OPEN LOOP POLICIES FOR SINGLE-LEG AIR-CARGO REVENUE MANAGEMENT

Birce Tezel

Submitted to the Graduate School of Engineering and Natural Sciences
in partial fulfillment of the requirements for the degree of
Master of Science

Sabancı University

August, 2012

OPEN LOOP POLICIES FOR SINGLE-LEG AIR-CARGO REVENUE MANAGEMENT

Approved by:

Assist. Prof. Dr. Nilay Noyan Bülbül ....
(Thesis Supervisor)

Assoc. Prof. Dr. J.B.G. Frenk ....
(Thesis Co-supervisor)

Assoc. Prof. Dr. Kerem Bülbül ....

Assist. Prof. Dr. Güvenç Şahin ....

Assoc. Prof. Dr. Koray Şimşek ....

Date of Approval:

to my family

Acknowledgements

Firstly, I would like to thank my thesis supervisor Dr. Nilay Noyan who supported, guided, encouraged and motivated me throughout my academic program. She taught me many valuable lessons and most importantly, she showed me how an ideal academician should be. This thesis would not have been completed without her.

Secondly, I am grateful to my thesis co-supervisor Dr. Hans Frenk for sharing his deep knowledge on Mathematics. I will always admire the enthusiasm and passion he has towards his research.

I would also like to show my gratitude to Gabor Rudolf for helping us to improve the exposition of my thesis. His efforts and support were invaluable especially during the last few stressful weeks.

My best friend and my dear fiancé, Semih Atakan never left my side and he always supported me no matter what. He always managed to put a smile on my face whenever I felt stressed and motivated me even in my most hopeless moments. My academic studies would have progressed in a much slower fashion without him.

My precious friends Nurşen, Mahir, Muzaffer, Mustafa, Çetin, Ceyda, Halil and Belma deserve a lot of credit for sharing their knowledge and experience, supporting me and cheering me up whenever I needed it the most. It always made me feel lucky knowing that I have such great people around me.

I would like to thank the Scientific and Technological Research Council of Turkey (TÜBİTAK) for providing the financial support.

Finally, I would like to thank each member of my family: İnci, Uğur, Bora, Müge and İnci Asel for their endless love, support and devotion. Although I could not manage to spend too much time with them during my academic studies, they were always understanding. If they did not set such good examples throughout my life, I would not have reached this point.

TEK BACAKLI HAVA KARGO GELİR YÖNETİMİ İÇİN AÇIK DÖNGÜ POLİTİKALARI

Birce Tezel

Endüstri Mühendisliği, Yüksek Lisans Tezi, 2012

Tez Danışmanları: Nilay Noyan Bülbül, J.B.G Frenk

Anahtar Kelimeler: hava kargo, gelir yönetimi, çokboyutlu kapasite, kapasite üstü rezervasyon, yer ayırtma limitleri, teklif fiyatlari, rassal programlama.

Özet

Kargo nakliyatı havayolları endüstrisinde belirgin bir gelir kaynağıdır. Bu sebeple, kargo işinin kendine mahsus zorluklarını hesaba katan yer ayırtma politikaları geliştirmek kritik bir öneme sahiptir. Bu zorluklar arasında çoğunlukla hacim ve ağırlık olarak ölçülen çok boyutlu kapasite yapısı ve rezervasyon yapılırken siparişin kapasite gereksinimlerinin genelde kesin olarak bilinememesi sıranalabilir. Yolcu gelir yönetiminde yöneylem araştırması methodlarının, kapasite üstü satım yüzünden ödenen ceza maliyetleri ile kapasite altı satım yüzünden oluşan fırsat maliyetleri arasındaki ödünleşimi göz önüne alarak kısıtlı kapasitenin etkin bir şekilde kullanılmasında oldukça faydalı olduğu görülmüştür. Bu tezde, benzer methodlar çeşitli kargo tiplerini taşıyan tek bacaklı uçuşların kapasite kontrol problemi için geliştirildi. Gelen rezervasyon taleplerini, yer ayırtma limitlerine veya teklif fiyatlarına bağlı olarak kabul eden veya reddeden açık döngü politikaları üzerinde çalışıldı. Uygun yer ayırtma limitlerini ve teklif fiyatlarını hesaplayabilmek için, belirsiz hacim ve ağırlık gereksinimleri varlığında, kapasite üstü satım maliyetlerini göz önünde bulunduran eniyileme modelleri geliştirildi. Önerilen modellerin yararlılığını değerlendirmek için kapsamlı bir sayısal çalışma yapıldı. Sayısal sonuçlar, politikalarımızın literatürdeki çeşitli yöntemlerle elde edilen göstergeler ile kıyaslandıklarında iyi bir performans sergilediklerini gösterdi.

OPEN LOOP POLICIES FOR SINGLE-LEG AIR-CARGO REVENUE MANAGEMENT

Birce Tezel

Industrial Engineering, Master's Thesis, 2012

Thesis Supervisors: Nilay Noyan Bülbül, J.B.G. Frenk

Keywords: air-cargo, revenue management, multi-dimensional capacity, overbooking, booking limits, bid-prices; stochastic programming.

Abstract

Transporting cargo is a significant source of revenue in the airline industry. It is therefore of critical importance to develop booking policies that address the unique challenges presented by the cargo business: the capacity is multi-dimensional, generally measured in terms of volume and weight, and the exact capacity requirements of a shipment are usually not known with certainty at the time of making booking decisions. Operations research methods have proven highly useful in passenger revenue management to effectively allocate a limited capacity while considering the trade-off between the penalty costs for oversold capacity and the opportunity costs for having unused capacity at the departure time. In this thesis, we develop similar methods for the capacity control problem over a single-leg flight with multiple cargo types. We study open loop policies that accept or reject a booking request for a certain type of cargo shipment based on booking limits or bid-prices. In order to compute suitable booking limits and bid-prices, we develop optimization models that incorporate off-loading costs under uncertain volume and weight requirements. We conduct a comprehensive computational study to evaluate the effectiveness of our proposed models. Numerical results demonstrate that our policies perform well compared to benchmarks established by various methods in the literature.

Contents

List of Figures

List of Tables

Chapter 1

Introduction

Transporting cargo, either on a dedicated cargo fleet or in the bays of passenger aircraft, is a significant and rapidly growing source of revenue in the airline industry. The International Air Transport Association (IATA) reports that system-wide global revenues from cargo in 2010 amounted to $49 billion, versus $371 billion from passengers (IATA, 2009). Moreover, during the same period cargo traffic volume has increased by 7%, versus a 4.5% increase in passenger traffic volume. Boeing's 2012 Current Market Outlook forecasts that the air-cargo industry will continue to grow at an average annual rate of 5.2% through 2031 (Boeing Company, 2012). Despite the obvious importance of the problem, only a relatively limited number of research studies have been dedicated to cargo revenue and capacity management, in sharp contrast to the extensive literature on passenger bookings.

Airlines typically sell cargo capacity either through allotment contracts, reserved for major customers, or on the spot market (also referred to as free sale), where there are no guaranteed capacities. In this thesis we focus on managing the capacity available for free sale. The main objective is to obtain booking policies that make accept/reject decisions as booking requests arrive over a booking period. The fundamental choice is between accepting a request for a relatively cheap shipment, and rejecting it to save capacity for a potential later arrival that could yield higher revenue. In this context, the capacity is perishable: unused (spoiled) capacity after the departure of a flight is worthless. Therefore, it is common practice to allow more bookings than the available capacity can accommodate, in order to compensate for late cancelations, no-shows, and overestimated capacity requirements of accepted shipments. The trade-off that underlies booking decisions is then between the denied service costs for oversold capacity (also known as off-loading costs), and opportunity costs for spoiled capacity at the departure time. As discussed in Kasilingam (1997), off-loading costs may include the costs of transporting excess cargo by alternative means, the costs of additional handling and storage, and the cost of lost goodwill.

The literature on the cargo revenue management highlights numerous essential differences between passenger and air-cargo services (see, e.g., Kasilingam, 1996):

- Capacity is not necessarily integer-valued, and it is multi-dimensional, generally measured in terms of volume and weight. Sometimes an additional dimension is also considered, namely, the number of container positions (see, e.g., Kasilingam, 1998). However, this third dimension is rarely mentioned in the literature, and, according to Pak and Dekker (2004), has no significant impact in practice.

- The exact volume and weight requirements of a cargo shipment are usually not known with certainty at the time of making booking decisions, and are observed only immediately prior to departure.

- Unlike in a passenger case, where each booking request is for a single uniform seat regardless of the fare class, different types of cargo have different capacity requirements. In addition, cargo types are also distinguished by their contents (e.g., flowers, clothes, electronics, or food), which affects shipping rates.

- The available capacity may also be uncertain until loading at the departure time, due to dependence on various factors including the capacity utilized by the allotment contracts, and the capacity requirements of passenger bags if the cargo is carried on a passenger aircraft.

These differences provide a significant incentive to develop booking policies that are specific to cargo capacity management, and address some of the unique challenges outlined above. The two main classes of booking policies commonly used in the revenue management literature are those based on booking limits, and those based on bid-prices. A booking limit is an upper bound on the number of requests than can be accepted for a particular type of product. According to a booking limit based policy, requests are accepted as long as booking limits are not reached. On the other hand, a bid-price policy specifies a threshold price that should be charged for a booking, and a booking request is accepted only if its net revenue exceeds the this price. Threshold prices for a shipment are usually set as the sum of the bid-prices of its expected capacity resource requirements, and the bid-prices themselves can be interpreted as the monetary opportunity costs associated with the resources consumed. These monetary values depend on factors such as the remaining capacity, the remaining time to departure, and expectations about future demand.

When the booking limits or bid-prices are allowed to change over time in response to such factors, they lead to dynamic booking policies that account for the behavior of the system over time. It is obvious that dynamic policies have the potential to perform better than their static counterparts. However, dynamic models are computationally challenging due to potentially intractable multi-dimensional state spaces, and solving them typically requires elaborate decomposition methods. For example, Levin et al. (2011) formulate the booking control problem on the spot market as a dynamic program, and use a Lagrangian-based decomposition strategy to approximate its value functions. We mention that there exist other, comparatively easier decomposition-based methods that provide approximate solutions for dynamic cargo booking control models, see, e.g., Amaruchkul et al. (2007). As an alternative, we focus on open-loop, or static, models, which are generally more tractable for practical use. Such methods can be used with a rolling time horizon approach, preserving the favorable computational properties of static models, while taking into the dynamic behavior of the booking system.

In this thesis we limit our attention to cargo bookings over a single-leg flight. Some airline companies, in particular charter airlines, only accept booking requests for single-leg flights. However, larger airline companies typically transport cargo through a network of locations connected by flights, and cargo booking requests specify an origin-destination pair (in contrast to passenger booking requests, which typically specify an itinerary of flights). The resulting network cargo capacity management problems are notoriously difficult, and solution methods often involve solving a series of single-leg subproblems. Similarly to the passenger case (see, e.g. Topaloglu, 2009), this means that efficient solution methods for single-leg problems are of high importance even in a network context.

The simplest booking limit policies (sometimes known as bucket allocations), partition the available capacity according to fare classes. However, in practice partitioned booking limits are rarely applied in a strict fashion. For instance, in a passenger context it is clearly not beneficial to reject a higher fare class request when there is available capacity for lower fare classes. Booking limits are therefore typically implemented in a nested, or hierarchical, manner. Under a nested policy, higher fare classes are allowed to use all the capacity reserved for lower fare classes. Since each accepted booking request consumes a single unit of resource (namely, a uniform seat), the nested structure can be specified solely on the basis of the net revenues associated with each fare class. However, in the cargo case, each shipment consumes different amounts of multi-dimensional capacity. Therefore, it is not trivial how to rank the cargo types when defining a nested structure. In this thesis, we propose various methods to develop nested cargo booking limits. To the best of our knowledge, this is the first such attempt in the cargo revenue management literature.

Our work on booking limits extends some of the passenger booking models proposed in Aydin et al. (2010) to cargo bookings. We first consider a two-phase method, where in the first phase we solve either a risk-based model or a service level-based model to determine a total booking limit. The risk-based model aims to maximize expected profits, while the service level-based one enforces a bound on the probability of overselling capacity. In the second phase we use an allocation method based on expected marginal seat revenue (EMSR) models to obtain nested booking limits. Our second-phase methods provide several ways to rank cargo types according to profitability. We also present a single-phase risk-based optimization model, which directly determines partitioned booking limits. These partitioned limits are then used in a nested fashion, using our EMSR-based ranking methods.

The booking limit approaches described above make the common assumption that off-loading costs follow a specific structure, namely, that they can be written as the sum of two convex functions, which represent the costs due to oversold volume and oversold weight (see, e.g., Amaruchkul et al., 2007; Huang and Chang, 2010). While this cost structure is more complex than overbooking costs in the passenger case (often assumed either to be constant (Chatwin, 1999), or to depend only on the fare class), the assumption that off-loading costs can be separated according to volume and weight is still somewhat restrictive. In addition to our booking limit policies, we also present two bid-price-based approaches, which do not rely on such assumptions. First, we adapt a traditional randomized linear programming (RLP) model that defines bid-prices for units of volume and weight capacity using the optimal dual variables associated with capacity constraints in the RLP formulations. We then present a two-stage RLP model, where booking decisions are made in the first stage, followed by off-loading decisions (which explicitly determine the shipments that are to be denied loading) in the second stage. The cargo off-loading problem we encounter in the second stage has previously been considered by Levin et al. (2011), while a similar two-stage approach has been proposed in the passenger literature by Kunnumkal et al. (2012).

We now briefly list the main contributions of this thesis.

- We develop new optimization models to compute booking limits and bid-prices for air-cargo capacity control on a single-leg flight. These models prove useful in developing computationally tractable and practical policies.
- We propose various methods to rank different cargo types, and thus obtain nested booking policies.
- We conduct a comprehensive computational study to evaluate the effectiveness of our proposed models. In particular, we compare our policies with those provided by various benchmark methods in the literature. Numerical results demonstrate that our policies perform well in general compared to the benchmarks.

The rest of the thesis is organized as follows. In Chapter 2 we review the literature on cargo revenue management, with a particular emphasis on mathematical programming based approaches. In Chapter 3 we describe the general problem setting, and present our optimization models. Section 4 is dedicated to implementation details, numerical results and managerial insights, while Section 5 contains our concluding remarks.

Chapter 2

Literature Review

Revenue management (RM), also known as yield management, has been one of the most successful application areas of operations research (Talluri and van Ryzin, 2005; Phillips, 2005). The primary objective of RM is to maximize revenues by selling the right product to the right customer at the right time for the right price[1]. Operations research methods have proven highly useful in airline passenger revenue management to effectively allocate a limited capacity while considering the trade-off between the penalty costs for oversold capacity and the opportunity costs for having unused capacity at the departure time. However, there is a less extensive literature on cargo RM in contrast to the passenger case. This can be partially attributed to the relatively higher complexity of cargo business as discussed in Kasilingam (1996) and Becker and Dill (2007). Despite these challenges, cargo RM has recently received increasingly more attention in the literature. Some of the existing approaches from the rich passenger revenue management literature have been and can be adapted to the cargo case. In this direction, it is essential to highlight the differences between cargo and passenger transportation as in Kasilingam (1996). Billings et al. (2003), Slager and Kapteijns (2004), and Becker and Dill (2007) also discuss the unique features of cargo RM and review the related operations and implementations from a practical point of view.

Many studies consider the cargo capacity management problem for a single-leg flight and the most popular issues include the two-dimensional capacity and random volume and weight requirements. Considering these issues Amaruchkul et al. (2007) formulate the booking control problem as a Markov decision process (MDP). However, due to the high dimensionality of this formulation, they cannot provide optimal policies. Instead,

[1] http://en.wikipedia.org/wiki/Revenue_management

they propose various heuristics and an upper bounding approach. Their best performing heuristic is based on the decoupling idea; decomposing the DP model over volume and weight dimensions. There are many papers which base their research on the dynamic model introduced by Amaruchkul et al. (2007). Huang and Chang (2010) tackle the same problem and develop an approximate algorithm which jointly estimates the expected revenue from weight and volume by sampling a limited number of points in the state space instead of decoupling the problem and estimating the expected revenue in a sequential manner as in Amaruchkul et al. (2007). Similarly, Zhuang et al. (2011) propose two heuristics but for a single-resource (one-dimensional capacity) problem. Huang and Hsu (2005) study uncertainty in supply; but they measure the capacity only in terms of weight and they ignore the off-loading costs. Kasilingam (1997) also considers the uncertainty in one-dimensional supply while trying to find the overbooking limit which minimizes the total expected off-loading and spoilage costs. Xiao and Yang (2010) consider the two-dimensional capacity, formulate the booking control problem as a continuous time MDP but for only two types of demand and propose a threshold policy under some concavity assumptions. Different than the above studies, Levin et al. (2011) present a model that integrates multiple allotment contracts and spot market bookings of an airline for a set of parallel flights. Unlike the existing studies, they also consider a off-loading problem to compute the boundary condition of the DP optimality equations which accounts for the total cost incurred at the departure time. As in Amaruchkul et al. (2007), they formulate the booking control problem on the sport market as a dynamic program. However, they construct approximations to its value functions using a Lagrangian approach to estimate the total expected profit from the spot market. Using these approximations and a cutting plane algorithm, they solve the allotment selection problem, which maximizes the sum of the profit from the allotments and the estimated total expected profit from the spot market. After this brief review of studies on dynamic models for the single-leg problem, we next focus on the static approaches which are particularly related to this thesis.

Although static models are widely studied in the passenger case, there are a few static models introduced for cargo RM. Among the heuristics proposed in Amaruchkul et al. (2007), there are two static methods that solve deterministic linear programs based on the expected values of the uncertain parameters. One is proposed to compute the bid-prices and the other one is used to obtain the partitioned booking limits. To the best of our knowledge, Amaruchkul et al. (2007) is the only study presenting a (partitioned) booking limit policy. Even if there has been little work on bid-price policies for controlling cargo booking, we can say that they are still the most common static policies. Therefore, we focus on the literature on bid-price policies. Han et al. (2010) model the air-cargo booking process as a discrete-time Markov chain for a single-leg flight by discretizing the volume and weight requirements and capacities. The expected revenue is written as a function of the bid-prices and the optimal bid-prices are obtained using the Markovian model. There are also bid-price policies for the network cargo capacity management. Pak and Dekker (2004) model the booking process as a two-dimensional on-line knapsack problem and use the greedy algorithm proposed in Rinnooy Kan et al. (1993) to solve the knapsack problem and compute the bid-prices. As in Han et al. (2010), it is assumed that no penalty is incurred when a booking request is rejected and the capacity requirements are known with certainty when a booking request arrives. On the other hand, Karaesmen (2001) introduces a LP based bid pricing model with a continuous attribute space for a simplified cargo booking control problem, where attributes represent the capacity requirements. Sandhu and Klabjan (2006) also present a mathematical programming formulation that provides bid-prices for controlling origin-destination cargo bookings on a network. However, they consider the fleet assignment model (FAM) which assigns a particular equipment type to each given flight-leg while maximizing profit. They develop a FAM that incorporates both passenger and cargo revenue; the model is obtained by combining the traditional leg-based FAM model with the passenger and cargo mix bid price models. Recently, Popescu et al. (2012) have developed optimization models to compute the bid-prices to control the booking over a network for a mixed demand pattern with individual and batch requests. They decompose the demand into small and largo cargo bookings. For the small and large cargo booking they use a probabilistic nonlinear program from passenger literature and a DP model to compute the bid-prices, respectively. However, the proposed model is based on itinerary-specific demand rather than the origin-destination-specific demand.

Another type of static policy is based on overbooking limits; if accepting a booking request for a cargo would bring the total volume and/or weight of the accepted cargoes above the specified overbooking limits, that cargo would be rejected. The overbooking strategy is meaningful in the existence of cancellations and no-shows. Luo et al. (2009) and Moussawi and Cakanyildirim (2005) allow no-shows and study two-dimensional cargo overbooking models to obtain a overbooking limit based policy. Moussawi and Cakanyildirim (2005) develop two (aggregate and detailed) types of models to obtain weight and volume overbooking limits maximizing the net profit. Their off-loading cost does not depend on the individual cargoes; it is a linear function of the maximum of the total off-loaded volume and weight. They express the showing up volume and weight in terms of the cargo density and provide equations to find an optimal overbooking curve parameterized by the cargo density, which is proved to be a box. The modeling approach used in Moussawi and Cakanyildirim (2005) is adapted from Luo et al. (2009). Differently, Luo et al. (2009) ignore the revenues and focus on minimizing the expected total spoilage and off-loading costs, which are additive over volume and weight dimensions.

Air-cargo RM problems feature some similarities to passenger RM problems with group (multiple seat) bookings. Van Slyke and Young (2000) study the finite-horizon stochastic knapsack problem and consider a single-leg passenger RM problem with group bookings as a special case of it. As emphasized in Amaruchkul et al. (2007), the algorithm proposed in Van Slyke and Young (2000) may be computationally impractical for solving large air-cargo booking control problems. Moreover, the capacity requirements and the available capacities are assumed to be integer. Due to the random consumption of the capacity, air-cargo booking control problems are related to the stochastic multi-dimensional knapsack problem. There are other studies on the dynamic stochastic knapsack problem (see, e.g., Kleywegt and Papastavrou, 1998; 2001), but they in general propose models that do not allow arrivals to have multi-dimensional capacity requirements.

Another stream of literature on cargo transportation is related to the network cargo RM. It is a fairly recent research topic investigated among others by Karaesmen (2001); Popescu (2006); Levina et al. (2011).

Chapter 3

Stochastic Optimization Models

In this chapter, we first describe the general setting for our problem of interest: determining booking policies for cargo capacity management in the presence of uncertain capacity requirements. We consider three types of modeling approaches and develop corresponding optimization models.

- We first consider a two-phase approach: in the first phase we solve either a risk-based or a service level-based model to determine a total booking limit. Then, in the second phase we use an *expected marginal seat revenue* (EMSR) based allocation method to obtain nested booking limits. In order to implement such a method it is necessary to rank different types of cargo in order to specify a nested structure. We introduce and discuss several such ranking heuristics.

- We next consider an optimization model which directly obtains partitioned booking limits for each cargo type, without the use of a predetermined total booking limit. Similarly to the first approach, these partitioned limits can be used in a nested fashion.

- The third modeling approach focuses on bid-price policies. We adapt two existing methods from the literature on passenger revenue management, which use randomized linear programming (RLP) techniques.

3.1 Problem Setting

We consider the problem of controlling cargo bookings for a single-leg flight which transports multiple types of cargo between a particular origin-destination pair. Our goal is to find booking policies that make accept/reject decisions for each cargo shipment request. In particular, we focus on open loop policies based on booking limits or bid-prices.

Booking requests typically specify the type of a cargo shipment, but not its exact volume and weight requirements. However, we assume that the joint distribution for the volume and weight of a shipment is available for each cargo type, and the exact volume and weight are observed immediately before the departure time. Let us denote the available volume and weight capacities of a flight by C_v and C_w, respectively. If these capacities are not sufficient to accommodate all reserved cargo, some shipments are off-loaded to be transported by alternative flights or other cargo carriers. In such situations the airline incurs a penalty cost, similar to the overbooking penalty incurred for passengers that are denied boarding. We note that in the literature off-loading is often considered in the context of overbooking, i.e., when requests can be accepted in excess of available capacities in order to compensate for potential cancelations and no-shows (Moussawi and Cakanyildirim, 2005; Luo et al., 2009). In contrast, in our models off-loading can occur even under conservative booking policies, as a consequence of stochastic volume and weight requirements.

To quantify off-loading costs we adopt a common approach (Amaruchkul et al., 2007; Huang and Chang, 2010), and consider the sum of two convex functions h_v and h_w, which represent the costs due to the oversold volume and weight, respectively. In the literature the following choice of convex functions is commonly used:

$$h_v(x_v) = \theta_v[x_v - C_v]_+, \quad h_w(x_w) = \theta_w[x_w - C_w]_+, \tag{3.1}$$

where θ_v and θ_w are non-negative constants, and the variables x_v and x_w represent the total volume and weight of accepted shipments, respectively. This approach implicitly assumes that cargo shipments are divisible, and can be partially off-loaded; Moussawi and Cakanyildirim (2005) provide a discussion on the conditions under which such an assumption is justified. Recently, Levin et al. (2011) have proposed an alternate method which explicitly solves an "off-loading problem" by identifying the individual shipments that are to be denied loading. To implement this idea, we develop a two-stage stochastic programming model which leads to an RLP formulation. While Kunnumkal et al. (2012) consider a similar model to control passenger bookings, to the best of our knowledge no analogous developments exist in the cargo literature.

We now introduce some additional notation used throughout the rest of the thesis. We consider booking requests for a single-leg flight; each request concerns a single shipment which belongs to one of m cargo types. For $i = 1, \ldots, m$ we let (V_i, W_i) denote a random vector whose two components have the same joint probability distribution as the volume and weight of a shipment which belongs to type i. More precisely, we denote the volumes and weights of individual type-i requests by $(V_{i1}, W_{i1}), (V_{i2}, W_{i2}), \ldots$, and assume that these vectors are mutually independent and identically distributed (i.i.d.) as (V_i, W_i). In our models the distributions of $(V_1, W_1), \ldots, (V_m, W_m)$ are assumed to be given, with respective expected values of $(\mu_1^v, \mu_1^w), \ldots, (\mu_m^v, \mu_m^w)$.

Remark 1 *While cancelations lie outside the scope of this thesis, our modeling approach can naturally incorporate no-shows by allowing the random vectors* (V_i, W_i) *to take value* $(0, 0)$ *with a positive probability.*

The *dimensional weight* of a shipment with volume v is v/γ, where γ is a constant (sometimes referred to as *inverse density*) defined by the IATA volumetric standard. The revenue (or margin) obtained from accepting a type-i booking request with volume v and weight w is given by $r_i(\max(w, v/\gamma))$, where $r_i : \mathbb{R} \to \mathbb{R}$ is a revenue function associated with the cargo type. The corresponding expected revenue is denoted by

$$\rho_i = \mathbb{E}[r_i(\max(W_i, V_i/\gamma))], \quad i \in \{1, \ldots, m\}. \tag{3.2}$$

We also use some standard mathematical notation and conventions. Random variables are typically denoted by uppercase letters, while vectors are denoted by lowercase boldface letters. The indicator random variable of an event A, which takes value 1 if the event A occurs and 0 otherwise, is denoted by $\mathbf{1}_A$. The cumulative distribution function (CDF) of a random variable X is denoted by F_X. If two random variables X and Y have the same distribution, we denote this fact by $X \overset{d}{=} Y$. The positive part of a number x is denoted by $[x]_+ = \max(x, 0)$. The set of natural numbers is denoted by $\mathbb{N} = \{0, 1, \ldots\}$, while the set of the first n positive integers is denoted by $[n] = \{1, \ldots, n\}$.

3.2 Booking Limit Policies

A booking limit is an upper bound on the number of requests than can be accepted for a particular type of product (for a fare class in the passenger case, and for a certain shipment type in the cargo case). According to a booking limit policy, requests are accepted as long as limits are not reached. There are two main types of booking limits: *partitioned* and *nested*. Partitioned booking limits are enforced in a strict fashion, where capacities reserved for a particular product type cannot be used to accommodate booking requests for a different type. However, such restrictive policies can lead to suboptimal results. For instance, in a passenger booking context it is not desirable to reject a higher fare class request when there is capacity available for lower fare classes. Therefore, booking limits are typically used in a hierarchical, or nested, manner. Under a nested policy, higher ranked classes are allowed to use the capacity reserved for lower ranked classes.

To the best of our knowledge, Amaruchkul et al. (2007) is the only study in the cargo revenue management literature which develops a partitioned booking limit based policy, and this thesis is the first to develop nested booking limits. We also remark that in a cargo context it is possible to establish booking limits in terms of volume and weight capacities (instead of the number of shipments). While this appears to be a natural approach, we are not aware of any existing studies featuring such booking limits.

3.2.1 A Two-Phase Method

In this section we describe a two-phase method to obtain a booking limit policy. In the first phase we determine a total booking limit, then use an EMSR-based capacity allocation method in the second phase to calculate nested booking limits for various cargo types. A similar two-phase scheme has been considered for controlling passenger bookings (see, e.g., Phillips, 2005; Aydin et al., 2010), and Kasilingam (1997) highlights the importance of such an approach for cargo bookings. However, as existing methods cannot be directly applied to the cargo case, we need to develop non-trivial extensions.

We note that the methods mentioned above tackle the slightly different problem of determining overbooking limits in the presence of no-shows (and sometimes cancelations). There are a number of papers in the cargo literature that focus on the initial phase of finding an overbooking limit in terms of capacity units (Kasilingam, 1997; Moussawi and Cakanyildirim, 2005; Luo et al., 2009). To the best of our knowledge, there are no corresponding studies that develop partitioned or nested policies in a two-phase framework.

3.2.1.1 First Phase: Total Booking Limit

In this section we detail two methods to determine a total booking limit. A total booking limit b can be used to define a greedy policy, which accepts any booking requests regardless of cargo type, as long as the total number of reservations is below b. Our goal is to find booking limits that lead to optimal performance under such a greedy policy.

In our model we consider booking requests that arrive according to a point process during the time period leading up to the departure of a flight. The total number of requests that arrive during this period is denoted by D; we assume that this non-negative integer random variable is bounded, and its distribution is known. Using the greedy policy outlined above, the total number of accepted booking requests is given by $N(b) := \min(b, D)$.

We denote the probability that an individual booking request is for cargo of type i by p_i, $i \in [m]$, and assume that the types of various requests are mutually independent. The probabilities p_i, which in our model are considered to be known, necessarily satisfy the equation $\sum_{i=1}^{m} p_i = 1$.

Observation 1 *Recalling that the volume of a type-i shipment is distributed as the random variable V_i, it is easy to see that the volume of a shipment associated with an individual booking request of undetermined type has a mixture distribution obtained from V_i, $i \in [m]$, with corresponding mixing weights p_i, $i \in [m]$. Formally, the volumes of shipments are i.i.d. as a random variable $\overline{V}$ with CDF $F_{\overline{V}} = \sum_{i=1}^{m} p_i F_{V_i}$. Analogously, the weights of shipments are i.i.d. as a random variable $\overline{W}$ with CDF $F_{\overline{W}} = \sum_{i=1}^{m} p_i F_{W_i}$.*

Let us denote the total number of accepted type-i requests by $N_i(b)$. Conditional on $N(b)$, the values $N_i(b)$, $i \in [m]$, follow binomial distributions, while their joint distribution is multinomial. More precisely, we have

$$\begin{aligned} N_i(b) \Big| (N(b) = n) &\stackrel{d}{=} \text{Binomial}(n, p_i) \;\text{ for } i \in [m], \\ (N_1(b), \ldots, N_m(b)) \Big| (N(b) = n) &\stackrel{d}{=} \text{Multinomial}(n, p_1, \ldots, p_m). \end{aligned}$$

If we aggregate shipments by type, the total volume of shipments corresponding to accepted booking requests can be expressed as $V^r = \sum_{i=1}^{m} \sum_{j=1}^{N_i(b)} V_{ij}$. On the other hand, Observation 1 provides an alternative way to compute the distribution of this total volume, leading to the following formula:

$$V^r = \sum_{i=1}^{m} \sum_{j=1}^{N_i(b)} V_{ij} \stackrel{d}{=} \sum_{j=1}^{N(b)} \overline{V}_j, \tag{3.3}$$

where the random variables $\overline{V}_j$ are i.i.d. as $\overline{V}$. The following analogous formula holds for the total weight:

$$W^r = \sum_{i=1}^{m} \sum_{j=1}^{N_i(b)} W_{ij} \stackrel{d}{=} \sum_{j=1}^{N(b)} \overline{W}_j, \tag{3.4}$$

where the random variables $\overline{W}_j$ are i.i.d. as $\overline{W}$. For the sake of completeness, in Appendix

A we also provide an analytical proof for the above results (stated as the essentially equivalent Lemma 6).

We now proceed to propose two stochastic optimization models that determine total booking limits; the choice between these two models depends on the decision maker's preferences. The first one is a risk-based model which considers the trade-off between the potential revenue from accepting an additional booking request, and the penalty cost of an additional off-loaded shipment. The second model aims to find the largest possible booking limit which still allows the airline to guarantee a certain level of service.

A Risk-Based Model

We now present an optimization problem, adapted from Aydin et al. (2010), where the goal is to find a total booking limit which maximizes the expected net revenue under the greedy policy outlined in the beginning of this section.

$$\max\left\{\sum_{i=1}^{m}\rho_i p_i \mathbb{E}[N(b)] - \mathbb{E}[h_v(V^r)] - \mathbb{E}[h_w(W^r)] \;:\; b\in\mathbb{N}\right\} \qquad (\mathrm{Risk\_TB})$$

We can utilize formulas (3.3)-(3.4) to reformulate the above problem. Let us introduce the function $f:\mathbb{N}\to\mathbb{R}$ given by

$$f(b) = \sum_{i=1}^{m}\rho_i p_i b - \mathbb{E}\left[h_v(\sum_{j=1}^{b}\overline{V}_j)\right] - \mathbb{E}\left[h_w(\sum_{j=1}^{b}\overline{W}_j)\right], \qquad (3.5)$$

where all $\overline{V}_j$ are i.i.d. as the random variable $\overline{V}$, while all $\overline{W}_j$ are i.i.d. as $\overline{W}$ (as introduced in Observation 1). Then we can write problem ($\mathrm{Risk\_TB}$) as

$$\max\left\{\mathbb{E}[f(N(b))] : b\in\mathbb{N}\right\}. \qquad (3.6)$$

The following two lemmas show that both the function f and the objective function $b\mapsto\mathbb{E}[f(N(b))]$ are discrete concave.

Lemma 1 *Let $X_1, X_2, \ldots$ be i.i.d. non-negative random variables with common CDF F_X, and let h be a non-decreasing convex function. Then the mapping $b\mapsto\mathbb{E}\left[h(\sum_{j=1}^{b}X_j)\right]$ is discrete convex.*

Proof. It is sufficient to show that $\mathbb{E}\left[h\left(\sum_{j=1}^{b+1} X_j\right)\right] - \mathbb{E}\left[h\left(\sum_{j=1}^{b} X_j\right)\right]$ is a non-decreasing function of b. Using the law of total expectation, we have

$$\begin{aligned}\mathbb{E}\left[h(\sum_{j=1}^{b+1} X_j)\right] - \mathbb{E}\left[h(\sum_{j=1}^{b} X_j)\right] &= \mathbb{E}\left[\mathbb{E}\left[h(X_{b+1} + \sum_{j=1}^{b} X_j) - h(\sum_{j=1}^{b} X_j)\,\middle|\, X_{b+1}\right]\right] \\ &= \int_0^{\infty} \mathbb{E}\left[h(x + \sum_{j=1}^{b} X_j) - h(\sum_{j=1}^{b} X_j)\right] dF_X(x).\end{aligned}$$

It follows from the convexity of h and the non-negativity of dF_X that the above function is non-decreasing in b, which completes our proof. ■

Lemma 2 *If f is a discrete concave function, then the mapping $b \mapsto \mathbb{E}[f(N(b))]$ is also discrete concave.*

Proof. Similarly to the previous lemma, it is sufficient to show that the difference $\mathbb{E}[f(N(b+1))] - \mathbb{E}[f(N(b))]$ is a non-increasing function of b. Since $D \leq b$ implies $N(b+1) = N(b) = D$, we have

$$E[f(N(b{+}1))] {-} \mathbb{E}[f(N(b))] = \mathbb{E}[f(N(b{+}1)) {-} f(N(b))] = \mathbb{P}(D \geq b{+}1)(f(b{+}1) {-} f(b)).$$

As the function f is discrete concave, $f(b+1) - f(b)$ is non-increasing in b. In addition, the probability $\mathbb{P}(D \geq b+1)$ is clearly also a non-increasing function of b, which implies the desired result. ■

Interestingly, under our assumptions the optimal total booking limit does not depend on the distribution of the number of booking requests. For the proof of the following result we refer the reader to Aydin et al. (2010).

Lemma 3 *If f is a discrete concave function and the problem $\max\{f(b) : b \in \mathbb{N}\}$ has a finite optimal solution b_{OPT}, then this is also an optimal solution of the problem $\max\{\mathbb{E}[f(N(b))] : b \in \mathbb{N}\}$.*

Since Lemmas 1 and 2 show that the objective function of (3.6) is discrete concave, we can obtain an optimal solution as follows.

$$b_{\text{OPT}} = \inf\{b \in \mathbb{N} : \mathbb{E}[f(N(b+1))] - \mathbb{E}[f(N(b))] < 0\}. \tag{3.7}$$

Taking into account Lemma 3, the above formula can be further simplified:

$$b_{\text{OPT}} = \inf\{b \in \mathbb{N} : f(b+1) - f(b) < 0\}. \tag{3.8}$$

We note that since the total number of booking requests is bounded from above, it is sufficient to consider a bounded range of possible booking limits. It follows that we can replace the $\inf$ operator in (3.8) by $\min$, and perform a discrete one-dimensional search to obtain an optimal solution b_{OPT}. To numerically evaluate the function f during this search, one can use Monte Carlo simulation, or, under certain additional assumptions, use analytical approximations. In Appendix C we provide additional details on how to perform the necessary calculations, and discuss a normal approximation.

A Service Level Based Model

Service level constraints are often considered in the passenger revenue management literature in order to control the extent of overbooking. For example, a type-I service level constraint imposes the requirement that the probability of overbooking be less than or equal to a specified value (see, e.g., Phillips, 2005). To the best of our knowledge, similar constraints have not yet been discussed in the cargo literature. In this section we aim to introduce this approach in a cargo context, taking into account the multi-dimensional capacity requirements. We propose a constraint that limits the probability of oversale, i.e., of the event that either the total volume or the total weight of accepted shipments exceeds the available capacity. This leads to the following alternative to the risk based model ($\mathrm{Risk\_TB}$):

$$\max\left\{ b \in \mathbb{N} \,:\, \mathbb{P}\left(\sum_{j=1}^{N(b)} \overline{V}_j \geq C_v \ \text{OR} \ \sum_{j=1}^{N(b)} \overline{W}_j \geq C_w \right) \leq 1-\alpha \right\}, \qquad (\mathrm{Service\_TB})$$

where α is a specified service level (such as 0.95). One can use a Monte Carlo simulation method to approximate the probability of oversale, which is typically hard compute otherwise.

3.2.1.2 Second Phase: EMSR-Based Heuristics

In passenger revenue management, booking limits are typically used in a nested fashion, where the capacity that is available for sale to a particular fare class can also be sold to a more expensive fare class. Littlewood's rule (Littlewood, 1972) provides a well-known method to optimally determine such booking limits for the case of two fare classes. Heuristics based on expected marginal seat revenue (EMSR) (Belobaba, 1987; 1989) extend Littlewood's rule to multiple classes, and are widely used to find nested booking limits. The popularity of EMSR-based methods is in a large part due to their intuitive and practical nature (see, e.g., Talluri and van Ryzin, 2005), which motivates us to develop similar heuristics for cargo bookings. Before we present our methods, we briefly outline the EMSR-based approach as it is used in the passenger literature, then discuss the challenges that arise when one attempts to adapt this methodology to a cargo context.

EMSR in Passenger Booking

Let us consider a passenger flight with C seats available for sale to m classes of passengers, and assume that passenger classes are indexed in decreasing order of revenue values, i.e., $\rho_1 \geq \cdots \geq \rho_m$. In accordance with common practice in the literature, instead of referring to booking limits we can equivalently describe booking controls in terms of *protection levels*. These levels can be viewed as the complements of booking limits with respect to the capacity available for sale, and represent the amount of capacity saved for various classes of products. More precisely, the jth protection level, which we denote by y_j, is the amount of capacity saved for sale to classes j and lower. Protection levels form an increasing sequence $y_1 \leq \cdots \leq y_m = C$, and thus define a nested structure.

There are two main types of EMSR heuristics to determine protection levels. EMSR-a first calculates protection levels by applying Littlewood's rule to successive fare classes, then aggregates these to obtain the protection levels which define the booking policy. Since EMSR-a ignores statistical averaging effects, it has a tendency to produce protection levels that are overly conservative. EMSR-b addresses this issue by aggregating the demand across classes (instead of aggregating protection levels). While some studies that compare these heuristics have shown mixed results (see, e.g., Talluri and van Ryzin, 2005), EMSR-b appears to be more popular in practice, and is considered to generally perform better than EMSR-a. Accordingly, in this thesis we focus on EMSR-b. Before attempting to adapt this heuristic to a cargo context, we provide a short formal description of the method in the passenger case.

Let D_i denote the random total demand for class-i seats. At stage j of the EMSR-b heuristic we compute how much capacity to protect for the classes $j, j-1, \ldots, 1$ as follows.

$$\hat{y}_j = \max \left\{ y \in \{0, \ldots, C\} \ : \ \rho_{j+1} - \overline{\rho}_j \mathbb{P} \left(\sum_{i=1}^{j} D_i \geq y \right) \leq 0, \right\}, \quad j \in [m-1], \tag{3.9}$$

where $\overline{\rho}_j$ denotes the weighted-average revenue, calculated as $\overline{\rho}_j = \frac{\sum_{i=1}^{j} \rho_i \mathbb{E}[D_i]}{\sum_{i=1}^{j} \mathbb{E}[D_i]}$. Since the $\hat{y}_j$ values are not guaranteed to form a non-decreasing sequence, we define the protection levels as

$$y_j = \max\{\hat{y}_1, \ldots, \hat{y}_j\}, \quad j \in [m-1].$$

The main challenge in applying EMSR-b is to calculate the distributions of the aggregated demands that appear in (3.9). We list here some approaches that lead to tractable formulations under appropriate modeling assumptions.

- If the demands $D_i, \ \ i \in [m]$, are i.i.d with Poisson or normal distribution, the distributions of the aggregated demands are of the same respective type.
- More generally, if $D_i, \ \ i \in [m]$, are independent, we can numerically calculate the distributions of the aggregated demands using the fast Fourier transform (FFT) method (see, e.g., Tijms, 2003).
- If the demands D_i are not independent, but have a multinomial structure (similar to the situation outlined in Section 3.2.1.1), then the aggregated demands follow binomial distributions.

Adapting EMSR Methodology to Cargo Booking

In the passenger case, every accepted booking request consumes one uniform seat, therefore fare classes with higher revenues are always more profitable. This property leads to a naturally defined nested structure, based solely on revenue values. In contrast, cargo shipments have capacity requirements in multiple dimensions. A shipment which brings higher revenue may consume more capacity, and therefore be less profitable, than another shipment which brings lower revenue. Defining a nested structure among cargo types is therefore a highly non-trivial problem. Analogously to EMSR-b, we aim to find appropriate coefficients ϱ_i, associated with each cargo type $i \in [m]$, that quantify the marginal profitability of type-i shipments.

We now turn our attention to the problem of finding suitable profitability coefficients. We take as our starting point the following two-dimensional knapsack problem, which provides capacity allocations based on expected demands and expected capacity requirements.

$$\begin{aligned} \max \sum_{i=1}^{m} \rho_i x_i & \\ \text{subject to} \sum_{i=1}^{m} \mu_i^v x_i \le C_v & \\ \sum_{i=1}^{m} \mu_i^w x_i \le C_w & \\ x_i \le \mathbb{E}[D_i] \qquad & i = 1, \ldots, m \\ x_i \in \mathbb{N} \qquad & i = 1, \ldots, m \end{aligned} \tag{KS\_Alloc}$$

We refer to the continuous relaxation of the above integer program as (RKS_Alloc). Similar knapsack-based allocation models are widely used in the passenger booking literature to obtain bid-prices (see Section 3.3). In a cargo context, Amaruchkul et al. (2007) consider the problem (RKS_Alloc), while Pak and Dekker (2004) utilize the 0-1 version of (KS_Alloc) in an on-line booking system. Along these lines, we propose three types of profitability coefficients based on knapsack formulations, which in turn define corresponding nested structures for our cargo booking policies. Intuitively, a profitability coefficient ϱ_i can be interpreted as the ratio of the net revenue and some scalar measure of the capacity requirements associated with shipments of type i.

Type 1: Based on effective capacity Akçay et al. (2007) propose a greedy algorithm to solve multi-dimensional knapsack problems. They consider the *effective capacity* for an item, which in our context can be computed for shipments of type i as $\min(\lfloor \frac{C_v}{\mu_i^v} \rfloor, \lfloor \frac{C_w}{\mu_i^w} \rfloor)$. Their greedy algorithm then ranks items based on the product of associated revenue and effective capacity. Accordingly, we introduce the following coefficients:

$$\varrho_i = \rho_i \min(\lfloor \frac{C_v}{\mu_i^v} \rfloor, \lfloor \frac{C_w}{\mu_i^w} \rfloor), \quad i \in [m]. \tag{3.10}$$

Note that the inverse of the effective capacity for a cargo type can be viewed as the "effective capacity requirement" of type-i shipments.

Type 2: Based on weighted sums of expected capacity requirements Another class of greedy algorithms to solve multi-dimensional knapsack problems, proposed by Rinnooy Kan et al. (1993), ranks items based on the ratio of their profit and a weighted sum of their capacity requirements. Accordingly, for any positive weights α^v and α^w, we can consider coefficients of the form

$$\varrho_i = \frac{\rho_i}{\alpha_v \mu_i^v + \alpha_w \mu_i^w}, \quad i \in [m]. \tag{3.11}$$

Note that under the non-restrictive assumption $\alpha_v + \alpha_w = 1$ the denominator becomes a weighted average of capacity requirements. Rinnooy Kan et al. (1993) propose a simple method, based on combinatorial enumeration, to determine weights that lead to optimal performance of the greedy algorithm. For the sake of completeness, in Appendix F we briefly describe how to obtain these optimal weights.

Remark 2 *Pak and Dekker (2004) use the optimal weights in a cargo context to obtain bid-prices for units of capacity. Along these lines, it is always possible to define profitability coefficients based on bid-prices. Given respective bid-prices λ_v and λ_w for units of volume and weight, one can calculate a scalar measure of the capacity requirements of type-i cargo as the weighted average $\frac{\lambda_v \mu_i^v + \lambda_w \mu_i^w}{\lambda_v + \lambda_w}$. Omitting a constant factor, this leads to the following coefficients:*

$$\varrho_i = \frac{\rho_i}{\lambda_v \mu_i^v + \lambda_w \mu_i^w}, \quad i \in [m].$$

Type 3: Based on a Lagrangian approach One-dimensional continuous knapsack problems can be solved optimally by a simple greedy approach, which ranks items according to the ratio of their value and either their volume or their weight. To make use of this natural ordering, we consider continuous Lagrangian relaxations of ($\mathrm{RKS\_Alloc}$), where one of the capacity constraints is dropped, and a term that penalizes its violation amount is added to the objective function. For example, if we relax the weight capacity constraint, we obtain the following Lagrangian relaxation:

$$\begin{aligned} \max \sum_{i=1}^{m} \rho_i x_i + \lambda_w \left(C_w - \sum_{i=1}^{m} \mu_i^w x_i \right) & \\ \text{subject to} \sum_{i=1}^{m} \mu_i^v x_i \leq C_v & \\ x_i \leq \mathbb{E}[D_i] & \qquad i = 1, \ldots, m. \end{aligned} \tag{LRP^w}$$

For any fixed value of the Lagrange multiplier λ_w the above linear program can be viewed as a continuous knapsack problem, where type-i shipments correspond to items of value

$\rho_i - \lambda_w \mu_i^w$ and volume μ_i^v. Accordingly, we can define profitability coefficients as

$$\varrho_i = \frac{\rho_i - \lambda_w \mu_i^w}{\mu_i^v}, \quad i \in [m]. \tag{3.12}$$

If we rank cargo types according to these coefficients, then, as mentioned above, we can find an optimal solution to problem (LRP^w) by using a greedy algorithm. As an alternative to (LRP^w), we can consider the Lagrangian relaxation obtained by dropping the volume capacity constraint. Analogously to the previous case, we arrive at profitability coefficients of the form

$$\varrho_i = \frac{\rho_i - \lambda_v \mu_i^v}{\mu_i^w}, \quad i \in [m]. \tag{3.13}$$

It remains to provide suitable values for the Lagrangian multipliers λ_w and λ_v. A natural choice is to use the optimal dual variables associated with the capacity constraints in the LP ($\mathrm{RKS\_Alloc}$). In this case both of the Lagrangian relaxations have the same optimal solution as ($\mathrm{RKS\_Alloc}$), in accordance with the theory of LP duality. Notice that the Lagrange multipliers can be interpreted as shadow prices. In the passenger literature it is common practice to use shadow prices from randomized LP formulations (see, e.g., Talluri and van Ryzin, 1999). Along similar lines, in Section 3.3.1 we outline a method to obtain Lagrange multipliers λ_w and λ_v by solving a randomized version of ($\mathrm{RKS\_Alloc}$).

If the profitability coefficients are given based on one of the three methods, one can use Algorithm 1 to obtain EMSR-type protection levels.

3.2.2 A Risk-Based Model for Partitioned Booking Limits

As an alternative to the two-phase method, we present a risk-based model, originally introduced for passenger bookings by Aydin et al. (2010), that obtains partitioned booking limits without relying on a predefined total booking limit. The goal is to maximize the expected total net revenue, defined as the difference between the expected revenue from the accepted booking requests, and the expected total off-loading cost paid as a penalty for not shipping booked cargo.

As before, we denote the number of type-i booking requests by D_i, $i \in [m]$, and assume that these random variables are bounded, and their distributions are known. However, due to our use of approximation methods, knowledge of the joint distribution is not necessary. If b_i denotes a booking limit for type-i cargo, the number of accepted type-i booking requests is given by $N_i(b_i) = \min(b_i, D_i)$. If we denote an upper bound of the random variable D_i by M_i then, as the inequality $b_i > M_i$ implies $N_i(b_i) = N_i(M_i)$,

Algorithm 1 Two Phase Method for Computing the Nested Booking Limits

1: [**INPUTS**] Cargo types are ordered according to their profitability coefficients, i.e., $\varrho_1 \geq \cdots \geq \varrho_m$. Denote the total number of type-i booking requests that arrive during the booking period by D_i. The joint distribution of $D_1, \ldots, D_m$ is given.

2: [**FIRST PHASE**] Define a total booking limit b. A suitable value can be found by solving either problem ($\mathrm{Risk\_TB}$) or problem ($\mathrm{Service\_TB}$).

3: [**SECOND PHASE**] Analogously to (3.9), compute protection levels via the following formula:

$$\hat{y}_j = \max\left\{ y \in \{0, \ldots, b\} \ : \ \varrho_{j+1} - \overline{\varrho}_j \mathbb{P}\left(\sum_{i=1}^{j} D_i \geq y \right) \leq 0, \right\}, \quad j \in [m-1],$$

where $\overline{\varrho}_j$ denotes the weighted-average profitability, calculated as $\overline{\varrho}_j = \frac{\sum_{i=1}^{j} \varrho_i \mathbb{E}[D_i]}{\sum_{i=1}^{j} \mathbb{E}[D_i]}$. To ensure that protection levels are non-decreasing, we again set

$$y_j = \max\{\hat{y}_1, \ldots, \hat{y}_j\}, \quad j \in [m-1].$$

---

we can restrict ourselves to only considering booking limit policies given by vectors $\mathbf{b} = (b_1, \ldots, b_m)$ in the set $\mathcal{B} = \{\mathbf{b} \in \mathbb{N}^m \ : \ b_1 \leq M_1, \ldots, b_m \leq M_m\}$. Using this notation, we can express the expected total net revenue under a booking policy given by some $\mathbf{b} \in \mathcal{B}$ as follows:

$$\phi(\mathbf{b}) = \sum_{i=1}^{m} \rho_i \mathbb{E}[N_i(b_i)] - \mathbb{E}\left[h_v \left(\sum_{i=1}^{m} \sum_{j=1}^{N_i(b_i)} V_{ij} \right) + h_w \left(\sum_{i=1}^{m} \sum_{j=1}^{N_i(b_i)} W_{ij} \right) \right], \quad (3.14)$$

However, the corresponding optimization model, given by

$$\max\left\{ \phi(\mathbf{b}) \ : \ \mathbf{b} \in \mathcal{B} \right\}, \quad (3.15)$$

is typically very difficult to solve, as ϕ is not a separable function of the booking limits. To overcome this issue, we now describe an upper bound for ϕ that gives rise to a separable formulation.

Proposition 4 *The function* $\phi^U : \mathcal{B} \to \mathbb{R}$ *given by*

$$\phi^U(\mathbf{b}) = \sum_{i=1}^{m} \rho_i \mathbb{E}[N_i(b_i)] - h_v \left(\sum_{i=1}^{m} \mathbb{E}[N_i(b_i)] \mu_i^v \right) - h_w \left(\sum_{i=1}^{m} \mathbb{E}[N_i(b_i)] \mu_i^w \right)$$

provides an upper bound for the function ϕ *defined in* (3.14)

Proof. Let us recall that the functions h_v and h_w are convex, and that, according to our notation, we have $\mathbb{E}[V_{ij}] = \mu_i^v$, $i \in [m], j \in [N_i(b_i)]$. Then Jensen's inequality implies that, for all $\mathbf{b} \in \mathbb{N}^m$, the following holds:

$$\mathbb{E}\left[h_v\left(\sum_{i=1}^{m}\sum_{j=1}^{N_i(b_i)} V_{ij}\right)\right] \geq h_v\left(\mathbb{E}\left[\sum_{i=1}^{m}\sum_{j=1}^{N_i(b_i)} V_{ij}\right]\right) = h_v\left(\sum_{i=1}^{m} \mathbb{E}[N_i(b_i)]\mu_i^v\right).$$

As an analogous inequality is valid for the weight penalty term, our claim follows. ■

If we now replace the net revenue function $\phi(\mathbf{b})$ by its upper bound $\phi^U(\mathbf{b})$ in (3.15), we arrive at an approximate problem:

$$\max\left\{\phi^U(\mathbf{b}) \ : \ \mathbf{b} \in \mathcal{B}\right\}. \tag{Risk_D}$$

When the off-loading cost functions h_v and h_w are defined as in (3.1), we can use a standard linearization of the positive part function to cast (Risk_D) as a mixed integer program. Let us introduce the binary decision variables x_{ij}, $i \in [m]$, $j \in \{0, \ldots, M_i\}$, to represent the indicators $\mathbf{1}_{b_i=j}$. Furthermore, to simplify our notation, let us define $a_{ij} = \mathbb{E}[N_i(j)] = \mathbb{E}\left[\min(j, D_i)\right]$ for all $i \in [m]$, $j \in \{0, \ldots, M_i\}$. Since the distributions of the random variables D_i are known, these expected values can easily be computed. Then, similarly to Aydin et al. (2010), we arrive at the following formulation:

$$\begin{aligned}
\max \quad & \sum_{i=1}^{m} \rho_i \sum_{j=0}^{M_i} a_{ij}x_{ij} - \theta_v\vartheta_v - \theta_w\vartheta_w && && (3.16)\\
\text{subject to} \quad & \vartheta_v \geq \sum_{i=1}^{m} \mu_i^v \sum_{j=0}^{M_i} a_{ij}x_{ij} - C_v && && (3.17)\\
& \vartheta_v \geq 0 && && (3.18)\\
& \vartheta_w \geq \sum_{i=1}^{m} \mu_i^w \sum_{j=0}^{M_i} a_{ij}x_{ij} - C_w && && (3.19)\\
& \vartheta_w \geq 0 && && (3.20)\\
& \sum_{j=0}^{M_i} x_{ij} = 1 && i = 1, \ldots, m && (3.21)\\
& x_{ij} \in \{0, 1\} && i = 1, \ldots, m, j = 0, \ldots, M_i. && (3.22)
\end{aligned}$$

Proposition 5 *Let functions h_v and h_w be defined as in* (3.1), *and let $(\mathbf{x}^*, \vartheta_v^*, \vartheta_w^*)$ be an* *optimal solution of the problem* (3.16)-(3.22)*. Then the booking limits* $b_i^{\mathbf{x}^*} = \sum_{j=1}^{M_i} j x_{ij}^*$*,* $i \in [m]$*, provide an optimal solution of* (Risk_D)*. In addition, the two problems have the same optimum value, i.e., if we let* $\hat{\phi}(\mathbf{x}, \vartheta_v, \vartheta_w)$ *denote the objective expression given in* (3.16)*, we have the equality* $\hat{\phi}(\mathbf{x}^*, \vartheta_v^*, \vartheta_w^*) = \phi^U(\mathbf{b}^{\mathbf{x}^*})$*.*

Proof. Assume that $\mathbf{x} = (x_{ij})_{i \in [m], j \in \{0,\dots,M_i\}}$ satisfies the constraints (3.21)-(3.22). It is easy to see that, for every $i \in [m]$, exactly one of the binary variables $x_{i0}, x_{i1}, \dots, x_{iM_i}$ takes value 1. It follows that the sum $b_i^{\mathbf{x}} = \sum_{j=1}^{M_i} j x_{ij}$ belongs to the set $\{0, \dots, M_i\}$, and thus the vector $\mathbf{b}^{\mathbf{x}}$ is a feasible solution of (Risk_D). Let us introduce the additional notation

$$\vartheta_v^{\mathbf{x}} = \left[\sum_{i=1}^{m} \mu_i^v \sum_{j=0}^{M_i} a_{ij} x_{ij} - C_v \right]_+, \quad \vartheta_w^{\mathbf{x}} = \left[\sum_{i=1}^{m} \mu_i^w \sum_{j=0}^{M_i} a_{ij} x_{ij} - C_w \right]_+,$$

and note that $(\mathbf{x}, \vartheta_v^{\mathbf{x}}, \vartheta_w^{\mathbf{x}})$ satisfies all of the constraints (3.17)-(3.22), and has an objective value of $\hat{\phi}(\mathbf{x}, \vartheta_v^{\mathbf{x}}, \vartheta_w^{\mathbf{x}}) = \phi^U(\mathbf{b}^{\mathbf{x}})$. In addition, constraints (3.17)-(3.20) imply that the inequalities $\vartheta_v \geq \vartheta_v^{\mathbf{x}}$ and $\vartheta_w \geq \vartheta_w^{\mathbf{x}}$ hold for any other feasible solution $(\mathbf{x}, \vartheta_v, \vartheta_w)$, therefore we have $\hat{\phi}(\mathbf{x}, \vartheta_v, \vartheta_w) \leq \hat{\phi}(\mathbf{x}, \vartheta_v^{\mathbf{x}}, \vartheta_w^{\mathbf{x}})$.

On the other hand, let us consider an arbitrary solution $\mathbf{b}$ of (Risk_D), and define $x_{ij} = \mathbf{1}_{b_i = j}$. It is clear that $\mathbf{x}$ satisfies the constraints (3.21)-(3.22), and $\mathbf{b}^{\mathbf{x}} = \mathbf{b}$ holds. Therefore, taking into account the optimality of $(\mathbf{x}^*, \vartheta_v^*, \vartheta_w^*)$, we can combine our previous results to prove our claim as follows:

$$\phi^U(\mathbf{b}) = \phi^U(\mathbf{b}^{\mathbf{x}}) = \hat{\phi}(\mathbf{x}, \vartheta_v^{\mathbf{x}}, \vartheta_w^{\mathbf{x}}) \leq \hat{\phi}(\mathbf{x}^*, \vartheta_v^*, \vartheta_w^*) \leq \hat{\phi}(\mathbf{x}^*, \vartheta_v^{\mathbf{x}^*}, \vartheta_w^{\mathbf{x}^*}) = \phi^U(\mathbf{b}^{\mathbf{x}^*}).$$

■

We note that the proposed formulation (3.16)-(3.22) can be efficiently solved by a standard mixed integer programming solver such as CPLEX as illustrated in Chapter 4.

3.3 Bid-Price Policies

Bid-price policies make accept/reject decisions for booking requests by comparing their net revenues to a threshold price. In a cargo context, these thresholds are based on bid-prices for units of volume and weight capacities, and can be interpreted as marginal values of the capacity resources. Given such bid-prices, one can obtain a threshold price for a given type of cargo by adding up the prices of expected volume and weight requirements of a shipment; see (3.23).

Bid-prices can be updated periodically during the booking process, based on the remaining available capacity, the time to departure, and expectations about the future demand. This widely used approach (see, e.g., Kunnumkal et al., 2012; Levin et al., 2011) leads to dynamic booking policies which lie outside of the scope of this thesis. However, in lieu of updates to the bid-price, it is necessary to introduce additional controls to prevent oversale. In our proposed policies we adopt the following rule: the expected capacity requirements of accepted shipments are not allowed to exceed available capacities.

Let λ^v and λ^w denote bid-prices for unit volume and weight capacities, respectively. Then, in accordance with the principles outlined above, an arriving type-i booking request is accepted if and only the following conditions hold:

$$\rho_i \geq \mu_i^v \lambda_v + \mu_i^w \lambda_w, \quad \mu_i^v \leq C_v - z^v, \text{ and } \mu_i^w \leq C_w - z^w, \tag{3.23}$$

where z^v and z^w denote the total expected volume and weight capacity requirements of already accepted shipments. Notice that the net revenue ρ_i is being compared to the threshold price $\mu_i^v \lambda_v + \mu_i^w \lambda_w$, which expresses the price of the expected capacity requirements of a type-i shipment.

In this section we first consider an approach based on a widely used method in the passenger literature (Simpson, 1992; Williamson, 1992), which computes bid prices as the optimal values of dual variables associated with the capacity constraints in a deterministic capacity assignment LP. Amaruchkul et al. (2007) propose the use of such an LP-based heuristic (not incorporating off-loading costs) in a single-leg cargo context. We extend their model by using a randomized method originally proposed by Talluri and van Ryzin (1999) for controlling passenger bookings over networks.

All of the models discussed so far either ignore off-loading costs, or make the common simplifying assumption that these costs can be separated in an additive fashion, as in (3.1). In contrast, Levin et al. (2011) propose an optimization problem which determines which shipments are to be off-loaded; a similar approach has also been suggested in the passenger literature by Bertsimas and Popescu (2003), and Kunnumkal et al. (2012). The latter study provides a two-stage framework for network revenue management, extending the RLP methods proposed by Talluri and van Ryzin (1999). In the second half of this section we describe a way to compute bid-prices using a similar RLP model, which allows us to consider off-loading costs as a more accurate function of the capacity requirements of accepted reservations.

We mention here two other relevant studies: Han et al. (2010) model the single-leg booking process by a discrete-time Markov chain and compute bid-prices that maximize expected revenue, while Pak and Dekker (2004) consider a two-dimensional on-line knapsack formulation for networks, and use the greedy algorithm proposed in Rinnooy Kan et al. (1993) to solve this problem and compute bid-prices. Both studies assume that no penalty is incurred when a booking request is rejected, and that capacity requirements are known with certainty when a booking request arrives. Due to their practicality, we consider the methods proposed in Amaruchkul et al. (2007) and Pak and Dekker (2004) as benchmarks in our computational study.

3.3.1 A Traditional Randomized Linear Programming Method

Deterministic LP formulations, based on the expected values of the random demands, have been widely used to compute bid-prices for passenger booking in a network context (Simpson, 1992; Williamson, 1992). Amaruchkul et al. (2007) consider a similar deterministic LP model for a single-leg cargo capacity control problem; their formulation is essentially equivalent to the problem ($\mathrm{RKS\_Alloc}$). This approach analyzes a scenario when various random variables take on their expected values, which might not be sufficient to capture the randomness inherent in the booking process. As an alternative to deterministic LPs, Talluri and van Ryzin (1999) propose the use of RLPs to obtain bid-prices for controlling passenger bookings in the absence of no-shows, i.e., under the assumption that all the passengers with a reservation show up at the departure time. We adapt this approach to a cargo context, and introduce an RLP-based method to compute bid-prices for volume and weight capacities. The underlying idea is to use a Monte Carlo simulation to estimate the total demands, instead of relying on expected values.

Suppose that $\mathbf{d}^k$, $k \in [K]$, are K independent samples of the random total demand vector $\mathbf{D} = \{D_i,\ i \in [m]\}$. To obtain the RLP under the kth sample, we replace the expected total demand $\mathbb{E}[D_i]$ by d_i^k for all $i \in [m]$ in the allocation problem ($\mathrm{RKS\_Alloc}$):

$$\max\{\sum_{i=1}^{m} \rho_i x_i \ : \ 0 \le x_i \le d_i^k,\ i \in [m],\ \sum_{i=1}^{m} \mu_i^v x_i \le C_v,\ \sum_{i=1}^{m} \mu_i^w x_i \le C_w\} \tag{Random\_RKS}$$

We solve the above RLP to find the optimal dual variables λ^{vk} and λ^{wk} associated with the capacity constraints. Then, bid-prices can be calculated by averaging over all samples:

$$\lambda^v = \frac{1}{K}\sum_{k=1}^{K}\lambda^{vk}, \qquad \lambda^w = \frac{1}{K}\sum_{k=1}^{K}\lambda^{wk}.$$

Remark 3 *Consider a discrete-time framework, where the booking horizon is divided in T time periods and T is sufficiently large so that there is at most one booking request arrives in each time period. Suppose that we are given the probabilities of observing a particular type of cargo at each time period: $P(D_{it} = 1) = p_{it}$ for all $i \in [m],\ t \in [T]$. Then, alternatively, we can generate independent samples of $\mathbf{D} = \{D_{it},\ i \in [m], t \in [T]\}$ instead of $\mathbf{D} = \{D_i,\ i \in [m]\}$. In this case, denoting the demands under kth sample by d_{it}^k we replace $\mathbb{E}[D_i]$ by $\sum_{t\in T} d_{it}^k$ for all $i \in [m]$. In our computational study, we assume that we are given p_{it} parameters. However, by using the FFT method, we can exactly compute the distributions of $D_i,\ i \in [m]$ and still generate samples of $\mathbf{D} = \{D_i,\ i \in [m]\}$.*

While the above model incorporates the randomness in the number of booking requests, it does not account for the uncertainty in the capacity requirements of individual shipments. In the next section we present a two-stage approach that addresses this issue.

3.3.2 A Two-Stage Randomized Linear Programming Method

In this section we develop a two-stage RLP model following the template laid out by Kunnumkal et al. (2012): booking decisions are made in the first stage, and off-loading decisions are made in the second stage. Using a Monte Carlo approach, we first generate K samples of the demand distribution, then solve a two-stage LP for each sample. Similarly to our previous RLP method, we compute bid-prices by averaging over all K samples the optimal dual variables associated with capacity constraints.

In order to arrive at a tractable formulation, we need to make additional assumptions about the demand structure. In accordance with common practice in the literature, we divide the booking horizon into T time periods, where departure occurs at the end of the Tth period. We make the standard assumption that T is sufficiently large so that no two booking requests arrive in the same time period. We denote the probability that a booking request for type-i cargo arrives in period t by p_{it}, for $i \in [m]$, $t \in [T]$. The random demand for type-i cargo in period t, denoted by D_{it}, then follows a Bernoulli distribution with success probability p_{it}. We note that the demands $D_{1t}, \dots, D_{mt}$ for a given time period t, together with the indicator of the event that no requests arrive in the period, follow a multinomial joint distribution. We next describe a two-stage model under a given sample realization $\left(d_{it}^k\right)_{i\in[m],t\in[T]}$ of these demands.

Booking decisions for shipments are made without knowledge of their exact future capacity requirements. At the departure time, when these requirements are realized, we determine which accepted shipments should be off-loaded. Let x_{it}^k represent the number of type-i shipments accepted in time period t, and let y_{it}^k represent the number of these shipments that are off-loaded. If the random volume and weight requirements are given by the pair of random vectors $(\mathbf{V}^k, \mathbf{W}^k)$, we have the following first-stage problem:

$$\max \quad \sum_{i=1}^{m}\sum_{t=1}^{T}\rho_i x_{it}^k - \mathbb{E}[Q(\mathbf{x}^k, \mathbf{V}^k, \mathbf{W}^k)] \tag{3.24}$$

$$\text{s. t.} \quad 0 \le x_{it}^k \le d_{it}^k \qquad i\in[m],\ t\in[T], \tag{3.25}$$

where $\mathbb{E}[Q(\mathbf{x}^k, \mathbf{V}, \mathbf{W})]$ denotes the expected second-stage off-loading costs. For given booking decisions $\mathbf{x}^k$ and a given realization $(\mathbf{v}^{ks}, \mathbf{w}^{ks})$ of the random capacity requirements $(\mathbf{V}^k, \mathbf{W}^k)$, the off-loading decisions and costs are given by the optimal solution of the following second-stage LP:

$$Q(\mathbf{x}^k, \mathbf{v}^{ks}, \mathbf{w}^{ks}) = \min \quad \sum_{i=1}^{m}\sum_{t=1}^{T} c_{it}^{ks} y_{it}^k \tag{3.26}$$

$$\text{s.t.} \quad \sum_{i=1}^{m}\sum_{t=1}^{T} v_{it}^{ks}(x_{it}^k - y_{it}^k) \le C_v \tag{3.27}$$

$$\sum_{i=1}^{m}\sum_{t=1}^{T} w_{it}^{ks}(x_{it}^k - y_{it}^k) \le C_w \tag{3.28}$$

$$0 \le y_{it}^k \le x_{it}^k \qquad i\in[m],\ t\in[T], \tag{3.29}$$

where c_{it}^{ks} denotes the penalty cost paid for off-loading a shipment of volume v_{it}^{ks} and weight w_{it}^{ks}. We point out that constraints (3.27)-(3.28) ensure that the total volume and weight requirements of boarded shipments do not exceed the respective available capacities of the flight.

Remark 4 *It is possible to approximate the standard off-loading cost function given in* (3.1) *by setting* $c_{it}^{ks} = \theta_v v_{it}^{ks} + \theta_w w_{it}^{ks}$*. On the other hand, Levin et al. (2011) consider penalty costs for type-*i *cargo to be proportional to the shipping rate* r_i*, and computed as* $c_{it}^{ks} = r_i(0.15 v_{it}^{ks} + 1.5 w_{it}^{ks})$*. We also consider the penalty cost* c_{it}^{ks} *to be a deterministic function of the capacity requirements* v_{it}^s *and* w_{it}^s*, hence the omission of* $\mathbf{c}^{ks}$ *from the* *arguments of the function Q. However, this is not a necessary assumption; our approach can accommodate arbitrary choices of c_{it}^{ks}. Similarly, we can incorporate uncertainty in available volume and weight capacities into our model by replacing C_w and C_v by C_w^{ks} and C_v^{ks}, respectively. However, it is potentially very challenging to generate scenarios that accurately represent the joint distributions of all random parameters .*

It is possible to obtain a point estimation of the expected off-loading costs $\mathbb{E}[Q(\mathbf{x}^k, \mathbf{V}^k, \mathbf{W}^k)]$ via Monte Carlo simulation as follows. Let us generate L samples $(\mathbf{v}^{ks}, \mathbf{w}^{ks})$, $s \in [L]$, of the random capacity requirements, then obtain corresponding off-loading costs $Q(\mathbf{x}^k, \mathbf{v}^{ks}, \mathbf{w}^{ks})$ by solving the second-stage LP, and finally take the average of these costs across all L samples. Accordingly, we can combine our first-stage problem (3.24)-(3.25) and our second-stage problem (3.26)-(3.29) into a single large-scale LP:

$$\max \quad \sum_{i=1}^{m}\sum_{t=1}^{T} \rho_i x_{it}^k - \frac{1}{L}\sum_{s=1}^{L}\sum_{i=1}^{m}\sum_{t=1}^{T} c_{it}^s y_{it}^{ks} \tag{3.30}$$

$$\text{s. t.} \quad 0 \le x_{it}^k \le d_{it}^k \qquad i \in [m],\ t \in [T], \tag{3.31}$$

$$\sum_{i=1}^{m}\sum_{t=1}^{T} v_{it}^{ks}(x_{it}^k - y_{it}^{ks}) \le C_v \qquad s \in [L], \tag{3.32}$$

$$\sum_{i=1}^{m}\sum_{t=1}^{T} w_{it}^{ks}(x_{it}^k - y_{it}^{ks}) \le C_w \qquad s \in [L], \tag{3.33}$$

$$0 \le y_{it}^{ks} \le x_{it}^k \qquad i \in [m],\ t \in [T],\ s \in [L]. \tag{3.34}$$

Let us solve the above two-stage model for each of the K demand realizations, and let $\hat{\lambda}_v^{ks}$ and $\hat{\lambda}_w^{ks}$ denote the optimal values of the dual variables corresponding to the capacity constraints (3.32) and (3.33), respectively. Then, similarly to the traditional RLP method, we can set the bid-prices for unit volume and weight capacities as

$$\lambda_v = \frac{1}{K}\sum_{j=1}^{K}\sum_{s=1}^{L} \hat{\lambda}_v^{ks}, \qquad \lambda_w = \frac{1}{K}\sum_{j=1}^{K}\sum_{s=1}^{L} \hat{\lambda}_w^{ks}.$$

3.3.2.1 Solving the Two-Stage Model

For a given demand sample, the proposed large-scale LP formulation given by (3.30)-(3.34) involves mLT decision variables and $O(mLT)$ constraints. Depending on the size of the problem instances, it can be computationally challenging to solve this problem. In our computational study, we use the Monte Carlo approach with $K = 25$ samples when drawing first-stage parameters, and $L = 200$ samples for each set of second-stage parameters. For instances with $m = 240$, we could easily solve the resulting problems using CPLEX. However, if solving the large-scale LP formulation eventually becomes a computational bottleneck, one can use the well-known L-shaped method (Van Slyke and Wets, 1969), a widely applied Benders-decomposition approach (Benders, 1962) to solve two-stage stochastic programming problems with the expected recourse functions for the case of a finite probability space. For a detailed discussion on the L-shaped method, we refer the reader to Van Slyke and Wets (1969), Birge and Louveaux (1997) and Prékopa (1995). In our setup, this decomposition based approach requires to solve the second-stage problem for each sample of volume and weight requirements in order to obtain the subgradient inequalities for the total off-loading cost function. Observe that using a change of variables ($\tilde{y}_{it} = x_{it} - y_{it}$) we can formulate the second-stage problem under each realization as a continuous relaxation of the multiple knapsack problem (MKP) with two constraints:

$$Q(\mathbf{x}, \mathbf{v}^s, \mathbf{w}^s) = c^s_{it} x_{it}$$
$$- \max\{\sum_{i=1}^{m}\sum_{t=1}^{T} c^s_{it}\tilde{y}_{it} \; : \; \sum_{i=1}^{m}\sum_{t=1}^{T} v^s_{it}\tilde{y}_{it} \leq C_v, \; \sum_{i=1}^{m}\sum_{t=1}^{T} w^s_{it}\tilde{y}_{it} \leq C_w, \; 0 \leq \tilde{\mathbf{y}} \leq \mathbf{x}\}.$$

The relaxed MKP problem can be solved using a off-the-shelf software such as CPLEX. One can also solve it using an alternative approach. For this special class of MKP, we can use the Lagrangian method penalizing the violation of one of the capacity constraints, which leads to the well-known continuous knapsack problem. Thus, for a given Lagrangian multiplier associated with a capacity constraint, we have an analytical expression for the optimal solution of a Lagrangian relaxation of the second-stage problem. Then, we can optimize over the single Lagrange multiplier to obtain an optimal solution of the second-stage problem. Such an approach has been proposed by Martello and Toth (2003) to solve the continuous relaxation of the MKP with two constraints.

Chapter 4

Implementation Details and Computational Study

In this chapter, we first discuss in detail how different booking policies are implemented. Then, we describe our simulation setup and explain how we set the values of the input parameters used in the presented models. We also briefly describe the policies used as benchmarks and provide insights about the performance of different policies.

4.1 Implementing Cargo Booking Policies

In this section we outline various ways to implement open loop cargo booking policies for use in practice, or for the purpose of evaluation by simulations. We also describe methods to convert between different types of booking controls.

4.1.1 General Implementation Notes

In Section 3.3 we introduce the rule that, when we employing a bid-price policy, we do not accept booking requests for shipments that would bring the total expected capacity requirements (either volume or weight) for the flight over the available capacity. In our implementations we adopt this rule for all booking policies. This practice has been suggested by Pak and Dekker (2004) and Amaruchkul et al. (2007); the latter study states that adopting it leads to improved performance. We have also observed that this practice, which considers the capacity constraints given in (3.23), has significantly improved the performance of our booking policies.

As we briefly touched upon in Section 3.3, open loop methods are often used with a rolling horizon scheme, where booking controls (i.e., booking limits, or bid-prices) are periodically updated to take into account changes in available capacity and/or changes in predicted demands. Such approaches, which occupy a position between static and fully dynamic booking policies, are outside the scope of our thesis.

In practice, partitioned booking limits are rarely implemented directly; instead, they are usually converted to a nested policy. This conversion can be performed naturally in the passenger case, since net revenues define a unique ranking between the fare classes. In Section 4.1.3 we discuss how to perform similar conversions for cargo booking limits. In our computational results we only report the performance of nested implementations of partitioned booking limits; the reason for this decision is that nested implementations consistently outperform partitioned ones to a significant degree.

4.1.2 Implementing Booking Limit Policies

We consider booking requests that arrive in sequence. When a request arrives, we make an accept/reject decision based on our current booking limits, and if the request is accepted, we update the booking limits to reflect the decrease in available capacity.

Partitioned booking limits

Let $b_1(t), \ldots, b_m(t)$ denote the booking limits for various cargo types after accepting t booking requests. A new request for type-i cargo is accepted if and only if we have $b_i(t) \geq 1$. If the request is accepted, we decrease type-i limit, and leave the other limits unchanged. That is, we set $b_i(t+1) = b_i(t) - 1$, and $b_j(t+1) = b_j(t)$ for $j \neq i$.

Nested booking limits

We note that there are two ways of implementing nested booking policies: standard nesting and theft nesting. Talluri and van Ryzin (2005) state that "*standard nesting is the norm in revenue management practice*". Accordingly, in our study, we only consider this more natural approach, and refer the reader to Haerian et al. (2006) for a detailed description of theft nesting. The nested booking limit $\bar{b}i$ denotes the maximum total number of booking requests that we intend to accept for cargo types $i, \ldots, m$.

Let $\bar{b}_1(t) \geq \cdots \geq \bar{b}_m(t)$ denote the nested booking limits after accepting t booking requests. A new request for type-i cargo is accepted if and only if we have $\bar{b}_i(t) \geq 1$. If the request is accepted, we decrease the booking limits for cargo types $1, \ldots, i$, and update other limits to preserve the nested structure. That is, we set $\bar{b}_j(t+1) = \bar{b}_j(t) - 1$ for $j \leq i$, and $\bar{b}_j(t+1) = \min(\bar{b}_j(t), \bar{b}_i(t+1))$ for $j > i$.

Capacity-based booking limits

Instead of considering limits on the number of accepted booking requests, it is natural to consider limits on the expected capacity requirements. Such limits represent volume and weight capacities that are made available to cargo of various types. Accordingly, upon accepting a booking request for a shipment, the appropriate limits are decreased by the expected volume and weight of this shipment.

Partitioned capacity limits Let $B_1^v(t), \ldots, B_m^v(t)$ and $B_1^w(t), \ldots, B_m^w(t)$ denote the volume and weight limits, respectively, for various cargo types after accepting t booking requests. A new request for type-i cargo is accepted if and only if we have $B_i^v(t) \geq \mu_i^v$ and $B_i^w(t) \geq \mu_i^w$. If the request is accepted, we decrease type-i limits, and leave the other limits unchanged. That is, we set

$$
\begin{aligned}
B_i^v(t+1) &= B_i^v(t) - \mu_i^v \\
B_i^w(t+1) &= B_i^w(t) - \mu_i^w \\
B_j^v(t+1) &= B_j^v(t) && j \neq i \\
B_j^w(t+1) &= B_j^w(t) && j \neq i.
\end{aligned}
$$

Nested capacity limits Let $\bar{B}_1^v(t) \geq \cdots \geq \bar{B}_m^v(t)$ and $\bar{B}_1^w(t) \geq \cdots \geq \bar{B}_m^w(t)$ the nested capacity limits after accepting t booking requests. A new request for type-i cargo is accepted if and only if we have $\bar{B}_i^v(t) \geq \mu_i^v$ and $\bar{B}_i^w(t) \geq \mu_i^w$. If the request is accepted, we decrease the limits for shipments of type $1, \ldots, i$, and update other limits to preserve the nested structure. That is, we set

$$
\begin{aligned}
\bar{B}_j^v(t+1) &= \bar{B}_j^v(t) - \mu_i^v && j \leq i \\
\bar{B}_j^w(t+1) &= \bar{B}_j^w(t) - \mu_i^w && j \leq i \\
\bar{B}_j^v(t+1) &= \min\left(\bar{B}_j^v(t), \bar{B}_i^v(t+1)\right) && j > i \\
\bar{B}_j^w(t+1) &= \min\left(\bar{B}_j^w(t), \bar{B}_i^w(t+1)\right) && j > i.
\end{aligned}
$$

4.1.3 Conversions Between Booking Controls

We have discussed several classes of booking controls, including partitioned and nested booking limits, expressed both in terms of the number of booking requests and in terms of capacity. We now describe some ways in which a cargo booking policy based on controls of a certain class can be converted to a related (but not necessarily equivalent) policy based on controls of a different class.

Conversion between nested booking limits and protection levels

In the EMSR literature, and accordingly in our related Section 3.2.1.2, nested booking policies are described in terms of protection levels $y_1 \leq \ldots y_m$. Here the level y_i denotes the maximal number of booking requests that can be accepted for cargo types $1, \ldots, i$. Protection levels can be interpreted as "protecting" available capacity for requests with high profitability. In contrast, nested booking limits express the amounts of capacity made available for requests of lower profitability. These two conventions provide equivalent descriptions of nested booking policies, and we can convert protection levels to nested booking limits via the following simple formulas:

$$\bar{b}_1 = y_m, \qquad \bar{b}_i = y_i - y_{i-1} \quad \text{for } i = 2, \ldots, m.$$

Conversion to capacity limits

Instead of considering limits on the number of accepted booking requests, it is natural to consider limits on the expected capacity requirements of accepted requests. If $b_1, \ldots, b_m$ are partitioned booking limits, we can define corresponding booking limits in terms of volume as $B_i^v = b_i \mu_i^v$, and in terms of weight as $B_i^w = b_i \mu_i^w$, for $i \in [m]$. Similarly, given nested booking limits $\bar{b}_1 \geq \cdots \geq \bar{b}_m$, we can define corresponding nested capacity limits as

$$\bar{B}_i^v = \bar{b}_m \mu_m^v + \sum_{j=i}^{m-1} (\bar{b}_{j+1} - \bar{b}_j) \mu_j^v, \quad i \in [m],$$
$$\bar{B}_i^w = \bar{b}_m \mu_m^w + \sum_{j=i}^{m-1} (\bar{b}_{j+1} - \bar{b}_j) \mu_j^w, \quad i \in [m].$$

Nested implementations of partitioned booking limits

If the various types of cargo are ranked in such a fashion that lower-indexed types are considered to be more preferable, then partitioned booking limits naturally give rise to nested booking limits. More precisely, given partitioned booking limits b_i, $i \in [m]$, we can define $\bar{b}_i = \sum_{j=i}^m b_j$. Analogously, for partitioned capacity limits B_i^v and B_i^w, $i \in [m]$, we define $\bar{B}_i^v = \sum_{j=i}^m B_j^v$ and $\bar{B}_i^w = \sum_{j=i}^m B_j^w$. In our numerical experiments we consider nested implementations of partitioned booking limits based on the rankings of cargo types implied by the profitability coefficients introduced in Section 3.2.1.2.

4.2 Simulation Setup and Parameters

Following the setup presented in Amaruchkul et al. (2007) we have set the following parameters: cargo types, volume and weight requirements, volume and weight capacities, revenue function, off-loading costs, number of decision periods and the demand arrival probabilities.

In all of our computational study we assume that each cargo shipment has deterministic weight and random volume requirements at the time of booking. This is because it is relatively easier for the shipper to measure the weight, however it requires more sophisticated tools to measure the volume. Therefore, volume is represented by a random variable which follows a log-normal distribution.

As in Amaruchkul et al. (2007), a shipment type is defined by two components: class and category. Class of the shipment is characterized by its content, e.g. flowers, clothes, electronics or fresh products. Therefore, class is the primary component that determines the rate which company will charge per chargeable unit $\hat{w}$ (See Table 4.2). On the other hand, category of the shipment is defined by its expected volume and weight (See Table 4.1). There are 24 categories and 10 classes. Consequently, the number of different cargo types becomes $m = 24 \times 10 = 240$.

Table 4.1: Weight (kg) and Expected Volume ($\times 10^4$ cm^3) for Category

Category	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**
Weight	50	50	50	50	100	100	100	100	200	200	200	250
Mean vol.	30	29	27	25	59	58	55	52	125	119	100	147
	13	**14**	**15**	**16**	**17**	**18**	**19**	**20**	**21**	**22**	**23**	**24**
Weight	250	300	400	500	1000	1500	2500	3500	70	70	210	210
Mean vol.	138	179	235	277	598	898	1488	2083	233	17	700	52

Table 4.2: Revenue Function for Classes

Class	1	2	3	4	5	6	7	8	9	10
$0 < \hat{w} \leq 90$	1.12	1.04	0.92	0.82	0.8	0.87	0.99	0.72	0.7	0.55
$90 < \hat{w} \leq 990$	1.11	1.03	0.91	0.81	0.79	0.86	0.98	0.71	0.69	0.54
$990 < \hat{w} \leq 1990$	1.09	1.01	0.89	0.79	0.77	0.84	0.96	0.69	0.67	0.52
$1990 < \hat{w}$	1.08	1.00	0.88	0.78	0.76	0.83	0.95	0.68	0.66	0.51

In our numerical experiments, the revenue function $r_i(.)$ appearing in (3.2) is taken to be a piecewise linear function as described in Table 4.2 and the inverse density constant γ is equal to 6 m^3/ton. Our models require the revenue obtained when a single booking request is accepted. However, because volume is taken to be a random variable, we estimated this immediate contribution by using $\mathbb{E}[r_i(\max\{w, V/\gamma\})]$ instead. Please refer to Appendix D for related calculations.

There are 60 decision periods in which at most one arrival occurs. Booking horizon starts at $t = 60$ and plane leaves at $t = 0$. We use the time dependent arrival probabilities presented in Amaruchkul et al. (2007) (for details, see Tables 4.3 and 4.4); each value is associated with the probability that an arriving booking request belongs to a certain class and category at a particular time period. The probability of observing a booking request arrival for type-i cargo at time t, denoted by p_{it}, is obtained by multiplying the arrival probabilities associated with the category and the class of cargo type-i.

Table 4.3: Arrival Probabilities for Classes

Periods	1-10	11-20	21-30	31-40	41-50	51-60
Class 1	0.02	0.03	0.04	0.04	0.05	0.05
Class 2	0.006	0.007	0.01	0.01	0.015	0.02
Class 3	0.005	0.005	0.05	0.07	0.065	0.08
Class 4	0.02	0.02	0.02	0.045	0.045	0.07
Class 5	0.025	0.025	0.025	0.025	0.025	0.03
Class 6	0.03	0.02	0.03	0.02	0.03	0.04
Class 7	0.05	0.05	0.05	0.05	0.05	0.06
Class 8	0.078	0.06	0.07	0.06	0.07	0.09
Class 9	0.03	0.035	0.04	0.045	0.05	0.055
Class 10	0.001	0.045	0.002	0.002	0.05	0.05

Table 4.4: Arrival Probabilities for Categories

Categories	1-10	11-16	17-20	21-24
Probability	0.072	0.04	0.009	0.001

Volume and weight capacities (C_v, C_w) are determined as fractions of the expected total demands $d_v = \sum_{t=1}^{T}\sum_{i=1}^{m} p_{it}\mu_i^v$ and $d_w = \sum_{t=1}^{T}\sum_{i=1}^{m} p_{it}\mu_i^w$. Basically, given the capacity demand ratios ($C_v/d_v, C_w/d_w$), we determine the volume and weight capacities.

As mentioned in Chapter 3 we consider two ways of modelling the off-loading costs. In the first approach, we assume that off-loading cost functions h_v and h_w are defined as in (3.1). On the other hand, the second approach calculates the off-loading cost by solving a net revenue maximization problem which identifies the individual shipments that are to be denied loading. In both approaches we assume that partial loadings are allowed. For the first approach, we need to specify the off-loading cost coefficients θ_v and θ_w per unit of off-loaded volume and weight, respectively. To do this, we first calculate the benchmark penalty costs (η_v, η_w) using the following equations:

$$\sum_{t=1}^{T}\sum_{i=1}^{m} p_{it}\rho_i = \eta_v d_v$$
$$\sum_{t=1}^{T}\sum_{i=1}^{m} p_{it}\rho_i = \eta_w d_w.$$

Then, given the penalty cost rate ratios $(\theta_v/\eta_v, \theta_w/\eta_w)$, we set the values of θ_v and θ_w. In our second approach, the cost of off-loading a type-i shipment at period t under scenario s is taken as $\theta_v v_{it}^s + \theta_w w_{it}^s$. Remark 4 explains the motivation behind our selection.

We utilized a Monte Carlo simulation for all our models while estimating hard-to-compute expressions. First, we used this approach to estimate the complicated expectation terms for finding the optimal solution of $(\mathrm{Risk\_TB})$ (see Appendix C) and we selected the sample size as $10,000$ which gave quite stable results among different samplings. Secondly, for our traditional randomized linear programming model we sampled 1000 demand realizations in order to estimate the dual variables. Finally, for our two stage stochastic linear programming model, we sampled $K = 25$ realizations of demand and $L = 200$ realizations of volume and weight.

Recall that the problem $(\mathrm{Risk\_TB})$ requires the probability $p_i,\ i \in [m]$, that a booking request is for type-i cargo. These probabilities are calculated by

$$p_i = \frac{\mathbb{E}[D_i]}{\sum_{i=1}^{m}\mathbb{E}[D_i]} = \frac{\sum_{t=1}^{T} p_{it}}{\sum_{i=1}^{m}\sum_{t=1}^{T} p_{it}}.$$

Using these multinomial probabilities, we generate the mixture random variables $\bar{V}_i$ and $\bar{W}_i$ in the corresponding Monte Carlo simulation.

In order to obtain the bid-prices by solving the traditional RLP, we generate samples of total demand for each cargo type. Thus, we need the joint distribution of $D_1, \ldots, D_m$. Obtaining this probability distribution is not very straightforward because D_i is the sum of T independent Bernoulli random variables each having a different probability of success. Under the assumption of independent total demands, we obtain the marginal distributions of $D_i,\ i \in [m]$ by using the Fast Fourier Transform (FFT), see Appendix E for details. We utilize the FFT also for the EMSRb heuristic, since it requires the distribution of $\sum_{i=1}^{j} D_i$ for all $j \in [m]$. Similarly, $\sum_{i=1}^{j} D_i$ is the sum of jT independent Bernoulli random variables. Therefore, calculations are quite similar to those for the distribution of D_i.

A single problem instance is defined by the combination of three sets of parameters. The first is the capacity demand ratios for volume and weight ($C_v/d_v, C_w/d_w$). Second is the coefficient of variation denoted by cv. The final parameter is the penalty cost rate ratios ($\theta_v/\eta_v, \theta_w/\eta_w$). From this point on, we will represent a single instance using the notation: $(C_v/d_v, C_w/d_w, cv, \theta_v/\eta_v, \theta_w/\eta_w)$. We generated 154 different instances for our computational experiments and we next present the details. Let us denote the set of values we used as capacity demand ratio, coefficient of variation and penalty cost rate parameters by C_1, C_2, and C_3 respectively. The generated instances can be divided into two groups. The first group involves tighter capacities on at least one dimension, whereas the second group involves more moderate capacities. There are 24 instances within the first group and the associated parameter values are as follows: $C_1 = \{(0.1, 1.0),(0.2, 1.0),(0.3, 0.3),(0.3, 1.0),(0.4, 0.4), (0.4, 1)\}$, $C_2 = \{0.2, 0.8\}$, $C_3 = \{(1.5, 1.5), (2.0, 2.0)\}$. The parameter values of the second group are: $C_1 = \{(0.5, 0.5),(0.5, 1.0),(1.0, 0.5),(0.75, 0.75),(0.75, 1.0), (1.0, 0.75),(0.9, 0.9), (0.9, 1.0),(1.0, 0.9), (1.0, 1.0), (1.1, 1.1),(1.1, 1.0), (1.0, 1.1)\}$, $C_2 = \{0.2, 0.8\}$, $C_3 = \{(0.8, 0.8), (1.0, 1.0), (1.2, 1.2), (1.5, 1.5),(2.0, 2.0)\}$. In order to estimate the expected revenue under each setting, we conducted simulation and for each instance we ran 1000 replications. Solving RM_{2P} takes less than 10 seconds, RM_D and EMSR based heuristics take less than 1 second, RLP-1 takes less than 1 minute on average. Solving 25 large scale linear programming models in order to obtain bid-prices for RLP-2, took around 15 minutes on average. Please note that given times are in terms of wall clock time all and the computational experiments were conducted on an Intel® Core™2 Quad, 2.33 GHz processor, 8 GB RAM (Windows 7, 64-bit) computer.

4.3 Benchmark Policies

We implemented three of the heuristics proposed by Amaruchkul et al. (2007), the algorithm proposed by Pak and Dekker (2004) and the first come first serve policy as benchmark policies. Amaruchkul et al. (2007)'s first heuristic develops a policy based on two approximate DP formulations whereas the other two heuristics propose bid-price and booking-limit policies. Pak and Dekker (2004)'s algorithm also provides bid-prices and utilizes these bid-prices during the decision process. First come first serve policy accepts all booking requests unless it results in exceeding the capacity.

The DP formulation of the cargo capacity control problem is presented in Amaruchkul et al. (2007). The state space of this DP formulation is a vector ($\mathbf{x}$) of size m denoting the number of accepted type-i cargoes. As a result, this problem becomes computationally intractable. Let $\mathbf{x} = (x_1, \ldots, x_m) \in \mathbb{Z}_+^m$ be the state vector with x_i representing the number of accepted requests for cargo type-i so far. The initial state, at the beginning of the booking horizon is the zero m-vector, denoted by $\mathbf{e}_0$, and $g_T(\mathbf{e}_0)$ gives the optimal function value where $g_t(x)$ denotes the recursive equations of the DP formulation.

$$g_t(\mathbf{x}) = \sum_{i=1}^{m} p_{it} \max\{\rho_i + g_{t-1}(\mathbf{x} + \mathbf{e}_i), g_{t-1}(\mathbf{x})\} + p_{0t} g_{t-1}(\mathbf{x}), \quad t = 1, \ldots, T, \quad (4.1)$$

$$g_0(\mathbf{x}) = -\mathbb{E}\left[\theta_v \left[\sum_{i=1}^{m}\sum_{j=1}^{x_i} V_{ij} - C_v\right]_+ + \theta_w \left[\sum_{i=1}^{m}\sum_{j=1}^{x_i} W_{ij} - C_w\right]_+\right], \quad (4.2)$$

where V_{ij} and W_{ij} are the volume and weight requirements of jth accepted type-i cargo booking and $\mathbf{e}_i$ denotes the m-dimensional unit vector with a 1 in the ith position and 0 anywhere else. Because it is computationally challenging to solve this high-dimensional DP problem, Amaruchkul et al. (2007) propose different approximations to the formulation above. We next briefly describe one of their heuristics that we used for benchmarking.

HD Heuristic

This heuristic is based on an approximation approach which formulates two separate DP problems (u_t^v, u_t^w) based on volume and weight dimensions. The state space for the DP based on volume is taken as the expected total volume accepted and for DP based on weight, it is expected total weight accepted so far. Immediate revenue for u_t^v is $f_i^v = \mathbb{E}\left[[r_i(V_i/\gamma) - r_i(w_i)]1_{\{V_i \geq \gamma w_i\}}\right]$ and for u_t^w, it is $f_i^w = \mathbb{E}[r_i(w_i)]$. Boundary equations for both dimensions are equal to the related dimension's expected off-loading costs. This approximation is used for both providing an upper bound on the expected net revenue value ($u_T^v(0) + u_T^w(0)$) and determining the decision policy of HD heuristic.

In HD heuristic, a type-i booking request arrival at time t is accepted when the states are $x = \sum_{i=1}^{m} x_i \mu_i^v$ and $y = \sum_{i=1}^{m} x_i \mu_i^w$ if

$$\rho_i \geq [u_{t-1}^v(x) + u_{t-1}^w(y)] - [u_{t-1}^v(x + \mu_i^v) + u_{t-1}^w(y + \mu_i^w)].$$

Partitioned Allocations (PA) Policy

Booking limits for PA heuristic are derived by solving the problem:

$$\xi(\mathbb{E}[\mathbf{D}]) = \max\{\phi(\mathbf{x}) : \mathbf{x} \leq \mathbb{E}[\mathbf{D}]\},$$

where

$$\phi(\mathbf{x}) = \sum_{i=1}^{m} \rho_i x_i - \theta_v[\sum_{i=1}^{m} \mu_i^v x_i - C_v]_+ - \theta_w[\sum_{i=1}^{m} \mu_i^w x_i - C_w]_+.$$

Let $\mathbf{z}^*$ be the optimal solution of the above optimization problem. Then, PA accepts a type-i booking request if and only if

$$x + \mu_i^v \leq C_v, \quad y + \mu_i^w \leq C_w, \quad \text{and} \quad x_i < \lceil z_i \rceil.$$

Bid-Price (BP) Policy

Dual variables (λ_v and λ_w) associated with the volume and weight constraints of the problem ($\mathrm{KS\_Alloc}$) are used as bid-prices for the BP heuristic. Then a type-i booking request is accepted if and only if

$$x + \mu_i^v \leq C_v, \quad y + \mu_i^w \leq C_w, \quad \text{and} \quad \rho_i \geq \mu_i^v \lambda_v + \mu_i^w \lambda_w. \tag{4.3}$$

Pak and Dekker's Bid-Price (PD) Policy

Pak and Dekker (2004) model the booking process as a two-dimensional on-line knapsack problem and obtain the bid-prices using the greedy algorithm proposed by Rinnooy Kan et al. (1993) (See Appendix F). A booking request is accepted or rejected according to the rule given (4.3), but the dual variables (λ_v and λ_w) are replaced by the bid-prices obtained by Rinnooy Kan et al. (1993)'s algorithm. In order to make a more fair comparison with our two stage stochastic linear programming model, we ran this algorithm for 1000 times and took the average of the bid-prices.

First Come First Serve (FCFS) Policy

The FCFS policy accepts all the booking requests as long as the expected total volume and weight of the already accepted bookings do not exceed the respective capacities. In other words, a type-i booking request is accepted if and only if

$$x + \mu_i^v \leq C_v \quad \text{and} \quad y + \mu_i^w \leq C_w.$$

Note that FCFS policy can also be considered as a bid-price policy where the bid-prices are equal to zero.

4.4 An Overview of Implemented Methods

In the computational study, we implemented the policies obtained by solving our models (presented in Chapter 3) and the benchmark policies (described in the previous section) according to the details explained in Section 4.1.2. In particular, we consider four types of booking limit polices and three types of bid-price policies.

As discussed in Chapter 3, the proposed two-phase method sets the available capacity to be equal to the total booking limit obtained by solving the problem ($\mathrm{Risk\_TB}$) or ($\mathrm{Service\_TB}$). Then, it uses a particular type of profitability coefficients to obtain the nested booking limits as summarized in Algorithm 1. We refer to this approach as $\mathrm{RM_{2P}}$. Our model Risk_D and the PA approach provide us with the partitioned booking limits. Then, we use the proposed nested structures to convert them to the nested ones. Thus, the nested booking limits are obtained for three models: $\mathrm{RM_{2P}}$, Risk_D and PA. For each model, we use three types of profitability coefficients to obtain the nested booking limits. With the randomized version of the third type of profitability coefficients, we consider six types of nested structures. For convenience, we introduce the abbreviations summarized in Tables 4.5 and 4.6.

Obtaining Bid-Price Policies	
RLP-1	Traditional Randomized Linear Programming Model
RLP-2	Two Stage Stochastic Linear Programming Model
PD	Pak and Dekker (2004)'s Bid-Price Policy
FCFS	First Come First Serve Policy
Obtaining Booking Limit Policies	
$\mathrm{RM_{2P}}$	Two-Phase Risk-Based Model
$\mathrm{RM_D}$	Risk-Based Model for Partitioned Booking Limits
PA	Partitioned-Allocations Heuristic of Amaruchkul et al. (2007)

Table 4.5: Implemented Models

	Profitability coefficients		
$*$-1	Type 1:	(3.10)	
$*$-2	Type 2:	(3.11)	
	Type 3:		LP used to estimate λ_w and λ_v
$*^w$-3		(3.12)	(KS_Alloc)
$*^v$-3		(3.13)	(KS_Alloc)
$*^w$-R3		(3.12)	(Random_RKS)
$*^v$-R3		(3.13)	(Random_RKS)
“ * ”: Stands for the model RM_{2P}, RM_D or PA			

Table 4.6: Implemented Nested Structures

4.5 Numerical Results and Insights

According to the numerical results presented in Amaruchkul et al. (2007), HD heuristic outperforms their all other heuristics. Therefore, we took HD as a benchmark while evaluating the performance of different heuristics that we consider in our computational study. We quantify the solution quality of different heuristics by relative percent difference with respect to HD heuristic and it is calculated as:

$$100 \times \frac{\bar{Z}_{HD} - \bar{Z}_{\pi}}{\bar{Z}_{HD}}, \tag{4.4}$$

where π represents one of the heuristics, and $\bar{Z}_{HD}$ and $\bar{Z}_{\pi}$ represent the net revenues (averaged over all replications) of the policies obtained by HD and π, respectively.

Abbreviations in Tables (4.12) and (4.16) stand for:

Rel. Dif.	Equation (4.4)
Utilization	$\left(100 \times \text{Avg}\left(\frac{\text{Total Volume Accepted}}{\text{Volume Capacity}}\right), 100 \times \text{Avg}\left(\frac{\text{Total Weight Accepted}}{\text{Weight Capacity}}\right)\right)$
Offloaded	$\left(100 \times \text{Avg}\left(\frac{\text{Offloaded Volume}}{\text{Total Volume Accepted}}\right), 100 \times \text{Avg}\left(\frac{\text{Offloaded Weight}}{\text{Total Weight Accepted}}\right)\right)$
Acc.	$100 \times \text{Avg}\left(\frac{\text{Number of Requests Accepted}}{\text{Total Number of Requests}}\right)$
OC	$100 \times \text{Avg}\left(\frac{\text{Offloading Cost}}{\text{Total Revenue}}\right)$.

4.5.1 Booking Limit Policies

Relative percent differences from HD heuristic of all the booking limit policies are presented in Tables (4.7)-(4.12) and Figures (4.2)-(4.3).

Like PA heuristic of (Amaruchkul et al., 2007), RM_D initially provides partitioned booking limits. When these partitioned booking limits are directly utilized in the decision process, mean net revenue obtained is 4% worse than the nested versions of RM_D on average. In almost all instances, using booking limits in a nested structure performed better than using partitioned booking limits. Therefore we did not represent the results of policies where partitioned booking limits were used. Figure (4.1) represents the average revenue over all instances for each booking limit model and it reveals that, out of six different nesting methods, $*$v-3 and $*$w-3 performed the worst. Therefore, results given by these nesting methods are not presented either.

In figure (4.2), we present how total booking limit responds to the penalty cost rate ratios under different capacity demand ratio values (i.e. each line corresponds to a different capacity demand ratio). Selected instances in this figure, have coefficient of variation 0.2 and equal volume and weight capacity demand ratios. It is clear from the figure that RM_{2P} is quite sensitive to the changes in penalty cost rate ratio as total booking limit decreases strictly with increasing penalty coefficients. This behaviour leads RM_{2P} to perform more conservatively resulting in small volume and weight capacity utilizations (See Table (4.12)). Because of unused capacity, the opportunity cost increases and the overall performance of RM_{2P} decreases.

Figures (4.3(a))-(4.3(n)) show that it is not possible to make strict comparisons between different models and different nesting structures. Each model and each nesting structure have proven useful under different setups. However, in all figures, there are a number of instances, where for a single model, different nesting structures give the same result. This event does not necessarily imply that the orderings given by different nesting methods are the same. This can also be due to large booking limits. So, if the non-zero booking limits are relatively larger, those booking requests which have a non-zero booking request are always accepted and remaining requests are rejected. Since the set of cargo types which have zero booking limit were almost the same for different nesting structures, their performances were quite close to each other. So, equal results of different nesting methods do not imply that the ordering of cargo types are also equal.

Figures (4.3(a)) and (4.3(b)) represent the performance of all models implemented using all nesting methods when all parameters are fixed except for the capacity demand ratios which are equal to each other for volume and weight. Figure (4.3(a)) show the results under low coefficient of variation, whereas Figure (4.3(b)) represent a setting with high coefficient of variation. These figures illustrate how variability effects the performance of all the models. Under both settings RM_{2P} performs poorly. PA and RM_D models performed quite similarly. The differences in solutions were resulted because of different nesting strategies. Under low coefficient of variation when the capacity demand ratio is low, $*^v$-R3 and $*^w$-R3 performed the best. However, when the capacity demand ratio is higher, $*$-1 gave the best results. High coefficient of variation on the other hand, favoured $*$-2 and $*^w$-R3 methods.

Remaining figures in this section are organized in the following way: For each parameter set, we present two figures, each comparing two different models. For instance Figure (4.3(c)) and (4.3(d)) are plotted using the same instances, however in the first figure, we compare RM_{2P} and RM_D and in the second figure, we compare RM_D and PA. This was done to emphasize the settings where each model performs better. Because PA was mostly outperformed by other models, we decided to make comparisons of PA separately. This way, it became easier to identify each model's behaviour under different settings.

We fixed all instance parameters and observed results under changing weight capacity demand ratio in Figures (4.3(c))-(4.3(f)). Coefficient of variation of the instances were 0.2 in Figures (4.3(c)) and (4.3(d)), 0.8 in Figures (4.3(e)) and (4.3(f)). Although RM_D mostly gives the best results, RM_{2P} performed the best among all models when the weight capacity demand ratio is equal to 0.5 and it performed better than PA when it is equal to 0.75. First type ($*$-1) of nesting gave the most satisfactory results under these instances.

Similarly in Figures (4.3(g))-(4.3(j)), we fix all parameters except for volume capacity demand ratio. RM_{2P} performed the best when the capacity demand ratio is lower. Under these instances PA also performed close to RM_{2P} and for all models type 2 ($*$-2) nesting method gave the best results.

Figures (4.3(k))-(4.3(n)) capture the effect of changing penalty cost rate ratios. Figures (4.3(k)) and (4.3(m)) directly illustrate the effect of penalty cost rate ratio on the performance of RM_{2P}. Increasing penalty rate causes RM_{2P} to perform too conservatively. However, when the penalty costs are decreased RM_{2P}'s results were quite competitive to RM_D. Among different, nesting methods $*^w$-R3 and $*$-1 respond to the changes in penalty cost rate ratio the best.

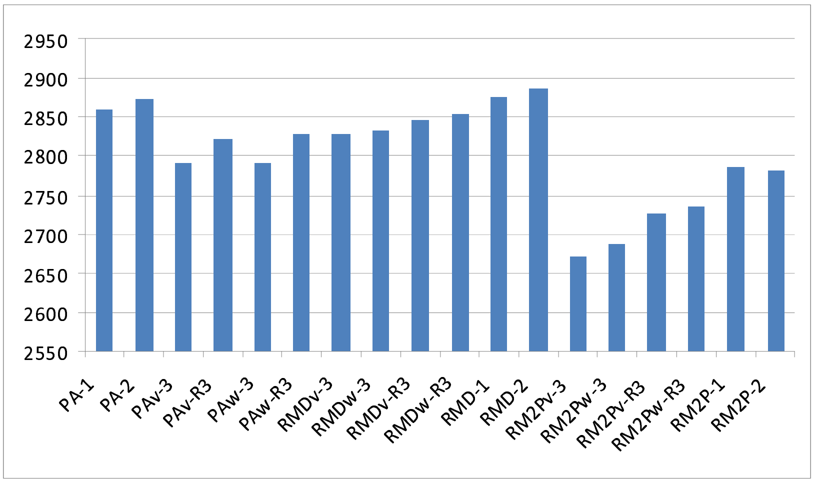


Figure 4.1: Net Revenues Averaged Over All Instances

Table 4.7: Relative Difference (%) of Booking Limit Policies

Instance	RM_{2P}-1	RM_{2P}-2	RM^{v}_{2P}-R3	RM^{w}_{2P}-R3	RM_{D}-1	RM_{D}-2	RM^{v}_{D}-R3	RM^{w}_{D}-R3	PA-1	PA-2	PA^{v}-R3	PA^{w}-R3	PA^{*}
(0.10 , 1.00 , 0.20 , 1.50 , 1.50)	**-4.55**	**-5.91**	21.48	**-4.55**	24.22	**-2.20**	24.22	24.22	30.05	**-2.56**	25.31	30.05	32.01
(0.10 , 1.00 , 0.20 , 2.00 , 2.00)	2.24	**-2.68**	25.59	2.24	21.36	**-4.79**	21.36	21.36	27.14	**-5.18**	22.42	27.14	28.99
(0.10 , 1.00 , 0.80 , 1.50 , 1.50)	**-11.15**	**-9.37**	17.46	**-11.15**	17.56	0.59	13.87	17.56	22.12	**-7.93**	22.05	22.12	23.87
(0.10 , 1.00 , 0.80 , 2.00 , 2.00)	**-11.13**	**-14.40**	17.38	**-11.13**	9.74	**-4.23**	5.66	9.74	15.12	**-13.29**	14.90	15.12	16.63
(0.20 , 1.00 , 0.20 , 1.50 , 1.50)	**-5.76**	**-5.78**	21.98	**-5.76**	3.44	**-5.56**	4.14	3.44	15.01	**-3.95**	15.02	15.01	18.01
(0.20 , 1.00 , 0.20 , 2.00 , 2.00)	**-1.66**	**-4.08**	25.10	**-1.66**	1.39	**-5.64**	2.22	1.39	12.90	**-5.53**	12.91	12.90	15.62
(0.20 , 1.00 , 0.80 , 1.50 , 1.50)	**-11.83**	**-11.62**	9.86	**-11.83**	7.24	**-11.58**	3.56	7.24	7.34	**-4.00**	4.29	7.34	10.43
(0.20 , 1.00 , 0.80 , 2.00 , 2.00)	**-5.72**	**-4.52**	19.34	**-5.72**	1.71	**-6.08**	**-2.08**	1.71	1.82	**-6.29**	**-1.17**	1.82	4.68
(0.30 , 0.30 , 0.20 , 1.50 , 1.50)	23.80	23.29	27.66	31.62	6.70	7.89	4.51	3.30	6.71	7.92	4.71	3.76	28.46
(0.30 , 0.30 , 0.20 , 2.00 , 2.00)	29.57	34.28	36.15	35.42	6.29	6.93	4.00	3.19	6.30	6.94	4.23	3.64	27.45
(0.30 , 0.30 , 0.80 , 1.50 , 1.50)	26.55	27.87	30.97	33.36	6.72	5.12	3.57	2.99	6.69	5.17	3.75	3.04	23.47
(0.30 , 0.30 , 0.80 , 2.00 , 2.00)	33.74	33.92	34.85	33.91	6.71	3.44	2.11	2.51	6.71	3.30	1.87	2.26	18.44
(0.30 , 1.00 , 0.20 , 1.50 , 1.50)	0.99	1.70	19.49	0.99	18.56	**-3.79**	18.40	18.56	18.54	**-1.94**	18.40	18.54	21.79
(0.30 , 1.00 , 0.20 , 2.00 , 2.00)	3.57	1.71	23.36	3.57	17.25	**-5.33**	17.08	17.25	17.21	**-3.68**	17.08	17.21	20.69
(0.30 , 1.00 , 0.80 , 1.50 , 1.50)	**-3.64**	**-3.82**	13.05	**-3.64**	2.38	**-2.88**	0.03	2.38	12.29	**-3.34**	12.18	12.29	15.65
(0.30 , 1.00 , 0.80 , 2.00 , 2.00)	**-5.98**	**-6.54**	12.65	**-5.98**	**-6.52**	**-5.32**	**-1.13**	**-6.52**	6.84	**-6.38**	6.72	6.84	10.48
(0.40 , 0.40 , 0.20 , 1.50 , 1.50)	23.16	22.44	23.94	27.40	6.77	4.38	4.34	6.79	6.51	7.70	5.92	8.32	26.78
(0.40 , 0.40 , 0.20 , 2.00 , 2.00)	34.90	33.37	37.82	41.46	6.40	4.31	4.13	6.32	6.05	6.92	5.36	7.56	25.62
(0.40 , 0.40 , 0.80 , 1.50 , 1.50)	19.19	18.60	20.63	22.87	6.47	2.90	3.57	4.92	4.88	3.51	3.58	5.40	22.40
(0.40 , 0.40 , 0.80 , 2.00 , 2.00)	29.69	28.91	31.43	35.74	8.66	4.62	4.60	8.71	4.72	2.72	2.22	4.94	20.03
(0.40 , 1.00 , 0.20 , 1.50 , 1.50)	1.74	1.19	15.60	1.74	15.57	3.17	15.39	15.57	15.59	**-0.58**	15.42	15.59	18.96
(0.40 , 1.00 , 0.20 , 2.00 , 2.00)	7.70	6.97	22.47	7.70	13.98	**-3.06**	13.92	13.98	14.28	**-1.83**	14.17	14.28	17.58
(0.40 , 1.00 , 0.80 , 1.50 , 1.50)	**-0.26**	**-2.23**	12.10	**-0.26**	10.36	**-3.75**	10.35	10.36	10.36	**-3.65**	10.35	10.36	13.42
(0.40 , 1.00 , 0.80 , 2.00 , 2.00)	**-0.83**	**-1.26**	12.79	**-0.83**	**-2.42**	**-4.42**	**-2.72**	**-2.42**	6.83	**-4.77**	6.89	6.83	9.93
(0.50 , 0.50 , 0.20 , 0.80 , 0.80)	6.67	6.69	8.63	9.67	4.68	4.71	4.64	4.91	3.30	13.81	13.25	14.92	23.06
(0.50 , 0.50 , 0.20 , 1.00 , 1.00)	11.40	11.70	12.92	13.67	4.43	3.29	3.27	3.55	3.08	15.07	14.35	16.07	23.81
(0.50 , 0.50 , 0.20 , 1.20 , 1.20)	14.12	13.51	14.45	17.88	4.16	4.37	4.11	4.54	2.77	14.61	13.76	15.52	22.88
(0.50 , 0.50 , 0.20 , 1.50 , 1.50)	20.72	21.04	24.00	26.30	2.66	13.32	13.01	14.43	2.65	13.91	13.16	14.85	22.49
(0.50 , 0.50 , 0.20 , 2.00 , 2.00)	31.88	30.27	32.00	36.10	4.18	3.57	3.46	3.72	2.46	13.63	13.01	14.45	22.45
(0.50 , 0.50 , 0.80 , 0.80 , 0.80)	6.21	6.30	7.77	8.58	5.16	4.21	5.71	7.34	4.30	6.38	9.29	9.30	21.46
(0.50 , 0.50 , 0.80 , 1.00 , 1.00)	9.35	9.85	10.93	11.05	4.36	3.46	5.12	6.42	3.51	5.56	9.07	9.08	20.91

*: The policy based on the partitioned booking limits is implemented.

Table 4.8: Relative Difference (%) of Booking Limit Policies (Continued)

Instance	RM_{2P}-1	RM_{2P}-2	RM^{v}_{2P}-R3	RM^{w}_{2P}-R3	RM_{D}-1	RM_{D}-2	RM^{v}_{D}-R3	RM^{w}_{D}-R3	PA-1	PA-2	PA^{v}-R3	PA^{w}-R3	PA^{*}
(0.50 , 0.50 , 0.80 , 1.20 , 1.20)	13.12	13.38	16.00	16.58	4.04	2.87	4.61	5.63	3.13	4.41	8.03	8.03	19.17
(0.50 , 0.50 , 0.80 , 1.50 , 1.50)	16.30	16.84	19.07	21.56	3.37	2.64	7.35	7.35	3.36	3.77	7.35	7.36	18.00
(0.50 , 0.50 , 0.80 , 2.00 , 2.00)	26.18	25.74	27.57	30.15	4.36	2.75	6.11	6.11	4.33	3.71	6.11	6.11	16.95
(0.50 , 1.00 , 0.20 , 0.80 , 0.80)	13.05	13.07	17.96	13.04	12.97	12.98	12.97	12.97	12.98	12.98	12.97	12.98	14.13
(0.50 , 1.00 , 0.20 , 1.00 , 1.00)	7.40	7.29	15.33	7.42	19.69	6.29	20.19	19.69	22.01	9.07	21.94	22.01	25.52
(0.50 , 1.00 , 0.20 , 1.20 , 1.20)	6.10	5.84	15.60	6.11	15.84	2.03	16.43	15.84	18.44	4.80	18.44	18.44	22.17
(0.50 , 1.00 , 0.20 , 1.50 , 1.50)	8.54	8.60	19.69	8.58	16.82	0.40	16.80	16.82	17.24	2.11	17.22	17.24	20.75
(0.50 , 1.00 , 0.20 , 2.00 , 2.00)	14.09	11.70	25.32	14.14	14.90	**-2.51**	14.84	14.90	15.28	0.44	15.22	15.28	18.60
(0.50 , 1.00 , 0.80 , 0.80 , 0.80)	16.84	16.95	20.69	16.84	16.77	16.81	18.22	16.77	16.77	16.81	17.95	16.77	17.81
(0.50 , 1.00 , 0.80 , 1.00 , 1.00)	9.44	9.16	15.26	9.44	18.71	8.92	21.99	18.71	21.79	10.04	21.37	21.79	25.00
(0.50 , 1.00 , 0.80 , 1.20 , 1.20)	4.59	4.63	12.60	4.68	12.29	2.25	15.89	12.29	15.60	3.34	15.10	15.60	19.15
(0.50 , 1.00 , 0.80 , 1.50 , 1.50)	3.75	4.21	13.67	3.78	6.88	3.11	11.59	6.88	11.51	**-2.10**	10.80	11.51	14.90
(0.50 , 1.00 , 0.80 , 2.00 , 2.00)	4.55	2.02	13.91	4.63	**-4.89**	**-6.36**	**-3.23**	**-4.89**	6.08	**-6.14**	5.47	6.08	8.97
(1.00 , 0.50 , 0.20 , 0.80 , 0.80)	14.18	14.14	14.17	19.97	13.98	13.98	13.98	14.03	13.98	13.98	13.98	14.03	15.03
(1.00 , 0.50 , 0.20 , 1.00 , 1.00)	9.86	10.08	10.05	17.70	24.05	19.45	24.05	24.05	24.08	19.66	24.08	24.08	27.49
(1.00 , 0.50 , 0.20 , 1.20 , 1.20)	9.22	9.33	9.40	19.81	13.41	16.90	13.41	15.36	21.72	17.10	21.72	21.72	25.11
(1.00 , 0.50 , 0.20 , 1.50 , 1.50)	11.39	12.63	11.92	25.52	12.98	16.85	12.98	15.22	21.57	17.00	21.57	21.57	24.65
(1.00 , 0.50 , 0.20 , 2.00 , 2.00)	17.64	17.40	18.40	29.35	12.64	16.23	12.64	14.81	20.66	16.38	20.66	20.67	23.80
(1.00 , 0.50 , 0.80 , 0.80 , 0.80)	10.42	10.39	10.41	16.05	10.37	10.21	10.37	10.37	10.37	10.21	10.37	10.37	11.34
(1.00 , 0.50 , 0.80 , 1.00 , 1.00)	6.17	5.99	6.27	13.43	16.25	16.29	16.25	17.44	20.39	16.89	20.39	19.36	23.89
(1.00 , 0.50 , 0.80 , 1.20 , 1.20)	5.86	6.07	5.88	14.46	14.43	14.22	14.43	15.64	18.63	14.75	18.63	17.66	22.17
(1.00 , 0.50 , 0.80 , 1.50 , 1.50)	10.73	11.35	10.78	22.37	15.59	15.43	15.59	16.67	19.84	15.88	19.84	18.84	22.95
(1.00 , 0.50 , 0.80 , 2.00 , 2.00)	20.60	21.64	20.35	30.36	14.95	14.67	14.95	15.73	17.99	14.99	17.99	16.93	21.23
(0.75 , 0.75 , 0.20 , 0.80 , 0.80)	3.69	3.77	4.00	4.01	2.94	7.30	7.47	5.42	2.94	8.38	8.54	7.39	12.02
(0.75 , 0.75 , 0.20 , 1.00 , 1.00)	6.58	6.42	9.21	9.34	2.73	7.87	7.33	8.21	2.81	8.76	8.83	7.90	12.42
(0.75 , 0.75 , 0.20 , 1.20 , 1.20)	8.77	9.68	10.68	12.30	2.17	8.58	8.31	7.70	2.27	8.31	8.35	7.47	12.08
(0.75 , 0.75 , 0.20 , 1.50 , 1.50)	17.13	18.62	19.24	19.57	1.48	7.94	7.71	6.81	1.54	7.71	7.82	6.62	11.86
(0.75 , 0.75 , 0.20 , 2.00 , 2.00)	23.83	23.20	24.10	25.09	1.58	8.08	7.59	6.88	1.68	7.59	7.66	6.42	11.47
(0.75 , 0.75 , 0.80 , 0.80 , 0.80)	3.22	3.34	3.52	3.46	2.88	4.13	3.66	2.96	2.84	7.65	5.52	3.22	11.20
(0.75 , 0.75 , 0.80 , 1.00 , 1.00)	3.65	3.50	4.30	4.68	2.27	5.89	3.33	2.26	2.27	7.12	4.82	2.51	10.61
(0.75 , 0.75 , 0.80 , 1.20 , 1.20)	6.50	7.95	8.64	9.07	1.61	6.08	4.11	1.83	1.65	6.83	4.36	2.33	10.40

Table 4.9: Relative Difference (%) of Booking Limit Policies (Continued)

Instance	RM_{2P}-1	RM_{2P}-2	RM_{2P}^{v}-R3	RM_{2P}^{w}-R3	RM_{D}-1	RM_{D}-2	RM_{D}^{v}-R3	RM_{D}^{w}-R3	PA-1	PA-2	PA^{v}-R3	PA^{w}-R3	PA^{*}
(0.75 , 0.75 , 0.80 , 1.50 , 1.50)	14.00	15.59	16.38	16.89	1.65	2.55	1.71	1.08	1.66	5.39	3.53	1.63	9.11
(0.75 , 0.75 , 0.80 , 2.00 , 2.00)	21.02	20.14	21.21	21.39	2.09	1.19	1.08	1.08	2.11	4.69	2.53	1.54	8.19
(0.75 , 1.00 , 0.20 , 0.80 , 0.80)	6.76	6.79	7.50	6.76	7.46	7.36	7.39	7.46	7.50	7.36	7.39	7.50	10.76
(0.75 , 1.00 , 0.20 , 1.00 , 1.00)	5.66	5.46	6.58	5.64	5.72	4.99	8.01	10.18	6.18	5.00	8.10	10.17	13.69
(0.75 , 1.00 , 0.20 , 1.20 , 1.20)	5.63	5.51	10.51	5.63	4.90	4.11	7.08	8.10	5.35	4.12	7.60	9.51	12.82
(0.75 , 1.00 , 0.20 , 1.50 , 1.50)	9.68	9.94	15.24	9.69	3.78	2.94	6.22	7.37	4.27	2.95	6.68	8.74	12.29
(0.75 , 1.00 , 0.20 , 2.00 , 2.00)	17.36	18.34	19.92	17.33	2.29	1.56	4.45	5.62	2.79	1.54	4.92	6.93	10.83
(0.75 , 1.00 , 0.80 , 0.80 , 0.80)	7.74	7.76	8.12	7.74	7.91	7.96	7.93	8.05	7.93	7.96	7.93	8.05	11.08
(0.75 , 1.00 , 0.80 , 1.00 , 1.00)	5.49	5.36	5.98	5.52	4.99	4.96	7.09	8.37	5.45	4.96	7.13	9.17	12.53
(0.75 , 1.00 , 0.80 , 1.20 , 1.20)	4.42	4.27	8.36	4.45	2.98	3.04	5.47	4.47	3.41	3.04	5.55	7.43	10.60
(0.75 , 1.00 , 0.80 , 1.50 , 1.50)	6.37	6.42	11.48	6.40	2.38	0.78	3.37	2.38	1.57	1.00	3.56	5.43	8.82
(0.75 , 1.00 , 0.80 , 2.00 , 2.00)	11.71	12.49	14.57	11.77	**-1.55**	**-1.36**	**-0.53**	**-0.46**	**-1.08**	**-1.38**	**-0.45**	1.01	4.56
(1.00 , 0.75 , 0.20 , 0.80 , 0.80)	5.56	5.50	5.55	6.34	6.20	6.19	6.53	6.19	6.20	6.19	6.53	6.19	9.81
(1.00 , 0.75 , 0.20 , 1.00 , 1.00)	5.14	5.44	5.27	7.82	5.11	7.20	11.54	8.15	6.74	7.69	12.04	8.85	15.56
(1.00 , 0.75 , 0.20 , 1.20 , 1.20)	6.07	6.07	6.24	10.98	4.49	7.40	11.20	8.36	6.07	7.31	11.06	8.28	14.49
(1.00 , 0.75 , 0.20 , 1.50 , 1.50)	8.80	10.75	9.23	16.26	3.62	6.08	9.02	6.66	5.20	6.30	9.74	7.17	13.41
(1.00 , 0.75 , 0.20 , 2.00 , 2.00)	15.96	19.72	17.07	21.31	2.75	5.58	8.91	6.27	4.32	6.02	9.91	7.04	13.70
(1.00 , 0.75 , 0.80 , 0.80 , 0.80)	3.16	3.12	3.17	3.81	3.46	3.64	4.20	3.65	3.48	3.64	4.24	3.65	7.75
(1.00 , 0.75 , 0.80 , 1.00 , 1.00)	2.64	2.55	2.75	4.07	2.28	4.99	9.22	5.17	4.35	5.00	9.22	5.18	12.72
(1.00 , 0.75 , 0.80 , 1.20 , 1.20)	3.43	3.27	3.52	8.00	1.74	4.37	8.37	4.54	3.63	4.37	8.36	4.55	11.94
(1.00 , 0.75 , 0.80 , 1.50 , 1.50)	5.95	7.60	6.20	12.58	1.21	3.77	7.50	3.95	2.91	3.78	7.50	3.96	11.43
(1.00 , 0.75 , 0.80 , 2.00 , 2.00)	16.14	18.03	16.09	20.44	0.81	3.37	8.40	3.52	2.66	3.40	8.40	3.55	12.31
(0.90 , 0.90 , 0.20 , 0.80 , 0.80)	2.64	2.58	2.71	2.63	2.37	3.60	3.60	3.60	2.47	3.58	3.58	3.60	7.23
(0.90 , 0.90 , 0.20 , 1.00 , 1.00)	3.92	3.91	4.51	4.44	2.46	3.69	3.69	3.69	2.54	3.92	3.92	3.69	7.31
(0.90 , 0.90 , 0.20 , 1.20 , 1.20)	4.37	4.40	4.94	4.75	2.16	3.31	3.31	3.31	2.22	3.72	3.72	3.31	7.11
(0.90 , 0.90 , 0.20 , 1.50 , 1.50)	10.39	11.38	12.91	12.73	1.70	2.99	2.99	2.99	1.71	3.19	3.19	2.99	6.97
(0.90 , 0.90 , 0.20 , 2.00 , 2.00)	15.53	17.77	18.97	18.91	1.11	2.42	2.42	2.42	1.13	2.49	2.49	2.42	6.26
(0.90 , 0.90 , 0.80 , 0.80 , 0.80)	2.56	2.49	2.61	2.47	2.39	3.30	3.26	3.07	2.42	3.37	3.32	3.07	6.73
(0.90 , 0.90 , 0.80 , 1.00 , 1.00)	2.77	2.79	3.50	3.42	1.68	2.65	2.59	2.33	1.71	2.71	2.60	2.34	6.21
(0.90 , 0.90 , 0.80 , 1.20 , 1.20)	3.19	3.26	3.63	3.59	1.66	2.16	2.13	1.97	1.70	2.31	2.15	2.01	5.60
(0.90 , 0.90 , 0.80 , 1.50 , 1.50)	6.99	8.28	9.12	9.18	0.41	1.26	1.24	1.04	0.43	1.26	1.14	1.09	4.88

Table 4.10: Relative Difference (%) of Booking Limit Policies (Continued)

Instance	RM_{2P}-1	RM_{2P}-2	RM^{v}_{2P}-R3	RM^{w}_{2P}-R3	RM_{D}-1	RM_{D}-2	RM^{v}_{D}-R3	RM^{w}_{D}-R3	PA-1	PA-2	PA^{v}-R3	PA^{w}-R3	PA^{*}
(0.90 , 0.90 , 0.80 , 2.00 , 2.00)	14.45	16.26	16.09	16.41	0.11	1.26	1.21	0.93	0.10	**-0.29**	0.15	0.93	3.36
(0.90 , 1.00 , 0.20 , 0.80 , 0.80)	3.98	3.98	4.05	3.98	4.80	5.36	5.36	5.49	4.80	5.36	5.36	5.54	8.83
(0.90 , 1.00 , 0.20 , 1.00 , 1.00)	3.84	3.78	4.67	3.82	4.26	4.63	4.63	4.60	4.27	4.63	4.63	4.81	8.31
(0.90 , 1.00 , 0.20 , 1.20 , 1.20)	5.18	4.96	6.61	5.18	3.58	4.25	4.25	3.87	3.64	4.25	4.25	4.32	7.75
(0.90 , 1.00 , 0.20 , 1.50 , 1.50)	7.92	8.15	11.96	8.31	3.44	4.12	4.12	3.69	3.50	4.12	4.12	4.09	7.69
(0.90 , 1.00 , 0.20 , 2.00 , 2.00)	14.63	13.65	18.61	15.30	2.71	3.19	3.19	3.11	2.72	3.19	3.19	3.20	6.85
(0.90 , 1.00 , 0.80 , 0.80 , 0.80)	4.02	4.02	4.11	4.02	3.98	4.99	4.99	5.13	4.10	4.99	4.99	5.14	8.43
(0.90 , 1.00 , 0.80 , 1.00 , 1.00)	3.02	2.90	3.45	2.98	2.46	3.31	3.33	3.52	2.66	3.31	3.33	3.69	7.24
(0.90 , 1.00 , 0.80 , 1.20 , 1.20)	2.77	2.66	3.73	2.89	1.60	2.54	2.56	2.62	1.74	2.54	2.56	2.74	6.17
(0.90 , 1.00 , 0.80 , 1.50 , 1.50)	3.81	3.65	5.92	3.80	1.36	1.87	1.88	1.86	1.43	1.87	1.88	2.13	5.31
(0.90 , 1.00 , 0.80 , 2.00 , 2.00)	13.79	12.26	16.87	14.13	**-0.25**	0.51	0.46	**-0.21**	**-0.26**	0.51	0.46	**-0.17**	3.46
(1.00 , 0.90 , 0.20 , 0.80 , 0.80)	3.07	3.04	3.09	3.09	3.82	4.26	4.65	4.25	3.89	4.26	4.63	4.25	7.86
(1.00 , 0.90 , 0.20 , 1.00 , 1.00)	3.44	3.39	3.48	4.59	3.68	4.20	4.56	4.20	3.73	4.20	4.86	4.20	8.45
(1.00 , 0.90 , 0.20 , 1.20 , 1.20)	4.19	4.33	4.32	5.34	3.23	4.00	4.24	3.99	3.39	4.00	4.52	3.99	8.09
(1.00 , 0.90 , 0.20 , 1.50 , 1.50)	7.76	8.02	8.23	12.68	3.34	3.87	3.98	3.87	3.43	3.87	4.24	3.87	7.97
(1.00 , 0.90 , 0.20 , 2.00 , 2.00)	11.64	14.18	12.71	19.01	2.43	3.15	3.46	3.15	2.58	3.15	3.71	3.15	7.32
(1.00 , 0.90 , 0.80 , 0.80 , 0.80)	2.48	2.44	2.48	2.48	2.40	3.50	4.10	3.50	2.49	3.50	4.15	3.50	7.37
(1.00 , 0.90 , 0.80 , 1.00 , 1.00)	2.05	1.98	2.09	2.84	1.48	2.70	3.79	2.70	1.55	2.70	3.97	2.70	7.66
(1.00 , 0.90 , 0.80 , 1.20 , 1.20)	2.45	2.39	2.66	3.41	1.02	2.31	3.35	2.31	1.08	2.31	3.50	2.31	7.16
(1.00 , 0.90 , 0.80 , 1.50 , 1.50)	3.78	3.48	3.77	6.45	0.69	1.61	2.64	1.61	0.76	1.61	2.82	1.61	6.37
(1.00 , 0.90 , 0.80 , 2.00 , 2.00)	12.90	13.72	12.42	17.81	**-0.29**	0.45	0.91	0.45	**-0.30**	0.45	1.79	0.45	5.57
(1.00 , 1.00 , 0.20 , 0.80 , 0.80)	2.36	2.36	2.37	2.36	2.31	2.31	2.31	2.31	2.31	2.31	2.31	2.31	5.85
(1.00 , 1.00 , 0.20 , 1.00 , 1.00)	2.34	2.29	2.40	2.33	2.04	2.04	2.04	2.04	2.04	2.04	2.04	2.04	5.69
(1.00 , 1.00 , 0.20 , 1.20 , 1.20)	3.63	3.82	4.40	4.21	1.54	1.54	1.54	1.54	1.54	1.54	1.54	1.54	5.08
(1.00 , 1.00 , 0.20 , 1.50 , 1.50)	5.66	5.57	6.40	6.04	1.54	1.54	1.54	1.54	1.54	1.54	1.54	1.54	4.97
(1.00 , 1.00 , 0.20 , 2.00 , 2.00)	10.94	11.91	12.30	13.51	1.29	1.29	1.29	1.29	1.29	1.29	1.29	1.29	4.87
(1.00 , 1.00 , 0.80 , 0.80 , 0.80)	2.24	2.24	2.25	2.23	2.19	2.19	2.19	2.19	2.19	2.19	2.19	2.19	5.61
(1.00 , 1.00 , 0.80 , 1.00 , 1.00)	1.73	1.63	1.84	1.66	1.43	1.43	1.43	1.43	1.43	1.43	1.43	1.43	4.97
(1.00 , 1.00 , 0.80 , 1.20 , 1.20)	2.32	2.35	3.06	3.00	0.76	0.76	0.76	0.76	0.76	0.76	0.76	0.76	4.22
(1.00 , 1.00 , 0.80 , 1.50 , 1.50)	2.91	2.94	3.31	3.22	1.02	1.02	1.02	1.02	1.02	1.02	1.02	1.02	4.22
(1.00 , 1.00 , 0.80 , 2.00 , 2.00)	7.30	9.20	10.07	9.85	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	3.37

Table 4.11: Relative Difference (%) of Booking Limit Policies (Continued)

Instance	RM_{2P}-1	RM_{2P}-2	RM^{v}_{2P}-R3	RM^{w}_{2P}-R3	RM_{D}-1	RM_{D}-2	RM^{v}_{D}-R3	RM^{w}_{D}-R3	PA-1	PA-2	PA^{v}-R3	PA^{w}-R3	PA^{*}
(1.10 , 1.10 , 0.20 , 0.80 , 0.80)	2.62	2.61	2.63	2.61	2.61	2.61	2.61	2.61	2.61	2.61	2.61	2.61	5.94
(1.10 , 1.10 , 0.20 , 1.00 , 1.00)	2.15	2.11	2.15	2.12	2.05	2.05	2.05	2.05	2.05	2.05	2.05	2.05	5.52
(1.10 , 1.10 , 0.20 , 1.20 , 1.20)	2.23	2.19	2.28	2.20	1.89	1.89	1.89	1.89	1.89	1.89	1.89	1.89	5.49
(1.10 , 1.10 , 0.20 , 1.50 , 1.50)	3.61	3.94	4.33	3.84	1.37	1.37	1.37	1.37	1.37	1.37	1.37	1.37	4.86
(1.10 , 1.10 , 0.20 , 2.00 , 2.00)	5.61	5.48	6.73	5.92	0.90	0.90	0.90	0.90	0.90	0.90	0.90	0.90	4.56
(1.10 , 1.10 , 0.80 , 0.80 , 0.80)	2.21	2.20	2.20	2.20	2.20	2.20	2.20	2.20	2.20	2.20	2.20	2.20	5.60
(1.10 , 1.10 , 0.80 , 1.00 , 1.00)	1.78	1.76	1.87	1.77	1.71	1.71	1.71	1.71	1.71	1.71	1.71	1.71	5.17
(1.10 , 1.10 , 0.80 , 1.20 , 1.20)	1.45	1.39	1.52	1.37	1.12	1.12	1.12	1.12	1.12	1.12	1.12	1.12	4.80
(1.10 , 1.10 , 0.80 , 1.50 , 1.50)	1.64	1.77	2.40	1.92	0.17	0.17	0.17	0.17	0.17	0.17	0.17	0.17	3.69
(1.10 , 1.10 , 0.80 , 2.00 , 2.00)	2.99	3.03	4.12	3.55	**-0.18**	**-0.18**	**-0.18**	**-0.18**	**-0.18**	**-0.18**	**-0.18**	**-0.18**	3.21
(1.10 , 1.00 , 0.20 , 0.80 , 0.80)	3.37	3.36	3.37	3.37	3.33	3.33	3.33	3.33	3.33	3.33	3.33	3.33	6.79
(1.10 , 1.00 , 0.20 , 1.00 , 1.00)	3.00	2.95	3.02	3.06	2.81	2.81	2.81	2.81	2.81	2.81	2.81	2.81	6.17
(1.10 , 1.00 , 0.20 , 1.20 , 1.20)	3.01	2.97	3.07	4.35	2.36	2.36	2.36	2.36	2.36	2.36	2.36	2.36	5.94
(1.10 , 1.00 , 0.20 , 1.50 , 1.50)	5.21	5.14	5.31	5.95	1.94	1.94	1.94	1.94	1.94	1.94	1.94	1.94	5.64
(1.10 , 1.00 , 0.20 , 2.00 , 2.00)	7.99	8.03	8.69	12.84	1.89	1.89	1.89	1.89	1.89	1.89	1.89	1.89	5.63
(1.10 , 1.00 , 0.80 , 0.80 , 0.80)	2.36	2.35	2.36	2.36	2.33	2.33	2.33	2.33	2.33	2.33	2.33	2.33	5.79
(1.10 , 1.00 , 0.80 , 1.00 , 1.00)	2.07	2.02	2.10	2.06	1.89	1.89	1.89	1.89	1.89	1.89	1.89	1.89	5.27
(1.10 , 1.00 , 0.80 , 1.20 , 1.20)	1.55	1.47	1.63	2.66	0.91	0.91	0.91	0.91	0.91	0.91	0.91	0.91	4.47
(1.10 , 1.00 , 0.80 , 1.50 , 1.50)	3.28	3.07	3.27	3.75	0.83	0.83	0.83	0.83	0.83	0.83	0.83	0.83	4.28
(1.10 , 1.00 , 0.80 , 2.00 , 2.00)	6.04	5.70	6.29	10.40	0.31	0.31	0.31	0.31	0.31	0.31	0.31	0.31	3.80
(1.00 , 1.10 , 0.20 , 0.80 , 0.80)	4.24	4.24	4.26	4.25	4.22	4.22	4.22	4.22	4.22	4.22	4.22	4.22	7.57
(1.00 , 1.10 , 0.20 , 1.00 , 1.00)	3.43	3.46	3.57	3.45	3.32	3.32	3.32	3.32	3.32	3.32	3.32	3.32	6.63
(1.00 , 1.10 , 0.20 , 1.20 , 1.20)	3.13	3.03	4.22	3.13	2.51	2.51	2.51	2.51	2.51	2.51	2.51	2.51	6.06
(1.00 , 1.10 , 0.20 , 1.50 , 1.50)	4.97	4.75	5.90	5.01	1.98	1.98	1.98	1.98	1.98	1.98	1.98	1.98	5.53
(1.00 , 1.10 , 0.20 , 2.00 , 2.00)	7.74	7.56	12.47	8.21	1.59	1.59	1.59	1.59	1.59	1.59	1.59	1.59	5.22
(1.00 , 1.10 , 0.80 , 0.80 , 0.80)	4.15	4.15	4.17	4.15	4.12	4.12	4.12	4.12	4.12	4.12	4.12	4.12	7.47
(1.00 , 1.10 , 0.80 , 1.00 , 1.00)	3.18	3.19	3.34	3.17	3.06	3.06	3.06	3.06	3.06	3.06	3.06	3.06	6.26
(1.00 , 1.10 , 0.80 , 1.20 , 1.20)	2.35	2.18	3.22	2.33	1.70	1.70	1.70	1.70	1.70	1.70	1.70	1.70	5.19
(1.00 , 1.10 , 0.80 , 1.50 , 1.50)	2.45	2.25	3.20	2.51	0.79	0.79	0.79	0.79	0.79	0.79	0.79	0.79	4.14
(1.00 , 1.10 , 0.80 , 2.00 , 2.00)	5.09	4.85	9.42	5.44	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	3.56

Table 4.12: Some Performance Measures of Booking Limit Policies

	RM$_{2P}$-2					RM$_{D}$-2					PA-2				
Instance	Rel. Diff.	Utilization	Offloaded	Acc	OC	Rel. Diff.	Utilization	Offloaded	Acc	OC	Rel. Diff.	Utilization	Offloaded	Acc	OC
(0.50 , 0.50 , 0.20 , 1.00 , 1.00)	11.70	(79.72 , 80.30)	(0.65 , 0.00)	59.16	5.65	3.29	(93.29 , 94.53)	(1.03 , 0.00)	70.52	1.04	17.75	(74.74 , 75.20)	(0.59 , 0.00)	54.05	4.45
(0.50 , 0.50 , 0.20 , 1.50 , 1.50)	21.04	(68.4 , 68.64)	(0.36 , 0.00)	48.83	2.60	13.32	(75.72 , 75.97)	(0.54 , 0.00)	55.35	0.77	16.15	(75.07 , 75.32)	(0.53 , 0.00)	54.30	4.60
(0.50 , 0.50 , 0.20 , 2.00 , 2.00)	30.27	(58.17 , 58.51)	(0.18 , 0.00)	40.07	1.40	3.57	(90.82 , 92.10)	(0.78 , 0.00)	68.03	1.59	15.79	(74.35 , 74.98)	(0.37 , 0.00)	54.34	3.70
(0.50 , 0.50 , 0.80 , 1.00 , 1.00)	9.85	(80.33 , 80.31)	(3.66 , 0.00)	59.14	10.35	3.46	(91.48 , 92.46)	(5.10 , 0.00)	68.15	5.59	5.89	(86.00 , 86.02)	(4.43 , 0.00)	63.15	12.70
(0.50 , 0.50 , 0.80 , 1.50 , 1.50)	16.84	(69.26 , 68.58)	(2.22 , 0.00)	48.75	6.75	2.64	(88.18 , 88.04)	(4.64 , 0.00)	65.79	7.32	3.91	(85.61 , 85.54)	(4.23 , 0.00)	63.33	13.35
(0.50 , 0.50 , 0.80 , 2.00 , 2.00)	25.74	(56.49 , 57.50)	(1.14 , 0.00)	40.26	3.55	2.75	(86.73 , 87.79)	(4.16 , 0.00)	65.66	8.76	3.85	(84.03 , 85.29)	(3.76 , 0.00)	62.92	11.95
(1.00 , 0.50 , 0.20 , 1.00 , 1.00)	10.08	(45.88 , 92.21)	(0.00 , 0.00)	68.02	0.00	19.45	(38.29 , 76.70)	(0.00 , 0.00)	51.97	0.00	24.48	(38.16 , 76.43)	(0.00 , 0.00)	51.88	0.00
(1.00 , 0.50 , 0.20 , 1.50 , 1.50)	12.63	(40.36 , 81.07)	(0.00 , 0.00)	58.81	0.00	16.85	(37.33 , 74.86)	(0.00 , 0.00)	52.61	0.00	20.47	(37.22 , 74.64)	(0.00 , 0.00)	52.57	0.00
(1.00 , 0.50 , 0.20 , 2.00 , 2.00)	17.4	(37.51 , 74.95)	(0.00 , 0.00)	52.47	0.00	16.23	(37.58 , 74.92)	(0.00 , 0.00)	52.68	0.00	19.58	(37.50 , 74.71)	(0.00 , 0.00)	52.63	0.00
(1.00 , 0.50 , 0.80 , 1.00 , 1.00)	5.99	(46.43 , 92.62)	(0.05 , 0.00)	67.41	0.20	16.29	(38.40 , 76.63)	(0.13 , 0.00)	51.93	0.15	20.32	(38.08 , 75.98)	(0.13 , 0.00)	51.41	0.35
(1.00 , 0.50 , 0.80 , 1.50 , 1.50)	11.35	(40.53 , 80.74)	(0.14 , 0.00)	58.90	0.35	15.43	(37.60 , 74.84)	(0.13 , 0.00)	52.65	0.23	18.88	(37.37 , 74.24)	(0.13 , 0.00)	52.03	0.35
(1.00 , 0.50 , 0.80 , 2.00 , 2.00)	21.64	(35.02 , 69.02)	(0.19 , 0.00)	48.57	0.45	14.67	(38.08 , 75.07)	(0.19 , 0.00)	52.92	0.44	17.64	(37.97 , 74.54)	(0.19 , 0.00)	52.36	0.45
(0.75 , 0.75 , 0.20 , 1.00 , 1.00)	6.42	(82.50 , 83.00)	(0.83 , 0.00)	78.63	7.45	7.87	(79.27 , 79.87)	(0.84 , 0.00)	77.73	0.84	9.60	(78.15 , 78.66)	(0.79 , 0.00)	75.14	6.55
(0.75 , 0.75 , 0.20 , 1.50 , 1.50)	18.62	(65.94 , 66.20)	(0.50 , 0.00)	64.72	3.60	7.94	(77.00 , 77.32)	(0.72 , 0.00)	74.98	1.06	8.36	(77.37 , 77.63)	(0.74 , 0.00)	74.86	5.95
(0.75 , 0.75 , 0.20 , 2.00 , 2.00)	23.20	(61.37 , 61.22)	(0.67 , 0.00)	56.90	3.20	8.08	(76.62 , 76.93)	(0.82 , 0.00)	75.06	1.62	8.22	(77.22 , 77.47)	(0.83 , 0.00)	75.05	5.35
(0.75 , 0.75 , 0.80 , 1.00 , 1.00)	3.50	(84.79 , 85.09)	(4.09 , 0.00)	80.33	12.10	5.89	(80.16 , 80.31)	(3.57 , 0.00)	78.34	3.88	7.67	(78.28 , 78.62)	(3.36 , 0.00)	74.85	10.05
(0.75 , 0.75 , 0.80 , 1.50 , 1.50)	15.59	(66.03 , 66.08)	(2.15 , 0.00)	64.86	5.75	2.55	(83.36 , 83.36)	(3.87 , 0.00)	79.28	6.65	5.69	(77.31 , 77.22)	(3.09 , 0.00)	74.91	9.10
(0.75 , 0.75 , 0.80 , 2.00 , 2.00)	20.14	(62.49 , 61.22)	(2.44 , 0.00)	57.00	5.10	1.19	(85.82 , 85.22)	(4.21 , 0.00)	82.39	9.61	4.92	(77.49 , 76.83)	(3.25 , 0.00)	74.99	8.15
(1.00 , 1.00 , 0.20 , 1.00 , 1.00)	2.29	(78.05 , 78.63)	(1.03 , 0.00)	91.47	6.20	2.04	(78.24 , 78.81)	(1.03 , 0.00)	91.75	1.13	2.08	(78.24 , 78.81)	(1.03 , 0.00)	91.75	6.20
(1.00 , 1.00 , 0.20 , 1.50 , 1.50)	5.57	(74.49 , 74.75)	(0.86 , 0.00)	83.52	5.70	1.54	(79.77 , 80.15)	(0.95 , 0.00)	91.68	1.54	1.57	(79.77 , 80.15)	(0.95 , 0.00)	91.68	6.65
(1.00 , 1.00 , 0.20 , 2.00 , 2.00)	11.91	(67.08 , 67.07)	(0.75 , 0.00)	78.62	4.65	1.29	(79.20 , 79.40)	(0.97 , 0.00)	92.09	2.06	1.31	(79.20 , 79.40)	(0.97 , 0.00)	92.09	6.90
(1.00 , 1.00 , 0.80 , 1.00 , 1.00)	1.63	(77.84 , 78.64)	(3.41 , 0.00)	91.46	8.15	1.43	(77.99 , 78.81)	(3.43 , 0.00)	91.75	4.16	1.45	(77.99 , 78.81)	(3.43 , 0.00)	91.75	8.25
(1.00 , 1.00 , 0.80 , 1.50 , 1.50)	2.94	(75.86 , 75.40)	(3.37 , 0.00)	85.73	8.95	1.02	(80.53 , 80.01)	(3.84 , 0.00)	91.63	6.89	1.03	(80.53 , 80.01)	(3.84 , 0.00)	91.63	10.05
(1.00 , 1.00 , 0.80 , 2.00 , 2.00)	9.20	(67.63 , 66.67)	(2.60 , 0.00)	78.86	6.00	0.01	(79.52 , 79.40)	(3.64 , 0.00)	92.09	8.53	0.01	(79.52 , 79.40)	(3.64 , 0.00)	92.09	9.90

4.5.2 Bid-Price Policies

Relative percent difference of the bid-price policies from HD heuristic are presented in Tables (4.13)-(4.16). Figure (4.4) present the relative percent differences of each bid-price policy from HD heuristic under selected instances. For each instance we generated 1000 realizations of our random parameters and evaluated net revenue of each policy under these individual realizations. In Figure (4.5), under four selected instances, we plot the histogram of realized differences in net revenues given by RLP-1 and BP. Similarly, in Figure (4.6), we plot the differences between RLP-2 and PD under the same instances.

Table (4.15) reveals that when the capacity demand ratio gets larger than 1 on either dimensions, all bid-price policies except for PD perform the same as FCFS (Also see figure (4.5(d))). This is because capacity constraints of RLP-1, RLP-2 and BP models gets looser leading to smaller bid-prices. Consequently, all incoming booking requests are accepted unless they violate the capacity constraints. PD on the other hand, gives larger bid-prices and therefore rejects some of the incoming requests.

In all instances with capacity demand ratio smaller than 1 on at least one of the dimensions, BP was outperformed by all other bid-price policies (See Figures (4.4)).

Although accepting more requests causes higher off-loading costs on average, because bid-price policies accept booking requests with marginal return larger than a threshold value, they mostly compensate the off-loading costs in our computational studies. The results showed that accepting requests so that a certain amount of offloading is allowed gave better net revenue values. This might be because the offloading costs are not high enough. Utilization, relative difference and offloading cost percentage columns of Table (4.16) also reveal that prioritizing the utilization of volume and weight capacities increases the overall performance under our parameters.

PD's bid-prices are more robust among different instances whereas bid-prices given by RLP-1 and RLP-2 are more responsive to the capacity-demand ratio. Unlike RLP-2, RLP-1 and PD do not incorporate the off-loading cost to their models, therefore bid-prices given by RLP-2 are also affected by the penalty cost rate ratios. Under some instances, this might result in conservative bid-prices causing RLP-2 policy to accept less. Average net revenue over all instances with penalty cost rate ratio greater than or equal to 1.5 was largest for RLP-2. Under instances with higher coefficient of variation, the best average net revenue is given by RLP-2. So, when there exists high penalty costs and high variability, RLP-2 performs satisfactorily.

RLP-2 mostly outperforms other policies however under some instances it's solution

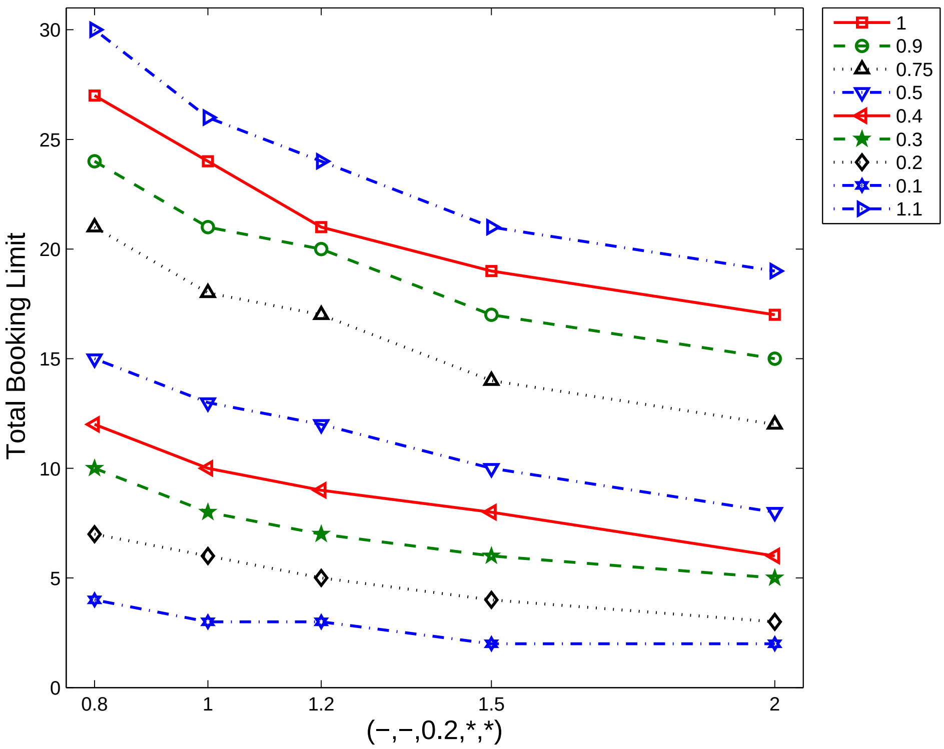
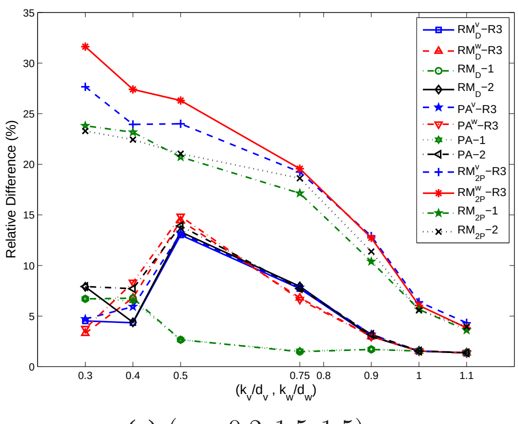
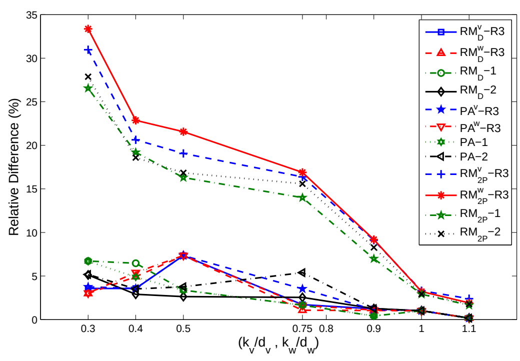


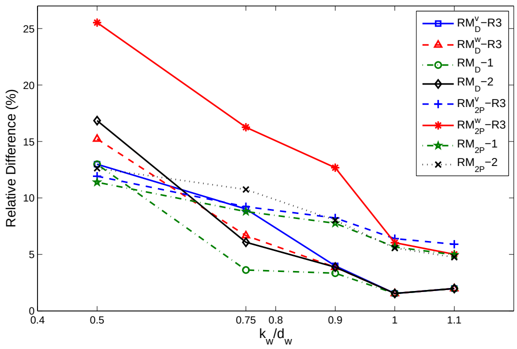
Figure 4.2: Total Booking Limits Obtained by RM_{2P}



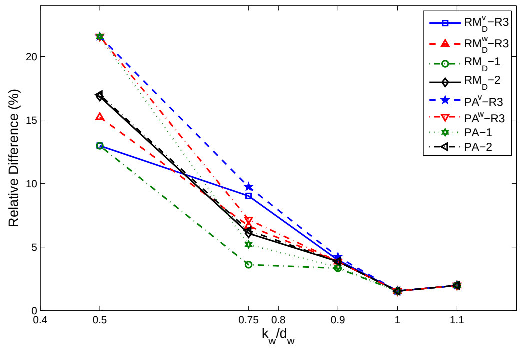
(a) $(*, *, 0.2, 1.5, 1.5)$



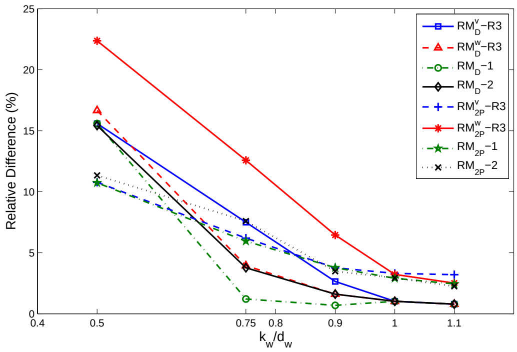
(b) $(*, *, 0.8, 1.5, 1.5)$



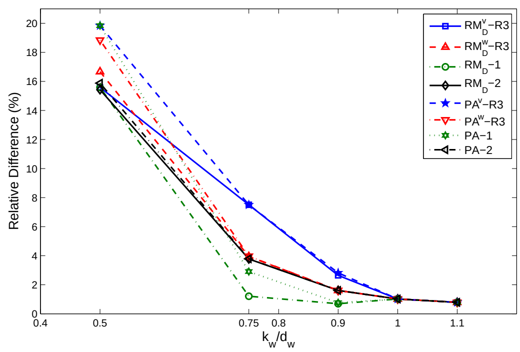
(c) $(1.0, *, 0.2, 1.5, 1.5)$



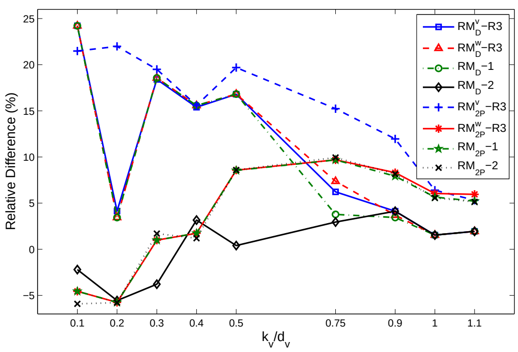
(d) $(1.0, *, 0.2, 1.5, 1.5)$



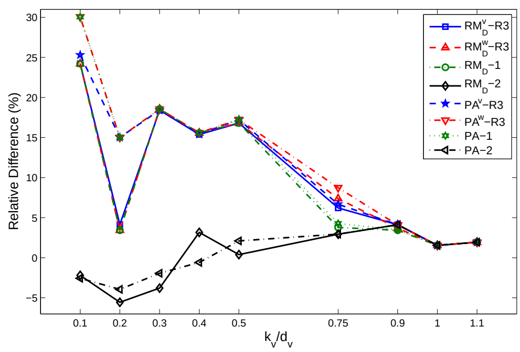
(e) $(1.0, *, 0.8, 1.5, 1.5)$



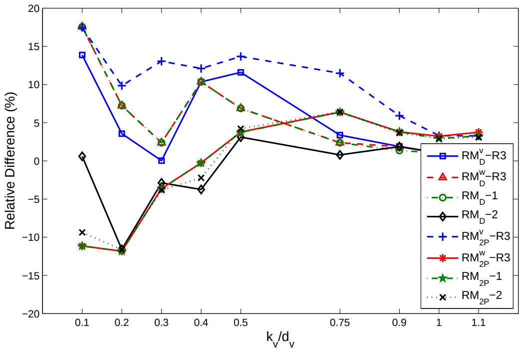
(f) $(1.0, *, 0.8, 1.5, 1.5)$



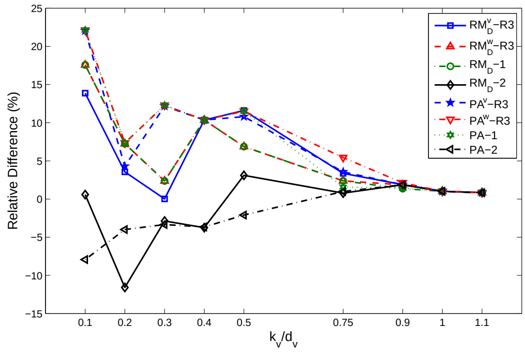
(g) $(*, 1.0, 0.2, 1.5, 1.5)$



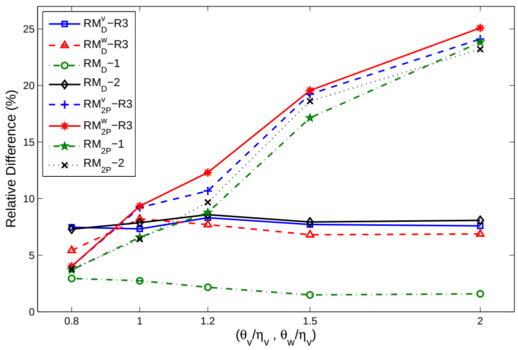
(h) $(*, 1.0, 0.2, 1.5, 1.5)$



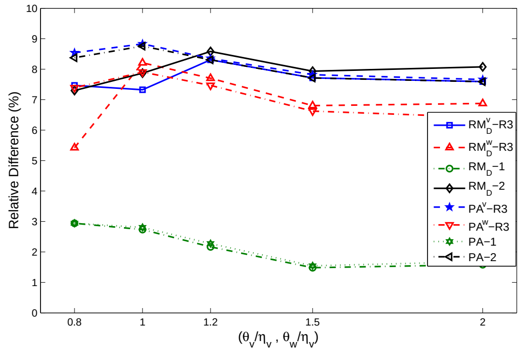
(i) $(*, 1.0, 0.8, 1.5, 1.5)$



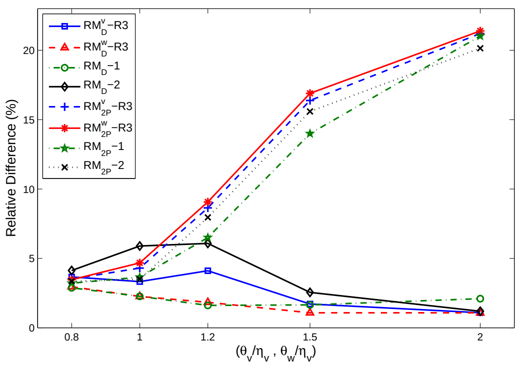
(j) $(*, 1.0, 0.8, 1.5, 1.5)$



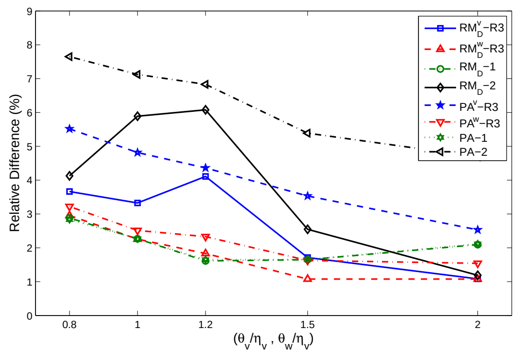
(k) $(0.75, 0.75, 0.2, *, *)$



(l) $(0.75, 0.75, 0.2, *, *)$



(m) $(0.75, 0.75, 0.8, *, *)$



(n) $(0.75, 0.75, 0.8, *, *)$

Figure 4.3: Relative Difference of Booking Limit Policies

quality decreases. When the capacity demand ratio is equal to 1 for volume and 0.5 or 0.75 for weight, RLP-1 mostly performs better than RLP-2. Our experimental results revealed that under those instances, bid-prices given by RLP-1 were smaller. PD on the other hand, outperforms all other bid-price policies when the capacity demand ratio is smaller than or equal to 0.3 on at least one of the dimensions (See Figure (4.4(a)) for an example). Although in terms of average net revenue RLP-2 is outperformed under these instances, Figure (4.5(d)) shows that out of 1000 replications, there are a large number of realizations where RLP-2 performs better than PD.

Bid-price policies performed better than the booking limit policies. This is caused by the fact that booking limit policies set a number limit on how many booking requests to accept from a certain class. However, arriving booking requests have different volume and weight values. Therefore, it is less logical to ignore the volume and weight requirements and accept booking requests solely based on a booking limit. Bid-price policies on the other hand, propose a more logical way to allocate capacity among different booking requests.

Table 4.13: Relative Difference (%) of Bid-Price Policies

Instance	RLP-1	RLP-2	BP	PD	FCFS	Instance	RLP-1	RLP-2	BP	PD	FCFS
(0.10 , 1.00 , 0.20 , 1.50 , 1.50)	9.76	**-9.49**	30.05	**-6.35**	3.30	(0.50 , 0.50 , 0.80 , 0.80 , 0.80)	9.31	8.38	17.90	5.96	5.35
(0.10 , 1.00 , 0.20 , 2.00 , 2.00)	7.22	**-12.78**	27.14	**-9.24**	0.86	(0.50 , 0.50 , 0.80 , 1.00 , 1.00)	9.08	2.64	17.68	5.33	4.46
(0.10 , 1.00 , 0.80 , 1.50 , 1.50)	8.32	**-13.52**	21.80	**-10.29**	4.88	(0.50 , 0.50 , 0.80 , 1.20 , 1.20)	8.03	2.54	16.17	4.27	4.15
(0.10 , 1.00 , 0.80 , 2.00 , 2.00)	**-0.19**	**-17.41**	14.73	**-18.24**	1.40	(0.50 , 0.50 , 0.80 , 1.50 , 1.50)	7.36	2.36	14.90	3.72	4.31
(0.20 , 1.00 , 0.20 , 1.50 , 1.50)	2.40	**-7.04**	15.00	**-8.89**	1.40	(0.50 , 0.50 , 0.80 , 2.00 , 2.00)	6.12	2.82	13.54	3.22	6.00
(0.20 , 1.00 , 0.20 , 2.00 , 2.00)	**-0.22**	**-9.69**	12.89	**-11.07**	0.10	(0.50 , 1.00 , 0.20 , 0.80 , 0.80)	15.72	12.77	26.71	13.09	14.12
(0.20 , 1.00 , 0.80 , 1.50 , 1.50)	**-6.14**	**-13.82**	7.24	**-13.55**	1.49	(0.50 , 1.00 , 0.20 , 1.00 , 1.00)	9.53	11.22	21.99	6.49	7.35
(0.20 , 1.00 , 0.80 , 2.00 , 2.00)	**-10.79**	**-15.30**	1.71	**-17.17**	**-1.51**	(0.50 , 1.00 , 0.20 , 1.20 , 1.20)	5.30	5.07	18.41	2.47	3.47
(0.30 , 0.30 , 0.20 , 1.50 , 1.50)	12.72	7.39	25.49	4.96	9.60	(0.50 , 1.00 , 0.20 , 1.50 , 1.50)	3.18	**-0.99**	17.23	**-0.47**	0.53
(0.30 , 0.30 , 0.20 , 2.00 , 2.00)	11.64	10.09	24.36	4.47	9.12	(0.50 , 1.00 , 0.20 , 2.00 , 2.00)	1.27	**-2.55**	15.27	**-2.14**	**-0.84**
(0.30 , 0.30 , 0.80 , 1.50 , 1.50)	10.11	2.37	20.32	3.47	10.25	(0.50 , 1.00 , 0.80 , 0.80 , 0.80)	18.64	17.98	28.08	16.86	18.26
(0.30 , 0.30 , 0.80 , 2.00 , 2.00)	6.79	2.28	15.43	1.50	10.57	(0.50 , 1.00 , 0.80 , 1.00 , 1.00)	10.82	8.47	21.77	8.90	10.25
(0.30 , 1.00 , 0.20 , 1.50 , 1.50)	3.01	**-3.49**	20.42	**-5.23**	1.09	(0.50 , 1.00 , 0.80 , 1.20 , 1.20)	3.97	2.08	15.59	2.69	4.01
(0.30 , 1.00 , 0.20 , 2.00 , 2.00)	0.54	**-5.69**	19.29	**-7.42**	**-0.25**	(0.50 , 1.00 , 0.80 , 1.50 , 1.50)	**-1.25**	**-3.10**	11.50	**-2.65**	**-0.89**
(0.30 , 1.00 , 0.80 , 1.50 , 1.50)	**-0.92**	**-7.76**	13.82	**-8.66**	0.62	(0.50 , 1.00 , 0.80 , 2.00 , 2.00)	**-6.32**	**-6.25**	6.08	**-7.44**	**-4.32**
(0.30 , 1.00 , 0.80 , 2.00 , 2.00)	**-7.29**	**-9.71**	8.80	**-13.45**	**-1.15**	(1.00 , 0.50 , 0.20 , 0.80 , 0.80)	19.73	17.03	27.30	18.56	15.22
(0.40 , 0.40 , 0.20 , 1.50 , 1.50)	9.86	4.55	23.29	5.55	6.79	(1.00 , 0.50 , 0.20 , 1.00 , 1.00)	15.46	9.04	24.05	14.02	10.05
(0.40 , 0.40 , 0.20 , 2.00 , 2.00)	8.97	3.55	22.37	5.17	6.42	(1.00 , 0.50 , 0.20 , 1.20 , 1.20)	12.51	6.80	21.69	11.11	7.32
(0.40 , 0.40 , 0.80 , 1.50 , 1.50)	8.07	2.30	18.75	3.60	8.41	(1.00 , 0.50 , 0.20 , 1.50 , 1.50)	12.11	17.85	21.56	10.52	5.24
(0.40 , 0.40 , 0.80 , 2.00 , 2.00)	6.82	2.52	16.84	2.31	8.69	(1.00 , 0.50 , 0.20 , 2.00 , 2.00)	11.77	18.89	20.71	10.13	4.74
(0.40 , 1.00 , 0.20 , 1.50 , 1.50)	3.10	**-1.85**	15.51	**-0.49**	0.62	(1.00 , 0.50 , 0.80 , 0.80 , 0.80)	15.05	17.58	23.19	14.50	11.37
(0.40 , 1.00 , 0.20 , 2.00 , 2.00)	1.72	**-3.14**	14.22	**-1.85**	**-0.29**	(1.00 , 0.50 , 0.80 , 1.00 , 1.00)	10.77	11.02	20.38	9.82	6.26
(0.40 , 1.00 , 0.80 , 1.50 , 1.50)	**-3.35**	**-4.23**	10.36	**-4.28**	**-1.17**	(1.00 , 0.50 , 0.80 , 1.20 , 1.20)	8.22	2.60	18.63	7.31	4.08
(0.40 , 1.00 , 0.80 , 2.00 , 2.00)	**-6.57**	**-4.36**	6.83	**-6.73**	**-1.31**	(1.00 , 0.50 , 0.80 , 1.50 , 1.50)	9.20	9.20	19.81	8.24	4.08
(0.50 , 0.50 , 0.20 , 0.80 , 0.80)	10.04	3.62	19.42	6.96	4.72	(1.00 , 0.50 , 0.80 , 2.00 , 2.00)	8.78	5.96	17.92	7.86	4.20
(0.50 , 0.50 , 0.20 , 1.00 , 1.00)	11.01	2.87	20.51	7.43	4.43	(0.75 , 0.75 , 0.20 , 0.80 , 0.80)	2.94	2.94	8.37	3.44	2.94
(0.50 , 0.50 , 0.20 , 1.20 , 1.20)	10.87	8.87	19.72	7.06	4.18	(0.75 , 0.75 , 0.20 , 1.00 , 1.00)	2.73	2.73	8.76	3.25	2.73
(0.50 , 0.50 , 0.20 , 1.50 , 1.50)	10.34	3.10	19.40	6.78	4.17	(0.75 , 0.75 , 0.20 , 1.20 , 1.20)	2.17	1.97	8.31	2.80	2.17
(0.50 , 0.50 , 0.20 , 2.00 , 2.00)	9.81	8.62	19.01	6.36	4.19	(0.75 , 0.75 , 0.20 , 1.50 , 1.50)	1.48	1.41	7.71	2.22	1.48

Table 4.14: Relative Difference (%) of Bid-Price Policies (Continued)

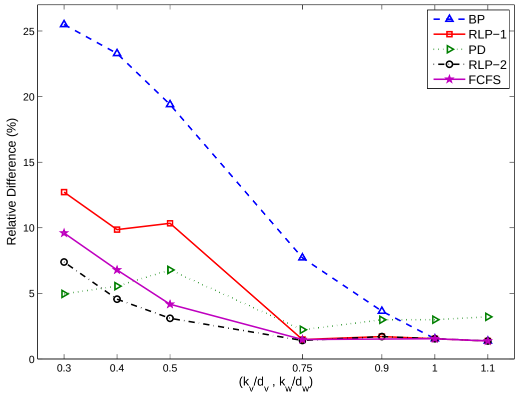
Instance	RLP-1	RLP-2	BP	PD	FCFS	Instance	RLP-1	RLP-2	BP	PD	FCFS
(0.75 , 0.75 , 0.20 , 2.00 , 2.00)	1.58	1.78	7.59	2.27	1.58	(0.90 , 0.90 , 0.20 , 1.50 , 1.50)	1.70	1.70	3.65	2.99	1.70
(0.75 , 0.75 , 0.80 , 0.80 , 0.80)	2.88	2.88	7.62	2.95	2.88	(0.90 , 0.90 , 0.20 , 2.00 , 2.00)	1.11	1.11	2.98	2.42	1.11
(0.75 , 0.75 , 0.80 , 1.00 , 1.00)	2.25	2.25	7.06	2.22	2.25	(0.90 , 0.90 , 0.80 , 0.80 , 0.80)	2.39	2.39	3.84	3.31	2.39
(0.75 , 0.75 , 0.80 , 1.20 , 1.20)	1.61	1.59	6.77	1.83	1.61	(0.90 , 0.90 , 0.80 , 1.00 , 1.00)	1.68	1.68	2.92	2.65	1.68
(0.75 , 0.75 , 0.80 , 1.50 , 1.50)	1.65	1.66	5.30	1.09	1.65	(0.90 , 0.90 , 0.80 , 1.20 , 1.20)	1.66	1.66	2.58	2.17	1.66
(0.75 , 0.75 , 0.80 , 2.00 , 2.00)	2.09	2.10	4.62	1.09	2.09	(0.90 , 0.90 , 0.80 , 1.50 , 1.50)	0.41	0.41	1.75	1.26	0.41
(0.75 , 1.00 , 0.20 , 0.80 , 0.80)	6.68	6.68	12.90	7.38	6.68	(0.90 , 0.90 , 0.80 , 2.00 , 2.00)	0.11	0.11	0.05	1.26	0.11
(0.75 , 1.00 , 0.20 , 1.00 , 1.00)	4.62	4.62	10.04	5.11	4.62	(0.90 , 1.00 , 0.20 , 0.80 , 0.80)	3.95	3.95	5.49	5.18	3.95
(0.75 , 1.00 , 0.20 , 1.20 , 1.20)	3.47	3.47	9.41	3.97	3.47	(0.90 , 1.00 , 0.20 , 1.00 , 1.00)	3.31	3.31	4.75	4.52	3.31
(0.75 , 1.00 , 0.20 , 1.50 , 1.50)	2.24	2.24	8.58	2.93	2.24	(0.90 , 1.00 , 0.20 , 1.20 , 1.20)	2.70	2.70	4.28	3.85	2.70
(0.75 , 1.00 , 0.20 , 2.00 , 2.00)	0.94	0.94	6.77	1.56	0.94	(0.90 , 1.00 , 0.20 , 1.50 , 1.50)	2.49	2.49	4.04	3.73	2.49
(0.75 , 1.00 , 0.80 , 0.80 , 0.80)	7.68	7.68	12.83	7.97	7.66	(0.90 , 1.00 , 0.20 , 2.00 , 2.00)	1.69	1.69	3.15	3.14	1.69
(0.75 , 1.00 , 0.80 , 1.00 , 1.00)	5.08	5.08	9.05	4.96	5.06	(0.90 , 1.00 , 0.80 , 0.80 , 0.80)	3.99	3.99	5.13	4.99	3.99
(0.75 , 1.00 , 0.80 , 1.20 , 1.20)	3.15	2.73	7.33	3.04	3.14	(0.90 , 1.00 , 0.80 , 1.00 , 1.00)	2.44	2.44	3.63	3.31	2.44
(0.75 , 1.00 , 0.80 , 1.50 , 1.50)	0.75	0.75	5.30	1.00	0.79	(0.90 , 1.00 , 0.80 , 1.20 , 1.20)	1.58	1.58	2.70	2.55	1.58
(0.75 , 1.00 , 0.80 , 2.00 , 2.00)	**-0.16**	**-0.16**	0.87	**-1.36**	**-0.18**	(0.90 , 1.00 , 0.80 , 1.50 , 1.50)	1.35	1.35	2.09	1.88	1.35
(1.00 , 0.75 , 0.20 , 0.80 , 0.80)	5.40	6.19	13.31	6.23	5.39	(0.90 , 1.00 , 0.80 , 2.00 , 2.00)	**-0.21**	**-0.21**	**-0.21**	0.51	**-0.21**
(1.00 , 0.75 , 0.20 , 1.00 , 1.00)	4.45	4.52	12.18	5.08	4.46	(1.00 , 0.90 , 0.20 , 0.80 , 0.80)	3.00	3.00	5.41	4.25	3.00
(1.00 , 0.75 , 0.20 , 1.20 , 1.20)	3.84	3.84	11.24	4.41	3.84	(1.00 , 0.90 , 0.20 , 1.00 , 1.00)	2.87	2.87	5.62	4.24	2.87
(1.00 , 0.75 , 0.20 , 1.50 , 1.50)	2.78	2.78	10.03	3.59	2.78	(1.00 , 0.90 , 0.20 , 1.20 , 1.20)	2.60	2.60	5.09	3.99	2.60
(1.00 , 0.75 , 0.20 , 2.00 , 2.00)	1.71	2.72	10.09	2.72	1.71	(1.00 , 0.90 , 0.20 , 1.50 , 1.50)	2.45	2.45	4.67	3.72	2.45
(1.00 , 0.75 , 0.80 , 0.80 , 0.80)	2.96	2.96	10.66	3.64	2.95	(1.00 , 0.90 , 0.20 , 2.00 , 2.00)	1.56	1.56	4.10	3.12	1.56
(1.00 , 0.75 , 0.80 , 1.00 , 1.00)	1.96	1.97	9.18	2.37	1.97	(1.00 , 0.90 , 0.80 , 0.80 , 0.80)	2.40	2.40	4.77	3.50	2.40
(1.00 , 0.75 , 0.80 , 1.20 , 1.20)	1.58	1.58	8.33	1.81	1.58	(1.00 , 0.90 , 0.80 , 1.00 , 1.00)	1.49	1.49	3.97	2.70	1.49
(1.00 , 0.75 , 0.80 , 1.50 , 1.50)	0.68	0.68	7.45	1.22	0.68	(1.00 , 0.90 , 0.80 , 1.20 , 1.20)	1.01	1.01	3.50	2.31	1.01
(1.00 , 0.75 , 0.80 , 2.00 , 2.00)	0.59	0.58	8.31	0.90	0.58	(1.00 , 0.90 , 0.80 , 1.50 , 1.50)	0.68	0.68	2.82	1.61	0.68
(0.90 , 0.90 , 0.20 , 0.80 , 0.80)	2.37	2.37	4.36	3.60	2.37	(1.00 , 0.90 , 0.80 , 2.00 , 2.00)	**-0.29**	**-0.29**	1.79	0.45	**-0.29**
(0.90 , 0.90 , 0.20 , 1.00 , 1.00)	2.46	2.46	4.55	3.69	2.46	(1.00 , 1.00 , 0.20 , 0.80 , 0.80)	4.22	4.22	4.22	5.82	4.22
(0.90 , 0.90 , 0.20 , 1.20 , 1.20)	2.16	2.16	4.25	3.31	2.16	(1.00 , 1.00 , 0.20 , 1.00 , 1.00)	3.32	3.32	3.32	5.01	3.32

Table 4.15: Relative Difference (%) of Bid-Price Policies (Continued)

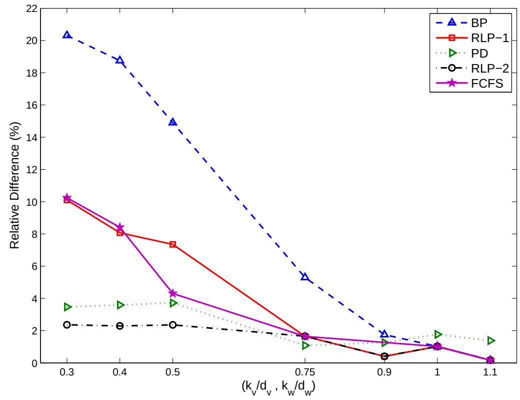
Instance	RLP-1	RLP-2	BP	PD	FCFS	Instance	RLP-1	RLP-2	BP	PD	FCFS
(1.00 , 1.00 , 0.20 , 1.20 , 1.20)	2.51	2.51	2.51	4.31	2.51	(1.10 , 1.00 , 0.20 , 1.00 , 1.00)	2.05	2.05	2.05	3.82	2.05
(1.00 , 1.00 , 0.20 , 1.50 , 1.50)	1.98	1.98	1.98	3.58	1.98	(1.10 , 1.00 , 0.20 , 1.20 , 1.20)	1.89	1.89	1.89	3.66	1.89
(1.00 , 1.00 , 0.20 , 2.00 , 2.00)	1.59	1.59	1.59	3.05	1.59	(1.10 , 1.00 , 0.20 , 1.50 , 1.50)	1.37	1.37	1.37	3.21	1.37
(1.00 , 1.00 , 0.80 , 0.80 , 0.80)	4.12	4.12	4.12	5.40	4.12	(1.10 , 1.00 , 0.20 , 2.00 , 2.00)	0.90	0.90	0.90	2.72	0.90
(1.00 , 1.00 , 0.80 , 1.00 , 1.00)	3.06	3.06	3.06	4.46	3.06	(1.10 , 1.00 , 0.80 , 0.80 , 0.80)	2.20	2.20	2.20	4.08	2.20
(1.00 , 1.00 , 0.80 , 1.20 , 1.20)	1.70	1.70	1.70	3.18	1.70	(1.10 , 1.00 , 0.80 , 1.00 , 1.00)	1.71	1.71	1.71	3.39	1.71
(1.00 , 1.00 , 0.80 , 1.50 , 1.50)	0.79	0.79	0.79	2.07	0.79	(1.10 , 1.00 , 0.80 , 1.20 , 1.20)	1.12	1.12	1.12	2.74	1.12
(1.00 , 1.00 , 0.80 , 2.00 , 2.00)	0.25	0.25	0.25	1.33	0.25	(1.10 , 1.00 , 0.80 , 1.50 , 1.50)	0.17	0.17	0.17	1.38	0.17
(1.10 , 1.10 , 0.20 , 0.80 , 0.80)	2.31	2.31	2.31	3.99	2.31	(1.10 , 1.00 , 0.80 , 2.00 , 2.00)	**-0.18**	**-0.18**	**-0.18**	0.91	**-0.18**
(1.10 , 1.10 , 0.20 , 1.00 , 1.00)	2.04	2.04	2.04	3.92	2.04	(1.00 , 1.10 , 0.20 , 0.80 , 0.80)	3.33	3.33	3.33	4.94	3.33
(1.10 , 1.10 , 0.20 , 1.20 , 1.20)	1.54	1.54	1.54	3.39	1.54	(1.00 , 1.10 , 0.20 , 1.00 , 1.00)	2.81	2.81	2.81	4.53	2.81
(1.10 , 1.10 , 0.20 , 1.50 , 1.50)	1.54	1.54	1.54	2.99	1.54	(1.00 , 1.10 , 0.20 , 1.20 , 1.20)	2.36	2.36	2.36	4.33	2.36
(1.10 , 1.10 , 0.20 , 2.00 , 2.00)	1.29	1.29	1.29	2.78	1.29	(1.00 , 1.10 , 0.20 , 1.50 , 1.50)	1.94	1.94	1.94	3.80	1.94
(1.10 , 1.10 , 0.80 , 0.80 , 0.80)	2.19	2.19	2.19	3.53	2.19	(1.00 , 1.10 , 0.20 , 2.00 , 2.00)	1.89	1.89	1.89	3.50	1.89
(1.10 , 1.10 , 0.80 , 1.00 , 1.00)	1.43	1.43	1.43	3.12	1.43	(1.00 , 1.10 , 0.80 , 0.80 , 0.80)	2.33	2.33	2.33	3.92	2.33
(1.10 , 1.10 , 0.80 , 1.20 , 1.20)	0.76	0.76	0.76	2.18	0.76	(1.00 , 1.10 , 0.80 , 1.00 , 1.00)	1.89	1.89	1.89	3.54	1.89
(1.10 , 1.10 , 0.80 , 1.50 , 1.50)	1.02	1.02	1.02	1.77	1.02	(1.00 , 1.10 , 0.80 , 1.20 , 1.20)	0.91	0.91	0.91	2.70	0.91
(1.10 , 1.10 , 0.80 , 2.00 , 2.00)	0.01	0.01	0.01	1.39	0.01	(1.00 , 1.10 , 0.80 , 1.50 , 1.50)	0.83	0.83	0.83	1.92	0.83
(1.10 , 1.00 , 0.20 , 0.80 , 0.80)	2.61	2.61	2.61	4.71	2.61	(1.00 , 1.10 , 0.80 , 2.00 , 2.00)	0.31	0.31	0.31	1.62	0.31

Table 4.16: Some Performance Measures of Bid-Price Policies

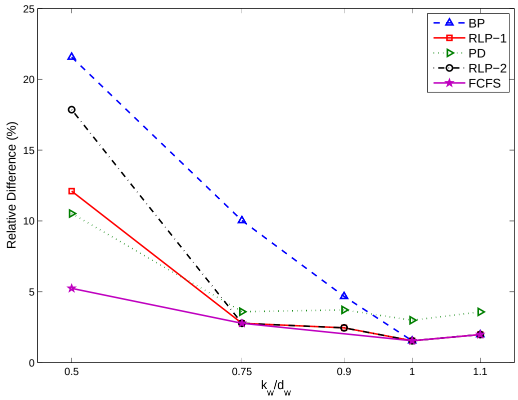
	RLP-2					PD				
Instance	Rel. Diff.	Utilization	Offloaded	Acc	OC	Rel. Diff.	Utilization	Offloaded	Acc	OC
(0.50 , 0.50 , 0.20 , 1.00 , 1.00)	2.87	(93.12 , 94.40)	(1.01 , 0.00)	71.49	1.02	7.43	(84.32 , 85.07)	(0.79 , 0.00)	65.57	0.76
(0.50 , 0.50 , 0.20 , 1.50 , 1.50)	3.10	(89.97 , 91.46)	(0.76 , 0.00)	69.92	1.14	6.78	(84.02 , 84.54)	(0.64 , 0.00)	65.65	0.93
(0.50 , 0.50 , 0.20 , 2.00 , 2.00)	8.62	(80.02 , 81.88)	(0.43 , 0.00)	60.39	0.81	6.36	(83.56 , 84.49)	(0.51 , 0.00)	65.39	0.97
(0.50 , 0.50 , 0.80 , 1.00 , 1.00)	2.64	(93.89 , 94.84)	(5.47 , 0.00)	71.36	6.04	5.33	(86.24 , 85.92)	(4.46 , 0.00)	63.40	4.72
(0.50 , 0.50 , 0.80 , 1.50 , 1.50)	2.36	(93.75 , 94.37)	(5.62 , 0.00)	71.60	9.27	3.72	(86.08 , 85.52)	(4.35 , 0.00)	63.57	6.86
(0.50 , 0.50 , 0.80 , 2.00 , 2.00)	2.82	(92.35 , 94.72)	(5.10 , 0.00)	71.08	11.28	3.22	(83.89 , 85.25)	(3.67 , 0.00)	63.4	7.65
(1.00 , 0.50 , 0.20 , 1.00 , 1.00)	9.04	(46.98 , 94.29)	(0.00 , 0.00)	67.95	0.00	14.02	(41.83 , 83.76)	(0.00 , 0.00)	59.15	0.00
(1.00 , 0.50 , 0.20 , 1.50 , 1.50)	17.85	(36.93 , 73.60)	(0.00 , 0.00)	50.82	0.00	10.52	(41.21 , 82.75)	(0.00 , 0.00)	60.03	0.00
(1.00 , 0.50 , 0.20 , 2.00 , 2.00)	18.89	(36.22 , 71.68)	(0.00 , 0.00)	49.43	0.00	10.13	(41.32 , 82.42)	(0.00 , 0.00)	59.84	0.00
(1.00 , 0.50 , 0.80 , 1.00 , 1.00)	11.02	(41.78 , 82.98)	(0.17 , 0.00)	56.72	0.20	9.82	(42.65 , 84.48)	(0.17 , 0.00)	60.11	0.20
(1.00 , 0.50 , 0.80 , 1.50 , 1.50)	9.20	(41.38 , 82.29)	(0.15 , 0.00)	58.73	0.26	8.24	(41.96 , 83.52)	(0.15 , 0.00)	61.00	0.26
(1.00 , 0.50 , 0.80 , 2.00 , 2.00)	5.96	(43.56 , 85.56)	(0.21 , 0.00)	62.12	0.49	7.86	(42.26 , 83.09)	(0.20 , 0.00)	61.10	0.47
(0.75 , 0.75 , 0.20 , 1.00 , 1.00)	2.73	(88.44 , 89.18)	(1.07 , 0.00)	86.34	1.15	3.25	(85.58 , 86.26)	(0.94 , 0.00)	82.38	0.98
(0.75 , 0.75 , 0.20 , 1.50 , 1.50)	1.41	(88.24 , 88.64)	(1.22 , 0.00)	86.45	1.96	2.22	(85.25 , 85.59)	(1.09 , 0.00)	82.57	1.70
(0.75 , 0.75 , 0.20 , 2.00 , 2.00)	1.78	(86.03 , 86.62)	(1.13 , 0.00)	85.09	2.35	2.27	(84.99 , 85.40)	(1.12 , 0.00)	82.61	2.33
(0.75 , 0.75 , 0.80 , 1.00 , 1.00)	2.25	(89.24 , 89.44)	(4.78 , 0.00)	85.88	5.56	2.22	(86.16 , 86.47)	(4.32 , 0.00)	82.09	4.86
(0.75 , 0.75 , 0.80 , 1.50 , 1.50)	1.66	(89.12 , 88.67)	(4.77 , 0.00)	86.30	8.50	1.09	(85.55 , 85.58)	(4.15 , 0.00)	82.56	7.12
(0.75 , 0.75 , 0.80 , 2.00 , 2.00)	2.10	(89.61 , 88.50)	(4.79 , 0.00)	86.33	11.33	1.09	(85.94 , 85.39)	(4.22 , 0.00)	82.6	9.62
(1.00 , 1.00 , 0.20 , 1.00 , 1.00)	2.04	(78.24 , 78.81)	(1.03 , 0.00)	91.75	1.13	3.92	(74.75 , 75.23)	(0.95 , 0.00)	87.05	1.02
(1.00 , 1.00 , 0.20 , 1.50 , 1.50)	1.54	(79.77 , 80.15)	(0.95 , 0.00)	91.68	1.54	2.99	(76.53 , 76.78)	(0.90 , 0.00)	86.45	1.42
(1.00 , 1.00 , 0.20 , 2.00 , 2.00)	1.29	(79.20 , 79.40)	(0.97 , 0.00)	92.09	2.06	2.78	(76.29 , 76.28)	(0.99 , 0.00)	86.83	2.04
(1.00 , 1.00 , 0.80 , 1.00 , 1.00)	1.43	(77.99 , 78.81)	(3.43 , 0.00)	91.75	4.16	3.12	(74.64 , 75.22)	(3.17 , 0.00)	87.04	3.78
(1.00 , 1.00 , 0.80 , 1.50 , 1.50)	1.02	(80.53 , 80.01)	(3.84 , 0.00)	91.63	6.89	1.77	(76.81 , 76.43)	(3.46 , 0.00)	86.61	5.98
(1.00 , 1.00 , 0.80 , 2.00 , 2.00)	0.01	(79.52 , 79.40)	(3.64 , 0.00)	92.09	8.53	1.39	(76.98 , 76.28)	(3.55 , 0.00)	86.83	8.17



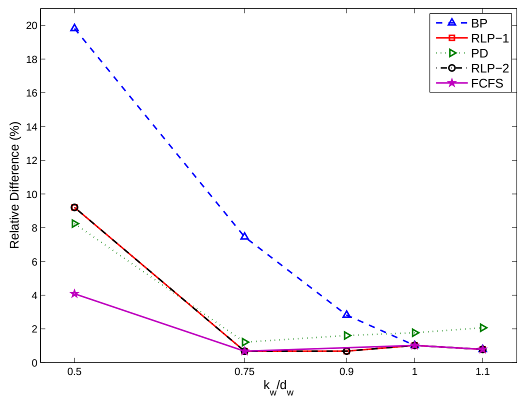
(a) $(*, *, 0.2, 1.5, 1.5)$



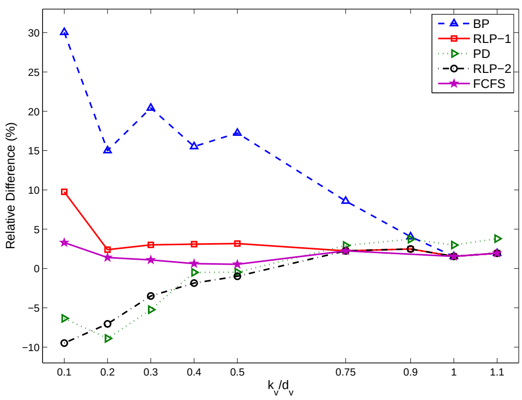
(b) $(*, *, 0.8, 1.5, 1.5)$



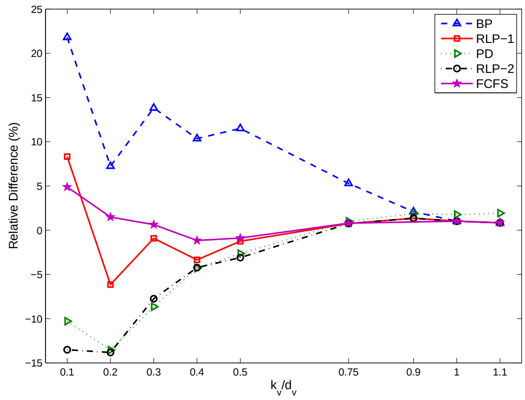
(c) $(1.0, *, 0.2, 1.5, 1.5)$



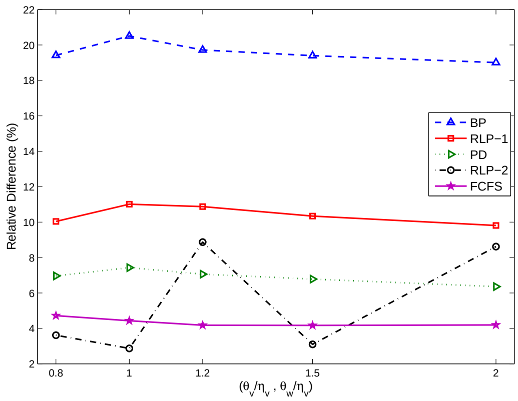
(d) $(1.0, *, 0.8, 1.5, 1.5)$



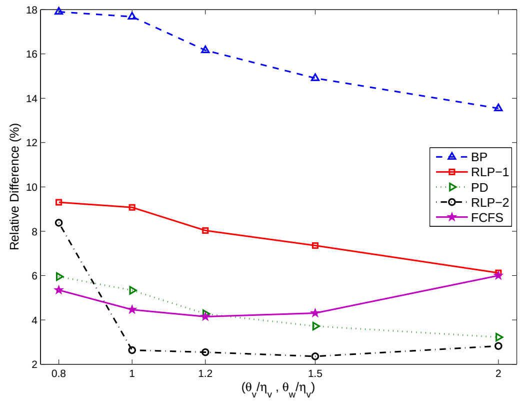
(e) $(*, 1.0, 0.2, 1.5, 1.5)$



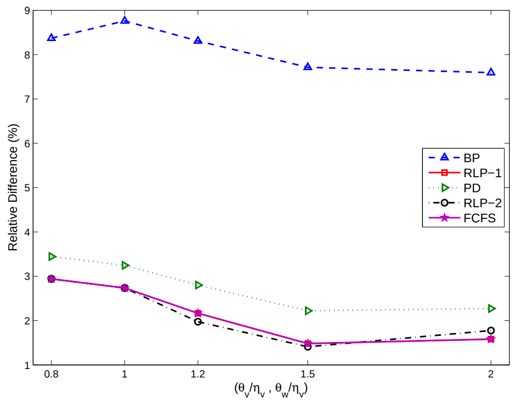
(f) $(*, 1.0, 0.8, 1.5, 1.5)$



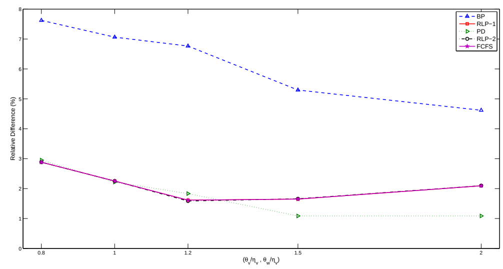
(g) $(0.5, 0.5, 0.2, *, *)$



(h) $(0.5, 0.5, 0.8, *, *)$

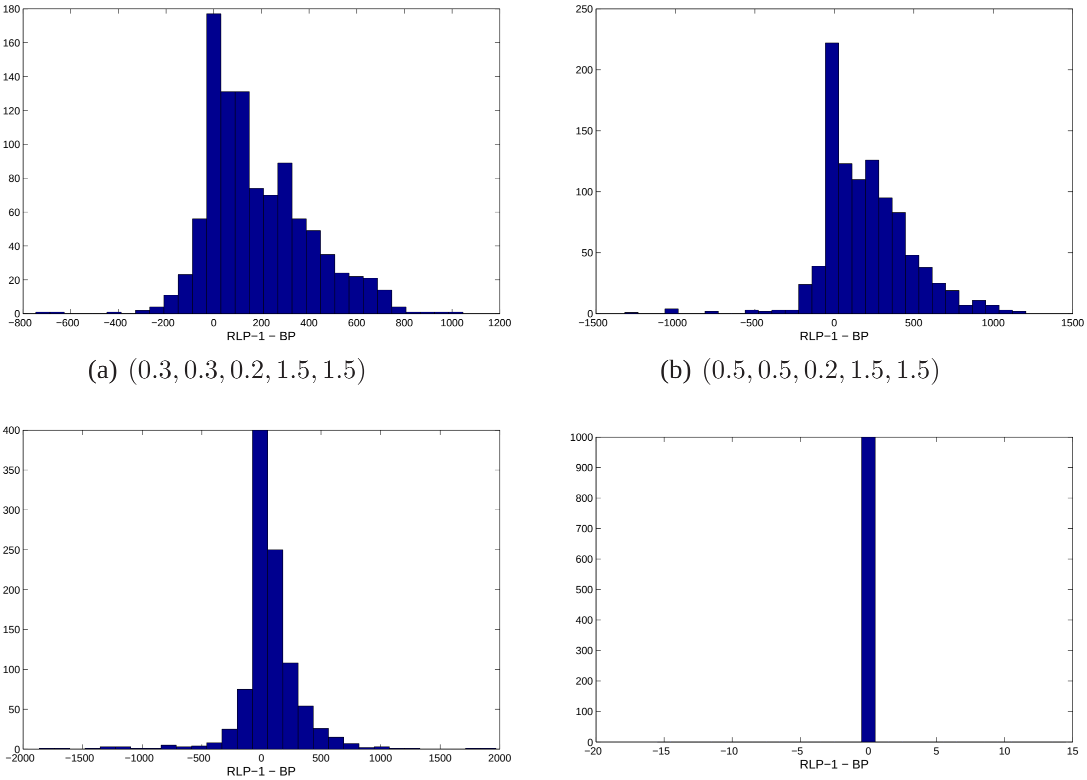


(i) $(0.75, 0.75, 0.2, *, *)$



(j) $(0.75, 0.75, 0.8, *, *)$

Figure 4.4: Relative Difference of Bid-Price Policies



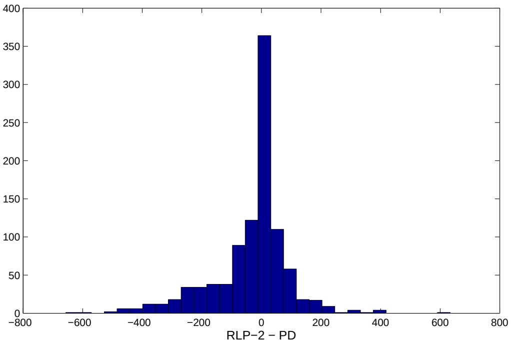
(a) $(0.3, 0.3, 0.2, 1.5, 1.5)$

(b) $(0.5, 0.5, 0.2, 1.5, 1.5)$

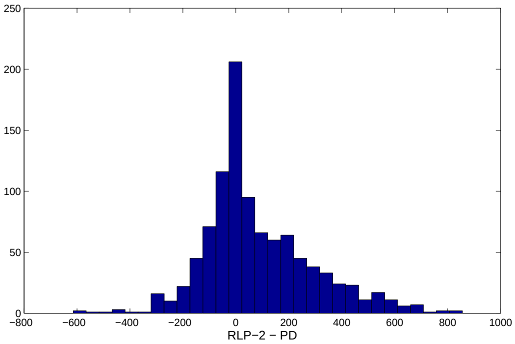
(c) $(0.9, 0.9, 0.2, 1.5, 1.5)$

(d) $(1.0, 1.0, 0.2, 1.5, 1.5)$

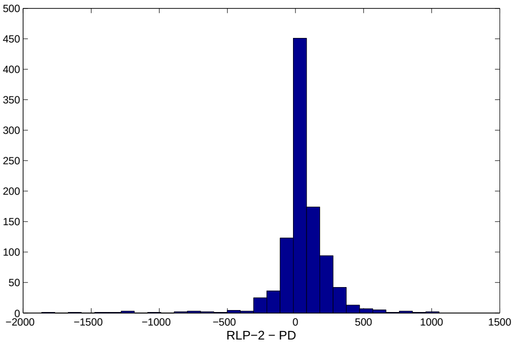
Figure 4.5: Difference between RLP-1 and BP



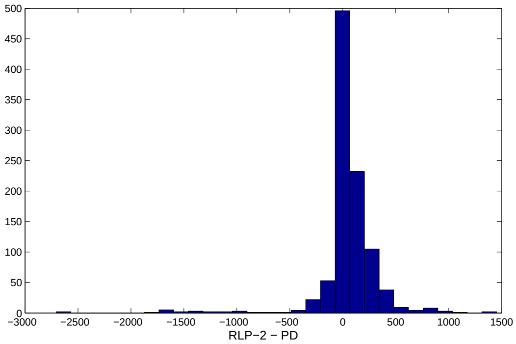
(a) $(0.3, 0.3, 0.2, 1.5, 1.5)$



(b) $(0.5, 0.5, 0.2, 1.5, 1.5)$



(c) $(0.9, 0.9, 0.2, 1.5, 1.5)$



(d) $(1.0, 1.0, 0.2, 1.5, 1.5)$

Figure 4.6: Difference between RLP-2 and PD

Chapter 5

Conclusions and Future Research

We have introduced new optimization models to develop open-loop booking limit and bid-price policies for air-cargo capacity control on a single-leg flight. While our exposition is presented in a setting which does not explicitly consider no-shows, they can be naturally incorporated into our models by allowing shipments to have zero capacity requirements. Our methods can therefore be useful in developing overbooking policies. We have conducted a comprehensive computational study to evaluate the effectiveness of our proposed models, and have illustrated that they are computationally tractable, and yield policies that perform well compared to the benchmarks established by various methods in the literature.

One of our main aims was to adapt existing methods from the extensive passenger literature to the relatively little-studied cargo case. Passenger booking methods often rely on a complete ranking of fare classes, which can be used to establish a nested structure. We therefore developed various novel methods to rank different types of cargo. To the best of our knowledge, these are the first rankings of this type in the cargo revenue management literature. However, in certain cases (in particular when volume and weight play a symmetrical role), any complete ranking of cargo types is necessarily arbitrary, and can therefore lead to suboptimal decisions. Consequently, in our future research we aim to develop booking policies with nested structures based on partial orderings of cargo types.

We note that our two-stage RLP model can accommodate randomness in the available volume and weight capacities. Since there is often significant uncertainty in the capacity utilized by allotment contracts, as well as in the capacity requirements of passenger bags, extending our other methods to similarly allow random capacities is also an important research goal. Finally, we mention that, as discussed in Section 4.1.3, our booking limits can be converted to limits on the expected volume and weight requirements of shipments. Since such capacity limits appear to be more natural in a cargo context than limits on the number of accepted requests, we plan to evaluate implementations of our booking limits based on this approach. Furthermore, if the results from the evaluation justify this capacity-based interpretation, we propose to directly develop separate booking limits in terms of volume and weight.

Bibliography

Akçay, Y., Li, H., and Xu, S. (2007). Greedy algorithm for the general multidimensional knapsack problem. *Annals of Operations Research*, 150:17–29.

Amaruchkul, K., Cooper, W. L., and Gupta, D. (2007). Single-leg air-cargo revenue management. *Transportation Science*, 41(4):457–469.

Aydin, N., Birbil, S. I., Frenk, J. B. G., and Noyan, N. (2010). Single-leg airline revenue management with overbooking. *Transportation Science, submitted*.

Becker, B. and Dill, N. (2007). Managing the of air cargo revenue management. *Journal of Revenue and Pricing Management*, 6(3):175–187.

Belobaba, P. P. (1987). Air travel demand and airline seat inventory management. Ph.D. thesis, Flight Transportation Laboratory, Massachusetts Institute of Technology, Cambridge, MA.

Belobaba, P. P. (1989). Application of a probabilistic decision model to airline seat inventory control. *Operations Research*, 37:183–197.

Benders, J. F. (1962). Partitioning procedures for solving mixed-variable programming problems. *Numerische Mathematik*, 54:238–252.

Bertsimas, D. and Popescu, I. (2003). Revenue management in a dynamic network environment. *Transportation Science*, 37(3):257–277.

Billings, J. S., Diener, A. G., and Yuen, B. B. (2003). Cargo revenue optimisation. *Journal of Revenue and Pricing Management*, 2(1):69–79.

Birge, J. and Louveaux, F. (1997). *Introduction to stochastic programming*. Springer, New York.

Boeing Company (2012). Long-term market – current market outlook 2012-2031. Technical report. http://www.boeing.com/commercial/cmo/index.html (Last accessed on July 12, 2012).

Chatwin, R. E. (1999). Continuous-time airline overbooking with time-dependent fares and refunds. *Transportation Science*, 33(2):182–191.

Haerian, L., de Mello, T. H., and Mount-Campbell, C. A. (2006). Modeling revenue yield of reservation systems that use nested capacity protection strategies. *International Journal of Production Economics*, 104(2):340 – 353.

Han, D. L., Tang, L. C., and C., H. H. (2010). A markov model for single-leg air cargo revenue management under a bid-price policy. *European Journal of Operational Research*, 200:800–811.

Huang, K. and Chang, K. (2010). An approximate algorithm for the two-dimensional air cargo revenue management problem. *Transportation Research Part E*, 46:426–435.

Huang, K. and Hsu, W. (2005). Revenue management for air cargo space with supply uncertainty. In *Proceedings of the Eastern Asia Society for Transportation Studies*, volume 5, pages 570–580.

IATA (2009). International air transport association, fact sheet: Industry statistics, december 2009. Technical report. http://www.iata.org/pressroom/facts_figures/fact_sheets/ (Last accessed on July 12, 2012).

Karaesmen, I. Z. (2001). Three essays on revenue management. Ph.D. thesis, Columbia University.

Kasilingam, R. G. (1996). Air cargo revenue management: Characteristics and complexities. *European Journal of Operational Research*, 96:36–44.

Kasilingam, R. G. (1997). An economic model for air cargo overbooking under stochastic capacity. *Computers and Industrial Engineering*, 32(1):221–226.

Kasilingam, R. G. (1998). *Logistics and Transportation: Design and Planning*. Springer.

Kleywegt, A. J. and Papastavrou, J. D. (1998). The dynamic and stochastic knapsack problem. *Operations Research*, 46:17–35.

Kleywegt, A. J. and Papastavrou, J. D. (2001). The dynamic and stochastic knapsack problem with random sized items. *Operations Research*, 49:26–41.

Kunnumkal, S., Talluri, K., and Topaloglu, H. (2012). A randomized linear programming method for network revenue management with product-specific no-shows. *Transportation Science*, 46(1):90–108.

Levin, Y., Nediak, M., and Topaloglu, H. (2011). Cargo capacity management with allotments and spot market demand. *Operations research*.

Levina, T., Levin, Y., McGill, J., and Nediak, M. (2011). Network cargo capacity management. *Operations Research*, 59(4):1008–1023.

Littlewood, K. (1972). Forecasting and control of passenger bookings. In *Proceedings of the Twelfth Annual AGIFORS Symposium*, Nathanya, Israel.

Luo, S., Cakanyildirim, M., and Kasilingam, R. G. (2009). Two-dimensional cargo overbooking models. *European Journal of Operational Reseach*, 197:862–883.

Martello, S. and Toth, P. (2003). An exact algorithm for the two-constraint 0-1 knapsack problem. *Operations Research*, 51(5):826–835.

Moussawi, L. and Cakanyildirim, M. (2005). Profit maximization in air cargo overbooking. Techinal Report, University of Texas at Dallas, TX.

Pak, K. and Dekker, R. (2004). Cargo revenue management: bid-prices for 0-1 multi knapsack problem. In *ERIM Report Series*. ERS-2004-055-LIS.

Phillips, R. L. (2005). *Pricing and Revenue Optimization*. Stanford University Press, Stanford, CA.

Popescu, A. (2006). Air cargo revenue and capacity management. Ph.D. thesis, Georgia Institute of Technology, Atlanta.

Popescu, A., Barnes, E., Johnson, E., and Keskinocak, P. (2012). Bid prices when demand is a mix of individual and batch bookings. *Transportation Science, Online First (June 22)*. doi:10.1287/trsc.1120.0420.

Prékopa, A. (1995). *Stochastic Programming*. Kluwer Academic, Dordrecht, Boston.

Rinnooy Kan, A., Stougie, L., and Vercellis, C. (1993). A class of generalized greedy algorithms for the multi-knapsack problem. *Discrete Applied Mathematics*, 42:279–290.

Sandhu, R. and Klabjan, D. (2006). Fleeting with passenger and cargo origin-destination booking control. *Transportation Science*, 40(4):517–528.

Simpson, R. W. (1992). Using network flow techniques to find shadow prices for market and seat inventory control. MIT flight transportation laboratory memorandum M89-1, MIT, Cambridge, MA.

Slager, B. and Kapteijns, L. (2004). Implementation of cargo revenue management at klm. *Journal of Revenue and Pricing Management*, 3(1):80–90.

Talluri, K. T. and van Ryzin, G. J. (1999). A randomized linear programming method for computing network bid prices. *Transportation Science*, 33(2):207–216.

Talluri, K. T. and van Ryzin, G. J. (2005). *The Theory and Practice of Revenue Management*. Springer, New York, NY.

Tijms, H. C. (2003). *A first course in stochastic models*. Wiley.

Topaloglu, H. (2009). Using lagrangian relaxation to compute capacity-dependent bid prices in network revenue management. *Operations Research*, 57(3):637–649.

Van Slyke, R. and Young, Y. (2000). Finite horizon stochastic knapsacks with applications to yield management. *Transportation Science*, 48(1):155–172.

Van Slyke, R. M. and Wets, R. (1969). L-shaped linear programs with applications to optimal control and stochastic programming. *SIAM Journal on Applied Mathematics*, 17(4):638–663.

Williamson, E. L. (1992). Airline network seat control. PhD thesis, MIT, Cambridge, MA.

Winkler, R. L., Roodman, G. M., and Britney, R. R. (1972). The determination of partial moments. *Management Science*, 19(3):290–296.

Xiao, B. and Yang, W. (2010). A revenue management model for products with two capacity dimensions. *European Journal of Operational Research*, 205:412–421.

Zhuang, W., Gumus, M., and Zhang, D. (2011). A single resource revenue management problem with random resource consumption. *Journal of the Operational Research Society*, pages 1–15.

Appendices

Appendix A

Mixture distributions

Here we provide an alternative analytical proof for Formula (3.3). A corresponding proof for Formula (3.4) can be obtained analogously.

Lemma 6 *Suppose that the random variable $\overline{V}$ has the following mixture cumulative distribution function (CDF)*

$$\mathbb{P}(\overline{V} \leq v) = \sum_{i=1}^{m} p_i \mathbb{P}(V_i \leq v).$$

Then it follows for every $n \in \mathbb{N}$ that

$$\sum_{i=1}^{m} \sum_{j=1}^{B(p_i,n)} V_{ij} \stackrel{d}{=} \sum_{j=1}^{n} \overline{V}_j, \tag{A.1}$$

where the random variables $\overline{V}_j$ are independent copies of the random variable $\overline{V}$.

Proof. We prove the assertion by showing that the Laplace-Stieltjes transform of both sides of the equation (A.1) are equal to each other. Let $\mathbf{B} = (B(p_1, n), B(p_2, n), ..., B(p_m, n))$ be a multinomially distributed random vector independent of the random variables $V_{ij},\ i \in [m],\ j \in \mathbb{N}$. By the total law of expectation, the Laplace-Stieltjes transform of $\sum_{i=1}^{m} \sum_{j=1}^{B(p_i,n)} V_{ij}$

is obtained as follows:

$$\begin{aligned}
&\mathbb{E}\left[\exp(-s\sum_{i=1}^{m}\sum_{j=1}^{B(p_i,n)} V_{ij})\right] \\
&= \sum_{\mathbf{k}\in K}\mathbb{E}\left[\exp(-s\sum_{i=1}^{m}\sum_{j=1}^{k_i} V_{ij})|\mathbf{B}=(k_1,k_2,...,k_m)\right]P(\mathbf{B}=(k_1,k_2,...,k_m)) \\
&= \sum_{\mathbf{k}\in K}\frac{n!}{\prod_{i=1}^{m}k_i!}\prod_{i=1}^{m}p_i^{k_i}\mathbb{E}\left[\exp(-s\sum_{i=1}^{m}\sum_{j=1}^{k_i} V_{ij})\right] \\
&= \sum_{\mathbf{k}\in K}\frac{n!}{\prod_{i=1}^{m}k_i!}\prod_{i=1}^{m}\left(p_i\mathbb{E}[\exp(-sV_i)]\right)^{k_i} \\
&= \left(\sum_{i=1}^{m}p_i\mathbb{E}[\exp(-s\mathbf{V}_i)]\right)^{n}, \qquad \text{(A.2)}
\end{aligned}$$

where $K := \{\mathbf{k} \in \mathbb{N}^m \ : \ k_1 + \cdots + k_m = n\}$.

Similarly, we also derive the Laplace-Stieltjes transform of $\sum_{j=1}^{n}\overline{V}_j$:

$$\begin{aligned}
\mathbb{E}\left[\exp(-s\sum_{j=1}^{n}\overline{V}_j)\right] &= \left(\mathbb{E}[\exp(-s\overline{V})]\right)^n \\
&= \left(\int_0^\infty \exp(-sv)f_{\overline{V}}(v)dv\right)^n \\
&= \left(\int_0^\infty \exp(-sv)\sum_{i=0}^{m}p_i f_{V_i}(v)dv\right)^n \\
&= \left(\sum_{i=0}^{m}p_i\int_0^\infty \exp(-sv)f_{V_i}(v)dv\right)^n \\
&= \left(\sum_{i=1}^{m}p_i\mathbb{E}[\exp(-s\mathbf{V}_i)]\right)^n. \qquad \text{(A.3)}
\end{aligned}$$

The assertion immediately follows from (A.2) and (A.3). ■

Appendix B

Partial expectations

Partial expectation of the random variable X having probability density function $f(.)$ is defined as

$$\mathbb{E}[X]_a^b := \int_a^b x f(x) dx.$$

Closed form of partial expectations is needed while calculating terms $\mathbb{E}[(X - y)_+]$ or similarly $\mathbb{E}[\max(X, y)]$. Therefore we will be using the equations below very frequently. For some of the most popular choices for the volume distribution, we will illustrate these calculations. (For more detailed study see Winkler et al. (1972))

<u>Normal Distribution</u>

Winkler et al. (1972) shows that, for X normally distributed with mean μ and variance σ^2, we have

$$\mathbb{E}[X]_y^\infty = \mu - \left[-\sigma\phi(\frac{y-\mu}{\sigma}) + \mu\Phi(\frac{y-\mu}{\sigma}) \right] \tag{B.1}$$

<u>Log-normal Distribution</u>

For X Log-normally distributed with parameters μ and σ^2,

$$\mathbb{E}[X]_y^\infty = \exp\left(\mu + \frac{1}{2}\sigma^2\right) \Phi\left(\frac{\mu + \sigma^2 - \ln y}{\sigma}\right) \tag{B.2}$$

<u>Mixtures</u>

For X having a mixture distribution, i.e. $\mathbb{P}(X \leq x) = \sum_{i=1}^{m} p_i \mathbb{P}(X_i \leq x)$ where X_i follows any of the distributions above, we have:

$$\mathbb{E}[X]_y^\infty = \int_y^\infty x f_X(x) dx = \sum_{i=1}^{m} p_i \int_y^\infty x f_{X_i}(x) dx = \sum_{i=1}^{m} p_i \mathbb{E}[X_i]_y^\infty \tag{B.3}$$

Appendix C

Calculations required for the risk based model

As in most of the existing studies, we assume that off-loading cost functions h_v and h_w are defined as in (3.1). In order to investigate whether it is possible to obtain a critical ratio rule as in newsvendor models, we derive the expression for $f(b+1) - f(b)$. Such a rule has been developed for the passenger case in Aydin et al. (2010). For ease of exposition let us introduce $S_b^v := \sum_{j=1}^{b} \overline{V}_j$ and $S_b^w := \sum_{j=1}^{b} \overline{W}_j$. Then, the following chain of equalities holds:

$$\begin{aligned} f(b+1) - f(b) &= \sum_{i=1}^{m} \rho_i p_i - \mathbb{E}\left[\theta_v \overline{V}_{b+1} \mathbf{1}_{\{S_b^v \geq C_v\}} + \theta_v (S_{b+1}^v - C_v) \mathbf{1}_{\{S_{b+1}^v \geq C_v \text{ and } S_b^v \leq C_v\}}\right] \\ &\quad - \mathbb{E}\left[\theta_w \overline{W}_{b+1} \mathbf{1}_{\{S_b^w \geq C_w\}} + \theta_w (S_{b+1}^w - C_w) \mathbf{1}_{\{S_{b+1}^w \geq C_w \text{ and } S_b^w \leq C_w\}}\right] \\ &= \sum_{i=1}^{m} \rho_i p_i - \theta_v \mathbb{E}\left[\overline{V}_{b+1}\right] \mathbb{P}(S_b^v \geq C_v) - \theta_v \mathbb{E}\left[(S_{b+1}^v - C_v) \mathbf{1}_{\{S_{b+1}^v \geq C_v \text{ and } S_b^v \leq C_v\}}\right] \\ &\quad - \theta_w \mathbb{E}\left[\overline{W}_{b+1}\right] \mathbb{P}(S_b^w \geq C_w) - \theta_w \mathbb{E}\left[(S_{b+1}^w - C_w) \mathbf{1}_{\{S_{b+1}^w \geq C_w \text{ and } S_b^w \leq C_w\}}\right]. \end{aligned} \tag{C.1}$$

The above difference function involves complicated expectations and convolution distributions, it is really hard to obtain an analytical form for it. Thus, unlike the passenger case this analysis does not lead to a critical ratio rule. Instead of calculating this difference, we can calculate the function $f(b)$ and search for the optimal total booking limit. However, it is still computationally challenging to calculate the expected off-loading costs. One can estimate these costs using approximation methods. For example, under the condition that b would be large enough, S_b^v and S_b^w may be assumed to be normally distributed by the Central Limit Theorem.

For $S_b^v := \sum_{j=1}^{b} \overline{V}_j$ we have

$$\begin{aligned}
\mathbb{E}[S_b^v] =& \mathbb{E}[\sum_{j=1}^{b} \overline{V}_j] = b\mathbb{E}[\overline{V}] \\
\mathbb{E}[\overline{V}] =& \int_0^\infty v f_{\overline{V}}(v) dv = \int_0^\infty v \sum_{i=1}^{m} p_i f_{V_i}(v) dv \\
=& \sum_{i=1}^{m} p_i \int_0^\infty v f_{V_i}(v) dv \\
=& \sum_{i=1}^{m} p_i \mu_i^v
\end{aligned}$$

$\therefore \mathbb{E}[S_b^v] = b \sum_{i=1}^{m} p_i \mu_i^v$

$$\begin{aligned}
\sigma^2(S_b^v) =& \sigma^2(\sum_{j=1}^{b} \overline{V}_j) = b\sigma^2(\overline{V}) \\
\sigma^2(\overline{V}) =& \int_0^\infty (v - \mathbb{E}[\overline{V}])^2 f_{\overline{V}}(v) dv \\
=& \int_0^\infty v^2 f_{\overline{V}}(v) dv - 2\mathbb{E}[\overline{V}] \int_0^\infty v f_{\overline{V}}(v) dv + \mathbb{E}[\overline{V}]^2 \\
=& \sum_{i=1}^{m} p_i \mathbb{E}[(V_i)^2] - \mathbb{E}[\overline{V}]^2
\end{aligned}$$

$\therefore \sigma^2(S_b^v) = b(\sum_{i=1}^{m} p_i \mathbb{E}[(V_i)^2] - (\sum_{i=1}^{m} p_i \mu_i^v)^2)$ Thus, under the normality assumption, we have $S_b^v \sim \text{Norm}\,(\mathbb{E}[S_b^v], \sigma^2(S_b^v))$. This implies $\frac{S_b^v - \mathbb{E}[S_b^v]}{\sigma(S_b^v)} = Y \sim \text{N}(0,1)$. Next, by using (B.1) we calculate $\mathbb{E}[\max\{S_b^v - C_v, 0\}]$.

$$\begin{aligned}
\mathbb{E}[\max\{S_b^v - C_v, 0\}] =& \mathbb{E}\left[\max\left(Y\sigma(S_b^v) + \mathbb{E}[S_b^v] - C_v, 0\right)\right] \\
=& \sigma(S_b^v)\mathbb{E}\left[\max\left(Y, \frac{C_v - \mathbb{E}[S_b^v]}{\sigma(S_b^v)}\right)\right] + \mathbb{E}[S_b^v] - C_v \\
=& \sigma(S_b^v)\left[\alpha(b)P(Y < \alpha(b)) + \mathbb{E}[Y]_{\alpha(b)}^{\infty}\right] + \mathbb{E}[S_b^v] - C_v \\
=& \sigma(S_b^v)\left[\alpha(b)\Phi\left(\alpha(b)\right) + \phi\left(\alpha(b)\right)\right] + \mathbb{E}[S_b^v] - C_v,
\end{aligned}$$

where $\alpha(b) = \frac{C_v - \mathbb{E}[S_b^v]}{\sigma(S_b^v)}$, $\Phi(.)$ and $\phi(.)$ are the cumulative and probability density functions of the standard normal distribution. Then we have:

$$f(b) = \sum_{i=1}^{m} \rho_i p_i b - \theta_v \left(\sigma(S_b^v)\left[\alpha(b)\Phi\left(\alpha(b)\right) + \phi\left(\alpha(b)\right)\right] + \mathbb{E}[S_b^v] - C_v\right)$$
$$-\theta_w \left(\sigma(S_b^w)\left[\beta(b)\Phi\left(\beta(b)\right) + \phi\left(\beta(b)\right)\right] + \mathbb{E}[S_b^w] - C_w\right),$$

where $\beta(b) = \frac{C_w - \mathbb{E}[S_b^w]}{\sigma(S_b^w)}$.

Appendix D

Expected revenue calculations

In this section, we show how to calculate $\mathbb{E}[r_i(\max\{w_i, V_i/\gamma\})]$ when $r_i(.)$ is a piece-wise linear function in chargeable weight ($\hat{W}_i$) with three kinks. Let I_n be the range of changeable weight where the slope of revenue function (α_n) is equal to c_n. Lower and upper limits of range I_n are equal to b_{n-1} and b_n respectively. (See Figure (D.1) for an illustration.) We calculate the expected revenue in the following way:

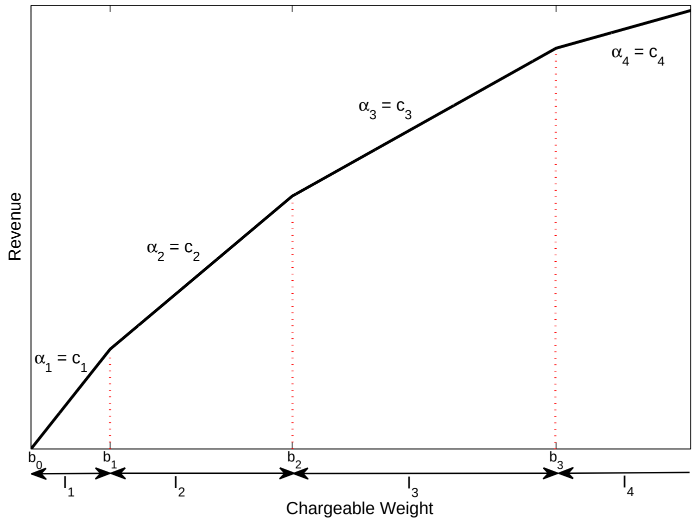


Figure D.1: An Illustrative Figure of Revenue Function

$$
\begin{aligned}
\rho_i &= \mathbb{E}[r_i(\hat{W}_i)] = \mathbb{E}[\mathbb{E}[r_i(\hat{W}_i)|\hat{W}_i \in I_n]] = \sum_{n=1}^{4} \mathbb{P}(\hat{W}_i \in I_n)\mathbb{E}[r_i(\hat{W}_i)|\hat{W}_i \in I_n] \\
&= \sum_{n=1}^{4} \mathbb{P}(\hat{W}_i \in I_n)\mathbb{E}[(\hat{W}_i - b_{n-1})c_n + \sum_{k=1}^{n-1}(b_k - b_{k-1})c_k] \\
&= \sum_{n=1}^{4} \mathbb{P}(\hat{W}_i \in I_n)\left[c_n\left(\mathbb{E}[\hat{W}_i] - b_{n-1}\right) + \sum_{k=1}^{n-1} c_k(b_k - b_{k-1})\right]
\end{aligned}
$$

where

$$
\begin{aligned}
\mathbb{E}[\hat{W}_i] &= \mathbb{E}[\max\{V_i/\gamma, w_i\}] = \frac{1}{\gamma}\mathbb{E}[\max\{V_i, w_i\gamma\}] \\
&= \frac{1}{\gamma}\left(\int_0^{w_i\gamma} w_i\gamma f(x)dx + \int_{w_i\gamma}^{\infty} xf(x)dx\right) = \frac{1}{\gamma}\left(w_i\gamma F_{V_i}(w_i\gamma) + \mathbb{E}[V_i]_{w_i\gamma}^{\infty}\right)
\end{aligned}
$$

and

$$
\begin{aligned}
\mathbb{P}(\hat{W}_i \leq x) &= \mathbb{P}(\max(\gamma w_i, V_i) \leq \gamma x) = 1_{\{x \geq w_i\}}\mathbb{P}(\max(\gamma w_i, V_i) \leq \gamma x) \\
&= 1_{\{x \geq w_i\}}[\mathbb{P}(\max(\gamma w_i, V_i) \leq \gamma x | V_i \leq \gamma w_i \leq \gamma x)\mathbb{P}(V_i \leq \gamma w_i \leq \gamma x) \\
&\quad + \mathbb{P}(\max(\gamma w_i, V_i) \leq \gamma x | \gamma w_i \leq V_i \leq \gamma x)\mathbb{P}(\gamma w_i \leq V_i \leq \gamma x) \\
&\quad + \mathbb{P}(\max(\gamma w_i, V_i) \leq \gamma x | \gamma w_i \leq \gamma x \leq V_i)\mathbb{P}(\gamma w_i \leq \gamma x \leq V_i)] \\
&= 1_{\{x \geq w_i\}}[\mathbb{P}(V_i \leq \gamma w_i) + \mathbb{P}(\gamma w_i \leq V_i \leq \gamma x)] = 1_{\{x \geq w_i\}}\mathbb{P}(V_i \leq \gamma x).
\end{aligned}
$$

Appendix E

Fast Fourier Transform

The discrete Fourier transform (DFT) is defined by:

$$X_k = \sum_{n=0}^{N-1} x_n e^{-i2\pi k \frac{n}{N}}$$

Fast Fourier Transform is an algorithm to compute DFT or its inverse. In our case, we will use it to obtain the inverse of DFT. In other words, we will compute the value of X_k and obtain x_n, $n = 0, \ldots, N-1$. We utilized FFT for calculating the probability distribution of random variables D_j and $\sum_{j=1}^{n} D_k$.

Recall that we consider a discrete-time framework, where the booking horizon is divided in T time periods and T is sufficiently large so that there is at most one booking request in each time period. The random demand for type-j cargo at time period $t \in T$, denoted by D_{jt}, is a Bernoulli random variable with success probability of p_{jt}. Then, the total demand for type-j cargo is the sum of T independent Bernoulli random variables with different success probabilities and it can take values of $0, 1, \ldots, T$. The characteristic function of D_j is given by

$$\varphi_{D_j}(z) = \mathbb{E}[e^{izD_j}] = \sum_{n=0}^{T} e^{izn} P(D_j = n). \tag{E.1}$$

We can easily calculate this function using the characteristic functions of independent

Bernoulli random variables:

$$\varphi_{D_j}(z) = \mathbb{E}[e^{izD_j}] = \prod_{t=1}^{T} \mathbb{E}[e^{izD_{jt}}] = \prod_{t=1}^{T} (e^{iz} p_{jt} + 1 - p_{jt}). \quad \text{(E.2)}$$

Basically, FFT method evaluates the characteristic function (E.2) at $z = \frac{-2\pi k}{T+1}$ for all $k = 0, 1, \dots, T$ and retrieves the probabilities $P(D_j = n)$ using (E.1) and (E.2). In other words, it solves the following set of equations to provide the probabilities $P(D_j = n)$ as output:

$$\varphi_{D_j}(\frac{-2\pi k}{T+1}) = \prod_{t=1}^{T} \mathbb{E}[e^{i\frac{-2\pi k}{T+1} D_{jt}}] = \sum_{n=0}^{T} P(D_j = n) \exp(\frac{-i2\pi kn}{T+1}), \quad k = 0, \dots, T. \quad \text{(E.3)}$$

Appendix F

Greedy Algorithm of Rinnooy Kan et al. (1993)

Let n denote the number of booking request arrivals, w_j and v_j be the observed volume and weight of jth booking request arrival. Then x_j and y_j are defined as w_j/C_w and v_j/C_v respectively, for all $j \in [n]$.

Algorithm 2 Algorithm for Obtaining Bid-Prices

Order the requests by increasing value of x_j.
for $j = 1$ to $n - 1$ **do**
    for $l = j + 1$ to n and $y_j \geq y_l$ **do**
        Let $\gamma := \frac{y_l - y_j}{x_l - x_j}$.
        for $h = 1$ to n **do**
            Let $\eta_h := y_h - \gamma x_h$.
            Order the requests by increasing value of η_h.
            Start accepting requests in this order until no more requests can be accepted.
            Let π be the profit obtained and η be the order value for the last request that is accepted.
        end for
    end for
end for
Find the maximum profit π^* over all orderings.
Let γ^* and η^* be the corresponding slope and order value.
return $\lambda_w = -\gamma^*/\eta^*$ and $\lambda_v = 1/\eta^*$.