Accounting for Customer Reactions
to
Customer Relationship Management
Initiatives

by

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ABSTRACT

Customer Relationship Management (CRM) is a holistic approach that unifies all points of customer interaction via various investments in people, technology and business processes. Cross-selling is a CRM tool that enables the companies to sell additional, complementary, or related products to the customers. The main focus of our study is on the implications of cross-selling in terms of customer reactions and especially the negative reactions. We investigate the impact of customer reactions on customer lifetime value to the firm and the firm's cross-selling policy to the customer. Under specific assumptions on how the cross-sell attempts affect a customer's arrival rate, churn/attrition rate and his/her likelihood to accept the offer provided by the company, we try to understand how the optimal policy on whether to cross-sell or not at each customer encounter will change. To this end in this thesis we propose a Markovian model. The model investigates the negative effect of cross-selling activities with a state space of number of cross-sell attempt failures and number of customer contacts when the additional number of products offered is limitless. The problem is being explored numerically using dynamic programming tools. Analytical results are also given on the values generated. The extension of the current model incorporates the positive impact of cross-selling. The model used in this case has additional information of number of cross-sell successes. However, in this case the number of additional products offered is limited. This model, though, is investigated only by numerical analysis.

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TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

NOMENCLATURE

CLV	Customer Lifetime Value
CRM	Customer Relationship Management
MDP	Markov Decision Process
MC	Markov Chain
RFM	Recency-Frequency-Monetary

Chapter 1

INTRODUCTION

Conventional marketing efforts are based on ‘Place’, ‘Product’, ‘Price’ and ‘Promotions’ in order to gain more of the market share and therefore increase profitability. However, the face of marketing is changing to become more customer-centered, and in today’s world profitability is not enough by itself [11]. In today’s world, because attracting new customers became more expensive than retaining the profitable customers, the primary objective is to increase the share of wallet from the profitable and loyal customers [17]. Especially in fast-moving and dynamic industries, loyal customers can be considered as the main assets of the firm, since the products change constantly, but the customers remain. In addition to this, long-term customers are generally more efficient to work with because of their familiarity with the firm and also they provide free advertising via word-of-mouth [19], [37]. To this end, relationship marketing is concerned with retaining profitable customers.

To come up with the marketing strategies related to customer retention, metrics such as customer’s lifetime value can be used. Customer lifetime value (CLV) for a company is the entire stream of purchases s/he makes from that company. Customer equity, on the other hand, is the sum of the acquisition cost and CLV [24], [3]. The key drivers for the customer equity are value equity, which encompasses price, quality and convenience, brand equity, which relies on image and meaning, and finally relationship or retention equity, which includes special programs to keep the customer loyal [26]. The calculation of these metrics can be obtained via customer databases, which should be formed by properly selected information about the customers [13].

To create more value, the firms should understand the stages of a relationship with the customer, which are Awareness, Exploration, Expansion, Commitment and Dissolution. For the continuity of a relationship formed between a firm and a customer, satisfaction for both parties play a vital role. From the customer’s perspective interactions with the personnel, the core service and also the organization in general should be satisfactory [2]. From the firm's perspective the customer should be profitable.

The relationship between customer and firm can be affected by many factors, which can cause feelings of satisfaction, dissatisfaction, and anger for the customer. Even though the relationship between satisfaction and loyalty varies across industries and competitive situations, as satisfaction increases loyalty also increases [24]. A customer is dissatisfied in cases of lack of service quality, service failure, and when the customer's expectations are not met. When the customer gets angry, customer is less likely to re-purchase again [6]. Whenever the customer's expectations are met and exceeded, customer is satisfied from the relationship. However, it is shown that negative events have a greater impact in a relationship context [47]. The negative impact of a bad experience should not be taken as irreversible, though. Kalyanaram and Winer [35] showed that customers typically have a short-term memory. Therefore, through a service recovery process, the relationships can be retained and in some cases even improved [30].

Focusing on customer retention may at least double the gross income, due to the future revenues to be generated from the customers [45]. The companies should try to retain the customers by investing time, personnel and by putting forth special programs as long as they provide a positive value, especially to the core customers that provide the most profit [17], [5]. In this sense, customer relationship management (CRM) provides a holistic approach that unifies all points of customer interaction via various investments in people, technology and business processes.

Using CRM applications, firms may affect customer's view on perceived quality and thus satisfaction. As Mithas et al. [31] also empirically showed using the data on CRM applications on a cross-section of large US firms during the 2001-2002 period, CRM applications improve firms' knowledge on customers and this in return affects the customers' satisfaction levels especially for the firms with greater supply chain integration.

A customer's decision on purchase and repurchase depends on the products/services offered, firm's brand and his/her relationship with the firm. To prevent customer defections there are many tools used, such as loyalty programs. However, a firm should not only try to retain its customers but also try to increase loyalty such that the customer equity also increases. To achieve this, firms may try to increase the relationship duration with the customers, and/or increase their profits by cross-selling, up-selling or add-on selling, or they can decrease their acquisition costs. Cross-selling enables the companies to sell to the customers additional, complementary, or related accessories or products during or just after their primary purchase so that the number of products sold increases, whereas up-selling is the technique of selling additional products to customers who only planned to buy one, or upgrading the original order. Therefore it leads to charging higher prices and reducing marginal cost of production. The add-on selling is different from cross-selling such that the firm can try to sell unrelated products as well [5]. Cross-selling also enables the firm to reduce churn [38]. Because as the firm succeeds the cross-selling activities, the switching cost for the customer increases. In addition to this, Kamakura et al. [23] also pointed out that cross-selling allows the firm to learn more about customer preferences and behaviors.

The main focus of our study is on improving customer equity by increasing revenues using cross-selling, up-selling and add-on selling. For the sake of unity we will mention only cross-selling strategy throughout the thesis. Too much cross-selling activities can cause customer annoyance [23]. As a result an annoyed customer may stop working with the company for good as long as s/he has alternatives in the market, or her/his likelihood to accept another offer during a future transaction may decrease, or his/her utilization of company services/products might be less. If the customer accepts and becomes satisfied with the offer, s/he may work with the firm more frequently, and her/his likelihood to accept other offers in the future might increase as her/his trust toward the firm increases. Previous research on cross-selling, up-selling and add-on selling did not model the implications of these strategies in terms of customer reactions. In our study, we model customer reactions, giving more emphasis on the negative reactions.

In this study, we assumed that the contacts are initiated by the customers. That is, contrary to direct mailing and outbound telemarketing cases the customer comes to the store/e-store or calls the call center. A service is provided to the customer whenever the customer contacts the firm. Because of this assumption, we explicitly assume that the relationship between a customer and a firm begins from the Exploration stage in our model, meaning that the customer is already aware of the firm and its products. This implies that the acquisition cost in our model is zero. We postulate that the customer is already satisfied with the firm's core service/product, and the firm itself, so even though the firm does not attempt any cross-selling activities, the customer continues to contact the firm. When the customer accepts the additional offers though, s/he may contact more frequently or may be more reluctant to quit her/his relationship with the firm. On the contrary, when the customer faces too many cross-selling attempts, this is annoying for the customer causing a negative impact on the service quality of the firm in customer's perception. However, as this negative impact is not due to a serious service failure and as the customers tend to have a relatively short-term memory, our model assumes a recovery period for the annoyed customer as well.

Our study is compromised of two parts. We mainly focus on the negative impacts of cross-selling providing both numerical and analytical analysis. Then, we incorporate the positive impacts to the model, which is investigated via only numerical analysis. In the setting, to which the models are applied, we use customer initiated contacts, even though, cross-selling is an applicable action in the contexts of direct mailing, and outbound telemarketing as well. Direct mailing has been so heavily used that response rates have started to drop significantly. Similarly, outbound telemarketing became intrusive to many customers, causing a decrease of response rates in many sectors. On the other hand, during the inbound calls, customers feel in control and therefore are more open to sale opportunities when compared to other methods [21].

We model the value of a single customer who is representative of the firm's customer base, or a certain segment of it. We use a Markovian model, which allows us to see the effects of cross-selling in terms of retention. The arrival rate in the model reflects the rate of contacts initiated by the customer. Death rate reflects the rate of termination of relationship with the firm, which can be as a result of a shift in the customer's perception, view or experience with the firm and/or the products/services offered.

The question explored in this study is the following: How do the special treatments to the customer in terms of cross-selling, up-selling, and add-on selling efforts affect the customer's retention and the revenue obtained during the entire relationship of the customer with the firm? To this end, we develop a Markov Decision Process (MDP) model and analyze optimal cross-selling policies to maximize the revenue generated from the customer throughout the lifetime of the customer based on two models. The first model is recency based and investigates the negative reactions and the second model explores the positive reactions on long-term basis. Both models operate in a transactional setting, at which the parameters change and the decisions are given at each transaction point initiated by the customer.

The rest of the paper is organized as follows. In Chapter 2, literature review on cross-selling, customer satisfaction and customer lifetime values and the models developed are given. Chapter 3 describes the cross-selling model where the information on the customer is limited to cross-sell attempt failure and total number of contacts. Whenever there is a successful cross-sell attempt in the system, the model is renewed so that the number of failures and the number of contacts become zero. These are followed by the analytical results related to the monotonicity of the value function and the computational experiments employed to test the performance of different policies upon application of cross-selling. To explore the impact of negative reactions, we defined a function such that the initial contacts in the relationship are more important, and whenever failures occur, the negative reactions increase, unless there occurs a renewal in the system. As the number of failures increase the values generated from the customer decrease. If the firm chooses not to use cross-selling activities, the function defined for failures allows for recovery period.

Because the model proposed is transaction based, we expect to find that the parameters that affect the instantaneous outcomes have the greater impact on both revenue generation and optimal policies. For this reason, when we investigate the negative reaction on failure probability, we expect to find the optimal policies to be more dynamic until the trust of the customer is gained. For all cases implored, as the failure probability increases, the system will be more reluctant to apply cross-selling activities.

In Chapter 4, two models are given with computational experiments, one with renewal in terms of failures and contacts whenever there is a successful cross-sell, which is also our primary concern among the two models given in this chapter and one by keeping all the information throughout the lifetime of the customer.

In this chapter, we incorporate both the negative and the positive reactions into the model with renewal in terms of failures and contacts when there is a success. We investigate the positive impact that leads to trust and commitment via mid-term and long-term parameters, such as contact rate and death rate and the negative impact via failure probability. Because of the transactional based nature of the models, we expect to find the potential loss by decreasing likelihood to accept offers offsets the potential gains via increased contact rate or decreased death rate.

Chapter 5 summarizes what has been done so far and compares two of the models described previously, finally giving the future research directions.

Chapter 2

LITERATURE REVIEW

This section reviews the prior research relevant to understanding the factors influencing the customer lifetime value (CLV) and also the research on how to incorporate effects of satisfaction and RFM (recency-frequency-monetary) framework into the CLV models as well as cross-selling activities.

As defined earlier, CLV is the total amount of profit a company can obtain from a customer during their relationship. CLV calculations are proved to be useful in pricing strategy problems, customer selection, media selection and balancing acquisition and retention expenses to prevent retaining unprofitable customers. [3], [43], [5], [15], [4], [46].

The CLV is calculated by projecting the expected net cash flows of a customer over time. The factors affecting the CLV are the duration of active relationship with the customer, related sales of new products and services and also the positive referrals of the satisfied customers. Satisfaction is related to customer loyalty, which is directly related to profit and growth [19]. For this reason, in our study, we investigate the effects of the additional sales in terms of customer reactions reflecting dissatisfaction and satisfaction in a Markovian setting on the CLV.

To estimate the cash flow of a customer, firms need to know which customers are still active in their databases. Schmittlein, Morrison and Colombo [39] calculated the probability that a customer still continues the relationship with the company with a combined purchase event and duration model (Pareto/NBD model), which will be referred as SMC model from this point on. Their calculations are individual based and their model is descriptive. In the NBD model, each customer makes Poisson purchases with rate λ. In the Pareto model, each customer's duration of relationship with the firm is exponential, i.e., Poisson events happen to trigger the end of the relationship. In our study, we model the situation in which the customer quits the relationship as a death. We use the Pareto/NDB model assumptions for the customer's arrival and death rates. Therefore the probability that the customer is active given the purchase history in the SMC model is equivalent to the probability that the customer is alive for our model. Therefore, our model computes the values generated by a customer as long as s/he is alive.

Later on Schmittlein and Peterson [40] calculated the expected dollar volume purchases based on SMC model. Fader et al. [16] proposed the beta-geometric/NBD (BG/NBD) model to pinpoint customer behavior. This model assumes customer dropouts occur immediately after a purchase and therefore is easier to implement when compared to the SMC model. When purchase frequency is not very low, BG/NBD is found to be very robust by empirical analysis.

Once the firm clears its database free of the customers that quit the relationship, the firm can calculate the customer equity based on the CLVs. CLV calculations differ for different customer types. There are primarily two main models on customer behavior that are built upon the work of Jackson [22], who grouped customers in industrial setting according to their behaviors so that proper marketing strategies could be chosen. An always-a-share type of customer can easily switch between vendors. This type of customers is found in sectors such as commodity chemicals, apartment building owners, buyers of customer terminals, mailing services and shipping services. A lost-for-good type of customer commits permanently to a vendor due to high switching costs and once the commitment is broken the customer is taken to be lost-for-good. This type of customers is found in sectors such as office automation systems, heavy construction equipments, magazine fulfillment services and aircraft engines. Dwyer [14] generalized these models to apply to all consumer markets. The lost-for-good type of behavior formed the basis for retention model. Even if some old customers return, in this model they are treated as new customers. The always-a-share type of behavior formed the basis of the migration model. In migration model, the possibility of repeat purchase is predicted by the recency of last purchase. In our model, once the customer is dead, the relationship ends. For this reason, it is a retention model. However, we also incorporated the repeat purchase behavior using the likelihood to buy as well.

Based on the migration and retention models proposed by Dwyer, Berger and Nasr [3] proposes five CLV calculations, four of which are based on cases with customer retention model and one is based on customer migration model.

Bolton [7] proposes a retention model to determine the duration of relationship between customer and the firm, in which duration times change for each customer with satisfaction and experience. Furthermore, based on the experiences customer satisfaction levels change in a cumulative manner. The model is supported with data obtained from a mobile phone operator service that provides continuous service to customers. It is empirically shown that for each customer when cumulative satisfaction level is high, the relationship duration is longer. The perceived loss affects the relationship duration negatively, however the opposite could not be found looking at the data for the perceived gains. In our model we also let the death rate, which determines the relationship duration, increase due to cumulative dissatisfaction levels while investigating the negative impact of cross-selling activities. While investigating the positive impact, we let the death rate decrease, increasing the lifetime, based on cumulative satisfaction levels.

Bolton's findings show that as customers gain more experience, the effect of prior customer satisfaction increases. The heterogeneity across customers leads to different tolerance levels for failures and costs. For this reason while working on customer reactions individuals or carefully selected segments of homogeneous customers should be investigated. In our study, as parameters may change for each customer, our model is taken to be individual based.

Pfeifer and Carraway [34] introduce Markov Chain (MC) models in CLV calculations based on the works of Blattberg and Deighton, and Berger and Nasr. MCs can handle both the customer retention and migration models, incorporate uncertainties surrounding the customer relationships, enable decision making, and also work nicely with the RFM (Recency-Frequency-Monetary) framework of marketing. RFM-based MC models can be formed by using purchase probabilities, net contribution from the customer and re-marketing expenditures. The authors apply the MCs to a migration model. In our study, we use the Markov Decision Processes (MDP) for customer retention case, which takes into account the purchase probabilities of a customer, as well as decision making part.

Reinartz and Kumar [36] empirically investigated the nature of customer lifetime duration and profitability for a migration model based on the SMC model, where the data is from a US catalog retailer with observable birth rates, purchases and defection rates. The authors found that the customer is more likely to continue the relationship with the company more frequently and with a larger probability of being alive. The customer lifetime duration does not explain the company profitability by itself and the CLV is more important in terms of profitability.

Lemon et al. [25] claims that expected future use and anticipated regret should be incorporated to the retention model as well as the satisfaction. To test the hypothesis about future considerations, the authors used data coming from an interactive TV service, where customers choose to subscribe and decide at their usage levels, and for which they pay a monthly fee. As a result, consumers are found to be forward-looking in terms of their decisions on continuing the service and this explains why some customers drop the service despite the fact that they are satisfied with it.

For a comparison of main CLV calculations based on retention and migration models, as well as algebraic and matrix computing methods, we refer the interested reader to the paper of Calciu and Salerno [10].

SMC model provided a framework to model the transitions among the phases of relationship development process. Oded et al. [32] similarly proposes a Hidden Markov Model, in which the Markovian states represent the relationship-level of customer and brand/company. Based on the interaction history, transitions among relationship states occur. This model enables companies to segment their customers and to evaluate the effectiveness of different marketing activities on moving the customer to higher relationship states. In this case the hidden Markov process describes the relationship transitions. A Hidden Markov Model is used when the stochastic process of interest can be observed through another stochastic processes.

Ho et al. [20] extend the SMC model by including the satisfaction in their model of predicting customer lifetime value, and characterize the optimal level of satisfaction based on the total expected dollar spending. It is shown that as customer satisfaction increases, the total number of purchases also increases. When the customer satisfaction is taken to be dependent on the last visit's outcome in terms of satisfied or not, a Hidden Markov Model similar to Oded et al. is used.

Ching et al. [12] proposes a stochastic dynamic programming model to capture the customer behavior. The authors solve the model to obtain the optimal allocation of promotion budget for maximizing the CLV in infinite and finite horizon cases.

Predictions of customer behavior in terms of what they will buy and when are not accurate given the investments on marketing activities, databases and new technology. Still it is important to understand customer behavior. Even though communicating with the customer in an excessive manner is costly for the firm and annoying for the customer, communicating too little is also harmful for the firm [44]. In our study, we try to balance this in terms of cross-selling activities of the firm.

The impact of cross-selling to firm's operations and profitability was analyzed by Akşin and Harker [1]. They show that cross-selling task can be undesirable for the service employees. Güneş and Akşin [18] modeled incentive contracts for the service employees who should cross-sell to a certain type of customer. Örmeci and Akşin [33] considered dynamic cross-sell policies. We also consider dynamic cross-selling policies. But we assume there is no incentive problem and the servers implement the firm's optimal policy correctly. Here we focus on the behavior of the customers in terms of reactions to cross-sell attempts.

Li, Sun and Wilcox [28] made an empirical study of cross-selling sequentially ordered products for financial services. They find that the overall satisfaction with a bank increases the bank's ability to sell products to existing customers. The higher the overall satisfaction with a bank for the customer is the higher the switching costs for the customer. They propose to model the type of usage, the flow of funds and cost to examine the cross-selling impact as a future research topic. In our case, we deal with the effects of cross-selling on retention, satisfaction and customer profitability for the firm.

We use dynamic programming applied to Markov process to understand the effects of cross-selling offers in customer lifetime value, death rate, arrival rate, and the likelihood of accepting the offer of each customer individually. We use a controlled transactional model to investigate the customer response given the place and the method of purchase to be the inbound calls of customers, time and frequency of purchases coming from exponential distributions. Discrete transactions may turn into durable relations through experiences that shape the expectations [15], [9]. For this reason, despite the transactional nature of our model we reflect the customer behaviors in a cumulative manner.

The possible reactions to cross-selling activities can be either annoyance or satisfaction. Therefore, the firm tries to make the best decision on value generation by taking into account the customer reaction. In this sense, the future expectations of the customer is incorporated into our model implicitly. We also incorporated the fact that customers may quit the relationship independent of the action the firm takes with the death rate in the system. Therefore, our model is different from the above mentioned models as it incorporates negative and positive customer reactions related to cross-selling activities with a dynamic control on the actions chosen by the firm with a forward looking structure.

Chapter 3

A DYNAMIC CROSS-SELLING MODEL WITH 2-D STATE SPACE WITH RENEWALS

In this section we present the model formulated to reflect customer reactions to cross-selling attempts of a firm and its possible effects on the expected value obtained from the customer over the customer's lifetime. The relationship duration between the customer and the firm changes for each customer individually and is considered to be the lifetime of the customer. The customer is considered to be alive as long as she has an active relationship with the firm. When the customer quits the relationship, she is considered to be dead.

We try to capture the value generated based on the customer reactions to CRM initiatives on an individual basis. The firm can decide to offer an additional product/service to the customer. We consider the relationship with the customer as 'lost for good' once the customer quits the relationship with the firm and therefore we deal with a retention model.

Call centers may provide an example for the proposed model. Call-centers are traditionally considered to be one of the cost-centers in a company, but they are increasingly being used to generate revenues and to increase the CLV. Cross-selling may be used for revenue generation purposes. Consider a call center, in which there is only in-bound calls. Customers may call for service or to buy a product. The firm receives a fixed revenue from each call, either due to the fact that a service request is handled or a product or service is sold. There is a possibility of an additional revenue from cross-selling at each in-bound call as well. However, it is not obvious how cross-selling attempts of the call-center will affect the customer, whether she is annoyed or rather satisfied from the attention and care.

Insurance companies may also provide an example. In this case, consider a customer contacting the firm as a precaution for the future. After the relationship with the customer begins with a contract on any insurance policy, additional revenue may be generated proposing different insurance policies and/or expanding the conditions of the current contract of the customer.

In the same manner, magazine and newspaper subscriptions may provide an example. Consider a customer contacting a firm to subscribe to a specific magazine and in return offered an additional magazine related to the customer, i.e. let the customer contact the firm for a scientific magazine and if the customer has children, the firm in return may offer a scientific magazine for children in addition to the customer's desired magazine. The relationship between the customer and the firm can be considered to be progressing through a set of stochastic transitions based on customer reactions in each of the above examples.

We use a Markov Decision (MD) Model, which is a powerful tool for analyzing probabilistic sequential decision processes. The control policy is formed upon the decision of the firm on whether to cross-sell to the customer or not. The performance of this control policy is measured by the expected value generated from the customer until the relationship between the customer and the firm ends. Our solution approach is the value-iteration algorithm that uses a recursive solution approach from dynamic programming and computes the maximum discounted profit.

In the following, we define the model and formulate it. In order to understand the impact of customer reactions, we first analyze a base case, where there is no customer reaction to cross-selling efforts. After that, we analyze customer reactions in terms of a change in the willingness to accept a cross-sell offer, in the frequency of contacts, and in the rate of quitting the relationship.

3.1 Schematic Representation of the Model

In our model a transaction occurs whenever the customer contacts the firm. A customer contacts the firm according to a Poisson process with rate λ and remains alive for a lifetime that is exponentially distributed with a death rate of μ. Each time a customer contact takes place, a revenue R is generated, and then the firm decides to attempt to cross-sell to the customer or not. If the decision is to attempt to cross-sell, then a cost of attempt, c_a, is incurred to the firm. Cost of attempt may include the training cost of an employee or the capacity cost per attempt. The customer declines the offer with probability P_f, in which case an additional cost of failure, c_f, is incurred. Cost of failure can be taken as the immediate cost of annoyance of the customer signaling how the dissatisfaction level of the customer affects the relationship. Otherwise, if the customer accepts the offer, which happens with probability $(1-P_f)$, a cross-sell revenue of r is generated, which is greater then the attempt cost. The revenues generated for possible outcomes can be seen in Figure 3.1.

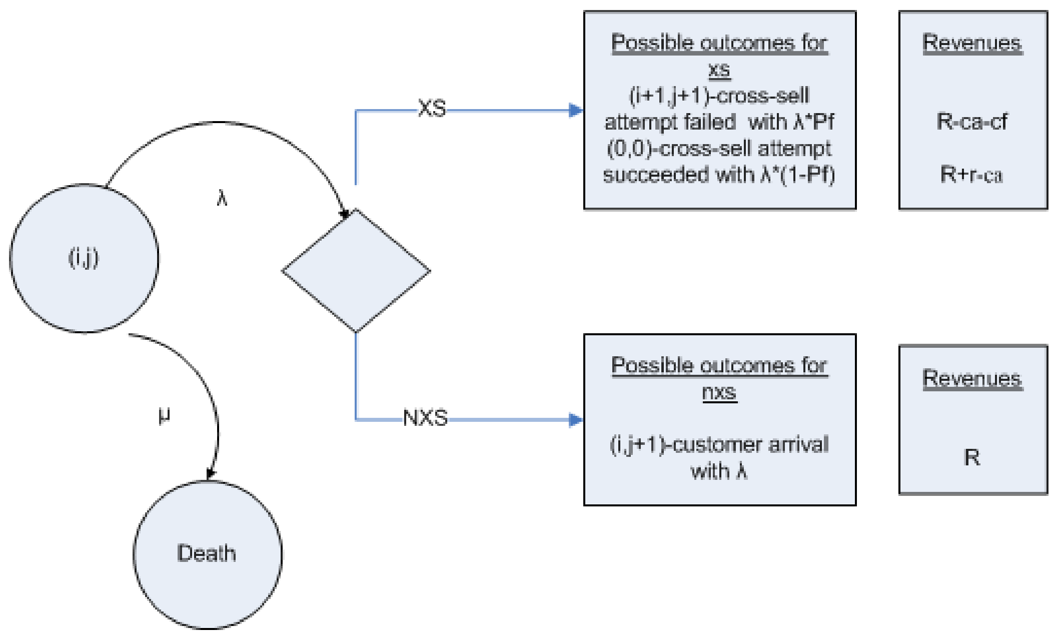

Figure 3.1: Possible Outcomes of the Model

The objective is to observe the effect of cross-sell attempts. We assume that failures have a cumulative effect, and when a customer declines an offer, this indicates annoyance or dissatisfaction. For this reason we want to keep track of the failures, and include the number of failures in the state representation.

The state space is taken to be infinite. The states are defined as (i, j), where i is the number of unsuccessful cross-sell attempts since the last successful cross-sell, and j is the number of the contacts initiated by the customer since the last successful cross-sell. Therefore, the state of the system at any time is represented by (i, j), where $i \leq j$ for all i and j. As shown in Figure 3.1, the transition among states is such that each time the customer contacts the firm, the number of contacts (j) increases by 1 unless there is a successful cross-sell attempt, and whenever the cross-sell attempt results in failure, the number of unsuccessful cross-sell attempts (i) also increases by 1. However, whenever the cross-sell attempt results in success, the system starts over from the initial state $(0, 0)$. As a result, the failures have a cumulative effect, but a successful sale cleans the cumulative

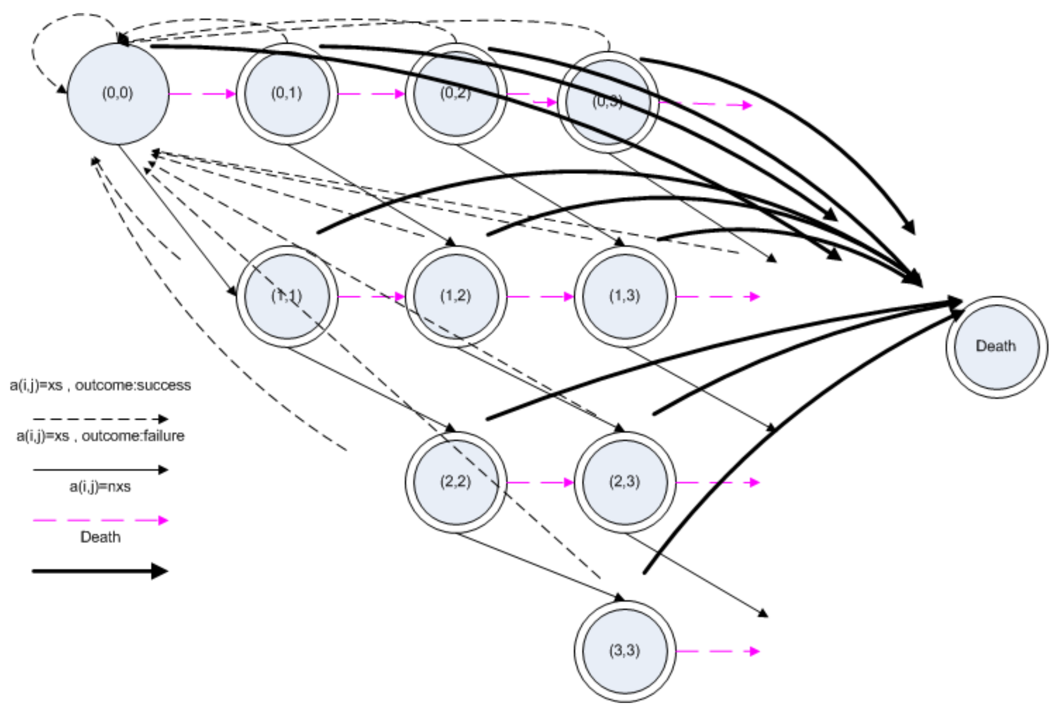

Figure 3.2: All possible paths a customer may take depending on action $a(i, j)$

effect of failures causing the system to be renewed. There is also a possibility of customer death independent of the firm's action. Based on the action chosen and the reaction of the customer, the customer may follow different paths during her relationship with the firm as shown in Figure 3.2.

3.2 Model Formulation

As described verbally above, the firm's decision on cross-selling activities may affect the customer behavior. The decision points are whenever the customer contacts the firm. We model the value generation on an individual basis. Given the parameter definition in Section 3.1, we can characterize the MDP as follows: State space is defined by $S = \{(i, j) : i = 0, 1, 2, ..., j; j = i, i + 1, ..., D\}$. The planning horizon ends when the customer quits the relationship with the firm, which is shown with state D. The action set in each state has two options, $a(i, j) = (xs, nxs)$, where xs represents the cross-sell attempt option of the firm and nxs represents the not attempt to cross-sell option. At each decision epoch, the expected net revenue is defined by the action chosen. For a particular state

(i, j), let $a(i, j) = nxs$. Then the expected revenue is the standard profit obtained from the customer R and due to the customer contact the state of the system changes from (i, j) to $(i, j+1)$. However for $a(i, j) = xs$, if the customer accepts the offer an extra revenue of $r - c_a$, as $r > c_a$, is gained and the system is renewed meaning that the state becomes $(0, 0)$. If the customer rejects the offer a cost of $c_a + c_f$ is incurred and the state of the system becomes $(i+1, j+1)$ as seen in Figure 3.1. Therefore:

$$revenue = \begin{cases} R + r - c_a & \text{with probability } 1 - P_f \\ R - c_a - c_f & \text{with probability } P_f \end{cases}$$

Let $v(i, j)$ be the value function that gives the profit earned at state (i, j) based on the action chosen by the firm. D is defined previously as the death state. $v(D)$ is the profit obtained from the state where the relationship between the firm and the customer ends, and therefore is zero. Based on these, the basic value function is defined as follows:

$$v(i,j) = R + \alpha\mu v(D) +$$
$$\alpha\lambda \max \begin{Bmatrix} (P_f(v(i+1, j+1) - c_a - c_f) + (1 - P_f)(v(0,0) + r - c_a)) \\ v(i, j+1) \end{Bmatrix}$$

A policy, π is a sequence of decision rules and basically forms a prescription for the firm whether to cross-sell to the customer or not at each decision epoch. A particular policy π is obtained based when a customer's expected value throughout his/her lifetime is maximized.

The exponentiality of the customer contacts and lifetimes allows us to use uniformization (see Lippman 1975 [29]). Using both uniformization and normalization we can turn the rates into probabilities. To adjust the time scale in cases, where we analyze how death rate, contact rate and failure probability of a cross-sell offer changes with the action chosen, we add a dummy rate, μ, with which at a particular state of the system, (i, j), the system remains in the same state, (i, j). Therefore, the above value function becomes:

$$v(i,j) = R + \alpha\mu v(D) + \alpha\mu v(i,j) +$$
$$\alpha\lambda \max \begin{Bmatrix} (P_f(v(i+1, j+1) - c_a - c_f) + (1 - P_f)(v(0,0) + r - c_a)) \\ v(i, j+1) \end{Bmatrix}$$

with

$$2\mu + \lambda = 1.$$

3.2.1 Base Case

We first describe the base case we consider. This is a simple model, where neither the decisions of the firm nor the reactions of the customer affect the parameters of the system. The base case is important since it sets a simple benchmark for the systems to be considered later on.

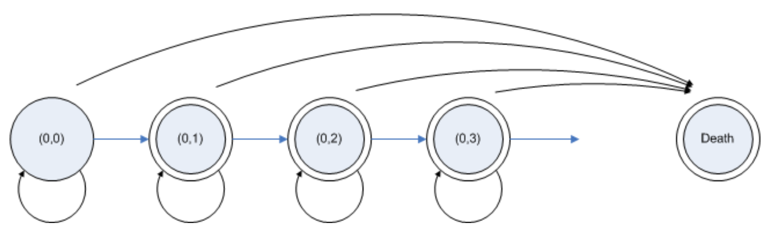

Figure 3.3: Transition Probabilities when $a(i, j) = nxs$

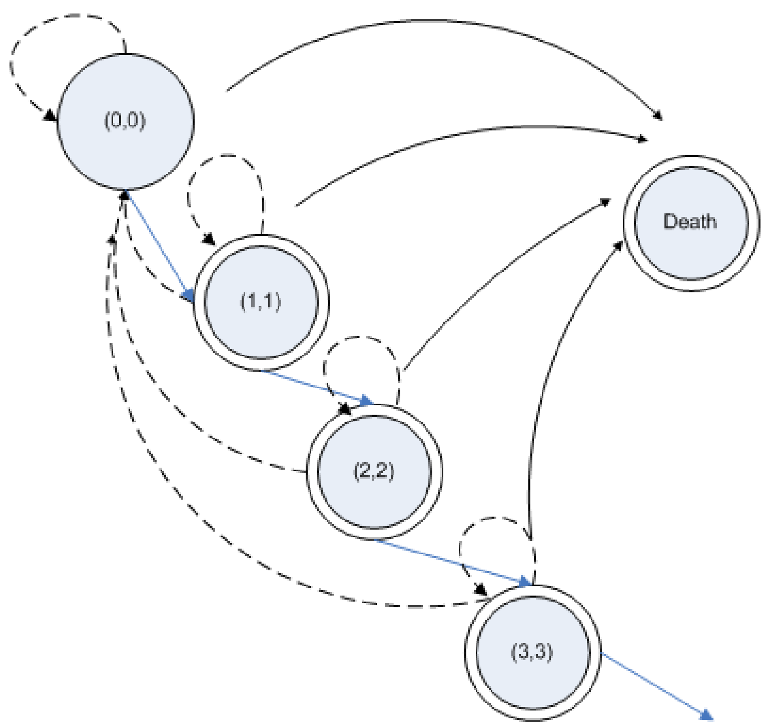

Figure 3.4: Transition Probabilities when $a(i, j) = xs$

We assume that the customer lifetime with the firm, T, is a random variable with an exponential distribution with rate μ. The customer initiates the contacts with the firm, where the time between the two contacts is also exponential with rate λ. By using uniformization and normalization, we turn the death and arrival rate of a customer into probabilities. Possible transitions differ for each action chosen and can be seen in Figure 3.3 when the action chosen is not to cross-sell and in Figure 3.4 when the action chosen is to cross-sell at state (i, j). In case of a cross-sell attempt the states that the system can go from state (i, j) are: $(0, 0)$ if success, $(i+1, j+1)$ if failure and (D) if death. In case of not cross-sell, the system can go from state (i, j) to death (D), or $(i, j+1)$.

Whenever the customer hits the state of death (D), s/he leaves the system. Therefore, $v(D) = 0$.

Proposition 1 *There exists a threshold value P_f^* such that, for any P_f, the optimal policy, π^*, is defined as follows:*

$$\pi^* = \begin{cases} \textit{always } xs & \textit{if } P_f < P_f^* \\ \textit{never } nxs & o.w. \end{cases}$$

The threshold value is given as, $P_f^* = \frac{r - c_a}{r + c_f}$. The optimal path the system follows is given in Figure 3.5 when the optimal policy π^* is never to cross-sell and in Figure 3.6 when the optimal policy π^* is always to cross-sell.

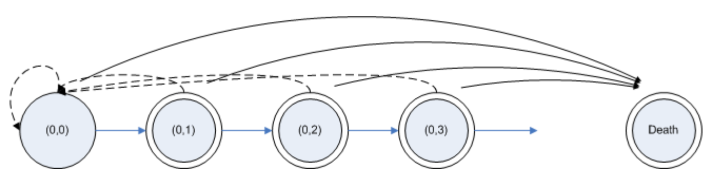

Figure 3.5: Optimal Path when $\pi^* = \textit{never nxs}$

Proof. *Since the parameters do not change over time, the optimal decision at any contact point will be optimal for all contact points. Hence, at each contact, the firm compares two actions to cross-sell or not to cross-sell:*

$$\text{Revenue of cross} - \text{sell} = R - c_a - P_f c_f + (1 - P_f) r$$

$$\text{Revenue of not to cross} - \text{sell} = R.$$

Then, simple algebra shows that the threshold value is $P_f^* = \frac{r - c_a}{r + c_f}$

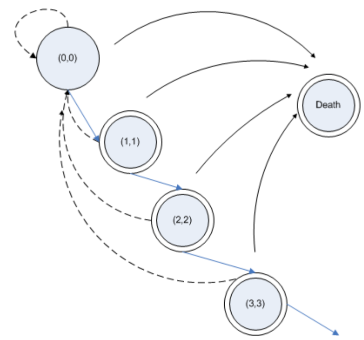

Figure 3.6: Optimal Path when $\pi^* = always\ xs$

In addition to the additional revenue obtained from cross-selling, cost of attempt and cost of failure have direct impact on the threshold value of the base case. When $r = c_a$, there is no incentive for the firm to bother with cross-selling even if the failure probability is zero. However, when $r + c_f$ value is really big compared to $r - c_a$ value the threshold becomes zero, meaning that the firm never attempts to cross-sell to the customer. When both of the costs are very small, say close to zero, then the firm always tries to cross-sell to the customer, as the additional revenue obtained will be quite large.

For the base case, as none of the parameters change over time or depending on the action chosen by the firm, it can be concluded that the optimal policy either always cross-sells or never cross-sells through the lifetime of a customer. Using the optimal policy, the average revenue between two contacts and the average revenue during a lifetime of a customer can be computed easily. In fact, it is straightforward to find the total expected discounted revenue.

$$v(0,0) = \begin{cases} \frac{R+\alpha\lambda(r-c_a-P_f(c_f+r))}{1-\alpha\lambda} & if\ \pi^* = xs\ \forall i,j \\ \frac{R}{1-\alpha\lambda} & if\ \pi^* = nxs\ \forall i,j \end{cases}$$

As the $v(0,0)$ values for both cases imply, as long as $r > \frac{c_a+P_f c_f}{1-P_f}$, the revenue obtained from cross-selling will be higher then the revenue obtained from not cross-selling. When the $P_f = P_f^*$ the $v(0,0)$ values for both cases become equal to $\frac{R}{1-\alpha\lambda}$. Also, since we model the negative reaction of the customer in the following cases, $v(0,0)$ values obtained for the base case is always better than the others.

3.2.2 Negative Reaction I: Increasing Death Rate (μ)

In this section, we consider the case in which the customer reactions to cross-sell attempts affect the customer lifetimes. If a firm attempts to cross-sell but fails, this causes an annoyance on the customer, affecting his/her relationship duration with the firm. This reaction is modeled as an increase in the expected death rate of the customer with each unsuccessful cross-sell attempt.

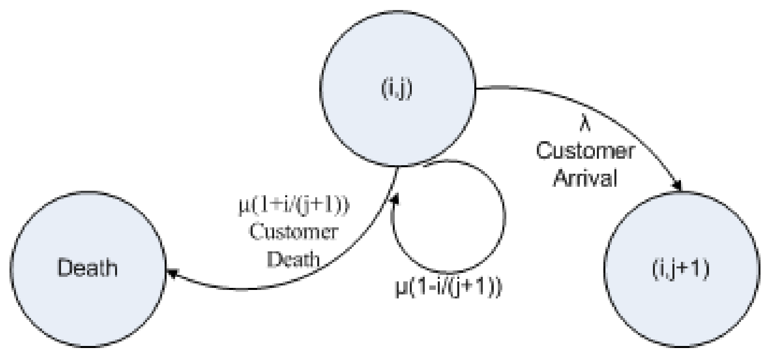

Figure 3.7: Transition Probabilities when $a(i,j) = nxs$

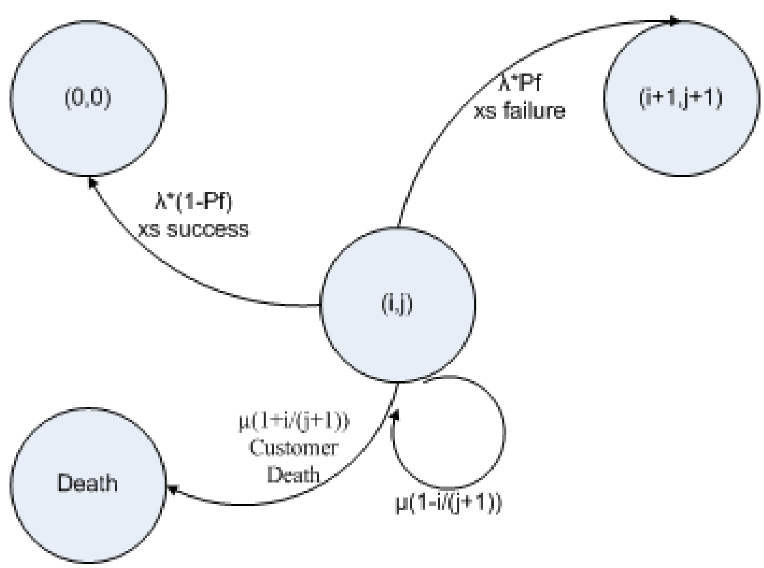

Figure 3.8: Transition Probabilities when $a(i,j) = xs$

We reformulated the value equation so that the death rate of the customer depends on the state of the system. State dependent death rates will enable us to understand the cumulative effect of cross-sell attempt failures on a customer over time. In our dynamic model we let the death rate change depending on the state of the system as

$$\mu(i,j) = \mu(1 + \frac{i}{j+1}).$$

The above formula ensures that as the unsuccessful cross-sell attempts increase, the probability of a customer to quit the relationship also increases. Besides with each successful cross-sell, the system is renewed, and the death rate becomes the initial value μ. This formula also ensures that without a successful cross-sell, the annoyance or the dissatisfaction caused by previous unsuccessful cross-sells are never erased. Once a cross-sell attempt is failed, even though the company never offers to cross-sell later on, the customer does not forget unless a success on cross-sell occurs.

For the varying death rate case, the transition probabilities for each action chosen are shown in Figure 3.7 when the action chosen is not to cross-sell and in Figure 3.8 when the action chosen is to cross-sell.

Then, the value function can be written as follows:

$$v(i,j) = R + \alpha\mu(1 + \tfrac{i}{j+1})v(D) + \alpha\mu(1 - \tfrac{i}{j+1})v(i,j) +$$

$$\alpha\lambda \max \left\{ \begin{array}{l} (P_f(v(i+1,j+1) - c_a - c_f) + (1-P_f)(v(0,0) + r - c_a)) \\ v(i,j+1) \end{array} \right\}$$

3.2.3 Negative Reaction II: Decreasing Contact Rate (λ)

Apart from the death rate of the customer, the annoyance of the customer due to cross-sell offers can be reflected in his/her incoming frequency. As the cross-sell attempt failures increase, the customer's contact rate decreases. We modeled this relationship by letting the contact rate change depending on the state of the system as:

$$\lambda(i,j) = \lambda(1 - \frac{i}{j+1}).$$

The above formula ensures that unless there is a renewal as a result of a successful cross-sell, the contact rate of the customer decreases with each failed cross-sell attempt. Even if after a single failed cross-sell attempt, the firm chooses never to attempt to cross-sell, the negative effect, in terms of the decreased contact rate, can not be erased totally. It can be considered as the customer's dissatisfaction can never be removed unless the firm succeeds to satisfy the customer with another offer.

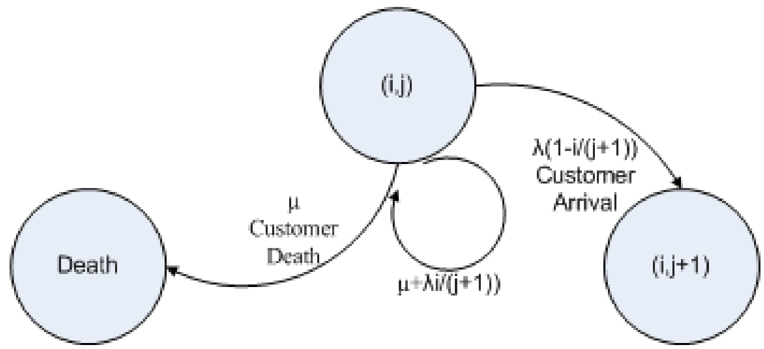

Figure 3.9: Transition Probabilities when $a(i,j) = nxs$

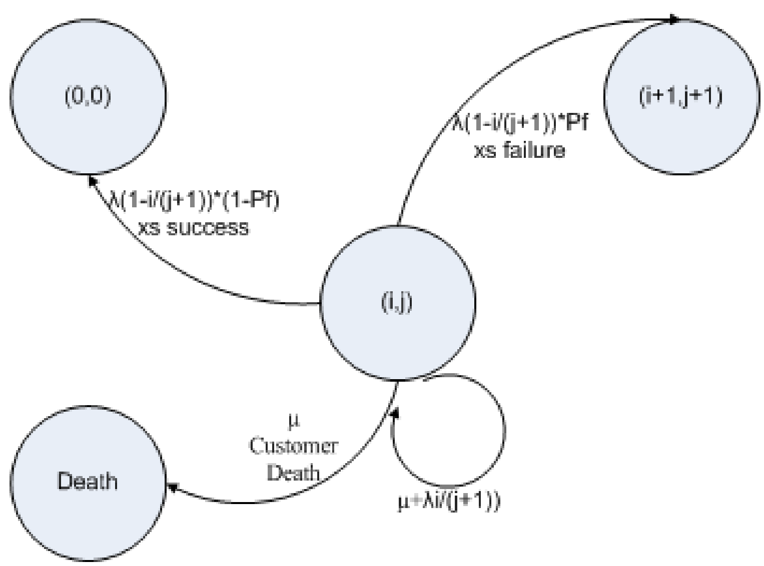

Figure 3.10: Transition Probabilities when $a(i,j) = xs$

We normalize the equations to turn the rates into probabilities. Figure 3.9 shows the transition probabilities among states where the action chosen is not to cross-sell and Figure 3.10 shows the transition probabilities when the action chosen is to cross-sell at some state (i, j).

In this case, the value function is found as follows:

$$v(i,j) = R + \alpha\mu v(D) + \alpha(\mu + \tfrac{\lambda i}{j+1})v(i,j) +$$

$$\alpha\lambda(1 - \tfrac{i}{j+1}) \max \left\{ \begin{array}{l} (P_f(v(i+1, j+1) - c_a - c_f) + (1 - P_f)(v(0,0) + r - c_a)) \\ v(i, j+1) \end{array} \right\}$$

3.2.4 Negative Reaction III: Increasing Death Rate (μ) & Decreasing Contact Rate (λ)

The annoyance of a customer due to cross-sell offers may be reflected both in his/her arrival rate and death rate at the same time. The number of times the customer contacts the company may decrease as well as his/her relationship duration with the company based on the state of the customer as in section 3.2.2 and 3.2.3. The death and arrival rates are normalized so that they represent probabilities in the value function equation that is given below:

$$v(i,j) = R + \alpha\mu(1 + \tfrac{i}{j+1})v(D) + \alpha(\mu - \mu\tfrac{i}{j+1} + \lambda\tfrac{i}{j+1})v(i,j) +$$

$$\alpha\lambda(1 - \tfrac{i}{j+1}) \max \left\{ \begin{array}{l} (P_f(v(i+1, j+1) - c_a - c_f) + (1 - P_f)(v(0,0) + r - c_a)) \\ v(i, j+1) \end{array} \right\}$$

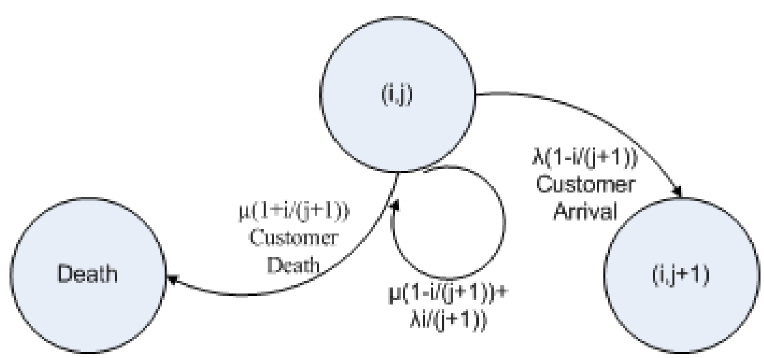

Figure 3.11: Trantision Probabilities when $a(i, j) = nxs$

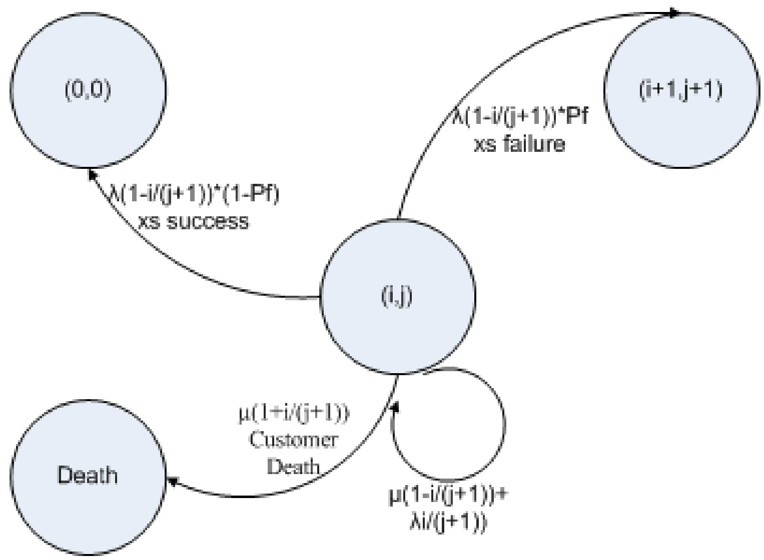

Figure 3.12: Transition Probabilities when $a(i,j) = xs$

The transition probabilities for any state (i,j) is given in Figure 3.11 when the action chosen is not to cross-sell and in Figure 3.12 when the action chosen is to cross-sell. For each possible action, the probabilities add up to 1.

3.2.5 Negative Reaction IV: Increasing Failure Probability (P_f)

In this case, a customer's likelihood to accept a cross-sell offer changes based on the state of the system. We change the failure probability from a constant P_f value to a state-dependent one, where

$$P_f(i,j) = P_f(1 + \frac{i}{j+1}).$$

In this case the customer's likelihood to accept a cross-sell decreases as the number of unsuccessful cross-sell attempts increase. This reflects the fact that if the customer refuses a cross-sell attempt before, his/her likelihood to accept it now gets lower. If the cross-sell attempt results in a success then the system is renewed, so that the failure probability becomes equal to the initial failure probability, which is the minimum within the system. If the customer does not accept the cross-sell offer, the annoyance caused by the offer can never be erased unless a renewal occurs.

The transition probabilities from a certain state (i,j) to other possible states for the action chosen as not to cross-sell is given in Figure 3.13 and for the action chosen as to

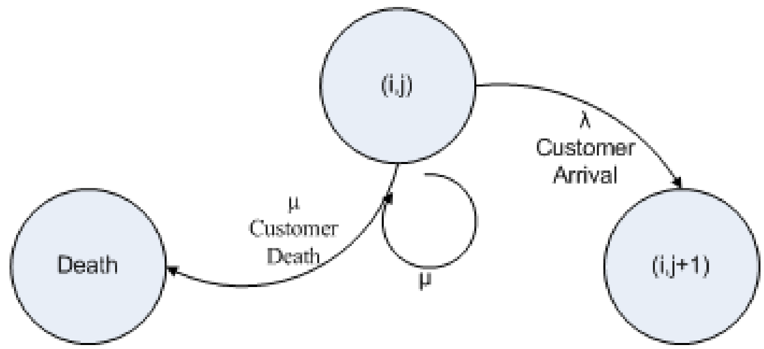

Figure 3.13: Transition Probabilities when $a(i, j) = nxs$

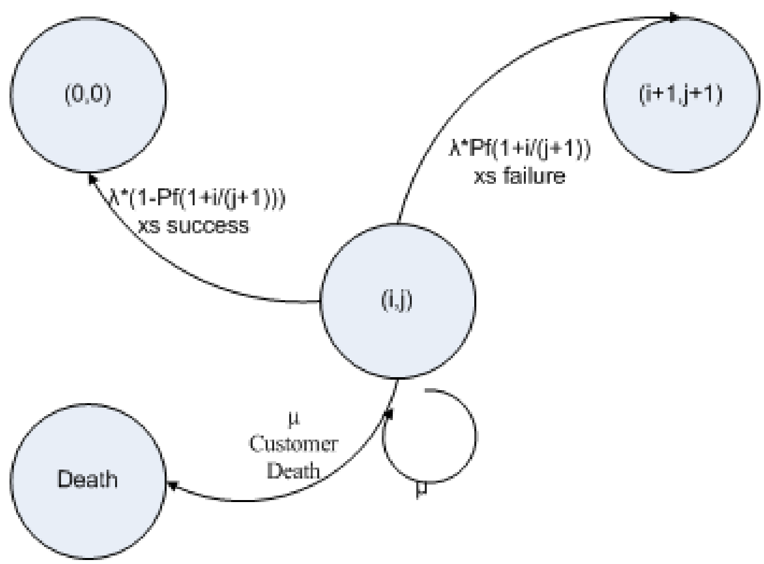

Figure 3.14: Transition Probabilities when $a(i, j) = xs$

cross-sell is given in Figure 3.14. The value function can be written as follows:

$$v(i,j) = R + \alpha\mu v(D) + \alpha\mu v(i,j) +$$

$$\alpha\lambda \max \left\{ \begin{array}{l} (P_f(1+\frac{i}{j+1})(v(i+1,j+1) - c_a - c_f) + (1 - P_f(1+\frac{i}{j+1}))(v(0,0) + r - c_a) \\ v(i,j+1) \end{array} \right\}$$

3.3 Theoretical Results

When the customer always accepts the cross-sell offer, meaning that the failure probability is zero, the firm should definitely choose to cross-sell to the customer at each contact, as long as the additional income obtained from cross-selling is greater than its costs. Because $v(i,j)$ is the maximal expected α-discounted reward of a system at state (i,j), where there is no bound on the possible number of interactions, the firm should decide to cross-sell to the customer when the failure probability is zero. This should be true for the varying contact and death rates as well.

Proposition 2 *If $P_f = 0$, then*

$$v(0,0) + r - c_a > v(i,j+1)\ \forall i,j$$

is true, and the optimal policy is always to cross-sell.

Proof.

We want to show that the optimal policy in this case is always to cross-sell if the above condition holds. We use the induction hypothesis to show this, where $v_n(i,j)$ is the value function for the finite horizon with n interactions remaining. As n goes to infinity, $v_n(i,j)$ becomes $v(i,j)$. We assume that for n interactions the inequality $v_n(0,0)+r-c_a-v_n(i,j+1) > 0$ holds. We check for the $n+1^{th}$ interaction. When the inequality is true for n interaction, and we write the equation for $v_{n+1}(0,0)$, we know that the optimal decision is to cross-sell because of the induction hypothesis. This is also true for $v_{n+1}(i,j+1)$. Based on these, we only need to check when both $v_{n+1}(0,0)$ *and* $v_{n+1}(i,j+1)$ has the optimal policy of to cross-sell. Below, we prove that the inequality holds for varying contact rate and death rate.

- **Decreasing contact rate $\lambda(i,j)$:**

$$\begin{aligned}
& v_{n+1}(0,0)+r-c_a-v_{n+1}(i,j+1)= \\
& R+\alpha\mu v_n(0,0)+\alpha\lambda\left(v_n(0,0)+r-c_a\right)+r-c_a \\
& -R-\alpha(\mu+\frac{\lambda i}{j+2})v_n(i,j+1)-\alpha\lambda(1-\frac{i}{j+2})(v_n(0,0)+r-c_a) \\
= \;& \alpha\mu(v_n(0,0)+r-c_a-v_n(i,j+1)) \\
& +\frac{\alpha\lambda i}{j+2}(v_n(0,0)+r-c_a-v_n(i,j+1)) \\
& +(1-\alpha\mu)(r-c_a) \\
\geq \;& 0
\end{aligned}$$

where the equalities follow as a consequence of the attempt to cross-sell action and by some algebra. The first two parts of the final inequality hold due to the induction hypotheses, where $v_n(0,0)+r-c_a-v_n(i,j+1)\geq 0$. We also know that $r-c_a$ value is greater than zero in all times, and also $1-\alpha\mu>0$. Hence, the first inequality is true for all i,j.

- **Increasing death rate $\mu(i,j)$:**

$$\begin{aligned}
& v_{n+1}(0,0)+r-c_a-v_{n+1}(i,j+1)= \\
& R+\alpha\mu v_n(0,0)+\alpha\lambda\left(v_n(0,0)+r-c_a\right) \\
& +r-c_a \\
& -R-\alpha\mu(1-\frac{i}{j+2})v_n(i,j+1)-\alpha\lambda(v_n(0,0)+r-c_a) \\
= \;& \alpha\mu(v_n(0,0)+r-c_a-v_n(i,j+1)) \\
& +\frac{\alpha\mu i}{j+2}v_n(i,j+1) \\
& +(1-\alpha\mu)(r-c_a) \\
\geq \;& 0
\end{aligned}$$

where the equalities follow as a consequence of the attempt to cross-sell action and by some algebra. The first part of the final inequality holds due to the induction hypotheses, where $v_n(0,0)+r-c_a-v_n(i,j+1)\geq 0$. Every expected value generated from a customer is at least zero. We also know that $r-c_a$ value is greater than zero in all times, and also $1-\alpha\mu>0$. Hence, the first inequality is true for all i,j.

Given the same number of contacts by a customer, if the number of failures of cross-sell attempts increases, we expect to have lower revenues due to the annoyance of the customer. In the same manner given the same number of cross-sell attempt failures, when the number of customer contacts increases, we expect to have more revenues as with each contact the affect of the previous failures fades, even if not completely disappears unless a renewal occurs. Proposition 3 defines this relationship between the revenues obtained from a customer. We expect this relationship to hold when the contact rate, death rate and probability of failure are varying as well. Below we formally present this result and also present the proof.

Proposition 3 *For the models with decreasing contact rate $\lambda(i,j)$, increasing death rate $\mu(i,j)$, and increasing failure probability $P_f(i,j)$, $v(i+1,j) \leq v(i,j) \leq v(i,j+1)$ for all i,j.*

Proof.

We will prove the statement first for the model with varying contact rate $\lambda(i,j)$, then for the model varying death rate $\mu(i,j)$ and finally for varying failure probability $P_f(i,j)$. In all our proofs, we will use induction on the number of interactions, n. For this we define $v_n(i,j)$ as the value function for the finite horizon, so that $v_n(i,j)$ is the maximal expected α-discounted reward of a system in state (i,j) when, after the present interaction, there will be still n interactions in the future.

To start the induction, we set $v_0(i,j) = 0$ for all i,j, therefore $v_n(i+1,j) \leq v_n(i,j) \leq v_n(i,j+1)$ hold for $n = 0$.

Assume that both inequalities are true for n, hence we have two statements as the induction hypotheses. Now using these hypotheses, we will show that the inequalities hold for $n+1$ in both systems with varying interaction rate and varying death rate:

1. **Decreasing contact rate $\lambda(i,j)$:**

   PART A: We show the second inequality namely, $v(i,j) \leq v(i,j+1)$ for all i,j, for which it is enough to show that this inequality holds for $n+1$.

   Let system I and system II start in states (i,j) and $(i,j+1)$, respectively. Now we will let system I follow an optimal policy, while system II will imitate all the actions of system I, which is always possible. Then:

$$v_{n+1}(i,j) - v_{n+1}(i,j+1) \leq v_{n+1}(i,j) - v^{II}_{n+1}(i,j+1),$$

where $v^{II}_{n+1}(i,j+1)$ is the value function of system II, and the inequality holds due to the optimality of $v_{n+1}(i,j+1)$. Then, if we show that $v_{n+1}(i,j) \leq v^{II}_{n+1}(i,j+1)$, then $v_{n+1}(i,j) - v_{n+1}(i,j+1)$ will also hold. The optimal action of system I is either to attempt cross-selling or not to. Hence, we have two cases to consider:

Case 1: Optimal action for System I is to cross-sell

$$\begin{aligned}
& v_{n+1}(i,j) - v^{II}_{n+1}(i,j+1) = \\
& R + \alpha\left(\mu + \frac{\lambda i}{j+1}\right) v_n(i,j) \\
& +\alpha\lambda\left(1 - \frac{i}{j+1}\right)(P_f(v_n(i+1,j+1) - c_a - c_f) + (1-P_f)(v_n(0,0) + r - c_a)) \\
& -R - \alpha\left(\mu + \frac{\lambda i}{j+2}\right) v_n(i,j+1) \\
& -\alpha\lambda\left(1 - \frac{i}{j+2}\right)(P_f(v_n(i+1,j+2) - c_a - c_f) + (1-P_f)(v_n(0,0) + r - c_a)) \\
= \;& \alpha\mu(v_n(i,j) - v_n(i,j+1)) \\
& + \frac{\alpha\lambda i}{j+2}(v_n(i,j) - v_n(i,j+1)) \\
& +\alpha\lambda\left(1 - \frac{i}{j+1}\right) P_f\left(v_n(i+1,j+1) - v_n(i+1,j+2)\right) \\
& + \left(\frac{\alpha\lambda i}{j+1} - \frac{\alpha\lambda i}{j+2}\right)(v_n(i,j) - P_f(v_n(i+1,j+2) - c_a - c_f) - (1-P_f)(v_n(0,0) + r - c_a)) \\
\leq \;& \frac{\alpha\lambda i}{(j+1)(j+2)}(v_n(i,j+1) - P_f(v_n(i+1,j+1) - c_a - c_f) - (1-P_f)(v_n(0,0) + r - c_a)) \\
\leq \;& 0
\end{aligned}$$

where the equalities follow as a consequence of the attempt to cross-sell action and by some algebra, the first inequality follows by the induction hypotheses for the two inequalities, and finally the last inequality holds since the optimal action of system A is to cross-sell, so that $v_n(i,j+1) \leq P_f(v_n(i+1,j+1) - c_a - c_f) + (1-P_f)(v_n(0,0) + r - c_a)$. Hence, $v(i,j) \leq v(i,j+1)$ is true for all i,j.

Case 2: Optimal action for System I is not to cross-sell

$$
\begin{aligned}
& v_{n+1}(i,j) - v^{II}_{n+1}(i,j+1) \\
= \;& R + \alpha\left(\mu + \frac{\lambda i}{j+1}\right) v_n(i,j) \\
& +\alpha\lambda\left(1 - \frac{i}{j+1}\right) v_n(i,j+1) \\
& -R - \alpha\left(\mu + \frac{\lambda i}{j+2}\right) v_n(i,j+1) \\
& -\alpha\lambda\left(1 - \frac{i}{j+2}\right) v_n(i,j+2) \\
= \;& \alpha(\mu + \frac{\lambda i}{j+1})(v_n(i,j) - v_n(i,j+1)) \\
& +\alpha\lambda(1 - \frac{i}{j+2})(v_n(i,j+1) - v_n(i,j+2)) \\
\leq \;& 0
\end{aligned}
$$

where the equality follows as a consequence of the not-to-cross-sell action and by some algebra. The inequality follows by the induction hypotheses as both $v_n(i,j) - v_n(i,j+1) \leq 0$ and $v_n(i,j+1) - v_n(i,j+2) \leq 0$. Hence, $v(i,j) \leq v(i,j+1)$ is true for all i,j.

PART B: As in PART A, we show the first inequality namely, $v(i+1,j) \leq v(i,j)$ for all i,j, holds by the induction hypothesis for which $v_{n+1}(i,j)$ imitates $v_{n+1}(i+1,j)$. As the second system imitates the first, it does not have to give optimal result. However due to the optimality of the first system the below inequality is valid:

$$v_{n+1}(i+1,j) - v_{n+1}(i,j) \leq v_{n+1}(i+1,j) - v^{II}_{n+1}(i,j),$$

where $v^{II}_{n+1}(i,j)$ is the value function of the system that imitates $v_{n+1}(i+1,j)$. If we show that $v_{n+1}(i+1,j) \leq v^{II}_{n+1}(i,j)$, then $v_{n+1}(i+1,j) - v_{n+1}(i,j)$ will also hold. To show this, we have two cases to consider namely, cross-sell or not-to-cross-sell:

Case 1: Optimal action for System I is to cross-sell

$$
\begin{aligned}
& v_{n+1}(i+1,j) - v_{n+1}^{II}(i,j) = \\
& R + \alpha(\mu + \frac{\lambda(i+1)}{j+1})v_n(i+1,j) \\
& +\alpha\lambda(1 - \frac{(i+1)}{j+1})(P_f(v_n(i+2,j+1) - c_a - c_f) + (1-P_f)(v_n(0,0) + r - c_a)) \\
& -R - \alpha(\mu + \frac{\lambda i}{j+1})v_n(i,j) \\
& -\alpha\lambda(1 - \frac{i}{j+1})(P_f(v_n(i+1,j+1) - c_a - c_f) + (1-P_f)(v_n(0,0) + r - c_a)) \\
= \quad & \alpha(\mu + \frac{\lambda i}{j+1})(v_n(i+1,j) - v_n(i,j)) \\
& +\alpha\lambda P_f(1 - \frac{i}{j+1})(v_n(i+2,j+1) - v_n(i+1,j+1)) \\
& +\frac{\alpha\lambda}{j+1}(v_n(i+1,j) - (P_f(v_n(i+2,j+1) - c_a - c_f) + (1-P_f)(v_n(0,0) + r - c_a)) \\
\leq \quad & 0
\end{aligned}
$$

where the equalities follow as a consequence of the attempt to cross-sell action and by some algebra. The first two parts of the final inequality hold due to the induction hypotheses.The optimal policy chosen for $v_{n+1}(i+1,j)$ is to cross-sell, then $P_f(v_n(i+2,j+1) - c_a - c_f) + (1-P_f)(v_n(0,0) + r - c_a) \geq v_n(i+1,j+1)$, but we also know from PART A that $v_n(i+1,j) - v_n(i+1,j+1) \leq 0$. Therefore, $v_n(i+1,j) - (P_f(v_n(i+2,j+1) - c_a - c_f) + (1-P_f)(v_n(0,0) + r - c_a)) \leq 0$ as well. Hence, the first inequality is true for all i, j.

Case 2: Optimal action for System I is not to cross-sell

$$
\begin{aligned}
& v_{n+1}(i+1,j) - v^{II}_{n+1}(i,j) \\
= \; & R + \alpha(\mu + \frac{\lambda(i+1)}{j+1})v_n(i+1,j) \\
& +\alpha\lambda(1 - \frac{(i+1)}{j+1})v_n(i+1,j+1) \\
& -R - \alpha(\mu + \frac{\lambda i}{j+1})v_n(i,j) \\
& -\alpha\lambda(1 - \frac{i}{j+1})v_n(i,j+1) \\
= \; & \alpha(\mu + \frac{\lambda i}{j+1})(v_n(i+1,j) - v_n(i,j)) \\
& +\alpha\lambda(1 - \frac{i}{j+1})(v_n(i+1,j+1) - v_n(i,j+1)) \\
& +\frac{\alpha\lambda}{j+1}(v_n(i+1,j) - v_n(i+1,j+1)) \\
\leq \; & 0
\end{aligned}
$$

where the equality follows as a consequence of the not-to-cross-sell action and by some algebra. The inequality follows by the induction hypotheses for the first two parts as both $v_n(i+1,j) - v_n(i,j) \leq 0$ and $v_n(i+1,j+1) - v_n(i,j+1) \leq 0$. The final part of the inequality follows from PART A, where $v_n(i+1,j) - v_n(i+1,j+1) \leq 0$. Hence, the first inequality is true for all i, j.

Proposition 3 is, therefore proved for the decreasing contact rate of the customer.

2. **Increasing death rate $\mu(i,j)$:**

PART A: As in the case of decreasing contact rate, we show the second inequality namely, $v(i,j) \leq v(i,j+1)$ holds for all i, j. For this, we again use the imitations of systems, and in this case $v_{n+1}(i,j+1)$ imitates the actions of $v_{n+1}(i,j)$. Then:

$$v_{n+1}(i,j) - v_{n+1}(i,j+1) \leq v_{n+1}(i,j) - v^{II}_{n+1}(i,j+1),$$

where $v^{II}_{n+1}(i,j+1)$ is the value function of the system that imitates $v_{n+1}(i,j)$, and the inequality holds due to the optimality of $v_{n+1}(i,j)$. If we show that $v_{n+1}(i,j) \leq v^{II}_{n+1}(i,j+1)$, then the first inequality will also hold. The optimal action for $v_{n+1}(i,j)$ is either to attempt cross-selling or not to. Hence, we have two cases to consider:

Case 1: Optimal action for System I is to cross-sell

$$
\begin{aligned}
& v_{n+1}(i,j) - v^{II}_{n+1}(i,j+1) = \\
& R + \alpha\mu(1 - \frac{i}{j+1})v_n(i,j) \\
& +\alpha\lambda\left(P_f(v_n(i+1,j+1) - c_a - c_f) + (1-P_f)(v_n(0,0) + r - c_a)\right) \\
& -R - \alpha\mu(1 - \frac{i}{j+2})v_n(i,j+1) \\
& -\alpha\lambda\left(P_f(v_n(i+1,j+2) - c_a - c_f) + (1-P_f)(v_n(0,0) + r - c_a)\right) \\
= \; & \alpha\mu(1 - \frac{i(j+1)}{(j+2)(j+1)})(v_n(i,j) - v_n(i,j+1)) \\
& +\alpha\lambda P_f(v_n(i+1,j+1) - v_n(i+1,j+2)) \\
& -\frac{\alpha\mu i v_n(i,j)}{(j+2)(j+1)} \\
\leq \; & 0
\end{aligned}
$$

where the equalities follow as a consequence of the attempt to cross-sell action and by some algebra. The first two parts of the final inequality is negative due to the induction hypotheses, where $v_n(i,j) - v_n(i,j+1) \leq 0$ and $v_n(i+1,j+1) - v_n(i+1,j+2) \leq 0$. We know that each and every expected value earned from a customer should be at least zero. Therefore the last part of the inequality with a minus sign guarantees that the inequality as a whole is less than or equal to zero. Hence, $v_{n+1}(i,j) - v_{n+1}(i,j+1)$ is true for all i, j.

Case 2: Optimal action for System I is not to cross-sell

$$
\begin{aligned}
& v_{n+1}(i,j) - v^{II}_{n+1}(i,j+1) \\
= \ & R + \alpha\mu(1 - \frac{i}{j+1})v_n(i,j) \\
& +\alpha\lambda v_n(i,j+1) \\
& -R - \alpha\mu(1 - \frac{i}{j+2})v_n(i,j+1) \\
& -\alpha\lambda v_n(i,j+2) \\
= \ & \alpha\mu(1 - \frac{i(j+1)}{(j+2)(j+1)})(v_n(i,j) - v_n(i,j+1)) \\
& +\alpha\lambda(v_n(i,j+1) - v_n(i,j+2)) \\
& -\frac{\alpha\mu i v_n(i,j)}{(j+2)(j+1)} \\
\leq \ & 0
\end{aligned}
$$

where the equality follows as a consequence of the not to cross-sell action and by some algebra. The inequality follows by the induction hypotheses as both $v_n(i,j) - v_n(i,j+1) \leq 0$ and $v_n(i,j+1) - v_n(i,j+2) \leq 0$. The last part of the inequality is already less than zero. Hence, the first inequality is true for all i, j.

PART B: We show the first inequality namely, $v(i+1,j) \leq v(i,j)$ for all i, j, for which it is enough to show that this inequality holds for $n+1$ and even when $v(i,j)$ imitates the actions of $v(i+1,j)$. Then:

$$
v_{n+1}(i+1,j) - v_{n+1}(i,j) \leq v_{n+1}(i+1,j) - v^{II}_{n+1}(i,j),
$$

where $v^{II}_{n+1}(i,j)$ is the value function of $v(i,j)$ when it imitates the actions of $v(i+1,j)$, and the inequality holds due to the optimality of $v_{n+1}(i+1,j)$. If we show that $v_{n+1}(i+1,j) \leq v^{II}_{n+1}(i,j)$, then the first inequality will also hold. The optimal action for $v(i+1,j)$ is either to attempt cross-selling or not to. Hence, we have two cases to consider:

Case 1: Optimal action for System I is to cross-sell

$$
\begin{aligned}
& v_{n+1}(i+1,j) - v^{II}_{n+1}(i,j) = \\
& R + \alpha\mu(1-\frac{i+1}{j+1})v_n(i+1,j) \\
& +\alpha\lambda(P_f(v_n(i+2,j+1)-c_a-c_f)+(1-P_f)(v_n(0,0)+r-c_a)) \\
& -R-\alpha\mu(1-\frac{i}{j+1})v_n(i,j) \\
& -\alpha\lambda(P_f(v_n(i+1,j+1)-c_a-c_f)+(1-P_f)(v_n(0,0)+r-c_a)) \\
= \; & \alpha\mu(1-\frac{i}{j+1})(v_n(i+1,j)-v_n(i,j)) \\
& +\alpha\lambda P_f(v_n(i+2,j+1)-v_n(i+1,j+1)) \\
& -\frac{\alpha\mu v_n(i+1,j)}{j+1} \\
\leq \; & 0
\end{aligned}
$$

where the equalities follow as a consequence of the attempt to cross-sell action and by some algebra. The first two parts of the final inequality hold due to the induction hypotheses, and the last part is already smaller than zero. Hence, the first inequality is true for all i, j.

Case 2: Optimal action for System I is not to cross-sell

$$
\begin{aligned}
& v_{n+1}(i+1,j) - v^{II}_{n+1}(i,j) \\
= \; & R + \alpha\mu(1-\frac{i+1}{j+1})v_n(i+1,j) \\
& +\alpha\lambda v_n(i+1,j+1) \\
& -R-\alpha\mu(1-\frac{i}{j+1})v_n(i,j) \\
& -\alpha\lambda v_n(i,j+1) \\
= \; & \alpha\mu(1-\frac{i}{j+1})(v_n(i+1,j)-v_n(i,j)) \\
& +\alpha\lambda(v_n(i+1,j+1)-v_n(i,j+1)) \\
& -\frac{\alpha\mu v_n(i+1,j)}{j+1} \\
\leq \; & 0
\end{aligned}
$$

where the equality follows as a consequence of the not to cross-sell action and by some algebra. The inequality follows by the induction hypotheses for the first two parts as both $v_n(i+1,j) - v_n(i,j) \leq 0$ and $v_n(i+1,j+1) - v_n(i,j+1) \leq 0$. The final part of the inequality is already smaller than zero. Hence, the first inequality is true for all i, j.

Proposition 3 is, therefore proved for the increasing death rate of the customer.

3. **Increasing failure probability** $P_f(i,j)$**:**

   PART A: We show the second inequality namely, $v(i,j) \leq v(i,j+1)$ for all i, j, for which it is enough to show that this inequality holds for $n+1$.

   Let system I and system II start in states (i,j) and $(i,j+1)$, respectively. Now we will let system I follow an optimal policy, while system II will imitate all the actions of system I. Then:

$$v_{n+1}(i,j) - v_{n+1}(i,j+1) \leq v_{n+1}(i,j) - v_{n+1}^{II}(i,j+1),$$

   where $v_{n+1}^{II}(i,j+1)$ is the value function of system II, and the inequality holds due to the optimality of $v_{n+1}(i,j+1)$. If we show that $v_{n+1}(i,j) \leq v_{n+1}^{II}(i,j+1)$, then the first inequality will also hold. The optimal action of system I is either to attempt cross-selling or not to. Hence, we have two cases to consider:

Case 1: Optimal action for System I is to cross-sell

$$\begin{aligned}
& v_{n+1}(i,j) - v^{II}_{n+1}(i,j+1) = \\
& R + \alpha\mu v_n(i,j) \\
& +\alpha\lambda\left(P_f(1+\frac{i}{j+1})(v_n(i+1,j+1)-c_a-c_f) + (1-P_f(1+\frac{i}{j+1}))(v_n(0,0)+r-c_a)\right) \\
& -R - \alpha\mu v_n(i,j+1) \\
& -\alpha\lambda\left(P_f(1+\frac{i}{j+2})(v_n(i+1,j+2)-c_a-c_f) + (1-P_f(1+\frac{i}{j+2}))(v_n(0,0)+r-c_a)\right) \\
= \; & \alpha\mu(v_n(i,j)-v_n(i,j+1)) \\
& +\alpha\lambda P_f(1+\frac{i}{j+2})(v_n(i+1,j+1)-v_n(i+1,j+2)) \\
& +\alpha\lambda P_f\frac{i}{(j+2)(j+1)}((v_n(i+1,j+1)-c_a-c_f)-(v_n(0,0)+r-c_a)) \\
\leq \; & \alpha\mu(v_n(i,j)-v_n(i,j+1)) \\
& +\alpha\lambda P_f(1+\frac{i}{j+2})(v_n(i+1,j+1)-v_n(i+1,j+2)) \\
& +(\frac{\alpha\lambda P_f i}{(j+2)(j+1)})\frac{(v_n(i+1,j+1)-v_n(i,j+1)-c_a-c_f)}{(1-P_f)} \\
\leq \; & 0
\end{aligned}$$

where the equalities follow as a consequence of the attempt to cross-sell action and by some algebra. The first two parts of the final inequality hold due to the induction hypotheses, where $v_n(i,j) - v_n(i,j+1) \leq 0$ and
$v_n(i+1,j+1) - v_n(i+1,j+2) \leq 0$. The optimal decision for $v_n(i,j)$ is to cross-sell, so we know that $v_n(0,0)+r-c_a > \frac{(v_n(i,j+1)-P_f(v_n(i+1,j+1)-c_a-c_f)}{(1-P_f)}$. When we rewrite the final part of the inequality replacing $v_n(0,0)+r-c_a$ by $\frac{(v_n(i,j+1)-P_f(v_n(i+1,j+1)-c_a-c_f)}{(1-P_f)}$, we see that even when a smaller number is subtracted from $v_n(i+1,j+1)-c_a-c_f$, the final part is less than or equal to zero if Part B of the Proposition 3 is true.

Case 2: Optimal action for System I is not to cross-sell

$$\begin{aligned}
& v_{n+1}(i,j) - v^{II}_{n+1}(i,j+1) \\
= \; & R + \alpha\mu v_n(i,j) + \alpha\lambda v_n(i,j+1) \\
& -R - \alpha\mu v_n(i,j+1) - \alpha\lambda v_n(i,j+2) \\
= \; & \alpha\mu(v_n(i,j)-v_n(i,j+1)) + \alpha\lambda(v_n(i,j+1)-v_n(i,j+2)) \\
\leq \; & 0
\end{aligned}$$

where the equality follows as a consequence of the not to cross-sell action and by some algebra. The inequality follows by the induction hypotheses as both $v_n(i,j) - v_n(i,j+1) \leq 0$ and $v_n(i,j+1) - v_n(i,j+2) \leq 0$. Hence, the first inequality is true for all i, j.

However, to show that Part A is true for every state, we need to make sure Part B is also true.

PART B: To show that $v(i+1,j) \leq v(i,j)$ for all i, j, we use induction and we let the $v(i,j)$ to imitate the action chosen by $v(i+1,j)$.Then:

$$v_{n+1}(i+1,j) - v_{n+1}(i,j) \leq v_{n+1}(i+1,j) - v^{II}_{n+1}(i,j),$$

where $v^{II}_{n+1}(i,j)$ is the value function that imitates the action of $v(i+1,j)$, and the inequality holds due to the optimality of $v_{n+1}(i+1,j)$. If we show that $v_{n+1}(i+1,j) \leq v^{II}_{n+1}(i,j)$, then the first inequality will also hold. We have two cases to consider for $v(i+1,j)$:

Case 1: Optimal action for System I is to cross-sell

$$\begin{aligned}
& v_{n+1}(i+1,j) - v^{II}_{n+1}(i,j) = \\
& R + \alpha\mu v_n(i+1,j) \\
& +\alpha\lambda(P_f(1+\frac{i+1}{j+1})(v_n(i+2,j+1) - c_a - c_f) + (1 - P_f(1+\frac{i+1}{j+1}))(v_n(0,0) + r - c_a)) \\
& -R - \alpha\mu v_n(i,j) \\
& -\alpha\lambda(P_f(1+\frac{i}{j+1})(v_n(i+1,j+1) - c_a - c_f) + (1 - P_f(1+\frac{i}{j+1}))(v_n(0,0) + r - c_a)) \\
= \;& \alpha\mu(v_n(i+1,j) - v_n(i,j)) \\
& +\alpha\lambda P_f(\frac{j+1+i}{j+1})(v_n(i+2,j+1) - v_n(i+1,j+1)) \\
& +\frac{\alpha\lambda P_f}{j+1}(v_n(i+2,j+1) - (v_n(0,0) + r - c_a) - c_a - c_f) \\
\leq \;& \alpha\mu(v_n(i+1,j) - v_n(i,j)) \\
& +\alpha\lambda P_f(\frac{j+1+i}{j+1})(v_n(i+2,j+1) - v_n(i+1,j+1)) \\
& +\frac{\alpha\lambda P_f}{j+1}(v_n(i+2,j+1) - v_n(i+1,j+1) - c_a - c_f) \\
\leq \;& 0
\end{aligned}$$

where the equalities follow as a consequence of the attempt to cross-sell action and by some algebra. The first two parts of the final inequality hold due to the induction hypotheses, and for the last part we used the fact that due to the optimal decision of $v_n(i+1,j)$, $v_n(0,0)+r-c_a > \frac{v_n(i+1,j+1)-P_f(v_n(i+2,j+1)-c_a-c_f)}{(1-P_f)}$. Therefore, the inequality holds in each state.

Case 2: Optimal action for System I is not to cross-sell

$$\begin{aligned}
& v_{n+1}(i+1,j) - v^{II}_{n+1}(i,j) \\
= \; & R + \alpha\mu v_n(i+1,j) + \alpha\lambda v_n(i+1,j+1) \\
& -R - \alpha\mu v_n(i,j) - \alpha\lambda v_n(i,j+1) \\
= \; & \alpha\mu(v_n(i+1,j) - v_n(i,j)) + \alpha\lambda(v_n(i+1,j+1) - v_n(i,j+1)) \\
\leq \; & 0
\end{aligned}$$

where the equality follows as a consequence of the not to cross-sell action and by some algebra. The inequality follows by the induction hypotheses as both $v_n(i+1,j) - v_n(i,j) \leq 0$ and $v_n(i+1,j+1) - v_n(i,j+1) \leq 0$. Hence, the first inequality is true for all i,j.

As Part B is proved for each state, we can conclude that Part A is also true. Thus, Proposition 3 holds for the increasing failure probability case as well.

When the number of cross-sell attempt failure increases as well as the number of contact, the increase in the contact number of the customer is not enough to erase the negative effect of the cross-sell attempt failure. Therefore the revenue obtained decreases when the cross-sell attempt failure increases even if the number of contact increases. This is due to the function that defines customer behavior in terms of negative effect of cross-sell attempts. Because of the choice of function the customer never forgets about bad experiences unless a renewal occurs. Proposition 4 formally presents this relationship:

Proposition 4 *For the models with decreasing contact rate $\lambda(i,j)$ and increasing death rate $\mu(i,j)$,*
$v(i+1,j+1) \leq v(i,j)$ for all i,j.

Proof.

We will prove the statement first for the model with decreasing contact rate $\lambda(i,j)$, and for the model with increasing death rate $\mu(i,j)$. In all our proofs, we will use induction on the number of interactions, n, as in Proposition 3. In the same manner, we define $v_n(i,j)$ as the value function for the finite horizon, so that $v_n(i,j)$ is the maximal expected α-discounted reward of a system in state (i,j) when, after the present interaction, there will be still n interactions in the future.

To start the induction, we set $v_0(i,j) = 0$ for all i, j, therefore $v_n(i+1, j+1) \leq v_n(i,j)$ hold for $n = 0$.

Assume that both inequalities are true for n, we will show that the inequalities hold for $n+1$ in both systems with decreasing interaction rate and increasing death rate:

1. **Decreasing contact rate $\lambda(i,j)$:**

   To show $v(i+1, j+1) \leq v(i,j)$ for all i, j, it is enough to that this inequality holds for $n+1$.

   Let system I and system II start in states $(i+1, j+1)$ and (i,j), respectively. Now we will let system I follow an optimal policy, while system II will imitate all the actions of system I. Then:

$$v_{n+1}(i+1, j+1) - v_{n+1}(i,j) \leq v_{n+1}(i+1, j+1) - v_{n+1}^{II}(i,j),$$

   where $v_{n+1}^{II}(i,j)$ is the value function of system II, and the inequality holds due to the optimality of $v_{n+1}(i+1, j+1)$. If we show that $v_{n+1}(i+1, j+1) \leq v_{n+1}^{II}(i,j)$, then the first inequality will also hold. The optimal action of system I is either to attempt cross-selling or not. Hence, we have two cases to consider:

Case 1: Optimal action for System I is to cross-sell

$$
\begin{aligned}
& v_{n+1}(i+1,j+1) - v^{II}_{n+1}(i,j) = \\
& R + \alpha\left(\mu + \frac{\lambda(i+1)}{j+2}\right) v_n(i+1,j+1) \\
& +\alpha\lambda\left(1 - \frac{(i+1)}{j+2}\right)(P_f(v_n(i+2,j+2) - c_a - c_f) + (1-P_f)(v_n(0,0) + r - c_a)) \\
& -R - \alpha\left(\mu + \frac{\lambda i}{j+1}\right) v_n(i,j) \\
& -\alpha\lambda\left(1 - \frac{i}{j+1}\right)(P_f(v_n(i+1,j+1) - c_a - c_f) + (1-P_f)(v_n(0,0) + r - c_a)) \\
= \; & \alpha\mu(v_n(i+1,j+1) - v_n(i,j)) \\
& + \frac{\alpha\lambda(j+1-i)}{j+2} P_f\,(v_n(i+2,j+2) - v_n(i+1,j+1)) \\
& + \frac{\alpha\lambda i}{j+2}\,(v_n(i+1,j+1) - v_n(i,j)) \\
& + \frac{\alpha\lambda}{j+2} v_n(i+1,j+1) - \frac{\alpha\lambda i}{(j+1)(j+2)} v_n(i,j) \\
& - \frac{\alpha\lambda((j+1-i)(P_f(v_n(i+1,j+1) - c_a - c_f) - (1-P_f)(v_n(0,0) + r - c_a))}{(j+1)(j+2)} \\
\leq \; & \frac{\alpha\lambda i}{(j+1)(j+2)}(v_n(i+1,j+1) - v_n(i,j)) \\
& + \frac{\alpha\lambda(j+1-i)}{(j+1)(j+2)}(v_n(i+1,j+1) - P_f(v_n(i+1,j+1) - c_a - c_f) - (1-P_f)(v_n(0,0) + r - c_a)) \\
\leq \; & \frac{\alpha\lambda(j+1-i)}{(j+1)(j+2)}((v_n(i+1,j+1) - v_n(i+1,j+2)) + P_f(v_n(i+2,j+2) - v_n(i+1,j+1))) \\
\leq \; & 0
\end{aligned}
$$

where the equalities follow as a consequence of the attempt to cross-sell action and by some algebra. Because of the induction hypothesis we know that $v_n(i+1,j+1) - v_n(i,j) \leq 0$ and $v_n(i+2,j+2) - v_n(i+1,j+1) \leq 0$, also from Proposition 3 $v_n(i+1,j+1) - v_n(i+1,j+2) \leq 0$ holds as well. Because we know that the number of the failures cannot exceed the number of contacts, that is $i \leq j$, we separate the $\frac{\alpha\lambda}{(j+2)} v_n(i+1,j+1)$ in two parts as $\frac{\alpha\lambda i}{(j+2)(j+1)} v_n(i+1,j+1)$ and $\frac{\alpha\lambda(j+1-i)}{(j+2)(j+1)} v_n(i+1,j+1)$. Finally because of the optimality of $v_{n+1}(i+1,j+1)$ we know that $(1-P_f)(v_n(0,0)+r-c_a) > v_n(i+1,j+2) - P_f(v_n(i+2,j+2) - c_a - c_f)$, and using these we show that the first inequality holds at each state.

Case 2: Optimal action for System I is not to cross-sell

$$
\begin{aligned}
& v_{n+1}(i+1,j+1) - v^{II}_{n+1}(i,j) \\
= \; & R + \alpha\left(\mu + \frac{\lambda(i+1)}{j+2}\right) v_n(i+1,j+1) \\
& + \alpha\lambda\left(1 - \frac{(i+1)}{j+2}\right) v_n(i+1,j+2) \\
& - R - \alpha\left(\mu + \frac{\lambda i}{j+1}\right) v_n(i,j) \\
& - \alpha\lambda\left(1 - \frac{i}{j+1}\right) v_n(i,j+1) \\
= \; & \alpha(\mu + \frac{\lambda i}{j+2})(v_n(i+1,j+1) - v_n(i,j)) \\
& + \alpha\lambda(1 - \frac{i+1}{j+2})(v_n(i+1,j+2) - v_n(i,j+1)) \\
& + \frac{\alpha\lambda((j+1)v_n(i+1,j+1) - i v_n(i,j))}{(j+1)(j+2)} \\
& - \frac{\alpha\lambda(j+1-i)v_n(i,j+1)}{(j+1)(j+2)} \\
\leq \; & 0
\end{aligned}
$$

where the equality follows as a consequence of the not to cross-sell action and by some algebra. The first two parts of the final inequality hold due to the induction hypotheses as both $v_n(i+1,j+1) - v_n(i,j) \leq 0$ and $v_n(i+1,j+2) - v_n(i,j+1) \leq 0$. The third part holds as well, because $i \leq j$ and when we replace $j+1$ with i, the third part holds due to the induction hypotheses. The last part is already smaller or equal to zero. Hence, the first inequality is true for all i, j.

Proposition 4 is, therefore proved for the decreasing contact rate of the customer.

2. **Increasing death rate** $\mu(i,j)$**:**

Let system I and system II start in states $(i+1,j+1)$ and (i,j), respectively. Now we will let system I follow an optimal policy, while system II will imitate all the actions of system I, which is always possible. Then:

$$v_{n+1}(i+1,j+1) - v_{n+1}(i,j) \leq v_{n+1}(i+1,j+1) - v^{II}_{n+1}(i,j),$$

where $v^{II}_{n+1}(i,j)$ is the value function of system II, and the inequality holds due to the optimality of $v_{n+1}(i+1,j+1)$. If we show that $v_{n+1}(i+1,j+1) \leq v^{II}_{n+1}(i,j)$, then the first inequality will also hold. The optimal action of system I is either to attempt cross-selling or not to. Hence, we have two cases to consider:

Case 1: Optimal action for System I is to cross-sell

$$\begin{aligned}
& v_{n+1}(i+1,j+1) - v^{II}_{n+1}(i,j) = \\
& R + \alpha\mu(1-\frac{i+1}{j+2})v_n(i+1,j+1) \\
& +\alpha\lambda(P_f(v_n(i+2,j+2)-c_a-c_f)+(1-P_f)(v_n(0,0)+r-c_a)) \\
& -R-\alpha\mu(1-\frac{i}{j+1})v_n(i,j) \\
& -\alpha\lambda(P_f(v_n(i+1,j+1)-c_a-c_f)+(1-P_f)(v_n(0,0)+r-c_a)) \\
= \;& \alpha\mu(j+1-i)(\frac{v_n(i+1,j+1)-v_n(i,j)}{j+2}) \\
& +\alpha\lambda P_f(v_n(i+2,j+2)-v_n(i+1,j+1)) \\
& -\frac{\alpha\mu(j+1-i)v_n(i,j)}{(j+1)(j+2)} \\
\leq \;& 0
\end{aligned}$$

where the equalities follow as a consequence of the attempt to cross-sell action and by some algebra. The first two parts of the final inequality hold due to the induction hypotheses, and the last part is already smaller than zero. Hence, the first inequality is true for all i, j.

Case 2: Optimal action for System I is not to cross-sell

$$\begin{aligned}
& v_{n+1}(i+1,j+1) - v^{II}_{n+1}(i,j) \\
= \;& R + \alpha\mu(1-\frac{i+1}{j+2})v_n(i+1,j+1) \\
& +\alpha\lambda v_n(i+1,j+2) \\
& -R-\alpha\mu(1-\frac{i}{j+1})v_n(i,j) \\
& -\alpha\lambda v_n(i,j+1) \\
= \;& \alpha\mu(j+1-i)(\frac{v_n(i+1,j)-v_n(i,j)}{j+2}) \\
& +\alpha\lambda(v_n(i+1,j+2)-v_n(i,j+1)) \\
& -\frac{\alpha\mu(j+1-i)v_n(i,j)}{(j+1)(j+2)} \\
\leq \;& 0
\end{aligned}$$

where the equality follows as a consequence of the not to cross-sell action and by some algebra. The inequality follows by the induction hypotheses for the first two parts as both $v_n(i+1,j) - v_n(i,j) \leq 0$ and $v_n(i+1,j+1) - v_n(i,j+1) \leq 0$. The final part of the inequality is already smaller than zero. Hence, the first inequality is true for all i, j.

Proposition 4 is, therefore proved for the increasing death rate of the customer.

Proposition 5 compares the expected values generated for cases when the reactions to cross-sell attempts affect the contact rate and the death rate.

Proposition 5 *When the expected reward generated for the $v(i,j)_{\mu(i,j)}$ and $v(i,j)_{\lambda(i,j)}$ cases are compared,*

$$v(i,j)_{\mu(i,j)} > v(i,j)_{\lambda(i,j)}$$

is true $\forall i, j$.

Proof.

Let system I and II start in states (i, j) for the decreasing contact rate $\lambda(i, j)$ and increasing death rate $\mu(i, j)$ respectively. Now we will let system I follow an optimal policy, while system II will imitate all the actions of system I, which is always possible. Then:

$$v_{n+1}(i,j)_{\mu(i,j)} - v_{n+1}(i,j)_{\lambda(i,j)} > v_{n+1}^{II}(i,j)_{\mu(i,j)} - v_{n+1}(i,j)_{\lambda(i,j)}$$

where $v_{n+1}^{II}(i,j)_{\mu(i,j)}$ is the value function where the death rate changes. To show that $v_{n+1}(i,j)_{\mu(i,j)} - v_{n+1}(i,j)_{\lambda(i,j)} > 0$, we use induction method so that we assume that this inequality holds for n. The optimal action of system I is either to attempt cross-selling or not to. Hence, we have two cases to consider: **Case 1: Optimal action for System I is**

to cross-sell

$$\begin{aligned}
& v_{n+1}(i,j)_{\mu(i,j)} - v_{n+1}(i,j)_{\lambda(i,j)} = \\
& R + \alpha\mu(1 - \frac{i}{j+1})v_n(i,j)_{\mu(i,j)} \\
& +\alpha\lambda(P_f(v_n(i+1,j+1)_{\mu(i,j)} - c_a - c_f) + (1-P_f)(v_n(0,0)_{\mu(i,j)} + r - c_a)) \\
& -R - \alpha(\mu + \frac{\lambda i}{j+1})v_n(i,j)_{\lambda(i,j)} \\
& -\alpha\lambda(1 - \frac{i}{j+1})(P_f(v_n(i+1,j+1)_{\lambda(i,j)} - c_a - c_f) + (1-P_f)(v_n(0,0)_{\lambda(i,j)} + r - c_a)) \\
= \; & \alpha\mu(v_n(i,j)_{\mu(i,j)} - v_n(i,j)_{\lambda(i,j)}) + \frac{\alpha\mu i v_n(i,j)_{\mu(i,j)}}{j+1} \\
& +\alpha\lambda(P_f(v_n(i+1,j+1)_{\mu(i,j)} - v_n(i+1,j+1)_{\lambda(i,j)}) + (1-P_f)(v_n(0,0)_{\mu(i,j)} - v_n(0,0)_{\lambda(i,j)})) \\
& +\frac{\alpha\lambda i}{(j+1)}(P_f(v_n(i+1,j+1)_{\lambda(i,j)} - c_a - c_f) + (1-P_f)(v_n(0,0)_{\lambda(i,j)} + r - c_a) - v_n(i,j)_{\lambda(i,j)}) \\
> \; & 0
\end{aligned}$$

where the equalities follow as a consequence of the attempt to cross-sell action and by some algebra. The first two parts of the final inequality hold due to the induction hypotheses, and the last part is larger than zero, because of the optimality of System I, so that $P_f(v_n(i+1,j+1)_{\lambda(i,j)} - c_a - c_f) + (1-P_f)(v_n(0,0)_{\lambda(i,j)} + r - c_a) \geq v_n(i,j+1)_{\lambda(i,j)}$ and also $v_n(i,j)_{\lambda(i,j)} \leq v_n(i,j+1)_{\lambda(i,j)}$. Hence, the first inequality is true for all i, j.

Case 2: Optimal action for System I is not to cross-sell

$$\begin{aligned}
& v_{n+1}(i,j)_{\mu(i,j)} - v_{n+1}(i,j)_{\lambda(i,j)} = \\
& R + \alpha\mu(1 - \frac{i}{j+1})v_n(i,j)_{\mu(i,j)} \\
& +\alpha\lambda v_n(i,j+1)_{\mu(i,j)} \\
& -R - \alpha(\mu + \frac{\lambda i}{j+1})v_n(i,j)_{\lambda(i,j)} \\
& -\alpha\lambda(1 - \frac{i}{j+1})v_n(i,j+1)_{\lambda(i,j)} \\
= \; & \alpha\mu(v_n(i,j)_{\mu(i,j)} - v_n(i,j)_{\lambda(i,j)}) + \frac{\alpha\mu i v_n(i,j)_{\mu(i,j)}}{j+1} \\
& +\alpha\lambda(v_n(i,j+1)_{\mu(i,j)} - v_n(i,j+1)_{\lambda(i,j)}) \\
& +\frac{\alpha\lambda i}{(j+1)}(v_n(i,j+1)_{\lambda(i,j)} - v_n(i,j)_{\lambda(i,j)}) \\
> \; & 0
\end{aligned}$$

where the equality follows as a consequence of the not to cross-sell action and by some algebra. With this, Proposition 5 is proved for all states i, j.

3.4 Computational Experiments and Results

3.4.1 Parameters Used

The parameters used are chosen to enable us to see how the cross-selling decision by the firm is affected. We let the cost of attempt to be 1 ($c_a = 1$), and one-time-cost of cross-sell attempt failure to be 2 ($c_f = 2$). The value for the failure probability is taken to be 0.4 ($P_f = 0.4$).

We created 6 different scenarios, where the ratios of $\frac{r}{R}$ and $\frac{\lambda}{\mu}$ change. The ratio of $\frac{r}{R}$ shows how much extra revenue we can obtain from each cross-selling attempt. On the other hand, the ratio of $\frac{\lambda}{\mu}$ shows how many times customer contacts the firm during his/her lifetime. While changing the ratios, for computational purposes we take the standard revenue R and the death rate μ to be constant and equal to 1 and we change the additional revenue obtained from cross-selling r and contact rate λ. The contact rate can be low, medium or high. The additional revenue obtained from successful cross-selling can be low or high. Note that in cases of varying contact and death rates and the failure probability the above mentioned parameters are only initial values.

For numerical calculations, we let our customers die on their 100^{th} consecutive contact with the firm given that the customer never accepts a cross-sell offer, which means that the system never renews itself. This created a boundary effect, i.e. the optimal policy switches to cross-sell close to the end. Because toward the end of a 100 state-space system, the system knowing that when the customer will quit the relationship, the system chooses to cross-sell to the customer. Such an upper bound on the number of contacts is justified, since the probability of reaching these states are very low under plausible policies.

In the following cases, under each scenario we found the threshold values for the failure probability P_f, the optimal policies and also the $v(0, 0)$ values.

		λ		
		low (10)	**medium (50)**	**high (100)**
r	**low (5)**	Scenario 1	Scenario 3	Scenario 5
	high (10)	Scenario 2	Scenario 4	Scenario 6

Table 3.1: Scenarios

3.4.2 Base Case

		λ/μ		
		10	**50**	**100**
r/R	**5**	0.5714286	0.5714286	0.5714286
	10	0.75	0.75	0.75

Table 3.2: Threshold Values for the Base Case

The threshold values for the failure probability P_f are given in Table 3.2. The threshold values are equal to $\frac{r-c_a}{r+c_f}$ value, which is independent of the customer contact rates.

		λ/μ		
		10	**50**	**100**
r/R	**5**	15.096349	30.70348	35.701553
	10	33.483475	70.844315	82.809006

Table 3.3: $v(0,0)$ Values for the Base Case

The expected values for $v(0,0)$ generated for each scenario can be seen in Table 3.3. The $v(0,0)$ values found for the 6 scenarios use a failure probability of 0.4 which is smaller than the $\frac{r-c_a}{r+c_f}$ value. Therefore in each of the 6 scenarios, the optimal policy π^* is to try to cross-sell to the customer at each state. When we compare the scenarios according to their expected values of $v(0,0)$ generated, we observe that as the customer contact rate and/or the additional revenue obtained from cross-sell increase, the expected revenue also increases as expected. However, increasing the additional revenue r twice is more effective than increasing the contact rate λ ten times on the value of $v(0,0)$. This might be due to the model's property of reflecting the decisions that are recency-based rather than the long-term ones. For this reason a change in the contact rate does not affect the value generated as much as the additional cross-sell revenue.

The threshold values found for the base case are valid for the infinite state-space and therefore they are long-run results. Once the failure probability becomes equal to or more than the threshold value the system always chooses not-to-cross-sell to the customer. In the numerical analysis, we implemented a finite-state space, which created a boundary effect. However, the boundary effect is found to be negligible. The difference between the $v(0,0)$ values obtained between the infinite state-space and the finite state-space is found to be 0.22% with the parameters given in Section 3.4.1 and also the probability of reaching these states are very low. So our analysis is a good approximation for the infinite state-space case.

Based on the numerical analysis we observe that the marginal increase in $v(0,0)$ values decreases as λ increases, i.e. When the contact rate increase from 10 to 50, $v(0,0)$ generated increases by 103.38%, and when the contact rate increases from 50 to 100, the $v(0,0)$ generated increases by 16.28%. Moreover, the impact of an increase in the additional revenue obtained from cross-selling is higher on the $v(0,0)$ values when the contact rate is high, because the firm has more to lose, i.e. When the contact rate is 10, if we increase the r from 5 to 10, the $v(0,0)$ generated increases by 121.80%, and when the contact rate is 50, this increase in the r is reflected by an increase of 130.74% in the $v(0,0)$ values.

3.4.3 State Dependent Death Rate Case

A customer reaction to the cross-selling attempts of the firm can be such that the customer's likelihood to quit the relationship may increase. We represent this, in the model with a state-dependent increase in the death rate of the customer.

Given the initial death rate after uniformization and the optimal policy under certain failure probabilities, the changes in μ for Scenario 1 can be seen in the graphs in Figure 3.15. For P_f small enough, in this case 0.4, the optimal policy is always to cross-sell to the customer. For this reason, the rate of increase in the death rate of the customer if the customer keeps rejecting cross-sell offers is high. When the failure probability is taken to be 0.53, the system chooses not to cross-sell to customer at the initial state of $(0,0)$ and then for the rest of the contacts chooses to cross-sell. The effect of a failure during the initial contacts is greater when compared to cases where the failure occurs after more contacts. Because the system chooses not to cross-sell initially, the rate of increase in the death rate for the rest of the contacts is not as big as the case where the optimal policy is always to cross-sell. The number of not-to-cross-sell decisions increase, as the failure probability

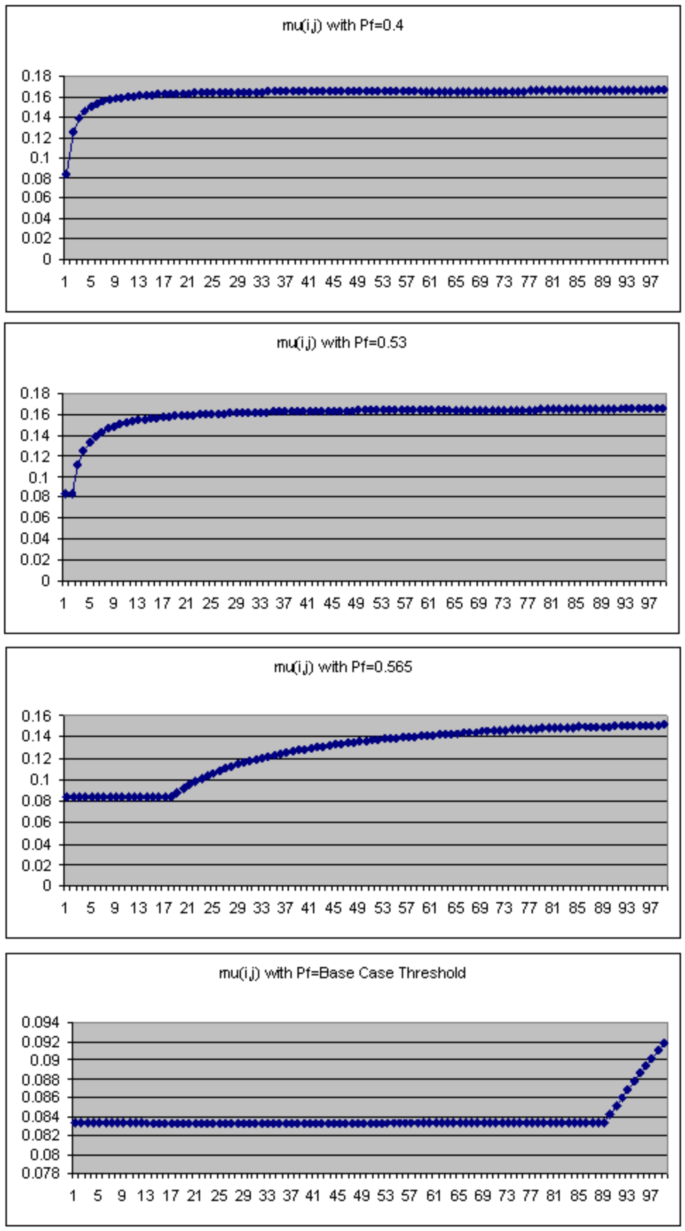

Figure 3.15: Comparison of $\mu(i, j)$ for different P_f values under Scenario 1, x-axis shows the number of contacts

approaches to the threshold value of the base-case, and finally when failure probability is taken to be threshold value of the base-case, the optimal policy becomes not-to-cross-sell.

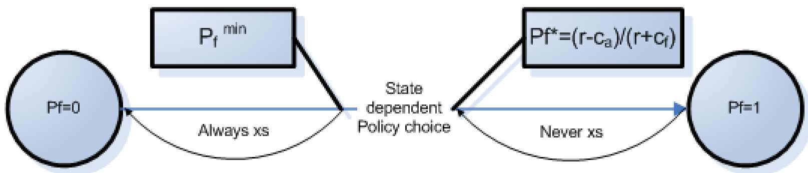

Figure 3.16: Change in the optimal policy based on P_f for $\mu(i,j)$ case

From the above analysis, we observe that when the customer death rate increases with the number of cross-selling failures, there is a P_f^{min}, below which to cross-sell to the customer is optimal in all states and there is a P_f^{max}, above which not-to-cross-sell is optimal in all states. For the failure probability values in between, $P_f^{min} \leq P_f < P_f^{max}$, the optimal policy is state-dependent displaying a dynamic structure. This relation can be seen in Figure 3.16. The threshold value of the base case, $P_f^* = \frac{r-c_a}{r+c_f}$, seems to be valid for the state dependent death rate case, such that above which the optimal policy becomes always not-to-cross-sell. Therefore, $P_f^{max} = P_f^* = \frac{r-c_a}{r+c_f}$.

		λ/μ		
		10	**50**	**100**
r/R	**5**	0.53	0.555	0.56
	10	0.72	0.735	0.74

Table 3.4: P_f^{min} values for the 6 scenarios considered

The P_f^{min} values are found computationally and given in Table 3.4. These values are rather close to the threshold values obtained for the base case, however the differences are not negligible. As can be seen in Table 3.4, for a fixed r value, the P_f^{min} values increase slightly as the contact rate increases, causing the gap between P_f^{min} and P_f^{max} to decrease. This decrease in the gap might be due to the fact that when the customer contact rate is very high compared to the death rate, this implies to the system, the customer will generate a large enough revenue even if the death rate increases, as a result of negative experiences of cross-selling attempts. So with each cross-sell offer the high contact rate of the customer makes up for the risk of higher rates of losing that customer for the firm.

Effect of State Dependent Death Rate on $v(0,0)$

		$\frac{\lambda}{\mu}$	**10**	**50**	**100**
$\frac{r}{R}$	**5**	**Base Case**	15.09635	30.70348	35.70155
		$\mu(i,j)$	13.55726	29.0557	34.53151
% Diff. from the Base Case			-10.20%	-5.37%	-3.28%
$\frac{r}{R}$	**10**	**Base Case**	33.48348	70.84432	82.80901
		$\mu(i,j)$	30.06931	67.04139	80.09395
% Diff. from the Base Case			-10.20%	-5.37%	-3.28%

Table 3.5: $v(0,0)$ Values for $\mu(i,j)$

The $v(0,0)$ values obtained for each of the 6 scenarios, when death rate is state dependent and failure probability is 0.4 is given in Table 3.5. As in the base case, the additional revenue obtained from cross-selling r is much more effective on the $v(0,0)$ values than the contact rate of a customer. Indeed when we double the r value the $v(0,0)$ values increase more than twice. The optimal policy is to cross-sell to the customer at each state as in the base case. However, as expected the $v(0,0)$ values obtained when death rate is state dependent are less than expected revenues obtained in the base case for each of the 6 scenarios.

The $v(0,0)$ values obtained for the P_f values between P_f^{min} and P_f^{max}, are not quite different from each other. When we compare the $v(0,0)$ values obtained for P_f^{min} and threshold of the base case, we see that the difference of $v(0,0)$ values is at most 6.01360% when $\lambda = 10$ and $v(0,0) = 7.741934$ for the threshold of the base case and $v(0,0) = 8.237292$ for the P_f^{min} value for Scenario 1. As the contact rate increases for a fixed r value, the difference between the $v(0,0)$ values decreases. There is also a slight decrease when the additional revenue is low for a fixed contact rate as can be seen in Table 3.6.

In the following subsections, in order to explore the gap between P_f^{min} and $P_f^{max} = P_f^*$ values, we investigate the effects of the additional revenue obtained from cross-selling per contact, number of contacts in a lifetime and also the cost of failure and attempt. We also

	$v(0,0) - forP_f^{min}$	$v(0,0) - forP_f^{max}$	**% Decrese**
Scenario 1	8.237292	7.741934	6.01%
Scenario 2	8.243113	7.741934	6.08%
Scenario 3	15.037281	14.647867	2.59%
Scenario 4	15.33642	14.647771	4.49%
Scenario 5	17.256439	16.859472	2.30%
Scenario 6	17.502474	16.859313	3.67%

Table 3.6: Difference between $v(0,0)$ values when $P_f = P_f^{min}$ *and* $P_f = P_f^{max}$

investigate the effect of predetermined policies on $v(0,0)$ values.

Effect of $\frac{\lambda}{\mu}$ *on Dynamic Policy*

The ratio of $\frac{\lambda}{\mu}$ is important as it shows the customer contacts during the lifetime of the customer. For small contact rate, even if the customer's probability of rejecting a cross-sell offer is small, the system does not choose always to cross-sell. Because with each rejection of the cross-sell offer, the annoyance caused by the offer increases the death rate of the customer. Especially when the customer contact rate is low, making the customer to leave the system quicker is not desirable.

		$\frac{\lambda}{\mu}$		
		1	**5**	P_f^*
$\frac{r}{R}$	**5**	0.525	0.525	0.571428571
	10	0.712	0.711	0.75
	100	0.965	0.964	0.970588235

Table 3.7: P_f^{min} values for different λ and r values

To see the dynamic change in the policy, we looked at the situations where the contact number of the customer in a lifetime is smaller and also the situation where the additional revenue obtained from cross-selling is very large compared to the Scenarios created. In

Table 3.7, we observe that for smaller contact rates the system becomes more dynamic in terms of optimal policy. We observe P_f^{min} values decrease for higher contact rate and smaller r values. This structure in the optimal policy is because of our choice of customer behavior indicating function. Any bad experience that occur within the initial contacts have larger effects, for this reason the system chooses not to cross-sell initially and then starts to cross-sell. Also as the r value increases, the gap between the P_f^{min} and P_f^{max} values decreases. When the gain from a successful cross-sell attempt is higher the system is more reluctant to stop offering additional products to the customer.

Effect of $\frac{c_a}{c_f}$ on Dynamic Policy

			P_f^*	P_f^{min}
$\frac{c_a}{c_f}$	**1**	$c_a = 0.5\ c_f = 0.5$	0.333333333	0.25
	$\frac{1}{10}$	$c_a = 0.5\ c_f = 0.05$	0.476190476	0.33
	$\frac{10}{1}$	$c_a = 0.05\ c_f = 0.5$	0.633333333	0.448
	1	$c_a = 0.1\ c_f = 0.1$	0.818181818	0.55
	1	$c_a = 0.01\ c_f = 0.01$	0.98019802	0.6

Table 3.8: P_f^{min} values for different c_a and c_f values

When the cost of cross-selling is rather insignificant, the gap between the P_f^{min} and P_f^{max} values changes based on the revenues obtained. As seen in Table 3.8, the gap is affected not by the ratio of $\frac{c_a}{c_f}$, but by the values of c_a and c_f. When the c_a and c_f values get smaller, the system becomes more dynamic, even if their ratio is equal. When the cost of failure is smaller than the cost of attempt, the decision depends on whether the attempt cost is tolerable by the system. However, for this to happen the costs should also be smaller than the additional revenue.

Effect of $\frac{r}{R}$ on Dynamic Policy

The additional revenue and the fixed revenue obtained from customer contacts are expected to have the greatest impact on the failure probability gaps. When the immediate revenue obtained from each contact is taken to be much larger than the additional revenue, the

			$\frac{\lambda}{\mu}$				
		P_f^*	**1**	**5**	**10**	**50**	**100**
$\frac{r}{R}$	1	0.4375	0.33	0.34	0.35	0.39	0.41
	$\frac{1}{5}$	0.4375	0.153	0.166	0.183	0.267	0.316
	$\frac{1}{10}$	0.4375	0.1	0.105	0.119	0.197	0.252
	$\frac{1}{100}$	0.4375	0.015	0.015	0.0164	0.036	0.0577

Table 3.9: P_f^{min} values for different r and R values

S 1-3-5: $P_f(0,0) = 0.565$ **S 2-4-6:** $P_f(0,0) = 0.745$	$v(0,0)_{\mu(i,j)}$ **Optimal**	$v(0,0)_{\mu(i,j)}$ **predetermined: xs**	**% Decrease**	$v(0,0)_{\mu(i,j)}$ **predetermined: nxs**	**% Decrease**
Scenario 1	7.743331	6.85285	11.49997%	7.741932	0.01807%
Scenario 2	7.742875	6.533563	15.61838%	7.741932	0.01218%
Scenario 3	14.68554	14.039993	4.39580%	14.636223	0.33582%
Scenario 4	14.691998	13.731628	6.53669%	14.636223	0.37963%
Scenario 5	16.960257	16.703689	1.51276%	16.822146	0.81432%
Scenario 6	16.987151	16.555019	2.54388%	16.822146	0.97135%

Table 3.10: $v(0,0)$ Comparison for Predetermined Policies

opportunity cost of losing this fixed, immediate revenue R becomes too big for the system. So, the system becomes more dynamic in terms of optimal decisions. It becomes harder for the system always to choose to cross-sell to the customer even though the failure probability is very small, when the immediate revenue obtained from the customer is really big. This can be seen in Table 3.9, where for different contact rates, the impact of different $\frac{r}{R}$ is investigated. As the contact rate decreases and the fixed revenue obtained from the customer at each contact increases, P_f^{min} values gets smaller compared to the threshold value of the base case, P_f^*.

Effect of Ignoring Customer Reactions

In Table 3.10, we compare the expected values for predetermined policies where we ignore the customer reaction with the optimal policies obtained for the 6 scenarios. For the predetermined policies, the firm decides either to cross-sell to the customer at each contact or never to cross-sell independent of customer reactions to the action chosen. The failure probabilities are taken between $[P_f^{min}, P_f^{max})$ values, where the system shows a dynamic structure for the $\mu(i,j)$ case. So optimal policy for the base case is always to cross-sell to the customer. If we pre-specify our policy to always cross-sell, the $v(0,0)$ values obtained show that there is a significant decrease in the expected values when we do not let the system choose its optimal policy of mixed actions. For high r value ($r = 10$) and small contact rate ($\lambda = 10$), which is scenario 2, this situation is obvious. When the predetermined policy is never to cross-sell, we see that as the contact rate increases, the decrease in $v(0,0)$ value becomes greater, however this is not as significant as in the situation where the predetermined case is always to cross-sell. Therefore if the firm does not know how the customer reacts to the cross-sell offers and does not know how to handle dissatisfaction when the cross-sell offers affect the death rate of the customer with a given failure probability, which is taken to be 0.565 for Scenarios 1, 3, and 5, and 0.745 for Scenarios 2, 4, and 6, the firm may choose always not to cross-sell over always to cross-sell policy. The firm can adjust its decisions looking at the $v(0,0)$ comparisons.

When $P_f = P_f^{min}$, the maximum difference between the $v(0,0)$ values for the predetermined never to cross-sell situation and the optimal value is found to be 6% and for the predetermined always to cross-sell situation and the optimal value is 1.93%.

3.4.4 State Dependent Arrival Rate Case

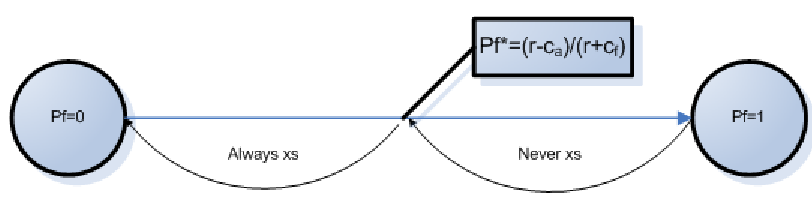

Figure 3.17: Change in the optimal policy based on P_f for $\lambda(i,j)$ case

The customer reaction to a cross-sell attempt can be such that the customer may decrease her contact rate with each cross-sell attempt failure. The threshold values for the state-dependent λ case is computationally found to be the same as in the base case. For this

		λ/μ		
		10	**50**	**100**
r/R	**5**	0.571428571	0.571428571	0.571428571
	10	0.75	0.75	0.75

Table 3.11: Threshold Values for $\lambda(i, j)$

reason, the policy chosen is either to always cross-sell or to never cross-sell. Any failure probability equal to or greater than the values given in the Table 3.11 results in action not to cross-sell. Even a slight change in the threshold values given in Table 3.11 result in an optimal policy of always to cross-sell to the customer as in the base case. This situation can be seen in Figure 3.17.

$\frac{\lambda}{\mu}$			**10**	**50**	**100**
$\frac{r}{R}$	**5**	**Base Case**	15.09635	30.70348	35.70155
		$\lambda(i, j)$	13.04324	25.41793	29.34649
% Diff. from the Base Case			-13.60%	-17.21%	-17.80%
$\frac{r}{R}$	**10**	**Base Case**	33.48348	70.84432	82.80901
		$\lambda(i, j)$	26.29653	52.34309	60.56405
% Diff. from the Base Case			-21.46%	-26.12%	-26.86%

Table 3.12: $v(0, 0)$ Values for $\lambda(i, j)$

$v(0, 0)$ values obtained for $\lambda(i, j)$ case is given in Table 3.12. As it can be seen, the expected values for the state dependent arrival rate case are less than the state dependent death rate case. The additional revenue affect $v(0, 0)$ values more than an increase in the contact rate. When the r value increases twice, $v(0, 0)$ values increase slightly more than twice the value they have when r is 5. This change in r, however, affects the value of $v(0, 0)$ more in the increasing death rate case.

3.4.5 State Dependent Arrival and Death Rate Case

In the previous sections, we have analyzed the cases where the contact rate and the death rate change separately. As a result of the cross-sell attempt, however, contact rate and death rate can be affected at the same time, where the contact rate may decline due to the dissatisfaction and the death rate may increase. In this case, the expected revenues obtained decrease significantly because of the greater impact of dissatisfaction on the system.

		λ/μ		
		10	**50**	**100**
r/R	**5**	0.5	0.51	0.53
	10	0.67	0.67	0.7

Table 3.13: P_f^{min} Values for $\lambda(i,j)$ & $\mu(i,j)$

In Table 3.13, P_f^{min} values are given. The gap between the threshold values of the base case and the values given in Table 3.13 is greater when compared to $\mu(i,j)$ case.

		$\frac{\lambda}{\mu}$	**10**	**50**	**100**
$\frac{r}{R}$	**5**	**Base Case**	15.09635	30.70348	35.70155
		$\lambda(i,j)\&\mu(i,j)$	11.43624	23.53735	27.9756
% Diff. from the Base Case			-24.25%	-23.34%	-21.64%
$\frac{r}{R}$	**10**	**Base Case**	33.48348	70.84432	82.80901
		$\lambda(i,j)\&\mu(i,j)$	23.28668	48.6545	57.85756
% Diff. from the Base Case			-30.45%	-31.32%	-30.13%

Table 3.14: $v(0,0)$ Values for $\lambda(i,j)$ & $\mu(i,j)$

The expected revenues obtained when the failure probability is fixed to 0.4 is given in Table 3.14. As expected for $P_f = 0.4$ the optimal policy is to cross-sell in each state.

S 1-3-5: $P_f(0,0) = 0.565$ **S 2-4-6:** $P_f(0,0) = 0.745$	$v(0,0)_{\mu(i,j)}$ **Optimal**	$v(0,0)_{\mu(i,j)}$ **predetermined: xs**	**% Decrease**	$v(0,0)_{\mu(i,j)}$ **predetermined: nxs**	**% Decrease**
Scenario 1	7.742949	6.5418	15.51281%	7.741932	0.01313%
Scenario 2	7.742559	6.287604	18.79166%	7.741932	0.00810%
Scenario 3	14.67488	13.418721	8.55993%	14.636223	0.26342%
Scenario 4	14.677111	13.085669	10.84302%	14.636223	0.27858%
Scenario 5	16.923791	16.096287	4.88959%	16.822146	0.60060%
Scenario 6	16.93568	15.836464	6.49053%	16.822146	0.67038%

Table 3.15: $v(0,0)$ Comparison for Predetermined Policies

Effect of Ignoring Customer Reactions

In Table 3.15, the comparison between $v(0,0)$ values for the optimal and predetermined policies are given when the failure probability is between P_f^{min} and P_f^{max} values. When the predetermined policy is chosen as always to cross-sell, we see that for high r and small contact rate values the decrease in expected value is greater. In each scenario though, the percentage decrease is significant in this situation. When the negative customer reaction is ignored, among the two options of always to cross-sell and never to cross-sell, the firm should choose in this case never to cross-sell. If the predetermined policy is never to cross-sell, the decrease in the $v(0,0)$ optimal is quite small. As the failure probabilities chosen gets closer to the minimum threshold values, the decrease in the $v(0,0)$ values for the never to cross-sell predetermined case can be as large as 24.26%, while the decrease for the predetermined always to cross-sell becomes 4% maximum. Therefore, the significance of the decrease and which policy is more effective depend on the failure probability as seen in Table 3.16.

3.4.6 State Dependent Probability of Failure Case

The probability that the customer rejects the cross-sell offer increases with each previous failure in this case. Based on this, the initial failure probabilities affect the optimal policy chosen on the later stages. For this reason P_f^{min} and P_f^{max} values are given for the initial failure probability, where the state of the system is $(0,0)$.

In Table 3.17, $P_f^{min}(0,0)$ values are given for the 6 scenarios. The system cannot go back to the initial probability levels given in Table 3.17 due to the state dependency unless

	$v(0,0) - forP_f^{min}$	$v(0,0) - forP_f^{max}$	**% Decrese**
Scenario 1	8.555439	7.741934	9.51%
Scenario 2	9.458753	7.741934	18.15%
Scenario 3	17.185915	14.647866	14.77%
Scenario 4	19.339395	14.647772	24.26%
Scenario 5	18.532537	16.859471	9.03%
Scenario 6	20.227881	16.859313	16.65%

Table 3.16: Difference between $v(0,0)$ values when $P_f = P_f^{min}$ *and* $P_f = P_f^{max}$

		λ/μ		
		10	**50**	**100**
r/R	**5**	0.35	0.35	0.35
	10	0.40	0.40	0.40

Table 3.17: Minimum initial failure probabilities

there occurs a renewal in the system.

As $P_f^{max}(0,0) = P_f^*$ of the base case, for $P_f(0,0)$ value in the state dependent failure probability case, even a slight decrease in the threshold of the base case results in a cross-selling attempt in the system. In between the $P_f^{min}(0,0)$ and $P_f^{max}(0,0)$ values, we observe a dynamic optimal policy depending on the state of the system, where the optimal path has a mixed action set of cross-sell and not to cross-sell decisions.

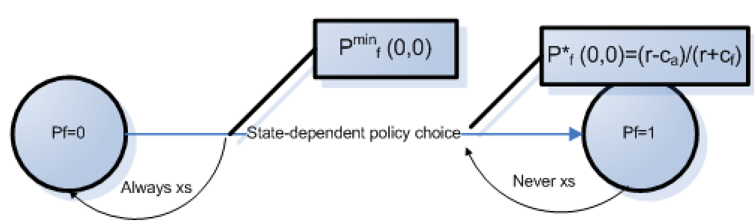

Figure 3.18: Change in the optimal policy based on $P_f(0,0)$ for $P_f(i,j)$ case

The values generated for $P_f(0,0) = 0.4$ are given in Table 3.18. The values obtained in

$\frac{\lambda}{\mu}$			**10**	**50**	**100**
$\frac{r}{R}$	**5**	**Base Case**	15.09635	30.70348	35.70155
		$P_f(i,j)$	11.5496	22.01735	25.36483
% Diff. from the Base Case			-23.49%	-28.29%	-28.95%
$\frac{r}{R}$	**10**	**Base Case**	33.48348	70.84432	82.80901
		$P_f(i,j)$	25.17348	50.24963	58.23279
% Diff. from the Base Case			-24.82%	-29.07%	-29.68%

Table 3.18: $v(0,0)$ values obtained for $P_f(0,0) = 0.4$

this case are smaller when compared to increasing death rate case in section 4.2 and varying state dependent arrival rate case in section 4.3, which means that failure of cross-selling attempts affect the system much more in the short-term in terms of revenue generation.

Optimal Path for Scenario 1

For the initial failure probabilities between the P_f^{min} and P_f^{max} values, there seems to occur a pattern in terms of cross-selling decisions. However, there is not a specific sequence of cross-sells and not cross-sells in each state. Table 3.19 provides an example for Scenario 1 where the initial failure probability is 0.4. The path formed by the bold 0s and 1s provide the optimal path. Using the optimal path, where 0 signifies not to cross-sell at that specific state (i,j) and 1 signifies to cross-sell, firm obtains the maximum profit in that system with the given parameters. In Table 3.19, the optimal path dictates to the firm that when the customer comes for the first time or when the customer contacts the first time after accepting a cross-selling offer, the firm should attempt to cross-sell. But if the customer refuses the offer, the next time customer contacts the firm, the firm should not annoy the customer and let the effects of the failed cross-selling attempt to decrease and so on. Doing this, the firm can obtain the maximum expected value generated from a single customer.

		j-number of customer contacts																	
		0	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**17**
i	**0**	**1**	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
-	**1**		**0**	**1**	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
#	**2**			0	**0**	**1**	1	1	1	1	1	1	1	1	1	1	1	1	1
of	**3**				0	0	**0**	**1**	1	1	1	1	1	1	1	1	1	1	1
xs	**4**					0	0	0	**1**	1	1	1	1	1	1	1	1	1	1
f	**5**						0	0	0	**0**	**1**	1	1	1	1	1	1	1	1
a	**6**							0	0	0	0	**1**	1	1	1	1	1	1	1
i	**7**								0	0	0	0	**0**	**1**	1	1	1	1	1
l	**8**									0	0	0	0	0	**1**	1	1	1	1
u	**9**										0	0	0	0	0	**1**	1	1	1
r	**10**											0	0	0	0	0	**0**	**1**	1
e	**11**												0	0	0	0	0	0	**1**

Table 3.19: Action Set for $P_f(i, j)$ with initial $P_f = 0.4$ for Scenario 1

Comparison of Different $P_f(0, 0)$ Values for Scenario 1

State dependency enables the system not to forget about the results of the previous actions taken that result in failure. The effects of the actions can lessen with time when the firm stops the cross-selling attempts, however, can never be erased unless a renewal occurs, which can be seen in Figure 3.19. In Figure 3.19 the change in the failure probabilities for the optimal path for scenario 1, where $r = 5$ and $\lambda = 10$, with different initial P_f values are given. No matter what the initial failure probability is if the first contact results in a failed cross-selling attempt the failure probability increases 50%. To decrease the effect of this change in P_f value becomes more difficult as the initial failure probability increases, and when the initial failure probability becomes the threshold of the base case, the effect of failed cross-selling attempt can not be compromised, therefore the system chooses not to cross-sell.

When the $P_f(0, 0)$ values increase, not to cross-sell decisions also increase among the

SCENARIO 1				
$P_f(0,0)$	$E[P_f]$	$E[v(0,0)]$	**# xs**	**# nxs**
0.3	0.3583801	17.0784	99	0
0.35	0.4343922	13.95201	98	1
0.4	0.4979294	11.5496	78	21
0.45	0.5502411	9.796126	51	48
0.5	0.5950751	8.647246	30	69
0.55	0.6314169	7.954458	14	85
$P_f^* = 0.5714292$	0.5714292	7.741934	8	91

Table 3.20: Comparison of Different Initial P_f Values for Scenario 1

optimal path as shown in Table 3.20. The threshold value obtained for the base case is shown as P_f^* in Table 3.20. For the infinite state-space when $P_f(0,0) = P_f^*$, the decision becomes always not to cross-sell, however in our numerical analysis we see that with a finite state-space we have 8 cross-selling decisions all given toward the end, where the system knows the customer will die. So that these 8 cross-selling decisions are due to the boundary effect. The expected values decrease as the initial failure probabilities increase, as we expect.

Expected Value of Probability of Failure ($E[P_f]$)

Given the optimal path for a specific scenario, we can compute the expected failure probabilities. We computed the expected failure probability when the system starts from the initial state of $(0,0)$ and then based on the optimal path goes to other states up until death. From the transient analysis we know how many times on average a customer is found in each state starting from the initial state, and we also know based on the optimal path how the failure probability changes. Based on these, we find the expected probability failure values for different $P_f(0,0)$ values when the system starts from $(0,0)$. The expected failure probabilities increase as the initial failure probabilities increase, up until the threshold value of the base case, at which the expected failure probability becomes equal to the threshold value of the base case as seen in Table 3.20. For the initial failure probabilities within the range of $[P_f^{min}(0,0), P_f^{max}(0,0))$, the increase in the expected failure probabilities is due to the state dependency of the failure probabilities.

Average # of Visits to States for Scenario 1

Based on the transaction matrix, we can find the average number of visits to each state starting from the initial state of $(0,0)$ in the long run for each of the initial probabilities given in Figure 3.19. The total number of customer contacts starting from state $(0,0)$ up until death is 12 on the average. Out of an average of 12-visit-relationship with the firm a customer's average number of visits to each state starting from the initial state of $(0,0)$ are given in Table 3.21 for different initial failure probabilities. As the initial failure probability increases, a firm attempts less to cross-sell to the customer. As renewal occurs only in case of successful cross-sells, an increase in the initial failure probability results in less visits to the initial state. During a 12-visit-relationship with the firm, the customer comes more frequently to the initial state due to renewals, however the number of times the $(0,0)$ state visited decreases as the initial failure probability increases. Even though, in the numerical analysis we have used a state space of (100x100), we see that in nearly 20 contacts the total number of customer contact to that specific state becomes 12. The customer stays in the initial states in most of her contacts. For this reason a state space of (100x100) is good enough for the numerical analysis of the model.

$P_f(0,0) = 0.3$			$P_f(0,0) = 0.35$			$P_f(0,0) = 0.4$			$P_f(0,0) = 0.45$			$P_f(0,0) = 0.5$			$P_f(0,0) = 0.55$		
States		**# of visits**	**States**		**# of visits**	**States**		**# of visits**	**States**		**# of visits**	**States**		**# of visits**	**States**		**# of visits**
0	**0**	8.0904	**0**	**0**	6.8662	**0**	**0**	5.3565	**0**	**0**	3.9432	**0**	**0**	2.81338	**0**	**0**	1.96442
1	**1**	2.2065	**1**	**1**	2.1847	**1**	**1**	1.94781	**1**	**1**	1.6131	**1**	**1**	1.27881	**1**	**1**	0.98221
2	**2**	0.9026	**2**	**2**	1.0427	**1**	**2**	1.77074	**1**	**2**	1.4665	**1**	**2**	1.16255	**1**	**2**	0.89292
3	**3**	0.4103	**2**	**3**	0.9479	**2**	**3**	0.85854	**1**	**3**	1.3332	**1**	**3**	1.05687	**1**	**3**	0.81174
4	**4**	0.1958	**3**	**4**	0.4524	**2**	**4**	0.78049	**1**	**4**	1.212	**1**	**4**	0.96079	**1**	**4**	0.73795
5	**5**	0.0961	**4**	**5**	0.2303	**3**	**5**	0.39734	**2**	**5**	0.595	**1**	**5**	0.87344	**1**	**5**	0.67086
6	**6**	0.0481	**5**	**6**	0.1221	**3**	**6**	0.36122	**2**	**6**	0.5409	**1**	**6**	0.79404	**1**	**6**	0.60987
7	**7**	0.0243	**6**	**7**	0.0666	**4**	**7**	0.18765	**3**	**7**	0.2845	**1**	**7**	0.72185	**1**	**7**	0.55443
8	**8**	0.0124	**7**	**8**	0.0371	**5**	**8**	0.10235	**3**	**8**	0.2586	**2**	**8**	0.36913	**1**	**8**	0.50403
9	**9**	0.0064	**8**	**9**	0.021	**5**	**9**	0.09305	**3**	**9**	0.2351	**2**	**9**	0.33557	**1**	**9**	0.4582
10	**10**	0.0033	**9**	**10**	0.012	**6**	**10**	0.05075	**4**	**10**	0.125	**2**	**10**	0.30506	**1**	**10**	0.41655
11	**11**	0.0017	**10**	**11**	0.007	**7**	**11**	0.02852	**4**	**11**	0.1137	**2**	**11**	0.27733	**1**	**11**	0.37868
12	**12**	0.0009	**11**	**12**	0.0041	**7**	**12**	0.02593	**5**	**12**	0.062	**2**	**12**	0.25212	**1**	**12**	0.34426
13	**13**	0.0005	**12**	**13**	0.0024	**8**	**13**	0.01451	**5**	**13**	0.0564	**3**	**13**	0.13223	**1**	**13**	0.31296
14	**14**	0.0002	**13**	**14**	0.0014	**9**	**14**	0.00829	**5**	**14**	0.0512	**3**	**14**	0.12021	**1**	**14**	0.28451
15	**15**	0.0001	**14**	**15**	0.0008	**10**	**15**	0.00482	**6**	**15**	0.0279	**3**	**15**	0.10928	**1**	**15**	0.25864
16	**16**	7E-05	**15**	**16**	0.0005	**10**	**16**	0.00438	**6**	**16**	0.0254	**3**	**16**	0.09935	**1**	**16**	0.23513
17	**17**	4E-05	**16**	**17**	0.0003	**11**	**17**	0.00253	**7**	**17**	0.0141	**4**	**17**	0.05313	**1**	**17**	0.21375
18	**18**	2E-05	**17**	**18**	0.0002	**12**	**18**	0.00148	**7**	**18**	0.0128	**4**	**18**	0.0483	**1**	**18**	0.19432
19	**19**	1E-05	**18**	**19**	0.0001	**13**	**19**	0.00088	**8**	**19**	0.0072	**4**	**19**	0.04391	**1**	**19**	0.17666
20	**20**	6E-06	**19**	**20**	7E-05	**13**	**20**	0.0008	**8**	**20**	0.0065	**4**	**20**	0.03991	**1**	**20**	0.1606

Table 3.21: Average # of Visits to States Starting from $(0,0)$ for Different $P_f(0,0)$ Values for Scenario 1

Effect of Ignoring Customer Reactions

In Table 3.22, we see the percentage decreases in the optimal $v(0,0)$ values when the policy is predetermined by the firm independent of the customer's reaction. For the initial failure probabilities chosen for the predetermined case, in case of no negative customer reactions, the optimal policy would be always to cross-sell to the customer. However, as seen in Table 3.22, the differences between the optimal policy and the predetermined policies are not negligible, for both predetermined policies: always to-cross-sell and always not-to-cross-sell.

S 1-3-5: $P_f(0,0)$=**0.4** **S 2-4-6:** $P_f(0,0)$= **0.45**	$v(0,0)_{P_f(i,j)}$ **Optimal**	$v(0,0)_{P_f(i,j)}$ **predetermined: xs**	**% Decrease**	$v(0,0)_{P_f(i,j)}$ **predetermined: nxs**	**% Decrease**
Scenario 1	11.549602	10.238955	11.34798%	7.741932	32.96798%
Scenario 2	20.069634	18.04605	10.08281%	7.741932	61.42465%
Scenario 3	22.017352	18.401757	16.42157%	14.636223	33.52414%
Scenario 4	39.044386	32.087894	17.81688%	14.636223	62.51389%
Scenario 5	25.36483	20.964339	17.34879%	16.822146	33.67925%
Scenario 6	45.071766	36.376792	19.29140%	16.822146	62.67698%

Table 3.22: $v(0,0)$ Comparison for the Predetermined Policies

3.4.7 Comparison of Cases

In this section we compare all the cases in terms of $v(0,0)$ values and P_f^{min} and P_f^{max} values.

	Scenario 1	**Scenario 2**	**Scenario 3**	**Scenario 4**	**Scenario 5**	**Scenario 6**
BASE CASE	15.096349	33.483475	30.70348	70.844315	35.701553	82.809006
$\mu(i,j)$	13.557258	30.069313	29.055698	67.041385	34.531509	80.093946
$\lambda(i,j)$	13.043239	26.29653	25.417925	52.343086	29.346494	60.564048
$\mu(i,j)\&\lambda(i,j)$	11.436237	23.286676	23.537347	48.654504	27.975595	57.857563
$P_f(i,j)$	11.549602	25.173477	22.017352	50.249626	25.36483	58.232789

Table 3.23: Comparison of $v(0,0)$ Values for $P_f = 0.4$

In Table 3.23, a comparison between the expected values when the failure probability is 0.4 is given. Except for the $P_f(i,j)$ case, in all the other cases the optimal policy is to cross-sell in each case. For the comparison of the different parameters, $P_f(i,j)$ is also added, but in this case only the initial failure probability starts from 0.4 and then changes based on the state of the system. Based on the Table 3.23, for always cross-sell policy, the effect of the cross-selling on different parameters affect the expected revenues in different magnitudes. The rank is found as follows:

$$v(0,0)_{basecase} > v(0,0)_{\mu(i,j)} > v(0,0)_{\lambda(i,j)} > v(0,0)_{P_f(i,j)} > v(0,0)_{\mu(i,j)\&\lambda(i,j)}$$

In the interval of $[P_f^{min}, P_f^{max})$, the optimal policy gradually changes having more not-to-cross-sell decisions initially and the number of not cross-sell decisions increase until the

	Scenario 1	**Scenario 2**	**Scenario 3**	**Scenario 4**	**Scenario 5**	**Scenario 6**
BASE CASE	0.571428571	0.75	0.571428571	0.75	0.571428571	0.75
$\mu(i,j)$	[0.53-0.571429)	[0.72-0.75)	[0.555-0.571429)	[0.735-0.75)	[0.56-0.571429)	[0.74-0.75)
$\lambda(i,j)$	0.571428571	0.75	0.571428571	0.75	0.571428571	0.75
$\mu(i,j)$&$\lambda(i,j)$	[0.5-0.571429)	[0.67-0.75)	[0.5-0.571429)	[0.67-0.75)	[0.53-0.571429)	[0.7-0.75)
$P_f(i,j)$	[0.35-0.571429)	[0.4-0.75)	[0.35-0.571429]	[0.4-0.75)	[0.35-0.571429]	[0.4-0.75)

Table 3.24: Comparison of Failure Probabilities for the Threshold Values

threshold of the base case is reached. From that point onward excluding the last states due to the boundary effect all decisions become always not to cross-sell when the failure probability becomes equal to P_f^{max}. When the P_f^{min} values are compared, the P_f^{min} value of the case where both death rate and arrival rate vary linearly based on states is lower than the rest of the cases. Even though the P_f^{min} values of the $\mu(i,j)$ and $\lambda(i,j)$ cases are lower than the threshold of the base case, there is not a specific relation between the $\mu(i,j)$ and $\lambda(i,j)$ cases. When P_f varies dynamically, optimal actions can also become state dependent, see Table 3.19. In fact, there is a threshold on the initial P_f value so that whenever P_f is lower than the threshold, it is still optimal to always cross-sell; but when the initial P_f exceeds this threshold, optimal cross-sell decisions vary with the state of the system, (i,j), see Table 3.20.

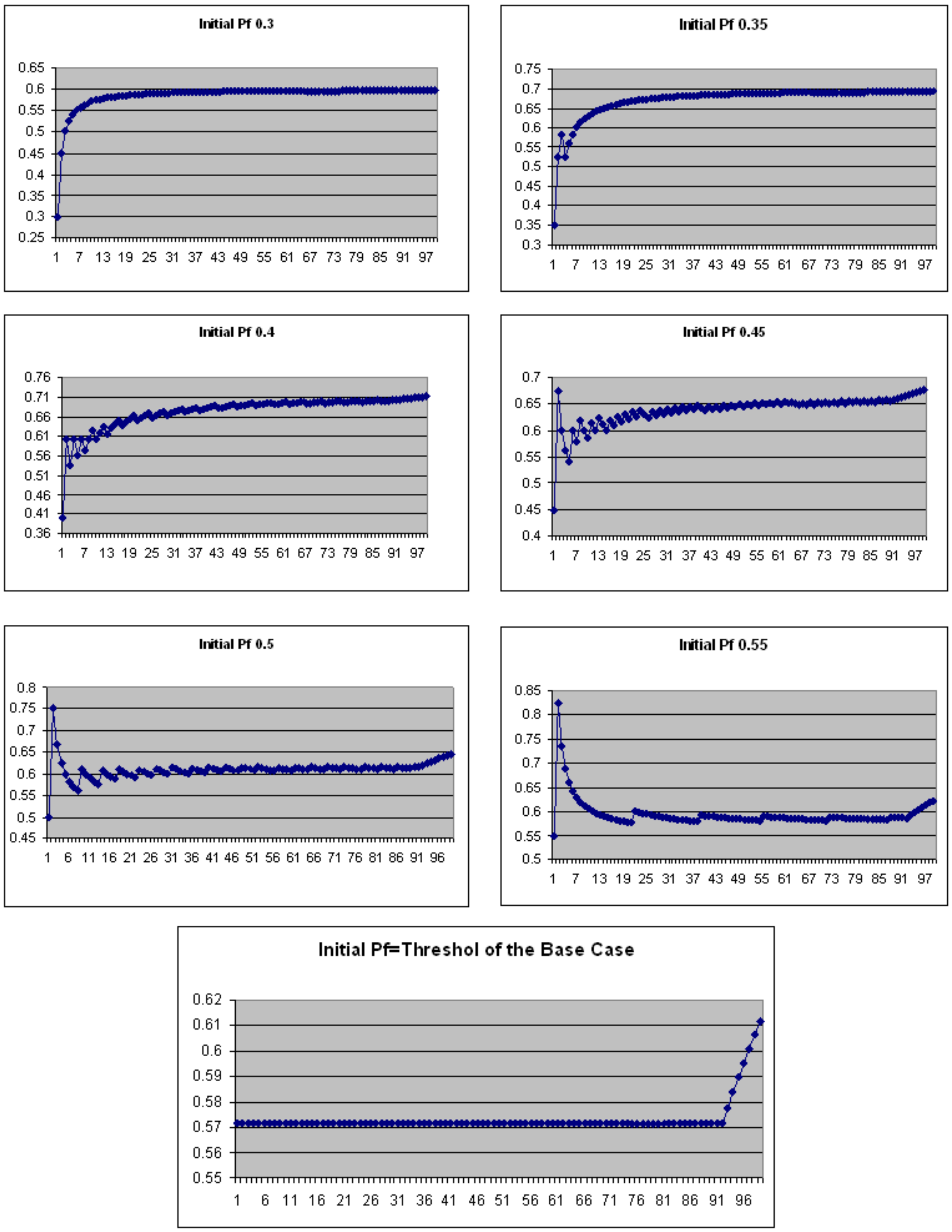

Figure 3.19: Comparison of $P_f(i,j)$ for Different Initial P_f Values under Scenario 1, x-axis shows the number of contacts, $P_f^* = 0.571428571$

Chapter 4

A DYNAMIC CROSS-SELLING MODEL WITH 3-D STATE SPACE

In this chapter we numerically investigate the effect of using additional data on cross-selling attempts of the firm. The information on the transactions related to cross-selling includes the number of successful and unsuccessful attempts and the total number of contacts.

In Section 4.1, we allow the system to be renewed whenever there is a successful cross-selling so that the number of failures and the total number of contacts become zero while the number of successful cross-sells increase. We use this model to compare the 2-dimensional and 3-dimensional state space models to have insights about the effect of keeping more information about customer purchase history. Allowing a renewal whenever there is a successful cross-selling attempt, we let the customers forget about the failures of the past. In 3-D state space, however, the system is not totally renewed as the customer does not forget about the successful cross-sell attempts.

In Section 4.1, to differentiate between the negative and positive impacts on parameters, we use the notations $\mu(k,i,j)^+$ or $\mu(k,i,j)^-$, $\lambda(k,i,j)^+$ or $\lambda(k,i,j)^+$ and $P_f(k,i,j)^-$, where positive reactions are symbolized by $+$, and negative reactions are symbolized by $-$.

In Section 4.2, a brief description of a modified model is given where there is no renewal in the system in terms of forgetting about the past failures. No matter how the cross-sell attempts end the customer/firm always remember the outcome and act accordingly.

4.1 Model with Renewals

In this model, different from the model described in the Chapter 3, we add an additional dimension to the state space, where we keep track of the successful cross-selling attempts. We again use a Markov Decision (MD) Model, where the control policy is formed upon the decision of the firm on whether to cross-sell to the customer or not. The performance of this control policy is measured by the discounted expected value generated from the customer until the relationship between the customer and the firm ends.

In the following, we formulate the model and conduct analysis in order to understand the impact of customer reactions in terms of customer frequency, death rate and the willingness to accept a cross-sell offer. To be able to compare these cases we use base case where there is no reaction. In this chapter not only the negative impact of cross-selling offers is investigated, but the positive effect is taken into account as well. Failure probability is chosen to reflect the negative reactions, while the contact frequency or the death rate is chosen to reflect the positive reactions.

Faced with the negative effect of an action, the customers are more likely to react on the point of interaction. In our study this can be due to the fact that they are annoyed with too much cross-selling offer. Customers may directly react as a reaction for the annoyance caused by too much cross-selling by being more reluctant for accepting cross-selling offers. For this reason, we use failure probabilities to demonstrate the negative impact. On the other hand, a relationship development based on trust and satisfaction requires time and more experience formation. As the number of encounters between the customer and the firm increases, due to the satisfaction and trust the customer will be less reluctant to quit the relationship or the customer may contact more [43]. For this reason, we used death rate and contact rate to show the positive effects in general.

We will implement these various effects in the model separately to see their impacts in isolation first.

4.1.1 Model Description

The parameters defined in Chapter 3 are valid in this model as well. The states are defined as (k, i, j), where k is the number of successful cross-sell attempts, i is the number of unsuccessful cross-sell attempts since the last successful cross-sell, and j is the number of the contacts initiated by the customer since the last successful cross-sell. Therefore, the state of the system at any time is represented by (k, i, j), where $i \leq j$ for all i and j. The total number of additional products/services that can be offered to the customer is limited to a certain number of K.

To model the positive impact of the cross-selling activities, we define $\lambda(k)$, that increases as the number of successful cross-sells increase, and $\mu(k)$, that decreases as the number of successful cross-sells increase. We assume that $\lambda(k)$ is a concave function of k, so that as the successful cross-sell attempts increase, the rate of increase in the contact rate decreases.

The negative impact is modeled by using increasing death rate, decreased contact rate and increased failure probability with the number of failures.

The possible events and the possible paths a customer may take at each state are given in Figures 4.1 and 4.2. However as there is additional information in the picture, the possible paths taken are more diversified when compared to the 2-D model. Based on these, the schematic representation for this model can be given as follows:

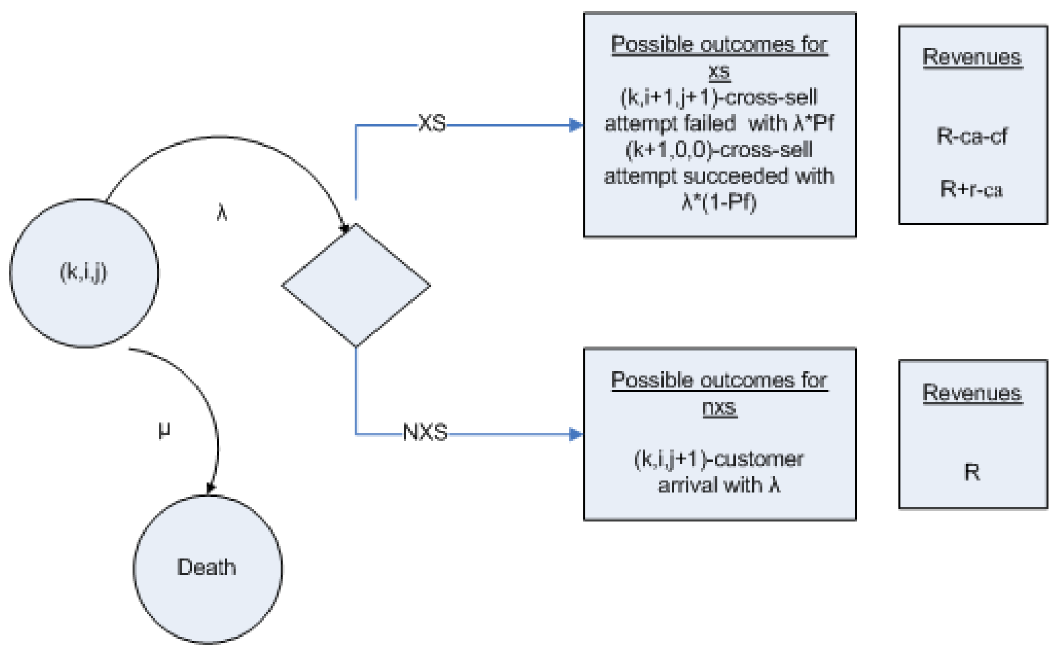

Figure 4.1: Possible Outcomes of the Model

As shown in Figure 4.1, the transition among states is such that each time the customer contacts the firm, the number of contacts (j) increases by 1. At each contact point the firm decides to try to cross-sell or not. Whenever the cross-sell attempt results in failure, the number of unsuccessful cross-sell attempts (i) increases by 1 whilst the number of successful cross-sell attempts (k) remains the same. Whenever the attempt is a success, the number of successful cross-sell attempts k increases by 1 whilst the number of unsuccessful cross-sell attempt and the number of contacts become zero indicating a renewal. The firm has the complete history on successful cross-selling attempts, and also the decisions made on

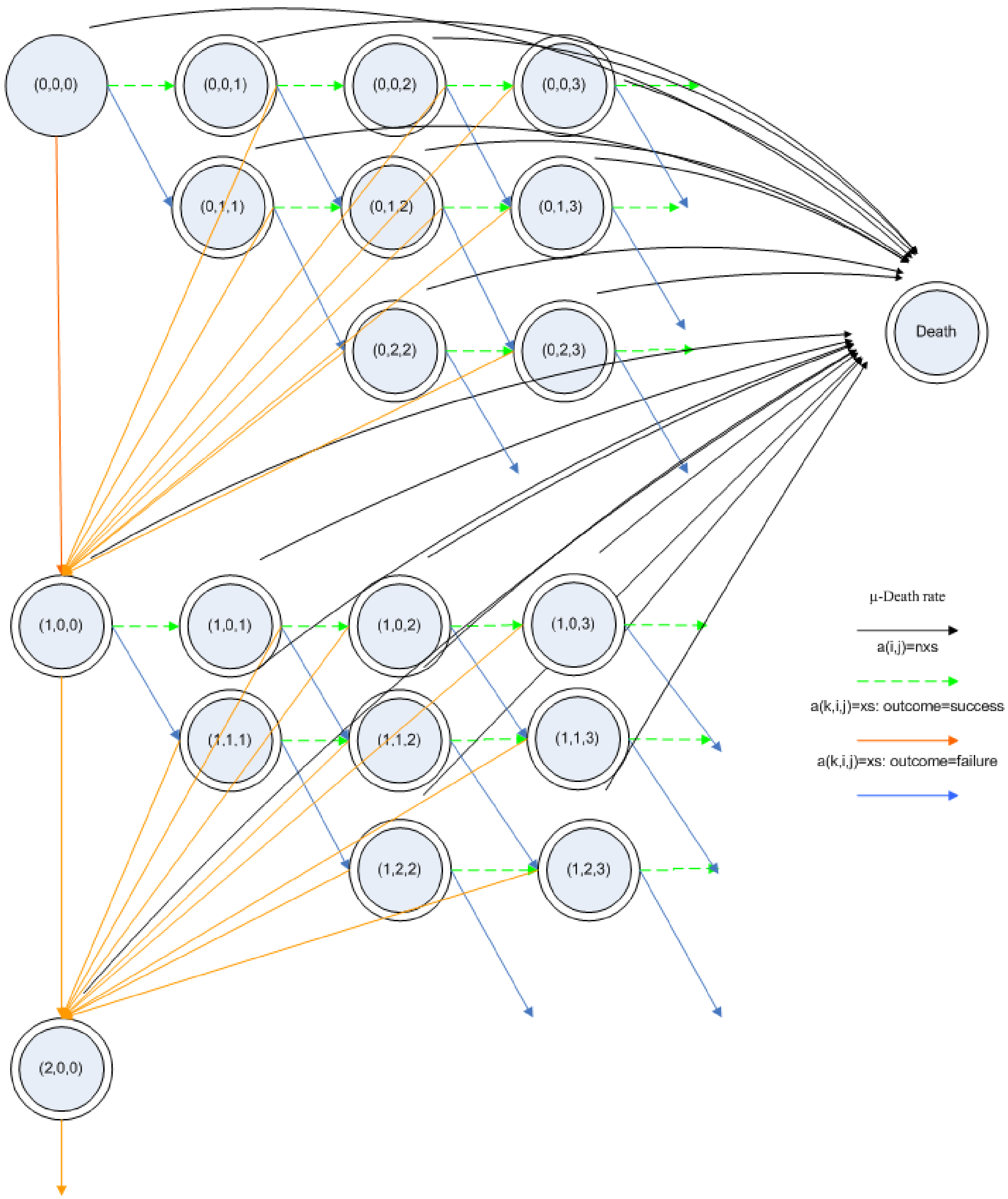

Figure 4.2: All possible paths a customer may take depending on action $a(k, i, j)$

cross-selling by the customer since the last successful cross-sell.

Based on the action chosen and the reaction of the customer, the customer may follow different paths during her relationship with the firm as shown in Figure 4.2.

The firm's decision on cross-selling activities may affect the customer behavior. The decision epochs are whenever the customer contacts the firm. We model the value generation on an individual basis. Given the parameter definition in Section 3.1, we can characterize the MDP as follows:

State space is defined by:

$S = \{(k, i, j) : k = 0, 1, ..., K; i = 0, 1, 2, ..., j; j = i, i + 1, ..., D\}$.

The planning horizon ends when the customer quits the relationship with the firm, which is shown with state D. The action set in each state has two options, $a(k, i, j) = (xs, nxs)$, where xs represents the cross-sell attempt option of the firm and nxs represents the not attempt to cross-sell option. At each decision epoch, the expected net revenue is defined by the action chosen and can be seen in Figure 4.1.

Let $v(k, i, j)$ be the value function that gives the discounted profit of the system starting from state (k, i, j) over an infinite horizon. D is defined previously as the death state. $v(D)$ is the profit obtained from the state where the relationship between the firm and the customer ends, and therefore is zero.

When $k = K$, as there is no more additional products to be sold to the customer, for states (K, i, j) for all i and j, the action set consists of only not to cross-sell. For $k = 0, 1, ..., K - 1$, when $v(k, i, j)$ is optimized, an optimal policy, π^* is obtained that forms a prescription for the firm whether to cross-sell to the customer or not at each decision epoch.

The exponentiality of the customer contacts and lifetimes allows us to use uniformization (see Lippman 1975 [29]). Using both uniformization and normalization we turn the rates into probabilities and we also adjust the time scale for comparison of cases, where we assume $2\mu + \lambda = 1$. Therefore, the above value function becomes:

c_a	1
c_f	2
P_f	0.4
R	1

Table 4.1: Parameters Used in Numerical Analysis of Section 4

$for\ k = 0, 1, .., K-1$

$$v(k,i,j) = R + \alpha\mu v(D) + \alpha\mu v(k,i,j) +$$

$$\alpha\lambda \max \left\{ \begin{array}{l} (P_f(v(k,i+1,j+1) - c_a - c_f) + (1-P_f)(v(k+1,0,0) + r - c_a)) \\ v(k,i,j+1) \end{array} \right\}$$

$for\ k = K$

$$v(K,i,j) = R + \alpha\mu v(D) + \alpha\mu v(K,i,j) + \alpha\lambda v(K,i,j+1)$$

For the numerical application of this general model, the state space for the number of customer contacts as well as the cross-sell attempt failures are taken to be 100 to be consistent with the previous chapter. In our calculations, we took K to be 11 as well, which puts a limit on the number of products to be sold to the customer. In each section, whenever all 11 products are sold to the customer, the optimal policy is set to be not-to-cross-sell.

The same parameter set used in Section 3.4.1 is used for the numerical analysis of the following cases. The fixed parameters are given in Table 4.1. For each case considered in the following section, for consistency in the numerical analysis, we look at the 6 Scenarios mentioned in Section 3.4.1, where the additional revenue obtained from cross-selling can be high or low, and where the contact rate can be low, medium and high.

Using the additional information, we investigate different cases. To see the effect of additional information and limited number of products provided to the customer we investigate the changes in terms of policy and revenue generated for the cases where death rate, contact rate and failure probability are state-dependent. To be able to compare the results with 2-D state space dynamic modeling proposed in Chapter 3, we let failure probability, death rate and the contact rate to reflect the negative impact of cross-selling as given in the previous chapter. However, because we have 11 products to sell by cross-selling, we can compare the first two scenarios of the 2-D and 3-D state space systems. For the rest of the scenarios, even though the customers have high contact rates during a lifetime we do not have enough products to offer, which leads to lower profits obviously. To see the positive impact of cross-selling we used contact rate and the death rate. Finally, we incorporated both the positive and the negative impact of cross-selling in the final cases considered.

4.1.2 Base Case

In the base case, neither the decisions of the firm nor the reactions of the customer affect the parameters of the system as similar to the base case defined in the previous chapter. However, in this case there are limited number of additional products as well as additional information on the number of successful cross-sell attempts. Within this section, all the comparisons between the base case are done for the 3-D state space system where the number of failures and contacts become zero depending on cross-sell success.

The base case is important since it sets a simple benchmark for the systems to be considered later on and also because no parameters change, the optimal policy is either to cross-sell at each state or not.

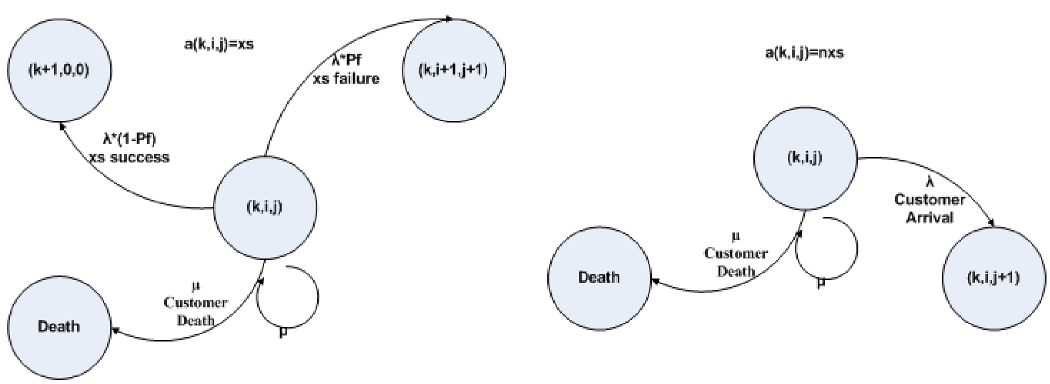

Figure 4.3: Transition Probabilities when $a(k,i,j) = nxs$ and $a(k,i,j) = xs$

We assume that the customer lifetime with the firm, T, is a random variable with an exponential distribution with rate μ. The time between the two contacts of the customer is also exponential with rate λ. The transition probabilities differ for each action chosen and can be seen in Figure 4.3 for both when the action chosen is not to cross-sell and when the action chosen is to cross-sell at state (k,i,j). In case of a cross-sell attempt, if the attempt is successful there occurs a renewal and the state of the system becomes $(k,0,0)$.

If the attempt fails, then the state of the system becomes $(k, i+1, j+1)$. In case of not to cross-sell, the state changes to $(k, i, j+1)$. For any action chosen the state of the system can also be (D), as the customer may leave the system whenever s/he wants independent of the action chosen.

		λ/μ		
		10	**50**	**100**
r/R	**5**	14.574416	26.287026	29.478059
	10	31.657089	55.392821	61.055303

Table 4.2: $v(0,0,0)$ Values for the Base Case

The expected values found in numerical analysis with the parameters mentioned previously are given in Table 4.2. As we can see, the impact of additional revenue obtained from cross-selling is greater when compared to the impact of contact rate.

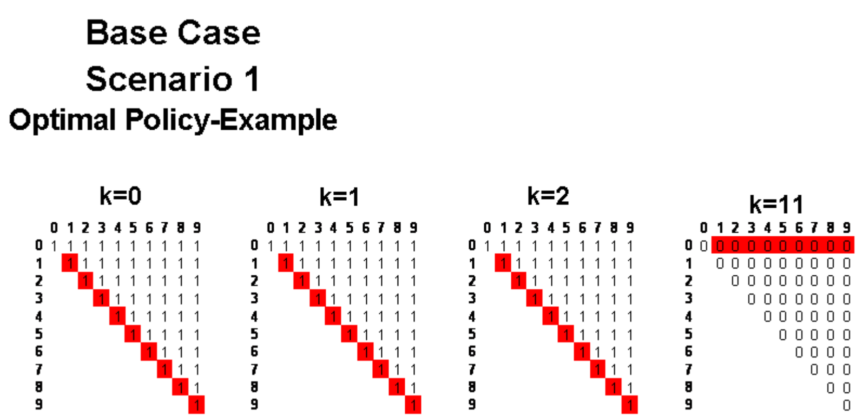

Figure 4.4: Optimal Policy Example for Base Case under Scenario 1

As long as the firm has additional products to sell, the optimal policy is always to cross-sell, irrespective of the remaining products to be sold in the future contacts. When the product limit is reached, in this case 11, the optimal policy is not-to-cross-sell. This situation can be seen in Figure 4.4 via a small example on Scenario 1. For the Base Case, in all Scenarios, the same optimal policy can be seen, where the policy is always to cross-sell until the limit on the additional products are reached. We can conclude that when there is no reaction to cross-selling attempts, having more information does not change the optimal policy, as expected.

4.1.3 Negative Reaction I: Increasing Failure Probability (P_f)

Failure probability changes such that a customer's likelihood to accept a cross-sell offer decreases as the number of cross-sell failures increases and when the cross-sell attempt is successful, the system is renewed in terms of cross-sell failures and customer contacts. Similar to the case in the previous chapter, we let the initial failure probability to be 0.4, and for the remaining states the failure probability changes as follows:

$$P_f(k,i,j)^- = P_f(1 + \frac{i}{j+1}).$$

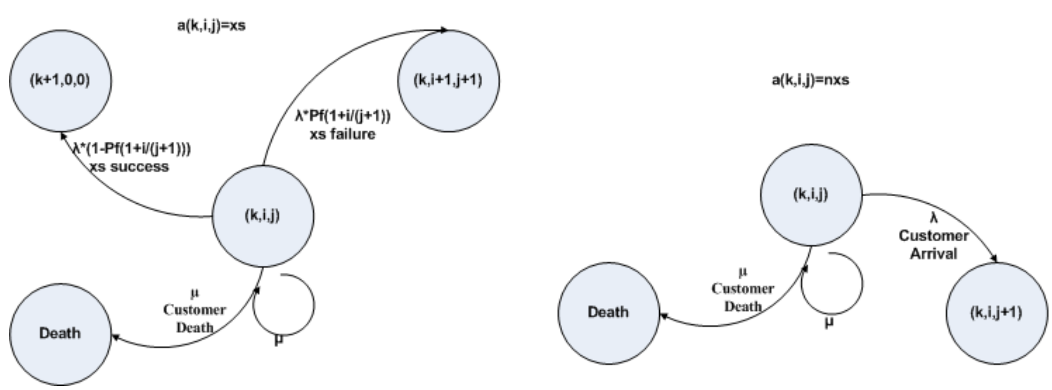

Figure 4.5: Transition Probabilities when $a(k,i,j) = nxs$ and $a(k,i,j) = xs$

The transition probabilities from a certain state (k,i,j) to other possible states depending on the action chosen is given in Figure 4.5. Based on these, the value function can be written as follows:

$for\ k = 0, 1, .., K-1$

$v(k,i,j) = R + \alpha\mu v(D) + \alpha\mu v(k,i,j) +$

$$\alpha\lambda \max \left\{ \begin{array}{l} (P_f(1+\frac{i}{j+1})(v(k,i+1,j+1) - c_a - c_f) + (1 - P_f(1+\frac{i}{j+1}))(v(k+1,0,0) + r - c_a) \\ v(k,i,j+1) \end{array} \right\}$$

$for\ k = K$

$v(K,i,j) = R + \alpha\mu v(D) + \alpha\mu v(K,i,j) + \alpha\lambda v(K,i,j+1)$

$\frac{\lambda}{\mu}$			**10**	**50**	**100**
$\frac{r}{R}$	**5**	**Base Case**	14.574416	26.287026	29.478059
		$P_f(k,i,j)^-$	11.466212	21.152079	24.05469
% Diff. from the Base Case			-21.33%	-19.53%	-18.40%
$\frac{r}{R}$	**10**	**Base Case**	31.657089	55.392821	61.055303
		$P_f(k,i,j)^-$	24.530113	43.983663	49.131506
% Diff. from the Base Case			-22.51%	-20.60%	-19.53%

Table 4.3: $v(0,0,0)$ Values for the $P_f(k,i,j)^-$ Case

Based on the above equations the values generated are given in Table 4.3. We observe that as the contact rate increases, the gap between the Base Case and the $P_f(k,i,j)^-$ Case decreases. This is due to the fact that when the contact rate is high the firm has more chance to make up for the previous failures. The gap between Base Case and the $P_f(k,i,j)^-$ Case slightly increases for high r value. When the additional revenue obtained from cross-selling is higher, the potential loss due to increasing failure probability will also be higher.

Just like in the Base Case, when the contact rate increases the values generated increase. In the same way as the additional revenue increases the $v(0,0,0)$ values increase. However, again the impact of additional revenue is greater than the impact of contact rate on value generation.

Because all the analyses are done for $P_f(k,0,0)^- = 0.4$, the optimal policies associated with the 6 Scenarios display a dynamic nature, in terms of cross-selling decisions. As more and more of the products are sold to the customer, the system gives more of not-to-cross-sell decisions at certain states.

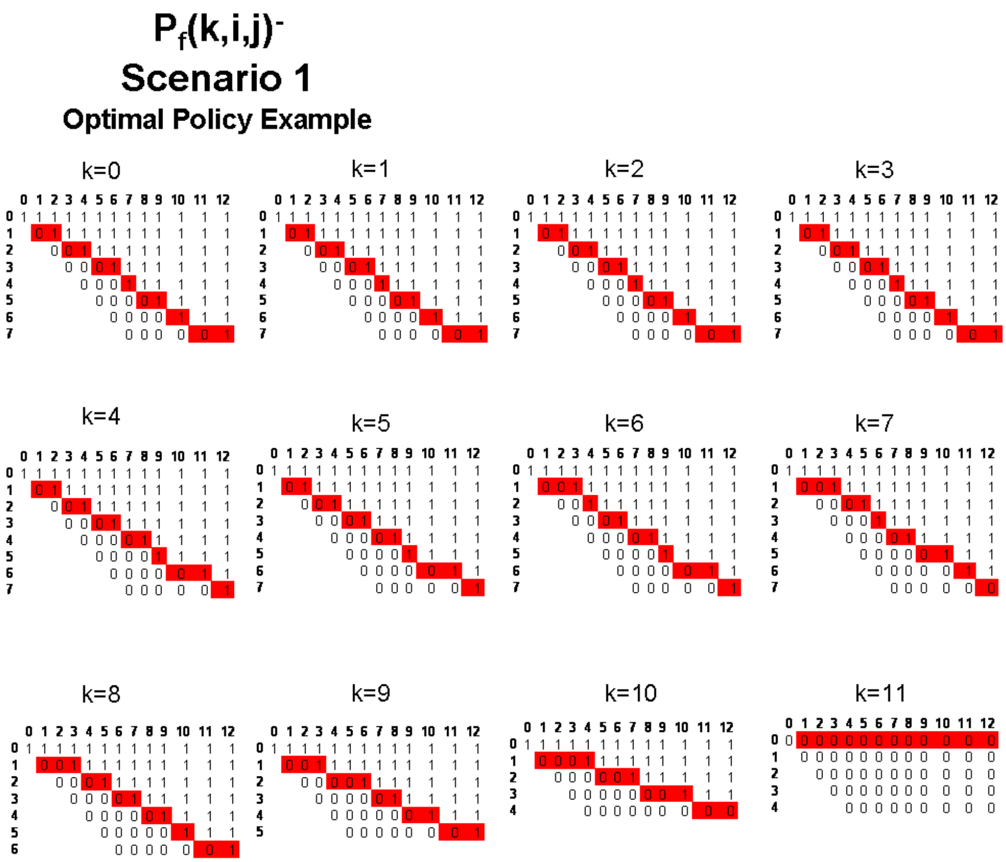

Figure 4.6: Optimal Policy Example for $P_f(k,i,j)^-$ Case under Scenario 1

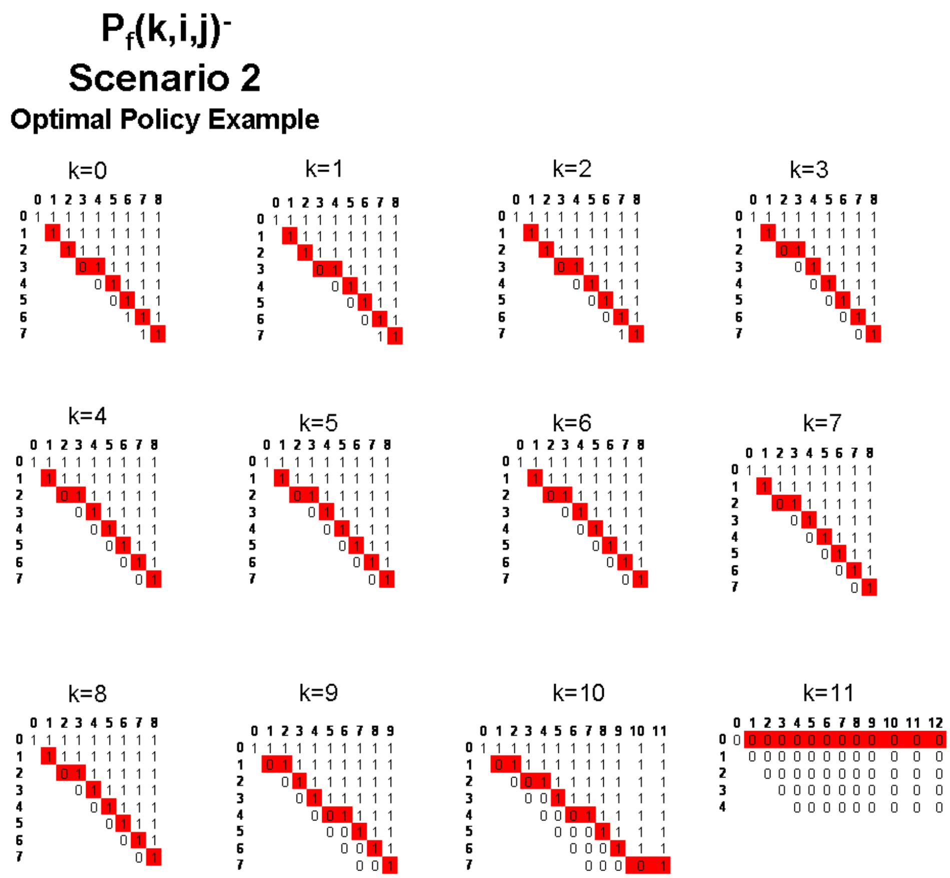

Figure 4.7: Optimal Policy Example for $P_f(k,i,j)^-$ Case under Scenario 2

When the additional revenue is low, the system becomes very dynamic. Yet, for the last two three products remaining to be sold, the system starts to wait more before attempting a cross-sell. To provide a small example for this situation, we choose Scenario 1, where both the contact rate and the additional revenue obtained from cross-selling is low. As can be seen from Figure 4.6, for Scenario 1, the optimal policy is more aggressive when the number of successful cross-sells is few. As the available number of products to sell to the customer decreases, the system becomes more cautious and becomes less aggressive. When all 11 products are sold to the customer, the optimal policy becomes not to cross-sell to the customer any more.

When the additional revenue is high, the optimal policy is much more aggressive and not as dynamic as the low additional revenue Scenarios. To provide an example for this, we choose Scenario 2, where the additional revenue from cross-selling is high, while the contact rate is low. The optimal policy is much more aggressive and not very dynamic as seen in Figure 4.7.

4.1.4 Negative Reaction II: Increasing Death Rate (μ)

Death rate is directly linked to the relationship duration of the customer and the firm. In the model we let the death rate increase as unsuccessful cross-sell attempts by the firm increase using the following function:

$$\mu(k,i,j)^{-} = \mu(1+\frac{i}{j+1}).$$

This implies that the system is renewed and death rate of the customer takes its minimum value when there is a successful cross-sell. The transition probabilities for each action chosen are shown in Figure 4.8. Based on these transaction probabilities, the value function can be written as follows:

$for\ k = 0, 1, .., K-1$

$$v(k,i,j) = R + \alpha\mu(1+\tfrac{i}{j+1})v(D) + \alpha\mu(1-\tfrac{i}{j+1})v(k,i,j) +$$

$$\alpha\lambda \max \left\{ \begin{array}{l} (P_f(v(k,i+1,j+1) - c_a - c_f) + (1-P_f)(v(k+1,0,0) + r - c_a)) \\ v(k,i,j+1) \end{array} \right\}$$

$for\ k = K$

$$v(K,i,j) = R + \alpha\mu v(D) + \alpha\mu v(K,i,j) + \alpha\lambda v(K,i,j+1)$$

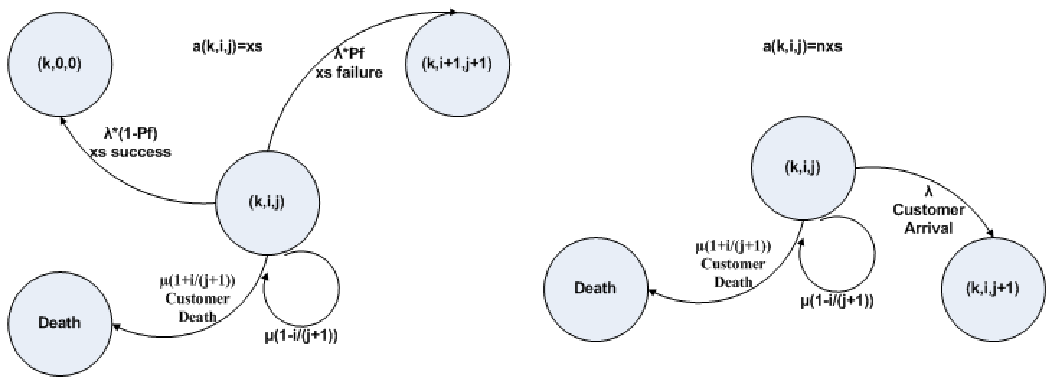

Figure 4.8: Transition Probabilities when $a(k,i,j) = nxs$ and $a(k,i,j) = xs$

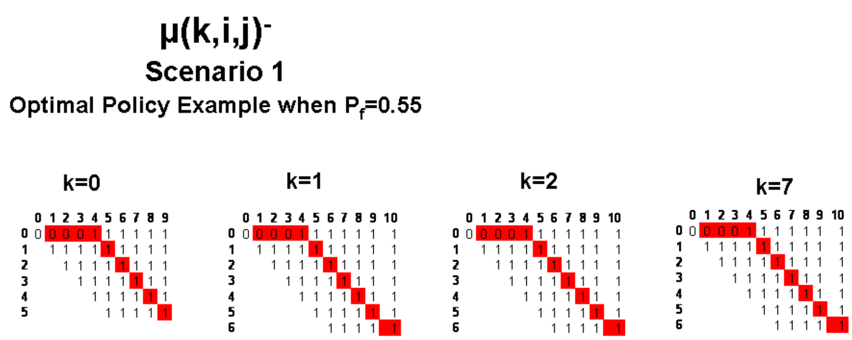

Figure 4.9: Optimal Policy Example for $\mu(k,i,j)^-$ under Scenario 1 and $P_f = 0.55$

$\frac{\lambda}{\mu}$			10	50	100
$\frac{r}{R}$	**5**	**Base Case**	14.574416	26.287026	29.478059
		$\mu(k,i,j)^-$	13.240378	25.35567	28.898114
% Diff. from the Base Case			-9.15%	-3.54%	-1.97%
$\frac{r}{R}$	**10**	**Base Case**	31.657089	55.392821	61.055303
		$\mu(k,i,j)^-$	28.852703	53.594216	59.966982
% Diff. from the Base Case			-8.86%	-3.25%	-1.78%

Table 4.4: $v(0,0,0)$ Values for the $\mu(k,i,j)^-$ Case

The values generated using the above equations are given in Table 4.4. The gap between the Base Case values and $\mu(k,i,j)^-$ values gets smaller when the contact rate increases. Even though customer death rate increases, as the number of contacts during his/her lifetime also increases, the decrease from the base case becomes very small. The gap slightly decreases when the r value increases as well. This is due to the fact that even if customer dies sooner, during his/her lifetime the revenue obtained by the firm increases. The effect of increasing additional revenue from cross-selling is greater compared to increasing contact rate. For this reason the increase in r makes up for the revenue loss up to a point, due to early quitting of the relationship.

The optimal policies for the 6 scenarios are similar to the Base Case. The optimal policy is to always cross-sell as long as there are still products to be sold to the customer. However, as the failure probability increases, while the remaining parameters are untouched, the optimal policy becomes more dynamic. Figure 4.9 provides an example for this case, when $P_f = 0.55$ under Scenario 1.

4.1.5 Negative Reaction III: Decreasing Arrival Rate (λ)

In this part we examine the situation, where as the cross-sell attempt failures increase, the customer's contact rate decreases as:

$$\lambda(k,i,j)^- = \lambda(1 - \frac{i}{j+1}).$$

The above formula ensures that when there is a successful cross-sell attempt, the contact rate takes its maximum value.

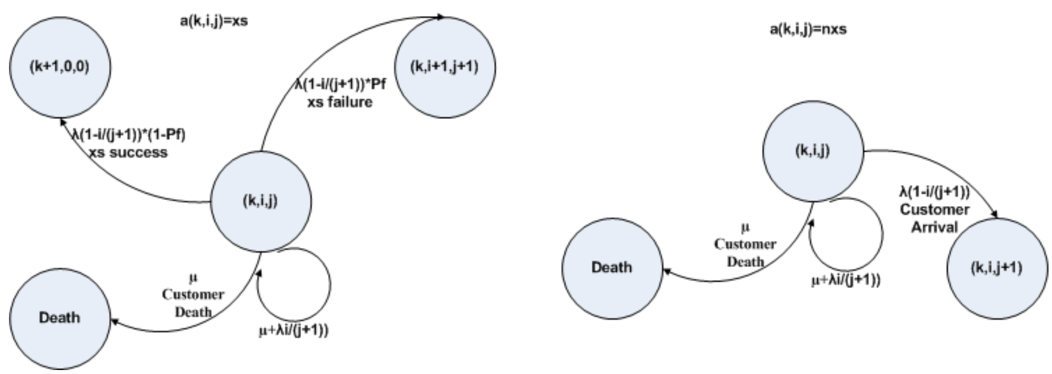

Figure 4.10: Transition Probabilities when $a(k,i,j) = nxs$ and $a(k,i,j) = xs$

Figure 4.10 shows the transition probabilities among states for each action chosen at some state (k,i,j). Based on the transaction states, the value function is given as follows:

$for\ k = 0, 1, .., K-1$

$$v(k,i,j) = R + \alpha\mu v(D) + \alpha(\mu + \tfrac{\lambda i}{j+1})v(k,i,j) +$$

$$\alpha\lambda(1 - \tfrac{i}{j+1}) \max \left\{ \begin{array}{l} (P_f(v(k,i+1,j+1) - c_a - c_f) + (1-P_f)(v(k+1,0,0) + r - c_a)) \\ v(k,i,j+1) \end{array} \right\}$$

$for\ k = K$

$$v(K,i,j) = R + \alpha\mu v(D) + \alpha\mu v(K,i,j) + \alpha\lambda v(K,i,j+1)$$

		$\frac{\lambda}{\mu}$	**10**	**50**	**100**
$\frac{r}{R}$	**5**	**Base Case**	14.574416	26.287026	29.478059
		$\lambda(k,i,j)^-$	12.885908	23.76185	26.902383
% Diff. from the Base Case			-11.59%	-9.61%	-8.74%
$\frac{r}{R}$	**10**	**Base Case**	31.657089	55.392821	61.055303
		$\lambda(k,i,j)^-$	25.747326	46.55239	52.029109
% Diff. from the Base Case			-18.67%	-15.96%	-14.78%

Table 4.5: $v(0,0,0)$ Values for the $\lambda(k,i,j)^-$ Case

λ(k,i,j)⁻ Case

Scenario 3
Optimal Policy-Example Pf=0.5714

k=0

	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1
1		0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1
2			0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1
3				0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1
4					0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1

k=5

	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1
1		0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1
2			0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1
3				0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1
4					0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1

Figure 4.11: Optimal Policy Example for $\lambda(k,i,j)^-$ under Scenario 3 and $P_f = 0.5714$

Given the value generation equations, $v(0,0,0)$ values found are listed in Table 4.5. We can observe that the values generated are mostly affected by the increase in additional revenue obtained from cross-selling. Especially when we compare the values with the base case, we see that the gap between the base case and $\lambda(k,i,j)^-$ case increases when r value increases. The possible revenues lost due to decreased contact rate have a stronger impact on value generation when the additional revenue obtained from cross-selling increases. When the contact rate increases the gap gets smaller. Each customer contact becomes more important when the contact rate is already small. For this reason any decrease in contact rates in those scenarios has a greater impact on values generated.

The optimal policies attached to these values for each Scenario and when $P_f = 0.4$ are similar to $\mu(k,i,j)^-$ and Base Case and therefore the system tries to cross-sell at each step, except when K is reached. However when the P_f value increases, the optimal policies begin to show a dynamic structure, which we haven't observed in the 2D State Space system. A small example to illustrate this is given in Figure 4.11 for Scenario 3-low r, medium λ and for $P_f = 0.5714$. The reason we have chosen this specific P_f value is related to the threshold of the base case for the 2-D state space model. The base case threshold for the P_f value for Scenario 3 in that case was 0.571429, for which the optimal policy becomes never to cross-sell. For the 3-D state space model under Scenario 3, when we use 0.571429 for the failure probability we also find that the optimal policy becomes never to cross-sell.

However, while in the 2-D state space model, omitting the last two digits makes the optimal policy to be always to cross-sell, in the 3-D state space model we see a dynamic structure. This may be due to the fact that in the 3-D state space model we have a limited number of products to be offered to the customer. Therefore, having more information on customer history makes the effect of changing customer contact rates reflected on the optimal policy as well.

4.1.6 Positive Reaction I: Increasing Death Rate (μ)

In this case, instead of showing the negative impact of the cross-sell attempt failures, death rate reflects the positive effect of successful cross-sells. This is due to the work of Bolton et al. [8], which shows that cross-selling and upgrading lead to lower propensity to quit the relationship due to higher switching costs. To reflect this fact, after each successful cross-selling attempt, we decrease the death rate as below:

$$\mu(k,i,j)^+ = \{1, 1, 0.995, 0.99, 0.98, 0.965, 0.925, 0.875, 0.825, 0.75, 0.5, 0.5\}$$

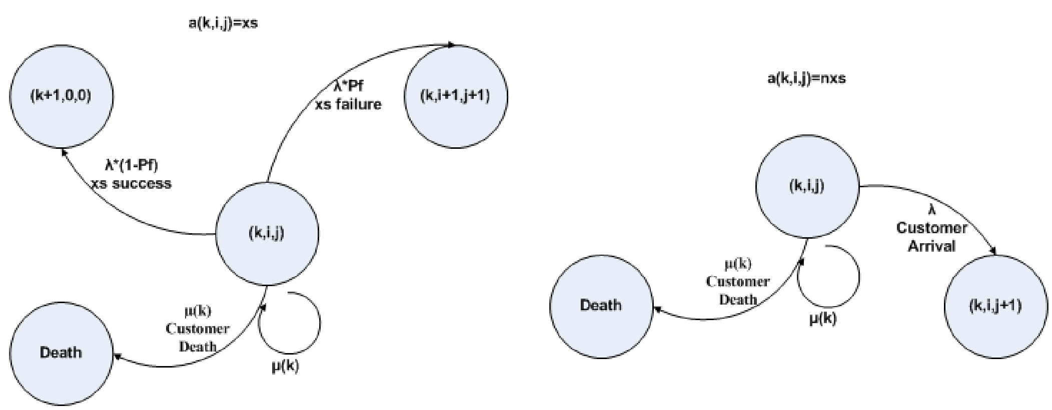

Figure 4.12: Transition Probabilities when $a(k,i,j) = nxs$ and $a(k,i,j) = xs$

For changing death rate, the transition probabilities from a specific state of (k,i,j) are given in Figure 4.12.

In this case for different k values, the fictitious rates should be adjusted accordingly. We define death rate to be a single dimensional array and let $\mu_{max} = \mu(0)$, which is the same value used for the Base Case. Then the uniformization and normalization condition becomes

$$2\mu + \mu_{max} = 1.$$

Based on these, the value function is as follows:

$for\ k = 0, 1, .., K-1$

$$v(k,i,j) = R + \alpha\mu v(D) + \alpha(2 * \mu_{max} - \mu)v(k,i,j) +$$

$$\alpha\lambda(k)\max\left\{\begin{array}{l}(P_f(v(k,i+1,j+1) - c_a - c_f) + (1-P_f)(v(k+1,0,0) + r - c_a)) \\ v(k,i,j+1)\end{array}\right\}$$

$for\ k = K$

$$v(K,i,j) = R + \alpha\mu v(D) + \alpha(2 * \mu_{max} - \mu)v(K,i,j) + \alpha\lambda(K)v(K,i,j+1)$$

		$\frac{\lambda}{\mu}$	**10**	**50**	**100**
$\frac{r}{R}$	**5**	**Base Case**	14.574416	26.287026	29.478059
		$\mu(k,i,j)^+$	15.124979	27.178299	30.114886
% Diff. from the Base Case			3.78%	3.39%	2.16%
$\frac{r}{R}$	**10**	**Base Case**	31.657089	55.392821	61.055303
		$\mu(k,i,j)^+$	32.414862	56.424468	61.774955
% Diff. from the Base Case			2.39%	1.86%	1.18%

Table 4.6: $v(0,0,0)$ Values for the $\mu(k,i,j)^+$ Case

The values generated using the above equations are given in Table 4.6. When the death rate decreases due to higher switching costs of the customer, the value generated from the customer increases. The gap between the Base Case values and the $\mu(k,i,j)^+$ values decreases as the contact rate increases. As the customer contacts the firm more during his/her lifetime, even though the lifetime does not increase, the firm generates a high value from the customer. The effect of death rate over value generation decreases when the contact rate is high.

The optimal policies for each of the 6 scenarios display a similar nature to that of Base Case, trying to cross-sell in every state until all the products are exhausted for $P_f = 0.4$.

4.1.7 Positive Reaction on Death rate (μ) & Negative Reaction on Failure Probability (P_f)

In this case, both the negative and the positive impact of cross-selling is modeled. Death rate of the customer is chosen to reflect the positive effect of successful cross-sells based on successful cross-selling attempts as in Section 4.1.6. Failure probability, on the other hand, reflects the negative impact of unsuccessful cross-sell attempts. P_f value changes as given in Section 4.1.3.

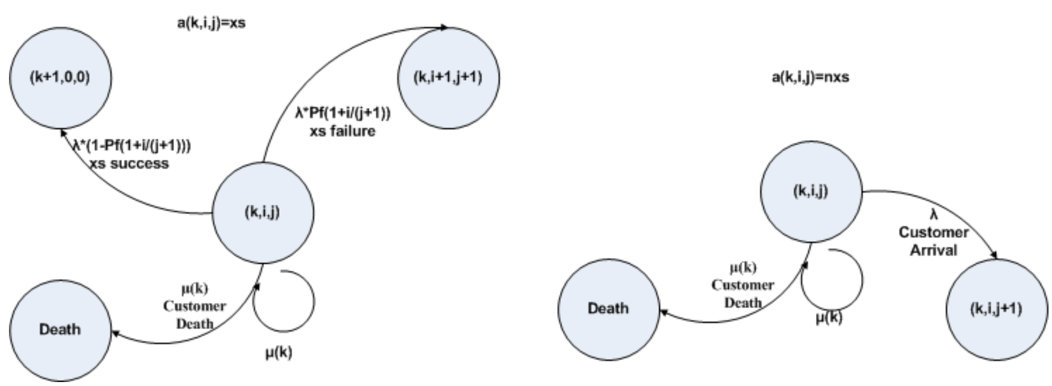

Figure 4.13: Transition Probabilities when $a(k,i,j) = nxs$ and $a(k,i,j) = xs$

For changing μ and P_f values, the transition probabilities from a specific state of (k,i,j) are given in Figure 4.17. With the uniformization and normalization equation adjusted, the value function is as follows:

$for\ k = 0, 1, .., K-1$

$v(k,i,j) = R + \alpha\mu v(D) + \alpha(2 * \mu_{max} - \mu)v(k,i,j)+$

$$\alpha\lambda(k)\max\left\{\begin{array}{l} (P_f(1+\frac{i}{j+1})(v(k,i+1,j+1) - c_a - c_f) + (1 - P_f(1+\frac{i}{j+1}))(v(k+1,0,0) + r - c_a) \\ v(k,i,j+1) \end{array}\right\}$$

$for\ k = K$

$v(K,i,j) = R + \alpha\mu v(D) + \alpha(2 * \mu_{max} - \mu)v(K,i,j) + \alpha\lambda(K)v(K,i,j+1)$

with

$$2\mu + \mu_{max} = 1.$$

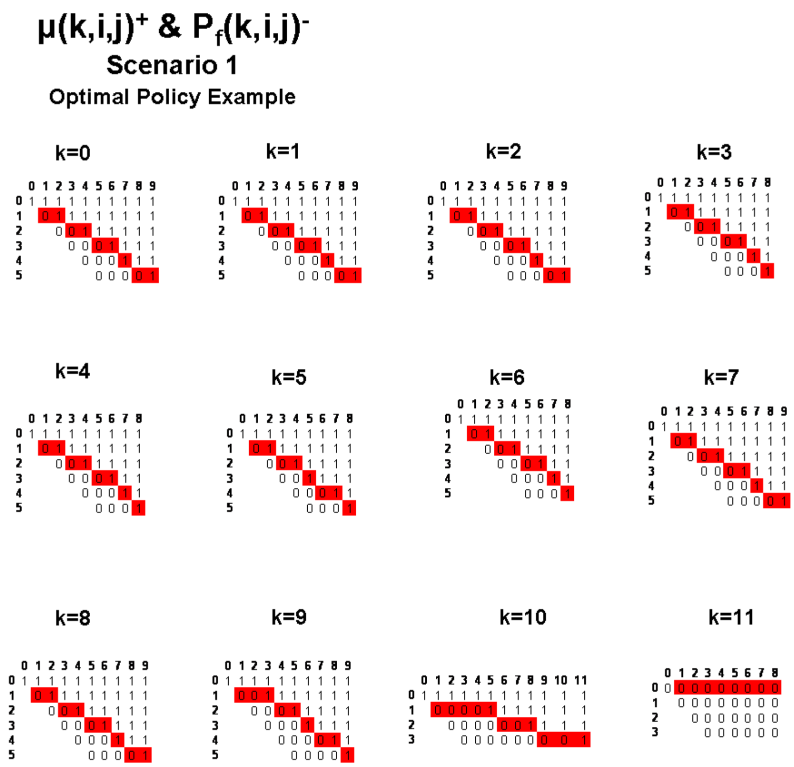

Figure 4.14: Optimal Policy Example for $\mu(k,i,j)^+$ & $P_f(k,i,j)^-$ for Scenario 1

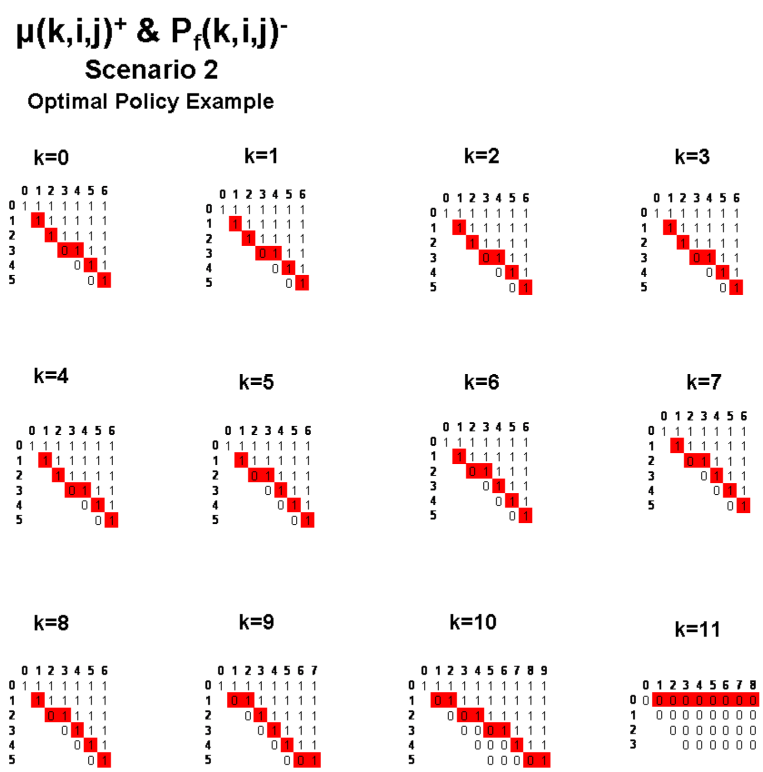

Figure 4.15: Optimal Policy Example for $\mu(k,i,j)^+$ & $P_f(k,i,j)^-$ for Scenario 2

$\frac{\lambda}{\mu}$			**10**	**50**	**100**
$\frac{r}{R}$	**5**	**Base Case**	14.574416	26.287026	29.478059
		$\mu(k,i,j)^{+}$ & $P_f(k,i,j)^{-}$	11.709557	21.627222	24.413911
% Diff. from the Base Case			-19.66%	-17.73%	-17.18%
$\frac{r}{R}$	**10**	**Base Case**	31.657089	55.392821	61.055303
		$\mu(k,i,j)^{+}$ & $P_f(k,i,j)^{-}$	24.991177	44.72502	49.659143
% Diff. from the Base Case			-21.06%	-19.26%	-18.67%

Table 4.7: $v(0,0,0)$ Values for the $\mu(k,i,j)^{+}$ & $P_f(k,i,j)^{-}$ Case

$v(0,0,0)$ values given in Table 4.7 for the 6 scenarios are compared with the values of the base case. Even when the positive effect is added to the model, it is not enough to compensate for the revenue loss due to negative reaction. However, it is observed that when the contact rate increases, the gap between the Base Case and the $\mu(k,i,j)^{+}$ & $P_f(k,i,j)^{-}$ Case decreases. When r is high, however, the gap increases compared with low r. When the additional income is higher, the system's loss in terms of value generation can not make up with the decrease in the death rate when the failure probability increases based on failures. Therefore, we conjecture that the negative reaction has a stronger impact on values generated.

The policies associated with the 6 Scenarios considered are similar in nature to the linearly changing failure probability case, in the case where both the negative and the positive effects are embedded, the optimal policy displays a dynamic nature. Different from the linearly changing failure probability case, however, the optimal policies become more aggressive, even when the number of products sold gets closer to K. For the scenarios with low r value the system assumes a more dynamic structure when compared to the scenarios with high r value, where the optimal policies become less dynamic and more aggressive. For all scenarios, the optimal policy becomes more dynamic as the number of products sold to the customer gets closer to K. Figures 4.14 and 4.15 provide an example for the dynamic nature of the optimal policies for both low and high r values under Scenario 1 and 2 respectively.

4.1.8 Positive Reaction II: Increasing Contact Rate (λ)

In this case, instead of showing the negative impact of the cross-sell attempt failures, arrival rate reflects the positive effect of successful cross-sells, such that λ increases as the number of successes increases preserving concavity, so that the rate of change decreases with each success. Because in this section we are interested in the analytical conclusions, we defined three λ sets, each of which has an initial point of 10, 50 and 100 and given below. The columns represent different λ sets, and the rows represent the number of products sold during cross-selling. λ values change based on the number of products sold as follows in each set:

$$\lambda(k,i,j)^{+} = \left\{ \begin{array}{rrr} 10 & 50 & 100 \\ 10 & 50 & 100 \\ 15 & 75 & 150 \\ 16.5 & 82.5 & 165 \\ 17.5 & 87.5 & 175 \\ 18.5 & 92.5 & 185 \\ 19 & 95 & 190 \\ 19.3 & 96.5 & 193 \\ 19.6 & 98 & 196 \\ 19.8 & 99 & 198 \\ 19.9 & 99.5 & 199 \\ 20 & 100 & 200 \end{array} \right\}$$

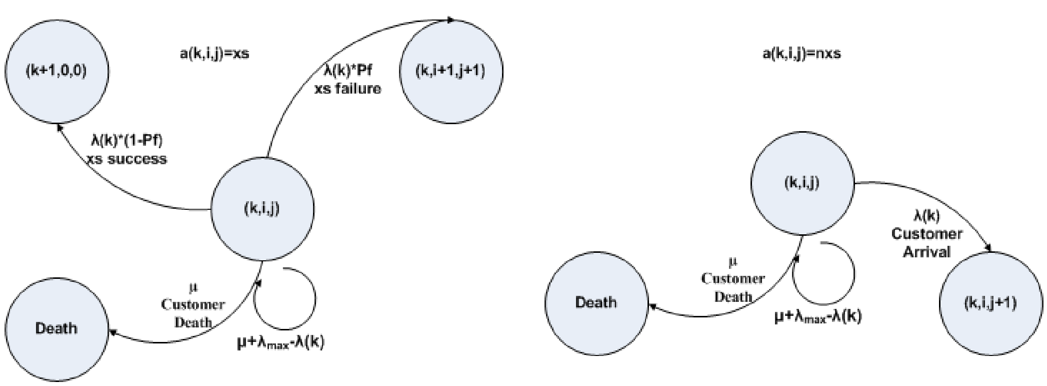

Figure 4.16: Transition Probabilities when $a(k,i,j) = nxs$ and $a(k,i,j) = xs$

For each of the λ sets, the transition probabilities from a specific state of (k, i, j) are given in Figure 4.16.

With the changing λ values, to be able to keep track of the value generation and policy sets with rates turned into probabilities, we need to re-define the uniformization and normalization condition such that there should not be a time-scale incompatibility. For each of the λ sets, we take the maximum of the set to use for the uniformization and normalization condition so that throughout the model the time-scale will be equal for each $\lambda(k, i, j)^+$ value.

For each of the sets being used in the numerical analysis, we define a single dimensional array for the λ values and accordingly let $\lambda_{max} = \lambda(K)$, then the uniformization and normalization condition becomes

$$2\mu + \lambda_{max} = 1.$$

Based on these, the value function is as follows:

$for\ k = 0, 1, .., K - 1$

$$v(k, i, j) = R + \alpha\mu v(D) + \alpha(\mu + \lambda_{max} - \lambda(k))v(k, i, j) +$$

$$\alpha\lambda(k) \max \left\{ \begin{array}{l} (P_f(v(k, i+1, j+1) - c_a - c_f) + (1 - P_f)(v(k+1, 0, 0) + r - c_a)) \\ v(k, i, j+1) \end{array} \right\}$$

$for\ k = K$

$$v(K, i, j) = R + \alpha\mu v(D) + \alpha\mu v(K, i, j) + \alpha\lambda(K)v(K, i, j+1)$$

$\frac{\lambda}{\mu}$			**10**	**50**	**100**
$\frac{r}{R}$	**5**	*Base Case*[a]	16.103153	25.147176	27.127996
		$\lambda(k, i, j)^+$	17.500342	27.021357	29.034574
% Diff. from the Base Case			8.68%	7.45%	7.03%
$\frac{r}{R}$	**10**	*Base Case*[a]	29.53201	45.866735	49.247058
		$\lambda(k, i, j)^+$	34.42135	52.449401	55.967402
% Diff. from the Base Case			16.56%	14.35%	13.65%

Table 4.8: $v(0, 0, 0)$ Values for the $\lambda(k, i, j)^+$ Case

The values generated using the above equations are given in Table 4.8. However to be able to compare the findings of the $\lambda(k, i, j)^+$ case, we need a time-scale adjustment at the base case values as well, meaning that the uniformization and normalization condition for the base case is taken to be $2\mu + \lambda_{max} = 1$ as well creating a fictitious rate for the system to return to its current with rate $\lambda_{max} - \lambda$ and let *Base Case*^{a} represent the time-scale adjusted base case in Table 4.8. When we compare the adjusted base case with the $\lambda(k, i, j)^+$ values, we see that as the contact rate increases the gap decreases, and when the r value increases the gap also increases.

The optimal policies for each of the 6 scenarios display a similar nature to that of Base Case, as even though after each successful cross-sell the λ values increase, until another successful cross-sell none of the parameters change.

4.1.9 Positive Reaction on Arrival rate (λ) & Negative Reaction on Failure Probability (P_f)

In this case, both the negative and the positive impact of cross-selling is modeled. Contact rate of the customer reflects the positive effect of successful cross-sells based on successful cross-selling attempts. The parameter sets used for the $\lambda(k)$ values are given in Section 4.1.8. Failure probability, on the other hand, reflects the negative impact of unsuccessful cross-sell attempts. P_f value changes as given in Section 4.1.3.

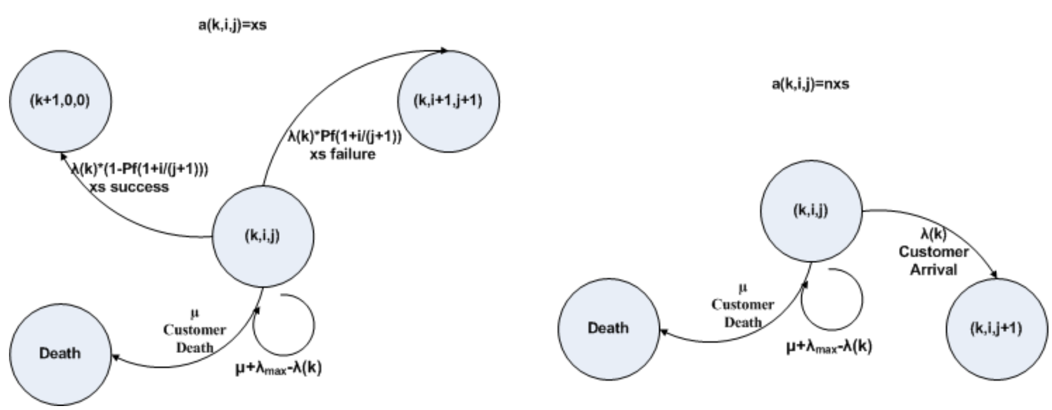

Figure 4.17: Transition Probabilities when $a(k, i, j) = nxs$ and $a(k, i, j) = xs$

For each of the λ sets and linearly changing P_f values, the transition probabilities from a specific state of (k, i, j) are given in Figure 4.17. Based on these, the value function is as follows:

$for\ k = 0, 1, .., K-1$

$$v(k,i,j) = R + \alpha\mu v(D) + \alpha(\mu + \lambda_{max} - \lambda(k))v(k,i,j) +$$

$$\alpha\lambda(k)\max\left\{\begin{array}{l}(P_f(1+\frac{i}{j+1})(v(k,i+1,j+1)-c_a-c_f)+(1-P_f(1+\frac{i}{j+1}))(v(k+1,0,0)+r-c_a)\\ v(k,i,j+1)\end{array}\right\}$$

$for\ k = K$

$$v(K,i,j) = R + \alpha\mu v(D) + \alpha\mu v(K,i,j) + \alpha\lambda(K)v(K,i,j+1)$$

$\frac{\lambda}{\mu}$			**10**	**50**	**100**
$\frac{r}{R}$	**5**	*Base Case*[a]	16.103153	25.147176	27.127996
		$\lambda(k,i,j)^+$ & $P_f(k,i,j)^-$	14.329882	22.355774	24.15515
% Diff. from the Base Case			-11.01%	-11.10%	-10.96%
$\frac{r}{R}$	**10**	*Base Case*[a]	29.53201	45.866735	49.247058
		$\lambda(k,i,j)^+$ & $P_f(k,i,j)^-$	27.192697	41.974787	45.051195
% Diff. from the Base Case			-7.92%	-8.49%	-8.52%

Table 4.9: $v(0,0,0)$ Values for the $\lambda(k,i,j)^+$ & $P_f(k,i,j)^-$ Case

Table 4.9 shows $v(0,0,0)$ values for the 6 scenarios comparing the values with the base case. However, due to changing λ values, we need to adjust the time-scale to be able to compare the current findings for this case with the base case. The adjustment of time-scale between the two models is described in Section 4.1.8.

It is observed that when both contact rate and the failure probability change, the percentage difference between Base Case and the $\lambda(k,i,j)^+$ & $P_f(k,i,j)^-$ case remains approximately the same for the same r values. However, all the values obtained is lower than the Base Case values. Based on our observations, we see that the positive impact of increasing contact rate is offset by the negative impact of increasing failure probability. Therefore, as in Section 4.1.7, we observe that the negative reaction has a stronger impact over the values generated when compared to the positive effect.

The policies attached to the 6 Scenarios considered are similar in nature to the linearly changing failure probability case, however in the case where both the negative and the positive effects are embedded, the optimal policy becomes more aggressive even though

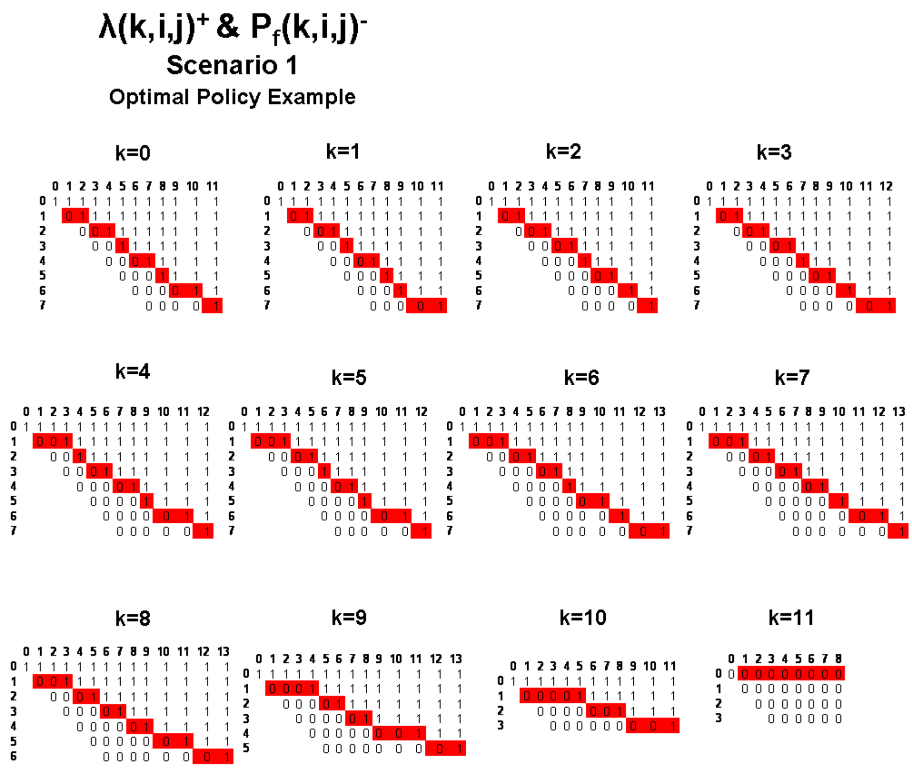

Figure 4.18: Optimal Policy Example for $\lambda(k, i, j)^+$ & $P_f(k, i, j)^-$ for Scenario 1

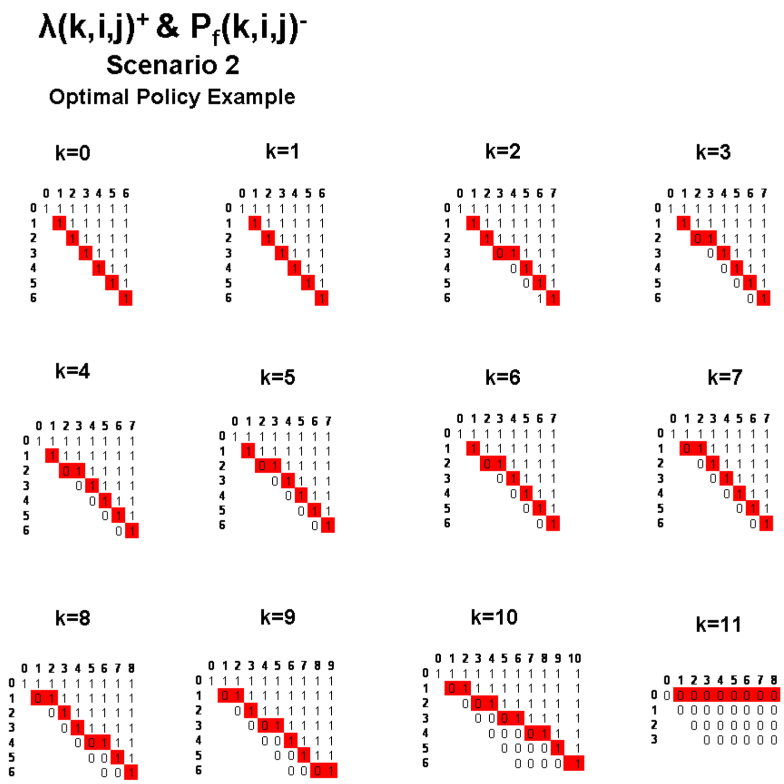

Figure 4.19: Optimal Policy Example for $\lambda(k,i,j)^{+}$ & $P_f(k,i,j)^{-}$ for Scenario 2

it preserves its dynamic nature. For the scenarios with low r value the system assumes a dynamic structure in terms of cross-selling policy and waits for another contact before cross-selling to a customer. However, for the scenarios with high r value, the optimal policies become less dynamic and more aggressive. For all scenarios, the optimal policy becomes more dynamic as the number of products sold to the customer gets closer to K. Figures 4.18 and 4.19 provide an example for the dynamic nature of the optimal policies for both low and high r values under Scenario 1 and 2 respectively.

4.2 Model without Renewals

In this section in addition to adding a third state we also relaxed the assumption of renewal depending on success.

4.2.1 Model Description

The objective is to observe the negative effects of cross-sell attempts when both the customer and the firm never forgets the results of the encounters on cross-sell successes and failures. Other than this fact, the parameter definitions and the action sets as well as policy definition are the same as in Section 4.1. Based on these, the schematic representation for this model can be given as follows:

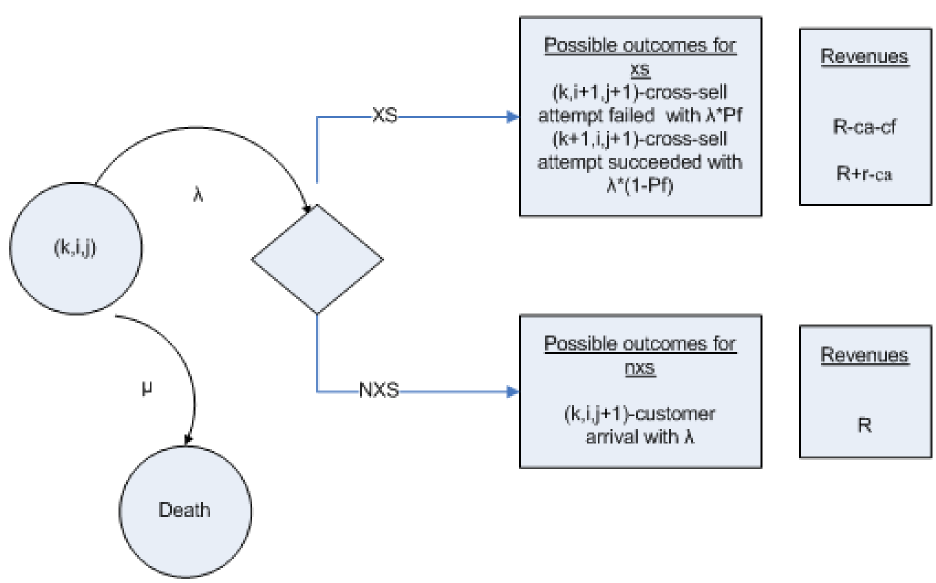

Figure 4.20: Possible Outcomes of the 3-D Model without Renewals

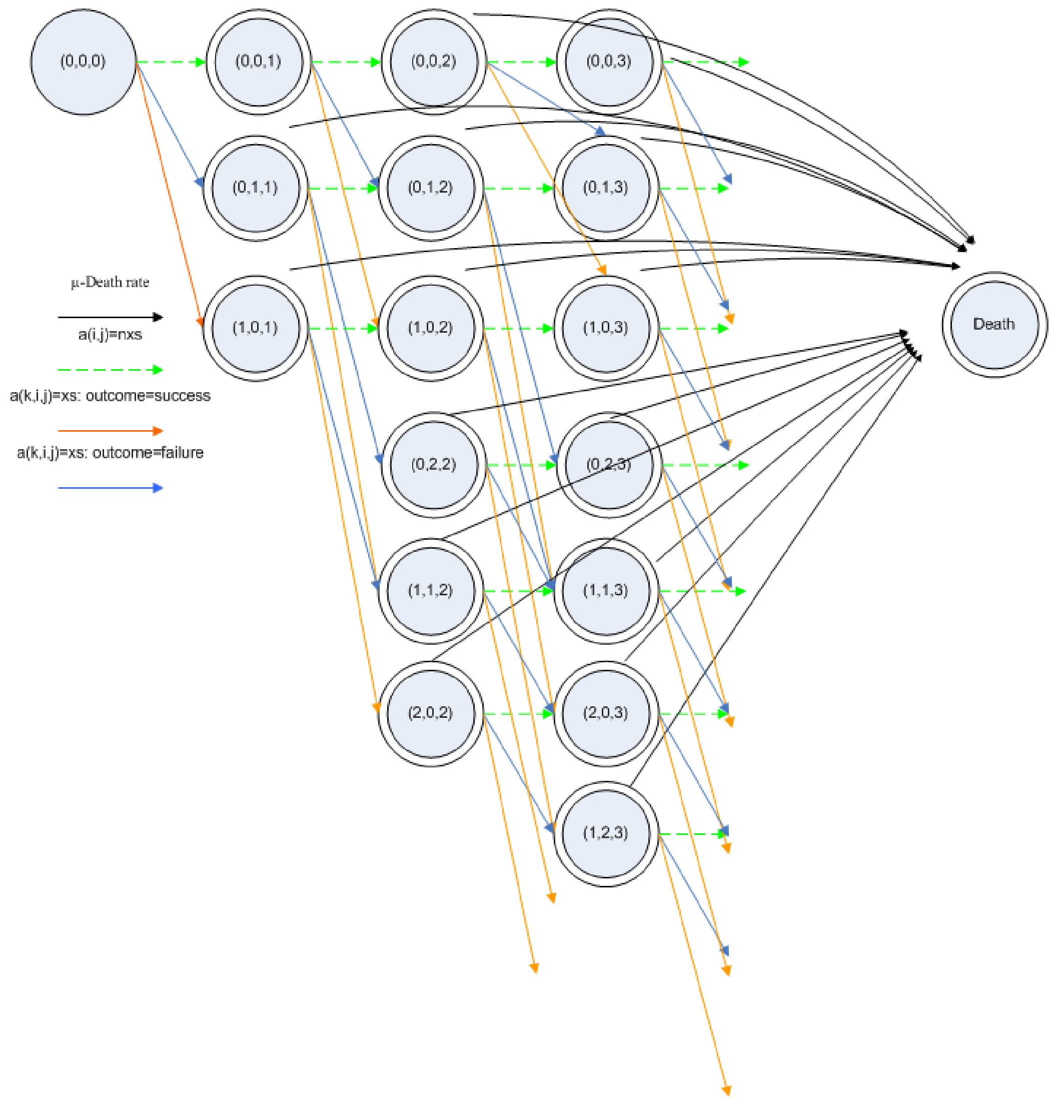

Figure 4.21: All possible paths a customer may take depending on action $a(k, i, j)$

The states are defined as (k, i, j), where k is the number of successful cross-sell attempts, i is the number of unsuccessful cross-sell attempts, and j is the number of the contacts initiated by the customer. Therefore, the state of the system at any time is represented by (k, i, j), where $k + i \leq j$ for all k, i, and j.

As shown in Figure 4.20, the transition among states is such that each time the customer contacts the firm, the number of contacts (j) increases by 1. If the firm tries to cross-sell and the outcome is a success, then the number of cross-sell successes k increases by 1. If the cross-sell attempt fails, then the number of unsuccessful cross-sell attempts (i) increases by 1. As a result, the firm has the complete history on cross-selling attempts, and also the decisions made on cross-selling both by the customer and the firm depend on the entire relationship history. Based on the action chosen and the reaction of the customer, the customer may follow different paths during her relationship with the firm as shown in Figure 4.21.

We can characterize the MDP as follows:

State space is defined by

$S = \{(k, i, j) : k = 0, 1, ..., K; i = 0, 1, 2, ..., j; j = i + k, i + k + 1, ..., D\}$.

The planning horizon ends when the customer quits the relationship with the firm, which is shown with state D. The action set includes, $a(k, i, j) = (xs, nxs)$, where xs represents the cross-sell attempt option of the firm and nxs represents the not attempt to cross-sell option. Cost structure is the same as in Chapter 3.

Let $v(k, i, j)$ be the value function that gives the discounted profit earned at state (k, i, j) based on the action chosen by the firm. D is defined previously as the death state. In this case as well $v(D)$ is zero. And also when $k = K$, as there is no more additional products to be sold to the customer, for states (K, i, j) for all i and j, the action set becomes not to cross-sell. Using the exponentiality of the customer contacts and lifetimes and uniformization in this case as well we can turn the rates into probabilities. To adjust the time scale we add a dummy rate, μ, with which at a particular state of the system, (k, i, j), the system remains in the same state, (k, i, j). Therefore, for $k = 0, 1, .., K - 1$ the value function is:

$$v(k, i, j) = R + \alpha\mu v(D) + \alpha\mu v(k, i, j) +$$

$$\alpha\lambda \max \left\{ \begin{array}{l} (P_f(v(k, i+1, j+1) - c_a - c_f) + (1 - P_f)(v(k+1, i, j+1) + r - c_a)) \\ v(k, i, j+1) \end{array} \right\}$$

and for $k = K$

$$v(K, i, j) = R + \alpha\mu v(D) + \alpha\mu v(K, i, j) + \alpha\lambda v(K, i, j + 1)$$

The parameters defined in Section 3.4.1 are valid in this model as well.

For numerical calculations, we let the number of products to be offered to customers to be 11 and as in the previous chapter let our customers die on their 100^{th} consecutive contact with the firm. However when there is no renewal in the system we can expect a decrease in the value generated as we both restrict the additional number of products offered and also shorten the relationship duration of the customer with the firm. We conducted our analysis by taking failure probability to be 0.4 for the rest of the analysis.

4.2.2 Base Case

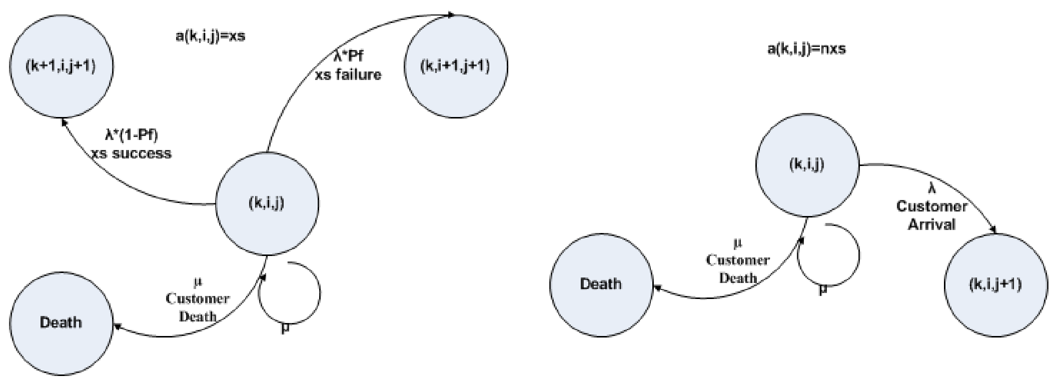

Figure 4.22: Transition Probabilities when $a(k, i, j) = nxs$ and $a(k, i, j) = xs$

We have more data concerning history of the relationship between customer and firm, yet the possible actions and the results are the same. The transition matrices of the base case for two actions are given as below in Figure 4.22.

Customer may die independent of the action chosen. When the action chosen is to cross-sell, and the cross-sell offer is accepted the state of the system changes to $(k + 1, i, j + 1)$, and when the offer is decline the system state becomes $(k, i + 1, j + 1)$.

For the base case, as none of the parameters change over time or depending on the action chosen by the firm, it can be concluded that the optimal policy either always cross-sells or never cross-sells through the lifetime of a customer. The expected values for $v(0, 0, 0)$

		λ/μ		
		10	**50**	**100**
r/R	**5**	14.574416	26.278515	29.453011
	10	31.657089	55.384379	61.030254

Table 4.10: $v(0, 0, 0)$ Values for the Base Case

generated for each scenario can be seen in Table 4.10. Using a failure probability of 0.4, we find that the optimal policy π^* for $k = 0, 1, .., K - 1 = 10$ is to try to cross-sell to the customer at each state.

When we compare the scenarios according to their expected values of $v(0,0,0)$ generated, we observe that despite the fact that the total number of times that a customer can contact is decreased, still the additional revenue obtained from cross-selling affects the system more than the contact rate of the customer. This situation can be seen in Table 4.10. A five times increase in the contact rate can not make up for the two times increase in the additional revenue in terms of value generated.

4.2.3 Negative Reaction I: Decreasing Contact Rate (λ)

To be able to see the effect of holding more data about the relationship, we also looked at the negative impact of failures on arrival rates. Therefore, as in the previous chapter, we let the contact rate change depending on the state of the system as:

$$\lambda(k,i,j)^{-} = \lambda(1 - \frac{i}{j+1}).$$

The above formula ensures that the contact rate of the customer decreases with each failed cross-sell attempt. However as there is not a renewal in this case, both the customer and the firm never forgets the experiences they had.

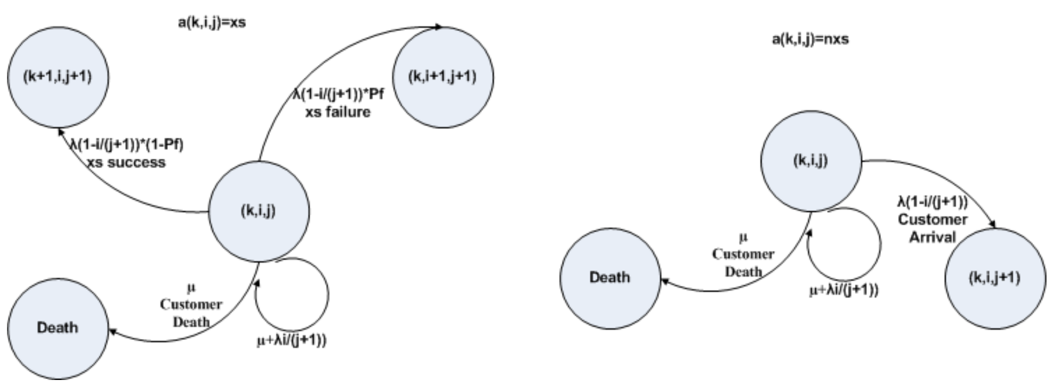

Figure 4.23: Transition Probabilities when $a(k,i,j) = nxs$ and $a(k,i,j) = xs$

We normalize the equations to turn the rates into probabilities. Figure 4.23 shows the transition probabilities among states separately where the action chosen is not to cross-sell and when the action chosen is to cross-sell at some state (k,i,j).

In this case, the value function is found as follows: For $k = 0, 1, .., K-1$,

$$v(k,i,j) = R + \alpha\mu v(D) + \alpha(\mu + \tfrac{\lambda i}{j+1})v(k,i,j) + \alpha\lambda(1 - \tfrac{i}{j+1}) \max \left\{ \begin{array}{l} (P_f(v(k,i+1,j+1) - c_a - c_f) + (1-P_f)(v(k+1,i,j+1) + r - c_a)) \\ v(k,i,j+1) \end{array} \right\}$$

For $k = K$,

$$v(K,i,j) = R + \alpha\mu v(D) + \alpha(\mu + \tfrac{\lambda i}{j+1})v(K,i,j) + \alpha\lambda(1 - \tfrac{i}{j+1})v(k,i,j+1)$$

$\frac{\lambda}{\mu}$			10	50	100
$\frac{r}{R}$	**5**	**Base Case**	14.57442	26.278515	29.45301
		$\lambda(k,i,j)^-$	12.88591	23.760782	26.89931
% Diff. from the Base Case			-11.59%	-9.58%	-8.67%
$\frac{r}{R}$	**10**	**Base Case**	31.65709	55.384379	61.03025
		$\lambda(k,i,j)^-$	25.74733	46.551321	52.02604
% Diff. from the Base Case			-18.67%	-15.95%	-14.75%

Table 4.11: $v(0,0,0)$ Values for $\lambda(k,i,j)^-$ Case

As we can see from Table 4.11, the negative impact of cross-sell attempt failures on the discounted value generated decreases as the customer contact increases. $v(0,0,0)$ values increase as the contact rate and r increases. However, an increase on the additional revenue has a stronger impact on the revenues generated from the customer as in the Base Case.

4.2.4 Negative Reaction II: Increasing Failure Probability (P_f)

A customer's likelihood to accept cross-sell offers decreases based on the previous failures of cross-selling attempts such that:

$$P_f(k,i,j)^- = P_f(1 + \frac{i}{j+1}).$$

As there are no renewals, the effect of the failed cross-selling attempts will remain throughout the relationship.

The transition probabilities from a certain state (k,i,j) to other possible states depending on the action chosen are given in Figure 4.24.

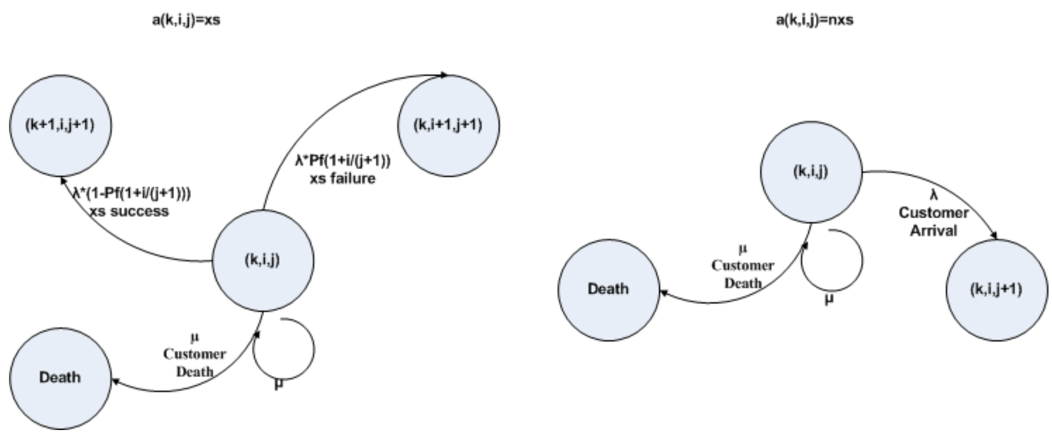

Figure 4.24: Transition Probabilities when $a(k,i,j) = nxs$

The value function can be written as follows:

For $k = 0, 1, ..., K-1$,

$v(k,i,j) = R + \alpha\mu v(D) + \alpha\mu v(k,i,j) +$

$$\alpha\lambda \max \left\{ \begin{array}{l} (P_f(1+\frac{i}{j+1})(v(k,i+1,j+1) - c_a - c_f) + (1 - P_f(1+\frac{i}{j+1}))(v(k+1,i,j+1) + r - c_a) \\ v(k,i,j+1) \end{array} \right\}$$

and for $k = K$,

$v(K,i,j) = R + \alpha\mu v(D) + \alpha\mu v(K,i,j) + \alpha\lambda v(K,i,j+1)$

Based on these equations, $v(0,0,0)$ values generated for the 6 scenarios described in Chapter 3 are given in Table 4.12. Failure probability has the greatest impact on revenue generation when compared to the contact rate. Even though as the contact rate increases, the negative impact on the values generated is lessened, the percentage difference between the base-case and the $P_f(k,i,j)^-$ case remains greater than 20%.

		$\frac{\lambda}{\mu}$	**10**	**50**	**100**
$\frac{r}{R}$	**5**	**Base Case**	14.57442	26.278515	29.45301
		$P_f(k,i,j)^-$	11.0787	20.110546	22.84335
% Diff. from the Base Case			-23.99%	-23.47%	-22.44%
$\frac{r}{R}$	**10**	**Base Case**	31.65709	55.384379	61.03025
		$P_f(k,i,j)^-$	23.64918	41.747625	46.53483
% Diff. from the Base Case			-25.30%	-24.62%	-23.75%

Table 4.12: $v(0,0,0)$ Values for $P_f(k,i,j)^-$ Case

4.3 Comparisons for the 3-D State Space Models

In this chapter, we have conducted numerical analysis on two models. In this section we provide $v(0,0,0)$ comparisons, as well as the optimal policies for each case considered for the model. However, our main focus is for the 3-d state space model with renewals on the number of failures and contacts based on cross-sell attempt successes. For this reason, we include further analysis on the effects of $P_f(k,i,j)^-$, $\lambda(k,i,j)^+$, and $\mu(k,i,j)^+$. We also compare 2-D state space model outcomes with the findings of Section 4.1, after which we present the $v(0,0,0)$ comparisons for the model given in Section 4.2.

4.3.1 Model with Renewals

	Scenario 1	**Scenario 2**	**Scenario 3**	**Scenario 4**	**Scenario 5**	**Scenario 6**
$\mu(k,i,j)^+$	15.124979	32.414862	27.178299	56.424468	30.114886	61.774955
BASE CASE	14.574416	31.657089	26.287026	55.392821	29.478059	61.055303
$\mu(k,i,j)^-$	13.240378	28.852703	25.35567	53.594216	28.898114	59.966982
$\lambda(k,i,j)^-$	12.885908	25.747326	23.76185	46.55239	26.902383	52.029109
$\mu(k,i,j)^+$ & $P_f(k,i,j)^-$	11.709557	24.991177	21.627222	44.72502	24.413911	49.659143
$P_f(k,i,j)^-$	11.466212	24.530113	21.152079	43.983663	24.05469	49.131506

Table 4.13: v(0,0,0) Value Comparisons for the Model with Renewals

In Table 4.13, $v(0,0,0)$ values for all cases are given, except for the ones where the contact rate reflects the positive effect. When we let λ value change based on the number of successes, a time-scaling problem occurs with the rest of cases. For this reason the cases including $\lambda(k,i,j)^+$ are separately given below.

One can easily see that death rate, contact rate and failure probability affect the values generated in different magnitudes. If we represent $v(0,0,0)$ as v, based on Table 4.13, we can rank different cases for each of the 6 scenarios as follows:

$$v_{\mu(k,i,j)^+} > v_{base} > v_{\mu(k,i,j^-} > v_{\lambda(k,i,j^-)} > v_{\mu(k,i,j)^+ \& P_f(k,i,j)^-} > v_{P_f(k,i,j)^-}$$

Base case is the only case where none of the parameters change. For this reason the values obtained in the base case are higher than the cases where the negative impact is explored. Cases, in which only the positive impact is explored provide with a higher revenue as expected. The greatest impact on values generated comes from $P_f(k,i,j)^-$ case. Even when we incorporate $\mu(k,i,j)^+$ into $P_f(k,i,j)^-$ case, the negative impact of the cross-sell failure is much stronger than the positive impact of success such that the values obtained slightly increase, yet not enough to be higher than the $\lambda(k,i,j)^-$ case. Any change in contact rate has a greater impact on the revenues then the change in death rates as well.

In terms of optimal policies though, base case, $\mu(k,i,j)^+$, $\mu(k,i,j)^-$, $\lambda(k,i,j)^+$ and $\lambda(k,i,j)^-$ display similar optimal policies when the failure probability is taken to be 0.4. However, when the failure probability increases the optimal policies behave differently for different cases. For the base case while the optimal policy is either always to-cross-sell or never to-cross-sell, for the cases which only deals with negative reaction, when the failure probability increases, the optimal policy becomes dynamic. For the $P_f(k,i,j)^-$ case, the optimal policy is very dynamic even when $P_f(k,0,0) = 0.4$.

Effect of $P_f(k,i,j)^-$

In Tables 4.14 and 4.15, we can see how much the failure probabilities affect the values generated when P_f value reflects the negative reaction in terms of cross-sell failures. When combined with the positive reaction, the impact on the values generated is slightly greater in terms of percentage differences.

In terms of optimal policies $P_f(k,i,j)^-$ case displays a very dynamic nature and less aggressive in terms of cross-selling attempts trying to balance the negative impact by giving

	BASE CASE	$P_f(k,i,j)^-$	**% Difference from Base Case**
S1	14.574416	11.466212	-21.3264%
S2	31.657089	24.530113	-22.5130%
S3	26.287026	21.152079	-19.5341%
S4	55.392821	43.983663	-20.5968%
S5	29.478059	24.05469	-18.3980%
S6	61.055303	49.131506	-19.5295%

Table 4.14: Effect of $P_f(k,i,j)^-$ on Base Case

	$\mu(k)$	$\mu(k,i,j)^+$ & $P_f(k,i,j)^-$	**% Difference from $\mu(k,i,j)^+$**
S1	15.124979	11.709557	-22.5813%
S2	32.414862	24.991177	-22.9021%
S3	27.178299	21.627222	-20.4247%
S4	56.424468	44.72502	-20.7347%
S5	30.114886	24.413911	-18.9308%
S6	61.774955	49.659143	-19.6128%

Table 4.15: Effect of $P_f(k,i,j)^-$ on $\mu(k,i,j)^+$ Cases

the customer time to recover. However, when the positive reaction is added in terms of decreased death rate, the policy, even though still dynamic, becomes more aggressive.

Effect of $\mu(k,i,j)^+$

	BASE CASE	$\mu(k,i,j)^+$	**% Difference from Base Case**	$\mu(k,i,j)^+$ & $P_f(k,i,j)^-$	**% Difference from Base Case**
S1	14.574416	15.124979	3.7776%	11.709557	-19.6568%
S2	31.657089	32.414862	2.3937%	24.991177	-21.0566%
S3	26.287026	27.178299	3.3905%	21.627222	-17.7266%
S4	55.392821	56.424468	1.8624%	44.72502	-19.2585%
S5	29.478059	30.114886	2.1603%	24.413911	-17.1794%
S6	61.055303	61.774955	1.1787%	49.659143	-18.6653%

Table 4.16: Effect of $\mu(k,i,j)^+$ Cases on Base Case

The impact of the positive reaction modeled via the death rate is given in Table 4.16. Even though when compared to the base case, the values generated increase with the positive impact of the decreasing death rate, when incorporated with the $P_f(k,i,j)^-$ case, the negative impact of $P_f(k,i,j)^-$ offset the positive impact significantly.

In terms of optimal policy, $\mu(k,i,j)^+$ case is aggressive in terms of cross-selling attempts. However, when the negative effect is taken into consideration with the positive effect, the optimal policy becomes less aggressive, and more dynamic even for very small failure probabilities.

Effect of $\lambda(k,i,j)^+$

In Table 4.17, the positive impact reflected on the increasing contact rate can be seen in terms of values generated. As similar to the $\mu(k,i,j)^+$ case, it is not enough to cover for the loss caused by the negative effect reflected via the failure probability. However, in this case, when we look at the percentage differences of the values, the positive impact of $\lambda(k,i,j)^+$ is stronger than the positive impact of $\mu(k,i,j)^+$.

In terms of policy, for $P_f(k,0,0) = 0.4$, the action chosen is to cross-sell in each case when only $\lambda(k,i,j)^+$ is taken into consideration. However, when the negative effect is

	BASE CASE[a]	$\lambda(k,i,j)^+$	**% Difference from Base Case**	$\lambda(k,i,j)^+ \& P_f(k,i,j)^-$	**% Difference from Base Case**
S1	16.103153	17.500342	8.6765%	14.329882	-11.0119%
S2	29.53201	34.42135	16.5561%	27.192697	-7.9213%
S3	25.147176	27.021357	7.4528%	22.355774	-11.1003%
S4	45.866735	52.449401	14.3517%	41.974787	-8.4853%
S5	27.127996	29.034574	7.0281%	24.15515	-10.9586%
S6	49.247058	55.967402	13.6462%	45.051195	-8.5200%

Table 4.17: Effect of $\lambda(k,i,j)^+$ on Base Case

incorporated, the optimal policy becomes dynamic. When compared with the optimal policy of $\mu(k,i,j)^+$ & $P_f(k,i,j)^-$, the optimal policy for $\lambda(k,i,j)^+$ & $P_f(k,i,j)^-$ is more aggressive in terms of cross-selling attempts.

Comparison of Cases between 2-D and 3-D State Space Models

To compare the 2-D and 3-D state space models, we use only the first two scenarios, because of the number of the additional products to be sold. For Scenarios 1 and 2, $\mu = 1$ and $\lambda = 10$. Given the exponentiality of the contact rate and the death rate and uniformization equation used in the system, a customer contacts the firm on the average 12 times, one of which is fictitious imposed by the uniformization equation. However, the average number of contact with the firm for a customer will be 52 for Scenarios 3 and 4, and 102 for Scenarios 5 and 6. Therefore, with the additional number of products limited to 11 for the 3-D state space model will apparently lower the values generated for these scenarios, because even though customer contacts more frequently, the firm does not have enough products to sell. For this reason we only give the comparison between the two models for Scenarios 1 and 2.

For the notation of the contact rate, death rate and failure probability that reflects the negative effect of cross-selling failures, we use the notation of Chapter 3 in Tables 4.18 and 4.19.

When we compare the values generated for both 2-D and 3-D state space models, we expect to see decreased values for the 3-D state space model due to the fact that the number of additional products sold to the customer is limited.

	$v(0,0)$	$v(0,0,0)$	**% Difference**
BASE CASE	15.096349	14.574416	3.4573%
$\mu(i,j)-\mu(k,i,j)^-$	13.557258	13.240378	2.3373%
$\lambda(i,j)-\lambda(k,i,j)^-$	13.043239	12.885908	1.2062%
$P_f(i,j)-P_f(k,i,j)^-$	11.549602	11.466212	0.7220%

Table 4.18: Comparison of $v(0,0,0)$ Values for Scenario 1

	$v(0,0)$	$v(0,0,0)$	**% Difference**
BASE CASE	33.483475	31.657089	5.4546%
$\mu(i,j)-\mu(k,i,j)^-$	30.069313	28.852703	4.0460%
$\lambda(i,j)-\lambda(k,i,j)^-$	26.29653	25.747326	2.0885%
$P_f(i,j)-P_f(k,i,j)^-$	25.173477	24.530113	2.5557%

Table 4.19: Comparison of $v(0,0,0)$ Values for Scenario 2

In Table 4.18, we compare the values generated for Scenario 1. The greatest percentage difference is for the base case comparisons of both models. In both models whenever there is a success, a renewal in terms of failures and contacts occur. However, for the 3-D state space model we hold additional information on cross-sell successes and the firm also has limited number of products to offer to the customer. When no parameters change, for a failure probability of 0.4, the firm has nothing to lose, therefore always tries to cross-sell. However, when the number of products offered is limited, the value generated is lower. When we incorporate the negative impacts on both models, this gap between two values decreases. This is the same for the values generated in Scenario 2 as well. The comparison of the values for Scenario 2 is given in Table 4.19. However, in this case as the additional revenue obtained from cross-selling is higher, when the number of products sold are limited, the percentage difference between the two models increases when compared to Scenario 1.

In terms of policies, base case, changing contact rate case and changing death rate case display a similar structure. When the failure probability change depending on the failures, we take the initial failure probability as 0.4. For the changing failure probability case, the optimal policy for the 3-D state space model behaves more aggressive when the successful cross-sells are few. As the number of products sold increases, getting close to the limit, the system becomes less aggressive. This is due to the fact that once all 11 products are sold to the customer the system can not be renewed in terms of failures and contacts.

4.3.2 Model without Renewals

In Table 4.20, a comparison between the expected values when the failure probability 0.4 is given. Except for the $P_f(k,i,j)^-$ case, in all the other cases the optimal policy is to cross-sell in each case. For $P_f(k,i,j)^-$ case, $P_f(k,0,0) = 0.4$. Based on the Table 4.20, the negative effect of cross-selling attempt failure can be ordered as follows:

$$v(0,0,0)_{basecase} > v(0,0,0)_{\lambda(k,i,j)^-} > v(0,0,0)_{P_f(k,i,j)^-}.$$

	SCENARIO 1	**SCENARIO 2**	**SCENARIO 3**	**SCENARIO 4**	**SCENARIO 5**	**SCENARIO 6**
BASE CASE	14.574416	31.657089	26.278515	55.384379	29.453011	61.030254
$\lambda(k,i,j)^-$	12.885906	25.747326	23.760782	46.551321	26.899313	52.026039
$P_f(k,i,j)^-$	11.078704	23.649179	20.110546	41.747625	22.843351	46.534829

Table 4.20: $v(0,0,0)$ Value Comparisons for without Renewal Model

Comparison between 3-D State Space Models

For base case and $lambda(k,i,j)^-$ case, the difference in terms of value generation is not significant between two models as given in Tables 4.21 and 4.22. This may be due to the fact that we let the customers die after 100 consecutive contacts, which gives close results to the infinite number of contact case when there is a renewal.

As the contact rate increases the gap between the two model values increases. As the failure probability has the stronger impact in terms of values generated, when there is not renewal in the system, the decrease in the values is also greatest when compared with the other cases.

In terms of optimal policy, when there is not a renewal, the system becomes less reluctant to cross-sell during the initial contacts, and only after succeeding in cross-selling becomes more aggressive, especially as the number of products sold gets closer to 11.

	BASE CASE		
	With Renewal	**Without Renewal**	**% Difference**
S1	14.574416	14.574416	0.0000%
S2	31.657089	31.657089	0.0000%
S3	26.287026	26.278515	0.0324%
S4	55.392821	55.384379	0.0152%
S5	29.478059	29.453011	0.0850%
S6	61.055303	61.030254	0.0410%

Table 4.21: Base Case Comparison

	$\lambda(k,i,j)^-$		
	With Renewal	**Without Renewal**	**% Difference**
S1	12.885908	12.885906	0.0000%
S2	25.747326	25.747326	0.0000%
S3	23.76185	23.760782	0.0045%
S4	46.55239	46.551321	0.0023%
S5	26.902383	26.899313	0.0114%
S6	52.029109	52.026039	0.0059%

Table 4.22: $\lambda(k,i,j)^-$ Comparison

	$P_f(k,i,j)^-$		
	With Renewal	**Without Renewal**	**% Difference**
S1	11.466212	11.078704	3.4978%
S2	24.530113	23.649179	3.7250%
S3	21.152079	20.110546	5.1790%
S4	43.983663	41.747625	5.3561%
S5	24.05469	22.843351	5.3028%
S6	49.131506	46.534829	5.5801%

Table 4.23: $P_f(k,i,j)^-$ Comparison

Chapter 5

SUMMARY & CONCLUSIONS

A firm's primary goal is to make profit. To achieve this goal, the firm tries to increase the amount and value of purchases customers make over time. In this sense, we focused on the effects of cross-selling, up-selling and add-on-selling on expected value of a customer by taking into account the customer reactions. Our main focus in terms of customer reactions is on the negative impact of cross-selling activities. However, as an extension, we also investigate the positive reactions numerically.

5.1 *Model for 2-D State Space*

We primarily formed a system defined by 2 dimensional states that include the number of cross-sell failures proposed by the firm and the number of customer contacts. At each customer contact firm has two alternative actions: to try to cross-sell to the customer or not. In case of a successful cross-sell, firm earns an additional revenue of r, while in case of failure there occurs a loss of cost of attempt and cost of failure, due to the annoyance caused by the offer. We have assumed that customer initiates the contacts, therefore causing transactions, for which a fixed revenue of R is obtained at each contact independent of the firm's decision on to cross-sell or not.

We analyze the effects of negative customer reactions on the total expected revenue and on optimal cross-sell policies. For this purpose, we first formulate the base case, which does not include any customer reaction. This case is fully analyzed, so that optimal cross-sell actions are determined for any set of parameters by a simple threshold of the probability of failing to sell an additional product to the customer, P_f^*. Therefore, an optimal CRM policy always attempts to sell additional products if P_f is lower than the threshold, and it never attempts otherwise. Moreover, this threshold value has a simple expression in terms of the other cost parameters independent of contact rate and death rate.

The negative reaction of a customer can be reflected in the customer contact/arrival rate, death rate or in the likelihood of a customer's accepting the cross-selling offer at the next contact. In order to model the negative customer reactions, we assume the system parameters change according to the result of a selling attempt; more specifically the parameter under consideration varies linearly with the ratio of the number of failed attempts to the number of total contacts since the last purchase. Obviously, including negative reactions in the model decreases the total revenue. In addition to this, as shown in Section 3.3, the expected reward obtained in any state for the varying death rate case is higher than the expected reward obtained for the varying contact rate case. The relationship between the $v(i,j)$ values of varying contact rate, death rate and base case is as follow:

$$v(i,j)_{base} \geq v(i,j)_{\mu(i,j)} \geq v(i,j)_{\lambda(i,j)} \geq v(i,j)_{\mu(i,j\&\lambda(i,j)}$$

also

$$v(i,j)_{base} \geq v(i,j)_{P_f(i,j)}$$

The cross-sell attempt failures affect the system negatively in each of these cases, so that as the number of failed cross-sell attempts increases the expected reward gained decreases for the same number of contacts and also when the number of contacts increases, while the number of failures remain the same, the death rate, contact rate and the failure probability will be less affected because of the negative reaction definition. For this reason, the value obtained for less contacts with the same number of failures will be lower. This is true for all cases considered. We can order the expected total reward gained at each state and for any case as follows:

$$v(i+1,j) \leq v(i,j) \leq v(i,j+1)$$

However, in addition to the above relationship for the changing death and contact rate cases, it is found that an increase in the contact rate cannot compensate the loss on value generated by a failure at the same time. For this reason for the changing death and contact rate cases, the following relationship is true:

$$v(i+1,j+1) \leq v(i,j)$$

When we take the failure probabilities to be the threshold values obtained from the base case, $\frac{r-c_a}{r+c_f}$, we observe that the optimal policies for all the cases considered become never to cross-sell. As the decision is always not to cross-sell, all of the parameters remain unchanged and thus the expected values obtained in each case are the same in the long run.

We have also investigated the effect of different failure probabilities for all cases. For the state dependent linearly changing death rate and the varying failure probability cases, there exists a gap $[P_f^{min}, P_f^{max} = P_f^* = \frac{r-c_a}{r+c_f})$ in between which there is a structure of optimal policies that includes actions of both cross-selling and not-cross-selling. Whenever $P_f < P_f^{min}$, the optimal policy is to always cross-sell, and whenever $P_f \geq P_f^{max} = P_f^*$, the optimal policy becomes never to cross-sell.

In between the gap starting from the P_f^{min}, for the varying death rate, the optimal policy starts to have more not-to-cross-sell actions starting from the $(0, 0)$ state and gradually increases as the failure probability gets closer to the threshold value of the base case. For the varying contact rate case, the gap does not seem to exist $P_f^{min} = P_f^{max} = P_f^*$. That is, when the negative reaction affects the failure probability and the death rate, the optimal policy becomes state dependent for some set of P_f values.

In the evaluation of these observations, we can rank the parameters of interest in terms of their effects on different time horizons. The death rate determines the mean customer lifetime, so that it reflects the long-term relationship of a customer with the firm. The arrival rate, on the other hand, is a measure of medium-term relations, since it shows the frequency of customer contacts during a lifetime. Finally, the probability of failure represents short-term relations, since it governs the behavior of a customer at each contact point.

The state space of the system does not include the total number of successful cross-sell attempts, or equivalently the total number of products a customer buys during his/her relationship. Moreover, we assume that whenever an attempt is successful, the customer starts his/her relation fresh. In other words, the state of the system has a short-term memory and a transaction based structure, so the model cannot grasp long-term effects. As a result, we observe that the structure of the optimal policy does not change significantly with the parameters related to medium and long term relations.

"Death" state is an absorbing state by definition, and all other states are transient in the system. For this reason we use transient analysis to find the average number of visits to the transient states. It can be concluded that as the renewal of the system depends on successful cross-sells, given an optimal path, a customer is more likely to visit the initial states during his/her lifetime.

For the changing failure probability case, the optimal policy has a dynamic structure. We observe that the system gives some time to the customer without cross-sell attempts, so that his/her likelihood to accept a cross-sell offer increases. As more time passes, the impact of the customer's negative experience decreases. However, the customer never totally forgets the negative experience unless there is a renewal in the system so that the failure probability becomes its initial value.

We also observe that as the customer spends more of his/her time on initial states, the expected failure probability is less than the P_f^{max} value. In terms of handling negative customer reactions, the structure of the optimal policy suggests that an aggressive selling policy will not be appropriate when the probability of failure increases with the negative experience of customers.

5.2 Model for 3-D State Space

Model With Renewals in terms of Failures and Contacts

Based on the model developed for the 2-D state space, we include another state space that represents the number of success. As opposed to the 2-D state space model, we assume that the firm has a limited number of additional products to offer to the customer throughout his/her lifetime. The renewal occurs only when there is a successful cross-sell attempt, however in this model the number of successes are not renewed. The model keeps the information on successful cross-selling all the time, while at each success the failures and the contacts become zero.

Using this model we try to understand how the customer reactions affect the value generation and the optimal policies attached to certain parameter sets. We numerically investigate the effects of contact rate, death rate and likelihood to accept offers when these parameters reflect the customer reactions.

For any kind of relationship, building trust and commitment requires experience. Under the setting our model operates, where the customers contact the firm, experiences can be either positive or negative given the customer reactions to cross-selling attempts of the firm. With each contact both the customer and the firm get to know more of each other in terms of expectations and behaviors, which in return lead to trust and commitment by both parties. We investigate the positive impact that leads to trust and commitment using mid-term and long-term parameters, such as contact rate and death rate and the negative impact via failure probability.

To be able to compare different cases in terms of value generated and optimal policies, we first look at the base case, in which none of the parameters change. As expected, when the customers do not react to cross-selling activities, having more information does not change the optimal policy, and when we take failure probability to be small enough to justify the costs associated with cross-selling, the policy is always to cross-sell.

To provide a ground for comparisons between the 2-D state space and 3-D state space models we also investigate the negative impact of cross-selling activities in terms of death rate, contact rate and failure probability. Base case, obviously provides the highest returns as there are no negative reactions. Among those three parameters, failure probability affects the system most when it incorporates the negative reaction. Contact rate is the second parameter affecting the values generated, however in terms of optimal policy, it preserves a similar structure to that of base case. Death rate affects the system least among the three parameters. This can be due to the fact that failure probability is the short-term parameter, contact rate is the mid-term parameter and the death rate is the long-term parameter in the system. Our model on the other hand is a transactional model, for this reason it is expected that failure probability has the greatest impact.

While we investigate the positive impact, first we let the death rate decrease as more of the additional products are sold to the customer. During the first contacts, customers basically form an opinion on the firm, and their relationship with the firm further develops by accepting offers, the customers become less reluctant to quit the relationship. To reflect this, we defined the decrease in the death rate numerically such that the rate of decrease increases as more of the additional products are sold. When we incorporate this to the model, we obtain higher values generated. In terms of optimal policy, however, as the other parameters do not change, the optimal policy behaves similar to the base case and always cross-sells to the customer. However, the calculation of a threshold on the failure probability is not as clear as the 2-D state space model. Because at each renewal point of $(k, 0, 0)$ the values obtained differ on the long-run.

To investigate the positive reactions, we used the contact rate as well. During the initial cross-sell successes the customers will form a buying behavior and based on satisfaction their incoming rate will increase more steeply. However, after a buying behavior formed, the rate of increase will decrease as the relationship pattern stabilize. To reflect this we defined the contact rate to increase numerically as the number of additional products sold in a concave manner. To be able to compare the values generated for the positively affected contact rate case, we need to adjust the time-scale of the base case. We observe that the revenues generated increase, as expected and also the optimal policies display a similar nature, which is always to cross-sell for the given parameters.

We also incorporate both the negative and the positive reactions to the model, where either the death rate or the contact rate reflects the positive impact, and failure probability reflect the negative impact. In both cases, we observe that the positive impact can not offset the potential loss that can be caused the negative impact if the system tries to cross-sell in an aggressive manner. For this reason the optimal policies are dynamic in nature, however more aggressive than the case when only failure probability reflects the negative impact.

Model for 3-D State Space without Renewals

We did not observe a significant difference when we relax the assumption that successful cross-sells renew the system in terms of failures and contacts using the same size for the dimensions of the states for the numerical calculations, we obviously decrease the possible number of contacts by the customer. The values generated are also lower because of this decrease in number of contacts.

In this model as well, the failure probability affects the values generated more than the death rate.

5.3 Comparison of 2-D and 3-D State Space Models

Finally, we compare the 2-D state space model with renewals with the 3-D state space model with renewals on failures and contacts whenever a success occurs based on values generated and optimal policies in terms of the negative reactions of customers reflected by failure probability, death rate and contact rate.

Having more information on the customer does not compensate the loss caused by the limited number of additional products assumed by the 3-D state space model. Therefore the values generated for the 3-D state space model are less than the values obtained from the 2-D state space model.

In both models, failure probability affect the values generated the most, followed by the contact rate and finally death rate. In terms of optimal policies, though, due to the limited number of products the policies of 3-D state space model differ from the 2-D state space model. In all cases, for the 3-D state space model, as the number of products sold approach the limit, the optimal policies become less aggressive in terms of cross-selling decisions.

When the death rate change based on failures, both the 2-D state space system and the 3-D state space system displays a similar structure in terms of optimal policies.

For the contact rate case, when the failure probability is small, the optimal policies are similar to each other. However, for the decreased contact rate case, for which the number of additional products sold is limited, the optimal policy obtained provides a dynamic nature in terms of cross-selling decisions especially when the number of products sold approach the limit. This situation can not be observed for the 2-D state space model.

When the failure probability change based on failures, the optimal policies are similar to each other, however for the 3-D state space model, as the number of products sold increase the system behaves less aggressive.

In conclusion, this thesis investigated the effects of negative and positive reactions of customers to CRM strategies like cross-selling attempts. We have found that there is a significant loss in revenues if the cross-sell strategies are determined ignoring the reactions.

Second, the additional revenues generated from the customer and the contact rate affect the values generated significantly. The magnitude of the impact of the negative reactions depend on the contact rate of the customer with the firm.

We found that negative reactions of customers have a stronger effect than the positive reactions on the values generated throughout the relationship. This is consistent with the literature on customer behavior and relationship marketing.

In terms of value generation, varying death rate has the least impact. Customer's contact frequency and likelihood of accepting cross-sell attempts affect the value generation more. Therefore, the firms should try to increase the customer contact rate and the customer's likelihood to accept offers. If correct marketing strategies are employed, firms can easily make up for the loss caused by the negative impact of CRM activities in terms of decreased lifetime. To generate more value from customers, the firm should try to retain its profitable customers using different CRM strategies and should also increase its sales via successful cross-selling attempts. Likelihood of accepting cross-sell offer has the most drastic impact on value generation. Therefore, firms should direct most of their efforts to estimate this likelihood and try to find the best cross-sell offers for the customers. In addition to these, firms should also try to increase the revenues obtained from the customers while keeping its costs down. However, in today's world, customer expectations are quite high, therefore keeping the costs in level with the quality of the service/product offered, firms should employ cross-selling techniques taking into account the customer reactions.

In terms of optimal policies associated with different parametric changes, again we see that likelihood to accept an offer is the most important. The optimal policy is very dynamic in this case, depending on the additional revenue obtained from cross-selling activities as well as the costs of cross-selling attempt and cross-selling failure. When the fixed revenue obtained from the customer is too big compared to the revenue obtained from cross-selling, then even the successful cross-selling hardly justifies the risk of losing the fixed revenue. Therefore, the firm should choose its additional products to offer to the customer very carefully. If the negative customer reactions are ignored in this case, firms' profits are severely affected. With the varying contact rate case, the optimal policy seems to be static, though. The optimal policies associated with the death rate become more dynamic whenever customer contacts are less frequent, fixed revenue obtained is greater than the cross-selling revenue and also when costs are low.

For future research, an empirical study can be conducted to validate the findings of our research.

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