ANALYSES OF FACTORS OF MARKET MICROSTRUCTURE: PRICE IMPACT, LIQUIDITY, AND VOLATILITY

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ABSTRACT

ANALYSES OF FACTORS OF MARKET MICROSTRUCTURE: PRICE IMPACT, LIQUIDITY, AND VOLATILITY

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First chapter of this thesis is an attempt to model the price impact by extending the model proposed by Kyle [54]. It is assumed that the market is not perfectly efficient so that it takes to time to adjust new equilibrium price. Thus, in order to model the price impact, two new concepts are introduced which are market resiliency and speed of price informativeness. It is showed that market resiliency and price impact tend to raise as speed of price information increases which emphasizes the fact that speed of information matters in financial markets and market resiliency is not a phenomenon that can be neglected.

In the second chapter, it is tried to stress the importance of the liquidity which is considered as the neglected dimension of the financial risk. To do that, a new approach called Liquidity Augmented Stochastic Volatility with Jump (LASVJ) model is introduced and it is compared with the Stochastic Volatility with Jump (SVJ) model in terms of stability and performance. This analysis includes both simulation and calibration analysis. The simulation results suggest that LASVJ model outperforms SVJ as it has lower bias and Root Mean Square Error. In the calibration part, ten companies listed in Dow-Jones 30 are used and it is found that the estimated probability of default and credit spread with LASVJ model are higher than those with SVJ model. The 2008 Crisis period is even aggravated this result. The findings imply that the probability of default and credit spread are underestimated if liquidity dimension of risk is neglected and this partly accounts for why 2007-2008 financial crisis and its full-scale effect could not be predicted.

In the third chapter, it is aimed to improve the volatility prediction which included in the financial risk management. As a well-performing volatility prediction sheds light on the uncertainty in the financial market, it is an important task to model it. To this end, GARCH-type models as well as SVR-GARCH model are used to model the volatility and the results are compared based on the performance metrics. In part of empirical analysis thirty stocks listed $S\&P$-500 are included and the period covered is between 01/01/2010-09/01/2019. The finding indicates that SVR-GARCH outperforms the traditional models in predicting volatility and also produce more reliable result in Value-at-Risk estimation based on Proportion of Failures and Basel's Traffic Light Backtesting approaches.

Keywords: Price Impact, Liquidity, Credit Risk, Volatility, Machine Learning, and Risk Management

ÖZ

MARKET MİKRO YAPISININ FAKTÖRLERİNİN ANALİZİ: FİYAT ETKİSİ, LİKİDİTE VE OYNAKLIK

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Bu tezin ilk bölümünde, Kyle[54] tarafından önerilen modeli genişletilerek fiyat etkisini modelleme çalışılmıştır. Yeni denge fiyatına ulaşılması zaman aldığında pazarın tam etkin olmadığı varsayılmaktadır. Bu nedenle, fiyat etkisini modellemek için, iki yeni kavram olan piyasa esnekliği ve fiyatın bilgilendiriciliği kavramları tanımlanmıştır. Sonuç olarak, fiyatın bilgilendiriciliği arttıkça piyasa esnekliği ve fiyat etkisinin artma eğilimi göstermiştir. Bu bulgu ise, finansal piyasalarda bilginin hızının önemli olduğu ve piyasa esnekliğinin ihmal edilebilecek bir olgu olmadığını vurgulamaktadır.

İkinci bölümde finansal riskin ihmal edilen boyutu olarak kabul edilen likiditenin önemi vurgulanmaya çalışılmaktadır. Bunun için, Likidite Etkili Zıplamalı Stokastik Volatilite Modeli (LASVJ) modeli tanıtılmış ve stabilite ve performans açısından Zıplamalı Stokastik Volatilite Modeli (SVJ) modeli ile karşılaştırılmıştır. Simülasyon sonuçları, LASVJ modelinin düşük sapma ve kök ortalama kare hatasına sahip olduğu için SVJ modelinden daha iyi performans gösterdiğini ortaya koymuştur. Kalibrasyon bölümünde Dow-Jones 30'da listelenen on şirket kullanılmıştır ve LASVJ modeliyle tahmini temerrüt ve kredi marjı olasılığının SVJ modeline göre daha yüksek olduğu bulunmuştur. Kriz dönemi bu sonuçlar daha net bir şekilde ortaya çıkmaktadır. Bulgular, riskin likidite boyutu ihmal edilirse temerrüt olasılığı ve kredi marjının olması

gerekenden daha düşük olduğunu ortaya koymaktadır ve bu da 2007-2008 finansal krizinin etkisinin tam olarak neden öngörülemediğini açıklamaktadır.

Üçüncü bölümde, oynaklık tahmin yöntemi geliştirilerek Riske Maruz Değer gibi finansal risk araçlarının daha etkin bir şekilde kullanılması amaçlanmaktadır. İyi performans gösteren bir oynaklık tahmini finansal piyasadaki belirsizliğe ışık tuttuğu için bunu modellemek önemli bir görevdir. Bu amaçla oynaklığı modellemek için GARCH tipi modeller ve bir Makine Öğrenmesi modeli olan SVR-GARCH modeli kullanılmıştır ve sonuçlar performans ölçütlerine göre karşılaştırılmıştır. Ampirik analizde, S&P-500 endeksinde yer alan otuz hisse ile 01/01/2010-09/01/2019 dönemini kapsanmıştır. Bulgular, SVR-GARCH'ın oynaklığı tahmin etmede geleneksel modellerden daha iyi performans gösterdiğini ve ayrıca POF ve Basel Trafik Işığı Geri Test yaklaşımlarına dayanan VaR tahmininde daha güvenilir sonuç verdiğini göstermektedir.

Anahtar Kelimeler: Fiyat Etkisi, Likidite, Kredi Riski, Oynaklık, Makine Öğrenmesi ve Risk Yönetimi

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CHAPTER 1

INTRODUCTION

The study of financial markets and how they operate are the subject of the Market Microstructure, which primarily deals with the price formation processes, trading behaviors, spreads and transaction costs. With the development, diversification and deepening of the securities markets, microstructures of the markets have been attracting the attentions of researchers. In this respect, a large body of literature in finance seeks to understand price impact in financial markets. Incoming new information and the process of information diffusion trigger the price changes, which cause price impact, illiquidity problems, and volatility.

Moreover, devastating effects of 2008 crisis highlight the importance of market efficiency, liqudity and volatility. Market Microstructure provides a framework to investigate the relationship between these structures and their nexus with financial market. At this conjecture, revisiting of these concepts is thought to be necessary. Neglected concepts like market resiliency, illiquidity, and volatility have gained importance. Their links need to be investigated.

Market liquidity is defined as the depth of buy and sell orders having an impact on price discovery processes. Liquidity and volatility go hand in hand in that the more liquid the market, the more reliable financial patterns it has.

The market microstructure theory suggests that markets with higher volumes are less volatile. Therefore, it can be derived from traditional market microstructure theories that a liquid market has a higher volume of trading and, therefore, is less volatile. A theoretically verified inverse relationship between liquidity and volatility can be established, indicating that greater volatility can lead to a reduction in the liquidity of a stock market and vice versa. Based on this relationship, the literature suggests that a serious liquidity bottleneck is accompanied by increased volatility and this led to the global financial crisis as well as the spread of contagion effects on the markets in 2007-2009. In this context, researchers try to capture the relationships between liquidity, volatility and trading activity in different markets.

This thesis starts with price impact analysis in chapter two by focusing on the components of price impact such as market resilience and price informativeness. By applying the Projection Theorem, the main variables, namely market resiliency, price impact, informativeness of price, error variance of price, and trade intensity, affecting price formation processes are derived as random variables. Besides, the effect of informativeness of price on these variables are graphed and dicussed.

In the third chapter, liquidity is studied by developing a new model called Liquidity Augmented Stochastic Volatility Model with Jumps. This new model is used to predict credit spreads and probability of defaults more accurately. A simulation analysis is conducted to observe the stability of the proposed model compared to Stochastic Volatility with Jump model and Stochastic Volatility model. Probability of default and credit risk spread are estimated using selected ten Dow-Jones listed companies.

Fourth chapter aims at improving volatility prediction using Machine Learning model called SVR-GARCH (Support Vector Regression GARCH). Accuracy of the SVR-GARCH model predictions are much better than those of traditional models based on Mean Absolute Error and Root Mean Squared Error. Using predictions obtained in the previous part, Value-at-Risk model is estimated. Moreover, backtesting is applied to check the accuracy of the Value-at-Risk results. Proportion of Failures and Basel Traffic Light Approach are used. It turns out that using SVR-GARCH model improves Value-at-Risk calculations and hence provides better financial risk management.

Chapter five concludes.

CHAPTER 2

MODELING THE PRICE IMPACT

2.1 Introduction

The price impact is at the heart of modern finance not solely for its role in price discovery mechanism but also for properly specifying execution cost of a trader. It is, basically, about how much a transaction can alter the market prices and has been intensively under scrutiny over the past two decades.

Despite the attention that the price impact has gained, market efficiency and speed of price informativeness have remained not investigated. With the absence of perfectly efficient market gives opportunity to investors to make profit up until equilibrium price is fully emerged. However, the increase of the Informativeness of price decreases the profit of insider traders. More specifically, the higher the informativeness of price, the quickier the insider traders respond and in turn the lower the profit.

The model suggested here extends the Kyle's [54] model to take into account the market resiliency and price informativeness accounting for market efficiency to some extent. That is, if a market is not efficient then profit opportunites emerge due to the irresiliency until price impact fully emerges. The other extension is to take into account the speed of information by which insiders and noise traders adjust their trade to gain profits.

As the resiliency amounts to market's ability to bounce back from non-equilibrium, it can be thought of a price recovery process. During this process, investors can gain profit until the process completely dies out. Thus, generally speaking, in this setting, two different mechanisms are at work: Market resiliency and price impact.

In Kyle's[54] model, the link between order flow and private information is established by β, which indicates how intensive the insider trade based on the private information she possesses. However, another question here is how informative the price is. Therefore, the model is extended so as to include the informativeness of the prices. Along with this, market marker is assumed to be risk neutral and traders are price sensitive. At the time insiders trade, it takes some time for market maker to adjust prices as it requires some time to reach market clearing prices, so insiders take advantage of this and gain profits. In other words, insiders' trade have an impact on order flow which in turn lead to increases in prices and profits, then noise traders, due to price sensitivity property, adjust their orders accordingly. Over time, as order flows increase market maker sets higher prices and market clearing condition, $\mathbb{E}(v|p_I) = p_I$, where ν is liquidation value and p_I is the price quoted by insiders, is satisfied then profit opportunities disappear until the next private information arrives into the market.

In order to fully understand the mechanism of the price formation process, two new concepts, somewhat related to each other, are needed to be brought into light: Market resilience and informativeness of prices.

The proposed model considers a market with noise traders and insider traders with private information in a multiperiod context. The model includes main components of price formation process, namely market resiliency, price informativeness, market impact, trade intensity, and error variance of price, so that it becomes clearer to comprehend the market dynamics, in particular, in the presence of insider traders.

Thus, this chapter attemps to address the following questions:

- What happens if markets cannot incorporate information into the prices almost instantenously?
- How deeply does informativeness of price affect the price formation process?
- What is the relationships between informativeness of price, price impact, market resilience, trade intensity, and market efficiency?

2.2 The Model

In this model, there are two types of traders: Insiders and noise traders. A trade occurs in N sequential auctions given a certain time period.

$$0 = t_0 < t_1 < \cdots < t_N = 1 \tag{2.1}$$

Assumption 1. *Liquidation value, ν, has a normal distribution with mean p_0 and variance Σ_0.*

$$\nu \sim \mathcal{N}(p_0, \Sigma_0) \tag{2.2}$$

Assumption 2. *Insiders know the liquidation value before trade takes place at time 0 but noise traders do not.*

p_n is the price of a security at n^{th} auction. x represents the quantity traded by insiders and u is the amount traded by noise traders. X_n is the total order flow of all agents at n^{th} auction.

$$X = x + u \tag{2.3}$$

Assumption 3. *Δu_n is the order flow submitted by noise traders at the n^{th} auction. Δu_n is serially uncorrelated and normally distributed with 0 mean and variance $\sigma_\epsilon^2 \Delta t_n$ where Δt is the time interval between auctions.*

$$\Delta u \sim \mathcal{N}(0, \sigma_\epsilon^2 \Delta t_n) \tag{2.4}$$

$$\Delta u_n = u_n - u_{n-1} \tag{2.5}$$

$$\Delta t_n = t_n - t_{n-1} \tag{2.6}$$

Assumption 4. *Insiders know liquidation value ν. There are J insiders, $j = 1, ..., J$ each insider j knows the liquidation value at time 0 and maximizes the profit level π at auctions 1,...,N.*

$$\pi_n = \sum_{i=1}^{N}(v - p_i)x_i \tag{2.7}$$

Assumption 5. *Let x_n and p_n denote the aggregate position of insiders and market clearing price, respectively. At each auction, insiders maximize their profit based on the liquidation value of the asset, which is already known by insiders. In addition, Δx_n represents the order flow of insiders at the n^{th} auction and insiders know both ν and past prices.*

$$\Delta x_n = x_n - x_{n-1} \tag{2.8}$$

Thus, insiders' position at time n is given by:

$$x_n = X_n(p_1, ..., p_{n-1}, v) \tag{2.9}$$

where n=1,...,N and X_n is a measurable function.

Assumption 6. *Insider traders' expected profit function has the following quadratic form:*

$$\mathbb{E}(\pi_n|p_1, ..., p_{n-1}, \nu) = \alpha_{n-1}(\nu - p_{n-1})^2 + \delta_{n-1} \tag{2.10}$$

Assumption 7. *There are two cases in one case there is instantenous equilibrium with price impact. In the other case, equilibrium does not occur instantenously. Once a trade is executed, market resiliency κ partially accounts for the deviation from previous equilibrium price and speed of reversion to the new equilibrium price. In the first case, the equilibrium is instantenous and price impact is permanent as in [54]. In the second case, the equilibrium does not occur instantenously but price impact is transient and we observe market resiliency.*

The equation of price takes the following form:

$$P(x+u) = p_0 + \lambda_n(x_n + u_n) \tag{2.11}$$

for all $n = 1, ...N$ and where n is the number of transactions, λ denotes the price impact.

Insiders also know the pricing rule of market maker during market resiliency and solve the following equation:

$$P(x+u) = p_0 + \kappa_n(\nu - p_{n-1})(x_{[0,n)} + u_{[0,n)}) \tag{2.12}$$

where κ represents the market resiliency.

Assumption 8. *During sequential auctions, informativeness of price at the presence of t insiders, τ_t, about ν is incorporated into prices and it affects the order flow by the following equation.*

$$\Delta X_n = \tau_t \beta_n(\nu - p_{n-1})\Delta t_n \tag{2.13}$$

where τ_t is the informativeness of price, t is the number of insider traders, and t=1,...,T.

Together with the market resiliency component, the speed of price informativeness at which information is fully incorporated into prices is ignored in price formation process. Inclusion of τ allows us to account for the price informativeness. The mechanism is as follows: Market marker sets prices based on the order book containing both insiders and noise traders' orders. As insiders have knowledge about ν, it gives insiders an advantage. Market marker has noisy signal where information coming from insiders is aggregated.

As the number of auctions increase price quotations become more predictable. To this end, insiders try to exploit the profit as fast as possible. Thus, private information is incorporated into prices fast and all this process is represented by τ.

Inclusion of the market resiliency term is one of the contributions of this study. It enables us to distinguish the price impact and market resiliency. More specifically, as the trade is executed, the assumption of instantenous market equilibrium set by market maker is relaxed in a way to show that there is time lag to reach a new equilibrium. This assumption implies that market efficiency is closely related to the market resiliency. Following [35], market efficiency is defined as the degree to which prices reflect all available information and the adjustment to the new prices occurs almost instantenously. However, in this study, by assuming the presence of market resiliency on the price formation process, the idea of "almost instantenous" is relaxed.

This idea is motivated by [15], who defines the efficient market as one in which price is within a factor of two. Shortly, as trading activity increases by a factor of eight, then market resilience increases by a factor of four and market efficiency increases by a factor of two.

Market resiliency and market efficiency, therefore, go hand in hand. To this respect, market resiliency defined as time elapsed to reach a new equilibrium after trade executions. This is modeled in a way to show the time elapsed to reach the full price impact.

That is to say, as the price impact is fully emerged, the magnitude of deviation from the previous equilibrium price and the speed of reversion to the new equilibrium are accounted for the market resiliency. Shortly, in the process of new price formation, price impact λ, market resiliency κ as well as informativeness of the price τ are the main factors.

During market resiliency, there exists a unique solution for optimal number of order x^*_{res} of a typical insider given in equation 2.15

$$\begin{aligned} \mathbb{E}(\pi_n|p_1,...,p_{n-1},v) &= \mathbb{E}((v-p_n(x+u))x \\ &= \mathbb{E}(v-h_\kappa-\kappa_n(v-p_{n-1})(x+u))x \end{aligned} \tag{2.14}$$

where $p_n = \kappa(v-p_{n-1})(x+u)+h_\kappa$, h_κ is the drift term, and, accordingly, $\alpha = \frac{-h_\kappa}{2\kappa(v-p_{n-1})}$ and $\beta = \frac{1}{2\kappa(v-p_{n-1})}$.

$$x^*_{res} = \text{argmax}_{x\in\mathbb{R}}\mathbb{E}(\pi_n|p_1,...,p_{n-1},v) = \frac{v-h_\kappa}{2\kappa(v-p_{n-1})} \tag{2.15}$$

Similarly, at time n, when price impact is in effect, there exists a unique solution for optimal number of order x^*_{impact} of insiders given in equation 2.17

$$\begin{aligned} \mathbb{E}(\pi|p_1,...,p_{n-1},v) &= \mathbb{E}((v-\lambda_n(x_n+u_n)-h_\lambda)x_n) \\ &= (v-\lambda_n x_n - h_\lambda)x_n \end{aligned} \tag{2.16}$$

$$x^*_{impact} = \text{argmax}_{x \in \mathbb{R}} \mathbb{E}(\pi | p_1, ..., p_{n-1}, v) = \frac{v - \mu}{2\lambda} \tag{2.17}$$

where $p_n = \lambda_n(x_n + u_n) + h_\lambda$. So, $\alpha_n = \frac{-h_\lambda}{2\lambda_n}$ and $\beta_n = \frac{1}{2\lambda_n}$.

As is known, p_0 is the equilibrium price and p_{n-1} is the price at time $n - 1$.

$$v - p_{n-1} = \theta + \epsilon \tag{2.18}$$

and

$$\mathbb{E}(v - p_{n-1}) = \eta \tag{2.19}$$

where θ represents the information observable to insider and ϵ is observable to noise traders. Both are random variables and $\theta \sim \text{Pois}(\delta)$ and $\epsilon \sim \mathcal{N}(0, \sigma^2)$.

$$\begin{aligned} p &= \mathbb{E}(\nu|X) \\ \mathbb{E}(\nu|X) &= \mathbb{E}(\nu) + \frac{cov(\nu, X)[(x - \mathbb{E}(X))]}{var(X)} \end{aligned} \tag{2.20}$$

where $X = x + u$ and $p_n = \mathbb{E}(\nu | x_1 + u_1, ..., x_n + u_n)$

$$\mathbb{E}(v|v - p_{n-1}, X) = \mathbb{E}(v) + \frac{\text{cov}(v, (v - p_{n-1})X_n)(v - p_{n-1} - \mathbb{E}(v - p_{n-1})(X_n - \mathbb{E}(X_n))}{var((v - p_{n-1})X_n)}$$

$$\begin{aligned} \text{cov}(v, (v - p_{n-1})X) &= \mathbb{E}[(v - p_0)((v - p_{n-1})(\alpha_n + \tau_t\beta_n v)) - \mathbb{E}((v - p_{n-1})(\alpha_n + \tau_t\beta_n v))] \\ &= \mathbb{E}[(v - p_0)(v\alpha_n + \tau_t\beta_n v^2 - p_{n-1}\alpha_n - p_{n-1}\tau_t\beta_n v) - (\theta_n + \epsilon_n)(\alpha_n + \tau_t\beta_n p_0)] \\ &= \mathbb{E}[v^2\alpha_n + v\tau_t\beta_n v^2 - vp_{n-1}\alpha_n - p_{n-1}\tau_t\beta_n v^2 - p_0 v\alpha_n - p_0\tau_t\beta_n v^2 + p_0 p_{n-1}\alpha_n \\ &\quad + p_0 p_{n-1}\tau_t\beta_n v - v\theta_n\alpha_n - v\tau_t\beta_n\theta_n p_0 + \theta_n p_0\alpha_n + \tau_t\beta_n\theta_n p_0^2] \\ &= p_0^2\alpha_n + p_0\tau_t\beta_n\mathbb{E}(v^2) - p_0 p_{t-1}\alpha_n - p_{t-1}\tau_t\beta_n\mathbb{E}(v^2) - p_0^2\alpha_n - p_0^3\tau_t\beta_n + p_{n-1}p_0\alpha_n \\ &\quad + p_0^2 p_{n-1}\tau_t\beta_n - p_0\theta_n\alpha_n - \beta_n\theta_n p_0^2 + \theta_n p_0\alpha_n + \beta_n\theta_n p_0^2 \end{aligned}$$

After cancelling out some of the terms,

$$\mathrm{cov}(\nu, (\nu - p_{n-1})X)) = p_n \tau_t \beta_n \mathbb{E}(\nu^2) - p_{n-1}\tau_t\beta_n\mathbb{E}(\nu^2) + p_0^2 p_{n-1}\tau_t\beta_n - p_0^3\tau_t\beta \tag{2.21}$$

$$= -\tau_t\beta_n p_{n-1}\Sigma_n + \tau_t\beta_n p_0\Sigma_n$$

where $\Sigma_n = \mathrm{var}(v|\Delta x_1 + \Delta u_1, ..., \Delta x_n + \Delta u_n)$

$$\mathrm{var}((v - p_{n-1})X) = (\sigma_\theta^2 + \sigma_\epsilon^2)(\beta^2\Sigma_n + \sigma_u^2) \tag{2.22}$$

When market resiliency rules the market, the price equation is given by:

$$p_n = p_{n-1} + \kappa_n(v - p_{n-1})(\Delta x_n + \Delta u_n) + h_\kappa \tag{2.23}$$

Market resiliency term turns out to be;

$$\kappa_n = \frac{-\tau_t\beta p_{n-1}\Sigma_n + \tau_t\beta p_0\Sigma_n}{(\sigma_\theta^2 + \sigma_\epsilon^2)(\tau_t^2\beta^2\Sigma_n\,\Delta t_n + \sigma_u^2)} \tag{2.24}$$

and

$$h_\kappa = p_0 + \kappa[(-\alpha + \beta p_0)(\theta + \epsilon)\eta\mathbb{E}(X)] \tag{2.25}$$

As price reverts back to the new position at which market is efficient then only effect that left out comes from order-related term represented by λ_n. Since, the effect of market resiliency term dies out, expectation of ex-post liquidation value conditioning only on order flow equals to price.

Assumption 9. *The market maker sets the price according to following pricing rule and earn zero profit. Depending on the market maker's aggreagate order flow $\Delta x_n + \Delta u_n$ observation, she sets p_n, the price at time n. Market marker does not observe only current order flow rather she observes previous order flows, too. Market marker solves recursive linear equilibrium in which $\lambda_1, ...\lambda_n$ exist for $n = 1, ...N$.*

$$p_n = p_{n-1} + \lambda(\Delta x_n + \Delta u_n) \tag{2.26}$$

$$p_n = p_{n-1} + \kappa_n(\nu - p_{n-1})(\Delta x_{[0,n)} + \Delta u_{[0,n)}) \tag{2.27}$$

So, the market marker should select best likely estimation of $\mathbb{E}(v|u+x)$ which is equal to price. Market marker applies maximum likelihood estimation where ν and X are normally distributed and β, λ, and κ are least square estimators obtained from the solution of following problem.

$$\mathbb{E}((v-p(X))^2) \tag{2.28}$$

Assumption 10. *Price impact comes from the private information of the insider and this widens the gap between the liquidation value and price. This gap, in turn, transmits a signal to some of the insider traders' about the value of ν conditional on the order flow at that auction. However, due to the market inefficiency, some insider traders react with a lag and this leads to a "sequential auction equilibrium".*

Similar to [46], noise traders are risk-neutral and they have ratinal rational expectations. They adjust their ' conditional on insiders' action at each auction. In this setup, profit is recursively determined by:

$$\pi_n = (\nu - p_n)\Delta X_n + \pi_{n+1} \tag{2.29}$$

So,

$$\mathbb{E}(\pi|p_1, p_2, ..., \nu) = \max_{\Delta_x}\mathbb{E}[(\nu - p_n)\Delta x + \alpha(\nu - p_n)^2 + \delta_n|p_1, p_2, ...p_n - 1, \nu] \tag{2.30}$$

When resiliency rules the market, the price equation is given by:

$$p_n = p_{n-1} + \kappa_n(\nu - p_{n-1})(\Delta x_n + \Delta u_n) + h \tag{2.31}$$

From here, it is easy to derive β_{n,κ_n}:

$$\begin{aligned}\mathbb{E}(\pi_n|p_1, p_2, ..., p_{n-1}, \nu) = &\max_{\Delta_x}\{(\nu - p_{n-1} - \kappa_n(v - p_{n-1})\tau_t\Delta_x - h)\Delta_x + \alpha_n(\nu - p_{n-1} \\ &- \kappa_n(\nu - p_{n-1})\tau_t\Delta_x - h)^2 + \alpha_n\lambda_n^2\sigma_u^2\Delta_{t_n} + \delta_n\}\end{aligned} \tag{2.32}$$

Maximization of the given problem above with respect to Δx yields:

$$\Delta x = \frac{(v - p_{n-1} - h)\big(1 - 2\alpha_n^2 \kappa \tau_t (v - p_{n-1})\big)}{(2\kappa\tau_t(v - p_{n-1}))\big(1 - \alpha^2 \kappa \tau_t (v - p_{n-1})\big)} \tag{2.33}$$

λ and μ are derived by applying projection theorem.

$$\begin{aligned} p_n &= p_0 + \frac{\tau_t \beta \Sigma_n}{\tau_t^2 \beta^2 \Sigma_n + \sigma_u^2}((x+u) - \mathbb{E}(x+u)) \\ &= \frac{\tau_t \beta \Sigma_n}{\tau_t^2 \beta^2 \Sigma_n + \sigma_u^2} X + \frac{-\alpha \tau_t^2 \Sigma_n + p_t \sigma_u^2}{\tau_t^2 \beta^2 \Sigma_n + \sigma_u^2} \end{aligned} \tag{2.34}$$

Price impact, denoted by λ, and the constant term takes the following form:

$$\lambda_n = \frac{\tau_t \beta_n \Sigma_n}{\tau_t^2 \beta_n^2 \Sigma_n \Delta t_n + \sigma_u^2} \tag{2.35}$$

and

$$h_\lambda = \frac{-\alpha_n \tau_t^2 \Sigma_n + p_n \sigma_u^2}{\tau_t^2 \beta_n^2 \Sigma_n + \sigma_u^2} \tag{2.36}$$

2.3 Equilibrium

Characteristics of the equilibrium is defined by α_n, β_n, δ_n, λ_n, τ_t, Σ_n, κ_n and they are derived in the following proposition.

Proposition 1.

$$\Delta X_n = \tau_t \beta_n (\nu - p_{n-1}) \Delta t_n \tag{2.37}$$

$$\Delta p_n = \lambda_n (\Delta x_n + \Delta u_n) + \tag{2.38}$$

$$\Delta p_n = \kappa_n (\nu - p_{n-1})(\Delta x_{[0,n)} + \Delta u_{[0,n)}) \tag{2.39}$$

$$\Sigma_n = var(\nu | \Delta x_1 + \Delta u_1, ..., \Delta x_n + \Delta u_n) \tag{2.40}$$

$$\mathbb{E}(\pi_n | p_1, ..., p_{n-1}, \nu) = \alpha_{n-1}(\nu - p_{n-1})^2 + \delta_{n-1} \tag{2.41}$$

for all $n = 1, 2, ...N$ and where n is the number of transactions, β shows the intensity of the trade, κ represents the market resiliency, λ denotes the price impact, p is the transaction price, ν is the liquidation value, and finally τ indicates the informativeness of price process.

Given Σ_0, the constants, τ_t, β_n, λ_n, κ_n, α, Σ_n, provide a unique solution to the system of equations given in the Proposition 1.

$$\kappa_n = \frac{-\tau_t \beta p_{n-1}\Sigma_n + \tau_t \beta p_0 \Sigma_n}{(\sigma_\theta^2 + \sigma_e^2)(\tau_t^2 \beta^2 \Sigma_0 \Delta t + \sigma_u^2)} \tag{2.42}$$

$$\lambda_n = \frac{\tau_t \beta \Sigma_0}{\tau_t^2 \beta^2 \Sigma_0 + \sigma_u^2} \tag{2.43}$$

$$\tau_t = \frac{1}{(1-\rho^2)\sigma_v^2} \tag{2.44}$$

$$\beta_{n,\kappa} = \frac{1 - 2\kappa \alpha_n \tau_t (v - p_{n-1})}{(2\kappa \tau_t (v - p_{n-1})[1 - \alpha \kappa \tau_t (v - p_{n-1})]} \tag{2.45}$$

$$\beta_{n,\lambda} = \frac{1 - 2\lambda_n \alpha_n \tau_t}{2\lambda_n \tau_t [1 - \lambda_n \alpha_n \tau_t]} \tag{2.46}$$

$$\alpha = \frac{\big(1 - 2\alpha_n^2 \kappa \tau_t (v - p_{n-1}\big)}{(2\kappa \tau_t (v - p_{n-1}))\big(1 - \alpha^2 \kappa \tau_t (v - p_{n-1})\big)} \tag{2.47}$$

$$p_n = \mathbb{E}(v | x_1 + u_1, ..., x_n + u_n, v - p_{n-1}) \tag{2.48}$$

$$\Sigma_n = (1 - \mathbb{I}_{[0,n)]} \tau_t \beta_\kappa \kappa_n \Delta_{t_n} - \tau_t \beta_\lambda \lambda_n \Delta_n)\Sigma_{n-1} \tag{2.49}$$

At this point, it is worth discussing the other contribution of this study that is the informativeness of price denoted by τ formulated by $\tau = [Var(v|p_I)]^{-1}$ which is posterior variance of price informativeness. This concept is first introduced by Vives[74] to formulate the speed of dispersed private information about the liquidation value.

2.4 Modeling the Price Informativeness

Informativeness of price, τ, is a latent variable. The task is to find the joint posterior distribution of state, $\sigma^2_{(v|p_{I,t})}$, and related parameters. As stated by [45] modeling τ is a partially observed process in which the density cannot be presented in an integrated form. Thus, conditional likelihood of the process can be derived by conditioning on parameters called hidden state of the process.

To estimate the informativeness of price, Bayesian approach is employed in that all unobserved variables can be modelled by their joint probability distribution.

$$\sigma^2_{(v|p_I)} = \frac{\sigma^2_{(p_I|v)}\sigma_v^2}{\sigma^2_{p_I}} \tag{2.50}$$

Or similarly

$$\sigma^2_{(v|p_I,\theta)} = \frac{\sigma^2_{(p_I|v,\theta)}\sigma^2_{(v|\theta)}}{\sigma^2_{(p_I|\theta)}} \tag{2.51}$$

where $\sigma_v^2 = \Sigma_0$, $\sigma^2_{(p_I)} = \Sigma_{p_I}$, and $\sigma^2(p_I|v) = \Sigma[p_I - \mathbb{E}(p_I|v)]^2\rho(p_I|v)$

- $\sigma^2_{(v|p_I,\theta)}$ observed posterior variance
- $\sigma^2_{(p_I|v,\theta)}$ variance of the state
- $\sigma^2_{(v|\theta)}$ prior variance

As the model described above is linear, then Kalman Filter technique can be good tool to employ. Kalman filter is an estimator in which parameters of interest can be measured recursively. It is an optimal estimator in the sense that it minimizes the mean square error of the estimated parameters.

To start with, all information of informed agents are pooled and labeled as ex-post liquidation value v that is unknown and normally distributed with mean p_0 and Σ_0. The ex-post liquidation value is an autoregressive process of order 1, AR(1), process.

State-space model (SSM) is employed to draw inference about the latent variable, i.e., price offered by insider. SSM is a useful tool for analyzing a dynamic system. The main idea behind the SSM is to capture the dynamics of an observed variable in terms of a latent variable known as a state vector. In these models, observed variables are presumed to be related to the state vector via a system of equations [43].

SSM outweighs linear regression model in some aspects one of which is to rely on the characteristics of state variables and the nexus between between observed and state variables. Linear regression employs exogenous variables to disaggregate the explained and unexplained variation.

A state–space model can be written in the following form:

$$\nu_t = \phi\nu_{t-1} + \sigma_2\eta_t \text{ where } \eta_t \sim \mathbb{N}(0, Q) \tag{2.52}$$

$$p_{I,t} = \nu_t + \zeta_{I,t} w_{I,t} + \sigma_1 \epsilon_t \text{ where } \epsilon_t \sim \mathbb{N}(0, R) \tag{2.53}$$

where $p_{I,t}$ is assumed to be conditionally independent given the state v_t and its distribution is $p(p_{I,t}|v_t, \theta)$. Parameter set is $\theta = (\phi, \sigma_2, \sigma_1, \zeta)$

Kalman Filter algorithm is as follows:

Projection $\hat{p'}_{I,k+1} = \phi \hat{p}'_{I,k}$

Update covariance $P_{k+1} = \phi P_k \phi^T + Q$

Residual covariance $S_k = \zeta_k P_{k|k-1} \zeta_k^T + R_k$

Kalman Gain $K_k = P'_k \zeta^T (\zeta P'_k \zeta^T + R)^{-1}$

Update estimate $\hat{p}_{I,k} = \hat{p'}_{I,k} + K_k(z_k - \zeta \hat{p'}_{I,k})$

Update covariance $P_k = (I - K_k \eta) P'_k$

Briefly, Kalman filter has time and measurement updates. In the time update phase, estimation is done based on the previous time step. As for the measurement updates, estimate procedure is conducted based on the current time step.

In this study, Nelder–Mead simplex method is applied to the following likelihood function:

$$f(\theta) = \sum_{t=1}^{T} ((\nu_{t+1} - Hp_{I,t+1|t})^T S_{t+1}^{-1} (\nu_{t+1} - Hp_{I,t+1|t}) + \log|S_{t+1}) \tag{2.54}$$

Before initating the simulation, initial values assigned to the parameters are:

$$(\phi^0, \eta^0, Q^0, R^0, v^0, P^0) = [2, 1, 0.2, 0.2, 5, 0.05]$$

$$(\hat{\phi}, \hat{\eta}, \hat{Q}, \hat{R}, \hat{v}, \hat{P}) = [0.9873, -0.0789, 0.8336, 0.8613, 21.2571, -0.0437] \tag{2.55}$$

By applying Kalman Filter, price offered by insider is measured. In order to calculate, $\sigma^2_{(v|p_I)}$, first $\sigma^2_{(p_I|v)}$ should be obtained. By assumption, both price offered by the in-

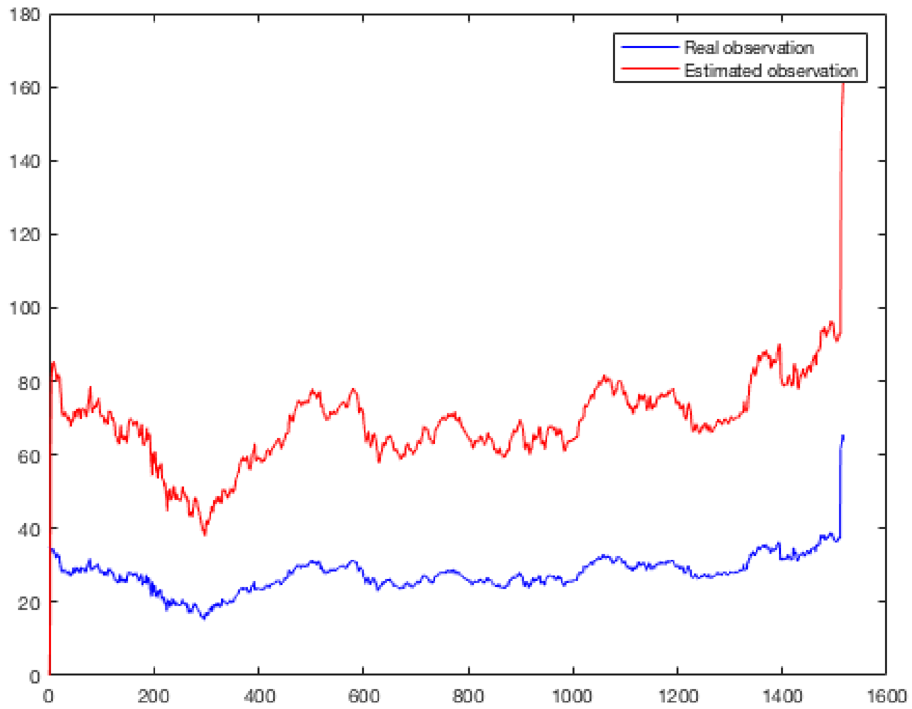


Figure 2.1: Simulation based on Kalman Filter and Comparison

sider and market value of the asset are normally distributed and therefore conditional distribution of p_I conditioning on the market liquidation value turns out:

$$P_I|v \sim \mathcal{N}(\mu_{P_I} + \rho\frac{\sigma_{P_I}}{\sigma_v}(v - \mu_v), \sigma^2(1 - \rho^2)) \tag{2.56}$$

Proof.

$$\sigma^2_{P_I|\nu} = \int_{-\infty}^{\infty} (P_I - \mu_{P_I|\nu})^2 g(p_I|\nu) dP_I \tag{2.57}$$

Substitute $\mu_{P_I|\nu}$ with $\mu_{P_I} + \rho\frac{\sigma_{P_I}}{\sigma_\nu}(\nu - \mu_\nu)$

$$\sigma^2_{P_I|\nu} = \int_{-\infty}^{\infty} (P_I - \mu_{P_I} + \rho\frac{\sigma_{P_I}}{\sigma_v}(v - \mu_\nu))^2 g(P_I|\nu) dP_I \tag{2.58}$$

Multiply both sides by $f_v(v)$, then it turns out:

$$\int_{-\infty}^{\infty}\sigma^2_{P_I|v}f_\nu(\nu)d\nu = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}(P_I-\mu_{P_I}-\rho\frac{\sigma_{P_I}}{\sigma_v}(\nu-\mu_v))^2 g(P_I|\nu)f_\nu(\nu)dP_I d\nu$$

$$\begin{aligned}
\sigma^2_{P_I|\nu}\int_{-\infty}^{\infty}f_\nu(\nu)d\nu &= \mathbb{E}[(P_I-\mu_{P_I})-(\rho\frac{\sigma_{P_I}}{\sigma_\nu}(\nu-\mu_\nu))]^2 \\
&= \mathbb{E}[(P_I-\mu_{P_I})^2]-2\rho\frac{\sigma_{P_I}}{\sigma_v}\mathbb{E}[(\nu-\mu_\nu)(P_I-\mu_{P_I})] \\
&+ \rho^2\frac{\sigma^2_{P_I}}{\sigma^2_v}\mathbb{E}[(\nu-\mu_\nu)^2] \\
&= \sigma^2_{P_I}-2\rho\frac{\sigma_{P_I}}{\sigma_v}\rho\sigma_v\sigma_{P_I}+\rho^2\frac{\sigma^2_{P_I}}{\sigma^2_v}\sigma^2_v
\end{aligned}$$

After simplification:

$$= \sigma^2_{P_I}(1-\rho^2)$$

Thus,

$$(\sigma^2_{(\nu|p_I)})^{-1} = \frac{1}{(1-\rho^2)\sigma^2_v} \tag{2.59}$$

□

To sum up, price informativeness is affected by two factors: Correlation between market liquidation value and price quoted by insider, that is variance of the market liquidation value σ^2_v. If correlation between market liquidation value and price offered by insider is positive and increasing, this boosts the price informativeness or vice versa. On the other hand, any increase in the variance of market liquidation value diminishes the price informativeness.

These findings confirm [74]'s propositions in which ρ and ν are defined as positive coefficient of risk aversion and ex-post liquidation value, respectively.

Information is revealed as soon as insider trades. Market maker observes the order flow while noise traders observe price.

$$p_n = \mathbb{E}\{\nu|x_1+u_1,...,x_n+u_n,p_0-p_{n-1}\} \tag{2.60}$$

$$x(\nu) = \tau_t\beta_n(\nu-p_0) \tag{2.61}$$

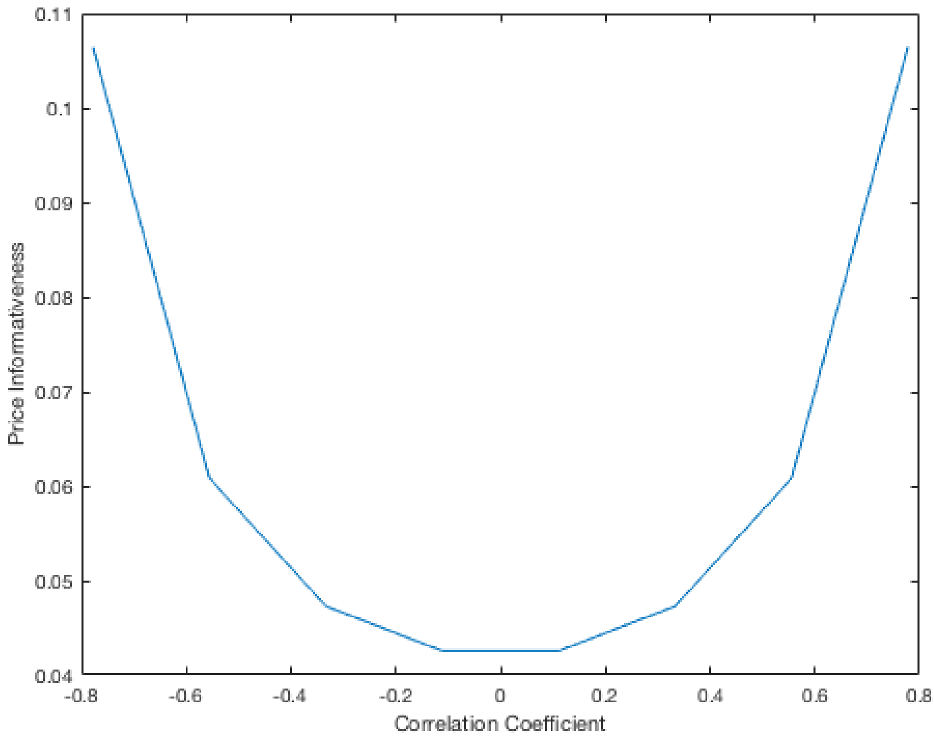


Figure 2.2: Reaction of Price Informativeness In the Presence of Correlation

where $\tau_t = var(\nu|p_I)^{-1}\Delta t$.

In the same vein,

$$\Sigma_n = \frac{\sigma_u^2 \Sigma_{n-1}}{\tau_t^2 \beta_n^2 \Sigma_{n-1} \Delta_{t_n} + \sigma_u^2} \tag{2.62}$$

for all trades n=1,...,n, as provided by [46], one of the boudary conditions is $\alpha_n = 0$. Then it turns out we are able to iterate Σ_n by simply assuming Σ_0 a exogenous variable. Now, other boundary conditions are:

$$\Sigma_n = (\frac{\nu - p_{n-1} - 1}{\nu - p_{n-1}})\Sigma_{n-1} \text{ where } \Sigma_0 = 1 \tag{2.63}$$

$$\beta_\kappa = \frac{1}{2\kappa\tau_t(\nu - p_{n-1})\Delta_{t_n}} \tag{2.64}$$

$$\kappa = \Big[\frac{(-\tau_t p_{n-1}\Sigma_n + \tau_t p_0 \Sigma_n)(2\tau_t(\nu - p_{n-1})) - (\tau_t^2 \Sigma_n \Delta t_n)(\delta + \sigma_e^2)}{4\tau_t^2(\nu - p_{n-1}\sigma_u^2)}\Big]^{1/2} \tag{2.65}$$

$$\beta_\lambda = \frac{1}{\lambda(\tau_t + 1)}\Delta_{t_n} \tag{2.66}$$

$$\lambda = \Big[\frac{\tau_t \Sigma_n(\tau_t + 1) - \tau_t^2 \Sigma_n \Delta_{t_n}}{\sigma_u^2(\tau_t + 1)^2}\Big]^{1/2} \tag{2.67}$$

2.5 Comparative Statistics

In this part of the study, how market resiliency, κ, price impact, λ, and error variance of prices, Σ response to the changes in informativeness of price. The results and illustrations are provided below.

- Given other parameters constant, price impact tends to increase as the informativeness of price decreases
- Given other parameters constant, trade intensity tends to decrease as the informativeness of price increase
- Given other parameters constant, error variance of price tends to increase as the informativeness of price decreases
- Given other parameters constant, market resiliency tends to increase as the informativeness of price increases

2.6 Illustrations of Comparative Analysis

Liquidation value as a business valuation is a controversial topic because there is a lack of unified framework about how to value it. Thus, [27]'s approach is embraced so that liquidation value is the book value of assets excluding intangible ones and it is adjusted with inflation.

Figure 2.6 exhibits the relationship between the informativeness of price, τ_t, and price impact, λ. This figure shows that there is positive correlation between these two variables but the slope of the relationship varies. More specifically, as the informativeness of price increases, the price impact raises about the same magnitude. However, the reaction of price impact declines as the informativeness of price increases simply due to the fact that the insider traders become very well informed with the high price informativeness and this, in turn, result in relative small change in prices. At the extreme, the informativeness of price has very small impact on the price.

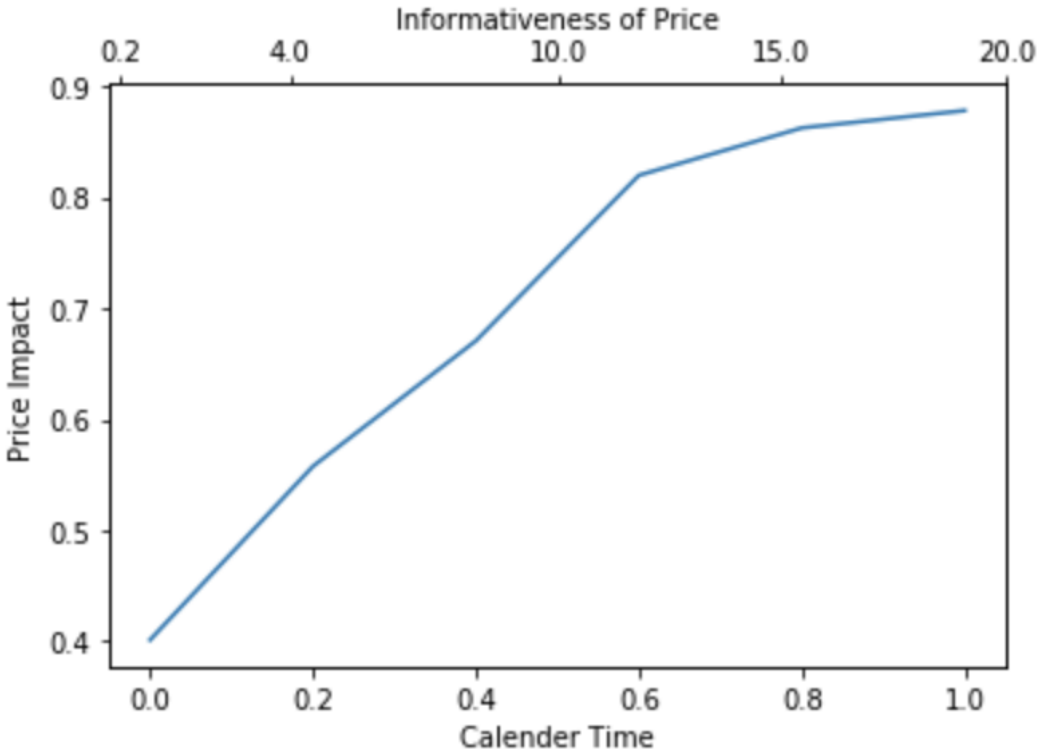


Figure 2.3: Price Impact and Price Informativeness

Figure 2.6 exhibits the nexus between the intensity of trade and the informativeness of price. As the information becomes public it is quite natural to expect an increase in the intensity of trade. More specifically, [37] states that if the focus is on the insider traders, the news informativeness induces them to trade more aggresively. Eventually, trading volume raises and the insider traders capture larger share of trading volume, which in turn increases the trade intensity among speculators. However, in this setup, trade intensity is defined as the intensity of insider's trade depending on the private information he has. Accordingly, an increase in price informativeness should result in less trade intensity because it is priorly assumed that likelihood of insider traders trading ahead of news is not considered as suggested by [37].

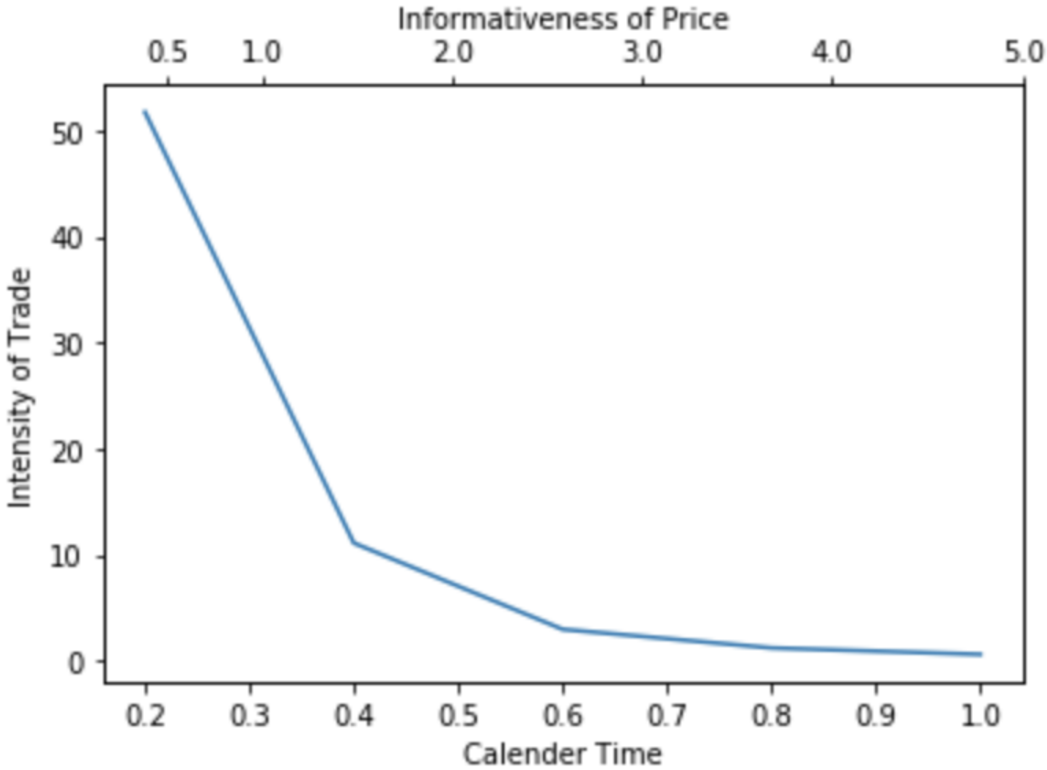


Figure 2.4: Trade Intensity and Price Informativeness

Theoretically, it is expected that the price informativeness results in lower error variance in prices, Σ, in that as the information about new prices travel around, insider traders revise her position, which led to a drop in error variance of price. Figure 2.6 confirms this observation and shows that as the price informativeness raises, error variance first decreases slowly, then it begins to drop sharply and at the extreme level of price informativeness, error variance of price becomes very low and so does the reaction to the price informativeness.

The main reason behind this asymmetric reaction of error variance of price against the informativeness of price may be the absorption of the information by insider traders. In other words, at the low price informativeness level, not all insider traders have the relevant information but as the number of insider traders endowed with the information increases, eror variance decreases in a very fast way. At the high level of price informativeness, due to the well endowed insider traders, the error variance reduces to a very low level and the magnitude of the reaction of error variance of price to the informativeness of price shrinks considerably.

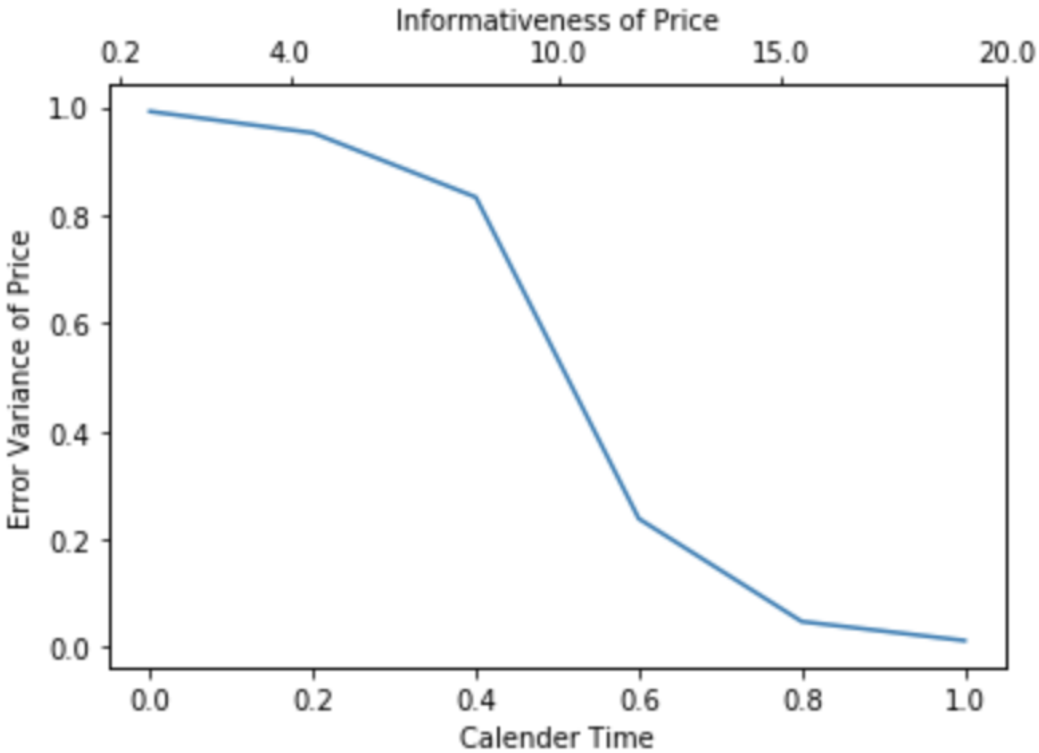


Figure 2.5: Error Variance of Price and Price Informativenesss

The informativeness of price is an indicator of market efficiency and as [55] denotes market efficiency and market resiliency are two sides of the same coin in that resiliency is greater with higher pricing accuracy or lower error variance. Aside from this indirect interpretation, intiuitively, market resiliency would be higher in a market where information travel fast because every traders in that market quickly responds to this public information.

In this regard, it is anticipated to have positive relationship between the informativeness of price and market resiliency. Figure 2.6 confirms this observation. Accordingly, market resiliency is low at low level of the price informativeness and conversely market resiliency is on the rise when price informativeness is high.

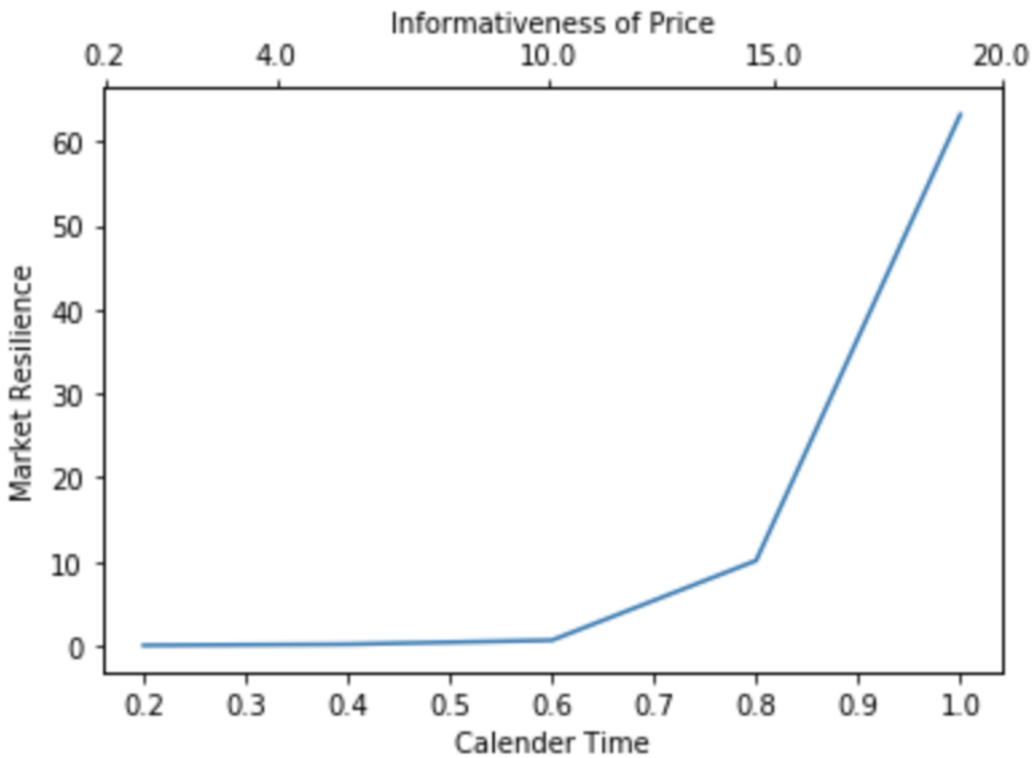


Figure 2.6: Market Resiliency and Price Informativeness

2.7 Conclusion

To the best of my knowledge, the model developed this chapter is the first model which takes into account both market resiliency and price informativeness together in the framework of an asymmetric information model. This model considers a market with insider traders and noise traders in a multiperiod context. The analyses of the model provides market resiliency, price informativeness, market impact, trade intensity, and error variance of price so that it becomes clearer to comprehend the market dynamics, in particular, in the presence of insider trader.

The results suggest that the informativeness of price is positively related to the price impact and market resiliency and negatively related to trade intensity and error variance of prices. However, the shape and the magnitude of these relationships vary and none of them is linear.

As the results of this chapter show that asymmetric information plays an important role in financial markets. When insiders dominate in financial markets they cause inefficiencies with high price impacts. This affects the efficiency in the market.

CHAPTER 3

CONSIDERING LIQUIDITY IN CREDIT SPREAD PREDICTON

3.1 Introduction

Risk management is an evolving process requiring the optimal management of highly complicated and challenging tasks in an uncertain dynamic environment. Identifying, characterizing and modelling the risks that financial decision makers face is essential in tackling with them. One of the most significant and prevalent risks that economies encounter in micro and macro scales is the credit risk caused by companies' defaults. Hence qualitative and quantitative modelling and predicting credit risk is essential to reduce this type of risk to its minimal levels. Besides, the recent financial crises revealed the importance of the liquidity risk which can lead to detrimental price distortions even when firms are solvent. These two types of risks are not independent from each other and in fact they are closely linked via multiple channels, see [71]. This study aims at modelling these risks together.

The standard asset pricing modelling is unrealistically characterized by perfectly liquid markets where unlimited amounts of assets are traded at no cost between the price taking financial decision makers [2]. However, the financial markets are very susceptible to illiquidity problems and liquidity risk is prevalent as the global mortgage crisis outbroken in 2007-2008 made it so visible. The debates about the lack of liquidity risk in asset price modelling have been intensified with the long running effects of the crises.

The most widely used asset valuation model is the Black Scholes model [16] and its improvements by Merton [61] led it to be also called Black Scholes Merton (BSM) model. This prominent model does not take into account illiquidity problems and hence liquidity risk. In this study we aim to tame the BSM model to incorporate the liquidity risk. In this direction, first we model liquidity process using Cox, Ingersoll and Ross [26] (CIR) model. Then we introduce a stochastic volatility jump model. Finally we integrate the liquidity process modelled with CIR [26] to the stochastic volatility jump model and we derive our *Liquidity Augmented Stochastic Volatility Jump (LASVJ) Model*.

By completing this task, not only practitioners such as banks and credit rating agencies, but also academics have a tool to estimate the probability of default and credit spread which goes hand in hand with credit risk in that a raise in credit risk would cause an increase in credit spread. Standard asset pricing models in which markets are assumed to be perfectly liquid that is another way of saying that markets are frictionless. Frictionless amount to trade at no cost and agents are price takers [76].The model proposed in this study helps explain how asset prices are affected by liquidity. To do that, liquidity augmented stochastic volatility with jump model is developed and it is emprically tested both via simulation and calibration techniques.

Traditionally, to predict the probability of default and credit risk spread, first market value of assets should be estimated which is done by the fair value of the assets at the end of measurement period. However, inclusion of liquidity dimension provides a tool to value an asset on a real time basis.

There have been numerous attempts to price an asset and some of them are frequently used in finance literature. The most prominent pricing model is the Black Scholes model[16] proposing easier way to pricing an option which is quite a complex task due to the number of factors affecting the option pricing. The attraction of this model lies in its usage in a broad range of areas in which credit risk is the central concern of this study. Black Scholes model first came up with partial differential equation but later on Merton [61] revisited and describe this model by stochastic calculus and the model turns out to be known as Black Scholes Merton (BSM) model.

As in many models, BSM model employ interest rate to discount future prices. However interest rate, to some extent, diverge from real market dynamics from time to time and it takes some time to settle in. Together with this, uncertainty is the solely source of risk in traditional asset pricing models however it is far from reality. To fill the gap between financial models and real market dynamics, liquidity dimension is introduced. Incorporating liquidity, model can adjust itself better to the developments in the financial markets in that liquidity both affects the required returns of assets and also the level of uncertainty. Thus, liquidity is quite an important dimension in estimating probability of default.

The importance of the liquidity has been highlighted and has been gaining much attention since global mortgage crisis outbroken in 2007-2008. During this crisis, most of the financial institutions hit hard by liquidity pressures and this results in several strict measures taken by regulatory authorities and Central Banks. Since then, debates over the need of inclusion of the liquidity, originating from the lack of tradable securities, have been intensified.

The remainder of this chapter is as follows. In the second part, literature review is given. In the third and fourth part, asset valuation model and Stochastic Volatility with Jump model are introduced. Liquidity is modeled via CIR process in the fifth part. Subsequent to fifth part, Liquidity Augmented Stochastic Volatility with Jump model is derived and discussed. In the seventh part, credit spread formula is derived based on the probability of default. In the eighth part simulation analysis is done to show the stability of the proposed Liquidity Augmented Stochastic Volatility with Jump model and in the nineth chapter empirical analysis are conduced to estimate the probability of default and credit spread both for Liquidity Augmented Stochastic Volatility with Jump and Stochastic Volatility with Jump model. In the last part concludes.

3.2 Literature Review

Despite the importance of liquidity, it has been a neglected topic until mortgage crisis hit in 2007-2008. Since then, liquidity has been gaining attention and ,thus, the liquidity literature has yet to mature. Before modelling asset price and credit spread by incorporating liquidity, it is worthwile discuss the concept of liquidity and how it has been employed in the literature.

As is known, liquidity enables investor to sell the asset at a competitive price. Though the liquidity has only recently incorporated into models and has a better understanding, it has been discussed over decades. [4] suggests that there is a positive relationship between price range and returns. However, this relationship is not linear rather the rate of return expected by investors for a certain increase in the price range increases as the holding periods of assets increase. [18] tries to establish the link between liquidity and return. They conclude that illiquidity of an asset and its return go in tandem. This result still holds irrespective of the characteristics of the type of the liquidity, that is variable and fixed components.

[63] examines the market microstructure of asset pricing and he states that the asset pricing models need to be revisited so as to include transaction costs of liquidity as well as the risk of price discovery. In this study, the effect of liquidity is defined indirectly via idiosyncratic properties of the firms and microstructure of the markets. More specifically, idiosyncratic properties of the firms and microstructure of the markets can affect the liquidity of the assets as long as the effect is high enough.

[28], by employing turnover rate, that is number of shares traded as a fraction of the number of shares outstanding, attempts to measure the effect of liquidity in stock returns. It is concluded that the liquidity has a role in stock returns.

Different from other researches, [1] focuses on the systematic sources of liquidity risk. In their study, liquidity level and liquidity risk are disaggregated in their role on stock return. The model called liquidity adjusted CAPM in which the level effect of expected liquidity is found limited compared to liquidity beta.

The study of [64] is another attemp to model liquidity as a factor in price formation process in CAPM. They find that average stock returns are sensitive to the market wide liquidity. [75] finds that liquidity premium, that is premium requested as assets cannot be converted into cash immediately, is correlated with the market cycles.

[67] contributes to the literature of liquidity risk and its relationship with the asset price. He separates the liquidity risk as variable and fixed and it turns out that variable component of liquidity risk is priced in return. [34] attempts to present the relation-

ship between liquidity and credit risk. Finding of this study indicates that liquidity spread goes hand in hand with the probability of default.

[59], have been published which discuss the liquidity risk. They find that the non-default components is somewhat related to the bond-specific illiquidity. Finally, [2] tries to model liquidity in fixed asset market. His model tries to capture both default and liquidity risk.

In the following section, our approach for modeling the value of an asset is discussed. Our starting point is stochastic volatility with jump model which, as its name suggests, has an jump and stochastic volatility part making it superior to other valuation models.

3.3 Modeling the Value of an Asset

Though Black-Scholes model has been widely used both by academics and practitioners, this log-normal process is not able to capture three emprical phenomena [77]:

- The asymmetric leptokurtic property of the distribution, that is, the return distribution has higher peak and skewed to the left and, as a characteric part of leptokurtic distribution, heavier tail compared to normal distribution.
- The volatility smile
- The large random fluctuation

Considering these deficiencies, [61] proposes a new model by using same Black-Scholes formula but with different approach. In Merton model, equity value of a company, E_T, is a call option on asset value,S_T, of the same company and the strike price is the debt, D_T at maturity which can be presented as:

$$E_T = \max(S_T - D, 0) \tag{3.1}$$

Based on this, value of an equity at time t turns out to be:

$$E_t = S_t N(d_1) - De^{-r(T-t)} N(d_2) \tag{3.2}$$

where

$$d_1 = \frac{\log(S_t/D) + (r + \frac{1}{2}\sigma^2)(T-t)}{\sigma\sqrt{(T-t)}} \tag{3.3}$$

$$d_2 = \frac{\log(S_t/D) + (r - \frac{1}{2}\sigma^2)(T-t)}{\sigma\sqrt{(T-t)}} \tag{3.4}$$

In both of these models as well as other models built on these, such as among others KMV-Merton model, [39] and [13], volatility is assumed to be constant and jump component is ignored. These led the models to be unable to capture the real dynamics of asset prices.

Many researches have been conducted to improve the accuracy of the asset models. Of them, [9]'s model known as stochastic volatility with jump-diffusion stands out with its strong empirical properties. [49], [36], [31], [48] are some of the researchers who advocate the empirical supremacy of the Stochastic Volatility with Jump model.

Thus, in this study, model proposed in [9] is employed in estimating the value of an asset, so that volatility and asset price formation mechanism can be decomposed and moreover this volatility process allows to capture the systematic volatility risk.

3.4 Stochastic Volatility with Jump Model

The stochastic volatility with jump model (hereinafter, SVJ) can also analytically be tractable and takes the form of:

$$\frac{dS_t}{S_t} = \mu dt + \sqrt{V_t} dW_t^s + dZ_t \tag{3.5}$$

$$dV_t = \kappa_v(\theta - V_t)dt + \sigma_v \sqrt{V_t} dW_t^\sigma \tag{3.6}$$

where S_t is the asset price at time t, μ is the return, V_t is the volatility at time t, σ_v is the volatility of the volatility, W_t^s is the Wiener process of asset value, W_t^σ is the Wiener process of volatility, Z_t is the compound poisson process, κ is the mean reversion parameter, and θ is the long term volatility.

The log-normal distribution of jump size, J_t, takes the form of:

$$\log(1+J) \sim \mathcal{N}(\log(1+\bar{J}) - \frac{1}{2}\sigma_j^2, \sigma_j^2) \tag{3.7}$$

where $\bar{J}$ is the mean of jump size. As accounting identity suggests market value of equity and debt give assets value of a firm.

$$S_t = E_t + D_t \tag{3.8}$$

In words, the sum of market value of equity and debt of firm amounts to asset value of a firm. Following [21], equity value of a firm can be modelled as follows. Because it is known that,

$$D_T = \min(S_T, F) \tag{3.9}$$

and

$$E_T = \max(S_T - F, 0) \tag{3.10}$$

So, default occurs when assets are less than the face value of debt, that is F. Thus E_t is modelled by call option pricing formula given as:

$$E_t = S_t P_1 - F e^{-r(T-t)} P_2 \tag{3.11}$$

where

$$P_1 = \frac{1}{2} + \frac{1}{\pi}\int_0^\infty Re(\frac{e^{-iulog(D)}\Phi_1(\log(S_t), V_t, \tau, u)}{iu})du \tag{3.12}$$

$$P_2 = \frac{1}{2} + \frac{1}{\pi}\int_0^\infty Re(\frac{e^{-iulog(D)}\Phi_2(\log(S_t), V_t, \tau, u)}{iu})du \tag{3.13}$$

$$\Phi_1(\log(S_t), V_t, \tau, u) = \exp(A_1 + B_1 V_t + i(u-i)\log(S_t)) \tag{3.14}$$

$$\exp(\lambda\tau(1+\mu)^{(u-i)+1/2}[(1+\mu)^{i(u-i)}e^{\sigma_j^2((u-i)^2 i + \frac{(i(u-i))^2}{2})} - 1] \tag{3.15}$$

$$- \lambda\tau\mu i(u-i))$$

$$\Phi_2(\log(S_t), V_t, \tau, u) = \exp(A_2 + B_2 V_t + iu\log(S_t)) \tag{3.16}$$

$$\exp(\lambda\tau(1+\mu)^{u+1/2}[(1+\mu)^{iu}e^{\sigma_j^2(u^2 i + \frac{(iu)^2}{2})} - 1] - \lambda\tau\mu iu)$$

$$A_1 = (u-i)(r-\lambda J)\tau + \frac{\tau\kappa_v(u-i)(-(i\rho\sigma_v u - \kappa_v))}{\sigma_v^2} - (\frac{2\kappa_v(u-i)}{\sigma_v^2}) \tag{3.17}$$

$$\log((\frac{-(i\rho\sigma_v u - \kappa_v)}{\gamma_{1,v}})\sinh(\frac{\gamma_{1,v}\tau}{2}) + \cosh(\frac{\gamma_{1,v}\tau}{2}))$$

$$A_2 = u(r-\lambda J)\tau + \frac{\tau\kappa_v u(-(i\rho\sigma_v u - \kappa_v))}{\sigma_v^2} - (\frac{2\kappa_v u}{\sigma_v^2}) \tag{3.18}$$

$$\log((\frac{-(i\rho\sigma_v u - \kappa_v)}{\gamma_{2,v}})\sinh(\frac{\gamma_{2,v}\tau}{2}) + \cosh(\frac{\gamma_{2,v}\tau}{2}))$$

$$B_1 = \frac{-((u-i)^2 + (u-i))}{-(i\rho\sigma_v(u-i) - \kappa_v) + \gamma_{1,v}(\frac{\cosh(\gamma_{1,v}\tau/2)}{\sinh(\gamma_{1,v}\tau/2)})} \tag{3.19}$$

$$B_2 = \frac{-(u^2 + u)}{-(i\rho\sigma_v u - \kappa_v) + \gamma_{2,v}(\frac{\cosh(\gamma_{2,v}\tau/2)}{\sinh(\gamma_{2,v}\tau/2)})} \tag{3.20}$$

$$\gamma_{1,v} = \sqrt{(i\rho\sigma_v(u-i) - \kappa_v)^2 - 2\sigma_v^2((u-i)^2 i - \frac{1}{2}(u-i)^2)} \tag{3.21}$$

$$\gamma_{2,v} = \sqrt{(i\rho\sigma_v u - \kappa_v)^2 - 2\sigma_v^2(u^2 i - \frac{1}{2}u^2)} \tag{3.22}$$

where λ is the compound Poisson process intensity, τ is T-t, and μ is r-λJ.

3.5 Modeling Liquidity

The ultimate aim of this study is to extend the asset pricing and probability of default estimation by incorporating liquidity dimension into the model. So, to do that, SVJ model is thought to be a baseline model and it is extended by liquidity.

As is briefly discussed in the previous parts, neoclassical asset pricing models does not allow for individual effect to incorporate. Among others, liquidity is one of them. Recent empirical findings confirm that return of an asset increases with its illiquidity. This finding also holds both for stocks and bonds [52].

In modeling liquidity as a risk factor, the model proposed by Cox-Ingersoll-Ross (CIR)[26] model is employed, an extended version of Vasicek model. CIR model is designed so as not to have negative short term interest rate that is the main drawback of the Vasicek model.

One of the main motivation in this part is to consider liquidity in estimating credit spread and asset valuation. As in known, an illiquid asset has higher cost of immediacy as well as opportunity cost, CIR model, however, providing an approximation considers these two costs. In this respect, [52] denotes that modeling liquidity via CIR model assumes that cost arising from illiquidity of an asset changes stochastically over time to represent bid-ask spread as well as changing repurchase rates.

Furthermore, the square root process of CIR model excludes negative values and finally, mean reversion characteristics imply that the cost of illiquidity oscillates around some long-run mean, that is κ. l_t refers to liquidity augmented discount of illiquid asset and the liquidity-based CIR model is given as follows:

$$dl_t = \kappa_L(\beta - l_t)d_t + \sigma\sqrt{l_t}dW_t \tag{3.23}$$

where W_t is Wiener process, κ is the mean reversion parameter, β is the long run interest rate, l_t is the pay-off realized due to the illiquidity, and σ is the volatility of the time change.

At this point, the liquidity dynamics, L(t), is given in equation 3.23:

$$L(t) = \int_0^t l(u)du \tag{3.24}$$

Following [45], the solution of Integrated CIR (ICIR) model is based on the characteristic function which is:

$$\mathbb{E}(e^{iuL(t)}) = A_{ICIR}(t,u)e^{B_{ICIR}(t,u)L(0)} \tag{3.25}$$

where

$$A_{ICIR}(t,u) = \frac{e^{(\frac{\kappa_L^2 \beta t}{\sigma^2})}}{(\cosh\frac{\gamma_L t}{2} + \frac{\kappa_L}{\gamma_L}\sinh\frac{\gamma_L t}{2}}) ^{2\kappa_L \beta/\sigma^2} \tag{3.26}$$

$$B_{ICIR}(t,u) = \frac{2iu}{\kappa_L + \gamma_L \coth(\frac{\gamma_L t}{2})} \tag{3.27}$$

$$\gamma_L = \sqrt{\kappa_L^2 - 2\sigma^2 iu} \tag{3.28}$$

If u is subsituted for -i, illiquidity model by one factor ICIR model is obtained:

$$\Phi_L(t,u) = A_{ICIR}(t,u)e^{B_{ICIR}(t,u)L(0)} \tag{3.29}$$

Needles to say, including the liquidity dimension enhances the complexity of the SVJ model. So, to deal with thise complexity, liquidity of an asset is determined exogenously by Principal Component Analysis, PCA for short , and uses as an input in the model.

3.5.1 Liquidity Augmented Stochastic Volatility with Jump Model

Thus far, SVJ and CIR model are separately derived but in this part, these two models are combined so that Liquidity Augmented Stochastic Volatility with Jump Model, LASVJ for short, is characterized.

It is assumed that the liquidity characteristics is independent of the dynamics of zero coupon bond which is treated as to be fully liquid. As it is discussed, the former part is modelled by CIR model and the latter part is modelled by the celebrated Stochastic Volatility with Jump Model.

In order to reconcile these two different models, characteristic functions of these two models are used which are derived in the previous sections. The characteristic function of SVJ is given in equation 3.4 and the characteristic function of CIR model is:

$$\Phi_L(\kappa, \beta, \sigma, l(0), u, t) = A_{ICIR}(t, u)e^{B_{ICIR}(t,u)L(0)} \tag{3.30}$$

where j=1,2. Following the assumption of independence of liquidity generating mechanism from the asset pricing process, the multiplication of these two characteristic functions give us the following:

$$\begin{aligned}\Phi_{LASVJ}(u) = &\exp(A_j + B_j V_t + iu\log(S))\exp(\lambda\tau(1+\mu)^{u+1/2} \\ &[(1+\mu)^{iu}e^{\sigma_j^2(u^2 i+\frac{(iu)^2}{2})} - 1] - \lambda\tau\mu iu)A_{ICIR}(t,u)e^{B_{ICIR}(t,u)L(t)}\end{aligned} \tag{3.31}$$

Complete version of liquidity augmented characteristic function of SVJ can be found in the Appendix. Equation 3.31 is one of the main equations that is used to estimate asset price and probability of default which is the basis of credit spread estimation. In order to get P_{1L} and P_{2L}, Φ_1 and Φ_L are integrating with respect to u over 0 to ∞:

$$P_{1L} = \frac{1}{2} + \frac{1}{\pi}\int_0^\infty Re\left(\frac{e^{-iu\log(D)}}{iu}\Phi_1(u)\Phi_L(u)du\right) \tag{3.32}$$

$$P_{2L} = \frac{1}{2} + \frac{1}{\pi}\int_0^\infty Re\left(\frac{e^{-iu\log(D)}}{iu}\Phi_2(u)\Phi_L(u)du\right) \tag{3.33}$$

Now, we are ready to derive credit spread that is the main aim of following part.

3.6 Credit Spread Measurement

Credit spread estimation is thought to be the most attractive application of the Liquidity Augmented Stochastic Volatility with Jump Model. The reason is two-fold: Firstly, credit spread is of considerable importance for investors and decision-makers in the process of portfolio allocation and policy decisions. The empirical literature on the credit spreads, to some extent, agrees on shape of the term structure but not on the corporate bond yield spreads which has led researchers to exert effort on this field as stated by [11].

Secondly, as [68] stresses studying credit spread makes sense due to high volatility of the assets' spread after 2007-2008 crisis, models lacking liquidity dimension cannot properly estimate the real asset spread.

Thus, the application of LASVJ model to the credit spread is believed to shed some light on the level of credit spreads and allow researchers to gain insights on the credit spreads. At this point, it is worthwhile to derive the credit spread. To start with, let's define the value of corporate bond at maturity:

$$B_T = S_T \mathbb{I}_{\{S_T < D\}} + D \mathbb{I}_{\{S_T \geq D\}} \tag{3.34}$$

with $\mathbb{I}$ is the indicator function showing that at maturity the firm's corporate bond payment corresponds either to face value of debt, D or to residual value of the firm's assets, S_T.

$$B_T = \min(S_T, D) = D - \max(D - S_T, 0) \tag{3.35}$$

Here, we have two conditions listed below:

- If $S_T \geq D$ at maturity, put option is not exercised and investor receives D only.
- If $S_T < D$ at maturity, $D - S_T$ is what the investors get.

As is discussed, pricing of the corporate bond is reduced to European option framework. Hence, discounting the right hand side of the equation 3.35, the price of risky corporate bond is defined as follows:

$$B_t = De^{-r(T-t)} - P_t^{Put} \tag{3.36}$$

where B_t is the price of risky corporate bond at time t, r is continuously compounded yield to maturity, T-t is time to maturity, D is the face value of the bond, P_t^{Put} represents the payoff of the put option on asset value S_t with strike price D. P_t^{Put} can be denoted as follows:

$$P_t^{Put} = De^{-r(T-t)}(1 - P_2^{Put}) - S_t(1 - P_1^{Put}) \tag{3.37}$$

The relationship between face value of a corporate bond and the price of it is:

$$e^{-r(T-t)}D = B_t \tag{3.38}$$

Taking logarithm of both sides gives us:

$$\log(e^{-r(T-t)}D) = \log(B_t) \tag{3.39}$$

Substituting B_t in equation 3.39 by B_t in equation 3.36, we have the following:

$$\log(e^{-r(T-t)}D) = \log(De^{-r(T-t)} - P_t^{Put}) \tag{3.40}$$

Eventually, the credit spread takes the form of:

$$s_t = -\frac{1}{T-t}\log(1 - \frac{P_t^{Put}}{De^{-r(T-t)}}) \tag{3.41}$$

3.7 Simulation

Thus far, the derivation of LASVJ model and its theoretical background is discussed. In this section, we conduct simulation analysis based on the LASVJ model and compare its performance with the celebrated SV and SVJ models. In comparison, bias and root mean squared error (RMSE) are employed as performance metric.

$$\text{Bias} = \frac{1}{n}\sum_{j=1}^{n}|y_j - \hat{y_j}| \tag{3.42}$$

$$\text{RMSE} = \sqrt{\frac{1}{n}\sum_{j=1}^{n}(y_j - \hat{y_j})^2} \tag{3.43}$$

The initial parameters are taken from [38] with $\mu = 0.034$, $\kappa = 13.93$, $\theta = 0.004$, $\sigma_v = 0.263$, $\rho = -0.7$, $F = 40$, $\lambda = 0.032$, J=0.0029, $\sigma_j = 0.3274$, $S_0 = 50$, $L = 1$. The simulation is design in a way to test and compare the sensitivity of the models. First, a simulation is run by using these initial values and it is called baseline. Then, the values of σ_v, λ, J, and finally L are changed and simulation exercise is repeated to examine the model performance. These changes in parameters basically amount to financial risk that firms are likely to expose.

Table 3.1 presents the values of old and new parameters. As is said, four main parameters are changed for the sake of sensitivity analysis.

Table 3.1: Old vs. New Parameters

Parameters	Old Values	New Values
σ_v	0.263	0.004
λ	0.0032	0.010
J	0.0029	0.010
L	1	0.5

where σ_v is the volatility of the volatility in stochastic volatility part, λ is the intensity of the jump process, J is the jump component, and L is the liquidity dimension.

In the Table 3.2, the performance of different models are reported based on bias and RMSE. To do that, first, stock price simulated based on the Geometric Brownian Motion and 1000 observation is obtained and first 700 observations are kept for training purpose and the last 300 are retained for the test purpose. Three models are utilized for comparison that are Stochastic Volatility and Stochastic Volatility with Jump Model, and Liquidity Augmented Stochastic Volatility Model.

Accordingly, firstly, the performance of the model is compared based on the baseline values of the parameters provided above. According to bias and RMSE metrics, LASVJ model has both lower bias and RMSE indicating that this model outweighs the others in terms of model stability. To be more specific, bias of LASVJ model is 1.17e-06 but the same metric is 6.52e-06 and 6.52e-06 for SV and SVJ models, respectively. Similarly, LASVJ model has a RMSE of 2.04e-05 and it is 0.002593989 for SV and 0.002593996 for SVJ model. Comparing to the SVJ and SV models, the smaller bias in the Liquidity Augmented SVJ model confirms that it yields more accurate prediction performance. Another observation is that SVJ is slightly superior to SV model based on the model performance in terms of bias and RMSE.

Before going into more detail, some points needs to be highlighted. [21] states that if models are almost identical then it is quite natural to have greater uncertainty in more complex models. However, if the data generating process of the asset is not identical, the more complex model, by and large, prevails. In this regard, it is anticipated to have lower bias and RMSE in LASVJ compared to SV and SVJ.

At this stage, the effects of changes in σ_v, λ, J and L are discussed. To begin with, σ_v is altered from 0.263 to 0.004, LASVJ model performs better than the other two models in that LASVJ has smaller bias and RMSE which are 3.036e-10 and 4.68e-08, respectively. The difference between SVJ and SV occurs in decimal points and SVJ has lower bias and RMSE.

λ parameter is changed from 0.0032 to 0.010 and this lead to a bias of 1.16e-06 and RMSE of 2.04e-05 in LASVJ model. LASVJ model has lower bias and RMSE and the difference of the other models are by a quite narrow margin.

Now, jump intensity parameter is changed to 0.010 and it turns out that Liquidity Augmented SVJ model is again superior than other models of interest. The bias and rmse of LASVJ model are 1.17e-06 and 2.04e-05, respectively.

Finally, lliquidity term, L, is the last parameter to discuss in the sensitivity analysis. Again, LASVJ model having lower bias and RMSE shows better performance compared to SV and SVJ models. In a nutshell, the stability of LASVJ Model outperforms other models in terms of bias and RMSE.

Table 3.2: Model Performance Comparison

Parameters	Criteria	SV Model	SVJ Model	LASVJ Model
Baseline	Bias	6.5201980e-06	6.5202132e-06	1.17683054e-06
	RMSE	0.0025939	0.0025939	2.0470606e-05
σ_v=0.004				
	Bias	6.5162040e-06	6.5162068e-06	3.0362974e-10
	RMSE	0.0025933	0.0025933	4.6808096e-08
$\lambda = 0.010$				
	Bias	6.5242445e-06	6.5242919e-06	1.1697261e-06
	RMSE	0.0025973	0.0025972	2.0420319e-05
$J = 0.01$				
	Bias	6.5201980e-06	6.5202510e-06	1.1750408e-06
	RMSE	0.0025939	0.0025939	2.0482782e-05
$L = 0.5$				
	Bias	6.5201980e-06	6.5202132e-06	1.1768305e-06
	RMSE	0.0025939	0.0025939	2.0470606e-05

3.8 Empirical Analysis

After running simulation analysis, an empirical analysis based on real data is conducted to assess and compare the LASVJ model's ability in credit spread and probability of default prediction. In the calibration analysis, some selected companies listed in Dow Jones Industrial Average (Dow Jones) Index are used. The data is gathered from The Center for Research in Security Prices (CRSP) for the year of 2012. The list of companies can be found below:

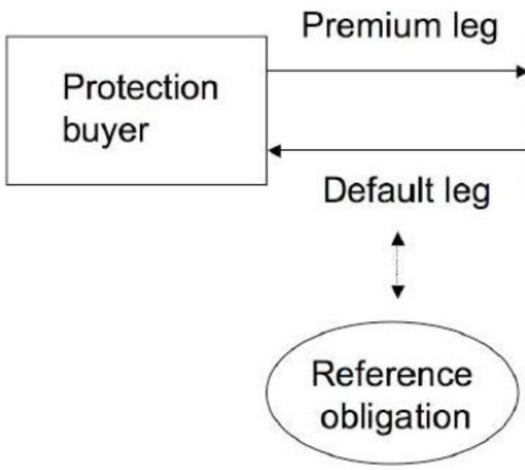
Table 3.3: Companies

3M	General Electric
Microsoft	Chevron Corp.
Intel	Home Depot
Exxon Mobil	Wal-Mart
McDonald's	AT&T

More specifically, in the empirical analysis part, firstly, it is aimed to estimate the model parameters. To do that, real credit default spread (CDS) data and least square optimization technique are utilized. CDS is a financial instrument designed to transfer credit risk from one part to another. A CDS contract involves two parties that agree to a contract that terminates at either maturity or credit event, whichever occurs earlier.

In CDS contract setting, the protection buyer makes a regular payments, that is the premium leg, to the protection seller until the earlier of maturity and credit event [65].

Figure 3.1: Credit Default Spread and Operations



As [21] states that CDS is appealing in modeling in that it tends to efficiently react to reflect in changing credit conditions. CDS, by its nature, is a pure pricing of default risk of the underlying company. Eventually, cds data is available publicly.

Before moving on, the descriptive statistics of 5-year credit default spread is provided in the Table 3.4 This analysis includes Dow Jones listed companies and, due to CDS data availability, ten companies are included in this study. Average of ten Dow Jones listed companies CDS mean are taken based on the maturities. According to the Table, mean values of CDS increase along with the maturities which is quite intitutive in that as maturity increases, CDS raises, too. More specifically, if a company sells an asset with a face value of $1000 and a 10-year maturity, it implies that a company is agreeing to pay its debt back the $1000 at the end of the 10-year period. As the issuer cannot guarantee to pay its debt back, all risks are on the investor which increases the CDS over maturity.

Standard deviation of the CDS raises ove maturity as well because the value of CDS becomes more volatile in longer maturities. Thus, as anticipated, minimum and maximum values of CDS behave in a similar fashion.

In the following part, market liquidity is attempted to estimate based on the Principal Component Analysis (PCA) following [60] and cross-sectional mean of the estimated

Table 3.4: Descriptive Statistics on CDS

	6 month	1 year	2 year	3 year	4 year	5 year	7 year	10 year
Mean	1132.31	1503.31	2393.71	3155.635	3907.46	4662.91	5747.60	6746.89
Std. Dev.	769.11	1033.72	1901.31	2396.08	2630.31	2844.56	2854.23	2919.65
Minimum	500.07	646.03	1068.36	1533.22	2072.57	2595.91	3693.85	4584.20
Maximum	3166.89	4261.84	7638.48	9788.65	11104.83	12380.73	13398.19	14467.34

liquidity suggested by [23]. In order to estimate the liquidity, five different bid-ask spreads are used which are Relative Bid Ask Spread, Buyer Initiated Bid Ask Spread, Seller Initiated Bid Ask Spread, Quoted Spread, and Effective Spread. Based on these bid ask spreads, PCA is conducted. As for the other liquidity measure proposed by [23], the cross-sectional average of these five bid-ask spreads is calculated.

3.8.1 A Brief Introduction to Principal Component as Marketwide Liquidity Measures

Measuring the liquidity has been long on the agenda in finance circle. [20] defines the market liquidity as "the ease with which it is traded" while funding liquidity is "the ease with which investors can obtain funding".

Here, in this study, the focus is on the market liquidity that varies between different assets requiring an additional premium to invest in asset with low level of liquidity. This, in turn, leads to lower asset price.

Liquidity is known as a latent variable, thus, it needs to be proxied to estimate. There have been several attempts to measure the different dimensions of liquidity. Of them, [60]'s and [23]'s approach are stood out. In[60]'s way of estimating liquidity, PCA is employed based on the most common bid ask spreads. However, [23], after calculating bid ask spread, simply calculates the cross-sectional average of the related bid ask spread to represents the market liquidity.

All liquidity proxies account for different aspects of liquidity. To take into account all these aspects and to extract information across these proxies, PCA is widely recognized approach. More precisely, each principal component represents a liquidity factor. Following [60], five liquidity measures are used in this study. These liquidity measures are standardized and put in the 5 x T matrix, where T is the time horizon in our data.

As it is mentioned above, in applying PCA, five liquidity proxies are chosen to represent market liquidity. These are relative bid ask spread, buyer and seller initiated bid ask spread, quoted bid ask spread, effective bid ask spread.

It is worth discussing the motivations in selecting these bid ask spread to estimate the market-wide liquidity. Several celebrated studies (see, [18] Brennan and Subrahmanyam, 1996 and [58]) prefers relative bid ask spread, a proxy of the trading cost aspect of liquidity, in estimating liquidity. Moreover, [3] states that relative bid ask spread support the predictions of the theoretical model. Thus, relative bid ask spread has gained remarkable attention among researchers and practitioners.

[44] splits bid ask spread into three components: Order processing components[1], inventory component[2], adverse-selection[3]. Morever, they stress that the most crucial issue in estimating of the bid-ask spread across these three components is the distinction among the adverse-selection and inventory components. To do that, difference between quoted bid ask spread and effective spread is highlighted.

The reason of making distinction between buyer-initiated and seller-initiated bid ask spread lies in the asymmetry between ask and bid components of spread about the efficient price as a function of the order flow. That is to say, an unanticipated excess of buyers relative to sellers tends to increase the ask price more than the bid price or vice versa [78].

Table 3.5: Liquidity Measures

Liquidity Measures	Formulas
Relative Bid Ask Spread	$(P_A - P_B)/P_M$
Buyer Initiated Bid Ask Spread	$(P - P_M)/P_M$
Seller Initiated Bid Ask Spread	$(P_M - P)/P_M$
Quoted Spread	$P_A - P_B$
Effective Spread	$2(\|P - P_M\|)$

where P is the transaction price, P_A is the ask price, P_B is the bid price, and finally P_M is the mid price, that is $(P_A + P_B)/2$

[1] A cost borne by market makers as they deliver market-making services.
[2] this risk is on the market makers incurring the risk of undesired inventory.
[3] This risk arises due to asymetric information the agent.

Table 3.6 exhibits the descriptive statistics of liquidity measures utilized in this study. It is denoted as nominal values on a daily basis. The sample data covers companies listed in Dow Jones and spans the period of 04/01/2010-12/29/2017. Results reveal that quoted bid ask spread has the highest mean and standard deviation that is simply the difference between ask and bid quotes. Additionally, buyer and seller initiated spread have the lowest spread in that these two spread is calculated as the difference between transaction price and mid-price relative to mid-price. These findings are quite intuitive in that quoted bid-ask spread and effective bid ask spread have no denominator that makes it larger in value compared to other liquidity indicators.

Table 3.6: Descriptive Statistics

Statistics	Rel.	Buyer Init.	Seller Init.	Quoted	Effective	Cross-Section
Count	2013	2013	2013	2013	2013	2013
Mean	0.0156	0.0023	0.0020	1.2771	0.7072	0.3972
Std. Deviation	0.0041	0.0016	0.2980	0.6558	0.6136	0.4815

In the Table 3.7, the correlation among the bid ask spreads used as liqudity measure are reported. It is readily observable that effective bid ask spread and quoted bid ask spread are highly correlated. The correlation coefficient among these measures is around %99 telling that there is almost one-to-one relationship between them. Conversely, the correlation coefficient of nearly %23.5 between quoted bid ask spread and buyer iniatiated bid ask spread is the lowest correlation.

Table 3.7: Correlation Between Liquidity Measures

	Rel.	Buyer Init.	Seller Init.	Quoted	Effective
Rel.	1.000000	0.808600	0.831070	0.273788	0.291015
Buyer Init	0.808600	1.000000	0.477220	0.235451	0.277372
Seller Init	0.831070	0.477220	1.000000	0.250365	0.289734
Quoted	0.273788	0.235451	0.250365	1.000000	0.989462
Effective	0.291015	0.277372	0.289734	0.989462	1.000000

In the below, the plots of the liquidity measures are given and it is observable that some measures follow similar pattern. For instance, relative bid ask spread, buyer and seller initiated spreads have same spikes at around 2014 however quoted bid ask spread, effective bid ask spread and cross sectional mean proposed by Chordia have spike at around 2011.

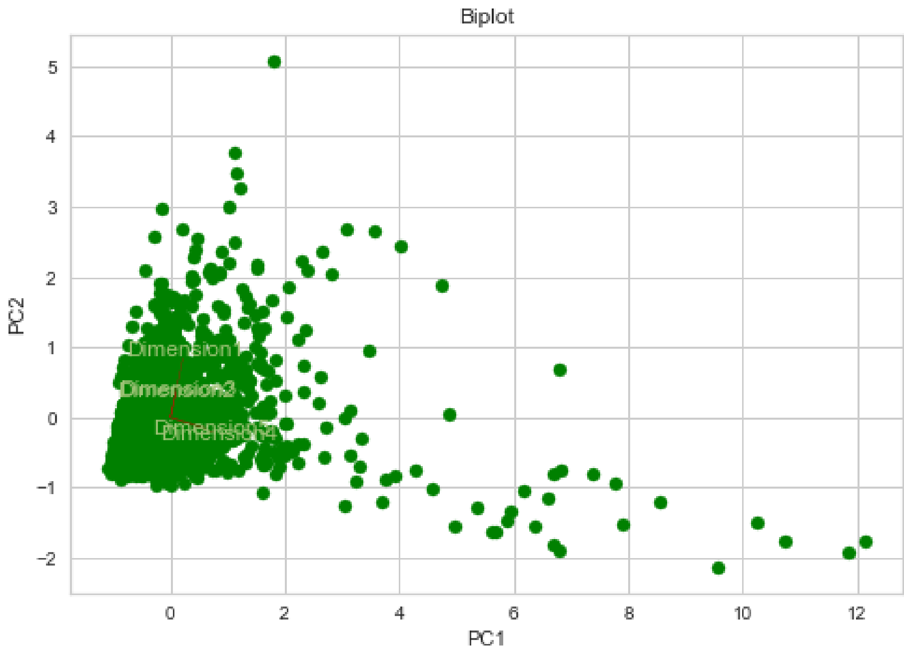
Figure 3.2: Bid-Ask Spreads Plots

PCA does not ignore the characteristics of the variables, instead, it shrinks the dimensionality which can be problematic. Principal components are the output of the dimension reduction process. Principal components account for the variation of the original data.

In the following figure, the biplot of obtained from PCA are exhibited. This figure basically tells us that dimension 4 and 5, corresponding the quoted and effective bid ask spread, have strong influence on Component 1 while, on the other hand, relative bid ask spread, buyer and seller initiated spread have something to say on Component

2. Accordingly, first two components are used to represent the market-wide liquidity.

Figure 3.3: Biplot



After obtaining the bid ask spreads, they are demeaned, standardized, and put in a matrix with 5xT, where T is the days spanned in this analysis. As the the spreads are standardized, it is higly likely that some of the spreads turn out to be negative.

The other approach used to measure liquidity is proposed by [23]. This approach allows us to take advantage of the bid ask spreads as a liquidity measure by simply taking the cross-sectional means of them. In this method, market-wide liquidity can be estimated as:

$$L_M, t = \frac{1}{N} \sum_{j=1}^{N} L_j, t \tag{3.44}$$

where $L_{j,t}$ is the liquidity of the individual stock, $L_{M,t}$ is the market-wide liquidity, and N is the number of stock considered.

After applying necessary preprocessing steps including standardization and demeaning, PCA can be employed. In the empirical application part, asset value of each stocks considered is predicted after optimizing the paramaters. Based on the asset values of the individual stocks, probability of default and credit spread are estimated.

3.8.2 Parameter Estimation

In this section, parameter estimations of ten Dow Jones listed companies are done. To do that, 5-year CDS data is used and the monthly sample period is 2012/01-2012/12. Data is extracted from Thompson Reuters database. As for the balance sheet information, the face value of debt of the companies together with the total asset and equity values are obtained from the Wharton Research Data Services (WRDS) CRSP. In order to calculate the face value of debt, total liabilities is employed. In the literature, lower bound of total liability, that is current liabilities and 0.5 long-term debt is prefered but, in this case, since the huge difference between liabilities and total asset might prevent convergence, total liability is chosen to proxy the face value of debt. In WRDS database, the equity value can be computed as the multiplication of closing price of equity by number of outstanding share of the same equity. The initial parameters used are $\theta, \kappa, \sigma_v, \lambda, \sigma_j$, J, L are 0.05,1.00, 0.2, 0.32,0.08,0.02,0, where J and L are jump and liqudity parameters.

In the Table 3.8 the parameter estimation of the ten selected Dow Jones listed companies are provided. These parameters are the input to the asset valuation model which is used to estimate the probability of default. The parameters estimated are $\sigma_t, \kappa, \theta, \sigma_v, \rho, \sigma_j, J, \lambda$.

Table 3.8: Parameter Estimation

Company	σ_t	κ	θ	σ_v	ρ	σ_j	J	λ
3M	0.016112683	1.000272707	0.003489919	0.224728369	0.698875723	0.248868945	0.106214472	0.265144209
Microsoft	0.006488108	1.021006035	-0.000771190	0.219509451	0.733171540	0.272756408	0.087935872	0.289219838
Intel	0.000500736	1.021773396	0.008107264	0.233721833	0.711406355	0.271229446	0.073795528	0.277627350
Exxon Mobil	0.006419183	1.019376067	0.001416090	0.228006643	0.752148963	0.253924404	0.094195244	0.277233655
General Electric	0.011317486	0.052301388	0.265762986	0.487116614	0.169196389	0.657656243	0.004255160	0.015118124
Chevron Corp.	0.002725939	1.023725809	0.003576732	0.220136924	0.736314112	0.255986654	0.083534668	0.286792819
Home Depot	0.001659094	1.010599729	0.007149109	0.233506999	0.721086937	0.251701329	0.070883688	0.324807601
Wal-Mart	0.008064077	1.010576117	0.007153164	0.224478982	0.717864356	0.256004934	0.080585424	0.284309271
McDonald's	0.003727153	0.696718616	0.031214366	0.041140789	1.262384840	0.236662914	-0.053848939	0.091828159
AT&T	0.002758863	1.014970736	0.005310352	0.236865799	0.731147245	0.258661088	0.070322521	0.295225581

In the Figure 3.4, based on the initial parameters of 3M company, the behavior of credit spread against maturity is provided. Accordingly, as time to maturity increases, credit spread react almost one-to-one up to a certain maturity. Around 7 years, the speed of increase in credit spread diminishes.

To compare, the probability of default is estimated based on the LASVJ and SVJ models. The probability of default for SVJ model is estimated via following formula:

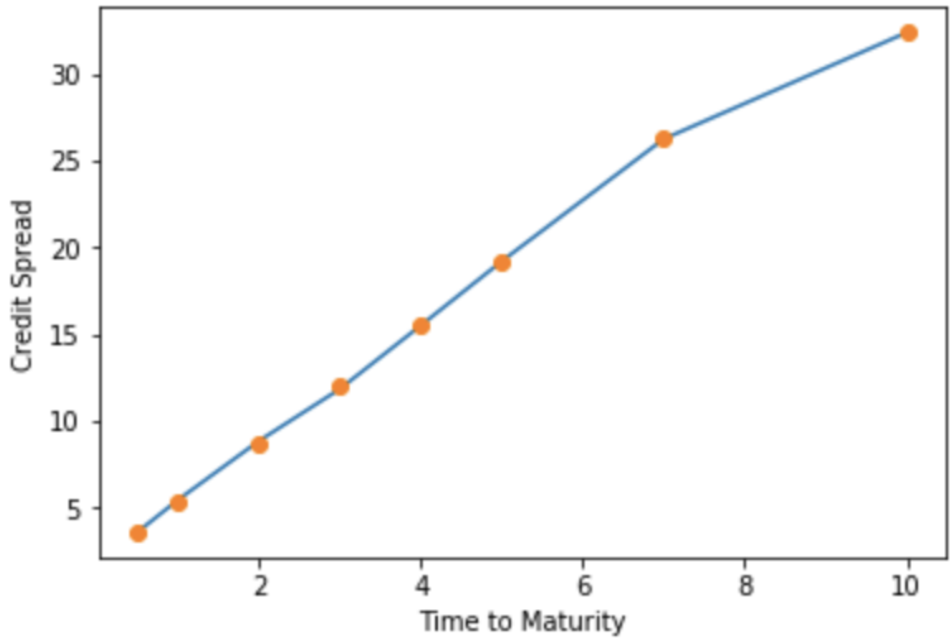


Figure 3.4: Credit Spread vs. Time to Maturity

$$P_2 = \frac{1}{2} + \frac{1}{\pi}\int_0^{\infty} Re\left(\frac{e^{-iu\log(K)}}{iu}\Phi_2(u)du\right)$$

Finally, A and B are defined in the equation 3.26 and 3.27, respectively. The probability of default for LASVJ model is given as follows:

$$P_{2L} = \frac{1}{2} + \frac{1}{\pi}\int_0^{\infty} Re\left(\frac{e^{-iu\log(K)}}{iu}\Phi_2\Phi_L(u)du\right)$$

Based on the above-given formulas, the probability of default for LASVJ and SVJ are estimated and the results are shown below in the Table 3.9. The probability of default after including the liquidity dimension increases considerably. For instance, the probability of default for General Electric is 26.6% in LASVJ model but it decreases to 2.3% in SVJ model. In a similar vein, Chevron Corporation has 0.029% in LASVJ model but this probability shrinks to 0.010% in SVJ model. LASVJ reveals that AT&T, a well-known telecomunicaton company, has a probability of default of 1.5% whereas it is only 0.46% in SVJ model.

The differences between probability of defaults are even exaggerated over maturity. At 10-year maturity, for instance, the difference between probability of defaults of 3M is nearly 63%. Except for General Electric, the difference of the probability of default of the companies are higher in LASVJ and SVJ models over maturity.

Thus, in all companies, the probability of default are higher in LASVJ model than that of SVJ model confirming the importance of the liquidity dimension in estimation probability of default. These differences in probability of defaults suggest that models ignoring liquidity dimension underestimate the probability of default which in turn lead to some adverse situation such mispricing and deteoriorated credit ratings.

Table 3.9: Probability of Default in LASVJ Model

Companies	6 months	1 year	2 years	3 years	4 years	5 years	7 years	10 years
3M	0.001137604	0.010861888	0.241979034	0.620303886	0.740949856	0.781511690	0.780718685	0.715481153
Microsoft	0.000961137	0.008054338	0.086346236	0.266602598	0.385459483	0.450054487	0.498078528	0.480062040
Intel	0.0001244475	0.0014920139	0.0289789722	0.1781420032	0.3709948705	0.4791671698	0.5447811738	0.5033872499
Exxon Mobil	0.00184648	0.01493410	0.26312577	0.65852602	0.76761206	0.79685080	0.79871468	0.72246298
General Electric	0.269830478	0.881924604	0.956994030	0.966605545	0.968500319	0.967264748	0.958476634	0.928617944
Chevron Corp.	0.0002969209	0.0034097753	0.0606221646	0.3334666667	0.5364702972	0.6193498505	0.6501774805	0.5926497511
Home Depot	0.008399570	0.048348369	0.616931176	0.806128643	0.854171533	0.866826150	0.860736044	0.789334395
Wal-Mart	0.012451694	0.075341447	0.747696469	0.857179443	0.890202411	0.899591466	0.886677872	0.820258164
McDonald's	0.004467926	0.024144307	0.463354254	0.692410162	0.762177119	0.780950856	0.762176808	0.671397189
AT&T	0.015021209	0.075432610	0.772958396	0.868992723	0.897573912	0.906014637	0.891617653	0.822169228

Table 3.10: Probability of Default in SVJ Model

Companies	6 months	1 year	2 years	3 years	4 years	5 years	7 years	10 years
3M	0.000204138	0.000673440	0.002553724	0.005974097	0.011269151	0.018705058	0.033442645	0.082231807
Microsoft	0.000219652	0.000661276	0.001890162	0.002221516	0.002465093	0.003156215	0.005381695	0.034693204
Intel	0.00006245471	0.00010669877	0.00052839569	0.00144068568	0.00301343700	0.00538840657	0.01555679828	0.03407392408
Exxon Mobil	0.00035967	0.00105738	0.00361927	0.00793746	0.01424813	0.02157772	0.05081320	0.08966156
General Electric	0.023192135	0.059300251	0.127867534	0.199051766	0.271326652	0.341564791	0.464914476	0.593031195
Chevron Corp.	0.0001019413	0.0001939191	0.0008920542	0.0022785364	0.0045539789	0.0077921340	0.0123060952	0.0410178295
Home Depot	0.002124143	0.005567631	0.016862873	0.034164676	0.056981096	0.082620646	0.145728562	0.246155841
Wal-Mart	0.003662867	0.009113872	0.025237681	0.048025536	0.076627642	0.110038966	0.173176672	0.302109014
McDonald's	0.001195598	0.002891324	0.008476639	0.017414801	0.032541682	0.053195473	0.097292259	0.151649908
AT&T	0.004675550	0.010942910	0.028526794	0.052100009	0.080544218	0.114393506	0.172183715	0.284227644

The estimation of probability of default is an intermediate step in calculating the credit spread estimation. Credit spread estimation is conducted for different maturities. The credit spread is estimated via the formula provided in equation 3.41.

Similar to estimation of probability of default, this analysis also starts from the 6-month maturity and extends to 10-year maturity. As theory suggests, the longer the maturity, the larger credit spread is. For example, in LASVJ model, 3M has credit spread of 9.10 for 6-month maturity and of 337.14 for 10-year maturity. Similarly, credit spread of Exxon Mobil is 14.77 and 341.06 for 6-month and 10-year maturity, respectively.

After estimating probability of default, credit spread can be predicted. In the Figure 3.5, credit spread of 3M company estimated by LASVJ model, raises quickly to a level of 600, then, it becomes declining as the maturity increases. This findings conforms to the finding of [56].

Table 3.11: Credit Spread Estimation with LASVJ

Companies	6 months	1 year	2 years	3 years	4 years	5 years	7 years	10 years
3M	9.102904344	43.542208761	509.009905238	950.602031926	878.791898142	749.691434195	534.834811751	337.141906424
Microsoft	7.690572413	32.269359635	175.745239176	375.889357561	418.632981289	396.955035883	317.404681025	213.223933625
Intel	0.995604798	5.969837389	58.296475258	246.410020066	401.589841506	425.562026491	351.126590234	224.838612093
Exxon Mobil	14.77725832	59.91555483	556.05635860	1019.08135826	916.97490406	767.62344655	549.86671244	341.06202296
General Electric	2284.26260968	4350.53315972	2412.79144797	1629.35618730	1225.10823297	978.47369235	690.74229344	464.33520918
Chevron Corp.	2.375508328	13.648410941	122.738552799	477.207946734	603.867523642	569.346381216	430.287189677	270.575757256
Home Depot	67.309699868	195.287992402	1416.939662610	1297.580262574	1045.117117425	851.530926215	602.919140109	379.408194300
Wal-Mart	99.862452475	306.000204500	1776.797531500	1399.587050562	1100.455704387	892.063602919	625.707493433	397.650619411
McDonald's	35.775389675	97.046611449	1024.932596616	1080.987849667	909.144049066	749.038834269	519.510629822	312.738547996
AT&T	120.532143211	306.376192996	1849.404214016	1423.643155247	1111.929865425	900.106668737	630.088301776	398.788979788

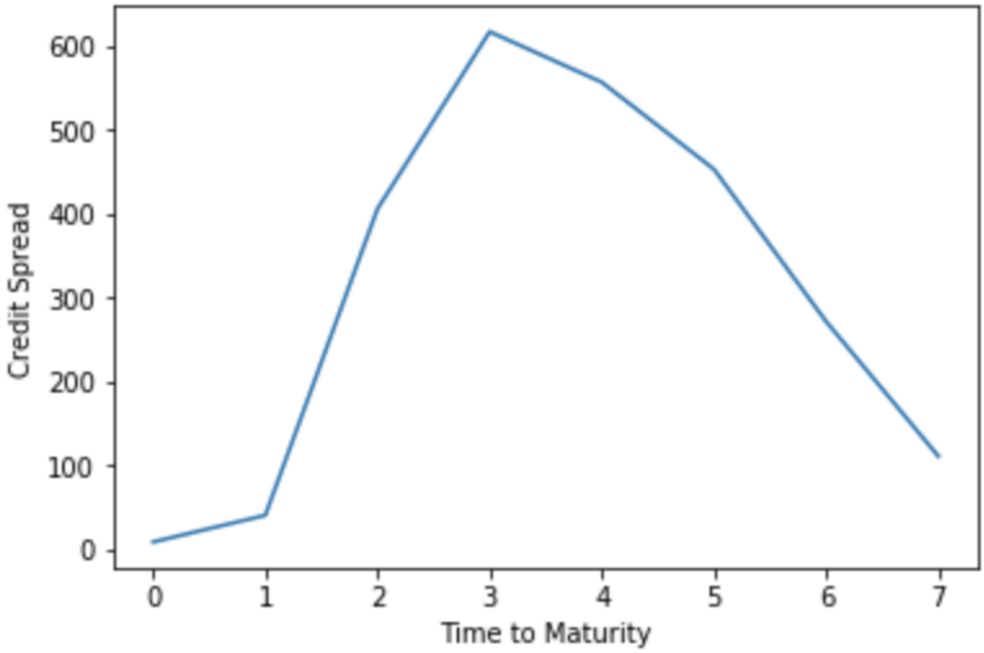


Figure 3.5: Credit Spread vs. Time to Maturity Based on LASVJ

Likewise, the credit spread is estimated for SVJ model. Due to the lack of liquidity dimension, SVJ model is assumed to have lower credit spread. Table 3.6 confirms the initial assumption by producing lower credit spread compared to LASVJ. To compare, at 6-month maturity, the credit spread of Microsoft is 1.75 in SVJ model whereas it is 7.69 in LASVJ model. Besides, the SVJ suggests a credit spread of 13.97 and LASVJ estimates a credit spread of 213.22 for the same company at 10-year maturity.

By and large, the difference between credit spreads are even higher over maturity between LASVJ and SVJ models implying that disregarding liquidity dimension undermines the sound functioning of the financial markets. To be more specific, aside from General Electric, Home Depot, and Wal-Mart, the credit spread grows at faster pace over maturity. This contradict with the claim of [34] whose idea based on the the positive correlation between liquidity and credit risk, that is to say, the relationship between liquidity and credit risk tends to be a decreasing function of time to maturity.

Table 3.12: Credit Spread Estimation with SVJ

Companies	6 months	1 year	2 years	3 years	4 years	5 years	7 years	10 years
3M	1.633172229	2.694121334	5.110058433	7.974995281	11.294626744	15.020307772	19.239052764	33.445851507
Microsoft	1.757290310	2.645453807	3.781753416	2.963338472	2.466309581	2.526567008	3.078568858	13.974471313
Intel	0.499643911	0.426804201	1.056903073	1.921467933	3.015254617	4.315377533	8.917373037	13.723304904
Exxon Mobil	2.87755094	4.23039553	7.24377498	10.60012240	14.28888252	17.33710701	29.33525869	36.52356074
General Electric	186.40303599	240.05951323	262.50703248	276.56472715	287.21058703	293.81442640	293.93269586	270.77576333
Chevron Corp.	0.815546720	0.775706340	1.784426742	3.039433782	4.558131735	6.243442188	7.049418850	16.543219366
Home Depot	17.000368968	22.295359703	33.840002828	45.867026043	57.640504215	67.213380085	85.799172080	103.653458290
Wal-Mart	29.324426003	36.522101280	50.731865644	64.657094413	77.826563439	90.027339572	102.552532970	128.792474006
McDonald's	9.567075167	11.571987540	16.982084185	23.300985669	32.755330020	43.015667011	56.706292971	62.577738862
AT&T	37.439423260	43.867720109	57.381596580	70.200737682	81.870251116	93.674689435	101.943029212	120.689695826

In the Figure 3.6, unlike others, the speed of increase of credit spread is getting even larger as maturity increases based on the SVJ model.

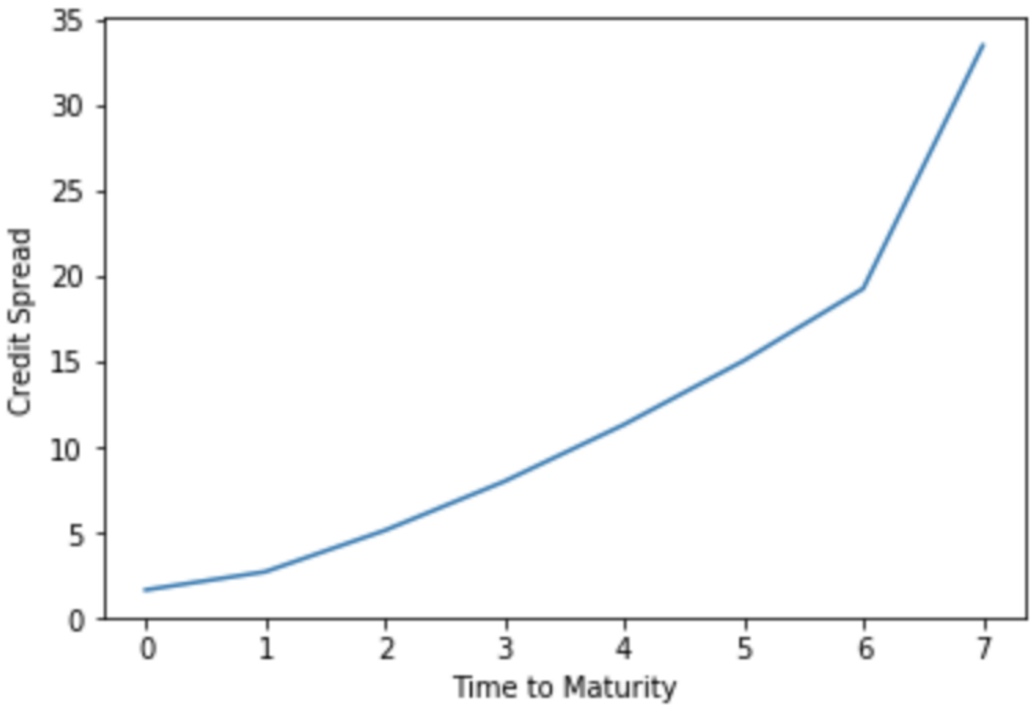


Figure 3.6: Credit Spread vs. Time to Maturity Based on SVJ

At this point, same analysis is done via the approach of [23]. From this starting point, [23] investigate total market bid-ask spreads, depths and transaction movements since there is, back then, no study examining why aggregate market liquidity changes over time . They come up that market-wide liquidity and trading movements were influenced by a variety of factors.

In order to fully capture the market-wide liquidity, differently from the Principal

Component Analysis, the analysis proposed by [23] is employed that is based on, given a measure of bid ask spread, daily liquidity given in equation 3.44 which is called cross-section averaging.

The probability of default and credit spread estimation are conducted based on the cross-section averaging and the result is given in the Table 3.13 and Table 3.14.

Table 3.13 exhibits the estimation result of probability of default and similar to the finding based on the PCA, the findings of the cross-sectional averaging approach confirms that the probability of default in LASVJ model is higher than that of SVJ model. Home Depot, for instance, has a probability of default of 0.85% in LASVJ and 0.21% in SVJ models at 6-month maturity. At 10-year maturity, the probability of default for the same company is 96% and 24% in LASVJ and SVJ models, respectively. Additionally, in LASVJ model, McDonald's has a probability of default of 0.4% and 92% at 6-month and 10-year maturity and, in the SVJ model, it is 0.2% and 15% at the same maturities. Hence, same conclusion can applies here which is, by and large, the difference between probability of default is getting larger among LASVJ and SVJ over maturity.

Table 3.13: Probability of Default in LASVJ Model

Companies	6 months	1 year	2 years	3 years	4 years	5 years	7 years	10 years
3M	0.001127131	0.011065507	0.275836714	0.768488635	0.870609712	0.915550670	0.944089405	0.948163168
Microsoft	0.000000183	0.001471758	0.235920469	0.557912261	0.716661834	0.792509658	0.850784420	0.860686075
Intel	0.00012029976	0.00150994844	0.03165547476	0.21834700147	0.54664343888	0.70457441082	0.81337676853	0.83864475757
Exxon Mobil	0.001830874	0.015235566	0.292000040	0.811666962	0.890924399	0.926843505	0.953344464	0.953105701
General Electric	0.258645552	0.898371222	0.966718619	0.977795126	0.982478691	0.984936600	0.986814640	0.985690511
Chevron Corp.	0.0003021356	0.0036774733	0.0675567463	0.5042803456	0.7645860076	0.8473221055	0.9020516864	0.9128029328
Home Depot	0.008590501	0.051298915	0.758293408	0.888327577	0.937618589	0.956678757	0.970341845	0.966448668
Wal-Mart	0.012707572	0.075299693	0.819313561	0.918307306	0.954265700	0.968667618	0.977204136	0.973813976
McDonald's	0.004564990	0.024320930	0.561814921	0.817508731	0.893158223	0.921247558	0.935976312	0.923722661
AT&T	0.004675550	0.010942910	0.028526794	0.052100009	0.080544218	0.114393506	0.172183715	0.284227644

In Table 3.14 , credit spread estimation with LASVJ models is provided. Analysis based on cross sectional averaging yields similar result with the previous analysis conducted based on the PCA. To be more specific, the estimated credit spreads in LASVJ model are higher than those in SVJ model for every company considered in this study. Other than the General Electric whose estimated credit spread poses thread to its financial sustainability, the rest of companies are deemed to be low risky companies.

In cross-sectional averaging method, the difference between estimated credit spread gets larger over maturity which is another way of saying that the difference in es-

Table 3.14: Credit Spread Prediction with LASVJ

Companies	6 months	1 year	2 years	3 years	4 years	5 years	7 years	10 years
3M	9.019084099	44.360275810	584.549691363	1224.320278230	1070.212109007	912.107621445	677.471094204	476.851449387
Microsoft	56.431100207	154.797846679	1551.161533617	1412.483980872	1146.670968734	949.550659766	693.592146181	481.643590235
Intel	0.962421201	6.041618465	63.715193183	304.635174122	616.853814158	662.097282001	562.231842385	408.657051484
Exxon Mobil	14.652354482	61.128721543	621.018108280	1308.496859872	1101.577196110	926.413248429	685.994072841	480.041489090
General Electric	2184.207686056	4452.696344795	2444.402999517	1653.769577558	1248.031114476	1001.666891410	717.248276141	501.331179871
Chevron Corp.	2.417230697	14.720722734	136.972627214	750.953391826	912.611836449	827.787586412	639.386042151	454.321122128
Home Depot	68.842355084	207.330173220	1807.126496283	1463.394399282	1175.198822034	964.707972553	701.780642115	488.704554016
Wal-Mart	101.919829573	305.827999501	1985.442136109	1525.983776886	1201.979147739	980.305074586	708.203860648	493.518903473
McDonald's	36.553306959	97.760019434	1272.693614540	1320.050462160	1105.050271664	919.311581135	670.041348802	461.224779517
AT&T	122.913460638	308.587097586	2050.713340903	1543.671559686	1209.648259529	984.880553583	710.382339916	494.630676492

timated credit spread between LASVJ and SVJ models is an increasing function of maturity. Again the sole exception to this is General Electric.

In this method, the behavior of credit spread for 3M company is quite similar to previous findings in that credit spread raises in a fast way up to nearly 3-year maturity then it starts to decline after this point.

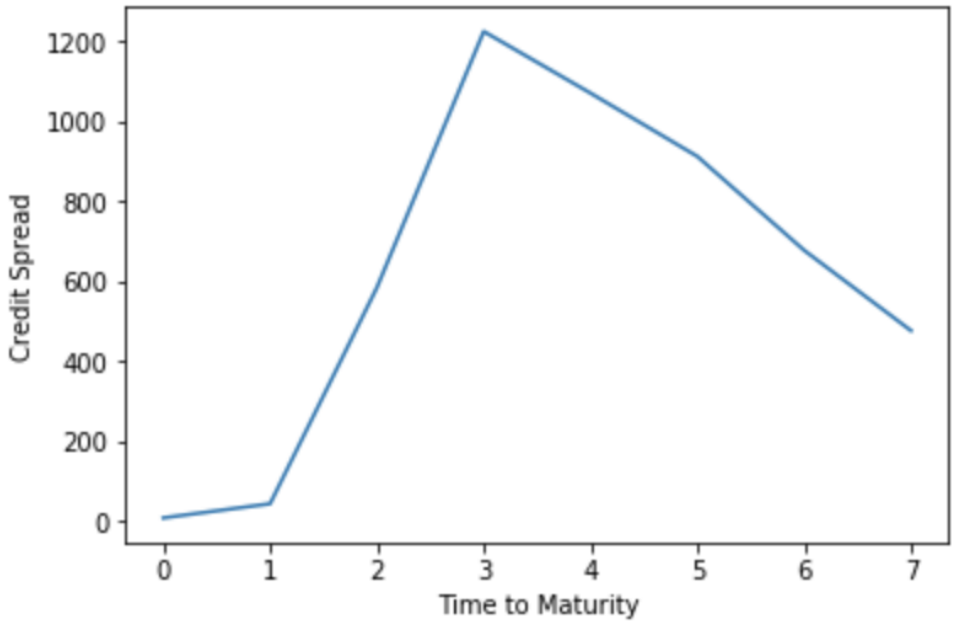


Figure 3.7: Credit Spread vs. Time to Maturity Based on LASVJ in Cross-Sectional Averaging Method

3.8.3 Credit Spread In the Presence of Crisis

The effect of the liquidity can be much more pervasive and observable during economic turbulence period in which bid ask spread might be wildly volatile. The mortgage crisis hit hard the global economy between 2007-2009 is a vivid example of this kind of situation. Hence, to fully capture the liquidity effect on asset pricing as well as credit spread, liquidity estimation analysis is extended as to cover the mortgage crisis period. With this aim, same five difference bid ask spreads are calculated for this period. To be more precise, the period declared by National Bureau of Economic Research. Accordingly, the crisis starts at 2007/12 and ends at 2009/06. So, the analysis spans this whole period.

Table 3.15 shows the descriptive statistics about the liqudity measure, that is bid ask spread. Similar to findings for the period of 04/01/2010-12/29/2017, this statistic imply, as expected, that quoted and effective bid ask spread have higher mean values indicating that these two spreads dominate that liquidity measure. The standard devation showing the volatility of the measures are high in these two spreads as well meaning that these two measure spans wide range.

Table 3.15: Liquidity Measures

Statistics	Rel.	Buyer Init.	Seller Init.	Quoted	Eff.	Cross-Section
Count	378	378	378	378	378	378
Mean	0.0156	0.00235	0.0019	1.5082	0.8550	0.4766
Std. Dev.	0.0041	0.0037	2.6757	1.8965	1.3450	0.3815

Table 3.16 gives the correlation table showing the commonality between bid ask spreads. Accordingly, focusing on the correlation above 90%, the correlation coefficient between quoted bid ask spread and effective bid ask spread and seller initiated bid ask spread and relative bid ask spread come forward. That is to say, the correlation coefficient between quoted bid ask spread and effective bid ask spread is 98% and it is 93% for seller initiated bid ask spread and relative bid ask spread. The rest of the correlation is lower than 90% level but they are still high as the lowest correlation coefficient is nearly 28% between quoted bid ask spread and seller initiated spread.

Table 3.16: Correlation Between Liquidity Measures During Crisis

	Rel.	Buyer Init.	Seller Init.	Quoted	Eff. Bid Ask
Rel.	1.000000	0.887999	0.933357	0.368576	0.391670
Buyer Init.	0.887999	1.000000	0.743946	0.399085	0.449743
Seller Init.	0.933357	0.743946	1.000000	0.281591	0.332334
Quoted	0.368576	0.399085	0.281591	1.000000	0.981629
Eff.	0.391670	0.449743	0.332334	0.981629	1.000000

As discussed before, liquidity during crisis becomes much more important and therefore its effect turns out to be more visible. As a stylized fact, it is expected to have even higher difference in probability of default as well as credit spread between LASVJ and SVJ models at the time of crisis.

The parameters estimated during this period and based on these parameters, probability of default and then credit spread are predicted.

The probability of default estimations are given in the Table 3.17. The estimated probability of default is higher for all companies in LASVJ model compared to SVJ model. To interpret, General Electric seems to have highest likelihood of default but conversely Intel has the lowest. Compared to the probability of default presented in the Table 3.9, except for Wal-Mart and McDonald's, the probability of default is, unsurprisingly, found higher during crisis at 6-month maturity. For instance, General Electric's estimated the probability of default is 50% in the crisis period which is around 2.3% between 04/01/2010-12/29/2017. In addition to that, Intel has an estimated probability of default of 0.17% according to LASVJ model whereas it is 0.0062% in SVJ model.

Thus, in LASVJ model, the estimated probability of default is even much higher during crisis period compared to SVJ model estimation. However, this effect vanishes over maturity along with the diminishing effect of crisis. In this regard, as of 1-year maturity, the estimated probability of default is higher during post-crisis period.

Table 3.17: Probability of Default in LASVJ Model

Companies	6 months	1 year	2 years	3 years	4 years	5 years	7 years	10 years
3M	0.001599927	0.017763688	0.393328329	0.627324198	0.684843496	0.685059213	0.611355653	0.433218869
Microsoft	0.008994660	0.057005526	0.609359841	0.753102894	0.772346116	0.753593117	0.669829525	0.459890276
Intel	0.00017548507	0.00259769512	0.06814588054	0.27131929611	0.39179339403	0.43347881807	0.41323272293	0.29670669078
Exxon Mobil	0.002542180	0.023132372	0.428094434	0.656213869	0.705716875	0.696547042	0.627441641	0.438681440
General Electric	0.507424633	0.918957531	0.960994698	0.962292197	0.955665384	0.943398365	0.900890117	0.780960121
Chevron Corp.	0.0004278782	0.0059412395	0.1516054836	0.4132557680	0.5169580945	0.5391649958	0.4907641606	0.3460235309
Home Depot	0.010845199	0.083644521	0.679305159	0.792492473	0.808739981	0.792740409	0.727716605	0.547752428
Wal-Mart	0.012153099	0.081687453	0.600643851	0.718221274	0.731785703	0.710849961	0.613648485	0.398245948
McDonald's	0.004355294	0.025688757	0.362092372	0.525776568	0.560683477	0.547231964	0.461933429	0.286057485
AT&T	0.018552634	0.206515913	0.794846653	0.855591027	0.858728121	0.842721880	0.769590449	0.583225620

In parallel with the probability of default estimation, credit spread prediction is expected to be higher during crisis at early maturities. Naturally, all companies have higher credit spread prediction in comparison with the period without crisis.

Figure 3.8 suggests that, during crisis, credit spread finds its peak at 2-year maturity, then the speed of this increase gets slower and after 4-year maturity, credit spread

Table 3.18: Credit Spread Prediction with LASVJ During Crisis

Companies	6 months	1 year	2 years	3 years	4 years	5 years	7 years	10 years
3M	12.803517211	71.308392702	855.907184050	963.074711426	800.297599094	640.475777911	400.616338675	190.298344127
Microsoft	72.087034240	230.662016739	1396.876257204	1194.832000572	923.815950468	717.460528939	445.545162198	203.287138977
Intel	1.403929797	10.396182641	138.183766724	382.936446930	426.132738942	380.848253623	258.106601062	126.337532100
Exxon Mobil	20.347789016	92.960231393	939.109924129	1014.898648950	829.212907908	653.175189046	412.835899749	192.944876281
General Electric	4537.255514270	4582.063116841	2425.772159756	1619.992879467	1204.243959572	947.571452488	638.347972493	374.524807024
Chevron Corp.	3.423318292	23.793241575	312.794108538	602.285547408	579.146875470	485.840653647	312.194659248	148.975077461
Home Depot	86.950326891	340.303273958	1585.362615562	1270.853370410	977.042267745	762.802449919	491.459947564	247.309422117
Wal-Mart	97.461879264	332.207297378	1373.878860932	1128.928607086	865.801416035	669.100126636	402.351690917	173.5184729958
McDonald's	34.872735279	103.286603325	782.315624722	787.052032178	634.887821268	494.085720003	291.841618083	121.515862264
AT&T	148.974534300	862.186350893	1913.178428571	1396.365663226	1052.048136828	822.228324473	525.618023604	265.64696921

shrinks very fastly. Thus, crisis period present another characteristics between credit spread and time to maturity.

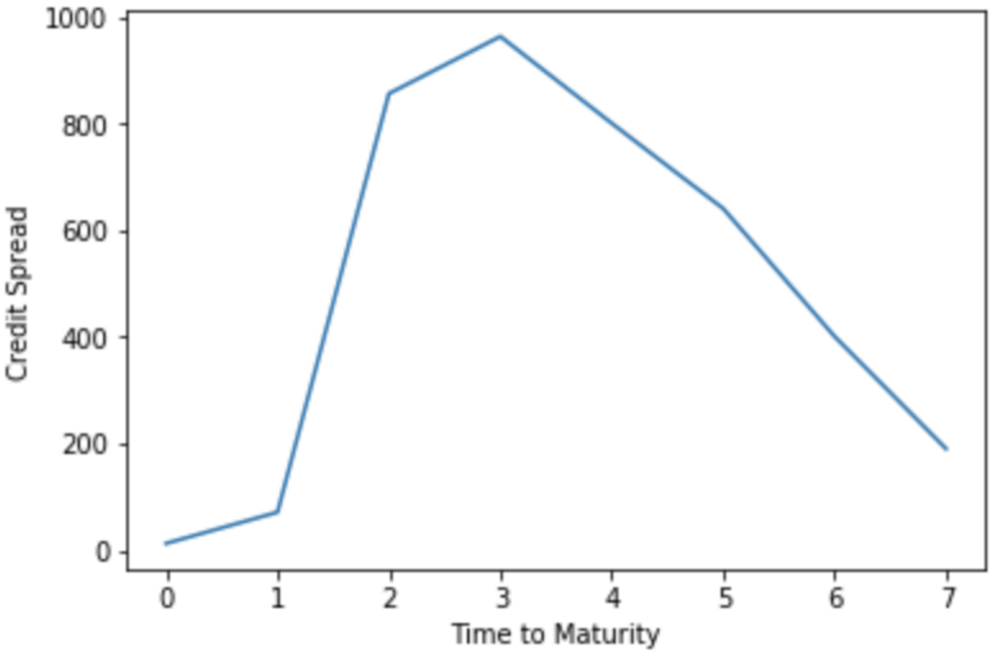


Figure 3.8: Credit Spread vs. Time to Maturity Based on LASVJ During Crisis

In the following part, cross-sectional averaging method is applied to the crisis period. The probability of default estimation in LASVJ and SVJ models based on the parameters obtained are provided in the Table 3.8. Differently, the estimated probabilities of default is higher in LASVJ model than that of SVJ model for all companies at 6-month maturity. Moreover,similarly, the difference between probability of default in LASVJ and SVJ model is even larger than the difference in PCA application. The reason is that cross-sectional averaging has higher liquidity measure value.

To interpret, Home Depot have probability of default of 1.06% and 0.21% in LASVJ and SVJ models, respectively. Exxon Mobil, an oil company, has an estimated probability of default of 0.24% and 0.035% in LASVJ and SVJ models, respectively. Even though Home Depot and Exxon Mobil is far from default and the probability of default at 6-month maturity, it turns out that probability of default is underestimated when liquidity dimension is neglected.

Table 3.19: The Probability of Default Estimation with Cross-Sectional Averaging in LASVJ During Crisis

Companies	6 months	1 year	2 years	3 years	4 years	5 years	7 years	10 years
3M	0.001562957	0.017208871	0.464199206	0.756678756	0.828906678	0.848868926	0.830354473	0.745184236
Microsoft	0.00887777968	0.05241867850	0.72363908683	0.86096924708	0.89131956599	0.89509666202	0.87044438296	0.76426551445
Intel	0.00017099690	0.00251695287	0.06115460650	0.33485148940	0.52711696900	0.60638326316	0.63206531444	0.55435534058
Exxon Mobil	0.002489312	0.022864640	0.510952364	0.789948427	0.849996125	0.861502211	0.845338984	0.750263002
General Electric	0.502489662	0.931961450	0.971837253	0.977452816	0.978113466	0.976415585	0.967324790	0.935243191
Chevron Corp.	0.0004162459	0.0057868266	0.1470670258	0.5272169582	0.6797454305	0.7280223688	0.7246679655	0.6363931753
Home Depot	0.010691908	0.071589255	0.770841503	0.877537183	0.905257012	0.908884373	0.892762842	0.811296992
Wal-Mart	0.015357357	0.185281850	0.840372453	0.909855083	0.929128164	0.931791933	0.913079057	0.838882358
McDonald's	0.005581037	0.053085959	0.638811241	0.804581463	0.843446306	0.847194943	0.813534127	0.705643189
AT&T	0.018362006	0.176061731	0.859380821	0.918328521	0.934608207	0.936694872	0.917193103	0.840623921

Table 3.20 gives the result of credit default prediction in LASVJ model via cross-sectional averaging. Similar to previous findings, the LASVJ model produces higher credit spread prediction compared to SVJ model. For example, Wal-Mart has credit spread prediction of 97.4 and 173.5 at 6-month and 10-year maturity with LASVJ model. Looking at Table 3.6, it can be observable that the credit spread prediction of 29.3 and 128.7 at the same maturities with SVJ model. Thus, in the cross-sectional averaging model, the credit spread prediction is higher in all maturities with LASVJ model indicating that credit spread which can be used as an proxy for credit risk is far from being fair when liquidity is overlooked.

Table 3.20: Credit Spread Prediction with Cross-Sectional Averaging in LASVJ model During Crisis

Companies	6 months	1 year	2 years	3 years	4 years	5 years	7 years	10 years
3M	12.507565118	69.073495006	1027.007398716	1201.662257267	1007.031590264	829.660359220	576.684838895	353.926857978
Microsoft	71.148640713	211.904106519	1708.619455381	1407.285563554	1102.191237854	886.454405330	611.404831154	364.860073446
Intel	1.368022014	10.072882945	123.830019741	479.339255507	591.986978595	555.604228124	416.367653687	250.697365286
Exxon Mobil	19.924417910	91.879359494	1143.173912638	1265.890673742	1038.782738554	845.021784769	589.563668747	356.825241834
General Electric	4487.783158939	4664.652249437	2461.122683486	1653.020068794	1240.850064711	990.450010348	698.965743245	468.5603137047
Chevron Corp.	3.330244414	23.174137784	303.140539006	789.484914922	793.285918018	688.389006504	489.004663924	293.7769687925
Home Depot	85.718691912	290.537030560	1843.279298723	1441.151599061	1123.945362117	903.710619484	631.105825815	392.329948696
Wal-Mart	123.237761996	770.028014070	2048.487620452	1508.218589882	1161.649893073	932.714245738	649.278724291	408.800078002
McDonald's	44.698209740	214.630762256	1475.376651899	1294.536990401	1028.878438306	827.633705409	562.365131146	331.644096671
AT&T	147.438167249	730.274554953	2106.085066699	1526.028486998	1170.386481334	938.976959865	652.987062359	409.849056142

Figure 3.9exhibits very similar pattern to previous LASVJ model output. Accordingly, after a rapid increase in credit spread against time to maturity, a sudden drop follows indicating that the effect of crisis disappears.

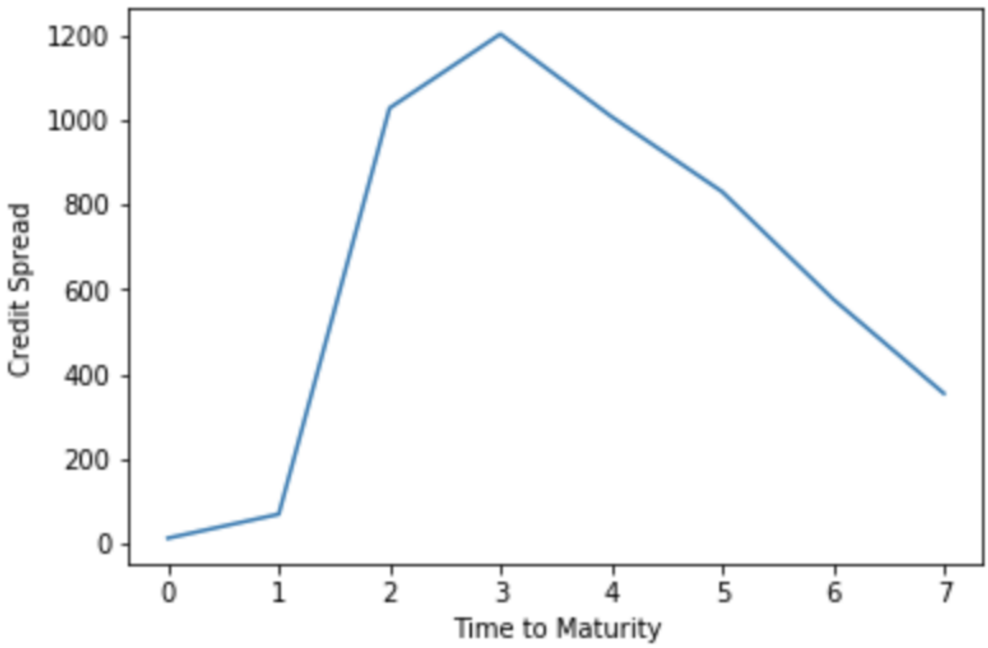


Figure 3.9: Credit Spread vs. Time to Maturity Based on LASVJ During Crisis by Cross Sectional Averaging Method

3.9 Conclusion

Liquidity has remained long as a controversial issue until the morgage crisis broke out. Since then, researchers try to incorporate liquidity in asset pricing and credit risk models among with others. It is, however, a formadible task to accomplish in that liquidity is an unobservable phenomenon. This study is an attemp to fulfill this task both theoretically and empirically.

By including liquidity into the SVJ model, model is able to capture liquidity dimension in the market by which it is expected to have more realistic estimation of probability of default and credit spread. Because, ignoring liquidity dimension during crisis period leads to some adverse situations such as mispricing the assets, misallocation of credit, and underestimation of credit risk. All these cases are likely to undermine the financial market and pave the way of a new financial crisis.

After theoretical derivation of the Liquidity Augmented Stochastic Volatility with Jump model, short for LASVJ, empirical analyses indicate that probabilities of default estimation are larger with LASVJ model indicating that ignoring liquidity dimension might lead to underestimation of risk in the market which in turn might trigger other key estimations in the financial markets as well.

Besides, credit spreads, calculated based on the probability of default, are, naturally, estimated larger with LASVJ model, too. Additionally, the gap between LASVJ and SVJ model is, generally, even more dramatic at longer maturities.

As empirically shown, crisis periods, during which liquidity drough prevails, the probability of default and credit spread estimation become even larger with LASVJ model compared to periods without crisis at early maturities but this effect dies out over maturity as the crisis period closes to an end.

In short, liquidity is an indispensable dimension and ignoring it might lead unrealistic and unfair financial estimations. Liquidity shocks or illiquidity shocks reflect themselves as volatile prices in markets. In this respect the next chapter provides comparative results of volatility models.

CHAPTER 4

VOLATILITY PREDICTION AND RISK MANAGEMENT: A MACHINE LEARNING APPROACH

4.1 Introduction

Increased integration of financial markets has led to a prolonged uncertainty in financial markets, which in turn stresses the importance of volatility. Volatility is used in measuring the degree of risk, which is one of the main engagements of the area of finance.

There is a large and growing body of literature regarding the volatility estimation that is used for assessing the various types of risks. After the ground-breaking studies of [14], [6], [29], [30], and [66], volatility estimation gained a great importance. The increasing significance of fluctuations in financial markets together with the availability rich data necessitate more accurate and reliable estimation, prediction and forecasting of volatility. This challenging task calls for new techniques to be integrated to volatility estimation. In this respect, this study incorporates Machine Learning (ML) approach and introduces a Support Vector Regression- GARCH (hereinafter SVR-GARCH) method.

The long tradition of GARCH-type models application, which quantitative model-based forecasts can provide financial institutions with a valuable assessment of future market trends. First quantitative model called Autoregressive Conditional Heteroskedasticity (ARCH) proposed by [32] and. This model was generalized by [17] and named as Generalized Autoregressive Conditional Heteroskedasticity (GARCH).

This model assumes a functional form of data generating process, error term, and moreover has low forecasting performance. Different variations of GARCH have been introduced to overcome the drawback by proposing improvement on the functional form, volatility proxy, and accuracy metrics [12].

In particular, volatility in financial assets returns can be affected differently from positive and negative news. Thus, volatility of financial assets returns can give asymmetrical responses to positive and negative shock and the timing of news might not be a surprise enabling more robust estimation of volatility. ARCH/GARCH models are not sufficient for modeling asymmetric reactions. In order to deal with these, Exponential GARCH (EGARCH) and Fractionally Integrated GARCH (FIGARCH) models are proposed.

Volatility does not only provide important insights about financial phenomena but also has a significant usage in risk management along with other areas such as pricing. Therefore, it is quite important to forecast the volatility in order to have solid risk management. As traditional models have empirically low forecast performance, it is expected that SVR-GARCH does not only boost the predictive performance but also improves the risk management. To apply volatility models in risk management, volatility predictions are used as an input in Value-at-Risk method and based on the backtesting of Value-at-Risk, it is expected to have improved risk management performance. In a nutshell, volatility prediction with high accuracy provides better risk management by shedding light on uncertainties of the future path of financial risks as well as increasing awareness about the risks that have yet to come.

To this end, in this thesis, traditional ARCH-type models in estimating and predicting volatility are employed and compared with a Machine Learning-based model. The results are compared using Root Mean Square Error and Mean Absolute Error metrics. From this point on, these volatility predictions serve as input in the Value-at-Risk as well as its backtesting.

The remainder of this chapter is as follows. In the second section, theoretical introduction of the volatility models are provided. In the third section, Machine Learning approach to volatility modeling are discussed. In the fourth section, volatility estimations with traditional and Machine Learning model are provided with the performance comparisons. In the fifth section, VaR applications of these models along with their backtesting are tested. The final chapter concludes this study.

4.2 Traditional Volatility Models

In this part, firstly the theoretical foundations of the GARCH model and its extensions are introduced.

4.2.1 GARCH Model

ARCH model proposed by [32] and improved by [17] and [72] by adding p number of delayed past conditional variance and it is called GARCH (p, q) model. GARCH models can be expressed as autoregressive moving average models for conditional variance [19].

Mathematically, ARCH models depends on the realized values of the squared error terms in previous time periods. The model has the following form:

$$y_t = u_t \tag{4.1}$$

$$u_t \sim \mathcal{N}(0, \sigma) \tag{4.2}$$

$$\sigma_t^2 = \alpha_0 + \sum_{i=1}^{q} \alpha_i u_{t-i}^2 \tag{4.3}$$

This model is known as ARCH(q) where q amounts to order of the lagged squared errors. As an extension, the structure of the GARCH model implies that conditional variance is determined by historical information implying market inefficiency. The term u_t in equation 4.3 is accepted as return and as the amount of news at time t so that positive u_t means good news or vice versa. In the GARCH model, the conditional variance of return is determined by the square of the error term and its lagged values.

$$\sigma_t^2 = \omega + \beta\sigma_{t-1}^2 + \alpha\epsilon_{t-1}^2 \tag{4.4}$$

where ω, β, and α are parameters and have restrictions: ω > 0, $\beta \geq 0$, and $\alpha \geq 0$. Moreover, in order for GARCH to be consistent, [47] states that β + α < 1.

The main reasons that GARCH are applied to financial phenomenon are returns are well fitted by GARCH model partly due to the volatility clustering and GARCH does not assume that the returns are independent that allows modeling the leptokurtic property of returns. Despite these useful properties and intuitiveness, GARCH is not able to asymmetric response of the shocks. Therefore, GJR-GARCH, or extended GARCH, is proposed by [40] to account for this asymmetric response to the shocks. Conversely, the effect of a negative shock indicates that the firm is more leveraged [10].

4.2.2 GJR-GARCH Model

As is discussed, GJR-GARCH tries to capture the asymmetry in the news impact function and to do that γ is included into the GARCH equation and it turns out:

$$\sigma_t^2 = \omega + \beta\sigma_{t-1}^2 + \gamma\epsilon_{t-1}^2 I(\epsilon_{t-1} < 0) \tag{4.5}$$

If γ=0, the response to the past shock is the same. If γ is greater than zero then the response to the past negative shock is stronger than that of a positive one. Thus, GJR-GARCH is designed to capture the asymmetric news impact curve, which is a function of ϵ_{t-1}.

4.2.3 EGARCH Model

GJR-GARCH model is not the only model proposed to capture the asymmetry in the news impact curve. Rather, [62] develops EGARCH model to account for this asymmetry and it takes the following form:

$$\log(h_t) = \omega + \sum_{j=1}^{p} \beta_j \log(h_{t-j}) + \sum_{i=1}^{q} \alpha_i \frac{|u_{i-1}|}{\sqrt{h_{t-i}}} + \sum_{i=1}^{q} \gamma_i \frac{u_{t-i}}{\sqrt{h_{t-i}}} \tag{4.6}$$

According to equation 4.6 , while ω grasps the asymmetric shocks of volatility and α capture the volatility clustering. Thus, similar to GJR-GARCH model asymmetric shocks of volatility and volatility of clustering are overcame by the EGARCH model.

In a nutshell, the reasons why EGARCH is superior model over GARCH model are as follows:

- EGARCH never gets negative values in that the conditional variance equation takes the logarithmic form. Thus, non-negativitiy condition of the GARCH model is no longer needed.
- As EGARCH allows us to take into account the positive and negative shocks, it is possible to examine the leverage effect with EGARCH model as opposed to GARCH model.

GARCH model and its extensions are insufficient to evaluate the long memory property defined as the hyperbolic rate decrease in the autocorrelation functions of the high frequency financial time series, long term dependence and tendency to slow reversion to return. Such time series exhibit hyperbolic decreasing autocorrelations and, if long memory is concerned, the impact of a shock on financial markets persists for a long time. The long memory process is therefore characterized by a fractional degree of integration rather than an integer degree of integration. Fractionally Integrated GARCH (FIGARCH) model is introduced by [7] and discussed in the following part.

4.2.4 FIGARCH Model

[33] first introduces the Integrated GARCH model known as IGARCH to model the long-term persistence. As stated by [69], any shocks to the conditional variance persist into the future and this property can be modeled by Integrated GARCH but it turns out IGARCH without drift converges to zero. Thus, the IGARCH model is considered as short-term volatility model.

In literature, many researchers, among others, [5], [41], and [42], have demonstrated that long memory features can be modeled by extending an integrated process into a fractional integrated process. If the stock market return volatility has a long memory characteristics, it is no longer a random process and can be predicted via historical data.

The derivation of FIGARCH is provided below. To start with, GARCH (1,1) can be written as:

$$\sigma_t^2 = \omega + \beta(L)\sigma_{t-1}^2 + \alpha(L)\epsilon_{t-1}^2 \tag{4.7}$$

where $\alpha(L) = \alpha_1 L^1 + \alpha_2 L^2 + ... + \alpha_q L^q$ and $\beta(L) = \beta_1 L^1 + \beta_2 L^2 + ... + \beta_p L^p$

Once $\upsilon \equiv \epsilon_t^2 - \sigma_t^2$, then it is possible to define GARCH(p,q) as follows:

$$[1 - \alpha(L) - \beta(L)]\epsilon_t^2 = \omega + [1 - \beta(L)]\upsilon \tag{4.8}$$

The FIGARCH model can be written as:

$$\phi(L)(1 - L)^d \epsilon_t^2 = \omega + [1 - \beta(L)]\upsilon \tag{4.9}$$

where $\phi(L) = [1 - \alpha(L) - \beta(L)](1 - L)^{-1}$

Equation 4.9 can be converted to the following equation, which is the standard representation of FIGARCH model:

$$[1 - \beta(L)]\sigma_t^2 = \omega + [1 - 1 - \beta(L) - \phi(L)(1 - L)^d]\epsilon_t^2 \tag{4.10}$$

4.2.5 SVR Model

Support Vector Machine (SVM) is a well-known machine learning algorithms based on convex optimization that operates according to the structural risk minimization principle and it is distribution-free learning algorithm [70]. Besides, the SVM is a controlled learning method used in classification and regression analysis that analyzes data and learns from samples. The SVM was first proposed by [73].

Both linearly distinguishable and non-distinguishable data sets can be classified with SVM. The disticntion feature of SVM lies in its n-dimensional application. With a nonlinear mapping, the n-dimensional data set is converted to a new d-size data set with d>n. With a suitable transformation, the data can always be divided into two classes with a hyperplane.

In this study, Support Vector Regression-based GARCH (SVR-GARCH) modeling is applied. SVR nonlinearly maps the input space into a high dimensional feature space and employs the linear regression in a high dimensional feature space. To show the theoretical background of SVR, let x_t and y_t be training dataset where $x_t \in \mathbb{R}^p$, $y_t \in \mathbb{R}^1$.

As the dataset used has a time-series structure, x_t can be considered as lagged values of y_t. Data is generated from a function

$$y_t = f(x_t) + \epsilon_t \tag{4.11}$$

At this point, it is needed to define a decision function f(x) as follows:

$$f(x_t) = w^T \phi(x_t) + b = \sum_{i=1}^{n} w_i \phi_i(x) + b \tag{4.12}$$

where $\phi(x) = [\phi_1(x), ..., \phi_n(x)]^T$ and $w = [w_1, ..., w_n]^T$ is a non-linear transformation to a higher dimension space. [73] suggests a ϵ-insensitive loss function, $L_\epsilon(x, y, f(x))$ and it is defined by:

$$L_\epsilon = \begin{cases} |y = f(x)| - \epsilon, & |y = f(x)| \geq \epsilon \\ 0, & \text{otherwise} \end{cases}$$

As [24] denotes that the loss function does not penalize errors below ϵ. In this case, error is ignored and no loss occurs. This implies that f(x) is obtained through data-point located on or outside the ϵproximity. This is called ϵ-insensitivity.

At this point, ξ and $\xi*$ are introduced as slack variable to describe the ϵ-insensitivit loss and ϵ-SVR is provided as:

$$\min_{w,b,\xi,\xi*} [\frac{1}{2}||w||^2 + C \sum_{t=1}^{n} (\xi + \xi^*)] \tag{4.13}$$

subject to

$$y_t - w'\phi(x_t) - b \leq \epsilon + \xi_t^* \tag{4.14}$$

$$w'\phi(x_t) + b - y_t \leq \epsilon + \xi_t^* \tag{4.15}$$

$$\xi_t, \xi_t^* \geq 0 \tag{4.16}$$

In equation 4.13, $\frac{1}{2}||w||^2$ measures the function flatness and the second term relates to ϵ-insensitive loss function. This problem can be solved using Langrangian approach.

$$L_p = \frac{1}{2}||w||^2 + C\sum_{t=1}^{n}(\xi + \xi^*) \tag{4.17}$$

$$- \sum_{t=1}^{n} \alpha_t(\epsilon + \xi_t - y_t + w'\phi(x_t) + b) - \sum_{t=1}^{n} \mu_t + \xi_t \tag{4.18}$$

$$- \sum_{t=1}^{n} \alpha_t^*(\epsilon + \xi_t^* - y_t + w'\phi(x_t) + b) - \sum_{t=1}^{n} \mu_t^* + \xi_t^* \tag{4.19}$$

Karush-Kuhn-Tucker condition makes it possible to have following equations:

$$\frac{\partial dL_p}{\partial dw} = w - \sum_{t=1}^{n}(\alpha_t - \alpha_t^*)\phi(x_t) = 0 \tag{4.20}$$

$$\frac{\partial dL_p}{\partial db} = \sum_{t=1}^{n}(\alpha_t - \alpha_t^*) = 0 \tag{4.21}$$

$$\frac{\partial dL_p}{\partial d\xi_t} = C - \alpha_t - \mu_t = 0 \tag{4.22}$$

$$\frac{\partial dL_p}{\partial d\xi_t^*} = C - \alpha_t^* - \mu_t^* = 0 \tag{4.23}$$

where the parameters $\alpha_t, \mu_t, \alpha_t^*, \mu_t^* \geq 0$. Additionally, the kernel function is of considerable importance role in forecasting performance of the SVR model. There are three types of kernel function, which are:

- Linear Kernel: $x_t x$
- Polynomial: $(x_t x + 1)^d$
- Gaussian: $e^{\frac{-||x-x_t||^2}{2\sigma^2}}$

4.2.6 SVR-GARCH Model

As it is discussed, SVM is a state-of-the-art method and can be applied in a wide range of areas. In this thesis, SVM is employed to model the volatility using the GARCH model. The main motivation of this rests upon the fact that SVR has no probability density function over returns [73].

SVR-GARCH equips researcher with a very powerful tool which is forecasting the volatility and in turn employing it in modeling risk. Traditional risk models such as VaR, for instance, uses standard deviation to account for the volatility in returns and then feeds the model. However, aside from the traditional volatility model, there is no many other tools to forecast volatility to be utilized in forecasting the risk. The SVR-GARCH model produces better-suited approach and provides more robust results.

Following structure specifies the SVR-GARCH process:

$$r_t = \log(P_t/P_{t-1}) \tag{4.24}$$

where P_t and r_t are price and return at time t, respectively. In order to find the squared residuals, the conditional mean estimation is used and it is given by:

$$r_t = g(r_{t-1}) + a_t \tag{4.25}$$

where g is the estimation function for mean equation estimated by SVR as suggested by [12].

Following [22], a volatility proxy is used due to the unobservability of the volatility.

$$\sigma_t^2 = (r_t - \overline{r})^2 \tag{4.26}$$

where σ_t is the conditional variance.

$$\sigma_t^2 = f(r_{t-1}^2, \sigma_{t-1}^2) \tag{4.27}$$

where f is the SVR decision function.

4.3 Empirical Application

In this part, empirical application is conducted over 30 stocks listed in S&P-500 between 2010/01-2019/01 corresponding to 2268 days. The first 90% of the data is allocated to the training set and the rest 10% is reserved for test set. The stock employed in this analysis is shown in the Table 4.1.

Table 4.1: Companies

Companies	Tickers	Companies	Tickers
Amgen Inc.	AMG	Lockheed Martin Corporation	LMT
Amazon.com Inc.	AMZN	Macy's Inc	M
AutoZone Inc.	AZO	Mettler-Toledo International Inc.	MTD
Booking Holdings Inc.	BKNG	Mylan N.V.	MYL
BlackRock Inc.	BLK	NIKE Inc. Class B	NKE
Duke Realty Corporation	DUK	O'Reilly Automotive Inc.	ORLY
Consolidated Edison Inc.	ED	People's United Financial Inc.	PBCT
Ford Motor Company	F	Prologis Inc.	PLD
Freeport-McMoRan Inc.	FCX	Regions Financial Corporation	RF
General Electric Company	GE	Raymond James Financial Inc.	RJF
Alphabet Inc. Class C	GOOG	Sherwin-Williams Company	SHW
Gap Inc.	GPS	TransDigm Group Incorporated	TDG
Huntington Bancshares Incor.	HBAN	Under Armour Inc.	UAA
Intercontinental Exchange Inc.	ICE	V.F. Corporation	VFC
Leidos Holdings Inc.	LDOS	WEC Energy Group Inc.	WEC

Using this data, return volatilities are modeled by GARCH, GJR-GARCH, E-GARCH, FIGARCH, and SVR-GARCH.

In SVR-GARCH application process, parameter optimization is conducted and based on this optimization, best parameters setting is found and used for each stocks. To do that C, ϵ, γ parameters spanned between 0 and 10. Based on the Akaike Information Criteria (AIC), the best model is chosen so that volatility is modeled by these parameters. Then, RMSE and MAE are calculated to compare the performance of the model.

Before moving forward, the returns of the stock considered along with the histograms and autocorrelation functions are depicted in Figure 4.1, 4.2, and 4.3, respectively. Return plots of the stocks included in the study are provided in Figure 4.1, by eyeballing, it indicates that returns oscillate around zero as anticipated though some stocks such as WEC Energy, Regions Financial Corporation, BlackRock show greater extent of volatility.

In the Figure 4.2, it is checked whether the returns are normally distributed and moreover normality test is run and its result revealed in the Table 4.2. Normality test indicates that stock returns are not normally distributed at conventional levels.

In the Table 4.3, p-values suggest all stock returns are stationary because p-values are less than 0.05. In the volatility estimation, autocorrelation function (ACF) plays an important role as it exhibits volatility clustering. Autocorrelation does not require that linear increases or decreases in returns be independent of each other. Rather, independence of any non-linear function of returns requires autocorrelation. In practice, simple nonlinear functions of returns show significant autocorrelation or persistency. This is the so-called volatility clustering. In the Figure 4.3, ACF of all stocks are provided, it is observed that autocorrelation coefficients, in almost all cases, remain outside the confidence interval implying that there exists volatility clustering.

4.3.1 Volatility Prediction Assessment

In this part of the study, volatility estimation via proposed models are conducted and these results are used as an input in the Value-at-Risk estimation. The proposed volatility models used in the out of sample evaluation are GARCH, GJR-GARCH, EGARCH, FIGARCH, and SVR-GARCH. Aside from SVR-GARCH, normal distribution, student-t and skewed distributions are employed in the model. In the SVR-GARCH, linear, Gaussian, and Polynomial extensions are utilized.

Data used are extracted from yahoo finance for the period of 01/01/2010-09/01/2019.

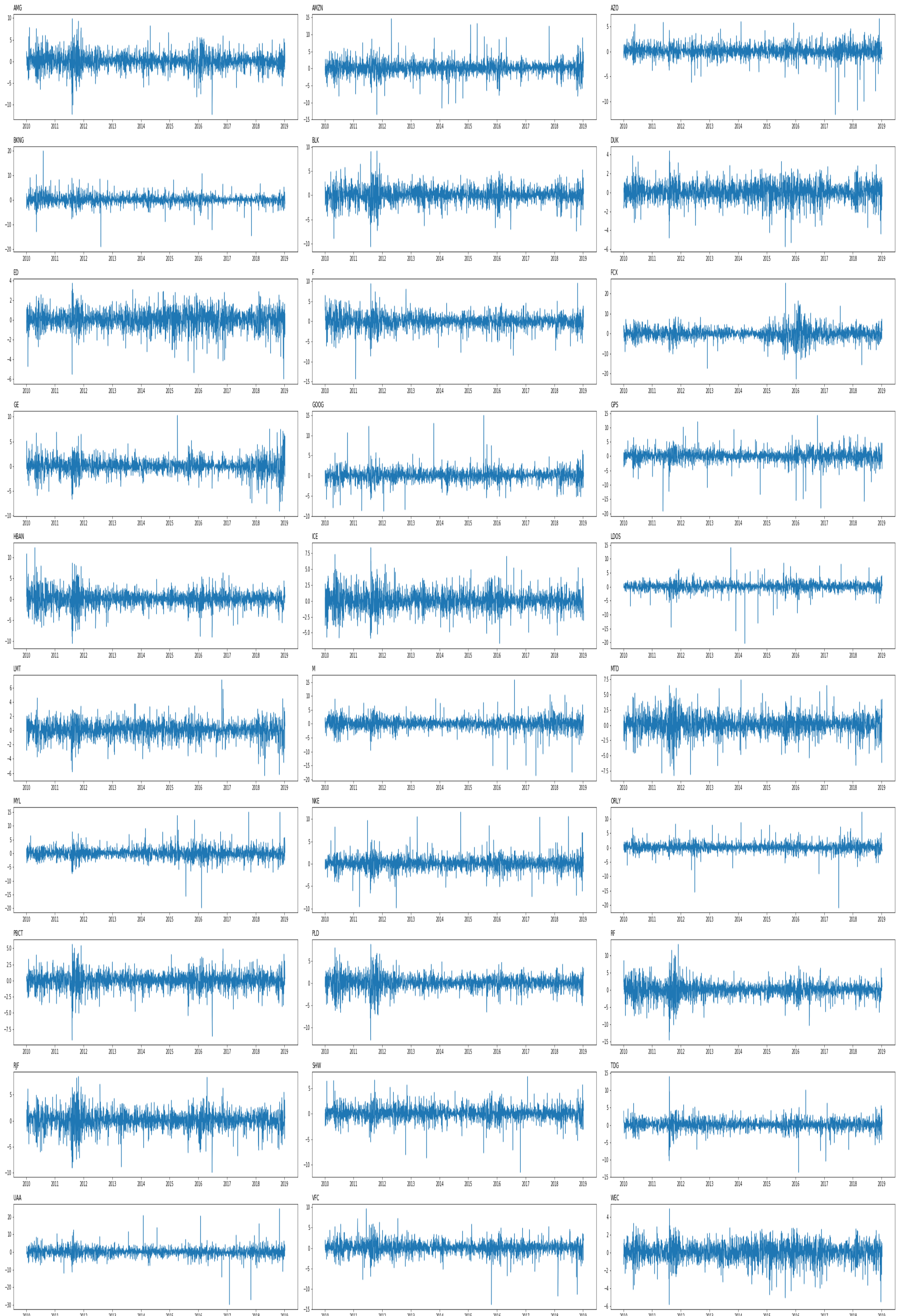


Figure 4.1: Returns

The returns are calculated by using closing price and 90% of the total reserved for training data and the remaining 10%, corresponding to 253 data points, belongs to test

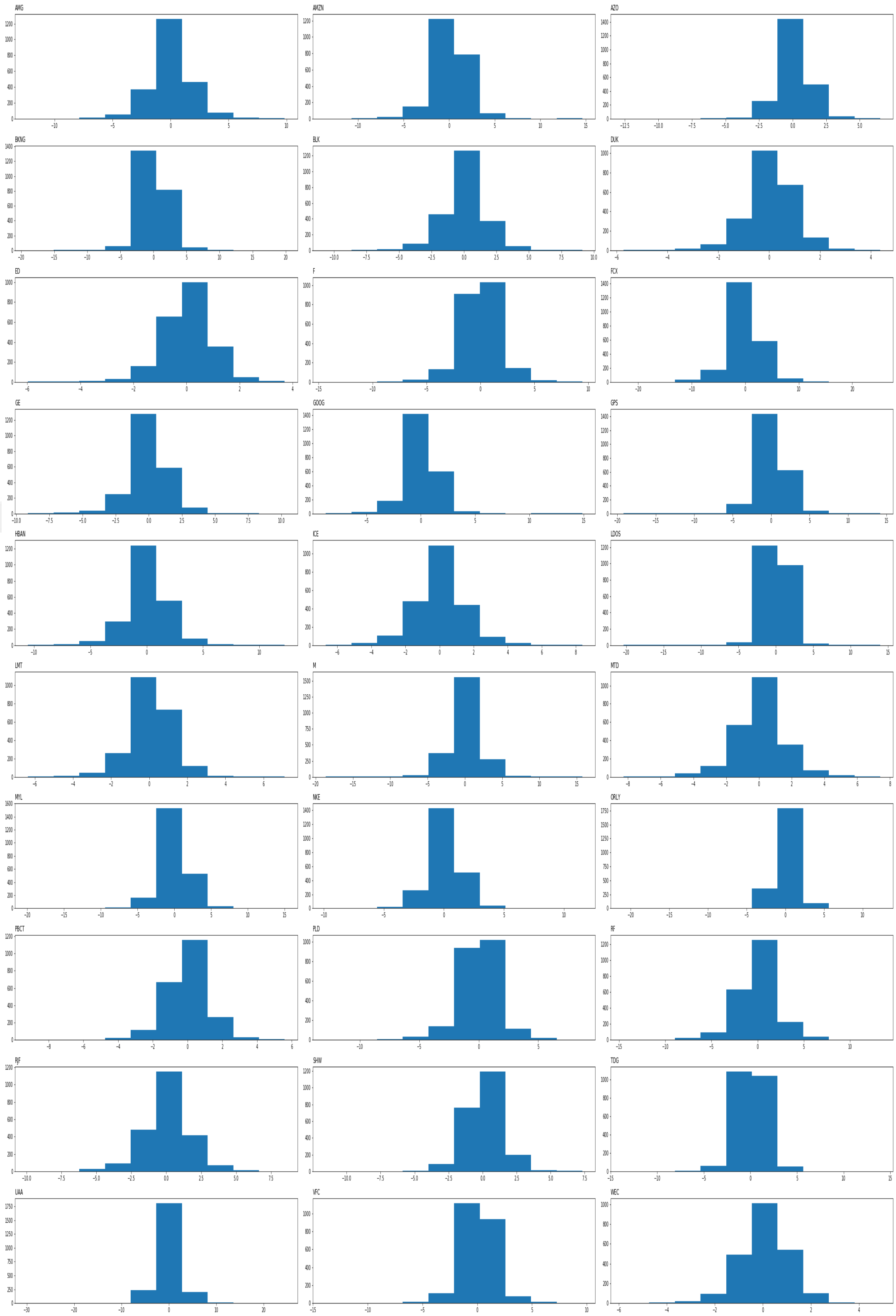


Figure 4.2: Histograms

data. According to the performance metrics of MAE and RMSE, SVR-GARCH models produces the best result compared to other traditional volatility models. To be in-

Table 4.2: The Results of the Normality Test

Companies	p-values	Companies	p-values
AMG	4.77e-47	LMT	8.85e-50
AMZN	1.159e-74	M	4.32e-137
AZO	1.9e-173	MTD	5.79e-45
BKNG	3.37e-105	MYL	2.43e-91
BLK	5.69e-49	NKE	6.74e-99
DUK	1.77e-45	ORL	6.76e-213
ED	2.41e-62	PBCT	2.54e-66
F	3.49e-64	PLD	4.50e-69
FCX	9.75e-67	RF	9.99e-53
GE	2.13e-54	RJF	6.16e-51
GOOG	1.58e-119	SHW	1.21e-77
GPS	1.10e-183	TGD	6.05e-99
HBAN	2.02e-45	UAA	2.59e-129
ICE	2.97e-30	VFC	1.23e-108
LDOS	1.35e-251	WEC	9.26e-49

Table 4.3: Stationarity Test

Companies	p-values	Companies	p-values
AMG	2.66e-17	LMT	0.0000
AMZN	0.0000	M	0.0000
AZO	0.0000	MTD	0.0000
BKNG	0.0000	MYL	0.0000
BLK	0.0000	NKE	0.0000
DUK	0.0000	ORLY	0.0000
ED	0.0000	PBCT	0.0000
F	0.0000	PLD	0.0000
FCX	4.21e-16	RF	0.0000
GE	4.66e-13	RJF	4.42e-21
GOOG	0.0000	SHW	0.0000
GPS	0.0000	TDG	6.45e-22
HBAN	1.08e-17	UAA	0.0000
ICE	0.0000	VFC	0.000000e+0
LDOS	0.0000	WEC	0.0000

terpret, SVR-GARCH-linear model has an MAE of 0.009 which is nearly one-fifth of the remaining traditional models. For instance, GARCH, GJR-GARCH, EGARCH, and FIGARCH with normal distribution have an MAE of 0.0052, 0.0054, 0.0057, and 0.0057, respectively. Similarly, SVR-GARCH-linear and EGARCH have the lowest

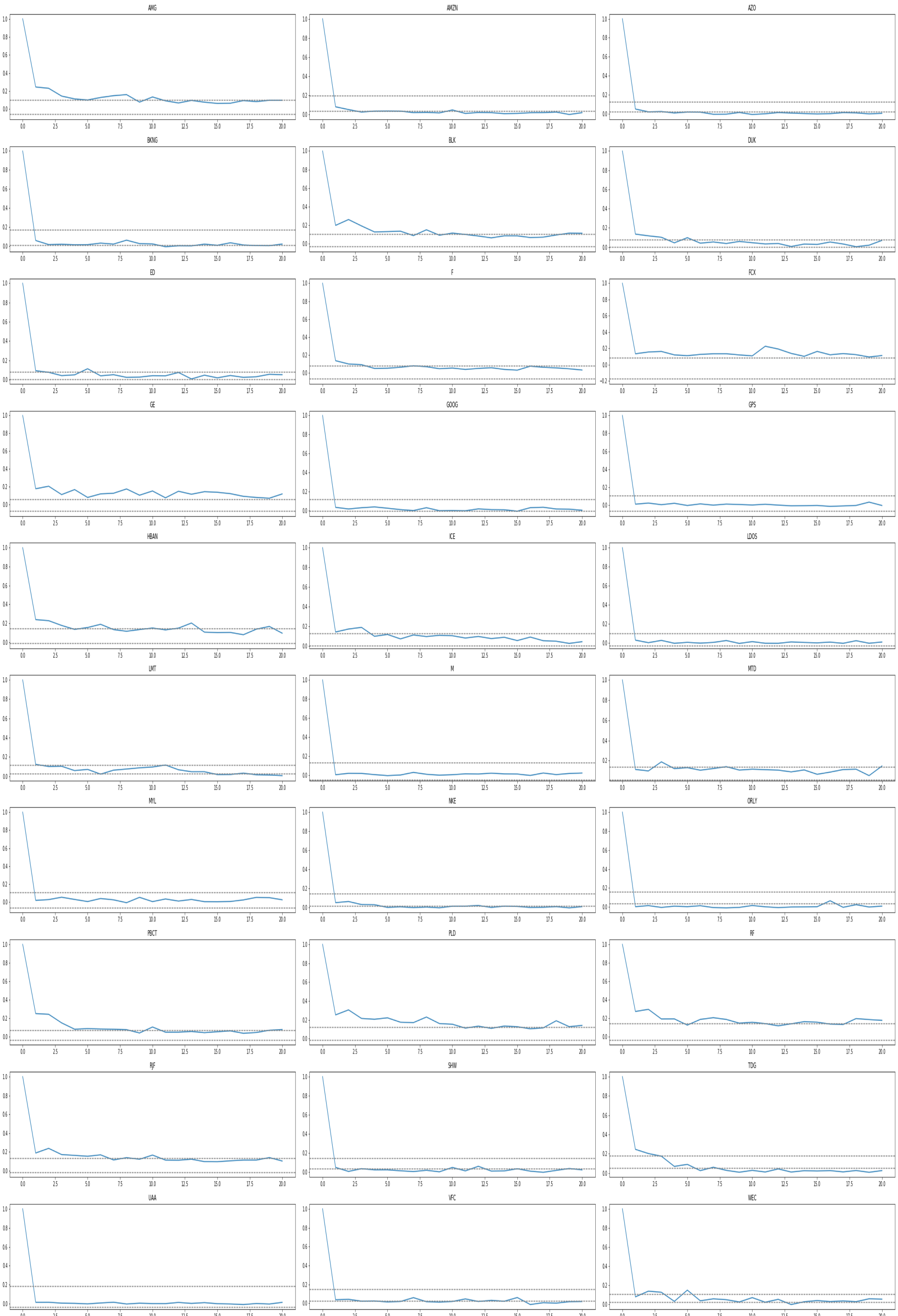


Figure 4.3: Autocorrelation Function

and highest RMSE, respectively.

Besides, the second and third best models in terms of performance metrics is SVR-GARCH-RBF and SVR-GARCH-Polynomial. These findings highlight that SVR-GARCH model outperforms the GARCH-type models at every confidence levels. Visualization based on these findings can be found in the Appendix B.

Table 4.4: Out-of-Sample Evaluation

Models	MAE	RMSE
SVR-GARCH-linear	0.0009	0.0013
SVR-GARCH-RBF	0.0013	0.0025
SVR-GARCH-Polynomial	0.0014	0.0029
GARCH-Normal	0.0052	0.0068
GARCH-Student t	0.0052	0.0068
GARCH-Skewed	0.0052	0.0068
GJR-GARCH-Normal	0.0054	0.0070
GJR-GARCH-Student t	0.0054	0.0070
GJR-GARCH-Skewed	0.0053	0.0070
EGARCH-Normal	0.0057	0.0075
EGARCH-Student t	0.0053	0.0071
EGARCH-Skewed	0.0053	0.0071
FIGARCH-Normal	0.0057	0.0075
FIGARCH-Student t	0.0053	0.0071
FIGARCH-Skewed	0.0053	0.0071

4.4 Risk Management

Risk management is an indispensable of the financial management that makes it possible to properly price the assets and helps institutions prepare the unexpected. In financial institutions, risk management is conducted by these four risks types: Market risk, liquidity risk, credit risk and operational risk.

In recent years, financial globalization, subsequent increase in competition, diversity in financial products and increasing transaction volumes have led institutions to form a complex and interconnected financial system. In such an environment, especially financial institutions have become more open to the risks arising from price movements in markets. To this end, institutions have intensified their efforts to identify, measure, control and update their risks and have been in search of an effective risk management system.

These developments have, therefore, even increased the importance of risk management and its tools. Over decades, financial risk measurements process which started with basic risk measurements, continued with the development of an internal model for the measurement of risks by many financial institutions. In a higly evolving and complex risk environment, the fact that institutions are exposed to diverse and intensified risks has led regulatory authorities to establish a standard and intuitive methodology. JP Morgan addresses this need by introducing the Riskmetrics system in 1994, which enables to calculate of Value at Risk (VaR).

4.4.1 Value-at-Risk Application

VaR is defined as a financial instrument that measures the expected and unexpected loss that can occur in a given time period in a given confidence interval as shown in the Figure [?] [50]. VaR method can be measured on a security and on a portfolio basis. Risks arising from different positions and risk factors may occur in the portfolio and it is calculated in terms of monetary basis. VaR can be computed as follows:

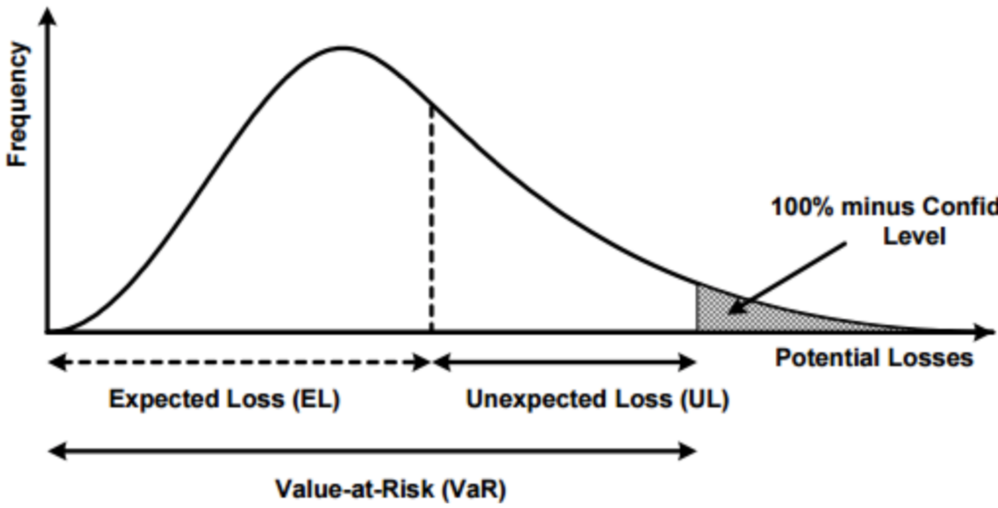


Figure 4.4: Value-at-Risk Representation

$$\text{VaR} = V_t * \sigma * \sqrt{t} * \alpha \tag{4.28}$$

whereV_t is the value of the portfolio, σ is the standard devation of portfolio, t is the holding period, α is the confidence interval. In this regard, it can be concluded that in the VaR estimation, four parameters are important:

- Portfolio diversification
- Distribution of the portfolio returns
- Holding time period
- Confidence interval

Differently, the standard VaR definition is [8]:

$$\mathbb{P}(P(T) - P(t) < -VaR_{\alpha}^{P/L}(t,T)) = 1 - \alpha \tag{4.29}$$

where P(T) represents the price of the portfolio. Using CDF of log-returns, it turns out:

$$\mathbb{P}(r(t,T) < -VaR_{\alpha}^{r}(t,T)) = 1 - \alpha \tag{4.30}$$

Given that $P(T) = P(t)e^{r(t,T)}$, VaR^r and $\text{VaR}^{P/L}$ can be redefined as:

$$\text{VaR}_{\alpha}^{r}(t,T)) = -\ln(1 - \frac{\text{VaR}_{\alpha}^{P/L}(t,T)}{P(t)}) \tag{4.31}$$

$$\text{VaR}_{\alpha}^{P/L}(t,T)) = P(t)(1 - e^{-\text{VaR}_{\alpha}^{r}(t,T)}) \tag{4.32}$$

Thus,

$$\text{VaR}_{\alpha}^{P/L}(t,T) \approx P(t)\text{VaR}_{\alpha}^{r}(t,T) \tag{4.33}$$

In this thesis, a parametric method of VaR called as Variance-Covariance VaR method is used. Using this method, VaR is calculated by multiplying the significance level, α value corresponding to the confidence level and the standard deviation (σ) by the market value (M) of the portfolio. This method is advantageous in terms of ease of calculation and computational efficiency compared to other methods.

4.4.2 Backtesting

Several models have been developed to calculate VaR estimates. Due to the high diversity of the VaR models, it is of considerable importance to test the validity of the method applied. The main reason why VaR is questioned is the various shortcomings of VaR models. VaR models are only strong when they have strong predictive power. Thus, it has become common practice to test the performance of VaR models called backtesting.

Backtesting is a statistical procedure where the deviations between the losses realized and the estimated losses during the backtesting process are calculated. For instance, if the confidence level set for calculating daily VaR is 90%, it is expected to occur ten exceptions in every 100 days on average. In addition, if the confidence level is 99%, one exception in every 100 days is expected on average. In a nutshell, in backtesting, it is statistically checked whether the frequency of exceptions over some specified time interval agrees with the related confidence level. This procedure is called as unconditional coverage tests [51]. Some well-known backtests can be listed as:

- Binomial test
- Traffic light test
- Kupiec's tests
- Christoffersen's tests
- Haas's tests

In this thesis, POF-Test (Proportion of Failures) proposed by [53] is applied and it measures whether the number of exceptions is in line with the confidence level. So, the test hypotheses can be stated as:

$$H_0 : p = \hat{p} = \frac{E}{T}$$

$$H_0 : p \neq \hat{p} = \frac{E}{T}$$

where E is the number of exception and T is the number of observations. This test assumes that the number of exceptions follows binomial distribution given below:

$$f(E) = \binom{T}{E} p^E (1-p)^{T-E} \tag{4.34}$$

Hence, POF-test tries to find out whether there is statistically significance between observed failure, $\hat{f}$, and the failure rate proposed by confidence level and likelihood ratio (LR) test is employed to decide the correctness of the model.

$$LR_{POF} = -2\ln\left[\frac{(1-p)^{T-E}p^E}{(1-\frac{E}{T})^{T-E}(\frac{E}{T})^E}\right] \tag{4.35}$$

LR test asymptotically follows χ^2 with one degree of freedom and model is true under null hypothesis of this test. The Table 4.5 shows the acceptance and rejection regions of the POF test.

Probability Level p	VaR Confidence Level	Nonrejection Region for Number of Failures N		
		T = 255 days	T = 510 days	T = 1000 days
0.01	99 %	N < 7	1 < N < 11	4 < N < 17
0.025	97.5 %	2 < N < 12	6 < N < 21	15 < N < 36
0.05	95 %	6 < N < 21	16 < N < 36	37 < N < 65
0.075	92.5 %	11 < N < 28	27 < N < 51	59 < N < 92
0.1	90 %	16 < N < 36	38 < N < 65	81 < N < 120

Figure 4.5: Acceptance-Rejection Regions for POF Test

Based on the number of fails obtained via POF test, it is possible to run Traffic Light backtest which is suggested by Basel Committee and this backtest is used as robustness test. According to the Traffic Light backtest, accuracy of a VaR forecast is assessed based on the number of VaR breaches using POF test. For example, according to the recommendations of the Basel Committee in Basel II, VaR values at 99% confidence level of the previous year are compared daily with actual losses. The Basel Committee tolerates up to 4 deviations per year and defines this level as a green zone. If the VaR violation exceeds 4 and remain lower than 10, it corresponds to yellow zone and finally violations at and over 10 defines red zone.

The VaR violation can be defined as follows:

$$E^{i}_{\text{VaR}}(\alpha) := \mathbb{I}_{L_i \leq \text{VaR}_i(\alpha)} = \begin{cases} 1 & \text{if } L_i \leq \text{Var}_i(\alpha) \\ 0 & \text{if } L_i > \text{Var}_i(\alpha) \end{cases} \tag{4.36}$$

where $E^{i}_{VaR}(\alpha) : [0,1] \rightarrow \{0,1\}$. Here, in this setup, X stores the violations happened within a trading day i. When it is generalized over the period under examination, it turns out [25]:

$$E^{N}_{\text{VaR}}(\alpha) := \sum_{i=1}^{N} \mathbb{I}_{L_i \leq \text{VaR}_i(\alpha)} \tag{4.37}$$

where $E^{N}_{VaR}(\alpha) : [0,1] \rightarrow \{0,1,2,3,...,N\}$. In addition, $\mathbb{E}[E^{N}_{\text{VaR}}] = N\alpha$. At this point, it is necessary to introduce cumulative probability to count the number of violation over a defined period of time. For given α and N, the cumulative probability of having a number of violation can be defined as [25] :

$$\Phi^{\alpha,N}_{\text{VaR}}(e) := \mathbb{P}(E^{N}_{\text{VaR}}(\alpha) \leq e) \tag{4.38}$$

In summary, three color zones can be formulated via cumulative probability given below:

- Green zone if $\Phi^{\alpha,N}_{\text{VaR}}(e) < 95\%$
- Yellow zone if $95\% < \Phi^{\alpha,N}_{\text{VaR}}(e) < 99\%$
- Red zone if $\Phi^{\alpha,N}_{\text{VaR}}(e) < 99.99\%$

Figure 4.6 shows these zones and number of corresponding violations[25]. Based on these violation, the performance of the VaR application and the corresponding volatility method is decided. Accordingly, if the number of VaR violations is less than 5, then backtesting suggest that VaR application performs well. If the VaR violations are greater than 4 and less than 10, it implies that the VaR approach should be treated with caution. Eventually, when the VaR violation is greater then 10, it implies that VaR method does not working well.

Basel Traffic Light Approach to VaR		
Zone	Breach Value	Cumulative Probability
Green	0	8.11 %
	1	28.58 %
	2	54.32 %
	3	75.81 %
	4	89.22 %
Yellow	5	95.88 %
	6	98.63 %
	7	99.60 %
	8	99.89 %
	9	99.97 %
Red	more than 10	99.99 %

Figure 4.6: Basel Traffic Light Approach

In the light of these two approaches, Kupiec's POF test and Basel Traffic Light, the validity of the VaR result incorporating volatility estimated via different models are discussed in the next part. First, failure rate and total number of violation are compared and then these violations are assessed considering Basel's Traffic Light approach.

4.4.3 Interpreting the Backtesting Result

After calculating VaR based on variance-covariance method, Kupiec's POF backtesting and Basel's Traffic Light Approach are embraced so that it is possible to compare the performance of the VaR application, which varies depending on the volatility model incorporated.

Table B.4 shows the POF test and the corresponding number of VaR violations based on the models used. To save space, only the name of the volatility models are kept in the Table B.4. As is known, the hypothesis testing of the POF test is conducted by LR test which can be found in the Appendix B.2.

Table B.4 reveals that VaR applications based on the SVR-GARCH models outperforms those with traditional models. To be interpret, when SVR-GARCH models are incorporated into the VaR application as a volatility model, it turns out that failure rate and corresponding number of violation significantly diminish compared to the traditional models. LR test also suggests that null hypotheses empirically determined probability ($\hat{p}$) is equal to the expected probability (p) for every single stock included is accepted at 1% level. This observation confirms the fact that Machine Learning models not solely increase the volatility prediction performance but also provide more consistent and reliable risk management application.

As for the VaR violation of traditional models, even if VaR estimation with FIGARCH is appeared to have better performance, LR test result of 10.17 in skewed and Student-t distributions tell that empirical probability is not equal to expected one. However, LR test suggests accepting the null hypothesis for the rest of the models at 1% level. In this case, Based on the failure rate, VaR application with EGARCH-Student-t distribution and with EGARCH-skewed distribution are second best performing model.

Table 4.5: Failure Test and Number of Violations

Volatility Models	Failure Rate	Total Number of Violations
SVR-GARCH-linear	0.0000	0
SVR-GARCH-RBF	0.0027	21
SVR-GARCH-Polynomial	0.0026	20
GARCH-Normal	0.0111	85
GARCH-Student t	0.0111	85
GARCH-Skewed	0.0114	87
GJR-GARCH-Normal	0.0090	69
GJR-GARCH-Student t	0.0125	95
GJR-GARCH-Skewed	0.0122	93
EGARCH-Normal	0.0092	70
EGARCH-Student t	0.0084	64
EGARCH-Skewed	0.0086	66
FIGARCH-Normal	0.0092	70
FIGARCH-Student t	0.0000	0
FIGARCH-Skewed	0.0000	0

Table 4.6 indicates the VaR performance based on the Regulatory Framework which is called Basel Traffic Light Approach. According to this approach, if a company falls into a green zone, it is highly unlikely that inaccurate model is accepted. Again, results imply that VaR application with SVR-GARCH volatility performs much better performance that those with traditional models. Because, 21 VaR violations of the total 30 stocks fall into the green zone whereas there is no other VaR violation falling

into other categories, namely yellow and red.

Conversely, there are violations both in the green and yellow zones in the rest of the models. For instance, VaR violations in the green zone with GARCH-normal, Student-t, and Skewed distrubutions are 77, 76, and 80, respectively and the violations in the yellow zone of the some models are 8, 9, and 7, respectively. Similarly, when GJR-GARCH models are applied as volatility model, VaR violations in the green zone are 65, 86, and 84, respectively and violations in the yellow zone are 4, 9, and 90, respectively. Based on the results, there is no VaR violations falling into the red zone.

Thus, considering the VaR violations falling into the yellow categories, Basel Committee would suggest monitoring and the model is considered to be more inaccurate than being accurate and moreover VaR measures need to be more heavily weighted in calculating required capital. Even if there is no violation falling into the red category, it provides a clear signal to the Regulatory Committe that the model is not working well and needs to be improved and investigated.

Table 4.6: Assesing the Violations Based on Basel Traffic Light Approach

Models	Violations in Green Zone	Violations in Yellow Zone	Violations in Red Zone
SVR-GARCH-linear	0	0	0
SVR-GARCH-RBF	21	0	0
SVR-GARCH-Polynomial	20	0	0
GARCH-Normal	77	8	0
GARCH-Student t	76	9	0
GARCH-Skewed	80	7	0
GJR-GARCH-Normal	65	4	0
GJR-GARCH-Student t	86	9	0
GJR-GARCH-Skewed	84	9	0
EGARCH-Normal	67	3	0
EGARCH-Student t	63	1	0
EGARCH-Skewed	64	2	0
FIGARCH-Normal	67	3	0
FIGARCH-Student t	0	0	0
FIGARCH-Skewed	0	0	0

4.5 Conclusion

As the volatility is a phenomenon that partly explains the value of financial assets, it is an indispensable part of pricing and risk management. However, it is equally important to have prediction as accurate as possible. Hence, this study is an attempt to improve the volatility prediction technique so that more accurate volatility prediction and forecasting can be incorporated which in turn enable researchers/practitioners conduct solid risk management as well as value an asset.

To do that SVR-GARCH approach is introduced as a Machine Learning algorithm and compare its performance with the traditional GARCH-type volatility models, namely GARCH, GJR-GARCH, EGARCH, and FIGARCH. Results suggest that SVR-GARCH outperforms the traditional models in terms of RMSE and MAE performance metrics.

Afterwards, volatility prediction obtained from the above-given models are incorporated into the Value-at-Risk model which is a frequently used financial risk management tool. As a VaR model, variance-covariance method is applied and the prediction obtained from volatility models are replaced by standard deviation of return. As a final step, in order to check the validity of the VaR results backtesting is conducted. Kupiec's POF test and Basel Traffic Light Approach are the test applied as backtesting. Again, the findings suggest that VaR applications with SVR-GARCH volatility model performs much better than those with traditional models in that failure rate of SVR-GARCH is quite lower and the VaR violations fall into the green zone in Basel's Traffic Light Approach.

Thus, this study hightlights the importance of application of Machine Learning model to further increase the performance of the volatility prediction as well as risk management. As the volatility gives insight about many phenomena in the field of finance, it is of considerable importance to have sound volatility prediction. So, the method introduced here is an attempt to accomplish this task.

CHAPTER 5

CONCLUSION

Market Microstructure is at the center of intricate relationships. Unfolding the relationships between price impact, liquidity, and volatility allow researchers to understand better the Market Microstructure.

This thesis addresses three prominent Market Microstructure components in three different chapters. In the second chapter, price impact is considered in the context of Kyle's model [54] . Another dimension of market impact is considered in terms of market resiliency and this dimension is not investigated in the literature well. Although it is an important deteminant of market efficiency. In all these considerations informativeness of price is the focus. In order to analyze informativeness of price State Space Models, specifically Kalman-Filter, are employed. This chapter analyzes price formation components: Market resiliency, price impact, informativeness of price, trade intensity, and error variance of prices. Furthermore, in the final part of this chapter, using graphs I look at comparative statistic characteristics of informativeness of price.

This chapter highlights the importance of informativeness of price in the price formation process. In addition to that, having derivations of these five variables pave the way for future research avenue.

The other neglected dimension Market Microstructure is liquidity. Disregarding the liquidity dimension in modeling financial issues might lead to severe consequences such as misvaluation and underestimating the risk. Even though liquidity proxied by bid-ask spread is at the heart of the third chapter, it has an important role in price formation process in the form of informativeness of price in the second chapter. Third chapter develops a new model in order to incorporate illiquidity riskand this new model is called "Liqudity Augmented Stochastic Volatility with Jump Model". In terms of credit spread models incorporating liquidity produces better result.

It turns out that probability of default and credit risk spread without liquidity dimension underestimate the risk. However, once liquidity is included in the model, both probability of default and credit risk spread are increased and during the crisis periods, it is even larger. Thus, the newly proposed model used in the third chapter enable us to consider liquidity in probability of default and credit risk spread estimations and this makes these estimations reliable and stable.

In the fourth chapter of this thesis, another component of Market Microstructure, namely volatility, is tried to be estimated using traditional and Machine Learning models. The aim is to improve the accuracy of the volatility estimation so that highly accurate volatility prediction can be used as an input in various models. To do that, after applying GARCH-type models, a new Machine Learning model called Support Vector Regression-GARCH model is proposed and it turns out that the Support Vector Regression-GARCH model outperforms the GARCH-type models.

In the subsequent phase, these volatility prediction used as a proxy of standard deviation in one of the well-known risk management tools named Value-at-Risk. Proportion of Failures and Basel Traffic Light Approach indicates that Support Vector Regression-GARCH model has way better Value-at-Risk calculation.

REFERENCES

[1] V. V. Acharya and L. H. Pedersen, Asset pricing with liquidity risk, Journal of financial Economics, 77(2), pp. 375–410, 2005.

[2] N. Aleksandrov and B. Hambly, Liquidity modelling and optimal liquidation in bond markets, preprint, 2010.

[3] A. Ali and S. Boadu, *Analysis of the bid-ask spread and its implication for portfolio returns*, Master's thesis, Nord universitet, 2016.

[4] Y. Amihud and H. Mendelson, Asset pricing and the bid-ask spread, Journal of financial Economics, 17(2), pp. 223–249, 1986.

[5] T. G. Andersen and T. Bollerslev, Heterogeneous information arrivals and return volatility dynamics: Uncovering the long-run in high frequency returns, The journal of Finance, 52(3), pp. 975–1005, 1997.

[6] T. G. Andersen and T. Bollerslev, Intraday periodicity and volatility persistence in financial markets, Journal of empirical finance, 4(2-3), pp. 115–158, 1997.

[7] R. T. Baillie, T. Bollerslev, and H. O. Mikkelsen, Fractionally integrated generalized autoregressive conditional heteroskedasticity, Journal of econometrics, 74(1), pp. 3–30, 1996.

[8] L. Ballotta and G. Fusai, A gentle introduction to value at risk, Available at SSRN 2942138, 2017.

[9] D. S. Bates, Jumps and stochastic volatility: Exchange rate processes implicit in deutsche mark options, The Review of Financial Studies, 9(1), pp. 69–107, 1996.

[10] L. Bauwens, C. M. Hafner, and S. Laurent, *Handbook of volatility models and their applications*, volume 3, John Wiley & Sons, 2012.

[11] M. Bedendo, L. Cathcart, and L. El-Jahel, The slope of the term structure of credit spreads: An empirical investigation, Journal of Financial Research, 30(2), pp. 237–257, 2007.

[12] P. C. S. Bezerra and P. H. M. Albuquerque, Volatility forecasting via svr–garch with mixture of gaussian kernels, Computational Management Science, 14(2), pp. 179–196, 2017.

[13] S. T. Bharath and T. Shumway, Forecasting default with the kmv-merton model, in *AFA 2006 Boston Meetings Paper*, 2004.

[14] F. Black, Studies of stock market volatility changes, 1976 Proceedings of the American Statistical Association Bisiness and Economic Statistics Section, 1976.

[15] F. Black, Noise, The journal of finance, 41(3), pp. 528–543, 1986.

[16] F. Black and M. Scholes, The pricing of options and corporate liabilities, Journal of political economy, 81(3), pp. 637–654, 1973.

[17] T. Bollerslev, Generalized autoregressive conditional heteroskedasticity, Journal of econometrics, 31(3), pp. 307–327, 1986.

[18] M. J. Brennan and A. Subrahmanyam, Market microstructure and asset pricing: On the compensation for illiquidity in stock returns, Journal of financial economics, 41(3), pp. 441–464, 1996.

[19] C. Brooks, Predicting stock index volatility: can market volume help?, Journal of Forecasting, 17(1), pp. 59–80, 1998.

[20] M. K. Brunnermeier and L. H. Pedersen, Market liquidity and funding liquidity, The review of financial studies, 22(6), pp. 2201–2238, 2008.

[21] D. Bu and Y. Liao, Corporate credit risk prediction under stochastic volatility and jumps, Journal of Economic Dynamics and Control, 47, pp. 263–281, 2014.

[22] S. Chen, W. K. Härdle, and K. Jeong, Forecasting volatility with support vector machine-based garch model, Journal of Forecasting, 29(4), pp. 406–433, 2010.

[23] T. Chordia, R. Roll, and A. Subrahmanyam, Commonality in liquidity, Journal of financial economics, 56(1), pp. 3–28, 2000.

[24] S. S. Chung and S. Zhang, Volatility estimation using support vector machine: Applications to major foreign exchange rates, Electronic Journal of Applied Statistical Analysis, 10(2), pp. 499–511, 2017.

[25] N. Costanzino and M. Curran, A simple traffic light approach to backtesting expected shortfall, Risks, 6(1), p. 2, 2018.

[26] J. C. Cox, J. E. Ingersoll Jr, and S. A. Ross, An intertemporal general equilibrium model of asset prices, Econometrica: Journal of the Econometric Society, pp. 363–384, 1985.

[27] A. Damodaran, *Damodaran on valuation: security analysis for investment and corporate finance*, volume 324, John Wiley & Sons, 2016.

[28] V. T. Datar, N. Y. Naik, and R. Radcliffe, Liquidity and stock returns: An alternative test, Journal of Financial Markets, 1(2), pp. 203–219, 1998.

[29] J. De Stefani, O. Caelen, D. Hattab, and G. Bontempi, Machine learning for multi-step ahead forecasting of volatility proxies., in *MIDAS@ PKDD/ECML*, pp. 17–28, 2017.

[30] N. Dokuchaev, Volatility estimation from short time series of stock prices, Journal of Nonparametric statistics, 26(2), pp. 373–384, 2014.

[31] D. Duffie, J. Pan, and K. Singleton, Transform analysis and asset pricing for affine jump-diffusions, Econometrica, 68(6), pp. 1343–1376, 2000.

[32] R. F. Engle, Autoregressive conditional heteroscedasticity with estimates of the variance of united kingdom inflation, Econometrica: Journal of the Econometric Society, pp. 987–1007, 1982.

[33] R. F. Engle and T. Bollerslev, Modelling the persistence of conditional variances, Econometric reviews, 5(1), pp. 1–50, 1986.

[34] J. Ericsson and O. Renault, Liquidity and credit risk, The Journal of Finance, 61(5), pp. 2219–2250, 2006.

[35] E. F. Fama, Efficient capital markets: A review of theory and empirical work, The journal of Finance, 25(2), pp. 383–417, 1970.

[36] H. Fang, Option pricing implications of a stochastic jump rate, University of Virginia, 2000.

[37] T. Foucault, J. Hombert, and I. Roşu, News trading and speed, The Journal of Finance, 71(1), pp. 335–382, 2016.

[38] A. Fulop and J. Li, Efficient learning via simulation: A marginalized resample-move approach, Journal of Econometrics, 176(2), pp. 146–161, 2013.

[39] R. Geske, The valuation of compound options, Journal of financial economics, 7(1), pp. 63–81, 1979.

[40] L. R. Glosten, R. Jagannathan, and D. E. Runkle, On the relation between the expected value and the volatility of the nominal excess return on stocks, The journal of finance, 48(5), pp. 1779–1801, 1993.

[41] C. Goodhart and L. Figliuoli, *The geographical location of the foreign exchange market: A test of an'islands' hypothesis*.

[42] C. A. Goodhart and M. O'Hara, High frequency data in financial markets: Issues and applications, Journal of Empirical Finance, 4(2-3), pp. 73–114, 1997.

[43] J. D. Hamilton, State-space models, Handbook of econometrics, 4, pp. 3039–3080, 1994.

[44] J. Hanousek and R. Podpiera, Informed trading and the bid–ask spread: evidence from an emerging market, Journal of comparative economics, 31(2), pp. 275–296, 2003.

[45] A. Hirsa, *Computational methods in finance*, CRC Press, 2012.

[46] C. W. Holden and A. Subrahmanyam, Long-lived private information and imperfect competition, The Journal of Finance, 47(1), pp. 247–270, 1992.

[47] J. C. Hull, *Options futures and other derivatives*, Pearson, 2009.

[48] R. A. Jarrow and E. R. Rosenfeld, Jump risks and the intertemporal capital asset pricing model, Journal of Business, pp. 337–351, 1984.

[49] P. Jorion, On jump processes in the foreign exchange and stock markets, The Review of Financial Studies, 1(4), pp. 427–445, 1988.

[50] P. Jorion, *Value at risk: the new benchmark for controlling market risk*, Irwin Professional Pub., 1997.

[51] P. Jorion, *Financial risk manager handbook 2001-2002*, volume 96, Wiley, 2001.

[52] A. Kempf and M. Uhrig-Homburg, Liquidity and its impact on bond prices, Schmalenbach Business Review, 52(1), pp. 26–44, 2000.

[53] P. Kupiec, Techniques for verifying the accuracy of risk measurement models, The J. of Derivatives, 3(2), 1995.

[54] A. S. Kyle, Continuous auctions and insider trading, Econometrica: Journal of the Econometric Society, pp. 1315–1335, 1985.

[55] A. S. Kyle and A. A. Obizhaeva, Market microstructure invariance: A dynamic equilibrium model, Anna A., Market Microstructure Invariance: A Dynamic Equilibrium Model (January 31, 2019), 2019.

[56] D. Lando, *Credit risk modeling: theory and applications*, Princeton University Press, 2009.

[57] J. Li, Studies of contingent capital bonds, 2015.

[58] C. Loderer and L. Roth, The pricing discount for limited liquidity: Evidence from swx swiss exchange and the nasdaq, Journal of Empirical Finance, 12(2), pp. 239–268, 2005.

[59] F. A. Longstaff, S. Mithal, and E. Neis, Corporate yield spreads: Default risk or liquidity? new evidence from the credit default swap market, The Journal of Finance, 60(5), pp. 2213–2253, 2005.

[60] L. Mancini, A. Ranaldo, and J. Wrampelmeyer, Liquidity in the foreign exchange market: Measurement, commonality, and risk premiums, The Journal of Finance, 68(5), pp. 1805–1841, 2013.

[61] R. C. Merton, On the pricing of corporate debt: The risk structure of interest rates, The Journal of finance, 29(2), pp. 449–470, 1974.

[62] D. B. Nelson, Conditional heteroskedasticity in asset returns: A new approach, Econometrica: Journal of the Econometric Society, pp. 347–370, 1991.

[63] M. O'Hara, Presidential address: Liquidity and price discovery, The Journal of Finance, 58(4), pp. 1335–1354, 2003.

[64] L. Pástor and R. F. Stambaugh, Liquidity risk and expected stock returns, Journal of Political economy, 111(3), pp. 642–685, 2003.

[65] W. F. J. G. X. Qiu and X. Y. D. Z. Y. Zhu, Credit risk modeling and cds valuation, 2012.

[66] M. Raju and A. Ghosh, Stock market volatility–an international comparison, Securities and Exchange Board of India, 2004.

[67] R. Sadka, Momentum and post-earnings-announcement drift anomalies: The role of liquidity risk, Journal of Financial Economics, 80(2), pp. 309–349, 2006.

[68] K. Schwarz, Mind the gap: Disentangling credit and liquidity in risk spreads, Available at SSRN 1486240, 2018.

[69] Y. Shi and Y. Yang, Modeling high frequency data with long memory and structural change: A-hyegarch model, Risks, 6(2), p. 26, 2018.

[70] K. Soman, R. Loganathan, and V. Ajay, *Machine learning with SVM and other kernel methods*, PHI Learning Pvt. Ltd., 2009.

[71] D. Y. Tang and H. Yan, Macroeconomic conditions, firm characteristics, and credit spreads, Journal of Financial Services Research, 29(3), pp. 177–210, 2006.

[72] S. J. Taylor, *Modelling financial time series*, world scientific, 2008.

[73] V. N. Vapnik, The nature of statistical learning theory. 1995, Statistical Learning Theory, 1998.

[74] X. Vives, The speed of information revelation in a financial market mechanism, Journal of Economic Theory, 67(1), pp. 178–204, 1995.

[75] A. Watanabe and M. Watanabe, Time-varying liquidity risk and the cross section of stock returns, The Review of Financial Studies, 21(6), pp. 2449–2486, 2007.

[76] A. Yakov, Illiquidity and stock returns: cross-section and time-series effects, Journal of financial markets, 5(1), pp. 31–56, 2002.

[77] G. Yan and F. B. Hanson, Option pricing for a stochastic-volatility jump-diffusion model with log-uniform jump-amplitudes, in *2006 American Control Conference*, pp. 6–pp, IEEE, 2006.

[78] M. Y. Zhang, J. R. Russell, and R. S. Tsay, Determinants of bid and ask quotes and implications for the cost of trading, Journal of Empirical Finance, 15(4), pp. 656–678, 2008.

APPENDIX A

EXTENSION OF THE BATES MODEL

In the presence of financial distress, it is important to reconsider the pricing of any asset having the stock price as the underlier. This modification might also be reflected in credit spread and probability of default. To model this phenomenon, it is assumed that liquidity behaves like ICIR process as assumed in [52] for bond prices.

$$L(t) = \int_0^t l(u)du \tag{A.1}$$

$$dl_t = \kappa(\beta - l_t)d_t + \sigma\sqrt{l_t}dW_t \tag{A.2}$$

Then our microstructure corrected asset price could be defined:

$$\tilde{S}_t = e^{-L(t)}S_t$$

So, illiquidity adjustment is used to discount the stock price since illiquidity is considered as a compensation factor in order to avoid loss or reduction in profits caused by thin trading [52]. At this stage, we revisit the option pricing model of stochastic volatility with jumps and option payoff for corrected asset price takes the form of:

$$E_t = \mathbb{E}(e^{-r(T-t)}(\tilde{S_T} - D)|\mathcal{F}_t)^+ \tag{A.3}$$

Under different measures which are $\mathbb{P}^S$ and risk neutral measure, $\mathbb{P}$, liquidity adjusted price is derived.

$$\mathbb{P}^s(\tilde{S}_T > D) = F_1, \quad \mathbb{P}(\tilde{S}_T > D) = F_2.$$

Using liquidity adjustment L_t term and setting $x = \log(L)$ we have:

$$F_{1L} = \frac{1}{2} + \frac{1}{\pi}\int_0^\infty \int_0^\infty Re\left(\frac{e^{-iu(\log(D)-x)}\Phi_1(u)f(x)}{iu}dudx\right) \tag{A.4}$$

$$F_{2L} = \frac{1}{2} + \frac{1}{\pi}\int_0^\infty \int_0^\infty Re\left(\frac{e^{-iu(\log(D)-x)}\Phi_2(u)f(x)}{iu}dudx\right). \tag{A.5}$$

where f(x) is the density of L(t) and Φ_1 represents the characteristic function of Bates model that can be defined as:

$$\Phi_1 = e^{A_{1,SVJ}(\tau)B_{1,SVJ}(\tau)V_t+i(u-i)\log(S_t)\lambda\tau[(1\mu)^{i(u-i)+1}exp(\frac{\sigma_j^2(i(u-i)-(u-i)^2)}{2})-(1+\mu)]} \tag{A.6}$$

$$A_1 = (u-i)(r-\lambda J)\tau + \frac{\tau\kappa((u-i)-i)(-x)}{\sigma_v^2} - (\frac{2\kappa((u-i)-i}{\sigma_v^2})\log((\frac{-x}{\gamma_1})\sinh(\frac{\gamma_1\tau}{2}) + \cosh(\frac{\gamma_1\tau}{2})) \tag{A.7}$$

$$B_1 = \frac{-((u-i)^2+u-i)}{-(i\rho\sigma_v(u-i)-\kappa)+\gamma_1(\frac{\cosh(\gamma\tau/2)}{\sinh(\gamma_1\tau/2)})} \tag{A.8}$$

As the L_t is a ICIR process, Fourier inverting charateristic function allows us to obtain the density. Applying Fubini theorem, we have the following:

$$F_{2,L} = \frac{1}{2} + \frac{1}{\pi}\int_0^\infty e^{iux}f(x)dx\int_0^\infty Re\left(\frac{e^{-iu\log(D)}}{iu}\Phi_2(u)\right)du$$

First element in the above-given double integral is actually the characteristic function of $x = \log(L_t)$ which is defined to be $\Phi_L(u)$. Further we can write the formula for probabilities as the product of two characteristic functions and Fourier inversion component as

$$F_{2,L} = \frac{1}{2} + \frac{1}{\pi}\int_0^\infty Re\left(\frac{e^{-iu\log(D)}}{iu}\Phi_2(u)\Phi_L(u)du\right)$$

and

$$F_{1,L} = \frac{1}{2} + \frac{1}{\pi}\int_0^\infty Re\left(\frac{e^{-iu\log(D)}}{iu}\Phi_1(u)\Phi_L(u)du\right).$$

The option price becomes:

$$E_t = S_tF_{1,L} - De^{-r(T-t)}F_{2,L}$$

APPENDIX B

VISUALIZATION OF PREDICTION RESULTS AND POF LR TEST RESULT

B.1 Visualization of Prediction Results

Following figures exhibit the prediction result of the each model used in this study. Thus, for every single model, 30 figures representing 30 stocks included are shown.

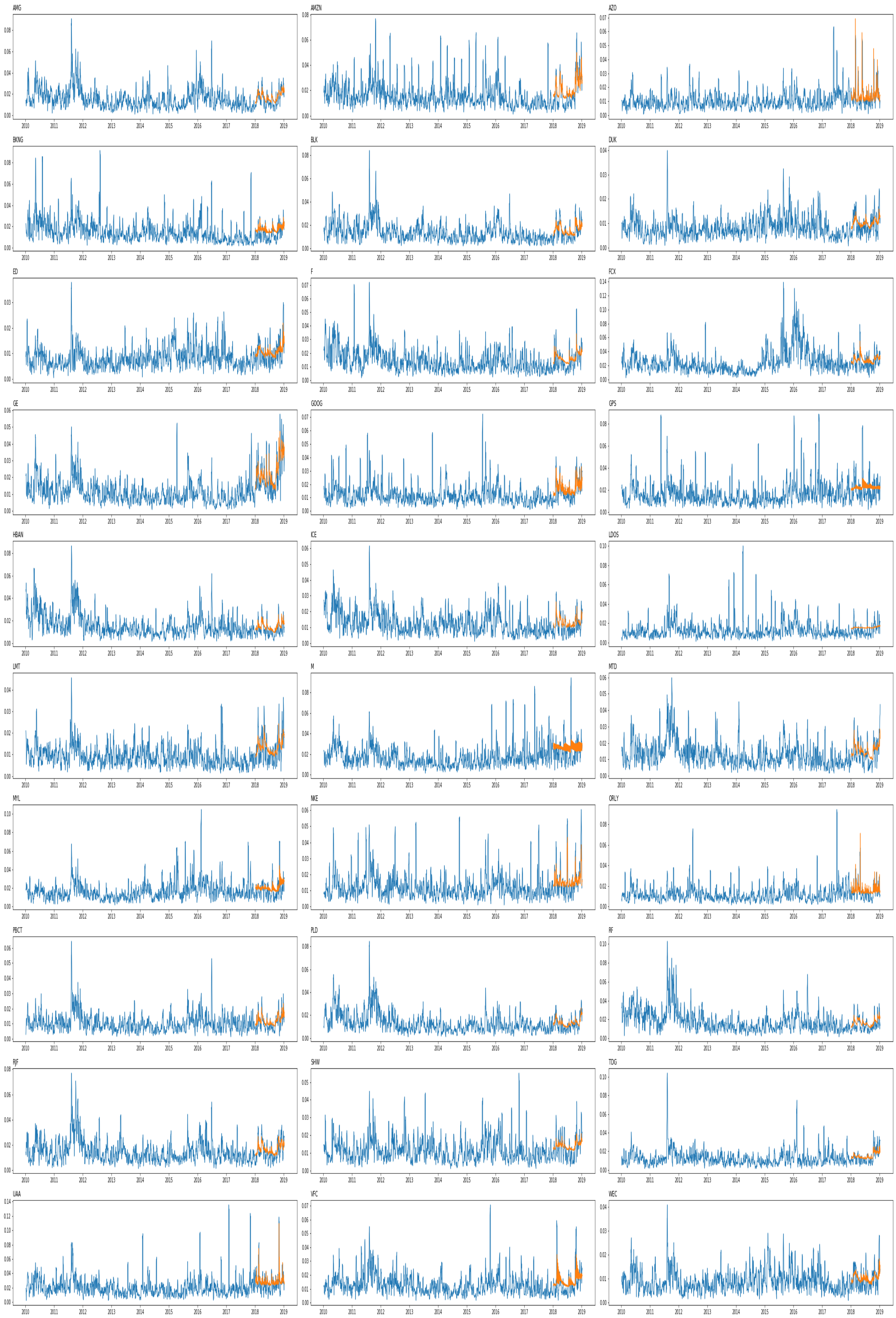


Figure B.1: GARCH-Normal Prediction Results

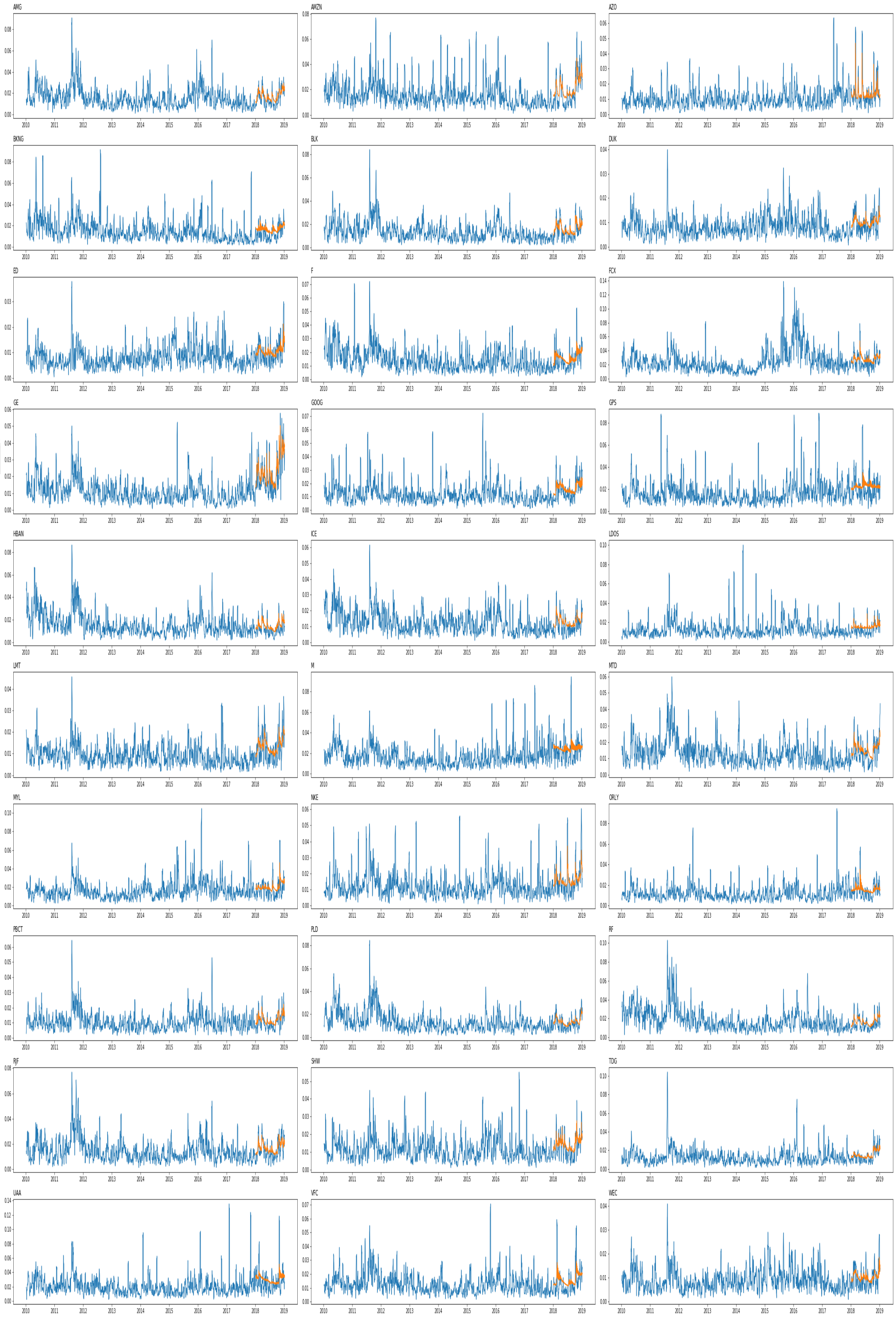


Figure B.2: GARCH-Student t Prediction Results

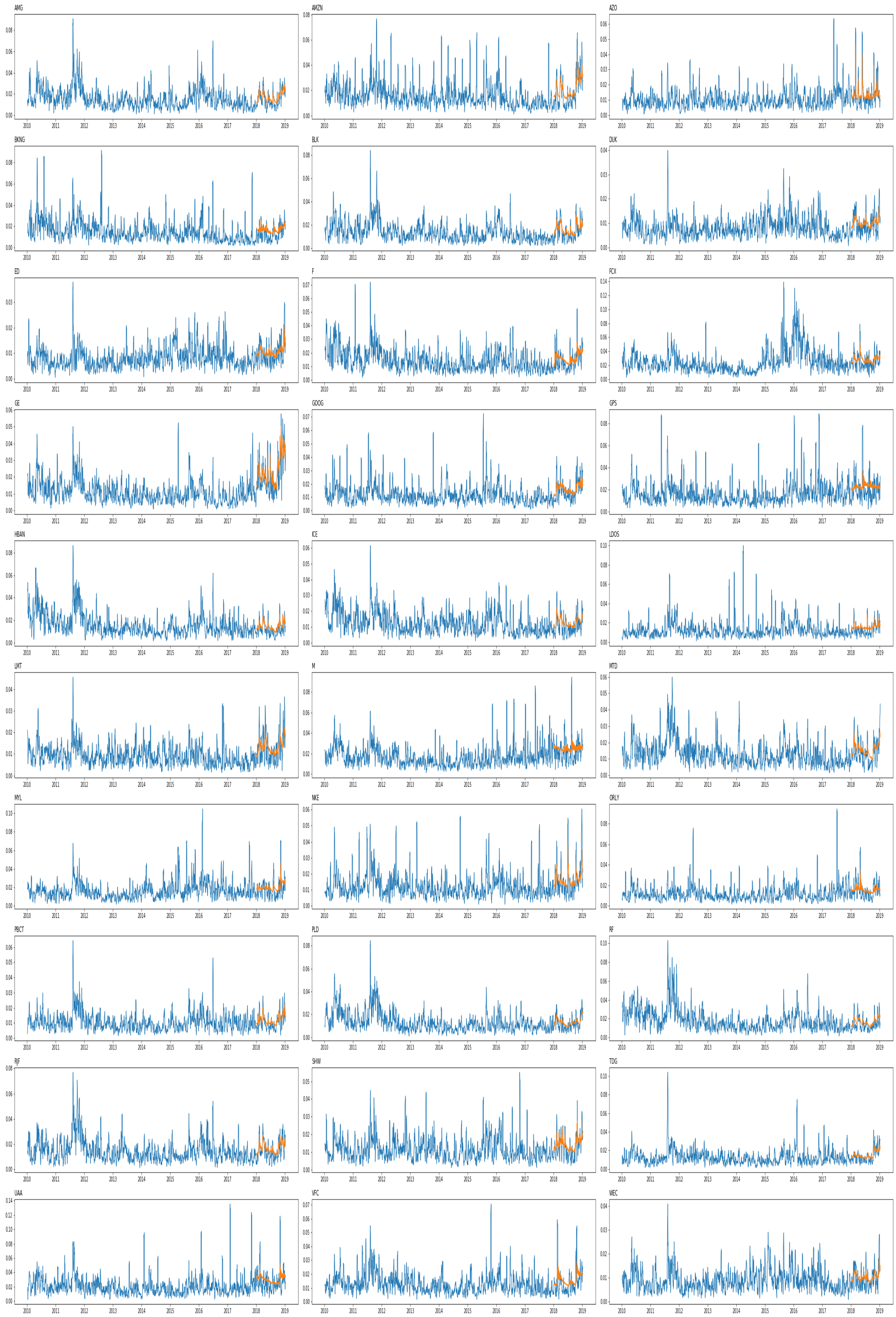


Figure B.3: GARCH-Skewed Prediction Results

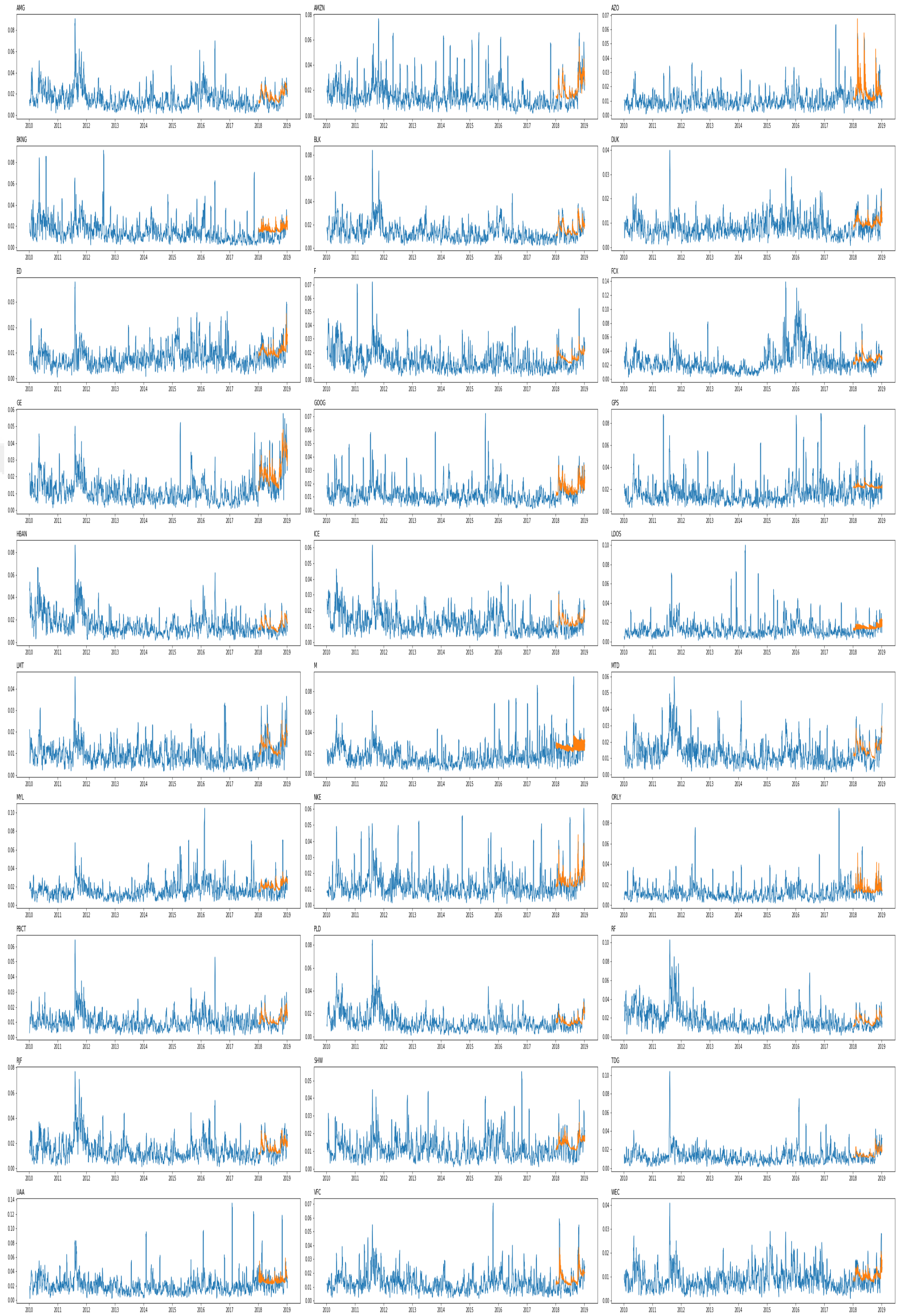


Figure B.4: GJR-GARCH-Normal Prediction Results

Figure B.5: GJR-GARCH-Student t Prediction Results

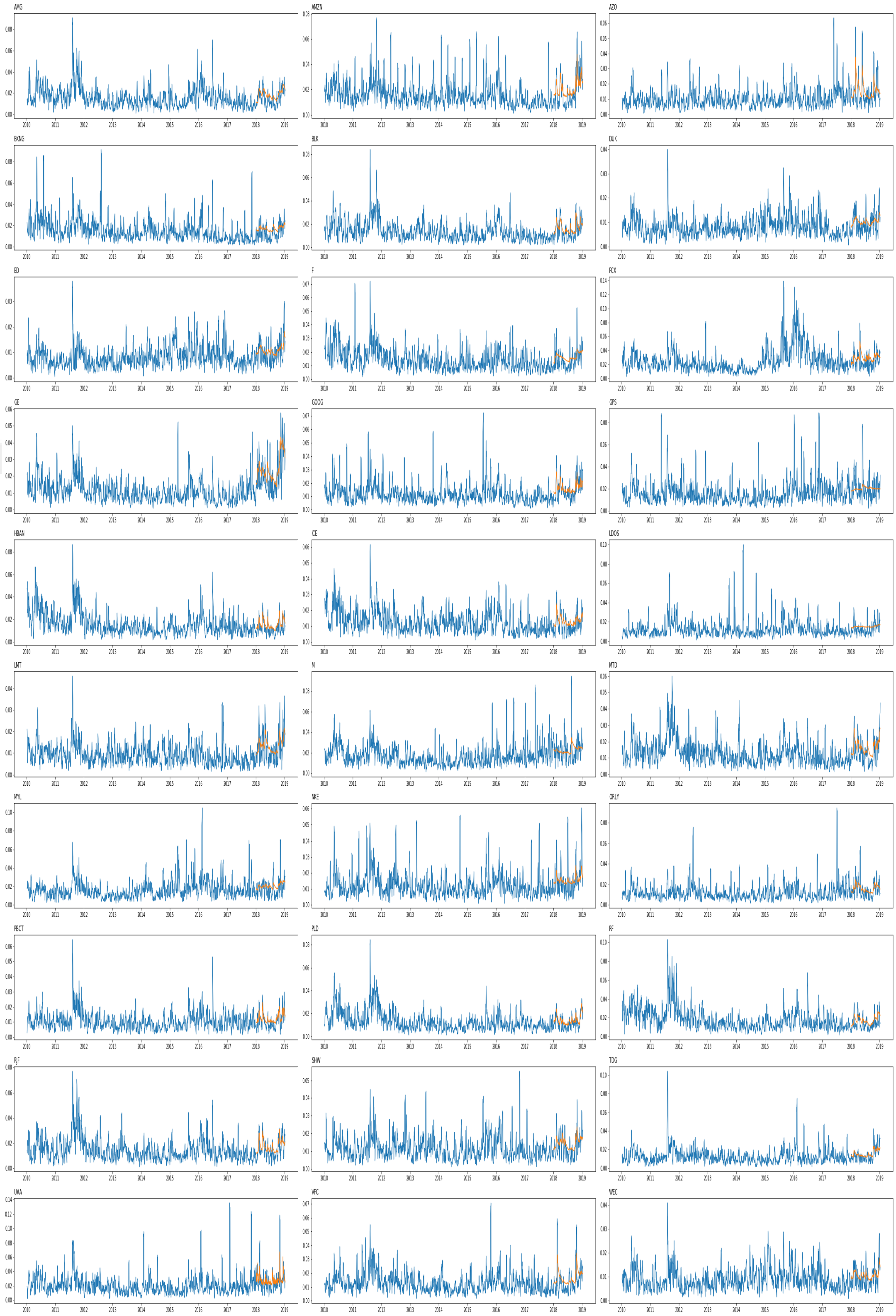


Figure B.6: GJR-GARCH-Skewed Prediction Results

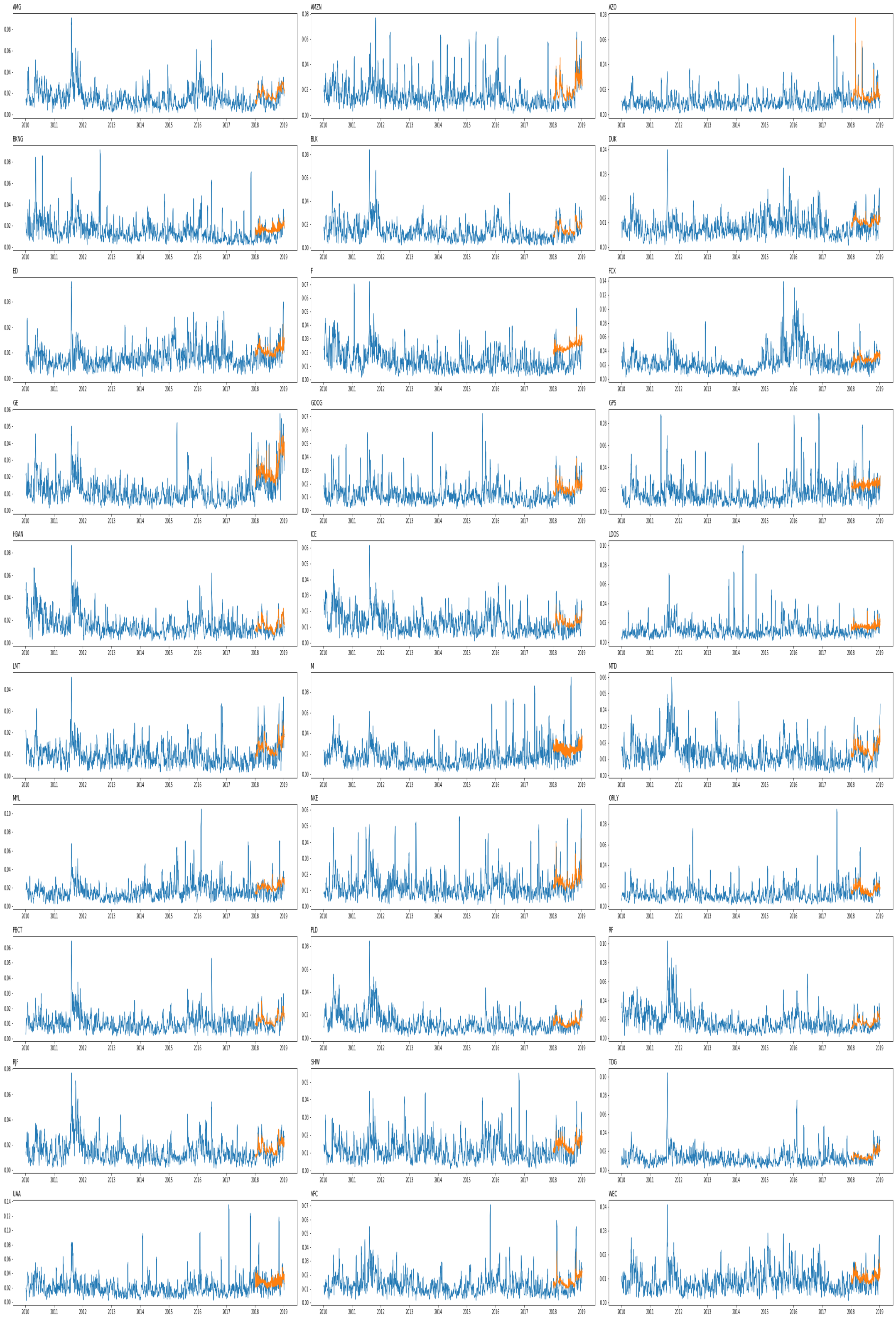


Figure B.7: EGARCH-Normal Prediction Results

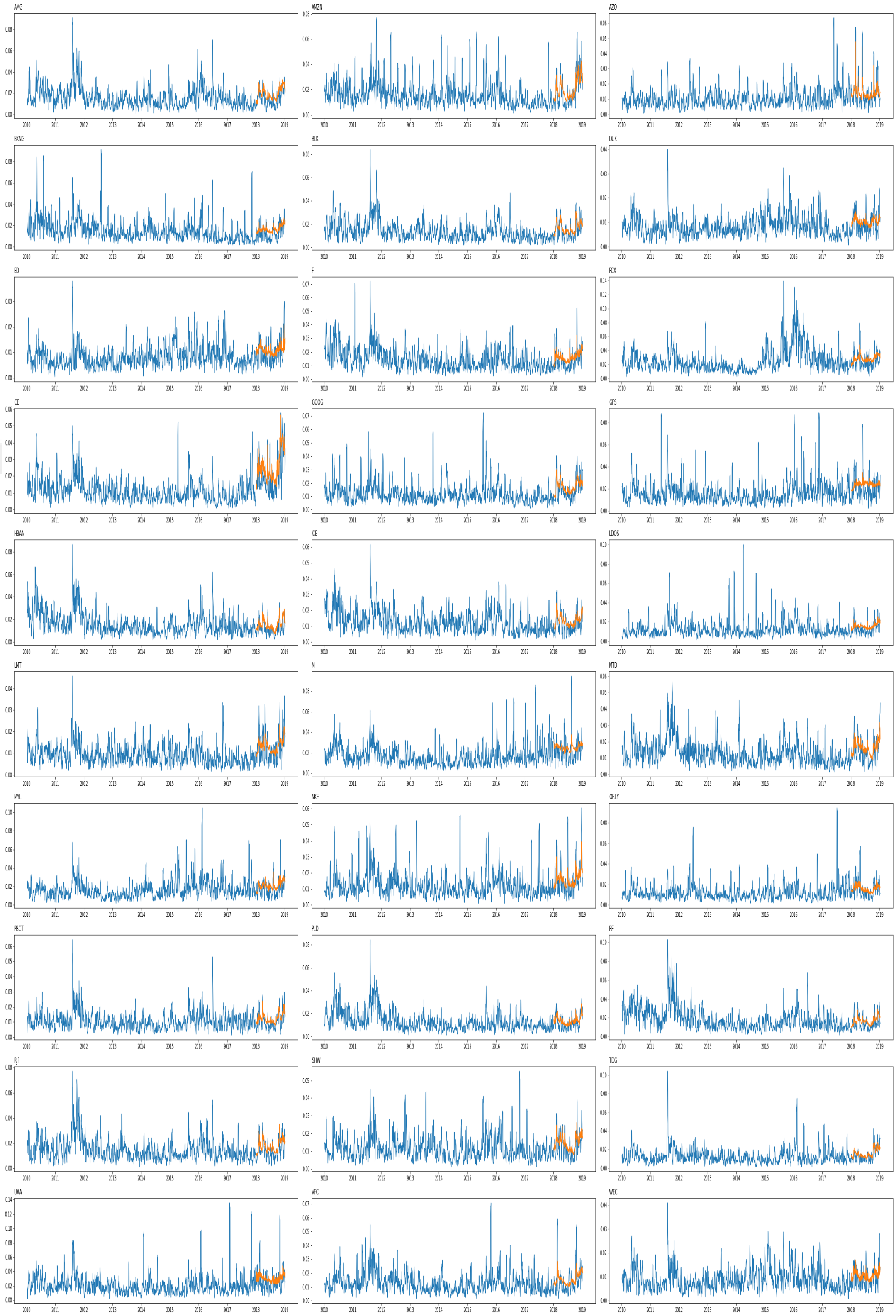


Figure B.8: EGARCH-Student t Prediction Results

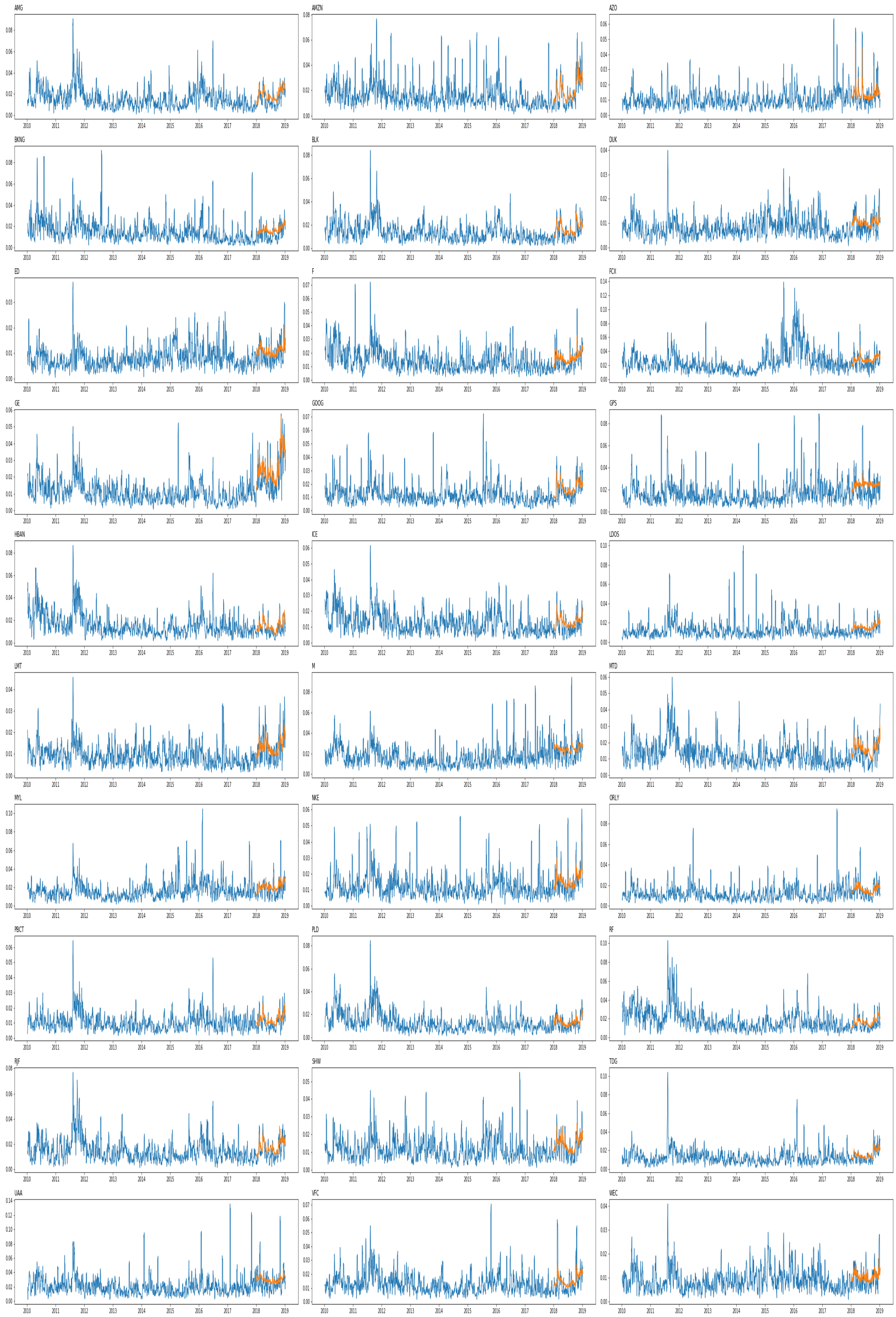


Figure B.9: EGARCH-Skewed Prediction Results

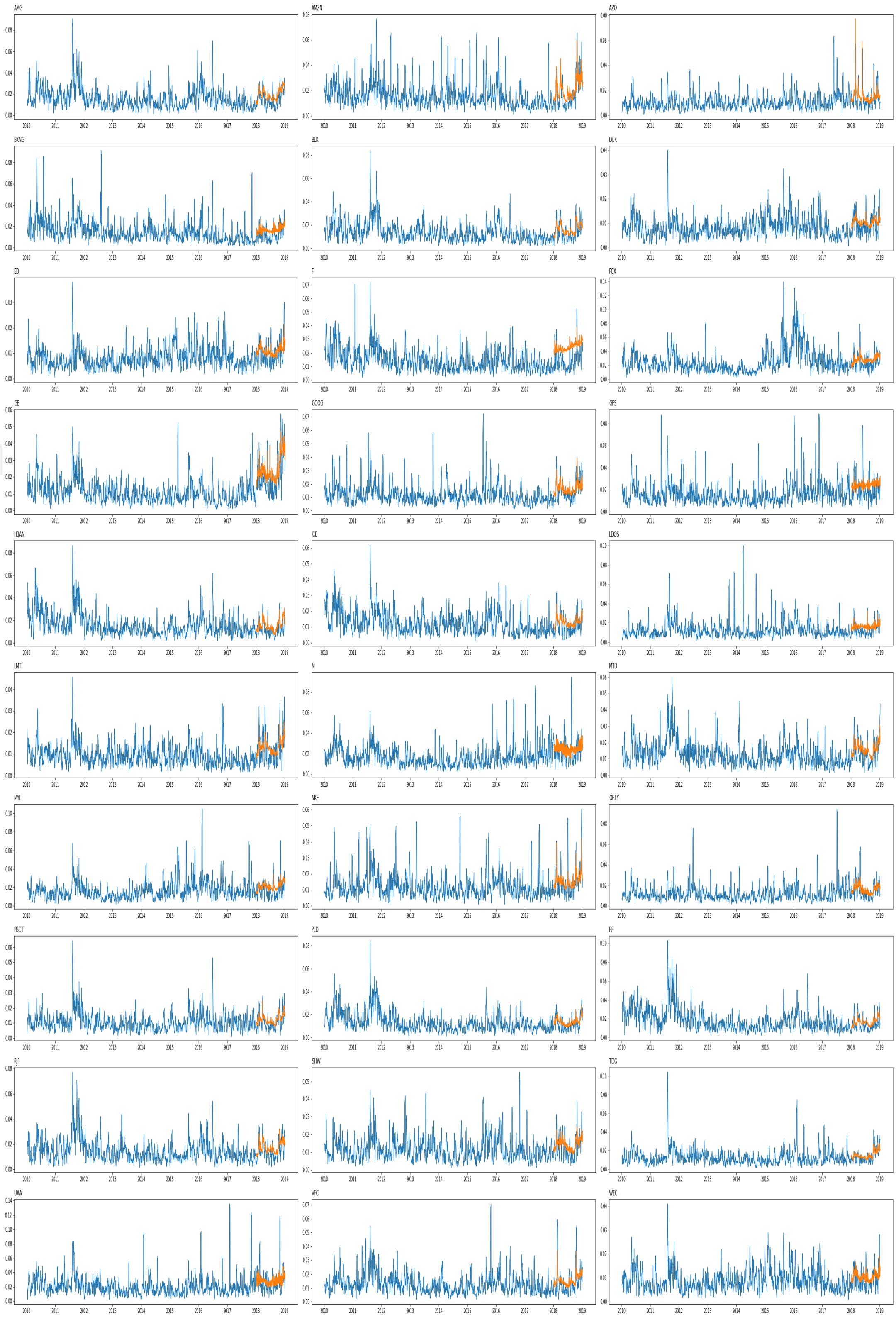


Figure B.10: FIGARCH-Normal Prediction Results

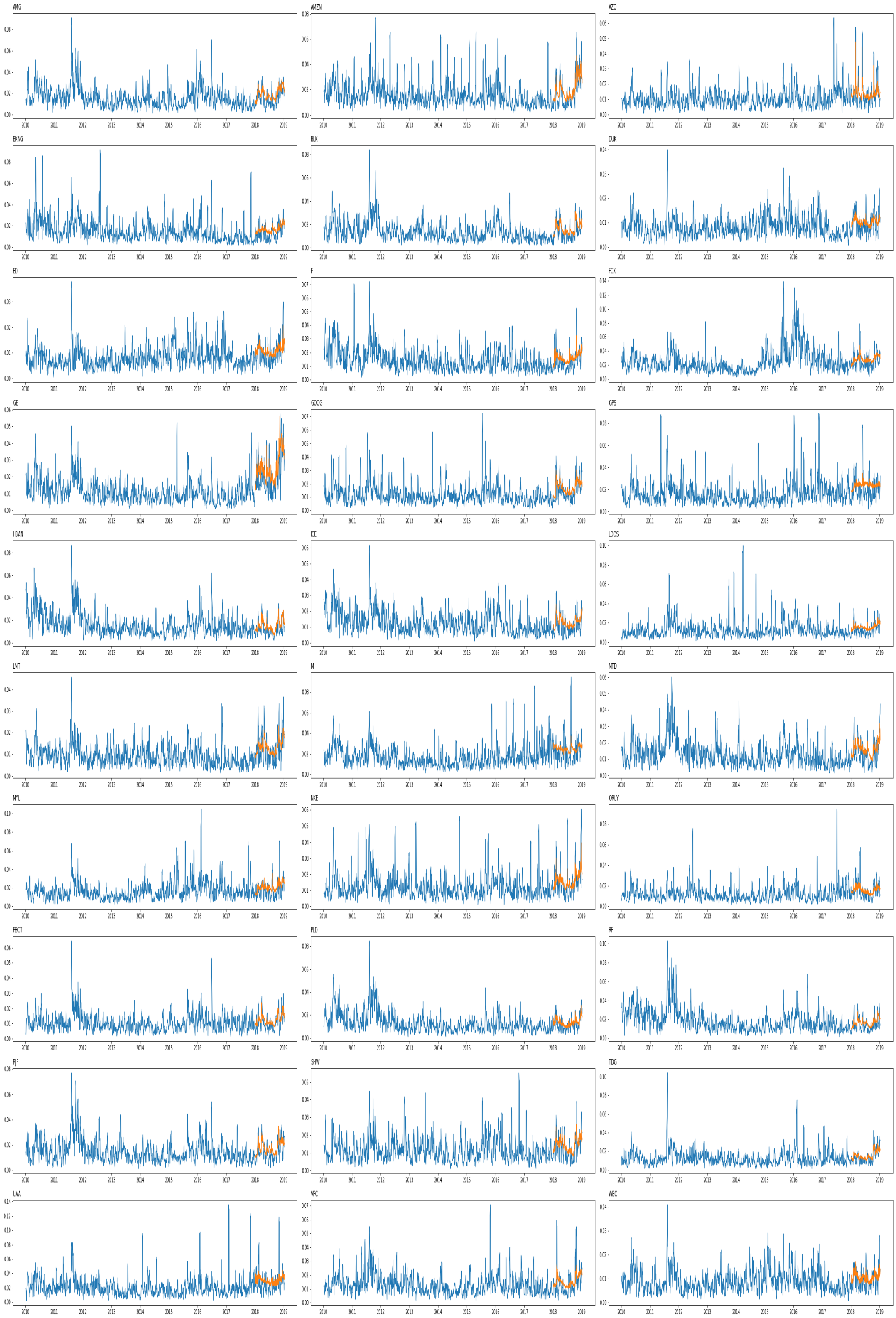


Figure B.11: FIGARCH-Student t Prediction Results

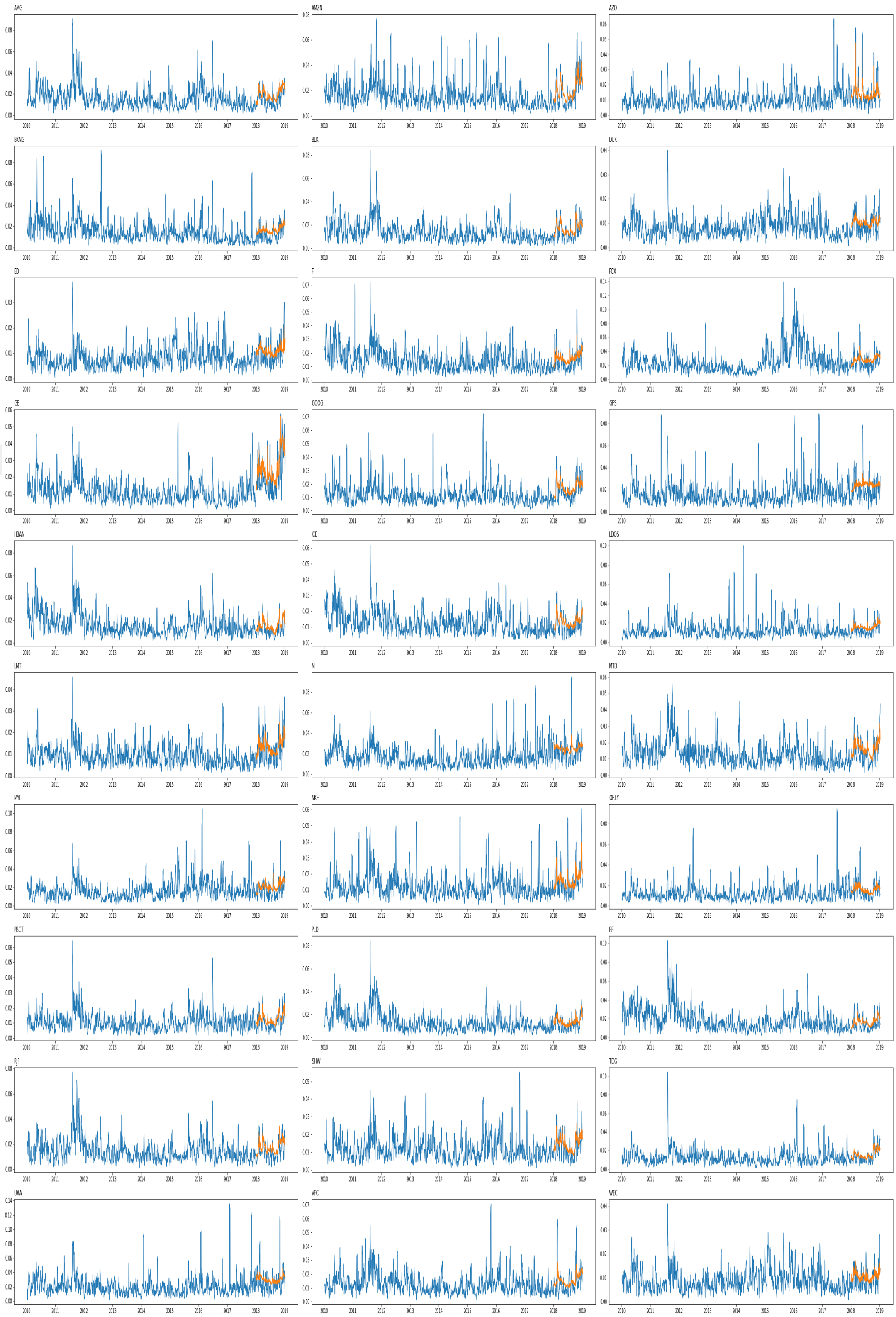


Figure B.12: FIGARCH-Skewed Prediction Results

Figure B.13: SVR-GARCH-Linear Prediction Results

Figure B.14: SVR-GARCH-RBF Prediction Results

B.2 POF LR Test Results

Table B.1: LR Test Result for GARCH

	Companies	Normal Dist.	Student- t Dist.	Skewed Dist.
0	AMG	5.085470	5.085470	5.085470
1	AMZN	5.085470	1.212888	1.212888
2	AZO	5.085470	0.083240	0.083240
3	BKNG	0.733245	1.212888	1.212888
4	BLK	0.083240	0.083240	0.083240
5	DUK	0.083240	0.083240	0.083240
6	ED	0.733245	0.733245	0.733245
7	F	0.120832	1.896624	3.470779
8	FCX	1.896624	1.896624	1.896624
9	GE	5.085470	5.085470	5.085470
10	GOOG	3.470779	3.470779	3.470779
11	GPS	1.896624	1.896624	0.733245
12	HBAN	5.085470	5.085470	5.085470
13	ICE	5.085470	1.212888	1.212888
14	LDOS	0.733245	5.085470	5.085470
15	LMT	0.083240	1.896624	0.733245
16	M	0.083240	0.083240	0.083240
17	MTD	3.470779	1.896624	1.896624
18	MYL	0.083240	0.120832	0.120832
19	NKE	1.212888	0.083240	0.083240
20	ORLY	1.896624	1.896624	1.896624
21	PBCT	1.896624	0.083240	0.083240
22	PLD	5.085470	5.085470	0.120832
23	RF	1.896624	1.896624	1.896624
24	RJF	0.083240	0.083240	0.733245
25	SHW	0.083240	5.085470	5.085470
26	TDG	1.212888	1.212888	1.212888
27	UAA	0.120832	0.733245	0.733245
28	VFC	0.733245	0.733245	0.733245
29	WEC	1.896624	1.896624	1.896624

Table B.2: LR Test Result for GJR-GARCH

	Companies	Normal Dist.	Student- t Dist.	Skewed Dist.
0	AMG	1.212888	5.085470	5.085470
1	AMZN	1.212888	5.085470	5.085470
2	AZO	5.085470	0.120832	0.120832
3	BKNG	3.470779	0.120832	0.120832
4	BLK	0.083240	0.083240	0.083240
5	DUK	0.083240	0.083240	0.083240
6	ED	0.083240	0.733245	0.733245
7	F	1.212888	0.083240	0.083240
8	FCX	3.470779	0.733245	1.896624
9	GE	1.212888	5.085470	5.085470
10	GOOG	0.733245	3.470779	3.470779
11	GPS	1.896624	0.733245	0.083240
12	HBAN	5.085470	5.085470	5.085470
13	ICE	5.085470	5.085470	5.085470
14	LDOS	1.212888	0.120832	0.120832
15	LMT	0.083240	0.733245	0.733245
16	M	0.733245	0.120832	0.120832
17	MTD	0.083240	0.083240	0.083240
18	MYL	1.212888	5.085470	5.085470
19	NKE	1.212888	1.212888	1.212888
20	ORLY	0.733245	0.083240	0.083240
21	PBCT	0.120832	1.212888	0.083240
22	PLD	5.085470	5.085470	5.085470
23	RF	0.083240	0.733245	0.733245
24	RJF	0.733245	0.733245	0.733245
25	SHW	1.212888	5.085470	5.085470
26	TDG	5.085470	5.085470	5.085470
27	UAA	1.212888	1.212888	1.212888
28	VFC	0.733245	0.733245	0.733245
29	WEC	0.733245	0.733245	0.733245

Table B.3: LR Test Result for EGARCH

	Companies	Normal Dist.	Student- t Dist.	Skewed Dist.
0	AMG	5.085470	5.085470	5.085470
1	AMZN	5.085470	5.085470	5.085470
2	AZO	0.083240	1.896624	1.896624
3	BKNG	0.120832	0.083240	0.083240
4	BLK	0.083240	0.083240	0.083240
5	DUK	0.083240	0.083240	0.083240
6	ED	1.896624	3.470779	3.470779
7	F	0.733245	3.470779	3.470779
8	FCX	1.896624	1.896624	1.896624
9	GE	1.212888	0.120832	0.120832
10	GOOG	1.212888	0.733245	0.733245
11	GPS	0.083240	7.599894	7.599894
12	HBAN	5.085470	5.085470	5.085470
13	ICE	5.085470	5.085470	5.085470
14	LDOS	0.733245	0.733245	0.733245
15	LMT	1.896624	3.470779	3.470779
16	M	0.120832	1.896624	1.896624
17	MTD	0.083240	0.083240	0.083240
18	MYL	0.120832	1.896624	1.896624
19	NKE	5.085470	0.733245	0.733245
20	ORLY	1.212888	0.083240	0.083240
21	PBCT	1.212888	5.085470	5.085470
22	PLD	5.085470	5.085470	5.085470
23	RF	0.733245	1.896624	1.896624
24	RJF	0.120832	0.120832	0.120832
25	SHW	0.733245	0.120832	0.120832
26	TDG	5.085470	1.212888	1.212888
27	UAA	0.120832	0.120832	5.085470
28	VFC	0.733245	0.733245	0.733245
29	WEC	1.896624	0.733245	0.733245

Table B.4: LR Test Result for FIGARCH

	Companies	Normal Dist.	Student- t Dist.	Skewed Dist.
0	AMG	1.212888	10.17094	10.17094
1	AMZN	1.212888	10.17094	10.17094
2	AZO	5.085470	10.17094	10.17094
3	BKNG	3.470779	10.17094	10.17094
4	BLK	0.083240	10.17094	10.17094
5	DUK	0.083240	10.17094	10.17094
6	ED	0.083240	10.17094	10.17094
7	F	1.212888	10.17094	10.17094
8	FCX	3.470779	10.17094	10.17094
9	GE	1.212888	10.17094	10.17094
10	GOOG	0.733245	10.17094	10.17094
11	GPS	1.896624	10.17094	10.17094
12	HBAN	5.085470	10.17094	10.17094
13	ICE	5.085470	10.17094	10.17094
14	LDOS	1.212888	10.17094	10.17094
15	LMT	0.083240	10.17094	10.17094
16	M	0.733245	10.17094	10.17094
17	MTD	0.083240	10.17094	10.17094
18	MYL	1.212888	10.17094	10.17094
19	NKE	1.212888	10.17094	10.17094
20	ORLY	0.733245	10.17094	10.17094
21	PBCT	0.120832	10.17094	10.17094
22	PLD	5.085470	10.17094	10.17094
23	RF	0.083240	10.17094	10.17094
24	RJF	0.733245	10.17094	10.17094
25	SHW	1.212888	10.17094	10.17094
26	TDG	5.085470	10.17094	10.17094
27	UAA	1.212888	10.17094	10.17094
28	VFC	0.733245	10.17094	10.17094
29	WEC	0.733245	10.17094	10.17094

CURRICULUM VITAE

PERSONAL INFORMATION

Surname, Name: Karasan, Abdullah
Nationality: Turkish
Date and Place of Birth: 23.10.1983, Berlin
Marital Status: Married

EDUCATION

Degree	Institution	Year of Graduation
M.A.	University of Michigan	2014
B.S.	Gazi University	2007
High School	Deneme Lisesi	2002

PROFESSIONAL EXPERIENCE

Year	Place	Enrollment
2017-Present	Datajarlabs	Senior Data Science Consultant
2018-Present	Thinkful	Data Science Mentor
2007-2017	Undersecretariat of Treasury	Treasury Controller

PUBLICATIONS

- Productivity vs. Institutions: An Empirical Evidence on Convergence (2014)

International Conference Publications

- The 17^{th} Workshop on Advances in Continuous Optimization (EUROPT), June 2019, Glascow, Scotland
- Market Microstructure Confronting Many Viewpoints 4, 6^{th}-9^{th} December, 2016, Paris, France
- 15^{th} Conference Research Network Macroeconomics and Macroeconomic Policies, 27-29 October 2011, Berlin, Germany
- 3^{rd} International Conference of Political Economy, 15-17 September 2011, Kocaeli, Turkey