Modeling and Analysis of an Auction-Based Logistics Market

by

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This is to certify that I have examined this copy of a master's thesis by

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and have found that it is complete and satisfactory in all respects,
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ABSTRACT

In this thesis, a logistics market that uses a reverse auction to match orders given by shippers that aim to transport their goods to various destinations with carriers is analyzed. Important performance measures for such a system are the average expected auction price, the average expected profit of carriers, the average number of orders and carriers, the probability of rejection of orders and carriers because of the capacity constraints and the proportion of carriers that take an order. The objective of this thesis is to analyze the effects of various system parameters, such as arrival and abandonment rates of orders and carriers, and the capacity of the system for carriers and orders, on the performance of the system in a stochastic environment. In this market, called as *logistics center*, a shipper opens an auction using an electronic reverse auction platform. All the carriers available at the logistics center at the time of the auction submit their bids for that order. Initially, in order to give qualitative insights on the real system, a descriptive statistics is performed with the actual data provided by the Eskisehir Chamber of Industry Guide Logistics Center. Then the system is modeled in two steps. First the auction is modeled in a static setting to determine the auction price and the profit of the carriers based on the number of carriers engaging in the auction and their cost distributions. Then a continuous time Markov chain model is developed to analyze the system in a dynamic setting with random arrivals and possible abandonment of orders and carriers. By combining these two models, the performance of the system in steady state is evaluated.

The main contribution of this thesis is to propose an analytical model that evaluates the performance of an auction-based logistics market in a dynamic setting. The model yields insights regarding the effects of the system parameters on the performance. It is shown that the average expected price is decreasing in the arrival rate of carriers and the abandonment rate of orders, and it is increasing in the arrival rate of orders and the abandonment rate of carriers. In addition, an estimation method is developed to find the cost distributions of the carriers from the observed bids. The proposed method is applied on the real life case data, and the results are found to be promising.

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NOMENCLATURE

$b_i(V_i)$	bid of carrier i with transportation cost v_i
$b_i(R_i)$	bid of carrier i with transportation cost r_i
P_M	market price
L	index for local carriers
B	index for in-transit carriers
C_L	capacity for local carriers
C_B	capacity for in-transit carriers
C_S	capacity for orders
λ_L	arrival rate of local carriers
λ_B	arrival rate of in-transit carriers
μ_S	arrival rate of orders
λ_{LA}	abandonment rate of local carriers
λ_{BA}	abandonment rate of in-transit carriers
μ_{SA}	abandonment rate of orders
V_i	Type L carrier i's transportation cost
R_i	Type B carrier i's transportation cost
$V_{(n)}$	n^{th} minimum of v_i's
$R_{(n)}$	n^{th} minimum of r_i's
$F_L(\cdot)$	cumulative distribution function of v_i's
$F_B(\cdot)$	cumulative distribution function of r_i's
$F_{(n)}(\cdot)$	cumulative distribution function of $v_{(n)}$
$f_{(n)}(\cdot)$	probability density function of $v_{(n)}$
$E[\cdot]$	expected value
p_j	the probability that there are j carriers available in the center
θ	the probability that the carrier is a local carrier

$N_L(t)$	number of local carriers at time t
$N_B(t)$	number of in-transit carriers at time t
$N_S(t)$	number of orders at time t
$p(l,b)$	expected auction price given l Type L bidders and b Type B carriers in the auction
$q(l,b)$	expected profit of the winner given l Type L and b Type B carriers in the auction
P_{av}	average expected auction price
Q_{av}	average expected profit of carriers
$\overline{N}_L$	average number of Type L carriers in steady state
$\overline{N}_B$	average number of Type B carriers in steady state
$\overline{N}_S$	average number of orders in steady state
K_L	the probability of rejecting a Type L carrier because of capacity constraint
K_B	the probability of rejecting a Type B carrier because of capacity constraint
K_S	the probability of rejecting an order because of capacity constraint
M_L	the proportion of Type L carriers that take an order
M_B	the proportion of Type L carriers that take an order
α	the scale parameter of the Weibull distribution
β	the shape parameter of the Weibull distribution
M_i	The i^{th} moment of the observed bids
M_i'	The i^{th} moment of the bids generated from the estimated cost distribution
w_i	The weight given to the difference of the i^{th} moments
cdf	cumulative distribution function
pdf	probability density function
ESO	Eskisehir Chamber of Industry Guide
LC	Logistics Center
ERA	Electronic Reverse Auction
YTL	New Turkish Liras
km	kilometers

Chapter 1

INTRODUCTION AND MOTIVATION

1.1. Introduction

Logistics is defined by the Council of Logistics Management as “the process of planning, implementing and controlling the efficient, effective flow and storage of goods, services, and related information from point of origin to point of consumption for the purpose of conforming to customer requirements” [1]. It is one of the important business processes in a supply chain that is perceived as a key factor for improving the performance of manufacturing and service organizations.

In logistics terminology, shippers are defined as buyers who pay the carriers for carrying their load and the carriers are the sellers who are paid for transportation service. A logistics marketplace is a virtual marketplace that bridges the procurement gap between the main two sets of players, shippers and carriers [2]. In general, the shippers post their requirements, carriers post their extra capacities and prices, and the negotiations between shippers and carriers are realized in the logistics market.

There are not many automated negotiation mechanisms currently used in logistics marketplaces and also there is no single negotiation model that can suit all these marketplaces. The reverse auction, which brings sellers together for the purpose of determining the price of the product or the service that a buyer will pay for, is the predominant negotiation mechanism in logistics marketplaces [3]. The aim of the reverse auction used in a logistics marketplace is to provide a match between shippers and carriers, so as to satisfy both shippers and carriers and to maximize the yield and resource utilization [2].

Several benefits for the buying firms as well as for the supplier can be derived from using a reverse auction. Shippers may reduce shipment costs and carriers may increase capacity utilization. Since the financial benefit of reverse auctions is simply too attractive for most shippers to ignore, auctions are being used as methods for procuring goods and services in recent years.

1.2. Motivation

This study is motivated by a logistics auction market, called the Logistics Center (LC), established in 2003 at the Eskisehir Organized Industrial Zone by the Eskisehir Chamber of Industry Guide (ESO) in Turkey. ESO reports that the transportation costs of the companies at the industrial zone that take service from the logistics center have decreased significantly, around 20-30%. Moreover, the carriers located at the LC state that their capacity utilizations have increased considerably.

Located almost at the center of the main route from the Western part to the Eastern part of Turkey, the ESO LC benefits from the imbalance between the West-East and East-West traffic. In addition to the local carriers that are based at the same place, the logistics market also attracts in-transit carriers that are based elsewhere and have available capacity. The logistics market provides an opportunity for those carriers that return empty to their base after delivering their original load. Attracting those carriers lowers the transportation costs for the companies, since some of the carriers are willing to accept lower rates.

The objective of this section is to give the quantitative insights on the real system. By using the ESO LC database that includes detailed information about auctions we perform a descriptive statistics. The following subsections are committed to the analysis of this data set.

1.2.1. Overview of the Data Set

The descriptive statistics focuses on the auctions that took place between December 2003 and May 2005. An auction is opened for one order that is defined as one truckload. During this period, 1717 transportation orders were opened by shippers for transportation from Eskisehir to 44 different cities and 1549 of these orders received bids from carriers. Moreover, 569 orders ended up with a successful auction, i.e. the order is taken by a carrier and the delivery is made, and there are 1126 bids given to these orders. A sample datasheet for the data set is given in Appendix A.

The data set contains the following information for each order: the date it is opened, its destination which is given by the license number of the city, how many carriers bid and their bids, and if it is realized or cancelled. Table 1.1 reports all the orders and bids. As seen in Table 1.1, we have transportation order information for 25 cities, and 10 of these cities are located at the eastern side of Eskisehir. The locations of these cities can also be seen in Figure 1.1. In addition, the average transportation price, the average transportation cost and distances from Eskisehir are given in Table 1.1 for each city.

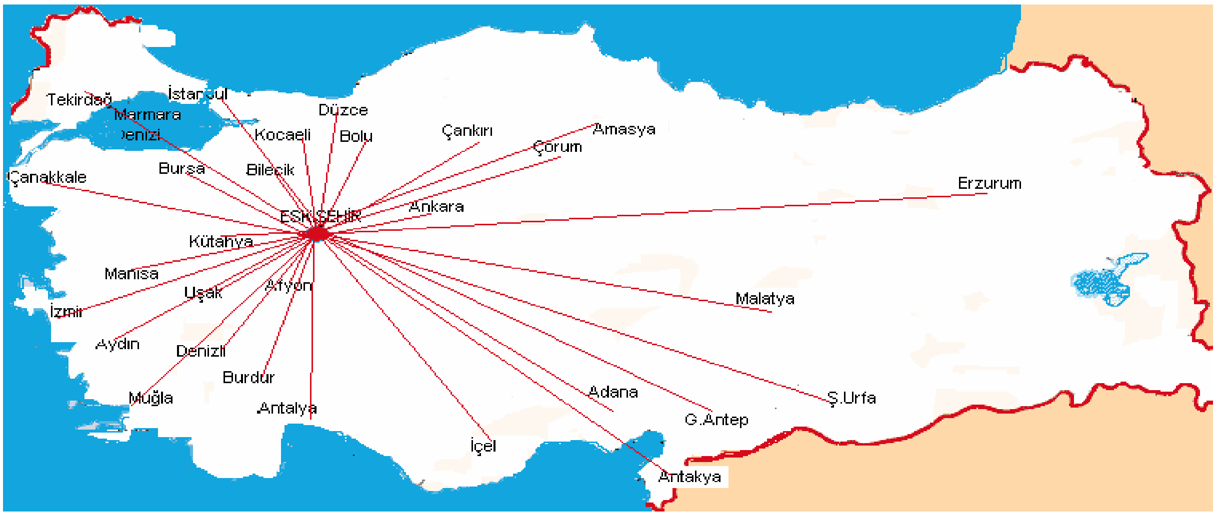

Figure 1.1. Map of the transportation from Eskisehir

Since the ESO LC has been providing service for only one year, there is limited data for doing a descriptive statistics. Moreover, while the arrival times of orders are available, the arrival times of carriers are not available which have great importance in the analysis. As a result we perform only restricted analysis and give insights on the order arrivals, bid distributions, price distributions, the relationship between the number of bids and the average price, and the total transaction volume for one of the cities.

City	*Average price (YTL)*	*Average bid (YTL)*	*Total number of orders*	*Total number of realized orders*	*Total number of orders that carriers give bids*	*Total number of bids*	*Average number of bids for each demand*	*Distance (km)*	*East/ West*
1	354	357	185	97	130	302	2.32	688	E
3	128	135	11	6	8	10	1.25	144	W
5	340	365	52	26	38	42	1.11	569	E
6	185	211	51	6	11	23	2.09	233	E
7	400	466	69	2	12	6	0.50	428	W
9	390	482	18	2	7	5	0.71	487	W
11	50	76	11	1	3	3	1.00	80	W
15	354	354	1	1	1	1	1.00	306	E
16	103	160	27	4	10	17	1.70	149	W
17	311	311	10	3	4	5	1.25	421	W
25	1150	1785	10	1	2	5	2.50	1109	E
26	95	146	2	2	2	4	2.00	0	-
27	504	544	80	32	49	85	1.73	894	E
34	216	227	257	78	108	4	0.04	330	W
35	238	255	90	49	60	119	1.98	412	W
41	181	185	393	176	223	306	1.37	219	W
42	345	375	22	3	3	9	3.00	338	E
43	90	95	4	1	2	2	1.00	78	E
44	675	775	16	1	5	4	0.80	883	E
45	225	233	13	6	7	15	2.14	394	W
48	415	565	64	1	12	11	0.92	506	W
59	283	301	28	14	19	48	2.53	462	W
63	700	700	1	1	1	2	2.00	1031	E
64	192	196	59	29	34	52	1.53	219	W
81	190	206	75	27	35	46	1.31	251	W
Overall	**324.56**	**380.20**	**1549**	**569**	**786**	**1126**	**1.51**	**425.24**	**E/W**

Table 1.1. Data list of the orders and bids

First, we analyze the order arrival process; and then by selecting two cities, that have enough data, we analyze the distributions of bids and the prices of the realized orders.

Moreover, numbers of bids – average transportation price and transportation cost – distance relationships are analyzed.

1.2.2. Order Arrivals

Initially, from the date of order entry, we count the number of orders opened for each day. The histogram for the frequency of daily orders is given in Figure 1.2. We analyze all the data for arrival of orders even for cancelled orders. In addition, we analyze the number of orders opened in an hour and the inter arrival time of orders in minutes, and give them in Figure 1.4 and Figure 1.3, respectively.

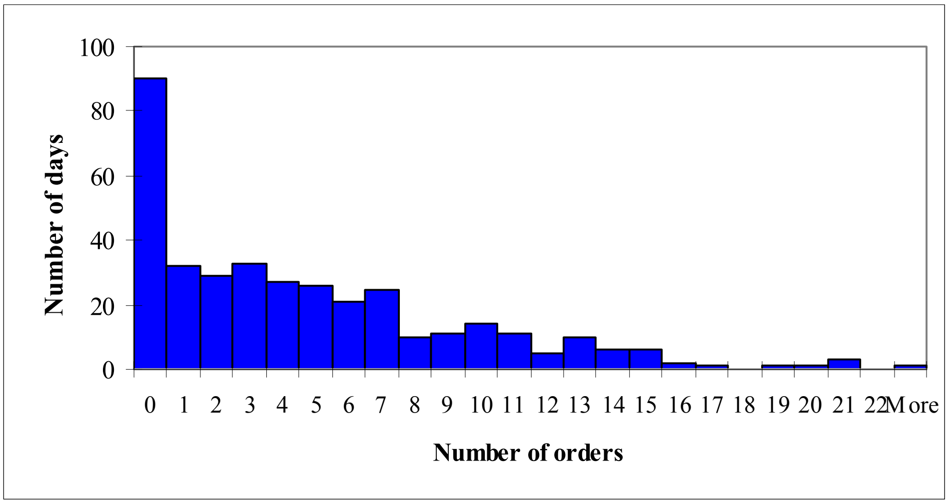

Figure 1.2. Daily number of orders

As seen from Figure 1.2 the minimum number of daily orders that arrive to the system is 0, and the maximum is 23. In most of the days we analyze, one to three orders are opened.

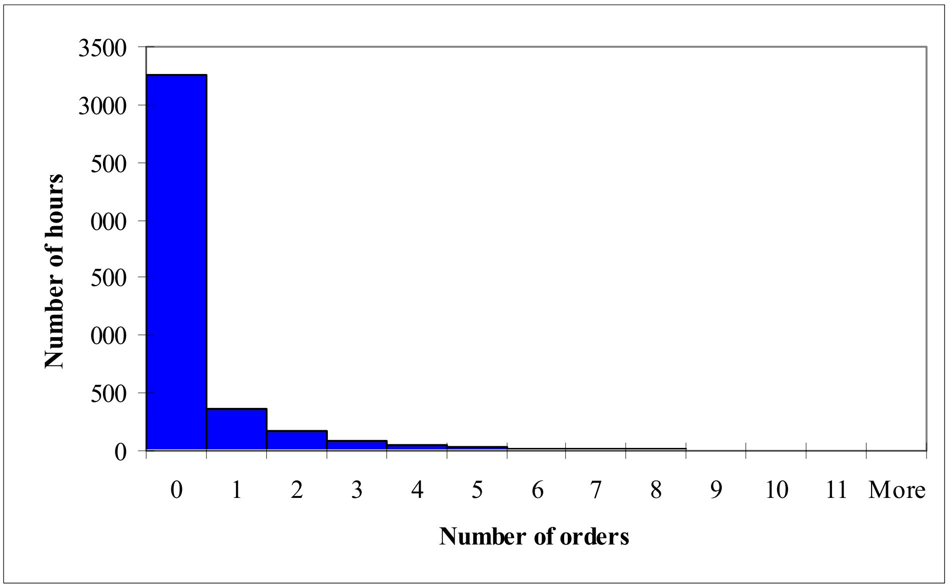

Figure 1.3. Hourly number of orders

Figure 1.3 and Figure 1.4 show that most of the time, the inter arrival time of orders is more than an hour. As a result the number of hours in which no auction opened is very high, e.g. around 3300.

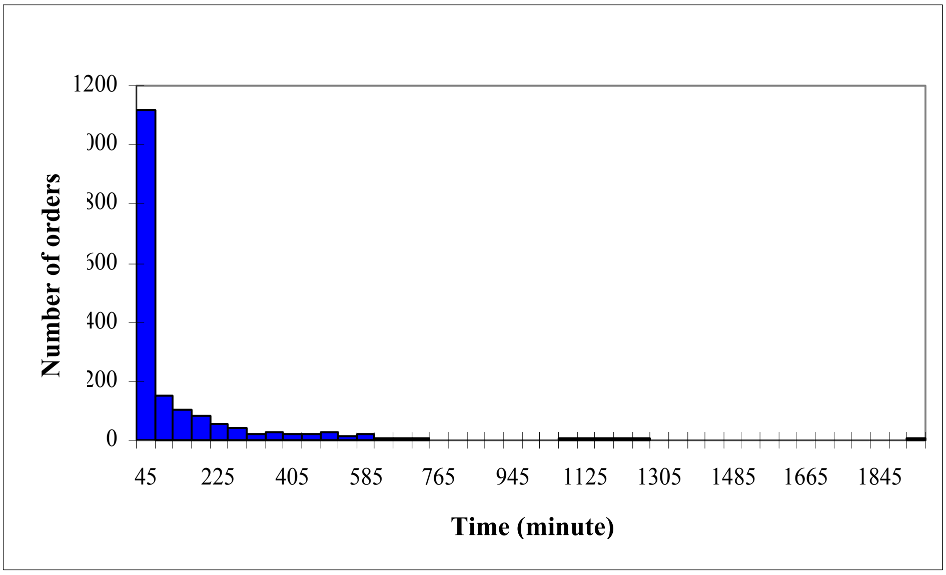

Figure 1.4. Inter arrival time of orders in minutes

1.2.3. Bid Distributions

As seen in Table 1.1, we have much of the data for only two cities, City 1 (Adana) and City 41 (Kocaeli). Since there is limited data for other cities, we focus on these two cities. There are 97 and 176 orders received by the system for transportation from Eskisehir to City 1 and 41, and 207 and 133 bids are given to these orders respectively. The histograms of bids given to orders for City 1 and 41 are given in Figures 1.5 and 1.6.

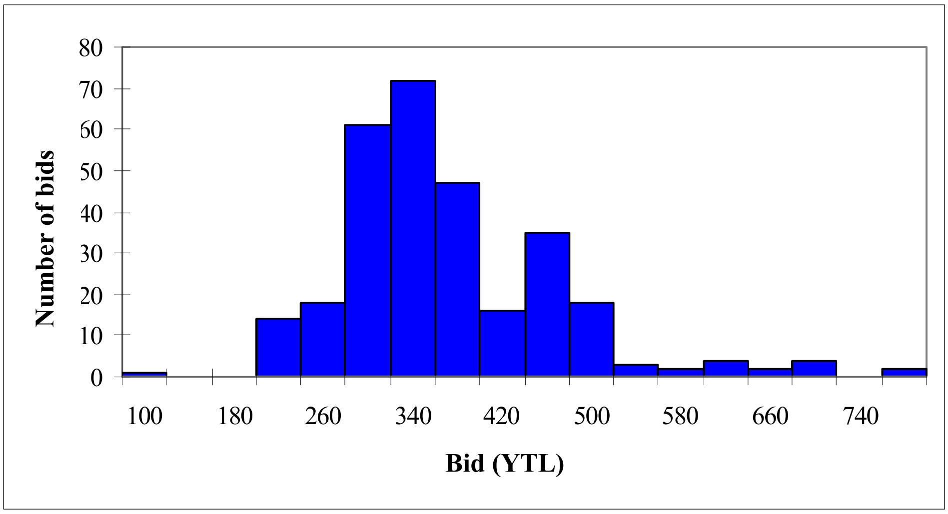

Figure 1.5. Bids Given to orders from Eskisehir to City 1

As seen from Figure 1.5, the minimum and the maximum bid values given for transportation from Eskisehir to City 1 are 100 YTL and 740 YTL respectively. The range of the bids seems to be very large as we expect beforehand because the logistics center attracts in-transit carriers which give low bids for orders. In addition, when there are a few carriers available to respond to orders, they give higher bids which explain the higher bids in Figure 1.5. Most of the bids are between 260 YTL and 360 YTL; also the median of the bids is between 340 – 360 YTL, and their mean is 317 YTL.

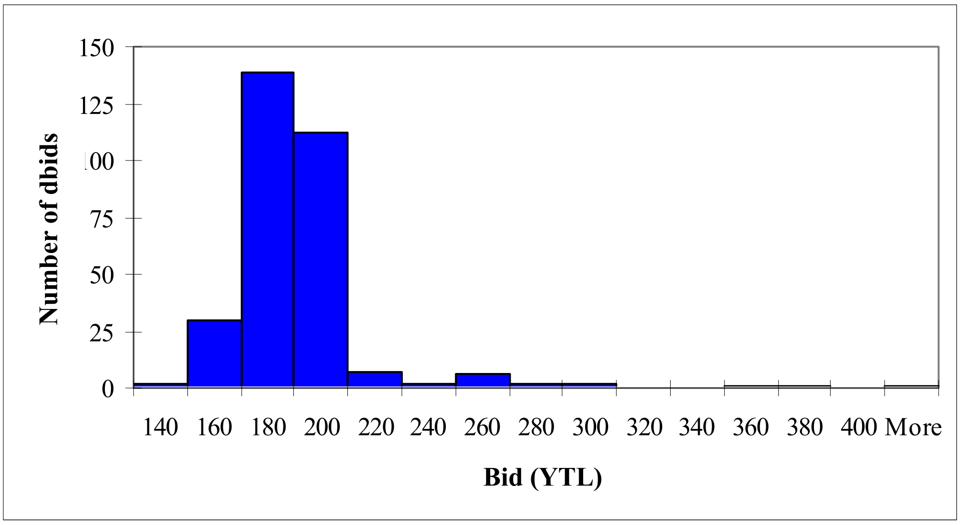

Figure 1.6. Bids Given to orders from Eskisehir to City 41

The same observations are also true for City 41. As seen from Figure 1.6, the bids lie between 140 YTL and 400 YTL. 180 YTL is the median and 179 YTL is the mean value of the bids.

1.2.4. Transportation Price Distributions

There are 185 orders opened and 97 of them are realized for transportation from Eskisehir to City 1, and the histogram of the prices that these orders are taken, i.e. winning bids, is given in Figure 1.7. Most of the orders are taken between 232 YTL and 562 YTL. Only one order is taken at a price of 100 YTL, and one at 700 YTL. The difference between these prices seems to be very high but it can be explained with the existence of in-transit and local carriers, i.e. lower prices are given by in-transit carriers and higher values are realized when there are a few carriers at the center.

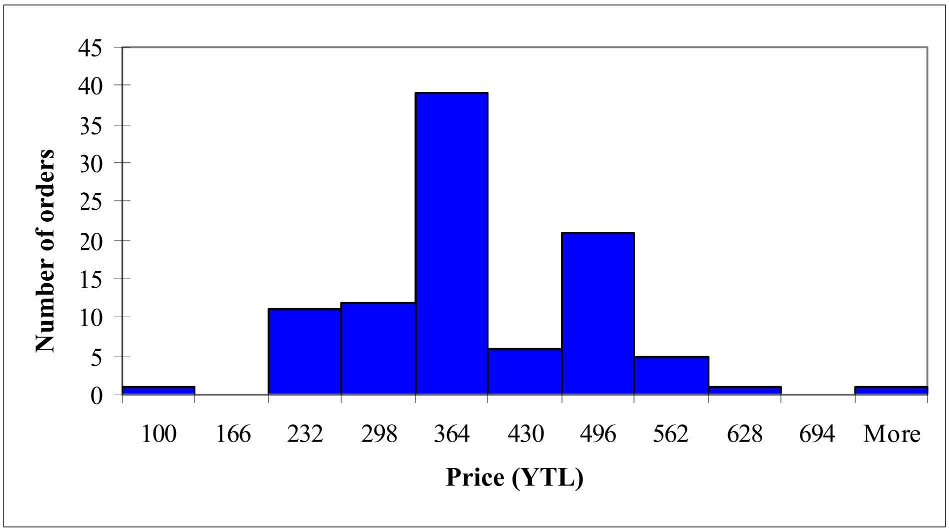

Figure 1.7. Auction Price from Eskisehir to City 1

There are 393 orders opened for transportation from Eskisehir to City 41, and 176 of these orders are realized. The histogram of transportation prices from Eskisehir to City 41 is given in Figure 1.8. The transportations are realized at prices between 150 YTL and 200 YTL and the median of the price is between 185-200 YTL.

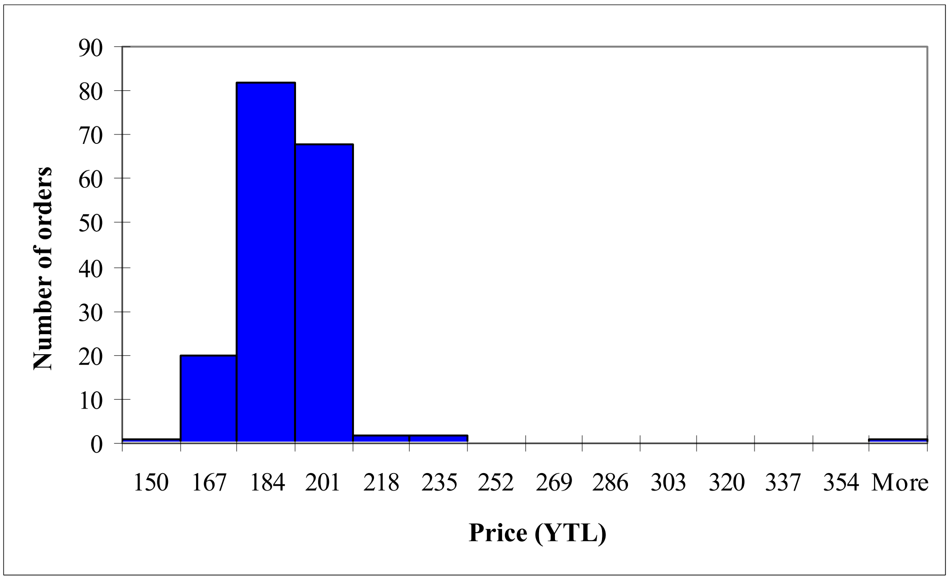

Figure 1.8. Auction price from Eskisehir to City 41

1.2.5. Number of Bids – Average Transportation Price Relationship

Reverse auctions are used to lower the price of a service; so it is expected that when there are a high number of bidders in the system, the price of the services decreases because of the competition between carriers. We try to see if this is realized in our logistics center by analyzing the number of bidders in auctions and average prices of those orders. Figures 1.9 and 1.10 show the average auction price according to the number of bidders joining the auction for City 1 and City 41.

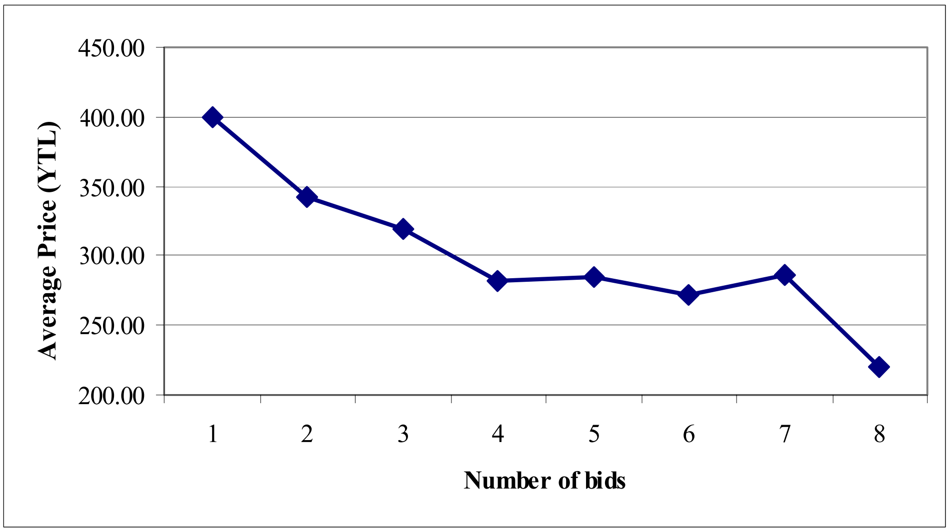

Figure 1.9. Number of bids and auction prices for City 1

As seen in Figure 1.9 and 1.10, there is a decrease in prices with an increase in the number of bidders joining the auction as expected. However, there are some points where the auction price increases. This increase can be again explained by local and in-transit carriers. Since in-transit carriers give lower bids, the auctions in which in-transit carriers mostly join end up with lower prices even if there are fewer carriers joining to the auction. In addition, if there are only local carriers at the center giving bids to an auction, the prices tend to be higher although there are more carriers engaging in the auction.

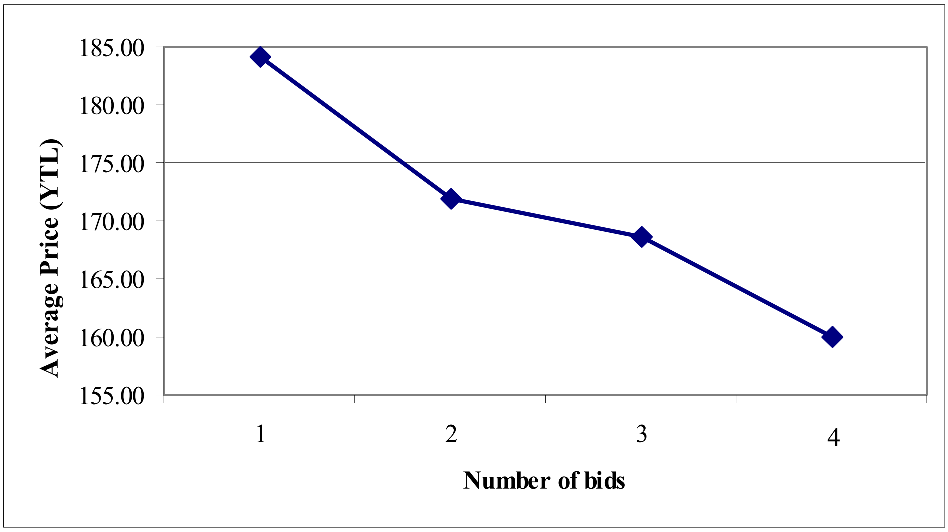

Figure 1.10. Number of bids and auction prices for City 41

For City 1, at most eight bids are given to a single order, and the transportations are realized at the minimum prices for these orders. The same case is valid for City 41 and the minimum transportation price is derived when there are four bids for an order.

1.2.6. Bids – Distance Relationship

Since the fuel consumption increases with distance, bids, a function of the transportation cost, also increase. The relationship between the distance and the bids is given in Figure 1.11. As seen from Figure 1.11, bids generally increase with the distance. In Turkey, there is an imbalance between the West-East and East-West traffic. Carriers usually go to the eastern part of Turkey full and go back to their home cities empty, so carriers generally carry goods to eastern parts at higher prices. We analyze the data separately for eastern and western parts; however, we do not get a specific result that supports this hypothesis. Maybe this situation takes place due to the fact that ESO LC attracts a high number of in-transit carriers that are willing to accept lower prices. Moreover, the lower bids given to longer distances in Figure 1.11 can be explained with the existence of in-transit carriers.

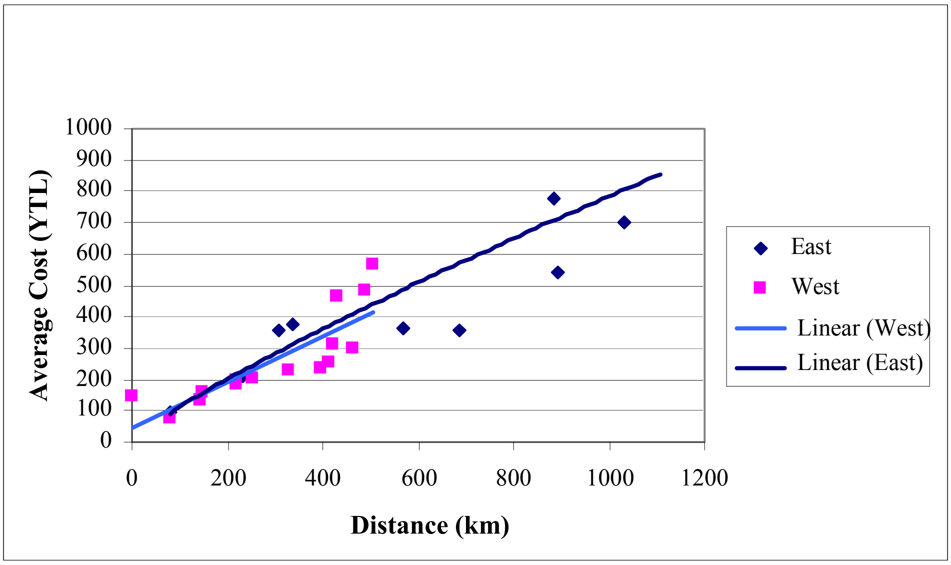

Figure 1.11. Average cost-Distance relationship

1.2.7. ESO LC Average Prices versus the Market Price

The prices realized at the ESO LC are compared with the market price value of the transportation, i.e. the price of Eskisehir Carriers' Cooperation. Table 1.2 shows the most popular cities that most of the transportation from Eskisehir is realized to. Moreover, the distances from Eskisehir to these cities, the fuel costs that are calculated according to distance, the market prices, the average prices that are realized in ESO LC and the percentage gains that are realized by using ESO LC rather than Carriers' Cooperation are shown.

City	*Distance (km)*	*Fuel Cost (YTL)*	*Market Price* (YTL)*	*ESO LC Average Transportation Price (YTL)*	*% Gain***
1	688	304.096	750	354	52.80
27	894	395.148	1100	533	51.55
34	330	145.86	350	217	38.00
35	412	182.104	450	250	44.44
41	219	96.798	250	181.5	27.40

Table 1.2. The data about the selected cities

* Carriers' Cooperation Transportation Price

** is calculated with (Market Price - ESO price)/Market Price*100

The main motivation is the reduction in the transportation price when using ESO LC. The average price realized in ESO LC, the market price and the fuel cost of transportation according to distance are given in Figure 1.12, and as it is seen that the average transportation price that is realized in ESO LC is very close to the fuel price and is much lower than the market price.

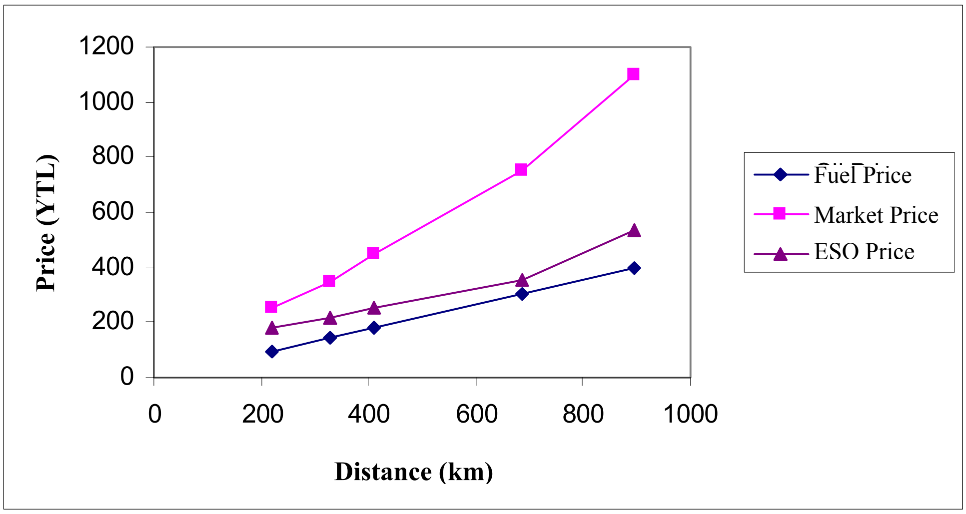

Figure 1.12. Fuel price, market price and ESO LC average transportation price according to distance

1.2.8. Analysis of the Total Transaction Volume of City 41

In this section the total transaction volume in terms of New Turkish Lira (YTL) for City 41 is calculated to give insights about the volume of the transaction. We analyze the auctions that took place between April 2004 and February 2005. The number of auctions, the average price, the total transaction volume realized in the LC and the total transaction volume that would be realized if the market price were used for City 41 for each month are given in Table 1.3. As seen in Table 1.3, the total number of auctions is 176, the average price is 182 YTL, and the total transaction volume realized for twelve months is 32,360.8 YTL.

Month	*Number of Auctions*	*Average Price (YTL)*	*Total Volume (YTL)*	*Total Volume with Market Price* (YTL)*	*Saving in terms of Volume (YTL)*
April 2004	5	179	895	1,250	355
May 2004	6	167.5	1,005	1,500	495
June 2004	4	166.4	665.6	1,000	334.4
July 2004	5	174	870	1,250	380
August 2004	17	176.5	3,000.5	4,250	1,249.5
September 2004	11	175	1,925	2,750	825
October 2004	30	189.1	5,673	7,500	1,827
November 2004	25	181.6	4,540	6,250	1,710
December 2004	24	187.5	4,500	6,000	1,500
January 2005	20	192.8	3,855	5,000	1,145
February 2005	29	187.3	5,431.7	7,250	1,818.3
TOTAL	**176**	**182.3**	**32,360.8**	**44,000**	**11,639.2**

Table 1.3. The data of auctions for City 41
*Market Price (Carriers Cooperation Price) for City 1 is taken as 250 YTL

Since the LC began to provide service at the end of December 2003, the number of auctions opened in the first months is small. As seen in Figure 1.13, even though some decreases that may be realized because of a seasonality effect, the number of auctions is generally increasing over time, and it is estimated that as firms realize the savings they will gain by using auctions, the number of auctions will increase.

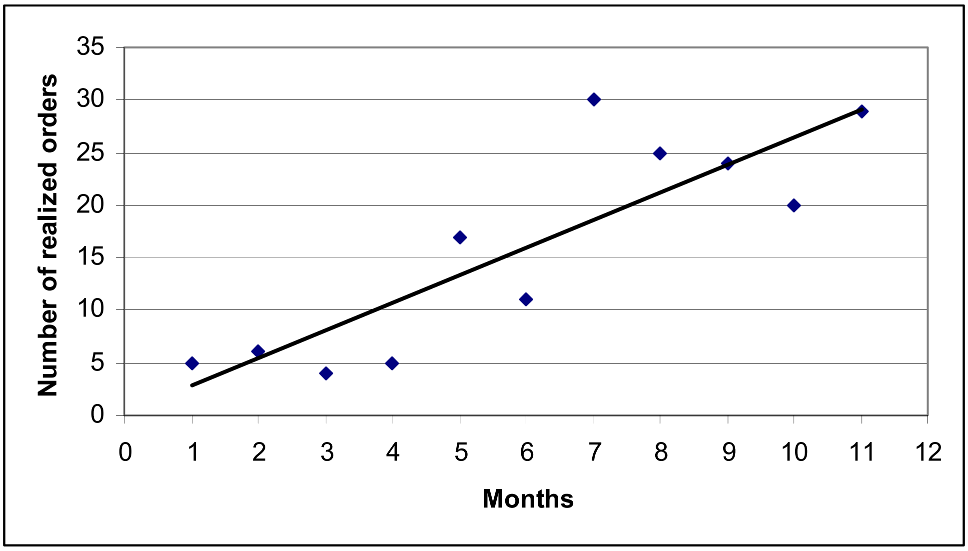

Figure 1.13. The number of auctions for City 41

The transaction volume is important for calculating the total saving that will be achieved by using auctions. Figure 1.14 shows the total transaction volume realized in each month from April 2004 to February 2005. As it is seen, the transaction volume also generally increases over time, and since it is only a year that the ESO LC began its process, it is likely to continue to increase in the near future.

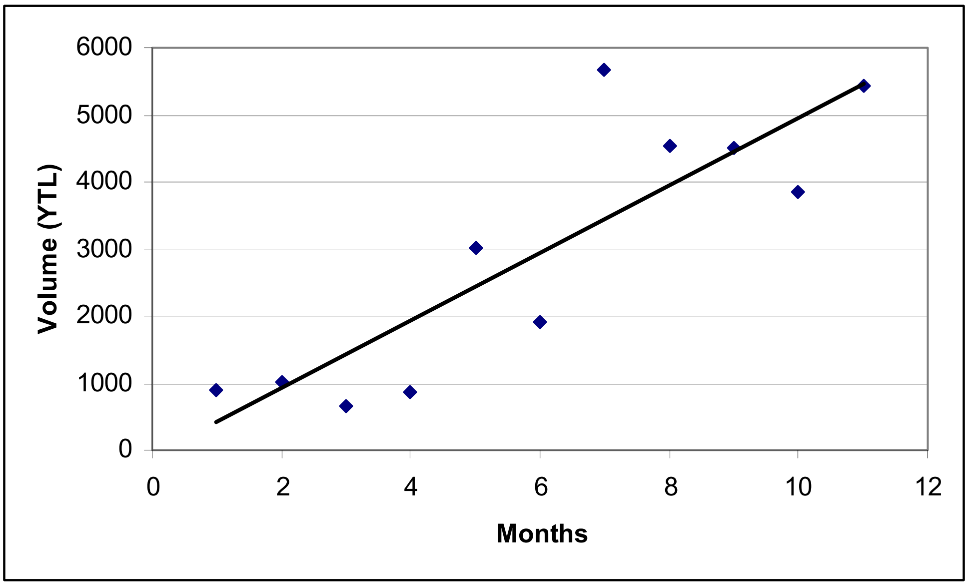

Figure 1.14. The transaction volume realized in each month for City 41

In addition, the total volume is graphed according to the number of auctions to see how the transaction volume changes with the number of auctions which is given in Figure 1.15. As it is expected, the total transaction volume is increasing in the number of auctions, and it is estimated to increase by the time.

Figure 1.15. The transaction volume according to the number of orders

Moreover, in order to establish the savings that will be obtained by using auctions, let us compare the total transaction volume with the market value of the transaction. As seen in Table 1.3, the total transaction volume realized at the LC in the first year for City 41 is 32,360.8 YTL. The market value of the transaction that is calculated with the market price is 44,000 YTL, and the total saving achieved by using auction at the ESO LC is 11,639.2 YTL. The percentage of the saving according to the market value of the transaction volume is 26.45%.

In summary, the prices realized at the ESO LC are much lower than the market price and are very close to the fuel price, some of the bids are even lower than the fuel price. In addition, the gain of the firms is very high. In one year, the transportation cost from Eskisehir to only one city, i.e. City 41, is lowered by 11,639.2 YTL. This amount may seem low; however, since the ESO LC has been providing service only for one year and the number of firms that take service from the ESO LC is very small when comparing with the number of firms located in Eskisehir, it is estimated that these savings will increase as the number of firms that use the reverse auction increases over time.

1.3. Framework of the Study

In this thesis, a logistics market where the orders from a number of producers with goods to transport to various destinations are matched with carriers through a reverse auction is considered. In order to analyze the effect of various system parameters, such as the order and the carrier arrival rates, the capacity of the system, and the availability of local and in-transit carriers, on the performance of the system, we develop a stochastic model.

In the market a company that wants to send its goods to a specific destination opens an auction by using an electronic reverse auction platform. All the carriers that are available at the logistics center at the time of the auction and are willing to go to the desired destination submit their bids. The order is given to the carrier who submitted the lowest bid. If no carriers are available or no bids are submitted, the company may cancel its auction, and either sends its goods by a logistics service provider or reopen the auction at a later time. An auction request can be rejected if the system capacity is reached.

We first model the auction in a static setting and determine the expected auction price and the profit of the carriers based on the number of carriers engaging in the auction and their cost distributions. We then develop a continuous-time Markov chain model to analyze the system in a dynamic setting with random arrivals and possible abandonment of orders and carriers. By combining these two models, we evaluate the performance measures such as the average expected price paid to the carriers, average expected profit of the carriers, average number of carriers and orders waiting at the logistics center in the steady state.

Moreover, we propose an estimation method to determine the cost distributions of carriers from the observed bids. This estimation method is practiced on the auctions opened for transportation from Eskisehir to two main cities. It is shown that the estimation procedure gives close results for these cities.

The main contribution of this study is evaluating the performance of an auction-based logistics market in a dynamic setting by using an analytical model. The model yields insights regarding the effects of the system parameters on the performance and it is validated with the results of the descriptive statistics. The other contribution is the estimation method that is proposed to determine the cost distributions from the observed bids.

The organization of the remaining part of the thesis is as follows: we summarize the necessary background and literature on the logistics contracts, auction mechanisms that are commonly used in procurement and logistics, and performance measures of auctions in Chapter 2.

In Chapter 3, we give the basic auction mechanisms, especially reverse auctions, and the basic analysis of bidding and auction processes that will be used to obtain the analytical results in this thesis.

In Chapter 4, we describe the problem, and give the general model with its assumptions and the performance measures for the general model.

In Chapter 5, the general model is analyzed analytically with uniform cost distribution assumption, and numerical experiments and the results of simulation are reported. All the performance measures are analyzed for different system parameters.

In Chapter 6, an estimation method for inferring cost distributions from the observed bid is proposed. Moreover, the method is tested on the real data for transportations from Eskisehir to two main cities.

Finally, the thesis concludes with a short summary of the performed study and future work.

Chapter 2

LITERATURE REVIEW

2.1. Overview

In this chapter, the literature on contracts that are used mostly in the logistics environment, the application of auctions in procurement and logistics, and the performance evaluation of auction-based systems are explained.

Logistics contract design problems have gained attention in both the economics literature and in the Operations Research/Management Science (OR/MS) literature [4]. Studies in the economics literature mostly deal with aspects such as the determination and the operation of the contract types and their implications on the parties involved in the contract, the motivation of contractual structures, the legal issues in contracting environments, and the selection of the contractor [4].

The OR/MS literature, on the other hand, focuses on some operational details such as the material flow among the parties, uncertainties in demand or supply, and penalties charged [4]. Tsay et al. [5] give a recent survey of supply chain contracts. Henig et al. [6], Yano and Gerchak [7], and Ernst and Pyke [8] are some examples of studies that deal with transportation contracts in supply chain environments.

Tyan et al. [9] introduce a new shipper-carrier partnership strategy, called collaborative transportation management (CTM) that includes the carrier as a strategic partner for information sharing and collaboration in a supply chain. In addition, Alp et al. [4] consider the problem of designing parameters of a given contract for the transportation activity. Lambert et al. [10] present a model that can be used to determine whether a partnership should be used, and also to define the most appropriate type of partnering for a given situation.

An increasing trend towards outsourcing in logistics activities has led to a growth in the third party logistics (3PL) industry. A 3PL provider is an external provider who manages, controls, and delivers logistics activities on behalf of a shipper [11]. There is a recent emphasis on 3PL and its practices in different sectors and countries in the literature on logistics [12, 13]. Relationships between shippers and 3PL providers are long term. Once established, they have a tendency to continue [11]. Most of the literature is devoted to long term contracts between shippers and carriers. The main difference of this study from the previous literature is considering a spot market where the auctions are established for only one trip. This means when a carrier wins an auction, he takes only that transportation order and if he wants to take another order, he has to join another auction again.

2.2. Auctions in Procurement and Logistics

In the supply chain literature, auction theory research has focused more closely on the competitive bidding parts of auctions, and mostly procurement auctions [14, 15, 16, 17] are analyzed. Goldstein's [14] application of linear programming to minimize the cost of a multi-unit procurement of a set of related items is presumably the earliest of the OR/MS auction related paper [18]. Stark and Rothkopf [15], in 1979, developed an extensive bibliography on competitive bidding with almost 500 titles. In 1994, Rothkopf and Harstad [19] updated this research. Moreover, they describe the conditions in which auctions develop and survey the theory of single, isolated auctions in their paper.

The use of online reverse auctions (or electronic reverse auctions) in sourcing activities is reviewed by Jap [16]. His research highlights four key aspects: (1) the differences from physical auctions and those of the theoretical literature, (2) the conditions for using online reverse auctions, (3) the methods for structuring the auctions, and (4) the evaluations of auction performance. In addition, he provides some empirical evidence on these issues. As pointed out by Lucking-Reiley [20], online auctions often decrease information, transaction and participation costs, as well as increase convenience for both sellers and buyers, and lead easy access to larger markets.

Wagner and Schwab [17] are also interested in electronic reverse auctions (ERA) and they try to give an answer to the question “Which conditions are suitable for conducting ERAs and how can purchasing managers influence these conditions in order to make ERAs successful?” in their paper. They are able to identify eight conditions by conducting a research on the literature and by performing interviews with academics and practitioners which will be put together to make up their research framework. Most of these conditions such as, ease of specifying demand, higher auction volume, low expense of switching suppliers, competition among suppliers participating and powerful buyer, match with the logistics market that we analyze.

Combinatorial auctions in procurement are studied by Elmaghraby and Keskinocak [21] and a case study of one of the largest home improvement retailers is given. Narahari and Biswas [22], propose an iterative auction model for efficiently solving the allocation problem in combinatorial exchanges which allow combinatorial bidding. Moreover, Dasgupta and Spulber [23] extend the standard fixed quantity auction that allows the buyer to vary the quantity of the good purchased based on bids by competing sellers. The combinatorial auctions are out of the scope of this study because only one auction can be opened at a time; however, an extension may be developed by taking combinatorial auctions into consideration

Chen et al. [24] consider multi-unit Vickrey auctions for procurement in supply chain settings and his paper is the first one that includes transportation costs into auctions in a complex supply network. Three incentive compatible auction mechanisms are proposed. Multi-attribute reverse auctions are studied by Talluri and Ragatz [25] and a framework for designing and performing multi-attribute reverse auctions in B2B exchanges is developed.

In recent years, the use of auctions in logistics marketplaces is being analyzed. Kameshwaran and Narahari [3] survey the negotiation models currently deployed in logistics marketplaces. Distinctive characteristics of the logistics marketplaces are presented and they propose four auction algorithms for the logistics marketplaces. Two negotiation models based on Dutch Auctions, a sequential-combinatorial auction and discriminatory call market are proposed. According to their survey, there are currently three different business models used in logistics marketplace: (i) reverse auctions, (ii) transportation quote request, and (iii) proprietary models. In addition, Narahari [2] provides a simple design of a logistics marketplace which can be used in Indian surface transportation. In this thesis, a reverse auction used in a logistics marketplace which is used in land transportation from Eskisehir to other cities in Turkey will be analyzed.

Ledyard et al. [26] study combined value auctions (CVAs), which gives bidders the opportunity to name their prices on combinations of items, and their paper is the first one that uses a combined value auction for transportation services. The use of CVAs are studied for transportation services for SLS (Sears Logistics Services) and it is said that SLS resulted in substantial saving by running combined-value auctions [26]. As it is mentioned before, single value auctions are analyzed in this thesis.

Song and Regan [27] analyze the for-hire truckload trucking industry in U.S. and they propose a new auction based carrier collaboration mechanism designed in which a group of small and medium-sized carriers can conduct post-market exchange and hence significantly improve their operational efficiency. Architecture for such a system is defined and its economic benefits are examined. We exclude the collaboration between carriers in this study.

Qi [28] prepared a case study that analyzes a neutral, double blind Internet platform for the exchange of surplus global ocean-going container space named CargoExchange.Net (CX). CX is defined as a neutral exchange platform that is based on matching bid and asks prices from multiple buyers and sellers, and serves both shippers and carriers. As the name implies, neutral exchanges did not favor either shipper or carrier, but simply allow carriers to post available cargo and the ‘ask price’ they are willing to take for it. Shippers in turn can post their demands for cargo space with a ‘bid price’ they are willing to pay. The logistics marketplace that we study mostly parallels to the study of Qi; however, we use reverse auction in our study rather than a neutral auction. Moreover, Qi gives only a case study and does not analyze the marketplace. This study analyzes the marketplace with a continuous time Markov chain and evaluates the performance of the system in steady state.

2.3. Performance Evaluation of the Auction-Based Systems

The performance evaluation of auction-based systems is studied in different branches like manufacturing systems, transportation systems and economics. Krishna and Morgan [29] study the war of attrition and the all-pay auction when players' signals are affiliated and symmetrically distributed. They examine the performance of these auctions in terms of the expected revenue accruing to the seller.

Parkes [30] compare the auction performance of two different agents one with hard local problems and the other with uncertain values for goods in terms of efficiency, revenue and average utility from participation.

Segev et al. [31] model an online auction in terms of a Markov Chain on a state space defined by the current price of the item and the number of bidders who have been previously eliminated. The model results are validated through a comparison with real-world online auction data. The paper answers the question "can we predict the number of items sold and the price at which they will likely sell given inputs, such as number of bidders and their behavior, the number of items for sale, market conditions and auction rules, to the auction. In this study, the average expected auction price, which is one of the performance measure that is analyzed, is predicted by giving the number of bidders in steady state.

A model to evaluate the performance of auction-based distributed shop-floor control schemes is provided by Veeramani and Wang [32]. They first analyze the associated queuing network approximately to identify control schemes and then use simulation to evaluate the performance of the system in detail. In a similar setting, Nandula and Dutta [33] use Petri nets to evaluate the performance of a manufacturing system that uses auctions as a control strategy. Three different models of an auction-based manufacturing system are discussed which helps the evaluation of various performance metrics like machine utilization, automated guided vehicle utilization, waiting times, work in process etc.

Vakrat and Seidmann [34] present a model for estimating the expected price as a function of the number of bidders, the distribution function of valuations, and the number of units to be sold in the auction. The analysis shows that the auction price may either decrease or increase with the dispersion in the bidders' values, depending on the bidders' overall arrival process, the length of the auction, and the number of units sold in the auction. The optimal auction length is calculated and it is shown that an auction's profit is a unimodal function of its duration and the number of units. This paper also defines several other economic tradeoffs that are important for the optimal design of Internet auctions. In this study, like Vakrat and Seidmann [34], the average expected price is estimated as a function of the number of bidders and the distribution function of costs of bidders.

Emiliani and Stec [35] discuss the savings that can result from online reverse auctions for the specific case of machined parts that have been designed by the buyer and examine if this savings is as great as advertised. Three terms are used to describe the savings that result from online reverse auctions: (i) identified savings, (ii) estimated savings, and (iii) achievable savings. The savings analyzed in this thesis are acquired by using an auction in terms of the average auction price.

In this study, the average expected auction price (as in [29, 30, 34]), and in addition the average expected profit of carriers, the average number of carriers and orders, and the probability of rejection of carriers and orders in the long run are analyzed as performance measures.

As a consequence, establishing an auction-based logistics market is a recent phenomenon, and there is little literature that considers auction-based logistics systems. The main contribution of this thesis is the modeling and analysis of a logistics market, which uses a reverse auction, in a dynamic setting and providing insights about the effects of the system parameters. To our knowledge, this is the first study that models and analyzes an auction-based logistics market.

Chapter 3

BACKGROUND ON AUCTION THEORY

3.1. Introduction

In this chapter, we summarize certain important results in auction theory that are relevant to our study. The purpose of this chapter is to give an overview about auctions. Some of the basic types of auctions and their rules, applicability and relative advantages and disadvantages are introduced.

An *auction* is defined by McAfee and McMillan [36] as a market institution with an explicit set of rules determining the allocation of resources and prices on the basis of bids from market participants. This set of rules control and manipulate the participants of auction for making and processing bids and determining the transaction price or prices for the buyers and sellers. Auction Theory is one of the subjects that received significant attention in the economics literature for a number of years.

In traditional auctions buyers are brought together in order to determine the price of a product or service that a seller will receive for. This type of auction is more common and usually called a *forward auction*. In a forward auction multiple buyers compete to purchase items from a single seller. Buyers bid, and the seller's goal is to push the price up and maximize her revenue. Another type of auction frequently used in sourcing activities is called a *reverse auction*. In a reverse auction, multiple sellers or suppliers compete to satisfy a buyer's needs. Sellers bid, and the goal of the buyer is to push the price down and minimize her cost [19]. The distinction between contexts in which bidders are competing to buy and to sell is relatively unimportant because there is an almost perfect correspondence in results [19]. Since our model is based on a reverse auction, in section 3.4 reverse auctions will be presented in detail.

3.2. The Basic Models of Auctions

A key feature of auctions is the presence of asymmetric information which means no bidder is perfectly informed and also the information that bidders have can show significant differences [37]. There are two qualitatively different auction settings depending on how a bidder's value of the item is formed. In *common value* auctions, a bidder's value of an item depends entirely on other bidders' values of it, which are identical to that of the agent by symmetry. On the other hand, in *private value* auctions, the value of the good depends only on the bidder's own preferences [34]. In this thesis, a private value auction is analyzed.

Most bidding theory papers discuss a single isolated auction of a *single* indivisible asset. There are a few papers discussing an isolated sale of a fixed number of identical assets to bidders each of who attach no value to a second purchase. These are typically called *multiunit* auctions [19]. In this thesis, a single-unit auction is considered.

Auctions can be either *open* or *closed*. In open auctions, prices are publicly announced by the auctioneer and bidders can indicate their willingness to buy the object at the particular prices. In closed auctions, bidders submit offers simultaneously, which can not be seen by other bidders, and these offers are then evaluated by the auctioneer [19]. A closed auction is studied in this thesis.

3.3. The Standard Auction Types

There are many different forms of auctions and several useful ways of classifying these variants. A *standard* auction means one in which the winner is the highest bidder among potential buyers, or the lowest bidder among potential sellers [19]. While the variants of auctions differ, only four traditional types of single-unit standard auctions exist:

(1) the ascending-bid auction (English Auction),

(2) the descending-bid auction (Dutch Auction),

(3) the first-price sealed-bid auction, and

(4) the second-price sealed-bid auction.

The *ascending-bid auction*, or *English auction*, is the auction form most commonly used for selling of goods. In the English auction, the price is successively raised until only one bidder remains and she wins the object at the final price [37]. This auction can be run by having an auctioneer announce prices, or by having bidders call out the bids themselves, or by having bids submitted electronically with the current best bid posted [37, 36]. The essential feature of the English auction is that, since it is an open system requiring that all bidders be made aware of all bids offered [19], at any point in time, each bidder knows the level of the current best bid [36].

The *descending-bid auction*, or *Dutch auction*, is the converse of the English auction. The auction begins with an initial high price set by the auctioneer, and then the auctioneer lowers the price continuously until one bidder accepts the current price. The first bidder who accepts the current price wins the object and pays that price [37]. The cut flowers in the Netherlands are mostly sold by using the Dutch auction.

The *first-price sealed-bid auction* requires each potential bidder to independently submit a single sealed-bid, and the bidder who makes the highest bid wins the object and pays his bid [38]. The basic difference between the first-price sealed-bid and the English auction is that, bidders are able to see their rival's bid and they can revise their own bids if they choose in the English auction; however in the sealed-bid auction, each bidder can submit a bid only once and cannot change his bid [36].

The *second-price sealed-bid auction*, mostly called a Vickrey auction, also requires bidders to independently submit sealed bids, without seeing the others' bids, having been told that the highest bidder wins the object but pay a price equal not to his own bid but to the second highest bid [36].

At the ESO LC, the auction is driven as a first-price sealed-bid auction. However since, as it will be discussed in Section 3.5.2, both first-price and second-price auctions are equivalent under some assumptions with the revenue equivalence theorem, in this thesis we analyze the auction as a second-price sealed-bid auction.

3.4. Reverse Auctions

Reverse auctions are an increasingly popular sourcing tool for many purchasing firms. These auctions hold the promise of enhancing competition among suppliers as well as reducing the cycle time for the sourcing process.

Reverse auctions have been used for buying a wide variety of products and services, with mixed results. There seems to be a consensus developing regarding the types of sourcing decisions for which reverse auctions might be an appropriate tool. Specifically, these are sourcing decisions where;

- Product or service specifications can be clearly and objectively stated
- Price is the major criteria
- Switching costs are relatively low
- There are many qualified suppliers in the market
- There is no well-established commodity market
- The product or service to be sourced is not considered "strategic". [25]

Since logistics is one of these sourcing activities, using reverse auctions in logistics contracts will be appropriate, and in this thesis the reverse auction used in a logistics market is analyzed.

As the Internet develops, the companies realize that Internet provides a powerful tool for the procurement of goods and services to support the supply chain. Firms have found that competitive procurements over the Internet, i.e. online auctions, have given them the opportunity to increase their purchasing power, attract larger group of suppliers, and, in general, upgrade the efficiency of their procurement processes [18].

An electronic auction is a special case of electronic negotiation and an electronic reverse auction (ERA) is a frequently used type of electronic auction in B2B commerce [17]. Although there is no formal definition of ERA, it is frequently considered as the application of internet technology throughout the purchasing process. In ERAs, suppliers compete for a buying companies business by offering successively lower bids via the internet, in a time limited bidding "event" [25].

Electronic reverse auctions offer the possibility of achieving favorable pricing and/or other terms by increasing competition among suppliers. Further advantages may be realized by reducing the time and overhead associated with the sourcing decision [25].

The discussion in this study will be restricted to the online reverse auctions with a single buyer that wants her goods to be transported and multiple carriers that compete to take this transportation order.

3.5. Literature Review of Auction Theory and Important Results

3.5.1. Early Literature and the Vickrey Auction

Although the prices at which goods will be bought and sold have been determined by auctions for a long time in certain markets, auctions entered the research literature only in 1950's and 1960's. Now, auction theory is a well-established area of research. It analyzes agents' strategies when an auctioneer wants to sell an item (or buy a service) and tries to achieve the highest (or the smallest) possible payment for it, where each bidder wants to acquire the item (or sell his service) at the lowest (or highest) possible price.

The economics literature takes auctions into consideration from both a theoretical and an empirical perspective. One of the earliest and the most important papers in the economics literature is written by Vickrey [39] in the early 1960's. In his paper, he takes auctions as a sub-field of economics. He proposes one of the most important auction mechanisms, called the second-price sealed-bid model, that is now referred as a Vickrey auction.

Vickrey considered the situation where a seller values an item at zero and she has the option to sell it to one of n risk-neutral bidders. According to the Vickrey auction, each bidder independently and privately picks a price and offers to buy the goods or supply the service at that price without knowing the others' bids. Each bidder alone knows his own valuation, *v*, which he places on a good. Because the seller and the other buyers are uncertain about this value, it appears to them to be a random variable drawn from a common distribution *F*(*v*), with $F(\underline{v}) = 0$ and $F(\overline{v}) = 1$. That is, everyone (including *i*) agrees that the probability that v_i is less than *v* is given by *F*(*v*) and everyone knows that everyone knows this. The one with the highest bid wins, but he pays or he is paid at the price of the second highest. In this auction, the seller's problem is designing an optimal auction that creates maximum revenue for her [39].

Vickrey auction encourages truth telling and it has been proved that the dominant strategy in a private value Vickrey auction is to bid one's true valuation [39]. This is because, in a forward auction, if a bidder bids more than his true valuation and wins he may end up with loss if the second highest bid is higher than his true value. In the mean time if he bids less, there is a smaller chance of winning, but the winning price is unaffected [34].

Following the paper of Vickrey, Griesmer et al. [40] analyze the equilibrium of a first-price auction in which contestants' valuations are drawn from uniform distributions with different supports. Wilson [41] introduces the common-value model and develops the closed-form equilibrium analysis. In this thesis, a logistics market that uses a Vickrey auction will be analyzed. Moreover, as a numerical example an auction where the costs of carriers are drawn from uniform distributions will be analyzed and closed-form equilibriums for a special case will be given.

3.5.2. Revenue Equivalence Theorem

The main contribution of Vickrey's study is the Revenue Equivalence Theorem, which states that expected seller revenue in equilibrium is independent of the auction mechanism. According to the Revenue Equivalence Theorem, "any auction mechanism in which (i) the object always goes to the buyer with the highest value (in reverse auctions it is the smallest value), and (ii) any bidder with value $\underline{v}$ expects zero surplus, yields the same expected revenue, and results in a buyer with value v making the same expected payment". See Appendix B for the proof. The principal assumptions of the Revenue Equivalence Theorem are (i) risk neutrality, (ii) independence of different buyers' private signals about the item's value, (iii) lack of collusion among buyers, and (iv) symmetry of buyers' beliefs [42].

Vickrey established the revenue equivalence of the standard auction mechanisms (first-price auctions, Dutch auctions, English auctions, and second-price auctions). In particular, the two most common auction institutions – the open "English" auction and the second-price sealed-bid auction – are equivalent despite their rather different strategic properties. Also the Dutch and the first-price sealed-bid auctions generate equal revenues for the seller. Moreover, it is shown that the first-price and the second-price sealed-bid auctions are equal to each other in terms of the expected revenue generated by the auctioneer.

This result was generalized in 1981, 20 years later than Vickrey, by Myerson [43], and in the same year independently by Riley and Samuelson [44]. Thus all the standard auctions yield the same expected revenue under the stated conditions, as do many non-standard auctions [37]. The result of the Revenue Equivalence Theorem is so fundamental that almost all of the auction theory literature is based on this theorem.

Over the last years, a number of papers have studied the implications of relaxing the assumptions of Revenue Equivalence Theorem. Holt [45], and Riley and Samuelson [44] relax the risk neutrality of buyers assumption and show that, when buyers are risk averse, the sealed-bid auction should be favored by a seller even if he also exhibits risk aversion. The assumption of independence of private signals of the item's value is relaxed by Milgrom and Weber [46] and it is shown that if reservation prices are "affiliated", the English auction generates higher expected revenue than the sealed-bid auction. At last, Graham and Marshall [47] and McAfee and McMillan [36] allow for the possibility that buyers collude, i.e. asymmetric auctions.

Since it is shown that if the assumptions of the Revenue Equivalence Theorem hold, the auction type does not affect the expected auction price, in this thesis under the appropriate assumptions, that will be discussed in Section 4.2.1, the auction is analyzed as a second-price sealed-bid auction.

3.5.3. Further Results

Although there were a number of studies on auctions following Vickrey's paper, the literature expanded mostly during the 1980s. For both the results obtained and the methods introduced, Myerson's [43] study is a basis for most of the studies about auctions. Myerson tries to design an optimal auction for a problem faced by a seller when he has imperfect information about the bids. He makes a contribution to the auction literature in two important ways. The first is to consider the case of asymmetric bidders where bidders privately known valuations of object are drawn from independent, but not necessarily identical, probability distributions. These distributions are assumed to be commonly known, so that all bidders and the seller know the distribution from which each bidder's value is drawn. The second extension was formulating an optimal auction by taking all possible ways of selling the good into consideration. The optimal auction problem can be defined as among all possible ways of selling the good, which one should the seller use if she wants to maximize her expected net revenues. Myerson gives an explicit formula and solves this problem for asymmetric auctions. He also extends Revenue Equivalence Theorem, proposed by Vickrey in 1961, to show that any two mechanisms that always lead to the same allocation of the good would result with the same expected revenue.

McAfee and McMillan [36] survey the developments in the theory of bidding mechanisms and they discuss the relevance of the theoretical results for auctions in practice. In addition, pitfalls for bidders, equivalences among auction institutions and comparison of auctions is researched by Milgrom [48]. More recently Klemperer [37] put a comprehensive review of the auction theory together by introducing and defining some of the critical papers. For more information about auction theory the readers are referred to this study.

Auctions with a stochastic number of bidders are analyzed by McAfee and McMillan [49]. It is proven that in a first-price sealed-bid auction with bidders having constant absolute risk aversion, the expected selling price is higher when the bidders do not know how many other bidders there are than when they do know this, which means the seller should not announce the number of bidders joining that auction. Moreover, the seller's maximum expected revenue with risk-neutral bidders having independent private values is the same whether or not the bidders know the set of bidders. In this study, we assume that the number of bidders is concealed by the auctioneer.

Most researchers have dealt mostly with symmetric auctions, where all the bidders' valuations are drawn from a single distribution, since in the symmetric case an explicit expression for the equilibrium bidding strategies can be obtained. However in many cases, bidders' valuations are drawn from different distribution functions. As Fibich et al. [50] states explicit expressions for asymmetric equilibrium strategies cannot be obtained except for very simple models, so the analysis of asymmetric auctions is more complex and relatively little is known about them at present.

One of the early papers that study asymmetric auctions is Wilson's [51] paper in which he analyzes the problem of competitive bidding under uncertainty when one of the groups is better informed. Since the basic work of Wilson [51], it has been recognized that auctions in which information about the value of the object being sold is symmetrically distributed among agents are qualitatively different from those in which information is asymmetrically distributed. Wilson's study is reanalyzed by Weverbergh in 1979 [52] and it is found that the value of the game is essentially zero for the party with incomplete information. In addition, Griesmer et al. [40] extend the work of Vickrey [39] to the case where the cost of bidders is asymmetrically distributed.

Myerson [43] showed that condition on the realization of the players' valuations, the probability of a player to win the object is independent of the auction mechanism, and the Revenue Equivalence Theorem remains true for asymmetric auctions. However, this condition is not usually true for asymmetric auctions. Indeed, it is well known that asymmetric auctions are not necessarily revenue equivalent, and also the issue of equilibrium existence is not as straightforward [53]. For example, Maskin and Riley [42] show that, with asymmetry, revenue equivalence no longer holds and that, under different assumptions about the nature of the heterogeneity, expected revenue in the sealed bid auction may be higher or lower than in the open auction.

Hendricks and Porter [54] examine federal auctions for drainage leases on the Outer Continental Shelf from 1959 to 1969 and they find that both informed and uninformed firms bid strategically according to the Bayesian-Nash equilibrium.

Fibich and Gavious [55] analyze an asymmetric auction and use perturbation analysis to calculate the equilibrium bid strategies in first-price auctions. Fibich et al. [50] analyze the effect of weak asymmetry on the seller's expected revenue by using perturbation analysis. Since it is known that asymmetric auctions are not revenue equivalent, the results shows that when asymmetry is weak, the revenues in the asymmetric case and in the corresponding symmetric case are essentially identical. Campo et al. [56] propose a simple method for estimating asymmetric first-price auctions with affiliated private values.

In this study we consider two different types of carriers, i.e. local and in-transit carriers, with different cost distributions. Different cost distributions among bidders make the auction asymmetric [42]; however, since the number of bidders is not revealed in this study, the auction turns out to be a symmetric auction [49].

Chapter 4

PROBLEM DESCRIPTION AND THE MODEL

4.1. Problem Description

In this chapter, the description of the problem that is analyzed, the assumptions that are made about the auction, the carriers and the orders, and the expected auction price and profit of the carriers in static setting is explained. Also, the general state-space model with the performance measures is introduced.

In this thesis, we consider a Logistics Center (LC) where a number of independent carriers are located and multiple producers that want their goods to be transported to different destinations open auctions. At the LC there are multiple carriers that respond transportation orders given by the shipper to send her goods to a specific destination. When the shipper has a transportation order, she opens an auction by using an electronic reverse auction platform, i.e. a web-page designed for this process. All the carriers, which are available at the time of auction, see the order and give their bids to that order. The order is given to the carrier who submitted the lowest bid. If no carriers are available or no bids are submitted, the company may cancel its auction, and either sends its goods by a logistics service provider or reopens the auction at a later time. Moreover, an auction request can be rejected if the system capacity is reached, and carriers who stop at the logistics center may leave after some time if they cannot get an order. Finally, an arriving carrier may be rejected if the physical capacity of the center is reached, e.g. if the parking lot is full.

At the ESO LC, the auction is designed as a first-price auction, which requires paying the winner his bid. However, since the dominant strategy in a second-price sealed-bid auction is telling the truth (so all the bidders bid their actual costs) and from the Revenue Equivalence Theorem both the first and second price auctions are equivalent (see Subsection 3.5.2), we model the auction as a second-price auction without changing the results from the perspective of the shippers and the carriers.

4.2. General Model

The mathematical model of the LC with local and in-transit carriers and producers who place orders is developed by using Continuous Time Markov Chain models. This model predicts the average expected price of the auction, the average expected profit of the carriers, the average number of carriers and orders, and the probability of rejection of carriers and orders in steady state.

The outline of the model is as follows:

1. The shipper opens the auction and this auction is announced to all bidders.
2. All bidders available at the time of the auction give their bids to that order. If there isn't any carrier available, the order is taken by the system. Then when a carrier arrives to the LC, it takes that waiting order at a designated market price without an auction, if the order is still valid.
3. At the end of the auction the shipper evaluates all the bids and chooses the bid with minimum price. Since a second-price sealed-bid auction is assumed the winning bidder is paid at the second lowest bid.
4. The winning bidder is announced and the auction is closed.

4.2.1. Model Assumptions

Some simplifying assumptions about the auction, the carriers and the orders are made for the system and given in the following parts.

4.2.1.1. Assumptions about the auction

A single-unit second-price sealed-bid auction, or the Vickrey auction, is used. It is assumed that the opening time of the auction is not regular, i.e. the process can start whenever the shipper wants to transport its goods. Also, only one auction can be opened at a time, i.e. shippers can not open more than one auction at the same time, nor can different firms open an auction at the same time. The auction does not advertise future auctions or items, nor does it contain any information about past ones.

Only the bidders that are available at the opening time of the auction give bids, and the bidders that come during the auction process cannot bid for that auction. The bid of a carrier is denoted with $b_i(.)$ and depends on the cost of the carrier.

If there are no carriers at the LC, the order is registered by the system and stays there until the order is cancelled by the shipper or given to the carrier that arrives first at the designated market price, P_M, without an auction.

4.2.1.2. Assumptions about the carriers

We assume that there are two types of carriers, one type (call as Type *L*) refers to local carriers that are based at the same region as the LC and the other type (call as Type *B*) refers to in-transit carriers that stop by the LC while traveling to their base. The Type *L* and Type *B* carriers are assumed to arrive randomly to the logistics center according to Poisson processes with rates λ_L and λ_B respectively.

The capacity of the LC for both types of trucks is limited, and C_L and C_B denotes the capacity of Type *L* and Type *B* carriers respectively. When these capacities are exceeded, the next arriving carrier is rejected.

A Type *L* carrier *i* has a transportation cost of V_i known only to him, which is considered by everyone (including the carrier *i*) to have been drawn from a distribution with cumulative density function (cdf) of F_L, $F_L(\underline{v}) = 0$, $F_L(\bar{v}) = 1$ with expectation $E[V]$.

Type B carrier i has a transportation cost of R_i which is distributed with cdf of F_B, $F_B(\underline{r})=0$, $F_B(\bar{r})=1$ with expectation $E[R]$.

The V_i and R_i for all the bidders of the same type are independent and identically distributed. For deriving analytical results, we assume the transportation costs of in-transit carriers are lower than of the local carriers and $\underline{v} > \bar{r}$.

We assume that the number of carriers engaging in an auction is concealed and all the carriers have the same belief about the probability distribution of the number of bidders joining an auction which is denoted by p_j, i.e. $p_j = Prob\{n=j\}$. Moreover, we assume that there is a common belief about the probability that each bidder is local (Type L), i.e. *Prob*{the carrier is a local carrier}=θ, or in-transit (Type B), i.e. *Prob*{the carrier is an in-transit carrier}=1-θ, and each bidder makes his bid according to this common belief. Since the probability distribution of the number of bidders joining an auction and the probability of the type of each bidder are common believes, the probability distribution that each bidder makes his bid according to is a function of the cost distributions of carriers, the number and the type of carriers joining an auction which is symmetric.

The bidders do not update their bids as the auction progresses and all the carriers that are in the LC bid for the auction when it opens.

A carrier can abandon the LC after waiting some time for receiving an order. We assume that this time is an exponentially distributed random variable with rate λ_{LA} for Type L carriers and λ_{BA} for Type B carriers.

The carriers are assumed to be risk-neutral and they all try to maximize their own profit. In addition, it is assumed that there is no collusion between carriers.

4.2.1.3. Assumptions about the orders

It is assumed that orders arrive randomly to the LC according to a Poisson process with rate μ_S. Since the auction is driven electronically, we thought that the software has a capacity for orders. The capacity for orders is denoted with C_S, and when this capacity is exceeded, the arriving order is rejected.

In addition, if there are no carriers available when an order arrives, the order can abandon after waiting some time. We assume that this time is exponentially distributed with rate μ_{SA}.

The auction is designed for one full truck transportation i.e. an order can not be split among different carriers.

As a consequence, the assumptions of the Revenue Equivalence Theorem, i.e. (i) risk neutrality, (ii) independence of different buyers' private signals about the item's value, (iii) lack of collusion among buyers, and (iv) symmetry of buyers' beliefs, are satisfied, we can analyze the auction as a second price auction.

4.2.2. Bids, Auction Price and Profit of Carriers in Static Setting

We look at the auction price as a function of the stochastic arrival process of orders and carriers to the system. Initially, suppose there are l independent Type L and b Type B carriers. Let $b_i(R_i)$ denote the final bid that Type B carriers i submits which equals to R_i, and $b_i(V_i)$ denote the final bid that Type L carrier i submits which equals to V_i. The marginal bidder in a Vickrey mechanism is the one who determines the auction uniform price and his bid is the smallest rejected bid. The total seller's cost in this case is that marginal bidder's bid.

Let $r_{(i)}$ and $v_{(i)}$ be the i^{th} smallest of the bids given by Type B and Type L carriers respectively. From the analysis above, the bidder with the cost of $r_{(1)}$ is the winning bidder and she will be paid a price equal to $r_{(2)}$.

Theorem 1: The expected auction price given *l* Type *L* and b Type B carriers in the auction, *p*(*l*,*b*), is

$$p(l,b) = \begin{cases} \underline{r} + \int_{\underline{r}}^{\bar{r}} [1-F_B(x)]^{b-1} bF_B(x)dx + \int_{\underline{r}}^{\bar{r}} [1-F_B(x)]^{b} dx & \text{if } b>1 \\ \underline{v} + \int_{\underline{v}}^{\bar{v}} [1-F_L(x)]^{l} dx & \text{if } l>0, b=1 \\ \underline{v} + \int_{\underline{v}}^{\bar{v}} [1-F_L(x)]^{l-1} lF_L(x)dx + \int_{\underline{v}}^{\bar{v}} [1-F_L(x)]^{l} dx & \text{if } l>1, b=0 \\ P_M & \text{otherwise} \end{cases}$$

(4.1)

The proof of the theorem that is derived from the standard auction given by Klemperer [37] is given in the Appendix B.

Note that when there is only one carrier, i.e. $l = 1$ and $b = 0$, or $l = 0$ and $b = 1$, or no carrier at the LC, i.e. $l = 0$ and $b = 0$, the arriving order is taken at the market price.

Theorem 2: The expected profit of the winner, *q*(*l*,*b*), when there are *l* Type *L* carriers and *b* Type *B* carriers at the LC is

$$q(l,b) = \begin{cases} \int_{\underline{r}}^{\bar{r}} b[1-F_B(x)]^{b-1} F_B(x)dx & \text{if } b>1 \\ \underline{v} + \int_{\underline{v}}^{\bar{v}} [1-F_L(x)]^{l} dx - E[R] & \text{if } l>0, b=1 \\ \int_{\underline{v}}^{\bar{v}} l[1-F_L(x)]^{l-1} F_L(x)dx & \text{if } l>1, b=0 \\ P_M - \mathrm{E}[V] & \text{if } l=1, b=0 \\ P_M - \mathrm{E}[R] & \text{if } l=0, b=1 \end{cases}$$

(4.2)

Proof: Since carriers bid their actual costs and they are paid at the second lowest bid, The expected profit of the winner, $q(l,b)$, is the difference between the second lowest bid and her actual cost, i.e. the lowest bid, when there are l Type L and b Type B carriers at the LC.

Let $r_{(1)} \le r_{(2)} \le r_{(3)} \le \ldots \le r_{(b)}$ be the order statistics defined on the actual reservation prices and $F_{(n)}(.)$ and $f_{(n)}(.)$ be the cdf and pdf of $r_{(n)}$ respectively. Then,

$$F_{(1)}(x) = 1 - P\{r_{(1)} > x\} = 1 - \left[1 - F(x)\right]^b \text{ where } F(x) = P\{r_i \le x\}$$

and

$$f_{(1)}(x) = \frac{\partial F_{(1)}(x)}{\partial x} = b\left[1 - F(x)\right]^{b-1} f(x)$$

Finally, the expected minimum price in the auction is

$$E\left[R_{(1)}\right] = \int_{\underline{r}}^{\bar{r}} x f_{(1)}(x)dx = \underline{r} + \int_{\underline{r}}^{\bar{r}} \left[1 - F_B(x)\right]^b dx \tag{4.3}$$

As a result the profit generated when there are l Type L and b Type B carriers or at least two Type B carriers at the LC equals to

$$q(l,b) = E\left[R_{(2)}\right] - E\left[R_{(1)}\right] = \int_{\underline{r}}^{\bar{r}} b\left[1 - F_B(x)\right]^{b-1} F_B(x)dx$$

When there is no Type B carrier and l Type L carriers, the profit is the same with equation (4.3) and written as

$$q(l,0) = E\left[V_{(2)}\right] - E\left[V_{(1)}\right] = \int_{\underline{v}}^{\bar{v}} l\left[1 - F_L(x)\right]^{l-1} F_L(x)dx$$

When there is just one carrier at the LC, the waiting order is taken at the market price. The profit of the carrier equals to the difference between market price and the actual cost of the carrier:

$$q(1,0) = P_M - E\left[V_i\right] \qquad\qquad q(0,1) = P_M - E\left[R_i\right]$$

Finally, when there is only one Type *B* carrier and *l* Type *L* carriers available, the profit is the difference between the minimum bid given by *l* Type *L* carriers, i.e. E[$V_{(1)}$] calculated from Equation 4.3, and the expected cost of the Type *B* carrier:

$$q(l,1) = E\left[V_{(1)}\right] - E\left[R\right] = \underline{v} + \int_{\underline{v}}^{\bar{v}} \left[1 - F_L(x)\right]^l dx - E\left[R\right]$$

Due to the random arrivals of orders and carriers, the number of carriers engage in an auction is also random. In order to determine the expected auction price and the expected profit in the long run, the steady state probabilities of the number of carriers engaging in an auction must be determined.

4.2.3. State Space Model

We model the system where carriers and orders arrive to the system according to a Poisson Process and abandon with exponentially distributed rates. The state of the system at time *t* is *S*(*t*). Let $N_S(t)$, $N_L(t)$, and $N_B(t)$ denote the number of orders, Type *L* and Type *B* carriers available at the logistics center at time *t* respectively. $N_S(t)$, $N_L(t)$, and $N_B(t)$ determine *S*(*t*), i.e. $S(t) = (N_L(t), N_B(t), N_S(t))$.

Since the arrival and abandonment times are exponential random variables, the process $\{S(t), t \geq 0\}$ is a Continuous-time Markov Chain and the steady state exists. The steady-state probabilities are defined as $\pi_{(l,b,s)} = \lim_{t \to \infty} \Pr[S(t) = (l,b,s)]$.

Since the steady state probabilities cannot be found in closed form in general, the state-space model needs to be solved numerically in order to evaluate the performance measures for a given set of system parameters.

There are $(C_L+1)(C_B+1)+C_S$ states in the state space where C_L, C_B and C_S denote the capacity of the system for Type *L* carriers, Type *B* carriers and orders respectively. Since we assume $\underline{v} > \bar{r}$, the transition from one state to another when an order comes, depends on the number of Type *B* carriers. That is if there is at least one Type *B* carrier at LC, the order is taken by this carrier. The state transition diagram is shown in Figure 4.1 and transition equations can be given as

$$\left(\lambda_L + \lambda_B + \mu_S + l\lambda_{LA} + b\lambda_{BA}\right)\pi_{(l,b,0)} = \lambda_L \pi_{(l-1,b,0)} + \lambda_B \pi_{(l,b-1,0)} + (\mu_S + (b+1)\lambda_{BA})\pi_{(l,b+1,0)} + (l+1)\lambda_{LA}\pi_{(l+1,b,0)} \quad l=1..C_L-1 \text{ and } b=1..C_B-1 \qquad (4.4)$$

When there are available capacity for both types of carriers and at least one carrier of each type, for example the state *S*(2,2,0) in Figure 4.1, the state changes with the arrival of carriers, orders and also with the abandonment of carriers.

$$\left(\lambda_L + \lambda_B + \mu_S + b\lambda_{BA}\right)\pi_{(0,b,0)} = \lambda_B \pi_{(0,b-1,0)} + (\mu_S + (b+1)\lambda_{BA})\pi_{(0,b+1,0)} + \lambda_{LA}\pi_{(1,b,0)} \quad b=1..C_B-1 \qquad (4.5)$$

$$\left(\lambda_L + \lambda_B + \mu_S + l\lambda_{LA}\right)\pi_{(l,0,0)} = \lambda_L \pi_{(l-1,0,0)} + (\mu_S + \lambda_{BA})\pi_{(l,1,0)} + (\mu_S + (l+1)\lambda_{LA})\pi_{(l+1,0,0)} \quad l=1..C_L-1 \qquad (4.6)$$

When there is no Type *L* carrier and an available capacity for Type *B* carriers provided that at least one Type *B* carrier available, for example the state *S*(0,2,0) in Figure 4.1, the states change with arrivals of both types of carriers and orders, and abandonment of Type *B* carriers. The same is valid for the situation when there is no Type *B* carriers and an available capacity for Type *L* carriers.

$$\left(\lambda_B + \mu_S + C_L\lambda_{LA} + b\lambda_{BA}\right)\pi_{(C_L,b,0)} = \lambda_L\pi_{(C_L-1,b,0)} + \lambda_B\pi_{(C_L,b-1,0)} + (\mu_S + (b+1)\lambda_{BA})\pi_{(C_L,b+1,0)}$$

b=1..C_B-1 (4.7)

$$\left(\lambda_L + \mu_S + l\lambda_{LA} + C_B\lambda_{BA}\right)\pi_{(l,C_B,0)} = \lambda_L\pi_{(l-1,C_B,0)} + \lambda_B\pi_{(l,C_B-1,0)} + (l+1)\lambda_{LA}\pi_{(l+1,C_B,0)}$$

l=1..C_L-1 (4.8)

When the capacity for Type *L* carriers is full and there is at least one Type *B* carrier, for example the state *S*(5,2,0) in Figure 4.1, the state changes with the arrival of Type *B* carriers and orders, and possible abandonment of both Type *L* and Type *B* carriers. Also, this situation is valid for the states with a full capacity for Type *B* carriers and available place for Type *L* carriers.

$$\left(\lambda_L + \lambda_B + \mu_S + l\lambda_{LA}\right)\pi_{(l,0,0)} = \lambda_L\pi_{(l-1,0,0)} + (\mu_S + \lambda_{BA})\pi_{(l,1,0)} + (\mu_S + (l+1)\lambda_{LA})\pi_{(l+1,0,0)}$$

l=1..C_L-1 (4.9)

$$\left(\lambda_L + \lambda_B + \mu_S + b\lambda_{BA}\right)\pi_{(0,b,0)} = \lambda_B\pi_{(0,b-1,0)} + \lambda_{LA}\pi_{(1,b,0)} + (\mu_S + (b+1)\lambda_{BA})\pi_{(0,b+1,0)}$$

b=1..C_B-1 (4.10)

When there is no Type *B* carrier (or Type *L* carrier) and at least one Type *L* carrier (or Type *B* carrier), for example the state *S*(2,0,0) (or *S*(0,2,0)) in Figure 4.1, the state changes with the arrivals of both types of carriers and orders, and abandonment of Type *L* carriers (or Type *B* carriers).

$$\left(\lambda_L + \lambda_B + \mu_S + s\mu_{SA}\right)\pi_{(0,0,s)} = (\lambda_L + \lambda_B + (s+1)\mu_{SA})\pi_{(0,0,s+1)} + \mu_S\pi_{(0,0,s+1)}$$

s=1..C_S-1 (4.11)

When there are waiting orders and available capacity for orders, for example the state *S*(0,0,2) in Figure 4.1, the states change with an arrival of carriers or orders or possible abandonment of waiting orders.

$$\left(\lambda_L + \lambda_B + \mu_S\right)\pi_{(0,0,0)} = (\mu_S + \lambda_{BA})\pi_{(0,1,0)} + (\mu_S + \lambda_{LA})\pi_{(1,0,0)} + (\lambda_L + \lambda_B + \mu_{SA})\pi_{(0,0,1)} \quad (4.12)$$

When the LC is empty, the state changes only with the arrivals of carriers or orders.

$$\left(\lambda_L + \mu_S + C_B \lambda_{BA}\right)\pi_{(0,C_B,0)} = \lambda_B \pi_{(0,C_B-1,0)} + \lambda_{LA}\pi_{(1,C_B,0)} \tag{4.13}$$

$$\left(\lambda_B + \mu_S + C_L \lambda_{LA}\right)\pi_{(C_L,0,0)} = \lambda_L \pi_{(C_L-1,0,0)} + (\mu_S + \lambda_{BA})\pi_{(C_L,1,0)} \tag{4.14}$$

$$\left(\mu_S + C_L \lambda_{LA} + C_B \lambda_{BA}\right)\pi_{(C_L,C_B,0)} = \lambda_L \pi_{(C_L-1,C_B,0)} + \lambda_B \pi_{(C_L,C_B-1,0)} \tag{4.15}$$

$$\left(\lambda_L + \lambda_B + \mu_S + C_S \mu_{SA}\right)\pi_{(0,0,C_S)} = \mu_S \pi_{(0,0,C_S-1)} \tag{4.16}$$

When the system capacity is full for one type of carrier, the state changes with possible abandonment and the arrival of the other type of carrier whose capacity is not full or the arrival of an order if there is a capacity for that order.

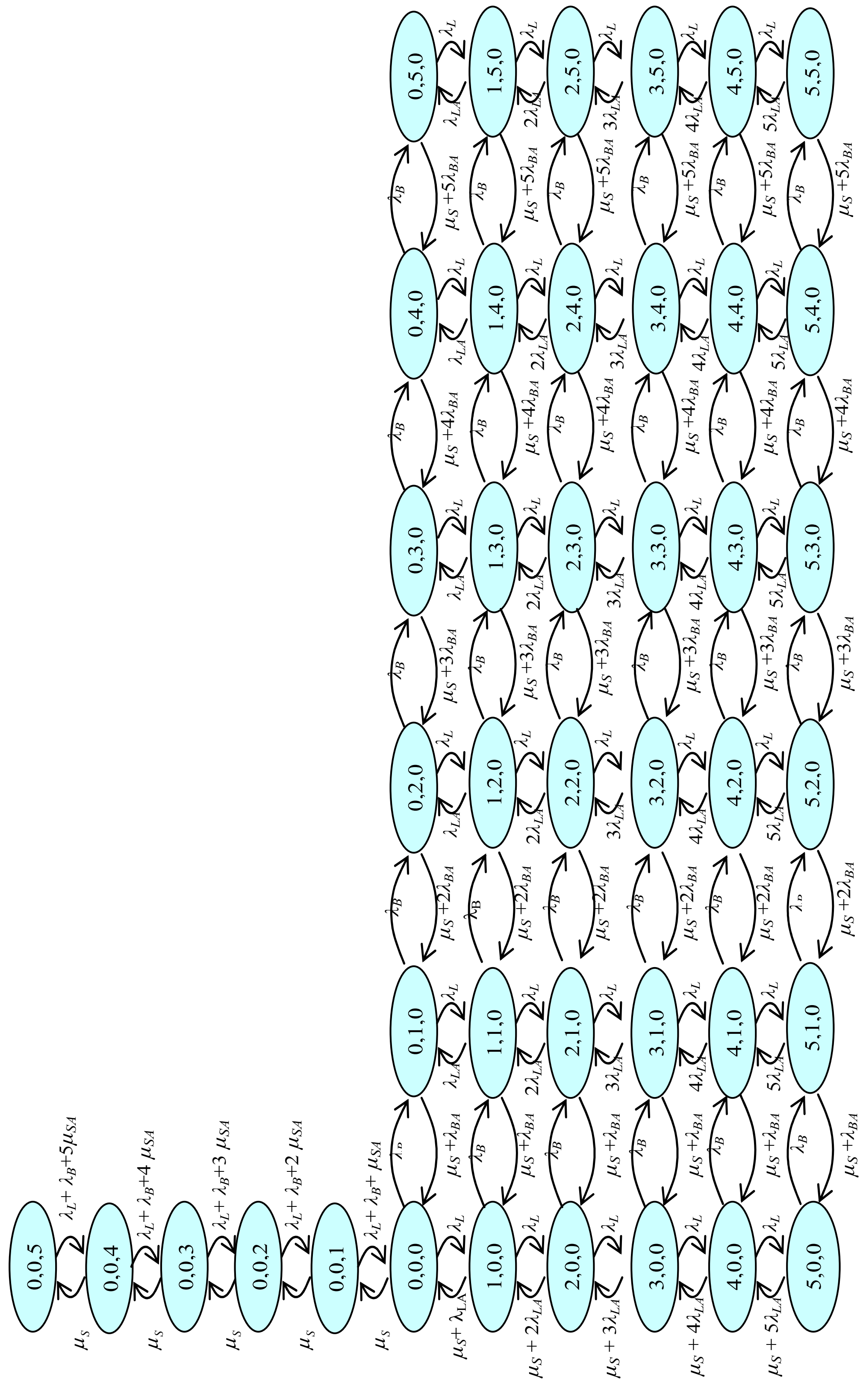

Figure 4.1. The state transition diagram

4.3. Performance Measures

We use the average expected auction price, P_{av}, as the main performance measure. The average expected auction price is the average auction price in steady state which is calculated by the summation of the product of the steady state probabilities and the average auction price realized in those states. P_{av} also determines average expected profit of the carriers, Q_{av}, which is the average profit of carriers in steady state. In addition, we determine the average number of Type L and Type B carriers waiting at the center, $\overline{N}_L$ and $\overline{N}_B$ respectively, the average number of waiting orders, $\overline{N}_S$, the probability of rejecting an order R_S, and the probability of rejecting carriers because of the capacity constraint, R_L and R_B for Type L and Type B carriers respectively in steady state.

4.3.1. Average Expected Auction Price

In Equation (4.1) the expected auction price is calculated by conditioning on the number of carriers engaging in the auction, i.e. $p(l,b) = E\left[R_{(2)} \middle| N_L(t) = l, N_B(t) = b\right]$; however, the number of carriers is also a random variable. Thus, in order to find the auction price to evaluate the performance of the system, we use the average expected auction price, P_{av}, which is calculated in the steady state as follows:

$$P_{av} = E[p(l,b)] = \sum_{b=1}^{C_B} \sum_{l=1}^{C_L} p(l,b)\text{Prob}\{N_L = l, N_B = b\} \tag{4.17}$$

We have four different situations in which the auction prices are calculated differently. First, when there are no carriers, i.e., $S(t) = (0,0,N_S(t))$, the carrier that arrives to the system takes one of the outstanding orders at a market price, i.e. $p(0,0)$, without an auction. Second, when there are only Type L carriers waiting in the LC, i.e. $S(t) = (N_L(t), 0,0)$, the arriving order is given to the winner of the auction at a price equals to p(l,0) given in Equation (4.1) where $l = N_L(t)$. Third, when there are no orders and both a number of Type *L* and Type *B* carriers are waiting, i.e., $S(t) = (N_L(t), N_B(t),0)$, an arriving order is given to the winner of an auction that has an expected price given by $p(l,b)$ in Equation (4.1) with $b = N_B(t)$. Finally, when there is only one Type *B* carrier and a number of Type *L* carriers, i.e. $S(t) = (N_L(t),1,0)$ then the winning carrier, i.e. Type *B* carrier, takes the order at a price that equals to the minimum bid given by Type *L* carriers, i.e. p(l,1). As a result the average expected auction price, P_{av}, can be written in a general form as:

$$P_{av} = \frac{\left((\lambda_L + \lambda_B)\sum_{s=1}^{C_S} \pi_{(0,0,s)} + \mu_S (\pi_{(1,0,0)} + \pi_{(0,1,0)}) \right) p(0,0)}{(\lambda_L + \lambda_B)\sum_{s=1}^{C_S} \pi_{(0,0,s)} + \mu_S \left(\sum_{l=0}^{C_L}\sum_{b=0}^{C_B} \pi_{(l,b,0)} - \pi_{(0,0,0)} \right)} + \frac{\mu_S \left(\sum_{l=1}^{C_L} \pi_{(l,1,0)} p(l,1) + \sum_{l=2}^{C_L} \pi_{(l,0,0)} p(l,0) + \sum_{l=0}^{C_L}\sum_{b=2}^{C_B} \pi_{(l,b,0)} p(l,b) \right)}{(\lambda_L + \lambda_B)\sum_{s=1}^{C_S} \pi_{(0,0,s)} + \mu_S \left(\sum_{l=0}^{C_L}\sum_{b=0}^{C_B} \pi_{(l,b,0)} - \pi_{(0,0,0)} \right)} \quad (4.18)$$

4.3.2. Average Expected Profit of Carriers

With the same setting, the average expected profit is calculated as follows: Firstly the profit of the carrier when there are waiting orders and the arriving carrier is Type *L* or only one Type *L* carrier is waiting at the LC is given by $q(1,0)$ in Equation (4.2). In addition, when the arriving carrier is Type *B* to take the waiting order or only one Type *B* carrier is waiting at the LC, the profit is calculated with $q(0,1)$. If there is only one Type *B* carrier and a number of Type *L* carriers, the profit of the carrier is the difference between the minimum bid given by Type *L* carriers and the expected bid (cost) of that carrier, i.e. $q(l,1)$. Finally, when there are only Type *L* carriers waiting, the profit equals to $q(l,0)$ given in Equation (4.2) and the profit is given with $q(l,b)$ when there are both Type *L* and Type *B* carriers waiting at the LC. We can find the average expected profit of carriers in steady state in a general form as:

$$Q_{av}=\frac{\left(\lambda_L\sum_{s=1}^{C_S}\pi_{(0,0,s)}+\mu_S\pi_{(1,0,0)}\right)q(1,0)+\left(\lambda_B\sum_{s=1}^{C_S}\pi_{(0,0,s)}+\mu_S\pi_{(0,1,0)}\right)q(0,1)}{(\lambda_L+\lambda_B)\sum_{s=1}^{C_S}\pi_{(0,0,s)}+\mu_S\left(\sum_{l=0}^{C_L}\sum_{b=0}^{C_B}\pi_{(l,b,0)}-\pi_{(0,0,0)}\right)}$$

$$+\frac{\mu_S\left(\sum_{l=1}^{C_L}\pi_{(l,1,0)}q(l,1)+\sum_{l=2}^{C_L}\pi_{(l,0,0)}q(l,0)+\sum_{l=0}^{C_L}\sum_{b=2}^{C_B}\pi_{(l,b,0)}q(l,b)\right)}{(\lambda_L+\lambda_B)\sum_{s=1}^{C_S}\pi_{(0,0,s)}+\mu_S\left(\sum_{l=0}^{C_L}\sum_{b=0}^{C_B}\pi_{(l,b,0)}-\pi_{(0,0,0)}\right)} \quad (4.19)$$

4.3.3. Average Number of Carriers and Orders

The average numbers of carriers are calculated with the summation of the products of the number of waiting carriers at the LC and their steady state probabilities. Moreover, the average number of orders is the summation of the product of the number of waiting carriers and their steady state probabilities. The average number of Type *L* carriers, $\overline{N}_L$, Type *B* carriers, $\overline{N}_B$, and orders, $\overline{N}_S$ are given as

$$\overline{N}_L=\sum_{l=0}^{C_L}\sum_{b=0}^{C_B}l\pi_{(l,b,0)}\,,\ \overline{N}_B=\sum_{l=0}^{C_L}\sum_{b=0}^{C_B}b\pi_{(l,b,0)}\,,\ \overline{N}_S=\sum_{s=1}^{C_S}\pi_{(0,0,s)}s \quad (4.20)$$

4.3.4. The Probability of Rejection of Carriers and Orders

The system rejects carriers and orders when the total capacities are reached. The probability of rejection of Type *L* carriers, K_L, Type *B* carriers, K_B, and orders, K_S in the long run are equal to

$$K_L=\sum_{b=0}^{C_B}\pi_{(C_L,b,0)}\,,\ K_B=\sum_{l=0}^{C_L}\pi_{(l,C_B,0)}\,,\ K_S=\pi_{(0,0,C_S)} \quad (4.21)$$

The average expected auction price, the average expected profit of the carriers and other performance measures are then determined by calculating the steady-state probabilities of $S(t) = (N_L(t), N_B(t), N_S(t))$ numerically. A special case with uniformly distributed cost of carriers will be analyzed analytically in Chapter 5.

4.3.5. The Proportion of Carriers that Take an Order

The arriving carriers can abandon the system after waiting some time that is exponentially distributed with λ_{LA} and λ_{BA} for Type L and Type B carriers respectively, so only a proportion of carriers that do not abandon and accepted by the system eventually awarded with an order and leave the system by taking that order. Let M_L and M_B be the proportion of Type L and Type B carriers that take an order, then M_L and M_B can be calculated as follows

$$M_L = \frac{\lambda_L - \lambda_L \sum_{b=0}^{C_B} p[C_L, b, 0] - \sum_{l=0}^{C_L} \sum_{b=0}^{C_B} l \lambda_{LA} p[l, b, 0]}{\lambda_L}$$

$$M_B = \frac{\lambda_B - \lambda_B \sum_{l=0}^{C_L} p[l, C_B, 0] - \sum_{l=0}^{C_L} \sum_{b=0}^{C_B} b \lambda_{BA} p[l, b, 0]}{\lambda_B}$$

4.4. Effects of the Parameters on Performance Measures

The arrival and abandonment rate of carriers and orders affect the performance of the system. Since Type B carriers have lower cost and mostly they determine the expected auction price, Type B carriers are the strong bidders in the system. Thus, in order to give insights about how the system parameters affect the performance of the system in terms of the average expected price, we analyze the arrivals and the abandonment of Type B

carriers. First let us define the stochastic order that will appear in the following Lemmas and the theorem.

Definition: Let *X* and *Y* be two random variables. *X* is said to be smaller than *Y* in the usual stochastic order (denoted by $X \leq_{st} Y$) if:

$$P\{X > u\} \leq P\{Y > u\} \text{ for all } u \in (-\infty, \infty).$$

Lemma 1: The expected auction price, *p*(*l*,*b*), is a decreasing function in the number of bidders joining the auction, *b*.

Proof: Let us look at the first order difference of the expected auction price;

$$p(l,b) - p_L(l,b+1) = \left(\underline{r} + \int_{\underline{r}}^{\bar{r}} [1-F_B(x)]^{b-1} bF_B(x)dx + \int_{\underline{r}}^{\bar{r}} [1-F_B(x)]^b dx \right)$$

$$- \left(\underline{v} + \int_{\underline{v}}^{\bar{v}} [1-F_L(x)]^l (l+1)F_L(x)dx + \int_{\underline{v}}^{\bar{v}} [1-F_L(x)]^{l+1} dx \right)$$

$$= \int_{\underline{v}}^{\bar{v}} [1-F_L(x)]^{l-1} lF_L(x)^2 dx \ \geq 0$$

Since $p_L(l) - p_L(l+1) \geq 0$, *p*(*l*,*b*) is a decreasing function in the number of carriers joining the auction.

The next Lemma is borrowed from Shaked and Shanthikumar [57].

Lemma 2: If $N_{L_1}(t) \geq_{st} N_{L_2}(t)$ and g is any decreasing function, then $E[g(N_{L_1})] \leq_{st} E[g(N_{L_2})]$.

Theorem 1: The average expected auction price is decreasing in the stochastically increasing number of bidders joining the auction.

Proof: From Lemma 1 the expected auction price is decreasing in the number of bidders. Since the average expected auction price that we use in this thesis is the expectation of the average auction price in steady state as given in Equation (4.17), this theorem directly follows from Lemma 2.

General Conditions:

The following conditions are similar to those proven by Bhattacharya and Ephremides for a queuing model with abandonment [58].

Condition 1: The stationary number of bidders joining the auction is stochastically increasing in the arrival rate of carriers.

Condition 2: The stationary number of bidders joining the auction is stochastically decreasing in the abandonment rate of carriers.

Condition 3: The stationary number of bidders joining the auction is stochastically decreasing in the arrival rate of orders.

Condition 4: The stationary number of bidders joining the auction is stochastically increasing in the abandonment rate of orders.

Properties: The following properties directly follow from Theorem 1 and the above conditions.

a) The average expected auction price is decreasing in the arrival rate of carriers.

b) The average expected auction price is increasing in the arrival rate of orders.

c) The average expected auction price is increasing in the abandonment rate of carriers.

d) The average expected auction price is decreasing in the abandonment rate of orders.

These properties are analyzed in Chapter 5 analytically for a special case.

The general model and the performance measures that will be used while analyzing the model are given in this chapter. The following chapter will use the performance measures to analyze the system analytically by making an assumption about the distribution of the cost of carriers.

Chapter 5

Numerical Analysis of the General Model

5.1. Overview

In this section we analyze the effects of the parameters on performance measures numerically by taking the cost distributions of the carriers as uniform, and give the simulation results of the model. First, in order to show the solution procedure, a special case is given in Section 5.2. Then the performance measures are analyzed for the general case.

Assume V_i are independent and identically distributed uniformly on $[\underline{v}, \overline{v}]$, where $\overline{v}$ is the market price for transportation and $(\overline{v} - \underline{v})$ measures the dispersion or the commonly perceived uncertainty with respect to the cost of the transportation being auctioned. In addition, assume R_i are independent and identically distributed uniformly on $[\underline{r}, \overline{r}]$. Then we can calculate all the performance measures with this information.

Firstly, let us calculate a single auction price where only Type B carriers engage in an auction, Since the cost of a Type B carrier is distributed uniformly on $[\underline{r}, \overline{r}]$, the cumulative distribution function equals to;

$$F(x) = \frac{x - \underline{r}}{\overline{r} - \underline{r}} \tag{5.1}$$

We know that the auction is a second-price auction, so the second minimum bid, i.e. second minimum of R_i, can be calculated by using Equation (4.1) and is given as follows:

$$p(l,b) = \underline{r} + \int_{\underline{r}}^{\bar{r}} \left[1 - \frac{x-\underline{r}}{\bar{r}-\underline{r}}\right]^{b-1} b \frac{x-\underline{r}}{\bar{r}-\underline{r}} dx + \int_{\underline{r}}^{\bar{r}} \left[1 - \frac{x-\underline{r}}{\bar{r}-\underline{r}}\right]^{b} dx = \underline{r} + \frac{2(\bar{r}-\underline{r})}{b+1} \tag{5.2}$$

Then, the auction price for all auctions is given as:

$$p(l,b) = \begin{cases} \underline{r} + \dfrac{2(\bar{r}-\underline{r})}{b+1} & \text{if } \quad b>1 \\ \underline{v} + \dfrac{(\bar{v}-\underline{v})}{l+1} & \text{if } \quad l>0,\ b=1 \\ \underline{v} + \dfrac{2(\bar{v}-\underline{v})}{l+1} & \text{if } \quad l>1,\ b=0 \\ \bar{v} & \text{otherwise} \end{cases} \tag{5.3}$$

As seen from Equation (5.3), the auction price decreases with the number of bidders that join an auction, and also decreases with the commonly perceived uncertainty with respect to the value of the transportation being auctioned, $\bar{r}-\underline{r}$.

The profit of a carrier in a single auction under the assumption of the uniform distribution can be given by using the Equations (4.2) and (5.1) as follows:

$$q(l,b) = \begin{cases} \dfrac{\bar{r}-\underline{r}}{b+1} & \text{if } \quad l>0,\ b>1 \\ \underline{v} + \dfrac{(\bar{v}-\underline{v})}{l+1} - E[R] & \text{if } \quad l>0,\ b=1 \\ \dfrac{\bar{v}-\underline{v}}{l+1} & \text{if } \quad l>1,\ b=0 \\ \bar{v} - E[V] & \text{if } \quad l=1,\ b=0 \\ \bar{r} - E[R] & \text{if } \quad l=0,\ b=1 \end{cases} \tag{5.4}$$

We see from Equation (5.4) that the expected profit of carriers decreases with the number of bidders that join an auction and with the commonly perceived uncertainty with respect to the value of the transportation being auctioned, $\bar{v} - \underline{v}$.

5.2. Analytical Results for a Special Case

In order to present the methodology, we first consider a simple case with only Type L carriers; no abandonment of orders and carriers; and no capacity constraint for arriving orders. That is $\lambda_B = 0$; $\mu_{SA} = 0$; $\lambda_{LA} = 0$; $C_S \to \infty$.

Since the logistics center can accommodate all the orders but a maximum of L carriers, let the state of the system be the number of outstanding orders at time t: $S(t) = N_S(t)$- $N_L(t) + C_L$, $S(t) = 0, 1,\ldots$. With this definition, the system is identical to an M/M/1 queue. As a result, the steady-state probabilities can be written as

$$\pi_i = (1-\rho)\rho^i \tag{5.5}$$

where $\rho = \dfrac{\mu_S}{\lambda_L}$.

5.2.1. Average Expected Auction Price

In this setting, we want to find the average expected price of the auction in steady state. When the state of the system is greater than C_L, i.e., $S(t) > N_S(t) + C_L$, there are no carriers at the LC, and when it is less than C_L, i.e. $S(t) =$ - $N_L(t) + C_L$ there are no orders and a number of carriers, $N_L(t)$, are waiting at the LC. As a result the average expected auction price, P_{av}, is written as

$$P_{av} = \frac{\mu_S \sum_{i=0}^{C_L-1} \pi_i p(C_L - i, 0) + \lambda_L \sum_{i=C_L+1}^{\infty} \pi_i p(0,0)}{\mu_S \sum_{i=0}^{C_L-1} \pi_i + \lambda_L \sum_{i=C_L+1}^{\infty} \pi_i}$$

$$= \frac{\mu_S (1-\rho) \sum_{i=0}^{C_L-1} \rho^i \left(\underline{v} + \frac{2(\bar{v} - \underline{v})}{C_L - i + 1} \right) + \lambda_L \rho^{C_L+1} \bar{v}}{\mu_S (1-\rho^{C_L}) + \lambda_L \rho^{C_L+1}}$$

$$= \frac{\mu_S \left(\underline{v}(1-\rho^{C_L}) + 2(\bar{v} - \underline{v})(1-\rho) \sum_{i=0}^{C_L-1} \frac{\rho^i}{C_L - i + 1}) \right) + \lambda_L \bar{v} \rho^{C_L+1}}{\mu_S (1-\rho^{C_L}) + \lambda_L \rho^{C_L+1}} \tag{5.6}$$

We calculate the average expected auction price for different values of capacities, rates and dispersion of the cost of the order, and also simulate the system. We take $\bar{v}$ and P_M as 100 euro and change the $\underline{v}$. Table 5.1 reports the comparison of simulation and analytical results for given parameters. The simulation results validate the analytical results.

C_L	ρ	$\bar{v}$ - $\underline{v}$ (YTL)	P_{av} (YTL) Model	Simulation Average	Simulation 95% C.I.	ρ	P_{av} (YTL) Model	Simulation Average	Simulation 95% C.I.
5	1/2	0	100	100	(100 100)	2/3	100	100	(100 100)
10	1/2	0	100	100	(100 100)	2/3	100	100	(100 100)
20	1/2	0	100	100	(100 100)	2/3	100	100	(100 100)
50	1/2	0	100	100	(100 100)	2/3	100	100	(100 100)
5	1/2	10	94.3334	94.3369	(94.326 94.348)	2/3	95.3745	95.3723	(95.359 95.386)
10	1/2	10	92.0678	92.0686	(92.063 92.075)	2/3	92.5225	92.5263	(92.518 92.535)
20	1/2	10	91.0061	91.0056	(91.003 91.009)	2/3	91.0828	91.0876	(91.084 91.091)
50	1/2	10	90.4003	90.4000	(90.399 90.401)	2/3	90.4093	90.4096	(90.408 90.411)
5	1/2	20	88.6667	88.6649	(88.643 88.687)	2/3	90.7489	90.7406	(90.714 90.768)
10	1/2	20	84.1356	84.1296	(84.118 84.142)	2/3	85.045	85.0689	(85.051 85.086)
20	1/2	20	82.0123	82.0151	(82.009 82.021)	2/3	82.1656	82.1646	(82.158 82.171)
50	1/2	20	80.8007	80.8001	(80.798 80.803)	2/3	80.8186	80.8174	(80.815 80.820)

Table 5.1. The average expected prices of the auctions for the special case

As seen in Table 5.1, the capacity of the system for carriers, the proportion of the arrival rate of carriers and orders, i.e. ρ, and the deviation of the cost of carriers from the market price affect the average expected auction price in steady state. When the capacity of the LC for carriers increases, the average expected auction price decreases because of the number of carriers that join in the auction increases. Moreover, an increase in the arrival rate of orders or a decrease in the arrival rate of carriers, which causes an increase in ρ, increases the average expected auction price. When the deviation of the cost of carriers from the market price increases, the distribution range of the cost of carriers become larger which in turn cause a decrease in the average expected price.

5.2.1.2. Average Expected Profit of Carriers

For the same setting, we can find the average expected profit of carriers in steady state as

$$Q_{av} = \frac{\mu_S \sum_{i=0}^{C_L-1} \pi_i q(C_L - i, 0) + \lambda_L \sum_{i=C_L+1}^{\infty} \pi_i q(1,0)}{\mu_S \sum_{i=0}^{C_L-1} \pi_i + \lambda_L \sum_{i=C_L+1}^{\infty} \pi_i}$$

$$= \frac{\mu_S (1-\rho) \sum_{i=0}^{C_L-1} \rho^i \frac{\bar{v} - \underline{v}}{C_L - i + 1} + \lambda_L \rho^{C_L+1} \bar{v} - \lambda_L (1-\rho) \sum_{i=C_L+1}^{\infty} \rho^i (\bar{v} - \frac{\bar{v} - \underline{v}}{2}))}{\mu_S (1-\rho^{C_L}) + \lambda_L \rho^{C_L+1}}$$

$$= \frac{\mu_S (1-\rho)(\bar{v} - \underline{v}) \sum_{i=0}^{C_L-1} \frac{\rho^i}{C_L - i + 1} + \lambda_L \rho^{C_L+1} \frac{\bar{v} - \underline{v}}{2}}{\mu_S (1-\rho^{C_L}) + \lambda_L \rho^{C_L+1}} \tag{5.7}$$

We calculate the average expected profit of the carriers for different values of capacities, rates and dispersion of the cost of the order. We again take $\bar{v}$ and P_M as 100 euro and change $\underline{v}$. The analytical results are given in Table 5.2 together with the simulation results.

C_L	ρ	$\bar{v}-\underline{v}$ (YTL)	Q_{av} (YTL) Model	Simulation Average	Simulation 95% C.I.	ρ	Q_{av} (YTL) Model	Simulation Average	Simulation 95% C.I.
5	1/2	0	0	0	(0 0)	2/3	0	0	(0 0)
10	1/2	0	0	0	(0 0)	2/3	0	0	(0 0)
20	1/2	0	0	0	(0 0)	2/3	0	0	(0 0)
50	1/2	0	0	0	(0 0)	2/3	0	0	(0 0)
5	1/2	10	2.1667	2.1631	(2.155 2.172)	2/3	2.6872	2.6789	(2.166 2.163)
10	1/2	10	1.0339	1.0330	(1.029 1.037)	2/3	1.2613	1.2621	(1.261 1.263)
20	1/2	10	0.503	0.5029	(0.501 0.505)	2/3	0.5414	0.5416	(0.540 0.542)
50	1/2	10	0.2002	0.2002	(0.199 0.201)	2/3	0.2047	0.2062	(0.204 0.208)
5	1/2	20	4.3334	4.3347	(4.318 4.352)	2/3	5.3744	5.3674	(5.359 5.375)
10	1/2	20	2.0678	2.0656	(2.057 2.074)	2/3	2.5225	2.5371	(2.530 2.547)
20	1/2	20	1.0062	1.0081	(1.004 1.012)	2/3	1.0828	1.0833	(1.080 1.086)
50	1/2	20	0.4003	0.4019	(0.400 0.404)	2/3	0.4093	0.4113	(0.408 0.414)

Table 5.2. The average expected profits of carriers for the special case

As seen in Table 5.2, like the average expected auction price, the average expected profit of carriers is decreasing in the capacity of the system for carriers. In addition, when the proportion of the arrival rate of carriers and orders, i.e. ρ, increases, the profit of carriers increases because of an increase in the average price in the long run. An increase in the deviation of the cost of carriers from the market price increases the profit of the carriers even though the average price of the auction increases because the decrease in the expected cost of carriers is more than an increase in the average price.

5.2.1.3. The Average Number of Carriers and Orders

Carriers wait at the logistics center when the outstanding order is less than L, i.e. $S(t) < L$. Then, the average number of carriers waiting at the center can be given as

$$\overline{N}_L = \sum_{i=0}^{C_L} \pi_i (C_L - i) = \sum_{i=0}^{C_L} (1-\rho)\rho^i (C_L - i) = C_L(1-\rho^{C_L+1}) + (C_L+1)\rho^{C_L+1} - \frac{\rho(1-\rho^{C_L+1})}{1-\rho}$$

(5.8)

When there is no carrier at the center, i.e. $S(t) > L$, orders wait. The average number of waiting orders equals

$$\overline{N}_S = \sum_{i=C_L}^{\infty} \pi_i (i - C_L) = \sum_{i=C_L}^{\infty} (1-\rho)\rho^i (i - C_L) = \frac{\rho^{C_L+1}}{1-\rho} \quad (5.9)$$

Average number of carriers and orders are calculated for different values of ρ and capacities of carriers, C_L. The analytical and simulation results are given in Table 5.3.

C_L	ρ	$\overline{N}_L$		$\overline{N}_S$	
		Model	***Simulation***	***Model***	***Simulation***
5	1/2	4.0312	4.0301	0.0313	0.0303
10	1/2	9.0009	8.9930	0.001	0.0012
20	1/2	19.0000	19.0187	0.0000	0.0000
50	1/2	49.0000	48.9939	0.0000	0.0000
5	1/2	4.6670	4.6640	0.0003	0.0004
10	1/2	9.6667	9.6713	0.0000	0.0000
20	1/2	19.6667	19.6677	0.0000	0.0000
50	1/2	49.6667	49.6634	0.0000	0.0000
5	2/3	3.2634	3.2759	0.2634	0.2617
10	2/3	8.0347	8.0549	0.0347	0.0331
20	2/3	18.0006	17.9916	0.0012	0.0014
50	2/3	48.0000	48.0110	0.0000	0.0000

Table 5.3. The average number of carriers and orders for the special case

The capacity of the system for carriers is the main parameter that affects the average number of carriers and orders in steady state. Since the arrival rate of carriers is always greater than the arrival rate of orders, when the capacity of the system for carriers increases, the average number of carriers also increases.

5.2.1.4. The Probability of Rejection of Carriers and Orders

The system rejects carriers when the total capacity for carriers is reached which means the time when the outstanding order equals to 0. This probability in the long run is equal to

$$M_L = \pi_0 = 1 - \rho \tag{5.10}$$

In this special case we assume that the system has no capacity for orders; however, when there is a capacity, say C_S, the probability of rejection of orders can be calculated as

$$M_S = \pi_{C_L + C_S} = (1 - \rho)\rho^{(C_L + C_S)} \tag{5.11}$$

The only parameter that affects the probability of rejection of carriers is the proportion of the arrival rate of carriers and orders, i.e. ρ. When ρ increases, i.e. the arrival rate of carriers decreases, the probability of rejection of carriers decreases. However, the probability of rejection of orders depends on ρ, and the capacity of the system for carriers and orders.

5.2.1.5. The Proportion of Carriers that Take an Order

Since for this special case the carriers cannot abandon the system, all the carriers that arrive the system take orders and thus the proportion of carriers that take an order equals to 1.

5.3. General Model

Since the closed form stationary distributions are not available for the general case, by solving linear Equations (4.5) – (4.16) numerically, we calculate the stationary probabilities of states. Then by using Equations (4.17) – (4.20), we derive the performance measures with the assumption of uniform distribution. We give the performance measures of the model for different values of rates which are given as different cases in Table 5.4.

Case	μ_S	λ_B	λ_L	μ_{SA}	λ_{BA}	λ_{LA}
1	**1**	2	4	1.5	1	0.5
2	**2**	2	4	1.5	1	0.5
3	**4**	2	4	1.5	1	0.5
4	**8**	2	4	1.5	1	0.5
5	3	**1**	4	1.5	1	0.5
6	3	**2**	4	1.5	1	0.5
7	3	**4**	4	1.5	1	0.5
8	3	**8**	4	1.5	1	0.5
9	3	2	**1**	1.5	1	0.5
10	3	2	**2**	1.5	1	0.5
11	3	2	**4**	1.5	1	0.5
12	3	2	**8**	1.5	1	0.5
13	3	2	4	**0.25**	1	0.5
14	3	2	4	**0.5**	1	0.5
15	3	2	4	**1**	1	0.5
16	3	2	4	**2**	1	0.5
17	3	2	4	1.5	**0.25**	0.5
18	3	2	4	1.5	**0.5**	0.5
19	3	2	4	1.5	**1**	0.5
20	3	2	4	1.5	**2**	0.5
21	3	2	4	1.5	1	**0.25**
22	3	2	4	1.5	1	**0.5**
23	3	2	4	1.5	1	**1**
24	3	2	4	1.5	1	**2**

Table 5.4. The analyzed cases for the numerical examples of the general model

5.3.1. Average Expected Auction Price

The analytical results of average expected auction price and the result of simulation are given in Table 5.5 We take C_B = 5, C_L = 5 and C_S = 5 for all experiments for the general case and the cost of carriers have a uniform distribution where $\underline{v}$ = 90, $\bar{v}$ = 100, $\underline{r}$ = 70, $\bar{r}$ = 80, and P_M = 100 while calculating the performance measures. As seen in Table 5.5, the simulation results validate the analytical results.

Case	***P_{av}* (YTL)**		
	Model	***Simulation***	
		Average	***95% C.I.***
1	86.8158	86.8199	(86.7376 86.9022)
2	89.7265	89.7163	(89.6964 89.7361)
3	93.7508	93.7531	(93.7405 93.7656)
4	98.3035	98.3037	(98.2795 98.3279)
5	94.9411	94.9545	(94.9219 94.9871)
6	91.9208	91.9219	(91.9076 91.9362)
7	85.1962	85.2003	(85.1833 85.2172)
8	77.2961	77.2912	(77.2766 77.3057)
9	95.2993	95.2900	(95.2713 95.3088)
10	93.9171	93.9166	(93.8992 93.9340)
11	91.9208	91.9219	(91.9076 91.9362)
12	90.6867	90.6858	(90.6739 90.6977)
13	92.1477	92.1400	(92.1255 92.1546)
14	92.0826	92.0654	(92.0505 92.0802)
15	91.9879	91.9941	(91.9386 92.0495)
16	91.8695	91.8817	(91.8664 91.8970)
17	89.0779	89.0692	(89.0484 89.0900)
18	90.3576	90.3486	(90.3288 90.3683)
19	91.9208	91.9219	(91.9076 91.9362)
20	93.4957	93.4987	(93.4855 93.5119)
21	91.3351	91.3340	(91.3188 91.3491)
22	91.9208	91.9219	(91.9076 91.9362)
23	92.9154	92.9251	(92.9103 92.9398)
24	94.1010	94.0969	(94.0803 94.1134)

Table 5.5. The average expected prices and profits for the analyzed cases of the general model

Since we measure the performance of the system by average expected values in steady state, the arrival and abandonment rates of the carriers and orders have great importance. In order to determine the effects on the average expected price, firstly we calculate the reductions in average price with respect to market price, and give them in Table 5.6.

μ_S	λ_B	λ_L	μ_{SA}	λ_{BA}	λ_{LA}	**$(P_M$-$P_{av})$ $/P_M$*100**
1	2	4	1.5	1	0.5	13.1842
2	2	4	1.5	1	0.5	10.2735
4	2	4	1.5	1	0.5	6.2492
8	2	4	1.5	1	0.5	1.6965
3	**1**	4	1.5	1	0.5	5.0589
3	**2**	4	1.5	1	0.5	8.0792
3	**4**	4	1.5	1	0.5	14.8038
3	**8**	4	1.5	1	0.5	22.7039
3	2	**1**	1.5	1	0.5	4.7007
3	2	**2**	1.5	1	0.5	6.0829
3	2	**4**	1.5	1	0.5	8.0792
3	2	**8**	1.5	1	0.5	9.3133
3	2	4	**0.5**	1	0.5	7.9174
3	2	4	**1**	1	0.5	8.0121
3	2	4	**1.5**	1	0.5	8.0792
3	2	4	**2**	1	0.5	8.1305
3	2	4	1.5	**0.5**	0.5	9.6424
3	2	4	1.5	**1**	0.5	8.0792
3	2	4	1.5	**1.5**	0.5	7.1401
3	2	4	1.5	**2**	0.5	6.5043
3	2	4	1.5	1	**0.5**	8.0792
3	2	4	1.5	1	**1**	7.0846
3	2	4	1.5	1	**1.5**	6.3842
3	2	4	1.5	1	**2**	5.8990

Table 5.6. The percentage reduction in the average expected price of the auction for the general model

As seen in Table 5.5 and Figure 5.1, the average expected auction price increases with an increase in the arrival rate of orders. This is the case that we expect, as it is stated in Subsection 4.4, when more orders arrive to the system, the number of carriers that give bids to an auction decreases which in turn cause an increase in the price. In contrast, the auction price decreases when the arrival rates of carriers increase. Since the transportation cost of Type *B* carriers are less than the cost of Type *L* carriers, as seen from Figure 5.2, the reduction in the average expected price is realized much more with an increase in the arrival rate of Type *B* carriers than an increase in the arrival rate of Type *L* carriers.

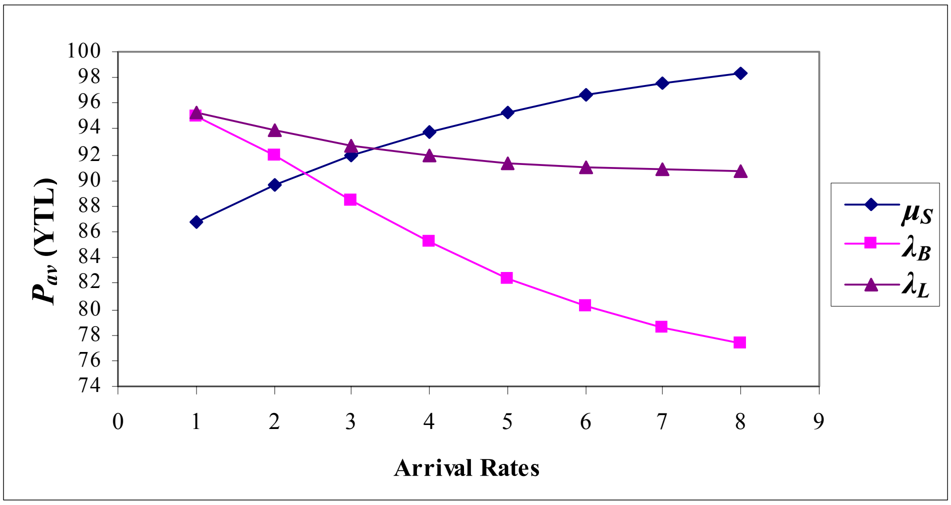

Figure 5.1. The effect of arrival rates on the average expected auction price

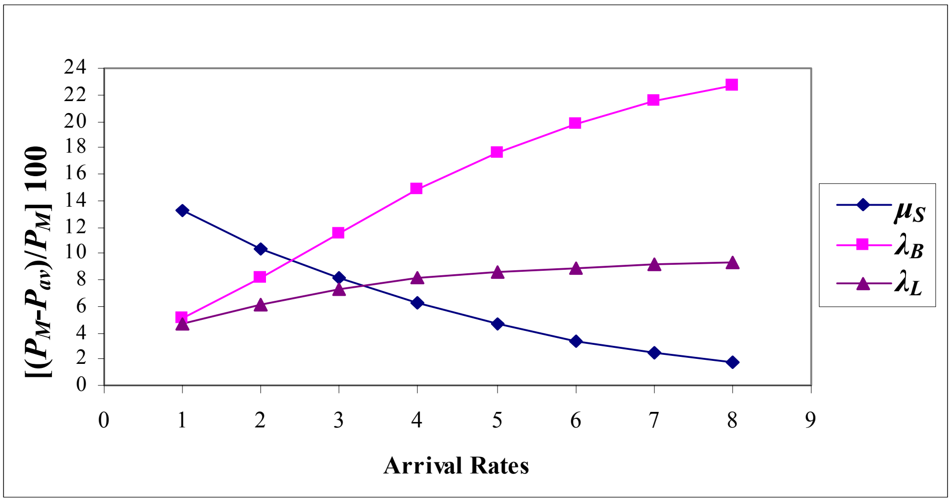

Figure 5.2. The effect of arrival rates on the reduction of the average expected auction price

Abandonment rates also affect the average expected price. Figures 5.3 and 5.4 show the effect of abandonment rates on the average expected price.

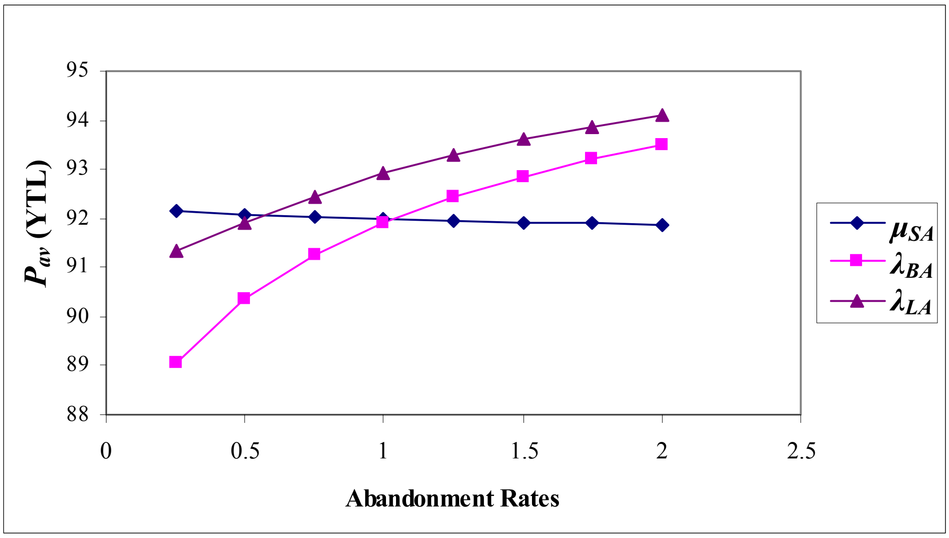

Figure 5.3. The effect of abandonment rates on the average expected auction price

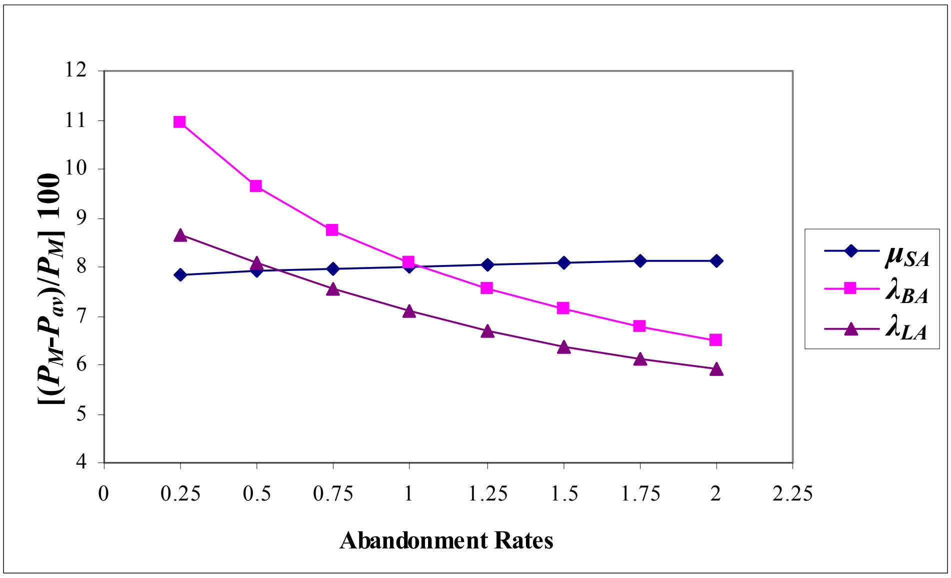

Figure 5.4. The effect of abandonment rates on the reduction of the average expected auction price

When the abandonment rate of the orders increase, since the waiting orders are taken at a market price by the first arriving carrier, the number of orders taken at the market price decreases. As a result the average expected price decreases with an increase in the abandonment rate of the orders; however the decrease rate is very small because the number of waiting carriers in steady state is very small. Conversely, as it is seen in the Figure 5.3 and Figure 5.4, the average expected price is increasing with the abandonment rates of the carriers because the number of carriers joining the auction decreases.

The capacities of the system for carriers and orders have different effects on performance measures. In order to analyze the effect of the capacity of the system for carriers and orders, we calculate the average expected auction price for different values of capacities. We take $\mu_S = 3$, $\lambda_B = 2$, $\lambda_L = 4$, $\mu_{SA} = 1.5$, $\lambda_B = 1$, $\lambda_L = 0.5$ for all experiments. Table 5.7 shows the parameters and the analytical and simulation results of average expected price in steady state.

C_B	C_L	C_S	P_{av} (YTL)		
			Model	**Simulation**	
				Average	**95% C.I.**
5	5	5	91.9208	91.9246	(91.9106 91.9385)
10	5	5	91.9067	91.9063	(91.8923 91.9203)
15	5	5	91.9067	91.8963	(91.8818 91.9108)
20	5	5	91.9067	91.9132	(91.8987 91.9277)
5	**5**	5	91.9208	91.9114	(91.8963 91.9264)
5	**10**	5	91.0940	91.0989	(91.0835 91.1143)
5	**15**	5	91.0259	91.0339	(91.0189 91.0488)
5	**20**	5	91.0243	91.0265	(91.0116 91.0415)
5	5	**5**	91.9208	91.9239	(91.9095 91.9384)
5	5	**10**	91.9212	91.8446	(91.8212 91.8679)
5	5	**15**	91.9212	91.8388	(91.8170 91.8606)
5	5	**20**	91.9212	91.8285	(91.8045 91.8525)

Table 5.7. The average expected auction price for different capacities of carriers

As seen in Table 5.7 and Figure 5.5 when the capacities for carriers are increased, the average expected price decreases because the number of carriers engaging in an auction increases. Since the arrival rate of Type *L* carriers is greater than both Type *B* carriers and orders, and the arrival rate of Type *B* carriers is less than the arrival rate of orders, after increasing the capacity for Type *B* carriers up to a point it does not have any effect on the average expected price; however, the average expected price continues to decrease with an increase in the capacity of Type *L* carriers.

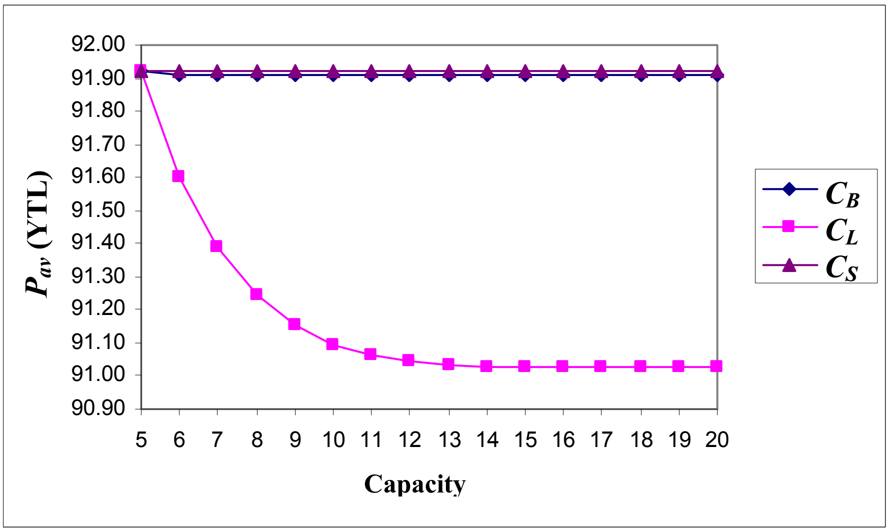

Figure 5.5. The effect of the capacities of carriers on the average expected auction price

In addition, the decrease in the average expected price is realized much more with an increase in the capacity of Type *L* carriers because the average number of Type *L* carriers become larger which makes the number of carriers engaging in an auction increase. Moreover, since the arrival rate of orders is less than the arrival rate of carriers, the average number of waiting carriers is not affected considerably with an increase in the capacity of orders.

5.3.2. Average Expected Profit of Carriers

The expected average profit of carriers also cannot be calculated in closed form, so we calculate them numerically. The analytical results of average expected profit of carriers and results of simulation are given in Table 5.8. As it is seen from Table 5.8, the simulation results validate the analytical results.

Case	**Q_{av} (YTL)**		
	Model	***Simulation***	
		Average	***95% C.I.***
1	7.1988	7.1909	(7.1794 7.2023)
2	7.3126	7.3094	(7.2995 7.3194)
3	7.9909	7.9855	(7.9762 7.9948)
4	10.3863	10.3816	(10.3731 10.3901)
5	6.4886	6.4814	(6.4732 6.4896)
6	7.5271	7.5232	(7.5148 7.5317)
7	6.7448	6.7419	(6.7335 6.7503)
8	3.7696	3.7689	(3.7612 3.7766)
9	14.1584	14.1487	(14.1315 14.1659)
10	10.4235	10.4171	(10.4052 10.4290)
11	7.5271	7.5232	(7.5148 7.5317)
12	6.4542	6.4506	(6.4439 6.4574)
13	7.6434	7.6479	(7.6392 7.6565)
14	7.61	7.6023	(7.5940 7.6106)
15	7.5615	7.5678	(7.5596 7.5759)
16	7.5001	7.5027	(7.4940 7.5114)
17	7.1294	7.1237	(7.1150 7.1324)
18	7.3725	7.3723	(7.3635 7.3811)
19	7.5271	7.5232	(7.5148 7.5317)
20	7.4246	7.4218	(7.4122 7.4314)
21	7.0191	7.0176	(7.0098 7.0253)
22	7.5271	7.5232	(7.5148 7.5317)
23	8.4779	8.4821	(8.4722 8.4921)
24	9.7946	9.7974	(9.7869 9.8079)

Table 5.8. The average expected profits of carriers for the analyzed cases of the general model

Like the average expected price, arrival rates affect the average profit of the carriers. Figure 5.6 shows the change in the average profit of carriers with changes in the arrival rates. As it is expected, when the arrival rate of the orders increase the average expected profit increases because of the increase in the average price. Moreover, an increase in the arrival rate of the Type *L* carrier decreases the average expected profit. However, the effect of the arrival rate of Type *B* carriers is different. First, an increase in the arrival rate of Type *B* carriers increases the average profit; however, after the arrival rate of the Type *B* carriers become larger than the arrival rate of the orders, the average expected profit begin to decrease. This is predictable since the transportation cost of Type *B* carriers are less than the other carriers, even the auction price decreases the average expected profit of the carriers increases up to the point where the number of Type *B* carriers joining an auction become larger, and then begin to decrease because of an increase in the competition between Type *B* carriers.

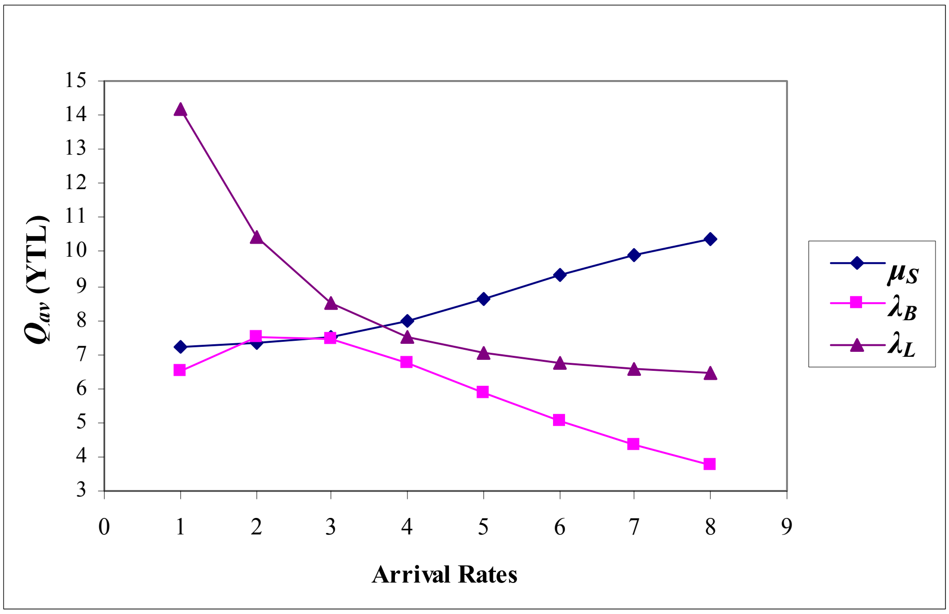

Figure 5.6. The effect of arrival rates on the average expected profit of the carriers

Moreover, the average expected profit of the carriers is affected with the changes in the abandonment rates of carriers and orders. As seen in Figure 5.7, since the average expected price decreases slightly with the abandonment rate of orders, the average profit also decreases slightly. Like the average expected price, an increase in the abandonment rate of the Type *B* carrier increases the average profit initially, and then decreases the profit.

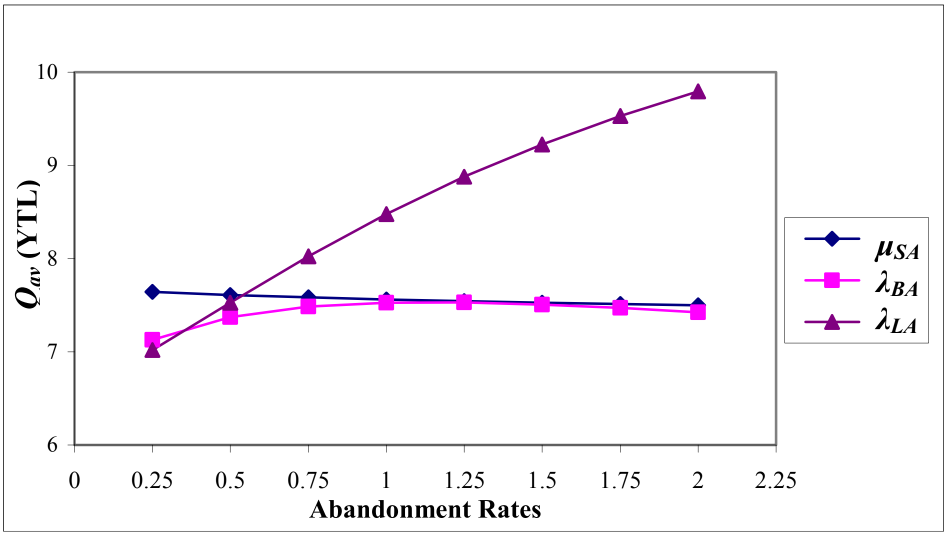

Figure 5.7. The effect of the abandonment rates on the average expected profit of the carriers

In addition, in order to analyze the effect of the capacity of the system for carriers and orders, we calculate the average expected profit of carriers for different values of capacities. We take $\mu_S = 3$, $\lambda_B = 2$, $\lambda_L = 4$, $\mu_{SA} = 1.5$, $\lambda_B = 1$, $\lambda_L = 0.5$ for all experiments. Table 5.9 shows the parameters and the analytical and simulation results of average expected profit of carriers in steady state.

C_B	C_L	C_S	Q_{av} (YTL)		
			Model	***Simulation***	
				Average	***95% C.I.***
5	5	5	7.5271	7.5308	(7.5218 7.5399)
10	5	5	7.5226	7.5179	(7.5085 7.5273)
15	5	5	7.5226	7.5182	(7.5098 7.5266)
20	5	5	7.5226	7.5176	(7.5080 7.5273)
5	**5**	5	7.5271	7.5212	(7.5125 7.5300)
5	**10**	5	6.9129	6.9085	(6.9004 6.9166)
5	**15**	5	6.8652	6.8589	(6.8512 6.8665)
5	**20**	5	6.8641	6.8689	(6.8601 6.8778)
5	5	**5**	7.5271	7.5239	(7.5156 7.5323)
5	5	**10**	7.5274	7.4909	(7.4784 7.5033)
5	5	**15**	7.5274	7.4898	(7.4787 7.5009)
5	5	**20**	7.5274	7.4801	(7.4660 7.4941)

Table 5.9. The average expected auction prices and the profits of carriers for different capacities of carriers

As it is seen in Table 5.9 and Figure 5.8, an increase in the capacity of Type *L* carriers causes a decrease in the average expected profit of carriers because the average expected price decreases. The increase in the capacity of Type *B* carriers slightly decreases the average expected profit of carriers, and the increase in the capacity of orders slightly increases the average expected profit of carriers because of the same reason that are stated in the previous section.

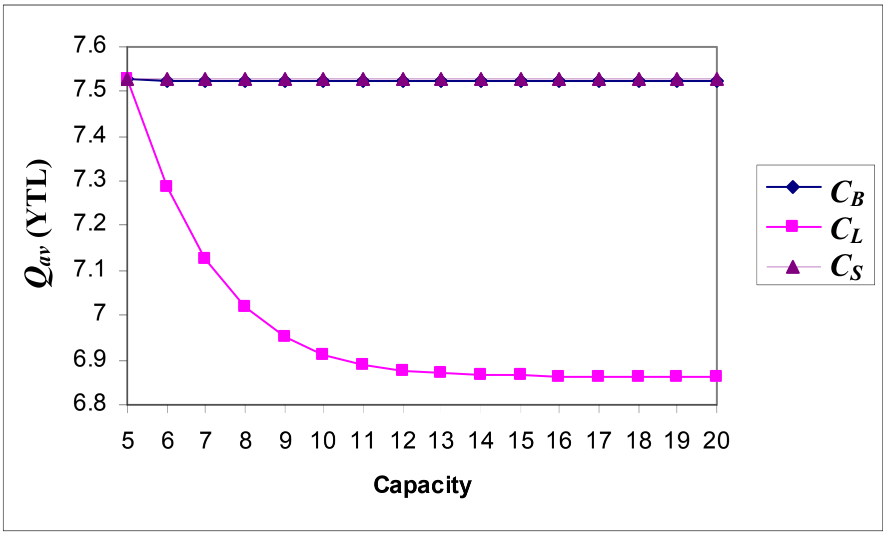

Figure 5.8. The effect of the capacities of carriers on the average expected profit of carriers in steady state

5.3.3. The Average Number of Carriers and Orders

The average number of carriers and orders in steady state are calculated, and the analytical and simulation results are given for the cases in Table 5.10.

Case	$\overline{N}_B$		$\overline{N}_L$		$\overline{N}_S$	
	Model	Simulation	Model	Simulation	Model	Simulation
1	1.2860	1.2854	3.9866	3.9874	0.0006	0.0006
2	0.8959	0.8965	3.5825	3.5826	0.0078	0.0078
3	0.4782	0.4781	2.3333	2.3331	0.1403	0.1402
4	0.1172	0.1171	0.4947	0.4949	1.3617	1.3607
5	0.2715	0.2714	2.5079	2.5055	0.0969	0.0968
6	0.6502	0.6503	3.0013	3.0006	0.0424	0.0425
7	1.6213	1.6212	3.6567	3.6562	0.0076	0.0076
8	3.2182	3.2185	4.0805	4.0810	0.0003	0.0003
9	0.4754	0.4755	0.5019	0.5029	0.4823	0.4819
10	0.5726	0.5725	1.3808	1.3850	0.2266	0.2261
11	0.6502	0.6503	3.0013	3.0022	0.0424	0.0422
12	0.6688	0.6694	4.1929	4.1917	0.0027	0.0027
13	0.6412	0.6426	2.9595	2.9587	0.0744	0.0744
14	0.6438	0.6439	2.9716	2.9706	0.0641	0.0640
15	0.6476	0.6472	2.9891	2.9875	0.0508	0.0507
16	0.6522	0.6518	3.0106	3.0104	0.0367	0.0367
17	1.0575	1.0599	3.2636	3.2643	0.0303	0.0304
18	0.8643	0.8651	3.1557	3.1555	0.035	0.035
19	0.6502	0.6503	3.0013	3.0006	0.0424	0.0425
20	0.4527	0.4526	2.8053	2.8051	0.0537	0.0536
21	0.6587	0.6580	3.5726	3.5715	0.0245	0.0245
22	0.6502	0.6503	3.0013	3.0006	0.0424	0.0425
23	0.6324	0.6325	2.1388	2.1385	0.0803	0.0801
24	0.6046	0.6046	1.2677	1.2688	0.1394	0.1393

Table 5.10. The average number of carriers and orders for the analyzed cases of the general model

The arrival rates of carriers and orders affect the average number of carriers and orders in steady state. Figure 5.9 and Figure 5.10 show the changes in the average number of Type *B* and Type *L* carriers respectively and Figure 5.11 shows the change in the average number of orders with changes in the arrival rates.

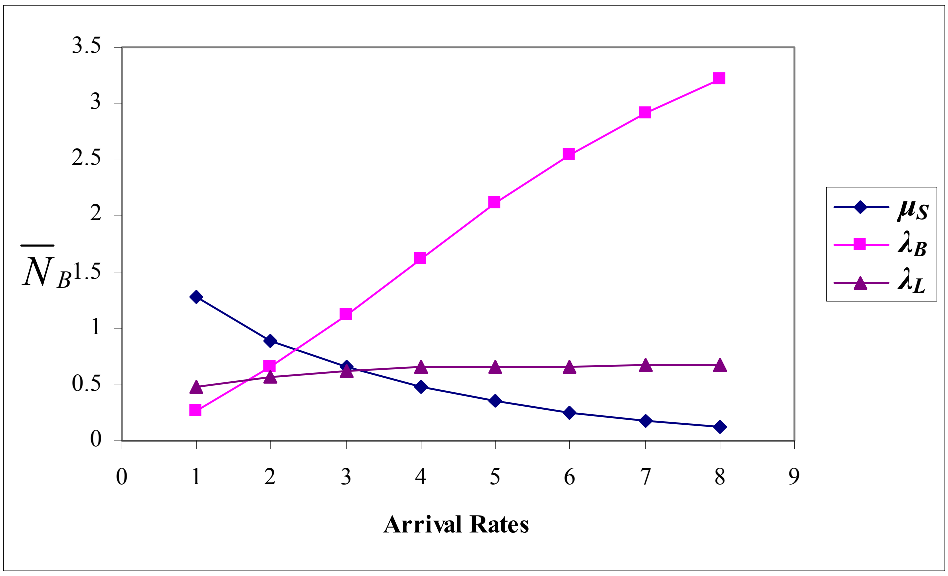

Figure 5.9. The effect of arrival rates on the average number of Type *B* carriers in steady state

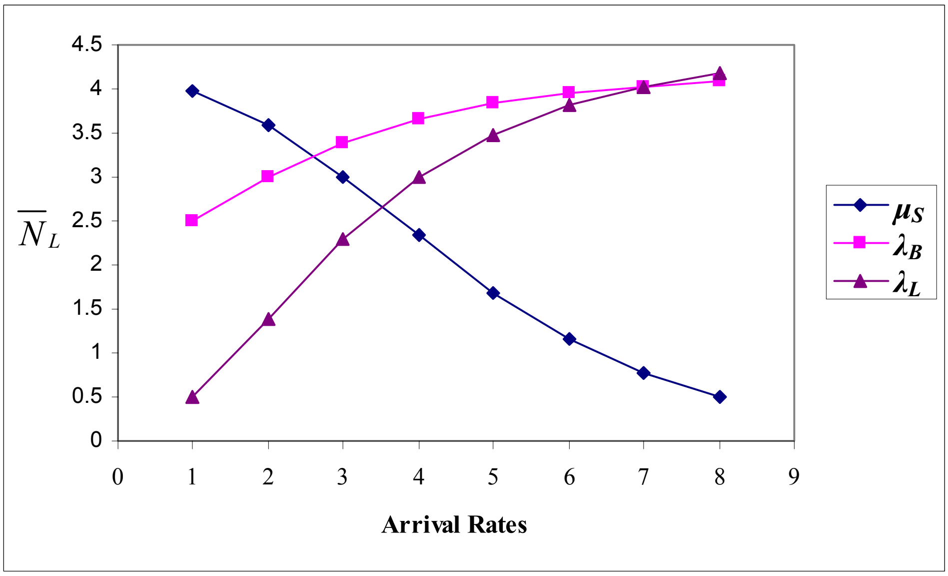

Figure 5.10. The effect of arrival rates on the average number of Type *L* carriers in steady state

As it is seen in Figure 5.9 and 5.10, when the arrival rate of the orders increase the average number of both Type *B* and Type *L* carriers decreases because of the increase in the number of auctions opened each of which ends up with a decrease in the number of carriers. An increase in the arrival rate of Type *B* carriers causes an increase in both the average number of Type *B* and Type *L* carriers in steady state. The number of Type *B* carriers increases because the number of Type *B* carriers arrive the system in a unit time increases. The reason of an increase in Type *L* carriers is the number of auctions that end up with a wining by a Type *L* carrier decreases. In addition, even Type B carriers have priority in winning the auctions an increase in the arrival rate of the Type *L* carrier cause a very small increase in the average number of Type *B* carriers. This is because when the arrival rate of Type *L* carriers increase the probability of taking a waiting order by a Type *L* carrier increases.

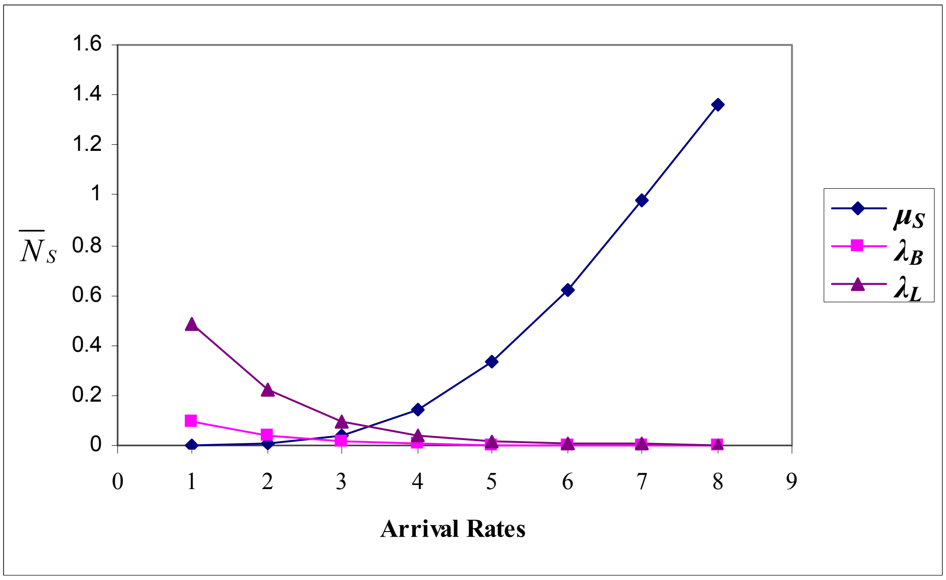

Figure 5.11. The effect of arrival rates on the average number of orders in steady state

An increase in the arrival rate of orders cause an increase in the average number of orders in steady state. As seen in Figure 5.11, when the arrival rate of orders become larger than the arrival rates of carriers, the average number of orders significantly increases. Moreover, an increase in the arrival rate of carriers decreases the average number of waiting orders.

In addition, the abandonment rates affect the average number of carriers and orders. As it is seen from Figures 5.12 and 5.13, the arrival rate of orders does not have any effect on the average number of both Type *B* and Type *L* carriers because the orders abandon when there are no carriers which has no effect on the waiting carriers. Also, when the abandonment rates of carriers increase, the average numbers of carriers decrease because of an increase in the abandonment of carriers.

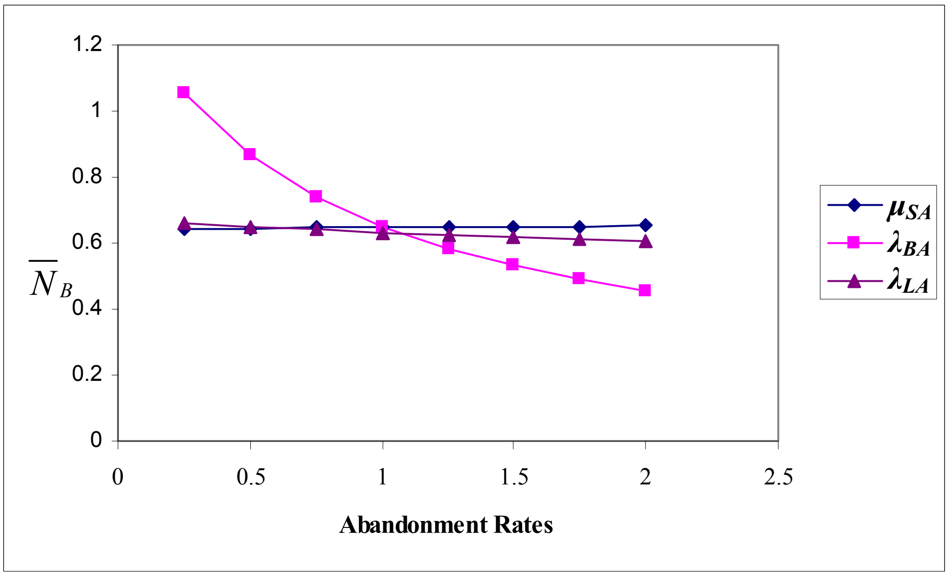

Figure 5.12. The effect of abandonment rates on the average number of Type *B* carriers in steady state

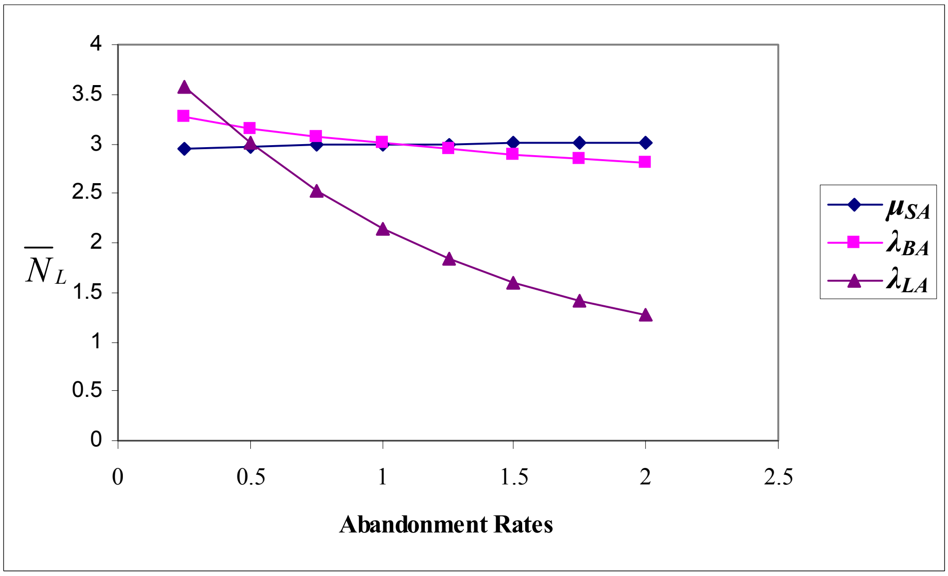

Figure 5.13. The effect of abandonment rates on the average number of Type *B* carriers in steady state

As it is expected when the abandonment rate of orders increases, average number of waiting orders also increases and an increase in the abandonment rate of carriers increase the average number of orders.

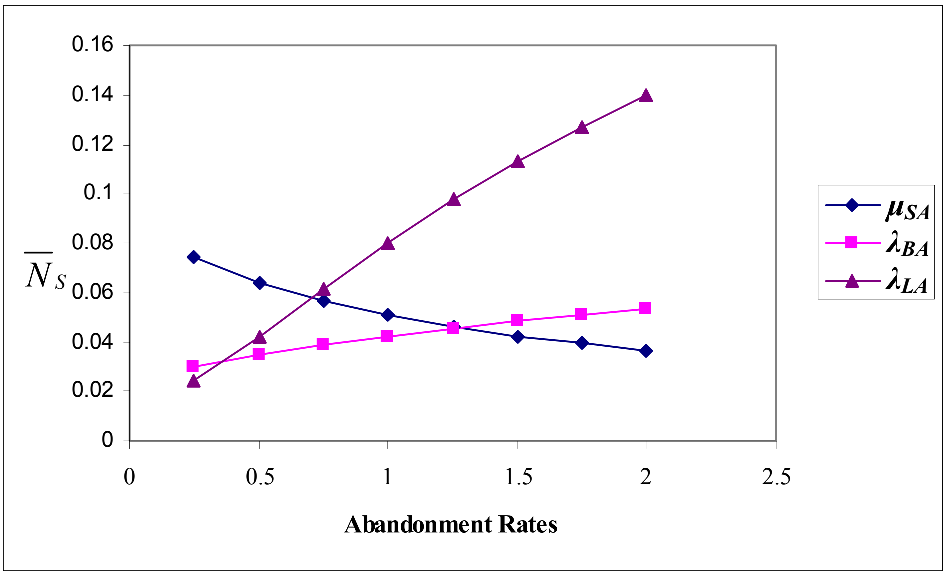

Figure 5.14. The effect of abandonment rates on the average number of orders in steady state

The capacity of the system for carriers and orders directly affects the average number of carriers. In order to analyze this effect, we calculate the average number of carriers in steady state for different capacities and give them in Table 5.11.

The average number of Type B carriers in steady state is affected by the capacity of both types of carriers. As it is seen in Figure 5.15, an increase in the capacity of Type L carriers cause an increase in the average number of Type B carriers. The reason can be explained as follows: when the capacity of Type L carriers increases, the average number of Type L carriers waiting at the LC also increases which in turns decreases the number of waiting orders at the LC. This means the probability of a waiting order to be taken by a Type B carrier decreases and the arriving Type B carriers wait at the LC for the next order. In addition, since the waiting order is taken by the first arriving carrier, the capacity of orders does not affect the average number of carriers.

C_B	C_L	C_S	$\overline{N}_B$		$\overline{N}_L$		$\overline{N}_S$	
			Model	***Simulation***	***Model***	***Simulation***	***Model***	***Simulation***
5	5	5	0.6502	0.6493	3.0013	3.0001	0.0424	0.0426
10	5	5	0.6542	0.6544	3.0022	3.0008	0.0424	0.0422
15	5	5	0.6542	0.6552	3.0022	3.0006	0.0424	0.0422
20	5	5	0.6542	0.6537	3.0022	3.0004	0.0424	0.0426
5	**5**	5	0.6502	0.6508	3.0013	3.0033	0.0424	0.0419
5	**10**	5	0.6568	0.6566	4.5229	4.5209	0.0285	0.0283
5	**15**	5	0.6572	0.6560	4.7505	4.7421	0.0276	0.0276
5	**20**	5	0.6572	0.6573	4.7582	4.7493	0.0276	0.0278
5	5	**5**	0.6502	0.6506	3.0013	2.9998	0.0424	0.0423
5	5	**10**	0.6502	0.6541	3.0012	3.0139	0.0426	0.0425
5	5	**15**	0.6502	0.6547	3.0012	3.0164	0.0426	0.0426
5	5	**20**	0.6502	0.6538	3.0012	3.0167	0.0426	0.0425

Table 5.11. The average number of carriers for different capacities of carriers

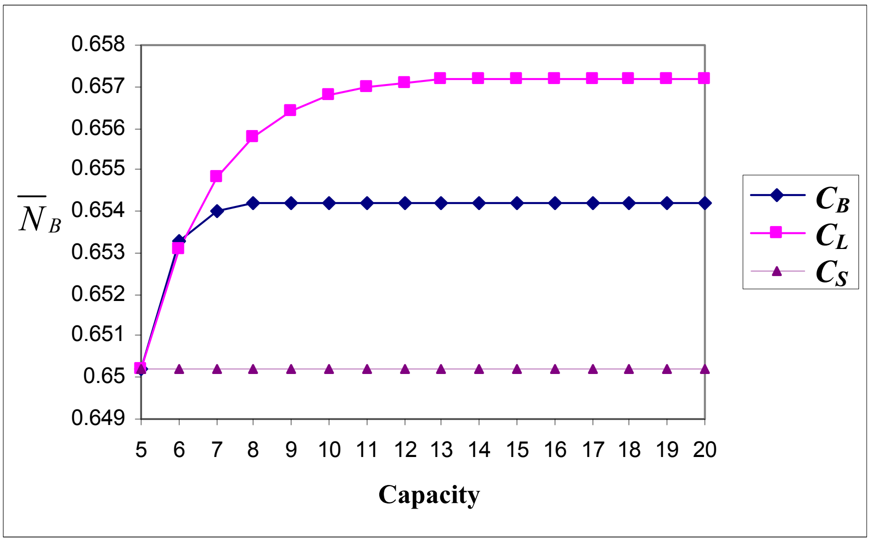

Figure 5.15. The effect of capacities on the average number of Type *B* carriers in steady state

As seen in Figure 5.16, when the capacity of Type *L* carriers increases, the average number of Type *L* carriers also increases because the probability of rejecting a Type *L* carrier decreases. Moreover, unlike the strong effect of the capacity of Type *L* carriers on the average number of Type *B* carriers, an increase in the capacity of Type *B* carriers slightly increases the average number of Type *L* carriers. The reason of this insignificant increase is the fact that the arrival rate of Type *B* carriers is much less than the arrival rate of Type *L* carriers.

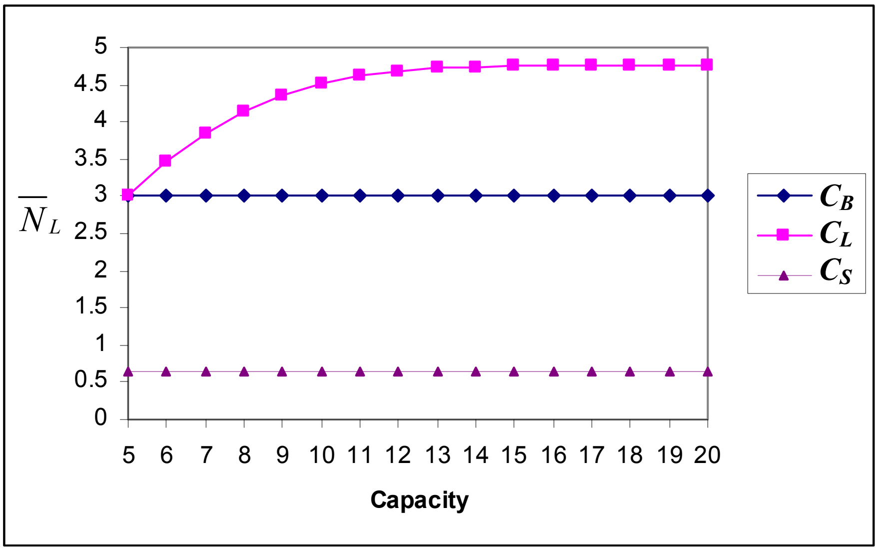

Figure 5.16. The effect of capacities on the average number of Type *L* carriers in steady state

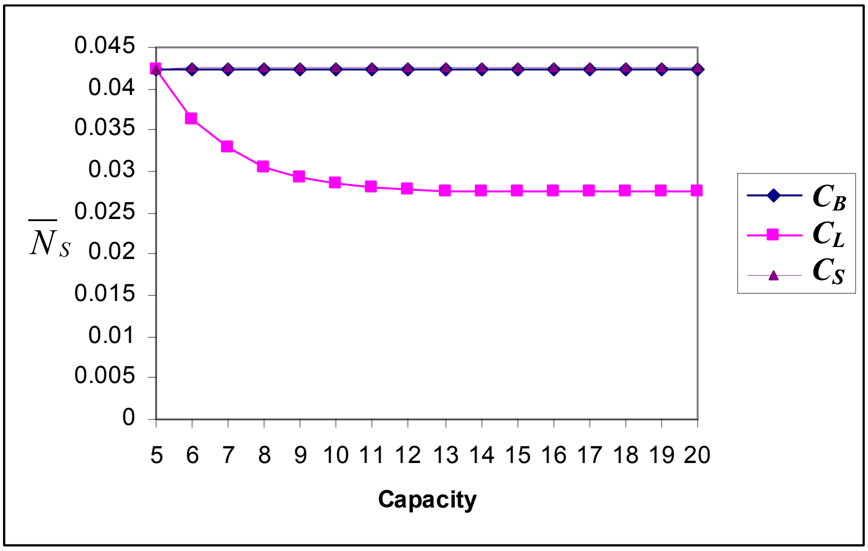

Figure 5.17. The effect of capacities on the average number of orders in steady state

The average number of orders is almost not affected by an increase in the capacity of Type *B* carriers because the arrival rate of Type B carriers is less than the arrival rate of the orders. However, since the arrival rate of Type *L* carriers is greater than the arrival rate of orders, an increase in the capacity of Type *L* carriers decreases the average number of carriers in steady state. In addition, an increase in the capacity of orders insignificantly decreases the average number of orders because the average number of orders in steady state is much less than the available capacity for orders.

5.3.4. The Probability of Rejection of Carriers and Orders

In this section, we calculate the probability of rejection of carriers and orders in steady state. Table 5.12 reports the comparison of simulation and analytical results for different cases. Simulation results validate the analytical results.

In addition, Figures 5.18, 5.19 and 5.20 show the change in the probability of rejection of orders and carriers in steady state.

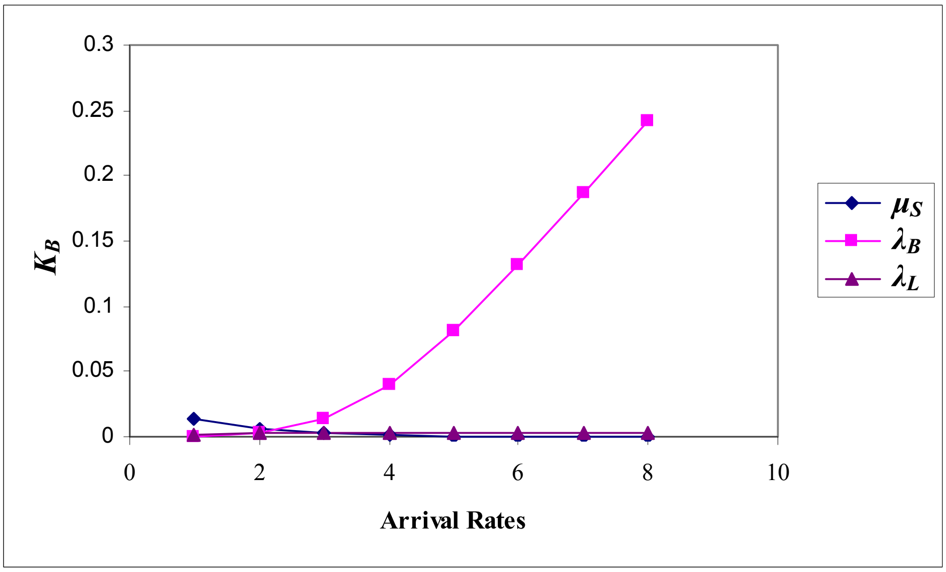

Figure 5.18. The effect of arrival rates on the probability of rejection of Type *B* carriers in steady state

Case	K_L		K_B		K_S	
	Model	***Simulation***	***Model***	***Simulation***	***Model***	***Simulation***
1	0.4234	0.4239	0.0140	0.0138	0	0
2	0.3283	0.3286	0.0058	0.0060	0	0
3	0.1416	0.1411	0.0012	0.0014	0.0009	0.0009
4	0.0111	0.0110	0	0	0.0458	0.0457
5	0.1552	0.1552	0.0001	0.0001	0.0003	0.0003
6	0.2270	0.2270	0.0025	0.0028	0.0001	0.0001
7	0.3505	0.3500	0.0399	0.0396	0	0
8	0.4533	0.4536	0.2411	0.2412	0	0
9	0.0026	0.0027	0.0019	0.0018	0.0031	0.0031
10	0.0377	0.0376	0.0023	0.0024	0.0010	0.0011
11	0.2270	0.2278	0.0025	0.0023	0.0001	0.0001
12	0.5292	0.5287	0.0027	0.0026	0	0
13	0.2238	0.2237	0.0025	0.0024	0.0009	0.0009
14	0.2247	0.2248	0.0026	0.0024	0.0005	0.0005
15	0.2261	0.2259	0.0026	0.0025	0.0002	0.0002
16	0.2277	0.2270	0.0026	0.0025	0.0001	0.0001
17	0.2780	0.2783	0.0189	0.0188	0	0.0001
18	0.2563	0.2566	0.0088	0.0086	0.0001	0.0001
19	0.2270	0.2272	0.0025	0.0025	0.0001	0.0001
20	0.1930	0.1930	0.0004	0.0004	0.0001	0.0001
21	0.3700	0.3702	0.0026	0.0025	0.0001	0.0001
22	0.2270	0.2268	0.0025	0.0025	0.0001	0.0001
23	0.0862	0.0864	0.0025	0.0025	0.0002	0.0002
24	0.0163	0.0163	0.0024	0.0024	0.0004	0.0004

Table 5.12. The probability of rejection of carriers and orders for the analyzed cases of the general model

As seen from Figure 5.18 and 5.19, when the arrival rate of the order increases, the probability of rejecting both Type *B* and Type *L* carriers decreases because the number of auctions increase which in turn makes the number of waiting carriers in the LC smaller.

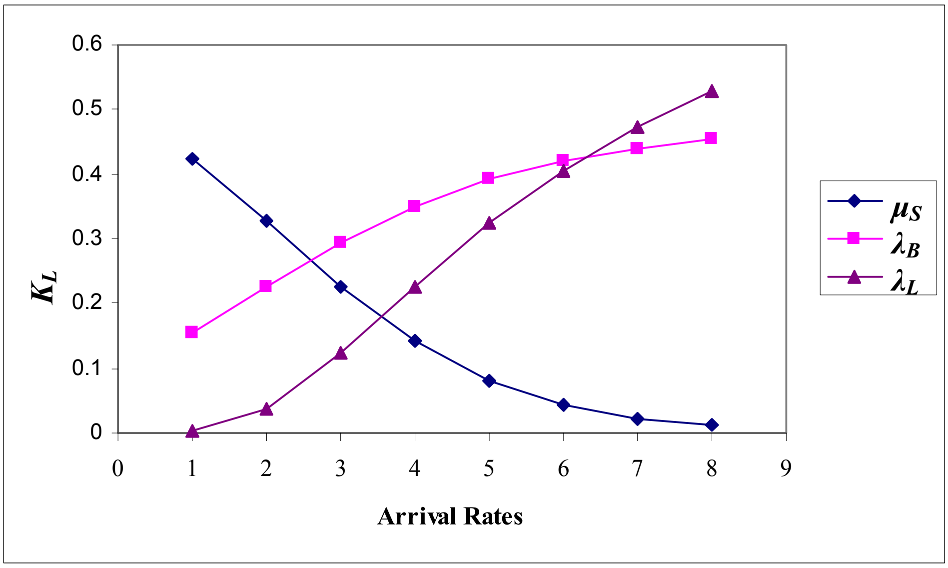

Figure 5.19. The effect of arrival rates on the probability of rejection of Type *L* carriers in steady state

An increase in the arrival rate of Type *L* carrier causes an increase in the probability of rejecting both Type *B* and Type *L* carriers. However, the arrival rate of Type *L* carrier does not have any increase or decrease in the probability of rejecting Type *B* carriers. Since Type *B* carriers have priority, this is the case that is expected.

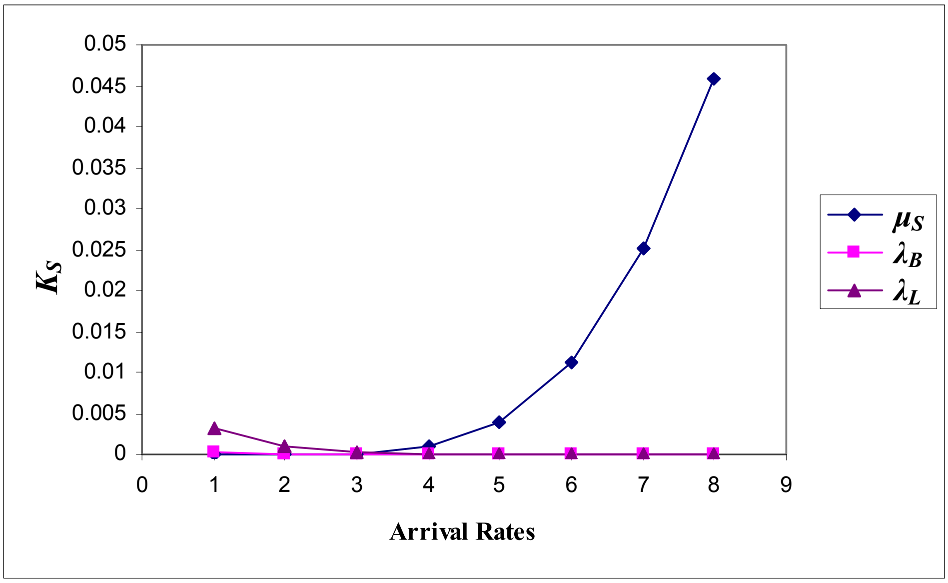

Figure 5.20. The effect of arrival rates on the probability of rejection of orders in steady state

When the arrival rate of orders increases, the probability of rejecting orders also increases. As seen in Figure 5.20 the probability of rejecting an order decreases with increase in the arrival rate of both Type *B* and Type *L* carriers because when more carriers arrive to the LC, the number of waiting orders decreases, which in turn decreases the probability of rejection.

In addition, abandonment rates affect the probability of rejection of carriers and orders. Figure 5.21, 5.22 and 5.23 show the effect of abandonment rates on the probability of rejection of Type *B* carriers, Type *L* carriers and orders respectively.

As seen in Figure 5.21 and 5.22 an increase in the abandonment rate of Type *B* carriers decreases the probability of rejecting both Type *B* and Type *L* carriers while an increase in the abandonment rate of Type *L* carriers does not have any effect on the probability of rejecting Type *B* carriers. Since Type *B* carriers have priority, abandonment of Type *L* carriers could not affect the number of Type *B* carriers; however, the number of Type *B* carriers is important for Type *L* carriers because Type *L* carrier can only win the auction when there is no Type *B* carrier available at the LC. Also, since when there are no carriers available, orders can wait, the abandonment rate of orders does not effect the probability of rejecting both Type *B* and Type *L* carriers.

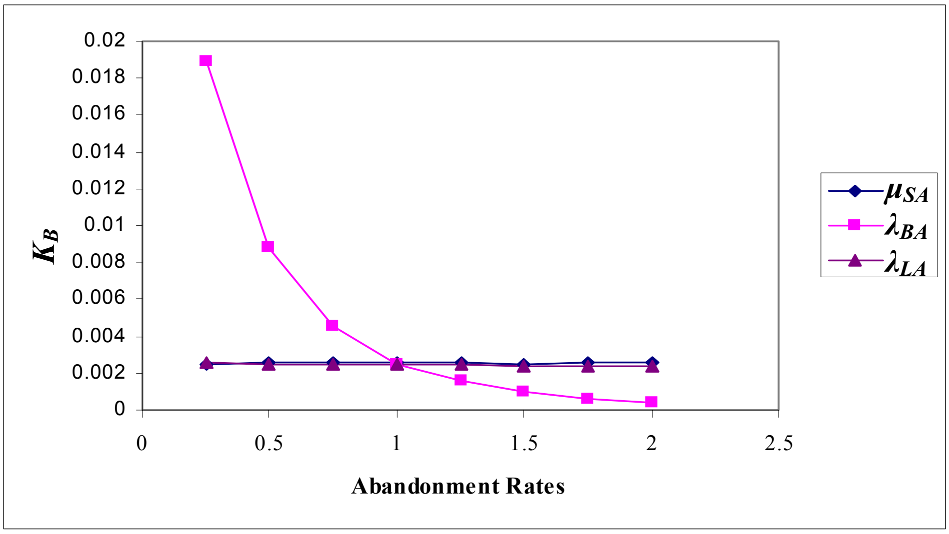

Figure 5.21. The effect of abandonment rates on the probability of rejection of Type *B* carriers in steady state

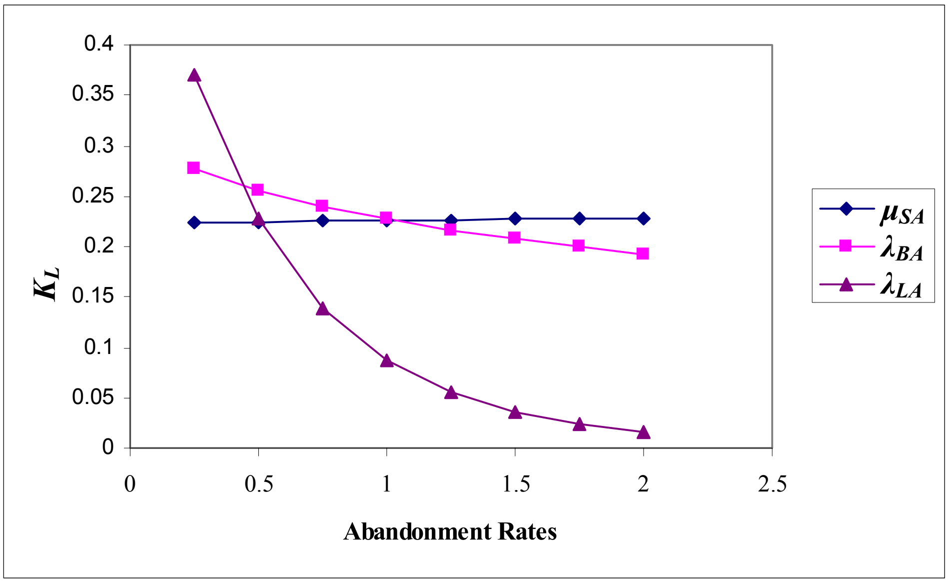

Figure 5.22. The effect of abandonment rates on the probability of rejection of Type *L* carriers in steady state

Moreover, as seen in Figure 5.23, when the abandonment rate of orders increase the probability of rejection of orders decreases because the arriving order can find an empty place in the capacity of orders. However, since when the abandonment rate of carriers increase, the number of waiting carrier increase, an increase in the abandonment rate of carriers increases the probability of rejecting orders.

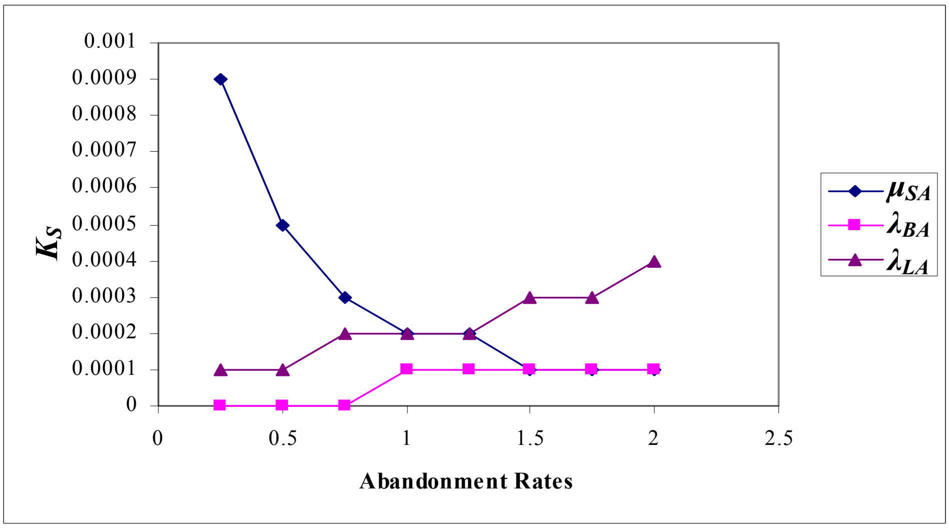

Figure 5.23. The effect of abandonment rates on the probability of rejection of orders in steady state

In order to analyze the effect of the capacity of the system for carriers, we calculate the probability of the rejection of carriers and orders for different capacity values. The analytical and simulation results of the probability of rejection of carriers and orders for different capacities and rates are given in Table 5.13.

C_B	C_L	C_S	K_B		K_L		K_S	
			Model	***Simulation***	***Model***	***Simulation***	***Model***	***Simulation***
5	5	5	0.0026	0.0025	0.2270	0.2267	0.0001	0.0001
10	5	5	0	0	0.2272	0.2273	0.0001	0.0001
15	5	5	0	0	0.2272	0.2272	0.0001	0.0001
20	5	5	0	0	0.2272	0.2273	0.0001	0.0001
5	**5**	5	0.0026	0.0025	0.2270	0.2267	0.0001	0.0001
5	**10**	5	0	0	0.0299	0.0301	0.0001	0.0001
5	**15**	5	0	0	0.0010	0.0009	0.0001	0.0001
5	**20**	5	0	0	0	0	0.0001	0.0001
5	5	**5**	0.0026	0.0025	0.2270	0.2267	0.0001	0.0001
5	5	**10**	0.0026	0.0026	0.2270	0.2269	0	0
5	5	**15**	0.0026	0.0024	0.2270	0.2271	0	0
5	5	**20**	0.0026	0.0025	0.2270	0.2270	0	0

Table 5.13. The probability of rejection of carriers and orders for different capacities of carriers

As seen in Table 5.13 and Figure 5.24, when the capacity of the system for Type *B* carriers increases, the probability of rejection of Type *B* carriers decreases and the other probabilities are not affected significantly.

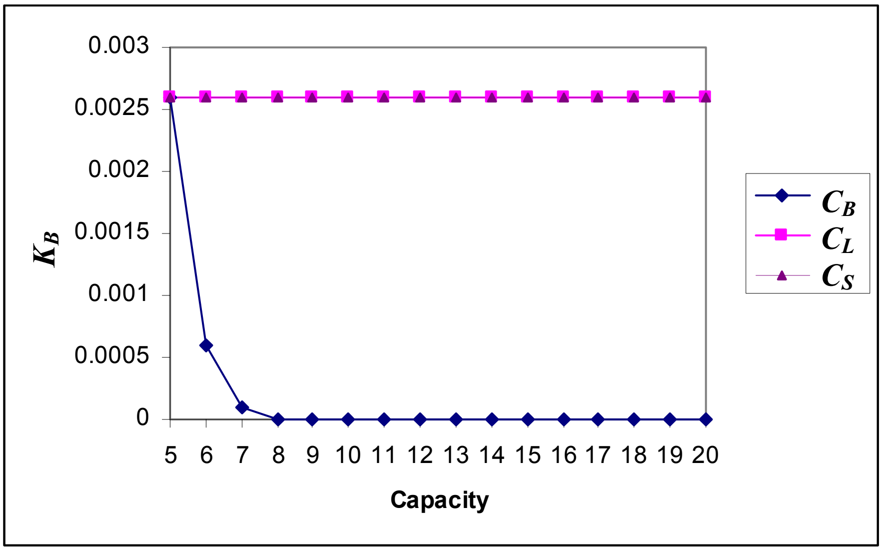

Figure 5.24. The effect of capacities on the probability of rejection of Type *B* carriers in steady state

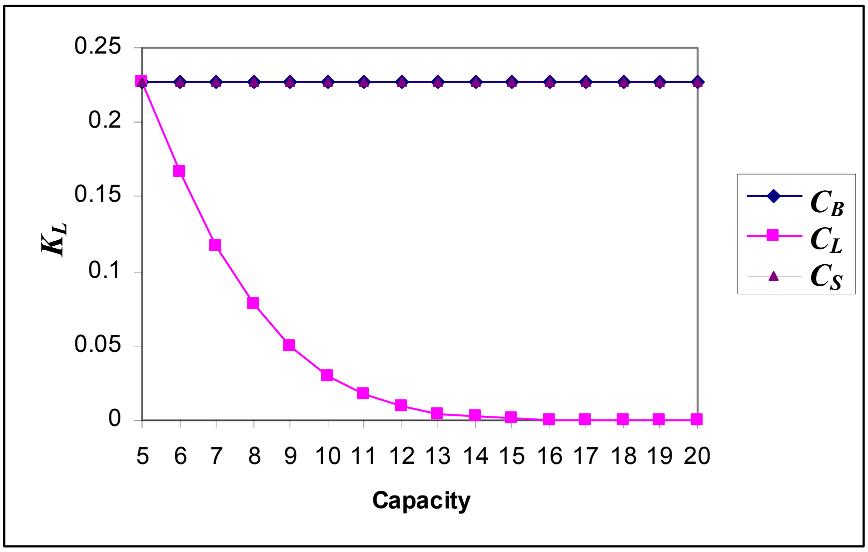

Figure 5.25. The effect of capacities on the probability of rejection of Type *L* carriers in steady state

As seen in Figure 5.25, an increase in the capacity of Type *L* carriers decreases the probability of rejection of Type *L* carriers because the arriving Type *L* carrier can find an empty place. Since the arrival rate of orders is less than the arrival rate of Type *L* carriers, the probability of rejecting orders does not affect the probability of rejecting orders considerably. When the capacity for orders increases, the probability of rejecting an order decreases because of the available capacity for the arriving order.

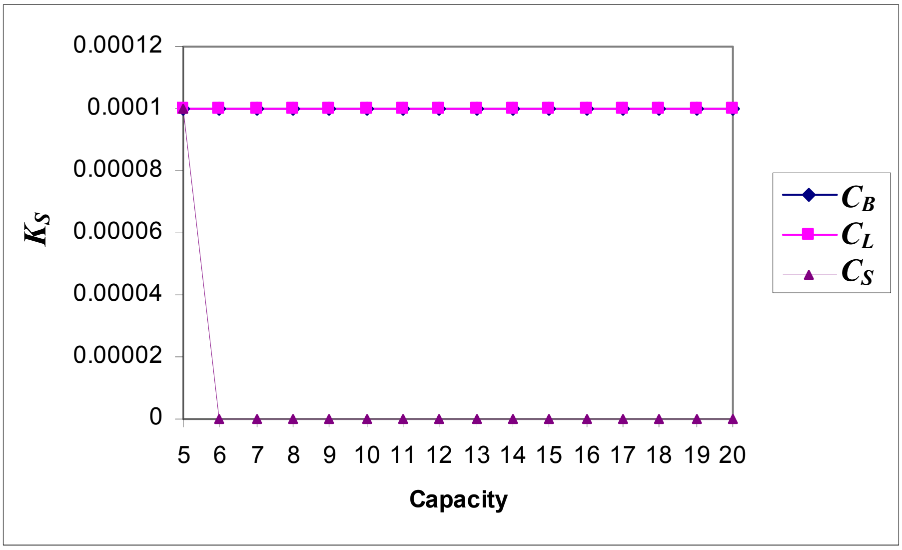

Figure 5.26. The effect of capacities on the probability of rejection of orders in steady state

5.3.5. The Proportion of Carriers that Take an Order

Since carriers can abandon after waiting some time, only a proportion of carriers leave the system by taking an order. In this section, we calculate the proportion of carriers that take an order in steady state and give them in Table 5.14 for different cases.

In addition, Figures 5.27, 5.28, 5.29 and 5.30 show the change in the proportion of carriers that take an order in steady state for different values of system parameters.

Case	M_L	M_B	***Case***	M_L	M_B
1	0.0783	0.343	**13**	0.4063	0.6769
2	0.2239	0.5463	**14**	0.4038	0.6756
3	0.5667	0.7597	**15**	0.4003	0.6736
4	0.9271	0.9413	**16**	0.396	0.6713
5	0.5313	0.7283	**17**	0.3141	0.849
6	0.3979	0.6723	**18**	0.3492	0.7752
7	0.1924	0.5547	**19**	0.3979	0.6723
8	0.0366	0.3566	**20**	0.4563	0.5469
9	0.7465	0.7604	**21**	0.4067	0.6681
10	0.6171	0.7113	**22**	0.3979	0.6723
11	0.3979	0.6723	**23**	0.3791	0.6813
12	0.2088	0.6629	**24**	0.3498	0.6953

Table 5.14. The proportion of carriers that take an order for different cases

As seen in Figures 5.27 and 5.28, when the arrival rate of orders increase, the proportion of carriers that take an order also increases because the number of auctions opened per time increases which then increases the proportion of carriers that take an order. In addition, an increase in the arrival rate of carriers decrease the proportion of carriers that take an order because the average number of carriers increase while the arrival rate of orders remain the same.

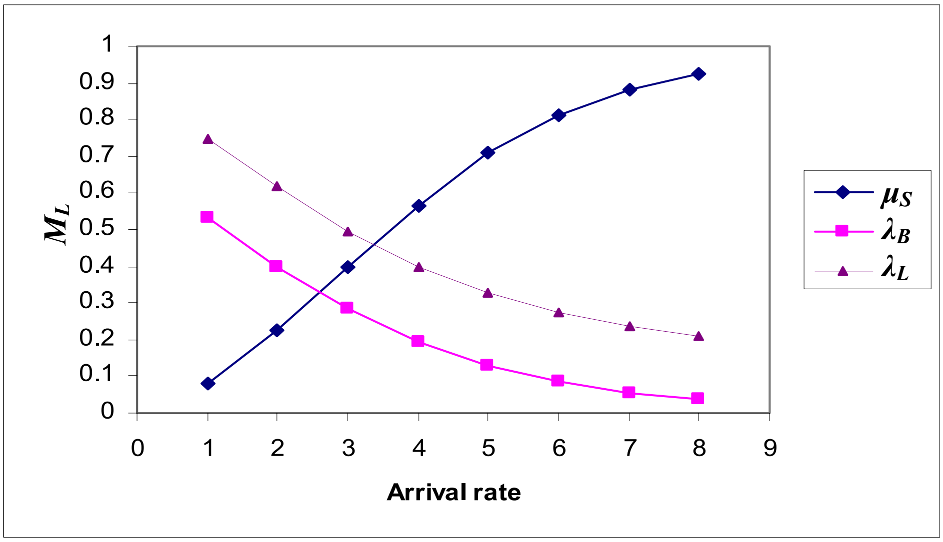

Figure 5.27. The effect of arrival rates on the M_L

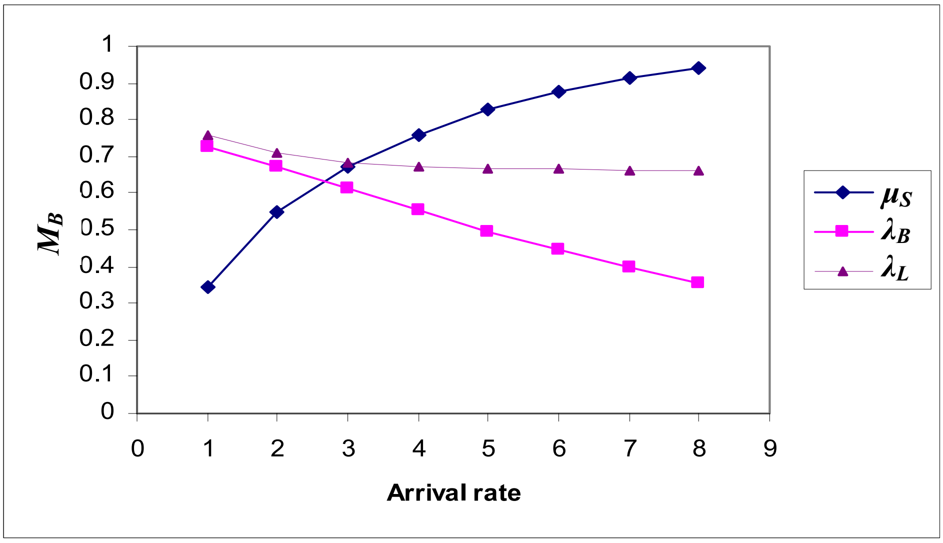

Figure 5.28. The effects of arrival rates on M_B

The effects of abandonment rates are given in Figures 5.29 and 5.30. As seen in figures, the proportion of carriers that take an order decreases insignificantly with an increase in the abandonment rate of orders. In addition, an increase in the abandonment rate of Type *B* carriers increases the proportion of Type *L* carriers that take an order because the number of Type *L* carriers that take an order increases. Moreover, when the abandonment rate of carriers increases, the proportion of carriers that take an order decreases.

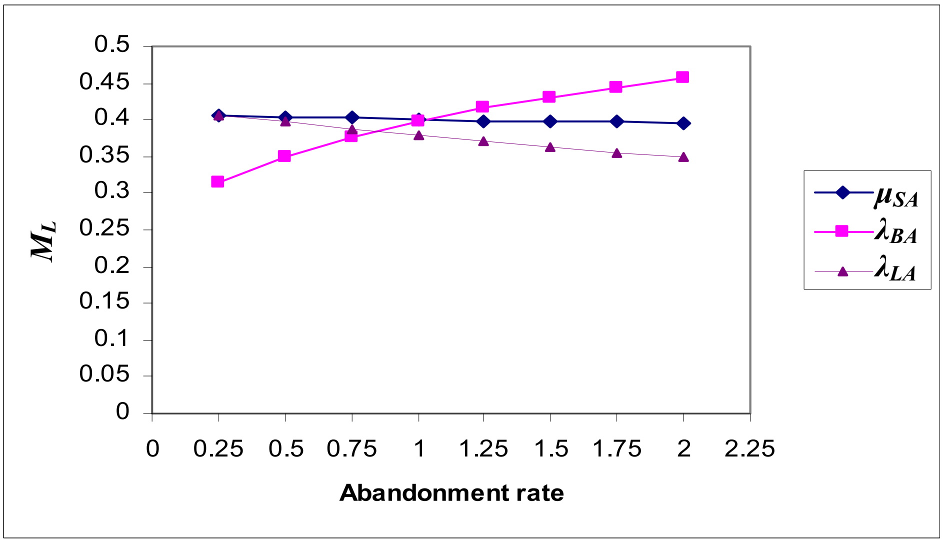

Figure 5.29. The effects of abandonment rates on M_L

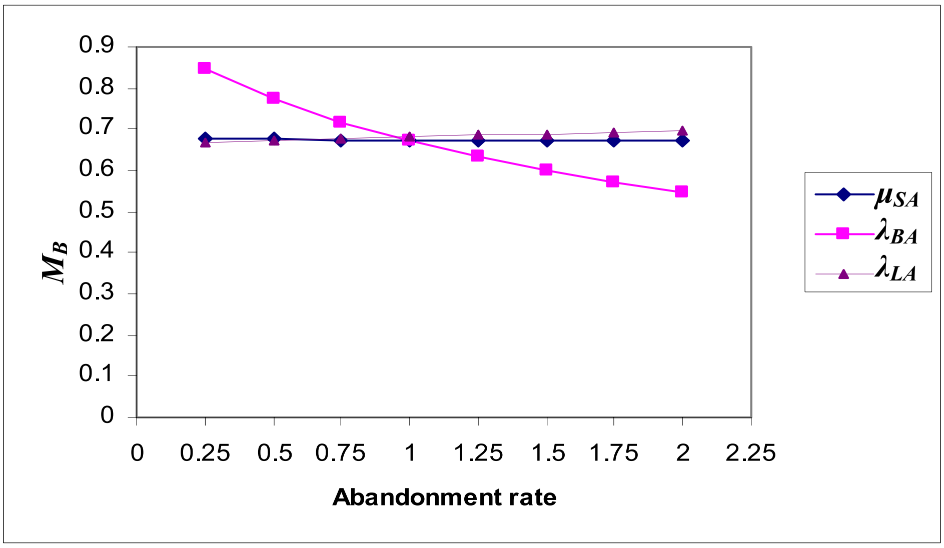

Figure 5.30. The effects of abandonment rates on M_B

The capacity limit of the system for carriers and orders also affects the proportion of carriers that take an order. We calculate these proportions for different capacity limits and give them in Table 5.15.

C_B	C_L	C_S	M_L	M_B
5	5	5	0.3979	0.6723
10	5	5	0.3976	0.6729
15	5	5	0.3976	0.6729
20	5	5	0.3976	0.6729
5	**5**	5	0.3979	0.6723
5	**10**	5	0.4048	0.669
5	**15**	5	0.4052	0.6688
5	**20**	5	0.4052	0.6688
5	5	**5**	0.3979	0.6723
5	5	**10**	0.3979	0.6723
5	5	**15**	0.3979	0.6723
5	5	**20**	0.3979	0.6723

Table 5.15. The proportion of carriers that take an order

As seen in Figures 5.31, when the capacity of Type L carriers increases, the number of carriers joining an auction increases which in turn increases the proportion of Type L carriers that take an order. In addition, the capacity limit for orders does not have any effect on the proportion of carriers that take an order because the average number of waiting orders is smaller than the capacity limit of orders.

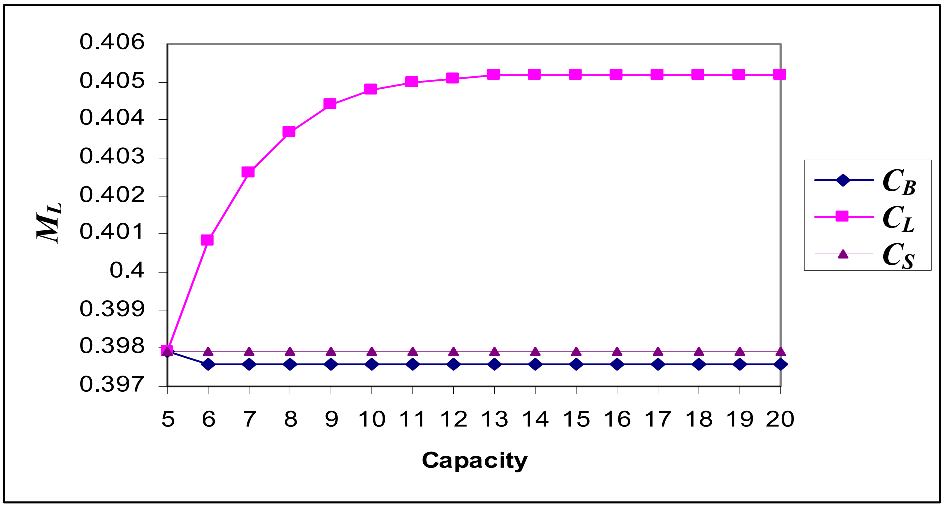

Figure 5.31. The effects of capacity limits on M_L

As seen in Figure 5.32, an increase in the capacity for Type *L* carriers increase the number of Type L carriers that take an order, so the proportion of Type *B* carriers that take an order decreases.

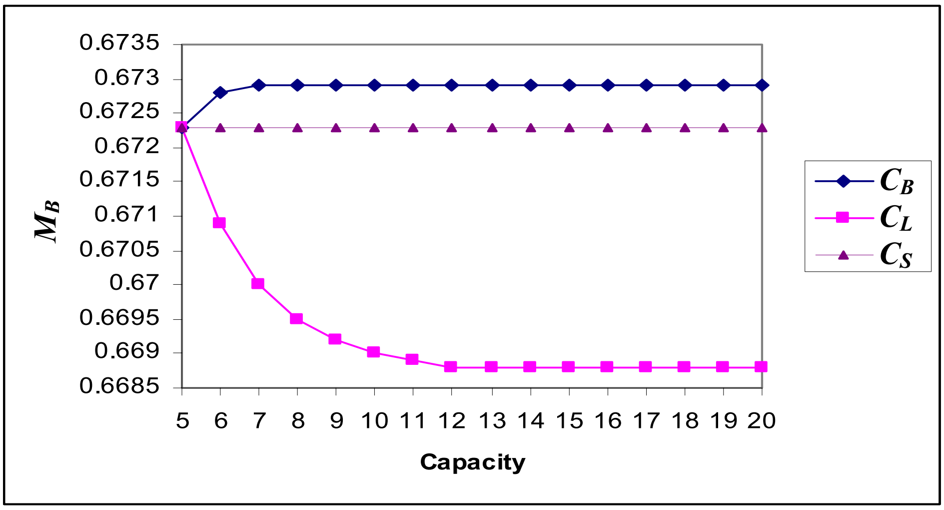

Figure 5.32. The effects of capacity limits on M_B

5.3.6. The Effect of Different Types of Carriers on the Performance of the System

One of the research questions of this thesis is if the different types of carriers affect the performance of the system. In order to find an answer to that question we analyze a special case where there is only one type of carrier, Type *L*, accepted by the system. In other words we take the arrival rate of Type *B* carriers zero, i.e. $\lambda_B = 0$.

When there is only one type of carrier that respond to the orders, the performance measures can be derived in closed form. The closed form equations for the performance measures are given in Appendix C. In order to show the effect of the types of carriers, we calculate the average expected price and profit of carriers for the special case where the cost distribution of the carriers is uniformly distributed on [90,100] and P_M = 100. The average expected price and the average expected profit of the carriers for different parameters is given in Table 5.16.

As seen in Table 5.16, when there is only one type of carrier, say local carriers, the average expected price increases. This is expected because the carriers that have lower cost are rejected by the system. Although the average expected price increases, the average expected profit of carriers decreases. The reason is the same with the average expected price. Since in the general model the carriers with lower cost are accepted, the average expected profit of the carriers in steady state is higher. As a result, different types of carriers that have lower cost should be accepted by the LC in order to decrease the average expected price.

As a consequence, the system parameters, such as the arrival and abandonment rates of orders and carriers, the capacity of the system and the types of carriers, have different effects on the performance measures as stated in this chapter. By changing these parameters the performance of the system can be increased. For instance, since when the arrival rates of carriers increase, the average expected price decreases, by giving high quality service to the carriers or making advertisements by putting signboards to the roads, the arrival rate of carriers can be increased. Also, by providing a place for staying to the carrier owners, the abandonment rate of carriers can be decreased which in turn will decrease the average expected price.

C_L	C_S	λ_S	λ_L	λ_{SA}	λ_{LA}	P_{av} (YTL)	Q_{av} (YTL)	P_{av}-P'_{av}* (YTL)	Q_{av}-Q'_{av}** (YTL)
5	5	**1**	4	1.5	0.5	95.3885	2.6943	8.5727	-4.5045
5	5	**2**	4	1.5	0.5	96.0590	3.0295	6.3325	-4.2831
5	5	**4**	4	1.5	0.5	97.1466	3.5733	3.3958	-4.4176
5	5	**8**	4	1.5	0.5	98.3203	4.1602	0.0168	-6.2261
5	5	3	**1**	1.5	0.5	99.1055	4.5527	3.8062	-9.6057
5	5	3	**2**	1.5	0.5	98.1718	4.0859	4.2547	-6.3376
5	5	3	**4**	1.5	0.5	96.6543	3.3272	4.7335	-4.1999
5	5	3	**8**	1.5	0.5	95.2160	2.6080	4.5293	-3.8462
5	5	3	4	**0.25**	0.5	96.7689	3.3693	4.6212	-4.2741
5	5	3	4	**0.5**	0.5	96.7517	3.3601	4.6691	-4.2499
5	5	3	4	**0.75**	0.5	96.7306	3.3504	4.7427	-4.2111
5	5	3	4	**1**	0.5	96.6972	3.3377	4.8277	-4.1624
5	5	3	4	1.5	**0.25**	96.2412	3.1206	4.9061	-3.8985
5	5	3	4	1.5	**0.5**	96.6543	3.3272	4.7335	-4.1999
5	5	3	4	1.5	**0.75**	96.9982	3.4991	4.0828	-4.9788
5	5	3	4	1.5	**1**	97.2852	3.6426	3.1842	-6.1520
10	5	3	2	1.5	0.5	95.6777	2.8388	4.5837	-4.0741
15	5	3	2	1.5	0.5	95.6308	2.8154	4.6049	-4.0498
20	5	3	2	1.5	0.5	95.6302	2.8151	4.6059	-4.0490
5	**10**	3	2	1.5	0.5	97.6725	3.8363	5.7513	-3.6911
5	**15**	3	2	1.5	0.5	97.6887	3.8443	5.7675	-3.6831
5	**20**	3	2	1.5	0.5	97.6887	3.8443	5.7675	-3.6831

Table 5.16. The average expected price and the average expected profit of the carriers with a single type carrier

*the average expected price for the general case

**the average expected profit of the carriers for the general case

In addition, the optimal capacity can be calculated for different parameters. As an example, when the arrival rate of carriers is 3, the arrival rates of carriers are 2 and 4 for Type *L* and Type *B* carriers respectively, and the abandonment rates are 1.5, 1, 0.5 for orders, Type *L* and Type *B* carriers respectively, the optimal capacities are 10 and 15 for Type *B* and Type *L* carriers. The following chapter will give an estimation method to define the cost distributions of carriers.

Chapter 6

A METHOD FOR INFERRING THE COST DISTRIBUTION FROM THE OBSERVED BIDS

6.1. Introduction

In this chapter, a method that estimates the cost distribution from the observed bids will be introduced. First, a short literature review on the empirical analysis of auctions will be presented, and then the method will be explained in detail. The cost distributions of carriers for two cities are estimated and presented as an example.

As it will be explained in the Subsection 6.2, inferring the valuations (costs) of the bidders from the observed bids is a hard problem which requires some assumptions to be made initially. In the proposed estimation method we assume that all the costs of carriers are drawn from the same distribution which is a common knowledge. In addition, we assume that the number of carriers joining an auction is a random variable with a probability function which is also commonly known.

The main difference of the proposed method is defining the cost distribution from all the bids given to an auction by using a moment matching algorithm, and involving a stochastic number of bidders for the cases where the number of bidders is not revealed. Moreover, although the estimation methods that are used in economics can be employed for large sample sizes, this method has only been tested for small sample sizes. It is shown that the method works well for the transportation from Eskisehir to other two cities.

6.2. Literature Review on Empirical Analysis of Auctions

Recently, game-theoretic models of auctions have been used by a number of empirical researchers to interpret actual field data. Hendricks and Paarsch [54] prepared a survey of empirical work about auctions. They first outline two complementary approaches to the empirical analysis of auctions, and then discuss recent developments in the empirical analysis of field data.

As Hendricks and Paarsch [54] stated, recent empirical research has had two main goals. The first one is to test the behavioral theory. This is done by trying to find an answer for the question "Do potential buyers bid according to a Bayesian-Nash equilibrium?" This approach is followed in experiments by choosing the probability law and drawing the valuations for potential buyers. The second goal is to identify the probability law that governs the valuations of potential buyers. In auctions, the bid strategy of a potential buyer in equilibrium is usually a function of his valuation. Therefore, if it is assumed that potential buyers bid according to Bayesian-Nash equilibrium strategies, then, by using bid data from auctions, the distribution of valuations may be estimated and this estimated distribution can be used to determine the revenue-maximizing selling mechanism. For example, Paarsch [59] design an optimal auction to sell timber cutting rights in British Columbia, Canada, after estimating the probability law of valuations from bid information from English auctions.

Hendricks and Porter [60] were among the first to recognize the empirical importance and richness of auctions. They have pursued the first goal of empirical research using data from auctions of drainage leases, which are adjacent to tracts upon which a deposit has been discovered, on the Outer Continental Shelf. The evidence suggests that neighbor firms, owning the adjacent tracts, are well informed about the value of the drainage lease and coordinate their bidding decisions. Non-neighbor firms are relatively uninformed. They find that the data are in agreement with the predictions of the Bayesian-Nash equilibrium model of bidding in first-price sealed-bid auction with asymmetric information.

The second goal, identifying the exact data-generating process, is studied by a number of researchers such as Paarsch [59]; Laffont, Ossard, and Vuong [61]; and Elyakime et al. [62]. In all of these papers it is assumed that the theory holds and the equilibrium behavioral relationships are imposed upon the data to determine the underlying probability law. The main difficulty with this approach, called as structural estimation, especially among econometricians, is the complexity of the equilibrium bid functions, which are often highly non-linear and, in some cases, do not have closed-form representations [56].

Donald and Paarsch [63] developed estimation procedures which require that the joint distributions belong to certain families of distributions that give closed-form solutions to the bid functions. Then, by using non-linear programming techniques empirical specifications can be estimated to obtain the maximum likelihood estimator.

Laffont, Ossard, and Vuong [61] have proposed a simulated non-linear least squares estimator, which can handle a larger class of distribution functions, for the empirical study of theoretical auction models. The method requires the exact computation of neither the equilibrium strategy nor the moments of the winning bid. It is based on simulations following McFadden [64] and Pakes and Pollard [65]. They adopt the private value paradigm and analyze the first-price sealed-bid and descending auctions. The proposed model is illustrated by studying a market of eggplants where descending auctions are used. Nevertheless, a specific distributional assumption must still be made initially. Zulehner [66] makes an empirical analysis of cattle auctions taking place in Amstetten, Austria by using the simulated non-linear least squares (SNLS) estimator proposed by Laffont et al.

More recently, to avoid the distributional assumption that has to be made in the SNLS estimator, Elyakime et al. [62] have proposed the non-parametric methods for estimating the probability law of valuations. This method essentially imposes no restrictions upon the admissible probability laws; however, in contrast to parametric methods, this method requires knowledge of all bids, not just the winning bid.

Structural estimation requires a much deeper level of commitment to the theoretical model and as noted in the conclusions of Hendricks and Porter [60], and Wilson [51]'s papers, it is difficult to develop an alternative behavioral model that can do as well in explaining the data.

In this thesis, we have all the bids given to all orders and also the winning bids. Since we have the exact moments of the bids, we propose an estimation method that uses exact moments of the bids. Like most of the empirical studies [61, 63, 66], we make an initial assumption on the cost distribution of the carriers. With this assumption, we determine the parameters of the cost distribution and it is given in Section 6.7. In addition, the proposed method does not take the sample size into account while making the estimation, so we can not give the statistical significance. Therefore this method is applicable for the cases where the sample sizes are high.

6.3. Proposed Estimation Method

The first-price sealed-bid auction is used while determining the winner in the ESO LC, so the focus of the proposed estimation method is the first-price sealed-bid auctions with independent private values.

The symmetric Bayesian-Nash equilibrium expresses the optimal bid as a function of the bidders' private value, the number of bidders and the distribution of private values [44]. In most of the auctions, only the bids are observed while the actual private values of bidders are not observed. These observed bids are functions of private values. Since the private values of bidders are random variables, the observed bids are also random with a distribution that is uniquely determined by the distribution of the valuations. Unfortunately, the optimal bid function in equilibrium, which is a function of the private values, is intractable. Thus, only very specific distributions of private values have been considered in empirical work [61].

6.3.1. The Basic Theoretical Model of First-Price Auctions

In a standard first-price sealed-bid auction a single and indivisible object is auctioned. All the bids are given simultaneously in sealed envelopes. The object is sold to the bidder that gives the highest bid, and he pays his bid. In this framework, the bidders do not know the others' bids while making their decision; however, it is assumed that the number of bidders is a common knowledge. The reverse auctions have the same properties with standard auctions, but the difference is that the bidder with the lowest bid wins the auction. The following calculations will be done for reverse auctions.

In the private value paradigm, each bidder has a private cost, r_i, for the service which is only known to him. However, each bidder knows that all private costs, including his own, have been drawn independently from a distribution which is assumed to be common knowledge. Let $F(\cdot)$ be the cdf of this distribution and $f(\cdot)$ be its density. Then, the bid given by a bidder with a cost of r_i, $b(r_i)$, in a first-price sealed-bid auction when there are n bidders joining the auction is given as

$$b(r_i) = r_i + \frac{\int_{r_i}^{\bar{r}} \left(1 - F(x)\right)^{n-1} dx}{\left(1 - F(r_i)\right)^{n-1}} \tag{6.1}$$

The proof of this equation is given in Appendix B.2.

Equation (6.1) is the equilibrium bid of bidder i when the number of bidders is commonly known. In some circumstances, the number of bidders may be concealed like in the ESO LC case. Thus, the Equation (6.1) can be modified for the situations where the number of bidders is not revealed but the distribution of the number of bidders joining the auction is commonly known.

Let p_j denote the probability of the number of bidders engaging an auction is j, i.e. $p_j = \Pr ob\{n = j\}$. Without driving the exact bids when the number of bidders joining an auction is not commonly known, we make an assumption that the bidders give their bids according to the following function.

$$b(r_i) = r_i + \sum_{j=1}^{N} \frac{p_j \int_{r_i}^{\bar{r}} \left(1 - F(x)\right)^{j-1} dx}{\left(1 - F(r_i)\right)^{j-1}} \tag{6.2}$$

where N is the maximum number of bidders joining the auction.

6.3.2. Steps of the Proposed Estimation Method

We propose a method that can be applied with a broad class of cost distributions (see Lam and Kelton [67] for the distributions). However, as it is mentioned in Section 6.2, in empirical analysis of auctions an assumption for the distribution of the valuations of the bidders has to be made initially. So, the method that we propose also requires making an initial assumption for the distributions of the costs of the bidders.

The proposed method is based on the matching of the first four moments of the observed bids, and also the moments of the bids that will be derived from the cost distribution that is assumed initially. Let M_1, M_2, M_3 and M_4 be the first, second, third and fourth moments of the bids, respectively, then the moments are calculated as follows:

$$M_1 = \sum_{x=1}^{m} p(x)t(x), \quad M_i = \sum_{x=1}^{m} p(x)\left(t(x) - M_1\right)^i \quad \text{for } i = 2, 3, 4. \tag{6.3}$$

where $t(x)$ is the observed bid value and $p(x)$ is its probability.

Now, we can give the steps of the method as follows:

Step 1. Calculate the moments of the observed bids: The moments of the observed bids, M_1, M_2, M_3 and M_4, are calculated as described above by using Equation (6.3)

Step 2. Calculate the moments of the bids from the cost distribution: In this step, first the bids have to be generated by using Equation (6.1) (or Equation (6.2), if the number of bidders is concealed). Then the moments of these bids, $M_1^{'}$, $M_2^{'}$, $M_3^{'}$ and $M_4^{'}$, have to be calculated as explained above by using Equation (6.3) with the estimated bids, $b(x)$.

Step 3. Formulating the optimization problem: The optimization problem that we formulate will minimize the total square of the differences of the first four moments of the actual bids calculated in Step 1, M_1, M_2, M_3 and M_4, and the moments calculated in Step 2, $M_1^{'}$, $M_2^{'}$, $M_3^{'}$ and $M_4^{'}$. Different weights are given for the moments relative to their importance. Since the first moment is of special interest in view of the Revenue Equivalence Theorem, the largest weight is given to the first moment and other weights are sequenced as $w_1 > w_2 > w_3 > w_4$. The objective function of the optimization problem is

$$Z = w_1\left(M_1 - M_1^{'}\right)^2 + w_2\left(M_2 - M_2^{'}\right)^2 + w_3\left(M_3 - M_3^{'}\right)^2 + w_4\left(M_4 - M_4^{'}\right)^2 \quad (6.4)$$

The purpose of this optimization problem is to find the parameters of the cost distribution that minimizes this objective function.

6.3.4. Application of the Proposed Method to the ESO LC Case

We define the cost distributions of two cities, City 1 and City 41, from the bids given to the orders that are opened for transportation from Eskisehir to these cities. Since the Weibull distribution is one of the most flexible distributions with its versatility, we assume that the transportation costs are distributed according to Weibull distribution, and then we perform the analysis with this assumption.

We use the two parameter Weibull distribution where α is the shape parameter and β is the scale parameter. The Weibull shape parameter, α, is also known as the slope because the value of α is equal to the slope of the regressed line in a probability plot. Different values of the shape parameter can have different effects on the behavior of the distribution. In fact, some values of the shape parameter will cause the distribution equations to reduce to those of other distributions. For example, when α = 1, the probability density function (pdf) of the Weibull distribution reduces to that of the exponential distribution. A change in the scale parameter, β, has the same effect on the distribution. An increase in β, while holding α constant has the effect of stretching out the pdf. In addition, the peak of the pdf curve will decrease with the increase of β.

Step 1: Calculating the moments of the Observed Bids

In this step we calculate the moments of the observed bid. Since the frequency distribution of the observed bids includes noise due to a low number of observations, we first fit a Weibull distribution to the observed bids. Next, we use the fitted distribution to determine the moments and then to estimate the cost distribution. As it is explained above, Weibull distribution is a flexible distribution, so this distribution is fitted to the observed bids with the procedure outlined by Law and Kelton [67]. According to this procedure, while fitting a distribution to a Weibull distribution, the following two equations must be satisfied:

$$\frac{\sum_{i=1}^{n} X_i^{\alpha_F} \ln X_i}{\sum_{i=1}^{n} X_i^{\alpha_F}} - \frac{1}{\alpha_F} = \frac{\sum_{i=1}^{n} \ln X_i}{n}, \quad \beta_F = \left(\frac{\sum_{i=1}^{n} X_i^{\alpha_F}}{n} \right)^{1/\alpha_F}$$

where n is the number of bids.

The first can be solved for α_F numerically by Newton's method, and the second equation then gives β_F directly. The general recursive step for the Newton iterations is

$$\hat{\alpha}_{k+1} = \hat{\alpha}_k + \frac{A + 1/\hat{\alpha}_k - C_k / B_k}{1/\hat{\alpha}_k^{\ 2} + (B_k H_k - C_k^{\ 2}) / B_k^{\ 2}}$$

where

$$A = \frac{\sum_{i=1}^{n} \ln X_i}{n}, \qquad B_k = \sum_{i=1}^{n} X_i^{\hat{\alpha}_k}\ , \ C_k = \sum_{i=1}^{n} X_i^{\hat{\alpha}_k} \ln X_i \qquad \text{and} \qquad H_k = \sum_{i=1} X_i^{\hat{\alpha}_k} (\ln X_i)^2$$

As a starting point for the iterations, the estimate

$$\hat{\alpha}_0 = \left\{ \frac{(6/\pi^2)\left[\sum_{i=1}^{n} (\ln X_i)^2 - \left(\sum_{i=1}^{n} \ln X_i \right)^2 / n \right]}{n-1} \right\}^{-1/2}$$

is used.

It is reported that an average of only 3.5 Newton iterations were needed to achieve four-place accuracy [67].

We fitted the bids given to Cities 1 and 41 by applying this procedure and found the parameters. These are given in Table 6.1.

City	$\hat{\alpha}$	$\hat{\beta}$
City 1	5.1674	293.02
City 41	21.4898	177.8

Table 6.1. Parameters of the Weibull distribution fitted to City 1 and 41

Figures 6.1 and 6.2 show the observed bids and the fitted distributions, e(x), for City 1 and City 41, respectively.

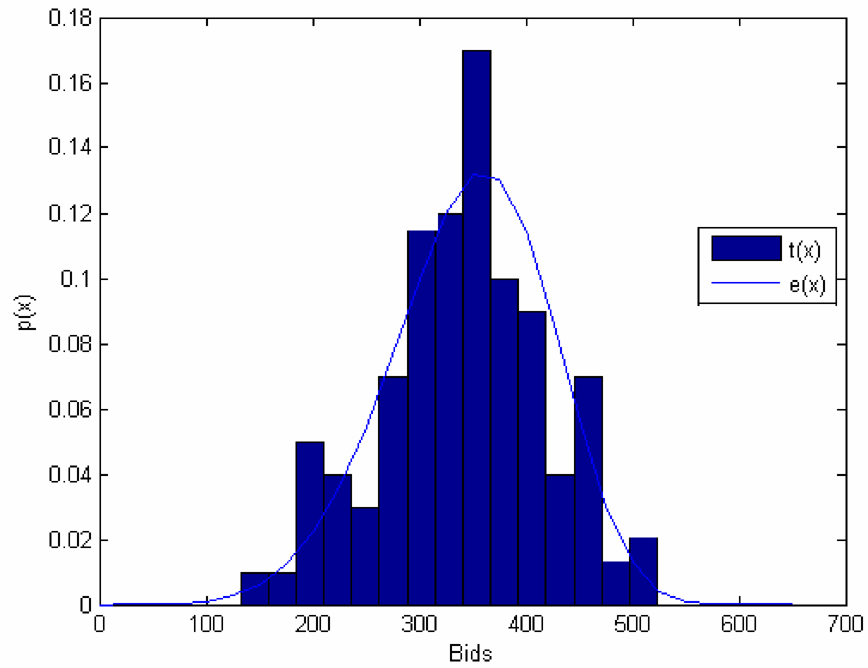

Figure 6.1. Observed bids given to City 1 and the fitted distribution

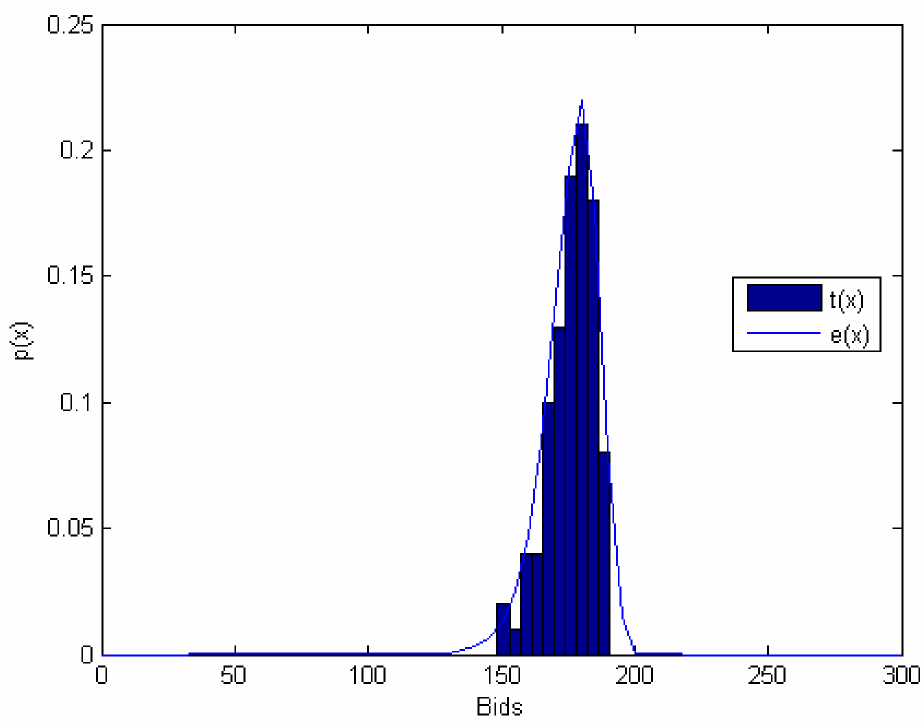

Figure 6.2. Observed bids given to City 41 and the fitted distribution

As seen from Figures 6.1 and 6.2, Weibull distribution gives a close fit for the bids. We calculate the moments for each city, and give them in Table 6.2

Cities	M_1	M_2	M_3	M_4
City 1	348.39	9.72	-0.34	0.47
City 41	178.38	0.54	-0.02	0.01

Table 6.2. The moments of the observed bids for City 1 and City 41

Step 2. Calculate the moments of the bids from the cost distribution: In this step, bids are generated from the cost distribution which is a Weibull distribution with parameters α_c and β_c. Then the moments, $M_1^{'}$, $M_2^{'}$, $M_3^{'}$ and $M_4^{'}$, are derived as given in Equation (6.3). The probabilities of the number of bidders are calculated from the observed bids and given in Table 6.3. Since the parameters of the cost distribution, i.e. α_c and β_c, are the variables, these moments are the equations with unknown parameters α_c and β_c.

The number of bidders (j)	***City 1***	***City 41***
	p_j	p_j
1	0.179	0.01
2	0.435	0.451
3	0.212	0.216
4	0.051	0.098
5	0.020	0.039
6	0.041	0.078
7	0.041	0.078
8	0.021	0.039

Table 6.3. The probabilities of the number of bidders joining auctions for City 41

Step 3. Formulating the optimization problem: We formulate the objective function by using the moments calculated in Step 1 and Step 2. Then, we solve the objective function by using "*fmincon*" optimization function in Matlab. "*fmincon*" attempts to find a constrained minimum of a scalar function of several variables starting at an initial estimate, and it uses a sequential quadratic programming (SQP) method. In this method, the function solves a quadratic programming (QP) sub problem at each iteration. An estimate of the Hessian of the Lagrangian is updated at each iteration using the BFGS formula (see Shanno[68] for more information on BFGS formula).

We find the parameters of the distributions of the transportation costs for the selected two cities and present them in Table 6.4.

Cities	***alpha (α)***	***beta (β)***	***mean (μ)***	***variance (σ^2)***
City 1	2.9071	293.02	261.28	16.6175
City 41	9.1133	167.9	159.08	2.18

Table 6.4. Parameters of the cost distributions

The observed bids given to orders for City 1 mostly range between 260 and 360, as it is seen in Figure 6.3, $t(x)$, and the cost distribution that is estimated for City 1 is a Weibull distribution where α=2.91, β=293.02 as it is shown in Figure 6.3 with r_i. In addition, the observed bids for City 41 mostly lie between 160 and 200, as seen in Figure 6.4, the estimated cost distribution is a Weibull distribution with α=9.11, β=167.9.

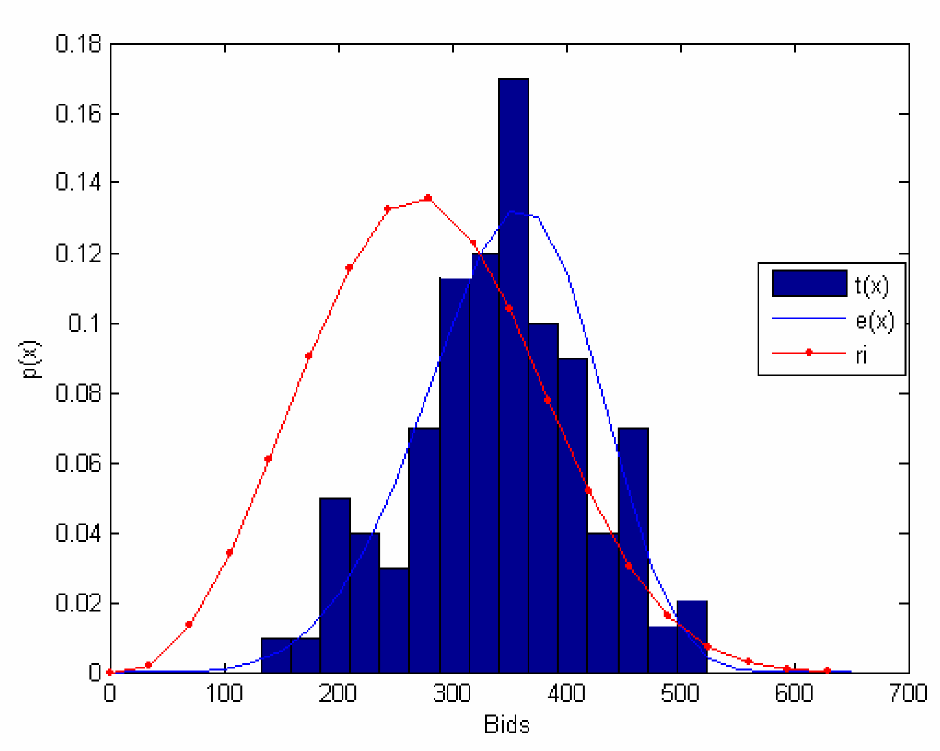

Figure 6.3. Observed bids, fitted distribution and the cost distribution for City 1

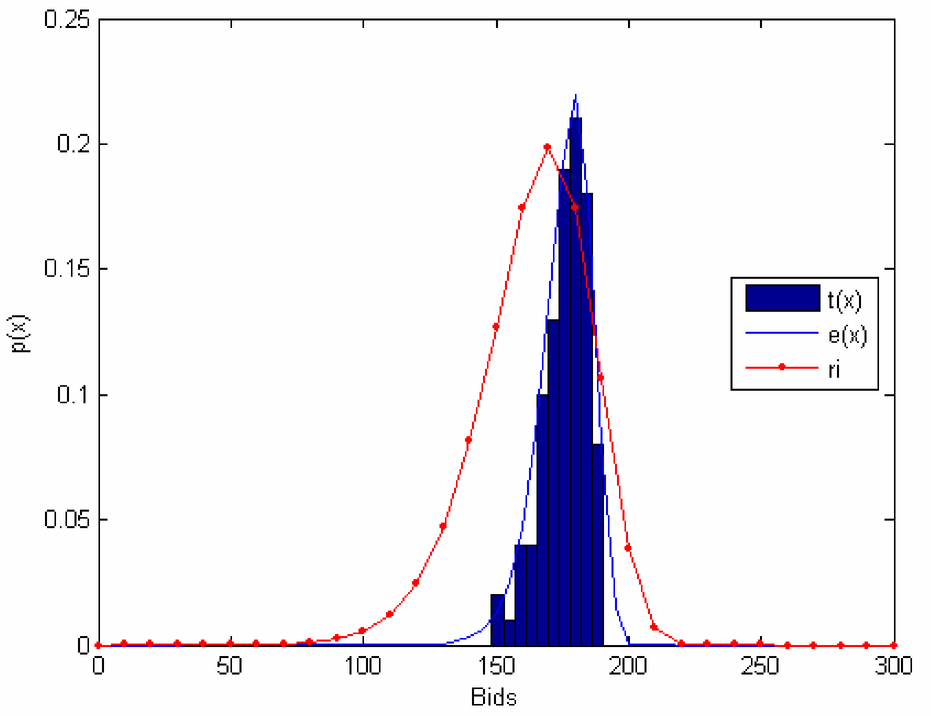

Figure 6.4. Observed bids, fitted distribution and the cost distribution for City 41

Moreover, in order to see how well the estimation method works, we generate bids from the estimated cost distribution and give them in Figure 6.5 and 6.6 for City 1 and City 41. As seen from figures, the estimation method gives very close bids with the observed bids.

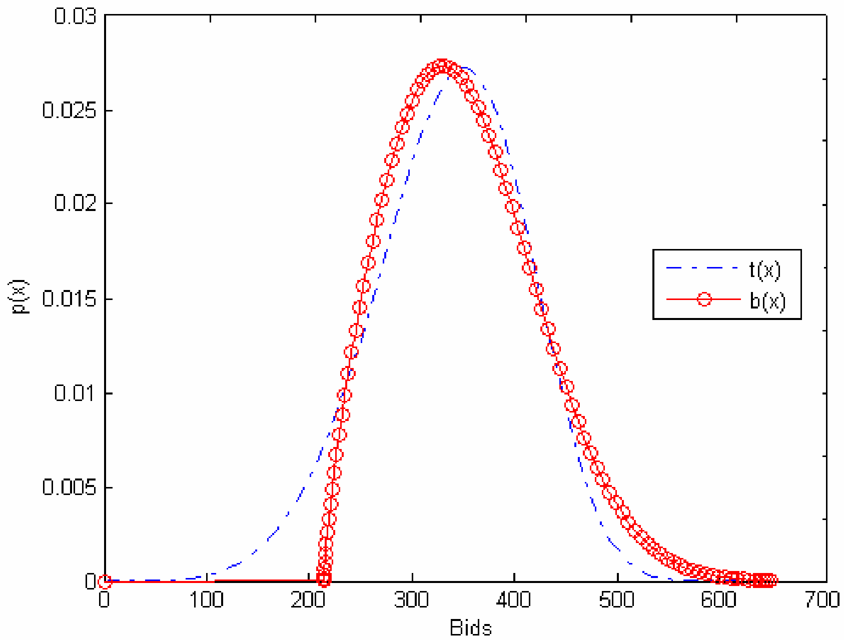

Figure 6.5. The fitted distribution of observed bids and the estimated bid distribution for City 1

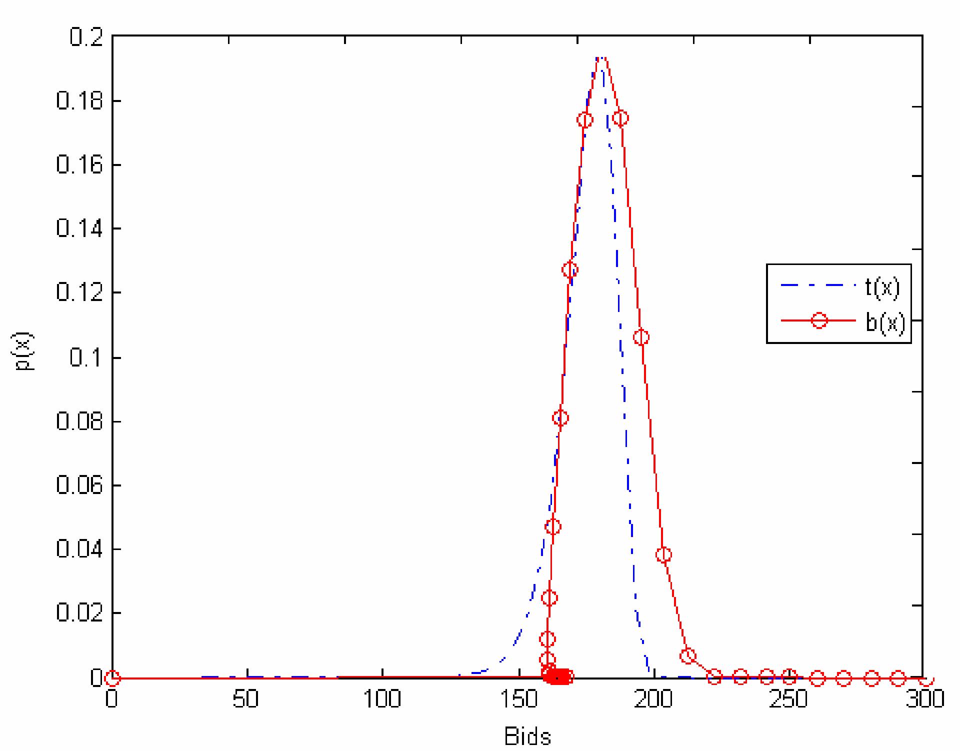

Figure 6.6. The fitted distribution of observed bids and the estimated bid distribution for City 41

Since the estimation method gives close bids to the real bids, if there were enough data available for all the transportation from Eskisehir to the other cities, the parameters of the cost distributions of the carriers for those cities could be calculated, and the model could be analyzed with the actual cost distributions. However, since we have very limited data, the analytical results are calculated with another type of cost distribution in Chapter 5.

Chapter 7

CONCLUSION

In this thesis, a logistics market, called as logistics center, where the orders from a number of shippers with goods to transport to various destinations are matched with carriers through a reverse auction is analyzed. Carriers are located in the logistics center and they give bids to orders that are opened by a shipper using an electronic reverse auction platform. A second-price sealed-bid auction is used to determine the winner which requires giving the order to the bidder with the lowest bid at the price equal to the second lowest bid.

The main objective of this study is to model the logistics market to analyze the effects of various system parameters, such as order and carrier arrival and abandonment rates, capacity of the system for carriers and orders, on the performance of the system.

This thesis is motivated by a logistics auction market, ESO Logistics Center, where a reverse auction is used to match carriers with shippers. The auctions that took place between December 2003 and May 2005 at the ESO LC are analyzed. By using the ESO LC database that includes detailed information about order arrivals, bid arrivals, bid values and the winning bids, the system parameters such as order arrivals, bid distributions and realized auction prices are estimated. Moreover, the numbers of bids – the average transportation price for the selected two cities are analyzed. It is found from the number of bids and the average transportation price that when there are high number of bidders in the system, the price of the services decrease.

In Turkey, there is an imbalance between the West-East and East-West traffic. Carriers usually go to eastern part of Turkey full and go back to their home cities empty, so the transportation prices from western parts to eastern parts of Turkey are higher than the reverse direction. In order to determine if this effect is true for the ESO LC, we analyzed the data separately for eastern and western parts; however, we did not get a meaningful result that supports this hypothesis. So, we concluded that the reason of the indifference between the transportation costs from the western part to the eastern part and from the eastern part to the western part may occur due to the fact that the ESO LC attracts a high number of in-transit carriers that have available capacity on their return trip to their bases and are willing to accept lower prices.

Modeling of the logistics center is achieved in two parts. First, the auction is modeled in a static setting to determine the auction price and the profit of the carriers based on the number of carriers engaging in the auction and their cost distributions. The auction used in the ESO LC is a first-price sealed-bid auction. From the Revenue Equivalence Theorem, under the appropriate assumptions the first-price and the second-price auctions are equal to each other in terms of revenue generated, so by satisfying these assumptions the second-price sealed-bid auction is used to determine the winner in the model proposed in this thesis. Since the optimal strategy of the bidders in a second-price sealed-bid auction is telling the truth about their valuations of the auctioned object or service, i.e. cost in our case, the auction price equals to the cost of the second lowest bidder. It is shown that the expected auction price is decreasing in the number of carriers engaging in the auction and also it depends on the carriers' cost distributions. In addition, the profit of the carriers decreases with the number of available carriers.

In the second part, a continuous time Markov chain model is developed to evaluate the performance of the system in a dynamic setting with random arrivals and possible abandonment of orders and carriers. It is assumed that there are two types of carriers that have different cost distributions available at the logistics center. Also, it is assumed that one type of carriers have lower costs than the other type. The order is given to the carrier with the lowest cost. Moreover, if there are no available carriers at the logistics center, the arriving order is registered to the system and waits until a carrier arrives to the system or the order abandons after waiting some time.

By combining these two models, various performance measures, such as the average expected auction price, the average expected profit of carriers, the average number of orders and carriers and the probability of rejecting an order and a carrier in the steady state are evaluated. It is found that the arrival and the abandonment rates of carriers and orders affect the performance of the system. In addition, the capacity of the system for carriers also affects the performance of the system.

Moreover, the average expected price is compared with the market price. It is shown that the reverse auction used in the logistics center significantly decreases the price. As the number of carriers increases over time, the price will continue to decrease.

Most of the logistics literature is devoted to long term contracts between shippers and carriers. The analysis of auction-based supply chain systems is very limited in the literature. The main contribution of this thesis is the modeling and analysis of an auction-based logistics market where a short term contract, i.e. a spot contract realized for only one full-truck transportation, is made between shippers and carriers. To our knowledge, this is the first study that models an auction-based logistics market with a continuous time Markov chain.

The other contribution is the estimation method that is constructed for defining cost distributions of the carriers from the observed bids. The proposed method handles a broad class of distribution types. The method is applied to the ESO LC case and the cost distributions of carriers for transportation from Eskisehir to two cities are obtained. It is shown that the method gives close results with the actual data.

As a conclusion, the reverse auction used in logistics significantly decreases the transportation cost. Since the system parameters affect the performance of the system, by changing the parameters the performance of the system can be increased.

In addition, the model gives insights about the effect of the capacity of carriers on the average price which provides to control the capacity of the system. As it is stated after some point an increase in the capacity does not affect the performance. By applying actual arrival rates to the model, the ESO LC can specify the optimal capacity.

Moreover, the distributions of the costs can be obtained by applying the proposed method, and then the system can be analyzed with these distributions by constructing different scenario analysis. The analysis will yield the average price and the savings that firms can obtain by using the system of the ESO LC. These savings can be used while determining the total savings over the total transaction volume.

An extension of this study is to relax the assumptions of the Revenue Equivalence Theorem, hence the assumptions of the auction. By relaxing the assumption of risk-neutrality of the bidders, the auction process can be analyzed with risk averse or risk seeking bidders which will change the bidding process.

In this thesis we assume there are two different types of carriers and the number of carriers that give bids is not revealed which helps to analyze the auction as a symmetric auction. As an extension, by assuming the number of carriers is revealed by the auctioneer, the auction process can be analyzed as an asymmetric auction.

Moreover, another attractive future work direction is relaxing the assumption of non-overlapping cost distributions of different types of carriers. Since the transitions from one state to another will change by considering overlapping distributions, analyzing the system under this assumption is not trivial.

Appendix A

Data Sheet for City 41

Order ID	Order Status	Order Arrival Date	City ID	Loading Type	Winning Bid ID
6	1	12/26/2003 10:15	41	1	NULL
12	3	1/8/2004 14:02	41	1	NULL
21	3	1/14/2004 8:52	41	1	NULL
221	3	4/8/2004 10:07	41	1	NULL
234	3	4/14/2004 9:04	41	1	NULL
247	3	4/15/2004 11:59	41	1	NULL
249	3	4/15/2004 12:04	41	1	NULL
279	3	4/17/2004 10:27	41	1	NULL
301	3	4/21/2004 10:16	41	1	NULL
303	7	4/21/2004 11:05	41	1	263
305	7	4/21/2004 11:38	41	1	266
313	3	4/22/2004 10:43	41	1	NULL
314	3	4/22/2004 10:45	41	1	NULL
321	7	4/23/2004 8:27	41	1	285
322	7	4/23/2004 8:40	41	1	287
325	7	4/23/2004 10:01	41	1	288
360	3	4/29/2004 9:10	41	1	NULL
361	3	4/29/2004 9:13	41	1	NULL
383	3	5/1/2004 10:21	41	1	NULL
405	7	5/6/2004 10:38	41	1	421
417	7	5/7/2004 12:11	41	1	443
428	7	5/8/2004 11:11	41	1	464
433	7	5/10/2004 10:52	41	1	483
436	7	5/10/2004 12:08	41	1	487
443	7	5/12/2004 11:17	41	1	518
444	7	5/12/2004 11:30	41	1	513
453	7	5/13/2004 11:50	41	1	537
457	3	5/13/2004 12:09	41	1	NULL
459	7	5/13/2004 13:13	41	1	548
465	7	5/14/2004 11:35	41	1	559
488	7	5/26/2004 11:55	41	1	590
495	3	5/27/2004 10:37	41	1	NULL
511	7	5/31/2004 9:25	41	1	617
515	7	6/1/2004 10:25	41	1	626
529	7	6/2/2004 11:00	41	1	640
530	7	6/2/2004 11:26	41	1	636
541	7	6/3/2004 10:56	41	1	654
542	7	6/3/2004 10:58	41	1	652
557	7	6/4/2004 13:24	41	1	678

Table D.1. Sample data sheet for City 41

Bid ID	*Order ID*	*Arrival Date*	*Amount*
250	297	4/20/2004 11:10	270
252	298	4/20/2004 13:02	300
253	298	4/20/2004 13:10	250
254	298	4/20/2004 13:13	240
255	298	4/20/2004 13:30	250
256	299	4/20/2004 13:55	975
257	299	4/20/2004 13:57	250
259	299	4/20/2004 14:40	190
260	300	4/21/2004 10:12	190
261	302	4/21/2004 11:05	400
267	302	4/21/2004 12:45	400
263	303	4/21/2004 11:21	175
264	304	4/21/2004 11:36	115
265	304	4/21/2004 11:56	240
266	305	4/21/2004 11:42	180
268	306	4/22/2004 8:55	250
269	306	4/22/2004 8:55	230
270	307	4/22/2004 9:05	230
271	306	4/22/2004 9:08	250
272	311	4/22/2004 10:46	340
280	317	4/22/2004 12:24	340
277	315	4/22/2004 11:16	500
275	312	4/22/2004 10:59	350
278	312	4/22/2004 11:18	270
281	318	4/22/2004 12:28	650
282	318	4/22/2004 12:42	800
283	318	4/22/2004 13:18	650
284	315	4/22/2004 13:52	325
285	321	4/23/2004 8:29	180
286	322	4/23/2004 9:14	180
287	322	4/23/2004 9:14	180
288	325	4/23/2004 10:02	180
289	324	4/23/2004 10:05	350
290	324	4/23/2004 10:18	340
291	326	4/23/2004 10:36	200
292	327	4/24/2004 8:22	1200
293	328	4/24/2004 8:22	450
294	329	4/24/2004 8:42	200
295	328	4/24/2004 8:43	500

Table D.2. Sample data sheet for bids

Order Status	*Explanation*
1	Active
2	Auction closed, waiting for confirmation
3	Cancelled
4	Confirmed
5	Delivered
6	Not delivered yet
7	Realized and filed in archive

Table D.3. Explanation of the order status

Appendix B

Proof of the Theorems

B.1. Proof of the Revenue Equivalence Theorem:

Since Revenue Equivalence Theorem is valid for both standard and reverse auctions, we can prove it for reverse auctions in the same way with Klemperer [37] as follows:

For simplicity, we focus on the "independent private values" model, in which n bidders compete for a single job. Bidder *i*'s cost for doing that job is v_i, and it is private information to her, but it is common knowledge that each v_i is independently drawn from the same continuous distribution $F(v)$ on $[\underline{v}, \bar{v}]$, with density $f(v)$. Consider any mechanism for allocating the job among *n* bidders. For this mechanism, and for a given bidder *i*, let $S_i(v)$ be the expected surplus that bidder *i* will obtain in equilibrium from participating in the mechanism and let $P_i(v)$ be her probability of getting the job in the equilibrium. So, $S_i(v)$ = *E*[*payment to the bidder by auctioneer*] – $vP_i(v)$. The following equation is the key:

$$S_i(v) \geq S_i(\tilde{v}) + (v - \tilde{v})P_i(\tilde{v}) \qquad \text{(A.1)}$$

The right-hand side is the surplus that player *i* would obtain if she had type *v* but deviated from equilibrium behavior, and instead followed the strategy that type $\tilde{v}$ of player *i* supposed to follow in the equilibrium of the game induced by the mechanism. So *v* gets the same utility that $\tilde{v}$ would get ($S_i(\tilde{v})$), except that in states in which $\tilde{v}$ would win the auction (which happens with probability $P_i(\tilde{v})$) type *v* values the job at $(v - \tilde{v})$ more than $\tilde{v}$ does, and so *v* obtains an extra $(v - \tilde{v})P_i(\tilde{v})$ more surplus in all. In equilibrium, *v* must prefer not to deviate from equilibrium behavior, so the left-hand side must (weakly) exceed the right-hand side. So, since type *v* mustn't want to mimic type *v*+*dv*, we have;

$$S_i(v) \geq S_i(v+dv) + (-dv)P_i(v+dv) \tag{A.2}$$

and since *v*+*dv* mustn't want to mimic type *v* we have;

$$S_i(v+dv) \geq S_i(v) + dvP_i(v) \tag{A.3}$$

Reorganizing (A.2) and (A.3) yields

$$P_i(v+dv) \geq \frac{S_i(v+dv) - S_i(v)}{dv} \geq P_i(v) \tag{A.4}$$

and taking the limit as $dv \to 0$ we obtain

$$\frac{dS_i}{dv} = P_i(v) \tag{A.5}$$

Integrating up,

$$S_i(v) = S_i(\underline{v}) + \int_{\underline{v}}^{\bar{v}} P_i(x)dx \tag{A.6}$$

Now consider any two mechanisms which have the same $S_i(\underline{v})$ and the same $P_i(v)$ functions for all *v* and for every player *i*. They have the same $S_i(v)$ functions. So, any given type, *v*, of player *i* makes the same expected payment in each of the two mechanisms. This means *i*'s expected payment averaged across her different types, *v*, is also the same for both mechanisms. Since this is true for all bidders, *i*, the mechanisms yield the same expected revenue for the auctioneer.

B.2. Proof of the Equivalence of the First-Price and Second-Price Auctions:

Also we can prove that first-price and second-price auctions are equivalent to each other with the same way with Klemperer [37] as follows;

There is a common equilibrium bidding strategy in which each buyer makes a bid b_i, which is a strictly decreasing function of his reservation value (cost) v_i, i.e.,

$$b_i = b(v_i) \quad i=1,\ldots,n \tag{A.7}$$

To examine the optimal choice of the buyer 1, let us begin by assuming his bid is $b(x)$. The amount he will be paid, p, given his own bid $b_1=b(x)$ and those by the other n-1 buyers is

$$p = p(b_1,b_2,\ldots,b_n) = p(b(x), b(v_2),\ldots, b(v_n)) \tag{A.8}$$

Therefore we can write the expected payment by buyer 1, given a bid of $b_1=b(x)$ as

$$P(x) = \underset{v_2,\ldots,v_n}{E}[p(b(x),b(v_2),\ldots,b(v_n))] \tag{A.9}$$

Also, the bid of $b(x)$ is the winning bid if and only if all other buyers have made higher bids, i.e. if all other valuations are greater than x. Since the probability buyer j has a reservation value higher than x is $1-F(x)$, buyer 1 wins with probability $[1-F(x)]^{n-1}$.

The expected gain of the winner, $\Pi(x,v_1)$, is the difference between the expected payment to the winner and cost of the winner times the probability that he will win the auction, and given as

$$\Pi(x,v_1) = P(x) - v_1[1-F(X)]^{n-1} \tag{A.10}$$

For $b(v)$ to be the equilibrium bidding strategy, buyer 1's optimal choice must be to select $x=v_1$ and bid $b(v_1)$. Then buyer 1's maximized expected gain is $\Pi(v_1, v_1)$ and the following first-order condition must be satisfied.

$$\frac{\partial \Pi}{\partial x}(x, v_1) = v_1 \frac{d}{dx}[1 - F(X)]^{n-1} - P'(x) = 0 \text{ at } x=v_1 \tag{A.11}$$

Setting $x = v_1$, it follows that the equilibrium expected payment to buyer 1 must satisfy the differential equation

$$P'(v_1) = v_1 \frac{d}{dv_1}[1 - F(v_1)]^{n-1} \tag{A.12}$$

Integrating and making use of a boundary condition, buyer 1's expected payment is therefore

$$P(v_1) = v_1[1 - F(v_1)]^{n-1} + \int_{v_1}^{\bar{v}} [1 - F(x)]^{n-1} dx \tag{A.13}$$

Since a buyer is paid if and only if he is the bidder with the lowest cost, his expected payment is

$$P(v) = Prob\{b(v) \text{ is the lowest bid}\} b(v) \tag{A.14}$$

But $b(v)$ is the lowest bid if and only if all other bidders have higher costs. Then $Prob\{b(v)$ is the lowest bid$\} = [1\text{-}F(v)]^{n-1}$ and then $b(v) = P(v)/[1\text{-}F(v)]^{n-1}$. By substituting $P(v)$ from Equation (A.14), we therefore have the equilibrium strategy of a typical buyer with cost v as

$$b(v) = v + \frac{\int_{v}^{\bar{v}} [1-F(x)]^{n-1}}{[1-F(v)]^{n-1}} \tag{A.15}$$

As a result a bidder with cost v will bid according to the equilibrium strategy (given in Equation [A.15]) which is the same as the second minimum of v_i with cdf of $F(.)$.

B.3. Proof of Theorem 1

Let $r_{(1)} \leq r_{(2)} \leq r_{(3)} \leq ... \leq r_{(l)}$ be the order statistics defined on the actual reservation prices and $F_{(n)}(.)$ and $f_{(n)}(.)$ be the cdf and pdf of $v_{(n)}$ respectively. Since the auction price will be the second minimum of bidders' valuations, we have to find the expected value of second minimum value of v_i. Then,

$$F_{(2)}(x) = P\{r_{(2)} \leq x\} = \sum_{i=2}^{b} \binom{b}{i} F^{i}(x)(1-F(x))^{b-i} \text{ where } F(x) = P\{r_i \leq x\}.$$

and

$$f_{(2)}(x) = \frac{\partial F_{(2)}(x)}{\partial x} = b(b-1)F(x)\left[1-F(x)\right]^{b-2} f(x) \text{ where } f(x) = \frac{dF(x)}{dx}$$

Finally, the expected auction price given l bidders in the auction is

$$p(l,b) = E\left[r_{(2)}\right] = \int_{\underline{r}}^{\bar{r}} x f_{(2)}(x) dx$$

$$= \underline{r} + \int_{\underline{r}}^{\bar{r}} \left[1-F(x)\right]^{b-1} bF(x) dx + \int_{\underline{r}}^{\bar{r}} \left[1-F(x)\right]^{b} dx \tag{A.16}$$

When there is no Type *B* carrier and *l* Type *L* carriers, the expected auction price is the second minimum of v_i's given *l* Type *L* carriers which is calculated with the same with Equation (A.16) and written as

$$p(l,0) = \underline{v} + \int_{\underline{v}}^{\bar{v}} \left[1 - F(x)\right]^{l-1} lF(x)dx + \int_{\underline{v}}^{\bar{v}} \left[1 - F(x)\right]^{l} dx \qquad \text{(A.17)}$$

Appendix C

Special Cases of the General Model

C.1. No Type *B* Carriers and without Capacity for Orders

In the general model we consider a logistics market with local and in-transit carriers. As a special case of the model we can consider a system with only local carriers. We analyzed this system in steady-state. Let the state of the system be the number of outstanding orders at time t: $S(t) = N_O(t) - N_L(t) + L$, $S(t)=0, 1, \ldots$. In this setting, we can write the performance measures in closed form.

By using the similar continuous time Markov chain we tried to find the steady-state probabilities and then analyze the performance measures. The expected auction price given in Equation (5.1) is valid for this case. If there is only one carrier, then the expected auction price is the market price. Also the waiting orders are taken by the first arriving carrier at a market price. Then the performance measures can be given as

$$P_{av} = \frac{\lambda_O \sum_{i=1}^{C_L-1} A(i)\left(\underline{v} + \frac{2(\bar{v}-\underline{v})}{C_L - i + 1}\right) + \lambda_L \bar{v} \sum_{i=C_L+1}^{\infty} D(i)}{\lambda_O \sum_{i=1}^{C_L-1} A(i) + \lambda_L \sum_{i=C_L+1}^{\infty} D(i)} \qquad \text{(A.18)}$$

where $A(i) = \prod_{j=0}^{i-1} \frac{\lambda_O + (C_L - j)\lambda_{LA}}{\lambda_L}$ and $D(i) = \prod_{j=1}^{i-C_L} \frac{\lambda_O}{\lambda_L + j\lambda_{OA}} \prod_{t=0}^{C_L-1} \frac{\lambda_O + (C_L - t)\lambda_{LA}}{\lambda_L}$

$$Q_{av} = \frac{\lambda_O \sum_{i=1}^{C_L-1} A(i) \frac{\bar{v}-\underline{v}}{C_L - i + 1} + \lambda_L \left(\frac{\bar{v}-\underline{v}}{2}\right) \sum_{i=C_L+1}^{\infty} D(i)}{\lambda_O \sum_{i=1}^{C_L-1} A(i) + \lambda_L \sum_{i=C_L+1}^{\infty} D(i)} \qquad \text{(A.19)}$$

$$L_{av} = \sum_{i=0}^{C_L} \pi_i (C_L - i) = \sum_{i=0}^{C_L} \pi_0 A(C_L - i) \tag{A.20}$$

$$O_{av} = \sum_{i=C_L}^{\infty} \pi_i (i - C_L) = \sum_{i=C_L}^{\infty} \pi_0 D(i - C_L) \tag{A.21}$$

$$R_L = \pi_0 \tag{A.22}$$

where $\pi_o = \dfrac{1}{\sum_{i=0}^{C_L} \prod_{j=0}^{i-1} \dfrac{\lambda_O + (C_L - j)\lambda_{LA}}{\lambda_L} + \sum_{i=C_L+1}^{\infty} \prod_{j=1}^{i-C_L} \dfrac{\lambda_O}{\lambda_L + j\lambda_{OA}} \prod_{k=0}^{C_L-1} \dfrac{\lambda_O + (C_L - k)\lambda_{LA}}{\lambda_L}}$

C.2. No Type *B* Carriers and without Abandonment

In the special case with no Type *B* carriers, waiting carriers can abandon or the waiting orders can be cancelled after waiting some time. As another special case, we assume once arriving to the center the carriers do not abandon, and also the orders are not cancelled.

We analyze this system in steady-state. The state of the system is still the number of outstanding orders at time *t*: $S(t) = N_O(t) - N_L(t) + L$, $S(t)=0, 1, \ldots$. In this setting we can find closed form steady-state probabilities.

$$\pi_i = \frac{(1-\rho)\rho^i}{1-\rho^{C_L + C_O + 1}} \tag{A.23}$$

where $\rho = \dfrac{\lambda_o}{\lambda_l}$.

The expected auction price given in Equation (4.1) is also valid for this case. Then the performance measures can be given as

$$P_{av} = \frac{\lambda_O (1-\rho)\sum_{i=0}^{C_L-1} \rho^i \left(\underline{v} + \frac{2(\bar{v}-\underline{v})}{C_L - i + 1} \right) + \lambda_L \bar{v} (\rho^{C_L+1} - \rho^{C_L+C_O+1})}{\lambda_O (1-\rho^{C_L}) + \lambda_L (\rho^{C_L+1} - \rho^{C_L+C_O+1})}$$

$$= \frac{\lambda_O \left(\underline{v}(1-\rho^{C_L}) + 2(\bar{v}-\underline{v})(1-\rho)\sum_{i=0}^{C_L-1} \frac{\rho^i}{C_L - i + 1}) \right) + \lambda_L \bar{v} (\rho^{C_L+1} - \rho^{C_L+C_O+1})}{\lambda_O (1-\rho^{C_L}) + \lambda_L (\rho^{C_L+1} + \rho^{C_L+C_O+1})} \quad \text{(A.24)}$$

$$Q_{av} = \frac{\lambda_O (1-\rho)(\bar{v}-\underline{v})\sum_{i=0}^{C_L-1} \frac{\rho^i}{C_L - i + 1} + \lambda_L (\rho^{C_L+1} - \rho^{C_L+C_O+1})(\frac{\bar{v}-\underline{v}}{2})}{\lambda_O (1-\rho^{C_L}) + \lambda_L (\rho^{C_L+1} - \rho^{C_L+C_O+1})} \quad \text{(A.25)}$$

$$L_{av} = \sum_{i=0}^{C_L} \pi_i (C_L - i) = \sum_{i=0}^{C_L} \frac{1-\rho}{1-\rho^{C_L+C_O+1}} \rho^i (C_L - i) = \frac{C_L(1-\rho) - \rho(1-\rho^{C_L})}{(1-\rho)(1-\rho^{C_L+C_O+1})} \quad \text{(A.26)}$$

$$O_{av} = \sum_{i=C_L}^{\infty} \pi_i (i - C_L) = \sum_{i=C_L}^{\infty} (\frac{1-\rho}{1-\rho^{C_L+C_O+1}}) \rho^i (i - C_L) = \frac{\rho^{C_L+1}}{(1-\rho)(1-\rho^{C_L+C_O+1})} \quad \text{(A.27)}$$

$$M_L = \pi_0 = \frac{1-\rho}{1-\rho^{C_L+C_O+1}}, \quad M_O = \pi_{C_L+C_O} = \frac{(1-\rho)\rho^{C_L+C_O}}{1-\rho^{C_L+C_O+1}} \quad \text{(A.28)}$$

We have solved the average expected auction price and profit for different values of capacities and rates, and also simulate the system. We take $\bar{v}$ as 100 euro and $\bar{v}-\underline{v}$ as 10 euro. The capacity for carriers and orders is denoted by C_L and C_O respectively. The results are given in Table A.1.

C_L	C_O	ρ	P_{av}			Q_{av}		
			Model	Simulation		Model	Simulation	
				Average	95% C.I.		Average	95% C.I.
5	5	1/2	94.3278	92.3204	(94.3093 94.3316)	2.1638	2.1548	(2.1464 2.1632)
10	5	1/2	92.0676	92.0675	(92.0616 92.0734)	1.0338	1.0337	(1.0294 1.0380)
20	5	1/2	91.0062	91.0064	(91.0035 91.0093)	0.5031	0.5026	(0.5005 0.5047)
50	5	1/2	90.4003	90.4001	(90.3989 90.4013)	0.2002	0.2014	(0.2006 0.2023)
5	**10**	1/2	94.3332	94.3252	(94.3140 94.3364)	2.1666	2.1608	(2.1524 2.1693)
5	**20**	1/2	94.3333	94.3264	(94.3153 94.3375)	2.1667	2.1622	(2.1537 2.1706)
5	**50**	1/2	94.3333	94.3262	(94.3151 94.3374)	2.1667	2.162	(2.1536 2.1704)
5	5	2/3	95.2929	95.2855	(95.2721 95.2989)	2.6464	2.6397	(2.6294 2.6499)
10	5	2/3	92.5054	92.5104	(92.5018 92.5190)	1.2527	1.2552	(1.2493 1.2661)
20	5	2/3	91.0824	91.0842	(91.0808 91.0877)	0.5412	0.5436	(0.5412 0.5460)
50	5	2/3	90.4093	90.4091	(90.4079 90.4103)	0.2047	0.2053	(0.2044 0.2061)
5	**10**	2/3	95.3639	95.3694	(93.3559 95.3829)	2.6819	2.6832	(2.6729 2.6935)
5	**20**	2/3	95.3743	95.3653	(95.3518 95.3787)	2.6872	2.6832	(2.6729 2.6935)
5	**50**	2/3	95.3745	95.3708	(95.3573 95.3843)	2.6872	2.6923	(2.6820 2.7027)
5	5	1/4	93.6016	93.6019	(93.5897 93.6142)	1.8008	1.8022	(1.7927 1.8118)
10	5	1/4	91.884	91.8864	(91.8792 91.8936)	0.942	0.9425	(0.9371 0.9479)
20	5	1/4	90.9688	90.9664	(90.9624 90.9703)	0.4844	0.4831	(0.4802 0.4859)
50	5	1/4	90.3948	90.394	(90.3924 90.3957)	0.1974	0.1977	(0.1965 0.1989)
5	**10**	1/4	93.6016	93.609	(93.5968 93.6212)	1.8008	1.8016	(1.7920 1.8111)
5	**20**	1/4	93.6016	93.6061	(93.5940 93.6183)	1.8008	1.7975	(1.7880 1.8070)
5	**50**	1/4	93.6016	93.6034	(93.5911 93.6156)	1.8008	1.801	(1.7914 1.8106)

Table A.1. The average expected auction price and the profit of the carriers for Special Case 2

The average number of carriers and orders, and the probability of rejection of carriers and orders can be calculated by using above equations. Table A.2. reports the values of parameters and analytical and simulation results of the performance measures.

C_L	C_O	ρ	L_{av}		O_{av}		R_L		R_O	
			Model	**Simulation**	**Model**	**Simulation**	**Model**	**Simulation**	**Model**	**Simulation**
5	5	1/2	4.0332	4.0355	0.0313	0.0267	0.5002	0.5007	0.0004	0.0005
10	5	1/2	9.0011	9.0067	0.0009	0.0009	0.5000	0.5005	0.0000	0.0000
20	5	1/2	19.0000	19.0003	0.0000	0.0000	0.5000	0.5012	0.0000	0.0000
50	5	1/2	49.0000	48.9929	00.000	0.0000	0.5000	0.4997	0.0000	0.0000
5	**10**	1/2	4.0313	4.0323	0.0312	0.0334	0.5000	0.4996	0.0000	0.0000
5	**20**	1/2	4.0313	4.0365	0.0312	0.0294	0.5000	0.5001	0.0000	0.0000
5	**50**	1/2	4.0313	4.0322	0.0312	0.0299	0.5000	0.5010	0.0000	0.0000
5	5	2/3	3.3015	3.3045	0.2665	0.2669	0.3372	0.3365	0.0058	0.0061
10	5	2/3	8.0469	8.0510	0.0347	0.0368	0.3338	0.3321	0.0007	0.0007
20	5	2/3	18.001	18.0020	0.0006	0.0006	0.3333	0.3332	0.0000	0.0000
50	5	2/3	48.0000	47.9897	0.0000	0.0000	0.3333	0.3345	0.0000	0.0000
5	**10**	2/3	3.2684	3.2597	0.2638	0.2665	0.3338	0.3322	0.0007	0.0007
5	**20**	2/3	3.2634	3.2586	0.2634	0.2645	0.3333	0.3312	0.0000	0.0000
5	**50**	2/3	3.2634	3.2584	0.2634	0.2644	0.3333	0.3341	0.0000	0.0000
5	5	1/4	4.6667	4.6775	0.0003	0.0003	0.7500	0.7497	0.0000	0.0000
10	5	1/4	9.6667	9.6710	0.0000	0.0000	0.7500	0.7487	0.0000	0.0000
20	5	1/4	19.6670	19.6680	0.0000	0.0000	0.7500	0.7503	0.0000	0.0000
50	5	1/4	49.6670	49.7010	0.0000	0.0000	0.7500	0.7510	0.0000	0.0000
5	**10**	1/4	4.6667	4.6686	0.0003	0.0003	0.7500	0.7498	0.0000	0.0000
5	**20**	1/4	4.6667	4.6675	0.0003	0.0003	0.7500	0.7504	0.0000	0.0000
5	**50**	1/4	4.6667	4.6654	0.0003	0.0003	0.7500	0.7511	0.0000	0.0000

Table A.2. The average number of carriers and orders in steady state for Special Case 2.

APPENDIX D

Static versus Dynamic Environment

We compare the average expected price in static setting and dynamic setting. Table D.1 shows the average expected prices for different number of carriers joining an auction when there is only one type of carriers.

$\overline{N}_L$	P_{av}	
	Dynamic	***Static***
3	95.29	95
8	92.5	92.2
18	91.08	91.05
48	90.4	90.4

Table D.1. P_{av} in static and dynamic setting with one type of carrier

As seen in Table D.1, the average expected prices in static and dynamic settings are close to each other when there is only one type of carrier. In addition we compare the average expected price when there are two types of carriers and give the values in Table D.2.

$\overline{N}_B$	$\overline{N}_L$	P_{av}	
		Dynamic	***Static***
1	3	82.37	76.6
2	4	78.5	75
3	4	88.49	80

Table D.2. P_{av} in static and dynamic setting with two types of carriers

As seen in Table D.2, the average expected price significantly differs in static and dynamic setting. Therefore, the system should be analyzed in dynamic setting to get more realistic results.

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