THE ESTIMATION OF ADOPTED MORTALITY AND MORBIDITY RATES USING MARKOV MODEL AND PHASE TYPE LAW: TURKISH CASE

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submitted by **FULYA AKAT** in partial fulfillment of the requirements for the degree of **Master of Science in Department of Financial Mathematics, Middle East Technical University** by,

Prof. Dr. Bülent Karasözen
Director, Graduate School of **Applied Mathematics** ________________

Assoc. Prof. Dr. Ali Devin Sezer
Head of Department, **Financial Mathematics** ________________

Assoc. Prof. Dr. A. Sevtap Selcuk Kestel
Supervisor, **Actuarial Sciences Department, METU** ________________

Examining Committee Members:

Assist. Prof. Dr. Şule Şahin
Actuarial Sciences Department, Hacettepe University ________________

Assoc. Prof. Dr. Sevtap Selcuk Kestel
Actuarial Sciences Department, METU ________________

Assoc. Prof. Dr. Yeliz Yolcu Okur
Financial Mathematics Department, METU ________________

Date: ________________

I hereby declare that all information in this document has been obtained and presented in accordance with academic rules and ethical conduct. I also declare that, as required by these rules and conduct, I have fully cited and referenced all material and results that are not original to this work.

Name, Last Name: FULYA AKAT

Signature :

ABSTRACT

THE ESTIMATION OF ADOPTED MORTALITY AND MORBIDITY RATES USING MARKOV MODEL AND PHASE TYPE LAW: TURKISH CASE

AKAT, Fulya
M.S., Department of Financial Mathematics
Supervisor : Assoc. Prof. Dr. A. Sevtap Selcuk Kestel

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Mortality estimation has an important role in determining future population structures and making economical decisions. However, mortality measures do not provide sufficient information about populations. Because of this reason, morbidity estimation of chronic diseases and disabilities has become more important. In this study, firstly Turkish Mortality Tables, such as TRSH 2010, TRH 2010 and SGK 2008, and CSO1980 are adopted to Markov Model to determine mortality rates with respect to phase type distribution for the purpose of justifying if these tables represent the Turkish population structure well. After that, ischemic heart disease (IHD), which is the most fatal illness in the Western countries, mortality and morbidity rates are found and analysed by utilizing the same approach. Using one absorbing state, Markov Model gives the opportunity of calculating time until absorbing by phase type distribution. We determine the time until absorbing of diseases and the time until death by this distribution. Moreover, ϵ factor in the phase type distribution is calculated. By the help of this factor, new risk factors can be applied to the model by modification.

Keywords : Markov Process, Phase Type Distribution, Ischemic Heart Disease, Morbidity, Mortality

ÖZ

TÜRKİYE DURUMU İÇİN MARKOV MODEL VE FAZ TİPİ DAĞILIM KULLANARAK UYARLANMIŞ ÖLÜMLÜLÜK VE HASTALIĞA YAKALANMA OLASILIKLARININ TAHMİNİ

AKAT, Fulya
Yüksek Lisans, Finansal Matematik Bölümü
Tez Yöneticisi : Doç. Dr. Sevtap Selcuk Kestel

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Populasyonun gelecekteki durumumun tahmini ve ekonomik kararlar için ölümlük değerinin tahmini önem taşımaktadır. Ancak ölümlülük değerleri populasyon için yeterli bilgiyi içermemektedir. Bu nedenlerle kronik hastalığa yakalanma olasılıkları ve sakatlanma olasılıkları daha önemli olmaktadır. Bu çalışmada öncelikle TRSH 2010, TRH 2010 ve SGK 2008 Türkiye Ölümlülük Tabloları ve CSO 1980 Ölümlülük Tablolarının ölümlülük oranlarını faz tipi dağılım ile göstermek için Markov modele adapte edilmiştir. Analizin amacı tabloların Türkiyenin demografik yapısını temsil edip etmediğini sorgulamaktır. Daha sonra aynı model kullanılarak batı ülkelerinde en ölümcül hastalık olan iskemik kalp hastalığının ölümlülük ve hastalığa yakalanma olasılıkları hesaplanmış ve analiz edilmiştir. Bir soğurucu adımlı Markov Model kullanmak bu soğurucu adıma kadar geçen sürenin faz tipi dağılım ile hesaplanması fırsatını vermektedir. Hastalığa maruz kalana kadar geçen süre ve ölüme kadar geçen süre faz tipi dağılım ile gösterilmiştir. Ayrıca ϵ faktör açıklanmıştır. Bu faktör yardımı ile yeni riskler modele modifiye edilerek eklenmektedir.

Anahtar Kelimeler : Markov Model, Faz Tipi Dağılım, İskemik Kalp Hastalığı, Hastalığa Yakalanma Olasılığı , Ölüm Olasılığı

To My Family

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TABLE OF CONTENTS

LIST OF FIGURES

LIST OF TABLES

LIST OF ABBREVIATIONS

ABBRV	Abbreviation
α	Initial distribution
$\mathbb{P}$	Probability
λ_{ij}	Force of transition from state i to state j
p_{ij}	Transition probability from state i to state j
τ	The time until death
q_i^ϵ	Modified mortality rates
$f(.)$	The density function
$S(.)$	The survival function
$F(.)$	The cumulative distribution function
$p_{t,t+1}$	Probability of transition over the cycle t to $t+1$
$P(t)$	The transition matrix
$P(s)$	The transition matrix with phase type notation
p_{ij}^s	Probability of transition from state i to state j over the cycle s
CSO 1980	Commissioners Standard Ordinary Mortality Table 1980
TRSH 2010	Turkish Insured Mortality Table 2010
TRH 2010	Turkish Mortality Table 2010
SGK 2008	Social Security Institute Mortality Table 2008
IHD	Ischemic Heart Disease
SSI	Social Security Institute
ABPRS	Address Based Population Registration System
TurkStats	Turkish Statistical Institute
TV	Table Value
EV	Estimated Value
MSE	Mean Square Errors

CHAPTER 1

Introduction

Mortality is useful in determining population health status, economical decisions and projections of the future population compositions. Furthermore, it is used for making a comparison of the survivability of people with other countries. Improvement of life standards and health conditions have caused a change in life expectancy. Mortality measures do not always provide sufficient information about the population, as the collection of the complete data may take over 100 years. Because of these reasons, morbidity estimation, especially that of chronic diseases and disabilities has become more important [15].

Addition to this, morbidity and mortality rates have important effects on actuarial valuation. Its estimation and efficiency are significant in economical decisions and reserves. Morbidity is also important for Social Security Institution (SSI) and insurance companies. The estimation of morbidity and mortality supplies a guiding principle to decision makers for the calculation of premiums and the estimation of reserves [20].

However, the estimation of the mortality and morbidity rates require mostly a long term collection of data, preferably in closed population. This is not achievable always because of the conditions they require. Many tables are found as resuls of estimation techiques.

The well-known models in estimating the mortality rates are DeMoivre, Gompertz, Makeham and Weibull, which depends on historical realizations of populations [6]. Additionally, recent developments, such as Lee-Carter, the mortality rate using a self-generating set of iterations, can be used when the past information related a population does not fit to the models above.

Kaplan-Meier, a non-parametric model, is also used to estimate morbidity rates. Moreover, incidence is a measure of the probability of the occurrence of a given medical condition in a population within a specified period of time. Prevalence represents the proportion of a population found to have a condition of a specific disease in morbidity studies [15]. A novel approach to estimate mortality and morbidity rates proposed by Lin (2007) and Kay (1986) is Markov model and phase type distribution. This method implements the stochastic structure into the estimation of the rates, as well as, enables researchers to calculate adopted probabilities. Additionally it allows introducing new states, especially, suc as, couses of the death or illnesses. As an example, we can distinguish with which probabilities one can swich to one phase to another in diseases, such as IHD.

Ischemic heart disease (IHD) is the most fatal disease especially, in the Western countries. It is estimated that, 20% of the causes of the total death come from this disease [9]. IHD is taken into account to determine the mortality and morbidity rates by using Markov process with an absorbing state, further to estimating time to failure by phase type distribution. Insufficient supply of blood and oxygen to the myocardium is the condition of ischemic heart disease. This disease occurs when there is an inequality between myocardial oxygen supply and demand [9].

1.1 Literature Survey

Newell explains methods and models in demography [25]. Also Heligman and Pollard propose a mathematical expression for the age pattern of mortality tables by applying to Australian National Mortality data. This model allows mortality comparisons regarding age and sex both among countries and within the same country over time [12]. These studies are used in this thesis to develop the methodology on estimation of adopted mortality and morbidity rates.

Additional to those models, Markov process can be utilized to determine mortality and morbidity rates. Markov process is a stochastic process satisfying the Markov property, which is held when the next step of the process depends on only the current state; not past steps. It is also known as 'memoryless property' in the literature [19]. This process has a countable set of states and there is a transition between states. The last state of Markov process can be defined as the absorbing state. A transition from a state to an absorbing state ends the process.

Phase type distribution is introduced as the distribution of absorbing times of the Markov process in the earlier studies [3]. The time until absorbing is defined as to follow phase type distribution [11]. This distribution reveals the advantages of the modification with error when absorbing rate has a constant change.

Steinsaltz and Evans explain the use of Markov process theory in mortality estimation. This study shows an appropriate choice of initial distribution [27]. Titman and Sharples indicate that Pearson-type goodness of fit test is not applicable to an absorbing state Markov process. Their study introduces modification of Pearson-type test to application of absorbing state cases [30]. Ellis gives some definitions and properties of two states of Markov process [10]. Wand and Pham define neutral ageing with degeneration of normal ageing. This study introduces a model with hidden Markov process [31]. Winkler introduces a theory of continuous Markov process [32]. Collins and Huzurbazar [8] review multi-state models, focusing on time-homogeneous semi-Markov processes, and then describe the statistical flowgraph framework, which comprises analysis methods and algorithms for computing quantities of interest such as the distribution of first passage times to a terminal state. They also use semi-Markov process and present uncensored and censored data cases separately [8].

Hubbard et al. [16] propose models in which the time scale of a non-homogeneous Markov process is transformed to an operational time scale on which the process is homogeneous. They develop a method for jointly estimating the time transformation and the transition intensity matrix for the time transformed homogeneous process. They assess maximum likelihood estimation using the Fisher scoring algorithm via simulation studies and compare performance of their method to homogeneous and piecewise homogeneous models. They apply this method to a study of delirium progression in a cohort of stem cell transplantation recipients and show that their method identifies temporal trends in delirium incidence and recovery [16].

Phase type distribution is used in the risk theory by Blandt at which basic definition of the distribution is given and phase type renewal theory is introduced [3]. Faddy uses this distribution to apply to some industrial data for the purpose of determining the failure times [11]. Bobbio et al. present a detailed study on discrete phase type distribution and its acyclic subclass. They use this model with canonical form and by the help of the resorting to the canonical form, minimal squared coefficient of variation on the mean is established [4].

Markov model is used to determine and analyse geriatric and elderly patients' home care and their behaviours. McClean and Millard use this model to determine the relationship between hospital and community cares for the older patients [24]. This study is extended by Taylor et al. Data collection of geriatric patients is applied to continuous time Markov model for the purpose of analysing patient behavior. Their approach is applied to the whole system of geriatric care not only to look at the time patients spend in the hospital but also the subsequent time patients spend in the community. The model is to fit using the method of maximum likelihood [29]. Xie et al. use continuous time Markov model to analyse transition within and between residential and nursing home cares. A procedure is introduced to determine the structure of the model and estimating parameters by maximum likelihood [33].

Markov model is also used to describe aging theory in the following studies. Stinsaltz et al. illustrate the theory and application of Markov models of aging. They use this model for defining the randomness of aging. They describe aging with stochasticity, they separate frail people from others [28]. Lisnianski and Frenkell apply non-homogeneous Markov model for reliability measures of aging [23]. Chacko et al. also comply with the Markov model to describe aging. They use this model with a multistate system having $m+1$ states $0,1,\ldots,m$ where *'0'* is the best state and *'m'* defines the worst state. The identification of the failure rate model of the first passage time distribution of a semi-Markov system is discussed [7].

Keyfitz and Rogers show the standard life contingency formulas with matrix analogues. The purpose of this notation is finding simple solutions to multiple contingency problems. Different risk factors are included in the analysis including transition from employment and unemployment, from marriage and divorce, from illness and healthy states and ect [21].

Yang uses the non-homogeneous Markov Model in the survival analysis illustrating comparison of Fix and Neyman (F-N) competing risks model with the Kaplan-Meier formulations. This comparison includes risks of recovery and calculation of patients' survival probability. There is an extantion to F-N model to the non-homogenous Markov process. This study introduces the explicit form of the survival probability to solve the system of Kolmogorov Equations for non-homogeneous Markov process [34].

Additionally, Jackson et al. use multistate Markov model for disease progression with classification error. This model based on Markov processes which is defined as a well-established method of estimating rates of transition between stages of disease [18]. Hoogenveen et al. introduce Markov-type multi-state model for deal with effects of changes in risk factors for chronic diseases on morbidity and mortality. This model includes risk factors and disease stages. Also heterogeneity and mortality risks are included to this model [14].

Boshuizen et al. developed model for Health Impact Assessment with Markov process to include the time required to add the impact of the risk factors by using micro simulation. Matrix transition probabilities contain the probabilities of changing from each current state to each possible next state. Diseases included in this model were ischemic heart disease, stroke, diabetes, colorectal cancer, esophageal cancer, and breast cancer. Diabetes was considered to be a causal disease for ischemic heart disease and stroke [5]. Ocana-Riola propose the non-homogenous Markov model for biomedical data analysis. The method has been applied to analyse breast cancer data [26]. In addition, survival model is adopted to the animal case in Hosgood study. In this study, survival time is described by the Markov model [15].

Lin and Liu use Markov process and phase type distribution in the mortality analyses. They add a health index called physiological age to this model. Aging is described as transition from one physiological age to another one. This model has an absorbing state. Transition from one state to absorbing state is described as a death or fatal disease. It is assumed that physiological age develops only in one direction. The development of health conditions is not taken into account in this study. Time of death is described by phase type distribution. Aging-independent hazard rate is included in the model with a background rate and a behavior-related accident rate. The model is fit to Swedish population cohort data. They emphasise that the fitting results are satisfactory [22]. Kay uses this process to analyse a disease with its stages. There is a series of states, the last one of which represents death and other ones represent disease periods taking the recovery of the disease into account [20]. Jones apply Markov Process to valuation to calculate the actuarial probabilities. He uses constant forces of transition rates [19]. Herrick et al. apply Markov Chain Monte Carlo model to analyse morbidity, mortality, illness duration and cost of salmonella disease. Transition rates were drawn from a meta-analysis. Pearson chi-squared goodness-of-fit tests were used to test approach similarity and model fit [13].

1.2 Scope of the Thesis

The aim of this thesis is to implement Markov model and phase type distribution to estimate adopted mortality rates for existing and currentlu used mortality tables and three state morbidity rates for selected disease.

This study aims to answer the following questions: How effective is to use the phase type distribution in estimating mortality rates for achieving a certain accuracy based on Markov model process assumptions? How informative to implement phase type distribution to determine a three state Markov model to estimate the morbidity rates for a chronic disease, such as IHD.

To achieve this, an application is performed on Turkish demographic and disease statistics collected from Social Security Institute (SSI) between 2007 and 2009 that years data collection is available. Compared to other diseases, the IHD is the most commonly observed, economically expensive and lethal. In methodology, recovery process is not included to the model as this illness has no recovery period and patients need to periodical treatment.

This thesis is composed of two parts in the frame of the application of Markov process and phase type distribution. One of them considers the application of the proposed model to Turkish Mortality data and the other focuses on the implementation of the proposed model to morbidity estimation of IHD based on the information on Social Security Institute (SSI) data.

Literature review in previous paragraphs shows that Markov process and phase type distribution are used to analyse disease periods and its mortality. Different from literature, we analyse morbidity and mortality within the same matrix analogue. In contrast to Herrick et al. [13](2012), this thesis estimates duration time by phase type distribution and healthy state morbidity rates are also added to the analyses.

There are two accurate aims of the thesis: one of them is estimation of adopted Turkish Mortality Rates by more sophisticated method, that is Markov model and phase type distribution. Phase type distribution is introduced for an alternative survival function, the other one is determining a precise morbidity rate for a certain disease and its mortality is based on the same method.

To the purpose of showing the implication of methodology on Turkish mortality rates, firstly, Turkish Mortality Tables and CSO1980 Mortality Table are investigated with Markov process. Transition and mortality rates are then compared. After that, we analyse the morbidity and mortality of IHD by using one absorbing state Markov process and calculate the time until exposure to a disease and death by phase type distribution. Even though Markov process refers to memoryless property considering the death cause of a disease, it depends on the duration of exposure or time of diagnosis in practice. We assume that there was no duration time of the exposure of this disease in past years to implement Markov process.

The organization of the thesis is as follows: Markov process and phase type distribution are given in Chapter 2. The estimation of adopted mortality is given in Chapter 3. Morbidity and mortality estimation of Ischemic heart disease (IHD) is analysed by three states; Markov process, time until exposure to the disease and time until death, that are given with phase type distribution. The outcomes and comparision of the results are given. The last chapter concludes the thesis.

CHAPTER 2

Preliminaries

This chapter includes the methodology proposed in this thesis. Markov process and phase type distributions are explained in details.

2.1 Markov Process

Markov process is a stochastic process that satisfies the Markov property, which is used in many fields for different purposes, such as finance, demography, insurance, industrial sectors and etc [11, 19, 21]. In literature, there are continuous and discrete, time-homogeneous and non-homogenous, finite state and infinite state versions and combinations of Markov process [15, 29, 32].

This process has countable sets of states and there is transition between states. If the state set is countably infinite, it refers to infinite state Markov process, otherwise finite state Markov process. If the transition probabilities depends on elapse time rather than absolute time, Markov process is time homogenous, otherwise, it is time non-homogenous. There is a discrete and continuous version of Markov process that refers to the discrete state space and time and that is called discrete time Markov process; otherwise, continuous time Markov process, respectively.

Definition 2.1. When n is time, $\{X_n : n \geq 0\}$ with a state space $S \in \{0, 1, 2, .., k\}$ is called a discrete-time Markov chain if and only if it has a Markov property.

$$\mathbb{P}\{X_{s+t} = j | X_s = i, X_u = x_u, 0 \leq u < s\} = \mathbb{P}\{X_{s+t} = j | X_s = i\} \tag{2.1}$$

which is also called memoryless property [19]. This property indicates that past and future states are conditionally independent. If past and the present states are known, then future state depends on present state, not past states.

The time of staying at state i before moving to state j follows an exponential distribution with parameter λ_i.

Definition 2.2. One-step Transition Matrix

$$P = \begin{pmatrix} p_{11} & p_{12} & \dots & p_{1k} \\ p_{21} & p_{22} & \dots & p_{2k} \\ \vdots & \vdots & \ddots & \vdots \\ p_{k1} & p_{k2} & \dots & p_{kk} \end{pmatrix}$$

P is stochastic matrix with $p_{ij} \geq 0$ and $\sum p_{ij} = 1$. $\lambda_{i,j}$ represents the force of transition from state i to state j.

Definition 2.3. Transition Rate Matrix

Let Q be a kxk matrix with (i, j) entry $\lambda_{i,j}$

$$Q = \begin{pmatrix} \lambda_{11} & \lambda_{12} & \dots & \lambda_{1k} \\ \lambda_{21} & \lambda_{22} & \dots & \lambda_{23} \\ \vdots & \vdots & \ddots & \vdots \\ \lambda_{k1} & \lambda_{k2} & \dots & \lambda_{kk} \end{pmatrix}$$

because of the Markov property for each row $\lambda_{ii} = -\sum \lambda_{ij}$ where $i \neq j$.

Definition 2.4. Chapman-Kolmogrov Equations

$$p_{ij}(s, s+t+u) = \sum_{l=1}^{k} p_{il}(s, s+t) p_{lj}(s+t, s+t+u)$$

where $i, j \in \{1, 2, ..., k\}$ stands for the state space.

The forces of transition and the transition probability functions are related by the Kolmogrov forward and backward equations.

Definition 2.5. Kolmogrov Forward and Backward Equations

$$\frac{\partial}{\partial t} p_{ij}(s, s+t) = \sum_{l=1}^{k} p_{il}(s, s+t) \lambda_{lj}(s, s+t) \quad (2.2)$$

$$\frac{\partial}{\partial s} p_{ij}(s, s+t) = -\sum_{l=1}^{k} \lambda_{lj}(s) p_{il}(s, s+t) \quad (2.3)$$

The assumption of constant force of mortality implies that the Kolmogorov differential equations 2.2, 2.3 can be written with the following form [19, 22].

$$P'(t) = P(t)Q \quad (2.4)$$

$$P'(t) = QP(t) \tag{2.5}$$

The equations 2.4 and 2.5 yield

$$P(t) = \exp(Qt) \tag{2.6}$$

2.2 Phase Type Distribution

Let $\{X(t)\}_{t\geq 0}$ be a Markov jump process on the finite state space $E = \{1, 2, .., k\}$ where the states $1, 2, .., k-1$ are the transient states and state k is the absorbing state. Then $\{X(t)\}_{t\geq 0}$ has an intensity matrix of the form

$$Q = \begin{pmatrix} \mathbf{T} & \mathbf{t} \\ \mathbf{0} & 0 \end{pmatrix}$$

T is $(k-1) \times (k-1)$ dimensional matrix, $\mathbf{t} = -\mathbf{T}e$ where $e' = (1 \quad 1 \ldots 1)$, and **0** is $k-1$ dimensional row vector of zeros. Let $\alpha = (\alpha_1 \quad \alpha_2 \ldots \alpha_k)$ denote the initial distribution of $(X(t))_{t\geq 0}$ over the transient states only where $\alpha_i = \mathbb{P}(X_0 = i)$, $\mathbb{P}(X_0 = k) = 0$ [3, 4, 11].

Definition 2.6. The time until death, τ , defined as

$$\tau = \inf\{t \geq 0 | X_t = k\}$$

is said to have **phase type distribution** and denoted as

$$\tau \sim PH(\alpha, \mathbf{T})$$

The set of parameters $(\alpha, \mathbf{T})$ is said to be a representation of the phase type distribution [22]. The $(\alpha, k-1)$ arenthe dimension of the phase type distribution.

Initial distribution, α, determines the probability mass function on the states [27].

Definition 2.7. Effect of constant change

If the absorbing rates have a constant change when other parameters are the same, then $(\alpha, \mathbf{T}^{\epsilon})$ denotes the phase type representation [22]. To illustrate, the adoption of Markov process estimates to phase type distribution, we define the mortality rates as

$$q_i^{\epsilon} = q_i + \epsilon \quad \text{where} \quad \epsilon \in [0, 1 - q_i)$$

$\mathbf{T}^{\epsilon}$ is defined in that form

$$\mathbf{T}^{\epsilon} = \mathbf{T} - \epsilon I$$

Thus,

$$\exp(\mathbf{T}^{\epsilon}t) = \exp((\mathbf{T} - \epsilon I)t) = \exp(-\epsilon t)\exp(\mathbf{T}t)$$

If there is a constant change in the absorbing rate, the survival function is

$$S^{\epsilon}(t) = \exp(-\epsilon t)S(t)$$

Also, transition rates can be adopted as

$$\lambda^{\epsilon}(t) = \lambda(t) + \epsilon$$

Theorem 2.1. *Let $f(.)$ be the density function, s be the age, α be the initial distribution and **T** and **t** come from the intensity matrix then,*

$$f(s) = \alpha \exp(\boldsymbol{T}s)\boldsymbol{t} \tag{2.7}$$

Proof. By using the approach given in Blandt [3].

$$\begin{aligned} f(s)ds &= \mathbb{P}(\tau\epsilon[s, s+ds)) \\ &= \textstyle\sum_{i=1}^{p-1} \alpha_i \sum_{j=1}^{p-1} p_{ij}^{s} t_j ds \\ &= \textstyle\sum_{i=1}^{p-1} \sum_{j=1}^{p-1} \alpha_i \exp(\mathbf{T}s)_{ij} t_j ds \\ &= \alpha \exp(\mathbf{T}s)\mathbf{t}ds \end{aligned}$$

Thus, we have

$$f(s) = \alpha \exp(\mathbf{T}s)\mathbf{t}$$

□

Theorem 2.2. *Let $S(s)$ be the survival function, then*

$$\begin{aligned} S(s) &= 1 - F(s) \\ &= \alpha \exp(\boldsymbol{T}s)_{ij}\boldsymbol{e} \end{aligned}$$

Proof. By using the approach given in Blandt [3].

$$
\begin{aligned}
1 - F(s) = S(s) &= \mathbb{P}(X(s) \in \{1, 2, .., k-1\}) \\
&= \sum_{i=1}^{p-1} \alpha_i \sum_{j=1}^{p-1} p_{ij}^s \\
&= \sum_{i=1}^{p-1} \sum_{j=1}^{p-1} \alpha_i \exp(\mathbf{T}s)_{ij} \\
&= \alpha \exp(\mathbf{T}s)_{ij} \mathbf{e}
\end{aligned}
$$

□

$$\lambda(s) = \frac{\alpha \exp(\mathbf{T}s)\mathbf{t}}{\alpha \exp(\mathbf{T}s)\mathbf{e}} \tag{2.8}$$

The probability of being alive at time t is defined as $p_t = \exp(-\lambda t)$ which is equivalent to the exponential survival function $S(t) = \exp(-\lambda t)$.

2.3 Two State Markov Process

Two state Markov process has one transition. This process is the simplest Markov process having two states. While applying this process, one of the states of it was used as the absorbing state. The last state represents absorbing. Therefore, there is only one transition.

The transition matrix over the cycle t to $t+1$ is

$$P_{t,t+1} = \begin{pmatrix} p_{t,t+1} & 1 - p_{t,t+1} \\ 0 & 1 \end{pmatrix}$$

$p_{t,t+1}$ is the probabilty of staying state 1 over the cycle t to $t+1$.

The corresponding transition rate matrix is

$$Q_{t,t+1} = \begin{pmatrix} -\lambda_{t,t+1} & \lambda_{t,t+1} \\ 0 & 0 \end{pmatrix}$$

The probability transition matrix is derived by spectral decomposition of Q.

If Q has distinct eigenvalues, d_1, d_2, then $Q = ADC$ where $C = A^{-1}$,
$D = diag(d_1, d_2)$ and the i-th column of A is the right-eigen vector associated with d_i.

$$P(t) = Adiag(d_1, d_2)C \tag{2.9}$$

Given the corresponding eigen vectors are $(1 \quad 0)'$ and $(1 \quad 1)'$

$$P(t) = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} \exp(-\lambda t) & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & -1 \\ 0 & 1 \end{pmatrix}$$

Thus,

$$P(t) = \begin{pmatrix} \exp(-\lambda t) & 1 - \exp(-\lambda t) \\ 0 & 1 \end{pmatrix}$$

Rewrite the transition matrix and the transition rate matrix with phase type formulation as using the same formula in Equation 2.6, matrix turns to in that form,

$$P(t) = \begin{pmatrix} \exp(-\lambda t) & 1 - \exp(-\lambda t) \\ 0 & 1 \end{pmatrix}$$

which becomes

$$P(s) = \exp(Qs) = \begin{pmatrix} \exp(-\mathbf{T}s) & \mathbf{e} - \exp(-\mathbf{T}s)\mathbf{e} \\ 0 & 1 \end{pmatrix}$$

$$p_{ij}^{s} = \mathbb{P}(X(s) = j \mid X(0) = i) = \exp(\mathbf{T}s)_{ij} \tag{2.10}$$

The probability of being state 1 at time t is $p_t = \exp(-\lambda t)$. This is also equivalent to the exponential survival function $S(t) = \exp(-\lambda t)$.

2.4 Three State Markov Process

Three state Markov process is used with last state represent absorbing. Transition is only one direction.

The transition matrix over the cycle t to $t+1$ is

$$P = \begin{pmatrix} p_{11} & p_{12} & p_{13} \\ 0 & p_{22} & p_{23} \\ 0 & 0 & 1 \end{pmatrix}$$

p_{ij} is the probability of staying state i over the cycle t to $t+1$. Last state represents absorbing, so transition rate matrix is;

$$Q = \begin{pmatrix} -(\lambda_{12} + \lambda_{13}) & \lambda_{12} & \lambda_{13} \\ 0 & -(\lambda_{23}) & \lambda_{23} \\ 0 & 0 & 0 \end{pmatrix}$$

because of the Markov property for each row $\lambda_{ii} = -\sum \lambda_{ij}$ where $i \neq j$.

The probability transition matrix is derived by spectral decomposition of Q.

If Q has distinct eigenvalues, d_1, d_2, d_3, then $Q = ADC$ where $C = A^{-1}$, $D = diag(d_1, d_2, d_3)$ and the i-th column of A is the right-eigen vector associated with d_i. We apply this Equation 2.9, we find the three state $P(t)$ matrix as follows,

$$P(t) = \begin{pmatrix} \exp(-(\lambda_{12} + \lambda_{13})t) & \exp(-\lambda_{12}t) & 1 - \exp(-\lambda_{13}t) \\ 0 & \exp(-\lambda_{23}t) & 1 - \exp(-\lambda_{23}t) \\ 0 & 0 & 1 \end{pmatrix}$$

Rewriting the transition matrix and the transition rate matrix with phase type notation $P(t) = \exp(Qt)$ yields following:

Let $\{X(t)\}_{t \geq 0}$ have an intensity matrix of the form

$$Q = \begin{pmatrix} \mathbf{T} & \mathbf{t} \\ \mathbf{0} & 0 \end{pmatrix}$$

$$\mathbf{T} = \begin{pmatrix} -(\lambda_{12} + \lambda_{13}) & \lambda_{12} \\ 0 & -(\lambda_{23}) \end{pmatrix}$$

$$\mathbf{t} = \begin{pmatrix} \lambda_{13} \\ \lambda_{23} \end{pmatrix}$$

then we use the same notations for three state like two state Eqaution 2.10. We find the probability of being state 1 and 2 at time t is $p_t = \exp(-\lambda \mathbf{t})$.

CHAPTER 3

Estimation of Mortality

3.1 Turkish Demography

If we look at the historical development of census of Turkish population, the first population census was carried out in 1927. Then, censuses were carried out every five years from 1935 to 1990. After 1990, population census was implemented in years ending with zero. Population census was carried out by de facto method in 2000. Person in the boundary of the country has been enumerated on the census day. People who were abroad for some reasons, have not been enumerated [1].

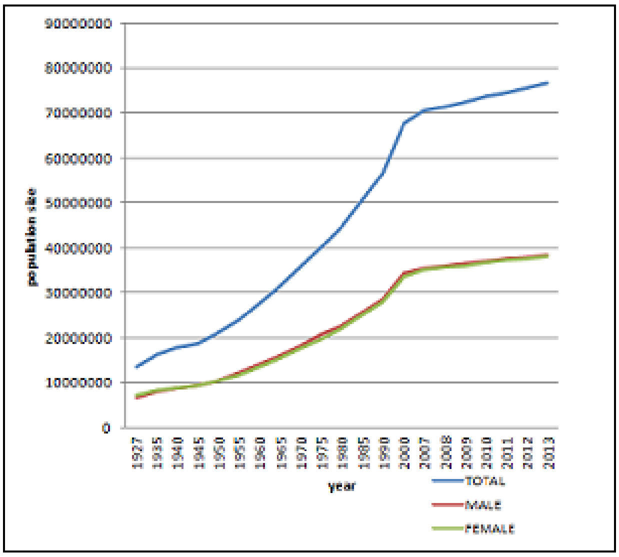
Address Based Population Registration System (ABPRS), by which basic population statistics have been announced annually, was implemented on 31 December 2007 [1]. This population census statistics and ABPRS data are given in the Table 3.1. As it is shown in Figure 3.1. and Table 3.1., population size increases in all years and male and female population size is close. There is a refraction in the 2007 data as the data comes from ABPRS.

As it is shown in the Figure 3.1, population increase rate is positive, but some facts,such as epidemic disease, natural disaster and etc., can also affect the census data. Also, World War I has affected the rate of increase in population all over the world. In Turkey case, the population rate has been affected by inclusion of Hatay to Turkish boundry in 1938.

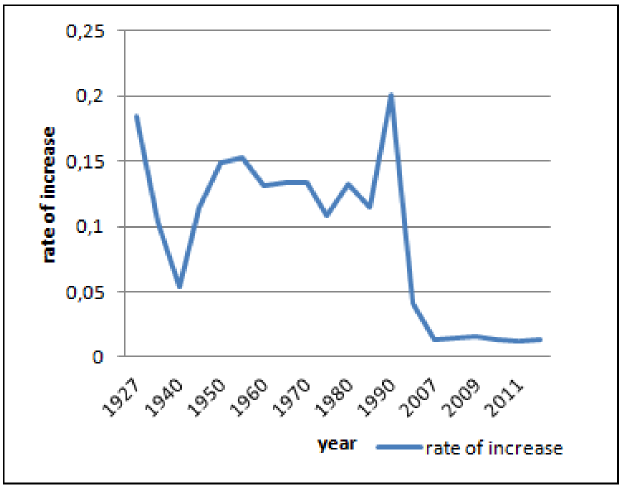
In addition to this, census data collected by ABPRS is given annually. Thus, that data is more reliable than past population census.

In past years, census of population was not carried out annually, but in particular years. ABPRS is very important progress in census estimations. However, the population statistics are given with range of ages, not on each age base. These facts make the study of demography more difficult and the given information about population is not enough to make prediction and projection about future. To exceed these restrictions, model life tables are used in some studies [35].

CSO 1980 Mortality Table is the most commonly used one in Turkey. Also, in 2010 Turkish Mortality tables are introduced; TRSH 2010 for insured, TRH 2010 for population and SGK 2008 for Social Security Insitute. These tables represent insured mor-



(a)



(b)

Figure 3.1: a) Trend in Turkish Population; b) Rate of Increase of The Turkish Population

Table 3.1: Turkish Population Data Between 1927 to 2013 [TurkStat,2011]

Census Year	Total Population	Male	Female
1927	13,648,270	6,563,879	7,084,391
1935	16,158,018	7,936,770	8,221,248
1940	17,820,950	8,898,912	8,922,038
1945	18,790,174	9,446,580	9,343,594
1950	20,947,188	10,527,085	10,420,103
1955	24,064,763	12,233,421	11,831,342
1960	27,754,820	14,163,888	13,590,932
1965	31,391,421	15,996,964	15,394,457
1970	35,605,176	18,006,986	17,598,190
1975	40,347,719	20,744,730	19,602,989
1980	44,736,957	22,695,362	22,041,595
1985	50,664,458	25,671,975	24,992,483
1990	56,473,035	28,607,047	27,865,988
2000	67,803,927	34,346,735	33,457,192
ABPRS Year	Total Population	Male	Female
2007	70,586,256	35,376,533	35,209,723
2008	71,517,100	35,901,154	35,615,946
2009	72,561,312	36,462,470	36,098,842
2010	73,722,988	37,043,182	36,679,806
2011	74,724,269	37,532,954	37,191,315
2012	75,627,384	37,956,168	37,671,216
2013	76,667,864	38,473,360	38,194,504

tality rates, total population mortality rates and social security beneficiary mortality rates, respectively [HAYMER,2010].

The range of mortality age in these tables given in age and gender do not yield the same values. Mortality tables age range is different and mortality rates are given with age and gender.

Table 3.2: The Age Range in Turkish Mortality Tables

	CSO 1980	TRSH 2010	TRH 2010	SGK 2008
Age Range	0-99	0-110	0-99	14-110

SGK 2008 Mortality Table starts from age 14, which is the minimum employment age in Turkey.

Although completion of these tables are taken as an important progress, the representation of the population mortality rates is a controversial issue and they are not used widely in Turkey. There is still a process of adaptation.

The key component of these mortality tables, q_x, is given for comparison in Table **??**.

Table 3.3: Comparison of Mortality Tables, $q_{x;male}$, $q_{x;female}$

Gender	Age	CSO 1980	TRSH 2010	TRH 2010	SGK 2008
Male	0	0.00370	0.01953	0.01953	-
	25	0.00108	0.00075	0.00091	0.00066
	45	0.00319	0.00278	0.00302	0.00268
	58	0.01102	0.01015	0.01205	0.01077
	65	0.02152	0.01930	0.02407	0.02146
Female	0	0.00245	0.00816	0.00816	-
	25	0.00053	0.00021	0.00028	0.00020
	45	0.00237	0.00080	0.00157	0.00119
	58	0.00635	0.00382	0.00588	0.00421
	65	0.01145	0.00868	0.01322	0.00868

TRSH 2010, TRH 2010 and SGK 2008 tables are formed by Preston-Bennet method, Turkish population data and model life table [HAYMER,2010]. In 2008, Turkish Social Security Institution (SSI) changed the retirement age by Law No. 5510. Based on this law, retirement age was raised. We give the data based on retirement ages, infant and young age for comparison.

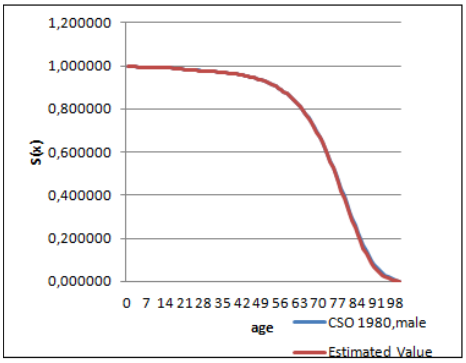
The rectangular shape of survival function indicates that the life expectancy has a great tendency to respond to a change in the force of mortality which is illustrated in Figure 3.2.

Mortality rates, q_x is different for each age, because risk factors change. Heligman and Pollard explain the age pattern of mortality through three components. One of them is infant high mortality, other one is young age mortality called accident hump and the last one is senescent mortality and considers the logatithm ($ln(100000 \cdot q_x)$) of the mortality rates drawn [12]. Figure 3.3. illustrates logarithm of mortality rates for these Mortality Tables.

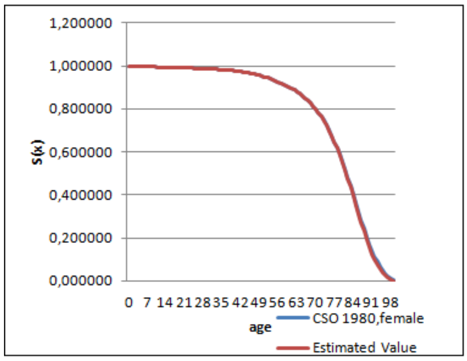
The uncertainty of mortality rates increases at young ages due to mortality rates increased because of a number of causes including AIDS, drug and alcohol abuse, violence and accidental death that are relatively common among younger adults. The improvements have clearly been substantial at all ages and there is a very clear correlation between age and the degree to which mortality has improved.

The graphs of survival function of current mortality table for Turkish population (Figure 3.2.) have a rectangular shape for males and females. In addition, the graphs of logarithm of mortality rates (Figure 3.3.) show that young age mortality rates are different from other ages.

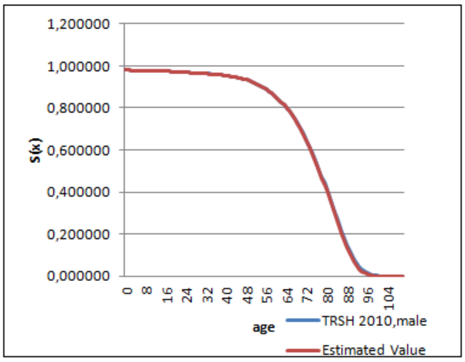
The improvement of medical area, such as the development of vaccines and the introduction of antibiotics, life standards, and so on has increased the expected life time and decreased the mortality rates. Because of the low fertility and longer lives, the populations of developed countries are growing older. Financial support, social support and health care have to be developed because of the rapidly growing population.



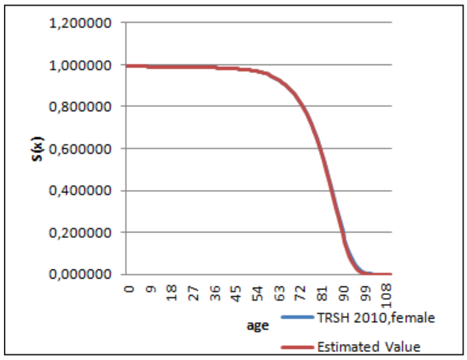
(a) CSO 1980,Male



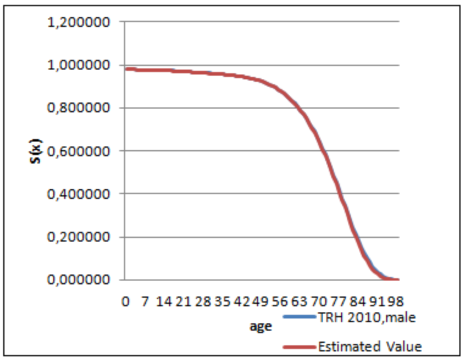
(b) CSO 1980,Female



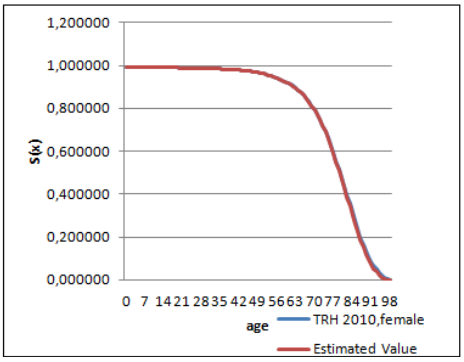
(c) TRSH 2010,Male



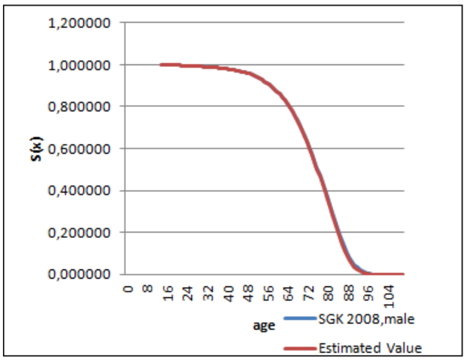
(d) TRSH 2010,Female



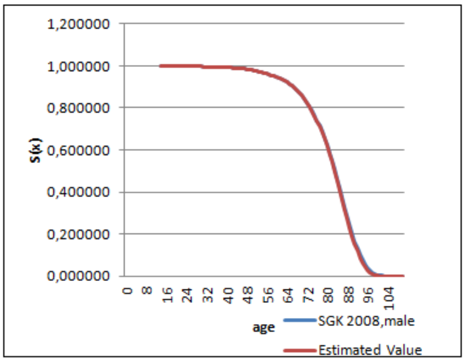
(e) TRH 2010,Male



(f) TRH 2010,Female

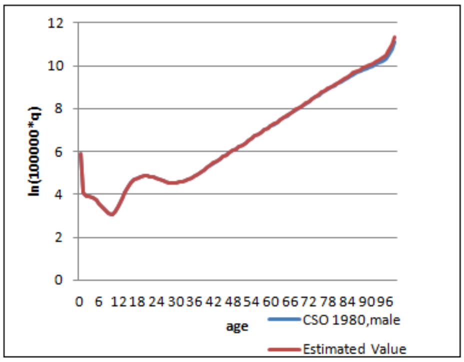


(g) SGK 2008,Male

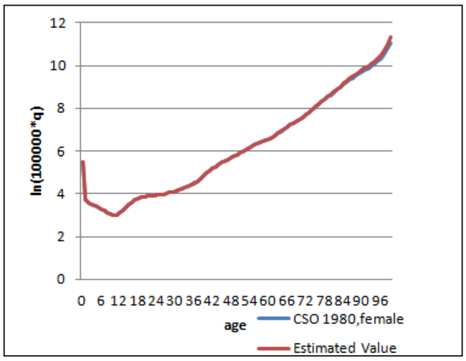


(h) SGK 2008,Female

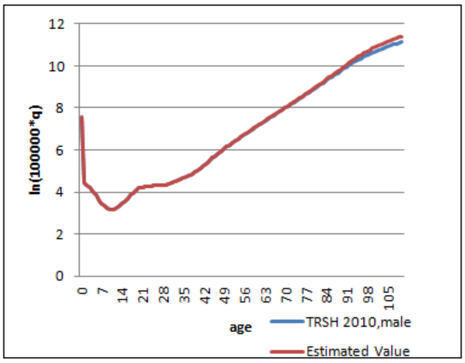
Figure 3.2: Survival Function of Mortality Tables [HAYMER,2010]



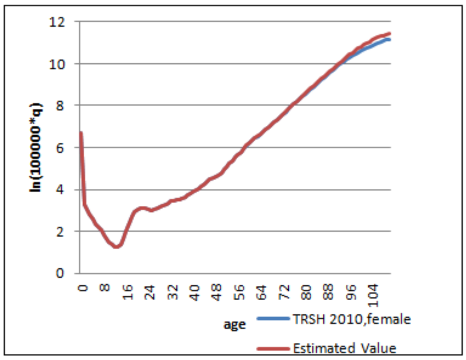
(a) CSO 1980,Male



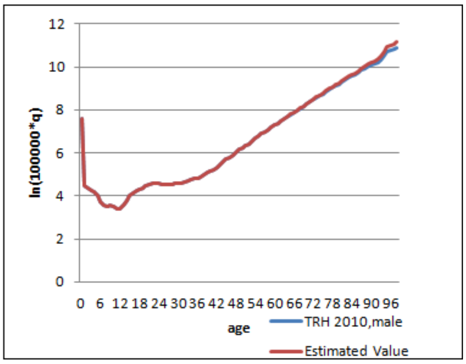
(b) CSO 1980,Female



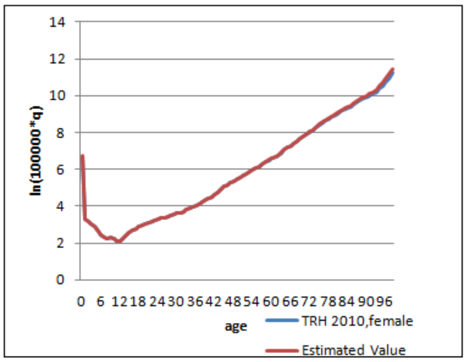
(c) TRSH 2010,Male



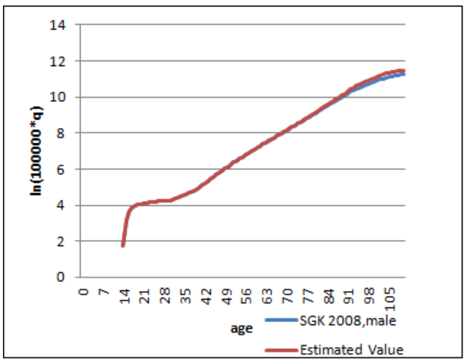
(d) TRSH 2010,Female



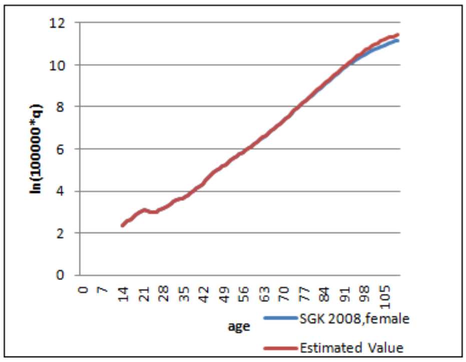
(e) TRH 2010,Male



(f) TRH 2010,Female



(g) SGK 2008,Male



(h) SGK 2008,Female

Figure 3.3: Mortality Rates of Mortality Tables [HAYMER,2010]

Mortality improvement means the reduction at the rate of mortality for a given age from one year to the next. Researches show that the improvement of the mortality rates is greater for women than men. However, there is no correlation between mortality rates and sex. But males suffer from higher mortality than females do.

Longevity risk is the risk that actual future mortality improvement and life expectancy are different. The progressive increase in lifetime duration leads to a potential increase in longevity risk.

As the improvement of the mortality rates and life expectancy rises, they cause an upward shift in the ages where additional reduction in mortality rates would be the most effective in increasing life expectancy.

The management of longevity risk requires life insurance companies and pension plans to model and measure it. Mortality projections should represent the increase of future expected life time. The main effect for annuity providers is an increase of the annuity payments period and actuarial liabilities. Life assurance companies need to have additional reserve on annuity portfolio. Because of the longevity risk, it is difficult to calculate the final salary pension scheme.

There are some studies about Turkish demographic structure and its change in years, carried out by TurkStat which can be found in 'Statistical Indicators 1923-2010' headline. This study provides information about Turkish demographic structure, which is important for using other researches and projections regarding the future of the population. It is announced by this study that population census and ABPRS data and other key components of the population [1].

TurkStat establish No. 13140 newsletter with "Turkish Demographic Structure and Its Future, 2010-2050" headline in July in 2014. This study announces the prediction of 2050 population statement. Based on the prediction, the total population will be 94 million 585 thousand, Turkish population growth rate will be 0.2%, and median age will be 40.2 in 2050 [2]. According to this prediction, Turkey will begin to have suffer longevity risk. Therefore, Social Security Institutes make the difference in their pension system by raising the retirement age.

3.2 Mortality Preliminaries

Cohort life table starts with certain population l_0 of individuals born. Mortality tables are defined with q_x, p_x, d_x and l_x values when x represents certain ages [6, 12, 25].

Definition 3.1. Probability of dying at a certain age is denoted by q_x value. When l_x is the number of lives at the age of x, d_x is defined as the number of death between ages x and $x + 1$.

$$d_x = l_x - l_{x+1} \tag{3.1}$$

Then,

$$q_x = \frac{d_x}{l_x} \tag{3.2}$$

The sum of the probability of living and dying at certain ages is equal to 1. So, the probability of living represents p_x value.

$$p_x = 1 - q_x \tag{3.3}$$

Definition 3.2. In distributional mortality analyses, $f(x)$, $F(x)$ and $S(x)$ values are used [6].

$f(x)$ is the density function defined as $\mathbb{P}(x \leq X < x + \varepsilon)$. When ε is too small. This value is used to define probability of dying at a certain age.

$F(x)$ is the cumulative distribution function defined as $\mathbb{P}(X \leq x)$. It indicates the probability of dying before certain ages.

$S(X)$ is survival function defined as $\mathbb{P}(X > x)$. The sum of cumulative function and survival function is equal to 1.

λ represents the hazard rate which is defined as follows:

$$\lambda_x = \frac{f(x)}{S(x)} \tag{3.4}$$

3.3 Methodology of Mortality Estimation of Turkish Population

We estimate the duration time until death and mortality rates obtained from tables of CSO 1980, TRSH 2010, TRH 2010 and SGK 2008 by Markov model and phase type distribution. To achieve this, primarily, all population data in Turkey regardless of the type of a disease; being alive and death have been investigated.

The improvement of medical technology and life standards cause higher expected life time and lower mortality rates leading to increasing population at older ages.

Turkey, like other countries, is also exposed to low fertility and longer life time. Mortality rates are the key components to make inferences and predictions on future developments in the population.

The rectangular shape of survival function indicates that the life expectancy has a great tendency to respond a change in the force of mortality.

Mortality projections should represent the change in future expected life time. For this reason, their precision plays an important role in actuarial modeling of life insurance and pension funds. Therefore, we use the forementioned mortality tables in estimating Markov process with two states.

The simplest Markov process has two states: 'alive'and 'dead'. There is only one transition. The first state represents being alive and the second one represents being dead, which is an absorbing state. This model is regarded as an alternative for survival analysis of Turkish mortality rates.

We use the same formulas in Chapter 2, Sub Section 2.3.

3.4 The Algorithm

1. Take the Mortality Tables p values
2. Calculate λ values
3. Determine P matrix
4. Determine Q matrix
5. Determine $f(x)$, $S(x)$, $F(x)$ and λ_x functions with phase type notations.

3.5 Results

Comparison of the transition rates for both gender is summarized in Table 3.5. SGK 2008 Mortality Table for the age zero is absent, as the SGK 2008 Mortality Table starts with age 14.

In Table 3.4, we show the Mortality Tables p_{12} values, this values indicate the probability of moving alive state to dead state, with gender and certain ages.

Table 3.4: Comparison of Mortality Tables, $p_{12,male}$, $p_{12,female}$

Gender	Age	CSO 1980	TRSH 2010	TRH 2010	SGK 2008
Male	0	0.0037	0.01953	0.01953	-
	25	0.00108	0.00075	0.00092	0.00066
	45	0.00319	0.00278	0.00302	0.00268
	58	0.01102	0.01015	0.01205	0.01077
	65	0.02152	0.01929	0.02407	0.02146
Female	0	0.00245	0.00816	0.00816	-
	25	0.00053	0.00021	0.00028	0.00020
	45	0.00237	0.00080	0.00157	0.00119
	58	0.00635	0.00382	0.00588	0.00421
	65	0.01145	0.00868	0.013220	0.00868

Table 3.4. shows that Mortality Tables gives the different mortality rates, this can be explained with the usage of the mortality tables, TRSH 2010 mortality table used for insured ones, TRH 2010 is for total population and SGK 2008 is for social sequrity beneficiaries.

In the Table 3.5. we show the Mortality Tables hazard rates, this value indicate how fast the transition from aile state to dead state, it is given gender and certain ages.

Table 3.5: Comparison of Mortality Tables, $\lambda_{12,male}$, $\lambda_{12,female}$

Gender	Age	CSO 1980	TRSH 2010	TRH 2010	SGK 2008
Male	0	0.00371	0.01992	0.01992	-
	25	0.00108	0.00075	0.00092	0.00066
	45	0.00320	0.00279	0.00303	0.00268
	58	0.01114	0.01025	0.01220	0.01088
	65	0.02199	0.01967	0.02466	0.02193
Female	0	0.00246	0.00823	0.00823	-
	25	0.00053	0.00021	0.00028	0.00011
	45	0.00238	0.00080	0.00158	0.00119
	58	0.00639	0.00384	0.00591	0.00423
	65	0.01158	0.00875	0.01340	0.00875

Also this values are vary each other.

As P matrix is stochastic, $p_{12} = 1 - p_{11}$. And there is an absorbing state, therefore $p_{21} = 0$ and $p_{22} = 1$. Because of the Markov property $\lambda_{11} = -\lambda_{12}$ and because of the fact that there is an absorbing state, $\lambda_{21} = \lambda_{22} = 0$.

It should be noted that phase type distribution can be performed as Chapter 2 and Sub Section 2.3

3.6 The Adopted Turkish Mortality Rates

Based on the tables taken into account, we compare the efficiency of the proposed model with the conventional tables. The Estimation of the rates (EV) by the model and the Table Values (TV) of CSO 1980, TRSH 2010, TRH 2010 and SGK 2008 are presented in Table 3.6.,3.7.,3.8. and 3.9. Also, mean square errors'(MSE) are given in Table 3.10. It can also be foolowed from Figures 3.2. and 3.4. estimated values (E.V.) and table values (T.V.) show discrepancy at the older ages. The age based estimated values (E.V.) are listed on Tables in Appendix A. Our estimated results agrees with the mortality tables. MSE is lower. TRSH 2010, TRH 2010 and SGK 2008 Mortality Tables' F(x) values increase faster in geriatric ages so that it increase the MSE. Because of that reason we cut the estimation before end of the tables. Even though our reasults are fit the tables' values. Tables in Appendix A illustrate estimated values for all ages. Also total MSE is given in Table 3.10.

Table 3.6: Comparison of CSO 1980 Mortality Table and Estimated Results

		q_x		$F(x)$		$S(x)$		$f(x)$		λ_x	
Gender	Age	T V	E V	T V	E V	T V	E V	T V	E V	T V	E V
Male	0	0.00370	0.00370	0.00370	0.00371	0.99630	0.99629	0.00370	0.00371	0.00371	0.00372
	25	0.00108	0.00108	0.02136	0.02137	0.97864	0.97862	0.00106	0.00106	0.00108	0.00108
	45	0.00319	0.00319	0.05142	0.05146	0.94858	0.94854	0.00304	0.00320	0.00304	0.00321
	58	0.01102	0.01108	0.12937	0.12969	0.87063	0.87031	0.00970	0.00975	0.01114	0.01121
	65	0.02152	0.02175	0.22418	0.22523	0.77582	0.77476	0.01706	0.01723	0.02199	0.02224
Female	0	0.00245	0.00245	0.00245	0.00245	0.99755	0.99754	0.00245	0.00245	0.00246	0.00246
	25	0.00053	0.00053	0.01130	0.01130	0.98870	0.98870	0.00052	0.00052	0.00053	0.00053
	45	0.00237	0.00237	0.03309	0.03311	0.96691	0.96689	0.00230	0.00230	0.00238	0.00238
	58	0.00635	0.00637	0.08547	0.08561	0.91453	0.91439	0.00584	0.00586	0.00639	0.00641
	65	0.01145	0.01152	0.13961	0.13998	0.86039	0.86002	0.00997	0.01002	0.01158	0.01165

Table 3.7: Comparison of TRSH 2010 Mortality Table and Estimated Results

		q_x		$F(x)$		$S(x)$		$f(x)$		λ_x	
Gender	Age	T V	E V	T V	E V	T V	E V	T V	E V	T V	E V
Male	0	0.01953	0.01973	0.01953	0.01973	0.98047	0.98028	0.01953	0.01973	0.01992	0.02012
	25	0.00075	0.00075	0.03173	0.03193	0.96827	0.96807	0.00073	0.00073	0.00075	0.00076
	45	0.00278	0.00278	0.05700	0.05721	0.94300	0.94279	0.00263	0.00263	0.00279	0.00279
	58	0.01015	0.01020	0.12875	0.12918	0.87125	0.87082	0.00893	0.00897	0.01025	0.01031
	65	0.01929	0.01948	0.21577	0.21680	0.78423	0.78320	0.01543	0.01556	0.01967	0.01987
Female	0	0.00816	0.00819	0.00816	0.00819	0.99184	0.99180	0.00816	0.00819	0.00823	0.00826
	25	0.00021	0.00021	0.01137	0.01140	0.98862	0.98860	0.00020	0.00020	0.00021	0.00021
	45	0.00080	0.00080	0.01943	0.01946	0.98057	0.98054	0.00079	0.00079	0.00080	0.00080
	58	0.00382	0.00383	0.04340	0.04346	0.95660	0.95654	0.00367	0.00368	0.00384	0.00385
	65	0.00868	0.00871	0.08557	0.08577	0.91443	0.91423	0.00800	0.00803	0.00875	0.00879

Table 3.8: Comparison of TRH 2010 Mortality Table and Estimated Results

		q_x		$F(x)$		$S(x)$		$f(x)$		λ_x	
Gender	Age	T V	E V	T V	E V	T V	E V	T V	E V	T V	E V
Male	0	0.01953	0.01973	0.01953	0.01973	0.98047	0.98028	0.01953	0.01973	0.01992	0.02012
	25	0.00092	0.00092	0.03485	0.03504	0.96515	0.96496	0.00089	0.00089	0.00092	0.00092
	45	0.00302	0.00303	0.06290	0.06311	0.93710	0.93689	0.00284	0.00284	0.00303	0.00303
	58	0.01205	0.01212	0.14320	0.14370	0.85680	0.85630	0.01045	0.01051	0.01220	0.01227
	65	0.02407	0.02436	0.24647	0.24784	0.75353	0.75216	0.01858	0.01878	0.02466	0.02497
Female	0	0.00816	0.00819	0.00816	0.00819	0.99184	0.99181	0.00816	0.00819	0.00823	0.00826
	25	0.00028	0.00028	0.01234	0.01238	0.98766	0.98762	0.00028	0.00028	0.00028	0.00028
	45	0.00157	0.00158	0.02562	0.02566	0.97438	0.97434	0.00154	0.00154	0.00158	0.00158
	58	0.00588	0.00589	0.06821	0.06833	0.93179	0.93167	0.00551	0.00552	0.00591	0.00593
	65	0.01322	0.01331	0.12722	0.12761	0.87278	0.87239	0.01169	0.01177	0.01340	0.01349

Table 3.9: Comparison of SGK 2008 Mortality Table and Estimated Results

		q_x		$F(x)$		$S(x)$		$f(x)$		λ_x	
Gender	Age	T V	E V	T V	E V	T V	E V	T V	E V	T V	E V
Male	0	-	-	-	-	-	-	-	-	-	-
	25	0.00066	0.00066	0.00591	0.00591	0.99409	0.99409	0.00066	0.00066	0.00066	0.00066
	45	0.00268	0.00268	0.02974	0.02976	0.97026	0.97024	0.00260	0.00261	0.00268	0.00269
	58	0.01077	0.01083	0.10543	0.10571	0.89458	0.89430	0.00974	0.00979	0.01088	0.01094
	65	0.02146	0.02169	0.20269	0.20373	0.79731	0.79627	0.01749	0.20373	0.01766	0.02217
Female	0	-	-	-	-	-	-	-	-	-	-
	25	0.00020	0.00020	0.00208	0.00208	0.99792	0.99792	0.00020	0.00020	0.00020	0.00020
	45	0.00119	0,00119	0.01209	0.01210	0.98791	0.98791	0.00118	0.00118	0.00119	0.00119
	58	0.00421	0.00422	0.04413	0.04417	0.95588	0.95583	0.00404	0.00405	0.00423	0.00424
	65	0.00868	0.00871	0.08647	0.08666	0.91353	0.91334	0.00799	0.00803	0.00875	0.00879

Table 3.10: MSE for Mortality Tables

	q_x		$F(x)$		$S(x)$		$f(x)$		λ_x	
MSE	Male	Female	Male	Female	Male	Female	Male	Female	Male	Female
CSO 1980	0.00066	0.00061	0.00002	0.00002	0.00002	0.00002	0.00001	0.00001	0.00076	0.00061
TRSH 2010	0.00238	0.00238	0.00001	0.00002	0.00001	0.00002	0.00001	0.00001	0.01206	0.01098
TRH 2010	0.00081	0.00090	0.00002	0.00002	0.00002	0.00002	0.00001	0.00001	0.01149	0.01960
SGK2008	0.00082	0.00028	0.00002	0.00002	0.00002	0.00002	0.00001	0.00001	0.01569	0.00215

CHAPTER 4

Morbidity and Mortality Estimation of IHD

4.1 Ischemic Heart Disease

Ischemic heart disease (IHD) occurs when there is an inadequate supply of blood and oxygen to a portion of the myocardium. The main reason is that there is an imbalance between myocardial oxygen supply and demand. Genetic factors, diet and smoking are related to that disease. Obesity, insulin resistat and type 2 diabetes are powerful risk factors and increase the mortality [9]. This disease is more deathful in developed countries.

IHD causes more deaths and disabilities compared to other diseases and incurs greater economic costs than any other illnesses do in the developed world. IHD is the most common, serious, chronic, life-threatening illness in the United States. As a result, the prevalence of the risk factors for and of IHD itself is both increasing so rapidly in those regions that a majority of the global burden of IHD occurs there. Population subgroups that appear to be particularly affected are men in South Asian countries, especially in India and in the Middle East. In light of the projection of large increases in IHD throughout the world, IHD is likely to become the most common cause of death worldwide by 2020 [9]. There is no recovery in this disease. Surgery is a necessary implementation. People even need a medical supervision and treatment annually.

In this chapter, the mortality and morbidity rates of IHD are determined by using the proposed methodology. The rates calculated are original in the sense that the joint transition probabilities reflect the lifetime remaining related to this disease.

4.2 Morbidity and Mortality Rates of IHD

4.2.1 Data and Descriptives

Data is taken from an earlier study on the estimation of IHD morbidity for Turkey [17]. The data set that is originally taken from SSI has information on every individual whose ID is anonymously kept. In addition, for each patient, his/her age, the city s/he lived, the city she/he was born, the date when she/he attended the hospital are available in the data set. However, the data collected between 2007-2009 is the only information accessible. For this reason, the existing collection of the data in the concent of

(i) application to the health institute first time in the year considered

(ii) the frequency of the repeated visits in the same year

(iii) the visit incurred in the consequative years

on individual basis are taken into consideration to determine the Markovian structure and transitions.

By the help of recoded ID numbers, the data provides the information about the date on which people has applied to the hospitals for the treatments of ischemic heart disease during 2007, 2008 and 2009 years. Moreover, it is comprised of nearly 5 million code lines including 1,251,260 female and 1,270,966 male personal information.

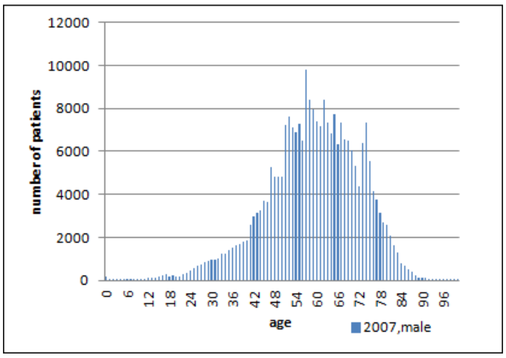
By the help of Macrocode in Excel, for 2007, 2008 and 2009, each patient is categorized regarding the fact that whether the hospital registrations for that specific year existed or not. That code has three digits, each of them representing obtained year situation. The consecutive registrations of the individuals are used to determine the state of the disease. When the individual appears for the considered year, it takes a code '1' to illustrate the disease that is still in action. For example, '010' code represents that patients appeared at the hospital only in 2008, but not in the years 2007 and 2008.

As mentioned in ischemic heart diseases section, patient suffering from IHD should be treated at a hospital annually, but when the data set is separated according to genders, ischemic heart disease patients are coded as 101 that forms 40% of 111 coded patients. This means that, 40% patients have refused treatment at hospitals or they might be treated abroad. Also, as understood from the literature, there is no information about the patients who recover from this disease. The purpose of this analysis is to estimate the probability of both healthy persons morbidity and patients mortality rates for that disease. To get a reliable result, data has been adjusted as considering these rates.

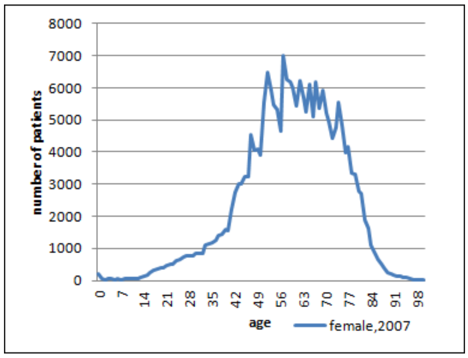
Based on the age and the gender, Figure 4.1. illustrate the distribution of the data set by years. When data is analysed carefully, it can be observed that there has been a huge agglomeration between 45 and 80 ages of females and males. The graph of the number of application for this disease at hospitals in 2007 and 2008 concerning gender and age is given in the Figure 4.1.

The number of patients is given in Table 4.1. The ages chosen as 0, 25, 45, 58, and 65 are determinant infant, young age and ages retirement system. The number of the patients having IHD concerning the age in 2007 is combined by codes 100, 110, 101 and 111. The number of the patients having IHD concerning the age in 2008 is combined by codes 010, 110, 011 and 111. Code 110 conserned with the two year because this patients applied to the hospital in 2007 and also 2008.

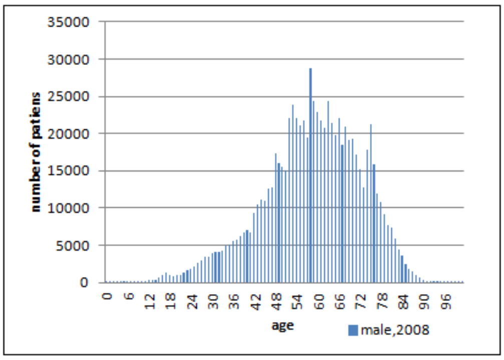
Code 100 shows that the person applied to the hospital for this disease in 2007 and there is not any application to hospitals in 2008 and 2009. The number of the people in that code is shown in Figure 4.2.



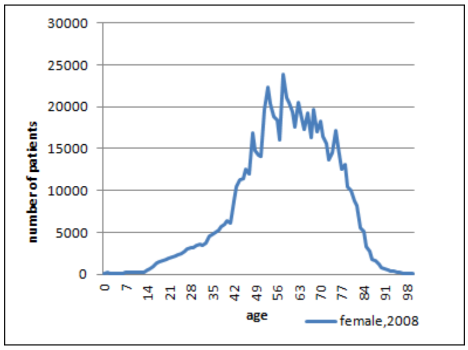
(a) 2007,Male



(b) 2007,Female

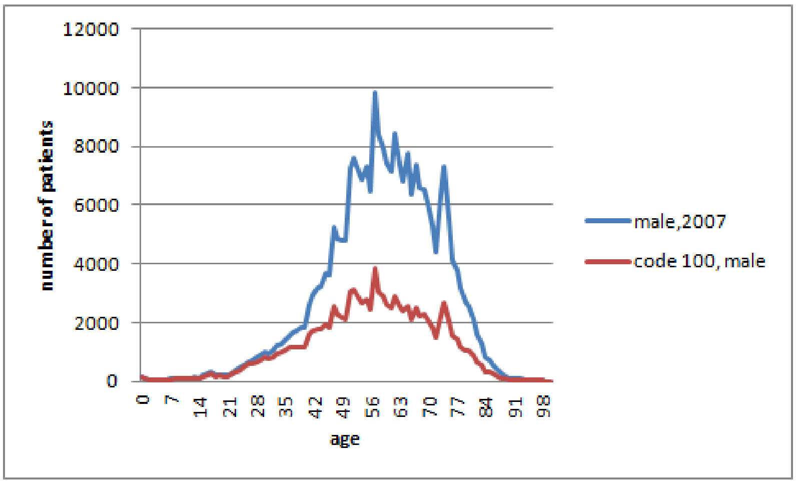


(c) 2008,Male

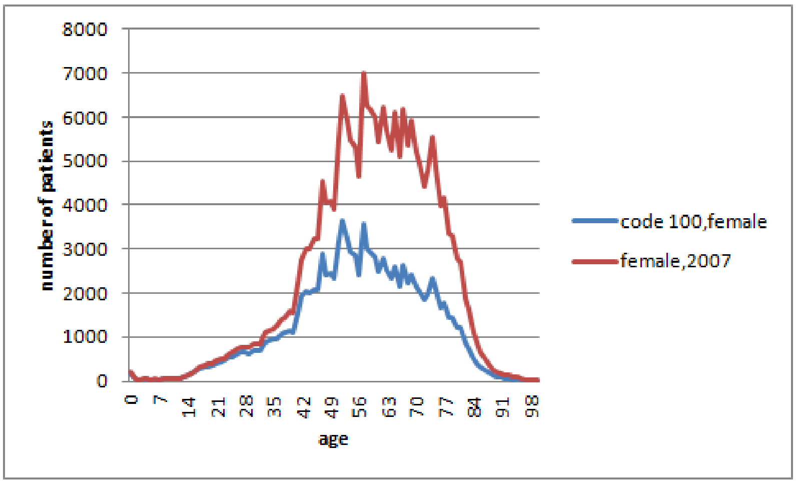


(d) 2008,Female

Figure 4.1: Number of Cases in IHD for Both Genders in Years 2007 and 2008



(a) Male



(b) Female

Figure 4.2: Comparison of Code 100 and Number of Patients

Table 4.1: IHD Age, Gender and Years Distribution

	2007		2008	
Age	Male	Female	Male	Female
0	171	191	159	139
25	538	665	2106	2432
45	3687	3234	11018	11387
58	8415	6274	28771	23840
65	7736	6093	19725	17246

As we see in the Figure 4.3., infant and older patients are frail and tend to die. On the other hand, middle age people continue to their treatment.

In the literature, the causes of the first three dead people are heart disease, canser and accident in developed contries [9]. For this reason, we assume the patients whose registrations disappear in 2009 are regarded as 'died' in the anayses. The total death number and its IHD patients in 2009 are given in figure according to age and gender.

Figure 4.3. is illustrated to compare the total number of patients and number of patients disappearing in 2009. This figure shows that infant patients are fragile and most of them do not appear in 2008 and 2009.

For the purpose of emphasizing the gender variable, the number of death in 2009 and the number of death of IHD people are given in Figure 4.4.

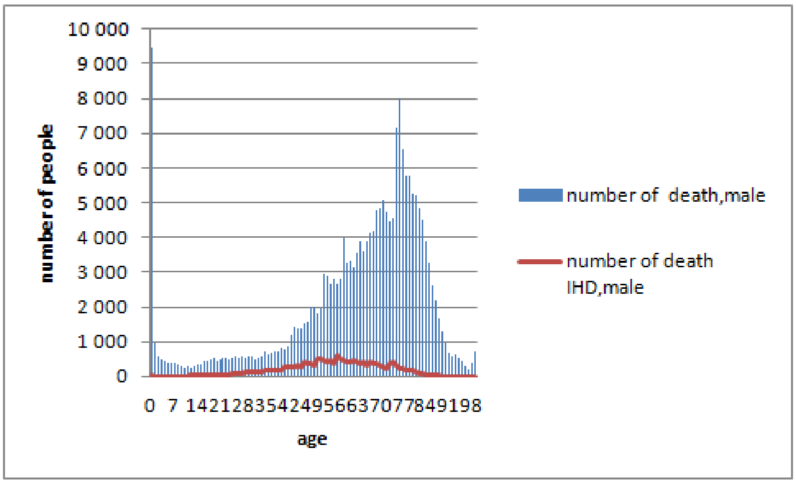
The tables of TRSH 2010, TRH 2010 and SGK 2008 have been constituted with the analysis of mortality study carried out in recent years in Turkey. The detailed information about these tables can be found in Chapter 2, but in this chapter, principally, the comparison of the numbers of living and dead patients has been carried out based on the information obtained from these tables. In this study, after two state Markov model was used, phase type distribution was applied to analyze the duration of time until death.

The information of registration to hospitals for this disease is shown in Table 4.2. The ages selected are purposedly chosen to represent mostly infants, young ages and alternating retirement ages in Turkey.

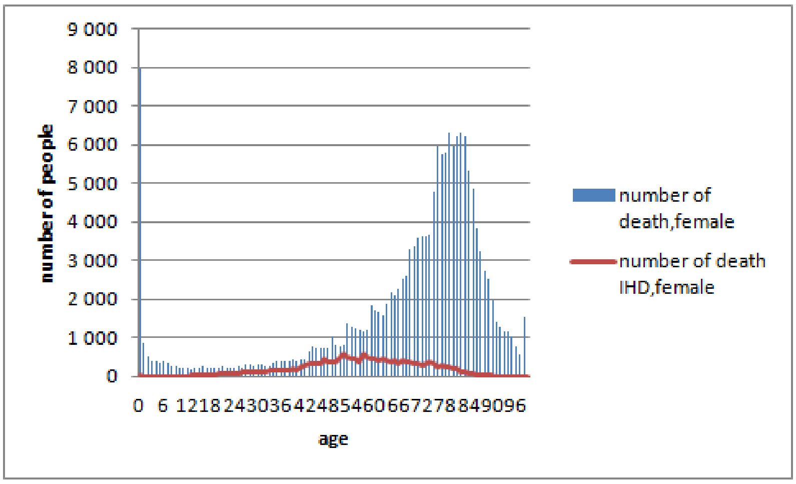
When we consider the total population size, these registration numbers might be considered as small but this disease is the most fatal disease in the Western countries and its disability rates are too high.

4.3 The Algorithm

1. Calculate p values from the data set,
2. Calculate λ values
3. Determine P matrix

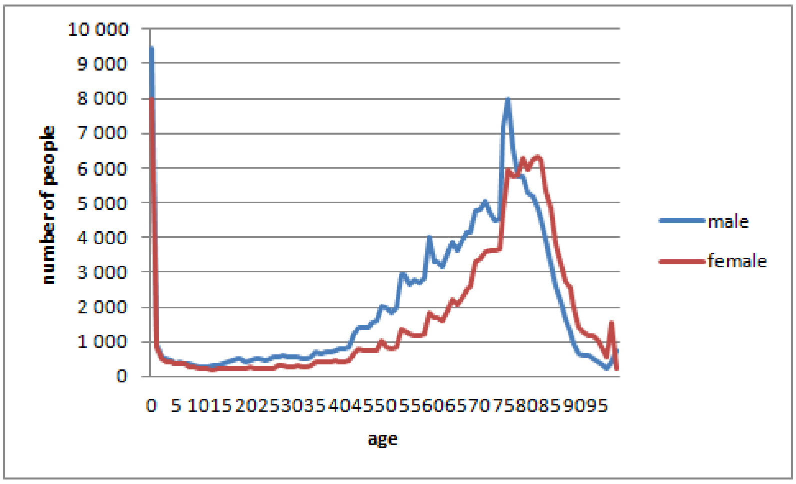


(a) Male

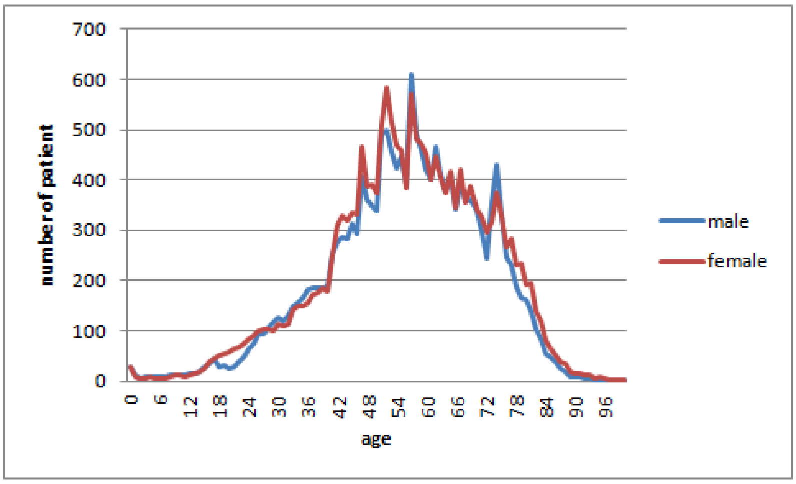


(b) Female

Figure 4.3: Number of Patients Disappearing in 2009 (Death)



(a) Number of death in 2009, male and female



(b) Number of Patient in 2009, Male and Female

Figure 4.4: Comparison of Gender Value

4. Determine Q matrix
5. Determine $f(x)$, $S(x)$, $F(x)$ and λ_x functions with phase type notations.

4.4 Two State Mortality Estimation for IHD

In this part, we estimate mortality rates for ischemic heart disease patients. In the literature, person who suffers from IHD has %40 of dying [9].

Two state Markov process is employed to determine state probabilities. The first column and row represent having IHD and the second column and row represent death.

Transition matrix is shown below.

$$P = \begin{pmatrix} p_{11} & p_{12} \\ p_{21} & p_{22} \end{pmatrix}$$

where p_{ij} are

p_{11}=$\mathbb{P}$(having IHD in year $t+1$ | having IHD in year t)

p_{12}=$\mathbb{P}$(death in year $t+1$ | having IHD in year t)

p_{21}=$\mathbb{P}$(having IHD in year $t+1$ | death in year t)=0

p_{22}=$\mathbb{P}$(death in year $t+1$ | death in year t)=1, and $\sum p_{ij} = 1$

The ages are chosen considering the infants, young ages and retirement age in Turkey.

Table 4.2: IHD Age, Gender and Years Distribution

	p_{12}		λ_{12}	
Age	Male	Female	Male	Female
0	0.15720	0.15414	0.18651	0.18222
25	0.13829	0.13498	0.16048	0.15604
45	0.08427	0.10350	0.09203	0.11545
58	0.05847	0.07694	0.06210	0.08335
65	0.05315	0.06817	0.05614	0.07316

We find the probability of being alive with IHD at time t as $p_t = \exp(-\lambda t)$. This is equivalent to the exponential survival function $S(t) = \exp(-\lambda t)$.

At the next step, we introduce the third state aspect of IHD by implementing the morbidity rate of IHD to the other states taken into account.

4.5 Three State Morbidity and Mortality Estimation for IHD

Data rearranged from SSI and the death statistics taken for TurkStat are used to analyse the morbidity and mortality for IHD patients.

First, we divide the total population into two parts, a part of which is a person having IHD and the other one is a person not having IHD. Then we use our IHD data set to find the number of the death for IHD. After that, we divide the total population dead number into two parts, one part of which shows IHD as the reason and the other one illustrates reasons assumed to be non IHD. Next, we refresh the mortality rates of the total population by separating non IHD people mortality and IHD people mortality rates. Later, using our data set of IHD we find non IHD person morbidity of IHD.

Three state transition matrix

$$P = \begin{pmatrix} p_{11} & p_{12} & p_{13} \\ p_{21} & p_{22} & p_{23} \\ p_{31} & p_{32} & p_{33} \end{pmatrix}$$

determines the probabilities p_{ij} as follows:

p_{11}=$\mathbb{P}$(no IHD in year $t+1$ | no IHD in year t)

p_{12}=$\mathbb{P}$(IHD in year $t+1$ | no IHD in year t)

p_{13}=$\mathbb{P}$(death in year $t+1$ | no IHD in year t)=0

p_{21}=$\mathbb{P}$(no IHD in year $t+1$ | IHD in year t)=1

p_{22}=$\mathbb{P}$(IHD in year $t+1$ | IHD in year t)=1

p_{23}=$\mathbb{P}$(death in year $t+1$ | IHD in year t)=1

p_{31}=$\mathbb{P}$(no IHD in year $t+1$ | death in year t)=1

p_{32}=$\mathbb{P}$(IHD in year $t+1$ | death in year t)=1

p_{33}=$\mathbb{P}$(death in year $t+1$ | death in year t)=1 , and $\sum p_{ij} = 1$

Furthermore, there is no recovery for this disease, $p_{21} = 0$ and the last state is absorbing so, $p_{31} = 0$, $p_{32} = 0$ and $p_{33} = 1$.

λ_{12} represents the transition rate from non IHD state to IHD state. λ_{13} is the transition rate from non IHD state to death state. λ_{23} is the transition rate from IHD state to death. There are no recoveries, so $\lambda_{21} = 0$, there are no transitions between death, so λ_{31}, λ_{32} and λ_{33} is equal to 0. In addition, because of the Markov property for each row $\lambda_{ii} = -\sum \lambda_{ij}$ where $i \neq j$. Thus, $\lambda_{11} = -(\lambda_{12} + \lambda_{13})$ and $\lambda_{22} = -\lambda_{23}$.

We need to define, α, initial distribution in order to define the phase type. If we take $\alpha = (0.5\ 0.5\ 0)$, it means the half of population has IHD.

Table 4.3: Transition probabilities, p_{ii}, for selected ages

Gender	Age	p_{12}	p_{13}	p_{23}
Male	0	0.00044	0.02906	0.15414
	25	0.00666	0.00110	0.13496
	45	0.03537	0.00348	0.10350
	58	0.11881	0.01499	0.07694
	65	0.11933	0.02613	0.06817
Female	0	0.00055	0.01505	0.15719
	25	0.00057	0.00072	0.13829
	45	0.00075	0.00255	0.08427
	58	0.00168	0.01136	0.05847
	65	0.00220	0.01898	0.05315

Table 4.4: Hazard rates, λ_{ii}, for selected ages

Gender	Age	λ_{12}	λ_{13}	λ_{23}
Male	0	0.00055	0.01528	0.18651
	25	0.00055	0.00071	0.16048
	45	0.00071	0.00248	0.09203
	58	0.00148	0.01097	0.06210
	65	0.00175	0.01813	0.05614
Female	0	0.00044	0.02993	0.18222
	25	0.00631	0.00104	0.15604
	45	0.03222	0.00234	0.11545
	58	0.08437	0.0039	0.08335
	65	0.04956	0.00279	0.07316

we will be able to calculate the λ values in Appendix B for all other ages.

It is known that, patients' mortality rates differ from each other based on the duration of the illness and the time since diagnosis. Moreover, IHD patient may have died by other death factors, such as car accidents and disasters and so on. This study assumes that, IHD patients die only as a consequence of this disease.

These λ values are shown with mortality tables values for comparison.

Table 4.5: λ_{12} Values

Gender	Age	IHD	CSO 1980	TRSH 2010	TRH 2010	SGK 2008
Male	0	0.18651	0.00371	0.01992	0.01992	-
	25	0.16048	0.00108	0.00075	0.00092	0.00066
	45	0.09203	0.00320	0.00279	0.00303	0.00268
	58	0.06210	0.01114	0.01025	0.01220	0.01088
	65	0.05614	0.02199	0.01967	0.02466	0.02193
Female	0	0.18222	0.00246	0.00823	0.00823	-
	25	0.15604	0.00053	0.00021	0.00028	0.00011
	45	0.11545	0.00300	0.00080	0.00158	0.00119
	58	0.08335	0.00639	0.00384	0.00591	0.00423
	65	0.07316	0.01158	0.00875	0.01340	0.00875

Exposure of the risk factors make the transition from IHD to death faster. Infant mortality rates for IHD are too high because of the frailty of this age period. Young age mortality rates are high because of the exposure of the risk factors as can be seen in Table 4.5. and Appendix B.

IHD makes death rates higher for all ages but infants and old people are so frailer than middle-aged people. Young people are exposed to the risk factors of IHD and it makes the transition rate faster.

There is no evidence about genders in the literature but it is seen in Turkish data that at the first ages, males are more fragile than females whereas after middle ages, females are more fragile. As the help of IHD data it can be directly said that, over 2,5 million people attended hospitals for this disease ,which makes the cost of the disease high. The duration of the time to absorbing by phase type distribution is calculated to make rate adjustment easy.

In this methodology, p_{12} value represents the morbidity rates for Turkish population. The verification of these probabilities needs a large-scale survey, which should be considered as a future study. As it was mentioned before, Social Security Institute data collection is used for morbidity analyses in the earlier study [IAM,2011]. In this section, we compare our outcomes through this study. Morbidity project provides the number of applications as a fraction of total population. On the contrary, our analyse provides the morbidity of Turkish population.

Therefore, there is a huge difference between these two studies' observations with respect to the number of the application. The morbidity calculated for IHD represents the rate adjusted to the overall population. Based on the same data set, the rates calculated in the thesis yield much lower rates. It is expected as the morbidity rates are determined on patient basis in the overall social sequrity institute population.

CHAPTER 5

Conclusion

This thesis utilizes Markov model and phase type distribution to determine the improvement in the mortality and morbidity rates based on an application to Turkish data. Phase type distribution is the effective way to determine mortality and morbidity estimation.

This study has two parts of analysis. One of them is mortality analysis of Turkish population. In this analysis, the most commonly used mortality table CSO 1980 and recently presented mortality tables TRSH 2010, TRH 2010 and SGK 2008 are adopted to the model. These tables are compared, adopted mortality rates are estimated. In this part, two state Markov process is used. Time until death is represented by phase type distribution. The other part is the morbidity analysis by Turkish Social Security data collection. We extend the mortality model of Markov process by adding a new state. Three state Markov process is used. The first state represents people who do not have IHD, the second state represents IHD patients and the last state represents death. We determine healthy people morbidity rates and their mortality rates. Also patients' mortality rates are determined.

Main contributions of this study are

(i) We use phase type distribution to estimate adopted mortality rates for each age and the morbidity rates of IHD

(ii) A real life data set on a group of social security insureds are considered to illustrate the methodology and estimate the transition probabilities

Ischemic heart disease is known as the most deathly illness all around the world. Differently from the literature, we analyse morbidity and mortality in the same matrix. It gives the opportunity to compare and reduce the computation time. We provide the mortality and morbidity rates regarding age and gender. We see that females are frail at early ages, while males' transition rates at younger ages are high. It can be explained by the exposure to the risk factor of this disease. Newborn patients have not change to live, as they are so frail. Middle-aged patients continue to their treatments. Futhermore, ϵ factor can be used to estimate mortality and morbidity so that authorities find solution to reduce mortality and morbidity related to the disease. One can see how the change of these rates effects the calculation.

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APPENDIX A

Adopted Mortality Rates for Turkish Case

Table A.1: Adopted Mortality Rates, Estimated Values for CSO 1980, Male

Age	q_x	p_x	$F(x)$	$S(x)$	$f(x)$	λ_x
0	0.00371	0.99629	0.00371	0.99629	0.00371	0.00372
1	0.00059	0.99941	0.00429	0.99571	0.00059	0.00059
2	0.00051	0.99949	0.00480	0.99520	0.00051	0.00051
3	0.00050	0.99950	0.00530	0.99470	0.00050	0.00050
4	0.00046	0.99954	0.00576	0.99424	0.00046	0.00046
5	0.00041	0.99959	0.00617	0.99383	0.00041	0.00041
6	0.00036	0.99964	0.00652	0.99348	0.00036	0.00036
7	0.00030	0.99970	0.00682	0.99318	0.00030	0.00030
8	0.00025	0.99975	0.00707	0.99293	0.00025	0.00025
9	0.00022	0.99978	0.00729	0.99271	0.00022	0.00022
10	0.00021	0.99979	0.00750	0.99250	0.00021	0.00021
11	0.00024	0.99976	0.00774	0.99226	0.00024	0.00024
12	0.00031	0.99969	0.00804	0.99196	0.00031	0.00031
13	0.00044	0.99956	0.00848	0.99152	0.00044	0.00044
14	0.00060	0.99940	0.00907	0.99093	0.00060	0.00060
15	0.00077	0.99923	0.00984	0.99016	0.00076	0.00077
16	0.00094	0.99906	0.01077	0.98923	0.00093	0.00094
17	0.00109	0.99891	0.01185	0.98815	0.00108	0.00109
18	0.00119	0.99881	0.01302	0.98698	0.00118	0.00119
19	0.00125	0.99875	0.01426	0.98574	0.00123	0.00125
20	0.00128	0.99872	0.01552	0.98448	0.00126	0.00128
21	0.00128	0.99872	0.01678	0.98322	0.00126	0.00128
22	0.00125	0.99875	0.01801	0.98199	0.00123	0.00125
23	0.00120	0.99880	0.01919	0.98081	0.00118	0.00120
24	0.00115	0.99885	0.02032	0.97968	0.00113	0.00115
25	0.00108	0.99892	0.02138	0.97862	0.00106	0.00108

Table A.2: Estimated Values for CSO 1980, Male (Continued)

Age	q_x	p_x	$F(x)$	$S(x)$	$f(x)$	λ_x
26	0.00102	0.99898	0.02238	0.97762	0.00100	0.00102
27	0.00098	0.99902	0.02334	0.97666	0.00096	0.00098
28	0.00095	0.99905	0.02426	0.97574	0.00093	0.00095
29	0.00094	0.99906	0.02518	0.97482	0.00092	0.00094
30	0.00094	0.99906	0.02610	0.97390	0.00092	0.00094
31	0.00096	0.99904	0.02703	0.97297	0.00094	0.00096
32	0.00099	0.99901	0.02800	0.97200	0.00096	0.00099
33	0.00104	0.99896	0.02901	0.97099	0.00101	0.00104
34	0.00110	0.99890	0.03008	0.96992	0.00107	0.00110
35	0.00118	0.99882	0.03122	0.96878	0.00115	0.00118
36	0.00128	0.99872	0.03246	0.96754	0.00124	0.00128
37	0.00141	0.99859	0.03383	0.96617	0.00137	0.00141
38	0.00155	0.99845	0.03533	0.96467	0.00150	0.00155
39	0.00172	0.99828	0.03699	0.96301	0.00166	0.00172
40	0.00191	0.99809	0.03883	0.96117	0.00184	0.00192
41	0.00213	0.99787	0.04088	0.95912	0.00205	0.00214
42	0.00236	0.99764	0.04315	0.95685	0.00227	0.00237
43	0.00262	0.99738	0.04566	0.95434	0.00251	0.00263
44	0.00289	0.99711	0.04842	0.95158	0.00276	0.00290
45	0.00320	0.99680	0.05146	0.94854	0.00304	0.00321
46	0.00351	0.99649	0.05478	0.94522	0.00333	0.00352
47	0.00385	0.99615	0.05842	0.94158	0.00364	0.00386
48	0.00420	0.99580	0.06237	0.93763	0.00395	0.00422
49	0.00459	0.99541	0.06668	0.93332	0.00430	0.00461
50	0.00502	0.99498	0.07137	0.92863	0.00469	0.00505

Table A.3: Estimated Values for CSO 1980, Male (Continued)

Age	q_x	p_x	$F(x)$	$S(x)$	$f(x)$	λ_x
51	0.00553	0.99447	0.07650	0.92350	0.00513	0.00556
52	0.00610	0.99390	0.08213	0.91787	0.00563	0.00614
53	0.00676	0.99324	0.08834	0.91166	0.00621	0.00681
54	0.00751	0.99249	0.09518	0.90482	0.00684	0.00756
55	0.00831	0.99169	0.10270	0.89730	0.00752	0.00838
56	0.00919	0.99081	0.11095	0.88905	0.00825	0.00928
57	0.01011	0.98989	0.11994	0.88006	0.00899	0.01021
58	0.01108	0.98892	0.12969	0.87031	0.00975	0.01121
59	0.01212	0.98788	0.14024	0.85976	0.01055	0.01227
60	0.01329	0.98671	0.15167	0.84833	0.01142	0.01347
61	0.01460	0.98540	0.16405	0.83595	0.01238	0.01481
62	0.01608	0.98392	0.17749	0.82251	0.01344	0.01634
63	0.01778	0.98222	0.19211	0.80789	0.01462	0.01810
64	0.01967	0.98033	0.20800	0.79200	0.01589	0.02007
65	0.02175	0.97825	0.22523	0.77477	0.01723	0.02224
66	0.02398	0.97602	0.24381	0.75619	0.01858	0.02457
67	0.02635	0.97365	0.26374	0.73626	0.01993	0.02706
68	0.02886	0.97114	0.28499	0.71501	0.02125	0.02972
69	0.03159	0.96841	0.30757	0.69243	0.02259	0.03262
70	0.03466	0.96534	0.33157	0.66843	0.02400	0.03590
71	0.03817	0.96183	0.35708	0.64292	0.02551	0.03969
72	0.04225	0.95775	0.38425	0.61575	0.02716	0.04411
73	0.04696	0.95304	0.41316	0.58684	0.02892	0.04927
74	0.05224	0.94776	0.44382	0.55618	0.03066	0.05512
75	0.05797	0.94203	0.47606	0.52394	0.03224	0.06153

Table A.4: Estimated Values for CSO 1980, Male (Continued)

Age	q_x	p_x	$F(x)$	$S(x)$	$f(x)$	λ_x
76	0.06405	0.93595	0.50961	0.49039	0.03356	0.06843
77	0.07036	0.92964	0.54412	0.45588	0.03451	0.07569
78	0.07685	0.92315	0.57915	0.42085	0.03503	0.08324
79	0.08371	0.91629	0.61438	0.38562	0.03523	0.09135
80	0.09120	0.90880	0.64955	0.35045	0.03517	0.10035
81	0.09958	0.90042	0.68445	0.31555	0.03490	0.11060
82	0.10916	0.89084	0.71889	0.28111	0.03444	0.12253
83	0.12007	0.87993	0.75264	0.24736	0.03375	0.13646
84	0.13208	0.86792	0.78532	0.21468	0.03267	0.15218
85	0.14488	0.85512	0.81642	0.18358	0.03110	0.16942
86	0.15819	0.84181	0.84546	0.15454	0.02904	0.18791
87	0.17183	0.82817	0.87201	0.12799	0.02656	0.20748
88	0.18573	0.81427	0.89578	0.10422	0.02377	0.22809
89	0.19987	0.80013	0.91661	0.08339	0.02083	0.24980
90	0.21438	0.78562	0.93449	0.06551	0.01788	0.27288
91	0.22944	0.77056	0.94952	0.05048	0.01503	0.29776
92	0.24548	0.75452	0.96191	0.03809	0.01239	0.32535
93	0.26333	0.73667	0.97194	0.02806	0.01003	0.35747
94	0.28470	0.71530	0.97993	0.02007	0.00799	0.39802
95	0.31309	0.68691	0.98621	0.01379	0.00628	0.45580
96	0.36180	0.63820	0.99120	0.00880	0.00499	0.56691
97	0.44062	0.55938	0.99508	0.00492	0.00388	0.78770
98	0.58920	0.41080	0.99798	0.00202	0.00290	1.43426
99	0.85235	0.14765	0.99970	0.00030	0.00172	5.77273

Table A.5: Estimated Values for CSO 1980, Female

Age	q_x	p_x	$F(x)$	$S(x)$	$f(x)$	λ_x
0	0.00245	0.99755	0.00245	0.99755	0.00245	0.00246
1	0.00042	0.99958	0.00287	0.99713	0.00042	0.00042
2	0.00036	0.99964	0.00323	0.99677	0.00036	0.00036
3	0.00034	0.99966	0.00357	0.99643	0.00034	0.00034
4	0.00032	0.99968	0.00389	0.99611	0.00032	0.00032
5	0.00030	0.99970	0.00419	0.99581	0.00030	0.00030
6	0.00027	0.99973	0.00446	0.99554	0.00027	0.00027
7	0.00025	0.99975	0.00471	0.99529	0.00025	0.00025
8	0.00023	0.99977	0.00493	0.99507	0.00023	0.00023
9	0.00021	0.99979	0.00514	0.99486	0.00021	0.00021
10	0.00020	0.99980	0.00534	0.99466	0.00020	0.00020
11	0.00020	0.99980	0.00554	0.99446	0.00020	0.00020
12	0.00022	0.99978	0.00576	0.99424	0.00022	0.00022
13	0.00025	0.99975	0.00601	0.99399	0.00025	0.00025
14	0.00029	0.99971	0.00630	0.99370	0.00029	0.00029
15	0.00033	0.99967	0.00663	0.99337	0.00033	0.00033
16	0.00037	0.99963	0.00699	0.99301	0.00037	0.00037
17	0.00041	0.99959	0.00740	0.99260	0.00041	0.00041
18	0.00044	0.99956	0.00784	0.99216	0.00044	0.00044
19	0.00047	0.99953	0.00830	0.99170	0.00047	0.00047
20	0.00048	0.99952	0.00878	0.99122	0.00048	0.00048
21	0.00049	0.99951	0.00927	0.99073	0.00049	0.00049
22	0.00050	0.99950	0.00976	0.99024	0.00050	0.00050
23	0.00051	0.99949	0.01027	0.98973	0.00051	0.00051
24	0.00052	0.99948	0.01078	0.98922	0.00051	0.00052
25	0.00053	0.99947	0.01131	0.98869	0.00052	0.00053

Table A.6: Estimated Values for CSO 1980, Female (Continued)

Age	q_x	p_x	$F(x)$	$S(x)$	$f(x)$	λ_x
26	0.00054	0.99946	0.01184	0.98816	0.00053	0.00054
27	0.00056	0.99944	0.01239	0.98761	0.00055	0.00056
28	0.00058	0.99942	0.01297	0.98703	0.00057	0.00058
29	0.00060	0.99940	0.01356	0.98644	0.00059	0.00060
30	0.00063	0.99937	0.01418	0.98582	0.00062	0.00063
31	0.00066	0.99934	0.01483	0.98517	0.00065	0.00066
32	0.00069	0.99931	0.01551	0.98449	0.00068	0.00069
33	0.00072	0.99928	0.01622	0.98378	0.00071	0.00072
34	0.00077	0.99923	0.01698	0.98302	0.00076	0.00077
35	0.00082	0.99918	0.01778	0.98222	0.00081	0.00082
36	0.00090	0.99910	0.01867	0.98133	0.00088	0.00090
37	0.00100	0.99900	0.01965	0.98035	0.00098	0.00100
38	0.00112	0.99888	0.02075	0.97925	0.00110	0.00112
39	0.00127	0.99873	0.02199	0.97801	0.00124	0.00127
40	0.00144	0.99856	0.02340	0.97660	0.00141	0.00144
41	0.00162	0.99838	0.02499	0.97501	0.00158	0.00162
42	0.00181	0.99819	0.02675	0.97325	0.00177	0.00181
43	0.00199	0.99801	0.02869	0.97131	0.00194	0.00200
44	0.00218	0.99782	0.03081	0.96919	0.00212	0.00219
45	0.00237	0.99763	0.03311	0.96689	0.00230	0.00238
46	0.00257	0.99743	0.03560	0.96440	0.00249	0.00258
47	0.00277	0.99723	0.03827	0.96173	0.00268	0.00278
48	0.00299	0.99701	0.04115	0.95885	0.00288	0.00300
49	0.00324	0.99676	0.04426	0.95574	0.00310	0.00325
50	0.00351	0.99649	0.04761	0.95239	0.00335	0.00352

Table A.7: Estimated Values for CSO 1980, Female (Continued)

Age	q_x	p_x	$F(x)$	$S(x)$	$f(x)$	λ_x
51	0.00380	0.99620	0.05122	0.94878	0.00362	0.00381
52	0.00412	0.99588	0.05513	0.94487	0.00391	0.00414
53	0.00449	0.99551	0.05937	0.94063	0.00424	0.00451
54	0.00487	0.99513	0.06396	0.93604	0.00458	0.00490
55	0.00527	0.99473	0.06889	0.93111	0.00494	0.00530
56	0.00567	0.99433	0.07417	0.92583	0.00528	0.00570
57	0.00603	0.99397	0.07975	0.92025	0.00558	0.00606
58	0.00637	0.99363	0.08561	0.91439	0.00586	0.00641
59	0.00672	0.99328	0.09176	0.90824	0.00615	0.00677
60	0.00714	0.99286	0.09824	0.90176	0.00648	0.00719
61	0.00767	0.99233	0.10515	0.89485	0.00692	0.00773
62	0.00836	0.99164	0.11264	0.88736	0.00749	0.00844
63	0.00927	0.99073	0.12087	0.87913	0.00823	0.00936
64	0.01034	0.98966	0.12996	0.87004	0.00909	0.01045
65	0.01152	0.98848	0.13998	0.86002	0.01002	0.01165
66	0.01275	0.98725	0.15095	0.84905	0.01097	0.01292
67	0.01398	0.98602	0.16281	0.83719	0.01187	0.01417
68	0.01517	0.98483	0.17552	0.82448	0.01270	0.01541
69	0.01645	0.98355	0.18908	0.81092	0.01357	0.01673
70	0.01795	0.98205	0.20364	0.79636	0.01456	0.01828
71	0.01979	0.98021	0.21940	0.78060	0.01576	0.02019
72	0.02213	0.97787	0.23668	0.76332	0.01728	0.02263
73	0.02506	0.97494	0.25580	0.74420	0.01913	0.02570
74	0.02855	0.97145	0.27705	0.72295	0.02125	0.02939
75	0.03251	0.96749	0.30055	0.69945	0.02350	0.03360

Table A.8: Estimated Values for CSO 1980, Female (Continued)

Age	q_x	p_x	$F(x)$	$S(x)$	$f(x)$	λ_x
76	0.03687	0.96313	0.32634	0.67366	0.02579	0.03829
77	0.04156	0.95844	0.35434	0.64566	0.02800	0.04336
78	0.04655	0.95345	0.38440	0.61560	0.03006	0.04882
79	0.05201	0.94799	0.41641	0.58359	0.03202	0.05486
80	0.05819	0.94181	0.45037	0.54963	0.03396	0.06178
81	0.06533	0.93467	0.48628	0.51372	0.03591	0.06990
82	0.07370	0.92630	0.52414	0.47586	0.03786	0.07956
83	0.08343	0.91657	0.56384	0.43616	0.03970	0.09103
84	0.09438	0.90562	0.60501	0.39499	0.04116	0.10421
85	0.10638	0.89362	0.64702	0.35298	0.04202	0.11904
86	0.11935	0.88065	0.68915	0.31085	0.04213	0.13552
87	0.13320	0.86680	0.73056	0.26944	0.04141	0.15367
88	0.14794	0.85206	0.77042	0.22958	0.03986	0.17362
89	0.16358	0.83642	0.80797	0.19203	0.03755	0.19557
90	0.18025	0.81975	0.84258	0.15742	0.03461	0.21988
91	0.19818	0.80182	0.87378	0.12622	0.03120	0.24716
92	0.21780	0.78220	0.90127	0.09873	0.02749	0.27844
93	0.23996	0.76004	0.92496	0.07504	0.02369	0.31573
94	0.26640	0.73360	0.94495	0.05505	0.01999	0.36314
95	0.30061	0.69939	0.96150	0.03850	0.01655	0.42983
96	0.34990	0.65010	0.97497	0.02503	0.01347	0.53823
97	0.42974	0.57026	0.98573	0.01427	0.01076	0.75360
98	0.57680	0.42320	0.99396	0.00604	0.00823	1.36295
99	0.84060	0.15940	0.99904	0.00096	0.00508	5.27339

Table A.9: Estimated Values for TRSH 2010, Male

Age	q_x	p_x	$F(x)$	$S(x)$	$f(x)$	λ_x
0	0.01973	0.98027	0.01973	0.98027	0.01973	0.02012
1	0.00085	0.99915	0.02056	0.97944	0.00083	0.00085
2	0.00076	0.99924	0.02130	0.97870	0.00074	0.00076
3	0.00066	0.99934	0.02194	0.97806	0.00064	0.00066
4	0.00056	0.99944	0.02249	0.97751	0.00055	0.00056
5	0.00047	0.99953	0.02295	0.97705	0.00046	0.00047
6	0.00039	0.99961	0.02333	0.97667	0.00038	0.00039
7	0.00033	0.99967	0.02365	0.97635	0.00032	0.00033
8	0.00028	0.99972	0.02393	0.97607	0.00028	0.00028
9	0.00025	0.99975	0.02418	0.97582	0.00025	0.00025
10	0.00024	0.99976	0.02441	0.97559	0.00023	0.00024
11	0.00024	0.99976	0.02464	0.97536	0.00023	0.00024
12	0.00025	0.99975	0.02488	0.97512	0.00024	0.00025
13	0.00028	0.99972	0.02515	0.97485	0.00027	0.00028
14	0.00031	0.99969	0.02545	0.97455	0.00030	0.00031
15	0.00035	0.99965	0.02580	0.97420	0.00034	0.00035
16	0.00041	0.99959	0.02619	0.97381	0.00040	0.00041
17	0.00047	0.99953	0.02665	0.97335	0.00046	0.00047
18	0.00055	0.99945	0.02719	0.97281	0.00053	0.00055
19	0.00061	0.99939	0.02778	0.97222	0.00060	0.00062
20	0.00067	0.99933	0.02843	0.97157	0.00065	0.00067
21	0.00069	0.99931	0.02910	0.97090	0.00067	0.00069
22	0.00071	0.99929	0.02978	0.97022	0.00069	0.00071
23	0.00072	0.99928	0.03048	0.96952	0.00070	0.00072
24	0.00073	0.99927	0.03120	0.96880	0.00071	0.00073
25	0.00075	0.99925	0.03193	0.96807	0.00073	0.00075
26	0.00077	0.99923	0.03267	0.96733	0.00075	0.00077
27	0.00078	0.99922	0.03342	0.96658	0.00075	0.00078
28	0.00077	0.99923	0.03417	0.96583	0.00075	0.00077
29	0.00078	0.99922	0.03492	0.96508	0.00075	0.00078
30	0.00080	0.99920	0.03570	0.96430	0.00077	0.00080
31	0.00084	0.99916	0.03650	0.96350	0.00081	0.00084
32	0.00089	0.99911	0.03736	0.96264	0.00085	0.00089

Table A.10: Estimated Values for TRSH 2010, Male (Continued)

Age	q_x	p_x	$F(x)$	$S(x)$	$f(x)$	λ_x
33	0.00095	0.99905	0.03827	0.96173	0.00091	0.00095
34	0.00102	0.99898	0.03925	0.96075	0.00098	0.00102
35	0.00108	0.99892	0.04029	0.95971	0.00104	0.00108
36	0.00114	0.99886	0.04139	0.95861	0.00109	0.00114
37	0.00121	0.99879	0.04254	0.95746	0.00116	0.00121
38	0.00129	0.99871	0.04378	0.95622	0.00124	0.00130
39	0.00141	0.99859	0.04513	0.95487	0.00135	0.00141
40	0.00156	0.99844	0.04662	0.95338	0.00149	0.00156
41	0.00174	0.99826	0.04828	0.95172	0.00166	0.00174
42	0.00196	0.99804	0.05014	0.94986	0.00186	0.00196
43	0.00220	0.99780	0.05224	0.94776	0.00209	0.00221
44	0.00247	0.99753	0.05458	0.94542	0.00234	0.00248
45	0.00278	0.99722	0.05721	0.94279	0.00263	0.00279
46	0.00311	0.99689	0.06014	0.93986	0.00293	0.00312
47	0.00345	0.99655	0.06339	0.93661	0.00325	0.00347
48	0.00383	0.99617	0.06697	0.93303	0.00359	0.00385
49	0.00425	0.99575	0.07094	0.92906	0.00396	0.00426
50	0.00470	0.99530	0.07530	0.92470	0.00437	0.00472
51	0.00520	0.99480	0.08011	0.91989	0.00480	0.00522
52	0.00574	0.99426	0.08539	0.91461	0.00528	0.00577
53	0.00633	0.99367	0.09118	0.90882	0.00579	0.00637
54	0.00697	0.99303	0.09751	0.90249	0.00634	0.00702
55	0.00766	0.99234	0.10442	0.89558	0.00691	0.00772
56	0.00843	0.99157	0.11197	0.88803	0.00755	0.00850
57	0.00927	0.99073	0.12021	0.87979	0.00823	0.00936
58	0.01020	0.98980	0.12918	0.87082	0.00897	0.01030
59	0.01119	0.98881	0.13892	0.86108	0.00974	0.01131
60	0.01226	0.98774	0.14948	0.85052	0.01055	0.01241
61	0.01344	0.98656	0.16090	0.83910	0.01143	0.01362
62	0.01478	0.98522	0.17330	0.82670	0.01240	0.01500
63	0.01626	0.98374	0.18674	0.81326	0.01344	0.01653
64	0.01783	0.98217	0.20124	0.79876	0.01450	0.01815

Table A.11: Estimated Values for TRSH 2010, Male (Continued)

Age	q_x	p_x	$F(x)$	$S(x)$	$f(x)$	λ_x
65	0.01948	0.98052	0.21680	0.78320	0.01556	0.01987
66	0.02123	0.97877	0.23343	0.76657	0.01663	0.02169
67	0.02319	0.97681	0.25121	0.74879	0.01778	0.02374
68	0.02536	0.97464	0.27020	0.72980	0.01899	0.02602
69	0.02774	0.97226	0.29044	0.70956	0.02024	0.02853
70	0.03033	0.96967	0.31196	0.68804	0.02152	0.03128
71	0.03319	0.96681	0.33480	0.66520	0.02284	0.03433
72	0.03645	0.96355	0.35904	0.64096	0.02424	0.03782
73	0.04002	0.95998	0.38469	0.61531	0.02565	0.04169
74	0.04397	0.95603	0.41174	0.58826	0.02705	0.04599
75	0.04820	0.95180	0.44010	0.55990	0.02835	0.05064
76	0.05289	0.94711	0.46971	0.53029	0.02961	0.05584
77	0.05804	0.94196	0.50048	0.49952	0.03078	0.06161
78	0.06382	0.93618	0.53236	0.46764	0.03188	0.06817
79	0.07007	0.92993	0.56513	0.43487	0.03277	0.07535
80	0.07702	0.92298	0.59862	0.40138	0.03349	0.08344
81	0.08485	0.91515	0.63268	0.36732	0.03406	0.09271
82	0.09414	0.90586	0.66726	0.33274	0.03458	0.10392
83	0.10461	0.89539	0.70206	0.29794	0.03481	0.11683
84	0.11600	0.88400	0.73663	0.26337	0.03456	0.13122
85	0.12782	0.87218	0.77029	0.22971	0.03366	0.14655
86	0.14050	0.85950	0.80256	0.19744	0.03227	0.16346
87	0.15432	0.84568	0.83303	0.16697	0.03047	0.18248
88	0.17011	0.82989	0.86144	0.13856	0.02840	0.20498
89	0.18837	0.81163	0.88754	0.11246	0.02610	0.23209
90	0.20982	0.79018	0.91113	0.08887	0.02360	0.26554
91	0.23482	0.76518	0.93200	0.06800	0.02087	0.30689
92	0.26149	0.73851	0.94978	0.05022	0.01778	0.35407
93	0.28865	0.71135	0.96428	0.03572	0.01450	0.40578
94	0.31444	0.68556	0.97551	0.02449	0.01123	0.45866
95	0.34170	0.65830	0.98388	0.01612	0.00837	0.51906
96	0.37124	0.62876	0.98986	0.01014	0.00598	0.59043

Table A.12: Estimated Values for TRSH 2010, Male (Continued)

Age	q_x	p_x	$F(x)$	$S(x)$	$f(x)$	λ_x
96	0.37124	0.62876	0.98986	0.01014	0.00598	0.59043
97	0.40421	0.59579	0.99396	0.00604	0.00410	0.67844
98	0.43925	0.56075	0.99661	0.00339	0.00265	0.78334
99	0.47597	0.52403	0.99823	0.00177	0.00161	0.90828
100	0.51414	0.48586	0.99914	0.00086	0.00091	1.05822
101	0.55351	0.44649	0.99962	0.00038	0.00048	1.23969
102	0.59373	0.40627	0.99984	0.00016	0.00023	1.46139
103	0.63438	0.36562	0.99994	0.00006	0.00010	1.73508
104	0.67500	0.32500	0.99998	0.00002	0.00004	2.07694
105	0.71507	0.28493	0.99999	0.00001	0.00001	2.50959
106	0.75401	0.24599	1.00000	0.00000	0.00000	3.06519
107	0.79125	0.20875	1.00000	0.00000	0.00000	3.79044
108	0.82623	0.17377	1.00000	0.00000	0.00000	4.75462
109	0.85842	0.14158	1.00000	0.00000	0.00000	6.06290
110	0.88737	0.11263	1.00000	0.00000	0.00000	7.87892

Table A.13: Estimated Values for TRSH 2010, Female

Age	q_x	p_x	$F(x)$	$S(x)$	$f(x)$	λ_x
0	0.00819	0.99181	0.00819	0.99181	0.00819	0.00826
1	0.00027	0.99973	0.00846	0.99154	0.00026	0.00027
2	0.00021	0.99979	0.00867	0.99133	0.00021	0.00021
3	0.00017	0.99983	0.00884	0.99116	0.00017	0.00017
4	0.00014	0.99986	0.00897	0.99103	0.00013	0.00014
5	0.00011	0.99989	0.00908	0.99092	0.00010	0.00011
6	0.00009	0.99991	0.00917	0.99083	0.00009	0.00009
7	0.00008	0.99992	0.00925	0.99075	0.00008	0.00008
8	0.00006	0.99994	0.00931	0.99069	0.00006	0.00006
9	0.00005	0.99995	0.00936	0.99064	0.00005	0.00005
10	0.00004	0.99996	0.00941	0.99059	0.00004	0.00004
11	0.00004	0.99996	0.00945	0.99055	0.00004	0.00004
12	0.00004	0.99996	0.00948	0.99052	0.00004	0.00004
13	0.00004	0.99996	0.00952	0.99048	0.00003	0.00004
14	0.00004	0.99996	0.00956	0.99044	0.00004	0.00004
15	0.00005	0.99995	0.00961	0.99039	0.00005	0.00005
16	0.00008	0.99992	0.00969	0.99031	0.00008	0.00008
17	0.00011	0.99989	0.00980	0.99020	0.00011	0.00011
18	0.00015	0.99985	0.00995	0.99005	0.00015	0.00015
19	0.00019	0.99981	0.01013	0.98987	0.00018	0.00019
20	0.00021	0.99979	0.01034	0.98966	0.00021	0.00021
21	0.00022	0.99978	0.01056	0.98944	0.00022	0.00022
22	0.00023	0.99977	0.01078	0.98922	0.00022	0.00023
23	0.00022	0.99978	0.01100	0.98900	0.00022	0.00022
24	0.00021	0.99979	0.01120	0.98880	0.00021	0.00021
25	0.00021	0.99979	0.01141	0.98859	0.00020	0.00021
26	0.00021	0.99979	0.01162	0.98838	0.00021	0.00021
27	0.00023	0.99977	0.01184	0.98816	0.00022	0.00023
28	0.00024	0.99976	0.01207	0.98793	0.00023	0.00024
29	0.00025	0.99975	0.01232	0.98768	0.00025	0.00025
30	0.00027	0.99973	0.01258	0.98742	0.00026	0.00027
31	0.00029	0.99971	0.01287	0.98713	0.00029	0.00029
32	0.00031	0.99969	0.01317	0.98683	0.00031	0.00031

Table A.14: Estimated Values for TRSH 2010, Female (Continued)

Age	q_x	p_x	$F(x)$	$S(x)$	$f(x)$	λ_x
33	0.00032	0.99968	0.01349	0.98651	0.00032	0.00032
34	0.00033	0.99967	0.01382	0.98618	0.00033	0.00033
35	0.00034	0.99966	0.01416	0.98584	0.00034	0.00034
36	0.00036	0.99964	0.01451	0.98549	0.00036	0.00036
37	0.00039	0.99961	0.01490	0.98510	0.00038	0.00039
38	0.00042	0.99958	0.01531	0.98469	0.00041	0.00042
39	0.00046	0.99954	0.01576	0.98424	0.00045	0.00046
40	0.00049	0.99951	0.01625	0.98375	0.00048	0.00049
41	0.00053	0.99947	0.01677	0.98323	0.00052	0.00053
42	0.00058	0.99942	0.01734	0.98266	0.00057	0.00058
43	0.00064	0.99936	0.01797	0.98203	0.00063	0.00064
44	0.00072	0.99928	0.01868	0.98132	0.00070	0.00072
45	0.00080	0.99920	0.01946	0.98054	0.00079	0.00080
46	0.00088	0.99912	0.02032	0.97968	0.00086	0.00088
47	0.00094	0.99906	0.02125	0.97875	0.00093	0.00095
48	0.00101	0.99899	0.02224	0.97776	0.00099	0.00101
49	0.00108	0.99892	0.02330	0.97670	0.00106	0.00108
50	0.00123	0.99877	0.02450	0.97550	0.00121	0.00124
51	0.00142	0.99858	0.02588	0.97412	0.00138	0.00142
52	0.00163	0.99837	0.02747	0.97253	0.00159	0.00164
53	0.00188	0.99812	0.02930	0.97070	0.00183	0.00189
54	0.00217	0.99783	0.03141	0.96859	0.00211	0.00218
55	0.00249	0.99751	0.03383	0.96617	0.00241	0.00250
56	0.00287	0.99713	0.03660	0.96340	0.00277	0.00288
57	0.00331	0.99669	0.03978	0.96022	0.00318	0.00332
58	0.00383	0.99617	0.04346	0.95654	0.00368	0.00385
59	0.00442	0.99558	0.04769	0.95231	0.00423	0.00444
60	0.00513	0.99487	0.05257	0.94743	0.00488	0.00515
61	0.00566	0.99434	0.05794	0.94206	0.00537	0.00570
62	0.00630	0.99370	0.06388	0.93612	0.00594	0.00634
63	0.00703	0.99297	0.07046	0.92954	0.00658	0.00708
64	0.00783	0.99217	0.07773	0.92227	0.00728	0.00789

Table A.15: Estimated Values for TRSH 2010, Female (Continued)

Age	q_x	p_x	$F(x)$	$S(x)$	$f(x)$	λ_x
65	0.00871	0.99129	0.08577	0.91423	0.00804	0.00879
66	0.00970	0.99030	0.09464	0.90536	0.00887	0.00980
67	0.01086	0.98914	0.10447	0.89553	0.00983	0.01098
68	0.01220	0.98780	0.11540	0.88460	0.01092	0.01235
69	0.01372	0.98628	0.12753	0.87247	0.01213	0.01391
70	0.01545	0.98455	0.14101	0.85899	0.01348	0.01569
71	0.01743	0.98257	0.15598	0.84402	0.01497	0.01773
72	0.01976	0.98024	0.17265	0.82735	0.01668	0.02016
73	0.02241	0.97759	0.19119	0.80881	0.01854	0.02292
74	0.02541	0.97459	0.21174	0.78826	0.02055	0.02607
75	0.02870	0.97130	0.23437	0.76563	0.02263	0.02955
76	0.03243	0.96757	0.25920	0.74080	0.02483	0.03352
77	0.03662	0.96338	0.28633	0.71367	0.02713	0.03801
78	0.04146	0.95854	0.31592	0.68408	0.02959	0.04326
79	0.04688	0.95312	0.34799	0.65201	0.03207	0.04918
80	0.05311	0.94689	0.38262	0.61738	0.03463	0.05609
81	0.06020	0.93980	0.41978	0.58022	0.03717	0.06406
82	0.06852	0.93148	0.45954	0.54046	0.03975	0.07356
83	0.07775	0.92225	0.50156	0.49844	0.04202	0.08430
84	0.08789	0.91211	0.54537	0.45463	0.04381	0.09636
85	0.09880	0.90120	0.59029	0.40971	0.04492	0.10964
86	0.11089	0.88911	0.63572	0.36428	0.04543	0.12471
87	0.12438	0.87562	0.68103	0.31897	0.04531	0.14205
88	0.13946	0.86054	0.72551	0.27449	0.04448	0.16206
89	0.15654	0.84346	0.76848	0.23152	0.04297	0.18559
90	0.17593	0.82407	0.80921	0.19079	0.04073	0.21348
91	0.19889	0.80111	0.84716	0.15284	0.03795	0.24827
92	0.22479	0.77521	0.88151	0.11849	0.03436	0.28997
93	0.25271	0.74729	0.91146	0.08854	0.02994	0.33816
94	0.28049	0.71951	0.93629	0.06371	0.02484	0.38983
95	0.30931	0.69069	0.95600	0.04400	0.01971	0.44784
96	0.34081	0.65919	0.97099	0.02901	0.01500	0.51700

Table A.16: Estimated Values for TRSH 2010, Female (Continued)

Age	q_x	p_x	$F(x)$	$S(x)$	$f(x)$	λ_x
97	0.37609	0.62391	0.98190	0.01810	0.01091	0.60280
98	0.41454	0.58546	0.98940	0.01060	0.00750	0.70805
99	0.45530	0.54470	0.99423	0.00577	0.00482	0.83589
100	0.49814	0.50186	0.99710	0.00290	0.00287	0.99258
101	0.54268	0.45732	0.99868	0.00132	0.00157	1.18667
102	0.58846	0.41154	0.99945	0.00055	0.00078	1.42992
103	0.63489	0.36511	0.99980	0.00020	0.00035	1.73889
104	0.68126	0.31874	0.99994	0.00006	0.00014	2.13737
105	0.72679	0.27321	0.99998	0.00002	0.00005	2.66025
106	0.77064	0.22936	1.00000	0.00000	0.00001	3.35993
107	0.81194	0.18806	1.00000	0.00000	0.00000	4.31732
108	0.84987	0.15013	1.00000	0.00000	0.00000	5.66082
109	0.88374	0.11626	1.00000	0.00000	0.00000	7.60110
110	0.91301	0.08699	1.00000	0.00000	0.00000	10.49598

Table A.17: Estimated Values for TRH 2010, Male

Age	q_x	p_x	$F(x)$	$S(x)$	$f(x)$	λ_x
0	0.01973	0.98027	0.01973	0,98027	0,01973	0.02012
1	0.00089	0.99911	0.02060	0.97940	0.00087	0.00089
2	0.00078	0.99922	0.02136	0.97864	0.00076	0.00078
3	0.00069	0.99931	0.02203	0.97797	0.00067	0.00069
4	0.00064	0.99936	0.02266	0.97734	0.00063	0.00064
5	0.00055	0.99945	0.02320	0.97680	0.00054	0.00055
6	0.00043	0.99957	0.02362	0.97638	0.00042	0.00043
7	0.00036	0.99964	0.02396	0.97604	0.00035	0.00036
8	0.00033	0.99967	0.02428	0.97572	0.00032	0.00033
9	0.00035	0.99965	0.02462	0.97538	0.00034	0.00035
10	0.00034	0.99966	0.02495	0.97505	0.00033	0.00034
11	0.00029	0.99971	0.02523	0.97477	0.00028	0.00029
12	0.00030	0.99970	0.02552	0.97448	0.00029	0.00030
13	0.00035	0.99965	0.02586	0.97414	0.00034	0.00035
14	0.00045	0.99955	0.02629	0.97371	0.00043	0.00045
15	0.00055	0.99945	0.02683	0.97317	0.00054	0.00055
16	0.00063	0.99937	0.02744	0.97256	0.00061	0.00063
17	0.00069	0.99931	0.02811	0.97189	0.00067	0.00069
18	0.00075	0.99925	0.02883	0.97117	0.00072	0.00075
19	0.00079	0.99921	0.02960	0.97040	0.00077	0.00079
20	0.00086	0.99914	0.03043	0.96957	0.00083	0.00086
21	0.00093	0.99907	0.03134	0.96866	0.00090	0.00093
22	0.00097	0.99903	0.03228	0.96772	0.00094	0.00097
23	0.00098	0.99902	0.03323	0.96677	0.00095	0.00098
24	0.00096	0.99904	0.03415	0.96585	0.00092	0.00096
25	0.00092	0.99908	0.03504	0.96496	0.00089	0.00092

Table A.18: Estimated Values for TRH 2010, Male (Continued)

Age	q_x	p_x	$F(x)$	$S(x)$	$f(x)$	λ_x
26	0.00091	0.99909	0.03592	0.96408	0.00087	0.00091
27	0.00091	0.99909	0.03679	0.96321	0.00087	0.00091
28	0.00092	0.99908	0.03768	0.96232	0.00089	0.00093
29	0.00096	0.99904	0.03860	0.96140	0.00092	0.00096
30	0.00097	0.99903	0.03954	0.96046	0.00094	0.00097
31	0.00098	0.99902	0.04048	0.95952	0.00094	0.00098
32	0.00101	0.99899	0.04145	0.95855	0.00097	0.00101
33	0.00107	0.99893	0.04247	0.95753	0.00102	0.00107
34	0.00115	0.99885	0.04357	0.95643	0.00110	0.00115
35	0.00120	0.99880	0.04472	0.95528	0.00115	0.00120
36	0.00124	0.99876	0.04591	0.95409	0.00118	0.00124
37	0.00132	0.99868	0.04717	0.95283	0.00126	0.00132
38	0.00146	0.99854	0.04855	0.95145	0.00139	0.00146
39	0.00164	0.99836	0.05012	0.94988	0.00156	0.00165
40	0.00177	0.99823	0.05180	0.94820	0.00168	0.00177
41	0.00186	0.99814	0.05357	0.94643	0.00177	0.00187
42	0.00205	0.99795	0.05551	0.94449	0.00194	0.00205
43	0.00233	0.99767	0.05771	0.94229	0.00220	0.00234
44	0.00271	0.99729	0.06027	0.93973	0.00256	0.00272
45	0.00302	0.99698	0.06311	0.93689	0.00284	0.00303
46	0.00327	0.99673	0.06618	0.93382	0.00307	0.00329
47	0.00364	0.99636	0.06957	0.93043	0.00340	0.00365
48	0.00411	0.99589	0.07340	0.92660	0.00383	0.00413
49	0.00470	0.99530	0.07776	0.92224	0.00436	0.00473
50	0.00517	0.99483	0.08253	0.91747	0.00477	0.00520

Table A.19: Estimated Values for TRH 2010, Male (Continued)

Age	q_x	p_x	$F(x)$	$S(x)$	$f(x)$	λ_x
51	0.00557	0.99443	0.08764	0.91236	0.00511	0.00560
52	0.00618	0.99382	0.09328	0.90672	0.00564	0.00622
53	0.00700	0.99300	0.09963	0.90037	0.00635	0.00705
54	0.00805	0.99195	0.10688	0.89312	0.00724	0.00811
55	0.00901	0.99099	0.11492	0.88508	0.00805	0.00909
56	0.00986	0.99014	0.12365	0.87635	0.00873	0.00996
57	0.01089	0.98911	0.13320	0.86680	0.00955	0.01101
58	0.01212	0.98788	0.14370	0.85630	0.01051	0.01227
59	0.01356	0.98644	0.15532	0.84468	0.01161	0.01375
60	0.01489	0.98511	0.16789	0.83211	0.01258	0.01512
61	0.01617	0.98383	0.18134	0.81866	0.01345	0.01643
62	0.01776	0.98224	0.19588	0.80412	0.01454	0.01808
63	0.01969	0.98031	0.21171	0.78829	0.01583	0.02008
64	0.02200	0.97800	0.22906	0.77094	0.01734	0.02250
65	0.02436	0.97564	0.24784	0.75216	0.01878	0.02497
66	0.02669	0.97331	0.26792	0.73208	0.02008	0.02743
67	0.02936	0.97064	0.28941	0.71059	0.02150	0.03025
68	0.03241	0.96759	0.31244	0.68756	0.02303	0.03350
69	0.03591	0.96409	0.33713	0.66287	0.02469	0.03725
70	0.04001	0.95999	0.36366	0.63634	0.02652	0.04168
71	0.04445	0.95555	0.39194	0.60806	0.02828	0.04651
72	0.04900	0.95100	0.42173	0.57827	0.02979	0.05152
73	0.05370	0.94630	0.45278	0.54722	0.03105	0.05674
74	0.05858	0.94142	0.48484	0.51516	0.03206	0.06223
75	0.06533	0.93467	0.51849	0.48151	0.03365	0.06989

Table A.20: Estimated Values for TRH 2010, Male (Continued)

Age	q_x	p_x	$F(x)$	$S(x)$	$f(x)$	λ_x
76	0.07359	0.92641	0.55392	0.44608	0.03543	0.07943
77	0.08144	0.91856	0.59025	0.40975	0.03633	0.08866
78	0.08867	0.91133	0.62659	0.37341	0.03633	0.09730
79	0.09491	0.90509	0.66203	0.33797	0.03544	0.10486
80	0.10432	0.89568	0.69728	0.30272	0.03526	0.11647
81	0.11785	0.88215	0.73296	0.26704	0.03567	0.13359
82	0.13110	0.86890	0.76797	0.23203	0.03501	0.15088
83	0.14332	0.85668	0.80122	0.19878	0.03326	0.16730
84	0.15300	0.84700	0.83164	0.16836	0.03041	0.18064
85	0.16565	0.83435	0.85952	0.14048	0.02789	0.19853
86	0.18568	0.81432	0.88561	0.11439	0.02608	0.22802
87	0.20797	0.79203	0.90940	0.09060	0.02379	0.26258
88	0.23194	0.76806	0.93041	0.06959	0.02101	0.30198
89	0.25539	0.74461	0.94818	0.05182	0.01777	0.34299
90	0.27258	0.72742	0.96231	0.03769	0.01412	0.37472
91	0.28836	0.71164	0.97318	0.02682	0.01087	0.40520
92	0.31118	0.68882	0.98152	0.01848	0.00835	0.45177
93	0.35237	0.64763	0.98803	0.01197	0.00651	0.54410
94	0.44277	0.55723	0.99333	0.00667	0.00530	0.79459
95	0.55038	0.44962	0.99700	0.00300	0.00367	1.22410
96	0.60304	0.39696	0.99881	0.00119	0.00181	1.51915
97	0.63382	0.36618	0.99956	0.00044	0.00075	1.73093
98	0.69150	0.30850	0.99987	0.00013	0.00030	2.24149

Table A.21: Estimated Values for TRH 2010, Female

Age	q_x	p_x	$F(x)$	$S(x)$	$f(x)$	λ_x
0	0.00819	0.99181	0.00819	0.99181	0.00819	0.00826
1	0.00028	0.99972	0.00847	0.99153	0.00028	0.00028
2	0.00024	0.99976	0.00870	0.99130	0.00023	0.00024
3	0.00020	0.99980	0.00890	0.99110	0.00020	0.00020
4	0.00019	0.99981	0.00909	0.99091	0.00019	0.00019
5	0.00014	0.99986	0.00923	0.99077	0.00014	0.00014
6	0.00012	0.99988	0.00935	0.99065	0.00011	0.00012
7	0.00010	0.99990	0.00944	0.99056	0.00010	0.00010
8	0.00009	0.99991	0.00954	0.99046	0.00009	0.00009
9	0.00010	0.99990	0.00963	0.99037	0.00010	0.00010
10	0.00009	0.99991	0.00973	0.99027	0.00009	0.00009
11	0.00008	0.99992	0.00981	0.99019	0.00008	0.00008
12	0.00008	0.99992	0.00989	0.99011	0.00008	0.00008
13	0.00010	0.99990	0.00999	0.99001	0.00009	0.00010
14	0.00012	0.99988	0.01010	0.98990	0.00011	0.00012
15	0.00014	0.99986	0.01023	0.98977	0.00013	0.00014
16	0.00015	0.99985	0.01038	0.98962	0.00015	0.00015
17	0.00016	0.99984	0.01054	0.98946	0.00016	0.00016
18	0.00018	0.99982	0.01072	0.98928	0.00018	0.00018
19	0.00019	0.99981	0.01091	0.98909	0.00019	0.00019
20	0.00021	0.99979	0.01111	0.98889	0.00021	0.00021
21	0.00023	0.99977	0.01134	0.98866	0.00022	0.00023
22	0.00024	0.99976	0.01158	0.98842	0.00024	0.00024
23	0.00026	0.99974	0.01183	0.98817	0.00025	0.00026
24	0.00027	0.99973	0.01210	0.98790	0.00027	0.00027
25	0.00028	0.99972	0.01238	0.98762	0.00028	0.00028

Table A.22: Estimated Values for TRH 2010, Female (Continued)

Age	q_x	p_x	$F(x)$	$S(x)$	$f(x)$	λ_x
26	0.00030	0.99970	0.01267	0.98733	0.00029	0.00030
27	0.00031	0.99969	0.01297	0.98703	0.00031	0.00031
28	0.00033	0.99967	0.01330	0.98670	0.00033	0.00033
29	0.00036	0.99964	0.01365	0.98635	0.00035	0.00036
30	0.00037	0.99963	0.01402	0.98598	0.00037	0.00037
31	0.00039	0.99961	0.01440	0.98560	0.00038	0.00039
32	0.00041	0.99959	0.01480	0.98520	0.00041	0.00041
33	0.00045	0.99955	0.01525	0.98475	0.00044	0.00045
34	0.00050	0.99950	0.01574	0.98426	0.00049	0.00050
35	0.00054	0.99946	0.01627	0.98373	0.00053	0.00054
36	0.00056	0.99944	0.01682	0.98318	0.00055	0.00056
37	0.00061	0.99939	0.01742	0.98258	0.00060	0.00061
38	0.00069	0.99931	0.01810	0.98190	0.00068	0.00069
39	0.00079	0.99921	0.01888	0.98112	0.00078	0.00079
40	0.00086	0.99914	0.01973	0.98027	0.00084	0.00086
41	0.00091	0.99909	0.02062	0.97938	0.00089	0.00091
42	0.00102	0.99898	0.02161	0.97839	0.00099	0.00102
43	0.00117	0.99883	0.02276	0.97724	0.00115	0.00118
44	0.00139	0.99861	0.02412	0.97588	0.00136	0.00139
45	0.00158	0.99842	0.02566	0.97434	0.00154	0.00158
46	0.00173	0.99827	0.02734	0.97266	0.00168	0.00173
47	0.00192	0.99808	0.02921	0.97079	0.00187	0.00192
48	0.00216	0.99784	0.03130	0.96870	0.00209	0.00216
49	0.00244	0.99756	0.03366	0.96634	0.00236	0.00244
50	0.00265	0.99735	0.03622	0.96378	0.00256	0.00266

Table A.23: Estimated Values for TRH 2010, Female (Continued)

Age	q_x	p_x	$F(x)$	$S(x)$	$f(x)$	λ_x
51	0.00284	0.99716	0.03896	0.96104	0.00274	0.00285
52	0.00313	0.99687	0.04198	0.95802	0.00301	0.00314
53	0.00352	0.99648	0.04535	0.95465	0.00338	0.00354
54	0.00401	0.99599	0.04918	0.95082	0.00383	0.00403
55	0.00441	0.99559	0.05338	0.94662	0.00419	0.00443
56	0.00475	0.99525	0.05787	0.94213	0.00449	0.00477
57	0.00524	0.99476	0.06281	0.93719	0.00494	0.00527
58	0.00589	0.99411	0.06833	0.93167	0.00552	0.00593
59	0.00671	0.99329	0.07458	0.92542	0.00625	0.00676
60	0.00728	0.99272	0.08131	0.91869	0.00673	0.00733
61	0.00775	0.99225	0.08844	0.91156	0.00712	0.00781
62	0.00865	0.99135	0.09632	0.90368	0.00788	0.00872
63	0.00997	0.99003	0.10533	0.89467	0.00901	0.01007
64	0.01175	0.98825	0.11585	0.88415	0.01052	0.01189
65	0.01331	0.98669	0.12761	0.87239	0.01177	0.01349
66	0.01464	0.98536	0.14038	0.85962	0.01277	0.01486
67	0.01647	0.98353	0.15454	0.84546	0.01416	0.01675
68	0.01885	0.98115	0.17048	0.82952	0.01593	0.01921
69	0.02182	0.97818	0.18858	0.81142	0.01810	0.02230
70	0.02476	0.97524	0.20867	0.79133	0.02009	0.02539
71	0.02760	0.97240	0.23051	0.76949	0.02184	0.02838
72	0.03099	0.96901	0.25436	0.74564	0.02385	0.03198
73	0.03504	0.96496	0.28049	0.71951	0.02613	0.03631
74	0.03986	0.96014	0.30916	0.69084	0.02868	0.04151
75	0.04595	0.95405	0.34091	0.65909	0.03175	0.04817

Table A.24: Estimated Values for TRH 2010, Female (Continued)

Age	q_x	p_x	$F(x)$	$S(x)$	$f(x)$	λ_x
76	0.05258	0.94742	0.37557	0.62443	0.03466	0.05550
77	0.05895	0.94105	0.41237	0.58763	0.03681	0.06264
78	0.06500	0.93500	0.45057	0.54943	0.03819	0.06952
79	0.07063	0.92937	0.48938	0.51062	0.03881	0.07600
80	0.07889	0.92111	0.52966	0.47034	0.04029	0.08565
81	0.08993	0.91007	0.57196	0.42804	0.04230	0.09881
82	0.10048	0.89952	0.61497	0.38503	0.04301	0.11170
83	0.11013	0.88987	0.65737	0.34263	0.04240	0.12376
84	0.11812	0.88188	0.69784	0.30216	0.04047	0.13394
85	0.13054	0.86946	0.73729	0.26271	0.03944	0.15014
86	0.14958	0.85042	0.77658	0.22342	0.03930	0.17589
87	0.16901	0.83099	0.81434	0.18566	0.03776	0.20338
88	0.18755	0.81245	0.84916	0.15084	0.03482	0.23085
89	0.20217	0.79783	0.87966	0.12034	0.03049	0.25340
90	0.21682	0.78318	0.90575	0.09425	0.02609	0.27684
91	0.24010	0.75990	0.92838	0.07162	0.02263	0.31595
92	0.26995	0.73005	0.94771	0.05229	0.01933	0.36976
93	0.30988	0.69012	0.96392	0.03608	0.01620	0.44903
94	0.36663	0.63337	0.97715	0.02285	0.01323	0.57885
95	0.44437	0.55563	0.98730	0.01270	0.01016	0.79977
96	0.55225	0.44775	0.99431	0.00569	0.00701	1.23338
97	0.71983	0.28017	0.99841	0.00159	0.00409	2.56925
98	0.94844	0.05156	0.99992	0.00008	0.00151	18.39617

Table A.25: Estimated Values for SGK 2008, Male

Age	q_x	p_x	$F(x)$	$S(x)$	$f(x)$	λ_x
14	0.00006	0.99994	0.00006	0.99994	0.00006	0.00006
15	0.00024	0.99976	0.00030	0.99970	0.00024	0.00024
16	0.00037	0.99963	0.00067	0.99933	0.00037	0.00037
17	0.00046	0.99954	0.00113	0.99887	0.00046	0.00046
18	0.00053	0.99947	0.00166	0.99834	0.00052	0.00053
19	0.00056	0.99944	0.00222	0.99778	0.00056	0.00056
20	0.00058	0.99942	0.00279	0.99721	0.00057	0.00058
21	0.00059	0.99941	0.00338	0.99662	0.00059	0.00059
22	0.00061	0.99939	0.00400	0.99600	0.00061	0.00061
23	0.00063	0.99937	0.00462	0.99538	0.00062	0.00063
24	0.00064	0.99936	0.00526	0.99474	0.00064	0.00064
25	0.00066	0.99934	0.00591	0.99409	0.00065	0.00066
26	0.00067	0.99933	0.00658	0.99342	0.00067	0.00067
27	0.00068	0.99932	0.00726	0.99274	0.00067	0.00068
28	0.00068	0.99932	0.00793	0.99207	0.00067	0.00068
29	0.00068	0.99932	0.00860	0.99140	0.00068	0.00068
30	0.00070	0.99930	0.00930	0.99070	0.00070	0.00070
31	0.00074	0.99926	0.01003	0.98997	0.00073	0.00074
32	0.00078	0.99922	0.01080	0.98920	0.00078	0.00078
33	0.00084	0.99916	0.01164	0.98836	0.00084	0.00085
34	0.00091	0.99909	0.01254	0.98746	0.00090	0.00091
35	0.00097	0.99903	0.01350	0.98650	0.00096	0.00097
36	0.00103	0.99897	0.01451	0.98549	0.00101	0.00103
37	0.00109	0.99891	0.01559	0.98441	0.00108	0.00109
38	0.00118	0.99882	0.01675	0.98325	0.00116	0.00118
39	0.00129	0.99871	0.01801	0.98199	0.00127	0.00129

Table A.26: Estimated Values for SGK 2008, Male (Continued)

Age	q_x	p_x	$F(x)$	$S(x)$	$f(x)$	λ_x
40	0.00144	0.99856	0.01942	0.98058	0.00141	0.00144
41	0.00162	0.99838	0.02101	0.97899	0.00159	0.00162
42	0.00184	0.99816	0.02281	0.97719	0.00180	0.00184
43	0.00209	0.99791	0.02485	0.97515	0.00204	0.00209
44	0.00236	0.99764	0.02715	0.97285	0.00230	0.00237
45	0.00268	0.99732	0.02976	0.97024	0.00261	0.00269
46	0.00302	0.99698	0.03268	0.96732	0.00293	0.00302
47	0.00338	0.99662	0.03595	0.96405	0.00327	0.00339
48	0.00378	0.99622	0.03959	0.96041	0.00364	0.00379
49	0.00422	0.99578	0.04365	0.95635	0.00405	0.00424
50	0.00471	0.99529	0.04815	0.95185	0.00450	0.00473
51	0.00524	0.99476	0.05314	0.94686	0.00499	0.00527
52	0.00583	0.99417	0.05866	0.94134	0.00552	0.00587
53	0.00648	0.99352	0.06477	0.93523	0.00610	0.00652
54	0.00719	0.99281	0.07149	0.92851	0.00673	0.00724
55	0.00795	0.99205	0.07888	0.92112	0.00739	0.00802
56	0.00882	0.99118	0.08700	0.91300	0.00812	0.00890
57	0.00977	0.99023	0.09592	0.90408	0.00892	0.00987
58	0.01083	0.98917	0.10570	0.89430	0.00979	0.01094
59	0.01195	0.98805	0.11640	0.88360	0.01069	0.01210
60	0.01319	0.98681	0.12805	0.87195	0.01165	0.01336
61	0.01456	0.98544	0.14074	0.85926	0.01269	0.01477
62	0.01612	0.98388	0.15460	0.84540	0.01385	0.01639
63	0.01786	0.98214	0.16970	0.83030	0.01510	0.01819
64	0.01972	0.98028	0.18608	0.81392	0.01638	0.02012

Table A.27: Estimated Values for SGK 2008, Male (Continued)

Age	q_x	p_x	$F(x)$	$S(x)$	$f(x)$	λ_x
65	0.02169	0.97831	0.20373	0.79627	0.01766	0.02217
66	0.02379	0.97621	0.22268	0.77732	0.01894	0.02437
67	0.02616	0.97384	0.24301	0.75699	0.02033	0.02686
68	0.02879	0.97121	0.26480	0.73520	0.02179	0.02964
69	0.03170	0.96830	0.28811	0.71189	0.02330	0.03273
70	0.03489	0.96511	0.31294	0.68706	0.02484	0.03615
71	0.03843	0.96157	0.33935	0.66065	0.02640	0.03996
72	0.04248	0.95752	0.36741	0.63259	0.02807	0.04437
73	0.04696	0.95304	0.39712	0.60288	0.02971	0.04928
74	0.05194	0.94806	0.42843	0.57157	0.03131	0.05478
75	0.05730	0.94270	0.46118	0.53882	0.03275	0.06079
76	0.06328	0.93672	0.49528	0.50472	0.03410	0.06756
77	0.06989	0.93011	0.53056	0.46944	0.03528	0.07514
78	0.07735	0.92265	0.56687	0.43313	0.03631	0.08384
79	0.08545	0.91455	0.60388	0.39612	0.03701	0.09344
80	0.09451	0.90549	0.64132	0.35868	0.03744	0.10437
81	0.10476	0.89524	0.67890	0.32110	0.03758	0.11702
82	0.11699	0.88301	0.71646	0.28354	0.03757	0.13249
83	0.13083	0.86917	0.75356	0.24644	0.03709	0.15052
84	0.14594	0.85406	0.78952	0.21048	0.03597	0.17088
85	0.16167	0.83833	0.82355	0.17645	0.03403	0.19285
86	0.17858	0.82142	0.85506	0.14494	0.03151	0.21740
87	0.19704	0.80296	0.88362	0.11638	0.02856	0.24540
88	0.21815	0.78185	0.90901	0.09099	0.02539	0.27902
89	0.24255	0.75745	0.93108	0.06892	0.02207	0.32022

Table A.28: Estimated Values for SGK 2008, Male (Continued)

Age	q_x	p_x	$F(x)$	$S(x)$	$f(x)$	λ_x
90	0.27116	0.72884	0.94977	0.05023	0.01869	0.37204
91	0.30438	0.69562	0.96506	0.03494	0.01529	0.43756
92	0.33959	0.66041	0.97692	0.02308	0.01187	0.51422
93	0.37518	0.62482	0.98558	0.01442	0.00866	0.60047
94	0.40863	0.59137	0.99147	0.00853	0.00589	0.69100
95	0.44359	0.55641	0.99526	0.00474	0.00378	0.79724
96	0.48094	0.51906	0.99754	0.00246	0.00228	0.92656
97	0.52189	0.47811	0.99882	0.00118	0.00129	1.09156
98	0.56446	0.43554	0.99949	0.00051	0.00066	1.29600
99	0.60789	0.39211	0.99980	0.00020	0.00031	1.55028
100	0.65164	0.34836	0.99993	0.00007	0.00013	1.87058
101	0.69510	0.30490	0.99998	0.00002	0.00005	2.27978
102	0.73760	0.26240	0.99999	0.00001	0.00002	2.81093
103	0.77841	0.22159	1.00000	0.00000	0.00000	3.51292
104	0.81684	0.18316	1.00000	0.00000	0.00000	4.45963
105	0.85220	0.14780	1.00000	0.00000	0.00000	5.76584
106	0.88392	0.11608	1.00000	0.00000	0.00000	7.61495
107	0.91158	0.08842	1.00000	0.00000	0.00000	10.30935
108	0.93492	0.06508	1.00000	0.00000	0.00000	14.36578
109	0.95392	0.04608	1.00000	0.00000	0.00000	20.69316
110	0.96877	0.03123	1.00000	0.00000	0.00000	30.60678

Table A.29: Estimated Values for SGK 2008, Female

Age	q_x	p_x	$F(x)$	$S(x)$	$f(x)$	λ_x
14	0.00011	0.99989	0.00011	0.99989	0.00011	0.00011
15	0.00012	0.99988	0.00022	0.99978	0.00012	0.00012
16	0.00013	0.99987	0.00035	0.99965	0.00013	0.00013
17	0.00015	0.99985	0.00050	0.99950	0.00014	0.00015
18	0.00016	0.99984	0.00066	0.99934	0.00016	0.00016
19	0.00018	0.99982	0.00084	0.99916	0.00018	0.00018
20	0.00020	0.99980	0.00104	0.99896	0.00020	0.00020
21	0.00022	0.99978	0.00125	0.99875	0.00022	0.00022
22	0.00022	0.99978	0.00147	0.99853	0.00022	0.00022
23	0.00021	0.99979	0.00169	0.99831	0.00021	0.00021
24	0.00020	0.99980	0.00189	0.99811	0.00020	0.00020
25	0.00020	0.99980	0.00208	0.99792	0.00019	0.00020
26	0.00020	0.99980	0.00228	0.99772	0.00020	0.00020
27	0.00022	0.99978	0.00250	0.99750	0.00022	0.00022
28	0.00024	0.99976	0.00274	0.99726	0.00024	0.00024
29	0.00025	0.99975	0.00299	0.99701	0.00025	0.00025
30	0.00027	0.99973	0.00326	0.99674	0.00027	0.00027
31	0.00031	0.99969	0.00357	0.99643	0.00031	0.00031
32	0.00034	0.99966	0.00390	0.99610	0.00034	0.00034
33	0.00036	0.99964	0.00426	0.99574	0.00036	0.00036
34	0.00037	0.99963	0.00463	0.99537	0.00037	0.00037
35	0.00039	0.99961	0.00501	0.99499	0.00039	0.00039
36	0.00041	0.99959	0.00542	0.99458	0.00041	0.00041
37	0.00046	0.99954	0.00588	0.99412	0.00045	0.00046
38	0.00051	0.99949	0.00638	0.99362	0.00050	0.00051
39	0.00057	0.99943	0.00694	0.99306	0.00056	0.00057

Table A.30: Estimated Values for SGK 2008, Female (Continued)

Age	q_x	p_x	$F(x)$	$S(x)$	$f(x)$	λ_x
40	0.00062	0.99938	0.00756	0.99244	0.00062	0.00062
41	0.00069	0.99931	0.00825	0.99175	0.00069	0.00069
42	0.00078	0.99922	0.00902	0.99098	0.00077	0.00078
43	0.00089	0.99911	0.00990	0.99010	0.00088	0.00089
44	0.00103	0.99897	0.01092	0.98908	0.00102	0.00103
45	0.00119	0.99881	0.01210	0.98790	0.00118	0.00119
46	0.00134	0.99866	0.01342	0.98658	0.00133	0.00134
47	0.00148	0.99852	0.01489	0.98511	0.00146	0.00149
48	0.00163	0.99837	0.01649	0.98351	0.00160	0.00163
49	0.00178	0.99822	0.01824	0.98176	0.00175	0.00178
50	0.00195	0.99805	0.02016	0.97984	0.00192	0.00196
51	0.00216	0.99784	0.02227	0.97773	0.00211	0.00216
52	0.00239	0.99761	0.02460	0.97540	0.00233	0.00239
53	0.00263	0.99737	0.02717	0.97283	0.00257	0.00264
54	0.00290	0.99710	0.02999	0.97001	0.00282	0.00291
55	0.00318	0.99682	0.03307	0.96693	0.00308	0.00319
56	0.00348	0.99652	0.03644	0.96356	0.00337	0.00349
57	0.00382	0.99618	0.04012	0.95988	0.00368	0.00384
58	0.00422	0.99578	0.04417	0.95583	0.00405	0.00424
59	0.00464	0.99536	0.04861	0.95139	0.00444	0.00467
60	0.00513	0.99487	0.05349	0.94651	0.00488	0.00515
61	0.00566	0.99434	0.05885	0.94115	0.00536	0.00570
62	0.00630	0.99370	0.06478	0.93522	0.00593	0.00634
63	0.00703	0.99297	0.07136	0.92864	0.00657	0.00708
64	0.00783	0.99217	0.07863	0.92137	0.00727	0.00789

Table A.31: Estimated Values for SGK 2008, Female (Continued)

Age	q_x	p_x	$F(x)$	$S(x)$	$f(x)$	λ_x
65	0.00871	0.99129	0.08666	0.91334	0.00803	0.00879
66	0.00970	0.99030	0.09552	0.90448	0.00886	0.00980
67	0.01086	0.98914	0.10534	0.89466	0.00982	0.01098
68	0.01220	0.98780	0.11625	0.88375	0.01091	0.01235
69	0.01372	0.98628	0.12838	0.87162	0.01212	0.01391
70	0.01545	0.98455	0.14184	0.85816	0.01346	0.01569
71	0.01743	0.98257	0.15679	0.84321	0.01495	0.01773
72	0.01976	0.98024	0.17346	0.82654	0.01666	0.02016
73	0.02241	0.97759	0.19197	0.80803	0.01852	0.02292
74	0.02541	0.97459	0.21250	0.78750	0.02053	0.02607
75	0.02870	0.97130	0.23511	0.76489	0.02260	0.02955
76	0.03243	0.96757	0.25992	0.74008	0.02481	0.03352
77	0.03662	0.96338	0.28702	0.71298	0.02710	0.03801
78	0.04146	0.95854	0.31658	0.68342	0.02956	0.04326
79	0.04688	0.95312	0.34862	0.65138	0.03204	0.04918
80	0.05311	0.94689	0.38321	0.61679	0.03459	0.05609
81	0.06020	0.93980	0.42034	0.57966	0.03713	0.06406
82	0.06852	0.93148	0.46006	0.53994	0.03972	0.07356
83	0.07775	0.92225	0.50204	0.49796	0.04198	0.08430
84	0.08789	0.91211	0.54581	0.45419	0.04377	0.09636
85	0.09880	0.90120	0.59068	0.40932	0.04488	0.10964
86	0.11089	0.88911	0.63607	0.36393	0.04539	0.12471
87	0.12438	0.87562	0.68134	0.31866	0.04527	0.14205
88	0.13946	0.86054	0.72578	0.27422	0.04444	0.16206
89	0.15654	0.84346	0.76870	0.23130	0.04293	0.18559

Table A.32: Estimated Values for SGK 2008, Female (Continued)

Age	q_x	p_x	$F(x)$	$S(x)$	$f(x)$	λ_x
90	0.17593	0.82407	0.80939	0.19061	0.04069	0.21348
91	0.19889	0.80111	0.84730	0.15270	0.03791	0.24827
92	0.22479	0.77521	0.88163	0.11837	0.03432	0.28997
93	0.25271	0.74729	0.91154	0.08846	0.02991	0.33816
94	0.28049	0.71951	0.93635	0.06365	0.02481	0.38983
95	0.30931	0.69069	0.95604	0.04396	0.01969	0.44784
96	0.34081	0.65919	0.97102	0.02898	0.01498	0.51700
97	0.37609	0.62391	0.98192	0.01808	0.01090	0.60280
98	0.41454	0.58546	0.98942	0.01058	0.00749	0.70805
99	0.45530	0.54470	0.99423	0.00577	0.00482	0.83589
100	0.49814	0.50186	0.99711	0.00289	0.00287	0.99258
101	0.54268	0.45732	0.99868	0.00132	0.00157	1.18667
102	0.58846	0.41154	0.99946	0.00054	0.00078	1.42992
103	0.63489	0.36511	0.99980	0.00020	0.00035	1.73889
104	0.68126	0.31874	0.99994	0.00006	0.00014	2.13737
105	0.72679	0.27321	0.99998	0.00002	0.00005	2.66025
106	0.77064	0.22936	1.00000	0.00000	0.00001	3.35993
107	0.81194	0.18806	1.00000	0.00000	0.00000	4.31732
108	0.84987	0.15013	1.00000	0.00000	0.00000	5.66082
109	0.88374	0.11626	1.00000	0.00000	0.00000	7.60110
110	0.91301	0.08699	1.00000	0.00000	0.00000	10.49598

APPENDIX B

Estimated λ_{ij} Values for IHD

Table B.1: λ_{ij} Values for Male

Age	λ_{11}	λ_{12}	λ_{13}	λ_{22}	λ_{23}
0	-0.01583	0.00055	0.01528	-0.18651	0.18651
1	-0.00053	0.00053	0.00000	-0.19048	0.19048
2	-0.00136	0.00053	0.00083	-0.17108	0.17108
3	-0.00128	0.00054	0.00074	-0.17801	0.17801
4	-0.00121	0.00055	0.00067	-0.16071	0.16071
5	-0.00112	0.00056	0.00057	-0.16279	0.16279
6	-0.00116	0.00056	0.00060	-0.16747	0.16747
7	-0.00111	0.00055	0.00055	-0.16618	0.16618
8	-0.00100	0.00052	0.00048	-0.17403	0.17403
9	-0.00091	0.00051	0.00040	-0.16853	0.16853
10	-0.00085	0.00051	0.00034	-0.17910	0.17910
11	-0.00087	0.00051	0.00036	-0.17143	0.17143
12	-0.00084	0.00052	0.00032	-0.17221	0.17221
13	-0.00095	0.00054	0.00042	-0.16395	0.16395
14	-0.00098	0.00054	0.00044	-0.17856	0.17856
15	-0.00100	0.00053	0.00047	-0.16365	0.16365
16	-0.00112	0.00054	0.00058	-0.15906	0.15906
17	-0.00121	0.00056	0.00065	-0.16865	0.16865
18	-0.00127	0.00056	0.00071	-0.16695	0.16695
19	-0.00132	0.00056	0.00076	-0.15731	0.15731
20	-0.00112	0.00055	0.00057	-0.15112	0.15112
21	-0.00123	0.00056	0.00067	-0.16004	0.16004
22	-0.00129	0.00056	0.00073	-0.16062	0.16062
23	-0.00127	0.00055	0.00072	-0.15365	0.15365
24	-0.00123	0.00056	0.00067	-0.15584	0.15584
25	-0.00127	0.00055	0.00071	-0.16048	0.16048

Table B.2: λ_{ij} Values for Male (Continued)

Age	λ_{11}	λ_{12}	λ_{13}	λ_{22}	λ_{23}
26	-0.00137	0.00056	0.00081	-0.16190	0.16190
27	-0.00133	0.00056	0.00077	-0.15370	0.15370
28	-0.00126	0.00051	0.00075	-0.14822	0.14822
29	-0.00125	0.00053	0.00072	-0.15037	0.15037
30	-0.00134	0.00055	0.00078	-0.15067	0.15067
31	-0.00138	0.00057	0.00081	-0.14933	0.14933
32	-0.00130	0.00059	0.00071	-0.14311	0.14311
33	-0.00146	0.00064	0.00082	-0.13996	0.13996
34	-0.00161	0.00068	0.00093	-0.14040	0.14040
35	-0.00170	0.00061	0.00108	-0.13481	0.13481
36	-0.00160	0.00063	0.00098	-0.13373	0.13373
37	-0.00178	0.00067	0.00112	-0.12832	0.12832
38	-0.00192	0.00070	0.00122	-0.12106	0.12106
39	-0.00200	0.00071	0.00128	-0.11401	0.11401
40	-0.00217	0.00073	0.00144	-0.11283	0.11283
41	-0.00225	0.00078	0.00146	-0.10816	0.10816
42	-0.00278	0.00090	0.00188	-0.10261	0.10261
43	-0.00334	0.00084	0.00250	-0.09979	0.09979
44	-0.00306	0.00066	0.00239	-0.09527	0.09527
45	-0.00319	0.00071	0.00248	-0.09203	0.09203
46	-0.00367	0.00081	0.00286	-0.08724	0.08724
47	-0.00407	0.00083	0.00324	-0.08378	0.08378
48	-0.00469	0.00094	0.00376	-0.08031	0.08031
49	-0.00510	0.00083	0.00426	-0.07789	0.07789
50	-0.00525	0.00087	0.00438	-0.07607	0.07607

Table B.3: λ_{ij} Values for Male (Continued)

Age	λ_{11}	λ_{12}	λ_{13}	λ_{22}	λ_{23}
51	-0.00582	0.00104	0.00479	-0.07287	0.07287
52	-0.00695	0.00116	0.00579	-0.07013	0.07013
53	-0.00886	0.00103	0.00783	-0.06840	0.06840
54	-0.00701	0.00082	0.00619	-0.06542	0.06542
55	-0.00808	0.00102	0.00706	-0.06492	0.06492
56	-0.00981	0.00118	0.00863	-0.06412	0.06412
57	-0.00989	0.00123	0.00866	-0.06618	0.06618
58	-0.01245	0.00148	0.01097	-0.06210	0.06210
59	-0.01427	0.00121	0.01305	-0.06130	0.06130
60	-0.01166	0.00118	0.01048	-0.06028	0.06028
61	-0.01475	0.00148	0.01328	-0.05896	0.05896
62	-0.01540	0.00160	0.01380	-0.05850	0.05850
63	-0.01952	0.00180	0.01773	-0.05886	0.05886
64	-0.01857	0.00155	0.01702	-0.05843	0.05843
65	-0.01988	0.00175	0.01813	-0.05614	0.05614
66	-0.02427	0.00196	0.02231	-0.05663	0.05663
67	-0.02429	0.00182	0.02247	-0.05680	0.05680
68	-0.02894	0.00211	0.02683	-0.05773	0.05773
69	-0.03055	0.00192	0.02863	-0.05838	0.05838
70	-0.03292	0.00200	0.03092	-0.05896	0.05896
71	-0.03530	0.00199	0.03330	-0.05915	0.05915
72	-0.03665	0.00215	0.03450	-0.05864	0.05864
73	-0.04047	0.00243	0.03805	-0.05867	0.05867
74	-0.04924	0.00281	0.04644	-0.06222	0.06222
75	-0.00779	0.00028	0.00751	-0.06324	0.06324

Table B.4: λ_{ij} Values for Male (Continued)

Age	λ_{11}	λ_{12}	λ_{13}	λ_{22}	λ_{23}
76	-0.03453	0.00110	0.03343	-0.06341	0.06341
77	-0.02820	0.00107	0.02714	-0.06532	0.06532
78	-0.02478	0.00104	0.02374	-0.06328	0.06328
79	-0.02467	0.00101	0.02366	-0.06570	0.06570
80	-0.02240	0.00099	0.02141	-0.06803	0.06803
81	-0.02195	0.00097	0.02098	-0.07054	0.07054
82	-0.02044	0.00095	0.01950	-0.06826	0.06826
83	-0.01889	0.00093	0.01796	-0.06949	0.06949
84	-0.01634	0.00091	0.01543	-0.07093	0.07093
85	-0.01367	0.00090	0.01277	-0.07741	0.07741
86	-0.01090	0.00089	0.01001	-0.07534	0.07534
87	-0.00919	0.00088	0.00831	-0.07412	0.07412
88	-0.00710	0.00088	0.00622	-0.07998	0.07998
89	-0.00555	0.00087	0.00468	-0.08543	0.08543
90	-0.00416	0.00087	0.00329	-0.09580	0.09580
91	-0.00301	0.00087	0.00214	-0.10329	0.10329
92	-0.00273	0.00087	0.00186	-0.09854	0.09854
93	-0.00284	0.00087	0.00198	-0.11395	0.11395
94	-0.00248	0.00087	0.00161	-0.09467	0.09467
95	-0.00210	0.00087	0.00123	-0.09075	0.09075
96	-0.00156	0.00087	0.00069	-0.16099	0.16099
97	-0.00123	0.00087	0.00037	-0.10294	0.10294
98	-0.00192	0.00087	0.00105	-0.12903	0.12903
99	-0.00321	0.00086	0.00235	-0.11940	0.11940

Table B.5: λ_{ij} Values for Female

Age	λ_{11}	λ_{12}	λ_{13}	λ_{22}	λ_{23}
0	-0.03036	0.00044	0.02993	-0.18222	0.18222
1	-0.00025	0.00025	0.00000	-0.16906	0.16906
2	-0.00205	0.00027	0.00178	-0.16060	0.16060
3	-0.00176	0.00027	0.00149	-0.16034	0.16034
4	-0.00169	0.00027	0.00142	-0.16487	0.16487
5	-0.00160	0.00033	0.00127	-0.15572	0.15572
6	-0.00176	0.00043	0.00133	-0.15467	0.15467
7	-0.00166	0.00041	0.00126	-0.15741	0.15741
8	-0.00142	0.00045	0.00098	-0.16487	0.16487
9	-0.00137	0.00048	0.00089	-0.16822	0.16822
10	-0.00123	0.00046	0.00077	-0.17493	0.17493
11	-0.00131	0.00056	0.00075	-0.16179	0.16179
12	-0.00150	0.00079	0.00071	-0.16174	0.16174
13	-0.00218	0.00138	0.00081	-0.15894	0.15894
14	-0.00286	0.00197	0.00089	-0.16839	0.16839
15	-0.00351	0.00262	0.00088	-0.16963	0.16963
16	-0.00439	0.00338	0.00101	-0.16857	0.16857
17	-0.00485	0.00377	0.00108	-0.16438	0.16438
18	-0.00521	0.00410	0.00111	-0.16091	0.16091
19	-0.00554	0.00440	0.00114	-0.15815	0.15815
20	-0.00551	0.00460	0.00091	-0.16226	0.16226
21	-0.00602	0.00495	0.00107	-0.15655	0.15655
22	-0.00646	0.00540	0.00106	-0.16018	0.16018
23	-0.00650	0.00547	0.00103	-0.16140	0.16140
24	-0.00689	0.00589	0.00100	-0.15924	0.15924
25	-0.00735	0.00631	0.00104	-0.15604	0.15604

Table B.6: λ_{ij} Values for Female (Continued)

Age	λ_{11}	λ_{12}	λ_{13}	λ_{22}	λ_{23}
26	-0.00857	0.00746	0.00111	-0.15911	0.15911
27	-0.00863	0.00750	0.00113	-0.15440	0.15440
28	-0.00787	0.00681	0.00107	-0.15023	0.15023
29	-0.00852	0.00751	0.00101	-0.14695	0.14695
30	-0.00899	0.00790	0.00109	-0.15574	0.15574
31	-0.00930	0.00816	0.00114	-0.15022	0.15022
32	-0.01005	0.00905	0.00100	-0.14875	0.14875
33	-0.01304	0.01196	0.00109	-0.14980	0.14980
34	-0.01444	0.01315	0.00129	-0.14793	0.14793
35	-0.01366	0.01225	0.00141	-0.14568	0.14568
36	-0.01426	0.01296	0.00130	-0.14147	0.14147
37	-0.01645	0.01504	0.00141	-0.14022	0.14022
38	-0.01802	0.01652	0.00150	-0.13860	0.13860
39	-0.01932	0.01770	0.00162	-0.12998	0.12998
40	-0.01905	0.01736	0.00168	-0.12821	0.12821
41	-0.02931	0.02765	0.00166	-0.12603	0.12603
42	-0.03977	0.03781	0.00196	-0.12652	0.12652
43	-0.03902	0.03643	0.00259	-0.12312	0.12312
44	-0.03018	0.02777	0.00240	-0.11841	0.11841
45	-0.03456	0.03222	0.00234	-0.11545	0.11545
46	-0.03855	0.03592	0.00263	-0.11524	0.11524
47	-0.05465	0.05202	0.00263	-0.11436	0.11436
48	-0.05331	0.05032	0.00299	-0.10599	0.10599
49	-0.04397	0.04067	0.00330	-0.10643	0.10643
50	-0.04603	0.04290	0.00313	-0.10621	0.10621

Table B.7: λ_{ij} Values for Female (Continued)

Age	λ_{11}	λ_{12}	λ_{13}	λ_{22}	λ_{23}
51	-0.07507	0.07202	0.00306	-0.10160	0.10160
52	-0.09306	0.08975	0.00331	-0.09864	0.09864
53	-0.07459	0.06998	0.00461	-0.09573	0.09573
54	-0.04823	0.04515	0.00308	-0.09413	0.09413
55	-0.05735	0.05406	0.00329	-0.09421	0.09421
56	-0.05798	0.05421	0.00377	-0.08966	0.08966
57	-0.08791	0.08447	0.00344	-0.08826	0.08826
58	-0.08824	0.08437	0.00387	-0.08335	0.08335
59	-0.06559	0.06141	0.00418	-0.08244	0.08244
60	-0.05234	0.04941	0.00292	-0.08172	0.08172
61	-0.05447	0.05126	0.00321	-0.07906	0.07906
62	-0.06456	0.06160	0.00297	-0.07747	0.07747
63	-0.06397	0.06043	0.00354	-0.07628	0.07628
64	-0.04724	0.04416	0.00309	-0.07652	0.07652
65	-0.05236	0.04956	0.00279	-0.07316	0.07316
66	-0.04416	0.04124	0.00292	-0.07269	0.07269
67	-0.04558	0.04300	0.00258	-0.07301	0.07301
68	-0.04017	0.03756	0.00260	-0.07112	0.07112
69	-0.03248	0.03017	0.00231	-0.06968	0.06968
70	-0.02709	0.02498	0.00211	-0.07009	0.07009
71	-0.02238	0.02047	0.00191	-0.07010	0.07010
72	-0.01815	0.01646	0.00170	-0.07156	0.07156
73	-0.01806	0.01654	0.00152	-0.07078	0.07078
74	-0.02006	0.01863	0.00143	-0.07216	0.07216
75	-0.00563	0.00499	0.00064	-0.07297	0.07297

Table B.8: λ_{ij} Values for Female (Continued)

Age	λ_{11}	λ_{12}	λ_{13}	λ_{22}	λ_{23}
76	-0.00432	0.00365	0.00067	-0.07211	0.07211
77	-0.00404	0.00351	0.00054	-0.07333	0.07333
78	-0.00302	0.00256	0.00046	-0.07338	0.07338
79	-0.00267	0.00222	0.00044	-0.07582	0.07582
80	-0.00220	0.00182	0.00038	-0.07377	0.07377
81	-0.00195	0.00159	0.00036	-0.07773	0.07773
82	-0.00131	0.00098	0.00033	-0.08070	0.08070
83	-0.00117	0.00087	0.00030	-0.07794	0.07794
84	-0.00077	0.00053	0.00024	-0.07751	0.07751
85	-0.00063	0.00042	0.00021	-0.08000	0.08000
86	-0.00043	0.00028	0.00016	-0.08872	0.08872
87	-0.00035	0.00022	0.00013	-0.08014	0.08014
88	-0.00028	0.00018	0.00010	-0.09071	0.09071
89	-0.00020	0.00011	0.00009	-0.08616	0.08616
90	-0.00015	0.00008	0.00007	-0.08429	0.08429
91	-0.00011	0.00006	0.00005	-0.08924	0.08924
92	-0.00009	0.00005	0.00004	-0.09052	0.09052
93	-0.00010	0.00006	0.00004	-0.09023	0.09023
94	-0.00007	0.00003	0.00004	-0.07904	0.07904
95	-0.00006	0.00003	0.00003	-0.09649	0.09649
96	-0.00004	0.00002	0.00002	-0.09890	0.09890
97	-0.00003	0.00001	0.00002	-0.09290	0.09290
98	-0.00005	0.00001	0.00004	-0.11241	0.11241
99	-0.00002	0.00000	0.00002	-0.09562	0.09562