PRICING AND HEDGING A PARTICIPATING FORWARD CONTRACT

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ABSTRACT

PRICING AND HEDGING A PARTICIPATING FORWARD CONTRACT

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We use the Garman-Kohlhagen model to compute the hedge and price of a participating forward contract on the US dollar that is written by a Turkish Bank. The algorithm is computed using actual market data and a weekly updated hedge is computed. We note that despite a weekly update and many assumptions made on the volatility and the interest rates the model gives a very reasonable hedge.

Keywords: Currency option, Brownian motion, participating forward, the Garman-Kohlhagen model, exotic derivatives

ÖZ

KATILIM VADELİ KONTRAT: FİYATLAMA VE KORUNMA

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Yerli bir bankanın ABD Doları üzerine yazdığı Katılım Vadeli Kontratın fiyatlama ve korunma prosedürünü Garman-Kohlhagen modelinden yararlanarak yaptık. Piyasadan gerçek datalar kullanıldı ve korunma portföyü haftalık yenilendi. Haftalık korunma yapılmış, oynaklık ve faizler üzerine birçok varsayım kabullenilmiş olmasına rağmen kullanılan model tutarlı bir korunma prosedürü sağlamıştır.

Anahtar Kelimeler: Döviz Opsiyonu, Brownian Hareketi, Katılım Vadeli Kontrat, Garman-Kohlhagen modeli, Egzotik Türevler

To my beloved mother

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CHAPTER 1

Introduction

In finance, an option is a contract that gives the buyer to exercise a right such as buying or selling an asset. The history of options goes back to ancient times. The first recorded user of options is Thales, the famous Greek mathematician and philosopher. He used options to secure low prices of olive presses just before harvest. From 2400 years ago to the present day, many societies used options in a great number of areas such as finance, marketing, transportation and tourism [12]. Today, people use options even in their daily life. When we buy a flight ticket, for example, we pay extra to get the cancellation right or the right to change the date of the flight; both of these are indeed option contracts.

When options were more integrated into business and finance, organized option markets started to develop in the world. In 1973, Chicago Board of Trade established the Chicago Board of Exchange (CBOE), the world's first (and now the largest) exchange market for option trading [9]. 1973 was not a random date for grand opening of CBOE, it was also the date when Black and Scholes delivered results for their master work, valuation of options. CBOE adopted immediately the Black-Scholes model to price options in 1973. Since then, options have been traded among investors as they were logically priced that the investors relied on.

Although the Black-Scholes model was the building block of stock option pricing, with the changing human needs, areas options used increased in numbers after 1973. Not surprisingly, options was also integrated into developing foreign exchange markets. Besides, valuing options written on currencies was another issue as they involve two currency interest rates (base and term currency interests) unlike the Black-Scholes model which involve the only one interest rate as it prices options on stocks. In 1983, Garman and Kohlhagen[4] developed a generalized Black-Scholes model to price the currency options which is still used today.

When “Foreign Currency Option Values”[4] first came out in 1983 currency options was a new market innovation at that time but now, foreign exchange market is the largest and the most liquid market in the world [10]. So, considering its liquidity and volume, it would not be wrong to say that valuing currency options is another crucial task. There are plenty of models to value currency options but in this thesis we will mention three of them to reach our goals: the Grabbe’s model, the Bigger and Hull model and the Garman-Kohlhagen model which is commonly used. Although the first two models will be considered in later chapters, our main focus in this thesis, is to price a specific financial contract, participating forward, using the Garman-Kohlhagen model.

A participating forward contract is a combination of currency options. It is an exotic derivative contract that is traded by a number of banks throughout the world. In Turkey, on the other hand, financial markets are not well developed. Yet, banks issue plenty of options to investors. They also offer some exotic derivatives such as participating forwards, par forwards, corridor options, etc., that are generally traded as zero cost products. In this thesis, we will price, analyze and give the hedging procedure for the participating forward contract written by a Turkish Bank using the Garman-Kohlhagen model by pricing its underlying components. We will also be finding the proportions of call and put options in the contract so that the contract would be zero cost.

By offering the hedging procedure and proportions we aim that banks and investors in Turkey could see and manage their risks that they are exposed to when they write or buy these type of financial products especially when VIOP[1] begins to take off.

[1] http://www.imkb.gov.tr/Uyeozel/SoftwareAndDocuments/FutureMarket.aspx

CHAPTER 2

Participating Forward Contract

2.1 The Contract at a Glance

2.1.1 The Outlook of the Participating Forward

A participating forward contract is a contract that consists of a foreign exchange (FX) call and FX put options on the same currency having the same strike and maturity date but different nominal values. It can be formed with either long call-short put or short call-long put options, depending on market conditions and expectations. The participating forward contract is sometimes called with a percentage like a 50% participating forward contract. Here 50% refers to the participation level of the holder of the contract; if the market goes in favor of the buyer, s/he has the right to exercise 50% of the notional amount of the contract. In this thesis we will only consider a 100% participating forward contract. At maturity, unless the spot rate is equal to the strike, one of the options will be exercised and an obligation will arise either for the buyer or for the writer of the contract. This is why the contract is called "forward." An example of an actual participating forward contract that is made available by a Turkish bank is given in Chapter 4. The main goal of the thesis is the hedging and pricing of this particular contract.

A useful interpretation of a participating forward contract is as follows. We are interested in, say, purchasing a number of call options. Purchasing a participating forward that contains this call position amounts to partially financing the long position on the call by having a short position on the put option that is in the participating forward. Commonly, the put and the call positions are selected so that the total value of the contract is zero; if this is the case, the contract is called "a zero cost product". If we continue the interpretation above, this corresponds to financing the long position in the call completely by a short position in the put. Interestingly, being zero cost usually is attractive to investors.

2.1.2 The Participating Forward for Investors

The participating forward contract is mostly attractive for importers and exporters who are exposed to FX exposures very frequently. Importers who desire to lower the cost of foreign payables as exporters who desire to increase the value of foreign receivables prefer the participating forward to hedge their FX risks[1]. The contract has several advantages and disadvantages for both the buyer and the seller. Some of the major disadvantages of the participating forward are as follows:

1. The exchange rate that you agree on the participating forward contract may not be as favorable as the one in the foreign exchange contract that you could enter instead of participating forward.
2. By entering a participating forward contract, the buyer takes a position either for bullish or bearish market.
3. Cancelling the contract will cost the investor.

On the other hand, the participating forward contract is convenient for those who wants to lower or get rid of exchange rate risks. Some of the major advantages of the contract are then:

1. The participating forward contract is zero cost instrument, which means there is no premium paid to enter the contract.
2. The participating forward contract provides full protection when the market goes unfovurably for the investor.
3. The participating forward contract may provide extra profit when the market goes in favor of the investor.

To clarify the benefits and harms, we take an example. Suppose an importer needs to buy \$1 million 3 months from now. He can buy a forward/futures contract in a simple way. Or, he

[1] http://finance.wharton.upenn.edu/ bodnarg/courses/readings/hedging.pdf

enters a participating forward contract and may get some more profit. By buying a forward contract, the importer just lockes himself at a rate agreed upon today. On the other hand, by entering a participating forward contract, the importer not only ensures to buy \$1 million, but he also plays for the bullish/bearish market. We assume that the contract on the Table 2.1 is traded in the market:

Table 2.1: Emprical Example

Underlying	USD/TRY
Expiration Date	3 months from now
Spot Rate	1,7800
Strike Rate	1,7900
Put Nominal	\$ 1,000,000
Call Nominal	\$ 1,500,000

He enters the contract by buying the call option and selling put. At maturity, two states can occur. If the USDTRY rate is less than 1,79, then the puts are exercised by the counterparty of the contract so that the importer gets \$1 million. If the USDTRY rate is greater than 1,79, then the importer exercises his call option to get \$1 million plus \$500k extra. He can immediately sell the amount of \$500k to the market so that he could benefit from this favourable move. In both cases, the importer guarantees to buy \$1 million. Moreover, he could increase his earnings when the spot moves in favor of him.
In global markets, a number of banks deliver participating forward contracts such as PNC, Dah Sing, Banksa, Westpac, HSBC, ING and a government bank in Turkey. In later chapters, we will focus on an actual participating forward contract written by the Turkish bank, calculate its value and give a delta hedging procedure.

2.2 Mathematics of the Contract

Although the participating forward is usually traded with no premium upfront, we will calculate the prices of its components so that we could get its theoretic price.

There are some models to price a currency option. Since a participating forward is a combination of currency options, we will benefit from a model to get a pricing formula for participating forwards. Because of the popularity and the simplicity of the Garman-Kohlhagen model in FX options, we will be using this model to price and hedge the bank's contract as we mentioned before.

Let the Garman-Kohlhagen model price of the call option in the contract is c with nominal n_1 and the Garman-Kohlhagen model price of the put option in the contract is p with nominal n_2. Depending on the positions in options, we will be having the prices of participating forward contract V are:

$$V = cn_1 - pn_2 : \qquad \text{Long call - Short put,}$$

$$V = pn_2 - cn_1 : \qquad \text{Long put - Short call.}$$

According to the $\frac{n_1}{n_2}$ ratio and the positions in the options, the payoffs of the contract differs. On Figures 2.1 and 2.2, we revealed the payoffs of the contract for each combinations: Options' prices derived from the Garman-Kohlhagen model are:

$$C = S\exp(-r_1 t)N(d_1) - X\exp(-r_1 t)N(d_2),$$

$$P = X\exp(-r_2 t)N(-d_2) - S\exp(-r_1 t)N(-d_1)$$

where

$$d_1 = \frac{\log(S/X) + (r_2 - r_1 + \sigma^2/2)t}{\sigma\sqrt{t}},$$

S = Domestic currency per unit of foreign currency,

X = Strike exchange rate,

r_1 = Continuously compounded foreign exchange rate,

r_2 = Continuously compounded domestic exchange rate,

t = Time in years until expiration,

σ = Implied Volatility,

N = Normal cumulative distribution function.

Note that on Figure 2.1, call nominal is greater than put's, and vice versa on Figure 2.2 assuming the put nominal is n, call nominal is k and the strike rate is X. Below table shows the payoffs of the contract at maturity date. Figures 2.1 and 2.2 are drawn with the data in this table.

	S = 0	$S < X$	$S > X$
Long Call	-	-	k(S-X)
Short Put	-nX	-n(X-S)	-
Contract Payoff	-nX	-n(X-S)	k(S-X)
Short call	-	-	-k(S-X)
Long Put	nX	n(X-S)	-
Contract Payoff	nX	n(X-S)	-k(S-X)

In the next chapter, we will analyze the model and derive the prices.

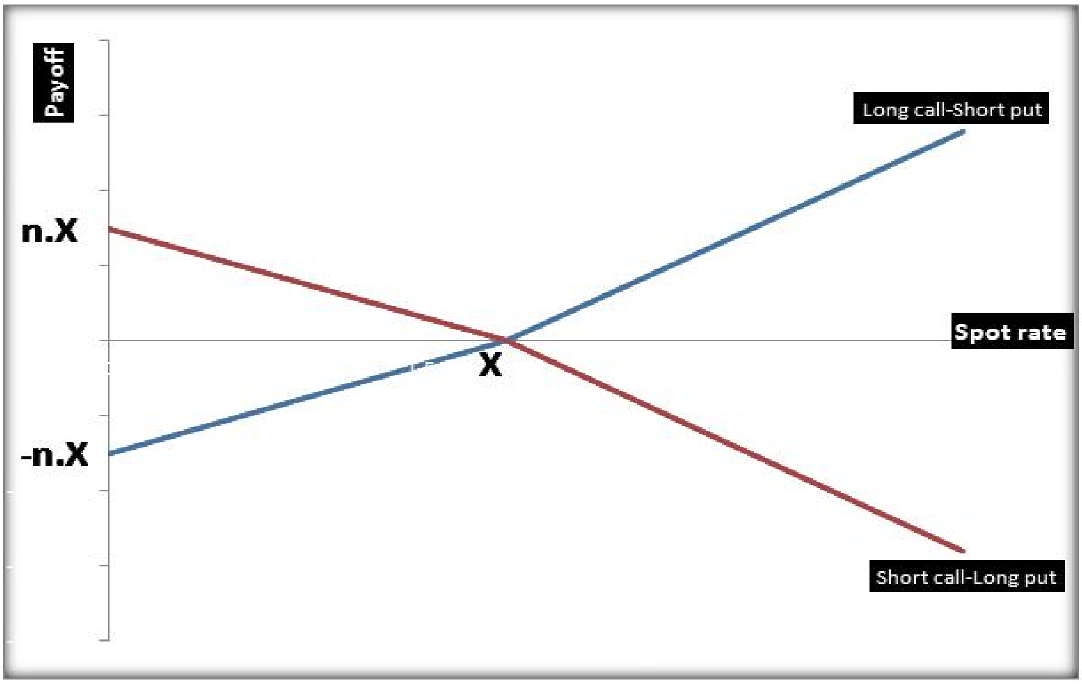

Figure 2.1: Contract payoff 1

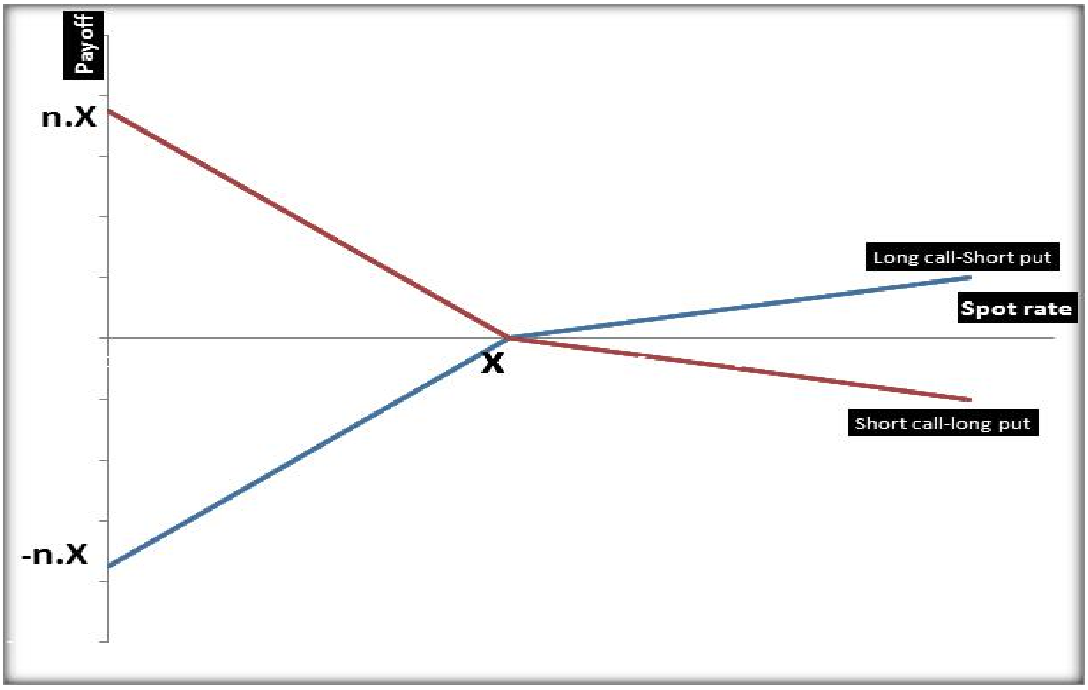

Figure 2.2: Contract payoff 2

CHAPTER 3

Valuation: The Garman-Kohlhagen Model

3.1 Model Assumptions

The Garman Kohlhagen model was developed to value European-style options on currencies. The model is a generalization of the Black-Scholes model having the following assumptions:

1. The returns are lognormally distributed.
2. Transactions cost and taxes are zero.
3. The exchange rate, the interest rates and the volatility are functions of time.
4. The exchange rates follows a continuous Ito process.

3.2 Derivation of the Model

To derive the hedging strategy and the prices of call and put FX options, we will follow the steps in problem 2 in the book Introduction to Stochastic Calculus Applied to Finance [7, problem 2, page 109]. Let S_t be the price of dollars at time t. By assumptions of the Garman Kohlhagen model:

$$\frac{dS_t}{S_t} = \mu dt + \sigma dW_t,$$

So

$$S_t = S_0 e^{\mu t - \frac{\sigma^2 t}{2} + \sigma W_t}$$

by Ito formula with $f(S_t) = log(S_t)$. The following proposition is a useful tool to go further steps and we obtain it and its proof from Lamberton's book.

Proposition 3.2.1 (14, proposition 3.3.3) . *If $(X_t)_{t\geq 0}$ is a standard F_t-Brownian motion, then:*

1. *X_t is an F_t-martingale.*
2. *$X_t^2 - t$ is an F_t-martingale.*
3. *$exp(\sigma X_t - (\sigma^2/2)t)$ is an F_t-martingale, for every $\sigma \in \mathbb{R}$.*

Proof. If $s \leq t$, then $X_t - X_s$ is independent of σ - algebra F_s. Thus $\mathbb{E}(X_t - X_s|F_s) = \mathbb{E}(X_t - X_s)$. Since a standard Brownian motion has an expectation equal to zero, we have $\mathbb{E}(X_t - X_s) = 0$. Hence, the first assertion is proved. To show the second one, we remark that

$$\begin{aligned}\mathbb{E}\left(X_t^2 - X_s^2|\, F_s\right) &= \mathbb{E}\left((X_t - X_s)^2 + 2X_s(X_t - X_s)|F_s\right), \\ &= \mathbb{E}\left((X_t - X_s)^2|F_s\right) + 2X_s\mathbb{E}\left((X_t - X_s)|F_s\right),\end{aligned}$$

and since $(X_t)_{t\geq 0}$ is a martingale, $\mathbb{E}(X_t - X_s|F_s) = 0$, whence

$$\mathbb{E}\left(X_t^2 - X_s^2|\, F_s\right) = \mathbb{E}\left((X_t - X_s)^2|F_s\right).$$

Because the Brownian motion has independent and stationary increments, it follows that

$$\begin{aligned}\mathbb{E}\left((X_t - X_s)^2|F_s\right) &= \mathbb{E}\left(X_{t-s}^2\right), \\ &= t - s.\end{aligned}$$

The last equality is due to the fact that X_t has a normal distribution with mean zero and variance t That yields $\mathbb{E}(X_t^2 - t|F_s) = X_s^2 - s$, if $s < t$. Finally, let us recall that if g is a standard normal variable, we have

$$\mathbb{E}\left(\mathrm{e}^{\lambda g}\right) = \int_{-\infty}^{+\infty} \mathrm{e}^{\lambda x}\, e^{\frac{-x^2}{2}}\, \frac{\mathrm{d}x}{\sqrt{2\pi}} = \mathrm{e}^{\frac{\lambda^2}{2}}.$$

On the other hand, if $s < t$,

$$\mathbb{E}\left(\mathrm{e}^{\sigma X_t - \frac{\sigma^2 t}{2}}\, |F_s\right) = \mathrm{e}^{\sigma X_s - \frac{\sigma^2 t}{2}}\, \mathbb{E}\left(\mathrm{e}^{\sigma(X_t - X_s)}\, |F_s\right),$$

because X_s is F_s measurable. Since $X_t - X_s$ is independent of F_s, it turns out that

$$\begin{aligned}\mathbb{E}\left(\mathrm{e}^{\sigma(X_t - X_s)}\, |F_s\right) &= \mathbb{E}\left(\mathrm{e}^{\sigma(X_t - X_s)}\right) \\ &= \mathbb{E}\left(\mathrm{e}^{\sigma(X_{t-s})}\right) \\ &= \mathbb{E}\left(\mathrm{e}^{\sigma g\sqrt{t-s}}\right) \\ &= \exp\left(\frac{1}{2}\sigma^2(t-s)\right).\end{aligned}$$

This completes the proof.

■

Now, for $s \leq t$

$$\begin{aligned}
\mathbb{E}(S_t|F_s) &= \mathbb{E}\left(S_0 e^{\mu t - \frac{\sigma^2 t}{2} + \sigma W_t} |F_s\right), \\
&= S_0 e^{\mu t}\, \mathbb{E}\left(e^{\sigma W_t - \frac{\sigma^2 t}{2}} |F_s\right), \\
&= S_0 e^{\mu t}\, e^{\sigma W_s - \frac{\sigma^2 s}{2}}, \\
&\geq S_s.
\end{aligned}$$

So (S_t) is submartingale. Similarly, let $U_t = \frac{1}{S_t}$ be the euro versus dollar exchange rate. Applying Ito formula to $f(X_t) = \frac{1}{X_t}$ with $K_s = \mu S_s$ and $H_s = \sigma S_s$, we get

$$\mathrm{d}U_t = ((\sigma^2 - \mu)dt - \sigma dW_t)U_t.$$

Again by Ito Formula with $K_s = (\sigma^2 - \mu)U_s$ and $H_s = -\sigma U_s$, we have

$$U_t = x_0 \mathrm{e}^{(\frac{\sigma^2}{2} - \mu)t - \sigma W_t}.$$

Thus, U_t is a submartingale as well.

Recall that we aim to price a European call on one dollar with maturity T and strike price K by using a Black-Scholes type method. We need to form a strategy having an initial wealth equal to Black-Scholes premium at trade date, and a final wealth equal to intrinsic value, $(S_t - K)_+$, at time T. We define a portfolio consist of H_t^0 Euros and H_t Dollars at time t.

At time t, the value in euros in portfolio made of H_t^0 Euros and H_t Dollars is as follows:

$$V_t = H_t^0 + H_t S_t.$$

A self financing strategy will be defined by an adapted process $(H_t^0, H_t)_{t\epsilon[0,T]}$ such that

$$dV_t = r_0 H_t^0 dt + r_1 H_t S_t dt + H_t dS_t, \tag{3.1}$$

where

$$r_0 \text{ : Domestic rate (Euro rate),}$$
$$r_1 \text{ : Foreign rate (Dollar rate).}$$

The above Equation (3.1) makes sense if

$$\int_0^t |H_t^0| d_s < \infty \text{and} \int_0^t |H_t^0|^2 d_s < \infty$$

$\mathbb{P}$ a.s. by Ito Theorem [7, Theorem 3.5.1]. Let $\tilde{V}_t = e^{-r_0 t} V_t$ be the discounted value of self financing portfolio (H_t^0, H_t). Then,

$$V_t = H_t^0 + H_t S_t,$$
$$dV_t = r_0 H_t^0 dt + r_1 H_t S_t dt + H_t dS_t,$$

where $dS_t = S_t(\mu dt + \sigma dW_t)$, $\tilde{V}_t = e^{-r_0 t} V_t$ implies

$$d\tilde{V}_t = -r_0 e^{-r_0 t} V_t dt + e^{-r_0 t} dV_t,$$
$$d\tilde{V}_t = -r_0 e^{-r_0 t} dt(H_t^0 + H_t S_t) + e^{-r_0 t}(r_0 H_0^t dt + r_1 H_t S_t dt + H_t S_t \mu dt + H_t S_t \sigma dW_t),$$
$$d\tilde{V}_t = H_t S_t e^{-r_0 t}(\mu + r_1 - r_0) dt + H_t e^{-r_0 t} S_t \sigma dW_t.$$

We wish to show that a probibility $\tilde{\mathbb{P}}$ equivalent to $\mathbb{P}$ exists under which the process $\tilde{W}_t$ is standart Brownian motion. Set $\theta_t = \frac{\mu + r_1 - r_0}{\sigma}$. $\int_0^T \theta_s^2 ds < \infty$ and we let

$$L_t = e^{\left(-\frac{\mu + r_1 - r_0}{\sigma} B_t - \frac{1}{2}\left(\frac{\mu + r_1 - r_0}{\sigma}\right)^2 t\right)},$$
$$\mathbb{E}(L_t | F_s) = \mathbb{E}\left(e^{\left(-\frac{\mu + r_1 - r_0}{\sigma} B_t - \frac{1}{2}\left(\frac{\mu + r_1 - r_0}{\sigma}\right)^2 t\right)} | F_s\right).$$

For simplicity, we let $\sigma' = \frac{\mu + r_1 - r_0}{\sigma}$, then

$$\mathbb{E}(L_t | F_s) = \mathbb{E}\left(e^{-\sigma' B_t - \frac{1}{2}\sigma'^2 t} | F_s\right),$$
$$= \mathbb{E}\left(e^{-\sigma'(B_t - B_s) - \sigma' B_s - \frac{1}{2}\sigma'^2 t} | F_s\right),$$
$$= e^{-\sigma' B_s - \frac{1}{2}\sigma'^2 t}\, \mathbb{E}\left(e^{-\sigma'(B_t - B_s)} | F_s\right)$$

since B_s is F_s measurable.

$$\mathbb{E}(L_t|F_s) = e^{-\sigma' B_s - \frac{1}{2}\sigma'^2 t}\, \mathbb{E}\left(e^{-\sigma'(B_t - B_s)}\right)$$

since $B_t - B_s \sim B_{t-s}$ is independent of F_s. We know

$$\mathbb{E}(e^{\lambda g}) = e^{\frac{\lambda^2}{2}} \tag{3.2}$$

if g is a standard normal rv. The Equation (3.2) implies $\mathbb{E}(e^{-r' B_{t-s}}) = e^{\frac{1}{2}\sigma'^2(t-s)}$. So,

$$\mathbb{E}(L_t|F_s) = e^{-\sigma' B_s - \frac{1}{2}\sigma'^2 t}\, e^{\frac{1}{2}\sigma'^2(t-s)} = L_s.$$

Thus, L_t is a martingale.

By Girsanov theorem [7, Theorem 4.2.2], $\tilde{W}_t = \frac{\mu + r_1 - r_0}{\sigma} t + W_t$ is a standard Brownian motion. We now wish to show that the discounted value $\tilde{V}_t$ is martingale under $\tilde{\mathbb{P}}$:

$$\begin{aligned}
\tilde{W}_t &= \left(\frac{\mu + r_1 - r_0}{\sigma} t\right) + W_t, \\
d\tilde{W}_t &= dW_t + \left(\frac{\mu + r_1 - r_0}{\sigma}\right) dt, \\
d\tilde{V}_t &= H_t e^{-r_0 t} S_t(\mu + r_1 - r_0)dt + H_t e^{-r_0 t} S_t(r d\tilde{W}_t - (\mu + r_1 - r_0)dt), \\
d\tilde{V}_t &= H_t e^{-r_0 t} S_t \sigma d\tilde{W}_t.
\end{aligned}$$

Here, $\tilde{V}_t$ is martingale as $d\tilde{V}_t$ is independent of dt. An admissible strategy replicates the call if it is worth $V_T = (S_T - K)_+$ at time T. Assuming our strategy replicates the call we will show that for any $t \leqslant T$ the value of the strategy at time t will be $V_t = F(t, S_t)$ where $F(t,x) = \tilde{E}(x e^{-(r_1 + (\frac{\sigma^2}{2}))(T-t) + r(\tilde{W}_T - \tilde{W}_t)} - K e^{-r_0(T-t)})_+$ where the symbol $\tilde{E}$ stands for expectation under $\tilde{\mathbb{P}}$. We first need to have S_t and dS_t under $\tilde{W}_t$:

$$\begin{aligned}
d\tilde{W}_t &= dW_t + \frac{\mu + r_1 - r_0}{\sigma} dt, \\
dS_t &= S_t(\mu dt + \sigma dW_t), \\
dS_t &= S_t\left(\mu dt + \sigma\left(d\tilde{W}_t - \frac{\mu + r_1 - r_0}{\sigma} dt\right)\right), \\
dS_t &= S_t((r_0 - r_1)dt + \sigma d\tilde{W}_t), \\
S_t &= x_0 + \int_0^t S_s(r_0 - r_1)dt + \int_0^t S_s \sigma d\tilde{W}_t, \\
K_s &= S_s(r_0 - r_1), H_s = \sigma S_s.
\end{aligned}$$

Let us apply the Ito Formula to $\log(S_t) = f(S_t)$:

$$\begin{aligned}
\log(S_t) &= \log S_0 + \int_0^t \frac{1}{S_s} ds + \frac{1}{2}\int_0^t -\frac{1}{S_s^2}\sigma^2 S_s^2 ds,\\
\log(S_t) &= \log S_0 + \int_0^t (r_0 - r_1) ds + \int_0^t \sigma d\tilde{W}_s - \frac{1}{2}\sigma^2 t,\\
\log(S_t) &= \log S_0 + (r_0 - r_1 - \frac{1}{2}\sigma^2)t + \sigma \tilde{W}_t,\\
S_t &= x_0 exp((r_0 - r_1 - \frac{1}{2}\sigma^2)t + \sigma \tilde{W}_t),\\
S_T &= S_t exp((r_0 - r_1 - \frac{1}{2}\sigma^2)(T-t) + \sigma \tilde{W}_{T-t}).
\end{aligned}$$

As we know that $\tilde{V}_t$ is martingale, we conclude;

$$\begin{aligned}
\tilde{V}_t &= \tilde{\mathbb{E}}(\tilde{V}_T | \tilde{F}_t) \text{ with } t \leqslant T\\
\Rightarrow V_t &= \tilde{\mathbb{E}}(e^{-r_0(T-t)} h | F_t).
\end{aligned}$$

If we write $h = f(S_T)$, then we would have

$$\begin{aligned}
V_t &= \tilde{\mathbb{E}}\left(e^{-r_0(T-t)} f(S_T) | F_t\right),\\
V_t &= \tilde{\mathbb{E}}\left(e^{-r_0(T-t)} f\left(S_t e^{((r_0 - r_1 - \frac{1}{2}\sigma^2)(T-t) + \sigma \tilde{W}_{T-t})}\right) | F_t\right).
\end{aligned}$$

We know S_t is F_t measurable under $\tilde{\mathbb{P}}$ and $W_T - W_t$ is independent of F_t. Hence, by Proposition A.2.5 in Lamberton we obtain $V_t = F(t, S_t)$, where

$$\begin{aligned}
F(t,x) &= \tilde{\mathbb{E}}\left(e^{-r_0(T-t)} f\left(x e^{(r_0 - r_1)(T-t)} e^{\sigma(\tilde{W}_T - \tilde{W}_t) - \frac{1}{2}\sigma^2(T-t)}\right)\right)\\
&= e^{-r_0(T-t)} \int_{-\infty}^{\infty} f\left(x e^{(r_0 - r_1 - \frac{\sigma^2}{2})(T-t) + \sigma y \sqrt{T-t}}\right) \frac{e^{-\frac{y^2}{2}}}{\sqrt{2\pi}} dy.
\end{aligned}$$

For the call option $f(x) = (X - K)_+$. When we adapt it, we will get

$$F(t,x) = \tilde{\mathbb{E}}\left(x e^{-(r_1 + \frac{1}{2}\sigma^2)(T-t) + \sigma(\tilde{W}_T - \tilde{W}_t)} - K e^{-r_0(T-t)})\right)_+$$

where $\tilde{\mathbb{E}}$ is expectation under $\tilde{\mathbb{P}}$.

We are close to get the results, though still have some work to do: Set $\theta = T - t, (W - T - W_t) \sim \sqrt{\theta} g$ where g is standard Gaussian variable. Then F(t,x) converts to

$$\begin{aligned}
F(t,x) &= \tilde{\mathbb{E}}\left(xe^{-(r_1 + \frac{\sigma^2}{2})(T-t) + \sigma(\tilde{W}_T - \tilde{W}_t)} - Ke^{-r_0(T-t)} \right) \\
&= \int_{-d_2}^{\infty} \left(xe^{\sigma\sqrt{\theta}y - (r_1 + \frac{\sigma^2}{2})\theta} - Ke^{-r_0\theta} \right) \frac{e^{-\frac{y^2}{2}}}{\sqrt{2\pi}} dy \\
&= \int_{-\infty}^{d_2} \left(xe^{\sigma\sqrt{\theta}y - (r_1 + \frac{\sigma^2}{2})\theta} - Ke^{-r_0\theta} \right) \frac{e^{-\frac{y^2}{2}}}{\sqrt{2\pi}} dy \\
&= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{d_2} xe^{\sigma\sqrt{\theta}y - r_1\theta - \frac{\sigma^2\theta}{2} - \frac{y^2}{2}} dy - \int_{-\infty}^{d_2} Ke^{-r_0\theta - \frac{y^2}{2}} dy
\end{aligned}$$

For the first integral of the above equality, set $z = y + \sigma\sqrt{\theta} \Rightarrow d_z = d_y, y = d_2 \Rightarrow z = d_1$ as $d_2 = d_1 - \sigma\sqrt{\theta}$. So, we convert the first integral function into $xe^{-r_1\theta}N(d_1)$where $N(d_1) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{d} e^{-\frac{x^2}{2}} dx$. Therefore, $F(t,x) = e^{-r_1(T-t)}xN(d_1) - Ke^{-r_0(T-t)}N(d_2)$.

If we replace x with S_t, we have the Garman-Kohlhagen call price.

We assumed that our strategy replicates the call. We now show that the option is effectively replicable:

Let $\tilde{S}_t = e^{(r_1 - r_0)t}S_t$. Then,

$d\tilde{S}_t = (r_1 - r_0)e^{(r_1 - r_0)t}S_t dt + e^{(r_1 - r_0)t}dS_t$. As we know that $dS_t = S_t((r_0 - r_1)dt + \sigma d\tilde{W}_t)$. If we plug dS_t into $d\tilde{S}_t$, we obtain

$$\begin{aligned}
d\tilde{S}_t &= (r_1 - r_0)e^{(r_1 - r_0)t}S_t dt + e^{(r_1 - r_0)t}(S_t(r_0 - r_1)dt + S_t\sigma d\tilde{W}_t) \\
&= e^{(r_1 - r_0)t}S_t\sigma d\tilde{W}_t \\
&= \tilde{S}_t\sigma d\tilde{W}_t.
\end{aligned}$$

Let $\tilde{F}$ be the function defined by $\tilde{F}(t,x) = e^{-r_0 t}F(t, xe^{(r_0 - r_1)})$. Let $C_t = F(t, S_t)$ and $\tilde{C}_t = e^{-r_0 t}C_t = \tilde{F}(t, \tilde{S}_t)$.

By Ito Formula for two variables, we recieve

$$\tilde{F}(t, \tilde{S}_t) = \tilde{F}(0, \tilde{S}_0) + \int_0^t \frac{\partial \tilde{F}}{\partial x}(u, \tilde{S}_u)d\tilde{S}_u + \int_0^t \frac{\partial \tilde{F}}{\partial t}(u, \tilde{S}_u)du + \int_0^t \frac{1}{2}\frac{\partial^2 \tilde{F}}{\partial x^2}(u, \tilde{S}_u)d\langle \tilde{S}, \tilde{S} \rangle_u$$

We have $d\tilde{S}_t = \sigma \tilde{S}_t dW_t$. So, the quadratic variation of $\tilde{S}$ with itself is

$$d\langle \tilde{S}, \tilde{S} \rangle_u = \sigma^2 \tilde{S}_u^2 du.$$

So, $\tilde{F}(t, \tilde{S}_t)$ can be written as $\tilde{F}(t, \tilde{S}_t) = \tilde{F}(0, \tilde{S}_0) + \int_0^t \sigma \frac{\partial \tilde{F}}{\partial x}(u, \tilde{S}_u) \tilde{S}_u dW_u + \int_0^t K_u du.$

Since $\tilde{F}(t, \tilde{S}_t)$ is martingale under $\tilde{\mathbb{P}}$, K_u is necessarily null!

Thus,

$$\tilde{F}(t, \tilde{S}_t) = \tilde{F}(0, \tilde{S}_0) + \int_0^t \frac{\partial \tilde{F}}{\partial x}(u, \tilde{S}_u) d\tilde{S}_u,$$
$$d\tilde{F}(t, \tilde{S}_t) = \frac{\delta \tilde{F}}{\delta x}(t, \tilde{S}_t) d\tilde{S}_t = \frac{\partial F}{\partial x}(t, S_t) \sigma e^{-r_0 t} S_t dW_t.$$

From $\tilde{F}(t, S_t)$ the candiate for the hedge ratio is

$$H_t = \frac{\delta \tilde{F}}{\delta x}(t, \tilde{S}_t) = \frac{\delta F}{\delta x}(t, S_t) = N(d_1) e^{-r_1(T-t)}.$$

If we set the $H_t^0 = \tilde{F}(t, S_t) - H_t \tilde{S}_t$, the portfolio (H_t^0, H_t) is self-financing and its discounted value is indeed $\tilde{V}_t = \tilde{F}(t, \tilde{S}_t)$.

We can easily get the formula for put option from put-call parity:

$$C(t) - P(t) = S(t) - K \exp -r(T-t),$$

where $C(t)$ is the value of the call, $P(t)$ is the value of the put, $S(t)$ is the spot price of the underlying at time t and K is the strike rate/price.

Hence,

$$C(t, x) = e^{-r_1(T-t)} S(t) N(d_1) - K e^{-r_0(T-t)} N(d_2),$$
$$P(t, x) = K e^{-r_0(T-t)} N(-d_2) - e^{-r_1(T-t)} S(t) N(-d_1),$$
$$Delta_{call} = e^{-r_1(T-t)} N(d_1),$$
$$Delta_{put} = -e^{-r_1(T-t)} N(-d_1).$$

These results and formulas can be found in the paper "Foreign Currency Option Values" as well.

3.3 Alternative models

There are alternative models valuing FX options, though here, we consider the Grabbe's model and the Biger&Hull model. Both models take Freiger and Jacquillat, Stulz and Black's previous works into account and are developed over the Black-Scholes model [3, 11]. In the paper 'Valuation of Currency Options' by Nahum Biger and John Hull, the authors approach the valuation procedure of FX options over pricing a currency option bond. A currency option bond is a bond that its payoffs (coupons and principal) are paid either in domestic currency or in foreign currency at the holder discretion. In this manner, we can see a currency option bond as a single currency bond along with a foreign currency option. Therefore,

$$P = B + cp,$$

where
P: the price of a bond paying either \$1 or p pound at T,
B: the price of pure discount bond paying \$1 at T,
c: the price of European call option to purchase 1 pound for a dollar price of $1/p$ at T.

Since the Black-Scholes model assumes that stock does not pay dividend, and as we noted before that FX options consider both risk free rates, domestic and foreign, BS model's direct application is not possible to price FX options. Biger and Hull state that the reason is that an investor who wish to hold foreign currency would prefer holding short term risk free foreign currency bonds rather than holding foreign currency in some non interest bearing account. They conclude that valuing a foreign currency option is the same as valuing stock option with continuous dividend. Assuming the dividend yield is constant, Smith [13] and Merton [8] found an explicit formula for pricing dividend paying option:

$$c = e^{-ST} S N\left(\frac{\log(\frac{S}{X} + [r - \delta + (\frac{\sigma^2}{2})]T}{\sigma\sqrt{T}}\right) - e^{-rT} X N\left(\frac{\log(\frac{S}{X} + [r - \delta - (\frac{\sigma^2}{2})]T}{\sigma\sqrt{T}}\right).$$

At this point, they switch to forward rates in the formula using interest rate parity. The main differences between the Garman-Kohlhagen model and these alternative models is that both models, the one of Biger&Hull and the one of Grabbe, assume that interest rate parity holds:

$$\frac{F}{S} = e^{(r-r^*)T},$$

where F is the forward rate. Replacing ln(F/S) with (r-r*)T in the above formula, they reached final formula:

$$c = e^{-rT} F N(\frac{\log(\frac{F}{X} + (\frac{\sigma^2}{2})T}{\sigma\sqrt{T}}) - e^{-rT} X N(\frac{\log(\frac{F}{X} - (\frac{\sigma^2}{2})T}{\sigma\sqrt{T}})$$

When we analyze the formula, we see that call and put option prices depend on F, X, sigma, T and r. This enables us to price FX options using forward rates and forming riskless hedge portfolios using forward contracts with a short call. In the paper "Pricing of a Call and Put Options on Foreign Exchange" by Orlin Grabbe, on the other hand, he used pure discount bonds to price the FX options. By a pure discount bond, we mean a bond paying 1 unit of currency at the expiration date T. He found the same formula with Biger and Hull model, but a slight difference:

$$c(t) = B(t,T)[F(t,T)N(d_1) - XN(d_2)],$$

where

$$d_1 = \frac{\log\frac{F}{X} + \frac{\sigma^2}{2}T}{\sigma\sqrt{T}},$$
$$d_2 = \frac{\log\frac{F}{X} - \frac{\sigma^2}{2}T}{\sigma\sqrt{T}},$$

$$\sigma^2 = \int_0^T \frac{1}{T}\sigma_F^2(t + T - u, u)du.$$

Consequently, these two alternative models are also generalization of the Black and Scholes model with one more assumption: Interest rate parity holds. In practice, all these models give close outcomes. On the other hand, the Garman-Kohlhagen model is more common in use because hedging with the spot market is fairly simple (The delta is just the first derivative with respect to S_t of the formula). Hedging with forward contracts is not handy for investors if the forward market in the country is not sufficiently deep. This makes it hard to hedge our position dynamically. In addition, $\frac{\partial C}{\partial F}$ is should not be taken as a hedge ratio as in Garman-Kohlhagen model. This is explained in Garman and Kohlhagen's paper in detail [4] .

CHAPTER 4

An example of participating forward

4.1 Pricing of the Contract

In this section, we will price and analyze a participating forward contract written by a Turkish Bank. It is delivered with zero cost. The terms of the contract is on the Table 4.1 :

Table 4.1: Contract Terms

Underlying	USD/TRY
Trade Date	14.11.2011
Expiration Date	15.05.2012
Spot Rate	1,7800
Strike Rate	1,9075
Put Nominal	$ 1,000,000
Call Nominal	$ 2,000,000
Volatility	14 %

The terms translate to

$$\text{Spot rate} = \mathrm{S},$$

$$\text{Strike rate} = \mathrm{K},$$

$$\text{Volatility} = \sigma$$

in the Garman-Kohlhagen model prices. Data for domestic and foreign risk free rates is derived from Bloomberg[1]. We take mean of the interest rates during the life of the option.

[1] http://www.bloomberg.com/enterprise/datasolutions/referencedata/

Spot rates after trade date are derived from Reuters[2]. As we explained in Chapter 2, the values of call and put components of the contract are calculated. Then, according the positions in call and put options, the price of the contract is calculated. In this example, we take the short position in call and long position in put options. We calculated the prices weekly, Table 4.2 shows the results:

[2] http://thomsonreuters.com/productsservices/financial/financialproducts/a-z/elektronrealtime

Table 4.2: Data and Prices

Date	Rem Day[3]	Spot	Dom Int(%)[4]	For Int(%)[5]	Call P[6]	Call D[7]	Put P[8]	Put D[9]	Cont P[10]	Cont D[11]
14.11.11	182	1,78	9,93	2,40	0,044395	-0,3805	0,104833	-0,6077	16,042,37 TL	-1,3688
21.11.11	175	1,8424	9,93	2,40	0,069819	-0,5107	0,071070	-0,4780	-68,568,10 TL	-1,4994
28.11.11	168	1,8637	9,93	2,40	0,078507	-0,5524	0,061175	-0,4368	-95,840,19 TL	-1,5415
05.12.11	161	1,8302	9,93	2,40	0,058742	-0,4705	0,077042	-0,5191	-40,442,17 TL	-1,4601
12.12.11	154	1,8569	9,93	2,40	0,069424	-0,5263	0,063792	-0,4638	-75,056,38 TL	-1,5163
19.12.11	147	1,8888	9,93	2,40	0,084495	-0,5951	0,049762	-0,3954	-119,228,82 TL	-1,5856
26.12.11	140	1,8924	9,93	2,40	0,083714	-0,5993	0,047900	-0,3916	-119,528,09 TL	-1,5903
02.01.12	133	1,8859	9,93	2,40	0,076918	-0,5795	0,050040	-0,4119	-103,797,24 TL	-1,5709
09.01.12	126	1,8718	9,93	2,40	0,066074	-0,5391	0,055680	-0,4527	-76,467,78 TL	-1,5310
16.01.12	119	1,8536	9,93	2,40	0,053855	-0,4846	0,064033	-0,5077	-43,677,95 TL	-1,4769
23.01.12	112	1,8291	9,93	2,40	0,040135	-0,4094	0,077161	-0,5834	-3,110,03 TL	-1,4021
30.01.12	105	1,7905	9,93	2,40	0,024107	-0,2948	0,102012	-0,6984	53,798,67 TL	-1,2880
06.02.12	98	1,7642	9,93	2,40	0,015359	-0,2170	0,121963	-0,7766	91,244,70 TL	-1,2107
13.02.12	91	1,7560	9,93	2,40	0,011961	-0,1841	0,129300	-0,8100	105,378,11 TL	-1,1782
20.02.12	84	1,7424	9,93	2,40	0,008209	-0,1413	0,141666	-0,8533	125,248,86 TL	-1,1358
27.02.12	77	1,7721	9,93	2,40	0,011464	-0,1885	0,117975	-0,8065	95,046,50 TL	-1,1835
05.03.12	70	1,7720	9,93	2,40	0,009646	-0,1705	0,118853	-0,8250	99,561,60 TL	-1,1660
12.03.12	63	1,7961	9,93	2,40	0,012275	-0,2125	0,100084	-0,7834	75,533,12 TL	-1,2084
19.03.12	56	1,8058	9,93	2,40	0,012110	-0,2189	0,092852	-0,7774	68,631,67 TL	-1,2153
26.03.12	49	1,7945	9,93	2,40	0,007773	-0,1633	0,102378	-0,8336	86,832,16 TL	-1,1601
02.04.12	42	1,7808	9,93	2,40	0,004170	-0,1051	0,115048	-0,8922	106,707,81 TL	-1,1024
09.04.12	35	1,7973	9,93	2,40	0,004336	-0,1160	0,101373	-0,8817	92,702,06 TL	-1,1137
16.04.12	28	1,8011	9,93	2,40	0,002991	-0,0932	0,098856	-0,9050	92,874,07 TL	-1,0914
23.04.12	20	1,7884	9,93	2,40	0,000761	-0,0334	0,112310	-0,9653	110,787,80 TL	-1,0321
30.04.12	14	1,7573	9,93	2,40	0,000028	-0,0020	0,144909	-0,9971	144,853,12 TL	-1,0011
07.05.12	7	1,7652	9,93	2,40	0,000000	0,0000	0,139641	-0,9995	139,640,79 TL	-0,9996
14.05.12	0	1,8072	9,93	2,40	0,000000	0,0000	0,100300	-1,0000	100,300,00 TL	-1,0000

On the Table 4.2, domestic and foreign interest rates are given in simple compounding. However, those rates are converted into continuous compound as in the GK model, continiuous coumpounded interest rates are used. Call, put prices and delta values are obtained from the GK model that we derived in the previous chapter. All the calculations are made in Microsoft Excel. The codes for the GK model is obtained from the book ”The complete guide to option pricing formulas” [6]. In the book, the writer puts VBA(Visual Basic) codes for many types of option prices and greeks. We used the Garman-Kohlhagen VBA codes, embeded them into Microsoft Excel. Then the formula “=GarmanKohlhagen” appears in formula box in Excel with inputs spot rate, strike rate, time to expiration, domestic interest rate, foreign interest rate and volatility. Plugging the inputs into formula, we obtained the prices and delta values for each week. After getting the prices and delta values of call and put options, the contract

[3] Remaining Day

[4] Domestic Interest Rate

[5] Foreign Interest Rate

[6] Call Price

[7] Call Delta

[8] Put Price

[9] Put Delta

[10] Contract Price

[11] Contract Delta

prices are calculated as:

$$\text{Contract Price} = \text{Put Price}.n_2 - \text{Call Price}.n_1,$$

where n_1 and n_2 are the nominal values of call and put options respectively. Similarly, delta values are of the contract are calculated as:

$$\text{Contract Delta} = \text{Put Delta} - 2.\text{Call Delta}.$$

For instance, on date 14.11.11, contact price and delta value occur as follows:

$$\begin{aligned}
\text{Contract Price} &= -2000000.0,044395 + 1000000.0,104833 \\
&= 16042,37, \\
\text{Contract Delta} &= -0,6077 - 0,3805.2 \\
&= -1,3688.
\end{aligned}$$

It is clear from the Table 4.2 that the contract is not worthless at the trade date. This is important because a positive valued option(or combinations of options) will create a position to manage. In this example, the whole contract size is $ 3.000.000 which is a huge amount even for banks. So, it is significant to hedge this position.

4.2 Hedging of the Contract

On Table 4.2 the total delta is calculated for the contract. Delta gives the change in value of the contract when the underlying, USDTRY here, goes up by one unit [14]. To have a long position in call option with delta 0.6 means the value of the option will arise 0.6 unit whenever the underlying increase in price by one unit. We wish to get delta neutral position to prevent losses from the small changes in USDTRY rate. We rebalance our position by selling/buying USDTRY currency each weeek. By saying buy/sell USDTRY currency, we mean buy/sell USD against TRY. Table 4.3 reveals the results:

Table 4.3: Hedge Portfolio

Contract Delta	USD/TRY Pos	Change in Pos	Cost of Change	Cumulative Cost of Change	Nuber of puts to be the zero cost
-1,3688	1.368.755,61	1.368.755,61	2.436.384,98	2.436.384,98 TL	0,423486082
-1,4994	1.499.360,49	130.604,88	240.626,43	2.681.668,41 TL	0,982399809
-1,5415	1.541.494,82	42.134,34	78.525,77	2.765.320,03 TL	1,283331201
-1,4601	1.460.073,75	-81.421,07	-149.016,85	2.631.588,92 TL	0,762468354
-1,5163	1.516.318,16	56.244,41	104.440,24	2.731.040,17 TL	1,088291164
-1,5856	1.585.556,52	69.238,37	130.777,43	2.867.037,81 TL	1,697989022
-1,5903	1.590.250,79	4.694,27	8.883,44	2.881.401,42 TL	1,747672862
-1,5709	1.570.936,61	-19.314,18	-36.424,61	2.850.484,44 TL	1,537148933
-1,5310	1.530.965,10	-39.971,51	-74.818,68	2.781.114,28 TL	1,186677458
-1,4769	1.476.944,53	-54.020,56	-100.132,52	2.686.297,70 TL	0,841059602
-1,4021	1.402.108,91	-74.835,63	-136.881,85	2.554.550,55 TL	0,520152934
-1,2880	1.288.041,45	-114.067,46	-204.237,78	2.355.195,63 TL	0,236312005
-1,2107	1.210.691,50	-77.349,95	-136.460,77	2.223.236,67 TL	0,125933698
-1,1782	1.178.223,34	-32.468,16	-57.014,09	2.170.472,17 TL	0,092504431
-1,1358	1.135.847,05	-42.376,30	-73.836,46	2.100.784,44 TL	0,057944421
-1,1835	1.183.501,87	47.654,82	84.449,11	2.189.249,08 TL	0,097174038
-1,1660	1.165.966,37	-17.535,50	-31.072,91	2.162.360,79 TL	0,081155721
-1,2084	1.208.397,32	42.430,95	76.210,23	2.242.704,24 TL	0,122651455
-1,2153	1.215.304,69	6.907,37	12.473,33	2.259.464,36 TL	0,130422816
-1,1601	1.160.078,69	-55.226,00	-99.103,06	2.164.680,13 TL	0,075922992
-1,1024	1.102.364,70	-57.713,99	-102.777,08	2.066.040,71 TL	0,036246447
-1,1137	1.113.714,29	11.349,60	20.398,63	2.090.388,45 TL	0,042768895
-1,0914	1.091.369,86	-22.344,44	-40.244,57	2.054.139,54 TL	0,030258083
-1,0321	1.032.116,18	-59.253,68	-105.969,28	1.952.096,62 TL	0,006778335
-1,0011	1.001.088,74	-31.027,44	-54.524,52	1.901.303,42 TL	0,000192498
-0,9996	999.589,72	-1.499,02	-2.646,08	1.902.291,57 TL	0
-1,0000	1.000.000,00	410,28	741,47	1.906.669,15 TL	0

On above table, we take the replication point of view. We replicate the option combination weekly by trading the USDTRY currency . For each contract delta value, we readjust the position in USDTRY currency so that we have zero delta value at the end of each rebalance act. Weekly USDTRY position is just -1000000.Contract Delta. Change in the hedge portfolio position is the USDTRY position difference between the current week and the previous one. For instance, on date 21.11.11, change in the position is

$$1499360,487 - 1368755,607 = 130.604,8797\text{TL}.$$

Rebalancing the position each week either charges or leaves profit. If the next USDTRY position is greater than the preceding, rebalancing act charges us. If it is smaller, then it gives some yield. Cost/Gain of change is calculated as the change in the position multiplied by the spot rate at that date. For example, on date 21.11.11, the cost of change in the position is:

$$130604,8797.1,8424 = 240.626,43\text{TL},$$

while the gain in the position change on date 05.12.11 is

$$-81421,0747.1,8302 = -149.016,85\text{TL},$$

where the spot rate on 21.11.11 is 1,8424 and on 05.12.11 is 1,8302. We reveal the costs with positive numbers in the table, so the profits are negative. The cumulative cost/profit of change in the position is calculated as follows:

Cumulative cost = Current cost of change + Previous cost of change shifted today.

By shifting, we mean the today's value of previous cost. For example, on date 28.11.11, the cumulative cost of change in the position is

$$240.626,430e^{\frac{0,0993}{52}} + 78.525,77 = 319.612,139\text{TL}.$$

The ratio that makes the contract zero cost is the number of put options for each call and is simply the price of the call divided by the price of put. For example, on date 28.11.11, the ratio is

$$\frac{0,078507}{0,061175} = 1,283331201.$$

At the date of expiration, the put option is exercised while the call option is out-the money. Since we buy the put option and sell the call, the market went in favor of us. At the exercise date, we have \$ 1.000.000 in our hedge portfolio as expcted. We sell \$1.000.000 at a rate of 1.9075 to get 1.907.500 TL. The cumulative cost of changes in position is 1.906.669 TL. The difference 830 TL represents the replication cost/gain. The gap between the cumulative cost/gain of changes in hedge portfolio position and the option position cost/gain is called tracking error [2]. If we rebalance our position frequently, the tracking error will decrease [5]. Continuous rebalancing act gives a perfect delta hedge. On the other hand, this example shows how important hedging is. In the actual contract, the bank(the counterparty) was long in call and short in put. So, if the bank attempted to hedge, it would have to take exactly the opposite positions listed in the tables 4.2 and 4.3. It costs only 830 TL if banks preserves itself from unfovurable moves. If the bank didn't hedge the position, it would cost the price of the contract at expiration date, 100.300 TL.

When pricing, we assumed that the domestic and foreign interest rates are constant during the life of the contract. This is an assumption of the Black-Scholes model, though in practice, the risk free rate is not constant. To reveal the difference, we also calculate the prices and the delta values of the contract with the up-to-date interest rates. The rates are drawn from Bloomberg. Table 4.4 gives the results:

Table 4.4: Data and prices with up-to-date interest rates

Date	Rem Day	Spot	Dom Int(%)	For Int(%)	Call P	Call D	Put P	Put D	Cont P	Cont D
14.11.2011	182	1,78	9,57	3,02	0,041386	-0,36173	0,110122	-0,62354	27.350,26 TL	-1,347
21.11.2011	175	1,8424	9,58	3,24	0,064904	-0,48653	0,076097	-0,4983	-53.711,62 TL	-1,47136
28.11.2011	168	1,8637	10	3,64	0,073189	-0,52771	0,065536	-0,45597	-80.841,01 TL	-1,51139
05.12.2011	161	1,8302	9,89	3,53	0,054539	-0,44709	0,081909	-0,53772	-27.169,88 TL	-1,43191
12.12.2011	154	1,8569	10,05	3,51	0,06546	-0,50617	0,067356	-0,47938	-63.564,07 TL	-1,49172
19.12.2011	147	1,8888	10,68	3,38	0,083025	-0,58801	0,05046	-0,39869	-115.589,32 TL	-1,57471
26.12.2011	140	1,8924	10,62	3,19	0,082881	-0,59508	0,048203	-0,39295	-117.558,57 TL	-1,5831
02.01.2012	133	1,8859	10,97	3,05	0,077873	-0,58328	0,049013	-0,40584	-106.733,84 TL	-1,57239
09.01.2012	126	1,8718	11,63	3,88	0,066059	-0,53803	0,055107	-0,44891	-77.010,00 TL	-1,52498
16.01.2012	119	1,8536	10,83	3,34	0,053415	-0,48156	0,064176	-0,50778	-42.654,72 TL	-1,47091
23.01.2012	112	1,8291	10,41	2,9	0,039956	-0,40791	0,077239	-0,58336	-2.673,51 TL	-1,39917
30.01.2012	105	1,7905	9,44	2,68	0,023057	-0,28516	0,104765	-0,70726	58.651,54 TL	-1,27758
06.02.2012	98	1,7642	9,25	2,07	0,015077	-0,21397	0,12328	-0,78054	93.125,77 TL	-1,20848
13.02.2012	91	1,756	9,07	1,83	0,011799	-0,18219	0,130375	-0,81329	106.777,21 TL	-1,17768
20.02.2012	84	1,7424	8,81	1,76	0,008003	-0,13844	0,143378	-0,85755	127.371,65 TL	-1,13444
27.02.2012	77	1,7721	8,36	2,83	0,010242	-0,17233	0,124008	-0,8218	103.523,27 TL	-1,16646
05.03.2012	70	1,772	9,61	1,63	0,009921	-0,17442	0,117633	-0,82248	97.792,03 TL	-1,17132
12.03.2012	63	1,7961	8,57	1,66	0,01196	-0,20827	0,101577	-0,78889	77.655,86 TL	-1,20544
19.03.2012	56	1,8058	8,76	1,89	0,011785	-0,21439	0,094247	-0,78274	70.677,31 TL	-1,21152
26.03.2012	49	1,7945	9,53	1,72	0,007895	-0,16533	0,10183	-0,83238	86.040,70 TL	-1,16305
02.04.2012	42	1,7808	9,18	1,59	0,004196	-0,10565	0,114945	-0,89253	106.552,81 TL	-1,10384
09.04.2012	35	1,7973	9,38	2,65	0,00419	-0,11276	0,102563	-0,88474	94.182,84 TL	-1,11025
16.04.2012	28	1,8011	9,51	2,4	0,002943	-0,09192	0,099369	-0,90626	93.484,06 TL	-1,0901
24.04.2012	20	1,7884	9,68	1,36	0,000788	-0,03443	0,111578	-0,96483	110.002,67 TL	-1,03369
30.04.2012	14	1,7573	8,97	1,25	2,82E-05	-0,00202	0,144791	-0,9975	144.734,22 TL	-1,00154
07.05.2012	7	1,7652	9,21	1,15	3,56E-07	-4,6E-05	0,139467	-0,99974	139.466,43 TL	-0,99983
14.05.2012	0	1,8072	9,43	1,44	0	0	0,1003	-1	100.300,00 TL	-1

Analyzing Table 4.4 shows us that the contract price and delta values during the life of the contract differ from the previous values where the interest rates are assumed to be constant. Yet, at expiration date, the value of the contract is same. So, for a speculator who buy this contract and who does not hedge himself is left with 100.300 TL again. On the other hand, hedging is more expensive than we did in the case of the contract with stagnant interests. Table 4.5 demonstrate the results:

Table 4.5: Hedge portfolio with up-to-date interest rates

Contract Delta	USD/TRY Pos	Change in Pos	Cost of Change	Cumulative Cost of Change
-1,3470	1.347.002,74	1.347.002,74	2.397.664,87 TL	2.436.384,98 TL
-1,4714	1.471.361,31	124.358,57	229.118,23 TL	2.670.160,21 TL
-1,5114	1.511.386,32	40.025,01	74.594,61 TL	2.749.858,68 TL
-1,4319	1.431.907,41	-79.478,91	-145.462,29 TL	2.609.652,57 TL
-1,4917	1.491.722,28	59.814,87	111.070,23 TL	2.725.710,99 TL
-1,5747	1.574.708,43	82.986,15	156.744,24 TL	2.887.665,27 TL
-1,5831	1.583.102,85	8.394,42	15.885,59 TL	2.909.070,46 TL
-1,5724	1.572.387,20	-10.715,65	-20.208,64 TL	2.894.422,33 TL
-1,5250	1.524.976,52	-47.410,67	-88.743,30 TL	2.811.211,55 TL
-1,4709	1.470.909,06	-54.067,47	-100.219,46 TL	2.716.365,55 TL
-1,3992	1.399.174,67	-71.734,39	-131.209,37 TL	2.590.348,35 TL
-1,2776	1.277.583,56	-121.591,11	-217.708,87 TL	2.377.590,77 TL
-1,2085	1.208.484,61	-69.098,95	-121.904,36 TL	2.260.231,03 TL
-1,1777	1.177.683,04	-30.801,57	-54.087,56 TL	2.210.463,76 TL
-1,1344	1.134.435,26	-43.247,78	-75.354,93 TL	2.139.334,01 TL
-1,1665	1.166.457,61	32.022,34	56.746,79 TL	2.200.170,01 TL
-1,1713	1.171.324,79	4.867,19	8.624,65 TL	2.213.000,15 TL
-1,2054	1.205.435,11	34.110,31	61.265,54 TL	2.278.495,70 TL
-1,2115	1.211.524,34	6.089,23	10.995,94 TL	2.293.846,85 TL
-1,1630	1.163.047,09	-48.477,26	-86.992,44 TL	2.211.238,97 TL
-1,1038	1.103.840,36	-59.206,72	-105.435,33 TL	2.110.030,28 TL
-1,1103	1.110.253,23	6.412,87	11.525,85 TL	2.125.589,33 TL
-1,0901	1.090.102,97	-20.150,26	-36.292,64 TL	2.093.359,63 TL
-1,0337	1.033.687,80	-56.415,16	-100.892,88 TL	1.996.468,08 TL
-1,0015	1.001.544,65	-32.143,15	-56.485,17 TL	1.943.799,04 TL
-0,9998	999.826,24	-1.718,41	-3.033,34 TL	1.944.481,16 TL
-1,0000	1.000.000,00	173,76	314,02 TL	1.948.511,94 TL

CHAPTER 5

Conclusion and Outlook

The fast paced life of financial markets requires innovations continuously as needs, circumstances and expectations are changing day by day. For this reason, new financial instruments are needed to spin the wheel of the markets. In this thesis, we focused on one of them, the participating forward contract. We analyzed its pricing and hedging techniques and applied those methods on a traded contract to make the concept concrete. The Garman-Kohlhagen(GK) model is the core part of all these analysis. We applied it to an FX option combination so that investors could adapt the techniques to other contracts consisting of FX option combinations such as par forwards, corridor options and so on. Of course, other models to price FX options could be used to get a pricing formulae for the FX based contracts. In the actual contract in Chapter 4, for example, the bank created a pricing method using Monte Carlo simulation embedded into the Black-Scholes formula. The bank simulated the USDTRY rate and plugged it into the formula. This is a good approximation to get the value of the contract. Yet, it does not provide a hedge ratio. So, as a discussion, we suggest the use of GK model for single FX options and FX options combinations.

Now, Turkey is being prepared to open VIOP (Vadeli Islem ve Opsion Piyasasi) in ISE(Istanbul Stock Exchange). Regulations are published in the official gazette of Republic of Turkey [1]. In that market, futures and option contracts on single stocks will be traded as a first step. We expect that the contracts will be diversified in time. Varied financial products are important to advance financial markets in developing countries as Turkey. At this point, we hope that our thesis will be helpful to understand the pricing and hedging of currency options, to manage FX risks and to apply to other FX-based products as a representative material.

REFERENCES

[1] Imkb vadeli islem ve opsiyon piyasasi yonetmeligi, 2012.

[2] D. Bertsimas, L. Kogan, and A.W. Lo. Hedging derivative securities and incomplete markets: An-arbitrage approach. *Operations Research*, 49(3):372–397, 2001.

[3] N. Biger and J. Hull. The valuation of currency options. *Financial Management*, pages 24–28, 1983.

[4] M.B. Garman and S.W. Kohlhagen. Foreign currency option values. *Journal of International Money and Finance*, 2(3):231–237, 1983.

[5] ER Grannan and GH Swindle. Minimizing transaction costs of option hedging strategies. *Mathematical finance*, 6(4):341–364, 1996.

[6] E.G. Haug. *The complete guide to option pricing formulas*, volume 2. McGraw-Hill New York, 2007.

[7] D. Lamberton and B. Lapeyre. *Introduction to stochastic calculus applied to finance*. Chapman & Hall/CRC, 2007.

[8] R.C. Merton. Theory of rational option pricing. *The Bell Journal of Economics and Management Science*, pages 141–183, 1973.

[9] G.J. Millman. Futures and options markets. *The Concise Encyclopedia of Economics*, page 1, 2008.

[10] J. Norris, T. Bell, and A. Gaskill. *Mastering the Currency Market: Forex Strategies for High and Low Volatility Markets*. McGraw-Hill Companies,Incorporated, 2009.

[11] J. Orlin Grabbe. The pricing of call and put options on foreign exchange. *Journal of International Money and Finance*, 2(3):239–253, 1983.

[12] G. Poitras. The early history of option contracts. In *Vinzenz Bronzin's Option Pricing Models. Exposition and Appraisal. Springer Verlag*, 2009.

[13] C.W. Smith. Option pricing: A review. *Journal of Financial Economics*, 3(1):3–51, 1976.

[14] S. Vine. *Options: trading strategy and risk management*, volume 288. Wiley, 2011.