The Location-Routing Problem with Inventory Control Considerations: Formulation, Solution Method and Computational Results

by

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ABSTRACT

The objective of this thesis is to develop mathematical models and solution algorithms for the multi-depot location-routing problem (MDLRP) in a distribution network with inventory control decisions. In a distribution network with a single plant, one type of product is sent from the plant via warehouses (i.e., distribution centers or DCs) to a set of customers each with uncertain demand. Each DC plays a direct intermediary role between the plant and the customers. Safety stock is kept at DCs to satisfy the desired level of customer service. Decisions related to the design of a distribution network include (i) facility location decisions that are considered as strategic decisions; (ii) transportation, or routing decisions that are considered as operational decisions; and lastly (iii) inventory decisions (e.g., levels of cycle and safety inventories at the depots) that are considered as tactical decisions. MDLRP with inventory control decisions involves facility (depot) location, warehousing, transportation (vehicle routing) and inventory control decisions in order to determine the location of depots, the optimal set of vehicle schedules and routes between depots and customers, quantity shipped from the plant (supply point) to depots and the inventory level at each depot.

The problem is formulated as a warehouse location-routing problem based on the model proposed by Perl and Daskin [1]. Since finding exact solutions for this problem is NP-hard, a heuristic approach, that is, a modified version of a two-phase tabu search algorithm which was proposed first for the solution of LRP by Tuzun *et al.* [2] is used. Finally, a set of test problems are performed to test and evaluate robustness and efficiency of the proposed method based on tabu search parameters, and computational benchmarking results and some concluding remarks are presented.

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NOMENCLATURE

WLRP	Warehouse Location Routing Problem
VRP	Vehicle Routing Problem
LAP	Location Allocation Problem
LRP	Location Routing Problem
TS	Tabu Search
GA	Genetic Algorithm
MINLP	Mixed Integer Non Linear Programming
MIP	Mixed Integer Programming
CSL	Customer Service Level
N	Number of customers
M	Number of potential DCs
P	Number of the existing plants
K	Number of vehicles
g, h	Indices for customers and DCs where $1 \leq g \leq N+M$, $1 \leq h \leq N+M$
i	Index for customers where i = 1,2,..,N
j	Index for depots where j = N+1, N+2,.., N+M
k	Index for vehicles where k = 1,2,..,K
p	Index for plants where p = 1,2,..,P
D_i	Demand of customer i, where i =1,.., N
C_{gh}	Distance between points g and h
K_k	Capacity of vehicle k
WC_j	Capacity of DC j
VC_j	Warehousing cost per unit throughput at DC j
DC_k	Transportation cost per kilometer of vehicle k
FC_j	Fixed cost of establishing DC j

CP_{pj}	Unit shipping cost for transferring goods from plant p to DC j
IC_j	Inventory cost for unit of good at DC j
bs_j	Minimum stock level to keep at DC j, i.e., customer service level
f_{pj}	The quantity of good shipped from the plant p to DC j
s_j	Stock level being kept at DC j
g, h	Indices for customers and DCs where $1\leq g \leq N+2M$, $1\leq h \leq N+2M$
I	Index for customers where i = 1,2,..,N
j_1	Index for departure depots where j_1 = N+1, N+2,…, N+M
j_2	Index for arrival depots where j_2 = N+M+1, N+M+2,..., N+2M
k	Index for vehicles where k = 1,2,..,K
AD_i	Average demand for customer i, where i =1,.., N
$EDL_{j1,k}$	Average demand for depot j_1 for route k during lead time
$R_{j1,k}$	Reorder point for replenishment for depot j_1 for route k
$f_L(x)$	Probability density function for customers' demand of each route during lead time L where x is the random demand during lead time
d_{gh}	Distance between points g and h
b	Capacity of vehicle
WC_{j1}	Capacity of DC j_1
VC_{j1}	Warehousing cost per unit throughput at DC j_1
cm	Transportation cost per unit distance
c	Cost of dispatching vehicles
FC_{j1}	Fixed cost of establishing DC j_1
h^+	Holding cost per unit time per unit goods
A	Ordering cost per each order
L	Lead time
$D_{j1,k}$	Average demand (in units of product per time unit) for depot j_1 on route k
$Q_{j1,k}$	Order quantity (units ordered by depot j_1 from the plant) for serving route k, or the truck load carried out from depot j_1
F_i	The truck load carried out from customer i
P_i	Delivery amount for customer i in each replenishment cycle

Chapter 1

INTRODUCTION

The concept of integrated logistics systems has emerged in today's competitive logistics environments for the last few decades. In order for companies to maintain efficiency in their distribution networks, more efficient solution techniques have been developed by using the concept of integrated logistics systems which emphasizes the interdependence between location and routing decisions.

Effective logistics management typically includes three levels of logistics decisions: (1) *strategic decisions* encompass major capital commitments over a relatively long time period; (2) *tactical decisions* involve plans with moderate capital investments for annual, semiannual, or seasonal time horizon; and (3) *operational decisions* deal with day-to-day operations with low capital investments. For instance, in the design of a distribution network, facility location decisions are considered as strategic decisions, transportation, or routing decisions are considered as operational decisions; and lastly inventory decisions (e.g., levels of cycle and safety inventories at the depots) are considered as tactical decisions.

In particular, warehouse location-routing problems have been studied in the distribution network literature as the problem of finding the optimal number and locations of depots, and the optimal set of vehicle schedules and routes with the objective of minimizing the total system costs. Namely, WLRPs deal with decisions belonging to two different levels: strategic and operational. However, inventory control decisions which may affect vehicle routing and depot location decisions are always ignored in WLRP. There exists only very few number of studies on the location-routing problem with inventory decisions.

This thesis introduces two mathematical models for the multi-depot warehouse location-routing problem with inventory control decision considerations. In the first model, the problem is formulated as a mixed integer programming problem for combined WLRP with simultaneous consideration of inventory decisions which aims to minimize the total system costs involving depot location, warehousing, routing and inventory costs while satisfying a specified level of customer service level in terms of an inventory level at each DC (note that throughout this thesis depot, warehouse and distribution center are used to represent the same facility type). Although the model considers the inventory costs and a minimum inventory level determination, some important questions cannot be answered for less deterministic distribution network structures. Therefore, the second model is established that considers the same goal with a slight difference in the customer service level. That is, the customer service level is interpreted as the probability of not stocking-out while serving the customers. Thus, by the presence of additional variables and constraints, the problem is more complicated, and it can be formulated as a mixed integer non-linear programming problem. In very small-size instances, the problem is solved to optimality with a general purpose code; however to obtain solutions for larger instances, a heuristic is shown to have more success. A modified version of a two-phase tabu search algorithm is applied to the problem and provides significant improvements. For finding good solutions, some additional randomness is included while choosing the customers in insertion and exchange procedures as well as the inventory costs are added into the cost evaluation part.

The remaining of the study is organized as follows. Chapter 2 provides the necessary background and the literature review on location-routing and inventory-location models. The fundamentals of LRPs, inventory analysis and previous models for LRP are reviewed. Following this review, Chapter 3 introduces two different mathematical models for multi-depot location routing problem with inventory decision considerations one of which is a *mixed integer programming model*, and the other one is a *mixed integer non-linear programming model*. All relevant assumptions together with the description of the variables and constraints are presented there in detail.

Since the second model is more challenging, but provides more comprehensive results on the inventory decisions, Chapter 4 is mainly devoted to finding a solution approach to the mixed integer non-linear programming model of the problem. The proposed solution methodology is described in two sections. In the first section, some additional variables and the corresponding constraints are added to the original formulation described in Chapter 3. Exact solutions for small instances problem are found using GAMS. The second section presents a tabu search heuristic solution method for larger instances.

The computational results of test problems are presented and discussed in Chapter 5. The performance of the algorithm is evaluated in two aspects: the solution quality (i.e., the best solution found) and the computational efficiency in CPU time. A set of common benchmarking problems is solved with our modified algorithm, and then the resulting best solutions found are compared with the results found in the past studies. Furthermore, the results are interpreted based on the changes in a group of problem parameters to better understand the interdependency between three different levels of decisions. Finally, Chapter 6 summarizes the concluding remarks and future research directions of this study.

Chapter 2

LITERATURE REVIEW

Location-routing problem (LRP) is the problem of finding the optimal number and locations of depots, and the optimal set of vehicle schedules and routes with the objective of minimizing the total system costs. It is made up of two sub-problems of location-allocation problem (LAP) and a vehicle routing problem (VRP) [3]. Location-allocation problems determine the optimal locations of depots from a given set of potential sites to minimize the sum of depot establishing cost and the cost of assigning customers to the depots; and vehicle routing problems determine the optimal delivery routes from a given depot to its assigned customers [4]. The interdependence between these two problems was not recognized until the 1970s [1].

Although research on LRPs is limited relative to the studies on pure location problems and VRPs, in the past two decades many LRP models have been studied. These LRP studies can be classified in two ways such as in terms of their problem perspectives and in terms of their algorithmic developments (i.e., exact solutions or heuristic methods) [5].

2.1 Literature Review for Heuristic Solution Methods

Since LRPs are even more complex than the traveling salesman problem (TSP) and the VRP, it also is in the NP-hard class of problems. Hence, rather than exact solution methods, heuristic methods have been most widely used for solving the LRPs.

Among the earliest studies in the OR literature, Burness and White [6] defined the traveling salesman location problem to locate a single new facility. To solve practical LRPs, other researchers Or and Pierskalla [7], and Jacobsen and Madsen [8], proposed new formulations and algorithms using some side constraints such as capacity limit and maximum cost/tour-length restriction. For instance, Or and Pierskalla [7] focused on the transportation location-allocation aspects of regional blood banking. Several algorithms were developed to decide how many blood banks to set up, where to locate them, how to allocate hospitals to the banks, and how to route periodic supply operations. The goal is to minimize total transportation costs (periodic and emergency supply costs) and the system costs. Jacobsen and Madsen [8] designed a newspaper distribution system as a combined location-routing problem and gave a comparison of three different procedures: *a tour construction method with implicit transfer point location, an alternate location-allocation procedure for transfer point location,* and *a savings procedure for routing*. As a benchmark problem, they simulated the presently operating distribution system by using exactly the same cost calculations as in the heuristics.

The first studies on the combined warehouse location and vehicle routing problem (or WLRP) include the work by Perl *et al.* in [1] and [9]. WLRP is first formulated in these two works as a mixed integer linear programming problem with capacity and maximum route distance constraints for a three level supply chain with suppliers, warehouses, and customers. The WLRP is modified to include only the warehouses and customers by eliminating the suppliers from the supply chain in (MWLRP) [1]. The MWLRP contains subtour elimination constraints to remove cycles that are added for every possible combination of customers. The prohibitively large number of subtour elimination constraints, even for small-scale problems, makes the MWLRP impossible to solve within acceptable computing times. Thus, they developed a heuristic method to solve MWLRP by decomposing the problem into three phases: multi-depot vehicle dispatch problem (MDVDP), warehouse location-allocation problem (WLAP), and multi-depot routing-allocation problem (MDRAP). Finally, a private sector application of the proposed method was presented. The heuristic method solves each problem sequentially and iteratively and generates good solutions; however, it does not guarantee optimality.

A similar work was performed in Hansen *et al.* [10]. In this paper, the WLRP model proposed by Perl and Daskin [1] was reformulated to deal with the subtour elimination constraints by introducing a set of flow-variables and flow-constraints.

Although this formulation can be solved optimally, the lower bound for the problem is very low and only small problems can be solved in reasonable time. They modified the heuristic method of [1], and improved solutions to benchmark problems significantly.

Madsen [11] gave an extensive survey of methods solving combined location-routing problems. Min *et al.* [5] synthesized and classified the past research and suggested some future research areas for the LRPs. They developed a two-way classification scheme to categorize location-routing problem studies in terms of the problem perspectives and the algorithmic developments. Lastly, they pointed out some promising future research opportunities in LRP studies.

Nambiar *et al.* [12] studied the problem of improving the collection, processing and marketing of the Malaysian rubber industry and solved this location routing problem with heuristics.

The study performed by Srivastava and Benton [13] focused on the impact of the external environmental factors on the performance of three location-routing heuristics; *savings-drop heuristic, savings-add heuristic and cluster-route procedure*. Their research showed that the performance of alternative location-routing heuristics was affected by various key environmental factors such as the spatial distribution of customers and the cost structure in terms of the ratio of location cost to routing cost.

Chien [14] proposed another heuristic approach for the LRP, in which two route length estimators are used in calculating the routing cost. Firstly, the feasible location-allocation schemes are generated and improved, and then the minimum-cost routes are found according to the location-allocation results. Their results showed that the sequential procedures can produce good solutions to the practical-sized problems in a reasonable amount of time.

Salhi and Fraser [15] presented an iterative heuristic method that alternates between the location phase and the routing phase until a suitable stopping criterion is met. Their heuristic method simultaneously finds the number of depots and their locations, determines the vehicle fleet combination, and the vehicle routes. In their study, a more practical version of the LRP with vehicles having different capacities was considered. Then, their results were compared with the sequential method (location-first, routing-second), and in all cases the solutions were not worse than the sequential method. Srivastava [16] also proposed three new location-routing problems and solved these problems by three new savings based heuristics. These heuristics are the same with those given in [13].

Beside the classical heuristic methods, some meta-heuristic methods were also applied to the LRPs. One of the meta-heuristic approaches was introduced by Tuzun and Burke [2]. They presented a two-phase tabu search architecture for solving the LRPs. The route-first, location-allocation-second approach was used sequentially, and an improvement search was performed based on the tabu method. The computational results showed the efficiency of the algorithm over another heuristic method. Moreover, in the work by Wu *et al.* [3], a different meta-heuristic method to solve the multi-depot location-routing problem was proposed. The problem was divided into two sub-problems of LAP and VRP, and then sub-problems were solved in a sequential and iterative manner by using a simulated annealing algorithm (SA). The results of test problems showed that the proposed method performed well in terms of solution quality and computation time needed.

Since the location literature mostly ignored the inventory and shortage costs as well as the demand uncertainty and the effects of reorder policies on these costs and shipping costs, due to the added complexity, no LRP studied in the literature to date has explored inventory decisions except for some cases. In Perl and Siriposonsilps [17], a mathematical model for explicitly representing the trade off among facility, transportation and inventory costs was proposed. This model differs from the existing LRP models only in the form of the objective function. In Nozick and Turnquist [18], a method was developed to estimate the inventory costs and to include them within a fixed-charge facility location model through re-estimating fixed facility-related costs.

The location-inventory problems literature also includes the research by Erlebacher and Meller [19] in which the authors presented a mixed integer non-linear model formulation and two new heuristic solution procedures. The computational experiments resulted in good solutions in the existence of the demand variation and the spatial dispersion.

There are some recent studies on location- inventory models, for example, Shen, Coullard and Daskin ([20], [21]). Their former paper introduced a DC location model that incorporates working inventory and safety stock inventory costs at the DCs. Firstly, they formulated the problem as a nonlinear integer programming problem, then they convert it into a set-covering IP model. Finally, they evaluated the proposed approach through a set of problems. In the study [20], the same problem was solved by a Lagrangian relaxation algorithm. The computational results showed that the computation time needed to solve the problem was less than that of the set partitioning method of the same model. It also suggested that as the fixed cost of placing orders decreases or the transportation cost increases, it is optimal to locate additional facilities.

In a later work [22], Ozsen, Daskin and Coullard studied a capacitated version of the location model with risk pooling, an uncapacitated version of which was previously studied in [20]. The model was formulated as a non-linear integer programming problem, and then solved with a Lagrangian relaxation algorithm.

A recent study on location-inventory models was by Miranda and Garrido [23]. Similar to the work in [22], to incorporate the risk pooling effect into the location models, a non-linear mixed integer model was developed. Based on the Lagrangian relaxation and the sub-gradient method, a heuristic solution approach was first proposed and its performance was evaluated. They showed that the reduction in the total system cost becomes more significant for high level of holding cost and high variability in demand.

The only study on the LRP with inventory decision considerations was by Liu and Lee [4]. They proposed a mathematical model for the single product multi-depot LRP taking inventory control decisions into consideration. They propose a two-phase heuristic method to find solutions for this problem. In phase 1, the initial solution based on routing-first location-allocation second approach was obtained, then in phase 2, an improvement heuristic search for a better solution for the initial solution found in phase 1 was developed. The comparisons based on computational tests showed that the proposed method is better than existing methods without taking inventory control decisions into consideration.

Lastly, Ambrossino and Scutella [24] studied some more complex distribution network design problems involving facility location, warehousing, and transportation and inventory decisions. Static and dynamic scenarios were developed and two different mathematical formulations were proposed based on the models in [1] and [10]. However, they only solved a limited set of instances via a general-purpose code. The optimal solution was obtained only for one instance, for the others the best solution found was quite far from the lower bound. Finally, they stated that heuristic approaches can be more promising for these types of problems.

2.2 Literature Review for Exact Solution Methods

Laporte *et al.* performed a series of important contributions to find exact methods for the LRPs [25]. Firstly, in [26] they formulated the location-routing problem as an integer programming model with degree constraints, generalized subtour elimination constraints, and chain barring constraints. Then an exact algorithm which uses initial relaxation of most of the problem constraints was developed. This algorithm achieved to solve problems optimally up to 20 sites within a reasonable number of iterations. Another study [25], the multi-depot VRP (MDVRP) and LRP was transformed into an equivalent constrained assignment problem by using an appropriate graph representation. The problem was then solved by the branch-and-bound method by which the LRPs with 80 nodes were solvable within a reasonable time.

Another study that was performed by Laporte and Dejax [27] presented two solution approaches to dynamic location-routing problems (DLRPs) in which locations of depots and vehicle routes are to be determined for multiple planning periods rather than single period planning (i.e., customer spatial distribution and demand change significantly over time). In the first one, an exact method for small size problems was proposed in which the problem was represented by a suitable network and then solved to optimality as an integer linear programming model. In the second approach, a global solution was obtained by determining a shortest path on a directed graph.

Lastly, Laporte *et al.* [28] and Chan *et al.* [29] worked on stochastic versions of the location-routing problems by decomposing the problems into location-allocation type problems.

One recent study by Gezdur [30] formulated the warehouse location-routing problem as a set partitioning problem and the column generation technique was applied. To handle the problem, the bounds were tightened using 2-path cuts and for sub-tour elimination a separation algorithm was used. Finally, the modified algorithm was evaluated using benchmark problems against three algorithms in [1], [3], [10] from the literature. The new algorithm was reported to yield better solutions than the others.

As summarized above, almost no study attempts to incorporate inventory control decisions into classical LRPs and to solve this type of problem. However, this thesis develops mathematical models and solution algorithms for the multi-depot location-routing problem (MDLRP) in a distribution network with inventory control decisions. The main contribution of this study is these newly developed models and solution algorithm to find good solutions for the problem. The corresponding models are presented in the next chapter.

Chapter 3

MODEL FORMULATION

3.1 WLRP Model with Inventory Control -M1

A distribution network consisting of three layers: plants (supply points), distribution centers (DCs), and customers (demand points) are considered for the problem description. In this logistics network, plants send goods to DCs and DCs serve customers. There is no direct shipment from plants to customers. The DCs function not only as a transfer point between plants and customers, but they also play a critical role by keeping a specified level of inventory to meet customer demand. Hence, the goal is to determine the best distribution system in order to minimize facility location, warehousing, transportation and inventory costs while satisfying a certain customer service level.

In classical WLRP, a company has to ship goods from a set of supply points (plants) to a certain number of depots via truck loads, and then it has to deliver the goods from the depots to a set of geographically dispersed customers. Our problem mainly differs from WLRP, in the aspect that there are inventory decisions at depots in addition to location, allocation and routing decisions to be determined while solving the problem.

The network is represented by a directed graph G = (N, A) where N is the set of nodes that are referred to as all the potential facilities (e.g., plants and depots) and customers; and A is the set of arcs that represents the flow of goods from plants to DCs and from DCs to customers.

We make the following assumptions:

i. Location and demand of each customer is known,
ii. Location and capacity of each potential facility site is known,

iii. Maximum number of vehicles available for the whole distribution network and the capacity of each vehicle are known.

Under these assumptions, the problem can be defined as the problem of making the following set of decisions:

(1) *Location decisions*: number of DCs and their locations,

(2) *Allocation decisions*: how to allocate the customers to open depots (DCs),

(3) *Routing decisions*: vehicle routes for serving customers starting from a DC,

(4) *Inventory decisions*: the quantity of goods shipped from plants to DCs, and inventory level at DC to satisfy capacity constraints at the facilities.

In this problem customer service level is expressed as a minimum stock level that has to be maintained at each open depot. Based on these assumptions, the WLRP for a single product, multi-depot, single period with deterministic demand and multiple-capacitated facilities and multiple-capacitated vehicles (non-homogeneous) with inventory decision considerations is modeled as follows:

Notation:

N: number of customers

M: number of potential DCs where N>M

P: number of the existing plants

K: number of vehicles

Indices:

g, h: indices for customers and DCs where $1 \leq g \leq N+M$, $1 \leq h \leq N+M$

i: index for customers where i = 1,2,..,N

j: index for depots where j = N+1, N+2,.., N+M

k: index for vehicles where k = 1,2,..,K

p: index for plants where p = 1,2,..,P

Parameters:

D_i: demand of customer i, where i =1,.., N

C_{gh}: distance between points g and h

K_k: capacity of vehicle k

WC_j: capacity of DC j

VC_j: warehousing cost per unit throughput at DC j

DC_k: transportation cost per kilometer of vehicle k

FC_j: fixed cost of establishing DC j

CP_{pj}: unit shipping cost for transferring goods from plant p to DC j

IC_j: inventory cost for unit of good at DC j

bs_j: minimum stock level to keep at DC j (if opened), i.e., customer service level

Decision Variables:

$$X_{ghk} = \begin{cases} 1, & \text{if g precedes h on route k} \\ 0, & \text{otherwise} \end{cases}$$

$$Y_{ij} = \begin{cases} 1, & \text{if customer i is assigned to DC j} \\ 0, & \text{otherwise} \end{cases}$$

$$Z_j = \begin{cases} 1, & \text{if DC j is opened} \\ 0, & \text{otherwise} \end{cases}$$

f_{pj}: the quantity of good shipped from the plant p to DC j

s_j: stock level being kept at DC j

M1:

Minimize

$$\sum_{j=N+1}^{N+M} \mathrm{FC}_j \mathrm{Z}_j + \sum_{j=N+1}^{N+M} \mathrm{VC}_j \sum_{i=1}^{N} \mathrm{D}_i \mathrm{Y}_{ij} + \sum_{p=1}^{P} \sum_{j=N+1}^{N+M} \mathrm{CP}_{pj} \mathrm{f}_{pj} + \sum_{j=N+1}^{N+M} \mathrm{IC}_j \mathrm{s}_j$$
$$+ \sum_{k=1}^{K} \sum_{g=1}^{N+M} \sum_{h=1}^{N+M} \mathrm{C}_{gh} \mathrm{DC}_k \mathrm{X}_{ghk} \quad (1)$$

s.t.

$$\sum_{k=1}^{K} \sum_{h=1}^{N+M} \mathrm{X}_{ihk} = 1, \forall \{i = 1, ..., N\} \quad (2)$$

$$\sum_{g=1}^{N+M} \mathrm{X}_{hgk} - \sum_{g=1}^{N+M} \mathrm{X}_{ghk} = 0, \forall \{k = 1, ..., K\}; \forall \{h = 1, ..., N+M\} \quad (3)$$

$$\sum_{g \in S} \sum_{h \in \overline{S}} \sum_{k=1}^{K} \mathrm{X}_{ghk} \geq 1, \forall S \in \{1, ..., N+M\} \text{ such that } \{N+1, ..., N+M\} \subseteq S \quad (4)$$

$$\sum_{i=1}^{N} \sum_{j=N+1}^{N+M} \mathrm{X}_{ijk} \leq 1, \forall \{k = 1, ..., K\} \quad (5)$$

$$\sum_{i=1}^{N} \mathrm{D}_i \sum_{h=1}^{N+M} \mathrm{X}_{ihk} \leq \mathrm{K}_k, \forall \{k = 1, ..., K\} \quad (6)$$

$$\sum_{p=1}^{P} \mathrm{f}_{pj} - \mathrm{WC}_j \mathrm{Z}_j \leq 0, \forall \{j = N+1, ..., N+M\} \quad (7)$$

$$\sum_{h=1}^{N+M} \mathrm{X}_{ihk} + \sum_{h=1}^{N+M} \mathrm{X}_{jhk} - \mathrm{Y}_{ij} \leq 1, \forall \{k = 1, ..., K\}; \forall \{i = 1, ..., N\} \quad (8)$$

$$\sum_{p=1}^{P} \mathrm{f}_{pj} - \sum_{i=1}^{N} \mathrm{D}_i \mathrm{Y}_{ij} = \mathrm{s}_j, \forall \{j = N+1, ..., N+M\} \quad (9)$$

$$\mathrm{s}_j \geq \mathrm{bs}_j, \forall \{j = N+1, ..., N+M\} \quad (10)$$

$$\mathrm{X}_{ghk}, \mathrm{Y}_{ij}, \mathrm{Z}_j \in \{0,1\} \quad (11)$$

$$f_{pj}, s_j \geq 0, \forall j, p \qquad (12)$$

Constraints set (1) is the objective function of the model (M1) that defines the objective as minimizing the sum of fixed cost of establishing DCs, warehousing cost for DCs, shipment cost from plants to DCs, variable transportation costs and inventory holding cost at DCs. Similar to the WLRP model of [1], constraints set (2) ensure that each customer must be assigned to exactly one route. Constraints (3) imply that every node entered by the vehicle should be left by the same vehicle. Constraint set (4) requires that every delivery route be connected to a DC. Constraints (5) state that a route cannot be operated from multiple DCs provided that exactly one DC must be visited on each route. Constraints (6) are the vehicle capacity constraints. Constraints (7) limit the flow through a DC to the DC capacity. Constraints (8) specify that a customer can be allocated to a DC only if there is a route from that DC going through that customer. Constraints (9) require that the difference between the flow into a DC from plants and the flow out of a DC to customers be equal to stock level being kept at that DC. Constraints (10) ensure that stock level at any DC should be greater than or equal to the minimum specified stock level to be kept at that DC. Lastly, constraints (11), (12) ensure the integrality and non-negativity of decision variables X_{ghk} (route design variables), Y_{ij} (allocation variables), and Z_j (location variables), as well as non-negativity of flow variables f_{pj} and inventory level variables s_j.

This problem can be reformulated by using flow variables and flow constraints as in the model proposed by Hansen *et al.* [10]. Hence, a new set of flow variables must be introduced as follows:

$fx_{gik} \geq 0$: quantity of good shipped through (g,i) on route k,

$$ZZ_{hk} = \begin{cases} 1, & \text{if point h on route k} \\ 0, & \text{otherwise} \end{cases}$$

After some relevant modifications are made as in the model of [10], all redundant constraints can be eliminated. Finally, although we do not formally represent the new form of the model here, the modified model based on the additional flow variables and flow constraints would be converted to a new form which is very similar to the model in [10].

Thus, our problem is formulated in this first model as a MIP; and is composed of the objective function (1) and a set of constraints between (2)-(12).

3.2 WLRP Model with Inventory Control –M2

Like in the first model, we again consider distribution networks consisting of plants (supply points), distribution centers (DCs), and customers (demand points) in which plants send goods to DCs and DCs serve customers, that is there is no direct shipments from plants to customers; DCs keep a specified level of inventory to meet customers' demand. However, for simplicity, we assumed that there is one plant from which goods are sent to all DCs. Hence, the goal is to determine the best distribution system in order to minimize facility location, warehousing, transportation and inventory costs while satisfying a desired customer service level.

This second model mainly differs from our first model. In the first model, the only inventory decision to be determined is stock level in each DC, however in the second model there are some additional values to consider in inventory decisions (e.g. order quantity, order frequency, etc.) to better reflect the interdependence among facility location, transportation and inventory decisions. To make these new decisions, the problem is modeled in a different manner with a large number of constraints and a large number of decision variables. In addition, the objective function is defined also differently.

We make the following assumptions:

i. there is a single-product, single-plant multi-depot location-routing problem,
ii. each customer is served by exactly one vehicle,
iii. each route is served by one vehicle, and each route begins and ends at the same depot,

iv. fleet type is homogeneous,

The following is known:

i. number and location of candidate depots,
ii. number and location of customers,
iii. demand of each customer, that is stochastic,
iv. vehicle and DC capacities (i.e., if DCs are assumed to have finite capacity),
v. ordering, fixed depot establishing and holding costs
vi. probability density function of customers' demand of each route during replenishment lead time,

Under these assumptions, the problem can be defined as the problem of making the following set of decisions:

(1) *Location decisions*: number of DCs and their locations,
(2) *Allocation decisions*: how to allocate the customers to open depots (DCs),
(3) *Routing decisions*: vehicle routes for serving customers starting from a DC,
(4) *Inventory decisions*: the order quantity shipped from DCs to customers, delivery to each customer from DCs and inventory (safety stock) level at DC to satisfy the desired level of customer service (CSL).

In this problem, customer service level is expressed as the probability of not stocking out in a replenishment cycle at each open depot.

A new formulation that eliminates some redundancies in the previous formulation is developed in this section. The proposed formulation becomes a two-index formulation for the WLRP with consideration of inventory control decisions. Thus, assuming that there is a single plant from which all depots are served, the number of variables for the first formulation is $K(N+M)^2 + NM + 3M$ decrease in the second formulation to $(N+2M)^2 + K(N+2M) + M + 2N + 2KM$. For instance, for a distribution network with 85 customers, 7 warehouses, and 15 vehicles, the problem formulation will consist of 127576 variables in the first model; whereas in the second model this number decreases to 11673.

Notation:

N: number of customers

M: number of potential DCs where N>M

K: number of vehicles (or routes)

Indices:

g, h: indices for customers and DCs where $1 \leq g \leq$ N+2M, $1 \leq h \leq$ N+2M

i: index for customers where i = 1,2,..,N

j_1: index for departure depots where j_1 = N+1, N+2,…, N+M

j_2: index for arrival depots where j_2 = N+M+1, N+M+2,..., N+2M

k: index for vehicles where k = 1,2,..,K

Parameters:

AD_i: average demand for customer i, where i =1,.., N

$EDL_{j1,k}$: average demand for depot j_1 for route k during lead time

$R_{j1,k}$: reorder point for replenishment for depot j_1 for route k

$f_L(x)$: probability density function for customers' demand of each route during lead time L where x is the random demand during lead time

d_{gh}: distance between points g and h

b: capacity of vehicle

WC_{j1}: capacity of DC j_1

VC_{j1}: warehousing cost per unit throughput at DC j_1

cm: transportation cost per unit distance

c: cost of dispatching vehicles

FC_{j1}: fixed cost of establishing DC j_1

h^+: holding cost per unit time per unit goods

A: ordering cost per each order

L: lead time (in time unit)

Decision Variables:

$$X_{gh} = \begin{cases} 1, & \text{if g precedes h} \\ 0, & \text{otherwise} \end{cases}$$

$$ZZ_{hk} = \begin{cases} 1, & \text{if node h is on route k} \\ 0, & \text{otherwise} \end{cases}$$

$$Z_{j1} = \begin{cases} 1, & \text{if DC } j_1 \text{ is opened} \\ 0, & \text{otherwise} \end{cases}$$

$D_{j1,k}$: average demand (in units of product per time unit) for depot j_1 on route k

$Q_{j1,k}$: order quantity (units ordered by depot j_1 from the plant) for serving route k, or the truck load carried out from depot j_1

F_i: the truck load carried out from customer i

P_i: delivery amount for customer i in each replenishment cycle

We adopted a continuous (Q, R) review policy as for inventory policy in which inventory is continuously tracked and an order for a lot size Q is placed when the inventory falls to the reorder point *R*. In continuous review, the size of the order does not change from one order to another; however, the time between orders may fluctuate given variable demand. This policy does not penalize unfulfilled demands; instead, it sets a reorder point. Once an order is submitted, the inventory level should cover the demand produced during lead time with a given probability 1-α (i.e., the desired level of customer service, CSL). Here, α represents the probability of stocking out during one replenishment cycle for any depot, thus CSL is the probability of not stocking out during a replenishment cycle.

One important reason to prefer a continuous review policy rather than periodic policy is that periodic review policies require more safety stock than continuous review policies for the same CSL. Hence, adopting a continuous policy would yield less inventory cost than a periodic policy would.

To incorporate the inventory problem into the classical WLRP problem, some relevant inventory costs: ordering cost and holding cost (i.e., cycle stock plus safety stock cost) are included in the objective function of the model as well as fixed depot establishing cost, variable warehousing cost and transportation cost. After these costs are discussed, the model is formulated as follows:

M2:

Minimize

$$\sum_{j_1=N+1}^{N+M} \mathrm{FC}_{j1}\mathrm{Z}_{j1} + \sum_{j_1=N+1}^{N+M} \mathrm{VC}_{j1}\sum_{k=1}^{K}\mathrm{D}_{j1k} + \sum_{k=1}^{K}\sum_{j1=N+1}^{N+M}\left[\sum_{g=1}^{N+2M}\sum_{h=1}^{N+2M}\left(\left(\mathrm{c}+\mathrm{cm}\,.\,\mathrm{d}_{gh}\right)\mathrm{X}_{gh}\frac{\mathrm{D}_{j1k}}{\mathrm{Q}_{j1k}}\right) + \left(\frac{\mathrm{Q}_{j1k}}{2}+\mathrm{R}_{j1k}-\mathrm{EDL}_{j1k}\right)h^{+}\right]+\frac{\mathrm{D}_{j1k}}{\mathrm{Q}_{j1k}}\mathrm{A} \quad (13)$$

s.t.

$$\mathrm{Q}_{j1k} \le \mathrm{b}, \forall\{k=1,...,K\}, \forall\{j_1=N+1,...,N+M\} \quad (14)$$

$$\sum_{i=1}^{N}\mathrm{AD}_i\mathrm{ZZ}_{ik} = \sum_{j_1=N+1}^{N+M}\mathrm{D}_{j1k}\mathrm{ZZ}_{ik}, \forall\{k=1,...,K\} \quad (15)$$

$$\sum_{h=1}^{N+2M}\mathrm{X}_{ih} = 1, \forall\{i=1,...,N\} \quad (16)$$

$$\sum_{g=1}^{N+2M}\mathrm{X}_{hg} - \sum_{g=1}^{N+2M}\mathrm{X}_{gh} = 0, \forall\{h=1,...,N\} \quad (17)$$

$$\sum_{g=1}^{N+2M}\mathrm{X}_{g,M+j1} - \sum_{g=1}^{N+2M}\mathrm{X}_{j1,g} = 0, \forall\{j_1=N+1,...,N+M\} \quad (18)$$

$$\sum_{k=1}^{K}\mathrm{Q}_{j1k} - \mathrm{WC}_{j1}\mathrm{Z}_{j1} \le 0, \forall\{j_1=N+1,...,N+M\} \quad (19)$$

$$\mathrm{X}_{gh} - \mathrm{X}_{hg} \le 1, \forall\{g,h=1,...,N\} \quad (20)$$

$$\mathrm{X}_{ih} + \mathrm{ZZ}_{ik} - \mathrm{ZZ}_{hk} \le 1, \forall\{i,h,k\} \quad (21)$$

$$\mathrm{X}_{hi} + \mathrm{ZZ}_{ik} - \mathrm{ZZ}_{hk} \leq 1, \forall\{i, h, k\} \tag{22}$$

$$\sum_{i=1}^{N} \mathrm{X}_{j1i} - \sum_{k=1}^{K} \mathrm{ZZ}_{j1k} = 0, \forall\{j_1 = N+1, ..., N+M\} \tag{23}$$

$$\sum_{k=1}^{K} \mathrm{ZZ}_{j1k} - \mathrm{K.Z}_{j1} \leq 0, \forall\{j_1 = N+1, ..., N+M\} \tag{24}$$

$$\sum_{k=1}^{K} \mathrm{K.ZZ}_{j1k} - \mathrm{Z}_{j1} \geq 0, \forall\{j_1 = N+1, ..., N+M\} \tag{25}$$

$$\sum_{j_1=N+1}^{N+M} \mathrm{ZZ}_{j1k} \leq 1, \forall\{k = 1, ..., K\} \tag{26}$$

$$\sum_{k=1}^{K} \mathrm{ZZ}_{ik} = 1, \forall\{i = 1, ..., N\} \tag{27}$$

$$\mathrm{F}_{i1} - \mathrm{F}_{i2} - \mathrm{P}_{i2}.\mathrm{X}_{i1,i2} - \mathrm{X}_{i1,i2}\sum_{u=1}^{N} \mathrm{P}_u \geq \sum_{u=1}^{N} \mathrm{P}_u, \forall\{i_1, i_2 = 1, ..., N\} \tag{28}$$

$$\sum_{j_2=N+M+1}^{N+2M} \mathrm{X}_{i,j2} \sum_{u=1}^{N} \mathrm{P}_u \mathrm{ZZ}_{uk} + \mathrm{F}_i \leq \sum_{u=1}^{N} \mathrm{P}_u \mathrm{ZZ}_u \mathrm{k}, \forall\{i, k\} \tag{29}$$

$$\sum_{j1=N+1}^{N+M} \sum_{k=1}^{K} \mathrm{AD}_i \mathrm{ZZ}_{ik} \frac{\mathrm{Q}_{j1k}}{\mathrm{D}_{j1k}} - \mathrm{P}_i = 0, \forall\{i = 1, ..., N\} \tag{30}$$

$$\mathrm{D}_{j1k} - \sum_{u=1}^{N} \mathrm{AD}_u \mathrm{ZZ}_{j1k} \leq 0, \forall\{j_1, k\} \tag{31}$$

$$\mathrm{ZZ}_{j1k} - \mathrm{ZZ}_{M+j1,k} = 0, \forall\{j_1 = N+1, ..., N+M\} \tag{32}$$

$$\sum_{g=1}^{N+2M} \sum_{j_1=N+1}^{N+M} \mathrm{X}_{g,j1} = 0 \tag{33}$$

$$\sum_{g=1}^{N+2M} \sum_{j_2=N+M+1}^{N+2M} \mathrm{X}_{j2,g} = 0 \tag{34}$$

$$\mathrm{X}_{gh}, \mathrm{Z}_{j1}, \mathrm{ZZ}_{hk} \in \{0,1\}; \forall\{g, h, j_1\} \tag{35}$$

$$\mathrm{F}_i, \mathrm{D}_{j1k}, \mathrm{Q}_{j1k}, \mathrm{P}_i \geq 0; \forall\{i, j_1, k\} \tag{36}$$

Constraints (14) state that the amount of each delivery from each depot on each route must be within vehicle capacity. The sum of the average customer demands served via a specified route must be equal to the sum of average demands for depots serving on that corresponding route, and this is handled in constraints (15). Constraint set (16) implies that each customer must be followed by exactly one node. Constraints (17) and (18) restrict that every point entered by the vehicle should be left by the same vehicle. Constraints (19) limit the total order quantities sent from a DC to customers on all routes to the corresponding DC capacity. Constraints (20) eliminate subtours for each pair of (g, h). Constraints (21) and (22) link the allocation and routing components using the propositional logic such that ($X_{ih} \vee X_{hi} \wedge ZZ_{ik}$)$\Rightarrow$ ZZ_{hk}. Constraints (23), (24), and (25) provide the connection between the allocation and routing variables for the warehouses, stating that every warehouse that is used must serve customers on at least one route. Constraints (26) require that for every route at most one warehouse must be assigned. Similarly, in constraints (27) it is stated that each customer must be assigned to exactly one route. Constraints (28) indicate that if customer i_2 follows i_1 then the sum of the outflow from i_2 and delivery amount for i_2 equals the outflow from i_1. Constraints (29) state that outflow from the last customer in any route must be zero. Constraints set (30) links the delivery amount for each customer in each replenishment cycle and average customer demands such that average demand for each customer over the number of replenishments equals the delivery amount for the corresponding customer. Constraints (31) state that if a route is not used by a DC then the outflow from that DC on that route (i.e., average demand met by the DC via that route) must be zero. Constraints (32) ensure that each route starts and ends at the same depot. Constraints (33) and (34) imply that there can be no arcs to a departure depot and no arcs originating from an arrival depot, respectively. Finally, the remaining constraints (35) and (36) are nonnegativity and integrality constraints.

In the objective function (13), inventory holding cost is composed of two costs: cycle stock and safety stock costs. The cycle stock level (i.e., Q/2) and the safety stock level (i.e., $SS_{jk} = R_{jk} - EDL_{jk}$) are obtained from the following equations: Given that

$$\text{Probability (Demand during lead time} \leq R_{jk}) = \text{CSL}$$

which implies that

$$F_L(R) = \text{CSL}$$

from which the reorder point R is found such that

$$F_L^{-1}(\text{CSL}) = R$$

and

$$EDL = \int_0^\infty x f_L(x) dL .$$

Then, safety stock level is found in such a way that

$$F_L(SS + EDL) = \text{CSL}.$$

Hence,

$$SS = F_L^{-1}(\text{CSL}) - EDL.$$

To simplify the calculation of safety stock level, customer demands are assumed to be normally distributed with a mean AD_i and a standard deviation σ_i for customer i. Then, the aggregated demand is normally distributed with a mean of D^c and standard deviation of σ_{D^c} such that

$$D^c = \sum_{i=1}^{N} AD_i \tag{37}$$

and

$$\sigma_{D^c} = \sqrt{\sum_{i=1}^{N} \sigma_i^2 + 2\rho_{ij} \sum_{i>j} \sigma_i \sigma_j} \tag{38}$$

where ρ_{ij} is the correlation coefficient of demands at customer i and j. If the demands of the N customers are independent, then

$$\rho_{ij} = 0$$

and

$$\sigma_{D^c} = \sqrt{\sum_{i=1}^{N} \sigma_i^2} \; .$$

Thus, in our formulation, if several customers are served from a certain distribution center *j*, then the combined standard deviation of those customer demands will be given by

$$\sigma_L^c = \sqrt{\sum_{i \in N} \sigma_i^2 \mathrm{L}_i}$$

assuming independence of the customer demands. In addition, if all customers experience the same lead time (i.e., $\mathrm{L}_i = \mathrm{L}$ for each $i \in \mathrm{N}$), then

$$\sigma_L = \sqrt{\mathrm{L} \sum_{i \in N} \sigma_i^2} \; .$$

For simplicity, in our formulation we also assume that all lead times are equal. Then, the safety stock level (that is, the inventory needed to maintain the desired customer service level CSL) can be found as

$$\mathrm{SS} = \mathrm{F_L}^{-1}(\mathrm{CSL}).\sigma_L$$

without the need for the use of R_{j1k} and EDL_{j1k} variables. Thus, the objective function is reformulated by replacing safety stock level with SS_{j1k} where

$$\mathrm{SS}_{j1k} = \mathrm{R}_{j1k} - \mathrm{EDL}_{j1k} \; .$$

Thus, the objective function is rewritten as follows:

Minimize

$$\sum_{j_1=N+1}^{N+M} \mathrm{FC}_{j1}\mathrm{Z}_{j1} + \sum_{j_1=N+1}^{N+M} \mathrm{VC}_{j1} \sum_{k=1}^{K} \mathrm{D}_{j1k} + \sum_{k=1}^{K} \sum_{j1=N+1}^{N+M} \left[\sum_{g=1}^{N+2M} \sum_{h=1}^{N+2M} \left(\left(\mathrm{c} + \mathrm{cm}.\mathrm{d}_{gh} \right) \mathrm{X}_{gh} \frac{\mathrm{D}_{j1k}}{\mathrm{Q}_{j1k}} \right) + \left(\frac{\mathrm{Q}_{j1k}}{2} + \mathrm{SS}_{j1k} \right) h^{+} \right] + \frac{\mathrm{D}_{j1k}}{\mathrm{Q}_{j1k}} \mathrm{A} \qquad (13')$$

Besides, two additional constraints are needed to reformulate the model such that

$$\sigma_{j1k} = \sqrt{\mathrm{L}\sum_{i \in N} \sigma_i^2 . ZZ_{j1k} . ZZ_{ik}} \; , \forall j_1 , k \tag{39}$$

$$\mathrm{SS}_{j1k} = \mathrm{F}_L^{-1}(\mathrm{CSL}) . \sigma_{j1k} \; , \forall j_1 , k \tag{40}$$

where constraint (39) defines the standard deviation of customer demands served by DC j_1 on route k for each pair of (j_1 , k); and constraint (40) defines the safety stock level on route k for DC j_1.

Thus, our problem is formulated in the second model as a MINLP; and is composed of the modified objective function (13′) and a set of constraints between (14)-(36) with additional constraints of (39) and (40).

Since finding exact solutions for this problem is NP-hard, we use a heuristic solution methodology which is presented in the next chapter.

Chapter 4

PROPOSED SOLUTION METHODOLOGY

In the previous chapter, two different mathematical models were presented for the same problem with slightly different objectives. The first model – *a mixed-integer model*- can answer only a part of the questions stated in the second one - *a mixed-integer non-linear model*. In the first model, we only consider the inventory level of depots to minimize the total system cost no matter how frequently and in which amount the orders will be given. However, all of these questions are examined in the second model in order to get a broader insight from the inventory consideration in terms of obtaining more effective and improved distribution systems. However, there is an obvious trade-off between obtaining a well integrated system and the resulting costs in the form of money, time, or distance. For instance, when even dealing with typical LRPs, such integrated models are complex and their design requires challenges in combining the short-term operational decisions of vehicle routing with the medium/long-term strategic issues of facility location.

At this point, we chose the way to find a well-designed system, and will concentrate on finding solutions to the second model which is more comprehensive and challenging. Although it seems more promising to find approximate and/or feasible solutions than to find exact solutions for our more complicated LRP problem with an additional inventory dimension in a reasonable amount of time, we first examine the model performance for small test problems that are coded in GAMS. Fortunately, the exact solution was obtained for an illustrative small-size test problem with specified parameter values and the result is presented in Section 4.1 in details.

However, since the problem size exponentially grows in terms of the variables and constraints size and the nonlinear nature of the problem, GAMS was not able to solve the larger test problems. Therefore, to solve large-sized problems we applied a modified tabu search heuristic which was previously proposed for classical LRPs in the literature by Tuzun *et al.*[2]. The modified version of the algorithm is described in Section 4.2.

4.1 Small-Sized Problems

In this section, to evaluate the proposed model formulation, a small-sized example is generated and the corresponding model is coded in GAMS. We assume that there are three depots (DCs) and four customers and two routes. Average yearly demand for each customer is randomly selected from a uniform distribution U[450,600]; the location (x- and y-coordinates) of each customer and candidate DC is randomly selected from a uniform distribution U[0,100]. The vehicle dispatching cost is 25 for each time; the traveling cost is 1 per unit distance. The holding cost is 0.5/unit/year; the ordering cost is 20 for each order; the desired CSL is set to 0.95. The vehicle capacity is 150 units; the DC capacity is 1000 units for each DC. Fixed depot establishing costs for DC 1, 2, and 3 are 209, 467 and 143, respectively. Standard deviation of customer demands is calculated from

$$\sigma_i = \sqrt{AD_i}$$

and the lead time L is set to 10 in days (i.e., 10/365 in years).

We encountered some difficulty while defining constraint (39) due to the existence of binary variables within the square root operator. That is, such definitions are not allowed in GAMS. To deal with this problem, that constraint is replaced by a group of new constraints and variables. According to this new formulation, a new parameter σ_{pjk} is defined as the standard deviations of combined demands for all possible combinations of customer groupings, where *p* is index for possible combinations of customer groupings, e.g. for our illustrative example p=15. Furthermore, a new set of binary variables T_{pjk} are defined as follows:

$$\mathrm{T}_{pjk} = \begin{cases} 1, & \text{if the } p^{th} \text{ combination of customers is served by DC j on route k} \\ 0, & \text{otherwise} \end{cases}$$

Thus, the MINLP model of this small-sized problem is formulated with modified objective function in Eqn. (13'), constraints (14)-(36), and the following additional constraints (41)-(61):

$$\sigma_{j1k} = \sum_{p=1}^{15} \sigma_{pj_1k} . \mathrm{T}_{pj_1k}, \forall j_1, k \tag{41}$$

$$\mathrm{SS}_{j1k} = \mathrm{F}_L^{-1}(\mathrm{CSL}).\sigma_{j1k}, \forall j_1, k \tag{42}$$

$$\sum_{p=1}^{15} \sum_{j_2=N+M+1}^{N+2M} \mathrm{T}_{pj_2k} = 0, \forall k \tag{43}$$

$$\sum_{p=1}^{15} \sum_{j_1=N+1}^{N+M} \mathrm{T}_{pj_1k} \leq 1, \forall k \tag{44}$$

$$ZZ_{j1k} + ZZ_{1k} - ZZ_{2k} - ZZ_{3k} - ZZ_{4k} - \mathrm{T}_{1j_1k} \leq 1, \forall j_1, k \tag{45}$$

$$ZZ_{j1k} + ZZ_{2k} - ZZ_{1k} - ZZ_{3k} - ZZ_{4k} - \mathrm{T}_{2j_1k} \leq 1, \forall j_1, k \tag{46}$$

$$ZZ_{j1k} + ZZ_{3k} - ZZ_{1k} - ZZ_{2k} - ZZ_{4k} - \mathrm{T}_{3j_1k} \leq 1, \forall j_1, k \tag{47}$$

$$ZZ_{j1k} + ZZ_{4k} - ZZ_{1k} - ZZ_{2k} - ZZ_{3k} - \mathrm{T}_{4j_1k} \leq 1, \forall j_1, k \tag{48}$$

$$ZZ_{j1k} + ZZ_{1k} + ZZ_{2k} - ZZ_{3k} - ZZ_{4k} - \mathrm{T}_{5j_1k} \leq 2, \forall j_1, k \tag{49}$$

$$ZZ_{j1k} + ZZ_{1k} + ZZ_{3k} - ZZ_{2k} - ZZ_{4k} - \mathrm{T}_{6j_1k} \leq 2, \forall j_1, k \tag{50}$$

$$ZZ_{j1k} + ZZ_{1k} + ZZ_{4k} - ZZ_{2k} - ZZ_{3k} - \mathrm{T}_{7j_1k} \leq 2, \forall j_1, k \tag{51}$$

$$ZZ_{j1k} + ZZ_{2k} + ZZ_{3k} - ZZ_{1k} - ZZ_{4k} - \mathrm{T}_{8j_1k} \leq 2, \forall j_1, k \tag{52}$$

$$ZZ_{j1k} + ZZ_{2k} + ZZ_{4k} - ZZ_{1k} - ZZ_{3k} - \mathrm{T}_{9j_1k} \leq 2, \forall j_1, k \tag{53}$$

$$ZZ_{j1k} + ZZ_{3k} + ZZ_{4k} - ZZ_{1k} - ZZ_{2k} - \mathrm{T}_{10j_1k} \leq 2, \forall j_1, k \tag{54}$$

$$ZZ_{j1k} + ZZ_{1k} + ZZ_{2k} + ZZ_{3k} - ZZ_{4k} - T_{11j_1k} \leq 3, \forall j_1, k \tag{55}$$

$$ZZ_{j1k} + ZZ_{1k} + ZZ_{2k} + ZZ_{4k} - ZZ_{3k} - T_{12j_1k} \leq 3, \forall j_1, k \tag{56}$$

$$ZZ_{j1k} + ZZ_{1k} + ZZ_{3k} + ZZ_{4k} - ZZ_{2k} - T_{13j_1k} \leq 3, \forall j_1, k \tag{57}$$

$$ZZ_{j1k} + ZZ_{2k} + ZZ_{3k} + ZZ_{4k} - ZZ_{1k} - T_{14j_1k} \leq 3, \forall j_1, k \tag{58}$$

$$ZZ_{j1k} + ZZ_{1k} + ZZ_{2k} + ZZ_{3k} + ZZ_{4k} - T_{15j_1k} \leq 4, \forall j_1, k \tag{59}$$

$$T_{pj_1k} \in \{0,1\}, \forall p, j_1, k \tag{60}$$

$$\sigma_{j_1k} \geq 0, \forall j_1, k \tag{61}$$

Finally, the above MINLP problem is solved optimally for the specified parameter values. In the optimal solution, the objective value is 5546.1106. The optimal assignments are as follows: on the 1st route customers 4, 1 and 3 are served consecutively by the DC#2; and on the second route only customer 2 is served by the DC#3.

To eliminate nonlinearity in some constraints, new set of decision variables and constraints are needed. For instance, in constraints (28) nonlinearity is encountered from the multiplication of two decision variables P_{i_2} and $X_{i_1i_2}$. To convert the constraint into linear form, we define a new decision variable $\delta_{i_1i_2}$ instead of

$$P_{i_2} \,.\, X_{i_1i_2}$$

and another one for

$$X_{i_1i_2} \,.\, \sum_{u=1}^{N} P_u$$

in the following manner:

$$\delta_{i_1i_2} = \begin{cases} P_{i_2}, & \text{if } X_{i_1i_2} \text{ equals 1} \\ 0, & \text{otherwise} \end{cases}$$

$$\theta_{i_1 i_2} = \begin{cases} \sum_{u=1}^{N} P_u \text{, if } X_{i_1 i_2} \text{ equals 1} \\ 0, \quad \text{otherwise} \end{cases}$$

Meanwhile, the corresponding new constraints are added using the big-M parameters and then constraints set (28) is modified.

$$\delta_{i_1 i_2} - \text{M.X}_{i_1 i_2} \leq 0, \forall i_1, i_2 \tag{28'a}$$

$$\delta_{i_1 i_2} - \text{P}_{i_2} - \text{M}(1 - \text{X}_{i_1 i_2}) \leq 0, \forall i_1, i_2 \tag{28'b}$$

$$\theta_{i_1 i_2} - \text{M.X}_{i_1 i_2} \leq 0, \forall i_1, i_2 \tag{28'c}$$

$$\delta_{i_1 i_2} - \sum_{u=1}^{N} \text{P}_u - \text{M}(1 - \text{X}_{i_1 i_2}) \leq 0, \forall i_1, i_2 \tag{28'd}$$

Thus, the modified constraints set (28) become:

$$\text{F}_{i1} - \text{F}_{i2} - \delta_{i_1 i_2} - \theta_{i_1 i_2} \geq -\sum_{u=1}^{N} \text{P}_u, \forall \{i_1, i_2 = 1, ..., N\} \tag{28'}$$

Similarly, constraints set (29) are reformulated to deal with its nonlinearity. For this reason, new decision variables are needed such that 1) define ν_{ik} instead of the multiplication term

$$\text{P}_i \text{ZZ}_{ik} \text{ ,}$$

2) define γ_{ik} instead of the term

$$\left(\sum_{j_2=N+M+1}^{N+2M} \text{X}_{ij_2}\right)\left(\sum_{u=1}^{N} \nu_{uk}\right).$$

That is,

$$\nu_{ik} = \begin{cases} \text{P}_i \text{, if } \text{ZZ}_{ik} \text{ equals 1} \\ 0, \ \text{otherwise} \end{cases}$$

$$\gamma_{ik} = \begin{cases} \sum_{u=1}^{N} \nu_{uk} \text{ , if } (\sum_{j_2=N+M+1}^{N+2M} \mathrm{X}_{ij_2}) \text{ equals 1} \\ 0, \qquad \text{otherwise} \end{cases}$$

Besides, the corresponding new constraints must be included in the model formulation such that

$$\nu_{ik} - \mathrm{M.ZZ}_{ik} \leq 0, \forall i, k \tag{29'a}$$

$$\nu_{ik} - \mathrm{P}_i - \mathrm{M}(1 - \mathrm{ZZ}_{ik}) \leq 0, \forall i, k \tag{29'b}$$

$$\gamma_{ik} - \mathrm{M}(\sum_{j_2=N+M+1}^{N+2M} \mathrm{X}_{i_1 i_2}) \leq 0, \forall i, k \tag{29'c}$$

Thus, the modified constraints (29) can be rewritten as

$$\gamma_{ik} + \mathrm{F}_i \leq \sum_{u=1}^{N} \nu_{uk}, \forall i, k \tag{29'}$$

Although these new modifications on the WLRP formulation reduce the nonlinearity, the resulting number of constraints would be larger.

For instance, even in our small-sized illustrative example with 4 customers, 3 DCs and 2 routes, there are 351 single equations (i.e., constraints) and 336 single variables. Furthermore, even slightly larger test problems could not be solved in GAMS due to the limits on the number of variables and constraints, such as one problem with 15 customers, 3 DCs and 5 routes.

Hence, the exponential growth of the number of constraints and the existence of binary variables within the square root operator in order to define safety stock level in the formulation make the exact solution methods limited to small and medium size instances (up to 20-50 customers) [25]. An efficient heuristic algorithm is presented in the next section for large-sized problem instances.

4.2 Large-Sized Problems

Due to the inherent complexity of LRP and its NP-hard nature, using exact optimization methods to solve this problem is difficult. Therefore, exact solution approaches to the LRP have been very limited in the literature. As the problem size increases, heuristic procedures seem to be a better alternative.

Metaheuristics such as tabu search, simulated annealing, genetic algorithms, neural networks and ant systems were introduced to handle the complexity of combinatorial optimization problems. All of these metaheuristics aim to search the solution space more effectively than conventional approaches using different strategies. They show great promise in solution of difficult combinatorial problems such as the LRP as well as TSP, VRP.

Since Tabu search (TS) has been applied to both the facility location problems and various forms of the VRP and "it yields the best solutions to the VRP instances ([31], [32], [33]) studied in the past", we decided that TS can be used to solve our LRP with inventory control decisions which combines three different levels of decisions in an integrated supply chain model. We will adopt ideas from a two-phase tabu search algorithm proposed by Tuzun *et al.*[2] in order to solve our problem.

The remaining of this section is organized as follows. Section 4.2.1 describes basic features and fundamentals of tabu search as the solution method. Then, section 4.2.2 gives a brief summary of the TS algorithms from the past studies in the VRP literature that provided us with important insights on how to apply tabu search methodology to our location-routing problem. Finally, in Section 4.2.3 we explain the solution framework of two-phase tabu search algorithm which is used to solve our problem and its distinct features.

4.2.1 Tabu Search

Tabu search has traditionally been used on combinatorial optimization problems and frequently has been applied to many integer programming problems, such as routing and scheduling, traveling salesman and others. The basic concept of Tabu search is presented by Glover [34] who described it as “a meta-heuristic superimposed on another heuristic”. The overall approach is to avoid cycles by forbidding or penalizing moves which take the solution, in the next iteration, to points in the solution space previously visited (hence “tabu”).

According to Glover [35], Tabu search is composed of three primary features: (1) the use of flexible attribute-based memory structures designed to permit evaluation criteria; (2) a control mechanism for employing the memory structures based on the interplay between conditions that restrict and free the search process; and (3) the incorporation of different time horizons, from short term to long term to implement strategies for intensifying and diversifying the search.

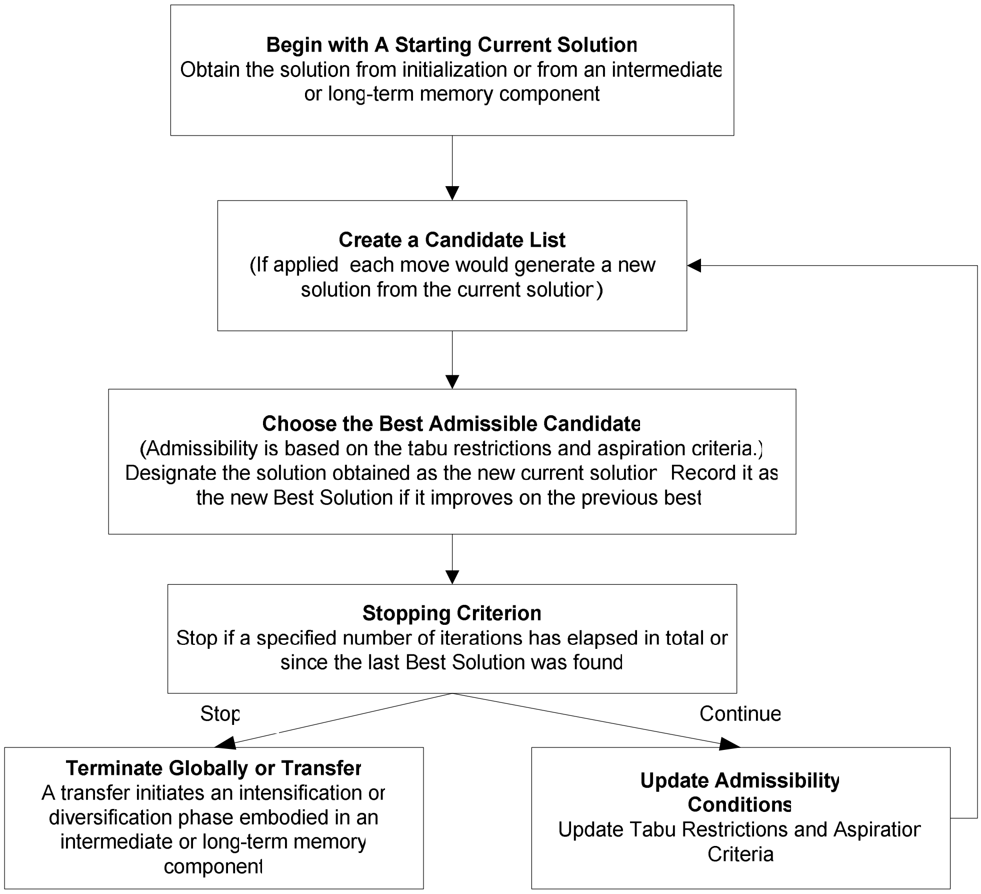

Figure 4.1 Basic Tabu Search Components (Adapted from Glover [35])

Tabu search begins by moving to a local minimum as shown in Figure 4.1. To avoid revisiting the steps used the method records recent moves in one or more tabu lists. The original aim of the list was not to prevent a previous move from being repeated, but rather to insure it was not reversed. Tabu lists are historical in nature and form the tabu search memory. The role of the memory can change as the algorithm proceeds. For initialization, in each iteration, the objective is to make a rough examination of the solution space, known as “diversification”, but as locations of the candidate solutions are identified, the search is more focused to produce local optimal solutions in a process of “intensification”. In other words, *diversification strategies* drive the search into new regions, while *intensification strategies* reinforce move combinations and solution features historically found good.

In many cases, various implementation models of the Tabu search method can be achieved by changing the size, variability of the tabu memory to a particular problem domain.

The main limitation of a local search method (i.e., the hill climbing procedure) is that it might stop at a local optima that might be far from the global optimum. As one of the heuristic approaches to overcome this shortcoming, Tabu search (TS) algorithm imitate an intelligent attitude by using an adaptive memory and can therefore avoid being entrapped at the local optima with the aid of a memory function.

In each iteration, tabu search explores the solution space by moving from a solution to the solution with the best objective function value in its neighborhood, even in the case that this might cause the deterioration of the objective. In order to avoid cycling, solutions that were recently examined are declared forbidden or "tabu" for a certain number of iterations (i.e., called *tabu tenure* or *tabu duration*) and associated attributes with the tabu solutions are also stored. The tabu status of a solution might be overridden if it corresponds to a new best solution, which condition is called "Aspiration criterion". There are groups of Tabu search methods that use either short term memory or intermediate and long term memory strategies. The recency-based memory functions require specifying the tabu tenure m and the frequency-based memory generally adds long term memory.

To define the basic steps of Tabu search algorithm, let the set $S(x)$ define a "neighborhood function" that consists of those moves from the current solution x to a next trial solution. Let T denote a subset of S that contain elements that are called "tabu moves" and "OPTIMUM" as the objective evaluation function. A basic version of the Tabu search algorithm without "aspiration" can be presented as follows:

Tabu Search Algorithm:

Step1. Select an initial $x \in X$ and let $x^* = x$.

Set the iteration counter $k = 0$.

Set the Tabu set $T = \phi$.

Step2. If $S(x) - T = \phi$, go to Step 4.

Otherwise, set $k = k+1$.

Select $s_k = S(x) - T$ such that $s_k(x) = \text{OPTIMUM}\ (s(x) | \ s \in S(x) - T\ )$.

Step3. Let $x = s_k(x)$.

If $c(x) < c(x^*)$, where x^* denotes the best solution currently found, let $x^* = x$.

Step4. If the number of iterations has reached the maximum user-defined iterations either in total or since x^* was last improved, or if $S(x) - T = \phi$ upon reaching this step directly from Step2, stop.

Otherwise, update Tabu set T and associated attributes and return to Step2.

Figure 4.2 Basic Tabu Search Algorithm in Pseudo-Code (Adapted from Glover [36])

As mentioned by Glover [36], by the preceding form of OPTIMUM, each execution in Step 2 moves from the current solution x to an s(x) that yields the greatest improvements, or if not improved, the least disimprovement in the objective function, subject to the restriction that only non-tabu moves are allowed. Put another way, tabu search algorithm makes a “best available move” at each step (like the greedy algorithms) (Figure 4.3).

Especially, when the “aspiration” is considered in a more advanced tabu search algorithm, if no improvements can be found in the current non-tabu lists but improvements can be made in the tabu moves lists, then one can allow tabu moves and let it override the rules.

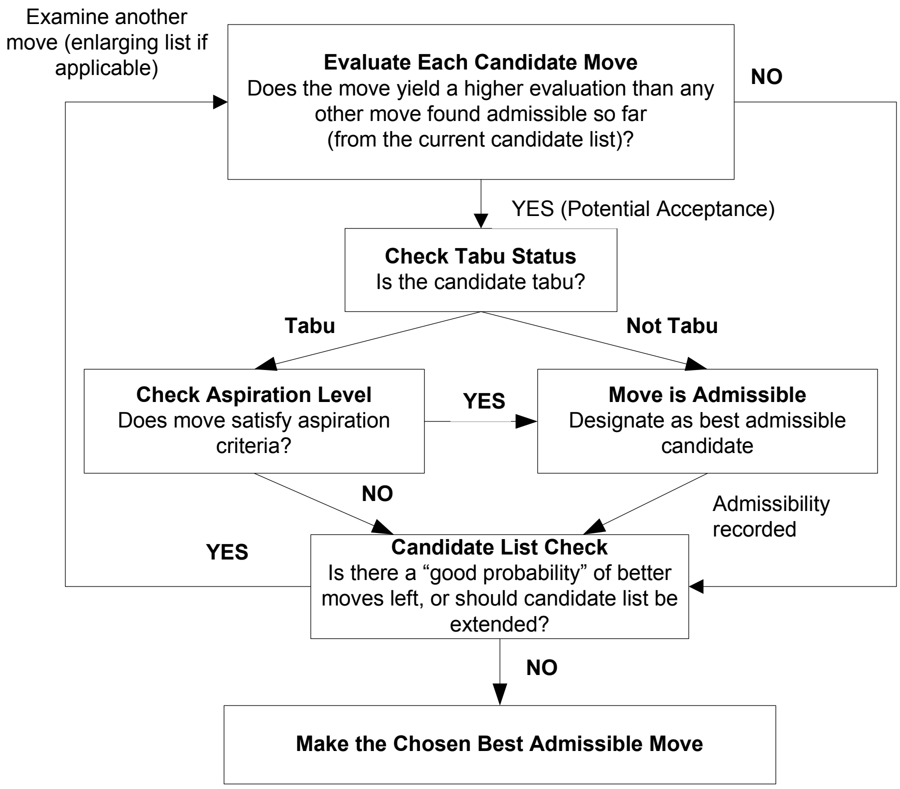

Figure 4.3 Selecting the best available move (Adapted from Glover [35])

For more advanced issues on tabu search, the study of Glover [37] can be referred. It involves new dynamic strategies to manage the tabu lists, and three newly developed methods to deal with integer programming problems as well as some new applications of tabu search.

4.2.2 Tabu Search Algorithms in the VRP literature

As mentioned in the first section, Tabu search methodology has a wide range of application areas in the O.R. literature. There are many applications of TS such as planning and scheduling, telecommunications, parallel computing, transportation, routing and network design, continuous and stochastic optimization and manufacturing. Detailed review of these applications can be found in Glover and Laguna [38].

TS methodology has been used for vehicle routing problems which are NP-hard problems. An extensive survey of the proposed VRP heuristic solution methods together with the comparisons of the computational results are presented in the book by Toth and Vigo [39].

In Osman [40], λ-interchange descent methods and new SA and TS algorithms on the best of the descent methods were developed. He concluded that TS is superior which facilitates a combination of 2-opt moves, vertex reassignments to different routes (insertion moves), and vertex interchanges between routes (swap moves). One of the important contributions to the VRP and TS literature was made by Taillard [31]. Here, two partition methods to speed up iterative search methods like the TS method were presented. The vertex set (customers) is divided into clusters separately through vertex moves from one route to another; and these clusters are updated during the algorithm execution. The experimental results were believed to be optimum for the given problems; besides, a later study by Gendreau *et al.* [32] provided also good solutions to the same problems using a newly developed algorithm called *TABUROUTE*. This new TS approach differs from the previous implementations in that in all the other algorithms an infeasible solution is never allowed, but with *TABUROUTE,* it is possible for a feasible solution to become infeasible in the next iteration in terms of the side constraints. However, there are two additional penalty cost terms to avoid the infeasibility in the solution. The results show that among the all existing ones, *TABUROUTE* produces highly competitive and good solutions on a set of common benchmark problems. With a later study, Taillard and Rochat [33] developed a probabilistic technique to diversify, intensify and parallelize a local search for the VRP. They showed that this technique can be applied to a wide variety of VRPs, and especially improves the TS approaches for VRPs. Cordeau *et al.* [41] proposed a tabu search heuristic to solve three different types of vehicle routing problems; the periodic vehicle routing problem, the periodic traveling salesman problem and the multi-depot vehicle routing problem. The algorithm is based on the GENI heuristic which is also a basic part of *TABUROUTE.*

Finally, the new algorithm developed in the study of Barbarosoglu and Ozgur [42], named with DETABA uses most of the tabu search principles developed previously, but introduces a new neighbour search procedure without any diversification and a new intensification criteria. Then, its performance is compared with the tabu search algorithms in the literature using the well-known benchmark problems. It was stated that DETABA in general overperforms all the algorithms except that of Taillard [31].

4.2.3 Two-Phase Tabu Search Solution Algorithm

As it is seen from the VRP literature, most promising algorithms facilitate the tabu search methodology. Since the VRP is one of the basic parts of the location-routing problems, we chose to apply a similar approach to our more complex problem as a heuristic solution method. Therefore, the solution algorithm for the LRP with inventory control considerations is a modified version of the TS algorithm proposed by [2] for typical LRPs.

To apply this algorithm to the different version of LRP with additional inventory control consideration, some relevant changes were made on the evaluation criteria of each phase in order to incorporate the effect of inventory cost on the solution selected in each iteration. Since the computational requirements of the algorithm in terms of CPU time increase as the problem size increases due to the somewhat recursive nature of it, we preferred to incorporate the inventory dimension into the existing two phase rather than implementing an additional phase. Additionally, in each phase while selecting a customer to perform either an insertion or a swap move, a randomization is added with an effort to make some jumps sooner and more accessible in the neighbourhood search, thus in each iteration a customer among the candidates is chosen randomly, not depending on any order.

Thus, the algorithm is still composed of two-phases: routing and location; however, in each phase, selection part is changed in terms of cost evaluation and the selection of customers. Together with these new features, the following issues must be pointed out to make the implementation clear.

i. the location and the average yearly demand and standard demand deviations of each customer is known (assume that each customer has a normally distributed demand),
ii. the location of each candidate depot site is known,
iii. the distribution system has a homogeneous fleet with a known capacity per vehicle,
iv. the system is relaxed in terms of depot capacities, that is, rather than depot capacity constraints, a new and more relaxed constraint is included, namely, the maximum vehicle service capacity,
v. the maximum service capacity of each vehicle is also known, which means that each vehicle can serve to an amount of customer demand up to this threshold value for one year, which equals the vehicle capacity times the maximum number of visits of a vehicle to the customers on a specified route.

Figure 4.4 represents the original form of the algorithm proposed by [2] on a flow chart. Moreover, the modified version of the algorithm with more detailed components is given in Appendix A on a flow chart.

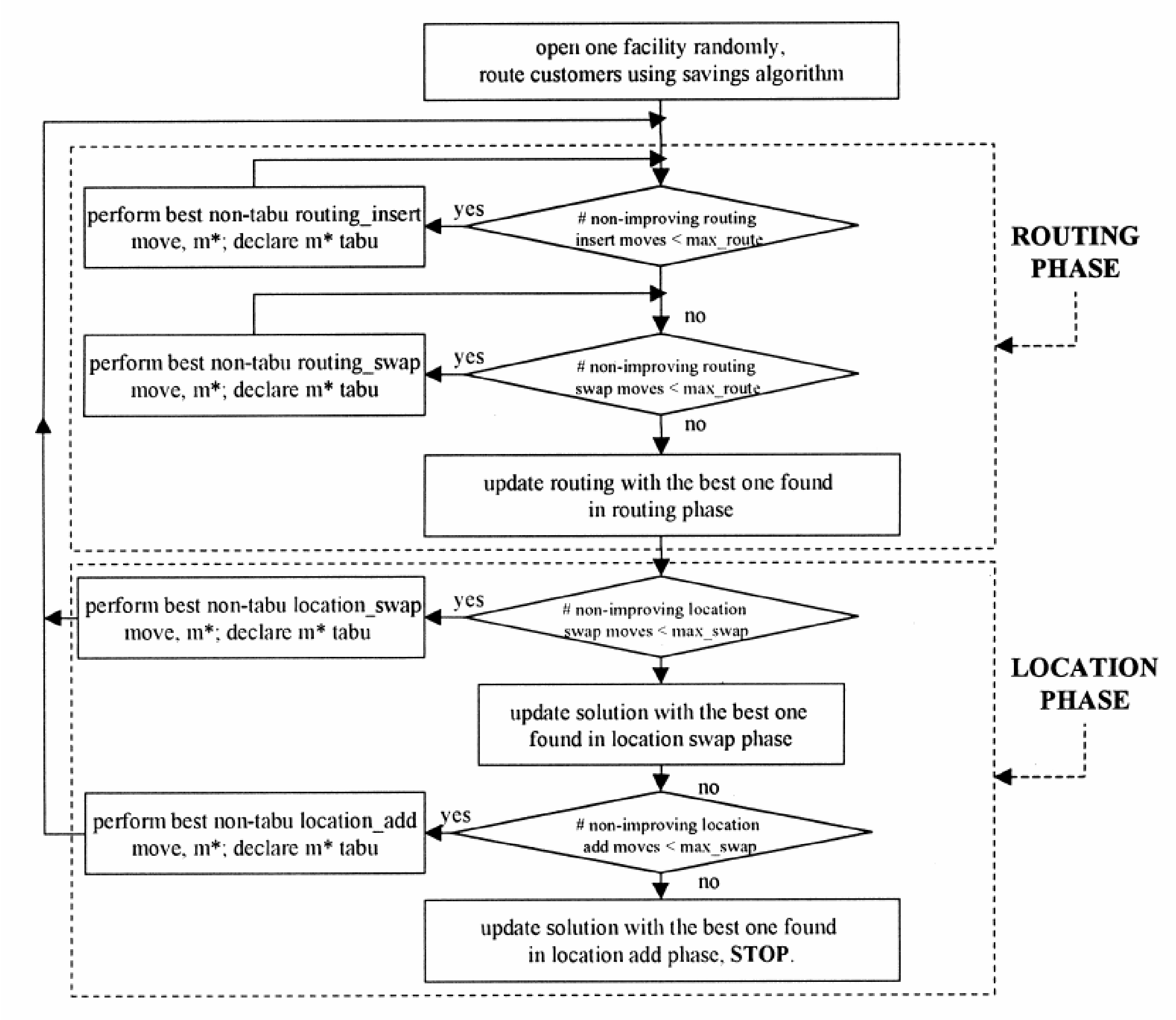

Figure 4.4 Flow chart representation of two-phase tabu search algorithm (Tuzun *et.al* [2])

In this algorithm, the route-first, location-allocation second approach is used first. Then, the improvement search is used based on the tabu search method. Tabu search is performed in two main phases to combine two main levels of decision: one is for location decisions and the other one is for routing decisions. Besides, the third level of decision is also considered within the main ones not as an additional main phase, but as a subphase encapsulated into the cost evaluation part of each phase. Since this is the case, the algorithm needs a large computational effort even in its two-phase form; we preferred to integrate inventory decisions not within a third phase, but within the existing two phases.

In the location phase of the algorithm, there are two consecutive sub-phases in each of which a TS is performed on the location variables to find a good constellation of depots to be used in the distribution. Then, consecutively, in two sub-phases of the routing phase, another TS is performed on the routing variables for each of the location configurations found in the location phase in order to obtain a good routing for the given configuration.

Tuzun *et al.* [2] state that “This two-phase approach offers a simple and natural representation, since the LRP is decomposed into two subproblems in terms of two different types of decision variables”. However, to deal with the ignorance of the interdependency among location, routing and newly added inventory decisions in sequential approach, in each phase neighbourhood search is coordinated so that the solution space can be efficiently explored.

Once a move is executed on the location phase, the routing phase is started in order to update the routing based upon the new depot arrangements. Fortunately, since only a certain part of the customer routings is influenced by the change in the depot arrangement, it is possible to reduce and limit the search with only this part. Thus, Tuzun *et al.* [2] say that “the routing phase is a localized search, as opposed to a global exploration of all routing moves”. This structure of the routing phase enables us to disregard a lot of unnecessary computation, thus to find good solutions within reasonable computation time.

One of the main differences in TS procedures applied in routing phase from the ones in the location phase is the presence of aspiration criteria. These criteria can be specified differently for different TS procedures, but in this algorithm for both subphases in the routing phase the aspiration criteria is as follows:

Aspiration criterion: A tabu routing move can be executed if it is a profitable move, i.e. if the cost of the move is negative. As it was stated in [2], an aspiration criterion for the location moves is not used since the move evaluation value is only estimation, and does not reflect the exact cost of the move.

Additionally, tabu attributes and tabu durations are defined separately for routing and location phases in the following manner.

Tabu attributes being recorded:

i. there is one tabu attribute for insert moves in the routing phase, that is the customer being inserted,
ii. there is one attribute for add moves in the location phase, that is the facility being added,
iii. there exist two tabu attributes for the swap moves in the routing phase, those are the two customers being swapped,
iv. there exist two tabu attributes for the swap moves in the location phase, those are the two facilities being swapped,

Besides, like in [2], a probabilistic tabu duration approach is used for the attribute(s) that are declared tabu to reduce the possibility of cycling. Both the location and the routing attributes are declared tabu for a tabu duration that is generated uniformly from an interval. However, the lengths of the intervals for the location and routing moves are different since the number of candidate depots is usually much smaller than the number of customers.

Tabu durations:

i. the tabu durations for the location attributes, which is referred to as *tabu_duration_location*, are set relatively shorter than those for the routing attributes which is referred to as *tabu_duration_routing*.

The detailed procedure for the tabu-search method is as follows:

Step 1. Open one depot randomly, route customers using savings algorithm,

Step 2. Is the number of non-improving routing insert moves less than max_route? If yes, go to Step 3. Otherwise, go to Step 4.

Step 3. Perform best non-tabu routing_insert move, m*, declare m* tabu and go back to Step 2.

Step 4. Is the number of non-improving routing swap moves less than max_route? If yes, go to Step 5. Otherwise, go to Step 6.

Step 5. Perform best non-tabu routing_swap move, m*, declare m* tabu and go back to Step 4.

Step 6. Update routing with the best one found in routing phase.

Step 7. Is the number of non-improving location swap moves less than max_swap? If yes, go to Step 8. Otherwise, go to Step 9.

Step 8. Perform best non-tabu location_swap move, m*, declare m* tabu and go back to Step 2.

Step 9. Update solution with the best one found in location swap phase.

Step 10. Is the number of non-improving add location moves less than max_swap? If yes, go to Step 11. Otherwise, go to Step 12.

Step 11. Perform best non-tabu location_add move, m*, declare m* tabu and go back to Step 2.

Step 12. Update solution with the best one found in location add phase.

Step 13. Stop.

Now, we can define the elements of the modified TS algorithm in detail.

a. Initialization

The algorithm is initialized making one randomly selected facility open, and all of the other candidate facilities closed. Because of it is speed and simplicity, this initialization has been chosen. In fact, any other initialization procedure may be used to initiate the TS algorithm. In particular, if the minimum number of facilities to open is known in advance, the algorithm may be initialized with this lower bound. Then, each customer is assigned to the nearest open facility. Given the allocation of the customers to the facilities, a separate routing problem is solved for each open facility.

In our case, the algorithm is initiated with one open facility, and then all customers are simply assigned to this facility. To obtain the initial routing for the open facility; we use the original savings algorithm of Clarke and Wright, [44] to allocate customers to routes, finally, to improve the resulting routes, a simple 2-opt procedure from Lin, [45] is used. The original savings algorithm is described in [46] briefly as follows:

Savings Algorithm:

This algorithm is used to assign customers to vehicles even when delivery time windows and other constraints exist. The steps of the algorithm are:

***Step* 1.** Identify the distance between every pair of locations to be visited given by $Dis(A,B)=\sqrt{(x_A-x_B)^2+(y_A-y_B)^2}$ where point A with coordinates (x_A , y_A) and point B with coordinates (x_B , y_B).

***Step* 2.** Construct the savings matrix in terms of distance. It is calculated by the following formula: S(x,y)= *Dist(DC,x)* + *Dist(DC,y)* - *Dist(x,y)* where *Dist*(.,.) is the distance matrix defined in Step 1.

***Step* 3.** Initially, assign each customer to a separate route. Then, combine two routes with the highest savings value into a feasible route if the total delivery amount of both routes does not exceed the vehicle's capacity. Continue, until no more combinations are feasible. Stop.

After finding feasible routes, customers in routes are sequenced in order to minimize the distance each vehicle must travel by 2-opt route improvement procedure. The basic features of this procedure can be summarized in the following section.

2-Opt Route Improvement Procedure:

Suppose a subtour consists of the following set of S vertices in the given order $S = \{v_0, v_1, v_2,...., v_k, v_0\}$, and let $X = \{(v_i, v_{i+1}); (v_j, v_{j+1})\}$ be a set of two edges in S which are to be replaced with edges $Y=\{(v_i, v_{i+1}); (v_i`1, v_{j+1})\}$ if this replacement will lead to an improvement. Here it is required that all the vertices under consideration be different from each other. Once the set X has been chosen, the set Y is directly determined. In a subtour consisting of k customer vertices and a depot, there are $[(k+1)(k-2)/2]$ possible edge combinations given by the set E. The 2-opt algorithm can be summarized as follows:

Step 1. For each $X \in$ E, calculate the improvement δ_x obtained by replacing X by the associated Y and given by $\delta_x = (c_{i,i+1} + c_{j,j+1}) - (c_{ij} + c_{i+1,j+1})$

Step 2. Calculate δ_{max} by $\delta_{max} = \max\{\delta_x\}$

Step 3. If $\delta_{max} > 0$, replace the two edges associated with δ_{max} and repeat this for all X in E.

Indeed, the resulting routes from replacement of all possible sets of X with all possible sets of Y are equivalent to the routes resulted from pairwise exchanges of each possible nodes (i.e., customers) in each route. At the end of the initialization part, an initial solution is obtained in which all customers are served by the same depot through different routes.

b. Routing Phase

At the end of the initialization phase, all customers are assigned to the only open depot through the routes found by savings and 2-opt algorithms. Since all the customers are served by a single depot, firstly all customers are potentially reroutable. However, in all the remaining part of the algorithm, only a set of customers which meet a pre-defined condition can be re-routed.

Each swap or add move in the location phase is followed by a routing move, since customer routes will change after the depot arrangements change. Then, the routing phase is started from the best routing found for the previous facility configuration in the corresponding location phase in order to update the routing according to the currently found facility configuration.

At the end of each location phase, each customer is reassigned to the closest open facility. For each open facility, the number of changes to the customer allocation that is referred to as Δc, is recorded. Δc includes the customers that are previously assigned to a different facility but currently reassigned to this facility, and the customers that are previously assigned to this facility but currently reassigned to another facility after the corresponding location move. Δc measures how much a facility is influenced by the location move. If Δc value is greater than a threshold value Δc_max, then the customers that are currently served by this facility should be included in the routing phase; i.e., its customers are considered for the routing moves. On the other hand, if Δc value does not exceed the threshold, customers newly assigned to this facility are simply inserted at the best position available, and the ones that are reassigned to other facilities are deleted from its routes. Furthermore, the routing for this facility with Δc value within limits remains unchanged during the routing phase. Thus, the threshold criterion disregards many irrelevant routing moves from consideration, and then results in the computation time reduction. Two set of moves are performed sequentially in the routing phase: insert moves, and swap moves.

b.1 Insert moves

Firstly, a customer, who is assigned to a facility that exceeds the threshold Δc_max, is randomly chosen, and inserted to a new position on a route originating from its current facility, or any other open facility within a specified proximity to the customer. That is, to consider a facility for insertion, the set of the f_max open facilities closest to the customer should involve this facility. Like the reason lying behind the usage of Δc_max threshold value, using this threshold value f_max eliminates many nonprofitable moves which try to insert the customer to facilities that are not close to the corresponding customer. Contrasting with the location moves, routing moves (i.e., insertion and swap moves) are very straightforward to evaluate. For instance, the actual cost of each move is calculated using the difference in the route length (i.e., new length minus old length) and the difference in the inventory cost provided that a customer is assigned to a new position on a route different from its current route, then the move which yields an improvement in the total system cost (i.e., cost reduction) is selected as the first best move and executed.

All admissible insertion moves (i.e., all non-tabu moves and some tabu moves which met the aspiration criteria) that insert randomly chosen customers, which are currently assigned to the facilities exceeding the threshold Δc_max, to one of their f_max closest facilities are evaluated with respect to the order of their appearance on the routes.

Osman [40] reports that the hill climbing approach overperforms the steepest descent approach for his TS implementation on a single depot VRP. Therefore, in our algorithm, once a profitable move is found, it is executed, and applying an insert move to this customer is declared tabu for a number of iterations which is referred to as *tabu_duration_routing*.

If no profitable move is found after all of the admissible moves are evaluated, the nontabu move with the least cost is performed. This first best admissible move approach is usually as effective as the steepest descent (i.e., the best admissible move) for TS applications, and it requires less computational effort.

As the stopping criteria in TS approach, after a pre-defined *max_route* number of iterations is performed without improvement after the best solution found, insertion moves are terminated. This subphase is followed by a set of swap moves.

b.2 Swap moves

In this subphase, the positions of any two customers that are currently assigned to a facility that exceeds the threshold Δc_max are swapped. A randomly chosen customer can only be swapped with one of its c_max closest customers to avoid doing unnecessary computation. Since these closest customers may be assigned to the same, as well as different facilities, swap moves can be between customers that are assigned to the same facility, or between those assigned to different facilities. All admissible swap moves (i.e., same with the insert moves, all non-tabu moves and some tabu moves which met the aspiration criteria are considered as admissible.) over the customers of the facilities with excess Δc values, with their c_max closest customers are evaluated in the order of their appearance on the routes. Similar to insertion move, choosing the first best admissible move approach is applied on the swap moves also. That is, a profitable move is performed once it is explored.

When a swap move is executed, swapping these two customers is declared tabu for a number of iterations which is equivalent to the *tabu_duration_routing* in the previous routing insert move. Finally, swap moves are also terminated after the previously defined stopping criteria; *max_route* numbers of routing iterations are performed without improvement.

After the swap moves are performed and the stopping criterion is met, the search updates the best solution found in the routing phase, and continues with the best solution, and resumes to the location phase with a location swap move.

c. Location Phase

Similar to the routing phase, there exist two different types of moves to go from one facility configuration to the other: these are swap moves and add moves. The location phase first performs swap moves, and then performs add moves for a given number of facilities. Swap moves are very similar to the swap moves of the routing phase with a difference in the swapping attributes, that is, two customers' positions are being swapped in the routing phase, whereas in the location phase two depots' status are being swapped. Similarly, add moves of the location phase is a bit different from the insert moves of the routing phase so that add moves make an additional depot open, while insert moves make the position of one customer different.

As previously mentioned, at the end of each location move, the customers are reassigned to the closest open facility. For each open facility, the number of changes to the customer allocation (Δc) is recorded. Δc is described in early parts of the section. Moreover, since the facility's routing will be mostly affected by the changes to its customer allocation, the customers assigned to this facility are rerouted using the savings algorithm at the end of the location phase.

c.1 Swap moves

This type of location moves close one of the open facilities, and open one that is currently closed simultaneously. While performing the swap moves the number of open facilities in the solution remains constant during the search of a good configuration for a certain number of facilities.

At each iteration, the location phase searches for the first best admissible swap move (i.e., only the non-tabu moves are permitted) to perform. Thus, in order to select the best move, the cost of a swap move is calculated as the sum of the difference in the fixed cost (i.e., when we open one facility and close another it equals the fixed cost of the facility to open – the fixed cost of the facility to close), the difference in the routing cost and the difference in the inventory cost. Unfortunately, the difference in the routing cost is difficult to estimate. To deal with this, we use the same approach taken from [2], in which each customer is assigned to the closest open facility, and then the difference in routing cost is estimated as the difference in the direct distance between the customer and the facility according to the new and old assignments. Once the new assignments are made, Δc value of each open facility is calculated and recorded. Thus, the swap move evaluation is the sum of this routing cost estimate, the difference in the fixed cost and the difference in the inventory cost (i.e., safety stock and cycle stock cost). The swap move which yields the first best evaluation is then performed, again if no profitable move is found after all of the admissible moves are evaluated, the nontabu move with the least cost is performed. Then, both the move and its reverse are declared tabu for a number of iterations, *tabu_duration_location*. After the swap move is performed, the search returns to the routing phase to update the routing according to the resulting swap move. Swap moves are applied until a *max_swap* number of nonprofitable moves (moves with positive cost) are completed. Then the swap moves are terminated and the best solution found at the end of the swap moves, are updated and the search continues on this best solution by entering the location add subphase.

c.2 Add moves

Applying an add move increases the number of facilities. An add move opens one of the currently closed facilities, and hence increases the number of facilities by one. The facility to be added is the one whose addition yields the minimum estimated cost at the end of the move evaluation process. Then, the customers are reassigned to the closest open facility. For each open facility, again Δc value is calculated and recorded.

Similar to the swap moves, the routing cost is estimated using the difference in direct distances for the customer assignments before and after the add move. Since opening a facility can only increase the routing cost estimate, this difference is always negative. The fixed cost of the facility to be opened and the difference in the inventory cost are then added to the routing estimate in order to calculate the overall cost estimate. Then, the search process with the best solution found in the add move returns to the routing phase in order to update the routing after the add move. After one add move, the search continues with a series of routing insert, routing swap and location swap moves until the termination criterion is satisfied. As in the swap move, once the stopping criterion is met after a *max_add* number of add moves without any improvement over the best objective function value are completed, the add moves are terminated as well as the overall TS algorithm. Finally, the best solution found in the location-add phase becomes the best solution found in the overall algorithm.

This termination criterion, *max_add* forces the search to terminate without exploring network configurations with more facilities than necessary. Since the number of facilities to be opened is not known at the beginning of the add moves, without such termination criterion, the search would continue until solutions with all candidate facilities open are explored. The number of candidate locations is typically much larger than the number of facilities required to be opened, therefore starting with one facility, and using the *max_add* threshold for termination, eliminates the search of many undesirable configurations.

In Tuzun *et al.* [2], it is stated that "An important feature of the location phase is the separation of the swap and add moves. Since the cost of a location move (swap or add) is only an estimate, it does not reflect the trade-off between the fixed cost of opening facilities, and the routing cost from those facilities to the customers. Therefore, if moves adding, dropping or swapping facilities are allowed at each iteration, without a precise estimate of the costs, the TS may lead to too few or too many facilities. Evaluating the moves that change the number of facilities separately from the rest reduces the error that is caused by cost estimation."

The aim of the TS algorithm introduced above is to explore the solution space of the LRP in accordance with an intelligent solution methodology. The exploration of nonpromising facility configurations is limited by terminating the process after a pre-defined number of nonprofitable location moves is performed. Secondly, each time the routing phase is started with a good routing solution for the previous facility configuration. Since only a part of the routing is changed by one location move, the routing phase does not require excessive computation each time it is restarted.

Chapter 5

COMPUTATIONAL RESULTS

This chapter focuses on assessing the effectiveness of the modified tabu search algorithm through the numerical experiments. The modified algorithm is coded in Java programming language.

The performance of the algorithm is evaluated in two dimensions: the solution quality, that is, the best solution found, and the computational efficiency in CPU time. A set of common benchmarking problems is solved with the help of our modified algorithm, and then the resulting best solutions found are compared with the results found in the past studies. Then, two average statistics are reported: % over best (i.e., percentage deviation over the best solution found) and computation times in minutes. Furthermore, the results are examined based on the changes in a group of problem parameters to have a better understanding on the interdependency between three different levels of decisions.

This chapter is organized as follows. Firstly, in Section 5.1, the effects of the additional inventory decisions and the related cost issues on the objective function and the network configuration found in the best solution are examined. Then, Section 5.2 presents the network representation and algorithm implementation details of the Java programming codes for the proposed methodology for solving the LRP with inventory decisions. Section 5.3 discusses the comprehensive numerical results of the proposed solution method over a set of the benchmark problems. Lastly, sensitivity analysis on the problem parameters is conducted in Section 5.4.

5.1 Effects of the Inventory Decisions

As mentioned before, although the interdependence among the location, transportation and inventory decisions was recognized, almost all warehouse location-routing problems ignored the effect of inventory decisions. Therefore, in this part, the presence of this relationship is examined through a small well-known test problem, which is the Perl's test problem with 12 customers and 2 possible warehouse sites. The locations of the warehouse sites and the customers are indicated in Figure 5.1 as below:

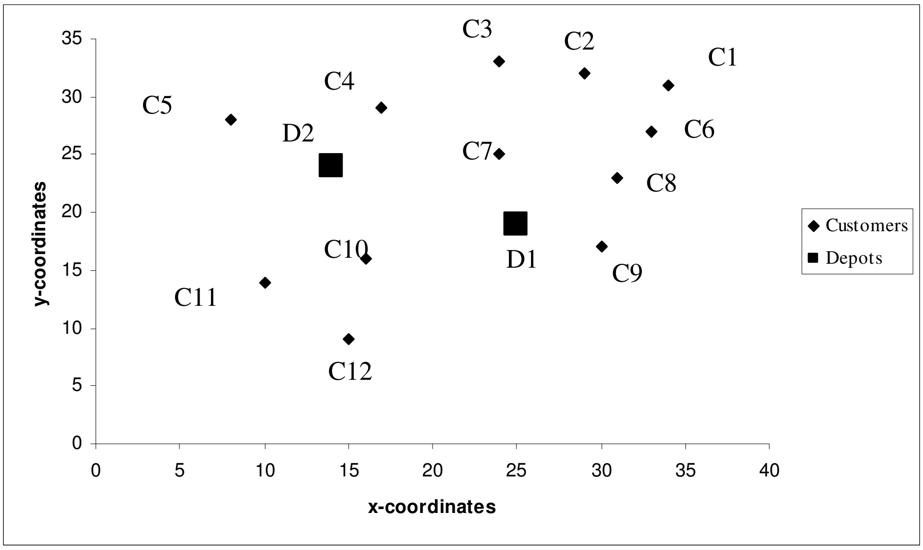

Figure 5.1 Location configuration for the test problem Perl-12

Table 5.1 gives the corresponding full coordinates of the customers and candidate depot sites.

Table 5.1 Coordinate data for 12 the test problem Perl-12

Customers/ Depots	*x-coordinate*	*y-coordinate*	*Demand*
C1	34	31	20
C2	29	32	20
C3	24	33	20
C4	17	29	20
C5	8	28	20
C6	33	27	20
C7	24	25	20
C8	31	23	20
C9	30	17	20
C10	16	16	20
C11	10	14	20
C12	15	9	20
D1	25	19	-
D2	14	24	-

Additionally, a summary of the other relevant parameters used in both the test problems with inventory and without inventory is given in Table 5.2.

Table 5.2 Parameters used in test problems

	Perl's Problem without inventory	Perl's Problem with inventory
Vehicle capacity	140	70
Vehicle service capacity	-	140
Depot capacity	280	-
Fixed establishing cost	100	100
Variable warehousing cost	0.74	0.74
Cost per mile	0.75	0.75
Cost per vehicle	-	25
Cost per order	-	20
Holding cost	-	0.5

In the above table, since some parameters are newly defined to incorporate inventory into the original Perl's problem in order to see the effects of them, these new parameters are not applicable for the original problem. On the other hand, since our algorithm considers a maximum service capacity rather than a depot capacity to better reflect the replenishment effect, and the vehicle capacity is used only for determination of the order quantity, some modifications are made on these three capacity parameters while solving the problem together with the inventory decisions.

The resulting best solutions found by the modified TS to the original problem without inventory and the problem with inventory, are shown respectively, on the following network configurations in Figure 5.2 and Figure 5.3.

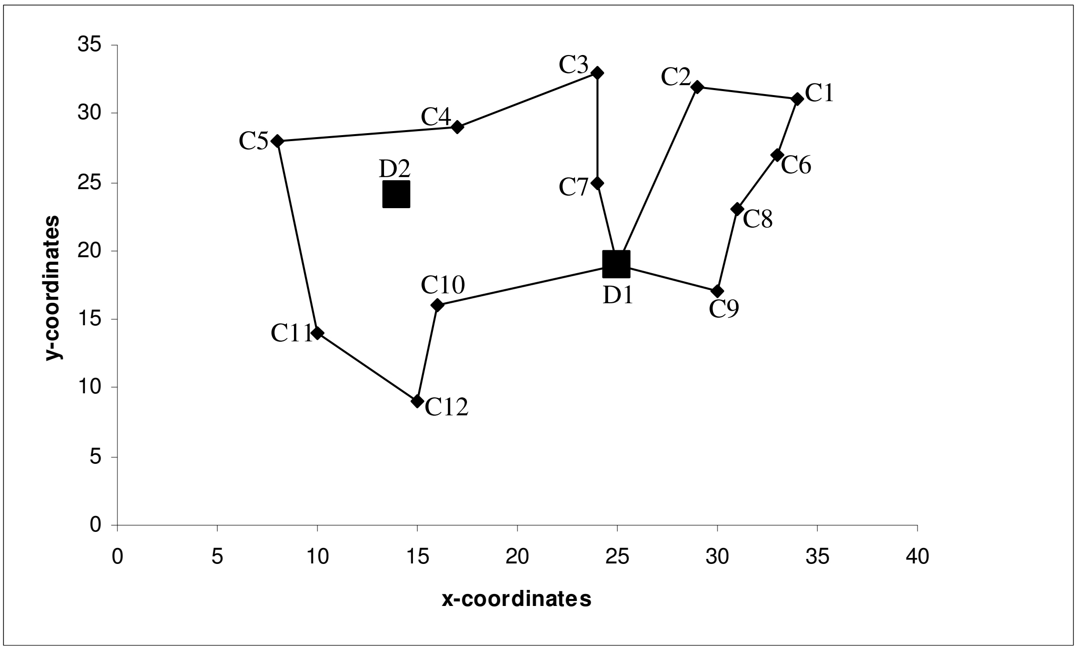

Figure 5.2 Best solution found to the problem Perl-12 without inventory

As seen in Figure 5.2, only one of the candidate depots, Depot#1 is used with two routes. The customer visits in each route can be summarized such that one of the routes originating from Depot #1 starts with customer 9, then visits customers 8, 6, 1 and 2 consecutively; the other route visits all the remaining customers 10, 12, 11, 5, 4, 3 and 7 sequentially. The objective function value is found as 360.446. This total cost value is composed of fixed establishing cost of 100, variable warehousing cost of 177.6, and routing cost of 82.85.

The problem is solved again after adding the inventory part with previously mentioned parameters. Then, the solution to this problem is given as follows:

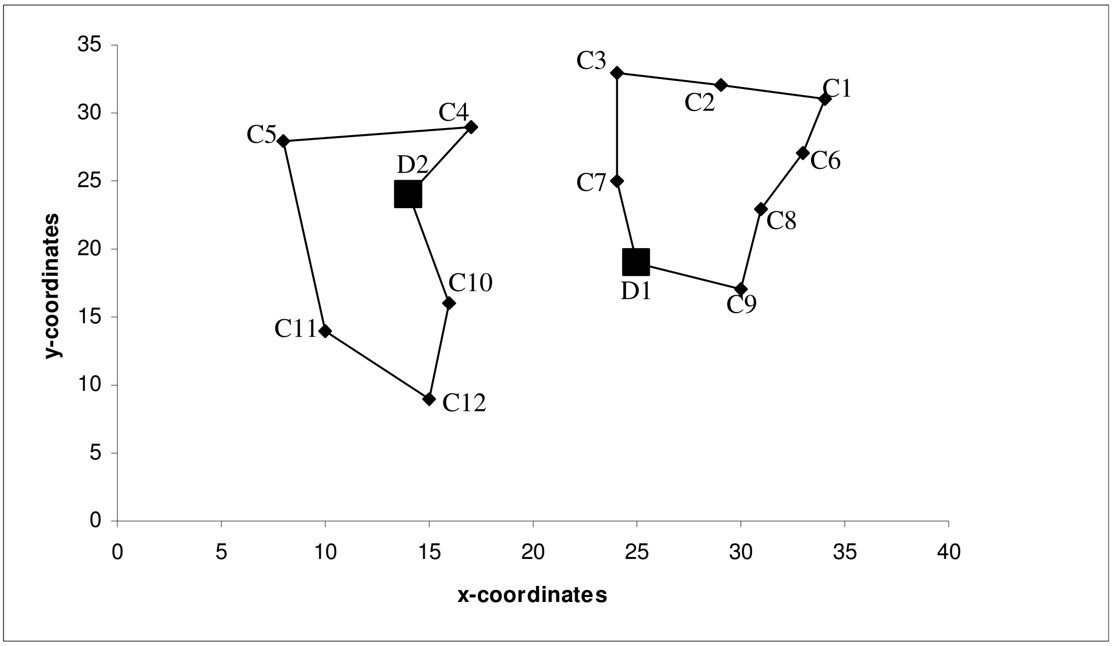

Figure 5.3 Best solution found to the problem Perl-12 with inventory

Unlike the solution to the original problem without inventory, the solution for the second one with inventory part given in the above figure uses both of the depots from each of which one route originates. Depot#1 serves to the customers 9, 8, 6, 1, 2, 3, and 7; and Depot#2 serves to the other customers in the order of customers 4, 5, 11, 12, 10 and 2. The objective value is 688.491. The fixed cost component is 200, the variable cost is again 177.6, inventory cost is 103.571, and as the last components, the routing cost is 207.320.

As it is seen from the results, the inventory cost component makes an important contribution to the total system cost as much as the other cost components. For instance, by incorporating the inventory component, the fixed depot establishing cost changes as well as routing cost. Furthermore, the solution found for the original problem differs from the solution for the problem with inventory part in terms of the final network configurations as well as open depots and the corresponding routes.

5.2 Computer Implementations

The fundamental network input data include all the information for customer locations and demands, and candidate depot location data for the current studied network. Given the above information, the data structures are developed to organize this data so that the Java program can be used to design the desired network as good as possible. To facilitate the implementation, eleven classes are defined in the Java programs:

- *Application:*

This class is the main class of the algorithm in order to execute the algorithm in which the customer set and depot set are generated according to the input data. Then, four basic methods are defined for *routing-insert*, *routing-swap*, *location-swap*, and *location-add* subphases. Finally, these methods are executed sequentially in the written order. Another important point is that in the location-swap method the previous two methods of routing phase are recalled; besides in the location-add method all of the other three methods are recalled.

- *Customer:*

A specific *numeric ID*, *x* and *y* coordinates, *mean demand*, *standard deviation*, *the closest depots set* and *the closest customers set* as well as *the depot data* (i.e., location, route number and position within the corresponding route) to which each customer is assigned are recorded for each customer. Additionally, a number of methods required to make operations over the customers are defined and stored in this class.

➤ *Depot:*

In this class, some data are stored as constant data; and all the remaining data are recorded as temporary data in the forms of *best solution set data*, *best of worst iteration data*, *iteration data*, and *previous iteration data*. Constant data are a specific *numeric ID*, *x* and *y* coordinates, *fixed* and *variable costs*; whereas *state data* to indicate whether the corresponding depot is open or not, and *routes* as double linked lists belonging to each depot are stored as temporarily.

➤ *Taboo:*

Four different tabu methods are defined here. In routing-insert tabu list the customer inserted in each iteration; whereas in routing-swap list two customers being swapped in each iteration are stored. Similarly, location-add list keeps the depot being open in each iteration, and in location-swap list two depots being swapped are kept. In addition, according to the pre-defined tabu durations for routing and location attributes, tabu iterations are stored.

➤ *Constants:*

Apart from the customer-specific and the depot-specific parameters defined in the previous classes, there are some relevant constant cost and capacity (i.e., vehicle capacity and vehicle service capacity) which are defined and recorded in this class.

➤ *SavingsMatrix:*

This class is defined for only a special method, called "Savings Algorithm to route the assigned customers to the corresponding depot" previously defined in *Depot* class.

Furthermore, the functions that are developed to implement the solution algorithms have been successfully tested and numerical results are presented in the next two sections.

5.3 Benchmark Results with Other Heuristics

In this section, numerical experiments were conducted to evaluate the performance of the modified TS algorithm; and the results were compared with three existing heuristic methods: Liu & Lee's algorithm in [4], Tuzun & Burke's algorithm in [2], and Srivasta's algorithm in [16]. The results are evaluated in terms of the solution quality (i.e., the best solution found) and computation time (in minutes). Since there does not exist a study on exactly the same problem with that considered in this thesis in the past studies except the Liu & Lee's study of [4], the other two algorithms which were used in comparison tests are algorithms proposed for solving typical LRPs, hence do not include inventory component within the defined problems.

The modified TS algorithm was coded in Java; whereas the results for all the other three heuristic methods were adapted from [4]. Tests were carried out on a PC: a Pentium(R) 4 of 1.00 GB RAM clocked at 1.70 GHz under the operating system Windows XP. The evaluation was performed based on all possible combinations of problem parameters and algorithmic parameters. The detailed parameter values are listed in Table 5.3.

In the experiments, the vehicle capacity is examined at two different levels of 150 and 300 units; the vehicle service capacity is examined at two levels: 3000 and 4000 units for each route. Furthermore, the number of customers was set at levels 100, 150 and 200, whereas the number of depots was set at 10 and 20. To observe the effect of different cost levels, two main cost structures as the combination of three different types of cost issues (i.e., fixed yearly depot establishing costs, cost per mile, holding cost) were generated such that "Low Cost Structure" as a combination of three cost components as 600:1:0.5; and "High Cost Structure" in the form of 1200:2:0.5.

Table 5.3 Parameters and their levels used in the experimental design

Parameters	Levels
Number of depots	20, 10
Number of customers	100, 150, 200
Vehicle service capacity	3000, 4000
Vehicle capacity	150, 300
Cost structure (Depot establishing cost: cost per mile: holding cost)	600:1:0.5 (Low) 1200:2:0.5 (High)

We consider all combinations of the above parameters. Totally there are 48 problem sets (i.e., 3 x 2^4) for each of which 3 test problems were generated. Thus, we obtained 144 problems in total. For all of these instances, the average yearly demand of each customer was generated from a uniform distribution in the range [450,600]. Standard deviation of customer demands is calculated by the formula of $\sigma_i = \sqrt{AD_i}$ where AD_i is the average yearly demand of customer *i*, and σ_i is the standard deviation of customer *i*'s demand, and the lead time L is set to 10 in days (i.e., 10/365 in years). The location (x- and y-coordinates) of each customer and candidate DC is randomly selected from a uniform distribution U[0,100]. The vehicle dispatching cost is 25 for each time. The holding cost is 0.5/unit/year; the ordering cost is 20 for each order; the desired customer service level (i.e., CSL) is set to 0.975 for which the corresponding $F^{-1}_L(CSL)$ value equals 1.96. Table 5.4 gives a summary of these mentioned constant cost values.

Table 5.4 Constant cost values used throughout all the problem instances

Cost Parameters	Values
Ordering cost	20
Vehicle dispatching cost (Cost per vehicle)	25
Holding cost	0.5

Additionally, since the TS algorithm requires the setting on the relevant threshold parameters, these threshold values were set to the corresponding values of *max_add* = 1, *max_swap* = 3, *max_route* = 1, *f_max* = 4, *c_max* = 8, and *Δc_max* = 0.5*(average number of customers per route). Tabu durations were drawn randomly in each iteration from a uniform distribution interval which was defined differently for location attributes (i.e., depots) and routing attributes (i.e., customers). The interval for the location attributes was defined as U[5,8], and for routing attributes as U[10,13].

To compare the results of the modified algorithm with three previously mentioned heuristic method, the average solution values of three test problems for each problem set were taken, and for the comparison of the computation times average computation time of three problems were taken for each problem set. The computational results of 48 problem set in terms of solution quality (i.e., the objective function value or the total system costs) and computation time are reported in Table 5.5. In this table, cost values given for each algorithm represent the total system costs. However, some components of total system costs vary with respect to the algorithms. That is, in our modified algorithm, the total system cost is composed of fixed depot establishing cost, variable warehousing cost, and transportation cost, inventory holding cost and ordering cost. Among other three algorithms, only Liu and Lee's algorithm has a similar total system cost structure in which the total system cost is the sum of fixed establishing cost, transportation cost, inventory holding cost, ordering cost and shortage cost. The difference of their algorithm from our algorithm is that there is a shortage cost component and they did not consider a variable warehousing cost. On the other hand, Tuzun and Burke's algorithm calculates the total system cost as the sum of transportation cost, fixed depot establishing cost and a vehicle dispatching cost. They did not consider variable warehousing cost, inventory holding cost and ordering cost in their cost calculations. Like Tuzun and Burke's algorithm, Srivasta's algorithm considers only fixed depot establishing cost and transportation cost, but disregards variable warehousing cost, inventory holding cost and ordering cost.

According to the results shown in Table 5.5, for all the problem sets except two sets of problem, the modified TS algorithm overperforms all the other three heuristic methods' results in terms of the total system cost found in the best solution. For the mentioned two problem sets, the algorithm were resulted in worse cost value than that of

Liu & Lee's algorithm, but still better than the original TS algorithm of Tuzun & Burke, and Srivastava's algorithm. Furthermore, due to the presence of randomization and additional cost computation of inventory part in each iteration, a significant difference exists in computational effort in CPU minutes, that is, for all cases the modified TS algorithm runs during a huge amount of time.

To better reflect this cost reduction over the best one, Table 5.6 gives the percent deviation (i.e., % savings achieved by the modified TS method) over the best solution value found in other three methods.

Table 5.5 Computational results of all the problem instances

Number of Depots	Number of Customers	Vehicle Service Capacity	Vehicle Capacity	Cost Structure	Modified TS		Liu & Lee's Algorithm		Tuzun & Burke's Algorithm		Srivasta's Algorithm	
					Cost	CPU	Cost	CPU	Cost	CPU	Cost	CPU
10	100	3000	150	Low	47515	20.60	68014	0.15	84628	0.18	87166	0.10
10	100	3000	300	Low	25773	28.39	37721	0.12	44055	0.15	45577	0.08
10	100	4000	150	Low	53639	16.79	78826	0.17	96639	0.27	99938	0.13
10	100	4000	300	Low	29401	24.78	41870	0.15	49710	0.23	51102	0.13
10	150	3000	150	Low	62582	47.39	94155	0.18	110828	0.28	114052	0.15
10	150	3000	300	Low	34567	49.79	51586	0.17	57961	0.37	59750	0.15
10	150	4000	150	Low	69227	59.95	107008	0.25	132494	0.42	136468	0.18
10	150	4000	300	Low	37416	55.43	56312	0.23	68170	0.47	70515	0.18
10	200	3000	150	Low	79578	79.46	118349	0.23	136686	0.42	141786	0.22
10	200	3000	300	Low	44030	65.87	64464	0.27	71663	0.33	75113	0.20
10	200	4000	150	Low	86633	67.62	132250	0.23	157479	0.47	162103	0.23
10	200	4000	300	Low	48009	67.97	70419	0.27	81270	0.45	83108	0.23
10	100	3000	150	High	74998	28.59	78638	0.12	100548	0.15	102565	0.08
10	100	3000	300	High	42637	25.25	45346	0.12	52853	0.15	54239	0.08
10	100	4000	150	High	90911	30.48	87832	0.17	109463	0.30	112847	0.13
10	100	4000	300	High	49399	30.70	49025	0.17	56841	0.22	58746	0.12
10	150	3000	150	High	102166	44.45	105950	0.18	143835	0.33	148950	0.15
10	150	3000	300	High	55122	38.94	60686	0.18	74966	0.30	77115	0.13
10	150	4000	150	High	116057	49.35	117510	0.23	158362	0.42	163012	0.18
10	150	4000	300	High	61566	56.61	64844	0.25	81784	0.37	83238	0.17
10	200	3000	150	High	126968	71.44	131333	0.28	185241	0.43	188798	0.22
10	200	3000	300	High	68016	81.50	75036	0.23	96246	0.38	97134	0.20
10	200	4000	150	High	140900	70.54	145126	0.28	200343	0.47	206353	0.23
10	200	4000	300	High	77137	60.54	81087	0.28	103206	0.45	103302	0.23
20	100	3000	150	Low	46050	78.46	64270	0.32	84005	0.75	85526	0.25
20	100	3000	300	Low	25395	82.05	35470	0.27	43700	0.50	44011	0.22
20	100	4000	150	Low	51602	82.66	73623	0.42	93422	0.67	95225	0.33
20	100	4000	300	Low	28322	73.55	40032	0.38	48058	0.73	48500	0.33
20	150	3000	150	Low	61533	108.08	86563	0.42	109380	0.77	110661	0.37
20	150	3000	300	Low	34484	110.26	47778	0.40	57129	0.68	57843	0.35
20	150	4000	150	Low	70144	110.52	99022	0.42	128535	0.72	130391	0.37
20	150	4000	300	Low	38036	117.66	53643	0.38	66162	0.65	68047	0.33
20	200	3000	150	Low	76894	117.03	106869	0.45	135689	0.75	138759	0.38
20	200	3000	300	Low	42487	113.69	59419	0.42	70987	0.85	72117	0.37
20	200	4000	150	Low	85125	105.96	122727	0.47	155907	0.85	160585	0.42
20	200	4000	300	Low	46867	104.19	65981	0.43	80416	0.83	81829	0.38
20	100	3000	150	High	76423	64.64	123399	0.25	174437	0.38	179670	0.20
20	100	3000	300	High	41799	62.67	70108	0.25	89667	0.43	91357	0.18
20	100	4000	150	High	83574	78.31	140025	0.28	190749	0.50	196472	0.23
20	100	4000	300	High	41490	58.09	77938	0.33	97564	0.53	100191	0.27
20	150	3000	150	High	89054	105.56	161200	0.33	240819	0.52	245044	0.27
20	150	3000	300	High	49448	103.89	91553	0.28	124137	0.45	125861	0.23
20	150	4000	150	High	100053	116.93	184460	0.40	276516	0.68	283812	0.35
20	150	4000	300	High	55620	114.06	103190	0.42	139897	0.78	143094	0.37
20	200	3000	150	High	107681	115.97	197001	0.43	288256	0.75	294904	0.38
20	200	3000	300	High	59883	118.91	113102	0.38	148495	0.70	151949	0.33
20	200	4000	150	High	124912	121.92	226471	0.43	333931	0.80	341949	0.37
20	200	4000	300	High	68295	111.84	125074	0.43	170438	0.72	172552	0.37

Table 5.6 Percent deviation over the best solution of the benchmarking methods

	43.14		4.85		39.57		61.47
	46.36		6.35		39.67		67.73
	46.96		-3.39*		42.68		67.55
	42.41		-0.76*		41.35		87.85
	50.45		3.70		40.68		81.01
1st 12-Problem Set	49.24	2nd 12-Problem Set	10.09	3rd 12 - Problem Set	38.55	4th 12-Problem Set	85.15
	54.58		1.25		41.17		84.36
	50.50		5.32		41.03		85.53
	48.72		3.44		38.98		82.95
	46.41		10.32		39.85		88.87
	52.66		3.00		44.17		81.31
	46.68		5.12		40.78		83.14

* denotes the results found by the modified algorithm which are worse than the best solution of the remaining three methods.

As seen from the above table, the percent deviation values are at the highest level shown in the rightmost column, for the last 12-problem set in which there are 20 candidate depots and the high cost structure was used. The reason behind that the highest deviation in cost values occurred for the last 12-problem set might be due to the high cost structure. That is, in our solution method rather than increasing the number of open depot, assigning as many customers as possible on a given route of one of the currently open depot is usually chosen. Therefore, the effect of high fixed depot cost is reduced. However, the routing cost becomes the greatest component of the total system cost. Secondly, the percent deviation values are almost at the same level for the first and third 12-problem sets, whereas for the second 12-problem set shown in the second column of the table, the deviation is not significant as much as that for the remaining 36 problem set. Moreover, two negative deviation values belong to this class of the problem sets which are shown with an asterix on the table imply that the results found by the modified algorithm are worse than the best solution of the remaining three methods. The presence of the random customer selection and our tabu threshold values might be the reason of these worse solution values.

The comparison results are also shown in a group of figures in order to better indicate the difference in the best solution values of each method in the following.

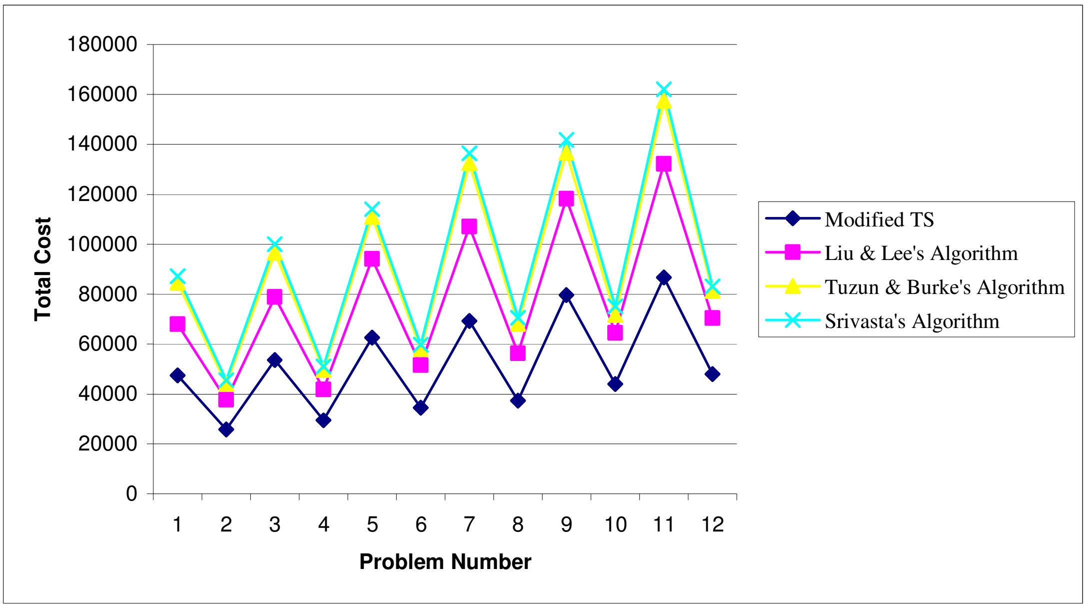

Figure 5.4 Comparison of the best solution found for the first 12-problem sets obtained from four different algorithms

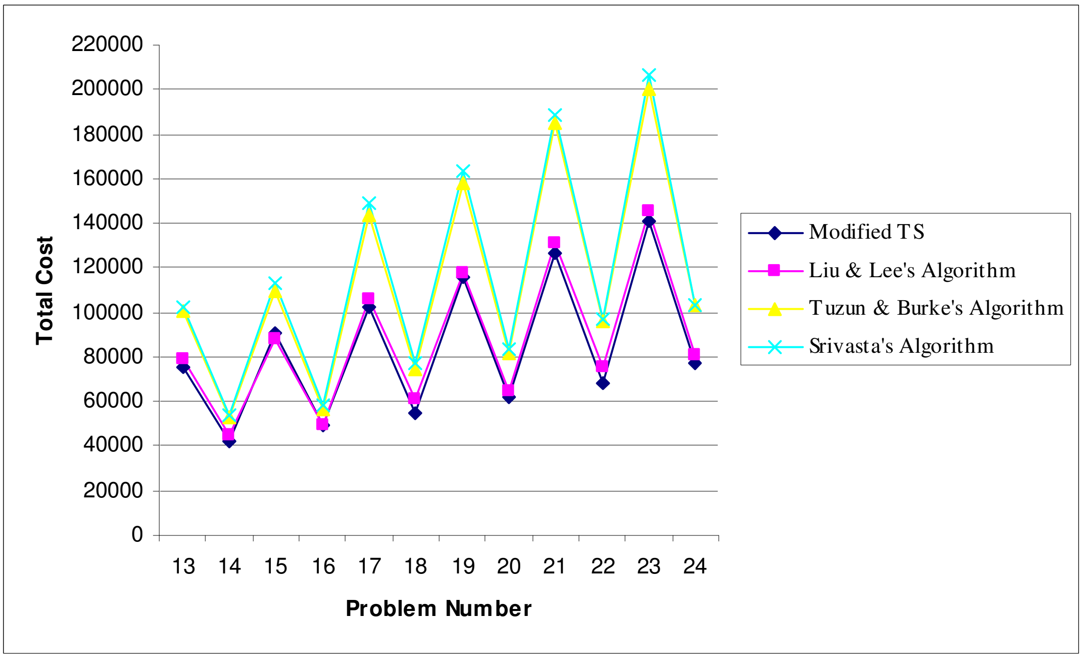

Figure 5.5 Comparison of the best solution found for the second 12-problem sets obtained from four different algorithms

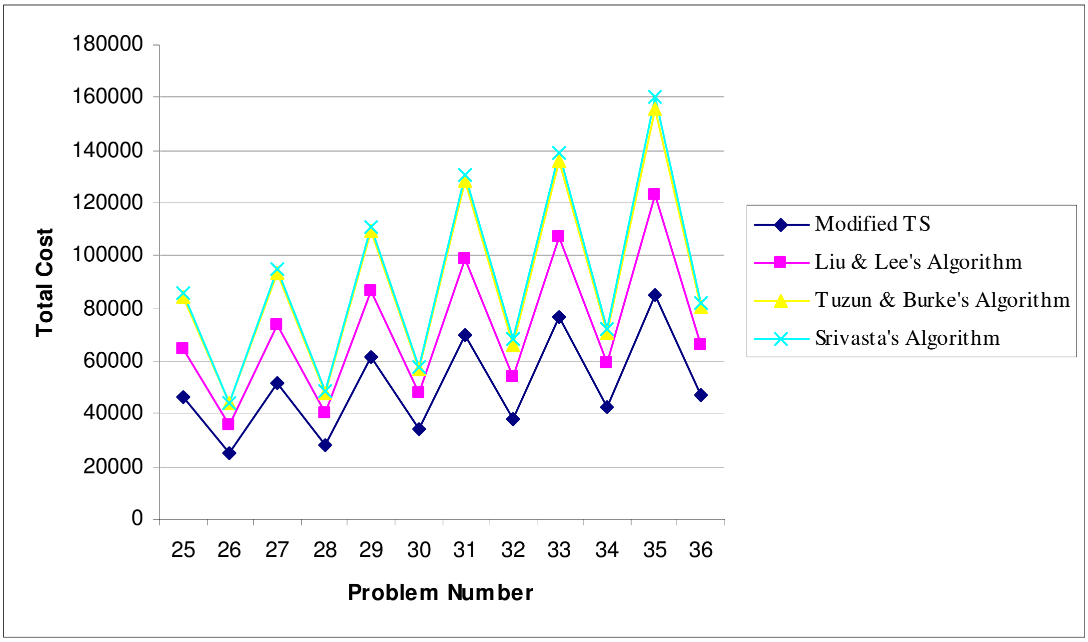

Figure 5.6 Comparison of the best solution found for the third 12-problem sets obtained from four different algorithms

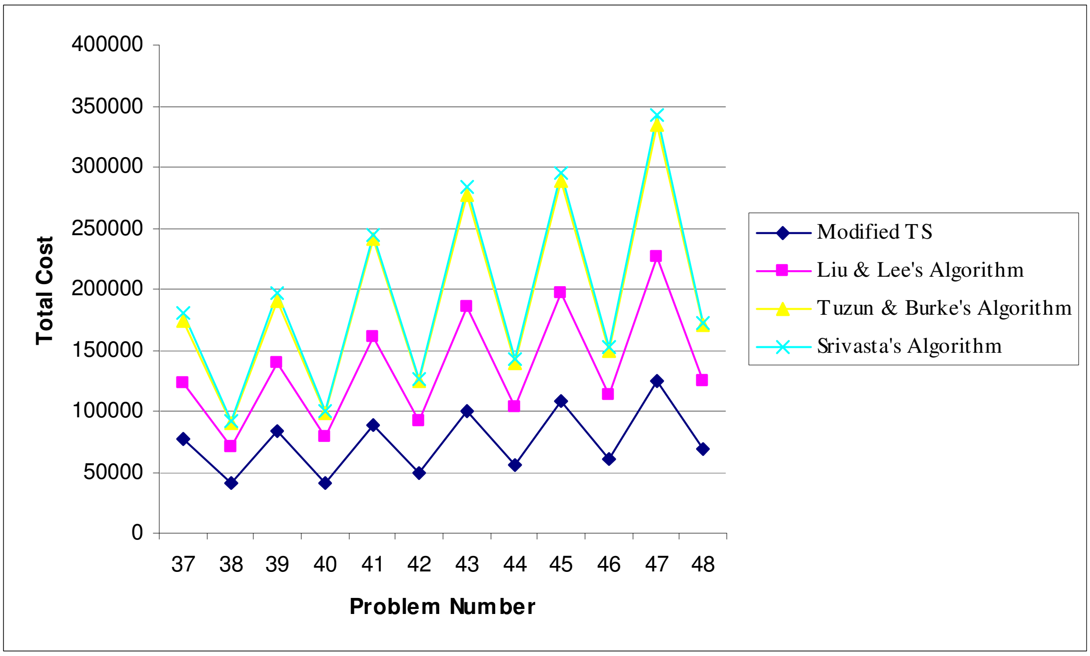

Figure 5.7 Comparison of the best solution found for the fourth 12-problem sets obtained from four different algorithms

As seen from the previous figures, from Figures 5.4 and 5.6, it can be said that the modified solution yields the best solution over the other ones. Similarly, Figure 5.7 shows that the modified solution method significantly overperforms the other solution methods. Although the savings obtained by the modified method are not as much as the previously-mentioned problem sets of 1, 3, and 4, almost all of the problems belonging to the second set- results shown in Figure 5.5- result in the best savings.

These figures also indicate the effects of the problem parameters. These are summarized as follows:

- *Effect of the number of customers:*

Each of the above four charts represents the resulting cost values belonging to a set of 12 problems. First four problems in each 12-problem set consist of 100 customers, and the next four consist of 150 customers, and the last four consist of 200 customers. As it can be observed from all charts, as the number of customers increases, the amount of customers demand to be satisfied increases, thus the total system costs for all the four heuristic solutions increase.

- *Effect of the number of depots:*

Figures 5.4 and 5.5 represents the results for the first 12-problem sets and the second 12-problem sets, respectively. The number of depots for both groups of problem sets is 10. Similarly, in Figures 5.6 and 5.7 the third and the fourth 12-problem sets are given with a number of depots of 20. These figures show that when the number of depots increases, the safety stock level to be kept at each open depot decreases, thus the total system cost decrease.

➢ *Effect of the vehicle service capacity:*

First two problems in each 12-problem set have a vehicle service capacity of 3000 units, and the next two have a vehicle capacity of 4000 units. Next four problems of the remaining part of the each 12-problem was designed in the same order of vehicle service capacity as well as the last four. The resulting cost trends indicate that, the higher the vehicle service capacity, the higher the inventory kept at each open depot, thus the higher the inventory holding cost and the total system costs obtained.

➢ *Effect of the vehicle capacity:*

Unlike the vehicle service capacity, first problem in each 12-problem set has a vehicle capacity of 150 units, and the next one has a vehicle capacity of 300 units. Then each pair of the problems has the same setting with respect to the vehicle capacity. In this framework, the resulting total system costs decrease for all the four heuristic solutions as the vehicle capacity increases.

➢ *Effect of the cost structure type:*

As mentioned previously, two different cost structures were designed while testing the performance of the modified algorithm as high cost structure and low cost structure. Figures 5.4 and 5.6 have a low cost structure, whereas the other Figures 5.5 and 5.7 have a high cost structure. According to the results drawn on the above four charts, when the cost per mile and the fixed yearly depot establishing costs increases (i.e., the ratio of routing cost and depot establishing cost to inventory cost gets higher) the total system cost for all heuristic solutions also increases.

Furthermore, concentrating on the modified TS algorithm, to observe the effects of some additional problem parameters, a sensitivity analysis is conducted in the next section.

5.4 Sensitivity Analysis on Problem Parameters

In the previous section, to compare the performance of the modified TS algorithm with three different algorithms, a group of 48 existing benchmarking problems were used. Other than this problem set, for a more comprehensive analysis on the effect of the problem parameters on the average cost values and the number of vehicles required in the best solution found, another set of 48 problems were generated under two different high-level cost scenarios. These additional problems parameters are exactly the same as in the first set of 48 problems generated except that the holding cost is set at levels 0.5 and 4, cost of dispatching a vehicle at levels 25 and 50 which were kept as constant in the previous problem set. All parameters together with their corresponding levels are listed in Table 5.7, and the cost results are given in Table 5.8.

Table 5.7 Parameters and their levels used in the second set of 48 problems

Parameters	Levels
Number of depots	20, 10
Number of customers	100, 150, 200
Vehicle service capacity	3000, 4000
Vehicle capacity	150, 300
Cost structure (Depot establishing cost: cost per mile: holding cost: cost per vehicle)	1200:2:4:25 (High level 1) 1200:2:0.5:50 (High level 2)

Table 5.8 Computational results of the second 48-problem set

Problem Set	Number of Depots	Number of Customers	Vehicle Service Capacity	Vehicle Capacity	Cost Structure	Cost
49	10	100	3000	150	HL 1	78940
50	10	100	3000	300	HL 1	52451
51	10	100	4000	150	HL 1	95286
52	10	100	4000	300	HL 1	56786
53	10	150	3000	150	HL 1	107760
54	10	150	3000	300	HL 1	71584
55	10	150	4000	150	HL 1	123327
56	10	150	4000	300	HL 1	73646
57	10	200	3000	150	HL 1	135752
58	10	200	3000	300	HL 1	91542
59	10	200	4000	150	HL 1	151919
60	10	200	4000	300	HL 1	95147
61	10	100	3000	150	HL 2	87200
62	10	100	3000	300	HL 2	48398
63	10	100	4000	150	HL 2	96490
64	10	100	4000	300	HL 2	54914
65	10	150	3000	150	HL 2	114535
66	10	150	3000	300	HL 2	61507
67	10	150	4000	150	HL 2	135343
68	10	150	4000	300	HL 2	70243
69	10	200	3000	150	HL 2	141580
70	10	200	3000	300	HL 2	77588
71	10	200	4000	150	HL 2	174334
72	10	200	4000	300	HL 2	86952
73	20	100	3000	150	HL 1	75906
74	20	100	3000	300	HL 1	53052
75	20	100	4000	150	HL 1	86114
76	20	100	4000	300	HL 1	54514
77	20	150	3000	150	HL 1	101639
78	20	150	3000	300	HL 1	71098
79	20	150	4000	150	HL 1	118662
80	20	150	4000	300	HL 1	74401
81	20	200	3000	150	HL 1	130673
82	20	200	3000	300	HL 1	90653
83	20	200	4000	150	HL 1	144871
84	20	200	4000	300	HL 1	91899
85	20	100	3000	150	HL 2	84293
86	20	100	3000	300	HL 2	48636
87	20	100	4000	150	HL 2	101569
88	20	100	4000	300	HL 2	53710
89	20	150	3000	150	HL 2	110167
90	20	150	3000	300	HL 2	61258
91	20	150	4000	150	HL 2	129876
92	20	150	4000	300	HL 2	71070
93	20	200	3000	150	HL 2	136318
94	20	200	3000	300	HL 2	74665
95	20	200	4000	150	HL 2	161999
96	20	200	4000	300	HL 2	84269

HL1 : High level 1; HL2 : High level 2

Since these problems are newly generated in order to concentrate only on the modified algorithm results, a number of new analyses were conducted on the parameters. The sensitivity analyses for the average costs and the number of vehicles required are presented as follows.

➢ *Average total system cost:*

As it is formulated in Chapter 3, the total system cost is composed of several cost components such as fixed depot establishing cost, transportation cost, inventory cost and order cost. On the other hand, to examine the effects of other unit cost parameters and capacity parameters on the total cost value, first each 12 set of problems were grouped together. The cost values of a total of 8 such sets of 12-problems are presented in the following Figure 5.8.

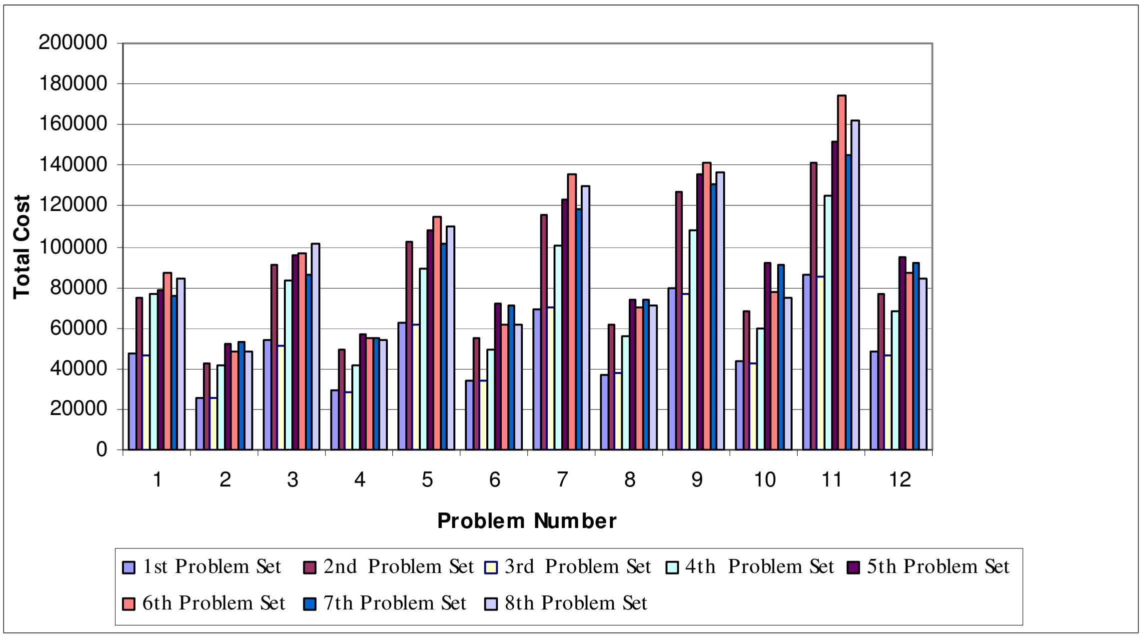

Figure 5.8 Bar chart representations of solution cost results for eight sets of 12-problems

As it is seen from the above figure, the total system costs show an increasing trend for the odd-numbered problems (i.e., problem number 1, 3, etc.,) and the even-numbered ones within each 12-problem set. This is the case, because first four problems consist of 100 customers; the next four includes 150 customers, and the last four includes 200 customers which has also an increasing trend. Moreover, within each four problems first two ones includes vehicles with service capacity of 3000 units, whereas the vehicles for the next two have a higher service capacity of 4000 units which implies also the reason of the increasing trend in total cost of the system.

Apart from this analysis, to determine the significance on the effects level of each parameter on the total cost, a regression analysis were performed. Using the experimental data, a good estimate *TC* of total cost was obtained via regressing the total cost values found on *number of depots (m)*, *number of customers (n)*, *vehicle service capacity (vsc)*, *vehicle capacity (vc)*, *fixed cost (f)*, *holding cost (h)* and *cost per vehicle dispatching (cpv).* Thus, the *TC* value can be estimated by:

$$TC = -\ 14019 - 398\ m + 380\ n + 9.57\ vsc - 288\ vc + 47.1\ f + 3851\ h + 605\ cpv$$

From this equation, the same interpretations can be reached like those obtained in the previous section only by comparing the results. Hence, only increase in two factors decreases the total cost estimate value these are *number of depots* and *vehicle capacity.* Furthermore, as a result of the ANOVA conducted, it can be said that all seven factors affect the total cost significantly at level 0.001. Additionally, R-square value implies that 93.3 % of all variations in total system cost estimate can be explained by the above multi-regression model. The details of the results of ANOVA and t-test values are also given in Appendix B.

- *Number of vehicles required:*

To examine whether the number of vehicles changes with respect to the problem type or not, the number of vehicles required for each set of 12 problems were grouped. Since we have 96 problem sets in total, the results are represented in eight different groups of 12-problem sets. Figure 5.9 demonstrates the number of vehicles of a total of eight such sets of 12-problems as follows.

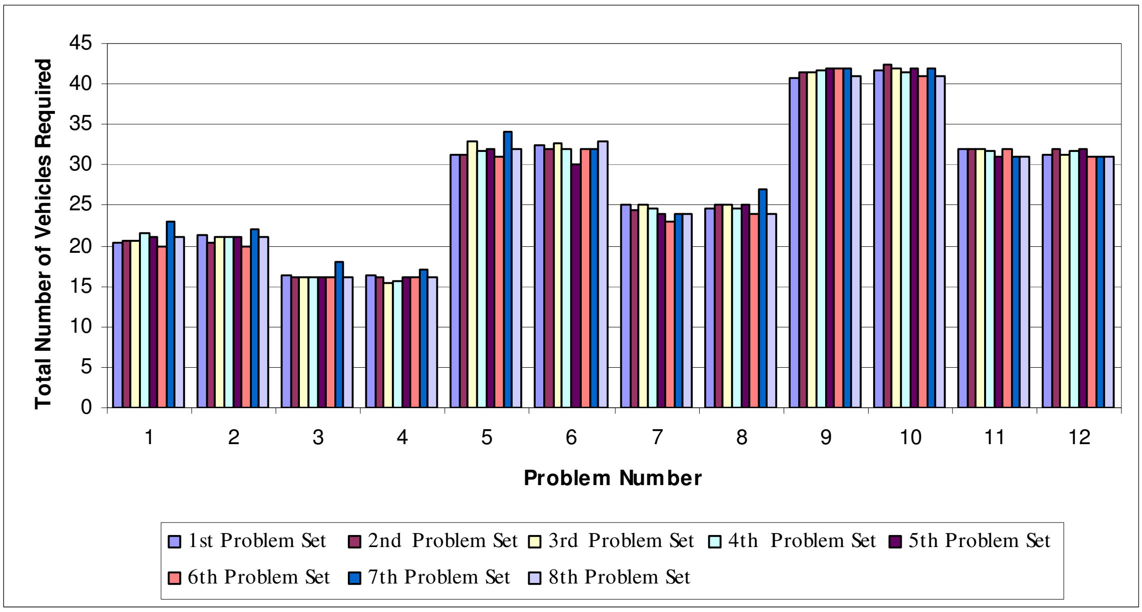

Figure 5.9 Bar chart representations of number of vehicles required for eight sets of 12-problems

According to the above figure, unlike the average cost trend, the number of vehicles required show a steady trend for each two-problem pair (i.e., problem number 1, and 2, 3 and 4, etc.) within each 12-problem set. For instance, the number of vehicles for the first two problems within each 12-problem set is at the same level, whereas for the next two problems the level is the same for each set at a lower value than that in the first two-problem set. Then, the same trend is seen in the second four problems group and the last four. The observed reason for this trend might be such that first four problems consist of 100 customers; the next four includes 150 customers, and the last four includes 200 customers which leads a probable increase in number of vehicles required. Another reason can be using vehicles with service capacity of 3000 units for within each four problems first two ones, whereas using vehicles for the next two with a higher service capacity of 4000 units in which each routes can serve to more customers than in the case of vehicles with service capacity of 3000 units, thus resulting in a smaller number of vehicles needed. A detailed list of the number of vehicles required can be seen from Appendix B1.

To better reflect the factors that have an influence and their influence level, like the analysis made for estimating the average total cost value depending on a number of parameters, a good estimate of the number of vehicles required - called *NOV*- of total number of vehicles required can be obtained by regressing the number of vehicles with respect to the same group of parameters including *number of depots (m)*, *number of customers (n)*, *vehicle service capacity (vsc)*, *vehicle capacity (vc)*, *fixed cost (f)*, *holding cost (h)* and *cost per vehicle dispatching (cpv).*

Thus, the *NOV* value can be estimated by:

$$NOV = 26.5 + 0.0437\, m + 0.180\, n - 0.00744\, vsc + 0.00014\, vc + 0.000069\, f + 0.083\, h - 0.0150\, cpv$$

From the calculated R-square value, 97.6% of all variation can be explained by the above regression model, and the ANOVA results show that all seven factors are significantly affect the number of vehicles required in the solution at level of 0.001. However, almost no directional effect of vehicle capacity and fixed depot establishing costs being observed on the number of vehicles required. Again, the detailed results of ANOVA and t-test can be seen in Appendix B2.

- *Relationship among inventory-transportation-fixed establishing cost:*

As mentioned before, the total system cost is composed of three main cost components including fixed depot establishing cost, transportation cost, inventory cost. To observe the contribution of each component to total cost, like in the previous analysis, first among 96 problem sets, each 12 set of problems were grouped together and the cost values for the resulting total of 8 such sets of 12-problems are demonstrated in Figures 5.10-5.17.

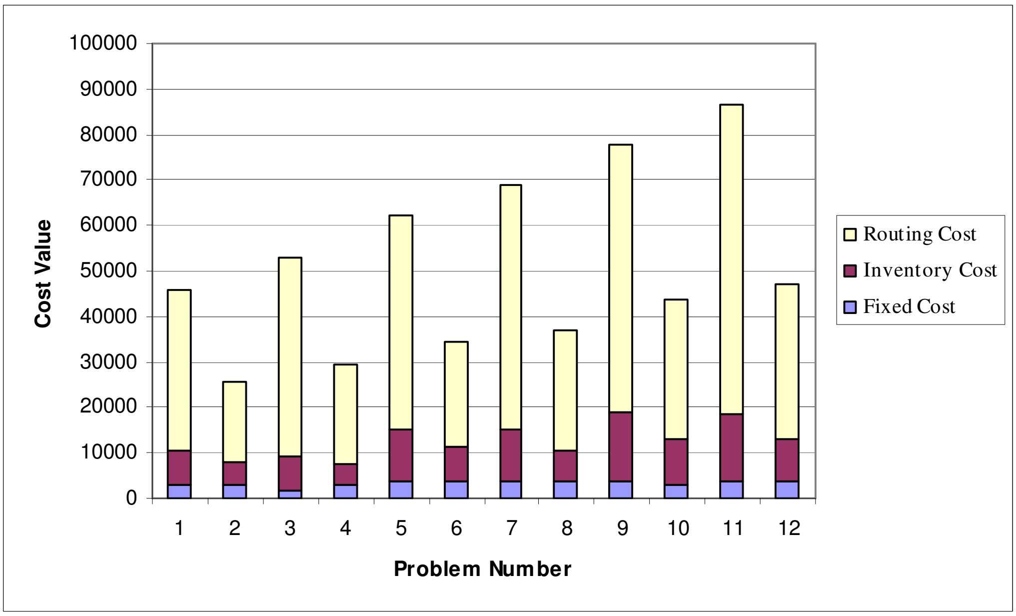

Figure 5.10 Bar chart representations of three main cost values for the first set of 12-problems

According to the above figure, provided that total fixed establishing cost directly proportional to the number of open depots and fixed cost per open depot in any problem instance, the transportation cost component yields the highest cost values, and the second highest values belong to the inventory cost part, and then it is followed by the fixed cost component. In any of the first 12 problem sets, inventory cost amount dominates the fixed establishing cost. Transportation cost ranges between 67-82 % as the percent of total cost; inventory cost ranges between 14 and 23%; and lastly, fixed cost lies between 3 and 12% of the total cost.

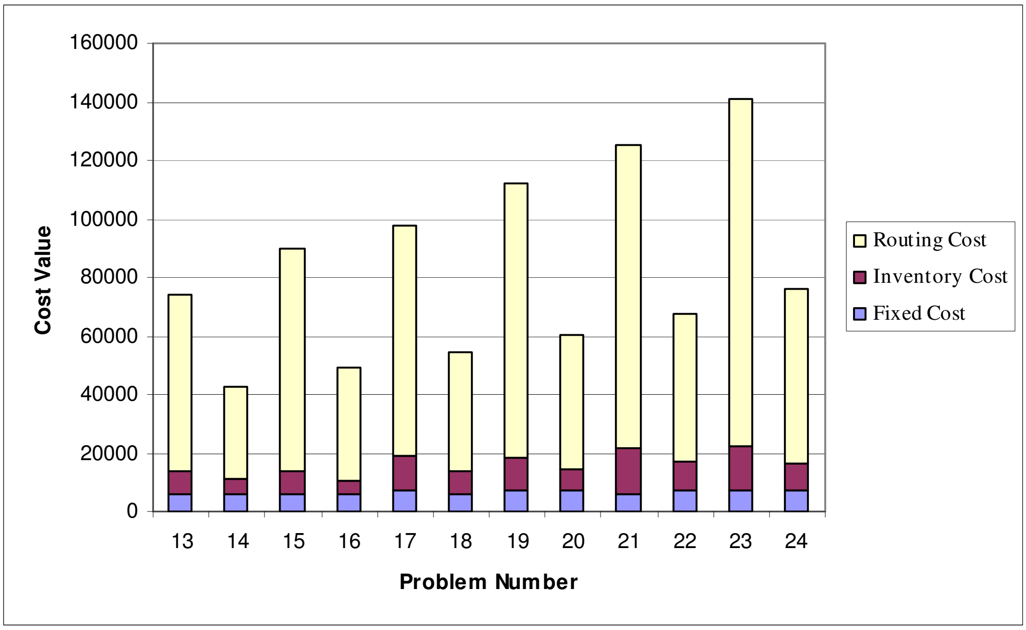

Figure 5.11 Bar chart representations of three main cost values for the second set of 12-problems

In the above figure, although a higher level of fixed establishing cost per open depot is used different from the first 12-problem sets, still domination exists between inventory and fixed establishing cost for any problem in the second 12-problem sets. Transportation cost ranges between 74-85 % as the percent of total cost; inventory cost ranges between 8 and 15%; and lastly, fixed cost lies between 5 and 14% of the total cost.

Like in Figure 5.10 and Figure 5.11, the same relation between three cost components can easily be observed from the following Figures 5.12 and 5.13 in which low level and high level fixed depot establishing costs are used, respectively. Again, the contribution of the inventory component is at significant level. For the third 12-problem sets, transportation cost ranges between 64-78 % as the percent of total cost; inventory cost ranges between 15 and 24%; and lastly, fixed cost lies between 5 and 13% of the total cost. Finally, as in Figure 5.13, transportation cost ranges between 63-84 % as the percent of total cost; inventory cost ranges between 9 and 22%; and lastly, fixed cost lies between 6 and 18% of the total cost.

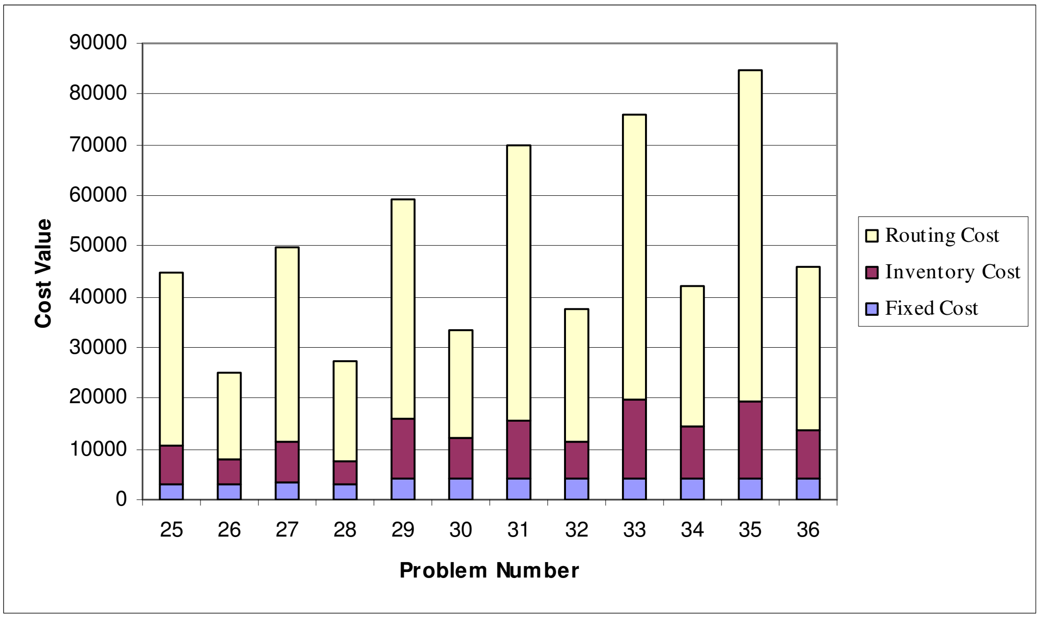

Figure 5.12 Bar chart representations of three main cost values for the third set of 12-problems

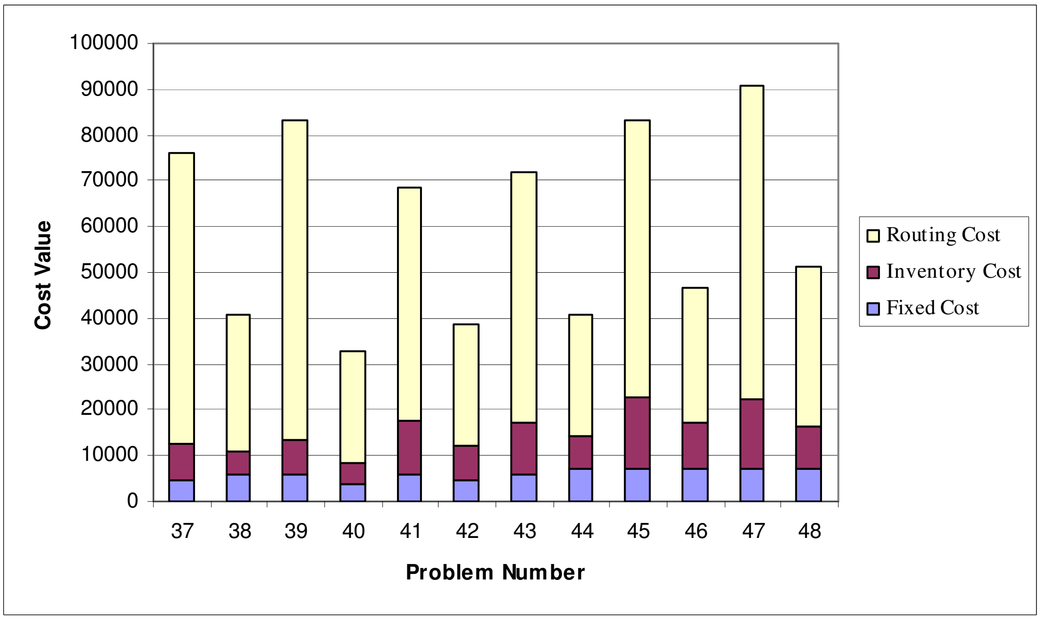

Figure 5.13 Bar chart representations of three main cost values for the fourth set of 12-problems

Unlike the first 48 problem sets, the remaining 48 problems were generated in two different high level cost scenarios. In all the remaining problems 49-96, the fixed depot establishing cost per open depot is at the same level of 1200, however in problems 49-60 and 73-84 unit inventory holding cost of 4 is higher than that of 0.5 in problems 61-72 and 85-96. Therefore, when higher level of holding cost is used, the resulting inventory cost portion becomes larger over the total cost. This relationship can be seen from the comparison of cost values presented in Figure 5.14 and Figure 5.15 as well as the comparison of cost values in Figure 5.16 and Figure 5.17. Moreover, the transportation cost component is the highest cost portions among all three components in overall 96 problems.

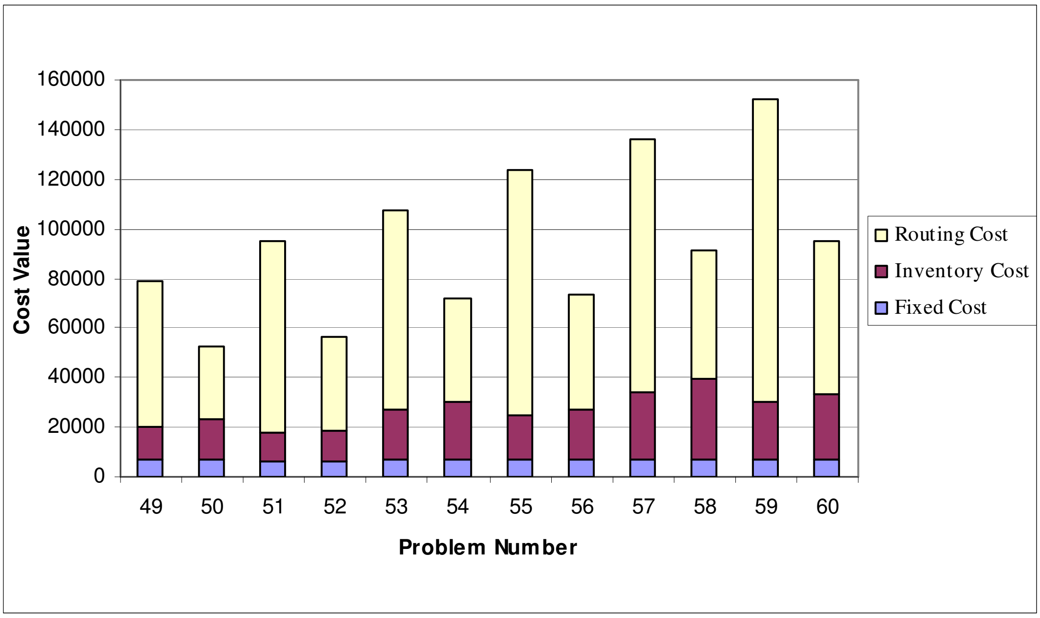

Figure 5.14 Bar chart representations of three main cost values for the fifth set of 12-problems

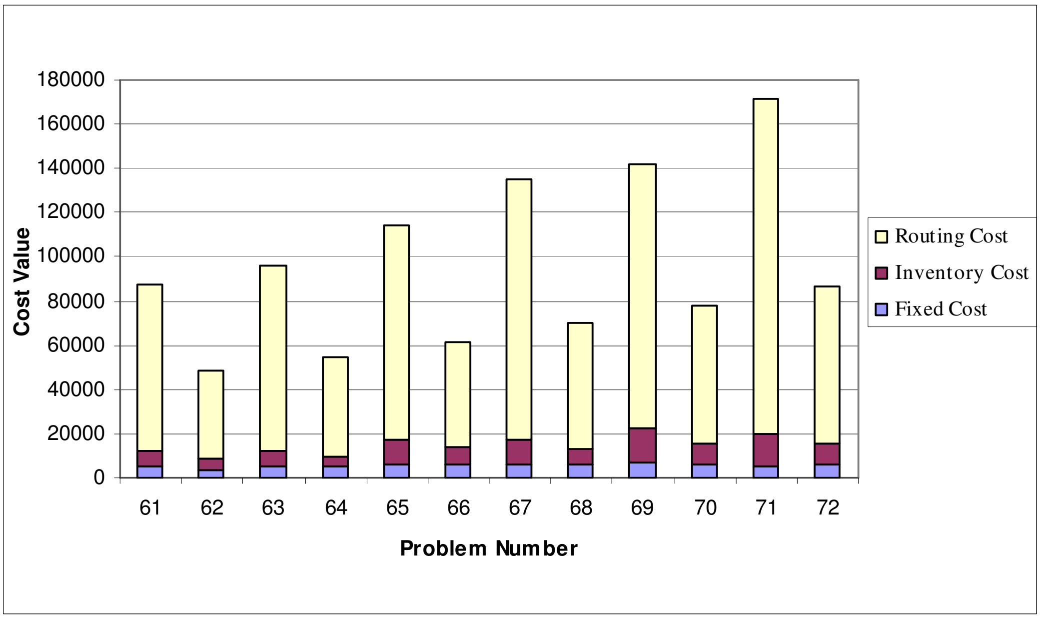

Figure 5.15 Bar chart representations of three main cost values for the sixth set of 12-problems

For the fifth 12-problem sets, transportation cost ranges between 56-81 % as the percent of total cost; inventory cost ranges between 12 and 35%; and lastly, fixed cost lies between 5 and 14% of the total cost. Secondly, as in Figure 5.15, transportation cost ranges between 78-88 % as the percent of total cost; inventory cost ranges between 8 and 13%; and lastly, fixed cost lies between 3 and 10% of the total cost.

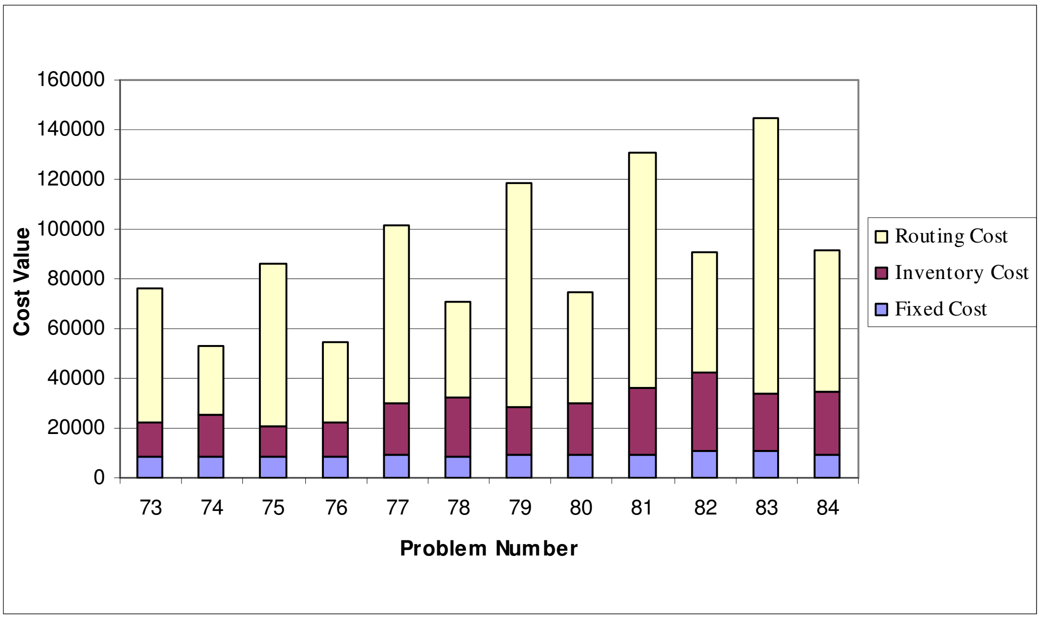

Figure 5.16 Bar chart representations of three main cost values for the seventh set of 12-problems

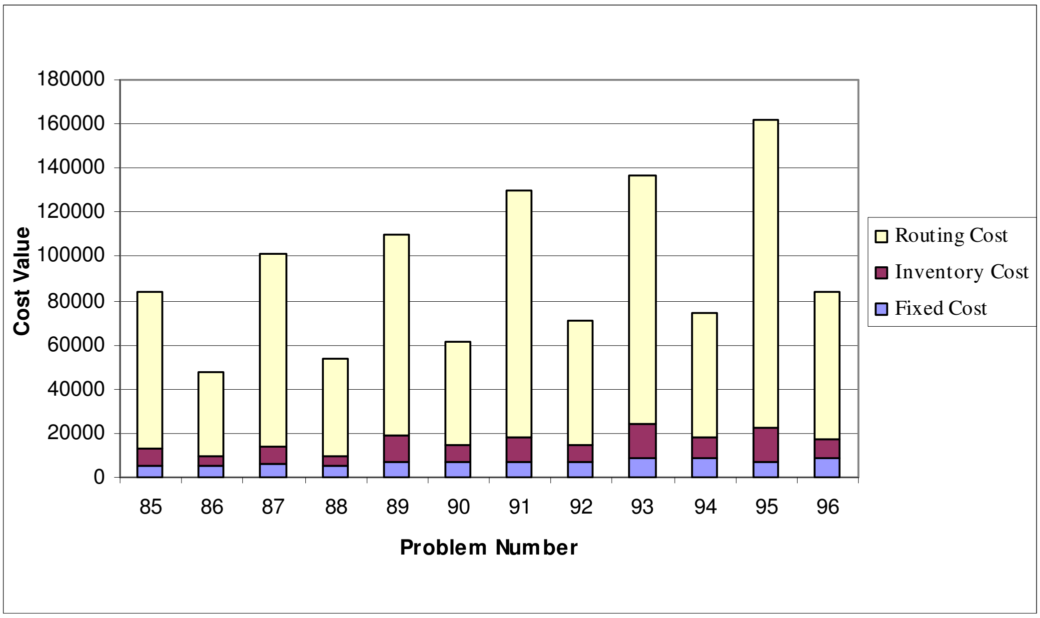

Figure 5.17 Bar chart representations of three main cost values for the eighth set of 12-problems

For the last two 12-problem sets, as seen from Figure 5.16, transportation cost ranges between 53-76 % as the percent of total cost; inventory cost ranges between 14 and 35%; and lastly, fixed cost lies between 7 and 16% of the total cost. From figure 5.17, transportation cost ranges between 75-87 % as the percent of total cost; inventory cost ranges between 7 and 13%; and lastly, fixed cost lies between 4 and 12% of the total cost.

From the above figures between Figure 5.10 - 5.17, the contribution of each cost component was examined, and it can be concluded that, in overall instances transportation cost gets the largest portion, and then it is followed by inventory, and then fixed establishing cost components, consecutively. More precisely, when unit holding cost increases, the difference between inventory and fixed establishing costs becomes more obviously observed.

Additionally, what percent of total cost belongs to each cost component was pointed out in a number of additional figures which are given in Appendix C.

Chapter 6

CONCLUSION

6.1 Conclusions

As mentioned throughout the study, the location-routing problem with inventory-control decisions is a combinatorial optimization problem in which decisions from three different levels are needed to be considered to design an effective and efficient distribution network while satisfying customer requirements.

In classical WLRP, a company has to ship goods from a set of supply points (plants) to a certain number of depots via truck loads, and then it has to deliver the goods from the depots to a set of geographically dispersed customers. On the other hand, the problem studied in this thesis is different from WLRP in the aspect that there are inventory decisions at depots in addition to location, allocation and routing decisions.

While defining this optimization problem, the objective is minimizing the total system costs incurred while satisfying the pre-defined customer requirements. That is, the goal is to determine the best distribution system in order to minimize facility location, warehousing, transportation and inventory costs while satisfying a certain customer service level. However, the problem might be differentiated from one another mainly in terms of cost components considered in the objective function, and decision parameters and variables as well as the constraints that exist in the mathematical model of the problem.

In this framework, two different mathematical formulations were presented. In the first formulation, the problem was formulated as a MIP, and the customer service level is expressed as a minimum stock level which has to be maintained at each open depot. On the other hand, the second model mainly differs from our first model. In the first model, the only inventory decision to be determined is stock level in each DC, however in the second model there are some additional values to consider in inventory decisions (e.g. order quantity, order frequency, etc.) to better reflect the interdependence among facility location, transportation and inventory decisions.

After formulating the problem in two different mathematical models, we first examined the second model performance for small test problems via the general purpose code-GAMS, and the exact solution was obtained. However, since the problem size exponentially grows in terms of the variables and constraints size and the nonlinear nature of it (i.e., the problem is NP-hard), GAMS was not able to solve the larger test problems. Therefore, to solve large-sized problems we developed a modified tabu search heuristic, which was previously proposed for classical LRPs in the literature by Tuzun *et al.*[2].

To evaluate the performance of the algorithm (coded in the Java programming language), test problems were run and their results were examined in two dimensions: the solution quality, that is, the best solution found, and the computational efficiency in CPU time. Firstly, a set of common benchmarking problems are solved with the help of our modified algorithm, and then the resulting best solutions found are compared with the results found in the past studies. According to these comparisons, for all the problem sets except two sets of problem, the modified TS algorithm yields the best results over all the other three heuristic methods' results in terms of the total system cost found in the best solution. However, due to the presence of randomization and additional cost computation of inventory part in each iteration, a significant increase was observed in computational effort in CPU minutes. On the other hand, although the computational time requirement is higher for the modified algorithm, the results found are significantly better than the best of all the other three algorithms. The analyses for all the four heuristic solutions showed that (1) as the number of customers increases, the total system costs increase, (2) when the number of depots increases, the total system decrease, (3) as the vehicle service capacity increases, the total system costs also increase, (4) the total system costs decrease as the vehicle capacity increases, and lastly, (5) as the ratio of routing cost and depot establishing cost to inventory cost gets higher, the total system cost also increases.

Secondly, to have a better understanding on the interdependency between three different levels of decisions, a group of sensitivity and regression analyses were conducted based on the changes in a group of problem parameters. According to the results, it can be concluded that total system cost is significantly influenced by number of depots, number of customers, vehicle service capacity, vehicle capacity, fixed cost, holding cost, and cost per vehicle dispatching. Furthermore, the number of vehicles is found to be strongly sensitive to the changes in the same group of parameters.

Finally, since the total system cost is composed of three main cost components including fixed depot establishing cost, transportation cost, inventory cost, the presence of a relationship between these cost components was examined, and as a result it can be said that, in overall cases the largest cost portion belongs to the transportation cost, and then it is followed by inventory, and then fixed establishing cost components. In addition, when unit holding cost increases the difference between inventory and fixed establishing costs gets higher.

6.2 Summary of Contributions

Classical LRPs have been studied in a wide range of studies in the literature, however the problem studied in this thesis has a very limited research example from the past studies. In fact, as this thesis being studied, there is no any study which exactly matches with this problem. From the point where the problem is re-defined with newly generated decision criteria, to the point where the results are obtained, some contributions which are made can be summarized as follows:

1. The location-routing problem with inventory decisions is re-defined, and two newly generated mathematical models are introduced including a MIP and a MINLP formulations,
2. To find good solutions to the defined problem, a modified tabu search algorithm was introduced with the modifications of

a. the inventory dimension is incorporated into the existing two phase rather than implementing an additional phase via changing the cost evaluation process
b. a randomization is added with an effort to make some jumps sooner and more possible in the neighbourhood search in each phase while selecting a customer to perform either an insertion or a swap move

3. The effects of the additional inventory decisions and the related cost issues on the objective function and the network configuration found in the best solution are examined and the results are presented
4. A comparison of the proposed solution method over a set of the benchmarking problems is made
5. To show the interdependency between three different levels of decisions, a group of sensitivity and regression analyses are performed with respect to the changes in the problem parameters.

6.3 Future Research

The problem studied in this thesis reflects a more relaxed distribution network system which is closer to the real-world circumstances since the inventory control decisions are tried to be incorporated although it seems ignored in most LRP research. In our problem, it was assumed that at the beginning not exact but an average yearly demand for each customer is known. Based on these average values, beside the decisions to be made in classical LRP, we tried to make inventory related decisions: 1) order quantity for each open depot on each of its route to serve its assigned customers, 2) inventory kept at each open depot to meet its assigned customers' demand, and 3) number of orders required by each open depot within the current year. While computing these values, the average yearly demands of the customers are considered only once at the beginning, whereas in a more realistic approach after the first replenishment occurred for a group of customers on a route, their corresponding average yearly demands might change, so some adjustments and recalculations of these values might be needed. Therefore, the same problem can be further analyzed in a multi-period and more stochastic environment.

Another important feature in our problem is that while solving the problem depot capacity issue is relaxed in an effort to better reflect the way of meeting customer demands in a group of order batches, not whole demand at one time. That is, at the beginning of solution algorithm each customer is assigned to depots until a capacity threshold is reached. If depots have a capacity, customers are assigned to a depot unless their demands exceed the corresponding depot's capacity. However, in our case customers' demands are not needed to be satisfied at one time, but in several replenishment cycles. Thus, it might not be reasonable to assign customers according to their yearly demand values and depot capacities. In fact, at the beginning, order quantity for any route is not known, and a direct assignment of customers to depots cannot be made based on depot capacity and order quantity. For these reasons, rather than using a depot capacity, we preferred using a vehicle service capacity for each vehicle while assigning customers to a specific route. (As mentioned in Chapter 4, vehicle service capacity means that each vehicle can serve to an amount of customer demand up to this threshold value for one year, which equals the vehicle capacity times the maximum number of visits of a vehicle to the customers on a specified route.) However, on the basis of problem scope, depot capacities can be further analyzed for the future decisions.

The solution method proposed provides us to find better solutions over a group of benchmark algorithms, but it is important that none of these algorithms considered exactly the same problem with us. However, since the optimal solution values for our test problems are not known, we could not evaluate the performance of our solution method in terms of objective function value with respect to the optimal solution values. Moreover, as the problem size increases the method might become less attractive due to the increased computation time. This increase might be resulted from the recursive feature of the algorithm. In addition, as the problem size increases, the exponential growth of the number of variables leads to an increase in calculations needed, thus an increase in computation time. For this reason, the modified algorithm can be enriched with several newly defined long term memory strategies or diversification and intensification strategies both to improve the solution quality and to reduce the computational effort, that is, currently the algorithm utilizes only short term memory. Furthermore, the computation time required to obtain the best solution might be reduced via using adaptive memory or granular tabu search approaches which have been shown to be useful for VRPs.

Alternatively, in order to obtain better solutions some newly studied metaheuristic approaches might be applied to this problem. For instance, solution recombination method used in genetic population search or ant systems methods are among these promising methods. They were resulted in the best solutions for VRPs among the methods ever studied in the past.

Finally, from the problem scope point, a single product, multi-depot LRP is considered here, whereas the same solution method can be further applied to other problems such as multi-product multi-depot problems to observe the effect of product types on the ordering and inventory decisions as well as on the other cost components of the system.

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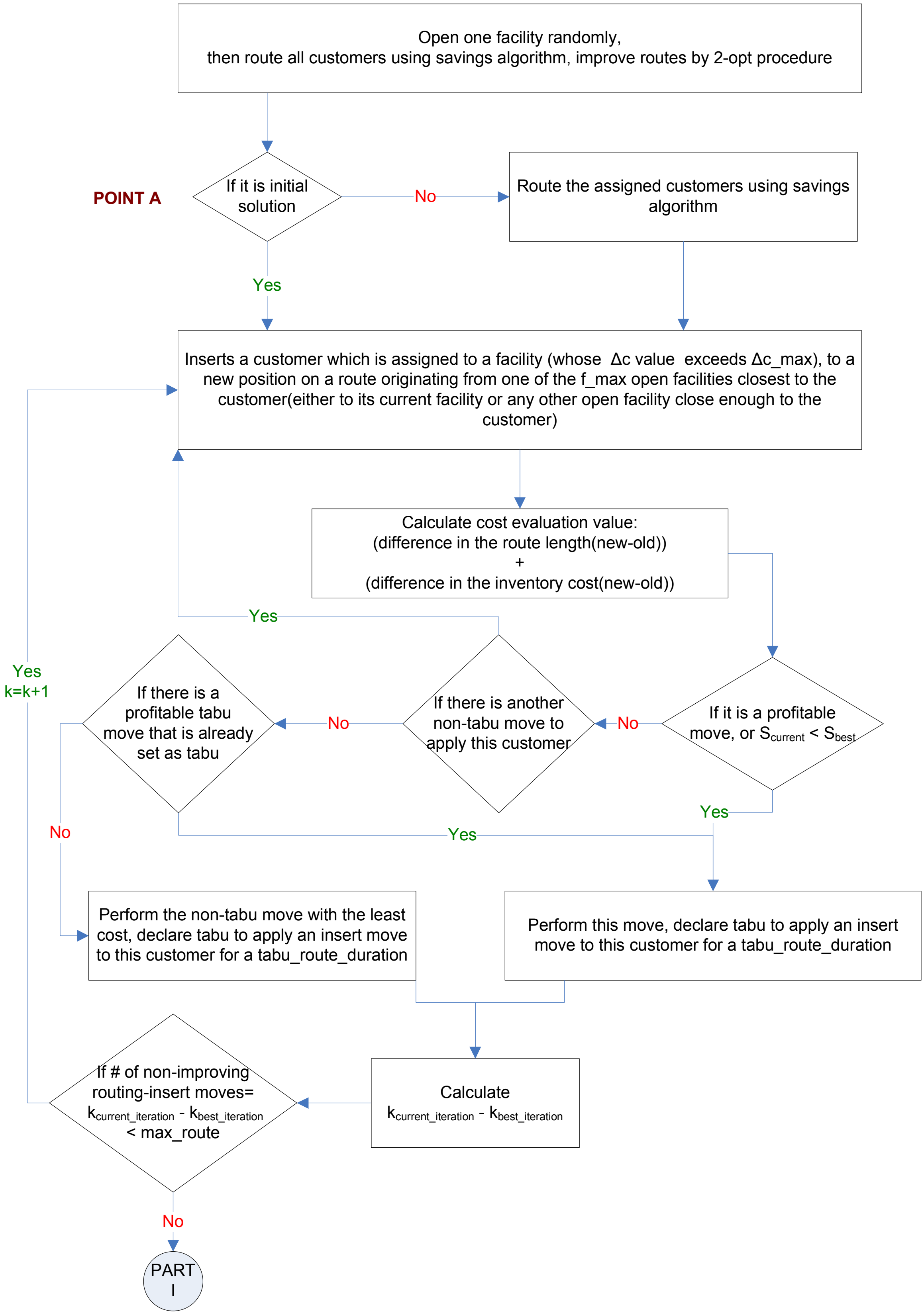
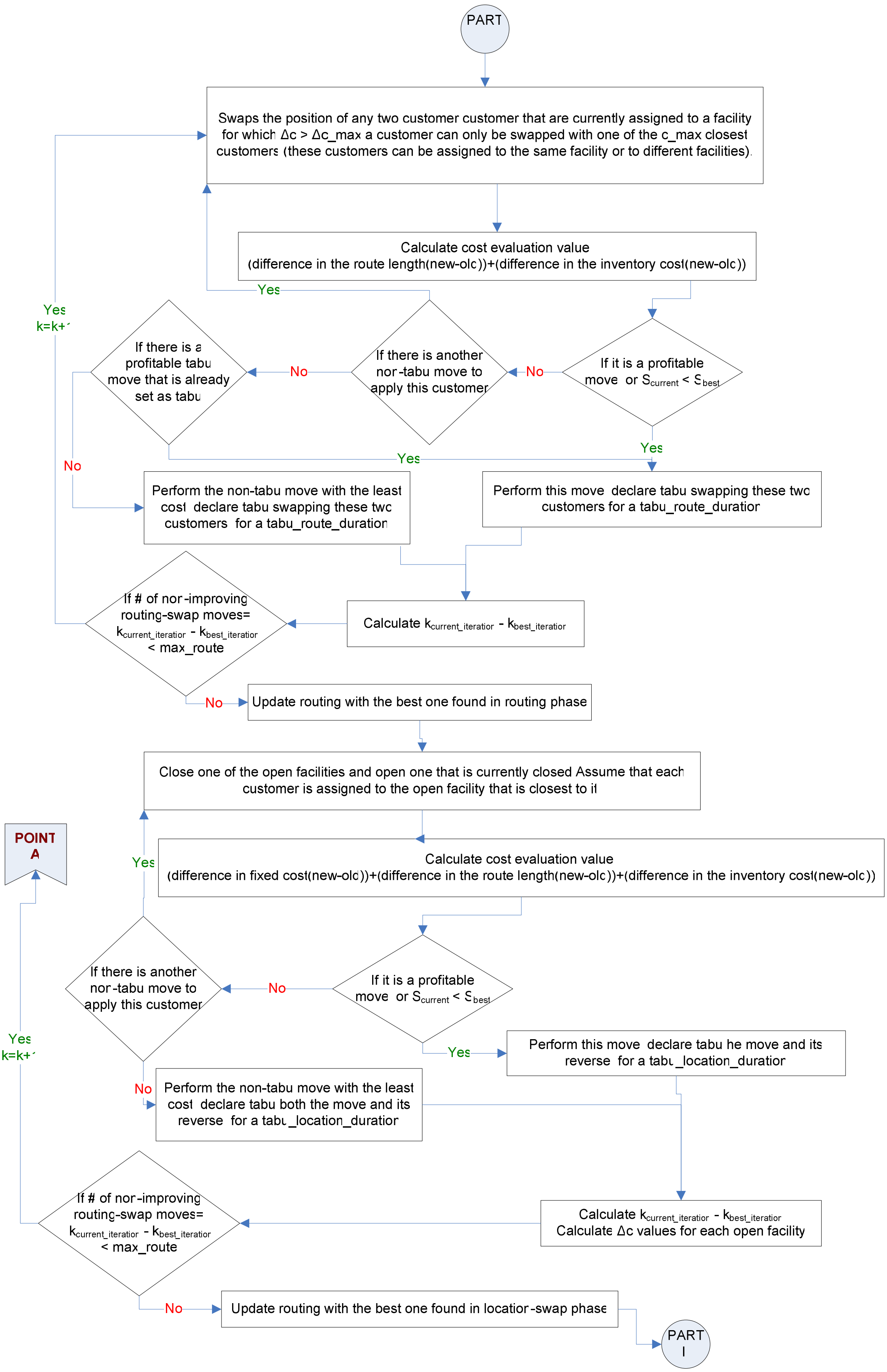
APPENDIX


Open one facility randomly,
then route all customers using savings algorithm, improve routes by 2-opt procedure
POINT A
If it is initial solution
No
Route the assigned customers using savings algorithm
Yes
Inserts a customer which is assigned to a facility (whose Δc value exceeds Δc_max), to a new position on a route originating from one of the f_max open facilities closest to the customer(either to its current facility or any other open facility close enough to the customer)
Calculate cost evaluation value:
(difference in the route length(new-old))
+
(difference in the inventory cost(new-old))
Yes
Yes
k=k+1
If there is a profitable tabu move that is already set as tabu
No
If there is another non-tabu move to apply this customer
No
If it is a profitable move, or S_current < S_best
Yes
No
Yes
Perform the non-tabu move with the least cost, declare tabu to apply an insert move to this customer for a tabu_route_duration
Perform this move, declare tabu to apply an insert move to this customer for a tabu_route_duration
If # of non-improving routing-insert moves= k_current_iteration - k_best_iteration < max_route
Calculate
k_current_iteration - k_best_iteration
No
PART I


PART
Swaps the position of any two customer customer that are currently assigned to a facility for which Δc > Δc_max a customer can only be swapped with one of the c_max closest customers (these customers can be assigned to the same facility or to different facilities).
Calculate cost evaluation value
(difference in the route length(new-old))+(difference in the inventory cost(new-old))
Yes
Yes
k=k+1
If there is a profitable tabu move that is already set as tabu
No
If there is another non-tabu move to apply this customer
No
If it is a profitable move or S_current < S_best
Yes
Yes
No
Perform the non-tabu move with the least cost declare tabu swapping these two customers for a tabu_route_duration
Perform this move declare tabu swapping these two customers for a tabu_route_duration
If # of non-improving routing-swap moves= k_current_iteration - k_best_iteration < max_route
Calculate k_current_iteration - k_best_iteration
No
Update routing with the best one found in routing phase
Close one of the open facilities and open one that is currently closed Assume that each customer is assigned to the open facility that is closest to it
POINT A
Yes
Calculate cost evaluation value
(difference in fixed cost(new-old))+(difference in the route length(new-old))+(difference in the inventory cost(new-old))
If there is another non-tabu move to apply this customer
No
If it is a profitable move or S_current < S_best
Yes
Perform this move declare tabu he move and its reverse for a tabu_location_duration
Yes
k=k+1
No
Perform the non-tabu move with the least cost declare tabu both the move and its reverse for a tabu_location_duration
If # of non-improving routing-swap moves= k_current_iteration - k_best_iteration < max_route
Calculate k_current_iteration - k_best_iteration
Calculate Δc values for each open facility
No
Update routing with the best one found in location-swap phase
PART I

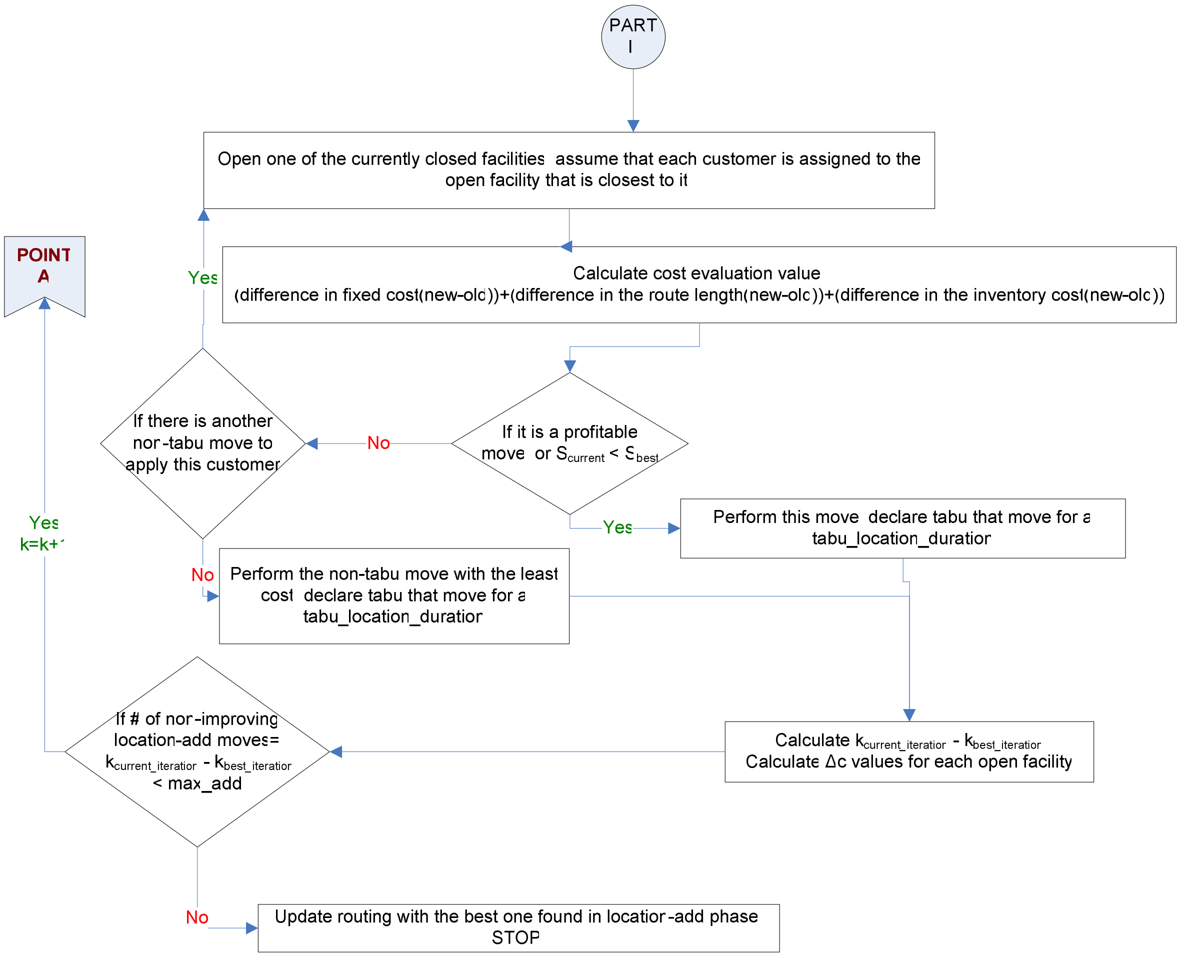

PART I
Open one of the currently closed facilities assume that each customer is assigned to the open facility that is closest to it
Calculate cost evaluation value
(difference in fixed cost(new-old))+(difference in the route length(new-old))+(difference in the inventory cost(new-old))
POINT A
Yes
If there is another non-tabu move to apply this customer
No
If it is a profitable move or $S_{current} < S_{best}$
Yes
Perform this move declare tabu that move for a tabu_location_duration
Yes
k=k+1
No
Perform the non-tabu move with the least cost declare tabu that move for a tabu_location_duration
If # of non-improving location-add moves= $k_{current\_iteration} - k_{best\_iteration}$ < max_add
Calculate $k_{current\_iteration} - k_{best\_iteration}$
Calculate Δc values for each open facility
No
Update routing with the best one found in location-add phase
STOP

Table B1.1 Computational results of 96-problem set in terms ofnumber of vehicles required

Number of Depots	Number of Customers	Vehicle Service Capacity	Vehicle Capacity	Cost Structure	Number of Vehicles Required
10	100	3000	150	L	20
10	100	3000	300	L	21
10	100	4000	150	L	16
10	100	4000	300	L	16
10	150	3000	150	L	31
10	150	3000	300	L	32
10	150	4000	150	L	25
10	150	4000	300	L	25
10	200	3000	150	L	41
10	200	3000	300	L	42
10	200	4000	150	L	32
10	200	4000	300	L	31
10	100	3000	150	HL0	21
10	100	3000	300	HL0	20
10	100	4000	150	HL0	16
10	100	4000	300	HL0	16
10	150	3000	150	HL0	31
10	150	3000	300	HL0	32
10	150	4000	150	HL0	24
10	150	4000	300	HL0	25
10	200	3000	150	HL0	41
10	200	3000	300	HL0	42
10	200	4000	150	HL0	32
10	200	4000	300	HL0	32
20	100	3000	150	L	21
20	100	3000	300	L	21
20	100	4000	150	L	16
20	100	4000	300	L	15
20	150	3000	150	L	33
20	150	3000	300	L	33
20	150	4000	150	L	25
20	150	4000	300	L	25
20	200	3000	150	L	41
20	200	3000	300	L	42
20	200	4000	150	L	32
20	200	4000	300	L	31
20	100	3000	150	HL0	22
20	100	3000	300	HL0	21
20	100	4000	150	HL0	16
20	100	4000	300	HL0	16
20	150	3000	150	HL0	32
20	150	3000	300	HL0	32
20	150	4000	150	HL0	25
20	150	4000	300	HL0	25
20	200	3000	150	HL0	42
20	200	3000	300	HL0	41
20	200	4000	150	HL0	32
20	200	4000	300	HL0	32

Number of Depots	Number of Customers	Vehicle Service Capacity	Vehicle Capacity	Cost Structure	Number of Vehicles Required
10	100	3000	150	HL 1	21
10	100	3000	300	HL 1	21
10	100	4000	150	HL 1	16
10	100	4000	300	HL 1	16
10	150	3000	150	HL 1	31
10	150	3000	300	HL 1	32
10	150	4000	150	HL 1	24
10	150	4000	300	HL 1	25
10	200	3000	150	HL 1	42
10	200	3000	300	HL 1	42
10	200	4000	150	HL 1	32
10	200	4000	300	HL 1	32
10	100	3000	150	HL 2	20
10	100	3000	300	HL 2	20
10	100	4000	150	HL 2	16
10	100	4000	300	HL 2	16
10	150	3000	150	HL 2	31
10	150	3000	300	HL 2	32
10	150	4000	150	HL 2	24
10	150	4000	300	HL 2	25
10	200	3000	150	HL 2	42
10	200	3000	300	HL 2	42
10	200	4000	150	HL 2	32
10	200	4000	300	HL 2	32
20	100	3000	150	HL 1	23
20	100	3000	300	HL 1	22
20	100	4000	150	HL 1	18
20	100	4000	300	HL 1	17
20	150	3000	150	HL 1	34
20	150	3000	300	HL 1	32
20	150	4000	150	HL 1	24
20	150	4000	300	HL 1	27
20	200	3000	150	HL 1	42
20	200	3000	300	HL 1	42
20	200	4000	150	HL 1	31
20	200	4000	300	HL 1	31
20	100	3000	150	HL 2	21
20	100	3000	300	HL 2	21
20	100	4000	150	HL 2	16
20	100	4000	300	HL 2	16
20	150	3000	150	HL 2	32
20	150	3000	300	HL 2	33
20	150	4000	150	HL 2	24
20	150	4000	300	HL 2	24
20	200	3000	150	HL 2	41
20	200	3000	300	HL 2	41
20	200	4000	150	HL 2	31
20	200	4000	300	HL 2	31

L:(600:1:0.5:25); HL0:(1200:2:0.5:25); HL1:(1200:2:4:25); HL2:(1200:2:0.5:50)

REGRESSION ANALYSIS

1. Effect of the problem parameters on the number of vehicles required

The regression equation is as follows:

Number of vehicles required = 26.5 + 0.0437 Number of Depots + 0.180 Number of Customers - 0.00744 Vehicle Service Capacity + 0.00014 Vehicle Capacity + 0.000069 FixedCost + 0.083 Holdingcost - 0.0150 Costpervehicle

Predictor	**Coef**	**SE Coef**	**T**	**P**
Constant	26.474	1.379	19.19	0.000
Number of Depots	0.04375	0.02749	1.59	0.115
Number of Customers	0.179687	0.003367	53.36	0.000
Vehicle Service Capacity	-0.0074375	0.0002749	-27.05	0.000
Vehicle Capacity	0.000139	0.001833	0.08	0.940
Fixed cost	0.0000694	0.0006481	0.11	0.915
Holding cost	0.0833	0.1111	0.75	0.455
Cost per vehicle	-0.01500	0.01555	-0.96	0.337

S = 1.34697 R-Sq = 97.6% R-Sq(adj) = 97.4%

PRESS = 191.066 R-Sq(pred) = 97.13%

Analysis of Variance

Source	DF	SS	MS	F	P
Regression	7	6503.58	929.08	512.08	0.000
Residual Error	88	159.66	1.81		
Total	95	6663.24			

2.Effect of the problem parameters on the average cost value found

The regression equation is as follows:

Cost-average = - 14019 - 398 Number of Depots + 380 Number of Customers
+ 9.57 Vehicle Service Capacity - 288 Vehicle Capacity + 47.1 FixedCost
+ 3851 Holdingcost + 605 Costpervehicle

Predictor	**Coef**	**SE Coef**	**T**	**P**
Constant	-14019	9234	-1.52	0.133
Number of Depots	-397.9	184.1	-2.16	0.033
Number of Customers	380.12	22.55	16.86	0.000
Vehicle Service Capacity	9.571	1.841	5.20	0.000
Vehicle Capacity	-288.00	12.27	-23.47	0.000
FixedCost	47.139	4.339	10.86	0.000
Holdingcost	3851.4	743.8	5.18	0.000
Costpervehicle	604.7	104.1	5.81	0.000

S = 9018.06 R-Sq = 93.3% R-Sq(adj) = 92.8%

PRESS = 8565570917 R-Sq(pred) = 91.97%

Analysis of Variance

Source	DF	SS	MS	F	P
Regression	7	99530091334	14218584476	174.84	0.000
Residual Error	88	7156628457	81325323		
Total	95	1.06687E+11			

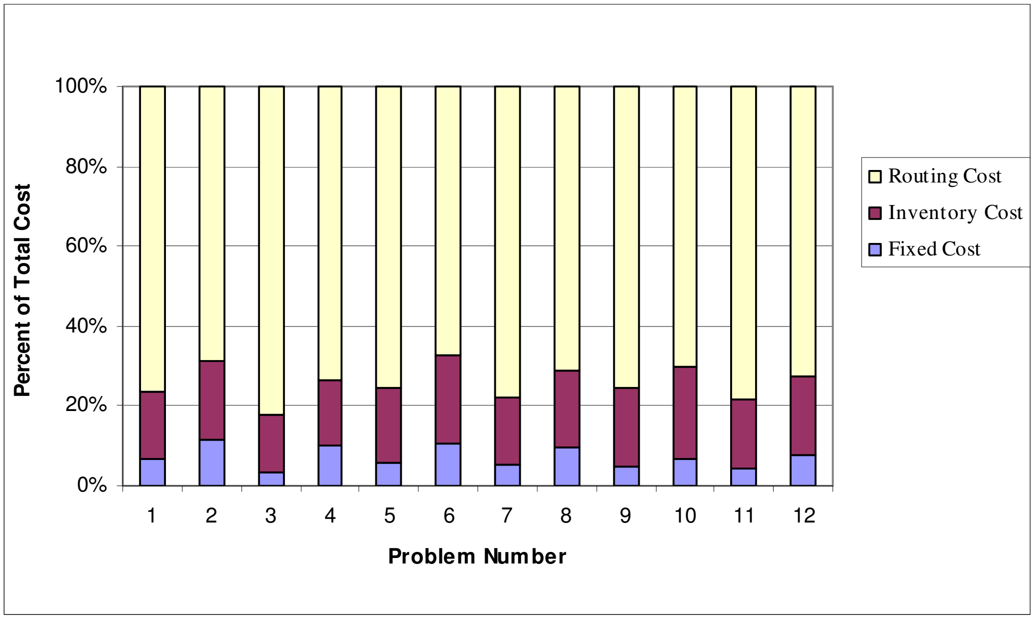


Figure C.1 Bar chart representations of three main cost values for the first set of 12-problems

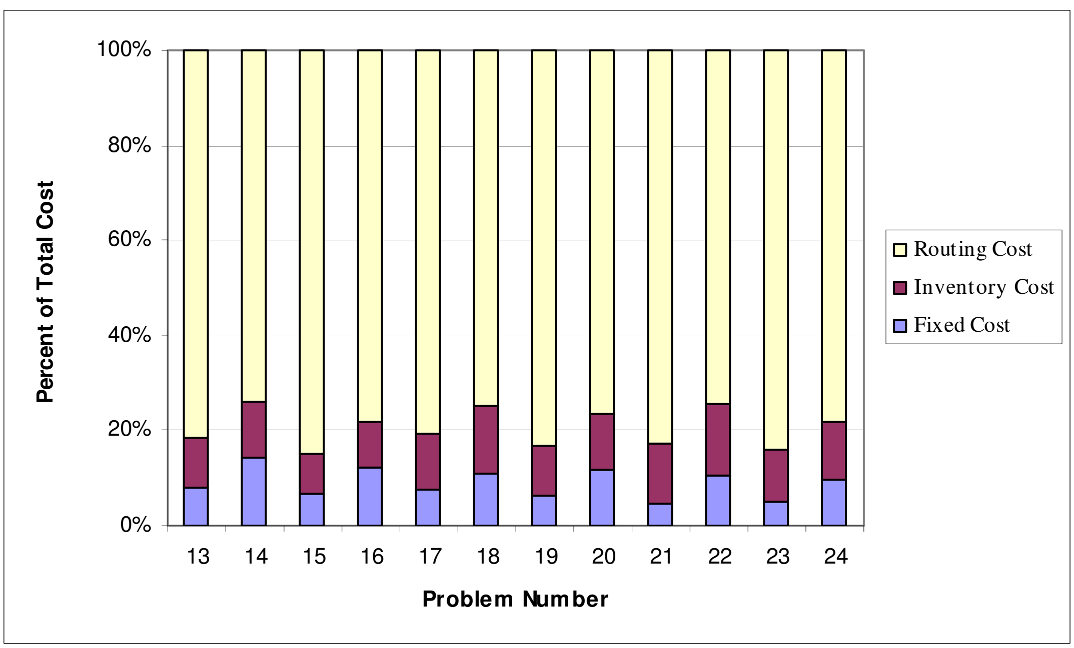


Figure C.2 Bar chart representations of three main cost values for the second set of 12-problems

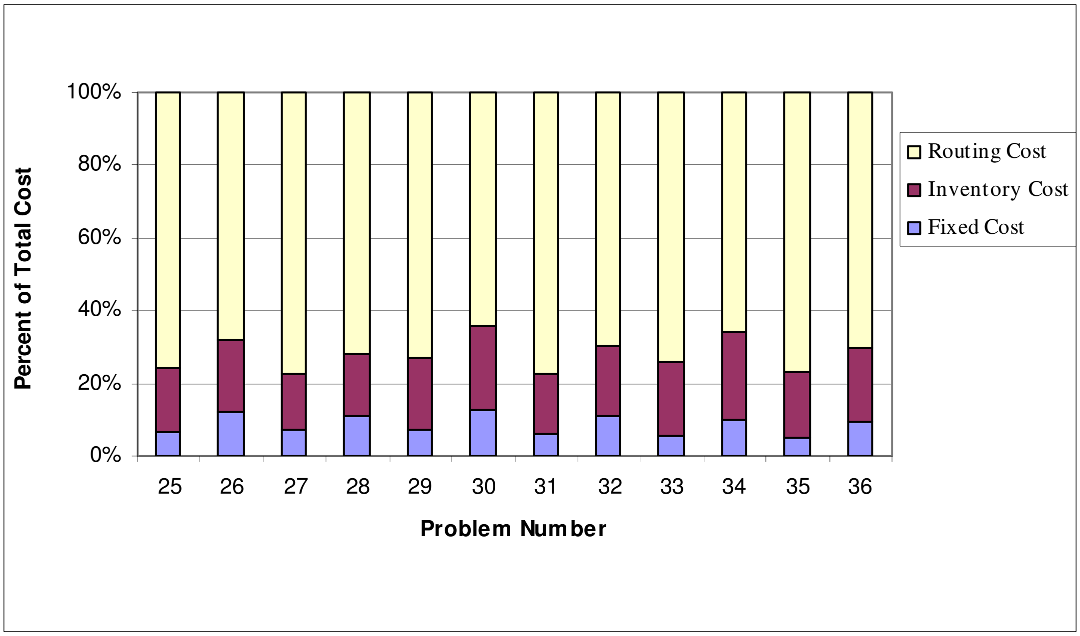


Figure C.3 Bar chart representations of three main cost values for the third set of 12-problems

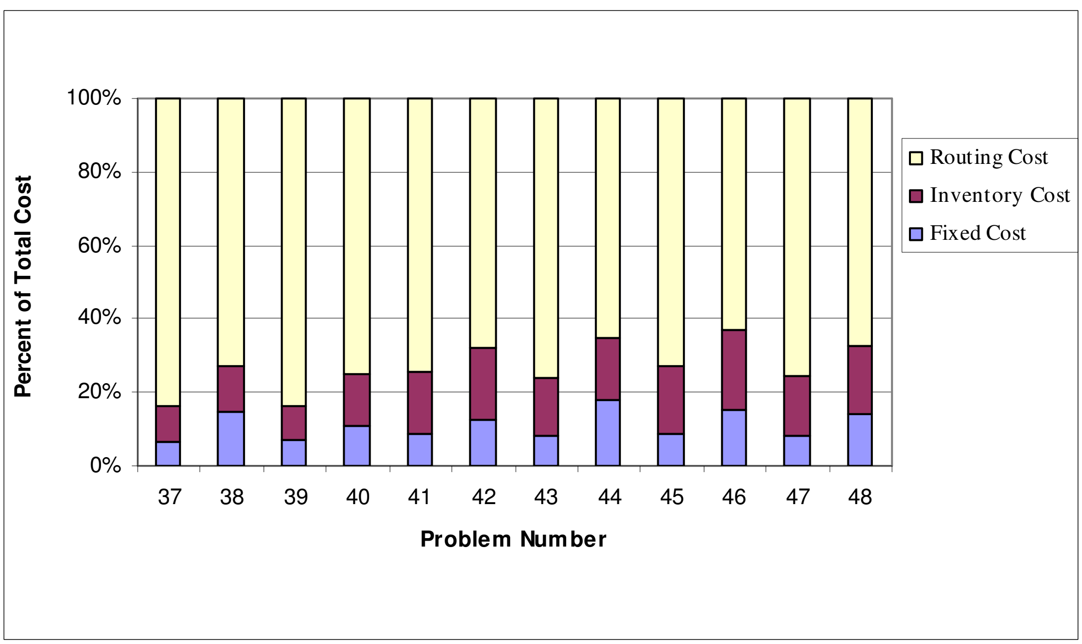


Figure C.4 Bar chart representations of three main cost values for the fourth set of 12-problems

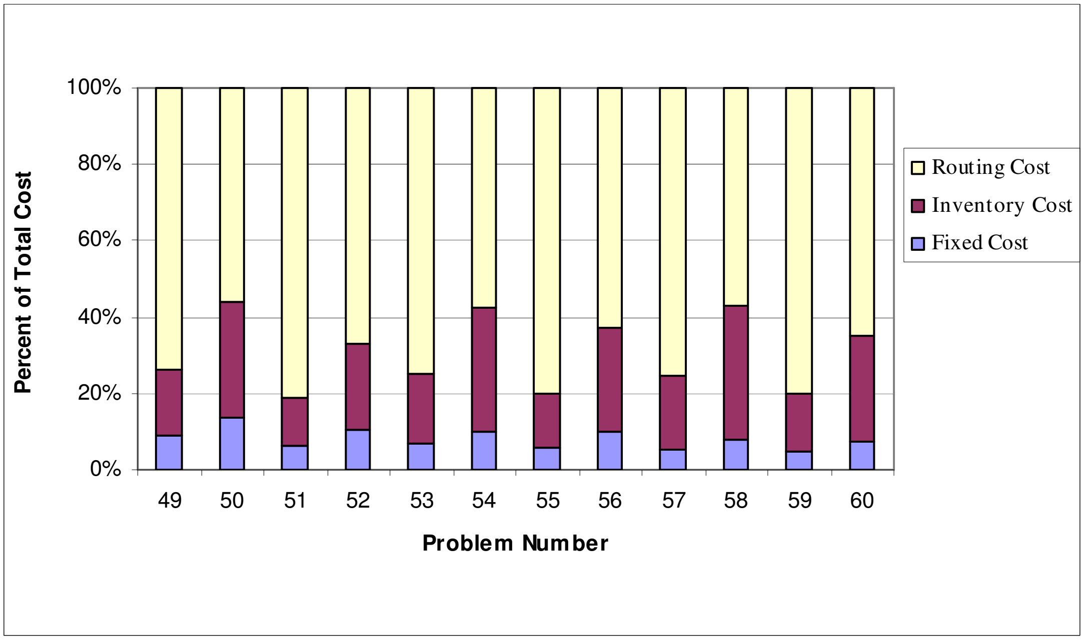


Figure C.5 Bar chart representations of three main cost values for the fifth set of 12-problems

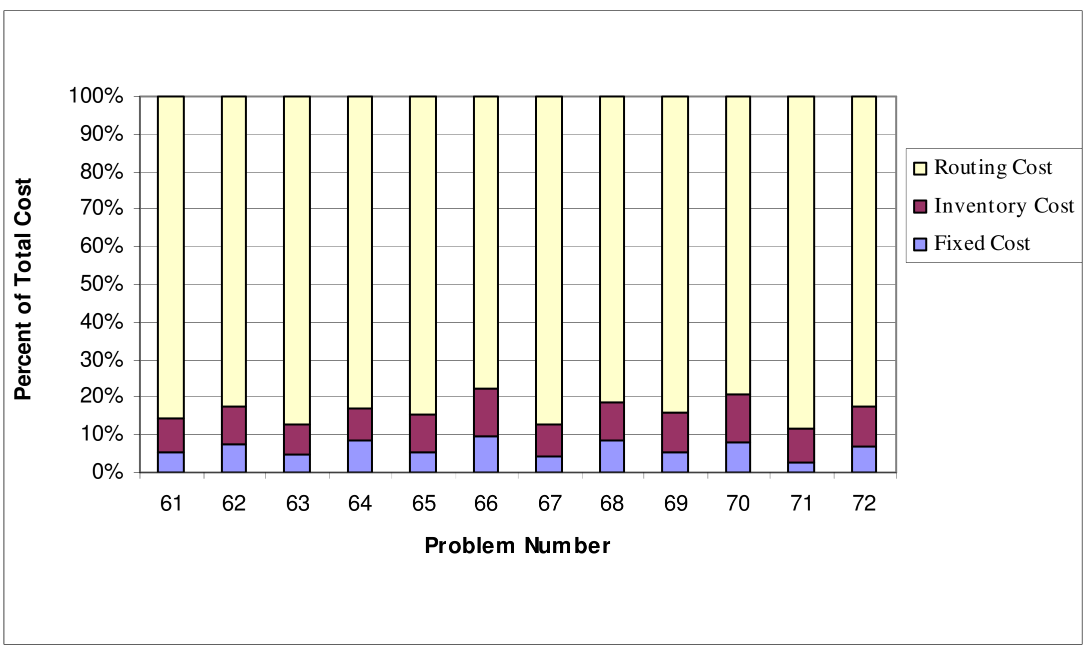


Figure C.6 Bar chart representations of three main cost values for the sixth set of 12-problems

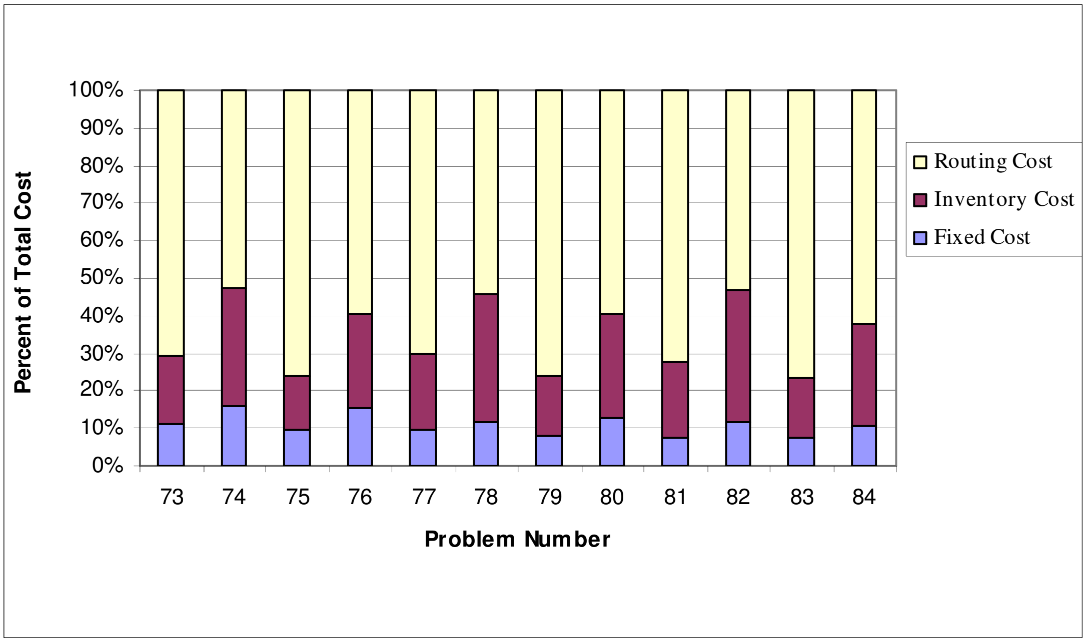


Figure C.7 Bar chart representations of three main cost values for the seventh set of 12-problems

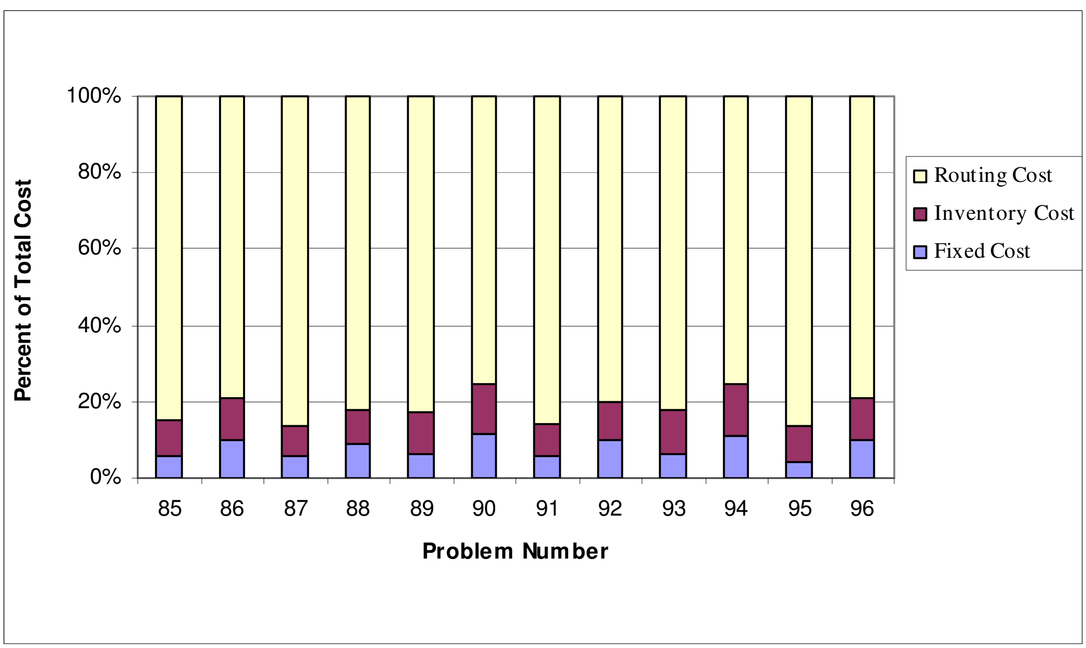


Figure C.8 Bar chart representations of three main cost values for the eighth set of 12-problems

VITA

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