ASSESSMENT OF SOLVENCY II REQUIREMENTS FOR TURKISH INSURANCE MARKET

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submitted by **MEHMET HÖBEK** in partial fulfillment of the requirements for the degree of **Master of Science in Department of Actuarial Sciences, Middle East Technical University** by,

Prof. Dr. Bülent Karasözen
Director, Graduate School of **Applied Mathematics** _______________

Assoc. Prof. Dr. A. Sevtap Kestel
Head of Department, **Actuarial Sciences** _______________

Assoc. Prof. Dr. A. Sevtap Kestel
Supervisor, **Actuarial Sciences, METU** _______________

Examining Committee Members:

Prof. Dr. G. Wilhelm Weber
Financial Mathematics, METU _______________

Assoc. Prof. Dr. A. Sevtap Kestel
Actuarial Sciences, METU _______________

Assoc. Prof. Dr. Fatih Tank
Statistics, Ankara University _______________

Date: _______________

I hereby declare that all information in this document has been obtained and presented in accordance with academic rules and ethical conduct. I also declare that, as required by these rules and conduct, I have fully cited and referenced all material and results that are not original to this work.

Name, Last Name: MEHMET HÖBEK

Signature :

ABSTRACT

ASSESSMENT OF SOLVENCY II REQUIREMENTS FOR TURKISH INSURANCE MARKET

Höbek, Mehmet
M.S., Department of Actuarial Sciences
Supervisor : Assoc. Prof. Dr. A. Sevtap Kestel

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Solvency II is the new capital regime being in force as of January 2016 in European Union (EU). It has brought profound changes to the previous one, namely Solvency I, by introducing new methods for the calculation of solvency capital requirement (SCR) of insurance and reinsurance companies. Besides the standard formula which is composed of sub-modules for the calculation of different risks, insurance companies are also allowed to use their partial or full internal models for the calculation of SCR. Since being also discussed recently in Turkish insurance market, this study analyzes the impact of Solvency II to Turkish insurance companies by comparing the standard formula and internal model results based on real data using copulas.

The study focuses on non-life premium and reserve risk calculation using both the standard formula and the internal model for three insurance companies of different sizes. Solvency II assumes that the premium and reserve risks for all segments are log-normal distributed and linearly correlated and aggregates the risks for the segments using predetermined correlation coefficients. Since companies are also allowed to use internal models and parameters based on their real data for the calculation of SCR, we used copulas to model the dependence between segments and calculate the SCR for the aggregated risks using Value-at-Risk (VaR) and Monte Carlo simulation. The proposed methodology is applied to motor vehicle liability (MTPL) and other motor (motor) segments' data over the years 2009-2015.

The internal model results for SCR are then compared for the three companies and the rest of the insurance sector with the results of Solvency II standard formula with respect to their size and the current Turkish solvency capital regime.

Keywords : Solvency II, solvency capital requirement, standard formula, internal model, copulas, VaR

ÖZ

TÜRK SİGORTACILIK SEKTÖRÜ İÇİN SOLVENCY II GEREKLİLİKLERİNİN DEĞERLENDİRİLMESİ

Höbek, Mehmet
Yüksek Lisans, Aktüerya Bilimleri Bölümü
Tez Yöneticisi : Doç. Dr. A. Sevtap Kestel

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Solvency II Avrupa Birliği ülkeleri için 1 Ocak 2016 itibarıyla yürürlüğe giren yeni sermaye rejimidir. Sermaye hesaplamasında yeni metodlar getiren Solvency II, önceki sermaye rejimi olan Solvency I'e göre önemli değişiklikler getirmiştir. Değişik risklerin hesabı için alt modüllerin yer aldığı standart formülün yanında Solvency II şirketlere sermaye yeterliliğinin hesabında kendi kısmi veya tam içsel modellerini de kullanma imkanı vermektedir. Son zamanlarda Türk sigorta sektöründe de tartışılmakta olan bir yöntem olması nedeniyle, bu çalışma Solvency II'nin Türk sigorta şirketlerine etkisini, gerçek şirket verileri kullanılarak bulunan standart formül ve içsel model sonuçlarını karşılaştırarak analiz etmektedir.

Çalışma değişik büyüklükteki üç şirket için standart formül ve içsel model kullanılarak hayat dışı prim ve rezerv risk hesabına odaklanmaktadır. Solvency II bütün branşlar için prim ve rezerv riskinin log-normal dağıldığını ve doğrusal korelasyona sahip olduğunu varsaymakta ve branşların risklerini önceden belirlenmiş olan korelasyon katsayıları kullanarak toplulaştırmaktadır. Şirketlere kendi gerçek verilerini kullanarak oluşturdukları içsel model ve parametreleri de kullanarak sermaye yeterliliklerini hesaplama izni verildiğinden, branşların arasındaki bağımlılığı modellemek için copulalar aracılığıyla toplulaştırılmış riskler için sermaye yeterliliği hesabı riske maruz değer yöntemi ve Monte Carlo simulasyonu kullanılarak yapılmıştır. Önerilen yöntem kara araçları sorumluluk (trafik) ve kara araçları (kasko) branşlarının 2009-2015 yılları verilerine uygulanmıştır.

İçsel model kullanılarak bulunan sermaye gerekliliği tutarları değişik büyüklükteki üç şirket ve sektörün geri kalanı için Solvency II standart formül sonuçlarıyla büyüklüklerine göre karşılaştırılmıştır.

Anahtar Kelimeler : Solvency II, sermaye yeterliliği, standart formül, içsel model, copula, VaR

To My Family

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The findings and comments presented in this thesis is not intended to impose nor shall it be construed as imposing any legally binding obligation to the Undersecretariat of Treasury.

TABLE OF CONTENTS

LIST OF FIGURES

LIST OF TABLES

CHAPTER 1

INTRODUCTION

Solvency II has become in force on January 1, 2016 in EU after postponing it several times since 2013. The ultimate objective of this project, as also was in Solvency I, is the protection of policy holders. What happens if a policy holder is not paid for his/her claims because of the inability of the insurance company to make a payment for its claims? This question lies behind all the motivation the regulatory and supervisory authorities have in designing the system of insurance. The probability of claims not being paid because of insolvent or bankrupt insurance companies can not be diminished at all, but can be minimized to a certain level. Is it reasonable to minimize this probability to 1/200, meaning that the insurance company would be insolvent once in every 200 years? Solvency II bets on this and sets rules for the capital requirement in order to make sure that the company has enough capital for paying its claims, which is described as Value-at-Risk (VaR) of the losses the company is expected to encounter within one year with 99.5% confidence level as described in EIOPA (2012) [9].

Solvency II redefines the capital regime for insurance and reinsurance undertakings in EU and comes up with new standards consisting of three pillars. It brings profound changes to the existing one, namely Solvency I by introducing new methods for the calculation of solvency capital requirement of insurance companies. Besides the standard formula which is composed of modules for different risks, insurance companies are also allowed to use their partial or full internal models and company specific parameters, provided that these are approved by the authorities. Since being discussed recently in Turkish insurance market, this study analyzes the impact of Solvency II to Turkish insurance companies by comparing the standard formula and a proposed internal model results based on real data of the companies.

The thesis focuses on non-life premium and reserve risks. Solvency Capital Requirement (SCR) is calculated using both the standard formula with predetermined correlation coefficients of Solvency II and an internal model based on distribution fitting of the loss ratios and using copulas for the dependence of the segments. SCR is calculated at a one-year horizon using real data of Turkish insurance companies. For confidentiality reasons the companies are not named explicitly and named as Large, Medium and Small denoting their sizes in terms of premium production and the remaining non-life insurance companies as Others. Instead of using predetermined correlation coefficients and the assumed distributions for the premium and reserve risks for different segments prescribed by Solvency II, the distribution of the risks are analyzed and the dependence between these are assessed using copulas at company level using real data. Using the joint distribution of the aggregated risks determined by the copulas the solvency capital requirement based on $VaR_{99.5\%}$ is calculated using simulation techniques. The Solvency II standard formula and the proposed internal model are compared with respect to the selected indicators.

1.1 Literature Survey

In literature several studies have focused on non-life premium and reserve risks and proposed various methods and models for the aggregation of the risks for different segments and the calculation of the SCR. The main distinctions among these studies are related to the duration of the calculation (e.g. multi- or one-year horizon), the definition of the risk (e.g. the loss ratio or the net underwriting result and the risk measures) and the modeling methods used such as linear regression, collective risk models, copulas etc [25, 1].

Ohlsson and Lauzeningks (2009) redefines one-year reserve risk concept using simulation approaches since the ultimo risk is in contrast to the short time horizon in internal risk models and the one-year risk perspective of Solvency II. They also discuss the relation between one-year premium risk and the premium reserve and discounting for the reserve and premium risk using cost-of-capital method [23].

Eling, Diers, Linde and Kraus (2011) model the insurance risk in a multi-year context for non-life insurance companies based on the fact that strategic management in an insurance company requires a multi-year time horizon for economic decision making in the context of internal risk models. They extend the simulation-based method for quantifying the one-year non-life insurance risk presented in Ohlsson and Lauzeningks (2009) to a multi-year perspective [11].

Savelli and Clemente (2013) analyze the risk profile of a multi-line non-life insurer using a simulation model and compare the results with Solvency II standard formula. The SCR results of this model are then compared with the results of the Solvency II standard formula. SCR is determined by using Gaussian copulas between different segments and between premium and reserve risk by assuming the same correlation coefficient provided by Solvency II. Numerical results are also presented for small and medium-large companies for premium and reserve risk. They concluded that SCR for large companies are reduced with the internal model but not for small companies [26].

Christiansen and Niemeyer (2012) compare different mathematical interpretations for the SCR found in the literature and introduce a mathematical modeling framework that allows a mathematically rigorous comparison. They also show similarities, differences, and properties such as convergence of the different SCR interpretations and generalize the SCR definition to future points in time based on a generalization of the value at risk allowing for a sound definition of the Risk Margin [6].

Alm (2015) proposes a simulation model that is able to generate SCR for nonlife insurance risk that only needs assumptions about the distribution of the payment patterns and ultimate claim amounts using motor insurance data from a Swedish insurer. He finds more or less the same best estimates, durations, SCR, risk margins and technical provisions for the aggregate method as for the individual-line method. He concludes that the SCR is markedly affected by the correlation but it is even more affected by not being able to predict a trend and assuming lognormal amounts instead of normal does not change the values much. The SCR values generated by the simulation model with different distributional assumptions are then compared to Solvency II standard formula. The uncertainty in prediction of the trend in ultimate claim amounts is found to be affecting the SCR substantially [2].

Gisler (2009) compared Solvency II with Swiss Solvency Test by focusing on parameter estimation and presented new parameter estimators [14].

Hürlimann (2009) estimates premium and reserve risk volatilities and correlation coefficients at segment level and developes Solvency II economic capital formula applying VaR and conditional VaR (CVaR) risk measures under a lognormal distribution of portfolio combined ratio which is defined as the ratio of incurred claims inclusive "run-off" to the premium and reserve volume. He decomposes the portfolio combined ratio in a weighted sum of the premium and reserve risk ratios as suggested in QIS3 and proposes simple weighted estimators for all volatilities and correlation coefficients [16].

Bermudez, Ferri and Gullen (2011) analyze different correlation assumptions between segments using an internal model based on Monte Carlo simulation of the net underwriting result at a one-year horizon and Gaussian copulas for correlations, and then compare the numerical results with Solvency II using insurance sector data of Spain. They also examine the cases where the segments are independent, totally dependent (comonotonicity) and correlated by the Solvency II correlation matrix. They find that for independence the standard formula underestimates SCR, for Solvency II correlation matrix and comonotonicity the standard model overestimates the SCR except in cases involving Student's t-distribution margins with fewer than ten degrees of freedom and assuming heavier tails. For different copulas, the independence case is found to invariably provide the smallest value and the comonotonicity case is found to invariably lead to the highest SCR. The Solvency II correlation matrix assumption is found to provide an SCR that lies between the independence and comonotocinity cases [3].

Frosberg (2010) studies the diffences between SST and Solvency II and explore some standard copulas used for aggregating loss distributions per risk type. He opposes the standard practice in the insurance industry of using the Gaussian copula and claimed that this copula is not really suitable in some aspects. He also emphasizes that the choice of copula has a large impact on the resulting solvency ratio and there is often a problem with fitting real data to a given model. He also analyzes the diversification of risk between companies within an insurance group [12].

Nguyen and Molinari (2011) analyze the method of risk aggregation via the proposed application of correlations. They find that modeling dependencies with copulas would incur significant costs for smaller companies that might outbalance the resulting more precise picture of the risk situation of the insurer and propose introducing incentives for those companies who use copulas such as reduced solvency capital requirements compared to those who do not use it, to push the deployment of copulas in risk modeling in general [22].

1.2 Aim of the Study

The aim of the study is to assess the suitability of Solvency II requirements for an emerging market such as Turkish insurance sector. Solvency II came into force at the beginning of 2016 in EU after several years of preparation and studies on its effects on insurance companies. In this study we focus on non-life premium and reserve risks of the Turkish insurance market since the non-life segments has been the dominant part for decades in Turkey. Using the real data of the Turkish non-life insurance companies we design an internal model based on the aggregation of the premium and reserve risks using copulas and compare its results with the standard formula of Solvency II. Since Solvency II allows the companies to calculate their solvency capital requirement using either the standard formula predetermined in Solvency II or partial or full internal models determined by the companies themselves, we compare the results of both methods using real company data of three insurance companies of different sizes and the remaining ones in Turkish non-life insurance market.

Unlike Solvency II, in this study we propose to use a single combined loss ratio for premium and reserve risks together for two segments motor vehicle liability (MTPL) and other motor (motor) segments. Since over 40% of the premium production of the non-life insurance companies come from these two segments, we believe that these segments also make up the most part of the solvency capital requirement for the non-life premium and reserve risks.

Instead of using the linear correlation coefficients between segments predetermined in Solvency II, we use copulas for modelling the dependence between segments. The capital requirement results calculated by the standard formula and an internal model using real company data for Turkish insurance market are then compared with each other.

This thesis includes five more chapters. Chapter 2 briefly summarizes Solvency II regime, its modules and the SCR calculation. In Chapter 3, the standard formula set forth in Solvency II technical documents is explained in detail. In Chapter 4 copulas are briefly explained. Chapter 5 starts with the assumed model lying behind Solvency II and the proposed internal model which is based on using company specific parameters for the distribution and correlation of the segments using copulas. Then SCR is calculated for the selected companies based on real data using the standard formula and the internal model via simulations. Chapter 6 concludes the study by explaining the findings and propositions for the Turkish insurance market.

CHAPTER 2

SOLVENCY CAPITAL REQUIREMENTS

As in every business entity, insurance companies are also required to start with a certain amount of capital to cover their expenses and to pay for the claims related to the segments they write policies. This amount is calculated as the sum of a fixed amount of capital plus incremental amounts for the segments the insurance company is licensed. Every year based on the underwriting and financial performance of the insurance company, the public authority may require the insurance company to inject more capital to be solvent for paying the claims of insured people.

As of January 2016, EU started a new capital regime for insurance companies, Solvency II which is composed of three pillars; Financial Requirements (Pillar 1), Governance and Supervision (Pillar 2) and Reporting and Disclosure (Pillar 3). Solvency II is a comprehensive set of rules and regulations for insurers, including authorization, corporate governance, supervisory reporting, public disclosure, risk assessment and management, and also solvency and reserving.

The main objectives of Solvency II are to improve policy holder protection, set a modernised supervision structure, extend and deepen EU insurance market integration and enhance the competitiveness of insurance companies in EU at global level.

2.1 Brief overview of Solvency II

In Solvency II the balance sheet of an insurance company is simply described as in Figure 2.1 [17]. On the asset side of the balance sheet we see two components as the assets covering technical provisions and the available capital (or own funds) which are presumed to be the capital source of paying for claims. The liability side consists of technical provisions (or reserves) and the solvency capital requirement calculated using the methods set or allowed by the public authority.

In principle, Solvency II provides a range of methods to calculate the SCR which allows insurance companies to choose a method that is proportionate to the nature, scale and complexity of the risk that are measured. These methods are described as full internal model, standard formula and partial internal model, standard formula with company-specific parameters, standard formula and simplification. This means that subject to the approval of the authority companies are allowed to modify or revise the standard formula partially or fully to meet their requirements concerning the SCR calculation based on the evidence that their risk profile is different from the one predetermined in Solvency II. This modification could either be a totally new internal model that is believed to better reflect the risk profile of the company or a new parameter such as a standard deviation or correlation coefficient predetermined in Solvency II.

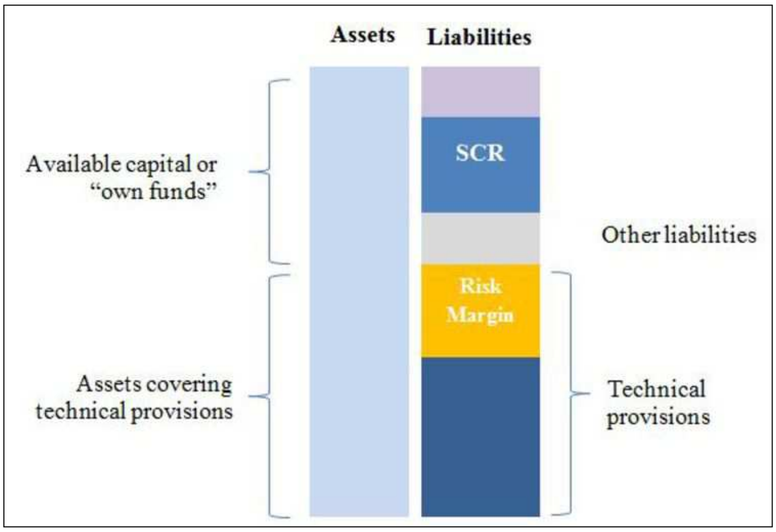


Figure 2.1: Simplified illustration of a Solvency II balance sheet[17]

2.2 Solvency Capital Requirement in Solvency II

The calculation of SCR according to the standard formula is divided into modules as shown in Figure 2.2 [9].

The SCR is determined as the sum of Basic Solvency Capital Requirement ($BSCR$), the capital requirement for operational risk (SCR_{op}) and adjustment for the risk absorbing effect of technical provisions and deferred taxes (Adj) given as

$$SCR = BSCR + Adj + SCR_{OP} \tag{2.1}$$

where $BSCR$ is the solvency capital requirement before any adjustments, combining the capital requirements for six major risk categories which are market risk (SCR_{mkt}), counterparty default risk (SCR_{def}), life underwriting risk (SCR_{life}), non-life underwriting risk (SCR_{nl}), health underwriting risk (SCR_{health}) and intangible assets risk ($SCR_{intangible}$).

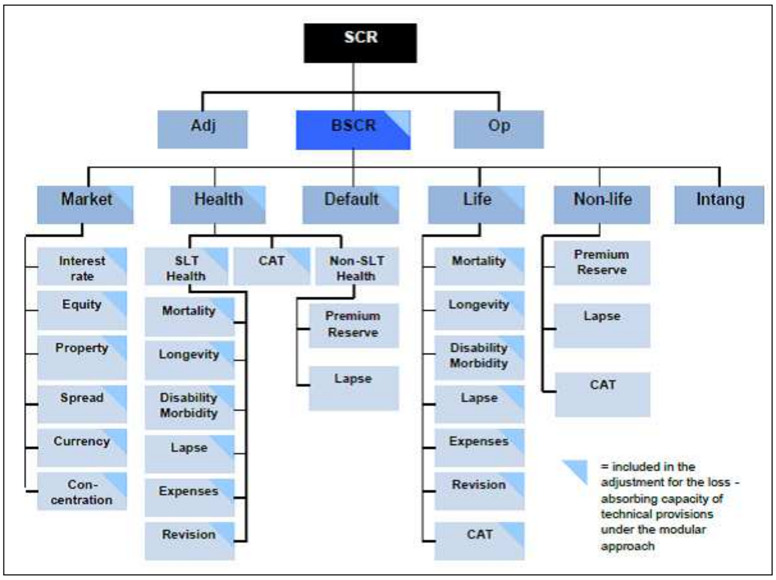


Figure 2.2: SCR modules in Solvency II [9]

$BSCR$ takes into account the correlation among these six major categories and the aggregate risk associated to them. The correlation pronounced by the solvency regime is set either 0 or 0.25 depending on the degree of association. Excluding the pair of segments (non-life, health) and (non-life, life) all other correlations are set to be 0.25. These values in terms of branches are shown in Table 2.1. The capital requirements for the five categories are aggregated using the correlation matrix in Table 2.1 and then capital requirement for intangible asset risk is added as follows:

$$BSCR = \sqrt{\sum_{ij} \rho_{ij}\ SCR_i\ SCR_j} + SCR_{IN} \tag{2.2}$$

The factor ρ_{ij} denotes the item set out in row i and column j of the following Table 2.1.

Table 2.1: The correlation matrix of six major risk categories in Solvency II [9]

	Market	Default	Life	Health	Non-life
Market	1				
Default	0.25	1			
Life	0.25	0.25	1		
Health	0.25	0.25	0.25	1	
Non-life	0.25	0.5	0	0	1

2.3 Capital requirement for insurance companies in Turkey

The capital requirement regime for insurance companies in Turkey also requires these companies to hold enough capital to protect the policy holders and maintain the financial stability of the insurance sector. The capital requirement valuation takes into account different risks such as asset, reinsurance, excessive premium increase, outstanding claims provision, underwriting and exchange rate risks. The calculation is simply based on multiplying the calculated risk volumes by predetermined risk coefficients as set out in "Regulation on Measurement and Assessment of Capital Requirements of Insurance and Reinsurance Companies and Pension Companies" published by Undersecretariat of Treasury. In this sense it resembles the risk-based capital regime in the United States as summarized in NAIC web site and documents.

Related to the on-going membership discussions of Turkey with EU and the dominant existence of foreign investors in Turkish insurance market, Solvency II is being discussed in Turkish insurance market for a long time. In 2011, similar with EU, Quantitative Impact Studies (QIS)[4] has been implemented to Turkish insurance companies. However, in recent years there had been major changes in Turkish insurance market threatening the insurance companies in several ways and enforcing them to increase their capital such as the factors forcing the insurance companies to increase their reserves dramatically. At this point one major concern about transitioning of the Turkish insurance sector to Solvency II is the expected upward shift in SCR of the insurance companies. The insurance companies which are owned by the insurance groups in EU are required to comply indirectly with the Solvency II within the context of group supervision.

The recent financial crisis and the dramatic decline in the interest rates markedly reduced the financial investment income of the insurance companies pushing them to focus on the technical profitability more than ever. The resulting losses in the financial tables of these companies have also led these companies experience higher capital requirements.

The discussion concerning the capital requirement regime relates also to the pricing of insurance products from the insurance companies' perspective as it affects the number of the companies operating in the market. According to some views, the capital regime should be serving to penalize the underpricing of the insurance products as this negatively impacts the sustainability of insurance business in the medium to long run.

Since the Turkish insurance market is mainly relying on the non-life part for a long time and the non-life part is being dominated by the MTPL and motor segments, we studied the effects of Solvency II requirements on these two non-life segments. Recent developments concerning the rising loss ratios and financial losses of the insurance companies and the increasing premiums as a result necessitate the assessment of Solvency II requirements for the Turkish insurance market.

CHAPTER 3

NON-LIFE PREMIUM AND RESERVE RISKS

The non-life underwriting risk in Solvency II is defined as the risks arising from non-life insurance obligations including the risk resulting from uncertainty in renewal or termination of the policies. The non-life underwriting risk module covers both existing insurance and reinsurance obligations and the new business expected to be written over the following 12 months [9].

The SCR for non-life underwriting risk, SCR_{nl} is composed of sub-modules which are used to calculate the solvency capital requirement for the non-life premium and reserve risks (NL_{pr}), the non-life catastrophe risk (NL_{cat}) and the non-life lapse risk (NL_{lapse}). SCR_{nl} is derived by aggregating the capital requirements for the three risks mentioned above. Similar to $BSCR$, the SCR_{nl} is calculated by

$$SCR_{nl} = \sqrt{\sum \rho_{rc} NL_r NL_c} \tag{3.1}$$

The correlation coefficients between the sub-risks of the non-life underwriting risk are predetermined in Solvency II technical documents given in Table 3.1. We can see that the highest correlation appears between NL_{pr} and NL_{cat}.

Table 3.1: The correlation matrix between the non-life underwriting risks [9]

	NL_{pr}	NL_{lapse}	NL_{cat}
NL_{pr}	1		
NL_{lapse}	0	1	
NL_{cat}	0.25	0	1

In this study we focus on the non-life premium and reserve risk, NL_{pr} as this constitutes the larger part of the non-life underwriting risk as a whole. We first explain how NL_{pr} is determined in Solvency II standard formula and then propose an internal model based on company-specific correlation coefficients and copulas for the dependence structure of the MTPL and motor segments.

3.1 Premium and reserve risks

The non-life premium and reserve risks only consider losses that occur regularly. Extreme risks are taken into account in the catastrophe risk category. Premium risk in Solvency II is defined as the risk that the claims including both amounts paid and claim provisions made being higher than the premiums received. It is also defined as the risk that premium provisions turn out to be insufficient for the claims to be paid during the next 12 months. It also includes the risk resulting from the volatility of expense payments. Expense risk is implicitly included as part of the premium risk. Reserve risk on the other hand results from fluctuations in the timing and amount of claim settlements. It mainly stems from two sources, the mis-estimation of claims provisions amount and the difference between the provisions and future claims payouts [9].

Solvency II states that for both premium and reserve risk analysis (based on premium risk type methods) models based on the assumption of a normal and lognormal probability distributions were used and based on various goodness-of-fit diagnostics and PP-plots of the underlying risks, the choice of distribution fitting between the normal and lognormal probability distributions were inconclusive. For reserve risk in addition to the premium risk type method which is a model based on financial year end data, it is said that a model based on runoff triangle accident year data was also tested but not used for calibration of the model parameters [10].

3.2 Valuation of SCR by standard formula

For the calculation of non-life premium and reserve risk, the standard formula requires the following variables to be determined for each segment, s;

- PCO_s: the best estimate for claims outstanding net of reinsurance
- P_s: estimate of the premiums to be earned during the following 12 months
- $P_{(last,s)}$: the premiums earned during the last 12 months
- $FP_{(existing,s)}$: the expected present value of premiums to be earned after the following 12 months for existing contracts
- $FP_{(future,s)}$: the expected present value of premiums to be earned for contracts where the initial recognition date falls in the following 12 months but excluding the premiums to be earned during the 12 months after the valuation date

The premium and reserve risk capital requirement, NL_{pr} is calculated as the product of the volume measure, V and the combined standard deviation for non-life premium and reserve risk, σ given as:

$$NL_{pr} = 3\sigma V \tag{3.2}$$

When describing the required capital, Solvency II considers one-year result and expects the company to hold enough capital to compensate for one-year random loss with a confidence level of α = 99.5% in value-at-risk (VaR) calculation.

In earlier versions of Solvency II documents [4] the equation for NL_{pr} was different than in equation (3.2). In the latest version of the document [9] the equation has been simplified by using 3σ instead of a function of σ by setting a risk capital charge consistent with the $VaR_{99.5\%}$ and assuming a lognormal distribution of the non-life premium and reserve risks as in equation (3.3).

$$NL_{pr} = \left(\frac{\exp(\Phi^{-1}(0.995)\sqrt{\log(\varphi^2+1)})}{\sqrt{\varphi^2+1}} - 1\right)V \tag{3.3}$$

Here NL_{pr} corresponds to the VaR of the basic own funds subject to a confidence level of 99.5% over a one-year period, V denotes the volume measure and σ is the combined standard deviation for non-life insurance portfolio. It can be easily seen that the coefficient of V in equation (3.3) is simplified to 3σ.

VaR is an estimate of the maximum loss over a target horizon within a given confidence interval. The estimation relies on two important parameters; horizon and confidence level. Solvency II requires the calculation of VaR over one-year horizon with a 99.5% confidence level. The result shows us the value that the loss at the end of one year would be smaller than 99.5% of the time. VaR is a popular method for calculating risks and widely used in finance. Basel II also requires banks to calculate their capital requirement using VaR.

There are three approaches for calculating VaR, variance covariance, historical simulation and Monte Carlo simulation. In variance covariance method, the variance and covariance is estimated using historical data and normal distribution of the risks are assumed. In historical simulation method, VaR is estimated by creating a hypothetical time series of the risk factors and computing the changes from the actual results. In Monte Carlo simulation, the probability distributions of the risk factors are used to get different values for the risk factors in each simulation run and the simulation results are ranked from highest to lowest to get the VaR estimate with a specified confidence level. The detailed information can be found in Jorion(2006) [18].

The volume measure, V, and the combined standard deviation, σ, are calculated in two steps: First, the standard deviation and volume measure for both premium and reserve risks are calculated for each segment. Second, these are aggregated to calculate the volume measure and the combined standard deviation for the whole portfolio. The same segmentation is also used for technical provisions. The only difference compared to technical provisions is the inclusion of proportional reinsurance based on the assumption that its risk profile is similar.

Table 3.2: Grouping of insurance segments in Solvency II [9]

	Segment
1	Motor vehicle liability insurance and proportional reinsurance (MTPL)
2	Other motor insurance and proportional reinsurance (motor)
3	Marine, aviation and transport insurance and proportional reinsurance
4	Fire and other damage to property insurance and proportional reinsurance
5	General liability insurance and proportional reinsurance
6	Credit and suretyship insurance and proportional reinsurance
7	Legal expenses insurance and proportional reinsurance
8	Assistance and its proportional reinsurance
9	Miscellaneous financial loss insurance and proportional reinsurance
10	Non-proportional casualty reinsurance
11	Non-proportional marine, aviation and transport reinsurance
12	Non-proportional property reinsurance

The volume measure for premium risk of each segment, $V_{(prem,s)}$ is

$$V_{(prem,s)} = max\{P_s, P_{(last,s)}\} + FP_{(existing,s)} + FP_{(future,s)} \tag{3.4}$$

The standard deviation for premium risk (gross of reinsurance) of each segment, $\sigma_{(prem,s)}$ is calculated as the product of the gross standard deviation and the adjustment factor for non-proportional reinsurance for each line of business, NP_{lob}, which allows undertakings to take into account the risk-mitigating effect of excess of loss reinsurance.

The standard deviation and the adjustment factors are predetermined in Solvency II as in Table 3.3. NP_{lob} is set for segments 1, 4 and 5 to 80% and for others to 100%.

Table 3.3: σ for premium and reserve risks in Solvency II [9]

Segments	$\sigma_{(prem,s)}$	$\sigma_{(res,s)}$
1	$10\%\ NP_{lob}$	9%
2	$8\%\ NP_{lob}$	8%
3	$15\%\ NP_{lob}$	11%
4	$8\%\ NP_{lob}$	10%
5	$14\%\ NP_{lob}$	11%
6	$12\%\ NP_{lob}$	19%
7	$7\%\ NP_{lob}$	12%
8	$9\%\ NP_{lob}$	20%
9	$13\%\ NP_{lob}$	20%
10	17%	20%
11	17%	20%
12	17%	20%

The volume measure for reserve risk of each segment, $V_{(res,s)}$ is assumed to be equal to PCO_s,

$$V_{(res,s)} = PCO_s \tag{3.5}$$

The standard deviation for reserve risk (net of reinsurance) of each segment is also predetermined in Solvency II. These are listed in Table 3.3. It can be observed that low variations are assumed for segment 1 (MTPL) and segment 2 (motor). The highest variations are assumed for segments 8 to 12.

In order to calculate the aggregated standard deviation for premium and reserve risks, σ for each segment, a pooled risk is used with the weights as volume measures for premium and reserve risks as follows:

$$\sigma_s = \frac{\sqrt{(\sigma_{(prem,s)}V_{(prem,s)})^2 + \sigma_{(prem,s)}\sigma_{(res,s)}V_{(prem,s)}V_{(res,s)} + (\sigma_{(res,s)}V_{(res,s)})^2}}{V_{(prem,s)} + V_{(res,s)}} \tag{3.6}$$

The aggregated volume measure for each segment, V_s, then becomes

$$V_s = (V_{(prem,s)} + V_{(res,s)})(0.75 + 0.25DIV_s) \tag{3.7}$$

where DIV_s stands for the diversification benefit and is calculated using equation (3.8). DIV_s is found as

$$DIV_s = \frac{\sum_j (V_{(prem,j,s)} + V_{(res,j,s)})^2}{(\sum_j (V_{(prem,j,s)} + V_{(res,j,s)}))^2} \tag{3.8}$$

The index j denotes the geographical segments as listed in Solvency II technical documents [9]. $V_{(prem,j,s)}$ and $V_{(res,j,s)}$ denote the volume measures for risks in the geographical segment j.

Since we aggregated volume measures and standard deviations for premium and reserve risks of all segments, we now need to calculate the aggregated standard deviation of the portfolio as a whole using the equation (3.9)

$$\sigma_{nl} = \frac{1}{V_{nl}} \sqrt{\sum_{s,t} \rho_{s,t}\sigma_s V_s \sigma_t V_t} \tag{3.9}$$

where s and t denote the segments, $\rho_{s,t}$ denotes the entries of correlation matrix in Table 3.4, V_s and V_t denote the volume measures for premium and reserve risk of segments, s and t, and σ_s and σ_t denote the standard deviations for non-life premium and reserve risk of segments, s and t.

Table 3.4: The correlation matrix, $\rho_{s,t}$ for the insurance segments in Solvency II [9]

	1	2	3	4	5	6	7	8	9	10	11	12
1	1											
2	0.5	1										
3	0.5	0.25	1									
4	0.25	0.25	0.25	1								
5	0.5	0.25	0.25	0.25	1							
6	0.25	0.25	0.25	0.25	0.5	1						
7	0.5	0.5	0.25	0.25	0.5	0.5	1					
8	0.25	0.5	0.5	0.5	0.25	0.25	0.25	1				
9	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	1			
10	0.25	0.25	0.25	0.25	0.5	0.5	0.5	0.25	0.25	1		
11	0.25	0.25	0.5	0.5	0.25	0.25	0.25	0.25	0.5	0.25	1	
12	0.25	0.25	0.25	0.5	0.25	0.25	0.25	0.5	0.25	0.25	0.25	1

As it can be seen from Table 3.4, the highest and lowest correlation coefficients are set to 0.50 and 0.25 respectively. Around 41% of the correlation coefficients ar set as 0.5 and show stronger relation than the rest.

CHAPTER 4

MEASURING DEPENDENCE BY COPULAS

In literature there has been numerous studies focusing on the definition of the dependence or association between random variables. Here we are interested in the joint distribution of different risks arising from premium and reserve of the segments MTPL and motor insurance. As suggested in Cherubini (2012) we have mainly two opposing alternatives for assessing the joint distribution of the aggregated risks; namely bottom-up or top-down approaches. The copula functions are used as the main tool for the bottom-up approach.

We first start with the basic definitions of association and dependence between two random variables and then give a brief overview of the copulas.

4.1 Types of association between variables

In literature there are various definitions for the measure of dependence and association of two random variables. The most known type of correlation is the Pearson correlation coefficient. The correlation measures the strength of linear dependence between two random variables. Pearson correlation coefficient is defined as the ratio of the covariance of the two variables to the product of their respective standard deviations.

$$\rho(X,Y) = \frac{Cov(X,Y)}{\sqrt{Var(X)Var(Y)}}$$

The correlation coefficient varies in between +1 and -1. If the correlation coefficient lies around +1 and -1, then it is said to be a perfect association between the two variables. If the correlation coefficient goes towards 0, the relationship between the two variables becomes weaker.

As noted in Nelsen (2006) the term measure of association is used for measures such as Kendall's τ and Spearman's ρ which are measuring the concordance between two random variables. Two random variables are said to be concordant if large values of one random variable tend to be associated with large values of the other random variable and vice versa.

Spearman's ρ is also known as Spearman's rank-order correlation coefficient and is a rank-based version of the Pearson's correlation coefficient. It can take values from +1 to -1. A value of +1 or -1 indicates a perfect association of ranks, respectively positive and negative whereas a value of zero indicates no association between ranks. Values closer to zero indicate weaker association between the ranks. For the sample data it is calculated as

$$\rho = \frac{\sum_{i=1}^{n}(rank(x_i)-rank(x))(rank(y_i)-rank(y))}{\sqrt{\sum_{i=1}^{n}(rank(x_i)-rank(x))^2 \sum_{i=1}^{n}(rank(y_i)-rank(y))^2}}$$

where $rank(x_i)$ and $rank(y_i)$ are the ranks of the observation in the sample.

For a sample of n values for two random variables, there would be $\binom{n}{2}$ distinct pairs of observations. Then each pair would be either concordant or discordant. If the number of concordant pairs is denoted as c and the number of discordant pairs as d, then Kendall's τ for the sample is defined as

$$\tau = \frac{c-d}{c+d}$$

The Kendall's τ for the population is defined as the probability of concordance minus the probability of discordance [21].

The correlation coefficients provide an easy way of understanding the relationship between the variables as they process all the information and put it in a single value. However, one of the major criticisms about the correlation coefficients is using a single value for all the range of the variables although these two variables behave differently in the lower or upper tails of their distributions. Unlike the correlation coefficients, copulas provide more information about the dependency between the variables varying across their range of distribution. Kole (2006) states that the Gaussian copula underestimates the probability of joint extreme downward movements in comparison with the Student's t copula and the Gumbel copula overestimates this risk.[19]

4.2 Copulas

As stated by Cherubini (2012) copulas are simply defined as the tool to separate the specification of marginal distributions from the dependence structure. By the probability integral transformation theorem we can define the cumulative distribution function, F_X of a continuous random variable, X as U which has uniform distribution in the unit interval (0,1). By the same approach we can define F_Y as V which is again uniformly distributed in the unit interval (0,1).

Then, the joint cumulative distribution function of X and Y can be defined as:

$$\begin{gathered} F(X,Y) = F(F_X^{-1}(U), F_Y^{-1}(V)) = C(U,V) \\ F(x,y) = F(F_X^{-1}(u), F_Y^{-1}(v)) = C(u,v) \end{gathered} \tag{4.1}$$

where $u = F_X(x)$ and $v = F_Y(y)$. The copula function $C(u, v)$ links the uniform variables to the joint distribution of X and Y. This approach for getting the copula function for two random variables may easily be extended to n-dimensional case.

According to Sklar's theorem, for the 2-dimensional case the following requirements should be satisfied by the copula which is defined as a function [5]

$C : [0, 1]^2 \rightarrow [0, 1]$

1. Grounded: $C(u, 1) = u, C(1, v) = v$
2. Uniform marginals: $C(0, v) = C(u, 0) = 0$
3. Non-negativity: $C(u_1, v_1) - C(u_1, v_2) - C(u_2, v_1) + C(u_2, v_2) \geq 0$

There are various copulas differing from each other by the dependence structure perceived in their copula function. These are mainly classified into two groups; Archimedean copulas and elliptical copulas. Frank, Clayton and Gumbel copulas are the most known Archimedean copulas. The elliptical copulas are mainly grouped into two: Gaussian and t-copulas.

The dependency structure of the copulas is one of the most important aspects when choosing the copula to use. Copulas differ from each other in the dependency perceived in the joint distribution between the marginal distributions especially in the lower and upper tails. For the selection of copulas best fitting the data and their marginal distributions, several tests such as the ones based on the probability integral transform, multivariate smoothing procedures and information criteria such as Akaike information criterion (AIC), Schwarz information criterion (SIC) also known as Bayesian information criterion (BIC) and Hannan-Quinn information criterion (HQIC) are used. The detailed information can be found in [22, 7, 13, 24, 20].

The information criteria statistics for AIC, SIC and HQIC are computed as follows:

$$AIC = (\tfrac{2n}{n-k-1})k - lnL_{max}^2$$

$$SIC = lnnk - lnL_{max}^2$$

$$HQIC = ln(lnk)^{2k} - lnL_{max}^2$$

where n is the number of observations; k is the number of parameters and L_{max} is the maximized value of the log-Likelihood for the estimated model. The copula with the lowest information criterion is selected as the best fitting copula.

Copulas show different characteristics in terms of dependence structure perceived in them. For instance, Gumbel copulas have more dependency in the upper tail but show independency in the lower tail. Thus they seem to offer a good alternative for modelling extreme events in insurance. Cook-Johnson copulas on the other hand show dependency only in the lower tail. Frank copulas resemble Gaussian copulas in terms of tail dependence but at a lower magnitude. Gaussian copulas since showing low dependence in both lower and upper tails are said not to provide a proper basis for modeling insurance risks. In contrast to Gaussian copulas, student-t (T) copulas foresee a higher dependence in the tails which increases with lower degrees of freedom and larger correlation [22, 5].

To capture the dependency structure of the segments, MTPL and motor, the elliptical copulas, Gaussian and T and the Archimedean copulas, Frank, Gumbel and Clayton are applied to the data.

4.2.1 Elliptical copulas

For an n-dimension multivariate Gaussian distribution of n standardized variables whose marginal distributions are also standard normal, we can define the joint normal distribution as:

$$C(u_1, ..., u_n) = \Phi_n(\Phi^{-1}(u_1), ..., \Phi^{-1}(u_n)) \tag{4.2}$$

where Φ denotes the cdf of Gaussian distribution. This function is the Gaussian copula and is the most widely used copula for the financial risks. It may be used even though the marginal distributions are not Gaussian. The dependence structure for this copula is radially symmetric and ignores any kind of tail dependence. Therefore the Gaussian copula is generally used for the aggregating the risks which are symmetric and light-tailed. Gaussian (normal) copulas lead to multivariate normal distribution with standard normal distributions as marginal distributions.

In order to take into account the fat tails, we have to substitute the normal distribution with the Student's t distribution whose degrees of freedom parameter increases as tails get fatter. So as in Gaussian copula function, the joint Student's t distribution may be defined as:

$$C(u_1, ..., u_n) = T_n(T_v^{-1}(u_1), ..., T_v^{-1}(u_n)) \tag{4.3}$$

The T copula is also symmetric as the Gaussian copula but has tail dependence. This type of copula fit well for the risks that tend to show extreme upward or downward movements.

4.2.2 Archimedean Copulas

Other than the elliptical copulas we have another type of copulas, namely Archimedean copulas. The definition of the copula function for Archimedean copulas is not straightforward as in the elliptical copulas where the probability integral transform is used conveniently by the use of multivariate distributions. Hofert (2007) addresses the challenge of efficiently sampling Archimedean copulas and with specific focus on large dimensions where methods involving generator derivatives are not applicable proposes direct sampling algorithms for some Archimedean families [15].

In defining the Archimedean copulas we use the generator function, $\phi(x)$ which is different for each copula function produced. The class of Archimedean copulas can be defined as follows [5]:

$$C(u_1, ..., u_n) = \phi^{-1}(\phi(u_1), ..., \phi(u_n)) \tag{4.4}$$

The three most known Archimedean copulas with their generator functions are defined as follows:

The Clayton copula has the generator function, $\phi(t) = \frac{1}{\theta}(t^{-\theta} - 1)$ for $0 < \theta < \infty$ and its copula function is defined as follows:

$$C(u_1, ..., u_n) = max((u_1^{-\theta} + ... + u_n^{-\theta} + n - 1), 0) \tag{4.5}$$

The Clayton copula is asymmetric and shows lower tail dependence.

The Frank copula with the generator function, $\phi(t) = -ln(\frac{exp(-\theta t)-1}{exp(-t)-1})$ for $\theta \neq 0$ is defined as follows:

$$C(u_1, ..., u_n) = -ln(1 + \frac{exp(-\theta u_1)...exp(-\theta u_n) - 1}{exp(-\theta) - 1}) \tag{4.6}$$

The Gumbel copula is generated by $\phi(t) = (-lnt)^\theta$ for $1 < \theta < \infty$ and is defined as follows:

$$C(u_1, ..., u_n) = exp(-((-lnu_1)^\theta) + ... + (-lnu_n)^\theta))^{\frac{1}{\theta}}) \tag{4.7}$$

The Gumbel copula is limited to positive dependence only and has upper tail dependence.

CHAPTER 5

CASE STUDY: MTPL AND MOTOR INSURANCE IN TURKEY

In this part of the study we investigate the suitability of Solvency II standard formula for the insurance sector of an emerging market such as Turkey. Since the Solvency II parameters for SCR calculation are determined using the data of the EU insurance market which is more developed compared to emerging markets, we analyze whether the correlation coeeficients predetermined in Solvency II for the aggregation of the risks are also convenient for the Turkish insurance market.

The analysis is performed in two parts based on the data collected for certain business lines in the market for prescribed companies. The first part determines capital requirements based on the Solvency II framework, taking into account the risk indicators such as VaR. The second part considers the quantification of SCR based on the internal model which incorporates the distributional approach. Additionally, along with the assumption on different statistical distributions, the correlation structure is investigated through different copula families. The findings are compared to determine the state of an emerging insurance market at certain business lines in Solvency II regime.

5.1 Turkish non-life insurance market

The insurance market in Turkey has been dominated by non-life segments for over a decade in contrast to the developed insurance markets such as US, Japan and some of the European countries. This is the usual case for most of the emerging market economies across all the world. One of the main reasons that is leading the Turkish insurance market to such a composition is the introduction of defined-contribution pension products offered by the pension companies since 2003. This has led to a tremendous amount of shift from the savings insurance products to pension products by also the incentives offered by the regulation.

Figure 5.1 shows the decomposition of the gross written premiums by life and non-life segments in Turkey for the period 2009-20015 using the public data provided by Turkish Insurance Association.

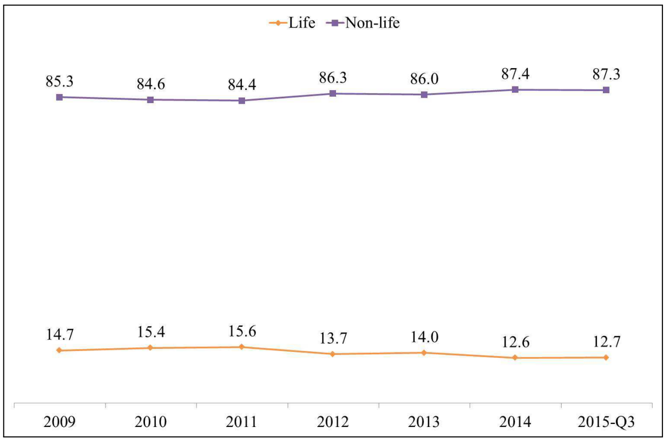
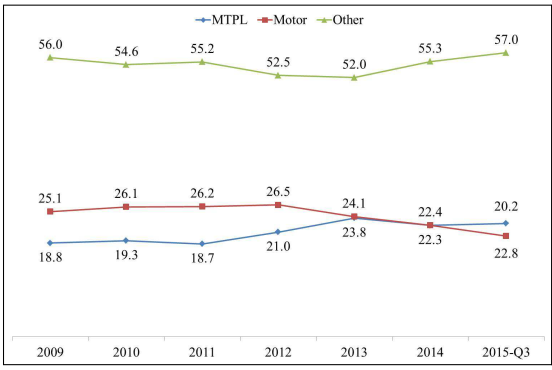
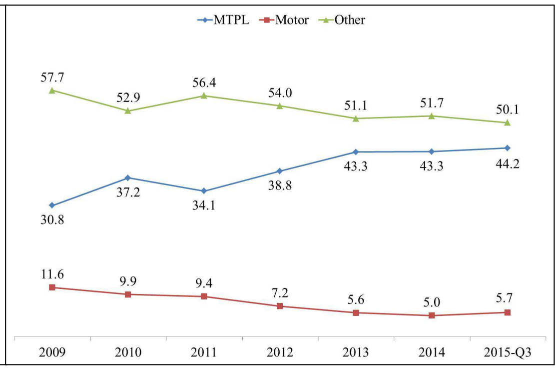


Figure 5.1: Share of life and non-life segments in Turkish insurance market (%)

In this sdtudy we focus on the non-life part and specifically motor vehicle liability (MTPL) and motor insurance segments as they together constitute 43% of the non-life premium production and 49.9% of outstanding claims provision in Turkish non-life insurance market. This pattern is shown in Figure 5.2a and Figure 5.2b. MTPL, which is a compulsory insurance segment in Turkey as in most of the other countries, and motor insurance are non-life insurance segments. MTPL serves for the compensation of the losses and damages to third parties caused by the policy holder while in traffic, whereas motor insurance provides cover for the damages of the policy holder's own car.



(a) Gross written premiums



(b) Outstanding claims provision including IBNR

Figure 5.2: Share of MTPL and motor segments in non-life (%)

Since these two segments constitute a considerable amount of the premiums of the insurance companies, there has been a fierce competition on price for these segments in Turkish insurance market. This can be easily seen from Figure 5.2a and Figure 5.2b as the premiums's share for MTPL is decreasing beginning in 2012 and its share in reserves is increasing for the same period. Since this was not sustainable from the actuarial point of view, the premiums for MTPL insurance products nearly doubled on average at the beginning of 2016.

The judicial decisions taken in 2012 by the Supreme Court led to the reopening of the old cases and opening the new ones. This resulted a huge rise in outstanding claims provision beginning by 2012 since new claims related to old cases are delivered to the insurance companies and also an upward shift in IBNR due to worsening loss ratios as a result of this court practice. For this reason, a high increase in premiums is experienced in 2016 leading also to a shift in premium risk.

5.2 Dependency modeling by copulas

A partial or full internal model in order to calculate the solvency capital requirement as stated in Solvency II requires the approval of public authority. To make the model design simpler it is better to make certain assumptions beforehand. First, we assume that the insurance company operates in one region meaning there is no diversification benefit. Second, there is no non-proportional reinsurance involved meaning all premiums and reserves are net of non-proportional reinsurance.

Our model is based on the underlying model in Solvency II which decomposes the risks into current and previous years' risks as stated in Gisler (2009) and Hürlimann (2009). The premium risk relates to current year's risk and the reserve risk relates to previous years' risk [14] [16] [8].

We decompose the non-life premium and reserve risk into premium (current) and reserve (past) risks as Hürlimann (2009) and Gisler (2009) suggest. Therefore, we define premium risk as the risk of being paid less premiums than claims incur for the next 12 months, and reserve risk as the risk of keeping less amount of reserves than the outstanding claims related to the policies already sold.

In Solvency II, the loss ratio related to premium risk is simply defined as the ratio of claims payments, C, plus expenses, E, over premiums, P. The reserve risk is also defined as the ratio of end of year reserves, R_1, to the beginning of year reserves, R_0, as in equations (5.1) and (5.2):

$$X_{prem} = \frac{C+E}{P} \tag{5.1}$$

$$X_{res} = \frac{R_1}{R_0} \tag{5.2}$$

The premium and reserve risks are then aggregated using the weighted volume measures of premium and reserves as

$$X_{prem+res} = \frac{PX_{prem} + R_0 X_{res}}{P + R_0} \tag{5.3}$$

Using the same analogy and the actuarial equivalence principle, the underwriting result may be defined as the equivalence of the inflows; sum of reserves at the beginning of year and premiums, to the outflows; sum of claims payments, expenses and reserves at the end of year. The outstanding claims reserve here includes claims incurred but not reported (IBNR). From a balance sheet point of view, the company's assets are the best estimate provisions for both reported and incurred but not reported claims, R_0 and premiums, P, for new business, its liabilities on the other hand are claim payments, C, expenses, E, and again the best estimate provisions for the outstanding claims, R_1.

$$R_0 + P = C + E + R_1 \tag{5.4}$$

Normally, this equivalence in equation (5.4) is expected to hold, assuming there are no shifts in the calculation of reserves such as judicial decisions concerning the coverage of policies as happened in Turkey in 2012. This illustrates that the insurance company estimates the expected value of the claims to be paid and the expenses, and requires this much amount of premium from the policy holders. This ratio, X is calculated in equation (5.5) using the framework used in Hürlimann (2009) and assumed to be equal to $X_{prem+res}$ as described in Solvency II.

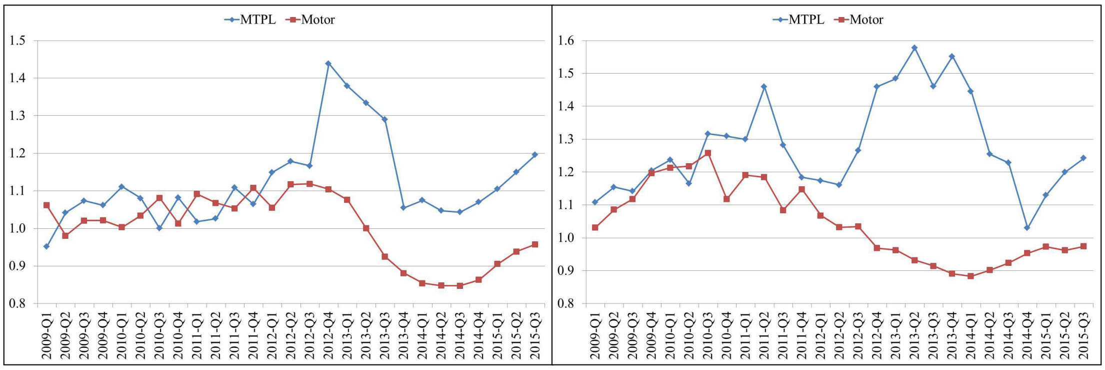
By actuarial equivalence principle, the expected value of this combined ratio for both premium and reserve risks, $E(X)$, should be 1. The deviation of this ratio from 1 results in a profit or loss for the segment which is also reflected in the capital accounts in the balance sheet of the insurance company. The risk for the insurance company can be defined as the misestimation of the premiums and reserves and thus making a loss for the segment.

$$X = \frac{C + E + R_1}{R_0 + P} \tag{5.5}$$

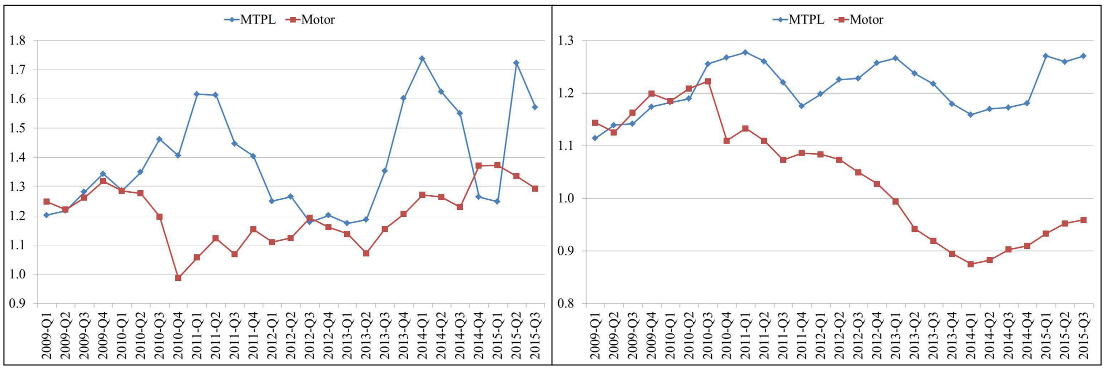
Therefore, this combined loss ratio, X, over the years and for each company can be taken as a random variable to determine the financial stability and solvency capital requirement of the insurance companies.

5.2.1 Data Description

Using quarterly data for the period 2009Q1-2015Q3, historical values of the combined loss ratios defined by equation (5.5) are calculated annually for the selected companies categorized as Large, Medium and Small sized and the remaining non-life insurance companies denoted as "Others". The historical values of the ratios, X, for MTPL and motor segments for Large, Medium, Small and Others are shown in Figure 5.3.

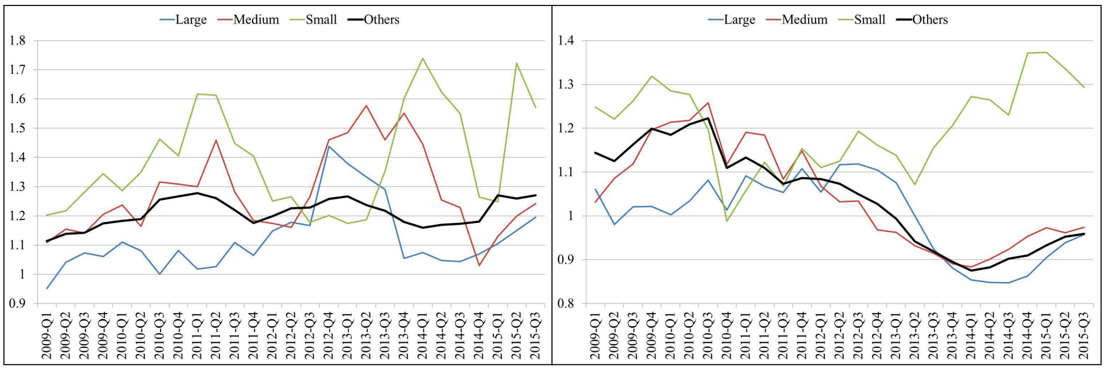


(a) Large (b) Medium



(c) Small (d) Others

Figure 5.3: Historical values of ratios, X, for MTPL and motor segments between 2009 and 2015



(a) MTPL (b) Motor

Figure 5.4: Historical values of X_{mtpl} and X_{motor} for selected companies

The premium and reserve data for the non-life insurance companies are taken from the publicly disclosed data set of Turkish Insurance Association and the name of the companies are concealed for confidentiality reasons. The analyses are done using statistical software R, Model Risk and Matlab. In Figure 5.3 the blue and red lines correspond to the historical values of the ratios, X_{mtpl} and X_{motor}, respectively for Large, Medium, Small and Others. Because of the reasons explained in previous section, after 2012 we see an upward shift in X_{mtpl} for all the companies. However, for X_{motor} there has been a downward trend from 2011 to 2014 and then a slight increase till 2015. This trend may also be seen in Figure 5.4 where the historical values of the ratios are depicted for MTPL and motor segments. In contrast to the general trend in motor segment there has been a significant upward shift for Small. Surprisingly, the historical values of X for MTPL segment seem to be rather stable for Other and fluctuates between 1.1 and 1.3 in the whole period.

5.2.2 Determination of statistical distributions

Solvency II assumes that the ratios X_{prem}, X_{res} and thus X are lognormally distributed. To verify whether this assumption is also valid for the Turkish insurance market, we check the statistical distributions of the ratios.

Table 5.1 illustrates the descriptive statistics and the normality test for the ratios, X_{mtpl} and X_{motor} determined for each company denoted by the indices l, m, s and o representing Large, Medium, Small and Others respectively. It can be seen that for the ratios except $X_{mtpl,l}$, $X_{motor,l}$ and $X_{mtpl,s}$ the normality assumption can not be rejected (i.e. p-value ≤ 0.05).

The ratios with minimum values greater than 1 indicates that the company has been experiencing losses for the relevant segment for the period 2009-2015. For MTPL segment all the companies except Large have minimum values greater than 1 and for motor segment all the companies experience minimum values of loss ratios less than 1 meaning that motor segment underwriting results are better than MTPL segment. The maximum value of the ratios for both MTPL and motor belongs to Small. The highest standard deviation for MTPL is 0.18 and for motor is 0.12 belonging to Small and Medium companies respectively.

Table 5.1: Descriptive statistics for X

	Min	Max	Mean	Std. dev	Skewness	Kurtosis	p-value
$X_{mtpl,l}$	0.91	1.44	1.12	0.12	1.35	4.45	0.002*
$X_{motor,l}$	0.85	1.12	1.00	0.09	-0.48	1.94	0.03*
$X_{mtpl,m}$	1.03	1.58	1.28	0.15	0.6	2.43	0.07
$X_{motor,m}$	0.88	1.26	1.05	0.12	0.31	1.78	0.08
$X_{mtpl,s}$	1.17	1.74	1.39	0.18	0.53	1.96	0.02*
$X_{motor,s}$	0.99	1.37	1.2	0.1	-0.19	2.44	0.87
$X_{mtpl,o}$	1.11	1.28	1.21	0.05	-0.19	1.87	0.07
$X_{motor,o}$	0.87	1.22	1.04	0.11	-0.03	1.62	0.07

Additionally, the histogram, CDF, Q-Q and P-P plots for each of the ratios are analyzed. Figure 5.5 shows that the distribution of $X_{mtpl,l}$ is right skewed and having a heavy tail.

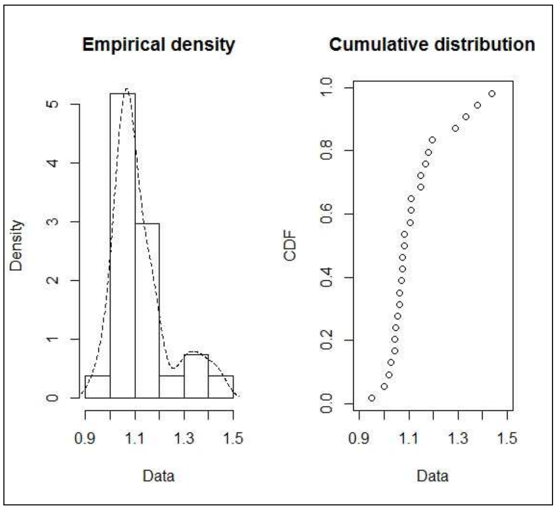


Figure 5.5: Empirical density and CDF of $X_{mtpl,l}$

A skewness-kurtosis plot in Figure 5.6 for the empirical distribution of $X_{mtpl,l}$ shows values for common distributions in order to fit the empirical data to these distributions. For some distributions (normal, uniform, logistic, exponential), there is only one value for the skewness and the kurtosis which are shown as a single point on the plot. For others, lines or areas of values are represented such as for gamma, lognormal and beta distributions.

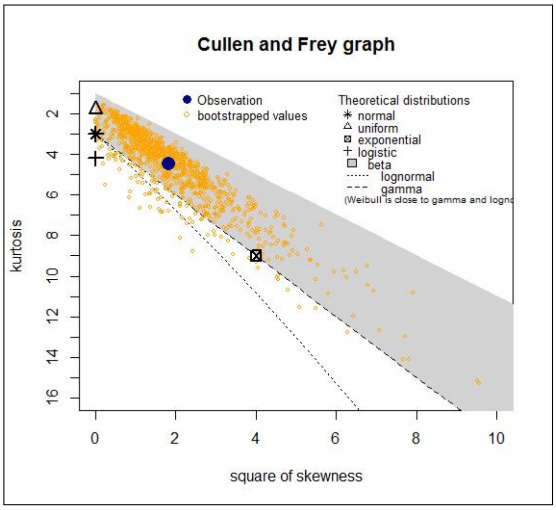


Figure 5.6: Cullen and Frey graph for $X_{mtpl,l}$

From the Cullen and Frey graph, the right skewed distributions Gamma, Lognormal and Weibull are chosen to fit to $X_{mtpl,l}$. In Figure 5.7 the theoretical densities of the fitted distributions, empirical and theoretical CDFs and Q-Q and P-P plots are presented. The Q-Q plot shows the lack of fit at the tails while the P-P plot focuses on the lack of fit at the center. As none of the distributions exactly fit the distribution of the real data, we test the goodness-of-fit of these distributions using three information criteria calculated by maximum likelihood method as shown in Table 5.2. Lognormal distribution is chosen as it gives the highest loglikelihood value and lowest AIC and BIC information criteria. Therefore, the ratio for MTPL segment of company Large, $X_{mtpl,l}$ is estimated to be lognormally distributed with mean of 0.1103 and standard deviation of 0.097.

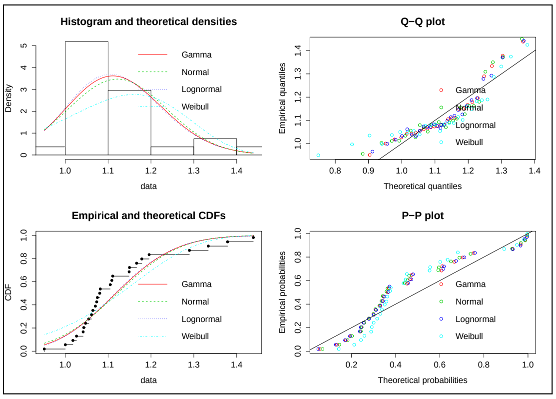


Figure 5.7: Goodness-of-fit plots for $X_{mtpl,l}$

Table 5.2: Goodness-of-fit test for $X_{mtpl,l}$

	Loglikelihood	AIC	BIC
Gamma	21.20	-38.40	-35.81
Lognormal	**21.69**	**-39.37**	**-36.78**
Normal	20.12	-36.25	-33.66
Weibull	14.77	-25.54	-22.87

The same procedure is followed for distribution fitting of the other ratios, $X_{motor,l}$, $X_{mtpl,m}$, $X_{motor,m}$, $X_{mtpl,s}$, $X_{motor,s}$, $X_{mtpl,o}$ and $X_{motor,o}$. The distribution fitting results and graphs related to these ratios are presented in Appendix. For all the ratios of MTPL and motor segments for the Large, Medium, Small and Others, the fitted distributions and their estimated parameters are listed in Table 5.3.

Table 5.3: Fitted distributions and their parameters for all ratios

	Fitted distribution	μ	σ
$X_{mtpl,l}$	Lognormal	0.110	0.097
$X_{motor,l}$	Weibull	14.31	1.04
$X_{mtpl,m}$	Lognormal	0.24	0.11
$X_{motor,m}$	Lognormal	0.038	0.107
$X_{mtpl,s}$	Lognormal	0.323	0.123
$X_{motor,s}$	Normal	1.204	0.098
$X_{mtpl,o}$	Weibull	30.43	1.23
$X_{motor,o}$	Normal	1.043	0.11

5.2.3 Internal model design using copulas

Solvency II requires the correlation coefficient between MTPL (segment 1) and motor (segment 2) segments to be determined as 0.5. This coefficient is estimated using the whole market data across EU. It aggregates the risks concerning the MTPL and motor segments given in equations (5.1) and (5.2). This defines a highly positive relationship between the risks of MTPL and motor segments.

Based on the data the Pearson, Spearman and Kendall correlation coefficients between MTPL and motor segments' ratios are shown in Table 5.4. It is remarkable that there exists negative correlation between two lines. Medium company yields higher negative correlation compared to Large, Small and Others, whereas Others yield the smallest correlation altering from negative to positive with respect to the type of correlation measure. These results do not coincide with the correlations predetermined in Solvency II and also conflicts with the direction of the correlation.

Table 5.4: The correlation coefficients between segments using real data

	Large	Medium	Small	Others
Pearson	0.1817	-0.2935	0.0772	-0.0801
Spearman	0.1044	-0.2546	0.0891	-0.0037
Kendall	0.0883	-0.1396	0.0655	0.0199

The linear correlation coefficients between the ratios for MTPL and motor segments are calculated using Pearson method and found to be very low compared to the predetermined correlation coefficient of 0.5 in Solvency II.

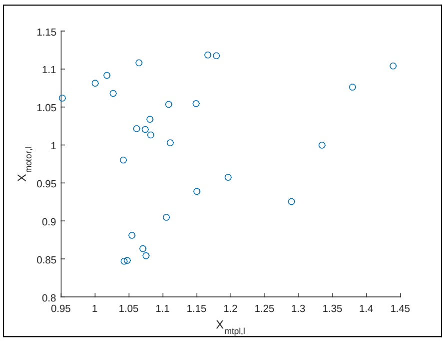
Since the linear correlation coefficient is not sufficient to reveal the association between any two variable, different copula families are employed to determine the association between the ratios of the segments. All the main copulas (e.g. Gaussian, T, Frank, Gumbel and Clayton) are evaluated for representing the dependence structure of the corresponding ratios for MTPL and motor segments for the companies selected. The choice of the best fitting copula is determined by the three information criteria; AIC, SIC and HQIC as shown in Table 5.5.

Based on the information criteria the best fitting copula for Large, Medium, Small and Others is determined as Gaussian, T, Gaussian and T copulas respectively. In addition to these selected copulas, another type of copula may also be taken into account; empirical copula. The difference between constructing an empirical copula and fitting an existing type of copula is that when fitting a copula, we determine the parameter of the copula that makes for a best fit to the data, but retaining the copula's functional form. However with the empirical copula, the functional form itself (not just the parameter) is based on the data, making it a flexible tool for capturing any type of association not captured by the selected functional form copula.

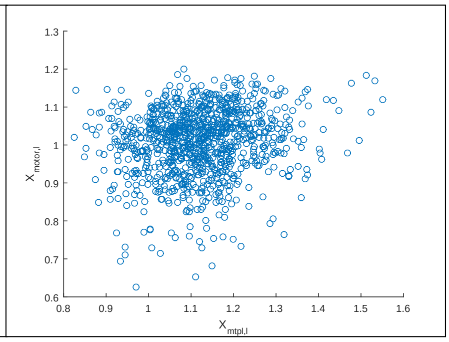
Table 5.5: The selection criteria of copula for the ratios

	Copula	SIC	AIC	HQIC
Large	**Gaussian**	**3.44**	**2.26**	**2.52**
	T	5.18	3.00	3.33
	Frank	6.51	4.32	4.66
	Gumbel	6.73	4.54	4.88
	Clayton	6.87	4.68	5.02
Medium	**T**	**1.62**	**-0.56**	**-0.23**
	Gaussian	3.47	2.29	2.54
	Clayton	6.44	4.25	4.59
	Gumbel	6.58	4.39	4.73
	Frank	6.65	4.47	4.80
Small	**Gaussian**	**3.16**	**1.98**	**2.23**
	Frank	4.68	2.49	2.83
	T	4.82	2.63	2.97
	Gumbel	5.64	3.45	3.79
	Clayton	6.73	4.54	4.88
Others	**T**	**1.53**	**-0.53**	**-0.22**
	Gaussian	3.27	2.16	2.39
	Clayton	6.07	4.01	4.33
	Gumbel	6.20	4.14	4.46
	Frank	6.27	4.21	4.52

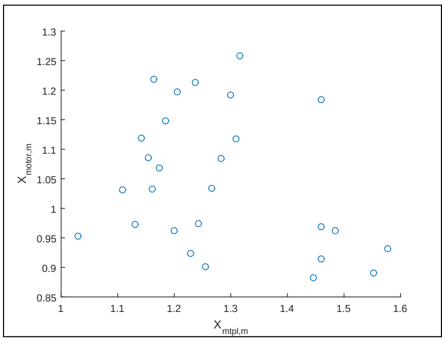
A copula consisting of two variables may be simply described as a probability distribution on these two random variables, each of whose marginal distributions is uniform on (0,1). These two variables may be independent, completely dependent ($u_1 = u_2$), or anything in between. The family of bivariate Gaussian copulas is parameterized by the linear correlation coefficient, r. u_1 and u_2 approach complete dependence as r approaches +/- 1, and approach complete independence as r approaches zero. To follow if this pattern holds, the scatter diagram of the ratios, $X_{mtpl,l}$ and $X_{motor,l}$ is plotted in Figure 5.8. The scatter diagram provides us the information about the dependence between $X_{mtpl,l}$ and $X_{motor,l}$. The linear correlation seems to be low based on the available data points. However this does not present the whole dependence structure between the variables. To figure out the dependence structure between the marginal distributions of $X_{mtpl,l}$ and $X_{motor,l}$, copulas provide us a simple method using the rank order and cumulative distribution function of the variables. The best fitting copula for $X_{mtpl,l}$ and $X_{motor,l}$ determined as Gaussian is simulated using the estimated parameters. The same procedure is applied to the Medium, Small and Others to illustrate the impact of copulas.



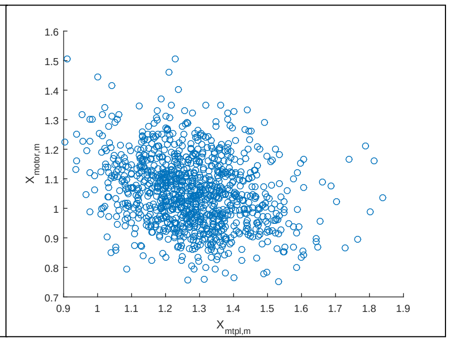
(a) Original values of Large



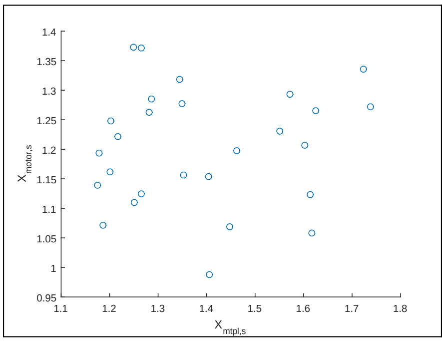
(b) Simulated using Gaussian



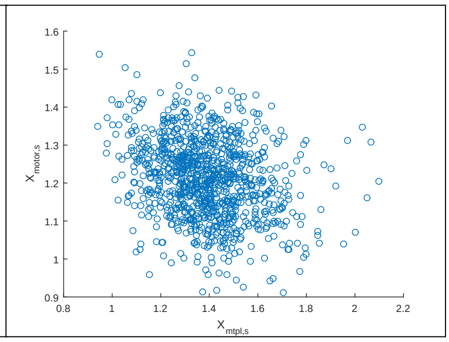
(c) Original values of Medium



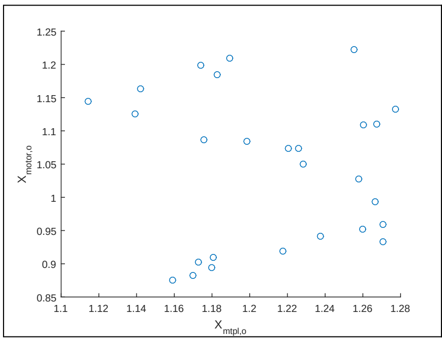
(d) Simulated using T



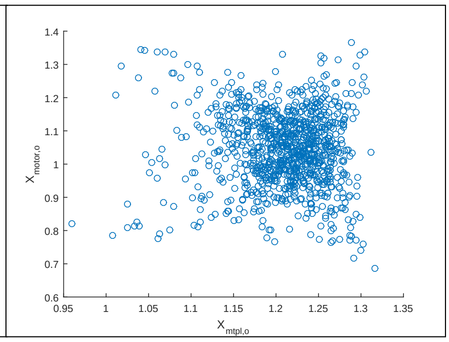
(e) Original values of Small



(f) Simulated using Gaussian



(g) Original values of Others



(h) Simulated using T

Figure 5.8: Original and simulated sets for all selected companies

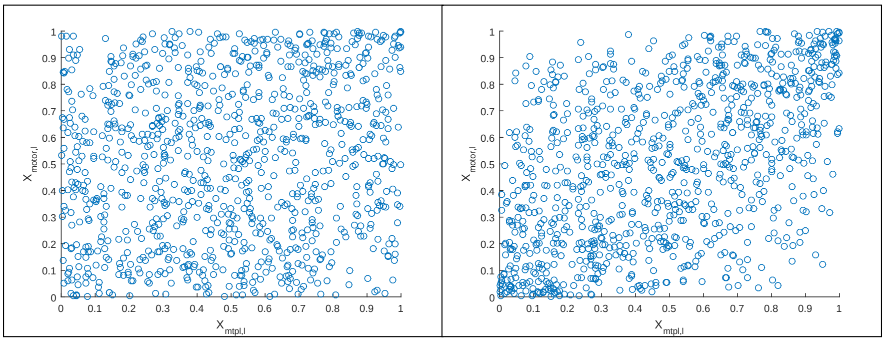
Figure 5.8b presents the 1000 simulated values of the ratios, $X_{mtpl,l}$ and $X_{motor,l}$, using Gaussian copula. In summary, we started with the 27 data points shown in Figure 5.8a and using copulas we estimated more data points (i.e. pair values of $X_{mtpl,l}$ and $X_{motor,l}$). Gaussian copula estimated the data points more concentrated and close to the historical data points. The same methodology is applied for the other ratios, ($X_{mtpl,m}$, $X_{motor,m}$), ($X_{mtpl,s}$, $X_{motor,s}$) and ($X_{mtpl,o}$, $X_{motor,o}$), and the resulting scatter diagrams are also presented in Figure 5.8.

The historical values of the ratios, $X_{mtpl,m}$ and $X_{motor,m}$ of company Medium is shown in Figure 5.8c. The linear correlation coefficient of $X_{mtpl,m}$ and $X_{motor,m}$ is calculated as -0.2935 and differs from the positive correlation coefficient of 0.5 predetermined in Solvency II. In Figure 5.8d the scatter diagram of the simulated values using the fitted T copula also incurs a correlation coefficient of -0.2998.

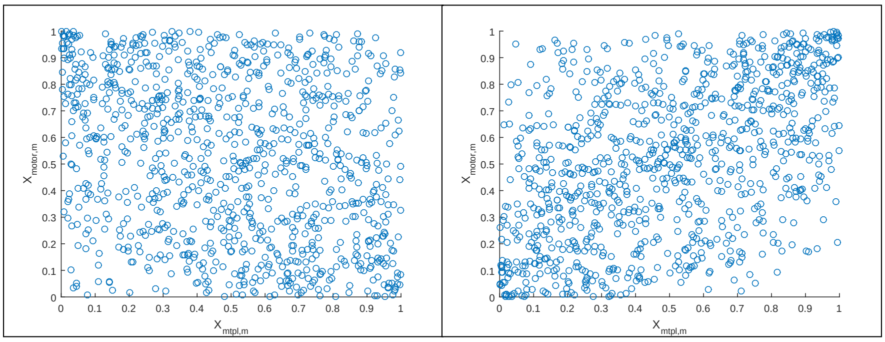
The historical values for Small company ratios are shown in Figure 5.8e. The linear correlation between $X_{mtpl,s}$ and $X_{motor,s}$ is calculated as 0.0772 which is again lower than 0.5 set in Solvency II. Figure 5.8f presents the simulated values using the fitted Gaussian copula which results in a correlation of 0.0676.

The historical values for the ratios, $X_{mtpl,o}$ and $X_{motor,o}$ for Others are shown in Figure 5.8g. We also see a negative linear correlation between the ratios similar to company Medium but at a lower magnitude. Since "Others" stand for the whole Turkish non-life insurance market excluding the three companies, Large, Medium and Small, we may conclude that MTPL and motor segments in Turkey are negatively correlated in contrast to EU as set in Solvency II. The simulated values for $X_{mtpl,o}$ and $X_{motor,o}$ using the fitted T copula are shown in Figure 5.8h.

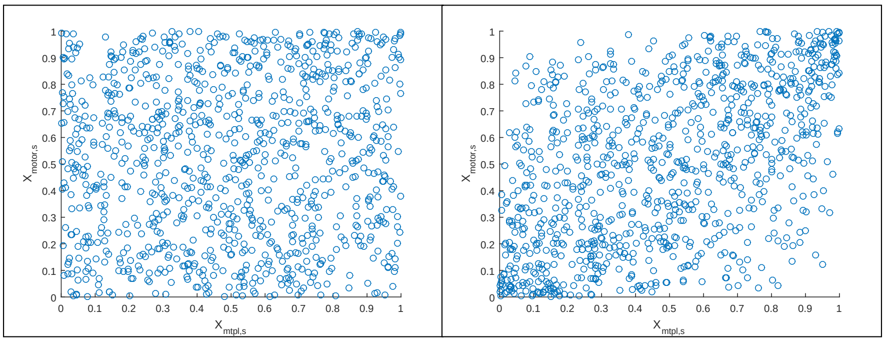
The simulated values of U_1 and U_2 for the fitted copulas corresponding to companies Large, Medium, Small and Others are presented in Figure 5.9 together with the linear coefficient of 0.5 as predetermined in Solvency II for comparison. These are helpful in visualizing the dependence structure estimated by the copulas and choosing the copula for the type of risks considered. As the linear correlation coefficient increases U_1 and U_2 gets more concentrated at the lower and upper tails for both Gaussian and T copulas. The concentration becomes stronger for T copulas than Gaussian.



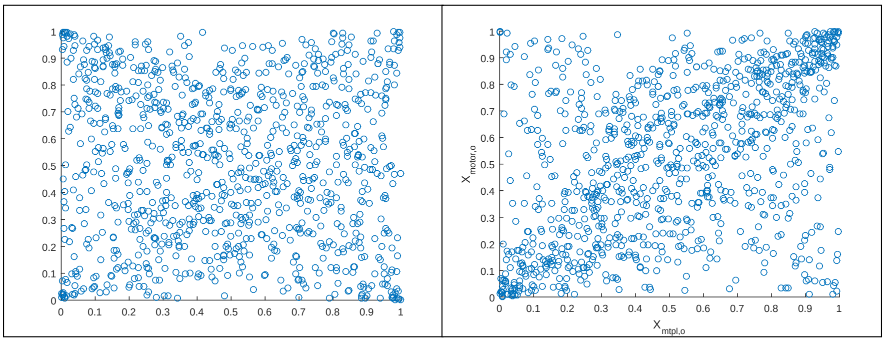
(a) Gaussian ($\rho = 0.1673$) for Large (b) Gaussian ($\rho = 0.5$) for Large



(c) T ($\rho = -0.2998$) for Medium (d) T ($\rho = 0.5$) for Medium



(e) Gaussian ($\rho = 0.0676$) for Small (f) Gaussian ($\rho = 0.5$) for Small



(g) T ($\rho = -0.0963$) for Others (h) T ($\rho = 0.5$) for Others

Figure 5.9: The simulated values of U_1 and U_2 for copulas

5.3 SCR by standard formula

In this section, SCR for non-life premium and reserve risks, NL_{pr} is calculated using the Solvency II standard formula as described in Section 3.2. In the calculation, the premium and reserve realizations of the non-life insurance companies as of 2014 year end are used. For the sake of simplicity and the reasons stated in the introduction, only two segments, MTPL and motor insurance are involved in the calculations. These two segments constitute a larger part of the total premium and reserve figures for the most of the companies.

The results and the details of the calculation are shown in Table 5.6. SCR for non-life premium and reserve risk, NL_{pr} for Large, Medium and Small companies as of 2014 year end are calculated as 695, 115 and 15 million TL respectively. NL_{pr} for the remaining companies, Others is also calculated as 2,979 million TL.

For comparability of the results with the internal model P_s is taken as equal to $P_{(last,s)}$.

Table 5.6: NL_{pr} values using Standard Formula

Company	Large		Medium		Small		Others	
Segments	Mtpl	Motor	Mtpl	Motor	Mtpl	Motor	Mtpl	Motor
P_s	1,289	714	100	185	21	27	4,404	4,791
$P_{(last,s)}$	1,289	714	100	185	21	27	4,404	4,791
$FP_{(existing,s)}$	0	0	0	0	0	0	0	0
$FP_{(future,s)}$	0	0	0	0	0	0	0	0
$V_{(prem,s)}$	1,289	714	100	185	21	27	4,404	4,791
$\sigma_{(prem,s)}$	0.1	0.08	0.1	0.08	0.1	0.08	0.1	0.08
PCO_s	1,076	53	236	29	20	5	4,412	532
$V_{(res,s)}$	1,076	53	236	29	20	5	4,412	532
$\sigma_{(res,s)}$	0.09	0.08	0.09	0.08	0.09	0.08	0.09	0.08
V_s	2,365	767	336	214	41	32	8,816	5,323
σ_s	0.0829	0.0774	0.0822	0.0751	0.0825	0.0745	0.0823	0.0763
V_{nl}	3,132		550		73		14,139	
σ_{nl}	0.074		0.070		0.069		0.070	
NL_{pr}	695		115		15		2,979	

5.4 SCR by internal model

Our aim is to calculate the SCR for non-life premium and reserve risk by first calculating the combined loss ratios, X, for MTPL and motor segments and fitting distributions for these ratios and then aggregating these risks for the two segments by using copulas. Based on the distributions chosen for the combined loss ratios of MTPL and motor segments of the company types, the estimated parameters are presented in Table 5.3. Solvency II assumes that these are lognormally distributed based on the estimation accomplished using the whole EU insurance market data [9]. However, the original data set yields different options for the segments as presented in the earlier sections. For the Turkish insurance market, the premium and reserve risks of MTPL segment follows lognormal distribution for Large, Medium and Small companies and Weibull for Others. For motor segment, the risks are distributed by Weibull for Large, lognormal for Medium and Normal for Small and Others.

As a second step we use copulas to identify the dependence between the MTPL and motor segments. To calculate the SCR using copulas we first generate the (U_1,U_2) pairs by simulating the fitted copula for 10,000 times. Then, we use the inverse cumulative distribution functions to calculate the ($X_{mtpl,l}$, $X_{motor,l}$), ($X_{mtpl,m}$, $X_{motor,m}$), ($X_{mtpl,s}$, $X_{motor,s}$) and ($X_{mtpl,o}$, $X_{motor,o}$) pairs for MTPL and motor segments of the companies Large, Medium, Small and Others.

For each pair (MTPL and motor) we calculate the SCR for non-life premium and reserve risk, NL_{pr} as follows. Let L/P denote the expected loss (or profit) calculated based on actuarial equivalence principle. The values of L/P and NL_{pr} are determined by

$$\begin{aligned} L/P &= (X_{mtpl} - 1)V_{mtpl} + (X_{motor} - 1)V_{motor} \\ NL_{pr} &= VaR_{\%99.5}(L/P) \end{aligned} \tag{5.6}$$

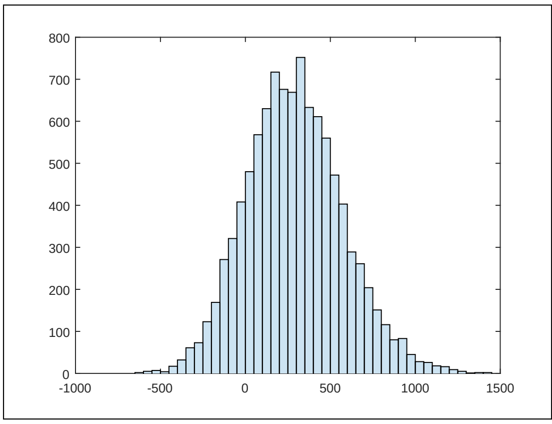
For a simulation set of 10,000 runs, $VaR_{\%99.5}$ is calculated and NL_{pr} values, consequently, SCR values are determined. The histogram of the SCR simulation results for Large, Medium, Small and Others by the fitted copulas are shown in Figure 5.10. The SCR simulations yield 1119, 241, 46 and 4,037 million TL of SCR for Large, Medium, Small and Others respectively. These values are much higher than 695, 115, 15 and 2,979 million TL, obtained using standard formula of Solvency II by 61%, 109%, 206% and 35% respectively.

The association of the ratios of the two segments, MTPL and motor can be evaluated by the correlation coefficients given in Table 5.7 for the real and simulated values of the ratios. We see that the linear correlation coefficient of the real ratios is preserved by the simulated ones reproduced by the fitted copulas. However, for Spearman and Kendall correlations, a rise occurs for Large, Medium and Others in the magnitude of correlation. For Small these correlations get lower for simulated ratios.

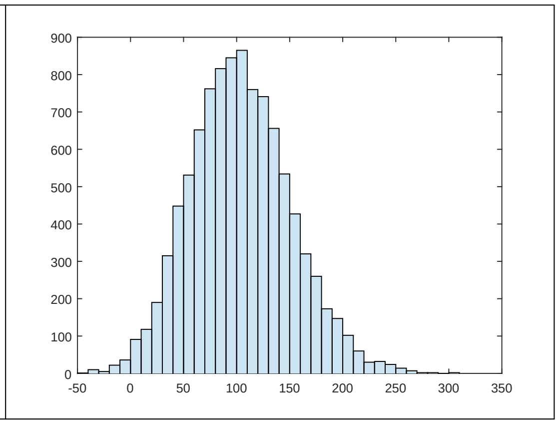
Table 5.7: The association of real and simulated ratios

	Ratios	Pearson	Spearman	Kendall
Large	Real	0.1817	0.1044	0.0883
	Simulated	0.1627	0.1626	0.1086
Medium	Real	-0.2935	-0.2546	-0.1396
	Simulated	-0.2868	-0.2869	-0.1932
Small	Real	0.0772	0.0891	0.0655
	Simulated	0.0673	0.0670	0.0446
Others	Real	-0.0801	-0.0037	0.0199
	Simulated	-0.0973	-0.0973	-0.0693

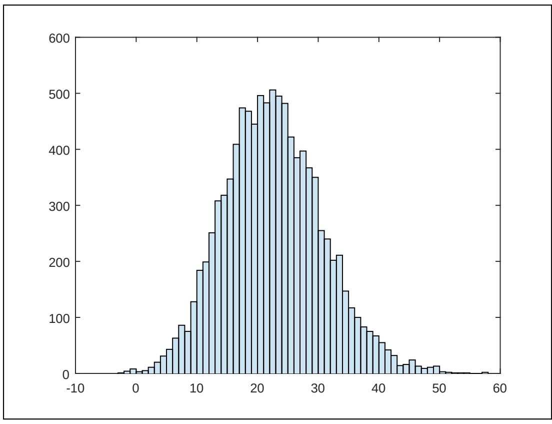
In Figure 5.10 the negative values of SCR stems from the loss ratios lower than 1 as the company makes profit in this case. Large, Medium and Small companies have more SCR values in the tails compared to Others for which the SCR has light-tailed distribution.



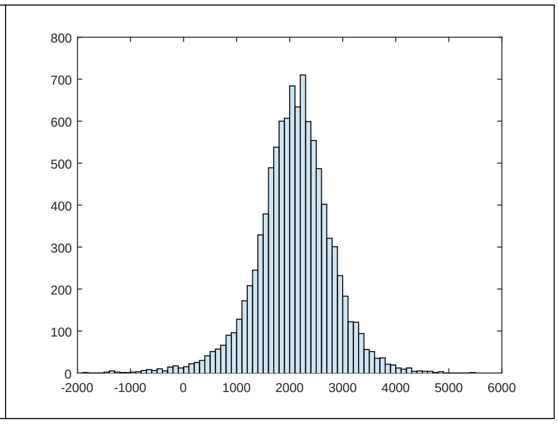
(a) Large - Gaussian



(b) Medium - T



(c) Small - Gaussian



(d) Others - T

Figure 5.10: Histogram of SCR results using fitted copula

5.5 Comparison of the SCR results

The SCR results calculated using Turkish capital requirement regime, Solvency II standard formula and the internal model constructed using copulas are summarized in Table 5.8. Compared to the Turkish capital requirement regime the standard formula overestimates the SCR by 90%, 53%, 38% and 42% for Large, Medium, Small and Others respectively. The highest discrepancy between SCR results is seen for Large. As the company size gets smaller the discrepancy becomes less. On the other hand, compared to the standard formula, the SCR is overestimated by all the copulas for all the companies although the correlation coefficient between MTPL and motor segments predetermined in Solvency II as 0.5 is greater than the calculated correlation coefficients for the real data of MTPL and motor segments.

For company Large, for which the standard formula results in an SCR of 695 million TL, the best fitting copula is chosen as Gaussian copula that yields an SCR of 1119 million TL which is 61% higher. The linear correlation coefficient for Large is calculated as 0.18 which is lower than 0.5 in Solvency II.

For company Medium the best fitting copula is T copula resulting in an SCR of 241 million TL which is 109% higher than the standard formula. The linear correlation coefficient of MTPL and motor segments for company Medium is -0.29 that is lower than the companies Large, Small and Others.

For company Small, Gaussian copula is estimated to be the best fitting copula. It results in an SCR of 46 million TL, almost three times of the standard formula. The linear correlation coefficient for Small is found to be 0.08 which is again lower than 0.5 in Solvency II.

For the remaining companies of the non-life insurance market, Others, T copula fits the best and yields an SCR of 4,037 million TL which is 35% higher than the standard formula. The linear correlation coefficient for Others is -0.08 which is again lower than 0.5 in Solvency II.

Table 5.8: Comparison of SCR results (million TL)

	Turkish SCR	Standard Formula	Internal Model			
			Copula	$VaR_{\%99.5}$	μ	σ
Large	365	695	Gaussian	1119	291	282
Medium	75	115	T	241	104	47
Small	11	15	Gaussian	46	22	8
Others	2100	2,979	T	4037	2084	705

While designing the internal model we aggregate the premium and reserve risks using equation (5.5) and fit distributions to each of these aggregated loss ratios instead of assuming that these are lognormal distributed as stated in Solvency II. In order to have comparable SCR results we used the same volume measures for the premium and reserve risks. In the end, the internal model designed by copulas results in higher SCR than the standard formula although the linear correlation coefficients of MTPL and motor segments are smaller than the one in Solvency II.

The overestimation of SCR by the internal model might be explained by the inability of the linear correlation coefficients in explaining the dependence between the risks concerning the two segments, MTPL and motor. The dependence between MTPL and motor segments seems to be higher in Turkey compared to the dependence implicitly assumed in Solvency II although the linear correlation coefficients for all the companies are much lower than the one predetermined in Solvency II.

Another reason for the higher SCR results would be related to the data as the recent increase in combined loss ratio for MTPL segment in Turkey has shifted upwards unexpectedly due to judicial decisions in 2012 which boosted the traffic claims for death and disability.

CHAPTER 6

CONCLUSION

In this study we focus on the SCR calculation of Solvency II for non-life premium and reserve risks using the standard formula. The aim is to design an alternative internal model for the SCR calculation of premium and reserve risks. Instead of relying upon the predetermined distributions and correlation coefficients recommended by Solvency II, we use copulas for assessing the dependence between two selected segments, MTPL and motor insurance.

The need for a new capital regime is becoming more evident for Turkish insurance market as the risks the insurance companies are facing get more diversified together with the worsening financial market conditions. The growing concern about the claims size and frequency for specific segments such as traffic insurance in Turkey has been influencing the insurance companies' financial strength recently. Solvency II emerged as a reaction for these concerns in EU and entered into force in the beginning of 2016.

The contribution of this thesis are summarized as follows:

i) The SCR requirements on the association between the lines of businesses are questioned. It has been shown that the correlation among two highly correlated segments may show lower association and vice versa.

ii) The distributional assumption on the main indicators, loss ratios, may not necessarily follow lognormal distribution. This causes theoretically difficulty in implementing the proposed methods.

This can be handled theoretically by employing copulas which at the same time pertain a better association and allow the use of different statistical distributions.

iii) The SCR by standard formula provides less security compared to the internal model imposing copulas. The current capital requirement regime in Turkey and the Solvency II allow unforeseen insolvency in the insurance companies especially in emerging countries with less stable loss ratios.

iv) Solvency indicators of Turkish insurance market, in two significant segments of MTPL and motor insurance, show varying responses to SCR calculation according to the size of company. Therefore, the implementation of same standards, despite seeming to be less penalizing compared to the internal model, may not be affordable and applicable int the long run.

v) In order to aggregate the premium and reserve risks, an indicator, loss ratio is defined and analyzed throughout the thesis based on the actuarial equivalence principle and the asset-liability framework.

The outcomes of this thesis can be used in assessing the needs of solvency regime in Turkey. As shown in this study, the linear correlation does not provide the full information concerning the dependence among selected segments. We believe that the methodology presented in this study would be of great use for both the authorities and the companies in analyzing the dependence structure between the insurance risks. The proposed model for SCR calculation is straightforward and robust as Solvency II standard formula. In addition to being easily designed and implemented it also helps the companies and the authorities to understand the risks and their dependence in a better way than the linear correlations. This study might also be helpful in assessing the dependence among the risks of other non-life insurance segments and evaluating the Solvency II framework of the capital requirement system for the Turkish insurance market.

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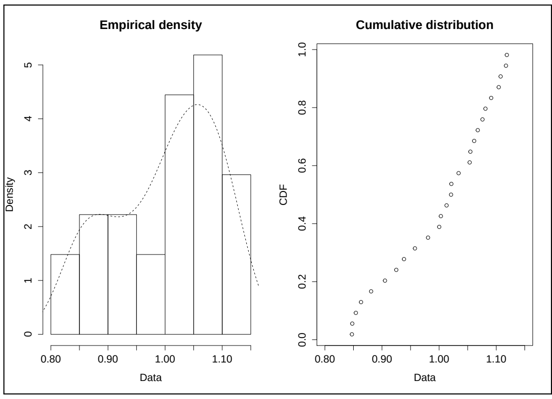
APPENDIX A

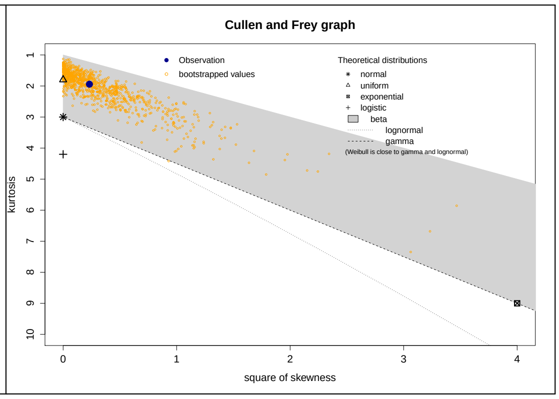
LOSS DISTRIBUTION FITTING FOR EACH SEGMENT

Table A.1: Goodness-of-fit test for $X_{motor,l}$

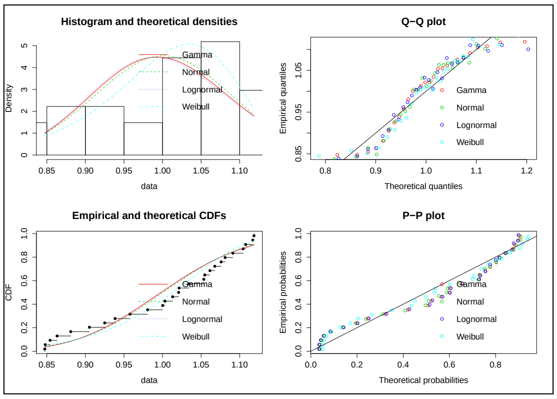
	Loglikelihood	AIC	BIC
Gamma	26.98	-49.96	-47.37
Lognormal	26.76	-49.53	-46.94
Normal	27.35	-50.70	-48.11
Weibull	28.65	-53.31	-50.71



(a) Empirical density and CDF



(b) Cullen and Frey graph

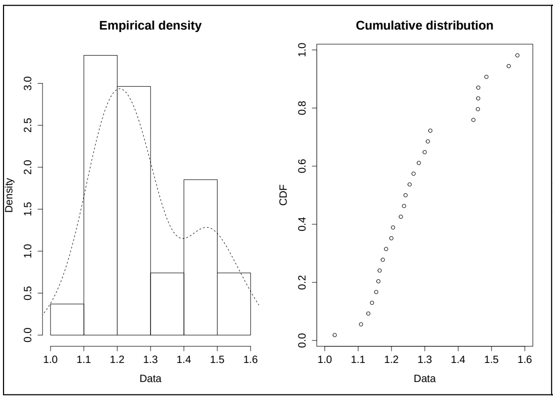


(c) Goodness-of-fit plots

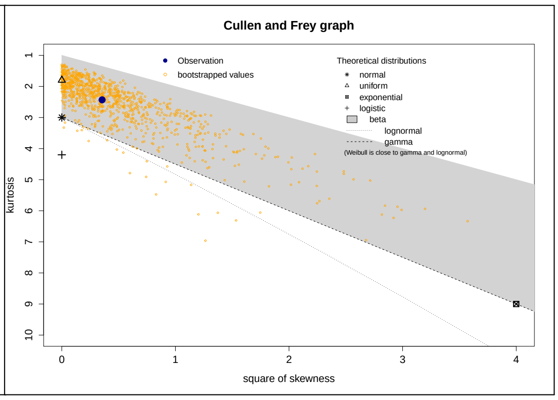
Figure A.1: Distribution fitting of $X_{motor,l}$

Table A.2: Goodness-of-fit test for $X_{mtpl,m}$

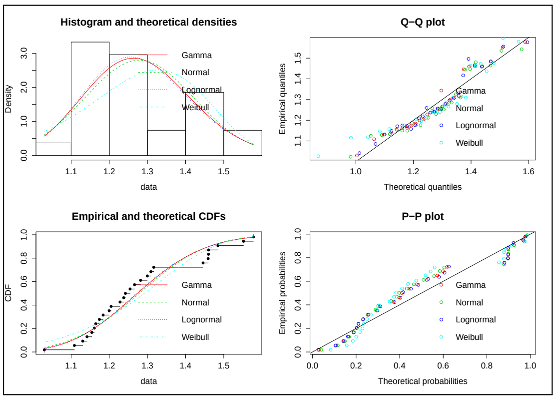
	Loglikelihood	AIC	BIC
Gamma	14.84	-25.67	-23.08
Lognormal	15.04	-26.08	-23.49
Normal	14.31	-24.63	-22.03
Weibull	12.33	-20.67	-18.08



(a) Empirical density and CDF



(b) Cullen and Frey graph

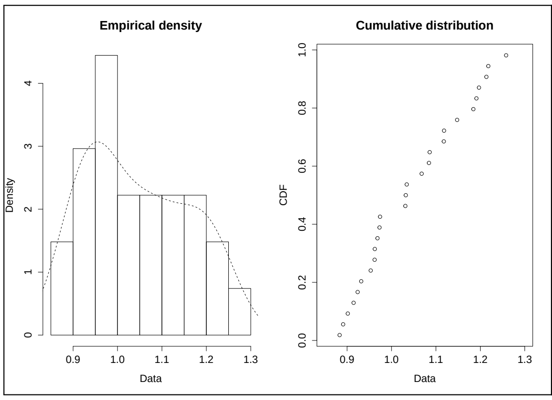


(c) Goodness-of-fit plots

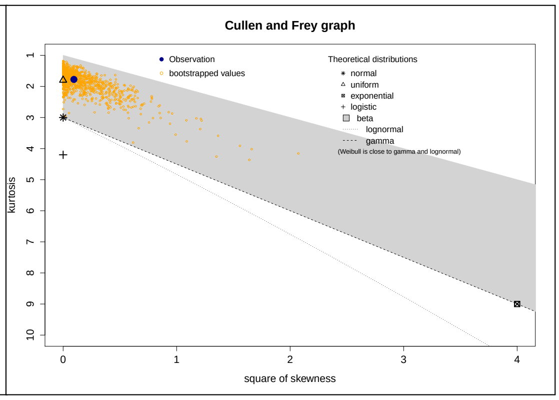
Figure A.2: Distribution fitting of $X_{mtpl,m}$

Table A.3: Goodness-of-fit test for $X_{motor,m}$

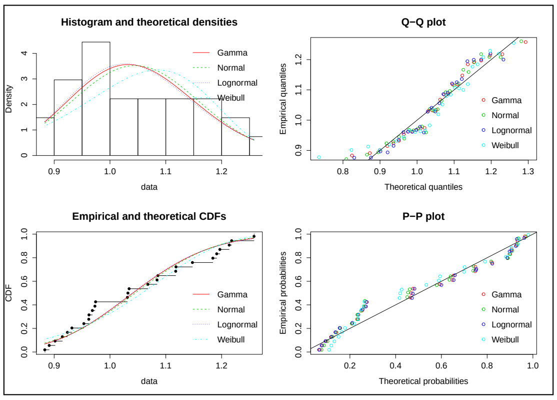
	Loglikelihood	AIC	BIC
Gamma	20.78	-37.57	-34.98
Lognormal	20.88	-37.76	-35.17
Normal	20.51	-37.01	-34.42
Weibull	19.38	-34.77	-32.18



(a) Empirical density and CDF



(b) Cullen and Frey graph

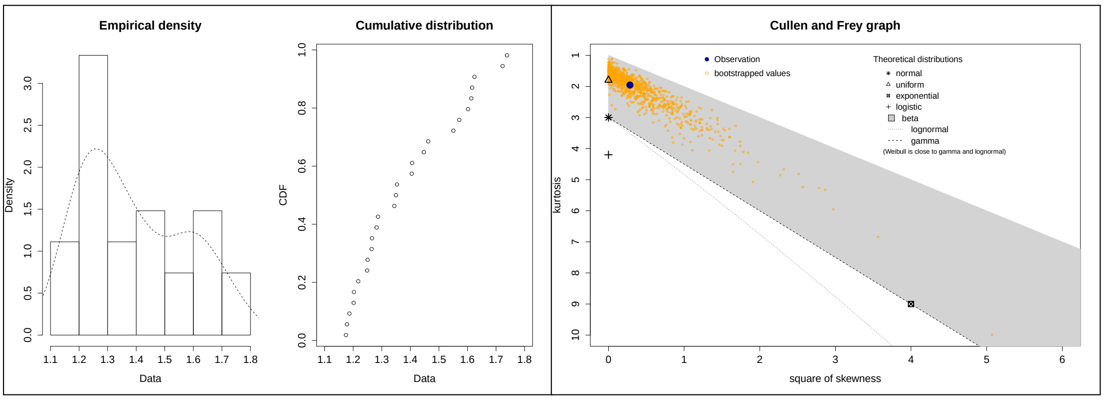


(c) Goodness-of-fit plots

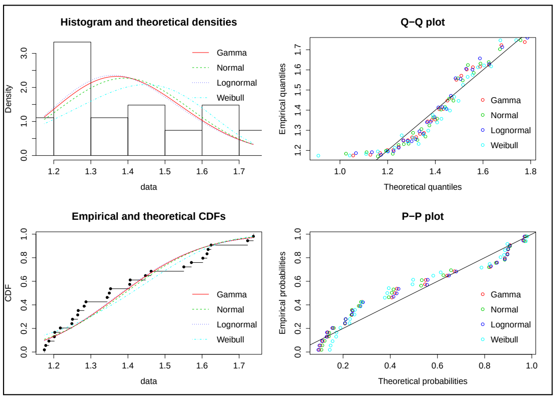
Figure A.3: Distribution fitting of $X_{motor,m}$

Table A.4: Goodness-of-fit test for $X_{mtpl,s}$

	Loglikelihood	AIC	BIC
Gamma	9.23	-14.46	-11.87
Lognormal	9.45	-14.91	-12.31
Normal	8.67	-13.34	-10.75
Weibull	7.12	-10.25	-7.66



(a) Empirical density and CDF (b) Cullen and Frey graph

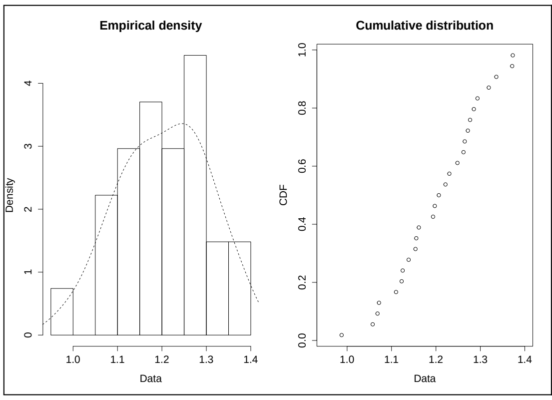


(c) Goodness-of-fit plots

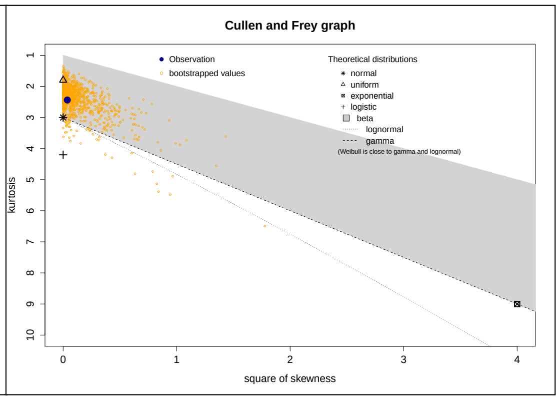
Figure A.4: Distribution fitting of $X_{mtpl,s}$

Table A.5: Goodness-of-fit test for $X_{motor,s}$

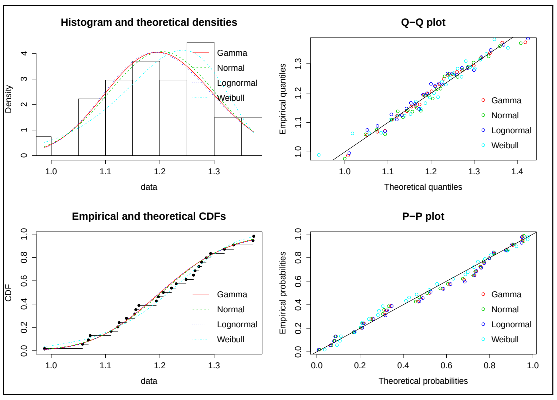
	Loglikelihood	AIC	BIC
Gamma	24.28	-44.55	-41.96
Lognormal	24.16	-44.32	-41.73
Normal	24.44	-44.88	-42.29
Weibull	24.30	-44.60	-42.01



(a) Empirical density and CDF



(b) Cullen and Frey graph

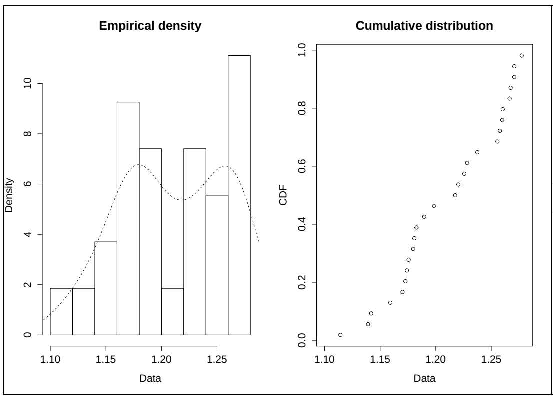


(c) Goodness-of-fit plots

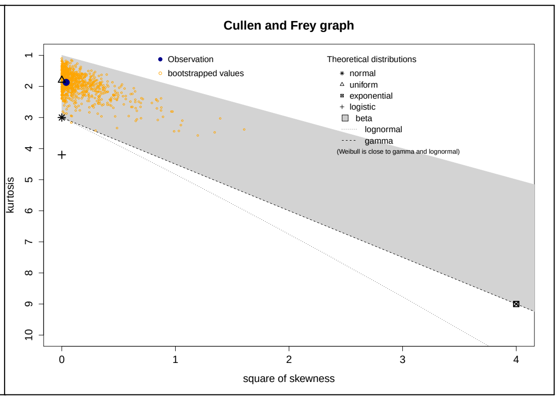
Figure A.5: Distribution fitting of $X_{motor,s}$

Table A.6: Goodness-of-fit test for $X_{mtpl,o}$

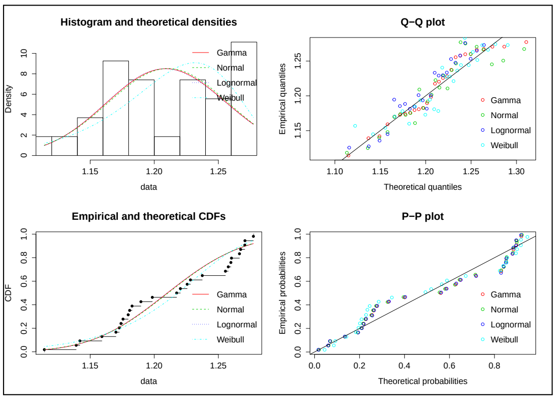
	Loglikelihood	AIC	BIC
Gamma	44.29	-84.58	-81.98
Lognormal	44.25	-84.50	-81.91
Normal	44.35	-84.71	-82.11
Weibull	44.55	-85.10	-82.51



(a) Empirical density and CDF



(b) Cullen and Frey graph

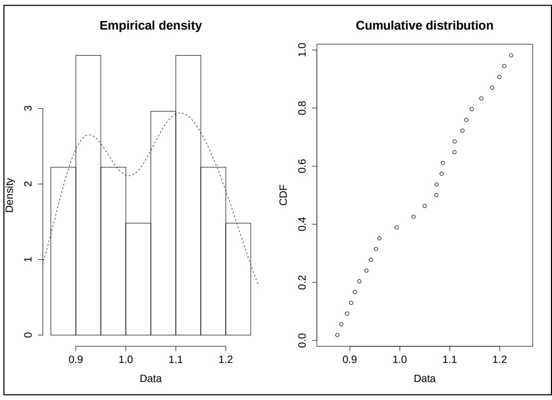


(c) Goodness-of-fit plots

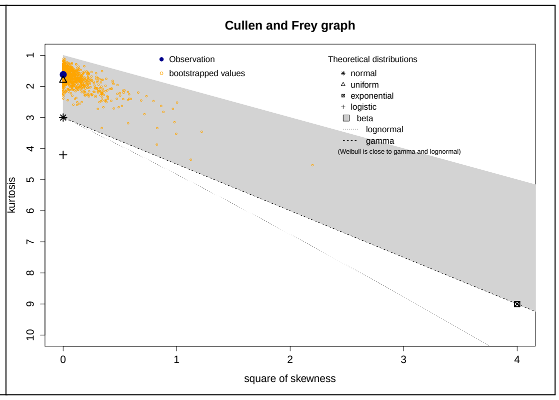
Figure A.6: Distribution fitting of $X_{mtpl,o}$

Table A.7: Goodness-of-fit test for $X_{motor,o}$

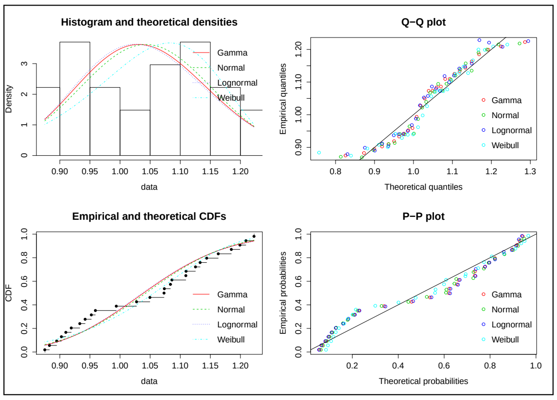
	Loglikelihood	AIC	BIC
Gamma	21.25	-38.50	-35.91
Lognormal	21.20	-38.39	-35.80
Normal	21.27	-38.54	-35.95
Weibull	21.13	-38.26	-35.67



(a) Empirical density and CDF



(b) Cullen and Frey graph



(c) Goodness-of-fit plots

Figure A.7: Distribution fitting of $X_{motor,o}$