A SINGLE PERIOD INVENTORY MODEL WITH MULTIPLE VENDORS AND RANDOM YIELD

by

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This is to certify that I have examined this copy of a master's thesis by

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To my parents

ABSTRACT

Inventory management requires handling uncertainties arising at procurement and production stages, transportation channels and demand attributes effectively and efficiently in order to have a robust operational control. In this thesis, a single period inventory model with unreliable suppliers and uncertain demand is analyzed. Supplier unreliability is modeled by stochastic proportions and/or random supplier capacity in related literature. We combine these two approaches where a stochastic proportion is applied to the minimum of the order amount and random supplier capacity. There are studies in the literature on random yield that primarily consider models with only a single vendor. The main contribution of this work is the extension to the case where there are multiple vendors. We consider cases involving distinct and identical servers to discuss a number of issues including order diversification. We show that the optimal ordering policy does not have a simple order-up-to or base stock structure. A number of numerical illustrations are given to discuss the structure of optimal policies.

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NOMENCLATURE

G	Cumulative distribution function for the periodic demand
F	Cumulative distribution function for vendor capacity
T	Cumulative distribution function for the stochastic proportion
D	Periodic demand
A	Vendor capacity
U	Stochastic proportion
q	Order quantity
P_q	Amount processed at the vendor's plant
Y_q	Actual (random) yield
L	Expected inventory cost
h	Unit holding cost
p	Unit shortage cost
c	Unit purchase cost
TC	Total expected cost
$\overline{q}(x)$	Solution of the unconstrained problem
S	Critical order level
μ	Rate for vendor capacity
λ	Rate for periodic demand
a	Lower bound for the density function of U
b	Upper bound for the density function of U
M	Minor of the Hessian Matrix

Chapter 1

INTRODUCTION

Optimizing the decisions on production and order quantities in inventory management is generally not a straightforward task. This may result from both the complex structure of the supply chain and some individual features of the components of this chain. In this study, we focused on a specific portion of a supply chain network instead of examining the entire chain. In fact, this thesis addresses the optimization of the ordering policy for a production plant or a retail store making decisions on a single item and working with several vendors.

In a fully-deterministic world, one would be sure about receiving or obtaining the exact amount ordered to the vendor or initiated in the production plant. However, obviously fully-deterministic modeling is not always applicable to most real-life circumstances since uncertainty is embedded in many components of the system and the level of this uncertainty increases as the scale of the system gets larger. Consequently, determination of optimum quantities becomes complicated by the inclusion of stochastic attributes in the modeling process.

The main source of uncertainty in such systems is on the vendor side since it is regarded as somewhat uncontrollable by the decision maker; here, the production plant or the retail store. A vendor has its own plant to produce semi-finished products within its capacity. The first source of uncertainty is this capacity limit, which may have a dynamic and stochastic structure changing each period. Different capacity realizations at successive periods may be caused by the following factors:

- *Machine breakdowns*: If the production at the vendor's side is dominated by machinery and there is a limited number of machines, then machine breakdowns becomes critical in the sense that they can decrease the periodic capacity by a considerable amount.

- *Tool wearing*: Some components of the machinery may become worn out, which affects the performance of the machines even if no breakdown occurs.

- *Unexpected maintenance duration*: Repair and maintenance of malfunctioning machines can take longer times than planned and some machinery may be out of operation when needed.

- *Utility loss*: Any kind of input (steam, electricity, etc.) may become absent due to external factors, which leads to interruption of the production process via capacity loss.

- *Chase strategy applied for labor force*: An aggressive hiring and firing policy causes a fluctuating labor force level, which also makes overall capacity more unstable.

The realized capacity as a consequence of all these factors becomes restrictive if the order given by the decision maker is higher than this realized capacity. As a result, only a portion of the order size, as much as the realized capacity, is actually processed in the plant of the vendor. In opposite case, the released order is fully processed at vendor's side. A common approach to model such vendors is to regard vendor capacity as a random variable that follows a known distribution. So the actual amount processed at the vendor's plant is the minimum of the order and the random capacity.

The second source of uncertainty is a result of all the processes that occur after the production was initiated at the vendor's plant until the orders are delivered to the decision maker. Therefore, this uncertainty is spread over the vendor's production process and transportation to the receiver. Due to imperfect production of the vendor, some portion of the produced amount goes to scrap. Additionally, some of the products become defective due to improper transportation. As a consequence of these two uncertainty factors, the decision maker can receive only a portion of the amount that was initially processed at the vendor's plant. These sources of uncertainty can be included into the model via applying a stochastic proportion to the amount initially processed. The stochastic proportion here is represented by a random variable that is assumed to be independent of the order size and realized capacity. The stochastic proportion is usually bounded by a finite value.

In this study, both of these two approaches are combined in order to observe the total effect of uncertainty on optimum order amounts. Once the order is given to the vendor, it is fully processed if and only if it is less than the realized capacity of the vendor. Otherwise, the portion equaling the amount of realized capacity is processed. Then, the processed amount is exposed to a stochastic proportion and final yield that is actually received is obtained. This order quantity is optimized to minimize the total expected holding and shortage costs that are driven by an uncertain demand, which is a random variable following a known distribution. Although random demand causes complications, it is required to model the real-life environment. A common approach in the literature is to study the single vendor case. However, we know that in real life making business with only one vendor has disadvantages besides its advantages. In order to benefit from the diversification effect and to reduce uncertainty costs, companies work with several vendors. Moreover, working with several vendors is advantageous when price and quality variations are concerned. The main contribution of this thesis is to find and analyze some results for multiple distinct and identical vendors. It is important to extract some features of total order quantity given to all vendors as some parameters such as capacity mean and number of vendors change.

The rest of this thesis begins with a comprehensive review of the related studies in this area presented in Chapter 2. The third chapter discusses the model with a single vendor in detail. Chapters 4 and 5 give the detailed analysis with multiple distinct and identical vendors respectively. Finally, there is a discussion about the future complementary work related to this thesis in Chapter 6.

Chapter 2

LITERATURE REVIEW

As mentioned in the previous section, there are mainly two approaches to model random yield in inventory models that are related to this study. The first approach is based on applying a stochastic proportion to the order quantity. The second one is driven by the idea that random yield is the result of the random capacity of the vendor or the production facility. However, some other approaches exist in literature that use binomial yields, interrupted geometric yields or take the stochastic proportion dependent on the order quantity. Yano and Lee [1] offer a relevant and comprehensive review of such approaches in literature. Following is the literature about the relevant two approaches classified under four groups.

2.1 Random Yield Models with Stochastic Proportion

The earliest studies concerning inventory models with random yields are Karlin [2], Silver [3], Shih [4], Mazzola et al. [5], Noori and Keller [6]. These studies explored a "stochastic proportion" in order to model the defective units in a lot. They generally represent the actual yield by $Y_q = UQ$ where Q is the amount of order given to the supplier or the production quantity initiated in the production plant. The general approach is to assign the random variable U a distribution such that it is bounded by 0 and 1 since it represents the proportion of defective items. On the other hand, there are articles in literature that assumes no upper-bound on U and justify this approach by potential measuring errors, scaling factors, poor communication between the vendor and the decision maker. Some other studies like Parlar and Berkin [7], Berk and Arreola [8], Gürler and Parlar [9] regard stochastic proportion as a binary variable that equals 1 if supply is available and 0 otherwise, i.e., the order is totally received if and only if the supplier is available at a specific time of realization and nothing is received otherwise.

Edhart and Taube [10] studies a random yield, random demand inventory model with a convex holding/shortage cost structure and linear ordering cost. They show that generalizations of base-stock and (s, S) policies are optimal. Lee and Yano [11] formulate the production system as a multistage serial system with proportional yield at each stage and deterministic demand. At each stage, a single critical level and the corresponding input level configures the optimal policy. If the available input is less than the target quantity, then the available level is utilized.

One of the important studies in relevant literature is made by Henig and Gerchak [12] in which the periodic review concept with a single production facility (analogy to single vendor case) with stochastically proportional yield was analyzed; however, they do not restrict the stochastic proportion by a finite value so that it may be higher than one. Variable cost is paid only for the actual realized yield. They build a recursive cost function and then prove that it is convex. Optimizing that function results in a non-order-up-to policy, in which no order is given when the initial inventory is higher than the critical order level for that period. The sum of the order quantity and the beginning inventory exceeds the critical order level since it is not an order-up-to policy. Finite period problem solutions converge to those of infinite-period problem.

Bassok and Akella [13] focus on the aggregate plan of the raw material procurement and production of a supply chain system. Stochastic proportion applies only to the raw material procurement. The aim is to find the optimum order quantities for procurement stage and the capacity to be allocated at the production facility to satisfy the random demand with this random yield in procurement stage. An optimum solution is hard to obtain due to the complexity of the cost function, so they propose some approximate solutions. Amihud and Mendelson [14] have a similar approach that jointly determines the sales and production quantities under uncertain output and demand. However in this approach, an additive variability is applied to the input level instead of using a multiplicative approach, i.e., stochastic proportion. Optimum solutions are reported to be insensitive to the length of the planning horizon.

Gerchak et al. [15] study a periodic-review production model with variable yield from single vendor and include uncertain demand into their formulation. Random yield is represented by a stochastic proportion applied to the input order level Q. They analyze two and n-period problems after obtaining a comprehensive characterization of the single period problem. In this model, the unit variable cost (unit production cost) is considered to be proportional to the actually realized yield so that defective units do not generate any cost. In the single period case, they state that the critical order point is unaffected by the yield distribution. However, due to the existence of the random yield, optimum quantity as a function of initial inventory is not a straight line and an order-up-to-level strategy does not emerge. For the n-period problem, they show that a critical order level can be found for each period so that no order is given if the initial inventory for that period is higher than that critical level.

There are also heuristic approaches for random yield problem in the literature. Bollapragada and Morton [16] propose three myopic heuristics for the multi-period inventory problem where a stochastic proportion is applied to the order released. An ordering cost is applied to both cases: cost can be proportional to either the ordered quantity or actually received quantity. The Newsvendor Heuristic obtains the order quantity Q as a function of initial inventory, which is nonlinear. The other two heuristics enable the user to make linear fits onto the function $Q(x)$, the order quantity if the inventory level is x. Via a computational study with different distributions and parameters, it is illustrated that the best of the heuristics has worst-case errors of 3.0 % and 5.0 % and average errors of 0.6 % and 1.2 % for the infinite and finite horizon cases respectively.

Random yield is also studied in the EOQ setting by Cheng [17], being one of the earliest works. In his model, an EOQ model is proposed with demand dependent unit production cost and unreliable supply process. Implicit form optimal solutions are obtained.

In the literature, there is a common assumption that defective items can be identified just after receiving. This obviously requires perfect inspection that is handled just before production or shipping to the customer. However, it is costly to implement such an inspection procedure and there is a trade-off between inspection and holding costs. Zhang and Gerchak [18] analyze a joint lot sizing and inspection policy under an EOQ model. Due to the cost of inspection, they find the optimum lot size and the portion of the received quantity to inspect so that total expected holding and inspection cost is minimized.

Many of the parameters of inventory models may change during the planning horizon, which is generally named "fluctuating environment" in literature. This concept is important in the sense that distributions and some other features of random variables change by the effects of the environment. Demand is an apparent example that is variable along the planning horizon. So, earliest studies dealing with random environments model the demand process environment dependent. Kalymon [19] models the related costs with a Markovian stochastic process where the demand distribution depends on cost parameters. In Feldman [20], demands are modulated by a compound Poisson process where parameters depend on the environment state which follows a continuous Markov process.

Özekici and Parlar [21] model random environment within a periodic review control policy. Supplier unreliability is represented by using a binary variable that equals 1 if supply is available in that period or 0 otherwise. The environmental process follows a Markov chain. In this setting, the environmental process affects demand, supply and all cost parameters. In general, they show that an environment dependent base-stock policy characterizes the optimum strategy. It is also shown that when a fixed ordering cost is included, the optimal strategy is the well-known (s, S) policy where the parameters depend on the state of the environment.

2.2 *Random Yield Models with Random Capacity*

Another approach for modeling random yield is to take the capacity random so that actual yield is the minimum of the realized capacity and the order released.

Ciarallo et al. [22] focus on random capacity, which is assumed to follow a known distribution. Parallel with the previous works, they find out that base-stock policy is the optimum strategy; however, the objective function is a non-convex but unimodal function due to the random capacity. Order quantity increases when uncertainty is included in the model and variability is increased; however, in the single period case uncertainty does not affect the size of optimum quantity. This is also observed by Henig and Gerchak [12].

Wang and Gerchak [23] analyze the effects of random capacity on the optimum policy where continuous review is applied. In fact two models are considered: EOQ and the order quantity/reorder point model with backlogging. Variable cost is applied to the actually received quantity, which is the minimum of the realized capacity and the order size. The expected cost per unit time comes out to be quasi-convex in the order size. If the mean capacity is allowed to be infinite, then the well-known EOQ formula is obtained. They show that random capacity makes the optimal order quantities and optimal reorder point for the second model higher. Also, for the specific case with exponentially distributed capacity, they propose a simplified method to reach the optimal quantity.

Güllü et al. [24] analyze a similar capacity uncertainty model and obtain a solution for the order up to level by utilizing queuing theory applications. Besides the $0-1$ availability of the supplier, yield also depends on the size of the order in this periodic review model. Demand is considered to be deterministic and dynamic over the finite planning horizon. As a result, they come up with optimality of an order up to policy that minimizes expected holding and backorder costs.

2.3 Random Yield Models with both Random Capacity and Stochastic Proportion

Wang and Gerchak [25] have a joint approach including variable production capacity and a stochastic proportion that are applied in an environment with single production plant (analogous to the single vendor case), uncertain demand and periodic review control policy. Capacity, stochastic proportion and demand are all assumed to follow a certain distribution. So when a production order is released, minimum of the order and the realized capacity for that period is actually obtained. In order to represent the final output of the plant, a stochastic proportion is applied to this value. A stochastic dynamic programming formulation is designed to explore some characterizations in the multi-period setting. They prove that, for the finite horizon problem, the objective function is quasi-convex and so attains a global minimum for a given initial inventory level. Optimal strategy is such that there is a single critical order level for each period, above which no order is given. Besides, the resulting strategy is not an order-up-to-level one. It is also proved that solution of the finite-horizon problem converges to that of the infinite-horizon problem. They give the expressions to be solved in order to find optimum quantities and critical order levels without a numerical illustration.

Matsumoto and Tabata [26] achieves a comprehensive study on periodic review and single product inventory control where random capacity, random yield, uncertain demand, and an environment that randomly fluctuates following a discrete time Markov chain are considered. A fixed ordering cost is included into the model and a unit variable cost is applied to the order amount released instead of only the actual yield. Due to the second one, critical order level comes out to be dependent on the mean of the stochastic proportion that represents random yield. They also show that finite-horizon objective function is quasi-convex and the optimal policy is not order-up-to type. Expressions to obtain optimal order quantities and critical order levels are provided.

2.4 Random Yield Models with Multiple Vendors

The studies mentioned above assume that there is a single vendor or production plant which is unreliable. However, in real life applications, it is obvious that working with multiple vendors is advantageous through many perspectives.

An important study about continuous review inventory models is by Gerchak and Parlar [27]. Stochastic proportions are applied to the order quantities of two distinct vendors having same prices but different yield rates with different means and variances. Variable cost is paid for the whole order size, no matter what the ratio of defective items is. So variability directly affects the variable cost besides the holding cost. The main outcome of this study is that the optimum order quantities from these two vendors are proportional to the ratio of means over ratio of variances. Interestingly, they observe that diversification over two vendors is advantageous if and only if the joint set up cost is less than the sum of the individual setup costs.

Parlar and Wang [28] study two distinct vendors having different yield distributions and unit variable costs. Applying different stochastic proportions to the orders given to each vendor, total cost function is shown to be convex for a wide range of parameters and the optimal order quantities are found explicitly.

Anupindi and Akella [29] focus on single and multiple-period problems with two distinct vendors and random demand. They model supplier unreliability via employing stochastic proportion. In fact they regard unreliability of the supply process as delayed deliveries. Since they focus on two vendors, optimum policy contains two different critical order levels. Orders are given to the both vendors if the initial inventory is less than both of the critical order level for that period. If the inventory is between the two critical levels, then order is given only to the cheaper vendor. Otherwise, no order is given to any vendor. They also show that optimum quantities are proportional to the same ratio of means and variances of the stochastic proportions, which was previously found by Gerchak and Parlar [27] but in a continuous review context.

Henig and Levin [30] study joint production planning and product delivery commitments. Based on the single vendor model by Bassok and Akella [13], they show that the profit function is concave which facilitates finding the optimum quantities by simple expressions. They also prove that among the unreliable vendors, there exists one whose yield distribution makes it relatively preferable.

One of the different approaches to supplier unreliability is to take lead time uncertain, which is studied in Lau and Zhao [31]. Besides determining the optimum order quantity, they propose methods to find the optimum splitting ratio between the suppliers so that annual holding and order costs are minimized subject to constraints for maximum allowable stockout risk.

A study that considers vendor unreliability with multiple vendors is by Erdem et al. [32] and Erdem et al. [33]. Erdem et al. [32] tries to see the effect of working with multiple identical vendors whose capacities are random variables and a potential diversification over those vendors within a continuous review EOQ setting. After giving the expression that solves for the optimum quantity, they show that in uniformly distributed capacity case total order quantity begins to decrease after some n, the number of identical vendors. On the other hand, in the exponential case, total order quantity always decreases as the number of identical vendors increases. In the single vendor case, optimal order quantity is higher than the classical EOQ level in a deterministic setting. In Erdem et al. [33], they obtain an implicit set of equations that gives the unique optimum set of optimal order quantities with distinct vendors. These equations are then simplified by taking uniformly and exponentially distributed capacities. As a consequence of optimality conditions, it is shown that the expected unsatisfied order amounts are same for all vendors.

In this study, we analyzed an inventory model with a periodic review policy. In the literature, the studies assuming periodic review generally include either random capacity or stochastic proportion in their model. Some include both of these approaches; however, those models are based on the single vendor case. Our approach differs from the literature by including both random capacity and stochastic proportion in our model and extending this approach with multiple vendors. After examining the model analytically, which turns out to be rather complicated for the multiple vendors case, we propose a solution procedure for this problem. We also show that the methodology greatly simplifies when the periodic demand is exponentially distributed.

Chapter 3

A SINGLE PERIOD INVENTORY MODEL WITH SINGLE VENDOR AND RANDOM YIELD

Before studying the multiple-vendors case, we will first discuss the single vendor case in order to benefit from the analogies between the two cases in terms of modeling. When working with only a single vendor, the decision maker is in a riskier position about material procurement when compared with working with multiple vendors. At this point any uncertainty or imperfection in the vendor's production process and transportation leads to delivery of the order with an unexpected quantity or lead time. In order to foresee the potential causes of uncertainties and take precaution against the unexpected results, the decision maker should analyze the course of the past data and extract some useful knowledge about the characteristics of the whole ordering cycle including mainly vendor's production capacity and quality realized at the moment of delivery to the decision maker. After detecting the sources of uncertainty, one should mathematically model the problem of optimizing the order quantity in this stochastic cycle where the sources of those uncertainties are represented by random variables.

In the previous section, several articles that deal with this problem with single vendor case were reported in detail. In this chapter of the thesis, the aim is to analyze the single vendor case with random yield and reveal some characterizations about the optimum policy of single-item inventory control. The problem is regarded as a single period problem. The main sources of the uncertainty are the capacity of the vendor and the imperfection of the processes that take place in the rest of the whole ordering cycle.

The rest of this chapter gives firstly the definition of the problem and the assumptions of the model. Then, the mathematical formulation of the problem follows. This chapter ends with some analytical outcomes and numerical results.

3.1 Problem Definition

The problem is faced by a single node of a supply chain network given in Figure (3.1), which means that the supply chain system is operated in a decentralized manner. The decision maker at this point is a production plant that has to satisfy an uncertain demand of a single product. It holds a single item inventory for production and procures this single item from a number of unreliable suppliers in the network. A single distributor or retailer may also be regarded as the decision maker that has to satisfy an uncertain demand coming directly from the customers and gives orders to the production plants (vendors in this case). So the model can be applied to any of these two stages. In the rest of this study, a single production plant will be regarded as the decision maker that gives orders to the unreliable suppliers. In other words, the interaction between the first (Supplier) and second (Production) stages of the supply chain is analyzed, while it may also be conceived as the relation between the second (Production) and the third (Distributors/retailers) stages. Periodic inventory control is implemented, but since the single period case is handled, a single decision on the order quantity is made at the beginning of the period. This decision is driven by the beginning inventory on hand. The following are the three sources of uncertainty, which are parametrized and assigned some cumulative distributions.

In real life, most of the problems emerge due to imperfect knowledge of demand. Since the customers are the least controllable portion of supply chains, nobody can propose an accurate demand in advance. However, using the past data, some features of the demand process can be extracted. In this study, demand is taken to be a random variable that is independent of any other parameter. Let's denote periodic demand by D. The cumulative distribution function of demand is

$$G(w) = P(D \leq w). \tag{3.1}$$

The probability density function of demand is represented by $g(w)$. Considering the amount of beginning inventory on hand, an order is released to the vendor. This amount of order is processed by the production facility of the vendor, whose capacity is not known in advance due to the many factors, some of which are listed below:

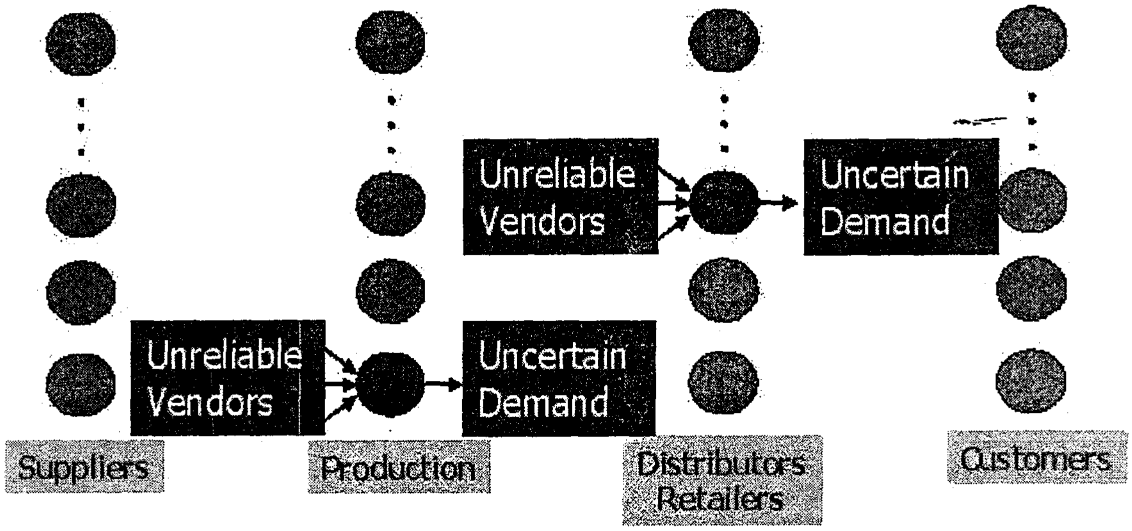

Figure 3.1: Parts of Supply Chain Network Where the Model is Applicable

- *Machine breakdowns*: If the production at the vendor's side is dominated by machinery and there is a limited number of machines, then machine breakdowns becomes critical in the sense that they can decrease the periodic capacity by a considerable amount.
- *Tool wearing*: Some components of the machinery may become worn out, which affects the performance of the machines even if no breakdown occurs.
- *Unexpected maintenance duration*: Repair and maintenance of malfunctioning machines can take times longer than planned period and some machinery may be out of operation when needed.
- *Utility loss*: Any kind of input (steam, electricity, etc.) may become absent due to external factors, which leads to interruption of the production process via capacity loss.
- *Chase strategy applied for labor force*: An aggressive hiring and firing policy causes a fluctuating labor force level, which also makes overall capacity more unstable.

The realized capacity as a consequence of all these factors becomes restrictive if the order given by the decision maker is higher than this realized capacity. As a result, only a portion of the order size, as much as the realized capacity, is actually processed in the plant of the vendor. In the opposite case, the released order is fully processed at vendor's side. Denoting this random capacity by a random variable A, the cumulative distribution of capacity can be given as

$$F(z) = P(A \leq z). \tag{3.2}$$

The probability density function of vendor's capacity is represented by $f(z)$. After the execution starts an uncertain cycle emerges, which is spread over the vendor's production process and the transportation to the receiver. Due to imperfect production of vendor, some portion of the produced amount goes to scrap. Additionally, some of the products become defective due to improper transportation. As a consequence of these two uncertainty factors, the decision maker can receive only a portion of the amount that was initially processed at the vendor's plant. These sources of uncertainty can be included into the model via applying a stochastic proportion U to the amount initially processed. Random variable U has the cumulative distribution function

$$T(u) = P(U \leq u) \tag{3.3}$$

with density function $t(u)$. Throughout this thesis we suppose that all cumulative distribution functions G, F, and T are differentiable with probability density functions g, f, and t. The density functions g, f, and t have some bounds on their domain. It is obvious that neither demand D nor the capacity A can be negative, which is also valid for U. In fact U should be also bounded above by 1, in addition to nonnegativity, since the uncertain events generally do not add to the order quantity but decrease the initial order by some amount.

3.2 Assumptions

In order for the model to have a logical structure, the whole decision process should satisfy some conditions. Besides, the formulation should include some specifications on parameters, distributions, etc. to state and justify the optimality conditions. Here is the list of the main assumptions made in this study:

- The vendor capacity is such that there is always a nonzero probability for the vendor to process the given order, whatever the order size is, i.e., $1 - F(z) > 0$ for all z.
- The density function of the periodic demand takes positive values for positive variables, i.e., $g(y) > 0$ for $y > 0$ and $g(y) = 0$ for $y \leq 0$.
- Decision is made for a single period.
- There is a single product.
- Orders are given to the vendor before capacity is realized. In the case of early information about capacity, random capacity modeling has no meaning.
- Procurement process contains only the variable cost of ordering. Transportation cost is assumed to be embedded in this variable cost.
- Production process of the decision maker does not comprise any uncertainty. So, actual yield received from the vendor is directly converted to finished goods to satisfy the demand without any loss in quantity.
- Lead time is short enough so that it causes no critical delay.
- Unit shortage cost is higher than unit variable cost. This assumption is common in related literature and required for some outcomes to make sense.
- Parameters related to costs and distributions are fixed throughout the decision period.
- Distribution of the stochastic proportion is independent of order quantity and realized capacity of the vendor.
- Unsatisfied demand is lost.

The first two assumptions on the distribution function F and G are needed for technical reasons to avoid some noncrucial difficulties in the analysis.

3.3 Model Formulation

An order of q is fully processed if it is less than the realized capacity of the vendor. Let's call this processed amount P_q . Recalling that the random capacity of the vendor is represented by A, P_q can be stated as

$$P_q = \min\{q, A\}. \tag{3.4}$$

The actual amount that is received from the vendor is found by applying the stochastic proportion U to P_q. We call it "actual yield" and denote by Y_q. Then, it is given by equation

$$Y_q = U \min\{q, A\} = UP_q. \tag{3.5}$$

The unit variable cost, which includes only the purchase cost per item in our model, is denoted by c; h is the unit holding cost and p is the unit shortage cost. Since we study a single period model, h can be regarded as the holding cost in the newsboy problem, which is equal to the unit purchase cost minus the salvage value of the item. Here it is important for p to have a value higher than c. At this point it is necessary to define a vital part of the cost function, the expected inventory cost. Suppose that after receiving the actual yield from the vendor, there is a total amount of y on hand together with the initial inventory. We use the common notation $L(y)$ in related literature to denote the expected total holding and shortage cost given that the sum of actual yield and initial inventory is y.

Recalling that demand is a random quantity with cumulative distribution function G and density g, we can express total expected holding and shortage cost as

$$\begin{aligned} L(y) &= E[h \max\{0, y-D\} + p \max\{0, D-y\}] && (3.6)\\ &= h\int_0^y (y-w)dG(w) + p\int_y^\infty (w-y)dG(w). && (3.7) \end{aligned}$$

In order to derive some characterizations about L, first and second derivatives are obtained as

$$L'(y) = (h+p)G(y) - p \tag{3.8}$$

and

$$L''(y) = (h+p)g(y). \tag{3.9}$$

As a matter of fact, L is strictly convex on $[0,+\infty)$ by our assumption that $g(y) > 0$ for all $y \geq 0$.

The expected total cost function TC can be thought as a two-variable function of initial inventory x and the corresponding order quantity q. There are mainly two components of TC; the variable ordering cost V_q and the expected inventory cost L. Modeling the variable ordering cost in similar studies may vary according to problem setting and perception of ordering cost. In some studies, "order" refers to the amount of production level that is aimed instead of order given to the vendor. When production is initiated, all of the input costs are incurred and the resulting scrapped items or any other inefficiencies increase the cost per finished good. Also several studies where "order" refers to order given to the vendor assume that variable cost is paid in advance so that it is not important whatever you receive since you pay what you exactly order. These two approaches implicitly claim that the more uncertain the ordering or production system, the more costly are the procurement or the production processes. Under such a setting, when order size is q, the variable ordering cost paid or incurred in advance V_q^a is simply given by

$$V_q^a = cq. \tag{3.10}$$

A model that is essentially identical to ours in this single period setting is discussed in detail by Wang and Gerchak [25]. Their model differentiates from our model with this application of unit variable cost. The relevant portion of the total cost is given by cP_q in their model. This approach makes sense since they apply the variable cost to the production process where the cost is generally incurred at the beginning of the processes. In this study, orders are given to a vendor and variable cost is the unit procurement cost. We believe that in real life applications, vendor is paid as much as the amount received, independent of the order initially given. In other words, if the decision maker receives 80% of its order, then an amount of $0.8cq$ is paid to the vendor and decision maker is not directly punished due to uncertainty. That final random yield is represented by Y_q in our model. So the random variable cost is

$$V_q = cY_q. \tag{3.11}$$

The second component of TC is $L(y)$ as mentioned above. Here, y refers to the total inventory on hand at the beginning, which is the sum of initial inventory and the actual yield. So, random demand is met by an inventory of $y = x + Y_q$. Then, the total expected cost function can be expressed as

$$\begin{aligned} TC(x,q) &= E[cY_q + L(x+Y_q)] && (3.12)\\ &= E[cUP_q + L(x+UP_q)] && (3.13)\\ &= E[cU(q \wedge A) + L(x+U(q \wedge A))] && (3.14) \end{aligned}$$

where $(q \wedge A) = \min\{q, A\}$.

Let $v_1(x)$ denote the optimal cost function with single vendor, given an initial inventory of x. So we can obtain $v_1(x)$ by minimizing $TC(x,q)$ in (3.12) for $q \geq 0$ so that

$$\begin{aligned} v_1(x) &= \min_{q\geq 0} TC(x,q) && (3.15)\\ &= \min_{q\geq 0} E[cU(q \wedge A) + L(x+U(q \wedge A))] && (3.16)\\ &= \min_{q\geq 0} \int E[cu(q \wedge A) + L(x+u(q \wedge A))]dT(u) && (3.17)\\ &= \min_{q\geq 0} \int [(1-F(q))(cuq + L(x+uq)) + \int_0^q (cuz + L(x+uz))dF(z)]dT(u) && (3.18) \end{aligned}$$

In order to derive some optimality conditions, first and second order derivatives with respect to q should be investigated for any given parameter x. The partial derivative of $TC(x,q)$ with respect to q is

$$\begin{aligned}
\frac{\partial TC(x,q)}{\partial q} &= \int [(1-F(q))(cu+uL'(x+uq))]dT(u) && (3.19)\\
&= E\left[cU+UL'(x+Uq)\right](1-F(q)) && (3.20)\\
&= (cE[U]+E[UL'(x+Uq)])(1-F(q)). && (3.21)
\end{aligned}$$

We know that for a given x and $U>0$, $L'(x+Uq)$ is a continuous and increasing function of q, and so $L(x+Uq)$ is convex in q. From the definition of L in (3.8) we can easily obtain

$$\begin{aligned}
\lim_{q\to+\infty} L'(x+Uq) &= h>0 && (3.22)\\
\lim_{q\to-\infty} L'(x+Uq) &= -p<0
\end{aligned}$$

for all $U>0$.

Considering the first assumption of our model, we have $(1-F(q))>0$ for any finite value of q. Under the light of this fact and the assumption that $p>c$, it can be claimed that for a unique value of $q=\overline{q}(x)$, the partial derivative in (3.21) becomes zero. Since $p>c$, the second term of the first factor, $E[UL'(x+Uq)]$, is increasing and takes values in the range $(-pE[U],hE[U])$, which includes $-cE[U]$. Therefore, for a given x, $\overline{q}(x)$ can be obtained by solving

$$cE[U]+E[UL'(x+U\overline{q}(x))]=0. \qquad (3.23)$$

The solution of this equation is unique, for example, when x is positive with $E[U]>0$ since then, $E[UL'(x+Uq)]$ is strictly increasing in the range $(-pE[U],hE[U])$ and it equals $-cE[U]$ at only one point. Otherwise, the solution may not be unique but increasing $E[UL'(x+Uq)]$ still guarantees a solution, which is optimal.

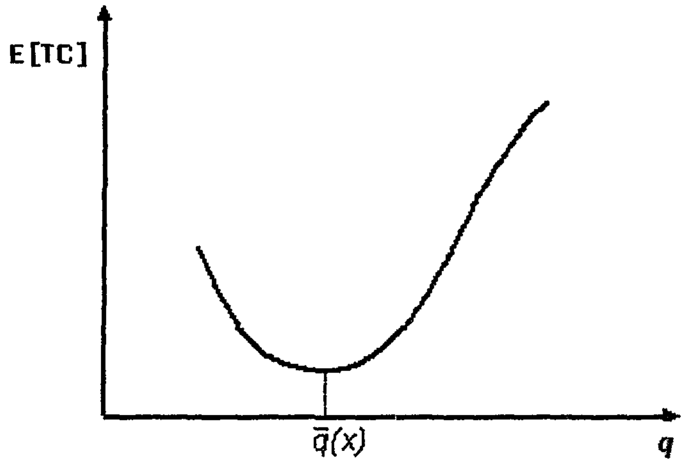

Figure 3.2: Total Expected Cost as a Function of q

According to the former arguments, the partial derivative of TC is negative for $q < \overline{q}(x)$ and nonnegative for $q > \overline{q}(x)$. In order to further analyze the characteristics of $TC(x,q)$, partial derivative of (3.21) with respect to q is investigated. It satisfies

$$\frac{\partial^2 TC(x,q)}{\partial q^2} = E\left[U^2 L''(x+Uq)\right](1-F(q)) - f(q)E\left[cU + UL'(x+Uq)\right]. \qquad (3.24)$$

The first term of (3.24) is always nonnegative since $L'' \geq 0$ and $(1-F(q)) > 0$ for all q. Since the left hand side of (3.23) is negative for the values of q smaller than $\overline{q}(x)$, we can claim that (3.24) is positive for $q \in (-\infty, \overline{q}(x))$. Here an important point is that $f(q)$ is defined to be 0 for $q \leq 0$ since the random capacity $A \geq 0$. Then the function TC turns out to be convex decreasing in this region. At the point where $q = \overline{q}(x)$, the second term of (3.24) is zero by (3.23) and second partial derivative is positive. Therefore $q = \overline{q}(x)$ is a minimal point for TC. In the region $q \in (\overline{q}(x), +\infty)$, TC increases since TC' is positive for q values that are greater than $\overline{q}(x)$. Figure (3.2) gives an illustration for this case.

In summary, TC is convex decreasing on $(-\infty, \overline{q}(x))$ and increasing on $(\overline{q}(x), +\infty)$. Then, TC turns out to be unimodular with respect to q for fixed x so that it attains its global minimum at $q = \overline{q}(x)$. So a nonnegative $\overline{q}(x)$ which satisfies the first order optimality condition (3.23) is also the solution of the constrained minimization problem (3.15). To make the notation more precise, we will let $q(x)$ denote the optimal solution of the constrained minimization problem while $\overline{q}(x)$ is the optimal solution of the unconstrained problem.

Given the inventory level x, optimum order quantity can be found by solving (3.23), which is obtained by equating first order partial derivative to zero. This expression can be stated as

$$cE[U] + \int u[(h+p)G(x+u\overline{q}(x)) - p]dT(u) = 0. \tag{3.25}$$

Proposition 1. *If $\overline{q}(x) < 0$, then the optimal order policy is to order nothing from the vendor so that $q(x) = 0$. Moreover, $q(x) = \overline{q}(x)$ if $\overline{q}(x) \geq 0$.*

Proof. For fixed x, the total expected cost function $TC(x,q)$ is unimodular in q where $\overline{q}(x)$ is the global minimum. In (3.15) we must choose $q \geq 0$ and the total expected cost function is increasing for $q > \overline{q}(x)$. Since $q \geq 0 > \overline{q}(x)$, the optimum solution is obtained at the minimum feasible point and $q(x) = 0$. It is clear that a nonnegative $\overline{q}(x)$ gives us the optimum order quantity since it is also a feasible order size, first order derivative is zero and second order derivative is positive at that point.■

Proposition 2. *The solution of the unconstrained problem $\overline{q}(x)$ is decreasing in x.*

Proof. We first take the derivative of (3.25) with respect to x in order to derive the structure of $\overline{q}(x)$. This yields

$$\int uL''(x+u\overline{q}(x))(1+u(\frac{d\overline{q}(x)}{dx}))dT(u) = 0 \tag{3.26}$$

which can be written as

$$(h+p)\int ug(x+u\overline{q}(x))(1+u(\frac{d\overline{q}(x)}{dx}))dT(u) = 0 \tag{3.27}$$

so that

$$\frac{d\overline{q}(x)}{dx} = -\frac{\int ug(x+u\overline{q}(x))dT(u)}{\int u^2 g(x+u\overline{q}(x))dT(u)} \leq 0. \tag{3.28}$$

Since the derivative of $\overline{q}(x)$ with respect to x is negative, the order quantity $\overline{q}(x)$ is decreasing in x. This is an intuitive result meaning that it is optimal to order less if you have more beginning inventory. ■

Let $S = \inf\{x : \overline{q}(x) = 0\}$. According to this definition, $\overline{q}(x)$ is greater than zero for values of $x \in (-\infty, S)$. At the point $x = S$, $\overline{q}(S) = 0$. This is the critical order level for the vendor and provides a characterization of the optimum ordering policy.

Theorem 3. *The resulting optimum ordering policy is not an order-up-to type (base stock) ordering policy. The optimum order quantity is given by*

$$q(x) = \begin{cases} \overline{q}(x), & x < S \\ 0, & x \geq S \end{cases} \tag{3.29}$$

where

$$S = G^{-1}\left(\frac{p-c}{p+h}\right). \tag{3.30}$$

Proof. For an order-up-to type or base stock policy $q(x) = S - x$ and the slope of this function is -1. However, in our case, $\overline{q}(x)$ is not equal to -1. This is because $0 < u < 1$ and, in turn, $u^2 < u$, which makes (3.28) less than -1. A slope that is less than -1 causes the sum of the initial inventory x and the released order size $q(x)$ to be greater than S for $x < S$ resulting in a non-order-up-to type ordering policy. A typical optimum policy is illustrated in Figure (3.3). The line with a slope of -1 has an equation of $q = S - x$ and represents the order-up-to policy. The resulting policy of our model is a curve whose slope is always less than -1. Clearly this curve lies above the order-up-to type policy line. In the region where $x \geq S$, our curve lies exactly on the x-axis meaning that we give no order to the vendor in this range.

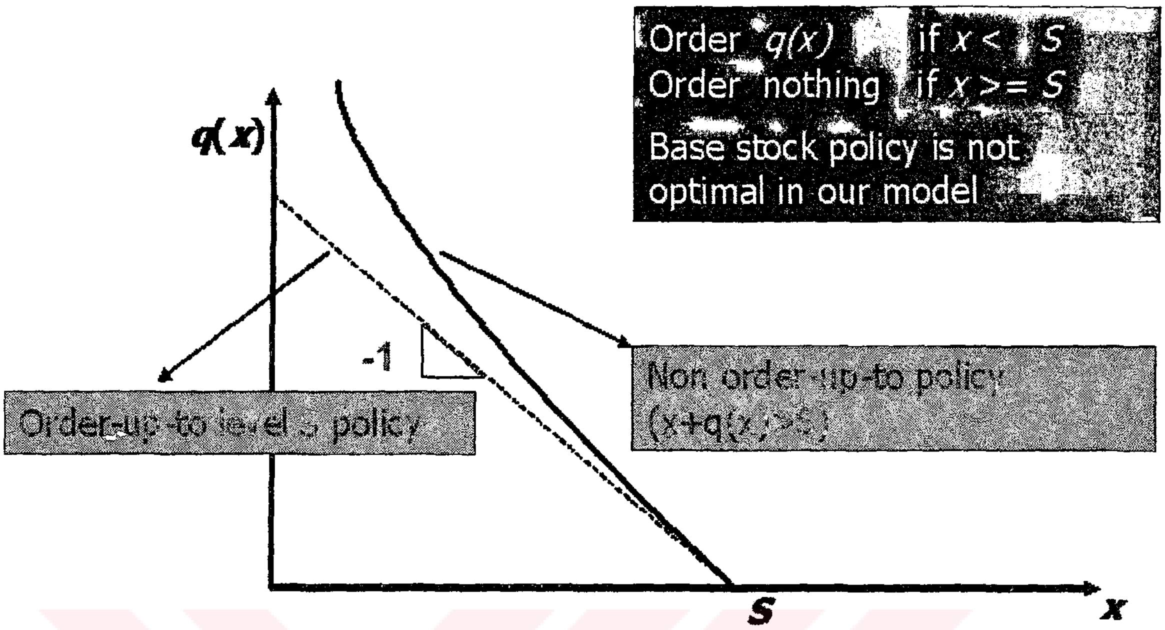

Figure 3.3: Single Vendor Ordering Policy

In the light of Proposition 1&2 and Theorem 3, it is obvious that after the critical order level S, $\overline{q}(x)$ becomes negative and no order is given beyond this level. Considering the definition of S, we substitute 0 for $\overline{q}(x)$ in (3.25) so that

$$\int u[(h+p)G(S+u\overline{q}(S))-p]dT(u) = -cE[U] \tag{3.31}$$

$$\int u[(h+p)G(S+0)-p]dT(u) = -cE[U] \tag{3.32}$$

which gives

$$G(S)=\frac{p-c}{p+h}.$$

■

An important outcome of the model is that the critical order level S is independent of both the distribution of vendor capacity and the distribution of the stochastic proportion. The relevant information is only the distribution of periodic demand and the parameters of unit variable ordering, holding, and shortage costs. So if we take only the demand uncertain and remove all the other uncertainties, we still begin ordering before the same critical order level S. The mathematical expression obtained for S explains why we make ~~the~~ assumption $p > c$. If we remove this assumption, the argument of G^{-1}may assume negative values but we know that G can take only the values that are on $[0, 1]$.

The outcomes of this chapter are almost the same as the results of Wang and Gerchak [25]. However they find the critical order level to be dependent on the mean of the stochastic proportion. This is not an unexpected result, because they apply the unit variable cost to the minimum of order release and realized capacity so that the amount of actual yield which is determined by the stochastic proportion has a direct effect on the actual variable cost incurred. In other words, when actual yield is half of the processed amount, then the actual variable cost becomes $2c$.

3.4 The Case of Exponentially Distributed Demand

Considering the assumptions on the distributions discussed above, exponential distribution is a good candidate for illustration purposes of vendor capacity and periodic demand. This is because this density function is defined for all positive values and is zero for negative values. We suppose that

$$g(y) = \begin{cases} \lambda e^{-\lambda y}, & y > 0 \\ 0, & y \leq 0 \end{cases} \tag{3.33}$$

with a mean of $1/\lambda$. Therefore, the exponential distribution satisfies the assumption on the periodic demand.

Using the exponential distribution, L is explicitly found as

$$L(y) = h \int_0^y (y - w)\lambda e^{-\lambda w} dw + p \int_y^\infty (w - y)\lambda e^{-\lambda w} dw \tag{3.34}$$

$$= \frac{1}{\lambda}[(h + p)e^{-\lambda y} + h\lambda y - h] \tag{3.35}$$

for $y \geq 0$ and

$$L(y) = -py + \frac{p}{\lambda} \tag{3.36}$$

for $y < 0$ so that

$$L'(y) = \begin{cases} (h+p)(1-e^{-\lambda y}) - p, & y \geq 0 \\ -p, & y < 0 \end{cases} \tag{3.37}$$

and

$$L''(y) = \begin{cases} (h+p)\lambda e^{-\lambda y}, & y \geq 0 \\ 0, & y < 0 \end{cases} \tag{3.38}$$

Skipping the detailed analysis of the model, we focus on the optimality equation (3.23) which becomes

$$E[U((h+p)(1-e^{-\lambda(x+U\bar{q}(x))}) - p)] = -cE[U]. \tag{3.39}$$

In (3.39), we used the form of L that is defined for the positive domain. Taking $x \geq 0$, we guarantee that the argument of L is positive since U and $\bar{q}(x)$ take positive values. In fact, we obtained all computational results for integer values of $x \in [0, 10]$ requiring the corresponding definition of L so that the optimality condition turns out to be as in (3.39). Actually L can be modeled for both positive and negative x values especially for a multi-period model; however, this creates a rather complex expression for L. Throughout the rest of this thesis, we assume positive x values and use the form of L for positive domain for illustration.

The selection of the capacity distribution has no effect since it does not exist in the equation. We write the equation as

$$E[U(1-e^{-\lambda(x+U\bar{q}(x))})] = \frac{p-c}{p+h}E[U] \tag{3.40}$$

which leads to

$$E[Ue^{-\lambda U\bar{q}(x)}] = e^{\lambda x}\frac{h+c}{h+p}E[U]. \tag{3.41}$$

Here, if we take U as a deterministic parameter so that $U = u$, then we have

$$\overline{q}(x) = \frac{S - x}{u} \tag{3.42}$$

where

$$S = -\frac{1}{\lambda} \ln\left(\frac{h+c}{h+p}\right). \tag{3.43}$$

So, for constant U, we have a linear ordering function having a slope of $-1/u$. In other words, we order $1/u$ units for each one unit of quantity needed. The value of S is a positive number by the assumption that $p > c$. As another special case, note that we obtain the base stock policy $\overline{q}(x) = S - x$ if $u = 1$ so that there are no defectives in the order.

Model with Exponential Vendor Capacity and Uniform Proportion

For illustration purposes, we now assume that vendor capacity is also exponentially distributed. Considering the assumption on vendor capacity, we observe that exponentially distributed capacity also satisfies the related assumption since

$$1 - F(z) = e^{-\mu z} > 0 \tag{3.44}$$

for all $z > 0$ with mean capacity of $1/\mu$. For the stochastic proportion, we select uniform distribution on $[a, b]$ since it has a density having finite bounds. We know that it is logical for U to take values between 0 and 1. Then,

$$t(u) = \begin{cases} \frac{1}{b-a}, & a \leq u \leq b \\ 0, & \text{elsewhere} \end{cases} \tag{3.45}$$

where $0 < a < b < 1$. Then, $TC(x, q)$ in (3.15) becomes

$$\begin{aligned} TC(x,q) &= \int_a^b \frac{1}{b-a} [e^{-\mu q}(cuq + L(x+uq)) \\ &\quad + \int_0^q (cuz + L(x+uz)\mu e^{-\mu z} dz] du. \end{aligned} \tag{3.46}$$

After the inclusion of all distributions and parameters, $\overline{q}(x)$ is found explicitly by the solving the equation

$$\frac{1}{(b-a)\lambda^2\overline{q}(x)^2}[e^{-\lambda\overline{q}(x)a}(1+\lambda\overline{q}(x)a) - e^{-\lambda\overline{q}(x)b}(1+\lambda\overline{q}(x)b)] = e^{\lambda x}\frac{(h+c)(a+b)}{2(h+p)}. \quad (3.47)$$

In order to find the critical order level S, we include the inverse of the exponential cumulative function

$$G^{-1}(x) = \frac{1}{\lambda}\ln\left(\frac{1}{1-x}\right) \quad (3.48)$$

into (3.30) so that we obtain

$$S = G^{-1}\left(\frac{p-c}{p+h}\right) = \frac{1}{\lambda}\ln\left(\frac{h+p}{h+c}\right) \quad (3.49)$$

which is the same value as in the case when $U = 1$.

3.5 Numerical Illustration and Results

In this section we give the outputs of the single vendor model when all input parameters are numerically included in the equations. We suppose that the variable cost c is \$2/unit and the holding cost h has a value of \$0.5/unit. The shortage cost p is \$5/unit considering the assumption that it should be higher than c. The parameter λ for the periodic demand is assigned to be 0.1, resulting in a mean periodic demand of 10 units, i.e., $D \sim \text{Exp}(\lambda)$. The stochastic proportion U is assumed to be uniformly distributed on (0.5, 0.8). The parameter μ for vendor capacity is taken to be 0.25 resulting a capacity mean of 4, i.e., $A \sim \text{Exp}(\mu)$. The value of μ has no effect on optimum order quantities; however, it affects the total expected cost function.

After setting up the optimality equation, we solved it by using Matlab 6.5. Table (3.1) shows the optimum order quantities that correspond to the different inventory levels. We tabulated the order quantities that are calculated each time when the inventory level is incremented by 1, 0 through 10. The critical order level S for this problem is calculated as

$$S = \frac{1}{0.1}\ln\left(\frac{0.5+5}{0.5+2}\right) = 7.88. \quad (3.50)$$

Table 3.1: Optimal Order Quantities for a Single Vendor

Initial Inventory (x)	$q(x)$	**Consecutive Difference**
0	12.00	-
1	10.47	-1.53
2	8.94	-1.53
3	7.41	-1.52
4	5.89	-1.52
5	4.37	-1.52
6	2.85	-1.52
7	1.34	-1.52
8	0.00	-
9	0.00	-
10	0.00	-
7.8	0.13	-

which is in accordance with the last row of Table (3.1). Up to the critical order level, $\overline{q}(x)$ is positive and optimum order size $q(x) = \overline{q}(x)$ on this range. Both functions are decreasing up to that point, which is previously stated in Proposition 2. At the points where $x \geq 7.88$, solving the optimality equation gave negative $\overline{q}(x)$ values resulting an optimum order size $q(x)$ that equals 0. The pattern of the optimum order size is best seen on Figure (3.4), despite the fact that non-order-up-to type policy is not obvious since the curve resembles a -1-slope straight line.

In order to gain insight on Theorem 3 and notice the non-order-up-to type policy, we should look at the 3rd column of Table (3.1) where the differences between the consecutive order sizes are tabulated. Since the incremental amounts on x all equal 1, this column gives a sound insight about the slope of the curve at discrete points. The values are less than one as stated in Theorem 3, which proves that the policy is a non-order-up-to type ordering policy. The last column gives a computational illustration of the theorem where it is observed that the sum of the initial inventory and the order size always exceeds the critical order level. In fact, the amount of difference increases as inventory level decreases

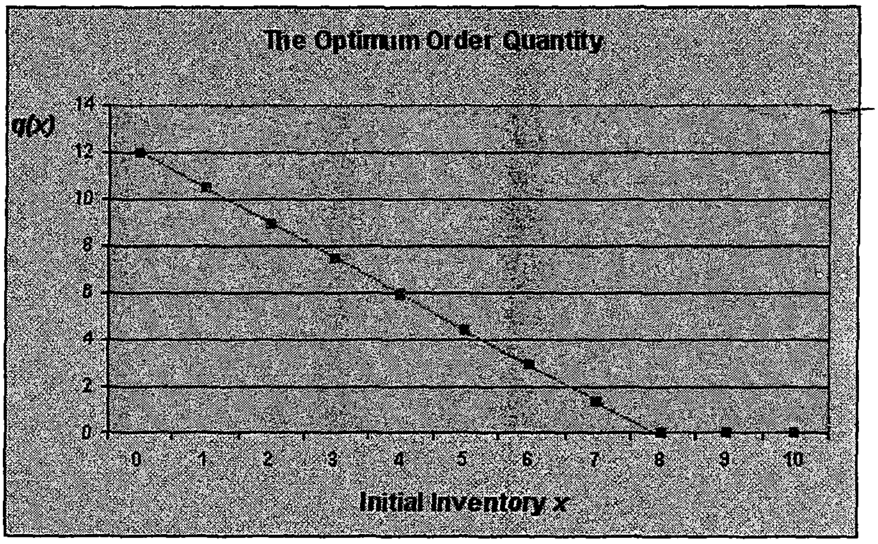

Figure 3.4: Optimum Order Quantities for Single Vendor

which is natural since the slope of the curve decreases as inventory level decreases.

Chapter 4

EXTENSION OF THE MODEL WITH MULTIPLE DISTINCT VENDORS

Examining the related literature on inventory models with random yield, we observe that most of the papers study the single vendor case. However, when the random yield concept is in question, working with a single vendor apparently means taking more risk. Making purchasing decisions using a single vendor may result in extremely unexpected outcomes despite the optimization of an objective using these decisions.

In order to acquire a more reliable position in the material procurement process, orders may be distributed among several vendors. Even if all vendors are unreliable and have uncertain processes, diversification effect will emerge and variability of the resulting outcome will decrease. A measure for this reduction in the variability of the whole ordering process may be the total expected cost. Obviously, decreasing variability is expected to decrease the total expected cost.

Besides the diversification effect, working with multiple vendors has some other advantages over the single vendor case. Negotiating with multiple vendors may decrease variable cost and increase the quality of the raw material due to competition among the vendors. However, these advantages are hard to incorporate in inventory models. In this study, it is assumed that decision maker works with multiple vendors having different attributes, i.e. variable costs, distribution of capacities, and distributions of stochastic proportions.

In this chapter, the inventory model discussed in previous chapter is extended to the multiple distinct vendors case. The aim is to see the effects of working with more than one vendor and differentiate between the vendors according to their attributes. Some vital questions may arise, like "Is it always optimal to release orders to all of the vendors we work with?"; "Should the decision maker always release his order to the cheapest vendor?";

"Are reliable but expensive vendors more preferable?". The chapter begins with analyzing the two vendors case before the generalization to n vendors.

4.1 Two Vendors Case

In the general multiple vendors case, the modeling approach in this thesis makes the related expressions fairly complex; consequently, it is hard to investigate some features of the optimum policy and extract some characterizations analytically. Therefore it is proper to focus on two vendors case first and then try to generalize some outcomes for the n vendors case.

This section is the application of the single vendor modeling approach to two vendors, so we keep all of the assumptions for both vendors and their processes. However it should be noted that, since distinct vendors are in question, parameters for each vendor are different. For instance, two vendors charge different variable costs c_1 and c_2 where $p > c_i$ for $i = 1, 2$. Besides, the capacity distributions F_1 and F_2 for the vendors are either different distributions or the same distribution with different parameters. Since the production processes are different and probably different transportation channels are used, stochastic proportions U_1 and U_2 should have distinct distributions or same distributions with different parameters.

Considering a single planning period, the problem is to obtain optimum order quantities q_1 and q_2, given the initial inventory level x. As in the single vendor case,

$$F_i(z) = P(A_i \leq z) \tag{4.1}$$

and

$$T_i(u) = P(U_i \leq u) \tag{4.2}$$

where $0 \leq A_i \leq +\infty$ and $0 \leq U_i \leq 1$ for $i = 1, 2$. In two vendors case, the decision maker decides on the order for each vendor and then receives the actual yield Y_{q_i} from each vendor. Actual yield of each vendor is obtained in the same manner as the single vendor case, applying the stochastic proportion to the processed amount so that

$$P_{q_i} = \min\{q_i, A_i\} \tag{4.3}$$

$$Y_{q_i} = U_i \min\{q_i, A_i\} = U_i P_{q_i} \tag{4.4}$$

for $i = 1, 2$. Then the total random yield received from these two vendors is the sum of individual yields

$$\text{Total random yield} = Y_{q_1} + Y_{q_2} \,. \tag{4.5}$$

Total inventory holding and shortage cost $L(y)$ is unchanged except for y. Together with the Y_{q_i}s, we have a total of $y = x + Y_{q_1} + Y_{q_2}$ on hand inventory to meet the random demand. Then, the total inventory holding and shortage cost in terms of actual yields is

$$\begin{aligned} L(x + Y_{q_1} + Y_{q_2}) &= h \int_0^{x+Y_{q_1}+Y_{q_2}} (x + Y_{q_1} + Y_{q_2} - w) dG(w) \\ &\quad + p \int_{x+Y_{q_1}+Y_{q_2}}^{\infty} (w - (x + Y_{q_1} + Y_{q_2})) dG(w). \end{aligned} \tag{4.6}$$

The next step is to derive the total expected cost function TC in terms of the beginning inventory x and quantities q_1 and q_2 ordered from each vendor. The same approach is valid here, TC is the expected sum of purchase costs from each vendor and the total inventory holding and shortage cost that is shaped by the initial inventory and the sum of actual yields form each vendor.

$$\begin{aligned} TC(x, q_1, q_2) &= E[c_1 Y_{q_1} + c_2 Y_{q_2} + L(x + Y_{q_1} + Y_{q_2})] && (4.7) \\ &= E[c_1 U_1 P_{q_1} + c_2 U_2 P_{q_2} + L(x + U_1 P_{q_1} + U_2 P_{q_2})] && (4.8) \\ &= E[c_1 U_1 (q_1 \wedge A_1) + c_2 U_2 (q_2 \wedge A_2) + L(x + U_1 (q_1 \wedge A_1) + U_2 (q_2 \wedge A_2))] && (4.9) \end{aligned}$$

Let $v_2(x)$ denote the optimal cost function with two vendors, given an initial inventory of x. So we can obtain $v_2(x)$ by minimizing $TC(x, q_1, q_2)$ in (4.7) for $q_1, q_2 \geq 0$ such that

$$\begin{aligned}
v_2(x) &= \min_{q_1,q_2\geq 0} TC(x,q_1,q_2) && (4.10)\\
&= \min_{q_1,q_2\geq 0} E[c_1U_1(q_1\wedge A_1)+c_2U_2(q_2\wedge A_2)\\
&\quad +L(x+U_1(q_1\wedge A_1)+U_2(q_2\wedge A_2))]. && (4.11)\\
&= \min_{q_1,q_2\geq 0} \int\int [(c_1u_1q_1+c_2u_2q_2+L(x+u_1q_1+u_2q_2))(1-F_1(q_1))(1-F_2(q_2))\\
&\quad +\int_0^{q_2}(c_1u_1q_1+c_2u_2z+L(x+u_1q_1+u_2z))(1-F_1(q_1))dF_2(z)\\
&\quad +\int_0^{q_1}(c_1u_1z+c_2u_2q_2+L(x+u_1z+u_2q_1))(1-F_2(q_2))dF_1(z)\\
&\quad +\int_0^{q_1}\int_0^{q_2}(c_1u_1z_1+c_2u_2z_2+L(x+u_1z_1+u_2z_2))dF_1(z_1)dF_2(z_2)]dT_1(u_1)dT_2(u_2)\\
& && (4.12)
\end{aligned}$$

The partial derivative of $TC(x, q_1, q_2)$ with respect to q_1 is obtained as

$$\begin{aligned}
\frac{\partial TC(x,q_1,q_2)}{\partial q_1} &= E\left[c_1U_1+U_1L'(x+U_1q_1+U_2(q_2\wedge A_2))\right](1-F_1(q_1)) && (4.13)\\
&= (c_1E[U_1]+E[U_1L'(x+U_1q_1+U_2(q_2\wedge A_2))])(1-F_1(q_1)). && (4.14)
\end{aligned}$$

We can observe from its definition that $L'(x + U_1q_1 + U_2(q_2 \wedge A_2))$ is a continuous and increasing function of q_1. Then, as in the single vendor case

$$\begin{aligned}
\lim_{q_1\to+\infty} L'(x+U_1q_1+U_2(q_2\wedge A_2)) &= h>0 && (4.15)\\
\lim_{q_1\to-\infty} L'(x+U_1q_1+U_2(q_2\wedge A_2)) &= -p<0 && (4.16)
\end{aligned}$$

for all $U_1, U_2 > 0$.

By our assumption, we know that $(1 - F_1(q_1))$ is always positive for finite q_1 values. So the first factor of (4.13) determines where the first partial derivative is zero. It is also obvious that, for a given q_2, second term of the first factor in (4.13) is also increasing and takes values between $-pE[U_1]$ and $hE[U_1]$ as in single vendor case. Therefore for a unique value of $q_1 = \bar{q}_1(x)$, the first partial derivative becomes zero. Given the initial inventory x and the order size for the second vendor q_2, $\bar{q}_1(x)$ can be obtained by solving the equation

$$c_1 E[U_1] + E[U_1 L'(x + U_1 \bar{q}_1(x) + U_2(q_2 \wedge A_2))] = 0. \tag{4.17}$$

The first partial derivative is negative for $q_1 < \bar{q}_1(x)$, and positive for $q_1 > \bar{q}_1(x)$. The next step is to analyze the second partial derivative of $TC(x, q_1, q_2)$ with respect to q_1. It can be derived as

$$\begin{aligned} \frac{\partial^2 TC(x, q_1, q_2)}{\partial q_1^2} &= E[U_1^2 L''(x + U_1 q_1 + U_2(q_2 \wedge A_2))](1 - F_1(q_1)) & (4.18) \\ &\quad - f_1(q_1)(c_1 E[U_1] + E[U_1 L'(x + U_1 q_1 + U_2(q_2 \wedge A_2))]). & (4.19) \end{aligned}$$

The first term is always positive since L'' and $(1 - F_1(q_1))$ are positive for all q_1 values. Also we know that the left hand side of (4.17) is negative for values of q_1 smaller than $\bar{q}_1(x)$. Then, it is concluded that for any fixed q_2 value, (4.18) is positive for $q_1 \in (-\infty, \bar{q}_1(x))$, which means that $TC(x, q_1, q_2)$ is convex and decreasing in q_1 in this interval. If $q_1 = \bar{q}_1(x)$, the second term in (4.18) is zero by (4.17), and (4.18) turns out to be positive at this point. This means that $TC(x, q_1, q_2)$ attains a minimal point at $q_1 = \bar{q}_1(x)$. In the region $(\bar{q}_1(x), +\infty)$, TC increases in q_1 since for given x and q_2, (4.13) is positive for q_1 values that are greater than $\bar{q}_1(x)$.

In summary, TC is convex and decreasing in q_1 on $(-\infty, \bar{q}_1(x))$ and increasing on $(\bar{q}_1(x), +\infty)$. Then, for a given q_2 value, $TC(x, q_1, q_2)$ is unimodular in q_1 and for this reason it attains its global minimum at $q_1 = \bar{q}_1(x)$. Since the cost function is symmetric in q_i, same arguments are also valid for q_2. Figure (4.1) gives the 3D graph of a typical $TC(x, q_1, q_2)$ for a fixed value of x.

We have shown that, $TC(x, q_1, q_2)$ is unimodular with respect to q_1 for fixed x and q_2 and unimodular with respect to q_2 for fixed x and q_1. If $q_1(x)$ and $q_2(x)$ are the optimum

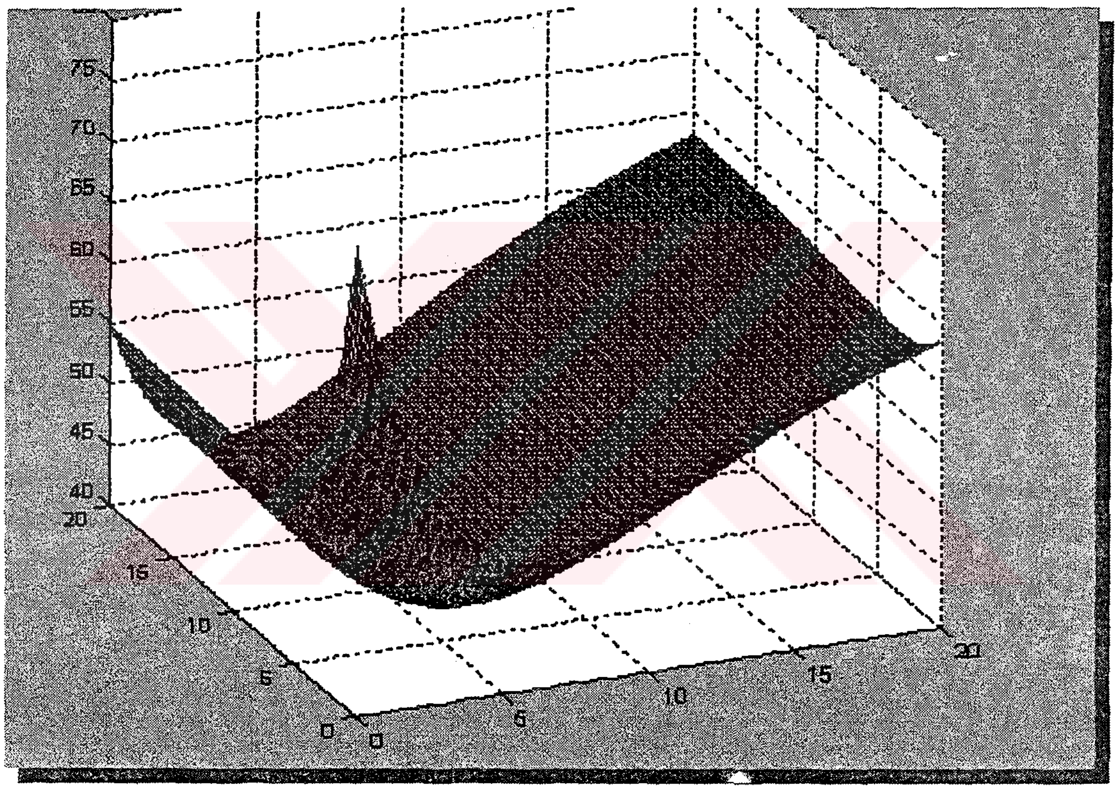

Figure 4.1: A Typical 3-D Graph of the Total Expected Cost Function

order quantities for a given initial inventory x and $q_1(x), q_2(x) > 0$, then they obviously satisfy the first order condition

$$\frac{\partial TC(x, q_1(x), q_2(x))}{\partial q_1} = 0 \qquad (4.20)$$

$$\frac{\partial TC(x, q_1(x), q_2(x))}{\partial q_2} = 0. \qquad (4.21)$$

The first order equations in explicit form are

$$\begin{aligned}
\frac{\partial TC(x, q_1, q_2)}{\partial q_1} &= (c_1 E[U_1] + E[U_1 L'(x + U_1 q_1 + U_2(q_2 \wedge A_2))]])(1 - F_1(q_1)) && (4.22)\\
&= (c_1 E[U_1] + E[U_1 L'(x + U_1 q_1 + U_2 q_2)](1 - F_2(q_2)) \\
&\quad + \int_0^{q_2} E[U_1 L'(x + U_1 q_1 + U_2 z)] dF_2(z))(1 - F_1(q_1)) && (4.23)\\
&= (c_1 E[U_1] + \int\int [u_1 L'(x + u_1 q_1 + u_2 q_2)(1 - F_2(q_2)) \\
&\quad + \int_0^{q_2} u_1 L'(x + u_1 q_1 + u_2 q_2) dF_2(z)] dT_1(u_1) dT_2(u_2))(1 - F_1(q_1)) \\
&= 0 && (4.24)
\end{aligned}$$

and

$$\begin{aligned}
\frac{\partial TC(x, q_1, q_2)}{\partial q_2} &= (c_2 E[U_2] + E[U_2 L'(x + U_1(q_1 \wedge A_1) + U_2 q_2)]])(1 - F_2(q_2)) && (4.25)\\
&= (c_2 E[U_2] + E[U_2 L'(x + U_1 q_1 + U_2 q_2)](1 - F_1(q_1)) \\
&\quad + \int_0^{q_1} E[U_2 L'(x + U_1 z + U_2 q_2)] dF_1(z))(1 - F_2(q_2)) && (4.26)\\
&= (c_2 E[U_2] + \int\int [u_2 L'(x + u_1 q_1 + u_2 q_2)(1 - F_1(q_1)) \\
&\quad + \int_0^{q_1} u_2 L'(x + u_1 z + u_2 q_2) dF_1(z)] dT_1(u_1) dT_2(u_2))(1 - F_2(q_2)). \\
&= 0 && (4.27)
\end{aligned}$$

Equations (4.24) through (4.27) state that, in order to find the optimum order quantities we should investigate the solutions $q_1 = \bar{q}_1(x)$ and $q_2 = \bar{q}_2(x)$ that satisfy

$$\begin{aligned}-c_1E[U_1] &= \int\int [u_1L'(x+u_1\bar{q}_1(x)+u_2\bar{q}_2(x))(1-F_2(\bar{q}_2(x))) \\ &+ \int_0^{\bar{q}_2(x)} u_1L'(x+u_1\bar{q}_1(x)+u_2z)dF_2(z)]dT_1(u_1)dT_2(u_2) \qquad (4.28)\end{aligned}$$

and

$$\begin{aligned}-c_2E[U_2] &= \int\int [u_2L'(x+u_1\bar{q}_1(x)+u_2\bar{q}_2(x))(1-F_1(\bar{q}_1(x))) \\ &+ \int_0^{\bar{q}_1(x)} u_2L'(x+u_1z+u_2\bar{q}_2(x))dF_1(z)]dT_1(u_1)dT_2(u_2). \qquad (4.29)\end{aligned}$$

Considering $TC(x, q_1, q_2)$ at the point $(q_1, q_2) = (\bar{q}_1(x), \bar{q}_2(x))$ and letting $H(i, j)$ be the entry of Hessian matrix corresponding to row i and column j, we obtain

$$\begin{aligned}H(1,1) &= (1-F_1(\bar{q}_1(x)))\int\int [(1-F_2(\bar{q}_2(x)))u_1^2L''(x+u_1\bar{q}_1(x)+u_2\bar{q}_2(x)) \\ &+ \int_0^{\bar{q}_2(x)} u_1^2L''(x+u_1\bar{q}_1(x)+u_2z)dF_2(z)]dT_1(u_1)dT_2(u_2) \qquad (4.30) \\ H(1,2) &= H(2,1) \\ &= (1-F_1(\bar{q}_1(x)))(1-F_2(\bar{q}_2(x))) \\ &\times \int\int u_1u_2L''(x+u_1\bar{q}_1(x)+u_2\bar{q}_2(x))dT_1(u_1)dT_2(u_2) \qquad (4.31) \\ H(2,2) &= (1-F_2(\bar{q}_2(x)))\int\int [(1-F_1(\bar{q}_1(x)))u_2^2L''(x+u_1\bar{q}_1(x)+u_2\bar{q}_2(x)) \\ &+ \int_0^{\bar{q}_1(x)} u_2^2L''(x+u_1z+u_2\bar{q}_2(x))dF_1(z)]dT_1(u_1)dT_2(u_2) \qquad (4.32)\end{aligned}$$

We know that if the Hessian matrix is positive definite at the point $(q_1, q_2) = (\bar{q}_1(x), \bar{q}_2(x))$, then it attains a minimum at that point. One of the ways for looking for this criteria is to analyze the determinants of the first minor M_1 and the second minor M_2. If both of these determinants are positive, then we say that Hessian matrix is positive definite at this point. The determinant of the first minor is

$$\det M_1 = H(1,1) \qquad (4.33)$$

which is positive by the definition of L''. We also have

$$\begin{aligned}
\det M_2 &= (1-F_1(\bar{q}_1(x)))^2(1-F_2(\bar{q}_2(x)))^2 \\
&\quad \times \int\int u_1^2 L''(x+u_1\bar{q}_1(x)+u_2\bar{q}_2(x))dT_1(u_1)dT_2(u_2) \\
&\quad \times \int\int u_2^2 L''(x+u_1\bar{q}_1(x)+u_2\bar{q}_2(x))dT_1(u_1)dT_2(u_2) \\
&\quad -(1-F_1(\bar{q}_1(x)))^2(1-F_2(\bar{q}_2(x)))^2 \\
&\quad \times [\int\int u_1u_2 L''(x+u_1\bar{q}_1(x)+u_2\bar{q}_2(x))dT_1(u_1)dT_2(u_2)]^2 \\
&\quad +(1-F_1(\bar{q}_1(x)))(1-F_2(\bar{q}_2(x)))^2 \\
&\quad \times \int\int u_1^2 L''(x+u_1\bar{q}_1(x)+u_2\bar{q}_2(x))dT_1(u_1)dT_2(u_2) \\
&\quad \times \int\int\int_0^{\bar{q}_1(x)} u_2^2 L''(x+u_1\bar{q}_1(x)+u_2 z)dF_2(z)dT_1(u_1)dT_2(u_2) \\
&\quad +(1-F_1(\bar{q}_1(x)))^2(1-F_2(\bar{q}_2(x))) \\
&\quad \times \int\int u_2^2 L''(x+u_1\bar{q}_1(x)+u_2\bar{q}_2(x))dT_1(u_1)dT_2(u_2) \\
&\quad \times \int\int\int_0^{\bar{q}_2(x)} u_1^2 L''(x+u_1 z+u_2\bar{q}_2(x))dF_1(z)dT_1(u_1)dT_2(u_2) \\
&\quad +(1-F_1(\bar{q}_1(x)))(1-F_2(\bar{q}_2(x))) \\
&\quad \times \int\int\int_0^{\bar{q}_1(x)} u_2^2 L''(x+u_1\bar{q}_1(x)+u_2 z)dF_2(z)dT_1(u_1)dT_2(u_2) \\
&\quad \times \int\int\int_0^{\bar{q}_2(x)} u_1^2 L''(x+u_1 z+u_2\bar{q}_2(x))dF_1(z)dT_1(u_1)dT_2(u_2). \quad (4.34)
\end{aligned}$$

At the first glance, (4.34) is very complex but it is observed that the last three of these five terms are positive by the definition of L''. Just for simplification purposes, we write the first two terms as

$$(1-F_1)^2(1-F_2)^2[\int\int u_1^2 L'' dT \times \int\int u_2^2 L'' dT - (\int\int u_1u_2 L'' dT)^2] \quad (4.35)$$

which is also positive by the Cauchy-Schwartz inequality. Therefore, $\det M_2$ is positive and the Hessian matrix is positive definite at the point $(q_1,q_2) = (\bar{q}_1(x), \bar{q}_2(x))$ and this minimizes the objective function.

Proposition 4. *If $\bar{q}_i(x) < 0$, then the optimal order policy is to order nothing from vendor i, so that $q_i(x) = 0$.*

Proof. Suppose without loss of generality that the solution of (4.28) and (4.29) gives $\bar{q}_1(x) < 0$. We know from the previous arguments that $TC(x, q_1, q_2(x))$ is unimodular with respect to q_1 given the initial inventory x and $q_2(x)$. This reveals that $\widetilde{TC}(x, q_1, q_2(x))$ decreases as q_1 decreases from positive values to the negative value $\bar{q}_1(x)$. However, we know that the optimum order quantity that will be released to the vendor 1 cannot take a negative value. So, the minimum feasible value for $TC(x, q_1, q_2(x))$ is attained at the boundary of the feasible region where $q_1(x) = 0$. The same arguments are also valid for $q_2(x)$.■

We now define the values $S_i = \inf\{x : \overline{q}_i(x) = 0\}$. According to this definition $\overline{q}_i(x)$, given as the solution of (4.28) and (4.29), is greater than zero for values of $x \in (-\infty, S_i)$. At the point $x = S_i$, $\overline{q}_i(S_i) = 0$. This is the critical order level for vendor i and provides a characterization of the optimum ordering policy.

Our computational analysis indicates that there is an inverse relation between c_i and S_i values. So, we have the conjecture that if the variable costs are ordered as $c_1 \geq c_2$, then the critical order levels are ordered as $S_1 \leq S_2$. Changing parameters while keeping $c_1 \geq c_2$ gives S_2 values which are always greater than S_1. In our computational results, the ordering policy comes out to be of the form

$$(q_1(x),\ q_2(x)) = \begin{cases} (q_1(x),\ q_2(x)), & x < S_1 \\ (0,\ q_2(x)), & S_1 \leq x < S_2 \\ (0,\ 0), & x \geq S_2. \end{cases} \tag{4.36}$$

The results show that, if we increase the initial inventory x from small values to larger values, $\overline{q}_1(x)$ and $\overline{q}_2(x)$ are both decreasing and $\overline{q}_1(x)$ first drops below zero. Considering Proposition 4, if $\overline{q}_1(x) < 0$, then in the optimal solution, order quantity $q_1(x)$ for the first vendor is zero. So for x values that are greater than S_1, we do not consider the first vendor in our calculation and regard the problem as in the single vendor case with only the second vendor, which is analyzed in Chapter 3 in detail. The details of the computational analysis is presented in Section 4.3.

4.2 The Case of Exponentially Distributed Demand

As in the single vendor case, we take exponential distribution for the distribution of periodic demand so that

$$D \sim \text{Exp}(\lambda). \tag{4.37}$$

The other parameters are unit holding cost h, unit shortage cost p, and unit variable costs c_1 and c_2 with $p > c_1 > c_2$. For the next step, we look at the optimality equations (4.28) and (4.29). Instead of analyzing these equations, we have a more compact approach that facilitate the creation of the optimality equations especially for the case of more than two vendors. This approach is driven by the simple structure of exponential density for the periodic demand as in the single vendor case. Focusing on (4.17) and considering that $L'(y) = (h+p)(1-e^{-\lambda y}) - p$ for $y \geq 0$, we have

$$E[U_1((h+p)(1-e^{-\lambda(x+U_1\overline{q}_1(x)+U_2(\overline{q}_2(x)\wedge A_2))}) - p)] = -c_1 E[U_1] \tag{4.38}$$

which leads to

$$E[U_1(1-e^{-\lambda(x+U_1\overline{q}_1(x)+U_2(\overline{q}_2(x)\wedge A_2))})] = \frac{p-c_1}{p+h}E[U_1]. \tag{4.39}$$

and so

$$E[U_1 e^{-\lambda(U_1\overline{q}_1(x)+U_2(\overline{q}_2(x)\wedge A_2)))}] = e^{\lambda x}\frac{h+c_1}{h+p}E[U_1]. \tag{4.40}$$

Since $U_1 e^{-\lambda U_1 \overline{q}_1(x)}$ and $U_2(\overline{q}_2(x) \wedge A_2))$ are independent random variables, we separate the left hand side and write

$$E[U_1 e^{-\lambda U_1\overline{q}_1(x)}]E[e^{-\lambda U_2(\overline{q}_2(x)\wedge A_2))}] = e^{\lambda x}\frac{h+c_1}{h+p}E[U_1]. \tag{4.41}$$

Considering the fact that the optimality equations are symmetric in $\overline{q}_i(x)$s, we can write (4.41) for $\overline{q}_2(x)$ as

$$E[U_2 e^{-\lambda U_2\overline{q}_2(x)}]E[e^{-\lambda U_1(\overline{q}_1(x)\wedge A_1))}] = e^{\lambda x}\frac{h+c_2}{h+p}E[U_2]. \tag{4.42}$$

Now, we have split the two equations into 6 parts which are

- $E[U_1 e^{-\lambda U_1 \bar{q}_1(x)}]$,
- $E[U_2 e^{-\lambda U_2 \bar{q}_2(x)}]$,
- $E[e^{-\lambda U_1 (\bar{q}_1(x) \wedge A_1))}]$,
- $E[e^{-\lambda U_2 (\bar{q}_2(x) \wedge A_2))}]$,
- $e^{\lambda x} \frac{h+c_1}{h+p} E[U_1]$, and
- $e^{\lambda x} \frac{h+c_2}{h+p} E[U_2]$.

Each of these pieces are easier to compute one by one, especially the last two. So, the solution of the problem is mainly obtained by computing the first 4 functions and then building the equations (4.41) and (4.42). In short, the exponential distribution assigned to periodic demand does not simplify solving the problem but it enables the user to build the optimality equations in an easier way.

Model with Exponential Vendor Capacity and Uniform Proportion

In order to obtain explicit functions here, we assume that vendor capacities are represented by exponentially distributed random variables and stochastic proportions are uniformly distributed random variables. Therefore, we have

$$A_1 \sim \text{Exp}(\mu_1) \tag{4.43}$$

$$A_2 \sim \text{Exp}(\mu_2). \tag{4.44}$$

and

$$U_1 \sim \text{Uniform}(a_1,\ b_1) \tag{4.45}$$

$$U_2 \sim \text{Uniform}(a_2,\ b_2). \tag{4.46}$$

Including the parameters, we have the total expected cost function as

$$\begin{aligned}
TC(x,q_1,q_2) \;=\; & \int_{a_1}^{b_1}\int_{a_2}^{b_2} \frac{1}{(b_2-a_2)(b_1-a_1)}[(c_1u_1q_1+c_2u_2q_2 \\
& \qquad + L(x+u_1q_1+u_2q_2))e^{-\mu_1q_1}e^{-\mu_2q_2} \\
& + \int_0^{q_2}(c_1u_1q_1+c_2u_2z+L(x+u_1q_1+u_2z))e^{-\mu_1q_1}\mu_2e^{-\mu_2z}dz \\
& + \int_0^{q_1}(c_1u_1z+c_2u_2q_2+L(x+u_1z+u_2q_1))e^{-\mu_2q_2}\mu_1e^{-\mu_1z}dz \\
& + \int_0^{q_1}\int_0^{q_2}(c_1u_1z_1+c_2u_2z_2+L(x+u_1z_1+u_2z_2)) \\
& \times\mu_1e^{-\mu_1z_1}\mu_2e^{-\mu_2z_2}dz_1dz_2]du_1du_2. \qquad (4.47)
\end{aligned}$$

The first item in the list above is computed as

$$E[U_1e^{-\lambda U_1\bar{q}_1(x)}]=\frac{1}{(b_1-a_1)\lambda^2\bar{q}_1(x)^2}[e^{-\lambda\bar{q}_1(x)a_1}(1+\lambda\bar{q}_1(x)a_1)-e^{-\lambda\bar{q}_1(x)b_1}(1+\lambda\bar{q}_1(x)b_1)] \qquad (4.48)$$

and the third item is

$$\begin{aligned}
E[e^{-\lambda U_1(\bar{q}_1(x)\wedge A_1))}] \;=\; & \frac{\mu_1}{\lambda(a_1-b_1)}[E_i(1,\bar{q}_1(x)(\mu_1+\lambda a_1)) \\
& -E_i(1,\bar{q}_1(x)(\mu_1+\lambda b_1))+\ln\left(\frac{\mu_1+\lambda a_1}{\mu_1+\lambda b_1}\right)] \\
& +\frac{1}{\lambda\bar{q}_1(x)(a_1-b_1)}(e^{-\bar{q}_1(x)(\mu_1+\lambda b_1)}-e^{-\bar{q}_1(x)(\mu_1+\lambda a_1)}) \qquad (4.49)
\end{aligned}$$

where $E_i(1,x)$ is the exponential integral defined as

$$E_i(1,x)=\int_{-\infty}^{x}\frac{1}{t}e^t. \qquad (4.50)$$

It is clear that the second and the fourth items above are in the same format as the first and the third items above respectively.

4.3 Numerical Illustration and Results

As in the single vendor case, this section contains the outputs of sample runs that are made after assigning numerical values to the parameters. We suppose that the variable cost c_1 for the first vendor is \$2.5/unit while the c_2 for the second vendor is \$2/unit. In fact we assume that the second vendor in this section is exactly the same as the one in the single vendor case. So, U_2 is assumed to be uniformly distributed on (0.5, 0.8) and the capacity $A_2 \sim \text{Exp}(\mu_2)$ where μ_2 is equal to 0.25 resulting in a mean capacity of 4. Besides, U_1 is assumed to be uniformly distributed on (0.8, 1.0) and $A_1 \sim \text{Exp}(\mu_1)$ where μ_1 is 0.10 with a mean of 10. It is observed that the first vendor is more reliable in the sense that mean values for the stochastic proportion and random capacity are higher than those for the second vendor. The costs h and p remain unchanged, and periodic demand is again a random variable that is exponentially distributed with the parameter $\lambda = 0.1$.

In the light of our conjecture on the order of S_1 and S_2, the critical order level S_1 can be found by setting $x = S_1$ in (4.41) and (4.42). Then, we obtain

$$E[U_1 e^{-\lambda U_1 \overline{q}_1(S_1)}]E[e^{-\lambda U_2(\overline{q}_2(S_1)\wedge A_2))}] = e^{\lambda S_1}\frac{h+c_1}{h+p}E[U_1] \tag{4.51}$$

$$E[U_2 e^{-\lambda U_2 \overline{q}_2(S_1)}]E[e^{-\lambda U_1(\overline{q}_1(S_1)\wedge A_1))}] = e^{\lambda S_1}\frac{h+c_2}{h+p}E[U_2]. \tag{4.52}$$

Considering the fact that $\overline{q}_1(S_1) = 0$, we have

$$E[e^{-\lambda U_2(\overline{q}_2(S_1)\wedge A_2))}] = e^{\lambda S_1}\frac{h+c_1}{h+p} \tag{4.53}$$

$$E[U_2 e^{-\lambda U_2 \overline{q}_2(S_1)}] = e^{\lambda S_1}\frac{h+c_2}{h+p}E[U_2] \tag{4.54}$$

with unknowns S_1 and $\overline{q}_2(S_1)$. Including the numerical values, the solution gives $S_1 = 4.1902$ and $\overline{q}_2(S_1) = 5.6021$. When $x > S_1$, we treat the second vendor as the single vendor and obtain $S_2 = 7.88$. Table (4.1) and Figure (4.2) gives the optimum order quantities for each vendor for discrete x values from 0 to 10. In the last row of this table, we observe that optimum order quantity for the first vendor hits the zero level just after the point $x = 4.15$, which is very close to $S_1 = 4.1902$. The order quantity $q_2(x)$ vanishes just after the point $x = 7.8$, which is close to $S_2 = 7.88$. So, the points where the optimum order quantities hit zero do not contradict with the critical values we found before computing the optimum order quantities.

Table 4.1: Optimum Order Quantities for 2 Distinct Vendors

Initial Inventory (x)	$q_1(x)$	$q_2(x)$	$x + q_1(x) + q_2(x)$
0	4.45	7.18	11.64
1	3.41	6.56	10.97
2	2.36	6.08	10.44
3	1.30	5.75	10.05
4	0.21	5.61	9.82
5	0.00	4.37	9.37
6	0.00	2.85	8.85
7	0.00	1.34	8.34
8	0.00	0.00	0.00
9	0.00	0.00	0.00
10	0.00	0.00	0.00
4.15	0.04	5.60	

Solutions for the unconstrained problem $\bar{q}_1(x)$ and $\bar{q}_2(x)$ both take positive values up to S_1 so that optimum order sizes are equal to these values. After S_1, $\bar{q}_1(x)$ takes negative values so we solve the single vendor problem after that point, considering only the second vendor. In the fourth column of Table (4.1), the sum of the initial inventory x and orders sizes $q_1(x)$ and $q_2(x)$ is given. This sum exceeds the largest critical order level S_2, which shows that the resulting policy is again not an order-up-to type ordering policy.

Table (4.2) gives the results obtained by directly minimizing the total cost with the constrained $q_i \geq 0$. In order to achieve this, we used constrained optimization functions of Matlab 6.5. It is clear that the results are extremely similar to those in Table (4.1), which confirms our methodology to find the optimum order quantities. There are tiny differences which are caused by utilization of different routines of Matlab having different termination criteria. Table (4.2) also gives the optimum total expected cost TC that is resulted by the optimum order quantities. Figure (4.3) provides a graph of this cost function with respect to the initial inventory x. It is observed that TC behaves as a convex function in x in the relevant range.

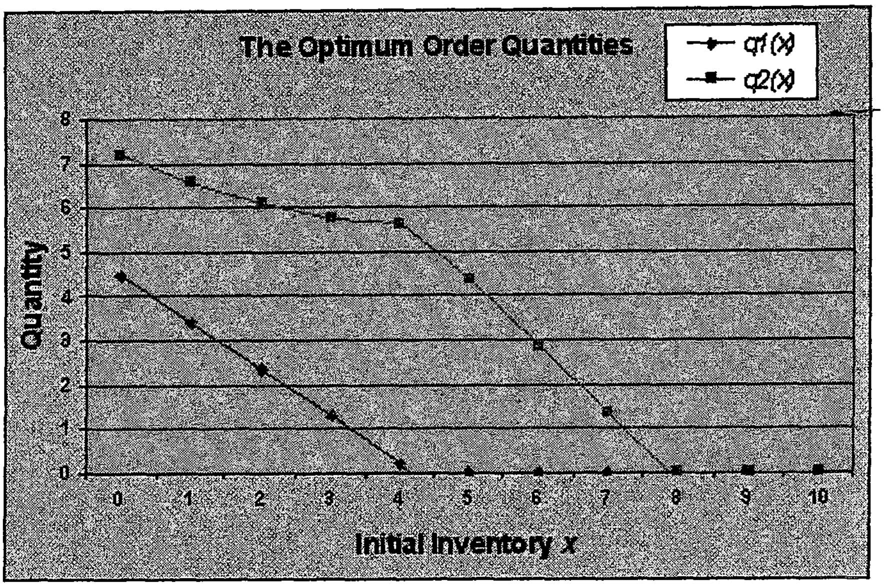

Figure 4.2: Optimum Order Quantities for 2 Distinct Vendors

4.4 *Multiple Vendors Case*

In this section, it is assumed that there are more than two vendors that make business with the decision maker. The approach used in the previous chapters is extended for the general n vendors case and, in turn, all of the assumptions related to the vendors and their processes are kept. The vendors analyzed in this section are distinct from each other so that parametric features of each vendor are different. In order to better differentiate among the vendors, each vendor has a variable cost c_i, where $c_i \neq c_j$ for $i, j = 1, 2, 3, ..., n$ and $i \neq j$. We also keep the assumption that $p > c_i$ for all i. Besides, the capacity distributions of each vendor are represented by F_i, where F_i and F_j are either different distributions or same distributions with different parameters for $i \neq j$. Since the production processes and transportation medium used by each vendor are different from each other, the previous argument is also valid for the distributions of stochastic proportions applied to each vendor

Table 4.2: Optimum Order Quantities for 2 Distinct Vendors Obtained by Constrained Optimization

Initial Inventory (x)	$q_1(x)$	$q_2(x)$	TC
0	4.45	7.19	42.86
1	3.41	6.57	40.14
2	2.36	6.08	37.53
3	1.29	5.75	34.98
4	0.21	5.61	32.47
5	0.00	4.37	30.04
6	0.00	2.84	27.81
7	0.00	1.31	25.72
8	0.00	0.00	23.71
9	0.00	0.00	21.84
10	0.00	0.00	20.12

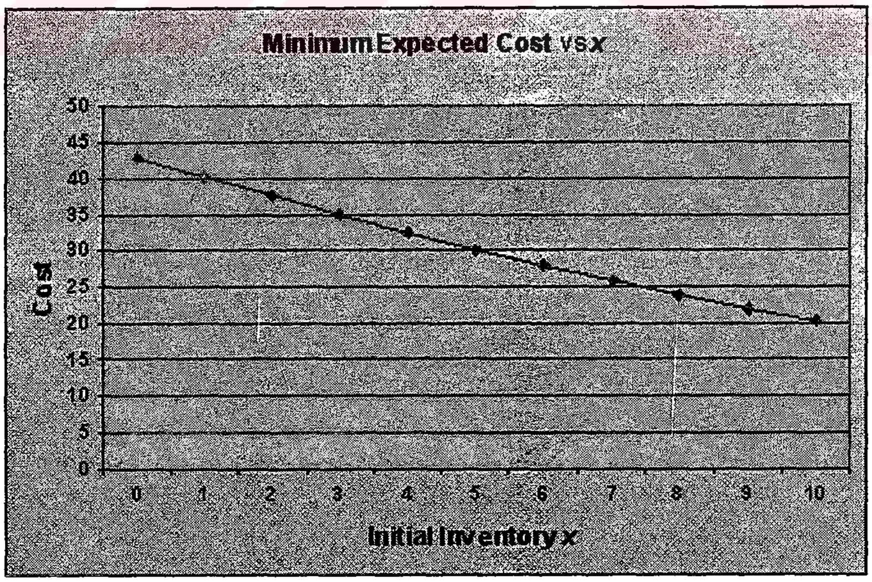

Figure 4.3: Minimum Expected Total Cost Values Corresponding to Different Inventory Levels

so that T_i and T_j are either different distributions or same distributions with different parameters for $i \neq j$.

The problem is to derive the optimum order quantity q_i for vendor i, given that the initial order level is x so that the expected total cost is minimized in the single period setting. Similar to the two vendors case,

$$F_i(z) = P(A_i \leq z) \tag{4.55}$$

and

$$T_i(u) = P(U_i \leq u) \tag{4.56}$$

where $0 \leq A_i \leq +\infty$ and $0 \leq U_i \leq 1$ for all i. In n vendors case, the decision maker releases an order for each vendor and then receives the actual yield Y_{q_i} from each vendor. Using the same notation for the processed amount, we obtain

$$\begin{aligned} P_{q_i} &= \min\{q_i, A_i\} && (4.57) \\ Y_{q_i} &= U_i \min\{q_i, A_i\} = U_i P_{q_i}. && (4.58) \end{aligned}$$

for $i = 1, 2, 3, ..., n$. Then the total random yield received from n vendors is the sum

$$\text{Total random yield} = Y_{q_1} + Y_{q_2} + Y_{q_3} + ... + Y_{q_n} = \sum_{i=1}^{n} Y_{q_i}. \tag{4.59}$$

Clearly, we have on-hand inventory $y = x + Y_{q_1} + Y_{q_2} + Y_{q_3} + ... + Y_{q_n}$ to meet the random demand. Then, the total inventory holding and shortage cost in terms of actual yields is obtained as

$$\begin{aligned} L(x + \sum_{i=1}^{n} Y_{q_i}) &= h \int_0^{x+\sum_{i=1}^{n} Y_{q_i}} (x + \sum_{i=1}^{n} Y_{q_i} - w) dG(w) \\ &\quad + p \int_{x+\sum_{i=1}^{n} Y_{q_i}}^{\infty} (w - (x + \sum_{i=1}^{n} Y_{q_i})) dG(w). \quad (4.60) \end{aligned}$$

Total expected cost function TC in terms of beginning inventory x and quantities q_i is obtained as in the two vendors case, so that

$$\begin{aligned} TC(x, q_1, q_2, ..., q_n) &= E[c_1 Y_{q_1} + c_2 Y_{q_2} + ... + c_n Y_{q_n} \\ &\qquad + L(x + Y_{q_1} + Y_{q_2} + ... + Y_{q_n})] \quad (4.61) \\ &= E[\sum_{i=1}^{n} c_i Y_{q_i} + L(x + \sum_{i=1}^{n} Y_{q_i})] \quad (4.62) \\ &= E[\sum_{i=1}^{n} c_i U_i(q_i \wedge A_i) + L(x + \sum_{i=1}^{n} U_i(q_i \wedge A_i))] \quad (4.63) \end{aligned}$$

Let $v_n(x)$ denote the optimal cost function with n vendors, given an initial inventory of x. Then, we can obtain $v_n(x)$ by minimizing $TC(x, q_1, q_2, ..., q_n)$ in (4.61) for $q_i \geq 0$ so that

$$\begin{aligned} v_n(x) &= \min_{q_i \geq 0} TC(x, q_1, q_2, ..., q_n) \quad (4.64) \\ &= \min_{q_i \geq 0} E[\sum_{i=1}^{n} c_i Y_{q_i} + L(x + \sum_{i=1}^{n} Y_{q_i})] \quad (4.65) \\ &= \min_{q_i \geq 0} \{ \int \int ... \int [(\sum_{i=1}^{n} c_i u_i q_i + L(x + \sum_{i=1}^{n} u_i q_i)) \prod_{i=1}^{n} (1 - F_i(q_i)) \\ &\quad + \sum_{i=1}^{n} [\int_0^{q_i} (c_i u_i z_i + \sum_{j=1, j \neq i}^{n} c_j u_j q_j \\ &\qquad + L(x + u_i z_i + \sum_{j=1, j \neq i}^{n} u_j q_j)) dF_i(z_i) \prod_{j=1, j \neq i}^{n} (1 - F_j(q_j))] \\ &\quad + \sum_{i=1}^{n} \sum_{j=1, j \neq i}^{n} [\int_0^{q_i} \int_0^{q_j} (c_i u_i z_i + c_j u_j z_j + \sum_{k=1, k \neq i,j}^{n} c_k u_k q_k \\ &\qquad + L(x + u_i z_i + u_j z_j + \sum_{k=1, k \neq i,j}^{n} u_k q_k)) \; dF_i(z_i) dF_j(z_j) \prod_{k=1, k \neq i,j}^{n} (1 - F_k(q_k))] \\ &\quad ... \\ &\quad + \int_0^{q_1} \int_0^{q_2} ... \int_0^{q_n} (\sum_{i=1}^{n} c_i u_i z_i + L(x + \sum_{i=1}^{n} u_i z_i)) dF_1(z_1) ... dF_n(z_n)] \\ &\qquad \times dT_1(u_1) ... dT_n(u_n) \} \quad (4.66) \end{aligned}$$

After some algebra, the partial derivative of $TC(x, q_1, q_2, ..., q_n)$ with respect to q_i is obtained as

$$\frac{\partial TC(x, q_1, q_2, ..., q_n)}{\partial q_i} = E[c_i U_i + U_i L'(x + U_i q_i + \sum_{j=1, j\neq i}^{n} U_j(q_j \wedge A_j))](1 - F_i(q_i)) \quad (4.67)$$

$$= (c_i E[U_i] + E[U_i L'(x + U_i q_i + \sum_{j=1, j\neq i}^{n} U_j(q_j \wedge A_j))])(1 - F_i(q_i)). \quad (4.68)$$

We can observe from its definition that $L'(x + U_i q_i + \sum_{j=1, j\neq i}^{n} U_j(q_j \wedge A_j))$ is a continuous and increasing function of q_i. Then,

$$\lim_{q_i \to +\infty} L'(x + U_i q_i + \sum_{j=1, j\neq i}^{n} U_j(q_j \wedge A_j)) = h > 0 \quad (4.69)$$

$$\lim_{q_i \to -\infty} L'(x + U_i q_i + \sum_{j=1, j\neq i}^{n} U_j(q_j \wedge A_j)) = -p < 0 \quad (4.70)$$

for all $U_i, U_j > 0$. It is clear by our assumption that $(1 - F_i(q_i))$ is always positive for all q_i values. Then, we should focus on the first factor of (4.68) since only it can make this first partial derivative zero. From (4.69) and (4.70) we see that, for any given finite q_j values with $j \neq i$, this second term is increasing in q_i and takes values between $-pE[U_i]$ and $hE[U_i]$. Therefore, for any given finite q_j values with $j \neq i$, there is a unique value of $q_i = \bar{q}_i(x)$ so that the second term takes the value $-c_i E[U_i]$ and (4.68) becomes zero since $p > c_i$. Given the initial inventory x and order sizes q_j for the other vendors, $\bar{q}_i(x)$ is obtained by solving the equation

$$c_i E[U_i] + E[U_i L'(x + U_i \bar{q}_i(x) + \sum_{j=1, j\neq i}^{n} U_j(q_j \wedge A_j))] = 0. \quad (4.71)$$

Then, it is clear that (4.68) is negative for $q_i < \bar{q}_i(x)$, and positive for $q_i > \bar{q}_i(x)$ for fixed q_j values with $j \neq i$. Second partial derivative of $TC(x, q_1, q_2, ..., q_n)$ with respect to q_i is obtained as

$$\frac{\partial^2 TC(x,q_1,q_2,...,q_i,...,q_n)}{\partial q_i^2} = E[U_i^2 L''(x+U_iq_i+\sum_{j=1,j\neq i}^{n} U_j(q_j\wedge A_j))](1-F_i(q_i)) \\ -f(q_i)(c_iE[U_i]+E[U_iL'(x+U_iq_i+\sum_{j=1,j\neq i}^{n} U_j(q_j\wedge A_j))]). \tag{4.72}$$

Arguments in the two vendors case are also valid for (4.72). The first term is positive since L'' and $(1-F_i(q_i))$ take positive values for all q_i. We also know from (4.71) that left hand side of (4.71) is negative for q_i values smaller than $\bar{q}_i(x)$. Therefore (4.72) is positive for $q_i \in (-\infty, \bar{q}_i(x))$ and any finite q_j values with $j \neq i$, which means that $TC(x,q_1,q_2,...,q_n)$ is convex and decreasing in q_i in this interval. At the point where $q_i = \bar{q}_i(x)$, second term of (4.72) is zero by (4.71) and so (4.72) is positive at this point. So we conclude that it is a minimal point. In the region $(\bar{q}_i(x), +\infty)$, TC is increases in q_i since for given x and q_j values with $j \neq i$, (4.68) is positive for q_i values that are greater than $\bar{q}_i(x)$.

In other words, TC is convex and decreasing in q_i on $(-\infty, \bar{q}_i(x))$ and increasing on $(\bar{q}_i(x), +\infty)$, which states that for fixed x and q_j values with $j \neq i$, $TC(x,q_1,q_2,...,q_n)$ is unimodular in q_i. This proves that given any finite initial inventory level x and q_j values with $j \neq i$, $TC(x,q_1,q_2,...,q_n)$ attains its global minimum at $q_i = \bar{q}_i(x)$. Same arguments hold for q_j with $j \neq i$ since the cost function is symmetric in all q's. Therefore, given any x, $TC(x,q_1,q_2,...,q_n)$ is unimodular in q_i for all i. This outcome reveals that a *nonzero* solution set $(q_1(x), q_2(x), ..., q_n(x))$ of the following equation system

$$\begin{aligned} \frac{\partial TC(x,q_1,q_2,...,q_n)}{\partial q_1} &= 0 \\ \frac{\partial TC(x,q_1,q_2,...,q_n)}{\partial q_2} &= 0 \\ &\dots \\ \frac{\partial TC(x,q_1,q_2,...,q_n)}{\partial q_n} &= 0 \end{aligned} \tag{4.73}$$

gives us the optimum order quantities. In more explicit terms, we should investigate the solution set of the system of equations

$$-c_iE[U_i] = \int\int ... \int E[U_iL'(x + U_i\bar{q}_i(x) + \sum_{j=1, j\neq i}^{n} U_j(\bar{q}_j(x) \wedge A_j))]dT_1(U_1)...dT_n(U_n) \tag{4.74}$$

for $i = 1, 2, ..., n$.

Proposition 5. *If $\bar{q}_i(x) < 0$, then the optimum order policy is to order nothing from vendor i, so that $q_i(x) = 0$.*

Proof. The same arguments in the two vendors case are valid here. Suppose that in the solution set of (4.74), we obtain $\bar{q}_i(x) < 0$. We know from the previous arguments that $TC(x, q_1(x), q_2(x), ..., q_i, ..., q_n(x))$ is unimodular with respect to q_i given the initial inventory x and $q_j(x)$ with $j \neq i$. This reveals that $TC(x, q_1(x), q_2(x), ..., q_i, ..., q_n(x))$ decreases as q_i decreases from positive values to the negative value $\bar{q}_i(x)$. Knowing that optimum order size cannot take a negative value, the minimum value for $TC(x, q_1(x), q_2(x), ..., q_i, ..., q_n(x))$ is attained at the boundary of the feasible region where $q_i(x) = 0$. ■

Remembering the definition for the critical order level $S_i = \inf\{x : \overline{q}_i(x) = 0\}$, it is clear that $\overline{q}_i(x)$, solution of (4.74), is greater than zero for values of $x \in (-\infty, S_i)$. At the point $x = S_i$, $\overline{q}_i(S_i) = 0$. Critical order levels are the main parameters in the expression of the optimum ordering policy.

As in the two vendors case, analyzing the results of our computations, we have the conjecture that if $c_i \geq c_j$, then $S_i \leq S_j$. As we increase the initial inventory from a small value to larger values, $\overline{q}_i(x)$s are all decreasing and the vendor having the largest unit variable cost is the first one for which the order quantity becomes zero. If the vendors are ordered such that $c_1 > c_2 > c_3 > ... > c_n$, our computational results indicate an optimal ordering policy which is of the form

$$(q_1(x), q_2(x), q_3(x), ..., q_n(x)) = \begin{cases} (q_1(x), q_2(x), q_3(x), ..., q_n(x)), & x < S_1 \\ (0, q_2(x), q_3(x), ..., q_n(x)), & S_1 \leq x < S_2 \\ (0, 0, q_3(x), ..., q_n(x)), & S_2 \leq x < S_3 \\ ... & \\ (0, 0, 0, ..., q_n(x)), & S_{n-1} \leq x < S_n \\ (0, 0, 0, ..., 0), & x \geq S_n. \end{cases} \tag{4.75}$$

During the computation process with k vendors where $1 < k \leq n$, any vendor i is removed from the calculation just after $\overline{q}_i(x)$ hits the zero level by Proposition 5. So, just after the point S_i where $\overline{q}_i(x)$ hits zero, the first order conditions are solved for $k-1$ vendors and this process goes on in this way until the last $\overline{q}(x)$ vanishes. The results show that the last vendor is the cheapest one. Making computations by increasing x value eliminates the vendors one by one and gives the idea of which vendors should be considered in the decision making process at a specific x value.

4.5 *The Case of Exponentially Distributed Demand*

In this section we will see once again how the exponentially distributed periodic demand facilitates the construction of the optimality equations since it has a separable structure. We again take

$$D \sim \text{Exp}(\lambda). \tag{4.76}$$

Then, optimality condition (4.74) is written as

$$E[U_i L'((h+p)(1 - e^{-\lambda(x + U_i \overline{q}_i(x) + \sum_{j=1, j\neq i}^{n} U_j(\overline{q}_j(x) \wedge A_j))}) - p)] = -c_i E[U_i] \tag{4.77}$$

for $i = 1, 2, ..., n$. In the same way as in the previous cases, we obtain

$$E[U_i e^{-\lambda(U_i \overline{q}_i(x) + \sum_{j=1, j\neq i}^{n} U_j(\overline{q}_j(x) \wedge A_j))}] = e^{\lambda x} \frac{h + c_i}{h + p} E[U_i]. \tag{4.78}$$

Since $U_i e^{-\lambda U_i \bar{q}_i(x)}$ and $e^{U_j(\bar{q}_j(x)\wedge A_j))}$ are all random variables that are independent from each other for $j = 1, 2, ..., n$ and $j \neq i$, we can separate the left hand side and take the advantage of exponential distribution to do that. Then, we obtain the set of optimality equations

$$E[U_i e^{-\lambda U_i \bar{q}_i(x)}] \prod_{j=1, j\neq i}^{n} E[e^{-\lambda U_j(\bar{q}_j(x)\wedge A_j))}] = e^{\lambda x} \frac{h + c_i}{h + p} E[U_i]. \tag{4.79}$$

for $i = 1, 2, ..., n$. Therefore, we have split the set of optimality equations into $3n$ pieces. The n of these are composed of the right hand side of (4.79) for all i, which are extremely easy to compute. Then the optimality set is constructed mainly by computing $2n$ functions which are $E[U_i e^{-\lambda U_i \bar{q}_i(x)}]$ and $E[e^{-\lambda U_i(\bar{q}_i(x)\wedge A_i))}]$ for $i = 1, 2, ..., n$. Upon computing these functions, (4.79) is constructed for all i and the resulting system is solved to reach to $\bar{q}_i(x)$ values.

Model with Exponential Vendor Capacity and Uniform Proportion

For illustration purposes, we assign as in the two vendors

$$A_i \sim \text{Exp}(\mu_i) \tag{4.80}$$

and

$$U_i \sim \text{Uniform}(a_i,\ b_i) \tag{4.81}$$

for $i = 1, 2, ..., n$.

The parameters are the same, i.e., unit holding cost h, unit shortage cost p, and unit variable costs c_i for $i = 1, 2, ..., n$. Including the parameters, we have the total expected cost function as

$$
\begin{aligned}
TC(x, q_1, q_2, ..., q_n) \; = \; & \int_{a_1}^{b_1}\int_{a_2}^{b_2} ... \int_{a_n}^{b_n} \prod_{i=1}^{n}\left(\frac{1}{(b_i - a_i)}\right)[(\sum_{i=1}^{n} c_i u_i q_i + L(x + \sum_{i=1}^{n} u_i q_i))e^{-\sum_{i=1}^{n}\mu_i q_i} \\
& + \sum_{i=1}^{n}[(\int_0^{q_i}(c_i u_i z_i + \sum_{j=1, j\neq i}^{n} c_j u_j q_j \\
& \quad + L(x + u_i z_i + \sum_{j=1, j\neq i}^{n} u_j q_j))\mu_i e^{-\mu_i z_i} dz_i)e^{-\sum_{j=1, j\neq i}^{n}\mu_j q_j}] \\
& + \sum_{i=1}^{n}\sum_{j=1, j\neq i}^{n}[(\int_0^{q_i}\int_0^{q_j}(c_i u_i z_i + c_j u_j z_j + \sum_{k=1, k\neq i,j}^{n} c_k u_k q_k \\
& \quad + L(x + u_i z_i + u_j z_j + \sum_{k=1, k\neq i,j}^{n} u_k q_k)) \\
& \times \mu_i e^{-\mu_i z_i}\mu_j e^{-\mu_j z_j} dz_i dz_j)e^{-\sum_{k=1, k\neq j,i}^{n}\mu_{k} q_k}] \\
& ... \\
& + \int_0^{q_1}\int_0^{q_2} ... \int_0^{q_n}(\sum_{i=1}^{n} c_i u_i z_i + L(x + \sum_{i=1}^{n} u_i z_i)) \\
& \times \prod_{i=1}^{n}(\mu_i e^{-\mu_i z_i}) dz_1 ... dz_n] du_1 ... du_n. \qquad (4.82)
\end{aligned}
$$

Remembering the two vendor case, the explicit form of the terms in (4.79) are

$$
E[U_i e^{-\lambda U_i \bar{q}_i(x)}] = \frac{1}{(b_i - a_i)\lambda^2 \bar{q}_i(x)^2}[e^{-\lambda \bar{q}_i(x) a_i}(1 + \lambda \bar{q}_i(x) a_i) - e^{-\lambda \bar{q}_i(x) b_i}(1 + \lambda \bar{q}_i(x) b_i)] \quad (4.83)
$$

and

$$
\begin{aligned}
E[e^{-\lambda U_i(\bar{q}_i(x)\wedge A_i))}] \; = \; & \frac{\mu_i}{\lambda(a_i - b_i)}[E_i(1, \bar{q}_i(x)(\mu_i + \lambda a_i)) \\
& - E_i(1, \bar{q}_i(x)(\mu_i + \lambda b_i)) + \ln\left(\frac{\mu_i + \lambda a_i}{\mu_i + \lambda b_i}\right)] \\
& + \frac{1}{\lambda \bar{q}_i(x)(a_i - b_i)}(e^{-\bar{q}_i(x)(\mu_i + \lambda b_i)} - e^{-\bar{q}_i(x)(\mu_i + \lambda a_i)}). \qquad (4.84)
\end{aligned}
$$

4.6 Numerical Illustration and Results

In this section we analyze a special case with $n = 3$ vendors. The first two vendors remain unchanged as in Section 4.3 where we introduce the third vendor with the variable cost $c_3 = \$2.25/\text{unit}$. Also $A_3 \sim \text{Exp}(\mu_3)$ where $\mu_3 = 0.20$ and $U_3 \sim$ Uni $(0.3,\ 0.5)$. All of the remaining distributions and parameters are kept unchanged.

We have $c_1 > c_3 > c_2$. Remembering our conjecture on the ordering of critical order levels, we can find S_1 and S_3 by solving the first order conditions obtained by setting $x = S_1$ (with 3 vendors) and $x = S_3$ (with only vendors 2 and 3). We have S_2 unchanged and equals to 7.88 since it is still the cheapest vendor. Therefore, S_1 is in the solution set of

$$E[e^{-\lambda U_2(\bar{q}_2(S_1)\wedge A_2))}]E[e^{-\lambda U_3(\bar{q}_3(S_1)\wedge A_3))}] = e^{\lambda S_1}\frac{h+c_1}{h+p} \quad (4.85)$$

$$E[U_2 e^{-\lambda U_2 \bar{q}_2(S_1)}]E[e^{-\lambda U_3(\bar{q}_3(S_1)\wedge A_3))}] = e^{\lambda S_1}\frac{h+c_2}{h+p}E[U_2] \quad (4.86)$$

$$E[U_3 e^{-\lambda U_3 \bar{q}_3(S_1)}]E[e^{-\lambda U_2(\bar{q}_2(S_1)\wedge A_2))}] = e^{\lambda S_1}\frac{h+c_3}{h+p}E[U_3] \quad (4.87)$$

and S_3 is found by solving

$$E[U_2 e^{-\lambda U_2 \bar{q}_2(S_3)}] = e^{\lambda S_3}\frac{h+c_2}{h+p}E[U_2] \quad (4.88)$$

$$E[e^{-\lambda U_2(\bar{q}_2(S_3)\wedge A_2))}] = e^{\lambda S_3}\frac{h+c_3}{h+p}. \quad (4.89)$$

The unknowns in (4.85)-(4.87) are $S_1, \bar{q}_2(S_1)$, and $\bar{q}_3(S_1)$ where the unknowns of (4.88) and (4.89) are S_3 and $\bar{q}_2(S_3)$. These systems give $S_1 = 2.9209$ and $S_3 = 5.3809$. We also obtain $\bar{q}_2(S_1) = 5.6021$, $\bar{q}_3(S_1) = 5.2503$, and $\bar{q}_2(S_3) = 3.7926$. The last 2 rows in Table (4.3) tell us that S_1 is located just after the point 2.90. Besides, we understand that S_3 is a little bit higher than 5.35. It is observed that the ranges for the critical values in Table (4.3) include the exact values and this does not cause any contradiction. The same is true for $\bar{q}_2(S_1), \bar{q}_2(S_3)$, and $\bar{q}_3(S_1)$. As in the two vendors case, the points where the optimum order quantities hit zero do not contradict with the critical values we found before computing the optimum order quantities.

Table 4.3: Optimum Order Quantities for 3 Distinct Vendors

Initial Inventory (x)	$q_1(x)$	$q_2(x)$	$q_3(x)$	$x + q_1(x) + q_2(x) + q_3(x)$
0	2.97	6.34	6.60	15.91
1	1.99	5.95	5.89	14.83
2	0.98	5.69	5.42	14.09
3	0.00	5.51	5.08	13.60
4	0.00	4.53	3.01	11.54
5	0.00	3.88	0.89	9.76
6	0.00	2.85	0.00	8.85
7	0.00	1.34	0.00	8.34
8	0.00	0.00	0.00	0.00
9	0.00	0.00	0.00	0.00
10	0.00	0.00	0.00	0.00
2.90	0.0231	5.6023	5.2506	—
5.35	0	3.7944	0.0749	—

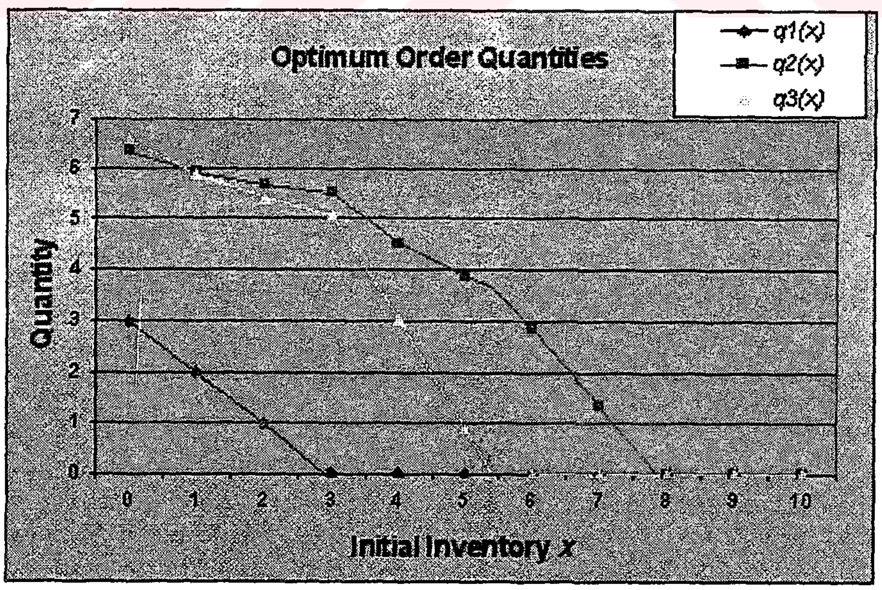

Figure 4.4: Optimum Order Quantities for 3 Distinct Vendors

When we look at the last column of Table (4.3), we realize that the resulting policy is not an order-up-to type since the total amount exceeds S_2. As in the other cases, the difference is higher for lower inventory values. Figure (4.4) gives the sketch of optimum order functions with respect to the initial inventory x. The introduction of the third vendor does not change S_2 but causes the first vendor to have a smaller critical level when compared to the two vendor case. This is driven by the cheaper position of the new vendor. Another outcome that is observed in Figure (4.4) is that the smallest valued function has a more smooth structure. This is observed especially for $\bar{q}_1(x)$. However after S_1, $\bar{q}_3(x)$ becomes the lower valued function and gets smoother through the points where $x = 3, 4$, and 5 when compared to $\bar{q}_2(x)$.

Table (4.4) gives the results obtained by directly minimizing the total cost with the constrained $q_i \geq 0$ again using the constrained optimization functions of Matlab 6.5. We obtained results that are extremely similar to those in Table (4.3) confirming our methodology to find the optimum order quantities. Several tiny differences are caused by utilization of different routines of Matlab as mentioned in the two vendors case. Table (4.4) also gives the optimum total expected cost TC which is sketched in Figure (4.5) with respect to x. As in the two vendors case, it is observed that TC behaves like a convex function in x in the relevant range.

Table (4.5) tabulates the optimum order quantities of these three vendors with only the modification that all of them has the same unit variable cost so that $c_1 = c_2 = c_3 = \$2$ /unit. The aim of creating this table is to support our conjecture about the vital dependency of the critical order levels on unit variable costs. The result is as we expected so that all of them have the same critical order level. In fact, they all equal 7.88, the critical order level of the second vendor having originally the unit variable cost of \$2/unit. Figure (4.6) gives the graphical illustration for this case.

Table 4.4: Optimum Order Quantities for 3 Distinct Vendors Obtained by Constrained Optimization

Initial Inventory (x)	$q_1(x)$	$q_2(x)$	$q_3(x)$	TC
0	2.97	6.34	6.60	42.36
1	1.99	5.95	5.89	39.77
2	0.98	5.69	5.41	37.74
3	0.00	4.49	5.03	34.74
4	0.00	4.02	3.01	32.33
5	0.00	2.90	0.81	30.03
6	0.00	1.30	0.00	27.82
7	0.00	0.00	0.00	25.72
8	0.00	0.00	0.00	23.71
9	0.00	0.00	0.00	21.89
10	0.00	0.00	0.00	20.25

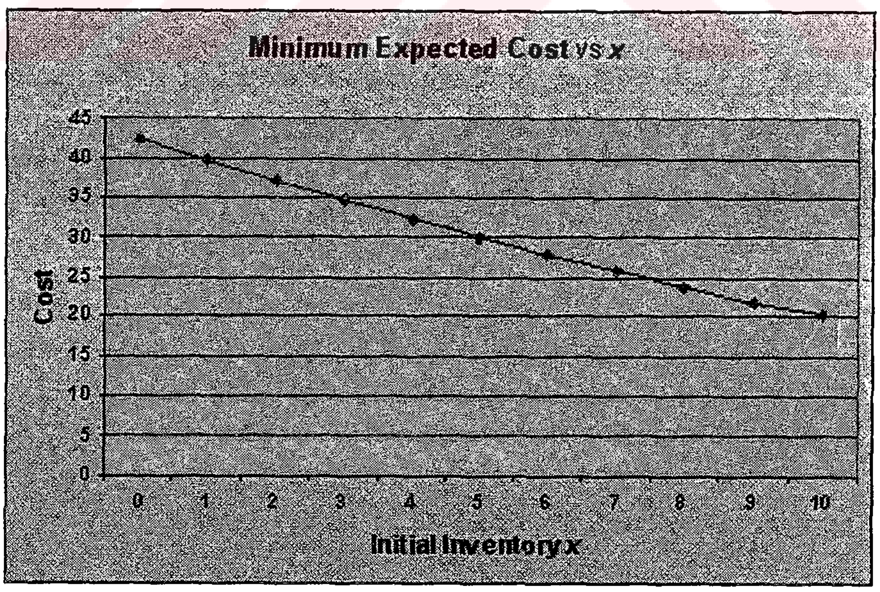

Figure 4.5: Minimum Expected Total Cost Values Corresponding to Different Inventory Levels

Table 4.5: Optimum Order Quantities for 3 Distinct Vendors with the Same Unit Variable Costs

Initial Inventory (x)	$q_1(x)$	$q_2(x)$	$q_3(x)$
0	5.32	4.39	6.45
1	4.52	3.66	5.34
2	3.77	2.99	4.35
3	3.05	2.37	3.44
4	2.37	1.81	2.61
5	1.72	1.29	1.85
6	1.11	0.80	1.15
7	0.52	0.36	0.51
8	0.00	0.00	0.00
9	0.00	0.00	0.00
10	0.00	0.00	0.00
7.80	0.0519	0.0412	0.0276

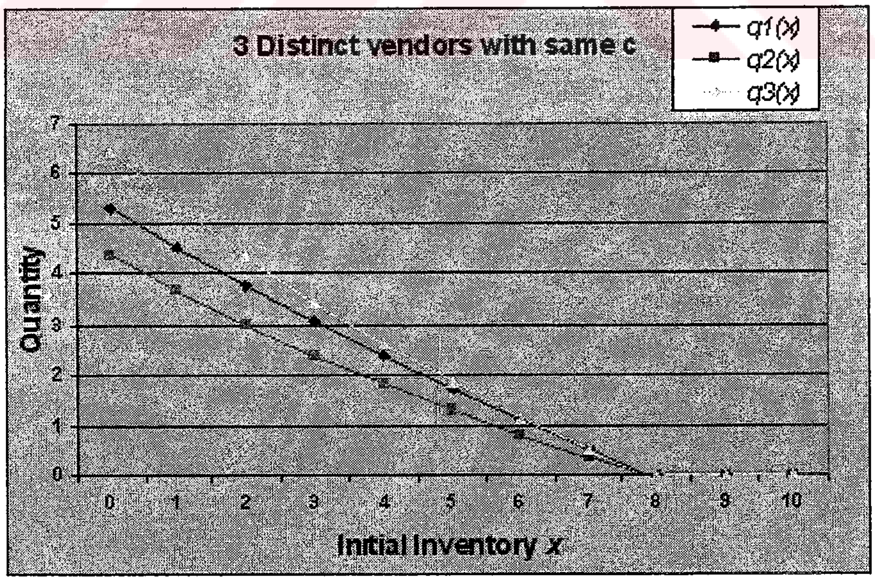

Figure 4.6: Optimum Order Quantities for 3 Distinct Vendors with the Same Unit Variable Costs

Chapter 5

THE MODEL WITH MULTIPLE IDENTICAL VENDORS

5.1 The Original Model with Identical Vendors

In this chapter we assume that there are n vendors as in the previous chapter. However the difference here is that vendors are not distinct from each other. In fact each vendor has the same characteristics with each other. For instance there is a single unit variable cost c that is charged by all of the vendors so that $c_i = c$, for all $i = 1, 2, ..., n$. Together with all of the previous assumptions, this c satisfies the inequality $p > c$. The cumulative capacity distributions for vendor i is represented by

$$F_i(z) = P(A_i \leq z) \ = F(z) \tag{5.1}$$

for all $i = 1, 2, ..., n$. In other words, the capacity for the different vendors are taken to be independent and identically distributed (IID) random variables. The same argument is also valid for the distribution of U_i's. Stochastic proportions applied to different vendors are also IID random variables so that

$$T_i(u) = P(U_i \leq u) = T(u) \tag{5.2}$$

for all $i = 1, 2, ..., n$. Actual yield from vendor i is defined as

$$Y_{q_i} = U_i \min\{q_i, A_i\} \tag{5.3}$$

from which we observe that for the same q_i's, actual yields received from different vendors are also IID random variables. Then, total random yield from n vendors is the sum of these IID random variables given by

$$\text{Total random yield} = Y_{q_1} + Y_{q_2} + Y_{q_3} + ... + Y_{q_n} = \sum_{i=1}^{n} Y_{q_i}. \tag{5.4}$$

Knowing the on-hand inventory $y = x + Y_{q_1} + Y_{q_2} + Y_{q_3} + ... + Y_{q_n}$, there is no change in the implicit form of $L(y)$ in (4.60). Since $c_i = c$, total expected cost function TC in terms of beginning inventory x and quantities q_i is obtained as

$$\begin{aligned} TC(x, q_1, q_2, ..., q_n) &= E[c(Y_{q_1} + Y_{q_2} + ... + Y_{q_n}) \\ &\qquad + L(x + Y_{q_1} + Y_{q_2} + ... + Y_{q_n})] \qquad (5.5) \\ &= E[c\sum_{i=1}^{n} Y_{q_i} + L(x + \sum_{i=1}^{n} Y_{q_i})] \qquad (5.6) \\ &= E[c\sum_{i=1}^{n} U_i(q_i \wedge A_i) + L(x + \sum_{i=1}^{n} U_i(q_i \wedge A_i))] \qquad (5.7) \end{aligned}$$

Considering this slight difference, we now let $v_n(x)$ denote the optimal cost function with n identical vendors. As in Chapter 4,

$$\begin{aligned} v_n(x) &= \min_{q_i \geq 0} TC(x, q_1, q_2, ..., q_n) \qquad (5.8) \\ &= \min_{q_i \geq 0} E[c\sum_{i=1}^{n} Y_{q_i} + L(x + \sum_{i=1}^{n} Y_{q_i})] \qquad (5.9) \\ &= \min_{q_i \geq 0}\{\int\int ... \int[(c\sum_{i=1}^{n} u_i q_i + L(x + \sum_{i=1}^{n} u_i q_i))\prod_{i=1}^{n}(1 - F_i(q_i)) \\ &\quad + \sum_{i=1}^{n}[\int_0^{q_i}(c(u_i z_i + \sum_{j=1, j\neq i}^{n} u_j q_j) \\ &\qquad + L(x + u_i z_i + \sum_{j=1, j\neq i}^{n} u_j q_j))dF_i(z_i) \prod_{j=1, j\neq i}^{n} (1 - F_j(q_j))] \\ &\quad + \sum_{i=1}^{n}\sum_{j=1, j\neq i}^{n}[\int_0^{q_i}\int_0^{q_j}(c(u_i z_i + u_j z_j + \sum_{k=1, k\neq i,j}^{n} u_k q_k) \\ &\qquad + L(x + u_i z_i + u_j z_j + \sum_{k=1, k\neq i,j}^{n} u_k q_k))\, dF_i(z_i)dF_j(z_j) \prod_{k=1, k\neq i,j}^{n} (1 - F_k(q_k))] \\ &\quad ... \\ &\quad + \int_0^{q_1}\int_0^{q_2} ... \int_0^{q_n}(c\sum_{i=1}^{n} u_i z_i + L(x + \sum_{i=1}^{n} u_i z_i))dF_1(z_1)...dF_n(z_n)] \\ &\qquad \times dT_1(u_1)...dT_n(u_n)\} \qquad (5.10) \end{aligned}$$

where $F_i = F$ and $T_i = T$ for all $i = 1, 2, ..., n$.

Due to the same arguments as in Chapter 4, a *nonzero* solution set $(q_1(x), q_2(x), ..., q_n(x))$ of the following set of equations gives the optimum order quantities

$$\begin{aligned} \frac{\partial TC(x, q_1, q_2, ..., q_n)}{\partial q_1} &= 0 \\ \frac{\partial TC(x, q_1, q_2, ..., q_n)}{\partial q_2} &= 0 \\ &\dots \\ \frac{\partial TC(x, q_1, q_2, ..., q_n)}{\partial q_n} &= 0. \end{aligned} \tag{5.11}$$

Considering the fact that there is a single unit variable cost c for all vendors and that the capacities and stochastic proportions are IID random variables, the equation system above becomes

$$-cE[U] = E[u_i L'(x + u_i \bar{q}_i(x) + \sum_{j=1, j \neq i}^{n} u_j(\bar{q}_j(x) \wedge A_j))] \tag{5.12}$$

for $i = 1, 2, ..., n$ where $E[U_1] = E[U_2] = ... = E[U_n] = E[U]$. Since all vendors are identical, a set of equal $\bar{q}_i(x)$ values solves this system. It is obvious that the optimum order quantities for each vendor should be the same. So there is a solution of this system that gives $\bar{q}_1(x) = \bar{q}_2(x) = ... = \bar{q}_n(x) = \bar{q}(x)$ and the optimality condition (5.12) can be written as

$$-cE[U] = E[U_1 L'(x + U_1 \bar{q}(x) + \sum_{j=1, j \neq 1}^{n} U_j(\bar{q}(x) \wedge A_j))] \tag{5.13}$$

which gives the single optimum order quantity $\bar{q}(x)$ that will be released to all vendors. Using the same arguments in Proposition 5, it is clear that if $\bar{q}(x)$ turns out to be a negative value, then optimum order quantity $q(x)$ is zero. In other words, if $\bar{q}(x) < 0$, then no order is given to the vendors.

Proposition 6. *When there are n identical vendors, the critical order level for each vendor is the same. The common critical order level S_n is given by*

$$S_n = G^{-1}\left(\frac{p - c}{p + h}\right). \tag{5.14}$$

Proof. Since all $\bar{q}_i(x)$ values are equal to each other for every x, they all hit zero at the same x value, which is S_n. In other words, this is the point where $\bar{q}(x)$ becomes zero. Then, setting $x = S_n$ and $\bar{q}(x) = 0$ in (5.13) gives us the critical order level so that

$$\begin{aligned} -cE[U] &= E[U_1 L'(S_n)] && (5.15) \\ &= L'(S_n)E[U_1] = [(h+p)G(S_n) - p]E[U] && (5.16) \end{aligned}$$

which leads to

$$S_n = G^{-1}\left(\frac{p-c}{p+h}\right). \blacksquare \tag{5.17}$$

After the arguments above, it is clear that an optimum order policy for the n identical vendors is given as

$$(q_1(x),q_2(x),q_3(x),...,q_n(x)) = \begin{cases} (\bar{q}(x), \bar{q}(x), \bar{q}(x), ..., \bar{q}(x)), & x < S_n \\ (0, 0, 0, ..., 0), & x \geq S_n \end{cases} \tag{5.18}$$

Now, we try to extract some characteristics of the optimum order quantity. In fact we will examine the effect of the number of vendors n and the capacity mean of the vendors on the optimum order quantity. Let $Q_n(x)$ be the optimum order quantity given to each vendor with the initial inventory x when we have n vendors. In the following propositions, parameters other than the mentioned ones remain unchanged.

Proposition 7. *As we increase the number of the vendors, we obtain smaller optimum order sizes for each vendor, i.e.,* $Q_{n+1}(x) \leq Q_n(x)$ for all x and $n = 1, 2, ...$

Proof. First of all, let's write (5.13) in the following compact way

$$E[UL'(x + UQ_n(x) + \sum_{j=1}^{n-1} W_j(Q_n(x) \wedge A_j))] = -cE[U] \tag{5.19}$$

where U stands for U_1 and W_j's represents the stochastic proportions for the $(n-1)$ vendors other than the first vendor. Now, we define the function $f_n(q)$ so that

$$f_n(q) = E[UL'(x + Uq + \sum_{j=1}^{n-1} W_j(q \wedge A_j))]. \tag{5.20}$$

Since L' is an increasing function, $f_n(q)$ is also increasing in q. Considering that the right hand side of (5.13) is independent of n, we have

$$f_1(Q_1(x)) = f_2(Q_2(x)) = f_3(Q_3(x)) = \ldots = f_n(Q_n(x)) = f_{n+1}(Q_{n+1}(x)) = -cE[U] \tag{5.21}$$

We can write

$$f_{n+1}(Q_n(x)) = E[UL'(x + UQ_n(x) + \sum_{j=1}^{n} W_j(Q_n(x) \wedge A_j))] \tag{5.22}$$

which leads to the inequality that

$$f_{n+1}(Q_n(x)) \geq f_n(Q_n(x)). \tag{5.23}$$

We know that $f_n(Q_n(x)) = f_{n+1}(Q_{n+1}(x))$. Therefore $f_{n+1}(Q_n(x)) \geq f_{n+1}(Q_{n+1}(x))$ and considering that f_{n+1} is an increasing function, $Q_n(x)$ turns out to be greater than or equal to $Q_{n+1}(x)$.■

5.2 *The Case of Exponentially Distributed Demand*

We again suppose that periodic demand $D \sim \text{Exp}(\lambda)$. Then, skipping the details about the total cost function and focusing again on the optimality condition, we can write (5.13) as

$$E[UL'(x + U\bar{q}(x) + \sum_{j=2}^{n} U_j(\bar{q}(x) \wedge A_j))] = -cE[U]. \tag{5.24}$$

Using the exponential distribution, we obtain

$$E[Ue^{-\lambda(U\overline{q}(x)+\sum_{j=2}^{n}U_j(\overline{q}(x)\wedge A_j))}] = e^{\lambda x}\frac{h+c}{h+p}E[U]. \tag{5.25}$$

As in Chapter 4,we separate the left hand side, which results in

$$E[Ue^{-\lambda U\overline{q}(x)}]\prod_{j=2}^{n}E[e^{-\lambda U_j(\overline{q}(x)\wedge A_j))}] = e^{\lambda x}\frac{h+c}{h+p}E[U]. \tag{5.26}$$

We know that all vendor are identical and, in turn, U_j and A_j are IID among themselves. Therefore, $E[e^{-\lambda U_j(\overline{q}(x)\wedge A_j))}]$ takes the same value for all j. Finally, the optimality condition simplifies to

$$E[Ue^{-\lambda U\overline{q}(x)}](E[e^{-\lambda U(\overline{q}(x)\wedge A))}])^{n-1} = e^{\lambda x}\frac{h+c}{h+p}E[U] \tag{5.27}$$

which means that, we have to determine the functions $E[Ue^{-\lambda U\overline{q}(x)}]$ and $E[e^{-\lambda U(\overline{q}(x)\wedge A))}]$ to construct the optimality equation. The rest is to put the right hand side, which is independent of the number of vendors and place the value $n-1$ so that the equation gives the $\overline{q}(x)$ value for n identical vendors.

Model with Exponential Vendor Capacity and Uniform Proportion

For illustration purposes, we again assume that

$$A_i \sim \text{Exp}(\mu) \tag{5.28}$$

and

$$U_i \sim \text{Uniform}(a,\ b) \tag{5.29}$$

for all $i = 1, 2, ..., n$ and keep all of the other parameters fixed. Then, we have to solve the equation

$$
\begin{aligned}
e^{\lambda x}\frac{h+c}{h+p}E[U] \quad = \quad & \frac{1}{(b-a)\lambda^2\bar{q}(x)^2}[e^{-\lambda\bar{q}(x)a}(1+\lambda\bar{q}(x)a) - e^{-\lambda\bar{q}(x)b}(1+\lambda\bar{q}(x)b)] \\
& \times[\frac{\mu}{\lambda(a-b)}[E_i(1,\bar{q}(x)(\mu+\lambda a)) - E_i(1,\bar{q}(x)(\mu+\lambda b)) \\
& +\ln\left(\frac{\mu+\lambda a}{\mu+\lambda b}\right)] + \frac{1}{\lambda\bar{q}(x)(a-b)} \\
& \times(e^{-\bar{q}(x)(\mu+\lambda b)} - e^{-\bar{q}(x)(\mu+\lambda a)})]^{n-1}. \qquad (5.30)
\end{aligned}
$$

We observe that changing the number of vendors does not make the problem more difficult when we have a periodic demand that is exponentially distributed since it does not require the computation of a new function but just changing the parameter n in the equation.

5.3 Numerical Illustration and Results

We assume that the vendors are identical to the vendor analyzed in the single vendor case. So, in this section we have $A_i \sim$ Exp(0.25) and $U_i \sim$ Uni (0.5, 0.8) for all i. By Proposition 6, all vendors have the same critical order level S, which is equal to 7.88. Table (5.1) gives the optimum order quantities versus both the inventory level x, and the number of vendors n. As mentioned in Proposition 7, we obtain smaller order quantities if we increase the number of vendors, which is also an intuitive result. In Table (5.2), we tabulate the total order quantity $nQ_n(x)$ released to all vendors. The point of this information is that it shows the effect of working with more vendors on the total order quantity released by the decision maker. There are two apparent outcomes in this table.

First, we see that the total order quantity first increases when we are working with more than a single vendor. We observe a sudden and sharp increase in the total order quantity when switching from a single vendor to two vendors. However this increase does not continue and the total quantity decreases later for some x and n values. The second observation, which is parallel to the first one, is that as n takes very large values, total order quantity again converges to the order quantity released for the single vendor, which is

Table 5.1: Optimum Order Quantities for Different Number of Identical Vendors

x	$Q_1(x)$	$Q_2(x)$	$Q_3(x)$	$Q_4(x)$	$Q_5(x)$	$Q_7(x)$	$Q_{10}(x)$	$Q_{50}(x)$	$Q_{500}(x)$
0	11.999	8.726	6.096	4.357	3.293	2.167	1.418	0.250	0.024
1	10.468	7.335	5.020	3.594	2.740	1.829	1.210	0.218	0.021
2	8.938	6.010	4.047	2.911	2.239	1.513	1.012	0.185	0.018
3	7.414	4.761	3.172	2.297	1.781	1.218	0.823	0.153	0.015
4	5.891	3.603	2.388	1.742	1.361	0.941	0.641	0.121	0.012
5	4.371	2.542	1.683	1.237	0.974	0.680	0.467	0.090	0.009
6	2.853	1.578	1.046	0.775	0.614	0.433	0.299	0.058	0.006
7	1.338	0.704	0.469	0.035	0.279	0.198	0.138	0.027	0.003
8	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
9	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
10	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000

easily observed from the last column of Table (5.2). These observations are hard to prove analytically due to the complexity of the model.

After observing the convergence of the total order quantity to the single vendor order quantity, we tried to investigate the effect of capacity on this convergence. To achieve that, we solved the problem for an extreme case where the capacity mean is taken to be 1000. This approach gives a sound idea about the infinite capacity case. Table (5.3) gives the results for this increased capacity case. The outcome is such that the total order quantity immediately converges to the single vendor quantity (i.e., the last two columns are almost the same as the first column). This is an intuitive result in the sense that when capacity constraint is removed, the initial quantity is expected to be directly shared among the identical vendors.

After focusing on the convergence point, a question related to this subject comes to mind: "When does the total order quantity begin to decrease?". Table (5.4) is obtained from Table (5.1) by taking the consecutive differences of the total order quantities. Negative values in this table reveals that for a fixed x value, the value of n determines where the decrease in total order quantity will begin and vice versa.

In order to observe the effect of mean capacity, we created Table (5.5) which gives the

Table 5.2: Total Order Quantities for Different Number of Identical Vendors

x	$n=1$	$n=2$	$n=3$	$n=4$	$n=5$	$n=7$	$n=10$	$n=50$	$n=500$
0	11.999	17.453	18.288	17.428	16.467	15.168	14.181	12.505	12.150
1	10.468	14.671	15.061	14.376	13.700	12.800	12.103	10.875	10.600
2	8.938	12.019	12.141	11.645	11.193	10.593	10.121	9.260	9.050
3	7.414	9.522	9.517	9.189	8.905	8.529	8.226	7.655	7.550
4	5.891	7.206	7.163	6.969	6.806	6.589	6.411	6.065	6.000
5	4.371	5.084	5.048	4.949	4.869	4.760	4.669	4.485	4.450
6	2.853	3.155	3.138	3.102	3.071	3.029	2.993	2.920	2.900
7	1.338	1.408	1.406	1.400	1.395	1.387	1.380	1.365	1.350
8	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
9	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
10	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Table 5.3: Optimum Order Quantities for Increased Capacity Case

x	$Q_1(x)$	$Q_2(x)$	$Q_3(x)$	$2Q_2(x)$	$3Q_3(x)$
0	**12.00**	6.04	4.03	**12.09**	**12.10**
1	**10.47**	5.27	3.52	**10.55**	**10.56**
2	**8.94**	4.50	3.00	**9.01**	**9.01**
3	**7.41**	3.74	2.49	**7.47**	**7.48**
4	**5.89**	2.97	1.98	**5.94**	**5.94**
5	**4.37**	2.20	1.47	**4.41**	**4.42**
6	**2.85**	1.44	0.96	**2.88**	**2.88**
7	**1.34**	0.67	0.45	**1.35**	**1.35**
8	**0.00**	0.00	0.00	**0.00**	**0.00**
9	**0.00**	0.00	0.00	**0.00**	**0.00**
10	**0.00**	0.00	0.00	**0.00**	**0.00**

Table 5.4: Consecutive Differences between the Total Quantities

x	$Q_1(x)$	(2-1)	(3-2)	(4-3)
0	12.00	5.45	0.84	**-0.86**
1	10.47	4.20	0.39	**-0.68**
2	8.94	3.08	0.12	**-0.50**
3	7.41	2.11	**-0.01**	**-0.33**
4	5.89	1.31	**-0.04**	**-0.19**
5	4.37	0.71	**-0.04**	**-0.10**
6	2.85	0.30	**-0.02**	**-0.04**
7	1.34	0.07	**0.00**	**-0.01**
8	0.00	0.00	0.00	0.00
9	0.00	0.00	0.00	0.00
10	0.00	0.00	0.00	0.00

results with a decreased capacity mean of 2 and all other parameters remain unchanged. It tabulates the consecutive differences of the total order quantities as in Table (5.4). Looking at the negative values, we observe that total order quantity begins to decrease at larger n values for fixed x when capacity mean is decreased. Therefore, the outcomes reveal that there is a negative correlation between the capacity mean and the value of n where the total order quantity begins to decrease. Another observation from both of the tables is that for a fixed value of n, total quantity tends to decrease at higher x values.

It is clear that working with multiple identical vendors does not necessarily create a diversification effect in terms of total order quantities. In other words, when we have multiple vendors, we release a total order quantity that is at least the quantity for the single vendor case. However, working with multiple vendors brings a diversification effect in terms of the total expected cost so that as we increase the number of identical vendors, computational results indicate that we incur smaller expected costs. Table (5.6) compares the total expected costs of working with 1, 2, and 3 identical vendors.

Table 5.5: Consecutive Differences between the Total Quantities with a Capacity Mean of 2

x	$Q_1(x)$	(2-1)	(3-2)	(4-3)	(5-4)
0	12.00	8.28	4.64	1.30	**-1.15**
1	10.47	6.77	3.24	0.32	**-1.31**
2	8.94	5.29	1.99	**-0.28**	**-1.10**
3	7.41	3.86	0.99	**-0.47**	**-0.74**
4	5.89	2.54	0.35	**-0.38**	**-0.42**
5	4.37	1.42	0.05	**-0.22**	**-0.20**
6	2.85	0.60	**-0.02**	**-0.08**	**-0.07**
7	1.34	0.13	**-0.01**	**-0.02**	**-0.01**
8	0.00	0.00	0.00	0.00	0.00
9	0.00	0.00	0.00	0.00	0.00
10	0.00	0.00	0.00	0.00	0.00

Table 5.6: Minimum Total Expected Costs for Different Number of Identical Vendors

x	$TC(n=1)$	$TC(n=2)$	$TC(n=3)$
0	45.12	42.38	41.05
1	41.43	39.44	38.56
2	38.12	36.76	36.22
3	35.15	34.30	33.89
4	32.47	32.00	31.85
5	30.04	29.83	29.77
6	27.81	27.74	27.73
7	25.72	25.72	25.71
8	23.71	23.71	23.65
9	21.86	21.86	21.75
10	20.23	20.23	20.20

5.4 Simplified Version of the Model

The aim of analyzing the multiple identical vendors case is to gain some insight about the characteristics of the optimum order quantity and total order quantity. It is critical to see the effect of the number of vendors on these quantities since we have unreliable suppliers and face some amount of risk in the procurement process. So it is beneficial to obtain some analytical outcomes before computation. Our model is rather complex due to the structure of random yield and it is hard make such analysis.

In this section, a simplified version of the model is briefly discussed to make some analytical analysis. The main simplification is to assume that all vendors have the same stochastic proportion U. More precisely, we remove *IID* U_is and use a single random variable U that is applied to all vendors. Therefore, $U = U_i$ for $i = 1, 2, ..., n$.

In order not to repeat the same arguments used for the previous model, we skip some details. It is found that a *nonzero* solution of the following equation gives us the optimum order quantity

$$-cE[U] \quad = \quad E[UL'(x + U(\bar{q}(x) + \sum_{j=1, j\neq 1}^{n} (\bar{q}(x) \wedge A_j)))]. \tag{5.31}$$

Let the right hand side of (5.31) be represented by

$$f_n^s(q) = E[UL'(x + U(q + \sum_{j=1}^{n-1}(q \wedge A_j)))]. \tag{5.32}$$

Proposition 8. *As the number of vendors increases, optimum order quantities gets smaller, i.e.,* $Q_{n+1}(x) \leq Q_n(x)$.

Proof. The same logic in Proposition 7 is also valid here and we can easily obtain the inequality

$$f_{n+1}^s(Q_n(x)) \geq f_n^s(Q_n(x)) = f_{n+1}^s(Q_{n+1}(x)) \tag{5.33}$$

to conclude $Q_{n+1}(x) \leq Q_n(x)$.■

Proposition 9. *For infinite vendor capacity case, the total order quantity equals to the single vendor order size, i.e.,* $nQ_n(x) = Q_1(x)$ for all x.

Proof. We can clearly state that if A_j is infinite, then $(Q_n(x) \wedge A_j) = Q_n(x)$. It reveals that

$$f_n^s(Q_n(x)) = E[UL'(x + UnQ_n(x))] = f_1^s(nQ_n(x)) = f_1^s(Q_1(x)) \tag{5.34}$$

which leads to the equality $nQ_n(x) = Q_1(x)$.■

Proposition 10. *If there is more than one vendor, the total order quantity given to all vendors is always greater than or equal to the order quantity in the single vendor case, i.e.,* $nQ_n(x) \geq Q_1(x)$ for all x and n.

Proof. Since $Q_n(x) \geq (Q_n(x) \wedge A_j)$, we can write

$$E[UL'(x + UnQ_n(x))] \geq E[UL'(x + U(Q_n(x) + \sum_{j=1}^{n-1}(Q_n(x) \wedge A_j)))] \tag{5.35}$$

which means

$$f_1^s(nQ_n(x)) \geq f_n^s(Q_n(x)) = f_1^s(q^1) = -cE[U] \tag{5.36}$$

So we have $f_1^s(nQ_n(x)) \geq f_1^s(q^1)$, which completes the proof since f_1^s is increasing. ■

5.5 The Case of Exponentially Distributed Demand

Assuming that all vendors are identical to the vendor analyzed in the single vendor case and that the demand is exponentially distributed, we focus on (5.31). Using the same approach in the previous section, we write it as

$$E[UL'(x + U(\bar{q}(x) + \sum_{j=1,j\neq 1}^{n} (\bar{q}(x) \wedge A_j)))] = -cE[U] \tag{5.37}$$

which leads to

$$E[Ue^{-\lambda(U(\bar{q}(x)+\sum_{j=1,j\neq 1}^{n}(\bar{q}(x)\wedge A_j)))}] = e^{\lambda x}\frac{h+c}{h+p}E[U]. \tag{5.38}$$

At this point we see that the simplified version of the model does not allow simplification during the computation. This is mainly caused by the removal of independence of the stochastic distributions. In the original model, the independence of U_is allowed us to separate the left hand side of (5.25), but we cannot make the same separation on the left hand side of (5.38) since the single U makes all vendors dependent to each other.

This fact complicates the solution procedure since it is hard to explicitly write the inside of the expectation in (5.38). For example, if we assume that there are two identical vendors, then (5.38) becomes

$$\int ue^{-\lambda u\bar{q}(x)}[(1-F(\bar{q}(x)))e^{-\lambda u\bar{q}(x)} + \int_0^{\bar{q}(x)} e^{-\lambda uz}dF(z)]dT(u). \tag{5.39}$$

It is observed that there are two extra terms inside the integral. This can be generalized so that there are 2^{n-1} additional terms inside the integral where n is the number of vendors. It is clear that working with large n makes the computation very difficult.

For illustration purposes, we computed the results of this model with two identical vendors, who are the same vendor as in Section 5. Table (5.7) gives the results of this two-vendor setting together with the original model. The outcomes are extremely close to those of the original model with two identical vendors, which is an expected result. In the last two columns of this table, we give the results of the same setting with large capacity mean, such as 1000, in order to illustrate the claim of Proposition 9. The last column, which is twice of the previous column, shows that with infinite capacity, the total order amount converges to the amount of the single vendor case.

Table 5.7: Optimum Order Quantities of the Simplified Model for 2 Identical Vendors

x	$Q_1(x)$	$Q_2(x)$	$Q_2(x)$, Simplified	$Q_2(x)$, Simplified, mean=1000	TQ
0	12.00	8.73	8.70	6.01	12.02
1	10.47	7.34	7.31	5.24	10.48
2	8.94	6.01	5.98	4.48	8.95
3	7.41	4.76	4.74	3.71	7.42
4	5.89	3.60	3.58	2.95	5.90
5	4.37	2.54	2.52	2.19	4.37
6	2.85	1.58	1.57	1.43	2.85
7	1.34	0.70	0.70	0.67	1.34
8	0.00	0.00	0.00	0.00	0.00
9	0.00	0.00	0.00	0.00	0.00
10	0.00	0.00	0.00	0.00	0.00

Chapter 6

CONCLUSION AND FUTURE RESEARCH

6.1 Contribution

Inventory management with random yield can be a hard task depending on the level of uncertainty in the whole process. In the literature, there are several different approaches to modeling uncertainty as mentioned in Chapter 2. One of the most common modeling approaches is to assume a random capacity for the vendor so that all orders can not be fully processed. The second approach is to apply a stochastic proportion so that only a portion of the released order can be received.

In the literature on periodic review inventory control, there are studies which analyze the two-vendor case such as Parlar and Wang (1993), Anupindi and Akella (1993); however, they include only one of the modeling approaches discussed above. Some works utilize both of the approaches in their models but they analyze the problem with a single vendor. At this point, the main difference of this study is to merge these two approaches in one model and try to solve the problem with multiple vendors.

6.2 Main Conclusions

We first applied our model to the single vendor case. For the single vendor case, the total expected cost function is not convex in the order quantity. However, it is convex on a specific region and attains a minimum in that region. Besides, it is increasing out of that region. So, the expected total cost function attains its global minimum on that convex region where the first derivative is zero.

The solution of the optimality condition can be regarded as the solution an unconstrained optimization problem. A nonnegative solution directly gives the optimum order quantity while a negative solution means that no order will be given to the vendor. The resulting policy is determined by a single critical order level above which no order is released. If the beginning inventory is lower than this critical level, then an amount of order is released, but the policy is not an order-up-to type ordering policy. In other words, the sum of beginning inventory and the order size exceeds the critical order level. Another important outcome of the single vendor case is that this critical order level depends on neither the existence of a stochastic proportion nor the distribution of the random capacity. It depends on merely the distribution of periodic demand and holding, shortage and unit variable costs.

After the single vendor case, we analyzed the same model for two vendors having all parameters different from each other. For a fixed beginning inventory level, the total expected cost is a function of q_1 and q_2, the order quantities for vendors 1 and 2, respectively. Analytical study of this function shows that it is unimodular in both q_1 and q_2 so that it has an extreme point where the first order partial derivatives are both zero. The Hessian matrix is positive definite at that point proving that it is a minimal point. The 3-D graph of this function obtained for some distributions and numerical values show that it is generally not convex but unimodal. Then, the problem is to solve a nonlinear system of two equations obtained by equating first order partial derivatives to zero.

Any nonnegative solution directly gives the optimum order quantities; however, when one element of the solution set drops below zero, just after the point of critical order level, then no order is released to the corresponding vendor in the optimal policy. At this point, it is kept out of consideration and the single vendor case is implemented for the remaining vendor. At the point of the second critical order level, the solution for remaining vendor also drops to zero and no order is released to this vendor as well. So the resulting policy is to order from both vendors if the beginning inventory is lower than the smaller of critical order levels; from one of the vendors if the beginning inventory is between the two critical levels and no order is given if the beginning inventory is higher than the higher of critical levels.

Computational results show that the order of critical order levels strictly depends on the unit variable costs. The cheaper vendor has a critical order level that is higher than the one belonging to the more expensive one. In other words, there is a range for the inventory level in which the decision maker releases an order to only the cheaper vendor. Another observation is that the resulting policy is again not an order-up-to type policy so that the sum of beginning inventory and the order amounts always exceeds the highest critical order level.

Extending the model to n-vendor case with distinct parameters, the problem gets more complicated for analytical investigation. For this case, the total expected cost is a function of n variables for a fixed x so that it is not feasible to illustrate the function visually. However, implicitly we have showed that the total expected cost function is unimodular in any q keeping all other q values fixed and attains its minimum at that single mode where first partial derivative is zero. So, the problem is handled by solving a nonlinear system of n equations obtained by equating the first partial derivatives of the cost function to zero.

As in the two vendor case, a solution set containing all nonnegative elements directly gives the optimum order sizes. However, when one of the elements first drops below zero, we know that no order will be given to that vendor. So, the corresponding vendor is kept out of computation and $(n-1)$- vendor case is considered with the remaining vendors. Whenever an element of the new solution set again drops below zero it is removed from the computation process and this goes on until no vendor remains.

Our computations revealed that the sequence of the critical order levels is in the inverse order of the unit variable costs. This is important in the sense that if we know the order of the critical levels, then we can calculate them by using the first order optimality conditions in advance. So knowing the beginning inventory and the critical order levels, only the vendors having a critical order level that is higher than the inventory size should be considered during the optimization process.

Another point that is worth to note is that exponentially distributed periodic demand greatly simplifies the computation process. Since the vendors are independent from each other, it allows the decision maker to establish the optimality equations with expectations in an easy way. The results of our model solved in this way are compared with the results obtained by constrained optimization routines of Matlab 6.5. They are almost the same with our previous results validating our methodology for finding the optimum order quantities.

We also analyzed the identical vendors case where all vendors have the same cost parameters and identical distributions for their capacities and stochastic proportions. The aim was to better observe the effect of the number of vendors and the mean capacity on the total order quantity. First of all, taking all vendors identical simplifies the optimality conditions for the n-vendor case since there is a single equation with a single unknown. This is due to the fact that all vendors should receive the same order size. Naturally, all vendors have the same critical order level, which can easily be computed.

We analytically proved that as the number of vendors increases, order size for each vendor decreases. In fact, no lower bound is detected so that in the limit, order size for each vendor is expected to hit the zero level. We observed that the total order quantity first increases when we shift to two vendors; however, this increase is not continuous and the total quantity begins to decrease after some n value for a fixed inventory level. In fact, for infinite number of vendors, the total order quantity is expected to converge to the single vendor order size, which is computationally validated by obtaining the results for 500 vendors. At this point it should be noted that, although working with multiple identical vendors has no diversification effect in terms of total order release, our results show that total expected cost decreases as the number of identical vendors increase.

It is hard to find where the decrease in total quantity will begin; however, solving the problem with a lower mean capacity, we observed that the decrease begins at a larger n for fixed x meaning a negative correlation. Taking this as a starting point, we asked what happens if we have infinite capacity. Considering the negative correlation, we expected a sudden convergence in the total order quantity so that increasing the number of vendors would have no effect on the total order quantity, which is the single vendor order quantity. Computational results obtained by taking capacity mean 1000 came out to be as we expected. In other words, total order released to two vendors or three vendors are all equal to original order size. Providing an analytical proof for this result is hard due to complexity. In order to analytically validate this result, we focused on a simplified version of the model which is thought to well approximate the original model. For this simplified version, it is almost impossible to make runs for a large number of vendors; however, it is simpler to investigate analytically so that it allows us to implicitly see that in the infinite capacity case, the total order quantity is always the same. Another analytical result of this simplified model is that in the normal setting, the total order quantity is higher than or equal to the single vendor order size for all n. This result is in accordance with the computational results of the original model.

To sum up, solving the inventory control problem by such a model is rather complex. The more critical point is that, it is extremely hard to analytically investigate the related functions and propose some characterizations. The same difficulty also remains after obtaining the computational results since the resulting policy does not allow the decision maker to get compact and useful insight about the optimum ordering policy.

6.3 Future Research

6.3.1 Multiperiod Setting

Our model obtains results for single-period setting; however, real life decision making processes about inventory management generally cover a multiple-period planning horizon. Wang and Gerchak [25] studies a very similar model to ours and gives analytical characterizations for a single vendor. In our case, combining the multiple vendor setting with multiple period planning horizon greatly complicates the total expected cost function and makes analytical investigation extremely hard.

Suppose that α is the one period discounting factor and $C(x, q_1, q_2, ..., q_n)$ is the one period realized cost when the decision maker releases orders $q_1, q_2, ..., q_n$ to n distinct vendors with an inventory level of x on hand. Also suppose that there is an m-period planning horizon to manage the inventory and the stochastic environment does not change from period to period so that demand and capacity are independent in consecutive periods. Now, we can define $TC_m^n(x)$ to be the minimum total expected cost for m periods with n vendors and having an inventory of x at the beginning of the first period. Then $TC_m^n(x)$ is written as

$$TC_m^n(x) = \min_{q_i \geq 0}\{C(x, q_1, q_2, ..., q_n) + \alpha TC_{m-1}^n(x + \sum_{i=1}^{n} Y_{q_i} - D)\} \quad (6.1)$$

where D is the random periodic demand in the first period and

$$C(x, q_1, q_2, ..., q_n) = E[\sum_{i=1}^{n} c_i Y_{q_i} + L(x + \sum_{i=1}^{n} Y_{q_i})]. \quad (6.2)$$

Handling $TC_m^n(x)$ analytically is extremely hard due to its complexity. In fact, when single vendor is assumed as in Wang and Gerchak (1996), mathematical induction helps in proving that this function is quasi-convex; however, the case of n vendors makes it very difficult to investigate the convexity of the function. Even considering two vendors raises many analytical difficulties.

At this point, a solution methodology for multiple period may be the repetition of the single period procedure for each period without considering the effect of current decisions on the future decisions. Actually this is a myopic approach and gives results that are not optimal over the whole planning horizon; however, considering the complexity of the multiple period problem, it may turn out to be a satisfactory policy especially when the stochastic features of each period are independent from those of other periods. So, it is obvious that the problem with m periods and n vendors is a challenging topic for future research.

6.3.2 Heuristic Approach

Determining the order quantities and critical order levels optimally is difficult with this model; moreover, there are no simple characterizations on the optimal ordering policy. The problem may be studied more and computational analysis may be achieved for different distributions and different ranges. However, it is clear that there are no simple and compact characterizations of the optimum strategy due to the complex structure of the modeling approach.

Therefore, heuristic approaches may be designed and implemented. According to our findings, every vendor has a critical value that determines whether that vendor will be given an order or not. Therefore, developing a heuristic approach that tries to determine the critical order levels is a potential research topic. Besides, the curves giving the optimum order quantities may be approximated by linear functions. So, a heuristic procedure that obtains both the critical order levels and the slopes of linear functions that approximate the optimum order curves may be developed to obtain satisfactory results that are close to optimal solution.

6.3.3 Inclusion of Fixed Ordering Cost

Our model assumes that there is no fixed cost for ordering. However, we know that in real life, a fixed amount of cost that is generally independent of the lot size is charged due to setup. The concept of fixed ordering cost becomes critical especially when working with multiple vendors. So, our model may be further studied with the inclusion of fixed ordering cost. A general approach is to denote the fixed cost of ordering to n vendors by K_n. The function K_n may be given by nK or any function that is increasing in n where K is the fixed cost of ordering to a single vendor.

With this setting, it is obvious that the total expected cost will not always decrease as in the results of Chapter 5. In other words, diversification effect of working with multiple identical vendors will be restricted by the increase in fixed ordering cost. Then, besides determining the optimum order quantities, the problem will have a dimension of analyzing the trade-off between the diversification effect and increase in fixed ordering cost. Fixed cost models generally result in an (s, S) policy; however, for our model, an order-up-to level clearly does not exist. A potential ordering policy may include the maximum number of vendors so that ordering to more vendors results in a higher expected total cost due to the increase in fixed ordering cost.

BIBLIOGRAPHY

[1] C. A. Yano and H. L. Lee, "Lot sizing with random yields: A review," *Operations Research*, vol. 43, no. 2, pp. 311–334, 1995.

[2] S. Karlin, "One stage model with uncertainty," 1958, in K. J. Arrow, S. Karlin and H. Scarf (Eds.) Studies in the Mathematical Theory of Inventory and Production, pp. 109-143 Stanford: Stanford University Press.

[3] E. A. Silver, "Establishing the order quantity when the amount received is uncertain," *INFOR*, vol. 14, pp. 32–39, 1976.

[4] W. Shih, "Optimal inventory policies when stockouts result from defective products," *International Journal of Production Research*, vol. 18, pp. 677–686, 1980.

[5] W. F. McCoy Mazzola, J. B. and H. M. Wagner, "Algorithms and heuristics for variable yield lot sizing," *Naval Research Logistics*, vol. 36, pp. 67–86, 1987.

[6] A. H. Noori and G. Keller, "Concepts, theory and techniques, the lot-size reorder-point model with upstream, downstream uncertainty," *Decision Sciences*, vol. 17, pp. 285–291, 1986.

[7] M. Parlar and D. Berkin, "Future supply uncertainty in EOQ models," *Naval Research Logistics*, vol. 38, pp. 107–121, 1991.

[8] E. Berk and A. Arreola-Risa, "Note on future supply uncertainty in EOQ models," *Naval Research Logistics*, vol. 41, pp. 129–132, 1994.

[9] Ü. Gürler and M. Parlar, "An inventory problem with two randomly available suppliers," *Operations Research*, vol. 45, pp. 904–918, 1997.

[10] R. Ehrhardt and L. Taube, "An inventory model with random replenishment quantities," *International Journal of Production Research*, vol. 25, pp. 1795–1803, 1987.

[11] H. L. Lee and C. A. Yano, "Production control in multistage systems with variable yield losses," *Operations Research*, vol. 36, pp. 269–278, 1988.

[12] I. M. Henig and Y. Gerchak, "The structure of periodic review policies in the presence of random yield," *Operations Research*, vol. 38, pp. 634–643, 1990.

[13] Y. Bassok and R. Akella, "Ordering and production decisions with supply quality and demand uncertainty," *Management Science*, vol. 37, no. 12, pp. 1556–1574, 1991.

[14] Y. Ahimud and H. Mendelson, "Multiperiod sales-production decisions under uncertainty," *Journal of Economic Dynamics and Control*, vol. 5, pp. 249–265, 1983.

[15] R. G. Vickson, Y. Gerchak, and M. Parlar, "Periodic review production models with variable yield and uncertain demand," *IIE Transactions*, vol. 20, pp. 144, 1988.

[16] S. Bollapragada and T. E. Morton, "Myopic heuristics for the random yield problem," *Operations Research*, vol. 47, no. 5, pp. 713–722, 1999.

[17] T. C. E. Cheng, "An economic order model with demand-dependent unit production cost and imperfect production cost," *IIE Transactions*, vol. 23, pp. 23–28, 1991.

[18] X. Zhang and Y. Gerchak, "Joint lot sizing and inspection policy in an EOQ model with random yield," *IIE Transactions*, vol. 22, pp. 41–47, 1995.

[19] B. Kalymon, "Stochastic prices in a single item inventory purchasing model," *Operations Research*, vol. 19, pp. 1434–1458, 1971.

[20] R. Feldman, "A continuous review (s, s) inventory system in a random environment," *Journal of Applied Probability*, vol. 15, pp. 754–659, 1978.

[21] S. Özekici and M. Parlar, "Inventory models with unreliable suppliers in a random environment," *Annals of Operations Research*, vol. 91, pp. 123–136, 1999.

[22] F. W. Ciarallo, R. Akella, and T. E. Morton, "A periodic review, production planning model with uncertain capacity and uncertain demand- optimality of extended myopic policies," *Management Science*, vol. 40, pp. 320–332, 1994.

[23] Y. Wang and Y. Gerchak, "Continuous review inventory control when capacity is variable," *International Journal of Production Economics*, vol. 45, pp. 381–388, 1996.

[24] R. Güllü, E. Önol, and N. Erkip, "Analysis of a deterministic demand production/inventory system under nonstationary supply uncertainty," *IIE Transactions*, vol. 29, pp. 703–709, 1998.

[25] Y. Wang and Y. Gerchak, "Periodic review production models wih variable capacity, random yield, and uncertain demand," *Management Science*, vol. 42, no. 1, 1996.

[26] H. Matsumoto and Y. Tabata, "A periodic review inventory model with variable capacity, random yield, and uncertain demand in a random environment," Working paper, Graduate School of Economics, Osaka University, 2003.

[27] Y. Gerchak and M. Parlar, "Yield randomness, cost tradeoffs and diversification in the EOQ model," *Naval Research Logistics*, vol. 37, pp. 341–354, 1990.

[28] M. Parlar and D. Wang, "Diversification under yield randomness in inventory models," *European Journal of Operation Research*, vol. 66, pp. 52–64, 1993.

[29] R. Anupindi and R. Akella, "Diversification under supply uncertainty," *Management Science*, vol. 39, pp. 944–963, 1993.

[30] I. M. Henig and N. Levin, "Joint production planning and product delivery commitments with random yield," *Operations Research*, vol. 40, pp. 404–409, 1992.

[31] H. S. Lau and L. G. Zhao, "Optimal ordering policies with two suppliers when lead times and demands are all stochastic," *European Journal of Operational Research*, vol. 68, pp. 269–278, 1988.

[32] A. S. Erdem, M. Fadıloğlu, and S. Özekici, "Order diversification with multiple vendors and random capacity," Working paper, Department of Industrial Engineering, Boğaziçi University, 2004.

[33] A. S. Erdem, M. Fadıloğlu, and S. Özekici, "An EOQ model with multiple vendors and random capacity," Working paper, Department of Industrial Engineering, Boğaziçi University, 2004.

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