Joint Flexibility and Capacity Design in Service and Manufacturing Systems

by

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This is to certify that I have examined this copy of a master's thesis by

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ABSTRACT

In this thesis, we analyze the capacity optimization problem with an objective of maximizing the profit in a multi-resource system under uncertain demand. For any realization of demand, the optimal capacity level is a function of the flexibility structure, the capacity cost and the prices. However, for unknown demand, the problem becomes more complicated. We solve the problem as a two-stage stochastic optimization problem. In the first stage, capacities are determined prior to the realization of demand. In the second stage, the realized demand is allocated to the resources. We propose and test a solution method based on the gradient estimation via perturbation analysis (GPA) technique. Our method is a simulation-based optimization procedure. After analyzing the legitimacy of the method through its theoretical properties, we apply the method to a few benchmark cases and confirm the accuracy of the method via numerical examples. Then we perform a numerical study to explore the link between capacity optimization and flexibility design considering the cost of the capacity. Despite the fact that literature asserts the advantage of the balanced flexibility structure, we show that through optimizing the capacity, some imbalanced structures can perform better in some cases.

ÖZET

Bu tez çalışmasında, belirsiz talep karşısında çok kaynaklı bir sistemin karını enbüyüklemek amaçlı kapasite eniyilemesi problemini analiz ediyoruz. Gerçekleşmiş talep için eniyi kapasite seviyesi; esneklik yapısı, kapasite maliyeti ve fiyat değerlerine bağlı bir fonksiyondur. Ancak belirsiz talep karşısında problem daha karmaşık bir yapıya sahip olur. Bu problemi iki aşamalı olasılıksal eniyileme problemi olarak çözüyoruz. İlk aşamada, talebin gerçekleşmesinden evvel kapasiteler belirleniyor. İkinci aşamada ise gerçekleşen talep kaynaklara dağıtılıyor. Bu problem için sarsım analizine dayalı gradyan kestirimi tekniği kullanan bir çözüm yöntemi öneriyor ve bu yöntemi test ediyoruz. Yöntemin meşruluğunu teorik özelliklerine dayalı olarak gösterdikten sonra, yöntemi denktaş problemlere uygulayarak sayısal örneklerle de doğruluğunu göstermiş oluyoruz. Daha sonra, kapasite eniyilemesi ve esneklik yapısı tasarımı arasındaki bağlantıyı maliyetleri göz önünde bulundurarak açıklamak üzere birtakım sayısal örnekler çözüyoruz.

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NOMENCLATURE

i	demand type index
j	resource index
r	index of the replenishment number in an iteration
z	index of iteration
K	capability matrix
k_{ij}	binary number that takes the value of 1 if resource j has skill j
d_i	amount of demand type i
c_j	capacity of resource j
x_{ij}	the amount of job i processed by resource j
p_i	unit price of demand type i
s_j	unit specialized capacity cost of department j
f_j	unit flexible capacity cost of department j
Ω	profit / performance criterion
Φ	revenue
M	a big number
$g_i(d_i)$	probability density function of demand type i.
G	cumulative demand distribution
β_j	portion of the flexible capacity in resource j.
$\mathbf{c}^z$	capacity vector at iteration z
b_z	step size at iteration z
∇	gradient
$\tilde{\nabla}$	gradient estimator

u	vector of shadow prices
$\mathbf{d}_{\mathbf{r}}^{\mathbf{z}}$	the vector of r^{th} demand realization at iteration z.
WW	maximum number of iterations
$\hat{p}$	highest contribution margin

PA	Perturbation analysis
GPA	Gradient estimation via perturbation analysis

Chapter 1

INTRODUCTION

Flexibility is a protection against uncertainties and variations. The benefits and the importance of flexibility are well-known. It enables usage of the resources for multiple tasks, and therefore increases the adaptability of any system against sudden changes in the system or in the market conditions. This thesis analyzes the capacity considerations in flexible service and manufacturing systems.

A resource is flexible if it can perform multiple operations; such as cross-trained workers or multi-task equipment. In response to the variations such as fluctuations of the demand or breakdown of some equipment, flexible resources can be allocated to different tasks. Through flexibility, the resources are shared between operations and the demand for different operations are aggregated. Consequently, the resources are better utilized and the production or service rate of the system is increased under uncertainty. De Groote (1994) defines flexibility as a hedge against diversity, where diversity indicates the variations in the system conditions. Moreover he claims that; as diversity increases, flexibility becomes more desirable; and as flexibility of the system increases, operating in diversity becomes more desirable.

Nevertheless, flexibility is expensive. It brings additional costs due to the need of more qualified personnel or equipment and increase in the operational complexity. Therefore, how much flexibility to add is a crucial decision. If flexibility is not added properly; the system cannot benefit from the resources as much as possible, and may even incur financial loss. The set of operations that the resource is capable to perform is the skill-set of that resource. The configuration of the system that shows the skill-sets of the resources, by specifying which operation can be performed by which operators, is the flexibility structure of the system. Within the spectrum of full-flexibility and dedication, a variety of limited-flexibility structures can be built. Flexibility design addresses the question of, which one of these structures performs the best. A full-flexible resource has the capability to perform all of the operations in the system while a dedicated resource is specialized in a single operation. Consequently, if a system is full-flexible, every resource can be allocated to any operation; and if it is dedicated, every resource can be allocated to one operation. Obviously, the productivity of the system is at the highest level when the system is full-flexible. Without considering the cost, full-flexibility seems ideal and saves the managers from the burden of designing the flexibility structure. However, our aim is to optimize the profit, so the cost of flexibility should be considered. Therefore the ideal flexibility structure is the one which maximizes the throughput with minimum additional flexibility cost.

Jordan and Graves (1995) is one of the seminal studies in the flexibility design area. They show that, in certain cases, some of the limited flexibility structures perform almost as good as the full-flexible structure under uncertain demand. Moreover, the general guidelines to design efficient flexibility structures are established by Jordan and Graves, which lead them to the chaining strategy. A "chain" is defined for the manufacturing system as "a group of products and plants which are all connected, directly or indirectly, by product assignment decisions". The details of the chaining strategy will be given in Section 2.3. In their analysis, Jordan and Graves consider fixed capacity. However, the performance of a flexible system is closely related to the capacity levels of the resources. Jordan and Graves state that, through chaining, the capacity can be shared between different operation types and therefore the maximum utilization of the available capacity can be achieved. We claim that, by optimizing the capacity, the system performance can improve considerably. Because, even with flexibility, the system may be holding unnecessary capacity, or the capacity may be kept at the wrong pool. A pool is a group of resources with the same skill set. Through capacity optimization, the ideal capacity of each resource pool and consequently the total capacity of the system is determined. In this study, we would like to investigate the effect of capacity optimization on the performance of the flexibility structure.

Unless the capacity is optimized, inefficiencies occur in a flexible system. Capacity optimization prevents holding unnecessary flexible capacity and consequently decreases the cost of the system since the level of flexibility increases the cost of the capacity. Moreover, in the absence of dedicated capacity, all the demand has to be assigned to the flexible capacity which is a waste of highly-qualified resources. Routing efficiency is also affected due to non-optimal capacity allocation. If enough capacity does not exist to perform the requested operation, the capacity of the relevant resource has to be made available through demand reallocation among the connected elements of the system, which incurs operational congestion.

The importance of flexibility can be well observed and analyzed in call centers. Therefore we focus on call-center motivated examples in one group of our numerical experiments. Call centers provide services by agents via telephone and can be used for multiple purposes. They can be used as sales centers, after-sales support centers, information centers, contact centers, etc. Due to many advantages of call centers, their number is rapidly growing worldwide. They speed up the processes, save the company from the burden of building and running offices and the customers from the burden of going to the offices and waiting in queues. The customers are led to the right operators by the voice menu, and if the call-center is organized well, the customers are served without waiting too long on the line. Moreover, a central call-center can provide service to the customers in a large area and decrease the service cost via economies of scale.

Figure-1.1 represents a flexible call-center as a bipartite directed graph. The left nodes stand for the customer types requesting different services and the right nodes stand for the agent pools with different skill-sets. The arriving calls are routed to the pools along the arcs. A call can be processed by any available agent in the corresponding pool. If an agent pool is flexible, more than one arc ends at that pool.

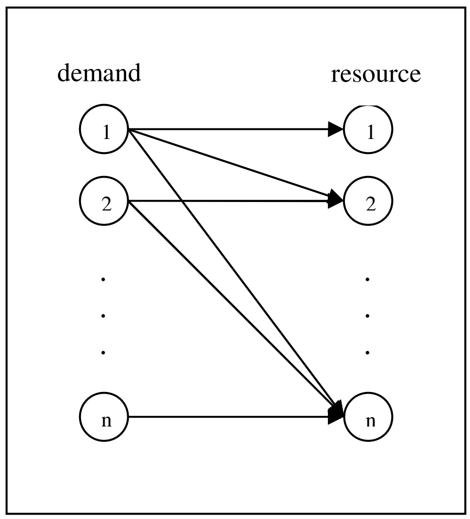

Figure 1.1: Configuration of a flexible multi-server call center

Consider a call center providing technical support in 3 languages. The agents provide the same service in different languages; therefore the skill set of an agent consists of the languages that he/she can speak. A dedicated agent can speak only one language while a full-flexible agent can speak all three. The cost of an agent increases as the number of languages in his/her skill set increases. If the number of the calls in each language is known in advance, there is no need for flexibility. Since the demand is unknown, the system should be designed in a way that balances the capacity cost and customer satisfaction.

There is a broad flexibility design literature. One stream of the related literature focuses on flexibility design given capacity while another focuses on capacity optimization given flexibility structure. In the capacity optimization literature, a limited set of flexibility structures are analyzed. However, with an objective of looking at capacity and flexibility interaction, we focus on capacity optimization problem for general flexibility designs. Our method can optimize the capacity of any flexibility structure. To accomplish this, we formulate the problem as a 2-stage newsvendor model and propose a solution method based on gradient estimation via perturbation analysis technique.

The remaining parts of this thesis are organized as follows. Chapter 2 is a review of the related literature. The GPA method is explained in Chapter 3. We present the model in Chapter 4. Chapter 5 introduces the solution method and analyzes the theoretical background of the method. Chapter 6 presents the numerical results. The results of the numerical studies are represented and analyzed in three subsections. Section 6.1 focuses on validation of the procedure. The capacity allocations by our method are compared to the optimal capacity assignments by the classical newsvendor solution for certain flexibility structures. Moreover, as a benchmark, the problems defined by Netessine et. al. (2002) are re-solved with our method and the results are compared. Section 6.2 provides the analysis of another benchmark. In this section, we show the appropriateness of using the newsvendor setting in our problem via comparing our results with the results of Harrison and Zeevi (2005). The experiments presented in Section 6.3, with an aim of exploring the interaction between capacity optimization and flexibility design, use the capacity optimization method to compare the performance of three flexibility structures under various system parameters. The final chapter summarizes the results and includes directions for future research.

Chapter 2

LITERATURE REVIEW

2.1 Overview

Flexible system design literature can be investigated under three main topics which are; flexibility design, capacity design and routing. There is a vast amount of literature on each topic and they are either analyzed together or independently. We are mainly concerned with the first two of them. It is possible to analyze the flexibility concept in many settings, such as manufacturing plants, telecommunication networks, computer systems and service operations as Gurumurthi and Benjaafar (2001) state. We consider the service and manufacturing systems with multiple operators which face multiple demand types. But the results are applicable to any context. The analyses exhibit slight differences between service and manufacturing systems. Flexibility is provided by multi-skill workers in a service system while it is provided by multi-function machines or production lines, as well as multi-skill workers in a manufacturing system.

Sethi and Sethi (1990) and Toni and Tonchia (1998) review the flexibility literature within the manufacturing systems. Toni and Tonchia stress three important characteristics of flexibility which are; acting as a buffer against the external changes, preserving the state of the system under random environment and having capability of adaptation. Many researchers analyze the capacity optimization, flexibility design and routing decisions in call-centers. Aksin et. al. (2005), Koole and Pot (2005), Gans et. al. (2003) provide detailed analysis of the flexibility literature within call centers. Aksin et. al. mention the increasing importance of the flexibility under random environment and state that, against diversifications in the demand types, the flexibility level should be increased.

2.2 Flexibility Design

First, we will review the flexibility design literature. This stream of the literature focuses on the efficiency of flexibility structures considering fixed capacity and deals with the decision of how much flexibility to add to each resource. Considering that the system is represented as a bipartite network as in Figure-1.1, through flexibility design, we can decide, between which nodes an arc should exist since arcs correspond to the skills.

Some basic notions about flexibility are established by Jordan and Graves (1995). Their main contribution is the introduction of the chaining concept, which will be explained in Section 2.3. Their research guides us in the sense that, they explore the relationship between capacity and flexibility and come up with some important results and guiding principles for designing the flexibility structure. Considering the capacity as fixed, they focus on the efficiency of the flexibility structures in the production systems with multiple plants. They give an insight into the relation of capacity and flexibility by investigating the relative benefits of adding flexibility on different capacity levels. Their results imply that the production rate can be increased either by adding capacity or increasing flexibility, but for the sake of the utilization, adding flexibility properly is more advantageous most of the times. They investigate that, the improvement in responsiveness gained by adding full flexibility can also be achieved through adding less flexibility properly. Aksin and Karaesmen (2004) provide a deeper understanding of the relationship between capacity and flexibility by analyzing the multi-departmental service system under uncertain demand. They also provide analytical justification for some earlier proposed flexibility principles. They represent the system as a directed graph where nodes stand for the demand types and departments. In a similar fashion, we build our model as a network flow problem. Aksin and Karaesmen (2004) show that, the effect of the capacity changes on the throughput level is dependent on the flexibility structure and additional flexibility has decreasing benefits for fixed capacity. Gurumurthi and Benjafaar (2001) support these results and state that; “flexibility exhibits diminishing returns”. They analyze the relationship between the performance and flexibility in a multi-server queuing system.

Iravani et. al. (2005), enable the comparison of the performance of any two flexibility structures satisfying certain conditions, via a flexibility index that they introduce. They analyze the systems, the departments of which have equal capacity and which has enough capacity to cover all of the demand. Different from the previously mentioned researchers, Pinker and Shumsky (2000), DeGroote (1994) and Sethi and Sethi (1990) include the cost of flexibility in their analysis. Pinker and Shumsky analyze the efficiency-quality trade-off of the flexible servers. They consider the career-path and analyze how the efficiency of a worker is affected as he/she moves to a higher level in the career path. When the cost of flexibility is considered, adding flexibility properly becomes more important. We also consider the flexibility cost in our analysis since our main concern is to balance the cost and efficiency of the system.

2.3 Chaining Strategy

Jordan and Graves (1995) introduce the chaining strategy. A chain, considering that the system is represented as a network, is the connected parts of a graph. The demand types which are the members of the same chain share their resources. Figure-2.1 represents a long chain on the left and shorter disjoint chains on the right. Jordan and Graves states that in a system, fewer and longer chains are better. Consequently, one complete chain where every operator has two skills and every operation can be allocated to two operators, such as the long chain structure in Figure-2.1, is superior to several shorter chains.

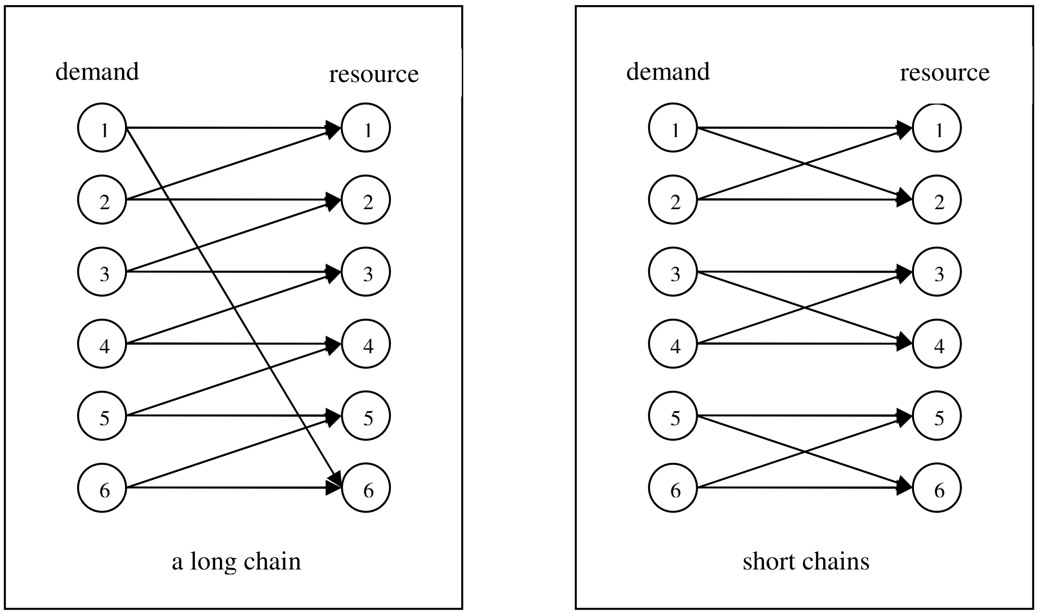

Figure 2.1: Chaining structures

Inman (2004) defines the characteristics of a complete chain as the following:

- There is a backup processor for every task
- At least one worker from each pool is cross-trained.
- All tasks are interconnected.

Complete chaining is advantageous because it brings perfect resource sharing with minimum flexibility. Graves and Tomlin (2003) develop the research of Jordan and Graves (1995) and make similar analyses for multi-stage supply chains. They consider K stages and I products where, each product is processed at each stage. They give the automotive supply chain as an example to a four-stage supply chain with the stages; component, engine, body and final assembly. Jordan and Graves (1995) analyze the flexibility of the system in one stage of the production process independent from the other stages. However Graves and Tomlin (2003) analyze the flexibility structure of consecutive production stages interdependently. They show the advantage of the chaining strategy for multi-stage supply chains.

Sheikzadeh et. al. (1998) discuss dedication, pooling and chaining strategies. They emphasize the importance of resource sharing which supports the chaining strategy. The more the resources are shared, the higher the responsiveness to the demand variations is achieved. In pooling, the resources that can process the same set of operations are grouped together, so that the demand can be processed by any resource in the pool that it is allocated to. A system with a single pool is fully-flexible. Sheikzadeh et. al. propose chaining structure instead of pooling and dedication. Later, Inman et. al. (2004), Gurumurthi and Benjafaar (2001), Jordan et. al. (2003) and Hopp et. al. (2001, 2004) also explored the benefits of chaining. Sheikzadeh et. al. and Jordan et. al. analyze chaining within manufacturing systems, Inman et. al. within assembly lines, Gurumurthi and Benjaafar within service systems, and Hopp et. al. within both service and manufacturing systems. Inman et. al. explains that, chaining is not an algorithm for optimizing the capacity, but only a guide for introducing the flexibility efficiently. Gurumurthi and Benjafaar consider a queuing system with abandonment. They define a priority-based routing scheme. Under this routing policy, they reveal that, chaining is not the optimal strategy in every case. They state that under equal demand arrival rates and homogeneous workers, symmetric chaining is the best strategy, but when the demand arrival rates are not equal, asymmetric flexibility performs better. However, in a queuing system the routing influences the performance of the system. Therefore the accuracy of the results of Gurumurthi and Benjaafar cannot be asserted in the newsvendor setting. Besides, Aksin and Karaesmen (2004) and Jordan and Graves (1995) show that, independent from the asymmetry of the demand, balanced flexibility performs better than unbalanced flexibility under the newsvendor setting. The analysis of Harrison and Lopez (1999) supports the chaining strategy. They propose a complete resource pooling strategy which is formed via the agents with overlapping skill-sets. The complete resource pooling strategy defined by Harrison and Lopez corresponds to the complete chaining strategy defined by Jordan and Graves (1995).

2.4 Capacity Optimization

In the previous section we analyzed the flexibility design literature with a focus on the chaining strategy. Second, we will review the capacity optimization literature. This stream focuses on optimal capacity allocation of the flexible systems, considering the effect of flexibility cost. In the capacity optimization literature, two approaches seem noteworthy; discrete time and continuous time. Considering discrete time periods, the problem can be modeled as a 2-stage newsvendor problem. In the continuous time setting, each department is modeled as a queuing system and time evolves continuously. The newsvendor approach is more popular due to the complexity of queuing systems. In the queuing approach the routing policy affects the performance of the system and becomes a major problem. Routing policy is the collection of rules that determine to which resource the arriving demand will be allocated. To be more specific, when a demand arrives and multiple resources are available, the operator to perform the requested operation is chosen according to the routing policy. If the routing decision is not made properly, customers wait longer in the queue because the waiting time in the queue depends on the processing times of the prior customers. Moreover, if the model considers abandonment, due to the long waiting times, the number of abandoning customers increase. However, in a discrete time setting, priority rules in the routing decision do not make any difference in the system performance.

Because all of the arrivals occur instantly, and since unsatisfied demand is lost, the demand is allocated to the resources so that the loss of demand will be minimized. The throughput of the system is dependent on the instant utilization of the capacity, which is independent from the priority rules. Therefore, priority rules are not important in newsvendor setting.

In the classic single-stage newsvendor problem, the vendor decides how many papers to buy at the beginning of the period under unknown demand. Capacity optimization problem resembles the newsvendor problem in the sense that, prior to the realization of the demand, the capacities should be set considering the contribution margins and costs under unknown demand. In this research, we consider the newsvendor setting. Aksin and Karaesmen (2002), analyzing the flexibility design problem, show that the basic behavior of the flexibility stays the same in both discrete and continuous time settings; therefore suggesting the possibility of using the newsvendor approach.

2.4.1 Newsvendor Approach

Even though some previous research proposes capacity optimization methods via newsvendor approach; they have restricted applications such as Netessine et. al. (2002). They consider a special flexibility structure in which, each server can have at most two skills. The detailed information about the structure defined by Netessine et. al. will be explained in Section 6.1 as one of our benchmarks. Harrison and Van Mieghem (1999) consider 2-product types manufactured in 2 dedicated lines and 1 final joint line in series. Fine and Freund (1990), Van Mieghem (1998), and Gupta et. al (1992), on the other hand, restrict their analysis to dedicated and full-flexible resources.

Harrison and Zeevi (2005), using the multi-dimensional newsvendor approach, propose a method for finding the optimal staffing level in call centers, which is based on linear programming and Monte Carlo simulation. They include abandonment and waiting time costs in the model.

2.4.2 Queuing Approach

Jennings et. al. (1996) and Grassman (1988) consider the time-dependent staffing decision in a multi-server service system under uncertain demand. The literature about capacity optimization in queuing systems is dominated by call-center applications. In the call-center literature, capacity optimization is referred to as “staffing”. Koole and Mandelbaum (2002) provide a review of the queuing models for call centers. In this section, we will review the literature on multi-class, multi-skill call centers considering the queuing approach. Koole and Pot (2006) review the literature on routing and staffing decisions in the multi-skill call centers. Chevalier et. al. (2004) study staffing decision in call centers via queuing approach considering only full-flexible and dedicated resources. They come up with an interesting result for staffing which states that, for symmetric call arrival rates, 20% of the budget should be spent on flexible servers. Aksin and Harker (2003) consider capacity sizing problem in the context of call center operations.

Borst et. al. (2004), Wallace and Whitt (2005), and Feldman et. al. (2005) generate methods for staffing call-centers using iterative simulation-based algorithms and the square-root principle, which was first introduced by Erlang. Feldman et. al. improve the analysis of Jennings et. al. and include abandonment in their model. Garnett et. al. (2002), model a call center with multiple identical agents and an infinite queue. Considering the abandonment, they optimize the staffing level. They modify the square root formula in order to include the abandonment in the model.

Wallace and Whitt (2005) addresses the routing and staffing decisions in a call-center. Flexibility literature is concerned with skill-based routing (SBR). Garnett and

Mandelbaum (2000) provide an introduction to skill-based routing. They represent a system with multiple demand types each of which has an independent queue and multiple pools of agents with different skill sets. The customers are routed to the agents according to their skill sets and some other rules. They state that, the routing decision may be made either depending on some priority rules, which is static, or depending on the state of the system, which is dynamic. Laws (1992), Kelly (1994) consider the dynamic routing decision and provide lower bound on the performance of the system under different routing policies.

Chapter 3

GRADIENT ESTIMATION VIA PERTURBATION ANALYSIS

3.1 Introduction

We solve the capacity optimization under uncertain demand problem using gradient estimation via perturbation analysis (GPA) method. The method is based on achieving the optimal value via small perturbations in the system parameter. Glasserman (1991) analyzes the perturbation analysis (PA) in discrete event dynamic systems. In this chapter, borrowing their notations, we will introduce the GPA method.

Perturbation analysis (PA) is a gradient estimation method used for the sensitivity analysis and optimization of discrete event systems under randomness. In discrete event systems, events occur at random time instances and the state of the system changes in response to these occurrences. The state of the system corresponds to its physical configuration such as the queue length, as Glasserman (1991) states. The state of the system and the gradients provide information regarding the performance of the system at any time instance. However, in general, the performance of such a stochastic system cannot be analyzed and optimized analytically. Therefore some gradient estimation methods are used for the performance analysis of stochastic systems.

Gradients are useful tools for sensitivity analysis and optimization. To be more specific, the gradient of a system's performance with respect to one of the system parameters shows how much the performance of that system is affected by a small change in the particular parameter. Let's consider a simple case, a service system with a single server and a single queue without abandonment the performance criteria of which is taken as the average waiting time. The gradient of the average waiting time with respect to the service rate shows how much the average waiting time will change if the service rate is increased or decreased. Let Ω represent the system performance and the scalar x represent the system parameter whose effect on the performance we want to observe. The gradient of Ω with respect to x, $\nabla_x \Omega(x)$, is formulated as the following;

$$\nabla_x \Omega(x) = \lim_{h \to \infty} \frac{\Omega(x+h) - \Omega(x)}{h} \qquad (3.1)$$

If the gradient is positive, it reveals that increasing the parameter in the direction of h improves the performance and vice versa. Consequently, if we want to maximize the performance criteria, we should change the current value of the parameter in the direction of the gradient and in the negative direction otherwise.

The performance of a stochastic system is in general expressed by the expectation of the performance criteria, such as the throughput or profit. Let L represent the performance criterion, $Z_t(\theta)$ the state of the system and θ, a system parameter. At any time instance t, L and Z are dependent on the value of θ. Hence, $L(\theta)$ represents the performance of the system when the parameter takes the value θ and the expected value of L over all possible values of θ, $E[L(\theta)]$, represents the performance of the system. As mentioned before, the performance analysis is possible via gradients however, the exact gradient of $E[L(\theta)]$ cannot be calculated. A popular gradient estimation method is via finite differences (FD). PA estimates the gradients via stochastic gradients instead of finite differences. In the next few paragraphs, the FD method will be summarized and its disadvantages will be explained.

In FD, the value of the system performance is measured at two different values of a system parameter via two independent simulations. Heidergot (1995) states that these single values generated by the simulations estimate the performance of the system at the corresponding parameter values. He defines a service system as a queuing model. Let $D_\theta(\Delta, n)$ represent the difference between the performance values at the parameter values θ and $\theta + \Delta$ at the n^{th} service. Heidergot defines $D_\theta(\Delta, n)$ as;

$$D_\theta(\Delta, n) = \frac{1}{\Delta}\left(E\left[\Omega_n(\theta+\Delta, w\right] - E\left[\Omega_n(\theta, w\right]\right), \qquad (3.2)$$

where w represents the randomness of the process, and introduces the gradient estimator via finite differences as the following:

$$\lim_{\Delta\to 0} D_\theta(\Delta), \qquad (3.3)$$

where, $D_\theta(\Delta)$ is defined as $\lim_{n\to\infty} D_\theta(\Delta, n)$.

Heidergot mentions two disadvantages of the FD method. The first one is the complication in the process of choosing a proper Δ. He states that, if Δ converges to zero, the variance and consequently the confidence interval of the estimator tend to infinity. Therefore, Δ should not be chosen too small. On the other hand, it should not be chosen too big so that, $D_\theta(\Delta, n)$ is close to the gradient. Therefore choosing Δ requires special attention and is time-consuming. The second disadvantage is the excessive computational effort. Because FD requires 2 separate simulations for estimating the gradient at each point.

3.2 Perturbation Analysis

Ho and Cao (1983) introduce the PA method. Ho defines PA in the foreword of Glasserman (1991) as "a technique for the efficient performance analysis of discrete event dynamic systems (DEDS) trajectories". According to Ho (1987), PA is advantageous over FD because it saves us from the burden of repeating the experiment for different values of a parameter. PA estimates the gradient via a single simulation. However, PA cannot be used at every setting. The system and the model has to satisfy certain conditions, which will be explained later.

Glasserman defines the gradient estimation problem via perturbation analysis as the following; the vector $\nabla_{\boldsymbol{\theta}} E[L(\boldsymbol{\theta})]$ represents the sensitivity of the system performance with respect to $\boldsymbol{\theta}$. The problem is finding a vector-valued function, $\xi(\boldsymbol{\theta})$, which provides an unbiased estimator of $\nabla_{\boldsymbol{\theta}} E[L(\boldsymbol{\theta})]$ and which satisfies the following condition;

$$E[\xi(\boldsymbol{\theta})] = \nabla_{\boldsymbol{\theta}} E[L(\boldsymbol{\theta})] \tag{3.4}$$

Glasserman states that if ξ is an unbiased estimator, that it satisfies 3.4, $D_n(\boldsymbol{\theta}) = \sum_{i=1}^{n} \frac{\xi_i(\boldsymbol{\theta})}{n}$ converges to $\nabla_{\boldsymbol{\theta}} E[L(\boldsymbol{\theta})]$ by the law of large numbers;

$$\lim_{n\to\infty} D_n(\boldsymbol{\theta}) = \nabla_{\boldsymbol{\theta}} E[L(\boldsymbol{\theta})] \tag{3.5}$$

As a result, the choice of ξ function is a major decision in GPA. Glasserman mentions two important characteristics of the candidate estimator. First, a general procedure for calculating the estimator should be defined and the estimator should be easy to compute. There are different methods to generate ξ. Infinitesimal perturbation analysis (IPA) is one of those methods. The others will not be mentioned here. IPA offers using $\nabla_{\boldsymbol{\theta}} L(\boldsymbol{\theta})$ as the estimator function **ξ(*θ*),** which satisfies both conditions mentioned previously. As Glasserman (1991) states, by IPA, "the derivative of a parametric random variable translates into the derivative of a performance measure". However, IPA is not applicable to every setting. It requires some structural properties which will be explained in section 3.4. Following the lead of Glasserman (1991), we use IPA in our solution algorithm and we will show that our model possesses the required structural properties in Section 5.1.

PA is a combination of two algorithms; perturbation generation and perturbation propagation. The system parameter is perturbed and its effect is propagated to the system performance. Glasserman (1991) defines a simulation as generation of a series of $\{X_i(\boldsymbol{\theta}), i=1,\ldots\}$ and transformation of them into a sample path of processes $\{Z_t(\boldsymbol{\theta}), t \geq 0\}$. $X_i(\boldsymbol{\theta})$ represents a random variable dependent on the system parameter $\boldsymbol{\theta}$, such as the service rate. The changes in $\boldsymbol{\theta}$ affects $X_i(\boldsymbol{\theta})$ and the changes in $X_i(\boldsymbol{\theta})$ affects $Z_t(\boldsymbol{\theta})$. In PA, the parameter perturbations are so small that they do not change the order of events.

Glasserman (1991) clarifies the gradient estimation via perturbation analysis procedure by explaining it from a simulation perspective and a mathematical perspective. From a simulation perspective, IPA can be summarized as the following;

S+1 simulations are made using a different seed each time. The first simulation is made at $\boldsymbol{\theta} = (\theta_1, \ldots, \theta_n)$. At the following simulations, one of the elements of the vector is perturbed and the system performance is measured. The i^{th} simulation is run using the $\amalg_i^{th}$ seed and the parameter becomes $\boldsymbol{\theta} = (\theta_1, \ldots, \theta_i + he_i, \ldots, \theta_n)$ where **e** is the unit vector and h is a small number. The system performance in simulation i is represented as $L(\boldsymbol{\theta}+he_i, \amalg_i)$. Then the effect of the perturbation in the i_{th} element of θ can be shown as the following:

$$L(\boldsymbol{\theta}+he_i, \amalg_i) - L(\boldsymbol{\theta}, \amalg_0) \qquad (3.7)$$

Using the same seed for each element of the vector, the formulation becomes;

$$L(\boldsymbol{\theta}+he_i, \amalg_0) - L(\boldsymbol{\theta}, \amalg_0) \quad (3.8)$$

Glasserman states that the second formulation is preferable since it measures the effect of each parameter under the exact same setting. Considering that the limit of the formulation above gives the partial derivative of the system performance:

$$\partial_{\theta i} L(\boldsymbol{\theta}, \amalg_0) = \lim_{h \to 0} \frac{L(\boldsymbol{\theta} + he_i, \amalg_0) - L(\boldsymbol{\theta}, \amalg_0)}{h} \quad (3.9)$$

Repeating the procedure S times where $S \rightarrow \infty$ with different $\boldsymbol{\theta}$ values and taking the average, the difference in the expected performance can be calculated; $E[L(\boldsymbol{\theta}+he_i)] - E[L(\boldsymbol{\theta})]$. Using the same seeds for the nominal and perturbed simulations, the gradient estimator vector can be represented as the following;

$$\nabla_{\boldsymbol{\theta}} L(\boldsymbol{\theta}, \amalg_0) = [\partial_{\theta i} L(\boldsymbol{\theta}, \amalg_0), \ldots, \partial_{\theta n} L(\boldsymbol{\theta}, \amalg_0)] \quad (3.10)$$

Mathematically the state of the system is defined as a function of three arguments, t, $\boldsymbol{\theta}$ and w. In this setting, w represents the randomness of the process. Then the state is represented as, $Z_t(\boldsymbol{\theta},w)$ and the formulation 3.9 which gives the partial derivative becomes;

$$\partial_{\theta i} L(\boldsymbol{\theta},w) = \lim_{h \to \infty} \frac{L(\theta + he_i, w) - L(\theta, w)}{h} \quad (3.11)$$

3.3 Optimization

Besides sensitivity analysis, IPA can be used for optimization when a system parameter cannot be optimized analytically. Glasserman (1991) states that, if the performance of the system changes smoothly with respect to a system parameter, than the information gathered from the gradient is valuable in terms of understanding its effect on the performance. The gradient estimator can be used in order to compare the performance of the system in different points and choose the point in which the system performs the best. The optimal point is found via small perturbations in the system parameter in the direction or in the negative direction of the gradient. As the algorithm approaches the optimal point, the perturbations get smaller. In other words, iteratively, the parameter of the system is updated. And using the new system parameter, the performance of the system is evaluated.

Robbins and Monro (1951) propose a stochastic approximation method for finding the optimal value of a parameter in a stochastic system. Via consecutive generations of the parameter value systematically, they approach an optimal point. They search for the x value where E[L(x)] takes the value of α. In other words they search for the x which satisfies the following condition; E[L(x)]= α where α is a fixed scalar. Robbins an Monro declare that, with any stochastic approximation method, a set of x values, x_1, x_2, …, x_r are chosen arbitrarily or systematically and E[L(x)] and $E'[L(x)]$ values at each x value are generated. They declare that, the speed of convergence and the computational efficiency determine the performance of the method. If $E[L(\varphi)] = \alpha$ and $\lim_{n\to\infty} x_n = \varphi$, then it can be claimed that the method works for the particular α. Robbins and Monro define a general procedure for generating successive x values. They generate the values according to the following formulation:

$$x_{n+1} - x_n = b_n\,(\alpha - L(x_n)), \tag{3.12}$$

where, b_n is a step size defined according to a certain rule. Glasserman (1991) declares that, if the gradient estimator, g_n, is unbiased, the following formulation is equivalant to 3.12;

$$\boldsymbol{\theta}_{n+1} = \boldsymbol{\theta}_n + b_n g_n \tag{3.13}$$

The formulation summarizes the optimization procedure via PA. The parameter is moved in the direction of the gradient with a certain step-size. As the parameter gets closer to the optimal value, the perturbations get smaller.

3.4 Structural Conditions of Perturbation Analysis

Two theoretical issues are important to guarantee the accuracy of our method which are; unbiasedness of the estimator and the guarantee of convergence.

3.4.1 Unbiasedness

The gradient estimator is unbiased if the expectation differentiation interchange can be done such as:

$$E[\nabla_{\boldsymbol{\theta}} L(\boldsymbol{\theta})] = \nabla_{\boldsymbol{\theta}} E[L(\boldsymbol{\theta})] \tag{3.14}$$

The conditions for unbiasedness are stated in Theorem 1.2 of Glasserman (1991). A system is defined as a family of vectors dependent on $\boldsymbol{\theta}$, $\{\mathbf{X}(\boldsymbol{\theta}) = [X_1(\boldsymbol{\theta}), X_2(\boldsymbol{\theta}), \ldots, X_n(\boldsymbol{\theta}), \boldsymbol{\theta} \in \Theta\}$. $f(\mathbf{X}(\boldsymbol{\theta}))$ is defined as the performance measure of the system. Therefore we are to estimate the derivative of $E[f(\mathbf{X}(\boldsymbol{\theta}))]$. Glasserman mentions two conditions to guarantee the unbiasedness of such a system;

Condition 1: Every element of the performance vector, $\{X_i,\ i = 1,\ldots,\ n\}$ is differentiable at $\boldsymbol{\theta}$ for all values of $\boldsymbol{\theta} \in \Theta$.

Condition 2: $f(\mathbf{X}(\ .\ ))$ is a.s. continuous and piecewise differentiable along Θ. $f(\mathbf{X}(.))$ is a.s. differentiable along Θ if $\lim_{h\to 0}\frac{f(\mathbf{X}(\theta+h))\text{-}f(\mathbf{X}(\theta))}{h}$ exists with probability one (Glasserman 1994).

Condition 2 provides a stronger statement regarding the continuous differentiability.

Theorem 1.2 (of Glasserman): If Condition 1 and Condition 2 holds along Θ, and $E[\sup|f'(\mathbf{X}(\boldsymbol{\theta}))|] < \infty$ in the subset of Θ where $\mathbf{X}(\boldsymbol{\theta})$ is continuously differentiable, then $f'(\mathbf{X}(\boldsymbol{\theta}))$ is an unbiased estimator of $E'[f(\mathbf{X}(\boldsymbol{\theta}))]$ so that, $E[f'(\mathbf{X}(\boldsymbol{\theta}))]=E'[f(\mathbf{X}(\boldsymbol{\theta}))]$.

Proof:

$\left|\frac{f(\mathbf{X}(\boldsymbol{\theta}+h)-f(\mathbf{X}(\boldsymbol{\theta}))}{h}\right| \le \sup\left|f'(\mathbf{X}(\boldsymbol{\theta})\right|$ is valid for the subset of Θ where $\mathbf{X}(\boldsymbol{\theta})$ is continuously differentiable considering the generalized mean value theorem. The right hand side is continuously differentiable. From the dominated convergence theorem;

$$E[f'(\mathbf{X}(\boldsymbol{\theta}))] = E\left[\lim_{h\to\infty}\frac{f(\mathbf{X}(\boldsymbol{\theta}+h)-f(\mathbf{X}(\boldsymbol{\theta}))}{h}\right] \tag{3.15}$$

$$= E'[f(\mathbf{X}(\boldsymbol{\theta}))].$$

Talluri and van Ryzin (1999), Karaesmen and van Ryzin (2004), Mahajan and van Ryzin (2000) and Ozdemir et. al. (2005) use gradient-based algorithms and provide additional analysis on the structural properties concerning gradient estimation. Talluri and van Ryzin introduce a method for computing the network bid prices. Karaesmen and van Ryzin consider an overbooking problem for substitutable inventory classes and Mahajan and van Ryzin consider a single period inventory model. Ozdemir et. al. solve a multi-location transshipment problem.

3.4.2 Convergence

The convergence of a stochastic approximation method is related to the step-size selection. Robbins and Monroe mentions the following two conditions regarding the step-size;

$$\sum_{n=1}^{\infty} b_n = \infty \qquad \sum_{n=1}^{\infty} b^2{}_n < +\infty \tag{3.16}$$

Glasserman (1991) clarifies the conditions. The first condition provides faster convergence and the second guarantees the convergence. Various step size selection rules can be defined satisfying 3.16. The most common step size selection rule is 1/z, where z represent the iteration.

Chapter 4

MODEL FORMULATION

The capacity of a department is defined as the number of operators and flexibility of a department is defined as the skill set of the operators in that department. Departments can be considered as pools as defined by Harrison and Zeevi (2005). The capacity of an operator is the number of requests that he/she can respond to in a period. For simplicity, we assume that each operator has the same capacity. Therefore, a department's capacity is proportional to the number of operators working in that department.

Each department is specialized in one demand type, which will be referred to as the main skill of that department, and the servers working in that department. Servers or departments do not always have one skill, i.e. they may be flexible. For a server, flexibility means having additional skills other than the main skill, so that the server is capable of serving more than one demand type. On the other hand, a department's flexibility means that, some or all of the servers working in that department are flexible.

When a demand arrives, it is directed to any available server who can handle that type of demand. If there is no available server, demand is lost. There is no priority rule for choosing a server or customer due to the reasons explained in Section 2.2.

Capacity design under uncertain demand is formulated as a two-stage stochastic optimization problem. The capacities of the resources are determined prior to the realization of the demand in the first stage and the demand is allocated to the resources in the second stage. We formulate the capacity optimization problem as a newsvendor problem. In this setting, it is assumed that all the demand is realized at the beginning of the period and the demand that cannot be processed immediately due to the capacity restriction is lost.

Consider a service or a manufacturing system with multiple parallel resources indexed by j = 1, …, n and the job types indexed by i = 1, …, m. The set of jobs that a resource can process will be referred as the skill-set of that resource. The skill-sets of the resources are different from each other and together, they form the flexibility structure of the system which is represented by the matrix K, where $k_{ij} = 1$ denotes that resource j has skill i and therefore demand i can be processed at resource j. The amount of capacity available in resource j is denoted by c_j and the amount of job i processed by resource j is denoted by x_{ij}. Demand is random and $\mathbf{d} = [d_1, \ldots, d_m]$ denotes the demand vector where demand for job i, d_i, has a probability density function $g_i(d_i)$. $\mathbf{c} = [c_1, \ldots, c_n]$ denotes the capacity vector where c_j is the amount of capacity available in resource j. The amount of job i processed by resource j is denoted by x_{ij}.

Each job i has an associated revenue p_i per unit and each specialized resource has an associated cost s_j per unit capacity. Similar to Chevalier at al. (2004), we assume that flexibility increases the cost of capacity in an amount proportional to the additional skills of the corresponding resource. Thereby, unit cost of a flexible resource is denoted by the expression $s_j + f_j\left(\sum_i k_{ij} - 1\right)$ where f_j denotes the cost of flexibility for each additional skill.

The problem is formulated as the following:

$$\text{Stage 1}: \max \Omega(\mathbf{c}) = \max_{\mathbf{c}} E_{\mathbf{d}}\left[\Phi(\mathbf{c},\mathbf{d}) - \sum_j c_j s_j - \sum_j c_j\left(\sum_i k_{ij} - 1\right) f_j\right] \quad (4.1)$$

$$\text{Stage 2}: \Phi(\mathbf{c},\mathbf{d}) = \max_{x} \sum_{i} \sum_{j} x_{ij} p_i \tag{4.2}$$

st

$$\sum_{i} x_{ij} \le c_j \quad \forall j \tag{4.3}$$

$$\sum_{j} x_{ij} \le d_i \quad \forall i \tag{4.4}$$

$$x_{ij} \le M k_{ij} \quad \forall i, j \tag{4.5}$$

x_{ij} is a decision variable in both stages while c_j becomes a parameter in the second stage. k_{ij} is given as a parameter. The capacity should be set at the beginning of the period so that the expected profit of the system, Ω, is maximized. The first term of 4.1 represents the revenue for a demand realization and given capacity. The second and the third terms represent the total cost of the capacity.

Since the capacity cost which is represented by the second and third terms of 4.1 is constant for any capacity value, the expectation only affects the first term of 4.1. Hence, the first stage problem can be re-formulated as the following:

$$\max \Omega(\mathbf{c}) = \max_{\mathbf{c}} \left[E_{\mathbf{d}}\left[\Phi(\mathbf{c},\mathbf{d})\right] - \sum_{j} c_j s_j - \sum_{j} c_j \left(\sum_{i} k_{ij} - 1 \right) f_j \right] \tag{4.6}$$

The second stage maximizes the revenue of the system for any demand realization and given capacity level. M denotes a big number. 4.3 guarantees that the number of jobs handled by any resource is not more than its capacity. 4.4 prevents the number of processed jobs from being more than the demand. 4.5 ensures that the jobs are assigned to the capable resources. The second stage becomes a maximum flow problem if the prices of the demand types are identical. The first stage is more complicated due to the uncertainty of the demand.

4.1 A different Interpretation in Relation to Harrison and Zeevi (2005)

Harrison and Zeevi (2005) solve a call-center staffing problem. Similar to our approach, they model the system as a two-stage newsvendor problem. Comparing the results of the newsvendor method to the results of simulation, they show the appropriateness of the discrete time approach for their problem. In this section, we will explain their problem and compare it to our problem. Consequently, we will show the appropriateness of using the discrete time-approach for our case.

Harrison and Zeevi (2005) consider a call center with multiple call types and multiple agent pools. Each call-type has its own queue with infinite capacity. The customers abandon after waiting a finite time in the queue. The routing decision is made dynamically according to the queue lengths and number of available servers. In this setting, Harrison and Zeevi optimize the pool capacities with an objective of minimizing the cost of the system which constitutes the capacity cost and the abandonment penalties. They convert the problem to a multi-dimensional newsvendor problem and solve it via linear programming and Monte-Carlo simulation.

Their model is formulated as follows: there are m customer classes indexed by $i=1,\ldots,m$; r server pools indexed by $k=1,\ldots,r$ and n activities indexed by $j=1, \ldots, n$. μ_j represents the mean service rate of activity j. i(j) represents the customer class and k(j) represents the pool involved in activity j. In order to clarify the notation, let the service that customer type 1 gets from server 2 be activity 3. Then, i(3) represents customer type 1 and j(3) represents server 2. R and A are defined as the matrices showing the mean service rates and skill sets respectively. If $i(j) = i$, $R_{ij} = \mu_j$, otherwise $R_{ij} = 0$. Similarly, if $k(j)=k$, $A_{kj}=1$, otherwise $A_{kj}=0$. $\mathbf{b} = (b_1, b_2, \ldots, b_r)$ represents the capacity vector where b_k stands for the number of agents in agent pool k. b_k is a continuous variable. Each abandoning customer of type i incurs an abandonment cost of p_i and each agent in pool k costs c_k. t denotes the time instant.

Harrison and Zeevi assume that the staffing decision is made at the beginning of each period, t = 0, which should be taken very short in order to enable frequent changes in the capacity in response to the changes in average arrival rates. T is the end of the time period. In the experiments they take a planning period as one working day of 480 minutes. The average demand arrival rates are time-dependent and mean arrival rate vector is represented by $\mathbf{\Lambda} = (\mathbf{\Lambda}(t): 0 \leq t \leq T)$. At any time instant t, the demand arrival rate vector is defined as $\mathbf{\Lambda}(t) = \boldsymbol{\lambda} = (\lambda_1, \lambda_2, \ldots, \lambda_m)$ where λ_i denotes the average arrival rate of customer type i. Harrison and Zeevi solve the problem via fluid approximation, which is used for large and very busy call centers. They ignore the queuing stochasticity. In other words, even though they include the abandonment in the system definition, they formulate the LP as a loss system. The fluid approximation is such that, the abandoning customers are equivalent to the lost customers in a loss system. Hence for the customers who cannot be served immediately, an abandonment cost is charged. Hence, the abandonment cost at any time instance is defined as $\pi = \mathbf{p}(\boldsymbol{\lambda}\text{-}\mathrm{R}\mathbf{x})$ and the expected abandonment cost along a planning period is estimated as $E\left\{\int_0^T \pi(\mathbf{\Lambda}(t), \mathbf{b})dt\right\}$ according to the fluid approximation. Note that, Harrison and Zeevi show the equivalence of the fluid approximation method to the queuing system.

The problem has two stages and is formulated as the following;

$$\text{Stage I: Min } \psi = cb + E\left\{\int_0^T \boldsymbol{\pi}^*(\mathbf{\Lambda}(t), \mathbf{b})dt\right\} \tag{4.7}$$

$$\text{Stage II: Min } \pi = \mathbf{p}(\boldsymbol{\lambda}\text{-R}\mathbf{x}) \tag{4.8}$$

s.t.

$$\text{R}\mathbf{x} \leq \boldsymbol{\lambda} \tag{4.9}$$

$$\text{A}\mathbf{x} \leq \mathbf{b} \tag{4.10}$$

$$\mathbf{x} \geq 0, \tag{4.11}$$

where, x_j denotes the number of servers dedicated to activity j.

First stage determines the optimal pool size. The first term of 4.7 represents the fixed capacity cost, and the second, as explained before, estimates the total abandonment cost. Second stage allocates the demand to the agent pools. The model has two constraints; 4.9 is the demand constraint and 4.10 is the capacity constraint.

The demand arrival rate changes along the period. For each day, a demand arrival pattern with respect to time is defined. The corresponding cumulative demand distribution function at any demand level, $\boldsymbol{\lambda}$, is defined as the percentage of the time that the demand is equal to or less than $\boldsymbol{\lambda}$;

$$\text{F}(\boldsymbol{\lambda}) := \frac{1}{\text{T}} \int_0^{\text{T}} \text{P}\{\boldsymbol{\Lambda}(\text{t}) \leq \boldsymbol{\lambda}\} \text{dt} \quad \text{for } \boldsymbol{\lambda} \in \text{R}_+^m , \tag{4.12}$$

Considering this demand arrival rate, 4.7 can be reformulated as:

$$\text{Min } \mathbf{cb} + \text{T} \int_{\text{R}_+^m} \boldsymbol{\pi}^*(\boldsymbol{\lambda}, \mathbf{b}) \text{dF}(\boldsymbol{\lambda}) \tag{4.13}$$

In our setting, we do not consider a demand pattern, however assume that all of the demand arrivals during one period occur at the beginning of that period and they come from a common probability space. F(λ) corresponds to the cumulative distribution function of the demand in our setting. Harrison and Zeevi state that, since 4.13 is a convex function on R_{+}^{m}, the gradient-descent method can be used in its solution via estimating its gradient.

The problem introduced above is similar to our capacity optimization problem. Its objective function minimizes the cost while our problem's objective maximizes the profit. The cost of the system is defined as the summation of the abandonment and the capacity costs and formalized as $\psi = cb + \mathbf{p}(\boldsymbol{\lambda}\text{-}R\mathbf{x})$ for any capacity level. The profit in our model is formalized as $\Omega = \sum_i \sum_j x_{ij} p_i - \sum_j c_j s_j - \sum_j c_j \left(\sum_i k_{ij} - 1 \right) f_j$. The second and third terms of Ω is the fixed capacity cost. The second term of Ω corresponds to the first term of ψ. The third term of Ω is the additional flexibility cost. The randomness is caused by the first term of Ω and the second term ψ. These terms constitute the objective function of the second stage problem in both settings. Therefore we are interested in the gradient of the expectation of these terms. The demand and capacity constraints exist in both models. The decision variable is **c** in our model and **b** in their model.

Harrison and Zeevi show that their method gives results similar to the simulation of the original queuing system with abandonment by comparing the optimal pool sizes at different settings and by comparing the paths showing the total average daily costs at different staffing levels. Since the total cost by the fluid approximation stays in the 95% confidence interval of the simulation results, Harrison and Zeevi conclude that fluid approximation provides a good estimator of the system performance.

4.2 An Extension

So far, it is assumed that if a resource is flexible, all of its capacity is flexible. However, it is possible to introduce partial flexibility to the system, which means that, only a proportion of the resource capacity is flexible and the rest is dedicated. With small modifications in our model, it is possible to introduce partial flexibility to the system. Let $\boldsymbol{\beta} = (\beta_1, \beta_2, \ldots, \beta_n)$ vector be a decision variable where β_j represents the percentage of resource j's capacity that is flexible. The third term of stage 1's objective function is multiplied with β_j and it becomes $\sum_j c_j \beta_j \left(\sum_i k_{ij} - 1 \right) f_j$. In the old version, if a resource is flexible, the additional flexibility cost was charged to all of the resource capacity. However, considering $\boldsymbol{\beta}$, the additional cost of flexibility is charged to only the β_i proportion of it. For the partial flexibility, a new constraint is added to the second stage problem which prevents using more than the β proportion of the capacity. Consequently the model can be reformulated as the following.

$$\text{Stage 1: } \max \Omega(c) = \max_{c,\beta} E_d \left[\Phi(c,d) - \sum_j c_j s_j - \sum_j c_j \beta_j \left(\sum_i k_{ij} - 1 \right) f_j \right] \quad (4.14)$$

$$\text{Stage 2: } \Phi(\mathbf{c},\mathbf{d}) = \max_x \sum_i \sum_j x_{ij} p_i \quad (4.15)$$

st

$$\sum_i x_{ij} \le c_j \quad \forall j \quad (4.16)$$

$$\sum_{i, i \ne j} x_{ij} \le \beta_j c_j \quad \forall j \quad (4.17)$$

$$\sum_j x_{ij} \le d_i \quad \forall i \quad (4.18)$$

$$x_{ij} \le M k_{ij} \quad \forall i, j \quad (4.19)$$

The solution of the problem considering partial flexibility requires further study. Two decision variables are multiplied on the right hand side of 4.17. Therefore the gradient of the objective function with respect to c_j or β_j cannot be calculated easily. In future work, this problem can be analyzed. In our numerical analysis in Section 6.3, of the three structures being compared, only the symmetric structure's performance would be influenced by β.

Chapter 5

THE METHOD

In this Chapter, we provide the details of the solution procedure and show that our model acquires the structural properties required for the IPA method. The algorithm of our solution method is similar to the gradient descent method. Gradient descent method, starting from an initial point, searches for the local maximum by moving in the direction of the gradient. Instead of the exact gradient, our method uses an estimator. Beginning from an initial capacity level, our method generates capacity vectors iteratively. Successive capacity vectors are generated via small perturbations in the direction of the gradient estimator with a certain step size.

We are mainly concerned with the first stage problem. Our objective is to find the optimum **c** vector. Let $\mathbf{c}^z$ denote the capacity level at iteration z. At every iteration z, we solve the second stage problem S times for different realizations of the demand for $\mathbf{c}^z$. After iteration z is completed, $\mathbf{c}^z$ is perturbed in the direction of the gradient estimator with a step size b_z. The gradient estimator of the first stage problem is calculated using the shadow prices gathered from the second stage problem.

The objective function of the first stage problem is $\Omega(\mathbf{c}) = E_{\mathbf{d}}\left[\Phi(\mathbf{c},\mathbf{d})\right] - \sum_j c_j s_j - \sum_j c_j \left(\sum_i k_{ij} - 1 \right) f_j$. Since the second and third terms of Ω are constant for a given capacity level, their gradients with respect to **c** can be calculated exactly. Therefore, we only estimate the gradient of the first term, which is the expectation of the second stage problem's objective function. According to IPA, we choose $\nabla_{\mathbf{c}} \Phi$ as the estimator. $\nabla_{\mathbf{c}} \Phi$ corresponds to the shadow prices of the second stage problem associated with the capacity constraints.

Let $\mathbf{u} = (u_1,\ldots, u_n)$ denote the vector representing the shadow prices of the second stage problem. u_j denotes the partial derivative of the total throughput with respect to the capacity of the j^{th} resource, $\partial \Phi(\mathbf{c},\mathbf{d})/\partial c_j$, for any demand realization. The average shadow price of S experiments is used as the estimator of the gradient. Hence, $(1/S)\sum_{r=1}^{S} u_j(\mathbf{c},\mathbf{d}_r^z)$ represents the gradient estimator of capacity j at iteration z, where $\mathbf{d}_r^z$ stands for the vector of r^{th} demand realization at iteration z. We will show that, $\nabla_{\mathbf{c}} \Phi$ provides an unbiased estimator of $\nabla_{\mathbf{c}} E_{\mathbf{d}}[\Phi(\mathbf{c},\mathbf{d})]$ which ensures that $(1/S)\sum_{r=1}^{S} u_j(\mathbf{c},\mathbf{d}_r^z)$ converges to $\nabla_{\mathbf{c}} E_{\mathbf{d}}[\Phi(\mathbf{c},\mathbf{d})]$ by the law of large numbers. Adding the gradients of the second and third terms of (1) to the gradient estimator of the first term, we find the gradient estimator of the first stage problem. The gradient of the second and third terms of 4.6 with respect to c_j are s_j and $\left(\sum_i k_{ij} - 1\right)f_j$ respectively. Let $\tilde{\nabla}^z$ represent the gradient estimator vector of stage 1 at iteration z. Then, $\tilde{\nabla}^z = (1/S)\sum_{r=1}^{S} u_j(\mathbf{c},\mathbf{d}_r^z) + s_j + \left(\sum_i k_{ij} - 1\right)f_j$.

The $\mathbf{c}$ vector is perturbed according to the formulation; $\mathbf{c}^{z+1} = \mathbf{c}^z + b^z \tilde{\nabla}^z$. We use two step-size selection rules in combination. Until certain conditions are satisfied, we use a constant step size of 1, and then begin decreasing it according to the rule $b^z = 1/z$. Step size selection is important in terms of the convergence guarantee. We will prove that our rule guarantees eventual convergence. The procedure is repeated for the successive capacity vectors until the perturbations get small enough or a predefined number of iterations is exceeded.

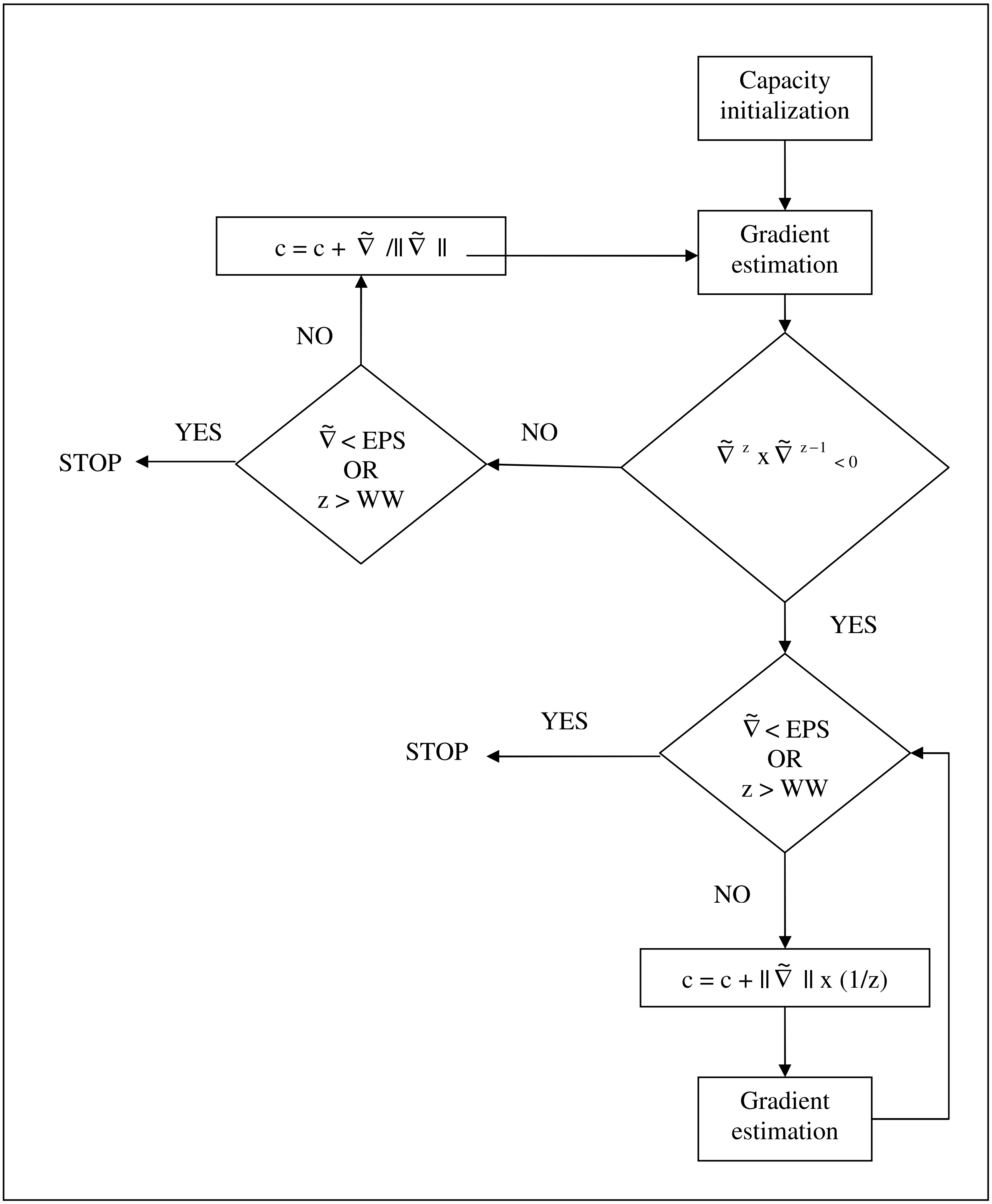

Figure 5.1: Flowchart of the solution procedure

Our stopping condition is $\|b^z \tilde{\nabla}^z\| \leq$ eps, where eps is a very small number. The procedure is summarized in Figure 5.1.

When the prices of different operations are identical, the second stage problem becomes a maximum flow problem. In that case, the problem is solved using the Ford Fulkerson algorithm in C. When the operation prices are not identical, the problem is solved with GAMS and C in coordination. The second stage problem is solved as a maximization problem in GAMS.

Using the Ford Fulkerson algorithm, the gradient is calculated via 2 simulations. In order to calculate the gradient of the objective function with respect to the capacity of resource j at any capacity level **c,** the maximum flow problem is solved at **c** and at $\mathbf{c}+0.01e_j$ where e represents the unit vector. The difference of the objective function divided by 0.01 gives the gradient at every differentiable point and a subgradient at the rest. Using the Ford Fulkerson algorithm, our method looses the advantage of less computational effort over the finite difference method since, instead of one simulation the gradient is calculated in two simulations. GAMS on the other hand, generates the shadow prices of the constraints automatically as a result of one simulation and the shadow price corresponding to the ^{jth} capacity constraint gives the gradient of the objective function with respect to c_j. However, the computational time increases significantly when the problem is solved by GAMS as a maximization problem. Therefore, using the Ford Fulkerson algorithm is time-efficient even though it requires more simulations. The Ford Fulkerson algorithm that we use makes breadth-first search. Both Ford Fulkerson and Breadth-First search algorithms can be found in Ahuja et al. (1956). The algorithm of our method solving the second stage problem as a maximum flow problem using the Ford Fulkerson algorithm is provided below:

The gradient calculation algorithm when the second stage problem is solved as a maximum flow problem:

INPUT

C ; // capacity vector.

OUTPUT

$\nabla_c \Phi(C)$ //gradient.

Line 1: Solve stage II problem by Ford Fulkerson;
Line 2: Maximum throughput = $\Phi(C)$;
Line 3: For(j=1; j≤n; j++)
Line 4: {
Line 5: $C' = C+0.01e_{j;}$
Line 6: Solve stage II problem at C' by Ford Fulkerson;
Line 7: Maximum throughput_2= $\Phi(C')$;
Line 8: $\nabla_{c_j} \Phi(C) = \dfrac{\Phi(C') - \Phi(C)}{0.01}$
Line 9: }

The gradient calculation algorithm when the second stage problem is solved as a maximization problem by GAMS:

INPUT

C ; // capacity vector.

OUTPUT

$\nabla_c \Phi(C)$ //gradient.

Line 1: Solve stage II problem by GAMS;
Line 2: Maximum throughput = $\Phi(C)$;

Line 3: $\nabla_{c_j}\Phi(C)$ = shadow price associated with the j[th] capacity constraint.

The algorithm of capacity optimization via IPA method:

INPUT

EPS: //tolerance level

S: 1000; //number of repetitions in one iteration

N : 1,000,000; //maximum number of iterations

C_0 ; // initial capacity vector.

OUTPUT

C* //optimal capacity vector.

Line 1: For(r=1; r $\leq$ S; r++)
Line 2:   Total_gradient += $\nabla_c\Phi(C_0)$;
Line 3: $\tilde{\nabla}_c\Phi(C_0)$ = Total_gradient/S;
Line 4: G(C)=$\|\tilde{\nabla}_c\Phi(C)\|$;
Line 5: if (G(C_0) $\leq$ EPS);
Line 6: {
Line 7:   Print(converged successfully);
Line 8:   STOP!
Line 9: }
Line 10: else
Line 11: {
    int z=1;
Line 12:   while(z$\leq$N)
Line 13:   {
        C_z=C_{z-1}+$\tilde{\nabla}_c\Phi(C_{z-1})$/G($C_{z-1}$);

Line 14: Calculate $\tilde{\nabla}_c \Phi(C_z)$;
Line 15: If($\|G(C_z)\|$<EPS)
Line 16: {
Line 17: print(converged successfully);
Line 18: STOP;
Line 19: }
Line 20: else
Line 21: {
Line 22: if($\nabla_{c_j} \Phi(C_z) \,.\, \nabla_{c_j} \Phi(C_{z+1}) < 0$ for any j),
Line 23: {
Line 24: t=1;
Line 25: while(t+z≤N)
Line 26: {
Line 27: step_size=1/t;
Line 28: C_{t+z}=C_{t+z-1}+step_size $\dfrac{\tilde{\nabla}_c \Phi(C_{t+z})}{G(C_{t+z})}$;
Line 29: if($\|$step_size. $\tilde{\nabla}_c \Phi(C_{t+z}) \| \leq$EPS)
Line 30: {
Line 31: STOP;
Line 32: converged successfully
Line 33: }
Line 34: else t=t+1;
Line 35: };
Line 36: Print Max. number of iterations exceeded.
Line 37: STOP.
Line 38: };
Line 39: else z=z+1;
Line 40: };
Line 41: };
Line 42: Stop.
Line 43: Max. number of iterations exceeded.
Line 44: }

5.1 Structural Properties

Glasserman (1991) draws our attention to the importance of two theoretical issues concerning the validity of the IPA method, which are unbiasedness and convergence. In our setting, if the estimator is unbiased, the following must hold;

$$E\left[\frac{1}{S}\sum_{r=1}^{S} u_j(\mathbf{c},\mathbf{d}_\mathbf{r}^\mathbf{z})\right] = \nabla_c E_d\left[\Phi(\mathbf{c},\mathbf{d})\right] \qquad (5.1)$$

Theorem 4.1 shows the unbiasedness of our estimator.

Theorem 4.1 $\frac{1}{S}\sum_{r=1}^{S} u_j(\mathbf{c},\mathbf{d}_\mathbf{r}^\mathbf{z})$ *is an unbiased estimator of* $\nabla_\mathbf{c} E_\mathbf{d}\left[\Phi(\mathbf{c},\mathbf{d})\right]$.

Proof. Given that X(θ) is a.s. differentiable at θ, $\nabla_\theta X(\theta)$ is an unbiased estimator of $\nabla_\theta E[X(\theta)]$ if X(θ) satisfies the Lipschitz condition (Glasserman [10]). X(θ) satisfies the Lipschitz condition, if a positive L exists such that:

$$|X(\theta + h) - X(\theta)| \le Lh, \qquad (5.2)$$

Since the objective function of a maximization problem is a piecewise linear concave function of the right hand side (Bertsimas and Tsitsiklis (1997)), $\Phi(\mathbf{c},\mathbf{d})$ is piecewise linear and concave with respect to **c**. This also justifies our use of the gradient descent algorithm. A general representation of $\Phi(\mathbf{c},\mathbf{d})$ versus c_j for different demand realizations is given in Figure 5.2. At the breakpoints, $\Phi(\mathbf{c},\mathbf{d})$ fails to be differentiable. But since the demand is continuous and the number of non-differentiable points is countable, $\Phi(\mathbf{c},\mathbf{d})$ is almost surely differentiable at c. At the non-differentiable points, the subgradient is not unique. But the procedure (when we do not use GAMS) picks a particular subgradient, therefore guarantees that the same subgradient is used at any capacity level and a certain demand arrival rate.

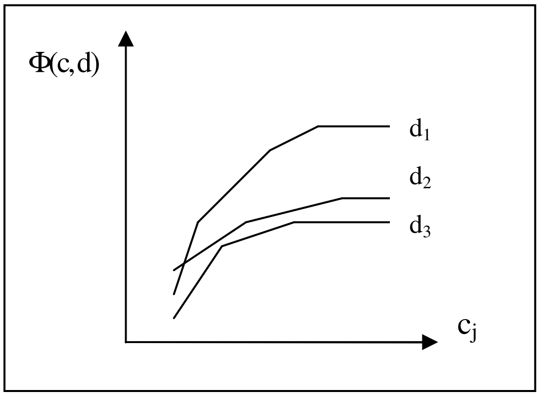

Figure 5.2: Throughput versus capacity for different demand realizations

At any capacity level, the effect of a small increase in the capacity cannot be less than the negative of the capacity cost incurred by the change in the amount of h and cannot be more than the profit gained by the same amount of increase in the throughput of the most expensive job in the skill-set of the corresponding resource. Let $\mathbf{h} = [h_1, \ldots, h_n]$ be the vector where h_j denotes the amount of increase in the capacity of resource j, $\hat{p}$ be the highest contribution margin among all the jobs and f_j+t_j be the unit capacity cost of resource j. Then the following holds for any resource j;

$$| \Phi(\mathbf{c} + h_j e_j, \mathbf{d}) - \Phi(\mathbf{c}, \mathbf{d})| \le \min(\hat{p} h_j, h_j(f_j+t_j)) \tag{5.3}$$

Therefore; $\Phi(\mathbf{c}; \mathbf{d})$ satisfies the Lipschitz condition for every j and since $\Phi(\mathbf{c}; \mathbf{d})$ is a.s. differentiable with respect to all c_j 's, $E[\frac{1}{S}\sum_{r=1}^{S} u_j(\mathbf{c}, \mathbf{d}_r^z)] = \nabla_c E_{\mathbf{d}}[\Phi(\mathbf{c}, \mathbf{d})]$.

using the following argument.

The second important theoretical issue is the convergence. Glasserman (1991) states that convergence is related to the step-size selection rule. Considering the possibility that the initial capacity is far from the optimal, we start with a big step size to accelerate the convergence. Until any of the gradients change sign at iteration v, which means that one of the capacity values passes over the optimal, a fixed step-size of 1 is used. When the sign of the gradient changes, we understand that the algorithm gets close to the optimal point. After that point, we begin decreasing the step-size according to the rule b_z=1/(z-v). Our step-size selection rule satisfies the conditions established by Robins and Monro (1951);

$$v + \sum_{z=v+1}^{\infty} (1/(z-v)) = \infty \quad , \qquad v + \sum_{z=v+1}^{\infty} (1/(z-v))^2 < +\infty \,. \tag{5.4}$$

Therefore we can claim that our method eventually converges.

Chapter 6

NUMERICAL EXPERIMENTS

In this Chapter, we present the results of some numerical experiments. The experiments are designed to test the accuracy of the method and to observe the effects of some problem parameters on the performance of the method. With this aim, we conducted a set of experiments and compared the results of our method with the optimal newsvendor problem results. Furthermore, we solved two benchmark problems which are defined by Netessine et. al. (2002). Then we validate the discrete time approach by applying our method to the problems defined by Harrison and Zeevi (2005). Finally, we used the method to compare the performances of three flexibility structures that are widely used in call-center applications. Under similar conditions, we tested their respective advantages and disadvantages. In all experiments we take S as 1,000. We tested the performance of the procedure for 1, 10 and 10,000 replications and saw that S = 1,000 is the best choice considering the accuracy of the results and the computational time. Moreover we used single seed in our model. The same demand set is used at each iteration since Glasserman (1991) suggests the advantage of using a single seed.

6.1 Model Validation

First, we focused on the two flexibility structures that are demonstrated in Figure 6.1 as bipartite graphs with nodes standing for the demand types and the resources; and arcs standing for the skills. The first structure represents a fully-specialized system where each resource has only one skill and the second structure represents a full-flexible system where each resource has all the skills. We compared the capacity assignments by our method to the capacity assignments by the newsvendor method. In the classical newsvendor problem, the optimal capacity of a resource is given by the formula $G_D^{-1}[(p-uc)/p]$, where G_D is the cumulative demand distribution, *uc* is the unit cost and *p* is the unit price. For finding the optimal newsvendor solution of the fully-specialized structure, each resource-demand pair is treated independently. In the full-flexible case, the resource capacities are aggregated and the whole system is treated like a single resource-demand pair.

For simplicity, the unit specialized capacity cost, of the resources are assumed to be identical. Also, the cost of unit flexible capacity with equal number of skills is assumed to be the same for each resource. The unit cost of the specialized capacity is *s* and the unit cost of the flexible capacity is s+f(m-1), where *m* is the number of the demand-types.

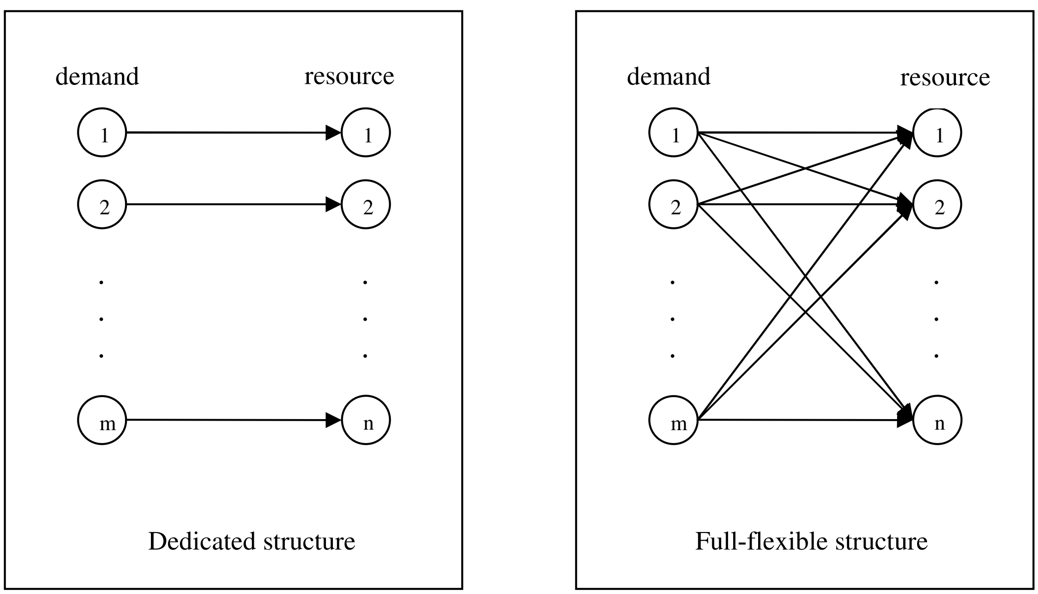

Figure 6.1: Dedicated and full-flexible structures

For combinations of the system parameters given in Table 6.1, we solved the capacity assignment problem for 4 cases; full-flexible with 2 resources, fully-specialized with 2 resources, full-flexible with 3 resources, fully-specialized with 3 resources. We considered the number of demand types to be equal to the number of resources in each case. The demand scenarios are created using a truncated normal distribution, $N(\mu,\sigma)$, to ensure positive demand values.

price (p)	Specialized capacity cost (s)	Additional skill cost (f)	Distribution normal(μ,σ)	Initial capacity (c_0)
50	15	5	(50,5)	0
40	10	2	(50,10)	50
30	5			100

Table 6.1: System parameters

The full-flexible structures are evaluated at a total of 3×3×2=18 cost scenarios which are formed by the combinations of p, s and f values shown in Table 6.1 , and the others are evaluated at 3×3=9 scenarios which are formed by the combination of p and s values for 3 initial capacity values which are 0, μ and 2μ. Appendix-A gives the resulting tables for the 4 cases mentioned above. The tables for full-flexible cases demonstrate the total capacity assignments while the dedicated cases demonstrate the individual capacity assignments. The percentage error values are calculated using the formula; 100×[(capacity by GPA)-(capacity by newsvendor)]/(capacity by newsvendor). The minimum, maximum and average errors are given below the tables. These values are also summarized in Table 6.2.

	3-3 flexible			**3-3 specialized**			**2-2 flexible**			**2-2 specialized**		
c₀	0	50	100	0	50	100	0	50	100	0	50	100
Min.	0.09	0	0.01	0.01	0	0.01	0.01	0	0.04	0.22	0	0.12
max.	0.7	3.48	0.7	2	1.71	2.27	1.4	1.97	1.53	1.76	1.62	1.83
Avg.	0.34	0.42	0.34	0.78	0.76	0.85	0.44	0.48	0.55	0.88	0.77	0.84

Table 6.2: Summary of percentage errors

The percentage errors are mostly less than 2%. It is observed that, the initial capacity value affects the convergence. However, a general pattern cannot be defined regarding the effect of an increase or decrease in the initial capacity level from the results. The small percentage errors show the robustness of our method. Under different demand scenarios and system parameters, our method converges to the capacity values that are close to the optimal.

Second, as a benchmark, the method is implemented to the structures determined by Netessine et. al. (2002), with slight modifications on the objective function of our model. They consider the optimal investment problem in a special flexibility structure. Their structure for two and three resources are represented in Figure 6.2. Different than our model, they define two types of capacity costs; F_j is the unit cost of capacity and V_j is the variable cost associated with the amount of capacity that is used. Also they consider a penalty cost PC_i per unit unsatisfied demand. p_i is defined as the profit per item i. They solve the problem for the structures shown in Figure-6.2 for a truncated normal demand distribution. Table 6.3 and Table 6.4 shows the parameters used by Netessine for the first and the second structures respectively.

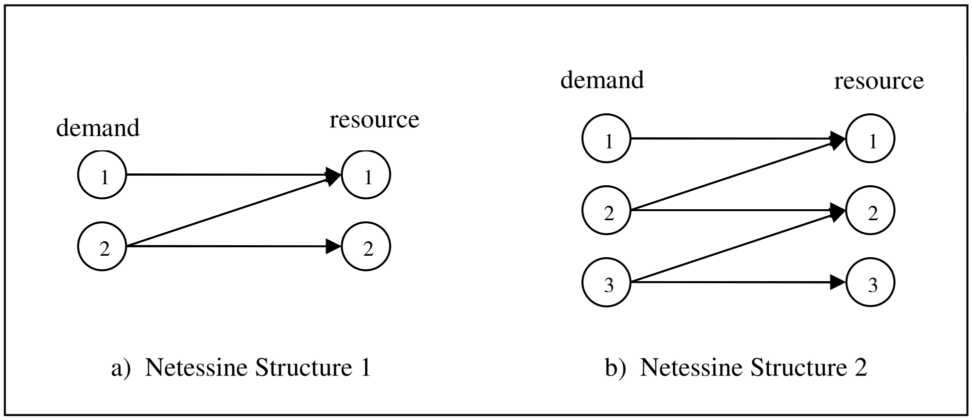

Figure 6.2: a) Netessine et al. structure 1, b) Netessine et al.structure 2

(Demand type, resource)	p_i	V_j	PC_i	F_j	(μ,σ)
(1,1)	*42*	18	12	20	(120,50)
(2,2)	*35*	10	7	18	(200,80)
(2,1)	*35*	18	7		

Table 6.3: Parameters defined by Netessine et al. for the first structure

(Demand type,resource)	p_i	V_j	PC_i	F_j	(μ,σ)
(1,1)	*70*	40	7	20	(120,50)
(2,2)	*50*	30	5	15	(165,80)
(3,3)	*35*	20	3	12	(220,100)
(2,1)	*50*	40	5		
(3,2)	*35*	30	3		

Table 6.4: Parameters defined by Netessine et al. for the second structure

A comparison of the results for the first and second structures can be seen in Table 6.5 and Table 6.6 respectively. The results show that, our algorithm converges to almost the same capacity level that was found optimal by Netessine et. al.

Server	Initial capacity	capacity by GPA	Capacity by Netessine	Percentage error
1	0	137.132095	138	0.6289167
2	0	170.790771	168	1.6611732

Table 6.5: Comparison of the results for the first problem of Netessine et al.

Server	Initial capacity	capacity by GPA	Capacity by Netessine	Percentage error
1	120	125.490883	127	1.1882811
2	165	155.770416	145	7.4278731
3	220	166.229034	165	0.74486909

Table 6.6: Comparison of the results for the second problem of Netessine et al.

We observed in the second problem that, when the demand arrival rates of different types are not equal, the initial capacity choice affects the performance of the algorithm significantly. We chose initial capacities equal to the expected demand of the corresponding demand type since various experiments with different initial capacity values revealed that, this approach gives better results faster.

6.2 Validating the Discrete Time Setting

In Section 4.1, we showed the equivalence of the discrete time setting to a fluid approximation in the queuing setting theoretically. In this section, we will provide numerical results. We solve the problems defined by Harrison and Zeevi (2005) and compare our results to the results of the simulation and the fluid approximation. In the remaining parts of this section, three numerical experiments introduced by Harrison and Zeevi will be explained using their notation and the numerical results will be provided. The notation is explained in Section 4.1. The first experiment considers a one pool one demand type system. The system parameters are defined as; c= \$240/day, p=\$2/customer, $\mu=1$ (service rate) customer/minute. When there is one pool and one call-type, the problem becomes a classical newsvendor problem. The expected cost of a classical newsvendor problem is formulated as the; $E[C(Q)]=c_u E[(D-C)^+] + c_o E[(C-D)^+]$ where Q represents the ordering level, D represents the demand realization, C represents the capacity, c_o represents the overage and c_u represents the underage costs. At the optimal Q value, the gradient of the expected cost is equal to zero. The objective of the LP is defined as the following for the first experiment:

$$\text{Min } \mathbf{cb} + \mathrm{T}\mathbf{p}\int_0^\infty (\boldsymbol{\lambda} - \mathbf{b}\boldsymbol{\mu})^+ \, dF(\boldsymbol{\lambda}) \tag{6.1}$$

The gradient of the function with respect to **b** is $\mathbf{c}\text{-P}(\boldsymbol{\lambda} \geq \mathbf{b}\boldsymbol{\mu})\mathrm{T}\mathbf{p}\mu$. Since $\text{P}(\boldsymbol{\lambda} \geq \mathbf{b}\boldsymbol{\mu})=1\text{-F}(\mathbf{b}\boldsymbol{\mu})$, the gradient of the objective function with respect to **b** becomes $\mathbf{b}\text{-}[1\text{-F}(\mathbf{b}\boldsymbol{\mu})]\mathrm{T}\mathbf{p}\boldsymbol{\mu}$, which reveals $\text{F}(b^*\mu) = 1 - \frac{c}{T p\mu}$.

The following graph represents the daily demand pattern for experiment 1. The demand comes from one of the curves with equal probability.

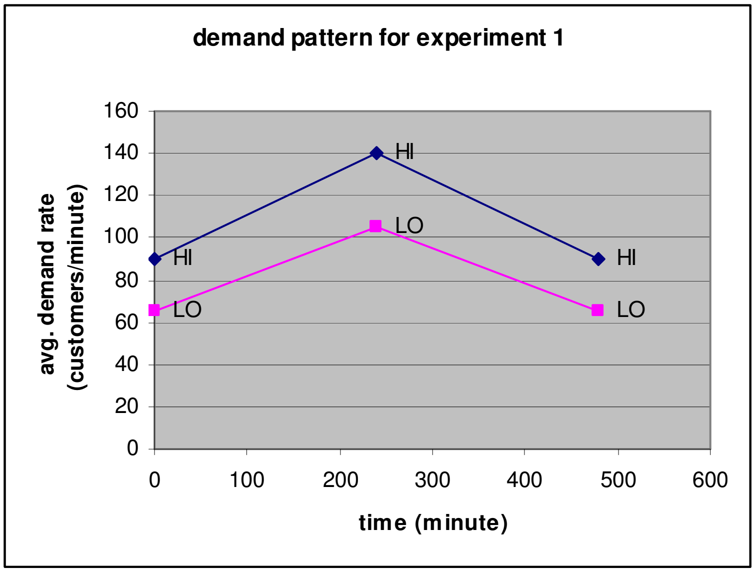

Figure 6.3: Demand pattern for Harrison and Zeevi experiment 1.

The following represents the corresponding cumulative distribution function:

$$F_1(\lambda) = \begin{cases} \frac{1}{2}\frac{(\lambda-65)}{40} & 65 \le \lambda \le 90 \quad (6.2) \\ \frac{1}{2}\left(\frac{\lambda-65}{40}+\frac{\lambda-90}{50}\right) & 90 \le \lambda \le 105 \quad (6.3) \\ \frac{1}{2}\left(1+\frac{\lambda-90}{50}\right) & 105 \le \lambda \le 140 \quad (6.4) \end{cases}$$

According to this demand distribution, $F(\mathbf{b}^*\boldsymbol{\mu})$ =0.75, which gives an optimal pool size of 115. The GPA method gives an optimal pool size of 113.8. The optimal pool size and cost comparisons of different methods for the three experiments are provided in Table 6.7.

The second numerical experiment considers one pool, two demand types and two activities. $\mu_j = 1$, abandonment penalties are p_1=1, p_2=2 and the daily personnel cost is 240. The third experiment considers two pools, two demand types and three activities. The first pool serves demand type 1 and the second serves both demand types. $\mu_j = 1$, the abandonment penalties are p_1=1, p_2=2 and the daily personnel cost is 160 and 240 for the first and second pools respectively.

The demand patterns defined for experiment 2 and 3 are provided in Figure 6.4. The corresponding cumulative distribution functions are as follows;

$$F_2(\lambda) = \begin{cases} \frac{1}{2}\frac{(\lambda - 35)}{20} & 35 \le \lambda \le 55 \quad (6.5) \\ \frac{1}{2}\left(1 + \frac{\lambda - 55}{40}\right) & 55 \le \lambda \le 95 \quad (6.6) \end{cases}$$

$$F_3(\lambda) = \begin{cases} \frac{1}{2}\frac{(\lambda - 25)}{20} & 25 \le \lambda \le 35 \quad (6.7) \\ \frac{1}{2}\left(\frac{\lambda - 25}{20} + \frac{\lambda - 35}{20}\right) & 35 \le \lambda \le 45 \quad (6.8) \\ \frac{1}{2}\left(1 + \frac{\lambda - 35}{20}\right) & 45 \le \lambda \le 55 \quad (6.9) \end{cases}$$

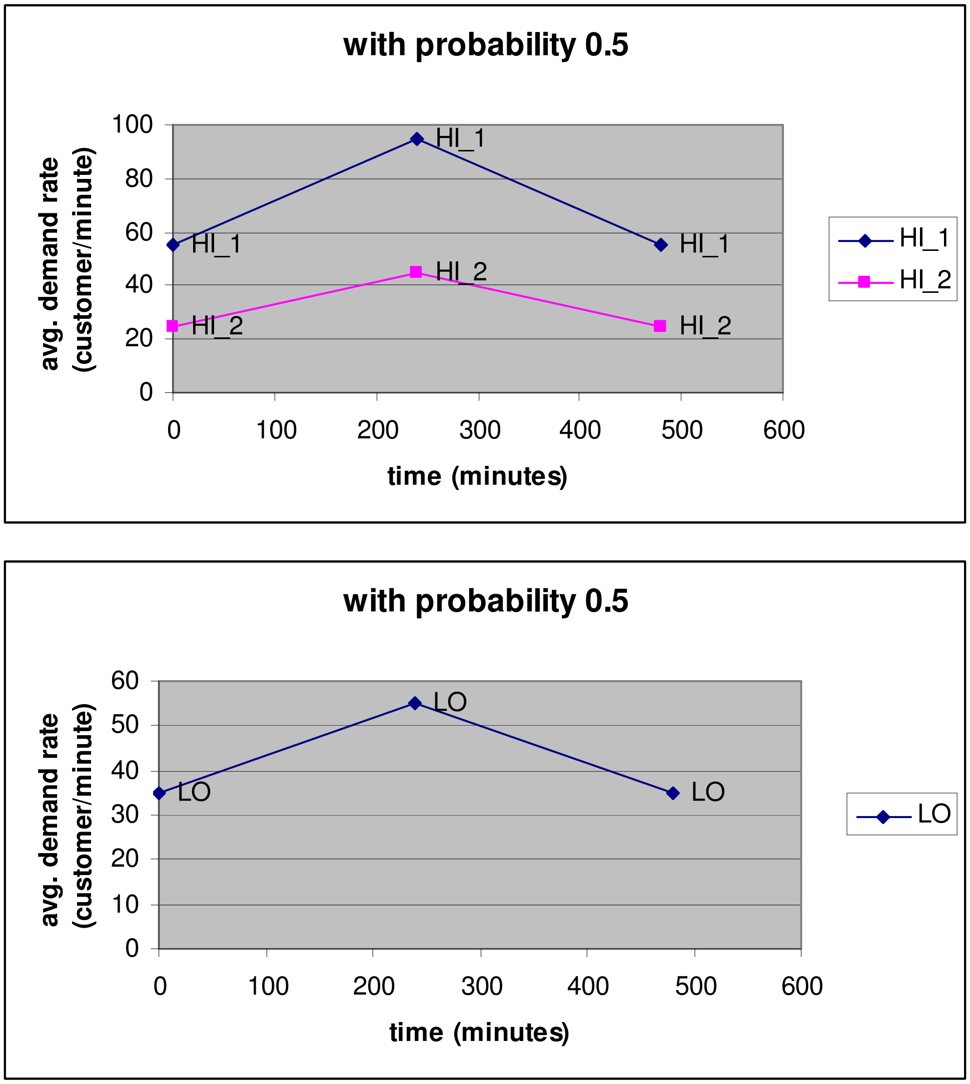

Figure 6.4: Demand pattern for Harrison and Zeevi experiments 2 and 3.

problem		Optimal pool size	personnel cost	avg. abandonment cost	average total cost	CI (total cost)
	fluid approximation	115	27,600	3,500	31,160	(29,340; 31,163)
	Simulation	116	27,840	3,220	31,060	(29,894; 31,804)
1	GPA	113.08	27,139	3,422	30,561	(29,482; 31,640)
	fluid approximation	93	22,320	5,000	27,320	(27,677; 28,775)
	Simulation	94	22,560	5,001	27,561	(26,706; 28,809)
2	GPA	94	22,560	5,197	27,757	(26,706; 28,809)
	fluid approximation	(54,52)	21,120	2,654	23,774	(22,933; 24,371)
	Simulation	(53,54)	21,440	2,499	23,939	(23,113; 24,479)
3	GPA	(52.28,49.50)	20,245	3,135	23,380	(22,554; 24,205)

Table 6.7: Comparison of the results for the problems defined by Harrison and Zeevi

The optimal pool sizes generated by the GPA method are close to the pool sizes generated by the other methods. Moreover, the results reveal that GPA provides a good estimator of the system performance since the total costs resulted from different methods are close to each other and the 90% confidence intervals of the total costs overlap. Confidence interval at any capacity level is calculated by 1000 independent replications at that capacity level. 90% confidence interval of 1000 Ω^r values is calculated as follows; $(\overline{\Omega} - 1.645\sigma/10, \overline{\Omega} + 1.645\sigma/10)$ where $\overline{\Omega}$ represents the mean and σ represents the standard deviation of Ω^r values.

6.3 Comparison of Different Flexibility Structures

We constructed some experiments to compare the performances of three flexibility structures; symmetric, overflow and nested for three and eight demand types. In this section, we will present the results of these experiments. The symmetric structure is frequently mentioned in the literature while the other two are encountered in practice in call centers. The flexibility literature suggests the superiority of the symmetric structure. After optimizing the capacity of each structure under various demand scenarios and system parameters, we compared their average profit values along with the associated throughputs in order to understand which structure outperforms the others under which circumstances.

The structures are represented in Figure 6.5 for three demand types; A, B and C. The call-center customers can be divided into groups according to their arrival rates and the type of service that they request from the system. Some standard services can be performed by any agent while some services require special training. Mostly, the demand for a standard service is higher than the demand for a sophisticated service. The three demand types in Figure 6.5, A, B and C, correspond to 3 customer profiles who request the highly sophisticated, sophisticated and standard operations. Let μ_i represent the mean arrival rate of demand type i, where i = {A, B, C}. Even though the demand arrival rate of a standard operation is higher than a sophisticated operation in most cases, in order to cover all possible cases, we analyzed three main cases regarding the mean arrival rates of A, B and C which are; $\mu_A < \mu_B < \mu_C$, $\mu_A > \mu_B > \mu_C$, $\mu_A = \mu_B = \mu_C$.

The first structure in Figure 6.5 represents the overflow structure. There are two groups of agents in this structure. The agents in department 1 are generalists who can give service to every customer and the other agents are specialists who can serve only one customer type. In this case, corresponding to each demand type, a specialized department exists. Due to the specialization, the capacities of these departments are cheap. Additionally, a full-flexible department exists to which, all of the excessive demand is allocated. However, its capacity is very expensive.

The second figure represents the nested structure in which, different levels of flexibility are available and which provides a career plan for the workers. According to the plan, an entering level worker has only one-skill. As the worker becomes more advanced in his/her job, he/she moves to the second level. In the figure, department 3 represents the entering level workers who can serve customer type C, 2 represents the middle level workers who can serve customer types B and C, and 3 represents the advanced level workers who can serve all customer types.

The third figure represents the symmetric structure. Symmetric is the most balanced structure in the sense that, every department has two skills and every demand type can be allocated to two departments. It is also possible to build a symmetric structure by shifting the arcs in the figure such that, department 1 has skills A and B, 2 has skills B and C and 3 has skills A and C. Even though the arcs are shifted, the average throughput and profit values do not change significantly. Therefore, independent from the skill sets of the departments, a symmetric system performs in a similar manner. In all our experiments, we determined the skill-sets according to the one shown in the figure.

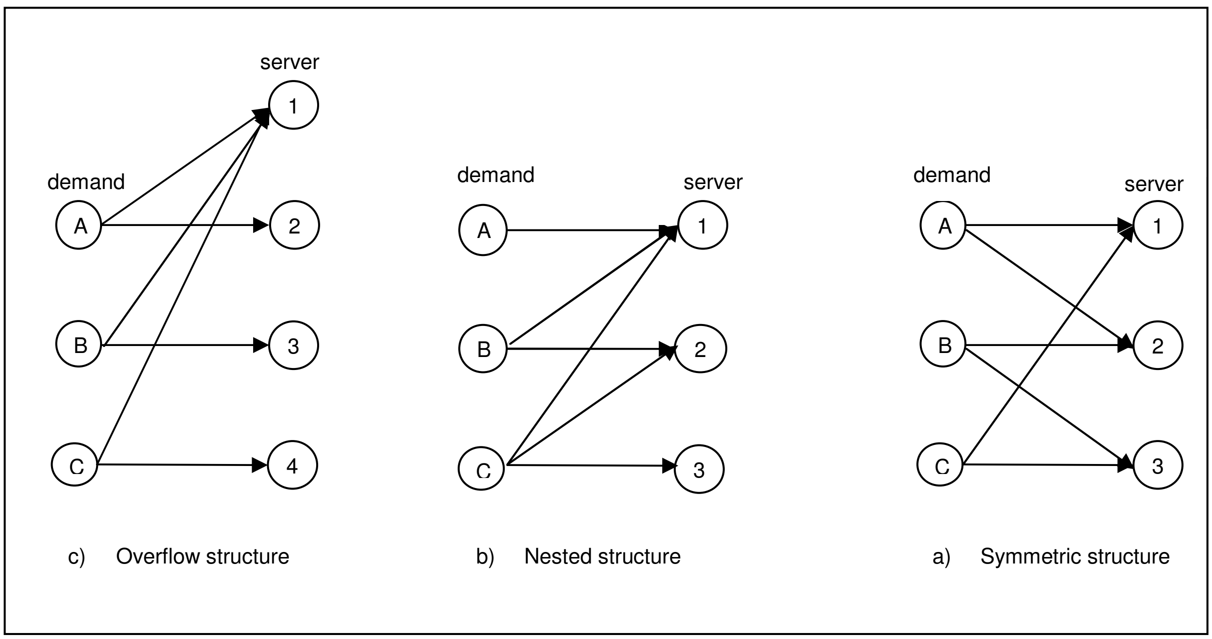

Figure 6.5: a) overflow structure, b) nested structure, c) symmetric structure

Aksin et. al. (2005) review the flexibility literature in a call-center setting. They state that, if the call-rates for different demand types are equal, balanced department capacities and balanced routing results in better performance (property 3), which implies the superiority of the symmetric structure in the case that the demand rates for A, B and C are equal. Moreover, Wallace and Whitt (2005) and Mazzuchi and Wallace (2004) show that, a call-center with servers having at most 2-skills perform almost as good as the full-flexible structure. This result suggests the superiority of the symmetric structure under any condition. The researchers mentioned above ignore the cost of capacity and flexibility in their analysis, therefore use throughput as the performance criterion. The aim of our analysis is to see whether we can explain the conflict between the literature and the real-life applications by including the effect of the cost structure in our analysis and optimizing the capacity with an objective of maximizing the profit.

6.3.1 Experiments for Three Demand Types

First, we designed experiments for three call-centers each of which serves three customer types as seen in Figure 6.5. We generated the demand using the normal distribution, N(μ, σ), considering the 18 sets of mean arrival rates ($\boldsymbol{\mu_A}$, $\boldsymbol{\mu_B}$, $\boldsymbol{\mu_C}$) that are shown in Table 6.8, 2 coefficient of variation values, $C_V = 1$ and $C_V = 2$ and 10 cost structures (p, s, f), which are shown in Table 6.9. Moreover, for 3 demand sets which are (10, 30, 90), (90, 30, 10) and (30, 30, 30), we analyzed the influence of the correlation between the demand types A and B considering that the demand for standardized operations is independent from the other demand types. We chose 4 correlation matrices besides the uncorrelated case, and defined the correlation matrices as follows;

$$\boldsymbol{\rho_1} = \begin{bmatrix} 1 & -1 & 0 \\ -1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \boldsymbol{\rho_2} = \begin{bmatrix} 1 & -0.5 & 0 \\ -0.5 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \boldsymbol{\rho_3} = \begin{bmatrix} 1 & 0.5 & 0 \\ 0.5 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \boldsymbol{\rho_4} = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

Besides $p_{12} = 1$, $p_{12} = -1$ and $p_{12} = 0$, we considered $p_{12} = -0.5$ and $p_{12} = 0.5$ in order to achieve a pattern between perfect positive and perfect negative correlation. When the correlation coefficient of demand type 1 and demand type 2 takes the value of $\dot{p}$, type 1 demand is generated using the formula $\sigma_1 Z_1 + \mu_1$, type 2 demand is generated using the formula $\sigma_2(\dot{p}Z_1 + \sqrt{(1-\dot{p}^2)}Z_2) + \mu_2$ and demand 3 is generated using the formula $\sigma_3 Z_3 + \mu_3$ where Z_1 and Z_2 represent independent standard normal random variables. Hence we constructed 18 x 2 x 10 = 360 experiments for uncorrelated demand and 3 x 2 x 4 x 10 = 240 experiments for correlated demand which makes a total 700 experiments for the system with three demand types.

$\mu_A < \mu_B < \mu_C$

Set	Demand type	μ
1	A	30
	B	40
	C	50
2	A	10
	B	30
	C	90
3	A	5
	B	15
	C	80
4	A	10
	B	40
	C	160
5	A	10
	B	50
	C	70
6	A	10
	B	50
	C	90

$\mu_A > \mu_B > \mu_C$

Set	Demand type	μ
7	A	50
	B	40
	C	30
8	A	90
	B	30
	C	10
9	A	80
	B	15
	C	5
10	A	160
	B	40
	C	10
11	A	70
	B	50
	C	10
12	A	90
	B	50
	C	10

$\mu_A = \mu_B = \mu_C$

Set	Demand type	μ
13	A	5
	B	5
	C	5
14	A	10
	B	10
	C	10
15	A	30
	B	30
	C	30
16	A	40
	B	40
	C	40
17	A	80
	B	80
	C	80
18	A	160
	B	160
	C	160

Table 6.8: Demand sets for three demand types

Structure	1	2	3	4	5	6	7	8	9	10
P	50	50	50	60	50	50	50	100	100	100
S	15	15	15	20	30	5	5	5	5	5
F	5	10	15	10	5	1	4	1	4	5

Table 6.9: cost structures

The results of the experiments for uncorrelated and correlated demand are given in Appendix-B and Appendix-C respectively. Even though the performance criterion is the average profit; besides the optimal capacity allocations and the average profit values at the optimal capacity level, we provide the associated average throughput levels. The optimal capacity allocations, the average profit values at the optimal capacity level and the associated throughput values allow us to analyze the effects of the demand arrival rates, specialized and flexible capacity costs and the demand correlation on the system performances and moreover will enable us to compare the overall system performances. The experiments are grouped in order to make the results easy to understand. The main groups, were determined according to the C_V's and the s/p (specialized capacity cost / price) ratios. (Considering the s/p ratios, the cost structures are divided into two groups. The first five structures will be referred as the high cost and the rest, as the low cost). According to this classification Appendix-B1 presents the results for low C_V -high cost (lv, hc), Appendix-B2 presents the results for low C_V -low cost (lv, lc), Appendix-B3 presents the results for high C_V -high cost (hv, hc) and Appendix-B4 presents the results for high C_V -low cost (hv, lc). Moreover, each group is divided into 3 subgroups according to the mean arrival rate rankings which are $\mu_A < \mu_B < \mu_C$, $\mu_A > \mu_B > \mu_C$, $\mu_A = \mu_B = \mu_C$.

The average profit values in Appendix-B give an idea regarding the performances of the three structures under different settings; however, we cannot assert that the performance rankings exactly match the average profit rankings. Because in some cases, the confidence intervals of different structures overlap and the average profit values may not reflect the reality. When the confidence intervals of the profit values associated with any two structures overlap, we cannot claim that any of them performs better even if the average profit of one structure is better than the other. Considering the possible inaccuracies, we compared the confidence intervals of the profit values under the optimal capacity as well. In the confidence interval analysis, we ignored the results unless the confidence intervals of the three structures were non-overlapping. The confidence intervals are calculated by 1000 independent experiments at the optimum capacity level. Table 6.10 summarizes the results of all experiments and Table 6.11 summarizes the results of the experiments in which the confidence intervals of the three structures are independent. The summary tables show in how many of the experiments, the performance (average profit at optimal capacity level) of which structure was ranked 1st, 2nd and 3rd. In Table 6.11, the overlapping cases are ignored. The tables filled with “0”s indicate that, in the corresponding set of experiments all the confidence intervals were overlapping, therefore the structures could not be ranked. Below each table, we provided the number of experiments in that group. In Table 6.11 these numbers are variable since all the experiments do not result in non-overlapping confidence intervals.

We will analyze the results according to the system performances under different demand arrival rates. For example Appendix B1_1, B2_1, B3_1, B4_1 represent the results of the experiments for the rankings; $\mu_A < \mu_B < \mu_C$. As seen in Table 6.10, nested structure performs the best and the overflow structure performs the worst in most of the experiments in this group (100 and 95 out of 120 respectively). Moreover, considering the independence of confidence intervals, the nested structure performs the best in 50 and the overflow structure performs the worst in 46 out of 55 experiments.

When $\mu_A > \mu_B > \mu_C$, the symmetric structure performs the best and the nested structure performs the worst in 95 and 120 out of 120 experiments respectively according to Table 6.10. The confidence interval analyses confirm these results. Considering the non-overlapping confidence intervals, the symmetric structure gives the best profit in 83 and the nested structure gives the worst profit in 99 out of 99 experiments as seen in Table 6.11. The detailed results for this group are given in Appendix B1_2, B2_2, B3_2, B4_2.

Appendix B1_3, B2_3, B3_3, B4_3 represents the results for $\mu_A = \mu_B = \mu_C$. Table 6.10 shows that, the overflow structure outperforms the other structures in all experiments and the symmetric structure performs the worst in 109 out 120 experiments. Table 6.11 reveals that, the confidence intervals of the three departments overlap in all experiments in this group; therefore we cannot be certain about the rankings.

The results reveal that, the performance of any structure is strongly related to the arrival rate rankings of different demand types. However, the C_V and the s/p ratios do not affect the performance as much as the arrival rates do. Analyzing the throughput levels under the optimal capacity which maximizes the profit gives more information about the performances of the structures. The summary tables for the throughput values in Appendix-B5 show that, the ranking of the structures are the same considering the throughput and the profit for $\mu_A < \mu_B < \mu_C$ and $\mu_A > \mu_B > \mu_C$. However when $\mu_A = \mu_B = \mu_C$, the throughput of the symmetric structure is better than the throughput of the overflow structure under high cost.

When $\mu_A = \mu_B = \mu_C$ the throughput of the overflow structure associated with the optimal capacity is the highest in 70 experiments and the throughput of the symmetric structure is highest in 40 experiments out of 120 when the demand arrival rates of A, B and C are equal. However, within the 5 experiments in which the confidence intervals are non-overlapping, the symmetric structure outperforms the others in terms of throughput in 3 experiments for (lv, hc) and in 2 experiments for (hv, hc). This result shows that, when the capacity is expensive, even though the throughput level of the symmetric structure is higher than the overflow structure, the profit of the overflow structure is better. The capacity of the symmetric structure is expensive with respect to the capacities of all departments in the overflow structure except one, which is the full-flexible department. Since different levels of flexibility do not exist in the symmetric structure, capacity has to be allocated to the departments which have two skills. However, as seen in Appendix-B, the algorithm allocates a little portion of the total capacity to the full-flexible department (at most 1/7) in the overflow structure. This result reveals that, when the arrival rates of different demand types are balanced, a high level of flexibility is not required. Therefore when $\mu_A = \mu_B = \mu_C$, even though introducing symmetric and balanced flexibility results in better throughput values, in terms of the profit, a high level of flexibility is unnecessary.

When $\mu_A < \mu_B < \mu_C$, the nested structure outperforms the others in terms of profit. Because, in the nested structure, the algorithm tends to minimize the capacity of department 1 and maximize the capacity of department 3 under any demand distribution due to their flexibility levels and consequently their capacity costs. Therefore $\mu_A < \mu_B < \mu_C$ is the ideal case for the nested structure. As seen in Appendix-B, the lowest level of capacity is assigned to department 1 and the highest level of capacity is assigned to 3 in this case. This structure takes advantage of the different flexibility levels. As the capacity required in each department decreases with the level of flexibility, the capacity cost is minimized while the throughput is maximized.

The symmetric structure outperforms the others when $\mu_A > \mu_B > \mu_C$. Due to the reasons explained previously, this case is not advantageous for the nested structure. For the overflow structure, $\mu_A < \mu_B < \mu_C$ and $\mu_A > \mu_B > \mu_C$ are equal, only the department-demand type matching alters. Since the departments have the same level of flexibility, the performance of the overflow structure does not change. The symmetric structure takes advantage of the balanced flexibility in this case. Appendix-B shows two cases regarding the capacity allocations. Either balanced capacities are allocated to each department or less capacity is assigned to department 2 while balanced capacities are allocated to department 1 and 2. The first case is seen when the demand arrival rates are close to each other. As the asymmetry increases between the arrival rates of demand type A and B, the demand is allocated in the second way. Because demand A is allocated to departments 1 and 3. And as its arrival rate increases, the effects of this increase is reflected to departments 1 and 3.

		$\mu_A<\mu_B<\mu_C$			$\mu_A>\mu_B>\mu_C$			$\mu_A=\mu_B=\mu_C$		
		O	**N**	**S**	**O**	**N**	**S**	**O**	**N**	**S**
(lv,hc)	**#1**	5	25	0	6	0	24	30	0	0
	#2	1	5	24	24	0	6	0	20	10
	#3	24	0	6	0	30	0	0	10	20
	Total	30	30	30	30	30	30	30	30	30
(lv,lc)	**#1**	5	25	0	5	0	25	30	0	0
	#2	0	5	25	25	0	5	0	29	1
	#3	25	0	5	0	30	0	0	1	29
	Total	30	30	30	30	30	30	30	30	30
(hv,hc)	**#1**	5	25	0	9	0	21	30	0	0
	#2	4	5	21	21	0	9	0	30	0
	#3	21	0	9	0	30	0	0	0	30
	Total	30	30	30	30	30	30	30	30	30
(hv,lc)	**#1**	5	25	0	5	0	25	30	0	0
	#2	0	5	25	25	0	5	0	30	0
	#3	25	0	5	0	30	0	0	0	30
	Total	30	30	30	30	30	30	30	30	30

Table 6.10: Summary profit values for three demand types.

If the structure is overflow or symmetric, interchanging the demand arrival rates of different demand types does not affect the optimal throughput and the profit. For example, the performance of the system is the same under the demand rates D_A=30, D_B=40, D_C=50 and D_A=50, D_B=40, D_C=30. Because in the overflow structure, each department is independent and when the demand arrival rates interchange, the corresponding department's capacity changes respectively but the total capacity stays the same, only the capacity is shifted from one department to another. Also, in the symmetric structure, all the departments are connected. Therefore, any change in the demand distribution of one type affects the capacity assignments of all the departments. But due to the symmetric structure, the effect is distributed in a balanced manner. While the capacity of one department decreases, the other one increases.

	$\mu_A<\mu_B<\mu_C$				$\mu_A>\mu_B>\mu_C$				$\mu_A=\mu_B=\mu_C$			
		O	**N**	**S**		**O**	**N**	**S**		**O**	**N**	**S**
(lv,hc)	**#1**	4	24	0	**#1**	5	0	24	**#1**	0	0	0
	#2	1	4	23	**#2**	24	0	5	**#2**	0	0	0
	#3	23	0	5	**#3**	0	29	0	**#3**	0	0	0
	Total	28	28	28	Total	29	29	29	Total	0	0	0
(lv,lc)	**#1**	0	6	0	**#1**	2	0	23	**#1**	0	0	0
	#2	0	0	6	**#2**	23	0	2	**#2**	0	0	0
	#3	6	0	0	**#3**	0	25	0	**#3**	0	0	0
	Total	6	6	6	Total	25	25	25	Total	0	0	0
(hv,hc)	**#1**	1	19	0	**#1**	8	0	19	**#1**	0	0	0
	#2	3	1	16	**#2**	19	0	8	**#2**	0	0	0
	#3	16	0	4	**#3**	0	27	0	**#3**	0	0	0
	Total	20	20	20	Total	27	27	27	Total	0	0	0
(hv,lc)	**#1**	0	1	0	**#1**	1	0	17	**#1**	0	0	0
	#2	0	0	1	**#2**	17	0	1	**#2**	0	0	0
	#3	1	0	0	**#3**	0	18	0	**#3**	0	0	0
	Total	1	1	1	Total	18	18	18	Total	0	0	0

Table 6.11: Summary profit values for three demand types considering non-overlapping confidence intervals.

The ranking of the demand arrival rates becomes important when the structure is nested. The nested structure is advantageous when $\mu_A< \mu_B< \mu_C$. And even when $\mu_A> \mu_B> \mu_C$ holds, by interchanging the demand department allocations, we achieve the same performance as the case where $\mu_A< \mu_B< \mu_C$. Therefore, unless the arrival rates of all the demand types are the same, a nested structure should be preferred. The demand with the lowest arrival rate should be allocated to the full-flexible department and the one with the highest rate should be allocated to the specialized department. The results of these experiments are useful in the sense that they give an idea to the managers for building the right flexibility structure under different conditions. In conclusion, when the demand arrival rates are equal, the overflow structure should be preferred and when the demand arrival rates are imbalanced, the nested structure should be preferred.

As stated before, we also considered the influence of the correlation over the system performance. We analyzed four cases regarding the correlation between demand type 1 and demand type 2. We made experiments for three demand sets, a representative from each main group, which are (10, 30, 90), (90, 30, 10) and (30, 30, 30). We chose the demand sets with high asymmetry since they represent the characteristics of the imbalanced demand better than the other sets. The detailed results the experiments in this group are given in Appendix-C. Table 6.11, Table 6.12, Table 6.13 summarizes the results of the experiments with non-overlapping confidence intervals for **p**. The results reveal that, the correlation of the demand types A and B do not affect the performance rankings of the structures. Under perfect correlation, the performance of overflow and nested increases relative to the symmetric structure when capacity cost is not high.

$\mu_A<\mu_B<\mu_C$

		O				N				S			
	p	-1	-0.5	0.5	1	-1	-0.5	0.5	1	-1	-0.5	0.5	1
(lv,hc)	#1	0	0	0	0	5	5	5	5	0	0	0	0
	#2	0	0	0	0	0	0	0	0	5	5	5	5
	#3	5	5	5	5	0	0	0	0	0	0	0	0
(lv,lc)	#1	0	0	0	0	0	0	0	0	0	0	0	0
	#2	0	0	0	0	0	0	0	0	0	0	0	0
	#3	0	0	0	0	0	0	0	0	0	0	0	0
(hv,hc)	#1	0	0	0	0	3	4	3	3	0	0	0	0
	#2	1	1	1	1	0	0	0	0	2	3	2	2
	#3	2	3	2	2	0	0	0	0	1	1	1	1
(hv,lc)	#1	0	0	0	0	0	0	0	0	0	0	0	0
	#2	0	0	0	0	0	0	0	0	0	0	0	0
	#3	0	0	0	0	0	0	0	0	0	0	0	0

Table 6.11: Summary profit values for different correlation matrices and non-overlapping confidence interval for $\mu A<\mu B<\mu C$.

$\mu_A > \mu_B > \mu_C$

		O				N				S			
	p	-1	0.5	0.5	1	-1	-0.5	0.5	1	-1	-0.5	0.5	1
(lv,hc)	#1	0	0	0	0	0	0	0	0	5	5	5	5
	#2	5	5	5	5	0	0	0	0	0	0	0	0
	#3	0	0	0	0	5	5	5	5	0	0	0	0
(lv,lc)	#1	0	0	0	0	0	0	0	0	5	3	3	3
	#2	5	3	3	3	0	0	0	0	0	0	0	0
	#3	0	0	0	0	5	3	3	3	0	0	0	0
(hv,hc)	#1	2	1	1	1	0	0	0	0	3	4	3	3
	#2	3	4	3	3	0	0	0	0	2	1	1	1
	#3	0	0	0	0	5	5	4	4	0	0	0	0
(hv,lc)	#1	0	0	0	0	0	0	0	0	2	3	3	1
	#2	2	3	3	1	0	0	0	0	0	0	0	0
	#3	0	0	0	0	2	3	3	1	0	0	0	0

Table 6.12: Summary profit values for different correlation matrices and non-overlapping confidence interval for $\mu A > \mu B > \mu C$.

$\mu_A=\mu_B=\mu_C$

		O				N				S			
	p	-1	-0.5	0.5	1	-1	-0.5	0.5	1	-1	-0.5	0.5	1
(lv,hc)	#1	5	4	4	4	0	0	0	0	0	0	0	0
	#2	0	0	0	0	0	0	0	0	5	4	4	4
	#3	0	0	0	0	5	4	4	4	0	0	0	0
(lv,lc)	#1	0	0	0	0	0	0	0	0	0	0	0	0
	#2	0	0	0	0	0	0	0	0	0	0	0	0
	#3	0	0	0	0	0	0	0	0	0	0	0	0
(hv,hc)	#1	4	4	1	0	0	0	0	0	0	0	0	0
	#2	0	0	0	0	0	0	0	0	4	4	1	0
	#3	0	0	0	0	4	4	1	0	0	0	0	0

Table 6.13: Summary profit values for different correlation matrices and non-overlapping confidence interval for µA=µB=µC .

The experiments under correlated demand shows that, the correlation of two demand types do not affect the relative performances of the structures. Finally, it can be observed form the appendices B and C that, as the demand variance increases, the capacity levels allocated by the method increases. Because the uncertainty increases and the department require more extra capacity to cover the unexpected jumps in the demand.

6.3.2 Experiments for Eight Demand Types

In order to observe how the system reacts as the network size is increased; we prepared experiments for eight demand types. We considered 3 basic cases where the demand rates are equal, the demand rates increase from demand type 1 to demand type 8 and the demand rates decrease from demand type 1 to demand type 8. The demand scenarios are shown in Table 6.12.

$\mu_A<\mu_B<\mu_C$

Set	Demand type	μ
1	A	80
	B	80
	C	80
	D	80
	E	80
	F	80
	G	80
	H	80

$\mu_A>\mu_B>\mu_C$

Set	Demand type	μ
2	A	5
	B	10
	C	20
	D	40
	E	80
	F	160
	G	320
	H	640

$\mu_A=\mu_B=\mu_C$

Set	Demand type	μ
3	A	640
	B	320
	C	160
	D	80
	E	40
	F	20
	G	10
	H	5

Table 6.14 demand sets for 8 demand types

Similar to the first set of experiments, we solved every problem for $C_V = 1$ and $C_V = 2$ for the 10 cost structures in Table 6.9. We found the average throughput and profit values, and calculated their confidence intervals under the optimal capacity. In total, we solved 3x2x10= 60 problems for 8 demand-type case.

Appendix-D gives the results in this group of experiments. Table 6.13 summarizes the results of the experiments and Table 6.14 summarizes the results considering the non-overlapping confidence intervals. As the number of demand types increases, the total number of skills required in the system increases too and new departments are added to the system. The capacity cost of the new departments of the symmetric structure and the overflow structure are the same as the existing ones. However, the capacity of the full-flexible department in the overflow structure increases corresponding to the additional skills in that department. Nested is the structure, of which the capacity cost is affected the most. Because each additional department has one more skill and therefore their capacities are more expensive than the existing ones.

When $\mu_A<\mu_B<\mu_C$, the confidence intervals are non-overlapping in all experiments. In overall, nested performs the best in 14 out of 20 experiments. But for (lv, hc), symmetric performs the best in 3 experiments and for (hv, hc) symmetric performs the best in 2 experiments and nested performs the best in 2 experiments out of 5. When $\mu_A>\mu_B>\mu_C$, overflow performs the best in 11 and symmetric performs the best in 9 out of 20 experiments. Moreover, overflow performs the best in 11 and symmetric performs the best in 8 out of 19 experiments with non-overlapping confidence intervals. The nested structure performs the worst in all experiments in this group. The results reveal that when $\mu_A=\mu_B=\mu_C$ overflow outperforms the others in 18 out of 20 experiments and 9 out of 11 experiments in which the confidence intervals are non-overlapping.

When $\mu_A<\mu_B<\mu_C$, the best structure is the nested structure for 8 demand types as well as 3 demand types. However for high capacity cost, the performance of the symmetric structure increases with respect to the smaller network. Because when the capacity cost is high, the flexibility is too expensive for the nested structure. However, the symmetric structure is not affected from the changes in the capacity cost. Therefore, becomes advantageous. Because the capacity cost of the symmetric structure stays the same while the capacity cost of the nested structure increases in average.

When $\mu_A>\mu_B>\mu_C$ the best structure becomes the overflow. The performance of the overflow increases while the performance of the symmetric decreases as the size of the system increases. When the capacity cost is high, the method does not allocate capacity to the full-flexible department. But since the capacity of the specialized departments are cheaper than the capacity of the symmetric structure, more capacity can be allocated to the overflow structure and therefore the throughput and the profit is maximized.

When $\mu_A=\mu_B=\mu_C$, the best structure does not change when the system size increases. But the performance of the symmetric structure increases with respect to the nested structure due to the advantage of less capacity and flexibility cost. The capacity of the departments with the highest level of flexibility is very expensive in the nested structure. Therefore, the algorithm cannot allocate enough capacity to these departments, therefore the structure cannot take advantage of the flexibility.

According to the results, as the network size grows, the performances become more sensitive to the cost structure. When $\mu_A<\mu_B<\mu_C$, as the capacity cost increases, the symmetric structure becomes more advantageous over the nested structure. This result can be seen in the tables for (lv, hc) and (hv, hc). Moreover as the variance increases, the capacity allocation level increases too, in order to be able to response the fluctuations in the demand.

In conclusion, the size of the network does not influence the performance rankings too much. But due to the changes in the capacity costs, in some cases the performance of overflow or symmetric structure improves over the nested structure. As the network size grows, under balanced demand, the overflow structure should be chosen. Under imbalanced demand arrival rates, if the capacity is not very expensive, the nested, otherwise the symmetric structure should be chosen. These results are valid for $\beta=1$ but they may change for $\beta<1$.

		A<B<C			A>B>C			A=B=C		
		O	N	S	O	N	S	O	N	S
(lv,hc)	**#1**	0	2	3	2	0	3	4	0	1
	#2	4	0	1	3	0	2	1	0	4
	#3	1	3	1	0	5	0	0	5	0
	Total	5	5	5	5	5	5	5	5	5
(lv,lc)	**#1**	0	5	0	2	0	3	5	0	0
	#2	2	0	3	3	0	2	0	0	5
	#3	3	0	2	0	5	0	0	5	0
	Total	5	5	5	5	5	5	5	5	5
(hv,hc)	**#1**	1	2	2	3	0	2	4	0	1
	#2	4	0	1	2	0	3	1	0	4
	#3	0	3	2	0	5	0	0	5	0
	Total	5	5	5	5	5	5	5	5	5
(hv,lc)	**#1**	0	5	0	4	0	1	5	0	0
	#2	4	0	1	1	0	4	0	0	5
	#3	1	0	4	0	5	0	0	5	0
	Total	5	5	5	5	5	5	5	5	5

Table 6.15 Summary profit values for eight demand types.

		A<B<C			A>B>C			A=B=C		
		O	**N**	**S**	**O**	**N**	**S**	**O**	**N**	**S**
(lv,hc)	**#1**	0	2	3	2	0	3	4	0	1
	#2	4	0	1	3	0	2	1	0	4
	#3	1	3	1	0	5	0	0	5	0
	Total	5	5	5	5	5	5	5	5	5
(lv,lc)	**#1**	0	5	0	2	0	3	1	0	0
	#2	2	0	3	3	0	2	0	0	1
	#3	3	0	2	0	5	0	0	1	0
	Total	5	5	5	5	5	5	1	1	1
(hv,hc)	**#1**	1	2	2	3	0	2	4	0	1
	#2	4	0	1	2	0	3	1	0	4
	#3	0	3	2	0	5	0	0	5	0
	Total	5	5	5	5	5	5	5	5	5
(hv,lc)	**#1**	0	5	0	4	0	0	0	0	0
	#2	4	0	1	0	0	4	0	0	0
	#3	1	0	4	0	4	0	0	0	0
	Total	5	5	5	4	4	4	0	0	0

Table 6.16 Summary profit values for eight demand types considering the non-overlapping confidence intervals.

Chapter 7

CONCLUSION

In this thesis, we analyze the problem of capacity optimization of a flexible multi-resource system under uncertain demand considering the capacity and the flexibility cost. We model the problem as a two-stage multidimensional newsvendor problem and propose a solution method based on gradient estimation method. Through comparison of our model and method to the model and the method of Harrison and Zeevi (2005), we show the appropriateness of using the newsvendor setting in our problem and the equality of the newsvendor setting to the queuing setting for our problem. We generate a tool that optimizes the capacity of any flexibility structure under uncertain demand. Unlike the previous research conducted on the problem, the scope of our method is not restricted to a specific flexibility structure. Via some benchmark problems we showed that our method converges to the optimal solution.

Our tool, besides optimizing the capacity, enables the comparison of the performances of any two structures. After optimizing their capacities, we can compare the performances of different flexibility structures which face the same demand distribution. We use our tool to compare the performances of three flexibility structures which are frequently referred to in the flexibility literature or used in the real-life applications which are the overflow, the nested and the symmetric structures. The previous research reveals the advantage of the symmetric structure under fixed capacity without considering the capacity cost. However adding flexibility to a system and optimizing its capacity becomes a more complicated problem when the effect of cost is considered. For example when the flexible capacity is very expensive, the system cannot take advantage of flexibility. Moreover, when the dedicated capacity is very cheap, the system does not need flexibility since it can allocate extra capacity to the dedicated departments instead of introducing flexibility. The results of our numerical experiments give an idea about the influence of the capacity and flexibility cost over the performance of the system. Our results show that, when the capacities are optimized with an objective of maximizing the profit, in certain cases the overflow or the nested structure performs better than the symmetric structure. For example, when the arrival rates of different demand types are balanced, the overflow structure performs the best and when they are imbalanced, if the nested structure is constructed in a proper way, it performs better than the symmetric structure. The nested structure outperforms the symmetric structure because it offers a variety of flexibility levels. The overflow outperforms the symmetric structure when a low level of flexibility is required. In that case, the symmetric structure keeps unnecessary flexible capacity which incurs cost. However, the overflow structure, keeping a small-sized full flexible department, covers the need for flexibility. The advantage of the symmetric structure is revealed when the size of the system increases due to the capacity cost advantage. The symmetric structure always offers perfect resource sharing and the capacity cost do not increase with respect to the increases in the number of total skills. However, the flexible capacity of other structures increase as the skill set expands. Hence either they do not keep flexible capacity, or they are charged for the high level of flexibility, which is not profitable in either case.

In conclusion, the method that we propose can be used for capacity optimization and performance analysis of any flexibility structure, and comparison of different flexibility structures. In this thesis we assumed that, if a resource is flexible, 100% of its capacity is flexible. However, it is possible to introduce partial flexibility. Section 4.2 provides the model for this case. In the future work, the capacity can be optimized considering partial flexibility via introducing the proportion of the flexible capacity as a decision variable. This may change some of our recommendations in terms of the three structures being considered.

Table-A1

Fully-specialized, 2demand types-2resources

	normal(50,5)	c^0=0		c^0=50		c^0=100		normal(50,10)	c^0=0		c^0=50		c^0=100	
(p, s)	capacity per resource by newsvendor	capacity by GPA	% error	capacity by GPA	% error	capacity by GPA	% error	capacity per resource by newsvendor	capacity by GPA	% error	capacity by GPA	% error	capacity by GPA	% error
(50,15)	52.62	52.36	0.51	52.64	0.03	52.59	0.06	55.24	54.76	0.87	55.17	0.13	55.08	0.30
		52.32	0.58	52.98	0.68	52.66	0.07		55.52	0.51	56.08	1.52	55.89	1.18
(50,10)	54.21	54.31	0.18	54.14	0.12	54.02	0.35	58.42	58.19	0.38	57.99	0.72	58.80	0.66
		54.57	0.66	54.34	0.24	54.06	0.27		59.16	1.28	58.95	0.91	59.28	1.47
(50,5)	56.41	56.50	0.16	56.34	0.13	56.82	0.74	62.82	62.13	1.09	61.93	1.42	63.48	1.06
		56.63	0.39	56.39	0.04	56.91	0.89		62.23	0.93	62.05	1.22	63.03	0.35
(40,15)	51.59	51.59	0.01	51.46	0.25	51.03	1.10	53.19	52.56	1.17	52.90	0.53	53.01	0.34
		51.62	0.05	51.36	0.45	51.49	0.20		53.47	0.53	54.05	1.62	53.69	0.95
(40,10)	53.37	53.75	0.70	53.43	0.10	53.27	0.20	56.74	56.10	1.14	55.92	1.46	56.53	0.39
		53.65	0.51	53.63	0.49	53.40	0.05		57.01	0.46	56.78	0.06	57.29	0.96
(40,5)	55.75	55.79	0.07	55.60	0.28	56.18	0.77	61.50	60.42	1.76	61.18	0.52	61.97	0.75
		55.92	0.30	55.71	0.07	56.14	0.70		61.19	0.51	61.34	0.27	61.71	0.33
(30,15)	50.00	50.24	0.48	50.00	0.00	49.93	0.15	50.00	49.27	1.46	50.00	0.00	50.12	0.24
		50.14	0.27	50.00	0.00	49.70	0.59		49.71	0.58	50.00	0.00	50.91	1.83
(30,10)	52.15	52.16	0.02	52.06	0.18	51.79	0.71	54.31	53.96	0.64	53.73	1.06	54.37	0.12
		52.43	0.53	52.17	0.03	52.06	0.17		54.90	1.10	54.69	0.71	55.17	1.60
(30,5)	54.84	54.33	0.92	54.88	0.08	55.40	1.03	59.67	58.98	1.17	58.79	1.48	60.46	1.31
		54.54	0.54	55.01	0.32	55.51	1.22		59.80	0.22	59.57	0.17	60.39	1.19
Minimum			0.01		0.00		0.05			0.22		0.00		0.12
Maximum			0.92		0.68		1.22			1.76		1.62		1.83
Average			0.38		0.19		0.52			0.88		0.77		0.84

Table-A2

Full-flexible, 2 demand types-2 resources

normal(50,5) **normal(50,10)**

		c^0=0		c^0=50		c^0=100			c^0=0		c^0=50		c^0=100	
(p, s, f)	total capacity by newsvendor	total capacity by GPA	% error	total capacity by GPA	% error	Total capacity by GPA	% error	total capacity by newsvendor	total capacity by GPA	% error	total capacity by GPA	% error	total capacity by GPA	% error
(50,15,5)	101.79	101.79	0.03	101.41	0.37	102.42	0.62	103.58	103.24	0.33	104.24	0.64	103.83	0.24
(50,15,2)	102.92	102.92	0.31	102.83	0.09	103.83	0.89	105.83	106.07	0.22	105.66	0.17	105.25	0.55
(50,10,5)	103.71	103.71	0.91	104.24	0.52	103.83	0.12	107.42	107.48	0.06	107.07	0.32	106.66	0.70
(50,10,2)	104.99	104.99	1.02	105.66	0.63	105.25	0.24	109.99	110.31	0.29	109.90	0.08	109.49	0.45
(50,5,5)	105.95	105.95	0.11	107.07	1.06	106.66	0.67	111.90	111.72	0.16	111.31	0.53	110.90	0.89
(50,5,2)	107.64	107.64	0.15	108.49	0.79	108.08	0.41	115.28	114.55	0.63	114.14	0.99	115.15	0.11
(40,15,5)	100.00	100.00	0.41	100.00	0.00	99.59	0.41	100.00	100.41	0.41	100.00	0.00	99.59	0.41
(40,15,2)	101.34	101.34	0.48	101.41	0.08	101.00	0.33	102.67	103.24	0.55	102.83	0.15	102.42	0.25
(40,10,5)	102.25	102.25	0.96	102.83	0.56	102.42	0.16	104.51	104.65	0.14	104.24	0.25	103.83	0.64
(40,10,2)	103.71	103.71	0.91	104.24	0.52	103.83	0.12	107.42	107.48	0.06	107.07	0.32	106.66	0.70
(40,5,5)	104.77	104.77	1.24	105.66	0.85	105.25	0.46	109.54	108.89	0.59	109.90	0.33	109.49	0.04
(40,5,2)	106.61	106.61	0.82	107.07	0.43	106.66	0.05	113.22	111.72	1.32	112.73	0.43	112.32	0.79
(30,15,5)	96.95	96.95	0.65	97.17	0.22	96.76	0.20	93.91	94.75	0.90	95.76	1.97	95.35	1.53
(30,15,2)	98.81	98.81	0.18	98.59	0.23	98.18	0.64	97.63	99.00	1.40	98.59	0.98	98.18	0.56
(30,10,5)	100.00	100.00	0.41	100.00	0.00	99.59	0.41	100.00	100.41	0.41	100.00	0.00	99.59	0.41
(30,10,2)	101.79	101.79	0.03	102.83	1.02	102.42	0.62	103.58	103.24	0.33	102.83	0.73	103.83	0.24
(30,5,5)	103.05	103.05	0.19	104.24	1.16	103.83	0.76	106.09	106.07	0.02	105.66	0.41	105.25	0.80
(30,5,2)	105.15	105.15	0.87	105.66	0.48	105.25	0.10	110.29	110.31	0.01	109.90	0.36	110.90	0.55
Minimum			0.03		0.00		0.05			0.01		0.00		0.04
Maximum			1.24		1.16		0.89			1.40		1.97		1.53
Average			0.54		0.50		0.40			0.44		0.48		0.55

Table-A3

fully-specialized, 3 demand types-3 resources

	Normal(50,5)	**c^0=0**		**c^0=50**		**c^0=100**		**normal(50,10)**	**c^0=0**		**c^0=50**		**C^0=100**	
(p, s)	capacity per resource by newsvendor	capacity by GPA	% error	capacity by GPA	% error	capacity by GPA	% error	capacity per resource by newsvendor	capacity by GPA	% error	capacity by GPA	% error	capacity by GPA	% error
(50,15)	52.62	52.57	0.10	52.28	0.65	52.16	0.89	55.24	54.78	0.84	53.60	1.19	55.05	0.34
		52.75	0.24	52.54	0.15	52.26	0.68		55.98	1.34	54.34	0.84	56.08	1.52
		52.26	0.68	52.08	1.04	51.87	1.43		55.42	0.31	54.14	0.07	55.55	0.55
(50,10)	54.21	54.21	0.00	54.12	0.17	53.76	0.82	58.42	57.53	1.51	55.33	1.71	58.03	0.65
		54.70	0.90	54.27	0.12	54.13	0.15		58.87	0.78	56.13	0.17	58.95	0.91
		53.74	0.86	53.71	0.92	53.58	1.17		58.42	0.01	55.82	0.30	58.37	0.08
(50,5)	56.41	55.90	0.89	55.72	1.22	56.06	0.61	62.82	62.22	0.95	57.11	1.44	62.41	0.65
		56.38	0.06	56.02	0.69	56.38	0.05		62.66	0.26	57.82	0.64	63.05	0.37
		55.69	1.28	55.56	1.51	55.95	0.81		62.08	1.17	57.56	1.37	62.04	1.23
(40,15)	51.59	51.35	0.47	51.74	0.29	51.55	0.08	53.19	53.00	0.35	52.43	0.72	53.24	0.09
		51.56	0.07	51.81	0.43	51.67	0.14		54.25	2.00	53.15	1.33	54.40	2.27
		51.21	0.73	51.57	0.05	51.30	0.56		53.62	0.82	53.02	0.72	53.85	1.25
(40,10)	53.37	53.17	0.38	53.63	0.49	53.28	0.17	56.74	56.02	1.27	54.21	0.57	56.05	1.23
		53.28	0.18	53.66	0.54	53.57	0.36		57.12	0.67	54.91	1.10	57.32	1.01
		52.88	0.93	52.97	0.75	52.90	0.89		56.52	0.40	54.70	0.30	56.87	0.21
(40,5)	55.75	55.37	0.69	55.18	1.03	55.49	0.46	61.50	60.83	1.09	55.99	1.62	61.71	0.34
		55.67	0.14	55.43	0.57	55.89	0.26		61.60	0.15	56.65	0.02	62.33	1.34
		55.20	0.98	54.96	1.43	55.29	0.84		61.10	0.66	56.39	1.08	61.69	0.30
(30,15)	50.00	50.03	0.07	50.00	0.00	49.79	0.42	50.00	49.69	0.62	50.00	0.00	50.01	0.01
		50.39	0.79	50.00	0.00	49.98	0.03		49.79	0.43	50.00	0.00	50.50	1.00
		50.11	0.23	50.00	0.00	49.55	0.90		49.46	1.09	50.00	0.00	50.65	1.30
(30,10)	52.15	51.88	0.52	51.72	0.84	52.13	0.05	54.31	53.61	1.28	52.58	0.77	54.47	0.29
		52.18	0.05	51.91	0.47	52.27	0.23		54.75	0.82	53.06	1.58	55.50	2.19
		51.79	0.69	51.55	1.15	51.85	0.58		54.32	0.03	52.99	0.67	54.98	1.24
(30,5)	54.84	54.84	0.00	54.65	0.33	54.42	0.75	59.67	59.03	1.08	54.86	1.50	59.96	0.48
		55.13	0.53	54.90	0.11	54.56	0.50		59.94	0.44	55.45	0.01	60.52	1.42
		54.55	0.53	54.28	1.02	54.26	1.04		59.27	0.68	55.25	0.80	60.04	0.61
Minimum			0.00		0.00		0.03			0.01		0.00		0.01
Maximum			1.28		1.51		1.43			2.00		1.71		2.27
average			0.48		0.59		0.55			0.78		0.76		0.85

Table-A4

Full-flexible,3demand types-3 resources

	normal(50,5)							normal(50,10)						
		$c^0=0$		$c^0=50$		$c^0=100$			$c^0=0$		$c^0=50$		$c^0=100$	
(p, s, f)	total capacity by newsvendor	total capacity by GPA	% error	total capacity by GPA	% error	Total capacity by GPA	% error	total capacity by newsvendor	total capacity by GPA	% error	total capacity by GPA	% error	total capacity by GPA	% error
(50,15,5)	150.00	148.96	0.70	150.00	0.00	149.31	0.46	150.00	150.00	0.46	150.00	0.00	149.31	0.46
(50,15,2)	152.65	152.42	0.15	151.73	0.60	151.04	1.05	155.29	155.29	0.38	155.20	0.06	154.51	0.50
(50,10,5)	152.19	150.69	0.99	151.73	0.30	151.04	0.76	154.39	154.39	0.15	155.20	0.52	154.51	0.08
(50,10,2)	155.05	154.15	0.58	153.46	1.02	154.51	0.35	160.10	160.10	0.47	160.39	0.19	159.70	0.24
(50,5,5)	154.54	154.15	0.25	153.46	0.70	154.51	0.02	159.08	159.08	0.17	158.66	0.27	157.97	0.70
(50,5,2)	157.93	157.62	0.20	156.93	0.63	157.97	0.03	165.85	165.85	0.25	165.59	0.16	166.63	0.47
(40,15,5)	147.24	147.22	0.01	146.54	0.48	147.58	0.23	144.48	144.48	0.70	144.80	0.22	144.12	0.25
(40,15,2)	150.54	148.96	1.05	150.00	0.36	149.31	0.82	151.09	151.09	0.26	151.73	0.43	151.04	0.03
(40,10,5)	150.00	148.96	0.70	148.27	1.15	149.31	0.46	150.00	150.00	0.46	150.00	0.00	149.31	0.46
(40,10,2)	153.34	152.42	0.60	151.73	1.05	152.78	0.37	156.67	156.67	0.50	156.93	0.16	156.24	0.28
(40,5,5)	152.76	152.42	0.22	151.73	0.67	151.04	1.12	155.52	155.52	0.24	155.20	0.21	154.51	0.65
(40,5,2)	156.54	155.88	0.42	156.93	0.25	156.24	0.19	163.08	163.08	0.17	162.12	0.59	163.17	0.05
(30,15,5)	141.62	142.03	0.29	141.34	0.20	140.65	0.69	133.24	133.24	0.09	137.88	3.48	133.72	0.36
(30,15,2)	147.05	147.22	0.12	146.54	0.35	145.85	0.82	144.10	144.10	0.24	144.80	0.49	144.12	0.01
(30,10,5)	146.27	145.49	0.53	144.80	1.00	145.85	0.29	142.54	142.54	0.36	143.07	0.37	142.38	0.11
(30,10,2)	150.72	150.69	0.02	150.00	0.48	149.31	0.94	151.45	151.45	0.64	151.73	0.19	151.04	0.27
(30,5,5)	150.00	148.96	0.70	148.27	1.15	149.31	0.46	150.00	150.00	0.46	150.00	0.00	149.31	0.46
(30,5,2)	154.54	154.15	0.25	153.46	0.70	152.78	1.14	159.08	159.08	0.17	158.66	0.27	157.97	0.70
min			0.01		0.00		0.02			0.09		0.00		0.01
max			1.05		1.15		1.14			0.70		3.48		0.70
Avg			0.43		0.62		0.57			0.34		0.42		0.34

Table-B1_1 (Low variance, high cost)

A<B<C			OVERFLOW			NESTED			SYMMETRIC		
Demand Normal (μ,σ)	**Cost Structure**	**Dept.**	**c***	**TH**	**PR**	**c***	**TH**	**PR**	**c***	**TH**	**PR**
d_A= (30,3)	1	1	22.33	118.27	3844.30	32.71	117.98	3531.20	40.70	118.13	3464.52
d_B= (40,4)		2	29.28			40.93			40.70		
d_C= (50,5)		3	35.67			48.76			40.70		
		4	35.79								
	2	1	13.90	117.49	3766.27	29.59	116.67	3019.11	40.13	117.38	2859.40
		2	30.37			40.67			40.13		
		3	38.33			50.79			40.12		
		4	39.44								
	3	1	4.78	115.88	3844.80	12.37	99.82	2466.12	39.56	116.45	2262.92
		2	31.10			39.77			39.55		
		3	40.23			51.66			39.53		
		4	44.28								
	4	1	16.13	117.49	4293.02	30.23	116.70	3582.51	40.12	117.38	3431.28
		2	29.93			40.39			40.13		
		3	37.45			49.93			40.12		
		4	38.17								
	5	1	16.69	114.75	2075.21	29.45	114.31	1737.99	38.78	114.96	1676.39
		2	28.26			39.65			38.78		
		3	35.54			47.05			38.78		
		4	36.01								
d_A= (10,1)	1	1	19.36	82.81	2690.51	34.45	127.11	3809.01	38.97	126.13	3695.93
d_B= (30,3)		2	10.54			44.04			45.77		
d_C= (90,9)		3	26.77			53.62			45.78		
		4	27.10								
	2	1	11.51	74.53	2364.45	23.29	125.54	3449.86	38.46	124.54	3037.89
		2	10.54			42.93			44.55		
		3	26.56			62.57			44.55		
		4	26.85								
	3	1	3.87	67.39	2228.10	9.67	123.05	3368.74	37.69	123.14	2399.17
		2	10.54			41.20			43.78		
		3	26.80			74.17			43.79		
		4	27.14								
	4	1	12.94	74.20	2693.71	26.23	125.72	4005.44	38.42	124.67	3647.41
		2	10.43			43.04			44.67		
		3	25.72			59.85			44.67		
		4	25.88								
	5	1	8.41	51.50	937.49	25.34	122.90	1946.85	37.16	120.91	1769.23
		2	9.71			41.52			42.51		
		3	16.83			57.71			42.51		
		4	16.83								
d_A= (5,0.5)	1	1	12.13	48.42	1557.36	25.45	97.00	2906.91	20.32	87.27	2595.68
d_B= (15,1.5)		2	5.30			33.78			34.04		
d_C= (80,8)		3	15.08			42.10			34.04		
		4	16.98								
	2	1	7.92	45.84	1427.51	17.26	95.84	2634.98	19.90	85.64	2120.97
		2	5.30			32.95			33.27		
		3	15.36			48.64			33.27		
		4	18.48								
	3	1	2.50	39.38	1286.21	6.61	92.80	2559.81	19.55	79.88	1582.98
		2	5.30			31.37			30.41		
		3	15.20			56.12			30.41		
		4	17.51								
	4	1	6.40	36.76	1334.80	19.64	95.92	3052.74	19.90	85.90	2552.60
		2	5.24			33.00			33.41		
		3	12.72			46.35			33.41		
		4	12.79								

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	5	1	4.20	25.73	468.31	19.13	93.54	1473.96	19.09	79.34	1178.60
		2	4.86			31.70			30.29		
		3	8.40			44.28			30.29		
		4	8.40								
d_A= (10,1)	1	1	21.56	91.71	2983.13	56.50	205.78	6153.33	50.93	191.85	5692.86
d_B= (40,4)		2	10.54			71.41			72.02		
d_C= (160,16)		3	30.17			86.32			72.02		
		4	30.18								
	2	1	12.64	81.40	2585.28	38.70	202.02	5533.96	49.83	187.73	4641.87
		2	10.54			68.96			69.98		
		3	29.48			99.25			69.98		
		4	29.48								
	3	1	4.47	76.77	2541.85	14.99	193.31	5318.98	48.84	181.96	3600.90
		2	10.54			64.94			67.20		
		3	31.25			114.93			67.20		
		4	31.26								
	4	1	13.26	76.07	2764.12	43.43	202.79	6440.63	49.82	187.92	5575.57
		2	10.43			69.39			70.08		
		3	26.52			95.38			70.08		
		4	26.52								
	5	1	8.40	51.45	936.53	42.01	197.69	3117.10	47.93	171.70	2549.84
		2	9.71			66.80			62.26		
		3	16.80			91.63			62.25		
		4	16.80								
d_A= (10,1)	1	1	20.20	86.57	2816.91	34.26	127.96	3843.96	44.17	127.89	3744.40
d_B= (50,5)		2	10.54			44.24			44.17		
d_C= (70,7)		3	28.28			54.20			44.17		
		4	28.28								
	2	1	11.77	76.47	2430.17	23.86	126.44	3468.95	43.59	127.10	3085.93
		2	10.54			43.25			43.59		
		3	27.45			62.44			43.60		
		4	27.45								
	3	1	4.46	76.78	2542.29	10.38	125.10	3291.28	42.72	125.67	2438.42
		2	10.54			47.26			42.72		
		3	31.26			71.93			42.73		
		4	31.26								
	4	1	14.59	82.73	3004.12	26.26	126.84	4045.26	43.59	127.10	3703.11
		2	10.43			43.44			43.59		
		3	29.19			60.56			43.59		
		4	29.19								
	5	1	8.40	51.45	936.53	26.00	123.90	1959.53	41.85	123.94	1801.98
		2	9.71			41.89			41.85		
		3	16.80			57.64			41.87		
		4	16.80								
d_A= (10,1)	1	1	20.20	86.57	2816.91	39.81	146.88	4408.98	50.76	146.95	4299.31
d_B= (50,5)		2	10.54			50.74			50.83		
d_C= (90,9)		3	28.28			61.67			50.83		
		4	28.28								
	2	1	11.77	76.47	2430.17	27.36	145.28	3988.32	49.84	145.80	3544.23
		2	10.54			49.60			50.00		
		3	27.45			71.86			49.99		
		4	27.45								
	3	1	4.46	76.78	2542.29	11.01	142.30	3882.72	49.22	144.86	2800.40
		2	10.54			48.77			49.43		
		3	31.26			84.91			49.43		
		4	31.26								
	4	1	14.59	82.73	3004.12	30.64	145.48	4634.69	49.84	145.80	4253.07
		2	10.43			49.73			49.99		
		3	29.19			68.83			49.99		
		4	29.19								
	5	1	8.40	51.45	936.53	30.07	142.17	2248.23	48.19	142.63	2069.35
		2	9.71			48.00			48.22		
		3	16.80			65.92			48.22		
		4	16.80								

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Table-B1_2 (A>B>C) (low variance high cost)

			OVERFLOW			NESTED			SYMMETRIC		
Demand Normal (μ,σ)	**Cost Structure**	**Dept.**	**c***	**TH**	**PR**	**c***	**TH**	**PR**	**c***	**TH**	**PR**
d_A= (50,5)	**1**	**1**	22.33	118.27	3844.30	35.86	103.67	3025.63	40.70	118.13	3464.52
d_B= (40,4)		**2**	35.79			39.81			40.70		
d_C= (30,3)		**3**	35.67			31.00			40.70		
		4	29.28								
	2	**1**	13.90	117.49	3766.27	23.82	90.58	2273.03	40.12	117.38	2859.40
		2	39.44			38.11			40.13		
		3	38.33			31.31			40.13		
		4	30.37								
	3	**1**	4.77	115.88	3844.79	9.59	75.62	1764.43	39.56	116.45	2262.92
		2	44.28			37.15			39.56		
		3	40.23			31.38			39.53		
		4	31.10								
	4	**1**	16.13	117.49	4293.01	26.85	93.72	2778.27	40.13	117.38	3431.28
		2	38.17			38.41			40.13		
		3	37.45			30.96			40.12		
		4	29.92								
	5	**1**	16.69	114.75	2075.20	21.82	82.62	1246.71	38.79	114.96	1676.39
		2	36.01			32.66			38.78		
		3	35.54			28.94			38.77		
		4	28.26								
d_A= (90,9)	**1**	**1**	21.88	90.48	2924.97	19.55	52.78	1523.27	45.78	126.13	3695.93
d_B= (30,3)		**2**	30.63			23.44			38.97		
d_C= (10,1)		**3**	28.98			10.54			45.77		
		4	10.54								
	2	**1**	14.63	87.36	2733.48	12.03	41.88	1013.69	44.55	124.54	3037.89
		2	34.12			20.05			38.46		
		3	30.17			10.54			44.55		
		4	10.54								
	3	**1**	4.10	70.03	2310.28	4.35	31.57	702.08	43.79	123.14	2399.16
		2	28.68			17.42			37.69		
		3	27.90			10.54			43.78		
		4	10.54								
	4	**1**	13.64	77.35	2803.05	13.21	42.80	1236.30	44.67	124.67	3647.42
		2	27.28			19.82			38.42		
		3	26.90			10.43			44.67		
		4	10.43								
	5	**1**	8.40	51.45	936.52	7.43	28.00	421.71	42.51	120.91	1769.21
		2	16.80			11.14			37.16		
		3	16.80			9.71			42.51		
		4	9.71								
d_A= (80,8)	**1**	**1**	12.13	48.42	1557.35	12.52	31.39	892.61	34.04	87.27	2595.68
d_B= (15,1.5)		**2**	16.98			14.30			20.32		
d_C= (5,0.5)		**3**	15.08			5.20			34.04		
		4	5.30								
	2	**1**	7.54	44.49	1389.45	6.99	23.54	562.08	33.27	85.64	2120.98
		2	17.58			11.64			19.90		
		3	15.21			5.30			33.27		
		4	5.30								
	3	**1**	2.92	42.85	1391.93	1.87	14.26	325.31	30.41	79.88	1582.99
		2	20.43			7.48			19.55		
		3	15.55			5.30			30.41		
		4	5.30								
	4	**1**	6.40	36.76	1334.80	5.53	18.72	548.50	32.72	84.59	2514.90
		2	12.79			8.29			19.91		
		3	12.72			5.24			32.72		
		4	5.24								
	5	**1**	4.20	25.73	468.31	4.61	16.25	240.13	30.29	79.34	1178.60
		2	8.40			6.92			19.09		
		3	8.40			4.86			30.29		
		4	4.86								
d_A= (160,16)	**1**	**1**	20.20	86.57	2816.91	19.56	52.82	1524.86	72.02	191.85	5692.87
d_B= (40,4)		**2**	28.28			23.47			50.93		
d_C= (10,1)		**3**	28.28			10.54			72.02		
		4	10.54								

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		1	12.93	83.05	2636.91	12.03	41.88	1013.71	69.98	187.73	4641.86
	2	**2**	30.16			20.05			49.82		
		3	30.16			10.54			69.97		
		4	10.54								
		1	4.26	73.76	2442.70	4.35	31.57	702.08	65.10	177.90	3523.48
	3	**2**	29.85			17.42			48.85		
		3	29.85			10.54			65.10		
		4	10.54								
		1	13.59	77.73	2824.12	13.23	42.84	1237.29	70.09	187.92	5575.55
	4	**2**	27.19			19.84			49.82		
		3	27.19			10.43			70.08		
		4	10.43								
		1	8.40	51.45	936.52	7.43	28.00	421.71	60.85	168.90	2508.60
	5	**2**	16.80			11.14			47.93		
		3	16.80			9.71			60.84		
		4	9.71								
d_A= (70,7)		**1**	20.20	86.57	2816.91	19.56			44.17		
d_B= (50,5)	**1**	**2**	28.28			23.47	52.82	1524.86	44.16	127.89	3744.40
d_C= (10,1)		**3**	28.28			10.54			44.17		
		4	10.54								
		1	12.93	83.05	2637.05	12.03			43.60		
	2	**2**	30.16			20.05	41.88	1013.71	43.60	127.10	3085.93
		3	30.16			10.54			43.58		
		4	10.54								
		1	4.26	73.76	2442.79	4.35			42.72		
	3	**2**	29.85			17.42	31.57	702.08	42.72	125.67	2438.42
		3	29.85			10.54			42.72		
		4	10.54								
		1	13.59	77.73	2824.12	13.23			43.59		
	4	**2**	27.19			19.84	42.84	1237.29	43.59	127.10	3703.11
		3	27.19			10.43			43.59		
		4	10.43								
		1	8.40	51.45	936.52	7.43			41.86		
	5	**2**	16.80			11.14	28.00	421.71	41.85	123.94	1801.98
		3	16.80			9.71			41.87		
		4	9.71								
d_A= (90,9)		**1**	20.20	86.57	2816.91	19.56			50.83		
d_B= (50,5)	**1**	**2**	28.28			23.47	52.82	1524.86	50.76	146.95	4299.31
d_C= (10,1)		**3**	28.28			10.54			50.83		
		4	10.54								
		1	12.93	83.05	2637.05	12.03			50.00		
	2	**2**	30.16			20.05	41.88	1013.71	49.83	145.80	3544.23
		3	30.16			10.54			49.99		
		4	10.54								
		1	4.26	73.76	2442.79	4.35			49.42		
	3	**2**	29.85			17.42	31.57	702.08	49.22	144.86	2800.40
		3	29.85			10.54			49.44		
		4	10.54								
		1	13.59	77.73	2824.12	13.23			49.99		
	4	**2**	27.19			19.84	42.84	1237.29	49.84	145.80	4253.07
		3	27.19			10.43			49.99		
		4	10.43								
		1	8.40	51.45	936.52	7.43			48.22		
	5	**2**	16.80			11.14	28.00	421.71	48.19	142.63	2069.35
		3	16.80			9.71			48.21		
		4	9.71								

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Table-B1_3 (A=B=C) (low variance high cost)

			OVERFLOW			NESTED			SYMMETRIC		
Demand Normal (μ,σ)	**Cost Structure**	**Dept.**	**c***	**TH**	**PR**	**c***	**TH**	**PR**	**c***	**TH**	**PR**
d_A= (5,0.5)		1	0.81	14.67	490.90	5.00			5.13		
d_B= (5,0.5)	1	2	4.94			5.21	14.61	421.70	5.13	14.58	421.04
d_C= (5,0.5)		3	4.94			5.30			5.13		
		4	4.94								
		1	0.00	14.74	499.34	4.71			4.96		
	2	2	5.28			4.93	14.31	347.58	4.96	14.33	344.73
		3	5.28			5.30			4.96		
		4	5.28								
		1	0.00	14.75	499.35	2.61			4.91		
	3	2	5.29			4.86	12.25	269.75	4.91	14.25	270.68
		3	5.29			5.30			4.91		
		4	5.29								
		1	0.00	14.69	567.60	4.67			5.07		
	4	2	5.23			4.99	14.28	415.97	5.07	14.50	413.59
		3	5.23			5.24			5.07		
		4	5.23								
		1	0.10	14.02	269.47	4.47			4.71		
	5	2	4.75			4.72	13.79	199.46	4.71	13.88	198.85
		3	4.75			4.86			4.72		
		4	4.75								
d_A= (10,1)		1	3.27	29.25	962.62	10.00			10.30		
d_B= (10,1)	1	2	9.29			10.31	29.12	841.54	10.30	29.16	840.11
d_C= (10,1)		3	9.29			10.54			10.30		
		4	9.29								
		1	1.34	29.09	966.87	9.21			9.91		
	2	2	9.79			9.97	28.46	693.30	9.91	28.64	688.44
		3	9.79			10.54			9.91		
		4	9.79								
		1	0.00	29.14	994.42	4.47			9.72		
	3	2	10.28			9.71	23.71	534.66	9.72	28.31	540.93
		3	10.28			10.54			9.72		
		4	10.28								
		1	1.75	29.04	1095.59	9.58			9.91		
	4	2	9.61			9.97	28.69	830.65	9.91	28.64	826.13
		3	9.61			10.43			9.91		
		4	9.61								
		1	1.48	28.10	525.69	7.88			9.53		
	5	2	9.11			9.45	26.57	391.15	9.53	27.94	396.85
		3	9.11			9.71			9.53		
		4	9.11								
d_A= (30,3)		1	15.14	87.45	2825.46	29.83			30.60		
d_B= (30,3)	1	2	25.96			30.67	86.99	2517.16	30.60	86.98	2512.91
d_C= (30,3)		3	25.96			31.55			30.60		
		4	25.96								
		1	8.97	86.39	2783.30	28.42			29.74		
	2	2	27.15			29.79	85.77	2075.76	29.74	85.78	2059.18
		3	27.15			31.55			29.72		
		4	27.15								
		1	2.78	83.79	2819.72	10.66			28.96		
	3	2	27.66			28.94	68.14	1585.89	28.96	84.50	1618.46
		3	27.66			31.54			28.97		
		4	27.66								
		1	10.31	86.52	3170.42	28.60			29.74		
	4	2	26.81			29.80	85.81	2484.99	29.74	85.78	2471.02
		3	26.81			31.26			29.72		
		4	26.81								
		1	10.02	83.82	1516.75	22.25			28.29		
	5	2	25.26			28.12	78.02	1158.01	28.29	83.19	1188.91
		3	25.26			28.94			28.29		
		4	25.26								
d_A= (40,4)		1	21.48	117.41	3791.32	39.93			40.99		
d_B= (40,4)	1	2	34.27			41.09	116.85	3390.79	40.99	116.89	3384.94
d_C= (40,4)		3	34.27			42.12			40.99		
		4	34.27								

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		1	12.90	115.56	3719.14	35.99			40.13		
	2	**2**	35.72			39.94	113.58	2788.72	40.13	115.72	2776.54
		3	35.72			42.14			40.13		
		4	35.72								
		1	4.22	111.46	3746.06	14.40			38.97		
	3	**2**	36.38			39.04	91.76	2136.78	38.97	113.82	2183.65
		3	36.38			42.14			38.98		
		4	36.38								
		1	14.83	115.87	4241.77	38.02			40.12		
	4	**2**	35.29			39.94	115.09	3349.35	40.13	115.72	3331.85
		3	35.29			41.86			40.12		
		4	35.29								
		1	14.14	112.70	2039.29	35.36			38.11		
	5	**2**	33.67			38.02	110.47	1607.33	38.11	112.14	1605.85
		3	33.67			39.05			38.10		
		4	33.67								
d_A= (80,8)		**1**	43.71	231.16	7466.70	79.53			81.41		
d_B= (80,8)	**1**	**2**	66.64			81.55	231.46	6696.89	81.41	231.41	6686.11
d_C= (80,8)		**3**	66.63			83.81			81.40		
		4	66.64								
		1	26.52	226.23	7275.93	73.68			79.37		
	2	**2**	69.05			79.34	226.73	5516.27	79.40	228.65	5478.67
		3	69.05			83.87			79.38		
		4	69.05								
		1	8.80	212.81	7150.83	26.56			77.63		
	3	**2**	68.75			77.52	179.75	4208.73	77.65	225.81	4301.91
		3	68.75			83.89			77.68		
		4	68.75								
		1	30.62	227.39	8316.41	76.06			79.39		
	4	**2**	68.37			79.32	228.18	6607.85	79.43	228.65	6574.40
		3	68.37			83.04			79.34		
		4	68.38								
		1	29.60	219.55	3972.85	59.34			75.63		
	5	**2**	64.67			74.90	207.96	3077.34	75.62	221.89	3153.21
		3	64.67			77.53			75.65		
		4	64.68								
d_A= (160,16)		**1**	88.89	466.13	15083.11	160.48			165.11		
d_B= (160,16)	**1**	**2**	133.36			165.25	468.43	13558.81	165.11	468.86	13535.83
d_C= (160,16)		**3**	133.36			169.70			165.15		
		4	133.35								
		1	54.35	451.42	14548.62	150.15			160.53		
	2	**2**	135.99			160.73	459.98	11177.00	160.45	462.57	11091.04
		3	136.00			169.91			160.52		
		4	136.00								
		1	18.24	425.21	14288.48	54.32			156.47		
	3	**2**	136.69			156.32	364.18	8525.39	156.48	455.87	8711.87
		3	136.69			169.95			156.44		
		4	136.70								
		1	62.79	455.37	16695.31	144.62			160.55		
	4	**2**	135.25			160.34	454.76	13332.01	160.55	462.57	13309.25
		3	135.25			167.94			160.40		
		4	135.26								
		1	60.56	441.48	8020.92	119.25			151.84		
	5	**2**	129.24			151.08	419.53	6227.15	151.83	446.78	6395.54
		3	129.23			156.37			151.85		
		4	129.21								

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Table-B2_1 (Low variance, low cost)

(A<B<C)

	Cost.S.	Dept.	OVERFLOW c*	TH	PR	NESTED c*	TH	PR	SYMMETRIC c*	TH	PR
d_A= (30,3)	6	1	30.81	119.66	5275.53	40.38	119.59	5215.27	42.72	119.59	5210.67
d_B= (40,4)		2	29.70			42.75			42.72		
d_C= (50,5)		3	34.33			45.03			42.72		
		4	34.33								
	7	1	25.88	119.47	5125.61	35.95	119.35	4877.27	42.15	119.34	4829.09
		2	30.81			42.30			42.15		
		3	35.73			48.45			42.15		
		4	35.77								
	8	1	32.13	119.83	11262.11	41.99	119.82	11200.63	43.59	119.82	11197.38
		2	30.22			43.62			43.59		
		3	34.53			45.23			43.59		
		4	34.53								
	9	1	29.08	119.78	11090.91	39.17	119.70	10838.27	43.01	119.69	10807.36
		2	31.04			43.10			43.02		
		3	35.41			46.94			43.01		
		4	35.41								
	10	1	28.10	119.76	11041.57	38.28	119.67	10724.74	43.01	119.69	10678.33
		2	31.14			43.01			43.01		
		3	35.76			47.64			43.01		
		4	35.76								
d_A= (10,1)	6	1	35.56	112.03	4951.44	44.78	129.32	5623.97	40.98	129.11	5611.22
d_B= (30,3)		2	11.37			47.02			49.85		
d_C= (90,9)		3	31.61			49.25			49.85		
		4	37.21								
	7	1	30.98	107.99	4592.72	39.38	128.86	5253.03	40.66	128.39	5185.36
		2	11.37			46.05			48.22		
		3	31.71			52.72			48.22		
		4	37.68								
	8	1	41.22	121.83	11462.29	46.22	129.64	12099.59	41.68	129.49	12082.92
		2	11.69			48.24			51.32		
		3	32.71			50.26			51.32		
		4	42.12								
	9	1	39.56	121.44	11190.91	42.85	129.48	11702.07	41.16	129.33	11650.79
		2	11.69			47.54			50.63		
		3	32.86			52.23			50.63		
		4	43.25								
	10	1	38.97	121.28	11102.48	42.23	129.40	11572.51	41.25	129.20	11505.37
		2	11.69			47.28			50.10		
		3	32.92			52.33			50.10		
		4	43.62								
d_A= (5,0.5)	6	1	17.00	54.38	2404.78	34.02	98.94	4297.62	21.62	93.39	4089.55
d_B= (15,1.5)		2	5.63			36.33			37.51		
d_C= (80,8)		3	15.61			38.64			37.51		
		4	17.79								
	7	1	15.99	55.12	2342.95	29.99	98.59	4013.80	21.30	92.31	3761.07
		2	5.63			35.57			36.82		
		3	15.98			41.16			36.82		
		4	19.44								
	8	1	30.95	82.39	7749.69	35.63	99.24	9252.57	21.98	97.31	9106.27
		2	5.80			37.52			41.04		
		3	17.08			39.41			41.04		
		4	31.62								
	9	1	27.07	76.51	7036.68	32.99	99.06	8944.68	21.73	96.22	8710.04
		2	5.81			36.71			39.81		
		3	17.01			40.43			39.81		
		4	29.56								
	10	1	27.35	77.79	7101.52	32.01	99.03	8850.92	21.69	95.59	8558.12
		2	5.81			36.61			39.21		
		3	17.05			41.21			39.21		
		4	30.57								
d_A= (10,1)	6	1	33.18	112.34	4981.52	73.33	210.11	9127.12	53.92	203.90	8910.95
d_B= (40,4)		2	11.37			77.00			80.03		
d_C= (160,16)		3	34.59			80.67			80.03		
		4	34.72								

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	7	1	28.20	106.57	4562.49	64.50	209.25	8518.27	53.22	201.54	8193.18
		2	11.37			75.17			78.06		
		3	34.20			85.85			78.06		
		4	34.30								
	8	1	51.74	154.25	14523.98	76.06	210.53	19642.67	54.77	207.66	19425.07
		2	11.74			78.62			84.39		
		3	43.24			81.19			84.39		
		4	52.85								
	9	1	59.08	173.41	15969.17	70.84	210.28	18988.59	54.24	206.18	18645.80
		2	11.74			77.51			82.44		
		3	44.56			84.17			82.44		
		4	64.51								
	10	1	57.57	171.75	15708.40	69.67	210.20	18778.66	54.14	205.60	18383.35
		2	11.74			77.26			81.78		
		3	44.55			84.84			81.78		
		4	64.34								
d_A= (10,1)	6	1	35.32	118.82	5267.41	44.07	129.75	5652.18	46.77	129.77	5646.93
d_B= (50,5)		2	11.36			46.70			46.77		
d_C= (70,7)		3	36.97			49.33			46.77		
		4	36.97								
	7	1	30.67	115.14	4928.24	39.19	129.37	5283.56	45.90	129.39	5230.14
		2	11.36			45.87			45.90		
		3	37.31			52.55			45.90		
		4	37.31								
	8	1	38.92	125.88	11859.28	45.93	130.07	12147.71	47.63	130.01	12143.46
		2	11.68			47.94			47.63		
		3	39.77			49.94			47.63		
		4	39.78								
	9	1	36.14	123.67	11443.95	42.90	129.89	11750.27	47.05	129.87	11716.25
		2	11.70			47.13			47.05		
		3	39.52			51.38			47.06		
		4	39.52								
	10	1	36.45	125.69	11555.04	41.75	129.85	11627.63	47.05	129.87	11575.09
		2	11.69			47.01			47.06		
		3	40.85			52.26			47.06		
		4	40.87								
d_A= (10,1)	6	1	35.39	119.42	5296.10	51.43	149.27	6498.07	53.74	149.29	6492.67
d_B= (50,5)		2	11.37			53.92			54.10		
d_C= (90,9)		3	37.03			56.41			54.10		
		4	37.04								
	7	1	30.70	115.32	4936.88	45.53	148.83	6070.78	53.02	148.90	6010.96
		2	11.37			52.97			53.16		
		3	37.33			60.40			53.16		
		4	37.33								
	8	1	44.24	142.29	13410.54	53.13	149.66	13973.45	55.29	149.66	13968.66
		2	11.69			55.39			55.49		
		3	44.91			57.65			55.49		
		4	45.21								
	9	1	42.22	142.05	13138.28	49.52	149.47	13514.27	54.31	149.47	13473.57
		2	11.70			54.53			54.69		
		3	45.74			59.54			54.68		
		4	46.17								
	10	1	41.52	141.94	13050.20	48.12	149.39	13372.22	54.16	149.38	13310.03
		2	11.70			54.27			54.32		
		3	46.00			60.41			54.32		
		4	46.49								

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Table-B2_2 (A>B>C) (low variance low cost)

			OVERFLOW			NESTED			SYMMETRIC		
	Cost.S.	**Dept.**	**c***	**TH**	**PR**	**c***	**TH**	**PR**	**c***	**TH**	**PR**
d_A= **(50,5)**	**6**	**1**	30.79	119.66	5275.58	48.39	116.57	5072.78	42.72	119.59	5210.67
d_B= **(40,4)**		**2**	34.34			42.62			42.72		
d_C= **(30,3)**		**3**	34.34			32.27			42.72		
		4	29.68								
	7	**1**	25.87	119.47	5125.66	44.83	113.74	4560.76	42.15	119.34	4829.09
		2	35.77			42.24			42.15		
		3	35.73			32.64			42.15		
		4	30.80								
	8	**1**	32.15	119.83	11262.03	52.40	118.82	11089.29	43.59	119.82	11197.38
		2	34.50			43.67			43.59		
		3	34.50			32.74			43.59		
		4	30.21								
	9	**1**	29.08	119.78	11090.90	51.71	118.55	10625.92	43.01	119.69	10807.36
		2	35.41			43.55			43.01		
		3	35.41			32.94			43.01		
		4	31.04								
	10	**1**	28.10	119.76	11041.56	50.61	118.04	10445.76	43.01	119.69	10678.33
		2	35.75			43.39			43.01		
		3	35.76			33.04			43.01		
		4	31.15								
d_A= **(90,9)**	**6**	**1**	34.89	110.63	4890.51	28.50	66.27	2887.05	49.85	129.11	5611.23
d_B= **(30,3)**		**2**	36.51			28.39			40.98		
d_C= **(10,1)**		**3**	31.45			11.31			49.85		
		4	11.37								
	7	**1**	29.74	105.14	4476.05	24.90	61.79	2464.20	48.68	128.59	5189.22
		2	36.17			27.22			40.44		
		3	31.36			11.34			48.68		
		4	11.37								
	8	**1**	40.56	120.76	11363.14	32.72	71.65	6695.60	51.32	129.49	12082.96
		2	41.44			30.48			41.66		
		3	32.61			11.59			51.32		
		4	11.69								
	9	**1**	39.56	121.44	11190.91	31.29	70.14	6276.66	50.63	129.33	11650.80
		2	43.25			30.27			41.16		
		3	32.86			11.60			50.63		
		4	11.69								
	10	**1**	38.97	121.28	11102.48	30.21	68.90	6079.17	50.09	129.20	11505.34
		2	43.62			29.94			41.25		
		3	32.92			11.62			50.10		
		4	11.70								
d_A= **(80,8)**	**6**	**1**	17.67	55.79	2466.55	15.67	35.03	1523.70	37.51	93.39	4089.55
d_B= **(15,1.5)**		**2**	18.49			14.99			21.62		
d_C= **(5,0.5)**		**3**	15.78			5.58			37.51		
		4	5.63								
	7	**1**	16.62	56.55	2401.57	14.99	34.42	1361.68	36.82	92.31	3761.08
		2	20.21			15.16			21.30		
		3	16.11			5.56			36.82		
		4	5.63								
	8	**1**	31.65	83.76	7877.93	19.22	38.96	3636.54	41.04	97.31	9106.27
		2	32.33			16.07			21.98		
		3	17.10			5.66			41.04		
		4	5.80								
	9	**1**	25.72	73.68	6779.14	17.98	37.67	3361.50	39.81	96.22	8710.03
		2	28.09			15.87			21.73		
		3	16.95			5.68			39.81		
		4	5.81								
	10	**1**	27.35	77.79	7101.51	17.88	37.57	3301.19	39.21	95.59	8558.12
		2	30.57			15.88			21.69		
		3	17.05			5.68			39.21		
		4	5.81								
d_A= **(160,16)**	**6**	**1**	35.50	118.92	5271.37	34.60	79.60	3469.96	80.03	203.90	8910.98
d_B= **(40,4)**		**2**	37.15			35.21			53.91		
d_C= **(10,1)**		**3**	36.69			11.36			80.03		
		4	11.37								

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	7	**1**	30.28	113.15	4840.75	31.55	76.18	3029.05	78.06	201.54	8193.19
		2	36.82			34.80			53.22		
		3	36.42			11.36			78.06		
		4	11.37								
	8	**1**	55.22	161.39	15192.34	38.42	85.71	8015.47	84.39	207.66	19425.09
		2	56.41			37.99			54.77		
		3	43.78			11.67			84.39		
		4	11.74								
	9	**1**	54.37	163.54	15071.14	36.91	84.07	7528.66	82.44	206.18	18645.79
		2	59.36			37.81			54.25		
		3	44.12			11.67			82.44		
		4	11.74								
	10	**1**	48.94	153.35	14051.40	36.66	83.92	7403.92	81.78	205.60	18383.35
		2	54.70			37.94			54.14		
		3	43.54			11.67			81.78		
		4	11.74								
d_A= (70,7)	**6**	**1**	34.24	115.73	5131.85	35.25			46.77		
d_B= (50,5)		**2**	35.84			36.07	81.28	3543.88	46.77	129.77	5646.93
d_C= (10,1)		**3**	35.83			11.37			46.77		
		4	11.36								
	7	**1**	29.18	110.12	4714.74	32.85			45.90		
		2	35.49			36.39	79.21	3148.85	45.90	129.39	5230.14
		3	35.49			11.37			45.90		
		4	11.37								
	8	**1**	38.92	125.88	11859.29	45.33			47.63		
		2	39.78			45.33	100.22	9373.90	47.63	130.01	12143.46
		3	39.77			11.71			47.63		
		4	11.68								
	9	**1**	37.08	125.74	11628.08	43.72			47.05		
		2	40.59			45.24	98.54	8819.67	47.05	129.87	11716.25
		3	40.58			11.72			47.05		
		4	11.69								
	10	**1**	36.45	125.69	11555.05	43.41			47.05		
		2	40.87			45.43	98.39	8674.73	47.05	129.87	11575.09
		3	40.85			11.72			47.05		
		4	11.69								
d_A= (90,9)	**6**	**1**	34.27	115.97	5142.87	35.25			54.10		
d_B= (50,5)		**2**	35.87			36.07	81.28	3543.88	53.74	149.29	6492.67
d_C= (10,1)		**3**	35.86			11.37			54.10		
		4	11.37								
	7	**1**	29.69	111.87	4789.72	32.85			53.17		
		2	36.11			36.39	79.21	3148.85	53.02	148.90	6010.96
		3	36.11			11.37			53.16		
		4	11.37								
	8	**1**	44.24	142.29	13410.55	45.33			55.49		
		2	45.21			45.33	100.22	9373.90	55.29	149.66	13968.66
		3	44.91			11.71			55.49		
		4	11.69								
	9	**1**	42.22	142.05	13138.28	43.72			54.69		
		2	46.17			45.24	98.54	8819.67	54.30	149.47	13473.58
		3	45.74			11.72			54.69		
		4	11.69								
	10	**1**	41.52	141.94	13050.20	43.41			54.32		
		2	46.49			45.43	98.39	8674.73	54.16	149.38	13310.03
		3	46.00			11.72			54.32		
		4	11.70								

Table-B2_3 (A=B=C) (low variance low cost)

			OVERFLOW			NESTED			SYMMETRIC		
	Cost.S.	**Dept.**	**c***	**TH**	**PR**	**c***	**TH**	**PR**	**c***	**TH**	**PR**
d_A= (5,0.5)	**6**	**1**	3.56	14.96	656.22	5.67			5.68		
d_B= (5,0.5)		**2**	4.47			5.66	14.97	646.32	5.68	14.97	646.28
d_C= (5,0.5)		**3**	4.47			5.67			5.68		
		4	4.47								
	7	**1**	1.65	14.93	649.92	5.33			5.48		
		2	5.01			5.49	14.88	597.09	5.48	14.89	596.60
		3	5.01			5.64			5.48		
		4	5.01								
	8	**1**	3.70	14.98	1404.86	5.79			5.77		
		2	4.51			5.81	15.00	1394.98	5.77	14.99	1395.03
		3	4.51			5.87			5.77		
		4	4.51								
	9	**1**	2.27	14.98	1393.94	5.60			5.68		
		2	4.96			5.65	14.97	1344.08	5.68	14.97	1343.66
		3	4.96			5.80			5.68		
		4	4.96								
	10	**1**	1.87	14.97	1393.24	5.58			5.68		
		2	5.07			5.64	14.97	1327.28	5.68	14.97	1326.62
		3	5.07			5.81			5.68		
		4	5.07								
d_A= (10,1)	**6**	**1**	6.79	29.88	1310.66	11.16			11.26		
d_B= (10,1)		**2**	9.05			11.25	29.85	1290.39	11.26	29.86	1290.30
d_C= (10,1)		**3**	9.05			11.33			11.26		
		4	9.05								
	7	**1**	4.50	29.82	1287.63	10.66			11.07		
		2	9.67			10.87	29.69	1191.16	11.07	29.78	1189.98
		3	9.67			11.36			11.07		
		4	9.67								
	8	**1**	8.16	29.97	2805.50	11.50			11.64		
		2	8.95			11.65	29.96	2786.80	11.64	29.96	2786.80
		3	8.95			11.69			11.64		
		4	8.95								
	9	**1**	6.17	29.95	2772.13	11.24			11.45		
		2	9.54			11.41	29.92	2683.85	11.45	29.92	2682.83
		3	9.53			11.77			11.45		
		4	9.53								
	10	**1**	5.62	29.95	2765.09	11.20			11.45		
		2	9.69			11.37	29.91	2650.04	11.45	29.92	2648.48
		3	9.69			11.76			11.45		
		4	9.69								
d_A= (30,3)	**6**	**1**	23.21	89.25	3910.68	33.23			33.49		
d_B= (30,3)		**2**	25.97			33.50	89.17	3856.24	33.49	89.17	3855.95
d_C= (30,3)		**3**	25.97			33.70			33.49		
		4	25.96								
	7	**1**	18.56	89.05	3806.22	31.86			32.91		
		2	26.99			32.76	88.82	3561.29	32.91	88.92	3557.31
		3	26.99			34.13			32.91		
		4	26.99								
	8	**1**	24.33	89.52	8381.57	34.42			34.64		
		2	26.70			34.61	89.49	8326.02	34.64	89.49	8325.83
		3	26.70			34.87			34.64		
		4	26.70								
	9	**1**	21.22	89.45	8258.94	33.49			34.06		
		2	27.37			34.03	89.36	8019.32	34.06	89.36	8016.39
		3	27.37			35.07			34.06		
		4	27.37								
	10	**1**	21.01	89.41	8216.95	33.05			33.77		
		2	27.23			33.78	89.28	7918.64	33.77	89.28	7914.46
		3	27.23			35.20			33.78		
		4	27.23								
d_A= (40,4)	**6**	**1**	30.70	119.75	5252.74	44.20			44.46		
d_B= (40,4)		**2**	34.66			44.42	119.59	5179.97	44.46	119.60	5179.63
d_C= (40,4)		**3**	34.65			44.70			44.46		
		4	34.66								

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	7	1	25.48	119.41	5106.54	42.67			43.59		
		2	35.50			43.66	119.21	4787.52	43.59	119.20	4783.05
		3	35.50			45.03			43.59		
		4	35.50								
	8	1	32.86	120.11	11249.42	45.80			46.19		
		2	35.43			46.16	120.07	11176.35	46.19	120.07	11176.09
		3	35.44			46.54			46.19		
		4	35.43								
	9	1	29.37	119.95	11076.06	44.45			45.32		
		2	35.83			45.43	119.88	10768.41	45.32	119.88	10764.17
		3	35.83			46.54			45.32		
		4	35.83								
	10	1	28.21	119.91	11027.24	44.26			45.03		
		2	36.04			44.98	119.81	10634.66	45.03	119.80	10628.52
		3	36.04			46.52			45.03		
		4	36.04								
d_A= (80,8)	6	1	62.09	237.08	10392.89	87.90			88.92		
d_B= (80,8)		2	68.44			88.89	236.95	10250.64	88.92	237.00	10249.72
d_C= (80,8)		3	68.44			89.68			88.90		
		4	68.43								
	7	1	52.15	235.77	10077.24	84.69			86.60		
		2	68.90			86.91	236.04	9469.80	86.60	235.96	9459.65
		3	68.90			89.78			86.60		
		4	68.90								
	8	1	66.12	238.01	22276.18	91.53			92.95		
		2	70.78			92.29	237.91	22130.41	92.96	238.03	22129.87
		3	70.78			93.17			92.95		
		4	70.78								
	9	1	59.44	237.46	21915.52	88.39			90.07		
		2	70.53			90.36	237.46	21316.34	90.07	237.38	21306.57
		3	70.53			93.47			90.07		
		4	70.53								
	10	1	57.96	237.14	21794.89	87.52			90.06		
		2	69.98			89.92	237.27	21050.07	90.06	237.38	21036.37
		3	69.99			93.03			90.08		
		4	69.98								
d_A= (160,16)	6	1	126.16	479.91	21045.09	178.15			179.55		
d_B= (160,16)		2	137.82			179.58	479.73	20758.82	179.56	479.80	20757.75
d_C= (160,16)		3	137.82			180.67			179.56		
		4	137.80								
	7	1	106.01	476.84	20387.91	171.36			175.51		
		2	138.38			175.25	477.65	19176.35	175.52	477.92	19157.18
		3	138.37			180.27			175.51		
		4	138.38								
	8	1	133.30	481.27	45082.81	183.54			184.75		
		2	140.72			184.88	481.17	44794.38	184.75	481.19	44793.78
		3	140.78			185.65			184.75		
		4	140.78								
	9	1	121.05	480.24	44342.26	179.19			181.86		
		2	140.54			181.71	480.49	43156.18	181.88	480.52	43141.30
		3	140.56			185.55			181.86		
		4	140.56								
	10	1	117.87	479.84	44110.37	177.61			181.29		
		2	140.40			180.80	480.18	42618.06	181.29	480.36	42596.99
		3	140.39			185.60			181.29		
		4	140.40								

Table-B3_1 (High variance high cost)(A<B<C)

			OVERFLOW			NESTED			SYMMETRIC		
	Cost.S.	**Dept.**	**c***	**TH**	**PR**	**c***	**TH**	**PR**	**c***	**TH**	**PR**
d_A= (30,3)	1	1	23.20	116.10	3681.27	34.33	115.82	3355.93	41.28	115.88	3317.26
d_B= (40,4)		2	29.19			42.65			41.28		
d_C= (50,5)		3	35.75			48.24			41.28		
		4	37.96								
	2	1	14.21	113.61	3564.51	29.81	113.51	2827.49	40.11	114.34	2707.37
		2	30.46			41.36			40.12		
		3	37.33			51.39			40.14		
		4	40.10								
	3	1	4.84	108.88	3555.35	17.63	102.76	2331.71	38.96	112.46	2115.48
		2	31.25			40.48			38.98		
		3	38.47			53.25			38.97		
		4	41.69								
	4	1	16.28	114.17	4071.85	30.91	113.86	3352.20	40.12	114.34	3248.85
		2	29.80			41.49			40.12		
		3	36.84			49.90			40.13		
		4	39.73								
	5	1	15.90	109.14	1899.69	29.26	108.26	1567.55	37.52	109.66	1542.45
		2	26.55			39.13			37.54		
		3	33.85			43.52			37.52		
		4	36.97								
d_A= (10,1)	1	1	22.84	90.96	2906.84	33.59	123.57	3623.52	37.74	121.83	3482.79
d_B= (30,3)		2	10.99			44.38			46.35		
d_C= (90,9)		3	28.34			55.18			46.35		
		4	32.01								
	2	1	15.28	87.95	2715.74	22.04	119.06	3230.39	36.38	119.84	2829.34
		2	10.99			41.49			45.06		
		3	29.71			60.92			45.06		
		4	35.74								
	3	1	5.50	81.49	2624.92	9.27	116.65	3147.25	35.18	116.71	2194.12
		2	11.00			39.18			43.09		
		3	30.51			72.87			43.10		
		4	38.62								
	4	1	18.01	90.90	3202.59	24.92	120.01	3760.75	36.34	119.90	3396.12
		2	10.81			42.02			45.13		
		3	29.62			59.12			45.13		
		4	36.13								
	5	1	14.77	78.32	1372.65	23.52	114.22	1761.27	33.77	113.00	1586.56
		2	9.57			39.10			41.16		
		3	25.95			54.68			41.16		
		4	29.56								
d_A= (5,0.5)	1	1	16.88	59.67	1881.67	25.64	95.11	2787.86	20.48	85.54	2505.86
d_B= (15,1.5)		2	5.57			34.23			34.04		
d_C= (80,8)		3	16.12			42.82			34.04		
		4	23.64								
	2	1	9.20	49.73	1521.38	16.70	91.97	2498.06	19.83	81.75	1992.73
		2	5.57			32.12			31.98		
		3	15.85			47.53			31.98		
		4	21.46								
	3	1	2.96	42.68	1371.15	6.14	85.47	2348.29	19.15	76.24	1487.80
		2	5.57			28.97			29.16		
		3	15.72			52.00			29.17		
		4	20.71								
	4	1	10.17	49.39	1730.64	18.80	92.53	2904.47	19.83	81.95	2396.48
		2	5.49			32.45			32.09		
		3	15.50			46.10			32.09		
		4	20.33								
	5	1	7.52	39.71	696.29	18.04	88.27	1360.79	18.33	74.27	1084.08
		2	4.73			30.24			28.40		
		3	13.20			42.42			28.40		
		4	15.03								
d_A= (10,1)	1	1	24.61	101.05	3257.80	55.23	198.81	5816.91	50.22	185.93	5396.74
d_B= (40,4)		2	11.00			71.43			72.38		
d_C= (160,16)		3	33.16			87.62			72.38		
		4	34.46								

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	2	**1**	14.61	90.44	2841.37	35.69	188.91	5128.93	48.43	178.05	4305.25
		2	11.00			65.50			67.73		
		3	32.88			95.34			67.72		
		4	34.08								
	3	**1**	5.07	83.14	2722.85	12.97	172.85	4748.11	46.46	172.56	3329.85
		2	11.00			58.38			65.07		
		3	33.91			103.95			65.07		
		4	35.50								
	4	**1**	23.66	117.20	4132.05	40.35	191.49	6001.77	48.43	178.23	5170.49
		2	10.83			66.86			67.84		
		3	39.55			93.40			67.84		
		4	47.32								
	5	**1**	13.91	77.91	1396.19	37.97	181.03	2800.25	44.58	164.21	2376.50
		2	9.57			61.79			61.06		
		3	27.37			85.65			61.05		
		4	27.82								
d_A= (10,1)	**1**	**1**	23.87	99.67	3222.16	34.07	124.56	3658.38	44.34	124.80	3554.73
d_B= (50,5)		**2**	10.98			44.68			44.96		
d_C= (70,7)		**3**	33.19			54.95			44.96		
		4	33.47								
	2	**1**	15.32	95.62	3012.73	22.84	120.66	3250.24	43.09	122.80	2892.44
		2	10.99			42.86			43.40		
		3	35.34			60.81			43.41		
		4	35.82								
	3	**1**	5.41	89.61	2944.64	9.89	118.99	3049.17	41.25	120.38	2253.67
		2	11.00			46.50			42.12		
		3	37.22			70.70			42.14		
		4	37.96								
	4	**1**	17.33	95.55	3442.47	25.45	121.47	3794.29	43.09	122.80	3470.92
		2	10.81			43.06			43.40		
		3	34.33			59.19			43.40		
		4	34.73								
	5	**1**	13.18	74.74	1344.50	24.37	116.06	1780.68	40.04	117.61	1638.16
		2	9.57			40.34			40.59		
		3	26.27			54.54			40.57		
		4	26.35								
d_A= (10,1)	**1**	**1**	23.92	99.99	3235.97	39.40	142.99	4208.28	48.98	143.66	4087.70
d_B= (50,5)		**2**	11.00			50.95			52.88		
d_C= (90,9)		**3**	33.21			62.47			52.90		
		4	33.50								
	2	**1**	15.97	98.97	3116.56	26.07	138.50	3756.57	47.42	141.31	3327.67
		2	11.00			48.28			51.03		
		3	36.60			69.93			51.05		
		4	37.26								
	3	**1**	5.43	89.72	2948.72	10.68	134.02	3577.98	45.74	138.43	2592.48
		2	11.00			47.66			49.26		
		3	37.21			80.85			49.30		
		4	37.99								
	4	**1**	19.80	106.77	3833.46	29.27	139.52	4378.80	47.46	141.32	3993.15
		2	10.81			48.76			51.03		
		3	38.55			67.92			51.04		
		4	39.66								
	5	**1**	13.18	74.74	1344.50	27.88	133.77	2064.26	43.78	134.74	1884.82
		2	9.57			45.91			47.41		
		3	26.27			63.39			47.44		
		4	26.35								

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Table-B3_2 (High variance high cost)(A>B>C)

			OVERFLOW			NESTED			SYMMETRIC		
	Cost.S.	**Dept.**	**c***	**TH**	**PR**	**c***	**TH**	**PR**	**c***	**TH**	**PR**
d_A= (50,5)	1	1	23.20	116.10	3681.27	38.90	103.99	2958.04	41.28	115.88	3317.26
d_B= (40,4)		2	37.97			39.96			41.28		
d_C= (30,3)		3	35.75			31.33			41.28		
		4	29.19								
	2	1	14.21	113.61	3564.53	26.27	90.65	2187.34	40.11	114.34	2707.37
		2	40.10			37.87			40.12		
		3	37.33			31.95			40.14		
		4	30.46								
	3	1	4.84	108.89	3555.33	10.31	73.54	1647.17	38.97	112.46	2115.48
		2	41.69			36.05			38.98		
		3	38.47			32.30			38.96		
		4	31.25								
	4	1	16.28	114.17	4071.86	28.54	92.57	2656.08	40.13	114.34	3248.85
		2	39.73			37.62			40.13		
		3	36.84			31.40			40.12		
		4	29.80								
	5	1	16.12	108.66	1893.48	23.84	83.10	1193.48	37.53	109.66	1542.45
		2	36.22			33.52			37.53		
		3	33.56			27.81			37.53		
		4	26.71								
d_A= (90,9)	1	1	22.84	90.96	2906.83	19.88	52.28	1489.68	46.35	121.83	3482.77
d_B= (30,3)		2	32.01			23.18			37.74		
d_C= (10,1)		3	28.34			10.91			46.35		
		4	10.99								
	2	1	15.65	89.32	2753.96	13.82	45.55	1069.26	45.06	119.84	2829.35
		2	36.63			22.42			36.38		
		3	29.97			10.93			45.06		
		4	10.99								
	3	1	4.97	76.65	2479.98	4.40	31.47	685.70	43.10	116.71	2194.12
		2	34.83			17.49			35.18		
		3	29.44			10.99			43.10		
		4	11.00								
	4	1	18.01	90.90	3202.59	13.96	43.98	1246.34	45.13	119.90	3396.10
		2	36.13			20.62			36.34		
		3	29.62			10.78			45.13		
		4	10.81								
	5	1	13.96	75.34	1326.14	10.41	34.90	498.19	41.16	112.99	1586.55
		2	27.92			15.54			33.77		
		3	25.25			9.56			41.16		
		4	9.57								
d_A= (80,8)	1	1	13.43	51.06	1622.28	11.35	28.72	811.20	34.04	85.54	2505.86
d_B= (15,1.5)		2	18.81			12.93			20.48		
d_C= (5,0.5)		3	15.29			5.50			34.04		
		4	5.57								
	2	1	8.42	47.00	1444.46	7.09	23.29	544.83	31.98	81.75	1992.71
		2	19.66			11.52			19.83		
		3	15.50			5.54			31.98		
		4	5.57								
	3	1	3.24	45.04	1442.48	2.88	18.79	390.45	29.17	76.24	1487.81
		2	22.66			11.21			19.15		
		3	16.03			5.55			29.16		
		4	5.57								
	4	1	9.73	48.00	1685.45	8.47	25.22	699.86	32.09	81.95	2396.48
		2	19.45			12.20			19.83		
		3	15.33			5.42			32.09		
		4	5.49								
	5	1	6.71	36.72	649.56	6.46	20.45	287.32	28.40	74.27	1084.07
		2	13.41			9.60			18.33		
		3	12.46			4.70			28.40		
		4	4.73								

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d_A= (160,16)	1	1	25.61	104.22	3355.54	21.61	56.90	1624.84	72.39	185.93	5396.76
d_B= (40,4)		2	35.85			25.77			50.22		
d_C= (10,1)		3	34.14			10.98			72.38		
		4	11.00								
	2	1	14.93	91.95	2886.00	13.66	45.95	1087.36	67.73	178.05	4305.26
		2	34.83			22.70			48.43		
		3	33.42			10.99			67.73		
		4	11.00								
	3	1	5.31	85.83	2805.93	4.88	33.98	730.31	65.07	172.55	3329.84
		2	37.14			19.48			46.45		
		3	34.99			11.00			65.07		
		4	11.00								
	4	1	21.55	109.98	3899.05	15.56	48.35	1364.95	67.84	178.23	5170.50
		2	43.09			23.27			48.43		
		3	37.99			10.80			67.84		
		4	10.83								
	5	1	13.91	77.91	1396.19	12.60	40.47	571.34	61.06	164.21	2376.52
		2	27.82			18.89			44.58		
		3	27.37			9.57			61.05		
		4	9.57								
d_A= (70,7)	1	1	23.87	99.67	3222.16	23.51			44.97		
d_B= (50,5)		2	33.47			28.12	61.21	1745.33	44.34	124.80	3554.73
d_C= (10,1)		3	33.19			10.99			44.96		
		4	10.98								
	2	1	14.15	89.35	2820.13	14.69			43.41		
		2	33.04			24.45	48.75	1147.26	43.09	122.80	2892.44
		3	32.78			11.00			43.40		
		4	11.00								
	3	1	5.10	85.38	2808.94	5.36			42.13		
		2	35.75			21.42	36.41	771.34	41.25	120.38	2253.67
		3	35.28			11.00			42.13		
		4	11.00								
	4	1	17.67	97.11	3496.52	17.77			43.40		
		2	35.41			26.60	53.91	1509.39	43.09	122.80	3470.92
		3	34.95			10.81			43.40		
		4	10.80								
	5	1	11.83	68.09	1225.39	11.49			40.58		
		2	23.66			17.22	37.69	535.28	40.04	117.61	1638.15
		3	23.62			9.57			40.58		
		4	9.57								
d_A= (90,9)	1	1	23.92	99.99	3235.97	23.51			52.88		
d_B= (50,5)		2	33.50			28.12	61.21	1745.33	48.98	143.65	4087.69
d_C= (10,1)		3	33.21			10.99			52.88		
		4	11.00								
	2	1	14.76	92.63	2922.03	14.69			51.05		
		2	34.44			24.45	48.75	1147.26	47.46	141.33	3327.64
		3	34.07			11.00			51.04		
		4	11.00								
	3	1	5.00	83.98	2764.07	5.36			49.27		
		2	35.04			21.42	36.41	771.34	45.78	138.45	2592.43
		3	34.61			11.00			49.29		
		4	11.00								
	4	1	19.81	106.77	3833.45	17.77			51.03		
		2	39.66			26.60	53.91	1509.39	47.43	141.30	3993.18
		3	38.55			10.81			51.03		
		4	10.81								
	5	1	11.83	68.09	1225.39	11.49			47.43		
		2	23.66			17.22	37.69	535.28	43.82	134.76	1884.78
		3	23.62			9.57			47.41		
		4	9.57								

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Table-B3_3 (High variance high cost)(A=B=C)

			OVERFLOW			NESTED			SYMMETRIC		
	Cost.S.	**Dept.**	**c***	**TH**	**PR**	**c***	**TH**	**PR**	**c***	**TH**	**PR**
d_A= (5,0.5)		**1**	0.87	14.34	463.62	4.97			5.29		
d_B= (5,0.5)	**1**	**2**	5.15			5.20	14.07	392.21	5.29	14.17	391.09
d_C= (5,0.5)		**3**	5.15			5.53			5.29		
		4	5.15								
		1	0.00	14.31	472.02	4.54			5.10		
	2	**2**	5.41			4.99	13.72	318.97	5.10	13.91	313.20
		3	5.41			5.58			5.10		
		4	5.41								
		1	0.00	14.36	472.46	3.18			4.72		
	3	**2**	5.46			4.72	12.42	252.79	4.72	13.29	239.97
		3	5.46			5.58			4.71		
		4	5.46								
		1	0.00	14.35	534.17	4.54			5.10		
	4	**2**	5.44			5.02	13.71	380.78	5.10	13.91	375.84
		3	5.44			5.49			5.10		
		4	5.44								
		1	0.00	13.26	239.96	4.11			4.52		
	5	**2**	4.70			4.38	12.64	172.43	4.52	12.91	170.86
		3	4.70			4.73			4.52		
		4	4.70								
d_A= (10,1)		**1**	3.43	28.62	914.32	10.06			10.49		
d_B= (10,1)	**1**	**2**	9.58			10.56	28.37	790.70	10.49	28.36	788.55
d_C= (10,1)		**3**	9.58			11.01			10.49		
		4	9.58								
		1	1.30	28.35	919.79	9.09			10.10		
	2	**2**	10.05			10.05	27.56	643.47	10.10	27.83	633.85
		3	10.05			11.01			10.10		
		4	10.05								
		1	0.00	27.14	921.60	5.46			9.53		
	3	**2**	9.67			9.57	24.06	505.52	9.53	26.88	486.48
		3	9.67			11.00			9.53		
		4	9.67								
		1	1.68	28.31	1037.94	9.31			10.10		
	4	**2**	9.89			10.05	27.65	768.56	10.10	27.83	760.63
		3	9.89			10.83			10.10		
		4	9.89								
		1	1.35	26.46	471.95	8.47			8.95		
	5	**2**	8.85			8.98	25.78	349.87	8.95	25.73	346.87
		3	8.85			9.52			8.95		
		4	8.85								
d_A= (30,3)		**1**	15.28	84.57	2657.27	29.84			31.46		
d_B= (30,3)	**1**	**2**	26.43			31.54	84.31	2344.73	31.48	84.50	2337.27
d_C= (30,3)		**3**	26.43			32.92			31.46		
		4	26.43								
		1	8.68	80.52	2559.68	26.49			30.03		
	2	**2**	25.83			29.77	81.51	1908.92	30.02	82.58	1877.18
		3	25.83			32.99			30.02		
		4	25.83								
		1	2.50	72.04	2416.76	14.25			28.29		
	3	**2**	23.84			28.20	69.46	1490.86	28.29	79.76	1441.84
		3	23.84			32.99			28.29		
		4	23.84								
		1	9.99	81.65	2934.49	27.08			29.74		
	4	**2**	26.08			29.75	81.68	2279.31	29.74	82.15	2252.74
		3	26.08			32.30			29.72		
		4	26.08								
		1	9.32	76.39	1335.22	24.74			26.56		
	5	**2**	23.46			26.70	76.15	1037.82	26.56	76.37	1029.86
		3	23.46			28.19			26.56		
		4	23.46								

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d_A= (40,4)		**1**	20.95	113.71	3580.27	40.27			42.15		
d_B= (40,4)	**1**	**2**	35.14			42.15	113.70	3168.55	42.14	113.74	3157.98
d_C= (40,4)		**3**	35.14			44.43			42.15		
		4	35.14								
		1	11.87	107.85	3432.85	35.58			40.41		
	2	**2**	34.31			40.37	110.06	2579.83	40.41	111.39	2538.33
		3	34.31			44.58			40.42		
		4	34.31								
		1	3.46	96.78	3247.87	19.52			38.11		
	3	**2**	31.90			38.17	94.17	2016.48	38.09	107.55	1948.24
		3	31.90			44.56			38.11		
		4	31.90								
		1	13.82	109.39	3937.13	36.68			40.42		
	4	**2**	34.56			40.41	110.57	3080.32	40.41	111.39	3046.00
		3	34.55			43.73			40.42		
		4	34.55								
		1	12.85	101.41	1787.04	33.43			35.79		
	5	**2**	30.77			36.05	102.94	1404.44	35.80	103.01	1392.06
		3	30.77			38.11			35.80		
		4	30.77								
d_A= (80,8)		**1**	42.28	223.26	7087.90	79.02			83.71		
d_B= (80,8)	**1**	**2**	67.07			83.66	225.75	6326.65	83.71	226.57	6305.65
d_C= (80,8)		**3**	67.07			87.47			83.72		
		4	67.07								
		1	24.36	211.61	6761.92	71.04			79.10		
	2	**2**	65.91			78.80	218.67	5164.16	79.09	220.33	5084.41
		3	65.91			87.56			79.10		
		4	65.91								
		1	7.36	188.15	6316.64	37.51			75.62		
	3	**2**	61.32			75.71	185.98	4024.64	75.63	214.61	3923.54
		3	61.32			87.66			75.64		
		4	61.32								
		1	28.50	214.18	7756.97	72.32			79.11		
	4	**2**	65.90			78.83	219.06	6167.67	79.08	220.33	6101.29
		3	65.90			85.90			79.09		
		4	65.90								
		1	26.37	202.70	3594.68	67.76			71.60		
	5	**2**	60.95			71.48	206.43	2839.11	71.61	206.71	2818.51
		3	60.95			75.68			71.56		
		4	60.95								
d_A= (160,16)		**1**	85.09	444.99	14086.82	159.69			167.43		
d_B= (160,16)	**1**	**2**	134.11			167.46	451.39	12570.73	167.42	451.41	12524.82
d_C= (160,16)		**3**	134.12			177.17			167.44		
		4	134.12								
		1	48.14	417.24	13316.24	141.70			159.93		
	2	**2**	130.24			159.92	437.06	10231.25	159.91	441.28	10069.51
		3	130.23			177.63			159.93		
		4	130.24								
		1	14.20	355.21	11926.37	79.19			151.23		
	3	**2**	115.45			151.62	375.72	8008.16	151.25	426.83	7727.62
		3	115.45			177.71			151.31		
		4	115.45								
		1	56.62	422.74	15287.46	145.03			159.96		
	4	**2**	130.21			159.81	438.20	12215.04	159.92	441.28	12083.41
		3	130.21			174.08			159.89		
		4	130.20								
		1	51.89	393.45	6973.09	126.69			142.62		
	5	**2**	118.04			142.48	402.98	5547.91	142.62	409.62	5507.62
		3	118.04			151.55			142.57		
		4	118.06								

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Table-B4_1 (high variance low cost)

	Cost.S.	Dept.	OVERFLOW c*	TH	PR	NESTED c*	TH	PR	SYMMETRIC c*	TH	PR
d_A= (30,3)	6	1	34.21	119.33	5202.69	43.71	119.25	5141.90	45.61	119.21	5139.71
d_B= (40,4)		2	30.86			46.17			45.61		
d_C= (50,5)		3	36.43			47.55			45.61		
		4	37.62								
	7	1	27.74	118.91	5038.71	38.71	118.61	4768.96	44.46	118.67	4733.12
		2	31.76			45.14			44.46		
		3	37.80			50.45			44.46		
		4	39.63								
	8	1	36.19	119.75	11181.43	46.22	119.73	11118.76	47.34	119.69	11117.02
		2	31.47			47.92			47.34		
		3	37.63			48.67			47.34		
		4	39.02								
	9	1	31.52	119.58	10994.41	42.45	119.47	10720.32	46.47	119.50	10694.88
		2	32.19			46.94			46.48		
		3	38.54			50.42			46.48		
		4	40.09								
	10	1	30.91	119.49	10933.01	41.40	119.39	10594.64	46.19	119.41	10555.72
		2	32.39			46.78			46.19		
		3	38.32			51.06			46.19		
		4	39.82								
d_A= (10,1)	6	1	39.60	116.31	5105.90	46.90	128.12	5507.59	41.65	127.12	5474.51
d_B= (30,3)		2	12.37			50.30			52.63		
d_C= (90,9)		3	32.59			53.69			52.63		
		4	41.54								
	7	1	35.77	115.39	4857.29	40.18	127.13	5119.12	40.52	126.38	5029.83
		2	12.40			48.20			51.36		
		3	33.15			56.21			51.35		
		4	43.93								
	8	1	45.12	122.73	11491.04	50.47	128.80	11932.95	42.83	128.29	11898.96
		2	12.94			52.86			56.07		
		3	34.06			55.25			56.08		
		4	46.21								
	9	1	43.10	122.45	11210.35	45.68	128.32	11499.51	42.11	127.76	11420.19
		2	12.96			50.89			54.26		
		3	34.34			56.10			54.26		
		4	47.59								
	10	1	42.39	122.34	11120.87	44.08	128.22	11369.46	41.79	127.72	11269.97
		2	12.96			50.57			54.21		
		3	34.43			57.06			54.21		
		4	48.05								
d_A= (5,0.5)	6	1	27.29	75.25	3308.72	36.46	99.23	4253.18	22.66	93.82	4074.81
d_B= (15,1.5)		2	6.28			39.71			40.01		
d_C= (80,8)		3	17.70			42.97			40.01		
		4	28.57								
	7	1	24.82	74.50	3130.81	31.18	98.19	3945.80	22.10	91.64	3698.04
		2	6.28			37.58			38.05		
		3	17.86			43.99			38.05		
		4	30.22								
	8	1	34.25	87.15	8173.93	39.14	99.79	9231.17	23.29	96.42	8985.28
		2	6.64			41.84			43.07		
		3	18.54			44.55			43.07		
		4	35.01								
	9	1	33.53	87.92	8046.04	35.59	99.43	8892.08	22.93	95.45	8586.35
		2	6.63			40.34			41.81		
		3	18.63			45.08			41.81		
		4	36.74								
	10	1	33.05	87.82	7974.69	34.58	99.29	8785.47	22.78	95.43	8478.99
		2	6.63			39.89			41.79		
		3	18.64			45.19			41.79		
		4	37.10								
d_A= (10,1)	6	1	50.83	151.78	6689.16	78.20	208.30	8932.63	55.82	202.21	8740.25
d_B= (40,4)		2	12.50			82.87			86.27		
d_C= (160,16)		3	43.08			87.55			86.27		
		4	53.21								

	7	**1**	46.11	150.31	6353.42	66.14	206.10	8281.48	54.32	199.34	7984.79
		2	12.51			78.64			82.95		
		3	43.90			91.14			82.95		
		4	56.12								
	8	**1**	61.53	171.39	16099.04	83.88	209.57	19388.93	57.73	206.49	19182.98
		2	13.14			87.54			93.34		
		3	45.91			91.20			93.34		
		4	62.88								
	9	**1**	58.40	169.55	15580.36	75.43	208.69	18669.60	56.71	205.10	18367.79
		2	13.15			84.02			90.67		
		3	46.12			92.60			90.67		
		4	63.86								
	10	**1**	57.55	169.32	15450.07	73.46	208.35	18440.12	56.43	203.50	18024.57
		2	13.15			83.00			88.04		
		3	46.23			92.54			88.04		
		4	64.44								
d_A= (10,1)	**6**	**1**	35.30	115.76	5111.11	46.75	128.52	5534.24	49.33	128.42	5527.44
d_B= (50,5)		**2**	12.33			49.89			49.81		
d_C= (70,7)		**3**	36.62			53.01			49.81		
		4	36.99								
	7	**1**	31.09	113.94	4853.26	40.15	127.49	5146.56	47.66	127.60	5086.10
		2	12.36			47.73			48.05		
		3	37.51			55.23			48.04		
		4	38.01								
	8	**1**	39.56	122.64	11521.11	49.12	129.03	11975.60	51.67	129.05	11970.16
		2	12.82			51.82			52.10		
		3	39.88			54.49			52.10		
		4	40.49								
	9	**1**	37.63	122.48	11284.56	45.07	128.74	11552.74	50.45	128.80	11508.59
		2	12.84			50.59			50.98		
		3	40.67			56.08			50.98		
		4	41.40								
	10	**1**	36.95	122.41	11209.03	43.95	128.61	11418.21	49.80	128.72	11356.65
		2	12.84			50.14			50.85		
		3	40.92			56.31			50.85		
		4	41.70								
d_A= (10,1)	**6**	**1**	39.49	128.50	5677.78	53.81	148.27	6385.35	54.69	148.19	6372.17
d_B= (50,5)		**2**	12.41			57.55			59.08		
d_C= (90,9)		**3**	40.35			61.29			59.08		
		4	41.36								
	7	**1**	35.94	129.02	5490.73	46.48	147.10	5933.39	53.76	147.57	5858.83
		2	12.42			55.25			57.55		
		3	42.33			64.01			57.55		
		4	43.89								
	8	**1**	46.32	140.42	13189.96	57.50	148.87	13814.76	56.34	148.83	13802.65
		2	12.96			59.85			61.89		
		3	45.19			62.21			61.89		
		4	47.39								
	9	**1**	44.15	140.16	12904.97	52.47	148.49	13323.36	55.85	148.53	13267.60
		2	12.97			58.21			60.15		
		3	45.95			63.95			60.16		
		4	48.54								
	10	**1**	43.40	140.05	12813.66	50.80	148.41	13174.15	55.12	148.44	13092.78
		2	12.98			57.95			60.00		
		3	46.20			65.10			60.00		
		4	48.93								

Table-B4_2 (high variance low cost) (A>B>C)

			OVERFLOW			NESTED			SYMMETRIC		
	Cost.S.	**Dept.**	**c***	**TH**	**PR**	**c***	**TH**	**PR**	**c***	**TH**	**PR**
d_A= **(50,5)**	**6**	**1**	34.20	119.33	5202.72	53.06	116.10	5000.52	45.61	119.21	5139.71
d_B= **(40,4)**		**2**	37.62			44.53			45.61		
d_C= **(30,3)**		**3**	36.42			33.20			45.61		
		4	30.83								
	7	**1**	27.74	118.91	5038.73	50.34	114.72	4514.91	44.46	118.67	4733.12
		2	39.63			44.13			44.46		
		3	37.80			33.85			44.46		
		4	31.76								
	8	**1**	36.19	119.75	11181.43	58.94	118.49	10988.14	47.34	119.69	11117.02
		2	39.02			46.38			47.35		
		3	37.63			34.05			47.34		
		4	31.47								
	9	**1**	31.52	119.58	10994.45	58.11	118.25	10482.95	46.48	119.50	10694.88
		2	40.10			46.07			46.48		
		3	38.54			34.40			46.47		
		4	32.19								
	10	**1**	30.90	119.49	10933.01	57.76	118.14	10315.23	46.19	119.41	10555.72
		2	39.82			45.93			46.19		
		3	38.32			34.55			46.19		
		4	32.39								
d_A= **(90,9)**	**6**	**1**	39.60	116.31	5105.91	32.54	69.81	3024.04	53.22	127.34	5479.58
d_B= **(30,3)**		**2**	41.54			29.63			41.48		
d_C= **(10,1)**		**3**	32.59			12.17			53.22		
		4	12.37								
	7	**1**	35.77	115.39	4857.30	28.71	65.50	2581.99	51.36	126.38	5029.84
		2	43.93			28.74			40.52		
		3	33.15			12.23			51.36		
		4	12.40								
	8	**1**	45.12	122.73	11491.04	39.73	78.23	7287.33	56.08	128.29	11898.94
		2	46.21			32.46			42.84		
		3	34.06			12.63			56.08		
		4	12.94								
	9	**1**	43.10	122.45	11210.33	37.37	75.70	6733.11	54.26	127.76	11420.22
		2	47.59			31.96			42.11		
		3	34.34			12.68			54.26		
		4	12.96								
	10	**1**	41.82	121.66	11065.71	36.25	74.48	6524.00	54.21	127.72	11269.96
		2	47.32			31.69			41.79		
		3	34.30			12.70			54.21		
		4	12.97								
d_A= **(80,8)**	**6**	**1**	27.29	75.25	3308.72	17.71	36.82	1593.19	40.01	93.82	4074.81
d_B= **(15,1.5)**		**2**	28.57			15.58			22.66		
d_C= **(5,0.5)**		**3**	17.70			6.09			40.01		
		4	6.28								
	7	**1**	24.82	74.50	3130.81	16.15	35.16	1379.52	38.05	91.64	3698.04
		2	30.22			15.36			22.10		
		3	17.86			6.11			38.05		
		4	6.28								
	8	**1**	34.91	88.16	8267.32	21.26	40.81	3800.10	43.07	96.42	8985.28
		2	35.70			16.76			23.29		
		3	18.57			6.35			43.07		
		4	6.63								
	9	**1**	33.53	87.92	8046.03	20.99	40.55	3599.26	41.81	95.45	8586.35
		2	36.74			16.78			22.93		
		3	18.63			6.35			41.81		
		4	6.63								
	10	**1**	33.05	87.82	7974.69	20.89	40.46	3532.68	41.79	95.43	8478.99
		2	37.10			16.78			22.78		
		3	18.64			6.36			41.79		
		4	6.63								
d_A= **(160,16)**	**6**	**1**	49.50	148.99	6568.08	37.56	82.16	3565.68	86.27	202.21	8740.25
d_B= **(40,4)**		**2**	51.81			36.27			55.82		
d_C= **(10,1)**		**3**	42.65			12.36			86.27		
		4	12.50								

	7	**1**	45.49	148.91	6296.52	33.63	77.59	3062.13	82.95	199.33	7984.80
		2	55.36			35.37			54.32		
		3	43.70			12.39			82.95		
		4	12.51								
	8	**1**	61.53	171.39	16099.07	46.08	93.44	8711.18	93.34	206.49	19182.98
		2	62.88			40.93			57.73		
		3	45.91			12.89			93.34		
		4	13.14								
	9	**1**	56.43	165.73	15237.35	44.56	91.86	8174.68	90.04	204.75	18343.90
		2	61.69			40.80			56.77		
		3	45.67			12.91			90.04		
		4	13.15								
	10	**1**	57.55	169.32	15450.09	43.43	90.57	7936.57	88.70	203.92	18053.97
		2	64.44			40.48			56.42		
		3	46.23			12.92			88.70		
		4	13.15								
d_A= (70,7)	**6**	**1**	35.30	115.76	5111.12	40.67			49.81		
d_B= (50,5)		**2**	36.99			40.42	89.99	3910.19	49.33	128.42	5527.44
d_C= (10,1)		**3**	36.62			12.39			49.81		
		4	12.33								
	7	**1**	31.56	115.19	4903.83	37.41			48.04		
		2	38.63			40.21	86.57	3418.13	47.66	127.60	5086.10
		3	38.08			12.41			48.05		
		4	12.35								
	8	**1**	39.57	122.65	11522.10	46.94			52.10		
		2	40.49			44.90	99.59	9296.51	51.67	129.05	11970.16
		3	39.89			12.94			52.10		
		4	12.82								
	9	**1**	37.63	122.48	11284.56	45.35			50.98		
		2	41.40			44.81	97.98	8740.65	50.45	128.80	11508.59
		3	40.67			12.97			50.98		
		4	12.84								
	10	**1**	36.95	122.41	11209.03	45.07			50.53		
		2	41.70			44.98	97.82	8591.41	49.60	128.63	11356.23
		3	40.92			12.97			50.53		
		4	12.84								
d_A= (90,9)	**6**	**1**	39.49	128.50	5677.78	40.67			59.06		
d_B= (50,5)		**2**	41.36			40.42	90.02	3911.69	54.68	148.18	6372.17
d_C= (10,1)		**3**	40.35			12.39			59.06		
		4	12.41								
	7	**1**	36.46	130.33	5543.46	37.41			57.55		
		2	44.55			40.21	86.59	3418.99	53.82	147.58	5858.78
		3	42.84			12.41			57.55		
		4	12.41								
	8	**1**	46.32	140.42	13189.96	46.98			61.93		
		2	47.39			44.87	99.74	9311.34	56.43	148.84	13802.59
		3	45.19			12.95			61.93		
		4	12.96								
	9	**1**	44.15	140.16	12904.96	46.20			60.15		
		2	48.54			45.35	99.28	8854.56	55.83	148.53	13267.63
		3	45.95			12.96			60.16		
		4	12.97								
	10	**1**	43.40	140.05	12813.66	45.11			60.03		
		2	48.93			44.96	97.94	8602.39	55.10	148.44	13092.84
		3	46.20			12.97			60.03		
		4	12.98								

Table-B4_3 (high variance low cost) (A=B=C)

			OVERFLOW			NESTED			SYMMETRIC		
	Cost.S.	**Dept.**	**c***	**TH**	**PR**	**c***	**TH**	**PR**	**c***	**TH**	**PR**
d_A= (5,0.5)	**6**	**1**	3.58	14.86	642.99	6.19			6.25		
d_B= (5,0.5)		**2**	5.01			6.24	14.88	631.61	6.25	14.88	631.52
d_C= (5,0.5)		**3**	5.01			6.29			6.25		
		4	5.01								
	7	**1**	1.71	14.83	636.11	5.76			5.87		
		2	5.56			5.95	14.76	577.90	5.87	14.70	576.53
		3	5.56			6.33			5.87		
		4	5.56								
	8	**1**	3.56	14.98	1391.01	6.58			6.64		
		2	5.50			6.62	14.97	1378.22	6.64	14.98	1378.14
		3	5.50			6.68			6.64		
		4	5.50								
	9	**1**	2.06	14.95	1381.06	6.27			6.45		
		2	5.83			6.40	14.94	1320.65	6.45	14.94	1319.65
		3	5.83			6.77			6.45		
		4	5.83								
	10	**1**	1.65	14.95	1380.85	6.18			6.35		
		2	5.94			6.31	14.92	1301.92	6.35	14.91	1300.59
		3	5.94			6.88			6.35		
		4	5.94								
d_A= (10,1)	**6**	**1**	7.72	29.74	1285.11	12.25			12.41		
d_B= (10,1)		**2**	9.87			12.39	29.72	1263.49	12.41	29.73	1263.27
d_C= (10,1)		**3**	9.87			12.53			12.41		
		4	9.87								
	7	**1**	5.03	29.64	1259.32	11.21			11.84		
		2	10.47			11.68	29.40	1157.09	11.84	29.49	1154.74
		3	10.47			12.37			11.84		
		4	10.47								
	8	**1**	8.57	29.92	2777.14	12.86			12.99		
		2	10.31			12.93	29.88	2754.86	12.99	29.89	2754.75
		3	10.31			13.17			12.99		
		4	10.31								
	9	**1**	6.44	29.88	2742.02	12.33			12.70		
		2	10.81			12.67	29.82	2640.98	12.70	29.82	2638.99
		3	10.81			13.33			12.70		
		4	10.81								
	10	**1**	5.79	29.85	2734.80	12.14			12.41		
		2	10.91			12.51	29.78	2604.17	12.41	29.73	2601.03
		3	10.91			13.30			12.41		
		4	10.91								
d_A= (30,3)	**6**	**1**	24.50	88.42	3817.93	35.89			36.37		
d_B= (30,3)		**2**	28.79			36.36	88.21	3756.72	36.37	88.22	3756.17
d_C= (30,3)		**3**	28.79			36.83			36.37		
		4	28.79								
	7	**1**	19.37	87.80	3703.76	34.08			35.22		
		2	28.96			35.24	87.76	3440.10	35.22	87.70	3433.97
		3	28.96			37.53			35.22		
		4	28.96								
	8	**1**	27.19	88.97	8253.77	38.39			39.26		
		2	30.22			39.16	88.94	8191.17	39.26	88.97	8190.32
		3	30.22			39.92			39.26		
		4	30.22								
	9	**1**	23.42	88.65	8111.71	36.23			37.53		
		2	29.92			37.55	88.60	7852.84	37.53	88.60	7846.39
		3	29.92			39.69			37.53		
		4	29.92								
	10	**1**	22.63	88.53	8067.10	35.78			37.24		
		2	29.73			36.99	88.49	7743.30	37.24	88.51	7734.25
		3	29.73			39.94			37.24		
		4	29.73								
d_A= (40,4)	**6**	**1**	33.88	119.57	5159.31	48.90			49.65		
d_B= (40,4)		**2**	38.81			49.59	119.39	5077.41	49.65	119.41	5076.61
d_C= (40,4)		**3**	38.81			50.41			49.65		
		4	38.81								

	7	1	26.85	118.16	4986.85	45.41			47.34		
		2	38.13			47.56	118.35	4648.14	47.34	118.31	4637.36
		3	38.13			50.26			47.35		
		4	38.13								
	8	1	36.72	120.31	11161.93	52.11			52.54		
		2	40.80			52.48	120.22	11076.88	52.54	120.22	11076.30
		3	40.80			53.00			52.54		
		4	40.80								
	9	1	32.12	119.90	10965.79	49.56			51.38		
		2	40.41			51.37	119.88	10616.23	51.39	119.97	10609.13
		3	40.41			53.00			51.38		
		4	40.41								
	10	1	30.91	119.72	10904.78	48.87			50.81		
		2	40.23			50.72	119.74	10467.12	50.81	119.81	10456.42
		3	40.23			53.40			50.81		
		4	40.23								
d_A= (80,8)	6	1	67.18	237.47	10261.13	96.87			98.13		
d_B= (80,8)		2	76.13			98.12	237.30	10101.25	98.16	237.33	10099.85
d_C= (80,8)		3	76.13			99.38			98.16		
		4	76.13								
	7	1	53.54	233.75	9886.47	89.67			94.68		
		2	73.66			94.42	235.22	9250.00	94.71	235.74	9230.51
		3	73.66			99.10			94.67		
		4	73.66								
	8	1	73.87	239.27	22198.96	103.24			105.67		
		2	80.77			104.95	239.17	22031.54	105.67	239.32	22030.68
		3	80.75			106.58			105.63		
		4	80.75								
	9	1	64.13	237.88	21778.95	97.83			101.04		
		2	78.35			101.51	238.34	21118.23	101.03	238.28	21099.65
		3	78.35			106.09			101.04		
		4	78.36								
	10	1	61.75	237.43	21648.87	96.51			99.89		
		2	77.82			100.27	238.05	20822.87	99.88	237.93	20796.27
		3	77.82			106.33			99.88		
		4	77.82								
d_A= (160,16)	6	1	135.43	473.89	20486.59	193.44			195.14		
d_B= (160,16)		2	150.66			195.48	473.80	20172.72	195.15	473.63	20169.02
d_C= (160,16)		3	150.67			198.06			195.14		
		4	150.64								
	7	1	107.94	467.26	19741.65	180.91			189.37		
		2	147.86			189.56	470.24	18463.87	189.38	470.87	18430.44
		3	147.86			198.04			189.36		
		4	147.88								
	8	1	147.19	477.68	44331.04	206.22			208.99		
		2	160.46			209.08	477.53	43998.54	208.99	477.58	43996.33
		3	160.47			211.33			209.02		
		4	160.45								
	9	1	129.06	474.49	43455.16	195.50			202.07		
		2	154.40			202.00	475.85	42173.96	202.09	475.97	42141.37
		3	154.41			210.30			202.06		
		4	154.39								
	10	1	124.33	473.69	43196.23	192.99			200.34		
		2	153.84			199.90	475.20	41582.10	200.35	475.47	41536.69
		3	153.84			208.83			200.34		
		4	153.84								

Table-B5_1 (all results summary)

$\mu_A<\mu_B<\mu_C$

		O	N	S
(lv,hc)	#1	3	15	12
	#2	2	10	18
	#3	25	5	0
	Total	30	30	30
(lv,lc)	#1	5	18	7
	#2	0	10	20
	#3	25	2	3
	Total	30	30	30
(hv,hc)	#1	1	13	16
	#2	4	12	14
	#3	25	5	0
	Total	30	30	30
(hv,lc)	#1	5	18	7
	#2	0	9	21
	#3	25	3	2
	Total	30	30	30

$\mu_A>\mu_B>\mu_C$

		O	N	S
(lv,hc)	#1	3	0	27
	#2	27	0	3
	#3	0	30	0
	Total	30	30	30
(lv,lc)	#1	5	0	25
	#2	25	0	5
	#3	0	30	0
	Total	30	30	30
(hv,hc)	#1	1	0	29
	#2	29	0	1
	#3	0	30	0
	Total	30	30	30
(hv,lc)	#1	5	0	25
	#2	25	0	5
	#3	0	30	0
	Total	30	30	30

$\mu_A=\mu_B=\mu_C$

		O	N	S
(lv,hc)	#1	17	1	12
	#2	8	8	14
	#3	5	21	4
	Total	30	30	30
(lv,lc)	#1	22	2	6
	#2	1	10	19
	#3	7	18	5
	Total	30	30	30
(hv,hc)	#1	12	0	18
	#2	4	16	10
	#3	14	14	2
	Total	30	30	30
(hv,lc)	#1	19	3	8
	#2	2	13	15
	#3	9	14	7
	Total	30	30	30

Table-B5_2
(non-overlapping CI)

$\mu_A<\mu_B<\mu_C$

		O	N	S
(lv,hc)	#1	0	11	1
	#2	0	1	11
	#3	12	0	0
	Total	12	12	12
(lv,lc)	#1	0	7	0
	#2	0	0	7
	#3	7	0	0
	Total	7	7	7
(hv,hc)	#1	0	9	2
	#2	1	1	9
	#3	10	1	0
	Total	11	11	11
(hv,lc)	#1	0	2	0
	#2	0	0	2
	#3	2	0	0
	Total	2	2	2

$\mu_A>\mu_B>\mu_C$

		O	N	S
(lv,hc)	#1	0	0	25
	#2	25	0	0
	#3	0	25	0
	Total	25	25	25
(lv,lc)	#1	0	0	25
	#2	25	0	0
	#3	0	25	0
	Total	25	25	25
(hv,hc)	#1	0	0	26
	#2	26	0	0
	#3	0	26	0
	Total	26	26	26
(hv,lc)	#1	0	0	25
	#2	25	0	0
	#3	0	25	0
	Total	25	25	25

$\mu_A=\mu_B=\mu_C$

		O	N	S
(lv,hc)	#1	2	0	3
	#2	3	0	2
	#3	0	5	0
	Total	5	5	5
(lv,lc)	#1	0	0	0
	#2	0	0	0
	#3	0	0	0
	Total	0	0	0
(hv,hc)	#1	1	0	2
	#2	1	1	1
	#3	1	2	0
	Total	3	3	3
(hv,lc)	#1	0	0	0
	#2	0	0	0
	#3	0	0	0
	Total	0	0	0

(The average throughput values associated with the optimal capacity levels which maximize the profit for 3 demand types.)

(tables for correlated demand arrival rate with 3 demand types)

Table-C1_1 (Low variance, high cost, p_{12}=-1)

p_{12}=-1											
			OVERFLOW			**NESTED**			**SYMMETRIC**		
Demand Normal (μ,σ)	**Cost Structure**	**Dept.**	**c***	**TH**	**PR**	**c***	**TH**	**PR**	**c***	**TH**	**PR**
d_A= (10,1)	**1**	**1**	17.96	77.92	2536.33	33.95	127.37	3824.43	39.46	125.85	3700.68
d_B= (30,3)		**2**	10.52			44.09			45.06		
d_C= (90,9)		**3**	25.06			54.24			45.07		
		4	25.15								
	2	**1**	10.61	69.83	2220.57	23.41	125.75	3455.95	38.99	124.71	3049.00
		2	10.52			42.97			44.23		
		3	24.69			62.53			44.23		
		4	24.76								
	3	**1**	3.36	60.18	1994.93	9.83	124.55	3401.74	38.51	123.53	2410.20
		2	10.52			41.48			43.52		
		3	23.50			75.94			43.52		
		4	23.52								
	4	**1**	11.25	66.03	2403.70	26.37	125.95	4013.04	38.92	124.83	3660.73
		2	10.41			43.09			44.36		
		3	22.49			59.82			44.36		
		4	22.50								
	5	**1**	10.06	59.81	1087.24	25.64	123.26	1950.74	38.02	122.21	1782.64
		2	9.77			41.63			42.82		
		3	20.12			57.64			42.82		
		4	20.13								
d_A= (90,9)	**1**	**1**	19.36	82.81	2690.74	16.99	47.19	1369.02	45.27	126.54	3745.68
d_B= (30,3)		**2**	27.10			20.38			38.51		
d_C= (10,1)		**3**	26.76			10.53			45.28		
		4	10.53								
	2	**1**	10.32	68.20	2169.42	10.48	37.76	926.73	44.67	125.70	3095.47
		2	24.07			17.47			38.25		
		3	24.03			10.53			44.67		
		4	10.53								
	3	**1**	3.46	61.65	2042.63	3.14	25.49	599.33	43.99	124.63	2452.48
		2	24.23			12.54			37.98		
		3	24.18			10.53			43.99		
		4	10.53								
	4	**1**	10.92	64.37	2343.44	12.12	40.08	1166.00	44.76	125.79	3716.56
		2	21.84			18.18			38.19		
		3	21.83			10.43			44.75		
		4	10.43								
	5	**1**	9.08	54.83	997.69	7.42	28.00	421.90	43.90	124.03	1827.26
		2	18.16			11.13			37.20		
		3	18.16			9.72			43.89		
		4	9.72								
d_A= (30,3)	**1**	**1**	15.05	89.26	2943.99	30.11	88.31	2598.79	30.31	89.22	2642.35
d_B= (30,3)		**2**	25.38			30.38			30.31		
d_C= (30,3)		**3**	25.14			30.44			30.31		
		4	25.67								
	2	**1**	8.55	88.90	2911.03	21.37	79.77	2021.07	30.02	88.83	2190.06
		2	27.16			30.25			30.02		
		3	27.12			30.87			30.02		
		4	28.02								
	3	**1**	2.45	88.11	2960.80	8.36	66.31	1590.38	29.74	88.36	1742.03
		2	29.39			29.39			29.73		
		3	29.31			31.14			29.73		
		4	30.25								

	4	1	10.47	88.93	3312.37	23.32	81.63	2445.98	30.02	88.83	2628.07
		2	26.58			30.24			30.02		
		3	26.47			30.60			30.02		
		4	27.18								
	5	1	10.48	87.89	1628.57	19.99	76.31	1152.95	29.54	88.00	1298.04
		2	25.87			28.50			29.55		
		3	25.70			28.84			29.53		
		4	26.65								

Table-C1_2 (Low variance, high cost, p_{12}=-0.5)

p_{12}=-0.5											
			OVERFLOW			**NESTED**			**SYMMETRIC**		
Demand Normal (μ,σ)	**Cost Structure**	**Dept.**	**c***	**TH**	**PR**	**c***	**TH**	**PR**	**c***	**TH**	**PR**
d_A= (10,1)	**1**	**1**	18.87	81.24	2642.40	34.06	127.54	3828.37	39.22	126.43	3709.84
d_B= (30,3)		**2**	10.52			44.16			45.68		
d_C= (90,9)		**3**	26.24			54.26			45.68		
		4	26.42								
	2	**1**	10.90	71.46	2271.75	23.54	125.93	3458.58	38.89	124.81	3049.67
		2	10.52			43.03			44.37		
		3	25.35			62.53			44.37		
		4	25.44								
	3	**1**	3.46	61.68	2044.29	9.83	124.09	3391.90	38.31	123.59	2410.14
		2	10.52			41.49			43.67		
		3	24.19			75.05			43.67		
		4	24.23								
	4	**1**	11.92	69.36	2522.94	26.50	126.13	4016.51	38.86	124.87	3660.58
		2	10.41			43.15			44.43		
		3	23.82			59.81			44.43		
		4	23.85								
	5	**1**	10.06	59.80	1087.06	25.74	123.43	1951.69	37.96	121.44	1776.61
		2	9.77			41.71			42.38		
		3	20.12			57.68			42.38		
		4	20.13								
d_A= (90,9)	**1**	**1**	16.15	71.14	2317.74	16.97	47.15	1367.89	45.20	126.24	3722.15
d_B= (30,3)		**2**	22.60			20.36			39.10		
d_C= (10,1)		**3**	22.59			10.53			45.20		
		4	10.53								
	2	**1**	10.61	69.83	2220.69	10.48	37.76	926.69	44.57	125.28	3071.61
		2	24.75			17.47			38.55		
		3	24.70			10.53			44.57		
		4	10.53								
	3	**1**	3.36	60.18	1994.75	3.14	25.49	599.33	43.84	124.00	2430.18
		2	23.53			12.54			37.98		
		3	23.50			10.53			43.84		
		4	10.53								
	4	**1**	13.63	77.42	2807.24	12.12	40.07	1165.92	44.67	125.38	3687.89
		2	27.26			18.17			38.49		
		3	26.95			10.43			44.67		
		4	10.43								
	5	**1**	9.07	54.78	996.63	7.42	28.00	421.90	43.17	122.63	1800.39
		2	18.14			11.13			37.42		
		3	18.13			9.72			43.17		
		4	9.72								
d_A= (30,3)	**1**	**1**	15.21	89.22	2930.79	30.05	88.13	2585.58	30.31	88.88	2625.52
d_B= (30,3)		**2**	25.53			30.79			30.31		
d_C= (30,3)		**3**	25.48			30.26			30.31		
		4	25.66								

	2	**1**	9.29	88.87	2887.92	21.24	79.82	2025.89	30.02	88.48	2172.44
		2	27.28			30.34			30.02		
		3	27.25			30.88			30.02		
		4	27.49								
	3	**1**	2.85	88.07	2949.93	8.20	66.35	1595.73	29.73	88.00	1724.17
		2	29.39			29.49			29.73		
		3	29.41			31.19			29.73		
		4	29.56								
	4	**1**	10.50	88.86	3298.30	23.20	81.69	2451.78	30.02	88.48	2606.93
		2	26.85			30.33			30.02		
		3	26.77			30.59			30.02		
		4	27.05								
	5	**1**	10.69	87.47	1610.96	19.88	76.37	1156.02	29.34	87.26	1281.32
		2	25.85			28.61			29.35		
		3	25.90			28.87			29.35		
		4	26.08								

Table-C1_3 (Low variance, high cost, p_{12}=0.5)

p_{12}=0.5											
			OVERFLOW			**NESTED**			**SYMMETRIC**		
Demand Normal (μ,σ)	**Cost Structure**	**Dept.**	**c***	**TH**	**PR**	**c***	**TH**	**PR**	**c***	**TH**	**PR**
d_A= (10,1)	**1**	**1**	18.41	79.61	2590.87	34.17	127.54	3823.07	39.07	126.74	3705.20
d_B= (30,3)		**2**	10.52			44.24			46.26		
d_C= (90,9)		**3**	25.68			54.32			46.26		
		4	25.77								
	2	**1**	10.60	69.85	2221.90	23.48	125.79	3453.52	38.46	125.20	3045.53
		2	10.52			43.03			45.06		
		3	24.71			62.56			45.06		
		4	24.74								
	3	**1**	3.36	60.19	1995.68	9.58	123.85	3390.65	37.69	123.97	2404.71
		2	10.52			41.50			44.38		
		3	23.51			75.06			44.38		
		4	23.51								
	4	**1**	10.92	64.40	2345.16	26.45	125.98	4009.87	38.42	125.27	3655.61
		2	10.41			43.14			45.14		
		3	21.84			59.83			45.13		
		4	21.84								
	5	**1**	9.40	56.47	1027.25	25.60	123.19	1946.56	37.14	121.83	1771.84
		2	9.77			41.65			43.14		
		3	18.79			57.70			43.14		
		4	18.79								
d_A= (90,9)	**1**	**1**	17.95	77.94	2537.17	16.99	47.19	1369.05	45.95	126.58	3695.86
d_B= (30,3)		**2**	25.13			20.38			39.75		
d_C= (10,1)		**3**	25.08			10.53			45.95		
		4	10.53								
	2	**1**	10.32	68.25	2171.44	10.48	37.76	926.73	45.21	125.54	3040.82
		2	24.07			17.47			39.04		
		3	24.06			10.53			45.21		
		4	10.53								
	3	**1**	3.56	63.16	2092.56	3.14	25.49	599.33	44.06	123.86	2396.21
		2	24.93			12.54			38.43		
		3	24.89			10.53			44.07		
		4	10.53								
	4	**1**	10.92	64.40	2344.66	12.12	40.08	1166.00	45.32	125.60	3649.84
		2	21.85			18.18			38.90		
		3	21.85			10.43			45.31		
		4	10.43								

	5	**1**	9.08	54.83	997.69	7.42	28.00	421.90	42.80	121.67	1764.97
		2	18.16			11.13			37.79		
		3	18.16			9.72			42.80		
		4	9.72								
d_A= (30,3)	**1**	**1**	15.40	88.91	2900.80	29.73	87.73	2565.03	30.60	88.58	2592.90
d_B= (30,3)		**2**	25.79			31.19			30.60		
d_C= (30,3)		**3**	25.80			30.29			30.60		
		4	25.71								
	2	**1**	9.13	88.41	2864.46	21.26	79.84	2025.88	30.02	87.78	2137.50
		2	27.56			30.34			30.02		
		3	27.56			30.92			30.02		
		4	27.31								
	3	**1**	2.91	87.33	2927.47	8.23	66.36	1595.74	29.54	86.99	1690.81
		2	29.12			29.46			29.54		
		3	29.17			31.20			29.54		
		4	28.95								
	4	**1**	10.33	88.42	3268.09	23.20	81.69	2451.45	30.02	87.78	2565.00
		2	27.17			30.33			30.02		
		3	27.14			30.60			30.02		
		4	26.90								
	5	**1**	10.71	86.46	1580.89	20.63	77.47	1166.93	29.15	86.25	1251.26
		2	25.78			29.09			29.16		
		3	25.77			28.77			29.16		
		4	25.58								

Table-C1_4 (Low variance, high cost, p_{12}=1)

p_{12}=1											
			OVERFLOW			**NESTED**			**SYMMETRIC**		
Demand Normal (μ,σ)	**Cost Structure**	**Dept.**	**c***	**TH**	**PR**	**c***	**TH**	**PR**	**c***	**TH**	**PR**
d_A= (10,1)	**1**	**1**	17.50	76.25	2483.16	34.01	127.24	3813.96	38.98	126.58	3695.40
d_B= (30,3)		**2**	10.52			44.16			46.35		
d_C= (90,9)		**3**	24.46			54.31			46.35		
		4	24.49								
	2	**1**	11.20	73.04	2320.60	23.39	125.56	3445.78	37.98	125.41	3040.35
		2	10.52			42.99			45.61		
		3	25.99			62.60			45.61		
		4	26.13								
	3	**1**	3.36	60.19	1995.60	9.51	122.97	3368.82	37.37	123.73	2396.16
		2	10.52			41.28			44.49		
		3	23.51			74.22			44.49		
		4	23.51								
	4	**1**	10.92	64.37	2344.12	26.35	125.75	4000.44	37.90	125.46	3649.38
		2	10.41			43.11			45.69		
		3	21.83			59.87			45.69		
		4	21.83								
	5	**1**	9.40	56.47	1027.25	25.46	122.99	1940.63	36.65	121.51	1764.31
		2	9.77			41.62			43.26		
		3	18.79			57.80			43.26		
		4	18.79								
d_A= (90,9)	**1**	**1**	17.49	76.24	2482.51	16.99	47.19	1369.05	46.23	126.47	3677.74
d_B= (30,3)		**2**	24.49			20.38			39.82		
d_C= (10,1)		**3**	24.46			10.53			46.24		
		4	10.53								
	2	**1**	10.32	68.24	2171.29	10.48	37.76	926.73	45.21	125.33	3021.28
		2	24.07			17.47			39.40		
		3	24.05			10.53			45.20		
		4	10.53								

	3	**1**	3.56	63.15	2092.32	3.14	25.49	599.33	44.17	123.88	2377.74
		2	24.93			12.54			38.86		
		3	24.88			10.53			44.18		
		4	10.53								
	4	**1**	10.92	64.40	2344.59	12.12	40.08	1166.00	45.18	125.30	3625.51
		2	21.85			18.18			39.39		
		3	21.85			10.43			45.19		
		4	10.43								
	5	**1**	9.08	54.83	997.69	7.42	28.00	421.90	42.91	121.63	1749.39
		2	18.16			11.13			37.95		
		3	18.16			9.72			42.91		
		4	9.72								
d_A= (30,3)	**1**	**1**	15.32	88.55	2883.43	29.46	87.45	2553.49	30.60	88.22	2574.79
d_B= (30,3)		**2**	25.84			31.28			30.60		
d_C= (30,3)		**3**	25.84			30.45			30.60		
		4	25.71								
	2	**1**	8.95	88.00	2850.43	21.35	79.83	2022.96	30.02	87.44	2120.40
		2	27.59			30.26			30.02		
		3	27.59			30.98			30.02		
		4	27.26								
	3	**1**	3.01	86.49	2900.23	8.34	66.35	1592.64	29.44	86.48	1673.87
		2	28.72			29.40			29.44		
		3	28.72			31.20			29.45		
		4	28.47								
	4	**1**	10.17	88.00	3249.70	23.29	81.66	2447.82	30.02	87.44	2544.48
		2	27.16			30.24			30.02		
		3	27.16			30.65			30.02		
		4	26.86								
	5	**1**	10.38	85.90	1567.42	20.73	77.41	1164.92	28.87	85.35	1236.43
		2	25.80			28.98			28.87		
		3	25.80			28.74			28.87		
		4	25.49								

Table-C2_1 (Low variance, low cost, p_{12}=-1)

p_{12}=-1											
			OVERFLOW			**NESTED**			**SYMMETRIC**		
Demand Normal (μ,σ)	**Cost Structure**	**Dept.**	**c***	**TH**	**PR**	**c***	**TH**	**PR**	**c***	**TH**	**PR**
d_A= (10,1)	**1**	**1**	36.88	114.73	5069.30	44.15	129.54	5634.97	40.78	128.92	5614.96
d_B= (30,3)		**2**	11.29			47.11			48.84		
d_C= (90,9)		**3**	31.93			50.07			48.84		
		4	38.60								
	2	**1**	30.94	107.98	4593.30	39.41	129.06	5262.45	40.44	128.26	5191.85
		2	11.29			46.06			47.63		
		3	31.76			52.72			47.63		
		4	37.64								
	3	**1**	43.12	124.64	11718.96	46.26	129.82	12118.39	41.30	129.48	12095.20
		2	11.57			48.16			50.43		
		3	32.97			50.07			50.43		
		4	44.08								
	4	**1**	41.36	124.30	11442.62	42.92	129.68	11721.20	41.08	129.10	11653.80
		2	11.58			47.54			49.26		
		3	33.09			52.16			49.26		
		4	45.28								
	5	**1**	40.74	124.17	11354.27	41.72	129.64	11598.67	40.95	129.08	11513.88
		2	11.58			47.41			49.21		
		3	33.13			53.10			49.21		
		4	45.69								

d_A= (90,9)	1	1	37.58	116.37	5142.13	29.29	67.47	2938.52	46.72	127.97	5598.70
d_B= (30,3)		2	39.33			28.96			39.84		
d_C= (10,1)		3	32.05			11.23			46.72		
		4	11.29								
	2	1	34.72	116.49	4943.20	25.64	63.03	2511.34	46.61	127.88	5201.42
		2	42.23			27.85			39.26		
		3	32.43			11.26			46.62		
		4	11.29								
	3	1	43.77	125.73	11823.98	32.67	71.73	6703.21	47.76	128.61	12047.22
		2	44.75			30.56			40.19		
		3	32.25			11.50			47.76		
		4	11.59								
	4	1	41.40	124.62	11476.65	31.24	70.21	6284.27	47.15	128.26	11617.24
		2	45.31			30.37			39.99		
		3	32.46			11.51			47.15		
		4	11.61								
	5	1	38.98	121.60	11136.48	31.04	70.05	6177.10	47.13	128.24	11482.81
		2	43.60			30.43			39.89		
		3	32.63			11.51			47.13		
		4	11.63								
d_A= (30,3)	1	1	21.75	89.86	3980.64	34.21	89.79	3921.04	31.18	89.84	3930.96
d_B= (30,3)		2	24.03			29.89			31.18		
d_C= (30,3)		3	23.92			29.98			31.18		
		4	24.11								
	2	1	17.47	89.78	3882.59	32.19	89.47	3626.53	30.89	89.71	3651.37
		2	25.24			30.22			30.89		
		3	25.19			31.29			30.89		
		4	25.47								
	3	1	22.89	89.96	8475.11	35.16	89.91	8415.75	31.66	89.96	8426.65
		2	24.05			30.00			31.66		
		3	23.96			29.87			31.66		
		4	24.11								
	4	1	20.10	89.93	8358.95	33.98	89.83	8117.29	31.27	89.88	8143.23
		2	24.81			29.88			31.27		
		3	24.73			30.93			31.27		
		4	24.96								
	5	1	19.33	89.92	8325.95	33.70	89.79	8019.76	31.27	89.88	8049.41
		2	25.03			29.80			31.27		
		3	24.97			31.21			31.27		
		4	25.23								

Table-C2_2 (Low variance, low cost, p_{12}=-0.5)

p_{12}=-0.5											
			OVERFLOW			NESTED			SYMMETRIC		
Demand Normal (μ,σ)	Cost Structure	Dept.	c*	TH	PR	c*	TH	PR	c*	TH	PR
d_A= (10,1)	1	1	36.84	114.80	5072.59	44.73	129.66	5641.66	41.11	129.08	5620.94
d_B= (30,3)		2	11.29			47.00			48.86		
d_C= (90,9)		3	32.00			49.27			48.86		
		4	38.56								
	2	1	29.67	105.23	4481.41	39.03	129.33	5275.61	40.57	128.69	5202.08
		2	11.29			46.25			48.20		
		3	31.50			53.46			48.19		
		4	36.09								
	3	1	42.47	123.93	11655.69	46.24	129.96	12133.80	41.45	129.75	12115.45
		2	11.58			48.11			50.90		
		3	32.97			49.98			50.90		
		4	43.40								

	4	1	41.33	124.43	11455.79	42.91	129.84	11736.95	41.20	129.44	11676.80
	4	2	11.58			47.55			49.80		
	4	3	33.16			52.20			49.80		
	4	4	45.26								
	5	1	40.72	124.31	11367.41	42.28	129.76	11607.23	41.04	129.41	11535.90
	5	2	11.58			47.28			49.73		
	5	3	33.21			52.28			49.74		
	5	4	45.67								
d_A= (90,9)	1	1	38.86	118.88	5251.02	29.24	67.56	2942.97	48.38	129.00	5626.04
d_B= (30,3)	1	2	40.68			29.02			40.54		
d_C= (10,1)	1	3	32.27			11.22			48.38		
	1	4	11.28								
	2	1	34.05	115.08	4885.95	26.34	64.30	2559.85	47.71	128.59	5210.28
	2	2	41.41			28.50			40.07		
	2	3	32.40			11.24			47.71		
	2	4	11.29								
	3	1	43.13	125.02	11757.83	31.72	70.75	6613.52	49.98	129.63	12117.41
	3	2	44.08			30.23			40.92		
	3	3	32.83			11.52			49.98		
	3	4	11.60								
	4	1	41.96	125.48	11549.75	30.30	69.22	6200.10	49.28	129.39	11685.81
	4	2	45.96			30.02			40.63		
	4	3	32.99			11.53			49.28		
	4	4	11.60								
	5	1	40.76	124.54	11390.74	30.10	69.06	6095.51	49.27	129.37	11547.49
	5	2	45.69			30.09			40.43		
	5	3	33.03			11.53			49.27		
	5	4	11.61								
d_A= (30,3)	1	1	22.18	89.97	3978.65	34.48	89.89	3918.71	31.66	89.96	3928.08
d_B= (30,3)	1	2	24.30			30.90			31.66		
d_C= (30,3)	1	3	24.28			29.83			31.65		
	1	4	24.34								
	2	1	18.15	89.90	3876.93	32.94	89.61	3621.94	31.27	89.78	3644.86
	2	2	25.47			30.63			31.27		
	2	3	25.44			30.99			31.27		
	2	4	25.51								
	3	1	22.61	90.08	8480.77	35.64	90.07	8420.21	32.04	90.06	8429.53
	3	2	24.56			31.14			32.04		
	3	3	24.60			30.07			32.04		
	3	4	24.63								
	4	1	19.96	90.06	8365.54	34.45	89.96	8116.66	31.85	90.02	8141.75
	4	2	25.40			30.74			31.85		
	4	3	25.33			31.07			31.85		
	4	4	25.45								
	5	1	19.20	90.05	8332.76	34.15	89.92	8017.06	31.75	89.99	8046.34
	5	2	25.60			30.67			31.75		
	5	3	25.52			31.29			31.75		
	5	4	25.64								

Table-C2_3 (Low variance, low cost, p_{12}=0.5)

p_{12}=0.5											
			OVERFLOW			**NESTED**			**SYMMETRIC**		
Demand Normal (μ,σ)	**Cost Structure**	**Dept.**	**c***	**TH**	**PR**	**c***	**TH**	**PR**	**c***	**TH**	**PR**
d_A= (10,1)	**1**	**1**	37.51	116.15	5131.52	44.53	129.71	5641.61	41.13	129.46	5626.90
d_B= (30,3)		**2**	11.29			47.17			49.95		
d_C= (90,9)		**3**	32.17			49.81			49.95		
		4	39.26								
	2	**1**	31.53	109.48	4655.59	39.13	129.31	5272.98	40.71	128.92	5202.16
		2	11.29			46.29			48.75		
		3	32.02			53.44			48.75		
		4	38.36								
	3	**1**	43.08	124.80	11734.89	46.50	130.00	12134.77	42.05	129.78	12114.83
		2	11.59			48.27			50.93		
		3	33.08			50.04			50.93		
		4	44.04								
	4	**1**	41.91	125.26	11527.31	43.15	129.86	11735.24	41.57	129.61	11680.83
		2	11.58			47.66			50.33		
		3	33.29			52.16			50.33		
		4	45.92								
	5	**1**	41.28	125.14	11438.61	41.90	129.80	11611.91	41.39	129.57	11538.58
		2	11.58			47.45			50.24		
		3	33.33			53.00			50.24		
		4	46.34								
d_A= (90,9)	**1**	**1**	36.84	114.92	5078.71	29.24	67.56	2943.05	50.23	129.56	5623.18
d_B= (30,3)		**2**	38.55			29.02			42.05		
d_C= (10,1)		**3**	32.05			11.22			50.22		
		4	11.29								
	2	**1**	35.25	117.65	4989.81	26.34	64.30	2559.92	49.07	129.07	5197.55
		2	42.91			28.49			41.43		
		3	32.67			11.24			49.07		
		4	11.28								
	3	**1**	43.08	124.90	11743.77	32.61	71.84	6714.61	52.16	130.03	12120.69
		2	44.04			30.65			42.80		
		3	33.23			11.52			52.16		
		4	11.60								
	4	**1**	41.33	124.57	11468.11	31.17	70.31	6294.35	51.07	129.83	11680.75
		2	45.26			30.45			42.59		
		3	33.36			11.54			51.07		
		4	11.61								
	5	**1**	40.72	124.43	11378.67	31.88	71.20	6275.57	51.04	129.80	11536.90
		2	45.66			30.88			42.21		
		3	33.40			11.51			51.04		
		4	11.61								
d_A= (30,3)	**1**	**1**	22.40	89.93	3965.10	34.70	89.91	3904.43	32.33	89.89	3912.48
d_B= (30,3)		**2**	24.98			33.27			32.33		
d_C= (30,3)		**3**	24.98			29.75			32.33		
		4	24.97								
	2	**1**	18.16	89.85	3863.63	32.55	89.48	3603.63	32.04	89.76	3622.91
		2	26.18			32.47			32.04		
		3	26.17			30.98			32.04		
		4	26.17								
	3	**1**	23.02	90.12	8467.07	35.48	90.09	8404.43	33.20	90.11	8413.38
		2	25.60			33.83			33.20		
		3	25.60			30.64			33.20		
		4	25.60								

	4	1	20.56	90.06	8347.46	34.20	89.95	8094.06	32.62	89.99	8117.86
		2	26.06			33.43			32.62		
		3	26.06			31.05			32.62		
		4	26.05								
	5	1	19.72	90.04	8314.22	33.72	89.87	7992.31	32.62	89.99	8020.00
		2	26.26			33.21			32.62		
		3	26.26			31.35			32.62		
		4	26.25								

Table-C2_4 (Low variance, low cost, p_{12}=1)

p_{12}=1					p_{12}=1						
			OVERFLOW			**NESTED**			**SYMMETRIC**		
Demand Normal (μ,σ)	**Cost Structure**	**Dept.**	**c***	**TH**	**PR**	**c***	**TH**	**PR**	**c***	**TH**	**PR**
d_A= (10,1)	**1**	**1**	38.20	117.37	5183.80	44.59	129.58	5632.72	41.32	129.32	5617.79
d_B= (30,3)		**2**	11.29			47.29			50.03		
d_C= (90,9)		**3**	32.19			50.03			50.03		
		4	39.98								
	2	**1**	32.17	110.79	4708.79	39.20	129.15	5262.78	40.73	128.74	5191.94
		2	11.30			46.36			48.82		
		3	32.07			53.52			48.82		
		4	39.14								
	3	**1**	43.10	124.69	11723.89	46.50	129.88	12119.85	41.96	129.82	12107.38
		2	11.59			48.45			51.87		
		3	33.00			50.41			51.87		
		4	44.06								
	4	**1**	41.93	125.15	11515.84	43.42	129.73	11717.56	41.59	129.60	11669.27
		2	11.59			47.80			50.89		
		3	33.20			52.18			50.89		
		4	45.95								
	5	**1**	41.31	125.03	11427.32	42.09	129.66	11594.01	41.35	129.55	11526.06
		2	11.59			47.57			50.78		
		3	33.24			53.04			50.78		
		4	46.37								
d_A= (90,9)	**1**	**1**	38.21	117.50	5189.99	27.68	65.14	2839.54	50.56	129.53	5609.99
d_B= (30,3)		**2**	39.99			27.86			43.28		
d_C= (10,1)		**3**	32.22			11.26			50.56		
		4	11.28								
	2	**1**	34.66	116.29	4934.13	25.61	63.08	2513.62	49.58	129.11	5181.59
		2	42.18			27.88			42.37		
		3	32.51			11.26			49.58		
		4	11.28								
	3	**1**	43.11	124.80	11734.09	32.65	71.76	6706.85	52.09	129.86	12097.59
		2	44.07			30.57			43.92		
		3	33.18			11.53			52.09		
		4	11.60								
	4	**1**	41.36	124.47	11458.62	31.22	70.25	6288.05	51.30	129.68	11655.95
		2	45.28			30.38			43.19		
		3	33.30			11.54			51.30		
		4	11.60								
	5	**1**	41.32	125.17	11439.86	31.02	70.08	6180.91	51.16	129.67	11510.10
		2	46.38			30.44			43.39		
		3	33.43			11.54			51.16		
		4	11.60								
d_A= (30,3)	**1**	**1**	22.31	89.75	3951.84	33.75	89.71	3891.77	32.62	89.71	3898.33
d_B= (30,3)		**2**	25.29			34.21			32.62		
d_C= (30,3)		**3**	25.29			30.39			32.62		
		4	25.28								

	2	1	18.40	89.63	3848.03	32.13	89.36	3592.19	32.04	89.45	3607.35
		2	26.28			33.62			32.04		
		3	26.28			31.09			32.04		
		4	26.26								
	3	1	23.54	89.96	8445.00	34.68	89.92	8384.49	33.49	89.94	8391.38
		2	25.78			35.00			33.49		
		3	25.78			30.96			33.49		
		4	25.77								
	4	1	21.12	89.91	8322.12	33.89	89.84	8074.43	33.20	89.88	8091.77
		2	26.28			34.62			33.20		
		3	26.28			31.57			33.20		
		4	26.28								
	5	1	20.25	89.89	8288.04	33.44	89.76	7972.78	32.91	89.80	7993.21
		2	26.49			34.46			32.91		
		3	26.49			31.38			32.91		
		4	26.49								

Table-C3_1 (high variance, high cost, p_{12}=-1)

p_{12}=-1										
			OVERFLOW			**NESTED**			**SYMMETRIC**		
Demand Normal (μ,σ)	**Cost Structure**	**Dept.**	**c***	**TH**	**PR**	**c***	**TH**	**PR**	**c***	**TH**	**PR**
d_A= (10,2)	**1**	**1**	22.81	90.97	2907.34	33.98	124.36	3651.11	38.47	121.84	3506.33
d_B= (30,6)		**2**	11.01			44.55			45.40		
d_C= (90,18)		**3**	28.43			55.11			45.41		
		4	31.98								
	2	**1**	15.97	90.67	2792.44	22.35	119.68	3251.60	37.55	119.77	2853.98
		2	11.01			41.57			43.91		
		3	30.36			60.73			43.91		
		4	37.45								
	3	**1**	5.86	85.02	2732.18	9.71	118.10	3178.99	36.58	117.52	2225.74
		2	11.03			39.48			42.55		
		3	31.21			73.65			42.55		
		4	41.44								
	4	**1**	15.90	83.91	2979.51	25.15	120.60	3788.03	37.52	119.84	3425.75
		2	10.81			42.08			43.99		
		3	28.26			58.98			43.99		
		4	31.86								
	5	**1**	11.62	65.98	1177.60	23.85	115.71	1787.56	35.69	114.95	1619.23
		2	9.54			39.58			41.13		
		3	22.43			55.27			41.13		
		4	23.25								
d_A= (90,18)	**1**	**1**	26.17	100.13	3184.08	20.57	53.59	1523.34	45.29	122.73	3587.38
d_B= (30,6)		**2**	36.64			23.87			36.89		
d_C= (10,2)		**3**	30.16			10.95			45.28		
		4	11.07								
	2	**1**	18.38	99.13	3032.62	12.71	43.04	1020.05	44.62	121.61	2947.76
		2	42.87			20.87			36.06		
		3	31.42			11.03			44.62		
		4	11.07								
	3	**1**	5.51	81.62	2628.87	4.66	32.70	705.47	43.37	119.55	2313.24
		2	38.55			18.48			35.40		
		3	30.66			11.05			43.37		
		4	11.07								
	4	**1**	18.47	92.61	3260.89	13.97	44.05	1247.66	44.71	121.71	3539.36
		2	36.93			20.67			36.00		
		3	30.05			10.83			44.71		
		4	10.87								

	5	**1**	13.16	72.23	1278.69	11.53	37.60	533.63	42.09	117.02	1696.09
		2	26.32			17.22			34.53		
		3	24.45			9.42			42.09		
		4	9.44								
d_A= (30,6)	**1**	**1**	15.71	88.11	2857.57	32.24	87.36	2517.57	30.50	88.30	2584.79
d_B= (30,6)		**2**	24.82			29.53			30.51		
d_C= (30,6)		**3**	24.77			30.23			30.50		
		4	27.43								
	2	**1**	9.87	87.33	2780.02	25.46	82.29	1983.97	30.02	87.64	2130.10
		2	27.05			30.78			30.02		
		3	26.83			31.32			30.02		
		4	28.87								
	3	**1**	3.38	85.69	2790.80	9.65	65.67	1496.46	29.44	86.68	1684.07
		2	29.59			29.10			29.44		
		3	29.50			32.00			29.45		
		4	30.35								
	4	**1**	11.65	87.37	3169.57	27.42	84.10	2409.97	30.02	87.64	2556.12
		2	26.22			30.87			30.02		
		3	26.04			30.63			30.02		
		4	28.06								
	5	**1**	11.90	85.20	1515.12	22.71	76.31	1091.94	28.96	85.75	1246.29
		2	24.37			28.60			28.97		
		3	24.19			27.13			28.96		
		4	27.06								

Table-C3_2 (high variance, high cost, p_{12}=-0.5)

p_{12}=-0.5											
			OVERFLOW			**NESTED**			**SYMMETRIC**		
Demand Normal (μ,σ)	**Cost Structure**	**Dept.**	**c***	**TH**	**PR**	**c***	**TH**	**PR**	**c***	**TH**	**PR**
d_A= (10,2)	**1**	**1**	22.74	91.15	2915.94	34.15	124.62	3658.77	38.21	122.56	3517.75
d_B= (30,6)		**2**	11.02			44.62			46.16		
d_C= (90,18)		**3**	28.64			55.07			46.16		
		4	31.89								
	2	**1**	14.45	85.37	2648.28	22.21	120.46	3272.86	37.03	120.49	2862.02
		2	11.03			41.93			44.73		
		3	29.46			61.62			44.73		
		4	33.82								
	3	**1**	5.19	79.28	2563.04	9.49	118.15	3186.53	36.21	117.63	2225.93
		2	11.03			39.64			42.82		
		3	30.31			73.64			42.82		
		4	36.50								
	4	**1**	15.84	84.03	2988.09	25.03	121.32	3809.28	36.98	120.55	3435.37
		2	10.81			42.42			44.81		
		3	28.46			59.80			44.80		
		4	31.74								
	5	**1**	13.08	72.37	1284.55	23.90	115.76	1789.03	35.06	114.68	1617.54
		2	9.54			39.59			41.28		
		3	24.64			55.25			41.28		
		4	26.17								
d_A= (90,18)	**1**	**1**	26.06	100.07	3184.01	20.50	53.73	1529.52	45.78	122.68	3552.63
d_B= (30,6)		**2**	36.52			23.98			37.51		
d_C= (10,2)		**3**	30.28			10.99			45.78		
		4	11.05								
	2	**1**	14.50	85.52	2653.01	13.77	45.69	1074.09	44.47	120.70	2900.52
		2	33.83			22.51			36.42		
		3	29.48			11.02			44.48		
		4	11.06								

Demand Normal (μ,σ)	Cost Structure	Dept.	OVERFLOW c*	TH	PR	NESTED c*	TH	PR	SYMMETRIC c*	TH	PR
	3	1	5.90	85.29	2743.16	5.17	35.10	744.58	43.11	118.38	2266.96
		2	41.32			20.41			35.52		
		3	31.34			11.04			43.11		
		4	11.07								
	4	1	16.69	87.03	3085.62	14.52	45.44	1283.37	44.58	120.82	3482.54
		2	33.39			21.50			36.38		
		3	29.16			10.84			44.58		
		4	10.86								
	5	1	13.10	72.33	1283.86	11.53	37.60	533.48	41.22	114.93	1650.13
		2	26.19			17.22			34.59		
		3	24.65			9.43			41.22		
		4	9.44								
d_A= (30,6)	1	1	15.91	88.27	2838.44	32.20	87.13	2486.92	30.89	88.09	2551.29
d_B= (30,6)		2	25.95			30.91			30.89		
d_C= (30,6)		3	25.80			29.76			30.89		
		4	26.75								
	2	1	9.76	87.34	2769.07	24.96	81.89	1981.55	30.13	87.08	2094.92
		2	27.77			30.83			30.12		
		3	27.73			31.26			30.11		
		4	28.24								
	3	1	3.32	85.25	2785.83	10.15	66.77	1512.47	29.44	85.97	1648.33
		2	29.46			29.71			29.44		
		3	29.54			31.86			29.45		
		4	29.52								
	4	1	11.58	87.33	3150.32	26.76	83.30	2392.26	30.12	87.08	2513.91
		2	26.91			30.80			30.12		
		3	26.94			30.57			30.11		
		4	27.47								
	5	1	11.55	84.43	1487.08	21.54	75.24	1087.02	28.58	84.27	1212.63
		2	25.06			28.45			28.58		
		3	25.09			27.27			28.57		
		4	25.61								

Table-C3_3 (high variance, high cost, p_{12}=0.5)

					p_{12}=0.5						
			OVERFLOW			**NESTED**			**SYMMETRIC**		
Demand Normal (μ,σ)	**Cost Structure**	**Dept.**	**c***	**TH**	**PR**	**c***	**TH**	**PR**	**c***	**TH**	**PR**
d_A= (10,2)	1	1	24.34	95.46	3042.51	34.20	124.45	3647.72	37.85	122.93	3506.88
d_B= (30,6)		2	11.02			44.65			47.07		
d_C= (90,18)		3	29.60			55.10			47.07		
		4	34.17								
	2	1	15.17	88.11	2724.54	22.34	119.61	3246.49	36.49	120.16	2841.67
		2	11.03			41.60			45.08		
		3	30.09			60.82			45.08		
		4	35.54								
	3	1	5.93	86.30	2774.69	9.00	117.04	3167.98	35.09	117.84	2210.60
		2	11.04			39.58			43.82		
		3	31.64			72.79			43.82		
		4	42.23								
	4	1	15.83	84.02	2987.69	24.98	121.10	3796.53	36.43	120.31	3411.97
		2	10.82			42.44			45.22		
		3	28.47			59.85			45.22		
		4	31.74								
	5	1	13.45	73.87	1308.53	23.71	115.42	1778.99	33.55	114.11	1599.55
		2	9.55			39.54			41.88		
		3	25.10			55.33			41.88		
		4	26.92								
d_A= (90,18)	1	1	22.76	91.30	2922.06	19.79	52.46	1497.12	47.03	123.15	3492.98

d_B= (30,6)		2	31.88			23.31			39.19		
d_C= (10,2)		3	28.65			10.99			47.02		
		4	11.05								
	2	1	16.34	92.39	2845.78	13.20	44.43	1050.56	45.15	120.68	2830.74
		2	38.21			21.74			37.83		
		3	30.85			11.03			45.15		
		4	11.05								
	3	1	5.07	78.13	2528.26	5.13	35.06	745.20	43.28	117.75	2193.68
		2	35.54			20.37			36.57		
		3	30.08			11.04			43.28		
		4	11.06								
	4	1	13.89	76.72	2748.47	13.93	44.14	1252.44	45.27	120.79	3397.79
		2	27.79			20.73			37.79		
		3	26.30			10.84			45.26		
		4	10.86								
	5	1	11.95	67.62	1207.97	12.06	39.00	552.40	41.30	113.95	1584.56
		2	23.91			18.06			34.92		
		3	23.14			9.43			41.30		
		4	9.44								
d_A= (30,6)	1	1	15.75	87.61	2793.14	31.12	86.14	2437.58	31.18	87.13	2485.80
d_B= (30,6)		2	26.83			32.25			31.18		
d_C= (30,6)		3	26.81			29.74			31.18		
		4	25.95								
	2	1	9.37	86.05	2730.04	23.74	80.53	1954.48	30.02	85.53	2024.98
		2	27.78			30.88			30.02		
		3	27.84			31.28			30.02		
		4	27.34								
	3	1	3.26	82.71	2724.99	10.26	66.82	1512.32	29.14	84.11	1581.51
		2	28.10			29.59			29.15		
		3	28.24			31.95			29.18		
		4	27.92								
	4	1	11.19	85.94	3097.00	26.25	82.57	2358.58	30.02	85.53	2429.97
		2	27.01			31.18			30.02		
		3	27.05			30.53			30.02		
		4	26.53								
	5	1	10.64	82.75	1451.86	21.37	75.03	1081.84	28.00	81.86	1152.98
		2	25.30			28.47			28.00		
		3	25.31			27.28			28.00		
		4	24.72								

Table-C3_4 (high variance, high cost, p_{12}=1)

p_{12}=1											
			OVERFLOW			**NESTED**			**SYMMETRIC**		
Demand Normal (μ,σ)	**Cost Structure**	**Dept.**	**c***	**TH**	**PR**	**c***	**TH**	**PR**	**c***	**TH**	**PR**
d_A= (10,2)	1	1	23.30	92.48	2953.00	34.04	124.01	3629.56	37.57	122.54	3487.12
d_B= (30,6)		2	11.02			44.61			47.22		
d_C= (90,18)		3	28.88			55.17			47.22		
		4	32.69								
	2	1	15.19	87.98	2718.83	22.14	119.11	3230.20	36.05	119.75	2822.77
		2	11.03			41.51			45.27		
		3	29.94			60.85			45.27		
		4	35.60								
	3	1	5.57	82.71	2664.58	8.83	114.29	3102.72	34.36	117.32	2193.77
		2	11.04			38.74			44.03		
		3	30.87			70.15			44.03		
		4	39.44								
	4	1	15.85	83.91	2981.82	25.03	120.12	3762.30	36.02	119.83	3388.30
		2	10.82			42.07			45.35		

		3	28.36			59.09			45.35		
		4	31.78								
	5	1	13.47	73.82	1306.02	23.61	114.29	1759.69	32.97	111.86	1574.06
		2	9.55			39.15			40.93		
		3	25.01			54.66			40.92		
		4	26.98								
d_A= (90,18)	1	1	23.32	92.66	2960.62	20.52	53.71	1528.72	47.48	123.32	3455.47
d_B= (30,6)		2	32.68			23.96			40.58		
d_C= (10,2)		3	28.89			10.97			47.48		
		4	11.05								
	2	1	14.89	86.81	2687.33	13.22	44.41	1049.27	45.45	120.76	2792.52
		2	34.76			21.72			38.92		
		3	29.66			11.03			45.44		
		4	11.06								
	3	1	5.08	78.03	2523.63	5.14	35.05	744.68	43.13	117.54	2155.12
		2	35.63			20.37			37.80		
		3	29.93			11.04			43.13		
		4	11.07								
	4	1	13.91	76.63	2743.86	13.92	44.09	1250.66	45.45	120.78	3351.01
		2	27.82			20.70			38.95		
		3	26.22			10.84			45.45		
		4	10.86								
	5	1	13.90	75.34	1330.66	12.07	39.00	552.25	40.64	113.05	1548.57
		2	27.81			18.06			35.98		
		3	25.43			9.43			40.64		
		4	9.44								
d_A= (30,6)	1	1	15.33	86.78	2762.72	30.29	85.62	2413.05	31.18	86.40	2449.60
d_B= (30,6)		2	26.93			32.98			31.18		
d_C= (30,6)		3	26.93			30.08			31.18		
		4	25.69								
	2	1	9.36	84.94	2694.14	24.46	81.04	1949.53	30.02	84.85	1990.79
		2	27.47			31.00			30.02		
		3	27.47			31.43			30.02		
		4	26.75								
	3	1	2.99	81.30	2689.82	10.50	66.85	1506.04	28.86	82.92	1547.72
		2	27.67			29.47			28.87		
		3	27.67			32.03			28.87		
		4	27.39								
	4	1	10.74	85.23	3069.33	26.67	82.75	2343.21	30.02	84.85	2388.94
		2	27.22			31.40			30.02		
		3	27.22			30.64			30.02		
		4	26.30								
	5	1	10.32	81.29	1426.49	21.37	74.74	1073.84	27.71	80.65	1122.87
		2	25.01			28.34			27.71		
		3	25.01			27.20			27.71		
		4	24.15								

Table-C4_1 (high variance, low cost, p_{12}=-1)

p_{12}=-1											
			OVERFLOW			**NESTED**			**SYMMETRIC**		
Demand Normal (μ,σ)	**Cost Structure**	**Dept.**	**c***	**TH**	**PR**	**c***	**TH**	**PR**	**c***	**TH**	**PR**
d_A= (10,2)	1	1	38.95	115.47	5072.78	47.17	129.05	5545.32	41.30	127.46	5502.32
d_B= (30,6)		2	12.42			50.79			51.92		
d_C= (90,18)		3	32.39			54.42			51.92		
		4	40.84								
	2	1	34.03	112.32	4739.96	40.29	127.81	5150.73	40.58	126.30	5050.61
		2	12.46			48.29			49.96		
		3	32.61			56.30			49.96		
		4	41.65								

Demand Normal (μ,σ)	Cost Structure	Dept.	OVERFLOW c*	TH	PR	NESTED c*	TH	PR	SYMMETRIC c*	TH	PR
	3	1	43.83	121.50	11385.72	50.01	129.59	12014.77	42.29	128.85	11965.45
		2	12.92			52.76			55.48		
		3	33.75			55.51			55.48		
		4	44.87								
	4	1	41.31	120.42	11043.46	45.67	129.32	11586.04	41.76	128.20	11479.84
		2	12.94			51.63			53.58		
		3	33.87			57.58			53.58		
		4	45.44								
	5	1	40.66	120.31	10956.76	44.43	129.17	11450.03	41.58	127.95	11320.06
		2	12.95			51.15			52.98		
		3	33.97			57.87			52.98		
		4	45.87								
d_A= (90,18)	1	1	40.32	118.63	5216.15	32.49	69.90	3028.22	47.94	125.55	5469.24
d_B= (30,6)		2	42.27			29.72			38.84		
d_C= (10,2)		3	31.89			12.22			47.94		
		4	12.45								
	2	1	36.48	117.65	4960.34	28.69	65.59	2585.87	47.26	124.98	5053.37
		2	44.67			28.82			38.35		
		3	32.43			12.28			47.26		
		4	12.48								
	3	1	43.93	122.49	11491.74	37.79	76.21	7101.65	49.72	126.77	11844.41
		2	44.95			31.92			39.33		
		3	31.93			12.68			49.72		
		4	13.03								
	4	1	42.06	122.19	11214.47	36.36	74.71	6649.50	49.05	126.34	11400.35
		2	46.26			31.72			38.95		
		3	32.19			12.72			49.05		
		4	13.04								
	5	1	41.39	122.05	11124.39	36.19	74.56	6531.89	48.38	125.88	11230.32
		2	46.68			31.77			38.97		
		3	32.28			12.72			48.38		
		4	13.05								
d_A= (30,6)	1	1	23.47	89.64	3942.81	37.28	89.51	3877.31	32.33	89.68	3901.90
d_B= (30,6)		2	24.29			31.03			32.33		
d_C= (30,6)		3	23.99			30.23			32.33		
		4	26.69								
	2	1	19.47	89.43	3826.49	34.69	88.96	3564.05	32.05	89.55	3612.50
		2	25.74			30.35			32.04		
		3	25.38			31.98			32.04		
		4	27.31								
	3	1	24.89	89.85	8430.43	39.05	89.78	8365.79	33.20	89.91	8393.52
		2	24.59			31.25			33.20		
		3	24.45			30.25			33.20		
		4	27.04								
	4	1	21.58	89.77	8301.66	37.51	89.62	8039.91	32.62	89.77	8096.72
		2	25.78			30.71			32.62		
		3	25.36			31.60			32.62		
		4	27.88								
	5	1	20.63	89.74	8265.56	37.20	89.56	7936.06	32.62	89.77	7998.86
		2	26.01			30.23			32.62		
		3	25.63			31.99			32.62		
		4	28.16								

Table-C4_2 (high variance, low cost, p_{12}=-0.5)

p_{12}=-0.5											
			OVERFLOW	overflow		**NESTED**	nested		**SYMMETRIC**	symmetric	
Demand Normal (μ,σ)	**Cost Structure**	**Dept.**	**c***	**TH**	**PR**	**c***	**TH**	**PR**	**c***	**TH**	**PR**
d_A= (10,2)	1	1	38.89	115.68	5082.98	47.59	129.29	5560.17	41.62	128.31	5527.47
d_B= (30,6)		2	12.42			50.56			53.19		
d_C= (90,18)		3	32.52			53.53			53.19		

		4	40.78								
	2	1	33.40	111.36	4704.34	40.35	128.41	5171.81	40.77	127.28	5073.90
		2	12.47			48.73			51.28		
		3	32.54			57.10			51.28		
		4	40.84								
	3	1	43.12	121.00	11344.07	50.06	129.93	12045.87	42.73	129.18	11995.05
		2	12.94			52.93			55.54		
		3	33.74			55.80			55.54		
		4	44.13								
	4	1	41.25	120.68	11070.47	45.92	129.55	11612.16	42.11	128.73	11518.33
		2	12.95			51.36			54.19		
		3	34.01			56.81			54.19		
		4	45.38								
	5	1	40.61	120.57	10983.74	44.73	129.37	11473.92	42.00	128.51	11358.88
		2	12.95			50.81			53.62		
		3	34.10			56.88			53.62		
		4	45.80								
d_A= (90,18)	1	1	39.61	117.31	5156.12	31.50	68.96	2990.21	51.56	127.84	5531.16
d_B= (30,6)		2	41.52			29.42			40.37		
d_C= (10,2)		3	32.43			12.20			51.56		
		4	12.44								
	2	1	35.81	116.33	4904.52	28.56	65.76	2594.96	49.67	126.65	5081.17
		2	43.86			28.97			39.69		
		3	32.99			12.25			49.67		
		4	12.46								
	3	1	43.85	122.42	11479.36	36.75	75.32	7021.24	54.55	129.11	12008.49
		2	44.87			31.70			41.28		
		3	33.31			12.67			54.55		
		4	13.03								
	4	1	41.97	122.13	11202.90	36.26	74.87	6666.26	52.65	128.39	11523.87
		2	46.17			31.83			40.79		
		3	33.58			12.69			52.65		
		4	13.04								
	5	1	41.31	122.00	11113.53	36.08	74.72	6548.47	52.00	128.08	11361.48
		2	46.59			31.88			40.67		
		3	33.68			12.69			52.00		
		4	13.05								
d_A= (30,6)	1	1	23.61	89.92	3945.32	38.46	89.83	3875.90	33.20	89.87	3896.17
d_B= (30,6)		2	25.52			32.57			33.20		
d_C= (30,6)		3	25.36			30.17			33.20		
		4	26.25								
	2	1	19.55	89.74	3831.53	35.64	89.28	3552.77	32.62	89.61	3599.80
		2	26.61			32.07			32.62		
		3	26.42			31.82			32.62		
		4	27.25								
	3	1	24.71	90.18	8449.10	40.17	90.12	8377.75	34.35	90.18	8399.41
		2	26.04			33.23			34.35		
		3	26.03			30.79			34.35		
		4	27.15								
	4	1	21.61	90.10	8322.06	38.46	89.96	8041.61	33.78	90.05	8093.47
		2	26.90			32.72			33.77		
		3	26.77			32.05			33.78		
		4	27.74								
	5	1	21.51	90.05	8278.29	38.07	89.90	7932.70	33.49	89.97	7992.68
		2	26.81			32.45			33.49		
		3	26.64			32.37			33.49		
		4	27.46								

Table-C4_3 (high variance, low cost, p_{12}=0.5)

p_{12}=0.5											
			OVERFLOW			**NESTED**			**SYMMETRIC**		
Demand Normal (μ,σ)	**Cost Structure**	**Dept.**	**c***	**TH**	**PR**	**c***	**TH**	**PR**	**c***	**TH**	**PR**
d_A= (10,2)	**1**	**1**	38.86	115.74	5085.72	47.76	129.42	5559.17	42.07	128.53	5527.52
d_B= (30,6)		**2**	12.46			51.01			53.89		
d_C= (90,18)		**3**	32.66			54.26			53.89		
		4	40.75								
	2	**1**	35.13	114.80	4839.58	40.56	128.37	5166.40	41.03	127.54	5071.30
		2	12.48			48.81			52.03		
		3	33.21			57.06			52.02		
		4	43.06								
	3	**1**	44.39	122.57	11483.86	50.85	130.01	12047.28	43.42	129.52	12010.33
		2	12.96			53.25			56.79		
		3	34.14			55.65			56.79		
		4	45.45								
	4	**1**	42.44	122.29	11206.11	45.99	129.64	11613.77	42.59	129.13	11531.17
		2	12.97			51.71			55.47		
		3	34.42			57.43			55.47		
		4	46.78								
	5	**1**	41.76	122.17	11117.01	44.75	129.49	11477.11	42.42	128.95	11371.86
		2	12.98			51.23			54.93		
		3	34.51			57.70			54.93		
		4	47.23								
d_A= (90,18)	**1**	**1**	38.79	115.71	5083.91	31.50	68.97	2990.39	54.61	128.85	5523.05
d_B= (30,6)		**2**	40.68			29.42			44.01		
d_C= (10,2)		**3**	32.92			12.19			54.61		
		4	12.43								
	2	**1**	35.07	114.82	4840.26	28.56	65.76	2595.09	52.69	127.93	5063.41
		2	43.01			28.96			42.77		
		3	33.46			12.23			52.68		
		4	12.45								
	3	**1**	43.04	121.14	11355.93	38.60	77.48	7220.13	59.02	130.03	12020.74
		2	44.04			32.41			45.60		
		3	34.30			12.65			59.02		
		4	13.02								
	4	**1**	41.78	121.63	11151.43	38.11	77.03	6851.29	56.75	129.55	11530.04
		2	46.00			32.52			44.77		
		3	34.75			12.68			56.75		
		4	13.03								
	5	**1**	41.12	121.53	11064.39	36.99	75.82	6640.65	56.21	129.38	11369.24
		2	46.44			32.26			44.44		
		3	34.84			12.70			56.21		
		4	13.04								
d_A= (30,6)	**1**	**1**	24.17	89.88	3920.53	38.42	89.73	3847.56	34.64	89.77	3864.94
d_B= (30,6)		**2**	27.25			36.07			34.64		
d_C= (30,6)		**3**	27.18			30.73			34.64		
		4	26.38								
	2	**1**	19.83	89.56	3805.07	35.40	89.10	3517.69	34.06	89.51	3555.86
		2	27.91			35.26			34.06		
		3	27.88			32.02			34.06		
		4	27.19								
	3	**1**	25.12	90.24	8424.87	40.49	90.16	8348.04	36.37	90.21	8366.78
		2	28.71			37.64			36.37		
		3	28.60			31.70			36.37		
		4	27.45								
	4	**1**	22.17	90.08	8293.49	38.36	89.89	7998.17	35.22	89.97	8045.69
		2	28.71			36.95			35.21		
		3	28.67			31.95			35.22		

Demand Normal (μ,σ)	Cost Structure	Dept.	OVERFLOW c*	TH	PR	NESTED c*	TH	PR	SYMMETRIC c*	TH	PR
		4	27.79								
	5	1	21.87	90.02	8250.13	37.61	89.78	7884.99	35.22	89.97	7940.04
		2	28.55			36.60			35.22		
		3	28.51			32.59			35.22		
		4	27.74								

Table-C4_4 (high variance, low cost, p_{12}=1)

p_{12}=1											
			OVERFLOW			**NESTED**			**SYMMETRIC**		
Demand Normal (μ,σ)	**Cost Structure**	**Dept.**	**c***	**TH**	**PR**	**c***	**TH**	**PR**	**c***	**TH**	**PR**
d_A= **(10,2)**	**1**	**1**	40.17	117.48	5155.43	48.13	129.13	5540.70	41.93	128.60	5516.94
d_B= **(30,6)**		**2**	12.47			51.22			55.12		
d_C= **(90,18)**		**3**	32.86			54.30			55.11		
		4	42.15								
	2	**1**	35.72	115.61	4868.37	40.53	127.95	5145.77	40.82	127.14	5051.14
		2	12.49			48.81			52.14		
		3	33.26			57.09			52.14		
		4	43.85								
	3	**1**	46.36	124.30	11630.93	50.94	129.79	12017.57	43.66	129.49	11990.99
		2	12.97			53.74			57.98		
		3	34.42			56.54			57.98		
		4	47.51								
	4	**1**	43.65	123.40	11293.32	46.29	129.36	11579.28	42.80	129.01	11504.20
		2	13.00			51.91			56.18		
		3	34.57			57.53			56.18		
		4	48.25								
	5	**1**	43.51	123.93	11254.50	45.12	129.20	11440.64	42.58	128.81	11343.66
		2	12.99			51.42			55.59		
		3	34.78			57.72			55.59		
		4	49.47								
d_A= **(90,18)**	**1**	**1**	38.81	115.49	5072.70	32.44	69.98	3032.27	55.72	128.96	5499.57
d_B= **(30,6)**		**2**	40.70			29.77			46.61		
d_C= **(10,2)**		**3**	32.91			12.21			55.72		
		4	12.43								
	2	**1**	35.09	114.62	4830.42	29.48	66.80	2631.28	53.50	128.04	5032.87
		2	43.05			29.35			45.14		
		3	33.42			12.25			53.50		
		4	12.45								
	3	**1**	43.66	121.66	11398.57	38.70	77.34	7206.49	58.95	129.74	11974.87
		2	44.69			32.26			48.54		
		3	34.64			12.70			58.95		
		4	13.01								
	4	**1**	41.16	120.59	11058.52	38.21	76.89	6837.69	57.36	129.42	11481.38
		2	45.30			32.36			47.54		
		3	34.66			12.73			57.36		
		4	13.04								
	5	**1**	41.10	121.27	11038.82	38.04	76.74	6715.74	57.01	129.30	11319.96
		2	46.44			32.40			46.99		
		3	34.95			12.74			57.00		
		4	13.04								
d_A= **(30,6)**	**1**	**1**	24.10	89.54	3894.64	37.12	89.35	3822.26	35.22	89.41	3836.67
d_B= **(30,6)**		**2**	27.93			37.91			35.22		
d_C= **(30,6)**		**3**	27.93			31.62			35.22		
		4	26.84								
	2	**1**	19.40	89.14	3781.89	34.35	88.75	3494.25	34.06	88.89	3524.71
		2	28.57			36.93			34.06		
		3	28.57			32.86			34.06		
		4	27.40								

	3	1	25.48	89.91	8382.60	39.26	89.84	8308.19	36.95	89.88	8322.76
		2	29.08			39.79			36.95		
		3	29.09			32.47			36.95		
		4	27.88								
	4	1	23.00	89.74	8244.98	37.42	89.62	7958.99	36.08	89.68	7994.14
		2	29.02			38.84			36.08		
		3	29.02			33.49			36.09		
		4	28.05								
	5	1	21.97	89.70	8207.51	36.85	89.52	7845.77	35.80	89.60	7886.43
		2	29.24			38.61			35.80		
		3	29.24			33.40			35.80		
		4	28.18								

(tables for 8 demand types) (Dept 9 corresponds to the full-flexible department)

Table-D1 (Low variance, high cost)

Demand Normal (μ,σ)	Cost Structure	Dept.	OVERFLOW c*	OVERFLOW TH	OVERFLOW PR	NESTED c*	NESTED TH	NESTED PR	SYMMETRIC c*	SYMMETRIC TH	SYMMETRIC PR
(80,8)	1	1	2.19	17.50	612.52	0.00	15.81	395.19	81.68	617.94	17829.75
(80,8)		2	2.19			0.56			81.68		
(80,8)		3	2.19			1.13			81.70		
(80,8)		4	2.19			1.69			81.69		
(80,8)		5	2.19			2.26			81.66		
(80,8)		6	2.19			2.82			81.65		
(80,8)		7	2.19			3.39			81.69		
(80,8)		8	2.19			3.95			81.62		
		9	0.00								
	2	1	83.83	623.90	21135.63	0.00	247.47	5611.36	79.20	608.97	14609.58
		2	83.83			0.00			79.20		
		3	83.83			0.00			79.20		
		4	83.83			0.00			79.19		
		5	83.82			23.91			79.18		
		6	83.83			69.90			79.17		
		7	83.83			79.28			79.19		
		8	83.83			83.84			79.22		
		9	0.00								
	3	1	83.82	623.89	21135.63	0.00	177.08	4196.20	77.55	601.95	11471.88
		2	83.82			0.00			77.53		
		3	83.82			0.00			77.59		
		4	83.82			0.00			77.54		
		5	83.82			0.00			77.62		
		6	83.82			24.09			77.63		
		7	83.82			77.20			77.65		
		8	83.82			83.84			77.74		
		9	0.00								
	4	1	83.01	621.81	24026.98	0.00	10.24	307.07	79.19	608.98	17531.50
		2	83.01			0.00			79.18		
		3	83.01			0.00			79.21		
		4	83.01			0.00			79.21		
		5	83.01			1.02			79.19		
		6	83.01			2.05			79.19		
		7	83.01			3.07			79.19		
		8	83.01			4.09			79.19		
		9	0.00								
	5	1	77.51	601.49	11471.92	0.00	7.72	115.74	75.54	590.82	8408.75
		2	77.51			0.00			75.56		
		3	77.51			0.00			75.54		
		4	77.51			0.00			75.35		
		5	77.51			0.77			75.52		
		6	77.51			1.54			75.57		
		7	77.51			2.31			75.33		
		8	77.51			3.09			75.36		
		9	0.00								

	(5,0.5)		**1**	2.19	17.50	612.52	0.00	15.81	395.19	15.07	160.08	4782.02
	(10,1)		**2**	2.19			0.56			19.73		
	(20,2)		**3**	2.19			1.13			21.05		
	(40,4)		**4**	2.19			1.69			21.05		
	(80,8)	**1**	**5**	2.19			2.26			21.05		
	(160,16)		**6**	2.19			2.82			21.05		
	(320,32)		**7**	2.19			3.39			21.05		
	(640,64)		**8**	2.19			3.95			21.05		
			9	0.00								
			1	5.30	108.87	3794.21	0.00	1142.86	29757.32	14.81	154.56	3846.06
			2	10.49			0.00			19.27		
			3	15.68			0.00			20.20		
			4	15.70			0.00			20.20		
		2	**5**	15.70			73.24			20.20		
			6	15.70			210.98			20.20		
			7	15.70			361.23			20.20		
			8	15.70			511.67			20.20		
			9	0.00								
			1	5.30	100.26	3493.58	0.00	1022.41	27420.11	14.54	148.83	2962.51
			2	10.43			0.00			18.69		
			3	14.26			0.00			19.34		
			4	14.26			0.00			19.34		
		3	**5**	14.26			0.00			19.35		
			6	14.26			96.28			19.34		
			7	14.26			339.07			19.34		
			8	14.26			613.01			19.35		
			9	0.00								
			1	5.24	90.89	3617.85	0.00	10.24	307.07	14.81	155.00	4628.26
			2	10.25			0.00			19.31		
			3	12.72			0.00			20.27		
			4	12.71			0.00			20.27		
		4	**5**	12.71			1.02			20.27		
			6	12.71			2.05			20.27		
			7	12.71			3.07			20.27		
			8	12.72			4.09			20.27		
			9	0.00								
			1	4.86	79.69	1582.59	0.00	7.72	115.74	14.24	142.86	2133.41
			2	9.59			0.00			18.06		
			3	10.94			0.00			18.47		
			4	10.93			0.00			18.47		
		5	**5**	10.94			0.77			18.47		
			6	10.94			1.54			18.47		
			7	10.93			2.31			18.47		
			8	10.93			3.09			18.47		
			9	0.00								
	(640,64)	**1**	**1**	2.19	17.50	612.52	0.00	15.77	394.08	21.05	160.08	4782.02
	(320,32)		**2**	2.19			0.56			21.05		
	(160,16)		**3**	2.19			1.13			21.05		
	(80,8)		**4**	2.19			1.69			21.05		
	(40,4)		**5**	2.19			2.25			21.05		

	(20,2)		**6**	2.19			2.82			19.73		
	(10,1)		**7**	2.19			3.38			15.07		
	(5,0.5)		**8**	2.19			3.94			21.05		
			9	0.00								
		2	**1**	17.22	117.86	4107.36	0.00	21.89	490.62	20.20	154.56	3846.07
			2	17.22			0.00			20.20		
			3	17.22			0.00			20.20		
			4	17.22			0.00			20.20		
			5	17.22			1.95			20.20		
			6	17.12			5.84			19.27		
			7	10.52			9.35			14.81		
			8	5.30			5.23			20.20		
			9	0.00								
		3	**1**	15.21	105.96	3692.63	0.00	14.92	336.20	19.35	148.83	2962.54
			2	15.21			0.00			19.35		
			3	15.21			0.00			19.35		
			4	15.21			0.00			19.34		
			5	15.21			0.00			19.34		
			6	15.21			2.01			18.69		
			7	10.47			8.00			14.54		
			8	5.30			5.29			19.34		
			9	0.00								
		4	**1**	12.32	88.53	3524.07	0.00	10.21	305.97	20.27	155.00	4628.26
			2	12.32			0.00			20.27		
			3	12.32			0.00			20.27		
			4	12.32			0.00			20.27		
			5	12.32			1.02			20.27		
			6	12.33			2.04			19.31		
			7	10.21			3.07			14.81		
			8	5.24			4.08			20.27		
			9	0.00								
		5	**1**	9.03	67.66	1347.07	0.00	7.72	115.74	18.46	142.80	2132.44
			2	9.03			0.00			18.46		
			3	9.03			0.00			18.46		
			4	9.03			0.00			18.46		
			5	9.03			0.77			18.46		
			6	9.03			1.54			18.05		
			7	8.84			2.31			14.24		
			8	4.86			3.09			18.46		
			9	0.00								

Table-D2 (Low variance, low cost)

			OVERFLOW			NESTED			SYMMETRIC		
Demand Normal (μ,σ)	**Cost Structure**	**Dept.**	**c***	**TH**	**PR**	**c***	**TH**	**PR**	**c***	**TH**	**PR**
(80,8)	**1**	**1**	78.64	628.93	27554.79	85.07	629.63	25586.86	88.75	631.84	27332.45
(80,8)		**2**	78.64			85.61			88.75		
(80,8)		**3**	78.63			86.10			88.75		
(80,8)		**4**	78.63			86.73			88.74		
(80,8)		**5**	78.64			87.38			88.74		
(80,8)		**6**	78.63			87.79			88.74		
(80,8)		**7**	78.64			88.48			88.75		
(80,8)		**8**	78.63			89.28			88.73		
		9	62.19								
	2	**1**	74.99	604.79	26385.65	75.75	616.26	18632.71	86.62	629.25	25225.80
		2	74.99			77.80			86.62		
		3	74.99			79.29			86.63		
		4	74.98			81.09			86.62		
		5	74.98			82.86			86.62		
		6	74.98			84.66			86.62		
		7	74.99			86.83			86.61		
		8	74.98			89.76			86.62		
		9	25.89								
	3	**1**	81.34	633.13	59208.18	88.86	633.47	57193.52	92.63	634.59	59013.00
		2	81.34			89.41			92.63		
		3	81.34			89.87			92.63		
		4	81.34			90.30			92.63		
		5	81.34			90.95			92.63		
		6	81.34			91.70			92.63		
		7	81.34			92.70			92.63		
		8	81.34			93.30			92.63		
		9	70.92								
	4	**1**	78.67	626.49	57827.28	83.19	628.83	49864.22	90.52	633.35	56818.20
		2	78.67			84.16			90.52		
		3	78.67			84.82			90.52		
		4	78.67			85.89			90.51		
		5	78.67			87.11			90.51		
		6	78.67			88.46			90.50		
		7	78.67			90.31			90.50		
		8	78.67			92.97			90.50		
		9	50.76								
	5	**1**	77.81	622.91	57379.64	81.64	626.97	47497.69	90.01	632.95	56097.14
		2	77.81			82.63			89.96		
		3	77.81			83.91			89.96		
		4	77.81			84.87			89.98		
		5	77.81			86.26			89.98		
		6	77.81			87.71			89.98		
		7	77.81			89.86			89.99		
		8	77.81			92.62			89.98		
		9	44.99								

	(5,0.5)		**1**	5.63	122.21	5397.79	148.99	1267.54	52221.82	15.92	180.82	7941.80
	(10,1)		**2**	11.05			154.23			21.06		
	(20,2)		**3**	15.67			159.45			24.37		
	(40,4)		**4**	15.68			164.66			24.37		
	(80,8)	**1**	**5**	15.68			169.89			24.37		
	(160,16)		**6**	15.68			175.09			24.37		
	(320,32)		**7**	15.68			180.33			24.37		
	(640,64)		**8**	15.68			185.56			24.37		
			9	13.24								
			1	5.63	119.00	5173.16	82.99	1235.56	41465.53	15.70	173.25	7085.87
			2	11.10			104.14			20.67		
			3	16.30			125.29			23.14		
			4	16.33			146.44			23.14		
		2	**5**	16.33			167.58			23.14		
			6	16.33			188.72			23.14		
			7	16.33			209.87			23.14		
			8	16.33			231.01			23.14		
			9	6.17								
			1	5.80	187.42	17617.43	161.20	1272.01	115690.96	16.16	192.46	18073.56
			2	11.61			164.00			21.56		
			3	21.40			166.79			26.29		
			4	25.77			169.60			26.29		
		3	**5**	25.77			172.41			26.29		
			6	25.77			175.21			26.29		
			7	25.77			178.01			26.29		
			8	25.77			180.81			26.29		
			9	23.87								
			1	5.81	190.84	17567.55	131.87	1262.63	102796.39	16.05	185.04	16814.36
			2	11.64			141.28			21.26		
			3	21.68			150.67			25.06		
			4	27.36			160.08			25.06		
		4	**5**	27.36			169.48			25.06		
			6	27.36			178.89			25.06		
			7	27.36			188.29			25.06		
			8	27.36			197.69			25.06		
			9	19.30								
			1	5.81	190.08	17426.27	121.91	1258.16	98937.57	16.00	185.01	16624.98
			2	11.64			133.67			21.24		
			3	21.71			145.42			25.06		
			4	27.58			157.15			25.06		
		5	**5**	27.58			168.89			25.06		
			6	27.58			180.67			25.06		
			7	27.58			192.39			25.06		
			8	27.58			204.14			25.06		
			9	17.42								
	(640,64)	**1**	**1**	15.68	122.21	5397.80	10.00	78.68	3241.35	24.37	180.82	7941.81
	(320,32)		**2**	15.68			10.26			24.37		
	(160,16)		**3**	15.68			10.53			24.37		
	(80,8)		**4**	15.68			10.79			24.37		
	(40,4)		**5**	15.68			11.05			24.37		

	(20,2)		**6**	15.67			11.32			21.06		
	(10,1)		**7**	11.05			10.35			15.92		
	(5,0.5)		**8**	5.63			5.55			24.37		
			9	13.24								
		2	**1**	15.52	113.89	4951.87	4.93	61.70	1965.19	23.14	173.28	7086.85
			2	15.52			6.09			23.14		
			3	15.52			7.25			23.14		
			4	15.52			8.41			23.14		
			5	15.52			9.57			23.14		
			6	15.51			10.73			20.67		
			7	11.03			10.37			15.70		
			8	5.63			5.54			23.14		
			9	5.86								
		3	**1**	24.95	182.50	17155.95	12.31	90.81	8268.58	25.88	190.00	17842.76
			2	24.95			12.44			25.88		
			3	24.95			12.58			25.88		
			4	24.95			12.72			25.88		
			5	24.95			12.86			25.88		
			6	21.23			13.00			21.48		
			7	11.60			10.81			16.17		
			8	5.80			5.66			25.88		
			9	23.11								
		4	**1**	27.36	190.85	17568.55	9.72	81.91	6564.38	25.06	185.04	16814.33
			2	27.36			10.30			25.06		
			3	27.36			10.88			25.06		
			4	27.36			11.46			25.06		
			5	27.36			12.04			25.06		
			6	21.68			12.62			21.26		
			7	11.64			10.77			16.05		
			8	5.81			5.67			25.06		
			9	19.30								
		5	**1**	26.30	182.79	16762.61	8.82	78.78	6048.58	25.06	185.01	16625.02
			2	26.30			9.55			25.06		
			3	26.30			10.28			25.06		
			4	26.30			11.02			25.06		
			5	26.30			11.75			25.06		
			6	21.51			12.49			21.24		
			7	11.62			10.76			16.00		
			8	5.81			5.67			25.06		
			9	16.61								

Table-D3 (high variance, high cost)

Demand Normal (μ,σ)	Cost Structure	Dept.	OVERFLOW c*	TH	PR	NESTED c*	TH	PR	SYMMETRIC c*	TH	PR
(80,8)	1	1	2.19	17.50	612.52	0.00	15.81	395.19	83.78	604.44	16815.24
(80,8)		2	2.19			0.56			83.78		
(80,8)		3	2.19			1.13			83.79		
(80,8)		4	2.19			1.69			83.82		
(80,8)		5	2.19			2.26			83.77		
(80,8)		6	2.19			2.82			83.84		
(80,8)		7	2.19			3.39			83.78		
(80,8)		8	2.19			3.95			83.78		
		9	0.00								
	2	1	87.55	614.92	20239.93	0.00	251.24	5326.94	78.86	586.54	13558.61
		2	87.55			0.00			78.84		
		3	87.55			0.00			78.82		
		4	87.55			0.00			78.87		
		5	87.55			32.63			78.87		
		6	87.55			70.93			78.85		
		7	87.55			78.83			78.84		
		8	87.55			87.58			78.77		
		9	0.00								
	3	1	87.54	614.89	20239.92	0.00	183.20	4011.29	75.71	572.42	10462.80
		2	87.54			0.00			75.70		
		3	87.54			0.00			75.72		
		4	87.54			0.00			75.66		
		5	87.54			0.00			75.59		
		6	87.54			34.79			75.61		
		7	87.54			75.63			75.65		
		8	87.54			87.61			75.63		
		9	0.00								
	4	1	85.83	610.50	22897.40	0.00	10.24	307.07	78.82	586.54	16270.33
		2	85.83			0.00			78.80		
		3	85.82			0.00			78.81		
		4	85.83			0.00			78.83		
		5	85.83			1.02			78.88		
		6	85.83			2.05			78.88		
		7	85.83			3.07			78.82		
		8	85.83			4.09			78.88		
		9	0.00								
	5	1	75.65	572.34	10462.79	0.00	7.72	115.74	71.36	550.23	7515.99
		2	75.65			0.00			71.38		
		3	75.64			0.00			71.43		
		4	75.64			0.00			71.48		
		5	75.64			0.77			71.48		
		6	75.64			1.54			71.32		
		7	75.64			2.31			71.35		
		8	75.64			3.09			71.50		
		9	0.00								

	(5,0.5)		**1**	2.19	17.50	612.52	0.00	15.81	395.19	14.88	169.44	5043.70
	(10,1)		**2**	2.19			0.56			19.66		
	(20,2)		**3**	2.19			1.13			22.79		
	(40,4)		**4**	2.19			1.69			22.82		
	(80,8)	**1**	**5**	2.19			2.26			22.82		
	(160,16)		**6**	2.19			2.82			22.82		
	(320,32)		**7**	2.19			3.39			22.82		
	(640,64)		**8**	2.19			3.95			22.82		
			9	0.00								
			1	5.57	134.43	4659.53	0.00	1088.84	27994.81	14.42	156.39	3876.07
			2	10.92			0.00			18.65		
			3	18.49			0.00			20.77		
			4	20.49			0.00			20.78		
		2	**5**	20.50			71.19			20.78		
			6	20.50			200.31			20.78		
			7	20.50			350.59			20.78		
			8	20.50			497.87			20.78		
			9	0.00								
			1	5.57	141.13	4890.16	0.00	1004.99	25828.87	13.92	150.61	2983.67
			2	10.95			0.00			18.08		
			3	19.04			0.00			19.92		
			4	21.75			0.00			19.93		
		3	**5**	21.78			0.00			19.93		
			6	21.78			108.44			19.93		
			7	21.78			344.23			19.93		
			8	21.78			614.27			19.93		
			9	0.00								
			1	5.49	129.49	5125.87	0.00	10.24	307.07	14.42	156.82	4664.21
			2	10.75			0.00			18.67		
			3	17.98			0.00			20.84		
			4	19.58			0.00			20.85		
		4	**5**	19.59			1.02			20.85		
			6	19.59			2.05			20.85		
			7	19.59			3.07			20.85		
			8	19.59			4.09			20.85		
			9	0.00								
			1	4.73	100.46	1980.29	0.00	7.72	115.74	13.44	142.10	2109.73
			2	9.49			0.00			17.26		
			3	14.28			0.00			18.66		
			4	14.58			0.00			18.67		
		5	**5**	14.59			0.77			18.67		
			6	14.58			1.54			18.67		
			7	14.58			2.31			18.67		
			8	14.58			3.09			18.67		
			9	0.00								
	(640,64)	**1**	**1**	2.19	17.50	612.52	0.00	15.61	387.61	22.82	169.47	5044.67
	(320,32)		**2**	2.19			0.56			22.82		
	(160,16)		**3**	2.19			1.13			22.82		
	(80,8)		**4**	2.19			1.69			22.82		
	(40,4)		**5**	2.19			2.26			22.79		

	(20,2)		**6**	2.19			2.82			19.66		
	(10,1)		**7**	2.19			3.38			14.88		
	(5,0.5)		**8**	2.19			3.84			22.82		
			9	0.00								
		2	**1**	18.91	125.91	4366.14	0.00	21.32	464.18	20.77	156.35	3875.09
			2	18.91			0.00			20.77		
			3	18.91			0.00			20.77		
			4	18.91			0.00			20.77		
			5	18.90			1.97			20.76		
			6	17.65			5.91			18.64		
			7	10.88			8.97			14.42		
			8	5.57			5.45			20.77		
			9	0.00								
		3	**1**	19.48	128.97	4471.35	0.00	14.24	313.80	19.93	150.61	2983.66
			2	19.48			0.00			19.93		
			3	19.48			0.00			19.93		
			4	19.48			0.00			19.93		
			5	19.47			0.00			19.92		
			6	17.97			1.96			18.08		
			7	10.89			7.57			13.92		
			8	5.57			5.53			19.93		
			9	0.00								
		4	**1**	19.59	129.49	5125.86	0.00	10.02	296.50	20.85	156.82	4664.21
			2	19.59			0.00			20.85		
			3	19.59			0.00			20.85		
			4	19.59			0.00			20.85		
			5	19.58			1.02			20.84		
			6	17.98			2.05			18.67		
			7	10.74			3.07			14.42		
			8	5.49			3.95			20.85		
			9	0.00								
		5	**1**	14.19	98.14	1934.66	0.00	7.66	114.43	18.67	142.10	2109.72
			2	14.19			0.00			18.67		
			3	14.19			0.00			18.67		
			4	14.19			0.00			18.67		
			5	14.19			0.77			18.66		
			6	13.94			1.54			17.27		
			7	9.48			2.31			13.44		
			8	4.73			3.05			18.67		
			9	0.00								

Table-D4 (high variance, low cost)

			OVERFLOW			NESTED			SYMMETRIC		
Demand Normal (μ,σ)	**Cost Structure**	**Dept.**	**c***	**TH**	**PR**	**c***	**TH**	**PR**	**c***	**TH**	**PR**
(80,8)	1	1	84.35	624.85	27101.06	90.78	628.94	25020.34	98.29	633.02	26933.14
(80,8)		2	84.35			92.22			98.30		
(80,8)		3	84.35			93.38			98.29		
(80,8)		4	84.35			94.84			98.28		
(80,8)		5	84.35			95.94			98.29		
(80,8)		6	84.35			97.06			98.29		
(80,8)		7	84.36			98.29			98.29		
(80,8)		8	84.35			99.75			98.29		
		9	63.94								
	2	1	72.26	570.32	24841.54	73.01	601.77	17788.05	94.74	628.74	24614.80
		2	72.26			76.27			94.73		
		3	72.26			78.78			94.75		
		4	72.26			82.87			94.74		
		5	72.26			85.69			94.75		
		6	72.26			89.60			94.75		
		7	72.26			94.61			94.75		
		8	72.26			99.26			94.80		
		9	23.76								
	3	1	90.40	634.52	58920.15	98.43	636.02	56698.49	105.36	638.05	58748.13
		2	90.39			99.54			105.35		
		3	90.40			100.50			105.36		
		4	90.40			101.71			105.35		
		5	90.40			102.43			105.38		
		6	90.40			103.37			105.38		
		7	90.40			105.13			105.35		
		8	90.40			106.63			105.34		
		9	76.36								
	4	1	82.00	616.77	56727.37	86.19	626.48	48691.43	101.48	635.73	56267.32
		2	82.00			88.47			101.47		
		3	82.00			90.20			101.49		
		4	82.00			92.83			101.48		
		5	82.01			95.37			101.47		
		6	82.00			97.84			101.46		
		7	82.01			101.76			101.46		
		8	82.00			106.78			101.45		
		9	50.59								
	5	1	80.40	610.65	56093.06	83.94	622.39	46180.49	100.05	634.62	55457.28
		2	80.40			85.42			100.05		
		3	80.40			87.68			100.03		
		4	80.40			90.15			100.07		
		5	80.40			93.24			100.06		
		6	80.40			96.35			100.09		
		7	80.39			99.61			100.06		
		8	80.39			104.10			100.05		
		9	43.89								

	(5,0.5)		**1**	6.29	201.82	8877.91	152.00	1256.37	51266.74	16.25	207.30	9093.90
	(10,1)		**2**	12.28			158.20			22.11		
	(20,2)		**3**	21.86			164.40			28.73		
	(40,4)		**4**	28.54			170.64			28.96		
	(80,8)	**1**	**5**	28.82			176.84			28.96		
	(160,16)		**6**	28.82			183.02			28.96		
	(320,32)		**7**	28.82			189.18			28.96		
	(640,64)		**8**	28.82			195.37			28.96		
			9	24.33								
		2	**1**	6.29	202.87	8757.86	78.69	1185.12	39612.21	15.89	190.01	7758.35
			2	12.34			99.79			21.21		
			3	22.42			120.89			26.00		
			4	31.03			142.01			26.10		
			5	31.57			163.12			26.10		
			6	31.58			184.26			26.10		
			7	31.58			205.34			26.10		
			8	31.58			226.44			26.10		
			9	11.92								
		3	**1**	6.68	268.21	25162.60	168.22	1265.86	114417.54	16.74	254.53	23881.66
			2	12.98			171.96			23.70		
			3	24.02			175.67			35.59		
			4	37.52			179.38			37.16		
			5	40.22			183.10			37.16		
			6	40.25			186.83			37.16		
			7	40.25			190.50			37.16		
			8	40.25			194.23			37.16		
			9	37.28								
		4	**1**	6.69	272.76	25004.49	132.01	1247.17	100783.67	16.44	223.55	20293.90
			2	13.03			142.64			22.77		
			3	24.31			153.25			31.23		
			4	39.06			163.88			31.71		
			5	42.87			174.51			31.71		
			6	42.90			185.14			31.71		
			7	42.91			195.76			31.71		
			8	42.91			206.38			31.71		
			9	30.26								
		5	**1**	6.69	271.54	24778.19	120.47	1239.31	96755.26	16.37	216.29	19415.38
			2	13.04			133.55			22.51		
			3	24.35			146.62			30.12		
			4	39.27			159.71			30.47		
			5	43.26			172.75			30.47		
			6	43.30			185.82			30.47		
			7	43.30			198.92			30.47		
			8	43.30			211.98			30.47		
			9	27.34								
	(640,64)	**1**	**1**	29.66	206.69	9090.94	12.32	93.23	3823.63	28.95	207.28	9092.91
	(320,32)		**2**	29.66			12.64			28.95		
	(160,16)		**3**	29.66			12.97			28.95		
	(80,8)		**4**	29.66			13.29			28.95		
	(40,4)		**5**	29.32			13.61			28.73		

	(20,2)		**6**	22.04			13.86			22.11		
	(10,1)		**7**	12.30			11.04			16.25		
	(5,0.5)		**8**	6.29			5.92			28.95		
			9	25.05								
		2	**1**	35.21	221.34	9545.66	5.94	71.15	2224.76	26.10	190.01	7758.38
			2	35.21			7.34			26.10		
			3	35.21			8.74			26.10		
			4	35.20			10.14			26.10		
			5	33.98			11.54			26.00		
			6	22.98			12.89			21.21		
			7	12.40			10.92			15.89		
			8	6.29			5.96			26.09		
			9	13.30								
		3	**1**	39.37	263.55	24727.16	15.53	109.98	9992.78	36.31	249.82	23440.69
			2	39.37			15.71			36.31		
			3	39.37			15.88			36.31		
			4	39.35			16.06			36.31		
			5	36.97			16.24			34.98		
			6	23.92			16.08			23.61		
			7	12.97			11.59			16.74		
			8	6.68			6.14			36.31		
			9	36.47								
		4	**1**	39.73	256.67	23543.76	12.24	98.67	7845.66	30.47	216.30	19636.64
			2	39.73			12.97			30.47		
			3	39.73			13.70			30.47		
			4	39.71			14.43			30.47		
			5	37.20			15.16			30.12		
			6	23.97			15.63			22.55		
			7	12.97			11.51			16.46		
			8	6.68			6.16			30.47		
			9	28.02								
		5	**1**	41.00	260.14	23749.36	11.21	95.52	7257.12	30.47	216.29	19415.32
			2	41.00			12.14			30.47		
			3	41.00			13.08			30.47		
			4	40.98			14.01			30.47		
			5	37.97			14.94			30.12		
			6	24.11			15.61			22.50		
			7	13.00			11.50			16.38		
			8	6.68			6.17			30.47		
			9	25.89								

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