MODELING FUTURE MORTALITY RATES USING BOTH DETERMINISTIC AND STOCHASTIC APPROACHES

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ABSTRACT

MODELING FUTURE MORTALITY RATES USING BOTH DETERMINISTIC AND STOCHASTIC APPROACHES

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This study is based on creating an alternative mortality rates projection method for constructing the future mortality tables of several countries including Turkey. A model, which can truly estimate the future mortality rates with a 95% confidence interval, is built. During the study, a general model which includes the dependent variable; logit function of death rate, and the independent variables; general tendency of the mortality evolution independent of age, the mortality steepness, additional effects of childhood, youth and old age, has been used. The model has two dimensions: the age of the target unit and the year of interest. Generalized linear model estimation, which is a deterministic approach, is used in order to estimate the parameters of the general model. Weighted Least Square Method with a stochastic term, which can be considered as both deterministic and stochastic approach, is used to project future values of one parameter. On the other hand, Random Walk with Drift is used on other parameters. Monte Carlo Simulation is used in order to make the model more stable. The study is applied on Hong Kong [6], USA [13], and Turkish Mortality Rates [23]. Eventually, the life insurance companies and Turkish Social Security Institution can take the advantage of future mortality projections obtained by using the model in this study for the calculation of premiums.

Keywords : Mortality Tables, Mortality Projection, Generalized Linear Model, Weighted Least Squares, Random Walk, Monte Carlo Simulation

ÖZ

DETERMİNİSTİK VE STOKASTİK YAKLAŞIMLARLA ÖLÜMLÜLÜK ORANLARI PROJEKSİYONU

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Bu çalışmada gelecek ölüm oranları tahmini için kullanılan yeni bir mortalite eğilim modeli kurulmuş ve bu model kullanılarak %95 güvenlik aralığında ölüm oranları projeksiyonu yapılmıştır. Çalışma süresince, bağımlı değişken olarak ölüm oranının logit fonksiyonunu, bağımsız değişkenler olarak da yaştan bağımsız genel mortalite eğimliliğini ve çocukların, gençlerin ve yaşlıların ek etkilerini içeren genel bir model hazırlanmıştır. Model zaman ve yaş olmak üzere iki boyut içermektedir. Modelin parametrelerini saptamak için deterministik bir yaklaşım olan Genelleştirilmiş Doğrusal Model yöntemi; bu parametrelerin ilkinin gelecek yıllardaki değerlerini bulmak için hem deterministik hem de stokastik yaklaşımları içeren Ağırlıklı En Küçük Kareler yöntemi kullanılmıştır. Diğer parametreler için de Yönlü Rassal Yürüyüş yöntemleri kullanılmıştır. Modelin daha kararlı bir yapıda olması için Monte Karlo Simülasyonu uygulanmıştır. Model Hong Kong [6], USA [13] ve Türk ölümlülük oranları [23] üzerinde uygulanmıştır. Sonuç olarak, genç yaştaki insanların ölümlülük eğilimlerinin genel ölümlülük oranlarına etkilerini daha belirgin olarak yansıtan ve bu nedenle özellikle Türkiye gibi genç nüfus yoğunluğu yüksek olan ülkelerde kullanılabilecek bir mortalite eğilim modeli oluşturulmuş, yukarıda belirtilen ülkelere ait gelecek ölümlülük oranları hesaplanmıştır.

Anahtar Kelimeler : Mortalite Tabloları, Mortalite Projeksiyonu, Genelleştirilmiş Doğrusal Modeller, Ağırlıklı En Küçük Kareler Yöntemi, Rassal Yürüyüş, Monte Carlo Simülaysonu

To My Family and Friends

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CHAPTER 1

INTRODUCTION

Mortality rates can be described as the probability that a person dies within a predetermined time interval. Values of mortality rates and consequently mortality tables change over time. A general decrease in mortality rates can be explained with the increase in people's cautiousness regarding their survival and developments in the health industry. Mortality tables consist of these mortality rates. The knowledge established through the mortalitiy tables of a country is very important for the society of that country.

Mortality tables are usually used in actuarial calculations of pension and life insurance products by the companies or governments. Expected life or the probability of surviving in a predetermined time interval of a person who is willing to get a pension or life insurance product must be known by the providers at the time of purchase.

Having well-constructed tables and accurate future mortality rate projections are very important for societies. Unless the rates are predicted accurately, there could be huge gaps between collected premiums and pension payments. This situation may lead to losses or even bankruptcies of pension companies. In addition, life insurance companies need accurate tables because the future mortality rates have the most important role in the price level of life insurance products. Possible losses and falling behind in the competition of the insurance market could be overcome by modeling the accurate future mortality rates.

Mortality tables can be considered as a matrix where years are represented by columns and ages by rows. Each unit of matrix represents the probability that aged (x) dies within a year over a period. If the data contain the number of people in a chosen population at each age in a time interval is available, the probability that a person at age x in year t lives one year can be calculated by dividing the number of people at age $x+1$ in year $t+1$ by the number of people at age x in year t under a closed demographic system. These tables are generally formed separately for men and women.

1.1 Literature Research

The future values of the mortality rates are projected by many methods like De Moivre [8], Gompertz [7], Helligman-Pollard [25], Lee-Carter [18]. The main idea of these studies is to model or systematize mortality rates from past to the future so that the actuarial calculations are proper for both present and the future. In addition to these basic methods, there are other models including trend analysis. Mortality trend modeling has been studied in the literature by many. Lee and Li (2005) proposed a multi-population mortality modeling as an extension of Lee-Carter method [19]. Their study was conducted to a group of population. The aim of the study was to improve the mortality forecasts for individual countries by considering the patterns in a large group. Every populaation have their own age pattern and level of mortality. Also, Jarner and Kryger (2011) studied a multi-population mortality model which includes long-term trend and short- to mid-term deviations [14]. In the study of Jarner and Kryger, a multivariate time series model which describes the deviations of mortality of small populations from the underlying mortality was fitted and forcasted. An application of Canada and US female mortality has been conducted by Li and Hardy (2011) with respect to basis risk in longevity index hedges [15]. They have considered four extensions to the Lee-Carter model which incorporate such dependence. Both Canada and USA populations are jointly driven by the same single time-varying index, the two populations are cointegrated, the populations depend on a common age factor, and there is an augmented common factor model in which a population-specific time-varying index is added to the common factor model with the property that it will tend toward a certain constant level over time. Boerger (2010) proposed one-year period longevity risk by considering the adequacy of Solvency II scenarios [2]. The adequacy of longevity shock has been analyzed by comparing the resulting capital requirement to the Value-at-Risk based on a stochastic mortality model. On the other hand, Plat (2011) modeled the changes of long-term mortality trend from the aspects of mortality and longevity risk [21]. One-year Value-at-Risk measure, which has been proposed by Plat, aims at covering the risk of the variation in the projection year as well as the risk of changes in the best estimate projection for future years. This study also explains how to determine this Value-at-Risk for longevity and mortality risk. Another important study was conducted by Richards et al. (2014) using different methods like Lee-Carter and Cairns-Blake-Dowd in order to determine one-year period longevity risk [22].

Boerger et al. (2014) proposed a new mortality trend model, which contributed to a better quantification of mortality and longevity risk over time, under modern solvency regimes [3]. Mortality trend model represents young and old age effects more precisely by using three variables separately for each group of age. They utilize the result of the model to compare the capital requirements with respect to Solvency 2 standard formula.

In Turkey, there have been several studies on mortality rate estimation and projection. Yıldırım (2010) modeled Turkish mortality with lee-carter and fuzzy Lee-Carter [24]. A fuzzy formulation of the extended Lee-Carter model was exercised based on Turkish mortality. Demircioğlu (2013) used Poisson Log-Bilinear approach to Lee-Carter modelling and its application for Turkish mortality [9]. In Demircioğlu's study, parameters of Lee-Carter model was estimated to age-specific death rates by gender in Turkey with both singular value decomposition method and Poisson Log-Bilinear regression approach and two methods were compared for 20 years of projection.

1.2 Aim of the Study

Mortality tables have always been a controversial issue in Turkey. Many tables in use are either of foreign origin which cannot reflect the Turkish mortality or they are national tables which are not justified. While insurance sector uses Commissioners Standard Ordinary (CSO) 2001, CSO 1980 Tables, Government and Institutions such as Social Security Institution (SSI) created and use their own tables. CSO Tables are constructed based on the mortality rates of European population. Furthermore, the methodology used in SSI tables do not include any trend analysis. These are the main disadvantages of mortality tables which are currently in use in Turkey.

In this thesis, in order to create a mortality trend model that has both stochastic and deterministic terms and is able to project future mortality rates accurately, a model in the literature is modified. Validation of modified mortality trend model is performed using some statistical techniques such as Mean Absolute Error, R-Squared and applied to mortality tables from several nationalities including Turkey. Constructing a model that includes additional effects of specific age groups is crucial for future of the Turkish Pension System because the different age groups have different effects on the trend of the model. If the parameters of specific age effects are not used, the model becomes less sensitive to inner trends of each age groups.

In this study, a childhood effect parameter is added in addition to parameters of Boerger's model in order to have more accurate results in analysis which includes younger ages. Results of mortality trend model and the modified mortality trend model are compared in analysis which includes younger ages.

Following the introduction chapter, Chapter 2 describes the literature models in detail. In Chapter 3, mortality trend model and modified trend model are explained in details. With Chapter 4, empirical and numerical analysis which are conducted to Hong Kong, USA and Turkish data are presented. In the last chapter, the thesis is concluded and the contributions are presented.

CHAPTER 2

BASIC METHODS

In the history of mortality rate modelling, there are several well-known parametric methods. Each method has its own algorithm and modelling. In 1729, De Moivre [8] proposed a survival function which determines the rates by using the current and terminal age ratio. In 1825, Gompertz used a force of mortality which is assumed to grow exponentially [7]. Furthermore, Heligman and Pollard in 1980 proposed a new model which seemed to fit Australian mortality trend [12]. Then, in 1992, Lee and Carter proposed another model which has three parameters to estimate and one random noise term [5]. In this chapter, these four methods are reviewed.

2.1 De Moivre's Law

According to De Moivre's Law (1729) occurence of deaths are uniformly distributed between starting age and the ultimate age [8]. In de Moivre's Law, the survival function is described as:

$$S(x) = 1 - \frac{x}{w}; \qquad w > 0, \;\; x > 0, \tag{2.1}$$

where w determines terminal age and x states current age [4] .

The survival function is used to determine the probability that a person who is at a specific age (x) lives $({}_hp_x)$ or dies $({}_hq_x)$ in a time interval h given as

$${}_hp_x = \frac{S(x+h)}{S(x)} = \frac{w-(x+h)}{w-x} \qquad 0 \leq h < w-x, \tag{2.2}$$

and

$${}_hq_x = \frac{S(x)-S(x+h)}{S(x)} = \frac{h}{w-x} \qquad (0 \leq h < w-x). \tag{2.3}$$

Using the survival function, force of mortality rate can also be determined as

$$\mu(x+h) = \frac{1}{w-(x+h)} \qquad 0 \leq h < w-x. \tag{2.4}$$

2.2 Gompertz Law

In Gompertz Law (1825), l_x denotes the number of people in a perfectly closed population at a specific age x. In a perfectly closed population, it is assumed that no one enters population from outside and the reduction in population is only caused by deaths.

By using l_x, a force of mortality is calculated as [7]:

$$\mu_x = -\frac{1}{l_x}\frac{dl_x}{dx} = -\frac{d\ln l_x}{dx}. \tag{2.5}$$

Since population reduces as age increases, $\frac{dl_x}{dx}$ becomes a negative value. In this model, μ_x has constant rate and grows exponentially such as:

$$\mu_x = ae^{bx} \qquad a, b > 0, \tag{2.6}$$

by taking the logarithms of both sides of Equation (2.6),

$$\ln \mu_x = \ln a + bx, \tag{2.7}$$

Equation (2.7) shows that the logarithm of μ_x is a linear function of x. By differentiating $\ln \mu_x$,

$$\frac{d\ln \mu_x}{dx} = \frac{1}{\mu_x}\frac{d\mu_x}{dx} = b. \tag{2.8}$$

It is proved that the growth rate of μ_x is a constant b.

Therefore, if a number of people in the population l_x, is as follows:

$$l_x = l_0\, e^{-(a/b)(e^{bx}-1)}, \tag{2.9}$$

then, the probability of surviving is calculated using Gompertz Law as:

$${}_hp_x = \frac{l_{x+h}}{l_x} = e^{-(a/b)[e^{b(x+h)}-1]}, \tag{2.10}$$

and consequently the probability of death is calculated as:

$${}_hq_x = 1 -_h p_x. \tag{2.11}$$

2.3 Heligman and Pollard Model

Heligman and Polard (2980) [12] created a mortality trend model having a form of

$$\frac{{}_hq_x}{1 -_h q_x} = A^{(x+B)^C} + D\,exp\{-E(lnx - lnF)^2\} + GH^x, \tag{2.12}$$

where ${}_hq_x$ is probability of death in one year at the age x and A, B, C, D, E, F, G are childhood mortality level, infant mortality, rate of mortality decrease in childhood, severity in the accident term, spread in the accident term, location in the accident term, base level of mortality at older ages, rate of increase in mortality at older ages, respectively[25].

2.4 Lee Carter Model

Lee-Carter Method (1992) is one of the most popular mortality estimation methods last decade. It has a mortality model which has two dimensions including time and age [5].

The mortality rate model in Lee-Carter Method [11] is given as:

$$m_{ah} = e^{\alpha_a + \beta_a \gamma_h + \epsilon_{ah}} \tag{2.13}$$

or

$$\ln[m_{ah}] = \alpha_a + \beta_a \gamma_h + \epsilon_{ah} \tag{2.14}$$

where α_a, β_a, γ_h are parameters to be estimated, ϵ_{ah} is the random noise term and c is the constant term which is used for transformation of parameters. α_a is general tendency in the trend of mortality rates. β_a tells which rates decline rapidly or which rates decline slowly in response. γ_h is an level of mortality index. ϵ_{ah} follows normal distribution with mean 0 and variance σ_ϵ^2 [18].

The method has several transformations on the parameters such as;

$$\beta_a \to c\beta_a \qquad , \gamma_h \to \frac{1}{c}\gamma_h, \qquad \forall \in \mathbb{R}, c \neq 0,$$
$$\alpha_a \to \alpha_a - \beta_a c, \qquad \gamma_h \to \gamma_h + c, \qquad \forall \in \mathbb{R},$$

and has two constraints such as;

$$\sum_h \gamma_h = 0 \qquad \text{and} \qquad \sum_a \beta_a = 0.$$

It is known that α_a is the average over time by making inference from the constraint $\sum_h \gamma_h = 0$. As a mathematical expression, we can take $\alpha_a = \bar{m}_a$ and the model is transformed to the mean centered log-mortality rate as $\tilde{m}_{ah} = m_{ah} - \bar{m}_a$.

As ϵ_{ah} comes from a normal distribution and consequently;

$$\tilde{m}_{ah} \sim \mathcal{N}(\bar{\mu}_{ah}, \sigma^2) \qquad \text{and} \qquad E(\tilde{m}_{ah}) \equiv \bar{\mu}_{ah} = \beta_a \gamma_h.$$

In order to obtain each γ_h, Lee and Carter employs random walk with drift as the method of forecasting with the form

$$\hat{\gamma}_h = \hat{\gamma}_{h-1} + \theta + \xi_h \qquad \text{where} \qquad \xi_h \sim \mathcal{N}(0, \sigma^2_{rw}).$$

Finally, the mortality rates are determined with respect to the new parameter values.

CHAPTER 3

Mortality Trend Model and Its Modification

3.1 Projection of Future Mortality Rates

The mortality trend model, Boerger (2014) which is based on the mortality rates, where x and t are corresponding age and year in $q_{x,t}$ respectively, is given as follows [3]:

$$logit\, q_{x,t} = \alpha_x + \kappa_t^{(1)} + \kappa_t^{(2)}(x - x_{center}) + \kappa_t^{(3)}(x_{young} - x)^+ + \kappa_t^{(4)}(x - x_{old})^+ + \gamma_{t-x} \tag{3.1}$$

$$\text{where} \quad logit(q_{x,t}) = \ln(\frac{q_{x,t}}{1 - q_{x,t}}) \quad \text{and} \quad x^+ = \max\{x, 0\}$$

This equation requires 4 major parameters to be estimated. First one is the general tendency of the mortality represented by $\kappa_t^{(1)}$ as independent of any age effect. Second one is mortality steepness represented by $\kappa_t^{(2)}$. Third and forth regresors -$\kappa_t^{(3)}$ and $\kappa_t^{(4)}$- are for special effects of young and old ages to mortality rates respectively. Young and old age terms provides a flexibility to determine mortality trends for different ages. Additionally, the parameters x_{center}, x_{young} and x_{old} are set with respect to the historical data analysis concerning different populations. In this study, the parameter values are determined as $x_{center} = 60$, $x_{young} = 55$ and $x_{old} = 85$ such as based in the Boerger's study.

After setting regressors and constant parameters, α_x is estimated:

$$\alpha_x = \frac{1}{t_{max} - t_{min} + 1} \sum_{t=t_{min}}^{t_{max}} logit\, q_{x,t}, \tag{3.2}$$

where t_{min} and t_{max} are respectively starting and ending year of the data to be analyzed.

3.1.1 Estimation of $\kappa^{(\cdot)}$

Estimation of $\kappa^{(\cdot)}$ parameters is performed through a Generalized Linear Modeling (GLM).

In a wide range of problems, standard linear regression approximations do not work as required in contravention of applied transformations. Therefore, generalized linear models are used as extension to linear models, since they are suitable in more complicated cases. Let

$$y_i = \beta_0 + \beta_1 X_{1i} + ... + \beta_p X_{pi} + \epsilon_i, \tag{3.3}$$

where the response y_i, $i = 1, ..., n$ is modelled by a linear function of explanatory variables; x_i, $j = 1, ..., p$ and an error term [20].

In ordinary least squares estimation, response variable y is assumed to come from a normal distribution and the assumption of normality is rarely applicable for categorical data.

GLM has three landmarks as Random Component, Systematic Component and Link Function in order to satisfy the limitations of least squares method. Response and the independent observations (y and $y_1, y_2, ..., y_N$) create the random component of GLM. If response follows a normal distribution, ordinary regression models can be applied. However, response variable can follow any distribution belonging to exponential family (Normal, Binomial, Poisson, Gamma and etc.). On the other hand, Systematic Component refers to explanatory variables ($X_1, X_2, ..., X_k$) where k covariates are combined to form linear predictor:

$$X_i\beta = \sum_j \beta_j X_{ij} = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + ... + \beta_k X_k, \tag{3.4}$$

Lastly, Link Component, which can be considered as the link between random and systematic component, makes a function or expectation be modeled as a combination of linear predictions. Link Function can be shown as $g(\mu_i)$=$\sum_j \beta_j X_{ij}$ where $i = 1, 2, ..., N$. Link Function is determined according to the type of underlying data. The choice of link function $g(.)$ provides the linearity between the expected value of response and the predictors.

Assumptions in GLM:

1. Correct Link Function,
2. Linear relationship between response and the predictors,
3. The residuals follows Normal distribution with zero mean and constant variance.

Therefore, by employing GLM, $\kappa_t^{(.)}$ parameters will be estimated for the year interval $[t_{min}, t_{max}]$. The GLM equation of the mortality trend model parameters are:

$$\begin{pmatrix} logit(q_{x_{min},t}) - \alpha_{x_{min}} \\ logit(q_{x_{min}+1,t}) - \alpha_{x_{min}+1} \\ \vdots \\ logit(q_{x_{max},t}) - \alpha_{x_{max}} \end{pmatrix} \approx M \begin{pmatrix} \kappa_t^{(1)} \\ \vdots \\ \kappa_t^{(4)} \end{pmatrix},$$

where M is the coefficient matrix of mortality trend model.

As following the generalized linear modeling, the next step is the projection of the future model parameters, coefficients and consequently the future mortality rates. There are two different projection methods for four $\kappa^{(\cdot)}$ parameters. These are $\kappa^{(1)}$, $\kappa^{(2)}$, $\kappa^{(3)}$ and $\kappa^{(4)}$ which are general tendency of the mortality, mortality steepness, special effect of young ages and special effects of old ages respectively.

3.1.2 Projection of $\kappa^{(1)}$

Weighted Least Squares (WLS) is a parameter estimation method which is related to other type of least squares methods such as Ordinary, Unweighted, Equally-Weighted and Regular Least Squares. As having same approach with other least squares methods, the main goal is to minimize the sum of the squared deviations between the observed values of responses and the estimation made by predictors. By having additional weights (w_i) for its each term, WLS differs from the other types of least squares methods. Weighted sum of squares can be showed as in Equation (3.5)

$$Q = \sum_{i=1}^{n} w_i[y_i - \hat{\beta}_0 + \hat{\beta}_1 X_{i1} + \hat{\beta}_2 X_{i2} + ... + \hat{\beta}_k X_{ik}], \tag{3.5}$$

Having the possibility of attaching importance to time, region or any other specification of the data is the main advantage of WLS. A stochastic term $\varepsilon_t^{(1)}(\bar{\sigma}^{(1)} + \sigma^{(1)})$ is used in WLS model as noise. Where $\sigma^{(1)}$ denotes the standard deviation of weighted sample's empirical errors and $\bar{\sigma}^{(1)}$ denotes optional volatility add-on [10].

Different from linear and non-linear least squares estimation; weighted least squares estimation does not depend on a particular type of function to represent the relationship between variables. However, it reflects the behavior of the random errors and can be used with either linear or non-linear functions.

Like the other least squares estimation methods, weighted least squares estimation is also sensitive to the impacts of outliers. In the case of unavailability of investigating the potential outliers and dealing with them inappropriately, it is possible to see a negative effect on the parameter estimation and other aspects of a weighted least squares estimation. On the other hand, in the case of increase in the influence of an outlier, weighted least squares can have worse results compared to some other least squares analysis.

The first step of projection process is fitting a weighted least square model to $\kappa_t^{(1)}$ where $l_{t_{max}}$ is regression line which is currently best estimate and w_t's are the weights given by

$$w_t = (1 + \frac{1}{h})^{t - t_{max}}, \tag{3.6}$$

which is used to make the data of last years more contributing compared to previous years in the projected model. The model for $\kappa_t^{(1)}$ is set by adding the stochastic term to the regression line $l_{t_{max}}$ as t in $[t_{min}, t_{min}]$. The forecast for $\kappa_t^{(1)}$ is:

$$\kappa_t^{(1)} = l_{t-1}(t) + \varepsilon_t^{(1)}(\bar{\sigma}^{(1)} + \sigma^{(1)}), \tag{3.7}$$

where $\varepsilon_t^{(1)} \sim^{iid} \mathcal{N}(0,1)$. The volatility $\sigma^{(1)}$ is the standard deviation of the empirical errors $\kappa_t^{(1)} - l_{t-1}(t)$ for $[t_{min}+2, t_{max}]$ obtained from the weighted least square estimation and the term $\bar{\sigma}^{(1)}$ is optional volatility which is assumed as 0.

The projection of $\kappa_t^{(1)}$ over time continuous iteratively by consisting each new member of projection in the next forecast model.

3.1.3 Projection of $\kappa_t^{(2)}$, $\kappa_t^{(3)}$ and $\kappa_t^{(4)}$

When a time series data follows a random and unpredictable path, it is called random walk process. Random walk is a non-stationary time series process [17]. Generally, it is used to describe the path of stock prices. It is assumed that stock price changes are from the same distribution and independent of each other. Therefore, prediction of future prices becomes impossible. However, all possible future paths of a specific stock price can be represented using random walk process. Random walk can be used in any other time series studies.

Random Walk With Drift (RWWD) is another non-stationary time series process. If the data, which follows a random and unpredictable path, have sustainable drifts over time -in other words, if the data have a trend-, random walk with drift is used to describe the data rather than random walk. In this study, since the $\kappa^{(2)}$, $\kappa^{(3)}$ and $\kappa^{(4)}$ parameters are non-stationary and has trends, RWWD is used to determine possible future paths of the parameters.

While applying random walk with drift method to terms $\kappa_t^{(2)}$, $\kappa_t^{(3)}$ and $\kappa_t^{(4)}$, the mean and standard deviation of differences between $\kappa_t^{(\cdot)}$ and $\kappa_{t-1}^{(\cdot)}$ are assumed to be the drift $\mu_t^{(\cdot)}$ and the volatility $\sigma^{(\cdot)}$ of the random walk with drift model, respectively. Three processes of $\kappa_t^{(2)}$, $\kappa_t^{(3)}$ and $\kappa_t^{(4)}$ are:

$$\kappa_t^{(2)} = \kappa_{t-1}^{(2)} + \mu_t^{(2)} + \epsilon_t^{(2)}, \tag{3.8}$$
$$\kappa_t^{(3)} = \kappa_{t-1}^{(2)} + \mu_t^{(3)} + \epsilon_t^{(3)}, \tag{3.9}$$
$$\kappa_t^{(4)} = \kappa_{t-1}^{(4)} + \mu_t^{(4)} + \epsilon_t^{(4)}, \tag{3.10}$$

where $\epsilon_t^{(\cdot)} \sim \mathcal{N}(0, (\sigma^{(\cdot)})$.

The correlations between $\kappa_t^{(1)}$ and separately $\kappa_t^{(2)}$, $\kappa_t^{(3)}$ and $\kappa_t^{(4)}$ are mostly insignificant. In addition, $\kappa_t^{(2)}$, $\kappa_t^{(3)}$ and $\kappa_t^{(4)}$ could be detected as binary correlated with respect to some populations.

3.1.4 Monte Carlo Simulation

Monte Carlo Simulation is a method that insights consisting of characteristics of statistic can be gained by repeatedly drawing random samples from the same population of interest and observing the behavior of the statistic and making inferences over the samples [1]. Distribution of a statistic is estimated by randomly sampling from the population and recording the value of the statistic for each sample. In monte carlo simulation, a pseudo-population is created with respect to the underlying population's parameters. The word pseudo means that the samples of the population are not observed but created using a computer.

Steps in Monte Carlo Simulation are as follows:

1. Determine the pseudo-population or model that represents the underlying population of interest,
2. Use a sampling procedure to sample from the pseudo-population,
3. Calculate a value for the statistic of interest and store it,
4. Repeat steps 2 and 3 for M trials,
5. Use the M values found in step 4 to study the distribution of the statistic.

M samples of future mortality rates are projected by monte carlo simulation and the average of M samples is calculated using Equation (3.11) to represent the future mortality rates.

$$\frac{\sum_{i=1}^{M}(q_{(.)}^{(i)})}{M}. \tag{3.11}$$

In addition, standard deviations of M samples of monte carlo simulation are used in the construction of the confidence intervals of projections.

3.2 Modification: Childhood Effect

The general model (3.1) proposed by Boerger (2014) has three parameters capturing additional effects of specific age groups which are young and old ages. In this study, another additional effect parameter is added to the general model in order to make model more sensitive to younger ages and cover ages smaller than 20 in the analysis. In Boerger's model, analysis is conducted at ages between 20 and 100 because including ages smaller than 20 disturbs the trend of the mortality rates evolution and consequently the errors of the model increase. In the interest of decreasing errors in the analysis including younger ages, childhood effect parameter is used. The modified model is shown below in Equation (3.12)

$$logit\, q_{x,t} = \alpha_x + \kappa_t^{(1)} + \kappa_t^{(2)}(x - x_{center}) + \kappa_t^{(3)}(x_{child} - x)^+ + \kappa_t^{(4)}(x_{young} - x)^+ + \kappa_t^{(5)}(x - x_{old})^+ + \gamma_{t-x}. \tag{3.12}$$

In Table 3.1, Mean Absolute Error values corresponding to years between 2006 and 2013 of Hong Kong Male Mortality Rates are listed. Errors are separately calculated with respect to different age boundaries. As making inference from the Table 3.1, it can be said that the different age boundaries lead to different level of errors. Decrease in the errors means more accurate projections made by model.

Table 3.1: Mean Absolute Error (MAE) values for age choice of additional childhood effect, ages between 10 and 35

Childhood effect age	2006	2007	2008	2009	2010	2011	2012	2013	Average
10	0.084384	0.075906	0.115740	0.108030	0.088837	0.083736	0.124027	0.159486	0.105018
11	0.086048	0.070481	0.115768	0.109216	0.083223	0.087878	0.126939	0.160337	0.104986
12	0.085940	0.075561	0.122336	0.104565	0.079954	0.081093	0.124937	0.156605	0.103874
13	0.087225	0.076641	0.125363	0.103752	0.082869	0.083008	0.107512	0.138389	0.100595
14	0.088228	0.072812	0.115553	0.096289	0.078066	0.073897	0.108446	0.131657	0.095618
15	0.091644	0.071437	0.115890	0.092161	0.075924	0.080312	0.115195	0.144269	0.098354
16	0.086123	0.068983	0.113069	0.095118	0.082462	0.077043	0.091636	0.125563	0.092500
17	0.088265	0.067688	0.119067	0.089378	0.075193	0.080459	0.092940	0.131877	0.093108
18	0.084914	0.063127	0.104715	0.087080	0.073280	0.074645	0.097914	0.129744	0.089427
19	0.083921	0.063506	0.107582	0.090769	0.080107	0.072829	0.096234	0.124756	0.089963
20	0.078233	0.061685	0.096748	0.084291	0.066682	0.061938	0.090601	0.124948	0.083141
21	0.080753	0.057320	0.106329	0.090842	0.073091	0.062149	0.077707	0.108874	0.082133
22	0.075317	0.054419	0.092671	0.078976	0.066559	0.056368	0.083743	0.122125	0.078772
23	0.075705	0.052122	0.083541	0.079842	0.072575	0.051818	0.081142	0.106824	0.075446
24	0.074211	0.051814	0.084767	0.079117	0.068414	0.051232	0.074757	0.111383	0.074462
25	0.068984	0.047088	0.075507	0.078492	0.058364	0.050338	0.091470	0.124577	0.074353
26	0.069143	0.043921	0.077797	0.088640	0.066170	0.051606	0.076286	0.115465	0.073628
27	0.070573	0.045815	0.068455	0.074221	0.069634	0.052726	0.096806	0.117270	0.074438
28	0.063984	0.043241	0.070971	0.073485	0.070456	0.057193	0.088044	0.120387	0.073470
29	0.063688	0.045435	0.071530	0.083810	0.068606	0.057470	0.088839	0.107104	0.073310
30	0.059609	0.041842	0.071885	0.078362	0.063071	0.068093	0.087159	0.114344	0.073046
31	0.057435	0.045179	0.083121	0.097669	0.078944	0.069127	0.086401	0.104839	0.077839
32	0.057844	0.047727	0.084194	0.072426	0.068931	0.079564	0.092854	0.125620	0.078645
33	0.061732	0.051755	0.089293	0.093276	0.088458	0.083806	0.089925	0.113072	0.083915
34	0.063160	0.056290	0.100876	0.088877	0.092523	0.090206	0.087466	0.117771	0.087146
35	0.068272	0.065464	0.105139	0.087128	0.093336	0.087181	0.100354	0.136521	0.092924

Errors relating to ages between 10 and 35 are compared in terms of average of mean square errors in each year. In order to decide best choice of childhood effect boundary, scatter plot of avarage of errors are created. As it is seen in Figure 3.1, the best choice is age 30 which gives minimum mean absolute error values with an average of 7.3%.

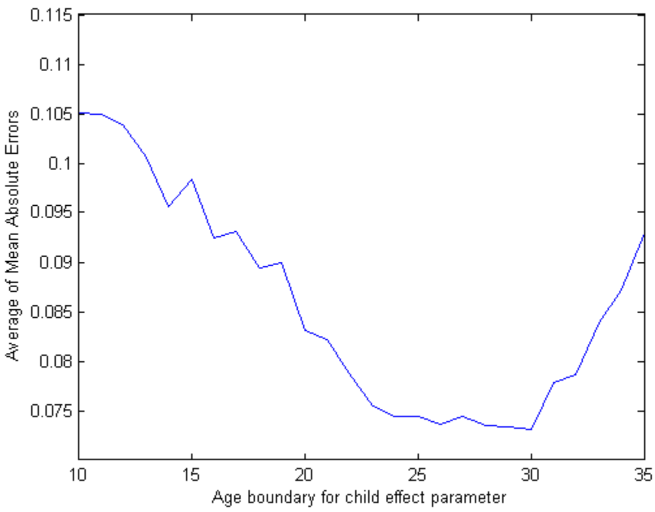


Figure 3.1: Age choice for childhood effect considering Mean Absolute Error values

In this case, age boundaries become 30, 55, 60 and 85 for child, young, center and old age boundaries respectively. Thus, the modified model is expressed as

$$logit\, q_{x,t} = \alpha_x + \kappa_t^{(1)} + \kappa_t^{(2)}(x-60) + \kappa_t^{(3)}(30-x)^+ + \kappa_t^{(4)}(55-x)^+ + \kappa_t^{(5)}(x-85)^+ + \gamma_{t-x}. \quad (3.13)$$

This model incorporates the impact of young ages into estimation and projection of mortality rates compared to the one given by Boerger (Equation 3.12). The sign of childhood effect is expected to be contributory, especially in the case of Turkey. This is because the population of young Turks is quite significant compared to the western countries.

The flowchart of the model is presented in Figure 3.2. As it can be seen from the chart it starts with GLM modeling to mortality rates which are transformed by logit function. After estimation of $\kappa^{(\cdot)}$ parameters, future values of the parameters are projected using weighted least squares and random walk with drift method. The future mortality rates of corresponding moratality table are projected by substituting the projected $\kappa^{(\cdot)}$ parameters into the mortality trend model. Finally, monte carlo simulation is conducted to have more stable results.

MATLAB is used in order to code the algoritm in Figure 3.2. All methods in the algorithm are combined in a single text code.

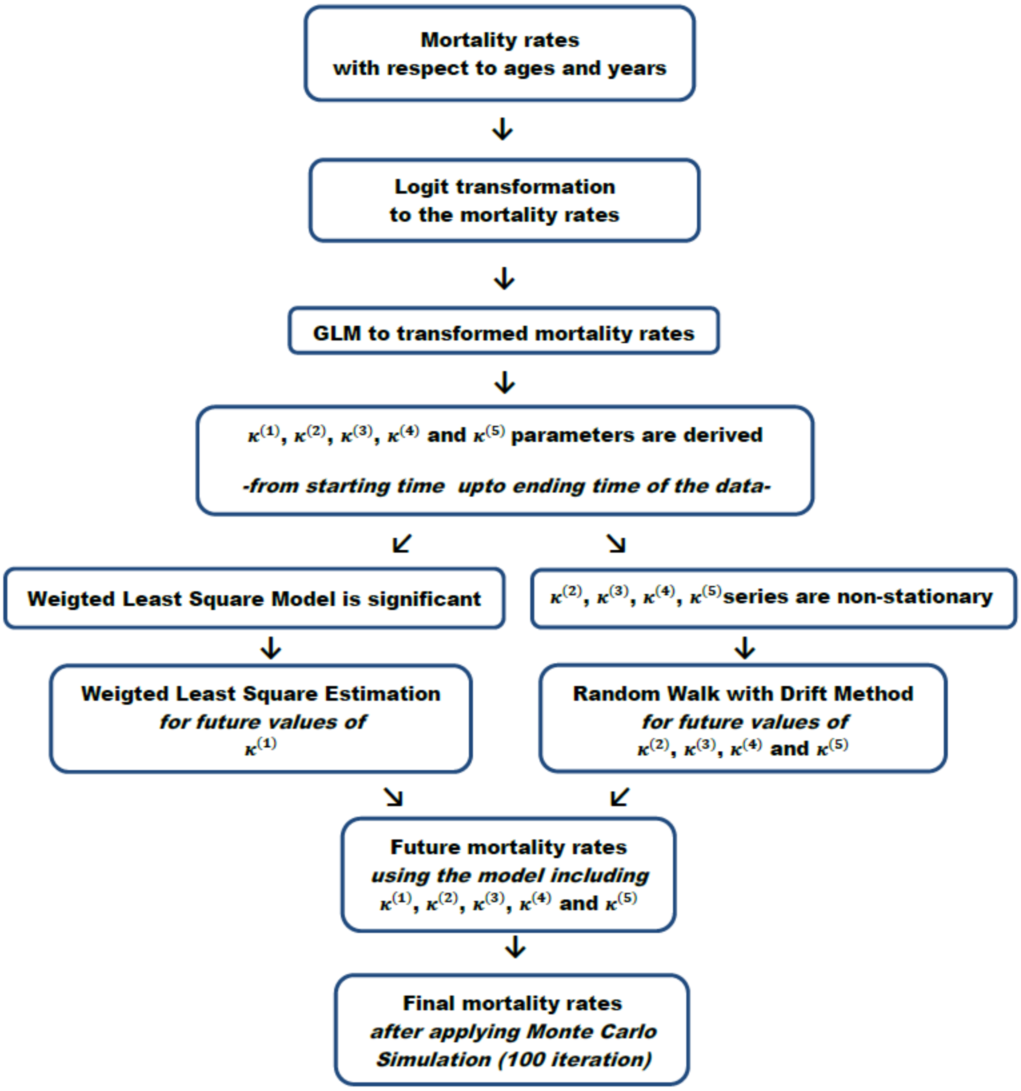


Figure 3.2: The Algorithm of the Methodology

CHAPTER 4

Empirical Analysis: Case Studies on Hong Kong, USA and Turkey

Application of mortality trend model and proposed modified mortality trend model to the data of selected countries are conducted and the results are compared.

Validation of the model is done on Hong Kong Male data as it is one of the well-constructed examples among other countries.

The reliability of the model should be satisfied through a validation process in order to be able to apply the model to the selected data. Throughout the validation process, 20% of the data is put aside and compared with the projected data. Mean Absolute Error and R-Squared indications are used in comparison of the observed versus projected data, and 95% confidence interval of projected values is constructed. Afterwards, 10 years of projections are made for Hong Kong, USA and Turkish Data.

4.1 Description of Data

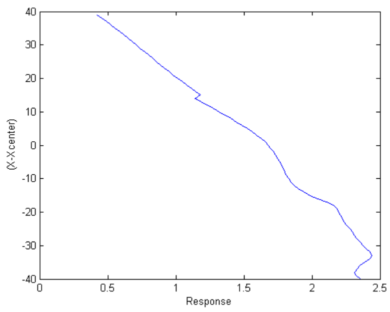
Three different data sets are used in this study. First one is Hong Kong table [6]. Hong Kong data consist of mortality rates of years between 1971-2013 and as well as ages 0-100. Second data set is USA mortality table [13]. USA data has wider intervals in terms of both time and ages. It is constructed within the years 1933-2010 and ages 0-110. The third data set is Turkish mortality table which is constructed by Yıldırım (2014) [23]. It has same age interval as Hong Kong data but different time interval as years between 1931-2015.

4.2 Analysis and Application on Hong Kong Mortality Rates

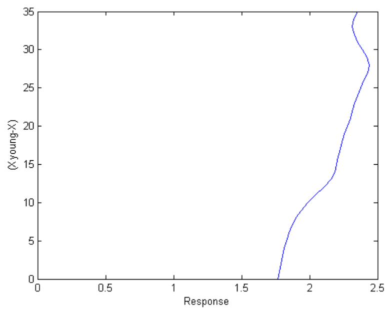
4.2.1 Modeling the Parameters

First step of the study is modelling the mortality rates. Since the response of the model -mortality rates- is not normally distributed, Generalized Linear Modeling is suitable method to apply. Distribution of mortality rates can be assumed as Bernoulli because there are only two options, which are being alive or dead in the end of time interval with the probabilities respectively p_x and q_x, with respect to response variable. Thus, the logit function[1] should be used as link in order to transform response into normal distribution.

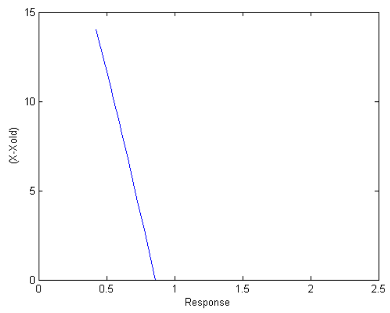
In the plots (Figures 4.1a, 4.1b, 4.1c) of response, which is $logit(q_x)$, and the regressors, there seem to be linear relationships. Observing the correlations between response and the regressors in Table 4.1, it can be inferred that: there is a strong negative linear relationship between response and the variable corresponding to center age $(X - X_{center})$; there is a positive linear relationship between response and the variable corresponding to young age $(X_{young} - X)$; and a negative linear relationship between response and the variable corresponding to old age $(X - X_{old})$. The assumption of linearity between response and regressors is satisfied.



(a) Response vs $(X - X_{center})$



(b) Response vs $(X_{young} - X)$



(c) Response vs $(X - X_{old})$

Figure 4.1: Linear relationships between GLM response ($logit(q_x)$) and variables

Table 4.1: Correlations between response and regressors

Response/Regressors	**(X-Xcenter)**	**(Xyoung-X)**	**(X_Xold)**
logit(qx)	**-0.9904**	**0.7998**	**-0.6431**

The HongKong Mortality Rates Data [6] consists the rates ranges from age 0 to 100 and within 1971 to 2013. This constitutes a matrix whose number of rows is 101 and number of columns is 43. A slice of data is presented in Table 4.2 for illustration.

[1] $logit(q_{x,t}) = ln(\frac{q_{x,t}}{1-q_{x,t}})$

Table 4.2: Hong Kong Male mortality rates, between 1971-2013 [6]

Age/Year	**1971**	**1972**	**.**	**.**	**2012**	**2013**
0	0.020159	0.018900	.	.	0.001403	0.001719
1	0.001343	0.001428	.	.	0.000214	0.000309
2	0.001110	0.001182	.	.	0.000189	0.000290
3	0.000923	0.000977	.	.	0.000168	0.000270
.	.	.	.	.	.	.
.	.	.	.	.	.	.
.	.	.	.	.	.	.
97	0.409479	0.467110	.	.	0.236638	0.252874
98	0.434969	0.496698	.	.	0.256317	0.275064
99	0.461269	0.527039	.	.	0.277375	0.298716
100+	1	1	.	.	1	1

Over 43 years data, the first 35 years' observations are used to estimate the model, whereas, the rest is employed as insample forecast validations. The values of $\kappa^{(\cdot)}$'s in time are shown in Table 4.3.

Table 4.3: Estimated $\kappa^{(\cdot)}$ parameters for Hong Kong data

Parameter/Year	**1971**	**1972**	**1973**	**.**	**.**	**2007**	**2006**	**2005**
$\kappa^{(1)}$	1.65257	1.62577	1.55721	.	.	0.72359	0.65537	0.64316
$\kappa^{(2)}$	-0.03191	-0.02595	-0.02793	.	.	-0.02145	-0.01950	-0.01635
$\kappa^{(3)}$	-0.01206	-0.00354	-0.00303	.	.	0.00260	0.00409	0.00628
$\kappa^{(4)}$	0.00194	0.00497	0.00940	.	.	-0.03807	-0.02402	-0.01198

To check whether the model residuals follow a normal distribution, a hypothesis test, H_0: $\varepsilon \sim \mathcal{N}(0, \sigma)$, is conducted.

Results of normality test for errors at each year are listed in Table 4.4. According to the test results, there is not enough evidence to reject that the errors are normally distributed with mean zero at a significance level of 5% for each year. Therefore, the assumption that the errors follow a normal distribution with zero mean is satisfied.

Table 4.4: P-values and T-stats of GLM modelling according to the years 1971-2005

Test/Years	**1971**	**1972**	**1973**	**1974**	**1975**	**1976**	**1977**	**1978**	**1979**
t-stat	-0.1553	-0.7632	0.3163	0.2752	0.4999	0.2858	0.0941	0.5606	0.6422
p-value	0.8770	0.4476	0.7526	0.7839	0.6186	0.7758	0.9253	0.5766	0.5226
Test/Years	**1980**	**1981**	**1982**	**1983**	**1984**	**1985**	**1986**	**1987**	**1988**
t-stat	0.6303	0.4570	0.7989	0.1919	0.4435	0.8576	0.0186	0.8253	0.5663
p-value	0.5303	0.6489	0.4267	0.8483	0.6586	0.3937	0.9852	0.4117	0.5728
Test/Years	**1989**	**1990**	**1991**	**1992**	**1993**	**1994**	**1995**	**1996**	**1997**
t-stat	-0.4444	-0.7900	-0.0920	-0.8870	-1.2113	-0.9583	0.6885	-0.7679	-0.8417
p-value	0.6579	0.4319	0.9269	0.3778	0.2293	0.3408	0.4931	0.4448	0.4024
Test/Years	**1998**	**1999**	**2000**	**2001**	**2002**	**2003**	**2004**	**2005**	
t-stat	-0.3683	-0.1473	-0.2838	-0.3859	-0.5487	-0.2029	-0.4627	0.5743	
p-value	0.7136	0.8833	0.7773	0.7006	0.5847	0.8397	0.6448	0.5674	

*Reject at 5% significance level

To check the constant variance we plot fitted values vs. residuals. Figure 4.2 does not show any pattern refusing constant variance.

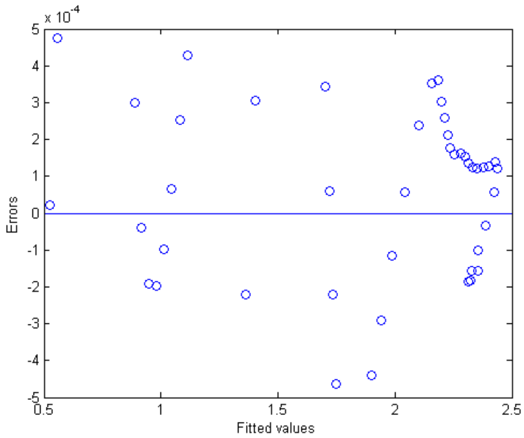


Figure 4.2: Plot of fitted values vs. residuals of GLM

By making inference from the Figure 4.3, it can be said that $\kappa^{(1)}$, which describes the total mortality trend of the population, is leading variable which changes dramatically over time from 1.65 to 0.64 compared to the other variables. In other words, Hong Kong male mortality rates generally decreases with respect to the major effect of $\kappa^{(1)}$ in the mortality model given in Equation (3.1).

$\kappa^{(2)}$, which represents the mortality steepness, shown in Figure 4.4b, has a tendency to increase in time however it has a specific decrease between years 1977 and 1984. $\kappa^{(3)}$ -additional young age effect- has a convexity as seen in Figure 4.4c. This convexity means the young population has a lowest level of contribution on the increase of mortality rates between 1983-1987. $\kappa^{(4)}$ -additional old age effect- shown in Figure 4.4d decreases over time. Thus, it has a sustainable decreasing effect on mortality rates.

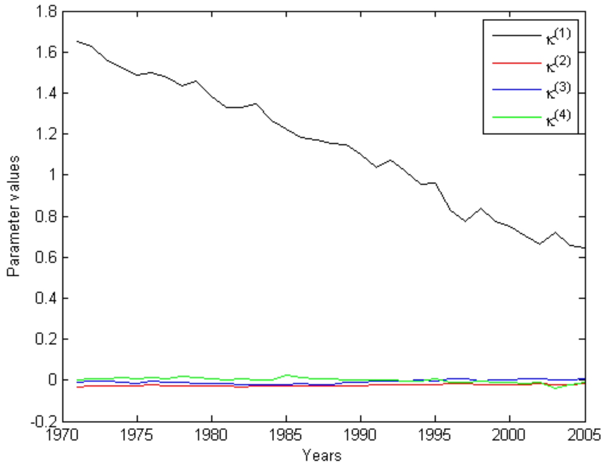
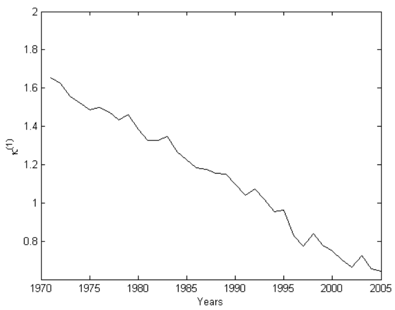
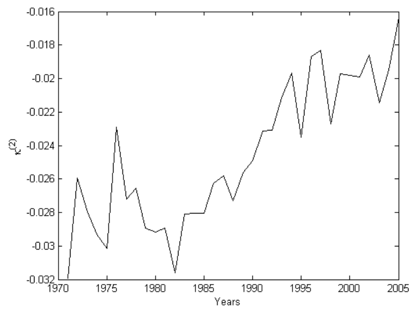


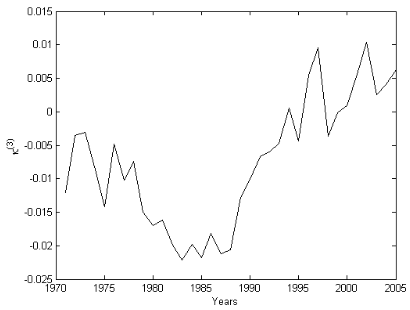
Figure 4.3: Estimates of $\kappa^{(\cdot)}$ parameters for Hong Kong Male between 1971-2005



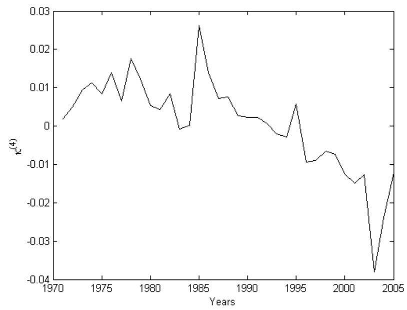
(a) Estimates of $\kappa^{(1)}$



(b) Estimates of $\kappa^{(2)}$



(c) Estimates of $\kappa^{(3)}$



(d) Estimates of $\kappa^{(4)}$

Figure 4.4: Estimation of each $\kappa^{(\cdot)}$ for Hongkong Male between 1971-2005

4.2.2 Projection of the Future Mortality Rates

Projection of future mortality rates for 2006-2013 achieved by estimating $\kappa^{(\cdot)}$ parameters for the same period. Projection of $\kappa^{(1)}$ is done using WLS. Whereas, projections of $\kappa^{(2)}$, $\kappa^{(3)}$ and $\kappa^{(4)}$ are done using random walk with drift.

4.2.2.1 Projection of $\kappa^{(1)}$

Weighted Least Square (WLS) Method is used in order to model future values of $\kappa^{(1)}$.

$$\text{WLS model is: } ((T' * W * T)^{-1}) * (T' * W * \kappa^{(1)}), \tag{4.1}$$

$$\text{where } W = (1 + \frac{1}{h}) \text{ represents weights } and \ T \text{ represents time vector.} \tag{4.2}$$

Graph of $\kappa^{(1)}$ by time shown in Figure 4.5 shows a strong linear relationship between time and $\kappa^{(1)}$.

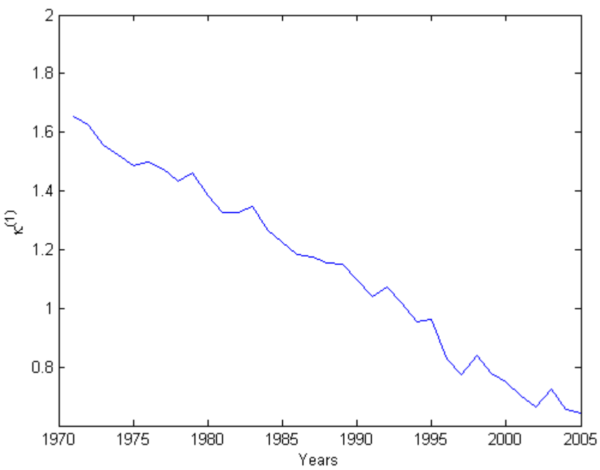


Figure 4.5: WLS: Years vs. $\kappa^{(1)}$

The linear model of years and $\kappa^{(1)}$ is significant with a very small p-value < 0.001 and has R^2=98%.

The errors of the model follow a normal distribution with zero mean according to the normality test with a p-value of 0.9933 (which is greater than 0.05 significance level).

To check the constant variance we plot fitted values vs. residuals. Figure 4.6 does not show any pattern that goes against the supposition that the errors have constant variance.

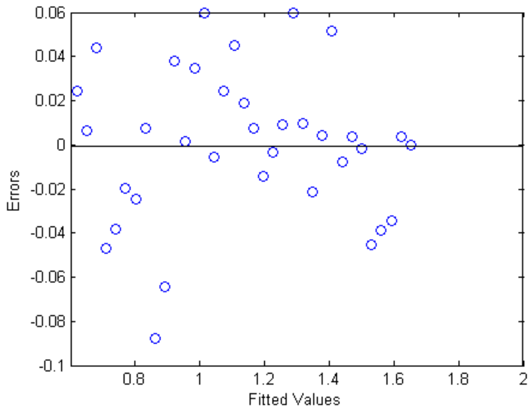


Figure 4.6: Plot of fitted values vs. residuals of WLS

WLS method is applied with five different values of weight factor h given in Equation (4.2) to detect the best choice of h. $\kappa^{(1)}$ projection values with the weight factors h=1, h=2, h=5, h=10 and h=20 are shown in Figure 4.7. By making inference from the Figure 4.7, the projections with $h = 5$, $h = 10$ and $h = 20$ seem to have more stable paths than the projections with $h = 1$ and $h = 2$ considering the estimated $\kappa^{(1)}$ values.

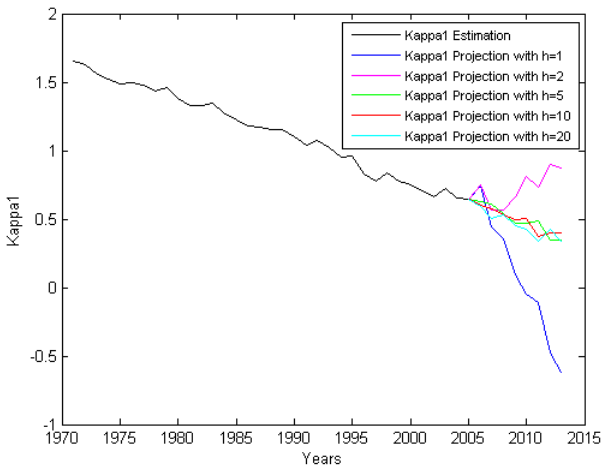


Figure 4.7: Effect of choice of h on $\kappa^{(1)}$ trend

In one modeling process, only one parameter, which is corresponding to next year, is estimated. Then, iteratively the next processes are obtained. Each projections consists all previous values of $\kappa^{(1)}$ -all projected values are included- in the model. In order to have k projected $\kappa^{(1)}$ parameters, the steps below are followed:

1. We have n estimated values of $\kappa^{(1)}$,
2. WLS to n values of $\kappa^{(1)}$,
3. We have n estimated and 1 projected values of $\kappa^{(1)}$,

4. WLS to $n+1$ values of $\kappa^{(1)}$,

5. We have n estimated and 2 projected values of $\kappa^{(1)}$,

6. WLS to $n+2$ values of $\kappa^{(1)}$,

⋮

(2k+1). We have n estimated and k projected values of $\kappa^{(1)}$.

4.2.2.2 Projection of $\kappa^{(2)}$, $\kappa^{(3)}$ and $\kappa^{(4)}$

Before projection of parameters, Kwiatkowski–Phillips–Schmidt–Shin (KPSS) test [16], which is a stationarity test, is conducted to $\kappa^{(2)}$, $\kappa^{(3)}$ and $\kappa^{(4)}$ series. KPSS test states the null hypothesis to be that the series is stationary around a deterministic trend.

According to the test results in Table 4.5, there is enough evidence to reject the null hypothesis for each parameter at the significance level of 5% so that parameters $\kappa^{(2)}$, $\kappa^{(3)}$ and $\kappa^{(4)}$ are not stationary around a deterministic trend.

Table 4.5: Stationarity Test (KPSS) of $\kappa^{(2)}$, $\kappa^{(3)}$ and $\kappa^{(4)}$

Test	$\kappa^{(2)}$	$\kappa^{(3)}$	$\kappa^{(4)}$
Test Statistic	0.2398	0.6013	0.3381
P-value	0.010*	0.010*	0.010*

*Reject at 5% significance level

Since the time series data of $\kappa^{(2)}$, $\kappa^{(3)}$ and $\kappa^{(4)}$ are non-stationary processes and have drifts, random walk with drift method is used in order to model future values of $\kappa^{(2)}$, $\kappa^{(3)}$ and $\kappa^{(4)}$. Average and standard deviation of differences between n^{th} and $(n+1)^{th}$ values in each series of $\kappa^{(\cdot)}$'s are used as drift and volatility respectively given by the equation 4.3.

$$\hat{\kappa}_{t+1}^{(\cdot)} = \kappa_t^{(\cdot)} + \mu^{(\cdot)} + \epsilon_t^{(\cdot)}, \tag{4.3}$$

where $\mu^{(\cdot)}$ is drift, $\sigma^{(\cdot)}$ is standard deviation and it is assumed that $\epsilon_t \sim \mathcal{N}(0, \sigma^{(\cdot)})$

The trend of $\kappa^{(2)}$, $\kappa^{(3)}$ and $\kappa^{(4)}$ for a time interval between 1971-2013 are shown in Figure 4.8.

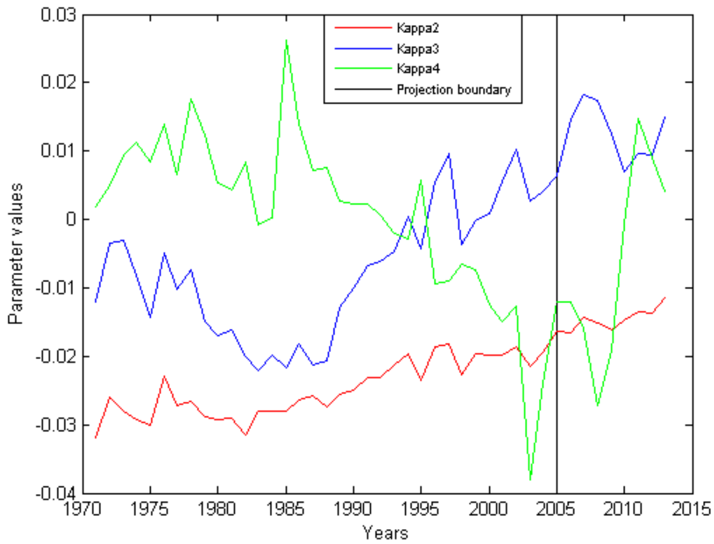


Figure 4.8: Projections of $\kappa^{(2)}$, $\kappa^{(3)}$ and $\kappa^{(4)}$ parameters using Random Walk with Drift for the test and the forecast periods

While interpreting the projections of $\kappa^{(3)}$ (additional effect of young ages) and $\kappa^{(4)}$ (additional effect of old ages) parameters, the sustainable decrease in $\kappa^{(1)}$ parameter, which states the general tendency of mortality evolution, should be considered as well. Although, there seem increases in the projections of parameters, it does not indicate the increases in the mortality rates of specific age groups, such as young and old ages. Since $\kappa^{(1)}$ parameter has a linear decrease as seen in Figure 4.5, it can be inferred that the rate of reduction in mortality rates of young and old ages decreases over time as values of $\kappa^{(3)}$ and $\kappa^{(4)}$ parameters increase.

4.2.2.3 Forecast Precision of the Model

After projection of parameters $\kappa^{(1)}$, $\kappa^{(2)}$, $\kappa^{(3)}$ and $\kappa^{(4)}$, mortality rates corresponding to last 8 years are projected by Monte Carlo Simulation with 100 trials performed for the random components in order to obtain more stable results.

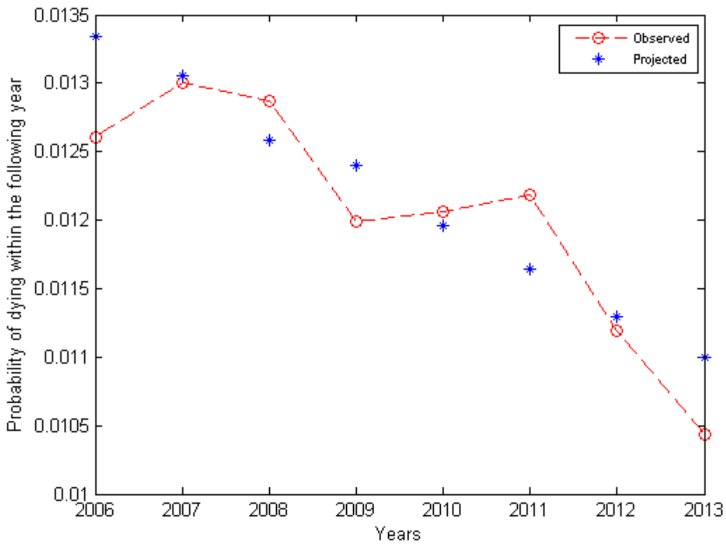


Figure 4.9: Observed and forecasted mortality rates for Hong Kong Data (Male, Age 65)

Figure 4.9 illustrates that the model works accurately in many of the years considered. However, to justify the precision, Mean Absolute Error (MAE) and R-Squared methods are used for the validation of model and 95% confidence interval of projected mortality rates is constructed.

After applying 100 Monte Carlo iteration MAE and R-Squared values are calculated as:

$$\text{MAE} = \frac{1}{n}\sum_{j=1}^{n}(|y_j - \hat{y_j}|), \tag{4.4}$$

$$R^2 = 1 - \frac{\sum_{j=1}^{n}(y_j - \hat{y_j})^2}{\sum_{j=1}^{n}(y_j - \bar{y_j})^2}. \tag{4.5}$$

Table 4.6: MAE and R-Squared values for Hong Kong Male between 2006-2013

	2006	**2007**	**2008**	**2009**	**2010**	**2011**	**2012**	**2013**	**Average**
MAE	0.048253	0.064504	0.078509	0.055188	0.065380	0.089623	0.070293	0.083492	0.069405
R-Squared	0.999974	0.999857	0.999527	0.999742	0.999351	0.999435	0.999834	0.999444	

Errors are between 4.8% and 8.9% and the average of all errors is 6.9% as shown in Table 4.6. All MAE values are smaller than 10%. In addition, R-Squared values are considerably high. The results indicate the well representation of the observed data.

The mortality rates of age 65 are used in the visualization of projection. Confidence interval given in Equation (4.6) on the mortality rate of age 65 is shown as Figure 4.10. As the observed mortality rates are at the confidence region, it can be said that the model is significant in order to represent future values of mortality rates at a significance level of 5% [2].

$$\bar{q}_x^{(i)} \mp t_{0.025}(\sigma_i). \tag{4.6}$$

Here, t-value is taken as 1.98, the mean and standard deviation of values obtained from 100 different trials of monte carlo simulation are used. As seen in Figure 4.10, all the observed mortality rates between 2006-2013 remain in the confidence interval.

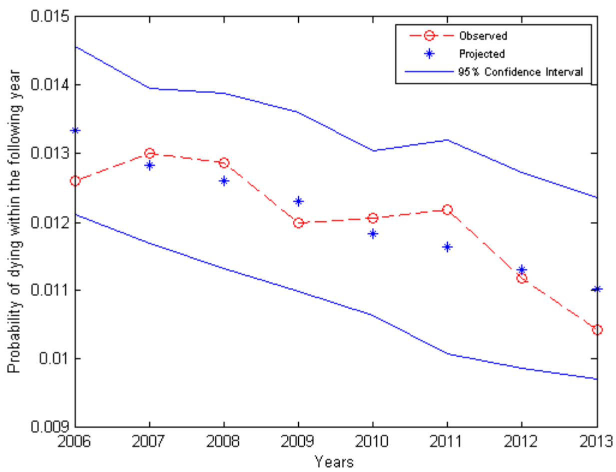


Figure 4.10: A 95% confidence interval for Hong Kong mortality rates and their point estimates (Male, Age 65)

[2] H_0: The model is significant at the significance level of 5%

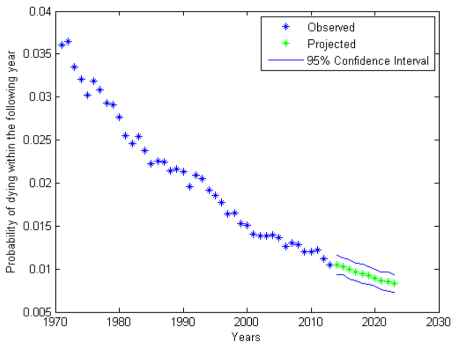
4.2.3 Ten Year Projection of Hong Kong Mortality Rates

The projection of 10 years -from 2014 to 2023- is determined by the proposed mortality trend model. 43 years of Hong Kong Male Mortality Rates are used in order to find future mortality rates. The observed mortality rates and the 10 years of projection are plotted in Figure 4.11a.

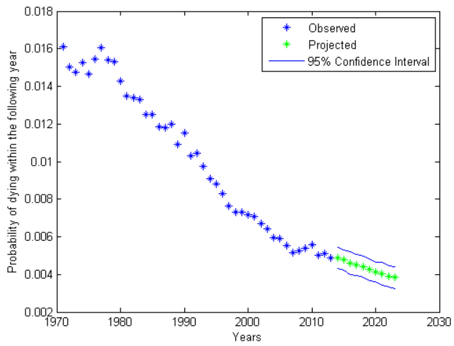
In the construction of confidence intervals for Hong Kong mortality rates projection, t-value is taken as 1.98, the mean and standard deviation of values obtained from 100 different trials of monte carlo simulation are used.

On the other hand, proposed Mortality Trend Model is applied on Hong Kong Female Mortality Rates. Estimates and future projections of the model parameters are reconstructed. Then, the future mortality rates of Hong Kong Female are forecasted. When female mortality rates is taken into consideration, trend of the rates looks similar to male mortality rates'. In Figure 4.11b, the trend and the values of Hong Kong Female Mortality Rates are given. An unusual increase in the mortality rates of females stands out between 1971-1977. The effect of this unexpected movement on the trend of the model was reduced through using weigted least squares method.

As they are trend models, the patterns in the forecasts agree with the original data.



(a) Male



(b) Female

Figure 4.11: Projection for Hong Kong Mortality Rates and 95% confidence interval (Age 65)

4.3 Application of the Model on Turkish Data

A custom-tailored mortality table, based on ADNK system, for Turkey has been studied since 2013. This means we have only two years of data available to us. Therefore, it is impossible apply the model to these observed data. On the contrary, there are mortality tables, based on estimations, which are representing the Turkish mortality. Yıldırım (2014) constructed a Turkish mortality table to analyze in that study [23]. The data constructed by Yıldırım is used in our study to apply the mortality trend model in case of Turkey.

Turkish mortality table was constructed as 5×1[3] type. Transformation of 5×1 type of data into 1×1[4] based on the interpolation techniques may lead to loss of information and biasness. Thus, application of Turkish case is performed by using 5×1 type.

In Turkish case, the model in which old age is arranged as 80 has smaller MAE values (shown in Table 4.7) and consequently a better forecast for the Turkish future mortality rates compared to the proposed model with old age 85. Therefore, the old age is determined as 80 for Turkish mortality rates.

Estimates and future projections of the model parameters are reconstructed for both female and male mortality rates separately. Then, the future mortality rates of Turkish male and female are forecasted.

Mean Absolute Error (MAE) values for Turkish males are between 5.3% and 9.9% and have 8.1% average. MAE values for Turkish females are between 5.1% and 9.4% and have 7.3% average. All MAE values are smaller than 10%. In addition, R-Squared values are considerably high. The results indicate the well representation of the observed data.

Table 4.7: MAE and R-Squared Values for Turkish Male

	2004	2005	2006	2007	2008	2009	
MAE, old age=80	0.05342	0.06296	0.06916	0.07186	0.07459	0.08150	
MAE, old age=85	0.03821	0.05137	0.06110	0.07429	0.09044	0.09940	
R-Squared	0.99933	0.99801	0.99673	0.99632	0.99441	0.99269	
	2010	**2011**	**2012**	**2013**	**2014**	**2015**	**Average**
MAE, old age=80	0.089434	0.085974	0.094375	0.097474	0.098536	0.099418	0.081557
MAE, old age=85	0.117014	0.132972	0.143444	0.163004	0.185417	0.201617	0.113189
R-Squared	0.991158	0.990208	0.989841	0.986965	0.985187	0.986449	

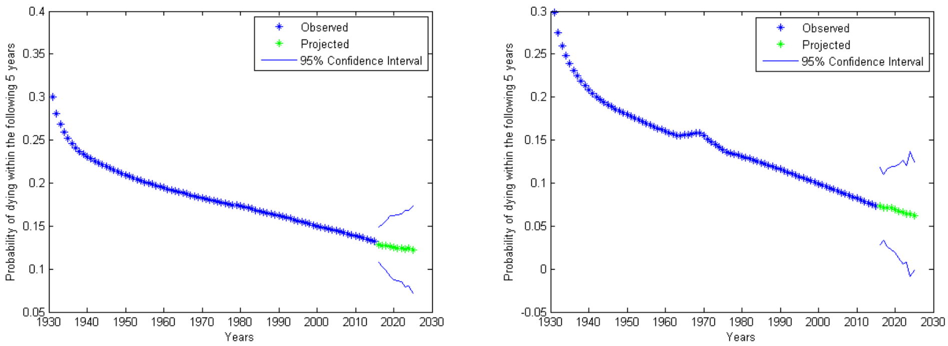
Table 4.8: MAE and R-Squared Values for Turkish Female

	2004	2005	2006	2007	2008	2009	
MAE	0.051719	0.057202	0.067072	0.068149	0.071668	0.075371	
R-Squared	0.999526	0.998896	0.998666	0.997257	0.997226	0.997111	
	2010	**2011**	**2012**	**2013**	**2014**	**2015**	**Average**
MAE	0.075516	0.076213	0.079473	0.080159	0.089059	0.094814	0.073868
R-Squared	0.996017	0.994874	0.993925	0.992560	0.992779	0.992212	

[3] consisting of each 5 years' cumulative mortality proportions
[4] consisting of each year's mortality proportions

4.3.1 Ten Year Projection of Mortality Rates: Turkish Male-Female Data

Mortality trend model is applied on both Turkish male and female mortality rates for the projections. 85 years of Turkish Mortality Rates are used in order to find future mortality rates. The projection of 10 years -from 2016 to 2025- is determined by the proposed mortality trend model. In the construction of 95% confidence interval for Turkish mortality rates projection, t-value is taken as 1.98, the mean and standard deviation of values obtained from 100 different trials of monte carlo simulation are used. The observed mortality rates and the 10 years of projections for Turkish male and female are plotted in Figures 4.12a and 4.12b, respectively.



(a) Male (b) Female

Figure 4.12: Projection for Turkish Mortality Rates and 95% confidence interval (Age 65)

Confidence interval of female projection is wider than male's at a given 5% significance level. It means the model performs a more accurate projection on male mortality rates in Turkish case.

4.4 Application of the Model on USA Data

USA mortality rates for both gender between 1933-2010 [13] is used in application of proposed mortality trend model. In Table 4.9, MAE and R-Squared values are listed. The MAE errors lies between 2.8% and 9.5% with an average of 6.6% in years between 2003-2010. All MAE values are smaller than 10%. On the other hand, R-Squared values are considerably high. The results indicate the well representation of the observed data.

Table 4.9: MAE and R-Squared Values for USA Mortality Rates

	2003	2004	2005	2006	2007	2008	2009	2010	Average
MAE	0.028003	0.044029	0.050699	0.069896	0.078553	0.073215	0.095700	0.089407	0.066188
R-Squared	0.999994	0.999975	0.999989	0.999968	0.999952	0.999961	0.999802	0.999797	

4.4.1 Ten Year Projection of USA Mortality Rates

Mortality trend model is applied on USA mortality rates for the projection. 78 years of USA mortality rates are used in order to project 10 years -from 2011 to 2020- by the model. In the construction of 95% confidence interval, t-value is taken to be 1.98, the mean and standard deviation of values obtained from 100 different trials of monte carlo simulation are used. Observed mortality rates between 1933-2010 and projection are plotted in Figure 4.13. As it is a trend model, the pattern in the forecasts agrees with the original data.

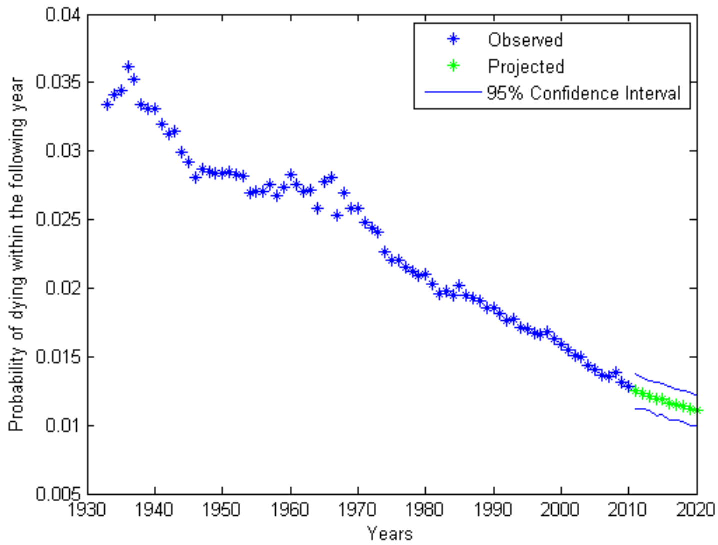


Figure 4.13: Projection for USA Mortality Rates and 95% confidence interval (Both gender, Age 65)

4.5 Application of Modified Mortality Trend Model on Hong Kong, USA and Turkish Data

In this section, the analysis is conducted at ages between 5 and 100 instead of ages between 20 and 100. After conducting two models and applying Monte Carlo Simulation with 100 trials, the results of modified mortality trend model (Modified MTM) and mortality trend model (MTM) are compared in terms of Mean Absolute Error values.

By conducting the t-comparison test, the MAE values are compared for applications of Hong Kong, USA and Turkish data, separately. Comparison test states the null hypothesis as there is no difference between means of MAE values of modified mortality trend model and mortality trend model.

Table 4.10: Modified Mortality Trend Model vs. Mortality Trend Model, MAE Comparison

DATA	*MODEL*	**Error 1**	**Error 2**	**Error 3**	**Error 4**	**Error 5**	**Error 6**	**Error 7**	**Error 8**	**Average**
Hong Kong Male	**MTM**	0.082074	0.068098	0.096997	0.119740	0.098116	0.082694	0.097944	0.144815	0.098810
Hong Kong Male	**Modified MTM**	0.060089	0.040717	0.078534	0.092192	0.070210	0.068413	0.089540	0.109197	0.076112
HK Female	**MTM**	0.096289	0.102775	0.078584	0.082917	0.164274	0.135761	0.189710	0.100518	0.118853
HK Female	**Modified MTM**	0.105911	0.113075	0.085006	0.086533	0.128598	0.124667	0.159464	0.102270	0.113191
Turkish Male	**MTM**	0.062808	0.069821	0.076597	0.082020	0.081155	0.083709	0.088781	0.093490	0.079798
Turkish Male	**Modified MTM**	0.045165	0.046847	0.051680	0.055261	0.060582	0.064222	0.065980	0.074903	0.058080
Turkish Female	**MTM**	0.057503	0.072173	0.081121	0.092510	0.112881	0.129507	0.142870	0.178688	0.108407
Turkish Female	**Modified MTM**	0.049974	0.060593	0.069314	0.085395	0.094637	0.121150	0.150956	0.169863	0.100235
USA Both Gender	**MTM**	0.083041	0.082900	0.092611	0.104941	0.107145	0.108290	0.118157	0.125951	0.102879
USA Both Gender	**Modified MTM**	0.052268	0.054177	0.059713	0.068034	0.067135	0.065251	0.082673	0.079599	0.066106

Errors (1-8) are placed corresponding to the last 8 years of each data.

Errors (1-8) in Table 4.10 are placed corresponding to the last 8 years of each data.

P-values of the comparison test at the 5% significance level are shown in Table 4.11.

Table 4.11: Modified Mortality Trend Model vs. Mortality Trend Model, Results of t-comparison test

DATA	**p-value**
Hong Kong Male	0.00015*
HK Female	0.40719
Turkish Male	<0.00001*
Turkish Female	0.01766*
USA Both Sex	<0.00001*

*Reject at 5% significance level

There is enough evidence to reject the null hypothesis for Hong Kong Male data at 5% significance level so that MAE values of MTM is greater than modified MTM's for Hong Kong Male mortality rates.

There is enough evidence to reject the null hypothesis for Turkish Male data at 5% significance level so that MAE values of MTM is greater than modified MTM's for Turkish Male mortality rates.

There is enough evidence to reject the null hypothesis for Turkish Female data at 5% significance level so that MAE values of MTM is greater than modified MTM's for Turkish Female mortality rates.

There is enough evidence to reject the null hypothesis for USA data at 5% significance level so that MAE values of MTM is greater than modified MTM's for USA mortality rates.

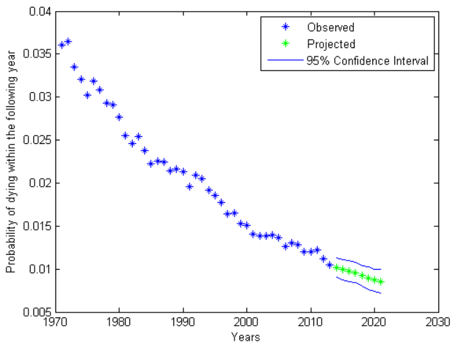
On the contrary, there is not enough evidence to reject the null hypothesis for Hong Kong Female data at 5% significance level so that there is no difference between MAE values of MTM and modified MTM's for Hong Kong Female mortality rates.

The results of comparison tests indicate that the Modified MTM generally has a better precision than MTM in the analysis which includes the ages between 5-20.

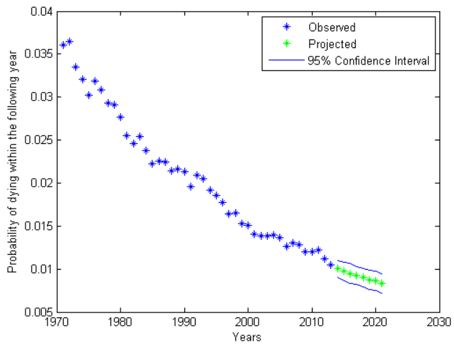
All the projected mortality rates are given in the Appendexes A.1,A.2, A.3, A.4, A.5, A.6, B.1, B.2, C.1 and C.2.

4.5.1 MTM vs. Modified MTM Comparison for Hong Kong Male Mortality Rates

MTM and Modified MTM are applied on Hong Kong Male mortality rates. Projections performed by both models have stable patterns at age 65. There is no difference in terms of patterns and confidence intervals between projections performed by MTM and Modified MTM.



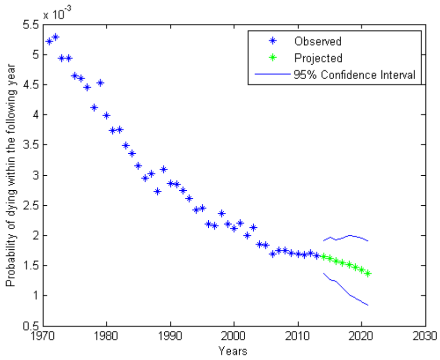
(a) MTM



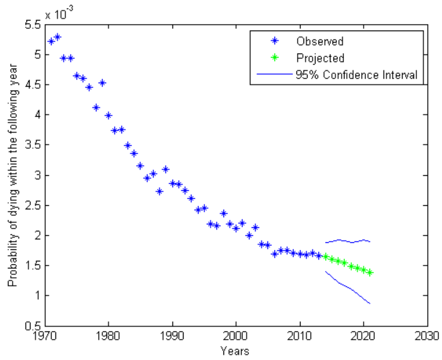
(b) Modified MTM

Figure 4.14: Projection for Hong Kong Mortality Rates and 95% confidence interval (Male, Age 65)

At age 45, standard deviations of projections performed by MTM and Modified MTM increase over time. It leads to wider confidence intervals. Wider confidence intervals increase the risk of obtaining less accurate forecasts. Therefore, accuracy of the forecast is reduced for both projections. There seems to be no difference in terms of patterns and confidence intervals of projections performed by MTM and Modified MTM.



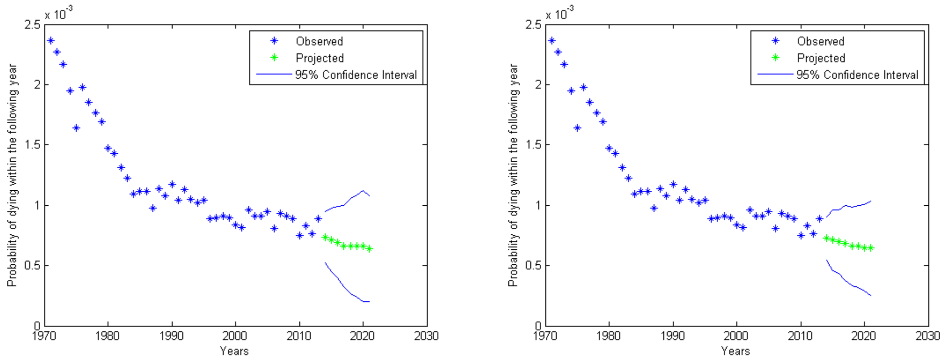
(a) MTM



(b) Modified MTM

Figure 4.15: Projection for Hong Kong Mortality Rates and 95% confidence interval (Male, Age 45)

At age 35, projection performed by Modified MTM has narrower confidence interval so that at a given confidence level of 5%, Modified MTM has less uncertainty on the projection compared to MTM.

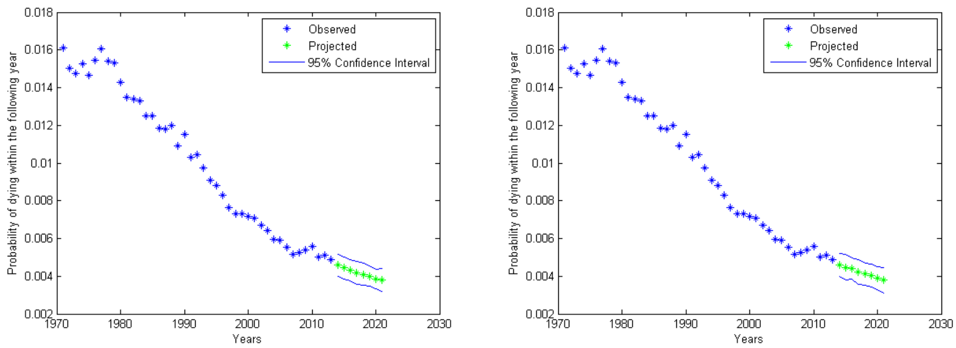


(a) MTM (b) Modified MTM

Figure 4.16: Projection for Hong Kong Mortality Rates and 95% confidence interval (Male, Age 35)

4.5.2 MTM vs. Modified MTM Comparison for Hong Kong Female Mortality Rates

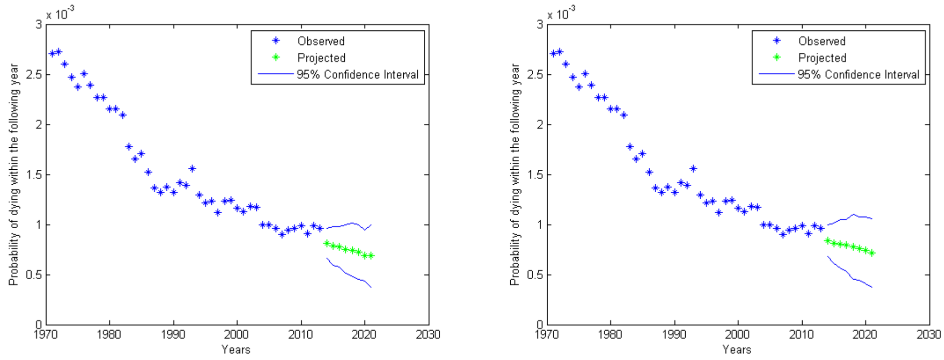
MTM and Modified MTM are applied on Hong Kong Female mortality rates. There is no significant difference between projections performed by modified MTM and MTM on Hong Kong Female mortality rates at any age.



(a) MTM (b) Modified MTM

Figure 4.17: Projection for Hong Kong Mortality Rates and 95% confidence interval (Female, Age 65)

At age 45, standard deviations of both projections increase over time as same as Hong Kong Male projections at age 45. However, there is no difference in terms of patterns and confidence intervals between Modified MTM and MTM.

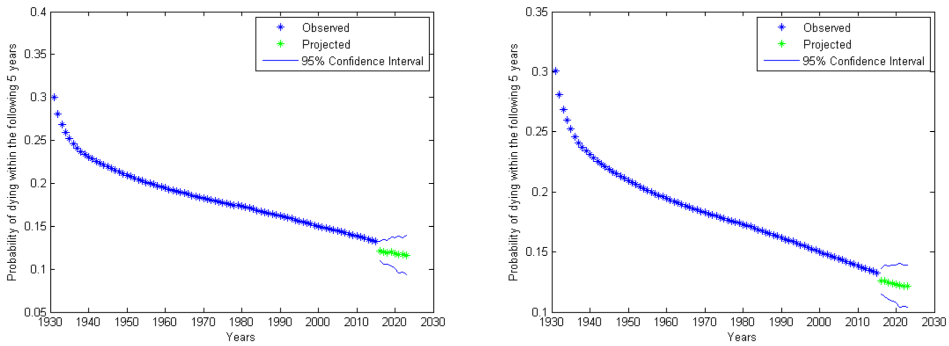


(a) MTM (b) Modified MTM

Figure 4.18: Projection for Hong Kong Mortality Rates and 95% confidence interval (Female, Age 45)

4.5.3 MTM vs. Modified MTM Comparison for Turkish Male Mortality Rates

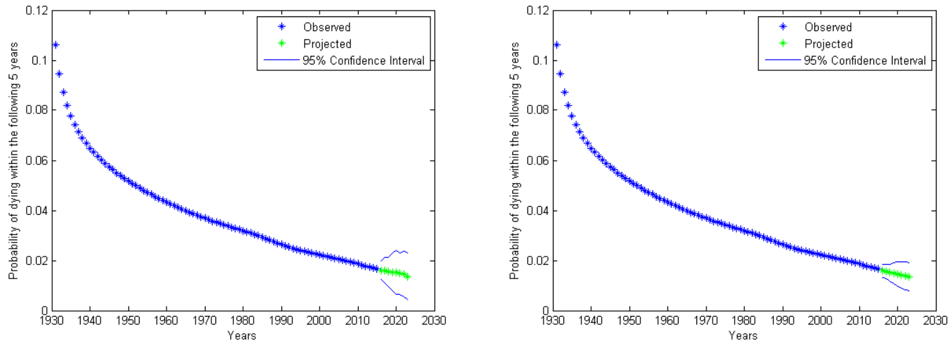
MTM and Modified MTM are applied on Turkish Male mortality rates. Both projections have similar patterns and confidence regions at age 65.



(a) MTM (b) Modified MTM

Figure 4.19: Projection for Turkish Mortality Rates and 95% confidence interval (Male, Age 65)

However, MTM projection has wider confidence interval than Modified Model's at age 45. Therefore, Modified MTM has less uncertainty on the projection compared to MTM at a given confidence level of 5% of age 45.

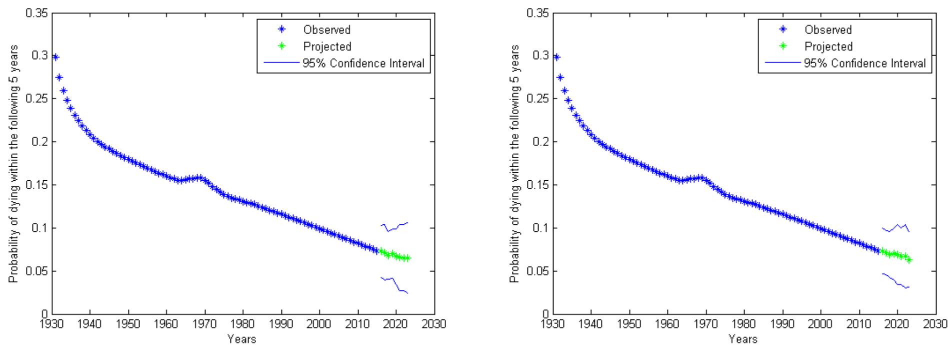


(a) MTM (b) Modified MTM

Figure 4.20: Projection for Turkish Mortality Rates and 95% confidence interval (Male, Age 45)

4.5.4 MTM vs. Modified MTM Comparison for Turkish Female Mortality Rates

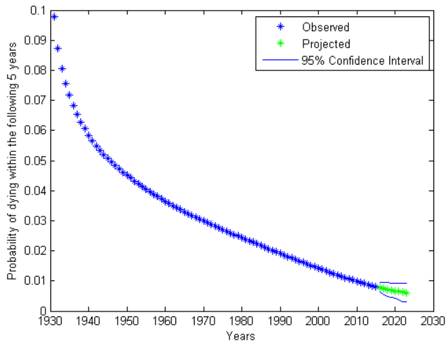
MTM and Modified MTM are applied on Turkish Female mortality rates. Both projections have similar patterns and confidence regions at age 65. Unlike the other mortality projections, Turkish Female mortality projections by both models have wide confidence intervals at age 65. However, at age 45, confidence intervals become narrower. Therefore, it can be inferred that models perform more accurate results at age 45 for Turkish Female case compared to the projections at age 65.



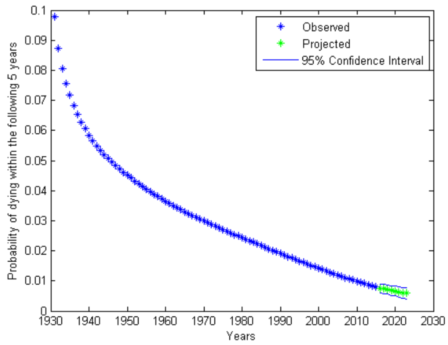
(a) MTM (b) Modified MTM

Figure 4.21: Projection for Turkish Mortality Rates and 95% confidence interval (Female, Age 65)

Confidence interval of MTM projection is wider than Modified MTM's. Thus, Modified MTM performs more accurate projections at 45, as in the cases of Hong Kong Male and Turkish Male.



(a) MTM

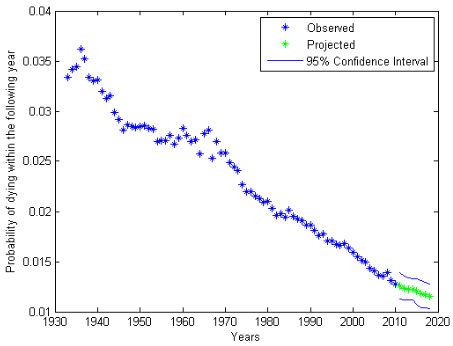


(b) Modified MTM

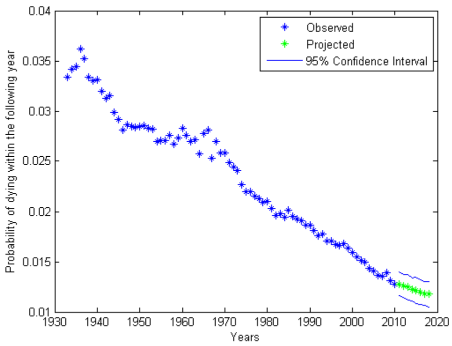
Figure 4.22: Projection for Turkish Mortality Rates and 95% confidence interval (Female, Age 45)

4.5.5 MTM vs. Modified MTM Comparison for USA Mortality Rates

MTM and Modified MTM are applied on USA mortality rates. Both projections have similar patterns and confidence regions at age 65 and 45.

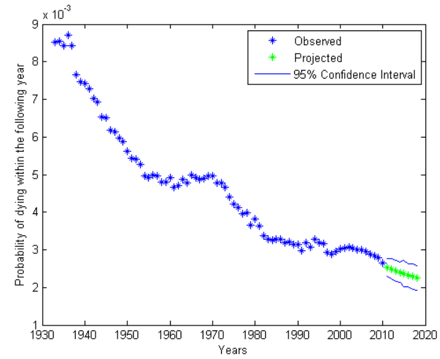


(a) MTM

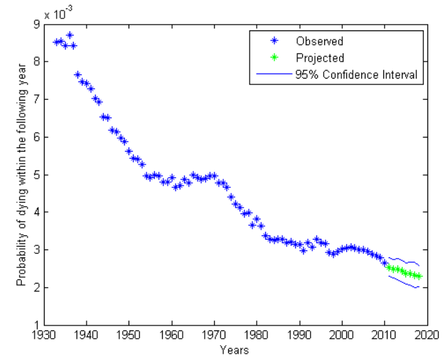


(b) Modified MTM

Figure 4.23: Projection for USA Mortality Rates and 95% confidence interval (Both gender, Age 65)



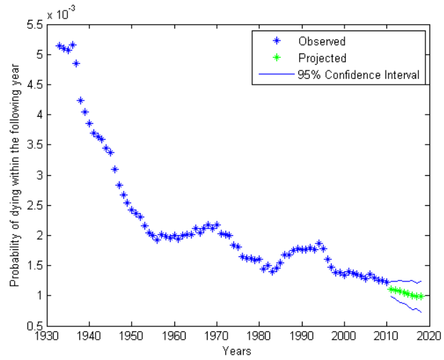
(a) MTM



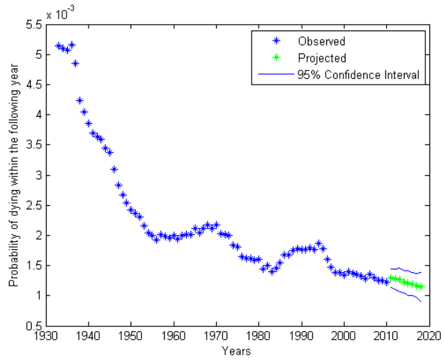
(b) Modified MTM

Figure 4.24: Projection for USA Mortality Rates and 95% confidence interval (Both gender, Age 45)

At age 35, both model projections have similar confidence intervals but different patterns. The pattern in the forecast of Modified MTM agrees with the original data slightly better than the MTM's.



(a) MTM



(b) Modified MTM

Figure 4.25: Projection for USA Mortality Rates and 95% confidence interval (Both gender, Age 35)

CHAPTER 5

CONCLUSION AND OUTLOOK

In this thesis, a mortality trend model is modified and applied to Hong Kong [6], USA [13] and Turkish mortality rates in order to project future mortality rates. While constructing the model, parameters and structure of the model which is proposed by Boerger (2014) [3] have been taken into consideration. In addition to the parameters of mortality trend model, which were constructed by Boerger, a new parameter which represents the childhood effect is added to the model and Monte Carlo Simulation is used to stabilize the projections and construct a confidence interval for projections. During the applications, both mortality trend model and modified mortality trend model are used and compared.

Generalized linear modeling (GLM) is used to model the mortality rates. Mortality rates are used as response variable and transformed by logit function in order to be linearized. Output of GLM gives values of $\kappa^{(\cdot)}$ parameters which are general tendency, mortality steepness, youth effect and old age effect. In addition, childhood effect parameter is also estimated in modified model. Different parameter values for each year are estimated. Modeling the trends of each parameter over time, estimated by GLM, is the key point of the study.

General tendency parameter is modeled using weighted least squares (WLS) method. In model, stochastic term $\varepsilon_t^{(1)}(\bar{\sigma}^{(1)} + \sigma^{(1)})$ is used as noise where $\varepsilon_t^{(1)}$ follows standard normal distribution, $\sigma^{(1)}$ is standard deviation of empirical errors and $\bar{\sigma}^{(1)}$ is optional volatility.

Mortality steepness, youth effect, old age effect and childhood effect parameters are not linear parameters. According to stationarity test results, none of them is stationary. Thus, most suitable method to model these parameters is random walk with drift (RWWD). The mean and standard deviation of differences between $\kappa_t^{(\cdot)}$ and $\kappa_{t-1}^{(\cdot)}$ are assumed to be respectively the drift $\mu_t^{(\cdot)}$ and the volatility $\sigma^{(\cdot)}$ of the random walk with drift model. Each parameter series are modeled separately.

After modeling the parameters, the future mortality rates are calculated using the future values of parameters in the mortality trend model. The process which starts with GLM and ends with projecting the future mortality rates is applied 100 times as a Monte Carlo Simulation and each trial is stored in a matrix. For each unit of mortality table, 100 different projections are obtained. Average and the standard deviation of these 100 trials are calculated for each unit in order to construct the confidence interval of future mortality rates.

Before projection of the future parameters and mortality rates, 20% of the observed mortality rates have been put aside for the validation of the model. Model is validated by calculation of Mean Absolute Error (MAE) and R-Squared values for each year of the validation. All MAE values are under 10% and all R-Squared values are above 90%. In addition, all observed mortality rates having been put aside for the validation are in the 95% confidence interval region of the projected mortality rates.

While comparing mortality trend model and modified mortality trend model, MAE values for two models are statistically compared with t-comparison test. In most of the cases, modified model performs more accurate results in the analysis which includes younger ages. In other words, if both models are conducted in a wider range of ages including childhood ages, modified model projects the future mortality rates with less margin of errors.

Since the mortality trend of young people has a different slope than the rest of population, forecasts with the analysis which includes younger ages, such as 5-20, have always been challanging in the mortality trend modeling. Projections performed by proposed modified mortality trend model, which has a childhood effect parameter, have narrower confidence intervals and more precise forecasts compared to mortality trend model in the projections which includes younger ages. Projection with a 95% confidence interval of future mortality rates for Hong Kong Male, Hong Kong Female, Turkish Male, Turkish Female and USA data can be considered as the output of the study. While producing a pension or a life insurance product for the people at childhood ages, the rates projected by modified mortality trend model would be more convenient.

Modified mortality trend model is very advantageous, especially for Turkey considering its sensivity to mortality tendencies of younger ages since the young population of Turkey is quite significant compared to the western countries.

For future studies, impact of the catastrophic events on mortality rates can be implemented to the modified mortality trend model. Turkish Statistical Institute has been constructing mortality tables, consisting of observed mortality rates, since 2013. A decade later, modified mortality trend model can be applied to 13 years of observed Turkish mortality rates which will have been constructed by Turkish Statistical Institute.

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APPENDIX A

Projected Mortality Rates for the Cases of Hong Kong and USA

Table A.1: Projection for Hong Kong (Male) mortality rates using MTM

Age/Year	2014	2015	2016	2017	2018	2019	2020	2021	2022	2023	Age/Year	2014	2015	2016	2017	2018	2019	2020	2021	2022	2023
20	0.000361	0.000383	0.000372	0.000384	0.000389	0.000398	0.000408	0.000417	0.000429	0.000447	61	0.007078	0.006886	0.006736	0.006539	0.006368	0.006192	0.006079	0.005853	0.005704	0.005536
21	0.000392	0.000414	0.000402	0.000415	0.000418	0.000427	0.000437	0.000446	0.000458	0.000475	62	0.007815	0.007607	0.007444	0.007225	0.007037	0.006845	0.006721	0.006475	0.006314	0.006129
22	0.000420	0.000442	0.000429	0.000441	0.000445	0.000453	0.000463	0.000470	0.000482	0.000499	63	0.008617	0.008392	0.008215	0.007971	0.007766	0.007557	0.007422	0.007154	0.006980	0.006777
23	0.000443	0.000465	0.000451	0.000463	0.000466	0.000473	0.000483	0.000490	0.000501	0.000517	64	0.009497	0.009253	0.009062	0.008791	0.008566	0.008340	0.008192	0.007901	0.007712	0.007491
24	0.000463	0.000485	0.000470	0.000482	0.000484	0.000490	0.000499	0.000505	0.000515	0.000531	65	0.010465	0.010202	0.009995	0.009694	0.009448	0.009202	0.009041	0.008724	0.008521	0.008279
25	0.000480	0.000501	0.000486	0.000497	0.000498	0.000503	0.000512	0.000516	0.000525	0.000540	66	0.011528	0.011243	0.011020	0.010686	0.010417	0.010150	0.009975	0.009631	0.009412	0.009147
26	0.000494	0.000514	0.000499	0.000509	0.000509	0.000514	0.000522	0.000525	0.000533	0.000547	67	0.012694	0.012387	0.012146	0.011776	0.011482	0.011193	0.011003	0.010628	0.010393	0.010104
27	0.000508	0.000527	0.000511	0.000520	0.000520	0.000523	0.000531	0.000533	0.000540	0.000552	68	0.013978	0.013647	0.013387	0.012977	0.012656	0.012343	0.012136	0.011730	0.011476	0.011161
28	0.000523	0.000541	0.000525	0.000533	0.000532	0.000534	0.000541	0.000542	0.000548	0.000559	69	0.015391	0.015035	0.014755	0.014302	0.013951	0.013612	0.013388	0.012947	0.012674	0.012330
29	0.000540	0.000557	0.000540	0.000547	0.000545	0.000547	0.000553	0.000552	0.000557	0.000568	70	0.016930	0.016548	0.016246	0.015745	0.015363	0.014996	0.014754	0.014275	0.013983	0.013608
30	0.000558	0.000575	0.000557	0.000563	0.000561	0.000561	0.000566	0.000564	0.000568	0.000578	71	0.018597	0.018186	0.017863	0.017310	0.016895	0.016499	0.016236	0.015720	0.015406	0.014998
31	0.000580	0.000595	0.000577	0.000582	0.000579	0.000578	0.000583	0.000579	0.000582	0.000591	72	0.020401	0.019961	0.019615	0.019006	0.018556	0.018129	0.017846	0.017288	0.016954	0.016511
32	0.000604	0.000618	0.000599	0.000604	0.000599	0.000597	0.000601	0.000597	0.000598	0.000606	73	0.022364	0.021894	0.021524	0.020854	0.020366	0.019907	0.019603	0.019001	0.018644	0.018164
33	0.000631	0.000644	0.000624	0.000627	0.000622	0.000619	0.000622	0.000616	0.000617	0.000623	74	0.024511	0.024010	0.023614	0.022878	0.022349	0.021857	0.021529	0.020881	0.020500	0.019980
34	0.000662	0.000674	0.000653	0.000656	0.000649	0.000645	0.000647	0.000640	0.000639	0.000644	75	0.027805	0.027253	0.026816	0.025979	0.025388	0.024841	0.024477	0.023755	0.023336	0.022753
35	0.000699	0.000710	0.000688	0.000690	0.000682	0.000676	0.000678	0.000669	0.000667	0.000671	76	0.030728	0.030135	0.029666	0.028740	0.028095	0.027505	0.027110	0.026327	0.025877	0.025242
36	0.000742	0.000751	0.000728	0.000728	0.000719	0.000712	0.000713	0.000702	0.000699	0.000701	77	0.033961	0.033326	0.032822	0.031798	0.031095	0.030458	0.030032	0.029183	0.028701	0.028008
37	0.000793	0.000801	0.000777	0.000775	0.000765	0.000756	0.000756	0.000743	0.000738	0.000740	78	0.037530	0.036850	0.036310	0.035178	0.034413	0.033725	0.033265	0.032346	0.031830	0.031075
38	0.000856	0.000862	0.000836	0.000833	0.000821	0.000810	0.000810	0.000794	0.000788	0.000787	79	0.041456	0.040730	0.040151	0.038901	0.038070	0.037329	0.036833	0.035838	0.035287	0.034466
39	0.000928	0.000933	0.000905	0.000901	0.000886	0.000874	0.000872	0.000853	0.000845	0.000843	80	0.045762	0.044987	0.044369	0.042991	0.042088	0.041293	0.040758	0.039684	0.039096	0.038205
40	0.001009	0.001012	0.000982	0.000975	0.000959	0.000944	0.000940	0.000919	0.000909	0.000905	81	0.050470	0.049646	0.048987	0.047469	0.046491	0.045637	0.045063	0.043906	0.043280	0.042314
41	0.001099	0.001099	0.001067	0.001058	0.001039	0.001022	0.001017	0.000992	0.000980	0.000973	82	0.055603	0.054728	0.054028	0.052359	0.051301	0.050388	0.049772	0.048527	0.047863	0.046818
42	0.001197	0.001194	0.001159	0.001147	0.001126	0.001106	0.001099	0.001071	0.001056	0.001047	83	0.061185	0.060260	0.059517	0.057685	0.056544	0.055569	0.054911	0.053575	0.052872	0.051743
43	0.001302	0.001296	0.001259	0.001244	0.001220	0.001196	0.001188	0.001155	0.001138	0.001126	84	0.067243	0.066268	0.065482	0.063476	0.062247	0.061209	0.060507	0.059076	0.058334	0.057118
44	0.001418	0.001409	0.001368	0.001350	0.001323	0.001296	0.001285	0.001248	0.001227	0.001212	85	0.073878	0.072852	0.072023	0.069827	0.068506	0.067402	0.066654	0.065124	0.064343	0.063035
45	0.001549	0.001536	0.001492	0.001470	0.001439	0.001408	0.001395	0.001353	0.001329	0.001310	86	0.081764	0.080581	0.079740	0.077303	0.075838	0.074650	0.073913	0.072341	0.071479	0.070101
46	0.001695	0.001676	0.001629	0.001603	0.001568	0.001533	0.001517	0.001469	0.001441	0.001418	87	0.090453	0.089098	0.088259	0.085561	0.083947	0.082673	0.081964	0.080361	0.079422	0.077973
47	0.001859	0.001834	0.001783	0.001752	0.001712	0.001672	0.001653	0.001599	0.001566	0.001539	88	0.100018	0.098477	0.097653	0.094674	0.092907	0.091547	0.090886	0.089267	0.088256	0.086735
48	0.002043	0.002011	0.001956	0.001919	0.001875	0.001829	0.001805	0.001745	0.001707	0.001674	89	0.110538	0.108795	0.108001	0.104721	0.102798	0.101354	0.100760	0.099143	0.098069	0.096478
49	0.002247	0.002207	0.002148	0.002104	0.002054	0.002002	0.001975	0.001907	0.001863	0.001823	90	0.122093	0.120132	0.119388	0.115784	0.113702	0.112176	0.111672	0.110079	0.108953	0.107294
50	0.002469	0.002420	0.002356	0.002305	0.002249	0.002190	0.002158	0.002081	0.002031	0.001985	91	0.134772	0.132574	0.131898	0.127949	0.125705	0.124103	0.123712	0.122169	0.121004	0.119280
51	0.002710	0.002650	0.002581	0.002522	0.002460	0.002393	0.002356	0.002270	0.002213	0.002158	92	0.148662	0.146209	0.145622	0.141306	0.138896	0.137224	0.136969	0.135504	0.134315	0.132532
52	0.002967	0.002896	0.002822	0.002754	0.002684	0.002609	0.002566	0.002470	0.002405	0.002342	93	0.163856	0.161128	0.160650	0.155945	0.153363	0.151629	0.151531	0.150178	0.148981	0.147144
53	0.003241	0.003157	0.003078	0.003000	0.002922	0.002839	0.002789	0.002682	0.002610	0.002538	94	0.180447	0.177420	0.177073	0.171954	0.169192	0.167407	0.167484	0.166279	0.165090	0.163206
54	0.003539	0.003440	0.003356	0.003267	0.003181	0.003088	0.003031	0.002912	0.002831	0.002748	95	0.198529	0.195177	0.194978	0.189423	0.186468	0.184644	0.184908	0.183888	0.182720	0.180801
55	0.003864	0.003749	0.003659	0.003558	0.003463	0.003359	0.003294	0.003163	0.003072	0.002977	96	0.218191	0.214485	0.214449	0.208435	0.205267	0.203419	0.203874	0.203077	0.201942	0.199998
56	0.004261	0.004135	0.004037	0.003925	0.003820	0.003708	0.003637	0.003493	0.003395	0.003291	97	0.239521	0.235428	0.235565	0.229067	0.225659	0.223803	0.224441	0.223907	0.222809	0.220854
57	0.004707	0.004570	0.004464	0.004338	0.004223	0.004100	0.004022	0.003865	0.003758	0.003644	98	0.262597	0.258081	0.258391	0.251387	0.247702	0.245855	0.246656	0.246422	0.245357	0.243407
58	0.005212	0.005063	0.004947	0.004807	0.004679	0.004545	0.004459	0.004287	0.004171	0.004045	99	0.287488	0.282508	0.282984	0.275451	0.271443	0.269620	0.270548	0.270647	0.269603	0.267678
59	0.005779	0.005617	0.005490	0.005332	0.005192	0.005044	0.004950	0.004762	0.004635	0.004496	100	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000
60	0.006400	0.006223	0.006085	0.005908	0.005753	0.005592	0.005489	0.005282	0.005145	0.004992											

Table A.2: Projection for Hong Kong (Male) mortality rates using Modified MTM

Age/Years	2014	2015	2016	2017	2018	2019	2020	2021	2022	2023	Age/Years	2014	2015	2016	2017	2018	2019	2020	2021	2022	2023
5	0.000200	0.000194	0.000214	0.000224	0.000237	0.000231	0.000260	0.000283	0.000290	0.000280	**53**	0.003448	0.003359	0.003216	0.003154	0.003054	0.002989	0.002889	0.002816	0.002769	0.002667
6	0.000167	0.000163	0.000178	0.000185	0.000194	0.000190	0.000212	0.000229	0.000234	0.000225	**54**	0.003770	0.003675	0.003517	0.003450	0.003342	0.003271	0.003162	0.003083	0.003029	0.002917
7	0.000144	0.000140	0.000152	0.000157	0.000164	0.000160	0.000177	0.000190	0.000194	0.000187	**55**	0.004123	0.004021	0.003847	0.003775	0.003658	0.003579	0.003462	0.003375	0.003315	0.003191
8	0.000128	0.000124	0.000134	0.000138	0.000143	0.000140	0.000153	0.000163	0.000166	0.000160	**56**	0.004544	0.004431	0.004240	0.004163	0.004035	0.003947	0.003819	0.003725	0.003657	0.003520
9	0.000119	0.000115	0.000123	0.000126	0.000130	0.000127	0.000138	0.000146	0.000149	0.000144	**57**	0.005016	0.004891	0.004681	0.004598	0.004458	0.004361	0.004220	0.004118	0.004042	0.003890
10	0.000121	0.000117	0.000124	0.000126	0.000130	0.000127	0.000137	0.000144	0.000146	0.000141	**58**	0.005552	0.005413	0.005182	0.005092	0.004939	0.004830	0.004675	0.004565	0.004479	0.004311
11	0.000128	0.000124	0.000131	0.000133	0.000136	0.000133	0.000142	0.000148	0.000150	0.000145	**59**	0.006152	0.005998	0.005743	0.005646	0.005478	0.005356	0.005186	0.005066	0.004969	0.004783
12	0.000141	0.000136	0.000142	0.000144	0.000147	0.000144	0.000152	0.000158	0.000159	0.000154	**60**	0.006809	0.006638	0.006357	0.006252	0.006068	0.005932	0.005746	0.005616	0.005507	0.005301
13	0.000157	0.000152	0.000158	0.000159	0.000161	0.000158	0.000165	0.000171	0.000172	0.000167	**61**	0.007526	0.007338	0.007028	0.006915	0.006714	0.006563	0.006359	0.006218	0.006096	0.005868
14	0.000176	0.000171	0.000176	0.000176	0.000178	0.000175	0.000182	0.000187	0.000188	0.000182	**62**	0.008305	0.008097	0.007757	0.007636	0.007417	0.007248	0.007025	0.006874	0.006736	0.006485
15	0.000199	0.000193	0.000197	0.000197	0.000198	0.000194	0.000201	0.000206	0.000206	0.000199	**63**	0.009152	0.008922	0.008550	0.008421	0.008181	0.007995	0.007751	0.007588	0.007435	0.007157
16	0.000224	0.000217	0.000221	0.000220	0.000221	0.000216	0.000222	0.000226	0.000226	0.000219	**64**	0.010080	0.009827	0.009419	0.009282	0.009021	0.008814	0.008548	0.008373	0.008202	0.007897
17	0.000252	0.000244	0.000247	0.000245	0.000245	0.000240	0.000244	0.000248	0.000247	0.000239	**65**	0.011101	0.010822	0.010376	0.010229	0.009945	0.009716	0.009426	0.009239	0.009048	0.008712
18	0.000280	0.000272	0.000273	0.000270	0.000269	0.000264	0.000267	0.000269	0.000269	0.000260	**66**	0.012221	0.011914	0.011425	0.011270	0.010961	0.010707	0.010392	0.010191	0.009978	0.009608
19	0.000309	0.000299	0.000299	0.000295	0.000293	0.000287	0.000289	0.000290	0.000290	0.000280	**67**	0.013448	0.013111	0.012577	0.012412	0.012077	0.011795	0.011452	0.011238	0.011001	0.010594
20	0.000335	0.000325	0.000323	0.000318	0.000315	0.000309	0.000309	0.000309	0.000308	0.000297	**68**	0.014799	0.014428	0.013845	0.013670	0.013306	0.012994	0.012621	0.012392	0.012129	0.011682
21	0.000359	0.000349	0.000345	0.000339	0.000334	0.000328	0.000327	0.000325	0.000324	0.000313	**69**	0.016285	0.015878	0.015240	0.015056	0.014661	0.014316	0.013911	0.013667	0.013374	0.012883
22	0.000380	0.000369	0.000364	0.000357	0.000350	0.000344	0.000341	0.000338	0.000336	0.000325	**70**	0.017902	0.017456	0.016760	0.016566	0.016137	0.015757	0.015316	0.015058	0.014732	0.014194
23	0.000397	0.000386	0.000378	0.000370	0.000363	0.000356	0.000352	0.000347	0.000345	0.000333	**71**	0.019651	0.019162	0.018404	0.018202	0.017737	0.017318	0.016841	0.016567	0.016206	0.015617
24	0.000410	0.000399	0.000389	0.000381	0.000372	0.000366	0.000359	0.000353	0.000351	0.000339	**72**	0.021543	0.021009	0.020184	0.019974	0.019471	0.019010	0.018494	0.018205	0.017806	0.017162
25	0.000541	0.000523	0.000517	0.000505	0.000495	0.000491	0.000482	0.000481	0.000484	0.000478	**73**	0.023600	0.023017	0.022121	0.021902	0.021359	0.020854	0.020296	0.019992	0.019551	0.018848
26	0.000546	0.000528	0.000520	0.000508	0.000496	0.000493	0.000481	0.000478	0.000480	0.000474	**74**	0.025848	0.025212	0.024239	0.024013	0.023427	0.022872	0.022270	0.021951	0.021464	0.020697
27	0.000551	0.000533	0.000522	0.000510	0.000497	0.000493	0.000480	0.000475	0.000477	0.000470	**75**	0.029300	0.028582	0.027491	0.027251	0.026597	0.025967	0.025296	0.024951	0.024395	0.023528
28	0.000556	0.000539	0.000526	0.000513	0.000499	0.000495	0.000480	0.000474	0.000475	0.000467	**76**	0.032358	0.031568	0.030375	0.030128	0.029417	0.028720	0.027990	0.027628	0.027009	0.026058
29	0.000563	0.000547	0.000531	0.000518	0.000502	0.000498	0.000482	0.000474	0.000475	0.000467	**77**	0.035737	0.034870	0.033566	0.033312	0.032540	0.031769	0.030977	0.030597	0.029910	0.028864
30	0.000572	0.000556	0.000537	0.000525	0.000507	0.000504	0.000486	0.000476	0.000477	0.000468	**78**	0.039463	0.038511	0.037086	0.036828	0.035990	0.035139	0.034279	0.033883	0.033119	0.031971
31	0.000595	0.000578	0.000559	0.000546	0.000528	0.000523	0.000505	0.000494	0.000494	0.000485	**79**	0.043558	0.042514	0.040960	0.040698	0.039790	0.038850	0.037918	0.037507	0.036659	0.035400
32	0.000620	0.000603	0.000582	0.000569	0.000550	0.000545	0.000525	0.000514	0.000514	0.000503	**80**	0.048045	0.046901	0.045208	0.044946	0.043962	0.042927	0.041917	0.041493	0.040554	0.039174
33	0.000649	0.000630	0.000609	0.000595	0.000575	0.000569	0.000548	0.000536	0.000536	0.000524	**81**	0.052946	0.051695	0.049852	0.049593	0.048529	0.047390	0.046299	0.045864	0.044824	0.043315
34	0.000682	0.000663	0.000640	0.000625	0.000604	0.000597	0.000576	0.000563	0.000562	0.000549	**82**	0.058284	0.056917	0.054915	0.054663	0.053515	0.052263	0.051086	0.050642	0.049494	0.047846
35	0.000722	0.000701	0.000677	0.000661	0.000638	0.000631	0.000608	0.000594	0.000593	0.000578	**83**	0.064083	0.062592	0.060422	0.060181	0.058943	0.057571	0.056303	0.055855	0.054589	0.052793
36	0.000767	0.000745	0.000718	0.000702	0.000678	0.000670	0.000646	0.000631	0.000628	0.000612	**84**	0.070370	0.068748	0.066399	0.066173	0.064843	0.063340	0.061977	0.061530	0.060136	0.058181
37	0.000821	0.000798	0.000769	0.000751	0.000726	0.000716	0.000690	0.000674	0.000671	0.000653	**85**	0.077247	0.075483	0.072944	0.072739	0.071310	0.069668	0.068204	0.067760	0.066228	0.064104
38	0.000888	0.000862	0.000831	0.000812	0.000784	0.000773	0.000745	0.000727	0.000723	0.000704	**86**	0.085615	0.083572	0.080747	0.080684	0.079108	0.077230	0.075733	0.075468	0.073831	0.071537
39	0.000965	0.000937	0.000902	0.000882	0.000852	0.000839	0.000809	0.000790	0.000785	0.000762	**87**	0.094844	0.092495	0.089361	0.089477	0.087745	0.085613	0.084101	0.084064	0.082332	0.079869
40	0.001050	0.001020	0.000982	0.000960	0.000927	0.000913	0.000880	0.000859	0.000852	0.000827	**88**	0.105011	0.102330	0.098861	0.099198	0.097301	0.094897	0.093394	0.093640	0.091826	0.089197
41	0.001146	0.001113	0.001071	0.001047	0.001011	0.000995	0.000960	0.000936	0.000928	0.000900	**89**	0.116200	0.113157	0.109328	0.109931	0.107860	0.105167	0.103700	0.104292	0.102410	0.099625
42	0.001250	0.001214	0.001168	0.001142	0.001103	0.001085	0.001046	0.001020	0.001011	0.000980	**90**	0.128495	0.125060	0.120845	0.121765	0.119512	0.116512	0.115109	0.116119	0.114188	0.111257
43	0.001362	0.001323	0.001272	0.001244	0.001202	0.001181	0.001139	0.001111	0.001100	0.001065	**91**	0.141985	0.138126	0.133501	0.134790	0.132347	0.129023	0.127715	0.129220	0.127260	0.124199
44	0.001486	0.001444	0.001388	0.001357	0.001312	0.001289	0.001243	0.001211	0.001199	0.001160	**92**	0.156763	0.152444	0.147384	0.149097	0.146457	0.142791	0.141611	0.143695	0.141728	0.138552
45	0.001627	0.001581	0.001518	0.001485	0.001436	0.001409	0.001360	0.001325	0.001310	0.001267	**93**	0.172920	0.168104	0.162585	0.164778	0.161933	0.157908	0.156885	0.159636	0.157683	0.154410
46	0.001783	0.001733	0.001664	0.001628	0.001574	0.001544	0.001490	0.001452	0.001435	0.001386	**94**	0.190547	0.185193	0.179196	0.181920	0.178861	0.174460	0.173621	0.177129	0.175207	0.171855
47	0.001958	0.001904	0.001827	0.001788	0.001729	0.001696	0.001637	0.001595	0.001574	0.001521	**95**	0.209732	0.203797	0.197303	0.200605	0.197324	0.192527	0.191891	0.196245	0.194365	0.190949
48	0.002155	0.002096	0.002011	0.001969	0.001904	0.001866	0.001802	0.001755	0.001732	0.001672	**96**	0.230558	0.223994	0.216988	0.220908	0.217393	0.212180	0.211755	0.217040	0.215203	0.211730
49	0.002375	0.002310	0.002215	0.002170	0.002099	0.002057	0.001986	0.001935	0.001907	0.001840	**97**	0.253100	0.245856	0.238328	0.242890	0.239129	0.233475	0.233256	0.239547	0.237739	0.234211
50	0.002614	0.002544	0.002438	0.002388	0.002311	0.002264	0.002186	0.002130	0.002099	0.002024	**98**	0.277420	0.269440	0.261386	0.266600	0.262575	0.256454	0.256417	0.263776	0.261965	0.258372
51	0.002873	0.002797	0.002680	0.002626	0.002541	0.002489	0.002404	0.002343	0.002307	0.002223	**99**	0.303568	0.294792	0.286213	0.292067	0.287760	0.281140	0.281240	0.289709	0.287843	0.284165
52	0.003150	0.003068	0.002938	0.002881	0.002788	0.002730	0.002638	0.002571	0.002530	0.002437	**100**	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000

Table A.3: Projection for Hong Kong (Female) mortality rates using MTM

Age/Year	2014	2015	2016	2017	2018	2019	2020	2021	2022	2023	Age/Year	2014	2015	2016	2017	2018	2019	2020	2021	2022	2023
20	0.000176	0.000176	0.000199	0.000206	0.000211	0.000219	0.000230	0.000246	0.000254	0.000259	**61**	0.003354	0.003224	0.003142	0.003068	0.002971	0.002876	0.002806	0.002746	0.002639	0.002550
21	0.000186	0.000186	0.000209	0.000215	0.000221	0.000229	0.000239	0.000255	0.000262	0.000267	**62**	0.003691	0.003549	0.003459	0.003379	0.003274	0.003169	0.003094	0.003026	0.002909	0.002811
22	0.000195	0.000195	0.000217	0.000224	0.000229	0.000237	0.000246	0.000262	0.000269	0.000273	**63**	0.004067	0.003911	0.003815	0.003728	0.003613	0.003497	0.003415	0.003339	0.003210	0.003103
23	0.000203	0.000203	0.000225	0.000231	0.000236	0.000244	0.000253	0.000268	0.000274	0.000278	**64**	0.004492	0.004321	0.004216	0.004122	0.003997	0.003868	0.003779	0.003693	0.003552	0.003435
24	0.000211	0.000210	0.000232	0.000238	0.000242	0.000249	0.000258	0.000273	0.000278	0.000281	**65**	0.004970	0.004782	0.004669	0.004566	0.004429	0.004286	0.004190	0.004093	0.003937	0.003808
25	0.000218	0.000217	0.000238	0.000244	0.000248	0.000254	0.000263	0.000277	0.000282	0.000284	**66**	0.005504	0.005297	0.005174	0.005063	0.004913	0.004754	0.004650	0.004540	0.004369	0.004227
26	0.000225	0.000224	0.000244	0.000249	0.000253	0.000259	0.000267	0.000280	0.000284	0.000286	**67**	0.006098	0.005871	0.005738	0.005617	0.005452	0.005276	0.005162	0.005039	0.004850	0.004694
27	0.000233	0.000231	0.000251	0.000256	0.000259	0.000264	0.000272	0.000285	0.000288	0.000289	**68**	0.006763	0.006513	0.006369	0.006238	0.006057	0.005861	0.005738	0.005599	0.005391	0.005219
28	0.000242	0.000239	0.000259	0.000263	0.000266	0.000271	0.000278	0.000290	0.000293	0.000293	**69**	0.007512	0.007236	0.007080	0.006937	0.006739	0.006521	0.006386	0.006230	0.006000	0.005811
29	0.000252	0.000249	0.000268	0.000272	0.000274	0.000279	0.000285	0.000297	0.000299	0.000299	**70**	0.008338	0.008034	0.007866	0.007711	0.007493	0.007251	0.007105	0.006930	0.006676	0.006468
30	0.000265	0.000261	0.000279	0.000283	0.000284	0.000289	0.000295	0.000306	0.000307	0.000306	**71**	0.009248	0.008914	0.008732	0.008565	0.008326	0.008057	0.007899	0.007702	0.007423	0.007193
31	0.000279	0.000274	0.000292	0.000295	0.000297	0.000300	0.000306	0.000317	0.000317	0.000315	**72**	0.010256	0.009888	0.009693	0.009511	0.009250	0.008952	0.008780	0.008559	0.008252	0.008000
32	0.000295	0.000289	0.000307	0.000310	0.000310	0.000314	0.000318	0.000329	0.000328	0.000326	**73**	0.011383	0.010979	0.010769	0.010573	0.010287	0.009955	0.009769	0.009521	0.009183	0.008906
33	0.000312	0.000307	0.000323	0.000326	0.000326	0.000329	0.000333	0.000343	0.000342	0.000339	**74**	0.012656	0.012210	0.011984	0.011773	0.011459	0.011090	0.010889	0.010609	0.010237	0.009932
34	0.000333	0.000326	0.000343	0.000345	0.000344	0.000346	0.000350	0.000359	0.000357	0.000354	**75**	0.014467	0.013963	0.013714	0.013479	0.013126	0.012704	0.012480	0.012157	0.011736	0.011390
35	0.000356	0.000349	0.000365	0.000366	0.000365	0.000366	0.000369	0.000378	0.000375	0.000371	**76**	0.016183	0.015624	0.015356	0.015101	0.014713	0.014240	0.013997	0.013632	0.013165	0.012783
36	0.000383	0.000374	0.000390	0.000391	0.000388	0.000389	0.000391	0.000400	0.000396	0.000391	**77**	0.018117	0.017499	0.017210	0.016934	0.016506	0.015977	0.015712	0.015300	0.014783	0.014360
37	0.000414	0.000404	0.000418	0.000419	0.000415	0.000416	0.000417	0.000425	0.000420	0.000414	**78**	0.020291	0.019607	0.019297	0.018998	0.018527	0.017934	0.017647	0.017182	0.016609	0.016140
38	0.000449	0.000437	0.000451	0.000451	0.000447	0.000446	0.000446	0.000454	0.000447	0.000440	**79**	0.022726	0.021968	0.021637	0.021314	0.020795	0.020132	0.019820	0.019295	0.018662	0.018143
39	0.000488	0.000476	0.000489	0.000487	0.000482	0.000480	0.000480	0.000487	0.000479	0.000470	**80**	0.025442	0.024604	0.024251	0.023903	0.023332	0.022590	0.022253	0.021661	0.020961	0.020388
40	0.000532	0.000518	0.000530	0.000528	0.000521	0.000518	0.000516	0.000523	0.000513	0.000503	**81**	0.028460	0.027535	0.027160	0.026786	0.026160	0.025331	0.024967	0.024301	0.023528	0.022895
41	0.000581	0.000564	0.000575	0.000572	0.000564	0.000559	0.000557	0.000562	0.000550	0.000539	**82**	0.031804	0.030785	0.030389	0.029987	0.029302	0.028377	0.027985	0.027236	0.026385	0.025688
42	0.000633	0.000614	0.000624	0.000619	0.000609	0.000603	0.000599	0.000604	0.000590	0.000577	**83**	0.035501	0.034379	0.033963	0.033534	0.032785	0.031754	0.031334	0.030494	0.029558	0.028791
43	0.000689	0.000668	0.000676	0.000669	0.000658	0.000650	0.000645	0.000648	0.000632	0.000618	**84**	0.039580	0.038348	0.037912	0.037456	0.036639	0.035493	0.035044	0.034103	0.033076	0.032233
44	0.000750	0.000726	0.000732	0.000724	0.000710	0.000701	0.000693	0.000695	0.000677	0.000661	**85**	0.044132	0.042780	0.042326	0.041842	0.040952	0.039677	0.039199	0.038145	0.037019	0.036094
45	0.000817	0.000790	0.000793	0.000784	0.000767	0.000756	0.000747	0.000747	0.000727	0.000708	**86**	0.049710	0.048121	0.047619	0.047016	0.046079	0.044685	0.044129	0.042886	0.041489	0.040530
46	0.000890	0.000860	0.000860	0.000849	0.000830	0.000816	0.000805	0.000803	0.000780	0.000759	**87**	0.056001	0.054147	0.053599	0.052856	0.051881	0.050362	0.049724	0.048265	0.046554	0.045564
47	0.000971	0.000937	0.000935	0.000920	0.000899	0.000882	0.000868	0.000865	0.000839	0.000816	**88**	0.063096	0.060946	0.060353	0.059451	0.058448	0.056799	0.056072	0.054371	0.052295	0.051281
48	0.001062	0.001024	0.001018	0.001001	0.000976	0.000956	0.000940	0.000934	0.000905	0.000879	**89**	0.071095	0.068615	0.067983	0.066898	0.065880	0.064099	0.063277	0.061302	0.058803	0.057773
49	0.001162	0.001120	0.001110	0.001090	0.001061	0.001038	0.001018	0.001010	0.000977	0.000948	**90**	0.080112	0.077266	0.076600	0.075305	0.074290	0.072375	0.071450	0.069170	0.066182	0.065146
50	0.001271	0.001224	0.001209	0.001185	0.001153	0.001125	0.001103	0.001092	0.001055	0.001022	**91**	0.090272	0.087021	0.086326	0.084795	0.083803	0.081753	0.080716	0.078098	0.074545	0.073517
51	0.001389	0.001337	0.001315	0.001288	0.001251	0.001219	0.001193	0.001179	0.001138	0.001102	**92**	0.101716	0.098016	0.097299	0.095501	0.094556	0.092371	0.091210	0.088220	0.084016	0.083012
52	0.001515	0.001458	0.001430	0.001398	0.001357	0.001320	0.001290	0.001272	0.001226	0.001186	**93**	0.114598	0.110399	0.109668	0.107571	0.106700	0.104379	0.103081	0.099683	0.094730	0.093771
53	0.001651	0.001587	0.001552	0.001516	0.001469	0.001427	0.001392	0.001371	0.001319	0.001275	**94**	0.129087	0.124336	0.123596	0.121167	0.120399	0.117941	0.116482	0.112645	0.106834	0.105943
54	0.001799	0.001728	0.001685	0.001644	0.001591	0.001543	0.001504	0.001478	0.001421	0.001372	**95**	0.145369	0.140006	0.139260	0.136465	0.135829	0.133227	0.131580	0.127272	0.120481	0.119684
55	0.001958	0.001881	0.001828	0.001781	0.001722	0.001667	0.001623	0.001593	0.001530	0.001476	**96**	0.163644	0.157600	0.156846	0.153652	0.153177	0.150419	0.148542	0.143737	0.135834	0.135160
56	0.002133	0.002049	0.001993	0.001942	0.001878	0.001818	0.001771	0.001737	0.001668	0.001610	**97**	0.184129	0.177327	0.176556	0.172929	0.172637	0.169703	0.167543	0.162219	0.153060	0.152545
57	0.002326	0.002234	0.002173	0.002119	0.002050	0.001985	0.001934	0.001895	0.001820	0.001757	**98**	0.207053	0.199405	0.198596	0.194508	0.194413	0.191266	0.188756	0.182897	0.172333	0.172015
58	0.002540	0.002441	0.002375	0.002317	0.002242	0.002170	0.002115	0.002072	0.001991	0.001922	**99**	0.232600	0.224009	0.223125	0.218554	0.218654	0.215242	0.212303	0.205897	0.193785	0.193706
59	0.002782	0.002673	0.002603	0.002539	0.002458	0.002379	0.002320	0.002272	0.002183	0.002108	**100**	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000
60	0.003051	0.002933	0.002856	0.002788	0.002700	0.002613	0.002549	0.002495	0.002398	0.002316											

Table A.4: Projection for Hong Kong (Female) mortality rates using Modified MTM

Age/Years	2014	2015	2016	2017	2018	2019	2020	2021	2022	2023	Age/Years	2014	2015	2016	2017	2018	2019	2020	2021	2022	2023
5	0.000127	0.000140	0.000148	0.000150	0.000172	0.000181	0.000214	0.000227	0.000233	0.000255	**53**	0.001691	0.001647	0.001614	0.001568	0.001504	0.001485	0.001451	0.001406	0.001363	0.001332
6	0.000105	0.000115	0.000120	0.000121	0.000137	0.000144	0.000169	0.000178	0.000182	0.000197	**54**	0.001841	0.001793	0.001754	0.001705	0.001635	0.001613	0.001576	0.001527	0.001480	0.001444
7	0.000089	0.000097	0.000101	0.000101	0.000113	0.000119	0.000137	0.000144	0.000147	0.000158	**55**	0.002002	0.001950	0.001905	0.001852	0.001776	0.001751	0.001711	0.001658	0.001607	0.001565
8	0.000079	0.000085	0.000088	0.000088	0.000098	0.000102	0.000117	0.000122	0.000124	0.000132	**56**	0.002182	0.002124	0.002075	0.002017	0.001935	0.001907	0.001863	0.001805	0.001750	0.001704
9	0.000074	0.000079	0.000082	0.000081	0.000089	0.000093	0.000105	0.000109	0.000110	0.000117	**57**	0.002379	0.002316	0.002261	0.002198	0.002110	0.002078	0.002029	0.001967	0.001907	0.001857
10	0.000079	0.000084	0.000086	0.000085	0.000092	0.000096	0.000108	0.000111	0.000112	0.000117	**58**	0.002600	0.002530	0.002469	0.002400	0.002306	0.002270	0.002215	0.002147	0.002082	0.002027
11	0.000084	0.000089	0.000090	0.000089	0.000096	0.000099	0.000111	0.000113	0.000113	0.000118	**59**	0.002848	0.002770	0.002703	0.002628	0.002526	0.002485	0.002424	0.002350	0.002280	0.002219
12	0.000090	0.000095	0.000097	0.000095	0.000101	0.000105	0.000116	0.000117	0.000117	0.000122	**60**	0.003125	0.003039	0.002964	0.002882	0.002772	0.002726	0.002658	0.002577	0.002500	0.002434
13	0.000099	0.000103	0.000105	0.000102	0.000108	0.000111	0.000122	0.000123	0.000123	0.000126	**61**	0.003436	0.003341	0.003258	0.003168	0.003048	0.002996	0.002920	0.002832	0.002748	0.002675
14	0.000107	0.000111	0.000112	0.000110	0.000115	0.000118	0.000128	0.000129	0.000128	0.000131	**62**	0.003782	0.003677	0.003584	0.003486	0.003356	0.003296	0.003213	0.003116	0.003024	0.002943
15	0.000116	0.000120	0.000121	0.000118	0.000122	0.000125	0.000135	0.000135	0.000133	0.000136	**63**	0.004169	0.004052	0.003949	0.003841	0.003700	0.003633	0.003539	0.003433	0.003333	0.003244
16	0.000126	0.000129	0.000130	0.000126	0.000129	0.000132	0.000142	0.000141	0.000139	0.000141	**64**	0.004606	0.004476	0.004362	0.004243	0.004090	0.004013	0.003908	0.003792	0.003682	0.003584
17	0.000136	0.000139	0.000140	0.000135	0.000138	0.000141	0.000149	0.000148	0.000146	0.000147	**65**	0.005098	0.004953	0.004826	0.004695	0.004528	0.004441	0.004324	0.004197	0.004075	0.003967
18	0.000147	0.000150	0.000150	0.000145	0.000146	0.000149	0.000157	0.000155	0.000153	0.000153	**66**	0.005648	0.005487	0.005344	0.005200	0.005019	0.004920	0.004788	0.004648	0.004515	0.004395
19	0.000158	0.000160	0.000160	0.000154	0.000155	0.000157	0.000165	0.000162	0.000159	0.000159	**67**	0.006260	0.006080	0.005921	0.005762	0.005565	0.005453	0.005305	0.005152	0.005005	0.004872
20	0.000168	0.000170	0.000169	0.000163	0.000163	0.000165	0.000172	0.000169	0.000165	0.000164	**68**	0.006945	0.006745	0.006568	0.006392	0.006177	0.006050	0.005885	0.005716	0.005554	0.005408
21	0.000178	0.000179	0.000178	0.000172	0.000170	0.000172	0.000178	0.000174	0.000170	0.000169	**69**	0.007717	0.007493	0.007295	0.007101	0.006867	0.006723	0.006537	0.006351	0.006173	0.006011
22	0.000187	0.000187	0.000186	0.000179	0.000176	0.000178	0.000183	0.000179	0.000174	0.000173	**70**	0.008569	0.008320	0.008099	0.007885	0.007630	0.007466	0.007259	0.007054	0.006858	0.006679
23	0.000195	0.000195	0.000193	0.000186	0.000182	0.000183	0.000187	0.000183	0.000178	0.000176	**71**	0.009507	0.009230	0.008984	0.008749	0.008471	0.008286	0.008054	0.007829	0.007613	0.007415
24	0.000202	0.000201	0.000199	0.000192	0.000187	0.000188	0.000191	0.000187	0.000181	0.000179	**72**	0.010547	0.010239	0.009965	0.009705	0.009404	0.009195	0.008935	0.008688	0.008451	0.008233
25	0.000232	0.000235	0.000241	0.000234	0.000236	0.000239	0.000241	0.000239	0.000239	0.000246	**73**	0.011710	0.011368	0.011062	0.010777	0.010449	0.010213	0.009922	0.009651	0.009390	0.009149
26	0.000239	0.000241	0.000247	0.000240	0.000240	0.000243	0.000244	0.000241	0.000241	0.000248	**74**	0.013024	0.012643	0.012302	0.011987	0.011631	0.011364	0.011038	0.010740	0.010452	0.010186
27	0.000247	0.000248	0.000253	0.000247	0.000245	0.000248	0.000248	0.000244	0.000243	0.000250	**75**	0.014893	0.014457	0.014067	0.013710	0.013312	0.013001	0.012626	0.012290	0.011964	0.011663
28	0.000255	0.000256	0.000261	0.000255	0.000252	0.000254	0.000253	0.000249	0.000247	0.000255	**76**	0.016665	0.016176	0.015739	0.015343	0.014909	0.014556	0.014134	0.013762	0.013400	0.013067
29	0.000266	0.000266	0.000271	0.000265	0.000260	0.000262	0.000261	0.000256	0.000254	0.000261	**77**	0.018663	0.018116	0.017626	0.017187	0.016713	0.016312	0.015836	0.015426	0.015024	0.014654
30	0.000278	0.000277	0.000283	0.000277	0.000271	0.000273	0.000271	0.000265	0.000262	0.000270	**78**	0.020910	0.020297	0.019748	0.019262	0.018744	0.018289	0.017753	0.017300	0.016854	0.016444
31	0.000292	0.000291	0.000296	0.000290	0.000283	0.000285	0.000282	0.000276	0.000273	0.000280	**79**	0.023426	0.022740	0.022126	0.021587	0.021023	0.020506	0.019902	0.019402	0.018907	0.018454
32	0.000309	0.000307	0.000311	0.000305	0.000297	0.000298	0.000295	0.000289	0.000285	0.000291	**80**	0.026234	0.025467	0.024780	0.024184	0.023569	0.022984	0.022304	0.021753	0.021204	0.020702
33	0.000327	0.000324	0.000329	0.000321	0.000313	0.000314	0.000310	0.000303	0.000299	0.000305	**81**	0.029355	0.028499	0.027732	0.027073	0.026404	0.025743	0.024978	0.024372	0.023763	0.023210
34	0.000348	0.000345	0.000349	0.000340	0.000331	0.000332	0.000328	0.000320	0.000315	0.000320	**82**	0.032814	0.031859	0.031005	0.030277	0.029552	0.028805	0.027947	0.027281	0.026607	0.025998
35	0.000372	0.000368	0.000372	0.000363	0.000352	0.000353	0.000348	0.000339	0.000334	0.000338	**83**	0.036639	0.035576	0.034625	0.033823	0.033039	0.032196	0.031235	0.030504	0.029761	0.029091
36	0.000399	0.000395	0.000398	0.000388	0.000376	0.000376	0.000371	0.000361	0.000355	0.000359	**84**	0.040859	0.039678	0.038622	0.037741	0.036895	0.035946	0.034871	0.034071	0.033251	0.032516
37	0.000431	0.000425	0.000427	0.000416	0.000403	0.000403	0.000397	0.000387	0.000379	0.000382	**85**	0.045570	0.044258	0.043086	0.042118	0.041206	0.040139	0.038937	0.038062	0.037157	0.036352
38	0.000467	0.000460	0.000462	0.000450	0.000435	0.000434	0.000428	0.000416	0.000408	0.000410	**86**	0.051343	0.049829	0.048481	0.047356	0.046370	0.045101	0.043772	0.042858	0.041843	0.040888
39	0.000507	0.000499	0.000501	0.000487	0.000471	0.000470	0.000462	0.000450	0.000440	0.000441	**87**	0.057854	0.056120	0.054581	0.053282	0.052222	0.050734	0.049268	0.048333	0.047204	0.046080
40	0.000552	0.000543	0.000543	0.000529	0.000510	0.000509	0.000500	0.000486	0.000475	0.000476	**88**	0.065198	0.063223	0.061478	0.059986	0.058855	0.057130	0.055518	0.054587	0.053343	0.052026
41	0.000602	0.000591	0.000591	0.000574	0.000554	0.000552	0.000542	0.000527	0.000514	0.000513	**89**	0.073477	0.071241	0.069276	0.067570	0.066373	0.064393	0.062629	0.061731	0.060371	0.058839
42	0.000656	0.000643	0.000641	0.000623	0.000601	0.000598	0.000587	0.000570	0.000556	0.000554	**90**	0.082809	0.080289	0.078087	0.076145	0.074891	0.072639	0.070716	0.069889	0.068416	0.066643
43	0.000713	0.000699	0.000695	0.000676	0.000651	0.000647	0.000635	0.000617	0.000601	0.000597	**91**	0.093322	0.090496	0.088040	0.085838	0.084536	0.081996	0.079909	0.079200	0.077617	0.075574
44	0.000775	0.000759	0.000754	0.000733	0.000705	0.000700	0.000687	0.000667	0.000649	0.000644	**92**	0.105158	0.102001	0.099270	0.096782	0.095447	0.092602	0.090349	0.089809	0.088121	0.085781
45	0.000843	0.000825	0.000818	0.000795	0.000764	0.000759	0.000744	0.000722	0.000702	0.000695	**93**	0.118472	0.114958	0.111927	0.109126	0.107775	0.104608	0.102187	0.101872	0.100085	0.097418
46	0.000917	0.000897	0.000888	0.000863	0.000829	0.000823	0.000806	0.000782	0.000760	0.000751	**94**	0.133435	0.129535	0.126172	0.123027	0.121680	0.118172	0.115583	0.115550	0.113669	0.110643
47	0.001000	0.000977	0.000966	0.000938	0.000901	0.000893	0.000875	0.000849	0.000824	0.000813	**95**	0.150231	0.145912	0.142173	0.138652	0.137330	0.133460	0.130702	0.131004	0.129032	0.125616
48	0.001092	0.001067	0.001053	0.001023	0.000982	0.000973	0.000952	0.000923	0.000896	0.000883	**96**	0.169058	0.164281	0.160109	0.156175	0.154900	0.150640	0.147710	0.148387	0.146323	0.142486
49	0.001195	0.001166	0.001149	0.001116	0.001071	0.001060	0.001038	0.001006	0.000976	0.000960	**97**	0.190126	0.184845	0.180162	0.175775	0.174568	0.169881	0.166771	0.167844	0.165679	0.161394
50	0.001305	0.001273	0.001253	0.001217	0.001168	0.001155	0.001130	0.001095	0.001062	0.001043	**98**	0.213657	0.207813	0.202523	0.197636	0.196511	0.191346	0.188045	0.189504	0.187219	0.182469
51	0.001425	0.001389	0.001365	0.001326	0.001272	0.001258	0.001230	0.001192	0.001156	0.001132	**99**	0.239817	0.233345	0.227323	0.221888	0.220852	0.215139	0.211628	0.213429	0.210993	0.205771
52	0.001553	0.001514	0.001485	0.001443	0.001384	0.001367	0.001337	0.001295	0.001256	0.001229	**100**	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000

Table A.5: Projection for USA mortality rates using MTM

Age/Years	2011	2012	2013	2014	2015	2016	2017	2018	2019	2020	Age/Years	2011	2012	2013	2014	2015	2016	2017	2018	2019	2020
20	0.000853	0.000825	0.000800	0.000786	0.000759	0.000742	0.000723	0.000681	0.000663	0.000652	**61**	0.009696	0.009514	0.009415	0.009291	0.009168	0.009042	0.008953	0.008765	0.008675	0.008668
21	0.000900	0.000870	0.000844	0.000830	0.000802	0.000784	0.000765	0.000721	0.000702	0.000690	**62**	0.010715	0.010514	0.010406	0.010269	0.010135	0.009995	0.009899	0.009695	0.009595	0.009588
22	0.000904	0.000875	0.000849	0.000835	0.000807	0.000789	0.000770	0.000726	0.000707	0.000696	**63**	0.011515	0.011298	0.011183	0.011037	0.010894	0.010744	0.010642	0.010426	0.010319	0.010312
23	0.000902	0.000872	0.000847	0.000833	0.000806	0.000788	0.000769	0.000726	0.000708	0.000697	**64**	0.012425	0.012191	0.012069	0.011911	0.011758	0.011597	0.011489	0.011260	0.011144	0.011138
24	0.000891	0.000863	0.000838	0.000824	0.000798	0.000780	0.000762	0.000720	0.000702	0.000691	**65**	0.013622	0.013365	0.013232	0.013060	0.012895	0.012718	0.012602	0.012355	0.012228	0.012222
25	0.000894	0.000866	0.000842	0.000828	0.000801	0.000784	0.000766	0.000725	0.000707	0.000696	**66**	0.014583	0.014307	0.014167	0.013984	0.013809	0.013619	0.013497	0.013237	0.013101	0.013097
26	0.000897	0.000869	0.000845	0.000831	0.000805	0.000788	0.000770	0.000729	0.000712	0.000702	**67**	0.015872	0.015571	0.015421	0.015222	0.015034	0.014827	0.014697	0.014420	0.014271	0.014268
27	0.000907	0.000879	0.000855	0.000841	0.000815	0.000797	0.000780	0.000739	0.000722	0.000712	**68**	0.017251	0.016924	0.016763	0.016547	0.016345	0.016121	0.015982	0.015686	0.015525	0.015523
28	0.000938	0.000910	0.000885	0.000871	0.000845	0.000826	0.000809	0.000767	0.000749	0.000740	**69**	0.018637	0.018284	0.018112	0.017881	0.017665	0.017423	0.017276	0.016962	0.016787	0.016787
29	0.000949	0.000921	0.000897	0.000882	0.000856	0.000838	0.000820	0.000779	0.000761	0.000751	**70**	0.020594	0.020204	0.020017	0.019762	0.019527	0.019259	0.019101	0.018760	0.018567	0.018569
30	0.000977	0.000948	0.000924	0.000909	0.000882	0.000864	0.000846	0.000804	0.000786	0.000777	**71**	0.022175	0.021754	0.021555	0.021283	0.021032	0.020745	0.020577	0.020218	0.020010	0.020014
31	0.001028	0.000998	0.000973	0.000957	0.000930	0.000911	0.000892	0.000849	0.000831	0.000821	**72**	0.024661	0.024193	0.023975	0.023674	0.023399	0.023080	0.022898	0.022506	0.022276	0.022282
32	0.001080	0.001049	0.001023	0.001006	0.000978	0.000958	0.000939	0.000895	0.000875	0.000865	**73**	0.026772	0.026264	0.026031	0.025706	0.025411	0.025065	0.024872	0.024455	0.024206	0.024215
33	0.001135	0.001103	0.001076	0.001059	0.001030	0.001009	0.000989	0.000944	0.000924	0.000913	**74**	0.029214	0.028661	0.028411	0.028057	0.027740	0.027364	0.027158	0.026713	0.026441	0.026454
34	0.001185	0.001152	0.001125	0.001107	0.001077	0.001056	0.001035	0.000989	0.000968	0.000958	**75**	0.032056	0.031448	0.031178	0.030793	0.030451	0.030039	0.029818	0.029340	0.029042	0.029059
35	0.001266	0.001231	0.001203	0.001183	0.001152	0.001130	0.001108	0.001060	0.001038	0.001027	**76**	0.034799	0.034140	0.033852	0.033437	0.033070	0.032624	0.032390	0.031883	0.031559	0.031581
36	0.001346	0.001310	0.001280	0.001260	0.001228	0.001204	0.001181	0.001131	0.001108	0.001097	**77**	0.037842	0.037127	0.036818	0.036370	0.035977	0.035493	0.035246	0.034706	0.034355	0.034383
37	0.001437	0.001399	0.001368	0.001347	0.001313	0.001288	0.001264	0.001211	0.001188	0.001177	**78**	0.041378	0.040598	0.040266	0.039779	0.039356	0.038829	0.038566	0.037989	0.037607	0.037641
38	0.001563	0.001522	0.001489	0.001466	0.001430	0.001403	0.001378	0.001322	0.001297	0.001285	**79**	0.045354	0.044501	0.044144	0.043614	0.043158	0.042583	0.042302	0.041685	0.041268	0.041309
39	0.001644	0.001602	0.001569	0.001544	0.001507	0.001479	0.001453	0.001396	0.001370	0.001358	**80**	0.050020	0.049081	0.048695	0.048116	0.047621	0.046990	0.046689	0.046026	0.045567	0.045617
40	0.001776	0.001731	0.001696	0.001670	0.001631	0.001600	0.001573	0.001512	0.001485	0.001473	**81**	0.054203	0.053189	0.052778	0.052156	0.051629	0.050948	0.050630	0.049930	0.049435	0.049494
41	0.001916	0.001868	0.001831	0.001803	0.001762	0.001730	0.001701	0.001637	0.001609	0.001596	**82**	0.060036	0.058917	0.058471	0.057789	0.057215	0.056466	0.056125	0.055371	0.054825	0.054896
42	0.002090	0.002039	0.002000	0.001969	0.001925	0.001891	0.001860	0.001792	0.001762	0.001749	**83**	0.066153	0.064926	0.064446	0.063701	0.063081	0.062261	0.061897	0.061089	0.060491	0.060574
43	0.002248	0.002194	0.002154	0.002121	0.002075	0.002039	0.002007	0.001935	0.001903	0.001890	**84**	0.072793	0.071450	0.070933	0.070123	0.069454	0.068558	0.068170	0.067307	0.066653	0.066751
44	0.002416	0.002359	0.002316	0.002281	0.002234	0.002195	0.002161	0.002087	0.002053	0.002040	**85**	0.079736	0.078274	0.077720	0.076843	0.076124	0.075151	0.074740	0.073822	0.073112	0.073225
45	0.002626	0.002565	0.002521	0.002483	0.002433	0.002391	0.002356	0.002277	0.002242	0.002228	**86**	0.089274	0.087706	0.087133	0.086206	0.085459	0.084424	0.084004	0.083054	0.082303	0.082469
46	0.002848	0.002783	0.002737	0.002696	0.002643	0.002599	0.002561	0.002478	0.002441	0.002427	**87**	0.099556	0.097886	0.097298	0.096328	0.095558	0.094465	0.094041	0.093067	0.092281	0.092510
47	0.003096	0.003027	0.002978	0.002935	0.002879	0.002831	0.002791	0.002704	0.002665	0.002651	**88**	0.110206	0.108443	0.107849	0.106843	0.106060	0.104919	0.104499	0.103515	0.102701	0.103001
48	0.003381	0.003307	0.003255	0.003208	0.003149	0.003098	0.003056	0.002963	0.002922	0.002908	**89**	0.122910	0.121042	0.120443	0.119399	0.118602	0.117408	0.116994	0.116002	0.115160	0.115544
49	0.003622	0.003544	0.003491	0.003441	0.003380	0.003326	0.003282	0.003186	0.003143	0.003130	**90**	0.137109	0.135137	0.134539	0.133460	0.132657	0.131414	0.131012	0.130023	0.129156	0.129639
50	0.003975	0.003892	0.003836	0.003782	0.003716	0.003659	0.003612	0.003510	0.003464	0.003452	**91**	0.148542	0.146516	0.145941	0.144860	0.144079	0.142822	0.142451	0.141502	0.140637	0.141220
51	0.004299	0.004211	0.004154	0.004095	0.004027	0.003966	0.003917	0.003810	0.003763	0.003751	**92**	0.164430	0.162319	0.161763	0.160667	0.159902	0.158617	0.158276	0.157362	0.156492	0.157195
52	0.004691	0.004597	0.004537	0.004474	0.004403	0.004338	0.004285	0.004173	0.004124	0.004112	**93**	0.179898	0.177727	0.177205	0.176113	0.175380	0.174087	0.173788	0.172935	0.172075	0.172908
53	0.005042	0.004944	0.004882	0.004815	0.004741	0.004673	0.004618	0.004502	0.004451	0.004441	**94**	0.195566	0.193354	0.192878	0.191802	0.191117	0.189831	0.189585	0.188813	0.187977	0.188946
54	0.005420	0.005317	0.005254	0.005183	0.005107	0.005035	0.004978	0.004858	0.004806	0.004797	**95**	0.217486	0.215203	0.214774	0.213705	0.213066	0.211778	0.211590	0.210903	0.210087	0.211221
55	0.005861	0.005753	0.005689	0.005612	0.005534	0.005458	0.005398	0.005274	0.005220	0.005213	**96**	0.236879	0.234570	0.234206	0.233172	0.232604	0.231345	0.231228	0.230658	0.229886	0.231183
56	0.006352	0.006233	0.006165	0.006082	0.006001	0.005915	0.005852	0.005720	0.005661	0.005654	**97**	0.257134	0.254817	0.254530	0.253543	0.253059	0.251845	0.251810	0.251378	0.250662	0.252130
57	0.006879	0.006751	0.006678	0.006589	0.006498	0.006409	0.006341	0.006200	0.006136	0.006128	**98**	0.278153	0.275846	0.275646	0.274719	0.274332	0.273180	0.273236	0.272960	0.272314	0.273957
58	0.007571	0.007429	0.007349	0.007251	0.007153	0.007054	0.006981	0.006828	0.006758	0.006750	**99**	0.299818	0.297540	0.297437	0.296583	0.296306	0.295231	0.295386	0.295287	0.294722	0.296544
59	0.008161	0.008008	0.007923	0.007818	0.007713	0.007606	0.007529	0.007366	0.007290	0.007283	**100**	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000
60	0.008990	0.008822	0.008729	0.008614	0.008499	0.008382	0.008298	0.008121	0.008038	0.008030											

Table A.6: Projection for USA mortality rates using Modified MTM

Age/Years	2011	2012	2013	2014	2015	2016	2017	2018	2019	2020	Age/Years	2011	2012	2013	2014	2015	2016	2017	2018	2019	2020
5	0.000137	0.000133	0.000129	0.000125	0.000122	0.000116	0.000111	0.000108	0.000103	0.000100	**53**	0.005464	0.005433	0.005340	0.005292	0.005243	0.005157	0.005071	0.005051	0.004951	0.004926
6	0.000128	0.000124	0.000121	0.000117	0.000114	0.000109	0.000104	0.000102	0.000097	0.000094	**54**	0.005883	0.005850	0.005751	0.005700	0.005648	0.005556	0.005464	0.005443	0.005336	0.005311
7	0.000120	0.000117	0.000114	0.000110	0.000108	0.000103	0.000099	0.000096	0.000092	0.000089	**55**	0.006371	0.006337	0.006230	0.006175	0.006120	0.006021	0.005922	0.005902	0.005787	0.005762
8	0.000118	0.000115	0.000112	0.000108	0.000106	0.000101	0.000097	0.000095	0.000091	0.000088	**56**	0.006872	0.006835	0.006720	0.006662	0.006602	0.006496	0.006390	0.006368	0.006244	0.006217
9	0.000115	0.000112	0.000109	0.000106	0.000103	0.000099	0.000095	0.000093	0.000089	0.000086	**57**	0.007409	0.007369	0.007246	0.007183	0.007119	0.007005	0.006890	0.006867	0.006734	0.006706
10	0.000114	0.000111	0.000108	0.000105	0.000103	0.000099	0.000095	0.000093	0.000089	0.000086	**58**	0.008115	0.008073	0.007937	0.007869	0.007798	0.007675	0.007550	0.007525	0.007379	0.007348
11	0.000123	0.000120	0.000116	0.000113	0.000111	0.000106	0.000102	0.000100	0.000096	0.000093	**59**	0.008708	0.008663	0.008517	0.008444	0.008369	0.008237	0.008103	0.008077	0.007920	0.007889
12	0.000138	0.000135	0.000132	0.000128	0.000125	0.000121	0.000116	0.000114	0.000109	0.000106	**60**	0.009549	0.009499	0.009340	0.009261	0.009178	0.009035	0.008888	0.008860	0.008688	0.008654
13	0.000165	0.000162	0.000158	0.000154	0.000151	0.000145	0.000140	0.000137	0.000132	0.000128	**61**	0.010251	0.010198	0.010028	0.009943	0.009854	0.009702	0.009544	0.009515	0.009330	0.009294
14	0.000208	0.000203	0.000198	0.000193	0.000190	0.000183	0.000176	0.000173	0.000166	0.000162	**62**	0.011276	0.011219	0.011031	0.010938	0.010842	0.010675	0.010502	0.010470	0.010267	0.010229
15	0.000269	0.000263	0.000257	0.000251	0.000246	0.000238	0.000230	0.000225	0.000217	0.000212	**63**	0.012062	0.012001	0.011801	0.011702	0.011599	0.011422	0.011237	0.011204	0.010986	0.010947
16	0.000365	0.000357	0.000349	0.000341	0.000335	0.000323	0.000313	0.000307	0.000296	0.000290	**64**	0.012955	0.012891	0.012676	0.012571	0.012460	0.012271	0.012073	0.012038	0.011805	0.011764
17	0.000444	0.000436	0.000426	0.000416	0.000409	0.000395	0.000383	0.000377	0.000363	0.000356	**65**	0.014138	0.014069	0.013835	0.013720	0.013600	0.013395	0.013179	0.013143	0.012888	0.012844
18	0.000545	0.000535	0.000523	0.000512	0.000503	0.000487	0.000472	0.000465	0.000449	0.000440	**66**	0.015066	0.014993	0.014744	0.014623	0.014494	0.014279	0.014049	0.014011	0.013739	0.013694
19	0.000596	0.000586	0.000573	0.000561	0.000552	0.000535	0.000519	0.000511	0.000495	0.000486	**67**	0.016323	0.016245	0.015975	0.015845	0.015706	0.015474	0.015226	0.015185	0.014891	0.014844
20	0.000632	0.000621	0.000608	0.000596	0.000587	0.000569	0.000553	0.000545	0.000528	0.000519	**68**	0.017660	0.017576	0.017285	0.017145	0.016995	0.016747	0.016479	0.016436	0.016117	0.016068
21	0.000690	0.000679	0.000665	0.000653	0.000643	0.000624	0.000607	0.000599	0.000581	0.000572	**69**	0.018993	0.018904	0.018591	0.018442	0.018281	0.018016	0.017728	0.017684	0.017341	0.017290
22	0.000717	0.000707	0.000692	0.000680	0.000670	0.000652	0.000635	0.000627	0.000609	0.000600	**70**	0.020892	0.020795	0.020452	0.020289	0.020113	0.019824	0.019508	0.019460	0.019084	0.019030
23	0.000740	0.000730	0.000715	0.000704	0.000694	0.000676	0.000659	0.000651	0.000633	0.000624	**71**	0.022393	0.022290	0.021923	0.021750	0.021562	0.021255	0.020918	0.020868	0.020465	0.020409
24	0.000757	0.000748	0.000733	0.000722	0.000712	0.000694	0.000678	0.000670	0.000653	0.000644	**72**	0.024791	0.024679	0.024274	0.024083	0.023876	0.023540	0.023167	0.023114	0.022668	0.022609
25	0.000788	0.000776	0.000759	0.000746	0.000734	0.000716	0.000698	0.000688	0.000665	0.000653	**73**	0.026792	0.026672	0.026236	0.026032	0.025808	0.025448	0.025047	0.024991	0.024510	0.024449
26	0.000818	0.000807	0.000790	0.000777	0.000765	0.000747	0.000729	0.000719	0.000697	0.000685	**74**	0.029106	0.028977	0.028505	0.028285	0.028043	0.027656	0.027222	0.027163	0.026641	0.026578
27	0.000856	0.000845	0.000827	0.000815	0.000803	0.000785	0.000767	0.000757	0.000735	0.000723	**75**	0.031794	0.031656	0.031141	0.030903	0.030640	0.030222	0.029749	0.029687	0.029118	0.029052
28	0.000916	0.000906	0.000887	0.000875	0.000863	0.000845	0.000826	0.000817	0.000794	0.000782	**76**	0.034362	0.034215	0.033661	0.033406	0.033123	0.032675	0.032166	0.032101	0.031487	0.031420
29	0.000960	0.000950	0.000931	0.000919	0.000907	0.000889	0.000871	0.000861	0.000838	0.000827	**77**	0.037203	0.037046	0.036448	0.036174	0.035870	0.035390	0.034841	0.034773	0.034110	0.034041
30	0.001022	0.001013	0.000994	0.000982	0.000970	0.000952	0.000933	0.000924	0.000901	0.000891	**78**	0.040502	0.040333	0.039686	0.039391	0.039060	0.038544	0.037949	0.037878	0.037158	0.037087
31	0.001078	0.001068	0.001048	0.001036	0.001023	0.001004	0.000984	0.000975	0.000951	0.000940	**79**	0.044203	0.044021	0.043317	0.042999	0.042640	0.042083	0.041437	0.041362	0.040579	0.040506
32	0.001133	0.001123	0.001102	0.001089	0.001076	0.001056	0.001036	0.001027	0.001001	0.000990	**80**	0.048542	0.048346	0.047577	0.047231	0.046840	0.046236	0.045530	0.045451	0.044594	0.044520
33	0.001193	0.001182	0.001160	0.001147	0.001133	0.001112	0.001091	0.001082	0.001055	0.001044	**81**	0.052380	0.052171	0.051345	0.050976	0.050556	0.049912	0.049155	0.049072	0.048151	0.048076
34	0.001248	0.001237	0.001214	0.001200	0.001186	0.001164	0.001142	0.001132	0.001105	0.001093	**82**	0.057776	0.057549	0.056645	0.056242	0.055783	0.055082	0.054252	0.054165	0.053153	0.053078
35	0.001335	0.001323	0.001299	0.001284	0.001269	0.001246	0.001223	0.001212	0.001183	0.001171	**83**	0.063404	0.063159	0.062173	0.061738	0.061237	0.060479	0.059575	0.059483	0.058378	0.058303
36	0.001422	0.001410	0.001384	0.001369	0.001353	0.001328	0.001303	0.001293	0.001262	0.001249	**84**	0.069490	0.069227	0.068154	0.067683	0.067139	0.066320	0.065336	0.065240	0.064036	0.063961
37	0.001521	0.001508	0.001480	0.001464	0.001447	0.001421	0.001394	0.001383	0.001351	0.001338	**85**	0.075820	0.075538	0.074376	0.073870	0.073281	0.072401	0.071336	0.071236	0.069930	0.069856
38	0.001656	0.001642	0.001612	0.001595	0.001577	0.001548	0.001520	0.001508	0.001473	0.001459	**86**	0.085181	0.084899	0.083654	0.083150	0.082519	0.081604	0.080456	0.080407	0.079004	0.078966
39	0.001745	0.001731	0.001699	0.001681	0.001662	0.001633	0.001603	0.001591	0.001554	0.001540	**87**	0.095317	0.095040	0.093714	0.093222	0.092551	0.091610	0.090379	0.090394	0.088898	0.088905
40	0.001888	0.001873	0.001839	0.001819	0.001799	0.001767	0.001735	0.001722	0.001683	0.001668	**88**	0.105869	0.105603	0.104205	0.103738	0.103032	0.102077	0.100770	0.100864	0.099284	0.099347
41	0.002039	0.002023	0.001987	0.001966	0.001944	0.001910	0.001875	0.001862	0.001820	0.001804	**89**	0.118474	0.118223	0.116742	0.116308	0.115564	0.114596	0.113204	0.113394	0.111720	0.111850
42	0.002228	0.002211	0.002171	0.002149	0.002125	0.002088	0.002050	0.002037	0.001991	0.001974	**90**	0.132606	0.132376	0.130816	0.130427	0.129645	0.128675	0.127197	0.127502	0.125736	0.125947
43	0.002401	0.002383	0.002340	0.002316	0.002291	0.002251	0.002211	0.002197	0.002148	0.002131	**91**	0.144121	0.143925	0.142326	0.142006	0.141208	0.140273	0.138747	0.139178	0.137369	0.137669
44	0.002583	0.002565	0.002519	0.002493	0.002467	0.002424	0.002381	0.002366	0.002314	0.002296	**92**	0.160055	0.159896	0.158235	0.157991	0.157167	0.156261	0.154664	0.155247	0.153368	0.153776
45	0.002813	0.002793	0.002743	0.002716	0.002687	0.002641	0.002594	0.002579	0.002523	0.002504	**93**	0.175662	0.175549	0.173848	0.173699	0.172861	0.172007	0.170359	0.171110	0.169188	0.169714
46	0.003055	0.003034	0.002980	0.002951	0.002920	0.002870	0.002820	0.002804	0.002743	0.002724	**94**	0.191543	0.191486	0.189760	0.189723	0.188880	0.188099	0.186414	0.187349	0.185405	0.186061
47	0.003326	0.003303	0.003245	0.003214	0.003181	0.003127	0.003072	0.003055	0.002990	0.002970	**95**	0.213670	0.213675	0.211903	0.211997	0.211140	0.210434	0.208692	0.209849	0.207860	0.208672
48	0.003637	0.003613	0.003550	0.003515	0.003480	0.003421	0.003362	0.003344	0.003274	0.003252	**96**	0.233397	0.233474	0.231694	0.231936	0.231085	0.230486	0.228722	0.230108	0.228120	0.229092
49	0.003902	0.003877	0.003809	0.003773	0.003736	0.003673	0.003610	0.003592	0.003517	0.003495	**97**	0.254061	0.254218	0.252447	0.252853	0.252017	0.251543	0.249774	0.251405	0.249439	0.250581
50	0.004289	0.004262	0.004188	0.004149	0.004109	0.004040	0.003971	0.003952	0.003871	0.003848	**98**	0.275561	0.275805	0.274058	0.274644	0.273834	0.273504	0.271746	0.273634	0.271712	0.273032
51	0.004645	0.004617	0.004538	0.004496	0.004453	0.004378	0.004304	0.004285	0.004198	0.004174	**99**	0.297777	0.298116	0.296410	0.297189	0.296413	0.296244	0.294515	0.296670	0.294813	0.296319
52	0.005076	0.005046	0.004960	0.004915	0.004868	0.004788	0.004707	0.004688	0.004593	0.004569	**100**	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000

APPENDIX B

Projected Mortality Rates for Turkish Male

Table B.1: Projection for Turkish (Male) mortality rates using MTM

Age/Years	2016	2017	2018	2019	2020	2021	2022	2023	2024	2025
20	0.003245	0.003093	0.002921	0.002719	0.002610	0.002522	0.002453	0.002360	0.002290	0.002197
25	0.003738	0.003577	0.003396	0.003181	0.003067	0.002970	0.002893	0.002792	0.002714	0.002611
30	0.004708	0.004526	0.004321	0.004074	0.003945	0.003829	0.003737	0.003619	0.003525	0.003401
35	0.006564	0.006341	0.006088	0.005781	0.005623	0.005473	0.005353	0.005202	0.005079	0.004915
40	0.010136	0.009841	0.009504	0.009090	0.008885	0.008675	0.008503	0.008297	0.008120	0.007885
45	0.016519	0.016122	0.015666	0.015100	0.014835	0.014532	0.014279	0.013994	0.013731	0.013383
50	0.027982	0.027459	0.026852	0.026091	0.025773	0.025339	0.024963	0.024582	0.024184	0.023669
55	0.046153	0.045499	0.044734	0.043787	0.043450	0.042855	0.042293	0.041827	0.041224	0.040486
60	0.073575	0.072874	0.072044	0.071045	0.070829	0.070105	0.069318	0.068867	0.068001	0.067033
65	0.120550	0.119965	0.119245	0.118455	0.118633	0.117866	0.116784	0.116559	0.115329	0.114125
70	0.197335	0.197222	0.197010	0.196995	0.198051	0.197511	0.196106	0.196536	0.194909	0.193568
75	0.313366	0.314205	0.315055	0.316554	0.318992	0.319097	0.317425	0.319019	0.317176	0.315869
80	0.469372	0.471351	0.473528	0.476785	0.480543	0.481499	0.479705	0.482487	0.480872	0.479641
85	0.668751	0.671181	0.673974	0.677835	0.681727	0.683054	0.681374	0.684314	0.683330	0.682002
90	0.789972	0.779095	0.752234	0.742027	0.720598	0.695062	0.684966	0.678679	0.653421	0.642075
95	0.896298	0.860473	0.815053	0.796122	0.761906	0.727982	0.706317	0.688892	0.659618	0.643560
100	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000

Table B.2: Projection for Turkish (Male) mortality rates using Modified MTM

Age/Years	2016	2017	2018	2019	2020	2021	2022	2023	2024	2025
5	0.001772	0.001753	0.001804	0.001745	0.001743	0.001744	0.001754	0.001710	0.001745	0.001688
10	0.001487	0.001465	0.001492	0.001444	0.001435	0.001425	0.001422	0.001378	0.001388	0.001340
15	0.002734	0.002682	0.002707	0.002623	0.002595	0.002560	0.002541	0.002449	0.002440	0.002351
20	0.004140	0.004050	0.004054	0.003937	0.003880	0.003812	0.003764	0.003616	0.003572	0.003435
25	0.003368	0.003301	0.003285	0.003202	0.003163	0.003091	0.003043	0.002907	0.002833	0.002725
30	0.004345	0.004251	0.004202	0.004109	0.004041	0.003948	0.003875	0.003702	0.003595	0.003452
35	0.006208	0.006074	0.005967	0.005861	0.005745	0.005619	0.005508	0.005268	0.005109	0.004900
40	0.009700	0.009503	0.009345	0.009187	0.009001	0.008831	0.008662	0.008325	0.008113	0.007806
45	0.015996	0.015700	0.015455	0.015213	0.014905	0.014670	0.014405	0.013920	0.013636	0.013165
50	0.027419	0.026964	0.026583	0.026210	0.025691	0.025369	0.024950	0.024251	0.023893	0.023155
55	0.047408	0.046730	0.046154	0.045598	0.044742	0.044333	0.043690	0.042733	0.042366	0.041226
60	0.075222	0.074347	0.073597	0.072861	0.071614	0.071195	0.070327	0.069173	0.068911	0.067389
65	0.122656	0.121577	0.120654	0.119723	0.117928	0.117633	0.116520	0.115264	0.115378	0.113428
70	0.199816	0.198635	0.197636	0.196571	0.194131	0.194249	0.192978	0.191946	0.192947	0.190706
75	0.315895	0.314849	0.313997	0.312938	0.309935	0.310843	0.309651	0.309408	0.311901	0.309785
80	0.471443	0.470790	0.470306	0.469358	0.466137	0.467945	0.467077	0.468061	0.472136	0.470644
85	0.669989	0.669756	0.669615	0.668737	0.665918	0.667997	0.667452	0.669247	0.673700	0.672963
90	0.784366	0.770343	0.763536	0.738296	0.704966	0.690407	0.666834	0.673120	0.643903	0.653586
95	0.893679	0.858970	0.844929	0.806739	0.746471	0.724823	0.695318	0.698730	0.654689	0.667415
100	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000

APPENDIX C

Projected Mortality Rates for Turkish Female

Table C.1: Projection for Turkish (Female) mortality rates using MTM

Age/Years	2016	2017	2018	2019	2020	2021	2022	2023	2024	2025
20	0.000788	0.000746	0.000702	0.000675	0.000638	0.000603	0.000578	0.000551	0.000533	0.000500
25	0.001191	0.001134	0.001073	0.001032	0.000981	0.000934	0.000895	0.000856	0.000830	0.000783
30	0.001804	0.001725	0.001641	0.001583	0.001513	0.001450	0.001389	0.001334	0.001298	0.001229
35	0.002851	0.002738	0.002622	0.002536	0.002437	0.002351	0.002252	0.002172	0.002119	0.002018
40	0.004709	0.004546	0.004381	0.004251	0.004109	0.003991	0.003825	0.003703	0.003623	0.003470
45	0.008278	0.008034	0.007793	0.007590	0.007378	0.007217	0.006921	0.006729	0.006606	0.006362
50	0.015208	0.014842	0.014496	0.014175	0.013861	0.013662	0.013113	0.012805	0.012613	0.012217
55	0.024745	0.024258	0.023832	0.023379	0.022968	0.022785	0.021870	0.021421	0.021144	0.020576
60	0.042606	0.041966	0.041479	0.040841	0.040312	0.040258	0.038663	0.037992	0.037589	0.036755
65	0.077345	0.076563	0.076135	0.075270	0.074651	0.075035	0.072168	0.071166	0.070589	0.069370
70	0.140276	0.139539	0.139546	0.138548	0.138037	0.139514	0.134580	0.133196	0.132467	0.130870
75	0.248417	0.248162	0.249258	0.248424	0.248485	0.251947	0.244256	0.242595	0.241894	0.240251
80	0.406284	0.406977	0.409618	0.409293	0.410458	0.416053	0.406155	0.404566	0.404250	0.403333
85	0.612894	0.614344	0.617830	0.617888	0.620024	0.626014	0.616296	0.615171	0.615428	0.615694
90	0.733445	0.692947	0.674195	0.660097	0.653947	0.632856	0.610729	0.591465	0.577584	0.573593
95	0.870834	0.802779	0.763934	0.736598	0.725731	0.691818	0.656961	0.637784	0.616313	0.613070
100	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000

Table C.2: Projection for Turkish (Female) mortality rates using Modified MTM

Age/Years	2016	2017	2018	2019	2020	2021	2022	2023	2024	2025
5	0.000340	0.000326	0.000308	0.000302	0.000285	0.000268	0.000256	0.000246	0.000246	0.000245
10	0.000336	0.000322	0.000304	0.000297	0.000282	0.000266	0.000254	0.000243	0.000241	0.000239
15	0.000643	0.000616	0.000582	0.000569	0.000542	0.000513	0.000488	0.000468	0.000462	0.000455
20	0.001092	0.001047	0.000991	0.000967	0.000927	0.000879	0.000835	0.000801	0.000789	0.000772
25	0.001084	0.001045	0.000992	0.000964	0.000935	0.000891	0.000852	0.000812	0.000804	0.000788
30	0.001734	0.001674	0.001592	0.001548	0.001507	0.001442	0.001377	0.001319	0.001303	0.001271
35	0.002894	0.002800	0.002670	0.002599	0.002542	0.002442	0.002334	0.002243	0.002218	0.002154
40	0.004563	0.004429	0.004238	0.004136	0.004050	0.003910	0.003745	0.003609	0.003576	0.003473
45	0.007656	0.007459	0.007163	0.007012	0.006877	0.006673	0.006405	0.006189	0.006148	0.005972
50	0.013436	0.013144	0.012669	0.012441	0.012226	0.011924	0.011477	0.011117	0.011074	0.010762
55	0.023718	0.023305	0.022552	0.022223	0.021890	0.021462	0.020718	0.020122	0.020100	0.019555
60	0.041439	0.040916	0.039758	0.039287	0.038831	0.038277	0.037103	0.036127	0.036220	0.035315
65	0.076315	0.075728	0.073917	0.073261	0.072671	0.072022	0.070135	0.068492	0.068909	0.067369
70	0.140280	0.139873	0.137205	0.136403	0.135794	0.135279	0.132425	0.129777	0.130949	0.128446
75	0.251184	0.251476	0.247993	0.247243	0.246942	0.247099	0.243286	0.239437	0.241988	0.238322
80	0.413483	0.414990	0.411354	0.410980	0.411421	0.412863	0.408817	0.404251	0.408378	0.403993
85	0.623553	0.625963	0.623203	0.623286	0.624442	0.626986	0.623872	0.619836	0.624409	0.620504
90	0.730132	0.693985	0.678128	0.657365	0.630714	0.614470	0.589762	0.563831	0.552296	0.538412
95	0.862742	0.795780	0.763881	0.732520	0.688821	0.658273	0.618635	0.583185	0.563309	0.548617
100	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000