A TWO-STAGE SUPPLY CHAIN MODEL WITH PRICE-DEPENDENT STOCHASTIC DEMAND AND LIMITED CAPACITY

by

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This is to certify that I have examined this copy of a master's thesis by

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ABSTRACT

In this thesis, we study a two-stage supply chain model with price-dependent stochastic demand and limited production capacity. The demand rate depends on the sales price and the replenishment of the retailer is instantaneous. The supplier prepares a contract by considering the parameters revealed by the retailer, his own parameters and his estimates of unrevealed parameters of the retailer. This model can be viewed as a mechanism to coordinate production and pricing in the chain. We first study the contract design problem of the supplier and obtain his optimal contract parameters under a deterministic price assumption. According to the deterministic price and the other parameters, the supplier offers a contract to the retailer that includes the cost values and capacity reservation quantities. The retailer then checks whether to accept the contract or not. Consequently, if the retailer accepts the contract the deal would take place The situation where the retailer reveals the demand-price relationship function parameters but hides the backorder cost value is studied in the first part of the analysis. In the second part, the condition that the retailer hides the demand-price relationship function parameters and the backorder cost value is examined. Our main contributions in this research are to analyze how the decentralized system operates, to understand the effects of limited information sharing on the decentralized system, and to benchmark the supply chain performance against the corresponding centralized system. To this end, we also study the contract design problem of the supplier and a contract design scheme that could operate with limited information

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Chapter 1

INTRODUCTION

The definition of Supply Chain Management according to the Council of Supply Chain Management Professionals (CSCMP) is that: "Supply Chain Management encompasses the planning and management of all activities involved in sourcing and procurement, conversion, and all logistics management activities. Importantly, it also includes coordination and collaboration with channel partners, which can be suppliers, intermediaries, third-party service providers, and customers" [1].

Different facilities in the supply chain frequently have different, conflicting objectives. Each facility in the chain focuses on its own profit and hence makes decisions with no regard to their impact on other supply chain partners. For instance, retailers focus on their own costs and profits and will try to reduce the cost as much as possible without considering the suppliers. Relationships between suppliers and retailers are established by means of supply contracts that specify pricing and volume discounts, delivery lead times, quality, returns and etc. In the recent years many academic researchers and industry practitioners have recognized that supply contracts are a powerful method that can be used for far more than to ensure adequate supply and demand for goods. Indeed, new contracts have been designed and used to enable supply chain parties to improve supply chain performance.

A typical two-stage supply chain consists of a retailer and a supplier. The sequence of events in such a supply chain is as follows. The retailer determines how many units to order from the supplier according to a particular price, and places an order to the manufacturer so as to optimize its own profit; the supplier reacts to the order placed by the retailer. This process can be defined as sequential supply chain optimization because decisions are made sequentially.

There are some mechanisms that enable supply chain entities to move beyond this sequential process and toward global optimization. Without any coordination mechanism, the retailer assumes all the risk of having more inventory than sales, whereas the supplier takes no risk. Thus, the supplier would like the retailer to order as much as possible, but the retailer limits its order quantity because of the risk. An effective coordination mechanism, or supply contract in other words, allocates profit to each partner in such a way that no partner can improve its profit by deciding to deviate from the optimal set of decisions.

Typically the two agents make decisions in order to optimize their own individual profit functions. The coordination of supply chains consisting of these agents (decision-makers), modeling the stochastic systems with multiple parameters, and the policies used in these models are important new research areas. In addition, increasing competition between the companies forces them to pay extra attention to the pricing, capacity and inventory management decisions. Actually, the supply chain efficiency is affected by the coordination scheme. Also information sharing is an important issue when supply chain participants attempt to coordinate the supply chain. Our main objectives in this research are to analyze how the decentralized system operates, to understand the effects of limited information sharing on the decentralized system, and to benchmark the supply chain performance against the corresponding centralized system. Finally, we also aim to study the contract design problem of the supplier and a contract design scheme that could operate with limited information.

Therefore we present and analyze a model that can be viewed as a mechanism to coordinate production and pricing. The demand rate depends on the sales price and the replenishment of the retailer is instantaneous. The supplier prepares a contract by considering the parameters revealed by the retailer, his own parameters and his estimates of unrevealed parameters of the retailer. According to the mechanism and the parameters, the supplier offers the contract to the retailer that includes the cost values and capacity reservation quantities. The retailer checks whether the contract is applicable for her or not. Consequently, if the retailer accepts the contract, the deal would take place.

First we study the centralized case with full information sharing to state the supply chain coordination issues formally. We also obtain closed form or numerical solutions to establish a basis for comparing the performance of the supply chain when decision makers operate with limited information sharing. We then examine the situation where the retailer reveals her demand parameters, and hides the backorder cost value to determine the significance of the backorder cost value and the effect to the supply chain performance. The condition when the retailer hides all of the information she has, is also studied to find out the effects of the backorder cost value and the demand parameters on supply chain performance, and their individual profits.

A review of the literature on pricing and supply chain coordination and the problem description are presented in Chapter 2. The definition of the centralized model, numerical examples of the model, the deterministic price assumption, and experiments for testing assumption performance by using both linear and exponential demand functions are explained in Chapter 3. The mechanism of the two-stage supply chain (decentralized model), the models of the supplier and the retailer, their contract process description, coordination of the case where the retailer hides the backorder cost, and the results of models with both linear and exponential demand functions are examined in Chapter 4. The second case where the retailer hides both the backorder cost and the demand function parameters and the outcomes of the model with the demand functions as the previous case are addressed in Chapter 5. The conclusion and the future research perspectives are presented in Chapter 6.

Chapter 2

LITERATURE SURVEY and PROBLEM DESCRIPTION

2.1. Literature Survey

The analysis of supply chain systems and their coordination is an important aspect in the recent supply chain research literature. We will give the literature survey in two parts; first is the pricing literature and the second is coordination and information asymmetry literature. Because the pricing is not directly related to our research only, a limited number of related articles are presented.

First, the pricing research can be classified according to the situations of a number of parameters such as demand type and its function, prices, number of periods, sales, capacity limits etc. Chan, Shen, Swann and Simchi [3] summarized the elements of classification system as seen on the Table 2.1:

Table 2.1 *Classifications of supply chain systems*

Factors	Alternatives
Price	Fixed or Dynamic
Demand Type	Deterministic or Stochastic
Demand Form	Linear, Exponential, Poisson
Demand Input	Price, Time, Inventory
Sales	Backlogged or Lost sales
Period	Single or Multiple
Capacity limit	Finite or Infinite

If we start from the period perspective, the single period models, called the newsvendor problem if the demand is stochastic, are examined since 1950s. Whitin [4] formulates a newsvendor model in which the selling price and the inventory quantity are set at the same time. He assumed a price-dependent demand, and so price is a decision variable unlike the original newsvendor problem that assumes pricing is an exogenous decision. In Whitin's [4] model the demand is deterministic and it is a linear function of price. Mills [5] reconsiders the model of Whitin [4] by adding some randomness into the demand. In his model, the demand is a decreasing linear function of price summed with a random variable defined in some range. Karlin and Carr [7] were the first to consider multiplicative price-demand relationship in a one period newsvendor problem. Petruzzi and Dada [8] firstly analyze a problem where a selling price and a stocking quantity are decided before the price-dependent random demand is realized in a single period. They examine both additive and multiplicative demand cases to show the value of information. Also, Lau and Lau [6] study a newsboy problem with price-dependent demand distribution. All these models are the price-dependent demand models with one period in the literature.

In Li's [20] model the demand and the production are Poisson processes and the demand is price-dependent. He assumes that unsatisfied demand is lost and the production and holding costs are linear. He demonstrates that a base-stock policy is optimal for both cases with single fixed price and with dynamically changing prices.

Zabel [11], Thowsen [12], Urban and Baker [9], Dana and Petruzzi [10] and Federgruen and Heching [18] investigate models where the demand depends on price. In Zabel's [11] model the price is dynamic and there are two types of costs, production cost and holding cost. He considers two demand models, a multiplicative model and an additive model as most of researchers do. Thowsen [12] extends Zabel's model by changing the lost sales to backlogged demand. Urban and Baker [9] develop a model where the demand is deterministic, but is a multivariate function of price, time and inventory level; contrary to most of the research that considers the demand to be a function of price alone as we do. In Dana and Petruzzi's [10] model, the uncertain demand depends on both price and inventory level. Federgruen and Heching [18] establish a periodic-review inventory pricing and replenishment model with stochastic demand and both limited and unlimited production capacity. They express that optimal profit can be provided by a base-stock list price policy for both finite and infinite horizon. There is more information about the dynamic pricing literature and current practices in the review paper of Elmaghraby and Keskinocak [31].

Price-dependent demand functions have been used in several papers and they are usually linear and multiplicative. Most of the models are extended to multi-period after the one period model is presented. In addition, a number of models add some randomness into the demand. Our research considers a multi-period model with linear and exponential price-demand relationship functions. Also, the demand arrives in single units according to a Poisson process and capacity is finite. On the contrary to many papers, we consider the capacity reservation cost for the retailer.

The second part of the literature survey is about coordination and information asymmetry in supply chain systems.

Chen [19] scrutinizes a supply chain model with lost sales and limited capacity. The customer demand is stochastic and price-dependent such that the random error can be changed according to the price. All parameters are nonstationary and the production cost and the holding cost are linear. He shows how an optimal pricing-replenishment strategy balances the costs and the revenues due to advanced demand information.

Cachon and Zipkin [13] examine an independently managed two stage serial system in which both the supplier and the retailer choose periodic-review base-stock policies that minimize their own expected costs, but they do not take into account the effect of price. They propose a linear transfer payment contract in which the transfer payment parameters depend upon the system performance. Also they showed that competition reduces efficiency and the value of cooperation is context specific.

Lee and Whang [14], Lee, So and Tang [15], Aviv [16], Gavirneni, Kapuscinski and Tayur [17], Chen and Simchi-Levi [21], Gupta and Weerawat [22] and Sharafali and Co [23] analyze a number of supply chain models and design contracts.

Lee and Whang [14] study two-stage single agent models in which only the upper stage incurs a holding cost while the lower stage incurs a backorder penalty. They propose a non-linear transfer payment contract to align the agents by utilizing an echelon inventory policy. Lee, So and Tang [15] analyze a supply chain consisting of a retailer and a supplier with a nonstationary demand process. The demand of period t depends on three independent random variables with a common normal distribution with mean zero and a variance. One of them is same for all periods, another one is a coefficient for previous period demand level and the last one is a random variable that is changeable to the period. Both retailer and supplier know the values of parameters of the demand process, but only the retailer knows the realized demand in each period.

Aviv [16] designs a supply chain model with one retailer and one supplier, too. Customer demands come to the retailer, who replenishes its inventory from the supplier. Both the supplier and the retailer independently forecast the customer demand for the future periods. These two members of supply chain collaborate except their specific forecasts. There are three cases: In first case, they do not use their forecasts in their replenishment policies, in second one they integrate their own forecast into their replenishment policies, in the last one they share their demand forecast and use the shared information in their decisions.

Gavirneni, Kapuscinski and Tayur [17] study a model in which the retailer faces independent and identical demands and replenishes his inventory by using the following base-stock policy. They check the inventory level at the beginning of each period and then they place an order to raise the inventory to base-stock quantity. The supplier satisfies the order if he can. If he has not enough on-hand inventory, a partial shipment is made to the retailer. The customer demand is backlogged if it cannot be satisfied. The supplier decides how much to produce for the period after delivering the retailers order. The supplier’s capacity is limited and he incurs linear inventory holding cost and penalty cost for retailer lost orders.

Chen and Simchi-Levi [21] formulate an infinite horizon, single product, periodic review model in which pricing and production/inventory decisions are made simultaneously. The demands of the different periods are independent and the distributions depend on price. They assume unsatisfied demand is backlogged and ordering cost includes both fixed and variable costs.

Gupta and Weerawat [22] study on supplier-manufacturer coordination in capacitated two-stage supply chains by comparing three different mechanisms that a manufacturer may use to affect its component supplier’s inventory. According to their model some processing is required at the manufacturer’s facility before the product is ready for the customer which is different from our model. Revenue per unit is lead-time sensitive and coordination is achieved by adjusting the share of the revenue that each player receives. The lateness penalty is shared by the two players and determined by the outcome of their game. They demonstrate with numerical experiments that, the component supplier benefits from having a high utilization of its production facility, whereas the manufacturer benefits from having excess production capacity.

Sharafali and Co [23] consider a two-stage supply chain with one retailer and one supplier. The demand at the retailer is random and distributed by Poisson with mean λ, stock outs are back-ordered and the lead time is constant. The retailer uses an (s,S) base-stock policy and replenishes from the supplier's inventory instantaneously. Sharafali and Co [23] explain a number of stochastic models of cooperation between the supplier and the retailer. For the price-dependent model they could not find an analytical solution, so they illustrate it only numerically.

More information can be found in the survey and classification chapter of Chan, Shen, Simchi-Levi, and Swann [**Error! Reference source not found.**] in which they summarized model-based research that coordinate pricing policies with inventory control and production decisions in the supply chain.

Jemai and Karaesmen [32] investigate a two-stage supply chain consisting of a capacitated supplier and a retailer that faces a stationary random demand. They study the determination of decentralized inventory decisions when the two parties optimize their individual inventory-related costs independently. The value of advance demand information on capacitated supply systems is investigated in the paper of Karaesmen, Liberopoulos and Dallery [33]. They model a single-stage manufacturing system operating in a make-to-stock mode where the manufacturing capacity is modeled by a single-server queuing system.

Corbett, Zhou and Tang [29] examine the value to a supplier of offering more general contracts and the value of obtaining better information about the cost structure of the retailer. They include a reservation profit level for the supplier, as we include in our model by declaring the capacity reservation cost for the retailer.

Yao, Chan and Yan [24] model a one supplier, one retailer supply chain with price-dependent stochastic demand as we do. They propose a novel contract to coordinate the decentralized system in which the retailer faces with price-dependent stochastic demand. The retailer can place a one-time order and the supplier produces and delivers the order to the retailer prior to the selling season. Before the retailer's order, the supplier announces the sales price, the return price or policy. According to this announcement the retailer decides his retail price to the customer and the ordering quantity. To achieve the coordination they define a profit sharing contract that wholesale price links with the supply chain profit margin, while the returns policy offsets a certain level of downward risk on the retailer side. Independent parameters from the stochastic demand distribution, linearly correlated profit between supplier and retailer and easy implementation for supplier are provided by this profit sharing contract. As a result the whole supply-chain profit is maximized while the retailer is maximizing his profit by optimally solving the newsvendor problem with pricing with this contract. Although it seems very similar to our model, there are many differences between these models. One of the different aspects of our model with respect to Yao, Chan and Yan's [24] model is that we have capacity constraint for the supplier and also there is a fixed cost of reserving one unit production capacity in addition to variable cost of producing a unit product. The other different aspect is that our model is a multi-period continuous model that the demand arrives in single units according to a Poisson Process with rate $\lambda(p)$ where p is the sales price of retailer, so there is no return material. The last and the most important difference from Yao, Chan and Yan's [24] model and most of the other supply chain models is our model that considers the asymmetric information between the agents of the supply chain and the supplier decides under limited information.

Caldentey and Wein [2] model a supply chain system that is running like an M/M/1 make-to-stock process. The backorder cost is shared by supplier and customer and all unsatisfied demand is backordered. The retailer earns fixed revenue per unit sold and that backorder allocation fraction is exogenously determined. There is a linear holding cost for the retailer and a linear capacity cost for the supplier. The retailer stocks finished products that require no further processing. He replenishes his inventory from an upstream supplier. The supplier's manufacturing facility behaves as a single server queue with exponential service times, so a product it produced corresponds to a service completed. The retailer's optimal inventory strategy is a base-stock policy. They use a continuous state approximation that can be justified by a heavy traffic approximation, to simplify their analysis. Our model framework is based on this article with some differences. We add price-dependent demand concept into the model and also there are two more cost parameters which are a variable cost of producing one unit and a reservation cost for one unit. In our model, all backorder costs charged to the retailer different from Caldentey and Wein's [2] model. As before, the main difference is the asymmetric information between the supply chain agents.

2.2. Problem Description

We want to analyze how the centralized and decentralized supply chain system operates under the conditions that the chain consists of several decision-makers and to understand the effects of limited information sharing on the supply chain performance. If this supplier and retailer are under different ownership or are independent divisions within the same firm, then their competing objectives can trigger problems about coordination [2]. The supplier typically wants the retailer to hold as much inventory as possible, while the retailer prefers to hold very little inventory and desires quick response from the supplier.

Our model is based on the contracts offered by the supplier that include the optimal base-stock level, optimal sales price and the optimal production capacity with considering the retailer's own problem. It is a basic supply chain model as an M/M/1 make-to-stock queue except some differences. The retailer faces a stationary random customer demand which is dependent on the sales price. The demand arrives in single units according to a Poisson Process with rate $\lambda(p)$ where p is the sales price of retailer. Besides, the retailer replenishes herself instantaneously from the supplier by using a base-stock policy and she can sell the products to the customer with negligible processing time and cost. The supplier produces or supplies his products with an exponentially distributed processing time with rate μ. There is no lost sale at both stages so unsatisfied demand of the customer will be backordered. Because the replenishment time of the retailer is zero and there is no processing, the holding costs of the retailer and the supplier can be assumed equal. This also leads us assume that the supplier does not have any stock in the centralized system. In addition, we will model the decentralized system like the centralized system that the supplier has no inventory.

To simplify our analysis we will use a continuous-state approximation, essentially replacing the geometric steady-state distribution of the M/M/1 queue by an exponential distribution with the same mean to convert the system from a discrete state space to a continuous one as

Caldentey and Wein [2] did. Another simplification, to be clarified later, is to normalize the backorder cost and the expected variable cost by dividing them by the holding cost. As a result we may assume the holding cost, h, is 1 and the backorder cost, b, and other cost parameters can be calculated by considering h.

Our main objectives in this research are to analyze how the decentralized system operates, to understand the effects of limited information sharing on the decentralized system, and to benchmark the supply chain performance against the corresponding centralized system. Finally, we also aim to study the contract design problem of the supplier and a contract design scheme that could operate with limited information.

Chapter 3

CENTRALIZED MODEL

In this chapter, we study the centralized model which will be used for benchmarking later. In the centralized system we consider a one stage system where the manufacturing and the retailing processes materialize in the same stage. There can be no doubt that this system can be supposed as a fully coordinated and a completely information sharing decentralized system. To our knowledge, this system has not been investigated in earlier research.

3.1. Notation

We summarize the notation to be used below:

s :	Base-stock level
p :	Sales price to the end customer
μ :	Production capacity
b :	Backorder cost
h :	Holding cost
c_1 :	Variable cost of producing a unit product
c_2 :	Fixed cost of reserving one unit production capacity
m & n :	The parameters to define the price function
IO:	The number of outstanding orders
$\lambda(p)$:	Demand rate (price dependent)
ρ :	Utilization rate
Π_C:	Profit function of the centralized model
Π_C' :	Profit function of the centralized model with deterministic price assumption
Π_S & Π_R:	Profit functions of the supplier and the retailer

3.2. Centralized Model

Our centralized supply chain consists of a facility that provides a single product and a retail outlet where the product is made available to the end customer. The demand of the product under consideration is modeled as a Poisson process with rate $\lambda(p)$ where p is the sales price to the end customer. The supply chain carries inventory to satisfy this demand and all unsatisfied demand is backordered. Because the production and retail processes operate as a single entity, the optimal replenishment policy is a $(s\text{-}1,s)$ base-stock policy. Under this policy in the beginning we have s units in the inventory; when a demand occurs, we deliver the order of the customer if the inventory is available and we release a production order to the manufacturing facility.

The manufacturing facility of the centralized system behaves as a single-server queue with service times that are exponentially distributed with rate μ. The server is only busy when customer orders are present in the queue. The retail outlet of the centralized system behaves as an M/M/1 queue, because the demand process is Poisson and a base-stock policy is used. When the production process is completed at the manufacturing facility, the product is ready for sale.

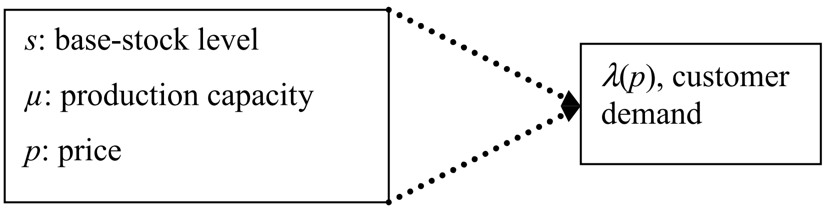

Figure 3.1 *Centralized system*

As seen in Figure 3.1, the decision variables of the centralized system are the production capacity, the base-stock quantity and the price of the product which are to be determined by the centralized system administrator. The demand rate is a function of the unit price charged. The parameters m and n define the price function. We consider both linear and exponential demand functions in our analysis, however it is not difficult to extend the model to other price functions.

$$\lambda(p) = \begin{cases} 0 & \text{o/w} \\ m - np & m/n \geq p \geq 0 \end{cases} \qquad \text{linear case}$$

$$\lambda(p) = m\, e^{-n\,p} \qquad \text{exponential case}$$

The cost of backordering one unit is given as b and the inventory holding cost is h per unit time of the production-inventory system. The variable cost of producing a single unit of product is defined by c_1, and the fixed cost of reserving a single unit production capacity is defined by c_2. Thus $c_2\,\mu$ represents the amortized cost per unit of time that the supply chain incurs for having a capacity of μ; this fixed cost rate is independent of the demand level.

To simplify our analysis we normalize all the cost parameters by dividing them by h, i.e. the holding cost, see Caldentey and Wein [2] for a similar application. Accordingly, the normalized cost parameters are as follows:

$$\tilde{h} = h/h = 1; \quad \tilde{b} = b/h; \quad \tilde{c}_1 = c_1/h; \quad \tilde{c}_2 = c_2/h; \quad \tilde{p} = p/h \tag{3.1}$$

In the remainder of the thesis, the tildes will be omitted to simplify the notation.

We assume that the capacity rate is always greater than the demand rate to satisfy the stability of the production-inventory system. The inventory process operates as shown in Figure 3.2. The base-stock level is s, and birth-death process runs with demand rate $\lambda(p)$ and production rate μ. Then, we can say N is geometrically distributed with rate 1-ρ, where ρ: $\lambda(p)/\mu$ is the utilization rate and N is the steady-state number of orders at the supplying facility.

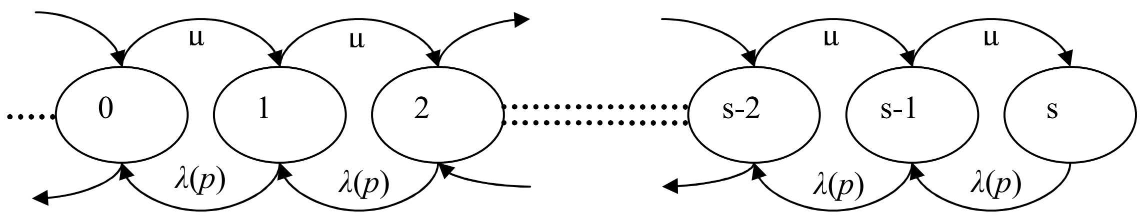

Figure 3.2 The *birth-death process of base-stock policy*

Another simplification for our analysis is a continuous-state approximation that can be justified by a heavy traffic approximation [25] to replace the geometric distribution by an exponential distribution or to convert the system from discrete to continuous in other words. Caldentey and Wein [2] also use this continuous-state approximation and it generates mean queue lengths that coincide with M/M/1 results for all server utilization levels. This approximation is known to be very accurate as ρ approaches 1 and can be demonstrated as shown below:

$$P(N \le n) = \sum_n P(N = n) = \sum_n (1-\rho)\rho^n \approx P\left(\widetilde{N} \le n\right) = 1 - e^{-n(1-\rho)}$$

where N is a geometrically distributed random variable corresponding to the steady state distribution of an M/M/1 queue and $\tilde{N}$ is an exponentially distributed random variable which corresponds to the steady state distribution of the limiting continuous state process.

If we assume that IO is the outstanding orders in the steady state, the expected profit of the centralized system can be written as:

$$E[\Pi_C] = (p-c_1)\,\lambda(p) - E[(s\text{-}IO)^+] - E[(IO\text{-}s)^+]\, b - c_2\,\mu \tag{3.2}$$

To find the expected profit, we can first calculate the cost functions:

$$E\left[(s\text{-}IO)^+\right] = \int_0^s P(s - IO > t)\,dt = \int_0^s P(IO < s - t)\,dt = s - \frac{1}{\rho} + \frac{e^{-s(1-\rho)}}{1-\rho}, \tag{3.3}$$

$$E\left[(IO - s)^+\right] = \int_s^\infty P(IO\text{ - }s > t)\,dt = \frac{e^{-s(1-\rho)}}{1\text{ - }\rho}. \tag{3.4}$$

As a result, the average centralized profit per unit time can be written as:

$$\Pi_C = (p\text{-}c_1)\,\lambda(p) - \left[s - \frac{1 - e^{-s(1-\frac{m-np}{\mu})}}{1 - \dfrac{m-np}{\mu}} + b\frac{e^{-s(1-\frac{m-np}{\mu})}}{1 - \dfrac{m-np}{\mu}} \right] - c_2\mu, \tag{3.5}$$

where the price function is assumed to be linear:

$$\lambda(p) = m - np. \tag{3.6}$$

In order to find the optimal solution, first we find the optimal value of s for given p and μ values.

The equation for the first order optimality condition and the optimal value of s are:

$$\frac{\partial E[\Pi_C]}{\partial s} = 0 \Leftrightarrow s^* = -\frac{\mu \log(1+b)}{m - np - \mu} \qquad (3.7)$$

After this point s is replaced by its optimal value in Equation 3.5 and we try to solve the equation. After taking into account the other two variables, μ & p, the optimum solution of the equation cannot be found analytically. If we find the optimum values of s and p then we are not able to find the optimal value of μ, similarly if we find the optimum values of s and μ then we are unable to find the optimal value of p. Since the equation could not be solved analytically when considering the price, p, base-stock level, s, and the capacity, μ, variables; we decide to examine the behavior of the function for numerical examples.

The optimal s and p values by using fixed capacity, and the optimal p and μ values by using fixed p are also found, but they are not shown here because the expressions are complex.

Hereafter we call to the analytical solution of the Equation 3.5: Problem C, and we refer to the optimal values of the price, the capacity, the base-stock level and the profit by referring to the solution of Problem C.

3.3. Numerical Examples

A number of numerical examples are considered with various parameters values to understand the behavior of the model. We will demonstrate two of these examples and present their results.

Case 1:

The parameters are given as follows:

$m = 100$, $n = 2$, $b = 5$, $c_1 = 1$, $c_2 = 5$.

The price value can be between 0 and 50 and $\lambda(p)$ can be between 0 and 100 because $m\text{-}np$ should be greater than 0. For different μ values, the optimal base-stock and the optimal price values are shown in Figure 3.3. We do not have to hold too much inventory for large capacity values as expected. An interesting observation is that the optimal price value is steady for large μ values.

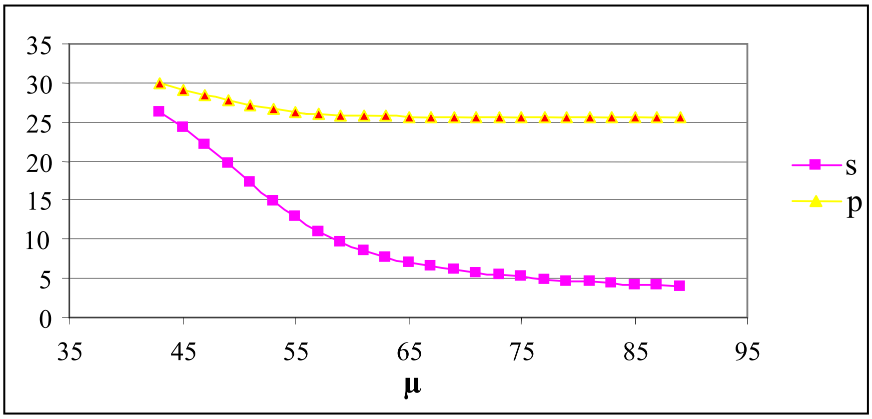

Figure 3.3 *The optimal base-stock and price values*

As seen in Figure 3.4 the optimal profit values are increasing first and then decreasing when capacity increases. As a result, the optimal profit can be found by choosing the specific μ value that gives the largest profit. Optimal capacity is 50, optimal base stock level is 20 and the optimal price is 27.5 for this particular example.

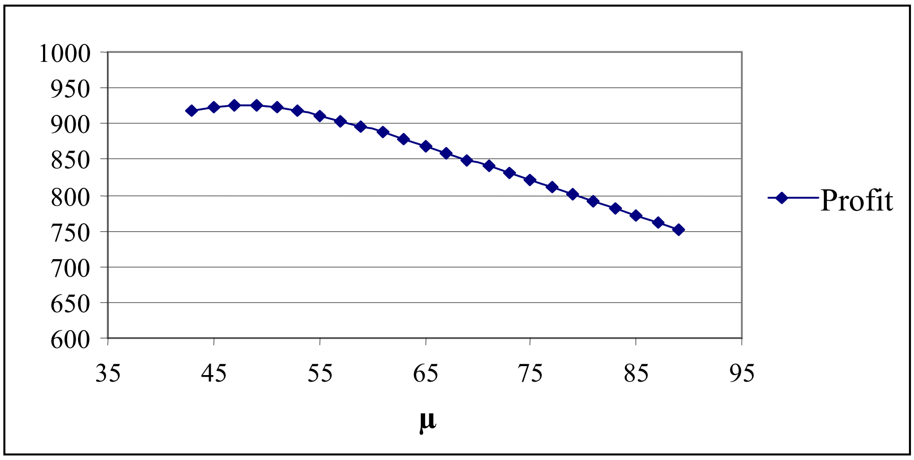

Figure 3.4 *Optimal profit values versus capacity*

Case 2:

The parameters are given as follows:

$m = 10$, $n = 1$, $b = 2$, $c_1 = 1$, $c_2 = 1$.

As seen in Figure 3.5 the optimal base-stock and price values decrease when μ increases. Although the rates of decrease are different, because of the demand parameters, the results are parallel with the first case.

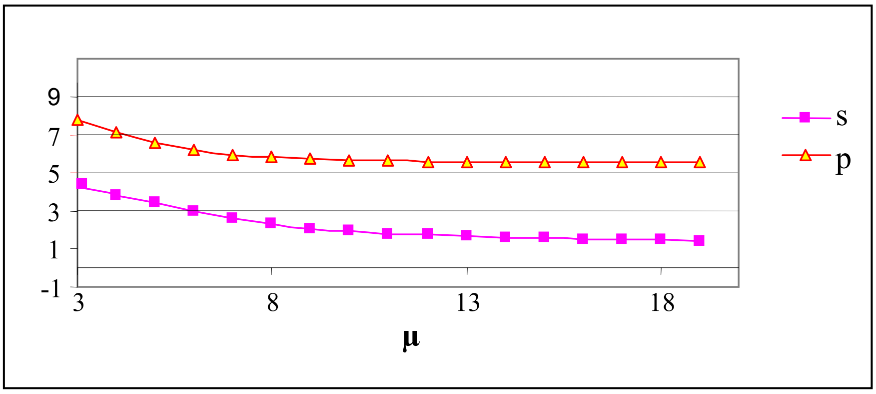

Figure 3.5 *Base-stock and price values versus capacity*

Figure 3.6 demonstrates the optimal profit values for different capacities. For large capacity values, the profit may go below zero because of the capacity cost.

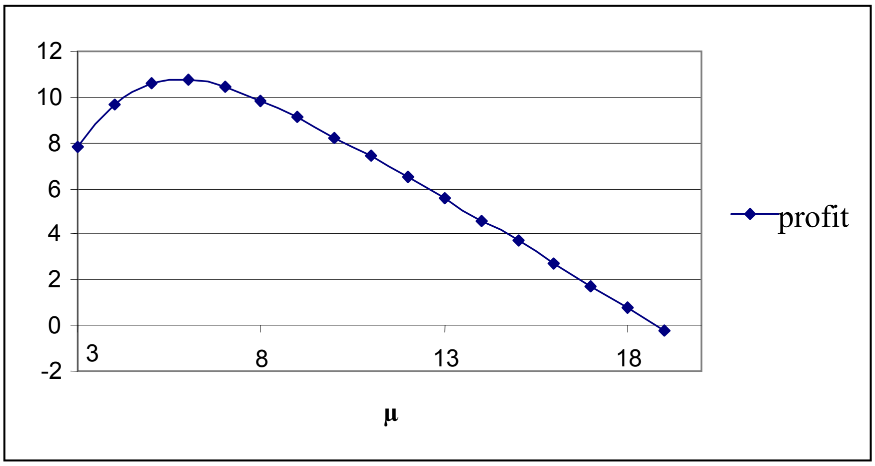

Figure 3.6 *Profit versus capacity*

We observe that the profit functions are convex for all numerical examples that have studied. We have performed the same analyses for the exponential price-demand relationship.

$$\lambda(p) = me^{-np} \tag{3.8}$$

All steps of the analyses with an exponential demand function are given in the Appendix A.

As a result, when the backorder cost increases without changing the other parameters, the price values also increase, and so the demand decreases because of the higher price. Thus, the base-stock levels and the profit values decrease as expected. In all cases it can be easily seen for large μ values the price will be steady. For this reason we develop a deterministic price assumption for all calculations.

3.4. The Deterministic Price Assumption

Because the optimal price seems to converge to a constant value as μ increases, the price can be assumed fixed for large enough capacity values. This simplifies the analysis considerably. In order to find the value of the deterministic price, we consider a case where the capacity is infinite and so neither the backorder cost nor the holding cost will be taken into account.

The expected profit function of the centralized system, without considering the backordering and the holding cost, can be written as:

$$E[\text{Profit}] = (p - c_1)\lambda(p) - c_2\mu \tag{3.9}$$

The first order optimality condition is:

$$\frac{dE[\text{Profit}]}{dp} = -2np + m + nc_1, \tag{3.10}$$

and the optimal deterministic price value is equal to

$$p_{\text{det}}^{\ *} = \frac{1}{2n}(m + nc_1). \tag{3.11}$$

We will use the optimal deterministic price in the analysis to lead the supplier while he is preparing the contract. So, only the contract parameters depend on the deterministic price, the effectiveness tests are done with numerical examples that are based on analytical formulations. To continue the analysis with the deterministic price we replace the variable p with the optimal deterministic price value.

Eventually the profit function becomes

$$\Pi_C{}' = \left(\frac{1}{2n}(m+nc_1)-c_1\right)\left(\frac{1}{2}(m-nc_1)\right)-\left[s-\frac{1-e^{-s\left(1-\frac{\frac{1}{2}(m-nc_1)}{\mu}\right)}}{1-\frac{\frac{1}{2}(m-nc_1)}{\mu}}+b\frac{e^{-s\left(1-\frac{\frac{1}{2}(m-nc_1)}{\mu}\right)}}{1-\frac{\frac{1}{2}(m-nc_1)}{\mu}}\right]-c_2\mu \quad (3.12)$$

3.5. Analytical Results with Deterministic Price Assumption

Proposition 3.1.

The optimal base-stock level, the optimal capacity and the optimal expected profit by using deterministic price can be written as

$$s^* = \log(1+b)+\sqrt{\frac{(m-nc_1)\log(1+b)c_2}{2}}, \quad (3.13)$$

$$\mu^* = \frac{1}{2}\left((m-nc_1)+\frac{\sqrt{2(m-nc_1)\log(1+b)c_2}}{c_2}\right), \quad (3.14)$$

$$E\left[\Pi_C{}'\right] = p(m-np)-\log(1+b)-2\sqrt{(m-np)\log(1+b)c_2}-(m-np)(c_1+c_2). \quad (3.15)$$

Proof :

When we assume the price is fixed, our model likes the model of Caldentey and Wein [2] and they demonstrate the solution of the model. By using the same approach, we will find the optimal solutions. The optimal *s, μ,* and the profit values for optimal deterministic price by using equation 3.12 are as follows

$$s = \frac{2\mu\ \log(1+b)}{m-2\mu-nc_1}, \quad (3.16)$$

$$\mu = \frac{\sqrt{2\log(1+b)\,(m-nc_1)}+m\sqrt{c_1}-nc_1\sqrt{c_2}}{2\sqrt{c_2}}. \quad (3.17)$$

By rewriting the value of μ in equation 3.16 the optimal s can be found as follows:

$$s^{*}=\frac{2\left(\frac{\sqrt{2\ \log(1+b)\ (m-nc_1)}+m\sqrt{c_1}-nc_1\sqrt{c_2}}{2\sqrt{c_2}}\right)\log(1+b)}{m-2\left(\frac{\sqrt{2\ \log(1+b)\ (m-nc_1)}+m\sqrt{c_1}-nc_1\sqrt{c_2}}{2\sqrt{c_2}}\right)-nc_1}$$

Similarly, by altering the values of s and μ in equation 3.12 the optimal expected profit can be found. ■

In comparison with the analytical results, Problem C, the approximation provides the simple results as seen in equations 3.13, 3.14 and 3.15. Moreover, the difference is small for large μ values. The behaviors of the profit functions with this deterministic price assumption for several cases are shown below.

Case 3.a:

The parameters are given as follows

$m=100$, $n=2$, $b=5$, $c_1=1$, $c_2=5$

The optimal profit values of Problem C, and the optimal profit values by using the deterministic price assumption for different μ values are compared in Figure 3.7. It can be seen easily that the difference between the optimum profit values are very small. Optimal profit is 926 and the optimal solution of the Problem C is 911, so the loss from the deterministic price is 0.16%. The capacity values which are smaller than 50 are not taken into consideration because the system is not stable for small μ values when a deterministic price is used.

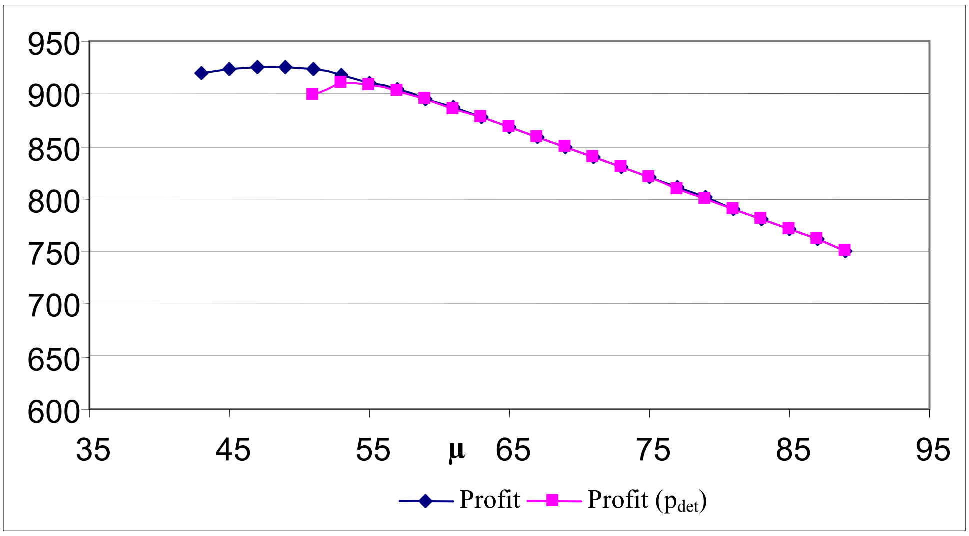

Figure 3.7 *Profit versus capacity with deterministic price assumption*

Case 3.b:

The parameters are given as follows

$m = 100$, $n = 2$, $b = 50$, $c_1 = 1$, $c_2 = 5$

Figure 3.8 illustrates the differences of the optimal profit values obtained by finding the solution of the Problem C, and by using the deterministic price assumption for different μ values. We can infer from the figure that the optimum profit value by using the deterministic price is close to the optimum profit that is found by solving the actual problem, Problem C.

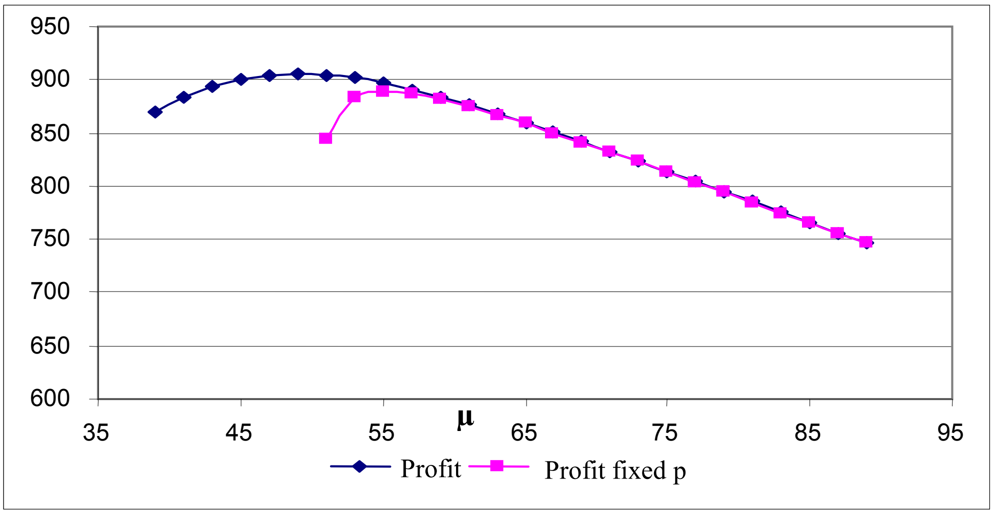

Figure 3.8 *Profit versus capacity with deterministic price assumption*

Case 3.c:

Besides the linear demand case, we also study the deterministic price assumption under the exponential demand function as well.

The parameters are given as follows

$m = 10$, $n = 1/10$, $b = 10$, $c_1 = 1$, $c_2 = 1$.

We can see the distinction of the optimal profit values by finding the solution of Problem C for the exponential case, and by using the deterministic price assumption for different μ values from Figure 3.9. Also, we can say that the optimum profit value with respect to the deterministic price is close to the optimum profit that is found by solving Problem C, as before.

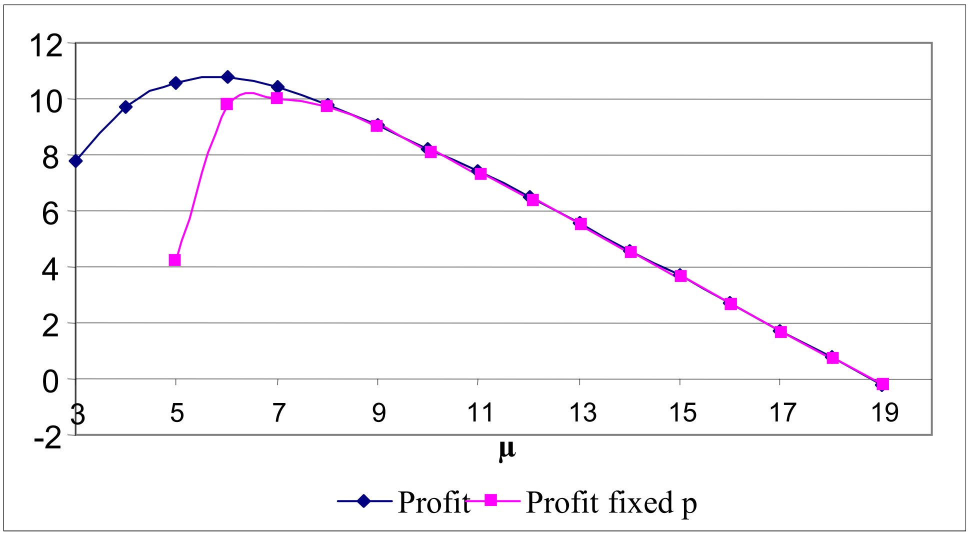

Figure 3.9 *Profit versus capacity with deterministic price assumption (exp case)*

In order to verify and generalize the quality of the deterministic price approximation, we test the accuracy of the profit functions for many different numerical examples for both linear and exponential demand cases.

3.6. Sensitivity Analysis

The results of the accuracy tests between the profits show that the optimal profit obtained by using the deterministic price is very close to the profit obtained by using the optimal price in many cases. The accuracy test strategy is based on the numerical calculations by assuming the variables in three dimensions. The first dimension is b, backorder cost, the second dimension is c_1, variable cost of producing a unit product and the third dimension is c_2, fixed cost of reserving one unit production capacity. The deviations are calculated for each value of b, c_1 and c_2 in order to each value of them. The backorder cost values that are used in the analysis are 10, 100 and 1000. Also, c_1 and c_2 values are changed between 1 and 500 by increments of 1. In addition, m values are 10, 100, and 1000.

According to Figure 3.10; the first recursion is on b, the second one is on c_1 and the third one is on c_2, and the graph demonstrates only the second and the third recursion for $m = 10$. Let's check the point in the circle; firstly the backorder cost is 10 for all the values on the graph, secondly c_1 is 5 (fifth cycle in the graph) and finally c_2 is 7 (seventh point in the fifth cycle), and the value of "deterministic price assuming profit / optimum profit" is 0.84 for this particular point. It is inferred from the whole graph (not shown here), the deviation is very little even if the backorder values are very large. It gives very good results in many cases with average "deterministic price assuming profit / optimal profit" between 98% and 99% with standard deviation between 0.1 and 0.0005. The worst result comes out when the c_1 and c_2 are the highest available values and the profit margin is the lowest value, approximately 0% - 1%. However this is a very special case in most industrial applications, we anticipate that different results and interpretations may be provided according to the different profit margin markets.

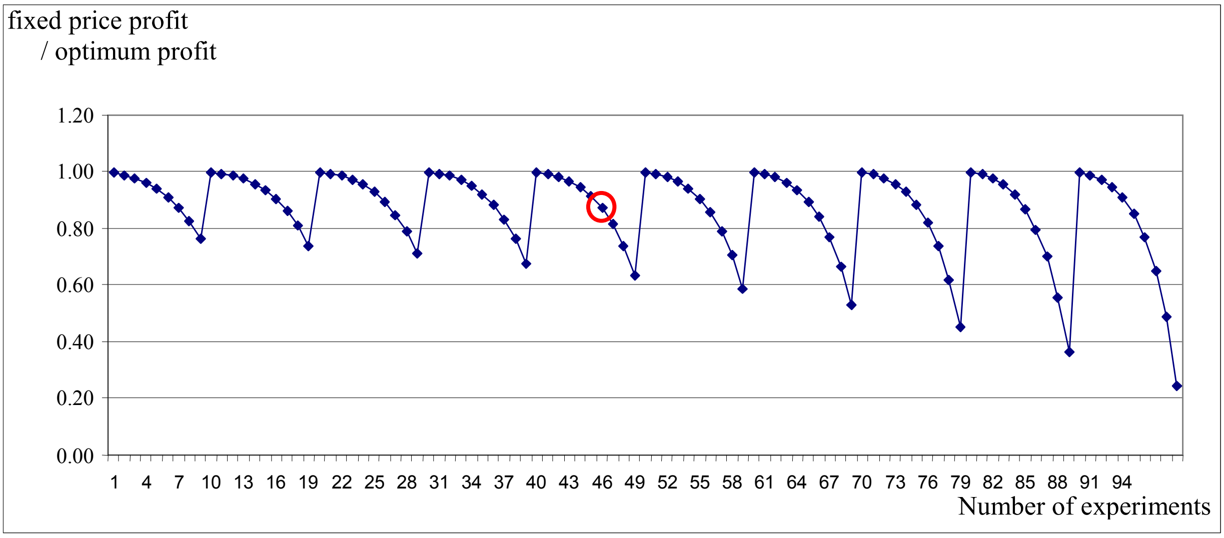

Figure 3.10 *Deviation results of deterministic price assumption*

In addition to the linear demand case, we also test the accuracy of the deterministic price assumption with an exponential demand function. After many numerical tests of the accuracy of the optimal profit, we observed that the results are better than the linear demand case. In summary, the average value of "deterministic price assuming profit / optimal profit" is 99% with a standard deviation of 0.01. The worst result comes out when the c_1 and c_2 are the highest values and the profit margin is the lowest value but the deviation is still 93% which is a reasonable value for the deterministic price assumption.

As a result, the deterministic price approximation may be used to determine the base-stock level, the capacity and the price values, hence we will try to prepare a contract by using this approximation in the decentralized system. The calculations are very simple and the results are good enough to decide, especially in the high profit markets. In addition, centralized analysis is a first step for studying the decentralized supply chain system.

Chapter 4

DECENTRALIZED MODEL CASE I : Unknown Backorder Cost

The objective of a typical supply chain is “to provide value to the end consumer in terms of products and services and for each channel participant to garner a profit in doing so” as mentioned in the article of Sahin and Robinson [26]. There are many interactions between the supply chain members to achieve this objective and these relations may be classified as financial, physical and information flows. The supply chain system should be identified, analyzed and coordinated to manage the system and the flows effectively and efficiently. Although it is difficult to integrate the conflicting objectives, the benefits of integration cannot be ignored. As discussed in the article of Stein and Sweat [27], sharing demand information vertically among supply chain members has achieved huge success in practice. According to Stein and Sweat, by ”exchanging information, such as inventory level, forecasting data, and sales trends, these companies are reducing their cycle times, fulfilling orders more quickly, cutting out millions of dollars in excess inventory, and improving customer service.”

In this chapter, we analyze the decentralized system to measure the performance of supply chain, to guide the members of the chain and to prepare a contract to integrate the decentralized system.

We can summarize the notations of the decentralized model to be used in the following list.

c_1 : Sales price of a unit product to the retailer

c_2 : Price of reserving one unit production capacity to the retailer

$c_1^{'}$:	Variable cost of producing a unit product for the supplier
$c_2^{'}$:	Fixed cost of reserving one unit production capacity for the supplier
μ :	Reserved production capacity
Π_S & Π_R:	Profit functions of the supplier and the retailer
$\Pi_S^{'}$ & $\Pi_R^{'}$:	Profit functions of the supplier and the retailer with deterministic price assumption

In the decentralized case there are two independent decision makers, i.e. the retailer and the supplier. As in Chapter 3, the retailer faces a stationary random customer demand which is dependent on the sales price. The demand arrives in single units according to a Poisson Process with rate $\lambda(p)$, call that p is the sales price of the retailer to the end customer. Besides, the retailer replenishes herself instantaneously from the supplier's inventory using a base-stock policy, and she can sell the products to the customer with negligible processing time and cost. The supplier produces or supplies his products one by one where the processing time is exponentially distributed with rate $\mu(c_2)$, as before c_2 is the fixed cost of reserving capacity μ. There are no lost sales at both stages, therefore unsatisfied demand of the customer will be backordered. We also assume that the supplier does not have any stock in our decentralized model. In other words, the supplier has neither inventory nor backorder cost, he offers only a series of contracts to the retailer that includes c_1, c_2 and μ, the reserved capacity. In addition, she has her own variable cost $c_1^{'}$ and fixed cost of reserving one unit of production capacity, $c_2^{'}$. It should be noted that the problem of the retailer is same as the centralized model by taking cost and capacity values from the offer of the supplier.

The sequence of events in our model is as follows. As shown in Figure 4.1 when the production process is completed on the side of the supplier, the product is ready for sale, because the transportation time is assumed to be negligible and the replenishment is instantaneous. When a unit demand arrives, the retailer satisfies the demand from her inventory, if the product is available in the inventory. The retailer then gives an order to the supplier whether or not the product is available in the inventory. If the demand cannot be satisfied from inventory, it is backordered, and when the supplier supplies the product it is then delivered to the customer. The backorder cost is paid by the retailer, and only the retailer carries inventory.

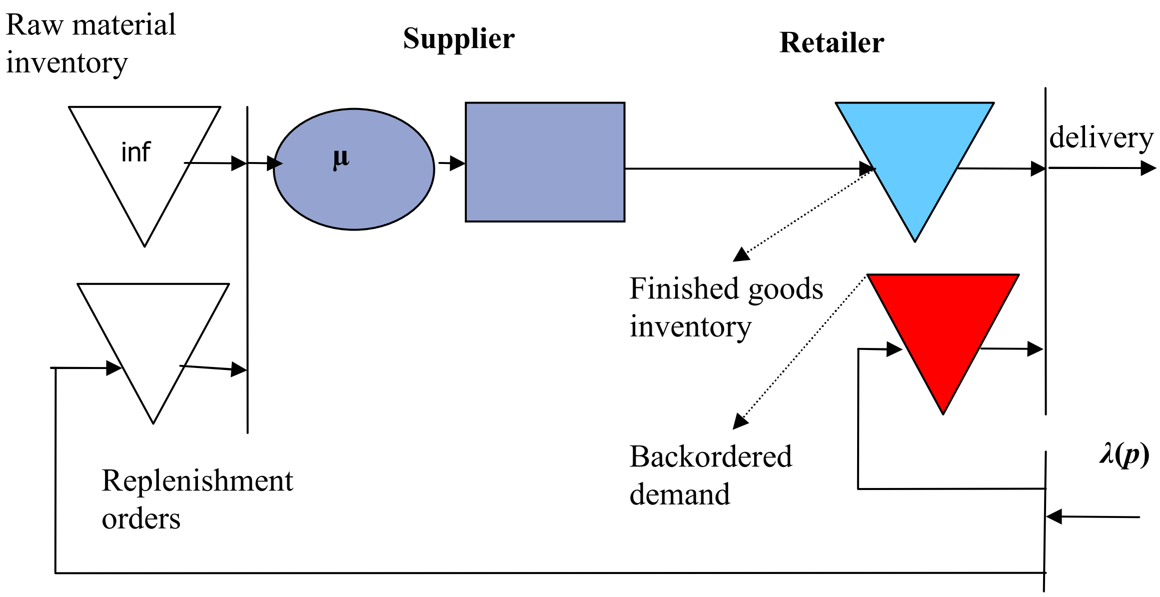

Figure 4.1 *Two stage supply chain mechanism*

4.1. Supplier's Objective Function

The contract offered by the supplier includes three variables: the reserved production capacity, the sales price of producing one unit of product and the capacity reservation price for one unit of capacity reservation. Besides these variables, he has his own cost values: The cost of producing one unit of product and the capacity reservation cost for one unit of capacity reservation. His objective is to maximize his profit by trading with the retailer. His objective function depends on the sales quantity of the retailer. He must consider the individual rationality and incentive compatibility of the retailer when he is offering a contract. Individual rationality means that the contract is beneficial for the agent; i.e., the expected profit of the retailer should be greater than zero. Incentive compatibility means that the contract is optimum for the agent with her own variables.

The profit function of the supplier is given as follows

$$E[\Pi_S] = (c_1 - c_1^{'})\lambda(p) + (c_2 - c_2^{'})\mu . \qquad (4.1)$$

4.2. Retailer's Objective Function

The model of the retailer is same as the centralized model by taking the cost and the capacity values from the offer of the supplier. Figure 4.2 illustrates the variables and the parameters of the supplier and the retailer. The process continues as follows: First the supplier offers contracts that include c_1, c_2, and μ values, that is to say each different combination of c_1, c_2, and μ corresponds a different contract. The retailer then chooses the particular contract that maximizes her own expected profit with variables price and base-stock quantity.

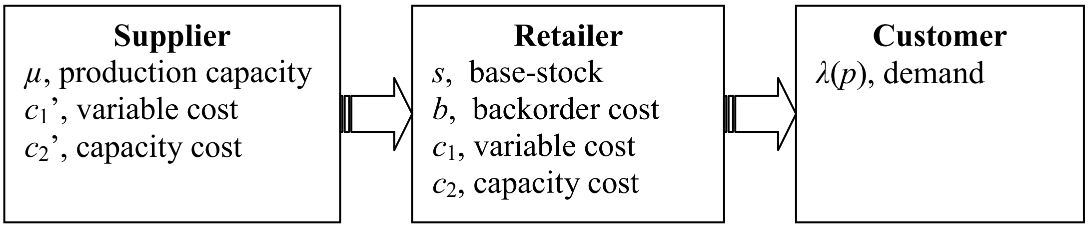

Figure 4.2 *Two stage supply chain system*

The retailer's expected profit is:

$$E[\Pi_R] = (p - c_1) E[\lambda(p)] - E\left[(s - IO)^+\right] - E\left[(IO - s)^+\right] b - c_2\mu \tag{4.2}$$

When we solve the equations, the profit function of the retailer becomes:

$$E[\Pi_R] = (p\text{-}c_1)\,\lambda(p) - \left[s - \frac{1 - e^{-s(1-\frac{m-np}{\mu})}}{1 - \dfrac{m-np}{\mu}} + b\frac{e^{-s(1-\frac{m-np}{\mu})}}{1 - \dfrac{m-np}{\mu}} \right] - c_2\mu \tag{4.3}$$

This profit function cannot be solved analytically as we explained in the centralized case in Chapter 3. Accordingly, it is not practical for the supplier and the retailer to use this function with three variables.

4.3. The Contract Design

The supplier is the contract designer and the leading player of the game in the model, therefore he needs to consider the parameters of the retailer before designing a contract. Similar contract design methodology is used in many articles such as the articles of Karabatı and Sayın [28], Corbett, Zhou and Tang [29] and Cachon [30]. In our model the supplier should give an (or more) offer(s) to the retailer that includes the various capacity, capacity cost and one unit variable cost information values (μ, c_2, c_1). The problem of the supplier is:

Maximize $(c_1 - c_1^{'})\lambda(p) + (c_2 - c_2^{'})\mu$

Subject to

$E[\Pi_R] > 0$ (Individual Rationality)

$s^* = \text{argmax}\ [\Pi_R\ (s)]$ (Incentive Compatibility)

Because the profit function of the retailer is very complex and could not be solved analytically, it is assumed that the supplier uses the deterministic price while he is preparing the contract but all of the analyses after the contract offer are done numerically without using deterministic price. The deterministic price assumption is an approximation from the viewpoint of the supplier and helps to the supplier only to prepare the contract. When the deterministic price value is replaced with the p in the equation, the profit function of the supplier becomes:

$$E\left[\Pi_R^{'}\right] = \frac{1}{2}(m - nc_1)(c_1 - c_1^{'}) + (c_2 - c_2^{'})\mu \qquad (4.4)$$

If we look the situation from the perspective of the retailer; after the offer of the supplier, she chooses a contract that maximizes her own profit by considering her own parameters. The supplier supposes that the profit maximization of the retailer is the same problem with minimizing her costs because the revenue function of the retailer depends on the contract and the fixed parameters.

4.4. Verifying c_1, the variable sales price of the supplier, should be 0

As a result of many numerical experiments, we see that the supplier should set $c_1 = 0$ to maximize his profit when c_2 values are in acceptable ranges. Due to the contract designer is the supplier in the model, he is free to set the parameter $c_1 = 0$ in order to simplify the calculations. As seen in Figure 4.3 the supplier can obtain the maximum profit when the value of $c_1 = 0$ for most of the c_2 values.

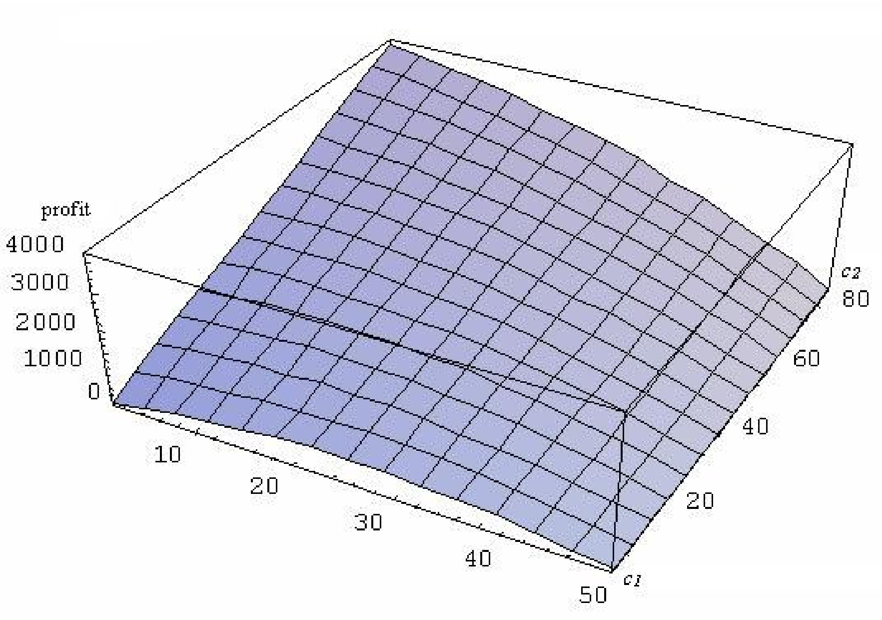

Figure 4.3 *Supplier's expected profit values versus capacity values*

4.5. The optimum contract offered by the supplier when the backorder cost of the retailer is unknown

The first analysis is the situation that the supplier knows the demand parameters (*m, n*) but he does not know the backorder cost value. First the supplier puts himself in place of the retailer to solve the model. The cost function of the retailer after setting $c_1 = 0$ is:

$$E[\text{RCost}] = s + \frac{be^{-s(1-\frac{m-np}{\mu})}}{1-\frac{m-np}{\mu}} - \frac{1-e^{-s(1-\frac{m-np}{\mu})}}{1-\frac{m-np}{\mu}} + \mu c_2 \tag{4.5}$$

The supplier solves the cost function of the retailer with fixed μ (he thinks μ is offered by himself) and an approximate b. The minimum cost can be found by the first order condition for s, because s is the only variable that the retailer decides individually. Eventually the optimal (minimum) cost is:

$$\mu(-\frac{2Log(1+b)}{m-2\mu} + c_2) \tag{4.6}$$

The supplier should offer a contract that satisfies the individual rationality and the incentive compatibility of the retailer so the contract parameters c_2 and μ must ensure a relationship that provides:

$$(m-np)p \geq \mu(-\frac{2Log(1+b)}{m-2\mu}+c_2) \tag{4.7}$$

The expression (*m-np*) *p* is a lower bound on the revenue of the retailer because the fixed p is used, so inequality 4.7 guarantees that the offered contract will be acceptable for the retailer. The relation that is extracted from the solution of inequality 4.7 is shown below when the equality is solved:

$$c_2 = \frac{m^3 - 2m^2\mu + 8n\mu Log(1+b)}{4n(m-2\mu)\mu} \tag{4.8}$$

By assuming that the supplier knows the backorder cost value, it can be said that all of the contracts that satisfy Equation 4.8 are acceptable contracts for the retailer. Thus, the problem of the supplier problem turns into finding the best contract among all feasible contracts.

The objective (profit) function of the supplier is:

$$E[\Pi_S] = -c_1^{'}\lambda(p) + (c_2 - c_2^{'})\mu \tag{4.9}$$

After writing the values of $\lambda(p)$ and c_2 the function can be written as:

$$E\left[\Pi_S^{'}\right] = \mu\left(\frac{m^3 - 2m^2\mu + 8n\mu Log(1+b)}{4n(m-2\mu)\mu} - c_2^{'}\right) - \frac{mc_1^{'}}{2} \tag{4.10}$$

To find the optimal solution as a function of c_2 and μ, the first derivatives are calculated and replaced in the Equation 4.10. As a result, the optimal c_2 and μ values are calculated as shown below:

$$\mu^* = \frac{\sqrt{2mLog(1+b)} + m\sqrt{c_2^{'}}}{2\sqrt{c_2^{'}}} \tag{4.11}$$

$$c_2^* = \frac{(\sqrt{2}(m^2 - 4nLog(1+b)) - 4\sqrt{mLog(1+b)c_2^{'}n})\sqrt{c_2^{'}}}{4\sqrt{mLog(1+b)}n + 2\sqrt{2}mn\sqrt{c_2^{'}}} \tag{4.12}$$

Now, if the supplier knew the backorder cost of the retailer, he could obtain most of the profit by offering the μ and c_2 combination by selecting the other parametric values according to Equation 4.11 and Equation 4.12. However, the supplier does not know the backorder value.

The expected profit values of the retailer according to the backorder costs guessed by the supplier are shown in Figure 4.4. If the supplier offers a (μ, c_2) combination by selecting a lower backorder cost value then the actual backorder cost value of the retailer, the retailer will not trade with the supplier because the expected profit of the retailer will be negative and her individual rationality constraint will not be satisfied.

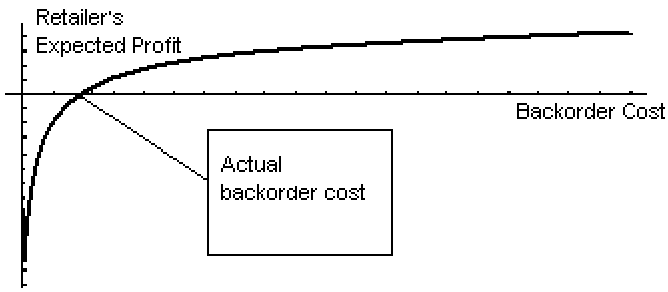

Figure 4.4 *Retailer's profit versus backorder cost*

The supplier guesses a maximum value for the retailer's backorder cost and he calculates other variables according to that particular *b*. Consequently, the contract offered by the supplier includes (μ^*, c_2^*, c_1=0) which are calculated according to the deterministic price assumption and an estimated backorder cost. The estimated backorder cost is randomly varied between 10 times to 20000 times of the holding cost in our numerical analyses which seems to be a reasonable interval.

4.6. Experimental Design of testing contract effectiveness

Because the contract is prepared using the deterministic price assumption, its effectiveness should be tested with respect to the solution without using the deterministic price assumption. The steps of the experimental process can be summarized as:

1) The supplier offers a (c_2, μ) combination that corresponds to a b_{max} value according to the deterministic price assumption.
2) The retailer accepts the contract because her individual rationality and incentive compatibility are satisfied.
3) The retailer sets the optimal *p* and *s* value to maximize her own profit.

This experimental process is performed on 50 random retailers, each of which with 50 random backorder costs. The parameters *m,* c_1' and c_2' are changed in acceptable ranges as seen in Table 4.1. Expected profits of the retailer and the supplier by trading by using the contract that is prepared by the supplier, expected centralized profit and optimum variable values for both centralized and decentralized system are calculated as the result of the experiments. The schema of the experimental set is shown in Table 4.1, and the experiments are performed for each *m,* c_1' and c_2' values. Some cases are infeasible because the sum of capacity cost and variables cost is greater than the price value. Calculations are repeated 50 times (for 50 random backorder cost values) for each feasible case. As a result 16000 experiments are feasible over 24300 possible combinations.

Table 4.1 *Experimental set for numerical calculations*

m	*n*	c_1'	c_2'
100	2	5	5
200		35	35
300		65	65
400		95	95
500		125	125
1000		155	155
		185	185
		215	215
		235	235

It should be noted that the probability of stock-out is equal to h / (h + b) in all supply chain systems considered here so the stock-out probability of our model is 1 / (1 + b). Also, 1 – P(stock-out) is equal to service level of the system. In our numerical experiments we change the backorder cost value from 10 to 20000 so that we scan all the service levels between 1 – (1 / 11) and 1 – (1 / 20001) that corresponds % 90.9 to %99.995 which is an acceptable range for most applications.

4.6. Results according to high / low profit margins, behavior of the players

We performed the experiments according to the above described procedure, and we will illustrate the results in three different aspects. Firstly, the effects of the backorder costs on the supply chain performance are examined and we see that the backorder cost values have no significant effect on supply chain performance in these particular cases. This can also be observed from Equation 3.7 where s^* changes as a function of $log(1+b)$. Although backorder values are changed on an exponential scale, the effects of these changes on the profit values are not considerable and also the variability of the backorder cost is unimportant for the supplier. Whereas, it can be easily seen that the profit margin is extremely important for interpreting the results as we expect from the deviation tests in Chapter 3. The profit margin is calculated according to c_1, c_2 and p values such that: profit margin = $[p - (c_1 + c_2)] / (c_1 + c_2)$. Most observations are based on the averages of 50 calculations for each case if the results are feasible.

Figure 4.5 shows the supply chain performance according to the different profit margin intervals. The supply chain performance can be defined as the ratio of the sum of the decentralized profits to the centralized profit. Efficiency loss is high in low profit margin markets and it is decreasing while the profit margin increases. If the profit margin is between 50 and 100 then the efficiency loss is very low according to the average of nearly 8000 observations. In addition efficiency loss is very high according to the average of nearly 500 observations if the profit margin is between 0 and 10. Because of the feasibility the number of observations in high profit margins is high but the number of observations in low profit margins is low.

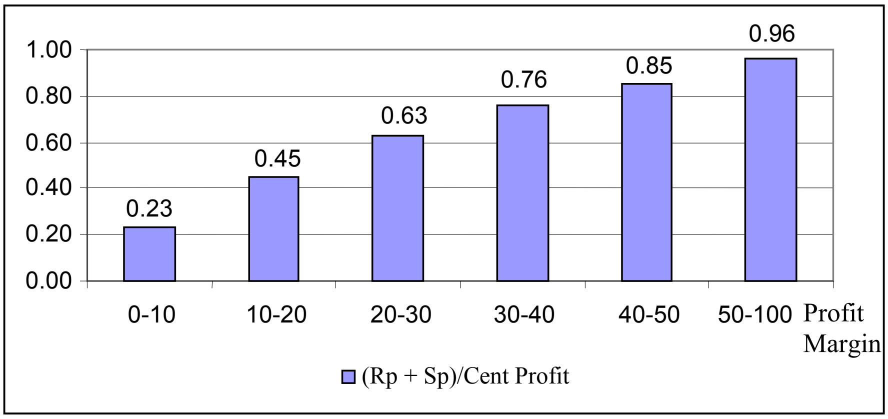

Figure 4.5 *Supply chain performances versus profit margins*

The profit ratios of the retailer over centralized profits according to the profit margins are shown in Figure 4.6. Although the profit ratios that are obtained by the retailer in high profit margins are higher than in the low profit margin markets, both are very low and this is a good reason not to reveal the demand parameter for the retailer.

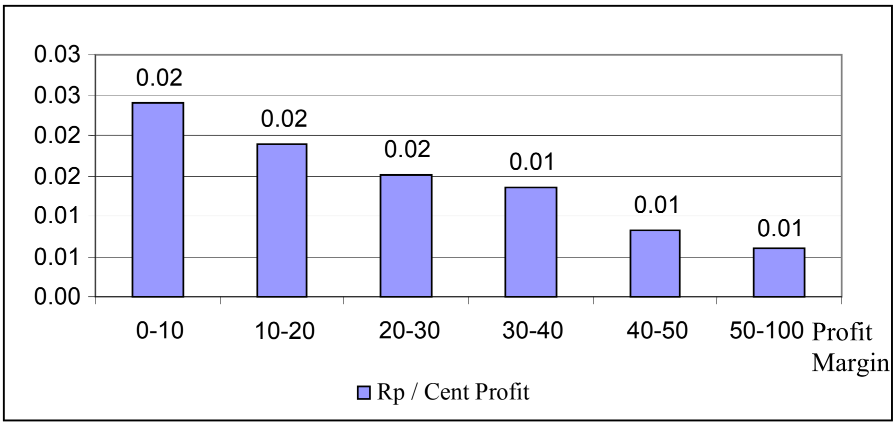

Figure 4.6 *Profit ratios of the retailer versus profit margins*

Figure 4.7 shows the profit rates of the supplier over the centralized profits according to the different profit margin markets. The rate is low in low profit margin markets and it is not a beneficial condition for both the supplier and the retailer but the supplier can obtain most of the profit in high profit margin markets.

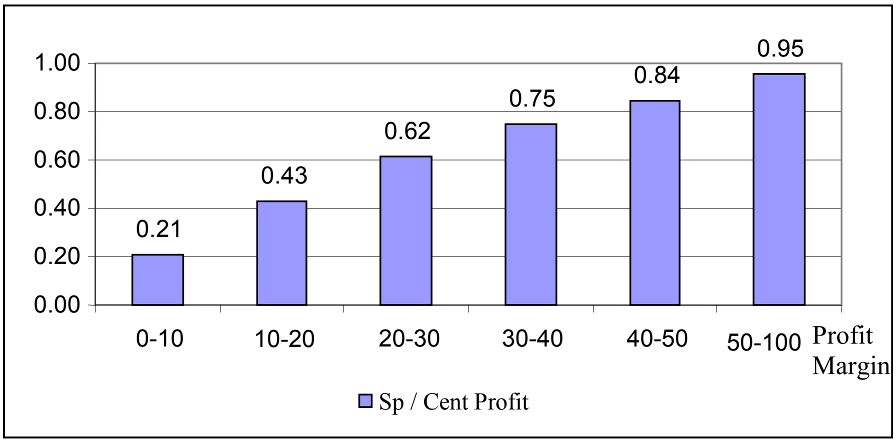

Figure 4.7 *Profit rates of the supplier versus profit margins*

After the general results, specific comments can be presented for high-low profit margin markets.

In high profit margin markets, the supplier can infer the real situation with little differences and he can get most of the profit of the supply chain, with an average of 94%. Thus, hiding the backorder cost information is not valuable when demand information is shared, so that the retailer should not reveal its demand function. In this case, loss of efficiency due to the backorder cost information is little.

The expected profits of the retailer and the supplier over the total decentralized supply chain profit are shown in Figure 4.8. In all markets the supplier can obtain most of the profit and the profit share of the retailer is very low.

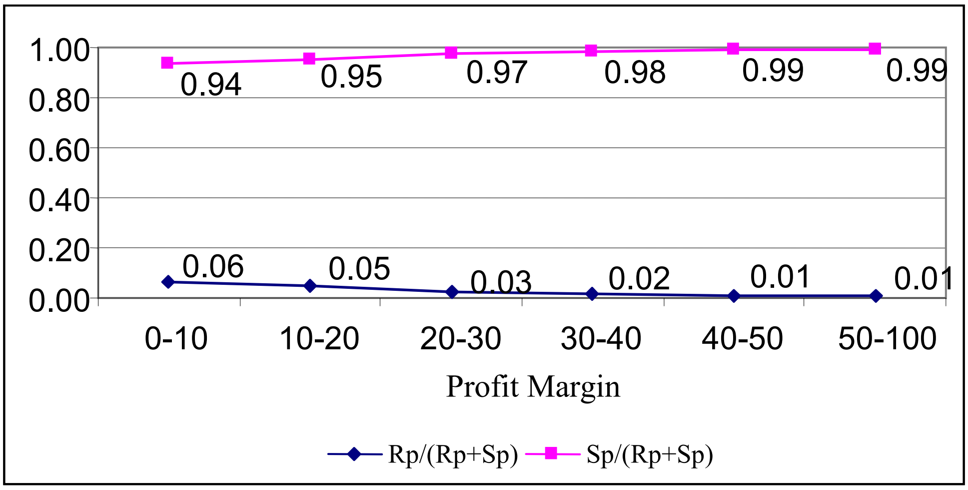

Figure 4.8 *Profit ratios of the supplier and the retailer*

Although the supplier still can get the most of the profit of the chain in low profit margin markets, this situation is not very lucrative for the supplier because the loss of efficiency is very high. Hiding the backorder cost information is worthy for the retailer because she may force the supplier to collaborate or to make an agreement with more advantageous terms.

In addition the linear demand case, a number of results can be given when an exponential demand function is used.

The effects of the backorder costs on the supply chain performance are not significant, parallel as the linear demand case. Although backorder values are changed exponentially, the effects of these changes on profits are not considerable.

Figure 4.9 shows the ratio of the total decentralized profit over centralized profit according to the different profit margin intervals. As seen efficiency loss is high in low profit margin markets.

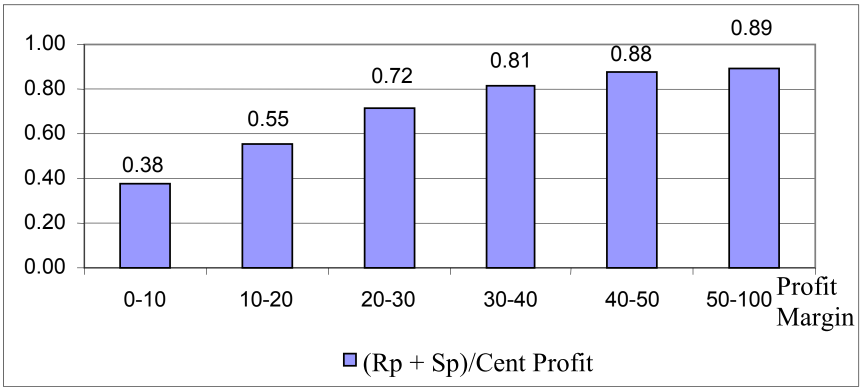

Figure 4.9 *Supply chain performances versus profit margins* (*exp*)

In the high profit margin markets, the supplier can see the real situation with little differences and he can get most of the profit. Hiding the backorder cost information is not valuable with sharing the demand information from the perspective of the retailer. As a result the retailer should not reveal the demand information in exponential demand case same as the linear demand case. Loss of efficiency from hiding the backorder cost is little.

Although the supplier still can get the most of the profit of the chain in the low profit margin markets, this situation is not very lucrative for him because the loss of efficiency is very high. Hiding backorder cost information is worthy for the retailer because she may force the supplier to collaborate or to make an agreement with better terms same as the linear demand case.

The general decentralized model and the case when the retailer reveals the demand information and hides the backorder cost value is studied in this chapter. As a result the deterministic price assumption can lead the supplier while he is preparing the contract. The results are different according to the profit margins and a number of comments are made by considering the supply chain efficiency.

Chapter 5

DECENTRALIZED MODEL CASE II: Unknown Backorder Cost & Demand Parameters

5.1. The Model

The case that the retailer hides demand parameters but reveals the backorder cost is studied in the previous chapter. We will discuss the case that the retailer hides all the information she has in this chapter. The models of the supplier and the retailer are the same with the previous case (chapter) and also the same notation will be used except some additions.

Some additional notations for this case are:

b_s :	backorder cost guessed by the supplier
b :	actual backorder cost of the retailer
m_s :	demand parameter guessed by the supplier
m :	actual demand parameter of the retailer

This analysis considers the situation when the supplier does not know the demand parameters (m, n) in addition to the backorder cost value. To simplify the functions all calculations executed on the parameters m and m_s, because parameter n can be adjusted according to parameters m and m_s.

The problem of the supplier is:

Maximize $-c_1^{'}\lambda(p)+(c_2-c_2^{'})\mu$

Subject to

$E[\,\Pi_R\,(s^*)] > 0$ (Individual Rationality)

$s^* = \text{argmax}\ E[\,\Pi_R\ (s)]$ (Incentive Compatibility)

As in Chapter 4, the supplier first puts himself in place of the retailer to solve the model.

The cost function of the retailer (according to the supplier) after setting $c_1 = 0$ is:

$$E[\text{RCost}] = s + \frac{b_s e^{-s(1-\frac{m_s - np}{\mu})}}{1-\frac{m_s - np}{\mu}} - \frac{1-e^{-s(1-\frac{m_s - np}{\mu})}}{1-\frac{m_s - np}{\mu}} + \mu c_2 \tag{5.1}$$

The supplier solves the cost function of the retailer with fixed μ (he thinks μ is offered by himself) and an approximate b_s. The minimum cost can be found by deriving this function according to s, because s is the only variable that the retailer decides individually. Eventually the optimal (minimum) cost is:

$$\mu(-\frac{2Log(1+b_s)}{m_s - 2\mu} + c_2) \tag{5.2}$$

The supplier should offer a contract that satisfies the individual rationality and the incentive compatibility of the retailer so the contract parameters c_2 and μ must ensure a relationship that provides:

$$(m_s - np)p \geq \mu(-\frac{2Log(1+b_s)}{m_s - 2\mu} + c_2) \tag{5.3}$$

The relation that is extracted from the solution of inequality 5.3 at the equality state is shown below:

$$c_2 = \frac{m_s^{\ 3} - 2m_s^{\ 2}\mu + 8n\mu Log(1+b_s)}{4n(m_s - 2\mu)\mu} \tag{5.4}$$

Consequently, acceptable contracts for the retailer would be created. Now, the supplier wants to find the particular contract that maximizes his expected profit.

After writing the values of $\lambda(p)$ and c_2 on the Equation 4.9 in the previous case, the objective (profit) function of the supplier is found. Then derivatives are calculated and replaced in the equation to find the optimal solution according to variables c_2 and μ. As a result the optimal c_2 and μ values are calculated as shown below:

$$\mu^* = \frac{\sqrt{2m_s Log(1+b_s)} + m_s\sqrt{c_2'}}{2\sqrt{c_2'}} \tag{5.5}$$

$$c_2^* = \frac{(\sqrt{2}(m_s^{\ 2} - 4nLog(1+b_s)) - 4\sqrt{m_s Log(1+b_s)c_2' n})\sqrt{c_2'}}{4\sqrt{m_s Log(1+b_s)n} + 2\sqrt{2}m_s n\sqrt{c_2'}} \tag{5.6}$$

By assuming the supplier knows the backorder cost and the demand parameters of the retailer, we can say that he can get all of the profit of the supply chain by offering the μ and c_2 combination with placing the other parametric values. However, the supplier does not know these values. If he prepares the contract according to not only a lower backorder value then the actual backorder cost of the retailer but also a demand parameter which is lower than the actual one; the retailer will not trade with the supplier because of her individual rationality which is the same as the previous case.

5.2. Experimental design of testing contract effectiveness

While preparing the contract, firstly the supplier defines the intervals for the demand parameter m with estimated maximum and minimum values. Since he does not know the real demand parameters; he calculates the expected profits according to the different m_s and m values. If $m_s \leq m$ then the retailer does not operate with the supplier. Table 5.1 demonstrates an example for this calculation. If the supplier offers a contract that considers m_s = 70 then the profit earned by him will be 0 if m < 70, the profit will be 184 if m = 70 and the profit will be 179 if m = 75 etc.

Table 5.1 *Experimental set for numerical calculations*

m_s \ m	55	60	65	70	75	80	85	90	95	100
55	31	26	21	16	11	6	1	1	1	1
60		73	68	63	58	53	48	53	48	43
65			120	115	110	95	100	105	90	95
70				184	179	164	159	154	159	154
75					244	239	234	229	224	219
80						311	306	301	286	291

85							383	378	373	368
90								463	458	463
95									558	553
100										650

By assuming that the probabilities for different m values are equal, the expected profits are calculated. After this, the supplier finds the m_s value that maximizes his profit and offers a contract that the parameters are calculated according to that particular m_s.

The steps of the experimental process can be summarized as:

1) The supplier offers a (c_2, μ) combination that corresponds to a b_{max} and an optimum m_s value according to the deterministic price assumption
2) The retailer accepts the contract because her individual rationality is satisfied (If $m_s \leq m$)
3) The retailer sets the optimal p and s value to maximize her own profit

This experimental process is performed on 50 random retailers with corresponding 50 randomly generated backorder costs. The parameters c_1' and c_2' are changed in acceptable ranges and also the demand parameters are changed between maximum and minimum values. The experiments are done with m values from 0 to 300 with ranges 5 to 50, and with c_1' and c_2' values from 5 to 150 with ranges 5 to 30. The actual expected profit of the retailer and the supplier, the expected centralized profit, the optimum variable values for both the centralized and the decentralized system are calculated as the results of experiments. The expected profits of the supplier according to m_s values are shown on Figure 5.1. The supplier can find the optimal m_s value that maximizes his own profit. The optimum m_s value is 85 and expected profit is 155 for this particular example. After finding the optimal m_s, the supplier continues his process as he knows the demand parameters: he prepares the contract by using the parameters m_s and b_{max}.

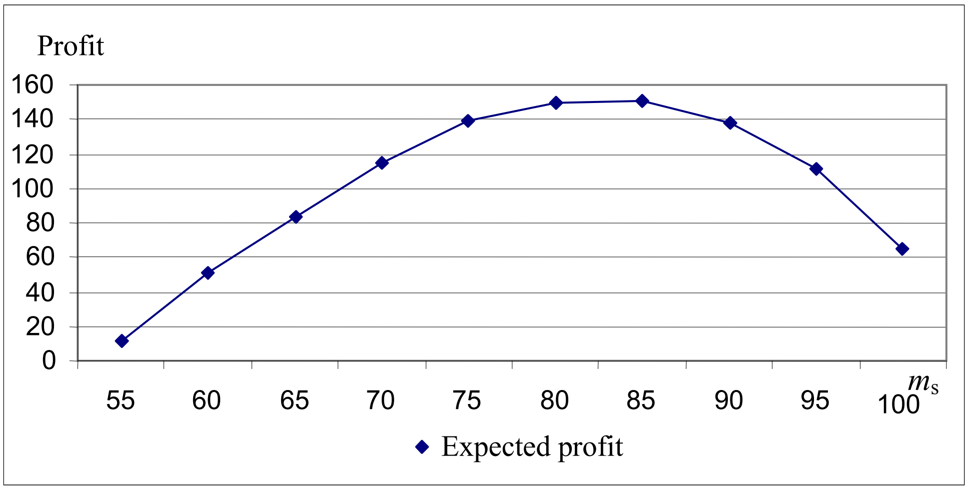

Figure 5.1 *Supplier's profit versus guessed demand*

5.3. Results according to high / low profit margins, behavior of the players

The supplier can obtain the optimal contract that maximizes his expected profit after carrying out the experiments. The effects of the backorder changes are not significant on the profits of both the supplier and the retailer similar to the previous case. The analysis depends on two factors: the disparity of profit margins and the accuracy of the m_s guesses of the supplier.

Figure 5.2 shows the supply chain performance, the ratio of the sum of the expected decentralized profits to the expected centralized profit, according to the different profit margin intervals. Loss of efficiency is very high in the low profit margin markets. If the profit margin is between 20 and 30 then the loss of efficiency is about 0.82.

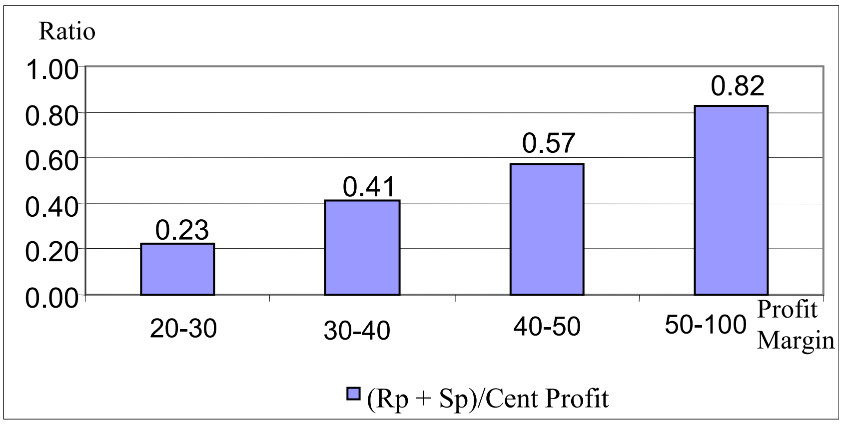

Figure 5.2 *Supply chain performances versus profit margins*

The profit ratios of the retailer according to centralized profit are shown in Figure 5.3. As seen, the profit rates are not as low as the previous case (Chapter 4), is even high for the high profit margin markets. Consequently, revealing or not revealing the demand information is a very important decision for the retailer and specific comments will be given below.

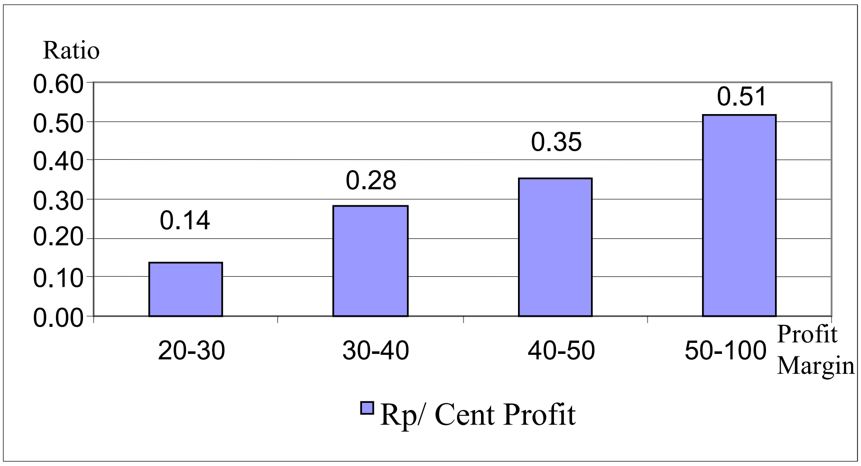

Figure 5.3 *Profit ratios of retailer versus profit margins*

The expected profit shares of the supplier according to the expected centralized profits for different profit margins are shown on Figure 5.4. As seen the share of the supplier from the earnable supply chain profit is very low.

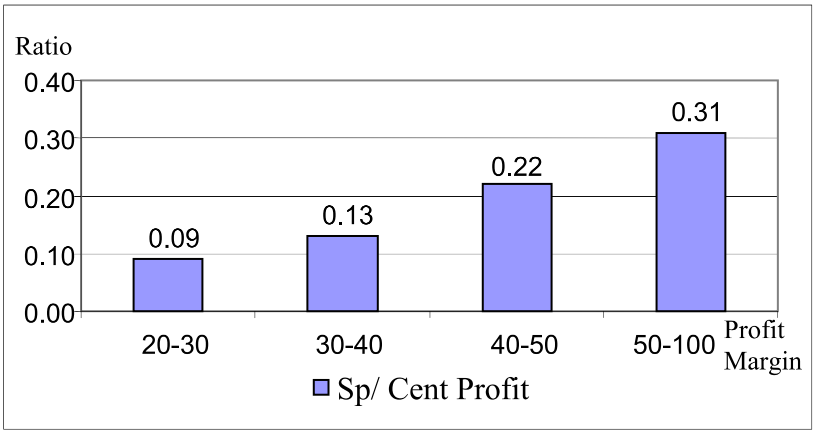

Figure 5.4 *Profit ratios of the supplier versus profit margins*

Loss of efficiency is about 30% on high profit margin markets with accurate m_s guesses of the supplier. Moreover, this rate decreases to 10% if the supplier cannot guess accurately. If the supplier guesses accurately he can get most of the profit of the chain, but a little deviation from the real demand parameters reverses the profit sharing rates. The more deviation from the real demand parameters, the less profit for the supplier. As a result hiding the demand parameters and the backorder cost value is beneficial for the retailer for high profit margin markets.

Figure 5.5 shows the profit ratios of the supplier and the retailer relatively. In contrary with the previous case the profit ratio of the retailer is greater than the ratio of the supplier. Moreover, these ratios are not differing from each other according to the various profit margins.

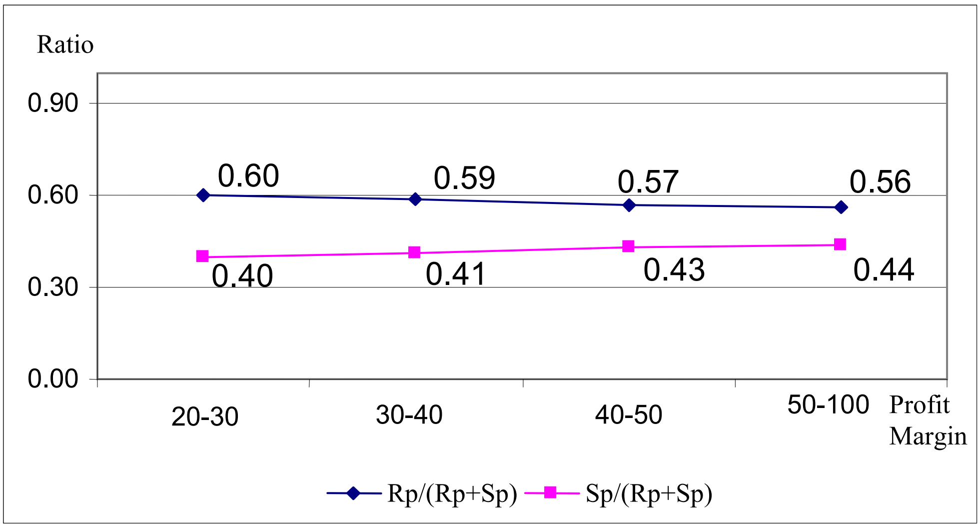

Figure 5.5 *Profit ratios of the supplier and the retailer relatively*

Although the supplier still can get the most of the profit of the chain in the low profit margin markets when he guesses m_s accurately, this situation is not very beneficial for the supplier because the loss of efficiency is very high as the previous chapter. Moreover, misestimation of the supplier decreases his earned profit. Consequently hiding backorder cost information and the demand parameters is worthy for the retailer because she may force the supplier to collaborate or to offer an agreement with better conditions.

The analyses of the exponential demand case for the situation where the retailer does not reveal the backorder cost value and the demand parameters are explained in Appendix C.

Figure 5.6 shows the supply chain performance according to the different profit margin intervals for the exponential demand case. As seen, loss of efficiency is very high in low profit margin markets.

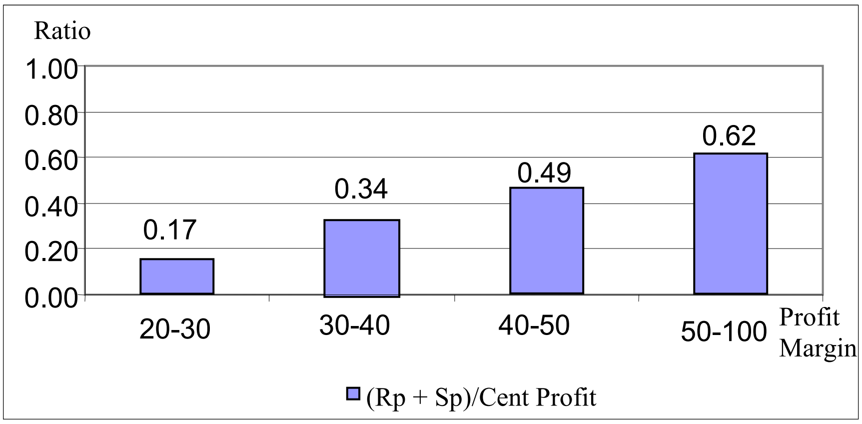

Figure 5.6 *Supply chain performances versus profit margins*

The results of the exponential demand case support the linear demand case as:

- The effects of backorder changes are not remarkable on the profits of both the supplier and the retailer similar to previous case.
- Loss of efficiency is about 35% for high profit margin markets and 70% for low profit margins.
- If the guess of the supplier deviates from the real demand parameters his profit ratio goes down.
- Hiding the demand parameters and the backorder cost value is valuable for the retailer for both high profit margin markets and low profit margins.

The model of the decentralized supply chain, when the retailer hides the demand information and the backorder cost value, is studied in this chapter. The decision process of the supplier and the retailer are explained and some comments are provided to interpret the behavior as a function of the problem parameters.

Chapter 6

CONCLUSION

In this thesis we have analyzed a two-stage supply chain model with an emphasis on mechanisms to coordinate production and pricing. Our model considers how the centralized and the decentralized supply chain system operate in the situations where the chain consists of several decision-makers. We have discussed both the centralized and the decentralized cases with both linear and exponential price-demand relationship functions. An assumption is developed that fixes the price using a deterministic approximation and helps the supplier while he is preparing the contract. We have studied on the effects of limited information sharing on supply chain performance.

Firstly, we established the centralized model which can be interpreted as a full information sharing two-stage supply chain model. Because the price-dependent model cannot be solved analytically with three variables, we tried to understand the behavior of the model with numerical examples. According to these examples, we saw that the price is steady for large capacity values. After this we developed a deterministic price assumption to calculate the optimal base-stock level, the shared capacity value and the sales price. We compared the optimal profit value when we use the deterministic price with the real optimal profit value. We observed that the deviation is very little if the profit margin is high but it is still acceptable when the profit margin is low. We have completed the same steps for the exponential demand price relationship function. As a result we have decided to use the deterministic price assumption to help to the supplier in the decentralized model.

We developed a strategy to coordinate the two-stage supply chain system. The supplier is the leading player of the game and he offers a contract that includes the unit price, the capacity reservation price and the reserved capacity to the retailer. He prepares his contract by using a deterministic price assumption. The situation that the retailer reveals the demand-price relationship function parameters and she hides the backorder cost value is studied in the first part of the analysis. In the second part, the condition that the retailer hides the demand-price relationship function parameters and the backorder cost value is examined. We verified the accuracy of the deterministic price assumption while preparing the contracts by comparing the numeric results. The results were different according to profit margin of the retailer. Finally, we completed the same steps for the exponential demand price relationship function as we did in the previous case.

Consequently, we can easily say that the deterministic price assumption can be used by the supplier while he is setting up a contract for the retailer. Although backorder values are changed exponential, the effects of these changes on the profit values are not considerable and also the variability of the backorder cost is unimportant for the supplier. We can summarize the results for the high profit and the low profit margin markets.

If the retailer shares the demand parameters, the supplier can see the real situation with little differences and he can get most of the supply chain profit in high profit margin markets. So hiding the backorder cost information is not valuable with sharing the demand information from the retailer’s perspective. Also, loss of efficiency from the limited information is little in the high profit margin markets.

Although the supplier still can get the most of the profit of the chain in the low profit margin markets, this situation is not very lucrative for the supplier because the loss of efficiency is very high. Hiding the backorder cost information is worthy for the retailer because she may force the supplier to collaborate or to make an agreement.

If the retailer hides the demand information and the backorder cost value, loss of efficiency is about %35 on high profit margin markets with accurate guesses of m_s of the supplier. If the supplier guesses accurate he can get most of the profit of the chain, but a deviation from the real demand parameters reverses the profit sharing rates. As a result hiding the demand parameters and backorder cost value is valuable for the retailer in high profit margin markets.

Although the supplier still can get the most of the profit of the chain in low profit margin markets when he guesses m_s accurately, this situation is not very profitable for the supplier because the loss of efficiency is very high. Moreover, misestimation of the supplier decreases his earned profit. Consequently hiding backorder cost information and the demand parameters is worthy for the retailer because she may force the supplier to collaborate or to make an agreement.

This model can be developed by considering multiple products. The supplier can supply many products for the retailer and the unit costs, the capacity costs and the capacity requirements may be different for each product. In these conditions the supplier may set up a contract for all products. Also, modeling with multiple suppliers or/and multiple retailers may be an interesting future research direction for implementing the model to the real life. Because of the difficulty of analytical solutions simulation techniques may be developed for the multiple product, multiple supplier and multiple retailer situations. In addition, the exact solution of the model, discrete case, can be found by simulation and then can be compared with the solution of our model to show the performance of the heavy traffic approximation. Another development may be the case where the designer of the contract is the retailer who then offers the contract to the supplier.

APPENDIX A

Centralized Model with Exponential demand Function

Because the most common price-demand relationship functions are linear and exponential ones, the same steps of the centralized supply chain analysis for the exponential case will be summarized, but we expect that the other price-demand function would yield with same results.

The same simplification assumptions about continuity and normalization are in use in the exponential case. The Equation A.1 shows the relationship between the price and the demand:

$$\lambda(p) = me^{-np} \tag{A.1}$$

The calculated expected profit function according to the exponential demand function is shown below:

$$E[\Pi_C] = (p-c_1)\lambda(p) - \left[s - \frac{1-e^{-s(1-\frac{me^{-np}}{\mu})}}{1-\frac{me^{-np}}{\mu}} + b\frac{e^{-s(1-\frac{me^{-np}}{\mu})}}{1-\frac{me^{-np}}{\mu}} \right] - c_2\mu \tag{A.2}$$

In order to find the optimal solution, the optimal value of s without regarding the variables p and μ is found first. The equation and the solution according to the derivative is given by

$$\frac{\partial E[\Pi_C]}{\partial s} = 0 \iff s^* = \frac{e^{np}\mu \ \log(1+b)}{-m+e^{np}\mu} \tag{A.3}$$

After this point s is replaced by its optimum value in Equation A.2 and we try to solve the equation. Although it could not be solved analytically when considering the price, p, and the capacity, μ, variables; the behavior of the function is examined numerically.

We see that the expected profit functions are convex for all numerical examples we examined. When the backorder cost increases without changing the other parameters, the optimal price values increases, too, but the base-stock quantities and the profit values decrease as expected. In all cases it can be easily seen for large μ values the price will be constant. For that reason we decide to use the deterministic price assumption for all calculations.

In order to find the value of the deterministic price, we consider a case where the capacity is infinite and so neither backorder cost nor holding cost will be taken into account.

The expected profit function can be given by:

$$E[\text{Profit}] = (p - c_1)\,\lambda(p) - c_2\mu \tag{A.4}$$

The first derivative of this expression with respect to p is:

$$\frac{dE[\text{Profit}]}{dp} = e^{-np}m - e^{-np}mn(p - c_1) \tag{A.5}$$

As a result the optimal deterministic price value is equal to:

$$p_{\text{det}}{}^{*} = \frac{1}{n}(1 + nc_1) \tag{A.6}$$

To continue the analysis with the deterministic price we replace the variable p with the optimal deterministic price value. The optimal base-stock level, the optimal capacity and the optimal profit values according to the deterministic price assumption are shown below:

$$s^{*} = Log(1+b) + e^{-\frac{(1+nc_1)}{2}}\sqrt{mLog(1+b)c_2} \tag{A.7}$$

$$\mu^{*} = \frac{e^{-1-nc_1}\sqrt{m}\left(e^{\frac{(1+nc_1)}{2}}\sqrt{Log(1+b)} + \sqrt{mc_2}\right)}{\sqrt{c_2}} \tag{A.8}$$

$$E[\text{Optimal Profit}] = -Log(1+b) - 2e^{-\frac{1-nc_1}{2}}\sqrt{mLog(1+b)c_2} + \frac{e^{-1-nc_1}(m - mnc_2)}{n} \tag{A.9}$$

APPENDIX B

Decentralized Model Case 1 with Exponential demand Function

The contract offered by the supplier includes three variables: the reserved production capacity, the sales price of producing one unit of product and the capacity reservation price for one unit of capacity reservation. His objective is to maximize his profit by trading with the retailer. His objective function depends on the sales quantity of the retailer. He must consider the individual rationality and incentive compatibility of the retailer when he is offering a contract.

The expected profit function of the supplier after with exponential demand function is:

$$E\left[\Pi_S\right] = (c_1 - c_1^{'})me^{-np} + (c_2 - c_2^{'})\mu \tag{B.1}$$

The model of the retailer is same as the centralized model by taking cost and capacity values from the offer of the supplier. The expected profit function of the retailer is:

$$E\left[\Pi_R\right] = \left(p - c_1\right)\lambda\left(p\right) - E\left[\left(s - IO\right)^{+}\right] - bE\left[\left(IO - s\right)^{+}\right] - c_2\mu \tag{B.2}$$

When we solve the equations with exponential demand; the profit function of the retailer turns into:

$$E\left[\Pi_R\right] = \left(p - c_1\right)me^{-np} - \left[s - \frac{1 - e^{-s(1 - \frac{me^{-np}}{\mu})}}{1 - \frac{me^{-np}}{\mu}} + b\frac{e^{-s(1 - \frac{me^{-np}}{\mu})}}{1 - \frac{me^{-np}}{\mu}} \right] - c_2\mu \tag{B.3}$$

This profit function cannot be solved analytically; accordingly both the supplier and the retailer could not use this function with three variables.

The supplier should give offers to the retailer that includes the capacity, capacity cost and one unit variable cost information (μ, c_2, c_1).

The supplier uses the deterministic price assumption while he is preparing the contract because the profit function of the retailer is very complex and could not be solved analytically. When the deterministic price value is replaced with the *p* in the equation, the profit function of the supplier turns into:

$$E[\Pi_S] = (c_1 - c_1^{'})me^{-1-nc_1} + (c_2 - c_2^{'})\mu \tag{B.4}$$

As a result of many numerical experiments, we see that the supplier must set $c_1 = 0$ to maximize his profit same as the linear case.

The model of the supplier with exponential demand function is:

Maximize $-c_1^{'}\lambda(p) + (c_2 - c_2^{'})\mu$

Subject to

$E[\,\Pi_R\,] > 0$ (Individual Rationality)

$s^* = \text{argmax}\,[\,\Pi_R\ (s)]$ (Incentive Compatibility)

Firstly the supplier thinks himself as the retailer to solve the model. The cost function of the retailer after setting $c_1 = 0$ is:

$$E[\text{RCost}] = s + \frac{be^{-s(1-\frac{m}{e\mu})}}{1-\dfrac{m}{e\mu}} - \frac{1-e^{-s(1-\frac{m}{e\mu})}}{1-\dfrac{m}{e\mu}} + \mu c_2 \tag{B.5}$$

The supplier solves the cost function of the retailer with fixed μ (he thinks μ is offered by himself) and an approximate *b*. The minimum cost can be found by deriving this function according to *s*, because *s* is the only variable that the retailer decides individually. Eventually the optimal (minimum) cost is:

$$\mu(-\frac{eLog(1+b)}{-m-e\mu} + c_2) \tag{B.6}$$

The supplier should offer a contract that satisfies the individual rationality and incentive compatibility of the retailer so the contract parameters c_2 and μ must ensure a relationship that provides:

$$(m-np)p \geq \mu(-\frac{eLog(1+b)}{-m-e\mu}+c_2) \tag{B.7}$$

The relation that is extracted from the solution of inequality B.7 is shown below:

$$c_2 = \frac{m^2-em\mu+e^2n\mu Log(1+b)}{en(m-e\mu)\mu} \tag{B.8}$$

By assuming that the supplier knows the backorder cost value, it can be said that all of the contracts that satisfy Equation B.8 are acceptable contracts for the retailer. Now, the supplier wants to find the best contract among all feasible contracts.

After writing the values of $\lambda(p)$ and c_2 in the objective (profit) function of the supplier, it can be written as:

$$E[\Pi_S] = \mu(\frac{m^2-em\mu+e^2n\mu Log(1+b)}{en(m-e\mu)\mu} - c_2^{'}) - c_1^{'}(m-1-nc_1) \tag{B.9}$$

To find the optimal solution as a function of c_2 and μ, the first derivatives are calculated and replaced in the equation. As a result the optimal c_2 and μ values are calculated as shown below:

$$\mu^* = \frac{\sqrt{emLog(1+b)}+m\sqrt{c_2^{'}}}{e\sqrt{c_2^{'}}} \tag{B.10}$$

$$c_2^* = \frac{(m-enLog(1+b)-\sqrt{emLog(1+b)c_2^{'}n})\sqrt{c_2^{'}}}{\sqrt{emLog(1+b)}n+mn\sqrt{c_2^{'}}} \tag{B.11}$$

Now, if the supplier knew the backorder cost of the retailer, he could obtain all of the profit by offering the μ and c_2 combination by selecting the other parametric values according to Equations B.10 and B.11. However, the supplier does not know the backorder value. If he offers a (μ, c_2) combination with placing a lower backorder value then the real backorder cost of the retailer, the retailer will not trade with the supplier because of her individual rationality.

The supplier guesses a maximum value for the backorder cost of the retailer and he calculates other variables according to that particular b same as the linear demand case. Consequently, the contract offered by the supplier includes (μ^*, c_2^*, $c_1=0$) which are calculated according to the deterministic price assumption and an estimated backorder cost.

Because the contract prepared according to the deterministic price assumption, its effectiveness should be tested. The steps of the experimental process, which will test the effectiveness of the contract with respect to the centralized solution, are same as the linear demand case. In Table B.1, the experimental set for numerical calculations to test the contract effectivity is shown.

Table B.1 *Experimental set for numerical calculations for exponential demand*

m	**n**	**c_1'**	**c_2'**
50	0.02	1	0.5
100		6	1
200		11	5
500		16	15
1000		21	20
		26	25
		31	
		36	
		50	

Calculations are repeated 50 times (for 50 random backorder) for each feasible case. As a result 9000 experiments are feasible over 13500 possible combinations.

The results of the exponential price-demand relationship are given in Chapter 4.

APPENDIX C

Decentralized Model Case 2 with Exponential demand Function

This analysis is the situation that the supplier does not know the demand parameters (*m, n*) and the backorder cost value with exponential demand function.

Firstly the supplier thinks himself as the retailer to solve the model.

The cost function of the retailer (according to the supplier) after setting $c_1 = 0$ is:

$$E\left[\mathrm{RCost}\right] = s + \frac{b_s e^{-s(1-\frac{m_s e^{-np}}{\mu})}}{1-\frac{m_s e^{-np}}{\mu}} - \frac{1-e^{-s(1-\frac{m_s e^{-np}}{\mu})}}{1-\frac{m_s e^{-np}}{\mu}} + \mu c_2 \tag{C.1}$$

Firstly, the supplier solves the cost function of the retailer with fixed μ and an approximate b_s. The minimum cost can be found by deriving this function according to s, because s is the only variable that the retailer decides individually. Eventually the optimal (minimum) cost is:

$$\mu(\frac{eLog(1+b_s)}{e\mu - m_s} + c_2) \tag{C.2}$$

The supplier should offer a contract that satisfies the individual rationality and incentive compatibility of the retailer so the contract parameters c_2 and μ must ensure a relationship that provides:

$$(m_s e^{-np})p \geq \mu(\frac{eLog(1+b_s)}{e\mu - m_s} + c_2) \tag{C.3}$$

The relation that is extracted from the solution of Equation C.3 is shown below:

$$c_2 = \frac{m_s^{\;2} - em_s\mu + e^2 n\mu Log(1+b_s)}{en\mu(m_s - e\mu)} \tag{C.4}$$

After writing the values of $\lambda(p)$ and c_2 on the expected profit function of the supplier, the derivatives are calculated and replaced in the equation to find the optimal solution according to the variables c_2 and μ. As a result the optimal c_2 and μ values are calculated as shown below:

$$\mu^* = \frac{\sqrt{em_s Log(1+b_s)} + m_s\sqrt{c_2^{'}}}{e\sqrt{c_2^{'}}} \tag{C.5}$$

$$c_2^* = \frac{(-enLog(1+b_s) - \sqrt{em_s Log(1+b_s)c_2^{'}n})\sqrt{c_2^{'}}}{\sqrt{em_s Log(1+b_s)}n + m_s n\sqrt{c_2^{'}}} \tag{C.6}$$

Now the supplier prepares and offers a contract by applying the same procedures as the linear demand case.

Because the contract prepared according to the deterministic price assumption, its effectiveness should be tested. The steps of the experimental process, which will test the effectiveness of the contract with respect to the centralized solution, are same as the linear demand case. In Table C.1, the experimental set for numerical calculations to test the contract effectivity is shown.

Table C.1 *Experimental set for numerical calculations for exponential demand*

m_s \ m	55	60	65	70	75	80	85	90	95	100
55	135	133	130	128	125	123	120	118	115	113
60		149	147	144	142	139	137	134	132	129
65			163	161	158	156	153	151	148	146
70				177	175	172	170	167	165	163
75					192	189	187	184	182	179
80						206	203	201	198	196
85							220	218	215	213
90								234	232	229
95									249	246
100										263

The results of the exponential price-demand relationship of the decentralized model Case 2 are given in Chapter 5.

Bibliography

[1] **http://www.cscmp.org**, Council of Supply Chain Management Professionals web site.

[2] Caldentey R. and L. M. Wein, Analysis of a Decentralized Production-Inventory System, *Manufacturing & Service Operations Management*, 5 (2003), 1-17.

[3] Chan, L. M., M. Shen, D. Simchi-Levi, and L. Swann, Coordination of Pricing and Inventory Decisions: A Survey and Classification, Working Paper, 2003.

[4] Within, T. M., Inventory Control and Price Theory, *Management Science*, 2 (1955), 61-68.

[5] Mills, E. S., Uncertainty and Price Theory, *Quart. J. Economics*. 73 (1959), 116-130.

[6] Lau, A., H. Lau, Maximizing the Probability of Achieving a Target Profit Level in a Two-Product Newsboy Problem, *Decision Sciences*, 19 (1988), 392-408.

[7] Karlin, S., and C. R. Carr, Prices and Optimal Inventory Policies, in Studies in Applied Probability and Management Science, K. J. Arrow, S. Karlin, and H. Scarf (eds.), 1962, Stanford University Press, Stanford, CA.

[8] Petruzzi, N., and M. Dada, Pricing and the Newsvendor Problem: A Review with Extensions, *Operations Research*, 47 (1999), 183-194.

[9] Baker, R. C., and T. L. Urban, A Deterministic Inventory System with an Inventory-Level-Dependent Demand Rate, *The Journal of the Operational Research Society*, 39 (1988), 823-831.

[10] Dana, J., and N. Petruzzi, Note: The Newsvendor Model with Endogenous Demand, *Management Science*, 47 (2001), 1488-1497.

[11] Zabel, E., Monopoly and Uncertainty, *The Review of Economic Studies*, 37 (1970), 205-219.

[12] Thowsen, G. T., A Dynamic, Nonstationary Inventory Problem for a Price/Quantity Setting Firm, *Naval Research Logistics*, 22 (1975), 461-476.

[13] Cachon, G. P., and P. H. Zipkin, Competitive and Co-operative Inventory Policies in a Two-Stage Supply Chain, *Management Science*, 45 (1999), 936–953.

[14] Lee, H., and S. Whang, Decentralized Multi-echelon Inventory Control Systems: Incentives and Information, *Management Science*, 45 (1999), 633–640.

[15] Lee, H., K. So, and C. Tang, The Value of Information Sharing in a Two-level Supply Chain, *Management Science*, 46 (2000), 626-643.

[16] Aviv, Y., The Effect of Collaborative Forecasting on Supply Chain Performance, *Management Science*, 47 (2001), 1326-1343.

[17] Gavirneni, S., R. Kapuscinski, and S. Tayur, Value of Information in Capacitated Supply Chains, *Management Science*, 45 (1996), 16-24.

[18] Federgruen, A. and A. Heching, Combined Pricing and Inventory Control Under Uncertainty, *Operations Research*, 47 (1999), 454-477.

[19] Chen, F., Market Segmentation, Advanced Demand Information and Supply Chain Performance, *Manufacturing and Service Operations Management*, 3 (2001), 53-67.

[20] Li, L., A Stochastic Theory of the Firm, *Mathematics of Operations Research*, 13 (1988), 447-466.

[21] Chen, X. and D. Simchi-Levi, Coordinating Inventory Control and Pricing Strategies with Random Demand and Fixed Ordering Cost: The Finite Horizon Case, Working Paper, MIT, 2002a.

[22] Gupta, D. and W. Weerawat, Supplier-Manufacturer Coordination in Capacitated Two-Stage Supply Chains, Technical Report, University of Minnesota, 2003.

[23] Sharafali, M. and H. Co, Some Models for Understanding the Cooperation between the Supplier and the Buyer, *International Journal of Production Research*, 38 (2000), 3425-3449.

[24] Yao, L., Y. Chen, and H. Yan, Analysis of a Supply Contract for Coordinating the Newsvendor with Price Dependent Demand, Working Paper, Chinese University of Hong Kong, 2004.

[25] Harrison, J. M., Brownian Models of Queuing Networks with Heterogeneous Customer Populations, W. Fleming, P. L. Lions, eds. Stochastic Differential Systems, *Stochastic Control Theory and Applications*, IMA 10 (1988), Springer-Verlag, New York.

[26] Sahin, F. and E. P. Robinson, Flow Coordination and Information Sharing in Supply Chains: Review, Implications and Directions for the Future, *Decision Science*, 33 (2002), 505-537.

[27] Stein, T. and J. Sweat, Killer Supply Chains, *Information Week*, (1998).

[28] Karabatı, S. and S. Sayın, Single-Supplier / Multiple-Buyer Supply Chain Coordination: Incorporating Buyers' Expectations under Vertical Information Sharing, Working Paper, Koç University, Istanbul, (2005).

[29] Corbett, C., D. Zhou, and C. Tang, Designing Supply Contracts: Contract Type and Information Asymmetry, *Management Science*, (50) 2004, 550-560.

[30] Cachon, G., Supply Chain Coordination with Contracts, *Handbooks in Operations Research and Management Science: Supply Chain Management*, 2001, editors S. Graves and T. de Kok, North Holland.

[31] Elmaghraby, W., and Keskinocak, P., Dynamic Pricing: Research Overview, Current Practices and Future Directions, Working Paper, ISYE, Georgia Institute of Technology, Atlanta, (2003).

[32] Jemai, Z., and Karaesmen, F., Decentralized Inventory Control In A Two-Stage Capacitated Supply Chain, Working Paper, June (2005).

[33] Karaesmen, F., Liberopoulos, G., and Dallery, Y., The Value of Advance Demand information in Production/Inventory Systems, *Annals of Operations Research*, 126 (2003), 135-157.

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