DISTRIBUTION PLANNING OF BULK LUBRICANTS AT AN ENERGY COMPANY

by

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Abstract

We address the distribution planning problem of bulk lubricants at an energy company operating in Turkey. The problem is a multi-product, multi-period, heterogeneous fleet management problem that involves the assignment of customer orders and routing of tank trucks by minimizing the routing costs. To solve this problem we develop a 0-1 mixed-integer linear programming model. Since the problem is intractable for real world data we propose two heuristic approaches and discuss their performances. The first approach is a linear programming relaxation-based algorithm while the second is a threshold accepting heuristic. We propose two variants of this heuristic, the first uses the distance priority whereas the second has a due date priority. The numerical results show that both threshold-accepting heuristics have competitive performance.

BİR ENERJİ ŞİRKETİNDE MADENİ YAĞ DAĞITIMI PLANLAMASI

Muhammed Furkan UZAR

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Özet

Bu tezde Türkiye'de faaliyet gösteren bir enerji firmasının madeni yağ dağıtımı planlaması problemi ele alınmıştır. Problem temel olarak birden çok ürünlü, birden çok dönemli, heterojen yapılı araç filosuna sahip bir yönetim problemi olarak, araçların rota maliyetlerinin enküçüklenerek araçların müşterilere atanması ve araç rotalarının kararlaştırılması karalarını içermektedir. Problemin çözümü için 0-1 karışık tamsayılı doğrusal programlama modeli geliştirilmiştir. Bu modelin gerçek verilerle çözümü yeterli zamanda sağlanamadığından tezde iki ayrı sezgisel yaklaşım önerilmiş ve gerçek verilerle denenmiştir. İlk yaklaşım doğrusal programlama gevşetmesine dayanırken ikincisi ise eşik değere bağlı çalışan bir sezgisel yöntemdir. Bu yöntemim iki farklı uygulaması önerilmiştir: birincisi uzaklık öncelikli iken ikincisi termin tarihi önceliğine sahiptir. Sayısal sonuçlar eşik değere bağlı yöntemin iyi sonuçlar verdiğini göstermiştir.

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Chapter 1 Introduction

The efficiency in transportation and distribution planning is a key success factor in the petroleum industry. Petroleum (crude oil) is processed in oil refineries to derive different products such as fuel oil, gasoline, diesel fuel, kerosene, liquefied petroleum gas (LPG), petrochemicals, lubricating oils, etc. The refined products are classified into two as light/white products like gasoline and heavy/black products like lubes. Ronen (1995) classifies the distribution of petroleum products into four categories: light products from refineries to tank terminals, light products from tank terminals to industrial customers, bulk lubes from lube plants to industrial customers, and packaged lubes from lube plants to industrial customers.

Petroleum products are mainly transported to the international markets by maritime transportation: approximately 60% of total petroleum produced is transported via sea lines (Rodrigue et al., 2009). The other modes of transportation are pipelines, trains, and trucks. Table 1 summarizes several properties of different transportation modes in the petroleum industry. In general, trucks are used to transport the end products to the industrial customers or to gas and service stations.

Table 1.1 Modes used in the transportation of petroleum products (Rodrigue et al., 2009).

	Pipeline	Marine	Rail	Truck
Volumes	Large	Very large	Small	Large
Materials	Crude / Products	Crude / Products	Products	Products
Scale	2 ML+	10 ML+	100 kL	5-60 kL
Unit costs	Very low	Low	High	Very high
Capital costs	High	Medium	Low	Very low
Access	Very limited	Very limited	Limited	High
Responsiveness	1-4 weeks	7 days	2-4 days	4-12 hours
Flexibility	Limited	Limited	Good	High
Usage	Long haul	Long haul	Medium haul	Short haul

In this study, we address the distribution planning problem of the lubricant (lube) production division of a global energy company operating in Turkey. With its specific characteristics and elements of the distribution system the problem differs from many of the transportation problems addressed in the literature. Although the oil industry has been a major source of applications, white papers and reports on those applications and the academic research in the field are rather scant (Ronen, 1995). Ronen (1995) provides a review of operations research (OR) applications in dispatching petroleum products and compares several applications in the oil industry. Among those, Bausch et al. (1995) consider the distribution problem of bulk and packaged lube oil in Mobil Oil Corporation. The setting is similar to our case with its product specifications and heterogeneous fleet structure. To solve this problem, Bausch et al. (1995) use an elastic set partitioning technique which selects a minimal set of schedules among the candidate schedules. Candidate schedules are obtained by generating trips with multiple stops using the sweep heuristic (Gillet and Miller, 1974).

Brown and Graves (1981) address the transportation problem of gasoline from a single bulk terminal to customers. They design and implement a centralized dispatching system where the objective is to minimize the transportation costs while maintaining equitable man and equipment workload, safety standards and customer service. Brown et al. (1987) extend this work by considering multiple sources. They first assign the orders to the tank trucks by using a search algorithm that uses pair wise interchange and try to find the best assignment. After the assignment, the routing problem becomes TSP and solved optimally. Then they use an integer mathematical program and run for each truck to find the loading scheme of the tanks of the tank truck. Franz and Woodmansee (1990) develop a rule based decision support system for a regional oil company. Their algorithm finds the drivers' schedule and the dispatching of the tank trucks for a single day and is implemented as a semi-automated system. Abdelaziz et al. (2002) propose a variable neighborhood search heuristic to dispatch the tank trucks with multiple compartments in the delivery of fuel. Their problem involves three set of decisions: the assignment of orders to delivery vehicles, the adjustment of order quantities to fit vehicle compartments and routing of the vehicles. They also model a mathematical program in a single period setting.

Vehicles with multiple compartments are also used in the transportation of food and grocery items. Chajakis and Guignard (2003) address such a problem using an approximation algorithm based on Lagrangean relaxation. They use 4 different Lagrangean Relaxations, a Langrangean substitution and a Langrangean Decomposition technique for finding lower bounds. They develop a Lagrangean heuristic to obtain feasible solutions and test the performance of their algorithms through a computational study. Bilgen and Ozkarahan (2007) propose a mixed integer linear programming model to solve the problem of shipping and blending the bulk grain in the maritime transportation. Fallahi et al. (2008) consider the classical vehicle routing problem (VRP) with identical vehicles having multiple compartments where each compartment can only hold a single product. They develop memetic and tabu search algorithms to solve this problem and evaluate their performance using the results of VRP instances from the literature. Mendoza et al. (2009) also present a memetic algorithm for the multi-compartment VRP where the demands are stochastic.

Our study considers the distribution of bulk lubes from a lube production plant to industrial customers. In our problem, the fleet is heterogeneous and consists of multi-compartment vehicles, i.e., tank trucks, where each compartment can only be assigned to a single product. The objective of the problem is to find a minimum cost transportation plan. The problem basically consists of loading the customer orders to tank trucks and finding the routes of the assigned tank trucks. The routing problem is an open vehicle routing problem (OVRP) where the company does not pay for the return trip of the trucks to the plant. In OVRP the vehicles either do not need to return to the depot or return to the depot by revisiting the customers visited in the reverse order (Sariklis and Powell 2000). Sariklis and Powell (2000) present a minimum spanning tree with penalties based heuristic method to solve OVRP. Brandao (2004), Fu et al. (2004) and Tarantilis et al. (2004) propose different tabu search algorithms to solve this problem. Tarantilis et al. (2004) and Tarantilis et al. (2005) propose two different threshold accepting heuristics for OVRP. Variable neighborhood search algorithms are also proposed for OVRP by Pisinger and Ropke (2005) and Fleszar et al. (2007). A review of the approaches developed for solving the OVRP may be found in Li et al. (2007).

The aim of this study is to develop a scientific approach to improve the bulk lubes distribution operations of a global energy company. We first formulate a mathematical programming model of the problem and then develop three heuristic algorithms to solve it. The remainder of the thesis is organized as follows: In Chapter 2 the problem is described and a 0-1 mixed integer programming model is presented. Chapter 3 is devoted to the description of the heuristics proposed for efficiently solving this problem. The numerical results and the comparison of the performance of the proposed heuristics are given in Chapter 4. Finally, the conclusions and directions for future research are provided in Chapter 5.

Chapter 2 Problem Description and Formulation

We address the distribution planning problem of bulk lube oils at an energy company operating in Turkey. The problem is a multi-product, multi-period, heterogeneous fleet management problem that involves the assignment of customer orders and routing of tank trucks. The elements of the distribution system can be classified into four categories: (i) the fleet which consists of multi-compartment tank trucks, (ii) the distribution network which includes the plant where the trucks are loaded and the cities where the customers are located, (iii) the products with their specific properties and (iv) the scheduling system, which has different constraints and flexibilities specific to this problem. In what follows, we provide further details on these elements of the problem and then formulate the mathematical model.

2.1. Elements of the problem

2.1.1 Tank Trucks

The company does not have its own fleet and uses a third party logistics (3PL) service provider for the distribution of the lubes. Every year it determines its fleet needs and makes a contract with the 3PL company. According to the contract, the 3PL provider dedicates a fixed number of tank trucks to the company. Therefore, determining the appropriate fleet size and type is an important decision for the company. In the case the contracted capacity is insufficient in any day the company can hire additional trucks from the spot market at an additional cost. Hence, the truck capacity can be considered as a loose constraint in that sense.

The tank trucks have 4 or 5 compartments (tanks) with different capacities. In addition to the tank capacity, the trucks have a maximum total load restriction imposed by the regulations of the General Directorate of Highways. The maximum load tonnage in a truck is determined according to its technical properties such as its number of wheels and engine power. The trucks in the fleet have different load restrictions and tank capacities, which makes the problem a heterogeneous fleet type distribution problem. In addition, the trucks in the fleet are classified as big- and small-sized trucks. Small trucks have a maximum total load capacity of 7 tons approximately and are used to serve the customers whose unloading area is not large enough to accommodate the big-size trucks. In the thesis, this type of customers is referred to as “small customers” whereas the customers that can be served with any truck are called as “large customers”.

A tank in a trunk can only be loaded with one single lube oil, unless the truck is equipped with a flow-meter. The flow-meter is the device used to measure the quantity of the lube loaded or unloaded. If the tank trucks do not have a flow-meter, then each tank must be dedicated to a single lube order. For instance, a tank truck with 5 tanks and without flow-meter can serve at most 5 customers. Currently, there is only one truck in the fleet which is equipped with a flow-meter. However, the flow-meter on that truck is seldom utilized because the customers usually require their orders to be officially measured and loaded in one single tank at the plant rather than the measurement and delivery being made at their site. Our observations on the delivery data show that the flow-meter is not being utilized at all according to the dispatching scheme. In addition, recent demand data reveal that there is no or little need for the use of this equipment in the deliveries. Hence, we assume that all trucks are identical in that sense and are not equipped with a flow-meter device.

2.1.2. Distribution Network

The distribution network consists of one plant in Bursa and 30 cities located in different regions of Turkey, as shown in Figure 2.1. The tank trucks are loaded at the plant according to the planned deliveries and visit the customers using a route such that the total distance until the last customer on the route is minimum. Once the loading decisions are made, the routing problem is easy to solve since a truck can at most visit 4 or 5 customers, if all tanks are filled. The routing is only made for the city-to-city network and the distances between the customers located in the same city are not taken into account. This is due the fact that the company is charged for long distance trips on a kilometer basis and pays a fixed cost for each additional customer served in the same city. For example, if a truck is loaded to serve 5 customers located in 2 cities (for example, 2 customers in the first and 3 in the second), it first goes to the closest city and makes the deliveries of the 2 customers and then goes to the next city to serve the remaining 3 customers. At the end of its trip, the truck returns to the plant. The total cost to the

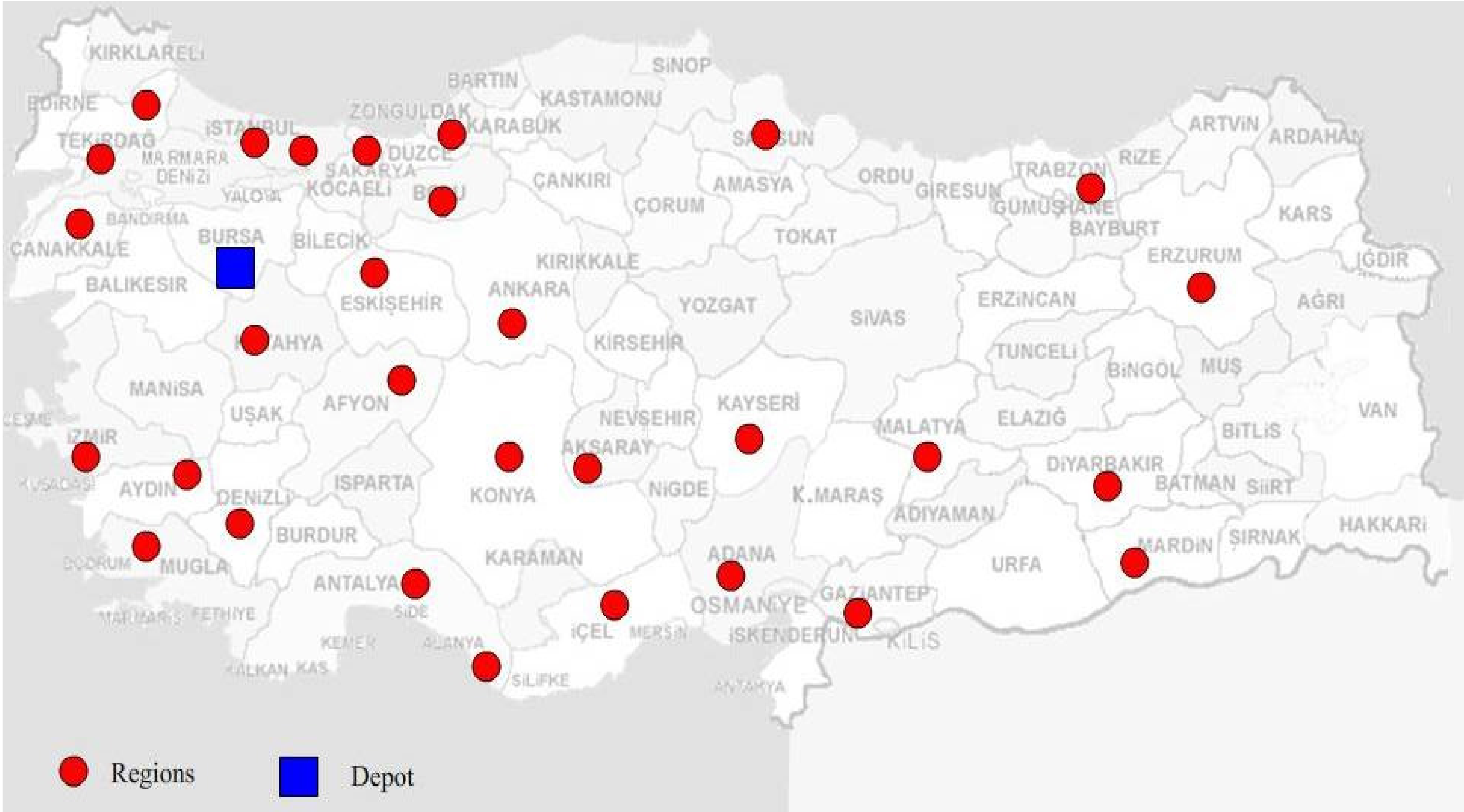

Figure 2.1 Regions of the customers

company is determined according to the distance of the first city visited to the plant and the extra number of customers served in that city; and the distance of the second city to the first city and the extra number of deliveries in that city. The company does not pay for the return trip of the truck to the depot, which makes the problem an OVRP. With this consideration the cost associated with the tour is also determined in a different way. The total distance made in the tour between cities is calculated and according to its value a cost is charged. In this thesis, we refer to the distance-related variable cost as the *routing cost* and the cost per each additional customer visited in a city as the *visit cost*.

2.1.3. Products and Setup Costs

The company produces and distributes 130 different products in total. There are 8 basic product families and each product family consists of product groups. Since the products are in liquid form two different products cannot be loaded within the same tank. In addition, the tank may require a cleaning operation depending on the type of lube oil last loaded in the tank. The cleaning is not product-dependent and its time (cost) is same for all product groups. According to our observations on the shipments, there are approximately 30 products which have been commonly demanded and delivered and setup is not frequently needed for the changeover from one lube to another. So, for simplicity, we exclude the setup cost due to the cleaning operation in our model formulation and leave it as a future research. However, we note that this setup nature can easily be accommodated in our heuristic approaches.

2.1.4. Scheduling

The Sales Department receives the orders on a daily basis and assigns each order with an estimated delivery date. However, the planned delivery date is finalized after an advanced payment from the customer has been confirmed. The company has flexibility in determining the delivery date for consolidation purposes. For instance, an order can be delivered 2 days before or after its planned delivery date. In this study, we refer to the latest day that the demand must be delivered as the due date of the order. That is, a demand with due date 5 can be satisfied in any of the days 1, 2, 3, 4 or 5. Therefore, the distribution problem is a multi-period problem which is solved on a rolling horizon basis.

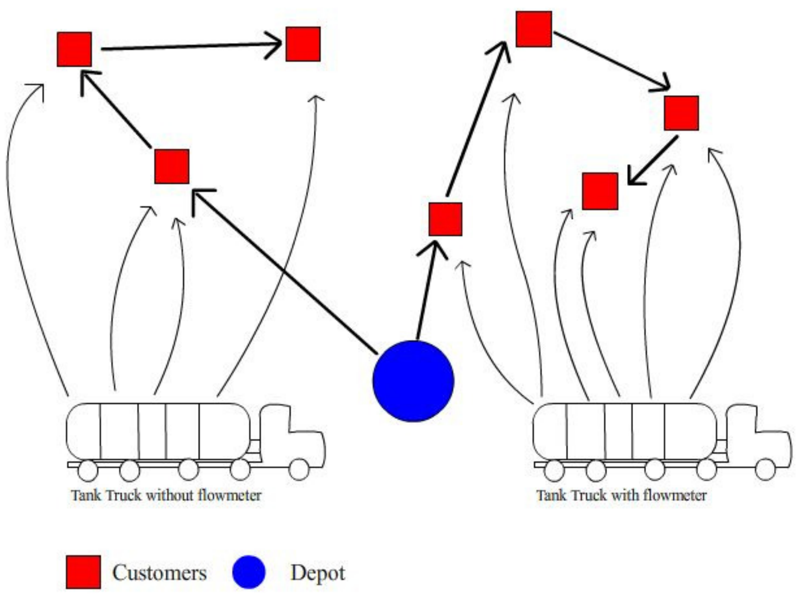

Figure 2.2. The representation of assignments and routes of the tank trucks

In summary, the problem we address basically consists of two integrated problems: the assignment problem of customer orders to the tanks of the trucks and the routing problem of the trucks. Figure 2.2 depicts an example loading and routing scheme for two different truck types, both equipped with 5 tanks. The objective of the problem is to minimize the total distribution cost over the planning horizon. However, the real total cost is calculated as the sum of the distribution costs of the first days since the problem needs to be solved every day to finalize the delivery schedule of the next day on a rolling horizon basis.

2.2. Model Formulation

In this section, a mixed integer linear programming model is developed in an attempt to obtain optimal distribution plans. The planning horizon is 5 days (i.e. 1 week since no delivery is made during weekends) and day 1 can be considered as tomorrow. An order must be delivered at latest on its due date and there are no penalty costs for early or tardy deliveries. The model is solved every day and the final distribution plan of the next day is determined as the solution of the first day and is frozen. The input data are updated next day and the model is resolved.

Indices and Sets

i index for tanks of tank trucks ($i = 1, \dots, I$)

j index for tank trucks ($j = 1, \dots, J$)

k index for customers ($k = 1, \dots, K$)

p index for products ($p = 1, \dots, P$)

t index for days ($t = 1, \dots, T$)

r index for cities ($r = 1, \dots, R$)

J_b set of big size tank trucks

J_t set of all available tank trucks in day t

K_r set of customers located in city r

K_s set of small customers

Parameters

Q_j total load restriction on tank truck j

cap_{ij} capacity of tank i of tank truck j

D_{kpt} demand of customer k for product p with due date t

$d_{rr'}$ distance from city r to city r'

C_v cost of visiting an additional customer in a city

C_r cost per km

Decision Variables

x_{ijkpt} fraction of tank i of truck j filled with product p ordered by customer k and due on day t

$$y_{ijkpt} = \begin{cases} 1, & \text{if } x_{ijkpt} > 0 \\ 0, & \text{otherwise} \end{cases}$$

$$q_{jkt} = \begin{cases} 1, & \text{if truck } j \text{ serves customer } k \text{ in day } t \\ 0, & \text{otherwise} \end{cases}$$

$$z_{jrt} = \begin{cases} 1, & \text{if truck } j \text{ goes to city } r \text{ in day } t \\ 0, & \text{otherwise} \end{cases}$$

$$p_{jt} = \begin{cases} 1, & \text{if truck } j \text{ is used in day } t \\ 0, & \text{otherwise} \end{cases}$$

$$v_{jrr't} = \begin{cases} 1, & \text{if truck } j \text{ visits city } r' \text{ immediately after city } r \text{ in day } t \\ 0, & \text{otherwise} \end{cases}$$

u_{jrt} sub-tour elimination variable

Mathematical Model

$$Min\ Z = \sum_{j}^{J}\sum_{t}^{T}\sum_{r}^{R} C_{visit}\left(-z_{jrt} + \sum_{k\in reg_r} q_{jkt}\right) + \sum_{j}^{J}\sum_{r}^{R}\sum_{r'}^{R}\sum_{t}^{T} C_r d_{rr'} v_{jrr't} \quad (1)$$

Subject to

$$\sum_{i}^{I}\left(\sum_{j}^{J}(cap_{ij} \times \sum_{m=1}^{t} x_{ijkpm})\right) \geq D_{kpt} \qquad \forall k,p,t \qquad (2)$$

$$x_{ijkpt} \leq y_{ijkpt} \qquad \forall i, j \in j_t, k, p, t \qquad (3)$$

$$\sum_{i}^{I}\sum_{k}^{K}\sum_{p}^{P} x_{ijkpt} \times cap_{ij} \leq Q_j \qquad \forall j_t, t \qquad (4)$$

$$\sum_{k=1}^{K}\sum_{p=1}^{P} y_{ijkpt} \leq 1 \qquad \forall i, j_t, t \qquad (5)$$

$$y_{ijkpt} \leq q_{jkt} \qquad \forall i, j \in j_t, k, p, t \qquad (6)$$

$$q_{jkt} \leq z_{jrt} \qquad \forall j \in j_t, k \in K_r, t, r \qquad (7)$$

$$q_{jkt} \leq p_{jt} \qquad \forall j \in j_t, k, t \qquad (8)$$

$$\sum_{t}^{T} p_{jt} \leq 1 \qquad \forall j \in j_t \qquad (9)$$

$$y_{ijkpt} = 0 \qquad \forall i, j \in j_b, k \in K_s, p, t \qquad (10)$$

$$z_{j0t} = 1 \qquad \forall j \in j_t, t \qquad (11)$$

$$\sum_{r'}^{R} v_{jrr't} = z_{jrt} \qquad \forall j \in j_t, r, t \qquad (12)$$

$$\sum_{r'}^{R} v_{jr'rt} = \sum_{r'}^{R} v_{jrr't} \qquad \forall j \in j_t, r, t \qquad (13)$$

$$u_{jrt} - u_{jr't} + 2v_{jrr't} \leq 1 \qquad \forall j \in j_t, r, r', t \mid r \neq r' \qquad (14)$$

$$x_{ijkpt} \geq 0 \qquad \forall i, j, k, p, t \qquad (15)$$

$$y_{ijkpt} \in \{0,1\} \qquad \forall i, j, k, p, t \qquad (16)$$

$$q_{jkt} \in \{0,1\} \qquad \forall j, k, t \qquad (17)$$

$$z_{jrt} \in \{0,1\} \qquad \forall j,r,t \qquad (18)$$

$$p_{jt} \in \{0,1\} \qquad \forall j,t \qquad (19)$$

$$v_{jrr't} \in \{0,1\} \qquad \forall j,r,r',t \qquad (20)$$

$$1 \le u_{jrt} \le R-1 \qquad \forall j,r,t \qquad (21)$$

The objective function (1) minimizes total routing costs and visit costs. Constraint set (2) makes sure that a customer order must be satisfied on its due date or earlier. Constraints (3) link the binary y continuous x assignment variables: if x_{ijkpt} takes a positive value then the corresponding binary variable y_{ijkpt} is forced to be 1. Constraint set (4) ensures that total load on truck does not exceed the maximum load restriction. Constraints (5) make sure that each tank on a truck is only filled with one product. Constraint set (6) enforces the binary variable q_{jkt} to take a value of 1 if the corresponding binary variable y_{ijkpt} is 1. In other words, if any tank of a tank truck j is used for customer k on day t then the tank truck j serves customer k on that day. Constraints (7) assure that if a customer is served by truck j on day t then the same truck visits the city where that customer is located on day t. Constraints (8) determine the days during which the trucks are assigned with a delivery. Since the returns of the trucks during the planning horizon are not considered constraint set (9) ensures that each tank truck can be dispatched at most once during the planning horizon. Note that the expected return days of the trucks on the road are taken into account in the data when solving the problem of the next day. Constraint set (10) makes sure that small customers are not serviced with the big trucks. Constraints (11) set the plant as the origin of all available trucks. Constraint sets (12) and (13) are the routing constraints which ensure that a truck should makes a feasible route starting from the plant and returning back to depot. In the distance matrix $d_{rr'}$ the return distance to the plant are set to zero to formulate the routing problem as OVRP. Finally, constraints (14) are sub-tour elimination constraints and constraints (15-21) define the decision variables.

Since this problem is intractable in the real-life industrial environment, an efficient heuristic approach is needed to obtain good quality solutions in reasonable computation time. Therefore, in the next section a greedy linear programming relaxation-based algorithm and a heuristic approach with two variants are developed in an attempt to solve this problem.

Chapter 3 Solution Methodology

We propose two different solution approaches to efficiently solve this large scale problem. The first is a Linear Programming (LP) relaxation-based algorithm and the second is a threshold accepting heuristic. We also present two different variants of the threshold accepting heuristic. As mentioned earlier, the distribution plan is made daily and the plan of the following day is implemented. So, the proposed algorithms are also designed to finalize the delivery schedule of the next day by iteratively solving them every day.

3.1. Linear Programming Relaxation-based Algorithm (LPH)

LP relaxation basically relaxes the binary variables by allowing them to take values between 0 and 1. By doing so, the problem may be solved easily; however, the solution becomes infeasible and cannot be implemented. This algorithm basically utilizes the LP relaxation with some rounding techniques and tries to find a feasible solution for the problem. Our initial experiments on the LP problem have shown that the existence of visit costs in the objective function causes inefficiently utilized tank trucks in the solutions. For this reason, we exclude the visit cost in the LP relaxation and construct our algorithm based on the routing costs only.

In the original model, recall that the *y* binary variable is used for the assignment of the tanks of the trucks and *x* variable is used for determining the utilization of the tanks. In this algorithm the *y*'s are the key variables because the algorithm first finds the loading scheme of the tanks with respect to the customer orders then routes the tank trucks with respect to the truck loads.

The primary idea is to satisfy the demands of the first period then to assign the remaining orders to the available tanks. To assign the tanks, firstly the y_{ijkpt} variables are set to one since they indicate that whether a tank is assigned or not. Then x_{ijkpt} variables are determined and set because if a tank is assigned to a demand then that tank should be filled with that demand obviously. Indeed one may expect that the x_{ijkpt} should automatically take its value when the LP is resolved after setting the corresponding y_{ijkpt} ; this is not necessarily the case. In other words, when we are faced with some *x's* equal to 0 although its corresponding y variable is 1 we force those x's to be positive and to take an appropriate value.

Step 0. Initialize the LP problem and solve it.
Step 1. Select a demand arbitrarily with due date 1.
If there is no demand left with due date 1, go to Step 4.
Step 2. Find the maximum y_{ijkp1} with the same indices with selected demand (D_{kp1})
Set maximum y_{ijkpt} to 1
Set corresponding x_{ijkpt} to 1 if $D_{kpt} \geq Cap_{ij}$, otherwise set x_{ijkpt} to D_{kpt} / Cap_{ij}
Step 3. Solve LP.
If no feasible solution exists, set y_{ijkpt} and corresponding x_{ijkpt} to 0.
Otherwise, check whether selected demand (D_{kp1}) is satisfied.
If demand is satisfied, go to Step 1.
Otherwise, go to Step 2.
Step 4. Select an assigned tank truck.
If there is no assigned truck left unselected, go to Step 7.
Step 5. In the selected tank truck find max y_{ijkpt} that is not equal to 1.
Set max y_{ijkpt} to 1.
Set corresponding x_{ijkpt} to 1 if $D_{kpt} \geq Cap_{ij}$, else set x_{ijkpt} to D_{kpt} / Cap_{ij}
Step 6. Solve LP.
If no solution exists, set y_{ijkpt} and corresponding x_{ijkpt} to 0.
Otherwise,
If all tanks of the selected tank truck are assigned to 1 or 0, then go to Step 4.
Otherwise, go to Step 5.
Step 7. For each tank truck that exists in the solution, determine the routes by the nearest neighbor heuristic

Figure 3.1. The steps of LPH

The main steps of the LP relaxation algorithm are given in Figure 3.1. Firstly, the data of the LP model is initialized and the model is solved using CPLEX v.11.0. Then algorithm selects a demand arbitrarily with a due date 1 and tries to satisfy it by assigning it to the tank having the maximum y_{ijkpt} value. After satisfying all demands with due date 1, the algorithm tries to fill the remaining available tanks of the already assigned tank trucks with the waiting orders in the planning horizon.

Once the loads of the tank trucks are determined, the routing is a relatively easy task since each truck will have an open route that the problem reduces to finding a Hamiltonian path originating from the plant. Furthermore, since a tank truck can visit at most 5 different cities, the optimal solution may be efficiently obtained even by complete enumeration. However, due to the nature of the problem we address, we have observed that the nearest neighbor algorithm is usually able to find the optimal routes because the cities to be visited are found to be in one direction. In addition, it is rarely the case that a tank truck visits more than 2 cities. Hence, we implemented the nearest neighbor algorithm in the routing phase of our algorithms.

For the next day's plan, the demand and availability of the tank trucks are updated according to the solution of the previous day and the additional data that may become available, and the algorithm is re-run.

3.2. Heuristic algorithms

The primary goal in our heuristic approach is to find a minimum cost distribution plan by satisfying the demands with due date 1, as in the case of LPH. We propose two variants which basically work in the same manner with slight differences.

3.2.1. Threshold Accepting Heuristic 1 (TAH1)

The steps of the Threshold Accepting Heuristic 1 (TAH1) are depicted in Figure 3.2. The threshold parameter λ is a parameter that is used for controlling the insertion of a new customer into an existing tour. After initializing the data and setting the value of λ, the algorithm first assigns the demands of the small customers. If there is no demand of small customer left, then the algorithm assigns the demands of the large customers in a similar way. Once the loads are determined, the routes are obtained again using the nearest neighbor algorithm.

Step 0. Initialize the data. Set the threshold parameter λ.
Step 1a. Select a demand (D_{kp1}) with due date 1 of a small customer farthest to the depot.
If no unsatisfied demand of a small customer left with due date 1, Go to Step 2a.
Step 1b. Select an available small tank truck that has the maximum total load limit.
Step 1c. Put the selected demand to the selected tank truck. (PutDemand (PD) Procedure).
Step 1d. Fill the selected tank truck with the unsatisfied demands of small customers. (FillTruck (FT) Procedure)
Step 1e. Fill the selected tank truck with the all unsatisfied demands. (FT)
Step 1f. Add the tank truck to the used tank truck list and update the availability of the tank truck. Go to Step 1a.
Step 2a. Select a demand (D_{kp1}) with due date 1 of a customer farthest to the depot.
If no unsatisfied demand left with due date 1, go to Step 3.
Step 2b. Select an available tank truck that has the maximum total load limit.
Step 2c. Put the selected demand to the selected truck. (PD)
Step 2d. Fill the selected tank truck with the unsatisfied demands of large customers. (FT)
Step 2e. Add the tank truck to the used tank truck list and update the availability of the tank truck. Go to Step 2a.
Step 3. Find the route of each used truck by nearest neighbor heuristic.
Step 4. Terminate

Figure 3.2 The steps of the Threshold-Accepting Heuristic 1.

In step 1 TAH1 assigns an arbitrary demand of a small customer with due date 1 to the selected tank truck, then tries to fill the empty tanks of the same truck with the demands of remaining small customers, if any. If there is still empty space in the small sized truck then it tries to fill it with other demands. After satisfying the demands of small customers in day 1, in step set 2 it plans the demands of the large customers in the same manner. It firstly puts an order from day 1 to the largest truck selected and then fills the empty tanks from the remaining orders list.

PutDemand (PD) Procedure

The PutDemand (PD) Procedure is used for determining to which tanks the demand will be assigned. Firstly, the selected order is put to the best fitted tank if the tank capacity is sufficient. If the tank capacity is not enough, the tank with the maximum capacity is filled fully with the selected demand and the remaining portion of the demand is assigned to a second tank using the same logic. The steps of PD are depicted in Figure 3.3.

Step 1. If the unassigned portion of demand is smaller than the maximum empty tank capacity of the tank truck, go to step 2.
Otherwise, go to step 3.
Step 2. If there is an empty tank and total load on the truck is less than the limit, Assign the demand to an empty tank which maximizes the tank utilization. (Best fit) Go to Step 4.
Otherwise, go to step 5.
Step 3. If there is an empty tank and total load on the truck is less than the limit, Assign the demand to the tank with maximum capacity. Go to step 1.
Otherwise, go to step 5.
Step 4. Add the demand to the satisfied demand list and remove it from the unsatisfied demand list.
Step 5. Terminate.

Figure 3.3. The steps of PutDemand Procedure.

Basically, PD utilizes the well-known best-fit heuristic used for solving the bin packing problem. By this way, it tries to maximize the utilization of the tank truck.

FillTruck (FT) Procedure

Given a set of demands that must be satisfied and a tank truck, FillTruck Procedure (FT) iteratively assigns those demands to the tank truck using PD. If the given tank truck is completely empty, then FT assigns the demand of a farthest customer to the plant with due date 1. If the tank truck is partially loaded, then FT attempts to assign the order of the nearest customer to the previously assigned customer(s). If there are any orders from customers located in the same city as the customers' whose orders have already been assigned, they are given priority. FT computes the extra cost of inserting a demand using the following formulation:

$$\text{Insertion cost} = \min\ \left[C_{rd}, (C_{rd} + C_{0d} - C_{0r})\right]$$

where C_{rd} is cost of going to city d from city r, r is the city where a customer order has been already loaded, d is the destination city and o is the plant (origin). The demand with the minimum insertion cost is selected if it is less than λ. The selected order is assigned to the tank truck using PD. The procedure is repeated until all orders have been assigned or the capacity of the truck has been used up. The steps of the FT procedure are given in Figure 3.4.

The parameter λ plays an important role in the performance of the heuristic. If it is set to be high then the utilization of the tank trucks are expected to increase; however the cost of transportation may increase as well. If λ is set to be low, then additional tank trucks may be needed because of the decrease in the utilization of the trucks, which may increase the transportation cost as well.

Step 1. If the tank truck is completely empty, go to step 2.
Otherwise, go to step 3.
Step 2. Select a demand of customer with due date 1 and farthest to the depot.
Put the selected demand with PD, go to step 3.
Step 3. If a demand exists within the same city with the previous demand assigned, go to step 4.
Otherwise, go to step 5.
Step 4. Select the demand with the earliest due date. Go to step 6.
Step 5. Compute the extra cost of all demands by $\min\ [C_{rd}, (C_{rd} + C_{0d} - C_{0r})]$
If the minimum extra cost is smaller than parameter λ,
Select the demand with minimum extra cost, go to step 6.
Otherwise, go to step 7.
Step 6. Put the selected demand with PD. Go to step 3.
Step 7. Terminate.

Figure 3.4. The steps of FillTruck Procedure.

3.2.2. Threshold Accepting Heuristic 2 (TAH2)

Similar to TAH1, TAH2 firstly assigns the demands of the small customers then satisfies the demands of large customers. After the assignment has been made, the routing is performed using the nearest neighbor algorithm. In TAH2, a selected truck is loaded by the orders with due date 1 using the FT procedure, as is the case in TAH1. Then the selected truck is tried to be loaded by the orders chronologically with due dates 2, 3, 4, and 5, in this order. This difference between TAH1 and TAH2 can be interpreted as TAH2 having a due date priority whereas TAH1 having a distance priority. The steps of TAH2 are given in Figure 3.5.

Step 0. Initialize the data. Set the threshold parameter λ.
Step 1a. Select an available small tank truck that has the maximum total load capacity.
Step 1b. Fill the selected tank truck with the demands of small customers that have due date 1. (FillTruck (FT) Procedure)
If there is no unsatisfied demand left with due date 1, go to step 6.
Step 1c. Fill the selected tank truck with the demands of small customers that have due dates 2, 3, 4, 5. (FT)
Step 1d. Fill the selected tank truck with the whole demands. (FT)
Step 1e. Add the tank truck to the used tank truck list and update the availability of the tank truck. Go to Step 1a.
Step 2a. Select an available tank truck that has the maximum total load capacity.
Step 2b. Fill the selected tank truck with the demands that have due date 1. (FT)
Step 2c. Fill the selected tank truck with the demands of large customers that have due dates 2, 3, 4, 5. (FT)
Step 2d. Add the tank truck to the used tank truck list and update the availability of the tank truck. Go to Step 2a.
Step 3. Find the route of each used truck by nearest neighbor heuristic.
Step 4. Terminate

Figure 3.5. The steps of Threshold-Accepting Heuristic 2

3.2.3. An Illustrative Example

Table 3.1. Distance matrix

	O	IST	KOC	SAK	BOL	ANK	ADA
O	0	875	475	572	982	1375	3000
IST	0	0	400	532	943	1630	3380
KOC	0	400	0	133	543	1231	2980
SAK	0	532	133	0	410	1098	2847
BOL	0	943	543	410	0	687	2437
ANK	0	1630	1231	1098	687	0	1764
ADA	0	3380	2980	2847	2437	1764	0

To illustrate the working mechanism of TAH1, we provide a small example with 6 cities and 13 orders to be planned. The distances between the cities are given in Table 3.1 and the customer orders are given in Table 3.2. The first 3 letters of the customer names indicates the city they are located in. The addition "(s)" denotes that only a small-size truck can be used for delivery to the associated customer. For instance, customer "KOC1" denotes customer #1 in Kocaeli which can be serviced by either big- or small-size truck whereas IST4(s) denotes customer #4 in Istanbul which can only be serviced by a small-size truck.

Table 3.2. Demand data for the example

Customer	Product	Quantity(tons)	Due date
IST1	P1	3.0	1
KOC1	P3	2.0	1
BOL1	P2	3.2	1
IST2	P3	5.0	1
ANK1	P4	4.5	1
SAK1	P1	1.5	1
ADA1(S)	P5	3.5	1
IST3	P2	1.0	2
ANK2	P4	2.2	3
SAK2	P3	2.0	3
IST4(S)	P1	2.0	4
ADA2	P2	3.0	5
BOL2	P3	1.0	5

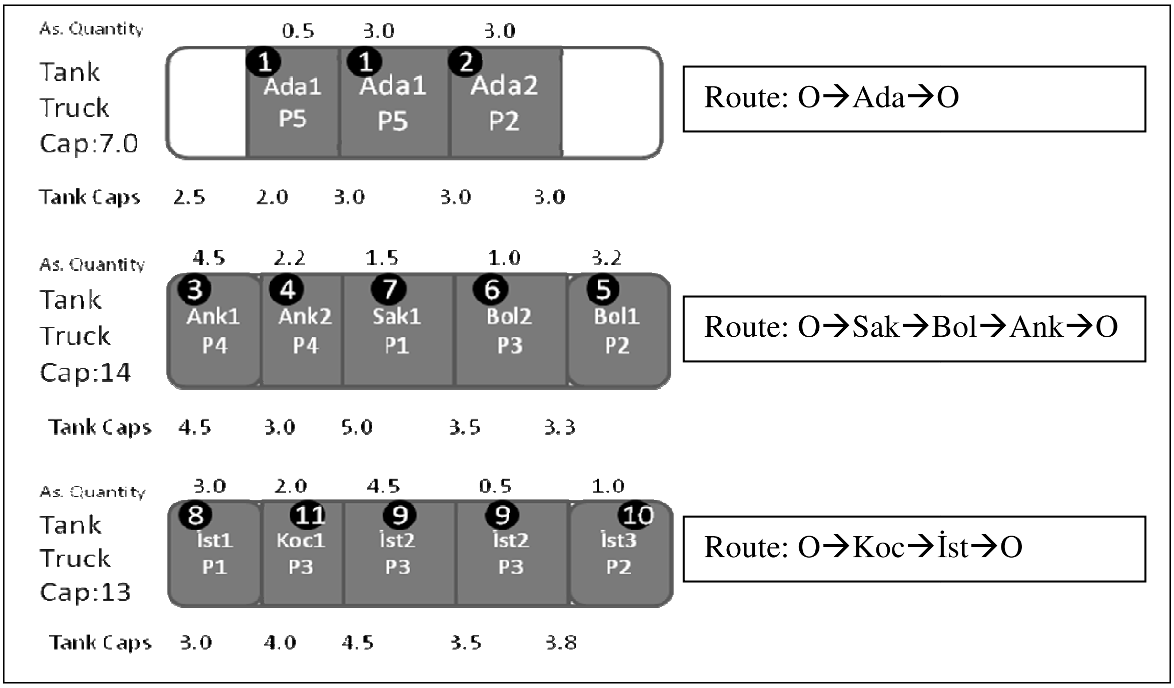

Figure 3.6. Solution of TAH1 for the example

The solution obtained by using TAH1 with parameter λ=500 is illustrated in Figure 3.6. The step-by-step explanation is as follows:

❶ Firstly order of a small customer with due date 1 which is farthest to the plant is selected: ADA1. Then a small truck with maximum load capacity is selected and the selected demand is assigned to the tank truck with the best fit. Since the order of ADA1 exceeds the capacity of all the tanks of the truck, it is assigned to two different tanks.

❷ IST4 is selected as the next small customer order and the cost of inserting it is calculated as follows:

$$min[C_{rd},(C_{rd}+C_{0d}-C_{0r})]=[3380,(3380+875-3000)]$$

Since 1255>500 and IST4 is due on day 4, it is not assigned to any of the trucks.

Since no other small customer order exists, select the customer that is located closest to the customer whose order has already been assigned: ADA2 is assigned to the tank truck with best fit. No more orders can be assigned due to the maximum load restriction.

❸ Select another demand of a customer farthest to the depot from period 1. ANK1 is selected and assigned to the tank truck with maximum capacity.

❹ Select another order of a customer nearest to ANK1. ANK2 is selected and assigned which is located in the same city.

❺ Select another demand of a customer nearest to ANK2. For BOL1 [687, (687+982-1375)] = 294<500. So BOL1 is selected and assigned.

❻ BOL2 is selected after BOL1 since it is in the same city with BOL1.

❼ After BOL2, SAK1 is selected as the nearest customer to BOL2. Extra cost is found as (410,410+572-982) = 0 which means SAK1 is on the way to BOL2. All the tanks are filled so select a different tank truck.

❽ Select an order of a customer farthest to the depot from period 1. IST1 is selected and assigned.

❾ IST2 is selected and assigned since it is located in the same city with IST1.

❿ IST3 is selected and assigned to the tank truck.

⓫ KOC1 is selected as the nearest customer to IST3 (min (400,400+475-875) = 0<500 so accept). IST4 is not selected because a big tank truck cannot serve a small customer. Since all the demands in period 1 are satisfied, the assignment step terminates. Next, the routes of the trucks are determined using the nearest neighbor heuristic.

The difference between TAH1 and TAH2 can be shown using the second tank truck with capacity 14 tons. Figure 3.7 shows the partially solution obtained by TAH2 for the second tank truck. Firstly ANK1 is assigned as in the TAH1. Then instead of looking at the whole demand list TAH2 looks only to the demands with due date 1. So the nearest customer is found as BOL1 to ANK1 and assigned. After that SAK1, KOC1 and IST1 is assigned to the tank truck in order.

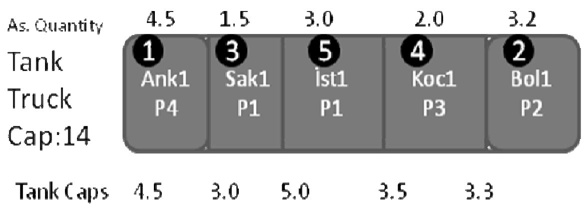

Figure 3.7. Assignment done by TAH2 for tank truck with capacity 14 tons.

Chapter 4 Numerical Results

In this chapter, the proposed algorithms are tested with the real data provided by the company and the results are discussed. Firstly the structure of the data is explained in the following section. Then, the results of the algorithms and the bounds obtained by CPLEX v.11 are reported and compared. The computational tests are performed on a notebook computer equipped with Intel Celeron 1.6GHz processor and 1 GB Ram. The algorithms are coded using Java programming language.

The data consist of capacities of tank trucks, cities where the customers are located at and order quantities with their due dates. However, the data we could obtain from the company is the shipments data which gives daily order deliveries. Thus, the exact due dates of the orders are not known. Therefore, we assume the delivery days as the due dates for experimental purposes. We have been also informed that the demand has reduced significantly due to the effect of the global economic crisis. To adjust this impact on the data, we combined the two month demand data into one month to better test the performance of the algorithms.

The tank trucks in the fleet of the logistics provider have different tank capacities and maximum total load limitations. The fleet consists of 10 tank trucks. As mentioned earlier, if additional truck capacity is needed it is hired from the spot market. Therefore, we have added 2 more trucks for capacity flexibility. Out of 12 tank trucks, 3 are small-size and 9 are big-size truck. The capacities of tanks and trucks are represented in tons.

The numerical results are reported in Table 4.1. The numbers are in monetary units that are kept fictitious for confidentiality reasons. Note that although we have monthly date the results include only the first 3 weeks of the month. This is due to the fact that the problem is solved on a rolling horizon basis and no solutions can be obtained for the 4^{th} week because 5^{th} week's data is not available.

Table 4.1. The results of the algorithms' runs with intensified data.

(λ=500)	TAH1	TAH2	LPH	CPLEX upper bound
Day1	14580	9463	14546	17069
Day2	4848	4492	6168	7768
Day3	2625	3607	5276	3643
Day4	475	2019	2372	1375
Day5	8516	7641	7850	7322
Week1 total	31044	27222	36212	37177
Day6	12635	13539	13520	18047
Day7	4000	4000	3500	5554
Day8	1750	2625	875	3769
Day9	875	3409	5097	4003
Day10	3100	2225	3372	3578
Week2 total	22360	25798	26364	34951
Day11	3733	1980	1375	3466
Day12	5421	5421	7307	4237
Day13	3214	3603	1979	17868
Day14	3413	2538	3412	6386
Day15	9529	7632	6814	6850
Week3 total	25310	21174	20887	38807
Total	**78714**	**74194**	**83463**	**110935**

The results show that the algorithm that gives the minimum transportation plan is different in each week. In week 1, TAH2 gives the minimum cost distribution plan whereas LPH provides the worst solution. TAH1 is the best in week 2 and LPH is slightly worse than TAH2. However LPH performs best in week 3 and is slightly better than TAH2. Because of the rolling horizon nature, the results obtained in the first few days may be misleading and an overall cost analysis may be more meaningful. First of all, we observe that both threshold accepting heuristics perform better than the LPH. Secondly, TAH2 outperforms TAH1 by 6%, which can be considered as a noticeable difference.

The computational times are also important in the comparison of the algorithms. TAH1 and TAH2 are both solved in a negligible time (less than 1 second) and their CPU time does not increase much with the increase in the size of the problem. However LPH needs more time to solve a given problem. The CPU time varied from 5 minutes to 1.5 hours in the runs shown in Table 4.1. The size of the problem determined by the active variables and constraints affects LPH significantly. Actually, the real benefit of TAH1 and TAH2 for the distribution planners is their solution time and ease of implementation. The data and model parameters can easily be modified to make sensitivity analyses. Furthermore, the heuristic system can be integrated into the company's database system effectively.

The CPLEX upper bounds are also observed for a comparison. The global time limit is set to be 2000 seconds to be able to obtain a feasible solution using CPLEX. However, in some cases CPLEX failed to find a feasible solution within this limit. In those cases, the global time limit is extended up to 3000 seconds. We did not consider the lower bounds since the optimality gap in the range of 90% in all of the runs and did not provide any meaningful information. As can be seen in Table 4.1, the upper bound by CPLEX is the worst among all methods. Furthermore, the difference between TAH2 and CPLEX upper bound is almost 50%.

We have noted that, in the 3rd week's demand data there are some demands from the 4th week as well. For instance, when the 15th day is being solved the 16th, 17th, 18th, 19th days are also considered and can be assigned to the tank trucks in the 15th day. Therefore, a heuristic may assign some of the orders due in the 4th week to the distribution plan in the 3rd week and the distribution cost in the 3rd week may be higher because of this reason. When we look at the remaining demands of TAH2 and TAH1, the number of remaining demands in TAH1 is less than TAH2. So, we cannot conclude that TAH2 is definitely better than TAH1.

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Table 4.2 Total costs achieved by TAH1 & TAH2 for different λ values.

λ	TAH1	TAH2
100	82087	83988
300	82897	76284
400	84942	77666
500	78756	74194
600	79534	74194
750	80625	74815
1000	80625	86965
1500	81756	89402

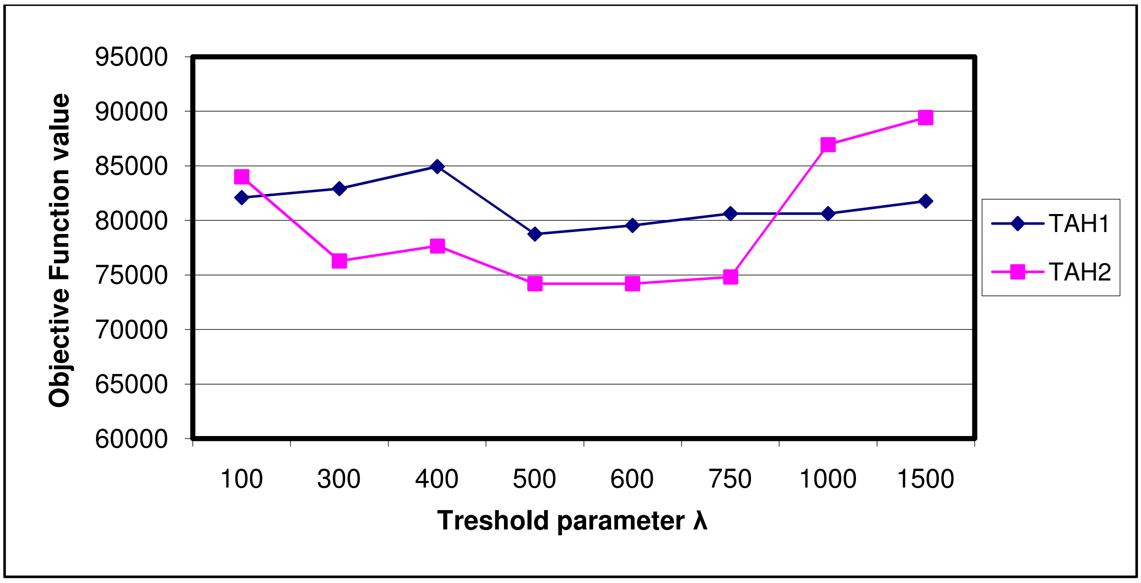

Figure 4.2 Line graph of the threshold parameter λ

We have also noted earlier that the threshold parameter λ is important for the performance of TAH1 and TAH2. In Table 4.2 a sensitivity analysis is performed for the parameter λ and it is found that both heuristics give the minimum cost distribution plan when λ is 500. The λ is set to be constant in this analysis through the 15 runs (3 weeks data that is analyzed in Table 4.1) and the results are given in terms of total cost of 3 weeks plan. In this analysis we can conclude that TAH2 is more sensitive to the threshold parameter. Indeed, this was an expected result because TAH2 attempts to assign the demands in period 1 firstly until the threshold parameter is not satisfied. Therefore, larger threshold parameter will cause longer trips and smaller ones will cause high tank truck usage and low utilization. The pattern of the line graphs in Figure 4.2 shows that TAH1 and TAH2 are both affected by λ in the same manner; however, TAH1 is affected slightly more. The small and large λ values result in high costs whereas intermediate λ values give better solution quality in both of the heuristics.

In the current system, the distribution plan is done by a worker in the logistic department. When the real plans are analyzed, it is easy to see the general tendency of the dispatcher, which is to generate single stop trips. On the other hand, the proposed algorithms are more likely to create trips with multiple stops. To make a comparison of the current practive to the proposed algorithms, we consider the data of a one full month and found the real cost of the distribution plans. The results show that in a month the cost of distribution excluding setup costs found to be 55375 TL. When the same month is solved by TAH1, TAH2 and LPH, it is found that TAH1 suggests a plan with cost of 49961, TAH2 gives a cost of 45334 and LPH finds a distribution plan with objective function value 49829. The details of the computational study are given in Table 4.3. Actually, this comparison does not show us the exact improvement because the real data of this month is not consistent for comparison because of lack of exact due dates.

Although we could not make a certain judgment about the level of improvement, we expect that the proposed heuristics will improve the current system significantly. Since the level of improvement of TAH2 is nearly 20 percent in the comparison mentioned above, the real improvement can be better than this percentage when the heuristics are adapted for the real problem and tested with consistent data.

Table 4.3. The results of the one month real data

(λ=500)	TAH1	TAH2	LPH	The current system
Day1	4075	4075	5193	6170
Day2	3697	2858	5010	2962
Day3	1857	2732	3419	2732
Day4	1144	1144	1144	1144
Day5	1159	1159	1375	3243
Week1	11932	11968	16141	16251
Day6	6677	5802	4568	4568
Day7	3125	4000	3931	4964
Day8	1750	875	2625	1750
Day9	0	0	0	0
Day10	875	875	1483	1750
Week2	12417	11552	12607	13032
Day11	875	875	2250	2250
Day12	1144	2019	1144	2019
Day13	4478	3603	608	1893
Day14	1663	1663	3012	2448
Day15	10669	6249	4912	5882
Week3	18829	14409	11926	14462
Day16	2225	2225	3975	3603
Day17	0	622	1497	2119
Day18	875	1750	875	2225
Day19	875	0	0	875
Day20	2808	2808	2808	2808
Week4	6783	7405	9155	11630
TOTAL	**49961**	**45334**	**49829**	**55375**

Chapter 5 Conclusions and Future Research

In this study, we considered the distribution planning problem of bulk lubricants at an energy company operating in Turkey. The problem had different properties and issues than the distribution problems studied in the literature. To solve the problem optimally, we developed a mixed integer mathematical model and tested it with the industrial data. Since the model is intractable in industry-size problems we proposed two heuristic approaches to efficiently solve it.

The proposed heuristics are tested with two different data sets, real data and intensified generated data. The numerical results show that threshold-accepting heuristics are very efficient in terms of computational times and are able to provide competitive results. On the other hand, the LP relaxation-based heuristic is rather inefficient in terms of computational time and solution quality in large-size problems; however it may provide reasonable results in small and medium size problems, with high computational effort though.

As for future research, the cleaning (setup) costs can be considered in the model and heuristic approaches may be improved accordingly. In that case, the loading procedure will need to consider the existing state of a tank while loading the lube oils. Furthermore, the impact of equipping the trucks with a flow-meter device may be investigated in more detail. A what-if type analysis may be performed to evaluate the benefit of installing the flow-meter to all or some of the tank trucks. Finally, in the current approach, the days are identical in terms of making the deliveries. However, the company would desire to deliver an order on the planned delivery date, as originally determined by the customer. Therefore, the model can be extended to involve penalty costs associated with the early and tardy deliveries.

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