

**A CLUSTERING BASED HEURISTIC FOR LOCATION ROUTING
PROBLEMS**

by
İLKAN SARIGÖL

Submitted to the Graduate School of Engineering and Natural Sciences
in partial fulfillment of
the requirements for the degree of
Master of Science

SABANCI UNIVERSITY
Spring 2007

© İLKAN SARIGÖL 2007
All Rights Reserved

A CLUSTERING BASED HEURISTIC FOR LOCATION ROUTING PROBLEMS

APPROVED BY:

Assistant Prof. Bülent Çatay
(Thesis Supervisor)

Assistant Prof. Tonguç Ünlüyurt.

Assistant Prof. Dilek Tüzün Aksu.

Assistant Prof. Güvenç Şahin
.....

Assistant Prof. Kerem Bülbül
.....

DATE OF APPROVAL:

ACKNOWLEDGMENTS

I would like to express my gratitude to all those who gave me the possibility to complete this thesis. First, I would like to extend my sincere gratitude to Dr. Bülent Çatay for his patience, guidance, and encouragement throughout this past year. I owe many thanks to my father for motivating me and my mother for always being there when I need her. To my sister, thank you for the joy and fun you bring to my life. Without the spiritual support of my family, it would have been harder to complete this thesis. I would like to thank to my colleague and roommate, Ersin Demirok, for his intelligent ideas and remarks. I wish to thank TUSSİDE directors, Necmettin Oktay and Necip Özçer, for their supports. I additionally owe many thanks to my friends at Sabanci University and TUSSİDE for their encouragements. Last, but not the least I thank my friend, Ayşegül Tunçelli for her invaluable support and guidance that made this thesis possible in the first place.

ABSTRACT

In this thesis we address the location routing problem (LRP) in which vehicle routing and warehouse location/allocation decisions are made simultaneously. LRP deals with determining the optimal number of warehouses as well as their locations while assigning customers to warehouses so that the shortest vehicle routes are achieved. The objective is to minimize total vehicle related costs (fixed vehicle cost and route cost) and the cost of operating the warehouses. To solve this NP-hard problem, we propose a clustering based heuristic approach which consists of three parts. Firstly, we determine the customer clusters based on vehicle capacities, i.e. all customers in the cluster are serviced by the same vehicle. Secondly, we solve a traveling salesman problem for each cluster. Thirdly, we attempt to improve the routes by using local search techniques. Finally, the routed clusters are assigned to warehouses.

Keywords: Clustering, Heuristics, Location-allocation, Location-routing, Vehicle routing

ÖZET

Bu tezde lokasyon rotalama problemleri (LRP) üzerinde durulmuştur. Problemin çözümü için kümeleme temelli sezgisel bir yöntem önerilmiştir. LRP iki zor problem olan lokasyon tahsisi ve araç rotalama problemlerinin birleşmesiyle oluşmaktadır. LRP ile bu iki probleme eş zamanlı çözüm üretilmektedir. Amaç fonksiyonu rotalama, araç kullanma ve depo maliyetinden oluşmaktadır ve enazlanmaya çalışılmaktadır. Literatürde genellikle sabit depo açılım maliyeti kullanılmaktadır. Bu tezde depo açma maliyeti kapasite belli değerlerin üzerine çıktığında artmaktadır ve genelden farklıdır. Maliyet fonksiyonunun bu yapısı ve LRP nin içsel kompleksliği birleştiğinde problemin çözümü zorlaşmaktadır. Önerdiğimiz sezgisel yöntemle göre araç kapasitelerine göre kümeler oluşturmuştur ve her küme içinde gezgin satıcı problemi çözülmüştür. Yerel tarama yöntemleri uygulanarak çözüm iyileştirilmiştir. Son olarak oluşturulan kümeler uygun depolara atanmıştır.

Anahtar Kelimeler: Araç Rotalama, Lokasyon Dağıtım, Lokasyon Rotalama, Sezgisel Yöntemler

TABLE OF CONTENTS

ACKNOWLEDGMENTS	iv
ABSTRACT.....	v
ÖZET	vi
TABLE OF CONTENTS.....	vii
LIST OF FIGURES	ix
LIST OF TABLES.....	x
1. INTRODUCTION	1
2. LITERATURE REVIEW	7
3. MODEL FORMULATION	11
3.1 Assumptions.....	12
3.2 Notation	13
3.3 Mathematical Model	14
4. SOLUTION METHODOLOGY	16
4.1 Clustering.....	13
4.1.1. Hierarchical Clustering Algorithm	18
4.1.2. K-means Clustering Algorithm.....	18
4.1.3. Clustering Heuristics.....	19
4.2 Traveling Salesman Problem	24
4.2.1. Branch-and-bound	25
4.2.2. Nearest Neighbor	29
4.2.3. Nearest Point to Cluster Median.....	29
4.3 Myopic and Look-Back Algorithm for Assignment.....	30
4.4 Local Search	32

5. COMPUTATIONAL STUDY.....	33
6. CONCLUSION AND FUTURE WORK.....	38
APPENDIX.....	40
REFERENCES	51

LIST OF TABLES

Table 3.1	Non-linear Cost of Warehouse.....	11
Table 4.1	Coordinates of Points	25
Table 4.2	Distance Matrix	25
Table 4.3	Reduced Matrix	26
Table 4.4	Cost After Exclusion	27
Table 4.5	Cost After Inclusion	27
Table 5.1	Max-Min Distance Heuristic with Improvement.....	35
Table 5.2	Clustering Heuristics	36
Table 5.3	TSP solution methods	36
Table 5.4	Non-Linear Cost Results.....	37

LIST OF FIGURES

Figure 4.1	Steps of Clustering Based Location Routing Heuristic	18
Figure 4.2	Max-Min Distance Heuristic	20
Figure 4.3	Max-Total Distance Heuristic	21
Figure 4.4	Nearest Point to Cluster Heuristic	21
Figure 4.5	Nearest Neighbor Heuristic	22
Figure 4.6	Gravity Force Heuristic	23
Figure 4.7	Branches	28
Figure 4.8	Branch-and-bound Traveling Salesman Problem (BAB-TSP)	28
Figure 4.9	BAB-TSP Explore Procedure	29
Figure 4.10	Nearest Neighbor Heuristic	30
Figure 4.11	Stem Distance.....	31
Figure 4.12	Myopic Heuristic	32
Figure 4.13	Look-Back Heuristic	33

CHAPTER 1

INTRODUCTION

Customer satisfaction is the primary target for most companies in today's business environment. Fierce competition in the market forces companies to improve their service levels, decrease costs, and increase delivery speed. Logistics activities are one of the vital cost drivers as nearly 15% of the product cost consists of logistic costs. They also affect delivery speed. Opening new plants, distribution centers, and warehouses have an impact on both the cost and the delivery speed. As the number of warehouses increases, delivery times are improved at the expense of increased fixed costs. The balance is crucially important for companies in order to be competitive and survive in the market. The Location Routing Problem (LRP) and its several variants address this motivation of balancing the trade-off.

The aim of this thesis is to briefly review the LRP and to present a solution approach. The definition of LRP contains both the Location-Allocation Problem (LAP) and the Vehicle Routing Problem (VRP). LRP focuses on finding optimal location, number, and capacity of distribution facilities serving more than one customer, optimizing vehicle numbers and finding the optimal set of vehicle schedules and routes. Tuzun and Burke (1999) describe the problem as follows:

“A feasible set of potential facility sites and locations and expected demands of each customer are given. Each customer is to be assigned to a facility which will supply its demand. The shipments of customer demand are carried out by vehicles which are dispatched from the facilities, and operate on routes that include multiple customers. There is a fixed cost associated with opening a facility at each potential site, and a distribution cost associated with any routing of vehicles that includes the cost of acquiring the vehicles used in the routing, and the cost of delivery operations. The cost of delivery operations is linear in the total distance traveled by the vehicles. The LRP is to determine the location of the facilities and the vehicle routes from the facilities to the customers to minimize the sum of the location and distribution costs such that the vehicle capacities are not exceeded.”

Additionally, Tansel *et al.* (2007) define LRP as a combination of Location/Allocation and Vehicle Routing Problems where location and routing decisions are taken simultaneously.

LAP is concerned with the provision of a service to satisfy a widely dispersed demand. It is impractical to satisfy the demand everywhere because of economic reasons which force us to build centralized servers. The problem has two components: location and allocation. The location problem deals with the number, capacities, and location of facilities whereas the allocation problems assign customers to facilities. Various application areas of LAP have been addressed in the literature, including the location of stores, restaurants, emergency facilities, ambulances, fire stations, schools, warehouses, regional offices of government departments, etc. In LAP it is assumed that a tour consists of only one customer. However, a tour serving more than one customer in most real-world settings renders this assumption unrealistic.

The most important difference between the LRP and the classical location-allocation problem is that LRP determines the location of customers/suppliers through making tours while the classical location-allocation problem goes on the radial or direct trips from the facility to the customers/suppliers. Hence the classical location-allocation problem causes the increased cost of distribution (Salhi and Rand, 1989). On the contrary, LRP establishes the optimal facilities using tours and designs the routes at the same time.

VRP determines k vehicle routes, where a route is a tour that begins at the warehouse, traverses a subset of the customers in a specified sequence and returns to the warehouse. Each customer must be assigned to exactly one of the k vehicle routes and the total size of deliveries for customers assigned to each vehicle must not exceed the vehicle capacity b . The routes should be chosen to minimize total travel cost. Although both LRP and VRP are dealing with routing problems, VRP does not deal with locating the distribution facilities and focuses on finding the best routes. In LRP, the primary objective is to find the optimum location, capacity, and number of facilities. While investigating this problem, routing costs are included.

Although many researchers are aware of the danger of sub-optimizing by separating warehouse location and vehicle routing problems, they often ignore this interrelation. The reason for small progress is the complexity of LRP. Berman *et al.*

(1995) observes that the facility must be central relative to the ensemble of demand points, as ordered by the (yet unknown) tour through all of them. By contrast, in classical problems, the facility must be located by considering the distance to individual demand points, thus making the problem more tractable.

From a mathematical point of view, LRP is a combinatorial optimization problem. The word combinatorial denotes that a finite number of alternative feasible solutions exist. Combinatorial optimization process searches one or more best (optimal) solutions in a well defined discrete problem space. The algorithms developed for combinatorial optimization solve instances of problems that are believed to be hard in general. They explore the large solution space of these instances. Combinatorial optimization algorithms are typically concerned with problems that are NP-hard. Such problems are not believed to be efficiently solvable in general. LRP consist of two NP-hard problems which are facility location and vehicle routing.

The location of facilities does not only affect the location cost but also it has a major impact on the routing cost. Salhi and Rand (1989) show the effect of ignoring routes. Due to the complexity of both location allocation and vehicle routing problems, they have been often solved independently. However, the researchers indicate that poor decisions may be made if routing cost is not considered.

Despite LRP solutions' more accurate location decision, there is some criticism. Balakrishnan *et al.* (1987) note that LRPs are essentially strategic decisions concerning facility location, whereas routing is a operational decision. Since LRP brings these strategic and operational decisions together, it is criticized for inconsistency. Despite the fact that routes can be recalculated frequently, the location decision of a warehouse is given for much longer periods. After investigations on this criticism, Salhi and Nagy (1999) point out that location routing could decrease cost over a long planning horizon, within which routes are allowed to change.

Types of LRP

Several classifications of LRP are provided in the literature. One of the most inclusive reviews is made by Salhi and Nagy (2006). Classifications are made according to parameters such as the type of input data, planning period, solution method, objective function, solution space, number of warehouses, number and types of vehicles, and route structure.

- The input data may be deterministic or stochastic. Most of the studies in the literature address the deterministic case. Stochastic papers generally consider customer demand as the stochastic parameter.
- The planning period may be single or multiple. Single-period problems are static and multiple period problems are dynamic. Most of LRP papers investigate the static case.
- The solution methods may be exact or heuristic. Although heuristic applications are more convenient, exact methods are successful for special cases of the LRP.
- The objective function generally minimizes total cost consisting of warehouse cost, vehicle cost, and routing cost. Some studies consider multiple objectives.
- The solution space may be discrete, network type, or continuous. In the literature LRP studies generally deal with discrete location.
- The number of warehouses may be single or multiple. Generally, the multiple warehouse case is considered in the literature. In addition, the number of warehouses is not given and selection may be made among the potential warehouses.
- The number of vehicles is fixed and homogeneous fleet is assumed in most LRP. On the other hand, a heterogeneous fleet is adopted in some studies.
- In LRP, vehicle starts out a warehouse and returns to the same warehouse. Vehicles may be allowed multiple trips. Routes may contain both deliveries and pickups.

A guiding review is given in Min *et al.* (1997). It classifies the problem into two main categories depending on the definition types and the solution techniques. The problem may be defined according to parameters as follows:

- Number of facilities: single or multiple.
- Hierarchical level: delivery or delivery-pickup.
- Size of fleet: single or multiple.

- Vehicle capacity: capacitated (homogenous or non-homogeneous), uncapacitated.
- Facility capacity: capacitated or uncapacitated.
- Facility cost: fixed cost or capacity increment cost (linear, non-linear).
- Planning horizon: single period (static) or multiple periods (dynamic).
- Objective: single or multiple.
- Time windows: unspecified, soft time, hard time.
- Nature of demand: stochastic or deterministic.
- Routing: single or multiple.

This thesis is organized as follows: Chapter 2 reviews the LRP literature. The definition of the problem and the mathematical model are presented in Chapter 3. In Chapter 4, the solution approach is described. The computational study is discussed in Chapter 5. Conclusions and future research ideas are given in Chapter 6.

CHAPTER 2

LITERATURE REVIEW

Since the 1970's, many researchers have studied LRP with different names and contents. However most of them have not noticed the connections between their work and LRP. The surveys published by Min *et al.* (1997) and Salhi and Nagy (2006) do help to gather the works together. A basic classification, heuristic approaches, optimal solutions and hybrid methods, depending on the solution method is made in their work.

The importance of routing decision in location is first noticed by Webb (1968) as well as Christofides and Eilon (1969). They imply that using route length estimation is more sophisticated than using radial distance. LRP is first modeled by Watson-Gandy and Dohrn (1973) in a warehouse location with a van salesman.

Heuristics are commonly used in the literature. Firstly, Gillett and Johnson (1976) build a single objective model with multiple uncapacitated facilities, capacitated vehicles, and deterministic customer demands. They apply multi-terminal sweep heuristics to solve the problem. Although deterministic demand is most widely applied in the models, Bumess and White (1976) contribute with stochastic customer demands. Their model includes single uncapacitated facility and vehicle. Later, Jacobsen and Madsen (1978) develop Gillett and Johnson's problem by extending hard time windows. In the solution, the heuristic method (location-allocation first, route-second using the savings and tree-tour methods) is used, and newspaper transfer point location-routing problem is solved in the private sector. Afterwards, Daskin (1987) contributed stochastic travel times. His model includes a single uncapacitated facility, and multiple uncapacitated vehicles. It is applied in emergency services. Bookbinder and Reece (1988) used the multi-commodity warehouse in their model. They apply heuristics in which location-allocation and routing are made respectively. Afterwards, the insertion method is first adapted by Chien (1993). Later, Srivastava (1993) use the clustering and saving method based heuristic for solving single stage, deterministic, static, single objective model with multiple capacitated vehicles, capacitated and hard time windows.

One of the most common methods, iterative heuristic method, is first used by Hansen (1994) to solve both location and routing phases. Afterwards, Tuzun and Burke, (1999) apply the two phase tabu search algorithm hierarchically for solution. In the problem, a feasible set of potential facility sites, locations, and expected demands of each customer exists. Each customer has to be assigned to a facility which will supply its demand. Vehicles supply multiple customer demand on routes and are dispatched from the facilities. The overall cost function consists of routing, facility, and vehicle cost. The objective function is single and minimization. Subsequent Wu *et al.* (2002) apply a combined tabu search, simulated annealing framework with a simpler neighborhood structure and sensible improvement is found when compared against a sequential method. Later on, Chan and Baker (2003) include vehicle range and multiple service-frequency requirements in the model and apply the heuristic consisting of minimum spanning forest (MSF) and a modified Clarke-Wright (MCW) procedure.

Branch-and-bound, integer programming and nonlinear programming are the most widely used to solve LRP optimally. Ghosh *et al.* (1981) use non-rectilinear distance location. The model is single stage, deterministic, single objective with a single uncapacitated facility and vehicle. They adopt nonlinear programming for an exact solution. Laporte *et al.* have great contributions on optimal solution of the LRP. Laporte and Nobert (1981) use two branching strategies to solve the model which is a deterministic, static, single objective, single uncapacitated facility and vehicles. Moreover, Laporte (1983) applies integer programming on single stage, deterministic, static problem with multiple uncapacitated facilities, vehicles, unspecified time windows, single objective. Additionally, Laporte *et al.* (1986) develop a formulation and an exact algorithm, integer programming, for the generic capacitated location-routing. Subsequent Laporte (1988) use a modified branch-and-bound method for solving the asymmetric, deterministic, static, single objective model with multiple capacitated facilities, multiple capacitated vehicles and hard time windows. Capacity and maximum cost restrictions are included in the model which is solved by using a graph transformation and branch-and-bound method. Additionally, Laporte (1989) develops an exact algorithm for the stochastic version of the single uncapacitated facility, multiple capacitated vehicles. In that paper, multiple uncapacitated vehicles and facilities, unspecified time windows, and a single objective are used. The problem is single stage, deterministic and solved by using integer programming. Dynamic location

routing problems is published by Laporte and Dejax (1989). Later on, Averbakh and Berman (1994) apply dynamic programming for an exact solution.

One of the most common solution techniques is hybrid methods. They are widely used for solving LRP and nearly fast as heuristic methods. The concept of iterating between locational and routing phases is first published by Perl and Daskin (1985). The location part is solved optimally by using implicit enumeration. It minimizes the sum of distances between warehouses and the end-points of routes found in the routing phase. Their model is single stage, deterministic, single objective with multiple capacitated facilities and vehicles. Later, sequential methods are introduced by Srivastava and Benton (1990). This method provides good quality solutions for some cases. Although both location and routing sub problems are optimal, it cannot guarantee an optimal solution to the combined problem. Afterwards, Albareda-Sambola *et al.* (2005) implement an effective graph transformation to the LRP. Linear programming with relaxation is applied for finding an initial solution. Additionally, the locational neighborhood search is made depending on the moves add, drop, and shift. Infeasible solutions are allowed but a penalty term is added to the objective function for the violation of warehouse capacity constraint. Tabu search is applied as a framework algorithm. Later on, Melechovsky *et al.* (2005) include nonlinear cost in the cost function. They apply the P-median approach to find the initial feasible solution and a combination of tabu search and neighborhood search as a hybrid metaheuristic to improve solution. The problem is static, deterministic with facilities having non-linear cost function, growing with the total demand supplied and capacitated vehicles.

LRP is widely applied to public and private sector. Madsen (1983) builds a model and develops a solution for newspaper dealers. In the model, the factory may send newspapers to transfer points or directly to customer. Transfer points distribute the newspaper to the customer. The problem consists of determining the locations of transfer points, designing a vehicle route through these points, allocating the customers to transfer points or directly to the factory. The multiple objective, single stage, static, deterministic model with multiple capacitated facilities, single uncapacitated vehicle, and unspecified time windows is formed and adopted to hazardous waste transportation and disposal by Zografos and Samara (1989). The objective function minimizes disposal/routing risk and travel time. The mixed-integer goal programming is applied as an exact algorithm. Later, a multi objective model including minimization of risk, cost

and equity is studied by List and Mirchandani (1991). They use the exact route generation method. The model is applied to the public sector by using real-world data on the field of hazardous material handling. Afterwards, ReVelle (1991) uses integer programming, shortest paths, and a weighting method for hazardous waste disposal site location routing. The model has two objectives: minimization of transportation cost and perceived risk. Subsequent, Bruns *et al.* (2000) study a problem arising in the parcel delivery operations of a postal service. In this system post offices send parcels directly to delivery centers. Vehicles carry these parcels to customers on a route by making multiple stops. The problem is to determine the locations of the delivery bases, their allocation to processing centers, and the allocation of customer areas to delivery bases. The flow from post offices to delivery bases is separate from the flow from bases to customers. The problem is reduced to LRP and branch-and-bound is applied in the solution. Later on, Lin *et al.* (2002) study bill delivery service. The relocation of the existing bill delivery warehouses and setting up these warehouses to existing company buildings is considered. The model searches where the facilities establishment, route formation, and routing sequence. They adopt the clustering heuristic approach for finding the initial warehouses location. Initial routes are found with Clarke Wright (CW) algorithm and solution is improved with threshold accepting (TA) and simulated annealing.

Similar problems are determined in the LRP literature. The problems given below have high similarities with LRP.

Min (1996) attends the location consolidation terminal problem in which goods from several supply sources are aggregated at warehouses before sent to customers. It is more complicated than the basic LRP because allocation of both customers and suppliers to terminals needs to be found. Customer clusters are formed according to vehicle capacity and assigned to warehouses from the centroid of each cluster.

Salhi and Nagy (1998) introduced many-to-many location-routing problem (MMLRP). In the problem, it is assumed that each customer sends a different commodity to every other customer. This is similar to a postal flow between locations. A network of hubs is to be located by considering costs. Although hub to customer routes are multi stop, the link between hubs are assumed to be direct. Both LRP and MMLRP investigate best locations facilities and hubs respectively. In the MMLRP, a pickup-and-delivery is allowed. The routing method considers both sending and

delivering goods. The problem is harder to solve than the VRP because the fluctuating load on the vehicles makes feasibility checks harder to perform. The hierarchical heuristic solution is presented.

Logistic problems are special cases of MMLRP. In the hub location problem, full-truckload routes are assumed. In freight transport, problem hubs are fixed. If there is no flow between hubs and all deliveries or pickups are zero, the problem becomes LRP.

Ghiani and Laporte (1999) study the arc routing problem. The usual route structure in LRP starts from a warehouse and returns to the same warehouse by visiting multiple customers. In the arc routing problem, vehicles may traverse given edges rather than nodes.

CHAPTER 3

MODEL FORMULATION

In this thesis, we address LRP where the objective is to select warehouse locations among potential warehouses to service a set of customers and determine their capacities by considering routing and vehicle acquisition costs. The customers are in a geographically dispersed area. Each customer has a deterministic demand and all demands must be satisfied. Moreover, each customer is serviced exactly once. Distance matrix is symmetric and all distances between customers and potential warehouses are known. Customers are served using a homogeneous fleet where multiple vehicles are available at each warehouse and each vehicle makes only one tour.

The objective function has three components: routing, vehicle operating, and warehouse operating costs. The routing cost is a function of the total distance traversed by all vehicles. The vehicle cost depends only on the number of vehicles used since the fleet is assumed to be homogenous and each vehicle has the same fixed cost. The warehouse cost has fixed and variable components. Opening a warehouse incurs a fixed cost. The variable cost component is related with the demand to be satisfied by the warehouse. Piece-wise linear capacity cost is assumed and given in table 3.1. As the amount of demand assigned the same warehouse increases, total cost increases after a certain quantity.

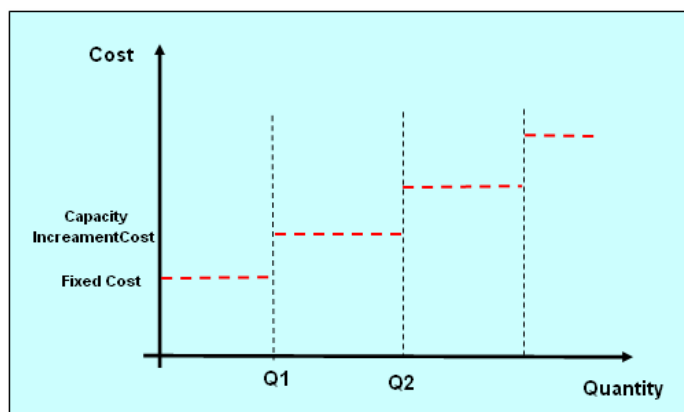


Table 3.1 Non-linear Cost of Warehouse

The problem is formulated as a mixed integer program.

3.1. Assumptions

In the design of the model, the following assumptions are made.

- Demand is deterministic.
- All customer demands are satisfied.
- Each customer is served by exactly one vehicle.
- Only delivery is made. Pick-ups are not considered.
- Number of product type is not important for the model. Since model deals with the transportation and location of product, different types of product may be denoted with same volume, capacity, etc.
- We consider a single planning period. The average demand of each customer for a multi period planning horizon is taken as the demand of a single period. This type of problem is called static problem in literature.
- There exists one facility layer. It means only the relationship between customer and warehouses are taken into consideration. In a two layer problem, connection between warehouses and plants or distributors would be considered.
- There exist multiple potential warehouses of which coordinates are known.
- Transportation between warehouses is not allowed.
- Distance cost is a linear function of range. Distance matrix is symmetrical.
- Warehouses are uncapacitated.
- Fixed cost of opening a warehouse and non-linear capacity increment cost are adopted to the model.
- There are no vehicle range constraints. No restriction exists on the route length.
- Capacitated vehicles are used. The total demand on each route is less than or equal to the capacity of a vehicle assigned to that route.
- Multiple vehicles serve to customers.
- There is no time windows constraint rod the delivery of demand.

- Each vehicle makes only one tour. A vehicle returns to its own warehouse. Each route begins and ends at the same warehouse.
- Vehicle acquisition cost is fixed.

3.2. Notation

We use the following notation to formulate the problem:

- D $\{r / r = 1, \dots, R\}$ is the set of R feasible sites of potential warehouses.
- C $\{i / i = R + 1, \dots, R + N\}$ is the set of N customers to be served.
- S $\{C\} \cup \{D\}$ is the set of all customers and potential warehouses.
- V $\{v_k / k = 1, \dots, K\}$ is the set of K vehicles available for routing from facilities.
- C_{ij} average annual cost of traveling from node i to node j , $i \in S, j \in S$.
- C_k annual cost of acquiring a vehicle k ($k=1, \dots, K$).
- F_r annual fixed cost of establishing a warehouse at site r ($r=1, \dots, R$).
- d_j average number of units demanded by customer j , $j \in C$.
- Q_k capacity of vehicle k ($k=1, \dots, K$).
- R_j rank of node j .

The decision variables are as follows:

$$X_{ijk} = \begin{cases} 1 & \text{if vehicle } k \text{ from node } i \text{ to node } j, i \in S, j \in S, k \in V, i \neq j. \\ 0 & \text{otherwise.} \end{cases}$$

$$Z_r = \begin{cases} 1 & \text{if a depot is established at site } r, r \in G. \\ 0 & \text{otherwise.} \end{cases}$$

$$Y_{kr} = \begin{cases} 1 & \text{if a vehicle is assigned to depot } r. k \in V, r \in D. \\ 0 & \text{otherwise.} \end{cases}$$

3.3. Mathematical Model

$$\text{Minimize } \left(\sum_{i \in S} \sum_{i \in S} \sum_{k \in V} C_{ij} X_{ijk} + \sum_{k \in V} \left(C_k \sum_{r \in D} \sum_{j \in C} X_{rjk} \right) + \sum_{r \in D} \left\{ \left(f \left(\sum_{k \in V} \sum_{j \in C} \sum_{i \in S} q_j X_{ijk} Y_{kr} \right) + FrZr \right) \right\} \right)$$

$$\text{s.t. } \sum_{k \in V} \sum_{i \in S} X_{ijk} = 1 \quad \forall j \in C \quad (1)$$

$$\sum_{j \in C} \sum_{i \in S} d_j X_{ijk} \leq Q_k \quad \forall k \in V \quad (2)$$

$$\sum_{i \in S} X_{ipk} - \sum_{j \in S} X_{pj k} = 0 \quad \forall k \in V, p \in S \quad (3)$$

$$\sum_{r \in D} \sum_{j \in C} X_{rjk} \leq 1 \quad \forall k \in V \quad (4)$$

$$\sum_{k \in V} X_{r m k} + Z_r + Z_m \leq 2 \quad \forall m = 1, \dots, R, r \in D \quad (5)$$

$$\sum_{k \in V} \sum_{j \in H} X_{rjk} - Z_r \geq 0 \quad \forall r \in D \quad (6)$$

$$\sum_{j \in H} X_{rjk} - Z_r \leq 0 \quad \forall k \in V, r \in D \quad (7)$$

$$R_i - R_j + (R + N) \sum_{k \in V} X_{ijk} \leq R + N - 1 \quad \forall i, j \in C, i \neq j \quad (8)$$

$$Y_{kr} = \sum_{j \in C} X_{rjk} \quad \forall r \in D \quad (9)$$

$$X_{ijk} = 0 \text{ or } 1 \quad \forall i, j \in C, k \in V \quad (10)$$

$$Z_r = 0 \text{ or } 1 \quad \forall r \in D \quad (11)$$

$$Y_{kr} = 0 \text{ or } 1 \quad \forall k \in V, r \in D \quad (12)$$

In this formulation, the objective function minimizes the total cost of transportation, vehicle acquisition, and warehouse operating cost. Constraints (1) ensure that each customer is serviced by exactly one vehicle. Constraints (2) guarantee that total demand assigned a vehicle does not exceed the vehicle capacity. Route

continuity is satisfied with Constraints (3): if a vehicle visits a customer it should also leave that customer. Constraints (4) assure that each vehicle is routed from at most one warehouse. Constraints (5) guarantee that there exists no link between two warehouses. In constraint set (6) if warehouse is open, at least one vehicle should be assigned to that warehouse. Constraints (7) make sure that a vehicle can serve a customer starting from a warehouse if and only if that warehouse is open. Constraints (8) ensure that each tour contains a warehouse None of the tours consists of only customers. R_i are continuous variables used in the sub-tour breaking constraints. In constraints (9) Y_{kr} is slack variable and takes 0 or 1 value. The remaining constraints are binary constraints for the decision variables.

CHAPTER 4

SOLUTION METHODOLOGY

Different solution techniques are applied to LRP. These solution techniques expressly categorized by Tansel *et al.* (2007) are given below.

1. Exact algorithms
 - a. Branch-and-bound
 - b. Dynamic programming
 - c. Integer programming
 - i. Branch and cut
 - ii. Constraint relaxation
 - iii. Cutting plane algorithms
 - iv. Benders' composition
 - d. Nonlinear programming
2. Heuristics
 - a. Tour construction heuristic
 - i. Locate first, route second
 - ii. Route first, locate second
 - iii. Cluster first, route second
 - b. Tour improvement heuristics
 - i. Add/drop heuristic
 - ii. k-exchange heuristic
 - c. Iterative methods
 - d. Nested methods

Since the problem we are dealing with is NP hard and has a non-linear cost function, a hybrid-hierarchical algorithm is adopted. This algorithm consists of a clustering part, a TSP solution method, improvement heuristics, and an assignment heuristic.

In the first part, clusters are formed depending on the vehicle capacity. Initially, the lower bound on the vehicles is found and farthest two points are selected. Different points are marked up as much as total number of vehicles with two methods. After clusters are formed, a traveling salesman problem is solved for each of cluster. Further local improvements like swap and move are applied in an attempt to obtain better solutions. Finally, clusters are assigned to the warehouse simultaneously. The steps of the algorithm are given in Figure 4.1.

Step 1: Find minimum number of vehicles.

$$VehicleNumb = \left\lceil \frac{\sum_{i \in U} Dem_i}{VehicleCapacity} \right\rceil$$

Step 2: Find $\max Dist_{ij}$ $i, j \in U$ (unassigned cluster) and move node i and j to from U to S (selected cluster). Insert node i to C_1 and node j to C_2 . Increase *Assigned Nodes* by two.

Step 3: Find means.

Case 1: Call Max-Min Distance Heuristic

Case 2: Call Max-Total Distance Heuristic

Step 4: Form clusters.

Case 1: Call Nearest Point to Cluster Heuristic

Case 2: Call Nearest Neighbor Heuristic

Case 3: Call Gravity Force Heuristic

Increase *VehicleNumb* and return to step 2 if clusters are not feasibly formed.

Step 5: Find routes in each cluster.

Case 1: Call nearest Point to Cluster Median Heuristic

Case 2: Call Nearest Neighbor Heuristic

Case 3: Call Branch-and-bound Traveling Salesman Problem Algorithm (BAB-TSP)

Step 6: Improve solution by local search methods.

Step 7: Assign clusters to open warehouses. Call Assignment Heuristic.

Figure 4.1 Steps of Clustering Based Location Routing Heuristic

4.1. Clustering

Clustering is the process of assigning discrete objects to groups with similar characteristics such as the grouping animals of different species in order to find species having most similarities. An important part of a clustering algorithm is the similarities between data points. One similarity criterion is the distance which is called distance-based clustering. In another type, conceptual clustering, objects are grouped according to their fit to descriptive concepts, not according to simple similarity measures. There exist many different clustering techniques. However, clustering algorithms may be mainly classified as hierarchical and k-means clustering algorithms.

4.1.1. Hierarchical Clustering Algorithm

It is first defined by Johnson (1967). In the model, a set of N items are clustered depending on a given distance (or similarity) matrix. Mostly hierarchical clustering merges clusters iteratively, it is called the agglomerative method. Divisive hierarchical clustering method starts with all objects in one cluster and subdivides them into smaller pieces. They rarely have been applied in the literature. In the first step, each item is assigned to different clusters. Hence, we have N clusters at the beginning. Secondly, the pair of items having highest similarity value is merged and number of cluster decreases by one. In the third step, similarities between clusters are found. Step 2 and 3 are repeated until all items are clustered into single cluster. If we need k clusters, we have to cut the $k-1$ longest links. Different methods may be applied in step 3 such as single-linkage, complete-linkage and average-linkage clustering. In single-linkage clustering, the shortest distance between any members of cluster to any member of other cluster is considered. In complete-linkage clustering, greatest distance between members in different clusters is considered. In average-linkage clustering, average distance between all members in different clusters are calculated.

4.1.2. K-means Clustering Algorithm

K -means is developed by Mac Queen (1967). The method is adopted when we need a fixed number of clusters. Firstly k points, centroids, are selected for each cluster. These centroids should be placed carefully inasmuch as different location causes different result. One of the most common methods is placing them far from each other.

In the next step, each point should be associated with the nearest centroid. As all points are assigned to clusters, early group age is completed. Later, k new centroids are recalculated. Points are checked whether they are assigned to nearest centroid. If not, they are deleted from previous cluster and inserted to the nearest cluster. This loop is carried out until centroids stand firm. That is to say, centroids do not move any more.

Actually, the objective of the algorithm is to minimize square of discrepancy between centroids and assigned points.

$$J = \sum_{j=1}^n \sum_{i=1}^n \|x_i^{(j)} - m_j\|^2 \quad x_i^{(j)}: \text{point}, m_j: \text{cluster centre}$$

4.1.3. Clustering Heuristics

Our clustering approaches are similar with k -means clustering. Since we build clusters depending on vehicle capacity, different methods are added in each heuristic.

Initially, a pair of nodes with the longest distance is found. These two points are selected as the centroid of two clusters. Later remaining centroids are selected with two different methods. In this first heuristic, Maximize Minimum Distance, shortest distance from unassigned nodes to centroids is found. The node having maximum distance is selected as the next centroid. This procedure continues until k centroids (number of vehicles) are assigned. In the second heuristic, Maximum-Total Distance, distance from unassigned nodes to centroids is summed for each node. The node having maximum value is selected as the next centroid. Algorithm terminates when k means are assigned. The algorithms are given in Figure 4.2 and 4.3.

WHILE *Assigned Nodes* \leq *VehicleNumb*

Find $\max\{\min(Dist_{ij})\} \mid i \in U, j \in S$

Increase *Assigned Nodes* by one

Move node i from U to S

Insert node i to C_k (cluster k) and increase k by one

ENDWHILE

Figure 4.2 Max-Min Distance Heuristic

```

WHILE Assigned Nodes ≤ VehicleNumb

    Find  $\max \left( \sum_{j \in S} Dist_{ij} \right) i \in U, j \in S \quad \forall i$ 

    Increase Assigned Nodes by one

    Move node i from U to S

    Insert node i to  $C_k$  (cluster k) and increase k by one

ENDWHILE

```

Figure 4.3 Max-Total Distance Heuristic

After assigning k means, we need to group other elements into clusters. Three different heuristic is presented for this part. In the nearest point to cluster heuristic, minimum distance to centroid is found starting from one centroid. The point is inserted to cluster if vehicle capacity is enough. New centroid is calculated by using inserted nodes. After completing the assignment of nodes to first cluster, the procedure starts for next centroid and carry outs until all nodes are assigned to nodes in the nearest point to cluster heuristic. It is presented in Figure 4.4.

```

VehicleRemainingCapacity is equal to VehicleCapacity initially.

FOR all vehicle k

    FOR node  $i \in U$ 

        Find  $\min(Dist_{im_k}) i \in U, m_k$  is the gravity center of  $C_k$ 

        IF  $VehicleRemainingCapacity_k \geq Dem_i$ 

            Move node i from U to S and insert node i to  $C_k$ 

             $VehicleRemainingCapacity_k = VehicleRemainingCapacity_k - Dem_i$ 

             $m_k = \sum i, i \in C_k$ 

        ENDIF

    ENDFOR

ENDFOR

```

Figure 4.4 Nearest Point to Cluster Heuristic

The only difference between Nearest Neighbor Heuristic and the Nearest Point to Cluster is the method for calculating the distance to centroid. The last inserted node is

taken as the centroid of cluster. Hence the nearest point to the last inserted node is found. Nearest Neighbor Heuristic is given in Figure 4.5.

```

VehicleRemainingCapacity is equal to VehicleCapacity initially.
FOR all vehicle k
    FOR node  $i \in U$ 
        Find  $\min(Dist_{ik})$   $i \in U$ ,  $k \in m_k$  and  $k$  is the last inserted node.
        IF  $VehicleRemainingCapacity_k \geq Dem_i$ 
            Move node  $i$  from  $U$  to  $S$  and insert node  $i$  to  $C_k$ 
             $VehicleRemainingCapacity_k = VehicleRemainingCapacity_k - Dem_i$ 
        ENDIF
    ENDFOR
ENDFOR

```

Figure 4.5 Nearest Neighbor Heuristic

Gravity Force Heuristic, given in Figure 4.6, is inspired by physical law. Gravity force is calculated for each node. This force is calculated by using the following formula.

$$force_{ij} = \frac{VehicleRemainingCapacity_i * Dem_j}{(Dist_{ij})^2}$$

Each median pulls nodes with a force. As the remaining capacity of vehicle decreases, the force decreases. This will help insert nodes to the empty clusters and balance demand distribution. Bigger demands have priority because it gets harder to insert them into clusters as remaining capacity decreases. It is obvious that increase in distance cuts down force. If the demand of a node is higher than the remaining capacity, negative force is applied. Iteratively, forces on nodes are recalculated and then, the node having the highest force is inserted in associated cluster. Iterations are carried out until all nodes are assigned to cluster. It is given in Figure 4.6.

PROCEDURE: Calculate All Forces

FOR node $i \in U$

FOR $j \in C_k$ and j is the first assigned node to cluster

Calculate *force*

$$force_{ij} = \frac{VehicleRemainingCapacity * Dem_j}{(Dist_{ij})^2}$$

ENDFOR

ENDFOR

ENDPROCEDURE

WHILE any node is unassigned.

FOR node $i \in U$

FOR $j \in C_k$ and j is the first assigned node to cluster.

Find max *force*_{ij}.

Move node i from U to S and insert node i to C_k

$$VehicleRemainingCapacity_k = VehicleRemainingCapacity_k - Dem_i$$

ENDFOR

ENDFOR

CALL Calculate All Forces

ENDWHILE

Figure 4.6 Gravity Force Heuristic

4.2. Traveling Salesman Problem

TSP is one of the most widely studied combinatorial optimization problems. Many articles are written on the TSP. Given a set of nodes and the distances between them, the shortest path starting from a given node, passing through all the other nodes and returning to the first node is determined. As a more formal definition, TSP finds a path through a weighted graph which starts and ends at the same vertex, includes every other vertex exactly once, and minimizes the total cost of edges. There exist $(N-1)!$ alternative solutions for the problem including N nodes. The main difference between TSP and VRP is the number of routes. While only one tour exists in TSP, more than one route may start from and end at the same node in VRP. Both problems are known to be NP-complete.

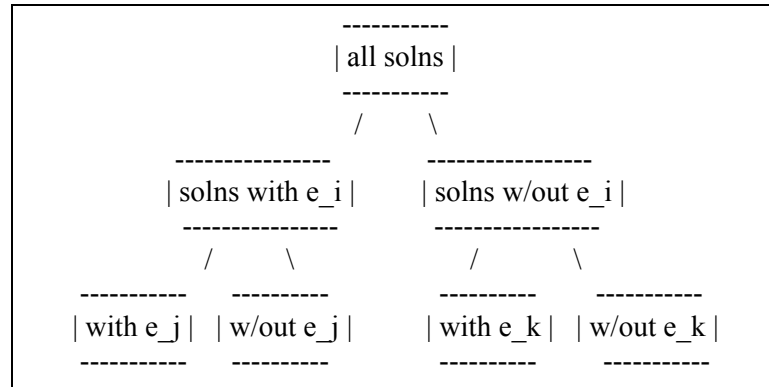
More formally, there is a graph $G=(N,V,C)$ consisting of a set of n nodes, a set of $V=(i,j)$ connecting cities and nonnegative weights C_{ij} . The graph is directed. If both edges exist, C_{ij} does not need to be equal to C_{ji} . The TSP consists of determining the minimum distance route passing through each node only once.

Although the problem looks simple, it has many application areas such as computer wiring, hole punching, job sequencing, etc. In computer wiring, we have n computers and we want to find the shortest cycle passing through computers in order to use least amount of cable. Metal sheet manufacturers often encounter the problem of drilling many holes on a sheet. TSP should be applied in order to find shortest path and reduce traveling time of the drill between nodes.

We solve the TSP using three methods described in the following sections.

4.2.1. Branch-and-bound

The following algorithm for the TSP is adopted from "Combinatorial Algorithms: Theory and Practice", by Reingold *et al.* (1977): A strategy for searching the solution space is to repeatedly divide it into two parts: those with a given edge and those without the edge. The search tree would unfold as follows:



Let us assume that the following customers and their coordinates are given in Table 4.1 for the traveling salesman problem:

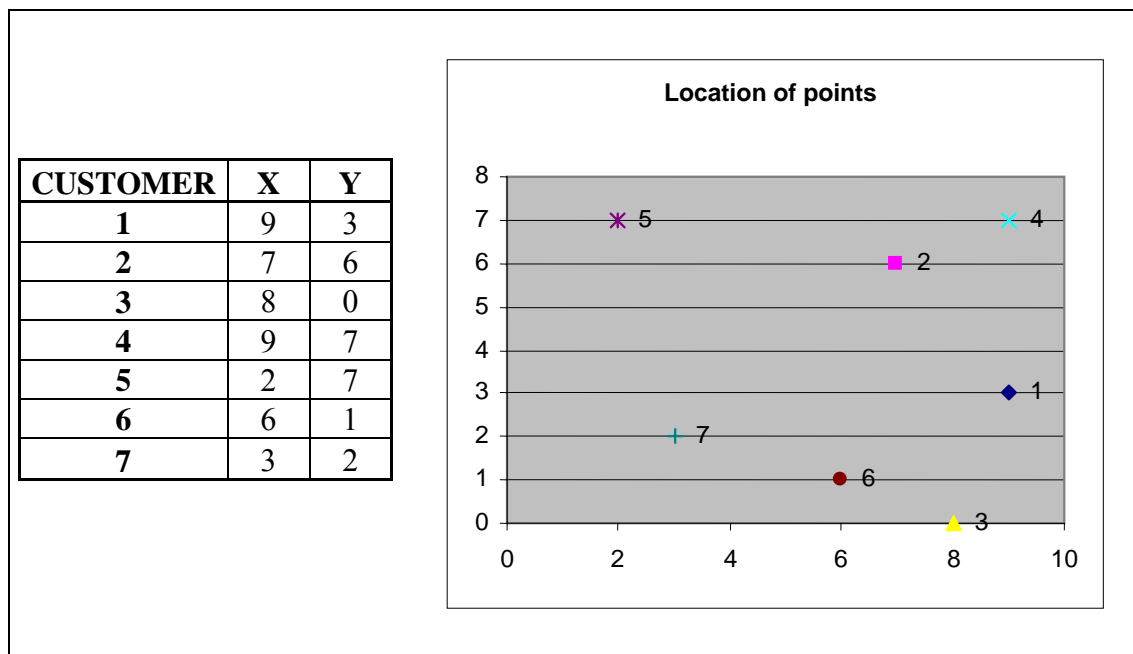


Table 4.1 Coordinates of Points

Assume that distance cost is equal to distance between nodes and symmetrical.

CUSTOMER	1	2	3	4	5	6	7	MIN
1	INF	3,6	3,2	4,0	8,1	3,6	6,1	3,2
2	3,6	INF	6,1	2,2	5,1	5,1	5,7	2,2
3	3,2	6,1	INF	7,1	9,2	2,2	5,4	2,2
4	4,0	2,2	7,1	INF	7,0	6,7	7,8	2,2
5	8,1	5,1	9,2	7,0	INF	7,2	5,1	5,1
6	3,6	5,1	2,2	6,7	7,2	INF	3,2	2,2
7	6,1	5,7	5,4	7,8	5,1	3,2	INF	3,2
							SUM	20,4

Table 4.2 Distance Matrix

Firstly a reduced cost matrix is found and cost is calculated. In the reduced cost matrix, there exist at least one zero on every column and row. Reduced cost matrix is given in Table 4.3. The sum of reduction values in reduced cost matrix gives the additional cost of including an edge in the tour. The lower bound is calculated by taking the sum of cheapest way to enter and leave each city. In order to reduce matrix, minimum element in each row is found and subtracted from all elements in that row. Additionally, same operations are carried out for each column. Calculations are given in Table 4.2 and 4.3.

CUSTOMER	1	2	3	4	5	6	7	MIN
1	INF	0,4	0,0	0,8	4,9	0,4	2,9	0,4
2	1,4	INF	3,8	0,0	2,9	2,9	3,4	0,0
3	0,9	3,8	INF	4,8	7,0	0,0	3,1	0,0
4	1,8	0,0	4,8	INF	4,8	4,5	5,6	0,0
5	3,0	0,0	4,1	1,9	INF	2,1	0,0	1,9
6	1,4	2,9	0,0	4,5	5,0	INF	0,9	0,0
7	2,9	2,5	2,2	4,6	1,9	0,0	INF	0,0
							SUM	2,9

CUSTOMER	1	2	3	4	5	6	7
1	INF	0,4	0,0	0,8	3,0	0,4	2,9
2	0,4	INF	3,8	0,0	0,9	2,9	3,4
3	0,0	3,8	INF	4,8	5,0	0,0	3,1
4	0,8	0,0	4,8	INF	2,8	4,5	5,6
5	2,0	0,0	4,1	1,9	INF	2,1	0,0
6	0,4	2,9	0,0	4,5	3,0	INF	0,9
7	2,0	2,5	2,2	4,6	0,0	0,0	INF

Table 4.3 Reduced Matrix

Since there exist at least one zero in each row and column, the matrix is reduced and the sum of the reduction values is 23, 3. The cost on rows shows the cost of leaving the city and cost on column shows cost of entering the city. The second step is to decide which edge to include or exclude. We will check zero values for this part, find exclusion costs and select maximum exclusion value. The exclusion cost of arc (3,1) is given in Table 4.4. If arc (3,1) is excluded, infinitive values inserted to cell (3,1) and (1,3). We can not return point 1 in order to prevent cycles.

CUSTOMER	1,0	2,0	3,0	4,0	5,0	6,0	7,0	Min
1	INF	0,4	INF	0,8	3,0	0,4	2,9	0,4
2	0,4	INF	3,8	0,0	0,9	2,9	3,4	0,0
3	INF	3,8	INF	4,8	5,0	0,0	3,1	0,0
4	0,8	0,0	4,8	INF	2,8	4,5	5,6	0,0
5	2,0	0,0	4,1	1,9	INF	2,1	0,0	0,0
6	0,4	2,9	0,0	4,5	3,0	INF	0,9	0,0
7	2,0	2,5	2,2	4,6	0,0	0,0	INF	0,0
Min	0,4	0,0	0,0	0,0	0,0	0,0	0,0	

Table 4.4 Cost After Exclusion

Cost increases 0,8 and exclusion cost of arc (3,1) becomes 24,1.

4 and 2	5 and 2	1 and 3	6 and 3	2 and 4	7 and 5	3 and 6	7 and 6	5 and 7
0,4	0	0,8	0	0,4	1,8	0	0	1,8

The highest exclusion cost is 1,8 on arc (5,7) or (7,5). Hence we select one of them. Matrix is given in Table 4.5 when arc (5,7) is included. The fifth row and seventh column are deleted. Arc (7,5) is excluded.

CUSTOMER	1	2	3	4	5	6	7
1	INF	0,4	0,0	0,8	3,0	0,4	INF
2	0,4	INF	3,8	0,0	0,9	2,9	INF
3	0,0	3,8	INF	4,8	5,0	0,0	INF
4	0,8	0,0	4,8	INF	2,8	4,5	INF
5	INF	INF	INF	INF	INF	INF	INF
6	0,4	2,9	0,0	4,5	3,0	INF	INF
7	2,0	2,5	2,2	4,6	INF	0,0	INF

CUSTOMER	1	2	3	4	5	6
1	INF	0,4	0,0	0,8	3,0	0,4
2	0,4	INF	3,8	0,0	0,9	2,9
3	0,0	3,8	INF	4,8	5,0	0,0
4	0,8	0,0	4,8	INF	2,8	4,5
6	0,4	2,9	0,0	4,5	3,0	INF
7	2,0	2,5	2,2	4,6	INF	0,0

Table 4.5 Cost After Inclusion

Figure 4.7 partially illustrates how algorithm works and Figure 4.8 and 4.9 show pseudo code of the algorithm. We will carry on excluding and including nodes until a solution is found. We apply depth first search. After finding a solution, we will check other branches. If lower cost is found on any branch, solution is carried out on that branch until a better solution is found. In the solution, the node having the highest exclusion cost is included to the route because we do not want to carry out different branches in order to run algorithm faster.

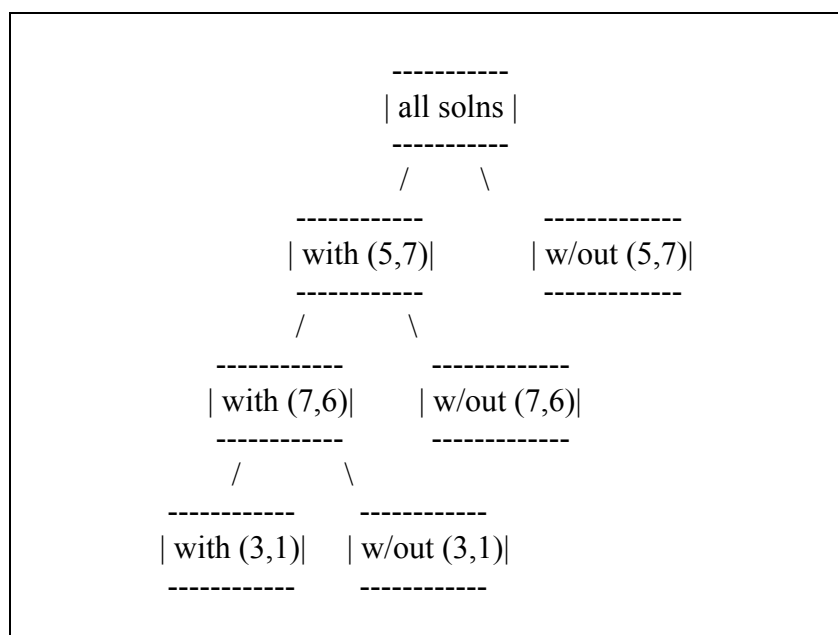


Figure 4.7 Branches

```

Lower Bound is equal to infinity
CALL Explore Nodes Procedure
FOR All nodes
    IF Exclusion cost of node is less than Lower Bound
        CALL Explore Nodes Procedure
    ENDIF
ENDFOR

```

Figure 4.8 Branch-and-bound Traveling Salesman Problem (BAB-TSP)

```

PROCEDURE: Explore Nodes
    WHILE assigned nodes is not equal to number of edges-2 and cost is
    less than lower bound
        Reduce Matrix
        Calculate cost by adding reduction values
        Find Best Edge (Maximum exclusion cost)
        Delete Included Edge
    ENDWHILE
    IF number of edges-2 nodes included to route
        Force remaining nodes to build a cycle
        Calculate Lower Bound
    ENDIF
END PROCEDURE

```

Figure 4.9 BAB-TSP Explore Procedure

4.2.2. Nearest Neighbor

This construction heuristic is simple and straightforward. The nearest city is always visited. Held-Karp lower bound is generally used to judge the performance of TSP heuristics. Johnson and McGeoch (1995) showed the Nearest Neighbor algorithm will often keep its tours within 25% of the Held-Karp lower bound.

4.2.3. Nearest Point to Cluster Median

This approach is similar with nearest neighbor heuristic. Cluster median is calculated by using coordinates at every iteration and nearest point to median is inserted to route.

The Nearest Neighbor and Nearest Point to Cluster Median heuristics are given in figure 4.10.

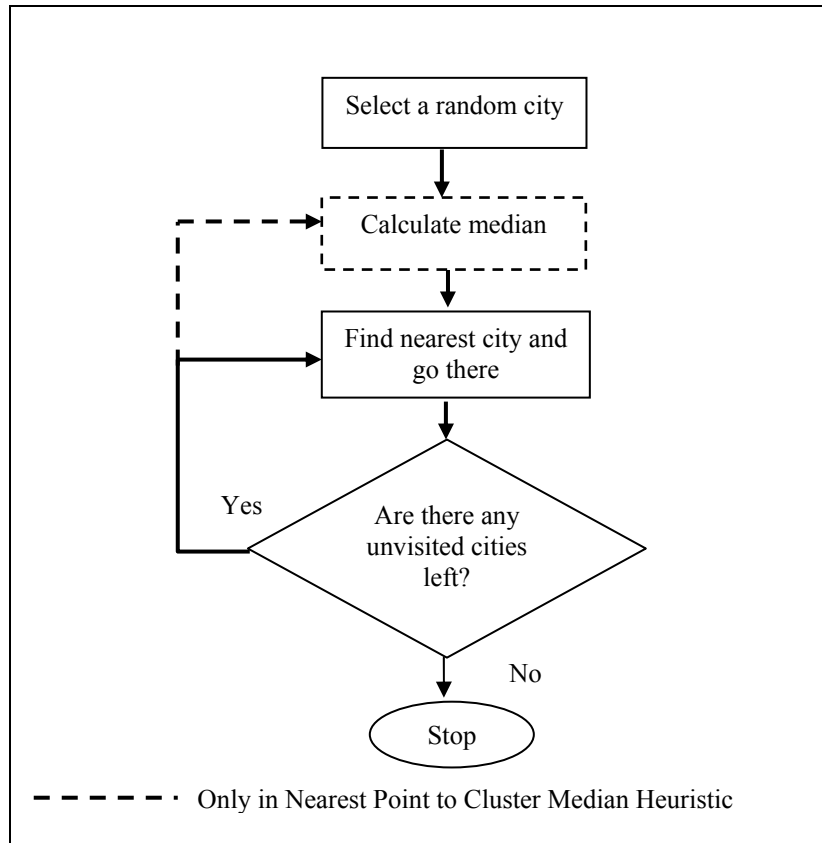


Figure 4.10 Nearest Neighbor Heuristic

4.3. Myopic and Look-Back Algorithm for Assignment

Assignment part of the problem is solved by using myopic approach. We have n clusters and k potential warehouses. Our objective is to determine the number and position of warehouses and assign n cluster to these warehouses with least cost. There are $\binom{n}{k}$ possible solutions. If there is only one facility, it is easy to find optimal solution by enumerating all solutions. However as the number of warehouses and clusters increase the problem becomes complicated. First of all we locate single facility optimally by enumerating all solutions. Secondly, we assume that previous solution is

given and test each possible location. Optimal solution is found for given condition and this procedure is carried out until total cost increases. Since clusters include many points, nearest points to warehouse is considered as stem distance. Myopic algorithm is applied when there is only a fixed cost for opening a warehouse. Figure 4.11 illustrates how algorithm works.

Stem distance denotes the minimum distance from cluster to potential warehouse.

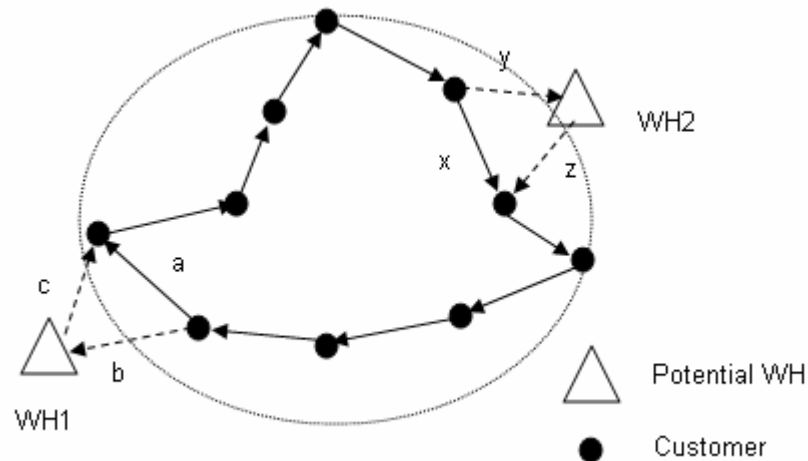


Figure 4.11 Stem Distance

Figure 4.11 show stem distance of a cluster to two potential WH. Stem distance form cluster to WH1 is equal to $b+c-a$. (Stem distance to WH2= $y+z-x$) Stem distance for a cluster may exist as much as number of potential warehouses.

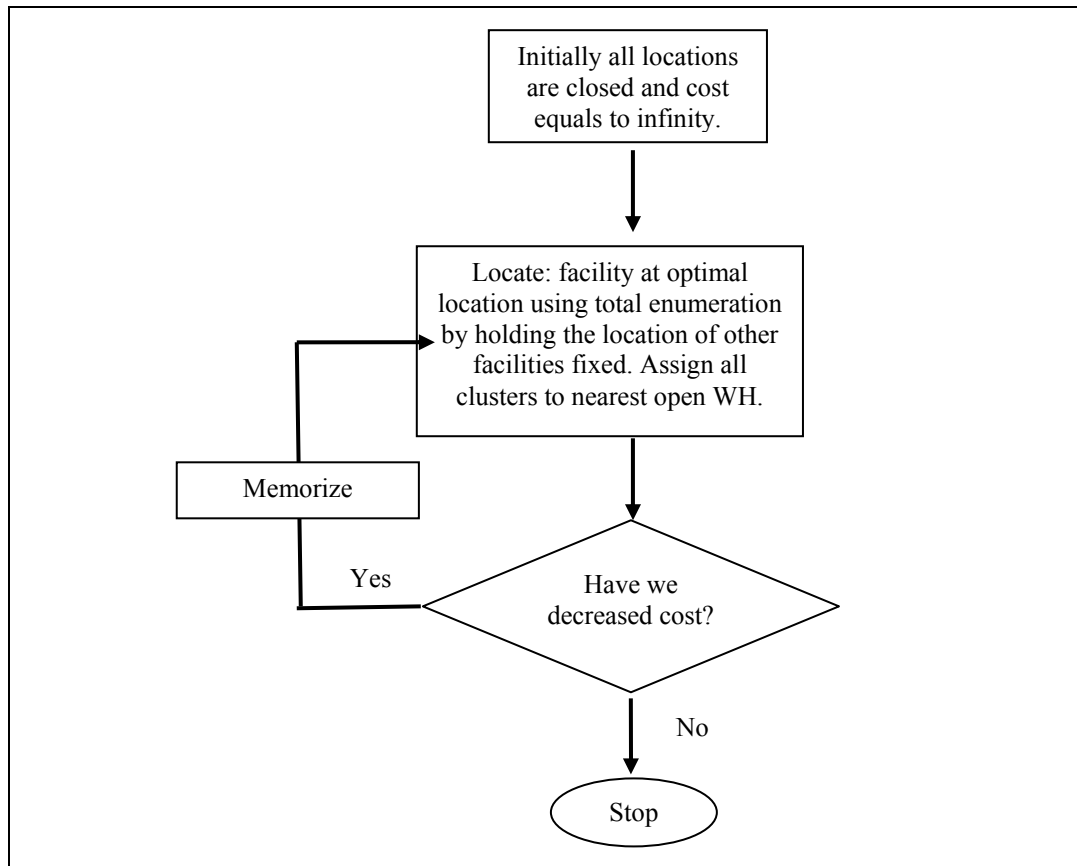


Figure 4.12 Myopic Heuristic

When facility costs are non-linear, the problem becomes more complicated. We adopt a method that uses a myopic approach. In this method, it is important to determine which cluster is assigned to which warehouse because of the non-linear warehouse opening cost. Hence we define cluster priority showing the assignment rank. This rank is simply related with total demand of cluster. Initially, we open nearest warehouse to the first cluster. Later, we calculate the cost of assigning cluster to nearest and open warehouse. The solution that gives the minimum cost is selected. If a new warehouse is opened in the new solution, the previous node is inserted to new warehouse and total cost is calculated. If total cost decreases, solution is accepted and previous node is inserted to new warehouse. Solution is carried out with same methodology until all clusters are assigned to warehouses. The work flow of the algorithm is given in Figure 4.12.

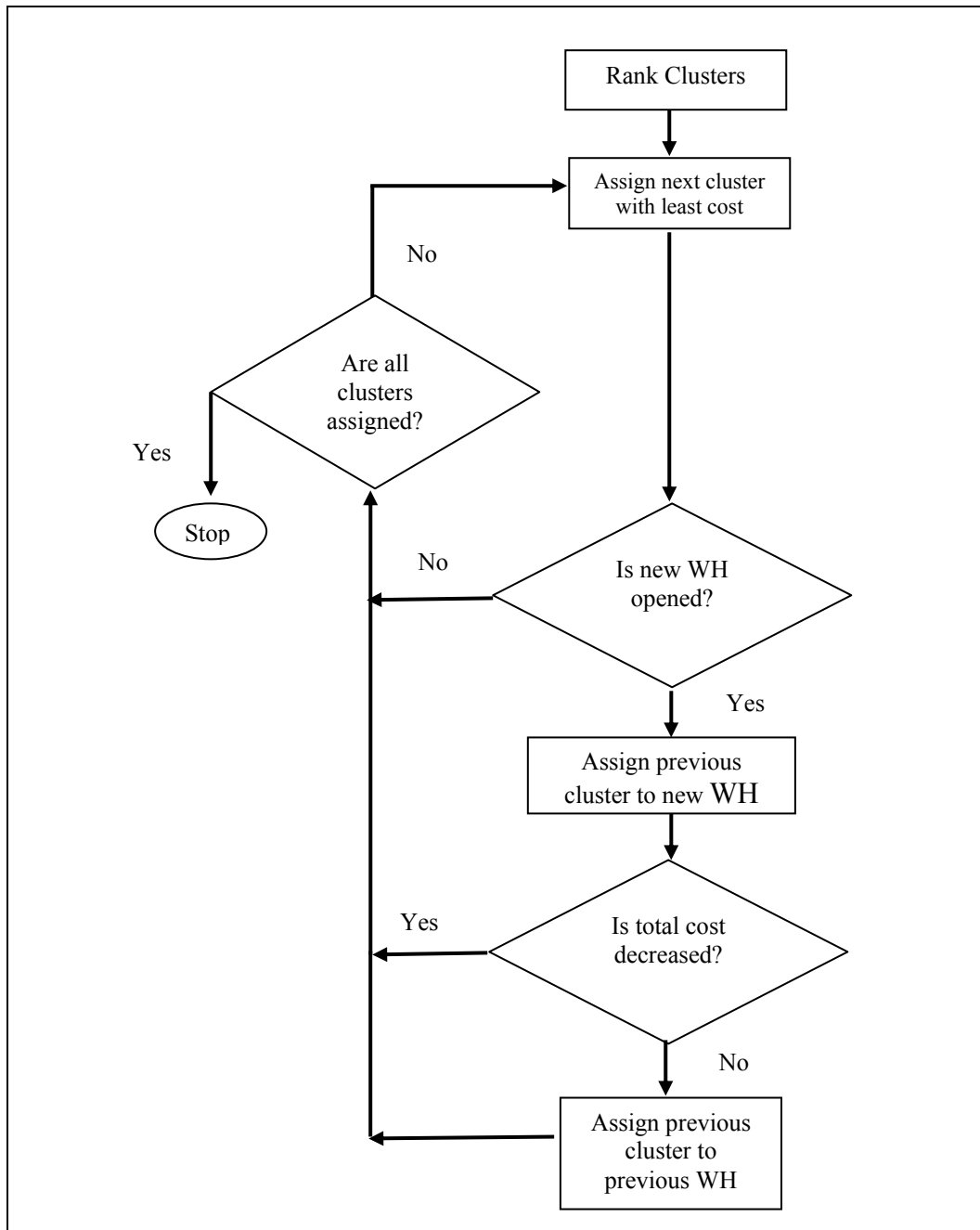


Figure 4.13 Look-Back Heuristic

4.4. Local Search

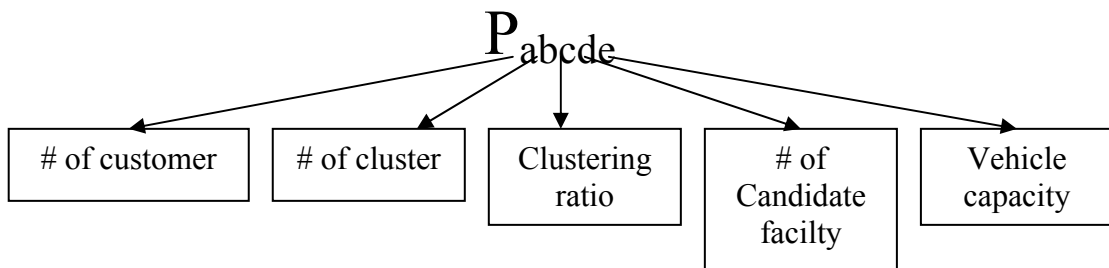
Swap and move searches are applied both inside and between clusters. Before using these methods between clusters, capacity check is made. Swap algorithm basically removes two edges from tour, and reconnects the two paths created. Move algorithm deletes an edge from a route and inserts it into a new route.

CHAPTER 5

COMPUTATIONAL STUDY

In this chapter, different clustering methods, TSP solution procedures and improvement heuristics are investigated. Additionally, we compare our solutions with the results of Tuzun and Burke (1999). Finally, we find the results with non-linear cost function.

We use the data generated by Tuzun and Burke (1999). They present two-phase tabu search architecture consisting location and routing phase. Characteristic of the problem instance may affect the performance of heuristic. A wide variety of problems are solved by clustering based heuristic in order to determine performance of the heuristic. The size of an LRP instance may affect both the solution quality and the computational requirements of the heuristic. Number of customers is set at 100, 150 and 200, and number of candidate facilities is set at 10 and 20. Customer demand is generated uniformly in the range [10, 20]. Vehicle capacity is set to 150 and 300. There are 10 and 20 customers on a route since the average demand is 15 for all problems. Number of clusters (cl), and clustering ratio (cl_ratio) is important for making point distribution. Number of clusters show the number of areas where the customer density is high. It is set at 3 levels: 0, 3, and 5, where level 0 refers to uniformly distributed customers. cl_ratio shows the ratio of number of customers that belong to a cluster to the total number of customers. This factor is at 2 levels: 75% and 100%.

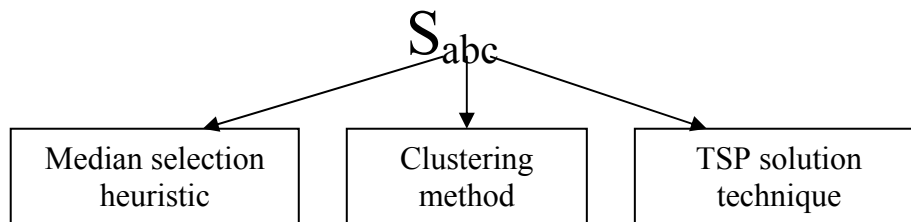


The algorithm is implemented in Visual C++ on an Intel Celeron 1.5 GHz processor with 512 MB RAM.

Calculations are made in order to determine median selection method, clustering heuristic and TSP solution technique. There exist two initial point selection procedures: max-min distance and max-total distance, three clustering method: nearest point to cluster, nearest neighbor, gravity force and three TSP solution technique: nearest point to cluster, nearest neighbor, branch-and-bound. After solving TSP in each cluster, improvements are applied in given order if branch-and-bound is not applied.

- 1) SWAP between clusters
- 2) MOVE between clusters
- 3) SWAP inside the cluster
- 4) MOVE inside the cluster

As branch-and-bound is applied for solving TSP, 3rd and 4th steps are removed and branch-and-bound is applied in each cluster instead.



S_{abc} represents the average results of solution techniques a-b-c. Detailed results are reported in the Appendix.

a=1: Max-min distance, a=2: Max-total distance

b=1: Nearest point to median, b=2: Nearest neighbor, b=3: Gravity force

c=1: Nearest point to median, c=2: Nearest neighbor, c=3: Branch-and-bound

Deviation of Results	Median Selection Methods	
	Max-Min Distance	Max-Total Distance
	14.8%	19.8%

Table 5.1 Max-Min Distance Heuristic with Improvement

Table 5.1 shows that max-min distance heuristic gives lower cost than max-total distance heuristic does on the average. All solutions are divided into two parts: max-min and max-total distance. From geometrical point of view, max-min distance method selects widely dispersed points. On the contrary, max-total distance heuristic selects close points as median. Hence we can conclude that selecting dispersed medians is important for building clusters and affects results 5 % on the average. We will use max-min distance heuristic for the remaining part of the thesis.

	S₁₁₁	S₁₁₂	S₁₁₃	Average
Nearest Point to Median	19.3%	20.1%	21.9%	20.4%
	S₁₂₁	S₁₂₂	S₁₂₃	Average
Nearest Neighbor	13.1%	13.5%	11.7%	12.8%
	S₁₃₁	S₁₃₂	S₁₃₃	Average
Gravity Force	12.9%	11.9%	9.0%	11.3%

Table 5.2 Clustering Heuristics

Table 5.2 shows the results of clustering methods with different TSP solution techniques. Gravity Force clustering method gives better results than Nearest Point to Median and Nearest Neighbor do for all TSP solution methods. Despite S₁₂₁ and S₁₃₁ are very close, discrepancy increases in other TSP solutions.

	S111	S121	S131	Average
Nearest Point to Median	19.3%	13.1%	12.9%	15.1%
	S112	S122	S132	Average
Nearest Neighbor	20.1%	13.5%	11.9%	15.2%
	S113	S123	S133	Average
Branch-and-bound	21.9%	11.7%	9.0%	14.2%

Table 5.3 TSP solution methods

Table 5.3 denotes effects of TSP solution techniques. Despite branch-and-bound finds better results on the average, it gives the worst result for the first clustering method. On the contrary, the minimum number in the table is found by applying branch-and-bound method.

As a result, max-min total distance heuristic gives best results for median selection, gravity force method with branch-and-bound finds minimum cost for building cluster and solving TSP in each of them. Hence we will apply only these three methods for the non-linear part.

In the non-linear cost function, there exist fixed opening and capacity increment costs. If any demand less than 200 is assigned to warehouse, only fixed cost of 100 is incurred. As demand exceeds 200, cost increases. The following non-linear cost function is used for the incremental capacity cost.

$$f(x) = \begin{cases} 100 + \left(\left\lceil \frac{\text{Total_Demand}}{200} \right\rceil - 1 \right) * 50, & \text{Total_Demand} > 0 \\ 0, & \text{Total_Demand} = 0 \end{cases}$$

Non-Linear Cost				Non-Linear Cost			
Problem	Cost	Time(seconds)	Open WH	Problem	Cost	Time(seconds)	Open WH
P11111.dat	2518.33	20.10	1,7,10	P21111.dat	4466.14	78.12	1,3,4,5,7
P11112.dat	2081.45	19.53	8,1	P21112.dat	3424.66	79.32	3,5,7,8
P11121.dat	2395.97	16.83	1,9,15	P21121.dat	4592.29	83.22	1,6,12,14
P11122.dat	1984.68	17.73	10,16,17	P21122.dat	3508.65	82.92	1,13,14,20
P11211.dat	2708.04	16.83	1,4,9,10	P21211.dat	4342.93	75.99	3,5,6,8
P11212.dat	1858.45	21.93	3,9	P21212.dat	3322.69	84.72	8,1
P11221.dat	2748.35	18.63	9,12,16	P21221.dat	4496.95	89.25	6,7,11,12,15
P11222.dat	2122.13	19.83	3,8,10	P21222.dat	3443.82	75.99	8,14,16,17
P12111.dat	2611.79	16.53	3,4	P22111.dat	5791.17	77.22	8,1
P12112.dat	1957.68	23.73	8,1	P22112.dat	3507.32	90.72	4,8
P12121.dat	2391.76	16.80	7,8	P22121.dat	3829.05	72.72	2,8,18
P12122.dat	1622.7	7.53	11,19	P22122.dat	3464.91	448.23	12,15,17
P12211.dat	1963.37	25.83	5,1	P22211.dat	3769.6	87.72	1,4,5
P12212.dat	1208.37	19.53	4,8	P22212.dat	2370.91	84.42	8,9
P12221.dat	1504.76	19.23	17,18	P22221.dat	3477.22	82.62	1,10,19
P12222.dat	1341.23	20.13	4,13	P22222.dat	2521.13	111.78	1,8,20
P13111.dat	2722.63	16.53	1,6,9	P23111.dat	4765.55	82.62	1,3,6
P13112.dat	1920.75	24.93	6,8,10	P23112.dat	3528.1	96.72	2,3,7,10
P13121.dat	2304.83	18.93	6,15,18	P23121.dat	5074.16	121.68	12,16,17
P13122.dat	1843.08	19.83	5,8,12,16	P23122.dat	3371.79	308.25	9,12,16,17,18
P13211.dat	2276.62	21.03	6,8	P23211.dat	3696.63	179.97	7,9
P13212.dat	1493.77	15.93	6,1	P23212.dat	2884.64	107.25	1,9
P13221.dat	1708.53	15.30	7,12,14,17	P23221.dat	3743.56	118.35	1,6,11,20
P13222.dat	1523.29	16.83	8,12,13	P23222.dat	2424.82	116.58	3,4,13,19,20

Table 5.4 Non-Linear Cost Results

In the non-linear part, medians are selected by applying max-min distance heuristic and clusters are formed by gravity force heuristics. In each cluster TSP is solved with branch-and-bound algorithm. At last assignment of cluster is made by using look-back heuristic with stem distance. Table 5.4 shows the results of non-linear cost function.

CHAPTER 6

CONCLUSION AND FUTURE WORK

In this thesis, we develop a hierarchical clustering-based heuristic for the LRP. We use the clustering approach to reduce the complexity of the problem in an attempt to obtain good feasible solutions fast. In the case of a non-linear cost function, LRP becomes more complicated and finding optimal, or even good, solution becomes extremely difficult. The developed heuristics aim at solving LRP with non-linear cost function efficiently with reasonable computational time. The hierarchical approach consists of three parts: clustering of the customers, building TSP routes for each cluster, and assigning routes to warehouses. The efficiency of the proposed heuristic is tested using the well-known instances in the literature. We conclude that the clustering based heuristic provides feasible solutions for complex problems with little computational effort. However, the solution quality for the linear cost case is inferior compared to the benchmark results.

In the clustering part, we tested two initial point selection techniques and three different clustering techniques (Section 4.1.3). The Max-Min distance approach with gravity force method gives best results on the average for building clusters. In each cluster, TSP is solved with three methods: nearest neighbor, nearest point to cluster median, branch-and-bound. The branch-and-bound method is adopted since the TSP solved for each cluster is rather small. Finally, the myopic and look-back heuristics are applied in the assignment part (Section 4.3).

The proposed method is a simple, common sense procedure which is based on clustering method. As a future research direction, a more extensive study may be conducted to develop a more sophisticated heuristic to improve solution quality. One of the most crucial part affecting results is clustering. New clustering methods may be applied and gravity force heuristic, which has been utilized for the first time in this context, may be improved. Furthermore, results may be improved by applying a k -opt local search procedure. In the assignment phase, intuitive heuristics such as myopic and

look-back heuristics are used; this can be improved for the non-linear cost structure specifically.

Finally, while building clusters, only the vehicle capacity is considered. The method may be easily adopted in the existence of vehicle distance constraints.

APPENDIX

S111				
Problem	Cost	Time(seconds)	Discrepancy	Open WH
P11111.dat	2268.06	8.51	9.90%	1,3,9,10
P11112.dat	1720.57	9.11	10.00%	4,8,10
P11121.dat	2253.68	8.61	10.80%	1,4,7,9,16
P11122.dat	1816.81	10.22	17.10%	14,17
P11211.dat	2346.37	99.44	7.30%	2,4,6,9
P11212.dat	1683.97	91.33	15.40%	2,8
P11221.dat	2398.17	85.12	15.90%	1,5,9,16
P11222.dat	1636.25	84.43	9.00%	10,12
P12111.dat	2322.29	105.15	14.40%	3,4,8
P12112.dat	1778.92	87.12	30.70%	6,7
P12121.dat	2199.25	84.92	26.40%	5,9,11
P12122.dat	1429.25	67.5	19.90%	5,18
P12211.dat	2199.71	77.71	35.30%	5,1
P12212.dat	9301.25	78.52	13.50%	4,8
P12221.dat	1426.16	139.3	13.90%	17,18
P12222.dat	14599	80.21	49.20%	4
P13111.dat	2786.47	60.99	23.90%	2,6,9
P13112.dat	1440.34	86.52	9.50%	8,9,10
P13121.dat	1969.58	72.81	14.10%	3,15,19,20
P13122.dat	1432.97	87.92	10.60%	6,11,13,16
P13211.dat	1799.04	75.11	17.00%	3,6,8,9
P13212.dat	14671.6	112.46	37.00%	1,4,6
P13221.dat	1653.98	73.21	30.80%	2,7,14,17
P13222.dat	1299.7	68.8	21.00%	11,13,15
S112				
Problem	Cost	Time(seconds)	Discrepancy	Open WH
P11111.dat	2282.75	7.91	10.50%	1,3,6,7
P11112.dat	1895.38	8.61	18.30%	4,8,10
P11121.dat	2255.93	9.41	10.90%	1,4,7,9,16
P11122.dat	1902.84	15.93	20.80%	3,17
P11211.dat	2365.88	91.43	8.10%	2,4,9,10
P11212.dat	1739.18	87.92	18.10%	2,8
P11221.dat	2386.99	89.93	15.50%	1,5,9,16
P11222.dat	1720.64	86.23	13.40%	10,12
P12111.dat	2295.18	110.56	13.40%	3,4,8
P12112.dat	1781.25	90.83	30.80%	6,7
P12121.dat	2204.19	75.5	26.60%	5,7,9,11

P12122.dat	1463.05	89.63	21.70%	11,19
P12211.dat	2199.85	89.53	35.30%	5,1
P12212.dat	9.218	84.12	12.70%	4,8
P12221.dat	1415.66	96.74	13.30%	17,18
P12222.dat	14444.9	83.32	48.60%	4
P13111.dat	2791.23	62.79	24.00%	2,6,9
P13112.dat	1482.43	81.52	12.10%	8,9,10
P13121.dat	1966.37	71.2	13.90%	3,15,19,20
P13122.dat	1451.23	76.11	11.70%	6,11,13,16
P13211.dat	1783.94	93.54	16.30%	3,6,8,9
P13212.dat	14321.1	73.4	35.50%	1,4,6
P13221.dat	1649.27	69.9	30.60%	2,14,17
P13222.dat	1276.72	91.83	19.60%	11,13,15
S113				
Problem	Cost	Time(seconds)	Discrepancy	Open WH
P11111.dat	2273.18	5.7	10.10%	1,3,9,10
P11112.dat	1735.3	5.71	10.80%	4,8,10
P11121.dat	2223.24	16.52	9.60%	1,4,7,9
P11122.dat	1695.43	22.43	11.10%	14,17
P11211.dat	2369.93	28.94	8.20%	2,4,9,10
P11212.dat	1658.56	33.84	14.10%	2,8
P11221.dat	2375.37	41.05	15.10%	1,5,9,16
P11222.dat	1635.42	45.66	8.90%	10,12
P12111.dat	2334.96	52.17	14.90%	3,4,8
P12112.dat	1728.43	57.18	28.70%	6,7
P12121.dat	2210.38	61.68	26.80%	5,9,11
P12122.dat	1351.97	67.59	15.30%	5,18
P12211.dat	2147.15	71	33.70%	5,1
P12212.dat	913.467	78.21	99.10%	4,8
P12221.dat	1415.66	82.61	13.30%	15,18,19
P12222.dat	14080.7	89.42	47.30%	4
P13111.dat	2771.22	94.53	23.40%	2,6,9
P13112.dat	1420.65	99.34	8.30%	8,9,10
P13121.dat	1975.13	104.85	14.30%	3,15,19,20
P13122.dat	1409.47	110.25	9.10%	6,11,13,16
P13211.dat	1826.04	117.16	18.30%	3,6,8,9
P13212.dat	14005.5	122.27	34.00%	1,4,6
P13221.dat	1642.14	127.68	30.30%	2,14,17
P13222.dat	1289.4	133.29	20.40%	11,13,15
S121				
Problem	Cost	Time(seconds)	Discrepancy	Open WH
P11111.dat	2155.34	10.11	5.20%	1,3,6,7
P11112.dat	1734.39	9.11	10.70%	4,8

P11121.dat	2159.57	11.72	6.90%	1,2,4,7,18
P11122.dat	1785.32	10.21	15.60%	11,14,17
P11211.dat	2432	96.04	10.60%	2,4,9,10
P11212.dat	1647.98	92.34	13.50%	1,2
P11221.dat	2289.27	85.42	11.90%	1,5,9,13,20
P11222.dat	1704.39	93.13	12.60%	10,12
P12111.dat	2074.97	87.93	4.20%	4,8,9
P12112.dat	1585.58	92.73	22.30%	2,6,7
P12121.dat	1830.93	94.74	11.60%	5,9,11,16
P12122.dat	1434.58	105.75	20.20%	5,18
P12211.dat	1634.45	70.5	12.90%	5,1
P12212.dat	9.528	65.9	15.50%	4,8
P12221.dat	1279.47	80.41	4.00%	1,17,18
P12222.dat	9.557	102.65	22.40%	4,13
P13111.dat	2397.08	71.1	11.50%	2,6,9
P13112.dat	1491.32	108.16	12.60%	2,6,10
P13121.dat	1786.4	115.26	5.20%	3,15,18,19,20
P13122.dat	1454.42	77.91	11.90%	6,13,16
P13211.dat	1706.59	76.72	12.50%	3,6,8,9
P13212.dat	11845	73.3	22.00%	1,4,6
P13221.dat	1499.4	132.29	23.70%	2,12,14,17
P13222.dat	1221.88	97.84	16.00%	4,13,15
S122				
Problem	Cost	Time(seconds)	Discrepancy	Open WH
P11111.dat	2159.19	9.8	5.30%	1,3,6,7
P11112.dat	1758.7	9.22	12.00%	3,4,8
P11121.dat	2147.75	12.12	6.40%	1,2,4,7,18
P11122.dat	1842.47	11.62	18.20%	11,14,17
P11211.dat	2442.61	89.83	10.90%	2,4,6,9
P11212.dat	1633.21	92.83	12.70%	2,8,9
P11221.dat	2291.62	91.13	12.00%	1,5,9,13,20
P11222.dat	1729.76	111.86	13.90%	10,12
P12111.dat	2095.03	96.44	5.10%	4,8,9
P12112.dat	1551.8	83.92	20.60%	2,6,7
P12121.dat	1835.57	81.42	11.80%	5,9,11,16
P12122.dat	1440.34	60.99	20.50%	5,18
P12211.dat	1630.61	75.1	12.70%	5,1
P12212.dat	9.389	70.6	14.30%	4,8
P12221.dat	1281.53	108.16	4.20%	1,17,18
P12222.dat	9.611	67.3	22.80%	4,13
P13111.dat	2405.67	69.1	11.80%	2,6,9
P13112.dat	1483.75	82.72	12.20%	8,9,10
P13121.dat	1801.4	60.08	6.00%	3,15,18,19,20

P13122.dat	1452.05	61.29	11.80%	6,13,16
P13211.dat	1731.02	67.4	13.80%	3,6,8,9
P13212.dat	12080.1	121.07	23.50%	1,4,6
P13221.dat	1486.95	62.49	23.00%	2,12,14,17
P13222.dat	1245.64	72.3	17.60%	10,15
S123				
Problem	Cost	Time(seconds)	Discrepancy	Open WH
P11111.dat	2159.06	98	5.30%	1,3,6,7
P11112.dat	1677.86	29.1	7.70%	3,4,8
P11121.dat	2136.63	90.2	5.90%	1,4,7,9,18
P11122.dat	1708.29	56	11.80%	11,14,17
P11211.dat	2410.81	91.2	9.80%	2,4,6,9
P11212.dat	1596.58	62.1	10.70%	1,2
P11221.dat	2308.49	67.1	12.60%	1,5,9,13,20
P11222.dat	1685.54	68.1	11.60%	3,7
P12111.dat	2073.91	59	4.20%	4,8,9
P12112.dat	1491.31	54.1	17.40%	2,6,7
P12121.dat	1822.31	45.1	11.20%	5,9,11,16
P12122.dat	1324.43	71.1	13.50%	5,18
P12211.dat	1599.48	70.1	11.00%	5,1
P12212.dat	9.1	55.1	11.60%	4,8
P12221.dat	1279.91	79.1	4.10%	1,17,18
P12222.dat	9.587	68.1	22.60%	4,13
P13111.dat	2382.2	60.1	10.90%	2,6,9
P13112.dat	1450.53	64.1	10.10%	8,9,10
P13121.dat	1801.04	67.1	6.00%	3,15,18,19,20
P13122.dat	1420.41	57	9.80%	6,13,16
P13211.dat	1741.23	76.1	14.30%	3,6,8,9
P13212.dat	11657	79.2	20.70%	1,4,6
P13221.dat	1485.27	62	22.90%	2,12,14,17
P13222.dat	1206.54	70.1	14.90%	10,15
S141				
Problem	Cost	Time(seconds)	Discrepancy	Open WH
P11111.dat	2125.7	7.8	3.90%	1,3,6,7
P11112.dat	1735.07	10.32	10.80%	4,8,10
P11121.dat	2092.22	7.31	3.90%	1,2,4,6
P11122.dat	1641.33	9.11	8.20%	3,17
P11211.dat	2311.32	57.89	5.90%	2,4,6,9
P11212.dat	1651.85	88.63	13.70%	2,8,9
P11221.dat	2182.25	88.12	7.50%	1,5,9,13,20
P11222.dat	1737.54	85.73	14.30%	10,12
P12111.dat	2127.07	88.22	6.60%	4,8
P12112.dat	1648.95	96.54	25.30%	2,7,10

P12121.dat	1844.63	76.21	12.20%	5,9,11,16
P12122.dat	1435.5	71	20.20%	5,18
P12211.dat	1632.87	109.16	12.90%	5,1
P12212.dat	9.321	139.7	13.60%	4,8
P12221.dat	1216.27	82.62	-1.00%	17,18
P12222.dat	12099	73.3	38.70%	4,13
P13111.dat	2451.04	54.58	13.40%	1,2,6,9
P13112.dat	1496.2	74.31	12.90%	8,1
P13121.dat	1735.67	58.08	2.50%	3,15,18,19,20
P13122.dat	1462.22	68.5	12.40%	4,6,13
P13211.dat	1666.04	66.5	10.40%	3,6,8,9
P13212.dat	13113	94.33	29.50%	1,4,6
P13221.dat	1374.11	80.02	16.70%	2,14,17
P13222.dat	1205.34	139.3	14.80%	4,13,15
S142				
Problem	Cost	Time(seconds)	Discrepancy	Open WH
P11111.dat	2128.04	9.1	4.00%	1,3,6,7
P11112.dat	1766.23	38.66	12.40%	4,8,10
P11121.dat	2087.86	77.62	3.70%	1,2,4,6
P11122.dat	1655.56	95.03	9.00%	3,17
P11211.dat	2255.83	69.1	3.60%	2,4,6,9
P11212.dat	1620.85	71.2	12.10%	2,8,9
P11221.dat	2184.69	79.72	7.60%	1,5,9,13,20
P11222.dat	1719.48	128.48	13.40%	10,12
P12111.dat	2137.7	69.8	7.00%	4,8
P12112.dat	1462.91	85.63	15.80%	2,6,7
P12121.dat	2023.44	74.7	20.00%	5,9,11,16
P12122.dat	1391.03	66.1	17.70%	5,18
P12211.dat	1596.08	90.33	10.90%	5,1
P12212.dat	9.682	82.22	16.90%	4,8
P12221.dat	1214.29	126.48	-1.10%	17,18
P12222.dat	10221.6	164.14	27.40%	4,13
P13111.dat	2443.6	115.56	13.20%	1,2,6,9
P13112.dat	1577.15	86.23	17.40%	9,1
P13121.dat	1739.07	97.54	2.70%	3,15,18,19,20
P13122.dat	1488.91	180.86	14.00%	4,6,13
P13211.dat	1652.54	112.06	9.70%	3,6,8,9
P13212.dat	11714.7	136.49	21.10%	1,4,6
P13221.dat	1309.86	140.61	12.60%	14,16,17
P13222.dat	1222.3	146.41	16.00%	4,13,15
S143				
Problem	Cost	Time(seconds)	Discrepancy	Open WH
P11111.dat	2114.11	58	3.30%	1,3,6,7

P11112.dat	1684.44	79.1	8.10%	3,4,8
P11121.dat	2136.37	41.1	5.90%	1,2,4,10,11
P11122.dat	1594.76	56.1	5.50%	3,17
P11211.dat	2303.77	46.1	5.60%	2,4,6,9
P11212.dat	1560.03	75.1	8.70%	1,2
P11221.dat	2202.15	51	8.40%	1,5,9,13,20
P11222.dat	1610.31	63.1	7.50%	7,19
P12111.dat	2112.66	49.1	5.90%	3,4,8
P12112.dat	1422.48	74.1	13.40%	6,7
P12121.dat	1828.27	39.1	11.50%	5,9,11,16
P12122.dat	1239.28	59	7.60%	5,18
P12211.dat	1611.05	50.1	11.70%	5,1
P12212.dat	9.09	63.1	11.50%	4,8
P12221.dat	1209.32	49.1	-1.50%	17,18
P12222.dat	8196.3	54.1	9.50%	4,13
P13111.dat	2378.64	48	10.80%	1,2,6,9
P13112.dat	1512.84	76.1	13.80%	7,9
P13121.dat	1755.05	55.1	3.60%	3,15,19,20
P13122.dat	1434.2	65.1	10.70%	4,6,13
P13211.dat	1646.82	48.1	9.40%	3,6,8,9
P13212.dat	10835.8	62.1	14.70%	1,4,6
P13221.dat	1371.25	41	16.50%	2,14,17
P13222.dat	1186.87	60.1	13.50%	4,13,15
S211				
Problem	Cost	Time(seconds)	Discrepancy	Open WH
P11111.dat	2506.38	5.8	18.50%	1,3,6,7
P11112.dat	1763.56	10.82	12.20%	4,8
P11121.dat	2656.14	8.61	24.30%	1,2,4,10
P11122.dat	1774.65	9.72	15.10%	10,17
P11211.dat	2788.5	110.85	22.00%	2,4,8,9
P11212.dat	1766.49	89.53	19.30%	2,8,9
P11221.dat	2609.29	95.24	22.70%	1,5,8,13
P11222.dat	1799.21	83.12	17.20%	10,12
P12111.dat	2497.39	96.14	20.40%	4,8
P12112.dat	1606.47	88.22	23.30%	6,8
P12121.dat	2103.65	81.22	23.00%	5,7,9,11
P12122.dat	1390.29	67.4	17.60%	5,18
P12211.dat	1945.77	72	26.90%	5,1
P12212.dat	12091.8	71.1	33.40%	4
P12221.dat	1509.9	111.06	18.70%	15,18,19
P12222.dat	11761.8	69.1	36.90%	4,13
P13111.dat	2795.02	55.98	24.10%	1,2,6,9
P13112.dat	1597.38	67.7	18.40%	2,1

P13121.dat	2265.45	82.72	25.30%	3,14,15,20
P13122.dat	1573.43	121.88	18.60%	4,6
P13211.dat	2000.4	87.42	25.40%	6,8,9
P13212.dat	16203.8	99.94	43.00%	1,4,6
P13221.dat	1845.8	76.92	38.00%	2,14,17
P13222.dat	1441.05	98.64	28.80%	4,7,15
S212				
Problem	Cost	Time(seconds)	Discrepancy	Open WH
P11111.dat	2552.44	8.1	19.90%	1,3,6,7
P11112.dat	1764.16	9.12	12.30%	4,8,10
P11121.dat	2603.87	9.01	22.80%	1,2,4,11
P11122.dat	1858.94	9.52	18.90%	17
P11211.dat	2778.7	124.38	21.70%	2,4,8,9
P11212.dat	1788.12	86.52	20.30%	2,8,9
P11221.dat	2653.84	89.33	24.00%	1,5,8,13
P11222.dat	1818.99	86.42	18.10%	4,12
P12111.dat	2508.33	91.93	20.80%	4,8
P12112.dat	1736.18	108.66	29.00%	6,8
P12121.dat	2095.86	93.13	22.80%	5,7,9,11
P12122.dat	1465.21	61.29	21.80%	5,11
P12211.dat	1946	84.82	26.90%	5,1
P12212.dat	12154.6	109.76	33.80%	4
P12221.dat	1500.15	98.74	18.10%	17,18
P12222.dat	11695.9	75.61	36.60%	4,13
P13111.dat	2750.97	67	22.90%	1,2,6,9
P13112.dat	1682.17	69.7	22.50%	2,1
P13121.dat	2266.71	76.51	25.30%	3,14,15,20
P13122.dat	1569.76	107.75	18.40%	4,6
P13211.dat	1995.35	108.86	25.20%	6,8,9
P13212.dat	17490.6	83.22	47.20%	1,4,6
P13221.dat	1790.48	55.48	36.10%	2,12,14,17
P13222.dat	1444.76	62.79	28.90%	4,7,15
S213				
Problem	Cost	Time(seconds)	Discrepancy	Open WH
P11111.dat	2544.61	57	19.70%	1,3,6,7
P11112.dat	1693.33	55.1	8.60%	4,8
P11121.dat	2680.69	69.1	25.00%	1,4,10,12
P11122.dat	1720.37	51.1	12.40%	2,17
P11211.dat	2771.48	79.1	21.50%	2,4,8,9
P11212.dat	1746.01	49.1	18.40%	2,8,9
P11221.dat	2574.53	50	21.60%	1,5,9,13
P11222.dat	1764.39	55.1	15.60%	4,12
P12111.dat	2499.74	56.1	20.50%	4,8

P12112.dat	1622.37	57.1	24.00%	6,8
P12121.dat	2172.06	45	25.50%	5,7,9,11
P12122.dat	1377.92	61.1	16.90%	5,18
P12211.dat	1946.54	39.6	26.90%	5,1
P12212.dat	11841	74.6	32.00%	4
P12221.dat	1499.16	62.1	18.10%	17,18
P12222.dat	11501.1	53.1	35.50%	4,13
P13111.dat	2691.27	38	21.20%	1,2,6,9
P13112.dat	1618.78	63.1	19.50%	2,1
P13121.dat	2284.1	44.1	25.90%	3,15,19,20
P13122.dat	1559.13	58.1	17.80%	4,6
P13211.dat	2038.72	51	26.80%	6,8,9
P13212.dat	17341.2	65.1	46.70%	1,4,6
P13221.dat	1823.06	46.1	37.20%	2,12,14,17
P13222.dat	1483.52	69.1	30.80%	7,10,15
S221				
Problem	Cost	Time(seconds)	Discrepancy	Open WH
P11111.dat	2312.07	7.8	11.60%	1,3,6,7
P11112.dat	1659.99	8.92	6.70%	4,8,10
P11121.dat	2366.23	11.82	15.00%	1,2,4,7
P11122.dat	1716.22	10.81	12.20%	3,17
P11211.dat	2457.18	97.04	11.50%	2,4,8,9
P11212.dat	1637.61	97.84	13.00%	2,8
P11221.dat	2242.87	92.84	10.00%	1,5,9,16,20
P11222.dat	1661.37	99.44	10.30%	3,7
P12112.dat	1424.73	7.81	13.50%	6,9
P12121.dat	1903.36	8.51	14.90%	5,7,9,11
P12122.dat	1331.87	8.61	14.00%	5,18
P12211.dat	1723.15	6.6	17.40%	5,1
P12212.dat	9.163	8.31	12.20%	4,8
P12221.dat	14738.8	8.81	91.70%	17,18
P12222.dat	8994.3	9.11	17.50%	4,13
P13111.dat	2341.93	7.81	9.40%	2,6,9
P13112.dat	1447.43	18.2	10.00%	1,9,10
P13121.dat	1973.86	10.12	14.20%	3,15,18,19,20
P13122.dat	1364.35	20.12	6.10%	4,13
P13211.dat	1799.28	94.24	17.00%	3,6,8,9
P13212.dat	12710.4	86.42	27.30%	1,4,6
P13221.dat	1483.83	86.23	22.90%	2,12,14,17
P13222.dat	1342.05	98.34	23.50%	11,13,15
S222				
Problem	Cost	Time(seconds)	Discrepancy	Open WH
P11111.dat	2300.89	9.2	11.20%	1,3,6,7

P11112.dat	1801.7	8.92	14.10%	4,8,10
P11121.dat	2339.27	11.42	14.10%	1,4,7,9
P11122.dat	1796.48	10.11	16.10%	3,17
P11211.dat	2470.59	104.05	11.90%	2,4,8,9
P11212.dat	1750.77	99.95	18.60%	1,2
P11221.dat	2301.46	90.43	12.30%	1,5,9,16
P11222.dat	1766.68	98.84	15.70%	10,12
P12112.dat	1480.18	10.31	16.70%	2,6,7
P12121.dat	1921.11	8.01	15.70%	5,7,9,11
P12122.dat	1320.6	8.61	13.30%	5,18
P12211.dat	1739.07	6.9	18.20%	5,1
P12212.dat	9.228	8.31	12.80%	4,8
P12221.dat	1466.23	8.61	16.30%	17,18
P12222.dat	8.922	9.01	16.90%	4,13
P13111.dat	2328.15	8.11	8.90%	2,6,9
P13112.dat	1459.67	21.6	10.70%	1,9,10
P13121.dat	1976.68	10.44	14.40%	3,15,18,19,20
P13122.dat	1424.83	52.48	10.10%	4,6,20
P13211.dat	1807.78	93.93	17.40%	3,6,8,9
P13212.dat	13307.4	91.63	30.50%	1,4,6
P13221.dat	1494.1	75.31	23.40%	2,12,14,17
P13222.dat	1444.8	77.61	29.00%	8,11,13
S223				
Problem	Cost	Time(seconds)	Discrepancy	Open WH
P11111.dat	2276.45	64	10.20%	1,3,6,7
P11112.dat	1699.93	58.1	8.90%	4,8,10
P11121.dat	2359.73	91.2	14.80%	1,4,7,9
P11122.dat	1649.09	69.1	8.60%	2,3,17
P11211.dat	2491.84	102.1	12.70%	2,4,8,9
P11212.dat	1557.46	55.1	8.50%	1,2
P11221.dat	2327.19	67.1	13.30%	1,5,9,16
P11222.dat	1673.35	56.1	11.00%	10,12
P12112.dat	1375.42	62	10.40%	6,7
P12121.dat	1891.65	67	14.40%	5,9,11,16
P12122.dat	1302.45	38	12.10%	5,18
P12211.dat	1712.66	40	16.90%	5,1
P12212.dat	9257.5	59	13.10%	4,8
P12221.dat	1505.47	59	18.40%	17,18
P12222.dat	8190	53	9.40%	4,13
P13111.dat	2355.8	58	9.90%	2,6,9
P13112.dat	1450.94	122	10.20%	1,9,10
P13121.dat	1979.78	72.2	14.50%	3,15,18,19,20
P13122.dat	1341.43	56.1	4.50%	4,11,13

P13211.dat	1793.96	92.1	16.80%	3,6,8,9,
P13212.dat	12613.1	132.2	26.70%	1,4,6
P13221.dat	1492.22	62.1	23.30%	2,14,17
P13222.dat	1332.35	62.1	23.00%	8,11,13,15
S241				
Problem	Cost	Time(seconds)	Discrepancy	Open WH
P11111.dat	2366.79	7.9	13.60%	1,3,6,7
P11112.dat	1679.17	10.42	7.80%	4,8,10
P11121.dat	2308.86	7.41	12.90%	1,2,4,12
P11122.dat	1669.73	8.91	9.80%	2,17
P11211.dat	2449.46	94.94	11.20%	2,4,8,9
P11212.dat	1834.73	94.04	22.30%	2,8,9
P11221.dat	2403.18	79.51	16.00%	1,3,5,9
P11222.dat	1761.92	94.74	15.40%	4,12
P12111.dat	2200.97	85.92	9.70%	4,8
P12112.dat	1456.17	86.73	15.40%	6,7
P12121.dat	1878.18	120.77	13.80%	5,7,9,11
P12122.dat	1361.19	127.18	15.80%	5,18
P12211.dat	1933.6	94.64	26.40%	5,1
P12212.dat	11147.7	124.68	27.80%	4,8
P12221.dat	1408.58	106.65	12.80%	15,18,19
P12222.dat	11822.6	84.02	37.30%	4,13
P13111.dat	2514.31	78.81	15.60%	2,6,9
P13112.dat	1470.26	70.51	11.40%	8,9,10
P13121.dat	2196.81	65.19	23.00%	3,15,18,19,20
P13122.dat	1539.73	62.69	16.80%	4,6,13
P13211.dat	1810.73	50.37	17.60%	3,6,8,9
P13212.dat	15844.7	96.74	41.70%	1,4
P13221.dat	1719.98	100.55	33.40%	2,14,17
P13222.dat	1265.45	72.2	18.90%	4,7,15
S242				
Problem	Cost	Time(seconds)	Discrepancy	Open WH
P11111.dat	2375.65	7.9	14.00%	1,3,6,7
P11112.dat	1761.68	11.32	12.10%	4,8,10
P11121.dat	2307.04	7.91	12.90%	1,2,4,12
P11122.dat	1706.72	8.92	11.70%	2,17
P11211.dat	2449.54	59.78	11.20%	2,4,8,9
P11212.dat	1800.75	111.16	20.90%	2,8
P11221.dat	2410.11	78.22	16.30%	1,5,9,16,20
P11222.dat	1877.19	109.45	20.60%	7,19
P12111.dat	2205.1	74.71	9.90%	4,8
P12112.dat	1538.12	82.42	19.90%	6,7
P12121.dat	1911.37	81.02	15.30%	5,7,9,11

P12122.dat	1421.85	68.49	19.40%	5,18
P12211.dat	1932.35	73.21	26.40%	5,1
P12212.dat	11160.4	74.41	27.90%	4,8
P12221.dat	1379.11	66.79	11.00%	15,18,19
P12222.dat	12086.6	82.72	38.60%	4,13
P13111.dat	2528.51	54.38	16.10%	2,6,9
P13112.dat	1434.08	77.91	9.10%	8,9,10
P13121.dat	2106.6	92.03	19.70%	3,15,18,19,20
P13122.dat	1497.07	74.11	14.40%	4,6,13
P13211.dat	1774.86	62.29	15.90%	3,6,8,9
P13212.dat	16729.8	64.49	44.80%	1,4
P13221.dat	1664.26	66.7	31.20%	2,14,17
P13222.dat	1250.7	63.59	17.90%	4,7,15
S243				
Problem	Cost	Time(seconds)	Discrepancy	Open WH
P11111.dat	2366.34	79	13.60%	1,3,6,7
P11112.dat	1665.38	52.1	7.00%	4,8,10
P11121.dat	2303.12	61.1	12.70%	1,4,7,9
P11122.dat	1613.07	55.1	6.60%	3,17
P11211.dat	2403.96	60.1	9.50%	2,4,8,9
P11212.dat	1706.81	81.1	16.50%	8,9
P11221.dat	2339.38	42.1	13.70%	1,5,9,16
P11222.dat	1734.94	65.1	14.10%	10,12
P12111.dat	2171.61	49	8.50%	4,8
P12112.dat	1430.61	66.1	13.90%	6,7
P12121.dat	1917.81	44.1	15.60%	5,9,11
P12122.dat	1332.11	55.1	14.00%	5,18
P12211.dat	1872.05	62.1	24.00%	5,1
P12212.dat	10915	60	26.30%	4,8
P12221.dat	1370.38	44.1	10.40%	15,18,19
P12222.dat	11657.5	84.1	36.40%	4,15
P13111.dat	2521.14	42.1	15.80%	1,2,6,9
P13112.dat	1422.9	68.1	8.40%	2,1
P13121.dat	2103.22	44.1	19.50%	11,13,15,19
P13122.dat	1523.35	71.1	15.90%	4,6,13
P13211.dat	1796.99	51	16.90%	3,6,8,9
P13212.dat	16425.2	63.1	43.70%	1,4
P13221.dat	1656.27	41.1	30.90%	2,14,17
P13222.dat	1228.05	131.2	16.40%	4,7,15

REFERENCES

- Albareda-Sambola, M., Di'az, J.A., Fern'andez, E., "A compact model and tight bounds for a combined location routing problem," *Computers and Operations Research* 32, 407–428, 2005.
- Averbakh, I., Berman, O., "Routing and location-routing p-delivery men problems on a path." *Transportation Science* 28 (2). 162-166, 1994.
- Baker S. F., Chan Y., "The multiple depot, multiple travelling salesmen problem: vehicle range vehicle frequency, and heuristic implementations," 2003.
- Balakrishnan. A., Ward, J.E., Wong, R.T., "Integrated facility location and vehicle routing models: Recent work and future prospects," *American Journal of Mathematical and Management Sciences* 7 (1&2), 35-61, 1987.
- Berman, O., Jaillet, P., Simchi-Levi, D., "Location-routing problems with uncertainty," In: Drezner, Z. (Ed.), *Facility Location: A Survey of Applications and Methods*. Springer-Verlag, New York, NY, pp. 427-453, 1995.
- Berman, O., Simchi-Levi, D., "Minisum location of a traveling salesman," *Networks* 16, 239–254, 1986.
- Bookbinder, J.H., Reece, K.E., "Vehicle routing considerations in distribution system design," *European Journal of Operational Research* 37, 204213, 1988.
- Bruns, A., Klose, A., Sta'hly, P., "Restructuring of Swiss parcel delivery services," *OR Spektrum* 22, 285–302, 2000.
- Bumess, R.C., White, J.A., "The traveling salesman location problem," *Transportation Science* IO (4). 348-360, 1976.
- Chien, T.W., "Heuristic procedures for practical-sized uncapacitated location-capacitated routing problems," *Decision Sciences* 24, 995–1021, 1993.
- Christofides, N., Eilon, S., "An algorithm for the vehicle dispatching problem," *Operational Research Quarterly* 20 (3), 1969.
- Daskin, M.S., "Location, dispatching, and routing models for emergency services with stochastic travel times," In: Ghosh, A., North-Holland Publishing, Amsterdam, Holland, pp. 163- 198, 1987.
- Ghiani, G., Laporte, G., "Eulerian location problems," *Networks* 34, 291–302, 1999.
- Ghosh, J.K., Sinha, S.B., Acharya, D., "A generalized reduced gradient based approach to round-trip location problem. in: Jaiswal, N.K. (Ed.)," *Scientific Management of Trans* 14, 1981.

- Gillett, B., Johnson, J., "Multi-terminal vehicle-dispatch algorithm," *Omega* 4 (6). 7 I I-718,1976.
- Hansen, P.H., Hegedahl, B., Hjortkjaer, S., Obel, B., " A heuristic solution to the warehouse location-routing problem," *European Journal of Operational Research* 76, 1 I I- 127, 1994.
- MacQueen, J., B., "Some Methods for classification and Analysis of Multivariate Observations," *Proceedings of 5-th Berkeley Symposium on Mathematical Statistics and Probability*, Berkeley, University of California Press, 1:281-297, 1967.
- Jacobsen, S.K., Madsen, O.B.G., "A comparative study of heuristics for a two-level routing-location problem," *European Journal of Operational Research* 5, 378-387, 1980.
- Johnson, D.S. and McGeoch, L.A., "The travelling salesman problem: A case study in local optimization", November 20, 1995.
- Laporte, G., " A survey of algorithms for location-routing problems," *Investigacion Operativa* 1, 93–123, 1989.
- Laporte, G., Nobert, Y., "An exact algorithm for minimizing routing and operating costs in depot location," *European Journal of Operational Research* 6, 224–226, 1981.
- Laporte, G., Nobert, Y., "Solving a family of multi-depot vehicle routing and location-routing problems," *Transportation Science* 22 (3). 161-172, 1988.
- Laporte, G., Nobert, Y., Pelletier, P., "Hamiltonian location problems," *European Journal of Operational Research* 12, 82– 89, 1983.
- Laporte, G., Dejax, P.J., "Dynamic location-routing problems," *Journal of the Operational Research Society* 40 (5), 471-482, 1989.
- Lin, C.K.Y., Chow, C.K., Chen, A., "A location-routingloading problem for bill delivery services," *Computers and Industrial Engineering* 43, 5–25, 2002.
- List, G.F., Mirchandani, P.B., " An integrated network/ planar multiobjective model for routing and siting for hazardous materials and wastes," *Transportation Science* 25, 146–156, 1991.
- Madsen, O.B.G., "Methods for solving combined two level location-routing problems of realistic dimensions," *European Journal of Operational Research* 12, 295-301, 1983.
- Melechovsky´, J., Prins, C., Wolfler Calvo, R., " A metaheuristic to solve a location-routing problem with non-linear costs," *Journal of Heuristics* 11, 375–391, 2005.
- Min, H., Jayaraman, V., Srivastava, R., "Combined location-routing problems: A synthesis and future research directions." *European Journal of Operational Research* 108, 1– 15, 1997.
- Nagy, G., Salhi S., "Location-routing: Issues, models and methods *European Journal of Operational Research*, 2006.

Nagy, G., Salhi, S., "The many-to-many location-routing problem," TOP 6, 261–275, 1998.

Perl, J., Daskin, M.S., "A warehouse location-routing problem," Transportation Research 19B (5). 381-396,1985.

Reingod E. M., Nievergelt J., Deo N. "Combinatorial algorithms: theory and practice," Englewood Cliffs, N.J. : Prentice-Hall, 1977.

ReVelle, C., Cohon, J., Shobrys, D., "Simultaneous siting and routing in the disposal of hazardous wastes," Transportation Science 25, 138–145, 1991.

Salhi, S., Nagy, G., "Consistency and robustness in location routing," Studies in Locational Analysis 13, 3–19, 1999.

Salhi, S., Rand, G.K., "The effect of ignoring routes when locating depots," European Journal of Operational Research 39, 150–156, 1989.

Srivastava, R., "Alternate solution procedures for the location-routing problem," Omega 21, 497–506, 1993.

Srivastava, R., Benton, W.C., "The location-routing problem: Considerations in physical distribution system design," Computers in Operations Research 17, 427–435, 1990.

Tansel B., Ahipasaoğlu S. D., Erdoğan G., "Location Routing Problems: A review and Assessment of Research Directions," Working Paper, 2007.

Tuzun, D., Burke, L.I., "A two-phase tabu search approach to the location routing problem," European Journal of Operational Research 116, 87–89, 1999.

Watson-Gandy, C.D.T., Dohrn, P.J., "Depot location with van salesmen – a practical approach," Omega 1, 321–329, 1973.

Webb, M.H.J., "Cost functions in the location of depots for multiple-delivery journeys," Operational Research Quarterly 19, 311–320, 1968.

Wu, T-H., Low, C., Bai, J-W., "Heuristic solutions to multidepot location-routing problems," Computers and Operations Research 29, 1393–1415, 2002.

Zografos, K.G., Samara, S., "Combined location-routing model for hazardous waste transportation and disposal," Transportation Research Record 1245, 52-SY, 1989.