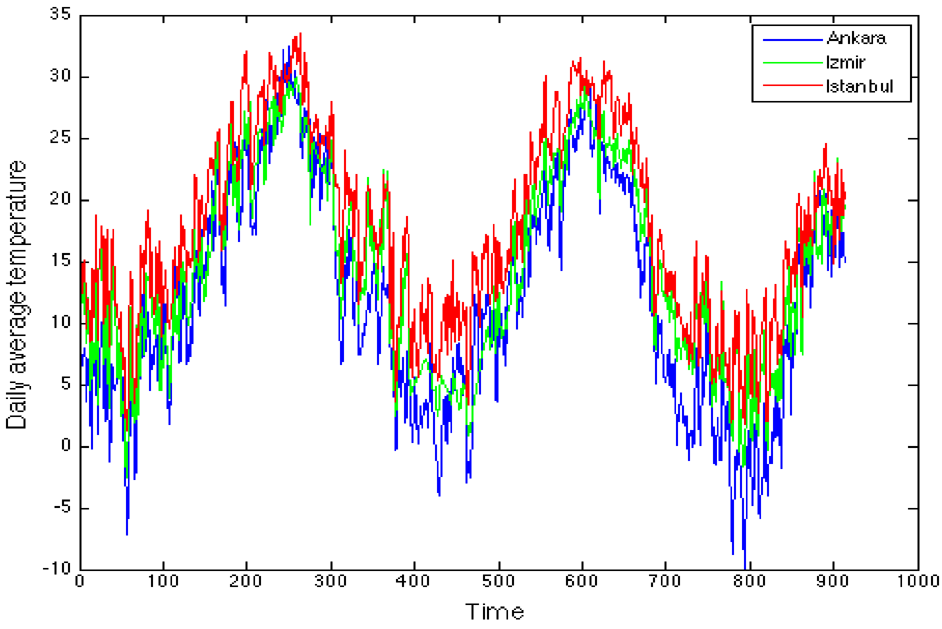


Figure 4.1: Daily average temperature of the cities.

## 4.2 Alternative models

Our initial approach included the following spatial averaging: take the averages of the daily temperature of the three cities to obtain a daily average temperature for Turkey. The following analysis uses this average. Later we have noticed a nontrivial problem with this idea. This problem is described at the end of this section. Still, an analysis based on this average was instrumental in developing our final approach (explained in Section 5.3) hence it is given below as is using the spatially averaged temperature.

To model temperature dynamics we start with

$$T_t = X_t + \Lambda_t,$$

where $T_t$ is the temperature and $\Lambda_t$ is a deterministic seasonal function.

We considered three alternatives to model $\Lambda_t$ of the daily average temperature data of Turkey.

1. The first approach we tried was to use a cosine function to model $\Lambda_t$. This is a well known idea to model seasonal trends, see, for example, [24]. To do this, one chooses

$(a_0, a_1, a_2, a_3)$ so that the square distance between $T$ and the following function is minimized:

$$\Lambda(t) = a_0 + a_1 t + a_2 cos(2\pi(t - a_3)/365). \tag{4.1}$$

Figure 4.2 shows the fitted curve of the function (4.1).

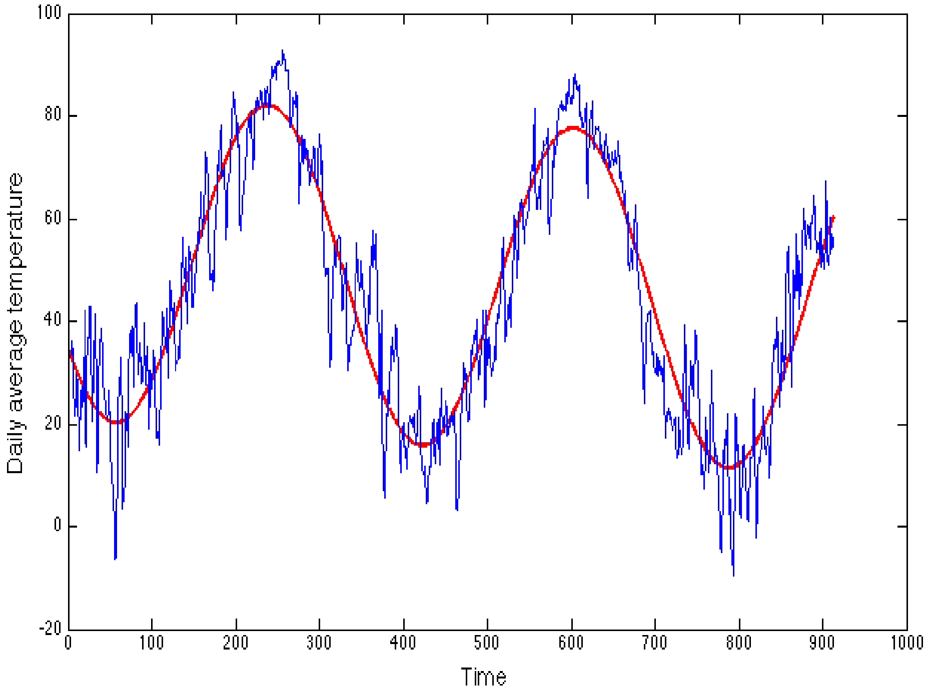


Figure 4.2: Mean temperature and the fitted curve of the function (5.1.).

Then one subtracts the seasonal part form $T_t$ in order to have a deseasonalized process $X_t$. Then the idea is to model $X_t$ as a stationary process. Without listing a detailed analysis, we would like to state that the resulting $X_t$ had a complicated structure to which simple time series models did not fit well. For this reason, we did not think the use of (4.1) appropriate to model the seasonality of temperature.

2. Local Linear Regression (LLR) is another alternative to model the seasonality of temperature, which is computed using

$$\arg\max_{e,f} \sum_{i=1}^{365} \{\bar{T}_t - e_s - f_s(t-s)\}^2 K\left(\frac{t-s}{h}\right),$$

where $\bar{T}_t$ is the mean of daily averages temperature, $h$ is the bandwidth, $K$ is a Kernel. We tried the Epanechnikov Kernel which is used in [64] to model temperature dynamics and bandwidth proposed by [9]. Figure (4.3) shows the LLR and the daily average temperature. This technique gives on the surface better results than the previous but it clearly overfits.

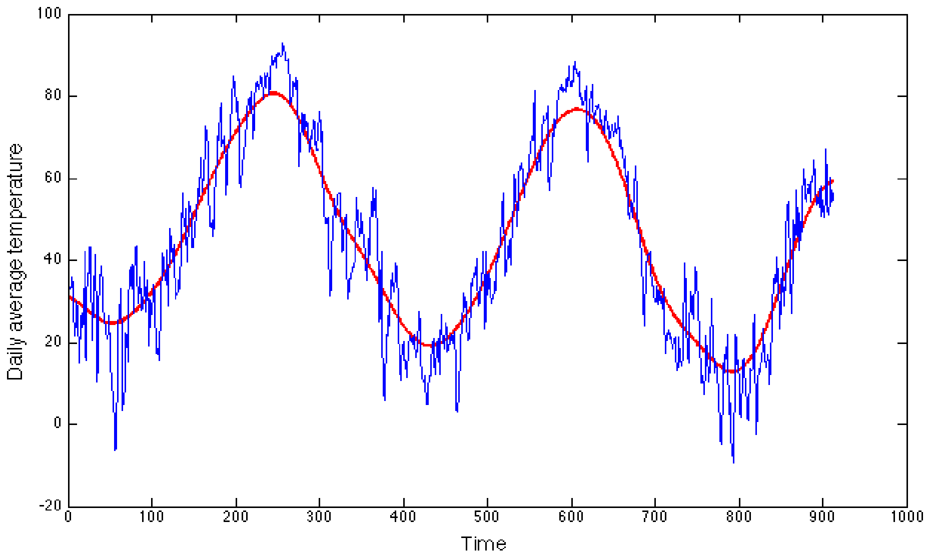


Figure 4.3: Mean temperature and the LLR.

3. Finally we tried a simple idea which seems to fit the best. Figure 4.1 suggests that every year consists of a warming period and a cooling period that are approximately of the same length. In order to determine the starting and the ending day of the cycle, we first compute the mean temperature of the two summer seasons.

   From Figure 4.4 it can be seen that temperature starts to decay at the end of July so we decide to end the first cycle at July 31,2010 which was started in December 1, 2009. The second cycle starts from August 1, 2010 till July 31, 2011 and the last cycle starts from August 1, 2011, till May 31, 2011. Figure 4.5 shows the temperature cycles of Turkey during the time interval for which we have data.

The figure 4.6 shows the mean of the cycles depicted in Figure 4.5. Figure 4.6 clearly suggests the use of a piecewise linear curve. We fit a linear curve of the form $y = a_i x + b_i$ for the decreasing and increasing part of the mean temperature (the variables are chosen so

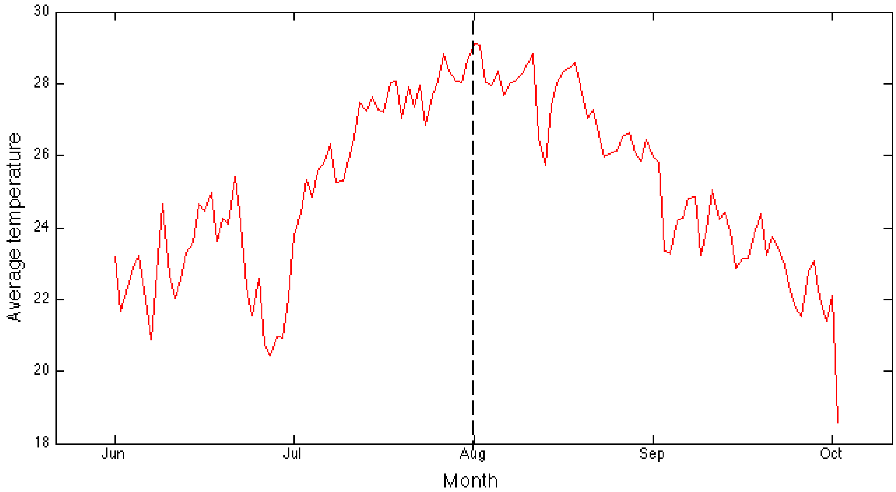


Figure 4.4: Mean temperature of the summer seasons.

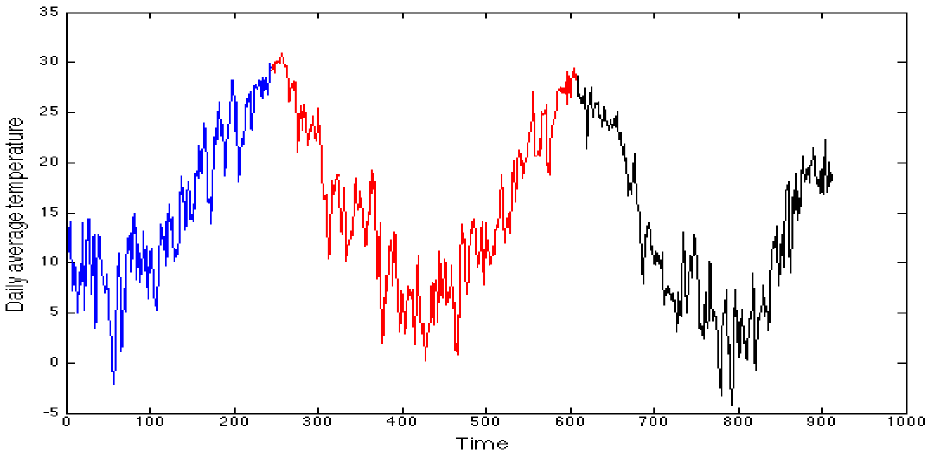


Figure 4.5: Temperature cycles of Turkey.

that the resulting piecewise linear curve is continuous). Figure 4.7 shows the fitted curve and mean temperature.

One obtains the deseasonalized temperature (or fluctuations in temperature) by taking the difference of the two curves in Figure 4.7. Although the results are not reported, we have been able to fit simple time series models to this difference.

These results inspired us to use the same approach without taking the mean over the years,

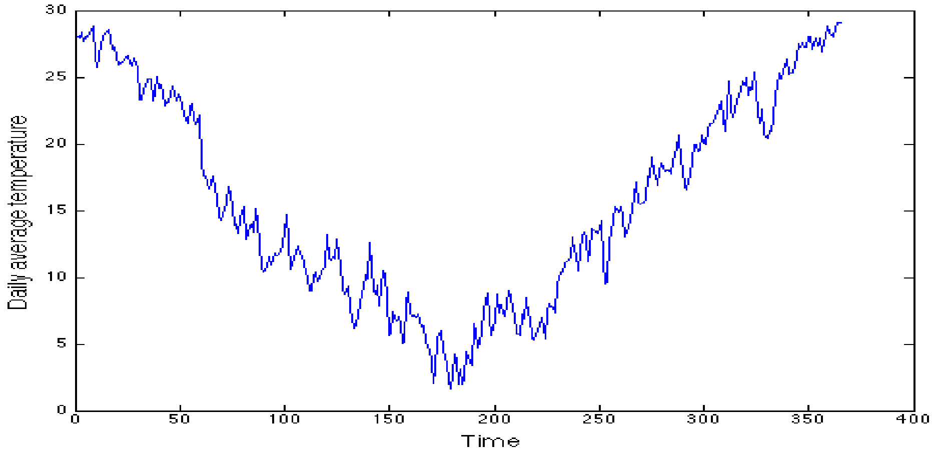


Figure 4.6: Mean temperature of the cycles.

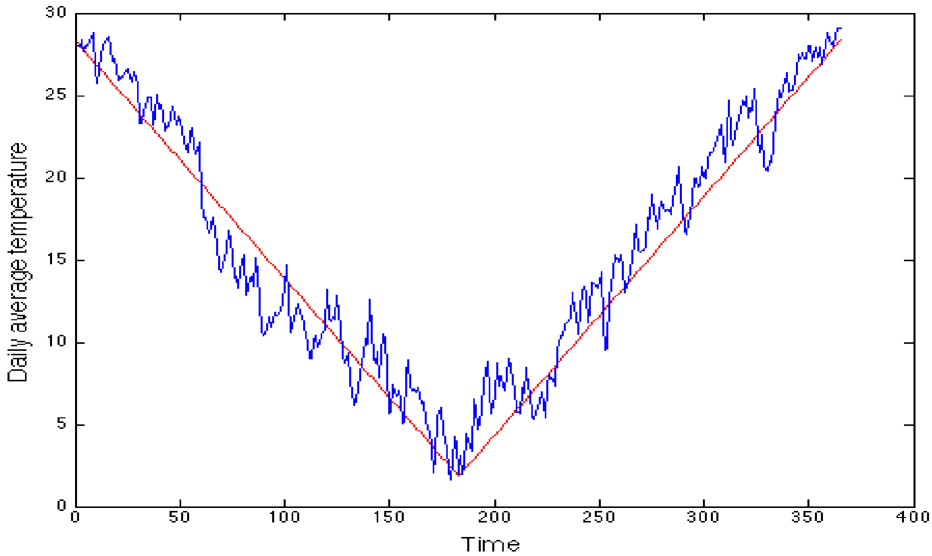


Figure 4.7: Fitted curve and mean temperature.

i.e., applying it directly to the temperature data of all of the three years. In doing this, one has to decide how to model the length of the periods. In the mean temperature one has to choose only one length, i.e., just decide when the cooling period ends and when the heating period starts (see Figure 4.7). In the analysis of the three years there are multiple cooling and heating periods and their lengths may change with the years. We settled with the simplest approach: use a fix constant length. Unfortunately this approach did not yield good results.

Without listing further numerical results we would like to state that the main problem is that the fluctuations that remain after subtracting off the trend have too complicated a structure.

*It is clear that to get a good global model of temperature, one has to take the lengths of the cooling and heating periods as random and model them as random variables.* This we leave to future work.

**Taking the spatial averages may not be a good idea** We have also noticed the following problem with taking spatial averages to compute an average temperature in Turkey. The next figure shows a sample of the average daily temperature from İzmir and Ankara.

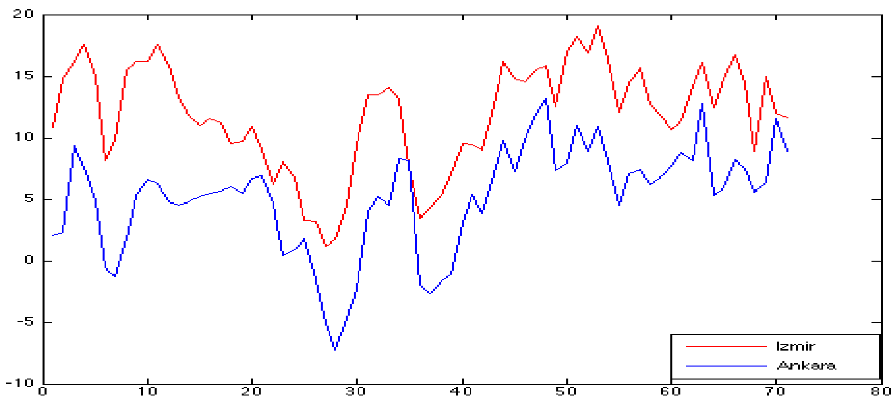


Figure 4.8: A sample form İzmir and Ankara.

Figure 4.8 suggests that the average daily temperature of İzmir and Ankara are highly correlated and the temperature of Ankara behaves like a shifted version of the İzmir's temperature. So when we calculate the average temperature of these two cities, we not only compute the mean over locations but also over time. After these considerations we have decided to use the temperature of a single city in our analysis. Another future work would be to model the temperature of several cities as a high dimensional stochastic processes whose dynamics correctly model the interdependencies.

## 4.3 Conditional Model

This time we are going to work with the two full cycles of Ankara's temperature date and we will divide the data into four separate parts. The first part and the third part are warming periods while the second part and the last part are cooling periods. In each part we fit a curve to remove the seasonal component and then we apply autoregression with degree two to the deseasonalized data. Figure 4.9 shows the four periods of Ankara's temperature data and the fitted curves.

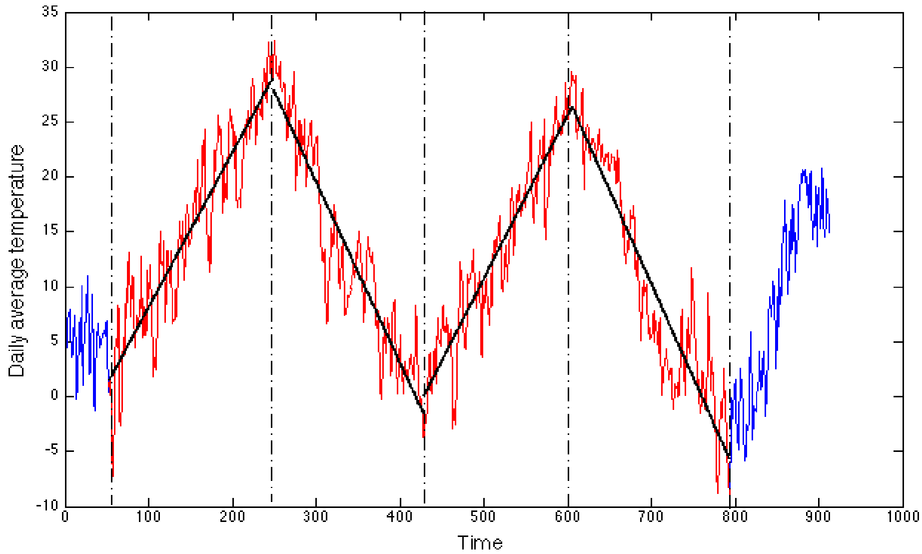


Figure 4.9: Four periods of Ankara's temperature data and the fitted curves.

The periods and their models are as follows (note that the lengths of the periods change from year to year, this change has *not* been modeled as a random variable in this thesis, but we hope to do it in future work):

- The first period starts from January 22, 2010 and ends on August 5, 2010 which is a warming period. We used the first 136 observations for calibration. The table figure shows the parameters and $R^2$ of the fitted line while Figure 4.10 shows the PACF (see for definition of PACF [10, p.222]) of the deseasonalized data.

Table 4.2: Fitted parameters and $R^2$ of the first period.

| a | b | $R^2$ |
|---|---|---|
| 0.1409 | 1.351 | 85% |

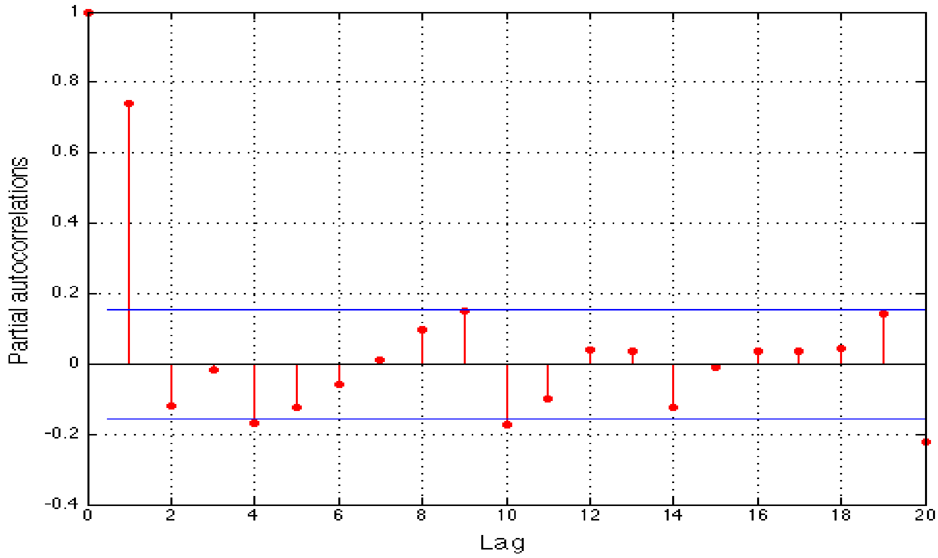


Figure 4.10: PACF of the desesonalized data for the first period.

Although Figure 4.10 and AIC suggests us to use AR(1), we use AR(2) to form a consistent model for Ankara, because in the next periods we will see that PACF and information criterions suggests us to use AR(2). The model is as follows:

$$x_t = 0.8287x_{t-1} - 0.1188x_{t-2} + \epsilon_t,$$

where $x_t$ is the deseasonalized temperature and $\epsilon_t$ is the white noise. The qq-plot of the AR(2) residuals showed in Figure 4.11.

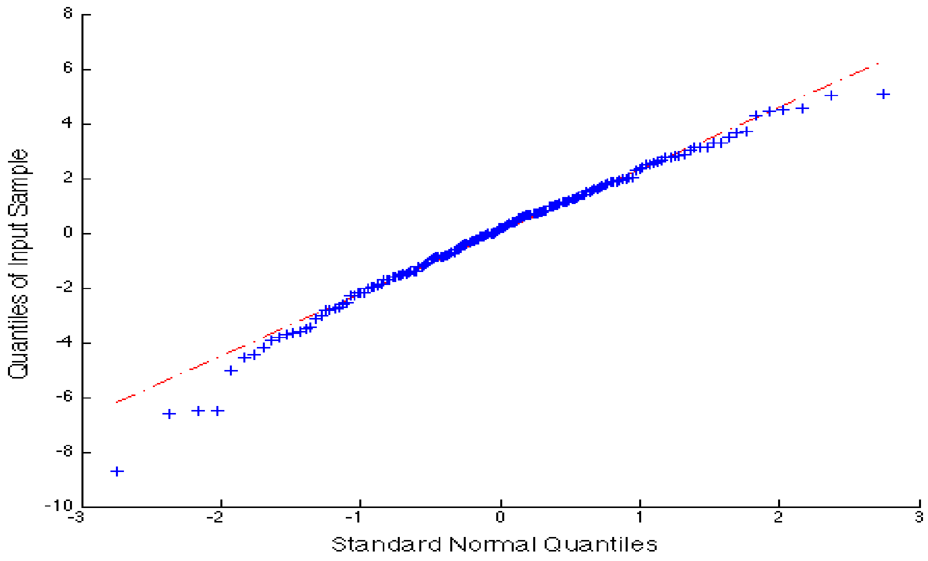


Figure 4.11: QQ-plot of the AR residuals for the first period.

Figure 4.11 shows that we have some outliers but when we drop the lowest three observation the data follows a normal distribution. Since this is a warming period, sudden and large temperature declines may cause disturbances in the normal distribution. The next figure shows the qq-plot of the residuals when the outliers were dropped.

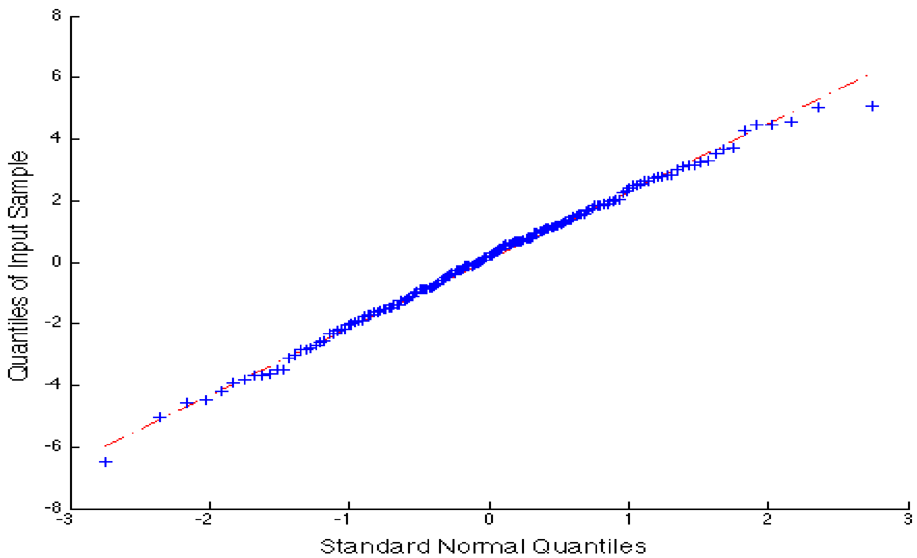


Figure 4.12: QQ-plot of the AR residuals for the first period when the outliers were dropped.

- The second period starts from August 6, 2010 till February 1, 2011 which is a cooling period. We used the first 150 observations for calibration. The table shows the parameters and $R^2$ of the fitted line while Figure 4.13 shows the PACF of the deseasonalized data.

Table 4.3: Fitted parameters and $R^2$ of the second period.

| a | b | $R^2$ |
|---|---|---|
| -0.1646 | 28.09 | 87% |

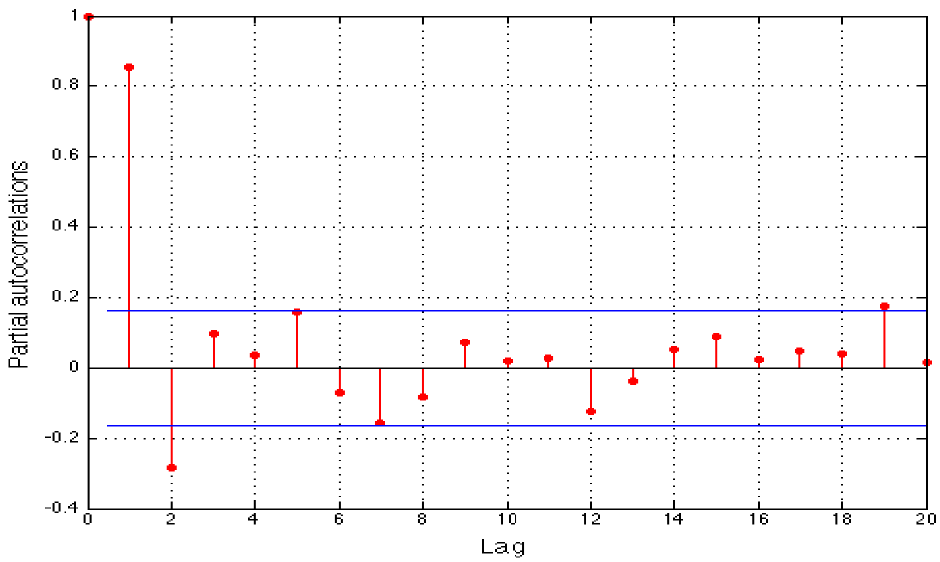


Figure 4.13: PACF of the desesonalized data for the second period.

AR(2) model for the second period is as follows:

$$x_t = 1.094x_{t-1} - 0.2833x_{t-2} + \epsilon_t,$$

where $x_t$ is the deseasonalized temperature and $\epsilon_t$ is the white noise.

Figure 4.14 shows the qq-plot of the AR(2) residuals.

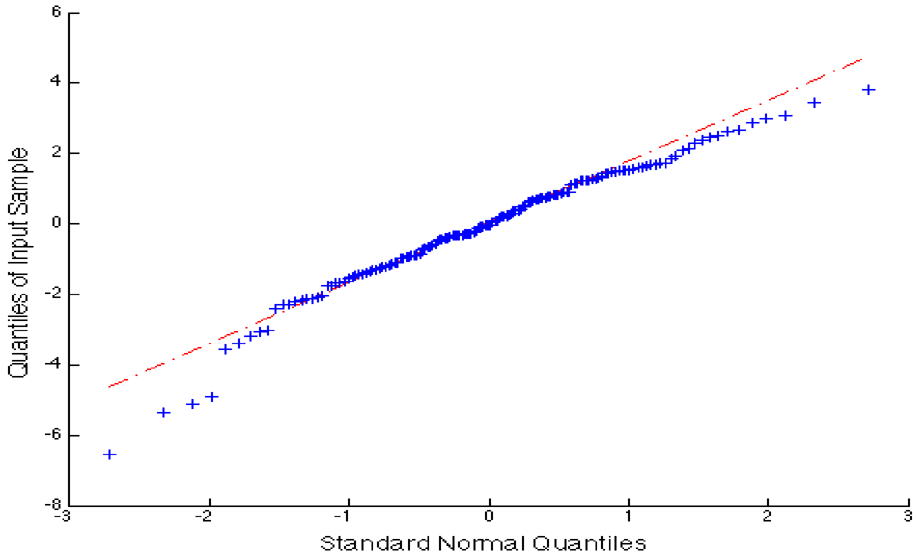


Figure 4.14: QQ-plot of the AR residuals for the second period.

Again we have outliers at the lower part of the qq-plot, this time we dropped the lowest three observations and from Figure 4.3 shows the qq-plot when the outliers were dropped. It can be seen that the residuals follows a normal distribution.

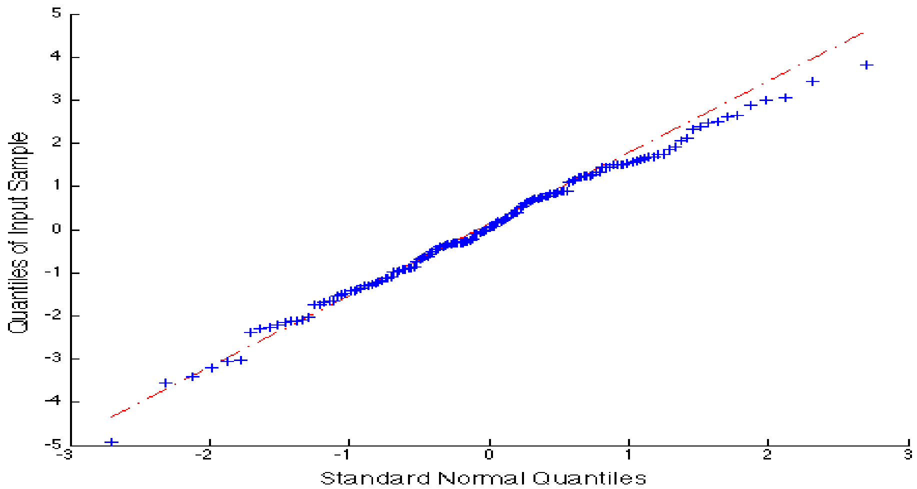


Figure 4.15: QQ-plot of the AR residuals for the second period when the outliers were dropped.

- The third period starts from February 2, 2011 and ends on July 28, 2011 which is a warming period. We used the first 147 observations to calibrate the model. The table figure shows the parameters and $R^2$ of the fitted line while Figure 4.16 shows the PACF of the deseasonalized data.

Table 4.4: Fitted parameters and $R^2$ of the third period.

| a | b | $R^2$ |
|---|---|---|
| 0.1491 | -0.02704 | 88% |

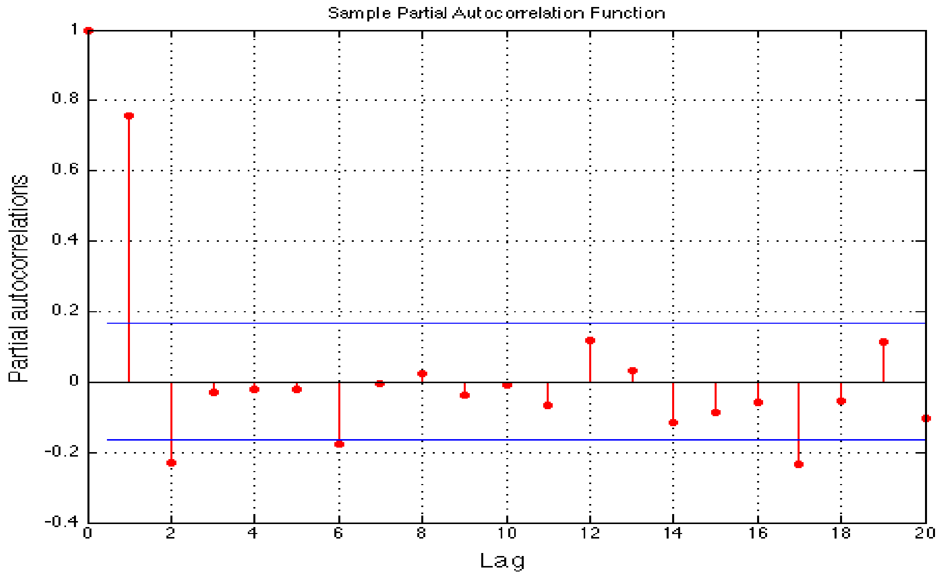


Figure 4.16: PACF of the desesonalized data for the third period.

Figure 4.16 suggests higher order autoregression with non-consecutive lags but we use AR(2) to create a simple and a general model for Ankara. The model for the third period is as follows:

$$x_t = 0.9144x_{t-1} - 0.2256x_{t-2} + \epsilon_t,$$

where $x_t$ is the deseasonalized temperature and $\epsilon_t$ is the white noise. Figure 4.3 shows the qq-plot of the AR(2) residuals.

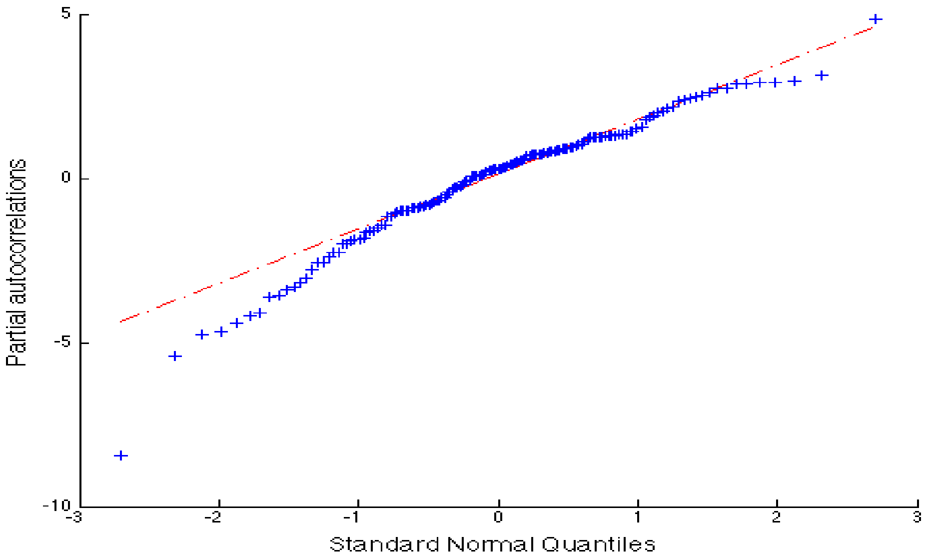


Figure 4.17: QQ-plot of the AR residuals for the second period.

Again we have outliers and we dropped the smallest three observations and the new qq plot is showed in Figure 4.3.

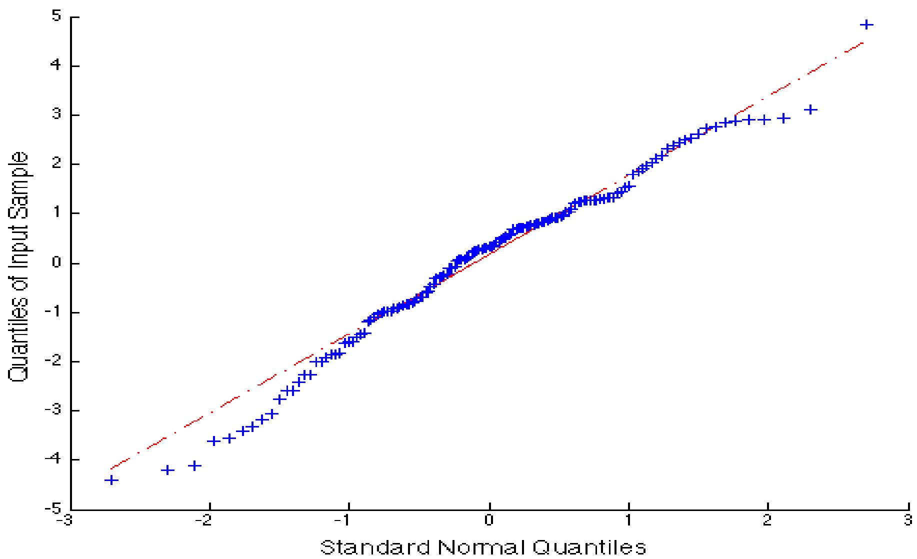


Figure 4.18: QQ-plot of the AR residuals for the third period when the outliers were dropped.

- The last period starts from July 28, 2011 till February 1, 2012 which is a cooling period. We used the first 153 observation to calibrate our model. The table figure shows the parameters and $R^2$ of the fitted line while the figure 4.19 shows the PACF of the deseasonalized data.

Table 4.5: Fitted parameters and $R^2$ of the last period.

| a | b | $R^2$ |
|---|---|---|
| -0.1706 | 26.47 | 87% |

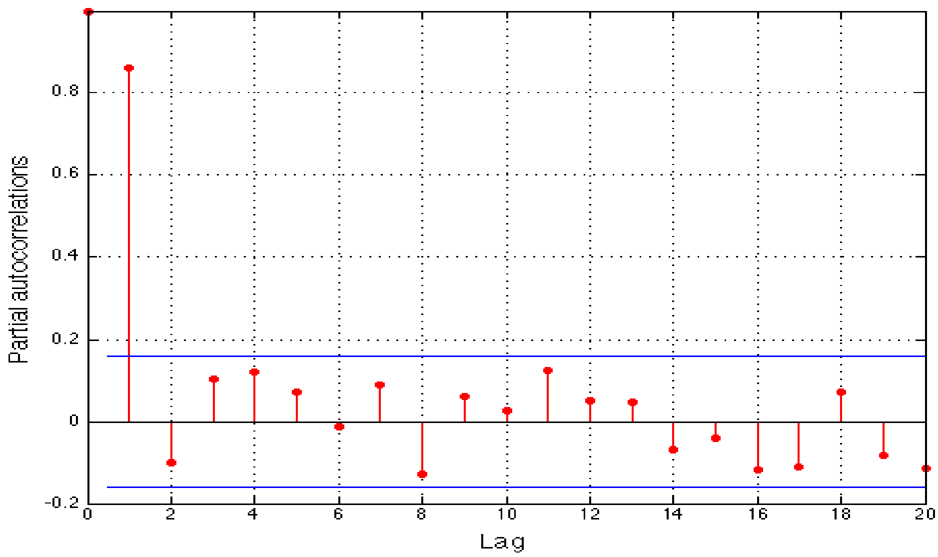


Figure 4.19: PACF of the desesonalized data for the third period.

The PACF again suggests AR(1) but we again use AR(2) to create a general model. The AR(2) model for the last period is as follows:

$$x_t = 0.9501x_{t-1} - 0.09718x_{t-2} + \epsilon_t,$$

where $x_t$ is the deseasonalized temperature and $\epsilon_t$ is the white noise. Figure 4.20 shows the qq-plot of the AR(2) residuals.

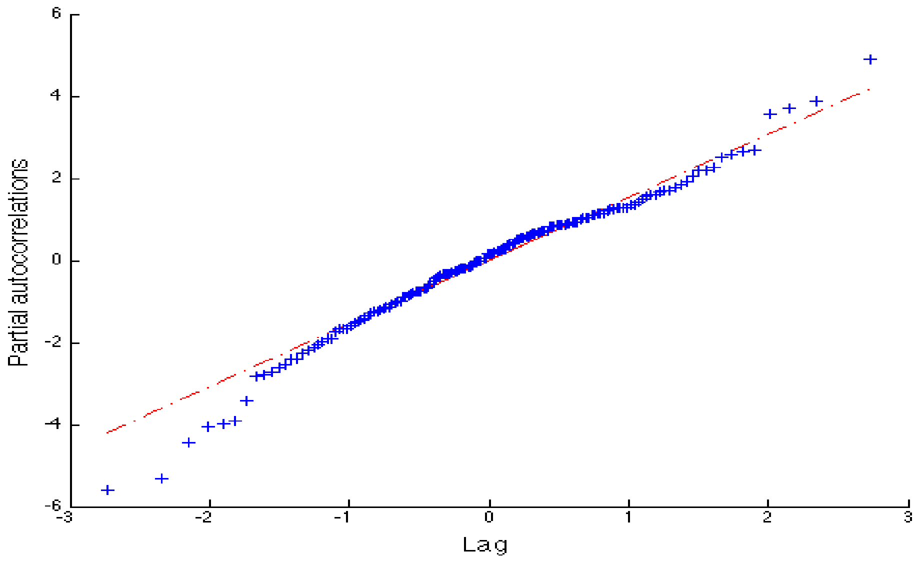


Figure 4.20: QQ-plot of the AR residuals for the last period.

We have outliers but when we drop the smallest two observations we have a qq-plot that looks like normally distributed. The next figure shows the qq-plot when the outlier were eliminated.

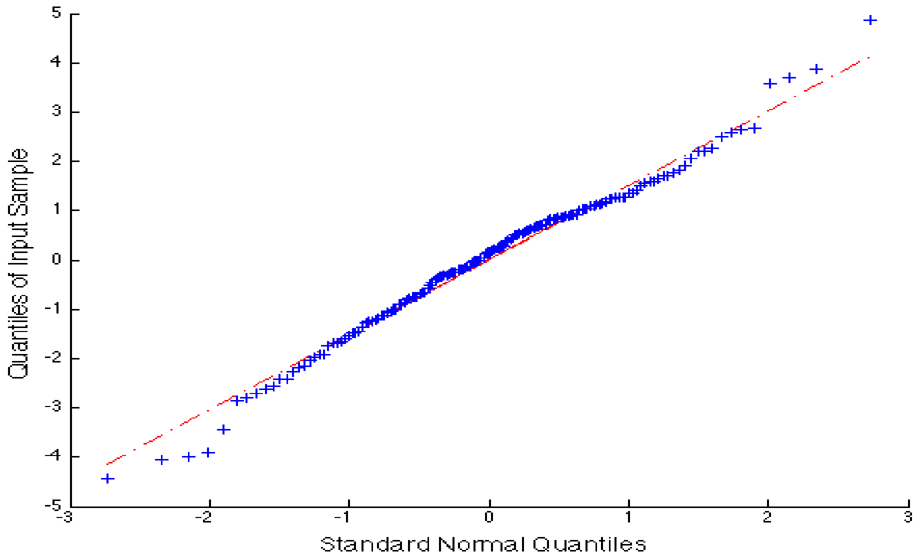


Figure 4.21: QQ-plot of the AR residuals for the third period when the outliers were dropped.

After removing the seasonal component we apply the ADF (see for definition of ADF [57, p.76]) test for all the periods and we reject the null hypothesis of unit root at 5% level of significance. We also compute the Jarque-Bera test (JBT) to check the normality of the residuals after eliminating the outliers and we failed to reject the null hypothesis at 5% level of significance for all the periods. In each periods sudden and large temperature falls create disturbances in the normal distribution, the reason for this can be the insufficient data. Also it can be caused from the cycle separations, the data can be divided into the cycles by using different methods. These facts can be explored in future studies.

### 4.3.1 Forecasting

We forecast the next days temperature for the each period defined in Section 5.3. The next figures shows the observed values, forecasted values and the confidence intervals for each period starting from the first till the end, respectively.

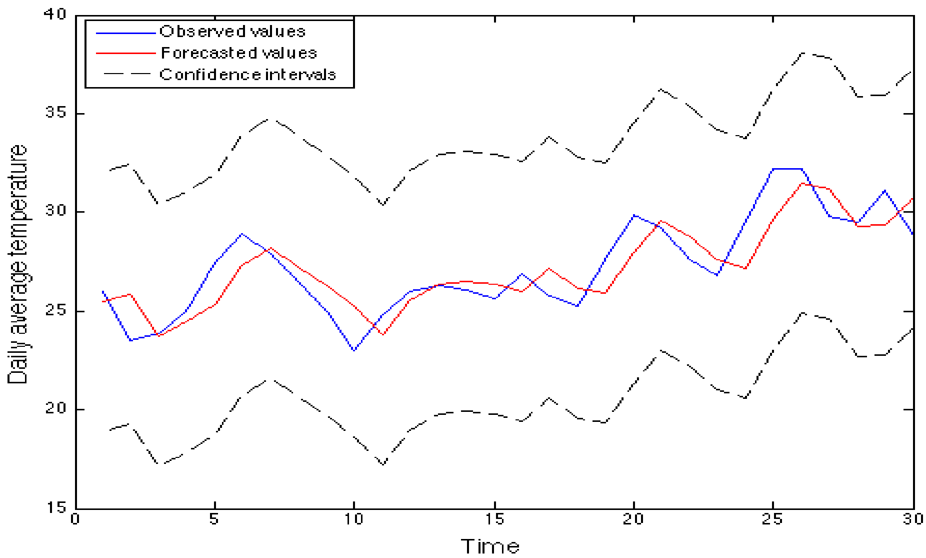


Figure 4.22: Forecasted values of the first period.

In the first period we use the first 136 observations for calibration and the last 30 observations are used to check the accuracy of the model. We find the mean squared forecast errors as 1.37. We also calculate the KST and the JBT for the errors and in both we failed to reject the null hypothesizes so we conclude that the errors follow a standard normal distribution.

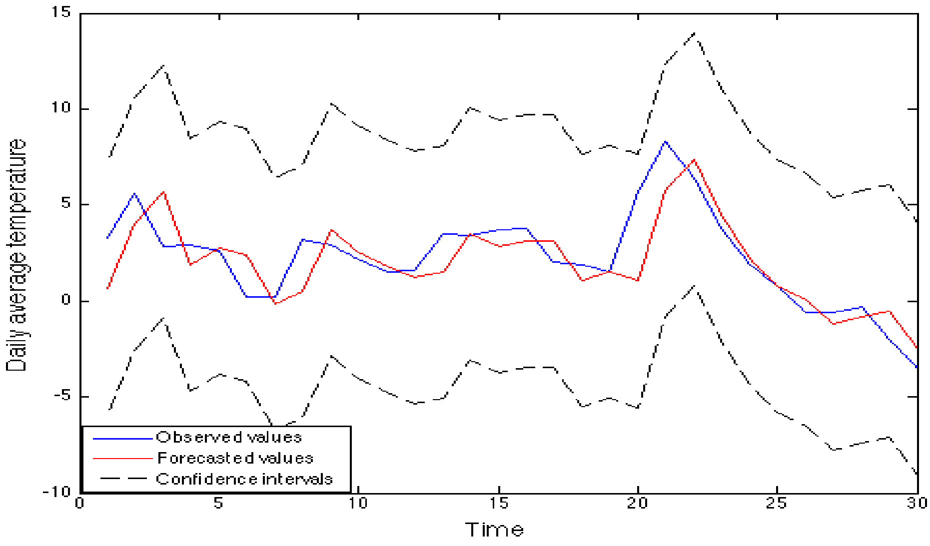


Figure 4.23: Forecasted values of the second period.

In the second period we use the first 150 observations to set the model and the last 30 observations are used to check the prediction power of our model. The MSE is 1.56 for this period so the accuracy of this period is a little worse than the first period. We again compute the KST and the JBT and in both we fail to reject the null hypothesizes so we have still standard normal errors.

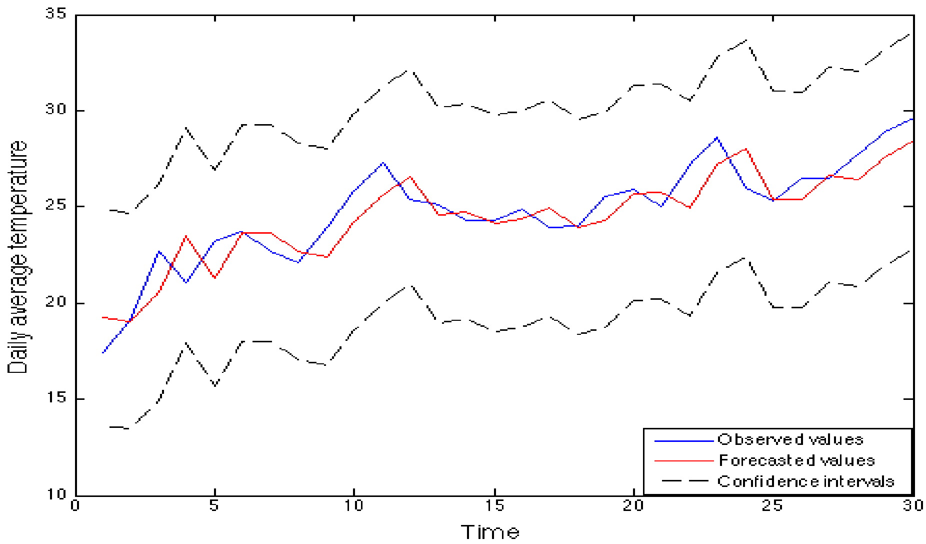


Figure 4.24: Forecasted values of the third period.

In the third period we use the first 147 observations for the calibration and the last 30 observations are used to validate our model. The MSE is 1.27 for this period which is the smallest of all periods. The qq-plot of this period had the worst shape but the model of this period has the smallest MSE so making decisions by only looking the qq-plots is not a good idea for our case. We compute the KST and the JBT and in both we failure to reject the null hypothesizes.

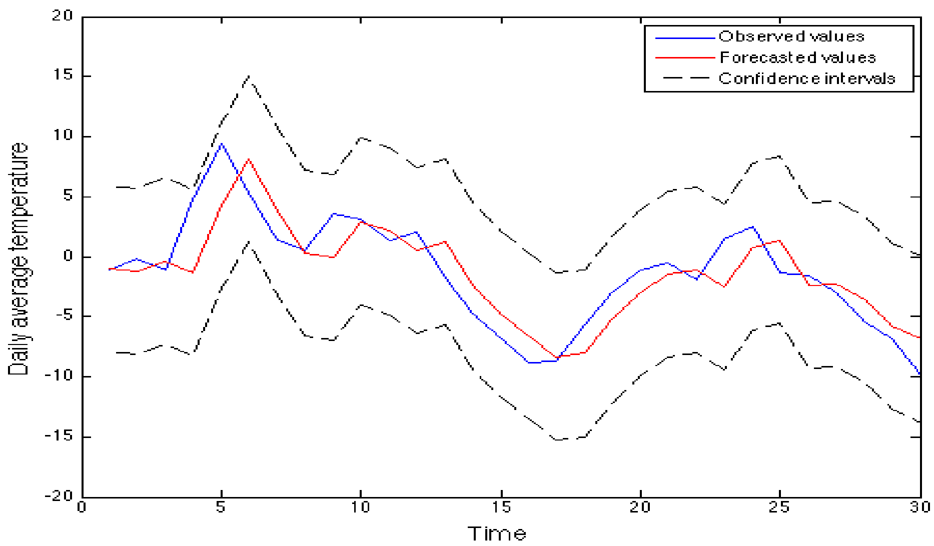


Figure 4.25: Forecasted values of the last period.

In the last period we use the first 153 observations to calibrate the model and the last 30 observations are used to check the accuracy of the model. The MSE is 2.41 which is the highest of all periods. We again compute the KST and the JBT and in both we failure to reject the null hypothesizes so we have still standard normal errors.

# CHAPTER 5

# MODELING the TURKISH DAY AHEAD ELECTRICITY PRICES

## 5.1 Introduction

To model the Turkish day ahead electricity market we use simple time series models. In Section 6.2 we discussed some properties of the price dynamics of Turkey and in Section 6.3 we model the Turkish electricity market and we explore the effect of temperature on the price. As discussed in Section 5.2 this effect can be studied on a global or a local level. The analysis of Section 6.3 conducts a local analysis. That is, conditioning on a particular period (mid February to late March) where the trends of temperature and price have simple structures, how much the fluctuations in temperature from its trend influence the fluctuations of the price from its respective trend? As shown in Section 6.3, this influence seems to be very little.

### 5.1.1 Data

The price data of our interest are the average day ahead electricity prices established at PMUM in the period between December 12, 2009 and September 2, 2012, which is taken from [51]. The price is measured in TRY/MWh. Table 6.1 shows the descriptive statistics of the electricity prices of Turkey while Figure 5.1 show the day ahead prices of the Turkish electricity market.

Table 5.1: Descriptive statistics of the average day ahead electricity prices of Turkey.

| City | N | Min | Median | Max | Mean | Std. Deviation | Variance |
|---|---|---|---|---|---|---|---|
| Ankara | 1007 | 28.75 | 138.6 | 703.97 | 135.24 | 38.944 | 1516.6 |

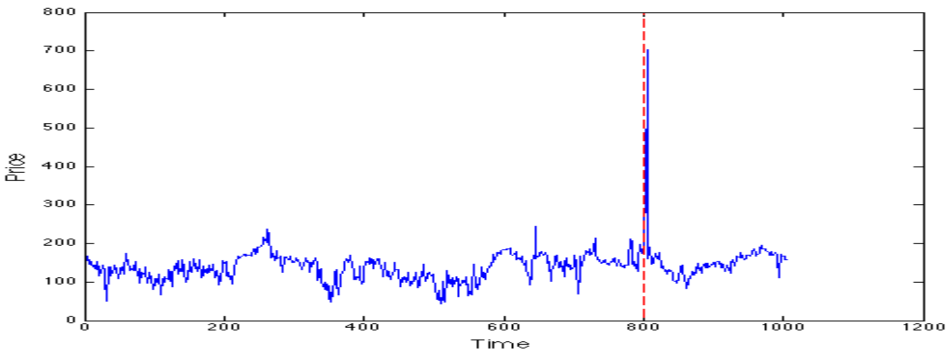


Figure 5.1: Day ahead prices of the Turkish electricity market.

## 5.2 Price Dynamics of the Turkish Electricity Market

The price of the Turkish electricity market shows all the characteristic behavior of the electricity markets defined in Subsection 2.1.3. We have a high volatility market which can be seen from Figure 5.1 clearly shows that market is highly volatile and exhibits many prices spikes. For example, on February 9, 2012, there is a price increase by almost 400 % which reverts back to its normal level in a couple of days. Figure 5.2 shows a 90 days sample from the Turkish electricity market which represents the seasonal behavior of the Turkish market.

On the weekends, electricity price starts to decrease and on mondays price came back to its normal level. This price movement repeats itself continuously so the price dynamics of the weekdays and the weekends have different characteristics. For the rest of this chapter, we will only consider the weekdays and the characteristics of the weekends can be explored in future work.

Figure 6.3 shows the logarithm of the electricity price, the official holidays and the religious holidays. The red dashed lines show the official holidays while the blue dashed lines show the beginning and the ending of the religious holidays. It can be clearly seen that the holidays effect the price of the Turkish electricity market. Before two or three days from the religious

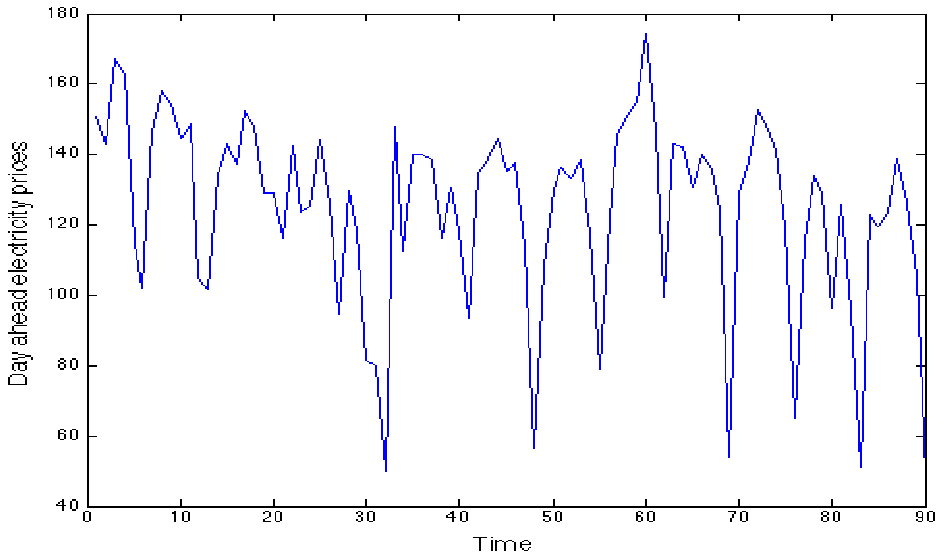


Figure 5.2: A sample from the Turkish electricity market.

holidays, price starts to decline until the last day of the holiday and the price reaches the lowest values of that month. It is well known that the dates of the religious holidays in Turkey shift backward every year. This creates a further challenge in the modeling of Turkish prices. To see the full effect of these holidays (in particular, how do the dynamics change when a holiday is in the summer or in the spring? and so on) on the prices many years have to pass to get a full set of data that covers all season/holiday interactions.

Except the year 2012 the electricity price declines on the official holidays. The decline in the industrial activities on holidays may cause price declines, this is reasonable but in the year 2012 the price does not fall, the reason for this can be investigated in future work.

In figure 6.4 we plot the price of the year 2010 and the price of the year 2011 together to show the similarities. It can be seen that the first 71 observations have similarities while the observations between the green dashed lines are almost the same. It is striking to see that from one year to the other the electricity prices follow similar paths. This is in stark contrast to the behavior of stock markets. As indicated in Chapter 2, this is a feature common to many electricity markets. To the best of our knowledge there is currently no clear explanation of this phenomenon and it is an interesting subject for future work.

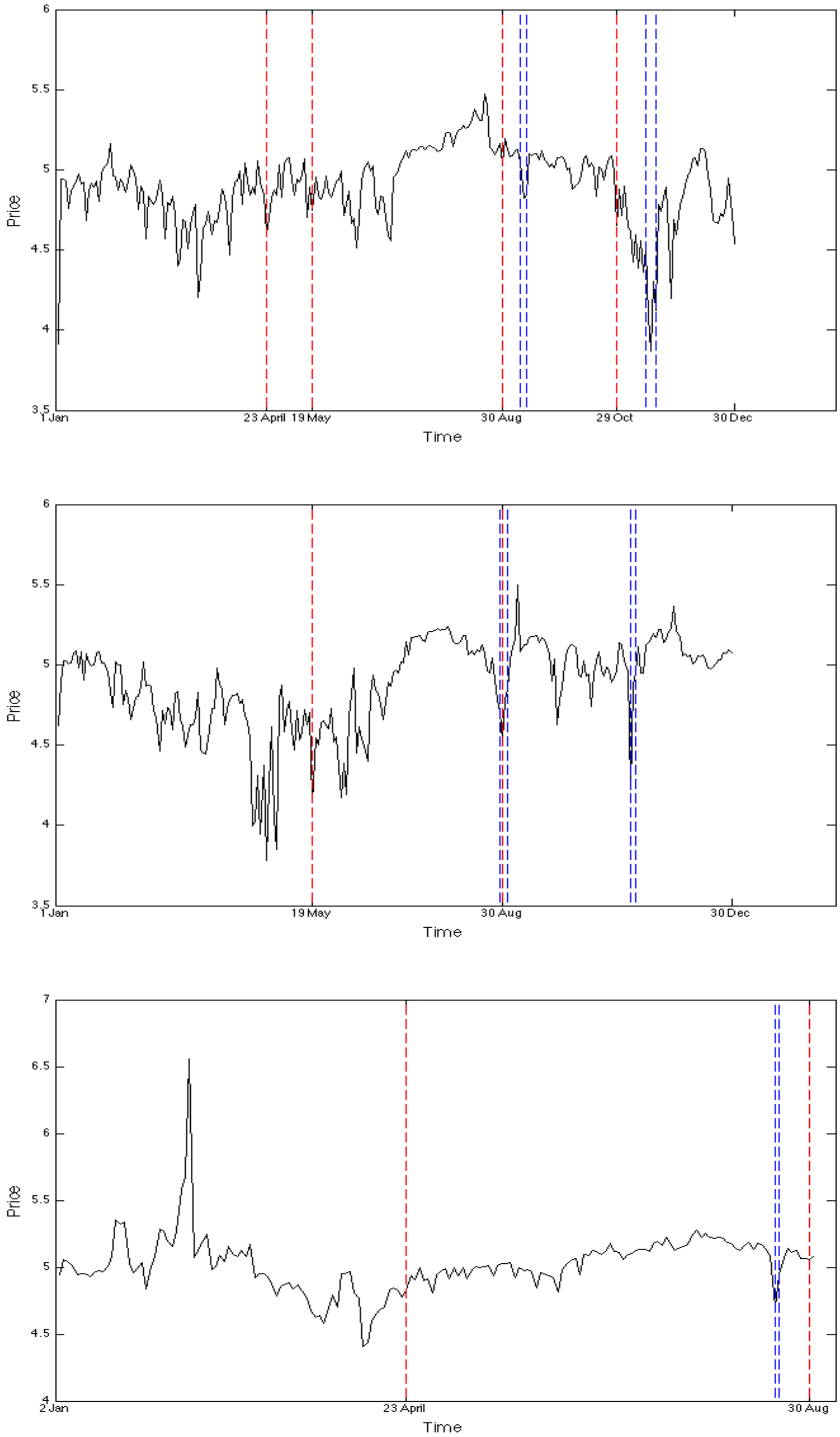


Figure 5.3: Day ahead price of the Turkish electricity market of 2010,2011,2012.

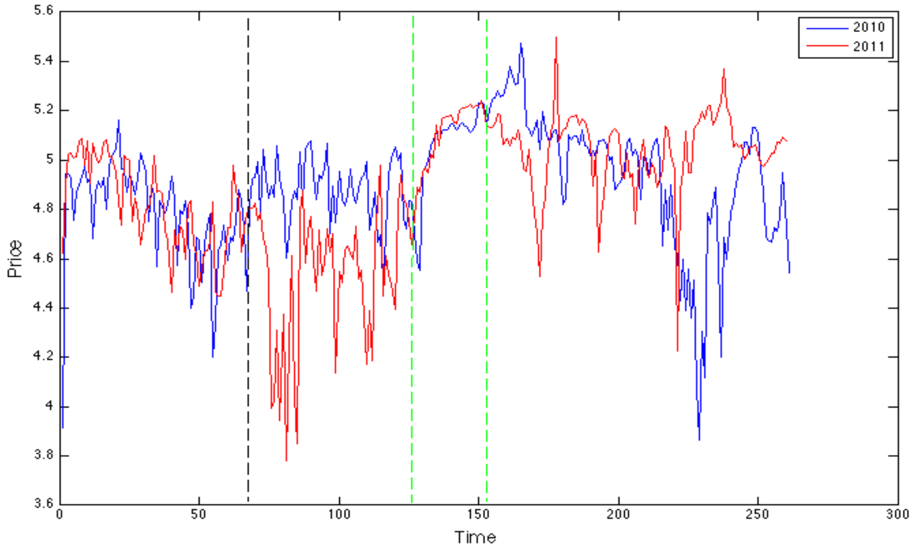


Figure 5.4: Day ahead price of the Turkish electricity market of the year 2010 and the year 2011.

## 5.3 Modeling the Price

In this section we model the price of selected periods from each year. Our main idea for a global model would be similar to the one we have developed for the temperature. However, this task is much more complicated and need to take into account the following considerations:

- Weekdays, weekends, religious holidays and official holidays have different price dynamics before the model this dynamics need to be understood fully.
- The reasons for the spikes and the behavior of this jumps need to be investigated. Proper modeling of the spikes need to be developed.
- we can refer to the change in dynamics of the price on the weekends and on holidays as a “phase change.” Then we have at least two types of phase changes: 1) phase changes whose dates are fixed: the weekends and the national holidays 2) phase changes whose dates shift earlier every year: these are the religious holidays. An inspection of data reveals that there maybe a third type of phase change: those whose starting date is random. The spikes can be seen as particular cases of this phase change. We think that data needs to be examined further whether this idea makes sense.

- The electricity market in Turkey is very young. Some of the features we currently observe may be transient.

### 5.3.1 The Model

In the price of electricity model, our aim is to investigate the effect of the temperature on the price. We compare AR with ARX where the $X$ represents the exogenous variable. We use the deseasonalized temperature as $X$ and we use three time periods to compare the models. The first period starts from February 2, 2010 ends on March 22,2010, the second period starts form February 2, 2011 till March 2 2011 and the last period contains the data between February 2, 2012 and March 22, 2012. In each period we dropped the weekends' prices and the temperature of the weekends. The reason we use these periods is the following: in this time period there seems to be linear decrease in prices. Furthermore, this time period has no holidays which ensure that no phase transitions of the first two types listed above occur in this period. This is a period of 35 days. We will use the first 30 observations for calibration and the last five observations used to check the accuracy of the models.

Figure 5.5 shows the price of the each period and the line on the figure represents the starting of the each period.

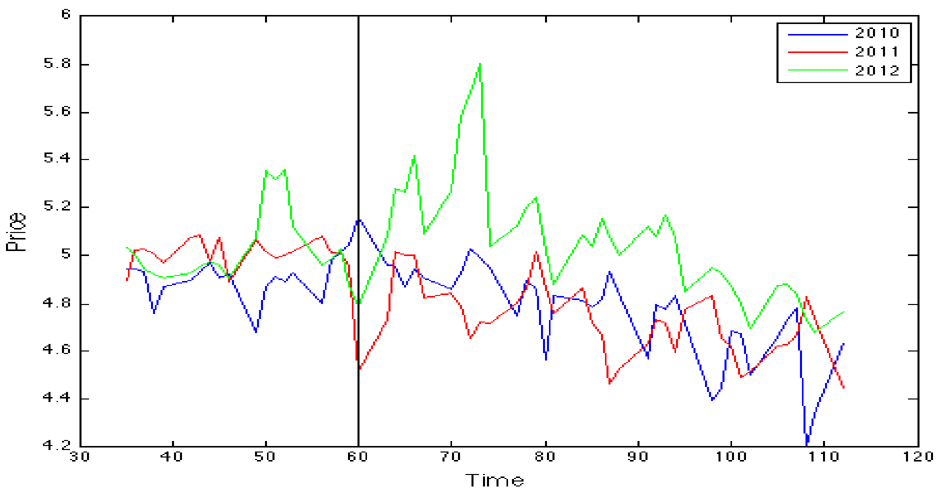


Figure 5.5: The price of the each periods.

We model all of the prices that are depicted in Figure 5.5 using a model of the form $\Psi_t + X_t$

where $\Psi$ is an affine function and $X$ a stationary process. We estimate the deterministic affine part using least squares regression.

Figure 5.6 shows the detrended (log price minus the estimated affine trend) price and the fitted lines of the each period.

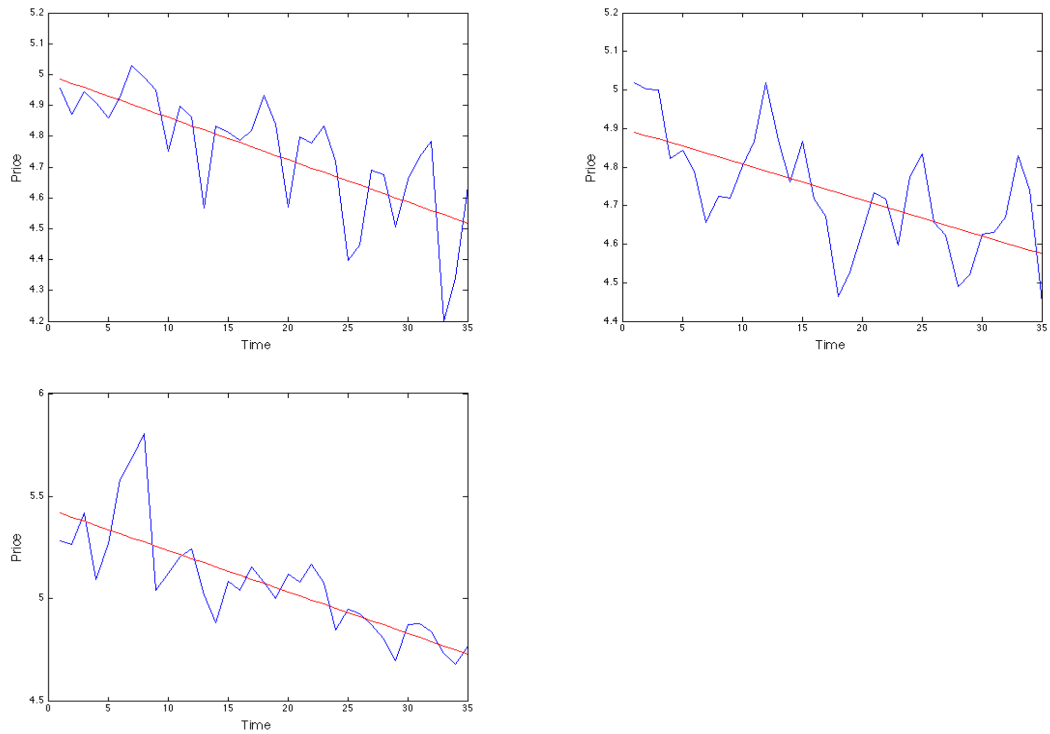


Figure 5.6: Price of the each period and the fitted lines.

In the next step we normalize the detrended price and detrended temperature by diving them their standard deviations. The same transformation will also be applied to the temperature process, the goal here is to have two processes that have the same scales . We plot the PACF of the each period to determine the order of AR and figure 5.7 show the PACF of the each period.

In the first period there are correlations at the second lag and the sixth lag, while in the second period and the last period there are correlations at the first lag and the sixth lag, so we decide to use same AR(6) with non consecutive lags to generate a common model for each period. The AR(6) model is as follows:

$$y_t = \theta_1 y_{t-1} + \theta_2 y_{t-2} + \theta_6 y_{t-6} + \epsilon_t,$$

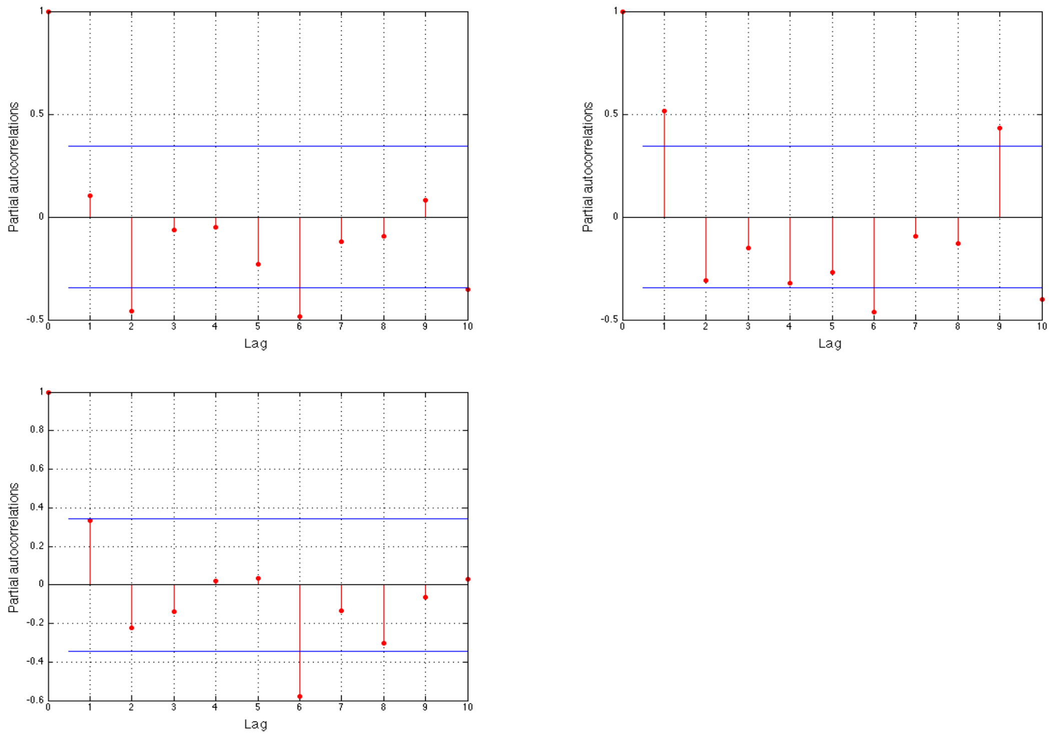


Figure 5.7: PACF of the each period.

where $y_t$ represent the price of electricity and $\epsilon_t$ is a white noise process. Our ARX(6) model is as follows:

$$y_t = \theta_1 y_{t-1} + \theta_2 y_{t-2} + \theta_6 y_{t-6} + \phi x_{t-1} + \epsilon_t,$$

where $x_t$ represents the temperature.

We use the temperature of the previous day because today's temperature can not effect the today's market price since we are working on the day ahead market. The next table shows the coefficients of the each period.

Table 5.2: Coefficients of each period.

| Period | Model | $\theta_1$ | $\theta_2$ | $\theta_6$ | $\phi$ |
|---|---|---|---|---|---|
| The First Period | AR | 0.1741 | -0.3445 | -0.2151 | |
| | ARX | 0.1742 | -0.3458 | -0.22002 | 0.04167 |
| The Second Period | AR | 0.4645 | -0.25535 | -0.5297 | |
| | ARX | 0.4584 | -0.25508 | -0.5249 | -0.031009 |
| The Third Period | AR | 0.44155 | -0.2787 | -0.43826 | |
| | ARX | 0.4511 | -0.2784 | -0.4368 | 0.1113 |

In each period we use the JBT and KST to see whether the residuals are coming form standard normal distribution or not and in both we failed to reject the null hypothesis at 5% significance level. Figure 5.8 shows the qq plot of the residuals of AR (left) and ARX (right), the first line represents the first period, the second lines represents the second period, while the last line represents the third period.

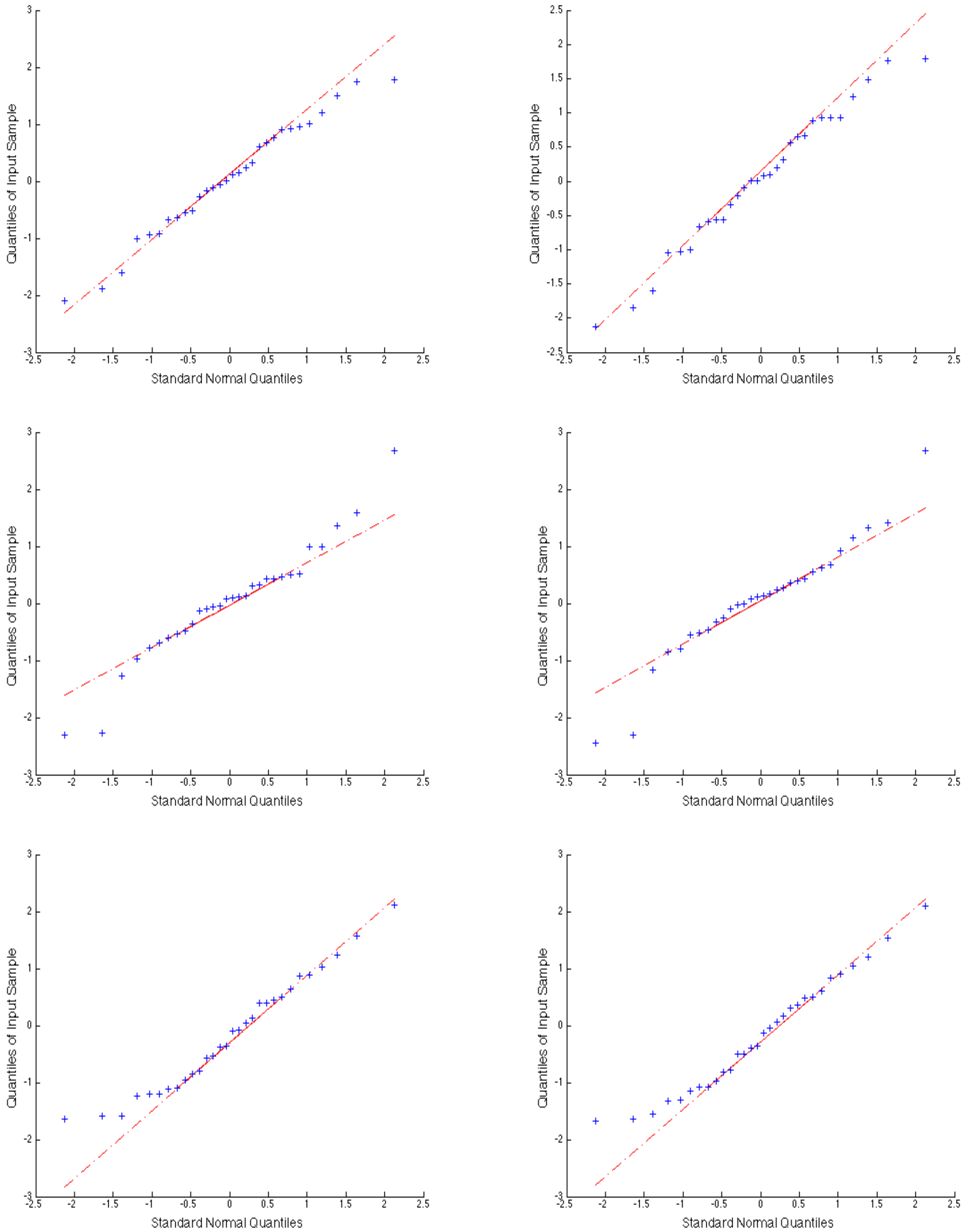


Figure 5.8: QQ plots of the residuals of the first period, the second year and the last periods.

### 5.3.2 Forecasting

Figure 5.9 shows the forecasted values with their confidence intervals.

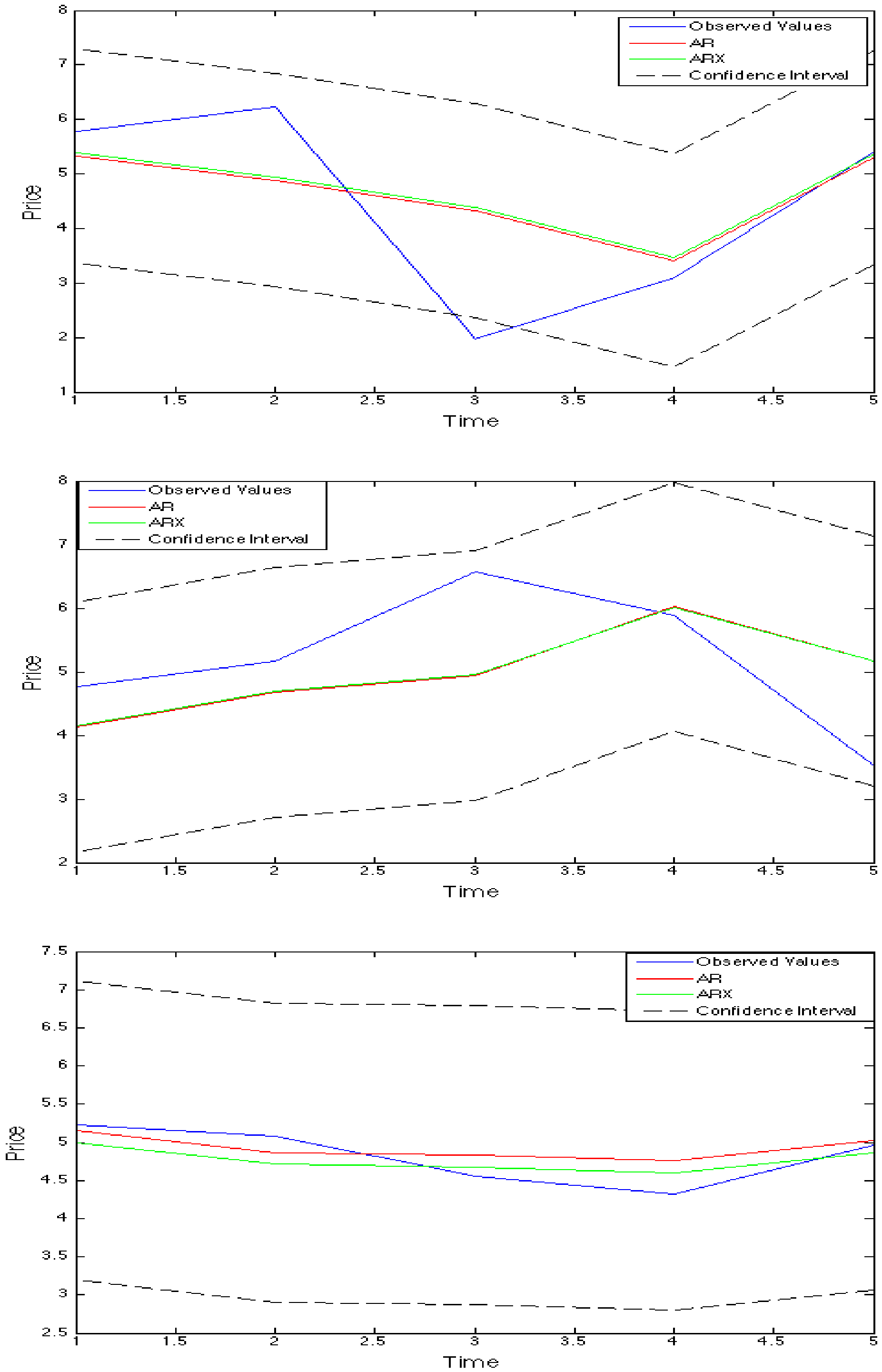


Figure 5.9: Forecasted price of the each period.

In the first figure the observation fall outside the confidence interval it is caused by a great decrease in the price since we determined the confidence interval according to normal distribution, the price in the first period may not follow a normal distribution.

Another observation about forecasts is that the temperature does not effect the price in the given periods. From Table 5.2 it can be seen that the coefficients of the temperature is very small. To check the relations between price and the temperature we plot the scatter plot where x axis represents the temperature while the y axis represents the price.

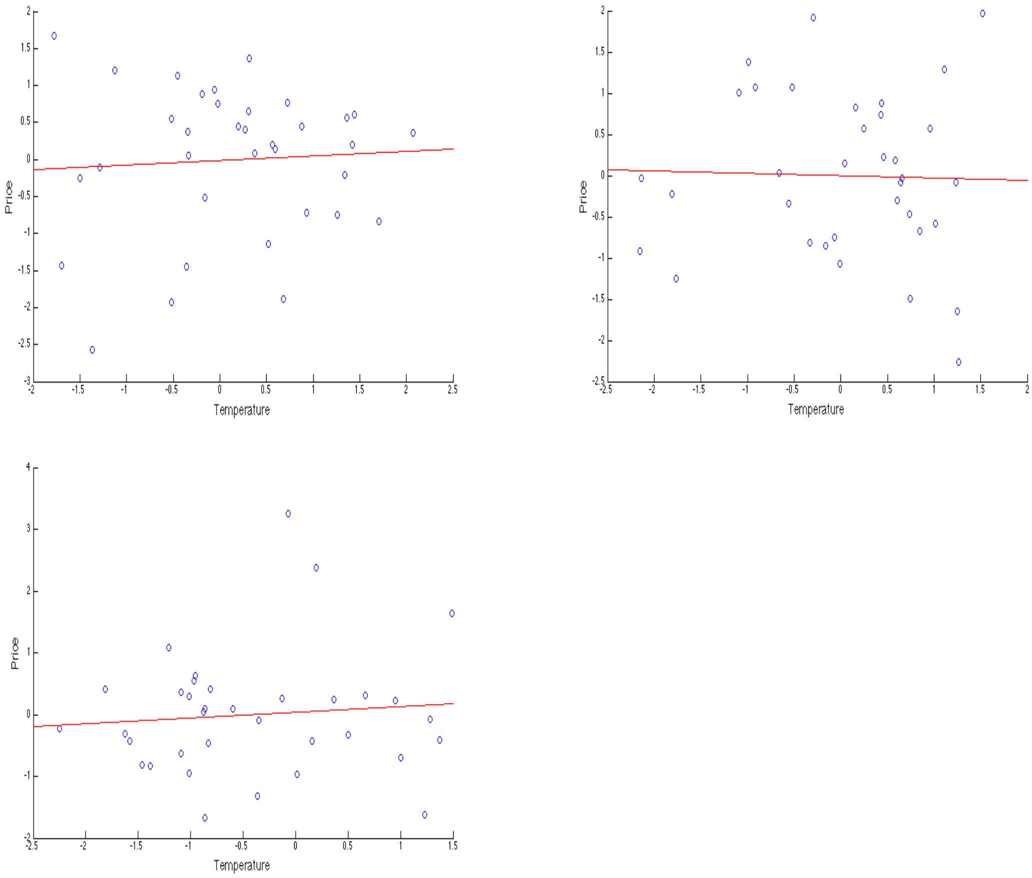


Figure 5.10: Scatter plots of temperature and price for each period.

In each period the red line represents the least squares line. We plot the least squares line to show the relations between the price and the temperature but the line in each period is almost parallel to $x$ axis so we concluded that there is a little relation between the price and temperature fluctuations .

For each period correlation coefficients and the $p$ values for testing the hypothesis of no correlation are as follows (for ease of notation we will refer to fluctuations in temperature simply as "temperature" and to fluctuations in the logarithm of the price process as "price."):

- The correlation coefficients between the price and the temperature in the first period is 0.0602 and the $p$ value is 0.731. In each case we assume that if the $p$ values are greater then 0.05 then the correlation is insignificant. For the first period we can conclude that there is no significant correlation since the $p$ value is 0.731.
- The correlation coefficients between the price and the temperature in the second period is $-0.02809$ and the $p$ value is 0.8727 which again suggests that there is no significant correlation in the second period.
- The correlation coefficients between the price and the temperature in the second period is 0.0945 and the $p$ value is 0.5889. Also in the last period there doesn't seem to be a significant correlation between price and temperature.

Note that these results are in agreement with Figure 5.10.

# CHAPTER 6

# CONCLUSION AND OUTLOOK

This thesis studies two stochastic processes in Turkey: temperature and day ahead electricity prices observed at PMUM. Our analysis of temperature is based on dividing time into alternating cycles, which represent warming and cooling periods. We observe that lengths of these cycles change from year to year and seem to be random. We defer an analysis of this randomness to future work and content ourselves to an analysis that fixes the period. Our analysis of temperature conditioned in this way consists of fitting a line to the data and using a simple time series model for the residuals. Our results indicate that this approach works fairly well. We find that the temperature of different cities may have correlations so when computing an average temperature for the whole country this situation must be taken into account. The temperature of the cities can be modeled by using a multidimensional stochastic process in future work.

In the electricity market part of this thesis we observed that the Turkish electricity market has many of the well known properties of the electricity markets in the world. Most significantly, electricity prices seem to be seasonal, that is from one year to the next prices seem to follow similar trajectories that include many trends. However, these trends are much more complicated than the trends we observe in temperature. A full analysis of these structures require further work. As significant is the multiphase behavior of these prices (weekends, holidays and random phases).

To simplify our analysis we focused on a particular period between february and march where the log prices have a linear trend. We deseanolized the price data of this period by fitting a line. We then fitted a simple autoregression to the residuals. Even this very simple model results in fairly good predictions. We also investigated the local effects of temperature on the electricity prices and found that deviations of the electricity price from its linear trend are not effected by the deviations of the temperature from its trend during this period.

There are many future directions, here we name some of them: 1) build a global model of electricity prices that allow random, deterministic and shifting phase changes and price spikes 2) remember that the fluctuations in temperature always had one or two outliers. It may be a good idea to use a distribution more general than normal to account for these outliers 3) build a global model for temperature 3) combine the global models to better understand the global effect of temperature on electricity prices.

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