DISTRICT-INDEX INSURANCE PROGRAM: BASIC MODEL AND PRICING

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ABSTRACT

DISTRICT-INDEX INSURANCE PROGRAM: BASIC MODEL AND PRICING

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The aim of this work is to build an inital model for and calculate the premium of an insurance scheme against drought proposed by TARSIM. The payoff of the insurance scheme is similar to that of a call option on the yield variable. We compute the premium using normal, gamma and beta distribution for the yield and examine how the premium depends on the parameters of these distributions. We find that, when we fix the mean and the variance of the yield, the premium computed under different distributions depends little on the distribution used for the yield.

Keywords : Agricultural insurance, drought, modeling,pricing

ÖZ

İLÇE-ENDEKS SİGORTA PROGRAMI : TEMEL MODEL VE FİYATLAMA

AYBERK, İDİL
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Bu çalışmanın amacı, TARSİM tarafından önerilen kuraklık riskine karşı sigorta programının başlangıç modelini kurup, prim hesaplamalarını yapmakır. Bu modeli kurarken ortalama verim için normal, gama ve beta dağılımlarını kullandık. Sigorta primini hesaplarken kuraklık olduğunu varsayarak primin farklı parametre değerlerinde nasıl davrandığını inceledik. Numerik hesaplamaların sonuçlarına göre, ortalamayı ve varyansı sabit tuttuğumuzda, farklı dağılımların prim üzerinde çok fazla bir etkisinin olmadığını söyleyebiliriz.

Anahtar Kelimeler : Tarım sigortası,kuraklık,modelleme,fiyatlama

To My Family

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CHAPTER 1

Introduction

Agriculture is an inherently risky business since it very much depends on weather conditions. Over the years, governments and the private sector have developed risk management tools that farmers may buy and use to protect themselves against weather risk. In Turkey, agriculture is one of the major sources of income. According to Worldbank Database [45], as of 2012, 24% of the country's workforce is employed in agriculture. Recent studies about Turkey reveal that climate will be more arid and warm and it is predicted that drought will be one of the major problems in the future [38, 33, 1]. With Law 5363 passed in 2005 [44], agricultural insurance has become supported by the Turkish government, and consequently, *Agricultural Insurance Pool (TARSIM)* has been established. Currently, Turkish government provides 50% premium subsidy for quality loss arising from hail, flood, storm, fire, landslide and earthquake, for crops, fruits, vegetables and cut flowers. Furthermore, the same coverage is supplied by the government according to a loss assessment for the greenhouses [46].

The land area effected by drought can vary widely and is typically very large. It is also difficult to delineate the boundaries of a drought: when/where does it begin and end? For these reasons drought risk is considered more difficult and potentially more costly to insure. Currently, drought risk is not insurable and not supported by the Turkish government nor by the private insurance companies in Turkey [22]. In the long run, one of the aims of TARSIM is to provide insurance coverage for the farmers against drought risk. In this thesis, our aim is to build a simple initial mathematical model of an insurance scheme against drought which is proposed in [20] and compute its premium for a single year. In the following paragraphs, we will explain the structure of this proposal which we will call District-Index insurance.

Throughout the world, there are two most commonly used insurance schemes: Multiple Peril Crop Insurance and Index-Based Insurance. In the application of Multiple Peril Crop Insurance program, yield losses are determined by a percentage of the actual production history of a farm. In the case of the Index-Based Insurance program, losses are determined as a percentage of a predetermined index such as amount of precipitation in an area or yield of an area.

District-Index insurance is an index based insurance where the index is computed at the district level. Turkey is divided into 81 provinces and these provinces are further divided into administration units called *districts* (ilçe in Turkish); there are 957 districts in Turkey after 2013 [49]. From each district a number of reference parcels are selected and their yield over the years is used to compute reference yield for the district. The proposal assumes that, for each district, there is a mechanism to decide whether a drought has taken place in that district in the present year. We will model this with the random variable D_i; $D_i = 1$ if a drought occurred in year i, it equals to 0 otherwise. In case of $D_i = 1$ the average of the yields of the reference parcels in the last five non-drought years is taken to be the reference yield R for the current year. A farmer who resides in the district and who wants to be insured against drought chooses an insurance level $\alpha \in [0.6, 0.8]$ and in case of drought he/she is paid $zP(\alpha R - Y_i)^+$ TL (Turkish Lira) where z is the size of his/her land and P is the unit price of the crop and Y is the average yield of the reference parcels for the current year. Therefore, the District-based insurance scheme is a put option on the reference yield of the district and its premium will show the characteristics of the price of a put option. The farmer who purchased the insurance specifies the strike of the put through his/her choice of α. Further details of this model is given in Section 3.2 of Chapter 3. The same chapter includes an initial analysis of the premium calculation. For this initial analysis we have limited ourselves to the case of a single district, a single crop and a single year. When all these are fixed, the only random quantity remaining in the model is essentially the distribution of the reference yield given that a drought occurs in the current year. The literature suggests three types of distributions for the yield: normal, gamma and beta (Section 3.2 includes a review of this literature). Section 3.3 gives formulas for the premium under these distributions and explains how one passes from one to the other so that the mean and the variance of the yield remain the same. In Section 3.4 we compare the premiums implied by these three distributions when we fix the mean yield and changing its variance.

The District-Index insurance program aims at balancing the cost and effectiveness of the two type of insurance schemes introduced above. First, it is an index-based program; therefore one need not measure the yield of each parcel of an insured farmer separately. This makes it less costly to implement. Second, the index is computed at a reasonably local level: districts in Turkey cover relatively small areas (compared to provinces and regions) and therefore one can expect that the average yield computed from the reference parcels and the drought indicator of the district accurately reflect the state of the parcels in the district. Finally, its being local enough assures that the premiums computed is suitable to the needs of each district.

Chapter 2 is a review of the existing literature on drought risk. Its first section gives a summary of the insurance programs against drought. Projects under construction in Turkey which aim to increase the efficiency of agricultural insurance programs are given in the second section. The Conclusion lists several directions for future work.

CHAPTER 2

Drought and Insurance Programs

Drought is considered to be a systemic risk. Miranda [26] defines systemic risk in agriculture as any weather event such as drought or extreme temperature levels which cause correlated reduction in the products among the farms in a given region or country.

There are several reasons why drought considered to be different than other weather risks. First of all, there is no significant evidence to determine the beginning and end of a drought. Second, the effect of a drought increases gradually and cumulatively with the low rainfall periods. Third, its effect range can be huge. Finally, it is complicated to determine the effect of drought for both individual farms and the whole agricultural sector [5, 6, 17]. There are mainly three types of drought: *meteorological drought*, *hydrological drought* and *agricultural drought*. Meteorological drought is defined as the degree of dryness and the continuation period of the dry term. Hydrological drought is the lack of underground water due to the low rainfall which eventually causes decrease in soil moisture. Agricultural drought is a combination of meteorological and hydrological drought that effects agriculture [5].

By the definition of insurance, several conditions must hold in order for risks to be insured. According to Miranda and Glauber [26], two of them are rather important:

1. Risks should be stochastically independent across insureds,

2. There should not be an asymmetric information between the agents.

However, in the case of agricultural insurance, these two conditions fail to hold. The first condition does not hold because drought strikes wide areas which increases correlation between farms in the same area. The second condition is a great challenge for all insurance programs. There are two problems associated with asymmetric information: *moral hazard* and *adverse selection*. Adverse selection occurs because of farmer's better knowledge about their own farm yields. If a farmer realizes that the expected indemnity he/she will get under an insurance is greater than the premium he/she pays, it is more likely for the farmer to buy insurance. On the other hand, when farmer change his/her behavior to increase the probability of getting an indemnity payment after he/she purchases an insurance, this violates the moral hazard [7, 47].

2.1 Insurance Programs against Drought

According to *International Labour Organization*'s report agricultural employment is 35% of the total global employment which suggests agriculture is one of the most important aspects of the world economy and even human life. As of 2012, this number is 24% of the total employment in Turkey as mentioned in Chapter 1. Especially for developing countries, importance of agriculture is undeniable. However, it's dependency to external factors, such as weather events, makes agricultural production risky and hard to insure.

From the viewpoint of insurers, weather events are hard to insure mainly for two reasons. First, occurrence of the event is not quantifiable. Second, presence of asymmetric information makes it difficult the assess the damage [18]. However, in order to increase agricultural production, involvement of insurance companies is necessary. Therefore, it is vital for governments to take precautions and necessary implementations to protect farmers who are employed in this field to enforce the economy. Offering subsidies is one of the policies that governments use to boost both insurance companies and farmers. In France, Spain, Norway and Switzerland, the governments offer subsidies to farmers which indicates that this is also an important issue for developed countries [9].

In addition to these, it is suggested in [2] that governments are substantial

1. To determine the legal framework,
2. To develop the infrastructure of the sector,
3. To meet the high start-up costs for development of the agricultural insurance,
4. To find reinsurance,
5. To help farmers for their premium payments,
6. To increase the awareness and notice of farmers.

On the other hand, offering subsidy is not enough to solve the entire problem. Optimum amount of subsidy has to be offered in order to provide incentive to act honestly. Skees argues that, insurance programs are too much subsidized making insurance programs inefficient in the USA. To illustrate, Skees collected the data of the 17557 Iowa corn farms between the years 1982 and 1994 that purchased crop insurance. He finds that there is a positive correlation between the subsidy rates and loss ratios [40].

Throughout the world, several crop insurance methods are used against drought. Among these, the most common ones are *Multiple Peril Crop Insurance* and *Index-Based Insurance*. In the next subsection, these programs are going to be discussed.

2.1.1 Multiple Peril Crop Insurance

This is the most widely applied insurance program in the world. With this program, farmers are insured against multiple sources of risk. Indemnity is calculated by multiplying the *indemnity price* and the difference between the *trigger yield* and realized yield, where trigger yield is the percentage of the average yields of an individual farm. For the indemnity price, farmer chooses one indemnity price level among several choices such as high, normal, or low depending on the insurance design. Average yield of an individual farm is the *actual production history* (*APH*) of a particular farm for ten or more years, and in the absence of *APH*, all calculations are based on geographical average yields [39, 43].

To formalize the scheme, let n denote the indemnity; y_g be the yield guarantee; y be the actual yield of the farm; y^* denotes the average yield of the farm, and p be the indemnity price. Then the indemnity payment for one particular insured will be:

$$n = (y^* \cdot y_g - y, 0)^+ \cdot p. \tag{2.1}$$

Despite the wide usage, asymmetric information is main drawback of this model. Because farmers have better knowledge than insurers, it is possible for farmers to influence their own yield distribution to get an indemnity payment [47]. Accroding to [7, 25, 42], on the other hand, adverse selection is a more serious issue than the moral hazard since farmers have more information about their own yields. Farmers who believe that the indemnity payments are going to exceed premiums will be more willing to buy a protection. This situation reduces the variability of the insurers' pool and eventually increases the loss ratios. In addition, since triggered yield is determined based on an individual farm's yield, administrative costs are high. Table 2.1.1 shows the financial performances of agricultural insurance schemes of seven countries which is taken from [18]. Without government support, insurer must satisfy $Z = \dfrac{A + I}{P} < 1$, where A represents average administrative costs, I represents average indemnities paid and P represents average amount of premiums collected from farmers . All of seven countries' Z ratios are much higher than 1 which indicates that amount of indemnity paid is above the premiums collected.

By considering those drawbacks, İçer suggests that financial sustainability of this program is impossible, even with government support in the case of drought [20] in Turkey.

2.1.2 Index-Based Crop Insurance

Since traditional agricultural insurance models experience losses due to information asymmetry, Index-Based Insurance model has been introduced to overcome adverse selection and moral hazard. Index-Based insurance differs from a traditional crop insurance by the estimation structure of the losses. In an Index-Based insurance, loss estimates are based on an index such as rainfall or the yield of an area rather than directly using the loss of an individual farm. If the predetermined index value falls

Country (Insurer)	Period	I/P	A/P	$(A+I)/P$
Brazil (PROAGRO)	1975-81	4.29	0.28	4.57
Costa Rica (INS)	1970-89	2.26	0.54	2.80
India (CCIS)	1985-89	5.11	n.a.	n.a.
Japan (agriculture)	1947-77	1.48	1.17	2.6
	1985-89	0.99	3.57	4.56
Mexico (ANAGSA)	1980-89	3.18	0.47	3.65
Philippines (PCIC)	1981-89	3.94	1.80	5.74
USA (FCIC)	1980-90	1.87	0.55	2.42

Table 2.1: Financial Performance of seven agricultural insurance programs

below the actual value, then the insured gets the indemnity payment. Usage of index in estimation process prevents farmers to get an extra indemnity payment by influencing the yield [10]. Another advantage is that the administrative costs are low because there is no need to measure each farm's loss separately. However, since indemnity payments are done by using an index, an insured may not get an indemnity payment even if there is a loss or, conversely, even if there is no loss, an insured may get an indemnity payment. This problem is called the *basis risk* [10, 41]. Generally, basis risk is a problem associated with the index-based insurance programs. However, Barnett et. al [3] suggest that this problem is also valid for MPCI, mentioned in Section 2.1.1, due to the sampling and measurement errors when calculating the expected yield. Besides the basis risk, there is another problem associated with the Index Insurance because of the time lag between the loss event and the indemnity payment. According to Parshad, Managing Director of *Agriculture Insurance Company of India (AIC)*, the reason for that delay is the elapsed time during the collection of the yield data [35].

Joint Research Centre (JRC) [12] reported that there are some features of an Index-Based insurance that should be taken into account. Firstly, it works well for homogeneous areas, i.e, all farms in the area should have correlated yields. Also, there should not be variations in the climate conditions. Secondly, yield time series should be available.

There are two types of Index-Based Insurance programs: *Area Yield Index-Bases Insurance* and *Weather Index-Based Insurance*.

Area Yield Index-Based Insurance: It was first proposed by Halcrow at 1949. According to this program, in a given area, indemnities will be paid if the actual area yield falls below the determined yield which is the percentage of the historical average yield of that area [15, 25]. With this method, moral hazard and adverse selection problems disappear since indemnity calculations are based on an area yield rather than the individual farm yield. Also, for the same reason, administrative costs are lower than the traditional crop insurances. This method has been used in the USA, Canada, Brazil and Mongolia [50]. In Turkey, Bilici and Zulauf determined the premiums for wheat in Konya based on an area yield insurance program [4].

In 1991, Miranda reconsidered and formulated Halcrow's proposed insurance scheme [25]. According to his formulation, a farmer i, where $i = 1, \ldots, N$, has a yield $\tilde{y}_i$ and the average area yield of that area across all farms is $\tilde{y}$. Miranda modeled yield as:

$$\tilde{y}_i = \mu_i + \beta_i \cdot (\tilde{y} - \mu) + \tilde{\epsilon}_i, \tag{2.2}$$

where

$$\beta_i = \mathrm{Cov}(\tilde{y}_i, \tilde{y}) \backslash \sigma_y^2, \tag{2.3}$$

$$\mathrm{E}[\tilde{\epsilon}_i] = 0, \quad \mathrm{Var}[\tilde{\epsilon}_i] = \sigma_{\tilde{\epsilon}_i}^2, \quad \mathrm{Cov}(\tilde{y}, \tilde{\epsilon}_i) = 0, \tag{2.4}$$

$$\mathrm{E}[\tilde{y}_i] = \mu_i, \quad \mathrm{Var}[\tilde{y}_i] = \sigma_{\tilde{y}_i}^2, \tag{2.5}$$

and

$$\mathrm{E}[\tilde{y}] = \mu, \quad \mathrm{Var}[\tilde{y}] = \sigma_{\tilde{y}}^2. \tag{2.6}$$

Let $\tilde{n}$ denote the indemnity and π be the premium. Then, a farmer will receive an indemnity

$$\tilde{n} = \max\{y_c - \tilde{y}, 0\}, \tag{2.7}$$

where y_c is the predetermined yield or critical yield. By assuming that the premium is actuarially fair, i.e., $\mathrm{E}[\tilde{n}] = \pi$, when farmer i obtains an insurance, his net yield is equal to

$$\tilde{y}_i^{net} = \tilde{y}_i + \tilde{n} - \pi, \tag{2.8}$$

and its variance which measures the yield risk is

$$\mathrm{Var}[\tilde{y}_i^{net}] = \sigma_{\tilde{y}_i}^2 + \sigma_{\tilde{n}}^2 + 2 \cdot \mathrm{Cov}(\tilde{y}_i, \tilde{n}), \tag{2.9}$$

where $\sigma_{\tilde{n}}^2 = \mathrm{Var}[\tilde{n}]$. Then, the reduction in his yield risk by getting an insurance will be

$$\Delta_i = \mathrm{Var}[\tilde{y}_i] - \mathrm{Var}[\tilde{y}_i^{net}] = -\sigma_{\tilde{n}}^2 - 2 \cdot \mathrm{Cov}(\tilde{y}_i, \tilde{n}). \tag{2.10}$$

Since $\tilde{y}_i$ and $\tilde{\epsilon}_i$, and $\tilde{\epsilon}_i$ and $\tilde{n}$ are uncorralated, it follows from Equation (2.2) that

$$\mathrm{Cov}(\tilde{y}_i, \tilde{n}) = \beta_i \cdot \mathrm{Cov}(\tilde{y}, \tilde{n}). \tag{2.11}$$

In addition, Miranda also defined the *critical beta* denoted by β_c such that:

$$\beta_c \doteq -\frac{\sigma_{\tilde{n}}^2}{2 \cdot \mathrm{Cov}(\tilde{y}, \tilde{n})}. \tag{2.12}$$

Then, by substituting Equation (2.12) into Equation (2.11), Δ_i can be rewritten as:

$$\Delta_i = \sigma_{\tilde{n}}^2 \cdot \left(\frac{\beta_i}{\beta_c} - 1\right). \tag{2.13}$$

We note that, Equation(2.13) is greater than 0 as long as $\beta_i > \beta_c$, i.e., Area Yield-Index Based insurance program is effective if and the only if β_i exceeds β_c. He also found that

$$0 \leq \beta_c \leq 0.5, \tag{2.14}$$

which shows us that, a farmer whose $\beta_i \geq 0.5$ will profit by this insurance.

Miranda also tested his theoretical findings by using data of 102 soy bean producers in western Kentucky. He found that β_c increases with the critical yield, and achieves its maximum 0.5 which is its theoretical maximum, and also the premium increases with the values of β_c.

According to İçer, this method is hard to implement in Turkey, since at least a 20-years data history based on village or district, is not available in Turkey [20].

Weather Index-Based Insurance: In this type of insurance, a weather related index such as rainfall is used to determine the indemnity. According to this program, if the actual rainfall falls below the threshold value, calculated by using historical rainfall data, then a predetermined amount is paid to the insured. Again, this method avoids asymmetric information by using an index . It has been used in the USA, in India and Mexico. In addition to these countries, several pilot programs continue in developing countries such as Malawi and Ghana [19, 29, 30]. On the other hand, in Ethiopia a pilot project was canceled, since only 30 policies were sold in 2006. The main reasons for this program to fail are the lack of weather data and the insufficient number of weather stations [10]. In Turkey, Evkaya modeled Weather Index-Based Insurance for Central Anatolia with three different index types in his thesis [32].

According to Rao, a Chief Risk Officer in *AIC* [37], there are three suggestions for Weather Index-Based Insurance to function properly:

1. This type of insurance program is more suitable for catastrophic losses than moderate losses,
2. It is more suitable for aggregate level than farm level,
3. There may be a combination of Weather Index-Based Insurance and an Area Yield-Index Based Insurance in the form of a double triggered product.

Rao also claims that in the case of optimal precipitation, other factors such as soil quality and fertilizer use have a great impact on crop yields; however, in the case of low rainfall, those other factors have a little impact on crop yields.

In his article, İçer states that lack of data availability and low correlation between yields and rainfall make this program unpreferable [20] in Turkey.

2.2 Latest Studies in Turkey

Because Multiple Peril Crop Insurance and Index-Based Insurance are not feasible risk management tools against drought in Turkey, Necati İçer proposed a new type of insurance program that aims to overcome drawbacks of traditional insurances [20]. According to this program, reference parcels are selected to represent districts in order to establish reference yield. Besides, the insured ones select a guaranteed yield which is the percentage of the reference yield. If the actual yield of that parcel falls below a guaranteed yield, then the insurer pays the insured an indemnity which is the product of the difference between the guaranteed yield and actual yield and a price.

General Directorate of Agricultural Reform which is linked to the Ministry of Food, Agriculture and Livestock conducts joint project with *Istanbul Technical University* (*ITU*) called *"Yield Model of Agricultural Parcels"*. Foundations of this project was established in 2006 with a protocol called *Information System of Agricultural Parcels (TARBIL)*. Purposes of TARBIL are:

- To digitize the cadastral agricultural parcels,
- Identification of land uses by orthophotograph and satellite images,
- Development of a software which integrates TARBIL and Ploughman Registration System *(ÇKS)*.

TARBIL project was started in 2006 in *Southeastern Anatolia Region Project (GAP)*, and it is planned that by the year of 2014, it will be used in the overall country. However as of 2015, it is still in progress. With this system, topographic measurements and satellite images are going to be gathered simultaneously. In the next step, by combining those two different data types, regional product progress and distribution are going to be obtained. Precisely, to get the best estimation of the crop yields, it is necessary to have historical data. Consequently, this project will be more effective in the long run.

According to Küsek, general director of *General Directorate of Agricultural Reform*, there are 52 million of digitalized parcels in Turkey; among these parcels, approximately 30 million of them are engaged in agriculture. With TARBIL project, the use of satellite images will be combined with the topographic measurements to construct agricultural data set. Moreover, those findings will be shared with the farmers through online network.

Throughout the world, Canada, China, Thailand, and India use satellite images to monitor plant growth. It is also noteworthy that these countries use the information gathered from the satellite images to practice Weather Index-Based Insurance schemes. The distinction between TARBIL and the above mentioned countries' projects is that TARBIL also observes phenological progress of the plants along with the meteorological data.

Integration of ÇKS and TARBIL are going to be as follows: With ÇKS, farmers will enter the inputs related with their own farm such as harvested area, live-stocks, seeds, and yields to construct database for monitoring and controlling purposes. With TARBIL, inputs which farmers' enter the system are going to be verified by using satellite images to check whether inputs are reported correctly or not. This mechanism aims to prevent farmers from reporting inaccurate information which is one of the main challenges of agricultural crop insurances. Therefore, application of this project is important to have efficiently working agricultural insurance programs.

CHAPTER 3

District-Index Insurance

3.1 District-Index Insurance Scheme and its Mathematical Model

The District-Index insurance scheme has properties of both yield-based insurance and index-based insurance. The first step of the district-index scheme is the determination of the *reference parcels*. The proposal suggests that 8-16 reference parcels be chosen from the villages of each district so that they include at most 3 different types of a given product, 3 different types of soil and 3 different types of altitudes; this choice tries to represent the variation in yield across the district. After the determination of the reference parcels, *reference yield* is determined as the k-years average (where k is at least 5) of the reference parcels' *normal* yields, that is, the years with loss are not included in the average calculation.

The insurance scheme defines drought as the 20% or more yield reduction than the reference yield for each crop due to high temperature, hot winds or lack of rainfall (random variable D_i is used to model drought). In the case of a drought, the insurer pays the indemnity to the insured by the amount up to *yield guarantee*. Yield guarantee is chosen by the farmer as a percentage, in our case between 60% and 80%, of the reference yield. The upper limit 80% comes from the definition of drought given above. These lower and upper bounds seem reasonable and have been suggested to us in our correspondence with TARSIM, other values can of course be used. When drought occurs, *drought indemnity* is calculated as the multiplication of the unit price and the difference between the yield guarantee and the actual yield. A mathematical formulation of these rules is given in the next section. When buying insurance under this program, a farmer needs to specify the size of the agricultural area to be insured and to choose his/her insurance level α.

3.1.1 Advantages of the District-Index Insurance Scheme

Districts in Turkey typically contain thousands of parcels. Therefore, it is difficult and expensive to measure the effects of drought on the yield of each parcel of a district. On the other hand, to run the insurance program one needs to only measure the yields of the reference parcels. Therefore, the administrative costs of running the insurance scheme is lower than it would be if it were based on the yield of each parcel in the district. The use of reference parcels ensure that: First, the insurance scheme does not suffer from moral risk or adverse selection. Second, basis risk is going to reduce; all of these are discussed in Chapter 2.

3.2 Modeling of the District-Index Insurance

To begin with a relatively simple and manageable model let us fix the district i.e, our model considers only one district of a province. In addition, let us assume that in our single district only one type of crop grows and therefore, the insurance scheme concerns only this one type of crop. Let N denote the number of farmers to use the insurance scheme. Let z^i denote the parcel size of the i^{th} farmer for $i = 1, 2, 3, \ldots, N$.

Let m denote the number of reference parcels and let Y^i_l, $i = 1, 2, 3, \ldots, m$," denote the yields of the i^{th} reference parcel in year j. If we consider the reference parcels together, we get the following m dimensional process $\mathbf{Y}_j \doteq (Y^1_j, Y^2_j, ..., Y^m_j)$. For each year j, let $D_j \in \{0, 1\}$ denote whether meteorologically the year j was a drought year (i.e., $D_j = 1$ if in year j there is a drop in crop yield due to hot winds, diminished rainfall, or other meteorological conditions related with drought). The sequence of reference yields R_j is defined as follows:

$$\bar{Y}_j \doteq \frac{1}{m}\sum_{l=1}^{m} Y^l_j, \quad R_j \doteq \frac{1}{5}\sum_{k=1}^{L_j}(1 - D_{j-k})\bar{Y}_{j-k},$$

where L_j satisfies

$$5 = \sum_{k=1}^{L_j}(1 - D_{j-k}).$$

Thus, the reference yield for year j is the arithmetic mean of the yields observed in the last five no-drought years R_j is computed at the beginning of the planting season of year j using yield data of earlier years, according to the formula above. Y^l_j on the other hand is measured from the reference parcels at the end of harvest in year j. Y^l_j, and therefore $\bar{Y}_j$ and R_j, are assumed to be yields per area. To convert these expressions to actual yields multiply them by the area of the parcel.

Let $\alpha^i_j \in [0.6, 0.8]$ denote the insurance level that the i^{th} farmer specifies in year j. For a real number a, let a^+ denote its positive part, i.e., $a^+ = \max(a, 0)$. The interval $[0.6, 0.8]$ can be replaced by another interval. Then, under this insurance program, the indemnity payment I^i_j payed to the i^{th} farmer in year j is defined as

$$I^i_j \doteq z^i D_j(\alpha^i_j R_j - \bar{Y}_j)^+ P_j, \tag{3.1}$$

where P_j is the unit price of the crop to be used in the insurance contract and z^i is the area to be insured by the i^{th} farmer. Even if the year j experiences a drought, it may happen that the yield $\bar{Y}_j$ is still above 80% of the reference level; if this occurs, $\alpha^i_j \leq 0.8$ implies that no farmer will be paid an indemnity. Thus, for insurance payments, the occurrence of a drought is not enough, it must also cause at least 80% reduction in yield as measured by the method above. The price P_j may be taken to be a function of the prices of the earlier years (similar to $\bar{Y}_j$ above). It may also be, for example, the spot price of the underlying product at a regulated market at the time of the sale of the contract. It is assumed that it is known at the time of the sale of the insurance contract and therefore before harvest.

Then the total indemnity payments I_j that the insurance company experiences in year j is

$$I_j \doteq \sum_{i=1}^{N} I_j^i = \sum_{i=1}^{N} z^i((\alpha_j^i R_j - \bar{Y}_j))^+ P_j D_j. \tag{3.2}$$

What we have given so far can be thought of as the skeleton of the model. To get a complete model, we need to specify the distributions of the processes that appear in the model. There are three processes here: the yield process $\mathbf{Y}$, the drought process D_j and the price process P_j. The modeling of commodity prices is a very well studied subject, see [14, 23, 48], and a model from this literature can be used for P. For the model of the yield process one can take two approaches. 1) One can specify the distribution of the m dimensional yield process $\{\mathbf{Y}_j\}$. Once this distribution is specified the distribution of the rest of the processes (R_j and $\bar{Y}_j$) are determined from that of $\mathbf{Y}$. 2) Instead of $\mathbf{Y}_j$ model directly the one dimensional average yield process $\bar{Y}_j$. The advantage of the first approach is that it will lead to more precise results but this will come with the disadvantage of more complicated mathematics and lengthier computations. The advantage/disadvantage tradeoff for the second approach is the reverse: we get a less precise but faster model. In this thesis, we are going to adopt the second approach. There is also a wide literature on modeling agricultural yield processes. The next paragraph is a brief review of this literature.

Just and Weninger [21] claim that crop distributions are normally distributed. Hay [11] and Ramirez et al. [36], on the other hand, suggest that crop yield distributions diverge from normality. In addition, Nelson and Preckel [28] find that beta distribution is applicable for corn yields, and also Gallagher [13] claimed that gamma distribution is suitable to capture the skewed nature of the yield. Moreover, Ozaki et al. [31] use both parametric and nonparametric statistical models to estimate the crop yields and conclude that nonparametric approach works well. Thus, in determining a model for the yield and price processes this literature should be studied.

On the other hand, there are a number of indices to quantify drought risk. Among them, the most widely used ones are Standardized Precipitation Index (SPI) and Palmer Drought Severity Index (PDSI). The literature for this process can be found in [8, 16, 24, 27, 34]. Obviously, the yield process, the drought process and the price process will be correlated, and therefore, they should not be fitted to historical data separately but together to capture this correlation.

In what follows we will limit ourselves to the consideration of a single year in isolation, which does not require a full model for the yield, drought and the price processes. Let us further focus on the case of a single farmer whose parcel size to be insured is z^i and whose chosen insurance level is α. Let us compute the expected value of the indemnity I defined in Equation (3.4) for this farmer (because we focus on a single year we drop the j subscript and because we focus on a single farmer we drop the i superscript); this value can serve as a basic reference for the pricing of the insurance product. The reference yield R is known at the time of the writing of the insurance contract; the same is also assumed for the price P. Thus, to compute this expectation we need the conditional density of the average reference yield given that the drought indicator D equals 1. This last distribution obviously depends on the model we use for the yield process; let us denote it by f_d. Assuming that the premium is actuarially fair, the expected total premium is

$$\mathrm{E}[I] = \int_0^\infty \left(z\left(\alpha R - \bar{Y}\right)^+\right) f_d(\bar{Y}) d\bar{Y}. \tag{3.3}$$

The variables z and P are constants and come out of the integral, this reduces the last display to

$$\mathrm{E}[I] = Pzp_d \int_0^\infty \left(\alpha R - \bar{Y}\right)^+ f_d(\bar{Y}) d\bar{Y}. \tag{3.4}$$

Note that the integral on the right is exactly the formula for the price of a put option on $\bar{Y}$. Therefore, one expects that the premium here will exhibit the characteristics of the price of a put option. One well known property of the price of a put is that it is convex in its strike. The corresponding property here is convexity in the insurance level. The next result directly shows that the premium is convex in α.

Proposition 3.1. $\mathrm{E}[I]$ *defined in Equation* (3.4) *is increasing and convex in* α.

Proof. First, let us show that $(\alpha R - \bar{Y})^+$ is convex and increasing in α. Define $I(\alpha) \doteq (\alpha R - \bar{Y})^+$ where $R, \bar{Y} \in \mathbb{R}^+$. To show that $I(\alpha)$ is increasing, let us assume that $\exists \alpha_1, \alpha_2$ such that $\alpha_1 < \alpha_2$. For $I(\alpha)$ to be increasing, one needs to show that $I(\alpha_1) \le I(\alpha_2)$. Assume to the contrary that $I(\alpha_2) < I(\alpha_1)$. Then,

$$I(\alpha_2) = (\alpha_2 R - \bar{Y})^+ = \max(\alpha_2 R - \bar{Y}, 0) < I(\alpha_1) = (\alpha_1 R - \bar{Y})^+ = \max(\alpha_1 R - \bar{Y}, 0).$$

For the first case, let $0 \le I(\alpha_2) = \max(\alpha_2 R - \bar{Y}, 0) \le \alpha_1 R - \bar{Y}$. This implies

$$\begin{aligned} \alpha_2 R - \bar{Y} &\le \alpha_1 R - \bar{Y} \\ \alpha_2 &\le \alpha_1, \end{aligned}$$

which contradicts with the assumption that $\alpha_1 < \alpha_2$. For the second case, we have $\alpha_2 R - \bar{Y} \le I(\alpha_2) = \max(\alpha_2 R - \bar{Y}, 0) \le 0$, i.e., $\alpha_2 R - \bar{Y} \le \alpha_1 R - \bar{Y} \le 0$ which implies $\alpha_2 \le \alpha_1$, contradiction. Therefore, $I(\alpha)$ is an increasing function in α.

To prove convexity of $I(\alpha)$ in α, we need to show that $\forall \alpha_1, \alpha_2$ and $\theta \in [0, 1]$, $I(\theta\alpha_1 + (1-\theta)\alpha_2) \le \theta I(\alpha_1) + (1-\theta) I(\alpha_2)$.

$$\begin{aligned} I(\theta\alpha_1 + (1-\theta)\alpha_2) &= \max\left((\theta\alpha_1 + (1-\theta)\alpha_2)R - \bar{Y}, 0\right) = \max\left((\theta\alpha_1 + (1-\theta)\alpha_2)R - \bar{Y}, 0\right) \\ &= \max\left(\theta\alpha_1 R + (1-\theta)\alpha_2 R - \bar{Y}, 0\right) \\ &= \max\left(\theta\alpha_1 R + (1-\theta)\alpha_2 R - \bar{Y} \pm \theta\bar{Y}, 0\right) \\ &= \max\left(\theta(\alpha_1 R - \bar{Y}) + (1-\theta)(\alpha_2 R - \bar{Y}), 0\right) \\ &\le \max\left(\theta(\alpha_1 R - \bar{Y}), 0\right) + \max\left((1-\theta)(\alpha_2 R - \bar{Y}), 0\right) \\ &= \theta I(\alpha_1) + (1-\theta) I(\alpha_2). \end{aligned}$$

Thus, $I(\alpha)$ is convex in α.

Let Δy be a discretization step that will be sent to 0. Let Y be the discretization of $\bar{Y}$ taking the values $\{0, k\Delta y, k = 1, 2, 3, ..., N_\Delta \doteq \lfloor \frac{1}{(\Delta y)^{3/2}} \rfloor\}$:

$$Y = k\Delta y \text{ when } k\Delta y < \bar{Y} \leq (k+1)\Delta y.$$

The distribution of Y will be

$$P(Y = y_k) = f(y_k)\Delta Y + O(|\Delta y|^2), \tag{3.5}$$

where $y_k = k\Delta y$. Define $I_\Delta \doteq (\alpha R - Y_\Delta)^+$. By definition

$$\mathrm{E}[I_\Delta] = \sum_{i=1}^{N_\Delta} (\alpha R - y_k)^+ P(Y = y_k). \tag{3.6}$$

The right side of the last display is a convex combination of convex functions in α and is hence convex, i.e., $\mathrm{E}[I_\Delta]$ is a convex function of α. On the other hand, (3.5) implies

$$\mathrm{E}[I_\Delta] = \left(\sum_{i=1}^{N_\Delta} (\alpha R - y_k)^+ f(y_k)\Delta y \right) + (\Delta y)^{1/2}.$$

The sum on the left is a Riemann sum for $\mathrm{E}[I]$ and the second term converges to 0 as $\Delta y \to 0$. These imply $\lim_{\Delta y \to 0} \mathrm{E}[I_\Delta] = \mathrm{E}[I]$. We know that the limit of a sequence of convex functions is convex; $\mathrm{E}[I_\Delta]$ is convex in α, then the limit $\mathrm{E}[I]$ must be so as well. □

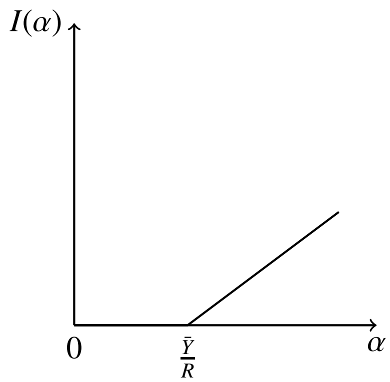

Figure 3.1: Convexity of $I(\alpha)$ in α

Convexity in α means the following: the yield guarantee that the farmer specifies has higher marginal cost as it increases. Thus if the farmer would like to pay the minimum possible marginal cost he will prefer $\alpha = 0.6$, the smallest possible yield guarantee. But in all of the computations we have done below we have seen that the convexity of the premium is not very marked and all yield guarantees have similar marginal costs.

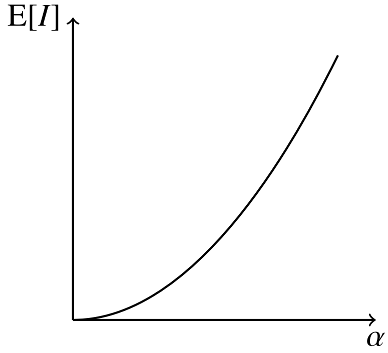

Figure 3.2: Convexity of $\mathrm{E}[I]$ in α

Figures 3.2 and 3.2 and shows the relation between $I(\alpha)$ and α and $\mathrm{E}[I]$ and α that are defined in Proposition 3.1.

As we have already noted, there are three prominent suggestions for the density Y in the current literature: normal, beta and gamma. The next section studies how the premium computed in (3.4) depends on this choice.

3.3 The effect of the yield distribution on the premium

Let us assume that the mean and the variance of the yield given a drought has occurred is known and they equal μ and σ^2. A well known and popular method to fit a distribution to a given data set is the method of moments, in which we select the distribution parameters so that the distributional mean and variance equals the observed ones. Supposing that this method is used we would like to see how our choice of distribution (normal, gamma or beta) effects the computed premium. To do so, we will fix the mean μ and variance σ^2 and use for the yield distribution normal, gamma and beta with these moments and compute the premium given by these distributions.

Let us begin with the normal distribution, i.e., we assume

$$Y = \sqrt{\sigma^2} \cdot X + \mu, \tag{3.7}$$

where $X \sim N(0, 1)$.

Then, assuming the normal distribution for the yield, the premium for our fixed farmer can be written as

$$\mathrm{E}[I] = zP\frac{1}{\sqrt{2\pi}} \int_{\frac{-\mu}{\sqrt{\sigma^2}}}^{\frac{\alpha R-\mu}{\sigma}} \left[\alpha R - \left(\sqrt{\sigma^2} \cdot X + \mu\right)\right]^+ \mathrm{e}^{-\frac{x^2}{2}} dx, \tag{3.8}$$

where z and P are defined as above.

Remark 3.1. The yield cannot be negative but the normal distribution does allow negative values. The restriction of the above integral to $Y > 0$ handles this problem by assigning an insurance payment of zero to these values. This would lead to an underestimation of the premium of $P(Y < 0)$ is relatively large. We will see an example of this below.

As suggested in [13], let average yield distribution X to be distributed as $Gamma(k, \theta)$. Mean and the variance of the gamma distribution is

$$\mathrm{E}[X] = k\theta, \tag{3.9}$$

$$\mathrm{Var}[X] = k\theta^2. \tag{3.10}$$

Then to match the moments of the gamma distribution to (μ, σ^2) we must have

$$k\theta = \mu \Rightarrow k = \frac{\mu}{\theta}, \tag{3.11}$$

$$k\theta^2 = \sigma^2 \Rightarrow \theta = \frac{\sigma^2}{\mu}, \tag{3.12}$$

and (3.4) becomes

$$\mathrm{E}[I] = zP \int_0^{\alpha R} [\alpha R - X]^+ \frac{1}{\Gamma(k)\theta^k} x^{k-1} \mathrm{e}^{\frac{-x}{\theta}} dx, \tag{3.13}$$

where $k > 0$, $\theta > 0$ and other variables are the same as before.

Finally, let average yield be distributed as $Beta(n, m)$ as suggested in [28]. However, beta is a distribution on the interval $[0, 1]$ and yield values do not necessarily have to take values in this interval. Then, to use the beta distribution for the yield, one has to change variables so that the changed variable can take values between 0 and 1. Suppose, X is a random variable whose distribution is not known but mean and variance is known, i.e., $X \sim (\mu, \sigma^2)$, such that $P(X < 0) \approx 0$ and $P(X > \mu + k\sigma) \approx 0$ for some $k > 0$. Then we can treat $\mu + k\sigma$ as an approximate upper bound on X. Then, the variable

$$Y = \frac{X}{\mu + k\sigma} \tag{3.14}$$

will almost with probability one fall in the interval $[0, 1]$ and one can attempt to fit a beta distribution to it. The mean of the transformed variable Y will be $\frac{\mu}{\mu+k\sigma}$ and its variance will be $\frac{\sigma^2}{(\mu+k\sigma)^2}$. Let us use this transformation to map yield to the interval $[0, 1]$.

The expected value and variance of the variable $Y \sim Beta(n, m)$ is

$$\mathrm{E}[Y] = \frac{n}{m + n}, \tag{3.15}$$

$$\mathrm{Var}[Y] = \frac{mn}{(m + n)^2(m + n + 1)}. \tag{3.16}$$

Then m and n must satisfy

$$\frac{n}{m+n} = \frac{\mu}{\mu + k\sigma} \Rightarrow m = \frac{kn\sigma}{\mu}, \tag{3.17}$$

$$\frac{mn}{(m+n)^2(m+n+1)} = \frac{\sigma^2}{(\mu + k\sigma)^2} \Rightarrow n = \frac{k\mu^2 - \sigma\mu}{k\sigma^2 + \mu\sigma}. \tag{3.18}$$

With this, we have introduced a further parameter k; in case there is actual yield data, a more natural approach is to perhaps use the max of the data points as the denominator of (3.14). With these, the formula for the premium under the beta distribution is

$$\mathrm{E}[I] = zP \int_0^{\frac{\alpha R}{\mu + k\sigma}} [\alpha R - X(\mu + k\sigma)]^+ \, x^{n-1}(1-x)^{m-1} \frac{\Gamma(n+m)}{\Gamma(n)\Gamma(m)} dx, \tag{3.19}$$

where $n > 0$ and $m > 0$.

3.4 Premium Calculations of the District-Index Insurance Scheme

As mentioned in Subsection 3.2, one can assume average yield distribution to be normal, gamma or beta. In this section, we will compute the premium based on the three distributions of the previous section.

We consider how variability of the average yield affects the premiums in case of drought. To do this, we hold the mean of the average yield constant and change the variance of the average yield.

Figure 3.3 shows the density functions of the normal, gamma and beta distributions when mean average yield is significantly smaller than the reference yield. All distributions are assumed to have the same mean ($\mu = 1.8$) and variance ($\sigma^2 = 0.5$) and the reference yield R is taken to be 4. Thus we assume that on average the drought causes more than 50% reduction in yield. Based on Figure 3.3, both gamma and beta distributions are positively skewed. For small values of x, the normal density function has heavier tail than the beta and gamma distributions on the left, and for bigger values of x, normal distribution has lighter tail than both gamma and beta distributions on the right. Gamma distribution has a longer tail on the right.

Figure 3.4 shows the (numerically computed) graphs of the premiums as a function of α given in equations (3.8), (3.13) and (3.19). To compute unit prices, we take the unit price $P = 1$ and the unit area $z = 1$; because we conditioned on a drought the probability of drought p_d is also omitted. In line with Theorem 3.1, in all of these graphs the premium is convex in α. Figure 3.4 shows that, when the variation is relatively small, premiums based on the normal, the gamma and the beta are close to each other.

Figure 3.5 focuses on the values of $\alpha \in [0.6, 0.7]$. For these α values, the premium based on the gamma distribution is more expensive than the other two premiums. However, there is not much difference between the premiums of the gamma and the beta distributions. The premium based on the normal distribution is the cheapest one. Similarly, Figure 3.6 shows that the premium based on the gamma distribution is greater

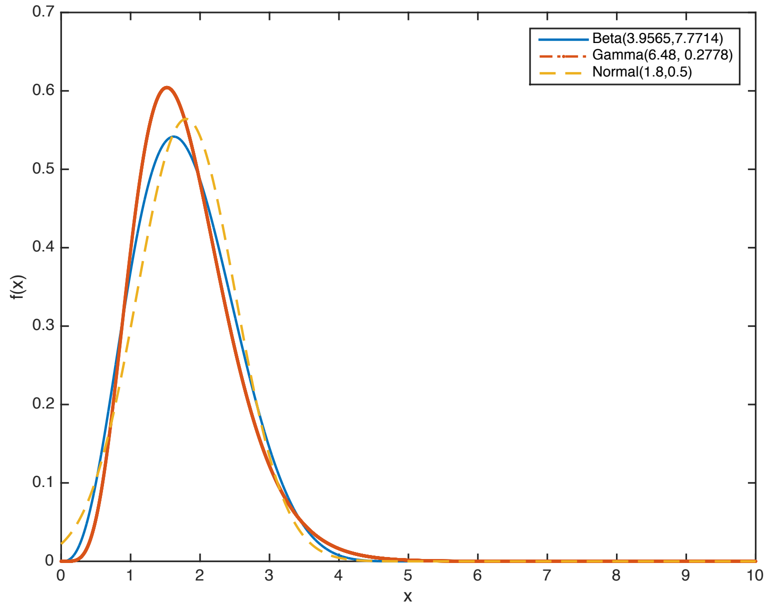

Figure 3.3: Comparison of the density functions of Normal, Gamma and Beta Distributions when $\mu = 1.8$, $\sigma^2 = 0.5$

than that based on the normal and beta distributions. The premium based on the gamma distribution is still the most expensive among the others. Even if premium based on normal distribution is still the cheapest one, it gets closer to other premiums.

Based on this analysis, when there is a little variation in the yield, the choice of distribution does not create big differences in the premiums, across all values of α.

To study how the increased variation of the average yield effects the premium, we hold the mean of the average yield as $\mu = 1.8$ and we let σ^2 to be 1.5. Figure 3.7 shows the density functions of the normal, the gamma and the beta distributions with these mean and variance. Again, the beta and the gamma distributions are positively skewed. In contrast to Figure 3.3, positive skewness is more pronounced when variance increased.

The underestimation of the normal distribution pointed out in Remark 3.1 is more pronounced when variance is increased. With $\mu = 1.8$ and $\sigma^2 = 1.5$, $P(Y < 0) \approx 0.07$ which is a relatively large value. To correct this, modified version of the total premium for a fixed year and farmer can be written as :

$$\mathrm{E}[I] = zP\frac{1}{\sqrt{2\pi}}\int_{\frac{-\mu}{\sqrt{\sigma^2}}}^{\frac{\alpha R-\mu}{\sigma}} \left[\alpha R - \left(\sqrt{\sigma^2}\cdot X + \mu\right)\right]^+ \mathrm{e}^{-\frac{x^2}{2}}dx + P(Y < 0)\alpha R. \tag{3.20}$$

In this modified version, we assign 0 to the negative values of x under the normal distribution.

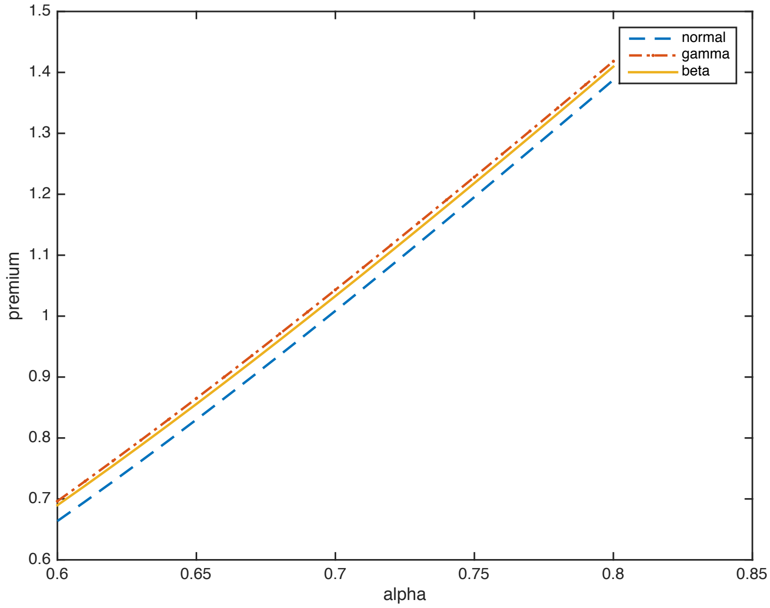

Figure 3.4: Premium calculation of the insurance scheme based on Normal, Gamma and Beta Distributions when $\mu = 1.8$, $\sigma^2 = 0.5$ and $R = 4$

The graphs (as a function of α) of the premiums listed in equations (3.20), (3.13) and (3.19) based on $\mu = 1.8$ and $\sigma^2 = 1.5$ is presented in Figure 3.8. Increased variability in the yield causes premium based on the normal distribution to differ slightly from the premiums based on the gamma and the beta distributions.

For values of $\alpha \in [0.6, 0.7]$, the premium based on the beta distribution is the most expensive one and the premium based on the normal distribution is the cheapest among the others as can be seen from Figure 3.9. When $\alpha \in [0.7, 0.8]$, premium based on the gamma distribution exceeds premium based on the beta distribution. Premium based on the normal distribution is the cheapest one.

To see how an increase in the mean effects the premiums, let $\mu = 2.5$ and $\sigma^2 = 0.5$ and $R = 4$. Figure 3.11 shows the density functions with given parameters above. When we increase the mean, only the gamma distribution is slightly skewed to the right. The normal distribution has lighter tail on the left and and heavier tail on the right.

Numerical premium calculations based on equations (3.20), (3.13) and (3.19) with $\mu = 2.5$, $\sigma^2 = 0.5$ and $R = 4$ is shown in Figure 3.12. When we increase the mean, convexity of the premiums is more apparent. Premium calculations with $\mu = 2.5$ and $\sigma^2 = 0.5$, cheaper than the premium calculations with $\mu = 1.8$, $\sigma^2 = 0.5$ as can be seen from Figure 3.4. For small values of α, premiums based on the normal, the gamma and the beta distributions slightly differ. For larger values of α, premium calculation based on the gamma distribution is the expensive among others and the premium based on the

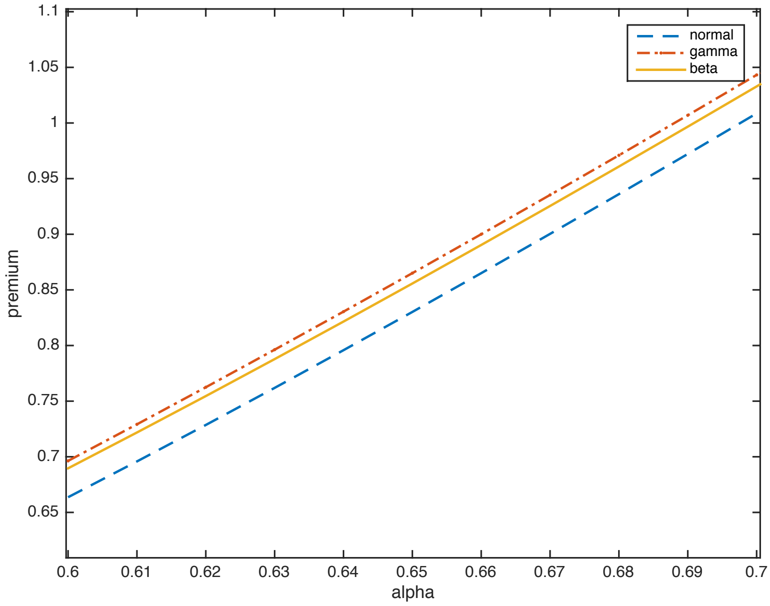

Figure 3.5: Comparison of premiums when $\alpha \in [0.6, 0.7]$, $\mu = 1.8$, $\sigma^2 = 0.5$ and $R = 4$

normal distribution is the cheapest one; however, this difference is not so pronounced.

Finally, densities of the normal, the gamma and the beta distributions with $\mu = 2.5$, $\sigma^2 = 1.5$ and $R = 4$ is given in Figure 3.13. Both the gamma and the beta distributions are negatively skewed but as apparent as in Figure 3.7.

As discussed in Remark 3.1, $P(Y < 0) \approx 0.02$ when yield is normally distributed. To correct this loss due to negative values of x under the normal distribution, we use (3.20). Approximate premiums based on these values are given in Figure 3.14. Unlike Figure 3.8, for small values of α, premiums based on the normal and the gamma distributions are close to each other and smaller than the premium based on the beta distribution. For larger values of α, premiums based on the gamma and the beta distributions are expensive than the premium based on the normal distribution.

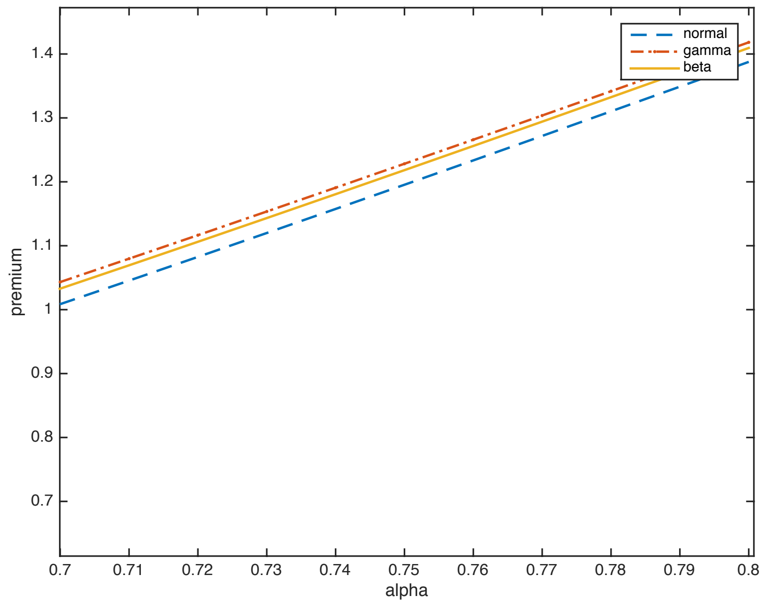

Figure 3.6: Comparison of premiums when $\alpha \in [0.7, 0.8]$, $\mu = 1.8$, $\sigma^2 = 0.5$ and $R = 4$

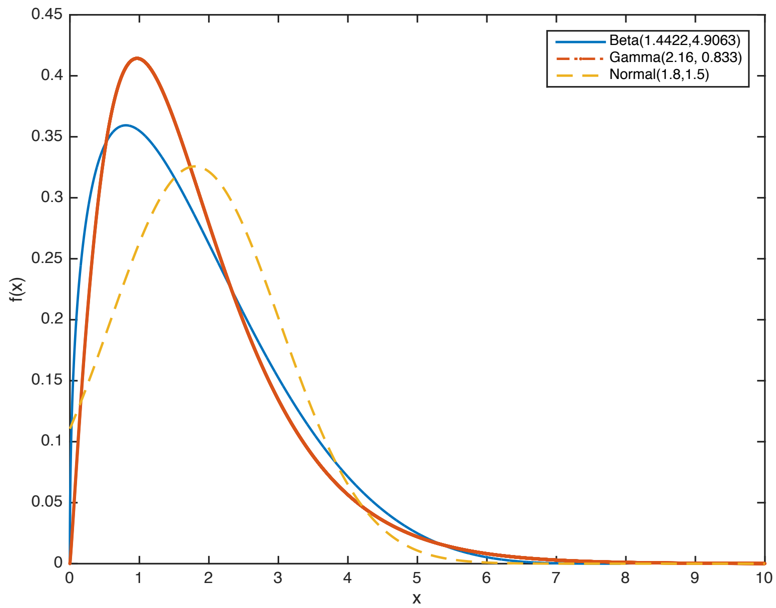

Figure 3.7: Comparison of the density functions of Normal, Gamma and Beta Distributions when $\mu = 1.8$, $\sigma^2 = 1.5$

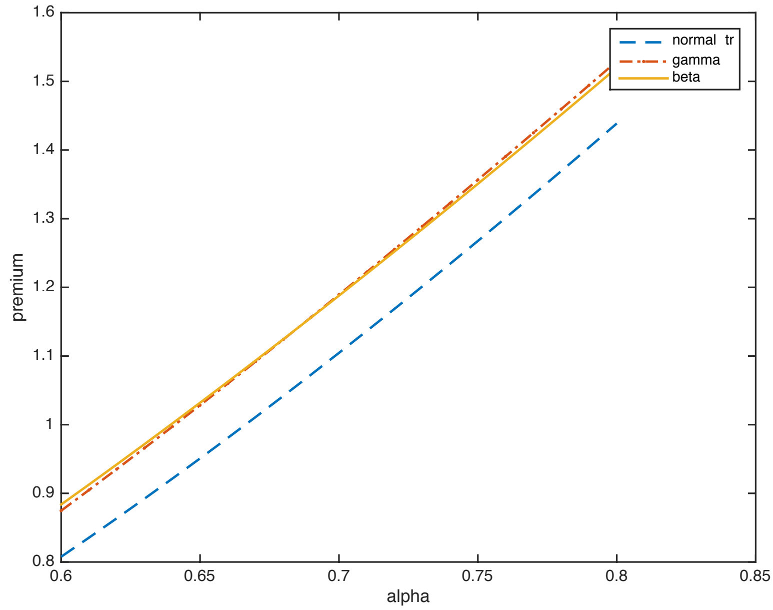

Figure 3.8: Premium calculation of the insurance scheme based on Normal, Gamma and Beta Distributions when $\mu = 1.8$, $\sigma^2 = 1.5$ and $R = 4$

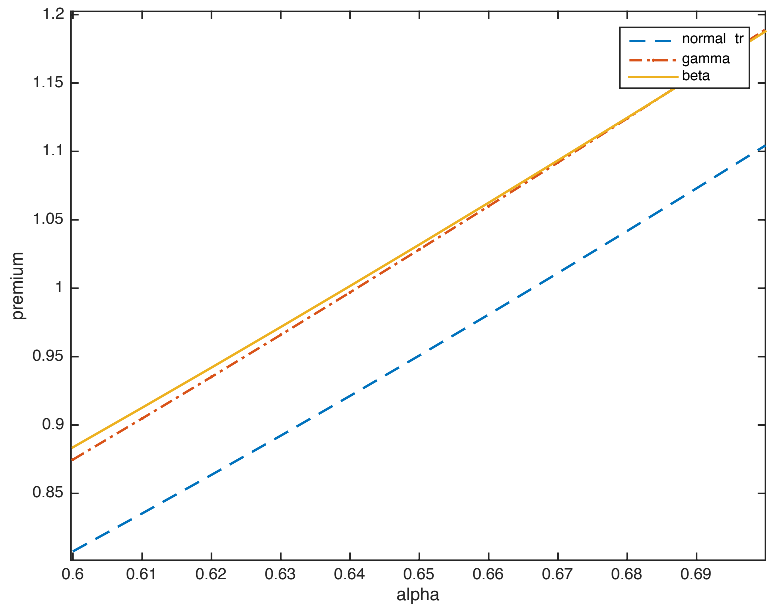

Figure 3.9: Comparison of premiums when $\alpha \in [0.6, 0.7]$, $\mu = 1.8$, $\sigma^2 = 1.5$ and $R = 4$

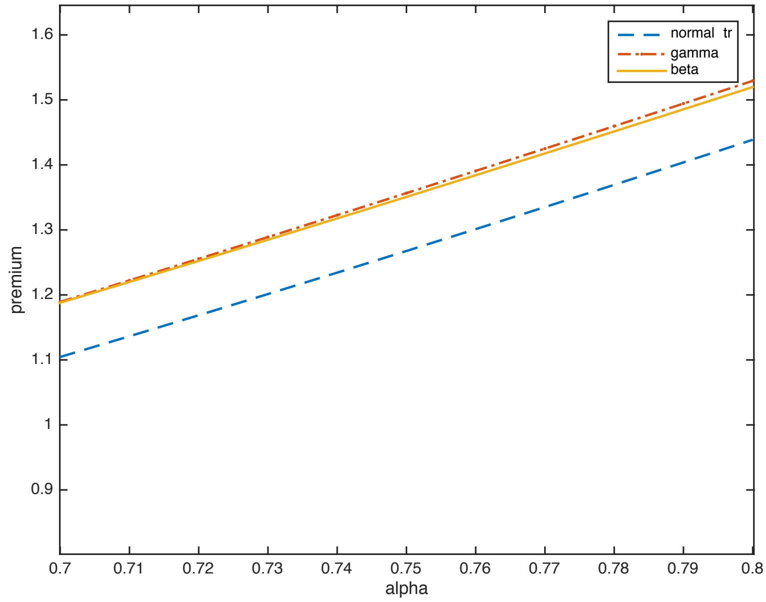

Figure 3.10: Comparison of premiums when $\alpha \in [0.7, 0.8]$, $\mu = 1.8$, $\sigma^2 = 1.5$ and $R = 4$

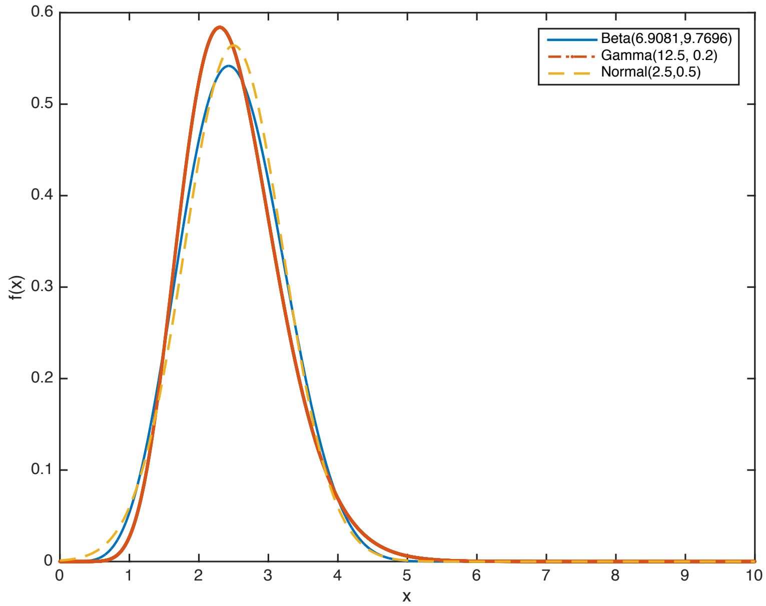

Figure 3.11: Comparison of the density functions of Normal, Gamma and Beta Distributions when $\mu = 2.5$, $\sigma^2 = 0.5$

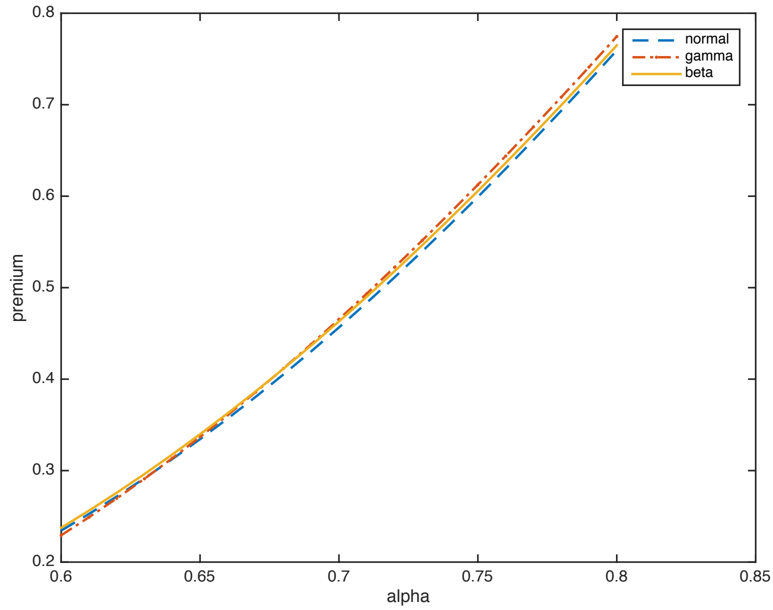

Figure 3.12: Premium calculation of the insurance scheme based on Normal, Gamma and Beta Distributions when $\mu = 2.5$, $\sigma^2 = 0.5$ and $R = 4$

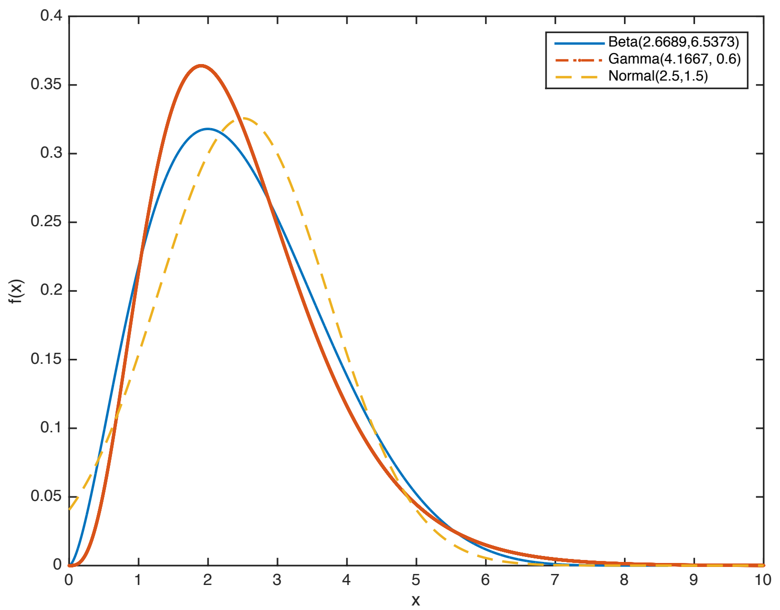

Figure 3.13: Comparison of the density functions of Normal, Gamma and Beta Distributions when $\mu = 2.5$, $\sigma^2 = 1.5$

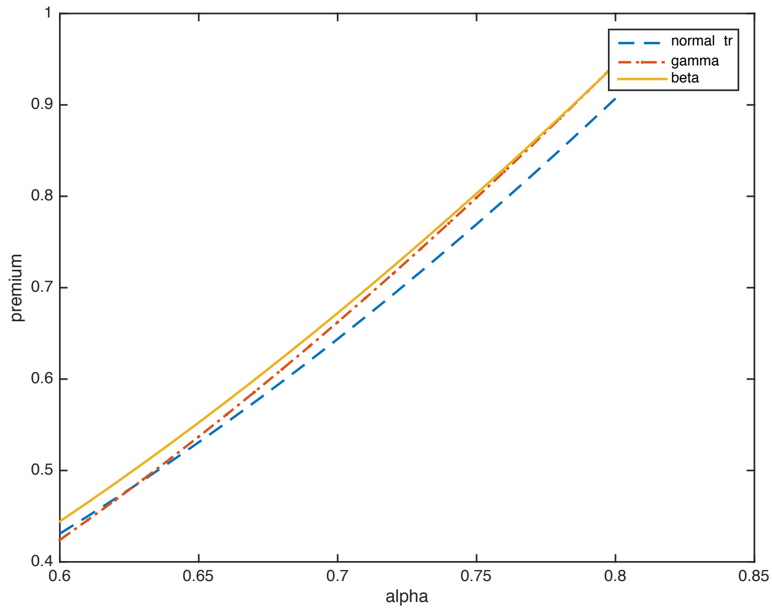

Figure 3.14: Premium calculation of the insurance scheme based on Normal, Gamma and Beta Distributions when $\mu = 2.5, \sigma^2 = 1.5$ and $R = 4$

CHAPTER 4

Conclusion

In this thesis, we considered a District-Index scheme to offer farmers insurance against drought. Implementation of this insurance program advances with the determination of reference parcels in each district. Then, 5-year yield average R of the chosen parcels is taken as a reference yield for this district. A farmer who wants to be insured chooses her/his insurance level $\alpha \in [0.6.0.8]$ and indicates the size z of the land to be insured. In case of a drought, the insured is paid $zP(\alpha R - Y)^+$, where P is the reference unit price of the crop (assumed to be known when the contract was signed). After building a general skeleton of this scenario we focused on the computation of the premium at the district level for a single crop; i.e., we fixed district, and the year and the crop to be insured.

We have observed that the insurance scheme is equivalent to a put option on the yield; therefore, the premium will exhibit the properties of the price of a put option. In particular, it will be convex in α.

There are three suggestions for the yield distribution in the literature: normal, gamma and beta. Our main analysis consisted of the dependence of the premium on this choice: for a fixed mean μ and variance σ^2 for the yield in a drought year we computed premiums for the three distributions above as a function of the insurance level α. Our main analysis focuses on two scenarios based on the variability of the yield. By holding mean of the yield constant, we examine how premiums are affected when the variance of the yield increases. We find that, when the variance of the yield is relatively small, there is no difference between the premiums based on the normal, the gamma and the beta distributions. When we increase the variance and correct the premium formula to avoid underestimation under the normal distribution, we find premiums are still close to each other based on these three distributions.

When we increase the mean of the yield, convexity of the premiums become more apparent. However, all three distributions' premium calculations give similar results.

What we have found in this study is that given that the drought occurred in a single district, choice of the distribution of the yield does not result in great differences in the premium calculations.

Our computations suffice from the point view of a farmer to be insured; for him/her the only variability is the yield in her/his district and this will determine his/her premium.

A much more difficult problem is an analysis from the side of the insurer. In particular, suppose that the insurer collected the premiums suggested by the computations of this thesis. What is the probability of its survival for a long period of time, or what is the level of initial reserves that will guarantee a long survival with high probability? To answer these and similar questions the distribution of all of the processes that appear in the model of Chapter 3 must be specified for a collection of districts and then estimated from available data. Perhaps the biggest challenge in specifying these distributions is to build a good model of the joint distributions of the D_i^k (drought indicator variables) where k varies over the districts of the country and then estimate them from available data. The same thing must be done also for the yields, which seems equally challenging. To make the model more realistic one must also consider a number of products (rather than just one). Future research may try to address these interesting and difficult problems.

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